



CAMBRIDGE

11

MATHEMATICS STANDARD

CambridgeMATHS NSW STAGE 6

SECOND EDITION

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1

Earning money

Australia has an ageing population ...

What does that mean?

We are all getting older every year, obviously, but the balance of younger people versus older people has been changing dramatically in the last 50 years.

There are several reasons for this, including:

- The Baby Boomers generation are getting older and reaching retirement age. When they were born, it was typical for families to have three or more children.
- More recently, people are more commonly having only one or two children.
- People are living longer.

Is that a problem?

It would be a problem if every Australian retiree was paid an age pension, funded by the Federal Government using money collected in taxes from working Australians. Income tax would need to increase in order to achieve this.

Why do I pay money into superannuation every time I get paid?

Back in 1992, the Australian Government made it compulsory for employers to pay 3% of every employee's income into a superannuation fund. That gradually increased to 12% in the year 2025. It is a compulsory form of saving which can be accessed after retirement, instead of the age pension.

Chapter contents

- 1A** Using percentages (**CONSOLIDATING**)
- 1B** Salaries and wages
- 1C** Annual leave loading and bonuses
- 1D** Penalty rates, overtime and special allowances
- 1E** Commission
- 1F** Piecework, royalties and government payments
- 1G** Gross pay, deductions from pay and net pay
- 1H** Allowable tax deductions which reduce taxable income
- 1I** Taxable income
- 1J** Calculating the Medicare levy and income tax payable

NSW Syllabus

In this chapter, a student:

- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- Solves financial problems involving earning money and taxation (MST-11-03)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

1A Using percentages

CONSOLIDATING

Learning intentions

- To revise and consolidate percentages and their applications
- To convert between fractions, decimals and percentages

Past, present and future learning

- Students should already be familiar with the contents of this section
- Percentages are used extensively throughout Years 11 and 12
- Questions involving percentages regularly appear in HSC Examinations

Many workplaces use percentages every day to calculate fees and prices. It is important that we recall how to use percentages.

Percentage means 'out of 100'. Percentages can be represented using % or as a fraction or a decimal. For example:

$$25 \text{ percent} = 25\% = \frac{25}{100} = \frac{1}{4} = 0.25$$



Example 1 Converting to a percentage

Convert each of the following to a percentage.

a $\frac{16}{25}$

b $\frac{5}{8}$

c 0.04

Solution

a

$$\frac{16}{25} \xrightarrow{\times 4} \frac{64}{100} = 64\%$$

So $\frac{16}{25} = 64\%$

b

$$\frac{5}{8} \times \frac{25}{1} = \frac{125}{2} = 62.5\%$$

So $\frac{5}{8} = 62.5\%$

c $0.04 \times 100 = 4$

So $0.04 = 4\%$

Explanation

Write the fraction using 100 as the denominator.

Multiply the fraction by 100.
Cancel common factors and simplify.

Multiply the decimal by 100.

Now you try

Convert each of the following to a percentage.

a $\frac{7}{20}$

b $\frac{3}{6}$

c 0.11



Example 2 Writing a percentage as a simplified fraction

Write each of the following percentages as a simplified fraction.

a 39%

b 62%

c $9\frac{1}{2}\%$

Solution

a $39\% = \frac{39}{100}$

b $62\% = \frac{62}{100}$
 $= \frac{31}{50}$

c $9\frac{1}{2}\% = \frac{9\frac{1}{2}}{100}$
 $= \frac{19}{200}$

Explanation

Write the percentage using 100 as the denominator.

Write the percentage using 100 as the denominator. Simplify by using the HCF of 62 and 100, which is 2.

Write the percentage using 100 as the denominator. To make the numerator a whole number, multiply by 2. Do the same to the denominator.

Now you try

Write each of the following percentages as a simplified fraction.

a 17%

b 75%

c $11\frac{1}{2}\%$



Example 3 Converting a percentage to a decimal

Convert these percentages to decimals.

a 91%

b 3%

c 40%

Solution

a $91\% = 91 \div 100$
 $= 0.91$

b $3\% = 3 \div 100$
 $= 0.03$

c $40\% = 40 \div 100$
 $= 0.4$

Explanation

Divide the percentage by 100. This is the same as moving the decimal point two places to the left.

Now you try

Convert these percentages to decimals.

a 11%

b 8%

c 70%



Example 4 Finding a percentage of a quantity

Find 32% of \$1600.

Solution

$$\begin{aligned} 32\% \text{ of } \$1600 &= 0.32 \times 1600 \\ &= \$512 \end{aligned}$$

Explanation

Remember 'of' means to multiply.
Change 32% to a decimal.
Multiply by the amount.

Now you try

Find 17% of \$1860.

Exercise 1A

BUILDING

1 Complete the following using the words *multiply* or *divide*.

- a To convert a decimal to a percentage _____ by 100.
- b To convert a percentage to a decimal _____ by 100.
- c To convert a fraction to a percentage _____ by 100.
- d To convert a percentage to a fraction _____ by 100.

2 Write each of the following fractions as a percentage.

- a $\frac{15}{25}$ b $\frac{2}{5}$ c $\frac{3}{4}$ d $\frac{3}{10}$
- e $\frac{3}{100}$ f $\frac{6}{24}$ g $\frac{14}{40}$ h $\frac{7}{8}$

3 Write these decimals as a percentage.

- a 0.19 b 0.77 c 0.43 d 0.07
- e 0.99 f 0.6 g 0.125 h 1.32

4 Write each percentage as a simplified fraction.

- a 73% b 40% c 55% d 66%
- e $19\frac{1}{2}\%$ f $9\frac{1}{4}\%$ g $12\frac{1}{4}\%$ h $81\frac{1}{2}\%$

5 Convert these percentages to decimals.

- a 78% b 42% c 30% d 4%
- e 16.5% f 150% g 12.25% h 0.8%

6 Find the following.

- a 10% of \$330 b 50% of \$220 c 80% of \$96 d 7% of \$63
- e 16.5% of 800 L f 120% of 300 g g 0.5% of 1000 mL h 1.1% of 42 km

DEVELOPING

7 A 750 g pizza contains 210 g of cheese, 180 g of ham and 150 g of pineapple.

- a What percentage of the pizza is cheese?
- b What percentage of the pizza is ham?
- c What percentage of the pizza is pineapple?

- 8** About 20% of a dog's weight is fur. If a dog weighs 45 kg, how many kilograms of fur does the dog have?
- 9** Dalton spends 9 hours sleeping, 7 hours at school, 2 hours travelling to and from school and 1 hour eating. He likes to play sport. What percentage of his day can he play sport? Answer to the nearest percent.

CHALLENGING

- 10** Jacob lost 11 kg and now weighs 89 kg. What percentage of his original weight did he lose?
- 11** Gabriel spends 53.6% of his earnings on entertainment. If he earns \$113.56 each month, how much does Gabriel have to spend on entertainment each year, to the nearest dollar?

Communicate your reasoning, understanding and fluency

- 12** The table shows the change in population from 2016 to 2021.

	2016	2021	Change over 5 years (% nearest whole number)
Sydney	4 321 535	4 698 656	
Newcastle	322 278	348 539	
Central Coast	307 742	325 255	
Wollongong	261 896	280 153	
Maitland	78 015	89 597	
Tweed Heads	59 776	63 721	
Albury	47 974	53 677	
NSW	7 480 228	8 072 163	

Source: Data taken from ABS census data. See https://en.wikipedia.org/wiki/List_of_places_in_New_South_Wales_by_population

- a** Calculate the percentage change for each town and NSW in the table.
- b** In 2021, what percentage of the NSW population lived in:
- i** Newcastle?
 - ii** Wollongong?
 - iii** Albury?
- c** Explain why the percentage increase is so different between Maitland and Sydney.

1B Salaries and wages

Learning intentions

- To compare and contrast salaries and wages
- To perform calculations involving salaries and wages

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about salaries and wages regularly appear in HSC Examinations

Comparing a salary to a wage

Most people are paid an annual salary or a weekly wage.

	Salary	Wage
Description	A payment for a year's work, which is then divided into equal monthly, fortnightly or weekly payments.	A payment for a week's work, calculated on an hourly basis.
Advantages	• Permanent employment • Superannuation, sick and holiday pay	• Permanent employment • Superannuation, sick and holiday pay • Overtime payments for extra work
Disadvantages	• No overtime for extra work • Hours are fixed	• Lower incentive to work hard each hour • Hours are fixed
Examples	People who are paid a salary include teachers and nurses.	People who are paid a wage include shop assistants, factory workers and mechanics.

Converting a salary or wage to another time period

Divide the annual salary by the number of time periods in a year.

Divide the weekly wage by the number of work days or hours in a week.

1 year = 52 weeks

1 year = 26 fortnights

1 year = 12 months



Example 5 Calculating from a salary

Wei earns a salary of \$65 208 per annum. He is paid fortnightly. How much does Wei receive each fortnight? Assume there are 52 weeks in the year.

Solution

$$\begin{aligned}\text{Fortnightly pay} &= 65\,208 \div 26 \\ &= \$2508.00\end{aligned}$$

Wei is paid \$2508 per fortnight.

Explanation

Write 'fortnightly pay' on the LHS of = sign.
Divide the salary by 26 fortnights in one year.

Now you try

Gordon earns a salary of \$84 000 per annum. He is paid monthly. What is Gordon's monthly pay?



Example 6 Calculating a wage

Jasmine is paid at a rate of \$1098 for a 40-hour week. How much does Jasmine earn per hour?

Solution

$$\begin{aligned}\text{Wage per hour} &= 1098 \div 40 \\ &= \$27.45\end{aligned}$$

Jasmine earns \$27.45 per hour.

Explanation

Write 'wage per hour' on the LHS of = sign.
Divide the pay by the number of hours worked.

Now you try

Koby is paid \$2025 per week. He works a 36-hour week. What is Koby's hourly rate of pay?

Exercise 1B

BUILDING

Example 5

- 1 Layla earns a salary of \$92 648. Write, to the nearest dollar, her salary as amounts per:
- a** week **b** fortnight **c** month.

- 2 The annual salary for four people is shown in the table. Calculate their weekly and fortnightly payments. Answer correct to the nearest dollar.

	Name	Salary
a	Aiko	\$57 640
b	Blake	\$78 484
c	Chloe	\$107 800
d	David	\$44 240

- 3 What is Zachary's fortnightly income if he earns a salary of \$43 056?
- 4 Find the annual salary for the following people.
- a** Amber earns \$580 per week. **b** Jun earns \$1520 per fortnight.
c Samuel earns \$3268 per month. **d** Ava earns \$2418 per week.
- 5 Harrison is a civil engineer who earns a salary of \$1500 per week.
- a** How much does he receive per fortnight?
b How much does he receive per year?



- 6** What is Yasmeen's annual salary if her salary per fortnight is \$1610?
- 7** Dylan receives a weekly salary payment of \$1560. What is his annual salary?
- 8** Stephanie is paid \$1898 per fortnight and Tahlia \$3821 per month. Calculate each person's equivalent annual income. Who earns more per week and by how much?
- 9** Laura is paid \$1235 per fortnight and Ebony \$2459 per month. Who receives the higher annual salary and by how much?
- 10** Tran is paid \$1898 per week and Jake \$8330 per month. Calculate each person's equivalent annual income. What is the difference between their annual salaries?
- 11** Arana works as a labourer and is paid \$25.50 an hour. How much does he earn for working the following hours?
a 35 hours **b** 37 hours **c** 40 hours **d** 42 hours
- 12** Lily earns \$29.75 an hour. If she works 6 hours each day during the week and 4 hours a day during the weekend, find Lily's weekly wage.
- Example 6** **13** Determine the wage for a 37-hour week for each of the following hourly rates.
a \$12.00 **b** \$9.50 **c** \$23.20 **d** \$13.83
- 14** Determine the income for a year for each of the following hourly rates. Assume 52 weeks in the year and 40 hours of work per week.
a \$7.59 **b** \$15.25 **c** \$18.78 **d** \$11.89
- 15** Suchitra works at the local supermarket. She gets paid \$22.50 per hour. Her time card is shown in the table. She is paid for her breaks, so they are not shown.
- | Day | In | Out |
|-----------|-----------|-----------|
| Monday | 9:00 a.m. | 5:00 p.m. |
| Tuesday | 9:00 a.m. | 6:00 p.m. |
| Wednesday | 8:30 a.m. | 5:30 p.m. |
| Thursday | 9:00 a.m. | 4:30 p.m. |
| Friday | 9:00 a.m. | 4:00 p.m. |
- a** How many hours did Suchitra work this week?
b Find Suchitra's weekly wage.
- 16** Grace earns \$525 in a week. If her hourly rate of pay is \$12.50, how many hours does Grace work in the week?
- 17** Kim is a plumber who earned \$477 for a day's work. He is paid \$53 per hour. How many hours did Kim work on this day?
- 18** Na is a hairdresser who earns \$24.20 per hour. She works an 8-hour day.
a How much does Na earn per day?
b How much does Na earn per week? Assume she works 5 days a week.
c How much does Na earn per fortnight?
d How much does Na earn per year? Assume 52 weeks in the year.
- 19** Alyssa is paid \$36.90 per hour and Connor \$320 per day. Alyssa works a 9-hour day. Who earns more per day and by how much?

DEVELOPING

- 20** Feng is retiring and will receive 7.6 times the average of his salary over the past three years. In the past three years he was paid \$84 640, \$83 248 and \$82 960. Find the amount of his payout.
- 21** Liam's salary is currently \$76 000. He will receive salary increases as follows: 5% increase from 1 July and then a 5% increase from 1 January. What will be his new salary from 1 January?
- 22** Create the spreadsheet shown.



IF =C5*D5

	A	B	C	D	E	F
1	Cambridge Mathematics Standard Year 11 Exercise 1B Question 22					
2	Worksheet to calculate wages of employees					
3						
4	<i>Family name</i>	<i>First name</i>	<i>Hours worked</i>	<i>Hourly pay rate</i>	<i>Weekly wage</i>	
5	Cini	Olivia	35	\$15.00	=C5*D5	
6	Croft	Liam		\$20.00		
7	Griffen	Lily		\$30.00		
8	Hong	Tin		\$26.00		
9	Lang	Molly		\$33.00		
10	Sitou	Noth		\$23.00		
11	Taylor	Nathan		\$26.00		
12	Woods	Joshua		\$15.00		
13						

- a** Cell E5 has a formula that multiplies cells C5 to D5. Enter this formula.
- b** Enter the hours worked for the following employees:
- | | |
|------------|---------------|
| Liam – 20 | Noth – 37.5 |
| Lily – 26 | Nathan – 42 |
| Tin – 38 | Joshua – 38.5 |
| Molly – 40 | |
- c** Fill down the contents of E5 to E12.
- d** Edit the hourly pay rate of Olivia Cini to \$16.50. Observe the change in E5.

CHALLENGING

- 23** Isabelle earns \$85 324 per annum. Isabelle calculated her weekly salary by dividing her annual salary by 12 to determine her monthly payment and then divided this result by 4 to determine her weekly payment. What answer did Isabelle get? What is the correct answer, and what is wrong with Isabelle's calculation?
- 24** Hika earns \$8 per hour and Ebony earns \$9 per hour. Last week they both earned at least \$150. What is the least number of hours that Hika could have worked last week?



Communicate your reasoning, understanding and fluency

- 25** Harry is a teacher who is paid a salary of \$95 000 per annum. Janet is a shop assistant who is paid a wage of \$35 per hour and normally works a 40-hour week.
- a** How many hours more in a week does Janet have to work to be paid more than Harry that week?
- b** Explain the major disadvantages of earning a salary instead of a wage.

1C Annual leave loading and bonuses

Learning intentions

- To understand the bonuses applied to wages and salaries
- To perform calculations involving leave loading and other bonuses

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about pay bonuses sometimes appear in HSC Examinations

Annual leave loading

Annual leave loading is a payment calculated as a fixed percentage of the normal pay (usually 17.5%) over a fixed number of weeks. It is usually paid at the beginning of the annual holidays to meet the increased expenses of a holiday.

Holiday loading = $17.5 \times \text{Normal weekly pay} \times \text{Number of weeks leave}$



Example 7 Finding the annual leave loading

Thomas works a 40-hour week at a rate of \$18.50 per hour. He receives 17.5% of 4 weeks normal pay as holiday loading. What is Thomas's pay for the holiday?

Solution

$$\begin{aligned} 4 \text{ weeks pay} &= (18.50 \times 40 \times 4) \\ &= \$2960 \end{aligned}$$

$$\begin{aligned} \text{Holiday loading} &= 17.5\% \times \text{Normal weekly pay} \\ &\quad \times \text{Number of weeks leave} \\ &= 17.5\% \text{ of } \$2960 \\ &= 0.175 \times 2960 \\ &= \$518 \end{aligned}$$

$$\begin{aligned} \text{Holiday pay} &= \$2960 + \$518 \\ &= \$3478 \end{aligned}$$

Thomas's holiday pay is \$3478.

Explanation

Write 4 weeks pay on the LHS of = sign.
Determine 4 weeks pay.

Write 'holiday loading' on the LHS of = sign.
Find 17.5% of 4 weeks pay.

Write 'holiday pay' on the LHS of = sign.
Add the 4 weeks pay and holiday loading.

Now you try

Oli works a 40-hour week at a rate of \$28 per hour. He receives 17.5% holiday loading for 4 weeks of his normal pay. What is Oli's holiday loading?

Bonus

A bonus is an extra payment or gift earned as reward for achieving a goal. It is paid in addition to the normal income. Bonuses are an incentive for employees to work harder. For example, an employee may receive a bonus of 5% of their annual salary or a flat payment of \$1000.



Example 8 Calculating a bonus

Amber's employer has decided to reward all employees with a bonus. The bonus awarded is 5% of their annual salary. What is Amber's bonus if her annual salary is \$68 560?

Solution

Bonus = 5% of \$68 560
 $= 0.05 \times 68\,560$
 $= \$3428$
 Amber receives a bonus of \$3428.

Explanation

Write 'bonus' on the LHS of = sign.
 Determine 5% of the salary.

Now you try

Paul's employee decides to pay him a 3% bonus on his annual salary of \$92 500. How much is Paul's bonus?

Exercise 1C

BUILDING

Example 7

- 1 A business pays 17.5% holiday loading on 4 weeks normal pay. Calculate the amount of holiday loading for these employees.
 - a Nicholas earns \$6240 normal pay for 4 weeks.
 - b Kumar earns \$5130 normal pay for 4 weeks.
 - c Samantha earns \$5320 per fortnight.
 - d Hamza earns \$2760 per fortnight.
 - e Bilal earns \$1680 per week.
- 2 The local government pays its employees 17.5% holiday loading on 4 weeks normal pay. Calculate the amount of holiday loading for these employees
 - a Paige earns an annual salary of \$105 560. Assume 52 weeks in a year.
 - b Jack earns an annual salary of \$58 760. Assume 52 weeks in a year.
 - c Riley earns \$32 per hour and works a 35-hour week.
 - d A'ishah earns \$41.50 per hour and works a 37-hour week.
- 3 Laura works a 37-hour week at a rate of \$20.50 per hour. When she takes her 4 weeks annual leave, she is paid a loading of 17.5%. What is Laura's holiday pay when she takes her leave?
- 4 Ethan is paid \$660 per week. He receives a holiday leave loading of 17.5% for three weeks holiday pay. What is his total holiday pay?
- 5 A bonus is awarded as a percentage of a person's annual salary. The percentage awarded depends on the person's achievements. Calculate the following bonuses.

a 6% of \$48 360	b 3% of \$96 540	c 2% of \$103 290
d 4.5% of \$65 420	e 2.5% of \$88 580	f 1.25% of \$164 400

Example 8

- 6** Grace received a bonus of 12% of her weekly wage. What was Grace's bonus if her weekly wage is \$1850?
- 7** Patrick's boss has decided to reward all employees with a bonus. The bonus awarded is 7.75% of their annual salary. What is Patrick's bonus if his annual salary is \$74 980?



DEVELOPING

- 8** Chen receives 17.5% of 4 weeks normal pay as leave loading. If Chen's leave loading was \$379.40, what was his normal weekly pay?
- 9** Create the spreadsheet shown.



IF =IF(AND(C5>10,D5>50),400,0)

	A	B	C	D	E
1	Cambridge Mathematics Standard Year 11 Exercise 1C Question 9				
2	Worksheet to calculate bonus				
3					
4	<i>Family name</i>	<i>First name</i>	<i>Years of service</i>	<i>Overtime</i>	<i>Bonus</i>
5	Cupac	Charlotte	15	121	=IF(AND(C5>10,D5>50),400,0)
6	Fraser	Dylan	10	24	\$0
7	Humes	Sienna	13	46	\$0
8	Huynh	Benjamin	6	90	\$0
9	Sengmany	Xay	8	50	\$0
10	White	Ava	12	119	\$400
11					

- a** The formula for cell E5 is '= IF(AND(C5 > 10, D5 > 50), 400, 0)'. It is the formula that calculates a \$400 bonus if the employer has more than 10 years of service and more than 50 hours of overtime. Fill down the contents of E6 to E10 using this formula.
- b** Edit the overtime amount for Sienna Humes to 52. Observe the changes in E7.
- c** Edit the years of service for Ava White to 10. Observe the changes in E10.
- d** Edit the overtime amount for Dylan Fraser to 60. Observe the changes in E6.
- e** Edit the years of service for Xay Sengmany to 20. Observe the changes in E9.
- f** Edit the overtime amount for Benjamin Huynh to 40. Observe the changes in E8.

CHALLENGING

10 Jim receives holiday loading of 17.5% of 4 weeks pay. His loading was \$996.80.

- a** Find Jim's normal weekly pay.
- b** Find Jim's normal hourly pay rate if he usually works a 40-hour week.

11 Chloe's annual salary is \$72 800.

- a** Calculate Chloe's weekly wage.
- b** Holiday loading is calculated at 17.5% of four weeks pay. Calculate Chloe's holiday loading.
- c** Chloe's employer is increasing her annual salary by 1%. What is Chloe's new annual salary?
- d** The increase in Chloe's annual salary is compensation for removing holiday loading. Explain why Chloe is worse off financially with the 1% increase.



Communicate your reasoning, understanding and fluency

12 a Does every employee in Australia get annual leave loading?

b Is annual leave loading always 17.5%?

13 Davis is paid a \$3600 bonus for his work last year. He also receives annual leave loading at 17.5% for 4 weeks holiday of \$540. Calculate Davis's:

- a** 4 weeks holiday pay including his leave loading
- b** weekly wage
- c** bonus as a percentage of his total annual income.

1D Penalty rates, overtime and special allowances

Learning intentions

- To understand the special pay rates applied to income
- To perform calculations involving penalty rates, overtime and allowances

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about special pay rates regularly appear in HSC Examinations

Penalty rates

Penalty rates are paid when we work at a time outside of normal working hours. These will include overtime, special allowances and receiving an increased hourly rate when doing casual work.

Overtime

Overtime rates apply when employees work beyond the normal working hours or days. Payment for overtime is usually more than the normal pay rate. For example, a person whose normal pay rate is \$10 an hour would receive \$20 ($\$10 \times 2$) an hour if they were paid overtime at double time. Another common rate is time-and-a-half, which is the normal pay rate multiplied by 1.5. Here a person would receive \$15 ($\$10 \times 1.5$) an hour.

Overtime rates

Time-and-a-half rate – normal pay rate $\times 1.5$

Double time rate – normal pay rate $\times 2$



Example 9 Calculating wages involving overtime

John works for a building construction company. Find John's wage during a week in which he works 40 hours at the normal rate of \$32 an hour, 3 hours at time-and-a-half rate and 1 hour at double time rate.

Solution

$$\begin{aligned}\text{Normal pay} &= 40 \times 32 \\ &= \$1280\end{aligned}$$

$$\begin{aligned}\text{Time-and-a-half pay} &= 3 \times 32 \times 1.5 \\ &= \$144\end{aligned}$$

$$\begin{aligned}\text{Double time pay} &= 1 \times 32 \times 2 \\ &= \$64\end{aligned}$$

$$\begin{aligned}\text{Wage} &= 1280 + 144 + 64 \\ &= \$1488\end{aligned}$$

Explanation

Determine normal pay.

Determine time-and-a-half pay.

Determine double time pay.

Calculate wage by adding all pay.

Now you try

Jane works for the local council. Find Jane's wage in a week when she works 38 hours at a normal rate of \$48 an hour, 2 hours at time-and-a-half rate and 4 hours at double time rate.

Special allowances

Employees receive an allowance if they work under difficult or dangerous conditions such as wet weather, extreme temperatures, confined spaces or isolated areas. Allowances are also paid when an employee has an expense related to their type of work, such as work uniform, meals, travel or tools.



Casual work

Casual work involves a set amount paid for each hour's work. It can be paid weekly or fortnightly.

Advantages

- Working hours are flexible
- Pay rate is often higher

Disadvantages

- No sick leave or holiday pay
- May lose job when not needed
- May not include superannuation



Example 10 Calculating casual pay

Milan is employed on a casual basis for a fast-food company. His rate of pay is \$15 per hour plus time-and-a-half on Saturday and double time on Sunday.

Last week Milan worked from 10:30 a.m. to 2:30 p.m. on Thursday, from 9:30 a.m. to 2 p.m. on Saturday, and from 12 noon to 4 p.m. on Sunday. How much did Milan earn last week?

Solution

$$\begin{aligned}\text{Normal pay} &= 4 \times 15 \\ &= \$60\end{aligned}$$

$$\begin{aligned}\text{Time-and-a-half pay} &= 4.5 \times 15 \times 1.5 \\ &= \$101.25\end{aligned}$$

$$\begin{aligned}\text{Double time pay} &= 4 \times 15 \times 2 \\ &= \$120\end{aligned}$$

$$\begin{aligned}\text{Wage} &= 60 + 101.25 + 120 \\ &= \$281.25\end{aligned}$$

Explanation

Determine normal pay.

Determine time-and-a-half pay.

Determine double time pay.

Calculate wage by adding all pay.

Now you try

Rafi works on a casual basis for a sales company. His rate of pay is \$17 per hour, plus time-and-a-half on Saturday before 12 noon and double time on Saturday after 12 noon.

Last week Rafi worked from 9:00 a.m. to 1:00 p.m. on Thursday, Friday and Saturday. How much did

Rafi earn last week?

Exercise 1D

BUILDING

- Calculate the payment for working 4 hours overtime at time-and-a-half given the following normal pay rates.
a \$18.00 **b** \$39.50 **c** \$63.20 **d** \$43.83
- Calculate the payment for working 3 hours overtime at double time given the following normal pay rates.
a \$37.99 **b** \$19.05 **c** \$48.78 **d** \$61.79
- Ibrahim earns \$32.50 an hour as a driver. He works 38 hours a week at normal time and 5 hours a week at double time. Find his weekly wage. Answer correct to the nearest cent.

Example 9



- Mei is a casual employee who worked 8 hours at normal time and 2 hours at time-and-a-half. Her normal rate of pay is \$12.30 per hour. What is her pay for the hours she worked?
- Oliver earns \$23.80 an hour. He earns normal time during week days and time-and-a-half on weekends. Last week he worked 34 hours during the week and 6 hours during the weekend. Find his weekly wage.
- George works in a take-away food store. He gets paid \$18.60 per hour for a standard 35-hour week. Additional hours are paid at double time. His time card is shown in the table.
a How many hours did George work this week?
b Find his weekly wage.
- Dave works for 5 hours at double time. He earns \$98. Find Dave's normal hourly rate.
- Ella works 3 hours at time-and-a-half and earns \$72. Find Ella's normal hourly rate.

Day	In	Out
Monday	8:30 a.m.	4:30 p.m.
Tuesday	9:00 a.m.	6:00 p.m.
Wednesday	8:45 a.m.	5:45 p.m.
Thursday	9:00 a.m.	6:30 p.m.
Friday	10:00 a.m.	8:00 p.m.

DEVELOPING

- Zahid is paid a set wage of \$774.72 for a 36-hour week, plus time-and-a-half for overtime. In one particular week he worked 43 hours. What were Zahid's earnings that week?
- Samantha is paid a set wage of \$962.50 for a 35-hour week, plus double time for overtime. In one particular week she worked 40 hours. What are Samantha's earnings?

- 11** A window washer is paid \$22.50 per hour and a height allowance of \$55 per day. In one week he works 9 hours each week day.

- a** Calculate the amount earned each week day.
b Calculate his total weekly earnings for five days of work.



- 12** Anna works in a factory and is paid \$18.54 per hour. When she operates the oven, she is paid a temperature allowance of \$4.22 per hour in addition to her normal rate. Find Anna's weekly pay when she works a total of 42 hours including 10 hours working the oven.
- 13** Scott is a painter who is paid a normal rate of \$36.80 per hour plus a height allowance of \$21 per day. In one week Scott works 9 hours per day for 5 days on a tall building. Calculate Scott's total earnings.
- 14** Kathy is a scientist who is working in a remote part of Australia. She earns a salary of \$86 840 plus a weekly allowance of \$124.80 for working under extreme and isolated conditions. Calculate Kathy's fortnightly pay.
- 15** Chris is a soldier and is paid \$27 per hour plus an additional allowance of \$12.50 per hour for disarming explosives. What is Chris's total weekly pay if he works from 6:00 a.m. to 2:00 p.m. for 7 days a week on explosives?



- 16** A miner earns a wage of \$46.20 per hour plus an allowance of \$28.20 per hour for working in cramped spaces. The miner worked a 10-hour day for 5 days in a small shaft. What was his weekly pay?

Example 10

- 17** Vien is employed on a casual basis. His rate of pay is shown in the table. Last week Vien worked from 11:30 a.m. to 3:30 p.m. on Thursday, from 8:30 a.m. to 2:00 p.m. on Saturday, and from 12 noon to 6:00 p.m. on Sunday. How much did Vien earn last week?

Rate of pay	
Weekdays	\$18.60 per hour
Saturday	Time-and-a-half
Sunday	Double time

- 18** A mechanic's industrial award allows for normal rates for the first 7 hours on any day. It provides for overtime payment at the rate of time-and-a-half for the first 2 hours and double time thereafter. Find a mechanic's wage for a 12-hour day if the normal pay rate is \$42.50 an hour.

- 19** Kaimi's timesheet is shown in the table. She gets paid \$12.80 per hour during the week, time-and-a-half for Saturdays and double time for Sundays. Kaimi is not paid for meal breaks.

Day	In	Out	Meal break
Monday	8:30 a.m.	5:30 p.m.	1 hour
Tuesday	8:30 a.m.	3:00 p.m.	1 hour
Wednesday	8:30 a.m.	5:30 p.m.	1 hour
Thursday	8:30 a.m.	9:00 p.m.	3 hour
Friday	3:00 p.m.	7:00 p.m.	No break
Saturday	8:00 a.m.	4:00 p.m.	No break
Sunday	10:00 a.m.	3:00 p.m.	30 minutes

- a** How much did Kaimi earn at the normal rate of pay during this week?
- b** How much did Kaimi earn from working at penalty rates during this week?
- c** What percentage of her pay did Kaimi earn by working at penalty rates?

CHALLENGING

- 20** Connor works a 35-hour week and is paid \$18.25 per hour. Any overtime is paid at time-and-a-half. Connor wants to work enough overtime to earn at least \$800 each week. What is the minimum number of hours of overtime that Connor will need to work?
- 21** Max works in a shop and earns \$21.60 per hour at the normal rate. Each week he works 15 hours at the normal rate and 4 hours at time-and-a-half.
- a** Calculate Max's weekly wage.
- b** Max aims to increase his weekly wage to \$540 by working extra hours at the normal rate. How many extra hours must Max work?
- c** Max's rate of pay increased by 5%. What is his new hourly rate for normal hours?
- d** What will be Max's new weekly wage, assuming he maintains the extra working hours?



Communicate your reasoning, understanding and fluency

- 22** List some of the reasons why people get paid penalty rates.
- 23** Joseph is paid \$32.56 per hour. Each week he works 12 hours at normal pay and 5 hours at time-and-a-half.
- a** Calculate Joseph's weekly earnings.
- b** He wants to save 20% of his earnings. How much will Joseph have saved in 12 months if he has 4 weeks off in a year?
- c** The inflation rate is at 7.5% per annum. To maintain the value of his earnings, how many extra hours in a year will Joseph need to work to cover the rise in costs due to inflation? Answer to the nearest hour.

1E Commission

Learning intentions

- To understand the concepts associated with commission
- To perform calculations involving commission

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about commission sometimes appear in HSC Examinations

Commission is usually a percentage of the value of the goods sold. People such as real estate agents and salespersons are paid a commission.

Advantages

- Higher sales increase the income
- May receive a small payment (retainer) plus the commission

Disadvantages

- Income may vary each week
- Competition for customers is usually high

Commission

Commission = Percentage of the value of the goods sold



Example 11 Finding the commission

Zoe sold a house for \$650 000. Find the commission from the sale if Zoe's rate of commission was 1.25%.



Solution

$$\begin{aligned}\text{Commission} &= 1.25\% \text{ of } \$650\,000 \\ &= 0.0125 \times 650\,000 \\ &= \$8125\end{aligned}$$

Commission earned is \$8125.

Explanation

Change 1.25% to a decimal.
Multiply by the amount.

Now you try

Chloe sold a unit for \$450 500. Find the commission from the sale if Chloe's rate of commission was 1.75%.

Retainer

Retainer = A set amount paid on top of commission


Example 12 Calculating pay involving a retainer and commission

An electrical goods salesman is paid \$570.50 a week as a retainer. He receives a 4% commission on all sales over \$5000 a week. Find his earnings in a week in which his sales amounted to \$6800.

Solution

$$\begin{aligned}\text{Sales} &= 6800 - 5000 \\ &= 1800\end{aligned}$$

$$\begin{aligned}\text{Earnings} &= 570.50 + (4\% \text{ of } \$1800) \\ &= 570.50 + (0.04 \times 1800) \\ &= \$642.50\end{aligned}$$

Earnings were \$642.50.

Explanation

Determine sales over \$5000.

Calculate earnings by adding the retainer to 4% of \$1800.

Now you try

Alf is paid a retainer of \$350 a week. He gets paid commission of 3.5% on all sales over \$400. He has sales of \$900 this week. How much is Alf's total pay for the week?

Exercise 1E
BUILDING
Example 11

- 1** Tama earns a commission of 4% on the sale price. What is Tama's commission on the following sales?

a \$8820

b \$16 740

c \$34 220

- 2** Michelle earns 2% on all sales as a real estate agent. Calculate Michelle's commission on these sales.

a \$456 000

b \$420 000

c \$285 500

d \$590 700

- 3** Olivia sold a car valued at \$54 000. Calculate Olivia's commission from the sale if her rate of commission is 3%.

Example 12

- 4** Sophie earns a weekly retainer of \$355 plus a commission of 10% on sales. What are Sophie's total earnings for each week if she made the following sales?

a \$760

b \$2870

c \$12 850

- 5** Chris earns \$240 per week plus 25% commission on sales. Calculate Chris's weekly earnings if he made sales of \$2880.

- 6** Ora is a salesperson for a cosmetics company. She is paid \$500 per week and a commission of 3% on sales in excess of \$800.

a What does Ora earn in a week in which she makes sales of \$1200?

b What does Ora earn in a week in which she makes sales of \$600?

- 7** A real estate agent charges a commission of 5% for the first \$20 000 of the sale price and 2.5% for the balance of the sale price. Copy and complete the following table.

	Sale price	5% commission on \$20 000	2.5% commission on balance
a	\$150 000		
b	\$200 000		
c	\$250 000		
d	\$300 000		

DEVELOPING

- 8** Jade is a real estate agent and is paid an annual salary of \$18 000 plus a commission of 2.5% on all sales. She is also paid a car allowance of \$50 per week. What was Jade's total yearly income if she sold \$1 200 000 worth of property?
- 9** The commission that a real estate agent charges for selling a property is based on the selling price, as shown in the table.
- What is the commission charged on properties with the following selling prices?
- a** \$100 000 **b** \$150 000 **c** \$200 000
- 10** Harry is a salesperson. He earns a basic wage of \$300 per week and receives commission on all sales. Last week he sold \$20 000 worth of goods and earned \$700. What was Harry's rate of commission?

Selling price	Commission
First \$20 000	5%
Next \$120 000	3%
Thereafter	1%

CHALLENGING

- 11** Caitlin and her assistant, Holly, sell perfume. Caitlin earns 20% commission on her own sales, as well as 5% commission on Holly's sales. What was Caitlin's commission last month when she made sales of \$1800 and Holly made sales of \$2000?
- 12** A real estate agency charges a commission for selling a property. The commission is based on the selling price shown in the table. Bailey is paid \$180 per week by the real estate agent plus 5% of the commission received by the real estate agency. This week, Bailey sold one property for \$290 000 and one for \$600 000. He sold no properties in the previous week.
- a** What is the commission paid to the real estate agency for the \$290 000 sale?
- b** What is the commission paid to the real estate agency for both properties?
- c** Calculate Bailey's pay for this week.
- d** What is Bailey's average weekly income for the two-week period?

Commission rates	
Up to \$300 000	4%
\$300 000 and over	5%



Communicate your reasoning, understanding and fluency

- 13** Wendy earns commission on sales on a sliding scale as listed in the table. Last month she had the following sales.
- Week 1 \$555
 - Week 2 \$1077
 - Week 3 \$2600
 - Week 4 \$2280
- a** How much did Wendy earn last month?
- b** Explain what the employer's purpose could be in using the sliding scale above.

Commission rates	
Up to 1000	3%
1001–2000	5%
Over 2001	7%

1F Piecework, royalties and government payments

Learning intentions

- To understand piecework, royalties and government payments
- To perform calculations involving these concepts

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about these forms of income sometimes appear in HSC Examinations

Piecework

Piecework is a fixed payment for work completed. People who are employed to complete a particular task, such as an electrician installing lights, are earning piecework.

Advantages

- Incentive to work hard. Income increases with more work completed
- Often flexible hours and work place

Disadvantages

- No permanent employment
- No sick leave or holiday pay
- May not include superannuation

Piecework

Piecework = Number of units of work $\times$ Amount paid per unit



Example 13 Calculating a piecework payment

Noah is a tiler and charges \$47 per square metre to lay tiles. How much will Noah earn for laying tiles in a room whose area is 14 square metres?

Solution

Earnings = $14 \times \$47$
 $= \$658$
 Noah earns \$658.

Explanation

Multiply the number of square metres (14) by the charge per square metre (\$47).

Now you try

Hamish makes socks and sells them for \$22 a pair. How much will Hamish earn in a week where he sells 42 pairs of socks?

Royalties

A royalty is a payment for the use of intellectual property such as a book or a song. It is calculated as a percentage of the revenue or profit received from its use. People such as creative artists and authors receive a royalty.

Advantages

- Incentive to work hard
- Income increases with a better product
- Flexible hours and work place

Disadvantages

- Income varies according to sales
- No superannuation, sick leave or holiday pay

Royalty

Royalty = Percentage of the goods sold or profit received

Example 14 Calculating a royalty

Hiroshi is an author and is paid a royalty of 12% of books sold. Find Hiroshi's royalties if there were 2480 books sold at \$67.50 each.

Solution

$$\begin{aligned}\text{Royalty} &= 12\% \text{ of } (2480 \times \$67.50) \\ &= 0.12 \times 2480 \times 67.50 \\ &= \$20\,088\end{aligned}$$

Hiroshi earns \$20 088 in royalties.

Explanation

Determine 12% of the total sales
($2480 \times \$67.50$).

Now you try

Yoko is paid a royalty of 14% of socks sold. Find the royalties if Yoko sells 1010 socks at \$24.99 each. Answer to the nearest dollar.

Government payments

Some people receive a pension, allowance or benefit from the government. For example, the age pension is payable for a person who has reached 67 years of age. The requirements for receiving these incomes may change according to the priorities of the current government.



Example 15 Calculating an income from government payments

Youth Allowance helps people studying, undertaking training or in an apprenticeship.

Status	Allowance per fortnight
Single, no children, younger than 18, and live at your parents' home	\$395.30
Single, no children, younger than 18, living away from your parents' home to study, train or look for work	\$639.00
Single, no children, 18 or older and live at your parents' home	\$455.20
Single, with children	\$806.00

How much Youth Allowance does Ryan receive in a year if he is over 18 and living at home while studying?

Solution

$$\begin{aligned}\text{Allowance} &= \$455.20 \times 26 \\ &= \$11\,835.20\end{aligned}$$

Explanation

Multiply the government allowance by the number of fortnights in a year (26).

Now you try

Joseph is over 18 and needs to live away from home to study. How much Youth Allowance is Joseph paid in a year?

Exercise 1F

BUILDING

Example 13

- 1 A dry cleaner charges \$24 to clean a dress. How much do they earn by dry cleaning:
 - a 250 dresses?
 - b 430 dresses?
 - c 320 dresses?
- 2 Sabika is an artist who makes \$180 for each large portrait and \$100 for each small portrait. How much will Sabika earn if she sells 13 large and 28 small portraits?
- 3 Angus works part-time by addressing envelopes at home and is paid \$23 per 100 envelopes completed, plus \$40 to deliver them to the office. What is Angus's pay for delivering 2000 addressed envelopes?
- 4 Emilio earns a royalty of 24% on net sales from writing a fiction book. There were \$18 640 net sales in the last financial year. What is Emilio's royalty payment?

Example 14

- 5 Calculate the royalties on the following sales.
 - a 3590 books sold at \$45.60 with a 8% royalty payment
 - b 18 432 DVDs sold at \$20 with a 10% royalty payment
 - c 4805 computer games sold at \$65.40 with a 5% royalty payment

Example 15

- 6 Austudy provides financial help if you are an eligible student or Australian apprentice who is studying full-time.

Status	Allowance per fortnight
Single, no children	\$639
Single, with children	\$806
A couple, no children	\$639
A couple, with children	\$691

- a How much does Madison receive in a year if she is single with a child and studying full-time? Madison is 29 years old.
- b How much does Oscar receive in a year if he is partnered with no children and studying full-time? Oscar is 35 years old.

DEVELOPING

- 7 Tahlia receives \$31 for delivering 125 brochures. She receives an additional \$50 per day when delivering in wet weather. How much does Tahlia receive for delivering:
 - a 600 brochures on a clear day?
 - b 1000 brochures on a clear day?
 - c 800 brochures on a wet day?
 - d 1400 brochures on a wet day?
- 8 Tiki works in a factory that makes key rings. Each key ring completed earns him \$0.34. Tiki also receives an additional \$25 if he works on the weekend. How much does Tiki earn for making:
 - a 420 key rings on Friday?
 - b 460 key rings on Wednesday?
 - c 380 key rings on Saturday?
 - d 230 key rings on Sunday?

- 9** Caitlin is a doctor and she charges each patient \$33.50 for a consultation. She works for 6 hours a day and usually sees 5 patients per hour.

- a** How much money does Caitlin receive each day?
- b** Caitlin also has costs of \$410 per day. What is the profit for the day?



- 10** A government payment is reduced by 50 cents for every dollar between \$62 and \$250 of fortnightly income. Tyler is 28 years of age, partnered and has one child. He is studying full-time but earning \$126 per fortnight in a part-time job. What will be Tyler's fortnightly Austudy payment? Use the Austudy table from question 6.

CHALLENGING

- 11** Anthony writes crime novels. He has just received the half-yearly statement of sales of his latest novel. He has been informed that 20 000 copies were printed and there are 8760 left in stock. Anthony receives 15% of the retail price as royalties.

- a** How many copies of Anthony's latest novel were sold?
- b** What is Anthony's royalty if the retail price of his latest novel is \$24.95?
- c** What is Anthony's royalty if the retail price of \$24.95 was discounted by 10%?



- 12** The maximum Youth Allowance is reduced by \$1 for every \$4 that the youth's parents' income is over \$31 400. By how much is Charlotte's Youth Allowance reduced if her parents earn a combined income of \$34 728?

Communicate your reasoning, understanding and fluency

- 13** Go to the Services Australia website and answer the following questions about the Child Care Subsidy.

- a** What are the 4 conditions parents must satisfy to be eligible for Child Care Subsidy?
- b** What are the 4 factors that contribute to how much Child Care Subsidy you get?
- c** Using the Payment Finder link, determine what government payments you may be eligible for given the following circumstances.
 - You are an Australian citizen who has always lived in Sydney.
 - You are married.
 - You and your partner are 28 years old.
 - Your rent is \$600 per week.
 - You are working full-time.
 - You are the principal carer of one dependent child aged 18 months.
 - You have no Aboriginal or Torres Strait Islander heritage.
 - You do not have a disability or provide care for someone who does.
 - Your gross income is \$3200 per fortnight.
 - You have assets of \$50 000.

1G Gross pay, deductions from pay and net pay

Learning intentions

- To understand the difference between gross pay and net pay
- To perform calculations involving gross pay, deductions from pay and net pay

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about these concepts sometimes appear in HSC Examinations

Gross pay is the total of an employee's pay, including all allowances, overtime pay, commissions and bonuses. It is the amount of money before any deductions are made. The amount remaining after deductions have been subtracted is called the net pay or 'take-home pay'.

Deductions from pay are a regular amount of money subtracted from a person's wage or salary, not to be confused with tax deductions. People have many different deductions subtracted from their gross pay such as:

- income tax – a charge that funds the government's operations
- superannuation – an investment for retirement. An employer must contribute 11.5% of the employee's wages into a superannuation fund
- health insurance – private insurance to cover medical and dental costs
- union fee – payment for union membership.

Gross pay, net pay and deductions

Net pay = Gross pay – Deductions



Example 16 Calculating net pay

Laura is a nurse who receives a gross weekly wage of \$2345. She has the following deductions taken from her pay:

- Income tax – \$861
- Health fund payments – \$48.25
- Superannuation – \$67.95

What is Laura's net pay?



Solution

$$\begin{aligned}\text{Net pay} &= \text{Gross pay} - \text{deductions} \\ &= 2345 - (861 + 48.25 + 67.95) \\ &= \$1367.80\end{aligned}$$

Laura's net pay is \$1367.80.

Explanation

Subtract all the deductions from the gross pay.

Now you try

Gary is a construction worker who earns a gross weekly pay of \$1250. He has the following deductions.

- Income tax – \$312
- Union fee – \$25
- Superannuation – \$150
- What is Gary's net pay?

**Example 17** Reading information from a pay slip

Oscar received the following pay slip. What amount is added or deducted this pay for:

- a** gross pay? **b** net pay? **c** superannuation? **d** income tax?

EAGLE ENTERPRISES					
Employee: Oscar Vargas No: 006914					
	Hours	Rate	Amount	This Pay	Year to Date
Ordinary time	26.00	\$25.00	\$650.00		
Annual holiday	0.0	\$25.99	\$ 0.00		
Total gross earnings				\$650.00	\$1300.00
Income tax				\$100.00	\$ 200.00
Social club				\$ 2.00	\$ 4.00
Higher education repayments				\$ 13.00	\$ 26.00
Superannuation				\$ 35.00	\$ 70.00
Less post-tax deductions				\$ 50.00	\$ 100.00
Net pay				\$450.00	\$ 900.00
Direct credit to account: 00 000 000					
Total payments				\$450.00	\$ 900.00

Solution

- a** Gross pay is \$650.
b Net pay is \$450.
c Superannuation is \$35.
d Income tax is \$100.

Explanation

Read each value from the payslip.

Now you try

From Oscar's payslip, what is:

- a** the value of Oscar's gross earnings so far this year?
b Oscar's higher education repayment for this pay?
c the total amount Oscar was paid this pay?

Exercise 1G**BUILDING****Example 16**

- 1** Calculate the weekly net pay for these people.
- a** Isabella receives a gross pay of \$1386 and has total deductions of \$875.
b Kim-Ly receives a gross pay of \$985 and has total deductions of \$265.
c Christopher receives a gross pay of \$715 and has total deductions of \$222.

2 Calculate the weekly net earnings for these people.

- a** Daniel receives a gross weekly wage of \$1056 and has deductions of \$294.75 for income tax, \$28.80 for superannuation and \$325.05 for loan repayments.
- b** Hannah receives a gross weekly wage of \$3042 and has deductions of \$1068 for income tax, health fund payments for \$50.85, superannuation for \$53.55 and savings for \$450.
- c** Kapil receives a gross weekly wage of \$2274. He has deductions of \$768 for income tax, \$28.95 for health insurance, \$49.02 for superannuation, \$15.30 for life insurance and \$450 for loan repayments.

Example 17

3 Jack's annual gross pay is \$48 750. The deductions are \$9150 for income tax, \$1462 for health insurance and \$5280 for superannuation.

- a** What are Jack's total deductions?
- b** What is Jack's annual net pay?



4 Calculate the weekly gross pay for these people.

- a** Aaron receives a net weekly pay of \$1245 and has deductions of \$374.15 for income tax, \$45.60 for superannuation and \$25.20 for union membership.
- b** Nani receives a net weekly pay of \$2645 and has deductions of \$1068 for income tax, \$53.95 for health fund payments and \$83.75 for superannuation.
- c** Ivan receives a net weekly pay of \$2511. He has deductions of \$913 for income tax, \$31.95 for health insurance, \$59.46 for superannuation, \$18.20 for life insurance and \$470 for loan repayments.

5 Harry's net pay is \$57 908. His deductions are \$12 580 for income tax, \$2087 for health insurance and \$6910 for superannuation. What is Harry's gross pay?

6 Joshua is on a working holiday picking fruit. He is paid per piece of fruit he picks. He picked fruit on a farm as follows:

Monday – 170
 Tuesday – 130
 Wednesday – 145
 Thursday – 210
 Friday – 190



- a** What is Joshua's gross salary at \$0.55 per piece of fruit?
- b** What is Joshua's net salary if he has total deductions of \$121?

7 Charlotte owns an investment property that is rented out for \$320 per week. She pays the real estate agent a fee of 3% for managing the property.

- a** How much does Charlotte pay the real estate agent each week?
- b** How much does Charlotte receive each week from the investment property?
- c** What is the net income received by Charlotte from this property over the year?

- 8** Nicholas receives a yearly gross salary of \$74 568. He pays 18% of his weekly gross salary in income tax. He contributes 9% of his weekly gross salary to his superannuation fund and has \$155 in miscellaneous deductions each week.
- a** What is Nicholas's gross weekly pay?
 - b** How much income tax is deducted each week from Nicholas's salary?
 - c** How much superannuation is Nicholas contributing each week?
 - d** What is the total amount of deductions Nicholas makes each week?
 - e** What is Nicholas's net weekly pay each week?
- 9** Lakshmi receives a fortnightly pay of \$2240. She pays 15% of her weekly gross salary in income tax. She contributes 9% of her weekly gross salary to her superannuation fund and has \$95 in miscellaneous deductions each week.
- a** What is Lakshmi's gross weekly pay?
 - b** How much income tax is deducted each week from Lakshmi's salary?
 - c** How much superannuation is Lakshmi contributing each week?
 - d** What is the total amount of deductions Lakshmi makes each week?
 - e** What is Lakshmi's net weekly pay each week?

DEVELOPING

- 10** Charlie is a building worker who receives \$48.50 per hour for a 38-hour week. In addition he receives an allowance of \$3.50 per hour for work on a multistorey development. Charlie is currently working on six-storey apartment block. Each week he has deducted from his pay a superannuation contribution of 9% of his gross pay and union fees of \$28.45. Because he only started working late in the financial year, he doesn't yet have to pay tax.
- a** What is Charlie's gross weekly pay this week?
 - b** How much superannuation is Charlie contributing each week?
 - c** What is Charlie's net weekly pay this week?
- 11** Amina receives a gross fortnightly salary of \$2703 and has deductions of \$891.75 for income tax, \$54.30 for health fund payments, \$753 for car loan payments and \$14.55 for union subscription.
- a** What is Amina's net income each fortnight?
 - b** What percentage of Amina's gross income is deducted for income tax? Answer correct to one decimal place.

CHALLENGING

- 12** Liam received a gross fortnightly salary of \$3795. His pay deductions were \$937.20 for income tax, \$215.25 for superannuation, \$21.45 for union fees and \$201 for a home loan repayment.
- a** What is Liam's net income each fortnight?
 - b** What was Liam's weekly net pay?
 - c** What percentage of Liam's gross income was deducted for income tax? Answer correct to one decimal place.
 - d** If Liam's loan repayment increased by 10%, what would be his new fortnightly net pay?

- 13** Jane normally works 37 hours a week at \$54 per hour. In one particular week she worked 42 hours and received overtime at the rate of time-and-a-half. Her deductions for the week were income tax \$602.20, medical fund \$49.60, superannuation \$74.40 and motor vehicle repayment \$417.40.

- a** What was Jane's gross weekly wage?
b What was Jane's net income for the week?
c What percentage of Jane's gross income is spent on the motor vehicle repayment? Answer correct to the nearest percent.



Communicate your reasoning, understanding and fluency

14

HOB'S WORKPLACE					
Employee: Mark King No: 3038761					
	Hours	Rate	Amount	This pay	Year to date
Ordinary time	30	\$34.55	\$1036.50		
Time-and-a-half	6	\$51.83	\$ 310.98		
Double time	0	\$69.10			
Annual holiday	0	\$35.00			
Total gross				\$1347.48	\$5389.92
Income tax				\$ 200.00	\$ 800.00
Union fees				\$ 30.00	\$ 120.00
Higher education repayments				\$ 30.00	\$ 120.00
Superannuation				\$ 100.00	\$ 400.00
Dry-cleaning				\$ 15.00	\$ 60.00
Net pay				\$ 972.48	\$ 3889.92
Account number: 0110 1110					
Total payments				\$ 972.48	\$ 3889.92

- a** From the table, find:
- i** how many hours Mark worked this week
 - ii** Mark's gross pay
 - iii** Mark's allowable tax deductions and their value
 - iv** the number of weeks Mark has worked this year.
- b** Calculate Mark's net pay as a percentage of his gross pay.

1H Allowable tax deductions which reduce taxable income

Learning intentions

- To understand the income tax system
- To appreciate that some expenses reduce taxable income
- To perform calculations involving allowable tax deductions

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about income tax regularly appear in HSC Examinations

Allowable tax deductions are amounts allowed by the Australian Taxation Office (ATO) to be deducted from gross income to calculate a lower taxable income. Details of allowable tax deductions are given on the ATO website (www.ato.gov.au).

Allowable tax deductions include:

- work-related expenses – costs incurred while performing your job
- self-education expenses – costs of education related to your work
- travel expenses – costs of travel directly connected with your work
- car expenses – costs of using your car related to your work
- clothing expenses – costs of work clothing and laundry
- tools – cost of work tools
- gifts and donations – gifts made to an eligible organisation.



Example 18 Recognising allowable tax deductions

Riley works as an information technology consultant. He is entitled to the following tax deductions:

- Equipment costs of \$1260
- Car expenses of \$1060
- Professional learning of \$985
- Union fees of \$650
- Charity donations of \$250
- Tax agent fee of \$212.

What is Riley's total allowable tax deduction?

Solution

$$\begin{aligned}\text{Tax deduction} &= 1260 + 1060 + 985 \\ &\quad + 650 + 250 + 212 \\ &= \$4417\end{aligned}$$

Explanation

Add all allowable deductions.

continued ...

Now you try

The following year, Riley has a different list of deductions:

- Equipment costs of \$1400
- Car expenses of \$1500
- Professional learning of \$750
- Union fees of \$660
- Donations to charities of \$300
- Tax agent fees of \$250.

How much are Riley’s total allowable tax deductions this year?



Example 19 Calculating allowable tax deductions

Ava has used her own car for a total of 7900 km on work-related travel this financial year. Calculate her tax deduction if Ava is entitled to claim 78 cents per kilometre.

Solution

Tax deduction = 7900×0.78
= \$6162

Explanation

Multiply the kilometres travelled by the rate per kilometre.

Now you try

Ezra has her own car and has travelled 15 000 km on work-related travel this financial year. Calculate her tax deduction if Ezra is entitled to claim \$0.76 per km.

Exercise 1H

BUILDING

- 1 Stephanie works as a waitress and is entitled to an allowance for the cost of her work clothing and laundry. Her clothing expenses are listed in the table. Calculate the total cost for each item of clothing and Stephanie’s allowable tax deduction for clothing.

	Work clothing	Quantity	Unit cost
a	Blue shirt	3	\$ 55
b	Black trouser	3	\$110
c	Belt	1	\$ 45
d	Tie	2	\$ 34
e	Dry-cleaning	4	\$ 32



Example 18

2 Zara is a childcare worker who is entitled to the following tax deductions:

- \$420 for union fees
- Tax agent fee of \$125
- Charity donations of \$160
- Self-education fee of \$840
- Stationery costs of \$46.

What is Zara's total tax deduction?

3 Chris is entitled to the following tax deductions: training courses of \$1460, motor vehicle expenses of \$1420, stationery costs of \$760, union fees of \$480, charity donations of \$310 and an accountant's fee of \$184. What is Chris's total allowable tax deduction?

Example 19

4 Car expense deductions can be claimed using the cents per kilometre method. Under this method, individuals can claim a tax deduction based on the number of business kilometres travelled in the financial year. This is shown in the table.

Maximum kilometres allowed per car	Tax deduction rate
5000	66 cents per kilometre

a Calculate the allowable tax deduction if the following business kilometres were travelled.

i 3560 km

ii 1280 km

iii 4580 km

iv 2340 km

v 3105 km

vi 6000 km

b The car expenses tax deduction rate has decreased to 60 cents per kilometre. Calculate the allowable tax deduction if the following business kilometres were travelled.

i 4350 km

ii 1289 km

iii 5500 km

iv 4000 km

v 80 km

vi 694 km

DEVELOPING

5 Harrison buys a new van costing \$42 560 for his business. He is entitled to claim a tax deduction of 12% of the cost of the vehicle if the motor vehicle travels more than 5000 business kilometres in a year. What is Harrison's allowable tax deduction for the van if he travelled 16 230 kilometres for business?

6 Joel is a wheat farmer. He has capital equipment on the farm worth \$240 000. The ATO allows a tax deduction for the depreciation of capital equipment based on a percentage of the current value. What is the tax deduction using the following rates of depreciation?

- a** 10% per annum
- b** 20% per annum
- c** 30% per annum
- d** 40% per annum



- 7** Syed has an investment property that contains furnishings valued at \$12 600. The furnishings are an allowable tax deduction with a rate of depreciation of 15% per annum. How much can Syed claim for depreciation over the year?
- 8** Xiang is a teacher who bought a \$2350 laptop for school use. The laptop is an allowable deduction with a rate of depreciation of 33% per annum of the current value. How much can Xiang claim for depreciation in the:
- a** first year? **b** second year? **c** third year?

CHALLENGING

- 9** Dylan is the owner of a newsagency in a shopping centre. He pays rent of \$860 per week, has an electricity bill of \$280 per quarter and a telephone bill of \$110 per month. These expenses are work related so he is entitled to a tax deduction. What is Dylan's total allowable tax deduction?
- 10** Chelsea has a small office in her home to run a business. The office in her home is 8% of the area of the house. The tax office allows 8% of the household bills as a tax deduction if it is a work-related expense. Calculate Chelsea's allowable tax deduction for the financial year on the following household bills.
- a** Electricity bill of \$360 per quarter
b Telephone bill of \$70 per month
c House insurance of \$684 per year
d Rent of \$440 per fortnight



Communicate your reasoning, understanding and fluency

- 11** At the end of the year, Gordon's logbook shows he travelled a total of 12 500 km of which 7000 km were for business-related purposes. Gordon's total expenses, including depreciation, are \$8400 for the year. He has the following options when claiming a tax deduction.
- Option 1:** He can claim \$0.88 per km travelled as a tax deduction for work-related travel up to a maximum of \$5000.
- Option 2:** He can work out what percentage of car travel he used for business-related purposes, then calculate this as a percentage of his total expenses.
- Show why Gordon should choose Option 2.

11 Taxable income

Learning intentions

- To understand the income tax system
- To appreciate that some expenses reduce taxable income
- To perform calculations involving gross pay, tax deductions and taxable income

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about income tax regularly appear in HSC Examinations

Each year people who earn an income are required to complete a tax return on a paper form or digitally, online. A tax return states a person's income, the amount of tax paid and any allowable tax deductions. Most taxpayers have PAYG (pay as you go) tax deducted from their wage or salary throughout the year. The PAYG tax can be greater or less than the required amount of tax to be paid.



Tax is calculated on the taxable income. The taxable income is the gross income minus any allowable deductions. The gross income is the total amount of money earned from all sources. It includes interest, profits from shares or any payment received throughout the year.

Taxable income

Taxable income = Gross income – Allowable tax deductions



Example 20 Calculating taxable income

Anthony is a businessman who earns a gross salary of \$93 250 per year. His accountant completed his tax return and calculated \$2890 in allowable tax deductions.

What is Anthony's taxable income?



Solution

$$\begin{aligned}\text{Taxable income} &= 93\,250 - 2890 \\ &= \$90\,360\end{aligned}$$

Explanation

Subtract deductions from gross income.

Now you try

Hamish earns a gross salary of \$122 000 per year. He has \$4444 in allowable tax deductions. What is Hamish's taxable income?



Example 21 **Calculating taxable income**

Emily is a journalist with a gross annual salary of \$87 620. She also made \$5680 from her share portfolio and received \$7320 in royalties. If Emily has tax deductions totaling \$6472, calculate her taxable income.

Solution

$$\begin{aligned} \text{Gross income} &= 87\,620 + 5680 + 7320 \\ &= \$100\,620 \end{aligned}$$

$$\begin{aligned} \text{Taxable income} &= 100\,620 - 6472 \\ &= \$94\,148 \end{aligned}$$

Explanation

Determine gross income by adding all income.

Determine taxable income by subtracting deductions from income.

Now you try

Jorgan earns a gross salary of \$115 600 per year. He receives rent of \$15 000 and \$3204 from shares. He has \$14 086 in allowable tax deductions. What is Jorgan’s taxable income?



Example 22 **Calculating taxable income**

Nicole is a scientist who earns a gross weekly pay of \$1624. She has allowable tax deductions of \$8 per week for dry-cleaning, \$60 for work-related travel per year, \$460 per year for union fees and she made donations to charities of \$620 throughout the year.

- a** What is Nicole’s gross yearly salary?
- b** What is Nicole’s total allowable tax deduction?
- c** Calculate Nicole’s taxable income.



Solution

a $\text{Salary} = 1624 \times 52$
 $= \$84\,448$

b $\text{Tax deduction} = (8 \times 52) + 60 + 460 + 620$
 $= \$1556$

c $\text{Taxable income} = 84\,448 - 1556$
 $= \$82\,892$

Explanation

Determine salary.

Determine tax deductions.

Determine taxable income by subtracting deductions from salary.

Now you try

Trevor is a research assistant who earns \$3816 each fortnight. He pays \$24 per week in dry-cleaning. He has \$40 week in work-related travel expenses. He paid \$720 in union fees throughout the year.

- a** What is Trevor’s gross annual salary?
- b** What is Trevor’s total allowable tax deduction?
- c** Calculate Trevor’s taxable income.

Exercise 11

BUILDING

Example 20

- 1** Benjamin has a gross income of \$84 000. What is Benjamin's taxable income given the following allowable tax deductions?

a \$5120	b \$9571	c \$4720
d \$24 104	e \$8205	f \$17 594
g \$12 520	h \$23 890	i \$34 560

Example 21

- 2** Chris earns a gross salary of \$67 840 per year. His tax deductions total \$3462. Calculate Chris's taxable income.

- 3** Jessica earned a gross income of \$75 480 in the last financial year.

- a** The table is a summary of her allowable tax deductions. What is Jessica's total allowable deduction?
- b** Calculate Jessica's taxable income in the last financial year.

Allowable tax deductions	Amount
Work-related expenses	\$1260
Self-education expenses	\$680
Travel expenses	\$940
Clothing expenses	\$320

Example 22

- 4** Eliza earned \$88 784 from her employer in the last financial year. She also earned bank interest of \$380. Eliza spent \$240 on books, \$520 on stationery and \$380 on a printer, all of which are needed for her work.

- a** What is Eliza's gross income?
- b** What is Eliza's total allowable tax deduction?
- c** Calculate Eliza's taxable income.

- 5** Daniel is a police officer who earned a gross income of \$63 620. He claimed a tax deduction for a utility belt (\$160), a pair of safety glasses (\$390), bulletproof vest (\$1240) and handcuffs (\$420).

- a** What is Daniel's total allowable tax deduction?
- b** Calculate Daniel's taxable income.



DEVELOPING

- 6** Nicholas earns a gross weekly pay of \$1120. He has tax deductions of \$1460.

- a** What is Nicholas's gross yearly salary?
- b** What is Nicholas's taxable income?

- 7** Ulani is a real estate salesperson who earns a commission of 2% on all sales. During the year Ulani sold real estate to the value of \$3 232 100.
- What is Ulani's gross annual income?
 - Ulani has calculated her tax deductions to be \$4320. What is her taxable income?
- 8** Isabelle works for a travel agency and earns a gross fortnightly pay of \$2780. She pays PAYG tax of \$602 per fortnight and has tax deductions of \$7 per week for dry-cleaning, \$80 for work-related travel per year and \$380 per year for union fees.
- What is Isabelle's gross yearly salary?
 - How much tax is deducted each week?
 - What is Isabelle's total allowable tax deduction?
 - Calculate Isabelle's taxable income.



CHALLENGING

- 9** Ahmed is a tradesman who receives a yearly gross salary of \$92 200. He also works part-time at TAFE for a wage of \$135 per week. Ahmed received \$360 in share dividends.
- What is Ahmed's gross annual income?
 - Ahmed is entitled to a tax deduction for travelling between his two places of employment. Ahmed has calculated that he travelled 340 km. The allowable deduction is 82 cents per kilometre. What is Ahmed's travel expense?
 - In addition to the above travel expenses, Ahmed is entitled to the following tax deductions: \$530 for union fees, tax agent fee of \$180, charity donations of \$280 and equipment costs of \$750. Calculate Ahmed's total allowable deduction.
 - What is Ahmed's taxable income?



Communicate your reasoning, understanding and fluency

- 10 a** Use the payslip from Section 1G, question **14** to calculate Mark's total tax deductions for one year.
- b** After 6 months Mark starts to make charitable donations of \$75 a month. What will be his taxable income at the end of the year? (Give your answer to the nearest dollar)

1J Calculating the Medicare levy and income tax payable

Learning intentions

- To understand the income tax system
- To appreciate that some expenses reduce taxable income
- To perform calculations involving the Medicare levy and tax payable

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- The Medicare levy is currently 2% of taxable income, but this could change
- Questions about income tax regularly appear in HSC Examinations
- An up-to-date tax payable table is provided in HSC questions

The amount of taxation varies according to the amount of money you earn. A tax return must be lodged with the ATO each year. It is statement of the income earned and the tax paid during the financial year from 1 July to 30 June. The ATO publishes a tax pack to assist people in completing their tax return. The Tax Pack provides information about the current income tax rates.



Personal income tax rates

The personal income tax rates are regularly changed to take into account government policy and inflation. The tax rates for 2024–25 are listed in the table. (Note: These may not be the current tax rates.)

Taxable income	Tax payable
0–\$18 200	Nil
\$18 201–\$45 000	16c for each \$1 over \$18 200
\$45 001–\$135 000	\$4288 plus 30c for each \$1 over \$45 000
\$135 001–\$190 000	\$31 288 plus 37c for each \$1 over \$135 000
\$190 001 and over	\$51 638 plus 45c for each \$1 over \$190 000

The tax payable is dependent on the taxable income. If the taxable income is \$18 200 or less then there is no tax payable. The tax rates then increase progressively. It starts out at 16 cents for every \$1, for amounts between \$18 201 and \$45 000. The highest tax rate is 45 cents for every \$1 over \$190 000. Most people have PAYG tax deducted from their wage or salary throughout the year. This PAYG tax can be greater or less than the required amount of tax to be paid. If a person pays more throughout the year than they are required to pay, they will receive a tax refund. If they pay less than the required amount of tax throughout the year, they will have tax owing.

Tax refund

Tax refund = Tax paid – Tax payable

Tax owing

Tax owing = Tax payable – Tax paid

**Example 23** Calculating the tax payable

Manjula has a taxable income of \$25 000. Using the tax table on page 41, find how much tax Manjula will have to pay. If Manjula has paid \$1650, what will be her refund?

Solution

$$\begin{aligned}\text{Tax payable} &= (25\,000 - 18\,200) \times 0.16 \\ &= \$1088\end{aligned}$$

$$\text{Tax refund} = \$1650 - \$1088 = \$562$$

Explanation

Identify \$25 000 is between \$18 201 and \$45 000.

Use the formula in Row 2 of the table.

Determine tax refund by subtracting the tax payable (\$1088) from the PAYG amount (\$1650).

Now you try

Tim earns a gross salary of \$43 000 per year. He has paid PAYG tax of \$4200. Using the tax table on page 41, calculate Tim's tax refund.

**Example 24** Calculating a tax refund

Joel has a taxable income of \$200 000 and has paid \$65 300 in tax instalments.

- Calculate the amount of tax payable by Joel. Use the tax rates on page 41.
- Will Joel receive a refund or will he have to pay more tax? Justify your answer.
- What percentage of his income does Joel pay in tax? Answer correct to one decimal place.

Solution

$$\begin{aligned}\text{a Tax} &= 51\,638 + (200\,000 - 190\,000) \times 0.45 \\ &= \$56\,138\end{aligned}$$

$$\begin{aligned}\text{b Tax refund} &= \$65\,300 - \$56\,138 \\ &= \$8862\end{aligned}$$

$$\begin{aligned}\text{c } \frac{\text{Tax paid}}{\text{Taxable income}} &= \frac{\$56\,138}{\$200\,000} \\ \text{Percentage} &= \frac{\$56\,138}{\$200\,000} \times 100 \\ &= 28.069 \\ &= 28.1\%\end{aligned}$$

Explanation

Identify \$200 000 is over \$190 000.

Use the formula in Row 5 of the table.

Determine tax refund by subtracting the tax payable from the PAYG amount.

Express the tax paid as a percentage of the taxable income.

Multiply by 100 to convert to a decimal and express to one decimal place.

Now you try

Faith earns \$155 545 each year. She pays \$40 300 in PAYG tax throughout the year.

- Use the tax table on page 41 to calculate the amount of tax payable by Faith. Answer to the nearest dollar.
- Will Faith receive a tax refund or will she have to pay tax? Justify your answer.
- What percentage of her income does Faith pay in tax? Answer to the nearest percent.

Medicare levy

The Medicare levy is an additional charge to support Australia's universal health care system. It ensures that all Australians have access to free or low-cost medical and hospital care. The Medicare levy is calculated at a rate of 2% of taxable income.

Medicare levy

Medicare gives Australian residents access to health care. It is partly funded by taxpayers who pay a Medicare levy of 2% of their taxable income.

Your Medicare levy is reduced if your taxable income is below a certain threshold. In some cases you may not have to pay the levy at all.

If you don't have private hospital health insurance, you may have to pay the Medicare levy surcharge (MLS) in addition to the Medicare levy. This depends on your income for MLS purposes.

If you do have an appropriate level of private hospital health insurance, you won't have to pay the MLS, and depending on your income you may be eligible for the private health insurance rebate. This rebate is an amount the government contributes towards the cost of your private hospital health insurance premiums.

Medicare levy

Additional charge for health services. It is calculated at 2% of the taxable income.



Example 25 Calculating the Medicare levy

The Medicare levy is 2% of the taxable income. What is the Medicare levy if the taxable income is \$90 600?

Solution

$$\begin{aligned}\text{Medicare levy} &= 2\% \text{ of } \$90\,600 \\ &= 0.02 \times 90\,600 \\ &= \$1812\end{aligned}$$

Explanation

Determine 2% of income.

Now you try

The Medicare levy is 2% of the taxable income. What is the Medicare levy if the taxable income is \$88 420?

Exercise 1J

BUILDING

- 1 The table shows the personal income tax rates for 2024–25.

Taxable income	Tax payable	Tax bracket
0–\$18 200	Nil	A
\$18 201–\$45 000	16c for each \$1 over \$18 200	B
\$45 001–\$135 000	\$4288 plus 30c for each \$1 over \$45 000	C
\$135 001–\$190 000	\$31 288 plus 37c for each \$1 over \$135 000	D
\$190 001 and over	\$51 638 plus 45c for each \$1 over \$190 000	E

Which tax bracket (A, B, C, D or E) from the table applies to these taxable incomes?

- a** \$24 000 **b** \$2500 **c** \$75 000
d \$122 500 **e** \$230 000 **f** \$80 000
g \$37 001 **h** \$180 000 **i** \$4500

- 2 Calculate the tax payable on the following taxable incomes by completing the tax payable expression.

- a** \$21 000 Tax payable = Nil + (21 000 – 18 200) × 0.19 =
b \$32 500 Tax payable = Nil + (32 500 – 18 200) × 0.19 =
c \$75 000 Tax payable = 3572 + (75 000 – 37 000) × 0.325 =
d \$122 600 Tax payable = 19 822 + (122 600 – 87 000) × 0.37 =
e \$230 000 Tax payable = 54 232 + (230 000 – 180 000) × 0.45 =
f \$80 000 Tax payable = 3572 + (80 000 – 37 000) × 0.325 =

- 3 Ava's taxable income was \$28 000. The first \$18 200 was tax-free and the balance was taxed at a marginal rate of 19%. Calculate the amount of tax payable.
 4 Tyler's taxable income was \$111 000. The tax payable on the first \$87 000 is \$19 822 and the balance was taxed at a marginal rate of 37%. Calculate the amount of tax payable.

Example 23

- 5 Use the tax table in Question 1 to calculate the tax payable on the following taxable incomes.

- a** \$18 201
b \$37 001
c \$87 001

Example 25

- 6 The Medicare levy is 2% of the taxable income. What is the Medicare levy on the following taxable incomes? Answer correct to the nearest cent.

- a** \$23 000 **b** \$88 541 **c** \$40 600
d \$46 906 **e** \$67 800 **f** \$200 592
g \$170 300 **h** \$15 790 **i** \$90 640

DEVELOPING

Use the tax table in question 1 to answer Questions 7 to 11.

7 What is the tax payable on the following taxable incomes?

a \$20 890	b \$73 966	c \$37 814
d \$115 900	e \$196 430	f \$53 410

Example 24

8 Calculate the tax refund or tax owing for the following.

- a** Taxable income of \$94 850 with tax instalments paid of \$24 678
- b** Taxable income of \$19 864 with tax instalments paid of \$220

9 Ebony's gross income is \$46 660 with allowable tax deductions of \$4120.

- a** What is Ebony's taxable income?
- b** Calculate the amount of tax due.
- c** Ebony has paid \$95 per week in tax. How much does Ebony owe in tax?

10 Charlie's gross income is \$155 730. His allowable tax deductions are \$8230.

- a** What is Charlie's taxable income?
- b** Calculate the amount of tax due.
- c** Charlie has paid \$860 per week in tax. How much tax has Charlie paid for the year?
- d** Will Charlie receive a refund or will he have to pay more tax? Justify your answer.

11 Ruby is a dentist with a taxable income of \$145 684.

- a** Find Ruby's tax payable on this amount.
- b** What percentage of Ruby's income is paid as tax? Answer correct to one decimal place.



12 Liam has a taxable income of \$56 400. He is required to pay \$9877 in tax plus a Medicare levy of 2% of his taxable income.

- a** How much is Liam's Medicare levy?
- b** Calculate the total amount of tax due, including the Medicare levy.

13 Mia works for a superannuation fund and received a taxable income of \$124 800.

- a** Medicare levy is 2% of the taxable income. How much is Mia's Medicare levy?
- b** Calculate the total amount of tax due, including the Medicare levy. Use the tax table from Exercise 1J question 1 to calculate Mia's tax.

Key ideas and chapter summary

Salary and wages	Salary – payment for a year’s work, which is then divided into equal monthly, fortnightly or weekly payments Wage – payment for work that is calculated on an hourly basis
Annual leave loading and bonuses	Annual leave loading – payment for going on holidays Holiday loading = $17.5\% \times \text{normal weekly pay} \times \text{weeks leave}$ Bonus – extra payment or gift earned as a reward
Overtime and other allowances	Overtime – work beyond the normal working day Casual rate – set amount paid for each hour’s work Time-and-a-half rate = $\text{normal rate} \times 1.5$ Double time rate = $\text{normal rate} \times 2$ Allowance – payment for difficult or dangerous conditions
Commission	Commission – percentage of the value of the goods sold Retainer – small payment in addition to the commission
Piecework, royalties and government	Piecework – payment for work completed Piecework = $\text{Number of units of work} \times \text{Amount paid per unit}$ Royalty – percentage of the goods sold or profit received Government income – pension, allowance or benefit
Allowable tax deductions	Deductions allowed by the Australian Taxation Office, which include work-related, self-education, travel, car, clothing, tools and donations
Taxable income	Tax – calculated on the taxable income Gross income – the total amount of money earned Taxable income = $\text{Gross income} - \text{Allowable tax deductions}$
Gross pay, net pay and deductions	Gross pay – total of the employee’s pay including allowances, overtime pay, commissions and bonuses Deductions – regular amount of money subtracted from a person’s wage or salary such as income tax Net pay = $\text{Gross pay} - \text{Deductions}$
Calculating tax	Personal income tax tables have an increasing rate of tax Tax refund = $\text{Tax paid} - \text{Tax payable}$ Tax owing = $\text{Tax payable} - \text{Tax paid}$
Medicare levy	Additional charge for health services Medicare is calculated at 2% of the taxable income

Chapter checklist with success criteria



Checklist

A printable version of this checklist is available in the Interactive Textbook.



- 1A 1** I can convert a fraction to a percentage. ☐
- e.g. Write each of the following as a percentage.
- a** $\frac{9}{25}$ **b** $\frac{4}{7}$ **c** 0.19
- 1A 2** I can write percentages as a simplified fraction. ☐
- e.g. Write each of the percentages below as a simplified fraction.
- a** 47% **b** 64% **c** $6\frac{1}{2}\%$
- 1A 3** I can write a percentage as a decimal. ☐
- e.g. Convert these percentages to decimals.
- a** 93% **b** 4% **c** 50%
- 1A 4** I can find the percentage of a quantity. ☐
- e.g. Find 23% of \$1975.
- 1B 5** I can calculate earnings from a salary. ☐
- e.g. Calculate the weekly pay of a person who earns a salary of \$52 052 per annum.
- 1B 6** I can make calculations using the details of a wage earner. ☐
- e.g. Jordan is paid \$590 each week. He works a 20-hour week. What is Jordan's hourly rate of pay?
- 1C 7** I can determine annual leave loading. ☐
- e.g. Ashton works a 38-hour week and is paid \$32.25 per hour. He receives 17.5% holiday loading for 4 weeks of his normal pay. How much is Ashton's holiday loading?
- 1C 8** I can find the amount of a bonus payment. ☐
- e.g. Grahame gets a 5% bonus on his annual salary of \$82 450. Calculate Grahame's bonus.
- 1D 9** I can calculate wages including overtime. ☐
- e.g. Cody works a 40-hour week and earns \$28 an hour. Last week he worked 4 hours at time-and-a-half and 2 hours of double time. How much did Cody earn last week?
- 1D 10** I can calculate casual pay. ☐
- e.g. Mohin is a casual employee who gets paid \$22 an hour on weekdays, time-and-a-half on weekends and double time on public holidays. Last week he worked 5 hours on Monday (a public holiday), 5 hours on Thursday and 6 hours on Saturday. How much did Mohin earn last week?

1E 11 I can find the amount of a commission earned. ☐

e.g. Gordon gets a 1.85% commission on the house he sold for \$990 000. How much is Gordon's commission?

1E 12 I can calculate pay that includes a retainer and commission. ☐

e.g. Lane gets a \$450 retainer as well as 3% commission on all sales over \$10 000. She makes sales of \$15 000. What are Lane's earnings?

1F 13 I can calculate a piecework payment. ☐

e.g. Audrey is a painter who gets paid \$15 per square metre she paints. How much will Audrey earn if she paints 35 square metres?

1F 14 I can calculate an income from royalties. ☐

e.g. Bailey is an author who gets paid a royalty of 11% on each book sold. How much are Bailey's royalties if he sells 1970 books at \$59.90 each book?

1F 15 I can calculate an income from the government. ☐

e.g. Use the table on page 25 to determine how much Youth Allowance Tegan receives in a year if she is 17 and living away from home.

1G 16 I can calculate net pay. ☐

e.g. Joe is a fire fighter who earns a gross weekly wage of \$1840. He has the following deductions:

- Tax – \$375
- Superannuation – \$75
- Health fund – \$50

Calculate Joe's net pay.

1G 17 I can read information from a payslip. ☐

e.g. Using the payslip on page 29, find the PAYG tax withheld:

- a** this pay **b** this year so far

1H 18 I can recognise allowable tax deductions. ☐

e.g. Dave works for a dry cleaner. He has the following tax deductions:

Car expenses of \$900 Union fees of \$712

Tax agent fees of \$450 Donations of \$400

Uniform of \$350

How much is Dave's total allowable deductions?

1H 19 I can calculate allowable tax deductions. ☐

e.g. Yoko uses her car for work and travels 15 067 km. Calculate Yoko's tax deduction if she is entitled to claim \$0.77 per km.

1I 20 I can calculate taxable income. ☐

e.g. Calculate the taxable income of a person who earns a salary of \$63 205 per annum and has \$4328 allowable tax deductions.

1I 21 I can calculate taxable income. ☐

e.g. Indigo earns a gross salary of \$97 600 per year. He receives rent of \$17 500 and \$8204 from shares. He has \$18 678 in allowable tax deductions. What is Indigo's taxable income?

1I 22 I can calculate taxable income. ☐

e.g. Jordan is paid \$575 each week. He also made \$4650 from shares and \$12 in interest. Jordan had \$60 a month in union fees, \$20 a week in dry-cleaning and \$125 a month in donations to charities.

- a** What is Jordan's gross annual salary?
- b** What is Jordan's total allowable deductions?
- c** Calculate Jordan's taxable income.

1J 23 I can calculate the tax payable. ☐

e.g. Lachlan earns a gross salary of \$73 000 per year. He has paid PAYG tax of \$13 000. Using the tax table on page 41, calculate Lachlan's tax refund.

1J 24 I can determine a tax refund. ☐

e.g. Pauline earns \$178 178 each year. She pays \$52 250 in PAYG tax throughout the year.

- a** Use the tax table on page 41 to calculate the amount of tax payable by Pauline. Answer to the nearest dollar.
- b** Will Pauline receive a tax refund or will she have to pay tax? How much will Pauline receive or have to pay?
- c** What percentage of her income does Pauline pay in tax? Answer to the nearest percent.

1J 25 I can calculate the Medicare levy. ☐

e.g. The Medicare levy is 2% of the taxable income. What is the Medicare levy if the taxable income is \$102 850?

Multiple-choice questions

- 1A 1** Convert $\frac{7}{20}$ to a percentage.
A 7% **B** 14% **C** 28% **D** 35%
- 1A 2** Convert 72% to a simplified fraction.
A $\frac{36}{50}$ **B** $\frac{72}{100}$ **C** $\frac{7}{20}$ **D** $\frac{2}{100}$
- 1B 3** Alyssa receives a salary of \$85 640. How much does Alyssa receive each fortnight?
A \$3293.84 **B** \$3293.85 **C** \$1646.92 **D** \$1646.93
- 1B** Hine works a 38-hour week at a rate of \$26.00 per hour. She receives 17.5% of 4 weeks normal pay as holiday loading. What is Hine's holiday loading?
A \$172.90 **B** \$691.60 **C** \$3952.00 **D** \$4643.60
- 1D 4** Christopher receives a normal hourly rate of \$22.60. What is Christopher's pay when he works 8 hours at normal rate and 3 hours at time-and-a-half?
A \$180.80 **B** \$248.60 **C** \$282.50 **D** \$316.40
- 1D 5** Qiang works for \$15.60 per hour. He works for 6 hours on Thursday and 8 hours on Friday. On Saturday he is paid at time-and-a-half for his work. Last week Qiang earned \$335.40 in total for his work on these three days. How many hours did Qiang work on Saturday?
A 5 hours **B** 6.5 hours **C** 7.5 hours **D** 11.25 hours
- 1E 6** Taylah earns a weekly retainer of \$425 plus a commission of 8% on sales. What was her weekly earnings in a week in which Taylah made sales of \$8620?
A \$34.00 **B** \$459.00 **C** \$689.60 **D** \$1114.60
- 1G 7** Alex is landscape gardener and entitled to the following tax deductions: union fees of \$484, donations of \$720, equipment cost of \$860, and car expenses of \$1455. What is Alex's total allowable tax deduction?
A \$484 **B** \$860 **C** \$1455 **D** \$3519
- 1H 8** Stephanie has an allowable tax deduction of \$4690. What is her taxable income if Stephanie's gross annual salary is \$43 720?
A \$39 030 **B** \$43 720 **C** \$48 410 **D** \$90 620
- 1I 9** Charlotte is a librarian who has a taxable income of \$69 410. The tax payable on the first \$37 000 is \$3572 and the balance was taxed at a rate of 32.5%. How much does Charlotte have to pay in tax, to the nearest dollar?
A \$3572 **B** \$14 105 **C** \$22 558 **D** \$37 000
- 1J 10** William has a taxable income of \$53 684. What is William's Medicare levy? Assume the Medicare levy is calculated at a rate of 2% of the taxable income.
A \$107.37 **B** \$1073.68 **C** \$2684.20 **D** \$52 610.32

Short-answer questions

- 1A 1** **a** Find 22% of 1400 kg. **b** Find 41% of \$900.
- 1B 2** Jake earns \$96 470.40 per annum and works on average 48 hours per week.
a What is Jake's average weekly wage?
b Calculate the Jake's hourly rate of pay.
- 1C 3** Carlo's employer has decided to reward all employees with a bonus. The bonus awarded is 6.25% of their annual salary. What is Carlo's bonus if his annual salary is \$85 940?
- 1C 4** The public service provides all employees with a 17.5% holiday loading on four weeks normal wages. Lucy works a 37-hour week for the public service in Canberra. She is paid a normal hourly rate of \$32.40.
a How much will Lucy receive in holiday loading?
b Calculate the total amount of pay that Lucy will receive for her holidays
- 1D 5** Alex works for a fast food company and is paid \$13.50 per hour for a 35-hour week. He gets time-and-a-half pay for overtime worked on weekdays and double time for working on weekends. Last week he worked a normal 35-hour week plus 3 hours of overtime during the week and 4 hours of overtime on the weekend. What was Alex's wage last week?
- 1E 6** Chelsea is a real estate agent and is paid the following commission on her sales: 3% on the first \$45 000, then 2% for the next \$90 000 and 1.5% thereafter.
a What is Chelsea's commission if she sells a property for \$240 000?
b How much would the owner of the property receive from the sale?
- 1F 7** Bailey is paid a royalty of 11.3% on the net sales of his book. The net sales of his book in the last financial year were \$278 420.
a What was Bailey's royalty payment in the last financial year?
b Net sales this financial year are expected to decrease by 15%. What is Bailey's expected royalty payment for this financial year?
- 1F 8** The maximum Youth Allowance is reduced by \$1 for every \$4 that the youth's parents income is over \$31 400. By how much is Hannah's Youth Allowance reduced if her parents earn a combined income of \$35 624?
- 1G 9** Ali works as a builder. His annual union fees are \$278.20. Ali has his union fees deducted from his weekly pay. What is Ali's weekly union deduction?
- 1H 10** Sienna owns a clothing store and is entitled to the following tax deductions: \$570 for union fees, tax agent fee of \$375, charity donations of \$390, information technology costs of \$3910 and printing costs of \$528. What is Sienna's total tax deduction?
- 1G 11** What is the taxable income if the gross salary is \$82 390 and the allowable tax deduction is \$4870?

- 1H 12** Ryan is a small business owner who bought a \$3850 computer for business use. The computer is an allowable tax deduction with a rate of depreciation of 33% per annum.
- How much can Ryan claim for depreciation in the first year?
 - What is the depreciated value of the computer after the first year?
- 1I 13** James works for a model agency and earns a gross fortnightly pay of \$3720. He pays PAYG tax of \$986 per fortnight and has tax deductions of \$15 per week for dry-cleaning, and yearly deductions of \$1450 for work-related travel and \$1000 for charities.
- What is James's gross yearly salary?
 - How much tax is deducted each week?
 - What is James's total allowable tax deduction?
 - Calculate James's taxable income.
- 1J 14** Ruby has a taxable income of \$134 830. Calculate Ruby's tax payable on this amount. Use the personal income tax rates on page 41.
- 1J 15** Cooper is a retail store manager. After he received a \$2300 annual pay rise, his salary become \$73 450. How much of the \$2300 pay rise was Cooper required to pay in tax? Use the personal income tax rates on page 41.

Extended-response questions

- 1D 1** Eliza is a casual employee and is paid a weekday rate of \$18 per hour, a Saturday rate of time-and-a-half and a Sunday rate of double time.
- 1J** This is her time card for last week.

Day	In	Out	Unpaid break
Friday	8:00 a.m.	3:30 p.m.	30 minutes
Saturday	9:00 a.m.	5:00 p.m.	1 hour
Sunday	9:00 a.m.	3:00 p.m.	30 minutes

- Calculate Eliza's gross pay for last week.
 - Eliza's employer withholds 20% of her weekly earnings as PAYG tax. Calculate Eliza's PAYG tax for last week.
- 1J 2** Nathan has a taxable income is \$106 770. He has paid \$980 per fortnight in tax.
- How much tax has Nathan paid for the year?
 - Calculate the amount of tax payable by Nathan. Use the tax rates on page 41.
 - Will Nathan receive a refund or will he have to pay more tax? Justify your answer.
 - What percentage of Nathan's income is paid as tax? Answer correct to one decimal place.



Formulas and equations

A fantastic formula

A polyhedron is a three-dimensional object with flat faces which are polygons. The table lists some polyhedra. Some are well-known and some are not so well-known. You can conduct an internet search to find pictures of them.

They all have:

- **FACES:** flat surfaces which are two-dimensional shapes
- **VERTICES:** points where the corners of three or more faces meet
- **EDGES:** lines where the edges of two faces meet.

For example, a square pyramid has:

- 5 faces
- 5 vertices
- 8 edges.

Make a copy of the table and place that information in the table:

Name of solid	Faces (F)	Vertices (V)	Edges (E)	$F + V - E$
Square pyramid				
Cube				
Triangular pyramid				
Triangular prism				
Octahedron				
Dodecahedron				
Icosahedron				

If you counted carefully, you would notice that the right column is the same for every solid.

$$F + V - E = 2$$

This equation, stated by Leonhard Euler in 1758, is known as Euler's polyhedron formula.

Chapter contents

- 2A** Substitution (**CONSOLIDATING**)
- 2B** Simplifying and expanding algebraic expressions (**CONSOLIDATING**)
- 2C** Linear equations (**CONSOLIDATING**)
- 2D** Using equations to solve worded problems
- 2E** Using formulas to solve problems
- 2F** Changing the subject of formulas
- 2G** Distance, speed, time and stopping distance
- 2H** Blood alcohol content
- 2I** Medication dosages

NSW Syllabus

In this chapter, a student:

- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- Selects and applies algebraic techniques to solve problems involving equations and formulas (MST-11-01)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

2A Substitution

(CONSOLIDATING)

Learning intentions

- To develop expertise with substituting numbers into algebraic expressions

Past, present and future learning

- Students should already be familiar with the contents of this section
- Substitution is used extensively throughout Years 11 and 12
- Questions involving substitution regularly appear in HSC Examinations

Substitution involves replacing pronumerals in an algebraic expression with numbers. The resulting numerical expression is evaluated and expressed to the specified level of accuracy.

Substitution of values

- Write the algebraic expression.
- Replace the variables in the expression with the numbers provided.
- Evaluate, using a calculator if necessary.
- Write the answer to the specified level of accuracy and correct units if necessary.

Note: On some calculators it is necessary to put negative numbers in brackets.

For example: If $x = -3$, $x^2 = (-3)^2 = 9$, not $x^2 = -3^2 = -9$.



Example 1 Substituting values into linear expressions

Evaluate $3a - 4b + c$, given $a = 2$, $b = 5$ and $c = -10$.

Solution

$$\begin{aligned} 3a - 4b + c &= 3 \times 2 - 4 \times 5 + -10 \\ &= 6 - 20 - 10 \\ &= -24 \end{aligned}$$

Explanation

Substitute the values.
Evaluate the expression.

Now you try

Evaluate $3a - 4b + c$, given $a = 2$, $b = -5$ and $c = 10$.



Example 2 Substituting values into non-linear expressions

Evaluate the following, given $a = 2$.

a $3a^3$

b $\sqrt{(2a + 5)}$

Solution

$$\begin{aligned} \text{a } 3a^3 &= 3 \times 2^3 \\ &= 3 \times 8 \\ &= 24 \end{aligned}$$

Explanation

Substitute the values.
Evaluate the expression.

$$\begin{aligned}\text{b } \sqrt{2a+5} &= \sqrt{2 \times 2 + 5} \\ &= \sqrt{9} \\ &= 3\end{aligned}$$

Substitute the values.
Evaluate the expression.

Now you try

Evaluate the following, given $a = -2$.

a $3a^2$

b $9 - a^2$



Example 3 Substituting values into non-linear expressions

Evaluate $a^2 + 5b^3 - c$, given $a = 4$, $b = 2$ and $c = -1$.

Solution

$$\begin{aligned}a^2 + 5b^3 - c &= 4^2 + 5 \times 2^3 - (-1) \\ &= 16 + 40 + 1 \\ &= 57\end{aligned}$$

Explanation

Substitute the values.
Evaluate the expression.

Now you try

Evaluate $x^2 + 2x - 5$, given $x = -3$.

Spreadsheet activity



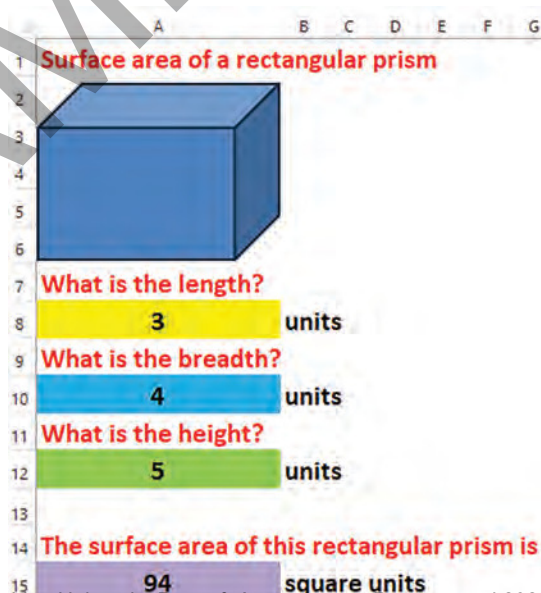
In this subject, you will often use the same formula, repeatedly, with different values. For example, the surface area of a rectangular prism requires length (l), breadth (b) and height (h).

Surface area formula:

$$A = 2(lb + lh + bh)$$

Using Microsoft Excel, a Google Sheet or some other spreadsheet software, there are numerous ways to set up a spreadsheet to do the calculation.

The snapshot shown is just one way to do this.



You can access this file from the online interactive version of this book.

You should:

- examine the formula in Cell A15
- try changing the numbers in the yellow, blue and green boxes
- adapt the spreadsheet for other frequently used formulas.

Exercise 2A

BUILDING

Example 1

1 Evaluate these expressions, given $a = 3$, $b = 4$ and $c = 8$.

a $5a + b$

b $a + 4b$

c $a - b + c$

d $a^2 + b^2$

e $4b \times (-2a)$

f $\sqrt{2ab}$

g $\frac{abc}{2}$

h $b^2 \div c$

i $\frac{2ab}{c}$

2 Find the value of $3a + 2b$ when:

a $a = 5$ and $b = 5$

b $a = 6$ and $b = -4$

c $a = 0$ and $b = 0$

d $a = -7$ and $b = -2$

e $a = \frac{2}{3}$ and $b = 1$

f $a = 2$ and $b = \frac{1}{2}$

Example 2a

3 Calculate the value of $e^2 - 3$ when:

a $e = 1$

b $e = 3$

c $e = 10$

d $e = 2$

e $e = -1$

f $e = -2$

g $e = \frac{1}{2}$

h $e = 3.1$

i $e = \frac{1}{5}$

4 Determine the value of $2r^2 + r + 1$ when:

a $r = 2$

b $r = 5$

c $r = -7$

d $r = -0.2$

e $r = \frac{2}{3}$

f $r = -\frac{3}{4}$

5 Calculate the value of mc^2 when:

a $m = 4$ and $c = 2$

b $m = 2$ and $c = 7$

c $m = -3$ and $c = -10$

d $m = 8$ and $c = 0.5$

e $m = -0.2$ and $c = 15$

f $m = 6.25$ and $c = -1.5$

g $m = \frac{7}{10}$ and $c = \frac{1}{5}$

h $m = -\frac{3}{8}$ and $c = 0.75$

i $m = \frac{7}{9}$ and $c = 0.3$

Example 2b

6 Find $\sqrt{\frac{2a}{b}}$ when:

a $a = 6$ and $b = 3$

b $a = 50$ and $b = 4$

c $a = -1$ and $b = -0.5$

d $a = 18$ and $b = 4$

e $a = -0.25$ and $b = -0.5$

f $a = 8$ and $b = 0.25$

g $a = \frac{1}{4}$ and $b = 8$

h $a = \frac{7}{8}$ and $b = \frac{4}{7}$

i $a = -\frac{1}{10}$ and $b = -125$

7 Determine the value of $\sqrt[3]{6c + d}$ when:

a $c = 1$ and $d = 2$

b $c = 10$ and $d = 4$

c $c = 4$ and $d = 3$

d $c = 0.5$ and $d = -2$

DEVELOPING

Example 3

8 Evaluate these expressions, given $x = 7$, $y = -5$ and $z = 21$.

a $x^2 + z^2 + y$

b $y^3 - 4x$

c $\sqrt{4x + z - 1}$

d $\sqrt{z + 4y^2}$

e $\frac{3xy^2}{z}$

f $\frac{6y^2}{5zx}$

9 The area of an annulus is evaluated using $\pi(R^2 - r^2)$ where R is the radius of the outer circle and r the radius of the inner circle. Find the area of an annulus if R is 8 cm and r is 4 cm. Answer correct to one decimal place.



10 Determine the value of $\frac{2}{3}p^2q$, given that $p = 4$ and $q = 6$.

11 Evaluate $\sqrt[3]{2y + 3}$, given that $y = 12$.

12 Evaluate $\sqrt[3]{x^2 + y}$, given that $x = 3$ and $y = 4$. Answer correct to two decimal places.

13 Find the value of $2\pi\sqrt{\frac{l}{g}}$ when $l = 2.6$ and $g = 9.8$. Answer correct to two decimal places.

14 Find the value of $\sqrt{u^2 + 2as}$ when $u = 6$, $a = 7$ and $s = 2$.

CHALLENGING

15 Find the value of $\frac{1}{2\pi fc}$ when $f = 10$ and $c = 2$. Answer correct to three decimal places.

16 Find the value of $\frac{3Rr}{R+r}$ when $R = 8.2$ and $r = 4.9$. Answer correct to two decimal places.

 **17** What is the value of $\frac{yA}{y+12}$ when $y = 9$ and $A = 15$? Answer correct to the nearest whole number.

Communicate your reasoning, understanding and fluency

18 William said: x^2 and $2x$ are always different, no matter what number you substitute.
Olivia said: x^2 and $2x$ are always the same, no matter what number you substitute.
Charlie said: x^2 and $2x$ can be equal, depending on the number you substitute.

Explain that Olivia is incorrect by finding two examples which prove her incorrect.

Explain that Charlie is correct by finding two examples which prove what he means.

2B Simplifying and expanding algebraic expressions (CONSOLIDATING)

Learning intentions

- To simplify algebraic expressions
- To expand algebraic expressions then simplify if possible

Past, present and future learning

- Students should already be familiar with the contents of this section
- These concepts are used extensively throughout Years 11 and 12
- Questions involving algebraic expressions regularly appear in HSC Examinations

Consider the **algebraic expression**: $2x - 3y + 5$

The **letters** x and y are **pronumerals**, sometimes called **variables** or **unknowns**.

The **coefficient** of x is 2.

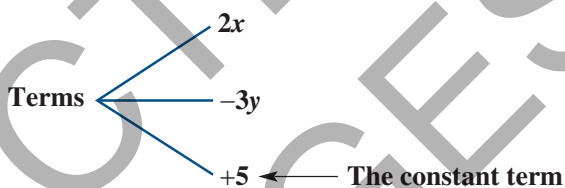
The **coefficient** of y is -3 .

Like terms: $2x$ and $5x$. Can be added or subtracted.

$$2x + 5x = 7x$$

Unlike terms: $2x$ and $3y$.

$2x + 3y$ cannot be simplified



Adding and subtracting like terms

- Find the like terms or the terms that have exactly the same pronumerals.
- Only like terms can be added or subtracted, unlike terms cannot.
- Add or subtract the coefficients or numbers before the pronumeral of the like terms.



Example 4 Determining true or false statements

Determine whether the following statements are true or false.

a x and $2x$ are like terms.

b $2x$ and x^2 are like terms.

c $2x$ and $2xy$ are like terms.

d $3x + 2x = 5x$

e $3x - 2x = x$

f $3x - x = 2$

Solution

a True

b False

c False

d True

e True

f False

Explanation

$3x$ and $2x$ have the same pronumeral.

x and x^2 are not the same pronumerals.

x and xy are not the same pronumerals.

Like terms can be added.

Like terms can be subtracted.

$3x$ take away $1x$ leaves $2x$, not 2.

Now you try

Determine whether the following statements are true or false.

a x and y are like terms.

b x and xy are like terms.

c $3x + x = 3xx$

d $6x - 5x = x$

e $9x - x = 8x$

f $2ab + 3c = 5abc$

**Example 5 Adding and subtracting like terms**

Simplify $5x^2 + 6x - 3x^2 - x$.

Solution

$$\begin{aligned} 5x^2 + 6x - 3x^2 - x &= 5x^2 - 3x^2 + 6x - x \\ &= 2x^2 + 5x \end{aligned}$$

Explanation

Add the like terms.

Now you try

Simplify:

a $6x + 3y + 4x + 5y$

b $6x + 3y - 4x - 5y$

c $6x - 3y + 4x - 5y$

To expand an algebraic expression using the distributive law, multiply the numbers or terms inside the grouping symbols by the number or term outside the grouping symbols.

Expanding algebraic expressions

1 Multiply the number or term outside the grouping symbol by the:

- a** first term inside the grouping symbol
- b** second term inside the grouping symbol.

2 Simplify and collect like terms if required.

$$\begin{aligned} a(b + c) &= a \times b + a \times c \\ &= ab + ac \end{aligned}$$

$$\begin{aligned} a(b - c) &= a \times b - a \times c \\ &= ab - ac \end{aligned}$$

Make sure you remember to multiply all the terms inside the grouping symbol by the number or term outside the grouping symbols.

**Example 6 Expanding algebraic expressions**

Expand $5(2y - 3)$.

Solution

$$\begin{aligned} 5(2y - 3) &= 5 \times (2y - 3) \\ &= 5 \times 2y + 5 \times (-3) \\ &= 10y - 15 \end{aligned}$$

Explanation

There are two terms in the brackets.
Multiply both of them by 5.

Now you try

Expand:

a $4(3y - 1)$

b $-4(3y - 1)$

c $-4(3y + 1)$


Example 7 Expanding algebraic expressions

Expand $-(m - 5)$.

Solution

$$\begin{aligned} -(m - 5) &= -1 \times (m - 5) \\ &= -1 \times m - 1 \times -5 \\ &= -m + 5 \end{aligned}$$

Explanation

Multiply both terms in the brackets by -1 .

Now you try

Expand $-(x + 5)$.


Example 8 Expanding and simplifying algebraic expressions

Remove the grouping symbols for $2(3x + 4) + 3(x - 1)$ and simplify if possible.

Solution

$$\begin{aligned} 2(3x + 4) + 3(x - 1) &= 6x + 8 + 3x - 3 \\ &= 9x + 5 \end{aligned}$$

Explanation

Expand the brackets.
Simplify by collecting like terms.

Now you try

Remove the grouping symbols for $2(3x - 4) + 2(x + 4)$ and simplify if possible.


Example 9 Expanding and simplifying algebraic expressions

Expand $y(5y + 1) - y(y - 6)$ and simplify if possible.

Solution

$$\begin{aligned} y(5y + 1) - y(y - 6) &= 5y^2 + y - y^2 + 6y \\ &= 4y^2 + 7y \end{aligned}$$

Explanation

Expand the brackets.
Simplify by collecting like terms.

Now you try

Expand $x(2x + 1) - x(2x - 1)$ and simplify if possible.

Exercise 2B
BUILDING
Example 4

1 Determine whether the following statements are true or false.

a $3x + 4x = 7x$

b $3x - 2x = x$

c $3x - 3x = 0$

d $3x - x = 3$

e $x + y = xy$

f $x^2 + x = x^3$

g $x - 2x = -x$

h $-x + 2x = x$

i $2x + 2 = 4x$

2 Simplify by collecting like terms.

a $6g + 2g$

b $5x + 12x$

c $8d - 7d$

d $5x^2 - x^2$

e $c^2 + 8c^2$

f $2r^3 - 5r^3$

g $-m + (-2m)$

h $21t + (-13t)$

i $9v^2 - (-3v^2)$

Example 5

3 Simplify by collecting like terms.

a $3w + 2 + 4w$

d $5x^2 + 4y - 2x^2$

g $3x^2 + 2y - 15x^2 - 3y$

b $2q + 3q - 8$

e $4b^2 + 6a - 5a$

h $5a^2 + b + 3a^2 - 2b$

c $1 + 4a + 11a$

f $n^3 + 2m - 4n^3$

i $5z^2 + y - z^2 + 8y$

4 Simplify by collecting like terms.

a $7u^2 - 4u^4 - u^2 + 9$

d $2x^2 + 6x^3 - 3x^3 - x^2$

g $3c + c^2 + 2 + c$

b $6q^3 + q^2 - 4q^2 + q^3$

e $p^2 + 3p + p^2 - p$

h $5 - k^2 + k^3 - 2$

c $5b + 9b^2 - 5b - 8b^2$

f $w + w^2 - 3w + w^2$

i $4y - 5 - 3y - 2y^2$

Example 6

5 Expand each of the following.

a $3(a + 2)$

d $2(3x + 4)$

g $4(5 + 2t)$

j $8(5e - 2d)$

b $2(d + 1)$

e $2(5x - 7)$

h $6(1 - 2w)$

k $5(4a + 9b)$

c $7(b - 2)$

f $4(9b + 1)$

i $5(3 + 9d)$

l $7(2h + 8g)$

Example 7

6 Expand each of the following.

a $-4(x + 3)$

d $-7(k - 2)$

g $-2(4 + 2q)$

b $-3(y + 5)$

e $-6(w - 1)$

h $-5(3 - 4r)$

c $-(b + 8)$

f $-2(x - 13)$

i $-7(8 - 2s)$

7 Expand each of the following.

a $y(y + 1)$

d $x(2x - 3)$

g $z(7e + 3f)$

b $v(v + 4)$

e $e(3e + 5)$

h $a(2b - 3c)$

c $n(n + 10)$

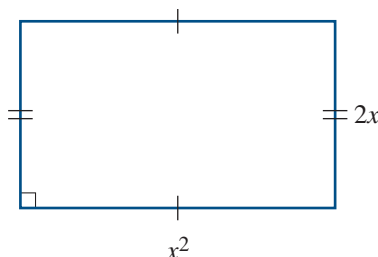
f $d(6d - 2)$

i $c(d + 4e)$

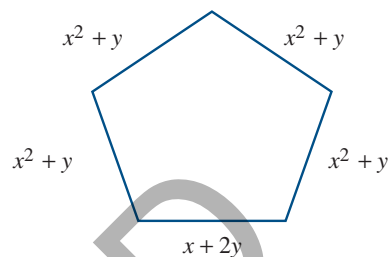
DEVELOPING

8 Copy and add like terms where possible to complete the table.

+	x^2	$3x^2 - 2x$	$x^2 + x$
$3x^2$			
$6x$			
$x^2 - 2x$			
$-2x$			

9 The perimeter of a rectangle is the distance around the outside of the rectangle. The rectangle has a length of x^2 and a breadth of $2x$. Write an expression for the perimeter of this rectangle by collecting like terms.10 Ava went shopping. She spent $\$4y^2$ on a jacket, $\$3y$ on a pair of shoes, and $\$2y^2$ on a skirt. If she had $\$500$, write an expression in simplified form for how many dollars she has left.

- 11** This pentagon has four sides of length $x^2 + y$ and one side of length $x + 2y$. Write an expression in simplified form for the perimeter of this pentagon by collecting like terms.

**Example 8**

- 12** Remove the grouping symbols and simplify if possible.

a $2(g + 1) + 4g$

b $7(s + 2) + s$

c $3(y - 9) - 2y$

d $5x - 4(x - 2)$

e $6z + 2(z - 1)$

f $3q - 7(q - 5)$

- 13** Remove the grouping symbols and simplify if possible.

a $4(x - 1) + 2x + 5$

b $7(3y - 2) + 4y - 2$

c $2(5b + 2) - b - 8$

d $4r + 17 + 5(r - 3)$

e $2n - 8 + 3(n + 2)$

f $5q + 2 - (q + 9)$

- 14** Expand each of the following.

a $4(y + 3)$

b $2(c + 1)$

c $5(a - 3)$

d $-3(4 + 3e^2)$

e $4(1 - 2b^2)$

f $-6(3 + 4g^2)$

- 15** Expand each of the following.

a $k(k + 3)$

b $b(2b - 6)$

c $y(5 - 3y)$

d $x(4x - 1)$

e $-a(b + 4c)$

f $p(5q - 3r)$

CHALLENGING**Example 8**

- 16** Simplify the following.

a $2(x + 1) + 5(x - 1)$

b $3(y + 2) + 2(y + 1)$

c $5(a + 2) + 3(a + 4)$

d $8(c - 3) + 5(c + 3)$

e $6(s + 6) + 2(2s - 1)$

f $5(h + 7) + 2(2h - 7)$

g $4(3x - 1) - 2(x - 2)$

h $9(z + 5) - 7(z - 2)$

i $5(2c - 4) - 3(c + 7)$

j $5(5g - 1) - 4(g - 2)$

k $7(2u - 3) - (u - 3)$

l $-(4d - 1) - 3(d - 3)$

- 17** Taylah was required to remove the grouping symbols and simplify. This was her solution.

$$\begin{aligned} 6(3x - 2) - 2(x + 3) &= 18x - 12 - 2x + 6 \\ &= 16x - 6 \end{aligned}$$

Where is the error in Taylah's working?

Example 9

- 18** Remove the grouping symbols and simplify if possible.

a $x(x - 5) + x(x + 2)$

b $b(b + 3) + b(b + 1)$

c $y(y - 3) + y(y + 8)$

d $g(2g + 3) - g(g + 3)$

e $v(v - 7) + v(6v + 4)$

f $b(5b - 1) - b(2 + 4b)$

g $2u(u - 2) + u(u + 9)$

h $4n(n - 6) - n(n + 1)$

i $3d(d + 7) + d(2d + 5)$

j $e(e + 2) - 7e(e - 9)$

k $6k(k - 3) + k(k + 3)$

l $t(5 - 3t) + 7t(2 - t)$

Communicate your reasoning, understanding and fluency

- 19** Tara made three errors in the first line of working. Find them and fix them, and then finish the simplification correctly.

$$3a(b + c) - a(a - c) = 3ab + 3c - 2a - ac$$

2C Linear equations

(CONSOLIDATING)

Learning intentions

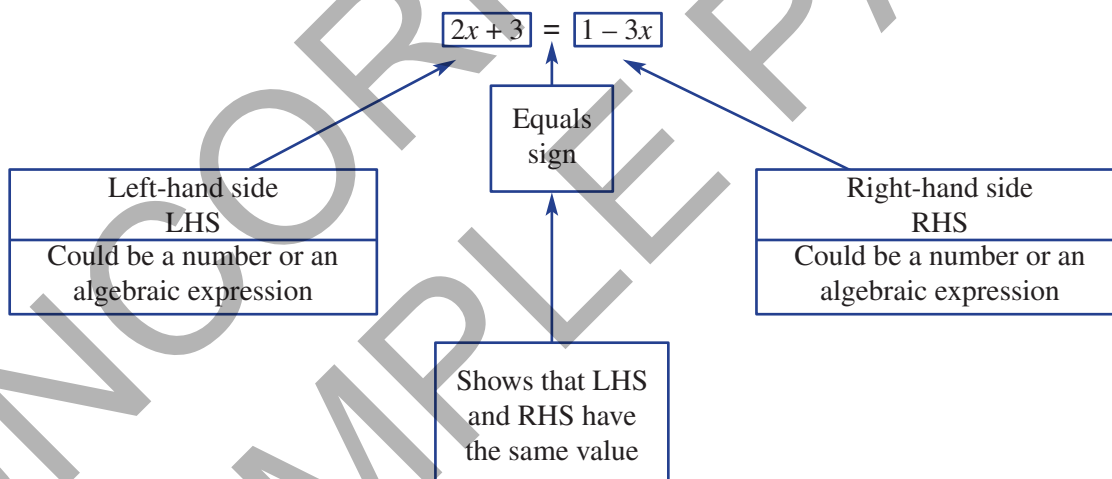
- To solve linear equations, including those involving fractions or brackets

Past, present and future learning

- Students should already be familiar with the contents of this section
- Equations are used extensively throughout Years 11 and 12
- Questions involving equations regularly appear in HSC Examinations

Consider the equation $2x + 3 = 1 - 3x$.

All equations contain:



Solving an equation

- Look to perform the opposite operation (+ is opposite to $-$, $\times$ is opposite to $\div$).
- Add or subtract the same number to both sides of the equation.
- Multiply or divide both sides of the equation by the same number.
- To solve two-step or three-step equations, repeat the steps 1 to 3 as required. It is often easier to first add or subtract the same number to both sides of the equation.
- When a solution has been reached, it can be checked. The solution of the equation must satisfy the equation. Always check your solution by substituting your answer into the original equation. The left-hand side of the equation must equal the right-hand side.


Example 10 Solving a two-step linear equation

Solve the equation $3x + 5 = -2$.

Solution

$$\begin{array}{rcl}
 3x + 5 & = & -2 \\
 -5 & & -5 \\
 \hline
 3x & = & -7 \\
 \div 3 & & \div 3 \\
 \hline
 \frac{3x}{3} & = & \frac{-7}{3} \\
 x & = & -2\frac{1}{3}
 \end{array}$$

Explanation

Subtract 5 from both sides.

Divide both sides by 3.

Check the solution by substituting it into the original equation.

Now you try

a Solve the equation $2x - 3 = -2$.

b Solve the equation $9 = 7x - 5$.


Example 11 Solving a linear equation with fractions

Solve the equation $\frac{y}{3} - \frac{y}{7} = 2$. Express your answer as a simple fraction.

Solution

Method 1: Multiply by LCM

$$\begin{array}{rcl}
 \frac{y}{3} - \frac{y}{7} & = & 2 \\
 \frac{y}{3} \times 21 - \frac{y}{7} \times 21 & = & 2 \times 21 \\
 7y - 3y & = & 42 \\
 \div 4 & & \div 4 \\
 \hline
 4y & = & 42 \\
 \frac{4y}{4} & = & \frac{42}{4} \\
 y & = & 10\frac{1}{2}
 \end{array}$$

Method 2: Make common denominators

$$\begin{array}{rcl}
 \frac{y}{3} - \frac{y}{7} & = & 2 \\
 \frac{y \times 7}{3 \times 7} - \frac{y \times 3}{7 \times 3} & = & \frac{y \times 21}{1 \times 21} \\
 \frac{7y}{21} - \frac{3y}{21} & = & \frac{42}{21} \\
 7y - 3y & = & 42 \\
 \div 4 & & \div 4 \\
 \hline
 4y & = & 42 \\
 y & = & 10\frac{1}{2}
 \end{array}$$

Explanation

The denominators are 3 and 7.

Their lowest common multiple is 21.

Multiply all three terms by 21.

Check solution by substituting.

Express all terms as fractions. LCM of 3, 7, 1 is 21.

Make all denominators 21.

Equate the numerators (i.e. ignore the denominators).

Solve equation.

Check solution by substituting.

Now you try

Solve the equation $\frac{x}{4} + \frac{x}{6} = 5$.


Example 12 Solving a four-step linear equation

Solve the equation $\frac{b}{2} + 5 = 3b + 7$.

Solution

$$\begin{array}{lcl}
 \frac{b}{2} + 5 = 3b + 7 & & \\
 \begin{array}{c} -5 \\ \times 2 \end{array} \left\{ \begin{array}{l} \frac{b}{2} + 5 = 3b + 7 \\ \frac{b}{2} = 3b + 2 \end{array} \right. \begin{array}{c} -5 \\ \times 2 \end{array} & & \\
 b = 6b + 4 & & \\
 \begin{array}{c} -4 \\ -b \end{array} \left\{ \begin{array}{l} 6b + 4 = b \\ 6b = b - 4 \end{array} \right. \begin{array}{c} -4 \\ -b \end{array} & & \\
 5b = 4 & & \\
 \begin{array}{c} \div 5 \end{array} \left\{ \begin{array}{l} 5b = 4 \\ b = \frac{4}{5} \end{array} \right. \begin{array}{c} \div 5 \end{array} & &
 \end{array}$$

Explanation

Subtract 5 from both sides.

Multiply both sides by 2.

Swap LHS and RHS.

Subtract 4 from both sides.

Subtract b from both sides.

Divide both sides by 5.

Now you try

Solve the equation $\frac{x}{5} + 1 = x - 7$.

Exercise 2C
BUILDING

1 Solve the following linear equations.

a $a + 7 = 15$

b $y + 12 = 23$

c $x + 4 = 14$

d $6 + g = 13$

e $7 + w = 3$

f $11 + z = -2$

g $b - 5 = 4$

h $m - 9 = 9$

i $c - 6 = 15$

j $12 - q = 5$

k $8 - h = 2$

l $-3 = n - (-2)$

2 Solve the following linear equations.

a $5n = 15$

b $6x = 30$

c $3c = -12$

d $2b = 15$

e $4m = -21$

f $-25 = 6b$

g $\frac{s}{2} = 8$

h $\frac{y}{5} = 6$

i $\frac{a}{11} = -7$

j $12 = \frac{g}{2}$

k $\frac{c}{-15} = -3$

l $3 = \frac{k}{-3}$

3 Eliza was required to solve the following equation on the blackboard. This was her solution.

$$6y + 5 = 7$$

$$6y = 12$$

$$y = 2$$

a Where is the error in Eliza's working?

b What is the correct solution to the equation?

c Check your solution by substituting your answer into the equation.



Example 10

4 Solve the following linear equations. All solutions are integers.

a $5w + 4 = 29$

b $2t + 6 = 12$

c $7x - 6 = 22$

d $5y - 10 = -15$

e $9m - 2 = -11$

f $12 = 3a - 9$

g $5 + 2e = 13$

h $4 + 3b = 7$

i $32 = 17 - 3k$

j $16 = 10 + 2w$

k $70 - 10d = 80$

l $98 = 38 - 10z$

5 Solve the following linear equations. Express your answer as a simplest fraction.

a $3q + 6 = 19$

b $4r + 2 = 27$

c $2x + 11 = 14$

d $31 = 4 + 4a$

e $18 = 17 + 5m$

f $12 = 29 + 2k$

g $2c - 7 = 6$

h $5e - 1 = 15$

i $6p - 4 = 11$

j $28 = 2r + 19$

k $37 = 3w - 7$

l $16 = 9c + 8$

6 Solve these equations.

a $4d = 1\frac{2}{3}$

b $3y = 4\frac{1}{5}$

c $7a = 4\frac{2}{3}$

d $\frac{x}{3} = 3\frac{2}{9}$

e $\frac{b}{2} = 3\frac{3}{5}$

f $\frac{n}{5} = 7\frac{1}{4}$

7 Solve these equations.

a $\frac{2z}{5} = 4$

b $\frac{6c}{2} = 12$

c $\frac{2m}{9} = 8$

d $7 = \frac{9x}{5}$

e $14 = \frac{5y}{6}$

f $15 = \frac{3a}{4}$

8 Twenty is added to a number divided by four and the result is 27.

a Write an equation using a to represent the number.

b Solve the equation for a .

9 Solve these equations.

a $\frac{g}{2} + 6 = 14$

b $5 + \frac{y}{3} = 17$

c $3 = \frac{c}{5} - 7$

d $8 = 1 - \frac{x}{7}$

e $7 = 1 + \frac{3b}{2}$

f $6 = \frac{2w}{5} - 9$

10 Solve these equations.

a $4(m + 3) = 20$

b $3(t - 3) = 3$

c $2(e + 10) = 12$

d $-2(3x + 1) = 6$

e $-5(d - 3) = -15$

f $(w + 5)7 = 28$

g $\frac{1}{5}(3 + 2a) = 2$

h $\frac{1}{6}(7 - 5g) = -4$

i $\frac{1}{3}(9 - 3r) = -2$

11 Solve these equations.

a $\frac{a - 4}{2} = 6$

b $\frac{3z - 2}{5} = 7$

c $\frac{5 + 2r}{3} = 2$

d $\frac{12 + 6x}{5} = -1$

e $\frac{b + 2}{8} = \frac{1}{2}$

f $\frac{3w + 2}{2} = \frac{3}{4}$

DEVELOPING

12 Solve the following linear equations.

a $4d + 4 = 3d + 9$

b $5x + 3 = 4x - 5$

c $7a + 2 = 4a - 10$

d $10y - 4 = 2 + 4y$

e $8m + 2 = (-17) + 3m$

f $7c - 4 = 7 + c$

g $7 + s = 8s - (-9)$

h $13 + 2z = 6z + 3$

i $1 + 5b = 8 - 2b$

j $-5 + 2x = -3 + 8x$

k $4 - (-2a) = 5a + 1$

l $2w - 7 + 4w = 8$

Example 12

13 Solve these equations.

a $\frac{x}{3} + 3 = 2x + 10$

b $8 + \frac{w}{2} = 3w - 10$

c $6y - 4 = \frac{y}{2} - 5$

d $\frac{e}{5} + 4 = 10 + 2e$

e $10m + 2 = 1 + \frac{m}{2}$

f $7 = \frac{b}{3} - 5b + 2$

g $2 = 5 + 2r - \frac{r}{5}$

h $\frac{4t}{3} + 3t - 5 = 0$

i $5n = \frac{2n}{7} - 6 + n$

14 Maya bought three dresses when she went shopping, each costing the same price, \$ c . Maya also paid \$15 for parking. The total cost for the day was \$261.

a Using c as the cost of one dress, write an equation showing the cost of the three dresses and parking.

b Use the equation to find the cost of each dress.



15 If 15 is subtracted from two times a certain number, the result is five times the number. Find the number.

Example 11

16 Solve these equations.

a $\frac{v}{3} + \frac{v}{4} = 7$

b $\frac{a}{5} + \frac{a}{2} = 10$

c $\frac{x}{4} + \frac{x}{8} = 5$

d $\frac{m}{5} - \frac{m}{15} = 7$

e $\frac{b}{6} - \frac{b}{7} = 2$

f $\frac{g}{6} - \frac{g}{8} = 4$

g $\frac{4y}{2} - \frac{y}{3} = 6$

h $\frac{4c}{3} - \frac{c}{4} = 3$

i $\frac{h}{2} - \frac{2h}{5} = 4$

j $\frac{n}{8} = \frac{n}{7} - 1$

k $\frac{3p}{10} = \frac{p}{2} + 4$

l $\frac{5z}{2} = \frac{2z}{5} + 1$

17 Solve these equations.

a $\frac{4}{x} = 12$

b $-6 = \frac{5}{a}$

c $\frac{2}{y} + 1 = 6$

d $7 + \frac{4}{d} = 3$

e $5 - \frac{8}{c} = -2$

f $4 + \frac{-2}{m} = 0$

18 Solve these equations.

a $\frac{3}{b+1} = 5$

b $7 = \frac{8}{1+y}$

c $\frac{4}{y-2} - 4 = 0$

d $\frac{2x}{x+1} = 3$

e $\frac{8c}{4c-2} = 1$

f $11 = \frac{3m}{4+m}$

19 Six more than twice a certain number is two more than the number itself.

a Write an equation using x to represent the number.

b Solve the equation for x .

20 My uncle is 8 years older than I, and 6 years ago he was double my age.

a Write an equation using x to represent my age.

b Solve the equation for x .



CHALLENGING

21 Solve these equations.

a $3(d + 7) = -2(d - 9)$

c $3(5x - 1) - 6(x - 5) = 11$

e $4(4m + 1) - (m - 5) = 3$

b $4(r + 2) + 1 = 3(r - 3)$

d $6(2a - 5) - 2(3 + a) = 20$

f $3(y - 2) = 10 - 2(3y - 1)$

22 Solve these equations.

a $\frac{a+2}{3} = 3a - 4$

d $\frac{b+5}{3} = \frac{b+8}{2}$

g $\frac{n+1}{6} + \frac{n+1}{2} = 7$

b $2m - 5 = \frac{m+1}{2}$

e $\frac{3x+2}{5} = \frac{2x+4}{2}$

h $\frac{k+4}{3} - \frac{2k-3}{2} = -3$

c $\frac{y+1}{4} - (y-1) = 2$

f $\frac{4c-2}{5} = \frac{2c+1}{3}$

i $\frac{3p-2}{5} - \frac{2p+1}{4} = 3$



Communicate your reasoning, understanding and fluency

23 Josh wrote the following, which is correct, but all the steps are missing:

$$2(2x - 1) + x - 6 = 0$$

$$x = 1.6$$

Solve the equation showing all the steps.

Change one number in the original equation so that the answer will be $x = 5$.

2D Using equations to solve worded problems

Learning intentions

- To represent a worded scenario as a linear equation
- To solve the equation and interpret the solution in the context of the scenario

Past, present and future learning

- Students should already be familiar with the contents of this section
- Equations are used extensively throughout Years 11 and 12
- Questions involving equations regularly appear in HSC Examinations

When faced with a worded problem such as the following, it is tempting to use ‘trial and error’, also known as ‘guess and check’ and ‘estimation and refinement’.

William and Olivia have the same birthday. William is 17 years older than Olivia. This year, on their birthday, the sum of their ages will be 99. How old will Olivia be?

However, sometimes it is much more efficient to:

- represent the scenario as an equation with only one unknown
- solve the equation
- interpret the solution in the context of the problem.

Using equations to solve worded problems

Read the worded scenario carefully, looking for:

- words which relate to mathematical operations, such as ‘sum’ for ‘addition’
- the things which are known
- the thing which is unknown and needs to be found.

To solve the problem:

- introduce one pronumeral for the unknown
- express the worded scenario as an equation
- solve the equation, either by inspection or systematically
- check that the answer satisfies the equation and the original problem
- make sure you answer the original question, including the units (e.g. dollars, cm).

**Example 13**

William and Olivia have the same birthday. William is 17 years older than Olivia. This year, on their birthday, the sum of their ages will be 99. How old will Olivia be?

Solution

Let x = Olivia's age

William's age is $x + 17$.

$$x + x + 17 = 99$$

$$2x + 17 = 99$$

$$2x = 82$$

$$x = 41$$

Olivia is 41 and William is $41 + 17 = 58$.

Olivia is 41 years old.

Explanation

Olivia's age is the unknown in this example.

William is 17 years older.

Form the equation.

Solve the equation.

Check: $41 + 58 = 99$

Answer the question.

Now you try

Two numbers have a sum of 150. One is 5 times greater than the other. What are the numbers?

**Example 14**

Charlie hires a bike from a shop which charges \$25 for the first hour then \$12 per hour, or part thereof, after that. He picked up the bike at 10:00 a.m. He has \$95 maximum to spend on this activity.

a What time will Charlie need to return the bike to the shop?

b If he rides for as long as possible, how much money will Charlie have left over?

Solution

a Let h = number of hours after first hour.

Every hour after first hour costs \$12.

$$25 + 12h = 95$$

$$12h = 70$$

$$h = 5.8333 \dots$$

Charlie can use the bike for 5 hours after the first hour, so 6 hours in total.

$$10:00 \text{ a.m.} + 6 \text{ hours} = 4:00 \text{ p.m.}$$

b First hour + 5 hours = $25 + 5 \times 12 = 85$

The cost will be \$85.

Charlie will have \$10 left over.

Explanation

Hours is the unknown in this example.

The first hour costs \$25 then every hour costs \$12.

Form the equation.

Solve the equation.

Charlie can't afford 6 extra hours, so round down to 5 hours.

The first hour plus 5 hours makes 6 hours.

Check: $25 + 5 \times 12 = 85$

Answer the question.

Now you try

A party venue charges \$300 for the room and \$45 per person for the catering. How many people can attend the party for \$1000?

Exercise 2D**BUILDING**

Match these worded scenarios (questions **1** to **12**) to the correct equation (A to H).

Use some letters more than once.

A $n - 5 = 20$	B $n + 5 = 20$	C $5 - n = 20$	D $\frac{n}{2} + 5 = 20$
E $5n = 20$	F $\frac{n}{5} = 20$	G $2n + 5 = 20$	H $2(n + 5) = 20$

- 1** A number was increased by 5 to give 20.
- 2** Five times a number is 20.
- 3** Dividing a number by 5 gives 20.
- 4** A number is doubled then 5 is added to give 20.
- 5** A number is increased by 5 then the result is doubled to give 20.
- 6** A number is subtracted from 5 to give 20.
- 7** The sum of a number and 5 is 20.
- 8** A number is halved then 5 is added to give 20.
- 9** The product of 5 and a number is 20.
- 10** One-fifth of a number is 20.
- 11** A number is decreased by 5 to give 20.
- 12** Five more than half of a number is 20.

DEVELOPING**Example 13**

- 13** Tara is 8 years younger than her brother Steve. The sum of their ages is 50. How old are Tara and Steve?
- 14** In a rectangle one side is 4 cm longer than the other. The perimeter is 60 cm. What is the length of the long side and the short side?
- 15** In a rectangle one side is 4 times longer than the other. The perimeter is 60 cm. What is the length of the long side and the short side?
- 16** In an isosceles triangle one side is 3 cm longer than the equal sides. The perimeter is 60 cm. How long are the sides?
- 17** The equal sides of an isosceles triangle are 12 cm long. The perimeter is 35 cm. How long is the other side?
- 18** Six more than twice a certain number is two more than the number itself. Use an equation to find the number.

CHALLENGING

Example 14

- 19** A plumber charges a \$90 call-out fee and \$80 per hour. He charged \$370 for one job. How many hours did the job take?
- 20** I can buy 3 rulers and 4 pencils for \$13.50. The pencils were \$1.50 each. What was the price for each ruler?
- 21** A rope 50 m long was cut into 4 pieces. Three of them are equal but the other was 6 m shorter. How long are the pieces?
- 22** The sum of three consecutive integers is 105. What are the integers?
- 23** Steve is 8 years older than Juliana. Six years ago, he was twice her age. Use an equation to find the current ages of Steve and Juliana.
- 24** In a triangle, one angle is double the smallest angle and the other is triple the smallest angle. How big are the three angles?
- 25** A hire car used for weddings costs \$850 plus \$156 per hour, or part thereof. A couple are willing to spend up to \$2000 on their wedding car. If they are going to be picked up at 1:15 p.m., what time will their car hire finish?

Communicate your reasoning, understanding and fluency

- 26** Josh likes to impress his friends with this ‘mathemagical’ trick.

Think of a positive whole number.

Add the next higher number to it.

Add 9.

Divide by 2.

Subtract the original number.

Explain why the answer will always be 5, no matter what number his friends choose.

2E Using formulas to solve problems

Learning intentions

- To substitute numbers into formulas
- To solve the equation and interpret the solution in the context of the scenario

Past, present and future learning

- Some aspects of this section may be new to students
- Formulas are used extensively throughout Years 11 and 12
- Questions involving formulas regularly appear in HSC Examinations

Formula

A formula is a mathematical relationship between two or more variables. For example:

- $a^2 = b^2 + c^2$ is Pythagoras' theorem for relating the sides of a right-angled triangle.
- $A = \pi r^2$ is a formula for relating the area and radius of a circle. A and r are the variables.

By substituting all the known variables into a formula, we are able to find the value of an unknown variable. If the unknown is not the subject of the equation, solve the equation.

Using a formula

- 1 Write the formula.
- 2 Replace the pronumerals in the formula with the numbers given in the question.
- 3 If the unknown is not the subject of the equation, solve the equation.
- 4 Evaluate using a calculator if necessary.
- 5 Write the answer to the specified level of accuracy and correct units if necessary.



Example 15 Substituting values

A traffic cone has a volume of 4000 cubic centimetres and a height of 40 centimetres.

Use the formula $r = \sqrt{\frac{3V}{\pi h}}$ to find the radius of the base of the cone. Answer correct to two decimal places.



Solution

$$\begin{aligned}
 r &= \sqrt{\frac{3V}{\pi h}} \\
 &= \sqrt{\frac{3 \times 4000}{\pi \times 40}} \\
 &= 9.772050238 \dots \\
 &= 9.77 \text{ cm (to 2 decimal places)}
 \end{aligned}$$

Explanation

Substitute the values.

Evaluate the formula.

Round off the answer.

Now you try

If $V = \pi r^2 h$, find the value of V when $r = 6$ and $h = 5$.


Example 16 Substituting values

If $P = 2L + 2B$, find B when $P = 50$ and $L = 8$.

Solution

$$P = 2L + 2B$$

$$50 = 2 \times 8 + 2B$$

$$50 = 16 + 2B$$

$$\begin{array}{rcl} 16 + 2B = 50 & & \\ -16 & \swarrow & \searrow -16 \\ 2B = 34 & & \\ \div 2 & \swarrow & \searrow \div 2 \\ B = 17 & & \end{array}$$

Explanation

Substitute the values.

Solve for B .

Subtract 16 from both sides.

Divide both sides by 2.

Now you try

If $P = 3L + 5B$, find L when $P = 60$ and $B = 8$.

Exercise 2E
BUILDING

- 1 If $A = 4\pi r^2$, find the value of A (correct to one decimal place) when:
 - a $r = 3$
 - b $r = 8$
 - c $r = 24$
- 2 If $A = \pi ab$, find the value of A (correct to two decimal places) when:
 - a $a = 10, b = 5$
 - b $a = 22, b = 15$
 - c $a = 8, b = 3$
- 3 If $I = Prn$, find the value of I (correct to one decimal place) when:
 - a $P = 10, r = 0.2, n = 3$
 - b $P = 45, r = 0.1, n = 4$
 - c $P = 84, r = 0.3, n = 2$
- 4 If $A = P(1 + r)^n$, find the value of A (correct to two decimal places) when:
 - a $P = 9, r = 0.05, n = 5$
 - b $P = 6, r = 0.04, n = 3$
 - c $P = 8, r = 0.03, n = 8$
- 5 Find the value of S (correct to one decimal place) in the formula $S = V_0 - Dn$ if:
 - a $V_0 = 25, D = 2.5$ and $n = 2$
 - b $V_0 = 17.5, D = 5$ and $n = 3$
- 6 Use the formula $V = \frac{4}{3}\pi r^3$ to find the value of V (correct to the nearest whole number) when:
 - a $r = 6$
 - b $r = 5.2$
 - c $r = 7.1$
- 7 If $z = \frac{x - \bar{x}}{s}$, find the value of z when:
 - a $x = 12, \bar{x} = 8$ and $s = 2$
 - b $x = 24, \bar{x} = 4.5$ and $s = 4$
- 8 The cost of hiring a rotary hoe is given by the rule $C = 25t + 900$ where C is the total cost in dollars and t is the number of hours for which the rotary hoe is hired. Find the cost of hiring a rotary hoe for:
 - a 6 hours
 - b 8.5 hours
 - c 24 hours

Example 15

- 9** The formula used to convert temperature from degrees Fahrenheit to degrees Celsius is $C = \frac{5}{9}(F - 32)$. Use this formula to convert the following temperatures to degrees Celsius. Answer correct to the nearest whole number.

a 60°F**b** 80°F**c** 100°F**DEVELOPING****Example 16**

- 10** The circumference of a circle is given by the formula $C = 2\pi r$ where C is the circumference and r is the radius. Evaluate the radius if the circumference is 26 cm. Answer correct to two decimal places.

- 11** A satellite is travelling at a speed of 22 500 km/h.

Use the formula $S = \frac{D}{T}$ to calculate the distance travelled by the satellite in:

a 24 hours**b** 1 week**c** 1 year (Assume 52 weeks in a year.)

- 12** Ohm's law is used to calculate the current (I), potential difference (V) and resistance (R). It is given by the formula $I = \frac{V}{R}$. Find the value of V , in volts, given that $I = 0.5$ amps and $R = 18$ ohms.
- 13** Use the formula $S = \frac{n}{2}(a + l)$ to evaluate n when $S = 720$ and $a = 21$ and $l = 3$.
- 14** The area of a trapezium is calculated using the formula $A = \frac{h}{2}(a + b)$ where the parallel sides are a and b , and height is h . Find the height of the trapezium when the area of the trapezium is 45 cm² and its parallel sides are 12 cm and 8 cm.
- 15** Use the formula $d = 5t^2$ to evaluate t when $d = 300$. Answer correct to two decimal places.
- 16** Pythagoras' theorem $a^2 = b^2 + c^2$ relates the sides of a right-angled triangle. Determine the value of b when $c = 20$ mm and $a = 35$ mm. Answer in mm correct to two decimal places.
- 17** Use the formula $P = 3r - sr^2$ to evaluate s , given that $P = 48$ and $r = -2$.
- 18** If $m = 2n^3 - 3$, what is the value of n when $m = 13$?
- 19** If $R = \frac{1}{R_1} + \frac{1}{R_2}$, what is the value of R_1 , given that $R = 1.15$ and $R_2 = 2.30$. Answer correct to two decimal places.

- 20** The entrance to the Louvre is a square pyramid with base length of x , volume V and perpendicular height h . The base length is given by the formula $x = \sqrt{\frac{3V}{h}}$. Find the value of h , given that $V = 8200$ and $x = 35$. Answer correct to the nearest whole number.



CHALLENGING

- 21** Use the formula $t = \sqrt{\frac{2(s - ut)}{a}}$ to find the value of s , given $u = 2$, $a = 5$ and $t = 15$. Answer correct to two decimal places.
- 22** Find the value of A in the formula $r = \sqrt{\frac{360A}{\pi\theta}}$ when $r = 2$ and $\theta = 20$. Answer correct to one decimal place.
-  **23** The radius r of a sphere with volume V is given by the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$. What is the volume of a sphere of radius 4 cm? Answer correct to the nearest whole number.

Communicate your reasoning, understanding and fluency

- 24** The formula for converting between temperature in degrees Celsius (C) and Fahrenheit (F) is $C = \frac{5}{9}(F - 32)$.
- a** On a hot day, the temperature may reach 40 degrees Celsius. What is the temperature in Fahrenheit?
 - b** On a cool day, the temperature may reach 50 degrees Fahrenheit. What is the temperature in Celsius?
 - c** There is one temperature which is the same on both scales. Solve this equation to find out this temperature:

$$C = \frac{5}{9}(C - 32)$$

2F Changing the subject of formulas

Learning intentions

- To rearrange linear and quadratic formulas to change the subject
- To solve problems and make decisions about solutions in the context of the problem

Past, present and future learning

- Some aspects of this section may be new to students
- Formulas are used extensively throughout Years 11 and 12
- Questions involving this skill regularly appear in HSC Examinations

Consider these formulas, which are identical:

$$S = \frac{D}{T}$$

S is the subject

$$T = S \times D$$

T is the subject

$$D = \frac{S}{T}$$

D is the subject

In this section, we will change the subject using two methods.

Method 1: Swap then solve

Make a the subject

$$V = u + at$$

$$\begin{aligned} u + at &= V & -u & \\ at &= V - u & -u & \\ a &= \frac{V - u}{t} & \div t & \end{aligned}$$

Method 2: Add something to both sides

Make a the subject

$$V = u - at$$

$$\begin{aligned} V &= u - at & -at & \\ V + at &= u & -at & \\ at &= u - V & -V & \\ a &= \frac{u - V}{t} & \div t & \end{aligned}$$

Note: Method 2 is preferable when the 'target' has a negative coefficient.



Example 17 Changing the subject of a formula

The total cost of a child's birthday party is given by the formula:

$$C = 40n + 75$$

where $C(\$)$ is the total cost and n is the number of children attending.

Make n the subject of the equation.

Solution

$$C = 40n + 75$$

$$\begin{aligned} 40n + 75 &= C & -75 & \\ 40n &= C - 75 & -75 & \\ n &= \frac{C - 75}{40} & \div 40 & \end{aligned}$$

Explanation

Identify the 'target' (n).

Swap LHS and RHS.

Subtract 75 from both sides.

Divide both sides by 40.

Now you try

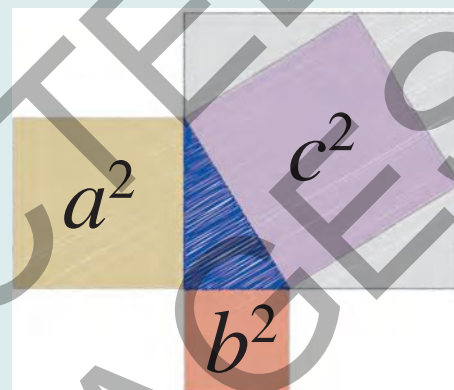
Make n the subject of $C = 5n - 6d + 50$.

**Example 18** Changing the subject of a formula

Pythagoras' theorem is given by the formula $c^2 = a^2 + b^2$ where a , b and c are the sides of a right-angled triangle.

- a** Make a the subject of the formula.
- b** What is the length of a given $c = 7$ and $b = 5$? Answer correct to one decimal place.

$$a^2 + b^2 = c^2$$

**Solution****a**

$$\begin{aligned}
 c^2 &= a^2 + b^2 \\
 a^2 + b^2 &= c^2 & -b^2 & \quad -b^2 \\
 a^2 &= c^2 - b^2 \\
 a &= \pm \sqrt{c^2 - b^2} \\
 \text{But } a \text{ is positive.} \\
 \therefore a &= \sqrt{c^2 - b^2}
 \end{aligned}$$

b

$$\begin{aligned}
 a &= \sqrt{c^2 - b^2} \\
 &= \sqrt{7^2 - 5^2} \\
 &= \sqrt{24} \\
 &= 4.8989 \dots \\
 &= 4.9 \text{ (to 1 decimal place)}
 \end{aligned}$$

Explanation

Identify the 'target' (a).

Swap LHS and RHS.

Subtract b^2 from both sides.

Take the square root of both sides.

a is the length of the side.

Ignore the negative answer.

Substitute the values.

Evaluate the formula.

Calculate the square root.

Round off the answer.

Now you try

Make E the subject of $F + V = E + 2$.



Example 19 Changing the subject of formulas

Make x the subject of $y = 3x^2 + 5$.

Solution

$$\begin{aligned}
 y &= 3x^2 + 5 \\
 -5 & \quad -5 \\
 3x^2 + 5 &= y \\
 3x^2 &= y - 5 \\
 \div 3 & \quad \div 3 \\
 x^2 &= \frac{y-5}{3} \\
 \sqrt{} & \quad \sqrt{} \\
 x &= \pm \sqrt{\frac{y-5}{3}}
 \end{aligned}$$

Explanation

Identify the 'target' x .

Swap LHS and RHS.

Subtract 5 from both sides.

Divide both sides by 3.

Take square root of both sides.

Now you try

Make x the subject of $y = 9 - 2x^2$.

Exercise 2F

BUILDING

- 1 Make x the subject of the following formulas.

a $y = x + 3$

b $y = 5x$

c $y = x - 7$

d $y = 2x + 9$

e $y = 4 - x$

f $y = -7 + 3x$

- 2 Make x the subject of the following formulas.

a $y = \frac{x}{8}$

b $y = \frac{x}{2} + 1$

c $y = \frac{x}{3} - 1$

d $y = 7 - \frac{x}{6}$

e $y = 5 + \frac{x}{5}$

f $y = -2 - \frac{x}{3}$

- 3 The distance a car travels is given by the formula $d = st$ where s is the speed and t is the time.

a Make t the subject of the formula.

b Make s the subject of the formula.

c Use the formula to find the speed of a car that has travelled a distance of 112 km in a time of 1.75 hours.

- 4 The area of a rectangle A with a length l and breadth b is given by the formula $A = lb$.

a Make l the subject of the formula.

b Make b the subject of the formula.

c Find the breadth of a rectangle with an area of 16 cm^2 and length of 2.5 cm .

d Find the length of a rectangle with an area of 42 mm^2 and breadth of 16 mm .

- 5 The force F on an object with mass m and acceleration a is given by the formula $F = ma$.

a Make m the subject of the formula.

b Make a the subject of the formula.

c Use the formula to find the acceleration when $m = 2.5$ and $F = 10$.

d Use the formula to find the mass when $a = 16$ and



Example 17

- 6 Make b the subject of the following formulas.

a $a = b + c$

b $a = 6b - c$

c $c = a - b$

d $a = 2c + 6b$

e $c = -4a + 7b$

f $b + 3c = 8a$

g $a + 2b + c = 0$

h $5b - a - c = 0$

i $0 = a + 4c - 9b$

- 7 The formula $y = mx + c$ represents a straight line.

a Make c the subject of the formula.

b Make m the subject of the formula.

c Make x the subject of the formula.

d Find x when $y = 24$, $c = 18$ and $m = 2$.

e Find m when $y = 12$, $c = 4$ and $x = 2$.

- 8 The perimeter of a rectangle is given by the formula $P = 2(l + b)$ where l is the length and b the breadth.

a Make length the subject of the formula.

b What is the length if the perimeter of a rectangle is 68 m and the breadth is 9 m?

c Make breadth the subject of the formula.

- 9 A supervisor of a business uses the formula $w = 35(a + 1.5b + 2c)$ to calculate the wages where a , b and c are hours worked.

a Make a the subject of the formula.

b Make b the subject of the formula.

c Make c the subject of the formula.

d If $a = 40$, $b = 3$ and $w = 1925$, what is the value of c ?



- 10 The area of an ellipse is $A = \pi ab$ where a is the length of the semi-major axis and b is the length of the semi-minor axis.

a Make b the subject of the formula.

b Calculate the length of the semi-minor axis if the area of the ellipse is 26 cm^2 and the length of the semi-major axis is 4 cm. Answer correct to two decimal places.

DEVELOPING

Example 18,19

- 11 Make x the subject of the following formulas.

a $y = x^2$

b $y = 3x^2 + 4$

c $y = 5 - 4x^2$

- 12 Write the formula $4m = 5n + 2p^2$ with p as the subject.

- 13 The volume of a cylinder is evaluated using $V = \pi r^2 h$ where h is the height and r is the radius.

a Write the formula with h as the subject.

b Write the formula with r as the subject.

c Calculate the radius of a cylinder if the volume of the cylinder is 54 cm^3 and the height is 6 cm. Answer correct to two decimal places.

- 14 Rearrange the formula for the surface area of a sphere $A = 4\pi r^2$ to make r the subject.

CHALLENGING

15 The body mass index is $B = \frac{m}{h^2}$ where m is the mass (kg) and h is the height (m).

a Make m the subject of the formula.

b Make h the subject of the formula.

c Find the height, correct to two decimal places, when:

i $B = 20$ and $m = 66$ kg

ii $B = 30$ and $m = 102$ kg

(Ext) 16 The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$ where r is the radius.

a Write the formula with r as the subject.

b What is the radius in metres of a spherical balloon with a volume of 8 m^3 ? Answer in metres correct to two decimal places.

17 Make h the subject of $A = 2\pi r^2 + 2\pi rh$

Communicate your reasoning, understanding and fluency

18 Using the formula $A = \frac{h}{2}(a + b)$, perform both methods as follows.

Method 1: Substitute $A = 30$, $h = 12$ and $a = 2$, then solve for b .

Method 2: Make b the subject, then substitute $A = 30$, $h = 12$ and $a = 2$.

Which method did you prefer: Method 1 or Method 2?

2G Distance, speed, time and stopping distance

Learning intentions

- To perform calculations involving speed, distance and time
- To perform calculations about the stopping distance of vehicles

Past, present and future learning

- Some aspects of this section may be new to students
- Rates are used extensively throughout Years 11 and 12
- Questions about distance, speed and time regularly appear in HSC Examinations
- The formula relating distance, speed and time is assumed knowledge and does not appear on the 2027 HSC Examination Reference Sheet
- A formula for stopping distance may be provided in HSC questions

Speed is a rate that compares the distance travelled to the time taken. The speed of a car is measured in kilometres per hour (km/h). The speedometer in a car measures the instantaneous speed of the car.



Distance, speed, and time

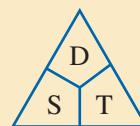
$$S = \frac{D}{T} \text{ or } T = \frac{D}{S} \text{ or } D = S \times T$$

D = Distance

S = Speed

T = Time

The memory aid on the right is used to recall the formulas. Cover the required quantity to determine the formula.



Example 20 Finding the distance, speed and time

- a** Find the distance travelled by a car with average speed of 65 km/h if the journey lasts 5 hours. Answer correct to the nearest kilometre.
- b** How long will it take a vehicle to travel 150 km at a speed of 60 km/h?

Solution

$$\begin{aligned} \text{a } D &= S \times T \\ &= 65 \times 5 \\ &= 325 \text{ km} \end{aligned}$$

Explanation

$$S = 65, T = 5$$

Substitute the values.
Evaluate the formula.

$$\begin{aligned} \text{b } T &= \frac{D}{S} \\ &= \frac{150}{60} \\ &= 2.5 \text{ h} \end{aligned}$$

$$D = 150, S = 60$$

Substitute the values.
Evaluate the formula.

Now you try

A marathon runner travelled 42.2 km in 2.25 hours. Find the average speed. Answer correct to one decimal place.

Stopping distance

The stopping distance is the distance a vehicle travels from the time a driver sees an event occurring to the time the vehicle is brought to a stop. It is calculated by adding the reaction distance and the braking distance. Reaction distance (or thinking distance) is the distance travelled by the vehicle from the time that a hazard, such as a pedestrian stepping into the road, first occurs to when the driver first commences braking. The reaction time averages 0.75 seconds for a fit and alert driver. The braking distance is affected by the road surface (wet, slippery, uneven or unsealed), the slope of the road (uphill or downhill), the weight of the vehicle and the condition of the brakes and tyres.

A formula for stopping distance

Stopping distance = Reaction distance + Braking distance

$$d = \frac{5Vt}{18} + \frac{V^2}{170} \text{ (formula is an approximation using average conditions)}$$

d = stopping distance in metres

V = velocity or speed of the motor vehicle in km/h

t = reaction time in seconds

Note: There are several other formulas you could be given in an examination.



Example 21 Calculating the stopping distance

Trevor was driving at a speed of 45 km/h and he had a reaction time of 0.75 seconds when a hazard occurred. Calculate the stopping distance using the formula $d = \frac{5Vt}{18} + \frac{V^2}{170}$.

Answer correct to the nearest whole metre.

Solution

$$\begin{aligned} d &= \frac{5Vt}{18} + \frac{V^2}{170} \\ &= \frac{5 \times 45 \times 0.75}{18} + \frac{45^2}{170} \\ &= 21.28676471 \end{aligned}$$

Stopping distance is 21 m. (to nearest whole metre)

Explanation

$$V = 45, t = 0.75$$

Substitute the values.
Evaluate the formula.

Now you try

What if Trevor slowed down to 40 km/h for a school zone? By how much does this reduce his stopping distance?

Spreadsheet activity



Build two spreadsheets which can be used for checking your answers to the following exercise.

	A	B
1	This spreadsheet does three things	
2	The orange cells need a formula	
3		
4	How long will the journey take?	
5	If the distance is	
6	And the speed is	
7	The time taken is	
8		
9	What is your average speed?	
10	If the distance is	
11	And the time taken is	
12	My average speed is	
13		
14	How far will you travel?	
15	If the speed is	
16	And the time is	
17	I will travel this distance	

	A	B
1	This spreadsheet calculates stopping distance	
2	The orange cell needs a formula	
3		
4	Velocity (speed)	
5	Reaction time (seconds)	
6	Stopping distance (metres)	

Exercise 2G

BUILDING

1 Find the average speed (in km/h) of a vehicle that travels:

a 180 km in 2 hours	b 485 km in 5 hours
c 360 km in 4.5 hours	d 21 km in $\frac{1}{4}$ hour
e 240 km in $2\frac{1}{2}$ hours	f 16 km in 20 minutes

Example 20

2 Find the distance travelled by a car whose average speed is 56 km/h over the following journey times. Answer correct to the nearest kilometre.

a 3 hours	b 7 hours
c 2.6 hours	d $1\frac{1}{4}$ hours
e $3\frac{1}{2}$ hours	f $2\frac{3}{4}$ hours

Example 20

3 How long will it take a vehicle to travel the following journeys? Answer correct to the nearest hour.

- a** 160 km at a speed of 80 km/h **b** 150 km at a speed of 60 km/h
c 120 km at a speed of 48 km/h **d** 225 km at a speed of 45 km/h
e 240 km at a speed of 40 km/h **f** 556 km at a speed of 69.5 km/h

4 The Melbourne Formula 1 track is 5.303 km in length. The track record is 1 minute and 24 seconds. What is the average speed (km/h) for the lap record? Answer correct to two decimal places.



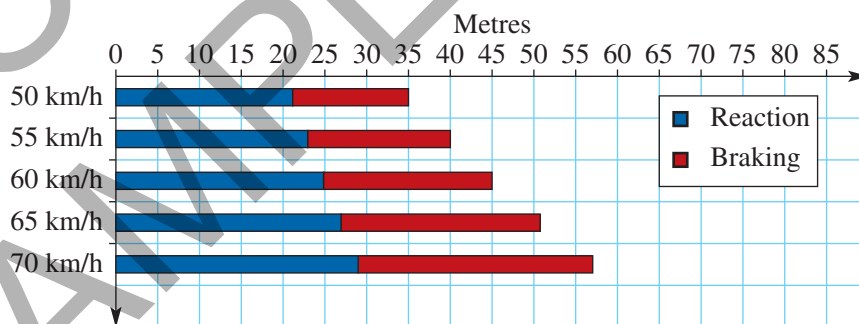
5 Caitlin lives in Wollongong and travels to Sydney daily. The car trip requires her to travel at different speeds. Most often she travels 30 kilometres at 60 km/h and 40 kilometres at 100 km/h.

- a** What is the total distance of the trip?
b How long (in hours) does the trip take?
c What is Caitlin's average speed (in km/h) when travelling to Sydney? Answer correct to two decimal places.

6 Mohammad drives his car to work 3 days a week. The length of the trip is 48 km. The trip took 43 minutes on Monday, 50 minutes on Tuesday and 42 minutes on Wednesday.

- a** Calculate the average time taken for Mohammad to travel to work.
b What is the average speed (in km/h) for the three trips?

7 The graph shows the reaction distance and the braking distance.



A car is travelling at 60 km/h.

- a** What is the reaction distance?
b What is the braking distance?
c What is the stopping distance?

8 What is the stopping distance for each of the following?

- a** Reaction distance of 25 metres and braking distance of 22 metres
b Reaction distance of 19 metres and braking distance of 30 metres

- 9** Michael is driving with a reaction time of 0.75 seconds. Calculate the stopping distance (to the nearest metre) for each of the following speeds. Use the formula $d = \frac{5Vt}{18} + \frac{V^2}{170}$.

a 30 km/h

b 50 km/h

c 70 km/h

d 90 km/h

e 110 km/h

f 130 km/h

- 10** Sarah was driving her car at 40 km/h through a school zone (reaction time is 0.50 seconds). A school student ran onto the road 12 metres in front of her.

- a** Do you think Sarah was able to stop without running over the child? Give a reason for your answer.
- b** What would have happened if Sarah had been driving her car at 60 km/h? Explain your answer.



Example 21

- 11** Oliver travels to work on the freeway. His reaction time is 0.60 seconds. Oliver usually drives at the speed limit of 110 km/h.

- a** What is the stopping distance on the freeway? Use the formula $d = \frac{5Vt}{18} + \frac{V^2}{170}$.
- b** Determine a safe distance (to the nearest metre) between cars that are travelling at 110 km/h on the freeway based on the NSW Roads and Maritime advice to keep three seconds behind the car in front.

DEVELOPING

- 12** Find the average speed (in km/h) of a vehicle that travels the following journeys. Answer correct to the nearest whole number.

a 500 km in 6 hours and 10 minutes

b 64 km in 1 hour and 30 seconds

c 36 000 m in 45 minutes

d 320 m in 10 seconds

- 13** Find the distance travelled by a car with an average speed of 68 km/h if the journey lasts the following times. Answer correct to the nearest kilometre.

a 30 minutes

b 2 minutes

c 1 hour and 20 minutes

d 4 hours 10 seconds

- 14** How long will it take a vehicle to travel the following journeys? Answer correct to the nearest minute.

a 450 km at a speed of 82 km/h

b 50 km at a speed of 60 km/h

c 250 km at a speed of 49 km/h

d 580 000 m at a speed of 62 km/h

e 24 000 m at a speed of 72 km/h

f 100 km at a speed of 1 km/min

- 15** The land speed record is 20.4 km/min.

a Express this speed in km/h.

b How far does this vehicle travel in 5 minutes?

c How far does this vehicle travel in 1 second?

d How long would it take for this vehicle to travel from Sydney to Brisbane (982 km)? Answer to the nearest minute.

- 16** The Bathurst 1000 motor race has a lap record of 2 minutes and 12.339 seconds. The length of the lap is 6.213 km.
- a** What is the average speed (to nearest km/h) for the lap record?
- b** How long is the race if the winning car travels the 161 laps at the average speed for the lap record? Answer to the nearest minute.



- 17** 'If you double your speed, you need to double your reaction distance.'

- a** Use $d = \frac{5Vt}{18}$ to complete the table. Assume a reaction time of 0.75 seconds.
- b** Do you agree with the above statement? Give a reason for your answer.

Speed (km/h)	Reaction distance (m)
10	
20	
40	
80	

- 18** 'If you double your speed, you need to quadruple your braking distance.'

- a** Use $d = \frac{V^2}{170}$ to complete the table.
- b** Do you agree with the above statement? Give a reason for your answer.

Speed (km/h)	Reaction distance (m)
10	
20	
40	
80	

CHALLENGING

19 Tahnee is driving at a speed of 30 km/h.

- a** Write the formula $d = \frac{5Vt}{18} + \frac{V^2}{170}$ with t as the subject.
- b** Find the value of t when $d = 10$ metres. Answer correct to one decimal place.
- c** Find the value of t when $d = 20$ metres. Answer correct to one decimal place.
- d** Find the value of t when $d = 30$ metres. Answer correct to one decimal place.

20 Liam is driving at a speed of 60 km/h.

- a** Use the formula $d = \frac{5Vt}{18}$ to complete the table.
- b** What effect does increasing the reaction time have on the stopping distance? Use the calculations in the table to justify your answer.

Reaction time (s)	Reaction distance (m)
0.50	
1.00	
1.50	



Communicate your reasoning, understanding and fluency

21 Advertisements in the media are warning that ‘casual speeding’ (i.e. going just a little bit over the speed limit) is very dangerous. They warn that ‘every k counts’ and ‘casual speeding is the biggest cause of deaths and serious injuries on NSW roads’.

They claim:

- Going 5 km/h over in a 60 km/h zone doubles your risk of having a crash.
- Going 10 km/h over in a 60 km/h zone means you are four times as likely to be involved in a crash.

a On a road with a 60 km/h speed limit, with a reaction time of 1 second, what is the stopping distance? Use this formula:

$$d = \frac{5Vt}{18} + \frac{V^2}{170}$$

b How much longer is the stopping distance if the speed is:

- i** 65 km/h?
- ii** 70 km/h?

Suggestion: Go outside and measure how much is added to the stopping distance by ‘casual speeding’.

2H Blood alcohol content

Learning intentions

- To apply formulas to calculate blood alcohol content
- To calculate the time taken for blood alcohol content to return to zero

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about blood alcohol content regularly appear in HSC Examinations
- The formulas for blood alcohol content do not appear on the 2027 HSC Examination Reference Sheet, but they should be provided in the question

Blood alcohol content (BAC) is a measure of the amount of alcohol in your blood. The measurement is the number of grams of alcohol in 100 millilitres of blood. For example, a BAC 0.05 means 0.05 g or 50 mg of alcohol in every 100 mL of blood. BAC is influenced by the number of standard drinks consumed in a given amount of time and a person's mass. Other factors that affect BAC include gender, fitness, health and liver function.

Blood alcohol content (BAC)

Blood alcohol can be estimated using a BAC formula.

$$\text{BAC}_{\text{Male}} = \frac{10N - 7.5H}{6.8M} \quad \text{or} \quad \text{BAC}_{\text{Female}} = \frac{10N - 7.5H}{5.5M}$$

BAC = Blood alcohol content

N = Number of standard drinks consumed

H = Hours drinking

M = Mass in kilograms



Example 22 Calculating the BAC

Josip is 82 kg and has consumed 7 standard drinks in the past two hours. He was stopped by police for a random breath test. What would be an estimate of Josip's BAC using the formula above? Answer correct to three decimal places.

Solution

$$\begin{aligned} \text{BAC}_{\text{Male}} &= \frac{10N - 7.5H}{6.8M} \\ &= \frac{10 \times 7 - 7.5 \times 2}{6.8 \times 82} \\ &= 0.098637 \end{aligned}$$

Josip's BAC is estimated to be 0.099. (to 3 decimal places)

Explanation

Choose the correct formula.

Substitute the values.

Evaluate the formula.

Now you try

If Josip only had 5 standard drinks, would he be under the legal limit of 0.05?

NSW has different blood alcohol limits for different types of drivers. Use the internet to find out what they are.

Hours to wait before driving

The number of hours you should wait for your BAC to reach zero can be estimated from your BAC by a formula.

$$\text{Time} = \frac{\text{BAC}}{0.015}$$



Example 23 Using BAC tables and a formula to estimate hours to wait before driving

The table shows the estimated BAC resulting from having 1 to 6 standard drinks, for five different body weights (kg).

Drinks	Body weight (kg)				
	45	55	65	75	85
1	0.008	0.007	0.006	0.005	0.004
2	0.041	0.033	0.028	0.025	0.022
3	0.074	0.060	0.051	0.044	0.039
4	0.106	0.087	0.074	0.064	0.056
5	0.139	0.114	0.096	0.083	0.074
6	0.172	0.140	0.119	0.103	0.091



Terry weighs 65 kg and consumes four standard drinks in an hour. Calculate the number of hours Terry has to wait for his BAC to reach zero. Answer to the nearest hour.

Solution

$$\begin{aligned} \text{Number of hours} &= \frac{\text{BAC}}{0.015} \\ &= \frac{0.074}{0.015} \\ &= 4.93333 \end{aligned}$$

Terry needs to wait 5 hours for his BAC to reach zero. (to nearest hour)

Explanation

Choose the correct number from the table.

Substitute the values.

Evaluate the formula.

Now you try

Terry doesn't need to drive for the next 8 hours. Can he have 6 drinks?

Spreadsheet activity



Build a spreadsheet which can be used for checking your answers to the following exercise.

	A	B	C
1	This spreadsheet does three things		
2	The orange cells need a formula		
3			
4	BAC for males		
5	Number of standard drinks		
6	Number of hours drinking		
7	Mass of person		
8	BAC is approximately		
9			
10	BAC for females		
11	Number of standard drinks		
12	Number of hours drinking		
13	Mass of person		
14	BAC is approximately		
15			
16	How long will it take for BAC to get back to zero?		
17	BAC is		
18	Number of hours		
19			

Exercise 2H

BUILDING

1 Calculate the BAC for the following females. Answer correct to two decimal places.

- a** Sarah is 48 kg and has consumed 4 standard drinks in the past 2 hours.
- b** Sienna is 59 kg and has consumed 3 standard drinks in the past hour.
- c** Alyssa is 81 kg and has consumed 6 standard drinks in the past 2 hours.
- d** Kayla is 65 kg and has consumed 8 standard drinks in the past 6 hours.
- e** Tahlia is 71 kg and has consumed 13 standard drinks in the past 3 hours.
- f** Mia is 55 kg and has consumed 9 standard drinks in the past 5 hours.

2 Calculate the BAC for the following males. Answer correct to two decimal places.

- a** Wei is 53 kg and has consumed 3 standard drinks in the past 3 hours.
- b** Riley is 64 kg and has consumed 5 standard drinks in the past hour.
- c** Thomas is 98 kg and has consumed 2 standard drinks in the past 2 hours.
- d** Zachary is 47 kg and has consumed 10 standard drinks in the past 5 hours.
- e** Charlie is 85 kg and has consumed 12 standard drinks in the past 4 hours.
- f** Jacob is 104 kg and has consumed 7 standard drinks in the past 6 hours.

3 James and Olivia are twins and both weigh 73 kg. At a party they each consume 6 standard drinks in two hours.

- a** What is James's BAC? Answer correct to two decimal places.
- b** What is Olivia's BAC? Answer correct to two decimal places.
- c** How long does James need to wait before he drives home?

4 Calculate the time to wait before driving. Answer to the nearest minute.

- a** BAC of 0.056
- b** BAC of 0.123
- c** BAC of 0.087
- d** BAC of 0.153
- e** BAC of 0.092
- f** BAC of 0.172

Example 23 5 The table shows the BAC after one hour for a range of drinks and body weights.

Drinks	Body weight (kg)							
	45	55	65	75	85	95	105	115
1	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.003
2	0.041	0.033	0.028	0.025	0.022	0.019	0.018	0.016
3	0.074	0.060	0.051	0.044	0.039	0.035	0.032	0.029
4	0.106	0.087	0.074	0.064	0.056	0.050	0.046	0.042
5	0.139	0.114	0.096	0.083	0.074	0.066	0.060	0.054
6	0.172	0.140	0.119	0.103	0.091	0.081	0.074	0.067

Calculate the time to wait before driving. Answer to the nearest minute.

- a Joshua weighs 85 kg and consumes 5 standard drinks in an hour.
 - b Mitchell weighs 115 kg and consumes 3 standard drinks in an hour.
 - c Harrison weighs 45 kg and consumes 6 standard drinks in an hour.
 - d Cooper weighs 65 kg and consumes 2 standard drinks in an hour.
 - e Zachary weighs 95 kg and consumes 4 standard drinks in an hour.
 - f Aarav weighs 75 kg and consumes 1 standard drink in an hour.
- 6 Use the table from question 5 to construct two separate column graphs for the following body weights. Make the number of drinks the horizontal axis and the BAC the vertical axis.
- a Body weight of 45 kg
 - b Body weight of 115 kg
- 7 The formula for calculating ‘standard drinks’ is $S = V \times A \times 0.789$ where S is the number of standard drinks, V is the volume of drink in litres and A is the percentage of alcohol. How many standard drinks are in each of the following drinks? Answer correct to one decimal place.
- a 345 mL bottle of full strength beer at 5.2% alcohol
 - b 750 mL bottle of champagne at 13.5% alcohol
 - c 150 mL glass of white wine at 12.5% alcohol
 - d Mixed drink with a 30 mL of brandy at 38% alcohol
 - e 360 mL can of light beer at 2.1% alcohol

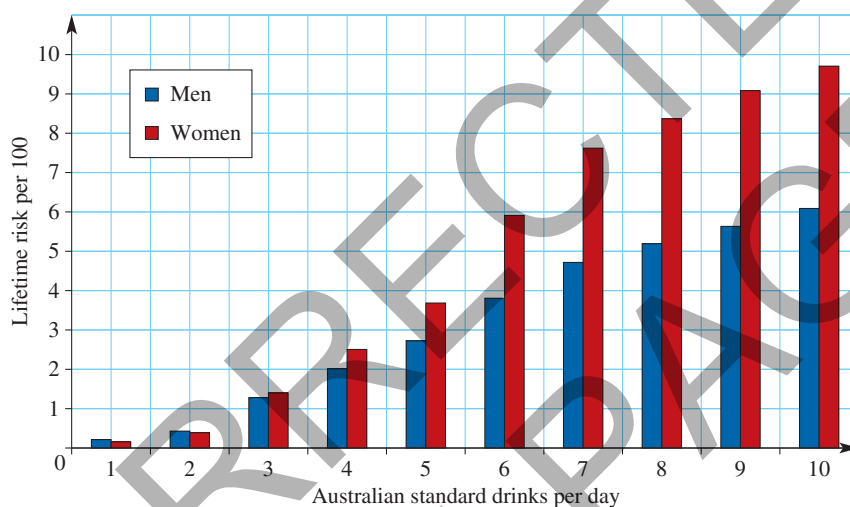


DEVELOPING

- 8 Find the value of BAC using the formula $\text{Number of hours} = \frac{\text{BAC}}{0.015}$, given the number of hours to wait before driving.
- a Number of hours to wait before driving is 5. Answer correct to three decimal places.
 - b Number of hours to wait before driving is 3. Answer correct to three decimal places.

CHALLENGING

- 9** Find the value of H in the formula $BAC_{\text{Male}} = \frac{10N - 7.5H}{6.8M}$ given:
- a** $BAC_{\text{Male}} = 0.066$, $M = 60$ and $N = 5$. Answer correct to the nearest minute.
- b** $BAC_{\text{Male}} = 0.050$, $M = 79$ and $N = 7$. Answer correct to the nearest minute.
- 10** Find the value of N in the formula $BAC_{\text{Female}} = \frac{10N - 7.5H}{5.5M}$ if:
- a** $BAC_{\text{Female}} = 0.066$, $M = 48$ and $H = 2$. Answer correct to one decimal place.
- b** $BAC_{\text{Female}} = 0.120$, $M = 57$ and $H = 4$. Answer correct to one decimal place.



- 11** The graph relates the lifetime risk of death to the number of standard drinks consumed per day.
- a** What is the lifetime risk for a female and a male who consume 7 drinks per day?
- b** Why is the effect of alcohol greater on a female than on a male?

Communicate your reasoning, understanding and fluency

- 12** Who can typically drink more for the same BAC: men or women? Use the BAC formulas in the following scenario.
- Two people: a man and a woman.
 - They have the same mass (65 kg).
 - They drink the same number of drinks (5) in the same amount of time (3 hours).

How different are their BACs?

Is it safe for one of them to drive but not the other?

2I Medication dosages

Learning intentions

- To apply formulas to calculate medication dosages for children and adults

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- The formulas for medication dosages do not appear on the 2027 HSC Examination Reference Sheet, but they should be provided in the question

Australians have access to free or low-cost medicines. When a medicine is prescribed, the amount per dose, the frequency of dosage and the dosages for different medication types are calculated. Medications involve concentrations that are expressed as mass per volume, for example 5 g/10 mL. A concentration is a measure of how much of a given substance is mixed with another substance.



Example 24 Converting a concentration

The concentration of a medicine is given as 100 mg per 50 mL. What is the dosage rate for this medicine in g/mL?

Solution

$$\begin{aligned} 100 \text{ mg}/50 \text{ mL} &= \frac{100 \text{ mg}}{50 \text{ mL}} \\ &= 2 \text{ mg/mL} \\ &= 2 \times 0.001 \text{ g/mL} \\ &= 0.002 \text{ g/mL} \end{aligned}$$

The concentration is 0.002 g/mL.

Explanation

Convert mg to g.
Evaluate.

Now you try

The concentration of a medicine is given as 50 mg per 100 mL. What is the dosage rate for this medicine in g/mL?

Formula for children and infants

The table shows the three formulas used to calculate the required medication dosages for children.

Name	Purpose	Formula
Fried's formula	Dosage for children 1 to 2 years	$\text{Dosage} = \frac{\text{Age (months)} \times \text{Adult dosage}}{150}$
Young's formula	Dosage for children 1 to 12 years	$\text{Dosage} = \frac{\text{Age of child (years)} \times \text{Adult dosage}}{\text{Age of child (years)} + 12}$
Clark's formula	Dosage for children	$\text{Dosage} = \frac{\text{Weight (kg)} \times \text{Adult dosage}}{70}$



Example 25 Calculating dosages for children

Young's formula is used to prescribe medicine for children aged 1 to 2 years. The formula is:

$$D = \frac{yA}{y + 12}, \text{ where:}$$

D = Dosage for children (in mg)

Y = Age (in years)

A = Adult dosage (in mg)

What is the dosage for a 3-year-old child if the adult dosage is 50 mg?



Solution

$$\begin{aligned} D &= \frac{yA}{y + 12} \\ &= \frac{3 \times 50}{3 + 12} \\ &= \frac{150}{15} \\ &= 10 \text{ mg} \end{aligned}$$

The child's dosage is 10 mg.

Explanation

Substitute the values.

Evaluate.

Now you try

How much of this medicine should be given to a 6-year-old?

Exercise 21

BUILDING

- 1 There are 250 capsules in a container labelled as containing 30 g of a drug. How many milligrams of the drug is in each capsule?



- 2 A medicine bottle has a capacity of 0.75 L.
 - a How many tablespoons (15 mL) of medicine does the bottle contain?
 - b How many teaspoons (5 mL) of medicine does the bottle contain?
 - c The dosage required is 20 mL, three times a day. For how many days will the bottle last?

- 3 Use the rate provided to answer the following questions.

- a The concentration of a drug is 25 mg/50 mL. How many mg are there in 10 mL?
- b The concentration of a drug is 100 mg/5 mL. How many mg are there in 2 mL?
- c The concentration of a drug is 18 g/100 mL. How many mg are there in 10 mL?

Example 24

- 4** Use Fried's formula to find the required infant dose.
- a** Mikayla aged 6 months, if the adult dose is 15 mL
 - b** Flynn aged 12 months, if the adult dose is 10 mL
 - c** Lara aged 15 months, if the adult dose is 20 mL
 - d** Toby aged 2 years, if the adult dose is 10 mL

Example 25

- 5** Use Young's formula to find the required dose for these children. Answer to the nearest millilitre.
- a** Christian who is 4 years old, if the adult dose is 10 mL
 - b** Indiana who is 8 years old, if the adult dose is 15 mL
 - c** Gemma who is 3 years old, if the adult dose is 10 mL
 - d** Alika who is 72 months old, if the adult dose is 30 mL
- 6** Use Clark's formula to find the required dose for these children. Answer to the nearest millilitre.
- a** Harper who weighs 40 kg, if the adult dose is 20 mL
 - b** Aaron who weighs 30 kg, if the adult dose is 10 mL
 - c** Eve who weighs 10 kg, if the adult dose is 15 mL
 - d** Tyson who weighs 25 kg, if the adult dose is 20 mL

DEVELOPING

- 7** The medical equipment shown inserts a fluid drip by drip into the blood. Use this formula for the following questions:

$$\text{Drip rate} = \frac{\text{Volume (mL)}}{\text{Time (h)}}$$

- a** Leo receives 1.5 litres of fluid over 8 hours. What is the required drip rate in mL/h?
 - b** Aanya receives 2 litres of fluid over 5 hours. What is the required drip rate in mL/h?
- 8** Use this formula for the following questions:

$$\text{Volume} = \frac{\text{Strength required (mg)}}{\text{Stock strength (mg)}}$$

where 'stock strength' is the strength of the medicine available.

- a** Alicia is prescribed 750 mg of medication. Tablets, each of 500 mg, are available. How many tablets should Alicia receive?
 - b** Hugo is prescribed 450 mg of medication. Tablets, each of 200 mg, are available. How many tablets should Hugo receive?
- 9** Use this formula for the following questions:

$$\text{Volume required (mL)} = \frac{\text{Strength required (mg)}}{\text{Stock strength (mg)}} \times \text{Volume of stock (mL)}$$

- a** Eden is prescribed 50 mg of a mild painkiller. The medication available contains 20 mg in 5 mL. How much medication should be given to Eden?
- b** Sofia is prescribed 40 mg of a mild painkiller. The medication available contains 50 mg in 15 mL. How much medication should be given to Sofia?



CHALLENGING

- 10** Alex buys a 200 mL bottle of cough mixture.
- a** The recommended dose for a 3-year-old child is 5 mL. What is the adult dose using Young's rule?
 - b** How many adult doses are contained in the bottle?
 - c** It is recommended the cough mixture be taken at most 4 times a day. How many days will the cough mixture last at this rate for an adult?
- 11** The drip rate is the total volume divided by total time. If Claire needed 300 mL of fluid to go through in 2 hours, calculate the drip rate (1 drop = 0.05 mL) in the following units:
- a** mL per hour
 - b** mL per minute
 - c** drops per hour
 - d** drops per minute

**Communicate your reasoning, understanding and fluency**

- 12** A patient is to receive 500 mL of fluid intravenously through a drip into a vein in their arm. Every mL is 20 drops. The fluid should be delivered over a 3-hour period. How many drops per minute should be given?

Key ideas and chapter summary

**Substitution**

- 1 Write the algebraic expression.
- 2 Replace the variables in the expression with the numbers.
- 3 Evaluate using the calculator.
- 4 Write the answer to the specified level of accuracy.

Distance, speed and time

$$S = \frac{D}{T} \text{ or } T = \frac{D}{S} \text{ or } D = S \times T \quad \begin{array}{l} D = \text{Distance, } S = \text{Speed} \\ T = \text{Time} \end{array}$$

Stopping distance = Reaction-time distance + Braking distance

Linear equations

- 1 Look to perform the opposite operation.
- 2 Add or subtract the same number from both sides.
- 3 Multiply or divide both sides of the equation by the same number.
- 4 To solve two- or three-step equations repeat the above steps.

Solving equations after substitution

- 1 Write the formula.
- 2 Replace the variables in the formula with the numbers.
- 3 Solve the equation if the unknown is not the subject.
- 4 Evaluate using the calculator.
- 5 Write the answer to the specified level of accuracy.

Changing the subject of the formula

Move the pronumerals and numbers, other than the pronumeral you want as the subject, to the right-hand side of the equation.

Blood alcohol content

$$\begin{aligned} \text{BAC}_{\text{Male}} &= \frac{10N - 7.5H}{6.8M} \text{ or} \\ \text{BAC}_{\text{Female}} &= \frac{10N - 7.5H}{5.5M} \\ \text{Number of hours} &= \frac{\text{BAC}}{0.015} \end{aligned}$$

BAC = Blood alcohol content
 N = Number of standard drinks
 H = Hours drinking
 M = Mass in kilograms

Medication

Medications involve concentrations expressed as mass/volume such as 5 g/10 mL. Fried's formula, Young's formula and Clark's formula are used to calculate the required dosages for children and infants.

Chapter checklist with success criteria



A printable version of this checklist is available in the Interactive Textbook.



- 2A** **1** I can substitute given numbers into algebraic expressions and calculate the value. ☐
 e.g. Substitute $a = 2$, $b = -3$ and $c = 4$ into these expressions.
- | | | |
|-------------------------------|-----------------------------|----------------------------|
| a $a + b + c$ | b $a - b - c$ | c $3a + 2b + 5c$ |
| d $a + b^2 + \sqrt{c}$ | e $\sqrt{b^2 + c^2}$ | f $\frac{a+b}{b+c}$ |
- 2B** **2** I can simplify expressions by combining like terms. ☐
 e.g. Simplify these expressions.
- | | |
|------------------------------|------------------------------|
| a $5x + 3y + 3x + 3y$ | b $5x + 3y - 3x - 3y$ |
| c $5x - 3y - 3x - 3y$ | d $5x - 3y - 3x + 3y$ |
- 2B** **3** I can expand brackets and then simplify. ☐
 e.g. Expand and simplify.
- | | |
|--------------------------------|--------------------------------|
| a $4(x + 3) + 2(5 - x)$ | b $4(x + 3) - 2(5 - x)$ |
|--------------------------------|--------------------------------|
- 2C** **4** I can solve linear equations and check that my solution is correct. ☐
 e.g. Solve these linear equations and substitute your answer back into the original equation to confirm it is correct.
- | | |
|--|-------------------------------------|
| a $5x + 3 = 33$ | b $4(x + 3) - 2(5 - x) = 20$ |
| c $\frac{x}{5} + \frac{x}{2} = 7$ | d $6 + \frac{x}{5} = 9$ |
- 2D** **5** I can convert worded problems into equations. ☐
 e.g. Write equations for these worded problems using n for the unknown number.
- | |
|---|
| a A number is doubled, then added to 5 to give 41. |
| b A number is added to 5, then the result is doubled to give 41. |
| c A number is doubled, then subtracted from 5 to give 41. |
| d Half of a number is subtracted from 5 to give 41. |
- 2D** **6** I can solve worded problems using equations. ☐
 e.g. The sum of two consecutive integers is 583. What are the integers?
- 2E** **7** I can substitute numbers into a formula then solve for the unknown. ☐
 e.g. Using the formula $D = ST$:
- | |
|---|
| a find D when $S = 12$ and $T = 0.5$ |
| b find S when $D = 12$ and $T = 0.5$ |
| c find T when $S = 12$ and $D = 0.5$ |

2F **8** I can change the subject of a formula. □

e.g. Change the subject of these formulas.

a Make h the subject of $A = \frac{b+h}{2}$

b Make h the subject of $A = \frac{b-h}{2}$

c Make h the subject of $A = \frac{b-h^2}{2}$

2G **9** I can use the formulas for distance, speed and time. □

e.g. A hippopotamus can run quickly for short distances, covering 20 metres in 3 seconds. What is its speed in km/h?

2H **10** I can calculate blood alcohol content (BAC). □

e.g. Karen is 65 kg and has consumed 5 drinks in 5 hours. What is her blood alcohol content (BAC)?

Use the formula $BAC = \frac{10N - 7.5H}{5.5M}$

2I **11** I can calculate medication dosages. □

e.g. Calculate the dosage (D) for a 6-year-old child, given that the adult dosage is 30 mg.

Use the formula $D = \frac{yA}{y+12}$

Multiple-choice questions

- 2A 1** Evaluate $\sqrt{x^2 + y}$ if $x = 5$ and $y = 1$. Answer correct to two decimal places.
- A** 5.00 **B** 5.09
C 5.10 **D** 6.00
- 2B 2** $4x - 3x + 2$ is equal to:
- A** -1 **B** 3
C $x + 2$ **D** $x - 2$
- 2C 3** What is the solution to $\frac{x}{3} = -8$?
- A** $x = -24$ **B** $x = -\frac{8}{3}$
C $x = \frac{8}{3}$ **D** $x = 24$
- 2C 4** Solve the equation $11v - 8 = 27$.
- A** $v = \frac{11}{35}$ **B** $v = \frac{11}{15}$
C $v = 1\frac{8}{11}$ **D** $v = 3\frac{2}{11}$
- 2D 5** William is 3 years older than Olivia. The sum of their ages is 27. Which equation represents this?
- A** $x + 3 = 27$ **B** $2x + 3 = 27$
C $2x = 27$ **D** $2(x + 3) = 27$
- 2E 6** If $p = \sqrt{q^2 + r^2}$, find the value of p when $q = 8$ and $r = 6$.
- A** $\sqrt{14}$ **B** $\sqrt{28}$
C 10 **D** 100
- 2F 7** Make w the subject of the equation $u = -5w + v$.
- A** $w = \frac{1}{5}u - v$ **B** $w = -\frac{1}{5}u - v$
C $w = \frac{u - v}{5}$ **D** $w = \frac{v - u}{5}$
- 2G 8** How long will it take a vehicle to travel 342 km at a speed of 70 km/h?
- A** 0.20 h **B** 2.394 h
C 4.89 h **D** 272 h
- 2H 9** Layla is 61 kg and has consumed 5 standard drinks in the past four hours. Calculate Layla's blood alcohol content using the formula $\text{BAC}_{\text{Female}} = \frac{10N - 7.5H}{5.5M}$.
- A** 0.007 **B** 0.048
C 0.059 **D** 0.060
- 2I 10** If $D = \frac{yA}{y + 12}$, find D if $y = 2$ and $A = 70$.
- A** 10 **B** 20
C 72 **D** 82

Short-answer questions

2A 1 Evaluate P if $P = 2l + 2b$ and:

a $l = 12$ and $b = 10$

b $l = 30$ and $b = 21$

2E 2 The circumference of a circle is given by the formula $C = 2\pi r$ where C is the circumference and r is the radius. Evaluate the circumference of a circle with radius 14 cm. Answer correct to two decimal places.**2B 3** Simplify by collecting like terms.

a $14x^2 - 2x - 6x^2$

b $6a + 2b - a + 7b$

2C 4 A car is travelling at 70 km/h and the reaction time of the driver is 0.50 seconds.**a** Find the stopping distance of this car using the formula $d = \frac{5Vt}{18} + \frac{V^2}{170}$.**b** How much further would it take to stop this car if the reaction time of the driver was 2 seconds?**2C 5** Solve the following linear equations.

a $e - 2 = 6$

b $6b = 27$

c $2 + w = 11$

d $2n + 5 = 17$

e $9 = 3a - 6$

f $8q + 4 = 36$

g $\frac{h}{7} = 14$

h $d + 1\frac{2}{5} = 2\frac{1}{10}$

i $7y = 3\frac{1}{2}$

2C 6 Solve the following linear equations.

a $7p + 2 = 6p - 6$

b $3 - 2s = 3s + 12$

c $1 + p = 2p - 1$

d $\frac{d}{2} + 5 = 4d + 2$

e $5(2y + 5) = 15$

f $9(1 - 3a) = 21$

2C 7 Solve these equations.

a $\frac{2c + 5}{3} = 6$

b $\frac{x}{2} + \frac{x}{6} = 1$

c $\frac{1}{3}(5u + 7) = 4$

2E 8 Use the formula $A = \sqrt[3]{\frac{4B}{2\pi}}$ to find the value of A (correct to two decimal places) when:

a $B = 5$

b $B = 20$

c $B = 150$

2F 9 The kinetic energy K (joules) of a particle of mass m kg with a velocity of v m/s is given by the formula $K = \frac{1}{2}mv^2$.**a** Find m when $K = 5.2$ joules and $v = 2$ m/s.**b** Find v when $K = 6.4$ joules and $m = 1.3$ kg. Answer correct to three decimal places.**2F 10** Make b the subject of the equation $A = \frac{b}{2}(a + b)h$.**2F 11** The formula $v^2 = u^2 + 2as$ is used to relate acceleration, velocity and distance.**a** Make s the subject of the formula.**b** Make u the subject of the formula.

2H 12 Levi weighs 74 kg and consumes five standard drinks in an hour.

a Calculate Levi's blood alcohol content: $BAC_{\text{Male}} = \frac{10N - 7.5H}{6.8M}$.

b What is the number of hours Levi will need to wait before driving? Number of hours = $\frac{BAC}{0.015}$.

Extended-response questions

2G 1 Anna and Bridget live 33 km apart. They cycle to meet each other. Anna travels at 10 km/h and Bridget travels at 12 km/h.

- a** How long until they meet each other?
b What distance have they each travelled?

2D 2 A landscaper charges \$120 up front and \$65 for each hour, h , that he works.

- a** Write a linear equation for the total charge, C , of any job.
b How much would a 4-hour job cost?



3

Applications of measurement

Fun facts about measurement!

Length

The Great Wall of China: The total length of the Great Wall of China is over **21 000 kilometres** when all sections are combined. That's about half the circumference of Earth!

Area

Smallest sovereign State: Vatican City is the smallest independent country, with an area of just **49 hectares** – smaller than many city parks. (Central Park in New York City is 340 hectares and Centennial Park in Sydney is 189 hectares).

Volume

A blue whale's mouth: A blue whale's mouth can hold about **64 cubic metres** of water – enough to fill a small swimming pool. However, it cannot swallow anything larger than a grapefruit due to its narrow throat.

Capacity

A teaspoon of honey: A teaspoon of honey, **5 millilitres**, represents the life's work of about 12 worker bees. Each bee produces just a twelfth of a teaspoon in its entire life, which is just six to eight weeks.



Chapter contents

- 3A** Pythagoras' theorem (**CONSOLIDATING**)
- 3B** Converting units of measurement (**CONSOLIDATING**)
- 3C** Scientific notation and significant figures
- 3D** Perimeter, circumference and arc length
- 3E** Perimeter of composite shapes
- 3F** Area of triangles and quadrilaterals
- 3G** Area of circles and sectors
- 3H** Area of composite shapes
- 3I** Trapezoidal rule
- 3J** Surface area of right prisms
- 3K** Surface area of cylinders and spheres
- 3L** Surface area of composite solids
- 3M** Volume of right prisms
- 3N** Volume of cylinders and spheres
- 3O** Volume of pyramids and cones
- 3P** Capacity
- 3Q** Volume of capacity and composite solids

NSW Syllabus

In this chapter, a student:

- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- Solves problems involving measurement in practical contexts (MST-11-05)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

3A Pythagoras' theorem

CONSOLIDATING

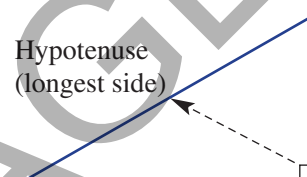
Learning intentions

- To understand that Pythagoras' theorem is a relationship between the three sides of every right-angled triangle
- To use two given sides of a right-angled triangle to calculate the unknown side
- To apply the theorem to solve problems involving right-angled triangles

Past, present and future learning

- Students should already be familiar with the contents of this section
- Pythagoras' theorem is used in various parts of Year 11 and 12
- Questions involving Pythagoras' theorem regularly appear in HSC Examinations
- The formula for Pythagoras' theorem is assumed knowledge and does not appear on the 2027 HSC Examination Reference Sheet

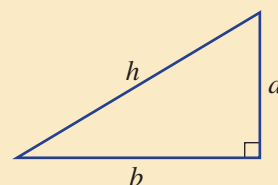
Pythagoras' theorem links the sides of a right-angled triangle. In a right-angled triangle the side opposite the right angle is called the hypotenuse. The hypotenuse is always the longest side.



Pythagoras' theorem

Pythagoras' theorem states that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$\begin{aligned} (\text{Hypotenuse})^2 &= (\text{side})^2 + (\text{other side})^2 \\ h^2 &= a^2 + b^2 \end{aligned}$$

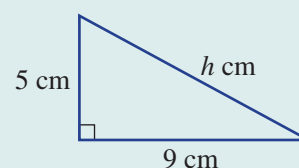


Pythagoras' theorem is used to find a missing side of a right-angled triangle if two of the sides are given. It can also be used to prove that a triangle is right angled.



Example 1 Finding the length of the hypotenuse

Find the length of the hypotenuse, correct to two decimal places.



Solution

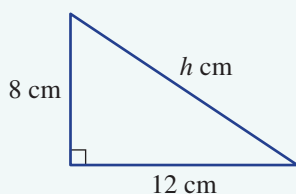
$$\begin{aligned} h^2 &= a^2 + b^2 \\ &= 5^2 + 9^2 \\ h &= \sqrt{5^2 + 9^2} \\ h &= \sqrt{106} \\ &\approx 10.30 \text{ cm} \end{aligned}$$

Explanation

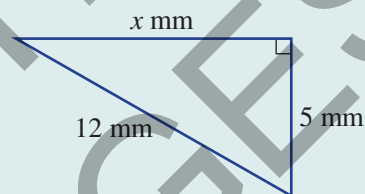
Write Pythagoras' theorem.
Substitute the length of the sides.
Calculate the value of h^2 .
Take the square root to find h .
Express answer correct to two decimal places.

Now you try

Find the length of the hypotenuse, correct to two decimal places.

**Example 2** Finding the length of a shorter side

What is the value of x , correct to one decimal place?

**Solution**

$$\begin{aligned} h^2 &= a^2 + b^2 \\ 12^2 &= x^2 + 5^2 \\ x^2 &= 12^2 - 5^2 \\ x &= \sqrt{12^2 - 5^2} \\ &\approx 10.9 \text{ mm} \end{aligned}$$

Explanation

Write Pythagoras' theorem.

Substitute the length of the sides.

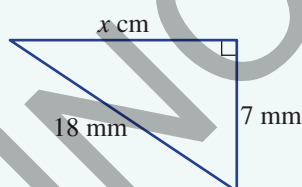
Make x^2 the subject.

Take the square root to find x .

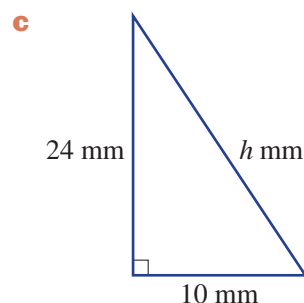
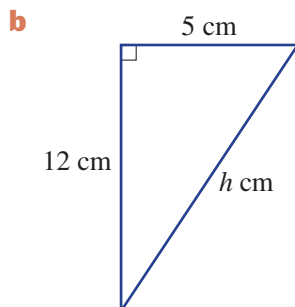
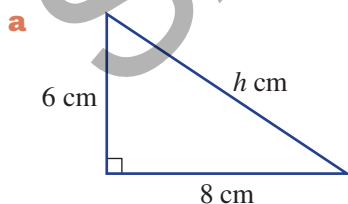
Express answer correct to one decimal place.

Now you try

What is the value of x , correct to one decimal place?

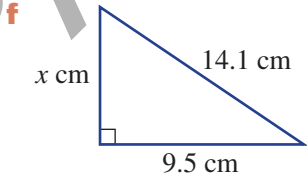
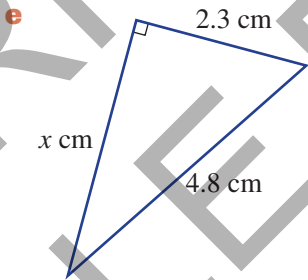
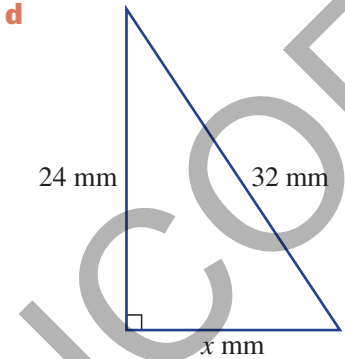
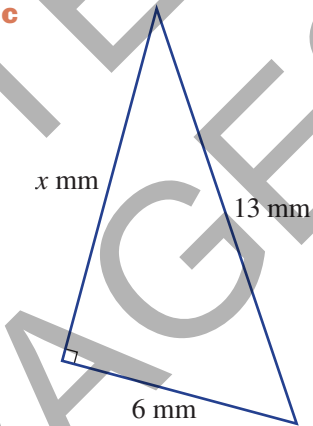
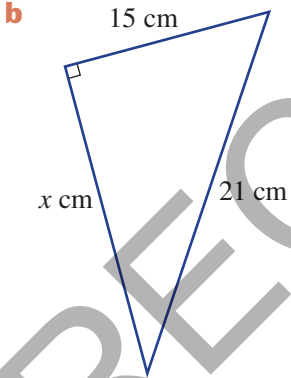
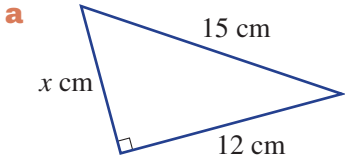
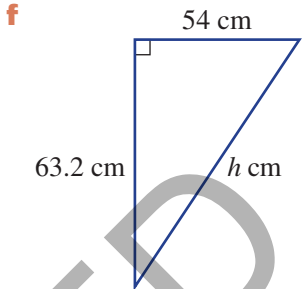
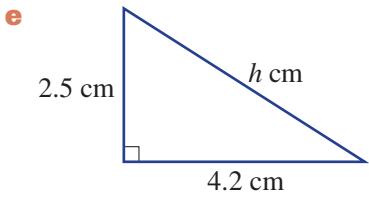
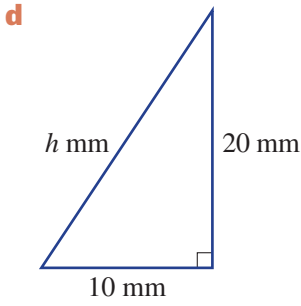
**Exercise 3A****BUILDING****Example 1**

1 Find the length of the hypotenuse, correct to one decimal place.



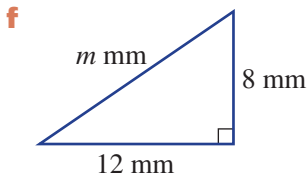
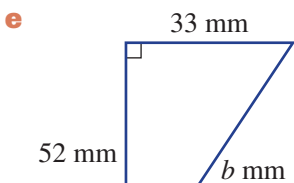
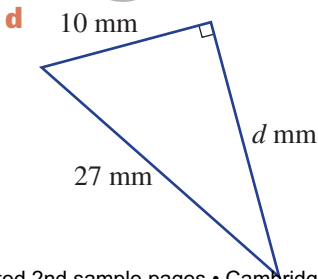
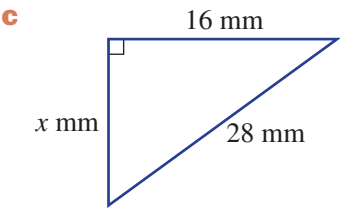
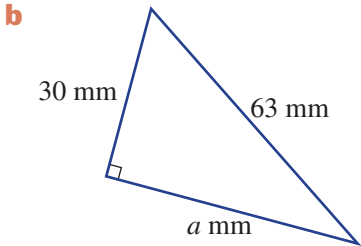
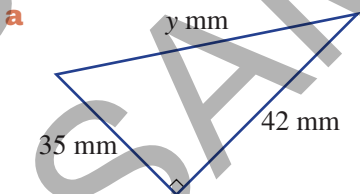
Example 2

2 Find the value of x , correct to two decimal places.

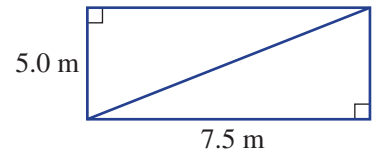


DEVELOPING

3 Calculate the length of the side marked with the pronumeral. Answer to the nearest millimetre.

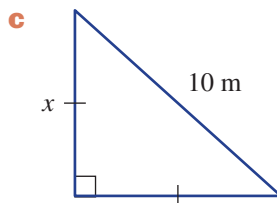
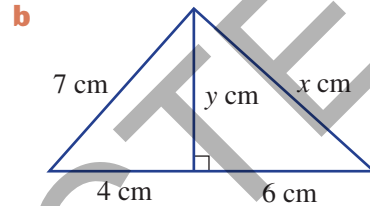
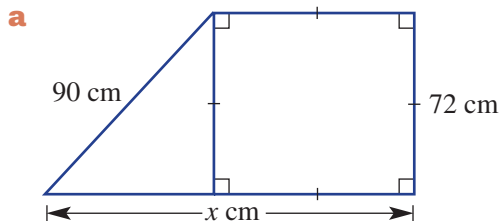


- 4 Find the length of the diagonal of a rectangle with dimensions 7.5 metres by 5.0 metres. Give the answer correct to one decimal place.

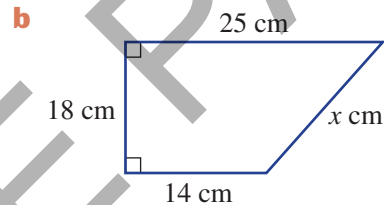
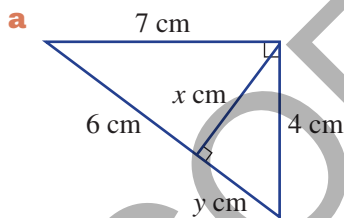


CHALLENGING

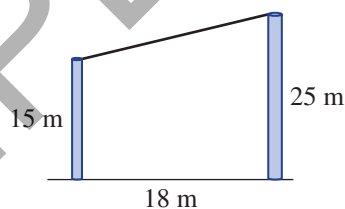
- 5 Find the value of the pronumerals, correct to two decimal places.



- 6 Calculate the length of the side marked with a pronumeral, correct to one decimal place.



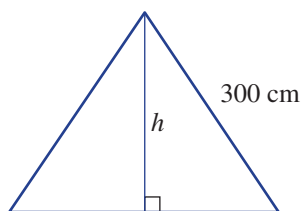
- 7 Two poles, 18 metres apart, are joined by a cable tied to their tops. What is the length of this cable? Answer to one decimal place.



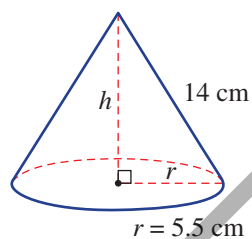
- 8 A TV is labelled based on its diagonal measurements. For a 105 cm TV, the diagonal measures 105 cm. If the length of this television is 90 cm, what is its breadth?
- 9 An airplane is flying from Sydney to Broken Hill. The airplane flies 413 km north to Tamworth, then 1030 km west to Broken Hill. How far is Sydney to Broken Hill directly, to the nearest km?

10 Find the perpendicular height, h , in these shapes, answer to one decimal place.

a An equilateral triangle with side length 300 cm



b Cone



Communicate your reasoning, understanding and fluency

11 Will a 9-metre plank fit into a square room of side length 7 metres?

12 The hypotenuse of a right-angled triangle is 10 cm. What might the lengths of the two shorter sides be?

13 A triangle has side lengths 9 cm, 41 cm and 40 cm. Is this a right-angled triangle? Use calculations to support your answer.

14 Matthew has completed this question. Can you spot any mistakes?

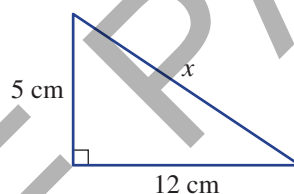
$$c^2 = a^2 + b^2$$

$$x^2 = 5^2 + 12^2$$

$$c^2 = 25 + 144$$

$$x^2 = 169$$

$$x = 84.5$$



3B Converting units of measurement

CONSOLIDATING

Learning intentions

- To know the commonly used units for length, area, mass, volume and capacity
- To convert measurements from one unit to another
- To solve problems involving measurements

Past, present and future learning

- Students should already be familiar with the contents of this section
- Units of measurement are used in various parts of Year 11 and 12
- Questions involving units of measurement regularly appear in HSC Examinations

Measurement is used to determine the size of a quantity. It usually involves using a measuring instrument. For example, to measure length, instruments that can be used include the rule, tape measure, caliper, micrometer, odometer, laser range finder and GPS. There are a number of systems of measurement that define their units of measurement. We use the SI metric system.

SI units

The SI ('Système International', or International System of Units of Measurement) is based on multiples of 10. It is a version of the metric system that allows easy multiplication when converting between related units. Units shown in black are SI and those in red (below) are non-SI units approved for everyday or specialised use alongside SI units.

Quantity	Name of unit	Symbol	Value
Length	Metre	m	Base unit
	Millimetre	mm	1000 mm = 1 m
	Centimetre	cm	100 cm = 1 m
	Kilometre	km	1 km = 1000 m
	Nautical mile	nm	1 nm = 1.852 km
Area	Square metre	m ²	Base unit
	Square centimetre	cm ²	10 000 cm ² = 1 m ²
	Hectare	ha	1 ha = 10 000 m ²
Volume	Cubic metre	m ³	Base unit
	Cubic centimetre	cm ³	1 000 000 cm ³ = 1 m ³
	Litre	L	1 L = 1000 cm ³
	Millilitre	mL	1000 mL = 1 L
	Kilolitre	kL	1 kL = 1000 L
Mass	Kilogram	kg	Base unit
	Gram	g	1000 g = 1 kg
	Milligram	mg	1000 mg = 1 g
	Tonne	t	1 t = 1000 kg

Mass

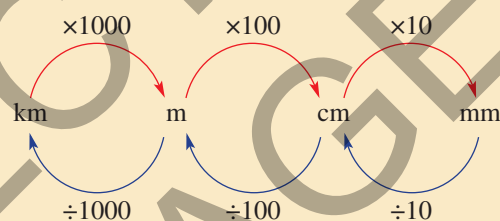
Mass is a measure of the amount of matter in a body. There is a difference between mass and weight. Weight is the measure of the amount of force acting on the mass due to gravity. However if you're on the surface of Earth and not moving, mass and weight can be considered to be the same in everyday contexts. If you change your location with respect to gravity, mass will remain unchanged, but weight will not.

Converting between SI units of the same type

The SI prefix to the unit indicates the conversion between units of the same type. It indicates a multiple of 10. Common prefixes are tera (10^{12}), giga (10^9), mega (10^6), kilo (10^3), centi (10^{-2}), milli (10^{-3}), micro (10^{-6}) and nano (10^{-9}).

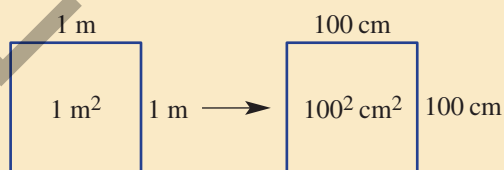
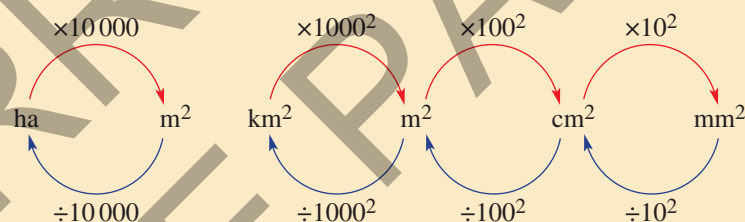
Converting units of length, area, volume and capacity

- The common metric units of **length** include the kilometre (km), metre (m), centimetre (cm) and millimetre (mm).



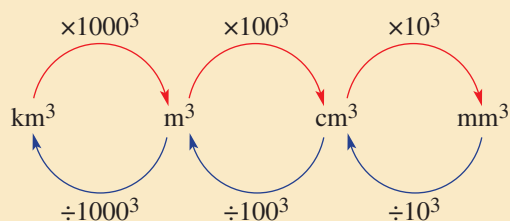
- The common metric units for **area** include:

- ▷ square millimetres (mm^2)
- ▷ square centimetres (cm^2)
- ▷ square metres (m^2)
- ▷ square kilometres (km^2)
- ▷ hectares (ha).



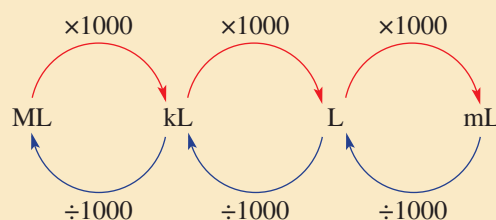
- The common metric units for **volume** include:

- ▷ cubic millimetres (mm^3)
- ▷ cubic centimetres (cm^3)
- ▷ cubic metres (m^3)
- ▷ cubic kilometres (km^3).

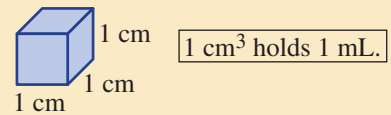


- The common units for **capacity** include:

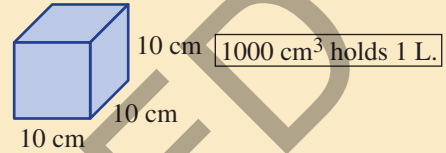
- ▷ megalitres (ML)
- ▷ kilolitres (kL)
- ▷ litres (L)
- ▷ millilitres (mL).



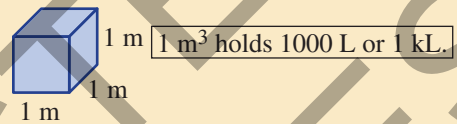
- 1 cm^3 holds 1 mL of liquid.



- This container of 1000 cm^3 holds 1 L of liquid.

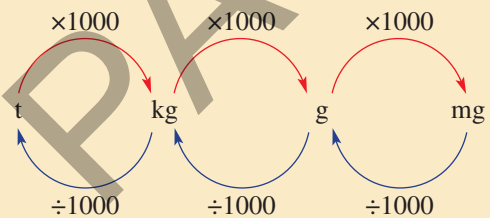


- 1 m^3 holds 1000 L or 1 kL.



Converting units of mass

- The **kilogram** (kg) is the basic unit for mass. 1 L of water has a mass that is very close to 1 kg.
- The common metric units for mass include:
 - ▷ milligram (mg)
 - ▷ gram (g)
 - ▷ kilogram (kg)
 - ▷ tonne (t).



Example 3 Converting units of length

Complete the following.

a $35 \text{ cm} = \square \text{ mm}$

b $4500 \text{ m} = \square \text{ km}$

Solution

a $35 \text{ cm} = 35 \times 10 \text{ mm}$
 $= 350 \text{ mm}$

b $4500 \text{ m} = 4500 \div 1000 \text{ km}$
 $= 4.5 \text{ km}$

Explanation

To change cm to mm multiply by 10.

To change m to km divide by 1000.

Now you try

Complete the following.

a $58 \text{ cm} = \square \text{ mm}$

b $3800 \text{ m} = \square \text{ km}$



Example 4 Converting units of mass

Complete the following.

a $5.2 \text{ kg} = \square \text{ g}$

b $3850 \text{ kg} = \square \text{ t}$

Solution

a $5.2 \text{ kg} = 5.2 \times 1000$
 $= 5200 \text{ g}$

b $3850 \text{ kg} = 3850 \div 1000$
 $= 3.85 \text{ t}$

Explanation

To change kilograms to grams multiply by 1000.

To change kilograms to tonnes divide by 1000.

Now you try

Complete the following.

a $3.7 \text{ kg} = \square \text{ g}$

b $5380 \text{ kg} = \square \text{ t}$



Example 5 Converting units of area

Convert the following area measurements into the units given in the brackets.

a $749 \text{ mm}^2 (\text{cm}^2)$

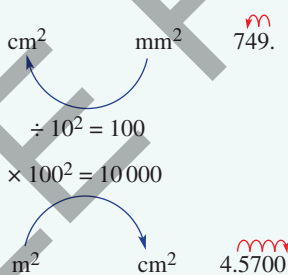
b $4.57 \text{ m}^2 (\text{cm}^2)$

Solution

a $749 \text{ mm}^2 = 749 \div 10^2 \text{ cm}^2$
 $= 7.49 \text{ cm}^2$

b $4.57 \text{ m}^2 = 4.57 \times 100^2 \text{ cm}^2$
 $= 45\,700 \text{ cm}^2$

Explanation



Now you try

Convert the following area measurements into the units given in the brackets.

a $42\,000 \text{ cm}^2 (\text{m}^2)$

b $0.3 \text{ km}^2 (\text{m}^2)$



Example 6 Converting units of volume

Convert the following volume measurements into the units given in the brackets.

a $3.5 \text{ m}^3 (\text{cm}^3)$

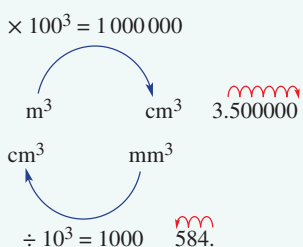
b $584 \text{ mm}^3 (\text{cm}^3)$

Solution

a $3.5 \text{ m}^3 = 3.5 \times 100^3 \text{ cm}^3$
 $= 3\,500\,000 \text{ cm}^3$

b $584 \text{ mm}^3 = 584 \div 10^3 \text{ cm}^3$
 $= 0.584 \text{ cm}^3$

Explanation



Now you try

Convert the following volume measurements into the units given in the brackets.

a $0.8 \text{ cm}^3 (\text{mm}^3)$

b $6\,350\,000 \text{ cm}^3 (\text{m}^3)$

Exercise 3B**BUILDING****Example 3**

1 Complete the following.

a $5 \text{ cm} = \square \text{ mm}$

b $78 \text{ m} = \square \text{ cm}$

c $2 \text{ km} = \square \text{ m}$

d $890 \text{ m} = \square \text{ cm}$

e $57 \text{ cm} = \square \text{ mm}$

f $6 \text{ km} = \square \text{ cm}$

g $9400 \text{ m} = \square \text{ km}$

h $600 \text{ mm} = \square \text{ cm}$

i $8100 \text{ cm} = \square \text{ m}$

j $49\,000 \text{ cm} = \square \text{ km}$

k $22\,000 \text{ m} = \square \text{ km}$

l $51 \text{ mm} = \square \text{ cm}$

Example 4

2 Complete the following.

a $3 \text{ t} = \square \text{ kg}$

b $45 \text{ kg} = \square \text{ g}$

c $76 \text{ t} = \square \text{ kg}$

d $8100 \text{ kg} = \square \text{ g}$

e $4 \text{ t} = \square \text{ g}$

f $0.52 \text{ t} = \square \text{ kg}$

g $6800 \text{ g} = \square \text{ kg}$

h $9\,300\,000 \text{ g} = \square \text{ t}$

i $45\,000\,000 \text{ g} = \square \text{ t}$

j $300 \text{ kg} = \square \text{ t}$

k $2300 \text{ g} = \square \text{ kg}$

l $60\,000 \text{ g} = \square \text{ kg}$

3 Complete the following.

a $2 \text{ L} = \square \text{ mL}$

b $12 \text{ kL} = \square \text{ L}$

c $9 \text{ kL} = \square \text{ mL}$

d $7800 \text{ kL} = \square \text{ L}$

e $50 \text{ L} = \square \text{ mL}$

f $300 \text{ kL} = \square \text{ mL}$

g $6100 \text{ L} = \square \text{ kL}$

h $400 \text{ mL} = \square \text{ L}$

i $2\,10\,000 \text{ mL} = \square \text{ kL}$

j $80 \text{ mL} = \square \text{ L}$

k $79\,000 \text{ mL} = \square \text{ kL}$

l $8\,000\,000 \text{ mL} = \square \text{ kL}$

4 What unit of length is most appropriate to measure each of the following?

a Length of a pen

b Height of a building

c Thickness of a credit card

d Distance from Sydney to Newcastle

e Height of a person

f Length of a football field

5 What unit of mass is most appropriate to measure each of the following?

a Weight of an elephant

b Mass of a mug

c Bag of onions

d Weight of a baby

e Mass of a truck

f Mass of a teaspoon of sugar

6 There are 20 litres of a chemical stored in a container.

a What amount of chemical remains if 750 mL is removed from the container? Answer in litres.

b How many containers are required to hold a kilolitre of the chemical?

7 The length of the Murray River is 2575 km. The length of the Hawkesbury River is 80 000 m. What is the difference in their

lengths? Answer in metres.



- 8** Christopher bought 3 kg of sultanas. What mass of sultanas remains if he ate 800 grams? Answer in kilograms.
- 9** A truck is loaded with 3000 bricks, each of mass 4 kg. Find the total mass (in t) of the bricks.
- 10** The mass of a box of cereal is 375 g. What is the total mass (in kg) of 15 boxes of cereal?
- 11** Find the total mass (in g) of five items of mass 250 mg, 1100 mg, 0.7 g, 5.95 g and 15.4 g.
- 12** An empty plastic box has mass 750 g. It is packed with 8 bottles of drink, each of mass 1.25 kg. Calculate the total mass (in g) of the packed box.
- 13** There are three tonnes of grain in a truck. What is the mass if another 68 kg of grain is added to the truck? Answer in kilograms.



- 14** Find the total mass (in kg) of the following contents of a shopping trolley:
- | | | |
|---------------|--------------------------|-----------------------|
| muesli, 600 g | instant coffee, 250 g | frozen peas, 1 kg |
| rice, 3 kg | self-raising flour, 2 kg | strawberry jam, 400 g |

DEVELOPING

- 15** The distance from John's home to Bankstown is 15.9 km.
- a** John travels to Bankstown and back 5 times each week. What is the total distance travelled?
- b** John's car can travel 8 km on 1 litre of petrol. How many litres of petrol will John use each week travelling to Bankstown and back?
- 16** A cyclist travels to and from work over a 1200-metre long bridge. Calculate the distance travelled in a week if the cyclist works for 5 days. Answer in kilometres.
- 17** Arrange 500 m, 0.005 km, 5000 cm and 5 000 000 mm in ascending order (smallest to largest).

Example 5

- 18** Complete the following.

a $1 \text{ km}^2 = \square \text{ m}^2$

c $1 \text{ cm}^2 = \square \text{ mm}^2$

e $2000 \text{ mm}^2 = \square \text{ cm}^2$

g $3.9 \text{ m}^2 = \square \text{ cm}^2$

i $4.7 \text{ m}^2 = \square \text{ mm}^2$

k $6500 \text{ mm}^2 = \square \text{ cm}^2$

b $1 \text{ m}^2 = \square \text{ mm}^2$

d $1000 \text{ cm}^2 = \square \text{ m}^2$

f $5000 \text{ m}^2 = \square \text{ km}^2$

h $310 \text{ km}^2 = \square \text{ m}^2$

j $74\,300 \text{ m}^2 = \square \text{ km}^2$

l $4000 \text{ cm}^2 = \square \text{ m}^2$

- 19** The area of a field is 80 000 square metres.
Convert the area units to the following.

- a** Square kilometres
b Hectares



CHALLENGING

- 20** Jackson swims 30 lengths of a 50-metre pool.
a How many kilometres does he cover?
b If his goal is 4 kilometres, how many more lengths must he swim?



Example 6

- 21** Complete the following.

- a** $7 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mm}^3$
c $8000 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ cm}^3$
e $6.2 \text{ m}^3 = \underline{\hspace{2cm}} \text{ cm}^3$
g $84\,000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ m}^3$
b $0.5 \text{ m}^3 = \underline{\hspace{2cm}} \text{ cm}^3$
d $500\,000 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ m}^3$
f $0.85 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mm}^3$
h $105\,000 \text{ mm}^3 = \underline{\hspace{2cm}} \text{ cm}^3$

- 22** Complete the following.

- a** $600 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mL}$
c $2.5 \text{ m}^3 = \underline{\hspace{2cm}} \text{ L}$
e $5000 \text{ L} = \underline{\hspace{2cm}} \text{ m}^3$
g $16\,500 \text{ L} = \underline{\hspace{2cm}} \text{ m}^3$
i $4300 \text{ mL} = \underline{\hspace{2cm}} \text{ cm}^3$
b $5200 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ L}$
d $8500 \text{ cm}^3 = \underline{\hspace{2cm}} \text{ mL}$
f $450 \text{ mL} = \underline{\hspace{2cm}} \text{ cm}^3$
h $8.5 \text{ kL} = \underline{\hspace{2cm}} \text{ m}^3$
j $250\,000 \text{ mL} = \underline{\hspace{2cm}} \text{ m}^3$

Communicate your reasoning, understanding and fluency

- 23** The surface area of Warragamba Dam is 75 km^2 .
a Convert this to m^2 .
b If 2 mm of rain falls onto the surface area of the dam (assuming no evaporation), how many kilolitres is this?
c If 15 mm of rain falls onto the surface area of the dam, how many megalitres is this?
- 24** Alan has been asked to convert 4 m^3 into cm^3 .
He says, “Since there are 100 centimetres in 1 metre, the answer is 400 cm^3 ”.
Is Alan correct? Explain with calculations.

3C Scientific notation and significant figures

Learning intentions

- To apply scientific notation to represent numbers involving standard prefixes
- To express quantities and measurements with the required number of significant figures

Past, present and future learning

- Some aspects of this section may be new to students
- Scientific notation and significant figures are used in various parts of Year 11 and 12
- Questions involving these concepts regularly appear in HSC Examinations

Scientific notation

Scientific notation is used to write very large or very small numbers more conveniently. It consists of a number between 1 and 10 multiplied by a power of 10. For example, the number 4 100 000 is expressed in scientific notation as 4.1×10^6 . The power of 10 indicates the number of tens multiplied together. For example:

$$\begin{aligned} 4.1 \times 10^6 &= 4.1 \times (10 \times 10 \times 10 \times 10 \times 10 \times 10) \\ &= 4\,100\,000 \end{aligned}$$

When writing numbers in scientific notation, it is useful to remember that large numbers have a positive power of 10 and small numbers have a negative power of 10.

Writing numbers in scientific notation

- 1 Find the first two non-zero digits.
- 2 Place a decimal point between these two digits. This is the number between 1 and 10.
- 3 Count the digits between the new and old decimal point. This is the power of 10.
- 4 Power of 10 is positive for larger numbers and negative for small numbers.



Example 7 Expressing a number in scientific notation

The land surface of Earth is approximately 153 400 000 square kilometres. Express this area more conveniently by using scientific notation.



Solution

1.534

1.5340000 – eight digits

Power of 10 is +8 or 8.

$$153\,400\,000 = 1.534 \times 10^8$$

Explanation

The first two non-zero digits are 1 and 5.

Place the decimal point between these numbers.

Count the digits from the old decimal point (end of the number) to the position of the new decimal point.

Large number indicates the power of 10 is positive. Write in scientific notation.

Now you try

Warragamba Dam is 750 000 000 square metres. Express in scientific notation.

Significant figures

Significant figures are used to specify the accuracy of a number. They are often used to round a number. Significant figures are the digits that carry meaning and contribute to the accuracy of the number. This includes all the digits except the zeros at the start of a number and zeros at the end of a number without a decimal point. These zeros are regarded as placeholders and only indicate the size of the number. Consider the following examples.

- 51.340 has five significant figures: 5, 1, 3, 4 and 0.
- 0.00871 has three significant figures: 8, 7 and 1.
- 56091 has five significant figures: 5, 6, 0, 9 and 1.

The significant figures in a number not containing a decimal point can sometimes be unclear. For example, the number 8000 may be correct to one, two, three or four significant figures. To prevent this problem, the last significant figure of a number can be underlined. For example, the number $8\underline{0}00$ has two significant figures. If the digit is not underlined the context of the problem is a guide to the accuracy of the number.

Writing numbers to significant figures

- 1 Write the number in scientific notation.
- 2 Count the digits in the number to determine its accuracy (ignore zeros at the end, except after a decimal point).
- 3 Round the number to the required number of significant figures.

**Example 8 Writing numbers to significant figures**

Write these numbers correct to the number of significant figures indicated.

- a** 153 400 000 (three significant figures)
b 0.000 657 (two significant figures)

Solution

- a** $153\,400\,000 = 1.534 \times 10^8$
 1.534 has 4 digits
 1.53 rounded to 3 sig. fig.
 $153\,400\,000 = 1.53 \times 10^8$
- b** $0.000\,657 = 6.57 \times 10^{-4}$
 6.57 has 3 digits
 6.6 rounded to 2 sig. fig.
 $0.000\,657 = 6.6 \times 10^{-4}$

Explanation

Write in scientific notation.
 Count the digits in the number.
 Round to three significant figures.
 Write the answer in scientific notation correct to three significant figures.

Write in scientific notation.
 Count the digits in the number.
 Round to two significant figures.
 Write the answer in scientific notation correct to two significant figures.

Now you try

Write these numbers correct to the number of significant figures indicated.

- a** 953 051 000 (two significant figures)
b 0.000 003 87 (one significant figure)

Standard prefixes

The metric system uses many standard prefixes that associate various powers of 10 with a standard unit.

The standard unit of measurement includes metre for length, gram for mass, second for time and litre for capacity. For example:

- the centi tells us 1 centimetre is 10^{-2} metres
- the milli tells us 1 milli second is 10^{-3} seconds
- the giga tells us 1 gigalitre is 10^9 litres.

Here is a table of standard prefixes.

Standard prefix	Symbol	Scientific notation
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}

Example 9 Apply scientific notation to represent numbers involving standard prefixes

Write using scientific notation.

- a** 5 microseconds (μs) = _____ s
b 6.4 gigalitres (GL) = _____ L
c 3.4 kilometres (km) = _____ m

Solution

- a** 5×10^{-6} seconds
b 6.4×10^9 litres
c 3.4×10^3 metres

Explanation

microseconds is 10^{-6} seconds
 gigalitres is 10^9 litres
 kilometres is 10^3 metres

Now you try

Write using scientific notation.

- a** 8 micrometres (μm) = _____ metres
b 6.4 terabytes (Tb) = _____ bytes

Exercise 3C

BUILDING

Example 7

1 Write these numbers in scientific notation.

a 7600

b 1 700 000 000

c 590 000

d 6 800 000

e 35 000

f 310 000 000

g 77 100 000

h 523 000 000 000

i 95 400 000 000

j 540

2 Write these numbers in scientific notation.

a 0.000 56

b 0.000 068 7

c 0.000 000 812

d 0.0043

e 0.000 058

f 0.000 003 12

g 0.26

h 0.092

i 0.000 000 000 167

j 0.000 06

3 Sharks existed 410 million years ago.

a Write this number in scientific notation.

b Express this number correct to one significant figure.



4 Write each of the following as a basic numeral.

a 1.12×10^5

b 5.34×10^8

c 5.2×10^3

d 8.678×10^7

e 2.4×10^2

f 7.8×10^9

g 3.9×10^6

h 2.8×10^1

i 6.4×10^4

j 3.5×10^4

5 Write each of the following as a basic numeral.

a 3.5×10^{-4}

b 7.9×10^{-6}

c 1.63×10^{-7}

d 5.81×10^{-3}

e 4.9×10^{-2}

f 9.8×10^{-1}

g 4.12×10^{-8}

h 6.33×10^{-5}

i 3.0×10^{-9}

j 7.134×10^{-9}

6 Convert a measurement of 5.81×10^{-3} grams into kilograms. Express your answer in scientific notation.

7 Evaluate the following and express your answer in scientific notation.

a $(2.5 \times 10^3) \times (5.9 \times 10^6)$

b $(4.7 \times 10^5) \times (6.3 \times 10^2)$

c $(7.1 \times 10^{-5}) \times (4.2 \times 10^{-2})$

d $(3.0 \times 10^{-4}) \times (6.2 \times 10^{-5})$

8 Evaluate the following and express your answer in scientific notation.

a $\frac{9.1 \times 10^5}{2.8 \times 10^{-2}}$

b $\frac{7.2 \times 10^7}{4.8 \times 10^{-3}}$

c $\frac{4.8 \times 10^{-4}}{3.2 \times 10^{-5}}$

Example 8

- 9** Write these numbers correct to the number of significant figures indicated.
- | | | |
|----------------------------------|----------------------------------|----------------------------------|
| a 1 561 231 (2 sig. fig.) | b 3 677 720 (4 sig. fig.) | c 789 001 (5 sig. fig.) |
| d 3 300 000 (1 sig. fig.) | e 777 777 (3 sig. fig.) | f 3 194 729 (5 sig. fig.) |
| g 821 076 (4 sig. fig.) | h 7091 (1 sig. fig.) | i 49 172 (2 sig. fig.) |
- 10** Write these numbers correct to the number of significant figures indicated.
- | | | |
|-------------------------------------|-----------------------------------|------------------------------------|
| a 0.0035 (1 sig. fig.) | b 0.191 785 (4 sig. fig.) | c 0.001 592 (3 sig. fig.) |
| d 0.111 222 33 (6 sig. fig.) | e 0.000 0271 (1 sig. fig.) | f 0.019 832 6 (5 sig. fig.) |
| g 0.008 12 (2 sig. fig.) | h 0.092 71 (3 sig. fig.) | i 0.000 419 (2 sig. fig.) |
- 11** A bacterium has a radius of 0.000 015 765 m. Express this length correct to two significant figures.



DEVELOPING

Example 9

- 12** Write the following using scientific notation.
- | |
|---|
| a 73 kilowatts to ____ watts |
| b 25 milligrams to ____ grams |
| c 550 nanometres to ____ metres |
| d 6.5 megalitres to ____ litres |
| e 1.2 micrometres to ____ metres |
| f 149.6 gigametres to ____ metres. |
- 13** The energy required to charge a battery is 45 megajoules (MJ). Express this in joules in scientific notation.
- 14** A strand of DNA has a breadth of approximately 2.5 nanometres (nm). Express this in metres in scientific notation.

- 15** If $y = \frac{1}{2}x^2$, find the value of y when:

a $x = 2.4 \times 10^3$

b $x = 9.8 \times 10^{-3}$

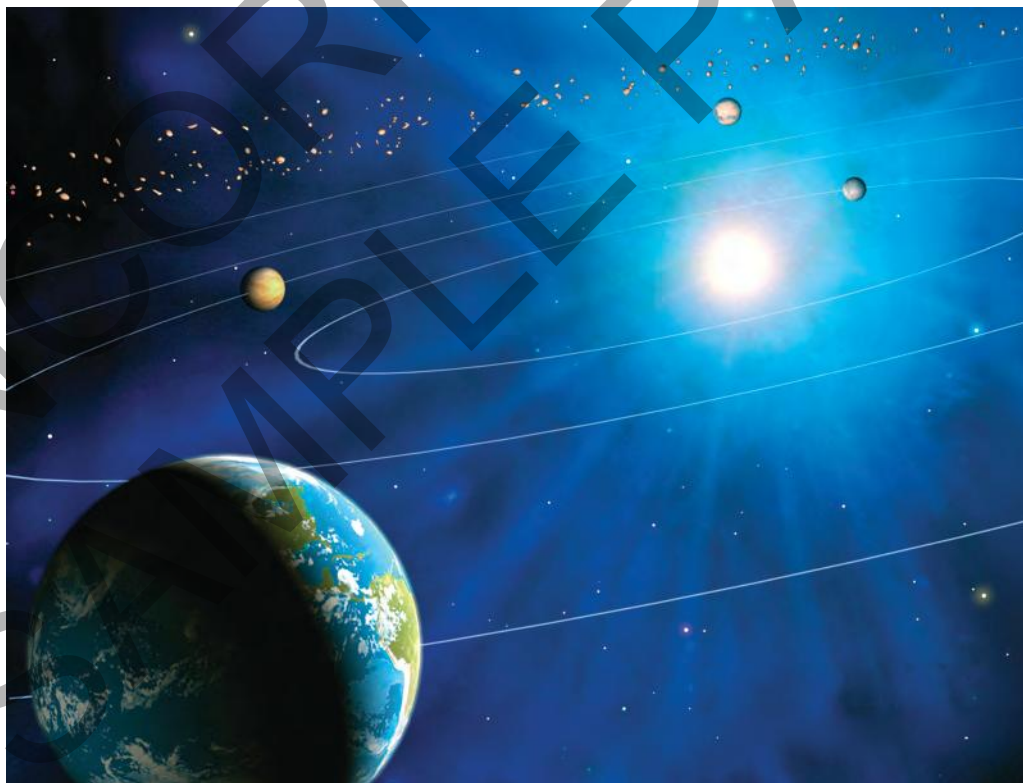
- 16** The arc length of a circle is $l = \frac{\theta}{360^\circ} 2\pi r$ where θ is the angle at the centre and r is the radius of the circle. Use this formula to calculate the arc length of a circle when $\theta = 30^\circ$ and $r = 7.4 \times 10^8$. Answer in scientific notation correct to one significant figure.

CHALLENGING

- 17** Use the formula $E = 3p - q$ to evaluate E , given that $p = 7.5 \times 10^5$ and $q = 2.5 \times 10^4$. Answer in scientific notation correct to one significant figure.
- 18** The volume of a cylinder is $V = \pi r^2 h$ where r is the radius and h is the height of the cylinder. Use this formula to calculate the volume of the cylinder when $r = 5.6 \times 10^4$ and $h = 2.8 \times 10^3$. Answer in scientific notation correct to three significant figures.

Communicate your reasoning, understanding and fluency

- 19** The distance from Earth to the Sun is approximately 149.6 gigametres (Gm). Express this distance in metres in scientific notation to two significant figures.



3D Perimeter, circumference and arc length

Learning intentions

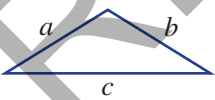
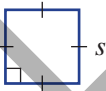
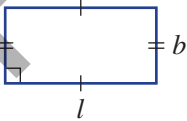
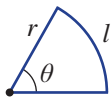
- To calculate perimeter, circumference and arc length
- To solve practical problems involving perimeter

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated or extended in Year 12
- Questions about these concepts regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the 2027 HSC Reference Sheet, but others are not

Perimeter is the total length of the outside edges of a shape. It is the distance of the boundary.

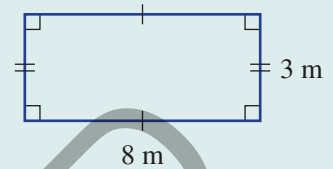
Perimeter formulas

Name	Shape	Perimeter
Triangle		$P = a + b + c$
Quadrilateral		$P = a + b + c + d$
Square		$P = 4s$
Rectangle		$P = 2(l + b)$
Circle		Circumference $C = 2\pi r$ $C = \pi d$
Sector		Arc length $l = \frac{\theta}{360} \times 2\pi r$



Example 10 Finding the perimeter of a rectangle

Find the perimeter of the following rectangle.



Solution

$$\begin{aligned} P &= 2(l + b) \\ &= 2 \times (8 + 3) \\ &= 22 \text{ m} \end{aligned}$$

Perimeter of the rectangle is 22 m.

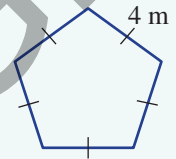
Explanation

The shape is a rectangle, so use the formula $P = 2(l + b)$. Substitute the values for l and b ($l = 8$ and $b = 3$). Evaluate.

Write the answer in words.

Now you try

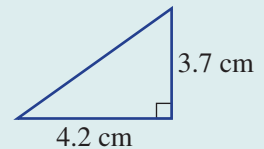
Find the perimeter of the regular pentagon with side length 4 m.



Example 11 Finding the perimeter of a triangle

Find the perimeter of the triangle.

Answer correct to one decimal place.



Solution

$$\begin{aligned} h^2 &= a^2 + b^2 \\ h^2 &= 4.2^2 + 3.7^2 \\ h &= \sqrt{4.2^2 + 3.7^2} \approx 5.6 \text{ cm} \\ P &= 3.7 + 4.2 + 5.6 = 13.5 \text{ cm} \end{aligned}$$

Perimeter of the triangle is 13.5 cm.

Explanation

Find the length of the hypotenuse or h .

Write Pythagoras' theorem.

Substitute the length of the sides.

Evaluate the value of h .

Add the lengths of sides to find the perimeter.

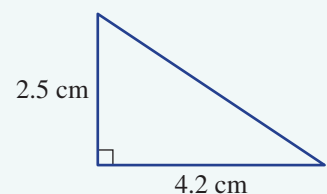
Express answer correct to one decimal place.

Write the answer in words.

Now you try

Find the perimeter of the triangle.

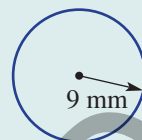
Answer correct to one decimal place.




Example 12 Finding the circumference of a circle

Find the circumference of a circle with a radius of 9 mm.

Answer correct to two decimal places.


Solution

$$\begin{aligned} C &= 2\pi r \\ &= 2 \times \pi \times 9 \\ &\approx 56.55 \text{ mm} \\ \text{Circumference of the circle is } 56.55 \text{ mm.} \end{aligned}$$

Explanation

The shape is a circle, so use the formula $C = 2\pi r$.
Substitute the value for r ($r = 9$).
Evaluate and express answer to two decimal places.
Write the answer in words.

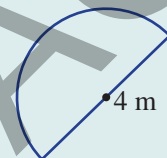
Now you try

Find the circumference of a circle with radius 12.5 cm. Answer correct to two decimal places.


Example 13 Finding the perimeter of a semicircle

Find the perimeter of a semicircle with a diameter of 4 m.

Answer correct to two decimal places.


Solution

$$\begin{aligned} C &= \frac{\pi d}{2} \\ &= \frac{\pi \times 4}{2} \\ &\approx 6.28 \text{ m} \\ P &= 6.28 + 4 \\ &= 10.28 \text{ m} \\ \text{Perimeter is } 10.28 \text{ m.} \end{aligned}$$

Explanation

The shape is a semicircle, so use the formula $C = \pi d \div 2$.
Substitute the value for d ($d = 4$) to find the curved distance.
Evaluate.
Add the curved distance to the diameter.
Evaluate and express answer to two decimal places.
Write the answer in words.

Now you try

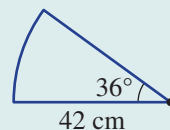
Find the perimeter of a semicircle with a diameter of 6.4 m. Answer correct to two decimal places.


Example 14 Finding the arc length and perimeter of a sector

Calculate:

- a** the arc length of the sector
- b** the total perimeter of the sector.

Answer correct to one decimal place.



Solution

a $\frac{36}{360} \times 2 \times \pi \times 42 = 26.389...$
Arc length is 26.4 cm.

b $26.4 + 42 + 42 = 110.4$ cm
Total perimeter is 110.4 cm.

Explanation

The arc length is a fraction of the circle.

Arc length = $\frac{\theta}{360} \times 2\pi r$ where r = radius
Write answer in words to one decimal place.

Total perimeter of a sector

= radius + radius + arc length

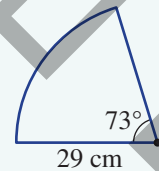
Write answer in words to one decimal place.

Now you try

Calculate:

- a** the arc length of the sector
b the total perimeter of the sector.

Answer correct to one decimal place.

**Perimeter of composite shapes**

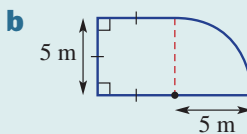
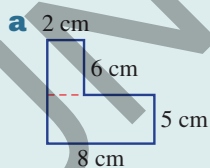
A composite shape is made up of two or more plane shapes. The perimeter of a composite shape is calculated by adding the lengths that make up the boundary of the shape.

Perimeter of a composite shape

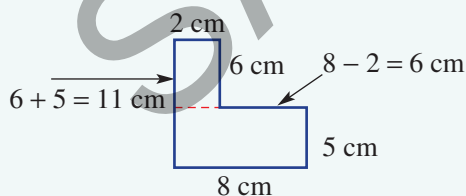
- 1 Sides with the same markings are of equal length.
- 2 Unknown side lengths are determined by using the given lengths of the other sides.
- 3 Pythagoras' theorem is used to find unknown side lengths involving a right triangle.
- 4 Lengths that are part of a circle are found using $C = 2\pi r$.
- 5 Add the lengths that make up the boundary of the shape to calculate the perimeter.

Example 15 Finding the perimeter of composite shapes

Find the perimeter of each of these shapes.

**Solution**

a



$$P = 2 + 6 + 6 + 5 + 8 + 11$$

$$= 38 \text{ cm}$$

Perimeter is 38 cm.

Explanation

Find the unknown side lengths using the measurements given in the question.

Add the lengths of all the edges to find the perimeter.
Evaluate.

Write the answer in words.

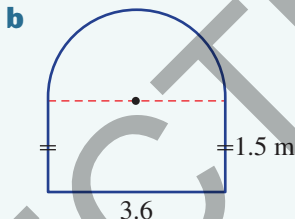
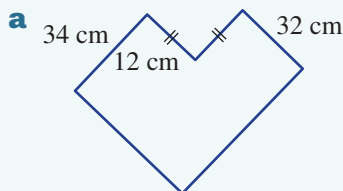
continued ...

$$\begin{aligned}
 \text{b } C &= \frac{2\pi r}{4} \\
 &= \frac{2 \times \pi \times 5}{4} \approx 7.85 \text{ m} \\
 P &= 7.85 + 5 + 5 + 5 + 5 \\
 &= 27.85 \text{ m} \\
 \text{Perimeter is } 27.85 \text{ m.}
 \end{aligned}$$

Use the formula $C = 2\pi r \div 4$ for the curved length.
 Substitute the value for r ($r = 5$).
 Evaluate.
 Add the curved length to other edges.
 Evaluate.
 Write the answer in words.

Now you try

Find the perimeter of each of these shapes.

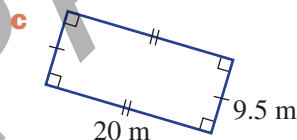
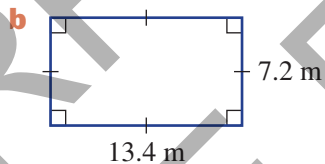
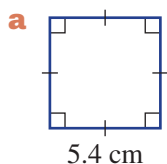


Exercise 3D

BUILDING

Example 10

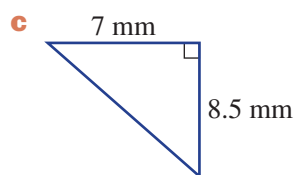
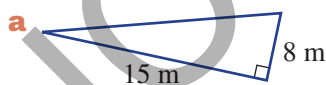
- 1 Find the perimeter of each quadrilateral. Answer correct to one decimal place.



- 2 Find the perimeter of a square with a side length of 12.3 m. Answer correct to one decimal place.

Example 11

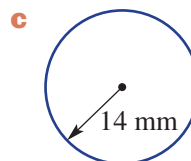
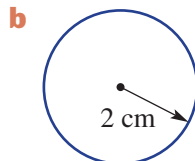
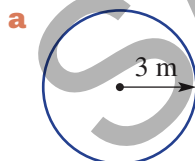
- 3 Find the perimeter of each right triangle. Answer correct to one decimal place.



- 4 Find the perimeter of a right triangle with a base of 10.25 cm and a height of 15.15 cm. Answer correct to two decimal places.

Example 12

- 5 Find the perimeter of each circle. Answer correct to one decimal place.



- 6** Find the circumference of each circle.
Answer correct to one decimal place.

- a** Radius of 4 cm
- b** Radius of 19 m
- c** Radius of 34 mm
- d** Diameter of 50 mm
- e** Diameter of 22 m
- f** Diameter of 6 cm

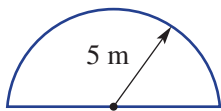


DEVELOPING

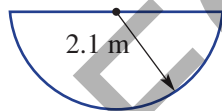
Example 13

- 7** Find the perimeter of each semicircle. Answer correct to one decimal place.

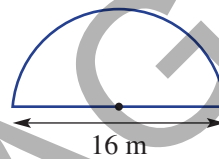
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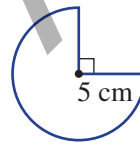
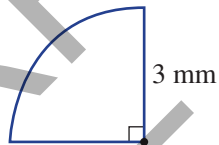
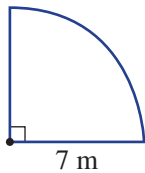
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c



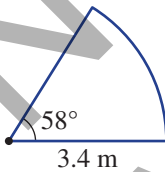
- 8** Find the perimeter of each shape. Answer correct to two decimal places.



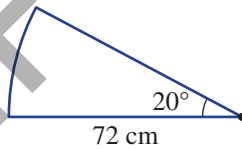
Example 14

- 9** Calculate:
- i** the arc length
 - ii** the total perimeter of the sector.
- Answer correct to two decimal places.

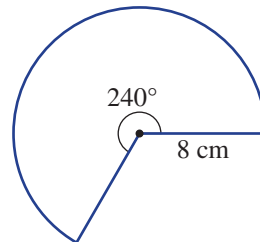
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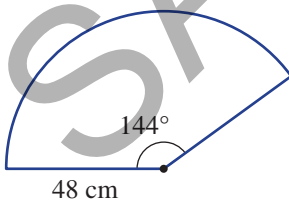
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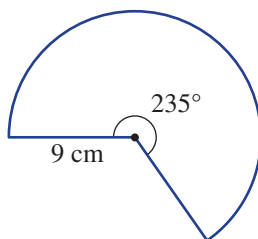
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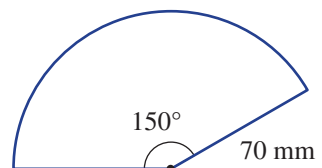
d



e

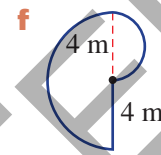
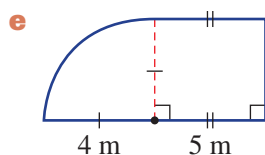
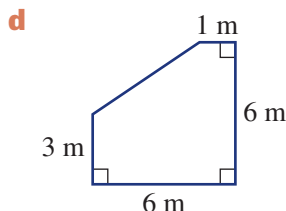
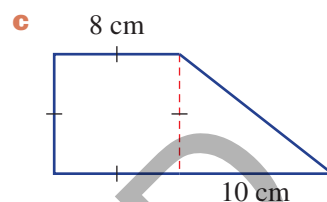
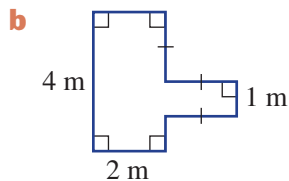
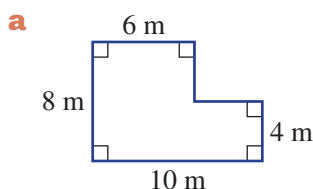


f



Example 15

10 Find the perimeter of each composite shape. Answer to the nearest whole number.



CHALLENGING

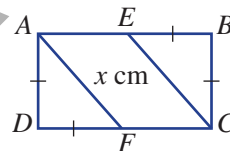
11 An annulus is a shape, like a doughnut or a tyre, between two circles with the same centre. Find the perimeter of an annulus if the inner diameter is 3 cm and the outer diameter is 6 cm. Answer correct to the nearest centimetre.



12 A rectangle $ABCD$ has length $AB = 12$ cm and a breadth of $BC = 6$ cm. Answer correct to two decimal places.

a Find the value of x .

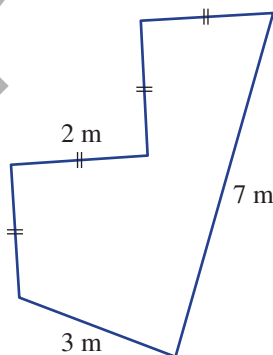
b Calculate the perimeter of quadrilateral $AECF$.



Communicate your reasoning, understanding and fluency

13 Andrew says that all rectangles with an area of 24 m^2 have the same perimeter. Show that Andrew is incorrect.

14 Kirsty wants to place a wire fence around her vegetable garden.



Wire fencing costs \$22 per metre.

What is the total cost of fencing the garden?

15 The length of a rectangular field is 120 metres longer than its breadth. If the length is 270 metres, find the perimeter of the field.

3E Perimeter of composite shapes

Learning intentions

- To calculate perimeter, circumference and arc length
- To solve practical problems involving perimeter
- To calculate perimeter of irregularly shaped blocks of land by dissection into triangles, rectangles and trapeziums

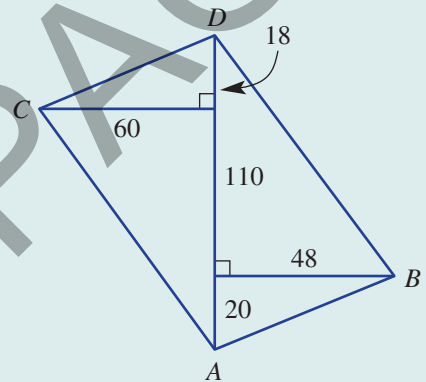
Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about perimeter regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the HSC Reference Sheet, but others are not



Example 16 Perimeter of irregular shapes

Find the perimeter of the irregular shape, all measurement are in metres. Answer to one decimal place.



Solution

$$DB = \sqrt{110^2 + 48^2} = 120.01666\dots$$

$$BA = \sqrt{20^2 + 48^2} = 52$$

$$CA = \sqrt{60^2 + 130^2} = 143.178\dots$$

$$CD = \sqrt{60^2 + 18^2} = 62.641\dots$$

Total perimeter

$$= 120.016\dots + 52 + 143.178\dots + 62.641\dots$$

$$= 377.8$$

The perimeter is 377.8 m.

Explanation

Use Pythagoras theorem to find DB .

$$DB^2 = 110^2 + 48^2$$

$$BA^2 = 20^2 + 48^2$$

$$CA^2 = 60^2 + (110 + 20)^2$$

$$CD^2 = 60^2 + 18^2$$

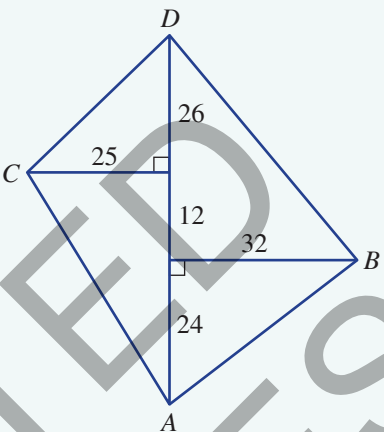
Add all the sides to find the perimeter.

Write your answer in words.

continued ...

Now you try

Find the perimeter of the irregular shape, all measurement are in metres. Answer to one decimal place.



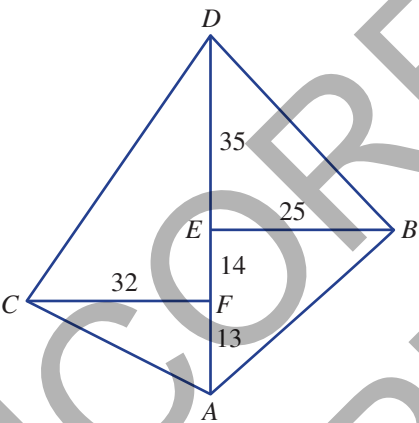
Exercise 3E

BUILDING

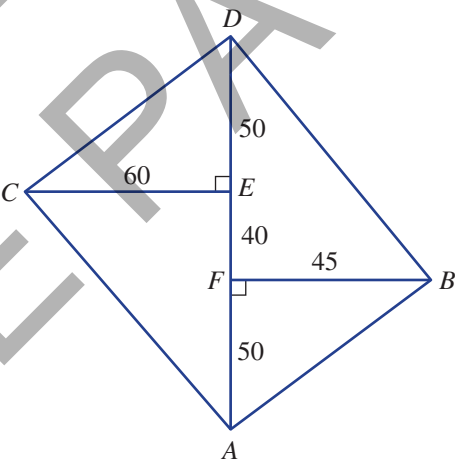
Example 16

1 Find the perimeter of the irregular shapes, all measurement in metres.

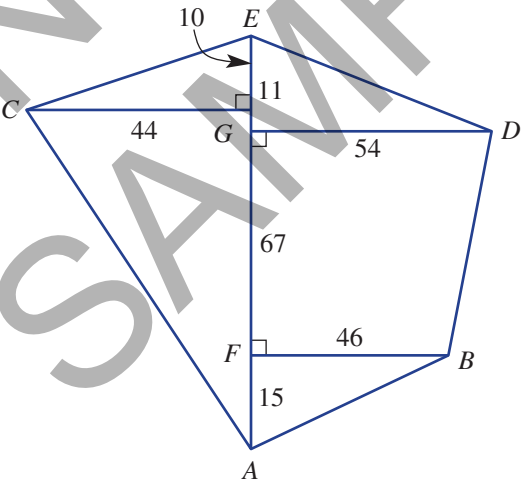
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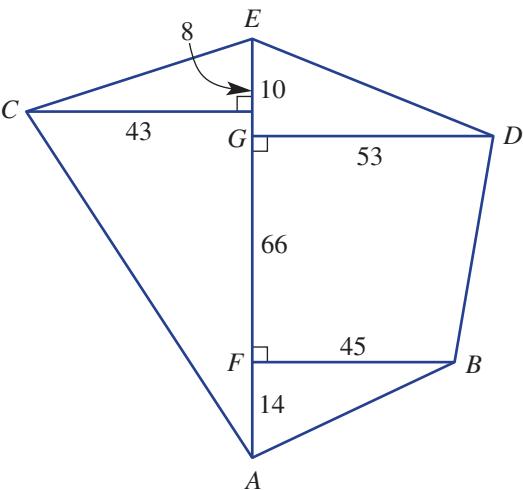
b



c



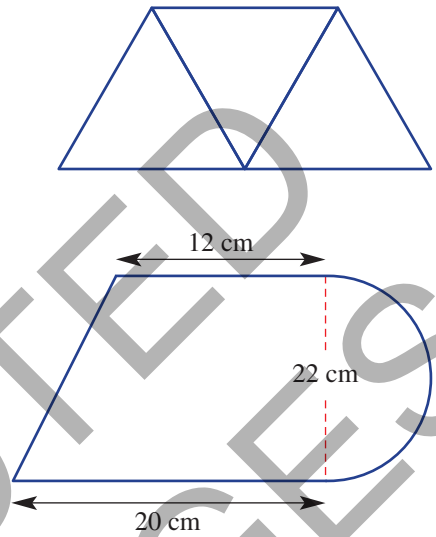
d



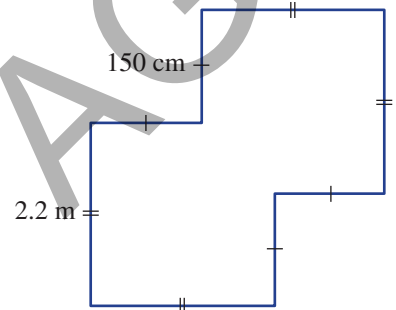
2 A rectangle has a perimeter of 54 cm. Write down a possible pair of values for its length and breadth.

DEVELOPING

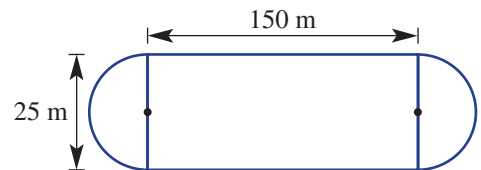
- 3** Tony wants to build this timber truss out of equilateral triangles of side length 1.2 m.
- a** How many metres of timber will he need?
- b** If a 1.2 m piece of structural pine (timber) costs \$5.30 each, how much will it cost Tony to build 4 of these trusses?
- 4** Find the perimeter of the following shape. Round your answer to two significant figures.



- 5** The diagram shows the shape and dimensions of a balcony. Find the perimeter of the balcony in metres. All angles are right angles.

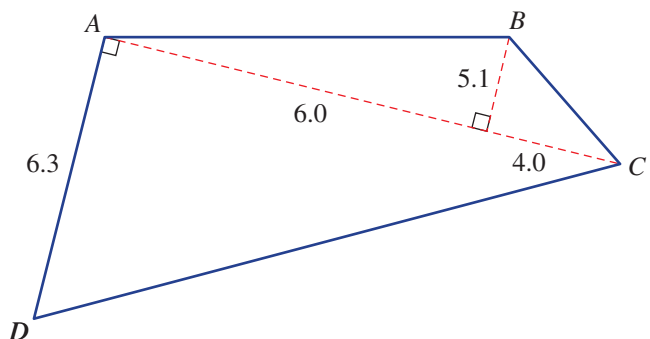


- 6** This is a plan of a running track.
- a** What is the distance a runner covers when they complete one lap around the outside of the track?
- b** If Christina runs 3 laps every morning for 5 days, how many kilometres has she run?

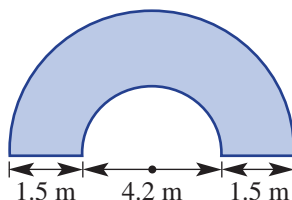


CHALLENGING

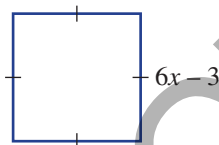
- 7** The diagram shows the shape of Mary's garden bed. All measurements are in metres.
- a** Find the perimeter of Mary's garden bed.
- b** Mary wants to install an edging around the garden bed. If it costs \$18 for a 6-metre roll of edging from the hardware shop, how many rolls of edging will Mary need to buy and how much will this cost her?



- 8 A semicircular archway is built, with the front face shown in the diagram. Find the perimeter of the archway to one decimal place.



- 9 Write an expression for the perimeter of the square shown in terms of x .



Communicate your reasoning, understanding and fluency

- 10 A rectangle has a perimeter of 40 cm. If the length and breadth of the rectangle are in the ratio of 3 : 2 respectively, find the dimensions of the rectangle.
- 11 Design an oval-shaped running track made up of a rectangle with semicircular ends, with a perimeter as close to 350 metres as possible.
- 12 The perimeter of a rectangular swimming pool is 120 metres. If the length is 3 times the breadth, find the dimensions of the pool.

3F Area of triangles and quadrilaterals

Learning intentions

- To calculate area of triangles, quadrilaterals and composite shapes
- To solve practical problems involving area

Past, present and future learning

- Students should already be familiar with the contents of this section
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about area regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the 2027 HSC Reference Sheet, but others are not

The area of a shape is the amount of surface enclosed by the boundaries of the shape. It is the number of squares that fit inside the shape. When calculating area, the answer will be in square units.

$$100 \text{ mm}^2 = 1 \text{ cm}^2 \quad 10\,000 \text{ cm}^2 = 1 \text{ m}^2$$

$$10\,000 \text{ m}^2 = 1 \text{ ha} \quad 1\,000\,000 \text{ m}^2 = 1 \text{ km}^2$$



To calculate the area of the most common shapes, we use a formula. These formulas are listed in the table.

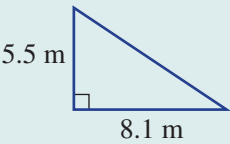
Area formulas

Name	Shape	Area
Triangle		$A = \frac{1}{2}bh$
Square		$A = s^2$
Rectangle		$A = lb$
Parallelogram		$A = bh$
Trapezium		$A = \frac{h}{2}(a + b)$
Rhombus		$A = \frac{1}{2}xy$



Example 17 Finding the area of a triangle

Find the area of the triangle.



Solution

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 8.1 \times 5.5 \\ &= 22.275 \text{ m}^2 \end{aligned}$$

The area of the triangle is 22.275 m².

Explanation

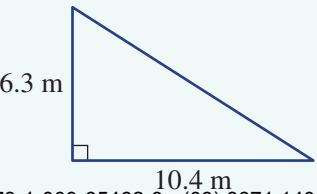
The shape is a triangle, so use the formula $A = \frac{1}{2}bh$.

Substitute the values for b and h ($b = 8.1$ and $h = 5.5$).
Evaluate.

Write the answer using the correct units.

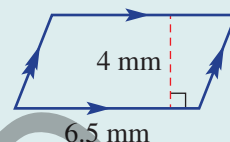
Now you try

Find the area of the triangle.




Example 18 Finding the area of a parallelogram

Find the area of the parallelogram.


Solution

$$A = bh$$

$$= 6.5 \times 4$$

$$= 26 \text{ mm}^2$$

The area of the shape is 26 mm^2 .

Explanation

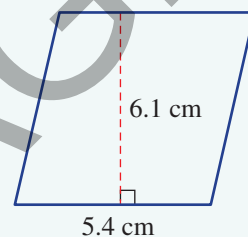
The shape is a parallelogram, so use the formula $A = bh$.

Substitute the values for b and h ($b = 6.5$ and $h = 4$). Evaluate.

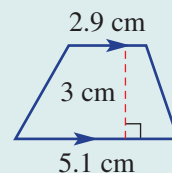
Write the answer using the correct units.

Now you try

Find the area of the parallelogram.


Example 19 Finding the area of a trapezium

Find the area of the trapezium.


Solution

$$A = \frac{h}{2}(a + b)$$

$$= \frac{3}{2}(2.9 + 5.1)$$

$$= 12 \text{ cm}^2$$

The area of the shape is 12 cm^2 .

Explanation

The shape is a trapezium, so use the formula

$$A = \frac{h}{2}(a + b).$$

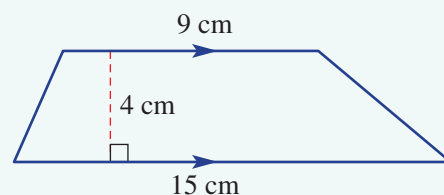
Substitute the values for a , b and h .

Evaluate.

Write the answer using the correct units.

Now you try

Find the area of the trapezium.



Area of composite shapes

A composite shape is made up of two or more plane shapes. The area of a composite shape is calculated by adding or subtracting the areas of simple shapes.

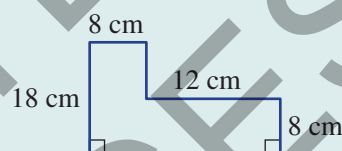
Area of composite shapes

- Composite shapes are made up of more than one simple shape.
- Area of a composite shape can be found by adding or subtracting the areas of simple shapes.



Example 20 Finding the area of a composite shape

Find the area of the composite shape.



Solution

$$A1: 8 \times 18 = 144$$

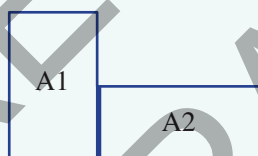
$$A2: 12 \times 8 = 96$$

$$\text{Total area} = 144 + 96$$

$$= 240 \text{ cm}^2$$

Total area is 240 cm^2 .

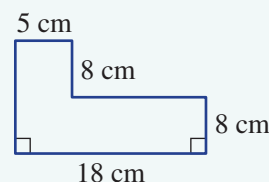
Explanation



To find the total area, break up the larger composite shape into two smaller regular rectangles, A1 and A2. Find the area of these separately and add them together.

Now you try

Find the area of the composite shape.

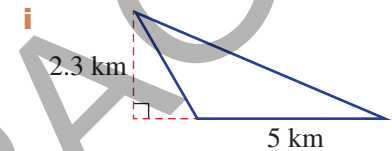
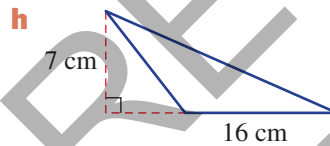
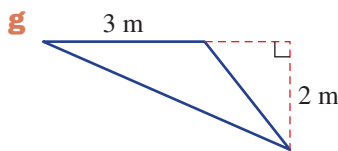
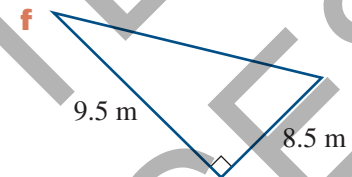
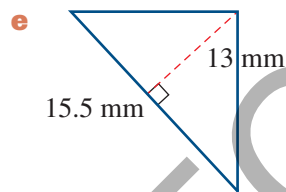
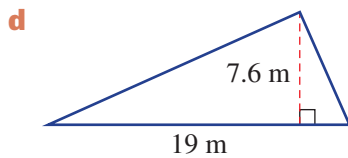
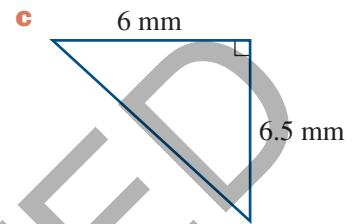
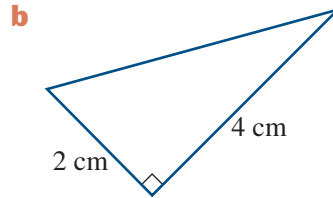
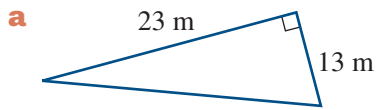


Exercise 3F

BUILDING

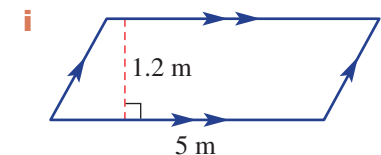
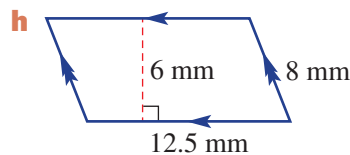
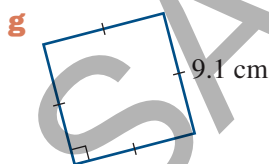
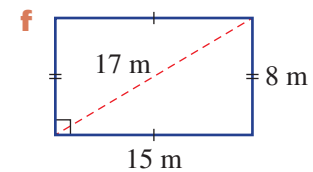
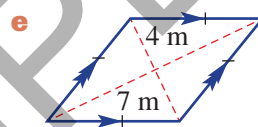
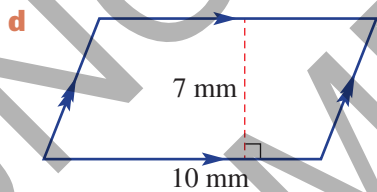
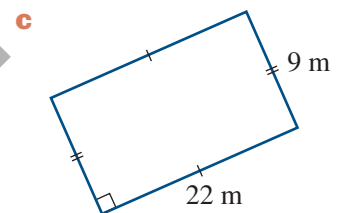
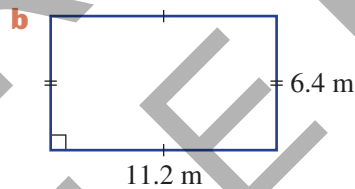
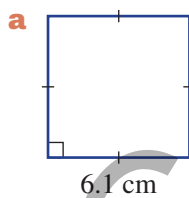
Example 17

- 1 Find the area of each triangle. Answer correct to one decimal place where necessary.



Example 18

- 2 Find the area of each shape. Answer correct to one decimal place where necessary.

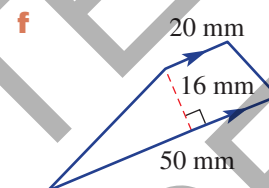
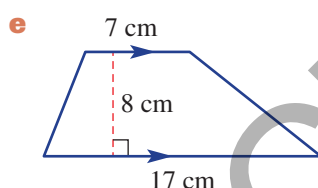
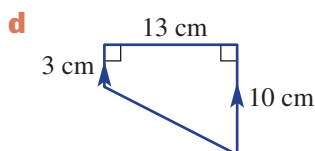
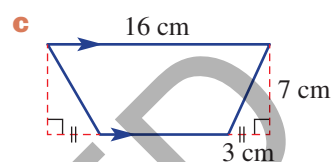
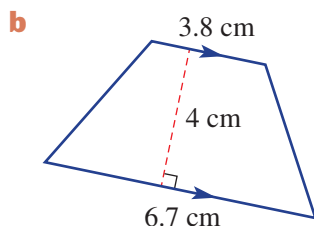
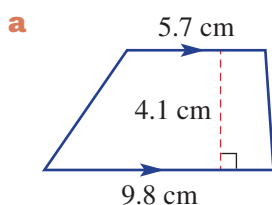


- 3 Find the area of a triangle with a base of 8.25 cm and a height of 10.15 cm. Answer correct to the nearest square centimetre.
- 4 Find the area of a square with a side length of 105.1 m. Answer correct to the nearest square metre.

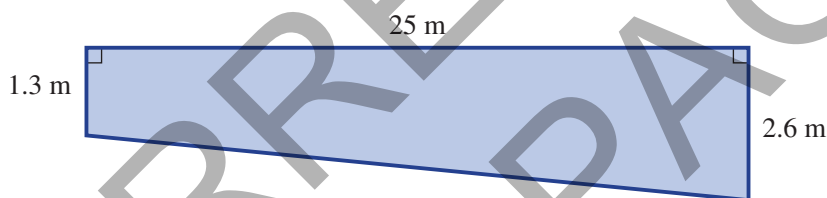
DEVELOPING

Example 19

- 5 Find the area of each shape. Give your answer correct to one decimal place where necessary.



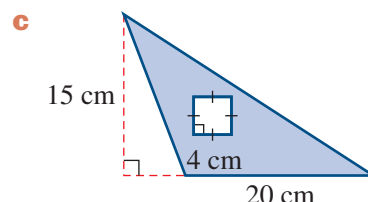
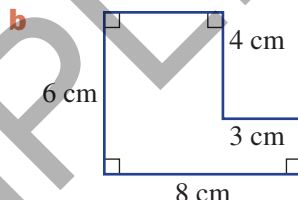
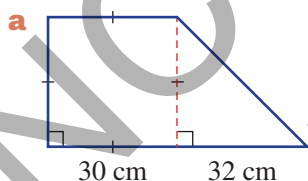
- 6 A 25 m swimming pool increases in depth from 1.3 m at the shallow end to 2.6 m at the deep end. Calculate the area of one side wall of the pool. Answer correct to the nearest square metre.



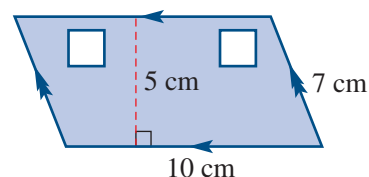
- 7 Philip wants to tile a rectangular area measuring 2.5 m by 3 m in his backyard. The tiles he wishes to use are 50 cm by 50 cm. How many tiles will he need? Give your answer as a whole number.

Example 20

- 8 Find the area of each composite shape.

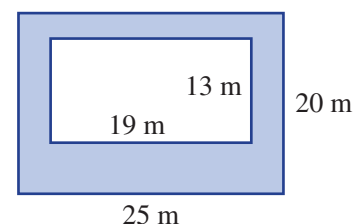


- 9 A metal parallelogram has two identical squares removed from its shape. The two squares have a side length of 2 cm. Find the shaded area. Answer correct to the nearest square centimetre.



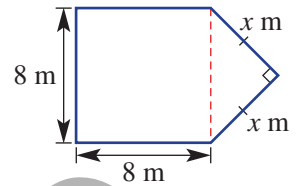
- 10 A lawn is to be laid around a rectangular garden bed.

- a** What is the amount of lawn required?
b Find the cost of the new lawn if the required turf costs \$20 per square metre.



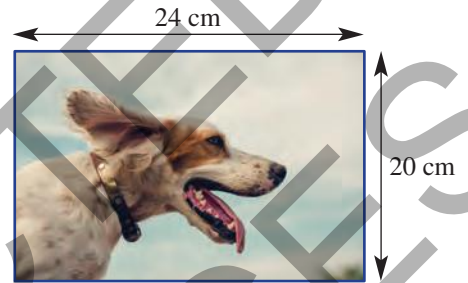
- 11** Decking for a house consists of a square and a triangle. The square has a side length of 8 metres and the triangle is isosceles.

- a** Use Pythagoras' theorem to find the value of x .
b Calculate the area of the shaded region.

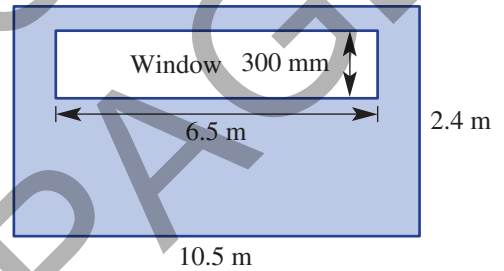


CHALLENGING

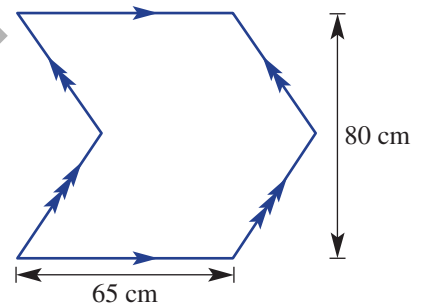
- 12** A photo is to have a 10 cm wide frame around it. What is the area of the frame around the photo?



- 13** A feature wall is to be created using wallpaper. If wallpaper costs \$18 per square metre, find the cost of the feature wall.

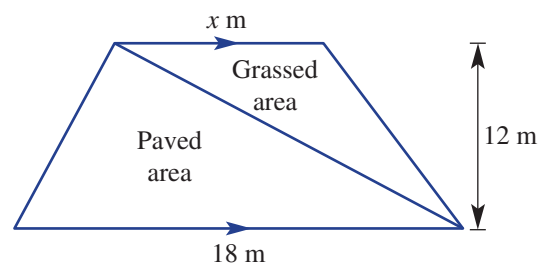


- 14** A logo is created using two congruent parallelograms. Calculate the area of the logo.



Communicate your reasoning, understanding and fluency

- 15** A garden is in the shape of a trapezium. There is a paved area and a grassed area. If the ratio of the paved area to grassed area is 9 : 5, find the length of x .



3G

Area of circles and sectors

- Learning intentions**
- To calculate area of circles, sectors and composite shapes
 - To solve practical problems involving area

- Past, present and future learning**
- Students should already be familiar with the contents of this section
 - This is a Year 11 concept which is not repeated or extended in Year 12
 - Questions about area regularly appear in HSC Examinations
 - Some measurement formulas for length, area, surface area and volume are provided on the 2027 Reference Sheet, but others are not

The area of a circle is calculated using the formula $A = \pi r^2$ where r is the radius of the circle. An annulus is the area between a large and a small circle with the same centre.

The area of an annulus is calculated using the formula $A = \pi(R^2 - r^2)$ where R is the radius of the large circle and r is the radius of the small circle.

A sector is part of a circle between two radii and an arc. The area of a sector is calculated using the formula $A = \frac{\theta}{360}\pi r^2$ where θ is the angle formed at the centre of the circle and r is the radius of the circle. The area of a semicircle ($A = \frac{1}{2}\pi r^2$) and the area of a quadrant ($A = \frac{1}{4}\pi r^2$) are special cases of the area of a sector formula.



Area formulas

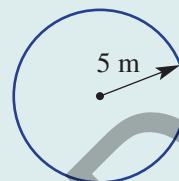
Name	Shape	Area
Circle		$A = \pi r^2$
Annulus		$A = \pi(R^2 - r^2)$
Sector		$A = \frac{\theta}{360}\pi r^2$

Name	Shape	Area
Semicircle		$A = \frac{1}{2}\pi r^2$
Quadrant		$A = \frac{1}{4}\pi r^2$


Example 21 Finding the area of a circle

Find the area of a circle with a radius of 5 metres.

Give your answer correct to one decimal place.


Solution

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 5^2 \\ &\approx 78.5 \text{ m}^2 \end{aligned}$$

The area of the circle is 78.5 m^2 .

Explanation

The shape is a circle, so use the formula $A = \pi r^2$.

Substitute the value for r ($r = 5$).

Evaluate correct to one decimal place.

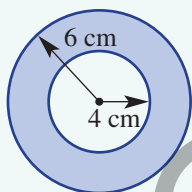
Write the answer using the correct units.

Now you try

Find the area of a circle with radius 7 metres. Give your answer correct to one decimal place.


Example 22 Finding the area of an annulus

Thomas draws two concentric circles whose radii are 4 cm and 6 cm. What is the area of the annulus formed to the nearest square centimetre?

Solution


$$\begin{aligned} A &= \pi(R^2 - r^2) \\ &= \pi \times (6^2 - 4^2) \\ &= 62.83185307 \end{aligned}$$

$$\approx 63 \text{ cm}^2$$

The area of the shape is 63 cm^2 .

Explanation

Draw a diagram to represent the information in the question.

The shape is an annulus, so use the formula

$$A = \pi(R^2 - r^2).$$

Substitute the values for R ($R = 6$) and r ($r = 4$).

Evaluate.

Express the answer correct to the nearest whole number.

Give your answer with the correct units.

Write the answer in words.

Now you try

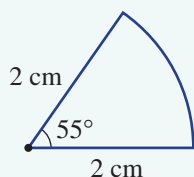
Two concentric circles have radii of 9.5 cm and 15.9 cm respectively. What is the area of the annulus formed to the nearest square centimetre?



Example 23 Finding the area of a sector

Find the area of a sector with an angle at the centre of 55° and radius 2 cm. Write your answer correct to one decimal place.

Solution



$$A = \frac{\theta}{360} \pi r^2$$

$$= \frac{55}{360} \times \pi \times 2^2$$

$$= 1.919862177$$

$$\approx 1.9 \text{ cm}^2$$

The area of the shape is 1.9 cm^2 .

Explanation

Draw a diagram to represent the information in the question.

The shape is a sector, so use the formula

$$A = \frac{\theta}{360} \pi r^2.$$

Substitute the values for θ ($\theta = 55$) and r ($r = 2$).

Evaluate.

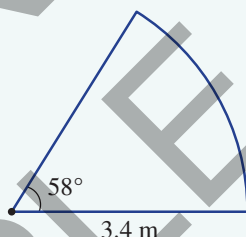
Express the answer correct to one decimal place.

Give your answer with the correct units.

Write the answer in words.

Now you try

Find the area of the sector correct to one decimal place.



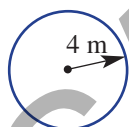
Exercise 3G

BUILDING

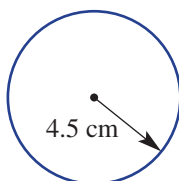
Example 21

1 Find the area of each circle. Answer correct to one decimal place.

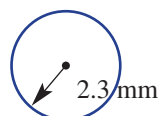
a



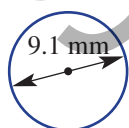
b



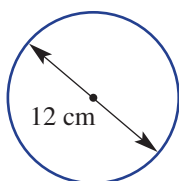
c



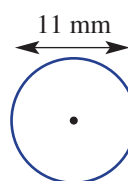
d



e



f



2 What is the area of the following circles? Answer correct to two decimal places.

a Radius of 25 mm

b Radius of 3 cm

c Radius of 10 m

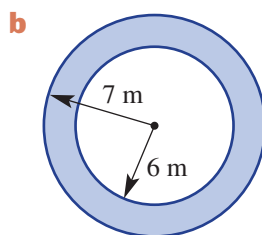
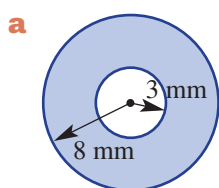
d Diameter of 18 m

e Diameter of 98 cm

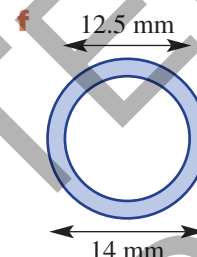
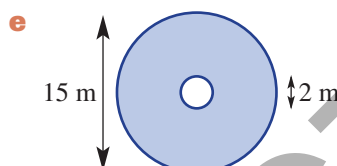
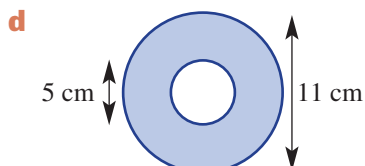
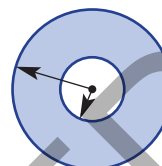
f Diameter of 2.8 mm

Example 22

- 3 Find the area of each annulus. Answer correct to one decimal place.



- c** Outer radius 3.2 m
Inner radius 0.6 m



- 4 What is the area of each annulus? Answer correct to two decimal places.

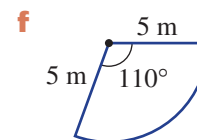
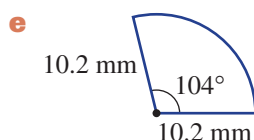
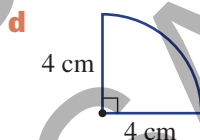
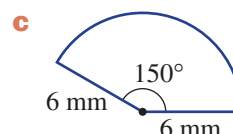
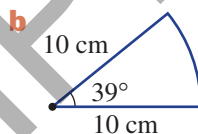
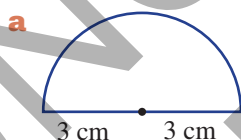
- a** Outer radius of 7 m and inner radius of 4 m
b Outer radius of 5.6 cm and inner radius of 2.1 cm
c Outer diameter of 12 cm and inner diameter of 8 cm
d Outer diameter of 44 mm and inner diameter of 38 mm

DEVELOPING

- 5 What is the area of an annulus with circles of radii 17 cm and 8.5 cm? Answer in square centimetres correct to one decimal place.
- 6 An annulus consists of two circles with the same centre. Find the area of an annulus that has an inner diameter of 6 cm and an outer diameter of 10 cm. Answer correct to the nearest square centimetre.

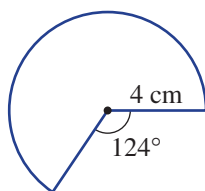
Example 23

- 7 Find the area of each sector. Answer correct to one decimal place.

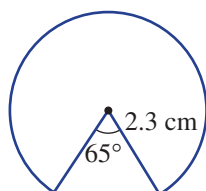


8 Find the area of each sector. Answer correct to two decimal places.

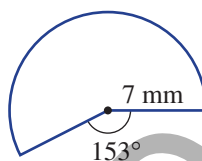
a



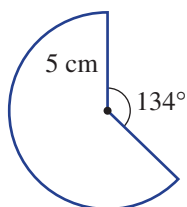
b



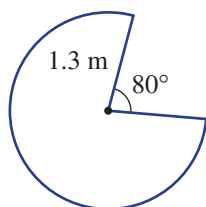
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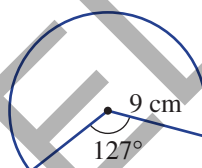
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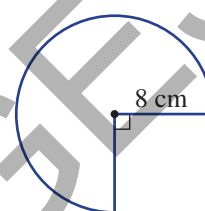
e



f



9 Calculate the area of the sector shown. Express your answer correct to the nearest square centimetre.



CHALLENGING

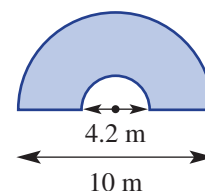
10 The cross-section of a piece of conduit is an annulus. It has an outer diameter of 60 mm and an inner diameter of 50 mm.

a Find the area of the cross-section to the nearest square millimetre.

b What is the increase in cross-sectional area if the outer diameter is increased to 65 mm? Answer correct to the nearest square millimetre.



11 An annulus with an inner diameter of 4.2 metres and an outer diameter of 10 metres is divided into two equal parts. What is the area of each part? Answer correct to three significant figures.



12 The area of a circular pond is 5.3 square metres. What is the radius of the pond? Answer correct to one decimal place.

13 What is the area of a quadrant if it has a radius of 8 mm? Answer correct to two decimal places.

14 An annulus has an area of 392.70 square metres. Calculate the inner radius of the annulus if the outer radius is 15 metres. Answer correct to the nearest whole number.

15 Find the angle at the centre of these sectors to the nearest degree.

a Area of 104.72 cm^2 and a radius of 10 cm

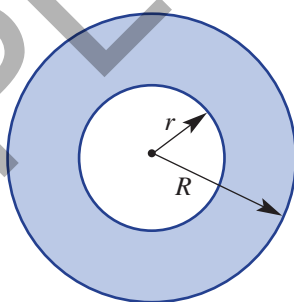
b Area of 139.63 m^2 and a radius of 40 m

Communicate your reasoning, understanding and fluency

- 16** An area to be landscaped is a sector with a radius of 19 metres and an angle at the centre of 145° . The area is to be covered with turf at \$12.50 per square metre and then top-dressed with soil at \$2.30 per square metre. Find the total cost of the landscaping to the nearest dollar.
- 17** A traffic roundabout has a circular garden in the centre and two lanes for traffic encircling the garden. The diameter of the garden is 16 metres and each lane is 3 metres wide. Each lane is to be resurfaced. Calculate the area to be resurfaced. Answer in square metres to the nearest whole number.



- 18** If $R = 100$ cm and the area of the annulus equals the area of the smaller circle, find r to four decimal places.



3H Area of composite shapes

Learning intentions

- To calculate area of composite shapes
- To solve practical problems involving area
- To calculate perimeter of irregularly shaped blocks of land by dissection into triangles, rectangles and trapeziums

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about area regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the 2027 HSC Reference Sheet, but others are not

A composite shape is made up of two or more plane shapes. The area of a composite shape is calculated by adding or subtracting the areas of each plane shape. In some questions it may be necessary to determine the length of a missing side.

Area of composite shapes

- 1 Divide the composite shape into two or more plane shapes.
- 2 Calculate the area of each plane shape.
- 3 Add or subtract the areas of the plane shapes.
- 4 Write the answer to the specified level of accuracy with the correct units.

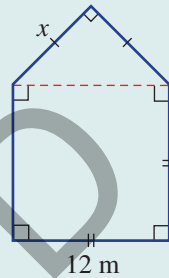


Example 24 Finding the area of a composite shape

A concrete area in the shopping centre consists of a square and a triangle.

The square has a side length of 12 metres and the triangle is isosceles.

- a** Use Pythagoras' theorem to find the value of x .
- b** Calculate the area of the composite shape.



Solution

a $a^2 + b^2 = c^2$

$$x^2 + x^2 = 12^2$$

$$2x^2 = 12^2$$

$$x^2 = 72$$

$$x = \sqrt{72}$$

b $A = s^2$ (Square)

$$= 12^2$$

$$= 144 \text{ m}^2$$

$$A = \frac{1}{2}bh \text{ (Triangle)}$$

$$= \frac{1}{2} \times \sqrt{72} \times \sqrt{72}$$

$$= 36 \text{ m}^2$$

Shaded area

= Area of square + Area of triangle

$$= 144 + 36$$

$$= 180 \text{ m}^2$$

Area of shaded region is 180 m^2 .

Explanation

Write Pythagoras' theorem.

Substitute the length of the sides.

Make x^2 the subject.

Take the square root to find x .

Divide the shaded region into a square and a triangle.

Use the formula $A = s^2$ to find the area of the square.

Substitute the value for s ($s = 12$) into the formula.

Evaluate. Give your answer using the correct units.

Use the formula $A = \frac{1}{2}bh$ to find the area of the triangle.

Substitute the values for b and h ($b = \sqrt{72}$, $h = \sqrt{72}$).

Evaluate. Give your answer using the correct units.

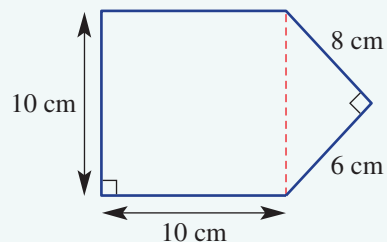
Add the areas of the two plane shapes to determine the area of the shaded region.

Express using the correct units.

Write the answer in words.

Now you try

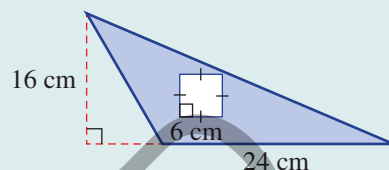
Find the area of the composite shape.





Example 25 Finding the area of a composite shape

A square with a side length of 6 cm is cut from a triangle with a base of 24 cm and a height of 16 cm. What is the shaded area?



Solution

$$A = \frac{1}{2}bh \text{ (Triangle)}$$

$$= \frac{1}{2} \times 24 \times 16$$

$$= 192 \text{ cm}^2$$

$$A = s^2 \text{ (Square)}$$

$$= 6^2$$

$$= 36 \text{ cm}^2$$

Shaded area

= Area of triangle – Area of square

$$= 192 - 36$$

$$= 156 \text{ cm}^2$$

Shaded area is 156 cm^2 .

Explanation

The shape is a triangle, so use the formula $A = \frac{1}{2}bh$.

Substitute the values for b and h ($b = 24$ and $h = 16$).

Evaluate and express using correct units.

The shape is a square, so use the formula $A = s^2$.

Substitute the value for s ($s = 6$).

Evaluate and express using the correct units.

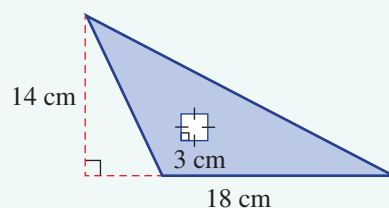
Subtract the area of the square from the area of the triangle to determine the area of the shaded region.

Express using the correct units.

Write the answer in words.

Now you try

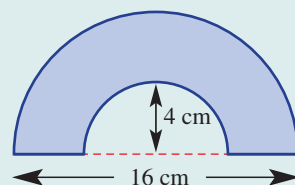
Find the area of the composite shape.



Example 26 Finding the area of a composite shape

The diagram shows a semicircle cut out of a semicircle.

- What is the area of the smaller semicircle?
- What is the area of the larger semicircle?
- What is the shaded area correct to one decimal place?



Solution

$$\text{a } A = \frac{1}{2}\pi r^2 \text{ (Semicircle)}$$

$$= \frac{1}{2} \times \pi \times 4^2$$

$$= 8\pi \text{ cm}^2$$

Explanation

The shape is a semicircle, so use the formula

$$A = \frac{1}{2}\pi r^2.$$

Substitute the value for r ($r = 4$).

Evaluate and express using correct units. Unless otherwise instructed leave as an exact answer (8π).

$$\begin{aligned} \text{b } A &= \frac{1}{2}\pi r^2 \text{ (Semicircle)} \\ &= \frac{1}{2} \times \pi \times 8^2 \\ &= 32\pi \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{c Shaded area} &= 32\pi - 8\pi \\ &= 75.39822369... \\ &\approx 75.4 \text{ cm}^2 \\ \text{Shaded area is about } 75.4 \text{ cm}^2. \end{aligned}$$

The shape is a semicircle, so use the formula $A = \frac{1}{2}\pi r^2$.
Substitute the value for r ($r = 8$).

Evaluate and express using the correct units.

Subtract the areas of the two plane shapes to determine the area of the shaded region.

Write the answer correct to one decimal place.

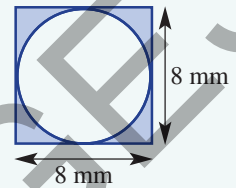
Express using the correct units.

Write the answer in words.

Now you try

The diagram shows a circle cut out of a square.

- What is the area of the circle?
- What is the area of the square?
- What is the shaded area correct to one decimal place?



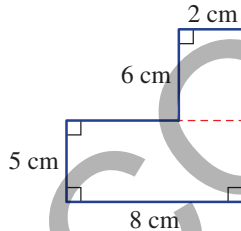
Exercise 3H

BUILDING

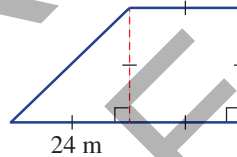
Example 24

- 1 Find the area of each composite shape. Answer correct to the nearest whole number.

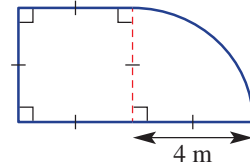
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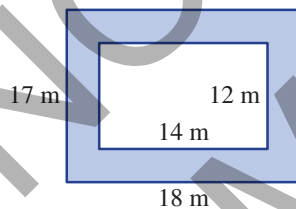
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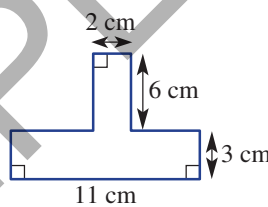
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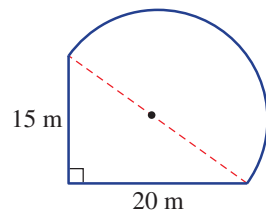
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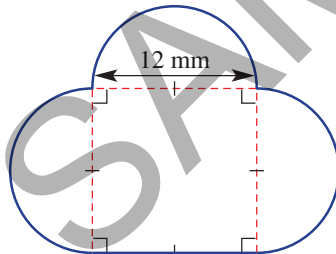
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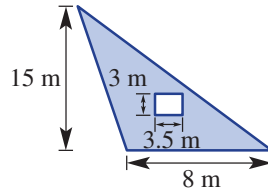
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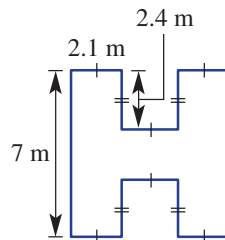
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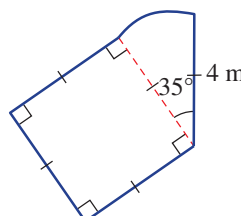
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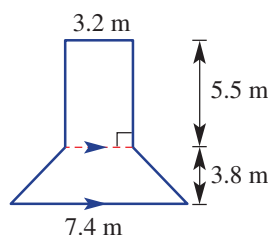
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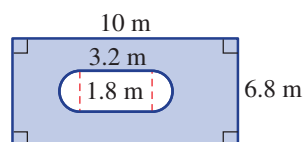
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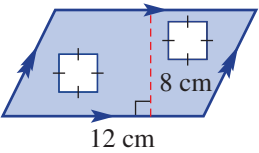


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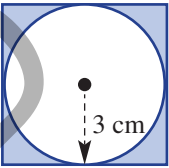
Example 25

- 2 A piece of cardboard consists of a parallelogram with two identical squares removed from its shape. The two squares have a side length of 2 cm. Find the shaded area. Answer correct to the nearest square centimetre.



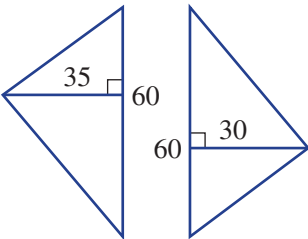
Example 26

- 3 A circle is removed from a square as shown in the diagram. The square has a side length of 6 cm and the radius of the circle is 3 cm. What is the shaded area? Answer correct to one decimal place.

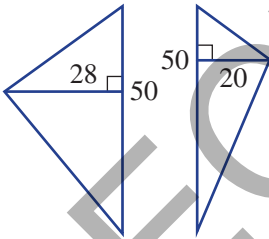


- 4 Find the area of the following irregular shaped blocks of land. Units are metres.

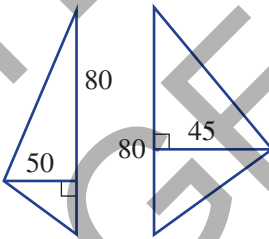
a



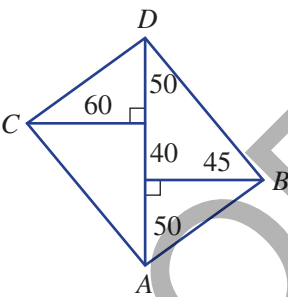
b



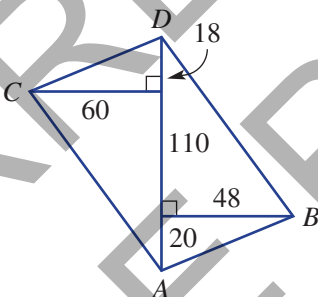
c



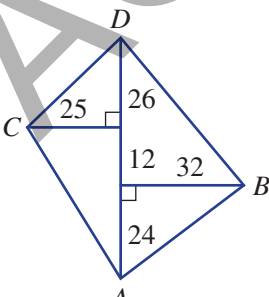
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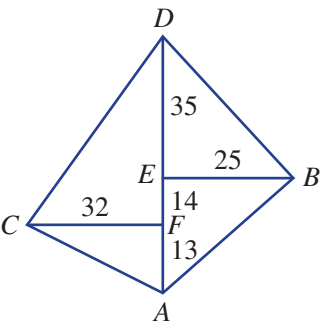


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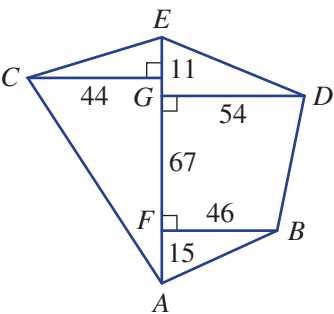


DEVELOPING

- 5 The diagram on the right shows a block of land that has been surveyed. All measurements are in metres.
- a Find the area of the quadrilateral $ABCD$.
- b What is the length of AB ? Answer correct to the nearest metre.

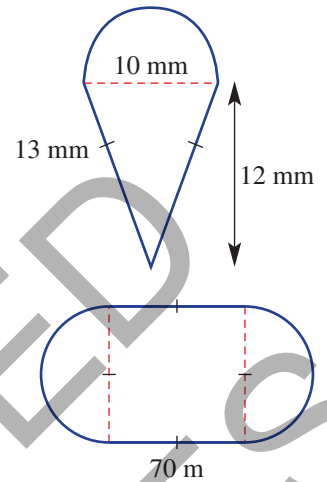


- 6 The diagram on the right shows a block of land that has been surveyed. All measurements are in metres.
- a Find the area of the triangle ABF .
- b Find the area of the triangle ACE .
- c Find the area of the triangle DGE .
- d Find the area of the trapezium $BFGD$.
- e What is the total area of the block of land?



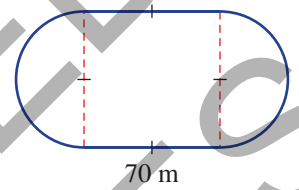
- 7** A composite plane shape consists of a semicircle and an isosceles triangle.

- a** What is the radius of the semicircle?
b What is the area of the semicircle? Answer correct to one decimal place.
c Calculate the area of the shape. Answer correct to one decimal place.

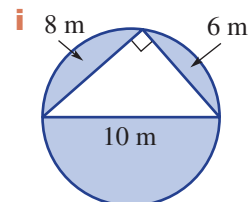
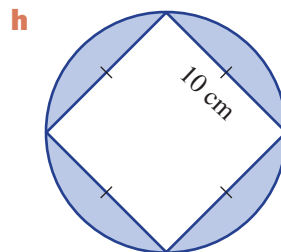
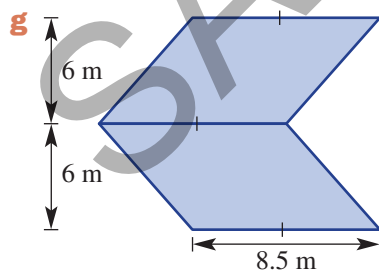
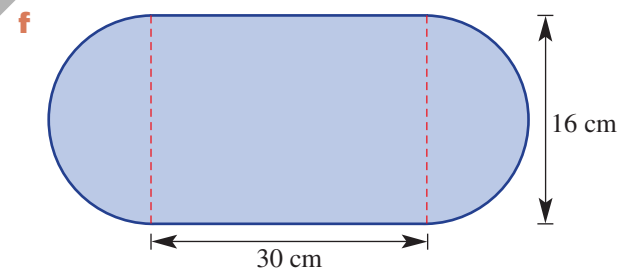
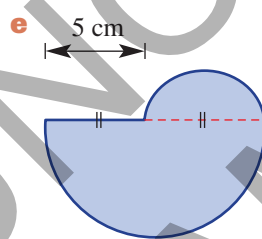
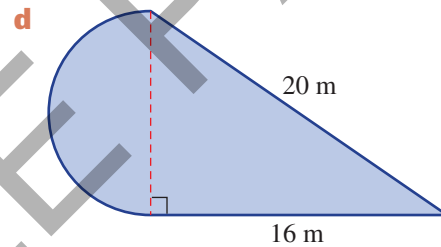
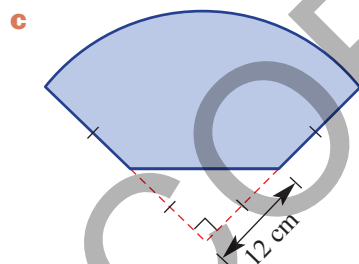
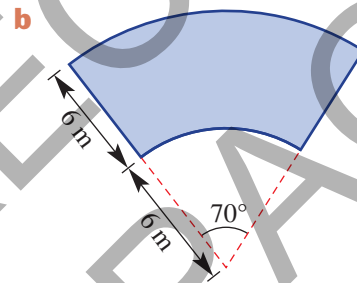
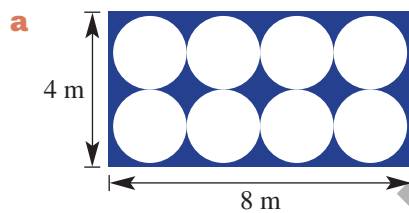


- 8** A sports ground consists of a square and two semicircles.

- a** What is the radius of the semicircular ends?
b Calculate the area of the sports ground in square metres correct to one decimal place.

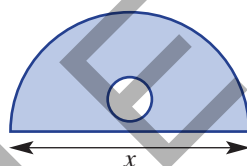
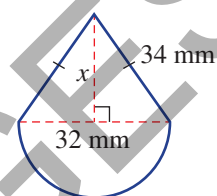


- 9** Find the area of each composite shape correct to one decimal place.

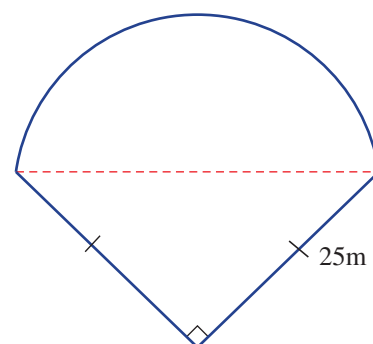


CHALLENGING

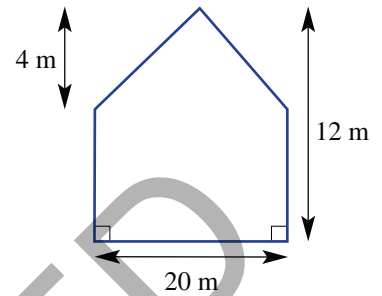
- 10** Lauren cuts circles with a radius of 4 cm from a rectangular piece of cardboard 8 cm by 16 cm.
- What is the area of the rectangular piece of cardboard?
 - How many circles can be cut from the piece of cardboard?
 - What is the area of the remaining cardboard after the circles have been cut? Answer correct to two decimal places.
- 11** Oliver wants to pave a rectangular area measuring 3.0 m by 2.5 m in his backyard. The pavers he wishes to use are 50 cm by 50 cm. How many pavers will he need? Answer correct to the nearest whole number.
- 12** Saanvi is using the composite shape on the right as a design element.
- What is the area of the semicircle? Answer correct to one decimal place.
 - Find the height of the triangle.
 - What is the area of the composite shape? Answer correct to one decimal place.
- 13** Samantha bought a block of land that is a composite shape. It consists of a square with an equilateral triangle on top. The side length of the square is 40 metres. What is the area of the block of land? Answer correct to the nearest square metre.
- 14** Aaron is using the composite shape shown as the basis for a garden. The composite shape is a semicircle with a circle of radius y removed from the centre.



- Find an expression for the area of the semicircle.
 - What is the shaded area of the composite shape?
 - Find the area of garden if $x = 6$ metres and $y = 0.5$ metres. Answer correct to the nearest square metre.
- 15** A yard is in the shape of an isosceles right-angled triangle and a semicircle.
- Find the area of the yard.
 - If it costs \$45 per square metre to supply and install paving, what is the cost to pave the yard?



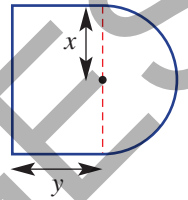
- 16** Jeff wants to paint the side of his house. One litre of paint covers 16 m^2 . How many litres of paint will Jeff need to buy to paint the side of his house?



Communicate your reasoning, understanding and fluency

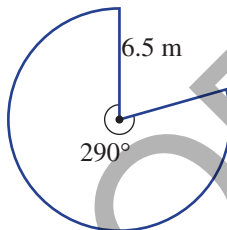
- 17** Aaron has decided to landscape a new lawn. The lawn is in the shape of a rectangle with a semicircle on one end.

- a** Explain why the expression for the area of the lawn is $\frac{1}{2}\pi x^2 + 2xy$.
b Find the area of lawn if $x = 4$ metres and $y = 5$ metres. Answer correct to the nearest square metre.

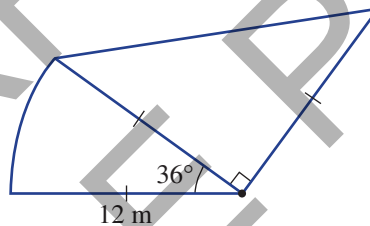


- 18** For chickens to be free-range, each chicken needs a minimum of 1 square metre of space. Find the area of each shape to calculate which design can accommodate the most number of chickens.

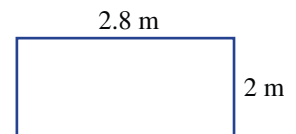
Design A



Design B



- 19** Finlay wants to tile his rectangular bathroom floor. If the tiles are 600 mm long $\times$ 300 mm wide, how many tiles will Finlay need to buy to tile the bathroom floor?



3I Trapezoidal rule

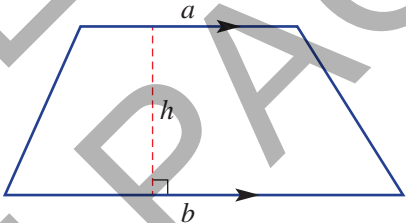
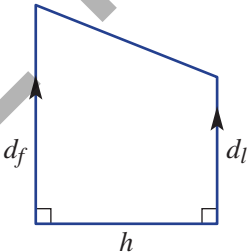
Learning intentions

- To understand that the trapezoidal rule provides an estimate for the area of an irregular shape with a curved side
- To apply the trapezoidal rule to solve practical problems involving area

Past, present and future learning

- The trapezoidal rule will be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions involving the trapezoidal rule regularly appear in HSC Examinations
- The formula for the trapezoidal rule is provided on the 2027 HSC Reference Sheet

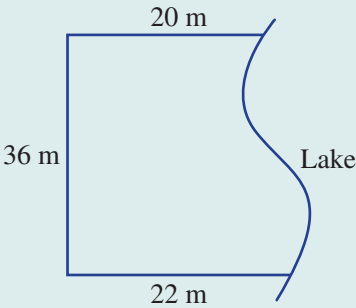
The trapezoidal rule is the area formula for a trapezium with different names for the variables. The variable h is the height or the distance between the parallel sides, while the variables d_f and d_l are the distances of the first and last parallel sides.

Area of trapezium	$A = \frac{h}{2}(a + b)$	
Trapezoidal rule	$A \approx \frac{h}{2}(d_f + d_l)$	

The trapezoidal rule is used to estimate the area of a shape with an irregular boundary, such as a lake.

Example 27 Using the trapezoidal rule to estimate the area

A lake forms a boundary for a block of land as shown. Apply the trapezoidal rule to approximate the area of the block. Answer to the nearest square metre.



Solution

$$A \approx \frac{h}{2}(d_f + d_l)$$

$$h = 36, d_f = 20, \text{ and } d_l = 22$$

$$\begin{aligned} A &= \frac{36}{2}(20 + 22) \\ &= 756 \text{ m}^2 \end{aligned}$$

Explanation

Write the trapezoidal rule $A \approx \frac{h}{2}(d_f + d_l)$.

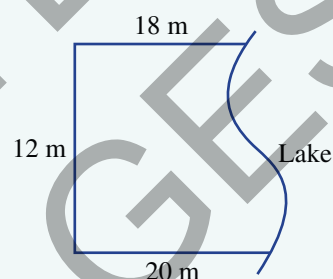
Write the values for h , d_f and d_l .

Substitute the values for h , d_f and d_l and evaluate.

Write the answer using the correct units.

Now you try

Apply the trapezoidal rule to approximate the area of the block. Answer to the nearest square metre.

**Trapezoidal rule**

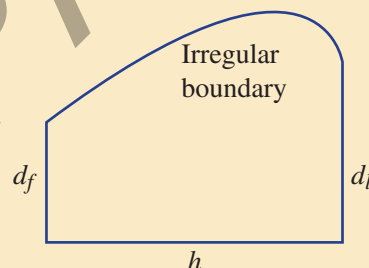
$$A \approx \frac{h}{2}(d_f + d_l)$$

A – Area of shape

h – Height or breadth between the parallel sides

d_f – Distance along first parallel side

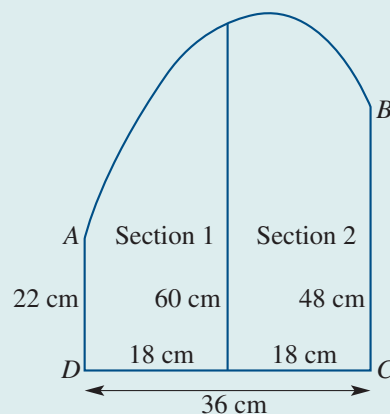
d_l – Distance along last parallel side

**Trapezoidal rule with two applications**

If the trapezoidal rule is applied more than once, it provides a better estimate for the area of the shape.

**Example 28 Using the trapezoidal rule to estimate the area**

Use the trapezoidal rule twice to estimate the area of $ABCD$ to the nearest square centimetre.



Solution

Section 1: $h = 18$, $d_f = 22$, and $d_l = 60$

Section 2: $h = 18$, $d_f = 60$, and $d_l = 48$

$$\begin{aligned} A &\approx \frac{h}{2}(d_f + d_l) + \frac{h}{2}(d_f + d_l) \\ &= \frac{18}{2}(22 + 60) + \frac{18}{2}(60 + 48) \\ &= 1710 \text{ cm}^2 \end{aligned}$$

Explanation

Divide the area $ABCD$ into two sections at 60. Label each section.

Write the trapezoidal rule for each

section: $A \approx \frac{h}{2}(d_f + d_l)$.

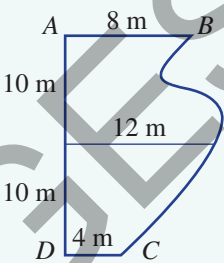
Substitute the values for h , d_f and d_l .

Evaluate.

Write the answer using the correct units.

Now you try

Use the trapezoidal rule twice to estimate the area of $ABCD$ to the nearest square metre.



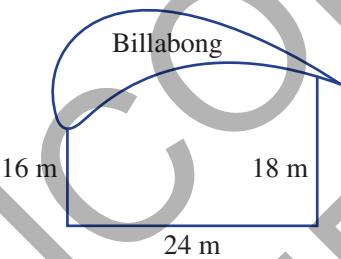
Exercise 3I

BUILDING

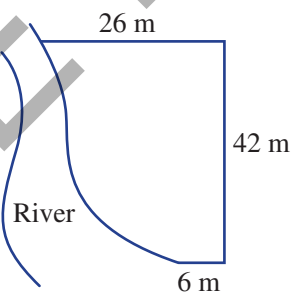
Example 27

1 Estimate the area by applying trapezoidal rule once for the following irregular fields. Answer correct to the nearest square metre.

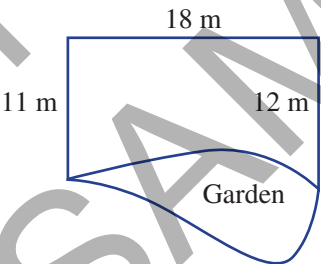
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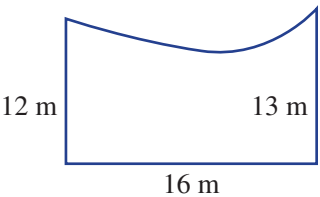
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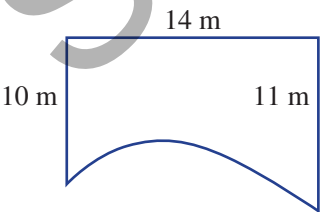
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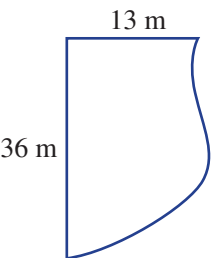
d



e

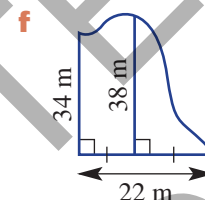
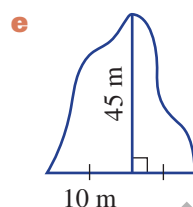
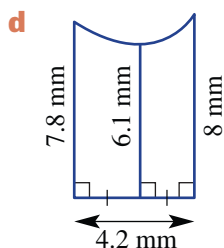
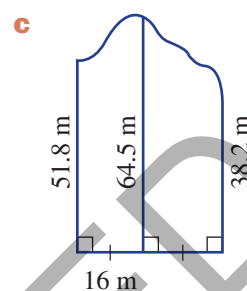
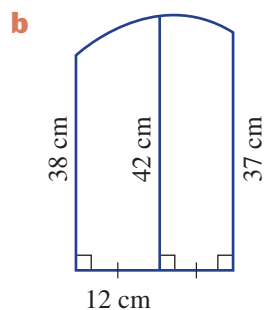
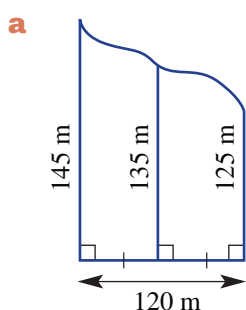


f



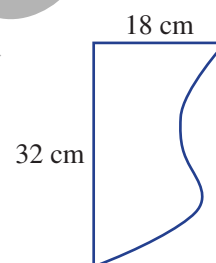
Example 28

- 2 Use two applications of the trapezoidal rule to estimate the area of each shape.



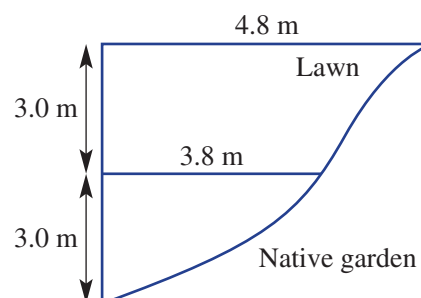
DEVELOPING

- 3 The cross-section of a piece of wood is shown. Use the trapezoidal rule to estimate the area of the piece of wood. Answer to the nearest square centimetre.



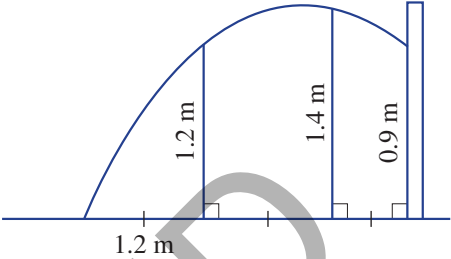
- 4 A portion of land 6 m by 4.8 m is divided into lawn and a native garden.

- a** Find the area of the lawn by applying the trapezoidal rule twice. Answer correct to one decimal place.
b What is the area of the native garden? Answer correct to one decimal place.

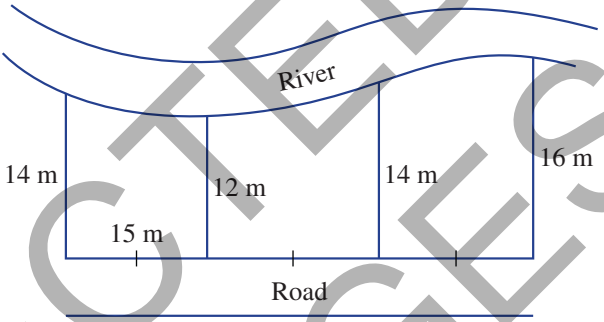


- 5 Connor took three measurements at 6 m intervals across an irregular-shaped field. The measurements were 10 m, 12 m and 8 m. Use two applications of the trapezoidal rule to calculate the area of the field. Answer correct to the nearest square metre.

- 6 The diagram shows a side view of a mound of dirt in a holding bin. Use three applications of the trapezoidal rule to estimate the area of the side view.

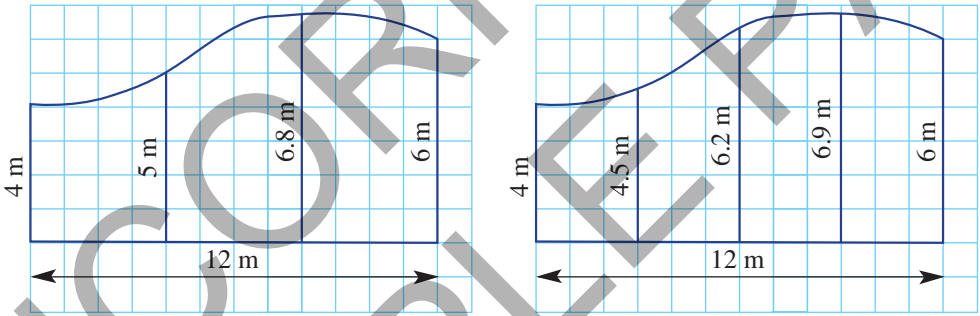


- 7 A park is bounded by a road and a river. Estimate the area of the park to the nearest square metre.



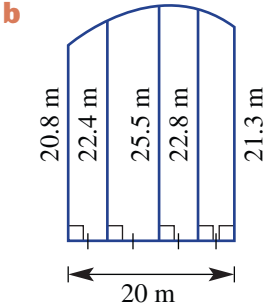
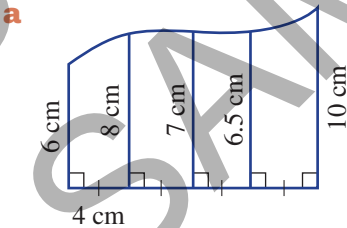
CHALLENGING

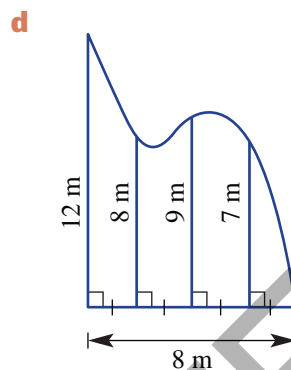
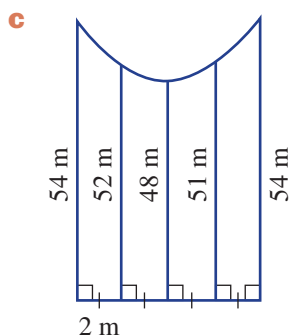
- 8 The more frequently the trapezoidal rule is applied, the more accurate the estimate for the area of the shape becomes.



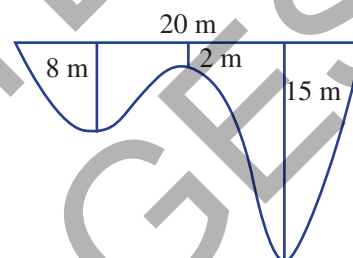
For the same shape in the first diagram, estimate the area using three and four applications of the trapezoidal rule.

- 9 Use four applications of the trapezoidal rule to estimate the area of each shape.

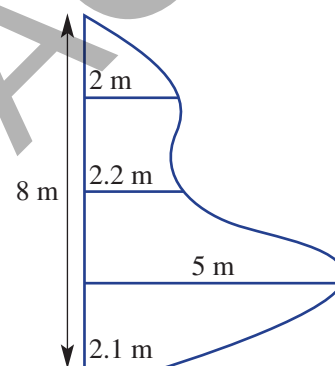




- 10** The diagram shows a vertical cross-section of the ocean. It is divided into four vertical strips of equal breadth by the dashed lines. Estimate the area of the cross-section by applying the trapezoidal rule four times. Answer correct to the nearest square metre.



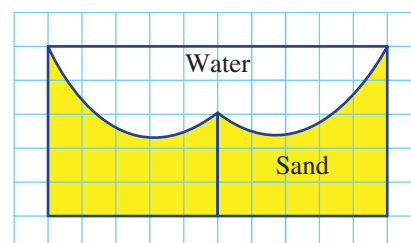
- 11** A uniform cross-section of a sculpture is shown. It is divided into four horizontal strips of equal breadth by the dashed lines. Apply the trapezoidal rule four times to estimate the area of the cross-section. Answer correct to the nearest square metre.



-  **12** A curved road has five offsets to a straight boundary line with distances of 50 m, 60 m, 30 m, 40 m and 30 m. The offsets are 20 m apart. Use four applications of the trapezoidal rule to calculate the area of the field. Answer correct to the nearest square metre.

Communicate your reasoning, understanding and fluency

- 13** The plan shown is a section of beach on a $5\text{ m} \times 10\text{ m}$ grid.
- Use two applications of the trapezoidal rule to estimate the area of the sand.
 - Is this answer in part **a** an overestimate or underestimate of the actual area of the sand? Give a reason for your answer.



3J Surface area of right prisms

Learning intentions

- To visualise the two-dimensional surfaces which form a three-dimensional solid
- To calculate the surface area of prisms
- To solve practical problems involving the surface area of prisms

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about surface area regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the 2027 HSC Reference Sheet, but others are not

To find the surface area of a prism, it is often useful to draw the net of the solid to ensure that all the sides have been added. A net is a plane figure that shows all the surfaces of a solid. When a net is folded it forms the solid. The net of a prism is identified by each polygon that represents a surface. The surface area (A) of a solid is the sum of the areas of each surface or polygon of the solid. It is common for the area of some of the surfaces to be equal. This makes the calculations easier.

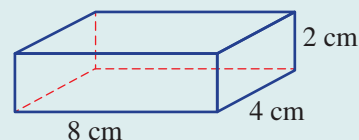
Finding the surface area of right prisms

- 1 Visualise the surfaces of the solid. If necessary draw the net of the solid.
- 2 Write a formula for the surface area, using the net as a guide.
- 3 Substitute the values into the formula for the surface area.
- 4 Use your calculator to find the surface area.
- 5 Write the answer to the specified level of accuracy with the correct units.

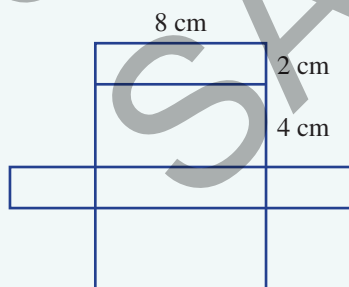


Example 29 Finding the surface area of a rectangular prism

Find the surface area of this rectangular prism.



Solution



Explanation

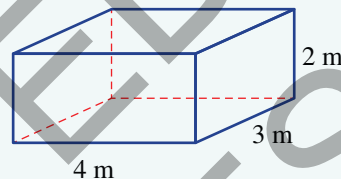
Draw a net of the rectangular prism.

$$\begin{aligned}
 A &= (2 \times l \times b) + (2 \times b \times h) + (2 \times l \times h) \\
 A &= (2 \times 8 \times 4) + (2 \times 4 \times 2) + (2 \times 8 \times 2) \\
 &= 112 \\
 &= 112 \text{ cm}^2 \\
 \text{Surface area is } 112 \text{ cm}^2.
 \end{aligned}$$

Write down a formula for surface area.
 Substitute the values into the formula.
 Evaluate.
 Give answer to the correct units.
 Write the answer in words.

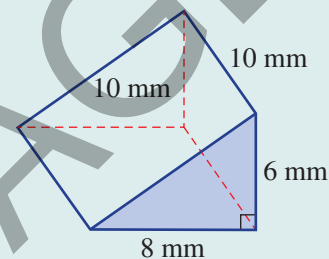
Now you try

Find the surface area of this rectangular prism.



Example 30 Finding the surface area of a triangular prism

Find the surface area of this triangular prism.



Solution

Triangular faces

$$\begin{aligned}
 A &= \frac{1}{2}bh \\
 &= \frac{1}{2} \times 8 \times 6 \\
 &= 24 \text{ mm}^2
 \end{aligned}$$

$$\begin{aligned}
 A &= (2 \times 24) + (10 \times 10) + (10 \times 6) \\
 &\quad + (10 \times 8) \\
 &= 288 \text{ mm}^2
 \end{aligned}$$

Surface area is 288 mm^2 .

Explanation

A triangular prism has two triangular faces and three rectangular faces.

The triangular face has a base of 8 mm and a height of 6 mm. Write the formula for the area of a triangle.

Substitute the values into the area of triangle formula.

Evaluate.

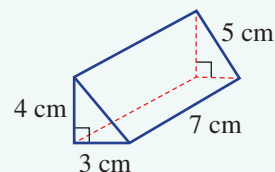
Write an expression for the surface area of the prism using the area of all the faces.

Evaluate and write the answer to the correct units.

Write the answer in words.

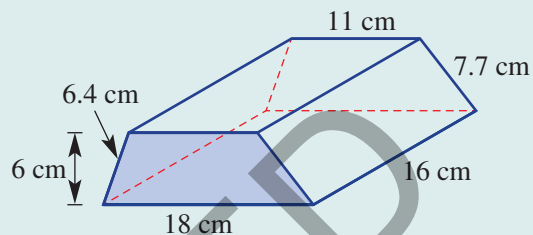
Now you try

Find the surface area of this triangular prism.




Example 31 Finding the surface area of a trapezoidal prism

Find the surface area of this trapezoidal prism.


Solution

Trapezoidal faces

$$A = \frac{1}{2}(a + b)h$$

$$= \frac{1}{2} \times (18 + 11) \times 6$$

$$= 87 \text{ cm}^2$$

$$A = (2 \times 87) + (11 \times 16) + (7.7 \times 16)$$

$$+ (18 \times 16) + (6.4 \times 16)$$

$$= 863.6 \text{ cm}^2$$

Surface area is 863.6 cm^2 .

Explanation

A trapezoidal prism has two trapezoidal faces and four rectangular faces.

The trapezoidal face has a perpendicular height of 6 cm and two parallel sides of 11 cm and 18 cm.

Substitute the values into the formula.

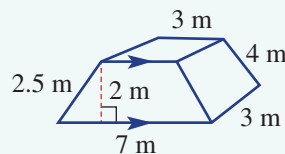
Evaluate.

Write down an expression for the surface area, using the area of all the faces.

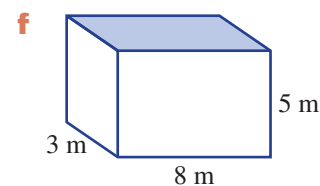
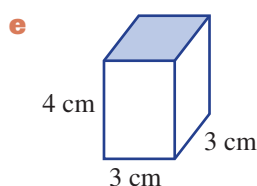
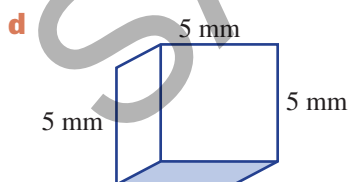
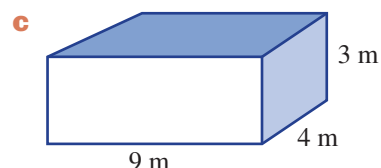
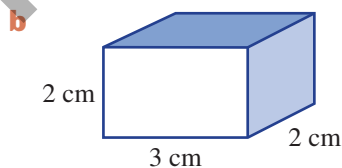
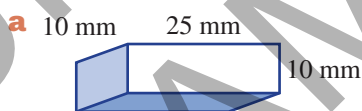
Evaluate and write the answer to the correct units. Write the answer in words.

Now you try

Find the surface area of this trapezoidal prism.


Exercise 3J
BUILDING
Example 29

1 Find the surface area of the following rectangular prisms.



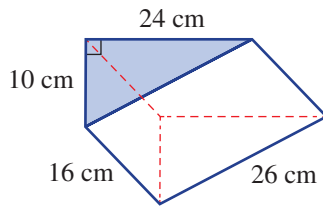
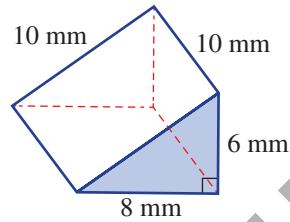
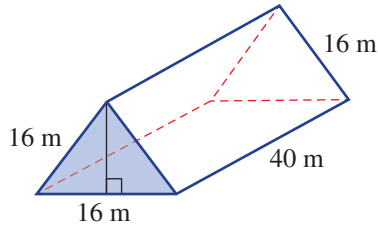
2 Find the surface area of rectangular prisms with the following dimensions.

a Length 7 m, breadth 4 m, height 2 m

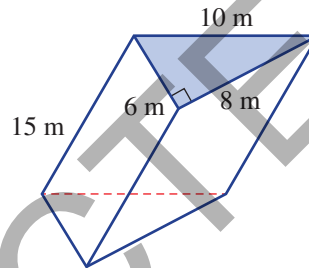
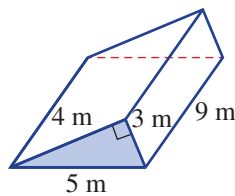
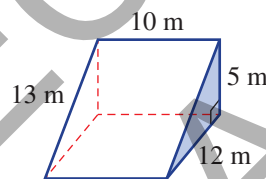
b Length 10 cm, breadth 9 cm, height 6 cm

Example 30

3 Find the surface area of the following triangular prisms.

a**b****c**

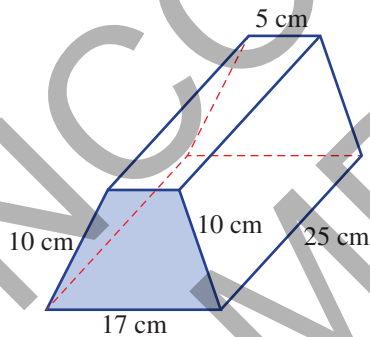
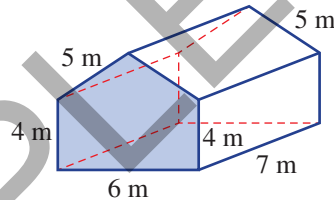
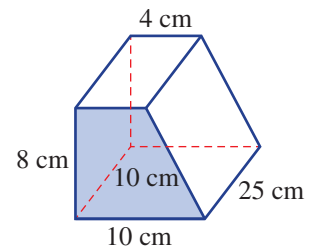
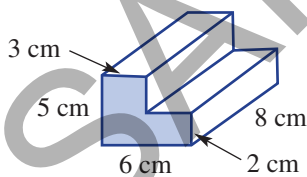
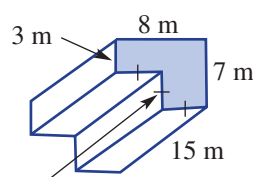
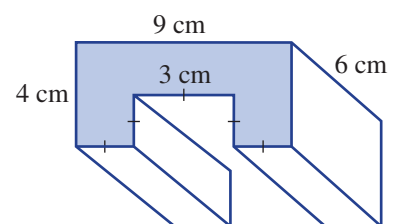
Area of shaded face (also called the base of the prism) is 110.8 m^2

d**e****f**

DEVELOPING

Example 31

4 Find the surface area of the following right prisms.

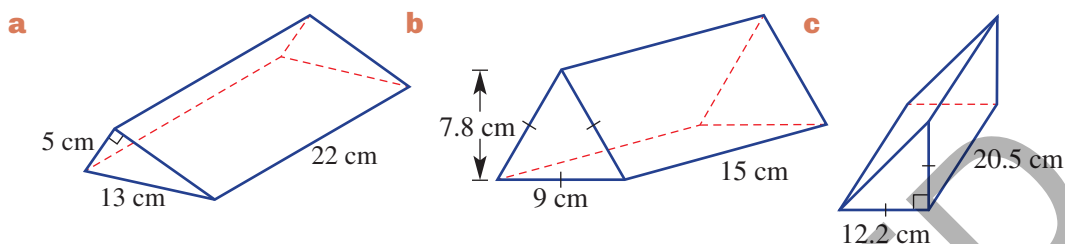
a Area of base is 88 cm^2 **b** Area of base is 36 m^2 **c** Area of base is 56 cm^2 **d****e****f**

5 A bathroom is 3.5 m long and 2.5 m wide and its walls are to be tiled to a height of 1.5 m.

a Find the area of the walls to be tiled, deducting 3 m^2 for the door and window.

b If the tiles cost \$150 per m^2 and the tiler charges \$120 per m^2 to lay them, find the total cost for tiling the bathroom walls.

- 6** Find the surface area of the following right prisms.



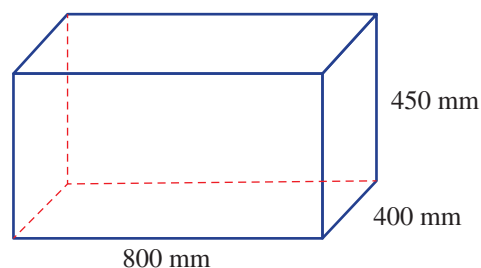
CHALLENGING

- 7** Find the surface area of a right triangular prism if the triangular base has a perpendicular height of 12 m and a base of 18 m. The height of 12 m and base of 18 m form the two short sides of the triangular base. The triangular prism has a length of 20 m.
- 8** The surface area of a cube is 162.24 cm^2 .
- a** How many faces make up the cube?
- b** Find the area of one face of the cube.
- 9** What is the height of a rectangular prism that has a surface area of 352 cm^2 , given that the length of the rectangular prism is 12 cm and its breadth is 4 cm?



Communicate your reasoning, understanding and fluency

- 10** Design a gift box in the shape of a rectangular prism with a surface area of 250 cm^2 .
- 11** Isla is constructing a fish tank in the shape of a rectangular prism, with dimensions 400 mm wide, 800 mm long and 450 mm high.
- a** There is no lid on the fish tank. Calculate the area of glass needed to make the fish tank.
- b** If 10 mm of glass costs \$80 per square metre, how much will it cost to make the fish tank?



3K Surface area of cylinders and spheres

Learning intentions

- To visualise the two-dimensional surfaces which form a three-dimensional solid
- To calculate the surface area of cylinders and spheres
- To solve practical problems involving the surface area of cylinders and spheres

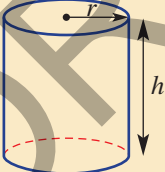
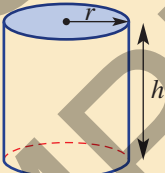
Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about surface area regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the 2027 HSC Reference Sheet, but others are not

Cylinders

The surface area (A) of a cylinder is the sum of the area of each surface of the solid. To calculate the surface area of cylinders we use the formulas in the table.

Surface area of a cylinder

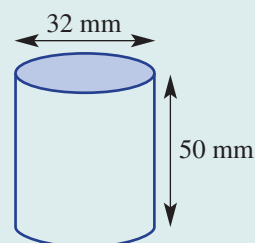
Open cylinder		$A = 2\pi rh$ Curved surface = $2\pi rh$
Closed cylinder		$A = 2\pi r^2 + 2\pi rh$ Area of top = πr^2 Area of bottom = πr^2 Curved surface = $2\pi rh$



Example 32 Finding the surface area of a cylinder

A closed cylinder has a diameter of 32 mm and a height of 50 mm.

- Find the area of the curved surface.
- Find the surface area of this cylinder. Answer in square millimetres correct to two decimal places.



continued ...

Solution

a $A = 2\pi rh$
 $= 2 \times \pi \times 16 \times 50$
 $= 5026.5482\dots$
 $\approx 5026.55 \text{ mm}^2$

Area of the curved surface is 5026.55 mm^2 .

b $A = 2\pi r^2 + 2\pi rh$
 $= 2 \times \pi \times 16^2 + 2 \times \pi \times 16 \times 50$
 $= 6635.043684$
 $\approx 6635.04 \text{ mm}^2$

Surface area of the cylinder is 6635.04 mm^2 .

Explanation

Write a formula for surface area of an open cylinder.
Substitute the values for r and h into the formula.
Write the answer correct to two decimal places.
Express using correct units.
Write the answer in words.
Write a formula for the surface area of a closed cylinder.
Substitute the values for r and h into the formula.
Write the answer correct to two decimal places.
Express using correct units.
Write the answer in words.

Now you try

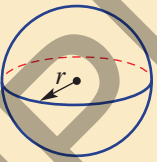
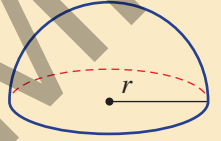
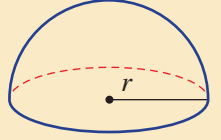
A closed cylinder has a diameter of 24 mm and a height of 35 mm.

- a** Find the area of the curved surface.
- b** Find the surface area of the cylinder. Answer correct to two decimal places.

Surface area of a sphere and hemisphere

The surface area (A) of a sphere is calculated using the formula listed in the table. The surface area of an open hemisphere is half that of the sphere. For a closed hemisphere, add the area of the base, which is a circle.

Surface area of a sphere and hemisphere

Sphere		$A = 4\pi r^2$
Open hemisphere		$A = 2\pi r^2$
Closed hemisphere		$A = 3\pi r^2$ Area of curved surface = $2\pi r^2$ Area of base = πr^2



Example 33 Finding the surface area of a sphere

What is the total surface area of one dozen tennis balls?

Each ball has a radius of 3.5 cm. Answer correct to the nearest square centimetre.



Solution

$$A = 4\pi r^2$$

$$= 4 \times \pi \times 3.5^2$$

$$\approx 153.93804 \text{ cm}^2$$

$$A \times 12 \text{ balls} = 153.93804 \times 12$$

$$= 1847.25648$$

$$\approx 1847 \text{ cm}^2$$

Surface area of 12 tennis balls is 1847 cm^2 .

Explanation

Write a formula for the surface area of a sphere.

Substitute the values for r into the formula.

Evaluate and express using correct units.

Multiply the surface area of one ball by 12.

Evaluate.

Write the answer correct to the nearest whole number and express using correct units.

Write the answer in words.

Now you try

What is the total surface area of 3 basketballs, if each ball has a radius of 12 cm?

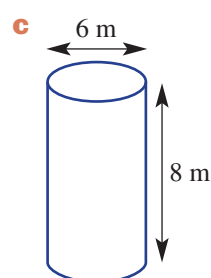
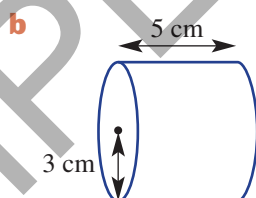
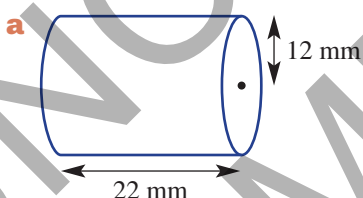
Answer correct to the nearest square centimetre.

Exercise 3K

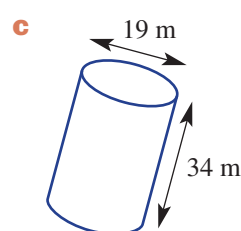
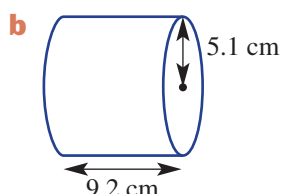
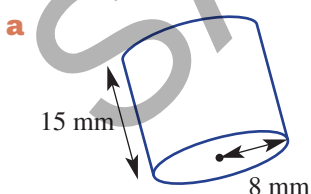
BUILDING

Example 32

- 1 Find the surface area of these open cylinders. Answer to the nearest whole number.

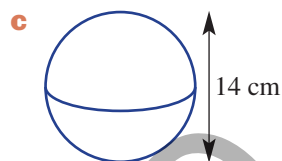
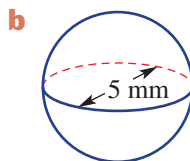
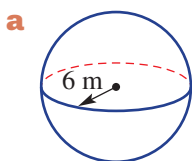


- 2 Find the surface area of these closed cylinders. Answer correct to one decimal place.

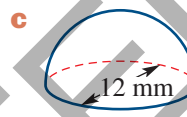
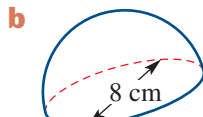
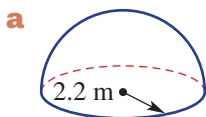


Example 33

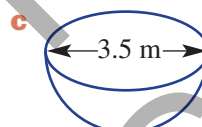
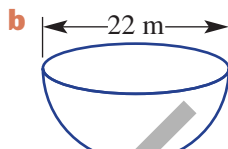
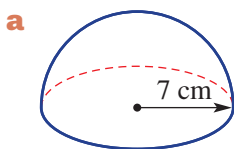
- 3** Find the surface area of these spheres. Answer correct to two decimal places.



- 4** Find the surface area of these open hemispheres. Answer correct to two decimal places.



- 5** Find the surface area of these closed hemispheres. Answer correct to two decimal places.



DEVELOPING

- 6** A can of soup has a 9 cm diameter and is 14 cm high.

- a** What is the radius of the can?
b Find the area of a label that wraps around the can. Answer correct to one decimal place.
c What is the surface area of the can including the top and bottom of the can? Answer correct to one decimal place.



- 7** The outer surface of a spherical ball is made from plastic. The diameter of the ball is 0.5 metres and the cost of plastic is \$24 per square metre.

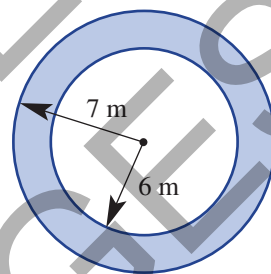
- a** What is the outer surface area of the ball? Answer correct to two decimal places.
b What is the cost of the plastic for one ball to the nearest cent?
c What is the cost of the plastic for a larger ball whose diameter is twice the size?



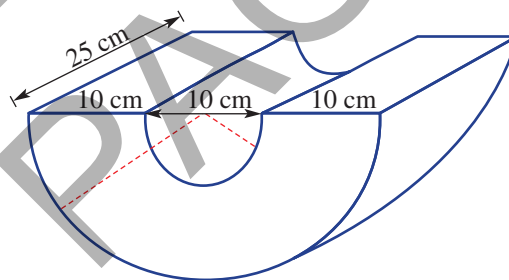
CHALLENGING

- 8** If the surface area of a sphere is 940 cm^2 , what is its diameter?

- 9** Throughout one Major League baseball game, 110 baseballs are used.
- a** The diameter of a baseball is 7.5 cm. How much leather is required for all 110 baseballs?
Give your answer in cm^2 .
- b** What is this in square metres?
- 10** If 1 litre of paint covers 10 m^2 , how many spheres of radius 6 cm can I paint?
- 11** The radius of Earth is 6400 km. If 71% of Earth is covered by water, find the total surface area of Earth that is covered by water.
- 12** This is a cross-section of a pipe to be painted. If the pipe is 12 metres long, calculate the inner and outer surface area of the pipe that needs to be painted.

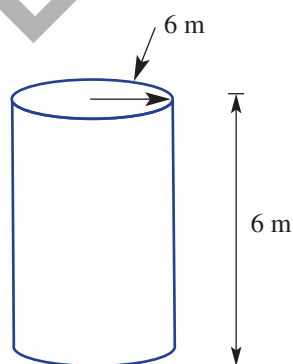
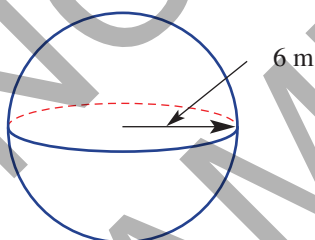


- 13** Find the surface area of the following solid.



Communicate your reasoning, understanding and fluency

- 14** Find the surface area of each solid.



What do you notice? Explain your answer

- 15** Find the surface area of a sphere with radius 12 cm. What happens to the surface area if I double the radius?

3L Surface area of composite solids

Learning intentions

- To visualise the two-dimensional surfaces which form a three-dimensional solid
- To calculate the surface area of composite solids
- To solve practical problems involving the surface area of composite solids

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about surface area regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the 2027 HSC Reference Sheet, but others are not

Surface area of composite solids

The surface area of a composite solid is calculated in the same way as for right prisms and pyramids. Visualise the surfaces of the solid, calculate the area of each face and then add these areas together.

Surface area of a composite solid

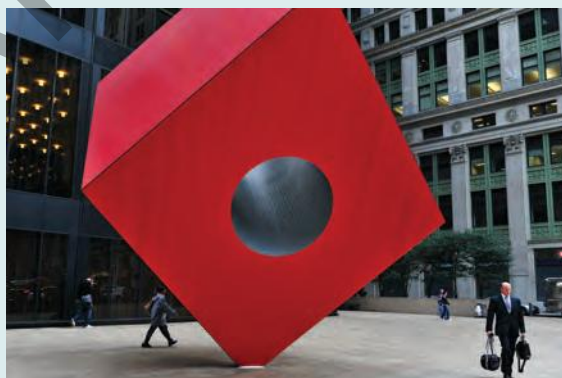
- 1 Visualise the surfaces of the solid.
- 2 Write a formula and calculate the surface area of each face.
- 3 Add the area of each surface.
- 4 Check that all surfaces have been counted.
- 5 Write the answer to the specified level of accuracy with the correct units.



Example 34 Finding the surface area of a composite solid

The red cube is a sculpture in New York consisting of a cube with a cylindrical hole removed from the centre. The side length of the cube is 8.53 m and the radius of the cylindrical hole is 1.42 m.

- a What is the surface area of the face with the circular hole, correct to two decimal places?
- b What is the surface area of the red cube? Include only the outer red surfaces. Answer correct to two decimal places.



Solution

$$\mathbf{a} \quad A = s^2 - \pi r^2$$

$$= 8.53^2 - \pi \times 1.42^2$$

$$= 66.4261... \approx 66.43 \text{ m}^2$$

$$\mathbf{b} \quad A = (2 \times 66.43) + (4 \times 8.53^2)$$

$$= 423.895...$$

$$\approx 423.90 \text{ m}^2$$

Surface area is 423.90 m^2 .

Explanation

Required face consists of a square with a circle removed, so use the formula for a square and a circle.

Substitute the values into formula.

Evaluate.

Composite solid has two faces with cylindrical holes and four faces that are squares.

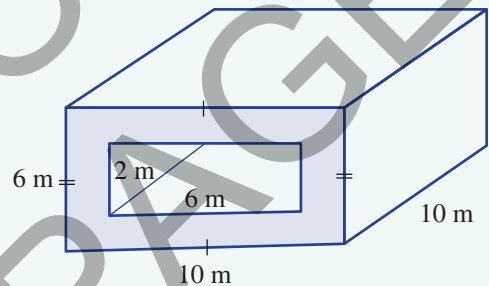
Write down an expression for surface area using the area of all the faces.

Evaluate.

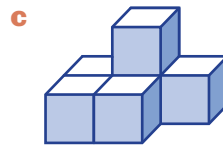
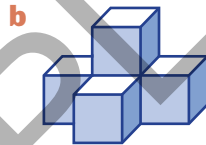
Write answer in the correct units.

Now you try

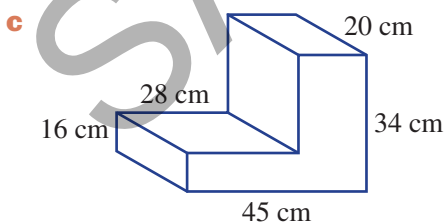
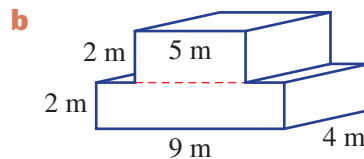
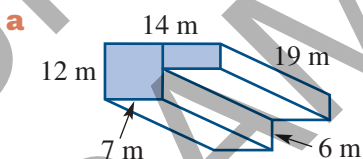
- a** What is the surface area of the face with the rectangular hole?
- b** What is the total **outer** surface area of this solid?
- c** What is the area of the **inner** surface of the rectangular prism?
- d** Add the outer surface area (part **b**) and inner surface area (part **c**) to find the total surface area.

**Exercise 3L****BUILDING**

- 1** Each of the following solids is made up of 1 cm cubes. What is the surface area of each solid?

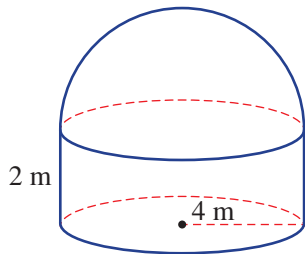
**Example 34**

- 2** What is the surface area of each solid? Answer to the nearest whole number.

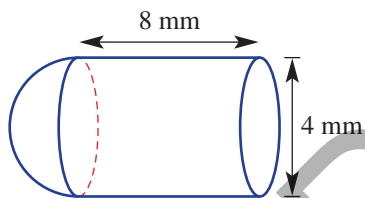


3 What is the surface area of each solid? Answer to the nearest whole number.

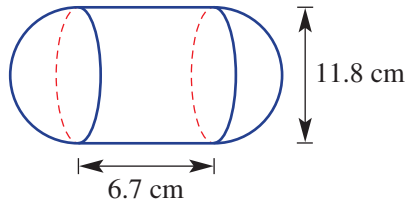
a



b



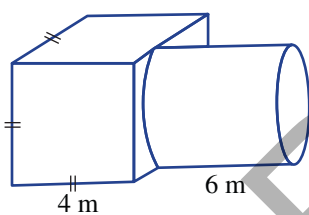
c



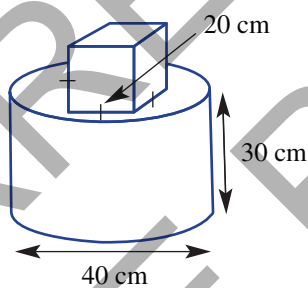
DEVELOPING

4 Find the surface area of these composite solids. Answer to one decimal place where necessary.

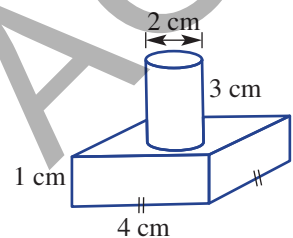
a



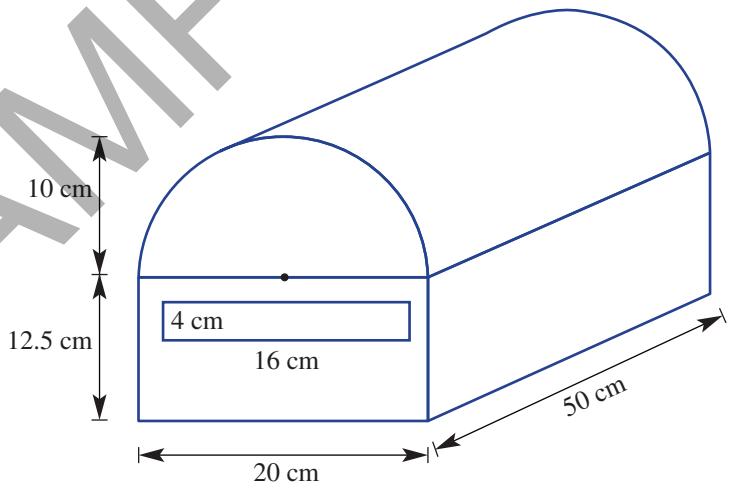
b



c



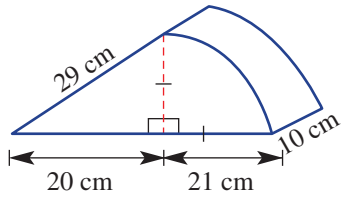
5 A letter box consists of a closed half-cylinder sitting on top of a rectangular prism. A rectangular gap measuring 4 cm by 16 cm is cut out to receive letters. The exterior of the letterbox will be painted red. Calculate the total surface area of the exterior of the letterbox to be painted, including the base.



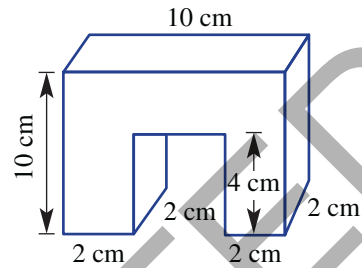
CHALLENGING

6 Find the surface area of these composite solids. Answer to one decimal place where necessary.

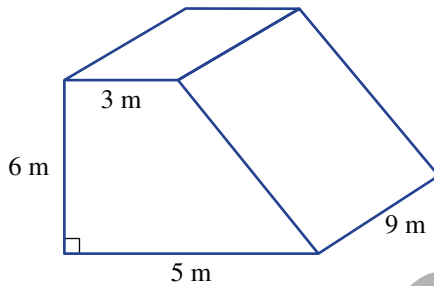
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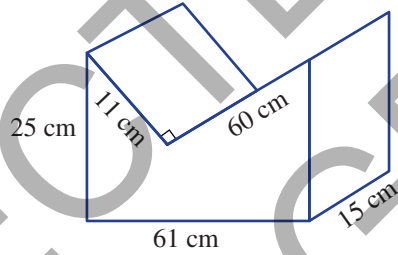
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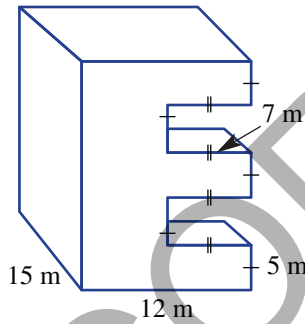
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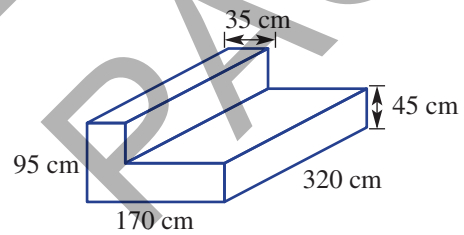
d



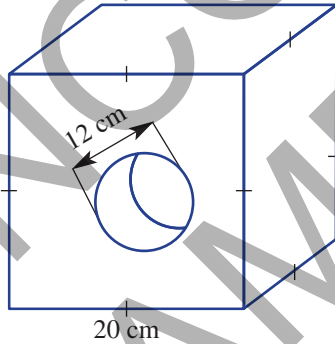
e



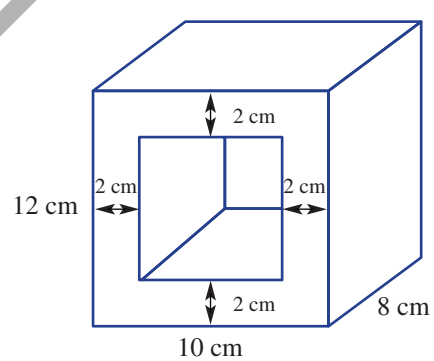
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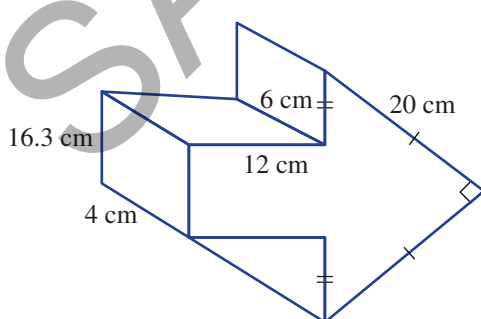
g



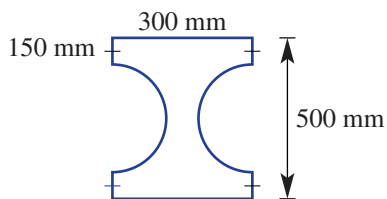
h



i

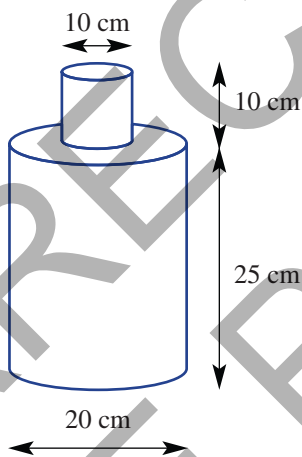


- 7 The image shows a cross-section of a steel beam being installed in a warehouse entry.



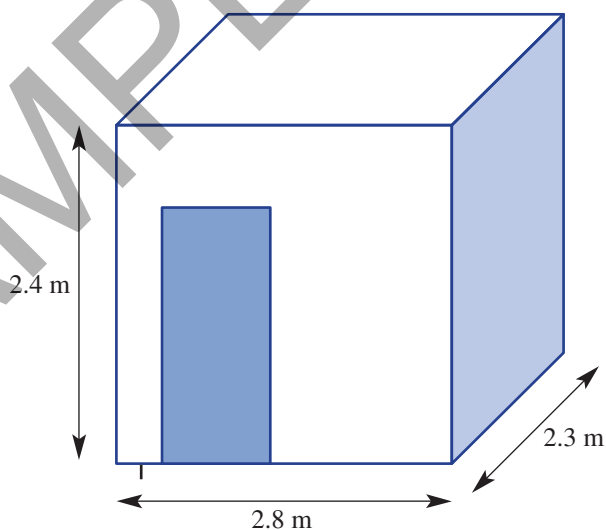
A 20-metre length of the steel beam is to be painted all over with a fire-retardant paint. Find the total surface area to be painted, giving your answer in square metres.

- 8 A birthday cake is made from two round cakes.



Calculate the visible surface area to be iced, in square centimetres.

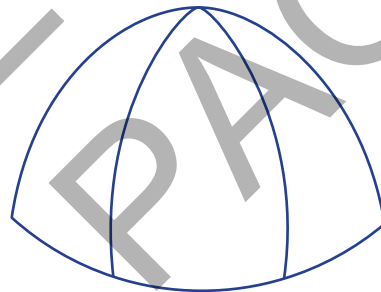
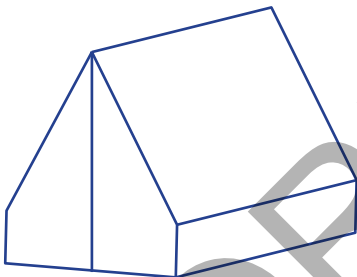
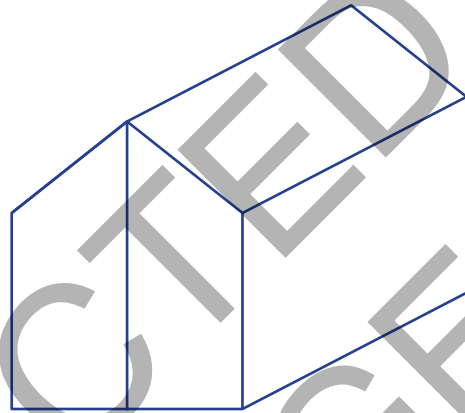
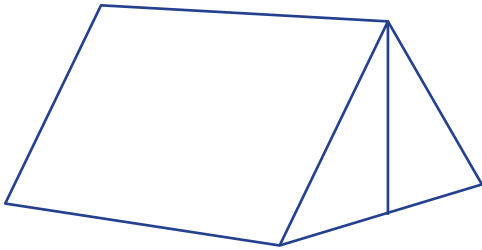
- 9 Max's bathroom is shown. He is painting the ceiling and tiling the floor and walls.



- Calculate the surface area to be tiled. The door is 1.2 m wide $\times$ 2.1 m high, and this is not tiled.
- If supply and installation of tiles costs \$120 per square metre, calculate the cost to tile the floor and walls of the bathroom.
- If painting the ceiling costs \$35 per square metre, find the cost to paint the ceiling.

Communicate your reasoning, understanding and fluency

- 10** You have 50 square metres of lightweight fabric to design a tent. The tent has no floor covering. What could the dimensions of the tent be? Show with calculations.



3M Volume of right prisms

Learning intentions

- To recognise a three-dimensional solid and choose the correct volume formula
- To calculate the volume of right prisms
- To solve practical problems involving the volume of right prisms

Past, present and future learning

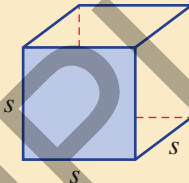
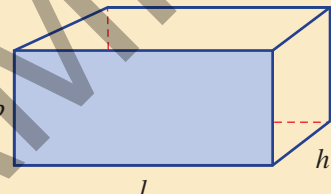
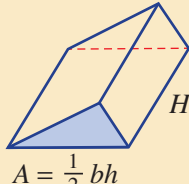
- Students should already be familiar with the contents of this section
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about volume regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the 2027 HSC Reference Sheet, but others are not

Volume is the amount of space occupied by a three-dimensional object. It is measured by counting the number of cubes that fit inside the solid. When calculating volume, the answer will be in cubic units.

$1000\text{ mm}^3 = 1\text{ cm}^3$
 $1\,000\,000\text{ cm}^3 = 1\text{ m}^3$
 $1\,000\,000\,000\text{ m}^3 = 1\text{ km}^3$

To calculate the volume of the most common solids, we use a formula. Some of these formulas are listed in the table. The volume of a prism is found by using its cross-sectional area. Prisms are three-dimensional objects that have a uniform cross-section along their entire length.

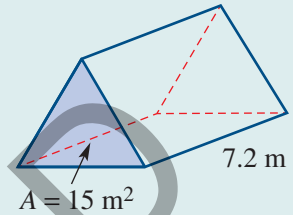
Volume of right prisms

Cube		$V = Ah$ $= (s^2) \times s$ $= s^3$
Rectangular prism		$V = Ah$ $= lb \times h$ $= lbh$
Triangular prism		$V = Ah$ $= \left(\frac{1}{2}bh\right) \times H$


Example 35 Finding the volume of a right prism

The base area of a triangular prism is 15 m^2 and the height is 7.2 m .

What is the volume of the triangular prism?


Solution

$$\begin{aligned} V &= Ah \\ &= 15 \times 7.2 \\ &= 108 \text{ m}^3 \end{aligned}$$

Explanation

Use the volume formula for a right prism $V = Ah$.
Substitute the values into the formula.
Evaluate.
Write answer to the correct units.

Now you try

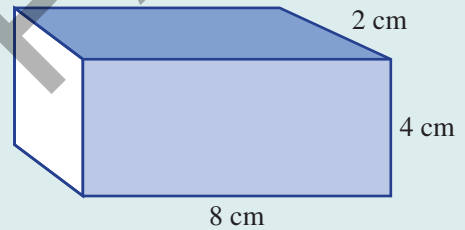
The base of a triangular prism is 32 cm^2 and the height is 10.8 cm .

What is the volume of the triangular prism?


Example 36 Finding the volume of a rectangular prism

A rectangular prism has a length of 8 cm , a breadth of 2 cm and a height of 4 cm .

Find the volume of this rectangular prism. Answer in cubic centimetres.


Solution

$$\begin{aligned} V &= Ah \\ &= lbh \\ &= 8 \times 2 \times 4 \\ &= 64 \text{ cm}^3 \end{aligned}$$

Explanation

Use the volume formula for a right prism $V = Ah$.
Determine the shape of the base and the formula to calculate the area of the base $A = lb$.
Substitute the values into the formula.
Evaluate.
Give the answer to the correct units.

Now you try

A rectangular prism has a length of 12 m , a breadth of 8 m and a height of 10 m .

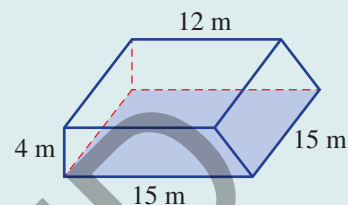
Find the volume of this rectangular prism. Answer in cubic metres.



Example 37 Finding the volume of a trapezoidal prism

The dimensions of a trapezoidal prism are shown.

Find the volume of this trapezoidal prism.



Solution

$$\begin{aligned} A &= \frac{h}{2}(a + b) \\ &= \frac{1}{2} \times (12 + 15) \times 4 \\ &= 54 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} V &= Ah \\ &= 54 \times 15 \\ &= 810 \text{ m}^3 \end{aligned}$$

Explanation

Find the area of the trapezoidal side (front face in the diagram.)

Use the area formula for a trapezium $A = \frac{h}{2}(a + b)$.

Substitute the values into the formula.

Evaluate.

Write answer to the correct units.

Use the volume formula for a right prism $V = Ah$.

Substitute the values into the formula.

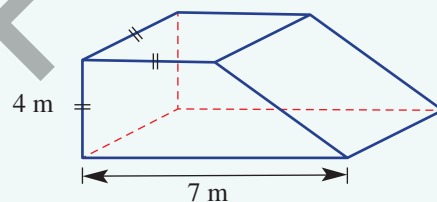
Evaluate.

Write answer to the correct units.

Now you try

The dimensions of a trapezoidal prism are shown opposite.

Find the volume of this trapezoidal prism.



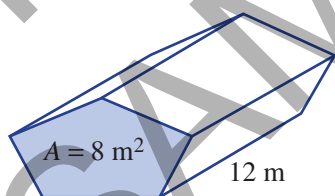
Exercise 3M

BUILDING

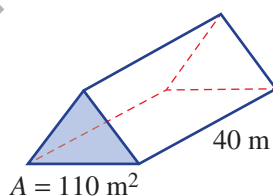
Example 35

- 1 Find the volume of the following prisms where A is the area of the base.

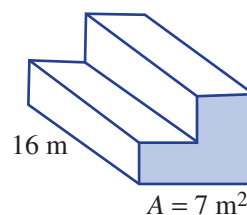
a



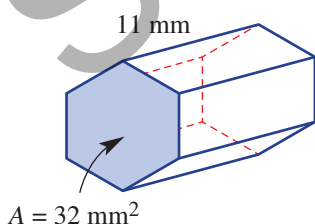
b



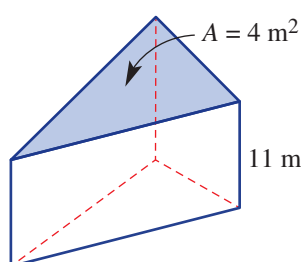
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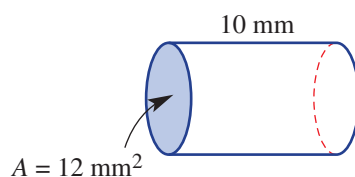
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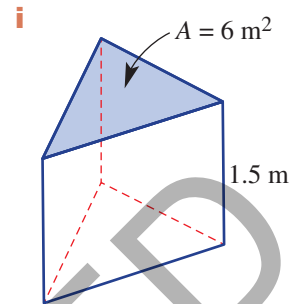
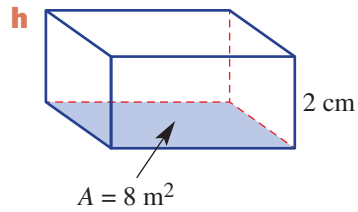
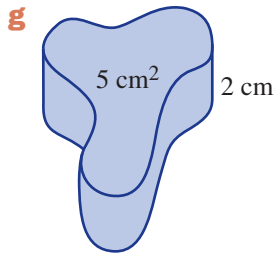


e



f



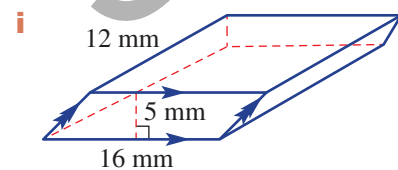
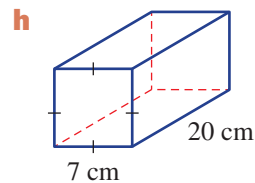
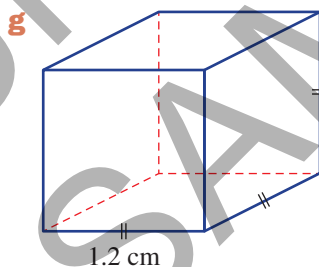
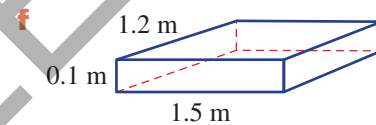
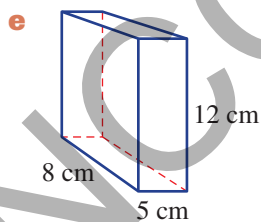
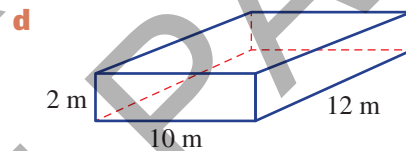
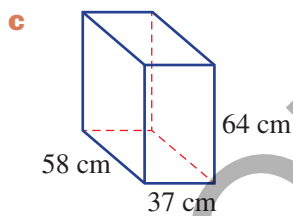
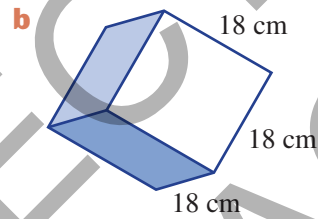
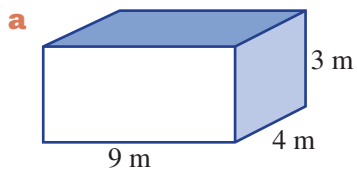


2 What is the volume of a rectangular prism with a base area of 15 mm^2 and a height of 11 mm ?

3 Find the volume of a triangular prism with a height of 15 m and a base area of 50 m^2 .

Example 36

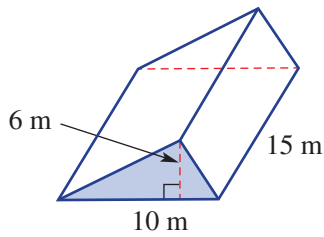
4 Find the volume of the following solids. Answer to the nearest whole number.



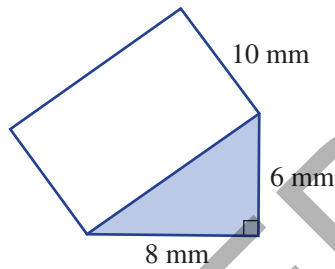
5 What is the volume of a rectangular prism with dimensions 4.5 cm by 6.5 cm by 10.5 cm ? Answer correct to one decimal place.

6 Find the volume of the following triangular prisms, to the nearest whole numbers.

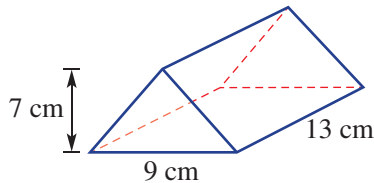
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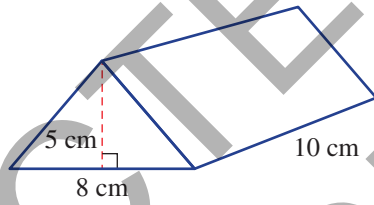
b



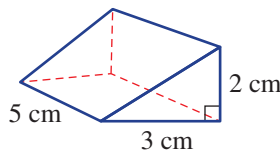
c



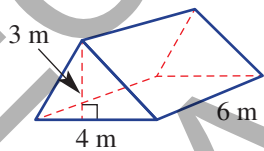
d



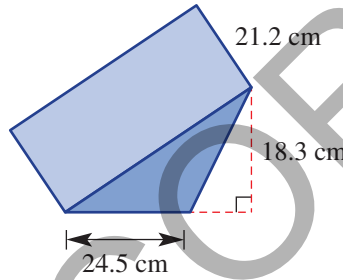
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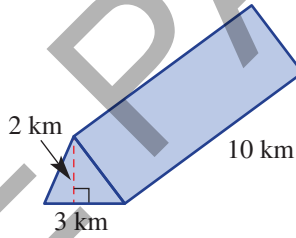
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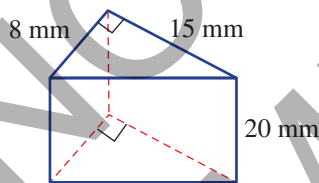
g



h



i

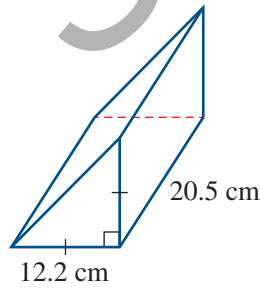


7 A triangular prism has a base length of 12 cm and base height of 20 cm. What is the volume of the prism if the height of the prism is 10 cm?

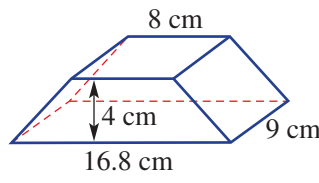
DEVELOPING

8 Find the volume of the following solids. Answer to the nearest whole number.

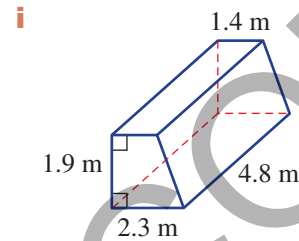
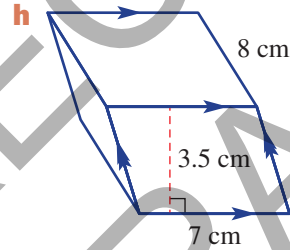
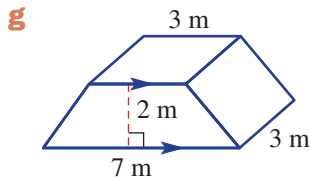
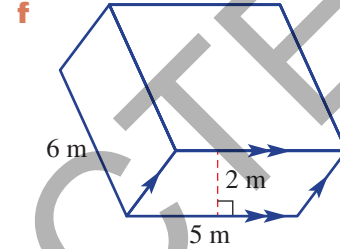
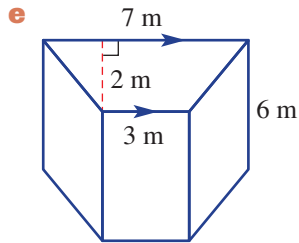
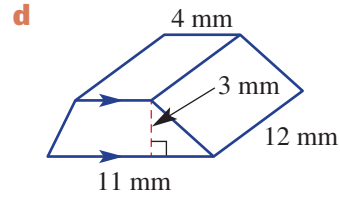
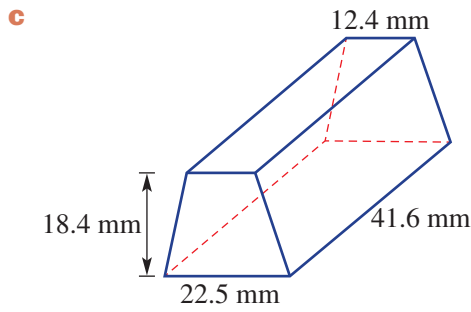
a



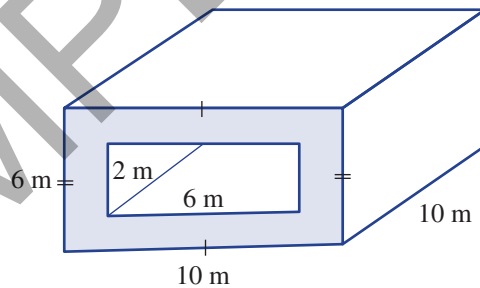
b



Example 37



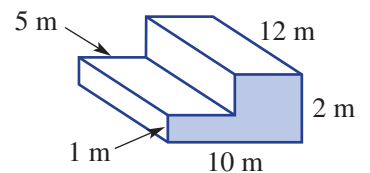
- 9** A hollow container is in the shape of a rectangular prism as shown.



- a** What would be the volume of the container if it were solid?
b What is the area of the shaded base?
c What is volume of the hollow container?

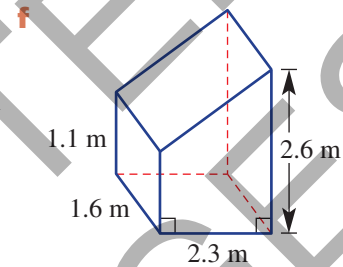
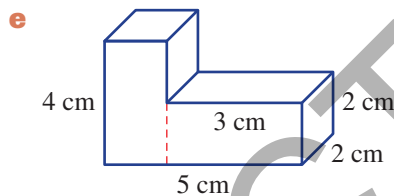
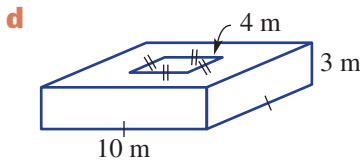
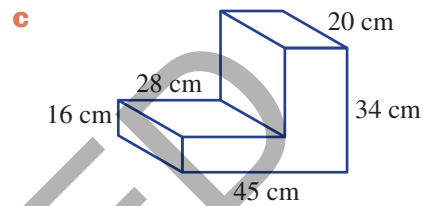
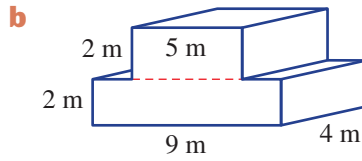
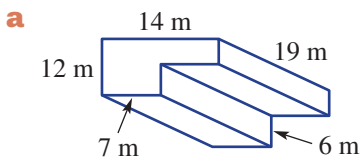
- 10** A step is shown.

- a** What is the area of the shaded base?
b Determine the volume of the step.



CHALLENGING

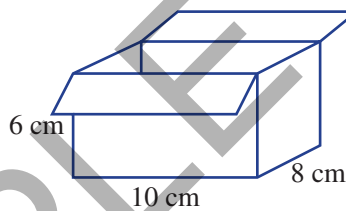
11 Find the volume of these composite solids.



Communicate your reasoning, understanding and fluency

12 Find the volume of an equilateral triangular prism with side lengths 3 cm and a depth of 10 cm. Answer correct to three decimal places.

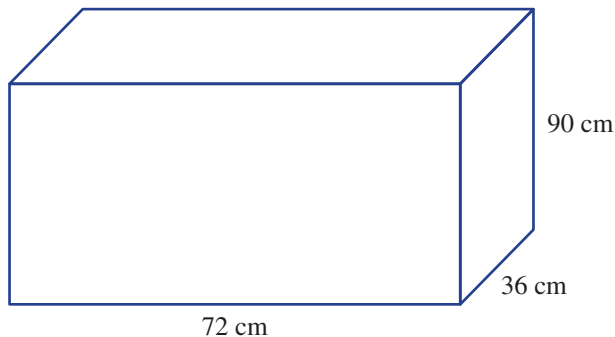
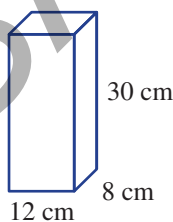
13 A vase with a volume of 200 cm^3 is packed into the cardboard box. The space around the vase is filled with foam to protect the vase from breaking. The parcel is sealed and posted.



a What is the volume of the foam?

b What is the area of cardboard on the surface of the box?

14 Packets of coffee are placed into a box. Each packet of coffee is a rectangular prism, and the box also is a rectangular prism. How many packets of coffee fit into the box?



3N Volume of cylinders and spheres

Learning intentions

- To recognise a three-dimensional solid and choose the correct volume formula
- To calculate the volume of cylinders and spheres
- To solve practical problems involving the volume of cylinders and spheres

Past, present and future learning

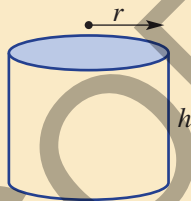
- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about volume regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the 2027 HSC Reference Sheet, but others are not

Cylinders

A cylinder is a right prism whose cross-section is a circle. Its volume is found using $V = Ah$ where $A = \pi r^2$. The volume of a cylinder is the product of π , the square of the radius and the height.

Volume of a cylinder

Cylinder

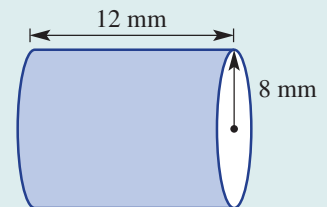


$$\begin{aligned} V &= Ah \\ &= (\pi r^2) \times h \\ &= \pi r^2 h \end{aligned}$$



Example 38 Finding the volume of a cylinder

A cylinder has a radius of 8 mm and a height of 12 mm. Find the volume of the cylinder. Answer correct to two decimal places.



Solution

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 8^2 \times 12 \\ &= 2412.743158 \\ &\approx 2412.74 \text{ mm}^3 \end{aligned}$$

Explanation

Use the volume formula for a cylinder $V = \pi r^2 h$. Substitute the $r = 8$ and $h = 12$ into the formula. Evaluate.

Write the answer correct number of decimal places and with correct units.

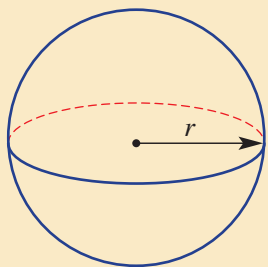
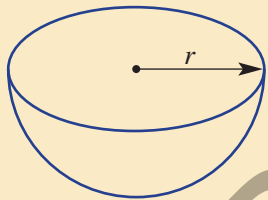
Now you try

A cylinder has a radius of 14 mm and a height of 35 mm. Find the volume of the cylinder. Answer correct to two decimal places.

Volume of a sphere

The volume of a sphere is four-thirds the product of π and the cube of the radius. The volume of a hemisphere is half the volume of the related sphere or two-thirds the product of π and the cube of the radius.

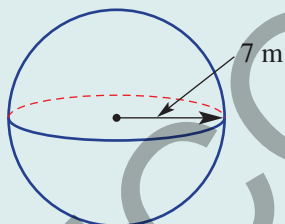
Volume of sphere

Sphere		$V = \frac{4}{3}\pi r^3$
Hemisphere		$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$ $= \frac{2}{3}\pi r^3$

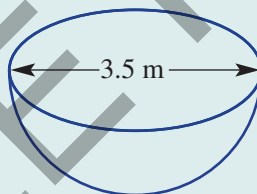
Example 39 Finding the volume of a sphere and a hemisphere

Find the volume of these solids. Answer correct to one decimal place.

a



b



Solution

$$\begin{aligned} \text{a } V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \pi \times 7^3 \\ &= 1436.75504 \\ &\approx 1436.8 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{b } V &= \frac{1}{2} \times \left(\frac{4}{3}\pi r^3 \right) = \frac{2}{3}\pi r^3 \\ r &= 3.5 \div 2 = 1.75 \text{ m} \\ V &= \frac{2}{3} \times \pi \times 1.75^3 \\ &= 11.2246... \\ &\approx 11.2 \text{ m}^3 \end{aligned}$$

Explanation

Use the volume formula for a sphere $V = \frac{4}{3}\pi r^3$.

Substitute $r = 7$ into the formula.

Evaluate.

Write the answer correct to one decimal place and with correct units.

Use the volume formula for a hemisphere.

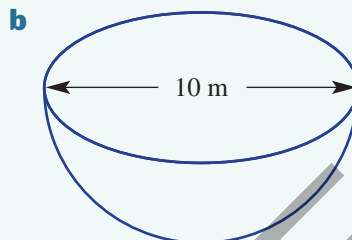
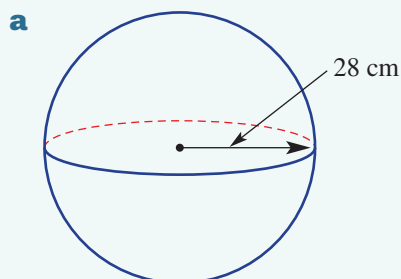
Find r and substitute the value into the formula.

Evaluate.

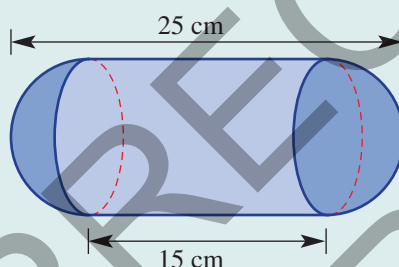
Write the answer correct to one decimal place and with correct units.

Now you try

Find the volume of these solids. Answer correct to one decimal place.

**Example 40 Finding the volume of a composite solid**

Find the volume of the composite solid. Answer to one decimal place.

**Solution**

Volume of cylinder + Volume of sphere

$$V = \pi r^2 h + \frac{4}{3} \pi r^3$$

$$V = \pi \times 5^2 \times 15 + \frac{4}{3} \times \pi \times 5^3$$

$$V = 1178.0972 \dots + 523.59877 \dots$$

$$V = 1701.7 \text{ cm}^3$$

Explanation

Break the solid into regular solids:
2 half spheres and middle cylinder.

Write up the formulas.

Substitute into the formula.

$$r = 5 \text{ cm}$$

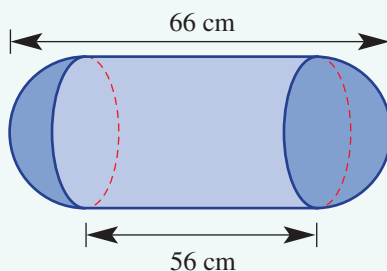
$$h = 15 \text{ cm}$$

Evaluate.

Write answer correct to one decimal place with the correct units.

Now you try

Find the volume of the composite solid. Answer to one decimal place.

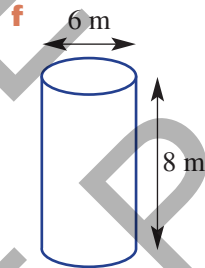
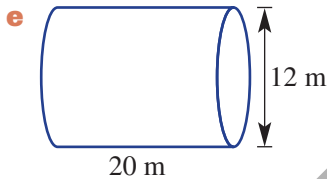
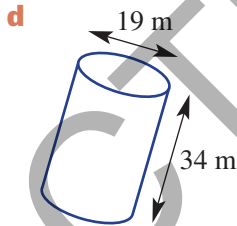
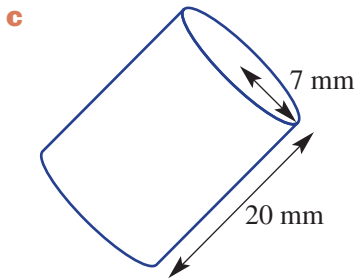
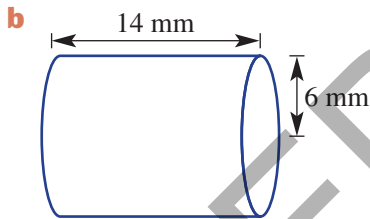
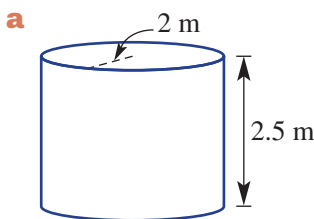


Exercise 3N

BUILDING

Example 38

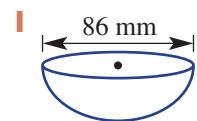
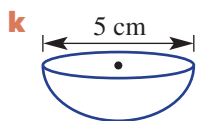
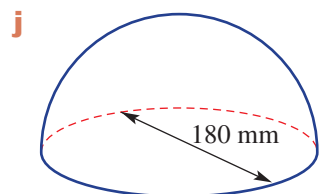
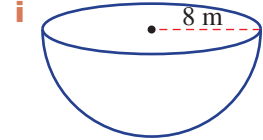
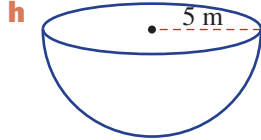
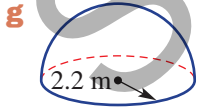
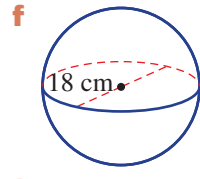
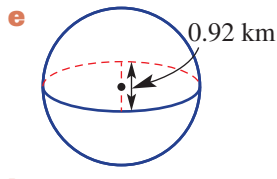
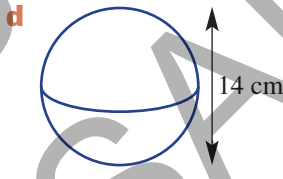
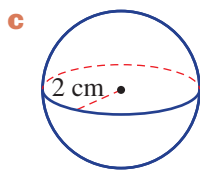
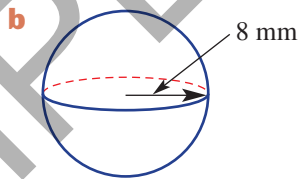
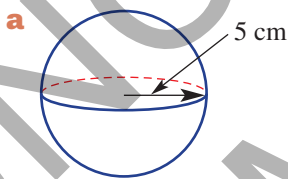
1 Find the volume of the following cylinders. Answer to the nearest whole number.



2 A closed cylindrical plastic container is 20 cm high and its circular end surfaces each have a radius of 5 cm. What is its volume, correct to two decimal places?

Example 39

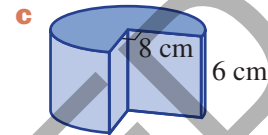
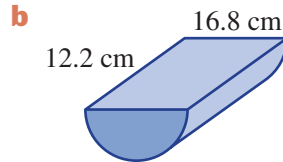
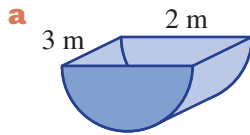
3 Find the volume of the following spheres and hemispheres. Answer to one decimal place where necessary.



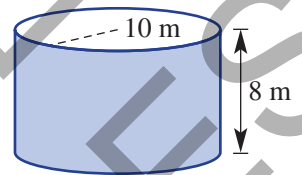
- 4 Find the volume of a sphere with a diameter of 12 mm. Answer correct to the nearest whole number.

DEVELOPING

- 5 Find the volume of the following solids. Answer correct to one decimal place.



- 6 A water tank is in the shape of a closed cylinder with a radius of 10 m and height of 8 m.

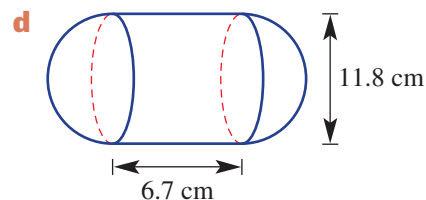
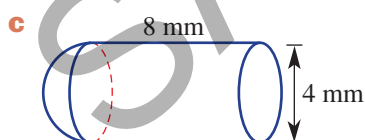
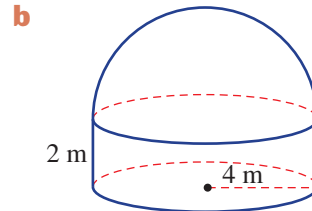
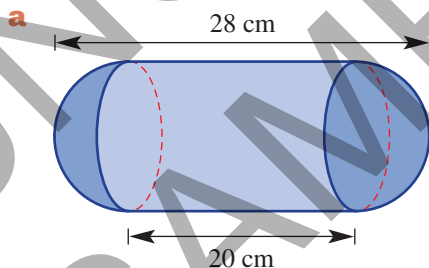


- a** What is the area of the top circular face of the water tank? Leave your answer in terms of π .
b Determine the volume of the water tank. Leave your answer in terms of π .
- 7 A cylindrical wheat hopper is 8 m across and 10 m high.
a Calculate its volume correct to one decimal place.
b Grain pours out at $48\,000\text{ cm}^3$ per second. How long would it take a full hopper to empty? Answer correct to the nearest second.
- 8 The circumference of Earth at the equator is about 40 075 km.
a Use the formula $C = 2\pi r$ to find the radius of Earth correct to the nearest kilometre.
b Use this radius to find the volume of Earth. Express your answer in standard notation using three significant figures.

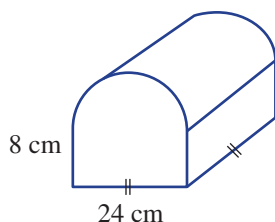
CHALLENGING

Example 40

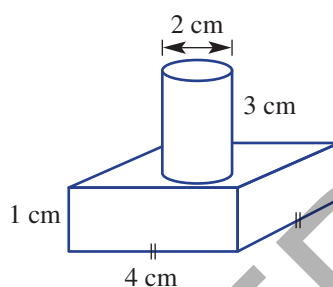
- 9 Find the volume of these composite shapes. Answer to one decimal place.



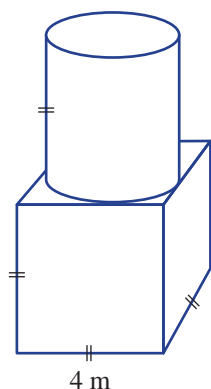
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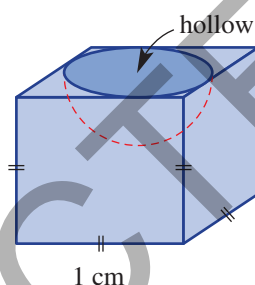
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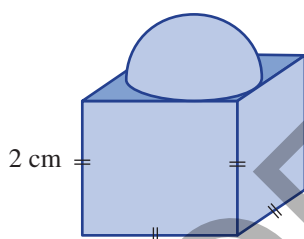
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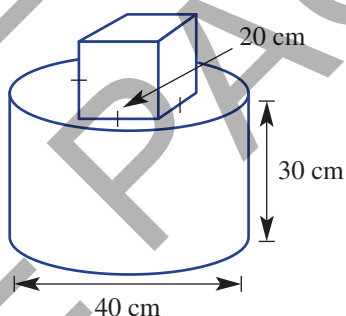
h



i



j



Communicate your reasoning, understanding and fluency

10 A cylinder has its radius doubled. How many times is its volume increased?

11 A can of tennis balls contains three balls squeezed in with no room for the balls to move. The container has a height of 21 cm. Answer these questions correct to one decimal place.

- What is the radius of a tennis ball?
- Calculate the volume of a ball.
- Calculate the volume of the container.
- What volume of the container is not occupied by the balls?
- Express this unused space as a percentage of the total container.



30 Volume of pyramids and cones

Learning intentions

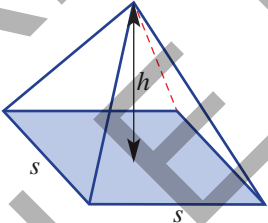
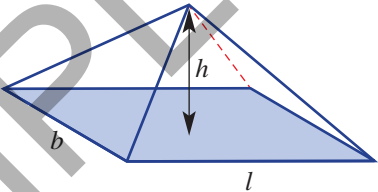
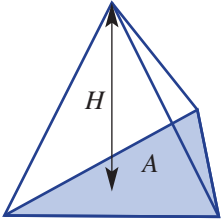
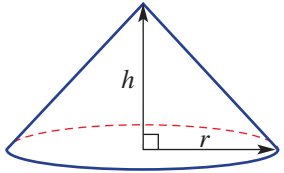
- To recognise a three-dimensional solid and choose the correct volume formula
- To calculate the volume of pyramids and cones
- To solve practical problems involving the volume of pyramids and cones

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about volume regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the 2027 HSC Reference Sheet, but others are not

A pyramid fits exactly inside a prism and occupies one third of the volume of the prism. Therefore the formula for a pyramid will be $V = \frac{1}{3}Ah$ where A is the area of the base and h is the height of the pyramid. Likewise, the volume of a cone is one third the volume of a cylinder.

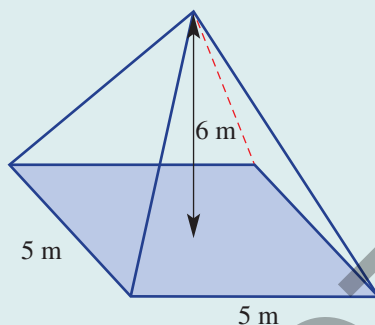
Volume formulas

Name	Solid	Volume
Square pyramid		$V = \frac{1}{3}Ah$ $= \frac{1}{3} \times s^2 \times h$
Rectangular pyramid		$V = \frac{1}{3}Ah$ $= \frac{1}{3} \times lb \times h$
Triangular pyramid		$V = \frac{1}{3}AH$ $= \frac{1}{3} \times \frac{1}{2}bh \times H$
Cone		$V = \frac{1}{3}Ah$ $= \frac{1}{3} \times \pi r^2 \times h$



Example 41 Finding the volume of a square pyramid

Find the volume of this square pyramid of height 6 m and base 5 m. Give your answer correct to the nearest cubic metre.



Solution

$$\begin{aligned} V &= \frac{1}{3}Ah \\ &= \frac{1}{3}s^2h \\ &= \frac{1}{3} \times 5^2 \times 6 \\ &= 50 \text{ m}^3 \end{aligned}$$

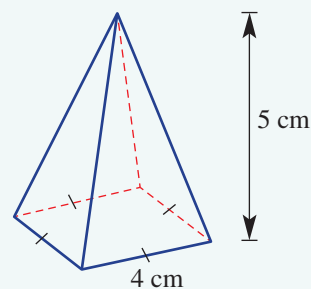
Volume of the pyramid is 50 m^3 .

Explanation

Use the volume formula for a pyramid $V = \frac{1}{3}Ah$.
Determine the shape of the base and the formula to calculate the area of the base: $A = s^2$.
Substitute the values into the formula.
Evaluate.
Give the answer to the correct units.
Write the answer in words.

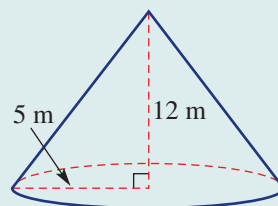
Now you try

Find the volume of a square pyramid of height 5 cm and a base of 4 cm.
Give your answer to the nearest cubic centimetre.



Example 42 Finding the volume of a cone

Find the volume of this cone. Answer correct to three significant figures.



Solution

$$\begin{aligned}
 V &= \frac{1}{3}Ah \\
 &= \frac{1}{3}\pi r^2 h \\
 &= \frac{1}{3} \times \pi \times 5^2 \times 12
 \end{aligned}$$

$$= 314.1592654\dots$$

$$\approx 314 \text{ m}^3$$

Volume of the cone is 314 m^3 .

Explanation

Use the volume formula for a cone $V = \frac{1}{3}Ah$.

Determine the shape of the base and the formula to calculate the area of the base: $A = \pi r^2$.

Substitute the values for r and h into the formula.

Evaluate.

Express the answer in the correct units.

Write the answer in words.

Now you try

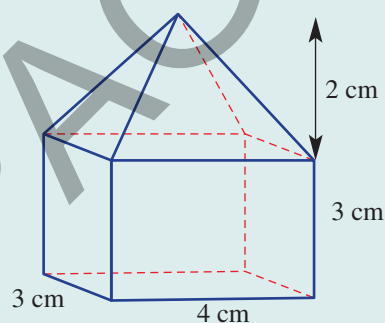
Find the volume of a cone with radius, $r = 12 \text{ cm}$, and height, $h = 22 \text{ cm}$, correct to one decimal place.

**Example 43 Volume of a composite solid**

Find the volume of the composite solid shown.

This solid consists of the rectangular prism and a rectangular pyramid.

Answer correct to the nearest cubic centimetre.

**Solution**

Volume of rectangular prism + volume of rectangular pyramid

$$V = lbh + \frac{1}{3}Ah$$

$$V = 3 \times 4 \times 3 + \frac{1}{3} \times 3 \times 4 \times 2$$

$$V = 48 + 8$$

$$V = 56 \text{ cm}^3$$

Volume of the composite solid is 56 cm^3 .

Explanation

Break the composite solid up into separate solids.

Write down the formulas associated with the volume of a rectangular prism and rectangular pyramid.

Substitute into the formula.

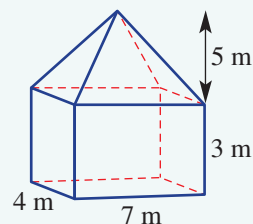
Evaluate.

Express the answer in the correct units.

Write the answer in words.

Now you try

Find the volume of the composite solid shown. Answer correct to the nearest cubic metre.



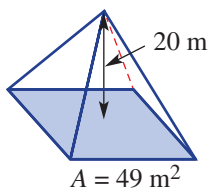
Exercise 30

BUILDING

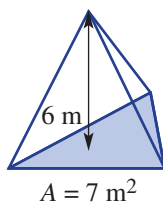
Example 41

- 1 Find the volume of the following pyramids. Answer to the nearest whole number.

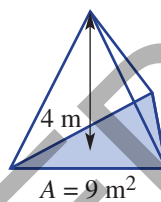
a



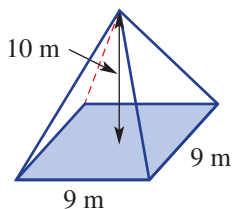
b



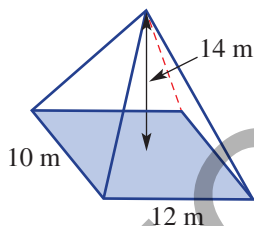
c



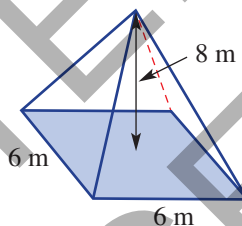
d



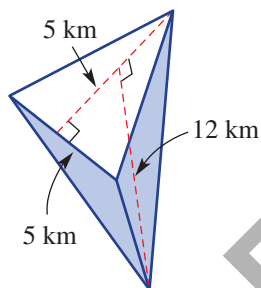
e



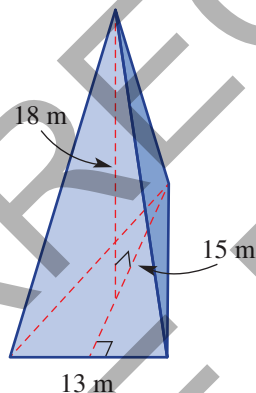
f



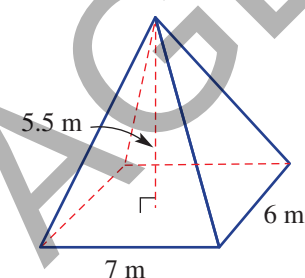
g



h



i



- 2 Find the volume of the solids with the following dimensions. Give the answers correct to the nearest whole number.

a Rectangular pyramid with base area 24 m^2 and height 20 m.

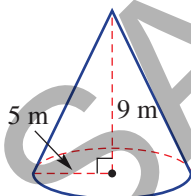
b Triangular pyramid with base area 120 mm^2 and height 50 mm.

- 3 A square pyramid has a base side length of 3 cm and a height of 4 cm. What is its volume? Answer correct to the nearest cubic centimetre.

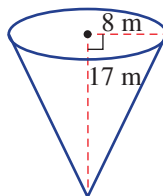
Example 42

- 4 Find the volume of these cones to the nearest whole number.

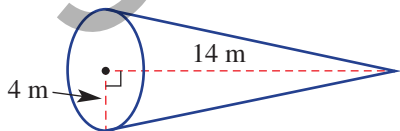
a



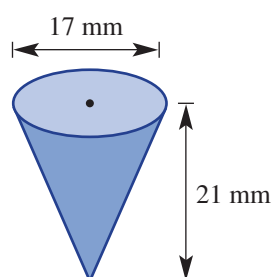
b

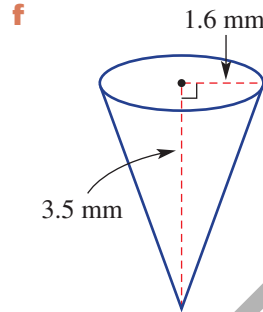
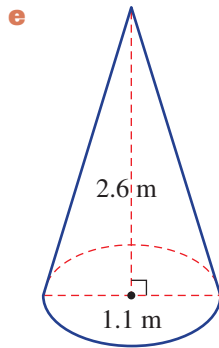


c

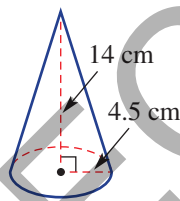


d





- 5** Find the volume of a cone with base radius 4.5 cm and height 14 cm. Answer correct to two decimal places.



DEVELOPING

- 6** The Red Pyramid is the third tallest pyramid ever built. It has a square base with a side length of 220 m and a height of 104 m.
- a** What is the area of the square base?
- b** Determine the volume of the Red Pyramid. Answer correct to two decimal places.

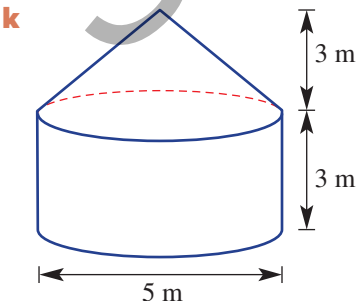
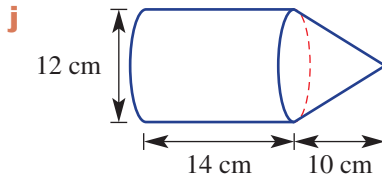
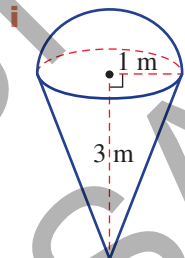
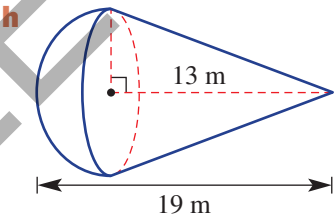
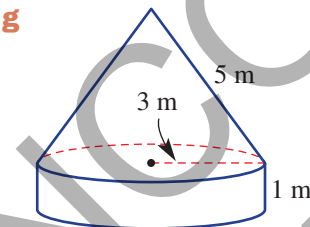
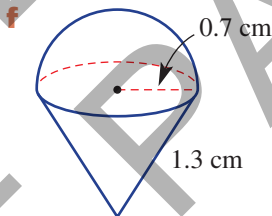
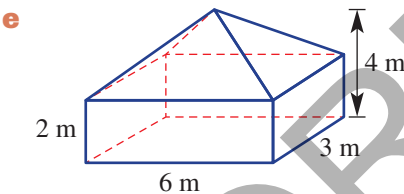
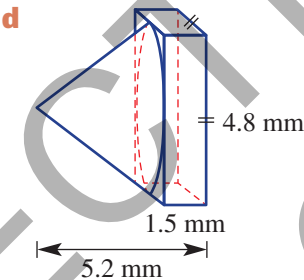
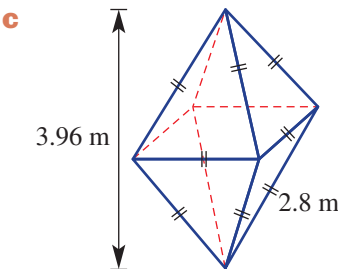
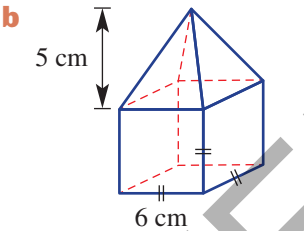
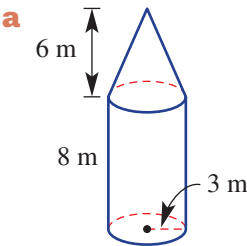


- 7** An ice-cream cone measures 4.6 cm in diameter and is 9 cm high. Calculate the volume of the cone. Answer correct to two decimal places.
- 8** A triangular pyramid has a base with a height of 15 cm and base length of 5 cm.
- a** What is the area of the base of the pyramid?
- b** What is the volume of the triangular pyramid if the height of the pyramid is 12 cm?

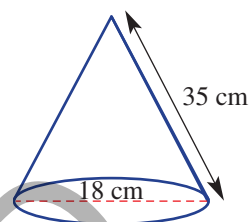
CHALLENGING

Example 43

9 Find the volume of these composite shapes. Answer to two decimal places where necessary.

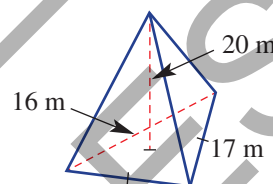


- 10** Calculate the volume of the cone. Answer to one decimal place.



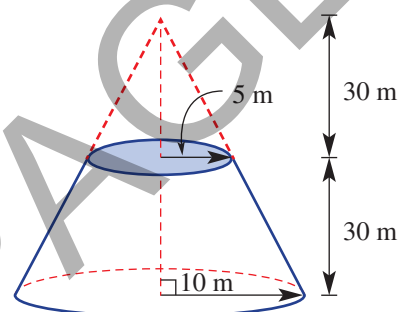
Communicate your reasoning, understanding and fluency

- 11** A triangular pyramid has a base with sides 16 m, 17 m and 17 m. The height of the pyramid is 20 m. What is the volume of the triangular pyramid?



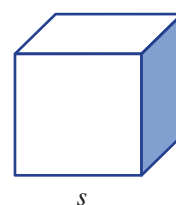
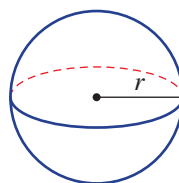
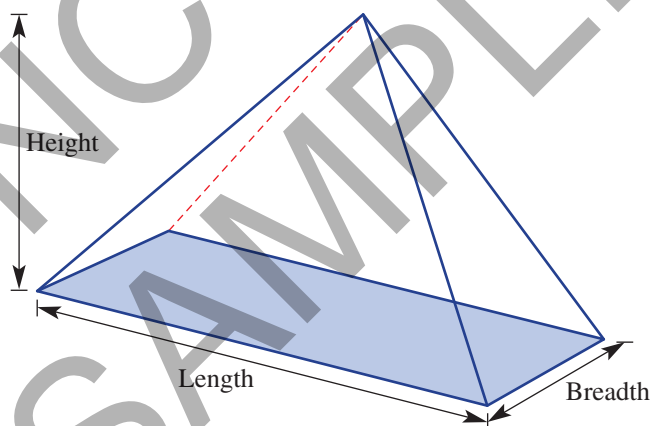
- 12** A frustum is a portion of a solid. The frustum is formed from a cone.

- a** What is the volume of the missing top cone?
Give answers correct to the nearest cubic metre.
b What was the volume of the entire cone?
c What is the volume of the frustum?



- 13** A rectangular-based pyramid is made of solid gold. The pyramid has length 20 cm, breadth 18 cm and height 25 cm. The pyramid is melted to form a sphere and cube with the same volume.

What is the radius of the sphere, r , and side length of the cube, s ?



3P

Capacity

- Learning intentions**
- To recognise a three-dimensional solid and choose the correct volume formula
 - To convert volume measurements to millilitres or litres
 - To calculate the volume and capacity of three-dimensional objects
 - To solve practical problems involving volume and capacity

- Past, present and future learning**
- Some aspects of this section may be new to students
 - This is a Year 11 concept which is not repeated or extended in Year 12
 - Questions about capacity regularly appear in HSC Examinations
 - Almost all the unit conversion facts are assumed knowledge and do not appear on the 2027 HSC Reference Sheet, except for the relationship between capacity and the cubic units of volume

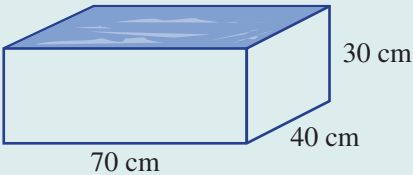
The capacity of a container is the amount of liquid it can hold. Some solids have both a volume and a capacity. For example, a can of soft drink is a cylinder that has a volume ($V = \pi r^2 h$) and a capacity (360 mL). The base unit for capacity is the litre (L). Three commonly used units for capacity are the megalitre (ML), kilolitre (kL) and millilitre (mL).



Capacity		
1 ML = 1000 kL	1 cm ³ = 1 mL	1 m ³ = 1 000 000 cm ³
1 ML = 1 000 000 L	1 cm ³ = 0.001 L	1 m ³ = 1 000 000 mL
1 kL = 1000 L	1000 cm ³ = 1 L	1 m ³ = 1000 L
1 L = 1000 mL		1 m ³ = 1 kL

Example 44 Finding the capacity of a container

- The container shown is filled with water.
- a Find the volume of the container in cubic centimetres.
 - b Find the capacity of the container in litres.



Solution

a $V = Ah$

$$= lbh$$

$$= 70 \times 40 \times 30$$

$$= 84\,000 \text{ cm}^3$$

b Capacity = $84\,000 \times 0.001 \text{ L}$

$$= 84 \text{ L}$$

$$\text{Capacity} = 84\,000 \times 1 \text{ mL}$$

$$= 84\,000 \text{ mL}$$

$$= 84 \text{ L}$$

Explanation

Use the volume formula for a right prism $V = Ah$.

Determine the shape of the base and the formula to calculate the area of the base $A = lb$.

Substitute the values into the formula.

Evaluate.

Give answer to the correct units.

To change cm^3 to L multiply by 0.001 ($1 \text{ cm}^3 = 0.001 \text{ L}$).

Alternative method is to convert to mL first ($1 \text{ cm}^3 = 1 \text{ mL}$).

Now you try

A container with dimensions 65 cm long, 30 cm wide and 30 cm high is filled with water.

a Find the volume of the container in cubic centimetres.

b Find the capacity of the container in litres.

Exercise 3P**BUILDING**

1 A can of soft drink has a capacity of 375 mL. How many cans of soft drink would it take to fill a 1.2 L bottle? How much would remain?

2 A medicine bottle has a capacity of 0.3 L.

a What is the capacity in millilitres?

b How many tablespoons (15 mL) does the bottle contain?

c How many teaspoons (5 mL) does the bottle contain?

d The correct dosage is 10 mL, 3 times a day.
How many doses does the bottle contain?

3 Complete the following.

a $4 \text{ cm}^3 = \square \text{ mL}$

c $70 \text{ cm}^3 = \square \text{ mL}$

e $900 \text{ cm}^3 = \square \text{ mL}$

g $43 \text{ m}^3 = \square \text{ kL}$

i $103 \text{ m}^3 = \square \text{ kL}$

k $5 \text{ m}^3 = \square \text{ kL}$

b $2000 \text{ cm}^3 = \square \text{ L}$

d $34\,000 \text{ cm}^3 = \square \text{ L}$

f $500 \text{ cm}^3 = \square \text{ L}$

h $30 \text{ m}^3 = \square \text{ L}$

j $7 \text{ m}^3 = \square \text{ L}$

l $8 \text{ m}^3 = \square \text{ mL}$

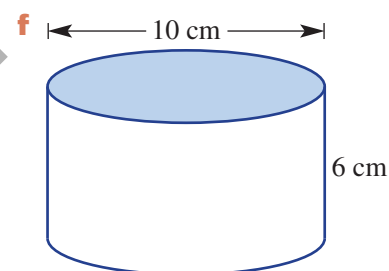
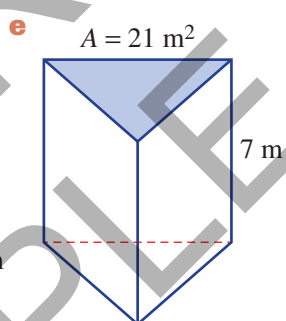
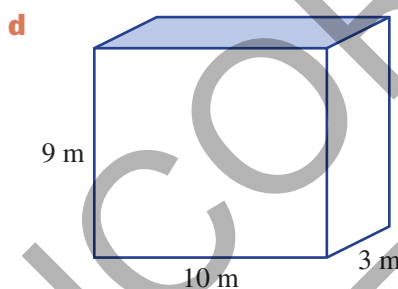
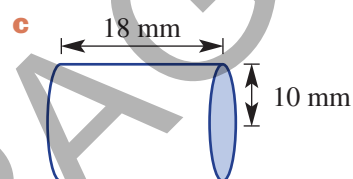
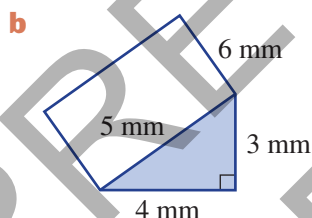
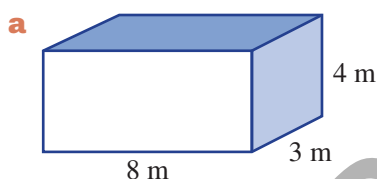


Example 44

- 4 What is the capacity of a rectangular prism whose base area is 20 cm^2 and height is 10 cm ? Answer correct to the nearest millilitre.
- 5 Find the capacity of a triangular prism with a height of 18 m and a base area of 40 m^2 . Answer in litres, correct to two significant figures.
- 6 Find the capacity of a rectangular pyramid whose base area is 12 cm^2 and height is 15 cm . Answer correct to the nearest millilitre.
- 7 Find the capacity of a cylindrical plastic container 16 cm high and with circular end surfaces of radius 8 cm . Answer correct to the nearest litre.
- 8 Find the capacity of a cube whose side length is 75 mm . Answer in millilitres, correct to two decimal places.

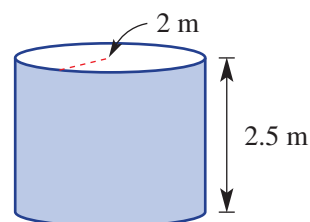
DEVELOPING

- 9 Find the capacity of the following solids in millilitres, correct to two decimal places where necessary.



- 10 A water tank is the shape of a cylinder with a radius of 2 m and height of 2.5 m .

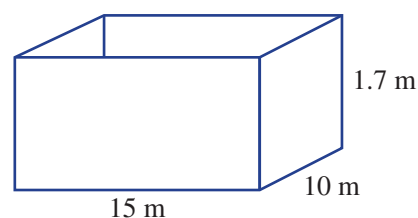
- a** What is the area of the top circular face of the water tank? Answer correct to one decimal place.
- b** Determine the volume of the water tank in cubic metres. Answer correct to one decimal place.
- c** What is the capacity of the tank, to the nearest kilolitre?



CHALLENGING

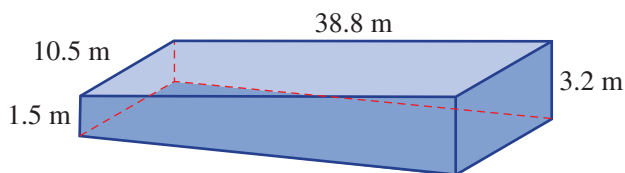
- 11 A swimming pool is the shape of a rectangular prism as shown. The swimming pool is filled to 25 cm from the top.

- a** What is the volume of water in cubic metres?
- b** How much water does the swimming pool contain, to the nearest kilolitre?



- 12** A swimming pool is the shape of a trapezoidal prism as shown.

- a** Find the volume of the pool in cubic metres.
- b** What is the capacity of the pool in kilolitres?
- c** In hot weather the level of the pool dropped by 75 cm. The pool was originally full. What volume of water, in litres, evaporated?



Communicate your reasoning, understanding and fluency

- 13** The surface area of Warragamba Dam is 75 km^2 .
- a** Convert this to m^2 .
- b** If 10 mm of rain falls onto the surface area of the dam (assuming no evaporation), how many kilolitres is this?
- 14** Which holds more water and by how much? Answer to the nearest litre.
- i** A spherical water tank with a diameter of 3 m.
- ii** A 2-metre-high cylindrical water tank with a radius of 1.4 m.

3Q Volume of capacity and composite solids

Learning intentions

- To visualise the three-dimensional solids which form a composite solid
- To convert volume measurements to millilitres or litres
- To calculate the volume and capacity of composite solids
- To solve practical problems involving volume and capacity

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about volume and capacity regularly appear in HSC Examinations
- Some measurement formulas for length, area, surface area and volume are provided on the 2027 HSC Reference Sheet, but others are not

Volume of composite solids

To calculate the volume of most common solids we use a formula. Some of these formulas are listed earlier in this chapter. Composite solids are made up of two or more common solids. The volume of a composite solid is calculated by adding or subtracting the volumes of each common solid.

Volume of composite solids

- 1 Divide the composite solid into two or more common solids.
- 2 Calculate the volume of each common solid.
- 3 Add or subtract the volumes of the common solids.
- 4 Write the answer to the specified level of accuracy with the correct units.



Example 45 Finding the volume of a composite solid

A grain silo consists of a cylinder with a cone on top, as shown. The diameter of the cylinder is 16 m and its height is 20 m. The diameter of the base of the cone is the same as that of the cylinder, and its height is 2 m. What is the volume of the silo correct to one decimal place?



Solution

$$\begin{aligned}
 V &= \frac{1}{3}\pi r^2 h \text{ (Cone)} \\
 &= \frac{1}{3} \times \pi \times 8^2 \times 2 \\
 &= 134.0412866 \dots \text{ m}^3
 \end{aligned}$$

$$\begin{aligned}
 V &= \pi r^2 h \text{ (Cylinder)} \\
 &= \pi \times 8^2 \times 20 \\
 &= 4021.238597 \text{ m}^3
 \end{aligned}$$

$$\begin{aligned}
 V &= \text{Cone} + \text{Cylinder} \\
 &= 134.0412\dots + 4021.238\dots \\
 &\approx 4155.3 \text{ m}^3
 \end{aligned}$$

Volume of silo is 4155.3 m^3 .

Explanation

Use the volume formula for a cone $V = \frac{1}{3}\pi r^2 h$.

Substitute $r = 8$ and $h = 2$ into the formula.

Evaluate.

Use the volume formula for a cylinder $V = \pi r^2 h$.

Substitute $r = 8$ and $h = 20$ into the formula.

Evaluate.

Add the volume of the cone to the volume of the cylinder.

Evaluate.

Write the answer correct to one decimal place and with correct units.

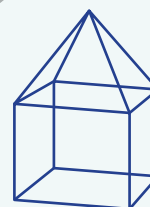
Now you try

A greenhouse consists of a square-base pyramid on top of a cube as shown.

The dimensions of the cube and base of the pyramid is 2 metres by 2 metres.

The height of the pyramid and cube are 2 metres each.

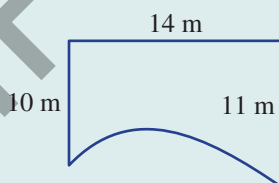
What is the volume of the greenhouse?



 Example 46 Using trapezoidal rule to estimate the capacity

The following irregular shape is the floor plan of a swimming pool with a uniform depth of 2 metres.

- Using the trapezoidal rule, find the approximate volume, in cubic metres, of the swimming pool.
- What is the capacity of the pool, in litres?

**Solution**

$$\begin{aligned}
 \text{a } A &\approx \frac{h}{2}(d_f + d_r) \\
 A &\approx \frac{14}{2}(10 + 11) \\
 A &\approx 147 \text{ m}^2 \\
 V &= Ah \\
 V &\approx 147 \text{ m}^2 \times 2 \text{ m} \\
 V &\approx 294 \text{ m}^3
 \end{aligned}$$

Explanation

Write the formula for trapezoidal rule.

Substitute the values into the formula.

Evaluate to find the approximate area.

Volume = area $\times$ depth of water

Substitute in the depth of the water.

Evaluate the approximate volume of the pool.

b Capacity = $294\text{ m}^3 = 294\text{ kL} = 294\,000\text{ L}$

The capacity of the pool is approximately 294 000 L.

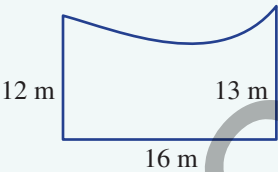
Convert $1\text{ m}^3 = 1\text{ kL} = 1000\text{ L}$

Answer in words.

Now you try

The following irregular shape is the floor plan of a swimming pool with a uniform depth of 2 metres.

- a** Using the trapezoidal rule, find the approximate volume, in cubic metres, of the swimming pool.
- b** What is the capacity of the pool, in litres?



Exercise 3Q

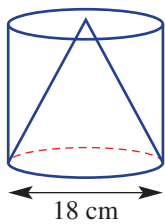
BUILDING

Example 45

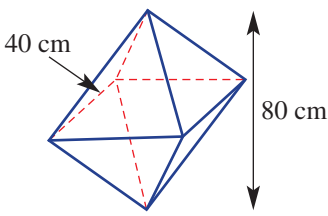
- 1** A hemisphere of ice cream is placed on a waffle cone. The waffle cone has a radius of 3.5 cm, a perpendicular height of 12 cm and a slant height of 12.5 cm.
Answer the following questions to nearest cubic centimetre.
 - a** What is the volume of the waffle cone?
 - b** What is the volume of the hemisphere of ice cream?



- 2** A cone sits exactly inside a cylinder as shown. The height of the cone is 16 cm.
 - a** What is the volume of the cylinder to the nearest cubic centimetre?
 - b** What is the volume of the cylinder not taken up by the cone? Answer to the nearest cubic centimetre.



- 3** A composite solid consists of two square pyramids with their bases touching, as shown. The side length of the base is 40 cm and the height of the solid is 80 cm. Find the volume of the solid correct to the nearest cubic centimetre.

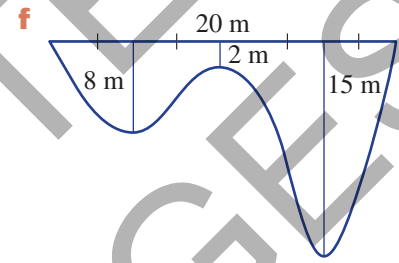
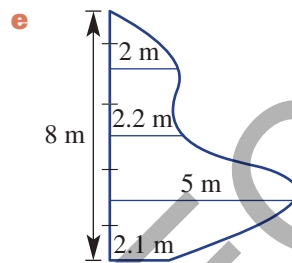
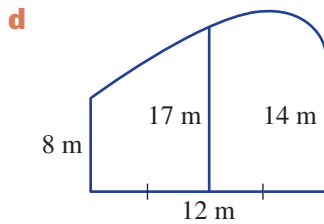
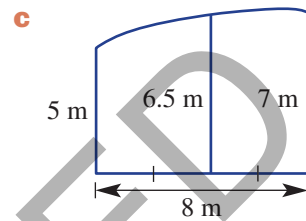
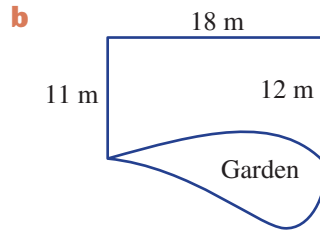
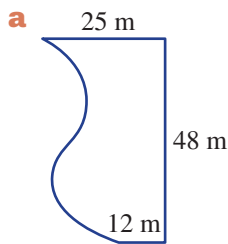


- 4** A cylindrical oil drum has a diameter of 80 cm and a height of 1.5 m. How many litres of oil can it hold, to the nearest litre?

DEVELOPING

Example 46

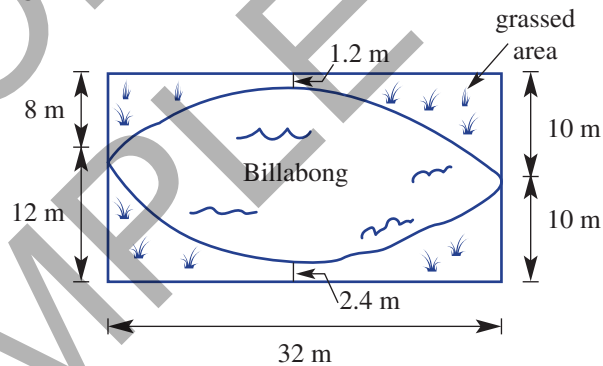
5 The following irregular shapes are plans for swimming pools.



- Use the trapezoidal rule to estimate the area of each pool.
- If each of the swimming pools have a uniform depth of two metres, estimate the volume of each swimming pool in cubic metres.
- How many litres does each pool hold?

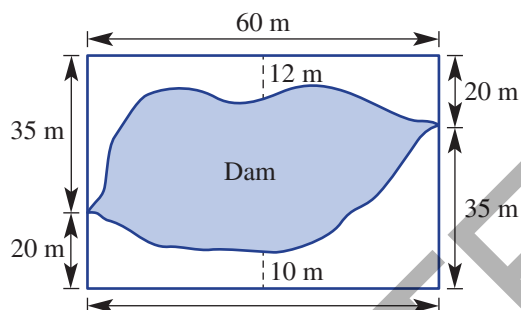
CHALLENGING

6 This is a plan of a billabong.

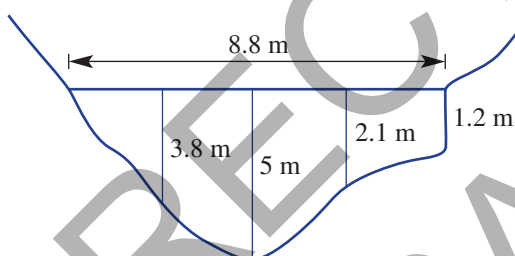


- Use four applications of the trapezoidal rule to estimate the grassed area around the billabong.
- Estimate the area of the billabong.
- If the billabong has a uniform depth of 95 cm, find the volume of the billabong in litres.

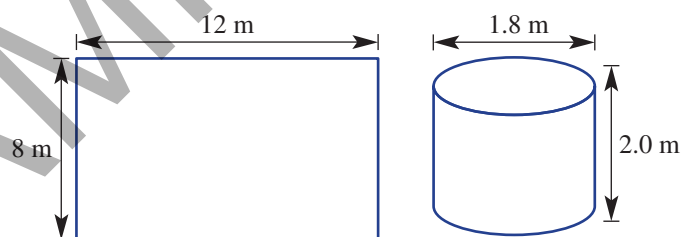
- 7 a** Using the trapezoidal rule, estimate the area of the dam.
- b** If 10 mm of rain falls overnight on the surface of the dam, approximately how many litres of water filled the dam?



- 8** This is a cross-section of a river.



- a** Use four applications of the trapezoidal rule to estimate the area of the cross-section of the river.
- b** If a part of the river is 120 metres long, estimate the volume of water in this section of the river, in cubic metres.
- 9** A cylindrical container with a diameter of 10 cm and a height of 20 cm is filled with water. A spherical ball with a diameter of 8 cm is then submerged completely in the water.
- a** How much water overflows from the container? Answer to the nearest millilitre.
- b** When the spherical ball is removed, what is the height of the water left in the cylindrical container?
- 10** A flat rectangular roof is connected to a cylindrical rainwater tank. All rainwater fallen onto the roof is collected into the tank.

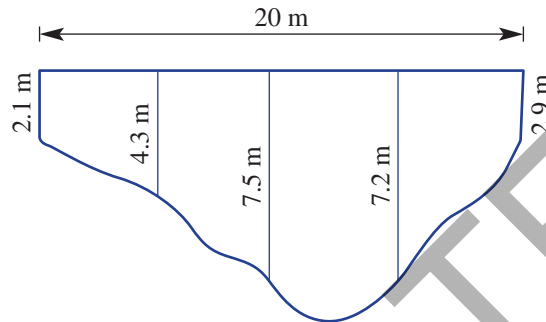


If 35 mm of rain falls on the roof overnight:

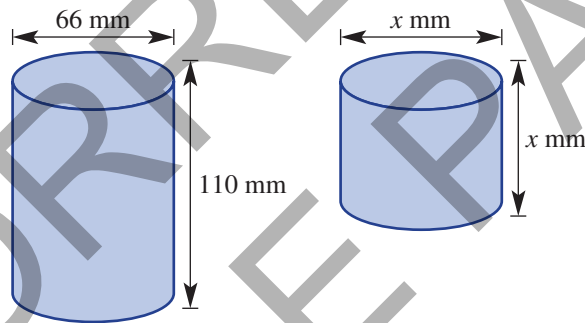
- a** how high does this fill the water tank?
- b** how full, to the nearest percent, is the rainwater tank?

Communicate your reasoning, understanding and fluency

- 11** Here is a cross-section of a canal that is 20 metres wide.



- a** Use four applications of the trapezoidal rule to estimate the area of the cross-section of the canal.
b The water in the canal flows at 0.5 metres per second. Calculate the approximate volume of water flowing through the cross-section of the canal in 10 seconds. Answer in kilolitres.
- 12** These are the dimensions of two 375 mL cans of soft drink.



- a** With the same volume of 375 mL, what would the dimensions of the second can be if the length of the diameter equals the height of the can? Answer to two decimal places.
b Find the curved surface area of each can.
c With the change in dimension, what is the percentage decrease in the curved surface area from the original dimensions to the new dimensions? Answer to the nearest percent.

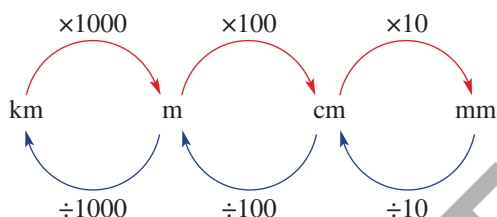
Key ideas and chapter summary

Pythagoras' theorem

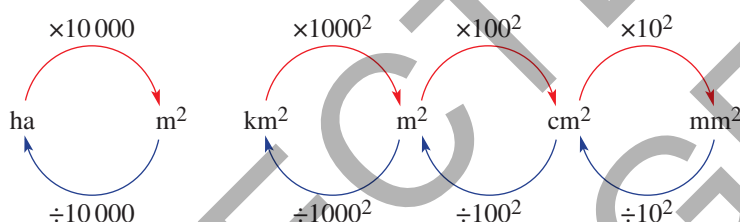
$$(\text{Hypotenuse})^2 = (\text{side})^2 + (\text{other side})^2$$

Converting measurements

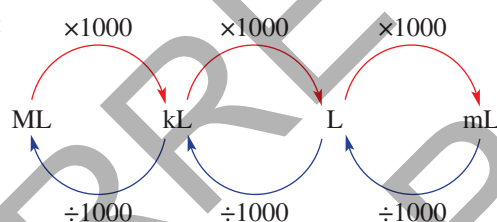
Length:



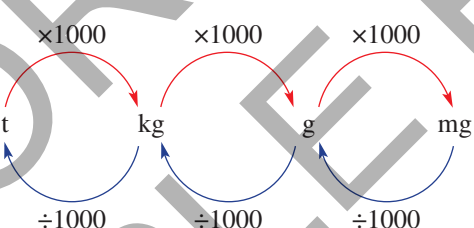
Area:



Volume:



Mass:



Scientific notation

Scientific notation is used to write very large or very small numbers more conveniently. It consists of a number between 1 and 10 multiplied by a power of 10.

Perimeter

Perimeter is the total length of the outside edges of a shape. It is the length of the boundary.

Area

Triangle $A = \frac{1}{2}bh$

Square

$A = s^2$

Rectangle $A = lb$

Parallelogram $A = bh$

Trapezium $A = \frac{h}{2}(a + b)$

Rhombus $A = \frac{1}{2}xy$

Area of circles and sectors

Circle $A = \pi r^2$

Semicircle $A = \frac{1}{2}\pi r^2$

Annulus $A = \pi(R^2 - r^2)$

Sector $A = \frac{\theta}{360}\pi r^2$

Area of composite shapes

- 1 Divide the composite shape into two or more plane shapes.
- 2 Add or subtract the areas of the plane shapes.

Trapezoidal rule

$$A \approx \frac{h}{2}(d_f + d_l)$$

A – Area of shape
 h – Height or distance between the parallel sides
 d_f – Distance of first parallel side
 d_l – Distance of last parallel side

Surface area of prisms, cylinders, spheres, pyramids and cones

Right prisms	Sum of the area of each surface.
Open cylinder	$A = 2\pi rh$
Closed cylinder	$A = 2\pi r^2 + 2\pi rh$
Sphere	$A = 4\pi r^2$
Square pyramid	$A = s^2 + 4 \times \left(\frac{1}{2}sl\right)$
Cone	$A = \pi r^2 + \pi rl$

Surface area of composite solids

- 1 Visualise the surfaces of the solid.
- 2 Calculate the area of each face and add them together.

Volume of prisms, cylinders, spheres, pyramids and cones

Cube	$V = Ah = (s^2) \times s = s^3$
Rectangular prism	$V = Ah = lb \times h = lbh$
Triangular prism	$V = AH = \left(\frac{1}{2}bh\right) \times H$
Cylinder	$V = Ah = (\pi r^2) \times h = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Square pyramid	$V = \frac{1}{3}Ah = \frac{1}{3} \times s^2 \times h$
Rectangular pyramid	$V = \frac{1}{3}Ah = \frac{1}{3} \times lb \times h$
Cone	$V = \frac{1}{3}Ah = \frac{1}{3} \times \pi r^2 \times h$

Capacity

The amount of liquid a container can hold. Base unit is the litre.

Volume of composite shapes

- 1 Divide the composite solid into two or more composite solids.
- 2 Add or subtract the volume of the common solids.

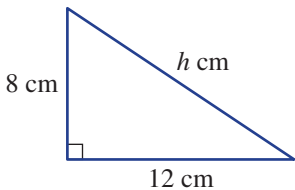
Chapter checklist with success criteria



Checklist

A printable version of this checklist is available in the Interactive Textbook.



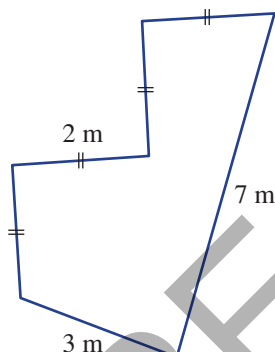
- 3A 1** I can use Pythagoras' theorem to find the missing side of a right-angled triangle. ☐
e.g. Find the length of the hypotenuse, correct to two decimal places.
- 
- 3A 2** I can apply Pythagoras' theorem to solve problems involving right-angled triangles. ☐
e.g. A TV is labelled based on its diagonal measurement. For a 155 cm TV, the diagonal measures 155 cm.
If the length of this television is 125 cm, what is its breadth?
- 3B 3** I can identify and convert between metric units of length. ☐
e.g. 5.4 m = _____ cm
- 3B 4** I can identify and convert between metric units of area. ☐
e.g. $0.5 \text{ m}^2 = \text{_____ cm}^2$
- 3B 5** I can identify and convert between metric units of volume. ☐
e.g. $0.085 \text{ m}^3 = \text{_____ cm}^3$
- 3B 6** I can identify and convert between metric units of capacity. ☐
e.g. 3500 mL = _____ L
- 3B 7** I can identify and convert between units of volume and capacity. ☐
e.g. $850 \text{ m}^3 = \text{_____ L}$
- 3B 8** I can identify and convert between metric units of mass. ☐
e.g. 8.93 kg = _____ g
- 3C 9** I can write numbers in scientific notation. ☐
e.g. Express 153 000 000 in scientific notation.
- 3C 10** I can write numbers in scientific notation, rounding using significant figures. ☐
e.g. Write 955 255 000 in scientific notation to two significant figures.
- 3C 11** I can apply scientific notation to represent numbers involving standard prefixes with and without the required number of significant figures. ☐
e.g. Using scientific notation, write 8 micrometres into metres.

- 3D 12** I can find the perimeter of triangles, rectangles, parallelograms, trapeziums, circles, sectors of circles and composite shapes. □

e.g. Find the perimeter of a semicircle with a diameter of 6.4 m. Answer correct to two decimal places.

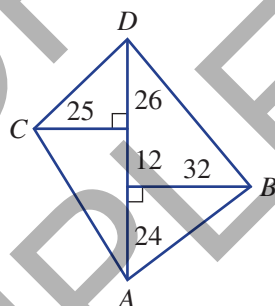
- 3E 13** I can solve practical problems requiring the calculation of perimeter of triangles, rectangles, parallelograms, trapeziums, circles, sectors of circles and composite shapes in a variety of contexts. □

e.g. Find the cost to install a fence around the garden at \$22 per metre.



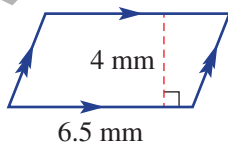
- 3E 14** I can calculate the perimeter of irregularly shaped blocks of land by dissection into triangles, rectangles and trapeziums. □

e.g. Find the perimeter of the irregular shape, if all measurements are in metres. Answer to one decimal place.



- 3F 15** I can find the area of triangles and quadrilaterals. □

e.g. Find the area of the parallelogram.

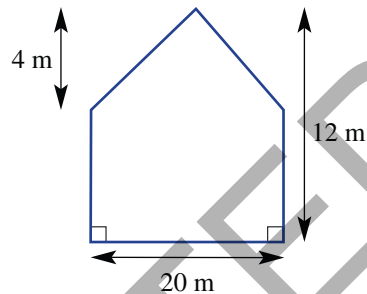


- 3G 16** I can find the area of circles, sectors and annulus. □

e.g. Find the area of a circle with radius 7 metres. Answer correct to one decimal place.

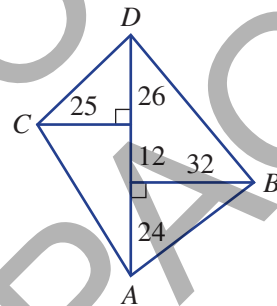
- 3H 17** I can solve practical problems requiring the calculation of the area of triangles, rectangles, parallelograms, trapeziums, circles, sectors of circles and composite shapes in a variety of contexts. □

e.g. Jeff wants to paint the side of his house.
1 litre of paint covers 16 m^2 . How many litres of paint will Jeff need to buy?



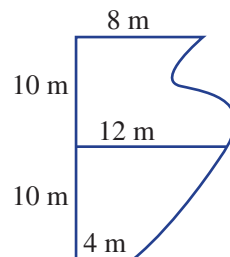
- 3H 18** I can calculate the area of irregularly shaped blocks of land by dissection into triangles, rectangles and trapeziums. □

e.g. Find the area of the irregular shape, if all measurements are in metres. Answer to one decimal place.



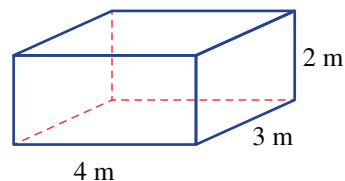
- 3I 19** I can apply the trapezoidal rule to solve a variety of practical problems involving area for up to four applications. □

e.g. Apply the trapezoidal rule twice to approximate the area of the block of land. Answer to the nearest square metre.



- 3J 20** I can find the surface area of right prisms. □

e.g. Find the surface area of this rectangular prism.



- 3J 21** I can solve practical problems involving the surface area of right prisms. □

e.g. Design a gift box in the shape of a rectangular prism with a surface area of 300 cm^2 .

3K 22 I can find the surface area of cylinders and spheres. □

e.g. A closed cylinder has a diameter of 18 mm and a height of 30 mm.

a Find the area of the curved surface.

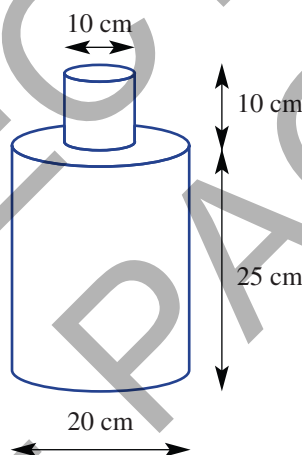
b Find the surface area of the cylinder to two decimal places.

3K 23 I can solve practical problems involving the surface area of cylinders and spheres. □

e.g. Find the surface area of a sphere with radius 12 cm. What happens to the surface area if I double the radius?

3L 24 I can solve practical problems involving the surface area of composite solids in a variety of contexts. □

e.g. A birthday cake is made from two round cakes. Calculate the visible surface area to be iced, in square centimetres.

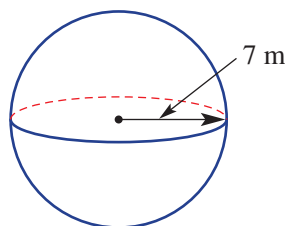


3M 25 I can find the volume of right prisms. □

e.g. A rectangular prism has a length of 12 m, a breadth of 8 m and a height of 10 m. Find the volume of this rectangular prism.

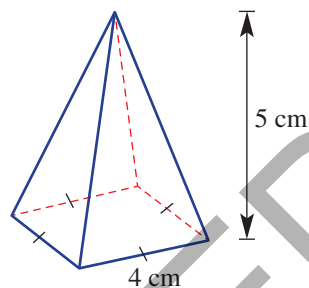
3N 26 I can find the volume of cylinders and spheres. □

e.g. Find the volume of the sphere. Answer correct to one decimal place.



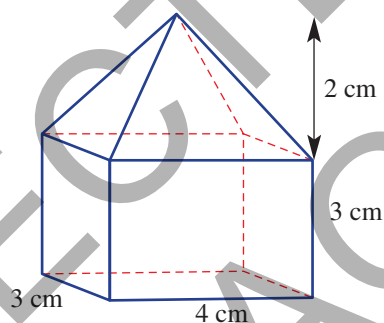
30 27 I can find the volume of pyramids and cones. □

e.g. Find the volume of a square pyramid of height 5 cm and a base of 4 cm. Answer to the nearest cubic metre.



30 28 I can find the volume of composite solids. □

e.g. Find the volume of the composite solid.



3P 29 I can find the capacity of a 3D solid. □

e.g. A container 65 cm long, 30 cm wide and 30 cm high is filled with water.

a Find the volume of the container in cubic centimetres.

b Find the capacity of the container in litres.

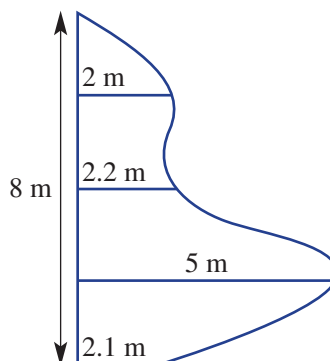
3Q 30 I can solve practical problems involving volume and capacity of prisms, cylinders, spheres, cones, pyramids and composite shapes in a variety of contexts. □

A cylindrical oil drum has a diameter of 80 cm and a height of 1.5 m.

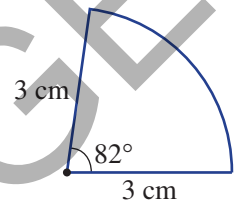
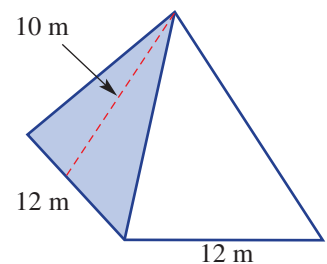
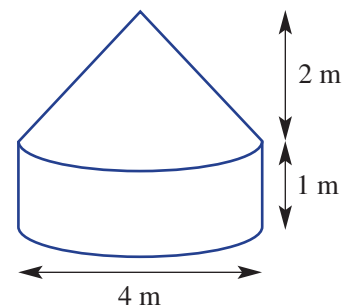
How many litres of oil can it hold, to the nearest litre?

3Q 31 I can apply the trapezoidal rule to solve a variety of practical problems involving volume and capacity for up to four applications. □

e.g. If the swimming pool has a uniform depth of two metres, estimate the volume of the swimming pool in litres.



Multiple-choice questions

3B 1 Convert 7.5 metres to millimetres.**A** 0.0075 mm**B** 75 mm**C** 750 mm**D** 7500 mm**3B 2** 6.2 m^2 is the same as:**A** 620 cm^2 **B** 0.0062 km^2 **C** $62\,000 \text{ cm}^2$ **D** 6200 mm^2 **3C 3** Write 4 500 000 in standard form.**A** 4.5×10^{-6} **B** 4.5×10^{-5} **C** 4.5×10^5 **D** 4.5×10^6 **3D 4** What is the perimeter of a quadrant with a radius of 5 mm?**A** 3.9 mm**B** 7.9 mm**C** 13.9 mm**D** 17.9 mm**3G 5** What is the area of the sector?**A** 1.6 cm^2 **B** 1.6 cm^2 **C** 6.4 cm^2 **D** 28.3 cm^2 **3J 6** What is the surface area of a rectangular prism with a length of 2.5 m, a breadth of 3 m and a height of 2.8 m?**A** 8.3 m^2 **B** 21 m^2 **C** 29 m^2 **D** 45.8 m^2 **3K 7** What is the surface area of a sphere with a diameter of 16 mm?**A** 804 mm^2 **B** 2145 mm^2 **C** 3217 mm^2 **D** $17\,157 \text{ mm}^2$ **3O 8** What is the volume of the square pyramid?**A** 120 m^3 **B** 384 m^3 **C** 480 cm^3 **D** 1440 cm^3 **3O 9** What is the volume of the composite solid?**A** 12.6 m^3 **B** 12.6 m^3 **C** 20.9 m^3 **D** 83.8 m^3 **3P 10** A cubic water tank has a side length of 6 m. What is the capacity of the tank?**A** 36 kL**B** 216 kL**C** 360 kL**D** 216 000 kL

Short-answer questions

3B 1 Complete the following.

a $500 \text{ cm}^2 = \square \text{ m}^2$

b $4000 \text{ cm}^2 = \square \text{ mm}^2$

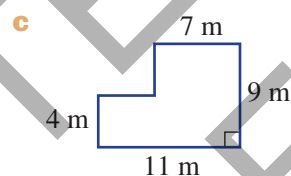
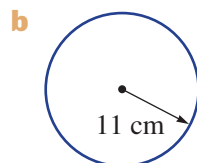
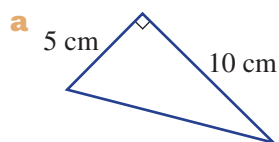
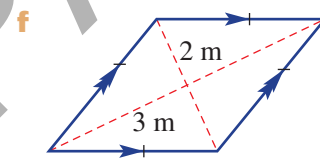
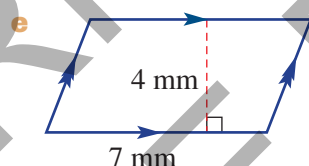
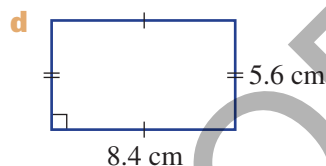
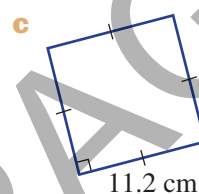
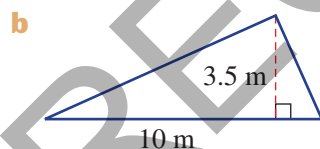
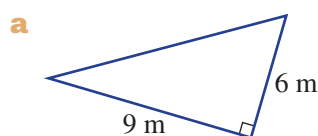
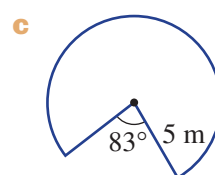
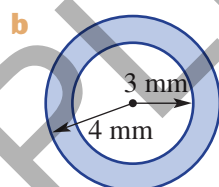
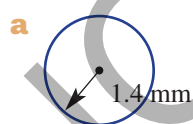
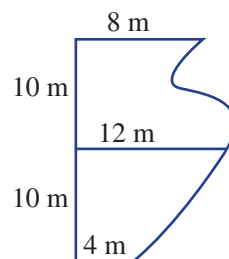
c $3 \text{ km}^2 = \square \text{ m}^2$

3C 2 Evaluate the following and express your answer in standard form.

a $(7.2 \times 10^5) \times (2.1 \times 10^4)$

b $\frac{4.6 \times 10^4}{2.3 \times 10^{-2}}$

c $\frac{5.8 \times 10^{-5}}{2.9 \times 10^4} \times (3.5 \times 10^6)$

3D 3 Find the perimeter of each shape. Answer correct to one decimal place.**3F 4** Find the area of each shape. Answer correct to one decimal place.**3G 5** Find the area of each shape. Answer correct to one decimal place.**3I 6** The diagram shows a vertical cross-section of an ornament. Estimate the area of the cross-section by applying the trapezoidal rule twice. Answer correct to the nearest square metre.**3K 7** A label covers the curved surface of a cylinder. The label does not overlap. The cylinder has a radius of 8 cm and a height of 12 cm. What is the area of the label? Answer correct to four significant figures.**3L 8** A car hose is an open cylinder with a radius of 3 cm and a length of 14 cm. Calculate the area of the outer surface, correct to two decimal places.

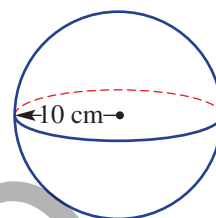
- 9 A sphere has a radius of 10 cm.

3K

a What is the surface area of the sphere? Answer correct to the nearest whole number.

3N

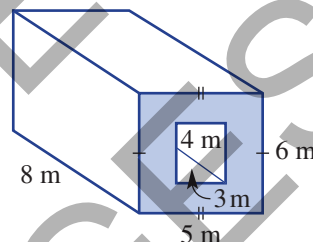
b What is the volume of the sphere? Answer correct to the nearest whole number.



- 3P 10 Find the capacity of a triangular prism with a height of 50 cm and a base area of 120 cm^2 . Answer in litres.

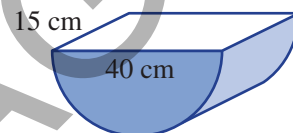
- 3Q 11 A composite solid is shown.

- a What is the volume of the outer rectangular prism?
 b What is the volume of the inner rectangular prism?
 c What is the volume of the composite solid?



- 3Q 12 A semicircular trough has a diameter of 40 cm and a breadth of 15 cm.

- a What is the cross-sectional area of the trough? Answer to the nearest square centimetre.
 b What is the volume of water for the trough? Answer to the nearest cubic centimetre.
 c How many litres of water will fill the trough? Answer correct to one decimal place.



Extended-response questions

- 1 A pool with dimensions 12 m by 5 m is surrounded by timber decking in the shape of a trapezium.

3H

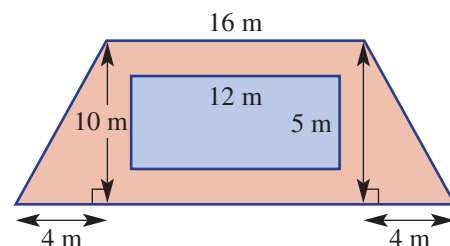
a What is the area of the timber decking?

3M

b The pool has a constant depth of 2 m. What is the volume of the pool?

3L

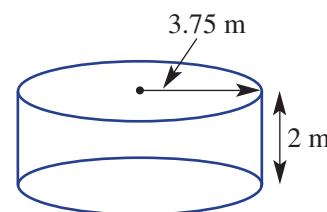
c The interior of the pool is to be painted blue. What is the surface area to be painted?



3Q

- 2 A cylindrical water tank has a radius of 3.75 metres and is 2 metres deep.

- a What is the volume of the cylindrical tank? Answer correct to the nearest cubic metre.
 b The tank provides water for the sprinklers to water the garden. Sprinklers use water at a rate of 5000 litres per hour. How long will it take the sprinklers to empty a full tank?



Data analysis 1

What is a sports statistician?

Professional sport is very much a results-driven industry. Every team in the major sports employs the services of sports statisticians to collect and analyse data which is reported to players, coaches and commentators.

According to salaryexpert.com, in the year 2025, sports statisticians in Australia began their career on \$100 000 per annum and senior statisticians earn more than \$170 000 plus bonuses.

Professional football players wear GPS tracking devices during training and games. These devices collect and transmit data that is used by statisticians to:

- provide feedback to players about their performance
- plan more effective training sessions
- design off-season programs to maintain fitness
- make tactical decisions during games using live performance data.

Chapter contents

- 4A The statistical investigation process
- 4B Classification of data
- 4C Population and sample
- 4D Dot plots and stem-and-leaf plots
- 4E Line graphs, sector graphs and divided bar charts
- 4F Ungrouped and grouped frequency tables
- 4G Cumulative frequency
- 4H Frequency and cumulative frequency graphs
- 4I Real-world applications

NSW Syllabus

In this chapter, a student:

- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- Displays and analyses datasets using summary statistics and graphical representations (MST-11-08)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

4A The statistical investigation process

Learning intentions

- To understand the process used for collecting, organising, displaying, summarising and analysing data in a responsible and ethical manner

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated in Year 12
- Data analysis is extended in Chapter 6 of this book and also in Year 12
- Questions about this process sometimes appear in HSC Examinations

Statistical investigation is the process of gathering statistics. The information gained from a statistical investigation is a vital part of our society. A statistical investigation involves four steps.

Four steps in a statistical investigation

1 Collect the data

Collecting data involves deciding what to collect, locating it and collecting it. The gathering of statistical data may take the form of a:

- census – data is collected from the whole population (explained in more detail in 4c)
- survey – data is collected from a smaller group of the population (explained in more detail in 4c)

It is important that procedures are in place to ensure the collection of data is accurate, up-to-date, relevant and secure. If the data collected comes from unreliable sources or is inaccurate, the information gained from it will be incorrect. When taking a sample, the data gathered must be representative of the entire population otherwise the information collected may be biased towards a particular outcome.

2 Organise the data

Organising data is the process of arranging, representing and formatting data. It is carried out after the data is collected. The organisation of the data depends on the purpose of the statistical investigation. For example, to store and search a large amount of data, the data needs to be categorised. Organising gives structure to the data.

3 Summarise and display the data

Displaying data is the presentation of the data and information. Information must be well organised, readable, attractively presented and easy to understand. Information is often displayed using graphs such as scatterplots, dot plots, histograms, line graphs, stem-and-leaf plots and box plots. Data is summarised using statistics such as the mean, median, mode and standard deviation.



4 Analyse the data

Analysing data is the process of interpreting data and transforming it into information. It involves examining the data and giving meaning to it.

Statistical investigation

A statistical investigation involves four steps: collecting data, organising data, summarising and displaying data, and analysing data.



Example 1 A statistical investigation

Uma has been asked to do a statistical investigation into the amount of screen time of students in Year 11 in one week. Uma performed the following steps.

- 1 Collecting the data: Uma used a short survey to collect the data.
- 2 Organising the data: Uma sorted the data by days of the week.
- 3 Summarising and displaying the data: Uma used the frequency table to display the data.

Day	Hours of screen time
Sunday	10
Monday	3
Tuesday	4
Wednesday	3
Thursday	5
Friday	7
Saturday	10
Total	42

4 Analysing the data:

- a Uma calculated the total hours of screen time in a week. Total hours of screen time = 42 hours
- b Uma calculated the mean hours of screen time each day.
 Mean hours of screen time each day
 $= \frac{42}{7} = 6 \text{ hours}$

Now you try

Use the data from Example 1 to answer these questions.

- a On which days did Year 11 students have 7 or more hours of screen time?
- b What is the mean screen time for weekdays?
- c What is the mean screen time for weekends?

Issues in a statistical investigation

A statistical investigation raises a number of ethical issues such as bias, accuracy, ethics privacy and responsiveness to diverse groups.

- Data needs to be free from bias. Bias means that the data is unfairly skewed or gives too much weight to a particular result. For example, if a survey about favourite music was only completed by teenagers, and the results were generalised to the entire population, it would have a bias. Several checks should be made to limit the impact of bias.
- The accuracy of the collected data is a vital ingredient of a statistical investigation. It depends on the source of the data and whether the data has been recorded correctly. The accuracy of the data is often difficult to check in a reasonable time. It is often necessary to compare data from a number of different sources and determine which data is accurate.
- Ethics deals with making appropriate moral decisions within an investigation as well as behaving within the scope of the copyright laws.

- Copyright is the right to use, copy or control the work of authors and artists. It is against the law to infringe copyright. You are not allowed to use or copy the work of another person without their permission. If data is collected from the internet, it should be assumed to be protected by copyright.
- Privacy is the ability of an individual to control personal data. Data collected on individuals is not always accurate. Inaccuracies can be caused by mistakes in gathering or entering the data, by mismatch of the data and the person or by information being out-of-date. Most people give information about themselves to selected parts of the outside world. Often people are quite willing to tell A something but would be shocked if B knew. But what prevents A telling B?
- Being responsive to diverse groups within our society is important in statistical investigation. Diverse groups cannot be ignored and should be taken into account and included in all investigations. For example, the investigation of student heritage becomes far more relevant and inclusive when all student data is included rather than just taking a sample.

Issues in a statistical investigation

A statistical investigation raises a number of ethical issues such as bias, accuracy, copyright and privacy.

Exercise 4A

BUILDING

1 Copy and complete the following sentences:

- A statistical _____ involves four steps: collecting data, organising data, summarising and displaying data, and analysing data.
- Census data is collected from the whole _____.
- When taking a _____ the data gathered must be representative of the entire population.
- Displaying data is the _____ of the data and information.
- When analysing _____ data the form, direction and strength of the association is determined.
- Bias means that the data is unfairly _____ or gives too much weight to a particular result.

2 Describe the following statements as true or false.

- A survey is when data is collected from a smaller group of the population.
- Data collected from unreliable sources results in incorrect information.
- Data is often displayed using graphs such as scatterplots, dot plots, histograms, line graphs, stem-and-leaf plots and box plots.
- Analysing data is the process that interprets data, transforms it into information.
- You are allowed to use or copy the work of another person without their permission.
- Data collected on individuals is always accurate.

3 The Australian Bureau of Statistics collects data for our society. Collecting data is one step in a statistical investigation. List all the four steps involved in a statistical investigation.

4 Explain the difference between a census and a sample.

DEVELOPING

- 5** How can you limit the impact of biased data?
- 6** Do an investigation into the most popular streaming services in your group of friends. Use a maximum of 10 people. Clearly outline the four stages of statistical investigation.
- 7** Explain how you considered the issues of bias, accuracy, ethics and responsiveness to diverse needs in your investigation in Question **6**. If you did not consider them, rework your response to Question **6** to take each into consideration.

CHALLENGING

- 8** The issue of privacy has been a concern for decades. Discuss this statement. What is your view on privacy? Use the internet to find information about privacy. Compare Australian and overseas data.

Communicate your reasoning, understanding and fluency**Example 1**

- 9** Implement a statistical investigation to solve these problems.
 - a** Students in Year 12 have decided to have a party after the HSC at a local venue. You have been given the responsibility of choosing the venue. Investigate a number of different venues and collect data such as the name and address of the venue; the cost per head; the initial deposit required; details of meals; and available dates. Design a survey for Year 12 students that will decide on the venue. Students will vote on the venue and if they are committed to attend. The survey should also decide on possible dates and meal arrangements.
 - b** The principal would like to promote the positive features of your school outside the local community. He is presenting these features at a conference centre. Develop a solution for the principal. Design a survey for Year 12 students that will decide on the best five features of your school and use the most appropriate method of presentation.

4B Classification of data

Learning intentions

- To understand that some datasets are categorical and some are numerical
- To understand the difference between categorical data which is nominal or ordinal
- To understand the difference between numerical data which is discrete or continuous

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated in Year 12
- Data analysis is extended in Chapter 6 of this book and also in Year 12
- Questions about data classification sometimes appear in HSC Examinations

There are many types of data that can be collected. For example, if you ask six friends how many pets they own, they might give you the following data: 1, 0, 2, 4, 1, 15. However not all data are numbers. For example, if you also record the gender of each of your friends, you get the following data: male, male, female, female, female, male. Data is divided into two broad classifications: categorical and numerical.

Categorical data

Categorical data represents characteristics such as a person's gender, marital status, address or music they like. Categorical data can take on numerical values (such as '0' for unsatisfactory and '1' for satisfactory), but those numbers don't have mathematical meaning.

Categorical data is further classified as nominal or ordinal.

- Nominal data uses a name or label that does not indicate order. For example, a student's gender could be classified as an 'F' for female and an 'M' for male.
- Ordinal data uses a name or label that does indicate order. For example, the quality of work could be classified as an 'A' for excellent, 'B' for good and 'C' for satisfactory. It shows a sequence A, B and C.



Categorical data has no quantity or amount associated with each category.

Numerical data

Numerical data indicates a quantity and is used to perform calculations. For example, if we asked each student in the class their height, we would expect to get a variety of answers. However each answer is a number.

Numerical data is further classified as discrete or continuous.

- Discrete data is data that can only take exact numerical values. For example, the number of sisters will give rise to numbers such as 0, 1 or 2. Counting a quantity often results in discrete data.
- Continuous data is data that can take any numerical value (depending on the degree of accuracy). For example, a student's height will give rise to numbers such as 171.2 cm and 173.5 cm. Measuring a quantity often results in continuous data.



Classification of data

- 1** Categorical data – data is classified by the name of the category it belongs to.
 - a** Nominal data – name does not indicate order.
 - b** Ordinal data – name does indicate order.
- 2** Numerical data – data indicates a quantity and is used to perform calculations.
 - a** Discrete data – data can only take exact numerical values.
 - b** Continuous data – data can take any numerical value.



Example 2 Classifying data as categorical or numerical

Classify the data from these situations as categorical or numerical.

- a** The heart rate of a group of personal trainers
- b** The most watched television show in Australia
- c** The number of people living in Smith Ave
- d** The reasons for people travelling to work by train

Solution

- a** The heart rate is numerical data.
- b** A television show is categorical data.
- c** The number of people living in Smith Ave is numerical data.
- d** The reasons for travelling to work is categorical data.

Explanation

Heart rate is a number.

Television shows are sorted in categories.

Number of people is a number.

The reason for travelling to work by train are sorted in categories.

Now you try

Classify the data from these situations as categorical or numerical.

- a** The hair colour of students in the class
- b** The number of chairs in each room at school
- c** The places travelled to in the holidays
- d** The time taken in days to complete the house



Example 3 Classifying data as nominal or ordinal

Classify the data from these situations as nominal or ordinal.

- a** School year
- b** Internet use at home

Solution

- a** School year is ordinal data.
- b** How the internet is used at home is nominal data.

Explanation

Year level at school is in order.

The types of internet use at home can be sorted into categories. However, the categories have no order, so internet use is nominal.

continued ...

Now you try

Classify the data from these situations as nominal or ordinal.

- a The shoe size of people in your family
- b The apps you have on your phone



Example 4 Classifying data as a discrete or continuous

Classify the data from these situations as discrete or continuous.

- a The number of pets in your family
- b The perimeter of the school grounds

Solution

- a The number of pets is discrete data.
- b The perimeter of the school is continuous data.

Explanation

Pets are counted as exact whole numbers.
Perimeter is a measurement of distance and so is continuous.

Now you try

Classify the data from these situations as discrete or continuous.

- a The number of strawberries in the garden
- b The height of the doors in your house

Exercise 4B

BUILDING

Example 2

- 1 Classify the data from these situations as categorical or numerical.
 - a The favourite colours of Jenny’s friends
 - b The number of people travelling in a car
 - c The weight of each student in the year in kilograms
 - d People rating their doctor on personal service (high, medium or low)
 - e The number of students in each class
 - f The IQ of a group of students
 - g Responses to a survey question (agree or disagree)
 - h A person’s lucky number
 - i A female’s favourite mobile phone
 - j The distance from Sydney to Wollongong
 - k The cost of bread at the supermarket
 - l The community’s preferred leader
 - m The number of computers in the school

Example 4

2 Classify the following numerical data as discrete or continuous.

- a** The price paid for a can of soft drink
- b** The number of people at a concert
- c** The time between trains
- d** The number of pages in the newspaper
- e** The amount of water used in the past month
- f** The number of people in your immediate family
- g** The numbers drawn in this week's lotto
- h** The length of the cricket pitch
- i** The distance measured for the long jump at the world championships
- j** The score achieved from a quiz consisting of 10 questions
- k** The height of the tallest person in the world



3 State whether the following is categorical, discrete or continuous data.

- a** The heights of members of a football team
- b** The distance to drive to the train station
- c** The different types of ice creams
- d** The quality of food in a restaurant
- e** The eye colour of a group of people
- f** The number of pets in a household
- g** The time to swim 50 metres
- h** The number of goals scored in the first match of the season
- i** Today's most fashionable style of dress
- j** The number of computers in the building
- k** Replies given to a questionnaire (Yes or No)
- l** The perimeter of Joel's block of land
- m** The breadth of the Anzac Bridge
- n** The number of people killed on the roads due to speed
- o** The stopping distance for a car travelling at 60 km/h
- p** The most popular type of car sold in the past 12 months

**Example 3**

4 A hospital measures the weight and length of every new baby.

- a** Classify the data as numerical or categorical.
- b** If the data is numerical, is this data discrete or continuous? If the data is categorical, is it nominal or ordinal?

5 A coffee shop is conducting a survey on the drinking habits of its customers. One of the questions was: 'How many cups of coffee do you drink each day?'

- a** State whether the data is numerical or categorical.
- b** If the data is numerical, is this data discrete or continuous? If the data is categorical, is it nominal or ordinal?

6 The government collected data on its latest policy proposal. The people surveyed answered 10 questions and were given three choices for each question: Agree, Disagree or Not sure. What type of data has the government collected?

DEVELOPING

- 7 In a survey customers were asked to rate the service they received by selecting one of the following:
outstanding, excellent, good, satisfactory or needs improvement

Describe the type of data that would result from this question.

- 8 A teacher marks a class’s assessment task and awards a mark out of 100 for each student.
Describe the type of data that that has been collected by the teacher.

- 9 A marketing poll was conducted that asked about a person’s employment status:
unemployed, receiving education, part-time job or full-time job

Describe the type of data that would result from this poll.

- 10 Emma is planning to build a new restaurant. She conducted a survey of the community. One of the questions asked was ‘How far in kilometres would you be prepared to travel to get to a good restaurant?’ The options given were:

5 km, 10 km, 20 km, 50 km

Describe the type of data that would result from this question.

- 11 The local community recorded the amount of rainfall each day for the past 3 months. Some of the data is shown in the table.
Describe the data collected by the local community.

Rainfall	
23rd April	10 mm
24th April	0 mm
25th April	25 mm

CHALLENGING

- 12 The police department collected data on fatal crashes. One of the questions it asked was: ‘What was the age of the driver involved in a fatal crash?’

a Describe the type of data collected by the police department.

b The question was modified to give the interviewee six choices:

17–24 years, 25–34 years, 35–44 years, 45–54 years, 55–64 years, 65+ years

Describe the type of data that would result from these choices.



Communicate your reasoning, understanding and fluency

- 13 The post office collects data on parcels it delivers. Categorise each type of data as categorical or nominal. If it is numerical also classify it as discrete or continuous.

- i a The colour of the parcel
- b The parcel size in kilograms
- c The distance the parcel has to travel
- d The postcode where the parcel is going
- ii Explain why the postcode in part d is difficult to categorise.

4C Population and sample

Learning intentions

- To understand the difference between a census and sample
- To understand different sampling strategies

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated in Year 12
- Data analysis is extended in Chapter 6 of this book and also in Year 12
- Questions about data sampling sometimes appear in HSC Examinations

Data collection involves deciding what data to collect, locating it and collecting it. Data comes from either primary or secondary sources.

- Primary sources – interviewing people, conducting questionnaires or observing a system in operation
- Secondary sources – data collected or created by someone else, such as information gathered from newspapers, books and the internet.

It is important that procedures are in place to ensure the collection of data is accurate, up-to-date, relevant and secure. If the data collected comes from unreliable sources or is inaccurate, the information gained from it will be incorrect.

Data collection methods are used for both a census and a sample survey.

- A **census** is a survey of every person in a population. For example, all the students in your school are regarded as the population. A census can be very expensive and time-consuming, if the population is large.
- A **sample** is only part of a population. For example, a sample of the school population is the students in your class. Estimates are made about the population based on the sample. A sample is cheaper than a census but not as accurate. Bias can also be an issue. A sample must be large enough to give a good representation of the population, but small enough to be manageable. There are many different types of sampling including a random sample, a stratified sample and a systematic sample.

Random sample

A random sample occurs when all members of the population have an equal chance of being selected. For example, six students are selected at random from the entire school population. Lotto is another good example of random sampling. A sample of 6 numbers is chosen from 40 numbers. Random samples are simple and easy to use for small populations. However, for large populations, it is possible to miss out on a particular group.

Stratified sample

A stratified sample occurs when categories or strata of a population are chosen and then members from each category are randomly selected. For example, one student is selected from each year 7, 8, 9, 10, 11 and 12. Each year group is a category in a stratified sample. Some other common types of categories are age, sex, religion or marital status. A stratified sample is useful when the categories are simple and easy to determine. However, care needs to be taken when selecting categories to avoid any bias in the data.

Systematic sample

Systematic sampling occurs when the population is divided into a structured sample size. For example, the students in the school population are put in alphabetical order and the 100th student, 200th student, 300th student, ... are selected. A systematic sample is often used by a manufacturer to ensure the machines are working correctly. Here the manufacturer might test a machine every 30 minutes or check the 50th item on a production line. Systematic sampling results in a gap between each selection.

Self-selected sample

In a self-selected sample, members of the population volunteer themselves. For example, six students from the entire school population offer to complete a questionnaire. A self-selected sample often occurs on the internet. There is the potential for self-selected sampling to be biased as only people who are motivated and have the time volunteer themselves.

Random sample	Stratified sample	Systematic sample	Self-selected sample
Members of the population have an equal chance of being selected.	Categories of a population are chosen. Members then are randomly selected from each category.	Population is divided into a structured sample size. Members are then selected in a certain order from this structure.	Members of the population volunteer themselves.



Example 5 Distinguishing sample types

A retirement village has 63 residents, 42 women and 21 men. Decide whether each sample of resident would be random, stratified, systematic or self-selected.

- a Every seventh resident
- b Six of the women and three of the men
- c Nine names picked from a hat containing the names of the residents
- d Residents sorted into alphabetical order and each ninth resident selected
- e Residents are divided into four age groups (51–60, 61–70, 71–80, 81–90) and two residents selected from each age group

Solution

- a Systematic sample
- b Stratified sample
- c Random sample
- d Systematic sample
- e Stratified sample

Explanation

The population has been divided into a structured sample size: 7th, 14th, 21st, ... 63rd.

The population has been divided into categories: women and men.

Sample is taken at random.

The population is divided into alphabetical order and a structured sample size taken.

The population has been divided into four age group categories and then randomly selected.

Now you try

A cricket club has 80 players: 16 are girls and 64 are boys. Decide whether each sample of the cricketers would be random, stratified, systematic or self-selected.

- a** The players who want to bowl
- b** Every sixth person
- c** Players sorted into teams and every third person selected
- d** 8 of the boys and 2 of the girls

Exercise 4C**BUILDING**

- 1** State whether a census or a sample is the most appropriate way to collect this data.

- a** Information on the shopping experience of people in the city
- b** John collecting the heights of his best friends
- c** The travelling habits of the Jones family to work
- d** Australians watching the grand final
- e** Number of people eating toast for breakfast
- f** Length of time every AAA battery lasts
- g** Number of people entering a gym between 5 p.m. and 6 p.m.
- h** Holly collecting the length of time students in her class spend on the internet
- i** The world's reaction to climate change
- j** Shop manager's reaction to a drop in sales

**Example 5**

- 2** State whether the sample is random, stratified, systematic or self-selected.

- a** A police officer breathalyses every tenth person.
- b** Each person is given a raffle ticket and the tickets are drawn out of a hat.
- c** Each person leaving a bus was given a survey form. We used the data from the people who sent it back.
- d** A business has 240 married and 120 unmarried employees. A sample was chosen to include 10 of the married and 5 of the unmarried employees.
- e** Students were sorted into alphabetical order and each third student selected.
- f** Individuals were randomly selected using their tax file number.
- g** Every 12th jogger was selected from an alphabetical list.
- h** Ten cards were randomly selected from a normal deck of cards.
- i** Ten girls and ten boys were randomly selected from a concert audience.
- j** Ten people who arrive at a shopping centre each day completed the survey.

- 3** Michael uses a random sample to survey 10% of the local community. In the local community there are 810 males and 920 females. How many people does Michael need to survey?

- 4** Amelia plans to conduct a random sample to survey the netball players in the local association. There are 2850 players in the local association and she plans to survey 171 players. What percentage of the population is her sample?



- 5** Paige uses a stratified sample to survey 5% of her school population. At the school there are 80 teachers and 1160 students.
- How many teachers should complete the survey?
 - How many students should complete the survey?
 - How many people should complete the survey?
- 6** Tyler uses a stratified sample to survey 25% of the members of his swimming club. He uses their sex as a category and selects a random group of female and male swimmers. There are 88 female swimmers and 112 male swimmers in his club.
- How many swimmers are in the entire population?
 - How many female swimmers are in the sample?
 - How many male swimmers are in the sample?
- 7** Osman uses a stratified sample to survey 7.5% of his chat room friends. He uses marital status as a category and selects a random group of married and unmarried friends. There are 200 married and 240 unmarried friends in his chat room.
- How many friends are in the entire population?
 - How many married friends are in the sample?
 - How many unmarried friends are in the sample?
- 8** Taylia uses a stratified sample to survey 20% of the senior students from her school. There are 205 Year 11 students and 180 Year 12 students. How many students should Taylia choose from Year 12?
- 9** Ming uses a stratified sample to survey $12\frac{1}{2}\%$ of the junior students from his school. There are 88 Year 7, 120 Year 8, 104 Year 9 and 128 Year 10 students.
- How many students are in the entire population?
 - How many students should Ming choose in following years?
 - Year 7
 - Year 8
 - Year 9
 - Year 10

DEVELOPING

- 10** A survey was conducted in a school on whether Australia should remain a constitutional monarchy or become a republic. The results are shown in the table.

	Male	Female	Total
Republic	51	79	130
Monarchy	23	47	70
Total	74	126	200

- a** How many males surveyed did not think Australia should change to a republic?
b What percentage of people favour changing to a republic?
c This survey is not a good random sample of all Australians. Why?
- 11** A sample of 30 students is taken from a primary school that has an enrolment of 420 students from kindergarten to Year 6. The sampling is designed so that the proportion of each year of the sample matches the population. There are 4 students from Year 1 in the sample. How many Year 1 students are there in the school population?
- 12** A store has 400 employees of which 208 are female and 192 are male. The store intends to survey 25 of its employees. A stratified survey is to be conducted.
a How many females should be surveyed?
b How many males should be surveyed?

CHALLENGING

- 13** Identify any possible issues with each of the following survey questions.
a Do you like the government's new policy? Yes/No
b Alan is a lazy boss who should be forced to pay his diligent workers more money. Agree or disagree?
- 14** Kayla surveyed a group of 15 people at the Tamworth country music festival on their music preferences. She used this data to draw conclusions for the entire population of NSW.
a Do you think her conclusions will be accurate? Give a reason.
b What would be a more appropriate method of sampling music preferences?



Communicate your reasoning, understanding and fluency

- 15** Cole created a survey with the response categories below. He surveyed 100 people.

1 strongly disagree, 2 disagree, 3 agree, 4 strongly agree

He assigned numbers to each of the above categories. He added the numbers and divided by 100 and concluded that most people agreed with his question.

Discuss the issues related to his conclusion and how he might refine his survey and conclusion.

4D

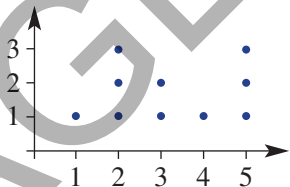
Dot plots and stem-and-leaf plots

- Learning intentions
- To be able to read, interpret and display data in dot plots
 - To be able to read, interpret and display data in stem-and-leaf plots

- Past, present and future learning
- Some aspects of this section may be new to students
 - These are Year 11 concepts which are not repeated in Year 12
 - Data analysis is extended in Chapter 6 of this book and also in Year 12
 - Questions about these data displays regularly appear in HSC Examinations

Dot plots

The simplest display of numerical data is a dot plot. A dot plot consists of a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other. Dot plots are a great way of displaying fairly small datasets in which the data takes a limited number of values.



Dot plot

It is a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other.

Example 6

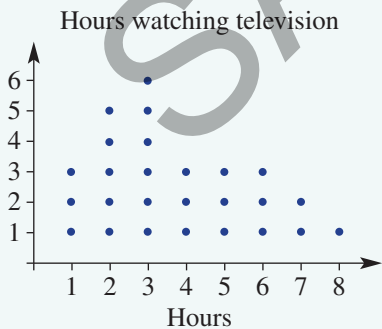
Constructing a dot plot

The number of hours spent watching television on the weekend is shown below. Construct a dot plot.

3 4 3 2 7 6 2 2 3 7 3 5 2
3 4 5 6 8 1 6 1 2 3 4 1 5



Solution



Explanation

Draw a number line, scaled to all the data values. Label the line with the variable being displayed.

The vertical axis indicates the frequency of the data value. The label may be omitted.

Plot each data value by marking in a dot above the corresponding value on the number line.

Now you try

The number of pets owned by each household is listed below. Construct a dot plot.

1 2 4 3 4 2 2 2 1 1 6

Stem-and-leaf plots

A stem-and-leaf plot or stem plot is used to present a small (less than 50 values) numerical dataset. The tens digit of the data values becomes the 'stem' and is written in numerical order down the page. The 'units' digits become the 'leaves' and are written in numerical order across the page.

Stem-and-leaf plot	Back-to-back stem-and-leaf plot
<pre> 0 5 7 1 2 2 5 5 7 Stem → 2 1 2 5 6 7 9 ← Leaf </pre>	<pre> Leaf → 9 0 0 8 5 3 2 1 1 2 5 5 9 Stem → 9 8 4 4 2 2 4 5 7 8 ← Leaf </pre>

**Example 7 Constructing a stem-and-leaf plot**

Use the following dataset to construct a stem-and-leaf plot.

24 7 19 19 15 18 27 28 11 19 29 30 9
 6 25 25 26 5 6 6 10 10 28 29 14

Solution

```

0 | 7 9 6 5 6 6
1 | 9 9 5 8 1 9 0 0 4
2 | 4 7 8 9 5 5 6 8 9
3 | 0

0 | 5 6 6 6 7 9
1 | 0 0 1 4 5 8 9 9 9
2 | 4 5 5 6 7 8 8 9 9
3 | 0

```

Explanation

The dataset has values from 5 to 30. This requires stems 0, 1, 2 and 3. Write these down from smallest to largest, followed by a vertical line. Attach the leaves. The first data value is '24'. It has a stem of '2' and a leaf of '4'. Opposite the 2 in the stem, write the number 4. Complete all the values.

Rewrite in increasing order.

Now you try

Use the following dataset to create a stem-and-leaf plot.

61 72 54 55 68 79 67 61 71 57



Example 8 **Constructing a back-to-back stem-and-leaf plot**

Use the following dataset to construct a back-to-back stem-and-leaf plot.

Girls	28	24	24	31	34	26	27	12	18	13	15	6	29	30	22
Boys	19	27	21	25	35	28	29	13	11	30	31	32	25	16	9

Solution

Girls		Boys
6	0	9
5 3 8 2	1	9 3 1 6
2 9 7 6 4 4 8	2	7 1 5 8 9 5
0 4 4 1	3	5 0 1 2

Girls		Boys
6	0	9
8 5 3 2	1	1 3 6 9
9 8 7 6 4 4 2	2	1 5 5 7 8 9
4 1 0	3	0 1 2 5

Explanation

The dataset has values from 6 to 35. This requires stems 0, 1, 2 and 3. Write these down from smallest to largest, followed by two vertical lines. Attach the leaves for the girls. The first data value is ‘28’. It has a stem of ‘2’ and a leaf of ‘8’. Attach the leaves for the boys. The first data value is ‘19’. It has a stem of ‘1’ and a leaf of ‘9’.

Rewrite in increasing order.

Now you try

Use the following dataset to create a back-to-back stem-and-leaf plot.

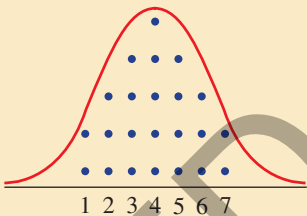
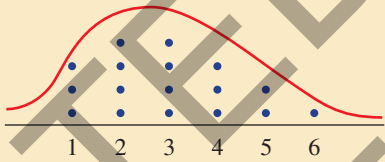
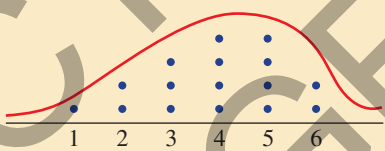
Soccer	23	24	28	30	30	12	17	23	11	22
Hockey	5	32	26	25	26	20	31	30	29	7

Symmetry or skewness

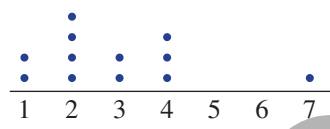
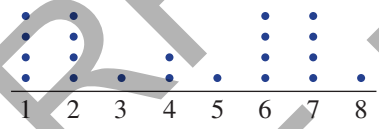
Data is symmetric if it forms a mirror image of itself when folded in the ‘middle’ along a vertical axis. Otherwise the data is skewed. Skewed data has more data on one side. There are two types:

- Positively skewed data has a long tail on the right-hand side.
- Negatively skewed data has a long tail on the left-hand side.



Shape	Description	Graph
No skew (Symmetric)	Data is symmetrical and balanced about a vertical line.	
Positively skewed	There is more data on the left side. The long tail is on the right side (positive side).	
Negatively skewed	There is more data on the right side. The long tail is on the left side (negative side).	

Outliers, clusters and gaps

		
Outliers	Clusters	Gaps
<pre> 0 7 7 8 8 9 1 2 4 6 8 2 3 1 </pre>	<pre> 0 1 1 1 2 2 1 3 2 3 3 3 1 1 2 2 4 6 8 </pre>	<pre> 0 1 1 5 5 1 2 3 6 8 9 9 4 7 </pre>

Outliers occur when a data point is a further away from most of the rest of the data. We will determine a way to formally identify outliers in Chapter 6.

Clusters are when most of the data is grouped in one or two places.

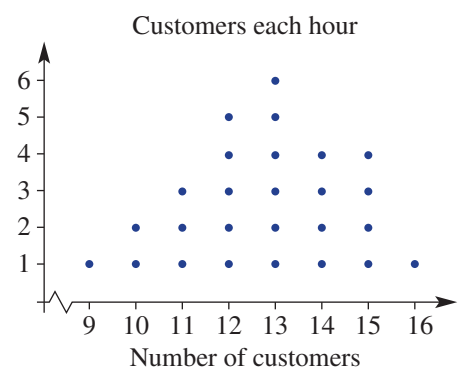
Gaps are when there is a relatively large space between parts of the data.

Exercise 4D

BUILDING

1 The dot plot represents the number of customers per hour.

- What is the highest number of customers?
- What is the most common number of customers per hour?
- What are the least common numbers of customers?
- Calculate the total number of customers.



Example 6

2 A group of 26 people were asked how many times in the last week they had shopped at a particular supermarket. Their responses were as follows:

1 3 3 2 1 1 4 5 4 2 1 3 4
6 2 1 1 1 3 3 2 2 1 5 6 1

- a Construct a dot plot of this data.
- b How many people were at the supermarket four times last week?
- c What is the difference between the highest and lowest number of visits to the supermarket?

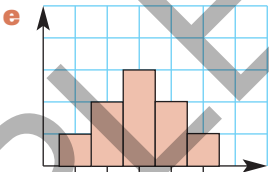
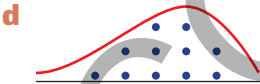
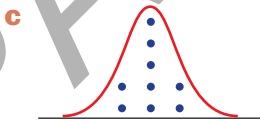
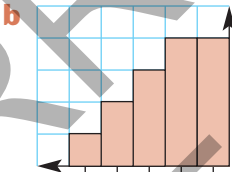
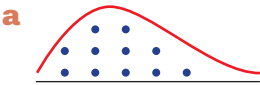
3 The goals scored in each match are listed below.

4 4 3 3 2 2 2
2 3 4 2 1 2 1
3 1 5 1 2 1
1 3 2 4 2 1

- a Construct a dot plot for this dataset.
- b How many matches resulted in 3 goals?
- c In how many matches were 4 or 5 goals scored?



4 Describe the skewness of the following graphs.



DEVELOPING

Example 7

5 The stem-and-leaf plot represents the results achieved by students in a test.

- a What is the highest score in this test?
- b Which score occurred the most number of times?
- c What is the difference between the highest and lowest scores?
- d How many students completed the test?
- e Describe the skewness of this dataset.

0	6
1	2358
2	2446789
3	014

6 An investigator recorded the life of 24 similar batteries in a toy. Her results (in hours) were:

41 25 37 46 17 4 33 31 28 34 19 26
40 24 31 27 30 22 33 20 21 27 30 26

- a Make a stem-and-leaf plot of these times.
- b How many of the batteries lasted for more than 25 hours?
- c Describe the skewness of this dataset.

CHALLENGING

Example 8

- 7** The following table shows the number of nights spent away from home in the past year by a group of 15 Australian tourists and by a group of 15 New Zealand tourists.

AUS	31	5	8	7	17	3	15	14	3	11	5	4	11	6	4
NZ	19	16	23	32	17	19	23	22	22	28	26	5	22	14	14

- a** Construct a back-to-back stem-and-leaf plot of these datasets.
- b** Compare the number of nights spent away by Australian and New Zealand tourists in terms of skewness or symmetry. Comment on any outliers, clusters or gaps.
- 8** The ages of patients admitted to a particular hospital during one week are given in the table.

Male	72	56	57	77	63	71	57	54	63	72	59	56	57	67	75
Female	61	55	58	78	65	68	71	78	79	72	73	64	68	66	69

- a** Construct a back-to-back stem-and-leaf plot of these datasets.
- b** Compare the ages at admission to the hospital for male and female patients in terms of skewness or symmetry. Comment on any outliers, clusters or gaps.



Communicate your reasoning, understanding and fluency

- 9** The following table shows the marks achieved by 20 students in PDHPE and English.

PDHPE	73	64	88	70	70	82	87	73	61	62	88	72	66	77	80	43	72	88	61	82
English	65	82	76	75	76	80	31	30	70	90	73	72	77	61	77	82	75	74	69	69

- a** Construct a back-to-back stem-and-leaf plot.
- b** Comment on the skewness of the marks in PDHPE and English.
- c** What conclusion can be made about the difficulty of each assessment?
- d** What are some of the factors that challenge the validity of your conclusion? What other factors should be considered?

4E

Line graphs, sector graphs and divided bar charts

- Learning intentions**
- To be able to read, interpret and display data in line graphs
 - To be able to read, interpret and display data in sector graphs
 - To be able to read, interpret and display data in divided bar charts

- Past, present and future learning**
- Some aspects of this section may be new to students
 - These are Year 11 concepts which are not repeated in Year 12
 - Data analysis is extended in Chapter 6 of this book and also in Year 12
 - Questions about these data displays regularly appear in HSC Examinations

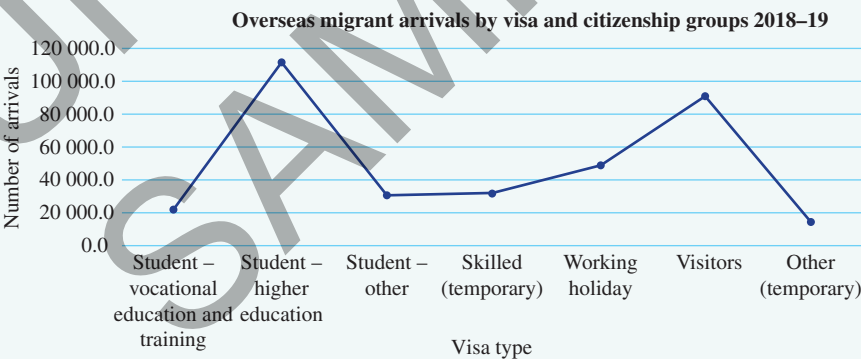


Example 9 Drawing a line graph

Represent the data from 2018–19 from the table as a line graph.

Overseas migrant arrivals by visa and citizenship groups		
	2018–19	2022–23
Student – vocational education and training	22 600.0	38 100.0
Student – higher education	111 300.0	177 400.0
Student – other	30 400.0	62 200.0
Skilled (temporary)	32 600.0	49 200.0
Working holiday	49 100.0	73 700.0
Visitors	91 200.0	108 200.0
Other (temporary)	13 400.0	47 600.0
Total temporary visas	350 700.0	556 600.0

Solution



Explanation

Start with a heading.
Create a key using the row labels from the table.
Place a dot at each value from the table, then join the dots.

Now you try

Represent the data from 2022–23 from the table in Example 9 as a line graph.

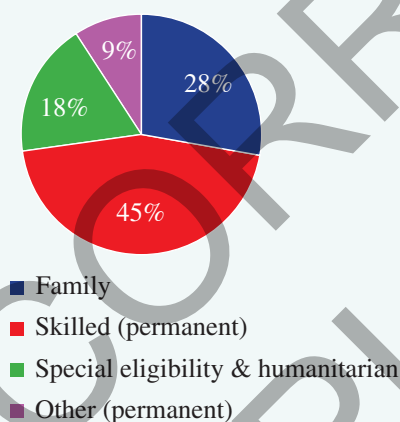

Example 10 Drawing a sector graph and divided bar graph

- a** Represent the data from 2018–19 from the table as a **sector graph**.
b Represent the data from 2018–19 from the table as a **divided bar graph**.

Overseas migrant arrivals by visa and citizenship groups			
		2018–19	2022–23
		pre-COVID	
Permanent visas			
	Family	24 300.0	21 100.0
	Skilled (permanent)	38 200.0	35 400.0
	Special eligibility & humanitarian	15 300.0	13 000.0
	Other (permanent)	7500.0	10 900.0
	Total permanent visas	85 400.0	80 400.0

Solution

- a** Overseas migrant arrivals by visa and citizenship groups 2018–19


Explanation

Start with a heading.

Create a key using the row labels from the table as categories.

Calculate the percentage of each part category.

$$\text{Family: } \frac{24\,300}{85\,400} \times \frac{100}{1} = 28\%$$

$$\text{Skilled: } \frac{38\,200}{85\,400} \times \frac{100}{1} = 45\%$$

Special eligibility & humanitarian:

$$\frac{15\,300}{85\,400} \times \frac{100}{1} = 18\%$$

$$\text{Other: } \frac{7500}{85\,400} \times \frac{100}{1} = 9\%$$

Ensure all percentages add to 100%.

Use each percentage to find the angle needed for each category in the sector graph.

$$\text{Family: } 28\% \text{ of } 360^\circ = \frac{28}{100} \times 360 = 101^\circ$$

$$\text{Skilled: } 45\% \text{ of } 360^\circ = \frac{45}{100} \times 360 = 162^\circ$$

Special eligibility & humanitarian:

$$18\% \text{ of } 360^\circ = \frac{18}{100} \times 360 = 65^\circ$$

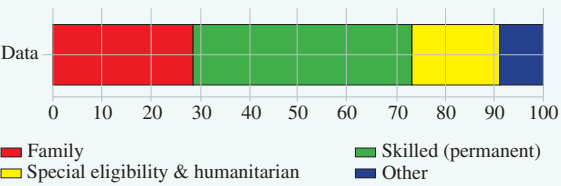
$$\text{Other: } 9\% \text{ of } 360^\circ = \frac{9}{100} \times 360 = 32^\circ$$

Make all angles add to 360°.

Use a protractor to create sector graph.

continued ...

b Overseas migrant arrivals by visa and citizenship groups 2018–19



Start with a heading.
Create a key using the row labels from the table as categories.

Calculate the percentage of each part category.

Family: $\frac{24\,300}{85\,400} \times \frac{100}{1} = 28\%$

Skilled: $\frac{38\,200}{85\,400} \times \frac{100}{1} = 45\%$

Special eligibility & humanitarian:
 $\frac{15\,300}{85\,400} \times \frac{100}{1} = 18\%$

Other: $\frac{7\,500}{85\,400} \times \frac{100}{1} = 9\%$

Make all percentages add to 100%.

Draw a 10 cm box on your page.
Use each percentage to find the length needed for each category in the divided bar graph.

Family: $28\% \text{ of } 10 \text{ cm} = \frac{28}{100} \times 10 = 28 \text{ mm}$

Skilled: $45\% \text{ of } 10 \text{ cm} = \frac{45}{100} \times 10 = 45 \text{ mm}$

Special eligibility & humanitarian:
 $18\% \text{ of } 10 \text{ cm} = \frac{18}{100} \times 10 = 18 \text{ mm}$

Other: $9\% \text{ of } 10 \text{ cm} = \frac{9}{100} \times 10 = 9 \text{ mm}$

Make sure all lengths add to 10 cm.

Divide box by adding in all lengths calculated.

Now you try

- a** Represent the data from 2022–23 from the table in Example 10 as a sector graph.
- b** Represent the data from 2022–23 from the table in Example 10 as a divided bar graph.

Exercise 4E

BUILDING

Example 9

- 1 The shoe sizes of students in Year 12 are shown in the table shown. Draw a line graph to represent this information.

Shoe sizes of Year 12 students	
Size	No. of students
5	7
6	12
7	6
8	8
9	16
10	14
11	5
12	2
13	1

- 2 The ages of residents in a retirement village are represented in the line graph shown. Create a table to represent this information.



- 3 Create a line graph to compare the numbers of school-age males and females who migrated to Australia in 2022-23, using the data in the table shown.

Overseas migrant arrivals – age and gender		
Age	2022-23 – Males ('000)	2022-23 – Females ('000)
4	3.42	3.32
5	3.31	3.15
6	3.13	2.97
7	2.93	2.85
8	2.83	2.73
9	2.73	2.57
10	2.61	2.41
11	2.59	2.33
12	2.35	2.25
13	2.18	2.16
14	2.33	2.23
15	2.37	2.39
16	2.31	2.21
17	2.90	2.80
18	3.35	3.19

DEVELOPING

- Example 10b
- 4

 Use the information from Question 1 to create a divided bar graph.
- 5

 Use the information from Question 3 to create separate divided bar graphs for males and females.

CHALLENGING

- Example 10a
- 6

 Represent the information from Question 1 as a sector graph.
- 7

 Represent the information for males from Question 3 as a sector graph.

Communicate your reasoning, understanding and fluency

8

Overseas migrant arrivals – visa and citizenship groups	
Years	Temporary visa holders ('000)
2017–18	327.30
2018–19	350.67
2019–20	313.66
2020–21	29.58
2021–22	266.21
2022–23	556.62
2023–24	464.78

- a

 Draw a line graph to represent this information.
- b

 Create a divided bar graph to represent this information.
- c

 Represent this information as a sector graph.
- d

 Which graph best helps you to analyse the data about overseas migrant arrivals from year to year? Why?

4F Ungrouped and grouped frequency tables

Learning intentions

- To be able to read, interpret and display data in frequency tables
- To be able to group data into classes

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated in Year 12
- Data analysis is extended in Chapter 6 of this book and also in Year 12
- Questions about these tables regularly appear in HSC Examinations

Frequency tables

A frequency table is a listing of the outcomes and how often (its frequency) each outcome occurs. The outcomes may also be called 'scores'. The tally of the frequency and the final count are listed in separate columns. When you create a frequency table from a set of data, you should include a tally column to record your count of how often each score occurs. The tally column may be omitted in a frequency table given to you that has already been created. A frequency table is also called a frequency distribution.

	Score	Tally	Frequency	
Lowest score →	17	I	1	← Lowest frequency
	18		6	
	19		5	
	20	II	7	← Highest frequency
Highest score →	21		3	

Frequency table

- 1 Scores or outcomes are listed in the first column in ascending order.
- 2 Tally column records the count of the number of times the score occurred (groups of 5s).
- 3 Frequency column is the total count of each outcome.



Example 11 Constructing a frequency table

The temperatures for 39 days are shown below. Construct a frequency table.

19 20 18 23 27 25 26 27 28 27 25 24 24 19 25 22 21 28 26 26
22 20 25 20 22 24 24 22 21 24 25 26 25 27 21 23 23 22 25

continued ...

Solution

Score	Tally	Frequency
18		1
19		2
20		3
21		3
22		5
23		3
24		5
25		7
26		4
27		4
28		2
	Total	39

Explanation

Draw a table with three columns and label them score, tally and frequency.
List the temperatures in the score column from the lowest (18) to the highest (28).
Record a mark in the tally column for each temperature.
Count the tally marks and write the total in the frequency column.
Add the frequency column to find the total number of scores. This should match the total number of temperatures (39).

Now you try

Use the following dataset to create a frequency table.

6 8 7 5 6 7 4 4 6 6 7 5 3

Grouped frequency tables

Data with a large range of values are often grouped into a small number of convenient intervals, called class intervals. When choosing class intervals ensure:

- every data value is in an interval
- intervals do not overlap
- no gaps exist between the intervals.

The choice of intervals can vary depending on that data. Recommendations:

- A division of 5 to 15 groups is preferred.
- Choose an interval that is easy to read such as 5 units, 10 units.
- Start the interval with an appropriate exact value. For example, choose intervals such as 0–9, 10–19, ... rather than 1–10, 11–20, ...

Grouped frequency tables

- 1 Classes or groups are listed in the first column in ascending order.
- 2 Class centre column is obtained by finding the middle of the class endpoints.
- 3 Tally column shows the number of times a score occurs in a class (groups of 5s).
- 4 Frequency column is total count of scores in each class.



Example 12 Constructing a grouped frequency table

Twenty-six people were asked to record how many cups of coffee they drank in a particular week. The results are listed below. Draw a frequency table to show the data.

0 33 6 14 0 32 0 25 10 0 2 9 23
0 34 5 17 3 0 23 1 32 0 8 0 2

Solution

Class	Class centre	Tally	Frequency
0–4	2		12
5–9	7		4
10–14	12		2
15–19	17		1
20–24	22		2
25–29	27		1
30–34	32		4
		Total	26

Explanation

Draw a table with 4 columns and label them class, class centre, tally and frequency.

The data ranges from 0 to 34. A class interval breadth of 5 results in 7 classes. Calculate the class centre (middle of each class).

Record a mark in the tally column for each data value in the class interval. Count the tally marks in the frequency column.

Add the frequency column to find the total number of scores.

Now you try

Use the following dataset to create a grouped frequency table.

26 20 3 5 0 29 13 15 21 28
2 17 22 12 2 6 9 15 16 20

Exercise 4F

BUILDING

Note: Include a tally column, where you are asked to create a frequency table in this exercise.

Example 11

- 1 The ages, in years, of players in a football team are recorded in a frequency table.

Score	Tally	Frequency
20		
21		
22		6
23		
24		2

- Copy and complete the table.
- What was the most common age?
- How many players are in the team?

- 2 A grouped frequency table is shown.
- a Copy and complete the table.
 - b How many class intervals have been used?
 - c Which class occurred the least?
 - d What is the total number of scores?

Class	Class centre	Tally	Frequency
3–7			4
	10		6
13–17	15		2
18–22	20		
	25		1
28–32			3
33–37	35		
	40		5

Example 11

- 3 The number of brothers and sisters for 30 students is recorded below. Construct a frequency table for this data.

2 1 1 5 3 4 1 0 2 1 5 2 6 1 0
3 1 0 4 3 3 4 2 3 0 2 0 1 0 2



Example 12

- 4 The heights of the 30 players in a netball club are recorded below. Construct a grouped frequency table using class intervals (170–174, 175–179, 180–184, 185–189, ...).

174 184 183 179 180 181 189 188
194 185 178 173 183 188 183 182
194 189 184 182 183 189 193
184 189 194 188 192 190 180



- 5 The assessment result for 30 students is recorded below. Construct a frequency table for this data.

96 97 97 95 92 94 97 98 91 97 95 91 96 97 94
92 92 98 94 93 93 94 95 93 92 91 98 96 92 91

DEVELOPING

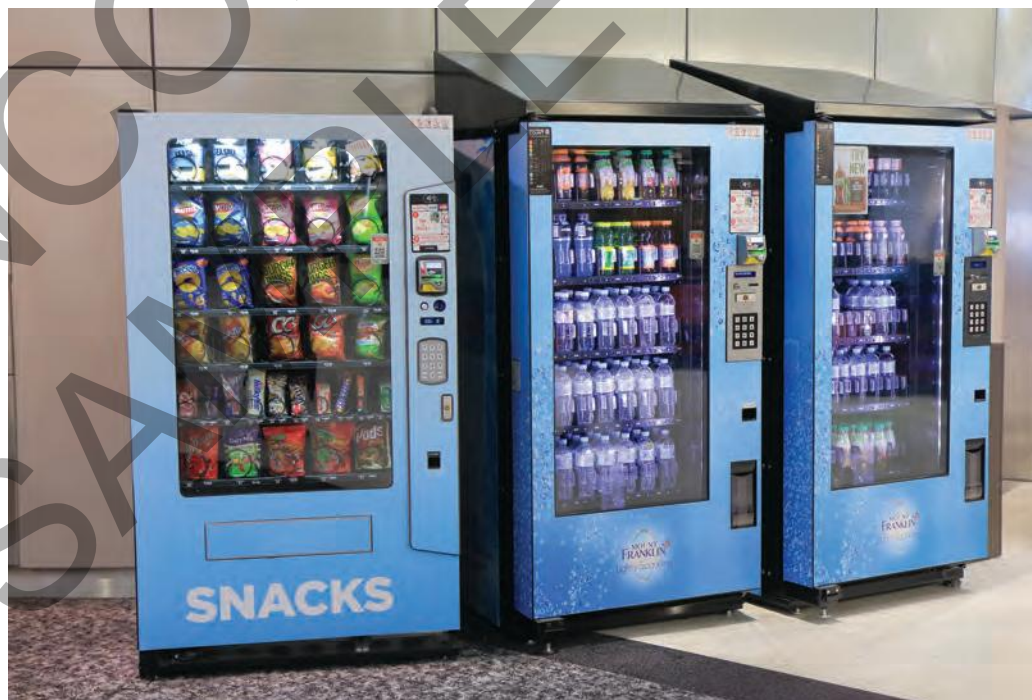
- 6** A die was rolled and the results listed below.

1	1	5	2	2	5	1	3	6	1	4	3	1	5	5	4	6
3	3	4	2	1	1	6	5	4	2	6	6	1	4	4	1	6
4	1	6	5	2	4	6	2	4	2	2	3	6	4	2	4	1

- Construct a frequency table.
 - How many times was the die rolled?
 - How many results are higher than 2?
 - What was the most common number rolled?
 - Do you think the die is biased? Give a reason for your answer.
- 7** A drink vending machine records the number of bottles sold each day.

13	22	26	34	21	33	32	31	33	26	30	33	37	27	36
28	35	24	31	3	35	39	26	13	34	21	29	36	24	33
25	9	39	19	29	36	38	29	38	38	37	38	31	37	37
28	40	34	29	18	35	22	20	35	25	31	39	39	18	36
38	35	8	29	35	20	34	30	37	33	27	32	32	36	16
39	30	14	29	20	22	12	24	17	21	18	17	38	28	25

- Decide on appropriate classes for a frequency table.
- Calculate the class centres for these classes.
- Construct a grouped frequency table using these class intervals.



CHALLENGING

- 8** Count the number of letters in each word of the paragraph below.

A frequency table is a listing of the outcomes and how often (frequency) each outcome occurs. The outcomes are often listed under a heading called 'score'. The tally of the frequency and the final count are listed in separate columns. Frequency tables are also called a frequency distribution.

- a** Create a frequency table for the length of words used in this paragraph.
b Using the frequency table, what is the most frequent word length in the English language?
c Comment on the fairness of the conclusion made in part **b**.

- 9** Jordan surveyed his friends to check the number of emails they saved on their computers. These are the numbers he recorded:

22, 9, 51, 6, 30, 18, 30, 4, 10, 5, 19, 23, 37, 17, 18, 12, 10, 24, 28, 25, 60, 45, 19, 17, 11, 8, 16, 1, 24, 3

- a** Decide on appropriate classes for a frequency table.
b Calculate the class centres for these classes.
c Construct a grouped frequency table using these class intervals.
d What percentage of friends had fewer than 20 emails saved on their computer?

Communicate your reasoning, understanding and fluency

- 10** The players' scores after the second round of a golf tournament are recorded below.

162	163	175	161	166	163	167	151	150	176
159	173	162	155	149	171	181	163	154	165
145	177	184	171	154	166	168	158	136	156
161	162	169	162	160	150	174	176	146	137

- a** Explain why it is better to represent this information in a grouped frequency table and not a frequency table.
b Decide on appropriate classes and class centres.
c Construct a grouped frequency table.
d Explain the process you used to minimise errors when transferring the data from the page to the grouped frequency table.
e What percentage of players shot less than 165?

4G Cumulative frequency

Learning intentions

- To be able to complete a cumulative frequency column in a frequency table

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated in Year 12
- Data analysis is extended in Chapter 6 of this book and also in Year 12
- Questions about cumulative frequency regularly appear in HSC Examinations

Cumulative frequency is the frequency of the score plus the frequency of all the scores less than that score. A cumulative frequency column is often inserted next to the frequency column in a frequency table.

Score	Frequency	Cumulative frequency
18	1	1
19	5	6
20	3	9
21	7	16

Cumulative frequency

The frequency of the score plus the frequency of all the scores less than that score



Example 13 Calculating the cumulative frequency

The frequency table shows the temperatures for 17 days. Complete the cumulative frequency column.

Score	Frequency	Cumulative frequency
18	1	
19	2	
20	3	
21	3	
22	5	
23	3	

Solution

Score	Frequency	Cumulative frequency
18	1	1
19	2	$1 + 2 = 3$
20	3	$3 + 3 = 6$
21	3	$6 + 3 = 9$
22	5	$9 + 5 = 14$
23	3	$14 + 3 = 17$

Explanation

The lowest score is 18 and it has a frequency of 1. Cumulative frequency of 18 is 1.

Add the frequency of the next score to the cumulative frequency of the previous score. Cumulative frequency of 19 = $1 + 2 = 3$.

continued ...

Now you try

Use the following dataset to create a cumulative frequency table.

6 8 7 5 6 7 4 4 6 6 7 5 3

Exercise 4G

BUILDING

Example 13

- 1 The frequency table shows the results of a test.
 - a Copy and complete the table.
 - b How many students scored 8?
 - c How many students scored more than 5?
 - d How many students scored less than 6?
 - e How many students completed the test?

Score	Frequency	Cumulative frequency
4	4	
5	6	
6	7	
7	10	
8	5	

- 2 The number of times an ambulance was called out each day is recorded in a frequency table.
 - a Copy and complete the table.
 - b On how many days was the ambulance called out 21 times?
 - c On how many days was the ambulance called out fewer than 25 times?
 - d On how many days was the ambulance called out more than 23 times?

Number of calls	Frequency	Cumulative frequency
20	4	
21	3	
22	10	
23	12	
24	6	
25	5	

- 3 The results of a survey are listed below.

13 11 11 10 10 10 9 10 9 11 9 10 12 11 13
10 9 12 11 13 10 11 10 10 10 13 8 8 10 9

Score	Tally	Frequency	Cumulative frequency
8			
9			
10			
11			
12			
13			

- a Copy and complete the table.
- b How many people completed the survey?
- c How many people scored 10 in the survey?
- d What is the difference between the highest and lowest scores?
- e Which score had the highest frequency?

DEVELOPING

- 4 The ages of children at the local park are shown below.

9 8 4 6 10 10 8 6 5 4 5 6 8 4 9
10 5 8 8 9 10 4 6 10 9 9 7 7 6 5

- a** Construct a frequency table to represent this data. Include a tally column.
b Add a cumulative frequency column.
c How many children are there altogether?
d How many children are 8 years old?
e How many children are 6 years old?
f How many children are older than 5?
g How many children are younger than 9?
h What are the most common ages?
i What is the least common age?
j What fraction are 7 years old?
k What fraction are 4 years old?
l What percentage are older than 5?
m What percentage are younger than 6?

CHALLENGING

- 5 Blake recorded his time (in seconds) to run the 400-metres hurdles throughout the year.

61 62 62 63 64 62 66 64 63 63 62 61 62 62 63
64 63 65 63 63 62 61 62 63 62 61 61 64 64 64
63 63 64 65 64 63 63 62 61 62 62 63 64 63 62

- a** Construct a frequency table to represent this data; include a tally column.
b Add a cumulative frequency column to the table.
c How many times were recorded?
d How many times did Blake run the 400 metres in 61 seconds?
e How many times are less than or equal to 63?
f What fraction of his 400-metre times is 62?
g What percentage of his 400-metre times is 65? Answer correct to two decimal places.
h What percentage of his times are less than or equal to 64? Answer correct to two decimal places.



Communicate your reasoning, understanding and fluency

- 6 Twenty-six cricket batting scores were recorded and are listed below.

4 33 13 140 4 101 2 17 0 9 11 0 31
91 6 11 12 4 38 50 26 45 31 54 71 110

- a** Construct a grouped frequency table for the this data, including a tally column and a cumulative frequency column.
b Why is it better to use a grouped frequency table to represent this data?
c What percentage of scores were 100 or above?
d What percentage of scores were 10 or less?
e What percentage of scores were 20 or above?
f If a batsman scores 20, what is their chance of scoring 50 or more? Explain your answer in words supported by working out.

4H Frequency and cumulative frequency graphs

Learning intentions

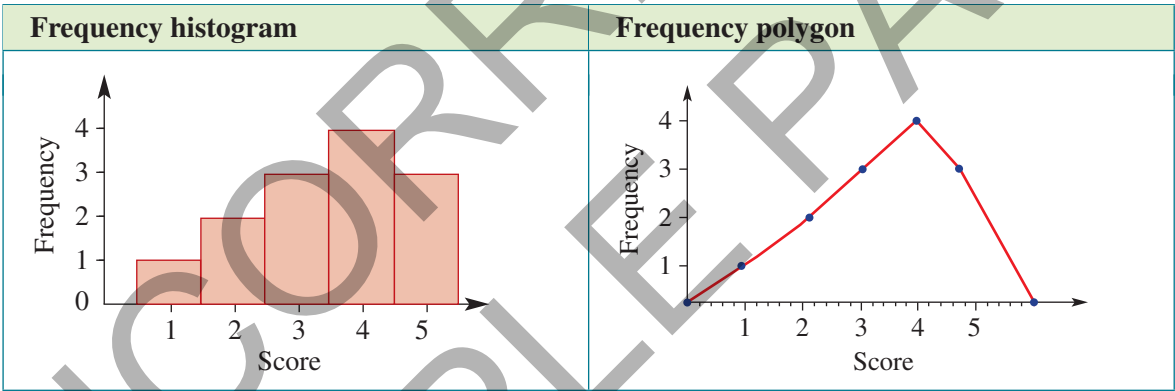
- To be able to read, interpret and display data in frequency histograms
- To be able to read, interpret and display data in cumulative frequency histograms

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated in Year 12
- Data analysis is extended in Chapter 6 of this book and also in Year 12
- Questions about these data displays regularly appear in HSC Examination

Frequency histogram and polygon

A frequency histogram is a graph of a frequency table in which equal intervals of the scores (or classes) are marked on the horizontal axis and the frequencies associated with these intervals are indicated by vertical rectangles. A frequency polygon is a line graph of the frequency table and can be constructed by joining the midpoints at the tops of the rectangles of a frequency histogram.

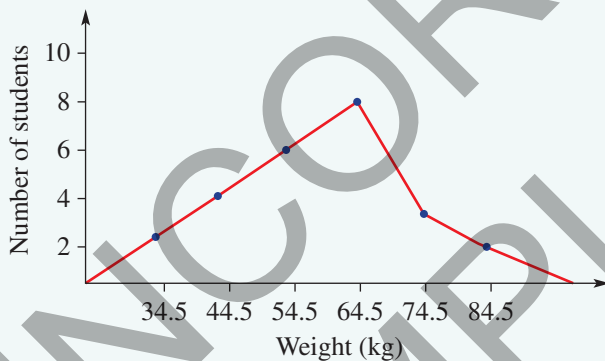
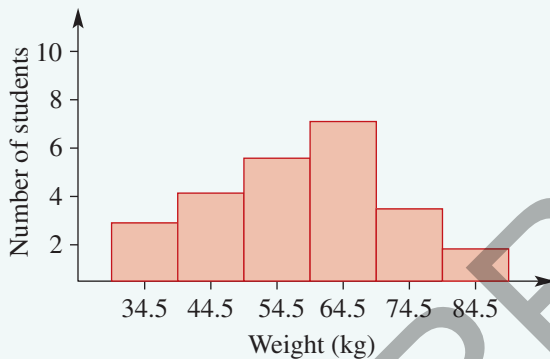


Example 14 Constructing a grouped frequency histogram and polygon

The weights of 30 students were recorded and displayed in a grouped frequency table. Construct a grouped frequency histogram and polygon.

Class	Class centre	Frequency
30–39	34.5	3
40–49	44.5	5
50–59	54.5	7
60–69	64.5	9
70–79	74.5	4
80–89	84.5	2

Solution



Explanation

Draw and label the horizontal axis with each class centre the same distance apart.

Leave half this distance at the start.

Add space for one extra class centre at the right-hand end of the axis.

Draw and label the vertical axis.

For frequency histogram

Use the frequency of each class to construct boxes for each one.

For frequency polygon

Starting at the intersection of the axes draw a line to join the centre of each box. End the line by connecting it to where the next class-centre would be on the vertical axis.

Now you try

Create a frequency histogram and polygon (on the same axes) from the frequency table.

Score	Frequency
3	1
4	4
5	7
6	4
7	1
8	5

Cumulative frequency graphs

A cumulative frequency histogram is constructed using equal intervals of the scores (or classes) on the horizontal axis and the cumulative frequencies associated with these intervals indicated by vertical rectangles. A cumulative frequency polygon or ogive is a line graph constructed by joining the top right-hand corner of the rectangles of a cumulative frequency histogram.



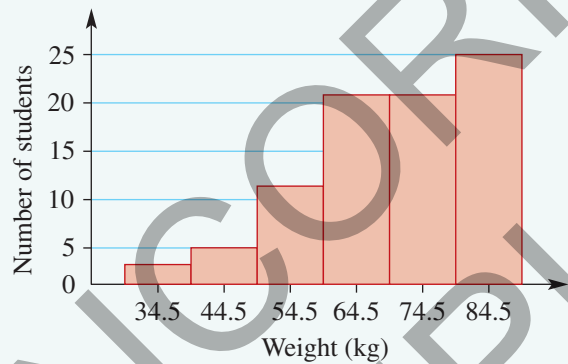
Example 15 Constructing a cumulative frequency histogram and polygon

Construct a cumulative frequency histogram and polygon or ogive.

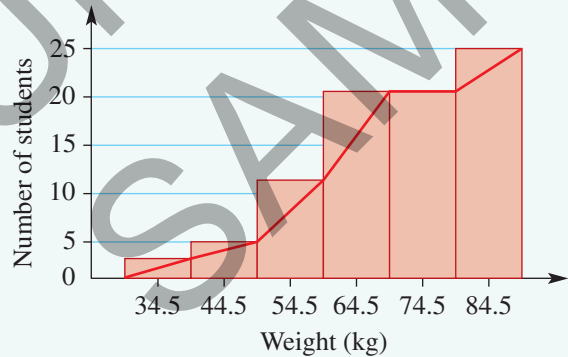
Class	Class centre	Frequency	Cumulative frequency
30–39	34.5	2	2
40–49	44.5	3	5
50–59	54.5	7	12
60–69	64.5	9	21
70–79	74.5	0	21
80–89	84.5	4	25

Solution

Cumulative frequency histogram



Cumulative frequency polygon



Explanation

Draw and label the horizontal axis with each class centre the same distance apart.
Leave half this distance at the start.
Add space for one extra class centre at the right-hand end of the axis.
Draw and label the vertical axis.

For cumulative frequency histogram

Use the cumulative frequency of each class to construct boxes for each one.

For cumulative frequency polygon

Starting at the left-hand bottom corner of the first box, draw a line to the right-hand top corner of the first box. Continue to draw a line connecting the top right-hand corner of each box. End the line at the top right-hand corner of the last box. (Do not return the line to the horizontal axis).

DEVELOPING

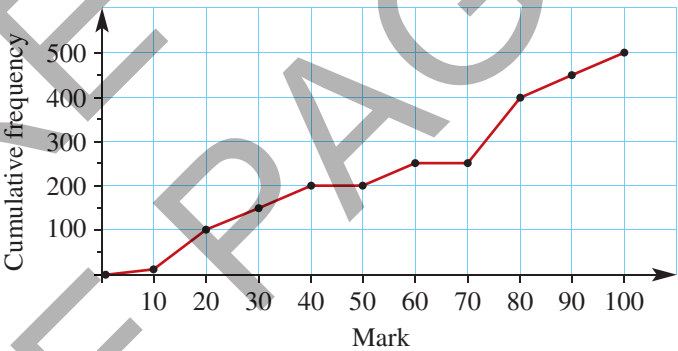
5 Jade owns five clothing stores that sell jackets. She recorded the total number of jackets sold each day for the month of April. This data is shown below.

61	66	67	67	60	63	67	63
64	62	65	65	67	66	61	62
65	61	63	63	67	67	60	
66	64	67	62	65	65	67	



- a Construct a frequency table with a cumulative frequency column.
- b Construct a cumulative frequency histogram for this data.
- c Construct a cumulative frequency polygon for this data.
- d On how many days was the number of jackets sold less than or equal to 63?
- e On how many days was the number of jackets sold less than or equal to 66?

6 The marks for a university exam are shown in the cumulative frequency polygon.



- a What was the frequency of 10?
- b What was the frequency of 20?
- c What was the frequency of 30?
- d What was the frequency of 40?
- e What was the frequency of 50?
- f What was the frequency of 60?
- g What was the frequency of 70?
- h What was the frequency of 80?
- i What was the frequency of 90?
- j What was the frequency of 100?
- k How many students completed this university exam?
- l Construct a frequency table from the cumulative frequency polygon.

CHALLENGING

7 The number of words in each of the first 30 sentences of a book were recorded.

22	20	22	23	23	24	24	25	25	21	21	20	20	22	26
22	20	24	26	20	21	21	23	22	23	22	20	20	21	21

- a Construct a grouped frequency table with a cumulative frequency column. Use classes as 20–21, 22–23, 24–25, 26–27.
- b Use the frequency table to construct a frequency histogram.
- c Use the frequency table to construct a frequency polygon.
- d Construct a cumulative frequency histogram and polygon for this data.

- 8** The percentage of female births, correct to the nearest whole number, is shown below. These birth percentages have been taken from 30 different hospitals.

38	56	57	59	58	60	43	52	49	61
47	38	41	50	51	55	45	50	49	53
54	48	51	43	55	53	42	42	44	46

- Decide on appropriate classes for a frequency table.
- Construct a grouped frequency table using these class intervals.
- Add a cumulative frequency column.
- Construct a frequency histogram for this data.
- Construct a frequency polygon for this data.
- Construct a cumulative frequency histogram and polygon for this data.



Communicate your reasoning, understanding and fluency

- 9** Twenty-six cricket batting scores were recorded and are listed below.

104	33	13	140	24	101	12	17	0	9	11	0	31
91	6	176	12	40	38	50	26	45	31	54	71	110

- Construct a grouped frequency table for the this data, including a tally column and a cumulative frequency column.
- Draw a frequency histogram and polygon on the same axes.
- What percentage of scores were 100 or above?
- What percentage of scores were 10 or less?
- What percentage of scores were 20 or above?
- Can you read the highest score achieved by the batsmen from the graph only? Explain.

4I

Real-world applications

- Learning intentions
- To examine, analyse and understand statistical data collected about real-world scenarios
- Past, present and future learning
- Some aspects of this section may be new to students
 - Data analysis is extended in Chapter 6 of this book and also in Year 12
 - Questions involving real-world data regularly appear in HSC Examinations

Motor vehicle statistics

Since there have been motor vehicles, there have been motor vehicle accidents that result in death, injury and damage. Governments collect, present and interpret data on road incidents to try to reduce the problem. There are many factors that may cause a road accident, such as poor driving, speeding, alcohol, fatigue, bad road design and lack of vehicle maintenance.



Example 16 Reading a table of motor vehicle accident statistics

The table shows the number of road accidents involving fatigue in the last four months of the year.

	Deaths	Injured
September	112	1488
October	197	4365
November	89	2019
December	134	3487



- a How many deaths occurred in road accidents during October involving fatigue?
- b Which month had the least number of injuries in road accidents involving fatigue?
- c How many deaths in road accidents involving fatigue occurred in the four months?
- d What is the percentage increase in number of injuries involving fatigue from November to December?

Solution

- a 197 deaths
- b September
- c Deaths = 112 + 197 + 89 + 134
= 532
- d Increase = 3487 – 2019
= 1468

Percentage = $\frac{1468}{2019} \times 100$
= 72.709...
 $\approx 73\%$

Explanation

Read the value from the table.
Read the value from the table.
Add the values from the table in the Deaths column.

Express the increase as a percentage of the original amount.

Now you try

The table shows motor vehicle accident statistics over a four-year period.

Year	Deaths	Injured
1900	207	2141
1901	370	1086
1902	456	1040
1903	789	3001

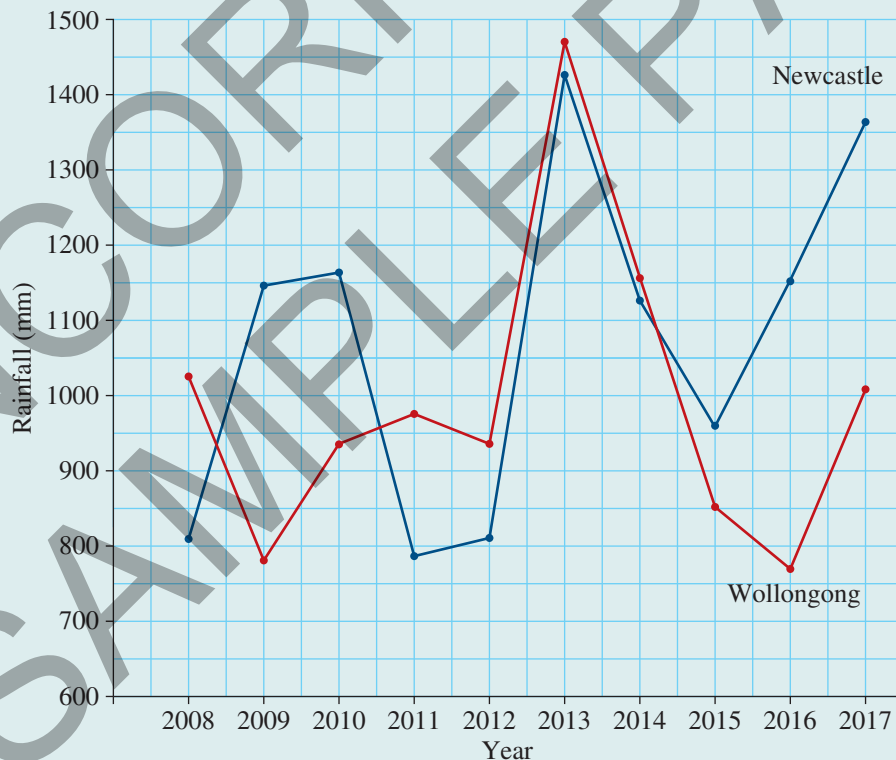
- How many deaths occurred in 1902?
- Which year had the least number of injuries caused by road accidents?
- How many injuries occurred due to motor vehicles from 1900 to 1903?
- What was the percentage increase in the number of deaths from 1900 to 1901?

Rainfall

Rainfall changes are occurring due to climate change as a result of the increased concentration of carbon dioxide in the atmosphere. We are also experiencing more droughts and floods. Australia's annual mean rainfall has increased slightly over the last century.

**Example 17** Interpreting rainfall graphs

The graph shows the annual rainfall data for Newcastle (—) and Wollongong (—) over 10 years.



- What was annual rainfall for Newcastle in 2015?
- Which was the wettest year during this 10-year period for Wollongong?
- Which year has the largest difference in annual rainfall?

continued ...

Solution

- a Newcastle’s rainfall in 2015 was 960 mm.
- b Wollongong’s wettest year was 2013.
- c Largest difference occurred in 2016.

Explanation

Read the value from the graph.
Read the value from the graph.
The year that the largest gap between the two lines occurs.

Now you try

- Using the graph in Example 17, answer the following questions.
- a What was the annual rainfall in Wollongong in 2016?
 - b Which was the driest year for Newcastle in these 10 years?
 - c Show (with working) which city received the most rainfall between 2008 and 2017?

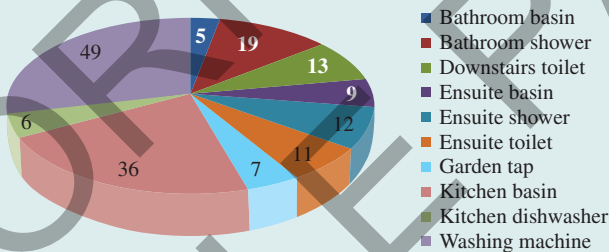
Household water usage

The amount of water that is available for local, state and national needs will vary from year to year. In recent years governments have imposed restrictions on our water use, to help conserve supplies in times of drought.



Example 18 Interpreting data about a household’s water usage

The pie chart shows the annual water usage in kilolitres for a family.



- a How many kilolitres of water were used by this family in a year?
- b What percentage of the total water use is in operating the washing machine?
- c A dual-flush toilet saves 60% of the water used in a conventional toilet. How many kilolitres per year would be saved by replacing the toilets with dual-flush toilets?

Solution

- a $\text{Water usage} = 36 + 6 + 49 + \dots$
 $= 167 \text{ kL}$
- b $\text{Washing machine} = \frac{49}{167} \times 100$
 $\approx 29.34\%$
- c $\text{Water usage} = 13 + 11 = 24 \text{ kL}$
 $\text{Water saving} = 60\% \text{ of } 24 \text{ kL}$
 $= 0.60 \times 24$
 $= 14.4 \text{ kL}$
Two dual-flush toilets save 14.4 kL a year.

Explanation

Add all the sector values from the graph.

Divide the amount for washing machines by the total amount and convert to a percentage by multiplying by 100.

Add the water usage from the ensuite and downstairs toilet.
Find 60% of this amount.

Now you try

Using the graph in Example 18, answer the following questions.

- What was the total water usage in the ensuite?
- What percentage of total water usage was used by the garden tap?
- A water saving shower head was installed in both showers, saving 30% of the water usage. How much water was saved?

Infographics are used to represent information using targeted information and pictures or graphics.



Example 19 Interpreting an infographic

Answer the following questions using the infographic about how Aussies reacted in a year of COVID-19.

- What percentage of men avoided public places?
- List 5 ways people reacted.
- What was the percentage increase of people working from home from February 2020 to February 2021?
- How many of us increased our screen time?



continued ...

Solution

- a 84%
- b Took precautions, focused on health, moved out of the city, worked from home, felt overwhelmed or lonely
- c 19%
- d 44%

Explanation

Read information from the infographic.

Now you try

Using the infographic in Example 19, answer the following questions.

- a What percentage of Aussies said we would get a vaccine?
- b What percentage of Aussies want to spend more time with family and friends?
- c Did more men or women avoid public spaces?

Exercise 4I

BUILDING

Example 16

- 1 The table shows the running costs for fuel, tyres and servicing as cents per km for four brands of motor vehicle.

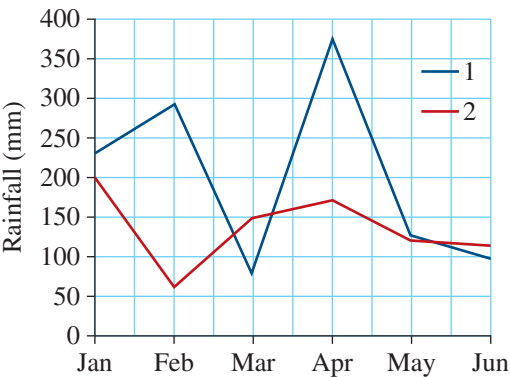
	Brand A	Brand B	Brand C	Brand D
Fuel	5.06	6.90	9.69	8.99
Tyres	1.03	1.18	0.88	1.28
Service	2.51	3.73	3.02	3.88

- a What is the cost of service for the Brand D vehicle?
- b Which of the cars has the best fuel economy?
- c Harry has driven his Brand C vehicle 7580 kilometres this year. What is the fuel cost of his vehicle for the year?
- d Calculate the difference in service costs between Brand A and Brand B, when both cars are driven 15 000 km in a year.
- e What is the difference in tyre costs between Brand D and Brand A, when they are both driven 100 000 km?

Example 17

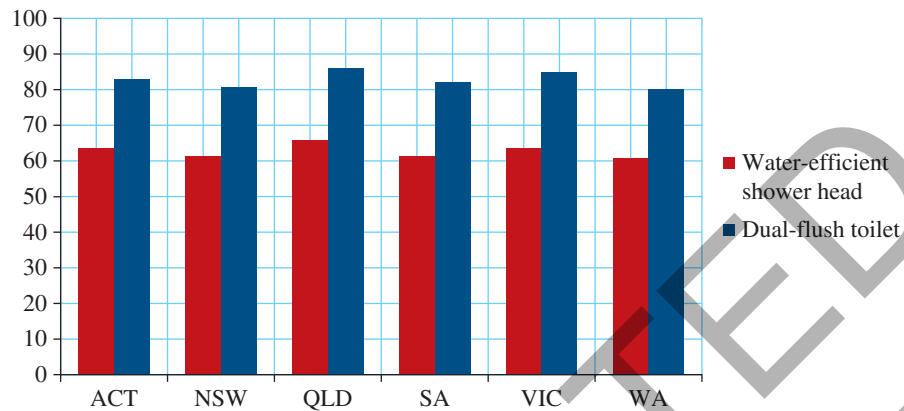
- 2 The rainfall (January to June) for two years at Byron Bay is shown in the graph.

- a What was the rainfall in year 1 for April?
- b Which month had the lowest rainfall for both years?
- c Which year had the lower rainfall for January to June?
- d In which month was there the biggest difference in rainfall between year 1 and year 2?



Example 18

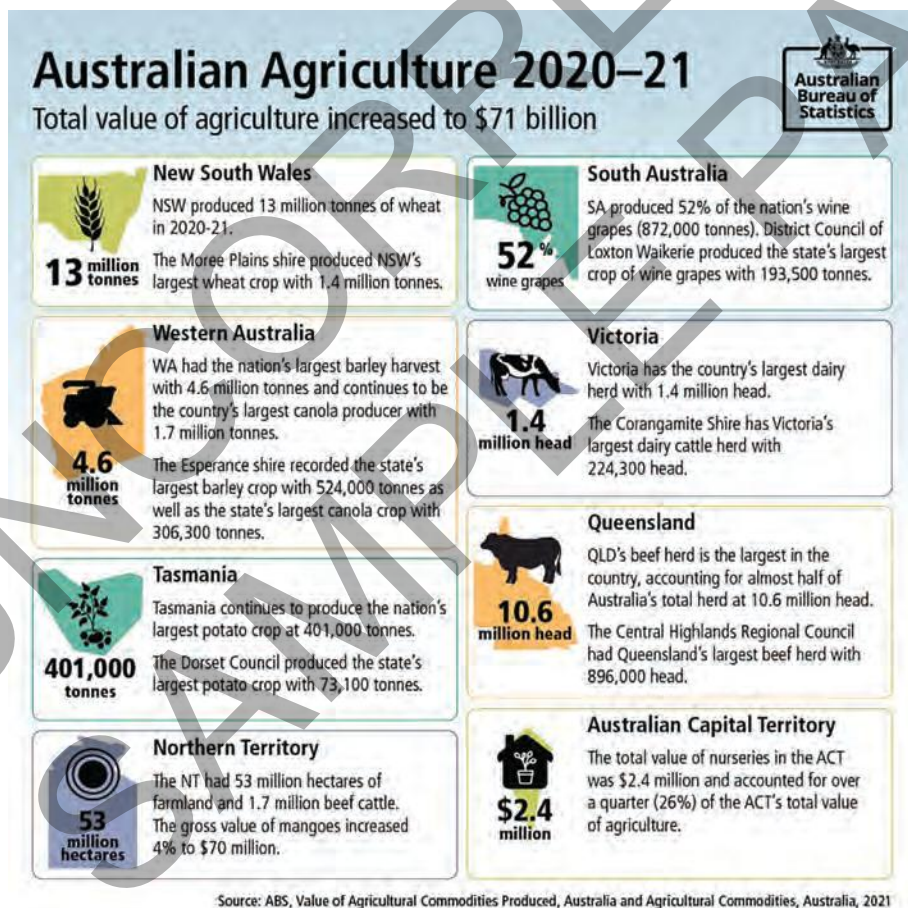
- 3 The column graph shows the percentage of households with water saving devices.



- Which state has the highest percentage of households with dual-flush toilets?
- Which state has the lowest percentage of households with water-efficient shower heads?
- Which state has the highest percentage of households with both water-saving devices?
- Which state has the lowest percentage of households with both water-saving devices?

Example 19

4



- How much wheat did NSW produce in 2020-21?
- Which shire in WA recorded the state's largest barley crop?
- How much is Australia's largest potato crop?
- How many beef cattle are there in Queensland?
- What accounts for over one quarter of the ACT's agricultural value?

- 5 The frequency table shows the number of motorbikes passing through a checkpoint each hour for the past 24 hours.

Motorbikes (x)	Frequency (f)	Frequency $\times$ Score (fx)
11	4	
12	7	
13	6	
14	5	

- a In how many hours did 13 motorbikes pass through the checkpoint?
b What was the greatest number of motorbikes passing through a checkpoint in an hour?
c How many motorbikes passed through the checkpoint?
d Find the mean of this data. Answer correct to one decimal place.
e What is the median of this data?
f What is the mode of this data?

DEVELOPING

- 6 The table shows Dylan’s average water use in one day.

- a How much water is Dylan using each day?
b How much water would Dylan use in a year?
c What percentage of his water usage is in taking a shower?
d Construct a line graph for this data.

Water usage	L/day
Shower	51
Toilet	32
Tap	22
Washing machine	40
Other	5

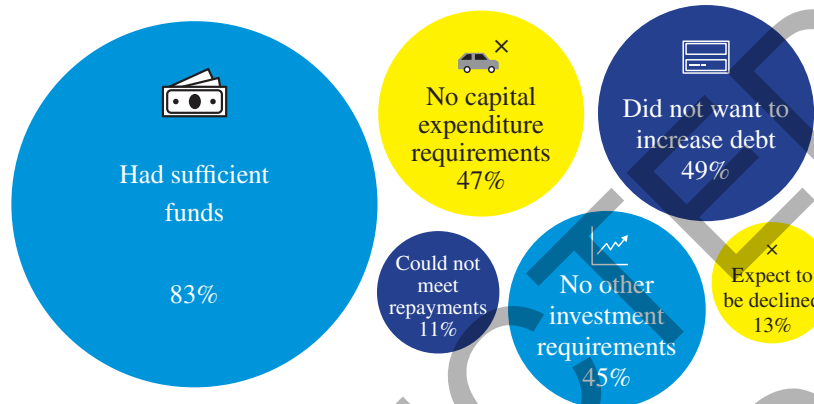
- 7 The table shows the annual rainfall (mm) at Prospect for the past 10 years.

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
January	76.9	69.5	83.0	109.9	45.1	83.2	22.4	34.0	68.5	140.6

- a What was the wettest year in this 10-year period?
b What was the driest year in this 10-year period?
c Find the mean and median rainfall for January.
d What is the range in rainfall for January?
e Construct a histogram to illustrate the changes in rainfall over time.
f Briefly comment on any trends that can be seen from the graph.

CHALLENGING

- 8** During COVID-19, almost three quarters (73%) of businesses did not seek additional funds. These businesses reported on reasons for not seeking funds.



- a** What was the most important reason that small businesses did not seek extra funds during COVID-19?
- b** What percentage of businesses did not seek government assistance because they did not want to increase debt?
- c** What percentage of businesses did not seek government assistance because they had no capital expenditure requirements?
- d** List two ways this infographic could be misleading.
- 9** The grouped frequency table shows the ages of drivers involved in fatal road accidents during the past year.

Class	Class centre (x)	Frequency (f)	$f \times x$
20–29		85	
30–39		72	
40–49		71	
50–59		55	
60–69		36	

- a** Copy and complete the grouped frequency table.
- b** How many fatal road accidents occurred in the past year?
- c** Find the mean of this data to the nearest whole number.
- d** What percentage of road accidents had a driver younger than 30? Answer correct to two decimal places.
- e** What percentage of road accidents had a driver older than 49? Answer correct to two decimal places.

10 A motor vehicle is bought for \$32 000. It depreciates at 16% per annum and is expected to be used for 5 years.

a How much does the motor vehicle depreciate in the first year?

b Copy and complete the following depreciation table for the first five years.

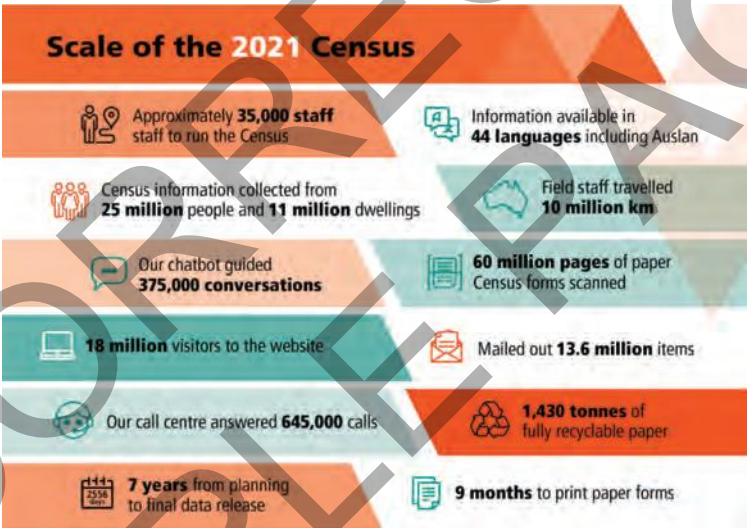
c Graph the value in dollars against the age in years.

Year	Current value	Depreciation	Depreciated value
1			
2			
3			
4			
5			



Communicate your reasoning, understanding and fluency

11 The infographic displays information about the 2021 Census in Australia.



a How many staff were needed to run the Census?

b How many chatbot-guided conversations were had in relation to the Census?

c How long did it take to plan the Census from start to final release?

d There were 645 000 call centre answered calls and 375 000 chatbot-guided conversations related to the 2021 Census. Discuss the employment implications that may be considered for the next Census?

e Why do you think the Census was made available in so many (44) languages? Do you think that this will need to increase for the next Census? Why?

STUDY
GUIDE

Key ideas and chapter summary

Categorical (Category)

Nominal Name does not indicate order
Ordinal Name does indicate order

Numerical (Numbers)

Discrete Data can only take exact numerical values such as 0 or 1
Continuous Data that can take any numerical value such as 71.25

Random sample

Members of the population have an equal chance of being selected.

Stratified sample

Categories of a population are chosen such as male/female. The members are randomly selected from each category.

Systematic sample

Population is divided into a structured sample size. The members are orderly selected from this structure, such as each 3rd person in order.

Self-selected sample

Members of the population volunteer themselves.

Dot plot

A number line with each data point marked by a dot.

Stem-and-leaf plot

Used to present a small (less than 50 values) numerical dataset. The 'tens' digit of the data values becomes the 'stem' and is written in numerical order down the page. The 'units' digits become the 'leaves' and are written in numerical order across the page.

Grouped frequency table

- 1 Classes are listed in ascending order.
- 2 Tally column records the number of times the score occurred.
- 3 Frequency column is a count of each outcome or class.

Cumulative frequency

The frequency of the score plus the frequency of all the scores less than that score.

Frequency and cumulative frequency graphs

- 1 Frequency histogram is a column graph that uses the score as the horizontal axis and frequency as the vertical axis.
- 2 Cumulative frequency histogram is a column graph with the score as the horizontal axis and cumulative frequency as the vertical axis.
- 3 Frequency polygon is a line graph of a frequency table. It can be constructed by joining the mid-points of the bars of the histogram.
- 4 Cumulative frequency polygon, or ogive, is a line graph constructed by starting at the left-hand bottom corner of the first rectangle and then joining the top right-hand corner of the rectangles in a cumulative frequency histogram.

Chapter checklist with success criteria



Checklist

A printable version of this checklist is available in the Interactive Textbook.



- 4A 1** I can use the four steps of a statistical investigation. ☐
 e.g. List the four steps of a statistical investigation.
- 4B 2** I can classify data as categorical or numerical. ☐
 e.g. Classify the data from these situations as categorical or numerical.
- a** The colour of pets in my street
 - b** The number of letters in each word on a Scrabble board
 - c** The places I visited while on holidays
- 4B 3** I can classify data as nominal or ordinal. ☐
 e.g. Classify the data from these situations as nominal or ordinal.
- a** The days of the week
 - b** The number of students who buy food from the canteen each day
- 4B 4** I can classify data as discrete or continuous. ☐
 e.g. Classify the data from these situations as discrete or continuous.
- a** The number of bananas in the shop near me
 - b** The weight of the pets in my street
- 4C 5** I can identify different types of samples. ☐
 e.g. A netball club has 100 players: 75 are girls and 25 are boys. Decide whether each sample would be random, stratified, systematic or self-selected.
- a** The players who want to play centre
 - b** Names drawn out of a hat
 - c** Players sorted into teams and every third person selected
 - d** 2 of the boys and 6 of the girls
- 4D 6** I can construct a dot plot. ☐
 e.g. Construct a dot plot for the following dataset.
 14 14 15 16 17 12 12 14 15 16
- 4D 7** I can construct a stem-and-leaf plot. ☐
 e.g. Use the data in Question 6 to construct a stem-and-leaf plot.

4D 8 I can construct a back-to-back stem-and-leaf plot. ☐

e.g. Construct a back-to-back stem-and-leaf plot for the following dataset.

A: 13 13 13 12 2 12 14 15 7 6

B: 14 4 15 16 7 12 12 14 5 16

4E 9 I can draw a line graph. ☐

e.g. Use the data in Question 6 to construct a line graph.

4E 10 I can construct a sector graph and a divided bar chart. ☐

e.g. A group of flags was collected and the colours in each flag recorded.

Red: 5 Green: 3 White: 10 Yellow: 3 Blue: 4

Construct a sector graph and divided bar chart to represent this data.

4F 11 I can construct a frequency table. ☐

e.g. Use the data below to construct a frequency table.

14 14 15 16 17 12 12 14 15 16

4F 12 I can construct a grouped frequency table. ☐

e.g. Use the data below to construct a grouped frequency table using classes of 0–4, 5–9, 10–14, 15–19.

17 9 15 0 5 6 8 3 2 8

4G 13 I can calculate the cumulative frequency. ☐

e.g. Add a cumulative frequency column to your frequency table in Question 11.

4H 14 I can construct a frequency histogram and polygon. ☐

e.g. Create a frequency histogram and polygon (ogive) from the data in Question 11.

4H 15 I can construct a cumulative frequency histogram and polygon. ☐

e.g. Create a cumulative frequency histogram and polygon (ogive) from the data in Question 11.

4I 16 I can read a table of motor vehicle statistics. ☐

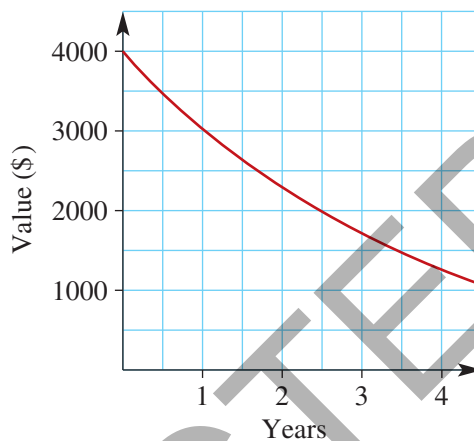
e.g. The table shows the number of road accidents involving speed that caused an injury in the first five months of the year. Find the following summary statistics.

- How many accidents involved speed in the first five months?
- What percentage of these road accidents occurred in January?
- What fraction of these road accidents occurred in March?
- Which month had the highest number of accidents? Can you suggest a reason for this month having the most accidents?

Month	Accidents
January	2814
February	1652
March	1786
April	1589
May	2182

4I 17 I can interpret real-world graphs and charts. □

e.g. The decrease in the price of a used car over four years is shown in the graph.



- a** What is the initial value of the used car?
- b** How much did the used car decrease in price during the first year?
- c** When is the value of the used car \$2000?
- d** When is the value of the used car \$1500?
- e** What is the value of the used car after 4 years?
- f** What is the value of the used car after $1\frac{1}{2}$ years?

4I 18 I can interpret information presented as an infographic. □

e.g. From the infographic in Question **11** of Exercise 4I about the 2021 Census, find:

- a** The distance field staff travelled
- b** The number of pages of paper used
- c** The number of dwellings that information was collected from

Multiple-choice questions

- 4A 1** Which of these is not one of the four steps in a statistical investigation?
A Organise the data **B** Review the data
C Analyse the data **D** Collect the data
- 4B 2** 'The number of rooms in your house'. What is the classification for this data?
A Categorical **B** Continuous **C** Discrete **D** Text
- 4B 3** What classification is a set of data that lists the height of windows in your house?
A Discrete **B** Nominal **C** Ordinal **D** Continuous
- 4C 4** A sample is designed to include four boys and four girls randomly selected from members of a surf club. How would you describe this type of sample?
A Self-selected **B** Random **C** Stratified **D** Systematic
- 4D 5** The female and male smoking rates, expressed as percentage are shown in the back-to-back stem-and-leaf plot.

Female	Male
9 7 7 6 5 1	7 9
8 6 5 5 6 2 1 0	2 2 2 4 4 5 6 7 7
	3 0 0 1 1 6 6 9

What is the lowest female smoking rate?

- A** 5% **B** 15% **C** 17% **D** 19%
- 4F 6** What is the frequency of 22 in this cumulative frequency table?
- A** 5 **B** 8
C 13 **D** 22
- 4G 7** What is the cumulative frequency of 23?
- A** 7 **B** 13
C 16 **D** 20
- 4I 8** Jesse's water usage in one day was 60 L shower, 45 L toilet, 30 L tap and 15 L washing machine. What percentage of Jesse's water usage is for the washing machine?
- A** 10% **B** 15% **C** 30% **D** 150%

Score	Frequency	Cumulative frequency
21	5	5
22		13
23	7	
24	3	23
	Total	

Short-answer questions

- 4B 1** Classify the data from these situations as numerical or categorical.

- a** Most popular student in the class
- b** Ava's favourite beach in Australia
- c** The call cost on a mobile phone
- d** Blake's school high jump record
- e** The amount of annual leave
- f** The hair colour of the students in your class
- g** The number of websites accessed today
- h** The digital download time for a 4 MB file
- i** The average age of the people living in NSW



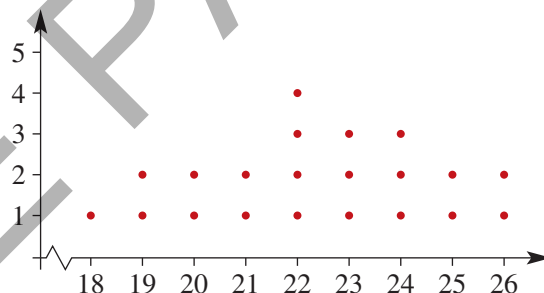
- 4D 2** The stem-and-leaf plot shown represents the results achieved by students in a test.

- a** What is the highest score?
- b** What is the difference between the highest and lowest scores?
- c** What is the median?

0	9
1	1 2 4 9
2	1 3 5 7 8 8
3	0 2 3 5

- 4D 3** Use the dot plot to answer these questions.

- a** What is the lowest score?
- b** What is the difference between the highest and lowest scores?
- c** What is the mode?
- d** What is the median?



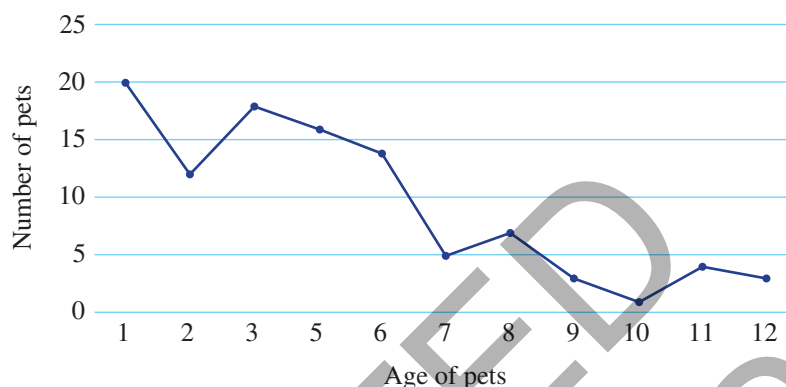
- 4G 4** The times (in hours) students spent completing an assessment task are listed in the table.

- a** Copy and complete the table.
- b** How many students completed the assessment task?
- c** How many students spent greater than 13 hours?
- d** What percentage of students spent less than 14 hours?

Class	Frequency	Cumulative frequency
4–8	5	
9–13	6	
14–18	8	
19–23	4	

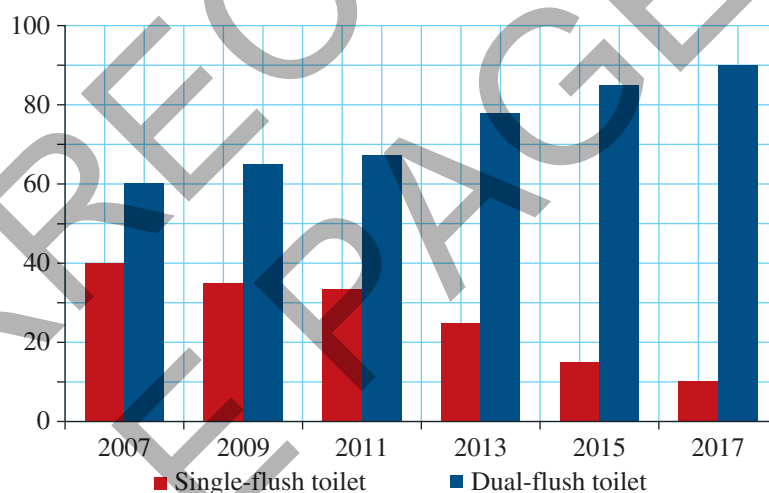
- 4F** 5 The line graph represents the ages of pets in a group of 120 pets.

- What is the age of the oldest pet in this dataset?
- What is the age of the youngest pet in this dataset?
- How many pets are 6 years old?
- What percentage of pets are 8 years old?
- What percentage of pets are less than 5 years old?



- 4I** 6 The percentage of households using single- and dual-flush toilets is shown in the graph.

- What percentage of households used a dual-flush toilet in 2007?
- In which year was the percentage of single-flush toilets half the percentage of dual-flush toilets?
- What is the trend shown by this column graph?
- There were 14.2 million toilets in 2017. How many of these toilets were dual flush?



Extended-response question

- 1 Alyssa recorded the following times (in minutes) running a cross-country course.

51 53 57 55 58 57 53 55 54 53 56 55 53 57 51 57 54
 58 55 56 51 53 53 54 52 56 52 54 54 53 56 56 53 54
 52 53 54 57 58 56 54 55 52 55 55 58 54 56 55 56 52

- 4F** a Construct a frequency table using a tally column.

- b How many times were recorded?

- c How many times were below 55 minutes?

- 4G** d Add a cumulative frequency column.

- e What is the Alyssa's fastest time?

- f What is the difference between Alyssa's fastest and slowest times?

- 4H** g Construct a frequency histogram.

- h Construct a cumulative frequency histogram and a cumulative frequency polygon.

Practice Paper 1

Section I

Attempt Questions 1–15 (15 marks)

Allow about 20 minutes for this section

- 1 Which of the following is the highest rate of pay?
A \$1442.22 per week
B \$2884.68 per fortnight
C \$6247.50 per month
D \$75 000 per annum
- 2 The colours of jelly beans in a packet are used to make a dataset. The data is:
A categorical, nominal
B categorical, ordinal
C numerical, discrete
D numerical, continuous.
- 3 What is the new price when \$60 is increased by 20% then decreased by 20%?
A \$38.40
B \$57.60
C \$60.00
D \$86.40
- 4 Stella sells shoes for a retail store and receives wages of \$1875 per month plus 4% commission on all her sales. What were her sales in a month in which she received a total pay of \$ 1953?
A \$878.00
B \$1946.88
C \$1950.00
D \$2190.12
- 5 When the decimal 2.97521 is rounded to two significant figures, the result is:
A 2.9
B 2.97
C 2.98
D 3.0
- 6 Arrange the numbers 4.8×10^{-2} , 4.0×10^{-1} , and 5.6×10^{-2} in ascending order.
A 4.8×10^{-2} , 5.6×10^{-2} , 4.0×10^{-1}
B 4.0×10^{-1} , 4.8×10^{-2} , 5.6×10^{-2}
C 5.6×10^{-2} , 4.8×10^{-2} , 4.0×10^{-1}
D 4.0×10^{-1} , 5.6×10^{-2} , 4.8×10^{-2}
- 7 What is the value of $2x - x^2$ if $x = -3$?
A -15
B -3
C 3
D 15
- 8 A wage sheet of a mobile phone business shows one employee's details.

Employee	Rate per hour	Normal hours	Overtime ($\times 2$)	Wage
Terry Brown	\$20.00	30	x	\$840

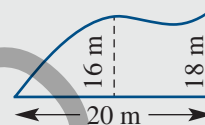
Terry worked some overtime at double-time rate but it is missing from the wage sheet. Using the information on the wage sheet, how many hours of overtime did Terry work?

- A** 4
B 5
C 6
D 8
- 9 The circumference of a bicycle wheel is 220 cm. How many revolutions will the wheel make if it travels a distance of 6.6 km?
A 30
B 33.3
C 300
D 3000
- 10 Find the value of m , correct to one decimal place, given $k = 24$ and the formula $m = \sqrt{\frac{k}{5}}$.
A 1.0
B 2.2
C 2.4
D 4.8
- 11 Find the value of x given $a = 32$, $y = 2$ and the formula $a = 2xy^2$.
A 3
B 4
C 5
D 6

- 12** Ann is x years old. Bill is 6 years older than Ann. Con is 3 years older than Bill. Their total age is 45 years. Which equation represents this information?

A $x + 9 = 45$ **B** $x + 15 = 45$ **C** $3x + 9 = 45$ **D** $3x + 15 = 45$

- 13** The irregular shape shown on the right has a breadth of 20 metres. The vertical dashed line divides it into two parts of equal breadth. Two applications of the trapezoidal rule were used to estimate the area. Which expression is correct?



A $\frac{10}{2}(16 + 18)$ **B** $\frac{10}{2}(32 + 18)$ **C** $\frac{20}{2}(16 + 18)$ **D** $\frac{20}{2}(32 + 18)$

- 14** What is the solution to the equation $9x - 8 = 27$?

A $x = \frac{9}{35}$ **B** $x = \frac{7}{3}$ **C** $x = 1\frac{8}{9}$ **D** $x = 3\frac{8}{9}$

- 15** A book was bought for $\$(x + 10)$ and sold for $\$(x - 20)$. Which of the following statements is true?

A There was a profit of $\$(x - 30)$. **B** There was a loss of \$30.
C There was a profit of $\$(x + 10)$. **D** There was a loss of $\$(x - 30)$.

Section II

Attempt Questions 16–18 (45 marks)

Allow about 70 minutes for this section

All necessary working should be shown for every question.

Question 16 (15 marks)

- a** A communication company pays overtime at a rate of time-and-a-half for the first 4 hours overtime and double-time thereafter. Natalie is employed as a personal assistant. During a normal week she works 35 hours at \$27.80 an hour.

i How much did Natalie earn in a week in which she worked 42 hours? (2 marks)

ii Natalie receives annual leave loading of $17\frac{1}{2}\%$ of 4 weeks basic pay. (2 marks)

1 What is the value of Natalie's leave loading?

2 Calculate the total amount Natalie is paid for her 4 weeks annual leave.

b

Taxable income	Tax payable
0–\$18 200	Nil
\$18 201–\$45 000	16c for each \$1 over \$18 200
\$45 001–\$135 000	\$4288 plus 30c for each \$1 over \$45 000
\$135 001–\$190 000	\$31 288 plus 37c for each \$1 over \$135 000
\$190 001 and over	\$51 638 plus 45c for each \$1 over \$190 000

Daniel earns a gross income of \$63 500 during the financial year.

He has allowable deductions of \$4500.

i What is Daniel's taxable income? (1 mark)

ii Calculate the tax payable on Daniel's income. (2 marks)

iii Daniel must pay 2% of his taxable income for the Medicare levy. Calculate how much Daniel pays in Medicare levy. (1 mark)

- iv** What is Daniel’s total tax payable including the Medicare levy? (1 mark)
- v** Daniel has paid \$12 255 in tax during the financial year. Determine whether Daniel receives a refund or whether he is required to pay more tax, and determine this amount. (2 marks)
- c i** Construct a frequency histogram for the dataset below. (2 marks)

Number of apps on my phone	Number of students
12	7
13	12
14	6
15	8
16	16
17	14

- ii** Add a cumulative frequency column to the table and represent this dataset as a cumulative frequency histogram and polygon. (2 marks)

Question 17 (15 marks)

- a** Logan records the details of each CD in his music collection. State the type of data that would be recorded for each of the following.
 - i** Title of the CD (1 mark)
 - ii** Number of tracks on the CD (1 mark)
 - iii** Playing time for each track (1 mark)
- b** An ornament is the shape of a cone with a 4.5 cm diameter and a height of 9 cm. Find the volume of the ornament. Answer correct to two decimal places. (2 marks)
- c** Solve the equation $\frac{2x - 5}{3} = 1$. (2 marks)
- d** A cylindrical aluminium can has a diameter of 22 cm and a height of 13 cm.
 - i** What is the total surface area of the cylindrical can? Answer correct to one decimal place. (2 marks)
 - ii** The curved surface of the cylindrical can is cut from an aluminium sheet 100 cm by 70 cm. How many curved surfaces can be cut? (1 mark)
- e** Olivia rolled two dice, then subtracted the lowest score from the highest score. For example, when she rolled a 5 and 1, she scored 4. Do this experiment 36 times.
 - i** Fill in the table.

Score	Frequency
0	
1	
2	
3	
4	
5	

- ii** Draw a frequency histogram and polygon. (2 marks)
- iii** Add a cumulative frequency column to the table. (1 mark)
- iv** Draw a cumulative frequency histogram and polygon. (2 marks)

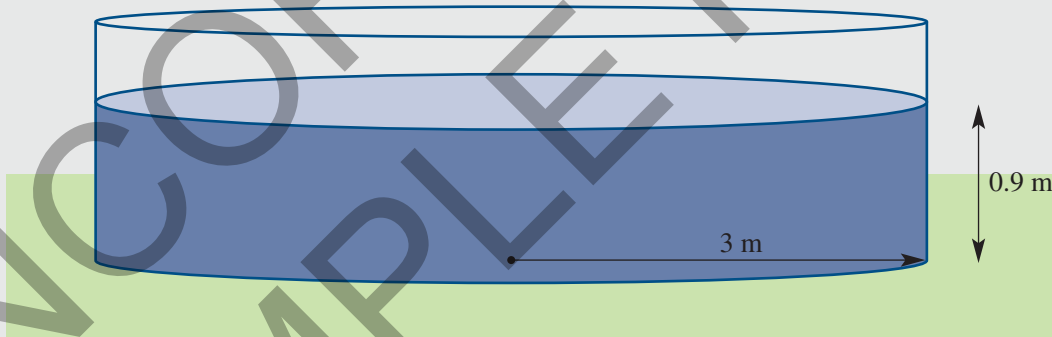
Question 18 (15 marks)

- a** The volume of a solid is given by the formula $V = \frac{4}{3}\pi r^3$ where r is the radius.
- i** Write the formula with r as the subject. (2 marks)
 - ii** What is the radius in metres of the solid, given it has a volume of 2 m^3 ? Answer correct to two decimal places. (1 mark)
- b** Henry's room measures 5850 mm by 4950 mm and needs carpeting.
- i** The cost of the carpet is \$90 per m^2 , and a tradesperson charges \$40 per m^2 to lay the carpet. What is the cost to have the room carpeted? (2 marks)
 - ii** Henry's room has a ceiling height of 2800 mm. He is considering buying heater A, B or C as shown in the table below. (2 marks)

Heater A	Rooms up to 70 m^3
Heater B	Rooms up to 80 m^3
Heater C	Rooms up to 90 m^3

Determine the most suitable heater and give a reason for your answer.

- c** An above-ground swimming pool is the shape of a cylinder. It has a radius of 3 metres and contains water to a uniform depth of 0.9 metres.
- i** What is the volume of water in the pool, in cubic metres? Answer correct to two decimal places. (2 marks)
 - ii** How many litres of water are in the pool? Answer correct to the nearest litre. (1 mark)
($1 \text{ m}^3 = 1000 \text{ L}$)



- d** What is the value of $\sqrt{\frac{a+3b}{4b}}$ if $a = 6.4$ and $b = 2.1$? Answer to two decimal places. (2 marks)
- e** Make x the subject of the equation $y = mx + c$. (1 mark)
- f** The number of 'standard drinks' in various glasses of wine is shown. (2 marks)

Number of standard drinks			
White wine		Red wine	
Small glass	Large glass	Small glass	Large glass
0.8	1.3	1.0	1.5

Jane weighs 66 kg and drinks two small glasses of white wine and three large glasses of red wine between 7:00 p.m. and midnight.

What would be her blood alcohol content (BAC) estimate at midnight?

Use the formula $BAC_{\text{female}} = \frac{10N - 7.5H}{5.5M}$

Answer correct to two significant places.

5

Linear relationships

Linear relationships are all around us!

Here are some everyday examples:

- Buying cans of drink for \$4 each: 1 for \$4, 2 for \$8, 3 for \$12 and so on.
- Saving money for a future purchase: add \$20 to your savings every week.
- Arranging for a plumber to fix your dripping taps: \$100 call-out fee plus \$120 per hour.
- A birthday party in a functions venue: \$500 for the room hire plus \$35 per person for food.
- Getting garden mulch delivered to your home from a nursery: \$25 delivery fee plus \$144 per cubic metre.
- Spending money for an overseas trip: you have saved up \$5000 but you plan to spend \$200 per day.

Chapter contents

- 5A** Straight-line graphs
- 5B** Gradient and y-intercept
- 5C** Using the gradient-intercept formula
- 5D** Linear modelling
- 5E** Direct variation

NSW Syllabus

In this chapter, a student:

- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- Models and interprets linear relationships to solve problems and make predictions in practical contexts (MST-11-02)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

5A Straight-line graphs

Learning intentions

- To draw straight-line graphs by constructing a table of values and plotting points on the Cartesian plane

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12
- Questions about linear relationships regularly appear in HSC Examinations
- Formulas for the equation of a straight line are assumed knowledge and do not appear on the 2027 HSC Examination Reference Sheet

Linear relationships: The big picture

This topic should be familiar to you from previous years of schooling.

In this chapter, the following concepts will be consolidated and extended, with emphasis on real-life scenarios which are linear relationships.

A linear relationship can be viewed and represented in several different ways, as follows.

A real-life scenario

Sophie has \$4 in her money box at present.

This is a single, one-off, once-only amount, at the beginning of a time period. The time is 0.

She decides to add \$3 every week. This is a repetitive increase.

This is a **linear relationship**, because she is always adding the same amount every week.

A number pattern

4, 7, 10, 13, ...

The numbers in the pattern are sometimes called **terms**.

The number 4 is the **present value**.

The numbers 7, 10, 13, ... are the **future values** of this investment.

A table of values

The numbers 4, 7, 10, 13, ... are placed in the bottom row of a table.

The top row tracks the weeks, starting from 0.

Weeks (<i>W</i>)	0	1	2	3
Dollars (<i>D</i>)	4	7	10	13

Variables

‘Weeks’ and ‘Dollars’ are called **variables**.

The top row is the **independent variable**.

The bottom row is the **dependent variable**.

An equation, sometimes called a ‘rule’ or a ‘function’

You may recall an activity called ‘guess the rule’, but guessing is really not required.

You just need to look in the correct places:

- Sophie is adding \$3, repeatedly. This is known as the **gradient, slope** or **rate of change**.
- She began with \$4. This is known as the **constant**, or the **vertical intercept**.

Choose any number in the top row. Multiply by 3 then add 4. That will give you the number in the bottom row.

$$W \times 3 + 4 = D$$

$$3W + 4 = D$$

The equation is usually written ‘backwards’.

$$D = 3W + 4$$

Note:

- The gradient (3) has a letter attached to it.
- The vertical intercept (4) is the constant term.

Every straight line can be written in the form:

$$y = mx + c, \text{ where:}$$

m is the **gradient**, sometimes called the **slope**

c is the **y-intercept**, also known as the **vertical intercept**.

A series of points on the Cartesian plane

Every column in the table is a point: (0, 4) then (1, 7), (2, 10), (3, 13), ...

A graph on the Cartesian plane

The points can be joined with one **straight line**.

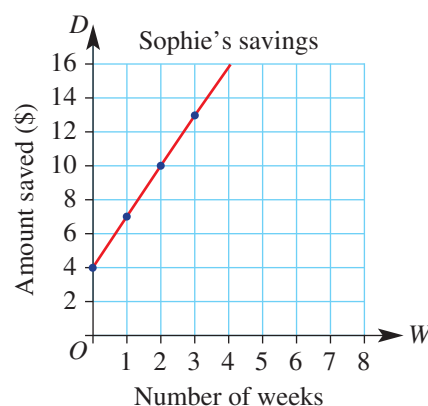
Sophie’s dollars are **increasing at a constant rate**.

The relationship between weeks and dollars is linear.

This is an example of a **linear relationship**.

Note:

- The top row of the table is the horizontal axis.
- The bottom row of the table is the vertical axis.
- The scale on the x -axis can be different to the scale on the y -axis.



Summary

Every linear relationship can be viewed and represented as a:

- worded real-life scenario
- number pattern
- table of values containing a series of points
- rule called an equation
- straight-line graph on the Cartesian plane.

In this chapter, you will:

- become more familiar with each of these
- learn how to move efficiently from one representation to the others
- solve problems involving real-life scenarios.

Drawing a straight-line graph

- 1 Construct a table of values with the independent variable as the top row and the dependent variable as the bottom row.
- 2 Draw a number plane with the independent variable on the horizontal axis and the dependent variable as the vertical axis.
- 3 Plot the points.
- 4 Draw a straight line through the points.

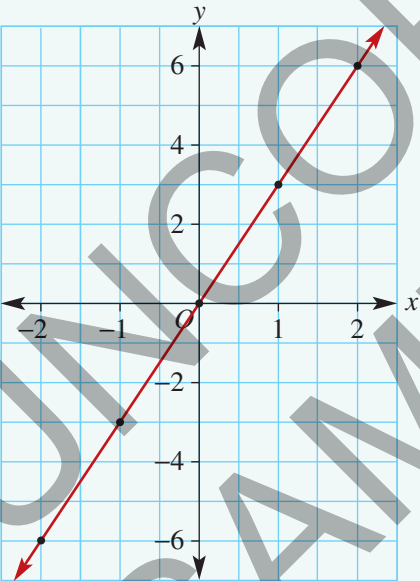


Example 1 Drawing a straight-line graph

Draw the graph of $y = 3x$.

Solution

x	-2	-1	0	1	2
y	-6	-3	0	3	6



Explanation

Choose some x -values.
Calculate y -values.

Draw a Cartesian plane.
Plot the points.
Draw a line through all the points.

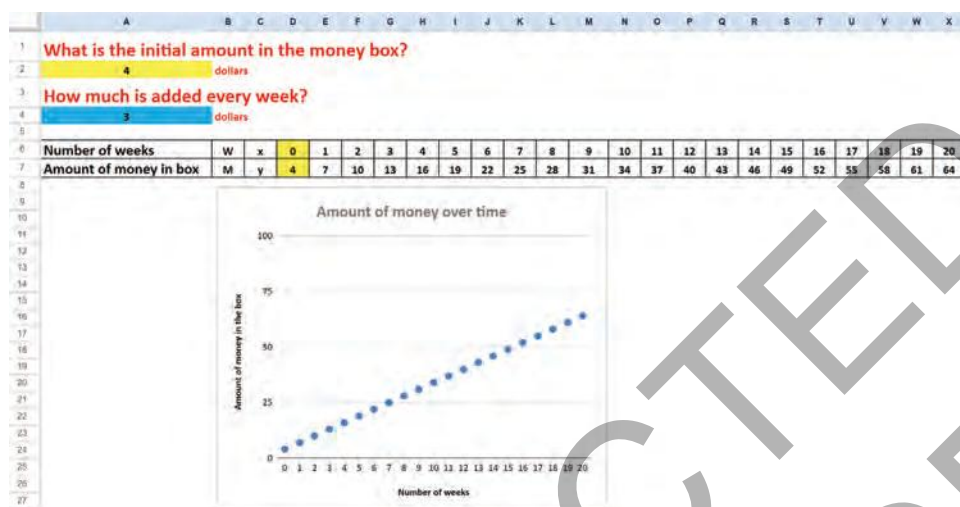
Now you try

Draw the graph of $y = 2x$.

Spreadsheet activity

 Using Microsoft Excel, a Google Sheet or some other spreadsheet software, there are numerous ways to create the representations of a linear relationship.

This snapshot is just one way to do this:



You can access this file from the online interactive version of this book.

You should:

- examine the formulas in the second row of the table of values
- try changing the numbers in the yellow box and blue box
- adapt the spreadsheet for other linear relationships.

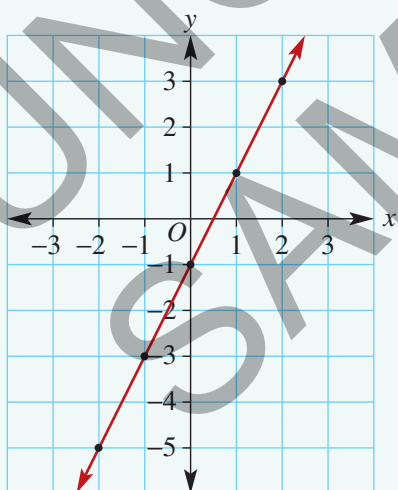


Example 2 Drawing a straight-line graph

Draw the graph of $y = 2x - 1$.

Solution

x	-2	-1	0	1	2
y	-5	-3	-1	1	3



Explanation

Choose some x -values.
Calculate y -values.

Draw a Cartesian plane.
Plot the points.
Draw a line through all the points.

Now you try

Draw the graph of $y = 2x + 1$.

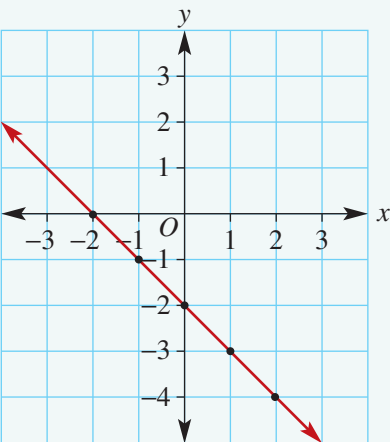


Example 3 Drawing a straight-line graph

Draw the graph of $y = -x - 2$.

Solution

x	-2	-1	0	1	2
y	0	-1	-2	-3	-4



Explanation

Choose some x -values.
Calculate y -values.

Draw a Cartesian plane.
Plot the points.
Draw a line through all the points.

Now you try

Draw the graph of $y = 4 - 2x$.

Exercise 5A

BUILDING

1 Plot the following points on a number plane and draw a straight line through them.

a

x	-2	-1	0	1	2
y	2	1	0	-1	-2

b

x	-2	-1	0	1	2
y	-3	-1	1	3	5

c

x	-2	-1	0	1	2
y	0	1	2	3	4

d

x	-2	-1	0	1	2
y	3	2	1	0	-1

2 Complete the following table of values for each equation.

a $y = 2x$

x	-2	-1	0	1	2
y					

b $y = -2x$

x	-2	-1	0	1	2
y					

c $y = x + 1$

x	-2	-1	0	1	2
y					

d $y = x - 1$

x	-2	-1	0	1	2
y					

e $y = 2x + 3$

x	-2	-1	0	1	2
y					

f $y = -x + 2$

x	-2	-1	0	1	2
y					

Example 1

Example 2

Example 3

3 Draw the graph of each straight line by first drawing a table of values.

a $y = x$

b $y = 2x + 2$

c $y = -x + 3$

d $y = -x - 1$

e $y = 3x - 1$

f $y = -2x + 3$

4 a Complete the following tables of values.

i $b = 3a + 4$

a	-2	-1	0	1	2
b					

ii $q = -p + 1$

p	-2	-1	0	1	2
q					

b Graph $b = 3a + 4$.

c Graph $q = -p + 1$.

DEVELOPING

5 Chocolates are sold for \$12 per kg. The table shows weight against cost.

Weight (w)	1	2	3	4	5
Cost (c)	12	24	36	48	60

a Which is the dependent variable?

b Which is the independent variable?

c Draw a graph of weight against cost.

d Use the graph to find c when $w = 1.5$.

6 Soraya conducted a science experiment and presented the results in a table.

Mass (m)	3	6	9	12	15
Time (t)	2.2	3.7	5.2	6.7	8.2

a Draw a graph of mass against time.

b Use the graph to find t when $m = 10$.

7 Dallas travels 4 km in 2 hours.

a Write a linear equation in the form $d = mt$ to describe this situation.

b Complete the table and draw the graph of d against t .

Time (t)	0	2	4	6	8
Distance (d)					

- 8 Describe the following statements as true or false.
- a The point (3, 0) lies on the line $y = 3x$.
 - b The line $y = x + 7$ passes through the point (-1, 6).
 - c The point (1, 4) lies on the line $x + 2y = 9$.
 - d The line $2x - 3y = 0$ passes through the point (0, 0).
 - e The point (1, -1) lies on the line $4x - y + 1 = 0$.
 - f The line $3x - 4y + 1 = 0$ passes through the point (2, 0).

CHALLENGING

- 9 Graph these lines by first drawing a table of values.

a $y + 2 = 3x$

b $x + y - 4 = 0$

c $y + \frac{1}{2}x = 1$

d $4x - y + 2 = 0$

e $\frac{1}{3}x - y = 1$

f $4 - y = 3x$



Communicate your reasoning, understanding and fluency

- 10 The table of values is all mixed-up in two different ways:

x	3	-2	0	2	1	-1	-3
y	11	-9	-3	5	1	-7	-15

- The numbers in the top row are out of order.
 - Two of the numbers in the bottom row are not on the same straight line as the other five numbers.
- a By rearranging the table, or drawing the graph, which two points are not on this straight line? How did you work that out?
- b For the two incorrect points, what should their coordinates be?
- c What is the equation of the straight line?

5B Gradient and y-intercept

Learning intentions

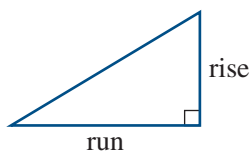
- To calculate the gradient of a straight line using the vertical rise and horizontal run from one point to another
- To identify the vertical intercept from a table of values, a graph and from the equation of the line

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12
- Questions about linear relationships regularly appear in HSC Examinations
- Formulas for the equation of a straight line are assumed knowledge and do not appear on the 2027 HSC Examination Reference Sheet

Gradient

The gradient of a line is the slope or steepness of the line. It is calculated by dividing the vertical rise by the horizontal run. The larger the gradient, the steeper the slope. The letter m is used to indicate gradient.

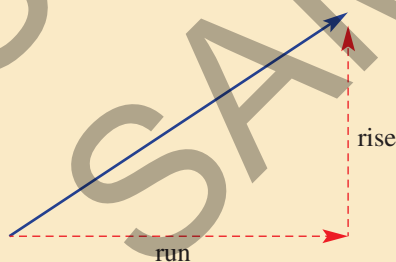


$$\text{Gradient of a straight line } m = \frac{\text{vertical rise}}{\text{horizontal run}}$$



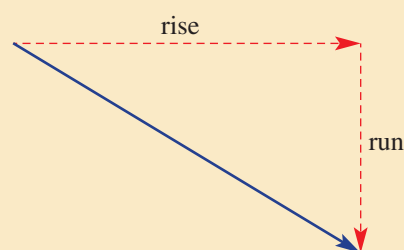
Like reading a book, it is best to view a line from left to right.

Positive gradient



Run is positive.
Rise is positive.
The line goes 'uphill'.

Negative gradient



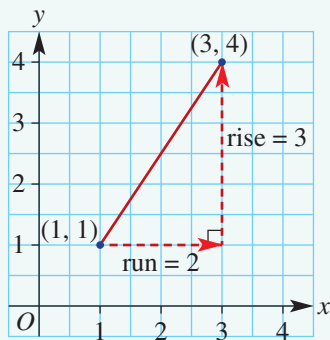
Run is positive.
Rise is negative.
The line goes 'downhill'.



Example 4 Finding the gradient of a line

Find the gradient of a line through the points (1, 1) and (3, 4).

Solution



$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{3}{2}$$

Explanation

Draw a Cartesian plane.
Plot the points.
Make a right-angled triangle.

Use the formula.

Now you try

Find the gradient of the line through the points (0, 0) and (3, 4).

The y-intercept

Lines can cross the x -axis and the y -axis at ‘intercepts’. The intercept on the vertical axis is called the y -intercept and is denoted by the letter c .

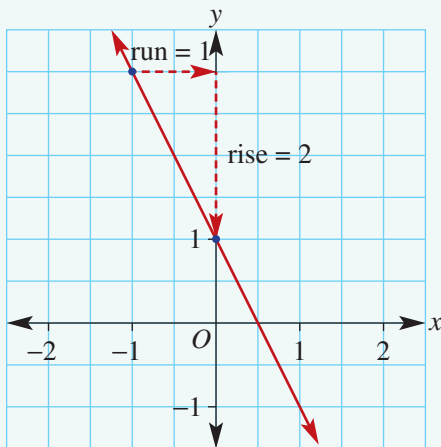
Gradient	Intercept
Gradient of a line is the slope of the line. $m = \frac{\text{rise}}{\text{run}}$	The intercept of a line is where the line cuts the axis. Vertical intercept is often denoted by c . In the table of values, the vertical intercept is always the number in the bottom row, under the 0.



Example 5 Finding the gradient and vertical intercept

Find the gradient and vertical intercept of this line.

x	-1	0	1
y	3	1	-1

Solution

$$\begin{aligned}
 m &= \frac{\text{rise}}{\text{run}} \\
 &= \frac{2}{1} \\
 &= 2
 \end{aligned}$$

Intercept on the vertical axis is 1.

Explanation

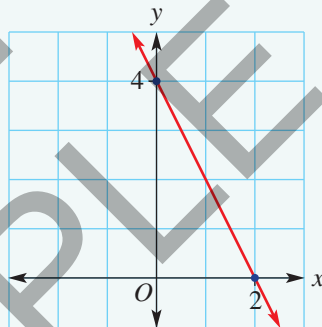
Plot the points.
Draw a line.
Choose two points.

The run is 1.
The rise is -2.

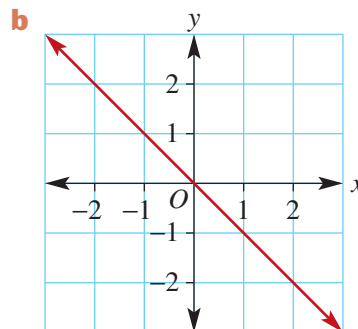
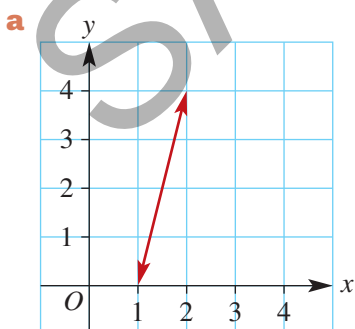
From the table or the graph.

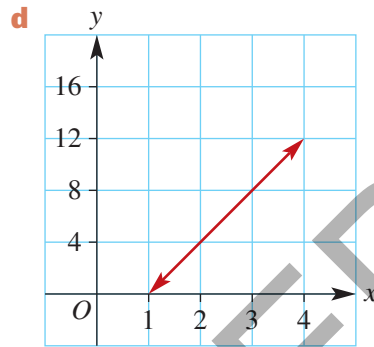
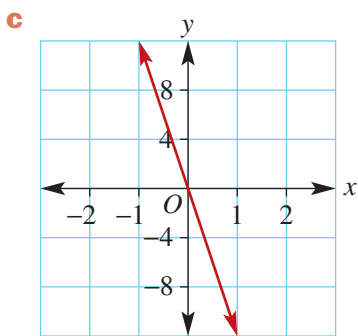
Now you try

Find the gradient and vertical intercept of the following graph.

**Exercise 5B****BUILDING**

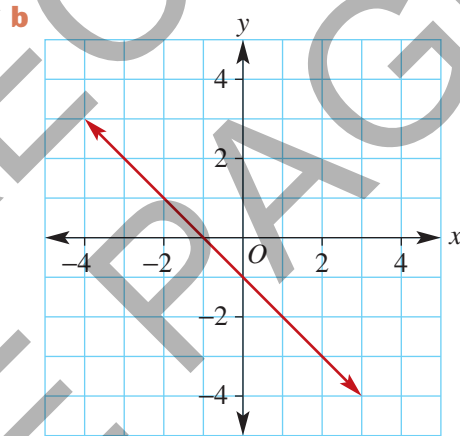
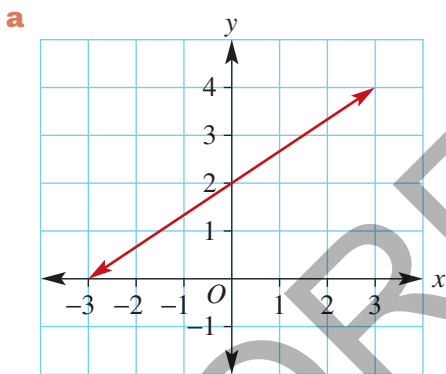
- 1** Find the gradient of the following lines.





Example 4

- 2** What is the gradient of the line that joins these points?
- a** (0, 1) and (2, 5) **b** (1, 3) and (2, -2) **c** (2, -1) and (4, -2)
- 3** What is the intercept on the vertical axis for the following lines?



DEVELOPING

Example 5

- 4** Plot the following points on a number plane and join them to form a straight line. Determine the gradient and y-intercept for each line.

a

x	0	1	2	3	4
y	2	4	6	8	10

b

x	-2	-1	0	1	2
y	3	1	-1	-3	-5

c

x	0	2	4	6	8
y	-1	1	3	5	7

d

x	0	3	6	9	12
y	0	1	2	3	4

- 5** Draw a graph of these lines and find the gradient (m) and the y-intercept (c).

a

x	0	3
y	-3	0

b

x	0	4
y	1	0

c

x	-3	0
y	0	3

d

x	-2	0
y	0	-2

- 6** The distance (d) a train travels in kilometres is calculated using the formula $d = 150t$ where (t) is the time taken in hours.
- Construct a table of values using 0, 1, 2, 3 and 4 as values for t . Calculate the distance (d).
 - Draw the graph of the distance (d) against the time (t).
 - What is the gradient of the graph?
 - What is the intercept on the vertical axis?



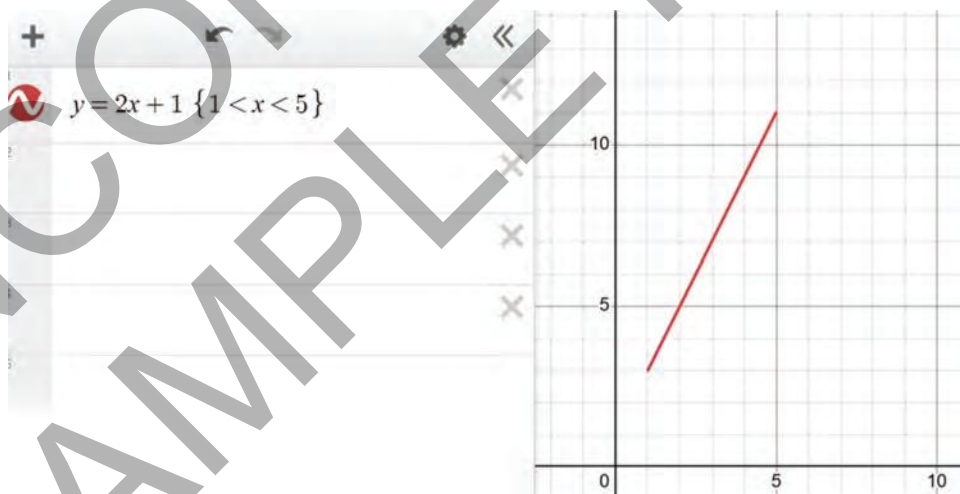
CHALLENGING

- 7** Meat is sold for \$16 per kilogram.
- Construct a table of values using 0, 1, 2, 3 and 4 as values for the number of kilograms (n). Calculate the cost (c) of the meat.
 - Draw the graph of the cost (c) against the number of kilograms (n).
 - What is the gradient of the graph?
 - What is the intercept on the vertical axis?



Communicate your reasoning, understanding and fluency

- 8** In the graphing software Desmos, it is possible to draw short pieces of lines, like this:



Use Desmos to make a new flag for a new country, using this one for inspiration. It must have some diagonal lines. It does not need to be coloured in.



5C Using the gradient–intercept formula

Learning intentions

- To understand the gradient–intercept formula for the equation of straight lines
- To use the equation of a straight line to identify the gradient and vertical intercept
- To draw straight lines using gradient and vertical intercept

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12
- Questions about linear relationships regularly appear in HSC Examinations
- Formulas for the equation of a straight line are assumed knowledge and do not appear on the 2027 HSC Examination Reference Sheet

When the equation of a line is written as $y = mx + c$, it is said to be in ‘gradient–intercept form’. The gradient is m , the coefficient of x , and the y -intercept is c , the constant term. The independent variable in the formula is x and the dependent variable in the formula is y .

Gradient–intercept form is useful in modelling relationships in many practical situations. However, the variables are often changed to reflect the situation. For example, the equation $c = 25n + 100$ has c as the cost of the event (\$) and n as the number of guests. These letters are the dependent and independent variables.

Using the gradient–intercept formula

$$y = mx + c$$

m = gradient of the line (vertical rise over the horizontal run)

c = y -intercept where the line cuts the y -axis or vertical axis

Example 6 Finding the gradient and y -intercept from the equation

Write the gradient and y -intercept for each of the following equations.

a $y = -2 + 5x$

b $y = 8 - x$

c $y = 6x$

d $y - 3x = 4$

Solution

a $y = -2 + 5x$
 $y = 5x - 2$
 Gradient is 5.
 y -intercept is -2 .

b $y = 8 - x$
 $y = -1x + 8$
 Gradient is -1 .
 y -intercept is 8.

Explanation

If the equation is written $y = mx + c$, the gradient is the coefficient of x .

The y -intercept is the constant term.

c $y = 6x$

$$y = 6x + 0$$

Gradient is 6.

y-intercept is 0.

d $y - 3x = 4$

$$y = 3x + 4$$

Gradient is 3.

y-intercept is 4.

Add $3x$ to both sides.

Now you try

Write the gradient and y-intercept of the line $2x + 3y = 6$.

Sketching a linear function using the gradient–intercept formula

Sketching a straight-line graph requires at least two points. When an equation is written in gradient–intercept form, one point on the graph is immediately available: the y-intercept. A second point can be quickly calculated by using the gradient or by substituting a suitable value of x into the equation.



Example 7 Sketching a straight-line graph from its equation

Draw the graph of $y = 3x + 1$.

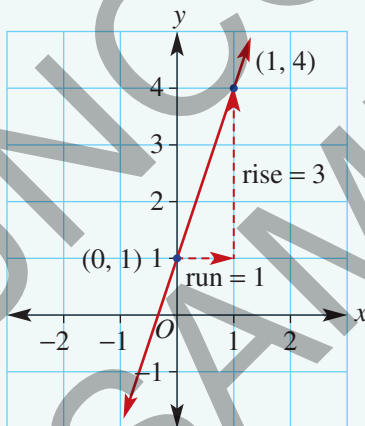
Solution

Method 1:

$$y = 3x + 1$$

Gradient is 3.

y-intercept is 1.



Method 2:

Let $x = 0$ in $y = 3x + 1$

$$y = 3 \times 0 + 1$$

$$y = 1$$

y-intercept = $(0, 1)$

Explanation

If the equation is $y = mx + c$:

the gradient is m

the y-intercept is c .

Substitute $x = 0$.

Solve to find y .

This gives the y-intercept.

continued ...

$$\text{Let } y = 0 \text{ in } y = 3x + 1$$

$$0 = 3x + 1$$

$$3x + 1 = 0$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$$x\text{-intercept} = \left(-\frac{1}{3}, 0\right)$$

Substitute $y = 0$.
Solve to find x .

This gives the x -intercept.

Now you try

Draw the graph of $y = 3x - 1$.



Example 8 Sketching a straight-line graph from its equation

Draw the graph of $y = -2x + 1$.

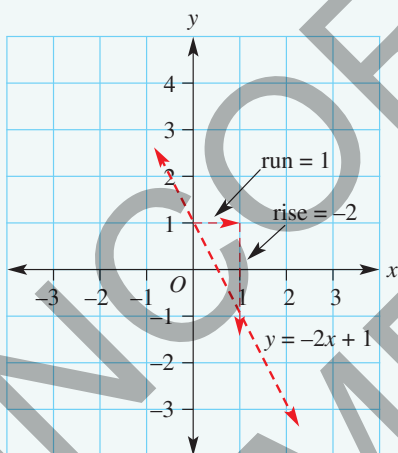
Solution

Method 1:

$$y = -2x + 1$$

Gradient is -2 .

y -intercept is 1 .



Method 2:

$$\text{Let } x = 0 \text{ in } y = -2x + 1$$

$$y = -2 \times 0 + 1$$

$$y = 1$$

$$y\text{-intercept} = (0, 1)$$

$$\text{Let } y = 0 \text{ in } y = -2x + 1$$

$$0 = -2x + 1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$x\text{-intercept} = \left(\frac{1}{2}, 0\right)$$

Explanation

If the equation is $y = mx + c$:

the gradient is m

the y -intercept is c .

Substitute $x = 0$.

Solve to find y .

This gives the y -intercept.

Substitute $y = 0$.

Solve to find x .

This gives the x -intercept.

Now you try

Draw the graph of $y = 2 - x$.

Exercise 5C**BUILDING****Example 6**

- 1** State the gradient and y-intercept for each of the following equations.

a $y = 4x + 2$

b $y = 3x - 7$

c $y = 5x + 0.4$

d $y = 1.5x - 2$

e $y = \frac{1}{2}x + 3$

f $y = 5 - 3x$

g $y = x$

h $y = 2 + 5x$

- 2** Write the equation of a line that has the following gradient and y-intercept.

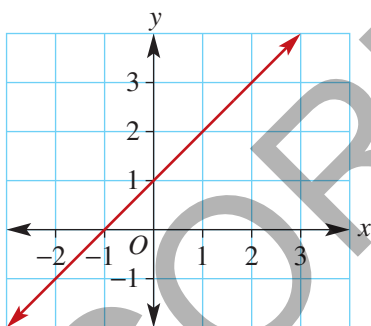
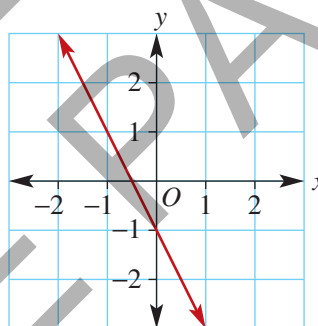
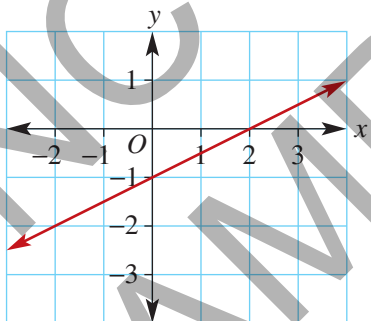
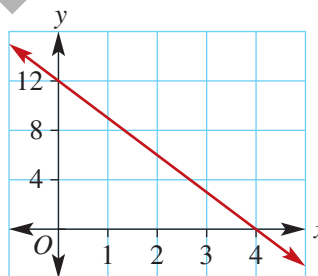
a Gradient = 3 and y-intercept = 2

b Gradient = -2 and y-intercept = 10

c Gradient = -4 and y-intercept = -1

d Gradient = 0.5 and y-intercept = 1

- 3** Find the equation of the following line graphs.

a**b****c****d****Example 7**

- 4** A straight line has the equation $y = 2x + 3$.

a What are the gradient and the y-intercept?

b Sketch the straight line on a number plane using the gradient and y-intercept.

DEVELOPING

- 5** When $x = 0$, $y = 0$. When $x = 4$, $y = 12$.

a Write a linear equation in the form $y = mx$ to describe this situation.

b Draw the graph of y against x .

- 6** Katrina gets paid by the hour. No work, no pay. For an 8-hour day she receives \$168.
- a** Write a linear equation in the form $p = mh$ to describe this situation.
- b** Draw the graph of p against h .



- 7** A bike is travelling at constant speed. It travels 350 km in 7 hours.
- a** Write a linear equation in the form $d = mt$ to describe this situation.
- b** Draw the graph of d against t .

Example 8

- 8** Sketch the graphs of the following equations on the same number plane.

a $y = 2x$

b $y = 2x + 1$

c $y = 2x + 2$

d $y = -x$

e $y = -x - 1$

f $y = -x - 2$

g What do you notice about these graphs?

- 9** Sketch the graphs of the following equations on the same number plane.

a $y = x + 1$

b $y = 3x + 1$

c $y = \frac{1}{2}x + 1$

d $y = -x - 2$

e $y = -2x - 2$

f $y = -3x - 2$

g What do you notice about these graphs?

CHALLENGING

- 10** Sketch the graphs of the following equations using the gradient–intercept formula.

a $y = \frac{2}{3}x + 2$

b $y = 0.25x - 3$

c $y = 2 - \frac{1}{3}x$

d $y = -0.5x - 3$

e $y + x = 5$

f $4x + y = 8$

g $2x + y + 6 = 0$

h $x + 4y = 0$

i $3x - y = -3$


Communicate your reasoning, understanding and fluency

- 11** What if you only know two points which are not on an axis? Can you determine the equation of the straight line?

Yes you can! Follow the steps and write down all the reasoning for the following question.

What is the equation of the straight line which passes through the points (2, 2) and (4, 6)?

Steps:

- 1** Plot the two points on the Cartesian plane.
- 2** Use rise and run to calculate the gradient (m).
- 3** In the equation $y = mx + c$,
 - substitute the value of m
 - substitute x and y from one of the given points
 - solve for c .
- 4** You now have m and c , so write the equation.
- 5** Check that both the points you were given satisfy the equation of the line.

5D Linear modelling

Learning intentions

- To use straight lines as models for real-life scenarios
- To solve problems involving real-life linear relationships

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12
- Questions about linear relationships regularly appear in HSC Examinations
- Formulas for the equation of a straight line are assumed knowledge and do not appear on the 2027 HSC Examination Reference Sheet

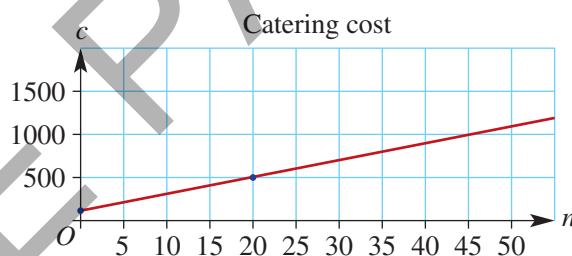
Linear modelling occurs when a practical situation is a linear relationship. For example, the gradient–intercept form of a straight-line graph can sometimes be used to model catering costs. A catering company charges a base amount of \$100 plus a rate of \$20 per guest. Using this information, we can write down a linear equation to model the cost of the event.

Let c be the cost of the event (\$) and n be the number of guests. We can then write $c = 20n + 100$.

Note: The number of guests (n) must be greater than zero and a whole number.

The graph of this linear model has been drawn. There are two important features of this linear model:

1. Gradient is the rate per guest (\$20).
2. The vertical-intercept is the base amount (\$100).



Some linear models have limitations and restrictions

In the catering cost scenario:

- the number of guests (n) must be a positive number, not negative
- the number of guests must be a whole number
- the venue for the event may have a limit of 100 people
- the catering company might only be able to cater for a maximum of 100 people.

If a water tank is being filled, there is a maximum amount which can fit in the tank, so the model only applies up to that moment. If a water tank is being emptied, the model stops when the tank is empty.

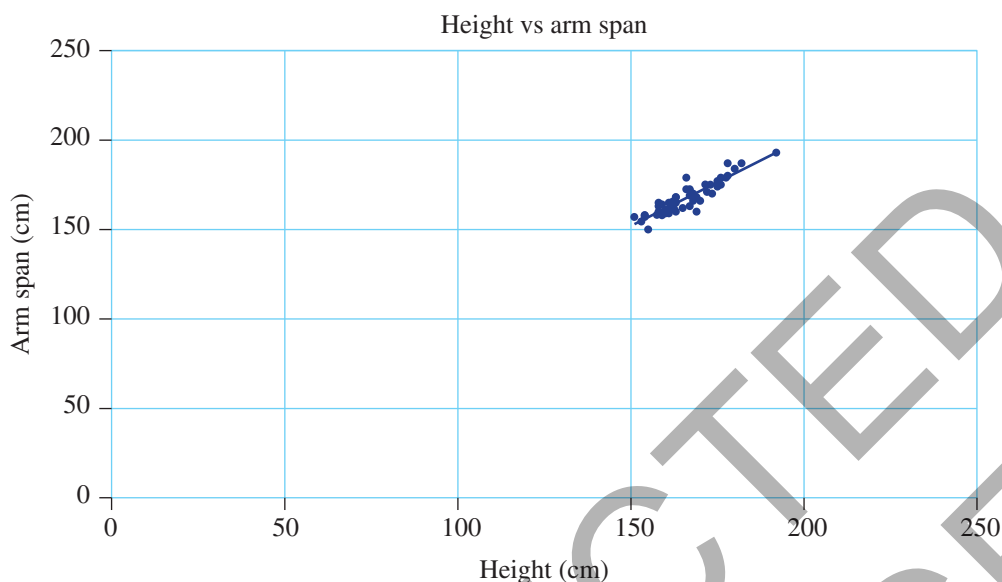
A teenager may be growing taller by 12 cm per year, but eventually they will stop growing taller, so the model only applies up to that time.

If you are spending the money you saved up for your holiday, the model stops when the money runs out.

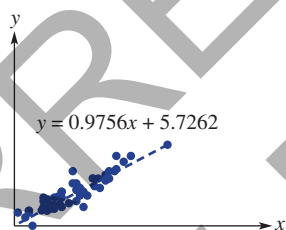
Interpolation and extrapolation

These concepts will be expanded upon in Year 12. This is a brief introduction.

Sometimes, the results of an experiment are points on the Cartesian plane which do not line up perfectly. For example, 50 people had their heights and arm spans measured, giving the 50 dots on this graph. One axis is height, the other is arm span.



Taking a closer look, we see a dotted line called a **line of best fit**.



The equation can be used to predict the height of a person using their arm span.

Using the equation to make a prediction within this data range is called **interpolation**.

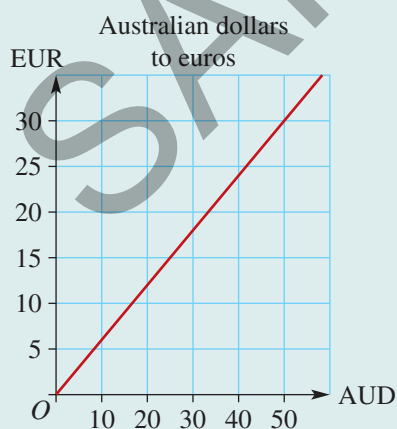
Using the equation for a much shorter or much taller person outside this data range is called **extrapolation**.

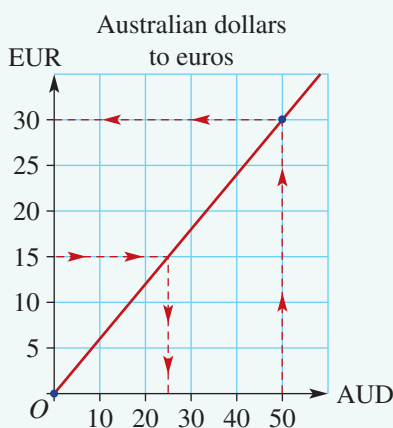


Example 9 Using graphs to make currency conversions

This graph is used to convert Australian dollars to euros. Use the graph to convert:

- a** 50 Australian dollars to euros
- b** 15 euros to Australian dollars.



Solution

- a** 30 euros (from graph)
b 25 Australian dollars (from graph)

Explanation

When $\text{AUD} = 50$, $\text{EUR} = 30$.

When $\text{EUR} = 15$, $\text{AUD} = 25$.

Now you try

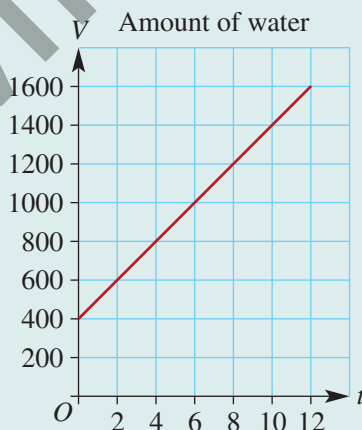
Using the graph in Example 9:

- a** convert 30 Australian dollars to euros
b convert 60 euros to Australian dollars.

Example 10 Interpreting linear models

Water is pumped into a partially full tank. The graph gives the amount of water V (in litres) after t minutes.

- a** How much water is in the tank at the start?
b How much water is in the tank after 10 minutes (i.e. $t = 10$)?
c The tank holds 1600 L. How long does it take to fill?
d Find the equation of the straight line.
e Use the equation to calculate the amount of water in the tank after 7 minutes.
f How many litres are pumped into the tank each minute?



continued ...

Solution**a** 400 L**b** 1400 L**c** 12 minutes

$$\begin{aligned} \mathbf{d} \quad m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{1600 - 400}{12 - 0} \end{aligned}$$

$$= 100$$

$$c = 400$$

$$y = mx + c$$

e $V = 100t + 400$

$$V = 100t + 400$$

$$= 100 \times 7 + 400$$

$$= 1100 \text{ L}$$

f 100 L/min**Explanation**

From graph

From graph

From graph

Choose two points on graph (0, 400) and (12, 1600).

Use rise and run to find gradient.

Substitute $t = 7$ or read from graph.

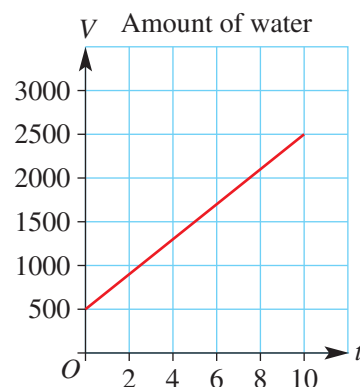
This is the gradient.

Now you try

A different tank started with 1600 L and was drained in 10 minutes. Find the equation of the straight line that represents this information.

**Exercise 5D****BUILDING****Example 10**

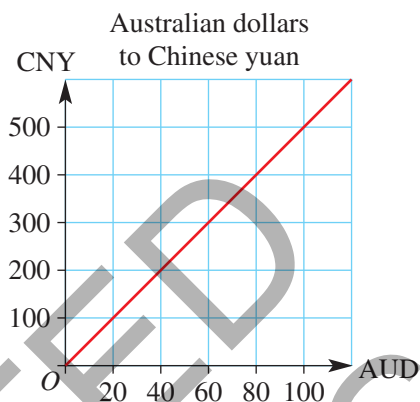
1 Water is pumped into a partially full tank. The graph gives the amount of water V (in litres) after t minutes.

a How much water is in the tank at the start?**b** How much water is in the tank after 5 minutes?**c** How much water is in the tank after 8 minutes?**d** The tank holds 2500 L. How long does it take to fill?**e** Use the graph to calculate the amount of water in the tank after 7 minutes.

Example 9

2 This conversion graph is used to convert Australian dollars to Chinese yuan. Use the graph to convert:

- a**
- i** 80 Australian dollars to yuan
 - ii** 50 Australian dollars to yuan
 - iii** 100 yuan to Australian dollars
 - iv** 350 yuan to Australian dollars
- b** What is the gradient of the conversion graph?



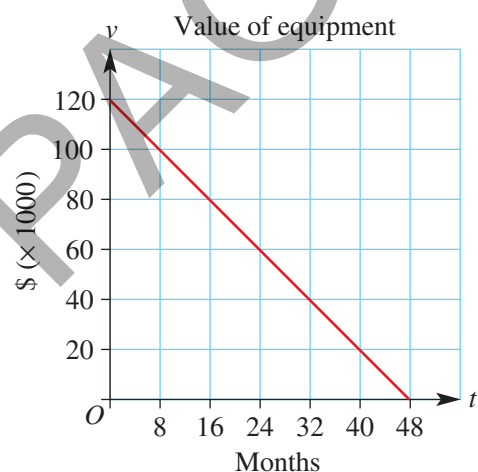
3 Sophie has \$4 in her money box. She decides to add \$3 every week.

- a** How much will she have at the end of 2 years (104 weeks)?
- b** How many weeks will it take her to reach \$1000?

DEVELOPING

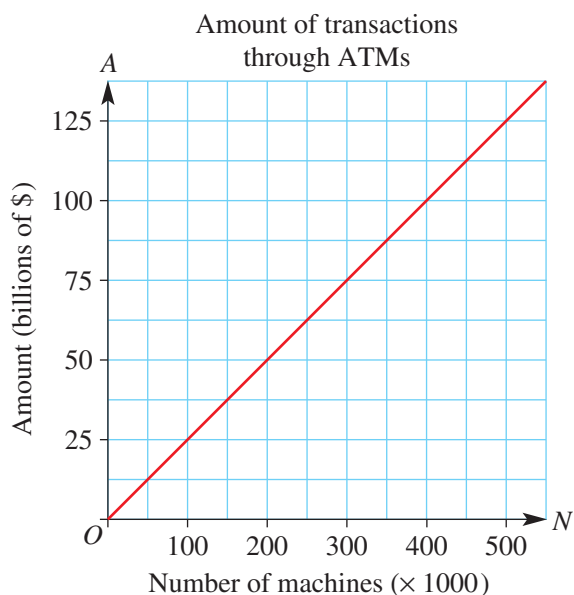
4 A new piece of equipment is purchased by a business for \$120 000. Its value declines or depreciates each month using the graph shown.

- a** What was the value of the equipment after 32 months?
- b** What was the value of the equipment after one year?
- c** When does the line predict the equipment will have no value?
- d** Find the equation of the straight line in terms of v and t .
- e** Use the equation to calculate the value of the equipment after 2 months.
- f** By how much does the equipment depreciate in value each month?



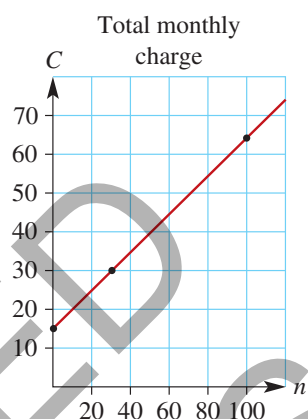
5 The amount of money transacted through ATMs has increased with the number of ATMs available. The graph shows this increase.

- a** What was the amount of money transacted through ATMs when there were 500 000 machines?
- b** How many ATM machines resulted in transactions of \$75 billion?
- c** Find the equation of the line in terms of amount of money transacted, A , and the number of ATMs, N .
- d** Use the equation to calculate the amount of money transacted when there were 350 000 machines.
- e** Use the equation to predict how much money will be transacted through ATM machines when there are 1 000 000 machines.



- 6** A phone company charges a monthly service fee plus the cost of calls. The graph gives the total monthly charge, C dollars, for making n calls. This includes the service fee.

- a** How much is the monthly service fee?
- b** How much does the company charge if you make 20 calls a month?
- c** How many calls are made if the total monthly charge is \$30?
- d** Find the equation of the line in terms of total monthly charge (C) and the number of calls (n).



CHALLENGING

- 7** The relationship for the age of a computer (t in years) to its current value (v in \$100) is $v = -5t + 30$.
- a** Construct a table of values for the age against current value ($t = 0, 1, 2, 3, 4$).
 - b** Draw the graph of the age (t) against current value (v).
 - c** What is the initial cost of the computer?
 - d** What will be the value of the computer after two years?
 - e** When will the computer be half its initial cost?
- 8** Tomas converted 100 Australian dollars to 40 British pounds.
- a** Draw a conversion graph with Australian dollars on the horizontal axis and British pounds on the vertical axis.
 - b** How many British pounds is 40 Australian dollars? Use the conversion graph.
 - c** How many Australian dollars is 10 British pounds? Use the conversion graph.
 - d** Find the gradient and vertical intercept for the conversion graph.
 - e** Write an equation that relates Australian dollars (AUD) to British pounds (GBP).
- 9** Emily was born on Jack's tenth birthday.
- a** Construct a table of values using 0, 1, 2, 3 and 4 as values for Emily's age (E) and calculating Jack's age (J).
 - b** Draw the graph of Emily's age (E) against Jack's age (J).



Communicate your reasoning, understanding and fluency

- 10** William had his height measured every second birthday. Here are some of the results:

William's age (years) (a)	6	8	10	12	14	16	18
William's height (cm) (h)	115	128	138	149	164	174	181

- a** Explain why this relationship is not linear.
- b** Plot the points on grid paper and draw a line which goes close to all the points.
- c** What is the equation of your line?
- d** Use the line to estimate William's height when he was 11 years old. This is called interpolation.
- e** Use the line to estimate William's height when he was 3 years old. This is called extrapolation.
- f** Explain why this is not a good model for estimating William's height for the years after he

5E Direct variation

Learning intentions

- To understand that direct variation can be a linear relationship in which the vertical intercept is zero
- To solve problems involving everyday scenarios which involve direct linear variation

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12
- Questions about direct variation regularly appear in HSC Examinations
- Formulas for the equation of a straight line are assumed knowledge and do not appear on the 2027 HSC Examination Reference Sheet

You may have noticed that some of the graphs in previous sections pass through the origin (0, 0). Their y-intercept is 0.

Here are some linear relationships which pass through the origin:

- Your bank account is empty, so you decide to save \$25 per week (0 weeks, 0 dollars).
- You are paid \$16 per hour for every hour you work (no work, no pay).
- Tickets for a show cost \$25 each (0 tickets, \$0).
- For every Australian dollar, you can buy 1.1 New Zealand dollars (\$0 AUD, \$0 NZD).
- Every kilometre you run takes 5 minutes (0 km, 0 minutes).

This is a special situation called **direct linear variation**, in which:

- the line passes through the origin, so the value of c (the y-intercept) is 0
- for the gradient, the letter k is often used as **the constant of variation**, so instead of $y = mx + c$, we use the equation $y = kx$.

There are two different phrases used to describe this relationship.

If $y = kx$, then:

- y varies directly with x , or
- y is directly proportional to x .

Solving a direction variation problem

If you are certain that the line passes through the origin, you only need one other point to find the gradient.

Substitute that point into $y = kx$.

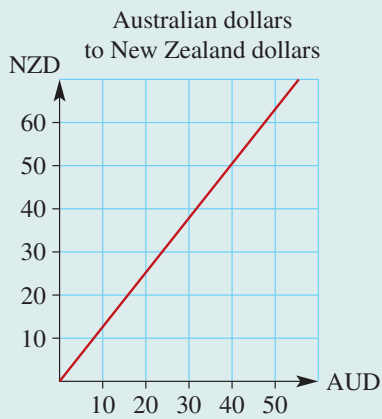
Use that equation to find the value for k .

That gives the equation of the straight line.

Example 11 Using direct variation for currency conversions

This graph is used to convert Australian dollars (AUD) to New Zealand dollars (NZD). Use the graph to convert:

- a 40 AUD to NZD
- b 25 NZD to AUD.



Solution

- a 50 NZD
- b 20 AUD

Explanation

From the graph
From the graph

Now you try

By extending the graph in Example 11, convert 120 AUD to NZD.

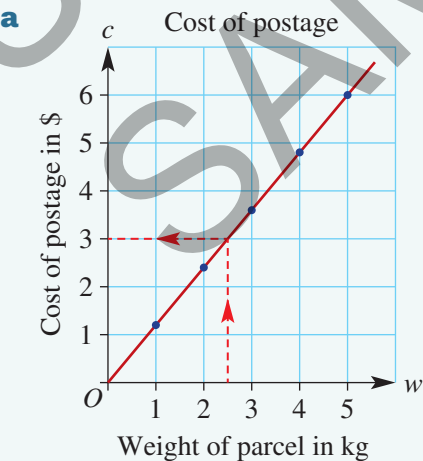
Example 12 Graphing direct variation from a table of values

The table shows the cost of postage (c) as a function of the weight of the parcel (w).

Weight (w) kg	1	2	3	4	5
Cost (c) \$	1.2	2.4	3.6	4.8	6.0

- a Draw a graph of cost (c) against the weight of the parcel (w).
- b Use the graph to determine the cost of a parcel if the weight is 2.5 kg

Solution



Explanation

Draw a Cartesian plane.
Plot the points.
Draw a line through all the points.

- b 2.5 kg would cost \$3

From the graph

Now you try

By extending the table or graph, find the cost of posting a 12 kg parcel.

**Example 13 Solving a direct variation problem**

The cost of printing booklets (C , in dollars) varies directly with the number of pages (p). A 50-page booklet costs \$40 to print.

- a** What is the constant of variation (k)?
- b** Write an equation connecting C and p .
- c** How much will a 172-page booklet cost to print?

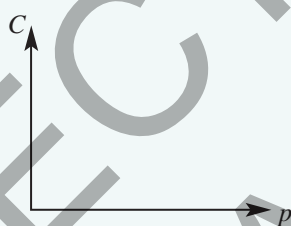
Solution

$$\begin{aligned}\mathbf{a} \quad C &= kp \\ 40 &= k \times 50 \\ k &= \frac{40}{50} \\ &= 0.8\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad C &= kp \\ C &= 0.8p\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad C &= 0.8 \times 172 \\ C &= 137.6\end{aligned}$$

It costs \$137.60 to print a 172-page booklet.

Explanation

$y = mx$ becomes $C = kp$
Substitute 40 for C and 50 for p .
Calculate k , the gradient.

Substitute $k = 0.8$

Substitute $p = 172$

Check:

p	50	172
C	40	137.6

Now you try

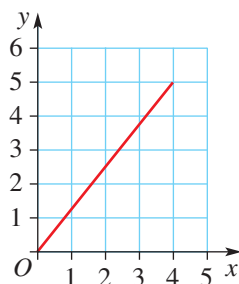
Using the equation in Example 13, how many booklets can be printed for \$600?

Exercise 5E

BUILDING

Example 11

- 1 a What is the equation of the line in this graph?



- b When $x = 2$, what is y ?
 c When $y = 2$, what is x ?

Example 12

- 2 One Australian dollar (AUD) was converted for 1.20 New Zealand dollars (NZD).

- a Construct a table of values using of 0, 10, 20, 30 and 40 as values for AUD and calculate the NZD using the above conversion.
 b Draw the graph of the AUD against NZD.

Example 13

- 3 y varies directly with x and $y = 24$ when $x = 8$.

- a Write an equation connecting y and x , using k as the constant of variation.
 b Calculate the constant of variation.
 c What is y when x is 4?
 d What is x when y is 15?

DEVELOPING

- 4 y varies directly with x and $y = 20$ when $x = 10$.

- a Write an equation connecting y and x , using k as the constant of variation.
 b Calculate the constant of variation.
 c What is y when x is 8?
 d What is x when y is 12?

- 5 It is known that y varies directly with x . When $x = 12$ then $y = 3$.

- a Write an equation connecting y and x , using k as the constant of variation.
 b Calculate the constant of variation.
 c What is y when x is 24?
 d What is x when y is 4?

- 6 It is known that a varies directly with b . If $a = 84$ then $b = 56$.

- a Write a linear equation connecting a and b , using k as the constant of variation.
 b Calculate the constant of variation.
 c Find the value of a when the value of b is 22.
 d Find the value of b when the value of a is 36.

CHALLENGING

- 7** Oscar's pay (p) is directly proportional to the number of hours (h) he works. For a 9-hour day he receives \$193.50.
- Write a linear equation to describe this situation.
 - Calculate the constant of variation.
 - What is Oscar's pay if he works for 11 hours?
 - What is Oscar's pay if he works for 6.5 hours?
 - How many hours does Oscar have to work to earn \$365.50?
 - How many hours does Oscar have to work to earn \$752.50?



- 8** The cost (c) of apples is \$2.50 per kilogram and is determined by the formula $c = 2.5w$ where (w) is the weight of the apples.
- Construct a table of values for the weight against cost. Use 0, 1, 2, 3 and 4 for w .
 - Draw the graph of the cost (c) against the weight (w).
 - How many kilograms of apples can be purchased for \$15?

Communicate your reasoning, understanding and fluency

- 9** Is it possible for the gradient to be negative in a direct variation scenario? Sort of, but it is rare. The temperature in the freezer is zero degrees Celsius. When the door is kept closed, the temperature drops by 3 degrees every hour.
- Draw a graph of the temperature for the first 5 hours.
 - What is the gradient of the line?
 - What is the equation of the line?

Key ideas and chapter summary

**Graphing straight lines**

- 1** Construct a table of values with the independent variable as the top row and the dependent variable as the bottom row.
- 2** Draw a number plane with the independent variable on the horizontal axis and the dependent variable on the vertical axis.
- 3** Plot the points.
- 4** Draw a line through the points.

Gradient and y-intercept

Gradient (m) of a line is the slope of the line.

$$m = \frac{\text{rise}}{\text{run}}$$

The y-intercept of a line is where the line cuts the vertical axis.

Gradient–intercept formula

Linear equation in the form $y = mx + c$, where:

m = slope or gradient of the line

c = y-intercept.

Sketching a straight-line graph requires at least two points. When an equation is written in gradient–intercept form, one point on the graph is immediately available: the y-intercept. A second point can be calculated using the gradient or by substitution.

Linear models

Linear modelling occurs when a practical situation is described mathematically using a linear function.

Direct variation

- 1** Write an equation relating the two variables, using k is the constant of variation.
When y is directly proportional to x the equation is $y = kx$.
- 2** Solve the equation for k by substituting values for x and y .
- 3** Write the equation with the solution for k (step 2) and solve the problem by substituting a value for either x or y .

Chapter checklist with success criteria



A printable version of this checklist is available in the Interactive Textbook.



5A

- 1** I can use an equation and a table of values to graph a straight line on the Cartesian plane. ☐

e.g. Draw these straight-line graphs.

a $y = 2x$

b $y = 2x - 1$

c $y = 4 - 2x$

5B

- 2** I can use any two points plotted on the Cartesian plane to calculate the gradient (m) of the line which passes through those points. ☐

e.g. Find the gradient (m) of the line which passes through:

a (1, 1) and (4, 3)

b (0, 5) and (6, 0)

c (0, 0) and (2, -1)

5B

- 3** I can find the value of the y-intercept (c), from the graph of a straight line or a table of values. ☐

e.g. What is the y-intercept (c) of the line which passes through the points (-1, 2) and (1, 0)?

5C

- 4** I can identify the gradient (m) and y-intercept (c) from the equation of a straight line. ☐

e.g. What is the gradient (m) and y-intercept (c) of these lines?

a $y = 2x - 5$

b $y = 5 - 2x$

c $2x + 3y = 6$

5C

- 5** I can use an equation of the form $y = mx + c$ to draw straight lines by using the gradient (m) and y-intercept (c), rather than using a table of values. ☐

e.g. Draw these straight lines using the gradient and y-intercept.

a $y = 2x$

b $y = 2x - 1$

c $y = 4 - 2x$

5D

6 I can use linear models to represent real-life scenarios.

e.g. Mia has \$1000 in her bank account. She withdraws \$45 per week.

a Complete this table of values.

Weeks (<i>W</i>)	0	1	2	3	4	5
Balance (<i>B</i>)						

- b Draw a graph which shows the first 10 weeks.
- c What is the equation of the graph?
- d For how many weeks can Mia continue to do this without going into debt?

5E

7 I can solve problems involving direct variation.

e.g. Brad can make 5 surfboards every 4 weeks.

- a Draw a graph with weeks on the horizontal axis and number of surfboards on the vertical axis.
- b What is the gradient of that straight line?
- c What is the equation of that straight line?
- d How many surfboards can he make in 120 weeks?
- e How many weeks will it take him to make 120 surfboards?

Multiple-choice questions

- 5A 1** Which equation does not pass through the point (1, 2)?

A $y = 2x$

B $y = x + 1$

C $x + y = 3$

D $y = x - 1$

- 5B 2** What is the gradient of the line shown?

A $\frac{2}{3}$

B $\frac{3}{4}$

C $\frac{4}{3}$

D $\frac{8}{5}$

- 5B 3** What is the y-intercept of the line shown?

A -2

B -1

C 1

D 2

- 5C 4** A straight line has the equation of $y = -3x + 1$. What is the y-intercept?

A -3

B -1

C +1

D 3

- 5D 5** The relationship of the age of a fax machine (t in years) to its current value (v) is $v = -40t + 150$. What is the value of the fax machine after two years?

A 70

B 108

C 150

D 230

- 5D 6** The cost (C) of a mobile phone plan for a certain number of months (t) is given by $C = 50t + 10$. What is the cost of the phone plan for 10 months?

A \$50

B \$60

C \$510

D \$600

- 5E 7** What is the constant of variation if a varies directly with b and when $a = 32$ then $b = 16$?

A -2

B -0.5

C 0.5

D 2

- 5E 8** What is the equation of the line shown in the graph?

A $c = n$

B $c = n + 30$

C $c = 30n$

D $c = 8n + 240$

- 5E 9** Using the graph from Question 8, what is the charge for 12 months?

A 24

B 36

C 240

D 360

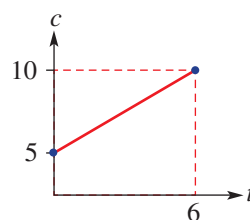
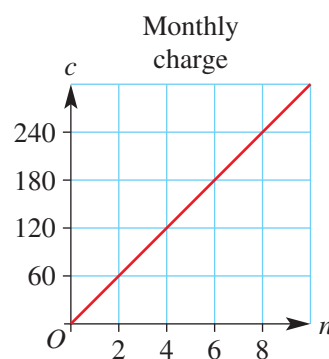
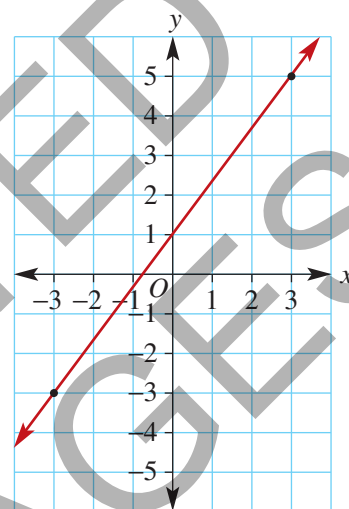
- 5E 10** Which is not the equation of this line?

A $y = \frac{6}{5}x + 5$

B $y = \frac{5}{6}x + 5$

C $6y = 5x + 30$

D $5x - 6y + 30 = 0$



Short-answer questions

- 5A 1** Complete the following table of values for each linear function.

a $y = x + 3$

x	-2	-1	0	1	2
y					

b $y = -2x - 1$

x	-2	-1	0	1	2
y					

- 5A 2** Draw the graphs for the two linear functions in Question 1.

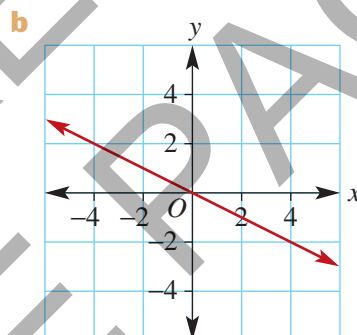
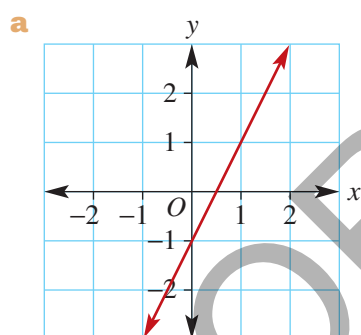
- 5B 3** What is the gradient of the line that joins these points?

a (1, 5) and (3, 7)

b (-2, 1) and (0, 4)

c (-3, -1) and (2, -11)

- 5C 4** Find the equation of the following line graphs.



- 5C 5** Draw a graph of these linear functions and find the gradient and y-intercept.

a $y = x + 1$

b $y = -2x + 5$

c $y = 3x - 2$

- 5D 6** The circumference of a circle (C) varies directly the diameter of the circle (d) using the formula $C = \pi d$.

a What is the constant of variation?

b What is the diameter (to nearest centimetre) if the circumference of a circle is 6 cm?

- 5D 7** An internet access plan charges an excess fee of \$8 per GB.

Data (d) GB	1	2	3	4	5	6
Cost (c)\$	8	16	24	32	40	48

a Which is the dependent variable?

b Which is the independent variable?

c Draw a graph of data against cost.

d Use the graph to find d if c is 20.

- 5E 8** One Australian dollar is 92.29 Japanese Yen.

a Convert 100 Australian dollars to Japanese Yen.

b Convert 100 Japanese Yen to Australian dollars.

Extended-response questions

For Questions 1 and 2, use 1 cm grid paper for drawing graphs.

- 5E 1** Town A has a population of 1000 and is shrinking by 45 people per year.
- a** Write the equation for the population of Town A, using P for population and t for years.
 - b** Complete a table of values from $t = 0$ to $t = 10$.
 - c** Draw, neatly and accurately, the population graph for Town A for these 10 years.
- 5E 2** Town B has a population of 500 and is growing by 25 people per year.
- a** Write the equation for the population of Town B, using P for population and t for years.
 - b** Complete a table of values from $t = 0$ to $t = 10$.
 - c** Using the same grid paper as Question 9, draw, neatly and accurately, the population graph for Town B for 10 years.
 - d** Using your tables of values and your graph, estimate, to one decimal place, the value of t for which the two towns have the same population.

Data analysis 2

Some interesting Australian statistics

More than 85 per cent of Australians live within 50 kilometres of the coast.

If you were to visit a new beach in Australia every day, it would take you over 27 years.

Tasmania has the cleanest air in the world.

In 1984, the average age of Australians was 33.05 years old.

By 2024, 40 years later, it had increased to 39.48.

By 2064, in another 40 years, it is predicted to be 43.33.

The world's oldest fossil, which is about 3.4 billion years old, was found in Australia.

Australia has over 750 different reptile species. This is more than any other country in the world.

Australia has three times more sheep than people.

Australia was the second country in the world to give women the right to vote in 1902.

The termite mounds that can be found in Australia are the tallest animal-made structures on Earth.



Chapter contents

- 6A** Measures of centre: mean, median and mode
- 6B** Measures of spread: range, interquartile range and standard deviation
- 6C** Outliers
- 6D** Describing the shape of a dataset
- 6E** Comparing datasets
- 6F** Parallel box plots

NSW Syllabus

In this chapter, a student:

- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- Displays and analyses datasets using summary statistics and graphical representations (MST-11-08)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

6A Measures of centre: mean, median and mode

Learning intentions

- To understand that the mean, median and mode may be used as measures of central tendency under certain circumstances
- To be able to read, interpret and display data in box plots
- To calculate the mean and locate median and mode

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated in Year 12
- Students will study bivariate data analysis in Year 12
- Questions about measures of centre regularly appear in HSC Examinations
- The formula for the mean of a dataset is assumed knowledge and does not appear on the 2027 HSC Examination Reference Sheet

Mean

Mean = $\frac{\text{Sum of scores}}{\text{Number of scores}}$

Mean

The mean is the measure of the spread of a dataset. The mean is commonly referred to as the average. It is calculated by adding all the scores and dividing by the number of scores.

For example, consider the scores 1, 6, 3, and 2. The mean is $\frac{1 + 6 + 3 + 2}{4} = 3$. The symbol $\bar{x}$ is used for the mean.



Example 1 Calculating the mean

The table shows the number of rainy days for the first six months.

J	F	M	A	M	J
12	15	13	8	8	10

Find the mean.

Solution

$$\bar{x} = \frac{\text{sum of scores}}{\text{number of scores } (n)}$$
$$\bar{x} = \frac{12 + 15 + 13 + 8 + 8 + 10}{6}$$
$$\bar{x} = 11$$

Explanation

Write the formula.

Sum the scores and divide by the number of scores.

Now you try

Gary scored 61, 48, 82 and 77 in four tests at school last term. Calculate his mean test score.

Median

The median is the middle score or value. To find the median, list all the scores in increasing order and select the middle one. For example, the median of 1, 4, 5, 7, 8 is 5. When there is an even number of scores, the median is the average of the two middle scores. For example, the median of 1, 1, 4, 5, 7, 8 is found by sorting the six scores and finding the average of 4 and 5 or 4.5.

Median

- 1 Arrange all the scores in increasing order.
- 2 Count the total number of scores. This is represented by the letter n .
- 3 Odd number of scores then the median is $\frac{n}{2} + 1$ score; that is, the middle score.
- 4 Even number of scores then the median is the average of the $\frac{n}{2}$ and the $\frac{n}{2} + 1$ score; that is, the average of the two middle scores.

Mode

The mode is the score that occurs the most. It is the score with the highest frequency. The mode is useful for categorical data that do not allow numerical calculations, such as when the data collected are colours. Modes may occur at the beginning or end of a range of values; therefore, conclusions based only on the mode may be inaccurate. It is common for data to have several modes. For example, if there are two modes the data is referred to as bimodal. When data is grouped into classes, the class that occurs the most is called the modal class.

Mode

- 1 Determine the number of times each score occurs.
- 2 Mode is the score that occurs the most number of times. If two or more scores occur the same number of times, they are both regarded as the mode.



Example 2 Calculating the median and mode

The dataset shows the change in temperature of a sick child.

37 38 39 38 40 40 40

- a Sort the dataset in ascending order.
- b Calculate the median.
- c Determine the mode.



Solution

- a 37, 38, 38, 39, 40, 40, 40
- b Median = $\frac{38 + 39}{2}$
= 38.5
- c Mode is 38 and 40

Explanation

Write the scores in ascending order.

6 is even, so the median is the average of middle two scores.

Both 38 and 40 occur the most (twice).

Now you try

Find the median and mode for the following datasets.

- a 28 28 28 30 30 30 41 41

- b 12 18 18 22 24

To calculate the mean from a grouped frequency table, the ‘class centre’ is used instead of the score (x). The ‘frequency $\times$ score’ (fx) column becomes the ‘frequency $\times$ class centre’ column. The same formula for the mean is used. The mean calculated from grouped data is an approximation.

Example 3 Finding the mean from a grouped frequency table

The points scored by a rugby team are shown in the grouped frequency table. The dataset represent all the team’s matches for the past two seasons.

What is the mean number of points scored over the two seasons?

Class	Class centre (x)	Frequency (f)	$f \times x$
0–5	3	2	
6–10	8	9	
11–15	13	11	
16–20	18	13	
21–25	23	10	
26–30	28	5	

Solution

x	f	fx
3	2	6
8	9	72
13	11	143
18	13	234
23	10	230
28	5	140
Total	50	825

$$\bar{x} = \frac{825}{50}$$
$$= 16.5$$

Mean number of points is 16.5.

Explanation

Complete the $f \times x$ column by multiplying the frequency by the class centre.

Mean is the total of fx column divided the total of the f column.

Now you try

Class	Class centre (x)	Frequency (f)	$f \times x$
0–5	3	5	
6–10	8	4	
11–15	13	6	
16–20	18	2	
21–25	23	5	

Use the grouped frequency table to calculate the mean for this dataset.

Exercise 6A

BUILDING

Example 1

1 Find the mean of each dataset.

a 6, 9, 9, 15, 9, 3, 7, 9, 13, 10

c 5, 5, 8, 12, 13, 3, 7, 9, 10

e 6, 5, 6, 17, 13, 6, 0, 9, 1

g 3, 5, 24, 19, 13, 13, 13, 9, 3, 8, 11

b 22, 28, 22, 31, 43, 22

d 7, 5, 11, 3, 0, 1, 2, 9, 7

f 39, 35, 39, 41, 47, 49, 44

h 55, 24, 14, 18, 13, 3, 2, 2, 2, 7, 14

2 Twenty people measured their heart rate using a heart-rate monitor.

The results were 64, 68, 64, 72, 75, 67, 91, 80, 77, 73, 68, 81, 73, 72, 60, 62, 74, 68, 55 and 62.

a What is the sum of these heart rates?

b Find the mean heart rate. Answer correct to two decimal places.

c Another person with a heart rate of 63 is included in this dataset. Find the mean of this dataset. Answer correct to the nearest whole number.



Example 2

3 There is an odd number of scores. Find the median of these numbers.

a 3, 9, 10

c 23, 28, 29, 30, 34, 45, 46, 49, 50

e 2, 2, 2, 2, 2, 2, 2, 9, 9

g 0, 0, 0, 1, 1, 1, 7, 8, 8, 8, 8

b 3, 4, 6, 7, 9, 10, 15

d 1002, 1010, 1100, 1120, 1160

f 14, 15, 100, 101, 102

h 2, 7, 9, 10, 10, 14, 18

Example 2

4 There is an even number of scores. Find the median of these numbers.

a 2, 5, 6, 8, 8, 9

c 20, 20, 20, 21, 22, 24

e 100, 110, 130, 140

g 10, 20, 22, 40, 60, 61, 70, 80

b 12, 14, 18, 22

d 3, 4, 5, 9, 10, 14, 16, 18, 18, 18

f 1, 1, 1, 1, 3, 3, 3, 5, 5, 5, 6, 6

h 3, 5, 8, 10, 11, 14, 18, 19

Example 2

5 Find the mode of each dataset.

a 13, 9, 3, 24, 19, 3, 5, 13, 8, 11, 13

c 15, 7, 9, 13, 6, 9, 9, 3, 9, 10

b 5, 8, 12, 13, 3, 7, 9, 5, 10

d 47, 49, 39, 41, 39, 35, 44

Example 3

6 The grouped frequency table shows the number of people by age at a Gold Coast resort.

Class	Class centre (x)	Frequency (f)	$f \times x$
25–29		25	
30–34		21	
35–39		35	
40–44		21	
45–49		26	
50–54		23	
55–59		32	



a Copy and complete the table by finding the class centre and fx column.

b How many people stayed at the Gold Coast resort?

c Find the mean of this data. Answer correct to the nearest whole number.

- 7 The grouped frequency table shows the weights of randomly selected motor vehicles. All motor vehicle weights are in kilograms.

Class	Class centre (x)	Frequency (f)	$f \times x$
2000–2004		9	
2005–2009		15	
2010–2014		18	
2015–2019		10	
2020–2024		8	
2025–2029		6	
2030–2034		3	
2035–2039		1	

- a Copy and complete the table by finding the class centre and the fx column.
b How many motor vehicles were weighed?
c Find the mean of this dataset.

DEVELOPING

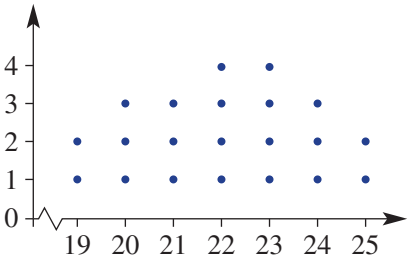
- 8 Use the stem-and-leaf plot to answer these questions.

- a How many scores are there?
b What is the lowest score?
c What is the median?
d Remove 42 from the dataset. What is the new median?

Stem	Leaf
0	7 8
1	2 3 5 9
2	1 2 6 7 9 9
3	0 1 7
4	2 7

- 9 Use the dot plot to answer these questions.

- a What is the highest score?
b What is the lowest score?
c Calculate the median.
d How many scores would need to be added to make the median 23?
e What is the median if a single score of 26 is included in the dataset?



- 10 Ten adults were surveyed on the number of hours they slept last night. Their answers were 8, 10, 9, 5, 5, 6, 9, 8, 4 and 12.
- a Find the mean number of hours slept.
b Another adult was then surveyed and the mean changed to 8. How many hours did this adult sleep?

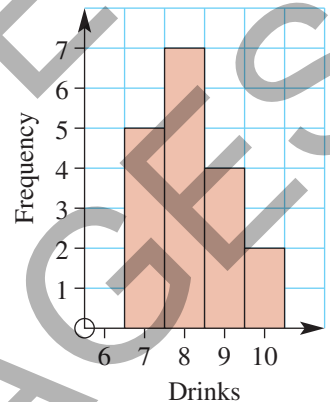


- 11** The table shows the ages of students at an acting academy.
- What is the median age?
 - What is the mean age? Answer correct to one decimal place.
 - Complete the cumulative frequency column.
 - The two oldest students leave the academy. What is the new median?

Age	Frequency	Cumulative frequency
18	2	
19	5	
20	6	
21	2	

- 12** The frequency histogram shows the number of drinks sold each day.

- What is the maximum number?
- What is the minimum number?
- Construct a frequency table.
- Calculate the median.
- The next day there were 14 drinks sold. Does the median change?

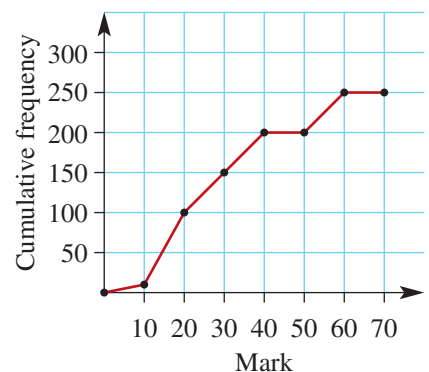


CHALLENGING

- 13** The mean height of five basketball players at the start of the game is 1.92 m. During the game a player who is 1.80 m tall is injured and replaced by a player who is 1.98 m tall. What is the mean height of the five players now? Answer correct to two decimal places.

- 14** The marks for a Mathematics exam are shown in the cumulative frequency polygon.

- What is the frequency of 70?
- What is the frequency of 30?
- How many students completed the exam?
- Estimate the median using the graph.



- 15** The number of mobile calls made by a phone company is shown.

80	84	87	70	68	83	85	86	70	90
68	90	78	83	75	69	82	89	89	82
80	85	90	74	74	70	90	80	75	75
70	90	83	68	79	79	82	79	91	80

- Decide on appropriate classes for a grouped frequency table.
- Calculate the class centres for these classes.
- Construct a grouped frequency table using these class intervals.
- How many pieces of data have been collected?



- e Using the class centres, find the mean of this dataset. Answer correct to one decimal place.
- f One of the scores entered was found to be incorrect. A score of 86 should have been 68. What is the new mean of this dataset? Answer correct to one decimal place.

Communicate your reasoning, understanding and fluency

16 A score was added to the set of scores: 15, 18, 20, 22, 24 and 26. The new mean is equal to 20. What score was added?

17 The number of plants sold in a nursery each day is shown.

21	20	20	24	21	23	21	21	23	22	22	22	20	20	26
22	23	20	22	26	21	24	24	25	25	22	21	20	20	23

- a Construct a frequency table with a cumulative frequency column.
- b Construct a cumulative frequency histogram and polygon for this dataset.
- c Use the cumulative frequency graphs to estimate the median.



6B Measures of spread: range, interquartile range and standard deviation

Learning intentions

- To understand that range, interquartile range and standard deviation may be used as measures of spread under certain circumstances
- To calculate range, interquartile range and standard deviation

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated in Year 12
- Students will study bivariate data analysis in Year 12
- Questions about measures of spread regularly appear in HSC Examinations

A measure of the spread is calculated to determine whether most of the values are clustered together or stretched out. The range, interquartile range and standard deviation are all measures of spread.

Range

The range is the difference between the highest and lowest scores. It is a simple way of measuring the spread of the data.

Range

Range = Highest score – Lowest score



Example 4 Calculating the range

The assessment results for two different tasks are shown in the table. Find the range for Task A and Task B.

Task A	3	5	10	13	14	14	17	22	24	24	27	27	28	33	38
	40	40	41	43	44	45	45	46	50	52	52	55	55	58	95
Task B	10	15	19	20	24	27	31	31	35	38	40	49	51	51	54
	55	58	62	62	68	68	71	72	76	78	79	79	86	88	90

Solution

Task A

Range = Highest – Lowest

$$= 95 - 3$$

$$= 92$$

Task B

Range = Highest – Lowest

$$= 90 - 10$$

$$= 80$$

Explanation

For both Task A and Task B

Write down the formula for range.

Subtract lowest score from highest score.

Note: The range for Task A is affected by the outlier (95) so is not a good indicator of the spread.

Now you try

Horrice has scores of 68, 61, 48, 82, 91 and 77 in six tests at school last term. Calculate the range of his test scores.

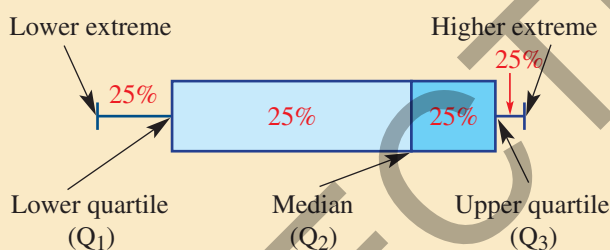
Five-number summary

A five-number summary of a dataset consists of lowest extreme, lower quartile (Q_1), median (Q_2), upper quartile (Q_3) and the highest extreme.

Box plot

A box plot is a graph that uses the data from a five-number summary. 25% of the dataset lies between each quartile as shown.

Box plot



Interquartile range

The interquartile range does not rely on the extreme values like the range. The dataset is arranged in increasing order and divided into 4 equal parts or quartiles. The interquartile range (IQR) is the difference between the first quartile and the third quartile. The first quartile cuts off the lowest 25% ($\frac{1}{4}$) of scores and the third quartile cuts off the lowest 75% ($\frac{3}{4}$) of scores.

Interquartile range

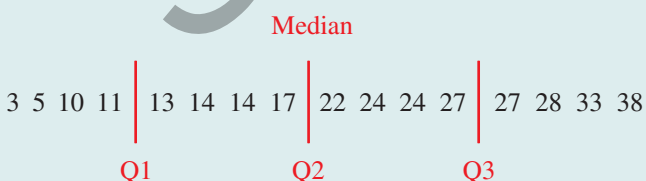
$$\begin{aligned} \text{IQR} &= \text{Third quartile } (Q_3) - \text{First quartile } (Q_1) \\ &= Q_3 - Q_1 \end{aligned}$$

- 1 Arrange the data in increasing order.
- 2 Divide the dataset into two equal-sized groups. If n is odd, omit the median.
- 3 Find Q_1 , the median of the first group.
- 4 Find Q_3 , the median of the second group.
- 5 Calculate the interquartile range (IQR) by subtracting Q_1 from Q_3 .



Example 5 Drawing a box plot from a five-number summary

Create a five-number summary and draw a box plot from the following dataset.



Solution

16 numbers in dataset

$$\text{Median} = \frac{\text{8th score} + \text{9th score}}{2} = \frac{17 + 22}{2} = 19.5$$

$$\begin{aligned}\text{Lower quartile (Q1)} &= \frac{\text{4th score} + \text{5th score}}{2} \\ &= \frac{11 + 13}{2} = 12\end{aligned}$$

$$\begin{aligned}\text{Upper quartile (Q3)} &= \frac{\text{12th score} + \text{13th score}}{2} \\ &= \frac{27 + 27}{2} = 27\end{aligned}$$

Five-number summary

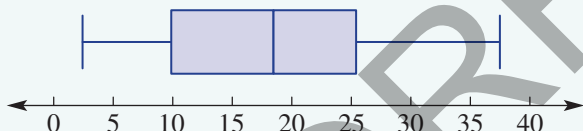
Lowest number = 3

Lower quartile (Q1) = 12

Median (Q2) = 19.5

Upper quartile (Q3) = 27

Highest number = 38

**Explanation**

Find the middle score of the dataset. Because there is an even number of scores in the dataset (16), the median is the average of the 8th and 9th score.

Find the median (middle) of the lower half.

Find the median (middle) of the upper half.

Create the box plot by using the values from the five-number summary as shown.

Now you try

Create a five-number summary and draw a box blot from the following dataset.

5 5 10 11 13 14 14 18 22 24 24 27 29 29 33 40

**Example 6** Calculating the interquartile range

Find the interquartile range (IQR) of the dataset from Example 5.

3 5 10 11 13 14 14 17 22 24 24 27 27 28 33 38

Solution**Five-number summary (from Example 5)**

Lowest number = 3

Lower quartile (Q1) = 12

Median (Q2) = 19.5

Upper quartile (Q3) = 27

Highest number = 38

$$\text{IQR} = \text{Q3} - \text{Q1}$$

$$= 27 - 12$$

$$= 15$$

Explanation

Using the information from the five-number summary, substitute Q1 and Q3 into

$$\text{IQR} = \text{Q3} - \text{Q1}$$

Now you try

Find the interquartile range (IQR) for the following dataset.

(You can use the five-number summary from the Now you try question after Example 5.)

5 5 10 11 13 14 14 18 22 24 24 27 29 29 33 40

Standard deviation

The standard deviation is a measure of the spread of data about the mean. It is worked out in STATISTICS mode on a calculator. The higher the standard deviation, the further a score is away from the mean.

Standard deviation

Standard deviation measures the spread of data about the mean.

σ – Standard deviation



Example 7 Calculating the standard deviation

What is the standard deviation of this dataset?

9, 11, 12, 12, 13, 13, 14, 15, 15, 16

Solution

$\sigma = 2$

Explanation

Place your calculator into STATISTICS mode.

Enter the dataset.

Find the standard deviation.

(Detailed instructions for commonly used calculators can be found in the online version.)

Now you try

What is the standard deviation of this dataset? Answer correct to one decimal place.

1, 3, 5, 5, 7, 7, 7, 8, 9, 10

Example 8 Calculating the standard deviation from a frequency table

The frequency table shows the number of customers at the local vegetable market for the past 22 days.

Score (x)	Frequency (f)
15	3
16	4
17	6
18	4
19	5



Find the standard deviation for this dataset.

Answer correct to two decimal places.

Solution

$$\sigma = 1.34$$

Explanation

Place your calculator into STATISTICS mode.

Turn on the frequency column.

Enter the dataset.

Enter the frequency column.

Find the standard deviation.

(Detailed instructions for commonly used calculators can be found in the online version.)

Now you try

What is the standard deviation of this dataset? Answer correct to two decimal places.

Score (x)	Frequency
11	5
12	6
13	7
14	9
15	2

Exercise 6B

BUILDING

Example 4

- 1 Find the range of each dataset.
- a** 4, 23, 6, 6, 9, 14, 32, 4, 4

b 14, 13, 13, 12, 23, 18, 13, 13, 15

c 12, 5, 0, 8, 5, 6, 7, 16, 0

d 12, 21, 18, 12, 33, 12

e 6, 33, 6, 1, 3, 6, 31, 11, 3, 7

f 5, 5, 5, 10, 7, 10, 15

Example 5

- 2 Aki scored the following marks in a spelling test: 4, 5, 5, 6, 7, 7, 8, 8, 9 and 9.
- a** Create a five-number summary.

b Draw a box plot.

- 3 The number of calls for help each hour after a storm are.

8	12	5	11	10	12
7	9	11	10	9	9
7	2	5	9	6	

- a** What is the range?

b What is the first quartile?

c What is the third quartile?

d What is the interquartile range?



Example 6

- 4 Find the five-number summary and interquartile range for each dataset.
- a** 13, 19, 8, 28, 16, 27, 11, 17

b 62, 51, 53, 64, 65, 53, 56, 61

c 29, 24, 30, 22, 32, 29, 24, 26, 25, 28

d 58, 59, 58, 34, 29, 29, 37, 57, 31, 39

e 31, 34, 38, 39, 43, 57, 57, 61, 41

f 59, 44, 40, 41, 46, 50, 46, 53, 46

- 5 The systolic blood pressure for a sample of 20 people is listed.

203	124	180	210	105	148	161	131	192	125
159	106	170	138	100	120	109	144	190	193

- a** What is the interquartile range?

b Draw a box plot.

Example 7

- 6 Find the standard deviation of each dataset. Answer correct to two decimal places.
- a** 5.3, 6.6, 4.4, 8.8, 3.3, 10.1

b 98, 112, 7, 4, 0, 100, 1

c 7, 7, 7, 7, 7, 7, 7, 7

d 130, 145, 145, 132, 124, 135, 156
- 7 The number of kilometres Stephanie walked each day while on her trek are 22, 26, 34, 28, 36, 30, 29, 31 and 28.
- a** What is the standard deviation, correct to two decimal places?

b The longest and the shortest distances were not included in this data. The longest distance was 44 km and the shortest distance was 17 km. What is the new standard deviation? Answer correct to two decimal places.



Example 8

- 8** Find the standard deviation for each of the following tables. Answer correct to two decimal places.

Score	24	25	26	27	28	29	30	31
Frequency	1	3	5	7	7	5	3	1

Score	55	56	57	58	59	60	61	62
Frequency	5	6	9	10	11	8	7	4

DEVELOPING

- 9** A grouped frequency table is shown.

Class	Class centre (x)	Frequency (f)
10–29		5
30–49		10
50–69		11
70–89		4

Copy and complete the class centre column in the table and find the standard deviation. Answer correct to one decimal place.

- 10** A railway attendant recorded the delay times of seven trains during peak hour: 0, 60, 35, 120, 10, 40 and 80. All times were measured in seconds.
- What is the range for these times?
 - What is the standard deviation for these times? Answer correct to two decimal places.
 - Create a five-number summary for this dataset.
 - Calculate the interquartile range for these times.

CHALLENGING

- 11** Joshua has been ill and only scored 36% in a recent English test. However, the results of his class were very good, with every other student scoring above 70%. Joshua's teacher has decided not to include Joshua's results in the class statistics. What effect would this have on the mean and standard deviation?

- 12** The results for Molly and Harry in their tests this term are listed in the table.

Molly	10	19	16	15	7	15	11	3	9	16
Harry	18	16	17	8	14	15	13	10	13	16

- What is the mean and standard deviation for Molly's results?
- What is the mean and standard deviation for Harry's results?
- Which student had the more consistent results throughout the term? Give a reason.

Communicate your reasoning, understanding and fluency

- 13** Charlie’s median score for cricket is 32 and the interquartile range of his scores is 5.
Thomas’s median score for cricket is 30 and the interquartile range of his scores is 10.
- a** Which player is the more consistent? Explain your answer.
 - b** Which player is more likely to make the higher score? Explain your answer.



- 14** The table shows Amber’s results for two Mathematics tests as well as the class mean and standard deviation.

Topic	Amber’s result	Class mean	Class standard deviation
Data	78	70	7
Algebra	83	74	10

Which is the better result for Amber compared to her class? Justify your answer.

- 15** Twelve teenagers were selected at random from a concert audience and their ages were recorded:
14 16 15 16 17 19 17 16 15 15 15 18
- a** Calculate the mean of their ages. Answer correct to two decimal places.
 - b** What is the standard deviation? Answer correct to one decimal place.
 - c** Explain the meaning of standard deviation.



6C Outliers

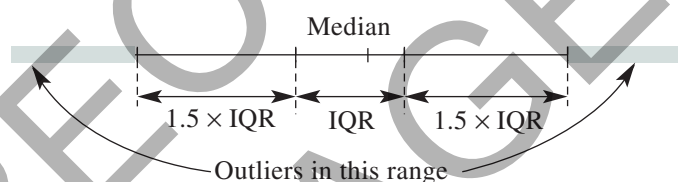
Learning intentions

- To understand the meaning of outliers and the effect they can have on the mean of a dataset
- To use a definition of an outlier to determine whether or not a data value is an outlier

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated in Year 12
- Students will study bivariate data analysis in Year 12
- Questions about outliers regularly appear in HSC Examinations
- A definition of an outlier is provided in a formula on the 2027 HSC Reference Sheet

An outlier is a score that is separated from the majority of the data. It is sometimes difficult to decide whether or not a score should be classified as an outlier. We define an outlier as any value more than 1.5 interquartile ranges above the third quartile ($Q_3 + 1.5 \times \text{IQR}$), or more than 1.5 interquartile ranges below the first quartile ($Q_1 - 1.5 \times \text{IQR}$).



Outliers have little effect on the mean, median and mode for large sets of data. However, in small datasets, the presence of an outlier will have a large effect on the mean, a smaller effect on the median and usually no effect on the mode.

Outlier

An outlier is a score that is separated from the majority of the data.

Use $Q_1 - 1.5 \times \text{IQR}$ and $Q_3 + 1.5 \times \text{IQR}$ as criteria to determine an outlier.



Example 9 Determining the effect of an outlier on a dataset

Task A	5	7	12	15	16	16	19	20	26	26	29	29	29	35	40
	40	42	43	45	46	47	47	48	52	54	54	57	57	60	91

a What is the outlier in Task A? Q_1 and Q_3 are in bold.

b What is the effect of the outlier on the mean, median and mode?

Solution

a $\text{IQR} = 48 - 20 = 28$

Lower limit $= 20 - (1.5 \times 28)$

$= -22$

No scores in the dataset below -22 , so no outlier.

Upper limit $= 48 + (1.5 \times 28)$

$= 90$

The outlier is 91.

Explanation

Determine IQR.

Find lower limit using $Q_1 - 1.5 \times \text{IQR}$.

There are no scores in the dataset less than -22 .

Find the upper limit using $Q_3 + 1.5 \times \text{IQR}$.

91 is the only score over 90.

- b** $\bar{x} = 36.9$ (with outlier)
 $\bar{x} = 35.0$ (without outlier)
 Mode = 29 (with outlier)
 Mode = 29 (without outlier)
 Median = 40 (with outlier)
 Median = 40 (without outlier)

Outlier has increased the mean but has had no effect on the median or the mode.

Calculate the mean for the dataset with and without the outlier using a calculator.

To calculate the mean ($\bar{x}$) after entering the dataset (using a Casio fx100 series calculator).

[SHIFT] [1]

[4]:VAR

[2]: $\bar{x}$

[=]

Calculate the mode for the dataset with and without the outlier.

Calculate the median for the dataset with and without the outlier.

Now you try

Task B 5 7 8 **10** 11 15 17 19 21 26 28 **30** 32 32 65

- a** With Q1 and Q3 in bold for Task B, determine if there is an outlier and what it is.
b What is the effect of the outlier on the mode and median?

Exercise 6C

BUILDING

- 1** List any outliers for each dataset by observation.

a 6, 5, 8, 8, 11, 16, 34, 6, 6

c 7, 5, 0, 7, 16, 0, 8, 5, 6

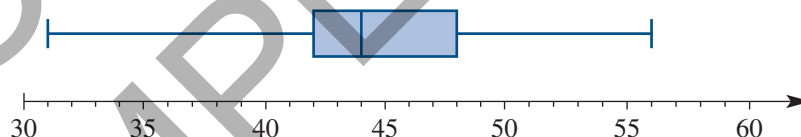
e 11, 3, 7, 6, 53, 6, 1, 6, 3, 1, 3

b 33, 33, 34, 33, 33, 32, 43, 48, 35

d 28, 22, 133, 22, 31, 22

f 13, 10, 13, 0, 8, 8, 8

- 2** The results of an assignment are represented in the following box plot.



a What is the lower extreme?

c What is the lower quartile?

e Find the interquartile range.

b What is the upper extreme?

d What is the upper quartile?

f Give the outlier if it exists.

- 3** Give the outlier in the following datasets, if it exists. Justify using the criteria $Q_1 - 1.5 \times \text{IQR}$.

a 1, 6, 7, 7, 8, 9, 9

c 20, 31, 32, 35, 35, 35, 35, 36, 36

e 56, 66, 68, 70, 70

b 69, 82, 82, 85, 86, 90, 91

d 10, 18, 19, 20, 21

f 0, 8, 8, 8, 10, 13, 13, 13

- 4** Give the outlier in the following datasets, if it exists. Justify using the criteria $Q_3 + 1.5 \times \text{IQR}$.

a 0, 0, 1, 3, 5, 5, 10

c 70, 72, 73, 74, 82

e 45, 45, 45, 47, 50, 50, 59

b 22, 23, 23, 24, 25, 25, 25, 27, 36

d 32, 33, 33, 33, 33, 34, 35, 43, 48

f 101, 102, 102, 102, 103, 115

Example 9

- 5** The ages of a group of people attending a musical was as follows:
29, 37, 27, 28, 40, 76, 26, 31, 42, 34, 25, 26, 30, 40, 27, 52, 36, 23, 31, 14, 27, 32, 28, 29, 40, 20.
- Find the lower quartile.
 - Find the upper quartile.
 - What is the interquartile range?
 - Give the outlier if it exists.
- 6** Seven students scored the following marks in an assessment task: 35, 49, 56, 59, 65, 69, 96. Is the mark of 96 an outlier for this set of data? Justify your answer with calculations.
- 7** The rainfall for the past nine months was as follows: 15, 20, 22, 14, 31, 19, 20, 16, 17. Is the 31 an outlier for this set of data? Justify your answer with calculations.

DEVELOPING

- 8** Jennifer scored the following points in her basketball games this season: 4, 5, 5, 6, 8, 8, 9, 10, 12 and 20.

- What is the range?
- What is the interquartile range?
- What is the value of the outlier?
- How does the outlier affect the range and the interquartile range?



- 9** Consider the following set of scores: 3, 5, 6, 13, 13, 19, 20, 20, 81.
- Calculate the mean and the median of this set of scores.
 - What is the effect on the mean and the median of removing the outlier?
- 10** **a** Calculate the mean, median and mode for each dataset. Answer correct to one decimal place when necessary.
- | | |
|---------------------------------------|---|
| i 26, 27, 21, 26, 57, 22, 23 | ii 38, 41, 26, 29, 34, 27, 27, 34, 116 |
| iii 95, 92, 91, 88, 88, 87, 42 | iv 21, 61, 62, 62, 65, 68, 69, 71, 73 |
- b** Calculate the mean, median and mode for each dataset after removing the outlier. Answer correct to one decimal place.
- 11** Consider the following set of scores: 12, 15, 16, 16, 18, 18, 19, 20, 20, 60.
- Calculate the mean and the median of the set of scores.
 - What is the effect on the mean and the median of removing the outlier?

CHALLENGING

- 12

Nine employees earned the following wages per week: \$680, \$590, \$490, \$990, \$660, \$520, \$300, \$510 and \$540.

a

Is the wage of \$990 an outlier for this set of data? Justify your answer with calculations.

b

Each employee receives a \$50 pay increase. What effect will this have on the standard deviation?
- 13

The players in Noah’s football team completed a survey on the number of text messages sent using their mobile phone within the last day. The results are summarised in the table.

a

What is the mean number of messages sent?
Answer correct to one decimal place.

b

Calculate the median number of messages sent.

c

What is the mode?

d

Calculate the mean, the median and the mode when the outlier is removed.

Number of messages	Frequency
0	10
1	8
2	5
3	4
6	1



Communicate your reasoning, understanding and fluency

- 14

The class scores for their latest assessment task are seen in this table.

Task 1	10	55	61	64	66	66	67	70	71	73	87

- a

Calculate the range and IQR.
- b

By working out the upper and lower limit, show which scores are outliers.
- c

An error was made in the marking and each student, except the top student, receives 5 more marks. Explain why there are now fewer outliers. Support your explanation with working out.
- d

Give reasons to explain which measure of spread (range or IQR) is more accurate in the dataset from part a.

6D Describing the shape of a dataset

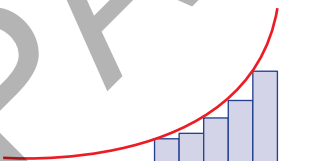
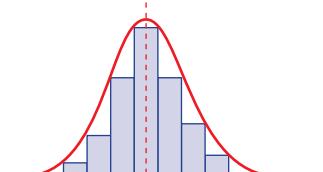
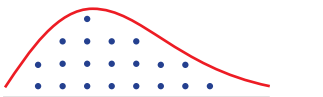
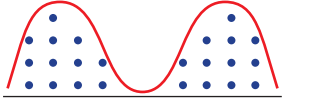
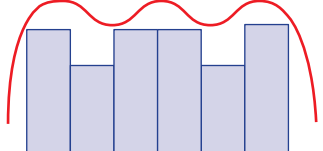
Learning intentions

- To understand and use terminology which describes the shape of a dataset

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated in Year 12
- Students will study bivariate data analysis in Year 12
- Questions about skewness appear in HSC Examinations

A curve can be used to describe the overall shape of the dataset. The general shape of a graph can be described in terms of smoothness, symmetry and the number of modes. **Smooth** curves have no breaks or jagged sections, and data gradually increases and/or decreases. **Symmetrical** graphs are balanced about a vertical line in the centre, and the mode is the score that occurs the most (highest frequency). **Unimodal** graphs have one mode. However, it is common for data to have several modes. **Bimodal** graphs have two modes and multimodal graphs have many modes.

Shape	Description	Graph
Smoothness	Data has no breaks or jagged sections.	
Symmetry	Data is symmetrical and balanced about a centre.	
Unimodal	Data has only one mode or peak.	
Bimodal	Data has two modes or peaks.	
Multimodal	Data has many modes or peaks.	

Symmetry or skewness

Data is symmetric if it forms a mirror image of itself when folded in the 'middle' along a vertical axis. Otherwise the data is skewed. Skewed data has more data on one side. There are two types:

- Positively skewed data has a long tail on the right-hand side.
- Negatively skewed data has a long tail on the left-hand side.

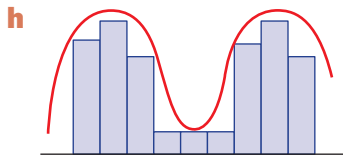
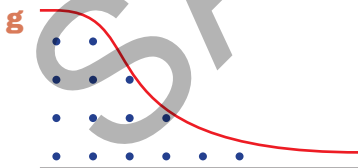
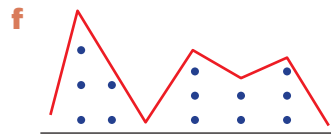
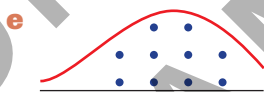
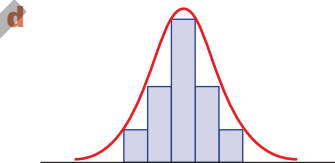
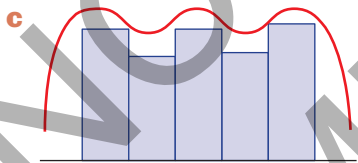
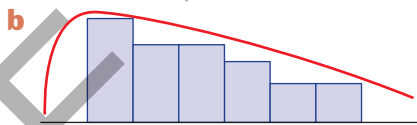
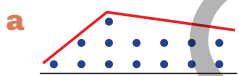


Shape	Description	Graph
No skew (Symmetric)	Data is symmetrical and balanced about a vertical line.	
Positively skewed	The data is more spread out on the left side. The long tail is on the right side (positive side).	
Negatively skewed	The data is more spread out on the right side. The long tail is on the left side (negative side).	

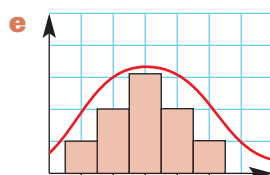
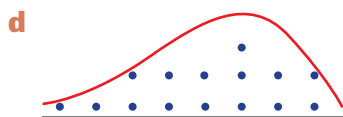
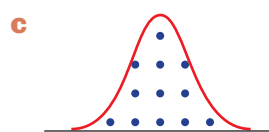
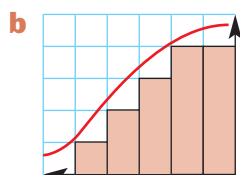
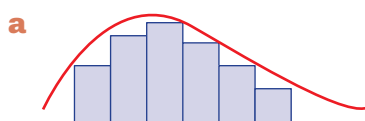
Exercise 6D

BUILDING

1 Describe the smoothness, symmetry and modality of the following graphs.



2 Describe the skewness of the following graphs.



DEVELOPING

3 For each dataset:

- a** draw a dot plot and then draw a smooth curve over the dot plot
- b** describe the smoothness, symmetry and modality of the dataset
- c** describe the skewness of the dataset.
 - i** 3 4 4 4 8 8 8 8 8 12 12 12 13
 - ii** 1 2 3 4 5 6 7 8 9 10 10 10 12 12 12

4 For each dataset:

- a** draw a frequency histogram and then draw a smooth curve over the histogram
- b** describe the smoothness, symmetry and modality of the dataset
- c** describe the skewness of the dataset.
 - i** 2 3 4 4 5 5 6 6 6 6 7 7 7 7 8 8
 - ii** 12 12 12 12 12 12 13 13 14 14 15 15 15 16 17 18 20

CHALLENGING

5 The grouped frequency table shows the age of people entering a competition.

Class	Class centre (x)	Frequency (f)	$f \times x$
5–9		1	
10–14		0	
15–19		2	
20–24		3	
25–29		7	
30–34		18	
35–39		16	

- a** Calculate the mean, median and modal class from the frequency table. Answer correct to one decimal place when necessary.
- b** What is the effect on the mean and the median of removing the outlier?
- c** Display the data in a frequency histogram.
- d** Draw a curve over the histogram, then describe the symmetry and modality of the data.
- e** Describe the skewness of the dataset.

Communicate your reasoning, understanding and fluency

- 6 There are eight students in a hospitality class. The students sat for a test and the results were 98, 95, 91, 94, 95, 11, 95 and 94.
- a Find the mean, the median and the mode. Answer correct to nearest whole number.
 - b Lily scored 91. She told her father that her result was above the average. Do you agree with Lily's statement? Give a reason for your answer.

- 7 High school students were surveyed and the results of two samples are shown in the tables.

Sample A

Score	1	2	3	4	5	6	7
Frequency	3	5	8	9	8	5	3

Sample B

Score	1	2	3	4	5	6	7
Frequency	3	9	9	7	6	5	1

- a For sample A (correct to one decimal place where necessary), find the:
 - i median
 - ii mean
 - iii mode
 - iv standard deviation.
- b For sample B (correct to one decimal place where necessary), find the:
 - i median
 - ii mean
 - iii mode
 - iv standard deviation.
- c Construct a frequency histogram for sample A.
- d Construct a frequency histogram for sample B.
- e Describe the symmetry and modality of sample A and sample B.
- f Describe the skewness of sample A and sample B.



6E Comparing datasets

Learning intentions

- To be able to compare and contrast two datasets using measures of location and spread and by describing the shape and skewness of both datasets

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated in Year 12
- Students will study bivariate data analysis in Year 12
- Questions involving comparison of datasets regularly appear in HSC Examinations

The selection and the use of the appropriate measure of location (mean or median) and measure of spread (range, interquartile range or standard deviation) depends on the nature of the dataset and the relative merits of each measure. For example, if the dataset contains one or two extreme scores then the value of the mean will greatly increase or decrease.

Measures of location	Advantages	Disadvantages
Mean	Easy to understand and calculate. Depends on every score. Varies least from sample to sample.	Distorted by outliers. Not suitable for categorical datasets.
Median	Easy to understand. Not affected by outliers.	May not be central. Dataset needs to be sorted. Varies more than the mean in a sample.
Mode	Easy to determine. Not affected by outliers. Suitable for categorical datasets.	There may be no mode or more than one mode. May not be central.
Measures of spread	Advantages	Disadvantages
Range	Easy to understand. Easy to calculate.	Dependent on the smallest and largest values. May be distorted by outliers.
Interquartile range	Easy to determine for small datasets. Easy to understand. Not affected by outliers.	Difficult to calculate for large datasets. Dependent on lower and upper quartiles. Dataset needs to be sorted.
Standard deviation	Depends on every score. Not significantly affected by outliers.	Difficult to determine without a calculator. Difficult to understand.



Example 10 Comparing two datasets

The table shows the number of participants in a fitness class for two instructors.

	M	T	W	T	F	S	S
Bec	8	5	4	8	8	4	5
Rita	10	9	12	14	8	10	1

- a Find the mean and median for each set of dataset.
Answer correct to one decimal place where necessary.
- b Find the range and interquartile range for each dataset.
- c Examine the summary statistics and outline any concerns.



Solution

- a Bec

$$\text{Mean } \bar{x} = \frac{42}{7} = 6$$

4, 4, 5, 5, 8, 8, 8

Median is 5.
- Rita

$$\text{Mean } \bar{x} = \frac{64}{7} = 9.1$$

1, 8, 9, 10, 10, 12, 14

Median is 10.

- b Bec

Range = $8 - 4 = 4$

IQR = $8 - 4 = 4$
- Rita

Range = $14 - 1 = 13$

IQR = $12 - 8 = 4$

- c There is an outlier (Rita’s class on Sunday: 1).
The outlier has decreased the mean and increased the range for Rita’s results.

Explanation

Complete for each dataset.
Determine the mean.
Determine the median by first placing the scores in ascending order.

Determine the range.
Determine Q1 and Q3 by using the ordered dataset in part **a**.
Calculate the IQR.

Now you try

The houses sold in two suburbs in a month are shown in the table.

- a Find the mean and median for each suburb.
- b Find the range and interquartile range for each suburb.
- c Examine the summary statistics and outline any concerns.
- d Is the mean or the median the best measure of what a homebuyer can expect when looking for a home? Why?

Suburb 1	Suburb 2
\$1 060 000	\$315 000
\$1 200 000	\$378 000
\$1 310 000	\$395 000
\$1 525 000	\$415 000
\$1 600 000	\$405 000
\$2 000 000	\$1 160 000

Exercise 6E

BUILDING

Example 10

1 Two classes in Year 11 were given a short quiz. The test was marked out of 10 and the results are listed in the table.

Score	Class A	Class B
7	4	1
8	3	3
9	5	6
10	6	8

Answer the following questions, correct to one decimal place when necessary.

- a How many students are there in each class?
- b What is the mean and median for class A?
- c What is the mean and median for class B?
- d What is the range and interquartile range for class A?
- e What is the range and interquartile range for class B?
- f What is the standard deviation for class A?
- g What is the standard deviation for class B?
- h Which class performed better in these short quizzes? Give reasons for your answer.
- i Compare and contrast the two datasets.

2 The number of males and females retiring in the past 15 years is displayed in the double stem-and-leaf plot.

Males		Females
6	0	9
8532	1	1369
9876442	2	155789
410	3	0125



Answer the following questions, correct to one decimal place when necessary.

- a Find the mean and median number of male retirees.
- b Find the mean and median number of female retirees.
- c Find the range and interquartile range for the number of male retirees.
- d Find the range and interquartile range for the number of female retirees.
- e Find the standard deviation for the number of male retirees.
- f Find the standard deviation for the number of female retirees.
- g Compare and contrast the two datasets.

DEVELOPING



3 Create the spreadsheet shown for the number of employees at two branches of a department store.

=SUM(B5:B11)				
	A	B	C	D
1	Cambridge Mathematics Standard Year 11 Exercise 6E Question 3			
2	Worksheet to analyse the number of employees			
3				
4	Department	City Branch	Bondi Branch	
5	Cosmetic	17	12	
6	Electrical	1	11	
7	Furniture	9	10	
8	Kitchen	11	8	
9	Men's	14	14	
10	Sport	9	12	
11	Women's	20	14	
12				
13	Sum	=SUM(B5:B11)	=SUM(D5:D11)	
14	Maximum	=MAX(B5:B11)	=MAX(D5:D11)	
15	Minimum	=MIN(B5:B11)	=MIN(D5:D11)	
16	Count	=COUNT(B5:B11)	=COUNT(D5:D11)	
17	Mode	=MODE(B5:B11)	=MODE(D5:D11)	
18	Median	=MEDIAN(B5:B11)	=MEDIAN(D5:D11)	
19	Average	=AVERAGE(B5:B11)	=AVERAGE(D5:D11)	
20	Standard deviation	=STDEV(B5:B11)	=STDEV(D5:D11)	

- a The formulae in cells B13 – B20 and D13 – D20 are shown in the spreadsheet. Enter these formulae.
- b Comment on the measures of location (mean, median and mode) for the city branch.
- c Comment on the measures of location (mean, median and mode) for the Bondi branch.
- d Comment on the measures of spread (range and standard deviation) for the city branch.
- e Comment on the measures of spread (range and standard deviation) for the Bondi branch.
- f Compare and contrast the two datasets.

CHALLENGING

4 The areas of eight suburbs of Sydney and Melbourne are listed in the table.

Sydney	9.0	7.4	8.2	11.5	6.4	6.3	5.6	7.3
Melbourne	3.0	7.5	9.5	25.2	4.5	3.6	2.0	14.5

Answer the following questions, correct to one decimal place when necessary.

- a Find the mean and median for each dataset.
- b Which is the better measure for the centre of this data? Explain your answer.
- c Find the range and interquartile range for each set of data.
- d Which is the better measure for the spread of this data? Explain your answer.
- e Compare and contrast the two datasets.

Communicate your reasoning, understanding and fluency

5

Task 1	10	55	61	64	66	66	67	70	71	73	87
Task 2	48	56	68	77	78	79	82	86	88	89	99

Use the test scores above to complete the following questions.

- a Find the mean and standard deviation for Task 1 and Task 2 (to one decimal place).
- b Find the range and IQR for Task 1 and Task 2.
- c Identify any outliers in Task 1.
- d Calculate the mean and standard deviation when the outlier is removed from Task 1.
- e Comment on the differences observed in the range, IQR, mean and median when the outlier is removed from Task 1.
- f Given what you have found, which is the best measure of spread when comparing Task 1 with and without the outlier? Explain your conclusion.



6F Parallel box plots

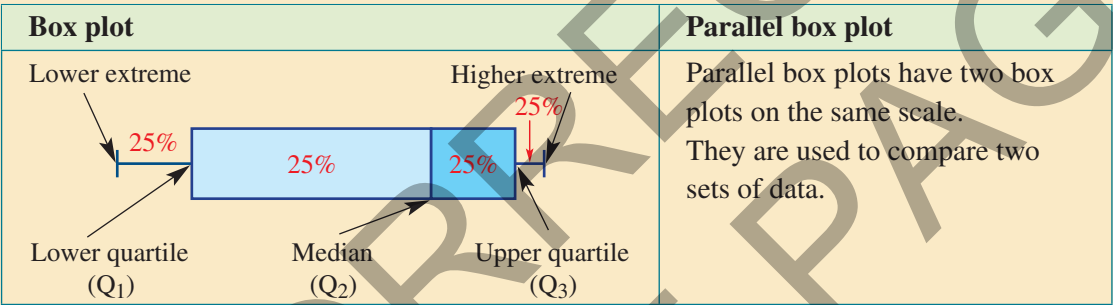
Learning intentions

- To be able to compare and contrast two datasets in parallel box plots using measures of location and spread and by describing the shape and skewness of both datasets

Past, present and future learning

- Some aspects of this section may be new to students
- These are Year 11 concepts which are not repeated in Year 12
- Students will study bivariate data analysis in Year 12
- Questions about parallel box plots regularly appear in HSC Examinations

Recall from Exercise 6B that a box plot is a graph created using the data from a five-number summary.



Using a parallel box plots to compare data

Parallel box plots are useful for comparing two or more sets of data collected on the same variable, such as the assessment results for two different groups of students. A double box plots allows the median and the spread to be easily identified and compared.

 Example 11 Using a parallel box plot to compare two datasets

The number of births in a city hospital each day is shown in the table.

Week A	13	13	15	15	16	17	18	18	19
Week B	8	9	11	12	12	13	16	16	20

- a Calculate the five-number summary for Week A and Week B.
- b What is the median (Q_2) for Week A?
- c What is Q_3 for Week B?
- d Display the five-number summaries from part a as a parallel box plot.
- e Compare and contrast the two datasets.

Solution**a Week A**

Lowest number = 13

Lower quartile (Q1) = 14

Median (Q2) = 16

Upper quartile (Q3) = 18

Highest number = 19

Week B

Lowest number = 8

Lower quartile (Q1) = 10

Median (Q2) = 12

Upper quartile (Q3) = 16

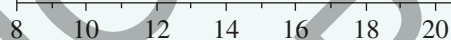
Highest number = 20

b Median = 16

$$\text{c } Q3 = \frac{7\text{th score} + 8\text{th score}}{2} = 16$$

d Week A

Week B



- e** The number of births each day is more consistent in Week A and the median is higher. However, the smallest and largest number of births occurred in Week B. The median number of births in Week A is the same as the upper quartile in Week B.

Explanation

Create a five-number summary for the Week A dataset.

Create a five-number summary for the Week B dataset.

Find the middle score (5th score).

Find the median (middle) of the upper half.

Now you try

The number of cars that passed by a country home is shown in the table.

Week A	130	130	150	150	160	170	180	180	190
Week B	85	90	110	124	124	130	160	160	207

- a** Calculate the five-number summary for Week A and Week B.
b What is the median (Q2) for Week B?
c What is Q1 for Week A?
d Display the five-number summaries from part **a** as a parallel box plot.
e Compare and contrast the two datasets.

Exercise 6F

BUILDING

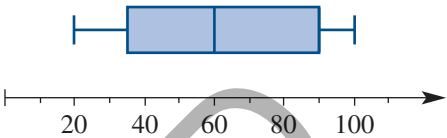
- 1 From the box plot, find the:
- a lower extreme

b higher extreme

c median

d lower quartile

e upper quartile.



- 2 Construct a box plot for each of the following five-number summaries.
- i Min = 3, Q_1 = 4, median = 5, Q_3 = 6 and max = 7

ii Min = 5, Q_1 = 7, median = 9, Q_3 = 11 and max = 13

Example 11

- 3 The heights (in centimetres) of people living in Olivia Parade and Hassle Street are listed in the table.

Olivia Pde	157	168	133	135	154	167	167	123	140	157	163	176	160	165
Hassle St	145	149	157	154	159	170	143	154	156	123	150	143	132	140

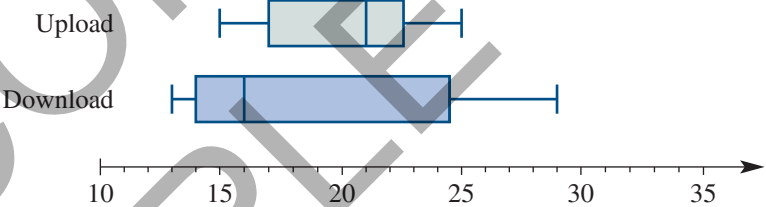
- a Create a five-number summary for Olivia Parade and Hassle Street.

b Construct a parallel box plot.

c Compare both datasets in terms of centre and spread.

DEVELOPING

- 4 The parallel box plot shows the amount of data uploaded and downloaded by Anthony each day for the past month.



- a What is the highest amount of data uploaded?

b What is the lowest amount of data downloaded?

c What is the median amount of data uploaded?

d Calculate the range for downloading data.

e What is the lower quartile for data downloaded?

f What is the upper quartile for data uploaded?

g What is the interquartile range for data uploaded?

h What is the interquartile range for data downloaded?
- 5 Construct a parallel box plot, using the following five-number summaries.
- Part 1: Min = 12, Q_1 = 14, median = 19, Q_3 = 22 and max = 26

Part 2: Min = 14, Q_1 = 19, median = 23, Q_3 = 26 and max = 29
- 6 Construct a parallel box plot using the following five-number summaries.
- Aaron: Min = 20, Q_1 = 21.5, median = 26, Q_3 = 30 and max = 35

Eliza: Min = 18.5, Q_1 = 26, median = 27.5, Q_3 = 34 and max = 35

- 7** The ages of residents who attended two community meetings are displayed in the double stem-and-leaf plot.

Meeting A	Meeting B
6	0
6521	1
987521	2
2100	3

- What is the five-number summary for meeting A?
- What is the five-number summary for meeting B?
- Construct a parallel box plot using the data in the double stem-and-leaf plot.
- What is the interquartile range for meeting A?
- What is the interquartile range for meeting B?

CHALLENGING

- 8** The TAFE results of two students are listed in the table.

Ryan	69	76	75	75	70	78	79	72	63	72	73	81
Tane	76	70	76	74	72	74	70	73	77	68	75	75

- What is Ryan's lower score?
- What is Tane's median?
- What is Tane's upper score?
- What is Ryan's median?
- What is Tane's lower quartile?
- What is Ryan's upper quartile?
- Construct a parallel box plot for the two datasets.
- Determine the range and interquartile range for each dataset.
- Who is the better student? Give a reason for your answer.

- 9** The approval ratings for two local politicians are listed in the table.

Jade	80	65	69	62	72	70	75	77	78	85	75	60
Ruby	75	85	66	35	92	94	40	30	45	54	77	80

- What is Ruby's lower extreme?
- What is Ruby's upper extreme?
- What is Jade's median?
- What is Jade's lower quartile?
- What is Jade's upper quartile?
- Construct a parallel box plot including each dataset.
- Compare and contrast the shapes of the box plots.
- Who is the more popular politician? Justify your answer.

Communicate your reasoning, understanding and fluency

- 10** The ages of the people attending an environmental conference are recorded in the table.

Female	35	63	28	30	27	42	53	50	52	63	61	62	45	18
Male	22	63	13	20	12	22	30	25	30	52	37	46	22	12

- What is the five-number summary for females attending the conference?
- What is the five-number summary for males attending the conference?
- Construct a parallel box plot including each dataset.
- Determine the range and interquartile range for each dataset.
- Compare and contrast the shapes of the box plots.
- What conclusions can you make about the ages of females and males attending the conference?



Key ideas and chapter summary

Measures of central tendency	Mean	Mean = $\bar{x} = \frac{\text{Sum of scores}}{\text{Number of scores}}$
	Median	The median is the middle score. To find the median, list all the scores in increasing order and select the middle one.
	Mode	The mode is the score that occurs the most. It is the score with the highest frequency.
Measures of spread	Range	Range = Highest score – Lowest score
	IQR	IQR = Third quartile – First quartile = $Q3 - Q1$
	Standard deviation	Standard deviation (σ) is the spread of data about mean.
Outliers	An outlier is a score that is separated from the majority of the data. Use $Q1 - 1.5 \times \text{IQR}$ and $Q3 + 1.5 \times \text{IQR}$ as criteria to determine an outlier.	
Describing the shape of a dataset	- Shape of the graph is described in terms of smoothness, symmetry and the number of modes. - Positively skewed data has a long tail on the right-hand side. - Negatively skewed data has a long tail on the left-hand side.	
Comparing datasets	The selection and the use of the appropriate measure of location (mean or median) and measure of spread (range, interquartile range or standard deviation) depends on the nature of the data and the relative merits of each measure.	
Parallel box plots	Two box plots on the same scale. They use the five-number summary of each dataset to compare them graphically.	

Chapter checklist with success criteria



A printable version of this checklist is available in the Interactive Textbook.



6A 1 I can calculate the mean. ☐

e.g. Mary scored 60, 42, 81, 80 and 74 in five tests at school last year. Calculate her mean test score. Answer correct to one decimal place.

6A 2 I can calculate the median and mode. ☐

e.g. Find the median and mode for the following datasets.

a 6 8 10 10 11 12 12

b 12 14 15 17 18 18

6A 3 I can find the mean from a grouped frequency table. ☐

e.g. Calculate the mean from the following grouped frequency table. Answer correct to two decimal places.

Class	Class centre (x)	Frequency (f)	$f \times x$
0–5	3	9	
6–10	8	4	
11–15	13	9	
16–20	18	12	
21–25	23	6	

6B 4 I can calculate the range. ☐

e.g. Farida recorded these times in training last week:

62 s, 61 s, 57 s, 72 s, 71 s and 60 s.

Calculate the range of her training times last week.

6B 5 I can draw a box plot from a five-number summary. ☐

e.g. Create a five-number summary and draw a box plot from the following dataset.

24 24 27 29 29 34 40 40 42 48 50 55

6B 6 I can calculate the interquartile range. ☐

e.g. Find the interquartile range (IQR) for the dataset in Question 5.

6B 7 I can calculate the standard deviation. ☐

e.g. Calculate the standard deviation for the dataset in Question 5. Answer correct to one decimal place.

6B 8 I can calculate the standard deviation using a frequency table. ☐

e.g. Calculate the standard deviation for the dataset in Question 3. Answer correct to two decimal places.

6C 9 I can identify an outlier and describe its effect on a dataset. □

e.g. Task D 5 7 9 **9** 11 15 17 19 21 21 21 **29** 32 32 68

- a** With Q1 and Q3 in bold for Task D, determine if there is an outlier and what it is.
- b** What is the effect of the outlier on the mode and median?

6D 10 I can describe the shape of a dataset. □

e.g. Draw a dot plot for the following dataset and then describe its smoothness, symmetry, modality and skewness.

1, 1, 1, 1, 2, 3, 4, 5, 6, 6, 6, 6, 7, 9, 9

6E 11 I can compare two datasets. □

e.g. A 6 8 10 10 11 12 12 12 14 B 12 14 15 17 18 18 20 20 20

- a** Find the median and mode for each of the dataset.
- b** Find the range and IQR for each of the dataset.
- c** Examine the summary statistics and outline any concerns.

6F 12 I can use parallel box plots to compare two datasets. □

e.g. Using the weekly attendance results in the table.

Week A	15	55	61	64	66	66	67	70	71	73	87
Week B	48	56	68	68	69	69	70	70	70	70	72

- a** Calculate the five-number summary for Week A and Week B.
- b** Display the five-number summaries from part **a** as a parallel box plot.
- c** Compare and contrast the two datasets.

Multiple-choice questions

- 6B 1** Sarah scored 48, 74, 89, 55, 64 and 89 in the Trial HSC. Which measure of location would Sarah prefer to tell her parents?

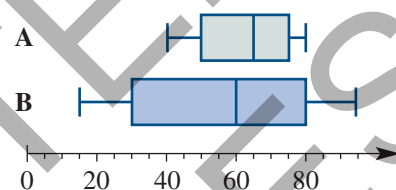
A Mean **B** Median **C** Mode **D** Range

- 6A 2** Find the median of this dataset: 8, 9, 10, 6, 7, 7, 3, 4, 5, 5, 4, 4, 10.

A 3 **B** 5.5 **C** 7 **D** 6

- 6B 3** Two examinations results are displayed in the box plot. What is the interquartile range for exam B?

A 25 **B** 40
C 50 **D** 60



- 6B 4** The table shows statistics of the results of five tests. Which student has the more consistent results?

Student	Mean	Standard deviation	Range
Alice	13.6	3.8	8
Hamza	13.9	4.2	8
Molly	13.8	2.7	8
Ryder	14.0	3.4	8

A Alice **B** Molly **C** Hamza **D** Ryder

- 6B 5** What is the difference between the range and interquartile range for the following dataset: 11, 14, 12, 10, 15, 16, 10, 15, 16, 20, 10

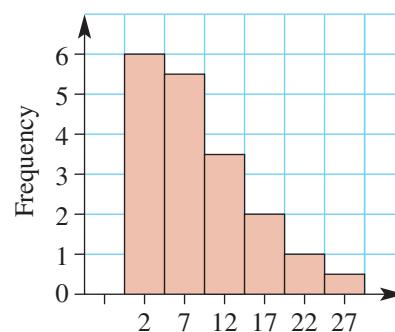
A 4 **B** 5 **C** 6 **D** 10

- 6C 6** There are 7 scores in a dataset. One of the scores is an outlier. Which of the mean, mode, median or interquartile range will be most influenced by the outlier?

A Mode **B** Mean
C Median **D** Interquartile range

- 6D 7** How would you describe the symmetry and skewness of the histogram?

A Positively skewed
B Negatively skewed
C Symmetric
D Symmetric with a clear outlier



Short-answer questions

6A 1 Find the missing number in these datasets.

- a** The scores 10, 11, 13, 15, $\square$, 16, 18, 20 and 20 have a median of 15.
- b** The scores 4, 4, $\square$, 6, 8 and 11 have a median of 5.
- c** The scores 5, 6, 6, 6, $\square$, 9, 10 and 11 have a median of 7.

6A 2 Members of a swimming club recorded the following times (in minutes) at a carnival.

46 48 48 47 44 46 44 45 42 45 45 45 44 46 48 42 42
 47 43 45 45 47 47 48 45 44 43 46 41 43 47 43 41 44
 43 45 44 41 46 43 44 42 46 42 44 46 43 46 43 43 44

- a** Construct a frequency table.
- b** How many times were recorded?
- c** Add an fx column.
- d** What is the mean for this dataset? Answer correct to one decimal place.

6A 3 Philip has completed three mathematics tests. His mean mark is 64%. What mark (out of 100) does he have to get in his next test to increase the mean to 65%?

6B 4 Find the interquartile range for each dataset.

- a** 18, 16, 19, 18, 16, 13, 12, 15
- b** 19, 22, 17, 18, 23, 15, 15, 13
- c** 32, 29, 24, 26, 25, 28, 29, 24, 30, 22
- d** 29, 37, 39, 57, 58, 34, 58, 59, 29, 31
- e** 43, 57, 39, 61, 31, 34, 57, 38, 41
- f** 40, 50, 46, 41, 46, 53, 59, 44, 46

6B 5 The number of service calls per day made by an air-conditioning technician is: 8, 1, 6, 8, 11, 4, 9, 11, 10, 8, 7, 10, 8, 4, 5, 6, 9

- a** What is the range?
- b** What is the first quartile?
- c** What is the third quartile?
- d** What is the interquartile range?



6B 6 The lifetimes in hours of six batteries of Brand A and Brand B are recorded in the table.

Brand A	74	57	61	90	89	79
Brand B	67	74	39	109	68	99

- a** What is the mean and standard deviation of each brand of battery?
- b** Which brand of battery has the more consistent lifetimes? Explain your answer.

6D 7 The time taken (in minutes) for Liam to travel to work each day for two weeks is shown in the table.

6F

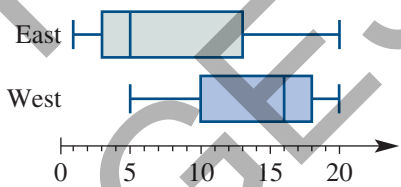
Week 1	32	31	35	33	35	59	40
Week 2	38	30	32	37	41	39	42

- a Draw a parallel box plot.
- b What is the median of the scores in Week 1?
- c What is the median of the scores in Week 2?
- d What is the interquartile range for Week 1?
- e What is the interquartile range for Week 2?
- f Comment on the skewness of the scores in Week 1.

6C 8 The results of a survey are shown in the parallel box plot.

6E

- a What is the range of the results in the east?
- b What is the range of the results in the west?
- c What is the median of the results in the east?
- d Describe the skewness of the results in the east.
- e Describe the skewness of the results in the west.
- f Is 20 an outlier for the East dataset? Show using calculations.



Extended-response questions

6E 1 A survey was conducted with 20 students to see if they took longer to perform a study task when listening to music than when not listening to music. The data is presented in the table.

6F

Five-number summary	Listening to music (min)	Not listening to music (min)
Lower extreme	17	13
Lower quartile	19	18
Median	24	21
Upper quartile	32	26
Upper extreme	33	30

- a Draw a parallel box plot.
- b Consider the following comment made by one of the students.
'It does not take any longer to complete a study task if you are listening to music.'
Is this a reasonable comment based on the dataset? Use the box plots to compare and contrast the datasets. Fully justify your response.



Time and location

Time travel in the Pacific, Samoa's leap to the future...

Crossing the International Date Line changes the calendar day by a full 24 hours. Travelling westward moves you forward by a day, while travelling eastward moves you back.

In 2011, Samoa made a significant change by shifting its position relative to the International Date Line to align its time zones with its neighbouring trading partners, Australia and New Zealand. As a result, Samoa skipped 30 December that year entirely. This change boosted its economic ties with its regional neighbours.

By moving Samoa to the western side of the International Date Line, Samoa adjusted its time zone from UTC -11 to UTC +13, effectively making it 24 hours ahead of American Samoa.

As a result, 30 December 2011 did not exist in Samoa. There were no births, no deaths, and no weddings recorded on that day. This shift also positioned Samoa as one of the first nations to welcome the New Year on 31 December, while American Samoa is the last country to do so.



Chapter contents

- 7A** Units of time
- 7B** Latitude and longitude
- 7C** Time zones
- 7D** Time difference using time zones
- 7E** Time difference using longitudes

NSW Syllabus

In this chapter, a student:

- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- Solves problems involving time and location in practical contexts (MST-11-06)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

7A Units of time

Learning intentions

- To use and understand the conventions of 12-hour and 24-hour time
- To convert between the commonly used units of time
- To perform calculations and solve problems involving time

Past, present and future learning

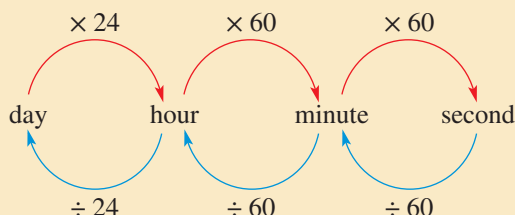
- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about time regularly appear in HSC Examinations

Time is the ongoing sequence of events taking place: the past, present and future. We measure time using seconds, minutes, hours, days, weeks, months and years. One year is the time it takes Earth to travel around the Sun. This is approximately $365\frac{1}{4}$ days. There are 30 days in September, April, June and November; all the rest have 31 except for February, which has 28 days, or 29 days in a leap year.



Units of time

- Time is measured in units such as **hours (h)**, **minutes (min)**, **seconds (s)**, **days**, **weeks**, **months** and **years**.
- To convert between units of time:
 - ▷ 1 h = 60 min
 - ▷ 1 min = 60 s
 - ▷ 1 day = 24 h
 - ▷ 1 week = 7 days
 - ▷ 1 year = 12 months
- **DMS conversion:** Most scientific calculators have a DMS (degrees, minutes and seconds) button $\left[\begin{smallmatrix} \circ \\ ' \\ '' \end{smallmatrix} \right]$ or function that converts time in fraction or decimal form to hours, minutes and seconds. e.g. 2.26 hours $\rightarrow 2^{\circ}15'36''$ meaning 2 hours, 15 minutes and 36 seconds.
- **Elapsed time** is the amount of time between the start and the end of an activity or event.
- Use **a.m.** to describe the 12 hours before noon, and **p.m.** to describe the 12 hours after noon.



- **24-hour time** is in the form 'hhmm'. The first two numbers show the hours after midnight, and the last two numbers show the minutes after midnight.
 - ▷ 0110 represents 1:10 a.m.
 - ▷ 1310 represents 1:10 p.m.



Example 1 Converting units of time

Complete the following.

a 3 h and 15 min = min

b 10 080 min = d

Solution

a 3 h 15 min = $3 \times 60 + 15$ min = 195 min

b 10 080 min = $10\,080 \div 60$
 = 168 h
 = $168 \div 24 = 7$ d

Explanation

To change hours to minutes multiply by 60.

To change minutes to hours divide by 60.

To change hours to days divide by 24.

Now you try

Complete the following.

a 6 h and 27 min = min

b 5760 min = days

24-hour time

The time of day is written in the 24-hour notation in the form 'hhmm', where 'hh' represents the hour and 'mm' represents the minutes. The day begins at 0000 (midnight) and the last minute of the day begins at 2359. The advantage of the 24-hour clock is that it clearly distinguishes the time in the morning and the afternoon without the a.m. and p.m. notation.

The 24-hour notation is used in many applications such as timetables, oven clocks and airport arrivals and departures.

12-hour clock	24-hour clock
12:00 midnight	0000
1:00 a.m.	0100
2:00 a.m.	0200
3:00 a.m.	0300
4:00 a.m.	0400
5:00 a.m.	0500
6:00 a.m.	0600
7:00 a.m.	0700
8:00 a.m.	0800
9:00 a.m.	0900
10:00 a.m.	1000
11:00 a.m.	1100
12:00 midday	1200

12-hour clock	24-hour clock
12:00 midday	1200
1:00 p.m.	1300
2:00 p.m.	1400
3:00 p.m.	1500
4:00 p.m.	1600
5:00 p.m.	1700
6:00 p.m.	1800
7:00 p.m.	1900
8:00 p.m.	2000
9:00 p.m.	2100
10:00 p.m.	2200
11:00 p.m.	2300
12:00 midnight	2400



Example 2 Using 24-hour time

- a** Convert 8:15 p.m. to 24-hour time.
b Write 2145 in 12-hour time.
c How long is it between 1800 and 2315?

Solution

- a** 2015
b 9:45 p.m.
c $2315 - 1800 = 0515$

It is 5 hours and 15 minutes between the two times.

Explanation

$$815 + 1200 = 2015$$

$$2145 - 1200 = 0945$$

Subtract the two times.

Write your answer in words.

Now you try

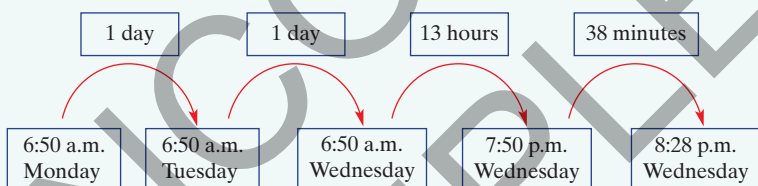
- a** Convert 6:55 p.m. to 24-hour time.
b Write 1455 in 12-hour time.
c How long is it between 1300 and 1720?



Example 3 Performing calculations involving elapsed time

Calculate the time, in days, hours and minutes, that has elapsed between 6:50 a.m. Monday and 8:28 p.m. the following Wednesday.

Solution



$$1 \text{ day} + 1 \text{ day} + 13 \text{ hours} + 38 \text{ minutes} = 2 \text{ days, } 13 \text{ hours and } 38 \text{ minutes}$$

The time elapsed is 2 days, 13 hours and 38 minutes.

Explanation

Use a timeline to break up the start and finish time into smaller increments.

Find the time between these and add them to calculate the total elapsed time.

Now you try

Calculate the time, in days, hours and minutes, that has elapsed between 9:20 a.m. Friday and 4:15 p.m. the following Sunday.

Exercise 7A

BUILDING

Example 1

- 1** Complete the following

a $2.5 \text{ h} = \square \text{ min}$

d $40 \text{ min} = \square \text{ s}$

g $720 \text{ min} = \square \text{ h}$

j $1080 \text{ h} = \square \text{ d}$

b $2 \text{ min} = \square \text{ s}$

e $4.5 \text{ d} = \square \text{ h}$

h $48\,000 \text{ s} = \square \text{ min}$

k $390 \text{ min} = \square \text{ h}$

c $20 \text{ d} = \square \text{ h}$

f $10 \text{ h} = \square \text{ min}$

i $96 \text{ h} = \square \text{ d}$

l $780 \text{ s} = \square \text{ min}$

- 2** What unit of time is most appropriate to measure each of the following?
- | | |
|---------------------------------|--|
| a Lesson at school | b Reheating a meal in a microwave |
| c Age of a person | d School holidays |
| e Accessing the internet | f Movie |
- 3** Use the DMS button/function on your calculator to convert the following to hours and minutes, rounding to the nearest minute where necessary.
- | | |
|-------------------------------|-------------------------------|
| a $1\frac{1}{2}$ hours | b $4\frac{1}{4}$ hours |
| c $2\frac{1}{3}$ hours | d 6.5 hours |
| e 3.75 hours | f 9.25 hours |
| g 7.05 hours | h 8.93 hours |
- 4** What time will it be:
- | | | |
|------------------------------------|-------------------------------------|---------------------------------------|
| a 4 hours after 5:30 p.m.? | b 7 hours before 1:00 p.m.? | c 17 hours before midday? |
| d 15 hours after 2:00 a.m.? | e 3.5 hours after 1:00 p.m.? | f 2.25 hours before 4:00 a.m.? |
- 5** The time required to bake a cake in an oven is 1 h 9 min. When will the cake be finished for each of the starting times shown?
- | | |
|--------------------|---------------------|
| a 5:00 a.m. | b 9:00 a.m. |
| c 8:20 p.m. | d 10:34 a.m. |
| e 5:44 p.m. | f Midnight |
| g 3:56 p.m. | h 10:51 a.m. |
- 
- 6** Add these times.
- | | | |
|----------------------------------|-----------------------------------|----------------------------------|
| a 2 h 24 min + 2 h 14 min | b 4 h 04 min + 1 h 46 min | c 5 h 30 min + 3 h 30 min |
| d 7 h 41 min + 5 h 50 min | e 11 h 29 min + 6 h 55 min | f 9 h 09 min + 3 h 59 min |
- 7** Subtract these times.
- | | | |
|-----------------------------------|----------------------------------|-----------------------------------|
| a 10 h 47 min – 2 h 22 min | b 4 h 52 min – 1 h 40 min | c 6 h 20 min – 5 h 08 min |
| d 8 h 10 min – 4 h 20 min | e 5 h 34 min – 2 h 18 min | f 10 h 46 min – 7 h 56 min |
- 8** Find the difference between these times.
- | | |
|-----------------------------------|-----------------------------------|
| a 8:00 p.m. and 11:00 p.m. | b 2:15 a.m. and 9:25 a.m. |
| c 1:43 p.m. and 12:40 p.m. | d 11:05 a.m. and 9:35 a.m. |
| e 1:30 p.m. and midnight | f 6:23 a.m. and 5:25 p.m. |
- 9** What is the time:
- | | |
|--|---|
| a 11 hours before 2:00 a.m.? | b 5 hours after 7:00 p.m.? |
| c $1\frac{1}{2}$ hours after 6:30 a.m.? | d $2\frac{1}{3}$ hours before 3:30 p.m.? |
| e 4 h and 11 min after 1:45 p.m.? | f 12 h and 50 min after 6:23 a.m.? |

- 10** Lauren is a cyclist who recorded the following training times at the gym.

1 h 21 min 59 min
 2 h 05 min 1 h 47 min
 45 min 1 h 35 min

- a** What is the total training time?
b What is the mean training time?
c What is the median training time?



Example 2

- 11** Write in 24-hour time.

a 2:00 a.m. **b** 10:30 p.m. **c** 8:41 p.m.
d 11:12 p.m. **e** 1:59 a.m. **f** Midnight
g 9:09 p.m. **h** 4:39 p.m. **i** 5:15 a.m.

- 12** Write in 12-hour time.

a 0318 **b** 1040 **c** 1200
d 1756 **e** 0911 **f** 2300
g 1845 **h** 0320 **i** 2050

- 13** What is the time 5 hours and 29 minutes after the following times?

a 0700 **b** 1319 **c** 2241

- 14** What is the time 6 hours and 20 minutes before the following times?

a 1545 **b** 1456 **c** 0230

DEVELOPING

Example 3

- 15** Calculate the time, in days, hours and minutes, that has elapsed between these times.

a 4:10 a.m. Monday and 4:50 p.m. Tuesday the following day
b 8:01 a.m. Wednesday and 9:10 a.m. the following Thursday
c 6:30 p.m. Tuesday and 4:28 a.m. the following Friday
d 0700 Saturday and 1300 on the following Sunday
e 1020 Tuesday and 1855 on the following Friday
f 0050 Wednesday and 1915 on the following Saturday

- 16** Ivy leaves home at 0928 to go shopping. She returns at 1348. How long, in hours and minutes, had she been away?

- 17** A game of football starts at 1412 and finishes at 1559. What is the length of the game? Answer in hours and minutes.

- 18** Alexis's dad is going to pick her up from the cinema once the film has finished. If the film starts at 1735 and is 99 minutes long, at what time will he need to pick her up?

- 19** Mason's dog ran away from home on Monday at 3:15 p.m. The dog was found the following Thursday at 1300. How much time elapsed before the dog was found?

CHALLENGING

- 20** Copy and complete the following table for a Blu-ray player that uses 24-hour time.

	Program (12-hour time)	Time settings (24-hour time)
a	6:00 a.m. to 7:38 a.m.	
b	10:25 a.m. to 12:35 p.m.	
c	3:20 p.m. to 4:54 p.m.	
d	6:30 p.m. to 8:05 p.m.	
e		0750 to 1000
f		1245 to 1410
g		0020 to 1549
h		2310 to 0207

- 21** A movie runs for 212 minutes.

- Convert this time to hours and minutes.
- It takes Sophie 34 minutes to drive to the cinema from home. What is the minimum time Sophie needs to leave home if the movie starts at 1608?
- What is the finishing time of the movie if the movie starts at 1435?
- What is the starting time if the movie finishes at 1110?

- 22** Sydney airport shows the arrival of a number of international flights.

- What is the arrival time of the London flight in 12-hour time?
- How long is there between the flight from Toronto and the flight from New York?
- How much time has elapsed since the flight from Rome arrived if the current time is 8:35 p.m.?
- How much time has elapsed since the flight from Dubai arrived if the current time is 9:45 p.m.?

Arrival			
FLIGHT	FROM	LANDED	GATE
DB692	LONDON	1632	A2
AK721	NEW YORK	1735	B6
CS169	DUBAI	1805	A5
AL379	ROME	1855	B2
AF4485	PARIS	1900	A1
QA034	MADRID	1910	B4
CA456	TORONTO	2030	B3

- 23** The Ghan is a legendary passenger train service that operates through the heart of Australia, offering one of the world's greatest train journeys.

The Ghan departs on Wednesdays at 9:30 a.m. from Darwin.

- If the Ghan arrives in Adelaide on Friday at 1355, how long is this journey?
- If the Ghan then continues on from Adelaide and takes 1 day, 20 hours and 40 minutes to travel to Alice Springs, what day and time will the Ghan arrive in Alice Springs?

Communicate your reasoning, understanding and fluency

- 24** A motorway closure due to roadworks starts on Friday 11:00 p.m. The motorway reopens on Monday 5:00 a.m. of the next week.
- a** For how many hours will the motorway be closed?
- b** Eliza lives a 5-minute drive from the motorway and must be at the airport at 7:00 a.m. on Monday. Travel time from Eliza's home to the airport is 2 hours 30 minutes.

Should Eliza take the motorway to get to the airport? Explain your answer.



7B Latitude and longitude

Learning intentions

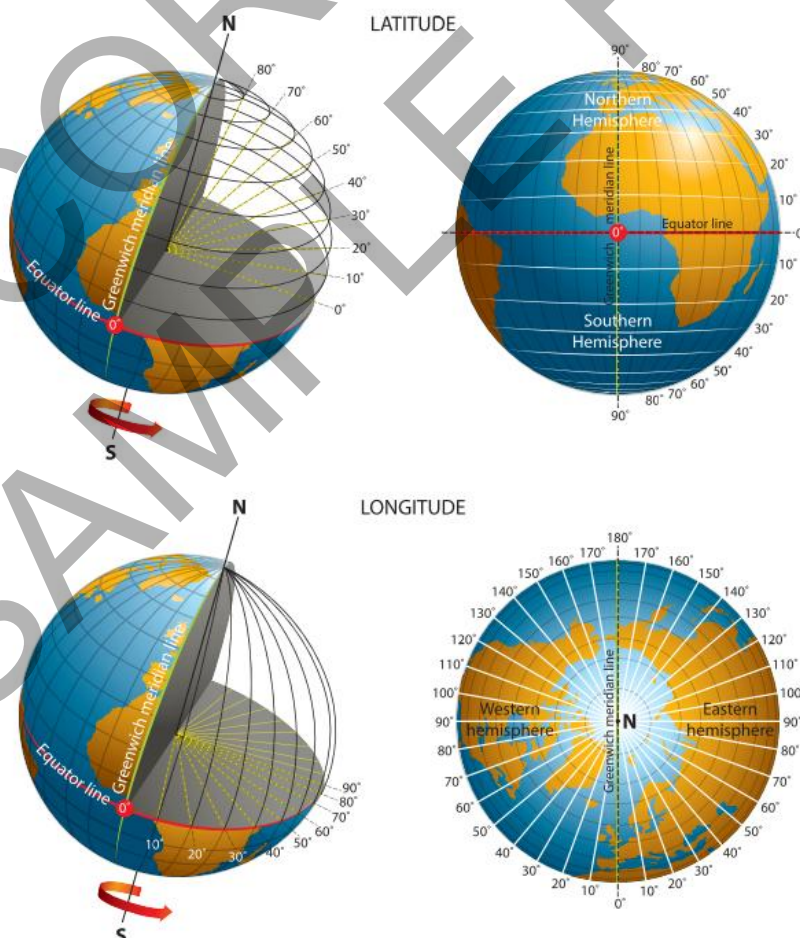
- To use and understand latitude and longitude
- To perform calculations and solve problems involving latitude and longitude

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about latitude and longitude regularly appear in HSC Examinations

Earth's surface is divided into parallels of latitude and meridians of longitude, which are imaginary lines on Earth's surface, forming circles and a grid. This imaginary grid is used to specify the position of a point on Earth's surface. Latitude is an angle north or south of the equator, and longitude is an angle east or west of the Greenwich meridian. The angle of latitude is the angle at Earth's centre between the plane of the equator and a line from the centre to the parallel of latitude. The angle of longitude is likewise formed at Earth's centre between the plane of the Greenwich meridian and a line from the centre of Earth to the meridian of longitude. The Greenwich or prime meridian is an imaginary north–south line that passes through the town of Greenwich (London).

Latitude is given first (N or S) and then the longitude (E or W). For example, the position of Sydney is (33°S , 151°E). It is 33° south of the equator and 151° east of the Greenwich meridian.

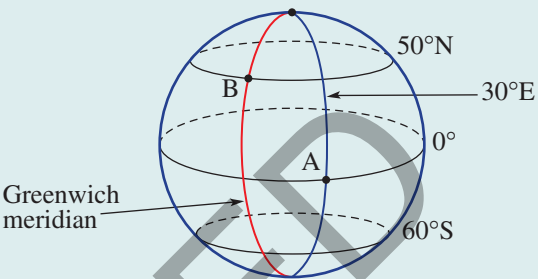




Example 4 Finding a position on Earth's surface

A diagram of Earth's surface is shown.

- a** What is the latitude and longitude of point A?
- b** What is the latitude and longitude of point B?
- c** What are the coordinates of a point 60° due south of point A?
- d** What are the coordinates of a point 10° due east of point B?



Solution

- a** Point A is at $(0^\circ, 30^\circ\text{E})$.
- b** Point B is at $(50^\circ\text{N}, 0^\circ)$.
- c** Point A is at $(60^\circ\text{S}, 30^\circ\text{E})$.
- d** Point B is at $(50^\circ\text{N}, 10^\circ\text{E})$.

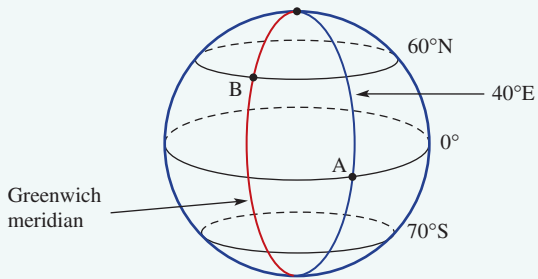
Explanation

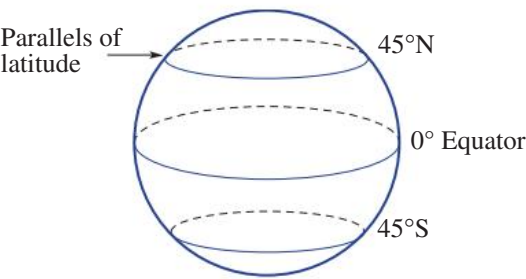
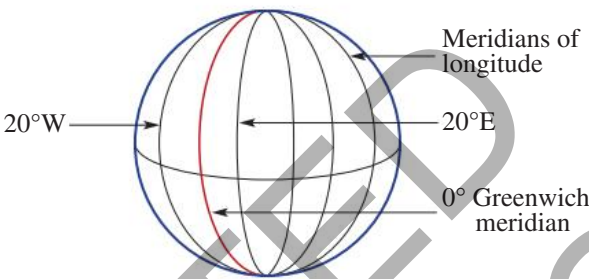
Latitude is the angle north or south of the equator (0°).
Point A is on the equator.
Longitude is the angle east or west of the Greenwich meridian (30°E).
Latitude is the angle north or south of the equator (50°N).
Longitude is the angle east or west of the Greenwich meridian (0°). Point B is on the Greenwich meridian.
Point A is on the equator (0°) hence the required point is 60°S . It has the same longitude as point A (30°E).
Point B is on the Greenwich meridian (0°) hence the required point is 10°E . It has the same latitude as point B (50°N).

Now you try

A diagram of Earth's surface is shown.

- a** What is the latitude and longitude of point A?
- b** What is the latitude and longitude of point B?
- c** What are the coordinates of a point 50° due south of point A?
- d** What are the coordinates of a point 20° due east of point B?



Latitude	Longitude
Measures the angular distance north and south of the equator (0°)	Measures the angular distance east and west of the Greenwich meridian (0°)
	



Example 5 Applying latitude and longitude

Brisbane is located at (27°S , 153°E). Perth is located at (31°S , 115°E).

- What is the difference in longitude between Brisbane and Perth?
- Which city is closer to the equator?



Solution

- Longitude difference = $153 - 115$
= 38°
- Brisbane is closer to the equator (smaller angle).

Explanation

Longitude of Brisbane is 153°E and Perth is 115°E . Both cities are east of the Greenwich meridian. Subtract the angles to find the difference in longitude.

Latitude of Brisbane is 27°S and Perth is 31°S . Both cities are south of the equator. Smaller angle is closer to the equator.

Now you try

Sydney is located at (34°S , 151°E). Auckland is located at (37°S , 175°E).

- What is the difference in longitude between Sydney and Auckland?
- Which city is closer to the equator?



Example 6 Describing positions of one point relative to another point

Describe the position of Cairo (30°N , 31°E) relative to Sydney (34°S , 151°E).

Solution

Comparing latitudes:

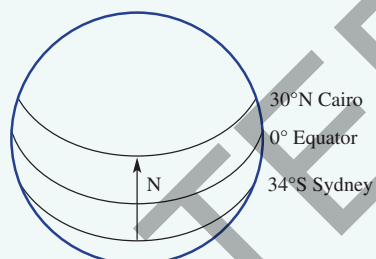
Cairo is 64° north of Sydney.

Comparing longitudes:

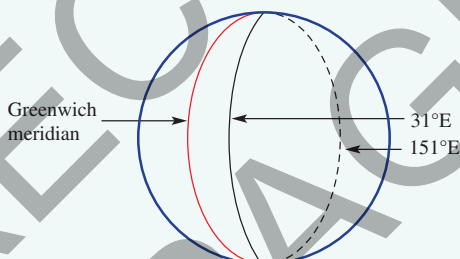
Cairo is 120° west of Sydney.

Explanation

Use a sketch to help with comparing.



$$34^\circ + 30^\circ = 64^\circ \text{ north}$$



$$151^\circ - 31^\circ = 120^\circ \text{ west}$$

Now you try

Describe the position of Rome (42°N , 12°E) relative to Sydney (34°S , 151°E).

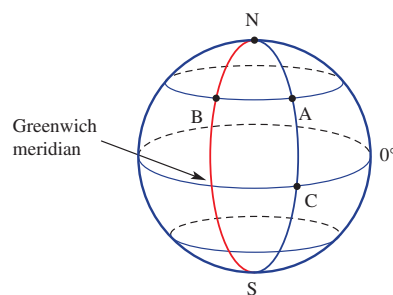
Exercise 7B

BUILDING

Example 4

- 1** A diagram of Earth's surface is shown right. The coordinates of point A are (60°N , 70°E).

- What are the coordinates of point B?
- What are the coordinates of point C?
- What is the maximum value possible for latitude? Where does it occur?



Example 5

- 2** Brussels is located at (50°N , 4°E). Lima is located at (12°S , 77°W).

- Which city is closer to the equator?
- What is the difference in latitude between Brussels and Lima?
- What is the difference in longitude between Brussels and Lima?

- 3** Tehran is located at (35°N , 51°E). Wellington is located at (41°S , 174°E).

- Which city is closer to the Greenwich meridian?
- What is the difference in latitude between Tehran and Wellington?
- What is the difference in longitude between Tehran and Wellington?

- 4** Nagoya is located at (35°N, 136°E). Determine the coordinates of a point that is:
- a** 20° due south of Nagoya
 - b** 20° due north of Nagoya
 - c** 20° due west of Nagoya
 - d** 20° due east of Nagoya.
- 5** Belgrade is located at (44°N, 20°E). Determine the coordinates of a point that is:
- a** 40° due south of Belgrade
 - b** 40° due north of Belgrade
 - c** 40° due west of Belgrade
 - d** 40° due east of Belgrade.
- 6** Munich has the coordinates (48°N, 11°E) and Paris has the coordinates (48°N, 2°E)
- a** What do these cities have in common in terms of their position on Earth's surface?
 - b** What is the longitude of a point 20° due south of Munich?
 - c** Which city is further from the Greenwich meridian?
 - d** What are the coordinates of a point 80° due east of Paris?

DEVELOPING

- 7** The following table shows the latitude and longitude of cities around the world.

City	Latitude	Longitude
Adelaide	34°S	139°E
Athens	37°N	23°E
Birmingham	52°N	1°W
Cairo	30°N	31°E
Montevideo	34°S	56°W
Rio de Janeiro	22°S	43°W
Sofia	42°N	23°E
Warsaw	52°N	21°E
Kuala Lumpur	3°N	101°E

- a** Which city is closest to the following latitudes?
 - i** 10°N
 - ii** 27°S
 - iii** Equator
 - iv** Latitude of Athens
- b** Which city is closest to the following longitudes?
 - i** 121°E
 - ii** 50°W
 - iii** Greenwich meridian
 - iv** Longitude of Sofia
- c** List the cities in the Northern Hemisphere.
- d** List the cities in the Western Hemisphere.
- e** Which cities have the same latitude?
- f** Which cities have the same longitude?
- g** Which city is closest to the North Pole?
- h** Which city is closest to the South Pole?

Example 6

8 Describe the following:

- a The position of the Athens (37°N , 23°E) relative to Cairo (30°N , 31°E).
- b The position of the Adelaide (34°S , 139°E) relative to Kuala Lumpur (3°N , 101°E).
- c The position of the Montevideo (34°S , 56°W) relative to Birmingham (52°N , 1°W).
- d The position of the Cairo (30°N , 31°E) relative to Rio de Janeiro (22°S , 43°W).

CHALLENGING

9 Hobart is located at (42°S , 147°E) and Port Moresby is located at (9°S , 147°E).

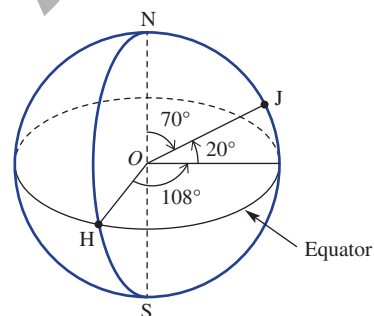
- a Describe the relative positions of these cities on Earth's surface.
- b What are the coordinates of a point halfway between these cities?
- c What are the coordinates of a point 65° due east of Hobart?
- d Describe the position of Hobart relative to Port Moresby.

10 Kingston is located at (17°N , 76°W). Jakarta is located at (6°S , 106°E).

- a What is the difference in latitude between Kingston and Jakarta?
- b What is the difference in longitude between Kingston and Jakarta?
- c What are the coordinates of a point 20° due south of Kingston?
- d What are the coordinates of a point 80° due east of Jakarta?
- e Describe the position of Jakarta relative to Kingston.

11 In this diagram of Earth, O represents the centre, and H is a point located on both the equator and the Greenwich meridian.

- a Find the coordinate of the point J .
- b Find the coordinate of the point that is:
 - i 12° due north of point J
 - ii 45° due south of point J
 - iii 55° due west of point J
 - iv 150° due west of point J
 - v 102° due east of point J .
- c Describe the position of point J relative to point H .
- d Toronto has coordinates (44°N , 80°W). Describe the position of Toronto relative to point J on the diagram.



Communicate your reasoning, understanding and fluency

- 12 The Galapagos Islands and Singapore are located at the same latitude. The longitude of the Galapagos Islands is 92°W . The longitude of Singapore is 104°E . You need to get from Galapagos Islands to Singapore as quickly as possible. Would you go east or west? Explain why.

7C Time zones

Learning intentions

- To use and understand Coordinated Universal Time (UTC)
- To use and understand time zones to calculate local times and time differences

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about time zones regularly appear in HSC Examinations

A time zone is a region of Earth that has a uniform standard time or local time. There are 24 time zones (from -12 to $+12$) roughly divided by longitude (see the table of international time zones on the next page). The time zone number indicates how many hours to add or subtract from the Coordinated Universal Time (UTC). The UTC reflects the mean solar time along Earth's prime meridian and is still widely known as the Greenwich Mean Time (GMT). The International Date Line is an imaginary line through the Pacific Ocean that corresponds to 180° longitude. It runs from the North Pole to the South Pole and defines the change of one calendar day to the next.

Time zone

Everywhere within a time zone has the same time.

- Positive time zone number – ahead of UTC or GMT. Add the time.
- Negative time zone number – behind of UTC or GMT. Subtract the time.

Australian time zones

Australia has three time zones called the Eastern Standard Time (+10), Central Standard Time (+9.5) and Western Standard Time (+8). On the map, DST stands for Daylight Saving Time.



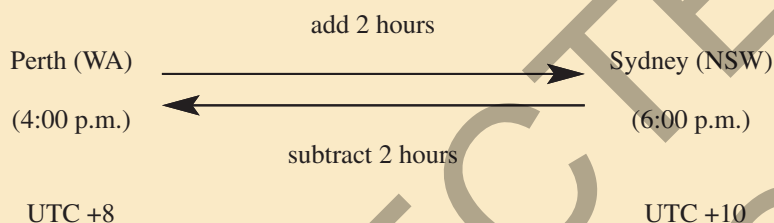
Table of international time zones

The standard time zones and the major places in each zone are shown in the table. This table does not take into account time differences as a result of daylight saving time.

Time zone	Major places
–12	Eniwetok
–11	Midway Island, American Samoa
–10	Cook Is, Honolulu, Tahiti
–9	Alaska
–8	Anchorage, Los Angeles, San Francisco, Seattle, Vancouver
–7	Alberta, Denver, Edmonton, Phoenix, Salt Lake City, Santa Fe
–6	Chicago, Dallas, El Paso, Guatemala, New Orleans, Mexico City
–5	Boston, Kingston, Lima, Miami, Montreal, New York, Quebec
–4	Caracas, Labrador, La Paz, Santiago
–3	Brasilia, Buenos Aires, Georgetown, Montevideo, Rio de Janeiro
–2	Mid-Atlantic
–1	Azores, Cape Verde Is
0	Belfast, Dublin, Edinburgh, Iceland, Lisbon, London, Morocco
+1	Amsterdam, Berlin, Madrid, Prague, Paris, Rome, Vienna
+2	Athens, Cairo, Cape Town, Greece, Harare, Helsinki, Istanbul
+3	Baghdad, Beirut, Kuwait, Moscow, Nairobi, Tehran
+4	Abu Dhabi, Baku, Kabul, Kazan, Muscat, Tehran, Volgograd
+5	Colombo, Islamabad, Kolkata, Madras, New Delhi
+6	Dhaka, Kathmandu, Colombo, Sri Lanka
+7	Bangkok, Hanoi, Jakarta, Phnom Penh
+8	Beijing, Hong Kong, Kuala Lumpur, Manila, Perth, Singapore
+9	Osaka, Seoul, Sapporo, Seoul, Tokyo
+9.5	Adelaide, Darwin, Alice Springs, Broken Hill
+10	Brisbane, Canberra, Hobart, Melbourne, Port Moresby, Sydney
+10.5	Lord Howe Is
+11	Magadan, New Caledonia, Solomon Is, Vanuatu
+12	Auckland, Christchurch, Fiji, Marshall Is, Wellington, Suva
+13	Samoa

Time zones

- Lines of longitude are generally used as the basis of time zones.
- Time is based on the time in a place called Greenwich, United Kingdom, and this is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT).
- Australia has three time zones:
 - ▷ Eastern Standard Time (EST), which is UTC plus 10 hours.
 - ▷ Central Standard Time (CST), which is UTC plus 9.5 hours.
 - ▷ Western Standard Time (WST), which is UTC plus 8 hours.



- Daylight saving is the practice of moving clocks forward, usually by one hour.
- The International Date Line separates one calendar day from the next, passing through the mid-Pacific Ocean with a longitude of 180° .
- The map on the next page shows the time zones all around the world.



Example 7 Solving problems using a time zone

Hobart is located in the Australian Eastern Standard Time zone (+10) and Greenwich is at the Coordinated Universal Time (UTC) in time zone (0). What is the time in:

- Hobart if it is 8:00 p.m. in Greenwich?
- Greenwich if it is 5:30 a.m. in Hobart?

Solution

- Hobart = 8:00 p.m. + 10 hours
= 6:00 a.m.
- Greenwich = 5:30 a.m. - 10 hours
= 7:30 p.m.

Explanation

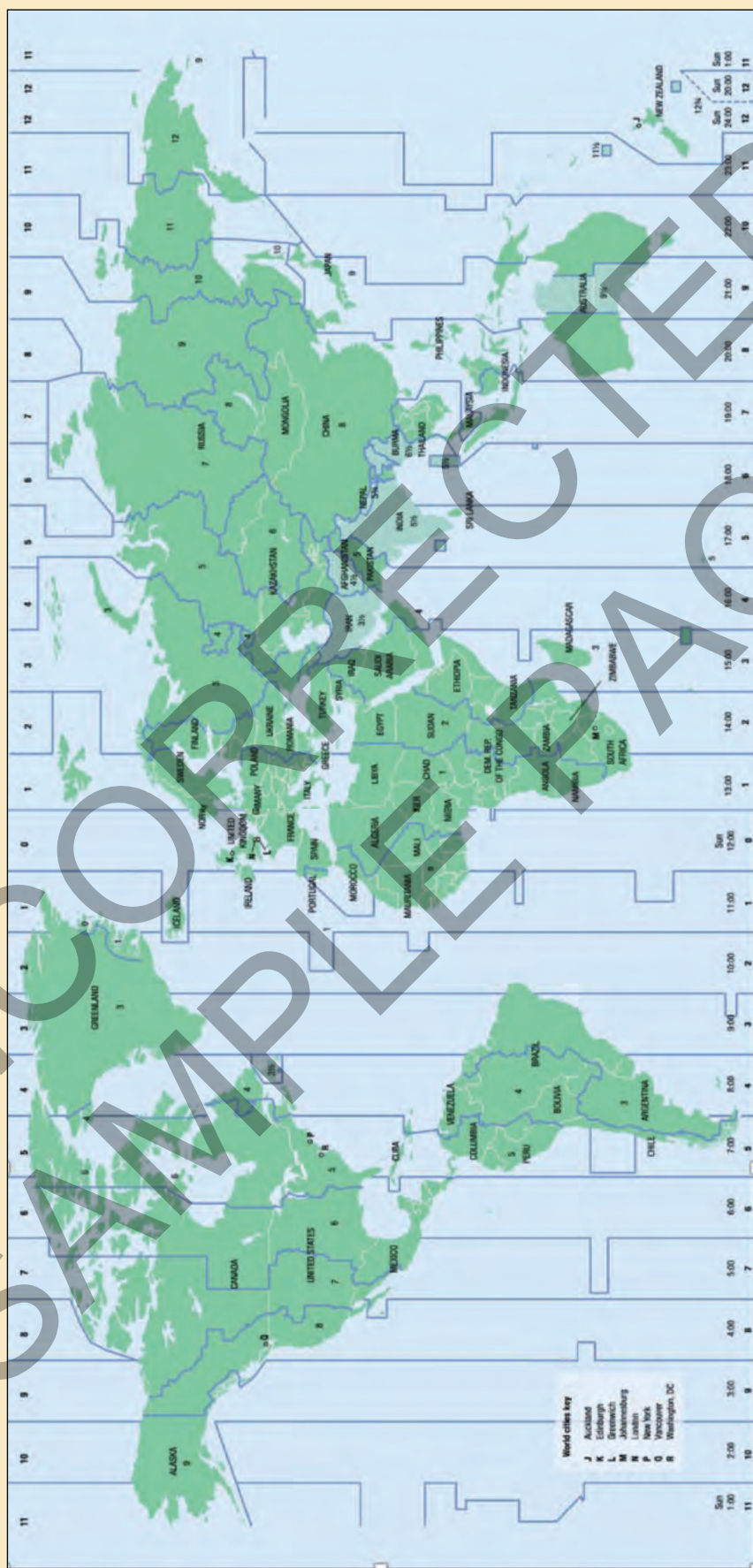
Hobart is in the EST (+10) zone and is 10 hours ahead of UTC at Greenwich. Add the time.

Greenwich is at UTC and is 10 hours behind Hobart, which is in EST (+10). Subtract the time.

Now you try

Perth is located in the Australian Western Standard Time zone (+8) and Greenwich is at the Coordinated Universal Time (UTC) in time zone (0). What is the time in:

- Perth if it is 11:00 a.m. in Greenwich?
- Greenwich if it is 9:30 a.m. in Perth?



Daylight saving time

Daylight Saving Time (DST) is the practice of advancing clocks one hour during the warmer months of the year. There are many countries that adopt daylight savings time to take advantage of the increased hours of sunlight. In Australia, daylight saving is observed in New South Wales, Victoria, South Australia, Tasmania and the Australian Capital Territory. Daylight saving is not observed in Queensland, the Northern Territory or Western Australia. Daylight saving time in Australia begins on the first Sunday in October, when clocks are put forward one hour. It ends on the first Sunday in April, when clocks are put back one hour.

Daylight saving time

For daylight saving time, add one hour.



Example 8 Solving problems using daylight saving time

Jack, in Sydney, wants to phone his mother in Perth. Sydney is located in UTC (+10) and Perth is in UTC (+8). Daylight saving time is operating in Sydney. When should Jack ring to reach his mother at 6:00 p.m. in Perth?

Solution

$$\begin{aligned}\text{Sydney} &= 6:00 \text{ a.m.} + 2 \text{ hours} \\ &= 8:00 \text{ p.m.}\end{aligned}$$

$$\begin{aligned}\text{Sydney} &= 8:00 \text{ p.m.} + 1 \text{ hour} \\ &= 9:00 \text{ p.m.}\end{aligned}$$

Jack should ring when it is 9:00 p.m. in Sydney.

Explanation

Sydney is in UTC (+10) and 2 hours ahead of Perth (+8). Add the time.

Sydney is in daylight saving time, so add one hour.

Write answer in words.

Now you try

Jill, in Melbourne, wants to phone her mother in Darwin. Melbourne is located in UTC (+10) and Darwin is in UTC (+9.5). Daylight saving time is operating in Melbourne.

When should Jill ring to reach her mother at 3:30 p.m. in Darwin?

Exercise 7C

BUILDING

- 1 Copy and complete this table to show the equivalent times in three Australian time zones.

WST (+8)	CST (+9.5)	EST (+10)
1:00 p.m.		
	6:30 a.m.	
		8:20 p.m.
4:19 a.m.		
	Midnight	
		9:12 a.m.

Example 7

- 2 What is the time in the following places if it is 6:30 p.m. in Sydney EST (+10)?

- a** Perth (WST+8) **b** Darwin (CST+9.5)
c Adelaide (CST+9.5) **d** Melbourne (EST+10)

3 What is the time in the following places if it is 9:40 a.m. in Alice Springs (CST +9.5)?

- a** Fremantle (WST+8) **b** Wollongong (EST+10)
c Port Adelaide (CST+9.5) **d** Hobart (EST+10)

4 What is the time in the following places if it is 2:00 p.m. in Greenwich?

- a** Athens (UTC +2)
b Mexico city (UTC –6)
c Singapore (UTC +8)
d Santiago (UTC –4)

5 What is the time in the following places if it is 1:00 a.m. Sunday in Greenwich?

- a** Kuwait (UTC +3)
b Alaska (UTC –9)
c Brisbane (UTC +10)
d Cape Verde Is (UTC –1)

6 Find the time difference between the following cities.

- a** Vanuatu (UTC+11) and Kolkata (UTC+5)
b Denver (UTC–7) and Montevideo (UTC–3)
c Bangkok (UTC+7) and Boston (UTC–5)
d Phoenix (UTC–7) and Madrid (UTC+1)
e Baghdad (UTC+3) and Christchurch (UTC+12)
f Chicago (UTC–6) and Cape Verde Is (UTC–1)
g Abu Dhabi (UTC + 4) and Edmonton (UTC–7)
h Samoa (UTC–11) and Rome (UTC+1)



This line and sculpture mark the Greenwich Meridian.

DEVELOPING

7 What is the time in the following cities if it is 8:00 a.m. in Cairo (UTC+2)?

- a** Abu Dhabi (UTC+4) **b** Brasilia (UTC–3)
c Berlin (UTC+1) **d** Edmonton (UTC–7)

8 What is the time in the following cities if it is 11:00 p.m. Monday in Dallas (UTC–7)?

- a** Honolulu (UTC–10) **b** Harare (UTC+2)
c New York (UTC–7) **d** Broken Hill (UTC+9.5)

9 Find the time in the following Australian cities if it is 4:00 p.m. in Sydney (UTC+10).

- a** Brisbane (UTC+10) **b** Perth (UTC+8)
c Darwin (UTC+9.5) **d** Lord Howe Island (UTC+10.5)

10 The time in Greenwich on the prime meridian is midnight on Monday 1 November. What is the time and day in Seattle (UTC–8)?

- 11** Newcastle is part of the Eastern Standard Time zone and is 10 hours ahead of UTC. What is the standard time in Greenwich when it is 5:00 p.m. Friday in Newcastle?

Example 8

- 12** Find the time in the following cities if it is 11:30 a.m. daylight saving time in Sydney (UTC+10).

a Perth (UTC+8)	b Brisbane (UTC+10)
c Malta (UTC+1)	d Suva (UTC+12)
e Vancouver (UTC−8)	f Belfast (UTC 0)

CHALLENGING

- 13** Sydney is 10 hours ahead of UTC and Seattle is 8 hours behind UTC.
- a** What is the time difference between the two cities?
- b** What is the time difference if Sydney is on daylight saving time and Seattle is on standard time?

- 14** What is the time difference between Tasmania and Sri Lanka if Tasmania is on daylight saving time and Sri Lanka is on standard time? Tasmania is 10 hours ahead of UTC and Sri Lanka is 5 hours ahead of UTC.

- 15** People in New South Wales (UTC+10) turn their clocks forwards one hour for daylight saving. Western Australia (UTC+8) does not adjust its clocks for daylight saving.

- a** In summer, what is the time in New South Wales if it is 3:10 p.m. in Western Australia?
- b** In summer, what is the time in Western Australia if it is 10:30 a.m. in New South Wales?

- 16** Daylight saving operates in the summer months in Victoria and New South Wales, but not in the Northern Territory, Queensland nor Western Australia.

Melbourne (Vic), Sydney (NSW) and Brisbane (Qld) are part of the Australian Eastern Standard Time (UTC+10).

Darwin (NT) is part of the Australian Central Standard Time (UTC+9.5).

Perth (WA) is part of Australian Western Standard Time (UTC+8).

Observing daylight saving time:

- a** what is the time in Sydney if it is 3:15 p.m. in Brisbane?
- b** what is the time in Darwin if it is 1:15 a.m. in Melbourne?
- c** what is the time in Perth if it is 6:20 p.m. in Brisbane?
- d** if it is 10:35 a.m. in Melbourne, what is the time in Perth?

Communicate your reasoning, understanding and fluency

- 17** Ariadne is on a holiday in Toronto (UTC−5). She wants to stream the Matildas' semi-final World Cup match played in Sydney (UTC+10) on Friday 16 August at 8:00 p.m.

Toronto is subject to daylight saving time in August, but Sydney is not.

What time and day in Toronto should Ariadne start streaming the match? Use calculations to support your answer.

7D Time difference using time zones

Learning intentions

- To use and understand Coordinated Universal Time (UTC)
- To solve practical problems using Coordinated Universal Time (UTC)

Past, present and future learning

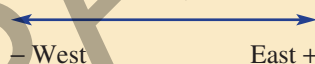
- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about time zones regularly appear in HSC Examinations

The international time zones can be used to calculate the local time in two locations. First, find the time difference between the two locations by subtracting the time zones. Then draw an east–west time line and place the locations on the time line. To find the local time you need to add or subtract the time difference. If the required time is east of the given time then add the time difference. Alternatively, if the required time is west of the given time then subtract the time difference.

Time difference and local time

To calculate the local time:

- Find the time difference between the two locations.
- Place the locations on an east–west time line.
- Find the local time by adding or subtracting the time difference.
 - East of location with the given time (moving west to east). Add the time difference.
 - West of location with the given time (moving east to west). Subtract the time difference.



Example 9 Finding the time difference and local time

Patrick is working in Seoul (UTC +9) and needs to travel to Chicago (UTC –6).

- What is the time difference between these places?
- What is the local time in Chicago if it is 5:00 p.m. in Seoul?
- A flight leaves Seoul at 5:00 p.m. and flies non-stop to Chicago. The flight takes 14 hours. What is the time in Chicago when the flight arrives?

Solution

- Time difference = $+9 - (-6)$
= 15 h
- Chicago

Seoul

Time in Chicago = 5:00 p.m. – 15 h
= 2:00 a.m.

Time in Chicago is 2:00 a.m.

- Time in Chicago = 2:00 a.m. + 14 h
= 4:00 p.m.

Time in Chicago is 4:00 p.m.

Explanation

Time difference is determined by the difference in longitude. Add the longitude of Chicago to that of Seoul.

Draw an east–west time line.

Mark Chicago (UTC –6) on the left-hand side and Seoul (UTC +9) on the right-hand side.

Require the time in Chicago. It is west of the location with the time information.

Subtract the time difference.

Write the answer in words.

Flight takes 14 hours. Add the time for the flight to the local time of the destination.

Write the answer in words.

Now you try

Deanna is working in Sydney (UTC +10) and needs to travel to New York (UTC –5).

- What is the time difference between these places?
- What is the local time in New York if it is 8:00 p.m. in Sydney?
- A flight leaves Sydney at 8:00 p.m. and flies non-stop to New York. The flight takes 18 hours. What is the time in New York when the flight arrives?

**Example 10 Finding the time difference and local time**

Moana is planning a trip from Suva in Fiji (UTC +12) to Miami in Florida (UTC –5).



- What is the time difference between these cities?
- What is the time and day in Miami if it is 10:10 a.m. on Wednesday 6 January in Suva?
- Moana boards a plane at 2:30 p.m. on 7 April Suva time and the flight lasts 16 hours. What is the local time and date in Miami when she arrives?

Solution

- a** Miami Suva
- ← West East →

$$\begin{aligned}\text{Time difference} &= 12 + 5 \\ &= 17 \text{ h}\end{aligned}$$

- b** Time in Miami = 10:10 a.m. – 17 h
= 5:10 p.m.

Miami's date is Tuesday 5 January and the time is 5:10 p.m.

- c** Time in Miami = 2:30 p.m. – 17 h + 16 h
= 1:30 p.m.

Local time in Miami is 1:30 p.m. on 7 April.

Explanation

Draw an east–west time line.

Mark Miami (–5) on the left-hand side and Suva (+12) on the right-hand side.

Time difference is determined by adding the time ahead of UTC (+12) and the time behind UTC (–5)

Require the time in Miami. It is west of the location with the time information, so subtract the time.

Subtracting 17 hours from 10:10 p.m. results in it being the previous day (5 January).

Write the answer in words.

Subtract the time difference (17 h) to find the local time in Miami.

Flight takes 16 hours. Add the time for the flight to the local time at the destination.

Write the answer in words.

continued ...

Now you try

Ava is planning a trip from Abu Dhabi (UTC +4) to Toronto (UTC –5).

- a** What is the time difference between these cities?
- b** What is the time and day in Toronto if it is 6:30 a.m. on Tuesday 13 February in Abu Dhabi?
- c** Ava boards a plane at 4:15 p.m. on 26 May from Abu Dhabi and the flight lasts 14 hours. What is the local time and date in Toronto when she arrives?

Exercise 7D**BUILDING****Example 9**

- 1** Lucas lives in Strasbourg and his sister Kayla lives in Vancouver.
 - a** Calculate the time difference between Strasbourg (UTC +1) and Vancouver (UTC –8).
 - b** What is the time in Strasbourg if it is 3:00 p.m. on a Monday afternoon in Vancouver?
- 2** It is 9:00 p.m. on Friday 30 March in Greenwich.
 - a** What is the date and local time in New Orleans (UTC –8)?
 - b** What is the date and local time in Canberra (UTC +10)?
- 3** Julia lives in Sydney (UTC +10) and her son Tyler lives in Los Angeles (UTC –8).
 - a** What is the time difference between Sydney and Los Angeles?
 - b** Julia would like to telephone Tyler at 0000 Monday for New Year's Eve. At what time and on what day in Sydney should Julia phone?
- 4** Patrick lives in Melbourne (UTC +10). He wants to watch a football game being played in Brazil (UTC –2) starting at 7:00 p.m. on Saturday. What is the time and day in Melbourne when the game starts?
- 5** A One Day International cricket game between Australia and the West Indies starts at 11:00 a.m. on 26 November, in Brisbane. Blake lives in Kingston, Jamaica, and wants to watch the game on television. Brisbane is 10 hours ahead of UTC and Kingston is 5 hours behind UTC. At what time and date should Blake switch on his television to watch the start of the game? Queensland does not have daylight saving.



- 6** Jasmine lives in Bathurst (UTC +11, as Bathurst is on daylight saving time), and wants to telephone her mother in London (UTC 0) so that she receives the call at 8:00 a.m. on Christmas Day. On what day and at what time in Bathurst should Jasmine make the call to London?
- 7** Andrew is in Moscow (UTC +3) and wants to record the USA Open tennis final. The tennis match is due to begin at 2:00 p.m. on Sunday in New York (UTC –5). On what day and at what time will Andrew need to start recording?

DEVELOPING

Example 10

- 8** A plane leaves Melbourne (UTC +10) and flies to Paris (UTC +1). The flight takes 23 hours. The plane leaves Melbourne at 4:00 p.m. Thursday. On what day and at what time does the plane arrive in Paris?
- 9** Na calls a friend in Beijing from Los Angeles. Beijing is 8 hours ahead of UTC and Los Angeles is 8 hours behind UTC. It is 8:15 p.m. in Los Angeles and they talk for 30 minutes. What is the time in Beijing at the end of the phone call?
- 10** Adelaide (UTC +9.5) is located in the Central Standard Time zone. Melbourne (UTC +10) is located in the Australian Eastern Standard Time zone.
- a** What is the time in Adelaide if it is 5:45 a.m. in Melbourne?
- b** What is the time in Melbourne if it is 2:10 p.m. in Adelaide?
- c** At what time will a train arrive in Adelaide if it departs Melbourne at 7:00 a.m. and travel time is 10 hours?
- 11** Andrew is planning a trip from Townsville (UTC +9.5) to Hong Kong (UTC +8).
- a** What is the time and day in Hong Kong if it is 7:30 a.m. on 5 June in Townsville?
- b** A flight leaves Townsville at 7:30 a.m. and flies non-stop to Hong Kong. The flight is in the air for 7 hours. What time and day is it in Hong Kong when the flight arrives?
- 12** A boat departs Brisbane on Tuesday 12 October at 12:00 p.m. and takes $15\frac{1}{2}$ days to travel west to Honolulu. Brisbane is 10 hours ahead of UTC and Honolulu is 10 hours behind UTC.
- a** What is the local time and day in Honolulu when the boat departs Brisbane?
- b** What is the local time and day when the boat arrives in Honolulu?



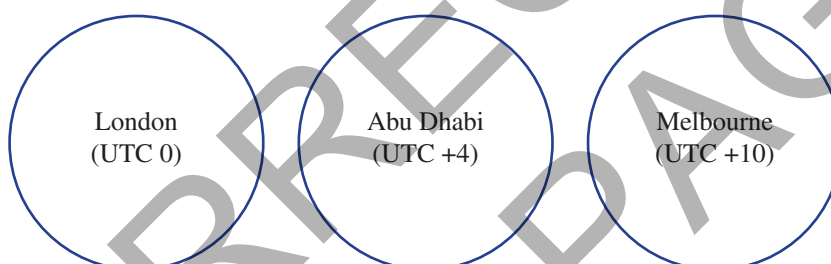
CHALLENGING

- 13** A plane leaves Sydney at 2:00 a.m. Tuesday 17 May for an 8 hour flight to Kabul. Sydney is 10 hours ahead of UTC and Kabul is 4 hours ahead of UTC.
- a** What is the local time and day when the plane arrives in Kabul?
- b** The return flight leaves Kabul at 2:00 p.m. on the next Friday. What is the time and day when the plane arrives in Sydney?
- 14** The Universal Coordinated Time (UTC) of Samoa is -11 hours. When it is 9:00 p.m. Wednesday in Samoa, it is 3:00 p.m. Thursday in Bangkok. What is the UTC of Bangkok?
- 15** A plane travels from Paris (UTC +1) to San Francisco (UTC -8). The flight from Paris to San Francisco takes 11 hours 30 minutes. If the flight leaves Paris on Wednesday at 1420, what day and time will the flight arrive in San Francisco?

- 16** Leanne is flying from Seoul (UTC +9) to Miami (UTC –5). The plane left Seoul 9:00 a.m. Tuesday (Seoul time).
- a** What is the time in Miami when the plane left Seoul?
- b** If the plane arrived in Miami 7:50 a.m. on Tuesday. What is the total flying time of the plane?
- 17** Nita is flying from Sydney (UTC +10) to Bangkok (UTC +7). Her plane leaves Sydney on Sunday 11:00 p.m. (Sydney time), stopped for 4 hours in Jakarta and arrives in Bangkok 9:50 a.m. on Monday (Bangkok time). What is the plane's total flying time?
- 18** A flight leaves Chicago (UTC –6) and lands in Johannesburg (UTC +2). The flight from Chicago to Johannesburg takes 18 hours. If the flight arrives in Johannesburg on Saturday 23 November at 10:00 a.m., what time and day did the plane leave Chicago?

Communicate your reasoning, understanding and fluency

- 19** A multinational company has offices in London, Abu Dhabi and Melbourne.



- a** What is the time difference between Melbourne and Abu Dhabi in July?
- b** What is the time difference between Melbourne and Abu Dhabi in December?
- c** You need to organise a 1-hour internet meeting from Melbourne with participants based in London and Abu Dhabi. Daylight saving time is operating in London only. People in London work from 8:00 a.m. until 5:00 p.m. People in Sydney work from 9:00 a.m. until 6:00 p.m. Explain the time you could organise the meeting from Melbourne. Include the local times in each city in your answer.
- d** Total flight time (including connections) from Melbourne to London is about 25 hours. Is it possible to depart from Melbourne and arrive in London on the same day? What about the way back? Explain your answers.

7E Time difference using longitudes

Learning intentions

- To calculate the difference in longitude between two locations
- To be able to use two longitudes to calculate time difference
- To solve practical time problems using longitude

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Time problems involving longitude regularly appear in HSC Examinations

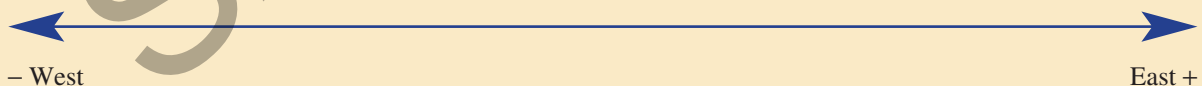
The time of day is dependent on the rotation of Earth about the north-south axis. A complete revolution (360° in longitude) of Earth takes 24 hours. This means that 15° of longitude takes 1 hour or 1° of longitude is equivalent to 4 minutes. Places on Earth with the same longitude will have the same local time unless they are in different time zones.

Time difference and local time

- **A 360° rotation in longitude on Earth takes 24 hours.** (This is because Earth completes a full rotation in 24 hours.)
- **15° of longitude corresponds to a 1-hour time difference.**
(Since $360^\circ \div 24 \text{ hours} = 15^\circ \text{ per hour}$)
- **1° of longitude corresponds to a 4-minute time difference.**
(Since $60 \text{ minutes} \div 15^\circ = 4 \text{ minutes per degree}$)
- You can approximate the time difference between two locations by calculating the difference in their longitudes. However, the actual time difference is determined by their positions relative to Coordinated Universal Time (UTC) on the globe, so the estimate may be slightly inaccurate.
- Latitudes are not useful.

To calculate the local time:

- 1 Find the time difference by multiplying the difference in longitude (degrees) by 4 minutes. (Convert to hours and minutes using DMS on the scientific calculator if necessary.)
- 2 Place the locations on an east-west time line.
- 3 Find the local time by adding or subtracting the time difference.
 - East of location with the given time – ADD the time difference
 - West of location with the given time – SUBTRACT the time difference





Example 11 Find the time difference and local time

Auckland is located at (37°S, 175°E) and Wollongong is located at (34°S, 150°E).

- Calculate the difference in longitude between these two cities.
- What is the time difference between these two cities?
- What is the time in Auckland if it is 2:00 p.m. in Wollongong?
- What is the time in Wollongong if it is 9:00 a.m. in Auckland?
- How does this compare to the UTC time difference given that Auckland is UTC +12 and Wollongong is UTC +10?

Solution

a Longitude difference = $175 - 150$
 $= 25^\circ$

b Time difference = $25^\circ \times 4 \text{ min}$
 $= 100 \text{ min} \div 60$
 $= 1 \text{ h } 40 \text{ min}$

c Auckland is 2:00 p.m. + 1 h 40 min
 $= 3:40 \text{ p.m.}$
 It is 3:40 p.m. in Auckland.

d Wollongong is 9:00 a.m. - 1 h 40 min
 $= 7:20 \text{ a.m.}$
 It is 7:20 a.m. in Wollongong.

e 20 minutes more

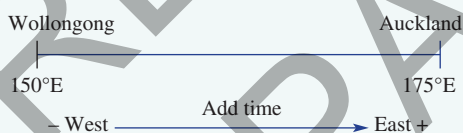
Explanation

Time difference is determined by the difference in longitude. Both locations are on the east side, so subtract.

Multiply the longitude difference by 4 minutes to find the time difference.

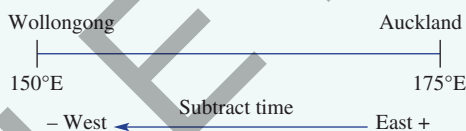
$100 \text{ minutes} \div 60 = \text{" "}$ to get hours and minutes

Use the " " button on the calculator to convert 100 minutes to hours and minutes.



Auckland is east of Wollongong.

ADD the time difference.



Wollongong is west of Auckland.

SUBTRACT the time difference.

Using UTC, it is a 2 hour difference, 20 minutes more.

Now you try

Christchurch is located at (44°S, 173°E) and Sydney is located at (34°S, 151°E).

- Calculate the difference in longitude between these two cities.
- What is the time difference between these two cities?
- What is the time in Christchurch if it is 3:30 p.m. in Sydney?
- What is the time in Sydney if it is 11:00 a.m. in Christchurch?
- How does this compare to the actual UTC time difference given that Sydney is UTC +10 and Christchurch is UTC +12?



Example 12 Find the time difference and local time

Chicago is located at (42°N, 88°W) and Seoul is located at (38°N, 127°E).

- What is the local time in Chicago if it is 5:00 p.m. in Seoul?
- A flight leaves Seoul at 5:00 p.m. and flies 14 hours non-stop to Chicago. What is the local time in Chicago when the flight arrives?

Solution

a Longitude difference = $88 + 127$
 $= 215^\circ$

Time difference = $215^\circ \times 4 \text{ minutes}$
 $= 860 \text{ minutes} \div 60$
 $= 14 \text{ h } 20 \text{ min}$

Time in Chicago = $5:00 \text{ p.m.} - 14 \text{ h } 20 \text{ min}$
 $= 2:40 \text{ a.m.}$

It is 2:40 a.m. in Chicago.

b Time in Chicago = $2:40 \text{ a.m.} + \text{flight time}$
 $= 2:40 \text{ a.m.} + 14\text{-hour flight}$
 $= 4:40 \text{ p.m.}$
 It is 4:40 p.m. when the flight lands in Chicago.

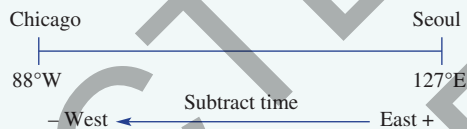
Explanation

Time difference is determined by the difference in longitude.

One location is east, the other west, so ADD longitude.

Multiply the longitude difference by 4 minutes to find the time difference.

$860 \text{ minutes} \div 60 = \boxed{''}$ to get hours and minutes
 Use the $\boxed{''}$ button on the calculator to convert 860 minutes to hours and minutes.



The flight takes 14 hours.

Add the flight time to the local time of the destination.

Now you try

New York is located at (40°N , 74°W) and Singapore is located at (1°N , 104°E).

- a** What is the local time in New York if it is 3:00 p.m. in Singapore?
b A flight leaves Singapore at 3:00 p.m. and flies 16 hours non-stop to New York. What is the local time in New York when the flight arrives?

Exercise 7E**BUILDING**

- What is the time:
 - 11 hours before 2:00 a.m.?
 - 5 hours after 7:00 p.m.?
 - 1 hour 30 minutes after 6:30 a.m.?
 - 2 hours 15 minutes before 3:30 p.m.?
 - 4 hours and 11 minutes after 1:45 p.m.?
 - 12 hours and 50 minutes after 6:23 a.m.?
- Use the DMS button/function on your calculator to convert the following to hours and minutes.
 - 100 minutes
 - 176 minutes
 - 344 minutes
 - 605 minutes
 - 231 minutes
 - 792 minutes
- Calculate the difference in longitude between the following points.
 - (72°S , 54°E) and (16°S , 14°E)
 - (25°N , 55°E) and (35°N , 104°E)
 - (40°N , 75°W) and (49°N , 123°W)
 - (15°S , 5°W) and (80°N , 68°W)
 - (25°N , 30°W) and (42°S , 10°E)
 - (61°S , 29°E) and (28°N , 7°W)

Example 11

- 4** Find the time difference between two cities given the following longitude difference. Express your answer in hours and minutes.
- | | | | |
|---------------|--------------|---------------|---------------|
| a 15° | b 90° | c 23° | d 200° |
| e 155° | f 35° | g 185° | h 120° |
- 5** What is the time difference between the following points? Express your answer in hours and minutes.
- | | |
|---|---|
| a (20°N, 40°E) and (12°S, 80°E) | b (25°N, 65°E) and (35°N, 104°E) |
| c (45°S, 20°W) and (45°N, 100°W) | d (15°S, 5°W) and (80°N, 78°W) |
| e (30°N, 23°W) and (40°S, 38°E) | f (49°S, 0°) and (4°N, 60°W) |
| g (25°N, 150°W) and (42°S, 10°E) | h (61°S, 85°E) and (28°N, 7°W) |
- 6** Find the difference in local time between:
- Port Moresby (9°S, 147°E) and Denpasar (9°S, 115°E)
 - Alice Springs (24°S, 134°E) and Greenwich
 - Nairobi (1°S, 37°E) and Rome (42°N, 12°E)
 - Nairobi (1°S, 37°E) and Shanghai (31°N, 121°W)
 - Strasbourg (49°N, 8°E) and Vancouver (49°N, 123°W)
 - Rome (42°N, 12°E) and Singapore (1°N, 105°E)
 - Vancouver (49°N, 123°W) and Honolulu (21°N, 158°W).

DEVELOPING

- 7** When it is noon in Greenwich, what is the local time in:
- | | |
|---------------------------------|----------------------------------|
| a Rome (42°N, 12°E)? | b Singapore (1°N, 105°E)? |
| c New York (40°N, 75°W)? | d Chicago (42°N, 88°W)? |
| e Sydney (34°S, 151°E)? | f Honolulu (21°N, 158°W)? |
- 8** When it is 2:00 p.m. in Santorini (36°N, 25°E), what is the local time in:
- | | |
|----------------------------------|-------------------------------------|
| a Dubai (25°N, 55°E)? | b Singapore (1°N, 105°E)? |
| c Auckland (37°S, 175°E)? | d Rome (42°N, 12°E)? |
| e Chicago (42°N, 88°W)? | f Los Angeles (34°N, 120°W)? |
- 9** When it is 11:00 a.m. in New York (40°N, 75°W), what is the local time in:
- | | |
|----------------------------------|-----------------------------------|
| a Greenwich? | b Malta (35°N, 14°E)? |
| c Dubai (25°N, 55°E)? | d Paris (49°N, 2°E)? |
| e Honolulu (21°N, 158°W)? | f Vancouver (49°N, 123°W)? |

CHALLENGING

- 10** Julia lives in Sydney (34°N, 151°E) and her brother Tyler lives in Los Angeles (34°N, 120°W).
- What is the time difference between Sydney and Los Angeles?
 - Julia would like to telephone Tyler at midnight on Sunday for New Years Eve. At what time and what day in Sydney should Julia call?
 - How does this time difference compare to the actual UTC time difference given that Sydney is UTC +10 and Los Angeles is UTC -7?

- 11** New York has coordinates (40°N , 75°W) and the coordinates for Naples are (40°N , 14°E).
- What is the time difference between New York and Naples?
 - It is 10:30 p.m. on Friday in Naples. What is the time and day in New York?
- 12** Patrick lives in Melbourne (38°S , 145°E). He wants to watch a football game being played in Brazil (13°S , 50°W) starting at 7:00 p.m. on Saturday. What is the time and day in Melbourne when the game starts?
- Example 12** **13** Singapore is located at (1°N , 105°E) and Canberra is located at (35°S , 149°E).
- Show that there is a 3-hour time difference between the two cities.
 - Harry lives in Canberra and wants to ring a friend in Singapore. What is the time and day in Singapore if it is 11:00 p.m. Friday in Canberra?
 - Harry's friend in Singapore sent him a parcel which takes 7 hours of flight time to reach him. It was sent at 9:00 a.m. Friday, Singapore time. What was the time and day in Canberra when Harry received the parcel?
- 14** The Gold Coast is located at (28°S , 153°E) and Bali is located at (8°S , 116°E).
- What is the time in Bali if it is 9:00 a.m. in the Gold Coast?
 - A flight leaves the Gold Coast at 11:00 a.m. and flies non-stop to Bali. The flight is in the air for 6 hours. At approximately what time does the flight arrive in Bali?
 - Given that the Gold Coast is UTC +10 and Bali is UTC +8, how close was the approximation you found in part **b**?
- 15** The standard time in Dubbo is based on the 149°E meridian and the standard time in Fremantle is based on the 116°E meridian.
- Isabella is in Dubbo and rings a friend in Fremantle at 8:00 p.m. on a Saturday. What day and time is it in Fremantle?
 - Isabella caught a plane at 8:00 p.m. on a Saturday from Dubbo to Fremantle. The travel time for the trip was 7 hours and 30 minutes. What is the time when Isabella arrives in Fremantle?

Communicate your reasoning, understanding and fluency

- 16** Molly flew from Sydney (34°N , 151°E) to Chittagong (22°N , 91°E). Her plane left Sydney at 8:30 a.m. Thursday (Sydney time), stopped for 4 hours in Singapore and arrived in Chittagong at 1:00 p.m. Thursday (Chittagong time). What was the total flying time?
- 17** Lucy is sailing across the Pacific Ocean from a position of (0° , 130°W) to Tahiti at (18°S , 150°W). At 8:30 a.m. on Saturday 9 July, she calls her sister in Tahiti. What is the local time and date in Tahiti when her sister receives the call?
- 18** Using the internet, can you find a flight that leaves and arrives at the same time and the same day? Is the plane flying east or west?

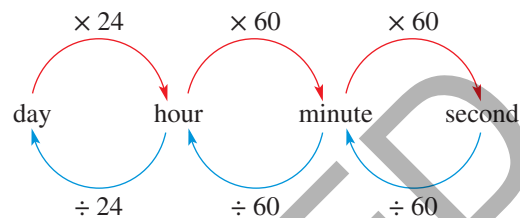
Key ideas and chapter summary

Units of time

1 min = 60 s

1 h = 60 min

1 day = 24 h

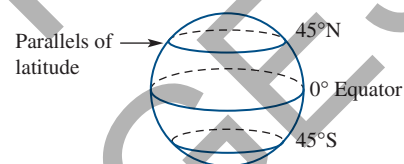


24-hour time

24-hour notation is given in the form 'hhmm'.
e.g. 1:10 a.m. is 0110 and 1:10 p.m. is 1310 in 24-hour time.

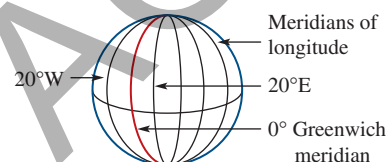
Latitude

Measures the angular distance north and south of the equator (0°)



Longitude

Measures the angular distance east and west of the Greenwich meridian (0°)



Time difference using time zones

To calculate the local time:

- 1 Find the time difference between the two locations.
- 2 Place the locations on an east–west time line.
- 3 Find the local time by adding or subtracting the time difference.
 - East of location with the given time (moving west to east): add the time difference.
 - West of location with the given time (moving east to west): subtract the time difference.

Time difference using longitudes

1° of longitude corresponds to a 4-minute time difference. To find the time difference between two locations, multiply the difference in longitude by 4 minutes.

Daylight Saving Time (DST)

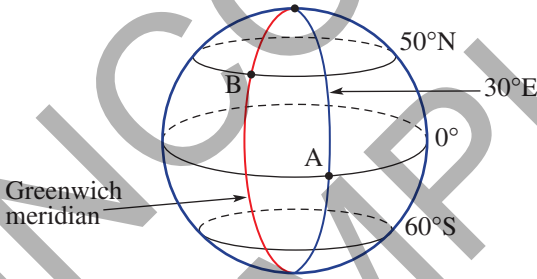
The practice of advancing clocks by 1 hour in the warmer months to take advantage of the longer daylight.

Chapter checklist with success criteria



A printable version of this checklist is available in the Interactive Textbook.



- 7A 1** I can convert between units of time. ☐
e.g. 720 minutes = ____ hours
- 7A 2** I can convert to hours, minutes and seconds. ☐
e.g. 6.85 hours = ____ hours ____ minutes
- 7A 3** I can calculate elapsed time. ☐
e.g. Calculate the elapsed time between 7:30 a.m. Monday and 11:30 a.m. the following Wednesday.
- 7A 4** I can add and subtract time. ☐
e.g. 10 h 12 min – 2 h 35 min = ____
- 7A 5** I can convert 12-hour time to 24-hour time. ☐
e.g. What is 6:20 p.m. in 24-hour time?
- 7A 6** I can convert 24-hour time to 12-hour time. ☐
e.g. What is 1440 in 12-hour time?
- 7B 7** I can find the latitude and longitude of a point on Earth's surface. ☐
e.g. A diagram of Earth's surface is shown.
- 
- Find the coordinate of a point 60° due south of point A.
- 7B 8** I can find the difference in longitude between two cities. ☐
e.g. What is the difference in longitude between Brisbane (27°S, 153°E) and Perth (31°S, 115°E)?
- 7B 9** I can find the position of one point relative to another point. ☐
e.g. Describe the position of Rome (42°N, 12°E) relative to Sydney (33°S, 151°E).
- 7C 10** I can find the time difference between locations using Coordinated Universal Time (UTC). ☐
e.g. Find the time difference between Vanuatu (UTC +11) and Montevideo (UTC –3).

- 7C 11** I can find the local time between locations using Coordinated Universal Time (UTC). ☐
e.g. If it is 4:00 p.m. in Sydney (UTC +10), what is the local time in New York (UTC -7)?
- 7C 12** I can find the local time between locations using Coordinated Universal Time (UTC) and daylight saving time. ☐
e.g. If it is 11:30 a.m. daylight saving time in Sydney (UTC +10), what is the local time in Suva, Fiji (UTC+12)?
- 7D 13** I can solve practical problems using Coordinated Universal Time. ☐
e.g. A plane leaves Melbourne (UTC +10) and flies to Paris (UTC +1). The flight takes 23 hours. The plane leaves Melbourne at 4:00 p.m. on Thursday. On what day and time does the plane arrive in Paris?
- 7E 14** I can calculate the difference in longitude between two locations. ☐
e.g. Calculate the difference in longitude between Auckland (37°S, 175°E) and Wollongong (34°S, 150°E).
- 7E 15** I can find the time difference using $1^\circ = 4$ minutes. ☐
e.g. Calculate the time difference between Auckland (37°S, 175°E) and Wollongong (34°S, 150°E).
- 7E 16** I can find the local time between locations using longitude. ☐
e.g. What is the time in Auckland (37°S, 175°E) if it is 2:00 p.m. in Wollongong (34°S, 150°E)?
- 7E 17** I can solve practical problems using longitude. ☐
e.g. A flight leaves Seoul (1°N, 104°E) at 3:00 p.m. and flies 16 hours non-stop to New York (40°N, 74°W). What is the local time in New York when the flight arrives?

Multiple-choice questions

- 7A 1** The school bus takes 49 minutes to get from Joseph's house to the school. If school starts at 8:30 a.m., what is the latest time Joseph can catch the bus to get to school on time?
A 7:19 a.m. **B** 7:41 a.m. **C** 7:51 a.m. **D** 9:19 a.m.
- 7A 2** A train leaves the station at 7:28 a.m. The journey to its final destination takes 6 hours and 54 minutes. What is the train's arrival time?
A 1:22 a.m. **B** 2:22 a.m. **C** 1:22 p.m. **D** 2:22 p.m.
- 7B 3** City A is at latitude 26°S and longitude 133°E . City B is 35° north of City A and 21° west of City A. What are the latitude and longitude of City B?
A (16°N , 112°E) **B** (16°N , 154°E) **C** (9°N , 112°E) **D** (9°N , 154°E)
- 7B 4** The coordinates of Dublin are (53°N , 6°W). What are the coordinates of Hamburg if it is 16° east of Dublin?
A (53°N , 10°E) **B** (53°N , 22°W) **C** (69°N , 6°W) **D** (37°N , 6°W)
- 7C 5** What is the time difference between Singapore (UTC +8) and Canberra (UTC +10)?
A 2 h **B** 3 h **C** 18 h **D** 80 h
- 7C 6** What is the time in St John's (UTC -3.5) if it is 4.00 a.m. in Dallas (UTC -6)?
A 1:30 a.m. **B** 2:00 a.m. **C** 6:30 a.m. **D** 7:00 a.m.
- 7C 7** The time in Greenwich on the prime meridian is 3:00 p.m. What is the standard time in Kuala Lumpur (UTC +8)?
A 11:00 p.m. **B** 6:00 a.m. **C** 11:00 a.m. **D** 6:00 p.m.
- 7C 8** When it is 4:30 p.m. daylight saving time in Adelaide (UTC +9.5), the local time in Brisbane (UTC +10), which does not have daylight saving time, is:
A 5:00 p.m. **B** 4:00 p.m. **C** 5:30 p.m. **D** 4:30 p.m.
- 7D 9** Perth, in Western Australia, is 8 hours ahead of Greenwich, in England. Athens, in Greece, is 2 hours ahead of Greenwich. What is the time in Athens when it is 1:00 p.m. in Perth?
A 3:00 a.m. **B** 7:00 a.m. **C** 7:00 p.m. **D** 11:00 p.m.
- 7E 10** The difference in local time between (43°N , 100°W) and (17°S , 15°W) is:
A 7 h 40 min **B** 5 h 40 min **C** 4 h **D** 1 h 44 min
- 7E 11** When it is noon in Greenwich, the local time in Vancouver (49°N , 123°W) is:
A 8:12 p.m. **B** 8:20 a.m. **C** 3:48 p.m. **D** 3:48 a.m.

Short-answer questions

7A 1 Complete the following.

a 120 min = s

b 3.5 days = h

c 7 h = min

d 630 min = h

e 24 000 s = min

f 120 h = days

g 1032 h = days

h 750 min = h

i 960 s = min

7A 2 What is the time 7 hours and 10 minutes after the following times?

a 0800

b 1325

c 2250

7A 3 Luke's football team starts its game at 10:15 a.m. The team plays two halves that are 25 minutes each and there is a 6-minute break at half time.

a Luke scores a goal 5 minutes after half time. At what time does he score his goal?

b At what time does the game finish?

7B 4 Karachi is located at (25°N, 67°E) and Salvador is located at (13°S, 39°W).

a Which city is closer to the Greenwich meridian?

b What is the difference in latitude between Karachi and Salvador?

c What is the difference in longitude between Karachi and Salvador?

7B 5 Monterrey is located at (26°N, 100°W) and Auckland is located at (37°S, 174°E).

a What is the difference in latitude between Monterrey and Auckland?

b What is the difference in longitude between Monterrey and Auckland?

c What are the coordinates of a point 50° due south of Monterrey?

d What are the coordinates of a point 50° due west of Auckland?

7C 6 What is the time in Boston (UTC −5) when it is 1:00 p.m. in Hanoi (UTC +7)?

7C 7 Perth (UTC +8) is located in the Western Standard Time zone. Brisbane (UTC +10) is located in the Eastern Standard Time zone.

a What is the time difference in hours between Perth and Brisbane?

b What is the time in Perth if it is 5:00 p.m. in Brisbane?

7D 8 A flight from Sydney (UTC +10) to Adelaide (UTC +9.5) takes two hours flying time.

a What is the time difference in minutes between Sydney and Adelaide?

b A plane leaves Sydney at 7:00 a.m. At what time will it arrive in Adelaide, taking into account the flying time?

7D 9 The Gold Coast's time zone is UTC +10 and Bali's time zone is UTC +8.

a What is the time in Bali if the time in the Gold Coast is 9:00 a.m.?

b A flight leaves the Gold Coast at 11:00 a.m. and flies non-stop to Bali. The flight is in the air for 3 hours. At what time does the flight arrive in Bali?

7E 10 When it is 1:30 p.m. in New York (40°N, 75°W), what is the local time in Seattle (47°N, 123°W)?

- 7E 11** Grace lives in Sydney (34°S, 151°E), and she needs to give an online presentation to her colleagues in Malta (36°N, 14°E) starting at 9:00 a.m. Monday, local time in Malta. What time and day in Sydney should Grace start her presentation?
- 7E 12** Julie lives in Rome (42°N, 12°E) and she wants to join the live online sale at her favourite shop in New York (40°N, 75°W), starting Friday 9:00 a.m., New York local time. What time and day in Rome can Julie join the live online sale in New York?

Extended-response questions

- 7D 1** Fremantle (UTC +8) is located in the Western Standard Time zone. Dubbo (UTC +10) is located in the Eastern Standard Time zone.
- a** Isabella, who is in Dubbo, rang a friend in Fremantle at 8:00 p.m. on a Saturday. What day and time was it in Fremantle?
- b** Isabella caught a plane, at 8:00 p.m. on a Saturday, from Dubbo to Fremantle. The travel time for the trip was 5 hours. When will Isabella arrive in Fremantle?
- 7D 2** Reece wants to call home to Sydney from Papeete, Tahiti when it is Saturday 10:00 a.m. in Sydney. Sydney is 10 hours ahead of UTC, while Papeete is 10 hours behind UTC. It is daylight saving time in Sydney.

What time and day in Papeete should Reece make the call to Sydney?

- 7E 3** This table shows the approximate coordinates for two cities.

City	Latitude	Longitude
Vancouver, Canada	50°N	123°W
Frankfurt, Germany	50°N	9°E

- a** What is the time difference between Vancouver and Frankfurt? (Ignore time zones).
- b** Ava lives in Frankfurt and her sister Lily lives in Vancouver. Ava wants to telephone Lily at 6:00 p.m. on a Saturday night, Vancouver time. What day and time should Ava make the call from Frankfurt?
- 7E 4** Melbourne has coordinates (38°S, 145°E) and the coordinates of Dubai are (25°N, 55°E). A plane leaves Melbourne on Sunday at 7:30 p.m. The flight time to Dubai is 15 hours. What time and day will it be in Dubai when the plane is scheduled to land?

8

Managing money

Why do we pay GST when we purchase goods and services?

The Goods and Services Tax (GST) was introduced in Australia in 2000. It increased the price of most goods and services by 10%, but at the same time some other taxes were abolished. This percentage rate has not changed since it was introduced, despite some politicians proposing that it should be raised to 15%. Some goods and services are exempt from GST.

Australian businesses collect GST and pass it on to the Federal Government, which then distributes it to the states and territories. It is estimated that this amount will be over \$108 billion dollars for the financial year 2027–2028. The division of GST between the states causes a great deal of debate on a regular basis.

Foreign tourists who visit Australia can claim a refund of the GST that they spent on some goods in Australia, as long as certain conditions are met.

Do other countries have GST?

Canada, Hong Kong, New Zealand and other countries also have a GST.

The United Kingdom has a Value Added Tax (VAT). The standard rate started at 10% then climbed to 20% over the following years.

When you travel to these countries from Australia, you may be able to claim a refund of GST or VAT at the airport before you leave, provided you keep all of your receipts.

Chapter contents

- 8A** Percentage increase or decrease
- 8B** Calculating GST and VAT
- 8C** Buying on terms and buy now pay later schemes
- 8D** Purchasing a car
- 8E** Car insurance
- 8F** Stamp duty
- 8G** Running and maintenance costs
- 8H** Household bills
- 8I** Personal budget

NSW Syllabus

In this chapter, a student:

- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- Solves financial problems involving budgeting and purchasing (MST-11-04)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

8A Percentage increase or decrease

Learning intentions

- To revise and consolidate skills and applications of percentages
- To solve problems involving percentage increase and decrease

Past, present and future learning

- Students should already be familiar with the contents of this section
- Percentages are used extensively throughout Years 11 and 12
- Questions involving percentages regularly appear in HSC Examinations

Percentage change involves increasing or decreasing a quantity as a percentage of the original amount of the quantity.

Percentage increase	Percentage decrease
1 Add the % increase to 100%.	1 Subtract the % decrease from 100%.
2 Multiply the above percentage by the amount.	2 Multiply the above percentage by the amount.



Example 1 Calculating the percentage change

The retail price of a toaster is \$36 and is to be increased by 5%. What is the new price?

Solution

$$\begin{aligned}
 100\% + 5\% &= 105\% \\
 \text{New price} &= 105\% \text{ of } \$36 \\
 &= 1.05 \times 36 \\
 &= \$37.80
 \end{aligned}$$

Explanation

Determine the overall percentage of the retail price (105%).
Determine the new price by multiplying the overall percentage by the retail price.

Now you try

The retail price of a microwave oven is \$96 and is to be increased by 8%. What is the new price?



Example 2 Calculating consecutive percentage changes

Increase \$75 by 20% and then decrease the result by 20%.

Solution

$$\begin{aligned}
 100\% + 20\% &= 120\% \\
 \text{New price} &= 120\% \text{ of } \$75 \\
 &= 1.20 \times 75 \\
 &= \$90 \\
 100\% - 20\% &= 80\% \\
 \text{New price} &= 80\% \text{ of } \$90 \\
 &= 0.80 \times 90 \\
 &= \$72
 \end{aligned}$$

Explanation

Determine the overall percentage of the retail price (120%).
Determine the new price by multiplying the overall percentage by the retail price.

Determine the decreased percentage of the new price (80%).

Calculate by multiplying the decreased percentage by the new price.

Now you try

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Increase \$400 by 15% and then decrease the result by 15%.

Exercise 8A

BUILDING

Example 1

- 1 What is the amount of the increase for each of the following?
 - a Increase \$48 by 10%
 - b Increase \$120 by 30%
 - c Increase \$66 by 15%
 - d Increase \$88 by 25%
 - e Increase \$1340 by 40%
 - f Increase \$196 by 36%
 - g Increase \$150 by 4.5%
 - h Increase \$24 by $\frac{1}{2}\%$
- 2 What is the amount of the decrease for each of the following?
 - a Decrease \$110 by 20%
 - b Decrease \$260 by 60%
 - c Decrease \$320 by 35%
 - d Decrease \$1096 by 75%
 - e Decrease \$50 by 6%
 - f Decrease \$36 by 32%
 - g Decrease \$640 by 12.5%
 - h Decrease \$56 by $1\frac{1}{4}\%$
- 3 The David Jones clearance sale has a discount of 30% off the retail price of all clothing. Find the amount saved on the following items.
 - a Men's shirt with a retail price of \$80
 - b Pair of jeans with a retail price of \$66
 - c Women's jacket with a retail price of \$450
 - d Boy's shorts with a retail price of \$22
 - e Jumper with a retail price of \$124
 - f Girl's skirt with a retail price of \$50
- 4 A manager has decided to award a salary increase of 6% to all employees. Find the new salary of employees currently earning following amounts.
 - a \$46 240
 - b \$94 860
 - c \$124 280
 - d \$64 980
- 5 Aya has a card that entitles her to a 12.5% discount at the store where she works. How much will she pay for the following items?
 - a Vase marked at \$190
 - b Cutlery marked at \$240
 - c Painting marked at \$560
 - d Pot marked at \$70

DEVELOPING

- 6 A used car is priced at \$18 600 and offered for sale at a discount of 15%.
 - a What is the discounted price of the car?
 - b The car dealer decides to reduce the price of this car by another 15%. What is the new price of the car?
- 7 Find the results after these repeated percentage changes.
 - a Increase \$100 by 20% and then decrease the result by 20%.
 - b Increase \$280 by 10% and then increase the result by 5%.
 - c Decrease \$32 by 50% and then increase the result by 25%.
 - d Decrease \$1400 by 5% and then decrease the result by 5%.
 - e Increase \$960 by 15% and then decrease the result by 10%.
 - f Decrease \$72 by 12.5% and then increase the result by $33\frac{1}{3}\%$.
- 8 An electronic store offered a \$30 discount on a piece of software marked at \$120. What percentage discount has been offered?

Example 2

CHALLENGING

- 9** The cost price of a sound system is \$480.
Retail stores have offered a range of successive discounts. Calculate the final price of the sound system at the following stores.
- a** Store A: Increase of 10% and then a decrease of 5%
 - b** Store B: Increase of 40% and then a decrease of 50%
 - c** Store C: Increase of 25% and then a decrease of 15%
 - d** Store D: Increase of 30% and then a decrease of 60%



- 10** The price of a clock has been reduced from \$200 to \$180.
- a** What percentage discount has been applied?
 - b** Two months later the price of the clock was increased by the same percentage discount. What is new price of the clock?
 - c** Explain why the new price of the clock is not \$200.

Communicate your reasoning, understanding and fluency

- 11** Jason wants to increase the selling price of his boat from \$20 000 to \$22 500.
- a** What will be the percentage increase?
 - b** He sells his boat to a friend, so he decides to decrease the \$22 500 by 5%. How much does his friend save?
 - c** Explain why Jason still received than \$20 000.

8B Calculating GST and VAT

Learning intentions

- To perform calculations involving prices of goods and services which have GST or VAT applied

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions involving the GST regularly appear in HSC Examinations

GST

The Australian Government collects a tax when people purchase goods and services. The tax is called the GST (Goods and Services Tax) and is 10% of the pre-tax price of the item. There are exceptions for basic food items and some medical expenses.

GST

To calculate the GST, find 10% of the pre-tax price.

To calculate the total cost of an item, add the GST to the pre-tax price. Alternatively, find 110% of the pre-tax price.

To calculate the pre-tax price given the total cost of an item, divide the total cost by 110%.



Example 3 Finding the GST

John bought a ticket with a pre-tax price of \$142 to see a concert at the Sydney Olympic Stadium. He was also required to pay a 10% GST.

- How much GST is payable?
- What was the total cost of his ticket including the GST?
- John bought a second ticket for \$149.60 including GST. What was the pre-tax price of the second ticket?



Solution

- $$\begin{aligned}\text{GST} &= 10\% \text{ of } \$142 \\ &= 0.10 \times 142 \\ &= \$14.20\end{aligned}$$
- $$\begin{aligned}\text{Total cost} &= \$142 + \$14.20 \\ &= \$156.20\end{aligned}$$
- $$\begin{aligned}\text{Pre-tax price} &= \$149.60 \div 110\% \\ &= \$149.60 \div 1.10 \\ &= \$136\end{aligned}$$

Explanation

Determine 10% of the pre-tax price.

Determine the total cost by adding the 10% (\$14.20) to the pre-tax price.

Final price is 110% of pre-tax price.
Calculate GST by dividing final price by 110%.

continued ...

Now you try

Chrestella bought a ticket with a pre-tax price of \$245. She also had to pay GST of 10%.

- a How much GST is payable?
- b What was the total price Chrestella paid?
- c Chrestella bought a second ticket for \$220 including GST. What was the pre-tax price of the second ticket?

VAT

In some countries the GST is called the VAT (Value Added Tax). The rate of the VAT ranges from 5% to 27%. The calculations for the VAT are similar to the calculations for the GST except the rate of taxation is different.

VAT

- To calculate the VAT, find the VAT rate of the pre-tax price.
- To calculate the total cost of an item, add the VAT to the pre-tax price. Alternatively, find $100\% + \text{VAT rate}$ of the pre-tax price.
- To calculate the pre-tax price given the total cost of an item, divide the total cost by $100\% + \text{VAT rate}$.



Example 4 Finding the VAT

Indonesia has a VAT of 11%. Olivia bought a microwave in Indonesia for \$275 plus VAT.

- a How much VAT is payable?
- b What was the total cost of her microwave including the VAT?
- c What was the pre-tax price of a different microwave if its total cost including VAT was \$672?

Solution

- a $\text{VAT} = 11\% \text{ of } \$275$
 $= 0.11 \times 275$
 $= \$30.25$
- b $\text{Total cost} = \$275 + \$30.25$
 $= \$305.25$
- c $\text{Pre-tax price} = \$672 \div 111\%$
 $= \$672 \div 1.11$
 $= \$605.41$

Explanation

Determine 11% of the cost of the microwave.

Determine the total cost by adding the 11% (\$30.25) to the cost of the microwave.

Total cost is 111% of the pre-tax price ($100\% + 11\% \text{ VAT}$). Calculate the pre-tax price by dividing the total cost by 111%.

Now you try

Canada has a VAT of 7%. Amuktha bought a painting in Canada for \$175 plus VAT.

- a How much VAT is payable?
- b What was the total cost of the painting?
- c What was the pre-tax price of a different painting if its total cost including VAT was \$275?

Exercise 8B

BUILDING

Example 3a

- 1 Calculate the GST payable on each of the following items. The GST rate is 10%.
- | | | |
|---------------------------|-------------------------|--------------------------|
| a Football at \$36 | b Shoes at \$140 | c Dinner at \$170 |
| d Bucket at \$3.20 | e Dress at \$490 | f Book at \$42 |
| g Belt at \$42.90 | h Ring at \$2600 | i Camera at \$370 |

Example 3b

- 2 Blake received a \$620 bill for electrical work and was required to pay a 10% GST.
- How much GST is payable?
 - What was the total cost of the electrical work including the GST?
- 3 Isabelle received an invoice for her gym membership of \$780. In addition, she was required to pay a 10% GST.
- How much GST is payable?
 - What was the total cost of her gym membership including the GST?

Example 4a

- 4 Ireland has VAT of 23% on clothing. How much VAT is payable on the following items?
- Football jumper with a pre-tax price of €150
 - Football boots with a pre-tax price of €80
 - Football shorts with a pre-tax price of €20
 - Football socks with a pre-tax price of €8
- 5 What is the VAT payable in the following countries on a car with a pre-tax price of \$42 000? Answer correct to the nearest whole number.
- | | |
|---------------------------------|---------------------------------|
| a Argentina – 21% VAT | b Canada – 7% VAT |
| c China – 9% VAT | d India – 12.5% VAT |
| e Ivory Coast – 18% VAT | f UAE – 5% VAT |
| g South Africa – 15% VAT | h Switzerland – 8.1% VAT |

DEVELOPING

Example 3c

- 6 The cost of the following items includes a 10% GST. What was the pre-GST price?
- | | |
|-------------------------|--------------------------------|
| a Pen at \$17.60 | b Calculator at \$24.20 |
| c Chair at \$99 | d Plant at \$15.40 |
| e Watch at \$198 | |

Example 4c

- 7 Saint Lucia has a VAT levied at 12.5%.
- Ata bought a wood carving in Saint Lucia for \$480 plus the VAT. What price did she pay for the wood carving?
 - A second wood carving costs \$390 including the VAT. What was the price of the wood carving before VAT was added? Answer correct to the nearest dollar.



8 Create the spreadsheet shown.

IF × ✓ ✗ =B5*0.1

	A	B	C	D	E	F
1	Cambridge Mathematics Standard Year 11 Exercise 8B Question 8					
2	Worksheet to calculate GST					
3						
4	<i>Item</i>	<i>Pre-tax price</i>	<i>GST</i>	<i>Total price</i>		
5	Basketball	\$50.00	=B5*0.1	\$55.00		
6	Cricket bat	\$180.00	\$18.00	\$198.00		
7	Dumbbell	\$55.00	\$5.50	\$60.50		
8	Golf shoes	\$120.00	\$12.00	\$132.00		
9	Hockey stick	\$160.00	\$16.00	\$176.00		
10	Tennis racquet	\$250.00	\$25.00	\$275.00		
11						

a Cell C5 has a formula that calculates a 10% GST. Enter this formula.

b The formula in cell D5 adds the cost price and the GST. Enter this formula. Fill down the contents of D6 to D10 using this formula.

CHALLENGING

9 After the 10% GST was added, the price of a mobile phone was \$362. What was the price without GST? Answer correct to the nearest cent.

10 What was the original cost of a notebook computer that has a GST-included price of \$1850? The rate of GST is 10%. Answer correct to the nearest cent.

Communicate your reasoning, understanding and fluency

11

	Yearly earnings	Computer before GST
Joseph	\$150 000	\$2200
Sandra	\$75 000	\$2200

a Joseph and Sandra both go shopping and spend the same amount when buying a computer, as shown in the table. How much GST did they each pay?

b What is the GST paid as a percentage of Joseph's income? Answer correct to three decimal places.

c What is the GST paid as a percentage of Sandra's income? Answer correct to three decimal places.

d From what you have found in parts b and c, explain this statement.

The GST is an unfair tax.

8C Buying on terms and buy now pay later schemes

Learning intentions

- To understand systems used to purchase goods using repeated partial payments
- To perform calculations and solve problems involving buy now pay later schemes

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about buying on terms regularly appear in HSC Examinations

We often want something before we can afford the entire amount. Saving up until we have enough money is one way to buy the item. Buying on terms is another. Buying on terms means either:

- 1 paying a deposit or
- 2 paying instalments (often monthly) until the terms are met.



Example 5 Buying on terms

Emma wants to buy a mobile phone for \$2250. She is required to pay a deposit of \$250 and then 24 monthly repayments of \$100.

- a How much do the 24 monthly repayments add up to?
- b How much does Emma pay for the phone?
- c How much extra does Emma pay for the phone?

Solution

$$\begin{aligned}\text{a Monthly repayments} &= 24 \times 100 \\ &= \$2400\end{aligned}$$

$$\begin{aligned}\text{b Phone} &= \text{deposit} + \text{repayments} \\ &= \$250 + \$2400 \\ &= \$2650\end{aligned}$$

$$\begin{aligned}\text{c Extra paid} &= \$2650 - \$2200 \\ &= \$450\end{aligned}$$

Explanation

Multiply the number of repayments by the amount of repayments.

Determine the total cost by adding the deposit to the repayments.

The extra paid is the price paid for the phone minus the original cost of the phone.

Now you try

Marcus wants to buy headphones for \$850. He has to pay a deposit of \$150 and then 12 monthly repayments of \$70.

- a How much do the 12 monthly repayments add up to?
- b How much does Marcus pay for the headphones?
- c How much extra does Marcus pay for the headphones?

Buy now pay later

Buy now pay later schemes are becoming more popular. When you choose *buy now pay later*, you set up an account with a provider, providing personal information and a method of payment. If you are approved, you usually pay a 25% deposit and pay off the rest in interest-free repayments. However, if you miss a payment you will have to pay extra interest and fees. You may be unable to use this service for future purchases.



Example 6 Buy now pay later

Haidar buys clothes for \$600 on 1 November. He takes up the option of buy now pay later and is approved. His deposit is 25%. He is required to make 4 payments over the next 6 weeks.

- a** How much is the deposit?
- b** How much is each repayment?
- c** What is the date that he must make his final payment?

Solution

- a** $\text{Deposit} = 600 \times 0.25$
 $= \$150$
- b** $\text{Repayments} = (600 - 150) \div 4$
 $= \$112.50$
- c** Final payment date is 12 December.

Explanation

Determine 25% of \$600.

Take the deposit away from the price and divide by 4.

6 weeks is $6 \times 7 = 42$ days. 30 days in November. $42 - 30$ is 12.

Now you try

Chi wants to buy books for \$400 on 15 June. She takes up the option of buy now pay later and is approved. The deposit is 25%. Chi is required to make 4 payments over the next 6 weeks.

- a** How much is the deposit?
- b** How much is each repayment?
- c** What is date that he must make his final payment?

Comparing methods of payment

There are a few options you have when buying an item. It is important to understand each method of payment and work out which is best for you.

- 1 Pay cash
- 2 Buying on terms
- 3 Buy now pay later schemes



Example 7 Comparing payment methods

Mustafa wants to buy a new TV for \$4200. He does not have the cash. The shop offers him two options.

Option 1 Pay a \$1200 deposit and then 12 monthly instalments of \$305.

Option 2 Pay \$1800 deposit and then 4 payments of \$600 over the next 6 weeks.

Which option should Mustafa choose?

Solution

Option 1:
Mustafa pays $\$1200 + 12 \times \$305 = \$4860$

Option 2:
Mustafa pays $\$1800 + 4 \times \$600 = \$4200$
Mustafa should choose option 2.

Explanation

Add the deposit and amount paid in 12 instalments.

Add the deposit and amount paid in the 4 payments.

Now you try

Mary wants to buy a new fridge for \$2200. She does not have the cash. The shop offers her two options.

- 1** Pay a \$500 deposit and then 12 monthly instalments of \$175.
- 2** Pay \$800 deposit and then 4 payments of \$350 over the next 6 weeks.

Which option should Mary choose?

Exercise 8C**BUILDING****Example 5**

- 1** Derek wants to buy a motor bike for \$3250. He is required to pay a deposit of 20% and then 24 monthly repayments of \$200.

- a** How much are the 24 monthly repayments?
- b** How much does Derek pay for the motor bike?
- c** How much extra does Derek pay for the motor bike?

- 2** The following items are purchased by paying on terms.

- a** Cost \$2200: Deposit 20%: 36 repayments of \$90.
- b** Cost \$4323: Deposit 25%: 24 repayments of \$210.
- c** Cost \$16 599: Deposit 10%: 60 repayments of \$350.
- d** Cost \$210: Deposit 40%: 12 repayments of \$15.

Answer the following questions for each item.

- i** How much are the monthly repayments?
- ii** How much is paid altogether?
- iii** How much extra than the advertised price is paid?

Example 6

- 3** Angela buys a watch for \$900 on 1 January. She takes up the option of buy now pay later and is approved. Her deposit is 25%. She is required to make 5 payments over the next 8 weeks.

- a** How much is the deposit?
- b** How much is each repayment?
- c** When must she pay her final payment?

- 4** The following items are purchased using a buy now pay later scheme.

- a** Cost \$2200: Deposit 25%: 12 monthly repayments.
- b** Cost \$4323: Deposit 30%: 24 monthly repayments.
- c** Cost \$600: Deposit 50%: 6 weekly repayments.
- d** Cost \$210: Deposit 40%: 8 weekly repayments.

Answer the following questions for each item.

- i** How much is the deposit?
- ii** How much is each repayment?

Example 7

- 5** Mandy wants to buy shoes for \$420. She does not have the cash. The shop offers her two options.

- 1** Pay a \$120 deposit and then 6 monthly instalments of \$60.
- 2** Pay \$140 deposit and then 4 payments of \$70 over the next 8 weeks.
- a** How much does Mandy pay if she chooses option 1?

- b** How much does Mandy pay if she chooses option 2?
- c** Which option should Mandy choose?

DEVELOPING

- 6** Nasser buys a shed valued at \$5500. He is given terms that state a 10% deposit and then 52 weekly payments of \$110.
 - a** How much does Nasser pay for the shed?
 - b** How much extra has he paid?
 - c** Calculate the interest rate per annum (year) that has been charged?
- 7** Victor has \$400 in savings. He has allowed \$200 a month from his earnings to buy a painting. The painting he likes is valued at \$2000. He is given the following options.
 - 1** Pay cash at a 10% discount.
 - 2** Pay a 20% deposit and then 12 monthly instalments of \$180.
 - 3** Pay a 25% deposit and 5 instalments of \$300 in 8 weeks.
 - a** Calculate the cost of each option.
 - b** Which is the only option that Victor can afford?

CHALLENGING

- 8** Julie wants to purchase a television valued at \$8000. She chooses a buy now pay later scheme that requires a 25% deposit and 10 monthly repayments of \$600. An interest rate of 0.8% per day on the outstanding balance is charged for late payments. Julie is 10 days late on her first repayment. How much extra does she have to pay?

Communicate your reasoning, understanding and fluency

- 9** Copy and complete the table, listing advantages and disadvantages of each method of payment.

	Advantages	Disadvantages
Buying on terms		
Buy now pay later		
Saving up and paying the total cost		

- 10** Landon bought play equipment for this four-year-old daughter for \$2100. He paid 30% deposit and 24 monthly instalments of \$100.
 - a** How much did Landon pay in total for the play equipment?
 - b** How much interest did Landon pay?
 - c** Why do you think Landon agreed to these terms and did not save up the money before making the purchase?

8D Purchasing a car

Learning intentions

- To investigate the various expenses incurred when buying a car
- To perform calculations and solve problems involving car expenses

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about purchasing a car sometimes appear in HSC Examinations

The cost of purchasing a motor vehicle depends on many factors. A motor vehicle is not an investment. It decreases in value immediately. In the first year of ownership, a new car can lose up to 20% of its value, and by the fifth year, your car will have decreased in price by over 65%.

The percentage decrease is determined by dividing the price decrease by the purchase price and multiplying the result by 100.



Example 8 Calculating the percentage decrease

A new vehicle is bought for \$25 000 and sold one year later for \$19 000. Calculate the percentage decrease in the value of the new vehicle.

Solution

$$\begin{aligned}\text{Percentage decrease} &= \frac{\$6000}{\$25\,000} \times 100 \\ &= 24\%\end{aligned}$$

Explanation

Determine the actual decrease in price: $\$25\,000 - \$19\,000 = \$6000$

Write the decrease as a fraction of the original price and express as a percentage by multiplying by 100.

Now you try

A car is bought for \$27 000 and sold for \$18 000 two years later. Calculate the percentage decrease in the value of the car.

Finance

Many people cannot afford to pay the entire purchase price of a new car at the time of purchase. A car buyer can arrange finance to pay for their new car over time at the car dealership. This is an example of buying on terms (see 8C). The total cost using finance to buy a car is always more than paying the entire purchase price at the time of purchase.

Buying on finance

Total cost = Deposit + Total repayments

Total repayments = Repayment $\times$ Number of repayments

Interest paid = Total cost – Sale price



Example 9 Calculating the cost of repayments

A new hybrid car is for sale at \$45 000. Finance is available at \$5000 deposit and monthly repayments of \$1470 for 5 years.

- a** What is the total of the repayments?
- b** What is the total cost of the finance?
- c** What is the interest paid?

Solution

- a** $\text{Total repayment} = 1470 \times 12 \times 5$
 $= \$88\,200$
- b** $\text{Total cost} = 5000 + 88\,200$
 $= \$93\,200$
- c** $\text{Interest paid} = 93\,200 - 45\,000$
 $= \$48\,200$

Explanation

Multiply the amount of each repayment by the number of repayments.

Add the deposit to the total repayment.

Subtract the sale price from the total costs.

Now you try

A new car is sold for \$52 000. Finance is available at \$7000 deposit and monthly repayments of \$1525 for 5 years.

- a** What is the total of the repayments?
- b** What is the total cost of the car using this finance?
- c** What is the interest paid?

Exercise 8D

BUILDING

Example 8

- 1** Calculate the percentage decrease in the price of a new vehicle after one year.
 - a** Purchase price is \$25 500. Market value after one year is \$21 420.
 - b** Purchase price is \$36 800. Market value after one year is \$27 600.
 - c** Purchase price is \$54 250. Market value after one year is \$48 825.
 - d** Purchase price is \$23 826. Market value after one year is \$20 900.
- 2** Calculate the price of the following cars after the trade-in.
 - a** Sale price is \$35 500. Trade-in is worth \$6000.
 - b** Sale price is \$16 850. Trade-in is worth \$2980.
 - c** Sale price is \$24 120. Trade-in is worth \$9460.
 - d** Sale price is \$64 870. Trade-in is worth \$11 820.
- 3** Calculate the amount of the deposit needed to purchase the following cars.
 - a** Sale price is \$21 400. Deposit is 25%.
 - b** Sale price is \$19 240. Deposit is 15%.
 - c** Sale price is \$45 100. Deposit is 35%.
 - d** Sale price is \$65 200. Deposit is 40%.

Example 9

- 4** Calculate the total repayments to purchase the following cars.
- a** Sale price is \$14 800. Monthly repayments of \$410 for 5 years.
 - b** Sale price is \$19 240. Monthly repayments of \$1120 for 2 years.
 - c** Sale price is \$45 100. Weekly repayments of \$360 for 3 years.
 - d** Sale price is \$85 200. Weekly repayments of \$610 for 4 years.
- 5** Charlotte has been offered terms to purchase a car. The price of the car is \$24 560 or 50% deposit and repayments of \$90 per week for 200 weeks.



- a** What is the amount of the deposit?
- b** Find the total cost of the repayments.
- c** What is the cost of purchasing the car on terms?

DEVELOPING

- 6** A utility vehicle is for sale at \$42 000. Finance is available at \$7500 deposit and monthly repayments of \$1280 for 5 years.

- a** What is the total cost of the repayments?
- b** How much will the car cost if you use the finance package?
- c** What is the interest paid?



- 7** Musa has seen a used car he would like to buy, priced at \$13 400. He has saved \$7000 towards the cost of the car. His parents have offered to lend him the balance to pay for it. Musa agrees to pay \$40 each week to repay his parents.
- a** How much will Musa need to borrow from his parents?
 - b** How long will it take Musa to repay the loan from his parents?
- 8** A used car is for sale at \$27 000. Finance is available at 10% deposit and monthly repayments of \$630 for 4 years.
- a** How much deposit is to be paid?
 - b** What is the total cost of the repayments?
 - c** How much will the car cost if you use the finance package?
 - d** What is the interest paid?

- 9** Emily has two choices of finance packages for a new car.
 Package A: Deposit of \$3000, \$1400 per month over 5 years.
 Package B: No deposit, \$1540 per month over 6 years.
- Determine the total cost of package A.
 - Determine the total cost of package B.
 - How much will be saved by selecting the cheaper package?
- 10** A prestige car is for sale at \$65 000. Finance from the car dealer is available at a deposit of 40% and weekly repayments of \$530 for 4 years. A personal loan of \$39 000 is available from the bank at 15% per annum simple interest for 4 years.
- How much deposit is required?
 - What is the interest paid using the finance from the car dealer?
 - What is the interest paid using the finance from the bank?

CHALLENGING

- 11** Ali is buying a used car for \$12 000. He is required to pay a transfer fee of \$26 and stamp duty of \$360. Finance from the car dealer is available at a deposit of 20% and monthly repayments of \$380 for 4 years. How much above the price is Ali paying the car dealer?

- 12** Personal loan calculators on lenders' websites can be used to determine the monthly repayments. Use a personal loans calculator with a monthly gross salary of \$5000 and monthly expense details of \$2000 to determine the maximum amount to be borrowed.

PERSONAL LOAN REPAYMENTS CALCULATOR

Our personal loans calculator can help you to understand what your repayments will be. You can adjust the rate and the amount to enter the calculation to meet your budget.

Take the next step:
Find the right personal loan to suit your needs.
Compare and apply

Loan Details

Type of Loan	Variable Rate
Loan Amount	
Loan Term	5 years
How often would you like to make your repayments?	Weekly
Interest Rate	10.0 % p.a.
Comparison Rate	14.71 % p.a.

Calculate

- Current variable rate of interest and a loan term of 4 years.
- Current variable rate of interest and a loan term of 2 years.
- Current fixed rate of interest and a loan term of 4 years.
- Current fixed rate of interest and a loan term of 2 years.

Communicate your reasoning, understanding and fluency

- 13** Jackson buys a new car for \$49 000. He uses finance to pay for the car over five years and ends up paying \$89 000 for the car.
- How much interest did Jackson pay in five years?
 - How much interest did Jackson pay in one year?
 - What is the interest rate charged by the finance company?

8E Car insurance

Learning intentions

- To investigate the various expenses incurred when maintaining a car
- To perform calculations and solve problems involving car insurance

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about car insurance sometimes regularly appear in HSC Examinations

Insurance is a major cost of keeping a motor vehicle on the road. There are three main types of insurance:

- Green slip insurance is compulsory. It is also called Compulsory Third Party insurance. It protects vehicle owners and drivers who are legally liable for personal injury to any other party in the event of a personal injury claim made against them by other road users.
- Owners of a vehicle may then choose to purchase Third Party Property insurance or Comprehensive insurance.
- Third Party Property insurance covers you for damage caused by your car to property owned by a third party in the event of an accident.
- Comprehensive insurance covers you for damage to your own vehicle as well as damage your car may cause to another person's vehicle or property.

An insurance premium is the cost of taking out insurance cover. Many insurance companies offer an online calculator for your vehicle insurance premium. It requires information on the make and model of car, your age, driving history, finance, modifications or accessories and location. The cost of insurance is affected by other factors:

- No-claim bonus is a discount on an insurance premium. This discount increases if no-claim is made on the policy until it reaches the maximum discount level.
- Excess is the amount the owner pays when a claim is made on the policy. The standard excess can be varied; there are also excesses for younger drivers.



Car insurance

The cost of insurance is affected by make and model of car, your age and driving history, finance, modifications or accessories and location. No-claim bonus and excess amount are major factors.



Example 10 Calculating the insurance premium

Elle has been quoted \$960 for comprehensive car insurance. She has a no-claim bonus of 40%. How much is Elle required to pay?

Solution

Premium = 60% of \$960
= 0.60×960

Elle is required to pay \$576.

Explanation

A no-claim bonus of 40% means you are required to pay $100\% - 40\% = 60\%$.

Determine 60% of \$960.

Now you try

Douglas has been quoted \$1130 for car insurance. He has a 50% no-claim bonus. How much is Douglas required to pay?

Exercise 8E

BUILDING

- Example 10
- 1

What is the cost of comprehensive car insurance for the following premiums?

a

Premium of \$1080 with a no-claim bonus of 60%

b

Premium of \$1690 with a no-claim bonus of 30%

c

Premium of \$880 with a no-claim bonus of 40%

d

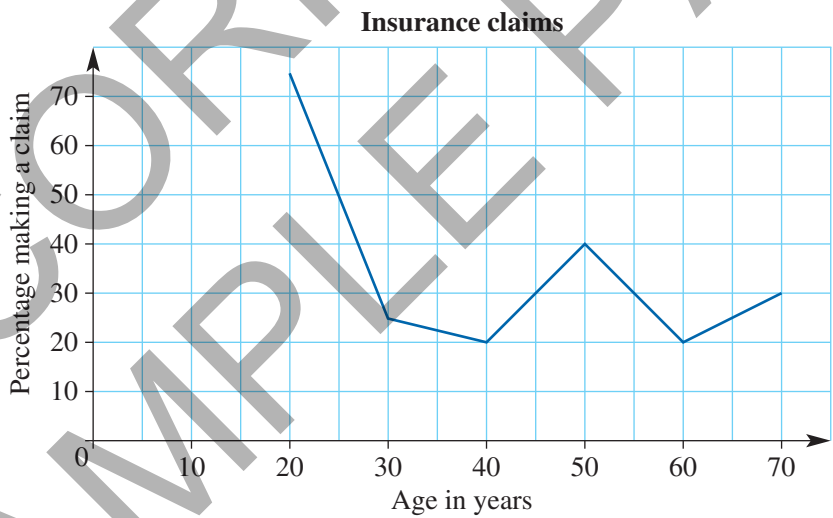
Premium of \$1320 with a no-claim bonus of 70%

e

Premium of \$2350 with a no-claim bonus of 50%

- 2

The graph shows the percentage of claims for each age group.



- a

What is the percentage of claims for people 50 years old?
- b

Which age group made the least number of claims?
- c

Calculate the gross percentage change in claims between the ages of 60 and 70.
- d

How do insurance companies cater for the large number of claims made by people 20 years old?
- 3

Dan is 20 years old and has received this quote for comprehensive insurance.

a

Calculate the cost of the insurance.

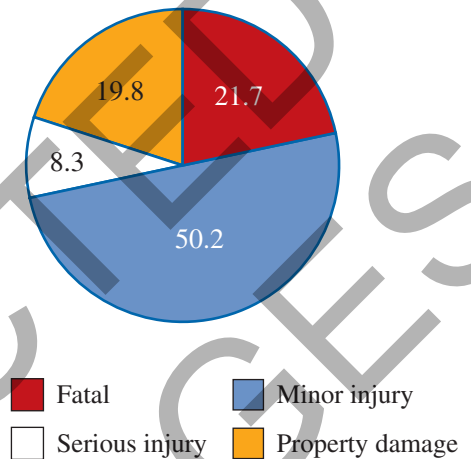
b

What is Dan's excess if he makes a claim?

Premium details	Excesses	Cost
12 month policy	Standard	\$500
\$678.00	Male under 21	\$1200
30% No-claim bonus	Female under 21	\$900

DEVELOPING

- 4 Connor has been quoted an insurance premium of \$980.60 by his insurance company. The company has given him a 20% no-claim bonus, as he had not made a claim in the previous year. What would the insurance premium have been without his no-claim bonus?
- 5 The sector graph shows the road crash costs according to categories. The total insurance cost was \$1.2 billion.
- What is the insurance cost for minor injury?
 - What is the insurance cost for serious injury?
 - What is the insurance cost for a fatal accident?
 - What is the insurance cost for property damage?



CHALLENGING

- 6 The premiums quoted are for clients with a maximum no-claim bonus. The car is owned outright by a mature-age driver and driven for private use.

Model of car	Agreed value	Premium A		Premium B	
		Mosman	Penrith	Mosman	Penrith
Brand A	\$20 100	\$540	\$605	\$600	\$760
Brand B	\$38 890	\$810	\$899	\$770	\$1500
Brand C	\$24 400	\$615	\$650	\$615	\$860

- Which suburb has the highest premium? Suggest a reason.
- Do expensive cars have higher premiums?
- What is the best quote for the Brand B?
- What is the best quote for the Brand A?
- Which model has the lowest premium?
- What is the average premium for the Brand B?
- What is the average premium for the Brand C?
- What is the average premium for Mosman?
- What is the average premium for Penrith?
- Premium A is being increased by 3%. What would be the new premium for a Brand A car at Mosman?



Communicate your reasoning, understanding and fluency

- 7 Give reasons why drivers are not able to register a car without a Green Slip (Compulsory Third Party insurance).

- 8 Why do you think it is more expensive for drivers under 25 years old to get car insurance?

8F Stamp duty

Learning intentions

- To investigate the various expenses incurred when buying a car
- To perform calculations and solve problems involving car expenses

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about stamp duty sometimes appear in HSC Examinations

Stamp duty is the tax paid to the government when registering or transferring ownership of a motor vehicle. The amount of stamp duty is based on the price of the motor vehicle. Some motor vehicles (like electric vehicles) are exempt from stamp duty. Stamp duty is calculated as \$3 per \$100 up to \$44 999 and \$5 per \$100 over \$45 000.

Stamp duty on vehicles

- 1 Round up the cost of the vehicle to the nearest \$100 (per \$100), \$200 (per \$200) etc.
- 2 Express the stamp duty as a fraction or decimal. (\$3 per \$100 is $\frac{3}{100}$ or 0.03.)
- 3 Multiply the answer obtained in step 1 by the fraction or decimal obtained in step 2.



Example 11 Calculating stamp duty on vehicles

A used car is bought for \$17 730. Calculate the stamp duty payable if the charge is \$3 per \$100 or part \$100.

Solution

$$\begin{aligned}\text{Value of vehicle} &= \$17\,800 \\ \text{Stamp duty} &= 17\,800 \times \frac{3}{100} \\ &= \$534\end{aligned}$$

Stamp duty payable is \$534.

Explanation

Round the price up to the next \$100.

Multiply the rounded up price by $\frac{3}{100}$.

Now you try

A new car is bought for \$13 925. Calculate the stamp duty if the charge is \$3 per \$100 or part of \$100.



Example 12 Calculating stamp duty on more expensive vehicles

Gary buys a new car for \$84 401. The stamp duty is \$3 per \$100 up to \$44 999 and \$5 per \$100 over \$45 000. How much stamp duty does Gary need to pay?

Solution

$$\begin{aligned}\text{Value for stamp duty calculation} &= \$84\,500 \\ \$84\,500 - \$45\,000 &= \$39\,500 \\ \text{Stamp duty} &= 45\,000 \times \frac{3}{100} + 39\,500 \times \frac{5}{100} \\ &= 1350 + 1975 \\ &= \$3325\end{aligned}$$

Stamp duty payable is \$3325.

Explanation

Round the price up to the next \$100.

Determine amount over 45 000.

Calculate the two parts of the stamp duty and add them together.

Now you try

Matt buys a new car for \$92 333. The stamp duty is \$3 per \$100 up to \$44 999 and \$5 per \$100 over \$45 000. How much stamp duty did Matt need to pay?

Exercise 8F**BUILDING****Example 11**

- 1** The table shows how to calculate the stamp duty payable on a vehicle.

Value of vehicle	Stamp duty payable
\$0–\$45 000	\$3 per \$100 or part \$100
More than \$45 000	\$1350 plus \$5 per \$100 (or part \$100) over \$45 000

Calculate the stamp duty payable on the following vehicles.

a \$32 600**b** \$26 500**c** \$45 000**d** \$13 790**e** \$35 521**f** \$23 802**g** \$52 700**h** \$98 435**i** \$120 080**Example 12**

- 2** The table shows how to calculate the stamp duty payable on a vehicle.

Value of vehicle	Stamp duty payable
\$0–\$60 000	\$5 per \$200 or part \$200
More than \$60 000	\$1500 plus \$7 per \$200 (or part \$200) over \$60 000

Calculate the stamp duty payable on the following vehicles.

a \$13 200**b** \$29 790**c** \$45 410**d** \$73 800**e** \$61 670**f** \$88 605**g** \$57 326**h** \$79 190**i** \$91 456

- 3** The table shows how used to calculate the stamp duty payable on a used vehicle.

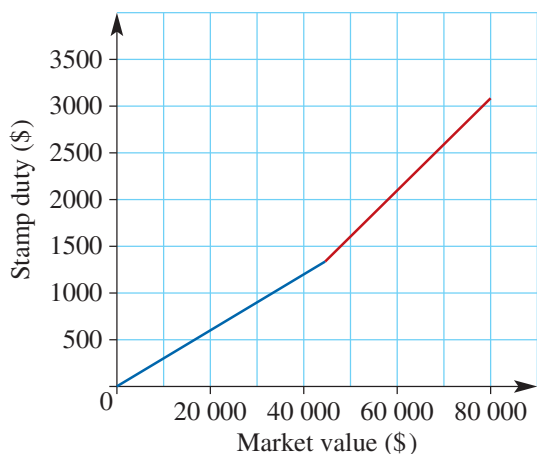
Value of vehicle	Passenger	Non-passenger
All prices	\$5 per \$300 (or part \$300)	\$7 per \$300 (or part \$300)

Calculate the stamp duty payable on the following vehicles.

a Passenger car \$21 300**b** Passenger car \$69 500**c** Non-passenger car \$45 880**d** Non-passenger car \$36 614

DEVELOPING

- 4 Stamp duty is calculated at 3% of the market value of a vehicle up to \$45 000, plus 5% of the value of the vehicle over \$45 000. Use the following graph to answer the questions.



- How much stamp duty is payable on a car whose market value is \$20 000?
- How much stamp duty is payable on a car whose market value is \$60 000?
- How much stamp duty is payable on a car whose market value is \$45 000?
- How much stamp duty is payable on a car whose market value is \$70 000?
- What is the market value of a car if the stamp duty paid was \$300?
- What is the market value of a car if the stamp duty paid was \$2300?

CHALLENGING

- 5 Construct a line graph to represent the following stamp duty charge. Stamp duty is calculated at 2.5% of the market value of a vehicle up to \$60 000, plus 4% of the value of the vehicle over \$60 000. Use your graph to answer the questions.

- How much stamp duty is payable on a car whose market value is \$30 000?
- How much stamp duty is payable on a car whose market value is \$60 000?
- How much stamp duty is payable on a car whose market value is \$80 000?
- What is the market value of a car if the stamp duty paid was \$500?
- What is the market value of a car if the stamp duty paid was \$1000?
- What is the market value of a car if the stamp duty paid was \$3000?



Communicate your reasoning, understanding and fluency

- 6 Stamp duty is calculated at 2.5% of the market value of a vehicle up to \$60 000, plus 4% of the value of the vehicle over \$60 000. Without using a graph, determine the market value of a car if the stamp duty paid was:

a i \$875

ii Explain how you calculated your answer.

b i \$1700

ii Explain how you calculated your answer.

8G Running and maintenance costs

Learning intentions

- To investigate the various expenses incurred when running and maintaining a car
- To perform calculations and solve problems involving car expenses

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about car expenses sometimes appear in HSC Examinations

Car running costs

Car running costs are a hidden, irregular cost and depend on the make, model and age of the car.

- Registration – cost paid every year to be allowed to drive your car on the road.
- Maintenance – includes oil changes, tune-ups, new tyres and brakes. Car parts need to be replaced after a certain period of time or after the car has travelled a specific number of kilometres.
- Repairs – damage to the car as a result of a car accident, scratch or dent. These costs are very unpredictable.
- Fuel – cost depends on the distance travelled by the car, the price paid for the fuel, the efficiency of the car and the type of driving.
- Improvements – additional items purchased to improve the car, such as seat covers.
- Parking – money required to park the car. This can be expensive in the major cities.
- Tolls – fee charges for travelling a tolled motorway.
- Car washes – cost of maintaining a clean car.
- Fines – traffic ticket issued by the police for breaking the law by speeding.



Car running costs

Car running costs involve registration, maintenance, repairs, fuel, improvements, parking, tolls, car washes and fines.



Example 13 Calculating the maintenance cost of a car tyre

Julia travels 20 000 km each year in her car. A set of tyres costs \$640 and lasts about 40 000 km. Calculate the cost of tyre wear:

- per kilometre
- per year.

Solution

$$\begin{aligned} \text{a Cost per km} &= \frac{\$640}{40\,000} \\ &= \$0.016 \end{aligned}$$

Explanation

Determine cost per km by dividing the cost of tyres by the kilometres travelled.

$$\text{b Years} = \frac{40\,000}{20\,000} = 2$$

$$\begin{aligned}\text{Cost per year} &= \$640 \div 2 \\ &= \$320\end{aligned}$$

Determine how many years the tyres will last.

Calculate the cost per year by dividing the cost of tyres by how many years the tyres will last.

Now you try

Yaseen travels 15 000 km each year in his car. A fuel filter costs \$240 and lasts about 45 000 km.

Calculate the cost of the fuel filter:

a per km

b per year.

Fuel consumption

A motor vehicle's fuel consumption rate (often referred to just as 'fuel consumption') is the number of litres of fuel it uses to travel 100 kilometres. The fuel consumption is calculated by filling the motor vehicle with fuel and recording the kilometres travelled from the odometer. When the motor vehicle is again filled with fuel, record the reading from the odometer and how many litres of fuel it takes to refill the tank. The distance travelled is the difference between the odometer readings.

Fuel consumption

$$\text{Fuel consumption} = \frac{\text{Amount of fuel (L)} \times 100}{\text{Distance travelled (km)}}$$



Example 14 Calculating fuel consumption

A medium-size car travelled 750 km using 60 L of petrol. What was the fuel consumption?

Solution

$$\begin{aligned}\text{Fuel consumption} &= \frac{\text{Amount of fuel} \times 100}{\text{Distance travelled}} \\ &= \frac{60 \times 100}{750} \\ &= 8.0 \text{ L/100 km}\end{aligned}$$

Fuel consumption is 8 L per 100 km.

Explanation

Write down the formula.

Substitute the numbers into the formula from the question.

Now you try

A car travelled 400 km using 30 L of petrol. What was the fuel consumption?

Exercise 8G

BUILDING

- 1 Engine oil costs \$60 for 5 litres. A car needs 5 litres of oil changed every 5000 km. Shane travels 15 000 km each year.
 - a How many oil changes are required each year?
 - b How many litres of engine oil are required each year?
 - c What is the total cost of engine oil per year?
 - d What is the cost of engine oil per kilometre?

Example 13

- 2 The table shows the running costs of a car in cents per kilometre.

Running costs	50 000 km	75 000 km	100 000 km
Fuel	10.4	10.4	10.4
Tyres	2.3	2.8	2.7
Services	7.8	7.5	8.6

- a What are the total running costs if the car travels 50 000 km?
- b What are the total running costs if the car travels 75 000 km?
- c What are the total running costs if the car travels 100 000 km?

Example 14

- 3 Calculate the fuel consumption (litres per 100 km) for each of the following vehicles.

- a Ahmed's car uses 38.2 litres of petrol to travel 400 km.
- b A sports car travelled 900 km using 79.38 litres of petrol.
- c Joel's sedan uses 30.36 litres of LPG to travel 600 km.
- d A small car uses 41.05 litres of petrol to travel 500 km.
- e Zahra's car uses 139.8 litres of petrol to travel 1500 km.
- f Max's motorbike uses 70 litres of LPG to travel 2000 km.

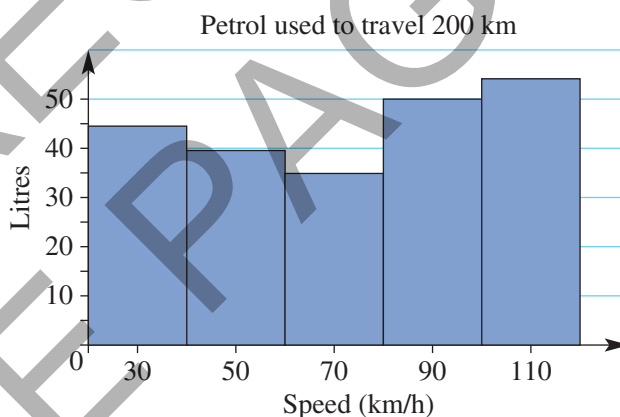


- 4 Chelsea has bought a used car whose fuel consumption is 10 litres of petrol per 100 kilometres. She is planning to travel around Australia. Calculate the number of litres of petrol Chelsea's car will use on the following distances.
 - a A trip of 2716 km from Perth to Adelaide
 - b A trip of 732 km from Adelaide to Melbourne
 - c A trip of 658 km from Melbourne to Canberra
 - d A trip of 309 km from Canberra to Sydney
 - e A trip of 982 km from Sydney to Brisbane
 - f A trip of 3429 km from Brisbane to Darwin
 - g A trip of 4049 km from Darwin to Perth

DEVELOPING

- 5** Stephanie travels 37 km to work and 37 km from work each day.
- How many kilometres does she travel to and from work in a 6-day working week?
 - Stephanie drives an SUV with a fuel consumption of 8.38 L/100 km. How many litres of petrol does Stephanie use travelling to and from work in a week? Answer correct to one decimal place.
 - What is Stephanie's petrol bill for travelling to work if petrol costs \$1.35 per litre?
- 6** A family car uses LPG at a rate of 15 L/100 km and the gas tank holds 72 litres. How far can the car travel on a tank of LPG?
- 7** Grace drives a four-wheel-drive vehicle with a petrol consumption of 15.2 L/100 km and a petrol tank capacity of 95 litres. She is planning a trip from Sydney to Bourke via Dubbo. The distance from Sydney to Dubbo is 412 km and from Dubbo to Bourke is 360 km. Grace filled her petrol tank at Sydney. How many times will she need to fill her tank before arriving at Bourke? Give reasons for your answer.

- 8** The graph shows a motor vehicle's fuel consumption at various speeds.
- How many litres of fuel were used at 70 km/h?
 - How many litres of fuel were used at 110 km/h?
 - What is the fuel consumption rate at 30 km/h?
 - What is the fuel consumption rate at 90 km/h?
 - What speed used fuel the most efficiently?
 - How many litres of fuel were saved by travelling at 90 km/h instead of 110 km/h?



- 9** Dylan owns a V8 car with a fuel consumption of 11 L/100 km in the city and 8 L/100 km in the country. Dylan travels 8000 km per year in the city and 10 000 km per year in the country. The average cost of petrol is \$1.50 per litre in the city and 10 cents higher in the country.
- Determine the cost of petrol to drive in the city for the year.
 - Determine the cost of petrol to drive in the country for the year.
 - What is the total cost of petrol for Dylan for one year?
 - What is the total cost of petrol for Dylan in one year if the average cost of petrol increased to \$1.80 per litre in the city and country prices stayed 10 cents higher?
- 10** Holly is planning a trip from Sydney to Brisbane using a car with a fuel consumption of 13 litres/100 km. The distance from Sydney to Brisbane via the Pacific Highway is 998 km and via the New England Highway it is 1027 km. The cost of LPG is 79.2 cents per litre.
- How much will the trip cost via the Pacific Highway?
 - How much will the trip cost via the New England Highway?
 - How much money is saved by travelling via the Pacific Highway?

- 11** Registration of a motor vehicle involves the payment of a fee. Note: Fees may change each year.

Size of vehicle	Tare weight	Registration fees	
		Private use	Business use
Cars, station wagons and trucks	up to 975 kg	\$218	\$321
	976 kg to 1154 kg	\$239	\$354
	1155 kg to 1504 kg	\$269	\$404
	1505 kg to 2504 kg	\$383	\$583
	2505 kg to 3504 kg	\$500	\$750
Trailers (including caravans)	up to 254 kg	\$52	\$133
	255 kg to 764 kg	\$143	\$200
	765 kg to 975 kg	\$218	\$321
	976 kg to 1154 kg	\$239	\$354
	1155 kg to 1504 kg	\$269	\$404
	1505 kg to 2499 kg	\$383	\$583
Motor cycle		\$101	\$101

- a** What is the cost of registering a car for private use, if the weight of the car is 1000 kg?
b What is the cost of registering a truck for business use, if the weight of the truck is 1500 kg?
c What is the cost of registering a car for business use, if the weight of the car is 925 kg?
d What is the cost of registering a motor cycle for private use?

CHALLENGING

- 12** Tyler buys a new car with a fuel consumption of 11.2 litres/100 km. Oscar buys the LPG version of Tyler's new car, with a fuel consumption of 15.4 litres/100 km. Both Tyler and Oscar average 300 km in a week in the same conditions. The average price of ULP is \$1.40 cents per litre and LPG is \$0.79 cents per litre.
- a** How many litres of fuel are used by Tyler in a week?
b How many litres of fuel are used by Oscar in a week?
c Calculate each car's yearly consumption of fuel.
d What is Tyler's yearly fuel bill?
e What is Oscar's yearly fuel bill?
f Oscar paid an additional \$1500 for the LPG version of the car. How many years will it take for the fuel savings to reach \$1500 or the break-even point? Answer correct to the nearest whole number.
g Research the current fuel prices of ULP and LPG. How long will it take for the fuel saving to exceed the initial costs?

Communicate your reasoning, understanding and fluency

-  **13** Investigate the costs for two common cars on a family trip in your local area. Calculate the cost for the return trip in each case. You will need to determine the distance of the trip, fuel consumption for each car and the average price of fuel in the local area.

8H Household bills

Learning intentions

- To investigate the various expenses incurred when maintaining a household
- To perform calculations and solve problems involving household expenses

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about household bills sometimes appear in HSC Examinations

Household bills are the costs associated with living in a home. They are a big percentage of the expenses in a budget. Household bills include council rates, electricity and gas bills, water and sewerage rates, telephone bills and internet connection.



Example 15 Reading household bills

Phone company

Tax Invoice
Phone Billing Services Pty. Ltd.
ABN 12 123 479 686
Invoice Number 234688731

Need Help?
Visit www.phone.com.au
Call 1300 123 569 for help
during Mon-Fri (8am-7pm)
Sat (9am-5pm) local time

Customer Account Number **2164 9142 0001 46**

Previous Account	Payments and Adjustments	GST Adjustments	Balance Forward
63.42	63.42CR	0.00	0.00

Summary of Charges

Recurring charges	52.66
Other charges and credits	0.58
Usage charges	15.44
Discounts	7.41CR
GST	6.13
New charges (including GST)	67.40
Balance forward	0.00
Total Amount Due (including GST)	\$67.40

Issue date 25 Feb
Account period 25 Jan to 24 Feb

New charges **\$67.40**
New charges due **11 Mar**
Total amount due **\$67.40**

Answer the following questions using the telephone account.

- What was the amount of the previous account?
- What is the due date?
- How much 'GST' is charged?

Solution

- a** Previous account is \$63.42.
b Due date is 11 March.
c GST charged is \$6.13.

Explanation

Read the amounts from the bill.

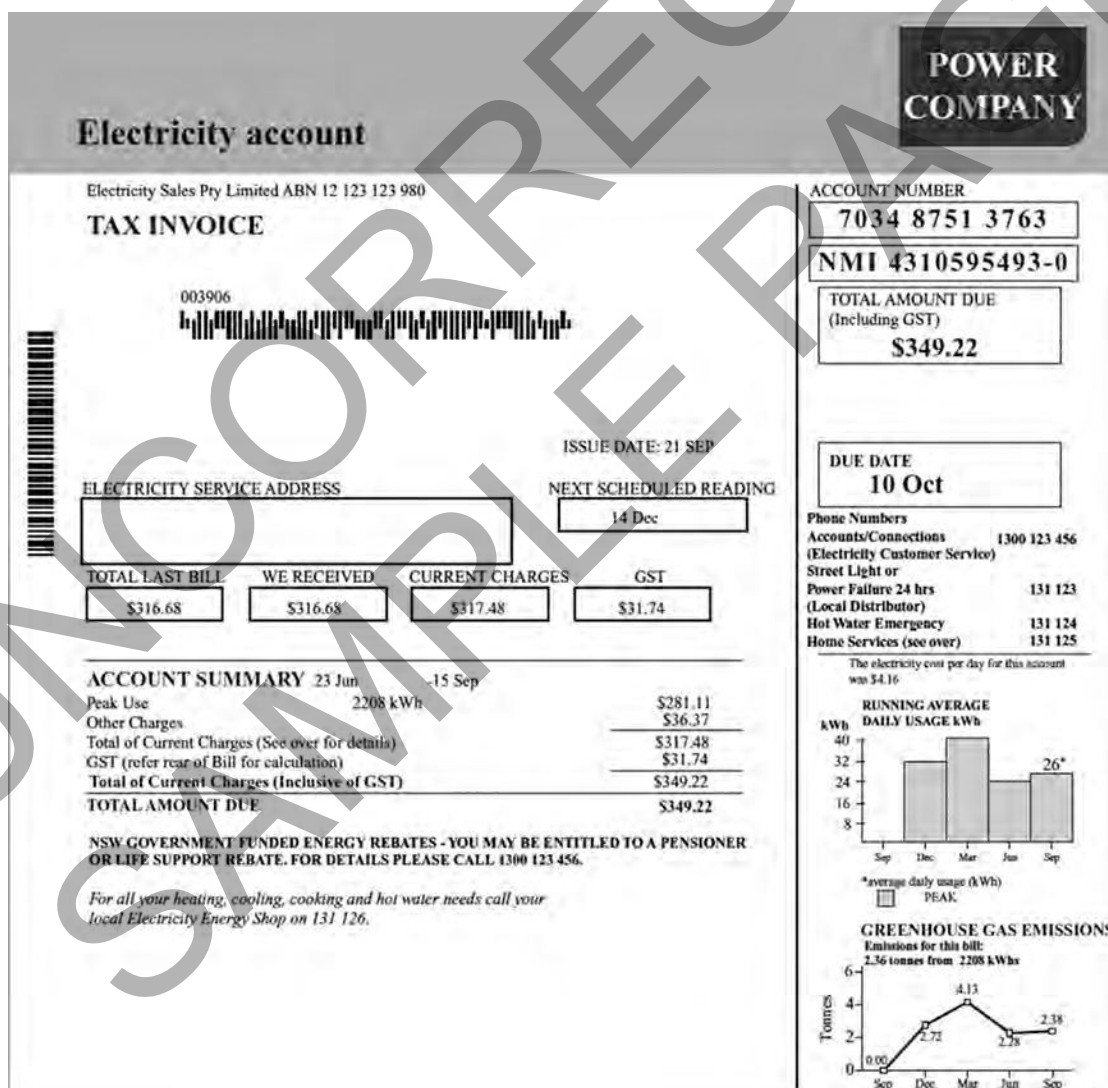
Now you try

From the telephone account in Example 15, find:

- a** the new charges (including GST)
b how much the account has increased from the previous account.

Exercise 8H**BUILDING****Example 15**

- 1** Answer the following questions using this electricity account.

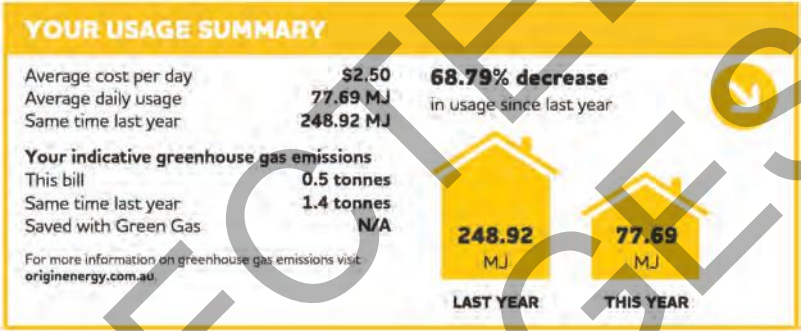


- a** When was this electricity account issued?
b What is the account number?

- c What is the total amount due?
- d What is the due date?
- e How much GST is charged?
- f What is the charge for the peak use of electricity?
- g How much was the last account?
- h By how much has the bill increased from the last bill?
- i Express the increase as a percentage of the last bill. Answer correct to two decimal places.

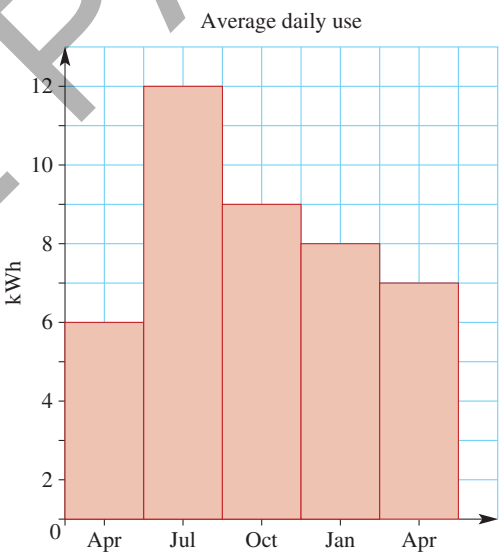
2 Answer the following questions using this summary of a quarterly gas account.

- a What was the average daily gas usage for the quarter?
- b What was the average daily gas usage at the same time last year?
- c What is the average cost per day for gas?
- d What was the percentage decrease in gas usage?
- e Describe the greenhouse gas emissions for this quarter.



3 The graph is part of an electricity account issued to a customer.

- a How many times per year is the electricity meter read?
- b What unit is used to measure electricity?
- c What was the average daily use in the October quarter?
- d Which quarter had the greatest usage of electricity?



DEVELOPING

- 4 Lucy received her natural gas account.
- a Calculate the cost of the first 5500 MJ. Answer to the nearest cent.
 - b Calculate the cost of the next 13 900 MJ. Answer to the nearest cent.
 - c What is the total charge? Answer to the nearest cent.

Amount used	Charges	Cost
First 5500 MJ	1.457 cents per MJ	
Next 13 900 MJ	1.360 cents per MJ	
Supply fee	\$40.00	

- 5 Local council rates increased by 4%. The old rates for a property were \$2200. What is the new rate for this property? Answer to the nearest dollar.
- 6 Amy uses a 1.5 kilowatt hair dryer for a total of 8 hours. She is charged at a rate of 14.81 cents per kilowatt-hour.
- How many kilowatt-hours were used by the hair dryer?
 - What is the cost of using the hair dryer? Answer to the nearest cent.
- 7 Council rates for a property valued at \$374 000 are calculated using the table.

Council rates	Charge
Residential rate	0.004155 cents in the dollar
Garbage rate	\$202.80
Water and sewerage service	\$114.99
Water usage charge	94 kL at \$1.3390 per kL

Find the amount payable to the nearest cent for:

- residential rate
- water usage
- garbage
- total rates payable on this property.

CHALLENGING

- 8 An energy company's charges for gas over a 3-month period are shown in the table.

Usage charge	First 750 MJ	1.3920 cents per MJ
	Additional MJ over 750	1.3330 cents per MJ

- Lucy used 2250 MJ of gas in this period. What is the cost of this gas? Answer to the nearest cent.
- What percentage of Lucy's gas usage was charged at the lower rate? Answer correct to two decimal places.



Communicate your reasoning, understanding and fluency

- 9 Look up the account details of each streaming service in the table and explain why Jordan and Jackson are paying different amounts for their streaming services.

Jackson		Ash	
Netflix	\$7.99	Netflix	\$18.99
Amazon prime	\$9.99	Amazon prime	\$12.99
Stan	\$27.00	Stan	\$22.00
Total	\$44.98	Total	\$53.98

8I

Personal budget

Learning intentions

- To create and balance budgets involving income and expenses
- To perform calculations and solve problems involving income and expenses

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is not repeated or extended in Year 12
- Questions about personal budgets sometimes appear in HSC Examinations

Budgeting involves balancing income and expenses. It is planning how to manage your income. Budgets are created for a specified time, such as weekly, monthly or yearly.

Creating a budget

- 1 List all the income categories.
- 2 List all the expense categories.
- 3 Calculate the total of the income and expenses categories.
- 4 Balance the budget by modifying the categories or by entering a balance category.



Example 16 Balancing a budget

Balance the following weekly budget.

Income		Expenses	
Salary	\$1726.15	Clothing	\$73.08
Bonus	\$20.00	Gifts	\$114.80
Investment	\$156.78	Groceries	\$467.31
Part-time work	\$393.72	Insurance	\$171.34
		Loan repayments	\$847.55
		Car	\$105.96
		Phone	\$38.26
		Power and heating	\$51.82
		Rates	\$54.82
		Recreation	\$216.79
		Work-related costs	\$68.76
		Balance	
Total		Total	

Solution

$$\begin{aligned}\text{Income} &= 1726.15 + \dots + 393.72 \\ &= \$2296.65\end{aligned}$$

$$\begin{aligned}\text{Expenses} &= 73.08 + \dots + 68.76 \\ &= \$2210.49\end{aligned}$$

$$\begin{aligned}\text{Balance} &= \text{Income} - \text{Expenses} \\ &= 2296.65 - 2210.49 \\ &= \$86.16\end{aligned}$$

Explanation

Add all the income.

Add all the expenses.

Balance = Income – Expenses.

Now you try

Replace the balance line with savings of \$100 in the Expenses column.

- a** How much extra money do we need to have?
- b** Suggest the most obvious item to reduce to find this amount.

Exercise 8I**BUILDING**

- 1** Oscar and Jill are living in a unit. Part of their budget is shown. Calculate the total amount paid over one year for:

- a** electricity
- b** insurance
- c** food
- d** rent.

Item	When	Cost
Electricity	Quarterly	\$384
Food	Weekly	\$360
Insurance	Biannually	\$1275
Rent	Monthly	\$1950

- 2** Sarah earns \$67 365 annually. She has budgeted 20% of her salary for rent. How much does she expect to pay to rent an apartment for one year?

- 3** Adam has constructed a yearly budget as shown.

- a** Calculate the total income.
- b** Calculate the total expenses.
- c** Balance the budget.

Income		Expenses	
Salary	\$63 829	Clothing	\$5635
Interest	\$673	Rates	\$2545
		Electricity	\$1951
		Entertainment	\$4400
		Food	\$17 550
		Gifts	\$6000
		Insurance	\$2300
		Loans	\$16 800
		Car	\$2451
		Mobile	\$1080
		Work-related costs	\$709
		Balance	
Total		Total	

- 4 Nathan has a net weekly income of \$1700. This table shows his weekly expenses.

Item	Cost
Rent	\$450
Car expenses	\$300
Food	\$250
Other expenses	\$200

- What are Nathan's total weekly expenses?
- How much can Nathan save each week?
- What percentage of his income is paid in car expenses?
- Nathan is planning an overseas holiday for \$10 000. For how many weeks does he need to save?



- 5 Create the spreadsheet shown.

VL... =C5/\$C\$7

	A	B	C	D	E	F
1	Mathematics Standard Year 11 Exercise	8I	Question 5			
2	Worksheet to calculate the monthly budget					
3						
4			Month	Week	Percentage	
5	Income	Full time pay	\$2,000	\$500.00	=C5/\$C\$7	
6		Part time pay	\$300	\$75.00	13%	
7		Total income	\$2,300	\$575.00	100%	
8						
9	Expenses	Board	\$450	\$112.50	20%	
10		Car expenses	\$100	\$25.00	4%	
11		Car loan repayment	\$350	\$87.50	15%	
12		Clothing	\$320	\$80.00	14%	
13		Eating out	\$200	\$50.00	9%	
14		Entertainment	\$400	\$100.00	17%	
15		Other expenses	\$180	\$45.00	8%	
16		Savings	\$300	\$75.00	13%	
17		Total expenses	\$2,300	\$575.00	100%	
18						
19		Balance	\$0	\$0.00		

- The formula for cell E5 is ' $=C5/\$C\$7$ '. It is the formula for relative percentage. Fill down the contents of E5 to E7 using this formula.
- Enter formulae in E9–E17 to calculate the relative percentages for expenses.
- Edit the amount spent per month on eating out from \$200 to \$240. Observe the changes.
- Edit the amount of savings per month from \$300 to \$360. Observe the changes.
- Edit the amount of car expenses per month from \$100 to \$150. Observe the changes.

- 6 Bella earns \$4000 each fortnight and pays 37% of this in tax and other deductions. This table shows her major fortnightly expenses.

Item	Cost
Mortgage	\$750
Utilities	\$250
Groceries	\$400
Entertainment	\$100

- What are Bella's total fortnightly expenses?
- What amount is Bella left with each fortnight after tax and other deductions?
- What is the maximum she can save each fortnight?
- Bella's utilities bill increased by 8%. How much will she be left with at the end of the fortnight?

- 7 Aiden lives with two friends and they agree to split the household costs evenly. For each week, rent is \$600; utilities are \$850 for each quarter; insurance is \$350 for each quarter; food is \$500 each fortnight; and cleaning and other expenses are \$200 each month.

- Find the household costs for the year.
- How much must Aiden contribute each week? Answer to the nearest dollar.

CHALLENGING

- 8** Patrick has a net annual salary of \$95 000. He pays \$1350 each month in rent and has regular bills that amount to \$4450 each year. Patrick estimates that, each month, food costs him \$625, entertainment is \$375 and other expenses are \$3250.
- a** How much will Patrick save each year?
- b** Patrick is considering taking out a loan to buy a house. Repayments will be \$2750 each month. Will he be able to afford the repayments? Justify your answer.

- 9** Hayley prepared a monthly budget as shown in the table.

- a** Calculate her total income.
- b** What is the difference between the fixed expenses and the variable expenses?
- c** How much can Hayley save each month?
- d** Hayley has no savings and is planning a trip to Europe in eight months time. She estimates the trip will cost \$30 000. Will Hayley be able to save enough to pay for the trip?

Income	Amount
Net wage	\$8750
Interest	\$300
Fixed expenses	Amount
Housing	\$2225
Transport	\$550
Utilities	\$420
Variable expenses	Amount
Clothing	\$670
Entertainment	\$320
Food	\$1125
Personal care	\$60

- 10** Dimitri had a total weekly income of \$104 made up of a part-time job earning \$74 and an allowance of \$30. He decided to budget his expenses in the following way: sport \$24, movies \$22, school \$16 and food \$20.
- a** Prepare a weekly budget showing income and expenses.
- b** What is the balance?
- 11** Ava has a net fortnightly pay of \$1896.
- a** Ava has a mortgage with an annual repayment of \$13 676. Calculate the amount that Ava must budget each fortnight for her mortgage.
- b** Ava has budgeted \$180 per week for groceries, \$60 per week for entertainment, \$468 per year for medical expenses and \$80 per week to run a car. Express these as fortnightly amounts and calculate their total.
- c** Ava has an electricity bill of \$130 per quarter, telephone bill of \$91 per quarter and council rates of \$1118 per annum. Express these amounts annually and convert to fortnightly amounts. What is the total of these fortnightly amounts?
- d** Prepare a fortnightly budget showing income and expenses.



Communicate your reasoning, understanding and fluency

- 12 a** Explain the difference between variable expenses and fixed expenses in Question 9.
- b** What will Hayley need to sacrifice to find an extra \$300 to spend on her holiday?

Key ideas and chapter summary

Percentage increase and decrease	Determine the percentage change by adding percentage increase to 100% or subtracting percentage decrease from 100%. Determine the new value by multiplying this percentage change by the original value.
GST	GST is equal to 10% of the pre-tax price. Total cost of an item is 110% of the pre-tax price. Pre-tax price is the cost of the item divided by 110%.
VAT	Value Added Tax. This works the same in other countries as the GST does in Australia, but the percentage rate varies.
Buying on terms	Buying on terms means either: 1 paying a deposit or 2 paying instalments (often monthly) until the terms are met.
Purchasing a car	$\text{Total cost} = \text{Deposit} + \text{Total repayments}$ $\text{Total repayments} = \text{Repayment} \times \text{Number of repayments}$ $\text{Interest paid} = \text{Total cost} - \text{Sale price}$
Car insurance	The cost of insurance is affected by make and model of car, your age and driving history, finance, modifications or accessories and location. No-claim bonus and excess amount are major factors.
Stamp duty	1 Round the cost of the vehicle up to the nearest \$100 (or as required). 2 Express the stamp duty as a fraction or decimal. 3 Multiply the answer in step 1 by the answer in step 2.
Car running and maintenance costs	Car running costs involve maintenance, repairs, fuel, improvements, parking, tolls, car washes and fines.
Household bills	Household bills are the costs associated with living in a home. They are a big percentage of the expenses in a budget. Household bills include council rates, electricity and gas bills, water and sewerage rates, telephone bills and internet connections.
Creating a budget	1 List all the income categories. 2 List all the expense categories. 3 Calculate the total of the income and expense categories. 4 Balance the budget by modifying the categories or by entering a balance category.

Chapter checklist with success criteria



Checklist

A printable version of this checklist is available in the Interactive Textbook.



8A 1 I can calculate the percentage change. ☐

e.g. Find the value when I:

- a** increase \$50 by 8% **b** decrease \$50 by 12%.

8B 2 I can find the amount of GST or VAT given a sale price. ☐

e.g. For the amount of \$110, calculate the:

- a** GST at 10%
b VAT at 22%.

8B 3 I can find the GST and pre-tax price of an item given the total cost. ☐

e.g. An item sells for \$132 including GST.

- a** Find the GST (10%) paid.
b Calculate the pre-tax price.

8C 4 I can work with a deposit and repayment when buying on terms. ☐

e.g. Sally wants to buy a mobile for \$850. She has to pay a deposit of \$150 and then 12 monthly repayments of \$65.

- a** How much do the 12 monthly repayments add up to?
b How much does Sally pay for the mobile?
c How much extra does Sally pay for the mobile?

8C 5 I can work with the details of a buy now pay later scheme. ☐

e.g. Dianne spends for \$500 on 10 June. She takes up the option of buy now pay later and is approved. The deposit is 25%. Dianne is required to make 4 payments over the next 6 weeks.

- a** How much is the deposit?
b How much is each repayment?
c What is the date that he must make his final payment?

8C 6 I can compare the total cost of different payment methods. ☐

e.g. Molly wants to buy a new TV for \$3000. She does not have the cash. The shop offers her two options.

Option 1 Pay a \$1000 deposit and then 12 monthly repayments of \$200.

Option 2 Pay \$600 deposit and then 12 monthly repayments of \$200.
Show with working which option Molly should choose.

8D 7 I can find the percentage decrease. ☐

e.g. A car is bought for \$32 300 and sold a year later for \$21 000. Find the

8D 8 I can find the cost of repaying a car using finance. □

e.g. A new car is sold for \$72 000. Finance is available at a \$7000 deposit and monthly repayments of \$1950 for 5 years.

- a** What is the total of the repayments?
- b** What is the total cost of the car using this finance?
- c** What is the interest paid?

8E 9 I can calculate an insurance premium. □

e.g. Jordan has been quoted \$975 for car insurance. He has a 40% no claim bonus. How much is he required to pay?

8F 10 I can calculate the stamp duty of a vehicle. □

- e.g. **a** Garnoor buys a new car for \$18 401. The stamp duty is calculated as \$3 per \$100. How much stamp duty did Garnoor need to pay?
- b** Mahtoof buys a new car for \$82 335. The stamp duty is calculated as \$3 per \$100 up to \$44 999 and \$5 per \$100 over \$45 000. How much stamp duty did Mahtoof need to pay?

8G 11 I can calculate the maintenance costs of a car. □

e.g. Victor travels 20 000 km each year in his car. A fuel filter costs \$285 and lasts about 40 000 km.

Calculate the cost of the fuel filter:

- a** per km
- b** per year.

8G 12 I can calculate the fuel consumption of a vehicle. □

e.g. A car travelled 700 km using 40 L of petrol. What was the fuel consumption?

8H 13 I can work with information from a household bill. □

e.g. Council rates increase by 5%. The old rates were \$3232. What are the new rates, correct to the nearest dollar?

8I 14 I can balance a household budget. □

e.g. Use this household budget to answer the questions.

- a** Calculate the total income.
- b** Calculate the total expenses.
- c** Balance the budget.

Income		Expenses	
Part-time job	\$2640	Entertainment	\$1600
Other	\$180	Gifts	\$250
		Mobile	\$105
		Balance	
Total		Total	

Multiple-choice questions

- 8A 1** What is the new price when \$80 is increased by 20% then decreased by 20%?
A \$51.20 **B** \$115.20 **C** \$80.00 **D** \$76.80
- 8B 2** An electrician quoted \$300, excluding GST, to complete a job. A GST of 10% is added to the job. What is the total cost of the job?
A \$303 **B** \$30 **C** \$310 **D** \$330
- 8B 3** After 16% VAT was added, the price of a DVD player was \$278. What was the price without VAT? Answer to the nearest cent.
A \$44.48 **B** \$239.67 **C** \$239.66 **D** \$322.48
- 8C 4** Cas buys a fridge on finance for \$1250. She pays a 10% deposit and 12 monthly repayments of \$110. How much did Cas pay for the fridge?
A \$1250 **B** \$2000 **C** \$1445 **D** \$1320
- 8D 5** Gary buys a cricket set for \$900. He uses a buy now pay later scheme. He pays \$225 deposit and makes 5 weekly payments. How much is each payment?
A \$135 **B** \$180 **C** \$900 **D** \$225
- 8E 6** Jake has been quoted \$1280 for comprehensive car insurance. He has a no-claim bonus of 60%. How much is Jake required to pay?
A \$512 **B** \$768 **C** \$1220 **D** \$1280
- 8F 7** A new car is bought for \$28 810. Calculate the stamp duty payable if the charge is \$3 per \$100 or part of \$100.
A \$840 **B** \$864 **C** \$867 **D** \$870
- 8G 8** Mia's car uses 8.25 litres per 100 km. How many litres of petrol will her car use on a trip of 1150 km from Broken Hill to Sydney?
A 94.875 L **B** 139.73 L **C** 1397.3 L **D** 9487.5 L
- 8H 9** The usage summary of a quarterly electricity account is shown. What was the percentage increase in electricity usage?
- | | |
|---|-------------------|
| Average cost per day | \$5.39 |
| Average daily usage | 21.55 kWh |
| Same time last year | 20.44 kWh |
| Your indicative greenhouse gas emissions | |
| Total for this bill | 1.8 tonnes |
| Same time last year | 1.8 tonnes |
| Saved with Green Power | N/A |
- A** 1.11%
B 5.15%
C 5.43%
D 21.55%
- 8H 10** Adam has the following bills: electricity \$250 per quarter, phone \$70 per month, rates \$1200 per year and rent \$300 per week. What is the total amount Adam should budget for the year?
A \$358 **B** \$1553 **C** \$1820 **D** \$18 640

Short-answer questions

8A 1 An electrician is buying a light fitting for \$144 at a hardware store. He receives a clearance discount of 15% then a trade discount of 10%. How much does the electrician pay for the light fitting?

8A 2 Jill has a shareholder card that entitles her to a 5% discount at a supermarket. How much will she pay for the following items? Answer to the nearest cent.

- a** Breakfast cereal at \$7.60
- b** Milk at \$4.90
- c** Coffee at \$14.20
- d** Cheese at \$8.40

8B 3 Ibrahim received a bill of \$286 for internet access. In addition he paid a 10% GST.

- a** How much GST is payable?
- b** What was the total cost of internet access including the GST?

8C 4 A new car is for sale at \$39 000. Finance is available at 20% deposit and monthly repayments of \$900 for 5 years.

- a** How much will the car cost if you use the finance package?
- b** What is the interest paid?

8D 5 David buys a new car with finance. The sale price of the car is \$42 000. David pays a \$10 000 deposit and makes monthly payments of \$1000 for 5 years.

- a** How much does David pay for the car?
- b** What is the total amount of interest that David paid?

8D 6 Conner uses a buy now pay later scheme to purchase a cubby house. The sale price is \$2200. He makes 4 monthly payments of \$500. How much was the deposit?

8E 7 Lucy is 18 years old and has received this quote for comprehensive insurance.

Premium details	Excesses	Cost
12-month policy \$850.00	Standard	\$600
10% No-claim bonus	Male under 21	\$1400
	Female under 21	\$1000

- a** Calculate the cost of the insurance.
- b** What is Lucy's excess if she makes a claim?

8G 8 Sienna filled her car with petrol. The odometer reading was 64 080 km at that time. When she next filled the petrol tank, the odometer reading was 64 605 km. The car required 42 L of petrol.

- a** How far has the car travelled between fill ups?
- b** What was the average fuel consumption in litres per 100 km?
- c** Sienna next filled the petrol tank at 65 200 with 45 L of petrol. What was the average fuel consumption in kilometres per litre from the last fill up? Answer correct to one decimal place.

8H 9 Using the table, what is the charge for water for the past month?

Water	
Usage	1200 kL
Cost per L 0–1000 kL	\$0.02
Cost per L above 1000 kL	\$0.05
Monthly charge	\$40.00
Total	

8I 10 Jett has constructed a yearly budget as shown in the table.

- a** Find the total income.
- b** Find the total expenses.
- c** Balance the budget.

Income		Expenses	
Salary	\$62 609.80	Christmas	\$6037.03
Interest	\$705.45	Clothing	\$4773.45
		Council rates	\$1589.86
		Electricity	\$2015.57
		Entertainment	\$5137.43
		Food	\$18 070.22
		Insurance	\$2418.56
		Loan repayments	\$17 293.02
		Motor vehicle costs	\$2532.67
		Telephone	\$857.97
		Work related costs	\$837.05
		Balance	
Total		Total	

Extended-response questions

- 8A 11** While shopping on Monday, Madeleine found a dress priced at \$350. On Wednesday, Madeleine noticed the dress was marked down by 20%.
- a** What is the marked-down price of the dress?
 - b** When Madeleine enquired about the dress, she was offered a further \$40 discount off the marked-down price. She bought the dress. What was Madeleine's saving as a percentage of the original price of the dress? Answer correct to one decimal place.
- 8G 12** Blake took out a loan to purchase a car and will make regular monthly payments over 5 years. However, owning and running a car is not cheap:
- Loan repayments are \$680 per month.
 - Car registration costs \$900 per year.
 - Servicing costs \$725 twice a year.
 - A green slip costs \$410 per year.
 - Comprehensive insurance costs \$190 per month.
 - Petrol costs \$80 per week.
- a** How much will Blake eventually pay on the loan?
 - b** What is the cost of the car, given that 40% of the loan repayments cover the interest?
 - c** What is the total annual cost during the purchase period?

Networks, paths and trees

The “Wood Wide Web”

The mycelium network is the largest natural network on Earth. This underground fungal system connects trees and plants, earning it the nickname “Wood Wide Web”. Spreading through the soil, these structures can extend for miles, acting as vast underground highways.

Through this network, the roots of plants and trees are linked, enabling them to share nutrients and even communicate. Trees can send chemical signals through the mycelium to warn nearby plants of pests or diseases,

helping neighbouring plants bolster their defences. Scientists refer to large, older trees as “mother trees” because they provide nutrients to saplings and other plants nearby through the fungal network.

In some forests, a single mycelium network can cover several square miles, making it one of the most extensive and intricate biological systems on the planet. Also, scientists have discovered a species of mycelium that is eating the radiation at Chernobyl.

Chapter contents

- 9A** Networks
- 9B** Names for journeys through networks
- 9C** Constructing a network diagram
- 9D** Eulerian circuits and Hamiltonian walks (EXTENDING)
- 9E** Solving problems involving network diagrams
- 9F** Spanning trees: Prim's algorithm and Kruskal's algorithm
- 9G** Solving minimal connector problems
- 9H** Shortest path through a network

NSW Syllabus

In this chapter, a student:

- Develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (**MAO-WM-01**)
- Applies network techniques to solve network problems (**MST-11-07**)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

9A Networks

Learning intentions

- To learn and understand the terminology associated with networks
- To interpret and draw network diagrams
- To solve problems involving networks

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12 Standard 2
- Questions about networks regularly appear in HSC Examinations

A network is a term to describe a group or system of interconnected objects. There are many situations in everyday life that involve connections between objects. Cities are connected by roads, computers are connected to the internet and people connect to each other through being friends on social media. Figure A shows the distances in kilometres between some NSW cities. This is referred to as a network or a network diagram. Note that the lengths of the lines in a network diagram are not generally drawn to scale.



 **Networks: Basic concepts** Watch the video in the Interactive Textbook for an illustration of the terms and concepts in action.

The common terms used in a network are described below.

- A **network diagram** is a representation of a group of objects called vertices that are connected together by lines. Network diagrams are also called **graphs**.
- A **vertex** (plural: vertices) is a point (or dot) in a network diagram at which lines of pathways intersect or branch. In Figure A, the names of the cities are the vertices. Vertices are also called nodes.
- An **edge** is the line that connects the vertices. In Figure B, the line marked with 560 is an edge. Edges can cross each other without intersecting at a node.

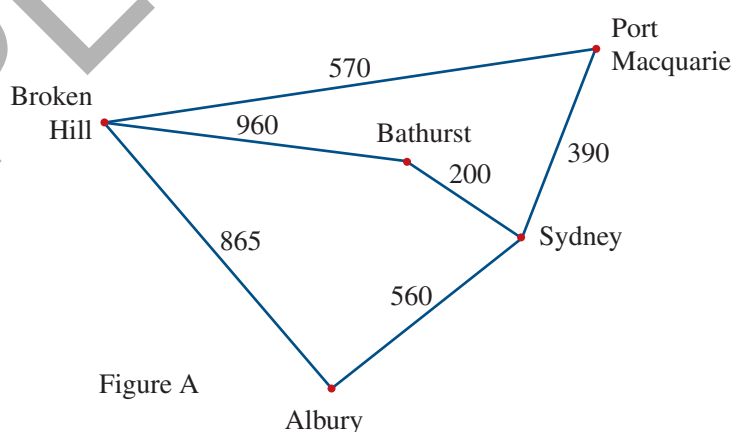


Figure A

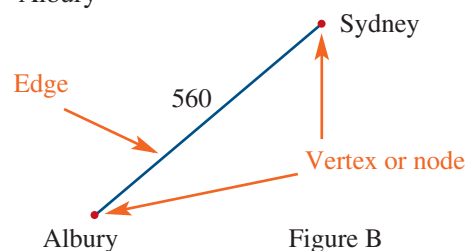


Figure B

- The **degree** of a vertex is the number of edges that are connected to it. In Figure C, the degree of the Broken Hill vertex is 3 because there are three edges attached to the vertex. This is written as $\text{deg}(\text{Broken Hill}) = 3$.

The degree of a vertex is either even or odd.

- The degree of a vertex is even if it has an even number of edges attached to the vertex. For example, in Figure A the vertices of even degree are Port Macquarie, Bathurst and Albury.
- The degree of a vertex is odd if it has an odd number of edges attached to the vertex. For example, in Figure A the vertices of odd degree are Sydney and Broken Hill.

- There can be multiple edges between vertices, as shown.

- A **loop** starts and ends at the same vertex as shown in Figure D. It counts as one edge, but it contributes two to the degree of the vertex.

- A **directed edge**, also called an arc, has an arrow and travel is only possible in the direction of the arrow. An **undirected edge** has no arrow and travel is possible in both directions. A network or graph may have both directed and undirected edges.

- In a **directed network** or graph all the edges are directed, as in Figure E, which has five vertices and six edges (arcs). It shows a path can be taken from A to B to C, however there is no path from C to B to A.

- In an **undirected network** or graph all the edges are undirected and travel on an edge is possible in both directions. Figure F is an undirected graph with five vertices and six edges. It shows that there is a path from A to B and from B to A.

- In a **simple network** like Figure F there are no multiple edges or loops.

- Labelling of vertices:** In addition to labelling vertices on a diagram, 'labelling of vertices' in a network means listing them all in curly brackets like this, using Figure F as an example: $V = \{A, B, C, D, E\}$.

- Labelling of edges:** An edge between vertex A and B would be labelled (A, B) . A loop at B would be (B, B) . A complete list of edges for Figure F would be $E = (A, B), (B, C), (B, D), (C, D), (C, E), (A, E)$.

- A **weighted edge** is an edge of a network diagram that has a number assigned to it that implies some numerical value such as cost, distance or time. Figure G shows a weighted edge that indicates a distance of 200 km between Sydney and Bathurst. See also Figure A.



Figure D A

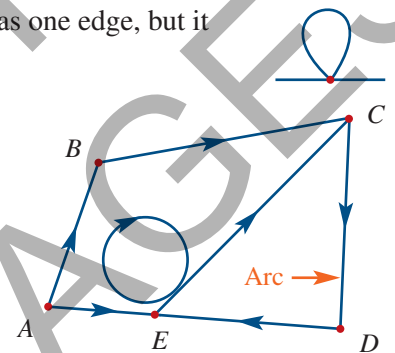


Figure E

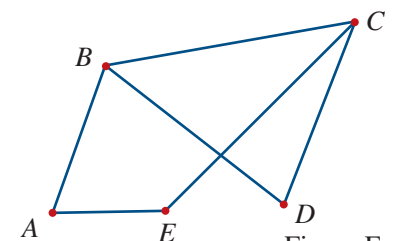


Figure F

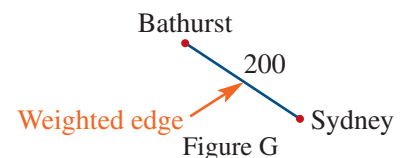


Figure G

Network

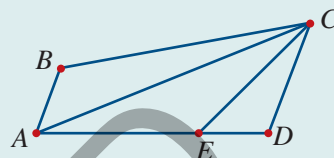
A network is a term to describe a group or system of interconnected objects. It consists of vertices and edges. The edges indicate a path or route between two vertices.



Example 1 Identifying and using network terminology

For the network shown, find the:

- a** number of vertices
- b** number of edges
- c** degree of vertex C
- d** number of vertices of odd degree.



Solution

- a** Five vertices
- b** Seven edges
- c** $\deg(C) = 4$
- d** $A3, B2, C4, D2, E3$
Two vertices of odd degree (A, E).

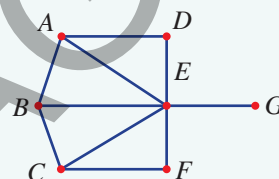
Explanation

Count the dots in the network diagram.
Count the lines in the network diagram.
Count the number of edges connected to C .
Count the number of edges for each vertex.
List the vertices of odd degree.

Now you try

For the network shown, find the:

- a** number of vertices
- b** number of edges
- c** degree of vertex C
- d** number of vertices of odd degree.



Exercise 9A

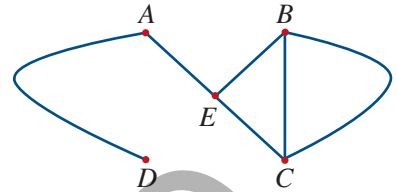
BUILDING

- 1** Copy and complete the following sentences.
 - a** A network is a term to describe a group or system of _____ objects.
 - b** In a network diagram the vertices are connected together by lines called _____.
 - c** A _____ is a point in a network diagram at which lines of pathways intersect.
 - d** The _____ of a vertex is the number of edges that are connected to it.
 - e** A directed graph is when the edges of a network have _____.
- 2** Describe the following statements as true or false.
 - a** Vertices are represented as a point or dot in a network diagram.
 - b** Directed networks are a connected sequence of the edges showing a route between vertices.
 - c** A loop starts and ends at the same vertex.
 - d** The degree of a vertex is either even or odd.
 - e** Degree of a vertex is odd if it has an odd number of vertices attached to the edges.
 - f** If an edge has a number assigned to it is called a directed edge.
 - g** The edges in a directed network are usually called arcs.

Example 1

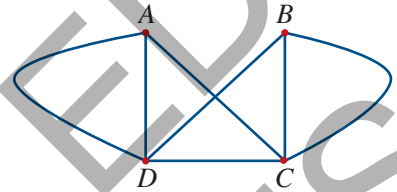
3 Using the network diagram shown find the:

a number of vertices	b number of edges
c degree of vertex A	d degree of vertex B
e degree of vertex C	f degree of vertex D
g degree of vertex E .	



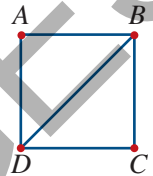
4 Using the network diagram shown find the:

a number of vertices	b number of edges
c degree of vertex A	d degree of vertex B
e degree of vertex C	f degree of vertex D .



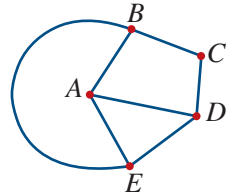
5 Using the network diagram shown find:

- | |
|---|
| a $\deg(A)$ |
| b $\deg(B)$ |
| c $\deg(C)$ |
| d $\deg(D)$ |
| e the sum of the degrees of all the vertices |
| f the number of edges. |



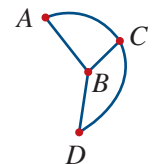
6 Using the network diagram shown find:

- | |
|---|
| a $\deg(A)$ |
| b $\deg(B)$ |
| c $\deg(C)$ |
| d $\deg(D)$ |
| e the sum of the degrees of all the vertices |
| f the number of edges. |



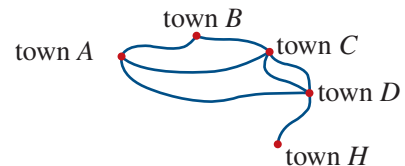
7 Using the network diagram shown find:

a $\deg(A)$	b $\deg(B)$	c $\deg(C)$	d $\deg(D)$
e the sum of the degrees of all the vertices			
f the number of edges.			



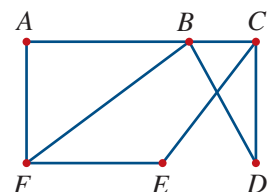
8 Find the degree of the following towns in the network diagram.

a A	b B	c C	d D



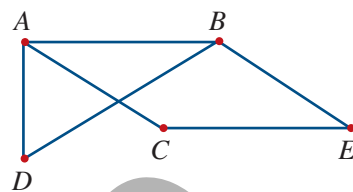
9 Using the network diagram shown find the:

- | |
|--|
| a vertex with the largest degree |
| b vertex with the smallest degree |
| c vertices with an even degree |
| d vertices with an odd degree. |



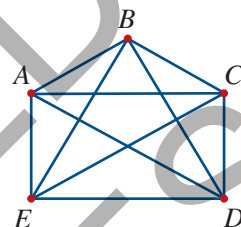
10 Using the network diagram shown find the:

- a** vertex with the largest degree
- b** vertex with the smallest degree
- c** vertices with an even degree
- d** vertices with an odd degree.



11 Using the network diagram shown find the:

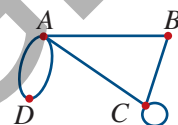
- a** vertex with the largest degree
- b** vertex with the smallest degree
- c** vertices with an even degree
- d** vertices with an odd degree.



DEVELOPING

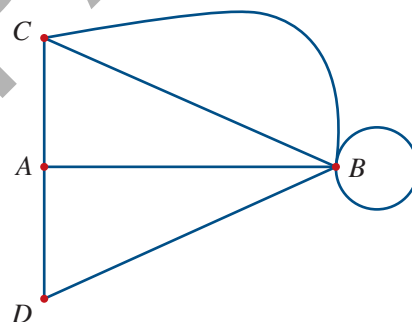
12 Using the network diagram shown find the:

- a** number of vertices
- b** number of edges
- c** degree of vertex A
- d** degree of vertex C.



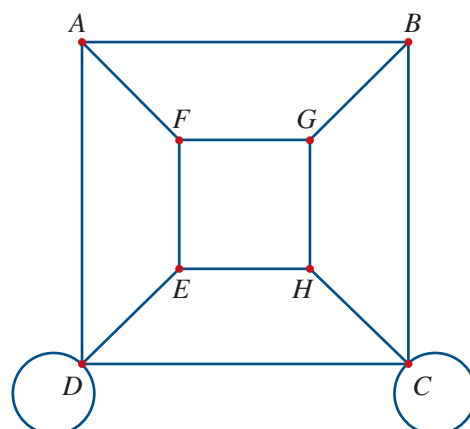
13 Using the network diagram shown find the:

- a** number of vertices
- b** number of edges
- c** degree of vertex B
- d** degree of vertex D
- e** number of vertices of odd degree
- f** number of vertices of even degree.



14 Using the network diagram shown find the:

- a** number of vertices
- b** number of edges
- c** degree of vertex A
- d** degree of vertex C
- e** degree of vertex F
- f** number of loops
- g** number of vertices of odd degree
- h** number of vertices of even degree.



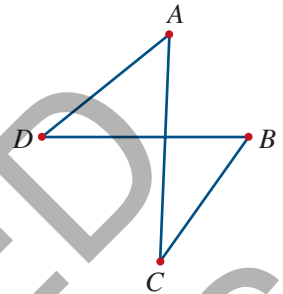
CHALLENGING

15 Consider the network diagram shown.

- a** How many edges in the diagram?
- b** What is the sum of the degrees of all the vertices?

A loop is now added at vertex A.

- c** How will this change the sum of the degrees of all the vertices?
- d** How many edges are added to the graph?

**Communicate your reasoning, understanding and fluency**

- 16** Draw a graph that has three vertices, two of which are of odd degree.
- 17** Why do you think that the sum of the vertex degrees of a graph will always equal twice the number of edges?
- 18** Can a graph be drawn with only one vertex of odd degree? If not, why not?

9B Names for journeys through networks

Learning intentions

- To learn and understand the terminology associated with networks
- To interpret and draw network diagrams
- To solve problems involving networks

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12 Standard 2
- Questions about networks regularly appear in HSC Examinations

Many practical problems, such as travel routes, that can be modelled by a network involve moving around a graph. To solve such problems you will need to know about a number of concepts to describe the different ways to travel a network.

- A **walk** is a connected sequence of the edges showing a route between vertices where the edges and vertices may be visited multiple times. When there is no ambiguity, a walk in a network diagram can be specified by listing the vertices visited on the walk.

For example, the network diagram shows a walk in a forest. The forest tracks are the edges (shown in blue) and the places in the forest are the vertices. The red arrows trace out a walk in the forest and is stated as:

$G1-B-F-G2-T-G2-F$

Note: ☒ repeated edges

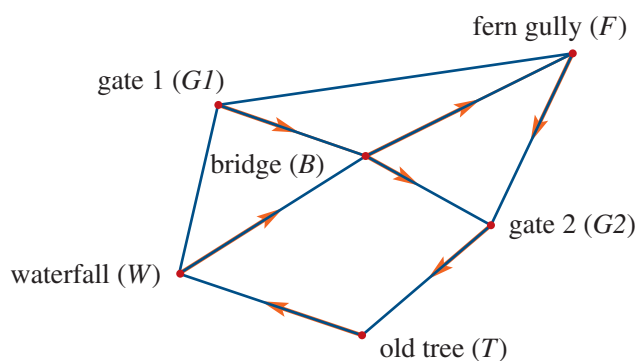
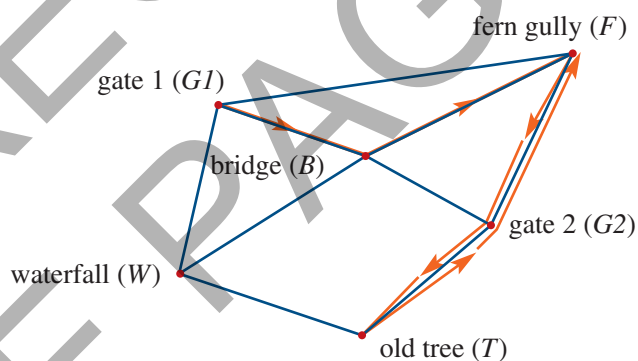
☒ repeated vertices

- A **trail** is a walk with no repeated edges. For example, the network diagram shows a trail in a forest. The red arrows trace out a trail in the forest and is stated as:

$G1-B-F-G2-T-W-B-G2$

Note: ☒ repeated edges

☒ repeated vertices



- A **path** is a walk with no repeated vertices. Open paths start and finish at different vertices while closed paths start and finish at the same vertex. Closed paths are also called cycles.

For example, the network diagram shows a path in a forest. The red arrows trace out a path in the forest and is stated as:

$G1-F-B-W-T-G2$

Note: ☒ repeated edges

☒ repeated vertices

- A **circuit** is a walk with no repeated edges that starts and ends at the same vertex. **Circuits are also called closed trails.** Alternatively, open trails start and finish at different vertices.

For example, the network diagram shows a circuit in a forest. The red arrows trace out a circuit in the forest and is stated as:

$G1-F-B-W-T-G2-B-G1$

Note: ☒ repeated edges

☒ repeated vertices (B)

☒ starts and ends at the same vertex

- A **cycle** is a walk with no repeated vertices that starts and ends at the same vertex. There are no repeated edges in a cycle as there are no repeated vertices. **Cycles are closed paths.**

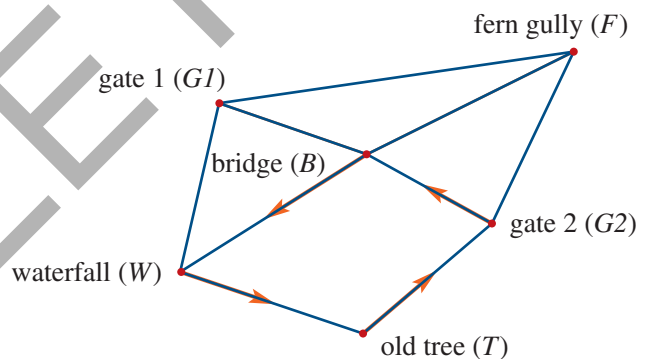
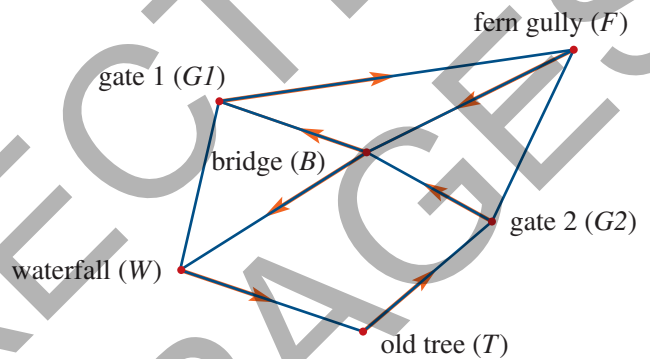
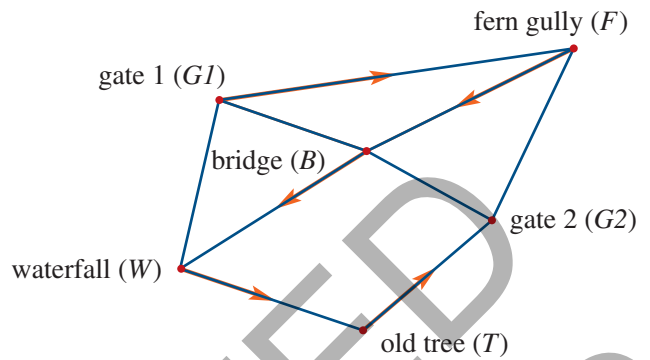
For example, the network diagram shows a cycle in a forest. The red arrows trace out a cycle in the forest and is stated as:

$G2-B-W-T-G2$

Note: ☒ repeated edges

☒ repeated vertices

☒ starts and ends at the same vertex



 **Travelling through a network** Watch the video in the Interactive Textbook to see the five types of routes that can be travelled through networks.

Travelling a network

Walk is a connected sequence of the edges showing a route between vertices and edges.

Trail is a walk with no repeated edges.

Path is a walk with no repeated vertices.

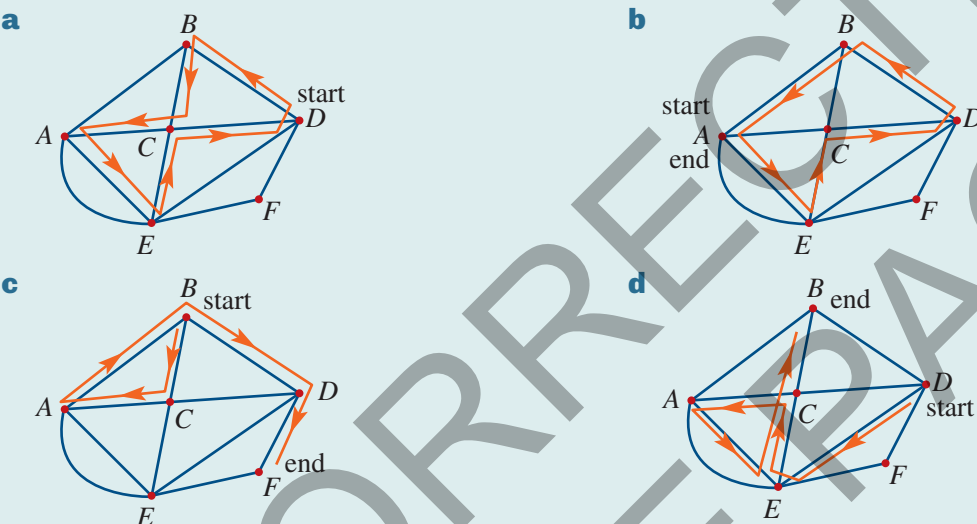
Circuit is a walk with no repeated edges that starts and ends at the same vertex.

Cycle is a walk with no repeated vertices that starts and ends at the same vertex.

Type of route	Are repeated edges permitted?	Are repeated vertices permitted?
Walk	Yes	Yes
Trail	No	Yes
Path	No	No
Circuit	No	Yes
Cycle	No	No (except first and last)

Example 2 Identifying walks, trails, paths, circuits and cycles

Identify the walk in each of the graphs shown as a trail, path, circuit, cycle or walk only.



Solution

a Circuit

b Cycle

c Trail

d Walk only

Explanation

This walk starts and ends at the same vertex without repeated edges so it is either a circuit or a cycle. The walk passes through vertex C twice without repeated edges, so it must be a circuit.

This walk starts and ends at the same vertex with no repeated edges so it is either a circuit or a cycle. The walk has no repeated vertex so it is a cycle.

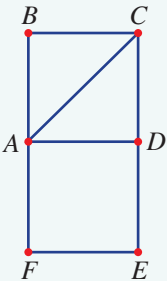
This walk starts at one vertex and ends at a different vertex, so it is not a circuit or a cycle. It has one repeated vertex (B) and no repeated edge, so it must be a trail.

This walk starts at one vertex and ends at a different vertex so it is not a circuit or a cycle. It has repeated vertices (C and E) and repeated edges (the edge between C and E), so it must be a walk only.

Now you try

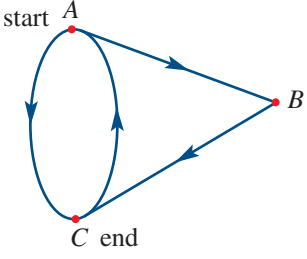
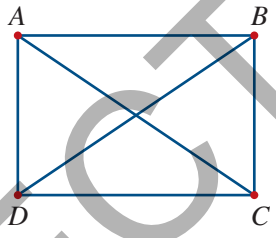
Use the graph, identify each walk as a trail, path, circuit, cycle or walk only.

- a** $ABCD A$
- b** $ABCA FED A$
- c** $FEDC$
- d** $ADCDA F$



Traversable graphs

Many practical problems involve finding a trail in a graph that includes every edge. You can trace out a trail on the graph without repeating an edge or taking the pen off the paper. Graphs that have this property are called traversable graphs.

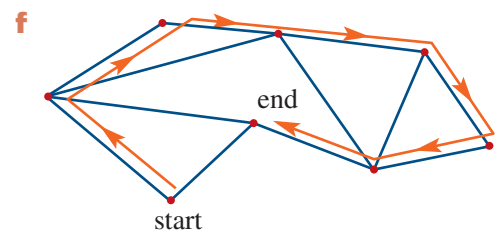
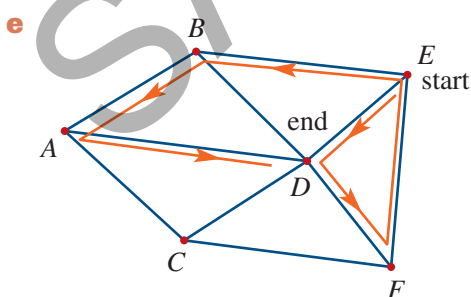
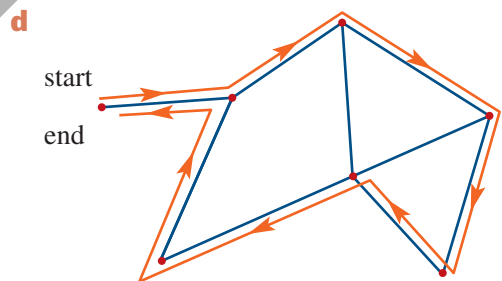
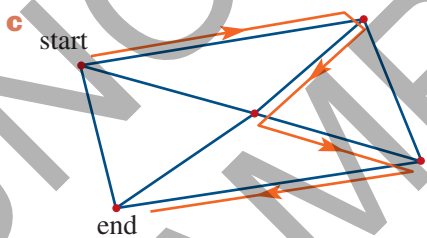
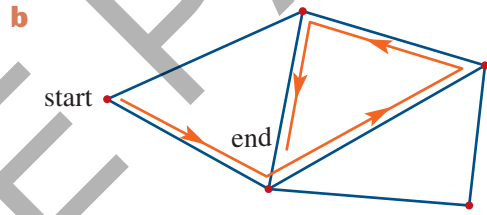
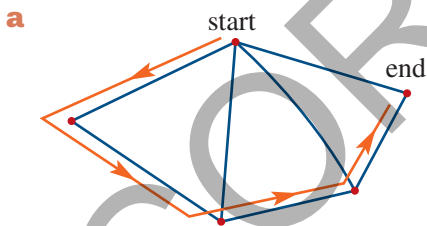
Traversable graph	Non-traversable graph
A traversable graph has a trail that includes every edge. The trail $A-B-C-A-C$ is one example. 	Not all graphs are traversable. It is impossible to find a trail in a non-traversable graph that includes every edge. 

Exercise 9B

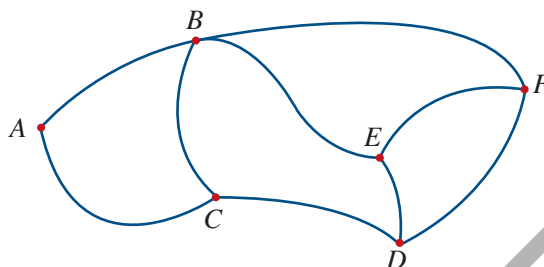
BUILDING

Example 2

- 1 Identify the walk in each of the graphs as a trail, path, circuit or walk only.



- 2 Using the graph shown, identify the walks listed as a trail, path, circuit, cycle or walk only.



- a $A-B-E-B-F$
- b $B-C-D-E-B$
- c $C-D-E-F-B-A$
- d $A-B-E-F-B-E-D$
- e $E-F-D-C-B$
- f $C-B-E-F-D-E-B-C-A$

DEVELOPING

- 3 The network diagram shows the pathway linking five animal enclosures in a zoo to each other and to the kiosk.

- a Which of the following represents a trail?

- A $S-L-K-M-K$
- B $G-K-L-S-E-K-M$
- C $E-K-L-K$

- b Which of the following represents a path?

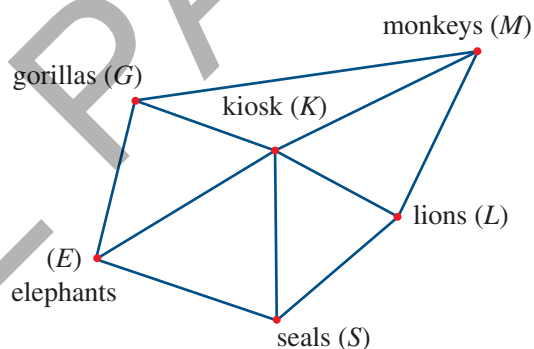
- A $K-E-G-M-L$
- B $E-K-L-M$
- C $K-S-E-K-G-M$

- c Which of the following represents a circuit?

- A $K-E-G-M-K-L-K$
- B $E-S-K-L-M-K-E$
- C $K-S-E-K-G-K$

- d Which of the following represents a cycle?

- A $K-E-G-K$
- B $G-K-M-L-K-G$
- C $L-S-E-K-L$



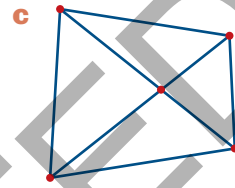
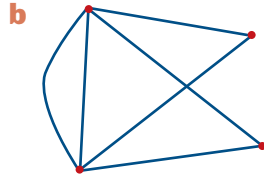
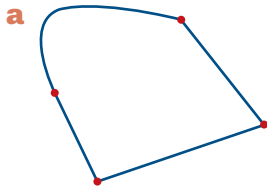
- 4 Identify the following sequence of vertices as either a trail or cycle.

- a $C-B-E-A-F-E-G-D$
- b $D-E-A-F-C-B-D$

CHALLENGING

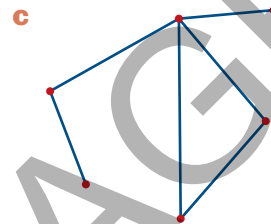
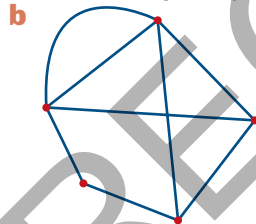
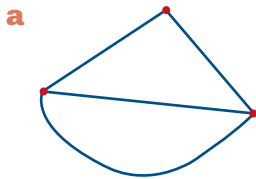
5 For each graph:

- i identify whether it is traversable
- ii if it is traversable, identify a circuit including every edge.



6 For each graph:

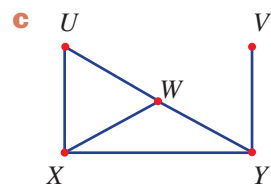
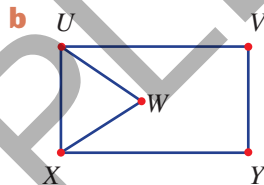
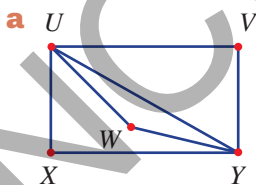
- i identify whether it is traversable
- ii if it is traversable, identify a circuit including every edge.



Communicate your reasoning, understanding and fluency

7 For each of the following graphs:

- i identify if it is traversable
- ii if it is traversable, find a trail or circuit including every edge
- iii find the number of odd-degree vertices.



8 Draw a non-traversable graph with exactly three vertices.

9C Constructing a network diagram

Learning intentions

- To learn and understand the terminology associated with networks
- To interpret and draw network diagrams and tables
- To solve problems involving networks

Past, present and future learning

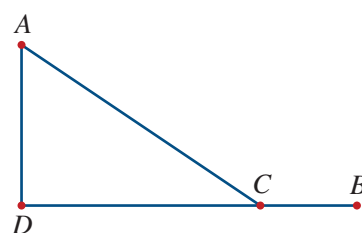
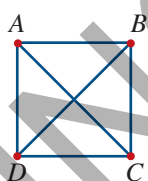
- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12 Standard 2
- Questions about network diagrams regularly appear in HSC Examinations

There are many situations in everyday life that involve connections between people or objects. Towns are connected by roads, computers are connected to the internet and families are connected to each other. The network diagram in the image demonstrates some of the connections on social media. When constructing a network, the graphs are either connected or not connected.

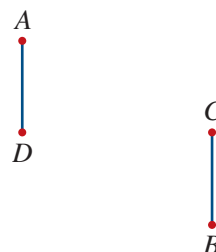
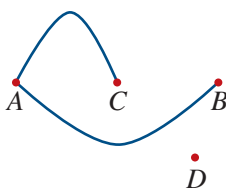
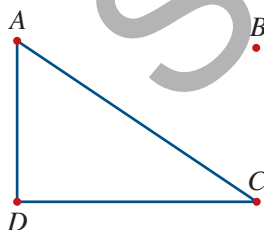


Connected graphs

A connected graph has every vertex connected to every other vertex, either directly or indirectly via other vertices. That is, every vertex in the graph can be reached from every other vertex in the graph. The three graphs shown below are all connected.



The graphs are connected because, starting at any vertex, say A, you can always find a path along the edges of the graph to take you to every other vertex. However, the three graphs below are not connected, because there is not a path along the edges that connects vertex A (for example) to every other vertex in the graph.

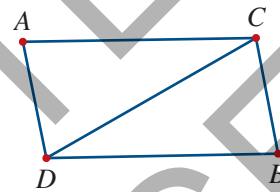
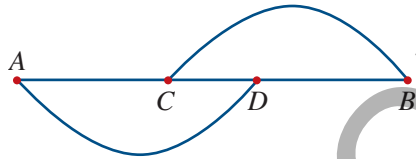
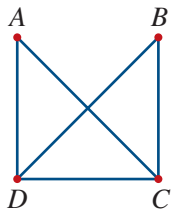


Connected graph

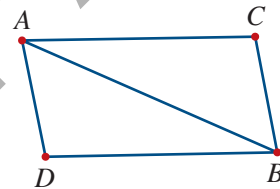
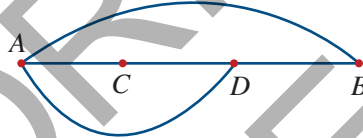
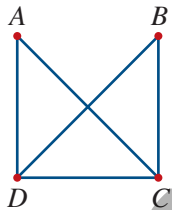
A graph is connected if every vertex in the graph is accessible from every other vertex in the graph along a path formed by the edges of the graph.

Isomorphic graphs

Different looking graphs can contain the same information. When this happens, we say that these graphs are equivalent or isomorphic. For example, the following three graphs look quite different but, in graphical terms, they are equivalent.



Each of the above graphs has the same number of edges (5), vertices (4), the corresponding vertices have the same degree and the edges join the vertices in the same way (A to C, A to D, B to C, B to D, and D to C). However, the three graphs below, although having the same numbers of edges and vertices, are not isomorphic. This is because corresponding vertices do not have the same degree and the edges do not connect the same vertices.



Isomorphic graphs

Two graphs are isomorphic (equivalent) if:

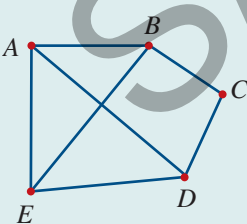
- they have the same numbers of edges and vertices
- corresponding vertices have the same degree and the edges connect to the same vertices.



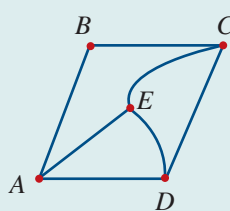
Example 3 Identifying an isomorphic graph

Which of the following graphs is not isomorphic to the other three graphs?

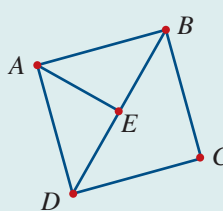
1



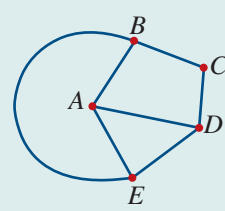
2



3



4



continued ...

Solution

Every graph has five vertices and seven edges.

In Graph 2, vertex B has degree 2 and C has degree 3; in all others, B has degree 3 and C has degree 2.

In graphs 1, 3 and 4, the edges are $A-B$, $A-D$, $A-E$, $B-C$, $B-E$, $C-D$ and $D-E$, so these graphs are isomorphic.

Graph 2 does not have edge $B-E$ and does have edge $C-E$, which does not appear in the other graphs, showing again that it is not isomorphic to the others.

Hence, graph 2 cannot be isomorphic to any of the other graphs shown.

Explanation

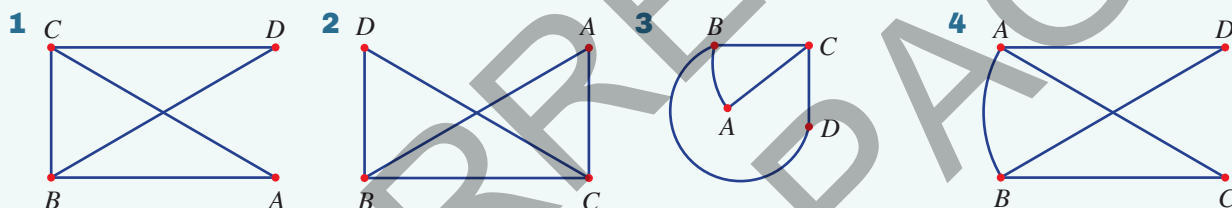
Check that each graph has the same number of vertices and edges.

Check that corresponding vertices have the same degree.

Check that edges connect to the same vertices.

Now you try

Which of the following graphs is not isomorphic to the other three graphs?



Example 4 Drawing a network diagram to represent a map

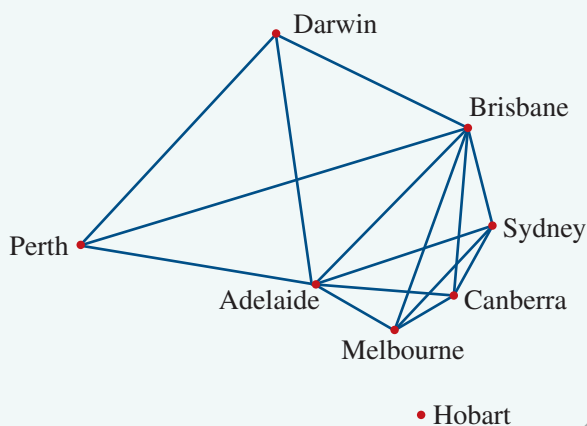


The map shows the main highways between the capital cities of the states and territories of Australia.

Construct a network diagram of the main highway connections between the cities. Let each capital city be a vertex and the highway route between the cities an edge.

Solution

The vertices will be Brisbane, Sydney, Canberra, Melbourne, Hobart, Adelaide, Perth and Darwin. Highway routes connect the cities with the exception of Hobart.

**Explanation**

Vertices are the dots of the network diagram. In this situation the capital city will be a vertex.

Edges are the connections or pathways between the vertices of the network diagram.

Start by drawing a dot for each vertex (capital city). Label each vertex with the name of the capital city. Draw a line to represent an edge if the capital cities have a highway route between them.

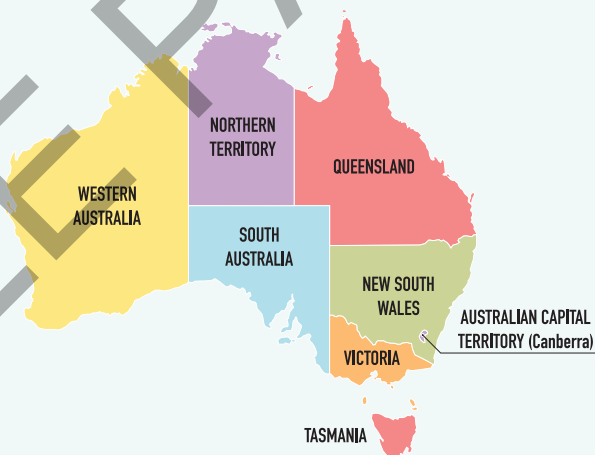
The network diagram is not connected, as Hobart does not have a highway linking it to any other city. The proportions of the network diagram do not have to match the real-life situation.

Now you try

The map shows the states and territories of Australia.

Construct a network diagram between the states and territories.

Let each state and territory be a vertex and the edges be a shared border between the states and territories.

STATES OF AUSTRALIA**Weighted graphs**

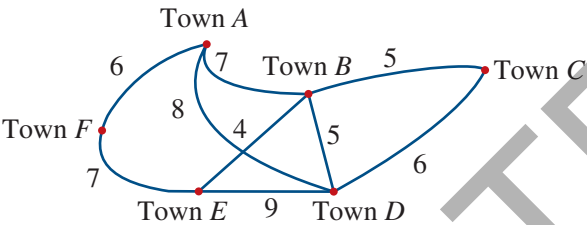
The edges of graphs represent connections between the vertices. Sometimes there is more information known about that connection. If the edge of a graph represents a road between two towns, we might also know the length of this road, or the time it takes to travel this road. Extra numerical information about the edge that connects vertices can be added to a graph by writing the number next to the edge. This is called a weighted edge. Graphs that have a number associated with each edge are called weighted graphs.

Weighted graph

A weighted graph is a network diagram that has weighted edges; that is, edges with numbers assigned that imply some numerical value such as cost, distance or time.

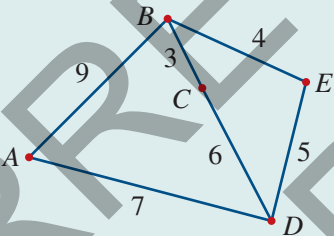
The weighted graph in the diagram shows towns, represented by vertices, and the roads between those towns, represented by edges. The numbers, or weights, on the edges are the distances along the roads. A problem often presented by this road network is, ‘What is the shortest distance between certain towns?’

While this question is easily answered if all the towns are directly connected such as Town A to Town B, the answer is not so obvious if we have to travel through towns to get there such as Town F to Town C.



Example 5 Identifying and constructing weighted graphs

Construct a table from this weighted graph.



Solution

Headings are A, B, C, D and E. Insert 9 in the cell for AB and BA.

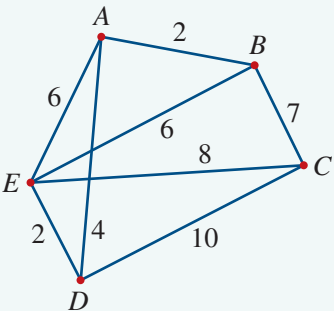
	A	B	C	D	E
A	–	9		7	
B	9	–	3	–	4
C	–	3	–	6	–
D	7	–	6	–	5
E	–	4	–	5	–

Explanation

The vertices are the headings for the rows and columns. A weighted edge is a value in the table.
Repeat the above step for each weighted edge until all the weighted edges have been entered into the table.

Now you try

Construct a table from this weighted graph.





Example 6 Drawing a weighted graph from a table

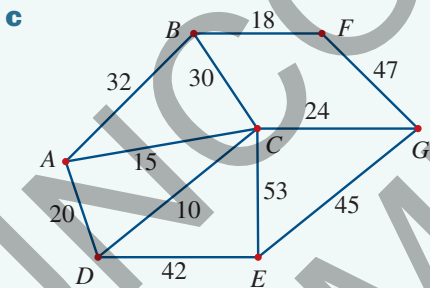
A network diagram is to be constructed to represent the table shown.

	A	B	C	D	E	F	G
A	–	32	15	20	–	–	–
B	32	–	30	–	–	18	–
C	15	30	–	10	53	–	24
D	20	–	10	–	42	–	–
E	–	–	53	42	–	–	45
F	–	18	–	–	–	–	47
G	–	–	24	–	45	47	–

- What will be vertices of the network diagram?
- What will be the values of the weighted edges in the network diagram?
- Draw a weighted graph to represent the information in the table.
- What is the total of the weighted path from A to E via B, C and G?

Solution

- Vertices are A, B, C, D, E, F and G.
- Weighted edges: $AB = 32$, $AC = 15$, $AD = 20$, $BC = 30$, $BF = 18$, $CD = 10$, $CE = 53$, $CG = 24$, $DE = 42$, $EG = 45$, and $FG = 47$



- $32 + 30 + 24 + 45 = 131$

Explanation

Vertices are the headings of the rows and columns in the table.

Edges are the values in each cell of the table.

Note: The table is symmetrical with 22 values. The number of weighted edges is 11. That is AB is the same BA.

Start by drawing a dot for each vertex (letter). Label each vertex.

Draw a line between two vertices if there is a weighted edge.

Label the weighted edge with the correct value.

Note: There can be slightly different weighted graphs drawn from the table. For example, a rotation or reflection of the diagram.

Add up the weights for (A, B), (B, C), (C, G), (G, E).

Now you try

A network diagram is to be constructed to represent the table shown.

- What will be the vertices of the network diagram?
- What will be the values of the weighted edges in the network diagram?
- Draw a weighted graph to represent the information in the table.

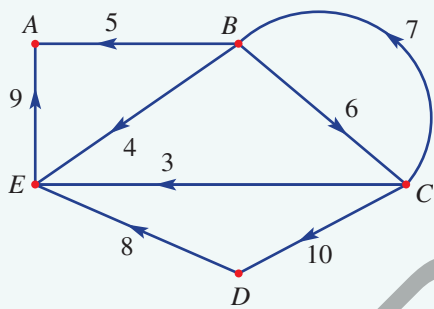
	L	M	P	Q	R
L	–	45	–	38	–
M	45	–	44	–	32
P	–	44	–	30	20
Q	38	–	30	–	35
R	–	32	20	35	–

Example 7 Constructing a directed weighted graph from a table

Construct a directed weighted network diagram to represent the information in the table.

		To				
		A	B	C	D	E
From	A					
	B	5		6		4
	C		7		10	3
	D					8
	E	9				

Solution



Explanation

Start with labelling your vertices A, B, C, D, E .
Draw a directed line:
From B to A weighted 5
From B to C weighted 6
From B to E weighted 4
From C to B weighted 7
From C to D weighted 10
From C to E weighted 3
From D to E weighted 8
From E to A weighted 9

Now you try

Construct a directed weighted network diagram to represent the information in the table shown.

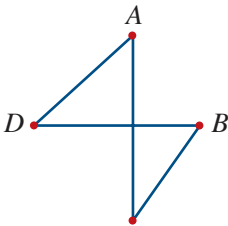
		To				
		A	B	C	D	E
From	A		12		15	7
	B					
	C		6			
	D		3	8		
	E	4				

Exercise 9C

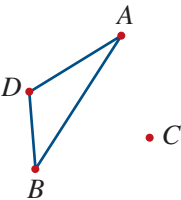
BUILDING

1 Which of the following graphs are connected in each question?

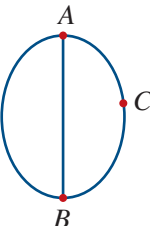
a Graph 1



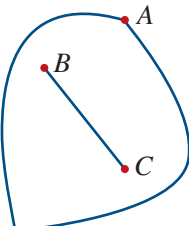
Graph 2

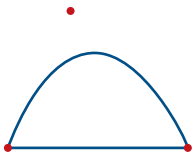


Graph 3

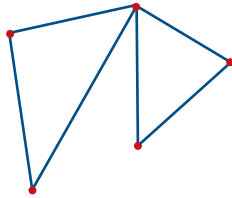


Graph 4

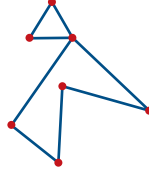


b Graph 1

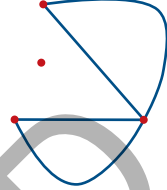
Graph 2



Graph 3



Graph 4



2 Draw a connected graph with:

- a** three vertices and three edges
- c** four vertices and six edges

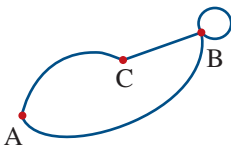
- b** three vertices and five edges
- d** five vertices and five edges.

3 Draw a graph that is not connected with:

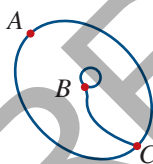
- a** three vertices and two edges
- c** four vertices and four edges

- b** four vertices and three edges
- d** five vertices and three edges.

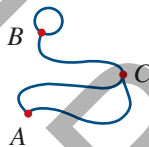
4 Which of the following graphs is not isomorphic to the other three graphs in each question?

a Graph 1

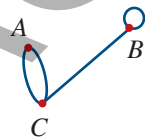
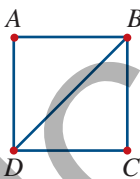
Graph 2



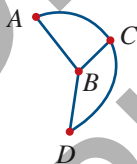
Graph 3



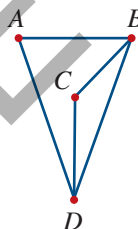
Graph 4

**b** Graph 1

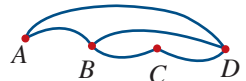
Graph 2



Graph 3



Graph 4

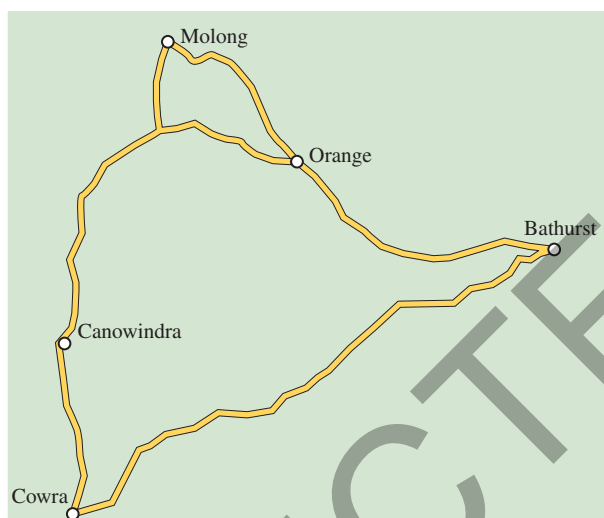


5 This is a network diagram for Sydney trains.

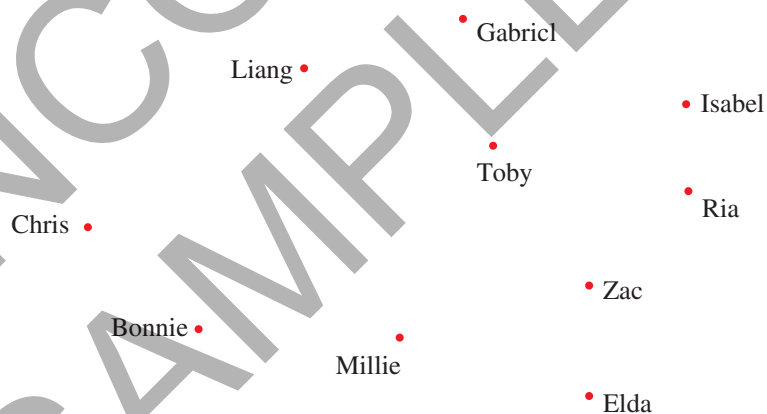
- a** What are the vertices of the network diagram?
- b** What are the edges in the network diagram?
- c** Is this network diagram connected?
- d** How many vertices are there in the city circle (shaded section)?
- e** Draw a network diagram to represent the city circle.



- 6** A map from a NSW region is shown.



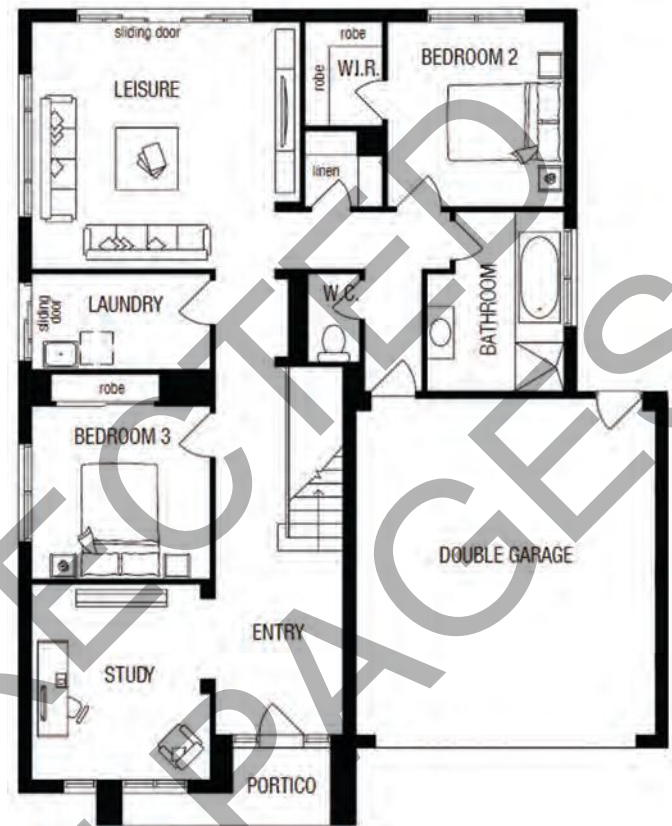
- a** What are the vertices of the network diagram?
b What are the edges in the network diagram?
c Is this network diagram connected?
d Draw an isomorphic graph to this network diagram.
- 7** A social network exists between ten students.
- Liang is friends with Millie, Bonnie, Chris, Gabriel and Toby.
 - Millie is friends with Liang, Elda and Zac.
 - Toby is friends with Liang, Isabel, Ria and Chris.
- a** Complete the network diagram shown by inserting the edges.



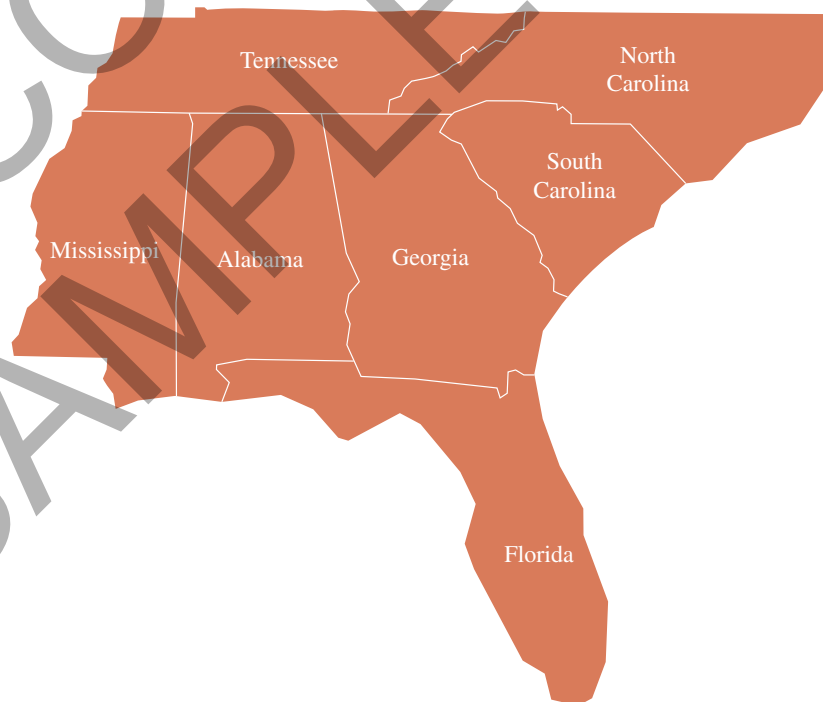
- b** What do the edges of the network diagram represent?
c Is this network diagram connected?
d How is Bonnie connected to Ria?

DEVELOPING

- 8 The first floor plan of a house is shown. Draw a network diagram by letting the rooms be the vertices and the doorways the edges. Make the hall be a vertex.



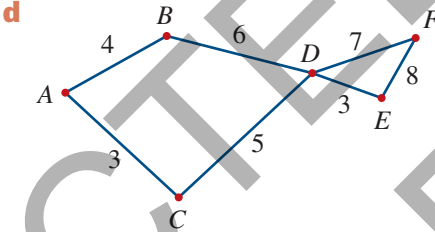
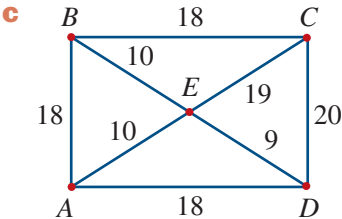
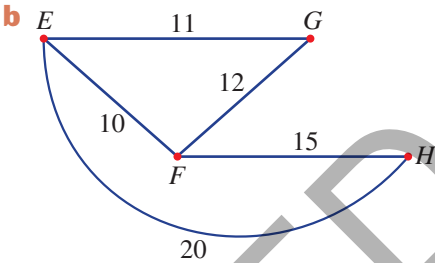
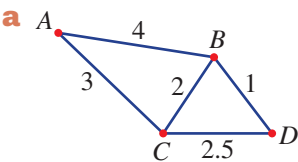
- 9 This is a map of the south-eastern states in the United States.



Using Tennessee, South Carolina, North Carolina, Georgia, Alabama, Mississippi and Florida as the vertices and the edges be a shared border between the states, construct a network diagram between the south-eastern states of the United States.

Example 5

10 Construct a table for each weighted graph.



Example 6

11 Four friends live close to each other. The table shows the friends and the number of minutes to walk between their homes.

- a Draw a network diagram to represent the information in the table.
- b What are the vertices of the network diagram?
- c What does a weighted edge represent in the network diagram?
- d Which friends do not have a direct path between their homes?
- e What is the shortest total walking time for Alex to leave home and visit Zac, Madan and Hui in that order, and to return home. Ignore any time spent in each house.

Friends		Minutes to walk between homes
Alex	Zac	1
Hui	Zac	3
Madan	Zac	2
Hui	Madan	4
Alex	Hui	2

12 There are six motorways between six cities labelled A, B, C, D, E and F . The table shows which cities are linked by the motorways and the length of each one in kilometres.

- a Draw a network diagram to represent the information in the table.
- b What are the vertices of the network diagram?
- c What does a weighted edge represent in the network diagram?
- d Which cities are not directly linked to city A ?
- e How would you travel from city F to city D ?
- f What is the shortest journey between city F and city D ?

	A	B	C	D	E	F
A	–	–	–	27	51	35
B	–	–	48	24	–	–
C	–	48	–	12	–	–
D	27	24	12	–	–	–
E	51	–	–	–	–	–
F	35	–	–	–	–	–

- 13** Construct a network diagram with weighted edges to represent the information in each table.

a

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>A</i>	–	4	–	6
<i>B</i>	4	–	7	–
<i>C</i>	–	7	–	4
<i>D</i>	6	–	4	–

b

	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>
<i>J</i>	–	16	8	12
<i>K</i>	16	–	10	6
<i>L</i>	8	10	–	–
<i>M</i>	12	6	–	–

c

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
<i>A</i>	–	40	–	38	28
<i>B</i>	40	–	35	–	50
<i>C</i>	–	35	–	20	20
<i>D</i>	38	–	20	–	15
<i>E</i>	28	50	20	15	–

d

	<i>L</i>	<i>M</i>	<i>P</i>	<i>Q</i>	<i>R</i>
<i>L</i>	–	7	8	9	–
<i>M</i>	7	–	6	–	–
<i>P</i>	8	6	–	10	–
<i>Q</i>	9	–	10	–	11
<i>R</i>	–	–	–	11	–

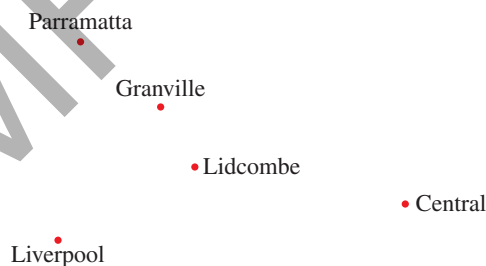
- 14** A regional airline operates flights in Queensland. Flights between connected towns are shown in the table in minutes.

	Cairns	Brisbane	Gold Coast	Townsville	Toowoomba
Cairns	–	140	145	55	–
Brisbane	140	–	–	120	40
Gold Coast	145	–	–	–	40
Townsville	55	120	–	–	–
Toowoomba	–	40	40	–	–

Draw a weighted network diagram to show how the towns are connected, using edges to show flight times.

CHALLENGING

- 15** This is a map of five train stations in the Sydney Rail network.



Use the information in the table to show how the stations are connected by a network diagram with weighted edges showing the travel times between stations, in minutes.

	Liverpool	Parramatta	Granville	Lidcombe	Central
Liverpool	–	–	22	26	68
Parramatta	–	–	4	–	–
Granville	22	4	–	6	27
Lidcombe	26	–	6	–	25
Central	68	–	27	25	–

16 The table describes the friendships between six people. For example, Natalie is friends with Amy, Harvey and Gabriel. Construct a network diagram to represent this table.

Person	Friendships		
Natalie	Amy	Harvey	Gabriel
Amy	Natalie	Bonnie	Louis
Bonnie	Harvey	Louis	Amy

Example 7 17 Construct a network diagram with weighted and directed edges from each table.

a

		To			
		A	B	C	D
From	A		8	7	9
	B				
	C	6			
	D	10			

b

		To				
		A	B	C	D	E
From	A		2	6		
	B				5	
	C					9
	D				8	7
	E					

c

		To				
		A	B	C	D	E
From	A		12			
	B			14	17	16
	C					13
	D					15
	E					

d

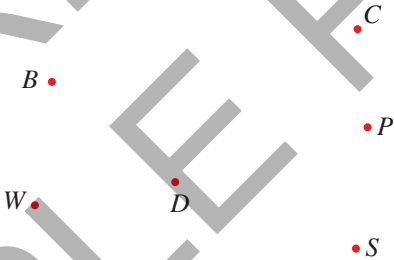
		To				
		A	B	C	D	E
From	A		6	6		
	B				8	9
	C					10
	D					
	E				12	

Communicate your reasoning, understanding and fluency

- 18 Draw a network diagram of a local area network (LAN) of four computers connected through a server to a printer and scanner.
- 19 The table shows distances, in kilometres, between various towns in NSW by car.

	Bourke (B)	Coffs Harbour (C)	Dubbo (D)	Port Macquarie (P)	Sydney (S)	Wilcannia (W)
Bourke		860	370			
Coffs Harbour	860			150	–	
Dubbo	370	–			390	560
Port Macquarie		150	–		395	
Sydney			390	395		
Wilcannia	–		560		–	

- a Use the information in the table to complete the network diagram where the edges are labelled with distances.



- b Marguerite wants to travel from Sydney to Bourke. Which is the quickest way to travel? Explain your answer.

- 20 The table shows the relationships between seven people. For example, Alex is friends with Charlie, Ethan and Fiona.

Person	Friends
Alex	Charlie, Ethan, Fiona
Danielle	Beatrice, Ethan, Charlie
Fiona	Alex, Beatrice, Gary

Draw a network diagram to represent this table.

9D

Eulerian circuits and Hamiltonian walks

(EXTENDING)

Learning intentions

- To learn and understand the terminology associated with networks
- To interpret and draw network diagrams
- To solve problems involving networks

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12 Standard 2
- Questions about networks regularly appear in HSC Examinations, but these concepts may be beyond the scope of the syllabus

Eulerian trails and circuits

An Eulerian trail is a trail that uses every edge of a graph exactly once. Eulerian trails start and end at different vertices. Similarly, an Eulerian circuit is a circuit that uses every edge of a graph exactly once. Eulerian circuits start and end at the same vertex. Eulerian trails and circuits are important for some real-life applications. For example, if a graph shows towns as vertices and roads as edges, then being able to identify a route through the graph that follows every road is important for mail delivery. A graph with an Eulerian trail is an example of a traversable graph.

An Eulerian trail will exist if the graph is connected and has exactly two vertices with an odd degree. These two vertices of odd degree will form the start and end of the Eulerian trail. Eulerian circuits will exist if every vertex of the graph has an even degree. These results were discovered by the Swiss mathematician called Leonhard Euler.

Eulerian trail	Eulerian circuit
A trail that uses every edge of a graph exactly once and starts and ends at different vertices. Eulerian trails exist if the graph has exactly two vertices with an odd degree.	A circuit that uses every edge of a network graph exactly once, and starts and ends at the same vertex. Eulerian circuits exist if every vertex of the graph has an even degree.

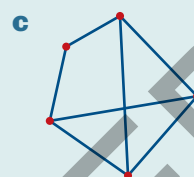
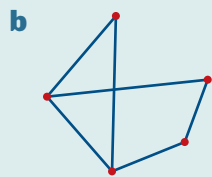
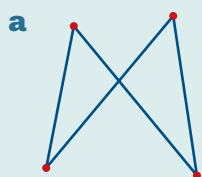


Eulerian trails and circuits Watch the video in the Interactive Textbook to see them in action.



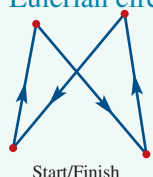
Example 8 Identifying Eulerian trails and circuits

For each graph, determine whether the graph has an Eulerian trail, an Eulerian circuit or neither. Show one example if the graph has an Eulerian trail or Eulerian circuit.

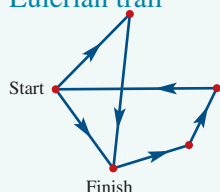


Solution

a Eulerian circuit



b Eulerian trail



c Neither

Explanation

All the vertices in the graph have an even degree (degree 2). Even degree indicates there is an Eulerian circuit.

Start and finish from the vertex on the bottom left-hand side and travel through each edge once.

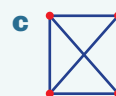
Two of the vertices in the graph have an odd degree (degree 3) and the remaining vertices have an even degree (degree 2). Two odd degrees indicates there is an Eulerian trail.

Start and finish from the vertices with the odd degrees.

Four vertices are odd and one is even. No Eulerian trail or circuit.

Now you try

For each graph, determine whether the graph has an Eulerian trail, an Eulerian circuit or neither. Explain why.



ENRICHMENT: Hamiltonian paths and cycles

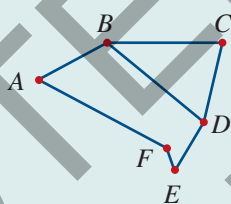
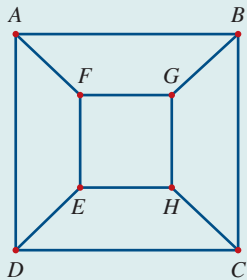
Eulerian trails and circuits are focused on the edges (though the degree of the vertices will tell you if walk is Eulerian). Hamiltonian paths and cycles are focused on the vertices. A Hamiltonian path passes through every vertex of a graph once and only once. It may or may not involve all the edges of the graph. A Hamiltonian cycle is a Hamiltonian path that starts and finishes at the same vertex. Hamiltonian paths and cycles have real-life applications where every vertex of a graph needs to be visited, but the route taken is not important. For example, if the vertices of a graph represent people and the edges of the graph represent email connections between those people, a Hamiltonian path would ensure that every person in the graph received the email message. Unlike Eulerian trails and circuits, Hamiltonian paths and cycles do not have a convenient rule or feature that identifies them. Inspection is the only way to identify them. Hamiltonian paths and cycles are named after an Irish mathematician called Sir William Hamilton.

Hamiltonian path	Hamiltonian cycle
A path passes through every vertex of a graph once and only once.	A Hamiltonian path that starts and finishes at the same vertex.



Example 9 Identifying a Hamiltonian path and cycle

- a** List a Hamiltonian path for the network graph shown that starts at *A* and finishes at *D*. **b** Identify a Hamiltonian cycle for the network graph shown starting at *A*.



Solution

a *A-F-G-B-C-H-E-D*

b *A-B-C-D-E-F-A*

Explanation

A Hamiltonian path involves all the vertices but not necessarily all the edges.
The solution *A-F-G-B-C-H-E-D* is not unique. There are other solutions such as *A-F-E-H-G-B-C-D*.
A Hamiltonian circuit is a Hamiltonian path that starts and finishes at the same vertex.
The solution *A-B-C-D-E-F-A* is not unique. There are other solutions such as *A-F-E-D-C-B-A*.

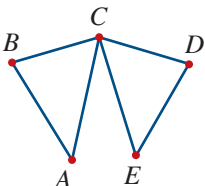
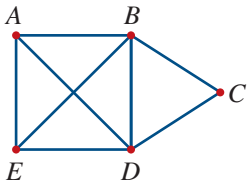
Now you try

- a** List a Hamiltonian path for the network graph in Example 9a that starts at *C* and finishes at *H*.
b Identify a Hamiltonian cycle for the network graph in Example 9b starting at *C*.

Exercise 9D

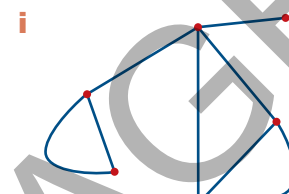
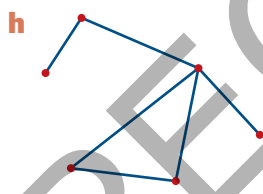
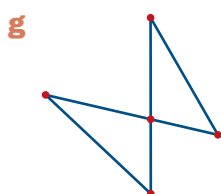
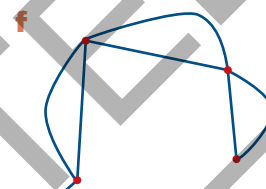
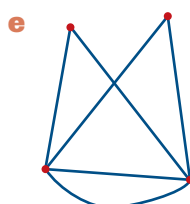
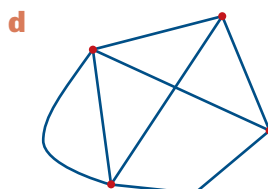
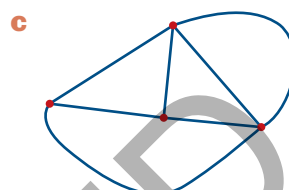
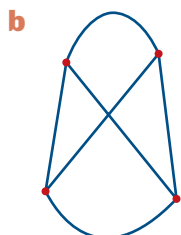
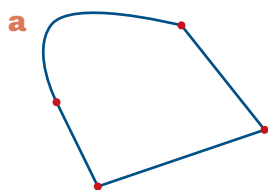
BUILDING

- 1** A network graph is shown.
a What is the degree of each vertex?
b Why does this graph have an Eulerian trail?
c List the Eulerian trail.
- 2** A network graph is shown.
a What is the degree of each vertex?
b Why does this graph have an Eulerian circuit?
c List an Eulerian circuit.



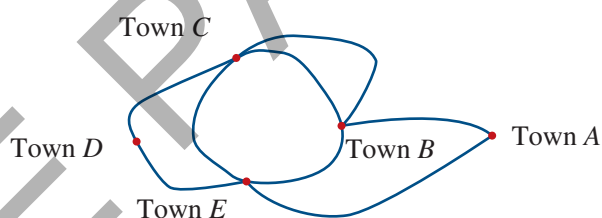
Example 8

- 3** For each graph, determine whether the graph has an Eulerian trail, an Eulerian circuit or neither. Show one example if the graph has an Eulerian trail or circuit.



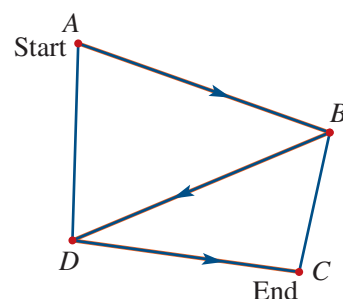
- 4** A road inspector lives in Town A and is required to inspect all roads connecting the neighbouring towns B, C, D and E.

- a** Is it possible for the inspector to travel over every road linking the five towns only once and return to Town A? Explain
b Show one possible route he can follow.



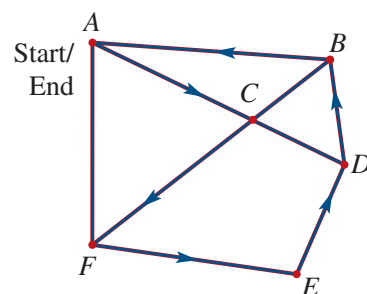
- 5** A network graph is shown.

- a** List the path shown in the graph.
b Does the path pass through every vertex?
c Does the path pass through every edge?
d ENRICHMENT: Why is this path a Hamiltonian path?
e ENRICHMENT: List another Hamiltonian path starting at A.



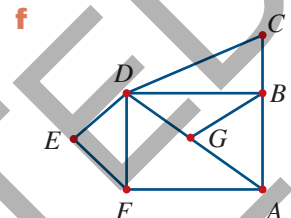
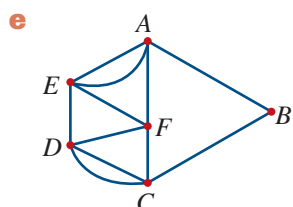
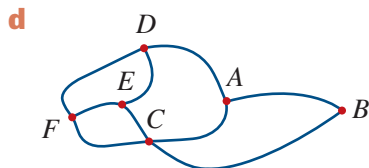
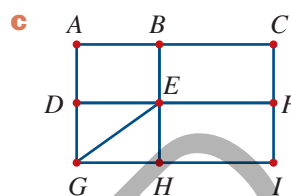
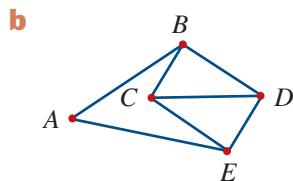
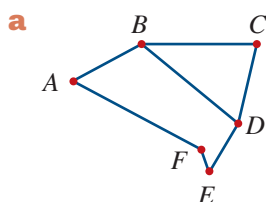
- 6** A network graph is shown.

- a** List the path shown in the graph.
b Does the path pass through every vertex?
c Does the path pass through every edge?
d ENRICHMENT: Why is this path a Hamiltonian cycle?
e ENRICHMENT: List another Hamiltonian cycle starting at A.



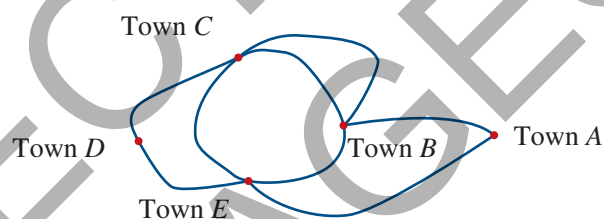
Example 9

7 ENRICHMENT: List a Hamiltonian cycle for each graph.



8 ENRICHMENT: A tourist wants to visit each of five towns shown in the graph only once. Identify one possible route for the tourist to start the tour at:

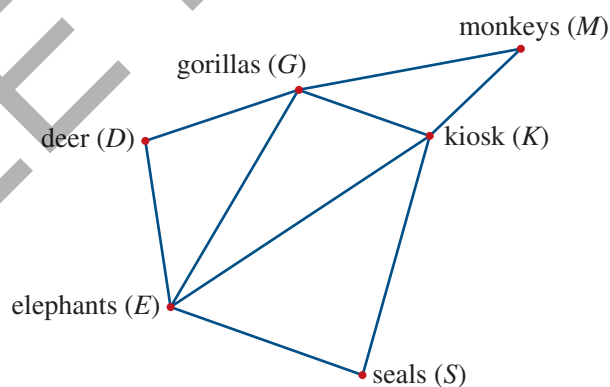
- a** C and finish at A. What is the mathematical name for this route?
b E and finish at E. What is the mathematical name for this route?



DEVELOPING

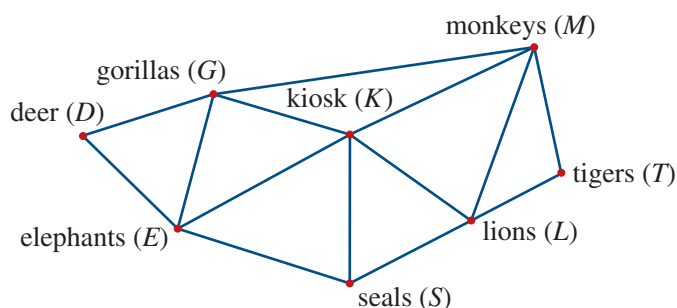
9 The graph models the pathways linking five animal enclosures in a zoo to the kiosk and to each other.

- a** Explain why it is possible for the zoo's street sweeper to follow a route that enables its operator to start and finish at the kiosk without travelling down any pathway more than once.
b List one route for the zoo's street sweeper.



10 The graph models the pathways linking seven animal enclosures in a zoo to the kiosk and to each other.

- a** List a possible route for a visitor to start and finish their visit at the kiosk and see all of the animals without visiting any animal enclosure more than once.
b List a possible route for a visitor to start their visit at the deer enclosure and finish at the kiosk without visiting the kiosk or any animal enclosure more than once.



9E Solving problems involving network diagrams

Learning intentions

- To interpret and draw network diagrams and tables
- To solve problems involving networks

Past, present and future learning

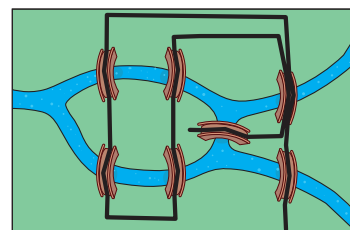
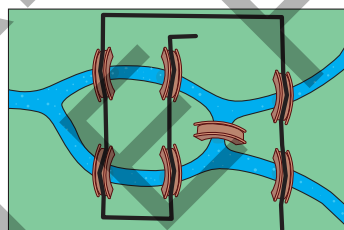
- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12 Standard 2
- Questions about networks regularly appear in HSC Examinations

Königsberg bridge problem

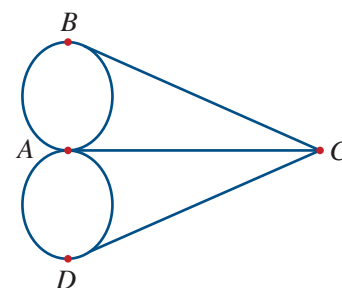
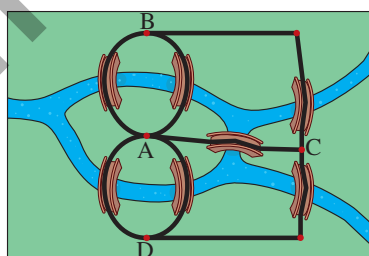
The town of Königsberg, or Kaliningrad as it is now known, is situated on the river Pregel in what used to be called Prussia. The old town had seven bridges linking two islands and the north and south banks of the river. A view of Königsberg is shown. The Königsberg bridge problem asked whether the seven bridges could all be crossed only once during a single trip that starts and finishes at the same place.



Put in another way, can a continuous walk be planned so that all the bridges are crossed only once? Whenever someone tried to walk the route, they either ended up missing a bridge or crossing one of the bridges more than once. Two such walks are marked on the diagrams. See if you can trace out a walk on the diagram that crosses every bridge, but only once.



The Königsberg bridge problem was well known in 18th century Europe and attracted the attention of the Swiss mathematician Leonhard Euler. He started analysing the problem by drawing a simplified diagram to represent the situation, as shown. We now call this type of simplified diagram a network diagram or graph.



Note: All four vertices in the Königsberg bridge graph have an odd degree.

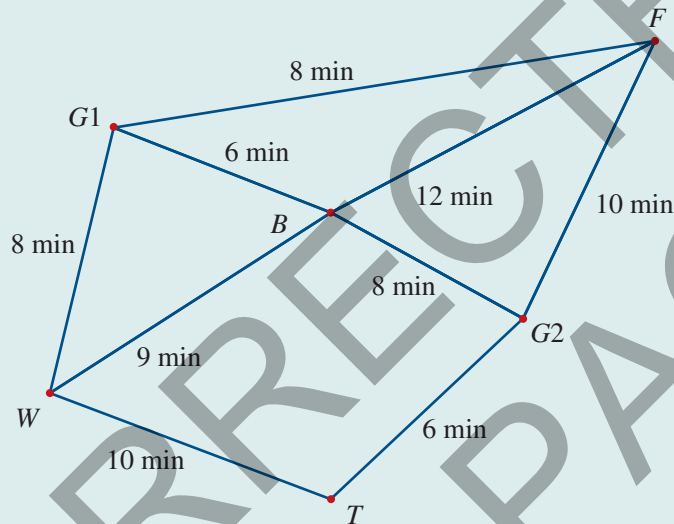
Euler was able to prove that a graph with all vertices of odd degree cannot be traced or drawn without lifting the pencil or going over the same edge more than once. Today we call this problem an Eulerian trail. The problem was solved by Euler. The seven bridges of Königsberg could not be crossed in a single walk without either missing a bridge or crossing one bridge more than once.



Example 10 Solving a practical network problem

The network diagram shown is used to model the tracks in a forest connecting a suspension bridge (B), a waterfall (W), a very old tree (T) and a fern gully (F). Walkers can enter or leave the forest through either gate 1 ($G1$) or gate 2 ($G2$). The numbers on the edges represent the times (in minutes) taken to walk directly between these places.

- Construct a table to represent the weighted graph.
- How long does it take to walk from the bridge directly to the fern gully?
- How long does it take to walk from the old tree to the fern gully via the waterfall and the bridge?



Solution

- Headings are W , $G1$, F , B , T and $G2$. Insert 8 in the cell for $WG1$ and $G1W$.

	W	$G1$	F	B	T	$G2$
W	—	8	—	9	10	—
$G1$	8	—	8	6	—	—
F	—	8	—	12	—	10
B	9	6	12	—	—	8
T	10	—	—	—	—	6
$G2$	—	—	10	8	6	—

- The edge is (B, F) . The time taken is 12 minutes.

- The path is $T-W-B-F$.
Time = $10 + 9 + 12 = 31$.
The time taken is 31 minutes.

Explanation

The vertices are the headings for the rows and columns.

A weighted edge is a value in the table.
Repeat the above step for each weighted edge until all the weighted edges have been entered into the table.

Identify the edge that directly links the bridge with the fern gully and read off the time.

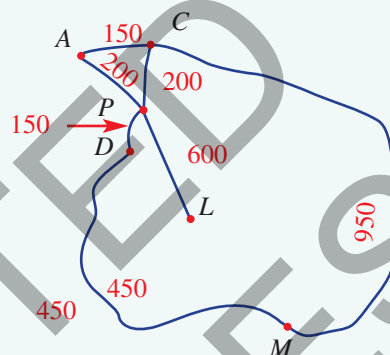
Identify the path that links the old tree to the fern gully, visiting the waterfall and the bridge on the way. Add up the times.

continued ...

Now you try

The network diagram shown is a simple map of Lizard Log Park, connecting the Amphitheatre (A), Carpark (C), Lizard Log playground (P), Grey Box Gum Dam (D), Hilltop Lookout (L) and the Moonrise Loop (M). All distances are in metres.

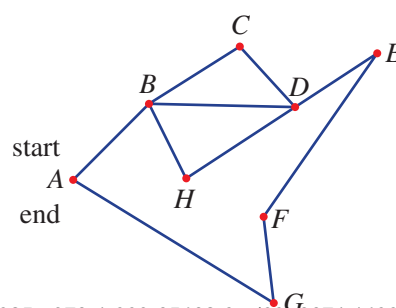
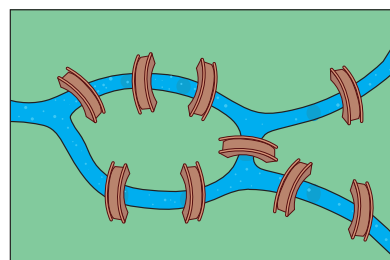
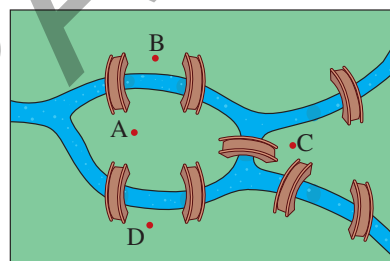
- a Construct a table to represent the weighted graph.
- b How long is the walk from the Carpark to the Amphitheatre?
- c How long is the walk from the Amphitheatre to Moonrise Loop via the Playground and Dam?



Exercise 9E

BUILDING

- 1 Investigate the Königsberg bridge problem when an eighth bridge is added as shown in the diagram.
 - a Construct a network diagram to represent this situation with eight bridges. Use the labelled dots shown in the diagram.
 - b How many vertices of odd degree are there in your network diagram?
 - c Does an Eulerian trail exist in your network diagram?
 - d List a continuous walk that crosses each bridge once or an Eulerian trail.
- 2 Investigate the Königsberg bridge problem if two bridges are added, making nine bridges as shown in the diagram.
 - a Construct a network diagram to represent this situation with nine bridges.
 - b How many vertices of odd degree in your network diagram?
 - c Does an Eulerian circuit exist in your network diagram?
 - d List a circuit that starts and ends at the same vertex, and crosses each bridge once or Eulerian circuit.
- 3 The roads for a garbage bin collection are shown in the network diagram. The garbage bin collection must start/end at A and travel every road only once.
 - a How many vertices of odd degree in this network diagram?
 - b Does an Eulerian circuit exist in this network diagram?
 - c Show one possible route for the garbage bin collection.
 - d List another route for the garbage bin collection.



Example 10

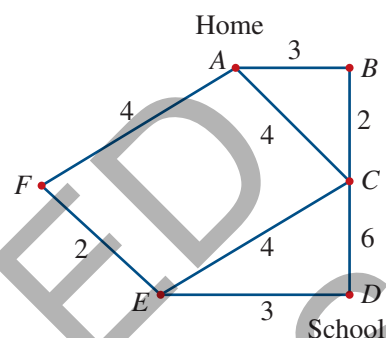
- 4 Evelyn has drawn a network diagram to represent several streets for travelling from her home to school. The numbers indicate the times in minutes.

a How long does it take to walk from home to school using the following paths?

- i $A-F-E-D$
- ii $A-F-E-C-D$
- iii $A-C-D$
- iv $A-B-C-D$
- v $A-C-E-D$

b Which of the walks is the longest journey?

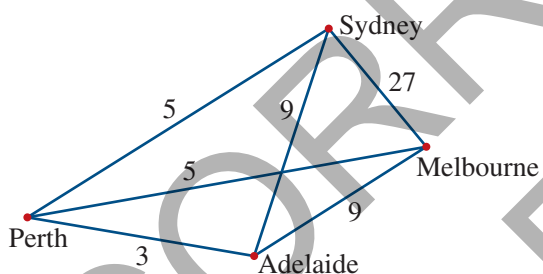
c Which of the walks is the shortest journey?



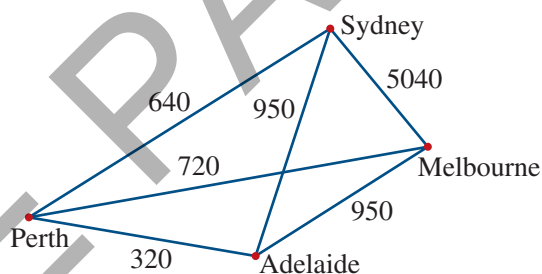
DEVELOPING

- 5 The network graph shows details about air travel between Australian cities. The first graph shows the number of flights in each direction between cities, and the second graph shows the maximum number of passengers per day in each direction.

Number of flights per day



Maximum number of passengers per day



- a How many flights are available per day from Sydney to Adelaide?
- b How many flights are available per day between these Australian cities?
- c What is the maximum number of passengers per day from Melbourne to Adelaide?
- d What is the maximum number of passengers per day that can fly out of Sydney?
- e Seth is wishing to fly from Perth to Melbourne but is told that the direct flights are fully booked. List any other ways of completing this journey.
- f What is the greatest number of people per day that can be flown from Perth to Melbourne?
- g Amy is wishing to fly from Sydney to Melbourne but is told that the direct flights are fully booked. List any other ways of completing this journey.
- h What is the greatest number of people per day that can be flown from Sydney to Melbourne?

CHALLENGING

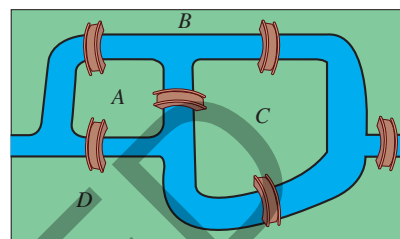
6 In this bridge problem there are six bridges as shown.

a Construct a network diagram to represent this situation with six bridges. Label the vertices A, B, C and D to represent the river banks and the two islands. Use the edges of the graph to represent the bridges.

b It is not possible to plan a walking route that passes over each bridge once only. Why not?

c Show where another bridge could be added to make such a walk possible. Draw a graph (a network diagram) to represent this situation.

d Explain why it is now possible to find a walking route that passes over each bridge once only. Mark one such route on your graph.



Communicate your reasoning, understanding and fluency

7 In this bridge problem, there are seven bridges as shown.

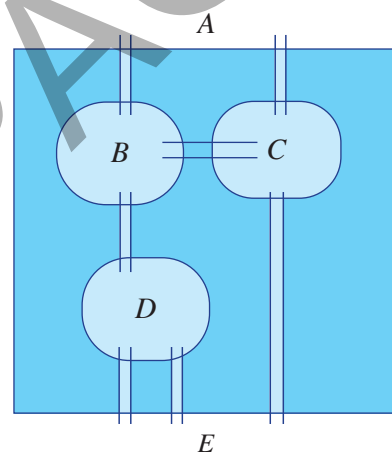
a Construct a network diagram to represent this situation, with A, B, C, D and E being the vertices and each bridge an edge.

b Is it possible to find a walking route that crosses each bridge only once? If not, why not?

c Another bridge will be built between islands C and D . Modify your network diagram accordingly.

d With the new bridge, is it possible to find a walking route that crosses each bridge only once? If so, what changed from your answer to part **b**?

e If you wanted to find a walking route that only crossed each bridge once and ended at the place where it started, where would you need a new bridge? Add the new edge to your network diagram.



8 A group of six towns have a bus network between them, with the cost to travel from one town to another shown by the table.

	A	B	C	D	E	F
A	–	20	–	10	–	–
B	20	–	30	30	45	40
C	–	30	–	–	20	10
D	10	30	–	–	25	30
E	–	45	20	25	–	15
F	–	40	10	30	15	–

a Draw a weighted network diagram representing this information.

b What is the cheapest way to travel from:

i A to E ?

ii A to F ?

iii A to C ?

c Is it possible to take every bus route once without taking any route twice?

d What is the cost of the route in part **c**?

9F Spanning trees: Prim's algorithm and Kruskal's algorithm

Learning intentions

- To learn and understand the terminology associated with networks
- To interpret and draw network diagrams
- To solve problems involving networks

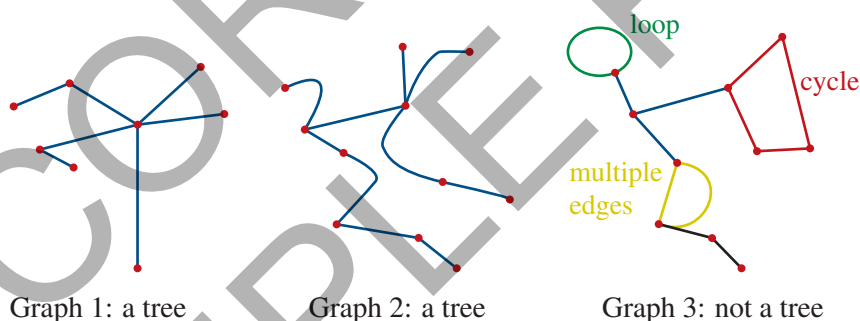
Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12 Standard 2
- Questions about spanning trees regularly appear in HSC Examinations

In some network problems it is important to minimise the number and weights of the edges to keep all vertices connected to the graph. For example, a number of towns might need to be connected to a water supply. The cost of connecting the towns can be minimised by connecting each town into a network of water pipes only once, rather than connecting each town to every other town. To solve these problems we need an understanding of trees.

Tree

A tree is a connected graph that contains no cycles, multiple edges or loops. The diagram shows two network graphs that are trees and one network graph that is not a tree.



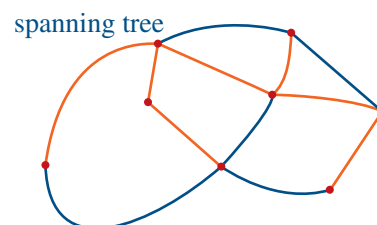
Graphs 1 and 2 are trees: they are connected and have no cycles, multiple edges or loops. Graph 3 is not a tree: it has several cycles (loops and multiple edges count as cycles). For trees, there is a relationship between the number of vertices and the number of edges.

- Graph 1, a tree, has 8 vertices and 7 edges.
- Graph 2, a tree, has 11 vertices and 10 edges.

In general, the number of edges is always one less than the number of vertices. In other words, a tree with n vertices has $(n - 1)$ edges.

Spanning trees

Every connected graph will have at least one subgraph that is a tree. If a subgraph is a tree, and if that tree connects all of the vertices in the graph, then it is called a spanning tree. An example of a spanning tree is shown. There are several other possibilities. Note: The spanning tree, like the network diagram, has 8 vertices. However it has only 7 edges ($8 - 1 = 7$).



Tree

A tree is a connected graph that contains no cycles, multiple edges or loops.

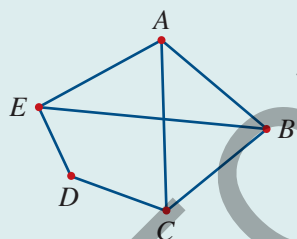
A tree with n vertices has $(n - 1)$ edges.

A spanning tree is a tree that connects all of the vertices of a graph.



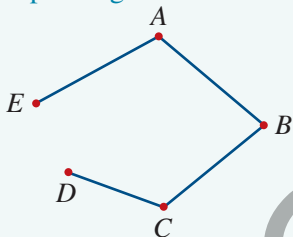
Example 11 Finding a spanning tree in a network

Find two spanning trees for the graph shown.

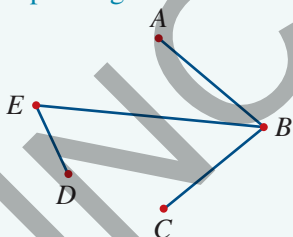


Solution

Spanning tree 1



Spanning tree 2



Explanation

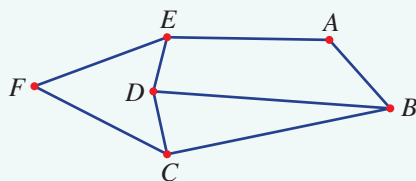
The graph has five vertices and seven edges. A spanning tree will have five (n) vertices and four ($n - 1$) edges.

To form a spanning tree, remove any three edges, provided that all the vertices remain connected, and there are no multiple edges or loops.

Spanning tree 1 is formed by removing edges EB , ED and CA .
Spanning tree 2 is formed by removing edges EA , AC and CD .
There are several other possible spanning trees.

Now you try

Find two spanning trees for the graph shown.

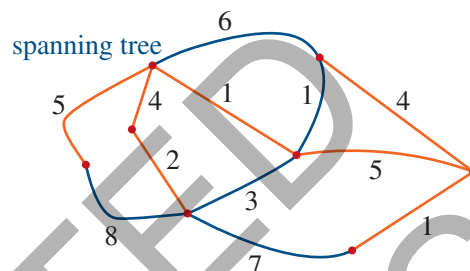


Minimum spanning trees

For a weighted graph, it is possible to determine the 'length' of each spanning tree by adding up the weights of the edges in the tree. For the spanning tree shown:

$$\begin{aligned}\text{Length} &= 5 + 4 + 2 + 1 + 5 + 4 + 1 \\ &= 22 \text{ units}\end{aligned}$$

A minimum spanning tree is a spanning tree of minimum length. It connects all the vertices together with the minimum total weighting for the edges.



Minimum spanning tree

A minimum spanning tree is a spanning tree of minimum length. It connects all the vertices together with the minimum total weighting for the edges.

 **A guide to trees** Watch the video in the Interactive Textbook to see trees, spanning trees and minimum spanning trees in action.

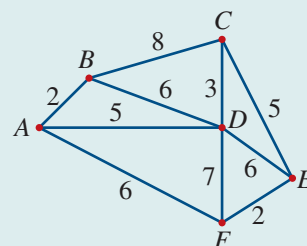
Prim's algorithm

Prim's algorithm is a set of rules to determine a minimum spanning tree for a graph.

- 1** Choose a starting vertex (any will do).
- 2** Inspect the edges starting from the starting vertex and choose the one with the lowest weight. (If there are two edges that have the same weight, it does not matter which one you choose). You now have two vertices and one edge.
- 3** Inspect all of the edges starting from both of the vertices you have in the tree so far. Choose the edge with the lowest weight, ignoring edges that would connect the tree back to itself. You now have three vertices and two edges.
- 4** Keep repeating step **3** until all of the vertices are connected.

Example 12 Finding the minimum spanning tree using Prim's algorithm

Apply Prim's algorithm to find the minimum spanning tree for the network graph shown. Calculate the length of the minimum spanning tree.

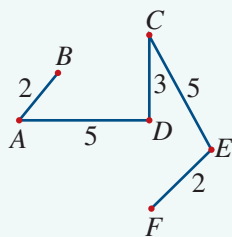


continued ...

Solution

$(A, B) = 2$
 $(A, F) = 6$
 $(A, B) = 2$ is lowest.
 $(A, D) = 5$
 $(A, F) = 6$
 $(B, C) = 8$
 $(B, D) = 6$
 $(A, D) = 5$ is lowest.
 $(C, D) = 3$ is lowest.
 $(C, E) = 5$ is lowest.

$(E, F) = 2$ is lowest.



Length = $2 + 5 + 3 + 5 + 2$
= 17 units

Explanation

Start with vertex A. List the weighted edges from vertex A and find the smallest.

Look at vertices A and B. List the weighted edges from vertex A and vertex B (apart from (A, B) which you have already found).

Repeat to find the smallest weighted edge from vertex A, B or D.

Repeat to find the smallest weighted edge from vertex A, B, D or C.

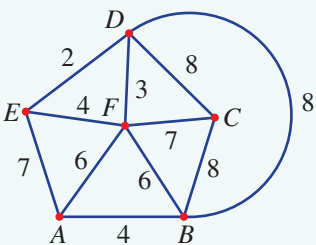
Repeat to find the smallest weighted edge from vertex A, B, D, C or E.

All vertices have been included in the graph. Draw the minimum spanning tree.

Find the length of the minimum spanning tree by adding the weights of the edges.

Now you try

Apply Prim's algorithm to find the minimum spanning tree for the network graph shown. Calculate the length of the minimum spanning tree.



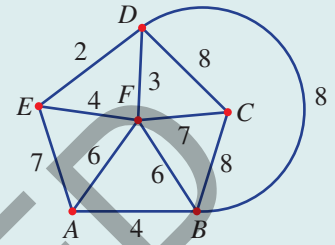
Kruskal's algorithm

Kruskal's algorithm is used for finding the minimum spanning tree of a network.

- 1 List the edges in order of weight.
- 2 Choose an edge of least weight. If there is more than one, it does not matter which you choose.
- 3 From the edges remaining, choose an edge of least weight which does not form a cycle with the already chosen edges.
- 4 Repeat the step 3 until $(n - 1)$ edges (n vertices) have been chosen, and all vertices are connected by chosen edges.

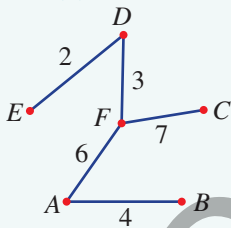
Example 13 Finding the minimum spanning tree using Kruskal's algorithm

Apply Kruskal's algorithm to find the minimum spanning tree for the network graph shown. Calculate the length of the minimum spanning tree.



Solution

DE (2) ✓
 DF (3) ✓
 EF (4) cycle ×
 AB (4) ✓
 AF (6) ✓
 FB (6) cycle ×
 EA (7) cycle ×
 CF (7) ✓ finished
 DC (8) ×
 BC (8) ×
 DB (8) ×



Length of minimum spanning tree
 $= 2 + 3 + 4 + 6 + 7 = 22$

Explanation

List edges in order of weight.

Work your way down the list and tick the edges you will be choosing, crossing out the ones you will not be choosing.

The edge of least weight is DE with a weight of 2.

Next smallest edge DF with a weight of 3.

Next smallest edge is EF . This cannot be chosen as it creates a cycle EDF .

The next smallest edge is AB with a weight of 4.

Two edges have weight 6, AF or FB . Choose either (AF chosen here).

FB cannot be chosen as this will create a cycle BAF .

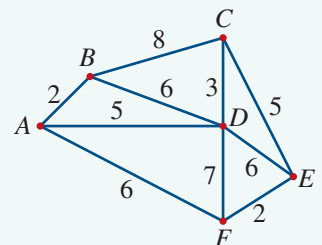
Next weight is 7. EA cannot be chosen as this will create a cycle. Choose CF instead.

All vertices have now been chosen to create the minimum spanning tree for the original network.

Length of minimum spanning tree is 22.

Now you try

Apply Kruskal's algorithm to find the minimum spanning tree for the network graph shown. Calculate the length of the minimum spanning tree.



Exercise 9F

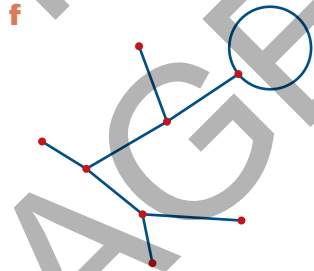
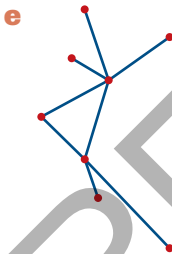
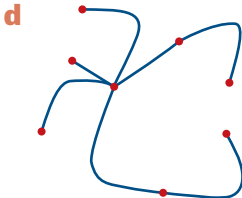
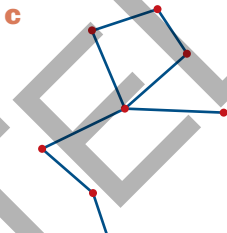
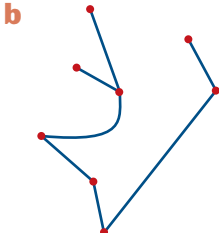
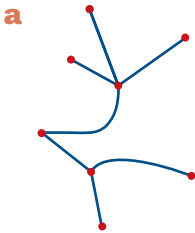
BUILDING

1 Copy and complete the following sentences.

- A tree is a connected graph that contains no cycles, multiple _____ or loops.
- A minimum spanning tree is a spanning tree of minimum _____.
- Prim's algorithm is a set of rules to determine a minimum _____ tree for a graph.
- A connected graph has eight vertices. Its spanning tree has _____ vertices.

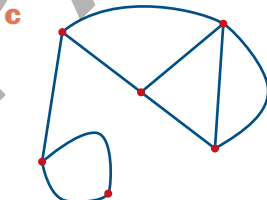
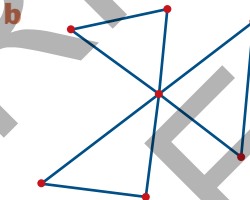
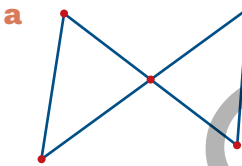
- 2 a How many edges are there in a tree with 15 vertices?
b How many vertices are there in a tree with 5 edges?
c Draw two different trees with four vertices.
d Draw three different trees with five vertices.

3 Which of the following graphs are trees?



Example 11

4 For each of the following graphs, draw three different spanning trees.



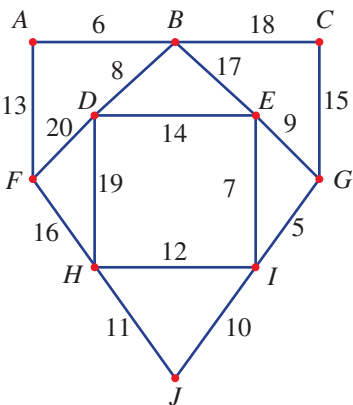
DEVELOPING

Example 12

5 Find the minimum spanning tree using:

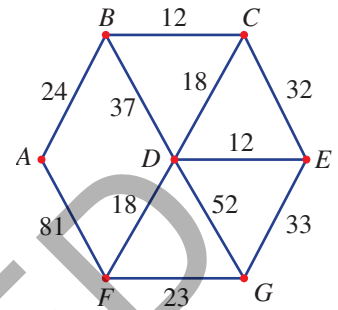
Example 13

- a Prim's algorithm
b Kruskal's algorithm.



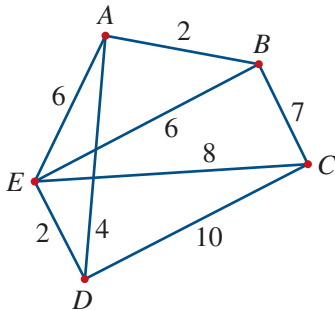
- 6** A company owns seven depots. The distances between them, in kilometres, by direct road links are shown in the diagram.

Communication cables are to be laid along these roads so that the depots are linked by cable, either directly or via other depots. What is the minimum length of cable that could be used?

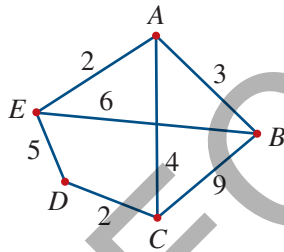


- 7** Find the minimum spanning tree and its length, for each of the following graphs, using any method.

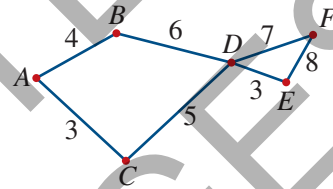
a



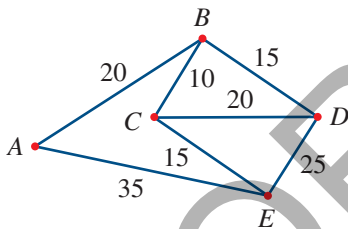
b



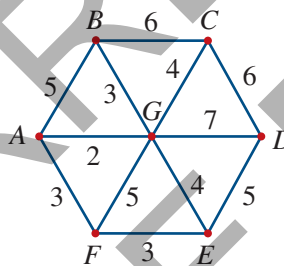
c



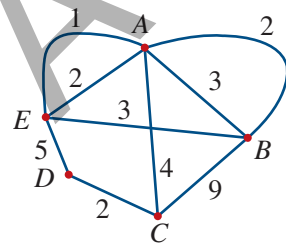
d



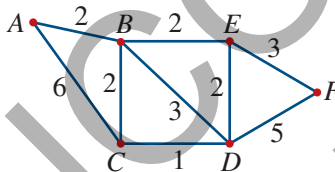
e



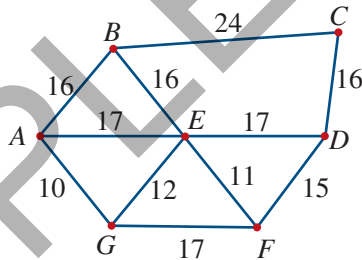
f



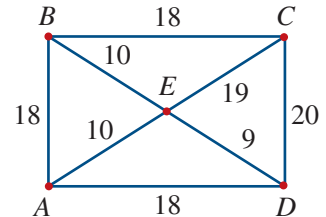
g



h

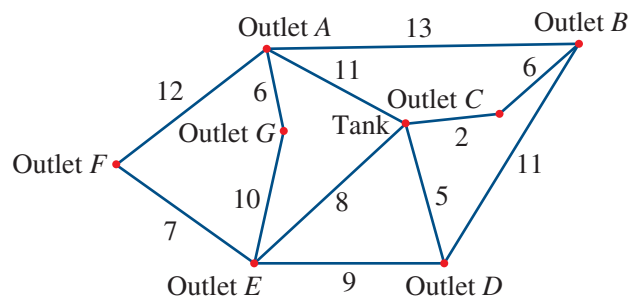


i

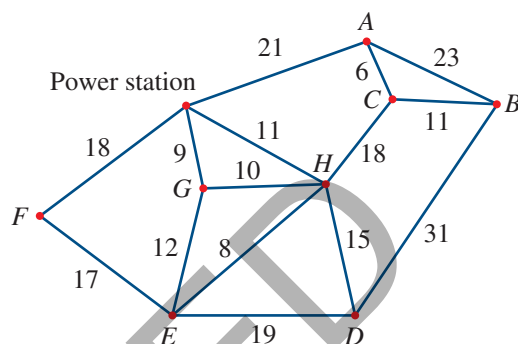


CHALLENGING

- 8** Water is to be piped from a water tank to seven outlets on a property. The distances (in metres) of the outlets from the tank and from each other are shown in the network. Starting at the tank, determine the minimum length of pipe needed.

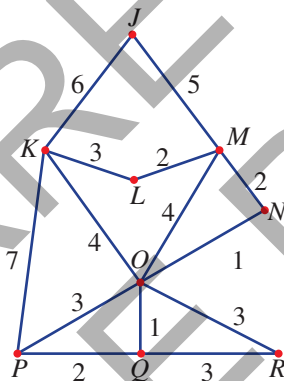


- 9** Power is to be connected by cable from a power station to eight substations (A to H). The distances (in kilometres) of the substations from the power station and from each other are shown in the network. Determine the minimum length of cable needed.

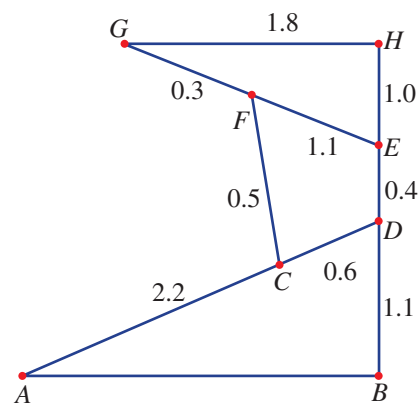


Communicate your reasoning, understanding and fluency

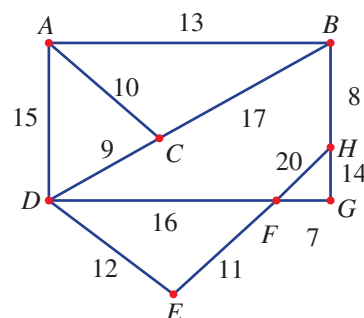
- 10** A house is being built, and each room in the house needs electricity. The distances (in metres) between each room in the house are shown in the network diagram. Starting from Room J , determine the minimum amount of electrical cable needed to provide the house with electricity.



- 11** A number of rural roads are being upgraded with powerlines. The network diagram shows eight intersections (A to H) and the distance (in kilometres) between them. The road between A and B already has powerlines on it and does not need to be upgraded. Determine the minimum distance of cable required for each intersection to have powerlines.



- 12** A network of gas pipelines is being planned to connect a gas supply (A) to seven towns (B to H). The network diagram shows the towns and the distances between them in kilometres. Determine the minimum distance of pipe needed to connect the gas supply to all seven towns.



9G Solving minimal connector problems

Learning intentions

- To interpret and draw network diagrams
- To solve problems involving networks

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12 Standard 2
- Questions about minimal connector problems regularly appear in HSC Examinations

Minimum spanning trees represent the least weight required to keep all of the vertices connected in the graph. If the edges of a graph represent the cost of connecting towns to a gas pipeline, then the total weight of the minimum spanning tree would represent the minimum cost of connecting the towns to the gas. This is an example of a connector problem, where it is important to make the cost of connecting towns or other objects together as low as possible.

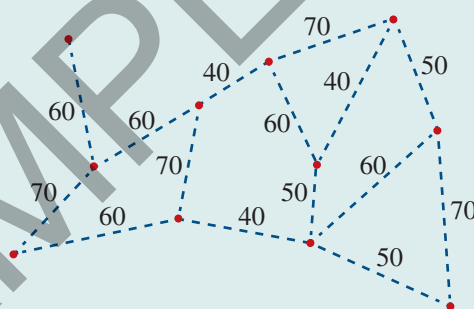
Connector problems

A connector problem uses a minimum spanning tree to find the least cost to link locations or objects.



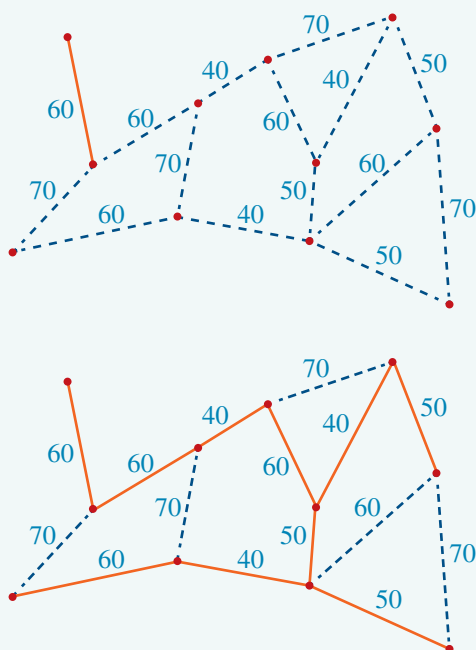
Example 14 Solving a minimal connector problem

There are 11 locations that require access to power. These locations are represented by vertices on the network diagram shown. The dashed lines on the network diagram represent possible cable connections between adjacent locations. The numbers on the dashed lines show the minimum length of cable, in metres, required to connect these locations. Find the minimum length of cable needed to provide power to each location.



continued ...

Solution



$$\begin{aligned} \text{Length} &= 60 + 60 + 40 + 60 + 50 \\ &\quad + 40 + 50 + 60 + 40 + 50 \\ &= 510 \text{ m} \end{aligned}$$

$\therefore$ Minimum length is 510 metres.

Now you try

A group of eight towns are planning a rail network between them. The towns and the distances (in km) between them are shown on the network diagram.

Draw a minimal spanning tree that will ensure all towns are connected using the minimum distance of rail.

What is the minimum length of rail required to connect all towns?

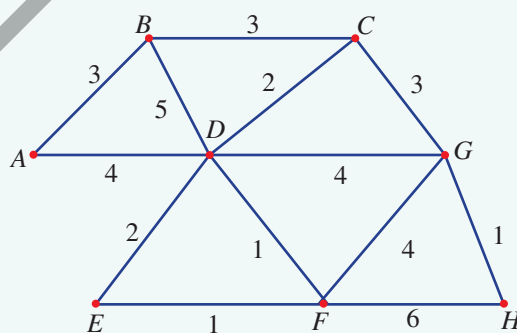
Explanation

The cables will be a minimum length if they are placed on the edges of the minimum spanning tree for the network. A good starting point for Prim's algorithm is the vertex that is connected by just one edge. This vertex must be connected to the minimum spanning tree by this edge.

Follow Prim's algorithm to find the minimum spanning tree or follow Kruskal's algorithm to find the minimum spanning tree.

Find the length of the minimum spanning tree by adding the weights of the edges.

Write answer in words.

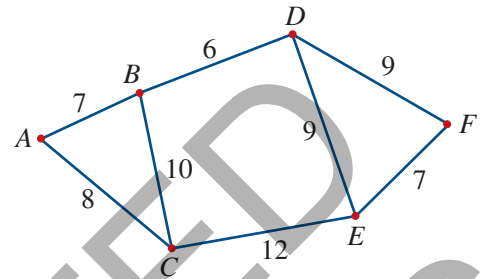


Exercise 9G

BUILDING

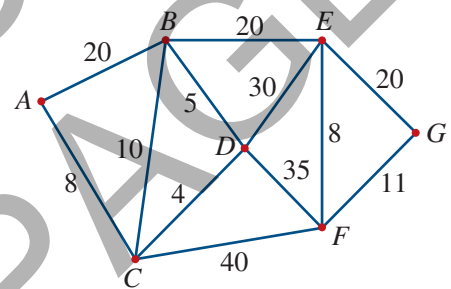
- 1** The network diagram shows possible water pipes connecting six towns. The numbers on each edge represent the distance (in km) between the towns

- Which town is closest to A? What is the distance?
- Which town is closest to either A or your answer in part **a**? What is the distance?
- Which town is closest to either A or your answer in part **a** or **b**? What is the distance?
- Which town is closest to either A or your answer in part **a** or **b** or **c**? What is the distance?
- Which town is closest to either A or your answer in part **a** or **b** or **c** or **d**? What is the distance?
- What is the shortest distance connecting water pipes to the six towns?



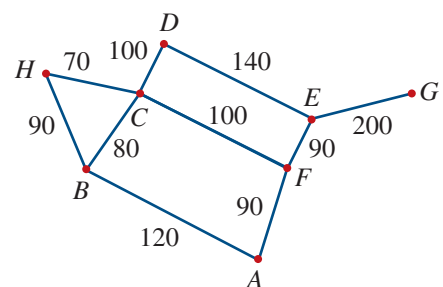
- 2** The network diagram shows possible railway lines connecting seven cities. The numbers on each edge represent the cost (in millions of dollars) in setting up a rail link.

- Which city is closest to A? What is the cost?
- Which city is closest to either A or your answer in part **a**? What is the cost?
- Which city is closest to either A or your answer in part **a** or **b**? What is the cost?
- Which city is closest to either A or your answer in part **a** or **b** or **c**? What is the cost?
- Which city is closest to either A or your answer in part **a** or **b** or **c** or **d**? What is the cost?
- Which city is closest to either A or your answer in part **a** or **b** or **c** or **d** or **e**? What is the cost?
- What is the minimum cost of connecting the seven cities with a railway line?



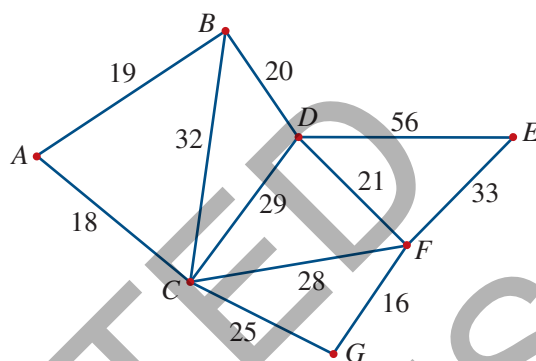
- 3** The network diagram shows the major roads connecting eight towns. The numbers on each edge represent the distance in kilometres between the towns.

- Which town is closest to A? What is the distance?
- Which town is closest to either A or your answer in part **a**? What is the distance?
- Which town is closest to either A or your answer in part **a** or **b**? What is the distance?
- Which town is closest to either A or your answer in part **a** or **b** or **c**? What is the distance?
- Which town is closest to either A or your answer in part **a** or **b** or **c** or **d**? What is the distance?
- Which town is closest to either A or your answer in part **a** or **b** or **c** or **d** or **e**? What is the distance?
- Which town is closest to either A or your answer in part **a** or **b** or **c** or **d** or **e** or **f**? What is the distance?
- What is the minimum distance connecting the eight towns?



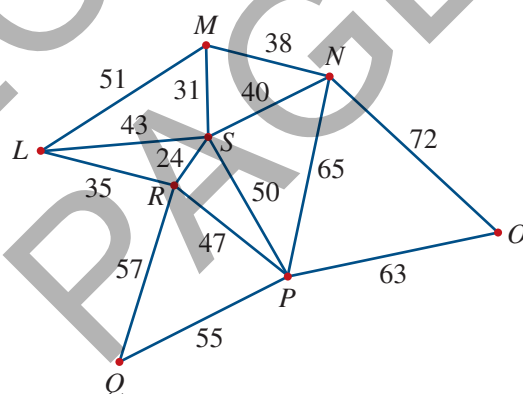
DEVELOPING

- 4** Seven towns on an island have been surveyed for transport and communications needs. The towns (labelled A, B, C, D, E, F, G) form the network shown here. The road distances between the towns are marked in kilometres. To establish a cable network for communications on the island, it is proposed to put the cable underground beside the existing roads.



- a** Draw a minimum spanning tree that will ensure that all the towns are connected to the network but that also minimises the amount of cable used.
- b** What is the minimal length of cable required here if back-up links are not considered necessary; that is, there are no loops in the cable network?

- 5** A number of towns need to be linked by pipelines to a natural gas supply. In the network shown, the existing road links between towns L, M, N, O, P, Q and R and to the supply point S , are shown as edges. The towns and the gas supply are shown as vertices. The distances along roads are given in kilometres.

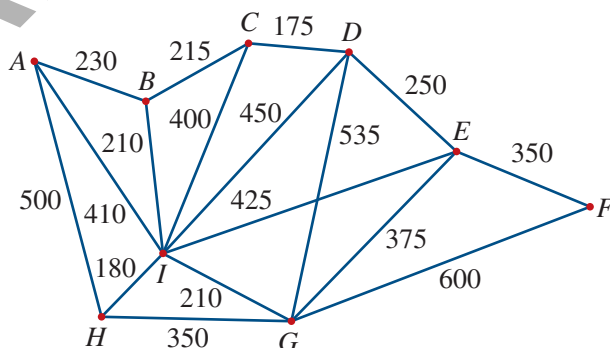


- a** Draw minimum spanning tree that will ensure that all the towns are connected to the network but that also minimises the amount of pipelines required.
- b** What is the minimum length of gas pipeline the company can use to supply all the towns?

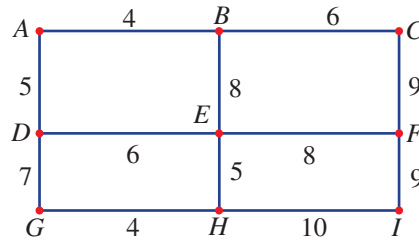
CHALLENGING

- 6** The company has constructed nine new faculty buildings for a university in a layout as shown. The minimum distances in metres between adjacent buildings in the university are shown on the edges. A computer network is to be built to serve the whole university.

- a** Draw a minimum spanning tree that will ensure that all the buildings are connected to the network but that also minimises the amount of cable used.
- b** What is the minimum length of cable required to connect all the buildings?



- 7 Eight houses (B, C, D, E, F, G, H, I) need to be connected to a power substation, A . The houses and the distances between them (in tens of metres) are shown on the network diagram.

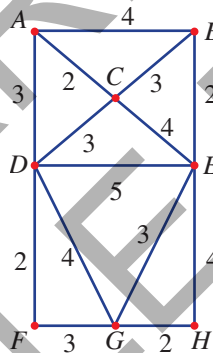


Draw a minimal spanning tree that will ensure all houses are connected using the minimum distance of cable. What is the minimum length of cable required to connect all houses?

Communicate your reasoning, understanding and fluency

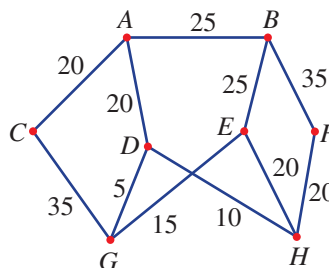
- 8 An electrical network is being planned for a house. The rooms in the house (A, B, C, D, E, F, G, H) and the distances between them (in metres) are shown on the network diagram.

Draw a minimal spanning tree that will ensure all rooms are connected with the minimum amount of cable. What is the minimum distance of cable needed for the house?



- 9 Eight towns (A to H) want to connect their water supplies together. The towns and the distances between them (in km) are shown on the network diagram.

Draw a minimal spanning tree that will ensure all towns are connected using the minimum distance of pipe. What is the minimum length of pipe required to connect all towns?



9H Shortest path through a network

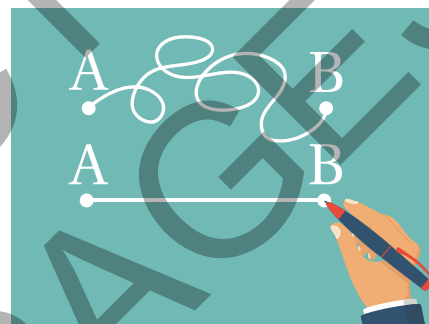
Learning intentions

- To interpret and draw network diagrams
- To solve problems involving networks

Past, present and future learning

- Some aspects of this section may be new to students
- This is a Year 11 concept which is extended in Year 12 Standard 2
- Questions about shortest paths regularly appear in HSC Examinations

The shortest path in a network is the path between two vertices where the sum of the weights of its edges is minimised. Finding the shortest path is often very useful. For example, if the weights of a network represent time, you can choose a path that will allow you to travel in the shortest time. If the weights represent distance, you can determine a path that will allow you to travel the shortest distance. However, be aware that travelling the shortest distance between two places is not necessarily the best path. For example, if shortest path has a speed limit of 60 km/h but another path has a speed limit of 110 km/h then the shortest path may take longer to reach the destination. In such a case, redraw the network diagram with time taken to travel each edge, rather than distance. While there are sophisticated techniques for solving shortest path problems (such as Dijkstra's algorithm in the next section) the method of inspection involves identifying and comparing the lengths of likely candidates for the shortest path. All of the possible paths should be listed and the length of the path calculated. When finding the shortest path it is important to be aware there can be more than one shortest path between two vertices and the shortest path may not pass through all of the vertices.



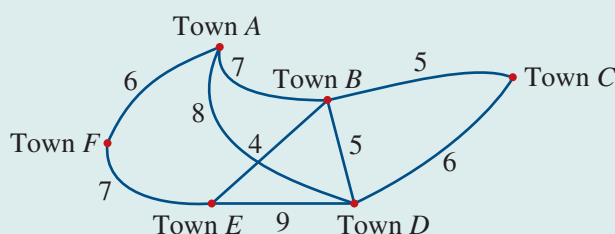
Shortest path

The shortest path between two vertices in a network is the path where the sum of the weights of its edges is minimised.



Example 15 Finding the shortest path by inspection

Find the shortest path between Town C and Town F.



Solution

$$\begin{aligned} C-D-E-F &= 6 + 9 + 7 \\ &= 22 \text{ km} \end{aligned}$$

$$\begin{aligned} C-B-E-F &= 5 + 4 + 7 \\ &= 16 \text{ km} \end{aligned}$$

$$\begin{aligned} C-B-A-F &= 5 + 7 + 6 \\ &= 18 \text{ km} \end{aligned}$$

The shortest path is $C-B-E-F$.

Explanation

Identify all the likely shortest routes between Town C and Town F and calculate their lengths. Note: Time can be saved by eliminating any route that ‘takes the long way around’ rather than the direct route. For example, when travelling from Town B to Town D , ignore the route that goes via Town A because it is longer.

Compare the different path lengths and identify the shortest path. Write your answer in words.

Now you try

Using the same diagram in Example 15, find the shortest path from Town B to Town F .

**Example 16** Finding the length of the shortest path

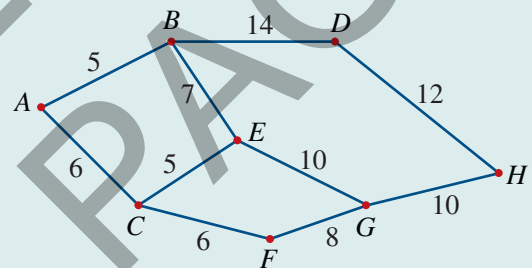
Find the length of the shortest path between the following vertices.

a A and E

b A and F

c A and G

d A and H

**Solution**

$$\begin{aligned} \text{a } A-C-E &= 6 + 5 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{b } A-C-F &= 6 + 6 \\ &= 12 \end{aligned}$$

$$\begin{aligned} \text{c } A-C-F-G &= 6 + 6 + 8 \\ &= 20 \end{aligned}$$

$$\begin{aligned} \text{d } A-C-F-G-H &= 6 + 6 + 8 + 10 \\ &= 30 \end{aligned}$$

Explanation

Identify the shortest paths between vertex A and vertex E .

Add the weighted edges to find the length of the path.

Identify the shortest paths between vertex A and vertex F .

Add the weighted edges to find the length of the path.

Identify the shortest paths between vertex A and vertex G .

Add the weighted edges to find the length of the path.

Identify the shortest paths between vertex A and vertex H .

Add the weighted edges to find the length of the path.

continued ...

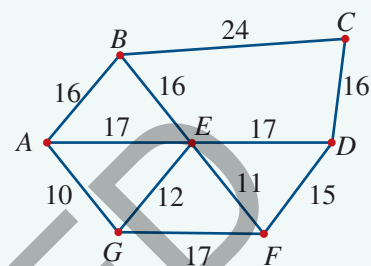
Now you try

Find the length of the shortest path between the following vertices.

a A and D

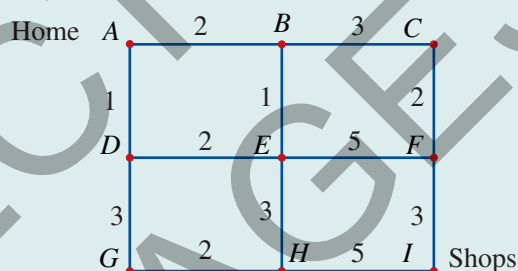
b B and D

c G and C



Example 17 Solving a shortest path problem

Darcy has drawn a network diagram to represent several streets for travelling from his home to the shops. The numbers indicate the times in minutes. Describe the shortest path and minimum travelling time.



Solution

$$\begin{aligned} A-D-E-H-I &= 1 + 2 + 3 + 5 \\ &= 11 \text{ min} \end{aligned}$$

$$\begin{aligned} A-D-G-H-I &= 1 + 3 + 2 + 5 \\ &= 11 \text{ min} \end{aligned}$$

$$\begin{aligned} A-B-E-H-I &= 2 + 1 + 3 + 5 \\ &= 11 \text{ min} \end{aligned}$$

$$\begin{aligned} A-B-C-F-I &= 2 + 3 + 2 + 3 \\ &= 10 \text{ min} \end{aligned}$$

The shortest path is $A-B-C-F-I$ and the time taken is 10 minutes.

Explanation

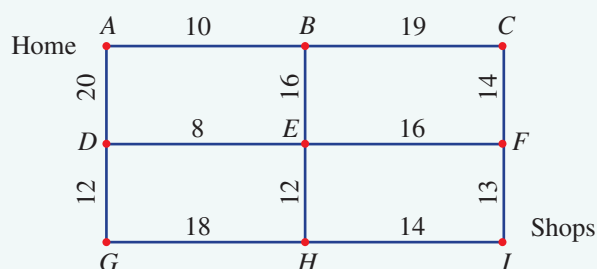
Identify all the likely shortest routes between home and the shops and calculate their lengths. Compare the different path lengths and identify the shortest path.

Write your answer in words.

Now you try

Julian has drawn a network diagram to represent several streets for travelling from his home to the shops. The numbers indicate the times in minutes.

Describe the shortest path and minimum travelling time.



Dijkstra’s algorithm

Dijkstra’s algorithm is a method to find a spanning tree that contains all the shortest paths from a starting vertex to every other vertex of a connected weighted network. It is used in larger networks where it can be difficult to consider all possible paths between two vertices.

Starting at any vertex, A, Dijkstra’s algorithm builds a tree of shortest paths originating from A. The process involves the following steps:

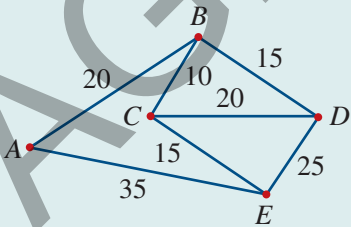
- 1 Start the tree with only the starting vertex A.
- 2 Create a list of all possible paths extending the tree by one edge from the vertex A. Select the path with the smallest total weight and add its corresponding vertex and edge to the tree.
- 3 Repeat step 2 until every vertex is included in the tree.

This is similar to Prim’s algorithm, except in step 2 you choose a new edge based on the total distance from vertex A rather than the next edge of least weight.

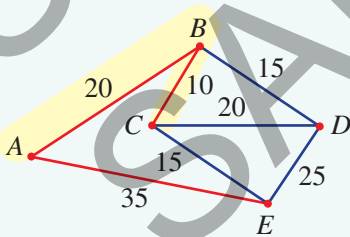
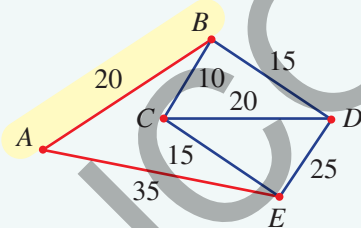


Example 18 Using Dijkstra’s algorithm to find the shortest path

Use Dijkstra’s algorithm to find the shortest path from A to D in the network shown.



Solution



Explanation

Start at Vertex A.
List the paths from A, adding one vertex at a time.

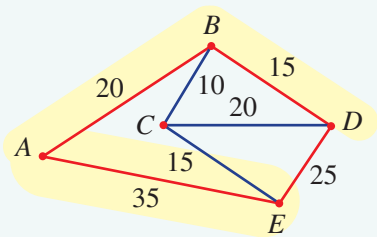
Path	AB	AE
Total weight	20	35

Path AB has minimum total weight, add this. The tree contains vertices A and B.

Path	ABC	ABD	AE
Total weight	30	35	35

The path ABC has minimum total weight.
The tree contains vertices ABC.

continued ...



The shortest path from A to D is $A-B-D$, length 35.

Path	ABD	$ABCD$	$ABCE$	AE
Total weight	35	50	45	35

The paths ABD and AE are shortest paths.

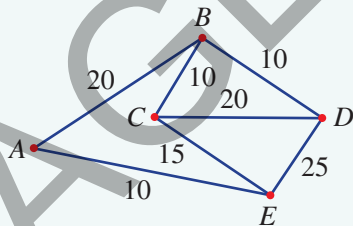
In summary, there is now a list of the shortest paths from Vertex A to every other vertex

$A-B$ (20), $A-B-C$ (30), $A-B-D$ (35), $A-E$ (35)

Note: This answer can also be found by inspection. Dijkstra’s algorithm is used in larger networks, a simple network was used for this example.

Now you try

Use Dijkstra’s algorithm to find the shortest path from A to D in the network shown.



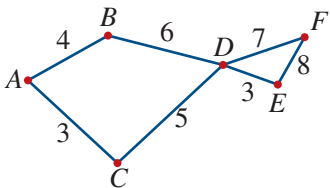
Exercise 9H

BUILDING

Example 15

1 The numbers in this weighted graph represent time in hours. Find the length of the shortest path between the following vertices.

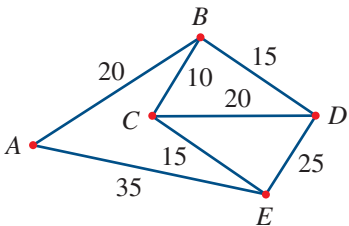
- a** A and D
- b** A and E
- c** A and F
- d** C and F



Example 16

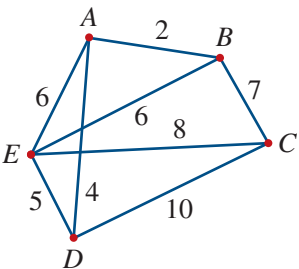
2 The numbers in this weighted graph represent length in metres. Find the length of the shortest path between the following vertices.

- a** C and D
- b** A and C
- c** A and D
- d** B and E



3 The numbers in this weighted graph represent cost in dollars. Find the length of the shortest path between the following vertices.

- a** E and C
- b** B and E
- c** C and D
- d** A and E
- e** A and C
- f** A and D
- g** B and D
- h** A to D to E



- 4 The numbers in this weighted graph represent time in minutes. Find the length of the shortest path between the following vertices.

a A and C

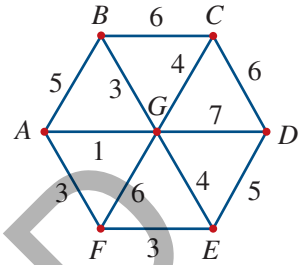
b A and E

c B and D

d B and F

e C and F

f C and E



- 5 The numbers in this weighted graph represent distance in metres. Find the length of the shortest path between the following vertices.

a X and C

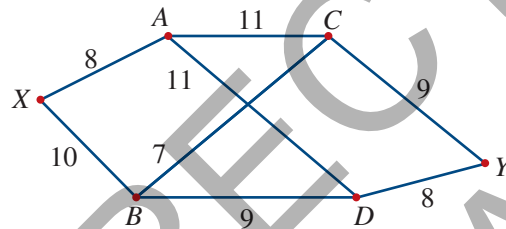
b X and D

c A and Y

d B and Y

e A and B

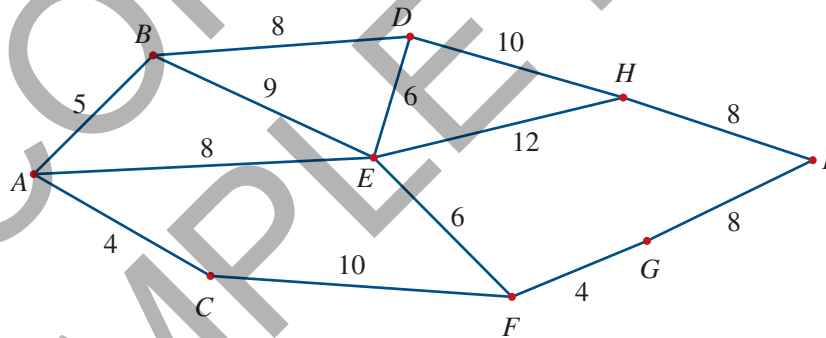
f X and Y



DEVELOPING

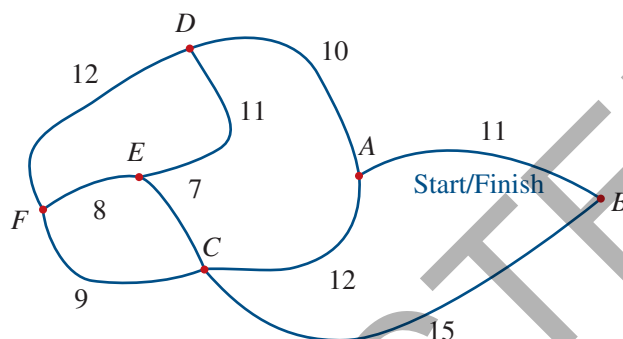
Example 17

- 6 The network shows the distance, in kilometres, along walkways that connect landmarks A, B, C, D, E, F, G, H and I in a national park.



- What distance is travelled on the path $A-B-E-H-I$?
- What distance is travelled on the path $I-G-F-E-D-B-A$?
- What distance is travelled on the circuit $F-E-D-H-E-A-C-F$?
- What distance is travelled on the circuit $D-E-A-B-E-F-G-I-H-D$?
- What is the distance travelled on the shortest cycle starting and finishing at E?
- What is the distance travelled on the shortest cycle starting and finishing at F?
- Find the shortest path and distance travelled from A to I.
- Find the shortest path and distance travelled from C to D.

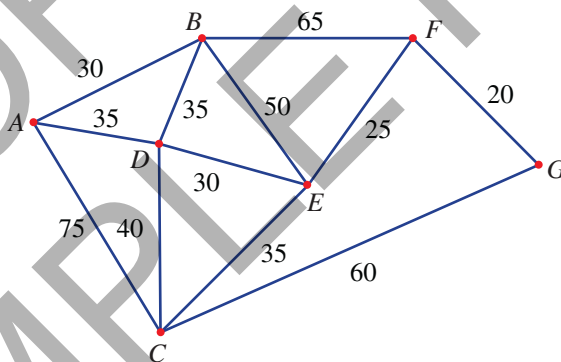
- 7** The graph shows a mountain bike rally course. Competitors must pass through each of the checkpoints, A , B , C , D , E and F . The average times (in minutes) taken to ride between the checkpoints are shown on the edges of the graph. Competitors must start and finish at checkpoint A but can pass through the other checkpoints in any order they wish.



- What is the average time travelled on the circuit $A-D-E-F-C-B-A$?
- What is the average time travelled on the circuit $A-B-C-F-E-D-A$?
- What is the average time travelled on the circuit $A-C-E-D-F-C-B-A$?
- Which path would have the shortest average time?
- Will the path with the shortest average time always be the best path? Explain your answer.

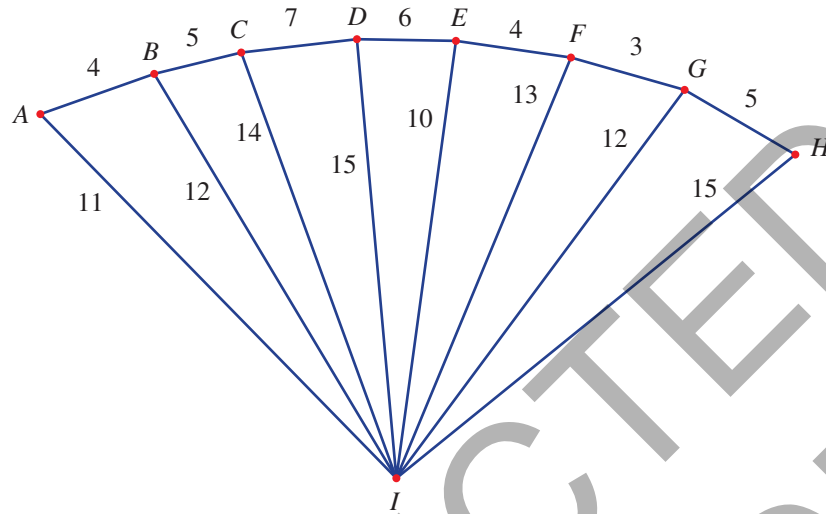
CHALLENGING

- 8** The network shows the time (in minutes) of train journeys between seven stations.

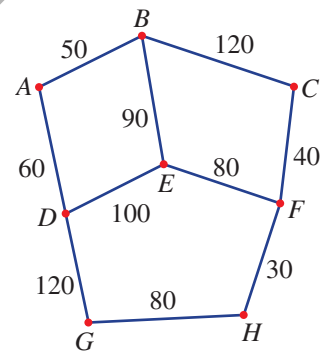


- What is the time taken to travel $A-B-E-D-A$?
- What is the time taken to travel $F-G-C-D-B-F$?
- Find the shortest time it would take to travel from A to G .
- Will the path with the shortest time always be the best path? Explain your answer.
- Find the shortest time it would take to travel from A to G if in reality each time the train passes through a station, excluding A and G , an extra 10 minutes is added to the journey.

- 9 The network shows the time (in seconds) to move between a number of seats in a hall.

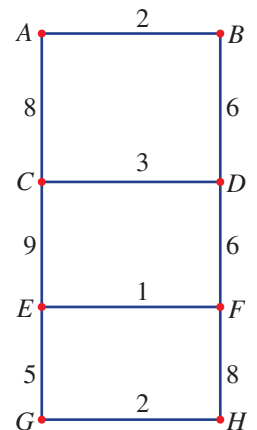


- a What is the fastest way to move from A to D?
 b What is the fastest way to move from A to E?
 c What is the fastest way to move from C to H?
 d If moving through I caused an additional three-second delay, what is the fastest way to move from B to G?
- 10 The network shows a number of houses and the distances between them in metres.
- a What is the shortest distance between A and F?
 b What is the shortest distance between C and D?



Communicate your reasoning, understanding and fluency

- 11 The network shows the time (in minutes) taken to drive along several roads.
- a What is the fastest way to drive from A to G?
 b If making a 90-degree turn at an intersection takes 2 minutes and going straight ahead takes 1 minute, does the fastest route from A to G change? Explain.



Key ideas and chapter summary

Network	A network is a term to describe a group or system of interconnected objects. It consists of vertices and edges that indicate a path or route between two objects.	
Travelling a network	A walk is a connected sequence of the edges showing a route between vertices and edges. A trail is a walk with no repeated edges. A path is a walk with no repeated vertices. A circuit is a walk with no repeated edges that starts and finishes at the same vertex. A cycle is a walk with no repeated vertices that starts and ends at the same vertex.	
Traversable graph	A trail that includes every edge. You can trace out this trail without repeating an edge or removing pen from paper.	
Drawing a network diagram	Connected graph	A graph is connected if every vertex in the graph is accessible from every other vertex in the graph along a path formed by the edges of the graph.
	Isomorphic graph	Two graphs are isomorphic (equivalent) if: • they have the same numbers of edges and vertices • corresponding vertices have the same degree and the edges connect to the same vertices.
Weighted graph	A weighted graph is a network diagram that has weighted edges or an edge with a number assigned to it that implies some numerical value such as cost, distance or time.	
Eulerian trails and circuits	Eulerian trail is a trail that uses every edge of a graph exactly once and starts and finishes at different vertices. A Eulerian trail only exists if the graph has exactly two vertices of odd degree. Eulerian circuit uses every edge of a network exactly once and it starts and finishes at the same vertex. Eulerian circuits only exist if all vertices are of even degree.	
Hamiltonian paths and cycles	Hamiltonian path passes through each vertex of a graph exactly once. Hamiltonian cycle is a Hamiltonian path that starts and finishes at the same vertex.	
Spanning trees: Prim's algorithm and Kruskal's algorithm	A tree is a connected graph that contains no cycles, multiple edges or loops. A tree with n vertices has $(n - 1)$ edges. A spanning tree is a tree that connects all of the vertices of a graph. A minimum spanning tree is a spanning tree of minimum length. It connects all the vertices together with the minimum total weighting for the edges. Prim's and Kruskal's algorithms are a set of rules to determine a minimum spanning tree for a graph.	
Connector problem	Problems using the minimal spanning tree to find least cost to link locations or objects.	
Shortest path	The shortest path between two vertices in a network is the path where the sum of the weights of its edges is minimised.	

Chapter checklist with success criteria



A printable version of this checklist is available in the Interactive Textbook.



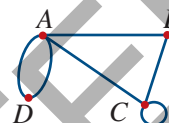
9A

- 1** I can describe and use network terminology including terms such as vertex, edge, degree, loop, directed networks and weighted edges.



e.g. Using the network diagram, find the:

- a** number of vertices
- b** number of edges
- c** degree of vertex A
- d** degree of vertex C.



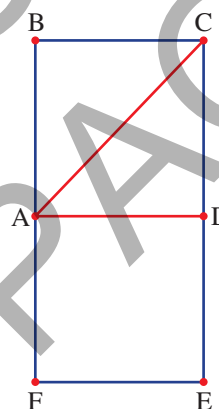
9B

- 2** I can describe different ways to travel around a network.



e.g. Using the graph, identify each walk as a trail, path, circuit, cycle or walk only.

- a** ABCDA
- b** ABCAFEDA
- c** FEDC
- d** ADCDAF



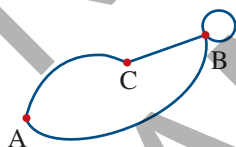
9C

- 3** I can draw and recognise isomorphic graphs.

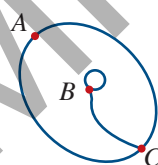


e.g. Which of the following graphs is not isomorphic to the other three graphs in each question?

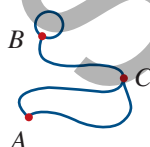
- a** Graph 1



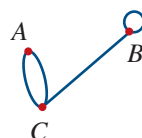
- Graph 2



- Graph 3



- Graph 4



9C

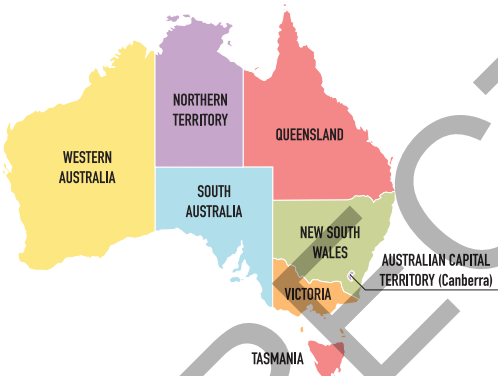
4 I can draw a network diagram to represent a map.

e.g. The map shows the states and territories of Australia.

Construct a network diagram between the states and territories.

Let each state and territory be a vertex and the edges be a shared border between the states and territories

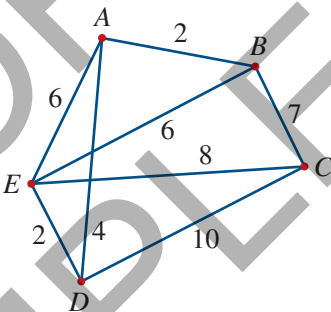
STATES OF AUSTRALIA



9C

5 I can construct a table from a weighted graph.

e.g. Construct a table from a weighted graph.



9C

6 I can construct a weighted graph from a table.

e.g. A network diagram is to be constructed to represent the table shown.

- a What will be the vertices of the network diagram?
- b What will be the values of the weighted edges in the network diagram?
- c Draw a weighted graph to represent the information in the table.

	L	M	P	Q	R
L	–	45	–	38	–
M	45	–	44	–	32
P	–	44	–	30	20
Q	38	–	30	–	35
R	–	32	20	35	–

9C

7 I can construct a directed and weighted graph from a table.



e.g. Construct a directed weighted network diagram to represent to table shown.

		To				
		A	B	C	D	E
From	A		12		15	7
	B					
	C		6			
	D		3	8		
	E	4				

9D

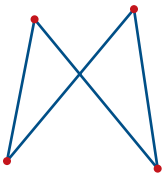
8 (EXTENDING)



I can identify Eulerian trails and circuits.

e.g. For each of the following graphs, determine whether the graph has an Eulerian trail, an Eulerian circuit or neither. Show one example if the graph has an Eulerian trail or Eulerian circuit.

a



b



c



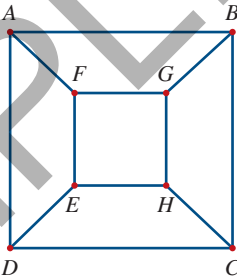
9D

9 (EXTENDING)



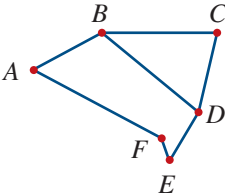
I can identify Hamiltonian paths and cycles.

e.g. a List a Hamiltonian path for the network graph shown that starts at A and finishes at D.



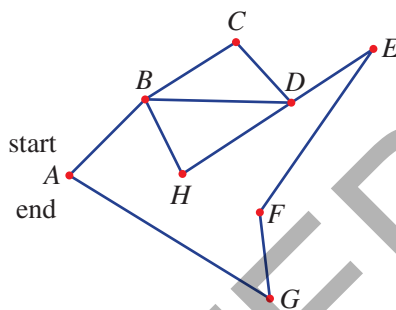
b

Identify a Hamiltonian cycle for the network graph shown starting at A.



9E 10 I can solve problems involving network diagrams in a variety of practical contexts. □

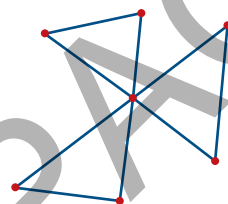
e.g. The roads for a garbage bin collection are shown in the network diagram. The garbage bin collection must start/end at A and travel every road only once.



- How many vertices of odd degree are in this network diagram?
- Does a Eulerian circuit exist in this network diagram?
- Show one possible route for the garbage bin collection.
- List another route for the garbage bin collection.

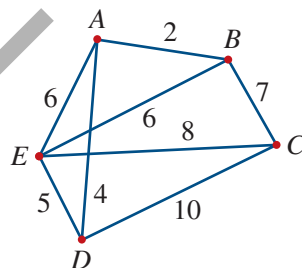
9F 11 I can find a spanning tree in a network. □

e.g. For the graph shown, draw two different spanning trees.



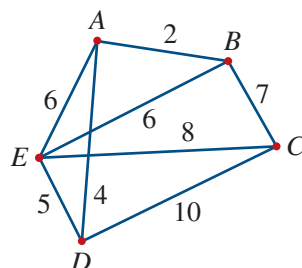
9F 12 I can find the minimum spanning tree in a weighted network graph using Prim's algorithm. □

e.g. Using Prim's algorithm, find the minimum spanning tree.



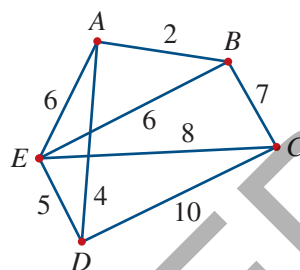
9F 13 I can find the minimum spanning tree in a weighted network graph using Kruskal's algorithm. □

e.g. Using Kruskal's algorithm, find the minimum spanning tree.



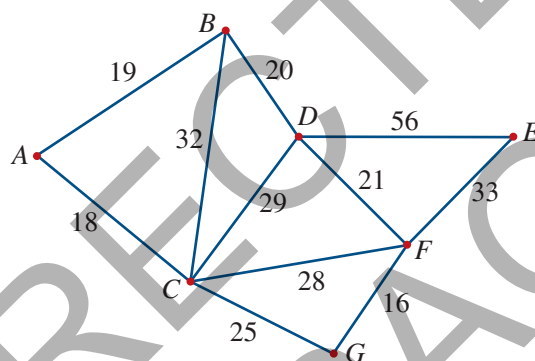
- 9F 14** I can determine the minimum spanning tree with weighted edges using any method. □

e.g. Using any method, find the minimum spanning tree.



- 9G 15** I can use minimum spanning trees to solve minimal connector problems. □

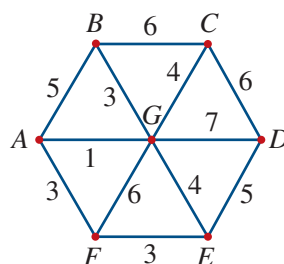
e.g. Seven towns on an island have been surveyed for transport and communications needs. The towns (labelled A, B, C, D, E, F, G) form the network shown. The road distances between the towns are marked in kilometres. To establish a cable network for communications on the island, it is proposed to put the cable underground beside the existing roads.



Draw a minimum spanning tree that will ensure that all the towns are connected to the network but that also minimises the amount of cable used.

- 9H 16** I can identify a shortest path through a network diagram. □

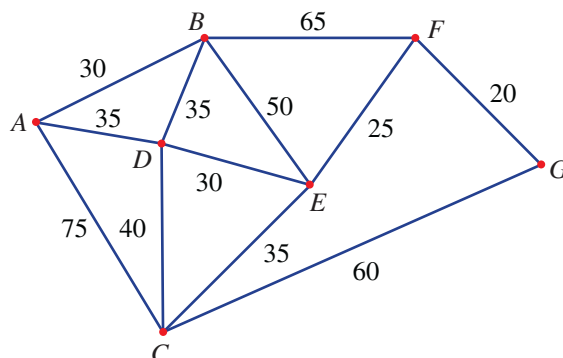
e.g. Find the length of the shortest path between A and E.



- 9H 17** I can solve problems involving the shortest path. □

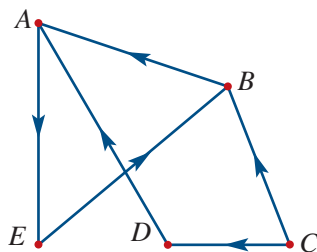
e.g. The network shows the time (in minutes) of train journeys between seven stations.

Find the shortest time it would take to travel from A to G if each time the train passes through a station, excluding A and G, an extra 10 minutes is added to the journey.



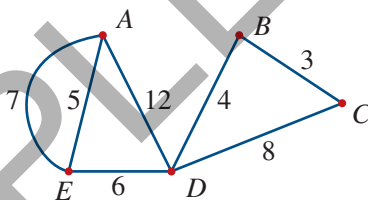
Multiple-choice questions

Questions 1 to 4 relate to this network diagram.



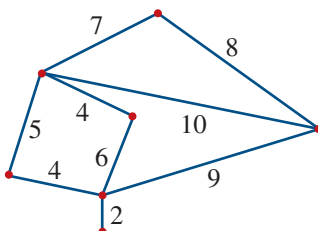
- 9A 1** Which of the following vertices has the highest degree?
A Vertex B **B** Vertex C **C** Vertex D **D** Vertex E
- 9A 2** How many edges are in the network diagram?
A 4 **B** 5 **C** 6 **D** 7
- 9B 3** Which of the following is a valid path?
A $A-E-B-C$ **B** $C-D-A-E-B-A$
C $C-D-A-E-B-C$ **D** $A-E-B$
- 9B 4** The sequence $C-D-A-E-B$ represents:
A a walk only **B** a path **C** a trail **D** a cycle

Questions 5 to 6 relate to this network diagram.



- 9D 5** Which one of the following paths is a Eulerian trail?
A $A-E-D-B-C$ **B** $E-D-A-B-C-E$
C $A-E-D-C-B-D-A-E$ **D** $E-D-C-B-D-C-A$
- 9H 6** What is the length of the shortest path from A to C in the network?
A 17 **B** 18 **C** 19 **D** 20

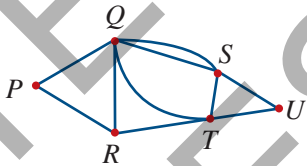
Questions 7 to 8 relate to this network diagram.



- 9F** 7 What is the length of the minimum spanning tree?
A 30 **B** 31 **C** 33 **D** 34
- 9H** 8 What is the length of the shortest path from the bottom vertex to the top vertex?
A 18 **B** 19 **C** 20 **D** 21

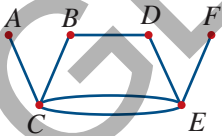
Short-answer questions

- 9A** 1 Find the degree of the following vertices in this network diagram.
a *P* **b** *Q*
c *R* **d** *S*
e *T* **f** *U*



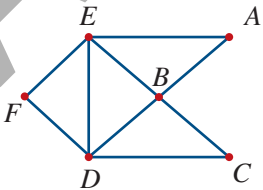
- 9A 9D** 2 This network graph has six vertices.
a What is the degree of each vertex?
b Why does this graph have an Eulerian trail?
c List an Eulerian trail.

Ext



- 9A 9D** 3 This network graph has six vertices.
a What is the degree of each vertex?
b Why does this graph have an Eulerian circuit?
c List an Eulerian circuit.

Ext

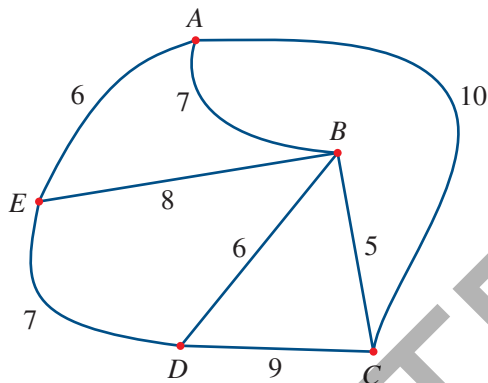


- 9C** 4 A chess tournament is completed between five players. Each game has two players competing against each other. This table shows the games and the players.

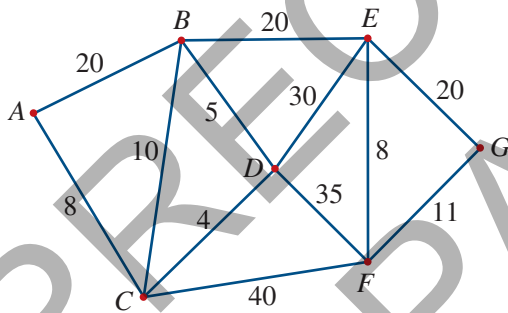
Match	Players	
1	José	Akira
2	Naomi	Beau
3	Naomi	José
4	Akira	Ellie
5	Beau	José
6	José	Ellie

- a** Draw a network diagram to represent the information in the table.
b What are the vertices of the network diagram?
c Which players have not played a game against each other?

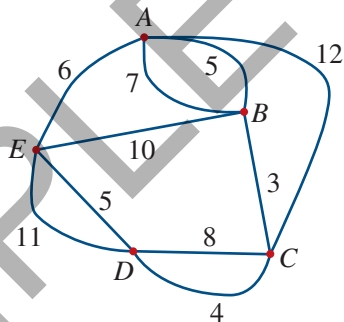
9F 5 For the weighted graph shown, determine the length of the minimum spanning tree.



9F 6 What is the length of the minimum spanning tree in this network?

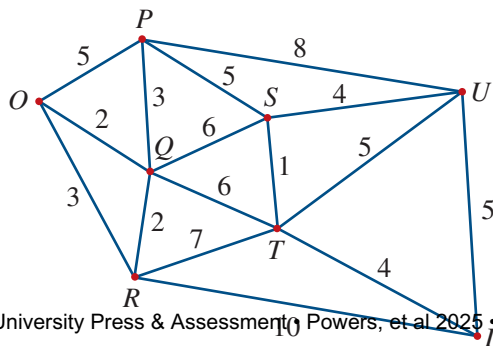


9H 7 In this network, the numbers on the edges represent distances in kilometres.



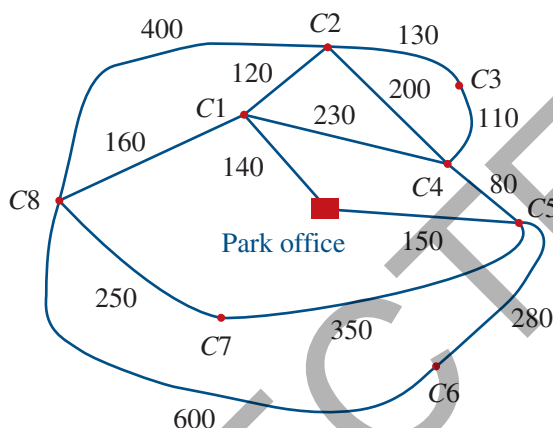
- 9F** Determine the length of:
- a** the shortest path between vertex *A* and vertex *D*
 - b** the length of the minimum spanning tree.

9H 8 What is the length of the shortest path between *O* and *D* in this network?



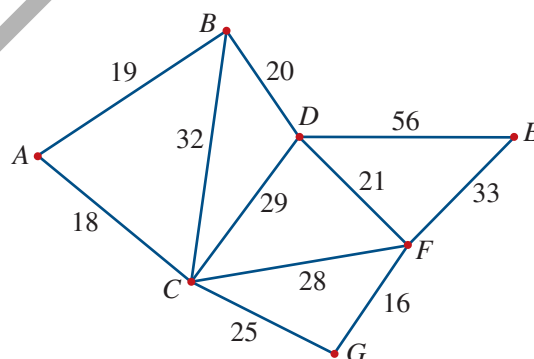
Extended-response questions

- 9G 1** This diagram shows the network of walking tracks in a small national park. These tracks connect the campsites to each other and to the park office. The lengths of the tracks (in metres) are also shown.



- A ranger at campsite C8 plans to visit campsites C1, C2, C3, C4 and C5 on her way back to the park office. What is the shortest distance she will have to travel?
- Each day, the ranger on duty has to inspect each of the tracks to make sure that they are all passable. Is it possible for her to do this starting and finishing at the park office without retracing her steps? Explain why or why not.
- Following a track inspection after wet weather, the head ranger decides that it is necessary to put gravel on some walking tracks to make them weatherproof. What is the minimum length of track that will need to be gravelled to ensure that all campsites and the park office are accessible along a gravelled track?

- 9G 2** Seven towns on an island have been surveyed for transport and communications needs. The towns (labelled A, B, C, D, E, F, G) form the network shown. The road distances between the towns are marked in kilometres. To establish a cable network for communications on the island, it is proposed to put the cable underground beside the existing roads.



- Draw a minimum spanning tree that will ensure that all the towns are connected to the network but that also minimises the amount of cable used.
- What is the minimal length of cable required here if back-up links are not considered necessary; that is, there are no loops in the cable network?

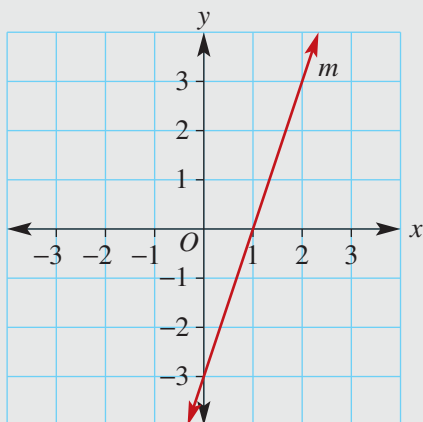
Practice Paper 2

Section I

Attempt Questions 1–15 (15 marks)

Allow about 20 minutes for this section

1



What is the equation of line m ?

A $y = 3x - 3$

B $y = \frac{x}{3} - 3$

C $y = -3x - 3$

D $y = -\frac{x}{3} - 3$

- 2 For the scores 20, 22, 22, 14, 19, 22, 20, 21, 24 and 12 consider the following statements.

i The median is greater than the mean.

ii The mode is less than the median.

A Both statements I and II are true.

B Both statements I and II are false.

C Statement I is false and statement II is true.

D Statement I is true and statement II is false.

- 3 When the price of an item is doubled, the percentage increase is:

A 2%

B 50%

C 100%

D 200%

- 4 What is the gradient of a line given by the equation $y = 2x - 4$?

A -4

B -2

C 2

D 4

- 5 A car mechanic charges a total of \$165 to repair a motor vehicle. The Goods and Services Tax (GST) of 10% was included in this total. Which of the following statements is correct?

A 90% of \$165 was the price of the repair before the GST was added.

B The total repair price included \$16.50 GST.

C The price before adding the GST is $\$165 \div 1.10$.

D The GST cannot be determined without knowing the original repair cost.

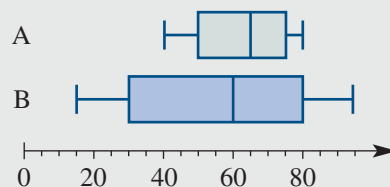
- 6 Two examinations results are displayed in the box plot. What is the interquartile range for exam A?

A 25

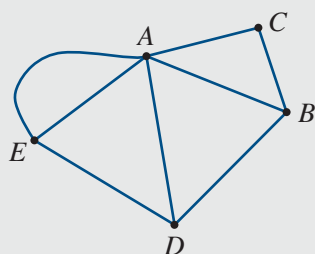
B 40

C 50

D 80



Use the following network graph to answer Questions 7 to 10.



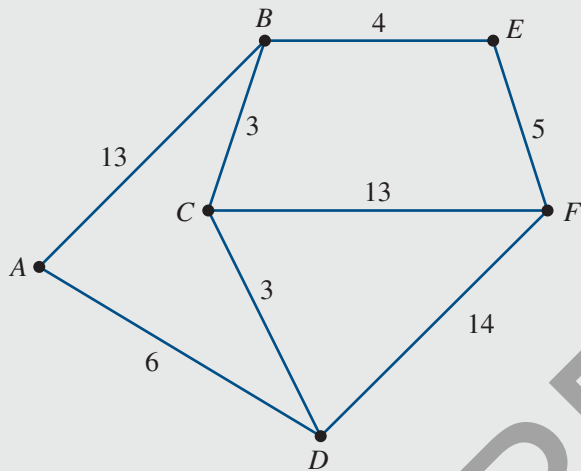
- 7 What is the degree of vertex B ?
A 1 **B** 2 **C** 3 **D** 4
- 8 How many edges are there in the graph?
A 5 **B** 6 **C** 7 **D** 8
- 9 How many even vertices are there in the network diagram?
A 1 **B** 2 **C** 3 **D** 4
- (Ext)** 10 The graph has:
A an Eulerian trail but not an Eulerian circuit.
B several Eulerian trails but no Eulerian circuits.
C an Eulerian circuit.
D neither an Eulerian trail nor an Eulerian circuit.
- 11 A car is travelling at a constant speed. It travels 60 km in 3 hours. This situation is described by the linear equation $d = mt$. What is the value of m ?
A 0.05 **B** 3
C 20 **D** 60
- 12 A cruise ship leaves the wharf at 6:48 a.m. The journey to its next port takes 13 hours and 16 minutes. When does the cruise ship arrive at this port?
A 7:32 a.m. **B** 8:04 a.m.
C 7:32 p.m. **D** 8:04 p.m.
- 13 If $C = 2n + 5$, by how much does C increase when n increases by 10?
A 5 **B** 10
C 15 **D** 20
- 14 What is the time difference between Ho Chi Minh (UTC +7) and Melbourne (UTC +10)?
A 3 h **B** 7 h
C 17 h **D** 70 h
- 15 Which equation correctly gives the relationship between x and y , from the table?
- | | | | | | |
|-----|----|----|----|----|----|
| x | 0 | 2 | 5 | 9 | 10 |
| y | -2 | 10 | 28 | 52 | 58 |
- A** $y = 5x - 2$ **B** $y = 4x + 2$
C $y = 6x - 2$ **D** $y = 10x + 2$

Section II

Attempt Questions 16–18 (45 marks)
Allow about 70 minutes for this section
All necessary working should be shown in every question.

Question 16 (15 marks)

Ext a A network graph is shown.



- i

Does this graph have an Eulerian circuit? Give a reason.

(1 mark)

ii

Find the minimum spanning tree and its length for the network.

(2 marks)

iii

What is the length of the shortest path from A to E in the network?

(1 mark)

iv

What is the length of the shortest path from A to F in the network?

(1 mark)

v

What is the length of the shortest Hamiltonian cycle for the network?

(2 marks)
- b

Nathan is an auto-electrician who is entitled to a 15% trade discount. In addition, he is given a 10% reduction on the discounted price if he pays cash. Nathan bought electrical equipment with a full retail price of \$480 and was given both discounts.

i

How much does Nathan pay for the electrical equipment?

(1 mark)

ii

How much money is saved using the discounts?

(1 mark)

iii

Express the overall savings as a percentage of the retail price.

(1 mark)
- c

Jacksonville is located at (30°N, 82°W) and Suva is located at (18°S, 178°E).

i

What is the difference in latitude between Jacksonville and Suva?

(1 mark)

ii

What is the difference in longitude between Jacksonville and Suva?

(1 mark)

iii

What are the coordinates of a point 45° due south of Jacksonville?

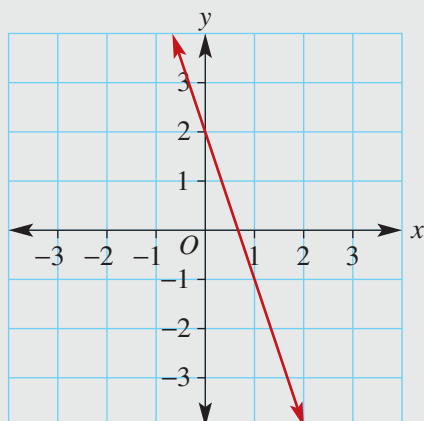
(1 mark)
- d

Raymond has been quoted \$1540 for comprehensive car insurance. He has a no-claim bonus of 40%. How much is Raymond required to pay?

(2 marks)

Question 17 (15 marks)

a



- i What is the gradient of this straight line? (1 mark)
- ii What is the y-intercept of this straight line? (1 mark)
- iii What is the equation of this straight line? (1 mark)
- iv What is the x-intercept of this straight line? (1 mark)

b Max has completed three tests. His mean mark is 64%. What percentage mark does he have to get in his next test to increase the mean to 65%? (2 marks)

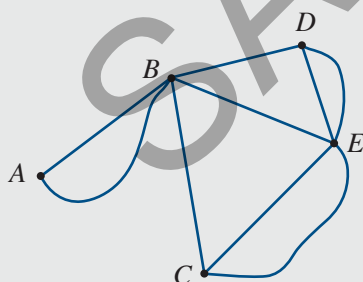
c Calculate the mean and standard deviation of this dataset: (2 marks)
12, 20, 23, 34, 46, 67, 78, 87, 54, 32, 29, 18, 17, 12, 11

d A Year 11 class is given a short quiz out of 10. Calculate the following measures of location and spread. Answer correct to one decimal place.

Score	Number of students
7	4
8	3
9	5
10	6

- i Mean (1 mark)
- ii Median (1 mark)
- iii Mode (1 mark)
- iv Range (1 mark)
- v Interquartile range (1 mark)

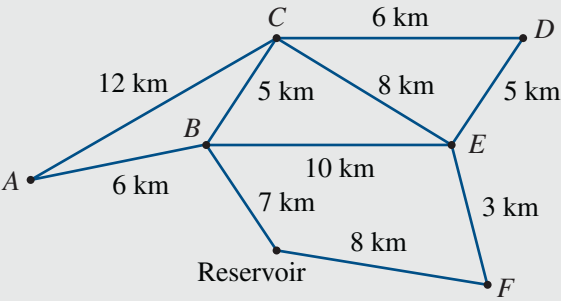
e A network diagram is shown.



- i What is the degree of vertex E ? (1 mark)
- ii Give an example of a Hamiltonian path in this graph. (1 mark)

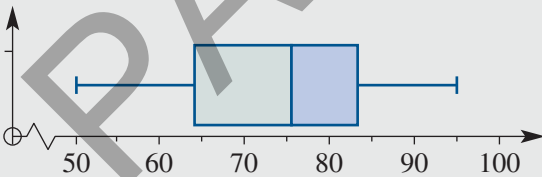
Question 18 (15 marks)

- a** The diagram shows the network of pipes providing water from a reservoir to six small settlements. These pipes connect the settlements to each other and to the reservoir. The lengths of the pipes (in km) are shown.

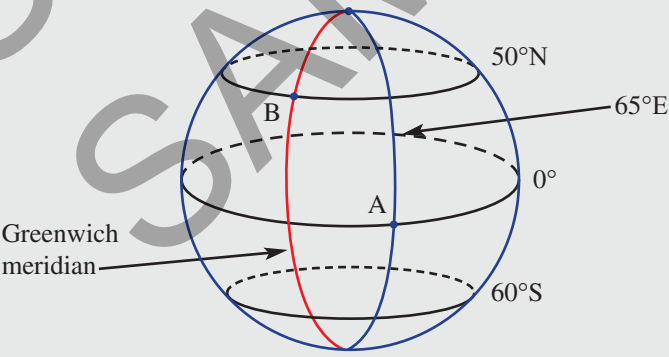


The pipe inspector plans to start her inspection at the reservoir, travel along each of the pipes and return to the reservoir without having to travel along any pipe section more than once.

- i** What is the technical name for the route she wants to follow? (1 mark)
 - ii** Explain why such a route is possible for this network of pipes. (1 mark)
 - iii** Name one such route she could follow. (1 mark)
 - iv** The system of pipes is due for replacement. What is the minimum length of pipe that can be used to service all of the settlements? (2 marks)
- b** The box plot shows the weights (kg) of 400 people.



- i** What is the range of weights? (1 mark)
 - ii** Determine the median weight. (1 mark)
 - iii** Determine the interquartile range of weights. (1 mark)
 - iv** How many people weighed less than 83 kg? (2 marks)
- c** The time zone of Dili is UTC +9 and Napier is UTC +12.
- i** What is the time in Dili if the time in Napier is 10:30 a.m.? (1 mark)
 - ii** A flight leaves Napier at 12:30 p.m. and flies non-stop to Dili. The flight is in the air for 6 hours. At what time does the flight arrive in Dili? (1 mark)
- d** A diagram of Earth's surface is shown.



- i** What is the latitude and longitude of point A? (1 mark)
- ii** What are the coordinates of a point 80° due south of point A? (1 mark)
- iii** What are the coordinates of a point 80° due west of point A? (1 mark)

Glossary

Note: The interactive textbook includes more pop-up definitions and a dictionary.

A

Absolute error The difference between the actual value and the measured value indicated by an instrument.

Algebraic expression Mathematical statement built up from numbers and pronumerals (variables) and connected by mathematical operations, but containing no equals sign.

Allowable deduction See Tax deduction.

Amount owed The result when the interest of a loan or investment is added to the principal amount (also known as future value).

Angular distance The distance between two points on a circle or sphere measured as the angle formed at the centre between radii at the points.

Annual leave loading A payment calculated as a fixed percentage of the normal pay over a fixed number of weeks. Annual leave loading is usually at the rate of $17\frac{1}{2}\%$.

Annulus Area between a large and a small circle with a common centre.

Appreciation An increase in value of an item over time. It is often expressed as the rate of appreciation.

Area The amount of surface enclosed by the boundaries of the shape.

B

BAC See Blood alcohol content.

Balance The amount of money in an account; the total amount an investment is worth at a point in time including amount invested and interest; or the amount still owing on a loan.

Bias When events are not equally likely.

Bimodal Data with two modes or peaks.

Blood alcohol content (BAC) A measure of the amount of alcohol in your blood.

Bonus An extra payment or gift earned as reward for achieving a goal.

Bound A limit to a measurement. See also Upper bound and Lower bound.

Box plot A graph that uses a five-number summary of a numerical data set.

Budget A plan used to manage money by listing a person's income and expenditure.

C

Calorie A non-SI unit of heat and energy, once commonly used to measure food energy.

Capacity The maximum volume of liquid or gas which a container could hold.

Car running costs Car costs, such as maintenance, repairs, fuel, improvements, parking, tolls, car washes and fines.

Casual rate An amount paid for each hour of casual work.

Categorical data Data that is divided into categories such as hair colour. It uses words not numbers.

Census Collecting data from the whole population.

Central tendency See Measure of central tendency.

Circumference The distance around the outside of a circle; the curved boundary.

Class A group of data.

Class centre Median or middle score of a class in a grouped frequency distribution.

Closed cylinder A cylinder with both circular bases. See Open cylinder.

Coefficient A numeral placed before a variable to indicate that the variable is multiplied by that number.

Commission A payment for services, mostly as a percentage of the value of the goods sold.

Common denominator A common denominator is always found by multiplying the denominators of two given fractions together. This is required before adding or subtracting fractions.

Common factors A number or term that divides exactly (no remainder) into each of the given numbers or terms.

Complementary event The outcomes that are not members of the event.

Composite shape A 2D shape made up of two or more 2D shapes.

Composite solid Two or more common solids.

Compound interest A type of interest that is paid on a loan or earned on an investment, which is calculated not only on the initial principal, but also on the interest accumulated during the loan/investment period.

Compounding annually A form of compound interest where the interest is calculated per year.

Compounding time period The compounding period is the time period for the calculation of interest for an investment or loan. Typical compounding periods are yearly, quarterly, monthly or daily.

Concentration The amount of an ingredient in a mixture or solution expressed as a ratio to the amount of the mixture or solution.

Cone A solid figure, with a circular base, that tapers to a point.

Constant of variation The constant coefficient related to pairs of variables that are in direct proportion.

Constant term The part of an equation or expression without any pronumerals.

Consumption Of fuel or energy, the rate expressed as amount used over time or distance or other variable.

Continuous data Numerical data obtained when quantities are measured rather than counted.

Conversion graph A graph used to change a quantity from one unit to another unit.

Coordinated Universal Time (UTC) *See* Greenwich Mean Time.

Corresponding Matching, for example in two or more similar figures, the sides or angles which match.

Cross-section The intersection of a solid with a plane.

Cumulative frequency The frequency of the score plus the frequency of all the scores less than that score. It is the progressive total of the frequencies.

Cumulative frequency histogram A histogram with equal intervals of the scores on the horizontal axis and the cumulative frequencies associated with these intervals shown by vertical rectangles.

Cumulative frequency polygon A line graph constructed by joining the top right-hand corner of the rectangles in a cumulative frequency histogram. Also called an ogive.

Cylinder A prism right with a circular cross-section. *See* Open cylinder and Closed cylinder.

D

Data Raw scores. Information before it is organised.

Daylight saving time Advancing clocks one hour during the warmer months of the year so that the increased hours of daylight fall in the evening more than in the early morning.

Decile Divides an ordered dataset into 10 equal groups.

Deduction A regular amount of money subtracted from a person's wage or salary. *See also* Tax deduction.

Dependent variable If two variables y and x are related in an equation, when a number is substituted for one variable, such as $x = 2$, then this variable is called the independent variable and the other is called the dependent variable.

Deposit An amount put into an account; an initial amount paid to reserve the goods or services; the first payment of a series of instalments.

Depreciation *See* Straight-line depreciation.

Direct variation The relationship between two quantities that increase or decrease at the same rate. Equation for direct variation: $y = mx$ where y and x are two variables in direct proportion to each other and m is the gradient of the graph of y against x .

Discrete data Data obtained when a quantity is counted. It can only take exact numerical values.

Distributive law A rule for expanding grouping symbols by multiplying each term inside the grouping symbol by the number or term outside the grouping symbol.

Dose The amount of a medication or drug given to a patient; it may refer to the amount of the active ingredient rather than total amount of medication itself.

Dot plot A graph that consists of a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other.

Double stem-and-leaf plot A stem-and-leaf plot that uses two sets of similar data together.

Double time A penalty rate that pays the employee twice the normal hourly rate.

E

Element of the sample space An outcome or data value in probability.

Energy The capacity to do work.

Energy consumption The amount of energy consumed per unit of time.

Equally likely outcomes Outcomes of an event that have the same chance of occurring.

Equation A mathematical statement that says that two things are equal.

Equator Imaginary horizontal line that divides Earth into two hemispheres. Latitude of the equator is 0° .

Equivalent fractions Fractions that can be reduced to the same basic fraction; fractions that have the same value but are written differently.

Evaluate Calculate the exact value of an expression.

Event In a probability experiment, the outcome or result. *See also* Multistage event.

Expand Remove the grouping symbols.

Expanded form An expression with no grouping symbols; a term shown without indices.

Expected frequency The number of times that a particular event should occur.

Expression A mathematical statement written in numbers and symbols, containing two or more numbers or variables connected by operations.

F

Factorise To break up an expression into a product of its factors.

Five-number summary A summary of a data set consisting of the lower extreme, lower quartile, median, upper quartile and upper extreme.

Flat interest See Simple interest.

Formula A mathematical relationship between two or more variables.

Fortnight Two weeks or 14 days.

Frequency The number of times a certain event occurs; also the number of times a data value occurs in a set.

Frequency distribution The division of a set of observations into a number of classes, together with a listing of the number of observations (the frequency) in that class. Frequency distributions can be displayed in the form of a frequency table, a two-way-table or in graphical form.

Frequency histogram A histogram with equal intervals of the scores on the horizontal axis, and the frequencies associated with these intervals shown by vertical rectangles.

Frequency polygon A line graph constructed by joining the midpoints at the tops of the rectangles of a frequency histogram.

Frequency table A table that lists the outcomes and how often (frequency) each outcome occurs. If the data is grouped it is a grouped frequency table.

Fuel consumption rate The number of litres of fuel a vehicle uses to travel 100 kilometres.

Function (mathematical) A mathematical relation where each element of a given set is associated with at most one element of another set.

G

General form A linear equation written in the form $ax + by + c = 0$.

Goods and Services Tax (GST) A tax added to the purchase price of each item. The GST rate in Australia is 10% of the purchase price of the item except for basic food items and some medical expenses.

Gradient The steepness or slope of the line. It is calculated by dividing the vertical rise by the horizontal run.

Gradient-intercept formula A linear equation written in the form $y = mx + b$.

Greenwich Mean Time Time at the Greenwich meridian.

Greenwich meridian Imaginary vertical line that passes through the town of Greenwich (London). Longitude of the Greenwich meridian is 0° .

Gross income The total amount of money earned from all sources. It includes interest, profits from shares and all payments received throughout the year.

Gross pay The total of an employee's pay including allowances, overtime pay, commissions and bonuses.

Grouped data Data organised into small groups or class intervals rather than as individual scores.

Grouped frequency table A table with classes listed in the first column in ascending order, a class centre column, an (optional) tally column and a frequency column with total count of scores in each class.

Grouping symbol Symbols used to indicate the order of operations such as parentheses () and brackets [].

H

Histogram A graph using columns to represent frequency or cumulative frequency. See Frequency histogram and Cumulative frequency histogram.

Hypotenuse The side in a right-angled triangle opposite the right-angle. It is the longest side.

I

Income tax Tax paid on income received.

Independent variable See Dependent variable.

Index form See Index notation.

Index notation A method to write expressions in a shorter way such as $a \times a = a^2$.

Inflation A rise in the price of goods and services or Consumer Price Index (CPI). It is often expressed as annual percentage.

Intercept The position where the line cuts the axis.

Interest The amount paid for borrowing money or the amount earned for lending money.

Interest rate The rate at which interest is charged or paid. It is usually expressed as a percentage.

International Date Line An imaginary line through the Pacific Ocean that corresponds to 180° longitude.

Interquartile range The difference between the first quartile and third quartile.

Interval (data) Describes the boundaries of a group of data.

Investment Money deposited into a bank or financial institution that grows at a particular interest rate for the time it is kept there.

Isosceles triangle A triangle where two sides have equal lengths and two angles are equal. The equal angles are the ones joining each same-length side to the third side.

J

Joule SI unit of energy.

K

Kilojoules SI unit of energy, commonly used to measure food and heat energy, equal to 1000 joules accepted measurement for food energy.

Kilowatt-hour Non-SI unit energy commonly used to measure electrical energy.

L

Latitude The angle or angular distance north or south of the equator.

Like term Terms with exactly the same pronumerals, such as $3a$ and $6a$ and the same powers.

Limit of reading See Precision.

Linear equation An equation whose variables are raised to the power of 1.

Linear expression A mathematical statement written in numbers and symbols See also Non-linear expression.

Linear function A function that when graphed on a number plane is a straight line. See also Non-linear function and Function (mathematical).

Linear modelling A mathematical description of a practical situation using a linear function.

Loan repayment An amount paid to a bank or financial institution at regular intervals to repay a loan with interest included.

Longitude The angle or angular distance east or west of the Greenwich meridian.

Lower bound Measurement minus absolute error.

Lower extreme Lowest score in the data set.

Lower quartile The lowest 25% of the scores in the data set.

Lowest common denominator For two or more fractions, the lowest common multiple of their denominators.

M

Mass The amount of matter within an object.

Mean A measure of the centre. It is calculated by summing all the scores and dividing by the number of scores.

Measurement Determining the size of a quantity.

Measures of central tendency Also known as measures of location. The most common measures are mean, median and mode.

Measures of spread Measures of spread include range, interquartile range and standard deviation.

Median The middle score or value. To find the median, list all the scores in increasing order and select the middle one.

Medicare levy An additional charge to support Australia's universal healthcare system.

Meridians of longitude Great imaginary circles east and west of the Greenwich meridian.

Modality The number of modes occurring in a set of data.

Mode The score that occurs the most. It is the score with the highest frequency.

Multimodal Data with many modes or peaks.

Multistage event Two or more events such as tossing a coin and rolling a die.

N

Negatively skewed Data more on the right side. The long tail is on the left side (negative side).

Net of a solid A drawing consisting of plane shapes that can be folded to form the solid.

Net pay The amount remaining after deductions have been subtracted from the gross pay.

Nominal data Categorical data whose name does not indicate order.

Non-linear An equation or expression which when graphed does not produce a straight line.

Number pattern A sequence of numbers formed using a rule. Each number in the pattern is called a term.

Number plane A graph with a vertical y-axis and a horizontal x-axis intersecting at right angles at the origin O (0, 0).

Numerator The top number in a fraction.

Numerical data Data that is measured using numbers.

O

Ogive See Cumulative frequency polygon.

Open cylinder A cylinder without a circular base. It is the curved part of the cylinder.

Order of operations The sequence in which computations are done.

Ordinal data Categorical data whose name does indicate order.

Outcome A possible result in a probability experiment.

Outlier Data values that appear to stand out from the main body of a data set.

Overtime Extra payment when a person works beyond the normal working day.

P

Parallel box plot A box plot that uses two sets of similar data together.

Parallelogram A quadrilateral with both pairs of opposite sides parallel.

Parallels of latitude Small imaginary circles north and south of the equator.

Pareto chart A graph that combines a frequency histogram and a cumulative frequency line graph. The histogram is plotted in decreasing order of relative frequency.

Pareto principle 20% of the invested input is responsible for 80% of the results obtained.

Pay As You Go (PAYG) Tax deducted from a person's wage or salary throughout the year.

Per annum Calculated or applied each year.

Percentage change The increase or decrease in the quantity as a percentage of the original amount of the quantity.

Percentage error The maximum error in a measurement as a percentage of the measurement given.

Percentile Divides an ordered data set into 100 equal groups.

Perpendicular height The height of a 2D shape or 3D solid when measured at right angles to the base.

Piecework A fixed payment for work completed.

Population The entire data set.

Population mean μ The mean of all values of a measure in the entire population.

Population standard deviation A calculation for the standard deviation that uses all the data or the entire population (σ_n).

Positively skewed Data more on the left side. The long tail is on the right side (positive side).

Power (Physics) The rate at which energy is generated or consumed. (Mathematics) Another name for index or exponent; a way to express a number multiplied by itself one or more times.

Precision The smallest unit on the measuring instrument, also known as the limit of reading.

Prefix The first part of a word. In measurement, it is used to indicate the size of a quantity.

Principal The initial amount of money borrowed, lent or invested.

Prism A solid shape that has the same cross-section for its entire length or height.

Probability The chance of something happening. The probability of the event is calculated by dividing the number of favourable outcomes by the total number of outcomes.

Pronumeral A letter or symbol used to represent a number.

Pyramid A solid shape with a plane shape as its base and triangular sides meeting at an apex.

Pythagoras' theorem The square of the hypotenuse is equal to the sum of the squares of the other two sides.
 $c^2 = a^2 + b^2$.

Q

Quadrant Quarter of a circle. The arc of a quadrant measures 90° .

Quantile A set of values that divide an ordered dataset into equal groups.

Quantitative data Numerical data. It is data that has been measured.

Quantile Divides an ordered data set into four equal groups. See Upper quartile and Lower quartile.

R

Random sample A sample that occurs when members of the population have an equal chance of being selected.

Range The difference between the highest and lowest scores. It is a simple way of measuring the spread of the data.

Rate A comparison of different quantities in definite order.

Rate of interest See Interest rate.

Relative error A measurement calculated by dividing the limit of reading (absolute error) by the actual measurement.

Relative frequency The frequency of the event divided by the total number of frequencies. It estimates the chances of something happening or the probability of an event.

Replacement In the selection of cards (or other objects) from a set in a probability experiment, 'with replacement' means the selected card is returned to the set before the next card is selected, so the set always contains the original cards. Selection without replacement means the set reduces with each selection.

Retainer A fixed payment usually paid to a person receiving a commission.

Rhombus A quadrilateral with both pairs of opposite sides parallel and all sides equal.

Royalty A payment for the use of intellectual property such as book or song. It is calculated as a percentage of the revenue or profit received from its use.

S

Salary A payment for a year's work, which is divided into equal monthly, fortnightly or weekly payments.

Sample A part of the population.

Sample mean $\bar{x}$ The mean of all values of a measure in a particular sample.

Sample space The set of all possible outcomes.

Sample standard deviation A calculation for the standard deviation when the data set is a sample (σ_{n-1}).

Scale factor The amount by which a shape is enlarged or reduced to get a similar figure of different size.

Scientific notation See Standard form.

Score A single value from a list of data.

Sector Part of a circle between two radii and an arc.

Self-selected sample Members of the population volunteer themselves.

Semicircle Half a circle. The arc of a semicircle measures 180° .

Significant figures A statement to specify the accuracy of a number. It is often used to round a number.

Similar figures Figures which are exactly the same shape but different sizes.

Simple interest A fixed percentage of the amount invested or borrowed and is calculated on the original amount.

Simulation A mathematical model that represents a real experiment or situation.

Skewed data Data that is not symmetrical. *See* Symmetrical, Positively skewed and Negatively skewed.

Slant height The shortest distance from the apex to an edge of the base in a pyramid or cone.

Smoothness Data whose graph has no breaks or jagged sections.

Sphere A perfectly round solid such as a ball.

Stamp duty Tax paid to the government when registering or transferring a motor vehicle.

Standard deviation A measure of the spread of data about the mean. It is an average of the squared deviations of each score from the mean.

Standard drink Any drink containing 10 grams of alcohol.

Standard form (also called scientific notation) A number between 1 and 10 multiplied by a power of 10. It is used to write very large or very small numbers more conveniently.

Stem-and-leaf plot A method of displaying data in which the first part of a number is written in the stem and the second part of the number is written in the leaves.

Stopping distance The distance a vehicle travels from the time a driver sees an event occurring to the time the vehicle is brought to a stop.

Straight-line depreciation The value of an item decreases by the same amount each period.

Strata A group within a population that reflects the characteristics of the entire population.

Stratified sample A sample using categories or strata of a population. Members from each category are randomly selected. For example, one student is selected from each year 7, 8, 9, 10, 11 and 12.

Subject of a formula or equation When a formula or equation has a pronumeral with no numbers on the left-hand side of the equal sign, such as $C = 40n + 75$ then C is the subject of the formula.

Substitution It involves replacing the pronumeral in an algebraic expression with one or more numbers.

Summary statistic A number such as the mode, mean or median that describes the data.

Superannuation Type of annuity in which money is invested for a person's retirement.

Surface area The sum of the area of each surface of the solid.

Symmetrical Data that forms a mirror image of itself when folded in the 'middle' along a vertical axis.

Symmetry Data evenly balanced about the centre.

Systematic sample A sample that divides the population into a structured sample size. For example, sorting the names of people in alphabetical order and selecting every 5th person.

T

Table of values A set of ordered pairs (arranged in table format) that can be used to plot the points to draw a graph.

Tally To use simple marks or strokes to track the number of times a value occurs; a cross stroke is often used to show groups of 5.

Tax deduction Expenses related to earning an income allowed by the Australian Taxation Office to be deducted from their income for the purposes of calculating taxable income, such as self-education, travel, car or clothing expenses.

Tax payable Tax owed to the taxation office, such as the amount of tax which an employee has to pay according to their taxable income.

Tax refund Money returned to an individual by the Australian Taxation Office when their tax return reveals that their PAYG tax paid exceeds their tax payable in a year.

Tax return A form filled out on paper or online and submitted to the Australian Taxation Office which presents details of an individual's tax claims, income and tax deductions.

Taxable income The gross income minus any allowable deductions.

Time 24-hour Time of day written in the form hhmm.

Time-and-a-half A penalty rate that pays the employee one and half times the normal hourly rate.

Time zone A region of Earth that has a uniform standard time or local time.

Timetable A list of times at which possible events or actions are intended to take place.

Trapezium A quadrilateral with at least one pair parallel sides.

Trapezoidal rule A formula to estimate the area of a shape with an irregular boundary.

Tree diagram A technique used to list the outcomes in a probability experiment. It shows each event as a branch of the tree.

U

Unimodal Data with only one mode or peak.

Upper bound Measurement plus absolute error.

Upper extreme Highest score in the data set.

Upper quartile The highest 25% of the scores in the data set.

V

Value Added Tax (VAT) A tax added to the purchase price of each item. VAT is used in many countries with the rate ranging from 2% to 25%.

Variable A symbol used to represent a number or group of numbers.

Variation A mathematical relationship between two variables.

Volume The amount of space occupied by a three-dimensional solid.

W

Wage A payment for work that is calculated on an hourly basis.

Watt SI unit for power.

X

x-intercept The point at which the graph cuts the x -axis.

Y

y-intercept The point at which the graph cuts the y -axis.