GENERAL MATHEMATICS

UNITS 1 & 2

CAMBRIDGE SENIOR MATHEMATICS
FOR QUEENSLAND

PETER JONES | KAY LIPSON | DAVID MAIN | BARBARA TULLOCH
KYLE STAGGARD
Consultants: Ray Minns | Steve Sisson

INCLUDES INTERACTIVE
TEXTBOOK POWERED BY
CAMBRIDGE HOTMATHS

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Introduction and overview

Cambridge Senior Mathematics for Queensland General Mathematics Units 1&2 has been written for the QCAA syllabus to be implemented in Year 11 from 2019. As well as covering all the subject matter of the Queensland General Mathematics syllabus, the package addresses its objectives, assessment, underpinning factors, formula sheet, and pedagogical and conceptual frameworks.

Its four components—the print textbook, the downloadable PDF textbook, the online Interactive Textbook and the Online Teaching Resource*—contain a huge range of resources, including worked solutions and revision of Year 10 material, available to schools in a single package at one convenient price. There are no extra subscriptions or per-student charges to pay.

**The Online Teaching Resource is included with class adoptions, conditions apply.*

► Overview of the print textbook (shown on the page opposite)

1. **Appendix 1 Review of Computation and practical arithmetic** can be used at the end of Year 10 or the beginning of Year 11 to prepare for the course and ensure that basic skills have been covered. It could also be used for review during the course.
2. Chapter **outcomes** are listed at the beginning of each chapter under the syllabus units and topics.
3. Each section and most exercises begin at the top of the page to make them easy to find and access.
4. Step-by-step **worked examples** with precise explanations and **video** versions encourage independent learning, and are linked to exercises.
5. Important concepts are formatted in boxes for easy reference.
6. **Degree of difficulty categories** are indicated for exercises and are featured in the **revision chapters**.
   Degree of difficulty classification of questions: in the exercises, questions are classified as ***simple familiar*** **SF**, ***complex familiar*** **CF**, and ***complex unfamiliar*** **CU** questions. The revision chapters described below also contain model questions for each of these categories, and tests are also provided in the teacher resources, made up of such categorised model questions.
7. **Problem-solving and modelling questions** are included in many exercises, and most chapters have specific problem-solving and modelling sections or exercises. QCAA guidelines have been followed to include both guided and unscaffolded problems and **investigations**, which can be used as assessment tasks.
8. Two **revision chapters** are provided, one for each unit. Each exercise covers a main context chapter and is divided into degree of difficulty categories, and problem-solving and modelling questions and investigations. **Multiple-choice questions** are provided in the Interactive Textbook for automatic marking.
9. Technology is supported via **scientific calculator** guidance, **spreadsheets** and **Desmos widgets**.
10. **Spreadsheet activities** are integrated throughout the text, with accompanying Excel files in the Interactive Textbook.
11. Chapter reviews contain a **chapter summary** and **multiple-choice**, **short-answer** and **extended-response questions**.

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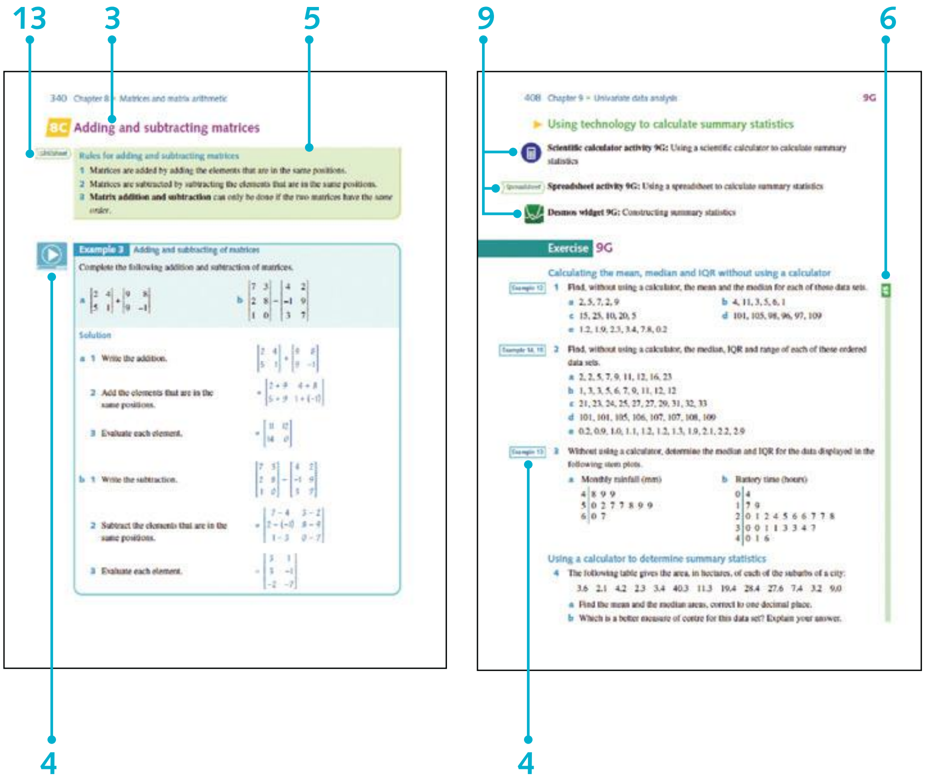
12 A comprehensive **glossary** is included.

13 Additional linked resources in the Interactive Textbook and Online Teaching Suite are indicated in the text, such as:

- problem-solving and modelling tasks and investigations
- skillsheets
- revision of Year 10
- Desmos widgets
- spreadsheet activities
- calculator activities.

PRINT TEXTBOOK

Numbers refer to the descriptions in the overview.



340 Chapter 8 • Matrices and matrix arithmetic

8C Adding and subtracting matrices

Skillsheet

Rules for adding and subtracting matrices

1 Matrices are added by adding the elements that are in the same positions.
2 Matrices are subtracted by subtracting the elements that are in the same positions.
3 **Matrix addition and subtraction** can only be done if the two matrices have the same order.

Example 3 Adding and subtracting of matrices

Complete the following addition and subtraction of matrices.

a $\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 9 & -1 \end{bmatrix}$ **b** $\begin{bmatrix} 7 & 3 \\ 2 & 8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 9 \\ 3 & 7 \end{bmatrix}$

Solution

a 1 Write the addition. $\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 9 & -1 \end{bmatrix}$

2 Add the elements that are in the same positions. $= \begin{bmatrix} 2+9 & 4+8 \\ 5+9 & 1+(-1) \end{bmatrix}$

3 Evaluate each element. $= \begin{bmatrix} 11 & 12 \\ 14 & 0 \end{bmatrix}$

b 1 Write the subtraction. $\begin{bmatrix} 7 & 3 \\ 2 & 8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 9 \\ 3 & 7 \end{bmatrix}$

2 Subtract the elements that are in the same positions. $= \begin{bmatrix} 7-4 & 3-2 \\ 2-(-1) & 8-9 \\ 1-3 & 0-7 \end{bmatrix}$

3 Evaluate each element. $= \begin{bmatrix} 3 & 1 \\ 3 & -1 \\ -2 & -7 \end{bmatrix}$

408 Chapter 9 • Univariate data analysis 9G

Using technology to calculate summary statistics

Scientific calculator activity 9G: Using a scientific calculator to calculate summary statistics

Spreadsheet **Spreadsheet activity 9G:** Using a spreadsheet to calculate summary statistics

Desmos widget 9G: Constructing summary statistics

Exercise 9G

Calculating the mean, median and IQR without using a calculator

Example 12 1 Find, without using a calculator, the mean and the median for each of these data sets.
a 2, 5, 7, 2, 9 b 4, 11, 3, 5, 6, 1
c 15, 25, 10, 20, 5 d 101, 105, 98, 96, 97, 109
e 1.2, 1.9, 2.3, 3.4, 7.8, 0.2

Example 14, 15 2 Find, without using a calculator, the median, IQR and range of each of these ordered data sets.
a 2, 2, 5, 7, 9, 11, 12, 16, 23
b 1, 3, 3, 5, 6, 7, 9, 11, 12, 12
c 21, 23, 24, 25, 27, 27, 29, 31, 32, 33
d 101, 101, 105, 106, 107, 107, 108, 109
e 0.2, 0.9, 1.0, 1.1, 1.2, 1.2, 1.3, 1.9, 2.1, 2.2, 2.9

Example 13 3 Without using a calculator, determine the median and IQR for the data displayed in the following stem plots.

a Monthly rainfall (mm)

4 | 8 9 9
5 | 0 2 7 7 8 9 9
6 | 0 7

b Battery time (hours)

0 | 4
1 | 7 9
2 | 0 1 2 4 5 6 6 7 7 8
3 | 0 0 1 1 3 3 4 7
4 | 0 1 6

Using a calculator to determine summary statistics

4 The following table gives the area, in hectares, of each of the suburbs of a city:

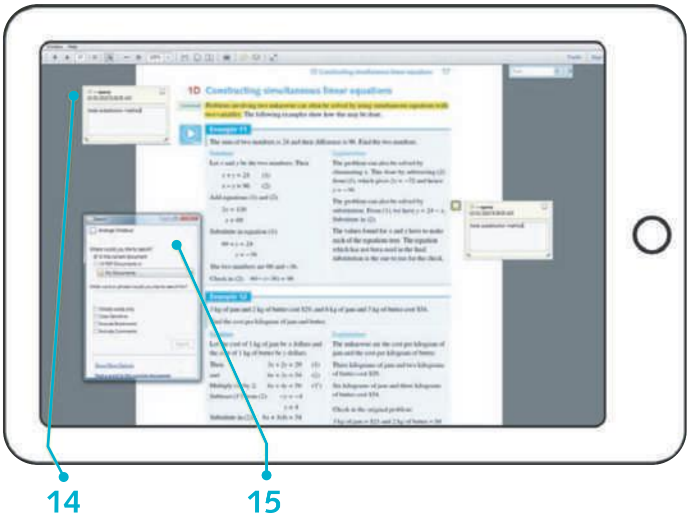
3.6 2.1 4.2 2.3 3.4 40.3 11.3 19.4 28.4 27.6 7.4 3.2 9.0

a Find the mean and the median areas, correct to one decimal place.
b Which is a better measure of centre for this data set? Explain your answer.

▶ Overview of the downloadable PDF textbook

14 The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.

15 **PDF annotation** and **search features** are enabled.



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▶ Overview of the Interactive Textbook (shown on the page opposite)

The **Interactive Textbook** (ITB), an online HTML version of the print textbook powered by the HOTmaths platform, is included with the print book or available as a separate digital-only product.

16 The material is formatted for on-screen use, with a convenient and easy-to-use navigation system and links to all resources.

17 **Extra problem-solving and modelling tasks and investigations** are provided as downloadable PDFs and editable Word documents.

18 The new **Workspaces** enable students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploading images of writing or drawing.

19 The new **self-assessment tools** enable students to check answers, mark their own work, and rate their confidence level in their work. This helps develop responsibility for learning, and communicates progress and performance to the teacher. Student accounts can be linked to the learning management system used by the teacher in the Online Teaching Suite.

20 Examples have **video versions** to encourage independent learning.

21 **Worked solutions** are included and can be enabled or disabled in the student accounts by the teacher.

22 Interactive **Desmos widgets** demonstrate key concepts and enable students to visualise the mathematics.

23 The **Desmos scientific calculator** and geometry tool is also available for students to use for their own calculations and exploration.

24 **Revision of prior knowledge** is provided with links to knowledge check quizzes and Year 10 **HOTmaths lessons**.

25 **Quick quizzes** containing automarked multiple-choice questions enable students to check their understanding.

26 **Definitions** pop up for key terms in the text, and are also provided in a **dictionary**.

27 Messages from teacher assign tasks and tests.

28 **Practice exam-style papers** are provided in downloadable PDF and Word files.

29 **Spreadsheets** are provided in Excel format.

30 **Calculator** guides are provided as PDFs.

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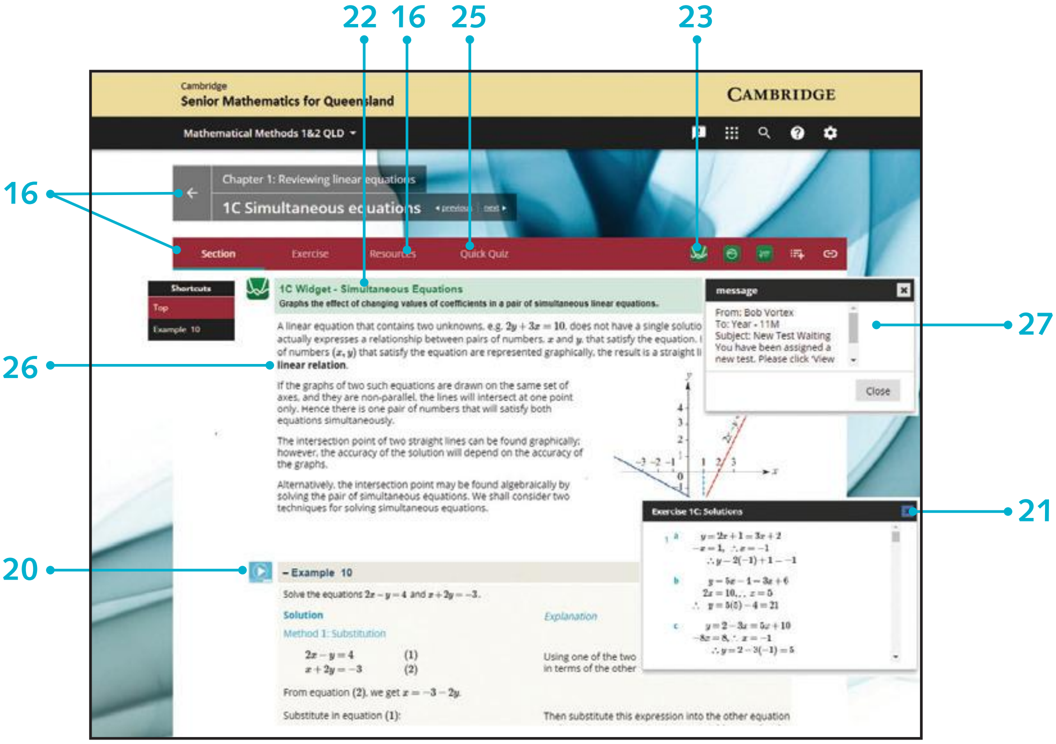
INTERACTIVE TEXTBOOK POWERED BY THE HOTmaths PLATFORM

A selection of features is shown. Screenshots are taken from the Mathematical Methods textbook in this series. Numbers refer to the descriptions on the opposite page. HOTmaths platform features are updated regularly.



1C Widget - Simultaneous Equations
Graphs the effect of changing values of coefficients in a pair of simultaneous linear equations.

A linear equation that contains two unknowns, e.g. $2y + 3x = 10$, does not have a single solutio actually expresses a relationship between pairs of numbers, x and y, that satisfy the equation. I of numbers (x, y) that satisfy the equation are represented graphically, the result is a straight li **linear relation**.

If the graphs of two such equations are drawn on the same set of axes, and they are non-parallel, the lines will intersect at one point only. Hence there is one pair of numbers that will satisfy both equations simultaneously.

The intersection point of two straight lines can be found graphically; however, the accuracy of the solution will depend on the accuracy of the graphs.

Alternatively, the intersection point may be found algebraically by solving the pair of simultaneous equations. We shall consider two techniques for solving simultaneous equations.

From: Bob Vortex
To: Year - 11M
Subject: New Test Waiting
You have been assigned a new test. Please click 'View

1 a $y = 2x + 1 = 3x + 2$
$-x = 1, \therefore x = -1$
$\therefore y = 2(-1) + 1 = -1$

b $y = 5x - 1 = 3x + 6$
$2x = 10, \therefore x = 5$
$\therefore y = 5(5) - 4 = 21$

c $y = 2 - 3x = 5x + 10$
$-8x = 8, \therefore x = -1$
$\therefore y = 2 - 3(-1) = 5$

- Example 10

Solve the equations $2x - y = 4$ and $x + 2y = -3$.

Solution

Method 1: Substitution

Explanation

$2x - y = 4$ (1)
$x + 2y = -3$ (2)

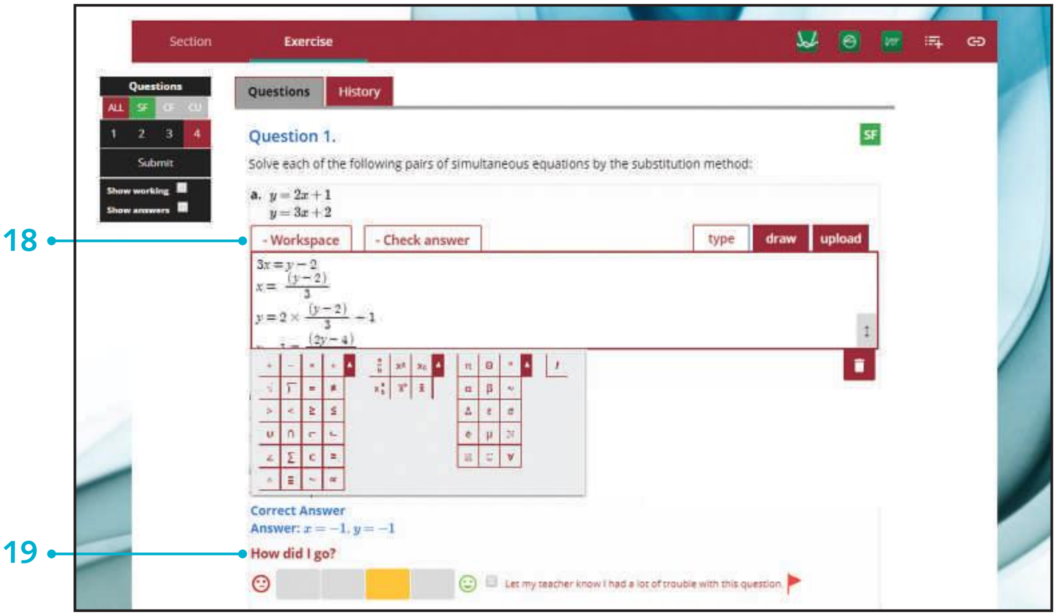
Using one of the two in terms of the other

From equation (2), we get $x = -3 - 2y$.

Substitute in equation (1):

Then substitute this expression into the other equation

WORKSPACES AND SELF-ASSESSMENT



Question 1.

Solve each of the following pairs of simultaneous equations by the substitution method:

a. $y = 2x + 1$
$y = 3x + 2$

$3x = y - 2$
$x = \frac{(y-2)}{3}$
$y = 2 \times \frac{(y-2)}{3} - 1$

Correct Answer
Answer: $x = -1, y = -1$

How did I go?

☐ Let my teacher know I had a lot of trouble with this question.

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Overview of the Online Teaching Suite Powered by the HOTmaths platform (shown below)

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the assets and resources are in one place for easy access. The features include:

31 The HOTmaths **learning management system** with class and student analytics and reports, and communication tools.

32 Teacher's view of a student's working, scores and self-assessment, which they can comment upon.

33 A HOTmaths-style **test generator**.

34 Chapter test **worksheets** and exam **practice papers**.

35 Editable **curriculum grids** and **teaching programs**.

ONLINE TEACHING SUITE POWERED BY THE HOTmaths PLATFORM

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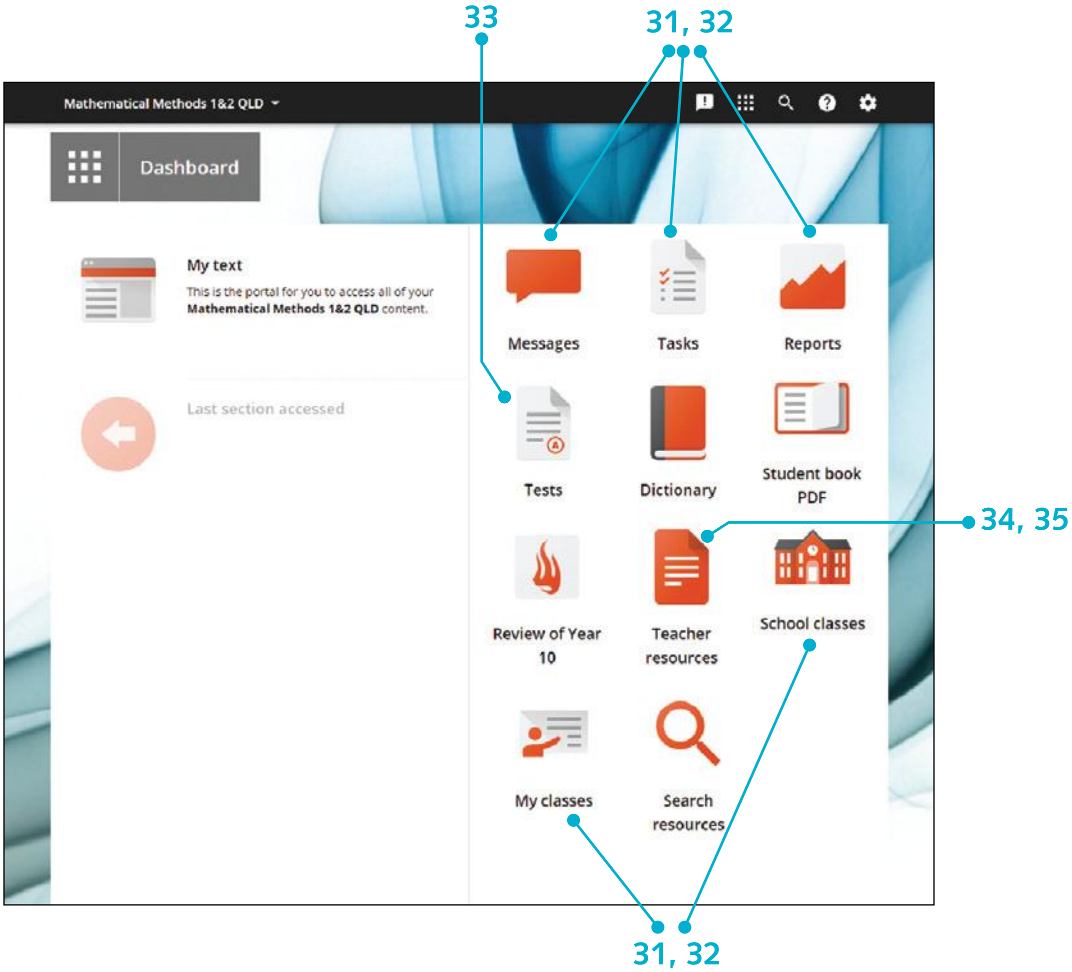

My text

This is the portal for you to access all of your **Mathematical Methods 1&2 QLD** content.

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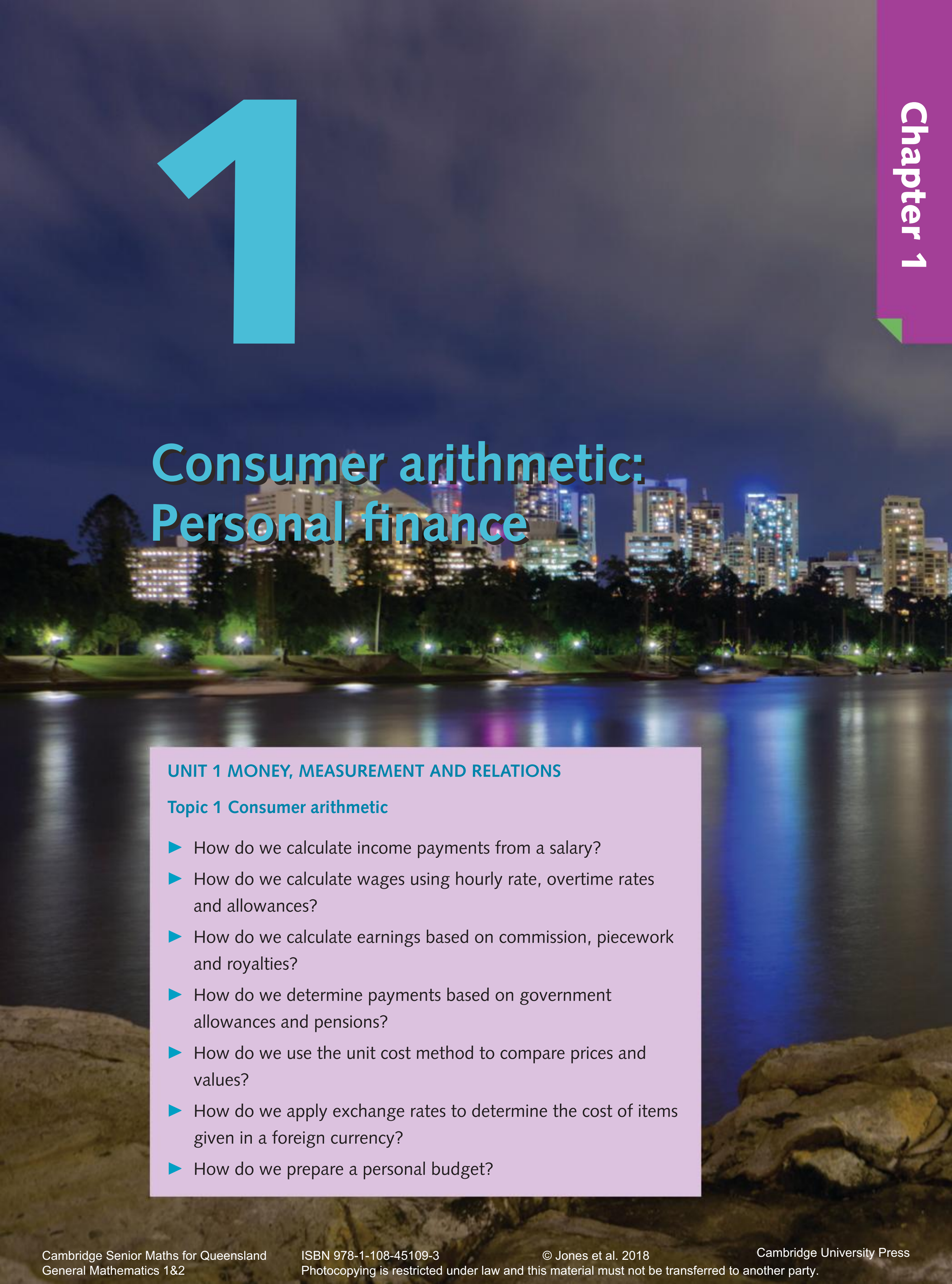

1

Consumer arithmetic: Personal finance

UNIT 1 MONEY, MEASUREMENT AND RELATIONS

Topic 1 Consumer arithmetic

- How do we calculate income payments from a salary?
- How do we calculate wages using hourly rate, overtime rates and allowances?
- How do we calculate earnings based on commission, piecework and royalties?
- How do we determine payments based on government allowances and pensions?
- How do we use the unit cost method to compare prices and values?
- How do we apply exchange rates to determine the cost of items given in a foreign currency?
- How do we prepare a personal budget?

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1A Salary and wages

The money paid to an individual for the work they carry out can be expressed as a **salary** or a **wage**.

▶ Salary

A **salary** is payment for a year's work, generally paid as equal weekly, fortnightly or monthly payments. People who earn a salary work a pre-agreed number of hours per week, which is generally from 36 hours to 40 hours for a full-time worker. They are also entitled to benefits such as sick leave and holiday pay, which are factored into the salary.

Example 1 Calculating from a salary

Mitchell earns a salary of $65 208 per annum. He is paid fortnightly. How much does he receive each fortnight? Assume there are 52 weeks in the year.

Solution

1 Write the quantity to be found.	Fortnightly pay
2 Divide the salary by the number of fortnights in a year (26).	= 65 208 ÷ 26
3 Evaluate and write using correct units.	= $2508.00
4 Write your answer in words.	Mitchell is paid $2508 per fortnight.

▶ Wages

A **wage** describes payment for work calculated on an hourly basis, with the amount earned dependent on the number of hours actually worked. There are no additional payments such sick leave or holiday pay.

Example 2 Calculating a wage

Jasmine is paid at a rate of $1098 for a 40-hour week.

a How much does Jasmine earn per hour?

b What wages will Jasmine receive for a week in which she works 38 hours?

Solution

a	**1**	Write the quantity to be found.	*Wage per hour*
	2	Divide the amount by the number of hours worked.	$= 1098 \div 40$
	3	Evaluate and write the answer correct to two decimal places.	$= \$27.45$ *Jasmine earns $27.45 per hour.*
b	**1**	Write the quantity to be found.	*Wage for 38 hours*
	2	Multiply the rate by the number of hours worked.	$= 27.45 \times 38$
	3	Evaluate and write using correct units.	$= \$1043.10$
	4	Write your answer in words.	*Jasmine receives $1043.10 for the 38-hour week.*

Salary

A payment for a year's work, which is then divided into equal monthly, fortnightly or weekly payments.

Wage

A payment for a week's work that is calculated on an hourly basis.

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Exercise 1A

SF

1 Emily earns a salary of $92 648. Write, to the nearest dollar, her salary as amounts per:

a week **b** fortnight **c** month.

2 The annual salary for 4 people is shown in the table below. Calculate their weekly and fortnightly payments. (Answer correct to the nearest dollar.)

	Name	Salary	Week	Fortnight
a	Abbey	$57 640		
b	Blake	$78 484		
c	Chloe	$107 800		
d	David	$44 240		

Example 1 **3** What is Zachary's fortnightly income if he earns a salary of $43 056?

4 Find the annual salary for the following people.

a Amber earns $580 per week. **b** Tyler earns $1520 per fortnight.

c Samuel earns $3268 per month. **d** Ava earns $2418 per week.

5 Harrison is a civil engineer who earns a salary of $1500 per week.

a How much does he receive per fortnight?

b How much does he receive per year?

6 What is Yasmeen's annual salary if her salary per fortnight is $1610?

7 Dylan receives a weekly salary payment of $1560. What is his annual salary?

CF

8 Stephanie is paid $1898 per fortnight and Tahlia $3821 per month. Calculate each person's equivalent annual income. Who earns more per year and by how much?

9 Laura is paid $1235 per fortnight and Ebony $2459 per month. Which person receives the higher annual salary and by how much?

10 Tran is paid $1898 per week and Jake $8330 per month. Calculate each person's equivalent annual income. What is the difference between their annual salaries?

SF

Example 2 **11** Joshua works as a labourer and is paid $25.50 an hour. How much does he earn for working the following hours?

a 35 hours **b** 37 hours **c** 40 hours **d** 42 hours

12 Lily earns $29.75 an hour. If she works 6 hours each day during the week and 4 hours a day during the weekend, find her weekly wage.

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SF

13 Determine the wage for a 37-hour week for each of the following hourly rates.

a $12.00 **b** $9.50 **c** $23.20 **d** $13.83

14 Determine the income for a year (52 weeks) for each of the following hourly rates. Assume 40 hours of work per week.

a $7.59 **b** $15.25 **c** $18.78 **d** $11.89

CF

15 Suchitra works at the local supermarket. She gets paid $22.50 per hour. Her time card is shown below.

Day	In	Out
Monday	9:00 a.m.	5:00 p.m.
Tuesday	9:00 a.m.	6:00 p.m.
Wednesday	8:30 a.m.	5:30 p.m.
Thursday	9:00 a.m.	4:30 p.m.
Friday	9:00 a.m.	4:00 p.m.

a How many hours did Suchitra work this week?

b Find her weekly wage.

SF

16 Grace earns $525 in a week. If her hourly rate of pay is $12.50, how many hours does she work in the week?

17 Zachary is a plumber who earned $477 for a day's work. He is paid $53 per hour. How many hours did Zachary work on this day?

18 Lucy is a hairdresser who earns $24.20 per hour. She works an 8-hour day.

a How much does Lucy earn per day?

b How much does Lucy earn per week? Assume she works 5 days a week.

c How much does Lucy earn per fortnight?

d How much does Lucy earn per year? Assume 52 weeks in the year.

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CF

19 Alyssa is paid \$36.90 per hour and Connor \$320 per day. Alyssa works a 9-hour day. Who earns more per day and by how much?

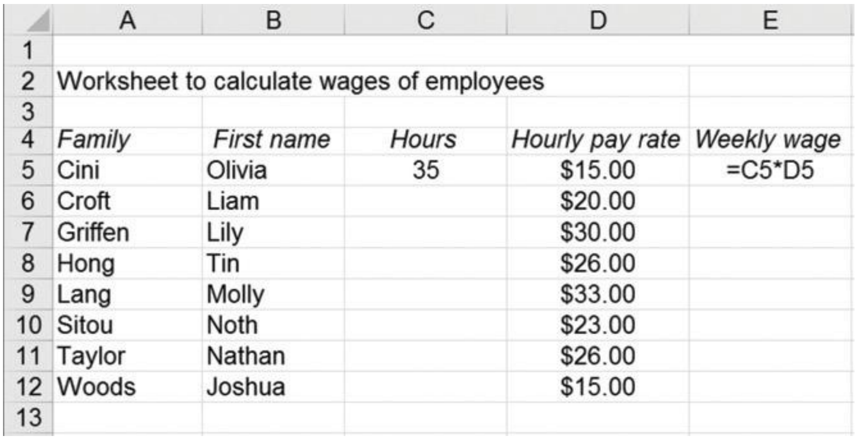
20 Feng is retiring and will receive 7.6 times the average of his salary over the past three years. In the past three years he was paid \$84 640, \$83 248 and \$82 960. Find the amount of his payout.

21 Liam's salary is currently \$76 000. He will receive salary increases as follows: a 5% increase from 1 July and then a 5% increase from 1 January. What will be his new salary from 1 January?

Spreadsheet
1AQ22

22 Create the spreadsheet below.

Spreadsheet guide A complete guide to spreadsheets is provided in the interactive textbook



	A	B	C	D	E
1					
2	Worksheet to calculate wages of employees				
3					
4	*Family*	*First name*	*Hours*	*Hourly pay rate*	*Weekly wage*
5	Cini	Olivia	35	\$15.00	=C5*D5
6	Croft	Liam		\$20.00	
7	Griffen	Lily		\$30.00	
8	Hong	Tin		\$26.00	
9	Lang	Molly		\$33.00	
10	Sitou	Noth		\$23.00	
11	Taylor	Nathan		\$26.00	
12	Woods	Joshua		\$15.00	
13					

a Cell E5 has a formula that multiplies cells C5 and D5. Enter this formula.

b Enter the hours worked for the following employees:

Liam – 20 Lily – 26 Tin – 38 Molly – 40
Noth – 37.5 Nathan – 42 Joshua – 38.5

c Fill down the contents of E5 to E12.

d Edit the hourly pay rate of Olivia Cini to \$16.50. Observe the change in E5.

CU

23 Isabelle earns \$85 324 per annum. Isabelle calculated her weekly salary by dividing her annual salary by 12 to determine her monthly payment and then divided this result by 4 to determine her weekly payment. What answer did Isabelle get, what is the correct answer, and what is wrong with Isabelle's calculation?

24 Lucy earns \$8 per hour and Ebony earns \$9 per hour. Last week they both earned at least \$150. What is the least number of hours that Lucy could have worked last week? What is the least number of hours Ebony could have worked?

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1B Overtime, penalty rates and allowances

▶ Overtime

Overtime rates apply when employees work beyond the normal working day. Payment for overtime is more than the normal pay rate, often paid at 150% (referred to as time-and-a-half) or 200% (double time). So a person whose normal pay rate is \$10 per hour receives:

Time and half: $\$10 \times 150\% = \$10 \times 1.5 = \$15$

Double time: $\$10 \times 200\% = \$10 \times 2.0 = \$20$

Example 3 Calculating wages involving overtime

John works for a building construction company. Find John's wage during one week in which he works 40 hours at the normal rate of \$16 an hour, 3 hours at time-and-a-half rates and 1 hour at double time rates.

Solution

1 Write the quantity to be found.	Wage
2 Normal wage is 40 multiplied by \$16.	$= (40 \times 16)$ normal pay
3 Payment for time-and-a-half is 3 multiplied by \$16 multiplied by 1.5.	$+(3 \times 16 \times 1.5)$ time-and-a-half pay
4 Payment for double time is 1 multiplied by \$16 multiplied by 2.	$+ (1 \times 16 \times 2)$ double time pay $= \$744.00$
5 Evaluate and write your answer in words.	John's wage is \$744.

▶ Penalty rates

As well as earning a higher pay rate when working overtime, employees often get a higher pay rate, called a **penalty rate**, for working weekends, public holidays, late night shifts or early morning shifts.

In Australia, penalty rates are determined by the Fair Work Commission, and can be found on their website. The following table shows the penalty rates payable for those working in the Fast Food Industry.

Fast-food award

Note: These rates are indicative only, they may change at any time.

Sunday penalty rates:

- 125% for full-time and part-time employees
- 150% for casuals.

Public holiday penalty rates:

- 225% for full-time and part-time employees
- 250% for casuals.

A 10% evening work penalty will apply from 10:00 p.m. until midnight and a 15% penalty after midnight for will apply hours worked between midnight and 6:00 a.m.

Note that the penalty rate is higher for **casual** workers, people who are paid an hourly rate, and who may be asked to work differing number of hours each week, as required by the business. There is generally no guarantee of ongoing work for a casual worker.

Example 4 Calculating penalty pay

Milan is employed on a casual basis for a fast-food company. His pay rate is $15 per hour with penalty rates as shown in the table above. Last week Milan worked from 6 p.m. until 10 p.m. on Thursday, from 8 p.m. until midnight on Friday and from 12 noon until 4 p.m. on Sunday. How much did Milan earn last week?

Solution

1 Write down the quantity to be found.	*Wages last week*
2 The number of normal hours worked is 4 hours on Thursday and 2 hours on Friday (from 6 p.m. until 8 p.m.)	$= 6 \times \$15$
3 The number of evening hours worked is 2 hours on Friday (from 10 p.m. until midnight).	$+2 \times \$15 \times 110\%$
4 The number of Sunday hours worked is 4.	$+4 \times \$15 \times 150\%$ $= \$213$

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▶ Allowances

Allowances are extra payments that may be made to employees who have a particular skill, use their own tools or equipment at work, or work in unpleasant or dangerous conditions.

Common allowances include:

- uniforms and special clothing
- tools and equipment
- travel and fares
- car and phone.

Example 5 Calculating pay including an allowance

Richard works as a builder and is paid $35 per hour, plus an extra $8.50 per hour when he supplies his own tools. Last week he worked 3 days on which the tools were supplied, and two days on which he supplied his own tools. If he worked 8 hours each day, how much did he earn?

Solution

1 Write down the quantity to be found.

Wages last week

2 Determine the number of normal hours = 8 hours on each of the three days.

$= 3 \times 8 \times \$35$

3 Determine the number of allowance hours = 8 hours on each of the two days.

$+2 \times 8 \times (\$35 + \$8.50)$

$= \$1536$

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Exercise 1B

SF

1 Calculate the payment for working 4 hours overtime at time-and-a half given the following normal pay rates.

a \$18.00 **b** \$39.50 **c** \$63.20 **d** \$43.83

2 Calculate the payment for working 3 hours overtime at double time given the following normal pay rates.

a \$37.99 **b** \$19.05 **c** \$48.78 **d** \$61.79

Example 3

3 Andrew earns \$32.50 an hour as a driver. He works 38 hours a week at normal time and 5 hours a week at double time. Find his weekly wage. Answer correct to the nearest cent.

4 Mei is a casual employee who worked 8 hours at normal pay rates and 2 hours at time-and-a-half. Her normal rate of pay is \$12.30 per hour. What is her pay for this time?

5 Oliver earns \$23.80 an hour. He earns normal rates during week days and time-and-a-half on weekends. Last week he worked 34 hours during the week and 6 hours during the weekend. Find his weekly wage.

Example 4

6 George works in a take-away food store. He gets paid \$18.60 per hour for a standard 35-hour week. Additional hours are paid at double time. His time card is shown below.

Day	In	Out
Monday	8:30 a.m.	4:30 p.m.
Tuesday	9:00 a.m.	6:00 p.m.
Wednesday	8:45 a.m.	5:45 p.m.
Thursday	9:00 a.m.	6:30 p.m.
Friday	10:00 a.m.	8:00 p.m.

a How many hours did George work this week?

b Find his weekly wage.

CF

7 Dave works for 5 hours at double time. He earns \$98.00. Find his normal hourly rate.

8 Ella works 3 hours at time-and-a-half and earns \$72.00. Find her normal hourly rate.

9 Zahid is paid a set wage of \$774.72 for a 36-hour week, plus time-and-a-half for overtime. In one particular week he worked 43 hours. What were Zahid's earnings?

10 Samantha is paid a set wage of \$962.50 for a 35-hour week, plus double time for overtime. In one particular week she worked 40 hours. What were Samantha's earnings?

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Example 5 **11** A window washer is paid \$22.50 per hour and a height allowance of \$55 per day. If he works 9 hours each week day on a high-rise building, calculate the:

a amount earned each week day

b total weekly earnings for five days of work.

12 Anna works in a factory and is paid \$18.54 per hour. If she operates the oven she is paid a temperature allowance of \$4.22 per hour in addition to her normal rate. Find her weekly pay if she works a total of 42 hours including 10 hours working the oven.

13 Scott is a painter who is paid a normal rate of \$36.80 per hour plus a height allowance of \$21 per day. If Scott works 9 hours per day for 5 days on a tall building, calculate his total earnings.

14 Kathy is a scientist who is working in a remote part of Australia. She earns a salary of \$86 840 plus a weekly allowance of \$124.80 for working under extreme and isolated conditions. Calculate Kathy's fortnightly pay.

15 Chris is a soldier and is paid \$27 per hour plus an additional allowance of \$12.50 per hour for disarming explosives. What is his total weekly pay if he works from 6 a.m. to 2 p.m. for 7 days a week on explosives?

16 A miner earns a wage of \$46.20 per hour plus an allowance of \$28.20 per hour for working in cramped spaces. The miner worked a 10-hour day for 5 days in a small shaft. What is his weekly pay?

17 Vien is employed on a casual basis. His rate of pay is shown below. Last week Vien worked from 11:30 a.m. until 3:30 p.m. on Thursday, from 8:30 a.m. until 2:00 p.m. on Saturday, and from 12 noon until 6:00 p.m. on Sunday. How much did Vien earn last week?

Rate of pay	
Weekdays	\$18.60 per hour
Saturday	Time-and-a-half
Sunday	Double time

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CF

18 A mechanic's industrial award allows for normal rates for the first 7 hours on any day. It provides for overtime payment at the rate of time-and-a-half for the first 2 hours and double time thereafter. Find a mechanic's wage for a 12-hour day if the normal pay rate is $42.50 an hour.

19 Abbey's timesheet is shown below. She gets paid $12.80 per hour during the week, time-and-a-half for Saturdays and double time for Sundays. Abbey is not paid for meal breaks.

Day	In	Out	Meal break
Monday	8:30 a.m.	5:30 p.m.	1 hour
Tuesday	8:30 a.m.	3:00 p.m.	1 hour
Wednesday	8:30 a.m.	5:30 p.m.	1 hour
Thursday	8:30 a.m.	9:00 p.m.	2 hour
Friday	4:00 p.m.	7:00 p.m.	No break
Saturday	8:00 a.m.	4:00 p.m.	No break
Sunday	10:00 a.m.	3:00 p.m.	30 minutes

a How much did Abbey earn at the normal rate of pay during this week?

b How much did Abbey earn from working at penalty rates during this week?

c What percentage of her pay did Abbey earn by working at penalty rates?

CU

20 Connor works a 35-hour week and is paid $18.25 per hour. Any overtime is paid at time-and-a-half. Connor wants to work enough overtime to earn at least $800 each week. What is the minimum number of hours of overtime that Connor will need to work?

21 Max works in a shop and earns $21.60 per hour at the normal rate. Each week he works 15 hours at the normal rate and 4 hours at time-and-a-half.

a Calculate Max's weekly wage.

b Max aims to increase his weekly wage to $540 by working extra hours at the normal rate. How many extra hours must Max work?

c Max's rate of pay increased by 5%. What is his new hourly rate for normal hours?

d What will be Max's new weekly wage, assuming he maintains the extra working hours?

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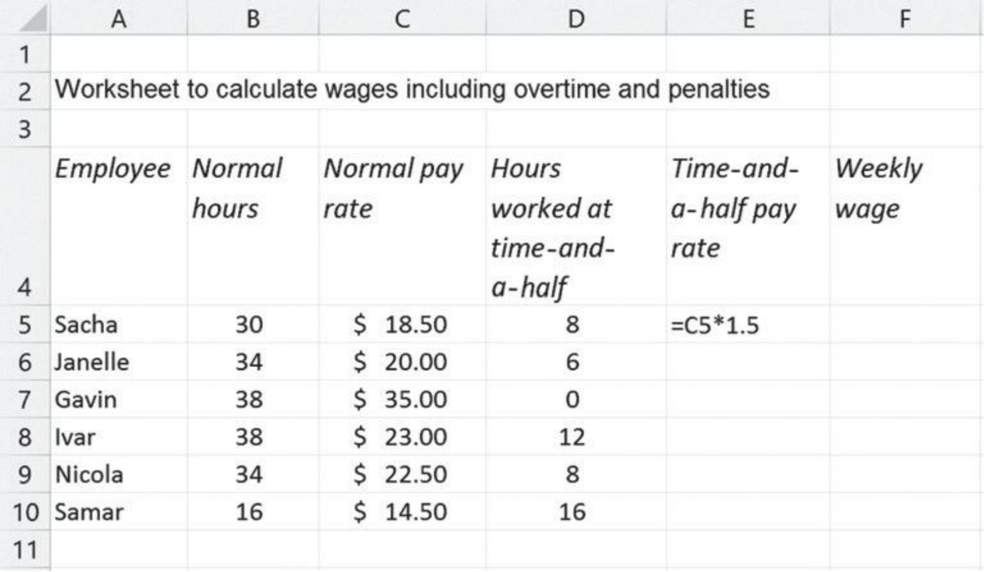
22 The information in the spreadsheet below gives the hours worked one week by a group of employees. Create the spreadsheet as shown.



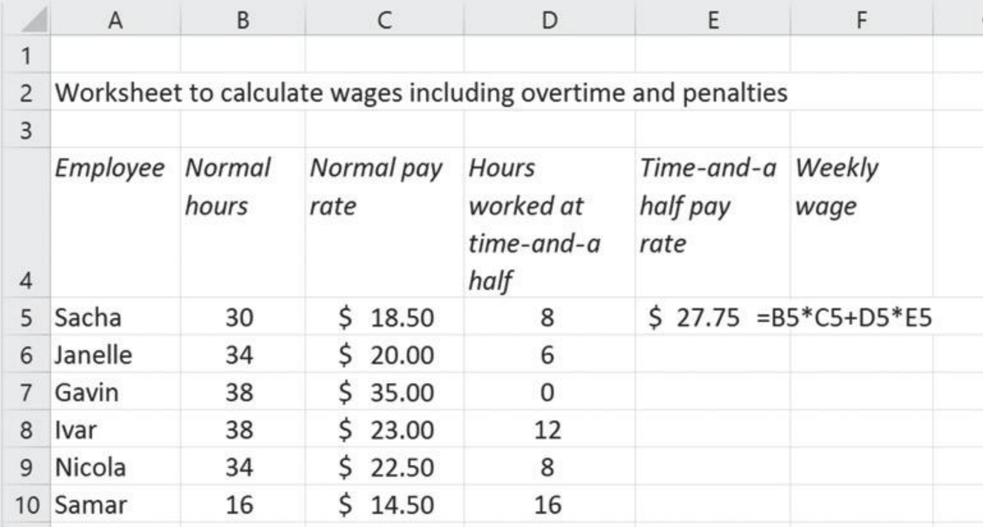
	A	B	C	D	E	F
1						
2	Worksheet to calculate wages including overtime and penalties					
3						
4	*Employee*	*Normal hours*	*Normal pay rate*	*Hours worked at time-and-a-half*	*Time-and-a-half pay rate*	*Weekly wage*
5	Sacha	30	$ 18.50	8	=C5*1.5	
6	Janelle	34	$ 20.00	6		
7	Gavin	38	$ 35.00	0		
8	Ivar	38	$ 23.00	12		
9	Nicola	34	$ 22.50	8		
10	Samar	16	$ 14.50	16		
11						

a Cell E5 has the formula that multiplies C5 by 1.5. Enter this formula, and fill down to E10.

b What is the hourly pay rate for Nicola when she is paid time-and-a-half?

c Cell F5 has the formula that multiplies B5 by C5, D5 by E5, and then adds these two amounts together. Enter this formula as shown below and fill the contents down to F10.

Spreadsheet
1BQ22



	A	B	C	D	E	F
1						
2	Worksheet to calculate wages including overtime and penalties					
3						
4	*Employee*	*Normal hours*	*Normal pay rate*	*Hours worked at time-and-a half*	*Time-and-a half pay rate*	*Weekly wage*
5	Sacha	30	$ 18.50	8	$ 27.75	=B5*C5+D5*E5
6	Janelle	34	$ 20.00	6		
7	Gavin	38	$ 35.00	0		
8	Ivar	38	$ 23.00	12		
9	Nicola	34	$ 22.50	8		
10	Samar	16	$ 14.50	16		

d What is Ivar's weekly wage?

e What is the total wage for this group of employees this week?

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1C Commission, piecework and royalties

Commission

Commission is a percentage of the value of the goods or services sold. People such as real estate agents and salespersons are paid a commission. The advantage of this is that a very good salesperson will be able to earn a higher income, but the disadvantage is that it is very hard to plan, as the income may vary from week to week.

Commission

Commission = Percentage of the value of the goods sold

Example 6 Finding the commission

Zoë sold a house for $650 000. Find the commission from the sale if her rate of commission was 1.25%.

Solution

1 Write the quantity (commission) to be found.	*Commission*
2 Multiply 1.25% by $650 000.	*= 1.25% of $650 000*
3 Evaluate and write using correct units.	*= 0.0125 × 650 000* *= $8125*
4 Write the answer in words.	*Commission earned is $8125.*

Example 7 Finding the commission

An electrical goods salesman is paid $570.50 a week plus 4% commission on all sales over $5000 a week. Find his earnings in a week in which his sales amounted to $6800.

Solution

1 Commission is paid on sales of over $5000; that is, on $1800.	*Sales = 6800 − 5000* *= 1800*
2 Write the quantity (earnings) to be found.	*Earnings*
3 Add the weekly payment and commission of 4% on $1800.	*= 570.50 + (4% of $1800)* *= 570.50 + (0.04 × 1800)*
4 Evaluate and write using correct units.	*= $642.50*
5 Write the answer in words.	*Earnings were $642.50.*

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Piecework

Piecework is when a worker is paid a fixed payment, called a **piece rate**, for each unit produced or action completed. For example, a dressmaker may be paid a fixed price for each dress produced, regardless of the time spent sewing each one. The advantage of piecework is that harder work is rewarded with higher pay, while the disadvantage is the lack of permanent employment and holiday and sick pay.

Piecework

Piecework = Number of units of work × Amount paid per unit

Example 8 Calculating a piecework payment

Noah is a tiler and charges $47 per square metre to lay tiles. How much will he earn for laying tiles in a room with an area of 14 square metres?

Solution

1 Write the quantity (earnings) to be found.	*Earnings*
2 Multiply number of square metres (14) by the charge ($47).	$= 14 \times \$47$
3 Evaluate and write using correct units.	$= \$658$
4 Write the answer in words.	*Noah earns $658.*

Royalties

A **royalty** is payment for the use of intellectual property such as a book or song. It is calculated as a percentage of the revenue or profit received from its purchase or use. People such as musicians and authors receive a royalty. The advantage of royalties is that income increases with a better, more popular song or book. The disadvantage is that income is entirely dependent on sales, and there is no holiday or sick pay.

Royalty

Royalty = Percentage of the goods sold or profit received

Example 9 Calculating a royalty

Andrew is an author and is paid a royalty of 12% of the value of books sold. Find his royalties if there were 2480 books sold at $67.50 each.

Solution

1 Write the quantity (royalty) to be found.	*Royalty*
2 Multiply 12% by the total sales or 2480 × $67.50.	$= 12\% \text{ of } (2480 \times \$67.50)$ $= 0.12 \times 2480 \times 67.50$
3 Evaluate and write using correct units.	$= \$20\,088$
4 Write the answer in words.	*Andrew earns $20 088 in royalties.*

Exercise 1C

SF

Example 6

1 Jake earns a commission of 4% of the sales price. What is the commission on the following sales?

a \$8820 **b** \$16 740 **c** \$34 220

2 Michael Tran is a real estate agent. He earns 2% on all sales. Calculate Michael's commission on these sales.

a \$456 000 **b** \$420 000 **c** \$285 500 **d** \$590 700

3 Olivia sold a car valued at \$54 000. Calculate Olivia's commission from the sale if her rate of commission is 3%.

Example 7

4 Sophie earns a weekly retainer of \$355 plus a commission of 10% on sales. What are Sophie's total earnings for each week if she made the following sales?

a \$760 **b** \$2870 **c** \$12 850

5 Chris earns \$240 per week plus 25% commission on sales. Calculate Chris's weekly earnings if he made sales of \$2880.

6 Ella is a salesperson for a cosmetics company. She is paid \$500 per week and a commission of 3% on sales in excess of \$800.

a What does Ella earn in a week in which she makes sales of \$1200?

b What does Ella earn in a week in which she makes sales of \$600?

7 A real estate agent charges a commission of 5% for the first \$20 000 of the sale price and 2.5% for the balance of the sale price. Copy and complete the following table.

	Sale price	5% commission on \$20 000	2.5% commission on balance
a	\$150 000		
b	\$200 000		
c	\$250 000		
d	\$300 000		

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CF

8 Jade is a real estate agent and is paid an annual salary of \$18 000 plus a commission of 2.5% on all sales. She is also paid a car allowance of \$50 per week. What was Jade's total yearly income if she sold \$1 200 000 worth of property?

9 The commission that a real estate agent is paid for selling a property is based on the selling price and is shown in the table.
What is the commission paid on properties with the following selling prices?

Selling price	Commission
First \$20 000	5%
Next \$120 000	3%
Thereafter	1%

a \$100 000 **b** \$150 000 **c** \$200 000

10 Harry is a salesperson. He earns a basic wage of \$300 per week and receives commission on all sales. Last week he sold \$20 000 worth of goods and earned \$700. What is Harry's rate of commission?

11 Caitlin and her assistant, Holly, sell perfume. Caitlin earns 20% commission on her own sales, as well as 5% commission on Holly's sales. What was Caitlin's commission last month when she made sales of \$1800 and Holly made sales of \$2000?

SF

Example 8 **12** A dry cleaner charges \$9 to clean a dress. How much do they earn by dry cleaning:

a 250 dresses? **b** 430 dresses? **c** 320 dresses?

13 Angus works part-time by addressing envelopes at home and is paid \$23 per 100 envelopes completed, plus \$40 to deliver them to the office. What is his pay for delivering 2000 addressed envelopes?

14 Abbey is an artist who makes \$180 for each large portrait and \$100 for each small portrait. How much will she earn if she sells 13 large and 28 small portraits?

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CF

15 Kristy earns $25 for making a skirt, $35 for a shirt, and $55 for making a dress.

a How much is she paid for making 30 skirts, 15 shirts and 10 dresses?

b If it takes Kristy on average 60 minutes to make a skirt, 90 minutes to make a shirt, and 140 minutes to make a dress, what are her hourly rates for each of these three items?

c If Kirsty wants to earn $1000 as quickly as possible, what should she make?

SF

16 Emilio earns a royalty of 24% on net sales from writing a fiction book. There were $18 640 net sales in the last financial year. What is Emilio's royalty payment?

Example 9

17 Calculate the royalties on the following sales.

a 3590 books sold at $45.60 with a 8% royalty payment

b 18 432 DVDs sold at $20 with a 10% royalty payment

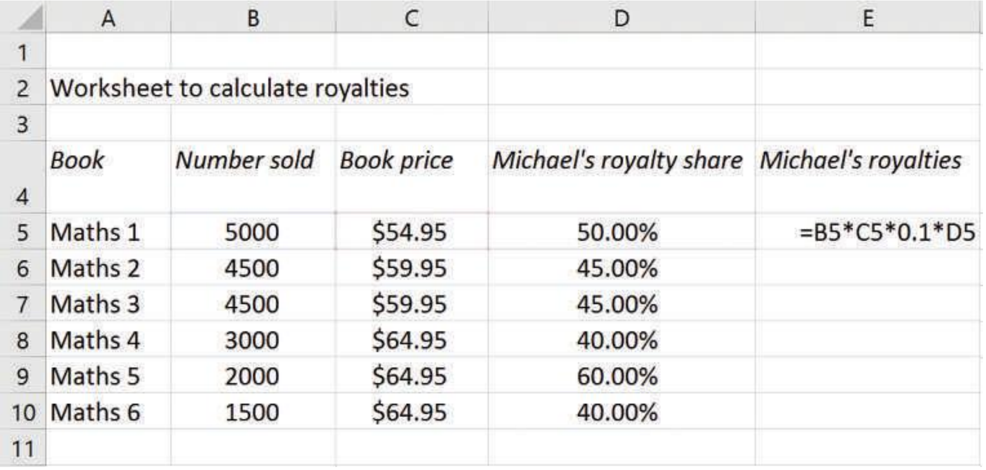
c 4805 computer games sold at $65.40 with a 5% royalty payment

CF

18 Michael is a member of a team that wrote a series of text books on which royalties are payable. There is a royalty of 10% payable on the purchase price of the books, which is then divided between the authors according to the amount of writing they have done on each book.

The information in the spreadsheet below gives the sales for each of the books, as well as the share of the royalties to which Michael is entitled. Create the spreadsheet as shown.

Spreadsheet
1CQ18



	A	B	C	D	E
1					
2	Worksheet to calculate royalties				
3					
4	*Book*	*Number sold*	*Book price*	*Michael's royalty share*	*Michael's royalties*
5	Maths 1	5000	$54.95	50.00%	=B5*C5*0.1*D5
6	Maths 2	4500	$59.95	45.00%	
7	Maths 3	4500	$59.95	45.00%	
8	Maths 4	3000	$64.95	40.00%	
9	Maths 5	2000	$64.95	60.00%	
10	Maths 6	1500	$64.95	40.00%	
11					

a Cell E5 has the formula that determines Michael's royalties, which is the number of books sold (B5) times the cost of the book (C5) times the author's commission (10% or 0.1) times Michal share of the royalties (D5). Enter this formula, and fill down to E10.

b How much royalty does Michael earn for the book Maths 5?

c What are Michael's total royalties for all six books?

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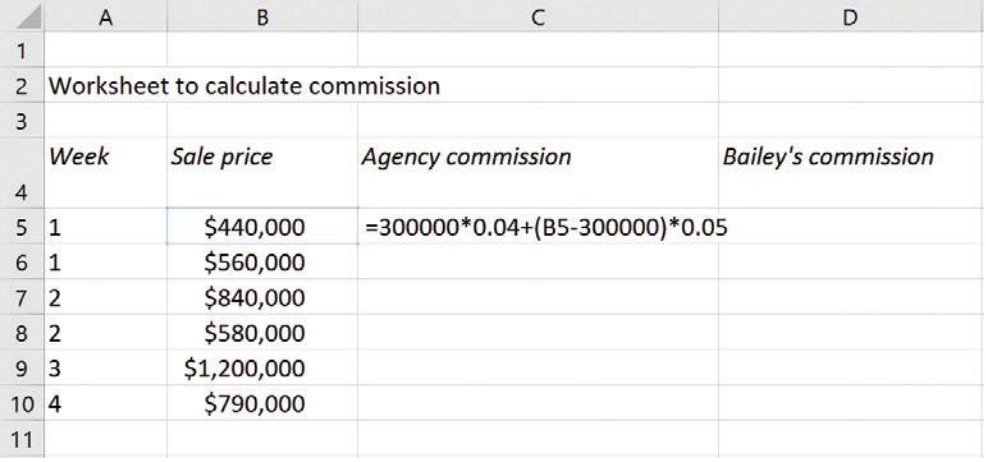
CF

19 The commission charged by a real estate agency for selling a property is based on the selling price below:

Commission rates	
Up to \$300 000	4%
\$300 000 and over	5%

Bailey is paid \$180 per week by the real estate agency plus 5% of the commission received by the agency.
The information in the spreadsheet below gives the sales made by Bailey in one month, together with their values and his commission rates. Create the spreadsheet as shown.

Spreadsheet
1CQ19



	A	B	C	D
1				
2	Worksheet to calculate commission			
3				
4	*Week*	*Sale price*	*Agency commission*	*Bailey's commission*
5	1	\$440,000	=300000*0.04+(B5-300000)*0.05	
6	1	\$560,000		
7	2	\$840,000		
8	2	\$580,000		
9	3	\$1,200,000		
10	4	\$790,000		
11				

a Cell C5 has the formula that determines the agency's commission. Enter this formula, and fill down to C10.

b How much commission does the agency receive from the properties sold by Bailey in week 3?

c In cell D5 enter a formula that multiplies C5 by 0.05 to give the amount of commission paid to Bailey, and fill the contents down to D10.

d How much total commission is paid to Bailey in week 1?

e How much total commission is paid to Bailey that month?

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1D Incomes from the government

Some people who are unable to work receive a pension, allowance or benefit from the government. The eligibility requirements, and the amount paid, vary from time to time according to the priorities of the current government.

▶ Youth Allowance

Subject to meeting requirements you may be eligible for Youth Allowance if you are:

- 18–24 years old and studying full-time
- 16–24 years old and undertaking a full-time Australian Apprenticeship
- 16–20 years old and looking for full-time work.

The Youth Allowance scale at the time of publication was as follows:

Status	Allowance per fortnight
Under 18, living at home	\$239.50
Under 18, living away from home	\$437.50
Over 18, living at home	\$288.10
Over 18, living away from home	\$437.50

Note: These scales may change at any time.

Example 10 Youth Allowance

Ryan is eligible for Youth Allowance. How much does he receive in a year if he is over 18 and living at home while studying?

Solution

1 Write the quantity to be found. *Yearly allowance*

2 Multiply the allowance per fortnight (\$288.10) by 26. *= \$288.10 × 26*

3 Evaluate. *= \$7490.60*

4 Write the answer in words. *Yearly youth allowance is \$7490.60.*

▶ Disability Support Pension

You may get a Disability Support Pension if you have a permanent and diagnosed disability or medical condition that stops you from working.

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At the time of publication the Disability Support Pension scale was as follows:

Status	Allowance per fortnight
Under 18, living at home	$364.20
Under 18, living independently	$562.20
18–20 years of age, living at home	$412.80
18–20 years of age, living independently	$562.20
Over 21, or under 21 with children	$814.00

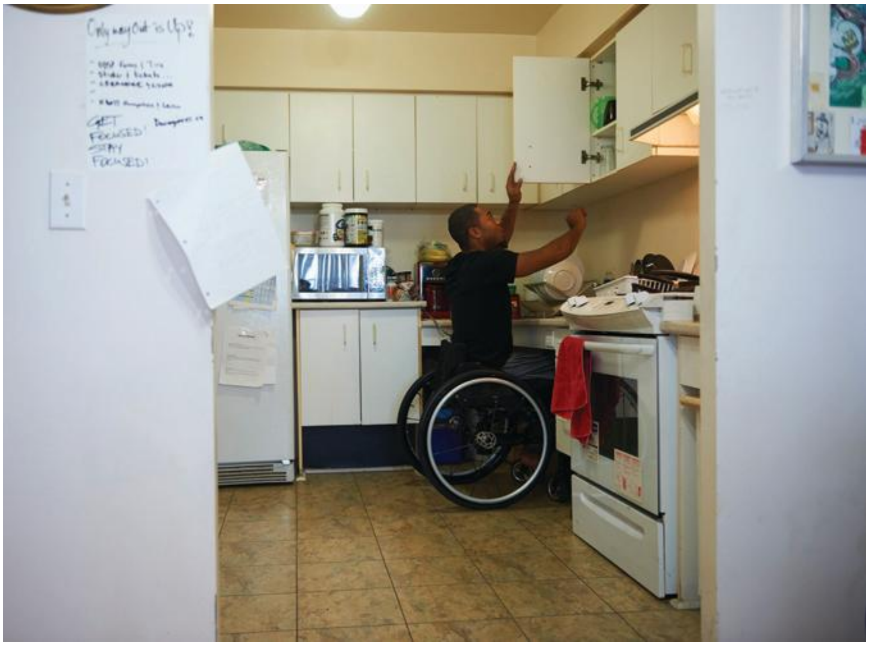
Note: Allowances change yearly

Example 11 Disability

Mike is 20 years old and living independently. How much more will he receive each fortnight from his Disability Support Pension after he turns 21?

Solution

1 Write the quantity to be found.	*Additional Disability Support Pension*
2 Subtract Mike's allowance when he is 20 ($562.20) from his allowance when he is 21 (814.00).	*= $814.00 − $562.20*
3 Evaluate.	*= $251.80*
4 Write the answer in words.	*Additional Disability Support Pension is $251.80 per fortnight.*



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Austudy

To be eligible for Austudy you must be aged 25 years or older and:

- studying full-time in an approved course at an approved educational institution, or
- undertaking a full-time Australian Apprenticeship or traineeship.

In 2017 the Austudy payment scale was as follows:

Status	Maximum payment per fortnight
Single	\$437.50
Single, with children	\$573.30
Couple, no children	\$437.50
Couple with children	\$480.50

Example 12 Austudy

Jen and Brad are both full-time students, living as a couple, with no children. What is their combined annual income from Austudy?

Solution

1 Write the quantity to be found.	*Annual Austudy allowance*
2 Determine what they receive in total each fortnight (\$437.50) and multiply by 26.	= (\$437.50 + \$437.50) × 26
3 Evaluate.	= \$22 750
4 Write the answer in words.	*Annual Austudy allowance* = \$22 750

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Exercise 1D

Use the scales given in the section to answer the following questions.

SF

Example 10 **1** Nikki is eligible for Youth Allowance. She is 17 years old and living away from home.

a How much does she receive each fortnight?

b How much does she receive over the year in total?

2 Ben begins receiving Youth Allowance payments on 1 January. He is 17 years old, and his birthday is on 1 July, when he will turn 18. He is living at home.

a How much does he receive between 1 January and 31 December in total?

b How much does he receive between 1 January and 31 December in total if he decided to move out of home on his 18th birthday?

Example 11 **3** Connie receives a Disability Support Pension. She is 19 years old and living at home.

a How much does she receive each fortnight?

b How much does she receive over the year in total?

Example 12 **4** Madison and Oscar are both eligible for Austudy.

a How much does Madison receive in a year if she is single and studying full-time? Madison is 29 years old.

b Oscar is partnered with no children and studying full-time. Oscar is 35 years old. How much does he receive in a year?

5 Martina is 24 years old, living independently and studying full-time.

a Which allowance is she eligible for?

b How much would she receive each fortnight?

c After she turns 25, which allowance would she be eligible for?

d How much would more she receive each fortnight after she turned 25?

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1E Comparing using the unit cost method

▶ Unit cost method

Many products in a supermarket are sold in packets containing multiple individual items. For example, chocolate truffles might be sold individually as well as in packets of 12 truffles.

A single chocolate truffle in a particular supermarket is sold for 85 cents.

A bag of 12 chocolate truffles is sold in the same supermarket for $5.28.

Would you buy 12 individual chocolate truffles or a bag of 12 chocolate truffles?

We can answer this question by calculating the **unit cost**, or the cost of one single chocolate truffle from the bag.

$$\text{One truffle from bag} = \frac{\text{bag cost}}{12} = \frac{\$5.28}{12} = \$0.44$$

It is obviously better value to buy a bag of truffles because the *unit cost* of a truffle from the bag is less than the individual price.

Example 13 Using the unit cost method

If 24 golf balls cost $86.40, how much do 7 golf balls cost?

Solution

1 Find the cost of one golf ball by dividing $86.40 (the total cost) by 24 (the number of golf balls).

$\$86.40 \div 24 = \$3.60$

2 Multiply the cost of one golf ball ($3.60) by 7. Write your answer.

$\$3.60 \times 7 = \$25.20$

7 golf balls cost $25.20

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▶ Using the unit cost method to compare items

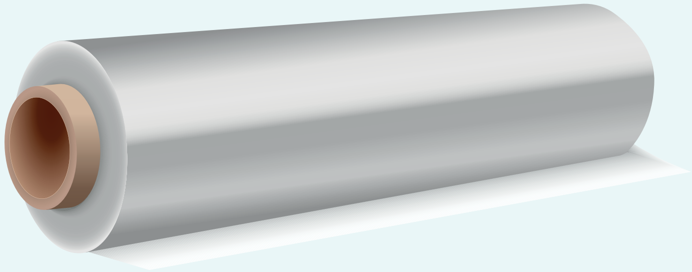
The unit cost method is used to compare the cost of items using the unit cost of the contents. This enables us to calculate which item is the best buy.

Example 14 Using the unit cost method to compare items

Two different brands of kitchen plastic wrap are sold in a shop.

- Brand A contains 50 metres of plastic wrap and costs $4.48.

- Brand B contains 90 metres of plastic wrap and costs $5.94.



Which brand is the better value?

Solution

The 'unit' in each pack is a metre of plastic wrap. The prices of both brands can be compared based on this unit.

1 Calculate the unit cost, per metre, for each brand by dividing the package cost by the number of units inside.

$$\text{Unit cost brand A} = \frac{\$4.48}{50\text{ m}} = \$0.0896 \text{ per metre}$$

$$\text{Unit cost brand B} = \frac{\$5.94}{90\text{ m}} = \$0.066 \text{ per metre}$$

2 Choose the brand that has the lower unit cost.

Brand B has the lower unit cost per metre of plastic wrap so it is the better value brand.

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Exercise 1E

SF

Example 13

1 Use the unit cost method to answer the following questions.

- **a** If 12 cupcakes cost \$14.40, how much do 13 cupcakes cost?
- **b** If a clock gains 20 seconds in 5 days, how much does the clock gain in three weeks?
- **c** If 17 textbooks cost \$501.50, how much would 30 textbooks cost?
- **d** If an athlete can run 4.5 kilometres in 18 minutes, how far could she run in 40 minutes at the same pace?

2 If one tin of red paint is mixed with four tins of yellow paint, it produces five tins of orange paint. How many tins of the red and yellow paint would be needed to make 35 tins of paint of the same shade of orange?

3 If a train travels 165 kilometres in 1 hour 50 minutes at a constant speed, calculate how far it could travel in the following times.

a 3 hours **b** $2\frac{1}{2}$ hours **c** 20 minutes
d 70 minutes **e** 3 hours and 40 minutes **f** $\frac{3}{4}$ hour

Example 14

4 Ice creams are sold in two different sizes. A 35 g cone costs \$1.25 and a 73 g cone costs \$2.00. Which is the better buy?

5 A shop sells 2 L containers of brand A milk for \$2.99, 1 L of brand B milk for \$1.95 and 600 mL of brand C milk for \$1.42. Calculate the best buy.

CF

6 A car uses 45 litres of petrol to travel 495 kilometres. Under the same driving conditions calculate:

- **a** how far the car could travel on 50 litres of petrol
- **b** how much petrol the car would use to travel 187 kilometres.

7 You need six large eggs to bake two chocolate cakes. How many eggs will you need to bake 17 chocolate cakes?

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1F Currency and exchange rates

Currency exchange

The money that you use to pay for goods and services in one country cannot usually be used in any other country. If you take Australian dollars to New Zealand, for example, they must be exchanged with, or converted to, New Zealand dollars. Even though the Australian dollar (AUD) and the New Zealand dollar (NZD) have the same name, they have different values.

The table below shows the rate of exchange between Australian dollars and other currencies on a particular day, rounded to five decimal places.

Currency exchange: Australian dollar (AUD)				
Country	Currency name	Symbol	Code	Units per AUD
United States of America	Dollar	$	USD	0.743 99
European Union	Euro	€	EUR	0.675 97
Great Britain	Pound	£	GBP	0.522 82
Japan	Yen	¥	JPY	84.648 03
South Africa	Rand	R	ZAR	11.417 48
Brazil	Real	R$	BRL	2.780 55
United Arab Emirates	Dirham	إ.د	AED	2.732 69

Source: *http://www.xe.com*

Note: Exchange rates may change on a daily basis

The numbers in the column 'Units per AUD' are the **exchange rates** for each currency and are used to convert between Australian dollars and other currencies, in a similar way to converting between units of measurement.

The units per AUD for the Japanese yen is 84.64803, which means that one AUD will be exchanged for 84.64803 yen in Japan.

$1 AUD = 84.64803 JPY

Ten Australian dollars would be exchanged for ten times this amount.

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Converting between currencies

An *exchange rate* between Australian dollars and other currencies is given as units per AUD.

Convert Australian dollars to other currencies by *multiplying* the amount by the exchange rate.

Convert other currencies to Australian dollars by *dividing* the amount by the exchange rate.

In Australia, the dollar consists of 100 cents. Most countries divide their main currency unit into 100 smaller units, and so it is usual to round currency amounts to two decimal places, even though the conversion rates are usually expressed with many more decimal places than this.

Example 15 Converting between Australian dollars and other currencies

Use the table of currency exchange for the Australian dollar (on page 27) to convert these currencies.

a 300 AUD into British pounds

b 2500 ZAR into Australian dollars

Solution

a 1 Write the exchange rate for AUD to GBP.	1 AUD = 0.522 82 GBP
2 Multiply 300 AUD by the exchange rate to convert to GBP.	$300 \text{ AUD} = 300 \times 0.522\,82 \text{ GBP}$ $= 156.846 \text{ GBP}$
3 Round your answer to two decimal places.	$300 AUD is converted to £156.85
b 1 Write the exchange rate for AUD to ZAR.	1 AUD = 11.417 48 ZAR
2 Divide 2500 ZAR by the exchange rate to convert to AUD.	$2500 \text{ ZAR} = \dfrac{2500}{11.417\,48} \text{AUD}$ $= 218.962\,503\,109$
3 Round your answer to two decimal places.	2500 ZAR is converted to $218.96 AUD.

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Exercise 1F

SF

Example 15 **1** Use the table of currency exchange values below to convert the following amounts into the currency in brackets. Round your answer to two decimal places.

Currency exchange: Australian dollar (AUD)				
Country	Currency name	Symbol	Code	Units per AUD
United States of America	Dollar	$	USD	0.743 99
European Union	Euro	€	EUR	0.675 97
Great Britain	Pound	£	GBP	0.522 82
Japan	Yen	¥	JPY	84.648 03
South Africa	Rand	R	ZAR	11.417 48
Brazil	Real	R$	BRL	2.780 55
United Arab Emirates	Dirham	إ.د	AED	2.732 69

Source: *http://www.xe.com*

a $750 AUD (EUR) **b** $4800 AUD (USD) **c** $184 AUD (BRL)

d €1500 (AUD) **e** R$8500 BRL (AUD) **f** إ.د 16 000 AED (AUD)

2 On a particular day, one Australian dollar was worth 8.6226 Botswana pula (BWP). How many pula would Tapiwa need to exchange if she wanted to receive $2000 AUD?

CF

3 On a particular day, $850 AUD could be exchanged to €581.40. How many euro would be exchanged for $480 AUD?

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1G Budgeting

The best way to manage your finances is to prepare a budget. A budget involves balancing income and expenses, ensuring that you have enough to pay the essential bills and start putting money towards your future goals. Expenses associated with essential living costs, such as rent and electricity, are considered **fixed spending**, and are less able to be varied. Expenses associated with activities such as entertainment or clothing are considered **discretionary spending**.

Here are some simple steps to preparing and using a budget:

1. Choose a time period for your budget that suits your lifestyle; for example, a week, a fortnight or a month.
2. List all the income for the time period. This should include income from work, investments, and any other allowances.
3. List all of your expenses for the time period. It helps to determine your annual expense in some categories, and put aside money for this each time period. For example, if your annual car insurance payment is \$900, and you are preparing a monthly budget, then you need to allow \$900/12 = \$75 in each calendar month for car insurance.
4. Calculate the total of the income and expenses.
5. Balance the budget, ensuring that you are not spending more than you have. This might mean that you need to modify spending in the categories over which you have control.

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Example 16 Balancing a budget

Balance the following weekly budget.

Income		Expenses	
Salary	$1726.15	Clothing	$ 73.08
Bonus	$ 20.00	Gifts and Christmas	$ 114.80
Investment	$ 156.78	Groceries	$ 467.31
Part-time work	$ 393.72	Insurance	$ 171.34
		Loan repayments	$ 847.55
		Motor vehicle costs	$ 105.96
		Phone	$ 38.26
		Power and heating	$ 51.82
		Rates	$ 54.82
		Recreation	$ 216.79
		Work-related costs	$ 68.76
		Balance	
Total		**Total**	

Solution

1 Add all the income.

$\text{Income} = 1726.15 + \ldots + 393.72$
$= \$2296.65$

2 Add the all the expenses excluding 'balance'.

$\text{Expenses} = 73.08 + \ldots + 68.92$
$= \$2210.49$

3 Subtract the total expenses from the total income.

$\text{Balance} = \text{Income} - \text{Expenses}$
$= 2296.6 - 2210.49$
$= 86.16$

4 Write the result of step **3** as the balance.

$= \$86.16$

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Example 17 Creating a budget

Maya and Logan have a combined weekly net wage of $954. Their monthly expenses are home loan repayment $1032, car loan repayment $600, electricity $102, phone $66 and car maintenance $120. Their other expenses include insurance $2160 annually, rates $1800 annually, food $180 weekly, petrol $48 fortnightly and train fares $36 weekly. Maya and Logan allow $72 for miscellaneous items weekly and need to save $84 per week for a holiday next year.

a Prepare a monthly budget for Maya and Logan. Assume there are four weeks in a month.

b What is the balance?

c How can Maya and Logan ensure they have their holiday next year?

Solution

a **1** Draw up a table with columns to list income and expenses.

2 List all the monthly income categories.

3 List all the monthly expenses categories.

Solution is shown below.

Income		Expenses	
Wage	$3816	Home loan repayment	$1032
		Car loan repayment	$600
		Electricity	$102
		Phone	$66
		Car maintenance	$120
		Insurance	$180
		Rates	$150
		Food	$720
		Petrol	$96
		Train fares	$144
		Miscellaneous	$288
		Holiday	$336
		Balance	−$18
	$3816		$3816

4 Calculate the total income and expenses categories.

Total income = $3816

Total expenses = $3834

b **5** Subtract the total expenses from the total income to calculate the balance.

Balance = $3816 − $3834

= −$18

c **6** The balance is −$18. A negative balance indicates a need to increase their income or reduce their expenses to ensure they have a holiday.

Maya and Logan need to increase income or reduce expenses by $18.

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Exercise 1G

SF

1 Oscar and Jill are living in a unit. Part of their budget is shown below. Calculate the total amount paid over one year for:

a electricity
b insurance
c food
d rent.

Item	When	Cost
Electricity	Quarterly	$ 384
Food	Weekly	$ 360
Insurance	Biannually	$1275
Rent	Monthly	$1950

Example 16, 17

2 Sarah earns $67 365 annually. She has budgeted 20% of her salary for rent. How much should she expect to pay to rent an apartment for one year?

3 Adam has constructed a yearly budget as shown below.

Income		Expenses	
Wage	$60 786.22	Clothing	$ 4634.42
Interest	$ 674.15	Council rates	$ 1543.56
		Electricity	$ 1956.87
		Entertainment	$ 4987.80
		Food	$17 543.90
		Gifts and Christmas	$ 5861.20
		Insurance	$ 2348.12
		Loan repayments	$16 789.34
		Motor vehicle costs	$ 2458.91
		Telephone	$ 832.98
		Work-related costs	$ 812.67
		Balance	
Total		**Total**	

a Calculate the total income.
b Calculate the total expenses.
c Balance the budget.

4 Dimitri had a total weekly income of $104 made up of a part-time job earning $74 and an allowance of $30. He decided to budget his expenses in the following way: sport – $24, movies – $22, school – $16 and food – $20.

a Prepare a weekly budget showing income and expenses.
b What is the balance?

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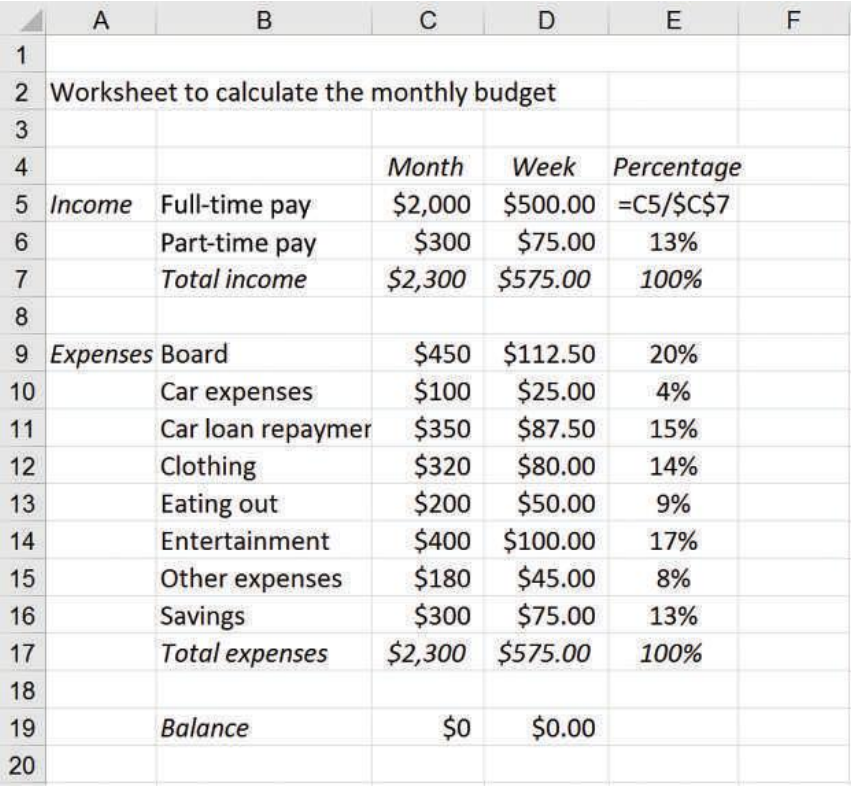
Spreadsheet
1GQ5

5 Create the spreadsheet below.

SF



	A	B	C	D	E	F
1						
2	Worksheet to calculate the monthly budget					
3						
4			*Month*	*Week*	*Percentage*	
5	*Income*	Full-time pay	$2,000	$500.00	=C5/$C$7	
6		Part-time pay	$300	$75.00	13%	
7		*Total income*	*$2,300*	*$575.00*	*100%*	
8						
9	*Expenses*	Board	$450	$112.50	20%	
10		Car expenses	$100	$25.00	4%	
11		Car loan repaymer	$350	$87.50	15%	
12		Clothing	$320	$80.00	14%	
13		Eating out	$200	$50.00	9%	
14		Entertainment	$400	$100.00	17%	
15		Other expenses	$180	$45.00	8%	
16		Savings	$300	$75.00	13%	
17		*Total expenses*	*$2,300*	*$575.00*	*100%*	
18						
19		*Balance*	$0	$0.00		
20						

a The formula for cell E5 is '=C5/$C$7'. It is the formula for relative percentage. Fill down the contents of E5 to E7 using this formula.

b Enter the formula in cell E9 to calculate the relative percentages for expenses. Fill down the content of E9 to E17.

c Edit the amount spent per month on eating out from $200 to $240. Observe the changes.

d Edit the amount of savings per month from $300 to $360. Observe the changes.

e Edit the amount of car expenses per month from $100 to $150. Observe the changes.

6 Ava has a gross fortnightly pay of $1896.

CF

a Ava has a mortgage with an annual repayment of $13 676. Calculate the amount that Ava must budget each fortnight for her mortgage.

b Ava has budgeted $180 per week for groceries, $60 per week for entertainment, $468 per year for medical expenses and $80 per week to run a car. Express these as fortnightly amounts and calculate their total.

c Ava has an electricity bill of $130 per quarter, a telephone bill of $91 per quarter and council rates of $1118 per annum. Express these amounts annually and convert to fortnightly amounts. What is the total of these fortnightly amounts?

d Prepare a fortnightly budget showing income and expenses.

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1H Focus on problem-solving and modelling

Exercise 1H

Buying from a store or online – which is cheaper?

1 Investigate the cost of buying clothes online compared to buying them in a shopping centre.

a Choose an outfit comprising at least three items, such as a shirt, jeans and jacket. Find the advertised price of similar items in two different stores in a shopping centre or high street, and from an online retailer in Australia and two others overseas, for which the prices are in foreign currencies.

b Draw up a table or spreadsheet comparing the retail prices including any sales discounts. For the overseas online stores, you will need to convert the currency to Australian dollars. Find the conversion rate you'll need to pay online, and describe in words the calculation needed to convert the foreign currency to Australian dollars.

c Next work out the additional expenses for each purchase. For the online stores this will be post and packing, and there may be other charges listed on the website for insurance or handling. For the shopping centre, include the cost of one return journey by public transport to the centre.

d Write a report saying which is the 'best buy', comparing not only the price of each retailer but also giving your views on the advantages of buying from Australian websites and whether this would influence your decision.

Renting a house

2 One day, when you have your own income, you may move out of home and into a house that you rent with friends. How much income will you need? How much can you afford to pay in rent, given all your other expenses? Devise a plan for renting your own place with friends, and draw up a spreadsheet to record your plan and carry out the calculations.

You will need to investigate the following issues:

a **How much do you have to budget for rent?**
Assume you are going to share with two friends and you each want a bedroom, so you will rent a three-bedroom apartment or house. Research the rates for renting properties in at least three suburbs and choose one. There are a number of websites that advertise rented accommodation. Google 'real estate rentals' to find one, and look up the prices. Work out a target rental rate for where you want to live. Assume you'll divide it evenly with your two friends.

b **What other expenses are associated with renting a property?**
Think about essential services you need to pay – water, power, gas. Assume your rent includes council rates and property taxes – i.e. the landlord pays them. You will also need to pay a bond – a lump sum held to guarantee payment of any damage during the rental, but you may assume that your family is willing to lend you this.

c **What other living expenses will you have?**

Draw up a budget for all your other expenses – food, clothes, household cleaning materials and toiletries, transport, travel, plus entertainment. Perhaps you should save something each month for luxuries and unexpected expenses?

d **How much do you have to earn to pay for items 1, 2 and 3?**

Work out the take-home pay – net income – you need after tax, then work out what gross salary or wages is needed pre-tax and other deductions to pay for it.

Creating an income tax calculator

3 These are the income tax rates at the time of publication. There are five tax brackets.

Taxable income	Tax rate	Tax payable
0–$18 200	0%	Nil
$18 201–$37 000	19%	19 cents for each $1 over $18 200*
$37 001–$87 000	32.5%	$3572 plus 32.5 cents for each dollar over $37 000
$87 001–$180 000	37%	$19 822 plus 37 cents for each dollar over $87 000
$180 001 and above	45%	$54 232 plus 45 cents for each dollar over $180 000

a Write down in words the numbered series of steps required to calculate the tax payable on any taxable income. There should be five steps, one for each tax bracket.

Hint: Use 'if/then' statements, the first one should be:

'If the taxable income is equal to or less than $18 200, then the tax payable is zero'.

Use decimal equivalents instead of percentages, i.e. use 0.19 for 19%.

b Write down the steps again but substitute mathematical statements where possible. Use arithmetical operators such as + and – and in particular use these terms:

For:	Use:
taxable income	Income
equal to or less than	≤
equal to or more than	≥
tax payable	Tax
times (multiplication)	*
is	=

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For example write the first step as:
1 If Income ≤ \$18 200 then Tax = 0
Write statements such as

‘If the taxable income is equal to or more than \$18 201 and equal to or less than \$37 000’

as

‘If Income ≥ \$18 201 and Income ≤ \$37 000’.

Write statements such as

‘0.19 times the amount of taxable income over \$18 200’

in the same way you would enter it into a calculator:

‘0.19*(Income–\$18 200)’

Spreadsheet 1HQ3

c Complete the spreadsheet activity that accompanies this question in the Interactive Textbook to create your own tax calculator. Building on your answer to part **b**, the spreadsheet shows how to use IF functions to determine the bracket and apply the relevant formula to calculate the tax payable.

How much work is needed to pay for a large expense?

4 Erin, a student, wants to save up to buy the latest smartphone and laptop computer for a grand total of \$4800. She would like to achieve this goal in three months’ time. She lives at home so she does not need to pay any other expenses such as rent, bills, food, cleaning, etc.; however, she does need to pay for her own entertainment and other expenses for which she has allocated \$100 in her weekly budget.
Erin has handed out her resume to a couple of local businesses and received offers for:

- part time work at the local supermarket for \$25 per hour, 14 hours per week
- delivering flyers around the local area for 4 hours a day, 5 days per week. The rate of pay is \$100 per day.

a How many whole weeks will it take for Erin to earn at least \$4800, if she works:

  i part time at the local supermarket?

  ii delivering flyers around the local area?

b What is the hourly rate of pay for delivering flyers around the local area?

c Will Erin be able to afford the phone and computer in 3 months, if she:

  i works at the local supermarket?

  ii delivers flyers around the local area?

Assume each month has 4 weeks.

d One of the jobs will not be sufficient for Erin to afford the goods she wants. State which job and how much she will be short.

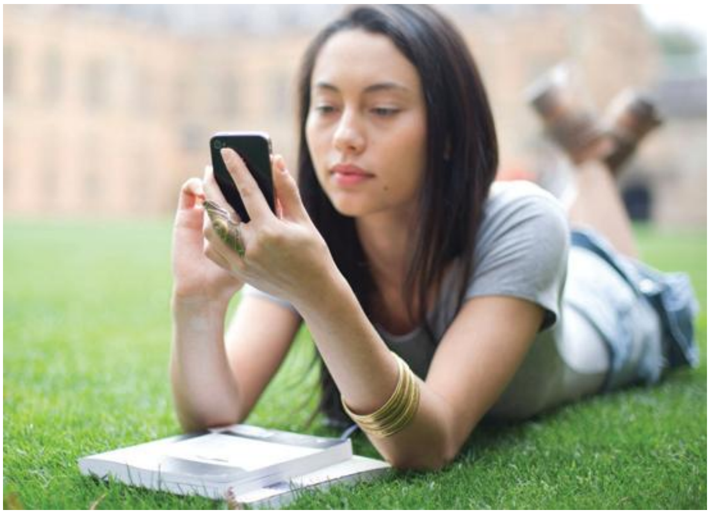
e Why does it take more time for Erin to buy the phone and computer working at the local supermarket than delivering flyers?

f Which work should Erin choose? Give your reasons.

A budget for a student away from home

5 Amina is 19 years old and is about to move away from home to start university. She receives a weekly allowance of $400 from her parents.

- **a** Investigate the cost of rent of student accommodation online. You may use websites such as http://realestate.com.au/ to help.
- **b** Investigate the cost of renting a room in a shared house online. You may use websites such as http://gumtree.com.au/ to help.
- **c** Suggest some positive and negative aspects of student accommodation or living in a shared house.
- **d** Find the approximate average cost of a student apartment. Use this for further calculations.
- **e** Amina is eligible for Youth Allowance.
  - **i** Find out how much Amina can receive.
  - **ii** What is Amina's total weekly income?
- **f** Amina has budgeted $150 towards food and $50 for entertainment per week. Using the cost of rent you determined in part **d**, calculate whether Amina can sustain her lifestyle or not.
- **g** How much does Amina have at the end of the week? Does she have enough to pay for all her expenses?
- **h** If it is not enough, how much can Amina afford to spend on accommodation per week?
- **i** Amina will need to spend $800 on books. In how many weeks can she save this amount? Assume rent is $200.
- **j** Investigate the various expenses that a student might need to consider when budgeting. You may use the internet or come up with your own ideas.



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Key ideas and chapter summary

Salary and wages	A **salary** is payment for a year's work, generally paid as equal weekly, fortnightly or monthly payments. A **wage** describes payment for work calculated on an hourly basis, with the amount earned dependent on the number of hours actually worked.
Overtime, penalty rates and allowances	**Overtime** is when employees work beyond the normal working day. A **penalty rate** is paid for working weekends, public holidays, late night shifts or early morning shifts. **Allowances** are extra payments for use of own tools, or for working in unpleasant or dangerous conditions.
Commission	**Commission** is a percentage of the value of the goods or services sold.
Piecework	Piecework is when a worker is paid a fixed payment, called a **piece rate**, for each unit produced or action completed.
Royalties	A **royalty** is a percentage of the price of intellectual property such as a book or song.
Youth Allowance	**Youth Allowance** may be paid by the government to 18–24 years olds studying full-time, 16–24 years olds undertaking a full-time Australian Apprenticeship, or 16–20 years old and looking for full-time work.
Disability Support Pension	A **Disability Support Pension** is paid to someone who has a permanent and diagnosed disability or medical condition that stops them from working.
Austudy	**Austudy** is an allowance paid to someone who is aged 25 years or older and studying full-time or undertaking a full-time Australian Apprenticeship or traineeship.
Unit cost method	The **unit cost method** is used to compare the cost of items using unit the unit cost of the contents.
Currency and exchange rates	**Exchange rates** allow you to convert the cost of items in another currency to the cost in Australian dollars, and vice-versa.
Budgeting	**Budgeting** involves balancing income and expenditure over a specified time frame, such as a week or month.

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Skills check

Having completed this chapter you should be able to:

- calculate income payments from an annual salary
- calculate income payments from an hourly rate
- calculate income payments when overtime, penalty rates and allowances are applied
- calculate income payments based on commission
- calculate income payments from piecework
- calculate income payments from royalties
- determine payments applied through Youth Allowance, Disability Support Pension and Austudy
- solve practical problems using the unit cost method
- solve practical problems involving converting foreign currency to AUD
- create and balance a personal budget taking into account fixed and discretionary spending
- apply spreadsheets to examples for any of the above situations.

Multiple-choice questions

SF

1 Alyssia receives a salary of $85 640. How much does she receive each fortnight?

A $3293.84 **B** 3293.85 **C** 1646.92 **D** $1646.93 **E** $7136.67

2 Christopher receives a normal hourly rate of $22.60 per hour. What is his pay when he works 8 hours at a normal rate and 3 hours at time-and-a-half?

A $180.80 **B** 248.60 **C** 282.50 **D** $296.60 **E** $316.40

3 Bonnie is employed on a casual basis. She earns $15 per hour normally, with time-and-a-half for Sundays. Last week Bonnie worked from 2 p.m. until 6 p.m. on Monday, Tuesday and Wednesday, and from 12 noon until 5 p.m. on Sunday. How much did she earn last week?

A $180 **B** $270 **C** $255 **D** $292.50 **E** $382.50

4 Taylah earns a weekly retainer of $425 plus commission of 8% on sales. What are her weekly earnings when she makes sales of $8620?

A $34 **B** $459.00 **C** $493.96 **D** 689.60 **E** $1114.60

5 Ahmet is a carpet layer and charges $37.50 per square metre of carpet laid. How much will he earn for laying carpet in a room that is 9 square metres?

A $37.50 **B** $46.50 **C** $112.50 **D** 225.00 **E** $337.50

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6 Isabelle earns a royalty of 18% on sales of her autobiography. There were sales worth $24 520 last year. What is Isabelle's royalty payment?

A $4413.60 **B** $9616.00 **C** $20 106.40 **D** $24 520.00 **E** $28 933.60

7 Three different brands of gift wrap are sold in a store. Brand A contains 10 metres of gift wrap and costs $12, brand B contains 12 metres of gift wrap and costs $15, and brand C contains 20 metres of gift wrap and costs $20. Putting the brands in order from cheapest to most expensive per metre we get:

A A, B, C **B** B, A, C **C** C, B, A **D** C, A, B **E** B, C, A

8 On a particular day, the exchange rate between the Australian dollar and the Thai baht (THB) is 26.305.

$550 Australian dollars, converted to Thai baht would be:

A 20.91 baht **B** 26.31 baht **C** 550 baht
D 576.31 baht **E** 14 467.75 baht

9 Mikki is planning a holiday to Bali. She has found some accommodation that will cost her 350 000 Indonesian rupiah (IDR) per night, and she intends to stay for 7 nights. If the exchange rate between the Australian dollar and Indonesian rupiah is 8863.12, then her total accommodation cost in Australian dollars is:

A $34.49 **B** $276.43 **C** $340.49 **D** $564.14 **E** $1266.16

10 Adam has the following bills: electricity $250 per quarter, phone $70 per month, petrol $1200 per year and rent $300 per week. What is the total amount Adam should budget for the year?

A $358 **B** $720 **C** $1553 **D** $6640 **E** $18 640

The following information relates to Questions 11 and 12.

Thomas wants to save up $10 000 for a new car. He earns $850 per week, and has expenses as shown in the table:

Electricity	$280 per quarter
Food	$185 per week
Rent	$1000 per month
Travel	$50 per week

11 Thomas's annual expenses are:

A $18 180 **B** $18 860 **C** $25 060 **D** $25 340 **E** $44 200

12 How many weeks will it take Thomas to save for the car?

A 12 weeks **B** 21 weeks **C** 26 weeks **D** 28 weeks **E** 363 weeks

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Short-answer questions

SF

1 Jake earns \$96 470.40 per annum and works an average of 48 hours per week.

a What is his average weekly wage?

b Calculate Jake's hourly rate of pay.

2 Alex works for a fast-food company and is paid \$13.50 per hour for a 35-hour week. He gets time-and-a-half pay for overtime worked on the weekdays and double time for working on weekends. Last week he worked a normal 35-hour week plus three hours of overtime during the week and four hours of overtime on the weekend. What was his wage last week?

3 Carlo's employer has decided to reward all employees with a bonus. The bonus awarded is $6\frac{1}{4}\%$ of their annual salary. What is Carlo's bonus if his annual salary is \$85 940?

4 The public service provides all employees with a $17\frac{1}{2}\%$ holiday loading on four weeks normal wages. Lucy works a 37-hour week for the public service in Canberra. She is paid a normal hourly rate of \$32.40.

a How much will Lucy receive in holiday loading?

b Calculate the total amount of pay that Lucy will receive for her holidays.

CF

5 Chelsea is a real estate agent and charges the following commission for selling the property: 3% on the first \$45 000, then 2% for the next \$90 000 and $1\frac{1}{2}\%$ thereafter.

a What is Chelsea's commission if she sold a property for \$240 000?

b How much would the owner of the property receive from the sale?

SF

6 Patrick is a comedian who makes \$120 for a short performance and \$260 for a long performance. How much will he earn if he completes 11 short and 12 long performances?

7 Bailey is paid a royalty of 11.3% on the net sales of his book. The net sales of his book in the last financial year were \$278 420.

a What was Bailey's royalty payment in the last financial year?

b Net sales this financial year are expected to decrease by 15%. What is the expected royalty payment for this financial year?

CF

8 The maximum Youth Allowance is reduced by \$1 for every \$4 that the youth's parents' income exceeds \$31 400. By how much is Hannah's youth allowance reduced if her parents earn a combined income of \$35 624?

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CF

9 William works as a builder. His annual union fees are \$278.20. William has his union fees deducted from his weekly pay. How much is William's weekly union deduction?

10 Quan received a gross fortnightly salary of \$2968. His pay deductions were \$765.60 for income tax, \$345.15 for superannuation and \$23.40 for union fees.

a What was his fortnightly net pay?

b What percentage of his gross income was deducted for income tax? (Answer correct to one decimal place.)

SF

11 Joel is a carpet layer and is paid \$16 per square metre to lay carpet. How much will he earn for laying carpet in a house with an area of 32 square metres?

12 Daniel has a gross monthly wage of \$3640. He has the following deductions taken from his pay: \$764 for income tax, \$71.65 for superannuation and \$23.23 for union membership. What is Daniel's net pay?

13 Hannah has budgeted \$210 per week for groceries, \$70 per week for leisure, \$23 per fortnight for medical expenses and \$90 per week to run a car. Calculate the monthly expenses. Assume 4 weeks in a month.

14 Amelie earns \$90 345 annually. She has budgeted 30% of her salary for a loan repayment. How much should she expect to pay for a loan repayment for one year?

15 On a particular day, one Australian dollar (AUD) can be exchanged for 0.7562 United States dollars (USD).

a What is the equivalent amount of USD for \$350.00 AUD?

b A tourist from the US is visiting Australia. A tour to Phillip Island will cost \$140.00 USD per person. What is the cost in AUD ?

16 An online shop sells computer equipment and lists the prices of items in Australian dollars, US dollars and British pounds (GBP).
One Australian dollar exchanges for \$0.842 USD and £0.53 GBP on a particular day.
If a hard drive is listed with a price of \$125.60 AUD, what is the price for a customer in:

a the US?

b Great Britain?

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Extended-response questions

1 Amy decides to create a budget so she can save for a holiday that will cost $3000. She records her major expenses for 6 months, and enters them into the spreadsheet shown.

SF

Spreadsheet

1ERQ1

	A	B	C	D	E	F	G	H
1								
2	*Amy's budget*							
3		January	February	March	April	May	June	
4	Rent	$800.00	$800.00	$800.00	$800.00	$800.00	$800.00	
5	Phone	$125.00	$50.00	$60.00	$80.00	$120.00	$100.00	
6	Food	$380.00	$240.00	$180.00	$240.00	$300.00	$370.00	
7	Entertainment	$230.00	$500.00	$440.00	$220.00	$450.00	$300.00	
8	Clothing	$500.00	$200.00	$340.00	$700.00	$430.00	$50.00	
9	Petrol	$50.00	$220.00	$200.00	$180.00	$150.00	$165.00	
10								

- **a** Create Amy's spreadsheet as shown.
- **b** Find the total of her expenses for each month. How much did she spend in total in
  - **i** January?
  - **ii** April?
- **c** Find the total of her expenses for each month, and then calculate the average she spends per month on each of the items in her budget.
- **d** Amy's income is $2950 per month.
  - **i** How much is she able to save per month on average?
  - **ii** How long will it take her to save for the holiday? Give your answer to the nearest month.
- **e** If Amy reduces her spending on entertainment to $200 per month, and on clothing to $300 per month, how long will it now take her to save for the holiday? Give your answer to the nearest month.

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2 Phil purchases a new car, costing $33 190. He is required to make repayments of $725 per calendar month for five years to pay off the loan. If he travels 15 000 km per year in the car, he must allow for petrol ($30 per week), tyres (one set during the five-year period, which will cost $572), servicing (one service every 6 months costing $620 each time), and insurance and registration ($2400 per year).

a Determine how much Phil's car will cost him per week to own and run.

b How much will Phil spend on the car over the five-year period?

c If Phil sells the car at the end of the five-year period for $12 500, how much will he have spent on the car in total?

d In order to help pay his weekly car costs (from part **a**), Phil takes on some additional overtime at work. His normal pay rate is $22.50 per hour and he earns time-and-a-half for overtime.

- **i** How many hours overtime per week should he work to earn the amount he pays for the car?
- **ii** How many hours overtime per week should he work to earn the amount he pays for the car, given that he pays 30% income tax on the overtime payment?

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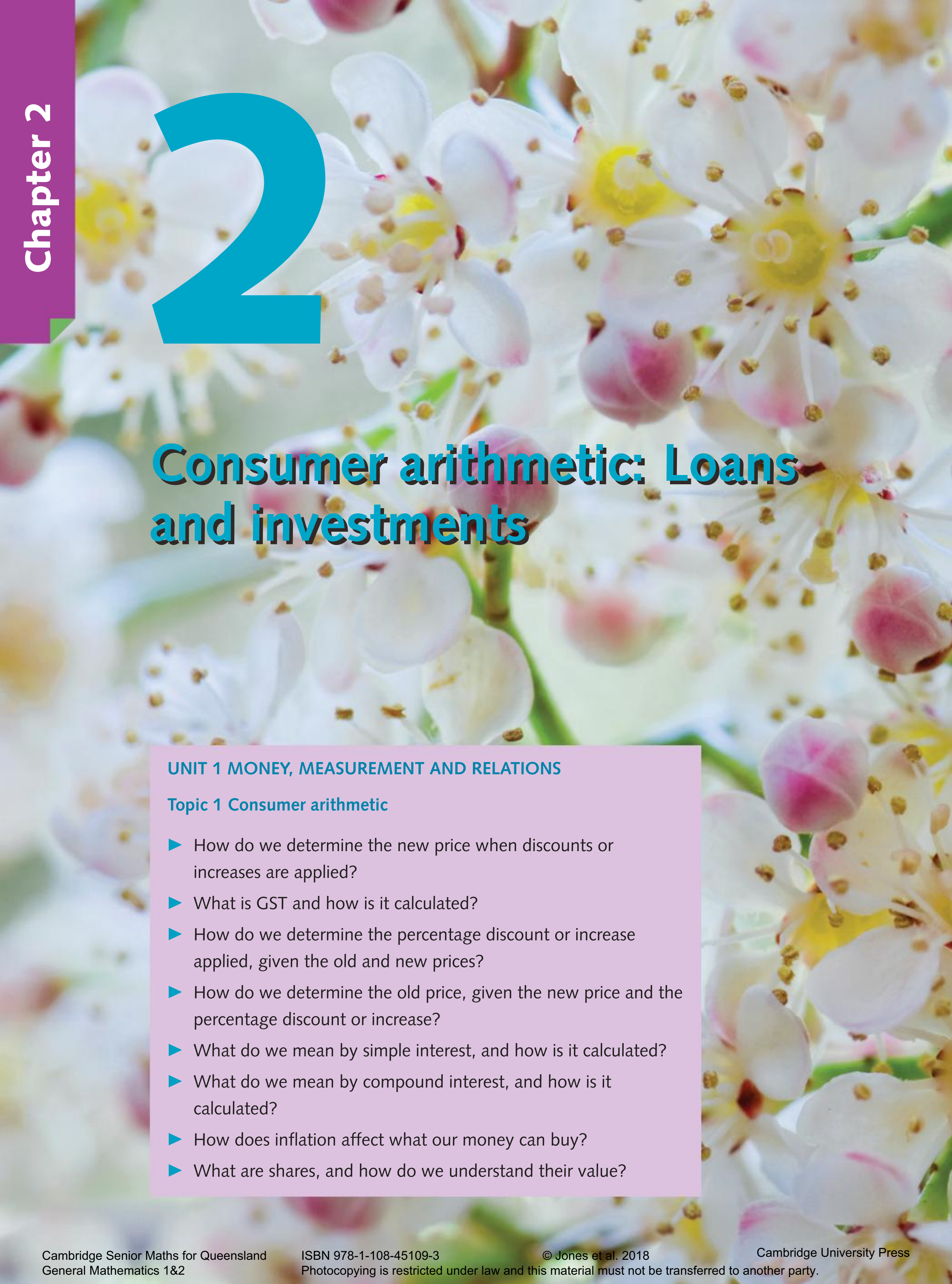

Chapter 2

2

Consumer arithmetic: Loans and investments

UNIT 1 MONEY, MEASUREMENT AND RELATIONS

Topic 1 Consumer arithmetic

- How do we determine the new price when discounts or increases are applied?
- What is GST and how is it calculated?
- How do we determine the percentage discount or increase applied, given the old and new prices?
- How do we determine the old price, given the new price and the percentage discount or increase?
- What do we mean by simple interest, and how is it calculated?
- What do we mean by compound interest, and how is it calculated?
- How does inflation affect what our money can buy?
- What are shares, and how do we understand their value?

Introduction

There is no doubt that an understanding of financial arithmetic will be the most useful life skill that you will develop in mathematics. Without this knowledge you could end up spending a lot of money unnecessarily.

If you need help with percentages and rates, the necessary skills are reviewed in Appendix 1 Computation and practical arithmetic.

2A Percentages and applications

Note: If you need help with percentages, the skills are covered in Appendix 1, p. 463.

▶ Discounts and mark-ups

Suppose an item is discounted, or **marked down**, by 10%. The amount of the **discount** and the new price are:

$$\begin{aligned}\text{discount} &= 10\% \text{ of original price}\\ &= 0.10 \times \text{original price}\end{aligned} \quad \textit{and} \quad \begin{aligned}\text{new price} &= 100\% \text{ of old price} - 10\% \text{ of old price}\\ &= 90\% \text{ of old price}\\ &= 0.90 \times \text{old price}\end{aligned}$$

Applying discounts

In general, if r% discount is applied:

$$\text{discount} = \frac{r}{100} \times \text{original price} \qquad \begin{aligned}\text{new price} &= \text{original price} - \text{discount}\\ &= \frac{(100 - r)}{100} \times \text{original price}\end{aligned}$$

Example 1 **Calculating the discount and the new price**

a How much is saved if a 10% discount is offered on an item marked $50.00?

b What is the new discounted price of this item?

Solution

a Evaluate the discount.

$$\text{Discount} = \frac{10}{100} \times \$50 = \$5.00$$

b Evaluate the new price by either:

- subtracting the discount from the original price

or

- calculating 90% of the original price.

$$\begin{aligned}\text{New price} &= \text{original price} - \text{discount}\\ &= \$50.00 - \$5.00 = \$45.00\end{aligned}$$

or

$$\text{New price} = \frac{90}{100} \times \$50 = \$45.00$$

Sometimes, prices are increased or marked *up*.
If a price is increased by 10%:

$$\begin{aligned}\text{increase} &= 10\% \text{ of original price}\\ &= 0.10 \times \text{original price}\end{aligned} \quad \textit{and} \quad \begin{aligned}\text{new price} &= 100\% \text{ of old price} + 10\% \text{ of old price}\\ &= 110\% \text{ of old price}\\ &= 1.10 \times \text{old price}\end{aligned}$$

Applying mark-ups

In general, if $r\%$ increase is applied:

$$\text{increase} = \frac{r}{100} \times \text{original price} \qquad \begin{aligned}\text{new price} &= \text{original price} + \text{increase}\\ &= \frac{(100 + r)}{100} \times \text{original price}\end{aligned}$$

Example 2 Calculating the increase and the new price

a How much is added if a 10% increase is applied to an item marked \$50?

b What is the new increased price of this item?

Solution

a Evaluate the increase.

$$\text{Increase} = \frac{10}{100} \times 50 = \$5.00$$

b Evaluate the new price by either:

- adding the increase to the original price, or
- calculating 110% of the original price.

$$\begin{aligned}\text{New price} &= \text{original price} + \text{increase}\\ &= \$50.00 + 5.00 = \$55.00\end{aligned}$$

or

$$\text{New price} = \frac{110}{100} \times 50 = \$55.00$$

Calculating the percentage change

Given the original and new price of an item, we can work out the **percentage change**.

Calculating percentage discount or increase

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original price}} \times \frac{100}{1}\%$$

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original price}} \times \frac{100}{1}\%$$

Example 3 Calculating the percentage discount or increase

a The price was reduced from \$50 to \$45. What percentage discount was applied?

b The price was increased from \$50 to \$55. What percentage increase was applied?

Solution

a **1** Determine the amount of the discount.

$$\text{Discount} = \text{original price} - \text{new price} = 50.00 - 45.00 = \$5.00$$

2 Express this amount as a percentage of the original price.

$$\text{Percentage discount} = \frac{5.00}{50.00} \times \frac{100}{1} = 10\%$$

b **1** Determine the amount of the increase.

$$\text{Increase} = \text{new price} - \text{original price} = 55.00 - 50.00 = \$5.00$$

2 Express this amount as a percentage of the original price.

$$\text{Percentage increase} = \frac{5.00}{50.00} \times \frac{100}{1} = 10\%$$

Calculating the original price

Sometimes we are given the new price and the percentage increase or decrease ($r\%$), and asked to determine the original price. Since we know that:

- for a discount, new price $= \dfrac{(100 - r)}{100} \times$ original price
- for an increase, new price $= \dfrac{(100 + r)}{100} \times$ original price

we can rearrange these formulas to give rules for determining the original price as follows.

Calculating the original price

When $r\%$ discount has been applied: original price = new price $\times \dfrac{100}{(100 - r)}$

When $r\%$ increase has been applied: original price = new price $\times \dfrac{100}{(100 + r)}$

Example 4 Calculating the original price

Suppose that Cate has a \$50 gift voucher from her favourite shop.

a If the store has a ‘10% off’ sale, what is the original value of the goods she can now purchase? Give the answer correct to the nearest cent.

b If the store raises its prices by 10%, what is the original value of the goods she can now purchase? Give the answer correct to the nearest cent.

Solution

a Substitute new price = 50 and $r = 10$ into the formula for an $r\%$ discount.

$$\text{Original price} = 50 \times \frac{100}{90} = \$55.555\ldots$$
$$= \$55.56 \text{ to nearest cent}$$

b Substitute new price = 50 and $r = 10$ into the formula for an $r\%$ increase.

$$\text{Original price} = 50 \times \frac{100}{110} = \$45.454\ldots$$
$$= \$45.45 \text{ to nearest cent}$$

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Goods and services tax (GST)

The **goods and services tax (GST)** is a tax of 10% that is added to the price of most goods (such as cars) and services (such as insurance). We can consider this a special case of the previous rules, where $r = 10$.

Consider the cost of an item after GST is added – this is the same as finding the new price when there has been a 10% increase in the cost of the item. Thus:

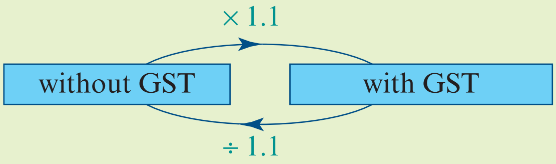
$$\text{cost with GST} = \text{cost without GST} \times \frac{110}{100} = \text{cost without GST} \times 1.1$$

Similarly, finding the cost of an item before GST was added is the same as finding the original cost when a 10% increase has been applied. Thus:

$$\text{cost without GST} = \text{cost with GST} \times \frac{100}{110} = \frac{\text{cost with GST}}{1.1}$$

Finding the cost with and without GST

- Cost with GST = cost without GST × 1.1

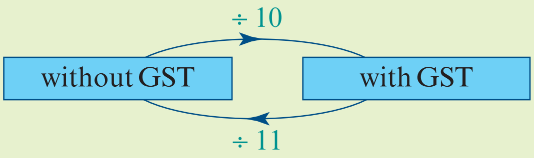


- Cost without GST = $\dfrac{\text{cost with GST}}{1.1}$

We can also directly calculate the actual amount of GST from either the cost without GST or the cost with GST.

Finding the amount of GST

- Amount of GST = $\dfrac{\text{cost without GST}}{10}$



- Amount of GST = $\dfrac{\text{cost with GST}}{11}$

Example 5 Calculating GST

a If the cost of electricity supplied in one quarter is \$288.50, how much GST will be added to the bill?

b If the selling price of a washing machine is \$990:

- **i** what is the price without GST?
- **ii** how much of this is GST?

Solution

a Substitute \$288.50 into the rule for GST from cost without GST.	$\text{GST} = 288.50 \div 10 = \$28.85$
b **i** Substitute \$990 into the rule for cost with GST.	$\text{Cost without GST} = 990 \div 1.1$ $= \$900$
ii We can determine the amount of the GST either by subtraction or by direct substitution into the formula.	$\text{GST} = 990 - 900 = 90$ or $\text{GST} = 990 \div 11 = \$90$

Profit and loss

Profit is income (e.g. from selling an item) minus costs (e.g. from buying or producing the item). Loss is the same kind of calculation, when costs are greater than the income, so the calculation is reversed – loss is cost minus income.

Profit and loss can be expressed in absolute terms (the amount of money) or percentage terms (the amount of money as a percentage of the income).

Example 6 Calculating profit and loss

The cost for you to make a cake is \$1.

a You sell each cake for \$1.20. What is your absolute profit and percentage profit?

b If you can only sell each cake for 80c, what is your absolute loss and percentage loss?

Solution

a Subtract cost from income.	$\$1.20 - \$1.00 = \$0.20c$
Write the answer for absolute profit.	The absolute profit is 20 cents.
Percentage profit is absolute profit divided by income, as a percentage.	$= 0.20/1.00 \times 100/1\% = 20\%$ The percentage profit is 20%
b Subtract income from cost.	$\$1.00 - \$0.80 = \$0.20$
Write the answer for absolute loss.	The absolute loss is 20 cents.
Percentage loss is absolute loss divided by income, as a percentage.	$= \$0.20/\$0.80 \times 100/1\% = 25\%$ The percentage loss is 25%.

Exercise 2A

Review of percentages

1 Calculate the following as percentages. Give answers correct to one decimal place.

a \$200 of \$410
b \$6 of \$24.60
c \$1.50 of \$13.50
d \$24 of \$260
e 30c of 90c
f 50c of \$2

2 Calculate the amount of the following percentage increases and decreases. Give answers to the nearest cent.

a 10% increase on \$26 000
b 5% increase on \$4000
c 12.5% increase on \$1600
d 15% increase on \$12
e 10% decrease on \$18 650
f 2% decrease on \$1 000 000

Discounts, mark-ups and mark-downs

Example 1

3 Calculate the amount of the discount for the following, to the nearest cent.

a 24% discount on \$360
b 72% discount on \$250
c 6% discount on \$9.60
d 9% discount on \$812

Example 2

4 Calculate the new increased price for each of the following.

a \$260 marked up by 12%
b \$580 marked up by 8%
c \$42.50 marked up by 60%
d \$5400 marked up by 17%

5 Calculate the new discounted price for each of the following.

a \$2050 discounted by 9%
b \$11.60 discounted by 4%
c \$154 discounted by 82%
d \$10 600 discounted by 3%
e \$980 discounted by 13.5%
f \$2860 discounted by 8%

Example 3

6 **a** The price of an item was reduced from \$25 to \$19. What percentage discount was applied?

b The price of an item was increased from \$25 to \$30. What percentage increase was applied?

Example 4

7 Find the original prices of the items that have been marked down as follows.

a Marked down by 10%, now priced \$54.00
b Marked down by 25%, now priced \$37.50
c Marked down by 30%, now priced \$50.00
d Marked down by 12.5%, now priced \$77.00

8 Find the original prices of the items that have been marked up as follows.

a Marked up by 20%, now priced \$15.96
b Marked up by 12.5%, now priced \$70.00
c Marked up by 5%, now priced \$109.73
d Marked up by 2.5%, now priced \$5118.75

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9 Mikki has a card that entitles her to a 7.5% discount at the store where she works. How much will she pay for boots marked at $230?

10 The price per litre of petrol was $1.80 on Friday. When Rafik goes to fill up his car on Saturday, he finds that the price has increased by 2.3%. If the tank holds 50 L of petrol, how much will he pay to fill the tank?

GST calculations

Example 5 **11** Find the GST payable on each of the following (give your answer correct to the nearest cent).

a A gas bill of $121.30 **b** A telephone bill of $67.55

c A television set costing $985.50 **d** Gardening services of $395

12 The following prices are without GST. Find the price of each after GST has been added.

a A dress priced at $139 **b** A bedroom suite priced at $2678

c A home video system priced at $9850 **d** Painting services of $1395

13 If a computer is advertised for $2399 including GST, how much would the computer have cost without GST?

14 What is the amount of the GST that has been added if the price of a car is advertised as $39 990 including GST?

15 The telephone bill is $318.97 after GST is added.

a What was the price before GST was added?

b How much GST must be paid?

Profit and loss

Example 6 **16** Andrew bought a rare model train for $450. He later sold the train for $600.

a Calculate the profit Andrew in dollars made on the sale of the train.

b Calculate the profit Andrew made as a percentage of the purchase price of the train, correct to one decimal place.

17 A bookseller bought eight copies of a book for $12.50 each. They were eventually sold for $10.00 each.

a Calculate the loss in dollars and cents that the bookseller made on the sale of the books.

b Determine the loss that the bookseller made as a percentage of the purchase price of the books.

18 The cost of producing a chocolate bar that sells for $1.50 is 60c. Calculate the profit made on a bar of chocolate as a percentage of the production cost of the bar.

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2B Simple interest

When you borrow money, you have to pay for the use of that money. When you invest money, someone else will pay you for the use of your money. The amount you pay when you borrow, or the amount you are paid when you invest, is called **interest**. There are many different ways of calculating interest. The simplest of all is called, rather obviously, **simple interest**. Simple interest is a fixed percentage of the amount invested or borrowed and is *calculated on the original amount*.

Suppose we invest \$1000 in a bank account that pays simple interest at the rate of 5% per annum. This means that, for each year we leave the money in the account, interest of 5% of the original amount will be paid to us. Remember 5% is equal to $\frac{5}{100}$ which is equal to 0.05.

In this instance, the amount of interest paid to us is 5% of \$1000 or $\$1000 \times 0.05 = \$50$

If the money is left in the account for several years, the interest will be paid yearly.

To calculate simple interest we need to know:

- the initial investment, called the **principal**
- the **interest rate**, as a decimal interest rate per annum (such as 0.1) or more/usually as % per annum (p.a.) (such as 10%)
- the length of time the money is invested.

Example 7 Calculating simple interest from first principles

How much interest will be earned if investing \$1000 at 5% p.a. (0.05 p.a. as a decimal) simple interest for 3 years?

Solution

1 Calculate the interest for the first year. Interest = 1000 × 0.05 = \$50

2 Calculate the interest for the second year. Interest = 1000 × 0.05 = \$50

3 Calculate the interest for the third year. Interest = 1000 × 0.05 = \$50

4 Calculate the total interest. Interest for 3 years = 50 + 50 + 50

= \$150

The same rules apply when simple interest is applied to a loan rather than an investment.

▶ The simple interest formula

Since the amount of interest in a simple interest investment is the same each year, we can apply a general rule.

Interest = amount invested or borrowed × interest rate (decimal rate per annum)
× length of time (in years)

This rule gives rise to the following formula.

Simple interest formula

To calculate the simple interest earned or owed:

$$I = Pin$$

where I = the total interest earned or paid, in dollars
P = the principal (the initial amount borrowed or invested), in dollars
i = the decimal interest rate per annum
n = the time in years of the loan or investment.

However interest rates are more often quoted as percentage interest rate per annum, $r\%$, where

$$i = \frac{r}{100}$$

So $I = \dfrac{P \times r \times n}{100}$.

Example 8 Calculating simple interest for periods other than one year

Calculate the amount of simple interest that will be paid on an investment of \$5000 at 10% simple interest per annum for 3 years and 6 months.

Solution

Apply the formula with $P = \$5000$, $i = \dfrac{r}{100} = \dfrac{10}{100} = 0.1\%$ and $n = 3.5$ (since 3 years and 6 months is equal to 3.5 years).

$$\begin{aligned} I &= Pin \\ &= 5000 \times 0.1 \times 3.5 \\ &= \$1750 \end{aligned}$$

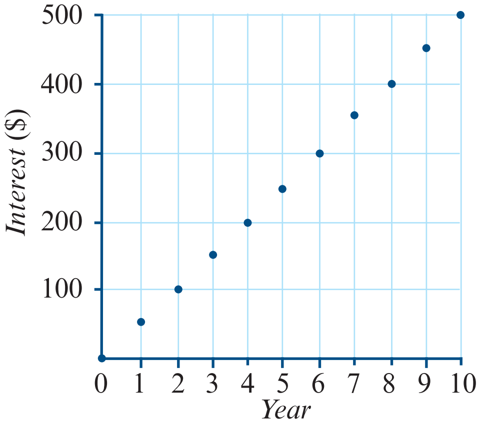
Write the answer.

The simple interest on the investment is \$1750

The graph below shows the total amount of interest earned after 1, 2, 3, 4, … years, when \$1000 is invested at 5% per annum simple interest for a period of years.

As we would expect from the simple interest rule, the graph is linear.

The slope of a line that could be drawn through these points is equal to the amount of interest added each year, in this case \$50.



Technology enables us to investigate the growth in simple interest with time, using tables and graphs.

Desmos widget 2B: Simple interest calculator

Spreadsheet **Spreadsheet activity 2B:** Calculating simple interest with a spreadsheet

Calculating the amount of a simple interest loan or investment

To determine the total value or amount of a simple interest loan or investment, the total interest amount is added to the initial amount borrowed or invested (the principal).

Total value of a simple interest loan

Total amount after t years (A) = principal (P) + total interest (I)

$$\text{or } A = P + I$$

Example 9 Calculating the total amount owed on a simple interest loan

Find the total amount owed on a simple interest loan of \$16 000 at 8% per annum after 2 years.

Solution

1 Apply the formula with $P = \$16\,000$, $i = \dfrac{r}{100} = \dfrac{8}{100} = 0.08$ and $t = 2$ to find the total interest accrued.

$$I = Pin$$
$$= 16\,000 \times 0.08 \times 2 = \$2560$$

2 Find the total amount owed by adding the interest to the principal.

$$A = P + I$$
$$= 16\,000 + 2560 = \$18\,560$$

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Interest paid to bank accounts

One very useful application of simple interest is in the calculation of the interest earned on a bank account. When we keep money in the bank, interest is paid. The amount of interest paid depends on:

- the rate of interest the bank is paying
- the amount on which the interest is calculated.

Generally, banks will pay interest on the **minimum monthly balance**, which is the lowest amount the account contains in each calendar month. When this principle is used, we will assume that all months are of equal length, as illustrated in the next example.

Example 10 Calculating interest paid to a bank account

The table shows the entries in Tom's bank account.

Date	Transaction	Debit	Credit	Total
30 June	Pay		400.00	400.00
3 July	Cash	50.00		350.00
15 July	Cash		100.00	450.00
1 August				450.00

If the bank pays interest at a rate of 3% per annum on the minimum monthly balance, find the interest payable for the month of July correct to the nearest cent.

Solution

1 Determine the minimum monthly balance for July.

The minimum balance in the account for July was $350.00.

2 Determine the interest payable on $350.00.

$$I = Pin$$
$$= 350 \times 0.03 \times \frac{1}{12} = 0.875$$
$$= \$0.88 \text{ or } 88 \text{ cents}$$

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Exercise 2B

Calculating simple interest

Example 8

SF

1 Calculate the amount of interest earned from each of the following simple interest investments. Give answers correct to the nearest cent.

	Principal	Interest rate	Time
a	\$400	5%	4 years
b	\$750	8%	5 years
c	\$1000	7.5%	8 years
d	\$1250	10.25%	3 years
e	\$2400	12.75%	15 years
f	\$865	15%	2.5 years
g	\$599	10%	6 months
h	\$85.50	22.5%	9 months
i	\$15 000	33.3%	1.25 years

Exploring the growth of interest in a simple interest loan or investment

2 A loan of \$900 is taken out at a simple interest rate of 16.5% per annum.

Spreadsheet

2BQ2

a Create the following spreadsheet to illustrate this investment. Enter the formula shown in B4 to multiple the amount borrowed by the interest rate per year, and fill down to B13.

	A	B
1		
2	*Simple interest*	
3	Year	Interest
4	1	=900*16.5/100*A4
5	2	
6	3	
7	4	
8	5	
9	6	
10	7	
11	8	
12	9	
13	10	
14		

b Use the table of values to determine the amount of interest owed after 5 years.

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Simple interest loans and investments

Example 9

SF

3 Calculate the total amount to be repaid for each of the following simple interest loans. Give answers correct to the nearest cent.

	Principal	Interest rate	Time
a	\$500	5%	4 years
b	\$780	6.5%	3 years
c	\$1200	7.25%	6 months
d	\$2250	10.75%	8 months
e	\$2400	12%	18 months

4 A simple interest loan of \$20 000 is taken out for 5 years.

a Calculate the simple interest owed after 5 years if the rate of interest is 12% per annum.

b Calculate the total amount to be repaid after 5 years.

5 A sum of \$10 000 was invested in a fixed term account for 3 years paying a simple interest rate of 6.5% per annum.

a Calculate the total amount of interest earned after 3 years.

b What is the total amount of the investment at the end of 3 years?

6 A loan of \$1200 is taken out at a simple interest rate of 14.5% per annum. How much is owed, in total, after 3 months?

7 A company invests \$1 000 000 in the short-term money market at 11% per annum simple interest. How much interest is earned by this investment in 30 days? Give your answer to the nearest cent.

CF

8 A building society offers the following interest rates for its cash management accounts.

	Interest rate (per annum) on term (months)				
Balance	1–<3	3–<6	6–<12	12–<24	24–<36
\$20 000–\$49 999	2.85%	3.35%	3.85%	4.35%	4.85%
\$50 000–\$99 999	3.00%	3.50%	4.00%	4.50%	5.00%
\$100 000–\$199 999	3.40%	3.90%	4.40%	4.90%	5.40%
\$200 000 and over	4.00%	4.50%	5.00%	5.50%	6.00%

Using this table, find the simple interest earned by each of the following investments. Give your answers to the nearest cent.

a \$25 000 for 2 months

b \$125 000 for 6 months

c \$37 750 for 18 months

d \$200 000 for 2 years

e \$74 386 for 8 months

f \$145 000 for 23 months

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Interest paid into bank accounts

Example 10

9 An account at a bank is paid interest of 4% per annum on the minimum monthly balance, credited to the account at the beginning of the next month.

Date	Transaction	Debit	Credit	Balance
1 October				5000.00
7 October	Cash	1000.00		4000.00
31 October	Cash		500.00	

a What was the balance of the account at the end of October?

b How much interest was paid for the month?

10 The minimum monthly balances for three consecutive months are:

$240.00 $350.50 $478.95

How much interest is earned over the three-month period if it is calculated on the minimum monthly balance at a rate of 3.5% per annum?

11 The bank statement below shows transactions for a savings account that earns simple interest at a rate of 4.5% per annum on the minimum monthly balance.

Date	Transaction	Debit	Credit	Balance
1 March				500.00
15 March	Cash		250.00	750.00
31 March	Cash		250.00	1000.00
1 April				1000.00

How much interest was earned in March?

12 The bank statement below shows transactions over a three-month period for a savings account that earns simple interest at a rate of 3.75% per annum on the minimum monthly balance.

Date	Transaction	Debit	Credit	Balance
1 March				650.72
8 April	Cash		250.00	900.72
21 May	Cash		250.00	1150.72
1 June				1150.72

a What were the minimum monthly balances in March, April and May?

b How much was earned over this three-month period?

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2C Rearranging the simple interest formula

Skillsheet

The simple interest formula can be rearranged to find any one of the four variables as long as the other three are known.

▶ Calculating the interest rate

Decimal interest rate

To find the decimal interest rate i given the values of P, I and n:

$$i = \frac{I}{Pn}$$

where P is the principal, I is the amount of interest accrued in n years.

Percentage interest rate

To find the percentage interest rate, $r\%$, given the values of P, I and n:

$$r = 100i = 100\left(\frac{I}{Pn}\right) = \frac{100I}{Pn}$$

Example 11 Calculating the interest rate

Find the rate of simple interest per annum if:

a a principal of \$8000 increases to \$11 040 in 4 years

b a principal of \$5000 increases to \$5500 in 9 months.

Solution

a **1** Find the amount of interest earned on the investment.

Interest:

$I = 11\,040 - 8000$

$= \$3040$

2 Apply the formula $i = \dfrac{I}{Pn}$ with $P = \$8000$, $I = \$3040$ and $n = 4$, then convert decimal rate to percentage ($r = 100i$).

$i = \dfrac{I}{Pn} = \dfrac{3040}{8000 \times 4}$

$= 0.095$

Interest rate is 9.5% per annum.

b **1** Find the amount of interest earned on the investment.

Interest:

$I = 5500 - 5000$

$= \$500$

2 Apply the same formula with $P = \$5000$, $I = \$500$ and $n = \dfrac{9}{12} = 0.75$ years.

$i = \dfrac{I}{Pn} = \dfrac{500}{5000 \times 0.75}$

$= 0.133$ to three decimal places

Note: Since the interest rate per annum is asked for, you need to convert the time in months to years before substituting into the formula.

3 Convert decimal rate to percentage.

$= 13.3\%$ to one decimal place

Interest rate is 13.3% per annum.

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Calculating the time period

Time period

To find the number of periods or term of an investment given P, I and i:

$$n = \frac{I}{Pi}$$

where P is the principal, I is the amount of interest and i is the decimal interest rate.

To use the percentage interest rate, $r\%$:

$$n = \frac{I}{Pi} = \frac{I}{P\left(\frac{r}{100}\right)} = \frac{100I}{Pr}$$

Example 12 Calculating the time period of a loan or investment

Find the length of time it would take for \$5000 invested at an interest rate of 12% per annum to:

a earn \$1800 interest

b earn \$404 interest.

For part **b** give the answer in days to the nearest day.

Solution

a Convert the percentage interest rate r to decimal interest rate i.

Apply the formula $n = \frac{I}{Pi}$ with $P = \$5000, I = \$1800$ and $i = 0.12$.

$$i = \frac{12}{100}$$

$$n = \frac{I}{Pi} = \frac{1800}{5000 \times 0.12}$$

$$= 3 \text{ years}$$

b Apply the same formula with $P = \$5000, I = \$404$ and $i = 12$.

We can convert years into days by assuming that there are 365 days in a year.

$$n = \frac{I}{Pi} = \frac{404}{5000 \times 0.12}$$

$$= 0.673\ldots \text{ years}$$

$$= 365 \times 0.673$$

$$= 245.766\ldots \text{ days}$$

$$= 246 \text{ days (to the nearest day)}$$

Calculating the principal P

Calculating the principal

- To find the value of the principal, P, given the values of I, i and n use the formula:

  $$P = \frac{I}{in}$$

  where I is the amount of interest accrued, i is the decimal interest rate and n is the number of time periods.

- To find the value of the principal, P, given the values of A, r and n:

$$P = \frac{A}{1 + in} \quad \text{where } i = \frac{r}{100}$$

where A is the amount of the investment or loan, $r\%$ is the annual percentage interest rate and n is the time in years.

- To find the value of the principal using the percentage interest rate:

$$P = \frac{I}{in} \quad \text{where } i = \frac{r}{100}$$

Example 13 Calculating the principal of a loan or investment

a Find the amount that should be invested in order to earn $1500 interest over 3 years at an annual interest rate of 5%.

b Find the amount that should be invested at an annual interest rate of 5% if you require the value of the investment to be $15 600 in 4 years' time.

Solution

a We are given the value of the interest, I, so use the formula $P = \frac{I}{in}$ with $I = \$1500$, $i = 0.05$ and $n = 3$ years.

$$P = \frac{I}{in} = \frac{1500}{0.05 \times 3}$$
$$= \$10\,000$$

b Here we are *not* given the value of the interest, I, but the value of the total investment, A.

$$P = \frac{A}{1 + in}$$

Use the formula $P = \frac{A}{1 + in}$

$$= \frac{15\,600}{1 + (0.05 \times 4)}$$

with $A = \$15\,600$, $i = 0.05$ and $n = 4$.

$$= \frac{15\,600}{1.2}$$
$$= \$13\,000$$

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Exercise 2C

Simple interest: calculating interest rate

Example 11 **1** Find the annual interest rate if a simple interest investment of \$5000 amounts to \$6500 in 2.5 years.

2 Find the annual interest rate if a simple interest investment of \$500 amounts to \$550 in 8 months.

SF

Simple interest: calculating time

Example 12 **3** Calculate the time taken for \$2000 to earn \$975 at 7.5% simple interest.

4 Calculate the time in days for \$760 to earn \$35 at 4.75% simple interest.

Simple interest: calculating principal

Example 13 **5** Calculate the principal that earns \$514.25 in 10 years at 4.25% simple interest.

6 Calculate the principal that earns \$780 in 100 days at 6.25% per annum simple interest.

Simple interest: mixed problems

7 Calculate the answers to complete the following table.

Principal	Rate	Time	Simple interest	Total investment
\$600	6%	5 years	**a**	**b**
\$880	6.5%	**c**	\$171.60	**d**
\$1290	**e**	6 months	\$45.15	**f**
g	10%	4 months	\$150.00	**h**
\$3600	**i**	200 days	\$98.63	**j**
\$980	7.5%	**k**	**l**	\$1200.50
m	7.25%	6 months	\$52.50	**n**

Applications

8 If Geoff invests \$30 000 at 10% per annum simple interest until he has \$42 000, for how many years will he need to invest the money?

CF

9 Josh decides to put \$5000 into an investment account that pays 5.0% per annum simple interest. If he leaves the money there until it doubles, how long will this take?

10 A personal loan of \$15 000 over a 3-year period costs \$500 per month to repay.

- **a** How much money will be repaid in total?
- **b** How much of the money repaid is interest?

2D Compound interest

We have seen that simple interest is calculated *only* on the original amount borrowed or invested. A more common form of interest, known as **compound interest**, calculates the interest on the original amount plus any interest accrued to that time.

▶ Calculating compound interest

Consider, for example, $250 invested at 10% per annum, with the interest added to the account each year.

After 1 year:

Skillsheet

$$\begin{aligned}\text{interest} &= \text{amount invested} \times \text{interest rate} \times \text{time}\\ &= \$250 \times 10\% \times 1\\ &= 250 \times \frac{10}{100} \times 1\\ &= \$25\end{aligned}$$

so that after one year, the amount of money in the account is:

$$\begin{aligned}\text{amount} &= \text{amount at the start of year} + \text{interest earned}\\ &= \$250 + \$25\\ &= \$275\end{aligned}$$

After 2 years:

$$\text{interest} = \$275 \times 10\% \times 1 = \$27.50$$

so that after two years, the amount of money in the account is:

$$\$275 + \$27.50 = \$302.50$$

After 3 years:

$$\text{interest} = \$302.50 \times 10\% \times 1 = \$30.25$$

so that after three years, the amount of money in the account is:

$$\$302.50 + \$30.25 = \$332.75$$

and so on.

If we tabulate this information, we will see that using compound interest, the amount of interest owed or paid increases each year.

After	Amount invested	Interest earned	Total amount of investment
1 year	$250	$25	$(250 + 25) = $275
2 years	$275	$27.50	$(275 + 27.50) = $302.50
3 years and so on	$302.50	$30.25	$(302.50 + 30.25) = $332.75

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Developing the compound interest formula

Calculating compound interest in this way can be very tedious. However, there is a pattern to the calculations that enables us to develop a formula.

Start by recalling that the multiplying factor to increase a quantity by 10% is $\left(1 + \frac{10}{100}\right) = 1.1$

Using this factor we have the value of the investment, A, is:

$$A = \$250 \times 1.1 = \$275 \qquad \text{(after 1 year)}$$

$$A = \$250 \times 1.1 \times 1.1$$

$$= \$250 \times (1.1)^2 = \$302.50 \qquad \text{(after 2 years)}$$

$$A = \$250 \times 1.1 \times 1.1 \times 1.1$$

$$= \$250 \times (1.1)^3 = \$332.75 \qquad \text{(after 3 years)}$$

and so on until, the value of the investment:

$$A = \$250 \times (1.1)^n \qquad \text{(after } n \text{ years)}$$

Thus the value of the investment after 10 years would be: $A = \$250 \times 1.1^{10} = \$648.44$

Following this pattern, we can write down a general formula for calculating the amount of a compound investment after a given amount of time.

The compound interest formula with an annual interest rate

The syllabus glossary formula:

In general, the amount, A, of a compound interest investment is given by:

$$A = P(1 + i)^n$$

- P is the initial amount invested (the principal)
- i is the decimal annual interest rate
- n is the number of years.
  If the percentage annual interest rate $r\%$ is given, convert to the decimal interest rate i first using $i = \frac{r}{100}$

Note: The formula can also be used to determine the amount of debt accrued by a compound interest loan.

To find the *total amount of interest* earned, subtract the initial investment from the final amount.

Determining the interest earned

Interest earned (I) = value of the investment (A) – initial amount invested (P)

$$I = A - P$$

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Example 14 Calculating the amount of the investment and interest

a Determine, to the nearest dollar, the amount of money accumulated after 3 years when \$2000 is invested at an interest rate of 8% per annum, compounded annually.

b Determine to the nearest dollar the total amount of interest earned.

Solution

a The annual percentage interest rate (r%) is given so convert to the decimal interest (i) rate first.

$$i = \frac{r}{100} = \frac{8}{100} = 0.08$$

Substitute $P = \$2000, i = 0.08, n = 3$, into the formula giving the amount of the investment. Enter the information into a scientific calculator and evaluate.

$$A = P \times (1 + i)^n = 2000 \times (1 + 0.08)^3$$

= \$2519 to the nearest dollar

```
2000 * (1 + 0.08)^3
              2519.424
```

b Subtract the principal from this amount to determine the interest earned.

$$I = A - P = 2519 - 2000$$
$$= \$519$$

The formula for compound interest can also be applied when money is borrowed, as shown in the following example.

Example 15 Calculating the amount of the debt and interest owed

a Determine, to the nearest dollar, the amount of money owed after 2 years when \$10 000 is borrowed at an interest rate of 10% per annum, compounded annually.

b Determine the amount of interest owed.

Solution

a The annual percentage interest rate (r%) is given so convert to the decimal interest (i) rate first.

$$i = \frac{r}{100} = \frac{10}{100} = 0.10$$

Substitute $P = \$10\,000, i = 0.10, n = 2$, into the formula giving the amount of the debt.

$$A = P \times (1 + i)^n = 10\,000 \times (1 + 0.1)^2$$
$$= \$12\,100$$

b Subtract the principal from this amount to determine the interest owed.

$$I = A - P$$
$$= 12\,100 - 10\,000$$
$$= \$2100$$

Other ways to determine compound interest, apart from using a scientific calculator as shown in Example 13, is to use the Desmos widget below, or to use a spreadsheet, provided in the interactive textbook.

Desmos widget 2D: Compound interest calculator

Spreadsheet 2D: Calculating compound interest

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▶ Compounding periods other than one year

All of the compound interest questions completed so far in this chapter have involved interest that is calculated and added every year. It is very common for interest to be calculated and added to an account, either loan or investment, more often than this. The compound interest formula is the same for any **compounding time period**, as long as the decimal interest rate is expressed for the same compounding period. The interest rate is still given as an annual one, but this is adjusted to take into account the more frequent interest calculations.

The compound interest formula used with any compounding period

In general, the amount, A, of a compound interest investment is given by

$$A = P(1 + i)^n$$

where:

- P is the initial amount invested or borrowed (the principal)
- i is the interest rate per compounding period written as a decimal
- n is the total number of compounding periods (sometimes called **'rests'**) for the loan or investment.

If the compounding period is less than a year but the given interest rate is an annual percentage interest rate, it can be converted to a percentage interest rate for the compounding period concerned. A particular bank loan might charge interest at an annual rate of 4.8%. However, if the bank calculates the interest owing and adds this to the loan after every month, the rate of interest will have to be a monthly one. Since there are 12 months in a year, the yearly interest rate is converted to a monthly rate by dividing by 12.

Annual percentage interest rate = 4.8% per year

$$\text{Monthly percentage interest rate} = \frac{4.8}{12} = 0.4\% \text{ per month}$$

Converting interest rates

Assume that, in one year, there are:

- 365 days (ignore the possibility of leap years)
- 52 weeks (even though there are slightly more than this)
- 26 fortnights (even though there are slightly more than this)
- 12 months
- 4 quarters

Convert an annual interest rate to another time period interest rate by dividing by these numbers.

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If an annual percentage interest rate $r\%$ is given, convert it to the decimal interest rate i before using the compound interest formula. Then calculate n, the number of compounding time periods, from the number of rests in a year k and the length of the loan, t years. The next box shows how to do this.

Calculating the decimal interest rate when an annual percentage rate is given and the compounding period is less than a year

Find the values of i and n separately using the following rules:

$$i = \frac{r}{100 \times k} \qquad n = k \times t$$

where

- r is the annual interest rate written as a percentage
- k is the number of rests (compounding periods per year)
- t is the time in years of the investment or loan

Example 16 Compound interest with compounding periods other than one year

Courtney invests \$50 000 in an account that earns interest at the rate of 4.5% per annum, compounding monthly (monthly rests). How much is in her account after three and a half years? Round your answer to the nearest cent.

Solution

1 Write the given information, the values of P, r, k and t.

$P = 50\,000$
$r = 4.5\%$
$k = 12$ interest rate compounding monthly
The investment lasts 3.5 years,
$t = 3.5$ (the investment lasts 3.5 years)

2 To solve this problem we will be calculating A (the amount of the investment) using the compound interest formula $A = P(1 + i)^n$ but we need to calculate the values of i and n first.

Calculate i using the rule $i = \frac{r}{100 \times k}$.

$$i = \frac{r}{100 \times k} = i = \frac{4.5}{100 \times 12} = 0.00375$$

3 Calculate n using the rule $n = k \times t$.

$$n = k \times t = 12 \times 3.5 = 42$$

4 Calculate A using $A = P(1 + i)^n$.

$$A = P(1 + i)^n$$
$$A = 50\,000\,(1 + 0.00375)^{42}$$
$$A = 58\,511.799\,17$$

Using a calculator:

$50\,000(1 + 0.00375)^{42}$
58 511.79917

5 Write your answer rounded to the nearest cent.

After three and a half years, the amount in Courtney's investment account is \$58 511.80.

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Exercise 2D

In the following exercises, give all answers correct to the nearest cent.

Compound interest investments with annual rests

Example 14

1 An amount of \$3500 is invested at 5% compound interest per annum for 5 years.

a Determine the final value of this investment.

b Calculate the total amount of interest earned.

SF

2 An amount of \$7000 is invested at 8% compound interest per annum for 4 years.

a Determine the final value of this investment.

b Calculate the total amount of interest earned.

CF

3 Calculate the difference between the simple interest and the compound interest on an investment of \$3000 at 7.9% per annum over 5 years.

Compound interest loans with annual compounding (annual rests)

SF

4 A person borrows \$1250 at 7.5% compound interest per annum for 3 years.

Example 15

a Determine the total amount of money owed after 3 years

b Calculate the amount of interest owed.

5 A person borrows \$1000 at 6.0% compound interest per annum for 5 years.

a Determine the total amount of money owed after 5 years

b Calculate the amount of interest owed.

CF

6 Calculate the difference between the simple interest and the compound interest on a loan of \$2000 at 7% per annum over 5 years.

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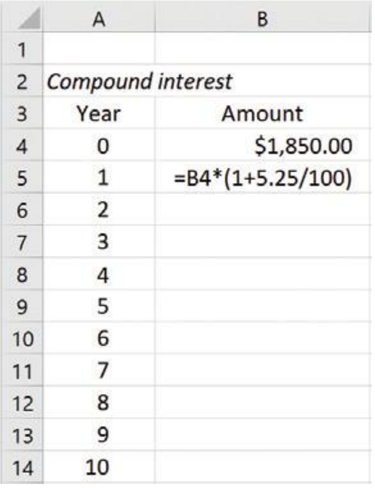
7 Ben decides to invest his savings of \$1850 from his holiday job for five years at 5.25% per annum.

a Create the following spreadsheet to illustrate this investment. Enter the formula shown in B5 which multiplies the amount in B4 (\$1850) by (1 + 5.25/100) to give the new amount with compound interest added at the end of year 1, and fill down to B14.



	A	B
1		
2	*Compound interest*	
3	Year	Amount
4	0	$1,850.00
5	1	=B4*(1+5.25/100)
6	2	
7	3	
8	4	
9	5	
10	6	
11	7	
12	8	
13	9	
14	10	

b Use the table of values to determine the amount of interest earned after 5 years.

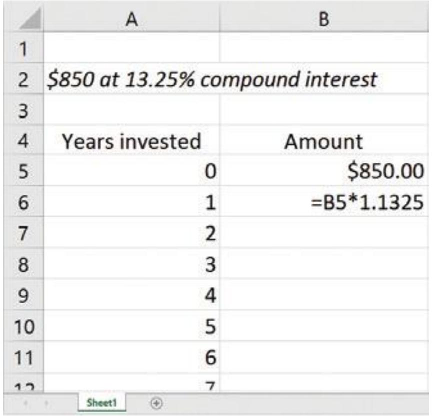
Exploring compound interest loans and investments with a Spreadsheet

8 \$850 is invested at 13.25% per annum compound interest for 8 years.

Spreadsheet

2DQ8

a Create a spreadsheet to illustrate this investment.



	A	B
1		
2	*$850 at 13.25% compound interest*	
3		
4	Years invested	Amount
5	0	$850.00
6	1	=B5*1.1325
7	2	
8	3	
9	4	
10	5	
11	6	

b Enter the formula shown, which multiplies the amount in B5 (\$850) by (1 + 13.25/100) to give the new amount with compound interest added at the end of year 1, and fill down.

c What is the total value of the investment after 5 years, and how much of that is interest?

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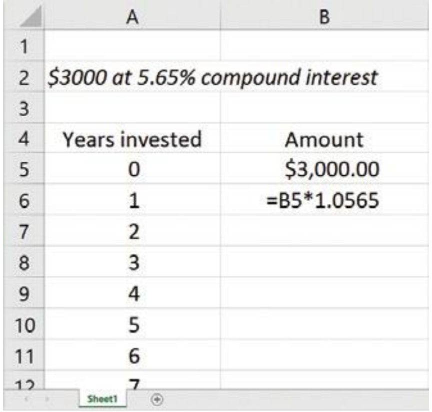
SF

9 Peter invests $3000 at 5.65% per annum compound interest for 10 years.

a Create following spreadsheet to illustrates this investment.



	A	B
1		
2	*$3000 at 5.65% compound interest*	
3		
4	Years invested	Amount
5	0	$3,000.00
6	1	=B5*1.0565
7	2	
8	3	
9	4	
10	5	
11	6	
12	7	

b Enter the formula shown which multiples the amount in B5 ($3000) by (1 + 5.65/100) to give the new amount with compound interest added at the end of year 1, and fill down.

c What is the total value of the investment after 4 years, and how much of that is interest?

Compound interest with compounding periods (rests) other than one year

CF

10 A bank offers a loan with a compound interest rate of 3.6% per annum, with monthly rests.

a What is the monthly interest rate for this loan?

b If $10 000 is borrowed for a period of 6 months, how much must be paid back to the bank?

c If $25 000 is borrowed for a period of 3 years, how much must be paid back to the bank?

Example 16

11 A bank will pay compound interest at the rate of 5.2% per annum, with fortnightly rests.

a What is the fortnightly interest rate for this investment?

b If $5000 is invested for a period of 5 years, what is the final value of the investment?

c How much interest is earned on this investment after 5 years?

12 Millicent invests $18 000 in an account that pays compound interest at the rate of 3.8% per annum.

a Calculate the amount in the account after 1 year if the interest compounds:

i monthly **ii** fortnightly **iii** daily

b How much extra will Millicent earn in interest over the first year if she chooses fortnightly rests instead of monthly rests?

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2E Inflation

▶ Effect of inflation on prices

Inflation is a term that describes the continuous upwards movement in the general level of prices. This has the effect of steadily *reducing* the **purchasing power** of your money; that is, what you can actually buy with your money.

In the early 1970s, inflation rates were very high, up to around 16% and 17%. Inflation in Australia has been relatively low in recent years.

- Since 1970, inflation has averaged 6.8% per year.
- Since 1990, it has averaged 2.1% per year.

Example 17 Determining the effect of inflation on prices over a short period of time

Suppose that inflation is recorded as 2.7% in 2012 and 3.5% in 2013 and that a loaf of bread cost \$2.20 at the end of 2011. If the price of bread increases with inflation, what was the price of the loaf at the end of 2013?

Solution

1 Determine the price of the loaf of bread at the end of 2012 after a 2.7% increase.

$$\text{Increase in price in 2012} = 2.20 \times \frac{2.7}{100} = 0.06$$

2 Calculate the price at the end of 2012.

$$\text{Price (2012)} = 2.20 + 0.06 = \$2.26$$

3 Determine the price of the loaf of bread at the end of 2013 after a further 3.5% increase.

$$\text{Increase in price (2013)} = 2.26 \times \frac{3.5}{100} = 0.08$$

4 Calculate the price at the end of 2013.

$$\text{Price (2013)} = 2.26 + 0.08 = \$2.34$$

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Although the difference in price seen in the previous example does not seem a lot, you will be aware from earlier compound interest examples that even if inflation holds steady at a low 2.1% per year for 20 years, prices will still increase a lot, as the following example shows.

Example 18 Determining the effect of inflation on prices over a long period

Suppose that a one-litre carton of milk costs $1.70 today.

a What will be the price of the one-litre carton of milk in 20 years' time if the average annual inflation rate is 2.1%?

b What will be the price of the one-litre carton of milk in 20 years' time if the average annual inflation rate is 6.8%?

Solution

a **1** This is the equivalent of investing $1.70 at 2.1% interest compounding annually, so we can use the compound interest formula, with *n* as the number of years, since we are dealing with annual inflation.

$A = P \times (1 + i)^n$

2 Convert the annual percentage interest rate to an annual decimal interest rate.

$i = \frac{r}{100} = \frac{2.1}{100} = 0.21$

3 Substitute $P = \$1.70$, $n = 20$ and $r = 2.1$ in the formula to find the price in 20 years.

$$\text{Price} = 1.70 \times (1 + 0.021)^{20}$$
$$= \$2.58 \text{ to the nearest cent}$$

b Substitute $P = 1.70$, $n = 20$ and $r = 6.8$ in the formula and evaluate.

$$\text{Price} = 1.70 \times (1 + 0.068)^{20}$$
$$= \$6.34 \text{ to the nearest cent}$$

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Effect of inflation on the purchasing power of money

Another way of looking at the effect of inflation on our money is to consider what a sum of money today would buy in the future. That is, to convert projected dollar numbers back into present-day values so you can think in today's money values.

Suppose, for example, that you put \$100 in a box under the bed and leave it there for 10 years. When you go back to the box, there is still \$100, but what could you buy with this amount in 10 years' time? To find out we need to 'deflate' this amount back to present-day purchasing power dollars.

We can do this using the compound interest formula.

Suppose there has been an average inflation rate of 4% over the 10-year period. Substituting $A = 100$, $r = 4$ and $t = 10$ gives:

$$100 = P \times \left(1 + \frac{4}{100}\right)^{10} = P \times (1 + 0.04)^{10}$$

Rearranging this equation gives:

$$P = \frac{100}{(1 + 0.04)^{10}} = \frac{100}{1.04^{10}}$$

$$= \$67.57 \text{ to the nearest cent}$$

```
100/(1.04^10)          67.556...
```

That is, the money that was worth \$100 when it was put away has a purchasing power of only \$67.56 after 10 years, if the inflation rate has averaged 4% per annum.

Example 19 Investigating purchasing power

If savings of \$100 000 are hidden under a mattress in 2016, what will be the purchasing power of this amount 8 years' later if the average inflation rate over this period is 3.7%? Give your answer to the nearest dollar.

Solution

1 Convert the annual percentage interest rate to an annual decimal interest rate.	$i = \frac{3.7}{100}$
2 Write the compound interest formula with P (the purchasing power, which is unknown), $A = 100\,000$ (current value), $i = 0.037$ and $t = 8$.	$A = P \times (1 + i)^n$ $100\,000 = P \times (1 + 0.037)^8$
3 Use your calculator to solve this equation for P and write your answer.	The purchasing power of \$100 000 in 8 years is \$74 777, to the nearest dollar.

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Exercise 2E

Effect of inflation on prices

Example 17

1 Suppose that inflation was recorded as 2.7% in 2017 and 3.5% in 2018, and that a magazine cost $3.50 at the end of 2016. Assume that the price increases with inflation.

SF

- **a** What was the price of the magazine at the end of 2017?
- **b** What was the price of the magazine at the end of 2018?

2 Suppose that Henry receives a salary increase at the end of each year equal to the rate of inflation for that year. Inflation was recorded as 3.2% in 2017 and 5.3% in 2018, and Henry's weekly salary was $825 at the end of 2016.

- **a** What was Henry's salary at the end of 2017?
- **b** What was Henry's salary at the end of 2018?

Example 18

3 Suppose that the cost of petrol per litre is $1.80 today.

- **a** What will be the price of petrol per litre in 20 years' time if the average annual inflation rate is 1.9%?
- **b** What will be the price of petrol per litre in 20 years' time if the average annual inflation rate is 7.1%?

4 A house is sold at auction for $500 000. If the price of the house increases with the inflation rate, what will be the price of the house in 12 years' time, if the average inflation rate over the 12-year period is:

- **a** 2.6%?
- **b** 6.9%?

Effect of inflation on purchasing power

Example 19

5 If savings of $200 000 are hidden in a mattress today, what will be the purchasing power of that money in 10 years' time, if the average inflation rate over the 10-year period is:

- **a** 3%?
- **b** 13%?

6 If Jo puts $1000 cash in her safe, what is its purchasing power in 20 years' time, if the average inflation rate over the 20-year period is:

- **a** 2.6%?
- **b** 6.9%?
- **c** 14.3%?

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2F Shares and dividends

Investors often choose to invest their money in shares. A **share** is a unit of ownership in a company. All shares are equal in value, and each share entitles the person who owns it to an equal claim on the company's **profits**.

For example:

- if there are 100 shares in a company and you own 20, then you own 20% of the shares in the company
- if the company makes a profit of \$100 000 in one year, then you are entitled to 20% of that profit (which would be \$20 000).

Example 20 Calculating profit from shares

There are 500 shares in the Kanz Construction Company. Richard owns 25 shares.

a What percentage of the company does Richard own?

b The company declares an annual profit of \$780 000. How much profit is Richard entitled to?

Solution

a We need to convert 25 out of 500 into a percentage.

Percentage ownership

$$= \frac{25}{500} \times \frac{100}{1}$$

$$= 5\%$$

b Richard is entitled to 5% of the profit.

$$\text{Profit} = 780\ 000 \times \frac{5}{100}$$

$$= \$39\ 000$$

Investors are not only interested in the amount of profit they are entitled to, they also want to interpret this profit in light of the amount that they have invested in shares. One measure of this is the **price-to-earnings ratio** of the shares.

Price-to-earnings ratio

$$\text{Price-to-earnings ratio} = \frac{\text{market share price per}}{\text{annual earnings per share}}$$

The lower the price-to-earnings ratio the less you are investing for each dollar of profit, which is better for the investor.

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Example 21 Price-to-earnings ratio

Suppose shares in Company A have a market price of \$20, and a 12-month earnings per share of \$1.85 is declared, while shares in Company B have a market price of \$50, and an annual earnings per share of \$3.30 is declared for the same 12-month period.

a What is the price-to-earnings ratio for each company for that time period? Give answers correct to one decimal place.

b Which shares have been a better investment?

Solution

a Substitute in the formula above.

A: price-to-earnings ratio

$$= \frac{20}{1.85} = 10.8$$

B: price-to-earnings ratio

$$= \frac{50}{3.30}$$

$$= 15.15\ldots = 15.2 \text{ to one d.p.}$$

b Compare the ratios.

Company A: price-to-earnings ratio lower.

In practice, companies do not share all of their profits with shareholders, but they do pay **dividends**. Dividends can be specified in one of two ways:

- as the number of dollars each share receives
- as a percentage of the current price of the shares, called the dividend yield.

Percentage dividend yield

$$\text{Dividend yield} = \frac{\text{dividend}}{\text{share price}} \times \frac{100}{1}\%$$

Example 22 Dividends

Miller has 3000 shares in Alphabet Childcare Centres. The current market price of the shares is \$3.50 each and the company has recently paid a dividend of 40 cents per share.

a How much does Miller receive in dividends in total?

b What is the percentage dividend yield for this share? Answer to one decimal place.

Solution

a Total dividend
= number of shares × dividend per share

$$\text{Total dividend} = 3000 \times 0.40$$

$$= \$1200$$

b Use the percentage dividend rule by substituting \$0.40 for the share dividend and \$3.50 for the share price.

$$\text{Dividend yield} = \frac{0.40}{3.50} \times \frac{100}{1}\%$$

$$= 11.4\% \text{ to one decimal place}$$

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Exercise 2F

Shares and dividends

SF

Example 20

1 Nick owns 500 of the 100 000 shares available in the Lucky Insurance Company.

a What percentage of the company does Nick own?

b If the company declares annual earnings of $2 500 000, how much of these earnings is Nick entitled to?

2 There are 50 000 shares available in the Get Rich Quick investment company, and Joe owns 4% of them.

a How many shares does Joe own?

b If the company declares annual earnings of $1.25 million dollars, how much will Joe get?

c How many more shares would Joe have to buy to get an annual earnings share of $80 000?

Example 21

3 Suppose shares in Company A have a market value of $42.50, and the company has annual earnings of $4.85 per share, while shares in Company B have a market value of $8, and has annual earnings per share of $0.80 for the same 12-month period.

a What is the price-to-earnings ratio for each company for that time period, correct to one decimal place?

b Which shares have been a better investment?

CF

4 After their last financial report the Bank of Brizbane had a price-to-earnings ratio of 20. If the annual earnings declared were $10 000 000, and there are 500 000 shares, what is the current price of the shares?

5 Nitika owns 5000 shares in Company A and 3000 shares in Company B.

a The price-to-earnings ratio for Company A is 5.5 and the annual earnings per share is $2.20.

i How much does Nitika receive in her dividend from Company A?

ii What is the share price for company A?

b The price-to-earnings ratio for Company B is 8 and the annual earnings per share is also $2.20.

i How much does Nitika receive in her dividend from Company B?

ii What is the share price for company B?

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6 Michael has \$5000 to invest in shares. He has decided to invest in either the Alpha Oil Company or Omega Mining.

a The price-to-earnings ratio for the Alpha Oil Company is 10. If the share price is \$5.00, what is the earnings per share?

b The price-to-earnings ratio for Omega Mining is also 10. If the share price is \$10.00, what is the earnings per share?

c Michael decides to spend his money equally between the two share investments. How many shares in each company does he buy?

d Suppose that in the next 12 months the share price of:

- Alpha Oil is expected to increase by 10% while the price-to-earnings ratio is expected to remain at 10
- Omega Mining is expected to increase by 8% while the price-to-earnings ratio is expected to reduce to 8.

What is the expected gain to Michael from these changes?

Example 22

7 Taj has 500 shares in Bunyip Plumbing Supplies. The current market price of the shares is \$4.60 each, and the company declares a dividend of 50 cents per share.

a How much does Taj receive in dividends in total?

b What is the percentage dividend yield for this share, correct to one decimal place?

8 Lacey has 2000 shares in the Yummy Chocolate Factory. The current market price of the shares is \$12.50, and the company recently paid a percentage dividend yield of 10%.

a How much dividend per share was paid?

b How much does Lacey receive in dividends in total?

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2G Focus on problem-solving and modelling

Exercise 2G

Dealer finance or a bank loan?

1 A truck is for sale at a price of $36 000. The dealer is offering finance terms of a 20% cash payment and repayments of $276 per week for 3 years, which includes simple interest and payment for the truck. Assume there are 52 weeks in the year.

a What is the deposit?

b Calculate the total cost of the truck if bought on the dealer's finance terms.

c What is the total interest paid?

d What is the simple interest rate for the loan, correct to one decimal place?

e Assume you have enough cash to pay 20% of the price of the truck. Suppose you borrow the difference from a bank that offers a loan with interest compounded monthly, and allows you to pay off the entire balance and interest in one lump sum after 3 years. What would the maximum annual compound interest rate have to be for the lump-sum repayment to be less than the total repayments to the dealer as calculated in part **b**?

CU

Invest or spend a windfall?

2 Suppose your share of the sale of a family property is $50 000 and you are unsure of how to spend the money.

One of your options is to put the money into a term deposit bank account paying 2.6% per annum, compounding monthly.

a What is the monthly interest rate offered by the bank, correct to three decimal places?

b After 2 years, what would be the value of your bank account?

c How many compounding periods are there in 2 years?

d What are the advantages and disadvantages of a term deposit? You may need to use the internet.

Alternatively, you could use the money to buy a brand new sports car for $50 000. Assume the sports car goes down in value by $10 000 per year, over the first 2 years.

e What is the value of the car after 2 years?

f What is your difference in financial position, after 2 years, between depositing the money into a bank account and buying a sports car? Answer to nearest cent.

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What effect does the duration of the rests have?

3 For this question, use technology such as the Desmos Compound Interest calculator in the interactive textbook, or an online compound Interest Calculator (use a Google search for the latter).
Give all answers to the nearest cent.

- **a** How much will you have if you deposit $30 000 into a bank account with an interest rate of 3.6% per annum and monthly rests for one year?
- **b** What happens if we change the rests to weekly? Work out the number of compounding periods first.
- **c** What happens if we change the rests to yearly? Work out the number of compounding periods first.
- **d** What effect do the rests have on the final amount?
- **e** Explain your answer to part **d**.

Setting up a business

4 Lei is thinking of setting up a food kiosk. To do this, she will need to buy a food kiosk and pay rent for the land on which it is situated. She has already made a list of expenses:

• Food kiosk	$50 000 fixed
• Ingredients and supplies	$200 per day
• Cleaning accessories and costs	$20 per week
• Maintenance	$832 per year
• Gas and electricity	$40 per week
• Rent	$200 per week
• Bank EFTPOS fees	1% of sales
• Other expenses e.g. transport	$40 per week

For this exercise, you may assume that there are no taxes involved.
Lei will also need to ensure that her business income exceeds her costs by a reasonable amount, so she has made a price list and estimated the sales:

Menu item	Price	Number of sales per week
Hot dog	$9	50
Burger meal	$14	180
Kebab	$11	145
Vegetarian option	$10	25
Extra sauces	$1	180
Extra cheese	$3	200
Extra meat	$6	20

Assume that Lei's business operates 7 days a week.

a Calculate Lei's total sales per week.

b Calculate the weekly EFTPOS fees.

c Convert all of the expenses into weekly rates, except for the kiosk.

d What is the sum of all expenses per week, apart from the kiosk?

Lei does not have enough money to buy the kiosk outright. She currently has $24 000 saved and plans to finance the remaining cost by taking out a loan of $26 000 from the bank with an interest rate of 4% per annum, compounding monthly. She decides to make weekly payments of $500 to pay off the loan.

e Calculate the amount she owes to the bank after the first week.

f Create a spreadsheet to calculate the amount owed to the bank after 10 payments.

g What is the total of the weekly expenses, including the weekly repayments for the food van?

h Is Lei's business profitable? How much does she take home (or lose) at the end of each week?

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Key ideas and chapter summary

Percentage increase or decrease	**Percentage increase or decrease** is the amount of the increase or decrease of value or quantity expressed as a percentage of the original value or quantity.
Profit and loss	Income (e.g. sales revenue) minus cost (e.g. cost or purchase or production) in the case of profit, or cost minus revenue in the case of loss.
Simple interest	**Simple interest** is paid on an investment or loan on the basis of the original amount invested or borrowed called the principal (P). The amount of simple interest is constant from year to year.
GST	**GST** (goods and services tax) is a 10% tax that is added to most purchases.
Minimum monthly balance	The lowest amount an account contains in each calendar month is its **minimum monthly balance**.
Compound interest	Under **compound interest**, the interest paid on a loan or investment is credited or debited to the account at the end of each period. The interest earned for the next period is based on the principal plus previous interest earned. The amount of interest earned increases each year.
Inflation	**Inflation** is the continuous upwards movement of the economy that increases prices over time or, conversely, decreases the spending power of money over time.
Shares	A **share** is a unit of ownership of a company.
Price-to-earnings ratio	The **price-to-earnings ratio** is a measure of the profit of a company, given by the market share price/earnings per share. A lower value of the price-to-earnings ratio may indicate a better investment.
Dividend yield	A **dividend** is paid by a company and is expressed as a percentage of the share price.

Skills check

Having completed this chapter you should be able to:

- calculate the amount of the discount and the new price when an $r\%$ discount is applied
- calculate the amount of the increase and the new price when an $r\%$ increase is applied
- calculate the percentage discount or increase given the old and new prices

- calculate the original price given the new price and the percentage discount or increase
- determine the new price of a good or service after the GST has been added
- determine the original cost of a good or service given the price inclusive of GST
- use the formula for simple interest to find the value of any one of the variables I, P, r or t when the values of the other three are known
- determine the interest payable on a bank account, paid on the minimum monthly balance
- calculate the amount of an investment after simple interest has been added
- use the formula for compound interest to solve problems involving investments and loans
- determine the new price of an item after a period of inflation
- determine the effect of inflation on the purchasing power of money
- determine the price-to-earnings ratio of shares
- determine the dividend yield of shares.

Multiple-choice questions

SF

1 The amount saved when a 10% discount is offered on an item marked \$120 is:

A \$20 **B** \$12 **C** \$1.20 **D** \$10.90 **E** \$10

2 If a 20% discount is offered on an item marked \$30, the new discounted price of the item is:

A \$10 **B** \$24 **C** \$6 **D** \$25 **E** \$28

3 If a 15% increase is applied to an item marked \$60, the new price of the item is:

A \$69 **B** \$9 **C** \$75 **D** \$67 **E** \$70

4 If GST is applied to an item marked \$121.50, the new price of the item is:

A \$131.50 **B** \$111.50 **C** \$12.15 **D** \$133.65 **E** \$110.45

5 The telephone bill including GST is \$318.97. The price before GST was:

A \$289.97 **B** \$31.90 **C** \$350.87 **D** \$29.00 **E** \$328.97

6 Shares in Company A have a market value of \$22.50. If the company makes a 12-month earnings of \$2.85 per share, the price-to-earnings ratio for that time period is:

A 12.67 **B** 0.13 **C** 7.89 **D** \$25.35 **E** \$19.65

7 How much interest is earned if \$2000 is invested for 1 year at a simple interest rate of 4% per annum?

A \$2080 **B** \$160 **C** \$8 **D** \$800 **E** \$80

8 The total value of an investment of \$1000 after 3 years if simple interest is paid at the rate of 5.5% per annum is:

A \$55 **B** \$1055 **C** \$1165 **D** \$1174.24 **E** \$3165

9 What is the interest rate, per annum, for a deposit of $1500 that earns interest of $50 over a period of 6 months?

A 0.56% **B** 5.45% **C** 5.55% **D** 6.45% **E** 6.67%

10 $2400 is invested at a rate of 4.25% compound interest, paid annually. The value of this investment after 6 years is closest to:

A $3080 **B** $3074 **C** $3012 **D** $680 **E** $674

11 $3600 is invested at a rate of 5% p.a. compounding annually. The value of the investment after 4 years is given by:

A $3600(1 + (5/100)^4)$ **B** $3600(1 + 5/100)^4$

C $3600(1 + 4 \times 0.05)$ **D** $3600 + 4 \times 3600 \times 0.05$

E $3600(5/100)^4$

12 The value of $8500 compounding annually for 5 years at 6% p.a. is closest to:

A $2550 **B** $10 731 **C** $11 050 **D** $11 375 **E** $11 700

13 The price of a newspaper is $2 today. If the price increases with inflation, what will be the price of the newspaper in 10 years' time if the average annual inflation rate is 1.8%?

A $2.39 **B** $2.10 **C** $2.18 **D** $20 **E** $5.57

The following information relates to Questions 14 and 15.

Janet buys a car costing $23 000. She pays a $5000 deposit and then makes payments of $440 per month for the next 5 years.

14 How many payments does Janet make under this arrangement?

A 5 **B** 12 **C** 20 **D** 40 **E** 60

15 How much interest does Janet pay under this arrangement?

A $3400 **B** $8400 **C** $3120 **D** $1600 **E** $1250

Short-answer questions

1 Rabbit Easter eggs were reduced from $2.99 to $2.37 because they were not selling quickly. Bilby Easter eggs were discounted from $4.79 to $3.83.

a Which type of Easter egg had the larger percentage reduction?

b Calculate the difference in the percentage rates.

2 After Christmas, all stock in JDs was discounted by 20%. The sale price of a pair of cross trainers was $110.

a Calculate the original marked price.

b If the cost of production of the cross trainers is $120, calculate the percentage profit or loss before and after they were discounted, correct to one decimal place.

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3 Farad takes out a loan of \$25 000, for which he has to make repayments of \$600 per month for 5 years. SF

a How much will he repay in total?

b How much of the money repaid is interest?

4 A bank offers a loan with a compound interest rate of 3.5% per annum, with monthly rests.

a What is the monthly interest rate for this loan?

b What is the total interest on a loan of \$50 000 after 3 years?

5 How much additional interest is earned if \$8000 is invested for 7 years at 6.5% compounded annually instead of simple interest paid at the same rate? CF

6 Gavin finds \$10 000 his grandfather had hidden in the attic 20 years ago. If the average inflation rate over that time period was 2%, what was the purchasing power of that money at the time that it was hidden? SF

7 William has 1000 shares in Capital Investments. The current price of the shares is \$22.80, and the company declares a dividend of \$1.10 per share.

a How much does William receive in dividends in total?

b What is the percentage dividend yield for this share, correct to one decimal place?

Extended-response questions

1 **a** The wholesale price of a digital camera is \$350. The maximum profit that a retailer is allowed to make when selling this particular camera is 75% of the wholesale price. Calculate the maximum retail price of the camera. SF

b The wholesale price of the camera increases at 1.3% per annum due to inflation for the next 5 years.

  i What is the new wholesale price of the camera?

  ii By how much will the wholesale price have increased at the end of 5 years?

  iii What is the new retail price of the camera (with 75% profit)? CF

  iv What percentage increase is this of the retail price determined in part **a**?

2 Suppose that you have \$3000 to invest and there are two alternative plans for investment:

Plan A offers 5.3% per annum simple interest.

Plan B offers 5.0% per annum compound interest, compounding annually.

a Create a spreadsheet to show the amount of interest earned each year for 10 years under Plan A.

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CF

b Create a spreadsheet to show the amount of interest earned each year for 10 years under Plan B.

c Which of the plans would you choose if the investment is for:

- **i** 3 years?
- **ii** 6 years?

SF

3 Alice invests $50 000 at 5% interest with monthly rests.

a Create a spreadsheet to show the amount in the account at the end of each month for 12 months.

b How much is in the account at the end of the 12 months?

Alice decides to take $100 out of the account at the end of each month after the interest has been paid.

CF

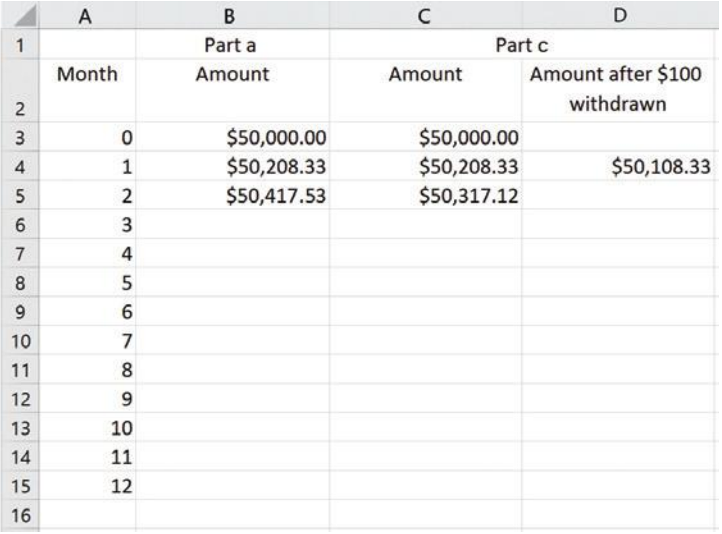
c Create the following spreadsheet which shows this.

In C4 enter the formula which gives the amount with interest at the end of month 1.	=C3*(1+5/1200)
In D5 enter the formula which gives the amount after $100 has been withdrawn.	=C4-100
In C5 enter the formula which gives the amount at the end of month 2, before the $100 is withdrawn.	=D4*(1+5/1200)

Fill down to C15 and D 15.

Spreadsheet

2ERQ3



	A	B	C	D
1		Part a	Part c	
2	Month	Amount	Amount	Amount after $100 withdrawn
3	0	$50,000.00	$50,000.00	
4	1	$50,208.33	$50,208.33	$50,108.33
5	2	$50,417.53	$50,317.12	
6	3			
7	4			
8	5			
9	6			
10	7			
11	8			
12	9			
13	10			
14	11			
15	12			
16				

d How much is now in the account at the end of the 12th month?

e What has been the net effect of Alice withdrawing $100 per month?

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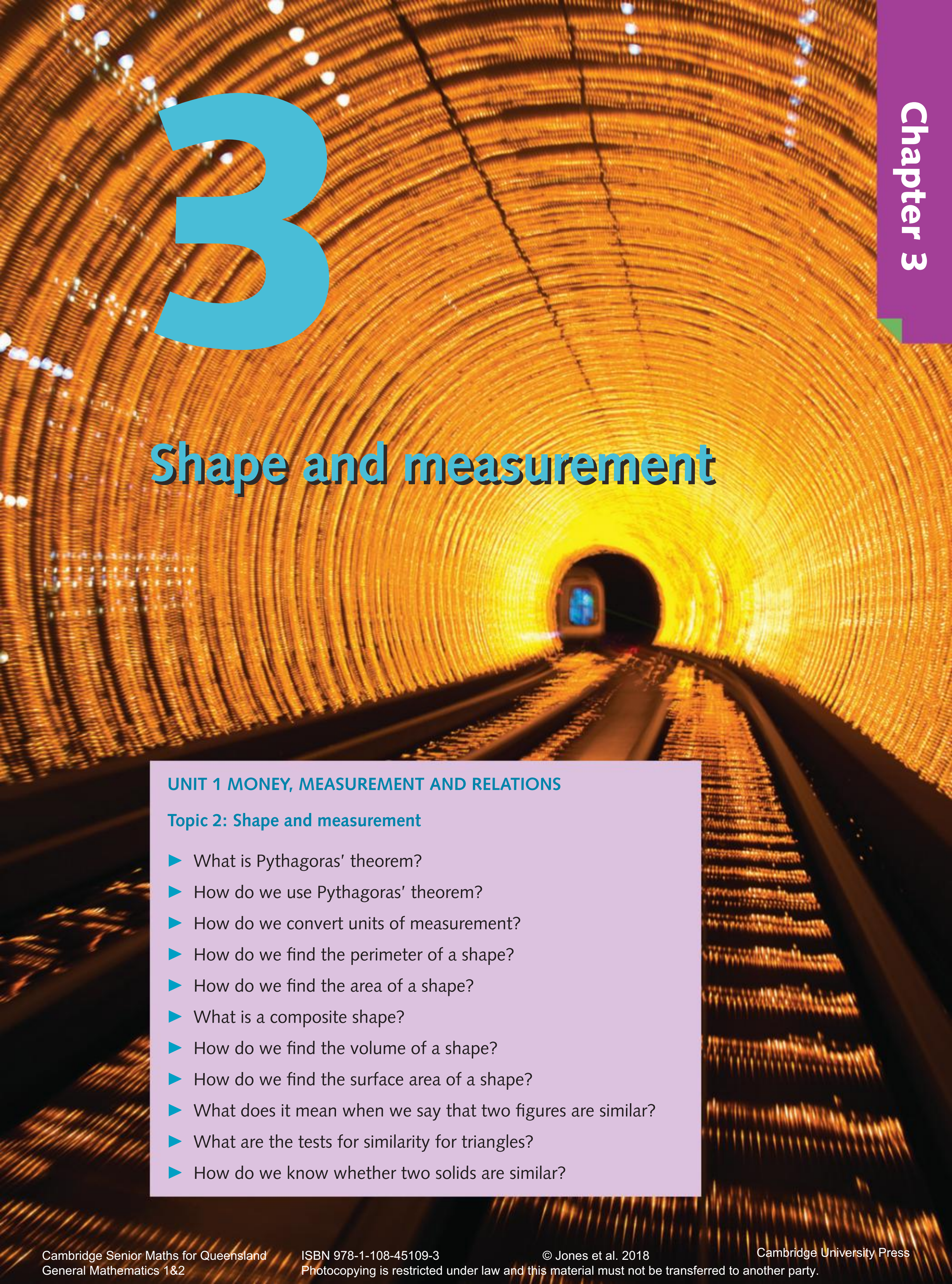


3

Shape and measurement

UNIT 1 MONEY, MEASUREMENT AND RELATIONS

Topic 2: Shape and measurement

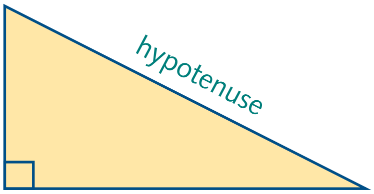
- What is Pythagoras' theorem?
- How do we use Pythagoras' theorem?
- How do we convert units of measurement?
- How do we find the perimeter of a shape?
- How do we find the area of a shape?
- What is a composite shape?
- How do we find the volume of a shape?
- How do we find the surface area of a shape?
- What does it mean when we say that two figures are similar?
- What are the tests for similarity for triangles?
- How do we know whether two solids are similar?

Introduction

This geometry chapter covers perimeter and area of two-dimensional shapes, and surface area and volume of three-dimensional solids. It also covers similarity within two-dimensional shapes and three-dimensional solids.

3A Pythagoras' theorem

Pythagoras' theorem is a relationship connecting the side lengths of a right-angled triangle. In a right-angled triangle, the side opposite the **right angle** is called the **hypotenuse**. The hypotenuse is always the longest side of a right-angled triangle.

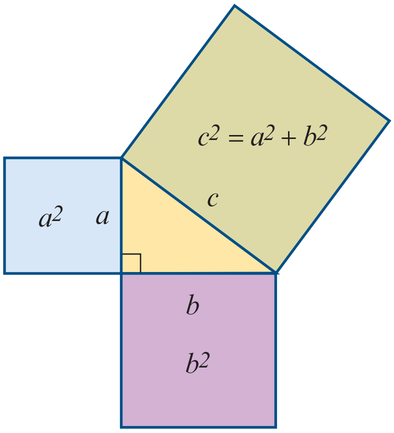


▶ Pythagoras' theorem

Pythagoras' theorem states that, for any right-angled triangle, the sum of the areas of the squares of the two shorter sides (*a* and *b*) equals the area of the square of the hypotenuse (*c*).

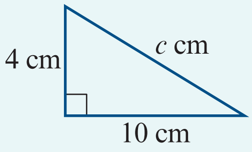
$$c^2 = a^2 + b^2$$

Pythagoras' theorem can be used to find the length of the hypotenuse in a right-angled triangle.



Example 1 **Using Pythagoras' theorem to calculate the length of the hypotenuse**

Calculate the length of the hypotenuse in the triangle opposite, correct to two decimal places.



Solution

1 Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
2 Substitute known values.	$c^2 = 4^2 + 10^2$
3 Take the square root of both sides, then evaluate.	$c = \sqrt{4^2 + 10^2}$ $= 10.770\ldots$
4 Write your answer correct to two decimal places, with correct units.	The length of the hypotenuse is 10.77 cm, correct to two decimal places.

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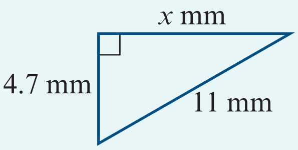
Pythagoras' theorem can also be rearranged to find sides other than the hypotenuse.

Example 2 Using Pythagoras' theorem to calculate the length of an unknown side in a right-angled triangle

Calculate the length of the unknown side, x, in the triangle opposite. Give the answer correct to one decimal place.



Solution

1. Write Pythagoras' theorem. $a^2 + b^2 = c^2$
2. Substitute known values and the given variable. $x^2 + 4.7^2 = 11^2$
3. Rearrange the formula to make x the subject, then evaluate. $x = \sqrt{11^2 - 4.7^2}$ $= 9.945\ldots$
4. Write your answer correct to one decimal place, with correct units. The length of x is 9.9 mm, correct to one decimal place.

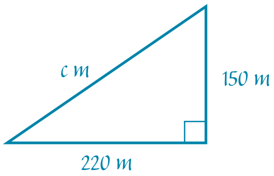
Pythagoras' theorem can be used to solve many practical problems.

Example 3 Using Pythagoras' theorem to solve a practical problem

A helicopter hovers at a height of 150 m above the ground and is a horizontal distance of 220 m from a landing pad. Find the direct distance of the helicopter from the landing pad, correct to two decimal places.

Solution

1. Draw a diagram to show which distance is to be found.



2. Write Pythagoras' theorem. $c^2 = a^2 + b^2$
3. Substitute known values. $c^2 = 150^2 + 220^2$
4. Take the square root of both sides, then evaluate. $c = \sqrt{150^2 + 220^2}$ $= 266.270\ldots$
5. Write your answer correct to two decimal places, with correct units. The helicopter is 266.27 m from the landing pad, correct to two decimal places.

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Exercise 3A

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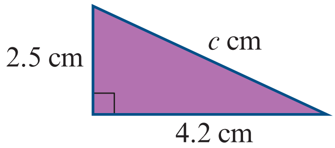
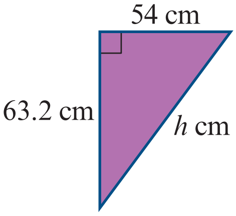
1 Find the length of the unknown side in each of these triangles, giving the answer correct to one decimal place.

Example 1, 2

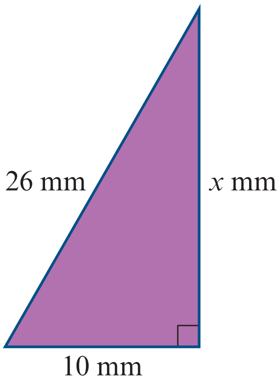
a



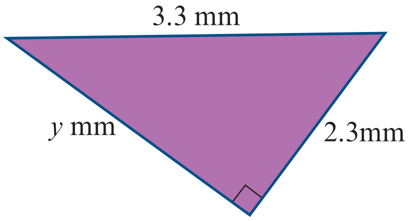
b



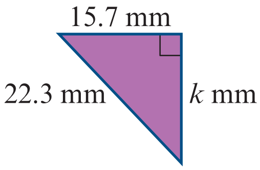
c



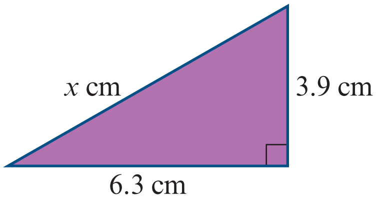
d



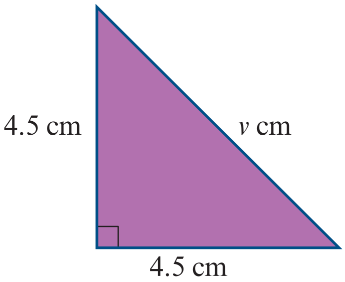
e



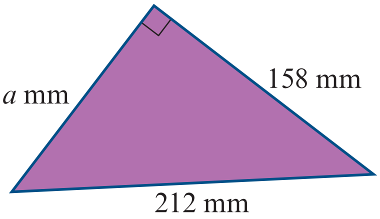
f



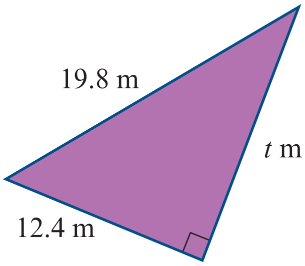
g



h



i



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Applications of Pythagoras' theorem

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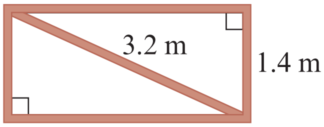
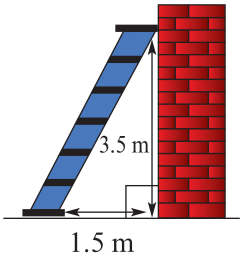
Example 3

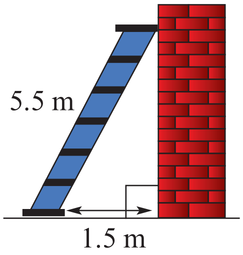
2 A farm gate that is 1.4 m high is supported by a diagonal bar of length 3.2 m. Find the width of the gate, correct to one decimal place.



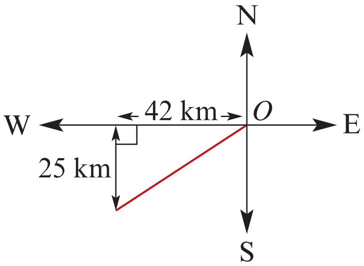
3 A ladder rests against a brick wall as shown in the diagram on the right. The base of the ladder is 1.5 m from the wall, and the top reaches 3.5 m up the wall. Find the length of the ladder, correct to one decimal place.

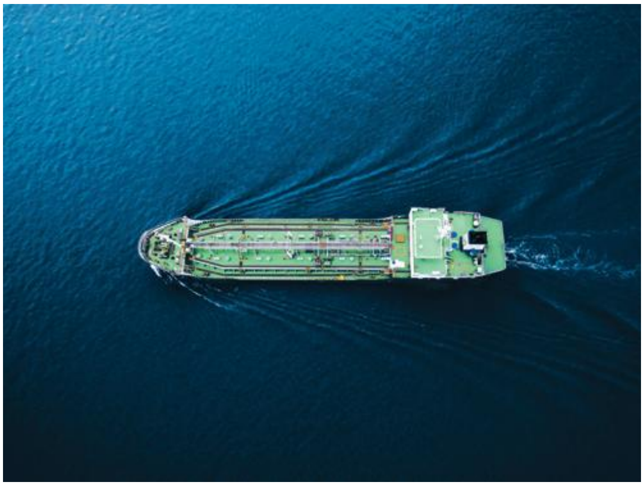


4 The base of a ladder leaning against a wall is 1.5 m from the base of the wall. If the ladder is 5.5 m long, find how high the top of the ladder is from the ground, correct to one decimal place.



5 A ship sails 42 km due west and then 25 km due south. How far is the ship from its starting point? Give the answer correct to two decimal places.





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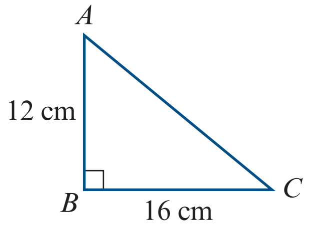
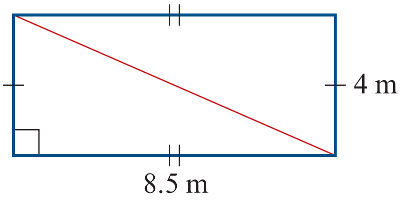
6 A yacht sails 12 km due east and then 9 km due north. How far is it from its starting point?

7 A hiker walks 10 km due west and then 8 km due north. How far is she from her starting point? Give the answer correct to two decimal places.

8 In triangle *ABC*, there is a right angle at *B*. *AB* is 12 cm and *BC* is 16 cm. Find the length of *AC*.

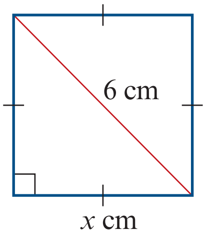


9 Find the length of the diagonal of a rectangle with dimensions 8.5 m by 4 m. Give the answer correct to one decimal place.



10 A rectangular block of land measures 28 m by 55 m. John wants to put a fence along the diagonal. How long will the fence be? Give the answer correct to three decimal places.

11 A square has diagonals of length 6 cm. Find the length of its sides, correct to two decimal places.



12 A flying fox on a school camp starts from a tower 25 m high and lands on the ground 100 m away. What is the distance from the top of the tower to the ground? Give the answer to the nearest metre.

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3B Pythagoras' theorem in three dimensions

When solving three-dimensional problems, it is essential to draw careful diagrams. In general, to find lengths in solid figures, we must first identify the correct right-angled triangle in the plane containing the unknown side. Remember, a plane is a flat surface, such as the cover of a book or a tabletop.

Once it has been identified, the right-angled triangle should be drawn separately from the solid figure, displaying as much information as possible.

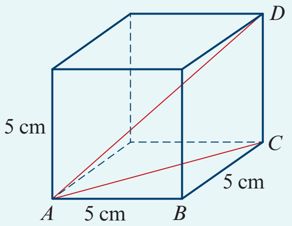
Example 4 Using Pythagoras' theorem in three dimensions

The cube in the diagram has sides of length 5 cm.

Find these lengths correct to two decimal places:

a AC

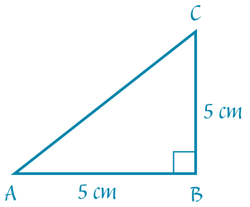
b AD



Solution

a **1** Locate the relevant right-angled triangle in the diagram.

2 Draw the right-angled triangle ABC that contains AC, and then mark in the known side lengths.



3 Using Pythagoras' theorem, calculate the length AC.

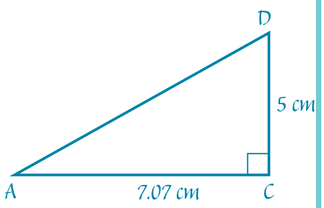
$$AC^2 = AB^2 + BC^2$$
$$\therefore AC = \sqrt{5^2 + 5^2}$$
$$= 7.071\ldots$$

4 Write your answer with correct units and correct to two decimal places.

The length AC is 7.07 cm, correct to two decimal places.

b **1** Locate the relevant right-angled triangle in the diagram.

2 Draw the right-angled triangle ACD that contains AD and mark in the known side lengths. (From part **a**, $AC = 7.07$ cm, correct to two decimal places.)



3 Using Pythagoras' theorem, calculate the length AD.

$$AD^2 = AC^2 + CD^2$$
$$\therefore AD = \sqrt{7.07^2 + 5^2}$$
$$= 8.659\ldots$$

4 Write your answer with correct units and correct to two decimal places.

The length AD is 8.66 cm correct to two decimal places.

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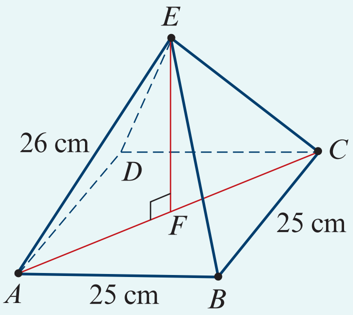
Example 5 Using Pythagoras' theorem in three-dimensional problems

For the square pyramid shown in the diagram, calculate:

a the length AC, correct to two decimal places

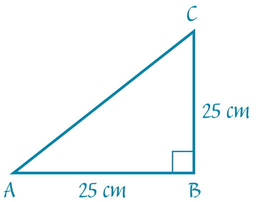
b the height EF, correct to one decimal place.



Solution

a **1** Locate the relevant right-angled triangle in the diagram.

2 Draw the right-angled triangle ABC that contains AC, and mark in known side lengths.



3 Using Pythagoras' theorem, calculate the length AC.

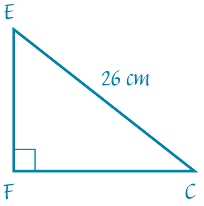
$$AC^2 = AB^2 + BC^2$$
$$\therefore AC = \sqrt{25^2 + 25^2}$$
$$= 35.355\ldots$$

4 Write your answer with correct units and correct to two decimal places.

The length AC is 35.36 cm, correct to two decimal places.

b **1** Locate the relevant right-angled triangle in the diagram.

2 Draw the right-angled triangle EFC that contains EF, and mark in known side lengths.



3 Find FC, which is half of AC. Use the unrounded value of AC calculated in part **a**.

$$FC = \frac{AC}{2}$$
$$= \frac{35.355}{2}$$
$$= 17.677\ldots$$

4 Using Pythagoras' theorem, find EF.

$$EF^2 = EC^2 - FC^2$$
$$\therefore EF = \sqrt{26^2 - 17.677^2}$$
$$= 19.066\ldots$$

5 Write your answer with correct units and correct to one decimal place.

The height EF is 19.1 cm, correct to one decimal place.

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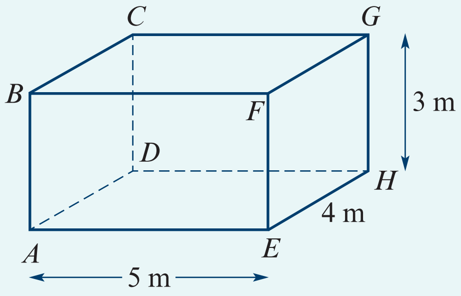
Example 6 Using Pythagoras' theorem in practical three-dimensional problems

A new home entertainment system needs to be set up in a room with dimensions 4 m × 5 m × 3 m as shown in the diagram. Expensive cabling is used to wire the room from corner *A* to corner *G*.



a What length of cabling is required to go from *A* to *E* to *H* to *G*?

b What length of cabling is required to go from *A* to *F* to *G*, correct to two decimal places?

c If cabling costs \$9.10 per metre, calculate the cost of cabling for each option and determine which is the cheaper option – *A* to *F* to *G* or *A* to *E* to *G*?

Solution

a **1** Add the distances from *A* to *E* (5 m), *E* to *H* (4 m) and *H* to *G* (3 m).

$5 + 4 + 3 = 12$

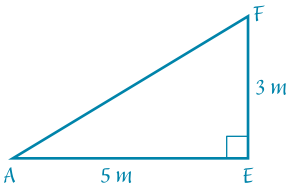
2 Write your answer with correct units.

The distance is 12 metres.

b **1** First find out the distance from *A* to *F*, by locating the relevant right-angled triangle in the diagram.

The triangle is AFE.

2 Draw the right-angled triangle *AFE* that contains *AF*, and then mark in the known side lengths.



3 Using Pythagoras' theorem, calculate the length *AF*.

$$AF^2 = AE^2 + EF^2$$

$$AF^2 = \sqrt{5^2 + 3^2}$$

$$AF = 5.8309\ldots$$

4 Add the lengths *AF* and *FG* and give your final answer with correct units and correct to two decimal places.

The total length (A–F–G)

$= 5.8309\ldots + 4$

$= 9.83$ m, correct to two decimal places

5 Write your answer with correct units.

The distance is 9.83 metres.

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c **1** Work out the cost of cabling from *A* to *F* to *G* by multiplying the length of cabling needed from *A* to *F* to *G* (9.8309 … m) by the cost of cabling per metre ($9.10).

$9.8309\ldots \times 9.10 = 89.461$

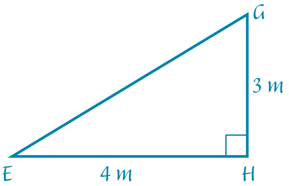
Cost of cabling from A to F to G is $89.46.

2 Work out the distance of cable needed to go from *A* to *E* to *G*. The distance from *A* to *E* is 5 m. Calculate the distance from *E* to *G* by first locating the relevant right-angled triangle in the diagram.

$AE = 5$ m

The triangle is EGH.

3 Draw the right-angled triangle *EGH* that contains *EG* and mark in known side lengths.



4 Using Pythagoras' theorem, calculate the length *EG*.

$$EG^2 = EH^2 + HG^2$$
$$EG = \sqrt{4^2 + 3^2}$$
$$EG = 5$$

5 Add the lengths *AE* and *EG* to give total distance from *A* to *E* to *G*.

$$\begin{aligned} A\text{–}E\text{–}G &= 5 + 5 \\ &= 10 \text{ m} \end{aligned}$$

6 Work out the cost of cabling from *A* to *E* to *G* by multiplying the length of cabling needed (10 m) by the cost of cabling per metre ($9.10).

$10 \times 9.10 = 91$

Cost of cabling from A to E to G is $91.00.

7 Compare the cost of cabling from *A* to *F* to *G* to the cost of cabling from *A* to *E* to *G* and decide which is the cheaper option.

Cost for A–F–G = $89.46
Cost for A–E–G = $91.00
Thus the cheaper option is to wire cabling from A to F to G.

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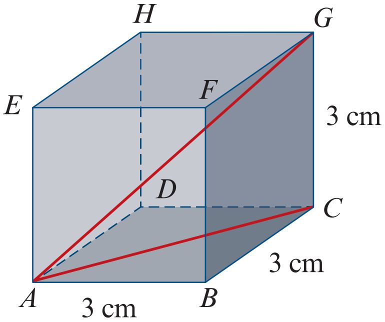
Exercise 3B

Pythagoras' theorem in three dimensions

Example 4

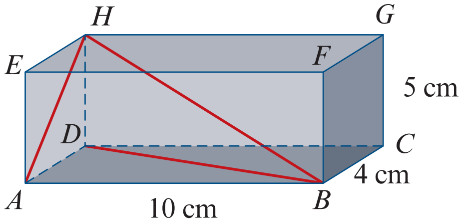
1 The cube shown in the diagram has sides of 3 cm. Find these lengths correct to three decimal places:

- **a** *AC*
- **b** *AG*



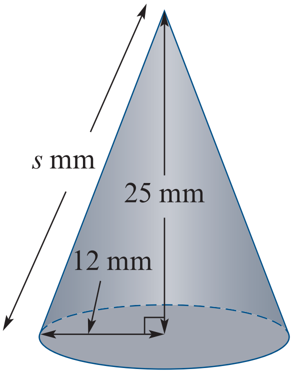
2 For this cuboid, calculate these lengths correct to two decimal places:

a *DB* **b** *BH* **c** *AH*

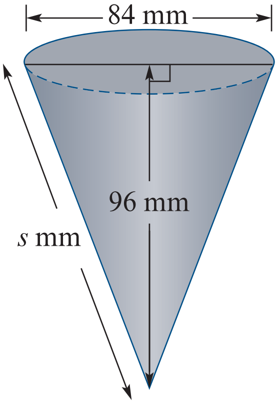


3 Find the sloping height, s, of each of the following cones, correct to two decimal places.

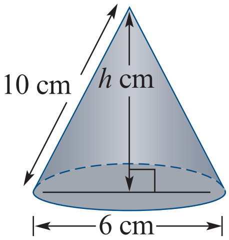
a



b



4 The slant height of this circular cone is 10 cm and the diameter of its base is 6 cm. Calculate the height of the cone, correct to two decimal places.



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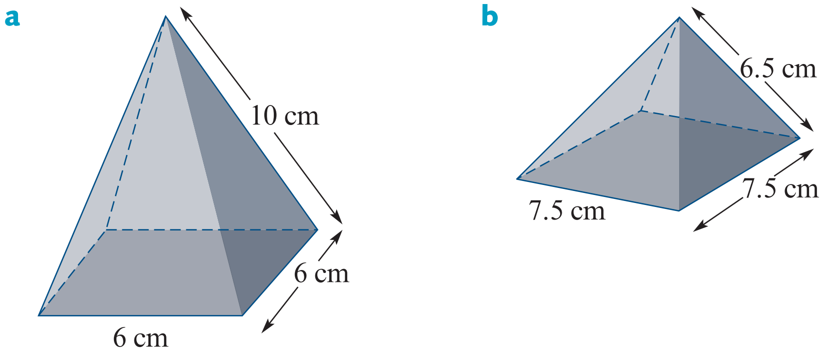
Example 5 **5** For each of the following square-based pyramids find, correct to one decimal place:

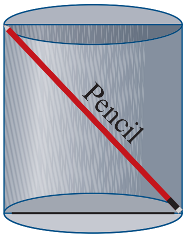
i the length of the diagonal on the base

ii the height of the pyramid.



Applications of Pythagoras' theorem in three dimensions

Example 6 **6** Find the length of the longest pencil that will fit inside a cylinder of height 15 cm and with circular base 8 cm in diameter.



7 Sarah wants to put her pencils in a cylindrical pencil case. What is the length of the longest pencil that would fit inside a cylinder of height 12 cm with a base diameter of 5 cm?

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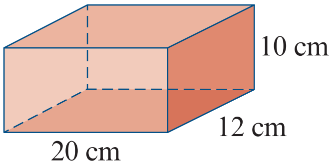
8 Chris wants to use a rectangular pencil box. What is the length of the longest pencil that would fit inside the box shown on the right? (Answer to the nearest centimetre.)



9 A broom is 145 cm long. Would it be able to fit in a cupboard measuring 45 cm by 50 cm and height 140 cm?

10 In order to check the accuracy of the framework and that a room is 'square', a builder often measures the length of the opposing diagonals. What is the distance, correct to two decimal places, from the bottom corner to the top corner diagonally opposite of a room that measures 6 m by 4 m by 3.5 m?

11 In the primate enclosure at the zoo, a rope is to be attached from the bottom corner of the enclosure to the opposite top corner for the monkeys to swing and climb on. If the enclosure measures 8 m by 10 m by 12 m, what is the length of the rope? Give your answer correct to two decimal places.

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3C Mensuration: perimeter and area

Mensuration is a part of mathematics that looks at the measurement of length, area and volume. It comes from the Latin word *mensura*, which means ‘measure’.

▶ Conversion of units

The modern metric system in Australia is defined by the International System of Units (SI), which is a system of measuring that has three main units.

The three main SI units of measurement

m	the *metre* for length
kg	the *kilogram* for mass
s	the *second* for time

Larger and smaller units are based on these by the addition of a prefix. When solving problems, we need to ensure that the units we use are the same. We may also need to **convert** our answer into specified units.

Conversion of units

To convert units remember to:

- use multiplication (×) when you convert from a larger unit to a smaller unit
- use division (÷) when you convert from a smaller unit to a larger unit.

The common units used for measuring *length* are kilometres (km), metres (m), centimetres (cm) and millimetres (mm). The following chart is useful when converting units of length, and can be adapted to other metric units.

$$\begin{tikzcd}
\text{\textit{Length}} & \text{km} \arrow[r, bend left, "\times 1000"] & \text{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \text{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \text{mm} \arrow[l, bend left, "\div 10"] \\
\text{\textit{Area}} & \text{km}^2 \arrow[r, bend left, "\times 1000^2"] & \text{m}^2 \arrow[l, bend left, "\div 1000^2"] \arrow[r, bend left, "\times 100^2"] & \text{cm}^2 \arrow[l, bend left, "\div 100^2"] \arrow[r, bend left, "\times 10^2"] & \text{mm}^2 \arrow[l, bend left, "\div 10^2"] \\
\text{\textit{Volume}} & \text{km}^3 \arrow[r, bend left, "\times 1000^3"] & \text{m}^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & \text{cm}^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & \text{mm}^3 \arrow[l, bend left, "\div 10^3"]
\end{tikzcd}$$

The common units for measuring *liquids* are kilolitres (kL), litres (L) and millilitres (mL).

$$\begin{aligned} 1 \text{ kilolitre} &= 1000 \text{ litres} \\ 1 \text{ litre} &= 1000 \text{ millilitres} \end{aligned}$$

The common units for measuring *mass* are tonnes (t), kilograms (kg), grams (g) and milligrams (mg).

$$\begin{aligned} 1 \text{ tonne} &= 1000 \text{ kilograms} \\ 1 \text{ kilogram} &= 1000 \text{ grams} \\ 1 \text{ gram} &= 1000 \text{ milligrams} \end{aligned}$$

Note: Strictly speaking the litre and tonne are not included in the SI, but are commonly used with SI units.

The following prefixes are useful to remember.

Prefix	Symbol	Definition	Decimal
micro	μ	millionth	0.000 001
milli	m	thousandth	0.001
centi	c	hundredth	0.01
deci	d	tenth	0.1
kilo	k	thousand	1000
mega	M	million	1 000 000
giga	G	billion	1 000 000 000

Example 7 Converting between units

Convert these measurements into the units given in the brackets.

a 5.2 km (m) **b** 339 cm^2 (m^2) **c** 9.75 cm^3 (mm^3)

Solution

a As there are 1000 metres in a kilometre and we are converting from kilometres (km) to a smaller unit (m), we need to multiply 5.2 by 1000.

$$\begin{aligned} & 5.2 \times 1000 \\ &= 5200 \text{ m} \end{aligned}$$

b As there are 100^2 square centimetres in a square metre and we are converting from square centimetres (cm^2) to a larger unit (m^2), we need to divide 339 by 100^2.

$$\begin{aligned} & 339 \div 100^2 \\ &= 0.0339 \text{ m}^2 \end{aligned}$$

c As there are 10^3 cubic millimetres in a cubic centimetre and we are converting from cubic centimetres (cm^3) to a smaller unit (mm^3), we need to multiply 9.75 by 10^3.

$$\begin{aligned} & 9.75 \times 10^3 \\ &= 9750 \text{ mm}^3 \end{aligned}$$

Sometimes a measurement conversion requires more than one step.

Example 8 Converting between units requiring more than one step

Convert these measurements into the units given in the brackets.

a 40 000 cm (km) **b** 0.000 22 km^2 (cm^2) **c** 0.08 m^3 (mm^3)

Solution

a As there are 100 centimetres in a metre and 1000 metres in a kilometre and we are converting from centimetres (cm) to a larger unit (km), we need to divide 40 000 by (100 × 1000 =) 100 000.

$40\,000 \div 100\,000$
$= 0.4$ km

b As there are 100^2 square centimetres in a square metre and 1000^2 square metres in a square kilometre and we are converting from square kilometres (km^2) to a smaller unit (cm^2), we need to multiply 0.000 22 by ($100^2 \times 1000^2$).

$0.000\,22 \times 100^2 \times 1000^2$
$= 2\,200\,000$ cm^2

c As there are 10^3 cubic millimetres in a cubic centimetre and 100^3 cubic centimetres in a cubic metre and we are converting from cubic metres (m^3) to a smaller unit (mm^3), we need to multiply 0.08 by ($10^3 \times 100^3$).

$0.08 \times 10^3 \times 100^3$
$= 80\,000\,000$ mm^3

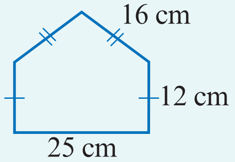
Perimeters of regular shapes

Perimeter

The **perimeter** of a two-dimensional shape is the total distance around its edge.

Example 9 Finding the perimeter of a shape

Find the perimeter of the shape shown.



Solution

To find the perimeter, add up all the side lengths of the shape.

Perimeter $= 25 + 12 + 12 + 16 + 16$
$= 81$ cm

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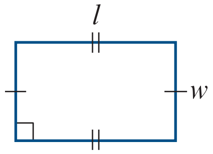
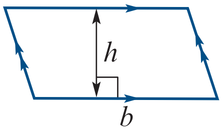
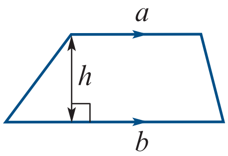
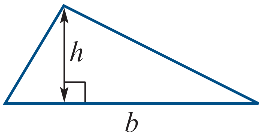
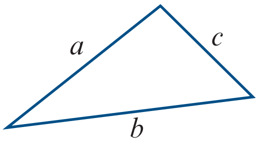
Areas of regular shapes

Area

The **area** of a shape is a measure of the region enclosed by its boundaries.

When calculating area, the answer will be in *square units*, i.e. mm^2, cm^2, m^2, km^2.

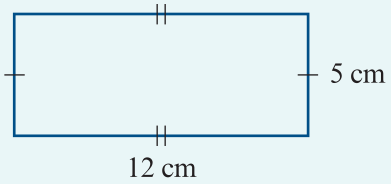
The **formulas for the areas** of some common shapes are given in the table below, along with the formula for finding the perimeter of a rectangle.

Shape	Area	Perimeter
Rectangle 	$A = lw$	$P = 2l + 2w$ or $P = 2(l + w)$
Parallelogram 	$A = bh$	Sum of four sides
Trapezium 	$A = \frac{1}{2}(a + b)h$	Sum of four sides
Triangle 	$A = \frac{1}{2}bh$	Sum of three sides
Heron's rule for finding the area of a triangle with three side lengths known 	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$ (s is the half perimeter)	$P = a + b + c$

Note: More work on Heron's rule is given in *Chapter 6 Applications of trigonometry* section 6G.

Example 10 Finding the perimeter of a rectangle

Find the perimeter of the rectangle shown.



Solution

1 As the shape is a rectangle, use the formula $P = 2l + 2w$. $P = 2l + 2w$

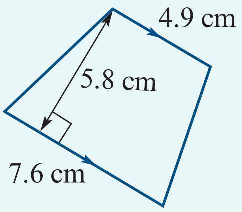
2 Substitute length and width values into the formula. $= 2 \times 12 + 2 \times 5$

3 Evaluate. $= 34$ cm

4 Give your answer with correct units. The perimeter of the rectangle is 34 cm.

Example 11 Finding the area of a shape

Find the area of the given shape.



Solution

1 As the shape is a trapezium, use the formula $A = \frac{1}{2}(a + b)h$. $A = \frac{1}{2}(a + b)h$

2 Substitute the values for a, b and h. $= \frac{1}{2}(4.9 + 7.6)5.8$

3 Evaluate. $= 36.25 \text{ cm}^2$

4 Give your answer with correct units. The area of the shape is 36.25 cm^2.

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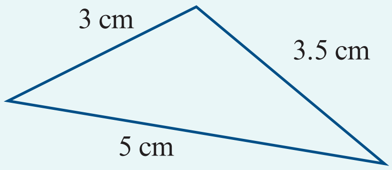
Example 12 Finding the area of a triangle using Heron's rule

Find the area of the following triangle. Give your answer correct to two decimal places.



Solution

1 As the height of the triangle is not given and the three side lengths are known, use Heron's rule.

2 Write down Heron's rule.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

3 Find the perimeter of the triangle by adding the three side lengths.

$$P = 3 + 3.5 + 5$$
$$= 11.5$$

4 Divide the perimeter by 2 to find s.

$$s = \frac{11.5}{2}$$
$$= 5.75$$

5 Substitute the value for s into Heron's rule to find the area of the triangle.

$$A = \sqrt{5.75(5.75-3)(5.75-3.5)(5.75-5)}$$
$$= 5.16562\ldots$$

6 Give your answer correct to two decimal places and with correct units.

The area of the triangle is 5.17 cm^2, correct to two decimal places.

The formulas for area and perimeter can be applied to many practical situations.

Example 13 Finding the area and perimeter in a practical problem

A display board for a classroom measures 150 cm by 90 cm.

a If ribbon costs \$0.55 per metre, how much will it cost to add a ribbon border around the display board?

b The display board is to be covered with yellow paper. What is the area to be covered? Give you answer in m^2, correct to two decimal places.

Solution

a **1** To find the length of ribbon required, we need to work out the perimeter of the display board. The display board is a rectangle so use the formula $P = 2l + 2w$.

$$P = 2l + 2w$$

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	2 Substitute $l = 150$ and $w = 90$ into the formula $P = 2l + 2w$ and evaluate to find the length of ribbon required.	$P = 2(150) + 2(90)$ $= 480$ The length of ribbon required is 480 cm.
	3 Convert from centimetres to metres by dividing the length of ribbon by 100.	$= 480 \div 100$ $= 4.8$ m
	4 To find the cost of the ribbon, multiply the length of the ribbon by \$0.55.	$4.8 \times 0.55 = 2.64$
	5 Evaluate and write your answer.	Cost of ribbon is \$2.64.
b	**1** To find the area of the board (which measures 150 cm by 90 cm), use the formula $A = lw$.	$A = lw$
	2 Substitute $l = 150$ and $w = 90$ and evaluate.	$= 150 \times 90$ $= 13\,500$ cm^2
	3 Convert your answer to m^2 by dividing by $(100 \times 100 = 10\,000)$.	$A = 13\,500 \div 10\,000$ $= 1.35$
	4 Write your answer with correct units.	Area to be covered with paper is 1.35 m^2.

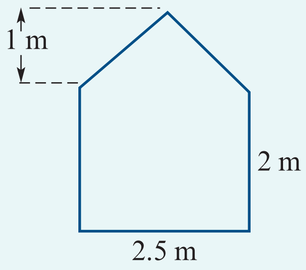
Composite shapes

Composite shape

A **composite shape** is a shape that is made up of two or more basic shapes.

Example 14 Finding the perimeter and area of a composite shape in a practical problem

A gable window at a reception venue is to have LED lights around its perimeter (but not along the bottom of the window). The window is 2.5 m wide and the height of the room is 2 m. The height of the gable is 1 m, as shown in the diagram.



a Calculate the length of LED lights needed, correct to two decimal places.

b The glass in the window needs to be replaced. Find the total area of the window, correct to two decimal places.

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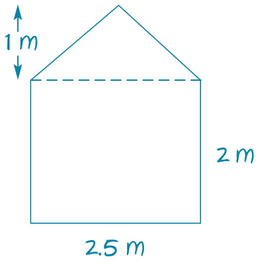
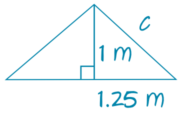
Solution

a The window is made of two shapes: a rectangle and a triangle. We need to find its perimeter and area.



1 First find the length of the slant edge of the triangle. Label it c on a diagram.



Use Pythagoras' theorem to find c.

Note: The length of the base of each triangle is 1.25 m ($\frac{1}{2}$ of 2.5 m).

$$c^2 = 1^2 + 1.25^2$$

$$\therefore c = \sqrt{1^2 + 1.25^2}$$

$$c = 1.6007\ldots$$

$c = 1.60$ m, correct to two decimal places

2 Add all the outside edges of the window but do not include the bottom length.

$$2 + 2 + 1.60 + 1.60 = 7.20$$

3 Write your answer with correct units.

The length of the LED lights is 7.20 m.

b **1** To find the total area of the window, first find the area of the rectangle by using the formula $A = bh$.

$$A = bh$$

2 Substitute the values for b and h.

$$= 2.5 \times 2$$

3 Evaluate and write your answer with correct units.

$$= 5 \text{ m}^2$$

4 Find the area of the triangle by using the formula $A = \frac{1}{2}bh$.

$$A = \frac{1}{2}bh$$

5 Substitute the values for b and h.

$$= \frac{1}{2} \times 2.5 \times 1$$

6 Evaluate and write your answer with correct units.

$$= 1.25 \text{ m}^2$$

7 To find the total area of the window, add the area of the rectangle and the area of the triangle.

$$\begin{aligned}\text{Total area} &= \text{area of rectangle} \\ &\quad + \text{area of triangle} \\ &= 5 + 1.25 \\ &= 6.25 \text{ m}^2\end{aligned}$$

8 Give your answer with correct units to two decimal places.

Total area of window is 6.25 m^2, correct to two decimal places.

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Exercise 3C

SF

Conversion of units

Example 7, 8

1 Convert the following measurements into the units given in brackets.

a 5.7 m (cm)
b 1.587 km (m)
c 8 cm (mm)
d 670 cm (m)
e 0.0046 km (cm)
f 289 mm^2 (cm^2)
g 5.2 m^2 (cm^2)
h 0.08 km^2 (m^2)
i 3700 mm^2 (cm^2)
j 6 m^2 (mm^2)
k 500 mL (L)
l 0.7 kg (g)
m 2.3 kg (mg)
n 567 000 mL (kL)
o 793 400 mg (g)
p 0.5 L (mL)

2 Convert the following measurements into the units indicated in brackets and give your answer in standard form.

a 5 tonne (kg)
b 6000 mg (kg)
c 27 100 km^2 (m^2)
d 33 m^3 (cm^3)
e 487 m^2 (km^2)
f 28 mL (L)
g 6 km (cm)
h 1125 mL (kL)
i 50 000 m^3 (km^3)
j 340 000 mm^3 (m^3)

3 Find the total sum of these measurements. Express your answer in the units given in brackets.

a 14 cm, 18 mm (mm)
b 589 km, 169 m (km)
c 3.4 m, 17 cm, 76 mm (cm)
d 300 mm^2, 10.5 cm^2 (cm^2)

4 A wall in a house is 7860 mm long. How many metres is this?

5 A truck weighs 3 tonne. How heavy is this in kilograms?

6 An Olympic swimming pool holds approximately 2.25 megalitres of water. How many litres is this?

7 Baking paper is sold on a roll 30 cm wide and 10 m long. How many baking trays of width 30 cm and length 32 cm could be covered with one roll of baking paper?

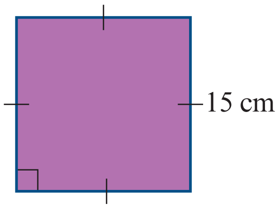
Perimeters and areas

Example 9, 10

8 For each of the following shapes, find, correct to one decimal place:

i the perimeter
ii the area

a



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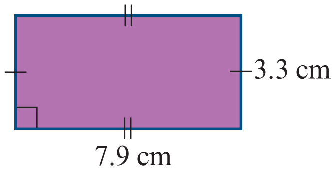

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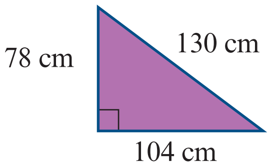

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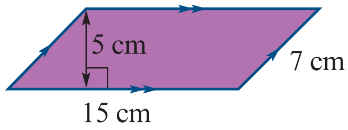

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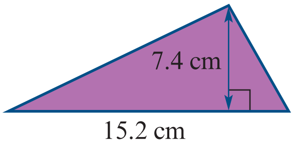
Example 11 **9** Find the areas of the given shapes, correct to one decimal place, where appropriate.

SF

a



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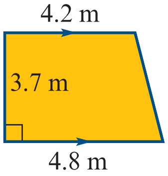

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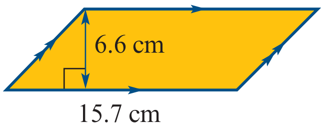

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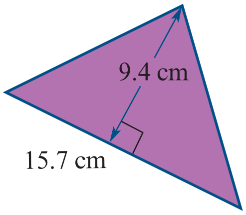

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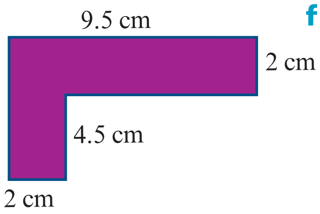

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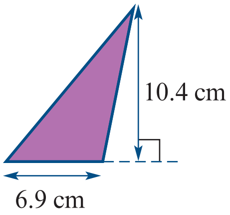

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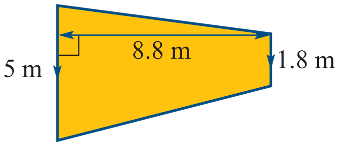

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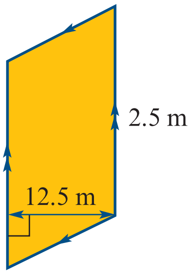
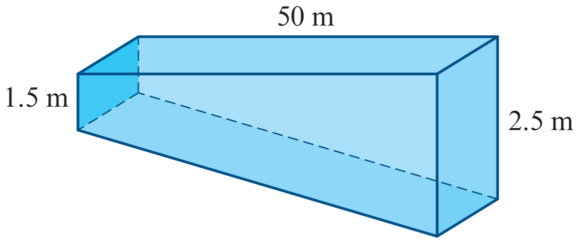


Applications of perimeters and areas

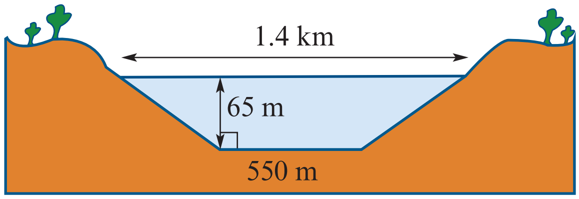
Example 13, 14 **10** A 50 m swimming pool increases in depth from 1.5 m at the shallow end to 2.5 m at the deep end, as shown in the diagram (*not* to scale). Calculate the area of a side wall of the pool.

CF



11 A dam wall is built across a valley that is 550 m wide at its base and 1.4 km wide at its top, as shown in the diagram (*not* to scale). The wall is 65 m deep. Calculate the area of the dam wall.

SF



12 Ray wants to tile a rectangular area measuring 1.6 m by 4 m. The tiles that he wishes to use are 40 cm by 40 cm. How many tiles will he need?

CF

13 One litre of paint covers 9 m^2. How much paint is needed to paint a wall measuring 3 m by 12 m?

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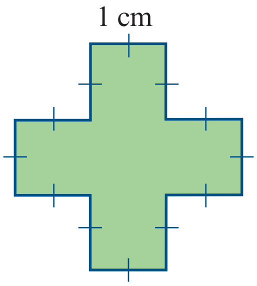
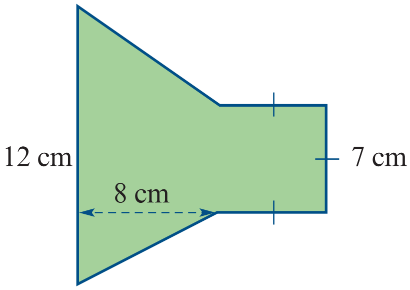
Composite shapes

14 Find the area of the following composite shapes.

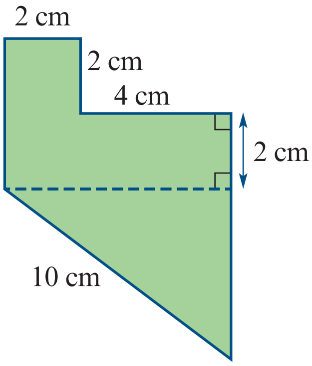
a



b

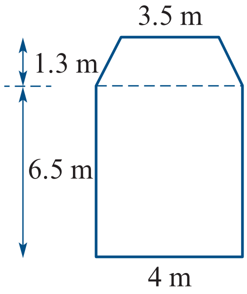


c

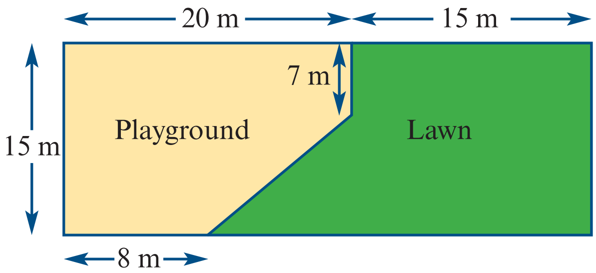


Applications of composite shapes

15 A driveway, as shown in the diagram, is to be paved. What is the area of the driveway, correct to two decimal places?



16 The council plans to fence a rectangular piece of land to make a children's playground and a lawn as shown. (Not drawn to scale.)



a What is the area of the children's playground?

b What is the area of the lawn?

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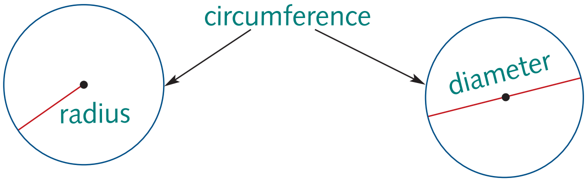
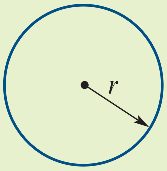
3D Circles

▶ The circumference and area of a circle

The perimeter of a circle is also known as the **circumference** (C) of the circle.



Formulas for area and circumference of a circle.

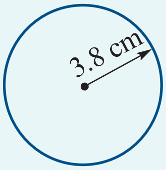
Circle	Area	Circumference
	$A = \pi r^2$ where r is the **radius**	$C = 2\pi r$ or $C = \pi d$ where d is the **diameter**

Example 15 Finding the circumference and area of a circle

For the circle shown, find, correct to one decimal place:

a the circumference

b the area.



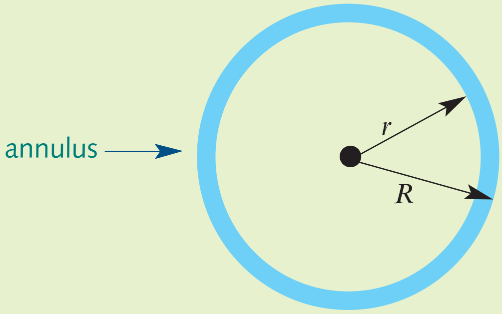
Solution

a **1**	For the circumference, use the formula $C = 2\pi r$.	$C = 2\pi r$
2	Substitute $r = 3.8$ and evaluate.	$= 2\pi \times 3.8$ $= 23.876\ldots$
3	Give your answer correct to one decimal place and with correct units.	The circumference of the circle is 23.9 cm, correct to one decimal place.
b **1**	To find the area of the circle, use the formula $A = \pi r^2$.	$A = \pi r^2$
2	Substitute $r = 3.8$ and evaluate.	$= \pi \times 3.8^2$ $= 45.364\ldots$
3	Give your answer correct to one decimal place and with correct units.	The area of the circle is 45.4 cm^2, correct to one decimal place.

The annulus

Annulus

An **annulus** is a flat ring shape bounded by two circles that have the same centre.

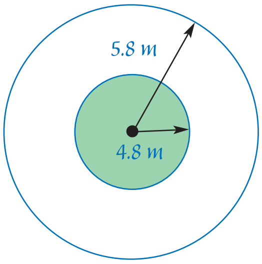


Example 16 Finding the area of an annulus in a practical problem

A path 1 metre wide is to be built around a circular lawn of radius 4.8 m. Find the area of the path, correct to two decimal places.

Solution

1 Draw a diagram to represent the situation. The two circles form an annulus.
The smaller circle has radius of 4.8 m.
With the path of width of 1 m, the larger circle has a radius of 5.8 m



2 Find the area of the larger circle using the formula $A = \pi r^2$.

$A = \pi r^2$

3 Substitute $r = 5.8$ and evaluate.

$A = \pi \times 5.8^2$

$A = 105.68$, correct to two decimal places

4 Find the area of the smaller circle using the formula $A = \pi r^2$.

$A = \pi r^2$

5 Substitute $r = 4.8$ and evaluate.

$A = \pi \times 4.8^2$

$A = 72.38$, correct to two decimal places

6 Subtract the area of the smaller circle from the area of the larger circle to give the required area (the area of the annulus).

Required area = area of large circle − area of small circle

= 105.68 − 72.38

= 33.30

7 Give your answer correct to two decimal places and with correct units.

Area of circular path is 33.30 m^2, correct to two decimal places.

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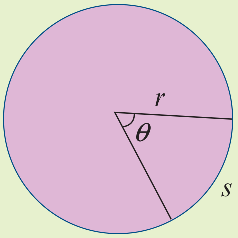
Arc length

Arc of a circle

An **arc** is the length of a circle between two points on the circle. The length of the arc, s, is given by:

$$s = r\left(\frac{\theta}{180}\pi\right)$$

where r is the radius of the circle and $\theta°$ is the angle subtended by the arc at the centre of the circle.

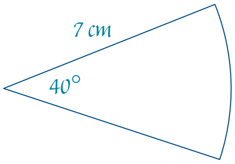


Example 17 Finding the length of an arc

Find, correct to two decimal places, the length of an arc that subtends an angle of 40° at the centre of a circle of radius 7 cm.

Solution

1 Draw a diagram to represent the situation.



2 Write down the formula for arc length, s.

$$s = r\left(\frac{\theta}{180}\pi\right)$$

3 Substitute $r = 7$ and $\theta = 40$ and evaluate.

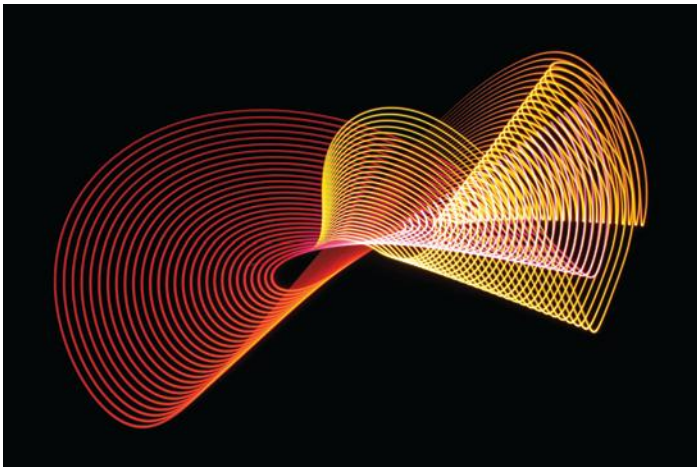
$$s = 7\left(\frac{40}{180}\pi\right)$$

4 Give your answer correct to two decimal places and with correct units.

$s = 4.8869\ldots$

$s = 4.89$, correct to two decimal places

The length of the arc is 4.89 cm.



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Exercise 3D

Finding the circumference and area of a circle

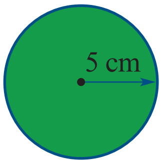
Example 15

1 For each of the following circles, find, correct to one decimal place:

i the circumference

ii the area.

a



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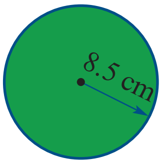

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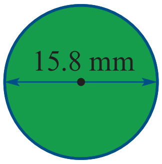
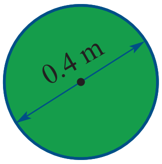


d



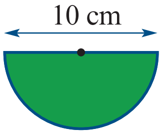
Finding the perimeter and area of shapes involving circles

2 For each of the following shapes, find, correct to two decimal places:

i the perimeter

ii the area.

a



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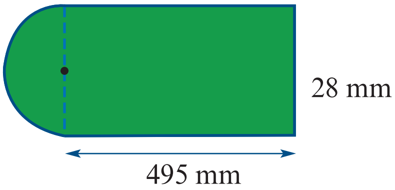

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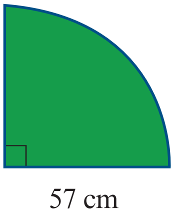

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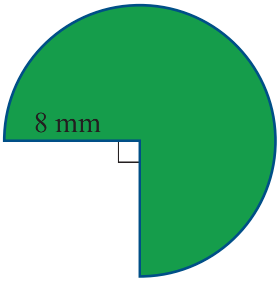
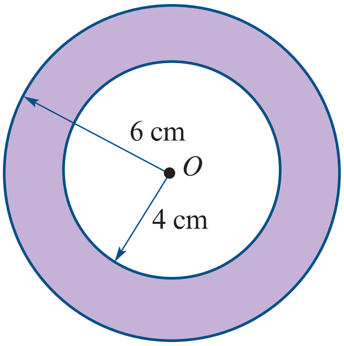


Example 16

3 The diagram shows two circles with centre O. The radius of the inner circle is 4 cm and the radius of the outer circle is 6 cm. What is the area of the annulus (shaded area) correct to two decimal places?



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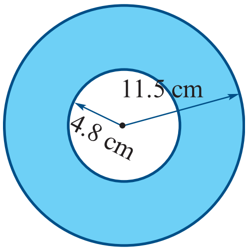
4 Find the shaded areas in the following diagrams, correct to one decimal place.

a



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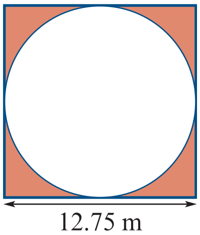

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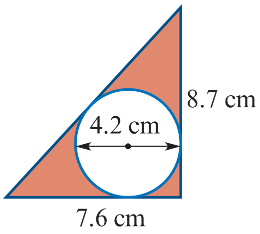

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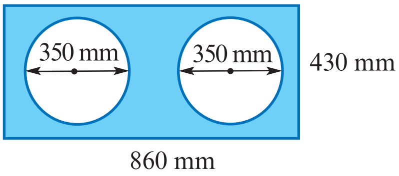


Applications of perimeters and areas involving circles

5 A fence needs to be built around an athletics track that has straights 400 m long and semicircular ends of diameter 80 m. Give answers correct to two decimal places.

a What length of fencing is required?

b What area will be enclosed by the fencing?

6 A couple wish to decorate an arch for their wedding. The width of the arch is 1.4 metres and the semicircle at the top begins at a height of 1.9 metres.

a Material is to be attached around the perimeter of the arch. What is this length, to the nearest metre?

b If material is to cover the whole arch so that you cannot see through the arch, what is the minimum number of square metres of material required, correct to one decimal place?

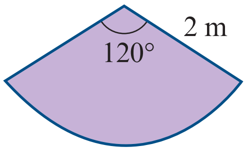
7 Three juggling rings cut from a thin sheet are to be painted. The diameter of the outer circle of the ring is 25 cm and the diameter of the inside circle is 20 cm. If both sides of the three rings are to be painted, what is the total area to be painted? (Ignore the inside and outside edges.) Round your answer to the nearest cm^2.

8 A path 1.2 m wide surrounds a circular garden bed whose diameter is 7 m. What is the area of the path? Give the answer correct to two decimal places.

Arc length and applications

Example 17 **9** A circle has a radius of 10 cm. An arc of the circle subtends an angle of 50° at the centre. Calculate the arc length correct to two decimal places.

10 Maria wishes to place edging around the perimeter of her garden bed. The garden bed is in the shape of a sector as shown. What is the perimeter of her garden bed, correct to two decimal places?

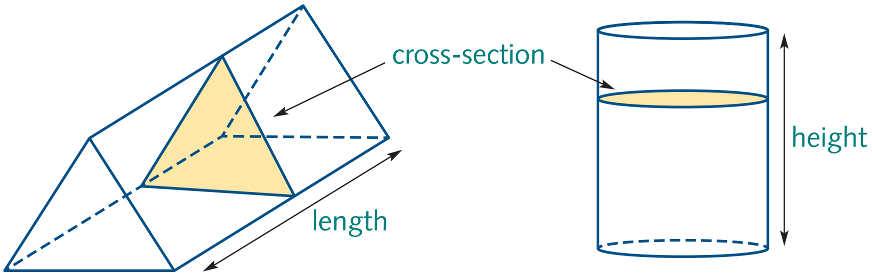


3E Volume of a prism

Volume

Volume is the amount of space occupied by a three-dimensional object.

Prisms and cylinders are three-dimensional objects that have a uniform cross-section along their entire length. The volume of a prism or cylinder is found by using its *cross-sectional area*.



For prisms and cylinders:

volume = area of cross-section × height (or length)

When calculating volume, the answer will be in *cubic units*, i.e. mm^3, cm^3, m^3.

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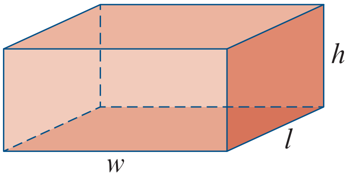
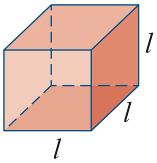
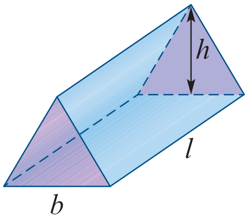
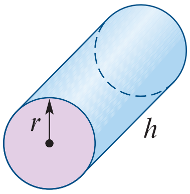
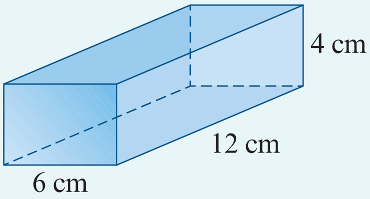
The **formulas for the volumes** of regular prisms and a cylinder are given in the table below.

Shape	Volume
Rectangular prism (cuboid) 	$V = lwh$
Square prism (cube) 	$V = l^3$

Shape	Volume
Triangular prism 	$V = \frac{1}{2}bhl$
Cylinder 	$V = \pi r^2 h$

Example 18 Finding the volume of a cuboid

Find the volume of this cuboid.



Solution

1 Use the formula $V = lwh$.	$V = lwh$
2 Substitute in $l = 12, w = 6$ and $h = 4$.	$= 12 \times 6 \times 4$
3 Evaluate.	$= 288 \text{ cm}^3$
4 Give your answer with correct units.	The volume of the cuboid is 288 cm^3.

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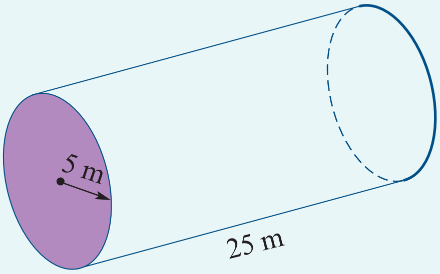
Example 19 Finding the volume of a cylinder

Find the volume of this cylinder in cubic metres. Give your answer correct to two decimal places.



Solution

1 Use the formula $V = \pi r^2 h$.

2 Substitute in $r = 5$ and $h = 25$ and evaluate.

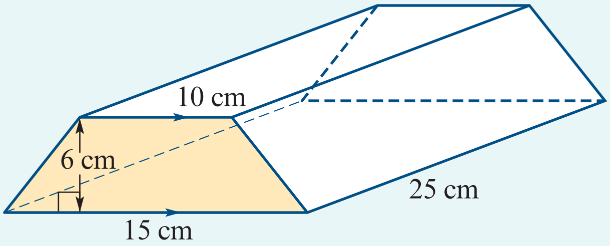
$$V = \pi r^2 h$$
$$= \pi \times 5^2 \times 25$$
$$= 1963.495 \ldots$$

3 Write your answer correct to two decimal places and with correct units.

The volume of the cylinder is 1963.50 m^3, to two decimal places.

Example 20 Finding the volume of a three-dimensional shape

Find the volume of the three-dimensional shape shown.



Solution

Strategy: To find the volume, find the area of the yellow shaded cross-section and multiply it by the length of the shape.

1 Find the area of the cross-section, which is a trapezium.

Use the formula $A = \frac{1}{2}(a + b)h$.

Substitute in $a = 10, b = 15$ and $h = 6$ and evaluate.

$$\text{Area of trapezium} = \frac{1}{2}(a + b)h$$
$$= \frac{1}{2}(10 + 15)6$$
$$= 75 \text{ cm}^2$$

2 To find the volume, multiply the area of the cross-section by the length of the shape (25 cm).

$$V = \text{area of cross-section} \times \text{length}$$
$$= 75 \times 25$$
$$= 1875 \text{ cm}^3$$

3 Give your answer with correct units.

The volume of the shape is 1875 cm^3.

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Capacity

Capacity

Capacity is the amount of substance that an object can hold.

For example, a bucket might have a capacity of 7 litres.

The difference between volume and capacity is that volume is the space available and capacity is the amount of substance that fills the volume.

Examples:

- A cube that measures 1 metre on each side has a volume of one cubic metre (m^3) and is able to hold 1000 litres (L) (capacity).
- A bucket of volume 7000 cm^3 can hold 7000 mL (or 7 L) of water.

Capacity conversions

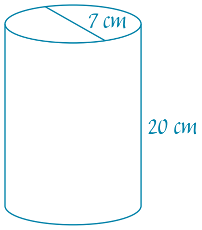
The following conversions are useful to remember.

$$\begin{aligned} 1\text{ m}^3 &= 1000\text{ litres (L)} \\ 1\text{ cm}^3 &= 1\text{ millilitre (mL)} \\ 1000\text{ cm}^3 &= 1\text{ litre (L)} \end{aligned}$$

Example 21 Finding the capacity of a cylinder

A drink container is in the shape of a cylinder. How many litres of water can it hold if the height of the cylinder is 20 cm and the diameter is 7 cm? Give your answer correct to two decimal places.

Solution

1 Draw a diagram to represent the situation.	
2 Use the formula for finding the volume of a cylinder $V = \pi r^2 h$.	$V = \pi r^2 h$
3 The diameter is 7 cm so the radius is 3.5 cm. Substitute $h = 20$ and $r = 3.5$.	$V = \pi \times 3.5^2 \times 20$
4 Evaluate to find the volume of the cylinder.	$V = 769.6902\ldots$ The volume of the cylinder is 769.69 cm^3.
5 As there are 1000 cm^3 in a litre, divide the volume by 1000 to convert to litres.	$\frac{769.69}{1000} = 0.76969\ldots$
6 Give your answer correct to two decimal places and with correct units.	Cylinder has capacity of 0.77 litres, correct to two decimal places.

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Exercise 3E

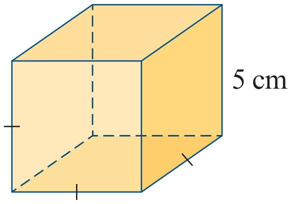
Volumes of prisms

Example 18–20

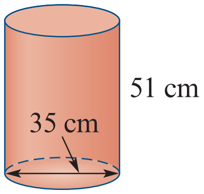
SF

1 Find the volumes of the following solids. Give your answers correct to one decimal place where appropriate.

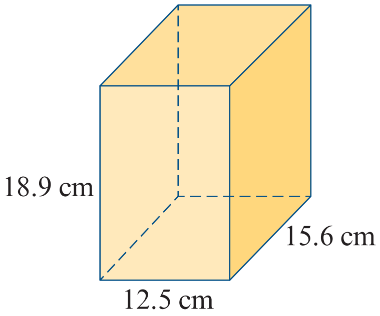
a



b



c



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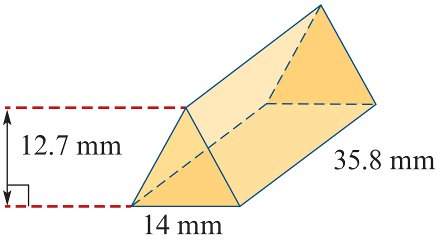
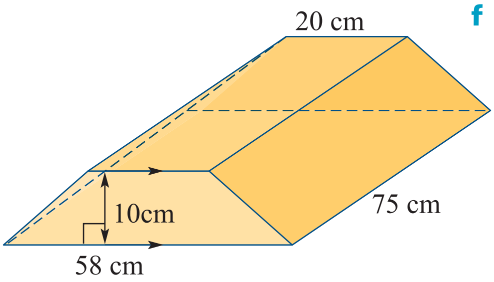
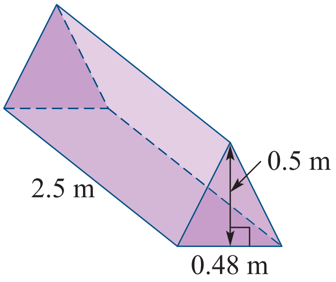


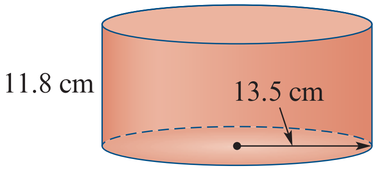
e



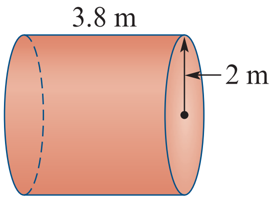
f



g



h



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2 A cylindrical plastic container is 15 cm high and its circular end surfaces each have a radius of 3 cm. What is its volume, to the nearest cm^3?

3 What is the volume, to the nearest cm^3, of a rectangular box with dimensions 5.5 cm by 7.5 cm by 12.5 cm?

Applications of volume and capacity

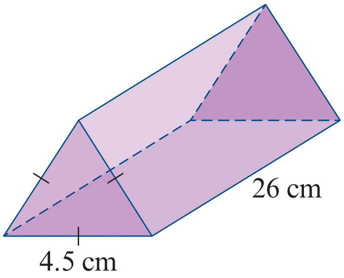
Example 21

4 How many litres of water does a fish tank with dimensions 50 cm by 20 cm by 24 cm hold when full?

5 **a** What is the volume, correct to two decimal places, of a cylindrical paint tin with height 33 cm and diameter 28 cm?

b How many litres of paint would fill this paint tin? Give your answer to the nearest litre.

6 The box for a chocolate bar is made in the shape of an equilateral triangular prism. What is the volume of the box if the length is 26 cm and the side length of the triangle is 4.5 cm? Give your answer to the nearest cm^3.



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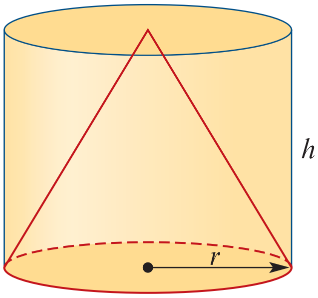
3F Volume of other solids

Volume of a cone

A cone can fit inside a cylinder, as shown in the diagram. The cone occupies one-third of the volume of the cylinder containing it.



Therefore, the formula for finding the volume of a cone is:

$$\text{volume of cone} = \tfrac{1}{3} \times \text{volume of its cylinder}$$

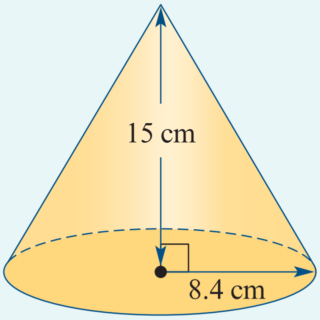
$$\text{volume of cone} = \tfrac{1}{3} \times \text{ area of base} \times \text{height}$$

$$V = \tfrac{1}{3}\pi r^2 h$$

The cone in the above diagram is called a right circular cone because a line drawn from the centre of the circular base to the vertex at the top of the cone is perpendicular to the base.

Example 22 Finding the volume of a cone

Find the volume of this right circular cone.
Give your answer to two decimal places.



Solution

1 Use the formula for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$.	$V = \frac{1}{3}\pi r^2 h$
2 Substitute $r = 8.4$ and $h = 15$ and evaluate.	$= \frac{1}{3}\pi(8.4)^2 \times 15$ $= 1108.353 \ldots$
3 Give your answer correct to two decimal places and with correct units.	The volume of the cone is 1108.35 cm^3, correct to two decimal places.

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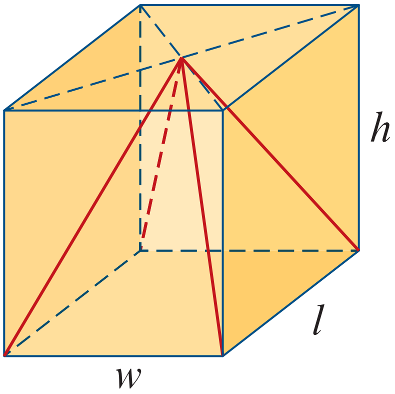
▶ Volume of a pyramid

A square pyramid can fit inside a prism, as shown in the diagram. The pyramid occupies one third of the volume of the prism containing it.



The formula for finding the volume of a pyramid is therefore:

$$\text{volume of pyramid} = \tfrac{1}{3} \times \text{volume of its prism}$$

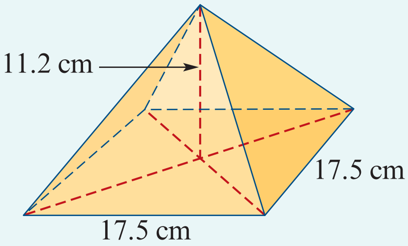
$$\text{volume of pyramid} = \tfrac{1}{3} \times \text{area of base} \times \text{height}$$

$$V = \tfrac{1}{3}lwh$$

This formula can be used for a pyramid with a base of any shape.

Example 23 Finding the volume of a square pyramid

Find the volume of a square right pyramid of height 11.2 cm and base 17.5 cm. Give your answer correct to two decimal places.



Solution

1 Use the formula:

$$V = \frac{1}{3} \times \text{area of base} \times \text{height} \qquad V = \frac{1}{3}lwh$$

2 Substitute the values for the area of the base (in this example, the base is a square) and height of the pyramid and evaluate.

$$= \frac{1}{3} \times 17.5^2 \times 11.2$$

$$= 1143.333\ldots$$

3 Give your answer correct to two decimal places and with correct units.

The volume of the pyramid is 1143.33 cm^3, correct to two decimal places.

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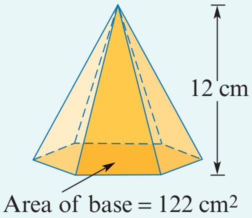
Example 24 Finding the volume of a hexagonal pyramid

Find the volume of this hexagonal pyramid that has a base of area 122 cm^2 and a height of 12 cm.



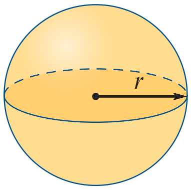
Solution

1. Use the formula: $V = \frac{1}{3} \times \text{area of base} \times \text{height}$ — $V = \frac{1}{3} \times \text{area of base} \times \text{height}$
2. Substitute the values for area of base (122 cm^2) and height (12 cm) and evaluate. — $= \frac{1}{3} \times 122 \times 12$
   $= 488\ \text{cm}^3$
3. Give your answer with correct units. — The volume of the pyramid is 488 cm^3.

Volume of a sphere

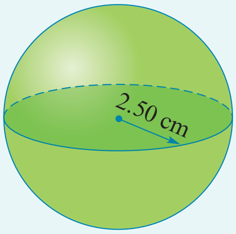
The volume of a sphere of radius r can be found by using the formula:

$$V = \frac{4}{3}\pi r^3$$



Example 25 Finding the volume of a sphere

Find the volume of this sphere, giving your answer correct to two decimal places.



Solution

1. Use the formula $V = \frac{4}{3}\pi r^3$. — $V = \frac{4}{3}\pi r^3$
2. Substitute $r = 2.5$ and evaluate. — $= \frac{4}{3}\pi \times 2.5^3$
   $= 65.449\ldots$
3. Give your answer correct to two decimal places and with correct units. — The volume of the sphere is 65.45 cm^3, correct to two

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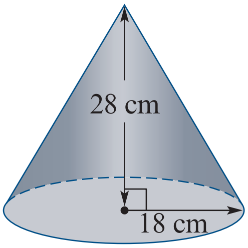
Exercise 3F

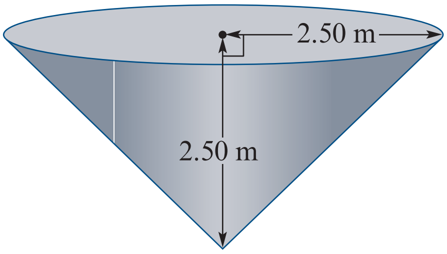
Volumes of cones

Example 22 **1** Find the volume of these cones, correct to two decimal places.

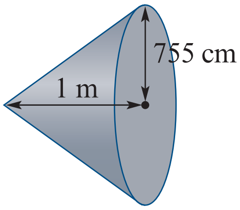
a



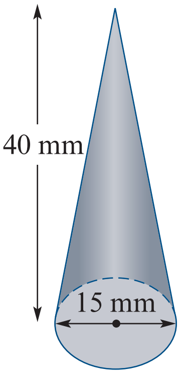
b



c



d



2 Find the volume (to two decimal places) of the cones with the following dimensions.

- **a** Base radius 3.50 cm, height 12 cm
- **b** Base radius 7.90 m, height 10.80 m
- **c** Base diameter 6.60 cm, height 9.03 cm
- **d** Base diameter 13.52 cm, height 30.98 cm

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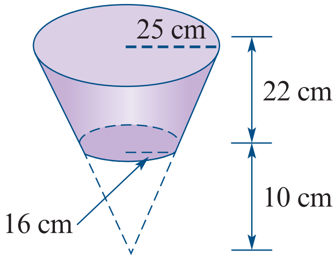
Applications

3 What volume of crushed ice will fill a snow cone level to the top, if the snow cone has a top radius of 5 cm and a height of 15 cm? Give your answer to the nearest cm^3.

4 A tepee is a conical-shaped tent. What is the volume, correct to two decimal places, of a tepee with height 2.6 m and diameter of 3.4 m?

5 How many litres of water, correct to 2 decimal places, can be poured into a conical flask with a diameter 2.8 cm and a height of 10 cm?

6 A solid figure is *truncated* when a portion of the bottom is cut and removed. Find the volume, correct to two decimal places, of the truncated cone shown in the diagram.



7 A flat-bottomed silo for grain storage is made of a cylinder with a cone on top. The cylinder has a circumference of 53.4 m and a height of 10.8 m. The total height of the silo is 15.3 m. What is the volume of the silo? Give your answer to the nearest m^3.

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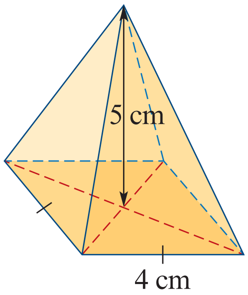
Volumes of pyramids

Example 23, 24

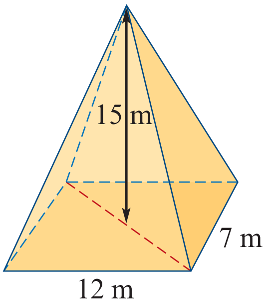
SF

8 Find the volumes of the following right pyramids, correct to two decimal places where appropriate.

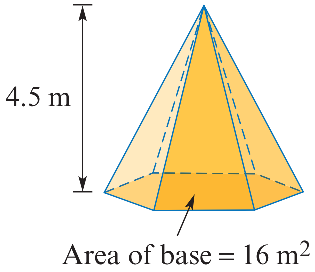
a



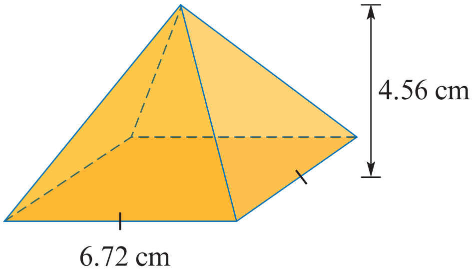
b



c



d



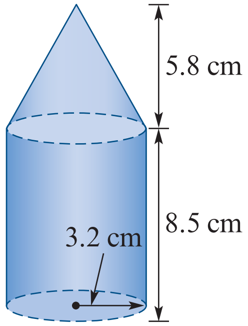
9 A square-based pyramid has a base side length of 8 cm and a height of 10 cm. What is its volume? Answer correct to three decimal places.

10 The first true pyramid in Egypt is known as the Red Pyramid. It has a square base approximately 220 m long and is about 105 m high. What is its volume?

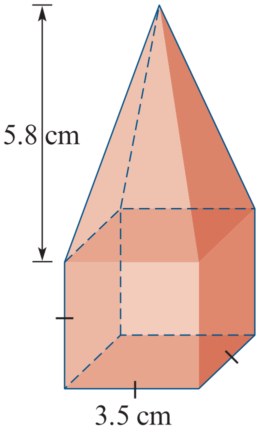
CF

11 Find the volumes of these composite objects, correct to one decimal place.

a



b



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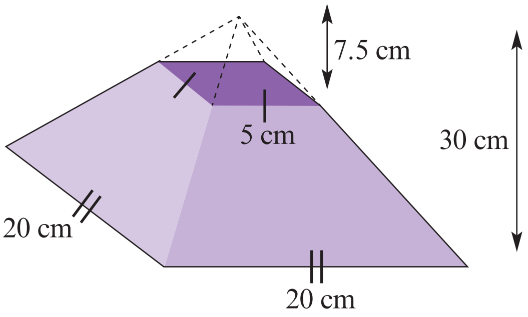
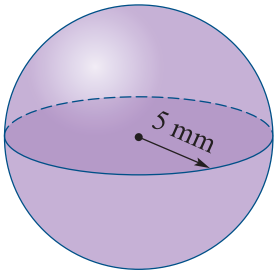
12 Calculate the volume of the following truncated pyramid, correct to one decimal place.



Volumes of spheres and hemispheres

Example 25 **13** Find the volumes of these spheres, giving your answers correct to two decimal places.

a



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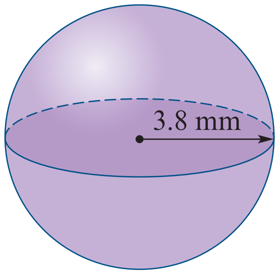

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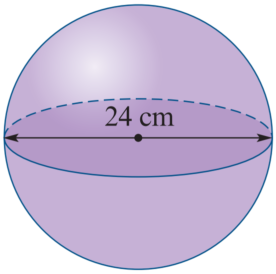


14 Find the volumes, correct to two decimal places, of the following balls.

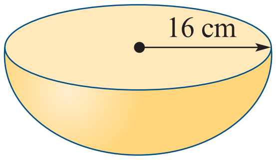
a Tennis ball, radius 3.5 cm

b Basketball, radius 14 cm

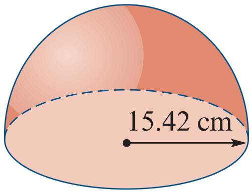
c Golf ball, radius, 2 cm

15 Find the volumes, correct to two decimal places, of the following hemispheres.

a



b



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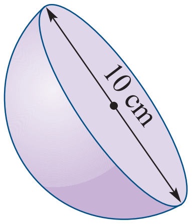

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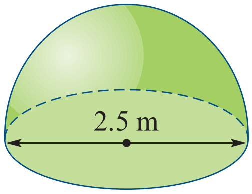

Applications

16 An orange is cut into quarters. If the radius is 35 mm, what is the volume of one quarter to the nearest mm^3?

17 Lois wants to serve punch at Christmas lunch in her new hemispherical bowl with diameter of 38 cm. How many litres of punch could be served, given that 1 millilitre (mL) is the amount of fluid that fills 1 cm^3? Answer to the nearest litre.

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3G Surface area

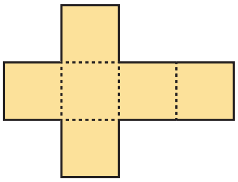
To find the **surface area** (*SA*) of a solid, you need to find the area of each of the surfaces of the solid and then add these all together.

▶ Solids with plane faces (prisms and pyramids)

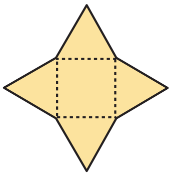
It is often useful to draw the *net* of a solid to ensure that all sides have been added.

A *net* is a flat diagram consisting of the plane faces of a polyhedron, arranged so that the diagram may be folded to form the solid.

The net of a cube and of a square pyramid are shown below.



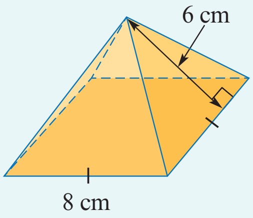
Net of a cube



Net of a square pyramid

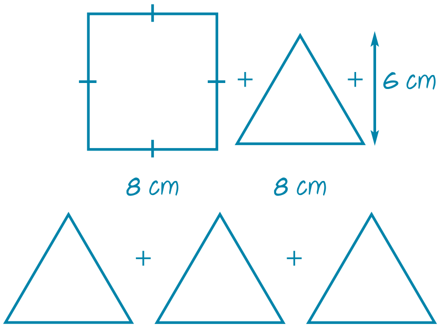
Example 26 Finding the surface area of a pyramid

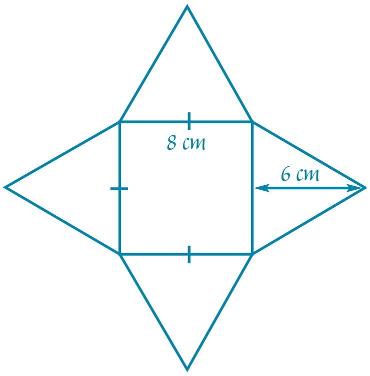
Find the surface area of this square-based pyramid.



Solution

1 Draw a net of the square pyramid. Note that the net is made up of one square and four identical triangles, as shown below.





2 Write down the formula for the total surface area, using the net as a guide, and evaluate.

$$\begin{aligned}\text{Total surface area} &= \text{area of } \square + 4\,\triangle \\ &= 8 \times 8 + 4 \times \left(\tfrac{1}{2} \times 8 \times 6\right) \\ &= 160\end{aligned}$$

The surface area of the square pyramid is 160 cm^2.

Note: To find the area of the square, multiply the length by the width (8 × 8).
To find the area of the triangles use $A = \frac{1}{2}bh$, where b is 8 and h is 6.

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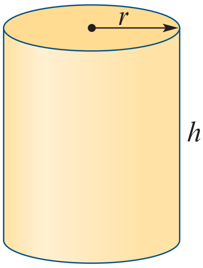
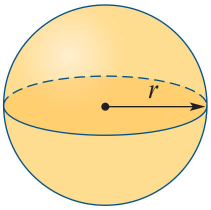
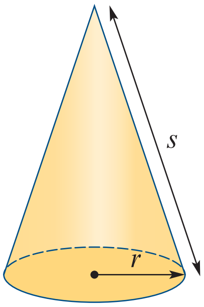
▶ Solids with curved surfaces (cylinder, cone, sphere)

For some special objects, such as the cylinder, cone and sphere, formulas to calculate the surface area can be developed.

The formulas for the surface area of a cylinder, cone and sphere are given below.

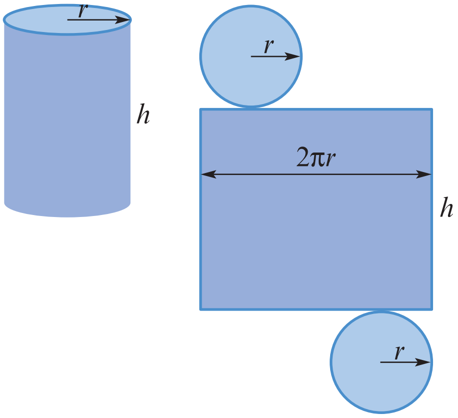
Shape	Surface area
Cylinder 	$SA = 2\pi r^2 + 2\pi rh$ $= 2\pi r(r + h)$
Sphere 	$SA = 4\pi r^2$

Shape	Surface area
Cone 	$SA = \pi r^2 + \pi rs$

To develop the formula for the surface area of a cylinder, we first draw a net, as shown.

The **total surface area (TSA)** of a cylinder can therefore be found using:

$$
\begin{aligned}
\text{TSA} &= \text{area of ends} + \text{area of curved surface}\\
&= \text{area of 2 circles} + \text{area of rectangle}\\
&= 2\pi r^2 + 2\pi rh\\
&= 2\pi r(r + h)
\end{aligned}
$$



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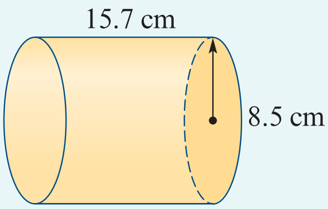
Example 27 Finding the surface area of a cylinder

Find the surface area of this cylinder, correct to one decimal place.

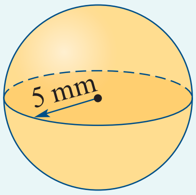


Solution

1 Use the formula for the surface area of a cylinder, $SA = 2\pi r^2 + 2\pi rh$.	$SA = 2\pi r^2 + 2\pi rh$
2 Substitute $r = 8.5$ and $h = 15.7$ and evaluate.	$= 2\pi(8.5)^2 + 2\pi \times 8.5 \times 15.7$ $= 1292.451\ldots$
3 Give your answer correct to one decimal place and with correct units.	The surface area of the cylinder is 12 92.5 cm^2, correct to one decimal place.

Example 28 Finding the surface area of a sphere

Find the surface area of a sphere with radius 5 mm, correct to two decimal places.



Solution

1 Use the formula $SA = 4\pi r^2$.	$SA = 4\pi r^2$
2 Substitute $r = 5$ and evaluate.	$= 4\pi \times 5^2$ $= 314.159\ldots$
3 Give your answer correct to two decimal places and with correct units.	The surface area of the sphere is 314.16 mm^2, correct to two decimal places.

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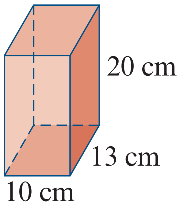
Exercise 3G

Surface areas of prisms and pyramids

Example 26

1 Find the surface areas of these prisms and pyramids. Where appropriate give your answer correct to one decimal place.

a



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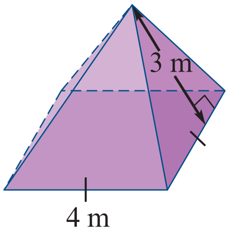
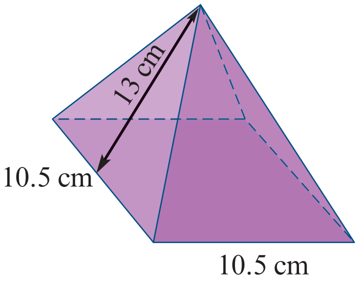
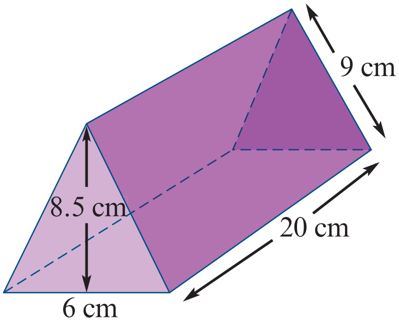


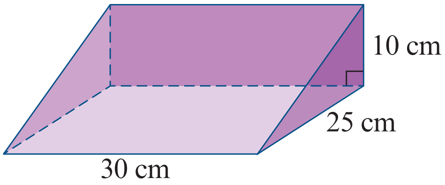
c



d



e



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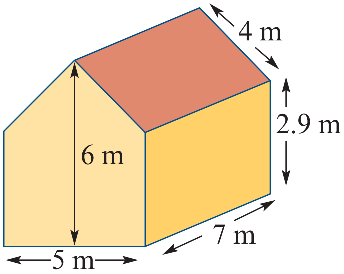
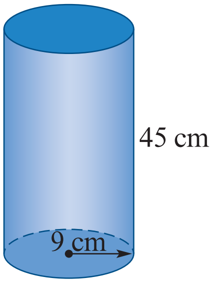


Surface area of curved surfaces

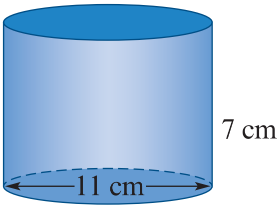
Example 27, 28

2 Find the surface area of each of these solids with curved surfaces, correct to two decimal places.

a



b



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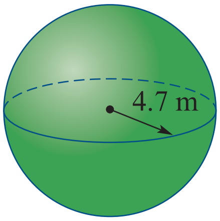

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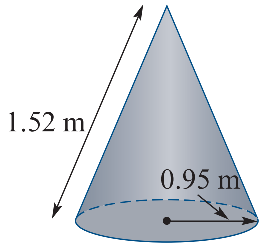

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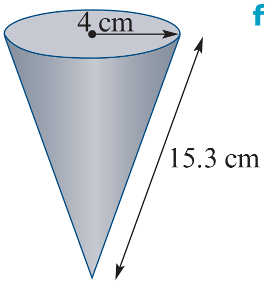

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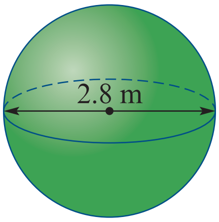

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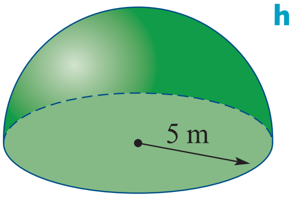

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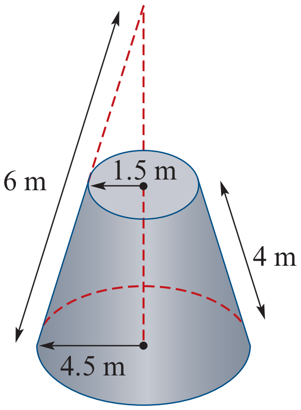


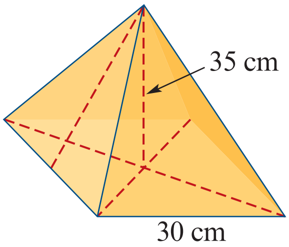
Applications of surface areas

3 A tennis ball has a radius of 3.5 cm. A manufacturer wants to provide sufficient material to cover 100 tennis balls. What area of material is required? Give your answer correct to the nearest cm^2.

4 A set of 10 conical paper hats are to be covered with material. The height of a hat is 35 cm and the diameter is 19 cm.

- **a** What amount of material, in m^2, will be needed? Give your answer correct to two decimal places.
- **b** Tinsel is to be placed around the base of the hats. How much tinsel, to the nearest metre, is required?

5 For a project, Mark has to cover all sides of a square-based pyramid with material (excluding the base). The pyramid has the dimensions as shown in the diagram. How much material will Mark need to cover the sides of the pyramid? Give your answer in square metres, correct to two decimal places.

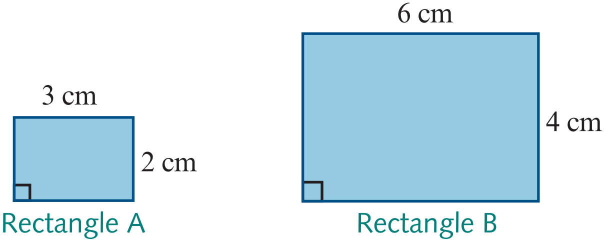


3H Similarity and scaling

Shapes that are similar have the same shape but are different sizes. These three frogs are **similar figures**.

Polygons (closed plane figures with straight sides), such as these rectangles, are similar if:

- corresponding angles are equal
- corresponding sides are proportional (each pair of corresponding side lengths are in the same ratio).



For example, the two rectangles above are similar as their corresponding angles are equal and their side lengths are in the same ratio.

$$\text{Ratio of side length} = 6 : 3 \text{ or } \frac{6}{3} = \frac{2}{1} = 2$$

$$\text{Ratio of side length} = 4 : 2 \text{ or } \frac{4}{2} = \frac{2}{1} = 2$$

When we enlarge or reduce a shape by a **scale factor**, the *original* and the *image* are similar. In the diagram above, rectangle A has been enlarged by a scale factor, $k = 2$, to give rectangle B.

We can also say that rectangle A has been scaled up to give rectangle B.

We can also compare the ratio of the areas of the rectangles.

$$\text{Area of rectangle A} = 6 \text{ cm}^2$$

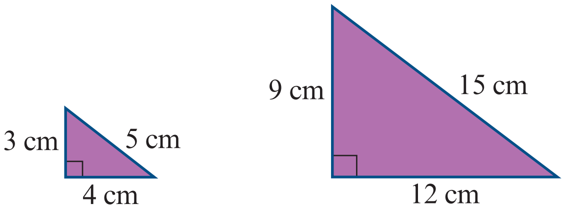
$$\text{Area of rectangle B} = 24 \text{ cm}^2$$

$$\text{Ratio of areas} = 24 : 6 = \frac{24}{6} = \frac{4}{1} = 4$$

The area of rectangle A has been enlarged by a scale factor, k^2, of 4 to give rectangle B. We notice that, as the length dimensions are enlarged by a scale factor of 2, the area is enlarged by a scale factor of $2^2 = 4$.

Scaling areas

When all the dimensions are multiplied by a scale factor of k, the area is multiplied by a scale factor of k^2.



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For example, the two previous triangles are similar as their corresponding side lengths are in the same ratio.

$$\text{Scale factor, } k = \text{Ratio of lengths} = \frac{15}{5} = \frac{9}{3} = \frac{12}{4} = \frac{3}{1} = 3$$

We would expect the area scale factor, k^2, or the ratio of the areas of the triangles to be $9 (= 3^2)$.

$$\text{Area of small triangle} = 6 \text{ cm}^2$$

$$\text{Area of large triangle} = 54 \text{ cm}^2$$

$$\text{Area scale factor, } k^2 = \text{Ratio of areas} = \frac{54}{6} = \frac{9}{1} = 9$$

Shapes can be scaled up or scaled down. When a shape is made larger, it is scaled up and when it is made smaller, it is scaled down.

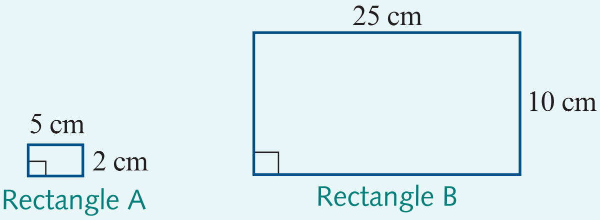
When working out scale factors, the numerator is the length of a side of the second shape and the denominator is the length of the corresponding side of the first (or original) shape.

Example 29 Finding the ratio (scale factor) of dimension and area

The rectangles shown are similar.

a Find the ratio of their side lengths.

b Find the ratio of their areas.



Solution

a **1** Since the rectangles are similar, their side lengths are in the same ratio. Compare the corresponding side lengths.

$$\frac{25}{5} = \frac{10}{2} = \frac{5}{1}$$

2 Write your answer.

Note: We can also say that the second rectangle has been scaled up by a factor of 5.

The ratio of the side lengths is $\frac{5}{1}$.

b **1** Since the dimensions are multiplied by a scale factor of 5, the area will be multiplied by a scale factor of 5^2. Square the ratio of the side lengths.

$$5^2 = 25$$

2 Write your answer.

Note: We can also say that the area of the second rectangle has been scaled up by a factor of 25.

The ratio of the areas is $\frac{25}{1}$.

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Example 30 Finding the scale factor

The two shapes shown are similar.

a Determine whether the first shape has been scaled up or down to give the second shape and find the scale factor.

b What is the scale factor for the areas?

Solution

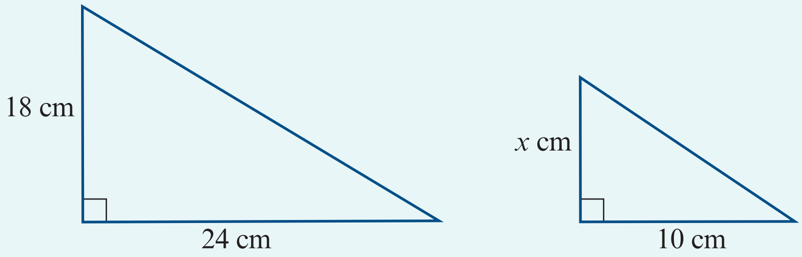
a **1** Since the shape is made smaller it has been scaled down.	Shape has been scaled down.
2 The shapes are similar so their side lengths are in the same ratio. Compare the corresponding side lengths.	$\frac{4}{6} = \frac{2}{3}$
3 Write your answer.	The scale factor is $\frac{2}{3}$.
b **1** The scale factor, k, for the shapes is $\frac{2}{3}$. As the scale factor for the area is k^2, square the value for k and evaluate.	$k = \frac{2}{3}$ $k^2 = \left(\frac{2}{3}\right)^2$ $= \frac{4}{9}$
2 Write your answer.	The scale factor for the area is $\frac{4}{9}$.

Example 31 Using a scale factor to find unknown values

The following two triangles are similar.

a Find the value of x.

b What is the scale factor?



Solution

a **1** As the triangles are similar, their side lengths are in the same ratio. Compare the corresponding side lengths.

Note: Make sure that corresponding sides are compared.

$$\frac{x}{18} = \frac{10}{24}$$

2 Solve for x. Multiply by 18.

$$\frac{x}{18} \times 18 = \frac{10}{24} \times 18$$

3 Evaluate and give your answer with correct units.

$$x = 7.5 \text{ cm}$$

b **1** Compare corresponding side lengths and simplify the fraction.

Remember: The numerator of the fraction is the length of a side of the second shape and the denominator is the length of the corresponding side of the first (or original) shape.

$$\frac{7.5}{18} = \frac{10}{24} = \frac{5}{12}$$

2 Write your answer.

Note: In this case, the triangle has been scaled down.

Triangle has been scaled down by a scale factor of $\frac{5}{12}$.

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Example 32 Using scaling in maps

A map has a scale of 1 : 20 000. If the measurement on the map between two towns is 5.4 cm, what is the actual distance between these two towns? Give your answer in kilometres, correct to two decimal places.

Solution

A scale of 1 : 20 000 means that 1 cm on the map represents 20 000 cm (or 200 m) on the ground.

Method 1

1 The map distance between the two towns is 5.4 cm, so multiply this distance by 20 000 to get the actual distance (in cm) between the two towns.

Map distance = 5.4 cm
Actual distance = 5.4 × 20 000
= 108 000 cm

2 Convert from centimetres to metres by dividing by 100.

108 000 cm ÷ 100 = 1080 m

3 Convert from metres to kilometres by dividing by 1000.

1080 m ÷ 1000 = 1.08 km

Method 2

1 Let x be the actual distance.
A scale of 1 : 20 000 can also be written as a scale factor of $\frac{1}{20\,000}$.
The ratio of the distance on the map to the actual distance will be the same as the scale factor of $\frac{1}{20\,000}$.
Write out the corresponding ratios.

Let x be the actual distance.

$$\frac{5.4}{x} = \frac{1}{20\,000}$$

2 Solve for x. (This can be done by cross-multiplying.)

$$5.4 \times 20\,000 = x$$
$$x = 108\,000 \text{ cm}$$

3 Convert to kilometres by dividing by 100 000.
Note: This is the same as dividing by 100 to convert to metres and then by 1000 to convert to kilometres.

108 000 cm ÷ 100 000 = 1.08 km

4 Write your answer.

The distance between the two towns is 1.08 km.

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Example 33 Using scaling in maps

A landscape gardener is planning the layout of a garden. She sketches a plan using a scale of 1:150.

a The largest feature of the garden is the rear fence, which is 30 metres long. How many centimetres long will it be on her plan?

b A shrub on her plan is drawn with a diameter of 3 cm. What is the diameter of the shrub in metres?

Solution

a	**1**	If the actual length of the fence is 30 m, then divide this distance by 150 to get the plan distance (in m).	Actual distance = 30 m Plan distance = 30 ÷ 150 = 0.2 m
	2	Convert from metres to centimetres by multiplying by 100.	0.2 m × 100 = 20 cm
	3	Write the answer.	The fence will be 20 cm long on the plan.
b	**1**	If the plan diameter of the shrub is 3 cm, then multiply this distance by 150 to get the actual diameter (in cm).	Plan distance = 3 cm Actual distance = 3 × 150 = 450 cm
	2	Convert from centimetres to metres by dividing by 100.	450 cm ÷ 100 = 4.5 m
	3	Write the answer.	The shrub has a diameter of 4.5 m.

Exercise 3H

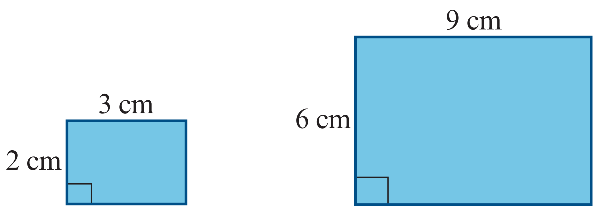
Similarity and ratios

Example 29

1 The following pairs of figures are similar.

i Find the ratio of their side lengths. **ii** Find the ratio of their areas.

a



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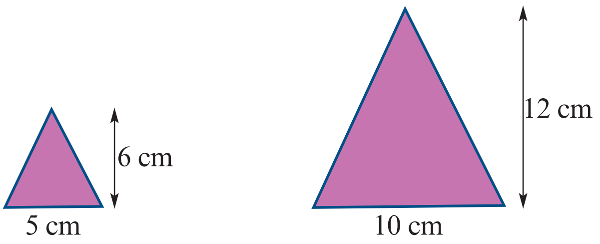

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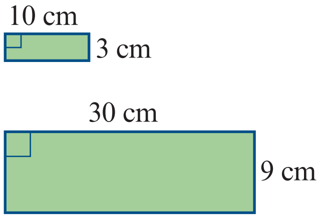
2 Which of the following pairs of figures are similar? For those that are similar, find the ratios of the corresponding sides.

a



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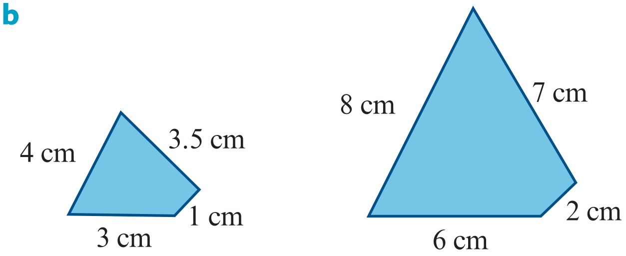

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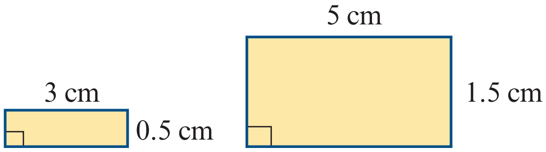


3 Which of the following pairs of figures are similar? State the ratios of the corresponding sides where relevant.

a Two rectangles 8 cm by 3 cm and 16 cm by 4 cm

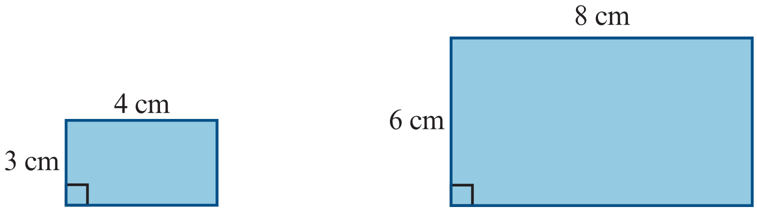
b Two rectangles 4 cm by 5 cm and 16 cm by 20 cm

c Two rectangles 4 cm by 6 cm and 2 cm by 4 cm

d Two rectangles 30 cm by 24 cm and 10 cm by 8 cm

e Two triangles, one with sides measuring 3 cm, 4 cm and 5 cm and the other 4.5 cm, 6 cm and 7.5 cm.

4 The following two rectangles are similar. Find the ratio of their areas.

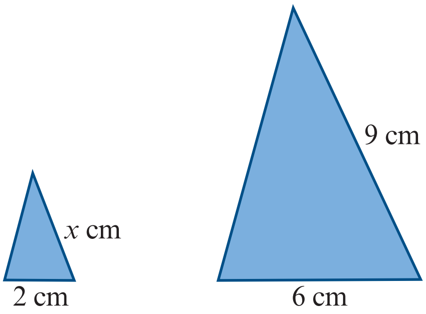


Example 31

5 The following triangles are similar.

a Find the value of x.

b Find the ratio of their areas.



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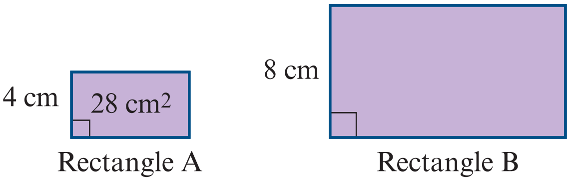
6 The two rectangles shown to the right are similar. The area of rectangle A is 28 cm^2. Find the area of rectangle B.



Applications of similarity and ratios

7 A photo is 12 cm by 8 cm. It is to be enlarged and then framed. If the dimensions are tripled, what will be the area of the new photo?

8 What is the scale factor if a photo has been enlarged from 15 cm by 9 cm to 25 cm by 15 cm? Give your answer correct to two decimal places.

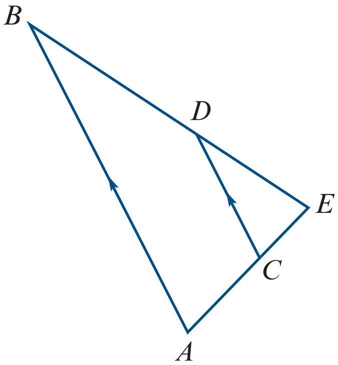
Example 32, 33

9 A scale on a map is 1 : 500 000.

a What is the actual distance between two towns if the distance on the map is 7.2 cm? Give your answer in kilometres.

b If the actual distance between two landmarks is 15 km, what would be the distance between them on the map?

10 In triangle *ABE*, point *C* lies on side *AE* and point *D* lies on side *BE*. The lines *CD* and *AB* are parallel. The length of *ED* is 5 cm, the length of *DB* is 7 cm and the length of *CD* is 6 cm. What is the length of *AB*?



CF

11 A plan of a rectangular shed has been drawn to a scale of 1 : 50.

a If the plan length of the shed is 8.5 cm, what is the actual length of the shed in metres?

b The actual perimeter of the shed is 25 m, what is the plan perimeter in centimetres?

12 A rectangular swimming pool has a width of 3.5 m and an area of 52.5 m^2. It is represented on a 1 : 25 scale plan. What is the plan area of the pool in cm^2?

13 A concreter reads a 1 : 120 scale plan showing a square helipad. The plan length of the side of the helipad is 12.5 cm.

a Find the actual side length of the helipad in metres.

b If the helipad is constructed to a depth of 15 cm, find the volume of concrete required in m^3.

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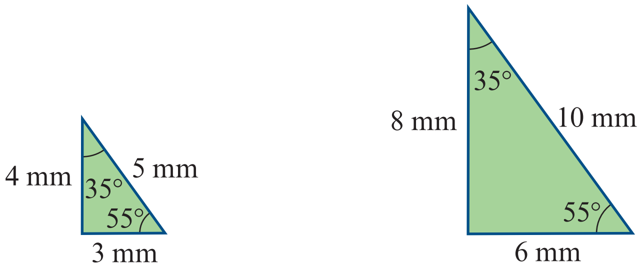
3I Similar triangles

In mathematics, two **triangles** are said to be **similar** if they have the same shape. As in the previous section, this means that corresponding angles are equal and the lengths of the corresponding sides are in the same ratio.

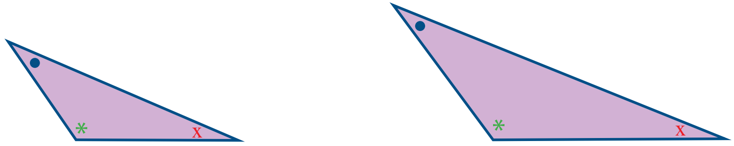
For example, these two triangles are similar.



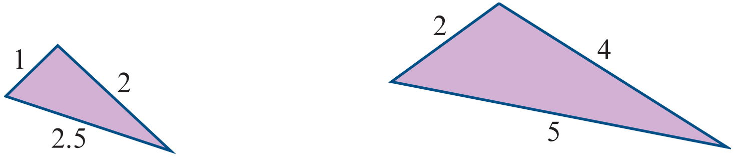
Two triangles can be tested for similarity by considering the following necessary conditions.

- Corresponding angles are equal (AA).

  Remember: If two pairs of corresponding angles are equal, then the third pair of corresponding angles is also equal.

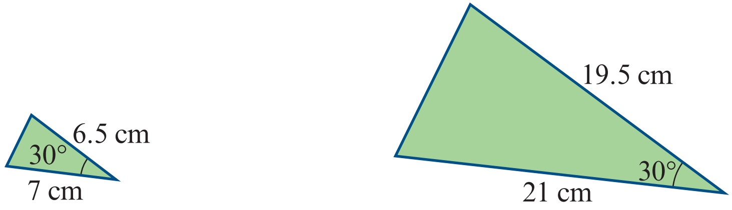


- Corresponding sides are in the same ratio (SSS).



$$\frac{5}{2.5} = \frac{4}{2} = \frac{2}{1} = 2$$

- Two pairs of corresponding sides are in the same ratio and the included corresponding angles are equal (SAS).



$$\frac{19.5}{6.5} = \frac{21}{7} = 3$$

Both triangles have an included corresponding angle of 30°.

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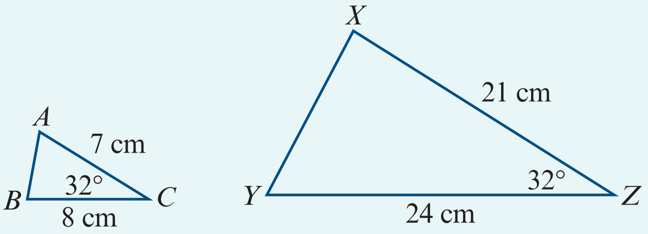
Example 34 Checking if triangles are similar

Explain why triangle *ABC* is similar to triangle *XYZ*.



Solution

1 Compare corresponding side ratios:

AC and *XZ* $\frac{XZ}{AC} = \frac{21}{7} = \frac{3}{1}$

BC and *YZ* $\frac{YZ}{BC} = \frac{24}{8} = \frac{3}{1}$

2 Triangles *ABC* and *XYZ* have an included corresponding angle.

32° is included and corresponding.

3 Write an explanation as to why the two triangles are similar.

Triangles ABC and XYZ are similar as they have two pairs of corresponding sides in the same ratio and the included corresponding angles are equal (SAS).

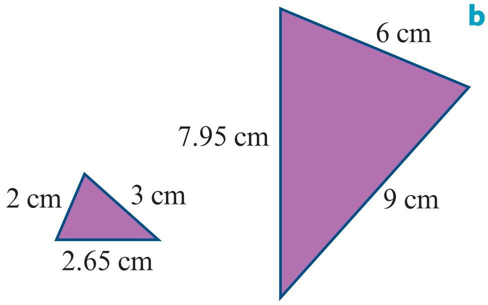
Exercise 3I

Similar triangles

Example 34

1 Three pairs of similar triangles are shown below. Explain why the triangles in each pair are similar.

a



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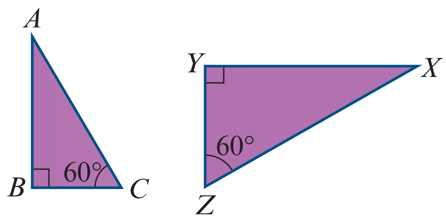
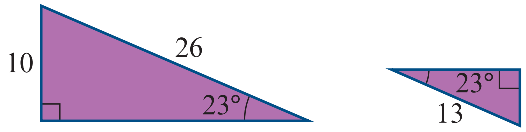


c



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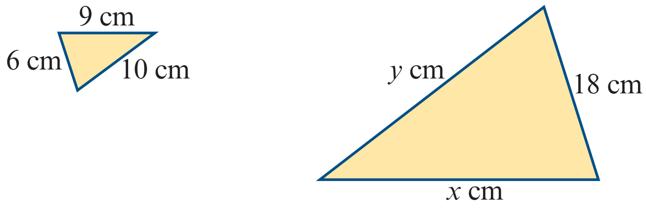
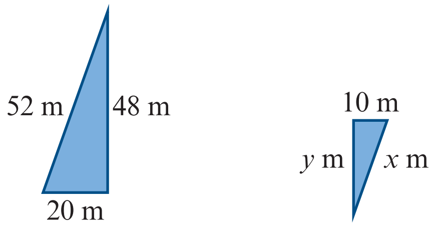
2 Calculate the missing dimensions, marked x and y, in these pairs of similar triangles.

a



b



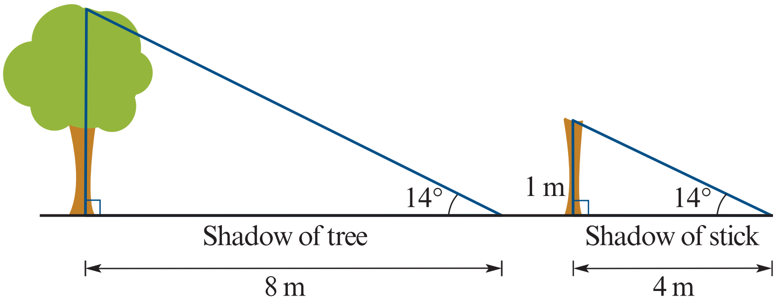
3 A triangle with sides 5 cm, 4 cm and 8 cm is similar to a larger triangle with a longest side of 56 cm.

- **a** Find the lengths of the larger triangle's other two sides.
- **b** Find the perimeter of the larger triangle.

Applications of similar triangles

4 A tree and a 1 m vertical stick cast their shadows at a particular time in the day. The shadow lengths are shown in the diagram below (*not* drawn to scale).

- **a** Give reasons why the two triangles shown are similar.
- **b** Find the scale factor for the side lengths of the triangles.
- **c** Find the height of the tree.



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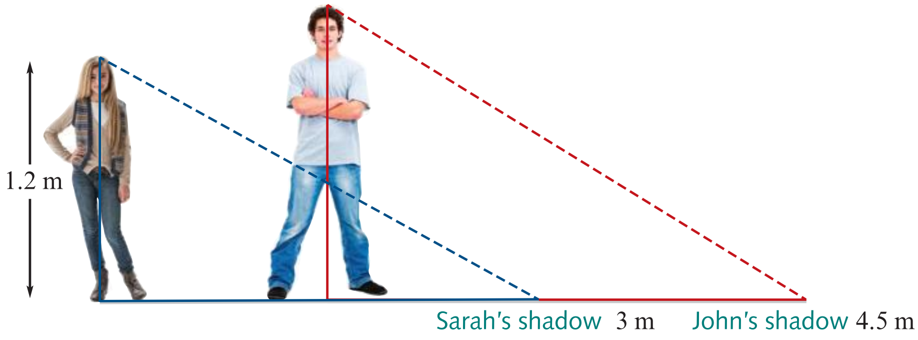
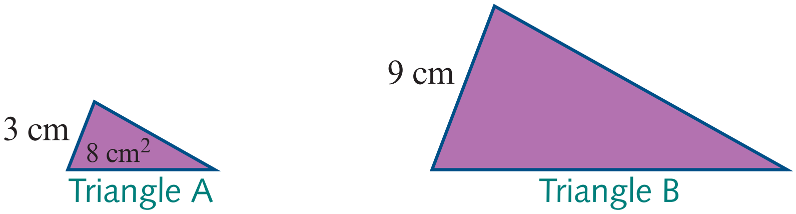
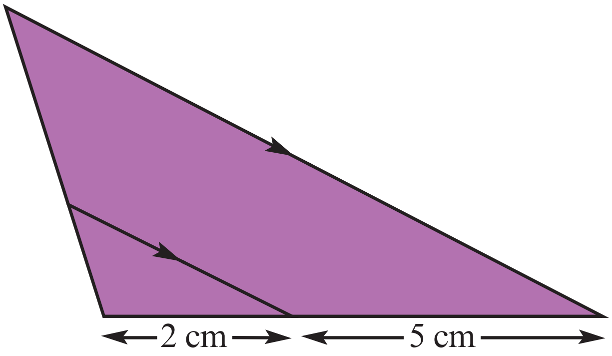
5 John and his younger sister, Sarah, are standing side by side. Sarah is 1.2 m tall and casts a shadow 3 m long. How tall is John if his shadow is 4.5 m long?



6 The area of triangle A is 8 cm^2. Triangle B is similar to triangle A. What is the area of triangle B?



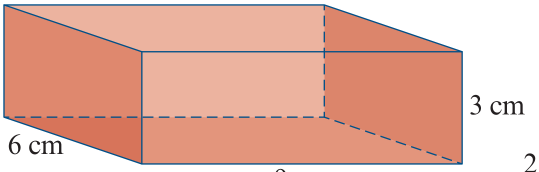
7 Given that the area of the small triangle in the following diagram is 2.4 cm^2, find the area of the larger triangle, correct to two decimal places.



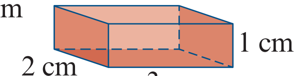
3J Similar solids

Two solids are similar if they have the same shape and the ratios of their corresponding linear dimensions are equal.

▶ Cuboids



Cuboid A



Cuboid B

The two cuboids are similar because:

- they are the same shape (both are cuboids)
- the ratios of the corresponding dimensions are the same.

$$\frac{\text{length of cuboid A}}{\text{length of cuboid B}} = \frac{\text{width of cuboid A}}{\text{width of cuboid B}} = \frac{\text{height of cuboid A}}{\text{height of cuboid B}}$$

$$\frac{6}{2} = \frac{9}{3} = \frac{3}{1} = \frac{3}{1}$$

$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{6 \times 9 \times 3}{2 \times 3 \times 1} = \frac{162}{6} = \frac{27}{1} = \frac{3^3}{1}$$

As the length dimensions are enlarged by a scale factor of 3, the volume is enlarged by a scale factor of $3^3 = 27$.

Scaling volumes

When all the dimensions are multiplied by a scale factor of k, the volume is multiplied by a scale factor of k^3.

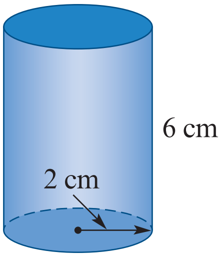
▶ Cylinders

These two cylinders are similar because:

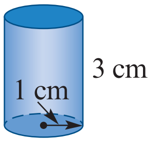
- they are the same shape (both are cylinders)
- the ratios of the corresponding dimensions are the same.

$$\frac{6}{3} = \frac{2}{1}$$

$$\frac{\text{height of cylinder A}}{\text{height of cylinder B}} = \frac{\text{radius of cylinder A}}{\text{radius of cylinder B}}$$



Cylinder A



Cylinder B

$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{\pi \times 2^2 \times 6}{\pi \times 1^2 \times 3} = \frac{24}{3} = \frac{8}{1} = \frac{2^3}{1}$$

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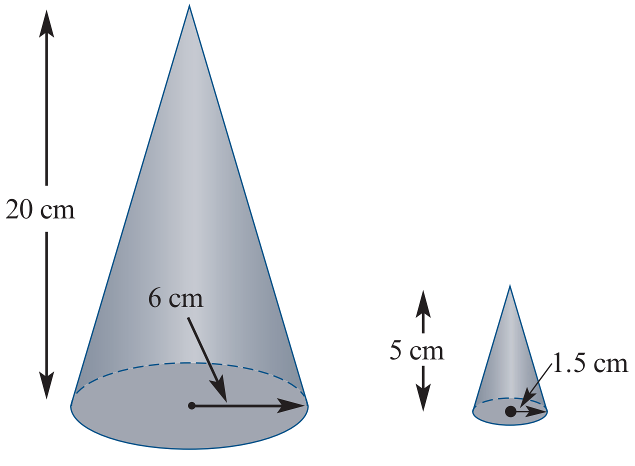
▶ Cones



Cone A Cone B

These two cones are similar because:

- they are the same shape (both are cones)
- the ratios of the corresponding dimensions are the same.

$$\frac{20}{5} = \frac{6}{1.5} = \frac{4}{1}$$

$$\frac{\text{height of cone A}}{\text{height of cone B}} = \frac{\text{radius of cone A}}{\text{radius of cone B}}$$

$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{\frac{1}{3} \times \pi \times 6^2 \times 20}{\frac{1}{3} \times \pi \times 1.5^2 \times 5} = \frac{720}{11.25}$$

$$= \frac{64}{1} = \frac{4^3}{1}$$

Example 35 Comparing volumes of similar solids

Two solids are similar such that the larger solid has all of its dimensions three times that of the smaller solid. How many times larger is the larger solid's volume?

Solution

1 Since all of the larger solid's dimensions are 3 times those of the smaller solid, the volume will be 3^3 times larger. Evaluate 3^3.

$3^3 = 27$

2 Write your answer.

The volume of the larger solid is 27 times the volume of the smaller solid.

Exercise 3J

Scaling volumes and surface areas

Example 35

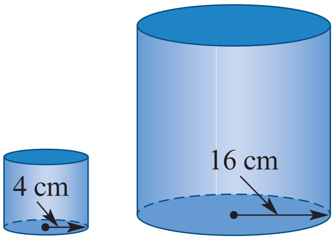
SF

1 Two cylindrical water tanks are similar such that the height of the larger tank is 3 times the height of the smaller tank. How many times larger is the volume of the larger tank than the volume of the smaller tank?

2 Two cylinders are similar and have radii of 4 cm and 16 cm respectively.

a What is the ratio of their heights?

b What is the ratio of their volumes?

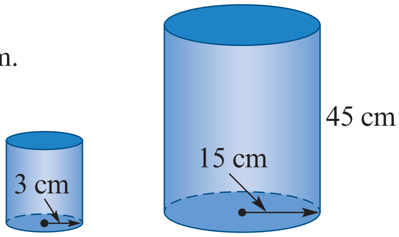


3 Find the ratio of the volumes of two cuboids whose sides are in the ratio $\frac{3}{1}$.

4 The radii of the bases of two similar cylinders are in the ratio $\frac{5}{1}$. The height of the larger cylinder is 45 cm.

a Calculate the height of the smaller cylinder.

b What is the ratio of the volumes of the two cylinders?

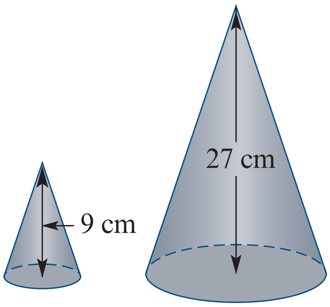


5 Two similar cones are shown at right. The ratio of their heights is $\frac{3}{1}$.

a Determine whether the smaller cone has been scaled up or down to give the larger cone.

b What is the volume scale factor?

c The volume of the smaller cone is 120 cm^3. Find the volume of the larger cone.



6 The radii of the bases of two similar cylinders are in the ratio 3 : 4. The height of the larger cylinder is 8 cm.

a Calculate the height of the smaller cylinder.

b What is the ratio of the volumes of the two cylinders?

7 A pyramid has a square base of side 4 cm and a volume of 16 cm^3.

a Calculate the height of the pyramid.

b Determine the height and the length of the side of the base of a similar pyramid with a volume of 1024 cm^3.

8 Two spheres have diameters of 6 cm and 12 cm respectively.

a Calculate the ratios of their surface areas.

b Determine the ratio of their volumes.

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3K Problem solving and modelling

Exercise 3K

1 Brandon is building a new rectangular deck that will measure 5 m by 3 m. He has to order concrete for the 20 stump holes that measure 350 mm by 350 mm by 500 mm.

a What amount of concrete (in m^3) will he need to order? Give your answer correct to two decimal places.

b If the decking boards are 90 mm wide, how many 3 m length boards should he order?

c The decking is to be stained. What is its area?

Brandon builds 4 wooden planter boxes to place on the decking. The boxes have a square base of length 40 cm and a height of 60 cm.

d What is the volume of dirt, in cubic metres, required to fill these boxes if he fills them up to 10 cm from the top?

e Brandon also stains the outside surface of the planter boxes, but not the base. What is the total surface area in m^2 of the planter boxes that he needs to stain? Give the answer correct to one decimal place.

2 Farmer Green owns an irregularly shaped paddock, *APQBRSTA*, as shown in the diagram.

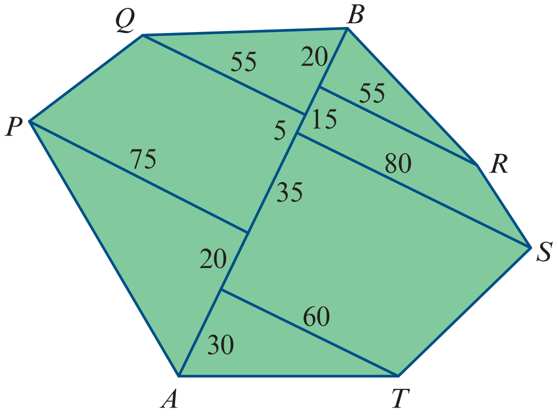
Starting at *A*, he has measured distances in metres along *AB* as well as the distances at right angles from *AB* to each other corner of his paddock.

Use this information to calculate:

a the total area of his paddock in hectares, correct to two decimal places

(Note: 1 hectare = 10 000 m^2)

b the length of fencing needed to enclose the paddock, correct to two decimal places.



3 A map is drawn to a scale of 1 : 20 000. A park drawn on the map has an area of 6 cm^2. Calculate the actual area of the park. Convert your answer into hectares. (1 hectare = 10 000 m^2)

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4 An architect uses a scale of 1 cm : 3 m for the plans of a house she is designing. On the plans, a room has an area of 3.3 cm^2.

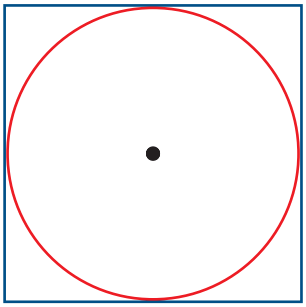
a Calculate the actual area of the room in m^2.

b Another room in the same house is to have an actual area of 3.5 m^2. What area, in cm^2, would this be on the plans?

5 A 1 m piece of wire is to be cut into two pieces, one of which is bent into a circle (red). The other piece is bent into a square around the circle (blue).

a What is the length of the side of the square (to the nearest centimetre)?

b What are the lengths of the two pieces of wire?



6 A steel pipe has an outside diameter of 100 mm and an inside diameter of 80 mm.

a What is the surface area of its cross section? Give your answer in square centimetres, correct to two decimal places.

b The pipe is 90 cm long.
What is the inside volume, in cubic centimetres, correct to one decimal place?

c What amount of water, in litres, can the pipe hold?

d The pipe needs to be coated on the outside with a protective material.
What is its surface area, correct to one decimal place?

7 A cubic box has sides of length 10 cm.

a How many times will you have to enlarge the box by a scale factor of two, before it is too big for a room that is 3 m high with length 4 m and width 3 m?

b What is the volume of the enlarged box?

c What is the surface area, in cm^2, of the enlarged box?

d Using the same cube of side length 10 cm, how many times can you reduce it by a scale factor of $\frac{1}{2}$ before it becomes smaller than 1 mm by 1 mm by 1 mm?

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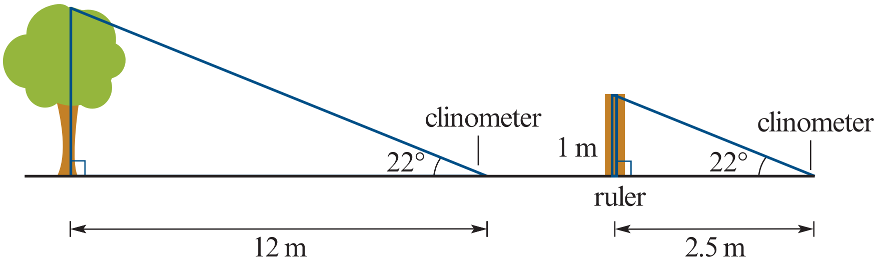
Using a clinometer

8 In this activity you will find the vertical height of a tree or other object using a clinometer, which measures angles. You can make one using directions given on the internet, such as those on the NRICH website (https://nrich.maths.org/5382). Alternatively, you can download a clinometer app for a smartphone – just search for ‘clinometer’ on an app store website. This converts the phone into a clinometer. You will also need a metre rule or a long straight stick or pole that you can measure and a long tape measure – at least 6 metres is best.

Using the clinometer from a convenient position, measure the angle between a horizontal line to the base of the tree and a sloping line to the top of the tree. Measure the distance from the clinometer to the tree. In the diagram below, the clinometer is 12 m from the tree and the angle to its top is 22°.

Set the metre rule vertically and place the clinometer level with the base of the ruler at a distance from it so the angle from the horizontal to the top of the rule is the same as the angle found to the top of the tree (22° in this case). In the diagram below right this has been done and the distance from the ruler was found to be 2.5 m.



Use similar triangles to find the height of your tree.

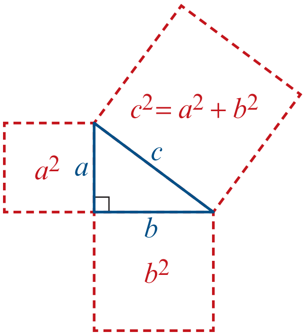
If you can’t do this activity, find the height of the tree in this diagram.

Note: The baselines of the two triangles does not have to be on the same level, but they have to be horizontal. If the ground to the tree is sloping, the clinometer has to be level with the base of the tree. The ruler could be set up on a level table.

Key ideas and chapter summary

Pythagoras' theorem

Pythagoras' theorem states that for any right-angled triangle, the sum of the areas of the squares of the two shorter sides (*a* and *b*) equals the area of the square of the hypotenuse (*c*): $c^2 = a^2 + b^2$



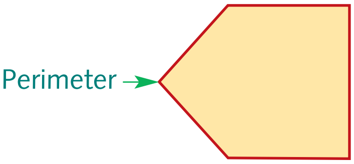
Conversion of measurements

$$\begin{tikzcd}
\mathit{Length} & \mathrm{km} \arrow[r, bend left, "\times 1000"] & \mathrm{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \mathrm{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \mathrm{mm} \arrow[l, bend left, "\div 10"] \\
\mathit{Area} & \mathrm{km}^2 \arrow[r, bend left, "\times 1000^2"] & \mathrm{m}^2 \arrow[l, bend left, "\div 1000^2"] \arrow[r, bend left, "\times 100^2"] & \mathrm{cm}^2 \arrow[l, bend left, "\div 100^2"] \arrow[r, bend left, "\times 10^2"] & \mathrm{mm}^2 \arrow[l, bend left, "\div 10^2"] \\
\mathit{Volume} & \mathrm{km}^3 \arrow[r, bend left, "\times 1000^3"] & \mathrm{m}^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & \mathrm{cm}^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & \mathrm{mm}^3 \arrow[l, bend left, "\div 10^3"]
\end{tikzcd}$$

1 kilolitre = 1000 litres
1 litre = 1000 millilitres
1 tonne = 1000 kilograms
1 kilogram = 1000 grams
1 gram = 1000 milligrams

Perimeter (P)

Perimeter is the distance around the edge of a two-dimensional shape.



Perimeter of rectangle

$P = 2l + 2w$

Circumference (C)

Circumference is the perimeter of a circle: $C = 2\pi r$

Area (A)

Area is the measure of the region enclosed by the boundaries of a two-dimensional shape.

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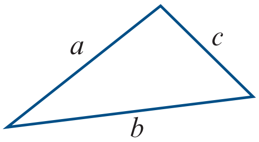
Area formulas

Area of rectangle = lw Area of parallelogram = bh

Area of triangle = $\frac{1}{2}bh$ Area of trapezium = $\frac{1}{2}(a+b)h$

Heron's rule

For the area of a triangle with three side lengths known:



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

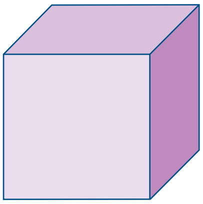
where

$$s = \frac{a+b+c}{2}$$

(s is the half perimeter)

Volume (V)

Volume is the amount of space occupied by a three-dimensional object.



- For prisms and cylinders:
  Volume = area of cross-section × height
- For pyramids and cones:
  Volume = $\frac{1}{3}$ × area of base × height

Volume formulas

Volume of cube = l^3 Volume of cuboid = lwh

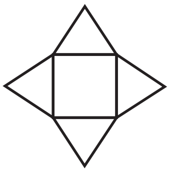
Volume of triangular prism = $\frac{1}{2}bhl$ Volume of cylinder = $\pi r^2 h$

Volume of cone = $\frac{1}{3}\pi r^2 h$ Volume of pyramid = $\frac{1}{3}lwh$

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area (SA)

Surface area is the total of the areas of all the surfaces of a solid. When finding surface area, it is often useful to draw the net of the shape.



Surface area formulas

Surface area of cylinder $2\pi r^2 + 2\pi rh$

Surface area of cone = $\pi r^2 + \pi rs$

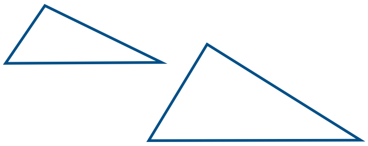
Surface area of sphere = $4\pi r^2$

Similar figures or solids

Similar figures or solids are the same shape but different sizes.

Similar triangles

Triangles are shown to be **similar** if:



- corresponding angles are equal (AA)
- corresponding sides are in the same ratio (SSS)
- two pairs of corresponding sides are in the same ratio and the included corresponding angles are equal (SAS).

Ratios of area and volume for similar shapes

When all the dimensions of similar shapes are multiplied by a scale factor of k, the areas are multiplied by a scale factor of k^2 and the volumes are multiplied by a scale factor of k^3.

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Skills check

Having completed this chapter you should be able to:

- understand and use Pythagoras' theorem to solve two-dimensional and three-dimensional problems
- convert units of measurement
- find the areas and perimeters of two-dimensional shapes
- find the volumes of common three-dimensional shapes
- find the volumes of pyramids, cones and spheres
- find the surface areas of three-dimensional shapes
- use tests for similarity for two-dimensional and three-dimensional figures.

Multiple-choice questions

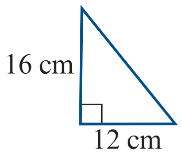
1 The three side measurements of four different triangles are given below. Which one is a right-angled triangle?

A 1, 2, 3 **B** 15, 20, 25 **C** 10, 10, 15 **D** 9, 11, 15 **E** 4, 5, 12

SF

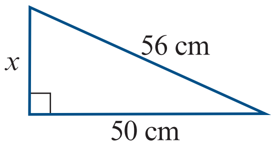
2 The length of the hypotenuse for the triangle shown is:

A 10.58 cm **B** 28 cm
C 7.46 cm **D** 20 cm
E 400 cm



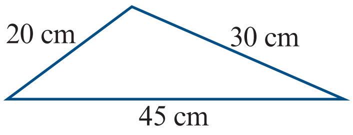
3 The value of x in the triangle shown is:

A 636 cm **B** 6 cm
C 75.07 cm **D** 25.22 cm
E 116 cm



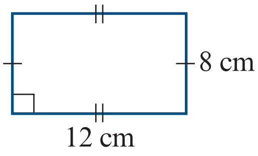
4 The perimeter of the triangle shown is:

A 450 cm **B** 95 cm
C 95 cm^2 **D** 90 cm
E 50 cm



5 The perimeter of the rectangle shown is:

A 40 cm **B** 20 cm
C 96 cm **D** 32 cm
E 28 cm



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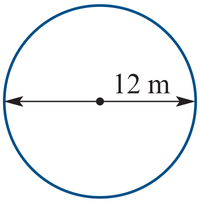
SF

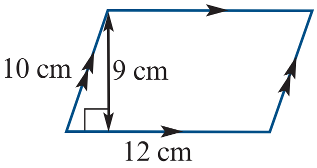
6 The circumference of a circle with diameter 12 m is closest to:

A 18.85 m **B** 37.70 m
C 453.29 m **D** 113.10 m
E 118.44 m



7 The area of the shape shown is:

A 90 cm^2 **B** 120 cm^2
C 108 cm^2 **D** 44 cm^2
E 180 cm^2



8 The area of a circle with radius 3 cm is closest to:

A 18.85 cm^2 **B** 28.27 cm^2 **C** 9.42 cm^2 **D** 113.10 cm^2 **E** 31.42 cm^2

9 The volume of a cube with side length 5 cm is:

A 60 cm^3 **B** 30 cm^3 **C** 150 cm^3 **D** 125 cm^3 **E** 625 cm^3

10 The volume of a box with length 11 cm, width 5 cm and height 6 cm is:

A 22 cm^3 **B** 44 cm^3 **C** 330 cm^3 **D** 302 cm^3 **E** 1650 cm^3

11 The volume of a sphere with radius 16 mm is closest to:

A 1072.33 mm^3 **B** 3217 mm^3 **C** 67.02 mm^3
D 268.08 mm^3 **E** 17 157.28 mm^3

12 The volume of a cone with base diameter 12 cm and height 8 cm is closest to:

A 1206.37 cm^3 **B** 904.78 cm^3 **C** 3619.11 cm^3
D 301.59 cm^3 **E** 1809.56 cm^3

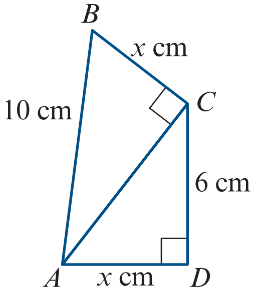
13 The volume of a cylinder with radius 3 m and height 4 m is closest to:

A 37.70 m^3 **B** 452.39 m^3 **C** 113.10 m^3
D 12 m^3 **E** 12.57 m^3

CF

14 In the diagram, angles *ACB* and *ADC* are right angles. If *BC* and *AD* each have a length of *x* cm, then *x* is closest to:

A 8.25 **B** 4 **C** 5
D 5.66 **E** 7.07



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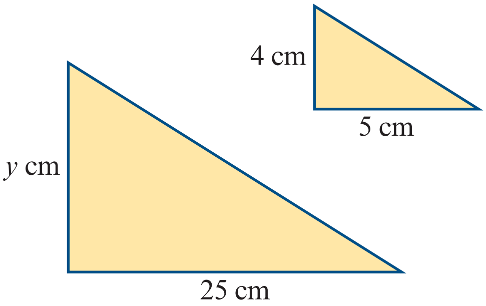
15 The two triangles shown are similar.
The value of y is:

- **A** 9 cm
- **B** 24 cm
- **C** 20 cm
- **D** 21 cm
- **E** 16 cm



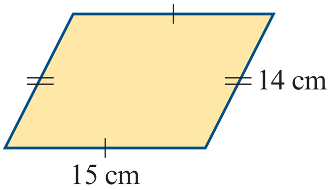
16 The diameter of a large sphere is 4 times the diameter of a smaller sphere. It follows that the ratio of the volume of the large sphere to the volume of the smaller sphere is:

- **A** 4 : 1
- **B** 8 : 1
- **C** 16 : 1
- **D** 32 : 1
- **E** 64 : 1

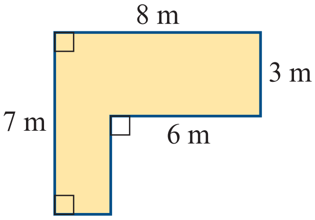
Short-answer questions

1 Find the perimeters of these shapes.

a



b

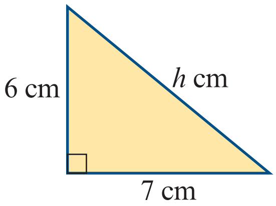


2 Find the perimeter of a square with side length 9 m.

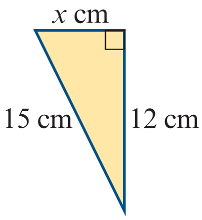
3 Find the perimeter of a rectangle with length 24 cm and width 10 cm.

4 Find the lengths of the unknown sides, correct to two decimal places, in the following triangles.

a

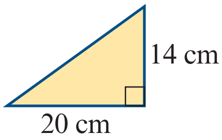


b

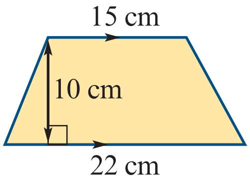


5 Find the areas of the following shapes.

a



b

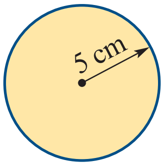


SF

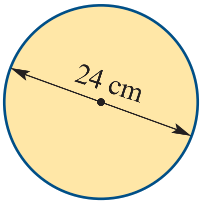
6 Find the surface area of a cube with side length 2.5 m.

7 Find the circumferences of the following circles, correct to two decimal places.

a



b



8 Find the areas of the circles in Question **7**, correct to two decimal places.

9 A soup can has a diameter of 7 cm and a height of 13.5 cm.

- **a** What area of metal sheeting, correct to two decimal places, is needed to make the can?
- **b** A paper label is made for the outside cylindrical shape of the can. How much paper, in m^2, is needed for 100 cans? Give your answer correct to two decimal places.
- **c** What is the capacity of one can? Give your answer in litres, correct to two decimal places.

10 A circular swimming pool has a diameter of 4.5 m and a depth of 2 m. How much water will the pool hold, to the nearest litre?

11 The radius of Earth is approximately 6400 km.

- **a** Calculate the surface area of Earth, correct to the nearest square kilometre.
- **b** Determine the volume of Earth, in standard form, correct to four significant figures.

12 The diameter of the base of an oil can in the shape of a cone is 12 cm and its height is 10 cm.

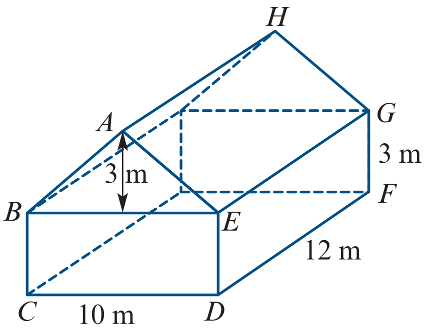
- **a** What is its volume in cubic centimetres, correct to two decimal places?
- **b** Find its capacity to the nearest millilitre.

13 A right pyramid with a square base of side length 8 m has a height of 3 m. Find the length of a sloping edge, correct to one decimal place.

CF

14 For the solid shown on the right, find correct to two decimal places:

- **a** the area of rectangle *BCDE*
- **b** the area of triangle *ABE*
- **c** the length *AE*
- **d** the area of rectangle *AEGH*
- **e** the total surface area.



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15 Find the volume of a rectangular prism with length 3.5 m, width 3.4 m and height 2.8 m.

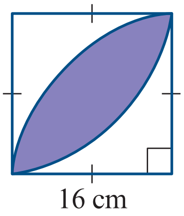
16 You are given a circle of radius r. The radius increases by a scale of factor of 2. By what factor does the area of the circle increase?

17 You are given a circle of diameter d. The diameter decreases by a scale factor of $\frac{1}{2}$. By how much does the area of the circle decrease?

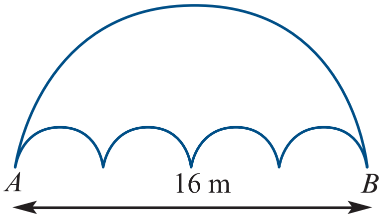
18 For the shaded region, find, correct to two decimal places:

a the perimeter

b the area of the shaded region shown.



19 Which is the shorter path from A to B? Is it along the four semicircles or along the larger semicircle? Give reasons for your answer.

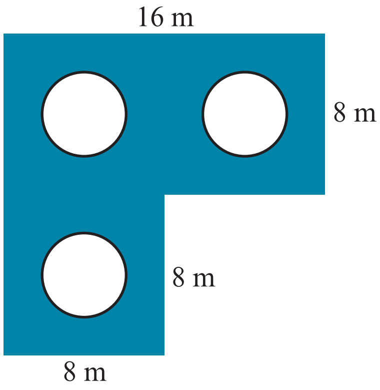


Extended-response questions

1 A lawn has three circular flowerbeds in it, as shown in the diagram. Each flowerbed has a radius of 2 m. A gardener has to mow the lawn and use a whipper-snipper to trim all the edges. Calculate:

a the area to be mown, correct to two decimal places.

b the length of the edges to be trimmed. Give your answer correct to two decimal places.



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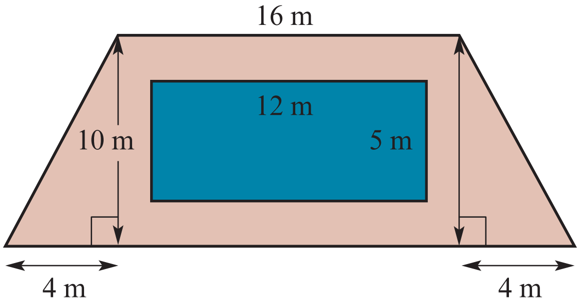
2 Chris and Gayle decide to build a swimming pool on their new house block. The pool will measure 12 m by 5 m and it will be surrounded by timber decking in the shape of a trapezium. A safety fence will surround the decking. The design layout of the pool and surrounding area is shown in the diagram.



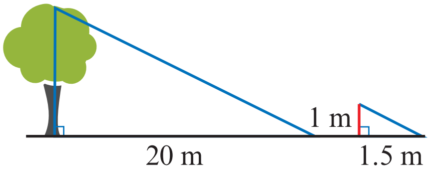
a What length of fencing is required? Give your answer correct to two decimal places.

b What area of timber decking is required?

c The pool has a constant depth of 2 m. What is the volume of the pool?

d The interior of the pool is to be painted white. What surface area is to be painted?

3 A biologist studying trees wanted to calculate the height of a particular tree. She placed a wooden stake vertically in the ground, sticking exactly 1 m above the ground, which was level with the base of the tree. The stake cast a shadow on the ground measuring 1.5 m. The tree cast a shadow of 20 m, as shown in the diagram below (*not* to scale). Calculate the height of the tree. Give your answer correct to two decimal places.



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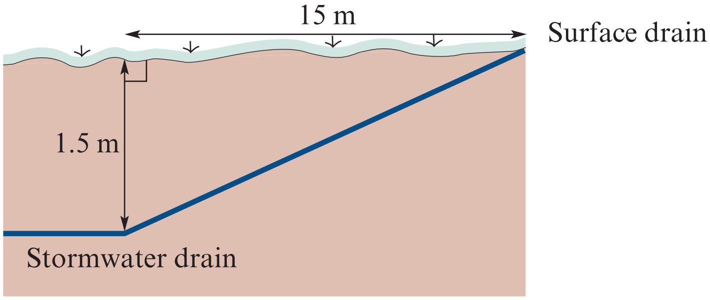
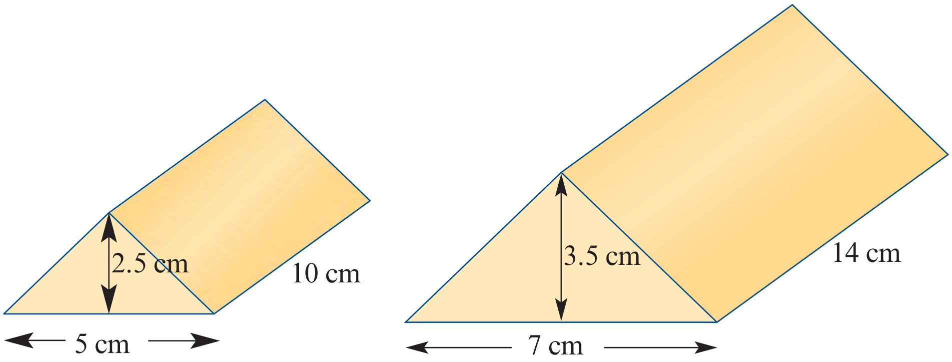
4 A builder is digging a trench for a cylindrical water pipe. From a drain at ground level, the water pipe goes 1.5 m deep, where it joins a stormwater drain. The horizontal distance from the surface drain to the stormwater drain is 15 m, as indicated in the diagram below (*not* to scale).



a Calculate the length of water pipe required to connect the surface drain to the stormwater drain, correct to two decimal places.

b If the radius of the water pipe is 20 cm, what is the volume of the water pipe? Give your answer correct to two decimal places.

5 Two similar triangular prisms are shown below.

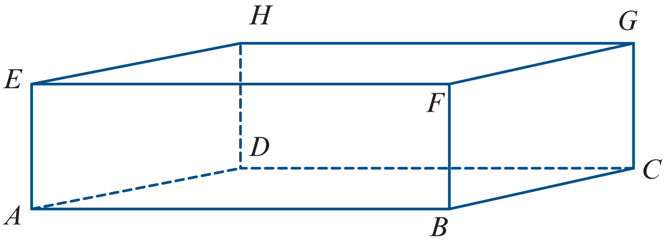


a Find the ratio of their surface areas.

b Find the ratio of their volumes.

c What is the volume of the smaller prism to the nearest cm^3?

6 The length of a rectangular prism is eight times its height. The width is four times the height. The length of the diagonal between two opposite vertices (AG) is 36 cm. Find the volume of the prism.



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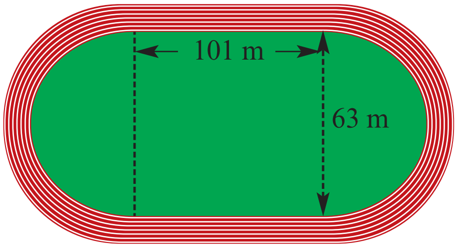
7 The volume of a cone of height 28.4 cm is 420 cm^3. Find the height of a similar cone whose volume is 120 cm^3, correct to two decimal places.

8 An athletics track is made up of a straight stretch of 101 m and two semicircles on the ends as shown in the diagram. There are 6 lanes each 1 metre wide.



a What is the total distance, to the nearest metre, around the inside lane?

b If 6 athletes run around the track keeping to their own lane, how far, to the nearest metre, would each athlete run?

c Draw a diagram and indicate at which point each runner should start so that they all run the same distance.

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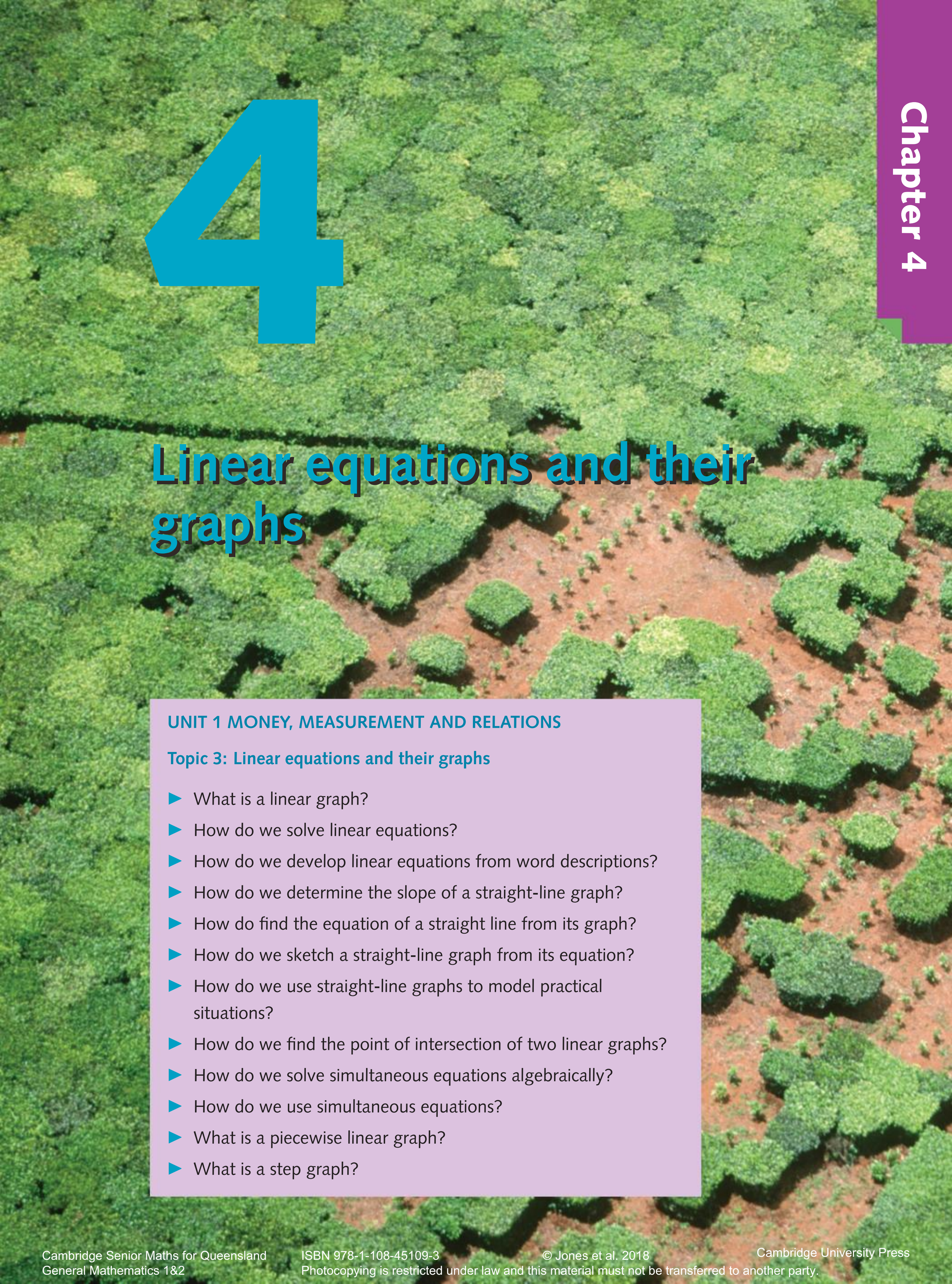

4

Linear equations and their graphs

UNIT 1 MONEY, MEASUREMENT AND RELATIONS

Topic 3: Linear equations and their graphs

- What is a linear graph?
- How do we solve linear equations?
- How do we develop linear equations from word descriptions?
- How do we determine the slope of a straight-line graph?
- How do find the equation of a straight line from its graph?
- How do we sketch a straight-line graph from its equation?
- How do we use straight-line graphs to model practical situations?
- How do we find the point of intersection of two linear graphs?
- How do we solve simultaneous equations algebraically?
- How do we use simultaneous equations?
- What is a piecewise linear graph?
- What is a step graph?

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Introduction

Many everyday situations can be described and investigated using a linear graph and its equation. Examples include the depreciating value of a newly purchased car, and the short-term growth of a newly planted tree. In this chapter, you will revise the properties of linear graphs and their equations and apply these ideas to modelling linear growth and decay in the real world.

4A Solving linear equations with one unknown

Practical applications of mathematics often involve the need to be able to solve **linear equations**. An *equation* is a mathematical statement that says that two things are equal. For example, these are all equations:

$$x - 3 = 5 \qquad 2w - 5 = 17 \qquad 3m = 24$$

Linear equations come in many different forms in mathematics but are easy to recognise because the powers on the unknown values are always 1. For example:

- $m - 4 = 8$ is a linear equation, with unknown value m
- $3x = 18$ is a linear equation, with unknown value x
- $4y - 3 = 17$ is a linear equation, with unknown value y
- $a + b = 0$ is a linear equation, with unknown values a and b
- $x^2 + 3 = 9$ is *not* a linear equation (the power of x is 2 not 1), with unknown value x
- $c = 16 - d^2$ is *not* a linear equation (the power of d is 2), with unknowns c and d.

The process of finding the unknown value is called *solving the equation*. When solving an equation, *opposite* (or *inverse*) operations are used so that the unknown value to be solved is the only **term** remaining on one side of the equation. Opposite or inverse operations are indicated in the table below.

Operation	+	−	×	÷	x^2 (power of 2, square)	$\sqrt{x}$ (square root)
Opposite or inverse operation	−	+	÷	×	$\sqrt{x}$ (square root)	x^2 (power of 2, square)

Remember: The equation must remain *balanced*. To balance an equation add or subtract the *same* number on *both* sides of the equation or multiply or divide *both* sides of the equation by the *same* number.

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Example 1 Solving a linear equation

Solve the equation $x + 6 = 10$.

Solution

Method 1: By inspection

Write the equation.

$$x + 6 = 10$$

What needs to be added to 6 to make 10?

The answer is 4.

$$\therefore x = 4$$

Method 2: Inverse operations

This method requires the equation to be 'undone', leaving the unknown value by itself on one side of the equation.

1 Write the equation.

$$x + 6 = 10$$

2 Subtract 6 from both sides of the equation. This is the opposite process to adding 6.

$$x + 6 - 6 = 10 - 6$$
$$\therefore x = 4$$

3 Check your answer by substituting the found value for x into the original equation. If each side gives the same value, the solution is correct.

$$\begin{aligned} \text{LHS} &= x + 6 \\ &= 4 + 6 \\ &= 10 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

Example 2 Solving a linear equation

Solve the equation $3y = 18$.

Solution

1 Write the equation.

$$3y = 18$$

2 The opposite process of multiplying by 3 is dividing by 3. Divide both sides of the equation by 3.

$$\frac{3y}{3} = \frac{18}{3}$$
$$\therefore y = 6$$

3 Check that the solution is correct by substituting $y = 6$ into the original equation.

$$\begin{aligned} \text{LHS} &= 3y \\ &= 3 \times 6 \\ &= 18 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

Example 3 Solving a linear equation

Solve the equation $4(x - 3) = 24$.

Solution

Method 1

1 Write the equation.

$$4(x - 3) = 24$$

2 Expand the brackets.

$$4x - 12 = 24$$

3 Add 12 to both sides of the equation.

$$4x - 12 + 12 = 24 + 12$$

4 Divide by 4.

$$\begin{aligned} 4x &= 36 \\ \frac{4x}{4} &= \frac{36}{4} \\ \therefore x &= 9 \end{aligned}$$

5 Check that the solution is correct by substituting $x = 9$ into the original equation (see 4 below).

Method 2

1 Write the equation.

$$4(x - 3) = 24$$

2 Divide both sides by 4.

$$\begin{aligned} \frac{4(x - 3)}{4} &= \frac{24}{4} \\ x - 3 &= 6 \end{aligned}$$

3 Add 3.

$$\begin{aligned} x - 3 + 3 &= 6 + 3 \\ \therefore x &= 9 \end{aligned}$$

4 Check that the solution is correct by substituting $x = 9$ into the original equation.

$$\begin{aligned} \text{LHS} &= 4(x - 3) \\ &= 4(9 - 3) \\ &= 4 \times 6 \\ &= 24 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

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Example 4

Solve the equations.

a $6 - 5b = 8$ **b** $3.4q + 2.5 = 2.2q + 3.7$

Solution

a

1 Write the equation.

$$6 - 5b = 8$$

2 Subtract 6 from both sides of the equation.

$$6 - 5b - 6 = 8 - 6$$
$$-5b = 2$$

3 Divide both sides of the equation by −5.

$$\frac{-5b}{-5} = \frac{2}{-5}$$
$$b = -\frac{2}{5}$$

4 Check that the solution is correct by substituting $b = -\frac{2}{5}$ into the original equation.

$$\text{LHS} = 6 - 5 \times -\frac{2}{5}$$
$$= 6 + 2$$
$$= 8 = \text{RHS}$$

$\therefore$ Solution is correct.

b

1 Write the equation.

$$3.4q + 2.5 = 2.2q + 3.7$$

2 Subtract $2.2q$ from both sides of the equation to collect like terms.

$$3.4q + 2.5 - 2.2q = 2.2q + 3.7 - 2.2q$$
$$1.2q + 2.5 = 3.7$$

3 Subtract 2.5 from both sides of the equation.

$$1.2q + 2.5 - 2.5 = 3.7 - 2.5$$
$$1.2q = 1.2$$

4 Divide both sides by 1.2.

$$\frac{1.2q}{1.2} = \frac{1.2}{1.2}$$
$$q = 1$$

5 Check that the solution is correct by substituting $q = 1$ into both sides of the equation.

$$\text{LHS} = 3.4q + 2.5$$
$$= 3.4 \times 1 + 2.5$$
$$= 5.9$$
$$\text{RHS} = 2.2q + 3.7$$
$$= 2.2 \times 1 + 3.7$$
$$= 5.9$$
$$\text{LHS} = \text{RHS}$$

$\therefore$ Solution is correct.

Exercise 4A

Example 1

1 Solve the following linear equations.

a $x + 6 = 15$
b $y + 11 = 26$
c $t + 5 = 10$
d $m - 5 = 1$
e $g - 3 = 3$
f $f - 7 = 12$
g $f + 5 = 2$
h $v + 7 = 2$
i $x + 11 = 10$
j $g - 3 = -2$
k $b - 10 = -5$
l $m - 5 = -7$
m $2 + y = 8$
n $6 + e = 9$
o $7 + h = 2$
p $3 + a = -1$
q $4 + t = -6$
r $8 + s = -3$
s $9 - k = 2$
t $5 - n = 1$
u $3 - a = -5$
v $10 - b = -11$

Example 2

2 Solve the following linear equations.

a $5x = 15$
b $3g = 27$
c $9n = 36$
d $2x = -16$
e $6j = -24$
f $4m = 28$
g $2f = 11$
h $2x = 7$
i $3y = 15$
j $3s = -9$
k $-5b = 25$
l $4d = -18$
m $\frac{r}{3} = 4$
n $\frac{q}{5} = 6$
o $\frac{x}{8} = 6$
p $\frac{t}{-2} = 6$
q $\frac{h}{-8} = -5$
r $\frac{m}{-3} = -7$
s $\frac{14}{a} = 7$
t $\frac{24}{f} = -12$
u $2a + 15 = 27$
v $\frac{y}{4} - 10 = 0$
w $13 = 3r - 11$
x $\frac{x + 1}{3} = 2$
y $\frac{3m}{4} = 6$
z $\frac{2x - 1}{3} = 4$

Example 3

3 Solve the following linear equations.

a $2(y - 1) = 6$
b $8(x - 4) = 56$
c $3(g + 2) = 12$
d $3(4x - 5) = 21$
e $8(2x + 1) = 16$
f $3(5m - 2) = 12$
g $\frac{2(a - 3)}{5} = 6$
h $\frac{4(r + 2)}{6} = 10$

Example 4

4 Solve these equations by first collecting all like terms.

a $2x = x + 5$
b $2a + 1 = a + 4$
c $4b - 10 = 2b + 8$
d $7 - 5y = 3y - 17$
e $3(x + 5) - 4 = x + 11$
f $6(c + 2) = 2(c - 2)$
g $2f + 3 = 2 - 3(f + 3)$
h $5(1 - 3y) - 2(10 - y) = -10y$

5 Solve the following linear equations.

a $3a + 5 = 16$
b $4b + 3 = 28$
c $2w + 5 = 10$
d $7c - 2 = 14$
e $3y - 5 = 18$
f $4f - 1 = 9$
g $3 + 2h = 16$
h $2 - 3k = 6$
i $2.5x - 1.4 = 3.2$
j $2 - 0.2y = 1.5$
k $2x - 3 = 4x - 4$
l $4 - 3x = 5x + 2$
m $1.8d + 2.7 = 1.4d - 1.5$
n $2(y + 3.5) = 15$
o $3(y + 1.2) = 5.4$

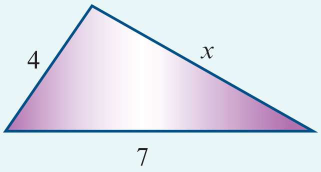
4B Developing a linear equation from a word description

In many practical problems, we often need to set up a linear equation before finding the solution to a problem. Some practical examples are given below showing how a linear equation is set up and then solved.

Example 5 Setting up a linear equation

Find an equation for the perimeter of the triangle shown.

Note: Perimeter is the distance around the outside of a shape.



Solution

1 Choose a variable to represent the perimeter. — Let P be the perimeter.

2 Add up all sides of the triangle and let them equal the perimeter, P.

$P = 4 + 7 + x$

$P = 11 + x$

3 Write your answer. — The required equation is $P = 11 + x$

Example 6 Setting up and solving a linear equation

If 11 is added to a certain number, the result is 25. Find the number.

Solution

1 Choose a variable to represent the number. — Let n be the number.

2 Using the information, write an equation.

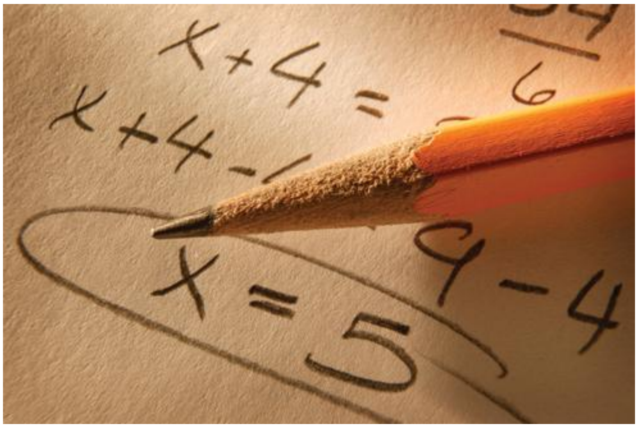
$n + 11 = 25$

3 Solve the equation by subtracting 11 from both sides of the equation.

$n + 11 - 11 = 25 - 11$

$\therefore n = 14$

4 Write your answer. — The required number is 14.



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Example 7 Setting up and solving a linear equation

At a recent show, Chris spent \$100 on 8 showbags, each costing the same price.

a Using x as the cost of one showbag, write an equation showing the cost of 8 showbags.

b Use the equation to find the cost of one showbag.

Solution

a **1** Write the cost of one showbag using the variable given.

Let x be the cost of one showbag.

2 Use the information to write an equation.
Remember: $8 \times x = 8x$

$$8x = 100$$

b **1** Write the equation.

$$8x = 100$$

2 Solve the equation by dividing both sides of the equation by 8.

$$\frac{8x}{8} = \frac{100}{8}$$

$$\therefore x = 12.5$$

3 Write your answer.

The cost of one showbag is \$12.50.

Example 8 Setting up and solving a linear equation

A car rental company has a fixed charge of \$110 plus \$84 per day for the hire of a car. The Brown family has budgeted \$650 for the hire of a car during the family holiday. For how many days can the car be hired?

Solution

1 Choose a variable (d) for the number of days for which the car is hired. Use the information to write an equation.

Let d be the number of days for which the car is hired.

$$110 + 84d = 650$$

2 Solve the equation.
First, subtract 110 from both sides of the equation.
Then divide both sides of the equation by 84.

$$110 + 84d - 110 = 650 - 110$$

$$84d = 540$$

$$\frac{84d}{84} = \frac{540}{84}$$

$$\therefore d = 6.428$$

3 Write your answer in terms of complete days.

Car hire works on a daily rate so 6.428 days is not an option. We therefore round down to 6 days to ensure that the Brown family stays within their budget of \$650. The Brown family could hire a car for 6 days.

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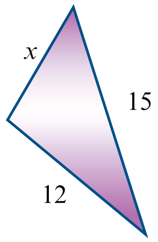
Exercise 4B

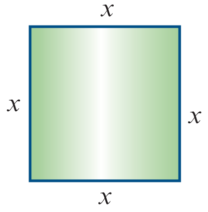
Example 5

1 Find an expression for the perimeter, P, of each of the following shapes.

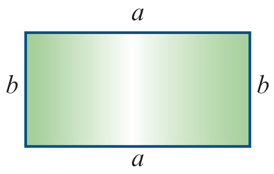
a



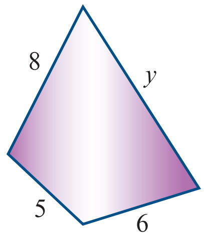
b



c

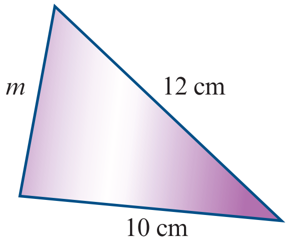


d



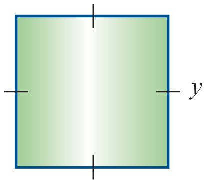
2 **a** Write an expression for the perimeter of the triangle shown.

b If the perimeter, P, of the triangle is 30 cm, solve the equation to find the value of m.



3 **a** Write an equation for the perimeter of the square shown.

b If the perimeter is 52 cm, what is the length of one side?



Example 6

4 Seven is added to a number and the result is 15.

a Write an equation using n to represent the number.

b Solve the equation for n.

5 Five is added to twice a number and the result is 17. What is the number?

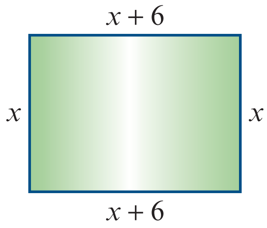
6 When a number is doubled and 15 is subtracted, the result is 103. Find the number.

7 The perimeter of a rectangle is 84 cm. The length of the rectangle is 6 cm longer than the width, as shown in the diagram.

a Write an expression for the perimeter, P, of the rectangle.

b Find the value of x.

c Find the lengths of the sides of the rectangle.



Example 7, 8

8 Year 11 students want to run a social. The cost of hiring a band is \$820 and they are selling tickets for \$15 per person. The profit, P, is found by subtracting the band hire cost from the money raised from selling tickets. The students would like to make a profit of \$350. Use the information to write an equation, and then solve the equation to find how many tickets they need to sell.

SF

9 The cost for printing cards at the Stamping Printing Company is \$60 plus \$2.50 per card. Kate paid \$122.50 to print invitations for her party. How many invitations were printed?

10 A raffle prize of \$1000 is divided between Anne and Barry so that Anne receives 3 times as much as Barry. How much does each receive?

CF

11 Bruce cycles x kilometres, then walks half as far as he cycles. If the total distance covered is 45 km, find the value of x.

12 Amy and Ben live 17.2 km apart. They cycle to meet each other. Ben travels at 12 km/h and Amy travels at 10 km/h.

a How long, to the nearest minute, until they meet each other?

b What distance, correct to one decimal place, have they each travelled?

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4C Developing a formula: setting up linear equations in two unknowns

Skillsheet It is often necessary to develop formulas so that problems can be solved. Constructing a formula is similar to developing an equation from a description.

Example 9 Setting up and solving a linear equation in two unknowns

Sausage rolls cost \$1.30 each and party pies cost 75 cents each.

a Construct a formula for finding the cost, C dollars, of buying x sausage rolls and y party pies.

b Find the cost of 12 sausage rolls and 24 party pies.

Solution

a **1** Work out a formula using x.

One sausage roll costs \$1.30.

Two sausage rolls cost $2 \times \$1.30 = \$2.60$.

Three sausage rolls cost $3 \times \$1.30 = \$3.90$ etc.

Write a formula using x.

x sausage rolls cost $x \times 1.30 = 1.3x$

2 Work out a formula using y.

One party pie costs \$0.75.

Two party pies cost $2 \times \$0.75 = \$1.50$.

Three party pies cost $3 \times \$0.75 = \$2.25$ etc.

Write a formula using y.

y party pies cost $y \times 0.75 = 0.75y$

3 Combine to get a formula for total cost, C.

$C = 1.3x + 0.75y$

b **1** Write the formula for C. $C = 1.3x + 0.75y$

2 Substitute $x = 12$ and $y = 24$ into the formula. $C = 1.3 \times 12 + 0.75 \times 24$

3 Evaluate. $C = 33.6$

4 Give your answer in dollars and cents. *The total cost for 12 sausage rolls and 24 party pies is \$33.60.*

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Exercise 4C

SF

Example 9

1 Balloons cost 50 cents each and streamers costs 20 cents each.

- **a** Construct a formula for the cost, C, of x balloons and y streamers.
- **b** Find the cost of 25 balloons and 20 streamers.

2 Tickets to a concert cost \$40 for adults and \$25 for children.

- **a** Construct a formula for the total amount, C, paid by x adults and y children.
- **b** How much money altogether was paid by 150 adults and 315 children?

3 At the football canteen, chocolate bars cost \$1.60 and muesli bars cost \$1.40.

- **a** Construct a formula to show the total money, C, made by selling x chocolate bars and y muesli bars.
- **b** How much money would be made if 55 chocolate bars and 38 muesli bars were sold?

4 At the bread shop, custard tarts cost \$1.75 and iced doughnuts \$0.70 cents.

- **a** Construct a formula to show the total cost, C, if x custard tarts and y iced doughnuts are purchased.
- **b** On Monday morning, Mary bought 25 custard tarts and 12 iced doughnuts. How much did it cost her?

5 At the beach café, Marion takes orders for coffee and milkshakes. A cup of coffee costs \$3.50 and a milkshake costs \$5.00.

- **a** Let x = number of coffees ordered and y = number of milkshakes ordered. Using x (coffee) and y (milkshakes), write a formula showing the cost, C, of the number of coffees and milkshakes ordered.
- **b** Marion took orders for 52 cups of coffee and 26 milkshakes. How much money did this make?

6 Joe sells budgerigars for \$30 and parrots for \$60.

- **a** Write a formula showing the money, C, made by selling x budgerigars and y parrots.
- **b** Joe sold 28 parrots and 60 budgerigars. How much money did he make?

CF

7 James has been saving fifty-cent and twenty-cent pieces.

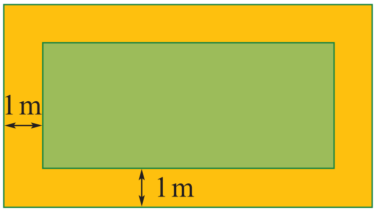
- **a** If James has x fifty-cent pieces and y twenty-cent pieces, write a formula to show the number, N, of coins that James has.
- **b** Write a formula to show the value, V dollars, of James' collection.
- **c** When James counts his coins, he has 45 fifty-cent pieces and 77 twenty-cent pieces. How much money does he have in total?

8 A tennis coach buys four cans of tennis balls and empties them into a large container that already has twelve balls in it. Altogether, there are now 32 tennis balls. How many tennis balls were in each can?

9 Maria is five years older than George. The sum of their ages is 37. What are their ages?

CU

10 A rectangular lawn is twice as long as it is wide. The lawn has a path 1 metre wide around it. The length of the perimeter of the outside of the path is 48 metres. What is the width of the lawn? Give your answer correct to the nearest centimetre.



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4D Drawing straight-line graphs

Plotting straight-line graphs

Relations defined by equations such as:

$$y = 2x + 1 \qquad y = 3x - 2 \qquad y = -5x + 10 \qquad y = 6x$$

are called *linear* relations because they generate *straight-line graphs*.

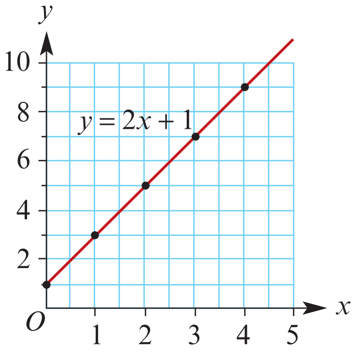
For example, consider the relation $y = 2x + 1$. To plot this graph, we can form a table.

x	0	1	2	3	4
y	1	3	5	7	9

We can then plot the values from the table on a set of axes, as shown opposite.

The points appear to lie on a straight line.

A ruler can then be used to draw in this straight line to give the graph of $y = 2x + 1$.



Example 10 Constructing a graph from a table of values

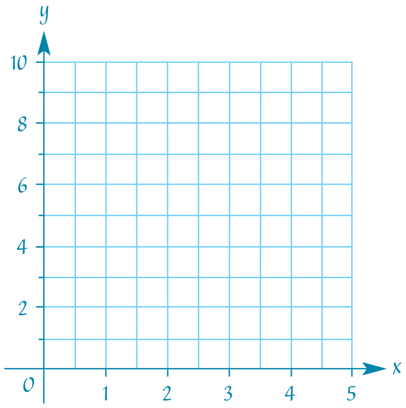
Plot the graph of $y = -2x + 8$ by forming a table of values of y using $x = 0, 1, 2, 3, 4$.

Solution

1 Set up table of values.
When $x = 0$, $y = -2 \times 0 + 8 = 8$.
When $x = 1$, $y = -2 \times 1 + 8 = 6$, and so on.

x	0	1	2	3	4
y	8	6	4	2	0

2 Draw, label and scale a set of axes to cover all values.



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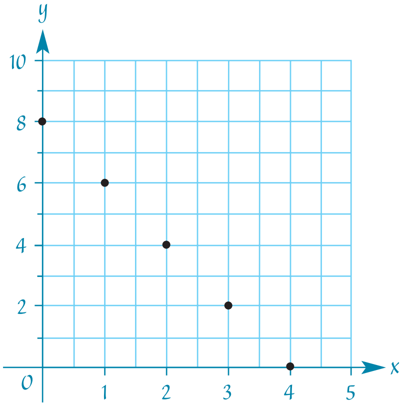
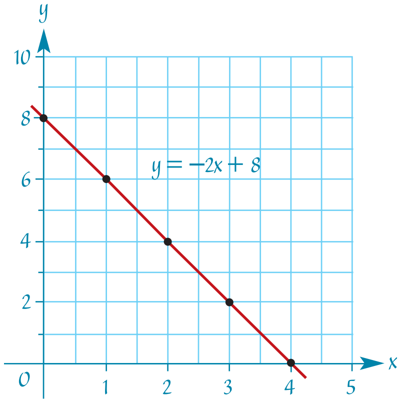
3 Plot the values in the table on the graph by marking each point with a dot (●). The first point is (0, 8). The second point is (1, 6), and so on.
(The table of values is copied here so you don't have to look back for it:)

x	0	1	2	3	4
y	8	6	4	2	0



4 The points appear to lie on a straight line. Use a ruler to draw in the straight line. Label the line $y = -2x + 8$.



Using technology to draw straight-line graphs

Several types of technology can be used to draw a straight-line graph from a linear equation. The Interactive Textbook covers three types:

Desmos widget 4D: Drawing a straight-line graph

Spreadsheet **Spreadsheet activity 4D:** Drawing a straight-line graph with Microsoft Excel

Drawing a straight-line graph with a graphing calculator

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Exercise 4D

Plotting by hand

Example 10 **1** Plot the graph of the linear equations below by first forming a table of values of y for $x = 0, 1, 2, 3, 4$.

a $y = 2x + 1$

b $y = x + 2$

c $y = -x + 10$

d $y = -2x + 9$

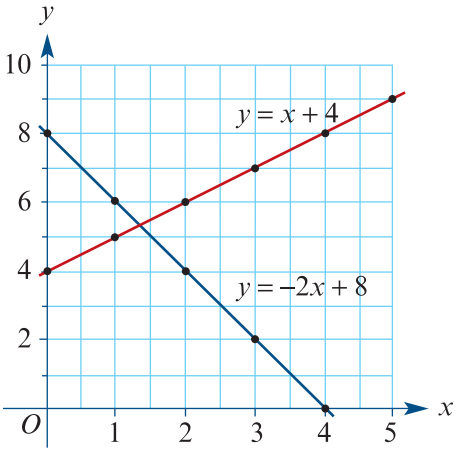
SF

Conceptual understanding

2 Two straight-line graphs, $y = x + 4$ and $y = -2x + 8$, are plotted as shown below.

a Reading from the graph of $y = x + 4$, determine the missing coordinates: (0, ?), (2, ?), (?, 7), (?, 9).

b Reading from the graph of $y = -2x + 8$, determine the missing coordinates: (0, ?), (1, ?), (?, 4), (?, 2).



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4E Determining the slope of a straight line

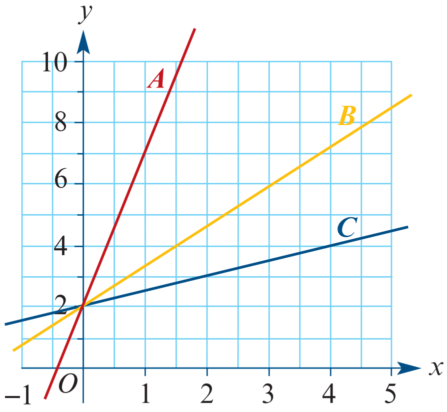
Positive and negative slopes

One thing that makes one straight-line graph look different from another is its steepness or slope. Another name for slope is **gradient**[1].

For example, the three straight lines on the graph opposite all cut the y-axis at $y = 2$, but they have quite different slopes.

Line A has the steepest slope and Line C has the gentlest slope. Line B has a slope somewhere in between.

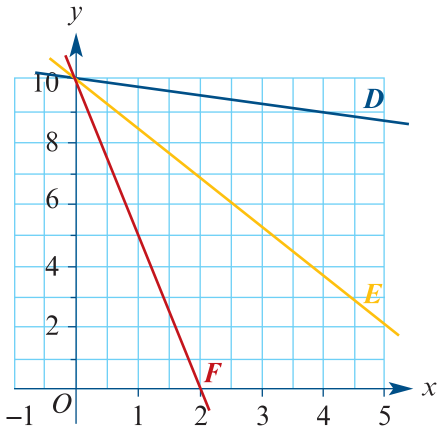
In all cases, the lines have **positive slopes**; that is, they rise from left to right.



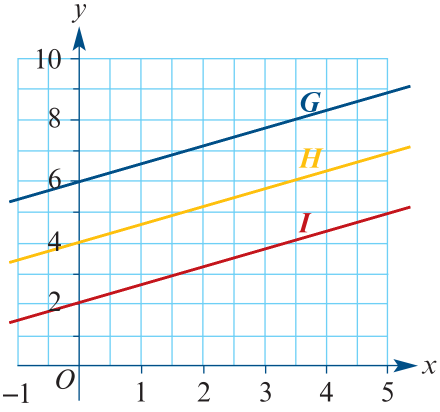
Similarly, the three straight lines on the graph opposite all cut the y-axis at $y = 10$, but they have quite different slopes.

In this case, Line D has the gentlest slope and Line F has the steepest slope. Line E has a slope somewhere in between.

In all cases, the lines have **negative slopes**; that is, they fall from left to right.



By contrast, the three straight lines G, H, I on the graph opposite cut the y-axis at different points, but they all have the *same* slope.



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[1] Note: For linear graphs, the terms slope and gradient mean the same thing. However, when dealing with practical applications of linear graphs and, most particularly, in statistics applications, the word 'slope' is preferred. For this reason, and to be consistent with the Queensland curriculum, this is the term used throughout this book.

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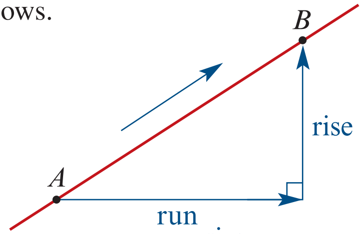
▶ Determining the slope

When talking about the **slope of a straight line**, we want to be able to do more than say that it has a gentle positive slope. We would like to be able to give the slope a value that reflects this fact. We do this by defining the slope of a line as follows.

First, two points *A* and *B* on the line are chosen.
As we go from *A* to *B* along the line, we move:

- up by a distance called the **rise**
- and across by a distance called the **run**.

The slope is found by dividing the rise by the run.



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

Example 11 Finding the slope of a line from a graph: positive slope

Find the slope of the line through the points (1, 4) and (4, 8).

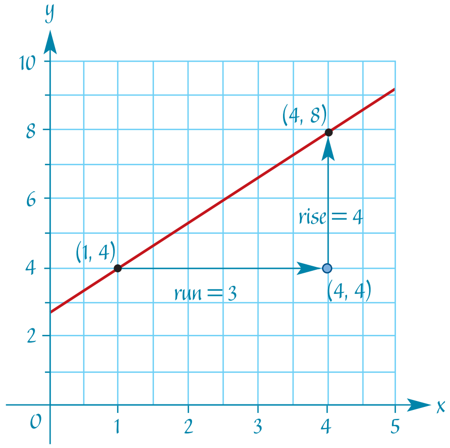
Solution

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{rise} = 8 - 4 = 4$$

$$\text{run} = 4 - 1 = 3$$

$$\therefore \text{slope} = \frac{4}{3} = 1.33 \text{ (to two decimal places)}$$



Note: To find the 'rise', look at the *y*-coordinates.
To find the 'run', look at the *x*-coordinates.

Example 12 Finding the slope of a line from a graph: negative slope

Find the slope of the line through the points (0, 10) and (4, 2).

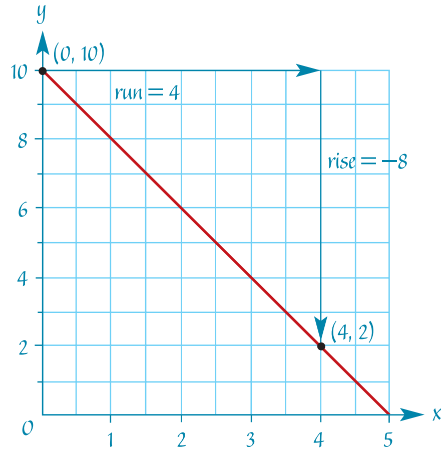
Solution

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{rise} = 2 - 10 = -8$$

$$\text{run} = 4 - 0 = 4$$

$$\therefore \text{slope} = \frac{-8}{4} = -2$$



Note: In this example, we have a negative 'rise', which represents a 'fall'.

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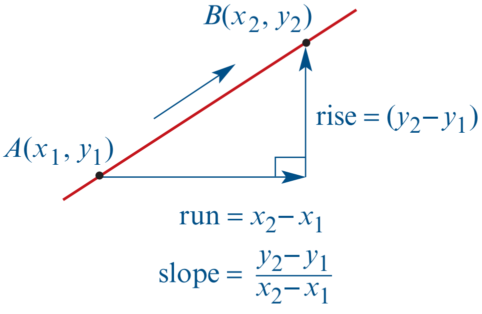
▶ A formula for finding the slope of a line

Although the ‘rise/run’ method for finding the slope of a line will always work, we can also use a formula to calculate the slope. The formula is derived as follows.

Label the coordinates of point A (x_1, y_1).

Label the coordinates of point B (x_2, y_2).



By definition: $\text{slope} = \frac{\text{rise}}{\text{run}}$

From the diagram: $\text{rise} = y_2 - y_1$

$\text{run} = x_2 - x_1$

By substitution: $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$

Example 13 Finding the slope of a line using the formula for the slope

Find the slope of the line through the points (1, 7) and (4, 2) using the formula for the the slope of a line. Give your answer correct to two decimal places.

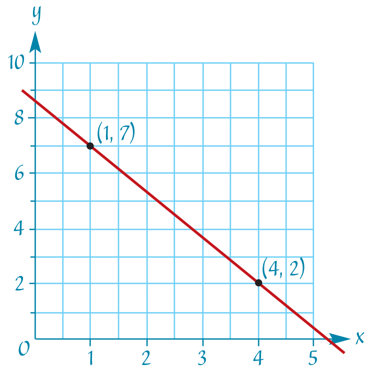
Solution

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Let $(x_1, y_1) = (1, 7)$ and $(x_2, y_2) = (4, 2)$.

$$\text{slope} = \frac{2 - 7}{4 - 1}$$

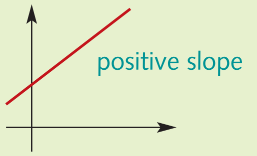
$$= -1.67 \text{ (to two decimal places)}$$



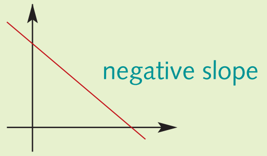
Note: To use this formula it does not matter which point you call (x_1, y_1) and which point you call (x_2, y_2), the rule still works.

The slope of straight-line graphs

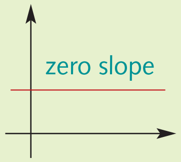
A straight-line graph that rises from left to right is said to have a **positive slope** (positive rise).



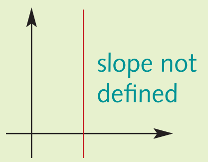
A straight-line graph that falls from left to right is said to have a **negative slope** (negative rise).



A straight-line graph that is horizontal has **zero slope** (‘rise’ = 0).



The slope is **undefined** for a straight-line graph that is vertical.



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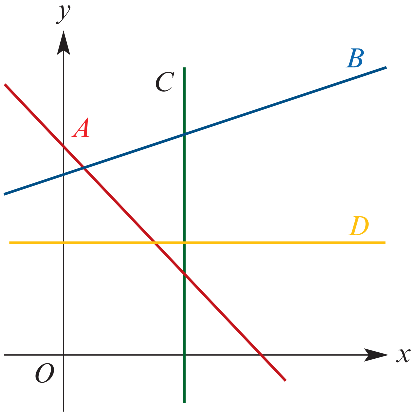
Exercise 4E

Basic ideas

1 Identify the slope of each of the straight-line graphs A, B, C and D as positive, negative, zero, or not defined.

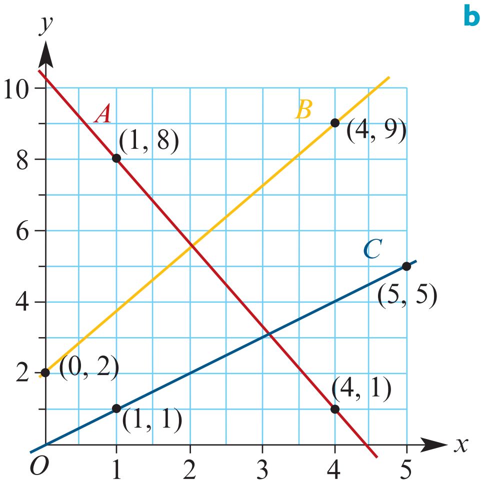


Calculating slopes of lines

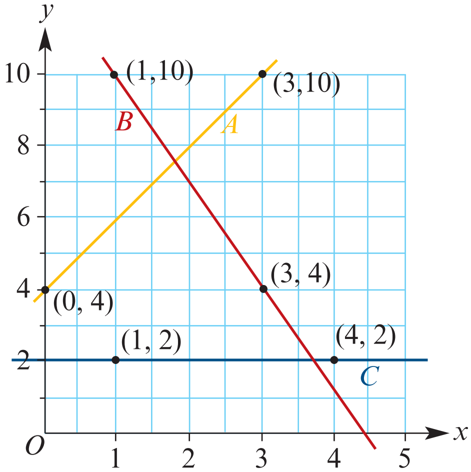
Example 11, 12

2 Find the slope of each of the lines (A, B, C) shown on the graphs below.

a



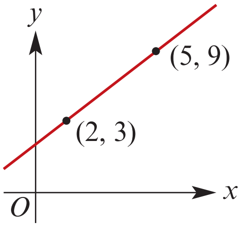
b



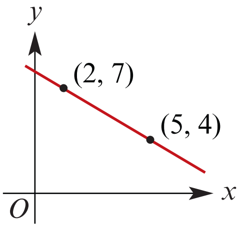
Example 13

3 Find the slope of each of the lines shown.

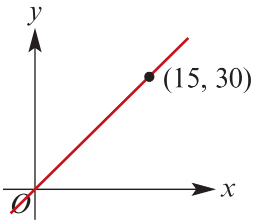
a



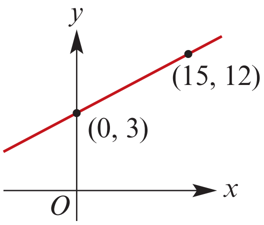
b



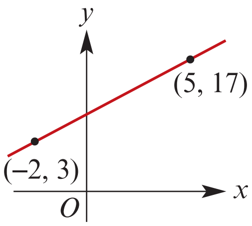
c



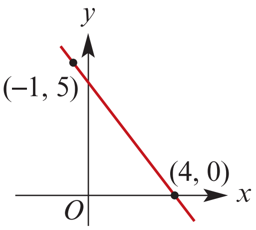
d



e



f



4F The slope–intercept form of the equation of a straight line

▶ Determining the slope and intercept of a straight-line graph from its equation

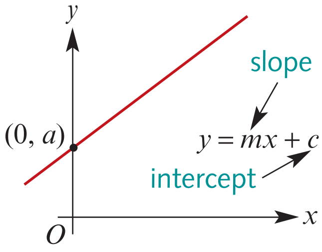
When we write the equation of a straight line in the form:[2]

$$y = mx + c$$

we are using what is called the **slope–intercept form of the equation** of a straight line.

We call $y = mx + c$ the slope–intercept form of the equation of a straight line because:

- m = the slope of the graph
- c = the **y-intercept** of the graph.



The slope–intercept form of the equation of a straight line is useful in modelling relationships in many practical situations. It is also the form used in **bivariate** (two-variable) statistics.

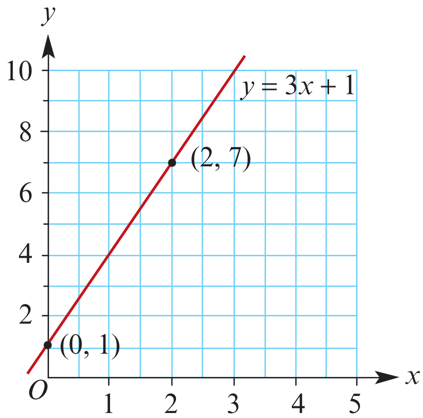
An example of the equation of a straight line written in slope–intercept form is $y = 3x + 1$.

Its graph is shown opposite.

From the graph we see that the:

$$\text{slope} = \frac{7-1}{2-0} = \frac{6}{2} = 3$$

$$y\text{-intercept} = 1$$



That is:

- the slope is given by the *coefficient* of x in the equation (slope = 3)
- the y-intercept corresponds to the (*constant*) term in the equation (intercept = 1).

---

[2] Note: The Queensland curriculum specifies using this form of the equation of a straight line: $y = mx + c$ in this topic. However, you may have seen a form using 'b' for the intercept and 'a' for the slope (rather than 'm' and 'c') so the equation is $y = a + bx$. Sometimes this form is more useful for modelling a real-world situation. Some calculators also use this form, particularly for statistics.

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Slope–intercept form of the equation of a straight line

If the equation of a straight line is in the slope–intercept form:

$$y = mx + c$$

then: m = the slope of the graph

c = the y-intercept of the graph (where the graph cuts the y-axis).

Example 14 Finding the slope and intercept of a line from its equation

Write down the slope and y-intercept of each of the straight-line graphs defined by the following equations.

a $y = 9x - 6$ **b** $y = -5x + 10$ **c** $y = -2x$ **d** $y - 4x = 5$

Solution

For each equation:

1 write the equation. If it is not in slope–intercept form, rearrange the equation.

2 write down the slope and y-intercept. When the equation is in slope–intercept form, write the value of:
m = the slope (the coefficient of x)
c = the y-intercept (the constant term).

a $y = 9x - 6$
slope = 9 y-intercept = −6,

b $y = -5x + 10$
slope = −5 y-intercept = 10,

c $y = -2x$ or $y = -2x + 0$
slope = −2 y-intercept = 0,

d $y - 4x = 5$ or $y = 4x + 5$
slope = 4 y-intercept = 5,

Example 15 Writing down the equation of a straight line given its *y*-intercept and slope

Write down the equations of the straight lines with the following slopes and y-intercepts.

a slope = 6, y-intercept = 9 **b** slope = −5, y-intercept = 2

c slope = 2, y-intercept = −3

Solution

The equation of a straight line is $y = mx + c$. In this equation, m = slope and c = y-intercept. Form an equation by inserting the given values of the slope and the y-intercept for m and c in the standard equation $y = mx + c$.

a slope = 6, y-intercept = 9
equation: $y = 6x + 9$

b slope = −5, y-intercept = 2
equation: $y = 2 + (-5)x$
or: $y = -5x + 2$

c slope = 2, y-intercept = −3
equation: $y = 2x - 3$

Sketching straight-line graphs

Because only two points are needed to draw a straight line, all we need to do is find two points on the graph and then draw a line passing through these two points. When the equation of a straight line is written in slope–intercept form, one point on the graph is immediately available: the y-intercept. A second point can then be quickly calculated by substituting a suitable value of x into the equation.

When we draw a graph in this manner, we call it a *sketch graph*.

Example 16 Sketching a straight-line graph from its equation

Sketch the graph of $y = 2x + 8$.

Solution

1 Write the equation of the line.

$y = 2x + 8$

2 As the equation is in slope–intercept form, the y-intercept is given by the constant term. Write the y-intercept.

y-intercept $= 8$

$\therefore (0, 8)$ is a point on the line.

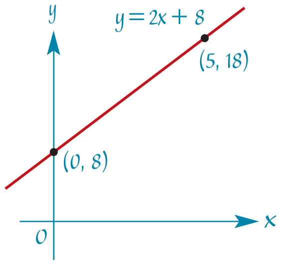
3 Find a second point on the graph. Choose an x value (not 0) that makes the calculation easy: $x = 5$ would be suitable.

When $x = 5$, $y = 2(5) + 8 = 18$

$\therefore (5, 18)$ is a point on the line.

4 Sketch the graph:

- Draw a set of labelled axes.
- Mark in the two points with coordinates.
- Draw a straight line through the points.
- Label the line with its equation.



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Exercise 4F

Finding slope and y-intercept of a straight-line graph from its equation

Example 14 **1** Write down the slopes and y-intercepts of the straight lines with the following equations.

a $y = 2x + 5$
b $y = -3x + 6$
c $y = -5x + 15$
d $y + 3x = 10$
e $y = 3x$
f $4y + 8x = -20$
g $x = y - 4$
h $x = 2y - 6$
i $2x - y = 5$
j $y - 5x = 10$
k $2.5x + 2.5y = 25$
l $y - 2x = 0$
m $y + 3x - 6 = 0$
n $10x - 5y = 20$
o $4x - 5y - 8 = 7$
p $2y - 8 = 2(3x - 6)$

Finding the equation of a straight-line graph given its slope and y-intercept

Example 15 **2** Write down the equation of a line that has:

a slope = 5, y-intercept = 2
b slope = 10, y-intercept = 5
c slope = 4, y-intercept = −2
d slope = −3, y-intercept = 12
e slope = −5, y-intercept = −2
f slope = −0.4, y-intercept = 1.8
g slope = −2, y-intercept = 2.9
h slope = −0.5, y-intercept = −1.5

Sketching straight-line graphs from their equation

Example 16 **3** Sketch the graphs of the straight lines with the following equations, clearly showing the y-intercepts and the coordinates of one other point.

a $y = 2x + 5$
b $y = 5x + 5$
c $y = -2x + 20$
d $y = 10x - 10$
e $y = 4x$
f $y = -2x + 16$

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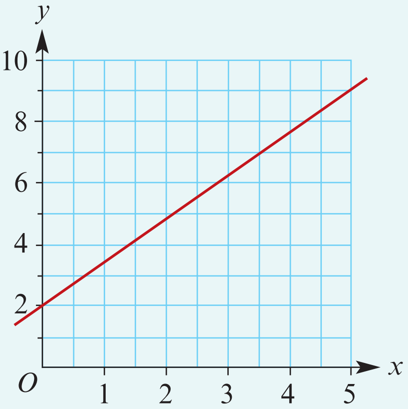
4G Finding the equation of a straight-line graph from its slope and intercept

Skillsheet

We have learned how to construct a straight-line graph from its equation. We can also determine the equation from a graph. In particular, if the graph shows the *y*-intercept, it is a relatively straightforward procedure.

Example 17 Finding the equation of a line: slope–intercept method

Determine the equation of the straight-line graph shown opposite.



Solution

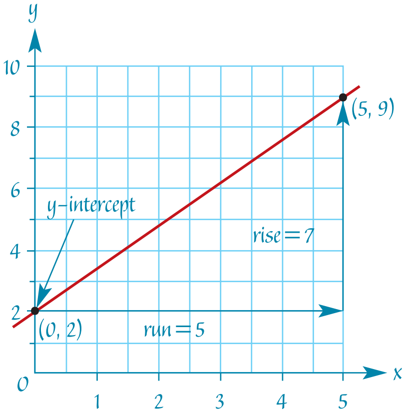
1 Write the general equation of a line in slope–intercept form.

$y = mx + c$

2 Read the *y*-intercept from the graph.

y-intercept $= 2$ so $a = 2$

3 Find the slope using two well-defined points on the line, for example, (0, 2) and (5, 9).



$\text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{7}{5} = 1.4$ so $b = 1.4$

4 Substitute the values of *a* and *b* into the equation.

$y = 1.4x + 2$

5 Write your answer.

$y = 1.4x + 2$ is the equation of the line.

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Finding the equation of a straight-line graph from its slope and intercept

To find the equation of a straight line in slope–intercept form ($y = mx + c$) from its graph:

1 identify the y-intercept (c)

2 use two points on the graph to find the slope (m)

3 substitute these two values into the standard equation $y = mx + c$.

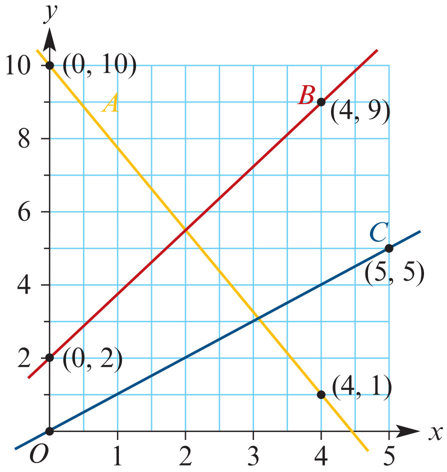
Note: This method *only works* when the graph *scale includes* $x = 0$.

Exercise 4G

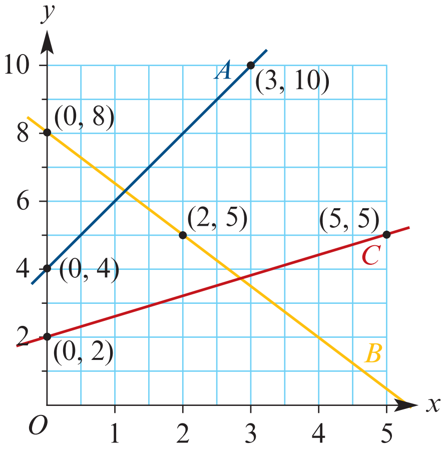
Finding the equation a line from its graph using the slope–intercept method

SF

Example 17 **1** Find the equation of each of the lines (A, B, C) shown on the graph below.



2 Find the equations of each of the lines (A, B, C) shown on the graph below.



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4H Finding the equation of a straight-line graph using two points on the graph

Skillsheet

Unfortunately, not all straight-line graphs show the y-intercept. When this happens, we have to use the two-point method for finding the equation of the line.

Finding the equation of a straight-line graph using two points

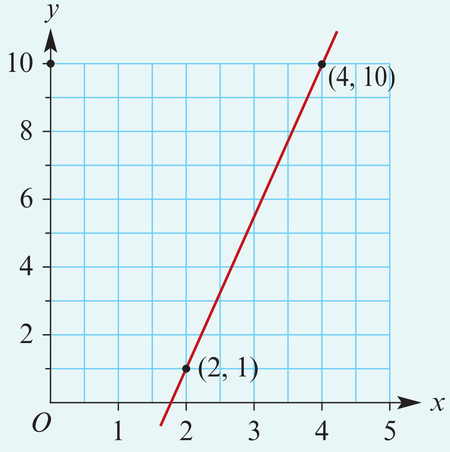
The general equation of a straight-line graph is $y = mx + c$.

1. Use the coordinates of the two points to determine the slope (m).
2. Substitute this value for the slope into the equation. There is now only one unknown, c.
3. Substitute the coordinates of one of the two points on the line into this new equation and solve for the unknown (c).
4. Substitute the values of m and c into the general equation $mx + c$ to obtain the equation of the straight line.

Note: This method works in all circumstances.

Example 18 Finding the equation of a straight-line using two points on the graph

Find the equation of the line that passes through the points (2, 1) and (4, 10).



Solution

1. Write down the general equation of a straight-line graph.

$y = mx + c$

2. Use the coordinates of the two points on the line to find the slope (m).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{10 - 1}{4 - 2} = \frac{9}{2} = 4.5$$

so $m = 4.5$

3. Substitute the value of m into the general equation.

$y = 4.5x + c$

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4 To find the value of c, substitute the coordinates of one of the points on the line (either will do) and solve for c.	Using the point (2, 1): $1 = 4.5 \times 2 + c$ $= 9 + c$ $c = -8$
5 Substitute the value of c into the general equation $y = mx + c$ to find the equation of the line.	Thus, the equation of the line is: $y = 4.5x - 8$.

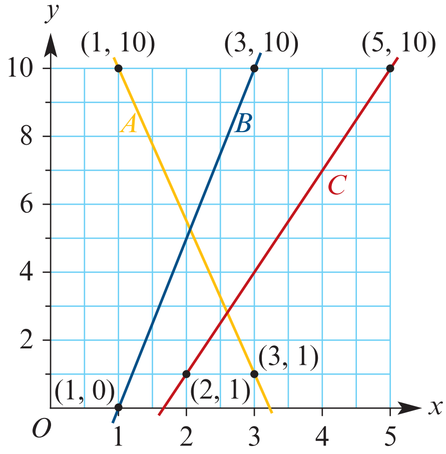
Desmos widget 4H: Finding the equation of a line from two points

Exercise 4H

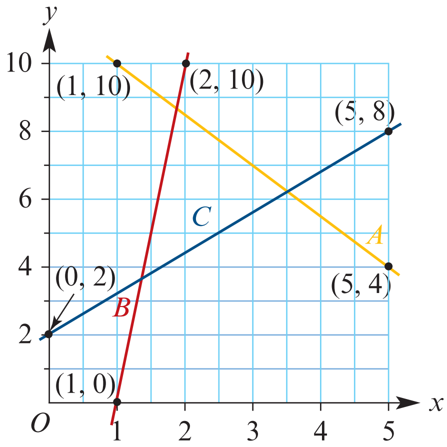
Finding the equation of a line given any two points on its graph

Example 18

1 Find the equation of each of the lines (A, B, C) on the graph below. Write your answers in the form $y = mx + c$.



2 Find the equations of each of the lines (A, B, C) on the graph below. Write your answers in the form $y = mx + c$.



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4I Linear modelling

Many real-life relationships between two variables can be described mathematically by linear (straight-line) equations. This is called *linear modelling*.

These linear models can be then used to solve problems such as finding the time taken to fill a partially filled swimming pool with water, estimating the depreciating value of a car over time or describing the growth of a plant over time.

▶ Modelling plant growth with a linear equation

Some plants grow remarkably quickly. At germination, the height of this plant was 5 cm.

The plant then grows at a constant rate of 6 cm per week for the next 10 weeks.

From this information, we can now construct a mathematical model that can be used to chart the growth of the plant over the following weeks and predict its height at any time during the first 10 weeks after planting.

Constructing a linear model

Let h be the height of the plant (in cm).

Let t be the time (in weeks) after it was planted.

For a linear growth model we can write:

$$h = a + bt$$

where:

- a is the initial height of the plant; in this case, 5 cm (in graphical terms, the y-intercept)
- b is the growth rate, the constant rate at which the plant's height increases each week; in this case, 6 cm per week (in graphical terms, the slope of the line).

Thus we can write: $h = 5 + 6t \quad \text{for } 0 \leq t \leq 10$

Note that we have used the alternative form $y = a + bx$ for the straight line, because it is easier to relate to the word description of the model:

"The height of the plant is the initial height plus the growth rate times the number of weeks after it was planted."

The graph for this model is plotted on the next page.

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Three important features of the linear model $h = 5 + 6t$ for $0 \leq t \leq 10$ should be noted:

- The *h-intercept* gives the height of the plant at the start; that is, its height when $t = 0$. The plant was 5 cm tall when it was first planted.
- The *slope* of the graph gives the growth rate of the plant. The plant grows at a rate of 6 cm per week; that is, each week the height of the plant increases by 6 cm.
- The graph is only plotted for $0 \leq t \leq 10$. This is because the model is only valid for the time when the plant is growing at the constant rate of 6 cm a week.

Notice that in slope–intercept form this model would be $h = 6t + 5$. So the only difference is that the model puts the y-intercept before the slope.

Note: The expression $0 \leq t \leq 10$ is included to indicate the range of number of weeks for which the model is valid. In more formal language this would be called the domain of the model.

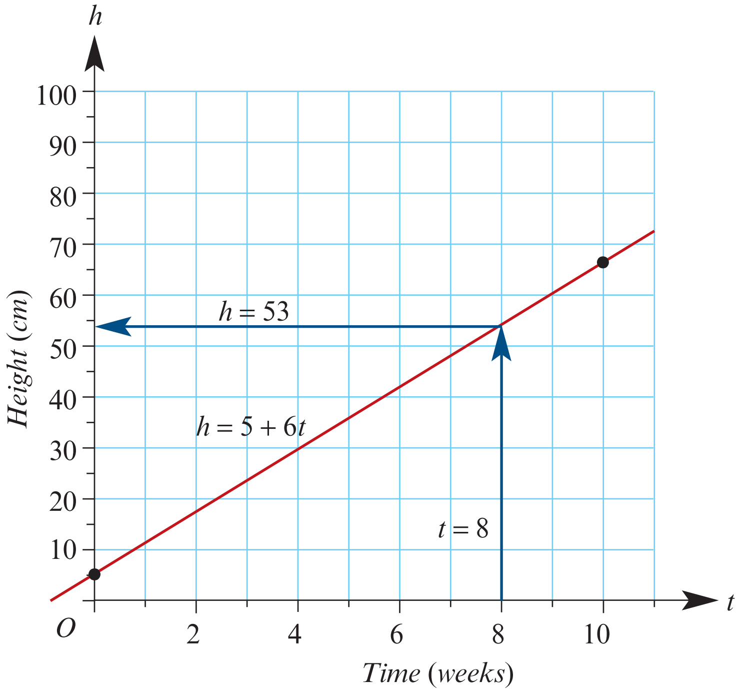
Using a linear model to make predictions

To use the mathematical model to make predictions, we simply substitute a value of t into the model and evaluate.

For example, after eight weeks growth ($t = 8$), the model predicts the height of the plant to be:

$$h = 5 + 6(8) = 53 \text{ cm}$$

This value could also be read directly from the graph, as shown below.



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Exercise 4I-1

Constructing and analysing linear models

1 A tree is 910 cm tall when first measured. For the next five years its height increases at a constant rate of 16 cm per year.

Let h be the height of the tree (in cm).

Let t the time in years after the tree was first measured.

a Write down a linear model in terms of h and t to represent this situation.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the height of the tree 4.5 years after it was first measured.

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2 An empty 20 L cylindrical beer keg is to be filled with beer at a constant rate of 5 litres per minute.

Let V be the volume of beer in the keg after t minutes.

a The beer keg is filled in 4 minutes; write down a linear model in terms of V and t to represent this situation.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the volume of beer in the keg after 3.2 minutes.

3 A home waste removal service charges \$80 to come to your property. It then charges \$120 for each cubic metre of waste that is removed. The maximum amount of waste that can be removed in one visit is 8 cubic metres.

Let c be the total charge for removing w cubic metres of waste.

a Write down a linear model in terms of c and w to represent this situation.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the cost of removing 5 cubic metres of waste.

4 A motorist fills the tank of her car with unleaded petrol, which costs \$1.57 per litre. Her tank can hold a maximum of 60 litres of petrol. When she started filling the tank, there was already 7 litres in it.

Let c be the cost of adding v litres of petrol to the tank.

a Write down a linear model in terms of c and v to represent this situation.

b Sketch the graph of showing the coordinates of the intercept and its end point.

c Use the model to predict the cost of filling the tank of her car with petrol.

5 A business buys a new photocopier for \$25 000. It plans to depreciate its value by \$4000 per year for 5 years, at which time it will be sold.

Let V be the value of the photocopier after t years.

a Write down a linear model in terms of V and t to represent this situation.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the depreciated value of the photocopier after 2.6 years.

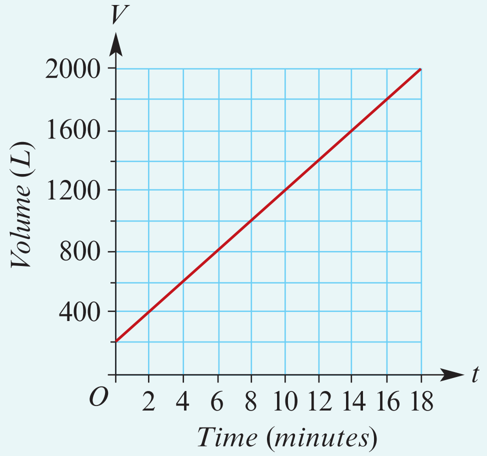
6 A swimming pool when full contains 10 000 litres of water. Due to a leak, it loses on average 200 litres of water per day.
Let V be the volume of water remaining in the pool after t days.

- **a** The pool continues to leak. How long will it take to empty the pool?
- **b** Write down a linear model in terms of V and t to represent this situation.
- **c** Sketch the graph showing the coordinates of the intercept and its end point.
- **d** Use the model to predict the volume of water left in the pool after 30 days.

Interpreting and analysing the graphs of linear models

Example 19 Graphs of linear models with a positive slope

Water is pumped into a partially full tank. The graph gives the volume of water V (in litres) after t minutes.



- **a** How much water is in the tank at the start ($t = 0$)?
- **b** How much water is in the tank after 10 minutes ($t = 10$)?
- **c** The tank holds 2000 L. How long does it take to fill the tank?
- **d** Find the equation of the line in terms of V and t.
- **e** Use the equation to calculate the volume of water in the tank after 15 minutes.
- **f** At what rate is the water pumped into the tank; that is, how many litres are pumped into the tank each minute?

Solution

a Read from the graph (when $t = 0$, $V = 200$). 200 L

b Read from the graph (when $t = 10$, $V = 1200$). 1200 L

c Read from the graph (when $V = 2000$, $t = 18$). 18 minutes

d The equation of the line is $V = a + bt$. $V = a + bt$

- a is the V-intercept. Read from the graph. $a = 200$
- b is the slope. Calculate using two points on the graph, say (0, 200) and (18, 2000). $b = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{2000 - 200}{18 - 0}$ $= 100$

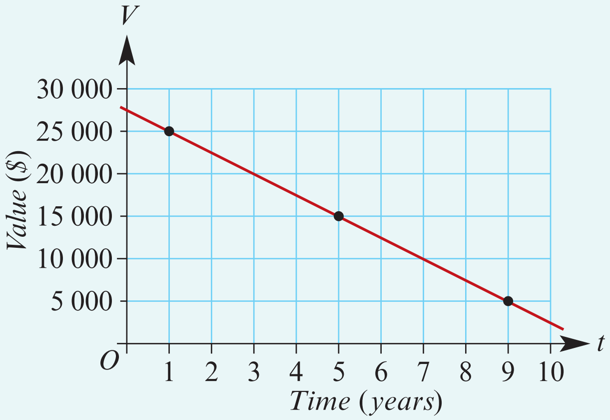
Note: You can use your calculator to find the equation of the line if you wish. $\therefore\ V = 200 + 100t\ (t \geq 0)$

e Substitute $t = 15$ into the equation. Evaluate. $V = 200 + 100(15) = 1700$ L

f The rate at which water is pumped into the tank is given by the slope of the graph, 100 (from **d**). 100 L/min

Example 20 Graphs of linear models with a negative slope

The value of new cars depreciates with time. The graph shows how the value V (in dollars) of a new car depreciates with time t (in years).



a What was the value of the car when it was new?

b What was the value of the car when it was 5 years old?

c Find the equation of the line in terms of V and t.

d At what rate does the value of the car depreciate with time; that is, by how much does its value decrease each year?

e When does the equation predict the car will have no (zero) value?

Solution

a Read from the graph (when $t = 0, V = 27\,500$).

$27 500

b Read from the graph (when $t = 5, V = 15\,000$).

$15 000

c The equation of the line is $V = a + bt$.

- a is the V-intercept. Read from the graph.
- b is the slope. Calculate using two points on the graph, say (1, 25 000) and (9, 5000).

Note: You can use your calculator to find the equation of the line if you wish.

$$V = a + bt$$
$$a = 27\,500$$
$$b = \text{slope} = \frac{25\,000 - 5000}{1 - 9}$$
$$= -2500$$
$$\therefore\ V = 27\,500 - 2500t \quad \text{for} \quad t \geq 0$$

d The slope of the line is –2500, so the car depreciates in value by $2500 per year.

$2500 per year

e Substitute $V = 0$ into the equation and solve for t.

$$0 = 27\,500 - 2500t$$
$$2500t = 27\,500$$
$$\therefore\ t = \frac{27\,500}{2500} = 11 \text{ years}$$

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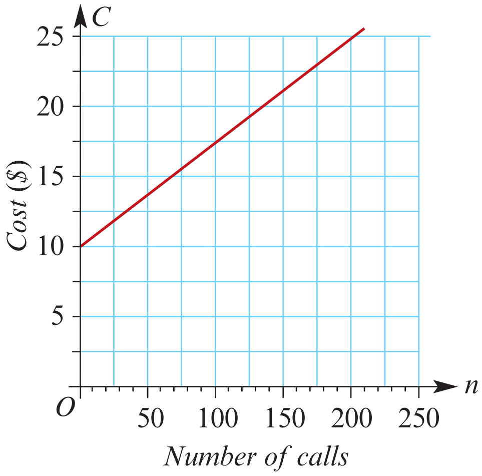
Exercise 4I-2

Interpreting the graphs of linear models in their context

Example 19 **1** A phone company charges a monthly service fee plus the cost of calls. The graph opposite gives the total monthly charge, C dollars, for making n calls. This includes the service fee.



a How much is the monthly service fee ($n = 0$)?

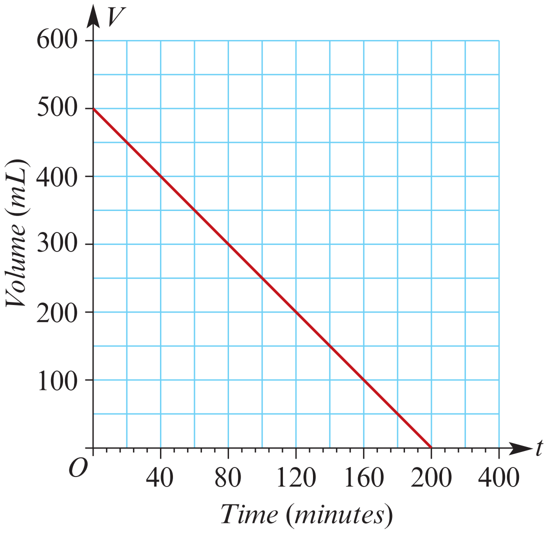
b How much does the company charge if you make 100 calls a month?

c Find the equation of the line in terms of C and n.

d Use the equation to calculate the cost of making 300 calls in a month.

e How much does the company charge per call?

Example 20 **2** The graph opposite shows the volume of saline solution, V mL, remaining in the reservoir of a saline drip after t minutes.



a How much saline solution was in the reservoir at the start?

b How much saline solution remains in the reservoir after 40 minutes? Read the result from the graph.

c How long does it take for the reservoir to empty?

d Find the equation of the line in terms of V and t.

e Use the equation to calculate the amount of saline solution in the reservoir after 115 minutes.

f At what rate (in mL/min) is the saline solution flowing out of the drip?

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3 The graph opposite can be used to convert temperatures in degrees Celsius (C) to temperatures in degrees Fahrenheit (F).

a Find the equation of the line in terms of F and C.

b Use the equation to predict the temperature in degrees Fahrenheit when the temperature in degrees Celsius is:

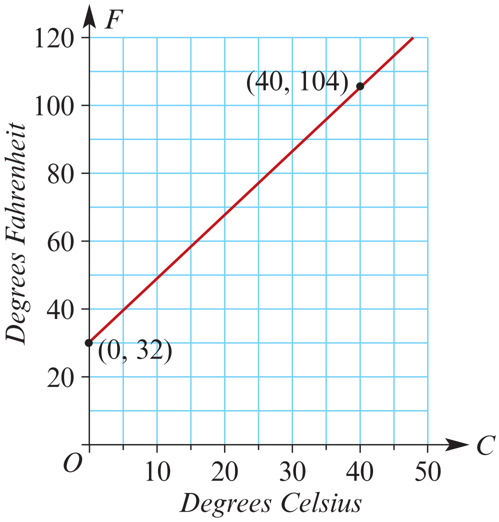
i 50°C

ii 150°C

iii –40°C

c Complete the following sentence by filling in the box.

When the temperature in Celsius increases by 1 degree, the temperature in Fahrenheit increases by ☐ degrees.



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4J Solving simultaneous linear equations algebraically

We now move on to the situation where we have two linear equations with two unknowns. Our task is to find the values of the two unknowns that make the two equations true at the same time (simultaneously).

We begin with the simplest algebraic method for solving equations simultaneously, the substitution method. Note that it may be necessary first to change some equations into the form $y = mx + c$.

Example 21 Solving simultaneous equations using substitution

Solve the pair of simultaneous equations $y = -2x + 5$ and $3x - 2y = 4$.

Solution

1 Number the two equations as (1) and (2).

$$y = -2x + 5 \quad (1)$$
$$3x - 2y = 4 \quad (2)$$

2 Substitute the y-value from equation (1) into equation (2).

Substitute (1) into (2)

$$3x - 2(-2x + 5) = 4$$

3 Expand the brackets and then collect like terms.

$$3x + 4x - 10 = 4$$
$$7x - 10 = 4$$

4 Solve for x. Add 10 to both sides of the equation.

Divide both sides of the equation by 7.

$$7x - 10 + 10 = 4 + 10$$
$$7x = 14$$
$$\frac{7x}{7} = \frac{14}{7}$$
$$\therefore x = 2$$

5 To find y, substitute $x = 2$ into equation (1).

Substitute $x = 2$ into (1).

$$y = 5 - 2(2)$$
$$y = 5 - 4$$
$$\therefore y = 1$$

6 Check by substituting $x = 2$ and $y = 1$ into equation (2).

$$\text{LHS} = 3(2) - 2(1)$$
$$= 6 - 2 = 4 = \text{RHS}$$

7 Write your solution.

$$x = 2, y = 1$$

Exercise 4J

Example 21

1 Solve the following pairs of simultaneous equations.

a $y = x - 1$
$3x + 2y = 8$

b $y = x + 3$
$6x + y = 17$

c $x + 3y = 15$
$y - x = 1$

d $x + y = 10$
$x - y = 8$

e $2x + y = 11$
$3x - y = 9$

f $3x + 5y = 8$
$x - 2y = -1$

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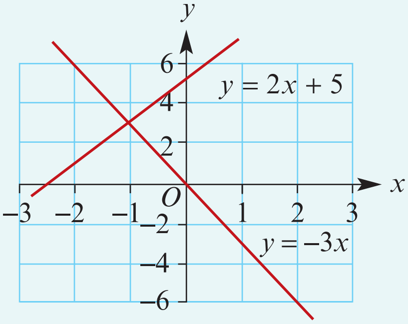
4K Solving simultaneous linear equations using technology

Two straight lines will always intersect unless they are parallel. The point at which two straight lines intersect can be found by sketching the two graphs on the one set of axes and then reading off the coordinates at the point of intersection. When we find the *point of intersection*, we are said to be **solving the equations simultaneously**.

Example 22 Finding the point of intersection of two linear graphs

The graphs of $y = 2x + 5$ and $y = -3x$ are shown. Write their point of intersection.



Solution

From the graph it can be seen that the point of intersection is (−1, 3).

Note: Graphing technology can also be used to find the point of intersection.

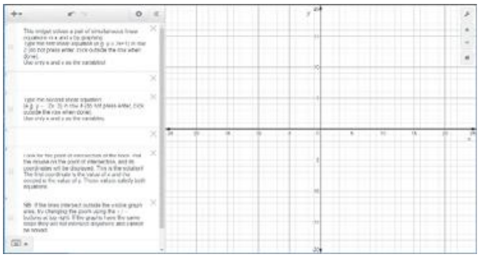
Desmos widget 4K: Finding the point of intersection of two linear graphs

Solving simultaneous equations in Desmos by finding the point of intersection

The online Desmos graphing calculator which is embedded in the Interactive Textbook will solve simultaneous linear equations by finding the coordinates of the point of intersection. Use Desmos widget 4K (above this box) to help guide the procedure.

Steps

1 Go to the Interactive Textbook, locate this section and page in it, and click on the Desmos widget icon above this box. Its window will open.



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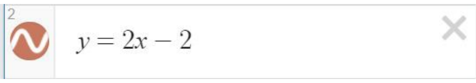
2 The simultaneous linear equations must have x and y as the only variables. Re-write the equations if this is not the case. For this example we will use:

$$y = 2x - 2$$

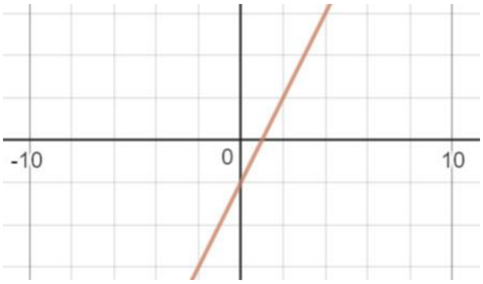
$$y = -x + 10$$

3 Type the first equation into row 2.

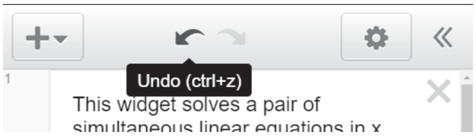


$y = 2x - 2$

Its graph will appear.

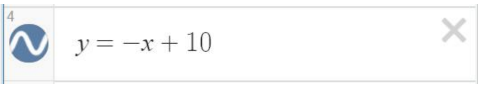


Don't press enter after typing, which will insert another row, just click anywhere outside the row to complete the equation. (There is an undo button at the top left of the Desmos window if you make a wrong move.)



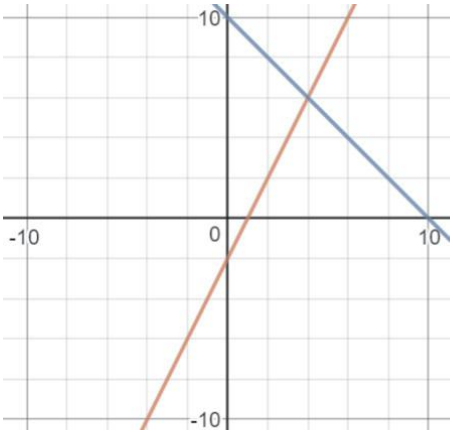
This widget solves a pair of
simultaneous linear equations in x

4 Type the second equation into row 4.

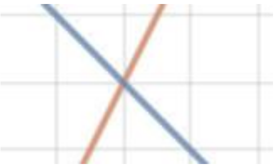


$y = -x + 10$

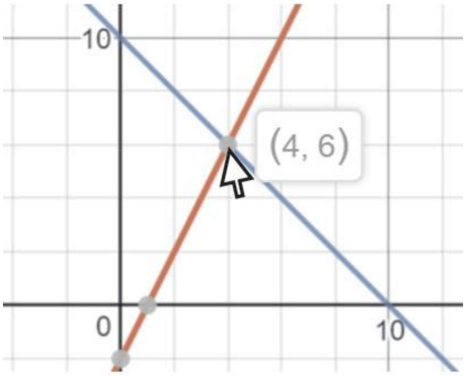
Its graph will also appear.



5 Look for the point of intersection of the two graphs.



6 Click the mouse on the point of intersection. Its coordinates will display. This is the solution.



The first coordinate is the value of x and the second is the value of y. These values satisfy both equations.

In this example, the solution of the simultaneous equations

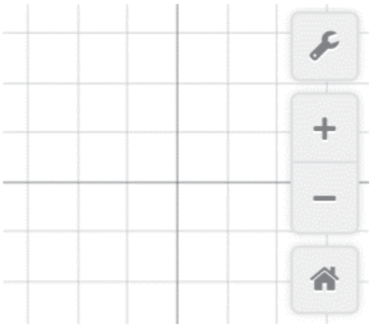
$$y = 2x - 2$$

$$y = -x + 10$$

is

$$x = 4, y = 6.$$

If the lines intersect outside the visible graph area, try zooming out using the − button at top right. Click the default zoom button ⌂ to return to the original zoom.



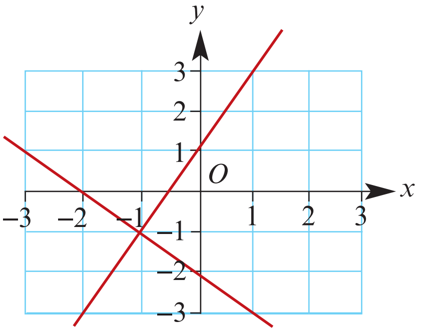
If the graphs have the same slope, they are parallel and will not intersect anywhere and cannot be solved.

Note: Once you have learnt the method, you can use a blank Desmos graphing calculator window to enter the two equations, rather than returning to the widget. Click on the Desmos icon at the top right of any page in a section of the Interactive Textbook.

Exercise 4K

Example 22 **1** State the point of intersection for each of these pairs of straight lines.

a



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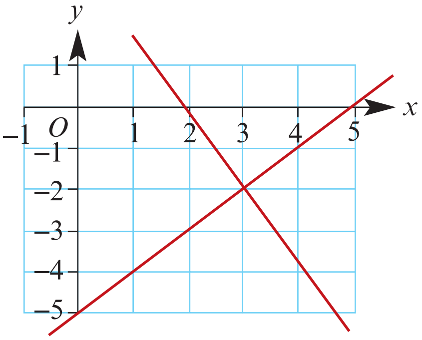

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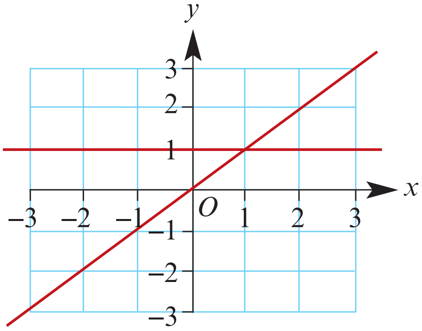

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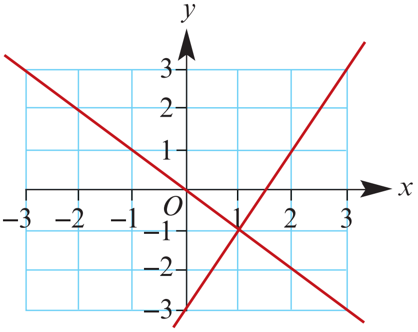

2 Using technology, find the point of intersection of each of these pairs of lines.

a $y = x - 6$ and $y = -2x$

b $y = x + 5$ and $y = -x - 1$

c $y = 3x - 2$ and $y = -x + 4$

d $y = x - 1$ and $y = 2x - 3$

e $y = 2x + 6$ and $y = x + 6$

f $x - y = 5$ and $y = 2$

g $x + 2y = 6$ and $y = -x + 3$

h $2x + y = 7$ and $y - 3x = 2$

i $3x + 2y = -4$ and $y = x - 3$

j $y = 4x - 3$ and $y = 3x + 4$

k $y = x - 12$ and $y = 2x - 4$

l $y + x = 7$ and $2y + 5x = 5$

m $y = 2x + 3$ and $y = 2x - 7$

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4L Problem-solving with simultaneous equations

Simultaneous equations can be used to solve problems in real situations. It is important to define the unknown quantities with appropriate variables before setting up the equations.

Example 23 Using simultaneous equations to solve a practical problem

Tickets for a movie cost $19.50 for adults and $14.50 for children. Two hundred tickets were sold, giving a total of $3265. How many children's tickets were sold?

Solution

Strategy: Using the information given, set up a pair of simultaneous equations to solve.

1 Choose appropriate variables to represent the cost of an adult ticket and the cost of a child ticket.	Let a be the number of adults' tickets sold and c be the number of children's tickets sold.
2 Write two equations using the information given in the question. Label the equations as (1) and (2). Note: The total number of adult and children's tickets is 200, which means that $a + c = 200$.	$a + c = 200$ (1) $19.5a + 14.5c = 3265$ (2)
3 Rearrange equation (1) to make a the subject.	$a = 200 - c$ (3)
4 Substitute a from (3) into equation (2).	$19.5(200 - c) + 14.5c = 3265$
5 Expand the brackets and then collect like terms.	$3900 - 19.5c + 14.5c = 3265$ $3900 - 5c = 3265$
4 Solve for c. Subtract 3900 from both sides of the equation. Divide both sides of the equation by −5.	$3900 - 5c - 3900 = 3265 - 3900$ $-5c = -635$ $\frac{-5c}{-5} = \frac{-635}{-5}$ $\therefore c = 127$
7 To solve for a, substitute $c = 127$ into equation (1).	$a + 127 = 200$ (1)
8 Subtract 127 from both sides.	$a + 127 - 127 = 200 - 127$ $a = 73$
9 Check by substituting, $a = 73$ and $c = 127$ into equation (2).	$127 + 73 = 200$
10 Write your solution.	127 children's tickets and 73 adults' tickets were sold.

Example 24 Using simultaneous equations to solve a practical problem

The perimeter of a rectangle is 48 cm. If the length of the rectangle is three times the width, determine its dimensions.

Solution

Strategy: Using the information given, set up a pair of simultaneous equations to solve.

Remember: The perimeter of a rectangle is the distance around the outside and can be found using $2w + 2l$.

Note: If the length, l, of a rectangle is three times its width, w, then this can be written as $l = 3w$.

1 Choose appropriate variables to represent the dimensions of width and length.

Let w = width
L = length

2 Write two equations from the information given in the question. Label the equations as (1) and (2).

$2w + 2L = 48 \quad (1)$
$L = 3w \quad (2)$

3 Solve the simultaneous equations by substituting equation (2) in equation (1).

Substitute $L = 3w$ into (1).
$2w + 2(3w) = 48$

4 Expand the brackets.

$2w + 6w = 48$

5 Collect like terms.

$8w = 48$

6 Solve for w. Divide both sides by 8.

$$\frac{8w}{8} = \frac{48}{8}$$
$\therefore w = 6$

7 Find the corresponding value for l by substituting $w = 6$ into equation (2).

Substitute $w = 6$ into (2).
$L = 3(6)$
$\therefore L = 18$

8 Give your answer in the correct units.

The dimensions of the rectangle are width 6 cm and length 18 cm.

Example 25 Break-even analysis

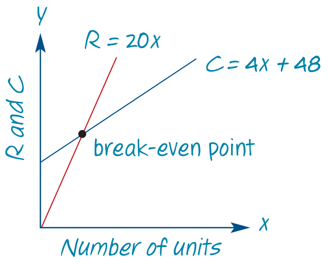
A firm sells its product at \$20 per unit. The cost of production (\$$C$) is given by the equation $C = 4x + 48$, where x is the number of units produced. Find the value of x for which the cost of the production of x units is equal to the revenue received by the firm for selling x units. This is called the break-even point.

Solution

1 Set up the equations.

Let the revenue from selling x units be \$$R$.
$R = 20x \qquad C = 4x + 48$

2 Draw the graphs of these equations.



3 Find the value of x at the point of intersection, when $C = R$.

$20x = 4x + 48$
$\therefore \quad x = 3$

Note: From the graph: for fewer than three items, cost is greater than revenue; for more than three items, cost is less than revenue; the point where cost equals revenue is called the break-even point. If the value of x is not a whole number, the break-even is considered to be the next whole number up, because you can't sell a part of a unit.

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Example 26

Anne-Marie requires a plumber to come to her house to unblock a drain. She contacts two different plumbers. The first plumber's cost, $\$C$, is given by $C = 80 + 40n$ where n is the number of hours the job takes. The second plumber charges a callout fee of \$65 to visit the house and then \$45 per hour for the work.

a Write an equation that gives the second plumber's cost, $\$C$, in the form $C = a + bn$ where n is the number of hours the job takes.

b Sketch the graph of C versus n for each plumber for $0 \leq n \leq 8$. This means the horizontal axis is n from 0 to 8. C is on the vertical axis; use the range 0 to \$500.

c After how many hours does the first plumber become the cheaper option?

Solution

a **1** Write the rule.

$C = a + bn$

2 Plumber 2 charges a callout fee of \$65 so $a = 65$.

$C = 65 + bn$

3 The plumber charges \$45 per hour so $b = 45$.

$C = 65 + 45n$

b **1** Write the first rule.

$C = 80 + 40n$

2 Find C when $n = 0$.

$$\begin{aligned} C &= 80 + 40n \\ &= 80 + 40(0) \\ &= 80 \end{aligned}$$

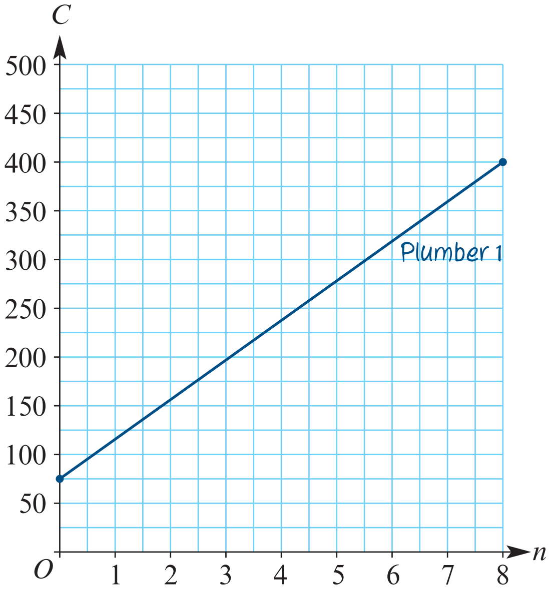
$\therefore$ (0, 80) is a point on the line.

3 Find C when $n = 8$.

$$\begin{aligned} C &= 80 + 40n \\ &= 80 + 40(8) \\ &= 400 \end{aligned}$$

$\therefore$ (8, 400) is a point on the line.

4 Plot the points (0, 80) and (8, 400) to construct the first graph.



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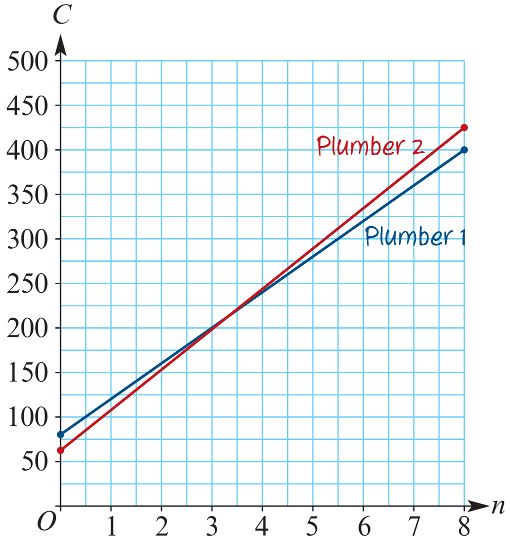
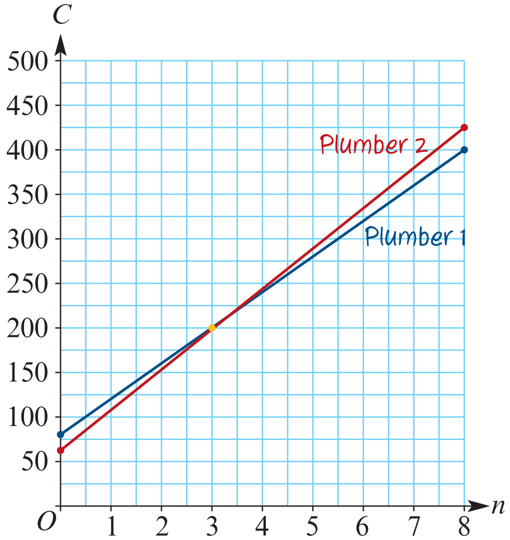
5 Write the second rule.	$C = 65 + 45n$
6 Find C when $n = 0$.	$C = 65 + 45(0)$ $= 65$ $\therefore (0, 65)$ is a point on the line.
7 Find C when $n = 8$.	$C = 65 + 45(8)$ $= 425$ $\therefore (8, 425)$ is a point on the line.
8 Plot the points $(0, 65)$ and $(8, 425)$ to construct the second graph.	



c **1** Find the point of intersection of the two lines.



Point of intersection (3, 200)

2 Determine on which side of the point of intersection the first line lies below the second. and write and answer to the question.

The first plumber becomes a cheaper option for more than 3 hours work.

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Exercise 4L

SF

Example 23

1 Jessica bought five textas and six pencils for \$12.75, and Tom bought seven textas and one pencil for \$12.30.

a Using t for texta and p for pencil, find a pair of simultaneous equations to solve.

b How much did one pencil and one texta cost each?

2 The weight of a box of nails and a box of screws is 2.5 kg. Four boxes of nails and a box of screws weigh 7 kg. Determine the weight of each box.

3 An enclosure at a wildlife sanctuary contains wombats and emus. If the number of heads totals 28 and the number of legs totals 88, determine the number of each species present.

4 Peter bought 50 litres of petrol and 5 litres of motor oil for \$109. His brother Anthony bought 75 litres of petrol and 5 litres of motor oil for \$146.

a Using p for petrol and m for motor oil, write down a pair of simultaneous equations to solve.

b How much did a litre of petrol and a litre of motor oil cost each?

5 Six oranges and ten bananas cost \$7.10. Six oranges and eight bananas cost \$6.40.

a Write down a pair of simultaneous equations to solve.

b Find the cost each of an orange and of a banana.

Example 24

6 The perimeter of a rectangle is 36 cm. If the length of the rectangle is twice its width, determine its dimensions.

7 The sum of two numbers x and y is 52. The difference between the two numbers is 8. Find the values of x and y.

8 The sum of two numbers is 35 and their difference is 19. Find the numbers.

9 Bruce is 4 years older than Michelle. If their combined age is 70, determine their individual ages.

10 A boy is 6 years older than his sister. In 3 years' time he will be twice her age. What are their present ages?

11 A chocolate thickshake costs \$2 more than a fruit smoothie. Jack pays \$27 for three chocolate thickshakes and four fruit smoothies. How much do a chocolate thickshake and a fruit smoothie cost each?

12 In 4 years' time a mother will be three times as old as her son. Four years ago she was five times as old as her son. Find their present ages.

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SF

13 The registration fees for a mathematics competition are \$1.20 for students aged 8–12 years and \$2 for students 13 years and over. One hundred and twenty-five students have already registered and an amount of \$188.40 has been collected in fees. How many students between the ages of 8 and 12 have registered for the competition?

CF

14 A computer company produces two laptop models: standard and deluxe. The standard laptop requires 3 hours to manufacture and 2 hours to assemble. The deluxe model requires $5\frac{1}{2}$ hours to manufacture and $1\frac{1}{2}$ hours to assemble. The company allows 250 hours for manufacturing and 80 hours for assembly over a limited period. How many of each model can be made in the time available?

15 The owner of a service station sells unleaded petrol for \$1.42 and diesel fuel for \$1.54. In five days he sold a total of 10 000 litres and made \$14 495. How many litres of each type of fuel did he sell? Give your answer to the nearest litre.

16 The perimeter of a rectangle is 120 metres. The length is one and a half times the width. Calculate the width and the length.

17 Three classes, A, B and C, in a school are such that class A has two thirds the number of students of class B and class C has five sixths the number of students in class B. If the total number of pupils in the three classes is 105, how many are there in each class?

Example 25 **18** A manufacturer sells his product at \$72 per unit, selling all that he produces. His fixed cost is \$45 000 and the cost per unit is \$22. Find:

a the break-even point

b the profit when 1800 units are produced

c the loss when 450 units are produced

d the sales volume required in order to obtain a profit of \$90 000.

19 A manufacturer of a product sells all that she produces. If her total revenue is given by $R = 7x$ (dollars) and her total cost is given by $C = 6x + 800$ (dollars), where x represents the number of units produced and sold, then:

a determine the level of production at the break-even point

b sketch the graph of C and R against x

c determine the level of production at the break-even point if the total cost increases by 5%.

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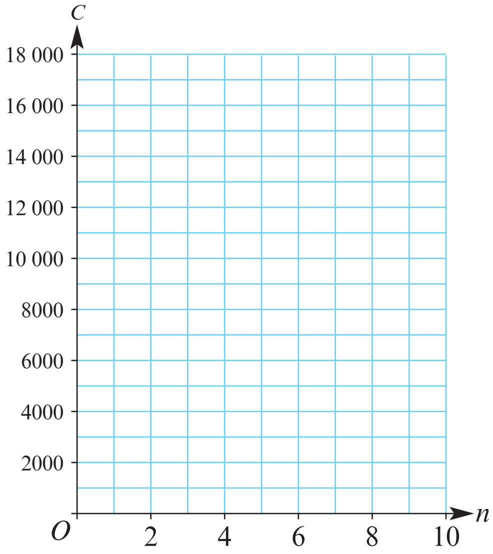
Example 26 **20** A machine hire company charges a $2000 delivery fee and a rental fee of $1500 per day for an excavator.

a Let C be the charge for hiring an excavator for n days. Find a rule for the charge of hiring the generator for n days.

b On the set of axes graph the relationship between C and n.

c A rival company charges a $4000 delivery fee and a rental fee of $500 per day. Let C be the charge for hiring an excavator for n days. Find a rule for the charge of hiring the excavator from this company for n days.

d Sketch the graph of this rule on the same set of axes as those used in **b**.



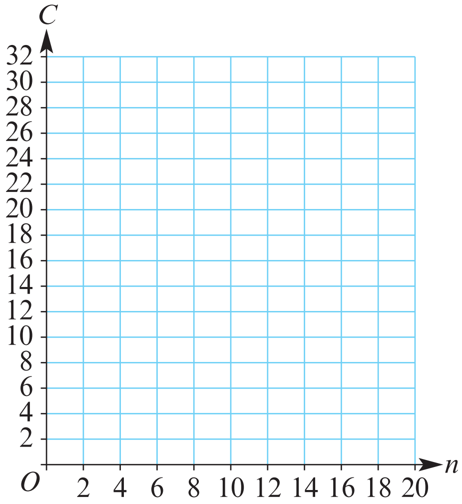
e For how many days hire do the two companies charge the same amount?

21 The Rabbitphone telephone company offers calls to a remote island for a connection fee of $14 and thereafter $1 per minute. The Foxphone telephone company offers calls to the same island for $2 per minute and no connection fee.

a Compare the cost of a 10-minute call to the island using each company.

b For the Rabbitphone company, write an equation for the total cost, C, of a call of duration n minutes.

c For the Foxphone company, write an equation for the total cost, C, of a call of duration n minutes.



d On the set of axes, graph the relationship between C and n for each company.

e For what call length do the two companies charge the same?

22 **a** A water tank initially contains 4800 litres of water. Water is drained from the tank at a rate of 200 litres per minute. Write a rule for the volume of water, V litres, in the tank at time, t minutes.

b The water from this tank is drained into an irrigation channel that initially contains 600 litres of water. Write a rule for the volume of water, V litres, in the irrigation channel at time, t minutes.

c After how many minutes do the tank and the channel contain the same volume of water?

4M Further problem-solving and modelling

Exercise 4M

1 As part of its urban renewal strategy, Camtown council makes one quarter of a hectare of land available for building middle-income homes. The project manager decides to build 10 houses on blocks of varying sizes.
There are five small blocks, three medium-sized blocks and two large blocks. The medium-sized blocks are 100 m^2 larger than the small blocks and the large blocks are 200 m^2 larger than the medium-sized ones.
What are the sizes of the blocks?
Note: 1 hectare = 10 000 m^2

2 A shop sells fruit in two types of gift boxes: standard and deluxe. Each standard box contains 1 kg of peaches and 2 kg of apples and each deluxe box contains 2 kg of peaches and 1.5 kg of apples. On one particular day the shop sold 12 kg of peaches and 14 kg of apples in gift boxes. How many of each kind of box were sold on the day?

3 A person's total body water is dependent on their gender, age, height and weight. Total body water (TBW) for males and females can be found by the following formulas:

$$\underset{\text{(litres)}}{\text{Male TBW}} = 2.447 - 0.09516 \times \underset{\text{(years)}}{\text{age}} + 0.1074 \times \underset{\text{(cm)}}{\text{height}} + 0.3362 \times \underset{\text{(kg)}}{\text{weight}}$$

$$\underset{\text{(litres)}}{\text{Female TBW}} = -2.097 + 0.1069 \times \underset{\text{(cm)}}{\text{height}} + 0.2466 \times \underset{\text{(kg)}}{\text{weight}}$$

a What is the TBW for a female of height 175 cm and weight 62 kg? Give your answer correct to two decimal places.

b Calculate the TBW, correct to two decimal places, for a 45-year-old male of height 184 cm and weight 87 kg.

c A healthy 27-year-old female weighs 62 kg and has a TBW of 32. What is her height, to the nearest cm?

d What would be the TBW, correct to two decimal places, for a 78-year-old man of height 174 cm and weight 80 kg?

e Over a period of a week, the 78-year-old man's weight rapidly increases to 95 kg due to fluid retention. What is his new TBW, correct to two decimal places?

f Construct a table showing the TBW for a 22-year-old male of height 185 cm with weights in increments of 5 kg from 60 kg to 120 kg. Give you answers correct to two decimal places.

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4 Kiera's water tank has sprung a leak and water leaks from the tank at a rate of 10 litres per hour. Her tank was initially full and contained 500 litres.

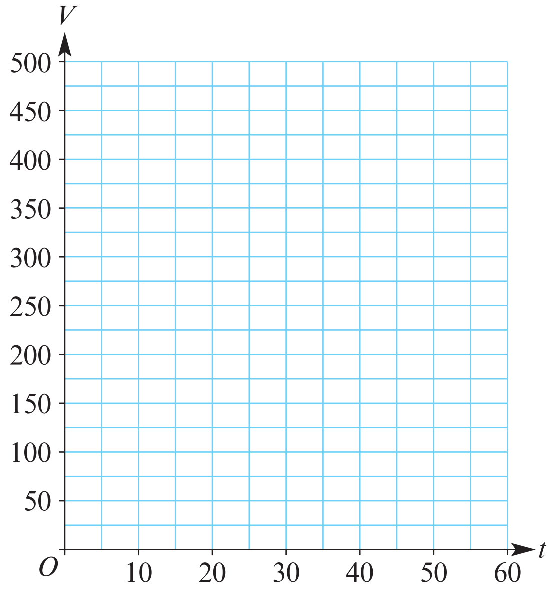
a A rule for the volume of water, V, remaining in the tank after t hours is $V = 500 - kt$. State the value of k.

b On the set of axes, graph the relationship between V and t.

c Find the volume of water left in the tank after 20 hours.

d After how many hours are there 150 litres remaining in the tank?

e Kiera is away on holiday and is due back 48 hours after the tank develops the leak. Will there be any water left in the tank when she arrives home? Give a reason for your answer.



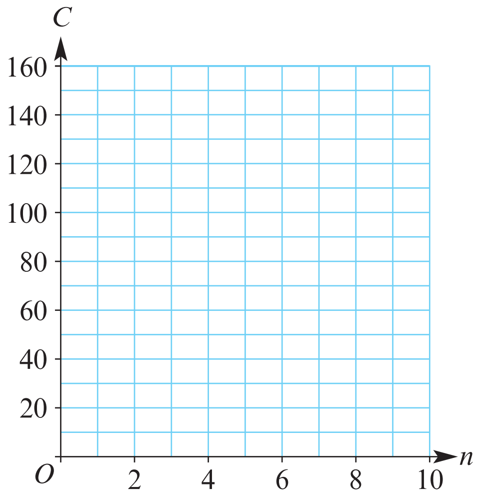
5 David's singing telegram service charges an \$80 appearance fee, and \$10 per minute for a singing telegram.

a Write an equation for the total cost, C, of a singing telegram in terms of the number of minutes sung, n.

b On the set of axes below graph the relationship between C and n.

c **i** What would be the charge for a 5-minute singing telegram?

ii The charge of a singing telegram is \$120, how long is it?



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4N Piecewise linear and step graphs

▶ Piecewise linear graphs

Sometimes a situation requires two linear graphs to obtain a suitable model. The graphs we use to model such situations are called **piecewise linear graphs**.

Example 27 Constructing a piecewise linear graph model

The amount, C dollars, charged to supply x m^3 of crushed rock is given by the equations:

$$C = 50 + 40x \qquad (0 \leq x < 3)$$
$$C = 80 + 30x \qquad (3 \leq x < 8)$$

a Use the appropriate equation to find the cost of supplying these amounts:

i 2.5 m^3 **ii** 3 m^3 **iii** 6 m^3

b Use the equations to construct a piecewise linear graph for $0 \leq x \leq 8$.

Solution

a **1** Write the equations.

$C = 50 + 40x \ (0 \leq x < 3)$
$C = 80 + 30x \ (3 \leq x \leq 8)$

2 For each case follow these steps:

- Choose the appropriate equation.
- Substitute the value of x and evaluate.
- Write your answer.

i When $x = 2.5$
$C = 50 + 40x$
$C = 50 + 40(2.5) = 150$
Cost for 2.5 m^3 of crushed rock is $150.

ii When $x = 3$
$C = 80 + 30x$
$C = 80 + 30(3) = 170$
Cost for 3 m^3 of crushed rock is $170.

iii When $x = 6$
$C = 80 + 30x$
$C = 80 + 30(6) = 260$
Cost for 6 m^3 of crushed rock is $260.

b The graph has two line segments.

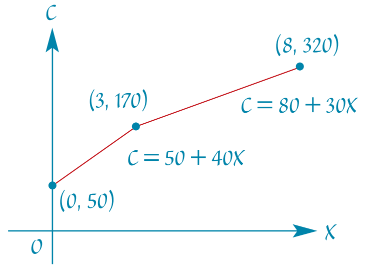
1 Determine the coordinates of the end points of each line.

$x = 0 : C = 50 + 40(0) = 50$
$x = 3 : C = 50 + 40(3) = 170$
$x = 3 : C = 80 + 30(3) = 170$
$x = 8 : C = 80 + 30(8) = 320$

2 Draw a set of labelled axes and mark in the points with their coordinates.

3 Join up the end points of each line segment with a straight line.

4 Label each line segment with its equation.



▶ Step graphs

A **step graph** can be used to represent information that is constant for particular intervals. One example of this is the cost of sending an airmail letter.

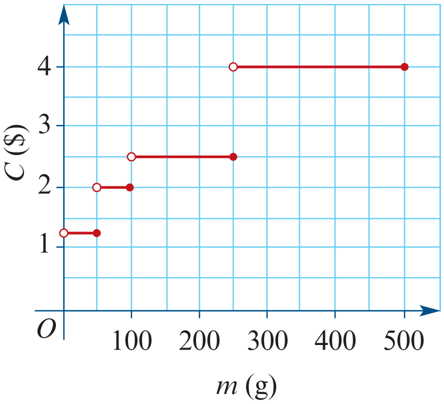
Mass	Cost
Up to 50 g	\$1.20
Over 50 g up to 100 g	\$2.00
Over 100 g up to 250 g	\$2.50
Over 250 g up to 500 g	\$4.00

The table on the right shows that the cost of sending a letter is constant for intervals of mass. For example, a letter will cost \$2.00 to send if it has a mass of more than 50 g but less than or equal to 100 g.

We must be careful when considering the cost of letters that have a mass at the end points of the mass interval.

A letter that has a mass of exactly 100 g belongs in the second interval because this interval includes masses up to and including 100 g. It does not belong in the third interval because this is for masses of more than 100 g, not equal to 100 g.

Each interval of mass from the table is shown as a horizontal line segment on the graph. The end points are open circles if the mass is *not* included in the interval and closed circles if the mass *is* included.



Example 28 Constructing a step-graph

The entry fee for a music competition depends on the age of the competitor, as shown in the table on the right.

Age of competitor	Entry fee
under 4 years	\$5.00
5 years to under 10 years	\$8.00
10 years to under 15 years	\$15.00
15 years to under 20 years	\$25.00
20 years and over	\$30.00

a What is the entry fee for a competitor who is 14 years and 9 months?

b Is the circle for the horizontal line segment that **ends** at 20 years open or closed?

c Sketch the step-graph that shows the *entry fee* against the *age of competitor*.

Solution

a 14 years and 9 months is over 10 years and under 15 years.

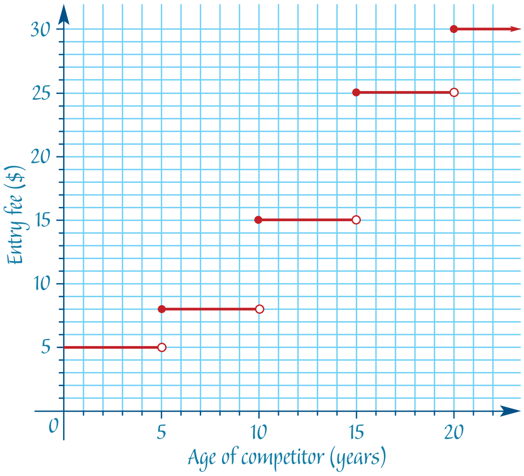
The entry fee for a competitor who is 14 years and 9 months old is \$15.00.

b The interval that ends at 20 years is 15 years to under 20 years.

Since 'under 20 years' does not include exactly 20 years, the circle will be open.

c For each interval of age, draw an appropriate line segment. The upper end of each interval will have an open circle.

Note: *The interval '20 years and over' does not have an exact end point. An arrow can be used to indicate that this interval extends beyond the graph.*



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Exercise 4N

SF

Example 27

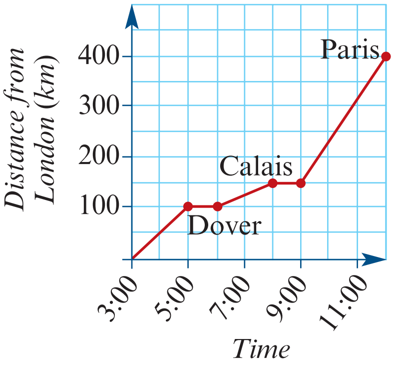
1 The graph shows a man's journey by train, boat and train from London to Paris.

a At what time does he:

- **i** arrive at Dover?
- **ii** arrive in Paris?
- **iii** leave Calais?
- **iv** leave Dover?

b For how long does he stop at Dover?

c At what time is he exactly halfway between Calais and Paris?



2 An empty tank is being filled from a mountain spring. For the first 30 minutes, the equation giving the volume, V, of water in the tank (in litres) at time t minutes is:

$$V = 15t \quad (0 \le t \le 30)$$

After 30 minutes, the flow from the spring slows down. For the next 70 minutes, the equation giving the volume of water in the tank at time, t, as given by the equation:

$$V = 150 + 10t \quad (30 < t \le 100)$$

a Use the appropriate equation to determine the volume of water in the tank after:

i 20 minutes **ii** 30 minutes **iii** 60 minutes **iv** 100 minutes

b Use the equations to construct a piecewise linear graph for $0 \le t \le 100$.

CF

3 For the first 25 seconds of the journey of a train between stations, the speed, S, of the train (in metres per second) after t seconds is given by:

$$S = 0.8t \quad (0 \le t \le 25)$$

For the next 180 seconds, the train travels at a constant speed of 20 m/s as given by the equation:

$$S = 20 \quad (25 < t \le 205)$$

Finally, after travelling for 205 seconds, the driver applies the brakes and the train comes to rest after a further 25 seconds as given by the equation:

$$S = 184 - 0.8t \quad (205 < t \le 230)$$

a Use the appropriate equation to determine the speed of the train after:

i 10 seconds **ii** 60 seconds **iii** 180 seconds **iv** 210 seconds

b Use the equations to construct a piecewise linear graph for $0 \le t \le 230$.

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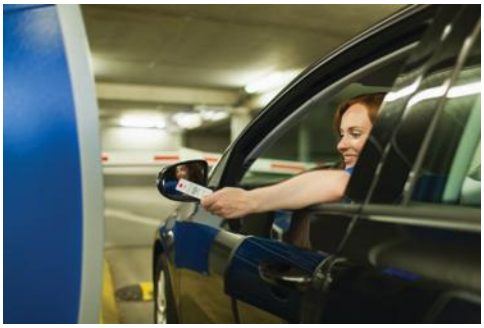
Example 28

4 The postal rates for letters for a particular country are shown in this table. Sketch a step graph to represent this information.

Weight not over	Rate	Weight not over	Rate
60 g	34c	350 g	$1.22
100 g	48c	450 g	$1.38
150 g	62c	600 g	$1.56
200 g	76c	750 g	$2.56
250 g	90c	1000 g	$3.40
300 g	$1.06		

5 A multistorey car park has tariffs as shown. Sketch a step graph showing this information.

First 2 hours	$5.00
2–3 hours	$7.50 (more than 2, less than or equal to 3)
3–4 hours	$11.00 (more than 3, less than or equal to 4)
4–8 hours	$22.00 (more than 4, less than or equal to 8)



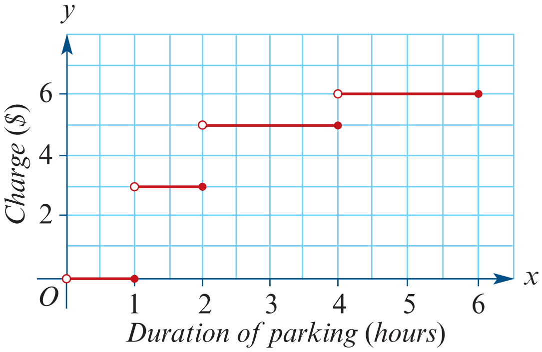
6 Suppose that Australia Post charged the following rates for airmail letters to Africa: $1.20 up to 25 g; $2.00 over 25 g and up to 50 g; $3.00 over 50 g and up to 150 g. Sketch a graph to represent this information.

7 This step-graph shows the charges for a market car park.

a How much does it cost to park for 40 minutes?

b How much does it cost to park for 2 hours?

c How much does it cost to park for 3 hours?



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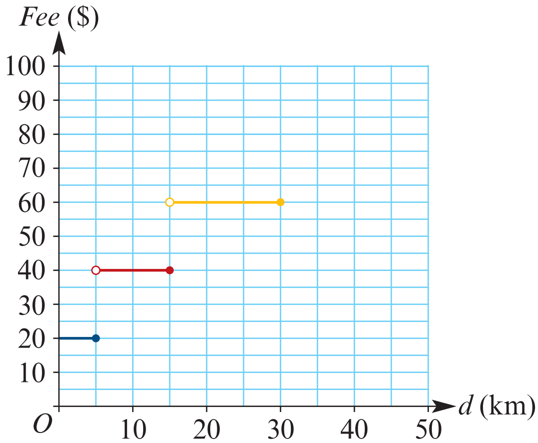
8 Brad operates a parcel delivery service. The fee charged depends on the distance he has to travel in order to deliver a parcel. The fees for distances up to 30 km are shown on the step graph below.

a What is the fee for a delivery requiring Brad to travel a distance of 20 km?

b What it the maximum distance Brad will travel for a delivery fee of $20?

c A fee of $90 is charged to deliver a parcel requiring Brad to travel more than 30 km and no more than 40 km. Draw this information on the step graph.



9 Marika prints designer t-shirts in large numbers. The revenue, $R, generated from the sale of t t-shirts is given by the rule $R = 12t$ for sales up to and including 500 t-shirts and $R = 6000 + 10(t - 500)$ for sales of more than 500 t-shirts.

a Calculate the revenue generated by the sale of the following number of t-shirts:

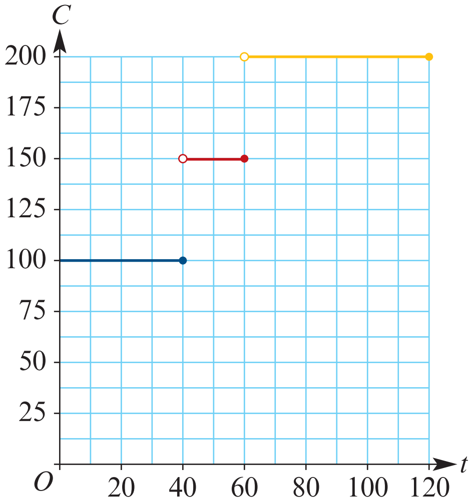
i 400 t-shirts **ii** 800 t-shirts

b Sketch a graph of revenue for $0 \leq t \leq 1000$.

10 The graph below shows the cost in dollars of home garden maintenance taking up to two hours by Kelvin the gardener.

a What is the cost of garden maintenance that takes 60 minutes?

b Kelvin does one job that takes 30 minutes and another that takes 100 minutes. How much does he earn altogether?



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Key ideas and chapter summary

Slope of a straight-line graph

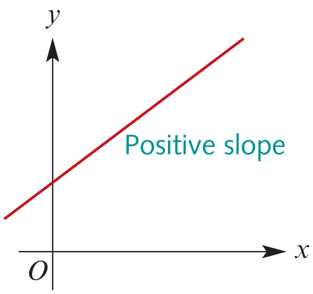
Slope of a straight-line graph is defined to be:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

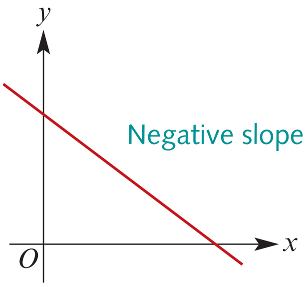
where (x_1, y_1) and (x_2, y_2) are two points on the line.

Positive and negative slope

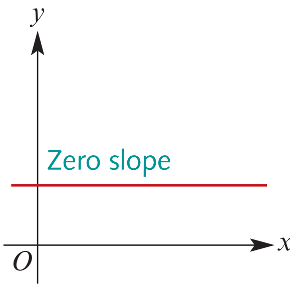
If the line rises to the right, the slope is **positive**.



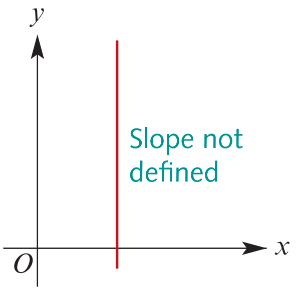
If the line falls to the right, the slope is **negative**.



If the line is horizontal, the slope is **zero**.



If the line is vertical, the slope is **undefined**.



Equation of straight-line graph: the slope–intercept form

The equation of a straight line can take several forms.

The **slope–intercept form** is:

$$y = mx + c$$

where m is the **slope** of the line and c is the **y-intercept**.

Linear model

A **linear model** has a linear equation or relation of the form:

$$y = a + bx \quad \text{where} \quad c \le x \le d$$

where a, b, c and d are constants.

Linear equation

A linear equation is one whose unknown values are always to the power of 1.

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Piecewise linear graphs	**Piecewise linear graphs** are used in practical situations where more than one linear equation is needed to model the relationship between two variables.
Step graphs	A **step graph** consists of one or more horizontal line segments. Step graphs can be used to graphically represent situations where the value of one variable is constant for intervals of another variable.
Simultaneous equations	Two straight lines will always intersect, unless they are parallel. At the point of intersection the two lines will have the same coordinates. When we find the point of intersection, we are solving the equations simultaneously. **Simultaneous equations** can be solved graphically, algebraically or by using technology. Example: $y = 3x + 6$ $y = 4x - 9$ are a pair of simultaneous equations.
Break even point	The number of units of an item that need to be sold for sales revenue to equal (or just exceed) cost of producing those units.

Skills check

Having completed this chapter you should be able to:

- solve linear equations
- recognise a linear equation written in slope–intercept form
- determine the slope and intercept of a straight-line graph from its equation
- determine the slope of a straight line from its graph
- determine the y-intercept of a straight line from its graph (if shown)
- determine the equation of a straight line, given its graph
- construct a linear model to represent a practical situation using a linear equation or a straight-line graph
- interpret the slope and the intercept of a straight-line graph in terms of its context and use the equation to make predictions
- construct a piecewise linear graph used to model a practical situation
- construct and interpret a step graph used to model a practical situation
- solve simultaneous equations graphically or algebraically.

Multiple-choice questions

1 The solution to $4x = 24$ is:

A $x = 2$ **B** $x = 6$ **C** $x = 20$ **D** $x = 96$

2 The solution to $\frac{x}{3} = -8$ is:

A $\frac{8}{3}$ **B** 24 **C** $-\frac{8}{3}$ **D** -24

3 The solution to $2v + 5 = 11$ is:

A $v = 8$ **B** $v = 3$ **C** $v = 6$ **D** $v = 16$

4 The solution to $3k - 5 = -14$ is:

A $k = 115.67$ **B** $k = 19$ **C** $k = 3$ **D** $k = -3$

5 The cost of hiring a car for a day is \$60 plus 0.25c per kilometre. Michelle travels 750 kilometres. Her total cost is:

A \$810 **B** \$187.50 **C** \$247.50 **D** \$188.10

6 The equation of a straight line is $y = 3x + 4$. When $x = 2$, y is:

A 2 **B** 3 **C** 4 **D** 6 **E** 10

7 The equation of a straight line is $y = 4x + 5$. The y-intercept is:

A 2 **B** 3 **C** 4 **D** 5 **E** 20

8 The area of a trapezium is given by $A = \frac{(a+b)h}{2}$.
An expression for h is:

A $\frac{2A}{a+b}$ **B** $\frac{A-2}{a+b}$ **C** $\frac{A-(a+b)}{2}$ **D** $2A - (a+b)$

9 If the formula $R = 5S - P$ is transposed to make S the subject then

A $S = \frac{R-P}{5}$ **B** $S = \frac{R+P}{5}$ **C** $S = \frac{R}{5} + P$
D $S = \frac{R}{5} - P$ **E** $S = \frac{P-R}{5}$

10 The equation of a straight line is $y = -3x + 10$. The slope is:

A -3 **B** 0 **C** 3 **D** 7 **E** 10

11 The equation of a straight line is $y - 2x = 3$. The slope is:

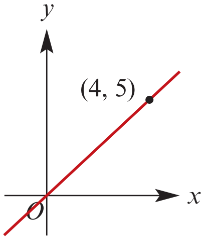
A -3 **B** -2 **C** 0 **D** 2 **E** 3

12 The slope of the line passing through the points (5, 8) and (9, 5) is:

A -1.3 **B** -1 **C** -0.75 **D** 0.75 **E** 1.3

13 The graph of $y = 5x + 4$ is:

A



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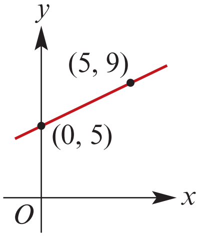

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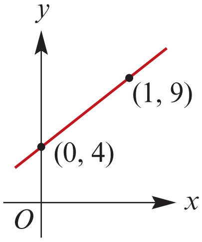

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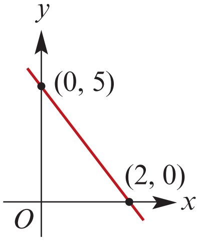
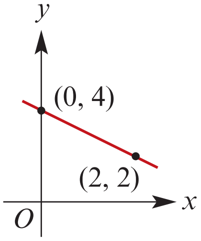
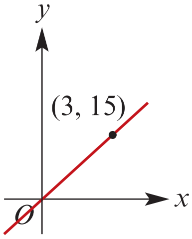


E



14 The graph of $y = -3x + 15$ is:

A



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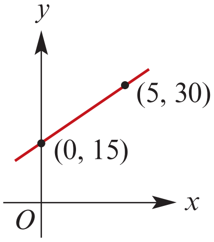

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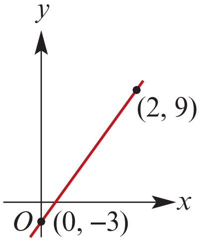
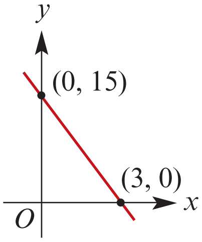
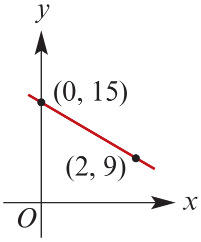


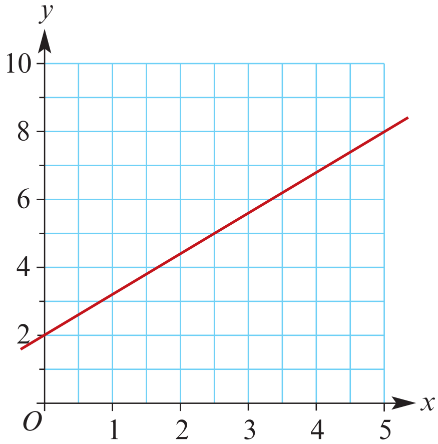
D



E



Questions 15 and 16 relate to the following graph.



15 The y-intercept is:

A –2 **B** 0 **C** 2
D 5 **E** 8

16 The slope is:

A 1.6 **B** 1.2 **C** 2
D 5 **E** 8

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17 The slope of the line in the graph shown opposite is:

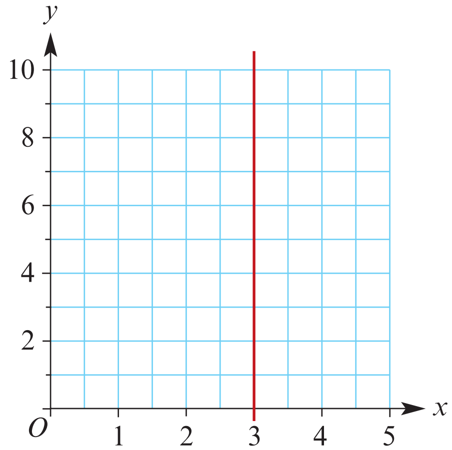
A negative

B zero

C positive

D three

E not defined



18 The equation of the graph shown opposite is:

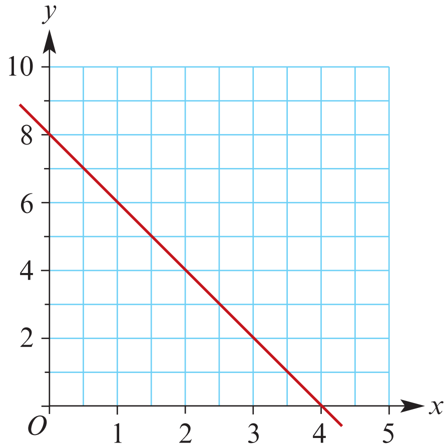
A $y = 8x + 2$

B $y = -2x + 4$

C $y = -2x + 8$

D $y = 2x + 4$

E $y = 2x + 8$



19 The equation of the graph shown opposite is:

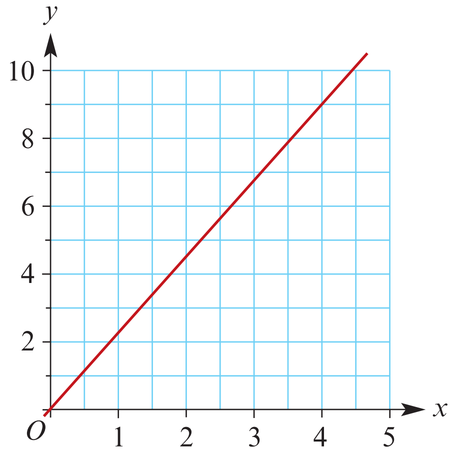
A $y = -2.25x$

B $y = 2.25x$

C $y = -9x$

D $y = 9x$

E $y = 2x + 1$



20 Which of the following points lies on the line $y = -5 + 10x$?

A (1, –5) **B** (1, 5) **C** (1, 15) **D** (2, 20) **E** (2, 23)

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21 The equation of the graph shown opposite is:

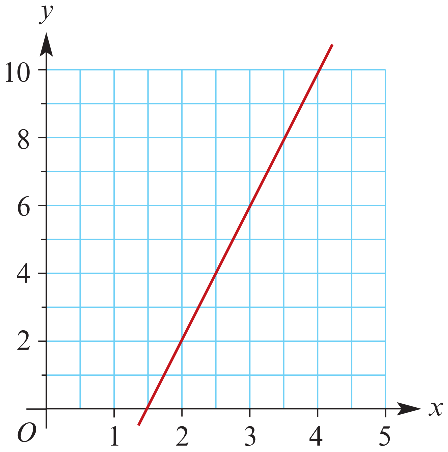
A $y = 4x - 6$

B $y = 4x - 8$

C $y = -4x - 4$

D $y = -4x + 2.5$

E $y = 2.5x - 4$



22 The graph opposite shows the height of a small sapling, h, as it increases with time, t. Its growth rate is closest to:

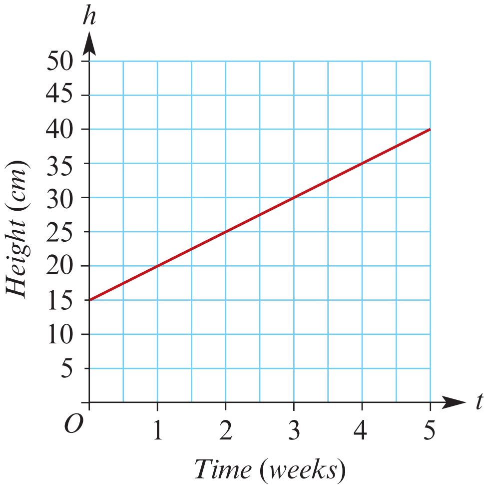
A 1 cm/week

B 3 cm/week

C 5 cm/week

D 8 cm/week

E 15 cm/week



23 The solution to the pair of simultaneous equations

$$y = 5x$$

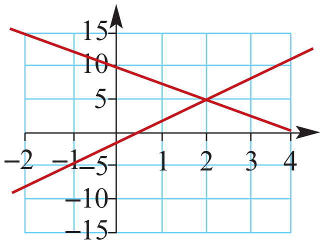
$$y = 2x + 6$$

is:

A $(-2, 0)$ **B** $(-1, -5)$ **C** $(3, 0)$ **D** $(2, 10)$

24 The point of intersection of the lines shown in the diagram is:

A $(5, 2)$ **B** $(0, 0)$ **C** $(0, 9)$ **D** $(2, 5)$



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25 The solution to the pair of simultaneous equations

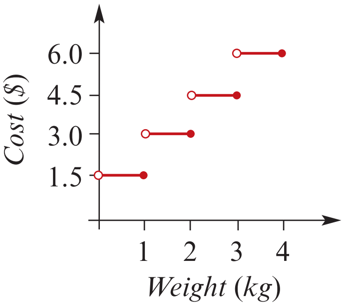
$$2x + 3y = -6$$
$$x + 3y = 0$$

is:

A $(-6, -2)$ **B** $(6, 2)$ **C** $(-2, 6)$ **D** $(-6, 2)$

26 The graph shows the cost of posting parcels of various weights. A parcel weighing 3 kg will cost:

A \$4.50 **B** \$6.00 **C** \$5.00
D \$4.00 **E** \$5.25



27 The piecewise linear graph shown represents the function with the rules:

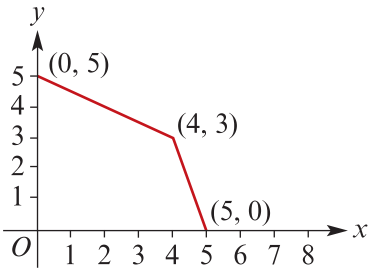
A $y = \frac{1}{2}x + 5 \quad 0 \le x \le 4$
$y = x - 5 \quad 4 < x \le 5$

B $y = \frac{1}{2}x + 5 \quad 0 \le x \le 4$
$y = -2x + 11 \quad 4 < x \le 5$

C $y = -\frac{1}{2}x + 5 \quad 0 \le x \le 4$
$y = -2x - 11 \quad 4 < x \le 5$

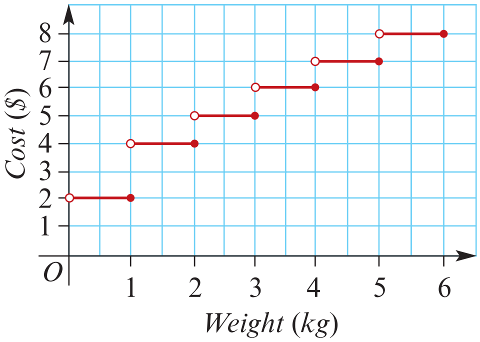
D $y = -\frac{1}{2}x + 5 \quad 0 \le x \le 4$
$y = -3x + 15 \quad 4 < x \le 5$

E $y = x + 5 \quad 0 \le x \le 4$
$y = -3x + 15 \quad 4 < x \le 5$



28 The graph shows the cost of posting parcels of various weights. A person posts two parcels, one weighing 3 kg and the other 1.5 kg. If each parcel is charged for separately, the cost of sending the two parcels is:

A \$4.50 **B** \$5.00 **C** \$7.00
D \$9.00 **E** \$10.00



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Short-answer questions

1 Solve the following equations for x.

a $x + 5 = 15$ **b** $x - 7 = 4$ **c** $16 + x = 24$

d $9 - x = 3$ **e** $2x + 8 = 10$ **f** $3x - 4 = 17$

g $x + 4 = -2$ **h** $3 - x = -8$ **i** $6x + 8 = 26$

j $3x - 4 = 5$ **k** $\frac{x}{5} = 3$ **l** $\frac{x}{-2} = 12$

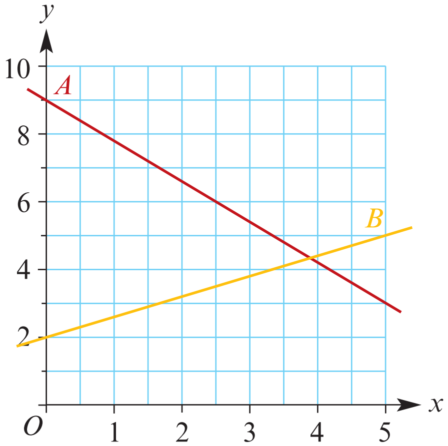
2 I think of a number, double it and add 4. If the result is 6, what is the original number?

3 Four less than three times a number is 11. What is the number?

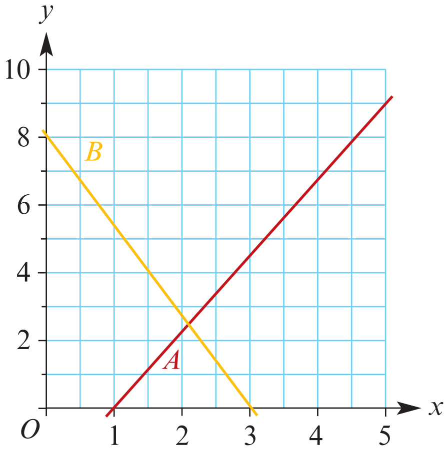
4 Plot the graphs of these linear relations by hand.

a $y = 5x + 2$ **b** $y = -x + 12$ **c** $y = 4x - 2$

5 Find the slope of each of the lines A and B shown on the graph below. Give the answers correct to one decimal place.



6 Find the slope of each of the lines A and B shown on the graph below, correct to two decimal places.



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7 A linear model for the amount C, in dollars, charged to deliver w cubic metres of builder's sand is given by $C = 95 + 110w$ for $0 \le w \le 7$.

a Use the model to determine the total cost of delivering 6 cubic metres of sand.

b When the initial cost of $95 is paid, what is the cost for each additional cubic metre of builder's sand?

8 **a** Sketch the piecewise linear graph defined by the rules:

$$y = x + 2 \quad 0 < x \le 2$$

$$y = 5 \quad 2 < x \le 4$$

$$y = -\frac{1}{2}x \quad 4 < x \le 6$$

b What is the value of y when $x = 5$?

9 An olive farm sells bottles of olive oil. The cost per bottle depends on the number of bottles purchased, as shown in the table below.

Number of bottles	Price per bottle
Up to 4	$6.50
5 up to 9	$6.00
10 up to 14	$5.50
15 up to 20	$5.00

a What is the price per bottle if 15 bottles of olive oil are purchased?

b How much will it cost to purchase 6 bottles of olive oil?

c Show the information from the table on a step graph.

Hint: You will need to show a dot for each point, rather than a solid line because only whole numbers of bottles can be purchased.

10 Find the point of intersection of the following pairs of lines.

a $y = x + 2$ and $y = -3x + 6$

b $y = x - 3$ and $2x - y = 7$

c $x + y = 6$ and $2x - y = 9$

11 Solve the following pairs of simultaneous equations.

a $y = 5x - 2$ and $2x + y = 12$

b $x + 2y = 8$ and $3x - 2y = 4$

c $2p - q = 12$ and $p + q = 3$

d $3p + 5q = 25$ and $2p - q = 8$

e $3p + 2q = 8$ and $p - 2q = 0$

12 Find the break-even point if sales revenue ($$S$) is given by the rule $S = 0.75x$, where x is the number of items sold and the cost ($$C$) is given by the rule $C = 0.25x + 100$.

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Extended-response questions

1 The perimeter of a rectangle is 10 times the width. The length is 9 metres more than the width. Find the width of the rectangle. SF

2 A secondary school offers three languages: French, Indonesian and Japanese. In Year 9, there are 105 students studying one of these languages. The Indonesian class has two-thirds the number of students in the French class and the Japanese class has five-sixths the number of students in the French class. How many students study each language? (No student is studying more than one language). CF

3 A new piece of machinery is purchased by a business for $300 000. Its value is then depreciated each month using the graph below. SF

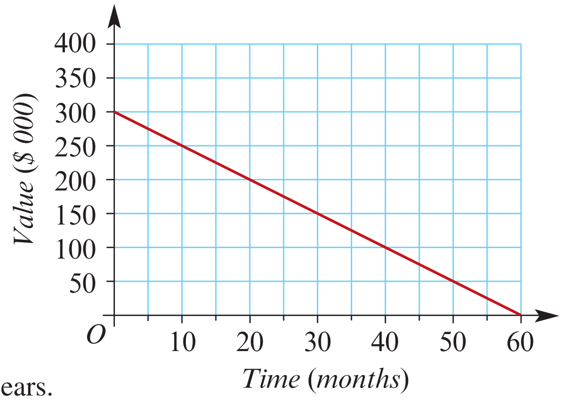
a What is the value of the machine after 20 months?

b When does the line predict that the machine will have no value?

c Find the equation of the line in terms of value, V, and time, t.

d Use the equation to predict the value of the machine after 3 years.

e By how much does the machine depreciate in value each month?

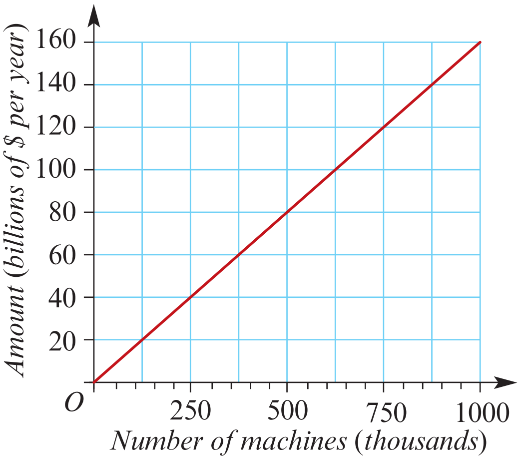


4 The amount of money transacted through ATMs has increased with the number of ATMs available. The graph charts this increase.

a What was the amount of money transacted through ATMs when there were 500 000 machines?

b Find the equation of the line in terms of amount of money transacted, A, and number of ATMs, N. (Leave A in billions and N in thousands).

c Use the equation to predict the amount transacted per year when there were 600 000 machines.



d If the same rule applies, how much money is predicted to be transacted through ATM machines when there are 1 500 000 machines?

e By how much does the amount of money transacted through ATMs increase with each 1000 extra ATMs?

5 Two families went to the theatre. The first family bought tickets for 3 adults and 5 children and paid \$73.50. The second family bought tickets for 2 adults and 3 children and paid \$46.50.

a Write down two simultaneous equations that could be used to solve the problem.

b What was the cost of an adult's ticket?

c What was the cost of a child's ticket?

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6 To conserve water, one charging system increases the amount people pay as the amount of water used increases. The charging system is modelled by:

$$C = 5 + 0.4x \ (0 \leq x < 30) \qquad C = -31 + 1.6x \ (x \geq 30)$$

C is the charge in dollars and x is the amount of water used in kilolitres (kL).

a Use the appropriate equation to determine the charge for using:

i 20 kL **ii** 30 kL **iii** 50 kL

b How much does a kilolitre of water cost when you use:

i less than 30 kL? **ii** more than 30 kL?

c Use the equations to construct a segmented graph for $0 \leq x \leq 50$.

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5

Revision of Unit 1 Chapters 1–4

UNIT 1 MONEY, MEASUREMENT AND RELATIONS

Topic 1 Consumer arithmetic

Topic 2: Shape and measurement

Topic 3: Linear equations and their graphs

The revision exercises are arranged by chapter with these categories of questions:

- Simple familiar question types
- Complex familiar question types
- Complex unfamiliar question types
- Problem-solving and modelling questions
- Problem-solving and modelling investigations

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5A Revision of Chapter 1 Consumer Arithmetic: Personal finance

Multiple-choice questions – see Interactive Textbook

▶ Simple familiar questions

1 Philip works as a builder and earns \$27.80 an hour. How much does he earn for working a 38 hour week?

2 Matt works in a department store. He gets paid \$25 per hour for the first 30 hours he works each week, then time and a half for additional hours. He is required to take a 30 minute break during each day that he works for which he is not paid. His time card for this week is as follows:

Day	Start	Finish
Monday	8.30	5.00
Tuesday	8.30	6.00
Wednesday	8.30	5.00
Friday	8.30	6.00

a How many hours did Matt work this week for which he will be paid?

b How much did he earn?

3 Fatima is a salesperson. She is paid \$26 000 per year, plus a commission of 1% of all sales she makes. Last week she sold \$64 900 worth of goods. How much did she earn that week?

4 If a pack of 12 batteries cost \$34.99, how much would 20 batteries cost (to the nearest cent)?

▶ Complex familiar questions

5 The supermarket has three brands of ground coffee for sale.

Brand A	250 g pack costing \$8.80
Brand B	610 g pack costing \$20.00
Brand C	200 g pack costing \$6.50

Compare the cost of each for 100 gm, and order the brands from cheapest to dearest.

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6 Various online store sells running shoes with the following prices listed for a range of countries:

Australia	\$149.95 (AUD)
US	\$129.99 (USD)
New Zealand	\$154.95 (NZD)
Great Britain	£85.00 (GBP)

The website notes that delivery is free in the USA but costs \$15 USD to anywhere else in the world. Miller can order from any of the online stores.

a Where do you recommend he purchase the shoes if on the day he wants to purchase them \$1 USD is equal to \$1.26 AUD, \$1.38 NZD, and £0.73 GBP?

b How much do the cheapest shoes cost in AUD?

7 Lacey has recorded all her expenses for a year as shown:

Expenses	
Rent	\$12 600.00
Electricity	\$923.40
Clothes	\$5240.00
Entertainment	\$12 459.00
Health Insurance	\$1479.40
Petrol &servicing	\$2120.60
Car insurance	\$1288.60
Telephone	\$972.00
Gifts	\$2500.00
Food & Groceries	\$5560.00

a Find the total of her expenses for the year, and then the average she spends per month on each of the items in her budget.

b If Lacey's income is \$3950 per month:

  i how much is she able to save per month on average?

  ii how much could she save during the year? Give you answer to the nearest dollar.

c Lacey really wants to save to buy a new car. She wants to increase the amount she earns by \$300 per week by working overtime in her current role. She can work extra hours at time and a half. If her annual salary is \$58 000, and she normally works 38 hours per week, how many hours overtime will she need to work each week to achieve her goal? Give your answer to the nearest half hour.

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▶ Complex unfamiliar questions

8 Tim wishes to change 2000 Singapore Dollars (SGD) to EUR. He decides to look around before changing his money. He gets information from three banks as shown in the table – commission is the amount charged by the bank for exchanging the currency.

Bank	Exchange rate	Commission
Eastern	0.57	1.7%
Centre	0.54	1.5%
Western	0.60	2%

a Where should Tim go to exchange his money? Explain your answer.

b On another trip Tim decides to exchange the money at the airport. He notices that they offer two different exchange rates:

- The **sell rate**, which according to the company ‘is the rate at which we sell foreign currency in exchange for local currency. For example, if you were heading to Europe, you would exchange Australian dollars for euros at the sell rate’.
- The **buy rate**, which according to the company ‘is the rate at which we buy foreign currency back from you into your local currency. For example, if you were returning from America, we would exchange your US dollars (USD) back into Australian dollars at the buy rate’.

This company also charges 1.75% commission, on both the buy and sell rates. Tim is traveling from Australia to the US, and then back to Australia. The exchange rates offered by the company are as follows:

Currency	Sell rate	Buy rate
USD United States Dollars	0.7636	0.8322

i If Tim changes $3000 AUD to US dollars on his way to the US, how much does he receive? Give your answer to the nearest cent.

ii If Tim exchanges $535 USD back to AUD on his return to Australia, how much money does he now have? Give your answer to the nearest cent.

iii How much has he spent in total on commission? Give your answer in AUD to the nearest cent.

9 Mark has just finished university and is looking for work. He owes a large amount on his credit card, and will need to pay $360 per fortnight to pay this debt off in 12 month. He also needs to earn enough so he can cover his living expenses, which add to $33 700 for the year.

a How much does Mark need each fortnight to cover his living costs, and to pay back the credit card debt in 12 months?

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b Mark knows that he will need to pay income tax. Income tax (annual) is payable as follows:

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$87 000	$3572 plus 32.5c for each $1 over $37 000
$87 001–$180 000	$19 822 plus 37c for each $1 over $87 000
$180 001 and over	$54 232 plus 45c for each $1 over $180 000

He is offered a job with an annual salary of $45 000. How much will he be paid each fortnight after income tax is deducted (assume there are no other deductions)?

c What is the minimum salary (in whole dollars) Mark needs to be paid annually before tax, in order to bring in enough each fortnight to cover his living costs and to pay back the credit card debt in 12 months?

▶ Problem solving and modelling questions

Going on a holiday Bali or Fiji?

10 Maggie is planning a holiday this June. She has her heart set on a tropical getaway in either Bali or Fiji. To help her make her decision she has decided to look at the relative cost of each holiday. She is planning to holiday for 7 nights in June, and will fly from Brisbane to her destination of choice.

a Flights: Maggie is happy to fly with any airline, so cost is her only consideration.

  i Using airline or other online flight-booking websites, find out and compare the cost of direct flights from Brisbane to Bali – don’t forget to add the cost of luggage and meals to make them comparable. Note which flight is the cheapest, and how much it costs.

  ii Find out and compare the cost of direct flights from Brisbane to Fiji – again adding the cost of luggage and meals to make them comparable. Note which flight is the cheapest, and how much it costs.

b Accommodation: Maggie wants to be comfortable, and would like accommodation which is comfortable and also close to the beach and restaurants. She’d like wifi in her room, and a hotel with at least one swimming pool

  i Using online hotel-booking websites, compare the price of three hotels in Bali which meet Maggie’s requirements, and choose one. Give reasons for your choice.

  ii Compare the price of three hotels in in Fiji that meets Maggie’s requirements, and choose one. Give reasons for your choice.

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c Spending money

  i List everything Maggie needs to cover when she is determining how much spending money she needs, identifying where you have made assumptions.

  ii How much spending money in Indonesian rupiah will Maggie need in Bali?

  iii How much spending money in Fijian dollars will Maggie need in Fiji?

d Compare the total cost of each holiday in terms of airfares, accommodation, and spending money. Where do you suggest Maggie should go? Give reasons.

▶ Problem solving and modelling and investigations

11 Investigate the cost of living away from home for a university student. Undertake appropriate research to inform yourself how much the student would need to pay for rent, and other living expenses, and use this information to produce a personal budget for the student, assuming that the student is able to access Youth Allowance, as well as to undertake some part-time work. Don't forget to consider the impact on the Youth Allowance of this additional income. Refine the budget to ensure that the student is able to study and live away from home.

5B Revision of Chapter 2 Consumer arithmetic: loans and finance

Multiple-choice questions – see Interactive Textbook

▶ Simple familiar questions

1 Calculate the new discounted price of an item costing \$49.95 if it is discounted by 5%.

2 An investment of \$4000 attracts an interest rate of 5% per annum simple interest for a period of 4 years. How much interest is earned in total?

3 Find the amount that should be invested in order to earn \$2400 over 5 years at an annual simple interest rate of 3.5%. Give your answer to the nearest dollar.

4 An amount of \$5000 is invested at 2% compound interest per annum for 2 years. Determine:

a The final value of this investment

b The amount of interest earned.

5 Kira has 1000 shares in an Art Gallery. The current market price of the shares is \$11.50 each, and the Gallery has recently paid a dividend of 83 cents per share.

a How much does Frankie receive in dividends in total?

b What is the percentage yield for this share? Give your answer correct to one decimal place.

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▶ Complex familiar questions

6 When Alan buys his shares in an IT company the purchase price is $35.50 per share, and he pays 1.8% commission on the purchase. He sells them 1 year later for $48.75, and again pays 1.8% commission on the sale.

a How much did he make per share?

b What is his percentage gain on the shares? Give your answer correct to two decimal places.

7 How much additional interest is earned if $10 000 is invested for 10 years at 3.5% when interested is compounded annually, as compared with simple interest paid at the same rate over the same time period? Give your answer to the nearest dollar.

8 Tom invests $8000 for 3 years at 2.4% per annum interest compounding monthly.

a How much does he have in the account after 3 years?

b At the time that he made the investment Tom could have used the money to buy a painting for the same amount ($8000). The price of the painting has increased according to inflation, which has averaged 2.5% per annum since then then.

i How much will the painting cost Tom if he decides to buy it at the end of the investment period, i.e. after 3 years?

ii Is he better off to have bought the painting for $8000 in the first place, or to have invested his money and bought it after 3 years?

▶ Complex unfamiliar questions

9 Ali planned to invest $8500 in a term deposit. He had two investment plans from which to choose.

Plan 1: Simple interest at 5% per annum
Plan 2: Compound interest at 5% per annum, calculated on the increasing balance and added to the account every month

a Calculate the total interest earned by Ali if he invested his money using Plan 1 for one year.

b Calculate, to the nearest cent, the total interest earned by Ali if he invested his money using Plan 2 for one year.

c Calculate the simple interest rate that would provide the same total interest as earned under Plan 2 for an investment of $8500 for one year. Give your answer correct to two decimal places.

d Determine the number of years it would take for the investment to double if it is invested under Plan 1.

e Ali sold his car and invested the proceeds in another investment plan. Under this plan this amount doubles in exactly nine years. Calculate the annual simple interest rate of this investment plan correct to one decimal place.

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10 Brad buys a coffee machine with an initial value of $12 000.

- **a** He thinks that the resale value of the coffee machine will decrease by 17% of the purchase price in year 1, 14% of the purchase price in Year 2 and 8% of the purchase price in Year 3. What is its value after three years? Write your answer correct to the nearest dollar.
- **b** His business partner James thinks instead that the value of the machine will decrease by 15% in each year. What is the value of the machine after three years? Write your answer correct to the nearest dollar.
- **c** If Brad sells 35 000 cups of coffee per year, calculate how much it costs him per coffee to cover the loss in value of the coffee machine after three years (give your answer in cents, correct to one decimal place):
  - **i** using Brad's figures for the value
  - **ii** using James's figures for the value.

Modelling and Problem Solving Question

11 Suppose that Liang and Lei borrow $410 000 from the Bayside Building Society at an interest rate of 3% per annum, compounded monthly, for 5 years.

- **a** How much interest is payable at the end of the first month?
- **b** What is the minimum payment they should make each month to ensure they don't owe more than $410 000 in any month?
- **c** If they make a payment of $1500 at the end of the first month, how much will they still owe?
- **d** Create a spreadsheet which allows to you to calculate the amount owing at the end of each month after they make a payment of $1500. To the nearest dollar, how much will they owe after six payments of $1500?
  They decide to make payments of $3000 per month.
- **e** To the nearest dollar, how much will they owe after six payments of $3000?
- **f** How long will it take them to pay back the loan if they pay $3000 per month?
- **g** To the nearest $1000, how much will they have paid in interest when the loan is paid off?
- **h** Investigate the change in amount of interest paid and time taken to pay back the loan by varying the interest rate and the amount of the monthly repayments.

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Modelling and Problem Solving Investigation

12 One of the first purchases you will most likely make is your first car. How much can you afford to pay for that car, and how will you go about financing the purchase? In this project you are going to use available resources to determine the best strategy. You will need to investigate each of the following themes:

- **What can you afford**?
  Assuming that you will need to finance the car, what can you afford to repay each week or fortnight? You will need to consider your likely salary, as well as you other living investments to determine this figure. Some of the major banks will give advice regarding this amount and include 'affordability' calculators on their websites.
- **How should you finance the car**?
  Compare some different forms of finance (such as variable interest personal loans, fixed interest personal loans, credit cards), different financial institutions, and finance offered directly by the car dealerships to determine your best option.
- **What car should you buy**?
  Cars often depreciate in value very quickly, especially if purchased new. In the worst case, you can end up owing more money on a car than its current market value! Using new and second-hand car sales websites, compare the price of two or three different brands of car purchased new, and at various ages, over the period of a loan you need to pay for it. How much will it be worth at the end?

5C Revision of Chapter 3 Shape and measurement

Multiple-choice questions – see Interactive Textbook

Simple familiar questions

1 A right-angled triangle has a hypotenuse of length 20 cm and another side of length 16 cm. What is the length of the remaining side?

2 A short cylinder has a volume of 216 cm^3 and a taller cylinder with the same radius has a volume of 512 cm^3. Given the taller cylinder has a height of 16 cm, find the height of the shorter cylinder.

3 Calculate the area of the following shapes

a A circle with diameter 7 cm

b A triangle with a base of 14.2 cm and height 7 cm

4 Calculate the volume of the following shapes

a A triangular pyramid with a base area of 32 cm^2 and height 9 cm

b A right square-based pyramid with a side length of 5 cm and height of 8 cm

c A hemisphere with radius 3 cm

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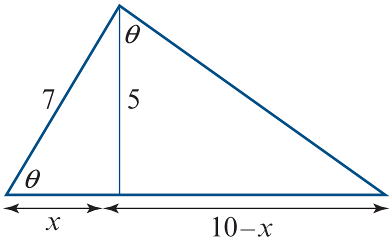
Complex familiar questions

5 The radius of the base of a cylinder is 4 cm and its height is 12 cm.

- **a** Calculate the volume of the cylinder in cm^3
- **b** Determine the amount of metal required to make the cylinder in cm^2

6 Consider this diagram.



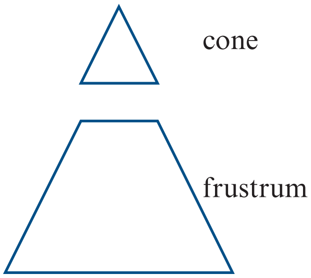
- **a** Identify an appropriate pair of similar triangles in the diagram, and set up an equation to find the unknown length x.
- **b** Solve the equation in part a to find the unknown length.

7 A rectangular prism has length 10 cm, width 5 cm and height 2 cm

- **a** Calculate the surface area of the object.
- **b** A similar object has a surface area of 640 cm^2.
- **c** Calculate the area scale factor k^2, and hence find the volume scale factor k^3.
- **d** Find the volume of the similar object.

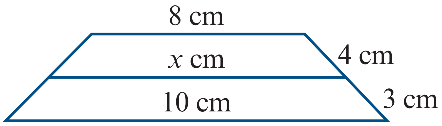
Complex unfamiliar questions

8 A right cone is sliced parallel to the base such that it forms a smaller cone and a frustum. The original cone has a height of 5 cm. The smaller cone has a height of 1 cm.



- **a** The smaller cone is similar to the original cone. Determine the scale factor, k.
- **b** Determine the volume scale factor, k^3.
- **c** Calculate the volume of the smaller cone, given that the frustum has a volume of 4 cm^3

9 Consider the diagram to the right.



- **a** Copy the diagram adding construction lines where necessary, to identify a pair of similar triangles.
- **b** Write an equation which, when solved, will give the value of x.
- **c** Find the value of x.

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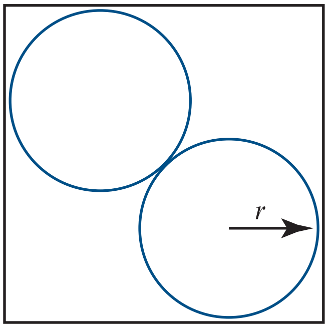
Modelling and problem-solving question

10 In the diagram, the two circles are congruent and have a radius of *r* cm.



a State the shortest distance between the centres of the two circles.

b Calculate the shortest distance of the centre of one circle to the nearest vertex of the enclosing square.

c Calculate the diagonal length of the enclosing square.

d Find the side length of the enclosing square.

e Hence, determine the ratio of the diagonal of the enclosing square to its side length.

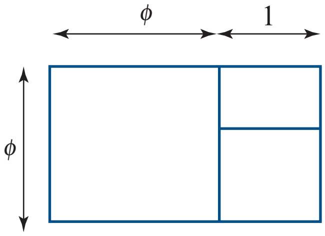
Now consider an arbitrary square of side length *l*.

f Using Pythagoras' theorem or otherwise, show that the diagonal length of the square is $\sqrt{2}l$.

g What do you notice about the ratio of the diagonal of the square to its side length? Is this always the case?

Modelling and problem-solving investigation

11 The 'golden rectangle' is shown in this diagram. It is a rectangle that can be divided into two squares and a smaller rectangle, which is a similar rectangle to the large one. The sides of the larger square are labelled with the symbol φ, which is the Greek letter phi.



To answer the questions below you may want to do a internet research using 'phi' and 'golden' as search terms.

a Construct an equation in terms of φ using similar rectangles.

b Solve this equation for φ using your calculator.

c Are any of the solutions for φ invalid? Explain why or why not.

d Investigate the significance of φ in mathematics and other fields such as architecture and art. What other names are given to φ?

e What is the value of φ to three decimal places?

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5D Revision of Chapter 4 Linear equations and their graphs

Multiple-choice questions – see Interactive Textbook

Simple familiar questions

1 Write down the equation of the line that:

- **a** has y-intercept -3, slope 7
- **b** passes through the points $(0, 3)$ and $(2, 5)$
- **c** has slope 2 and passes through $(1, 3)$

2 Sketch the following graphs:

- **a** $2y - x = 4$
- **b** $x = y + 3$
- **c** $\frac{y}{2x} + 3 = 1$
- **d** $y = \frac{1}{3}x - \frac{2}{3}$

3 Solve the following pairs of simultaneous equations

- **a** $x = 2y + 3$ and $2x + 3y = 7$
- **b** $x - y = 7$ and $3x + 2y = 1$

4 State the y-intercept and slope of the graphs of the following equations:

- **a** $y = 4x + 1$
- **b** $7x = 4y + 2$
- **c** $x = -\frac{1}{3}y + 4$
- **d** $8x + 4y = 12$

Complex familiar questions

5 The formula for converting Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. If the temperature in degrees Celsius increases by one degree, how much does the temperature in Fahrenheit increase by?

6 A graduate student has a starting salary of \$52 000. Their salary increases by \$4000 every year.
Let S be the salary after t years.

- **a** Write down a linear model in S and t to represent this situation.
- **b** Sketch the graph, showing the coordinates of the intercept and endpoint. Assume the model stops at 10 years.

7 Consider the simultaneous equations $3x + 2y = 7$ and $x - 3y = -5$.

- **a** Graph the two equations on the same set of axes and find the point of intersection.
- **b** Solve the equations algebraically to confirm your answer to part **a**.

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▶ Complex unfamiliar questions

8 Daniel buys a new car for \$40 000. Its value depreciates by \$8000 per year for four years, at which time it will be sold.
Let V be the value of the car after t years.

- **a** Write down a linear model in V and t to represent this situation.
- **b** Sketch the graph, showing the coordinates of the intercept and its endpoint.
- **c** Use the equation to predict the depreciated value after 2.8 years.
- **d** How much is the car worth at the time it is sold?
- **e** If the model were to be extended past four years, when will the car have zero value according to the model?
- **f** Is your answer to part e valid? Consider whether the model has any limitations or not.

9 A food delivery driver earns a base amount of \$3, and \$7 for each delivery they make. Let T be their total earnings after n deliveries.

- **a** Write down a linear model in terms of T and n to represent this situation.
- **b** How many deliveries does the driver need to make to earn at least \$50?

The food delivery company charges a 20% fee of all earnings to delivery drivers.

- **c** Write down the linear model representing the earnings, T_1, of the delivery driver.
- **d** If overhead costs are \$1 per delivery, what is the linear model representing the net earnings, T_{net} of the driver?
- **e** With all factors mentioned above, how much does a delivery driver earn, per delivery, in the end?

▶ Modelling and problem-solving question

10 Aeroplanes on long haul flights often climb to an initial cruising altitude for several hours and then proceed to a higher altitude later during the flight. An plane initially climbs to 31 000 ft. It maintains this altitude for 5 hours. After 5 hours, the plane takes 20 minutes to climb to 37 000 ft. (The international aviation industry uses feet not metres for altitude.
Let h be the height of the airplane and t be the time in hours after initially reaching 31 000 ft.

- **a** At what time does the plane start ascending and when does it stop?

Once the plane reaches 37 000 ft, it maintains this height for 3 hours and afterwards it descends to the ground over 1 hour.

- **b** Sketch a graph of h against t.
- **c** Find the equation relating h and t when the plane is climbing from 31 000 ft to 37 000 ft.
- **d** Find the equation relating h and t when the plane is descending from 37 000 ft to the ground.

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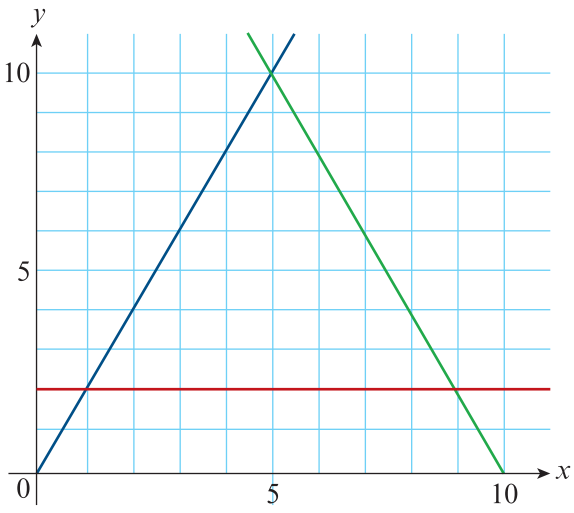
▶ Modelling and problem-solving investigation

Straight lines can be modelled by linear equations and graphed.

Here are three lines that create a triangular shape:



a What are the equations of the three lines red, blue and green?

b On a graph with axes from 0 to 12 for both x and y, plot the graphs of linear equations that intersect to form these shapes, and list the equations you have used:

- **i** A square with sides parallel to the axes
- **ii** A diamond with a vertical orientation
- **iii** A trapezium
- **iv** A hexagon

c In the section on piecewise linear graphs you used ‘domain’ statements such as $5 \leq x \leq 10$ that create line segments. Repeat part b above but this time add domains so that the lines end where they intersect., resulting in 2D geometric shapes being graphed without any extending lines.

d Enter the equations and domains you created in part c to draw the same geometric shapes in a graphing program such as Desmos (accessible in the interactive textbook).

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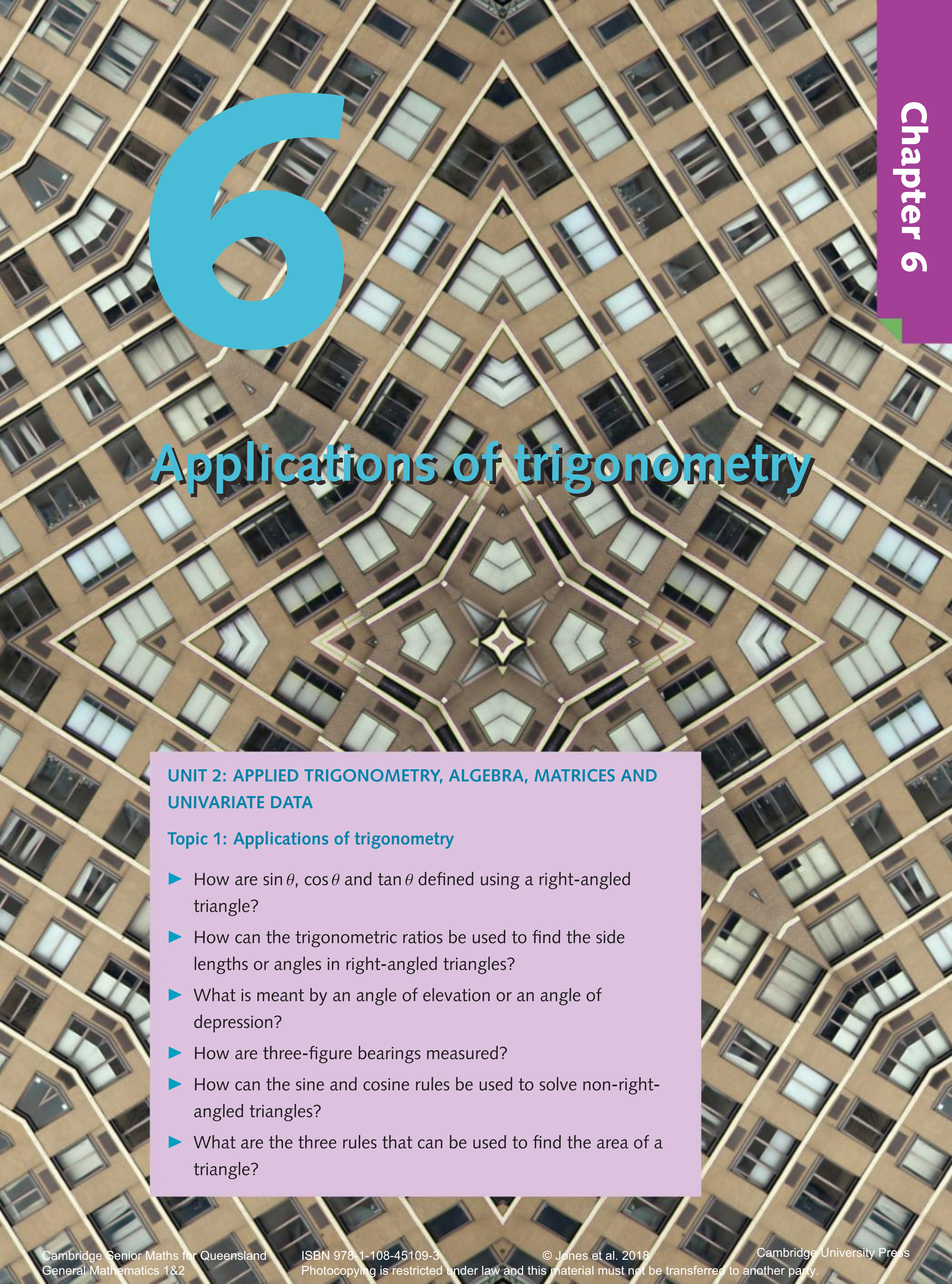


6

Applications of trigonometry

UNIT 2: APPLIED TRIGONOMETRY, ALGEBRA, MATRICES AND UNIVARIATE DATA

Topic 1: Applications of trigonometry

- How are $\sin\theta$, $\cos\theta$ and $\tan\theta$ defined using a right-angled triangle?
- How can the trigonometric ratios be used to find the side lengths or angles in right-angled triangles?
- What is meant by an angle of elevation or an angle of depression?
- How are three-figure bearings measured?
- How can the sine and cosine rules be used to solve non-right-angled triangles?
- What are the three rules that can be used to find the area of a triangle?

Introduction

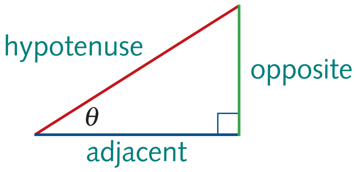
Trigonometry can be used to solve many practical problems. How high is that tree? What is the height of the mountain we can see in the distance? What is the exact location of the fire that has just been seen by fire spotters? How wide is the lake? What is the area of this irregular-shaped paddock?

6A Review of basic trigonometry

Although you are likely to have studied some trigonometry, it may be helpful to review a few basic ideas.

▶ Naming the sides of a right-angled triangle

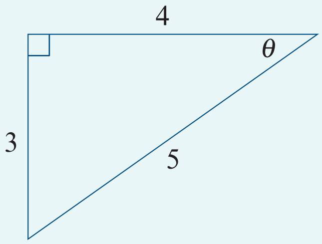
- The *hypotenuse* is the longest side of the right-angled triangle and is always opposite the right angle (90°).
- The *opposite* side is directly opposite the angle θ.
- The *adjacent* side is beside the angle θ, but it is not the hypotenuse. It runs from θ to the right angle.



The opposite and adjacent sides are located in relation to the position of angle θ. If θ was in the other corner, the sides would have to swap their labels. The letter θ is the Greek letter *theta*. It is commonly used to label an angle.

Example 1 **Identifying the sides of a right-angled triangle**

Give the lengths of the hypotenuse, the opposite side and the adjacent side in the triangle shown.



Solution

The hypotenuse is opposite the right angle.	The hypotenuse, $h = 5$
The opposite side is opposite the angle θ.	The opposite side, $o = 3$
The adjacent side is beside θ, but is not the hypotenuse.	The adjacent side, $a = 4$

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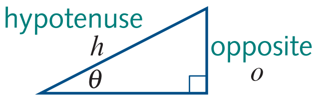
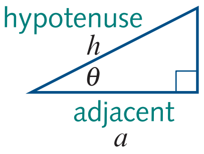
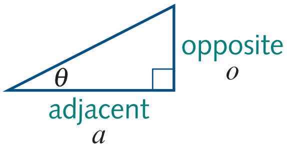
The trigonometric ratios

The **trigonometric ratios sin *θ*, cos *θ*** and **tan *θ*** can be defined in terms of the sides of a right-angled triangle. Note that we are using abbreviations here – 'sin' for sine, 'cos' for cosine, and 'tan' for tangent.

		
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \dfrac{o}{h}$	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{a}{h}$	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \dfrac{o}{a}$

"SOH — CAH — TOA"

The mnemonic **SOH-CAH-TOA** is often used by students to help them remember the rule for each trigonometric ratio.

In this mnemonic:

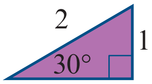
- **SOH** reminds us that sine equals opposite over hypotenuse
- **CAH** reminds us that cosine equals adjacent over hypotenuse
- **TOA** reminds us that tan equals opposite over adjacent.

Or you may prefer:

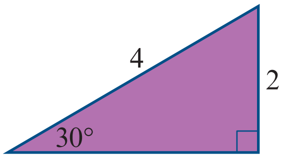
'**S**ir **O**liver's **H**orse **C**ame **A**mbling **H**ome **T**o **O**liver's **A**rms'

The meaning of the trigonometric ratios

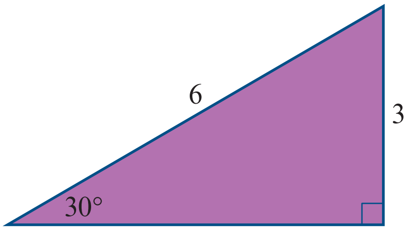
Using a calculator, we find, for example, that sin 30° = 0.5. This means that in *all* right-angled triangles with an angle of 30°, the ratio of the length of the side opposite the 30° to the length of the hypotenuse, is always 0.5.



$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} = 0.5$$



$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{2}{4} = 0.5$$



$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{6} = 0.5$$

Try drawing any right-angled triangle with an angle of 30° and check that the ratio:

$$\frac{\text{opposite}}{\text{hypotenuse}} = 0.5$$

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Similarly, for *any* right-angled triangle with an angle of 30° the ratios cos 30° and tan 30° always have the same values:

$$\cos 30^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ is always } \frac{\sqrt{3}}{2} = 0.8660 \text{ (to four decimal places)}$$

$$\tan 30^\circ = \frac{\text{opposite}}{\text{adjacent}} \text{ is always } \frac{1}{\sqrt{3}} = 0.5774 \text{ (to four decimal places).}$$

A calculator gives the value of each trigonometric ratio for any angle entered.

▶ Using a scientific calculator to evaluate trigonometric ratios

Warning!

Make sure that your calculator is set in DEGREE mode before attempting the following example.

On a scientific calculator the DEGREE mode can usually be set using the **mode** key.

Scientific calculator guide This is available in the interactive textbook.

Example 2 Finding the values of trigonometric ratios

Use your calculator to find, correct to four decimal places, the value of:

a sin 49° **b** cos 16° **c** tan 27.3°

Solution

1 Ensure that the mode is set in **Degree**.
To change, press $\boxed{\text{SHIFT}}$ $\boxed{\text{MOD}}$ $\boxed{\text{DEG}}$.

2 Press $\boxed{\sin}$, type **49**, then $\boxed{=}$.

3 Press $\boxed{\cos}$, type **16**, then $\boxed{=}$.

4 Press $\boxed{\tan}$, type **27.3**, then $\boxed{=}$.

5 Write your answer correct to four decimal places.

a $\sin 49^\circ = 0.7547$
b $\cos 16^\circ = 0.9613$
c $\tan 27.3^\circ = 0.5161$

Spreadsheet **Spreadsheet activity 6A:** Using a spreadsheet to find the values of trigonometric ratios

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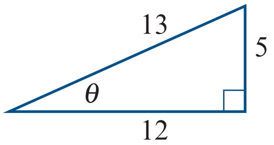
Exercise 6A

Example 1 **1** State the values of the hypotenuse, the opposite side and the adjacent side for each triangle.

a



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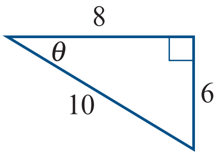

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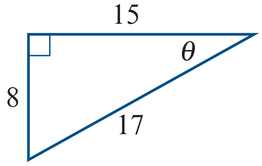

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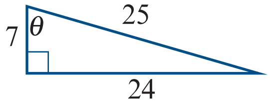

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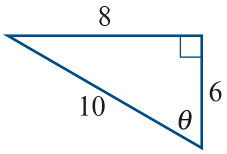

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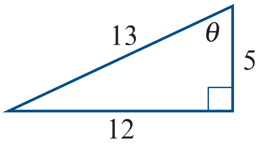


2 Write the ratios for sin θ, cos θ and tan θ for each triangle in Question **1**.

Example 2 **3** Find the values of the following trigonometric ratios, correct to four decimal places.

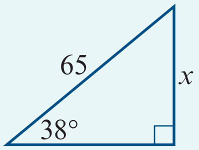
a $\sin 27°$
b $\cos 43°$
c $\tan 62°$
d $\cos 79°$
e $\tan 14°$
f $\sin 81°$
g $\cos 17°$
h $\tan 48°$
i $\sin 80°$
j $\sin 49.8°$
k $\tan 80.2°$
l $\cos 85.7°$

6B Finding an unknown side in a right-angled triangle

The trigonometric ratios can be used to find unknown sides in a right-angled triangle, given an angle and one side. When the unknown side is in the *numerator* (top) of the trigonometric ratio, proceed as follows.

Example 3 **Finding an unknown side**

Find the length of the unknown side x in the triangle shown, correct to two decimal places.



Solution

1. The sides involved are the opposite and the hypotenuse, so use sin θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

2. Substitute in the known values.

$$\sin 38^\circ = \frac{x}{65}$$

3. Multiply both sides of the equation by 65 to obtain an expression for x. Use a calculator to evaluate.

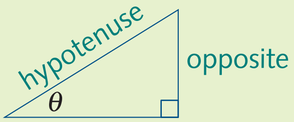
$$65 \times \sin 38^\circ = x$$
$$x = 65 \times \sin 38^\circ$$
$$= 40.017\ldots$$

4. Write your answer correct to two decimal places.

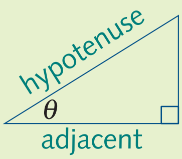
$$x = 40.02$$

Finding an unknown side in a right-angled triangle

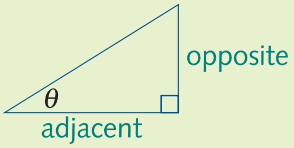
1. Draw the triangle and write in the given angle and side. Label the unknown side as x.
2. Use the trigonometric ratio that includes the given side and the unknown side.
   - **a** For the opposite side and the hypotenuse, use $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$



   - **b** For the adjacent side and the hypotenuse, use $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$



   - **c** For the opposite and the adjacent sides, use $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$



3. Rearrange the equation to make x the subject.
4. Use your calculator to find the value of x to the required number of decimal places.

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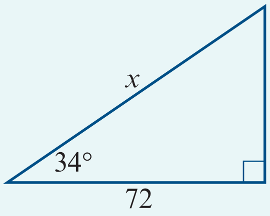
An extra step is needed when the unknown side is in the *denominator* (at the bottom) of the trigonometric ratio.

Example 4 Finding an unknown side which is in the denominator of the trigonometric ratio

Find the value of x in the triangle shown, correct to two decimal places.



Solution

1 The sides involved are the adjacent and the hypotenuse, so use cos θ.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

2 Substitute in the known values.

$$\cos 34^\circ = \frac{72}{x}$$

3 Multiply both sides by x.

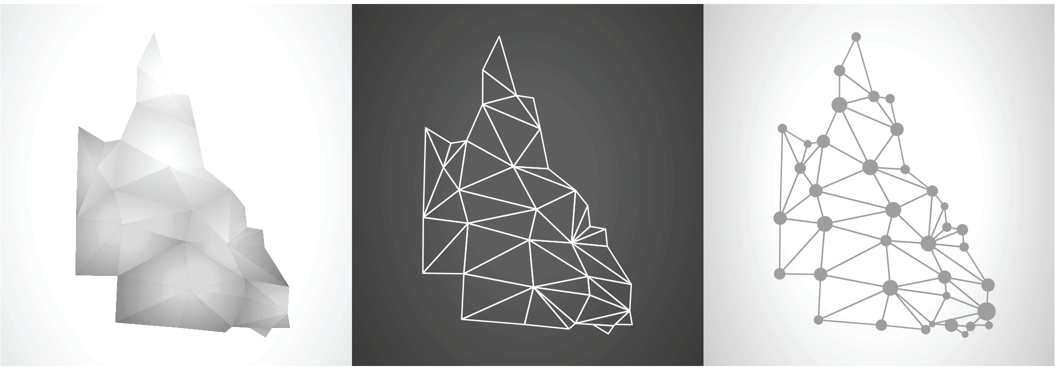
$$x \cos 34^\circ = 72$$

4 Divide both sides by cos 34° to obtain an expression for x. Use a calculator to evaluate.

$$x = \frac{72}{\cos 34^\circ}$$
$$= 86.847\ldots$$

5 Write your answer correct to two decimal places.

$$x = 86.85$$



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Exercise 6B

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Example 3 **1** In each right-angled triangle below:

- decide whether the sin θ, cos θ or tan θ ratio should be used
- then find the unknown side x, correct to two decimal places.

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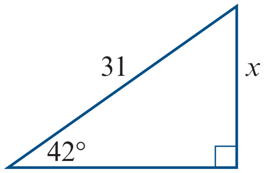

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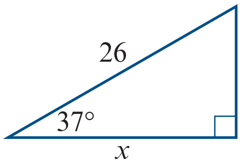

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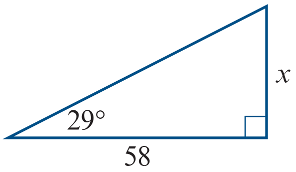

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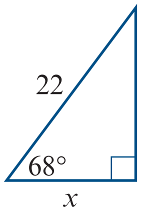

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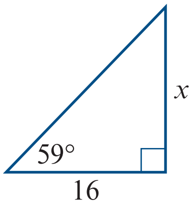

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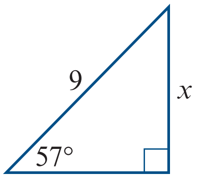

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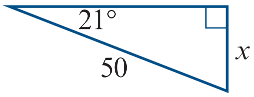

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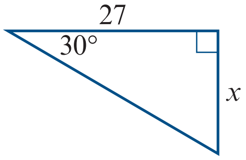

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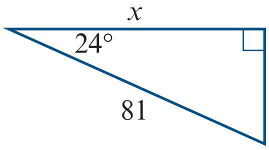

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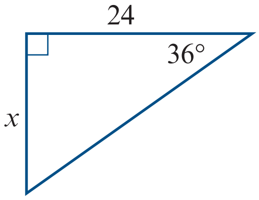

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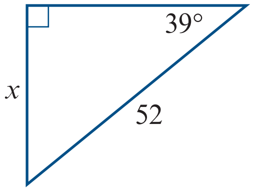

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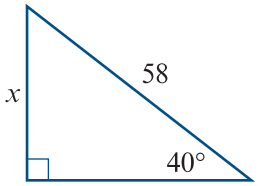

Example 4 **2** Find the unknown side x in each right-angled triangle below. Give the answer correct to two decimal places.

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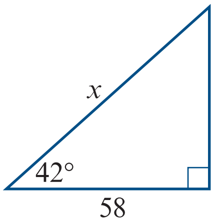

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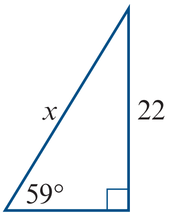

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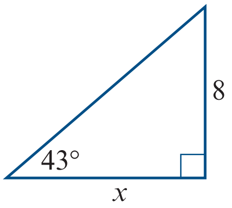

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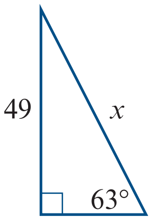

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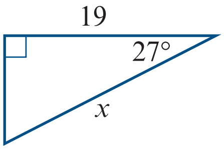

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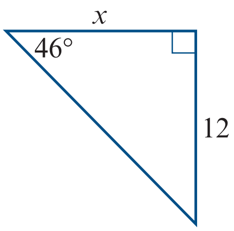

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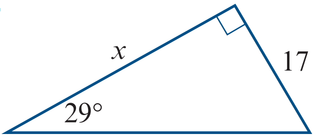

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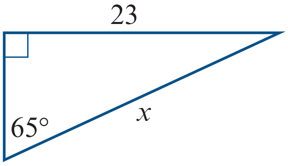

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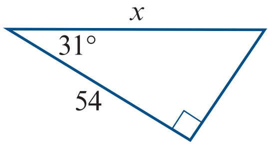

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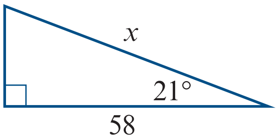

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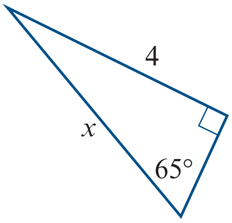

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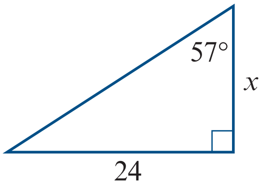

3 Find the length of the unknown side shown in each triangle. Give the answer correct to one decimal place.

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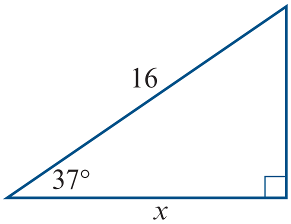

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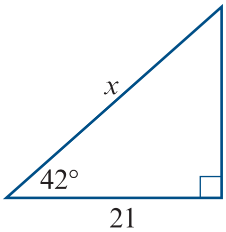

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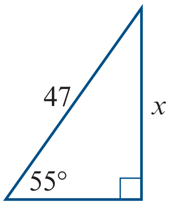

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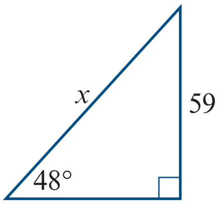

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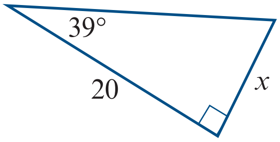

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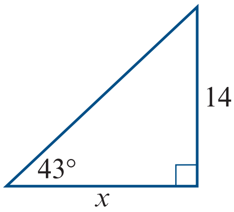

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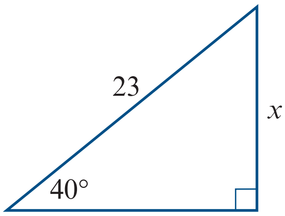

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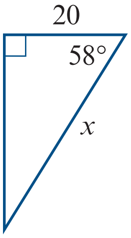

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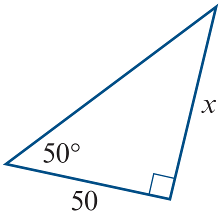

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6C Finding an angle in a right-angled triangle

▶ Finding an angle from a trigonometric ratio value

Before we look at how to find an unknown angle in a right-angled triangle, it will be useful to see how to find the angle when we know the value of the trigonometric ratio.

Suppose a friend told you that they found the sine value of a particular angle to be 0.8480 and challenged you to find out the mystery angle that had been used.

This is equivalent to saying:

$\sin\theta = 0.8480$, find the value of angle θ.

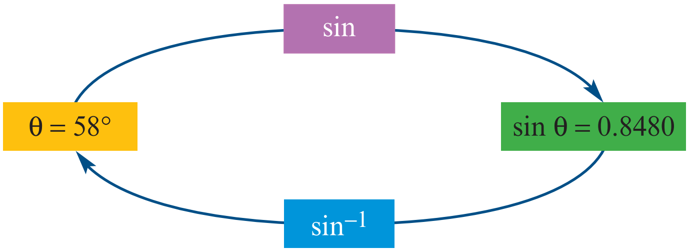
To do this, you need to work backwards from 0.8480 by undoing the sine operation to get back to the angle used. It is as if we have to find reverse gear to undo the effect of the sine function.

The reverse gear for sine is called the inverse of sine, written $\sin^{-1}$. The superscript -1 is not a power. It is just saying let us undo, or take one step backwards from using, the sine function.

The request to find θ when $\sin\theta = 0.8480$ can be written as:

$$\sin^{-1}(0.8480) = \theta$$

This process is summarised in the following diagram.



- The *top arrow* in the diagram corresponds to ‘given θ, find $\sin\theta$’. We use the sine function on our calculator to do this by entering sin 58° into a calculator to obtain the answer 0.8480.
- The *bottom arrow* in the diagram corresponds to ‘given $\sin\theta = 0.8480$, find θ’. We use the $\sin^{-1}$ function on our calculator to do this by entering $\sin^{-1}(0.8480)$ to obtain the answer 58°.

Similarly:

- The *inverse of cosine*, written as $\cos^{-1}$, is used to find θ when $\cos\theta = 0.5$ (e.g.).
- The *inverse of tangent*, written $\tan^{-1}$, is used to find θ when $\tan\theta = 1.67$ (e.g.).

You will learn how to how to use the $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ functions of your calculator in the following example.

Example 5 Finding an angle from a trigonometric ratio

Find the angle θ, correct to one decimal place, given:

a $\sin\theta = 0.8480$ **b** $\cos\theta = 0.5$ **c** $\tan\theta = 1.67$

Solution

a We need to find $\sin^{-1}(0.8480)$.

1. Press **2nd** (or **SHIFT**), **sin**, type **0.848**, then press **enter** (or =).

```
sin⁻¹(0.848)        57.9948
```

2. Write your answer correct to one decimal place.

$\theta = 58.0^\circ$

b We need to find $\cos^{-1}(0.5)$.

1. Press **2nd** (or **SHIFT**), **cos**, type **0.5**, then press **enter** (or =).

```
cos⁻¹(0.5)          60
```

2. Write the answer correct to one decimal place.

$\theta = 60^\circ$

c We need to find $\tan^{-1}(1.67)$.

1. Press **2nd** (or **SHIFT**), **tan**, type **1.67**, then press **enter** (or =).

```
tan⁻¹(1.67)         59.0867
```

2. Write the answer correct to one decimal place.

$\theta = 59.1^\circ$

Spreadsheet **Spreadsheet activity 6C:** Finding an angle from a trigonometric ratio.

Getting the language right

The language we use when finding an angle from a trigonometric ratio is difficult when you first meet it. The samples below are based of the results of Example 5.

- When you see $\sin 58^\circ = 0.8480$ think 'the sine of the angle 58° equals 0.8480'.
- When you see $\sin^{-1}(0.8480) = 58^\circ$ think 'the angle whose sine is 0.8480 equals 58°'.
- When you see $\cos 60^\circ = 0.5$ think 'the cosine of the angle 60° equals 0.5'.
- When you see $\cos^{-1}(0.5) = 60^\circ$ think 'the angle whose cosine is 0.5 equals 60°'.
- When you see $\tan 59.1^\circ = 1.67$ think 'the tan of the angle 59.1° equals 1.67'.
- When you see $\tan^{-1}(1.67) = 59.1^\circ$ think 'the angle whose tan is 1.67 equals 59.1°'.

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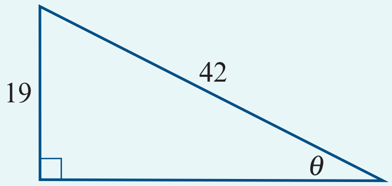
Finding an angle given two sides

Example 6 Finding an angle given two sides in a right-angled triangle

Find the angle θ, in the right-angled triangle shown, correct to one decimal place.



Solution

1 The sides involved are the opposite and the hypotenuse, so use $\sin\theta$.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

2 Substitute in the known values.

$$\sin\theta = \frac{19}{42}$$

3 Write the equation to find an expression for θ. Use a calculator to evaluate.

$$\theta = \sin^{-1}\left(\frac{19}{42}\right) = 26.896\ldots$$

4 Write your answer correct to one decimal place.

$$\theta = 26.9^{\circ}$$

The three angles in a triangle add to 180°. As the right angle is 90°, the other two angles must add to make up the remaining 90°. When one angle has been found, just subtract it from 90° to find the other angle. In Example 6, the other angle must be $90^{\circ} - 26.9^{\circ} = 63.1^{\circ}$.

Finding an angle in a right-angled triangle

1 Draw the triangle with the given sides shown. Label the unknown angle as θ.

2 Use the trigonometric ratio that includes the two known sides.

- If given the opposite side and the hypotenuse, use $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
- If given the adjacent side and the hypotenuse, use $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
- If given the opposite side and the adjacent side, use $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

3 Divide the side lengths to find the value of the trigonometric ratio.

4 Use the appropriate inverse function key to find the angle θ.

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Exercise 6C

Example 5

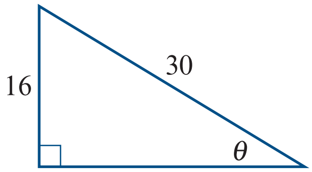
1 Find the angle θ, correct to one decimal place.

- **a** $\sin\theta = 0.4817$
- **b** $\cos\theta = 0.6275$
- **c** $\tan\theta = 0.8666$
- **d** $\sin\theta = 0.5000$
- **e** $\tan\theta = 1.0000$
- **f** $\cos\theta = 0.7071$
- **g** $\sin\theta = 0.8660$
- **h** $\tan\theta = 2.500$
- **i** $\cos\theta = 0.8383$
- **j** $\sin\theta = 0.9564$
- **k** $\cos\theta = 0.9564$
- **l** $\tan\theta = 0.5774$
- **m** $\sin\theta = 0.7071$
- **n** $\tan\theta = 0.5000$
- **o** $\cos\theta = 0.8660$
- **p** $\cos\theta = 0.3414$

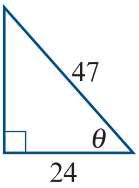
Example 6

2 Find the unknown angle θ in each triangle, correct to one decimal place.

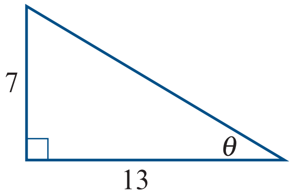
a Use $\sin^{-1}$ for this triangle.



b Use $\cos^{-1}$ for this triangle.



c Use $\tan^{-1}$ for this triangle.



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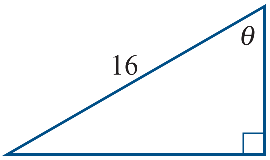

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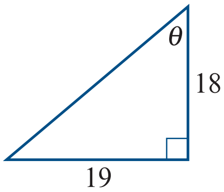

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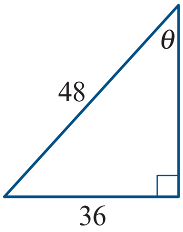

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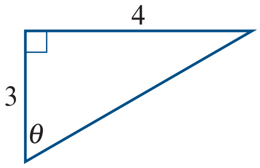
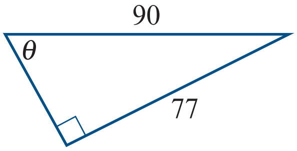
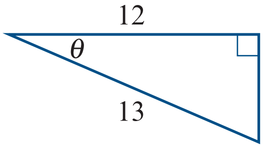


h



i



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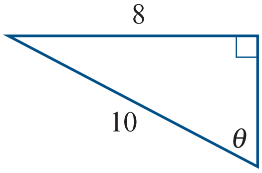

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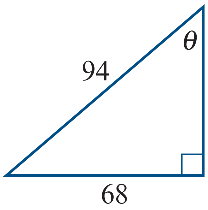

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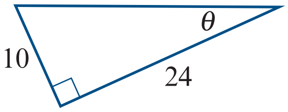
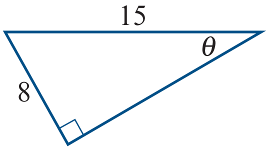


m



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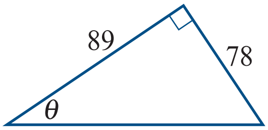
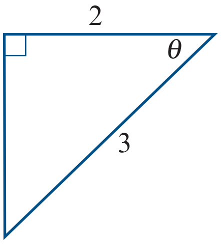


o



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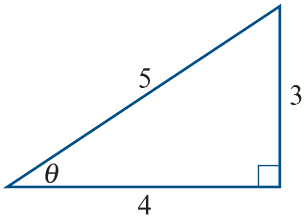
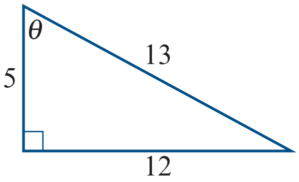
3 Find the value of θ in each triangle, correct to one decimal place.

a



b



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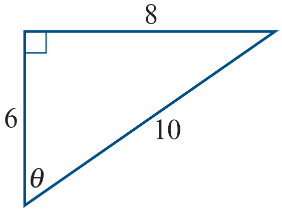

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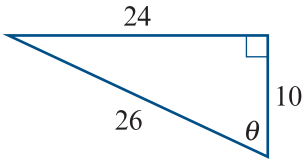
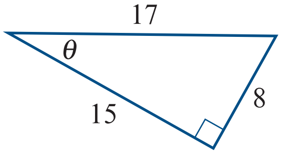


e



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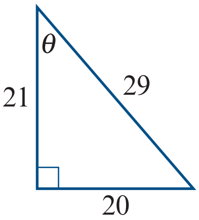
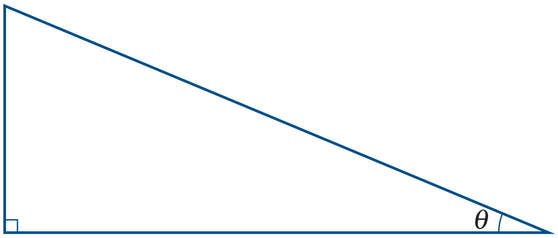


4 Use your ruler to measure appropriate side lengths of these triangles, and find the value of θ in each triangle, correct to the nearest degree.

a



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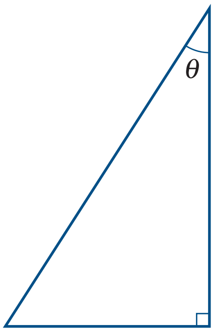

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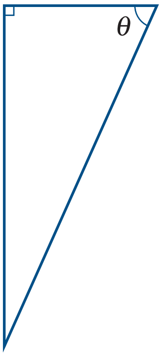
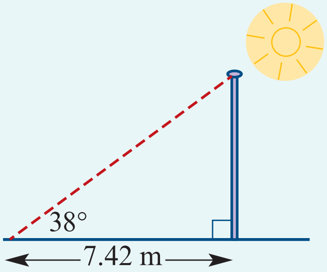


6D Applications of right-angled triangles to problem-solving

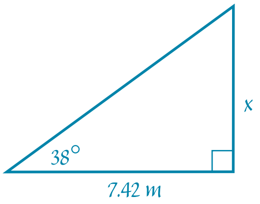
Example 7 Application requiring a length

A flagpole casts a shadow 7.42 m long. The sun's rays make an angle of 38° with the level ground. Find the height of the flagpole, correct to two decimal places.



Solution

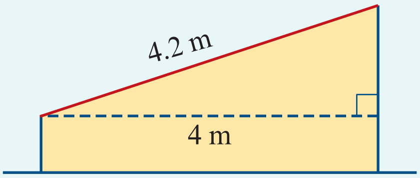
1. Draw a diagram showing the right-angled triangle. Include all the known details and label the unknown side.



2. The opposite and adjacent sides are involved, so use tan θ.
   $$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
3. Substitute in the known values.
   $$\tan 38^\circ = \frac{x}{7.42}$$
4. Multiply both sides by 7.42.
   $$7.42 \times \tan 38^\circ = x$$
5. Use your calculator to find the value of x.
   $$x = 5.797\ldots$$
6. Write your answer correct to two decimal places.
   The height of the flagpole is 5.80 m.

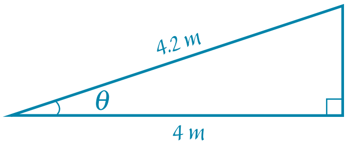
Example 8 Application requiring an angle

A sloping roof uses sheets of corrugated iron 4.2 m long on a shed 4 m wide. There is no overlap of the roof past the sides of the walls. Find the angle the roof makes with the horizontal, correct to one decimal place.



Solution

1. Draw a diagram showing the right-angled triangle. Include all known details and label the required angle.



2 The adjacent and hypotenuse are involved so use cos θ.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

3 Substitute in the known values.

$$\cos\theta = \frac{4}{4.2}$$

4 Write the equation to find θ.

$$\theta = \cos^{-1}\left(\frac{4}{4.2}\right)$$
$$\theta = 17.752\ldots$$

5 Use your calculator to find the value of θ.

$\cos^{-1}\left(\frac{4}{4.2}\right)$ 17.7528

6 Write your answer correct to one decimal place.

The roof makes an angle of 17.8° with the horizontal.

Warning!

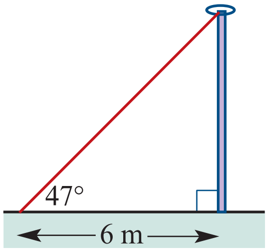
Always evaluate a mathematical expression as a whole, rather than breaking it into several smaller calculations. Rounding-off errors accumulate as more approximate answers are fed into the calculations.

Exercise 6D

SF

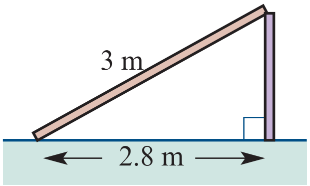
Example 7

1 A pole is supported by a wire that runs from the top of the pole to a point on the level ground 6 m from the base of the pole. The wire makes an angle of 47° with the ground. Find the height of the pole, correct to two decimal places.

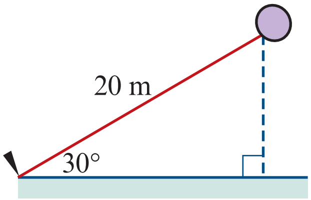


Example 8

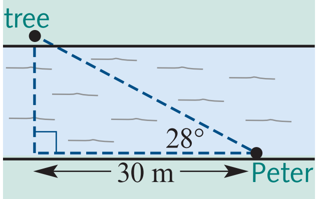
2 A 3 m log rests with one end on the top of a post and the other end on the level ground 2.8 m from the base of the post. Find the angle the log makes with the ground, correct to one decimal place.



3 A balloon is tied to a string 20 m long. The other end of the string is secured by a peg to the surface of a level sports field. The wind blows so that the string forms a straight line making an angle of 30° with the ground. Find the height of the balloon above the ground.

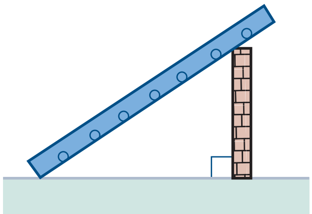


4 Peter noticed that a tree was directly opposite him on the far bank of the river. He then walked 30 m along his side of the river and found that his line of sight to the tree made an angle of 28° with the riverbank. Find the width of the river, to the nearest metre.



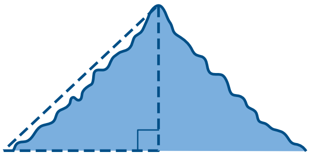
5 A ladder rests on a wall 2 m high. The foot of the ladder is 3 m from the base of the wall on level ground.

- **a** Copy the diagram and include the given information. Label as θ the angle the ladder makes with the ground.
- **b** Find the angle the ladder makes with the ground, correct to one decimal place.



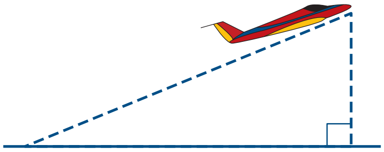
6 The distance measured up the sloping face of a mountain was 3.8 km. The sloping face was at an angle of 52° with the horizontal.

- **a** Make a copy of the diagram and show the known details. Show the height of the mountain as x.
- **b** Find the height of the mountain, correct to one decimal place.



7 An aeroplane maintains a flight path of 17° with the horizontal after it takes off. It travels for 2 km along that flight path.

- **a** Show the given and required information on a copy of the diagram.
- **b** Find, correct to two decimal places:
  - **i** the horizontal distance of the aeroplane from its take-off point
  - **ii** the height of the aeroplane above ground level.



8 A 3 m ladder rests against an internal wall. The foot of the ladder is 1 m from the base of the wall. Find the angle the ladder makes with the floor, correct to one decimal place.

9 The entrance to a horizontal mining tunnel has collapsed, trapping the miners inside. The rescue team decide to drill a vertical escape shaft from a position 200 m further up the hill. If the hill slopes at 23° from the horizontal, how deep does the rescue shaft need to be to meet the horizontal tunnel? Answer correct to one decimal place.

10 A strong rope needs to be fixed with one end attached to the top of a 5 m pole and the other end pegged at an angle of 60° with the level ground. Find the required length of the rope, correct to two decimal places.

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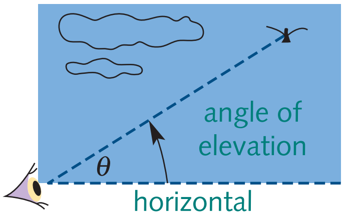
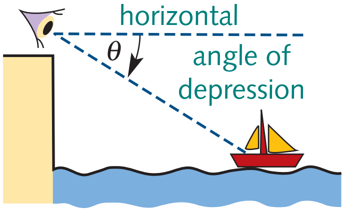
6E Angles of elevation and depression

The **angle of elevation** is the angle through which you *raise* your line of sight from the horizontal when you are looking *up* at something.



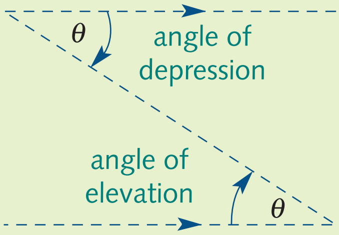
The **angle of depression** is the angle through which you *lower* your line of sight from the horizontal when you are looking *down* at something.



Angles of elevation and depression

angle of elevation = angle of depression

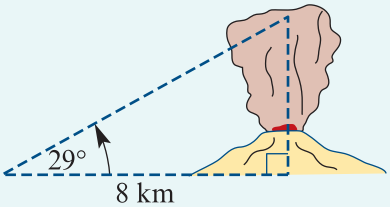
The diagram shows that the angle of elevation and the angle of depression are alternate angles ('Z' angles), so they are equal.



▶ Applications of angles of elevation and depression

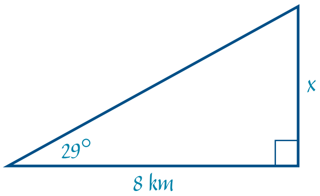
Example 9 Angle of elevation

A park ranger measured the top of a plume of volcanic ash to be at an angle of elevation of 29°. From her map she noted that the volcano was 8 km away. She calculated the height to the top of the plume to be 4.4 km. Show how she might have done this. Give your answer correct to one decimal place.



Solution

1 Draw a right-angled triangle showing the given information. Label the required height x.



2 The opposite and adjacent sides are involved, so use tan θ.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

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3 Substitute the known values in $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$.

$$\tan 29° = \frac{x}{8}$$

4 Multiply both sides by 8.

$$8 \times \tan 29° = x$$

5 Use your calculator to find the value of x.

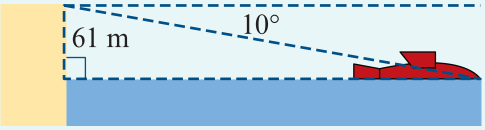
$$x = 4.434\ldots$$

6 Write your answer correct to one decimal place.

The height to the top of the ash plume was 4.4 km.

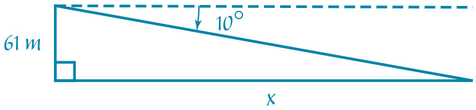
Example 10 Angle of depression

From the top of a cliff 61 m above sea level, Chen saw a capsized yacht. He estimated the angle of depression to be about 10°. How far was the yacht from the base of the cliff, to the nearest metre?

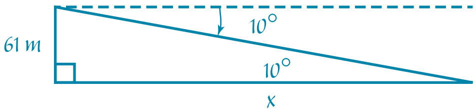


Solution

1 Draw a diagram showing the given information. Label the required distance x.



2 Mark in the angle at the yacht corner of the triangle. This is also 10°, because it and the angle of depression are alternate (or ‘Z’) angles.



Warning: The angle between the cliff face and the line of sight is *not* 10°.

3 The opposite and adjacent sides are involved, so use $\tan \theta$.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

4 Substitute in the known values.

$$\tan 10° = \frac{61}{x}$$

5 Multiply both sides by x.

$$x \times \tan 10° = 61$$

6 Divide both sides by tan 10°.

$$x = \frac{61}{\tan 10°}$$

7 Do the division using your calculator.

$$x = 345.948\ldots$$

8 Write your answer to the nearest metre.

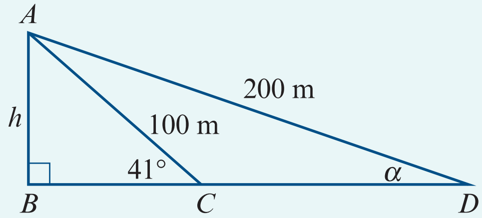
The yacht was 346 m from the base of the cliff.

Example 11 Application with two right-angled triangles

A cable 100 m long makes an angle of elevation of 41° with the top of a tower.

a Find the height, *h*, of the tower, to the nearest metre.

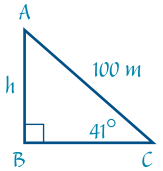
b Find the angle of elevation, α, to the nearest degree, that a cable 200 m long would make with the top of the tower.



Solution

Strategy: Find *h* in triangle *ABC,* then use this value to find α in triangle *ABD*.

a **1** Draw triangle *ABC* showing the given and required information.



2 The opposite and hypotenuse are involved, so use sin *θ*.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

3 Substitute in the known values.

$$\sin 41^\circ = \frac{h}{100}$$

4 Multiply both sides by 100.

$$h = 100 \times \sin 41^\circ$$
$$h = 65.605\ldots$$

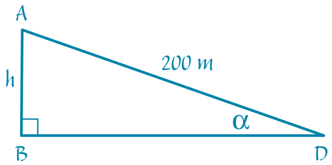
5 Evaluate 100 sin 41° using your calculator and store the answer as the value of the variable *h* for later use.

$100 \cdot \sin(41^\circ) \rightarrow h$ 65.6059

6 Write your answer to the nearest metre.

The height of the tower is 66 m.

b **1** Draw triangle *ABD* showing the given and required information.



2 The opposite and hypotenuse are involved, so use sin α.

$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

3 Substitute in the known values. In part **a** we stored the height of the tower as *h*.

$$\sin\alpha = \frac{h}{200}$$

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4 Write the equation to find α.

$\alpha = \sin^{-1}\left(\frac{h}{200}\right)$

5 Use your calculate to evaluate α.

$100 \cdot \sin(41°) \rightarrow h$	65.6059
$\sin^{-1}\left(\frac{h}{200}\right)$	19.1492

$\alpha = 19.149\ldots$

6 Write your answer to the nearest degree.

The 200 m cable would have an angle of elevation of 19°.

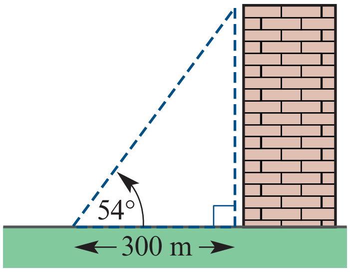
Exercise 6E

Example 9

Skillsheet

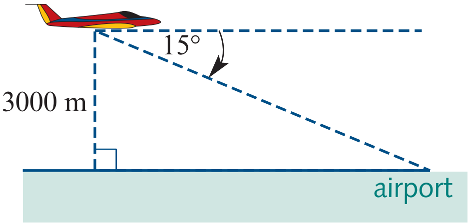
SF

1 From 300 m away from the base of a tall building, on level ground, Elise measured the angle of elevation from the ground to the top of the building to be 54°. Find the height of the building, to the nearest metre.



Example 10

2 The pilot of an aeroplane saw an airport at sea level at an angle of depression of 15°. His altimeter showed that the aeroplane was at a height of 3000 m. Find the horizontal distance of the aeroplane from the airport, to the nearest metre.



3 The angle of elevation measured from ground level to the top of a tall tree was 41°. The distance of the measurer from the base of the tree was 38 m. How tall was the tree? Give your answer to the nearest metre.

4 When Darcy looked from the top of a cliff, 60 m high, he noticed Elizabeth at an angle of depression of 20° on the ground below. How far was Elizabeth from the cliff? Answer correct to one decimal place.

5 From the top of a mountain, I could see a town at an angle of depression of 1.4° across the level plain. From my map I calculated that the town was 10 km away. Find the height of the mountain above the plain, to the nearest metre.

6 What would be the angle of elevation to the top of a radio transmitting tower 100 m tall and 400 m from the observer? Answer to the nearest degree.

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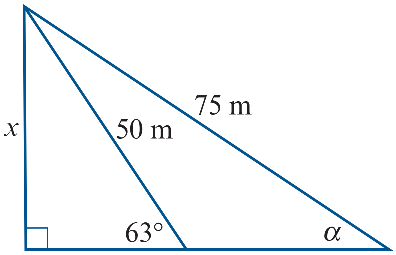
Example 11

7 **a** Find the length x, correct to one decimal place.

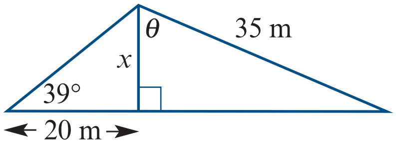
b Find the angle α, to the nearest degree.



SF

8 **a** Find the length x, correct to one decimal place.

b Find the angle θ, to the nearest degree.

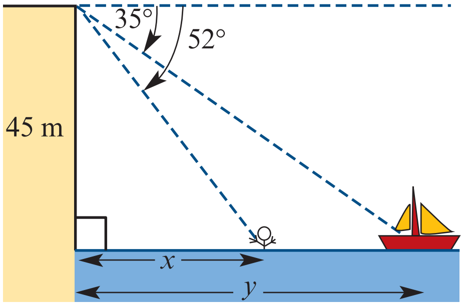


9 From the top of a cliff 45 m high, an observer looking along an angle of depression of 52° could see a man swimming in the sea. The observer could also see a boat at an angle of depression of 35°. Calculate, to the nearest metre:

a the distance x of the swimmer from the base of the cliff

b the distance y of the boat from the base of the cliff

c the distance from the man to the boat.



CF

10 A police helicopter hovering in a fixed position at an altitude of 500 m moved its spotlight through an angle of depression of 57° onto a lost child. The pilot sighted the rescue team at an angle of depression of 31°. If the terrain was level, how far, to the nearest metre, was the rescue team from the child?

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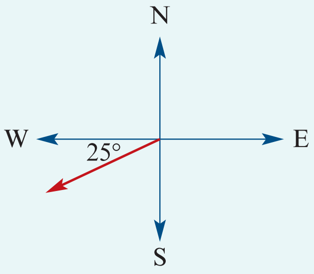
6F Bearings and navigation

True or three-figure bearings

A **true bearing** is the angle measured clockwise from north around to the required direction. True bearings are also called **three-figure bearings** because they are written using three numbers or figures. For example, 090° is the direction measured 90° clockwise from north, better known as east! In this section the old style compass bearings such as S 20°E are not used.

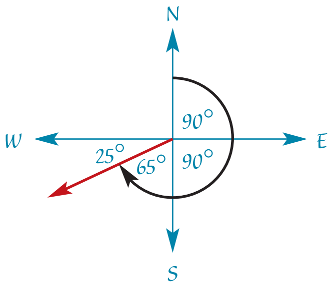
Example 12 Determining three-figure bearings

Give the three-figure bearing for the direction shown.



Solution

1 Calculate the total angles swept out clockwise from north. There is an angle of 90° between each of the four points of the compass.



2 Write your answer.

The angle from north = 90° + 90° + 65 = 245°
or 270° − 25° = 245°
The three-figure bearing is 245°.

Navigation problems

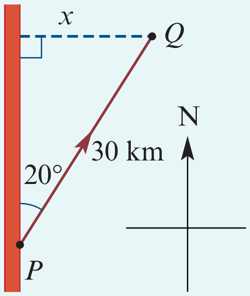
Navigation problems usually involve a consideration of not only the *direction* of travel, given as a bearing, but also the *distance* travelled.

Example 13 Navigating using a three-figure bearing

A group of bushwalkers leave point P, which is on a road that runs north–south, and walk for 30 km on a bearing 020° to reach point Q.

a What is the shortest distance x from Q back to the road, correct to one decimal place?

b Looking from point Q, what would be the three-figure bearing of their starting point?



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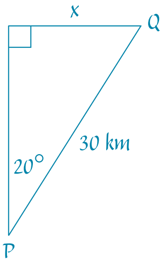
Solution

a **1** Show the given and required information in a right-angled triangle.
The question asks for the value of *x*.



2 The opposite and hypotenuse are involved, so use sin θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

3 Substitute in the known values.

$$\sin 20^\circ = \frac{x}{30}$$

4 Multiply both sides by 30.

$$30 \times \sin 20^\circ = x$$

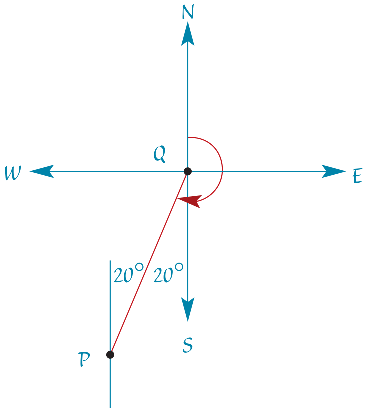
5 Find the value of *x* using your calculator.

$$x = 10.260\ldots$$

6 Write your answer correct to one decimal place.

The shortest distance to the road is 10.3 km.

b **1** The question asks for the bearing of *P* from *Q*.
Draw the compass points at *Q*.



2 Enter the alternate angle 20°.

3 Standing at *Q*, add all the angles when facing north and then turning clockwise to look at *P*. This gives the three-figure bearing of *P* when looking from *Q*.

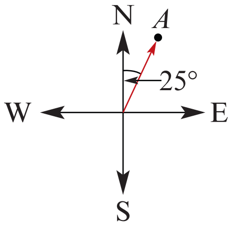
The angle from north is $180^\circ + 20^\circ = 200^\circ$
The three-figure bearing is 200°.

Exercise 6F

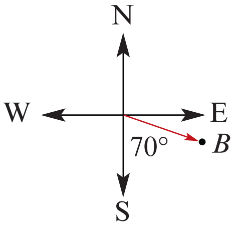
Example 12

1 State the three-figure bearing of each of the points *A*, *B*, *C* and *D*.

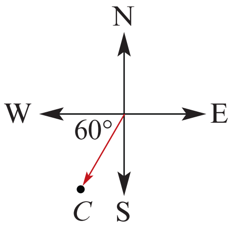
a



b



c



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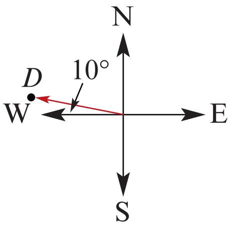
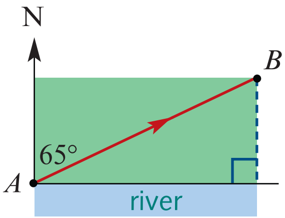


Example 13

2 Eddie camped overnight at point *A* beside a river that ran east–west. He walked on a bearing of 065° for 18 km to point *B*.

a What angle did his direction make with the river?

b What is the shortest distance from *B* to the river, correct to two decimal places?



3 A ship sailed 3 km west, then 2 km south.

a Give its three-figure bearings from an observer who stayed at its starting point, correct to the nearest degree.

b For a person on the ship, what would be the three-figure bearings looking back to the starting point?

4 An aeroplane flew 500 km south, then 600 km east. Give its three-figure bearing from its starting point, to the nearest degree.

5 A ship left port and sailed east for 5 km, then sailed north. After some time an observer at the port could see the ship on a bearing of 050°.

a How far north had the ship travelled? Answer correct to one decimal place.

b Looking from the ship, what would be the three-figure bearing of the port?

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6 A woman walked from point A for 10 km on a bearing of 060° to reach point B. Then she walked for 15 km heading south until she was at point D. Give the following distances correct to one decimal place and directions to the nearest degree.

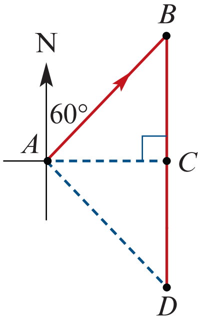
a Find the distances walked from A to B and from B to D.

b How far south did she walk from B to C?

c Find the distance from A to C.

d What is the distance from C to D?

e Find the three-figure bearing and distance she would need to walk to return to her starting point.



7 A ship left port P and sailed 20 km on a bearing of 230°. It then sailed north for 30 km to reach point C. Give the following distances correct to one decimal place and directions to the nearest degree.

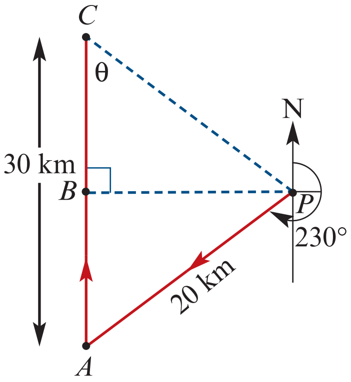
a Find the distance AB.

b Find the distance BP.

c Find the distance BC.

d Find the angle θ at point C.

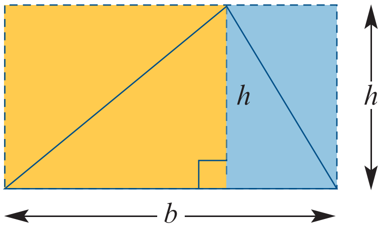
e State the three-figure bearing and distance of the port P from the ship at C.



6G The area of a triangle

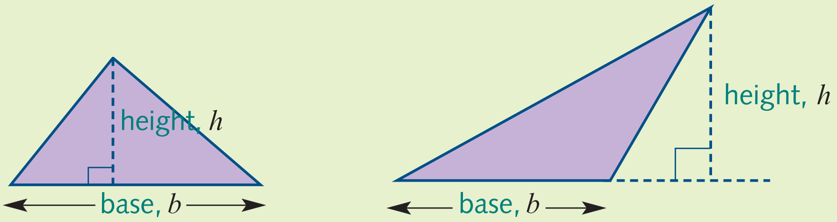
▶ Area of a triangle = $\frac{1}{2}$ × base × height

From the diagram, we see that the area of a triangle with a base b and height h is equal to half the area of the rectangle $b \times h$ that it fits within.



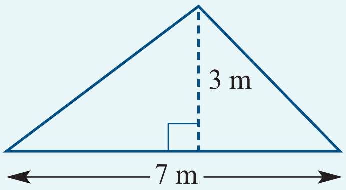
Area of a triangle

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times b \times h$$



Example 14 Finding the area of a triangle using $\frac{1}{2}$ × base × height

Find the area of the triangle shown, correct to one decimal place.



Solution

1 As we are given values for the base and height of the triangle, use Area = $\frac{1}{2}$ × base × height	Base, $b = 7$ Height, $h = 3$ Area of triangle = $\frac{1}{2} \times b \times h$
2 Substitute the given values.	$= \frac{1}{2} \times 7 \times 3$
3 Evaluate.	$= 10.5\ m^2$
4 Write your answer.	The area of the triangle is $10.5\ m^2$.

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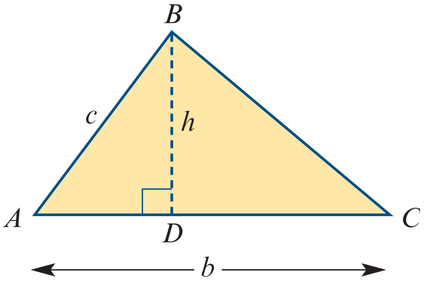
▶ Area of a triangle = $\frac{1}{2}bc \sin A$



In triangle ABD,

$$\sin A = \frac{h}{c}$$

$$h = c \times \sin A$$

So we can replace h with $c \times \sin A$ in the rule:

$$\text{Area of a triangle} = \frac{1}{2} \times b \times h$$

$$\text{Area of a triangle} = \frac{1}{2} \times b \times c \times \sin A$$

Similarly, using side c or a for the base, we can make a complete set of three rules:

Area of a triangle

$$\text{Area of a triangle} = \tfrac{1}{2}\, bc \sin A$$

$$\text{Area of a triangle} = \tfrac{1}{2}\, ac \sin B$$

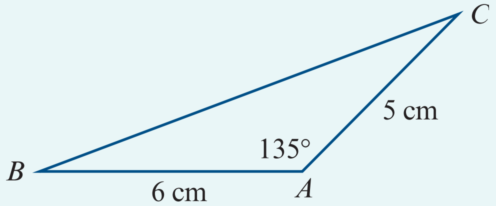
$$\text{Area of a triangle} = \tfrac{1}{2}\, ab \sin C$$

Notice that each version of the rule follows the pattern:

$$\text{Area of a triangle} = \tfrac{1}{2} \times (\text{product of two sides}) \times \sin(\text{angle between those two sides})$$

Example 15 Finding the area of a triangle using $\frac{1}{2}bc \sin A$

Find the area of the triangle shown, correct to one decimal place.



Solution

1 We are given two sides b, c and the angle A between them, so use:	$b = 5, c = 6, A = 135°$
Area of a triangle = $\frac{1}{2}$ $bc \sin A$	Area of triangle = $\frac{1}{2}bc \sin A$
2 Substitute values for b, c and A into the rule.	$= \frac{1}{2} \times 5 \times 6 \times \sin 135°$
3 Use your calculator to find the area.	$= 10.606\ldots$
4 Write your answer correct to one decimal place.	The area of the triangle is 10.6 cm^2.

Heron's rule for the area of a triangle

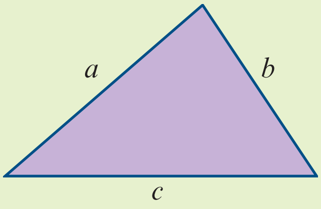
Heron's rule can be used to find the area of any triangle when we know the lengths of the three sides.

Heron's rule for the area of a triangle

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

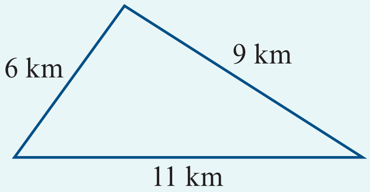
where $s = \frac{1}{2}(a+b+c)$

The variable s is called the *semi-perimeter* because it is equal to half the sum of the sides.



Example 16 Finding the area of a triangle using Heron's rule

The boundary fences of a farm are shown in the diagram. Find the area of the farm, to the nearest square kilometre.



Solution

1 As we are given the three sides of the triangle, use Heron's rule. Start by finding s, the semi-perimeter.

Let $a = 6, b = 9, c = 11$

$$s = \frac{1}{2}(a + b + c)$$
$$= \frac{1}{2}(6 + 9 + 11) = 13$$

2 Write Heron's rule.

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

3 Substitute the values of s, a, b and c into Heron's rule.

$$= \sqrt{13(13-6)(13-9)(13-11)}$$
$$= \sqrt{13 \times 7 \times 4 \times 2}$$

4 Use your calculator to find the area.

$$= 26.981\ldots$$

5 Write your answer.

The area of the farm, to the nearest square kilometre, is 27 km^2.

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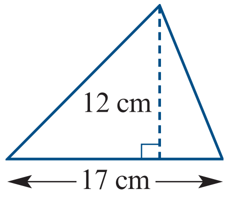
Exercise 6G

In this exercise, calculate areas correct to one decimal place where necessary.

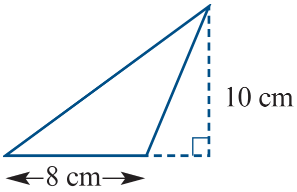
Finding areas using $\frac{1}{2}$ × base × height

Example 14 **1** Find the area of each triangle.

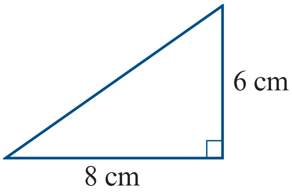
a



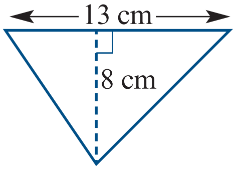
b



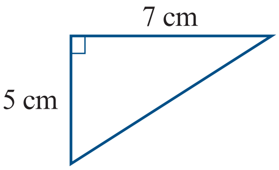
c



d



e



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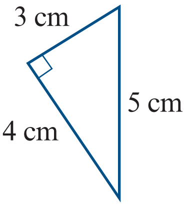
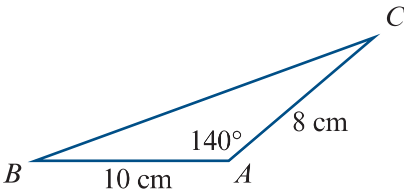


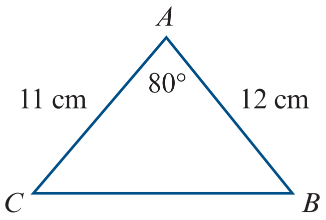
Finding areas using $\frac{1}{2}$ *bc* sin *A*

Example 15 **2** Find the areas of the triangles shown.

a



b



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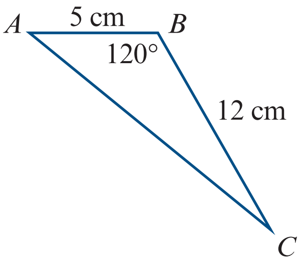

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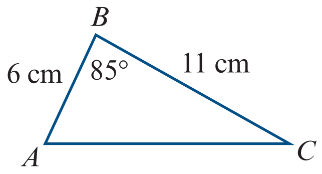

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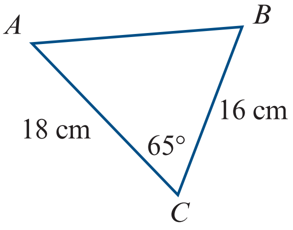

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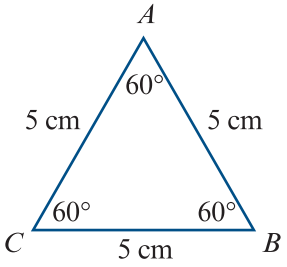

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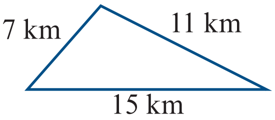
Finding areas using Heron's rule

Example 16

SF

3 Find the area of each triangle.

a



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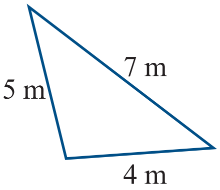
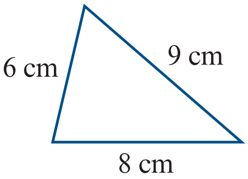


c



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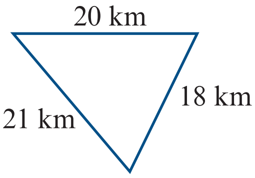

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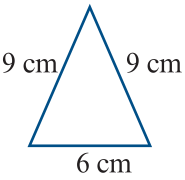

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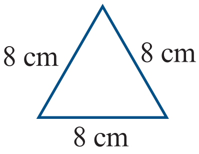


Mixed problems

4 For each triangle below, choose the rule for finding its area from rules **i** to **iv**:

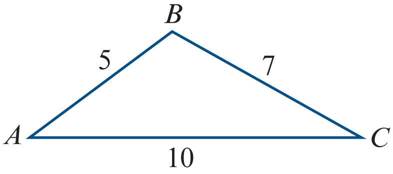
i $\frac{1}{2}$ *base* × *height*

ii $\frac{1}{2}bc\sin A$

iii $\frac{1}{2}ac\sin B$

iv $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$

a



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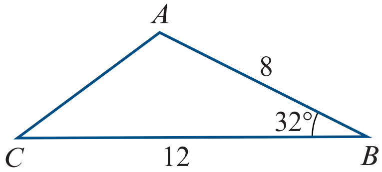

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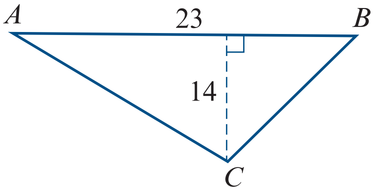

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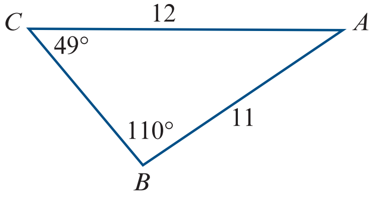
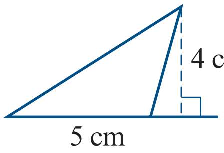


5 Find the area of each triangle shown.

a



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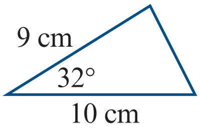

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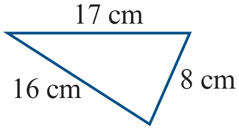

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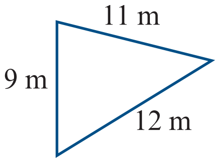

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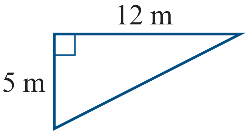

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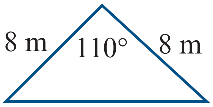

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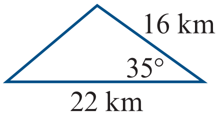

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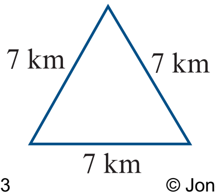
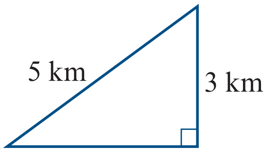


i



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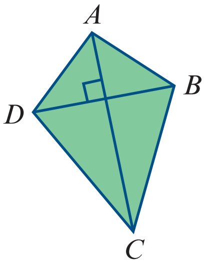
6 Find the area of a triangle with a base of 28 cm and a height of 16 cm.

7 Find the area of triangle *ABC* with side *a* 42 cm, side *b* 57 cm and angle *C* 70°.

8 Find the area of a triangle with sides of 16 km, 19 km and 23 km.

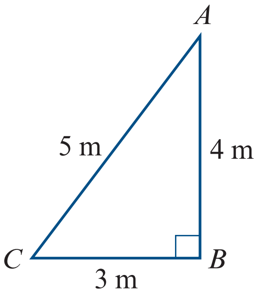
Applications

9 The kite shown is made using two sticks, *AC* and *DB*.
The length of *AC* is 100 cm and the length of *DB* is 70 cm.
Find the area of the kite.



10 Three students stretched a rope loop 12 m long into different triangles. Find the area of each triangle.

a



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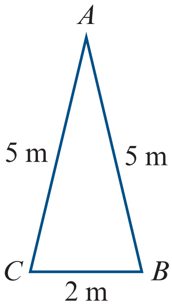

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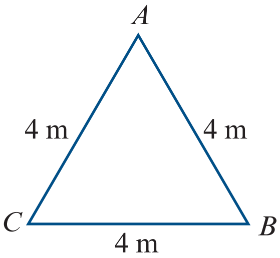

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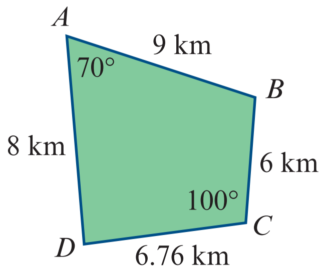
11 A farmer needs to know the area of his property with the boundary fences as shown. Write all answers correct to two decimal places.

Hint: Draw a line from B to D to divide the property into two triangles.

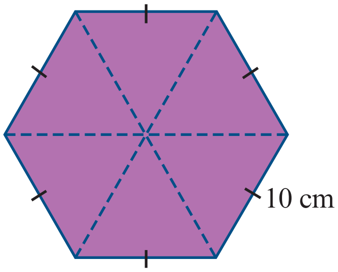


a Find the area of triangle ABD.

b Find the area of triangle BCD.

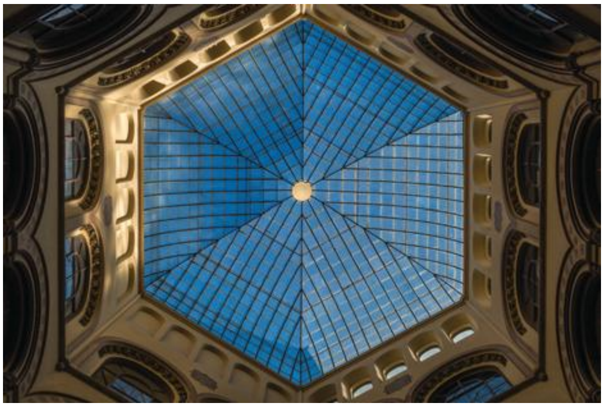
c State the total area of the property.

12 A regular hexagon with sides 10 cm long can be divided into six equilateral triangles. (Remember, an equilateral triangle has all sides of equal length.)

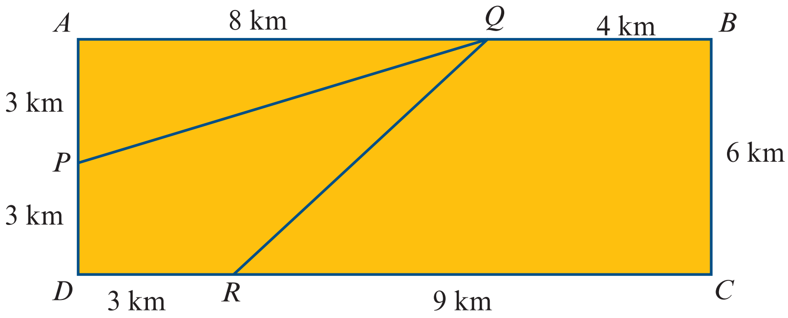


a Find the area of each triangle.

b What is the area of the hexagon?



13 A large rectangular area of land, $ABCD$ in the diagram, has been subdivided into three regions as shown.



a Find the area of:

i region PAQ **ii** region $QBCR$ **iii** region $PQRD$.

b Find the size of angle PQR, correct to one decimal place.

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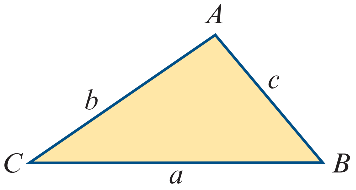
6H The sine rule

Standard triangle notation

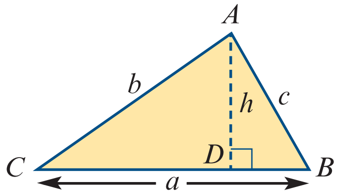
The convention for labelling a non-right-angled triangle is to use the upper case letters A, B and C for the angles at each corner. The sides are named using lower case letters, so that side a is opposite angle A, and so on.



This notation is used for the sine rule and cosine rule (see next section). Both rules can be used to find angles and sides in triangles that do not have a right angle.

How to derive the sine rule

In triangle ABC, show the height h of the triangle by drawing a perpendicular line from A to D on the base of the triangle.



In triangle ADC, $\sin C = \frac{h}{b}$

So (1) first rule for h: $h = b \times \sin C$

In triangle ABD, $\sin B = \frac{h}{c}$

So (2) second rule for h: $h = c \times \sin B$

Make (1) = (2): $b \times \sin C = c \times \sin B$

Divide both sides by sin C. $b = \frac{c \times \sin B}{\sin C}$

Divide both sides by sin B. $\frac{b}{\sin B} = \frac{c}{\sin C}$

If the triangle was redrawn with side c as the base, then using similar steps we get:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

We can combine the two rules as shown in the following box.

The sine rule

In any triangle ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

The **sine rule** is used to find sides and angles in a non-right-angled triangle, given:

- two sides and an angle opposite one of the given sides
- two angles and one side.

Note: If neither of the given angles is opposite the given side, find the third angle using $A + B + C = 180°$.

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The sine rule can take the form of any of these three possible equations:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \frac{a}{\sin A} = \frac{c}{\sin C}$$

Each equation has two sides and the two angles opposite those sides. If we know three of the parts, we can find the fourth.

So if we know two angles and a side opposite one of the angles, we can find the side opposite the other angle. Similarly, if we know two sides and an angle opposite one of those sides, we can find the angle opposite the other side.

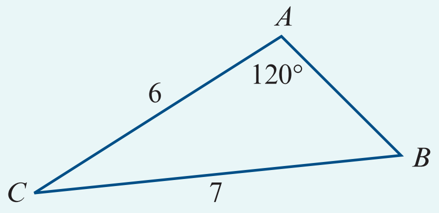
Although we have expressed the sine rule using triangle *ABC*, for any triangle, such as *PQR*, the pattern of fractions consisting of 'side / sine of angle' pairs would appear as:

$$\frac{p}{\sin P} = \frac{q}{\sin Q} \qquad \frac{q}{\sin Q} = \frac{r}{\sin R} \qquad \frac{p}{\sin P} = \frac{r}{\sin R}$$

▶ Using the sine rule

Example 17 Using the sine rule given two sides and an opposite angle

Find angle *B* in the triangle, correct to one decimal place.



Solution

1 We have the pairs $a = 7$ and $A = 120°$
$b = 6$ and $B = ?$
with only *B* unknown.

So use $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

2 Substitute in the known values.

$$\frac{7}{\sin 120°} = \frac{6}{\sin B}$$

3 Cross-multiply.

$$7 \times \sin B = 6 \times \sin 120°$$

4 Divide both sides by 7.

$$\sin B = \frac{6 \times \sin 120°}{7}$$

5 Write the equation to find angle *B*.

$$B = \sin^{-1}\left(\frac{6 \times \sin 120°}{7}\right)$$

6 Use your calculator to evaluate the expression for *B*.

$$B = 47.928\ldots°$$

7 Write your answer correct to one decimal place.

Angle B is 47.9°.

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In Example 7, now that we know that $A = 120°$ and $B = 47.9°$, we can use the fact that the angles in a triangle add to 180° to find C.

$$\begin{aligned} A + B + C &= 180° \\ 120° + 47.9° + C &= 180° \\ 167.9° + C &= 180° \\ C &= 180° - 167.9° = 12.1° \end{aligned}$$

As we now know that $A = 120°$, $a = 7$ and $C = 12.1°$, we can find side c using

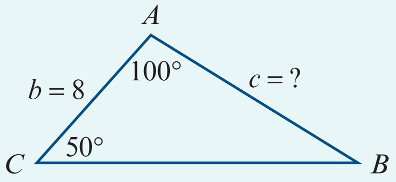
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

The steps are similar to those in the example.

Finding all the angles and sides of a triangle is called solving the triangle.

Example 18 Using the sine rule given two angles and one side

Find side c in the triangle shown, correct to one decimal place.



Solution

1 Find the angle opposite the given side by using $A + B + C = 180°$.

$$\begin{aligned} A + B + C &= 180° \\ 100° + B + 50° &= 180° \\ B + 150° &= 180° \\ B &= 30° \end{aligned}$$

2 We have the pairs $b = 8$ and $B = 30°$, $c = ?$ and $C = 50°$ with only c unknown. So use $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

3 Substitute in the known values.

$$\frac{8}{\sin 30°} = \frac{c}{\sin 50°}$$

4 Multiply both sides by sin 50°.

$$c = \frac{8 \times \sin 50°}{\sin 30°}$$

5 Use your calculator to find c.

$$c = 12.256\ldots$$

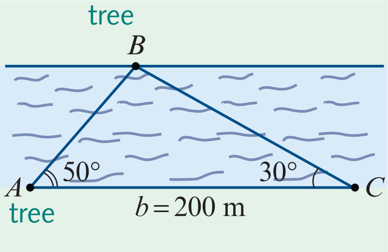
6 Write your answer correct to one decimal place.

Side c is 12.3 units long.

Example 19 Application of the sine rule

Leo wants to tie a rope from a tree at point *A* to a tree at point *B* on the other side of the river. He needs to know the length of rope required.

When he stood at *A*, he saw the tree at *B* at an angle of 50° with the riverbank. He then walked 200 metres east to *C*, and the tree was seen at an angle of 30° with the bank.



Find the length of rope required to reach from *A* to *B*, correct to two decimal places.

Solution

1 To use the sine rule, we need two angle–side pairs with only one item unknown.

The unknown is the length of the rope, side *c*. Angle $C = 30°$ is given.

So one part of the sine rule equation will be:

$$\frac{c}{\sin C}$$

2 We know side $b = 200$ and need to find angle *B* to use the sine rule equation.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

3 Use $A + B + C = 180°$ to find angle *B*.

$$A+B+C = 180^\circ$$
$$50^\circ + B + 30^\circ = 180^\circ$$
$$B = 100^\circ$$

4 We have the pairs:
$b = 200$ and $B = 100°$,
$c = ?$ and $C = 30°$
with only *c* unknown.

So use $\dfrac{c}{\sin C} = \dfrac{b}{\sin B}$.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

5 Substitute in the known values.

$$\frac{c}{\sin 30^\circ} = \frac{200}{\sin 100^\circ}$$

6 Multiply both sides by sin 30° .

$$c = \frac{200 \times \sin 30^\circ}{\sin 100^\circ}$$

7 Use your calculator to find *c*.

$$c = 101.542\ldots$$

8 Write your answer correct to two decimal places.

The rope must be 101.54 m long.

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Tips for solving trigonometry problems

- Always make a rough sketch in pencil as you read the details of a problem. You may need to make changes as you read more, but it is very helpful to have a sketch to guide your understanding.
- In any triangle, the longest side is opposite the largest angle. The shortest side is opposite the smallest angle.
- When you have found a solution, re-read the question and check that your answer fits well with the given information and your diagram.
- Round answers for each part to the required decimal places. Keep more decimal places when the results are used in further calculations; otherwise, rounding off errors accumulate.

Exercise 6H

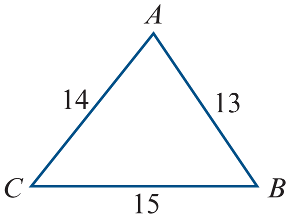
In this exercise, calculate lengths correct to two decimal places and angles correct to one decimal place where necessary.

Basic principles

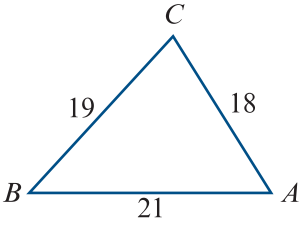
SF

1 In each triangle, state the lengths of sides a, b and c.

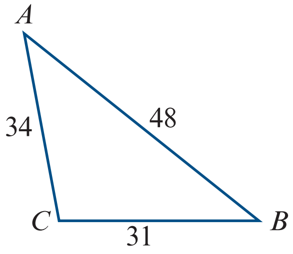
a



b

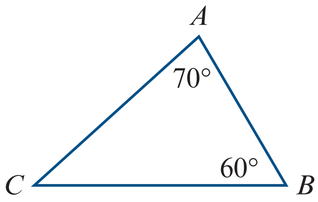


c



2 Find the value of the unknown angle in each triangle. Use $A + B + C = 180°$.

a



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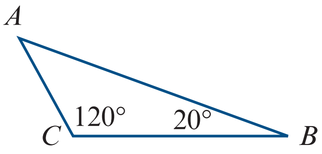

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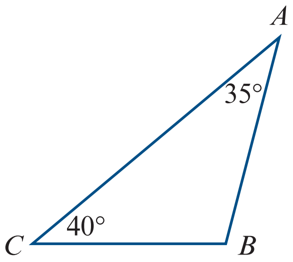

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3 In each of the following, a student was using the sine rule to find an unknown part of a triangle, but was unable to complete the final steps of the solution. Find the unknown value by completing each problem.

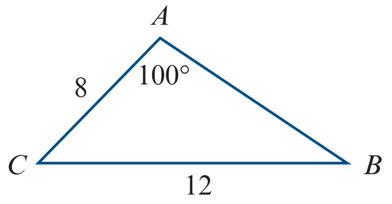
a $\dfrac{a}{\sin 40°} = \dfrac{8}{\sin 60°}$ **b** $\dfrac{b}{\sin 50°} = \dfrac{15}{\sin 72°}$ **c** $\dfrac{c}{\sin 110°} = \dfrac{24}{\sin 30°}$

d $\dfrac{17}{\sin A} = \dfrac{16}{\sin 70°}$ **e** $\dfrac{26}{\sin B} = \dfrac{37}{\sin 95°}$ **f** $\dfrac{21}{\sin C} = \dfrac{47}{\sin 115°}$

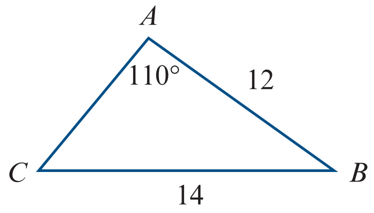
Using the sine rule to find angles

Example 17

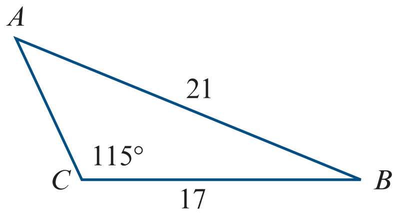
4 **a** Find angle B.



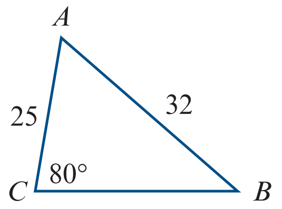
b Find angle C.



c Find angle A.



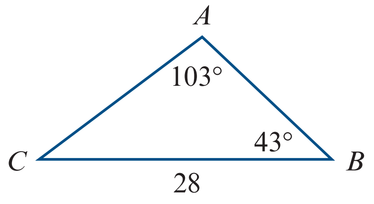
d Find angle B.



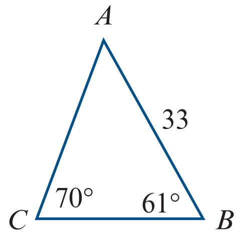
Using the sine rule to find sides

Example 18

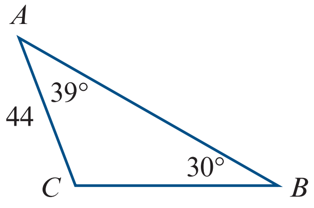
5 **a** Find side b.



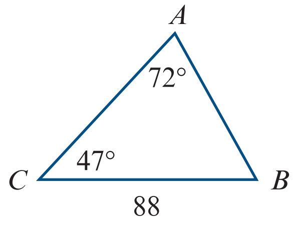
b Find side b.



c Find side a.



d Find side c.



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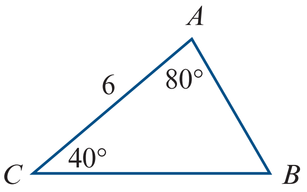
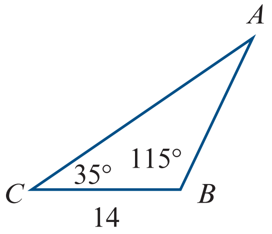
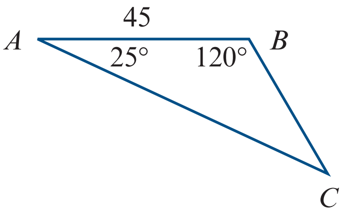
6 **a** Find side c.



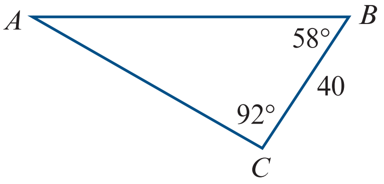
b Find side c.



c Find side b.



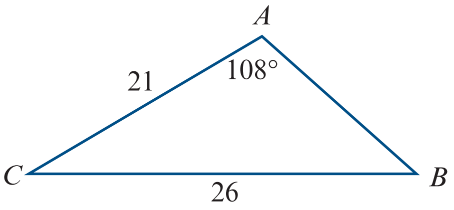
d Find side b.



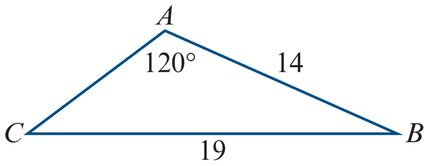
Solving triangles using the sine rule

7 Solve (find all the unknown sides and angles of) the following triangles.

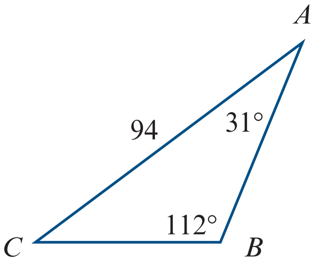
a



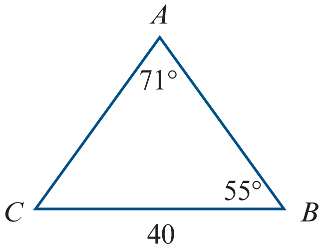
b



c



d



8 In the triangle $ABC, A = 105°$, $B = 39°$ and $a = 60$. Find side b.

9 In the triangle $ABC, A = 112°$, $a = 65$ and $c = 48$. Find angle C.

10 In the triangle ABC, $B = 50°$, $C = 45°$ and $a = 70$. Find side c.

11 In the triangle ABC, $B = 59°$, $C = 74°$ and $c = 41$. Find sides a and b and angle A.

12 In the triangle ABC, $a = 60$, $b = 100$ and $B = 130°$. Find angles A and C and side c.

13 In the triangle ABC, $A = 130°$, $B = 30°$ and $c = 69$. Find sides a and b and angle C.

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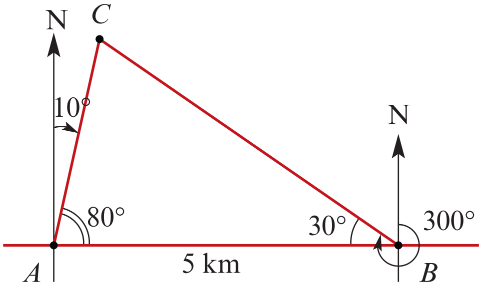
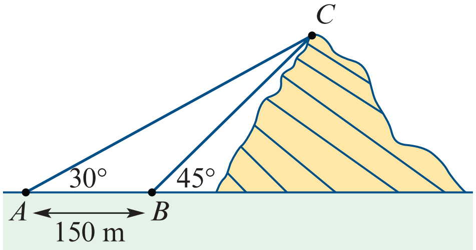
Applications

Example 19 **14** A fire-spotter located in a tower at A saw a fire in the direction 010°. Five kilometres to the east of A, another fire-spotter at B saw the fire in the direction 300°. Find the distance of the fire from each tower.



15 A surveyor standing at point A measured the angle of elevation to the top of the mountain as 30°. She moved 150 m closer to the mountain and at point B measured the angle of elevation to the top of the mountain as 45°.
There is a proposal to have a strong cable from point A to the top of the mountain to carry tourists in a cable car. What is the length of the required cable?



16 A naval officer sighted the smoke of a volcanic island on a bearing of 044°. A navigator on another ship 25 km due east of the first ship saw the smoke on a bearing of 342°.

a Find the distance of each ship from the volcano.

b If the ship closest to the volcano can travel at 15 km/h, how long will it take it to reach the volcano?

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17 An air-traffic controller at airport A received a distress call from an aeroplane low on fuel. The bearing of the aeroplane from A was 070°. From airport B, 80 km north of airport A, the bearing of the aeroplane was 120°.

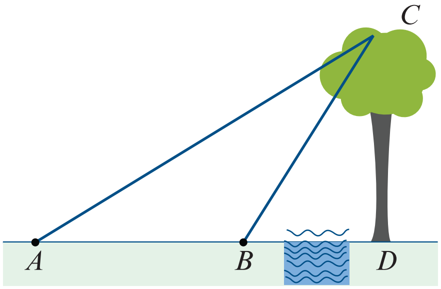
a Which airport was closer for the aeroplane?

b Find the distance to the closer airport.

c The co-pilot estimates fuel consumption to be 1525 litres per 100 km. The fuel gauge reads 1400 litres. Is there enough fuel to reach the destination?

18 Holly was recording the heights of tall trees in a State forest to have them registered for protection. A river prevented her from measuring the distance from the base of a particular tree.

She recorded the angle of elevation of the top of the tree from point A as 25°. Holly walked 80 m towards the tree and recorded the angle of elevation from point B as 50°.



a Copy the diagram shown and add the given information.

b Find the angle at B in triangle ABC.

c Find the angle at C in triangle ABC.

d Find the length b (from A to C).

e Use the length b as the hypotenuse in right-angled triangle ADC, and the angle at A, to find distance DC, the height of the tree.

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6I The cosine rule

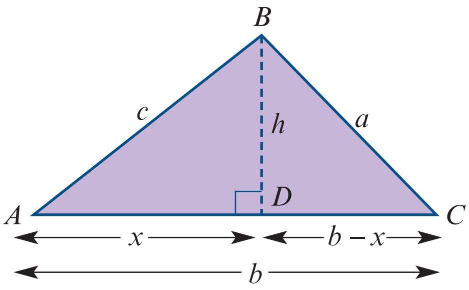
The **cosine rule** can be used to find the length of a side in any non-right-angled triangle when two sides and the angle between them are known. When you know the three sides of a triangle, the cosine rule can be used to find any angle.

▶ How to derive the cosine rule

In the triangle *ABC*, show the height *h* of the triangle by drawing a line perpendicular from *D* on the base of the triangle to *B*.

Let $AD = x$

As $AC = b$, then $DC = b - x$.



In triangle *ABD*, $\cos A = \dfrac{x}{c}$

Multiply both sides by c. $x = c\cos A$ ①

Use Pythagoras' theorem in triangle *ABD*. $x^2 + h^2 = c^2$ ②

Use Pythagoras' theorem in triangle *CBD*. $(b - x)^2 + h^2 = a^2$

Expand (multiply out) the squared bracket. $b^2 - 2bx + x^2 + h^2 = a^2$

Use ① to replace x with $c \cos A$. $b^2 - 2bc\cos A + x^2 + h^2 = a^2$

Use ② to replace $x^2 + h^2$ with c^2. $b^2 - 2bc\cos A + c^2 = a^2$

Reverse and rearrange the equation. $a^2 = b^2 + c^2 - 2bc\cos A$

Repeating these steps with side c as the base, we get: $b^2 = a^2 + c^2 - 2ac\cos B$

Repeating these steps with side a as the base, we get: $c^2 = a^2 + b^2 - 2ab\cos C$

The three versions of the cosine rule can be rearranged to give rules for $\cos A$, $\cos B$ and $\cos C$.

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The cosine rule

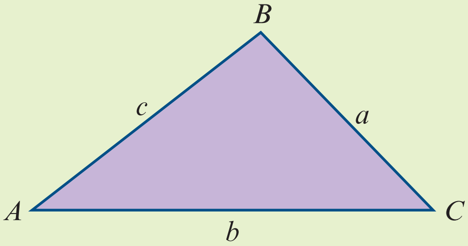
In any triangle *ABC*, the cosine rule can be used.

- When given two sides and the angle between them, the third side can be found using one of the equations:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



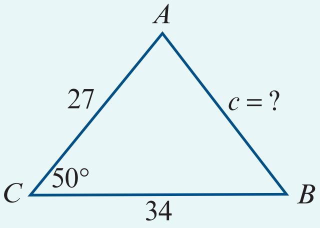
- When given three sides, any angle can be found using one of the following rearrangements of the cosine rule:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Using the cosine rule

Example 20 Using the cosine rule given two sides and the angle between them

Find side *c*, correct to two decimal places, in the triangle shown.



Solution

1 Write down the given values and the required unknown value. $a = 34, b = 27, c = ?, C = 50^\circ$

2 We are given two sides and the angle between them. To find side *c* use $c^2 = a^2 + b^2 - 2ab\cos C$. $c^2 = a^2 + b^2 - 2ab\cos C.$

3 Substitute the given values into the rule. $c^2 = 34^2 + 27^2 - 2 \times 34 \times 27 \times \cos 50^\circ$

4 Take the square root of both sides. $c = \sqrt{(34^2 + 27^2 - 2 \times 34 \times 27 \times \cos 50^\circ)}$

5 Use your calculator to find *c*. $c = 26.548\ldots$

6 Write your answer correct to two decimal places. The length of side *c* is 26.55 units.

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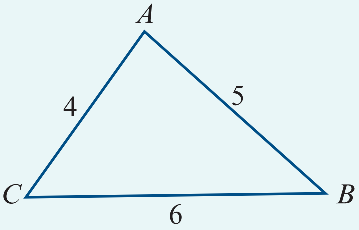
Example 21 Using the cosine rule to find an angle given three sides

Find the largest angle, correct to one decimal place, in the triangle shown.



Solution

1 Write down the given values.

$a = 6, b = 4, c = 5$

2 The largest angle is always opposite the largest side, so find angle A.

$A = ?$

3 We are given three sides. To find angle A use $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

4 Substitute the given values into the rule.

$$\cos A = \frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5}$$

5 Write the equation to find angle A.

$$A = \cos^{-1}\left(\frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5}\right)$$

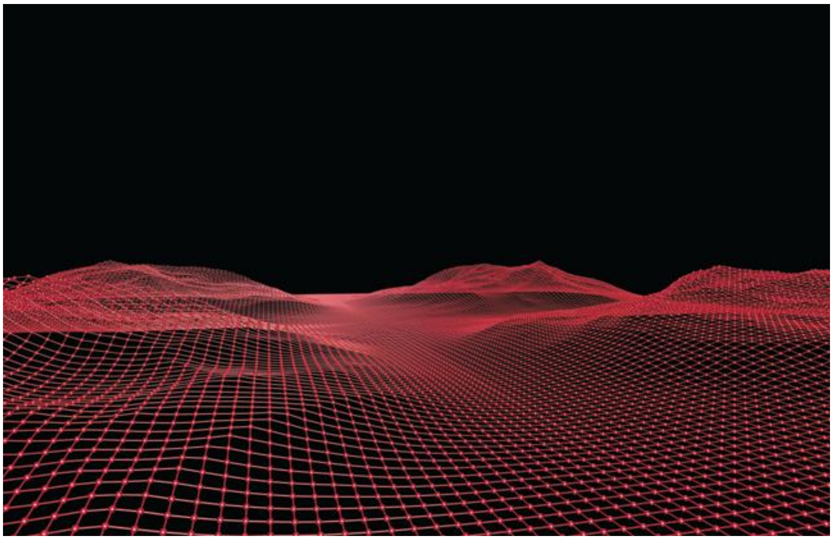
6 Use your calculator to evaluate the expression for A. Make sure that your calculator is in DEGREE mode.

Tip: Wrap all the terms in the numerator (top) within brackets. Also put brackets around all of the terms in the denominator (bottom).

$A = 82.819\ldots^\circ$

7 Write your answer.

The largest angle is 82.8°.



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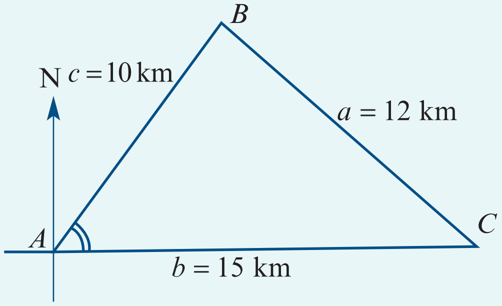
Example 22 Application of the cosine rule: finding an angle and a bearing

A yacht left point A and sailed 15 km east to point C. Another yacht also started at point A and sailed 10 km to point B, as shown in the diagram. The distance between points B and C is 12 km.



a What was the angle between their directions as they left point A? Give the angle correct to two decimal places.

b Find the bearing of point B from the starting point A to the nearest degree.

Solution

a **1** Write the given values.

$a = 12, b = 15, c = 10$

2 Write the form of the cosine rule for the required angle, A.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

3 Substitute the given values into the rule.

$$\cos A = \frac{15^2 + 10^2 - 12^2}{2 \times 15 \times 10}$$

4 Write the equation to find angle A.

$$A = \cos^{-1}\left(\frac{15^2 + 10^2 - 12^2}{2 \times 15 \times 10}\right)$$

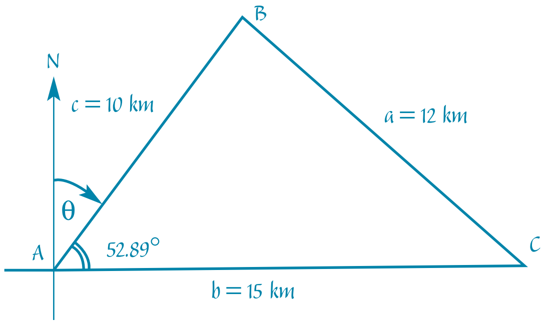
5 Use your calculator to evaluate the expression for A.

$A = 52.891°$

6 Give the answer to two decimal places.

The angle was 52.89°.

b **1** The bearing θ, of point B from the starting point A, is measured clockwise from north.



2 Consider the angles in the right-angle at point A.

$\theta + 52.89° = 90°$

3 Find the value of θ.

$$\begin{aligned}\theta &= 90° - 52.89° \\ &= 37.11°\end{aligned}$$

4 Write your answer.

The bearing of point B from point A is 037°.

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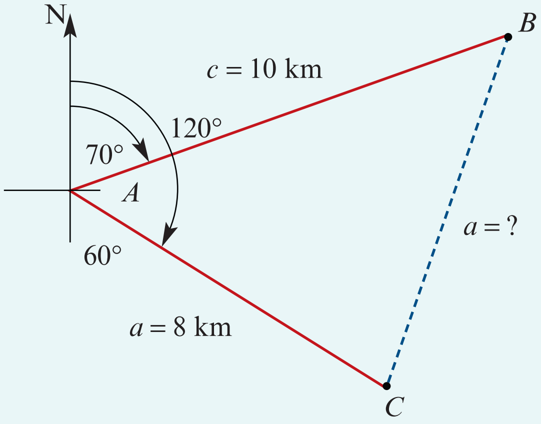
Example 23 Application of the cosine rule involving bearings

A bushwalker left his base camp and walked 10 km in the direction 070°.

His friend also left the base camp but walked 8 km in the direction 120°.

a Find the angle between their paths.

b How far apart were they when they stopped walking? Give your answer correct to two decimal places.



Solution

a **1** Angles lying on a straight line add to 180°.	$60^\circ + A + 70^\circ = 180^\circ$ $A + 130^\circ = 180^\circ$ $A = 50^\circ$
2 Write your answer.	The angle between their paths was 50°.
b **1** Write down the known values and the required unknown value.	$a = ?,\ b = 8,\ c = 10,\ A = 50$
2 We have two sides and the angle between them. To find side a use $a^2 = b^2 + c^2 - 2bc\cos A$	$a^2 = b^2 + c^2 - 2bc\cos A$
3 Substitute in the known values.	$a^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 50^\circ$
4 Take the square root of both sides.	$a = \sqrt{(8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 50^\circ)}$
5 Use your calculator to find the value of a.	$a = 7.820\ldots$
6 Write your answer correct to two decimal places.	The distance between them was 7.82 km.

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Exercise 6I

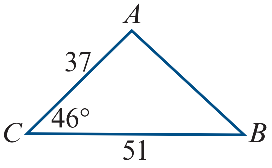
Calculate lengths correct to two decimal places and angles correct to one decimal place.

Using the cosine rule to find sides

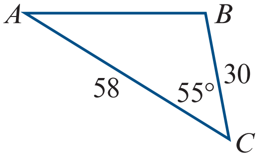
Example 20

1 Find the unknown side in each triangle.

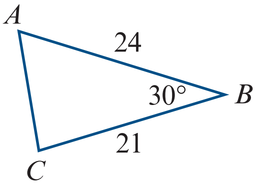
a



b



c



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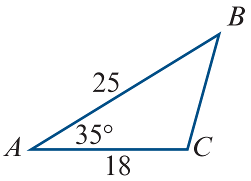
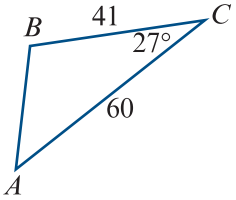


e



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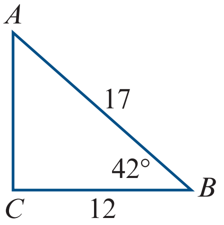


2 In the triangle ABC, $a = 27$, $b = 22$ and $C = 40°$. Find side c.

3 In the triangle ABC, $a = 18$, $c = 15$ and $B = 110°$. Find side b.

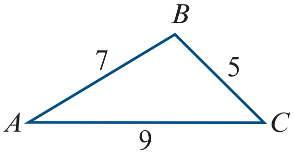
4 In the triangle ABC, $b = 42$, $c = 38$ and $A = 80°$. Find side a.

Using the cosine rule to find angles

Example 21

5 Find angle A in each triangle.

a



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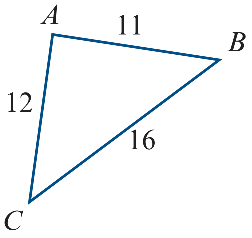

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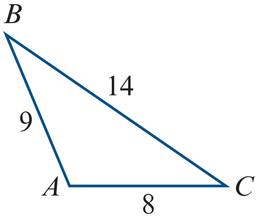

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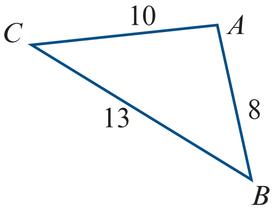

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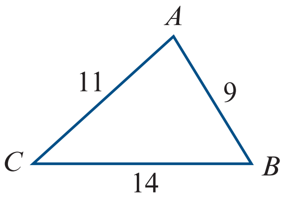

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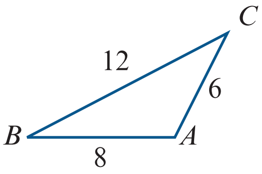


6 In the triangle ABC, $a = 9$, $b = 10$ and $c = 11$. Find angle A.

7 In the triangle ABC, $a = 31$, $b = 47$ and $c = 52$. Find angle B.

8 In the triangle ABC, $a = 66$, $b = 29$ and $c = 48$. Find angle C.

9 Find the smallest angle in the triangle ABC, with $a = 120$, $b = 90$ and $c = 105$.

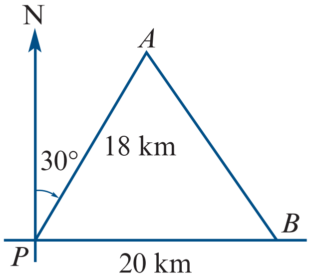
10 In the triangle ABC, $a = 16$, $b = 21$ and $c = 19$. Find the largest angle.

Applications

Example 22, 23

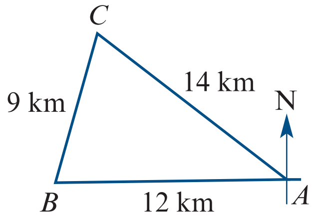
11 A farm has a triangular shape with fences of 5 km, 7 km and 9 km in length. Find the size of the smallest angle between the fences. The smallest angle is always opposite the smallest side.

12 A ship left the port P and sailed 18 km on a bearing of 030° to point A. Another ship left port P and sailed 20 km east to point B. Find the distance from A to B, correct to one decimal place.



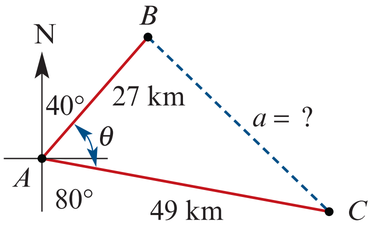
13 A bushwalker walked 12 km west from point A to point B. Her friend walked 14 km from point A to point C, as shown in the diagram. The distance from B to C is 9 km.

- **a** Find the angle at A, between the paths taken by the bushwalkers, correct to one decimal place.
- **b** What is the bearing of point C from A? Give the bearing correct to the nearest degree.

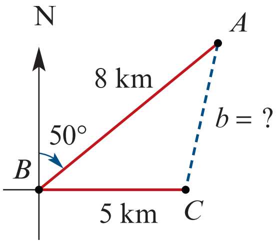


14 A ship left port A and travelled 27 km on a bearing of 40° to reach point B. Another ship left the same port and travelled 49 km on a bearing of 100° to arrive at point C.

- **a** Find the angle θ between the directions of the two ships.
- **b** How far apart were the two ships when they stopped?



15 A battleship B detected a submarine A on a bearing of 050° and at a distance of 8 km. A cargo ship C was 5 km due east of the battleship. How far was the submarine from the cargo ship?



16 From a lookout tower A, a fire-spotter saw a bushfire B at a distance of 15 km in the direction 315°. A township C was located 12 km on a bearing of 265° from the tower. How far was the bushfire from the township?

17 Passengers, who are travelling in a car west along a road that runs east–west, see a mountain 9 km away on a bearing of 290°. When they have travelled a further 5 km west along the road, what will be the distance to the mountain?

18 At a point A on the ground, the angle of elevation to the top of a radio transmission tower is 60°. From that point, a 40 m cable was attached to the top of the tower. At a point B, a further 10 m away from the base of the tower, another cable is to be pegged to the ground and attached to the top of the tower. What length is required for the second cable?

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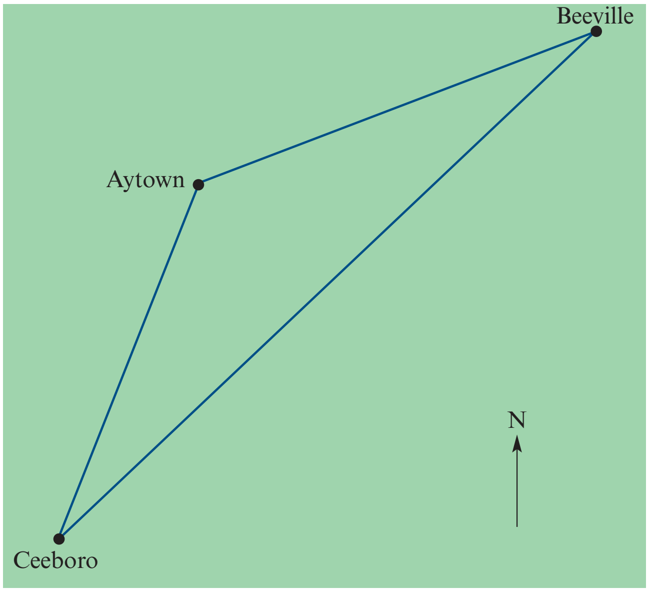
6J Further problem-solving and modelling

Exercise 6J

Alice operates a charter flight service. She is planning a round trip from her base at Aytown to Beeville, Ceeboro and then returning to Aytown.



a The direct distance from Aytown to Ceeboro is known to be 92 km. By measuring the lengths of the blues lines between the three cities calculate the direct distance from:

- **i** Aytown to Beeville
- **ii** Beeville to Ceeboro.

b Her Piper Archer TX has a range of 522 nautical miles. One nautical mile equals 1.85 km. The fuel gauge indicates the tank is one-third full. Is there enough fuel to complete the round trip?

c The average cruising speed is 147 miles per hour. One mile equals 1.61 km. Find the total flight time for the round trip.

d Fuel consumption cost is 95 cents per km. What will be her fuel cost for the trip?

e
- **i** If a vertical line points north, use the map and a protractor to find the bearing that Alice must fly from Aytown to Beeville.
- **ii** Measure the bearing that Alice must fly from Beeville to Ceeboro.
- **iii** Measure the bearing that Alice must fly from Ceeboro to Aytown.

f Give the direction and distance flight information that Alice will need to use on the Beeville to Ceeboro stage of the circuit.

g Use your answers to part **e** to find the angle between the flight paths to and from Beeville.

h Show how a trigonometry distance rule can be applied to the Aytown–Beeville and Beeville–Ceeboro distances with the angle at Beeville to find the Ceeboro–Aytown distance. Confirm that the distance is approximately 92 km.

i Find the area enclosed by the flight paths.

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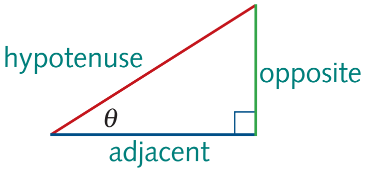
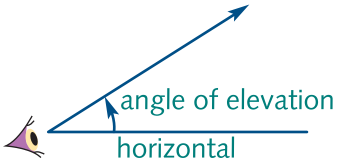
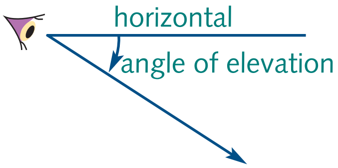
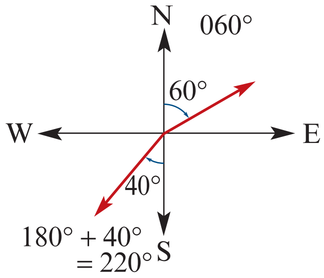
Key ideas and chapter summary

Right-angled triangles

Naming the sides of a right-angled triangle	The **hypotenuse** is the longest side and is always opposite the right angle (90°). The *opposite* side is directly opposite the angle θ (the angle being considered). The *adjacent* side is beside angle θ and runs from θ to the right angle. 
Trigonometric ratios	The **trigonometric ratios** are sin θ, cos θ and tan θ: $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
SOH-CAH-TOA	This helps you to remember the trigonometric ratio rules.
Degree mode	Make sure your calculator is in DEGREE mode when doing calculations with trigonometric ratios.
Applications of right-angled triangles	Always draw well-labelled diagrams showing all known sides and angles. Also label any sides or angles that need to be found.
Angle of elevation	The **angle of elevation** is the angle through which you *raise* your line of sight from the horizontal, looking *up* at something. 
Angle of depression	The **angle of depression** is the angle through which you *lower* your line of sight from the horizontal, looking *down* at something. 
Angle of elevation = angle of depression	The angles of elevation and depression are alternate ('Z') angles so they are equal.
Three-figure bearings	**Three-figure bearings** are measured clockwise from north and always given with three digits, e.g. 060°, 220°. 

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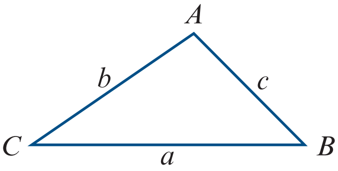
Non-right-angled triangles

Labelling a non-right-angled triangle

Side a is always opposite angle A, and so on.



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use the **sine rule** when given:

- two sides and an angle opposite one of those sides
- two angles and one side.

If neither angle is opposite the given side, find the third angle using $A + B + C = 180°$.

Cosine rule

The **cosine rule** has three versions. When given two sides and the angle between them, use the rule that starts with the required side:

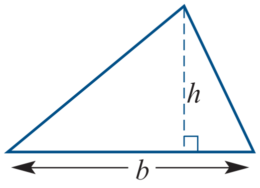
$$a^2 = b^2 + c^2 - 2bc\cos A$$

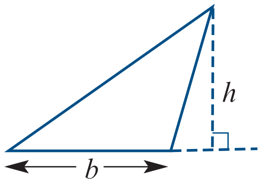
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

- Use the formula area of triangle $= \frac{1}{2} \times b \times h$ if the base and height of the triangle are known:





- Use the formula area of triangle $= \frac{1}{2} \times bc\sin A$ if two sides and the angle between them are known.
- Use Heron's rule if the lengths of the three sides of the triangle are known.

Heron's rule

Area of triangle $= A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ and a, b and c are the sides of the triangle.

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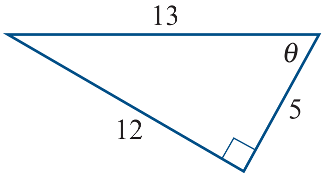
Skills check

Having completed this chapter you should be able to:

- use trigonometric ratios to find an unknown side or angle in a right-angled triangle
- show the angle of elevation or angle of depression on a well-labelled diagram
- show directions on a diagram by using three-figure bearings
- use the sine rule and cosine rule in non-right-angled triangles to find an unknown side or angle
- use the appropriate rule from the three rules for finding the area of a triangle
- solve practical problems involving right-angled and non-right-angled triangles.

Multiple-choice questions

1 In the triangle shown, $\sin \theta$ equals:

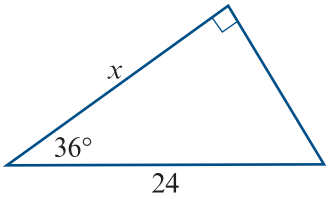


A $\frac{5}{12}$ **B** $\frac{5}{13}$

C $\frac{13}{12}$ **D** $\frac{12}{13}$

E $\frac{12}{5}$

2 The length x is given by:

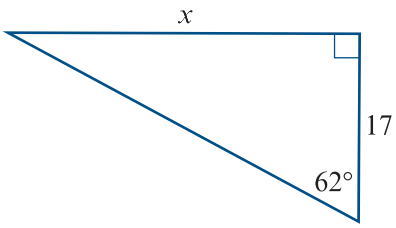


A $24 \sin 36°$ **B** $24 \tan 36°$

C $24 \cos 36°$ **D** $\frac{\sin 36°}{24}$

E $\frac{\cos 36°}{24}$

3 To find length x we should use:



A $17 \sin 62°$ **B** $17 \tan 62°$

C $17 \cos 62°$ **D** $\frac{\tan 62°}{17}$

E $\frac{\sin 62°}{17}$

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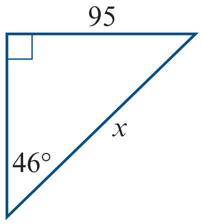
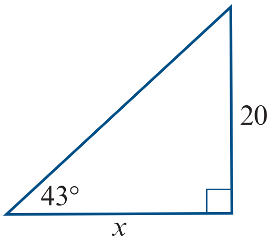
4 The side x is given by:



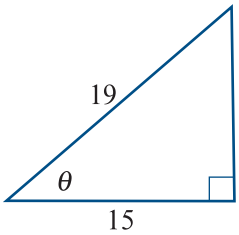
A $95 \tan 46°$
B $\dfrac{95}{\cos 46°}$
C $\dfrac{\sin 46°}{96}$
D $95 \sin 46°$
E $\dfrac{95}{\sin 46°}$

5 To find the side x we need to calculate:



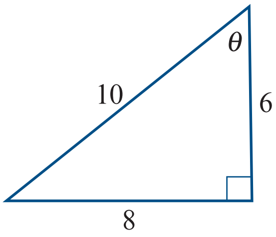
A $\dfrac{\tan 43°}{20}$
B $\dfrac{20}{\tan 43°}$
C $20 \tan 43°$
D $20 \cos 43°$
E $20 \sin 43°$

6 To find the angle θ we need to use:



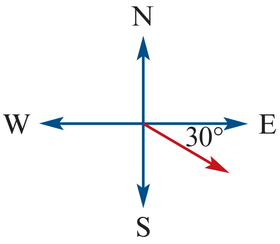
A $\cos^{-1}\left(\dfrac{15}{19}\right)$
B $\cos\left(\dfrac{15}{19}\right)$
C $\sin^{-1}\left(\dfrac{15}{19}\right)$
D $15 \sin(19)$
E $19 \cos(15)$

7 The angle θ, correct to one decimal place, is:



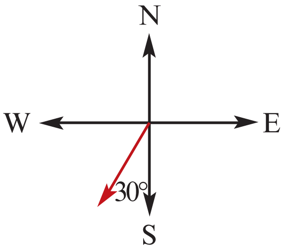
A 53.1°
B 36.9°
C 51.3°
D 38.7°
E 53.3°

8 The direction shown has the three-figure bearing:



A 030°
B 060°
C 120°
D 210°
E 330°

9 The direction shown could be described as the three-figure bearing:



A 030°
B 060°
C 120°
D 210°
E –030°

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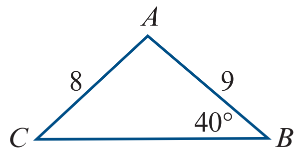
10 Correct to one decimal place, angle C in this triangle equals:

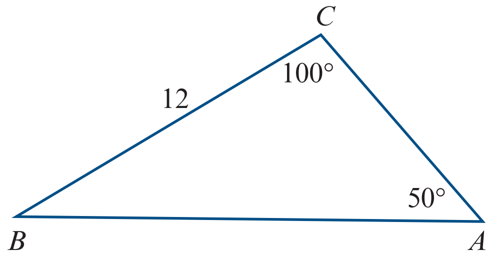


A 34.8° **B** 46.3°

C 53.9° **D** 55.2°

E 86.1°

11 To find length c in triangle ABC we should use:

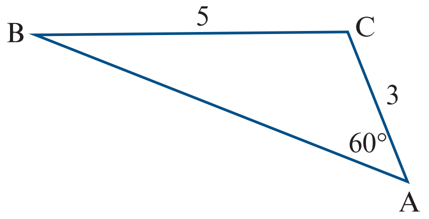


A $\dfrac{12\sin 100°}{\sin 30°}$ **B** $\dfrac{12\sin 50°}{\sin 100°}$

C $\dfrac{\sin 50°}{12\sin 100°}$ **D** $\dfrac{12\sin 100°}{\sin 50°}$

E $\dfrac{\sin 100°}{12\sin 50°}$

12 In triangle ABC, sin B equals:

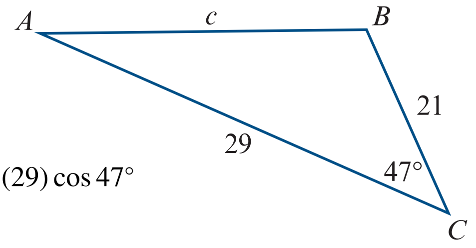


A $\dfrac{3}{5}$ **B** $\dfrac{3\sin 60°}{5}$

C $\dfrac{3}{5\sin 60°}$ **D** $\dfrac{5\sin 60°}{3}$

E $\dfrac{5}{3\sin 60°}$

13 Which expression should be used to find length c in triangle ABC?

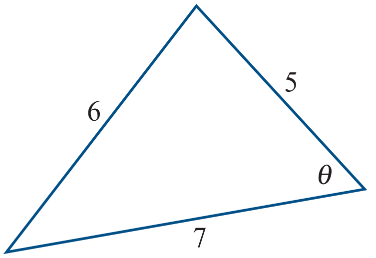


A $\dfrac{1}{2}(21)(29)\cos 47°$ **B** $\cos^{-1}\left(\dfrac{21}{29}\right)$

C $\sqrt{21^2 + 29^2}$ **D** $21^2 + 29^2 - 2(21)(29)\cos 47°$

E $\sqrt{21^2 + 29^2 - 2(21)(29)\cos 47°}$

14 For the given triangle, the value of cos θ is given by:



A $\dfrac{6^2 - 7^2 - 5^2}{2(7)(5)}$ **B** $\dfrac{7^2 + 5^2 - 6^2}{2(7)(5)}$

C $\dfrac{5}{7}$ **D** $\dfrac{7^2 - 5^2 - 6^2}{2(5)(6)}$

E $\dfrac{5^2 - 6^2 - 7^2}{2(5)(6)}$

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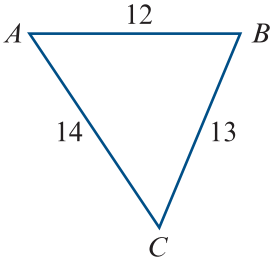
15 To find angle C we should use the rule:

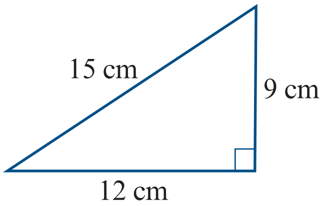


A $\cos C = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ **B** $\sin C = \dfrac{\text{opposite}}{\text{hypotenuse}}$

C $\cos C = \dfrac{a^2 + c^2 - b^2}{2ac}$ **D** $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

E $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

16 The area of the triangle shown is:

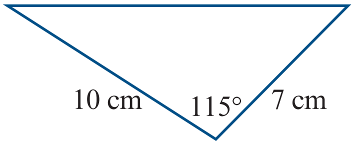


A 108 cm^2 **B** 54 cm^2

C 36 cm^2 **D** 90 cm^2

E 67.5 cm^2

17 The area of the triangle shown, correct to two decimal places, is:

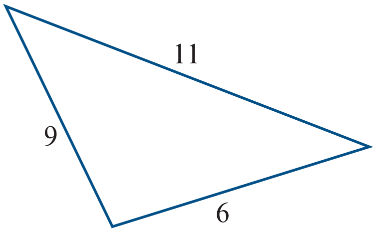


A 35.00 cm^2 **B** 70.00 cm^2

C 14.79 cm^2 **D** 31.72 cm^2

E 33.09 cm^2

18 The area of the triangle shown is given by:



A $26(26 - 6)(26 - 9)(26 - 11)$

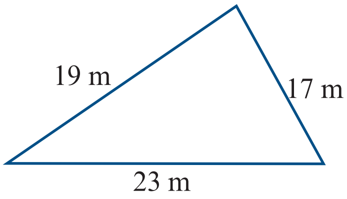
B $\sqrt{26(26 - 6)(26 - 9)(26 - 11)}$

C $\sqrt{13(13 - 6)(13 - 9)(13 - 11)}$

D $\sqrt{6^2 + 9^2 + 11^2}$

E $13(13 - 6)(13 - 9)(13 - 11)$

19 The area of the triangle shown, correct to one decimal place, is:



A 29.5 m^2 **B** 218.5 m^2 **C** 195.5 m^2

D 161.5 m^2 **E** 158.6 m^2

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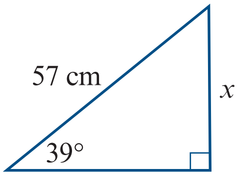
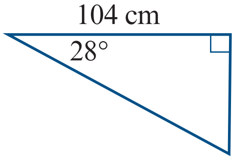
Short-answer questions

SF

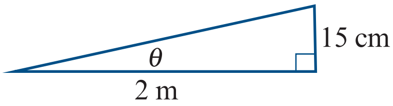
1 Find the length of x, correct to two decimal places.



2 Find the length of the hypotenuse, correct to two decimal places.



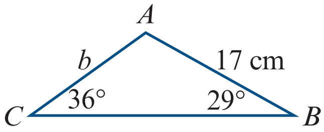
3 A road rises 15 cm for every 2 m travelled horizontally.
Find the angle of slope θ, to the nearest degree.



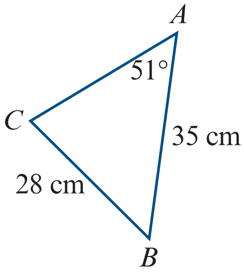
4 **a** Find the sides of a right-angled triangle for which $\cos\theta = \dfrac{72}{97}$ and $\tan\theta = \dfrac{65}{72}$.

b Hence find $\sin\theta$.

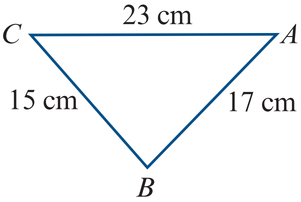
5 Find the length of side b, correct to two decimal places.



6 Find angle C, correct to one decimal place.



7 Find the smallest angle in the triangle shown, correct to one decimal place.



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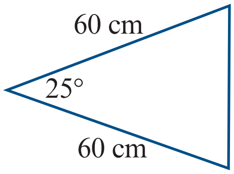
SF

8 A car travelled 30 km east, then travelled 25 km on a bearing of 070°. How far was the car from its starting point? Answer correct to two decimal places.

9 A pennant flag has the dimensions shown. What is the area of the flag? Answer correct to one decimal place.

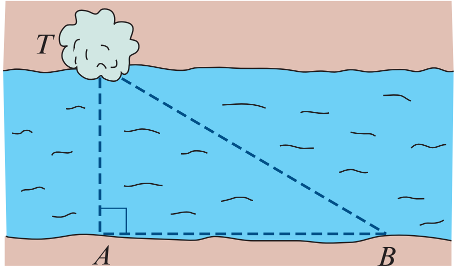


10 Find the area of an equilateral triangle with sides of 8 m, correct to one decimal place.

Extended-response questions

SF

1 Tim was standing at point A and looking at a tree, T, directly opposite him on the far bank of the river. He walked 100 m along the riverbank to point B and noticed that his line of sight to the tree now made an angle of 27° with the riverbank. Answer the following correct to two decimal places.



a How wide was the river?

b What is the distance from point B to the tree?

c Standing at B, Tim measured the angle of elevation to the top of the tree to be 18°. Draw a clearly labelled diagram showing distance TB, the height of the tree and the angle of elevation, then find the height of the tree.

2 One group of bushwalkers left a road running north–south to walk along a bearing of 060°. A second group of walkers started at a point 3 km further north. They walked on a bearing of 110°. The two groups met at point C, where their paths intersected.

a Find the angle at which their paths met.

b Find the distance walked by each group, correct to two decimal places.

c The bushwalkers decided to return to the road by walking back along the path that the second group of walkers had taken. What bearing should they follow?

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3 A yacht P left port and sailed 45 km on a bearing of 290°.
Another yacht Q left the same port but sailed for 54 km on a bearing of 040°.

- **a** What was the angle between their directions?
- **b** How far apart were they at that stage (correct to two decimal places)?

4 A triangular shade cloth must have sides of 5 m, 6 m and 7 m to cover the required area of a children's playground.

- **a** What angle is required in each of the corners (correct to one decimal place)?
- **b** The manufacturer charges according to the area of the shade cloth. What is the area of this shade cloth (correct to two decimal places)?
- **c** The cost of shade cloth is $29 per square metre. What will be the cost of this shade cloth?

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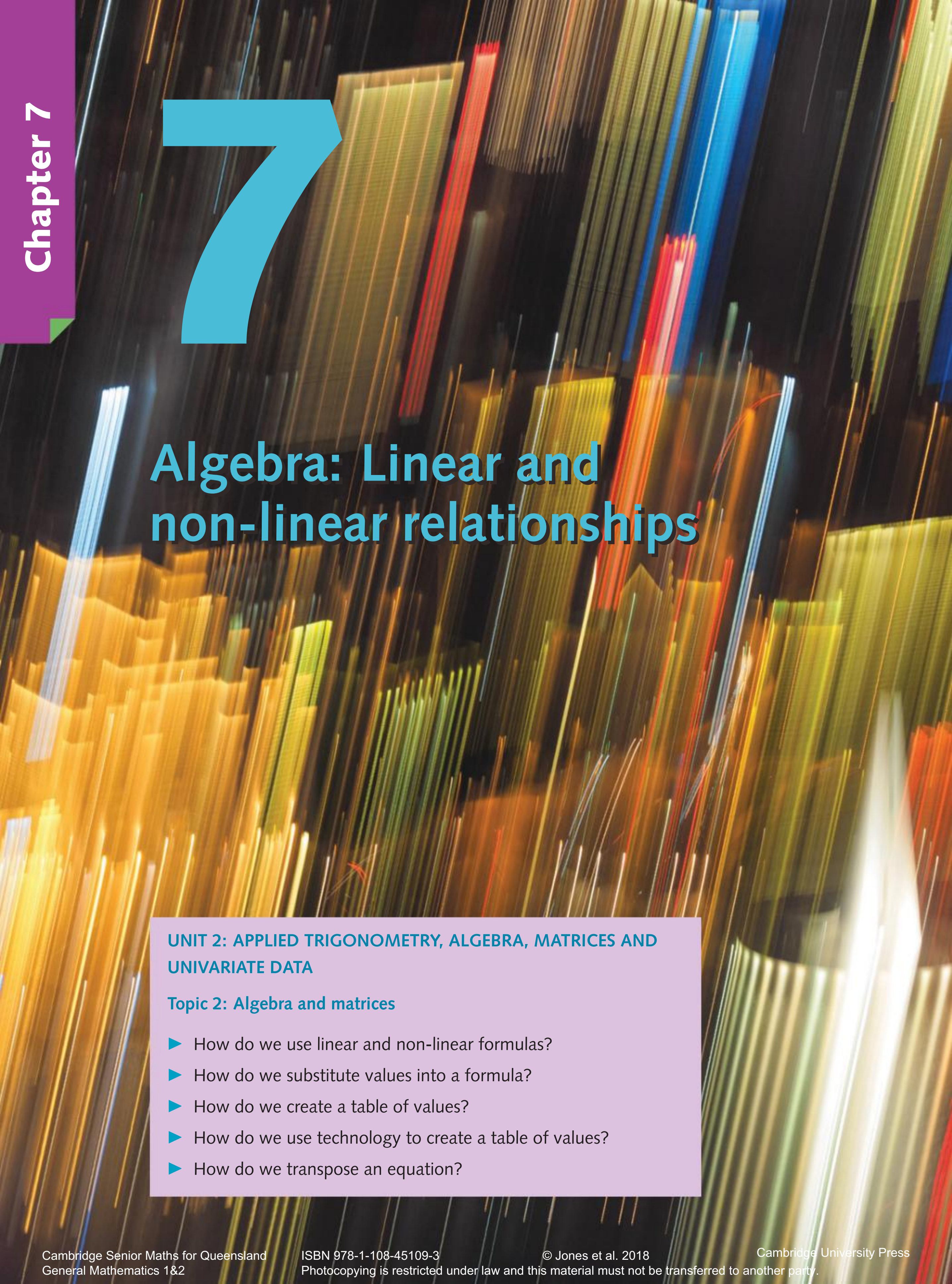


Chapter 7

7

Algebra: Linear and non-linear relationships

UNIT 2: APPLIED TRIGONOMETRY, ALGEBRA, MATRICES AND UNIVARIATE DATA

Topic 2: Algebra and matrices

- ► How do we use linear and non-linear formulas?
- ► How do we substitute values into a formula?
- ► How do we create a table of values?
- ► How do we use technology to create a table of values?
- ► How do we transpose an equation?

Introduction

Linear relationships and equations connect two or more variables such that they yield a straight line when graphed. The unknown values are always to the power of 1. Linear relationships have many applications in technology, science and business.

A non-linear relationship or equation is one whose unknown values are not all to the power of 1.

7A Substitution of values into an algebraic expression

An **algebraic expression** (or formula) is a mathematical relationship connecting two or more variables.

For example:

- $C = 45t + 150$ is an algebraic expression for relating the cost, C dollars, of hiring a plumber for t hours. C and t are the variables. It is a linear relationship.
- $P = 4L$ is an algebraic expression for finding the perimeter of a square, where P is the perimeter and L is the side length of the square. P and L are the variables. It is a linear relationship.
- $A = \pi r^2$ is an algebraic expression for the area of a circle, where A is the area, r is the radius and π is a constant approximately equal to 3.142. It is a non-linear relationship.

By substituting all known variables into an algebraic expression, we are able to find the value of an unknown variable.

Example 1 Using a linear algebraic expression

The cost of hiring a windsurfer is given by the rule:

$$C = 40t + 10$$

where C is the cost in dollars and t is the time in hours. How much will it cost to hire a windsurfer for 2 hours?

Solution

1 Write the algebraic expression.

$$C = 40t + 10$$

2 To determine the cost of hiring a windsurfer for 2 hours, substitute $t = 2$ into the algebraic expression.

Note: 40(2) means 40 × 2

$$C = 40(2) + 10$$

3 Evaluate.

$$C = 90$$

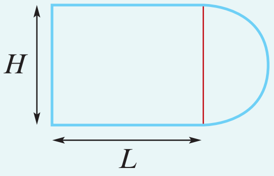
4 Write your answer.

It will cost $90 to hire a windsurfer for 2 hours.

Example 2 Using a linear algebraic expression

The perimeter of this shape can be given by the expression:

$$P = 2L + H\left(1 + \frac{\pi}{2}\right)$$



In this expression, *L* is the length of the rectangle and *H* is the height. Find the perimeter correct to one decimal place, if $L = 16.1$ cm and $H = 3.2$ cm.

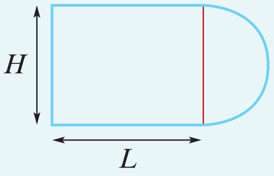
Solution

1 Write the expression.	$P = 2L + H\left(1 + \frac{\pi}{2}\right)$
2 Substitute values for *L* and *H* into the expression.	$P = 2 \times 16.1 + 3.2\left(1 + \frac{\pi}{2}\right)$
3 Evaluate.	$P = 40.4$ (correct to one decimal place)
4 Give your answer with correct units.	The perimeter of the shape is 40.4 cm.

Example 3 Using a non-linear algebraic expression (polynomial)

The area, *A*, of the shape shown can be given by the expression:

$$A = HL + \frac{1}{2}\pi\left(\frac{H}{2}\right)^2$$



In this expression, *L* is the length of the rectangle and *H* is the height. Find the area correct to one decimal place, if $H = 5.2$ cm and $L = 18.4$ cm.

Solution

1 Write the expression.	$A = HL + \frac{1}{2}\pi\left(\frac{H}{2}\right)^2$
2 Substitute values for *H* and *L* into the expression.	$A = 5.2 \times 18.4 + \frac{1}{2}\pi\left(\frac{5.2}{2}\right)^2$
3 Evaluate.	$A = 106.3$ (correct to one decimal place)
4 Give your answer with correct units.	The area of the shape is 106.3 cm^2.

Example 4 Using a non-linear algebraic expression (inverse proportion)

The current, I amperes, that flows in an electrical appliance depends on the resistance of the appliance, R ohms, where $I = \frac{240}{R}$. Find the current that flows through an appliance with a resistance of 100 ohms.

Solution

1 Write the expression. $I = \frac{240}{R}$

2 Substitute $R = 100$. $I = \frac{240}{100}$

3 Evaluate. $I = 2.4$

4 Give your answer with the correct units. The current that flows in the appliance is 2.4 amperes.

Example 5 Using a non-linear algebraic expression

Use the expression $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ to find f when $u = 5$ and $v = 3$.

Solution

1 Write the expression. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

2 Substitute $u = 5$ and $v = 3$ into the expression then simplify.

$$\begin{aligned}\frac{1}{f} &= \frac{1}{5} + \frac{1}{3}\\ &= \frac{3}{15} + \frac{5}{15}\\ &= \frac{8}{15}\end{aligned}$$

3 Invert both sides. $\frac{f}{1} = \frac{15}{8}$

4 Write your answer. $f = \frac{15}{8}$

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Exercise 7A

SF

Example 1 **1** The cost of hiring a dance hall is given by the rule:

$$C = 50t + 1200$$

where C is the total cost in dollars and t is the number of hours for which the hall is hired.
Find the cost of hiring the hall for:

a 4 hours **b** 6 hours **c** 4.5 hours

2 The distance, d km, travelled by a car in t hours at an average speed of v km/h is given by the expression:

$$d = v \times t$$

Find the distance travelled by a car travelling at a speed of 95 km/h for 4 hours.

3 Taxi fares are calculated using the expression:

$$F = 1.3K + 4$$

where K is the distance travelled in kilometres and F is the cost of the fare in dollars. Find the costs of the following trips.

a 5 km **b** 8 km **c** 20 km

4 The circumference, C, and area, A, of a circle with radius r can be calculated using the expressions:

$$C = 2\pi r \qquad A = \pi r^2$$

For the following circles, find, correct to two decimal places:

i the circumference **ii** the area

a A stained glass window with $r = 25$ cm
b An earring with $r = 3$ mm
c A DVD with $r = 5.4$ cm
d A circular garden bed with $r = 7.2$ m

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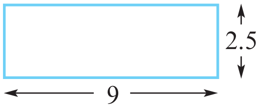
Example 2

5 The perimeter of a rectangle is given by $P = 2(L + W)$. Find the value of P for the following rectangles.

a $L = 3$ and $W = 4$

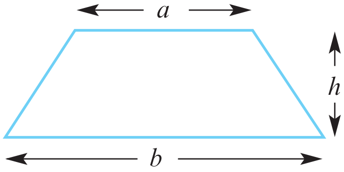
b $L = 15$ and $W = 8$

c



Example 3

6 The area of a trapezium as shown is $A = \frac{1}{2}h(a + b)$. Find A when:



a $h = 1, a = 3, b = 5$

b $h = 5, a = 2.5, b = 3.2$

c $h = 2.7, a = 4.1, b = 8.3$

7 The formula for calculating simple interest is:

$$I = \frac{Prt}{100}$$

where P is the principal (amount invested or borrowed), r is the interest rate per annum and t is the time (in years). Find the simple interest for the following, giving your answers to the nearest cent.

a Frank borrows \$5000 at 12% for 4 years. How much interest will he pay?

b Chris borrows \$1500 at 6% for 2 years. How much interest will he pay?

c Jane invests \$2500 at 5% for 3 years. How much interest will she earn?

d Henry invests \$8500 for 3 years with an interest rate of 7.9%. How much interest will he earn?

8 The formula used to convert temperature from degrees Fahrenheit to degrees Celsius is:

$$C = \frac{5}{9}(F - 32)$$

Use this formula to convert the following temperatures to degrees Celsius. Give your answers correct to one decimal place.

a 50°F

b 0°F

c 212°F

d 92°F

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9 In Australian Rules football, a goal is worth 6 points and a behind is worth 1 point. The total number of points is given by:

total number of points = 6 × number of goals + number of behinds

a Find the number of points for the following scores:

  i 2 goals and 3 behinds

  ii 5 goals and 7 behinds

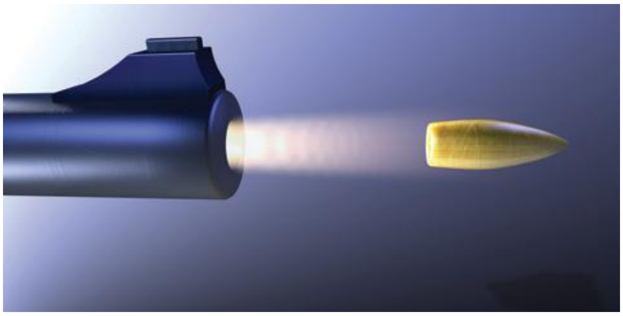
  iii 8 goals and 20 behinds

b In a match, Redteam scores 4 goals and 2 behinds and Greenteam scores 3 goals and 10 behinds. Which team wins the match?

10 In Rugby League, a try is worth 4 points, a conversion or penalty kick is worth 2 points, and a field goal (drop kick) is worth one point. In a match the Gold team scores 4 tries, 2 conversions, a penalty kick and a field goal. The Silver team scores 3 tries, 3 conversions and two penalty kicks. What is the final score in the match?

11 In Rugby Union, a try is worth 5 points, a conversion kick is worth 2 points, and a penalty goal and field goal are both worth 3 points. What would the score in the previous question have been if the Gold and Silver teams were playing Rugby Union?

12 The speed v in metres per second of a bullet fired from a gun after t seconds is given by $v = 6600–61t + 3t^2$. Find the speed of the bullet after 2 seconds.



13 The surface area, A, and volume of a sphere, V, can be calculated using the rules:

$$V = \frac{4}{3}\pi r^3 \qquad A = 4\pi r^2$$

where r is the radius of the sphere.

For the following spheres, find, correct to two decimal places:

i the volume **ii** the surface area

a A basketball with radius $r = 12.1$ cm

b A soap bubble with radius $r = 12.5$ mm

c A plastic sphere with radius $r = 1.35$ m

d An orange with radius $r = 6.3$ cm

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Example 4 **14** The current, I amperes, that flows through a toaster of resistance R ohms is given by $I = \dfrac{240}{R}$. Find the current that flows through a toaster of resistance 150 ohms.

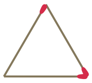
15 The average speed of a car, s km/h, that travels a distance of d kilometres in t hours is given by $s = \dfrac{d}{t}$. A car travels 250 km in 3.5 hours. Find the average speed of the car correct to one decimal place.

Example 5 **16** Use the expression $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ to find f when:

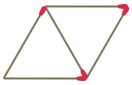
a $u = 4$ and $v = 7$

b $u = 6$ and $v = 10$

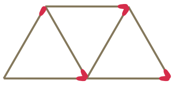
17 The number of matchsticks used for each shape below follows the pattern 3, 5, 7, …



Shape 1



Shape 2



Shape 3

The rule for finding the number of matches used in this sequence is:

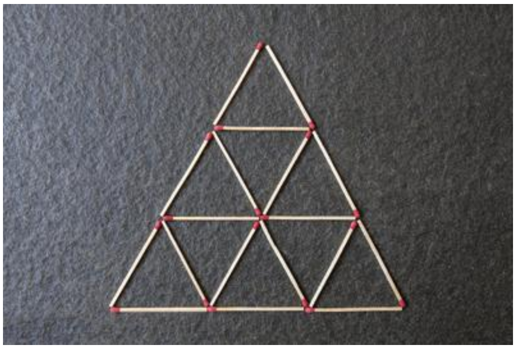
number of matches $= a + (n - 1)d$

where a is the number of matches in the first shape ($a = 3$), d is the number of extra matches used for each shape ($d = 2$) and n is the shape number.
Find the number of matches in the:

a 6th shape

b 11th shape

c 50th shape



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18 Suggested cooking times for roasting x kilograms of meat are given in the following table.

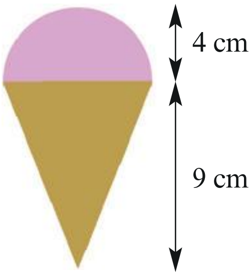
Meat type	Cooking time
Chicken (well done)	45 min/kg + 20 min
Lamb (medium)	55 min/kg + 25 min
Lamb (well done)	65 min/kg + 30 min
Beef (medium)	55 min/kg + 20 min
Beef (well done)	65 min/kg + 30 min

a How long, to the nearest minute, will it take to cook:

- **i** a 2 kg chicken?
- **ii** a 2.25 kg beef roast well done?
- **iii** a piece of lamb weighing 2.4 kg cooked well done?
- **iv** a 2.5 kg beef roast cooked medium?

b At what time should you put a 2 kg leg of lamb into the oven to be able to serve it medium at 7:30 p.m.?

19 An ice cream consists of a wafer in the shape of a cone and a sphere of ice cream on the top, as shown in the diagram on the right. The circular end of the cone has a radius of 4 cm. The formulas for the surface area and volume of cones and spheres are shown below:



$$\text{surface area of cone} = \pi r\left(r + \sqrt{h^2 + r^2}\right)$$

$$\text{volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{surface area of sphere} = 4\pi r^2$$

$$\text{volume of sphere} = \frac{4}{3}\pi r^3$$

where r is the radius of the cone or sphere and h is the height of the cone. Use the formulas to find the following, correct to two decimal places.

a The surface area of ice cream that is above the cone

b The surface area of the wafer

c The volume of ice cream

d The volume of air in the cone (assume all of the bottom half of the sphere of ice cream is inside the cone)

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7B Constructing a table of values

We can use an algebraic expression or formula to construct a *table of values*. This can be done by substitution (by hand) or with technology.

Example 6 Constructing a table of values

The formula for converting degrees Celsius to degrees Fahrenheit is given by:

$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F using values of C in intervals of 10 between $C = 0$ and $C = 100$.

Solution

Substitute values of $C = 0, 10, 20, 30, \ldots$ into the formula $F = \frac{9}{5}C + 32$, to find values of F.

If $C = 0, F = \frac{9}{5}(0) + 32 = 32$

If $C = 10, F = \frac{9}{5}(10) + 32 = 50$

and so on.

Construct the table.

C	0	10	20	30	40	50	60	70	80	90	100
F	32	50	68	86	104	122	140	158	176	194	212

How to construct a table of values using a spreadsheet

Spreadsheet

Guide

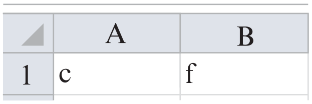
The formula for converting degrees Celsius to degrees Fahrenheit is given by:

$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F using values of C in intervals of 10 between $C = 0$ and $C = 100$.

1 Open a new spreadsheet (Excel is shown here).

2 In cell A1 type ‘c’ and in cell B1 type ‘f’.



	A	B
1	c	f

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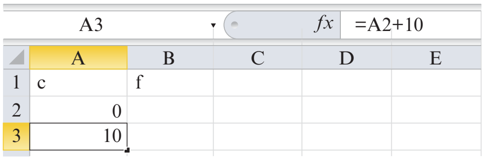
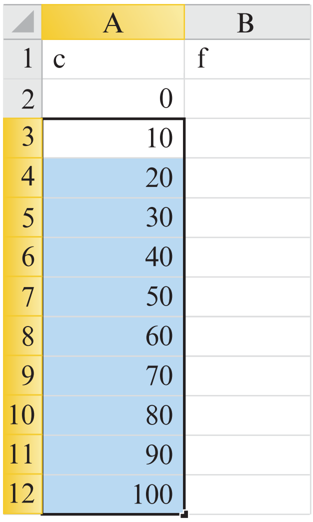
3 In cell A2 type ‘0’ and then in cell A3 type ‘= a2 + 10’.



A3 =A2+10

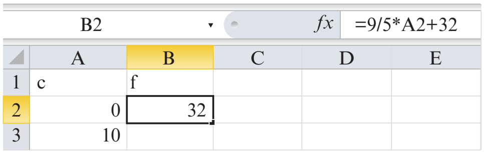
	A	B	C	D	E
1	c	f			
2	0				
3	10				

4 Highlight cells A3 to A12 then press CRTL D to fill down.



	A	B
1	c	f
2	0	
3	10	
4	20	
5	30	
6	40	
7	50	
8	60	
9	70	
10	80	
11	90	
12	100	

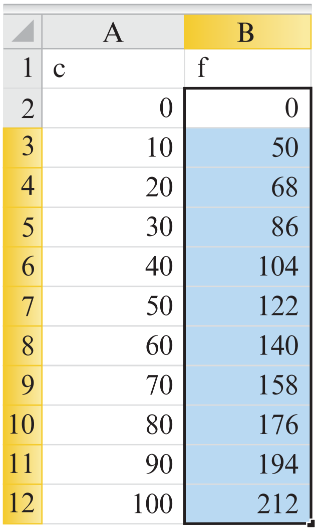
5 In cell B2 type ‘= 9/5 ∗ a2 + 32’.



B2 =9/5*A2+32

	A	B	C	D	E
1	c	f			
2	0	32			
3	10				

6 Highlight cells B2 to B12 and press CTRL D to fill down.



	A	B
1	c	f
2	0	0
3	10	50
4	20	68
5	30	86
6	40	104
7	50	122
8	60	140
9	70	158
10	80	176
11	90	194
12	100	212

Desmos widget 7B: Constructing a table of values

Example 7 Constructing a table of values

A car travels a fixed distance of 480 km. A formula for finding the average speed of the car, s km/h, when it takes t hours to travel this distance is $s = \frac{480}{t}$.

Use this formula to construct a table of values for s using values of t in intervals of 1 between $t = 4$ and $t = 12$. Give answers correct to one decimal place.

Solution

Substitute values of $t = 4, 5, 6, \ldots$ into the formula $s = \frac{480}{t}$, to find values of s.

$$\text{If } t = 4,\ s = \frac{480}{4} = 120$$

$$\text{If } t = 5,\ s = \frac{480}{5} = 96$$

and so on.

Construct the table.

t	4	5	6	7	8	9	10	11	12
s	120	96	80	68.6	60	53.3	48	43.6	40

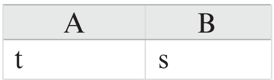
Alternatively use an Excel spreadsheet to construct this table of values

Spreadsheet

Guide

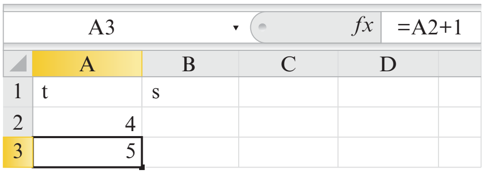
1 Open a new Excel spreadsheet.

2 In cell A1 type ‘t’ and in cell B1 type ‘s’.



A	B
t	s

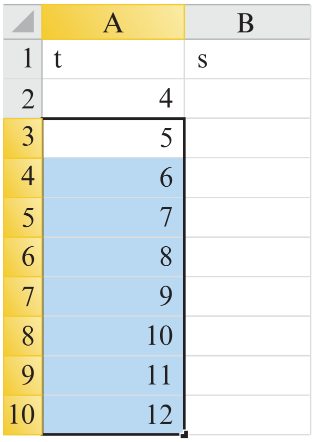
3 In cell A2 type 4 then in cell A3 type ‘=a2 + 1.’



A3 *fx* =A2+1

	A	B	C	D
1	t	s		
2	4			
3	5			

4 Highlight cells A3 to A10 then press CRTL D to fill down.



	A	B
1	t	s
2	4	
3	5	
4	6	
5	7	
6	8	
7	9	
8	10	
9	11	
10	12	

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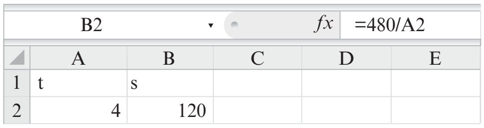
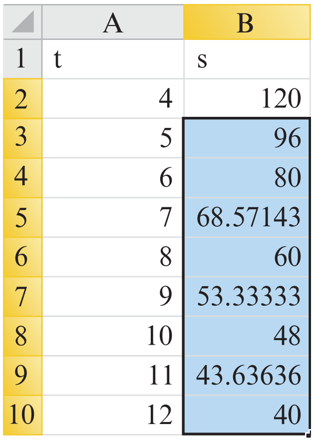
5 In cell B2 type ‘= 480/a2’.



B2 *fx* =480/A2

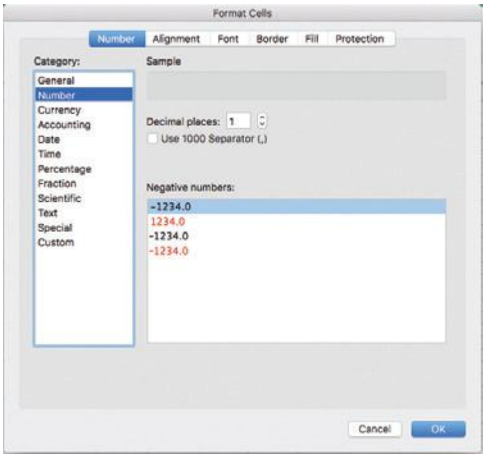
	A	B	C	D	E
1	t	s			
2	4	120			

6 Highlight cells B2 to B10 and press CTRL D to fill down.



	A	B
1	t	s
2	4	120
3	5	96
4	6	80
5	7	68.57143
6	8	60
7	9	53.33333
8	10	48
9	11	43.63636
10	12	40

7 Cells can be formatted to give answers correct to one decimal place as shown.

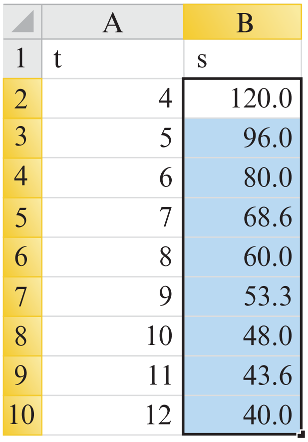


Decimal places: 1

☐ Use 1000 Separator (,)

Negative numbers:

-1234.0
1234.0
-1234.0
-1234.0



	A	B
1	t	s
2	4	120.0
3	5	96.0
4	6	80.0
5	7	68.6
6	8	60.0
7	9	53.3
8	10	48.0
9	11	43.6
10	12	40.0

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Example 8 Constructing a table of values with two variables

The formula for the body mass index, or BMI, of a person is:

$$\text{BMI} = \frac{m}{h^2}$$

where m is the mass of the person in kilograms and h is the height of the person in metres.

a Use the formula to construct a table of values for BMI using values of m in intervals of 10 between $m = 50$ and $m = 100$, and values of h in intervals of 0.2 between $h = 1$ and $h = 2$.

b Use the table to find the BMI for a man with mass 80 kg and height 1.8 m.

Solution

a **1** Determine the values of m to go in the table.

The values of m will start at 50 and increase by 10, up to 100:
m = 50, 60, 70, 80, 90, 100

2 Determine the values of h to go in the table.

The values of h will start at 1 and increase by 0.2, up to 2:
h = 1, 1.2, 1.4, 1.6, 1.8, 2.0

3 Draw up a table with one variable in the rows and one variable in the columns.
It won't matter which way it is arranged. Here, the mass is in the rows and height is in the columns.

BMI		Height (m)					
		1	1.2	1.4	1.6	1.8	2.0
Mass (kg)	50						
	60						
	70						
	80						
	90						
	100						

4 Calculate the BMI for each pair of mass and height values and enter them into the table.
For example, the value in the shaded box has been calculated as

$$\text{BMI} = \frac{70}{1.6^2} = 27.34$$

Round answers to two decimal places.

BMI		Height (m)					
		1	1.2	1.4	1.6	1.8	2.0
Mass (kg)	50	50	34.72	25.51	19.53	15.43	12.50
	60	60	41.67	30.61	23.44	18.52	15.00
	70	70	48.61	35.71	27.34	21.60	17.50
	80	80	55.56	40.82	31.25	24.69	20.00
	90	90	62.50	45.92	35.16	27.78	22.50
	100	100	69.44	51.02	39.06	30.86	25.00

b Read the value in the row for 80 kg and the column for 1.8 m, as indicated by the red lines on the table.

The BMI for a man of mass 80 kg and height 1.8 m is 24.69.

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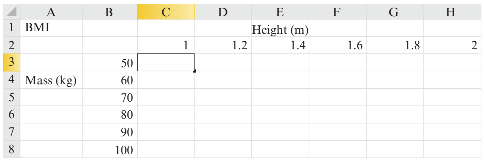
Using a spreadsheet to construct a table of values with two variables

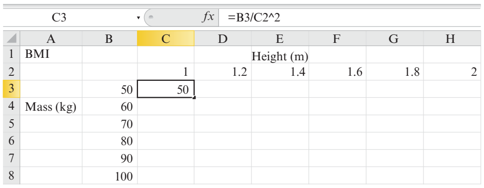
Spreadsheet

1 Set up a table with masses in the rows and heights in the columns as previously outlined.



	A	B	C	D	E	F	G	H
1	BMI				Height (m)			
2			1	1.2	1.4	1.6	1.8	2
3		50						
4	Mass (kg)	60						
5		70						
6		80						
7		90						
8		100						

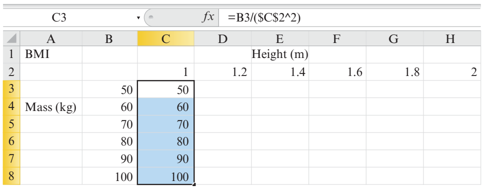
2 In cell C3 type the formula '= b3/(c2^2)'.



C3 fx =B3/C2^2

	A	B	C	D	E	F	G	H
1	BMI				Height (m)			
2			1	1.2	1.4	1.6	1.8	2
3		50	50					
4	Mass (kg)	60						
5		70						
6		80						
7		90						
8		100						

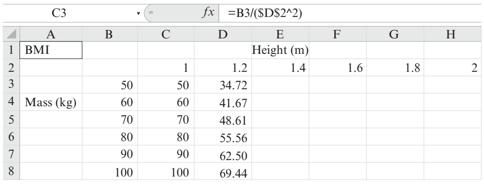
3 Highlight cells C3 to C8 and press CTRL D to fill down.



C3 fx =B3/($C$2^2)

	A	B	C	D	E	F	G	H
1	BMI				Height (m)			
2			1	1.2	1.4	1.6	1.8	2
3		50	50					
4	Mass (kg)	60	60					
5		70	70					
6		80	80					
7		90	90					
8		100	100					

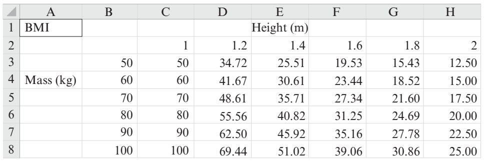
4 In cell D3 type the formula '= b3/(d2^2)' and then highlight cells D3 to D8 and fill down. Answers can be given correct to two decimal places by highlighting cells and changing them to 'Number' format.



C3 fx =B3/($D$2^2)

	A	B	C	D	E	F	G	H
1	BMI				Height (m)			
2			1	1.2	1.4	1.6	1.8	2
3		50	50	34.72				
4	Mass (kg)	60	60	41.67				
5		70	70	48.61				
6		80	80	55.56				
7		90	90	62.50				
8		100	100	69.44				

5 In cell E3 type the formula '= b3/(e2^2)' and then highlight cells E3 to E8 and fill down. Continue this process until columns F, G and H are all filled.



	A	B	C	D	E	F	G	H
1	BMI				Height (m)			
2			1	1.2	1.4	1.6	1.8	2
3		50	50	34.72	25.51	19.53	15.43	12.50
4	Mass (kg)	60	60	41.67	30.61	23.44	18.52	15.00
5		70	70	48.61	35.71	27.34	21.60	17.50
6		80	80	55.56	40.82	31.25	24.69	20.00
7		90	90	62.50	45.92	35.16	27.78	22.50
8		100	100	69.44	51.02	39.06	30.86	25.00

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Exercise 7B

Example 6

SF

1 A football club wishes to purchase pies at a cost of $2.15 each. If C is the cost ($) and x is the number of pies, complete the table showing the amount of money needed to purchase from 40 to 50 pies.

x	40	41	42	43	44	45	46	47	48	49	50
C ($)	86	88.15	90.3								

2 The circumference of a circle is given by:

$$C = 2\pi r$$

where r is the radius. Complete the table of values to show the circumferences of circles with radii from 0 to 1 cm in intervals of 0.1 cm. Give your answers correct to three decimal places.

r (cm)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C (cm)	0	0.628	1.257	1.885							

3 A phone bill is calculated using the formula:

$$C = 40 + 0.18n$$

where C is the total cost and n represents the number of calls made. Complete the table of values to show the cost for 50, 60, 70, . . . 130 calls.

n	50	60	70	80	90	100	110	120	130
C ($)	49	50.80	52.60						

4 The amount of energy (E) in kilojoules expended by an adult male of mass M at rest, can be estimated using the formula:

$$E = 110 + 9M$$

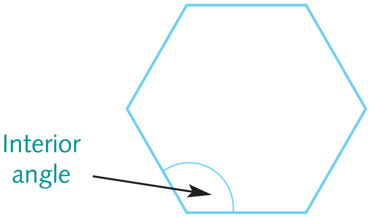
Complete the table of values in intervals of 5 kg for males of mass 60–120 kg to show the corresponding values of E.

M (kg)	60	65	70	75	80	85	90	95	100	105	110	115	120
E (kJ)	650	695											

5 The sum, S, of the interior angles of a polygon with n sides is given by the formula:

$$S = 90(2n - 4)$$

Construct a table of values showing the sum of the interior angles of polygons with 3 to 10 sides.



n	3	4	5					
S	180°	360°						

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CF

6 Use the rule $P = M \times T$ to complete the table on the right.

P		T			
		1	2	3	4
M	1				
	2				
	3				
	4				

7 Use the rule $H = \dfrac{R+2}{Z}$ to complete the table on the right. Write the table values as fractions.

H		Z			
		1	2	3	4
R	1				
	2				
	3				
	4				

SF

8 Taxi fares are calculated using the formula:

$$F = 1.3K + 4$$

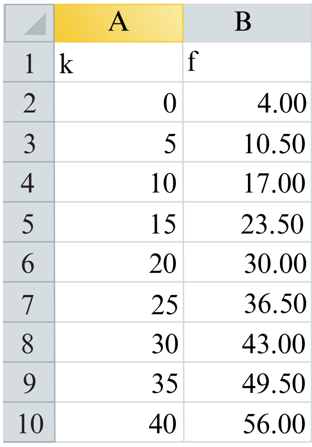
where K is the distance travelled in kilometres and F is the cost of the fare in dollars.

Spreadsheet Guide

The spreadsheet shows a table of values created using this formula.

a Write the formula that was used in cell B2 to calculate this value.

b What is the fare for a taxi journey of 30 kilometres?



	A	B
1	k	f
2	0	4.00
3	5	10.50
4	10	17.00
5	15	23.50
6	20	30.00
7	25	36.50
8	30	43.00
9	35	49.50
10	40	56.00

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Example 7 **9** A car salesman's weekly wage, E dollars, is given by the formula:

$$E = 60n + 680$$

where n is the number of cars sold.

a Use a spreadsheet to construct a table of values to show how much his weekly wage will be if he sells from 0 to 10 cars.

b Using your table of values, if the salesman earns \$1040 in a week, how many cars did he sell?

Spreadsheet
7BQ10

10 Anita has \$10 000 that she wishes to invest at a rate of 4.5% per annum. She wants to know how much interest she will earn after 1, 2, 3, . . . 10 years. Using the formula:

$$I = \frac{Prt}{100}$$

where P is the principal, and r is the interest rate (%) and t is the number of time periods. Construct a table of values with a spreadsheet to show how much interest, I, she will have after $t = 1, 2, 3, \ldots 10$ years.

11 The formula for finding the amount, A, accumulated at compound interest is given by:

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

where P is the principal, r is the interest rate per period (%) and t is the number of time periods. Construct a table of values using a spreadsheet showing the amount accumulated when \$5000 is invested at a rate of 5.5% over 5, 10, 15, 20 and 25 years. Give your answers to the nearest dollar.

12 Kaela uses a geometry package on a computer to construct rectangles of area 3200 cm^2. Let L cm be the length of a rectangle and W cm be the width of this rectangle. The formula $W = \frac{3200}{L}$ can be used to find the width of one of these rectangles if its length is known. Complete the following table of values that gives possible lengths and widths of Kaela's rectangles.

L	10	20	40	80	100	320	640
W							

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13 The height, h, of a triangle of base length b and area A is given by the formula $h = \frac{2A}{b}$.

SF

a Rewrite this formula for triangles of area 24 cm^2.

b Use this formula to complete the table of values that gives the height of triangles of area 24 cm^2.

b	2	4	8	12	16	24	48	96
h								

Example 8 **14** The formula for the cost of sending messages using a particular mobile phone plan, \$$C$, is:

CF

$$C = 0.05t + 0.2p$$

where t is the number of text messages sent and p is the number of photo messages sent.

a Use the formula and a spreadsheet to construct a table of values for the cost of sending messages using values of t in intervals of 2 between $t = 0$ and $t = 10$ and values of p in intervals of 2 between $p = 0$ and $p = 10$.

b Use the table to find the cost of sending 8 text messages and 4 photo messages.

15 David would like to borrow \$5000 from a bank. The amount of simple interest charged on this loan is given by the formula:

CU

$$I = 50r \times t$$

where I is the interest charged, r is the annual interest rate and t is the number of years before the loan is repaid.

a Use the formula and a spreadsheet to construct a table of values for the interest charged on the loan using values of r in intervals of 0.2 between $r = 3$ and $r = 4$ and values of t in intervals of 1 between $t = 1$ and $t = 5$.

b Use the table to find the interest charged if the loan has an annual interest rate of 3.8% and is repaid after 4 years.

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7C Transposition of equations

The formula $C = 2\pi r$ can be used to find the circumference, C, of a circle of radius r. In this formula C is the subject.

Dividing both sides of this equation by 2π gives $r = \dfrac{C}{2\pi}$ and now r is the subject of the formula.

The process of rearranging a formula in order to make a different pronumeral the subject of the equation is called **transposition**.

Example 9

The cost of hiring a windsurfer is given by the rule $C = 40t + 10$, where C is the cost in dollars and t is the time in hours.

a Rearrange the equation to make t the subject.

b For how long could a windsurfer be hired for a cost of $130?

Solution

a **1** Write the formula.

$$C = 40t + 10$$

2 Subtract 10 from both sides of the equation.

$$C - 10 = 40t + 10 - 10$$
$$C - 10 = 40t$$

3 Divide both sides of the equation by 40.

$$\frac{C - 10}{40} = \frac{40t}{40}$$
$$t = \frac{C - 10}{40}$$

b **1** Write the formula.

$$t = \frac{C - 10}{40}$$

2 Substitute $C = 130$ into the formula.

$$t = \frac{130 - 10}{40}$$

3 Evaluate.

$$t = \frac{120}{40}$$
$$t = 3$$

4 Write your answer.

The windsurfer can be hired for 3 hours for a cost of $130.

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Example 10

The formula $S = 4\pi r^2$ can be used to calculate the surface area, S, of a sphere of radius r. Find the radius of a sphere, correct to one decimal place, which has a surface area of 36.2 cm^2.

Solution

1 Write the formula.

$$S = 4\pi r^2$$

2 Rearrange the formula to make r the subject:

- Divide both sides of the equation by 4π.

$$\frac{S}{4\pi} = \frac{4\pi r^2}{4\pi}$$

$$r^2 = \frac{S}{4\pi}$$

- Take the square root both sides of the equation.

$$\sqrt{r^2} = \sqrt{\frac{S}{4\pi}}$$

$$r = \sqrt{\frac{S}{4\pi}}$$

3 Substitute $S = 36.2$ into the formula.

$$r = \sqrt{\frac{36.2}{4\pi}}$$

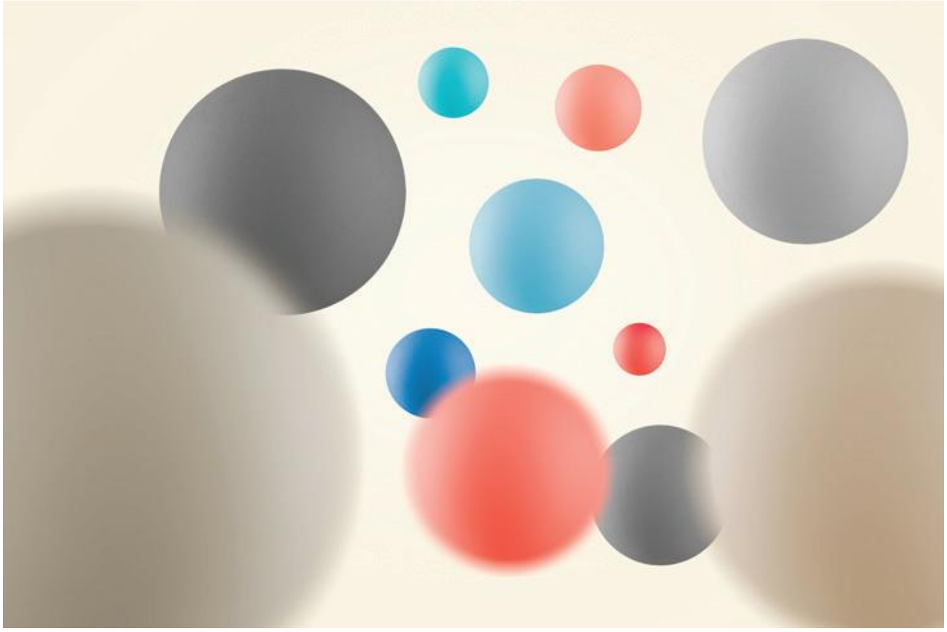
4 Evaluate and round to one decimal place.

$r = 1.6972\ldots$

$r = 1.7$ correct to one decimal place

5 Write your answer.

A sphere with surface area 36.2 cm^3 has a radius of 1.7 cm.



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Example 11

The formula for calculating simple interest is

$$I = \frac{Prt}{100}$$

where P is the principal (amount invested or borrowed), r is the interest rate per annum and t is the time (in years).

Daniel invests \$4000 at 6% per annum and earns \$720 in interest. For how long was this investment?

Solution

1 Write the formula.

$$I = \frac{Prt}{100}$$

2 Rearrange the formula to make t the subject.

- Multiply both sides of the equation by 100.

$$100 \times I = 100 \times \frac{Prt}{100}$$

$$100I = Prt$$

- Divide both sides of the equation by Pr.

$$\frac{100I}{Pr} = \frac{Prt}{Pr}$$

$$t = \frac{100I}{Pr}$$

3 Substitute $P = 4000$, $r = 6$ and $I = 720$ into the formula.

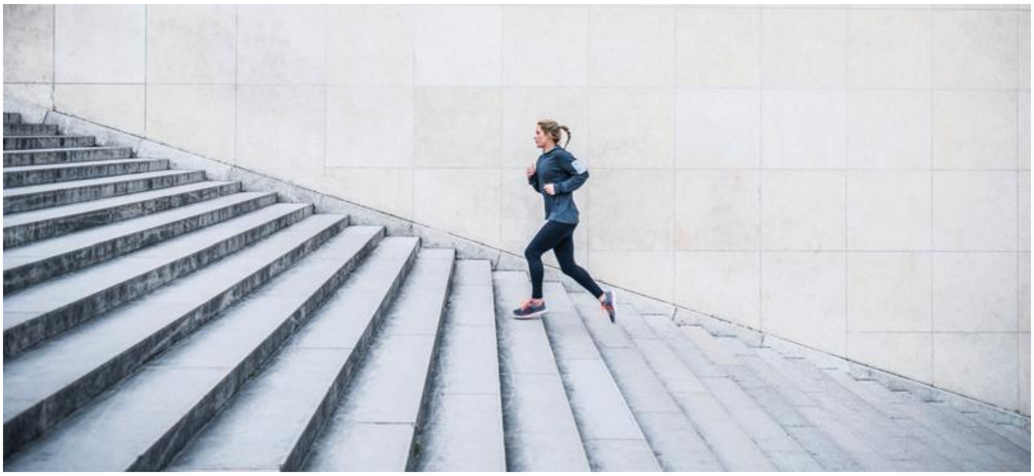
$$t = \frac{100 \times 720}{4000 \times 6}$$

4 Evaluate.

$$t = 3$$

5 Write your answer.

The length of the investment was 3 years.



Example 12

The current, I amperes, that flows in an electrical appliance depends on the resistance of the appliance, R ohms: $I = \frac{240}{R}$.

a Transpose the formula to make R the subject.

b If a kettle has a current of 1.2 amperes running through it, find the resistance of the kettle.

Solution

a **1** Write the formula.

$$I = \frac{240}{R}$$

2 Multiply both sides of the equation by R.

$$I \times R = \frac{240}{R} \times R$$

$$IR = 240$$

3 Divide both sides of the equation by I.

$$\frac{IR}{I} = \frac{240}{I}$$

$$R = \frac{240}{I}$$

b **1** Write the formula.

$$R = \frac{240}{I}$$

2 Substitute $I = 1.2$ into the formula.

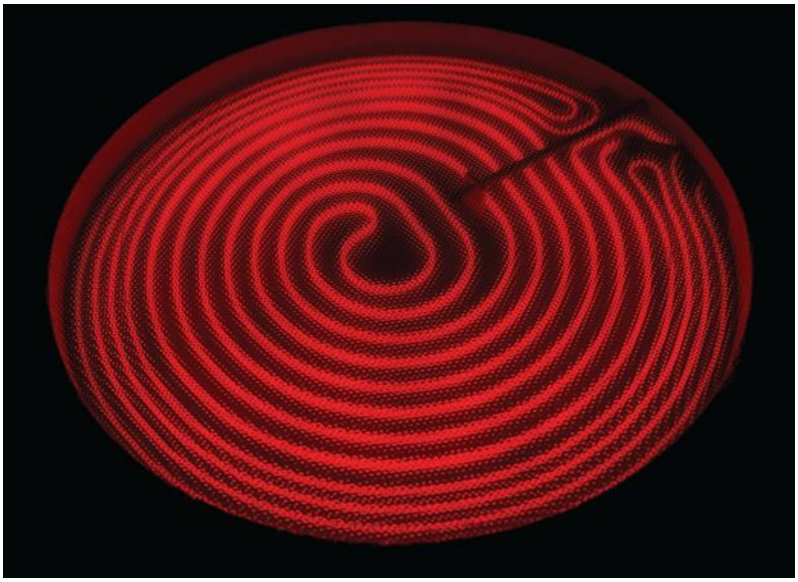
$$R = \frac{240}{1.2}$$

3 Evaluate.

$$R = 200$$

4 Write your answer.

A kettle that has a current of 1.2 amperes running through it has a resistance of 200 ohms.



Exercise 7C

SF

Example 9 **1** The cost of hiring a dance hall is given by the rule

$$C = 50t + 1200$$

where C is the total cost in dollars and t is the number of hours for which the hall is hired.

a Rearrange the equation to make t the subject.

b For how many hours could the dance hall be hired for a total cost of \$1450?

2 The distance, d km, travelled by a car in t hours at an average speed of v km/h is given by the formula

$$d = v \times t$$

a Transpose the formula to make t the subject.

b A car travels 275 km at 100 km/h, how long does the journey take? Give your answer in hours and minutes.

3 Taxi fares are calculated using the formula

$$F = 1.3K + 4$$

where K is the distance travelled in kilometres and F is the cost of the fare in dollars.

a Transpose the formula to make K the subject.

b The fare for a taxi ride was \$32.60, what distance was travelled?

Example 10 **4** The formula $S = 4\pi r^2$ can be used to calculate the surface area, S, of a sphere of radius r. Find, correct to the nearest whole number, the radius of a baseball that has a surface area of 452.4 cm^2.

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5 The circumference, C, of a circle of radius r can be calculated using the formula $C = 2\pi r$. Find the radius of a circular plate with a circumference 81.7 cm. Give your answer correct to the nearest whole number.

6 The area, A, of a circle of radius r can be calculated using the formula $A = \pi r^2$. A mysterious crop circle has an area of 725.83 m^2. Find the radius of this crop circle, correct to one decimal place.

7 The perimeter of a rectangle is given by $P = 2L + 2W$.

a Transpose this formula to make W the subject.

b Find the width of a rectangle of length 16 m which has a perimeter of 84 m.

Example 11

8 The formula for calculating simple interest is

$$I = \frac{Prt}{100}$$

where P is the principal (amount invested or borrowed), r is the interest rate per annum and t is the time (in years).

a Madeleine borrows \$8000 at a rate of 5.5% per annum and pays \$2200 in interest. How long was the loan for?

b Kyley invests an amount of money for 3 years at 2.3% per annum and earns \$372.60 in interest. How much did she invest?

c Samuel invests \$2000 for 4 years and earns \$240 in interest. What was the interest rate, per annum, for this investment?

9 In Rugby League football, a try, T, is worth 4 points and a conversion, C, is worth 2 points. If no penalties or field goals are scored in a game, the total number of points, P, is given by $P = 4T + 2C$. In one game, the Far North Salties scored a total of 44 points, which included 4 conversions and no penalties or field goals. How many tries did they score in this match?

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Example 12 **10** The current, I amperes, that flows in an electrical appliance depends on the resistance of the appliance, R ohms, where

$$I = \frac{240}{R}.$$

a Transpose the formula to make R the subject.

b If an oven has a current of 1.5 amperes running through it, what is the resistance of the oven?

11 A car travels a fixed distance of 260 km. A formula for finding the average speed of the car, s km/h, when it takes t hours to travel this distance is

$$s = \frac{260}{t}.$$

Find the time taken for a car to travel this distance at a speed of 100 km/h. Give your answer in hours and minutes.

12 The formula used to convert temperature from degrees Fahrenheit to degrees Celsius is:

$$C = \frac{5}{9}(F - 32)$$

A recipe requires a cake to be cooked at 180°C for 40 minutes. Andy's oven has a temperature scale marked in degrees Fahrenheit. At what temperature should he set his oven to cook this cake? Give your answer to the nearest 10 degrees.

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Key ideas and chapter summary

Algebraic expression	An **algebraic expression**, also called a formuala, is a mathematical relationship connecting two or more variables.
Substitution	**Substitution** involves replacing variables in a formula with values.
Transposition	**Transposition** involves making a different pronumeral the subject of a formula.
Linear equation	A **linear equation** is an equation whose unknown values are always to the power of 1.
Non-linear equation	A **non-linear equation** is one whose unknown values are *not* all to the power of 1.

Skills check

Having completed the current chapter you should be able to:

- substitute values in linear and non-linear expressions (formulas)
- transpose equations
- construct tables of values from given expressions (formulas).

Multiple-choice questions

1 If $a = 4$, then $3a + 5 =$

A 39 **B** 12 **C** 17 **D** 27

2 If $b = 1$, then $2b - 9 =$

A −11 **B** −7 **C** 12 **D** 21

3 If $C = 50t + 14$ and $t = 8$, then $C =$

A 522 **B** 1100 **C** 72 **D** 414

4 If $P = 2L + 2W$, then the value of P when $L = 6$ and $W = 2$ is:

A 48 **B** 16 **C** 12 **D** 30

SF

5 If $x = -2$, $y = 3$ and $z = 7$, then $\frac{z-x}{y} =$

A 3 **B** $\frac{5}{3}$ **C** $-\frac{5}{3}$ **D** -3

6 If $a = 2$, $b = 5$, $c = 6$ and $d = 10$, then $bd - ac =$

A 38 **B** 24 **C** 7 **D** 484

7 The area of a circle is given by $A = \pi r^2$. If $r = 6$ cm, then the area of the circle is:

A 18.84 cm^2 **B** 37.70 cm^2 **C** 35 531 cm^2 **D** 113.10 cm^2

8 The area of a trapezium is given by $A = \frac{(a+b)h}{2}$.
An expression for h is:

A $\frac{2A}{a+b}$ **B** $\frac{A-2}{a+b}$ **C** $\frac{A-(a+b)}{2}$ **D** $2A - (a+b)$

9 If the formula $R = 5S - P$ is transposed to make S the subject then:

A $S = \frac{R-P}{5}$ **B** $S = \frac{R+P}{5}$ **C** $S = \frac{R}{5} + P$
D $S = \frac{R}{5} - P$ **E** $S = \frac{P-R}{5}$

Short-answer questions

SF

1 If $P = 2l + 2b$, find P when:

a $l = 12$ and $b = 8$ **b** $l = 40$ and $b = 25$

2 If $A = \frac{1}{2}bh$, find A when:

a $b = 6$ and $h = 10$ **b** $b = 12$ and $h = 9$

3 The formula for finding the circumference of a circle is given by $C = 2\pi r$, where r is the radius. Find the circumference of a circle with radius 15 cm, correct to two decimal places.

4 The formula for the volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the circle cross-section of the cylinder and h is the height of the cylinder. Find the volume of a cylinder with a cross-section radius of 3 cm and a height of 15 cm. Round your answer to two decimal places.

5 Consider the equation $y = 33x - 56$.

a Construct a table of values for values of x in intervals of 5 from -20 to 25.
b For what value of x is $y = 274$?
c When $y = -221$, what value is x?

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6 For the equation $k = 2h + g$, construct a table of values for values of h and g in intervals of 1 from -2 to 2. SF

7 The curved surface area, S, of a cylinder of radius r and height h is given by the formula $S = 2\pi rh$. Transpose the formula to make r the subject and hence find the radius of a cylinder with surface area 225 cm^2 and height 20 cm. Give your answer correct to one decimal place.

Extended-response questions

1 The cost, C, of hiring a boat is given by $C = 8h + 25$ where h represents hours. SF

a What is the cost if the boat is hired for 4 hours?

b For how many hours was the boat hired if the cost was \$81?

2 A phone bill is calculated using the formula $C = 25 + 0.50n$, where n is the number of calls made.

a Complete the table of values below for values of n from 60 to 160.

n	60	70	80	90	100	110	120	130	140	150	160
C											

b What is the cost of making 160 phone calls?

3 A ticket to see a play costs \$89 per adult and \$42 per child. The total price, \$$P$, for a adult tickets and c child tickets is calculated using the formula $P = 89 \times a + 42 \times c$.

a Construct a table of values that shows the price of play tickets for values of a and c in intervals of 1 between 0 and 5.

b What is the total price of tickets to the play for 3 adults and 1 child?

4 An electrician charges \$80 up front and \$45 for each hour, h, that he works.

a Write a linear equation for the total charge, C, of any job.

b How much would a 3-hour job cost?

5 A model rocket is fired from a height of 1 metre above the ground. The height of the rocket, h metres, at time t seconds after it is fired is given by the equation $h = 1 + 20t - 5t^2$. CF

a Use a spreadsheet to complete the following table of values.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
h									

b At what time does the rocket reach its greatest height?

c What is the greatest height reached by the rocket?

d At what times is the rocket 16 metres above the ground?

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8

Matrices and matrix arithmetic

UNIT 2: APPLIED TRIGONOMETRY, ALGEBRA, MATRICES AND UNIVARIATE DATA

Topic 2: Algebra and matrices

- What is a matrix?
- How is the order of a matrix defined?
- How are the positions of the elements of a matrix specified?
- What are the rules for adding and subtracting matrices?
- How do we multiply a matrix by a scalar?
- What is the method for multiplying a matrix by another matrix?
- How is the power of a matrix evaluated?
- What are the properties of an identity matrix?
- How can communications be represented by matrices?

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Introduction

A **matrix** (plural matrices) is a rectangular group of numbers set out in rows and columns. Matrices can be used to store information such as databases, solve sets of simultaneous equations, find optimal solutions in business, analyse networks, transform shapes in geometry, encode information and devise the best strategies in game theory. We will explore some of these applications while learning the basic theory of matrices.

8A The basics of a matrix

A market stall operates on Friday and Saturday. Sales could be recorded using matrix A.

Matrix A

$$A = \begin{array}{r|ccc|l} & \textit{Shirts} & \textit{Jeans} & \textit{Belts} & \\ \textit{Friday} & 6 & 8 & 4 & \textit{row } 1 \\ \textit{Saturday} & 3 & 7 & 1 & \textit{row } 2 \\ & \textit{column } 1 & \textit{column } 2 & \textit{column } 3 & \end{array}$$

Rows	Columns
Friday sales are listed in row 1.	The number of shirts sold is listed in column 1.
Saturday sales are listed in row 2.	The number of pairs of jeans sold is listed in column 2.
	The number of belts sold is listed in column 3.

We can read the following information from the matrix:

- On Friday, 8 pairs of jeans were sold.
- On Saturday, 1 belt was sold.
- The total number of items sold on Friday was $6 + 8 + 4 = 18$.
- The total number of belts sold was $4 + 1 = 5$.

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Order of a matrix

The **order** (or size) **of a matrix** is written as:

number of rows × number of columns

$$\begin{array}{r} \textit{row } 1 \\ \textit{row } 2 \end{array}\begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix} \qquad \begin{array}{ccc} 6 & 8 & 4 \\ 3 & 7 & 1 \\ \textit{column } 1 & \textit{column } 2 & \textit{column } 3 \end{array}$$

Think: ‘rows in a cinema’

Think: ‘columns of the Parthenon’

The order of matrix A in the market stall example given earlier is 2×3; that is, 2 rows × 3 columns. It is called a ‘two by three’ matrix.

In writing down the order of a matrix, the number of rows is always given first, then the number of columns. *Rows first*, then *columns*.

Remember: When you walk into a cinema, you go to your *row first.*

Matrices are usually named using capital letters such as A, B, O.

Elements of a matrix

The numbers within a matrix are called its **elements**.

Locating an element in a matrix

a_{ij} is the element in *row i*, *column j*.

For example, in the matrix:

$$A = \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix}$$

- element a_{13} is in row 1, column 3 and its value is 4
- element a_{22} is in row 2, column 2 and its value is 7.

Example 1 Interpreting the elements of a matrix

Matrix B shows the number of boys and girls in years 10 to 12 at a particular school.

$$B = \begin{array}{c|cc|} & \textit{Boys} & \textit{Girls} \\ \textit{Year 10} & 57 & 63 \\ \textit{Year 11} & 48 & 54 \\ \textit{Year 12} & 39 & 45 \end{array}$$

a Give the order of matrix B.

b What information is given by the element $b_{1\,2}$?

c Which element gives the number of girls in Year 12?

d How many boys in total?

e How many students in Year 11?

Solution

a Count the rows, count the columns.
Remember: Order is rows × columns.

The order of matrix B is 3 × 2.

b The element $b_{1\,2}$ is in row 1 and column 2. This is where the Year 10 row meets the Girls column.

There are 63 girls in Year 10.

c Year 12 is row 3. Girls are column 2.

The number of Year 12 girls is given by $b_{3\,2}$.

d The sum of the Boys column gives the total number of boys.

The total number of boys is 144.

e The sum of the Year 11 row gives the total number of students in Year 11.

There are 102 students in Year 11.

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Row matrices

A **row matrix** has a *single row* of elements.

In matrix A, the Friday sales from the market stall can be represented by a 1 × 3 *row* matrix.

$$A = \begin{array}{c} \\ \textit{Friday} \\ \textit{Saturday} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Shirts} & \textit{Jeans} & \textit{Belts} \end{array} \\ \begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 1 \end{bmatrix} \end{array} \qquad \begin{array}{cccc} & \textit{Shirts} & \textit{Jeans} & \textit{Belts} \\ \textit{Friday} & 6 & 8 & 4 \end{array}$$

Column matrices

A **column matrix** has a *single column* of elements.

In matrix A, the sales of jeans from the market stall can be represented by a 2 × 1 *column* matrix.

$$\begin{array}{cc} & \textit{Jeans} \\ \textit{Friday} & 8 \\ \textit{Saturday} & 7 \end{array}$$

Although they appear to be very simple, row and column matrices have useful properties that will be explored in this chapter.

Square matrices

In **square matrices** the number of *rows* equals the number of *columns*.

Here are three examples.

$$[9] \qquad \begin{bmatrix} 5 & 4 \\ 4 & 2 \end{bmatrix} \qquad \begin{bmatrix} 0 & 4 & 3 \\ 8 & 1 & 6 \\ 2 & 0 & 7 \end{bmatrix}$$

1 × 1 2 × 2 3 × 3

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Exercise 8A

SF

Example 1 **1** Matrix C is shown on the right.

$$C = \begin{bmatrix} 2 & 4 & 16 & 7 \\ 6 & 8 & 9 & 3 \\ 5 & 6 & 10 & 1 \end{bmatrix}$$

a Write down the order of the matrix C.

b State the value of:

i c_{13} **ii** c_{24} **iii** c_{31}

c Find the sum of the elements in row 3.

d Find the sum of the elements in column 2.

2 For each of the following matrices:

i state the order **ii** find the values of the required elements.

a $A = \begin{bmatrix} 5 & 6 & 8 \\ 4 & 7 & 9 \end{bmatrix}$ Find a_{12} and a_{22}

b $B = \begin{bmatrix} 6 & 8 & 2 \end{bmatrix}$ Find b_{13} and b_{11}

c $C = \begin{bmatrix} 4 & 5 \\ 3 & 1 \\ 8 & -4 \end{bmatrix}$ Find c_{32} and c_{12}

d $D = \begin{bmatrix} 8 \\ 6 \\ 9 \end{bmatrix}$ Find d_{31} and d_{11}

e $E = \begin{bmatrix} 10 & 12 \\ 15 & 13 \end{bmatrix}$ Find e_{21} and e_{12}

f $F = \begin{bmatrix} 8 & 11 & 2 & 6 \\ 4 & 1 & 5 & 7 \\ 6 & 14 & 17 & 20 \end{bmatrix}$ Find f_{34} and f_{23}

3 Name the matrices in Question **2** that are:

a row matrices **b** column matrices **c** square matrices.

4 For matrix D on the right, give the values of the following elements.

$$D = \begin{bmatrix} 3 & 4 & 6 & 11 & 2 \\ 5 & 1 & 9 & 10 & 4 \\ 8 & 7 & 2 & 0 & 1 \\ 6 & 8 & 5 & 8 & 2 \end{bmatrix}$$

a d_{23} **b** d_{45} **c** d_{11}

d d_{24} **e** d_{42}

5 Use technology to enter matrix D from Question **4**, then use the technology to find the required elements.

CF

6 Students were asked which of four sports they preferred to play and the results were entered in the following matrix.

$$S = \begin{array}{c} \\ \textit{Year 10} \\ \textit{Year 11} \\ \textit{Year 12} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Tennis} & \textit{Basketball} & \textit{Football} & \textit{Hockey} \end{array} \\ \begin{bmatrix} 19 & 18 & 31 & 14 \\ 16 & 32 & 22 & 12 \\ 21 & 25 & 5 & 7 \end{bmatrix} \end{array}$$

a How many Year 11 students preferred to play basketball?

b Write down the order of matrix S.

c What information is given by s_{23}?

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7 $A = \begin{bmatrix} 2 & 7 & -1 \\ 0 & 5 & 3 \\ -3 & 4 & 8 \\ 7 & 6 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$ $C = \begin{bmatrix} 8 & -2 \end{bmatrix}$ $D = \begin{bmatrix} 4 & -3 & 0 & 1 & 9 \\ 6 & 11 & 2 & 7 & 5 \end{bmatrix}$

a Write down the order of each matrix A, B, C and D.

b Identify the elements $a_{32}, b_{21}, c_{11}$and d_{24} of matrices A, B, C and D respectively.

8 Matrix F shows the number of hectares of land used for different purposes on farms X and Y.

Row 1 represents Farm X and row 2 represents Farm Y. Columns 1, 2 and 3 show the amount of land used for wheat, cattle and sheep (W, C, S) respectively, in hectares.

$$F = \begin{array}{cc} & \begin{array}{ccc} W & C & S \end{array} \\ & \begin{bmatrix} 150 & 300 & 75 \\ 200 & 0 & 350 \end{bmatrix} \begin{array}{c} X \\ Y \end{array} \end{array}$$

a How many hectares are used on:

- **i** Farm X for sheep?
- **ii** Farm X for cattle?
- **iii** Farm Y for wheat?

b Calculate the total number of hectares used on the two farms for wheat.

c Write down the information that is given by these elements.

i f_{22} **ii** f_{13} **iii** f_{11}

d Which element of matrix F gives the number of hectares used:

- **i** on Farm Y for sheep?
- **ii** on Farm X for cattle?
- **iii** on Farm Y for wheat?

e State the order of matrix F.

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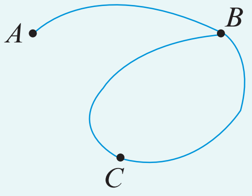
8B Using matrices to model (represent) practical situations

A **network** diagram is made of points (vertices) joined by lines (edges). It can be used to show **connections** or relationships. The information in network diagrams can be recorded in a matrix and used to solve related problems.

Example 2 Using a matrix to represent connections

The network diagram drawn shows the ways to travel between three towns, A, B and C.



a Use a matrix to represent the connections. Each element should describe the number of ways to travel *directly* from one town to another.

b What information is given by the sum of the second column of the matrix?

Solution

a As there are three towns, A, B and C, use a 3×3 matrix to show the direct connections.

There are 0 roads directly connecting any town to itself. So enter 0 where column A crosses row A, and so on.

$$\begin{array}{ccc} & & \text{TO} \\ & & \begin{array}{ccc} A & B & C \end{array} \\ \text{From} & \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & & \\ & 0 & \\ & & 0 \end{bmatrix} \end{array}$$

If there was a road directly connecting town A to itself, it would be a loop from A back to A.

There is one road directly connecting B to A (or A to B). So enter 1 where column B crosses row A and where column A crosses row B.

$$\begin{array}{ccc} & & \text{TO} \\ & & \begin{array}{ccc} A & B & C \end{array} \\ \text{From} & \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{bmatrix} \end{array}$$

There are no direct roads between C and A. So enter 0 where column C crosses row A and where column A crosses row C.

$$\begin{array}{ccc} & & \text{TO} \\ & & \begin{array}{ccc} A & B & C \end{array} \\ \text{From} & \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & \\ 0 & & 0 \end{bmatrix} \end{array}$$

There are 2 roads between C and B. Enter 2 where column C crosses row B and where column B crosses row C.

$$\begin{array}{ccc} & & \text{TO} \\ & & \begin{array}{ccc} A & B & C \end{array} \\ \text{From} & \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \end{array}$$

b The second column shows the number of roads directly connected to town B.

The sum of the second column is the total number of roads directly connected to town B.

$1 + 0 + 2 = 3$

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Exercise 8B

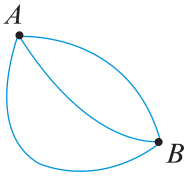
SF

Example 2

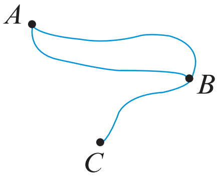
1 The networks show roads connecting towns.

a In each case use a matrix to record the number of ways of travelling *directly* from one town to another.

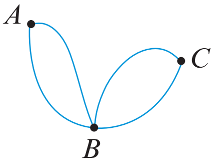
i



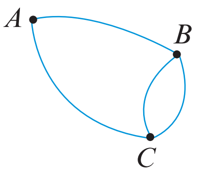
ii



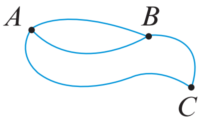
iii



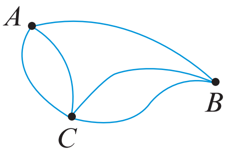
iv



v



vi



b What does the sum of the second column of each matrix represent?

2 The matrices below record the number of ways of going directly from one town to another.

a In each case draw graphs to show the direct connections between towns A, B and C.

i

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

ii

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

iii

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \end{array}$$

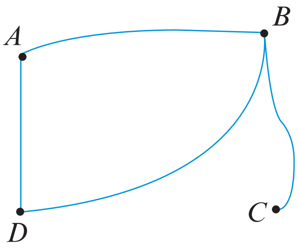
iv

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \end{array}$$

b State the information that is given by the sum of the first column in the matrices of part **a**.

CF

3 This network diagram has lines showing who of the four people A, B, C and D have met.



a Represent the graph using a matrix. Use 0 when two people have *not* met and 1 when they have met.

b How can the matrix be used to tell who has met the most people?

c Who has met the most people?

d Who has met the least number of people?

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8C Adding and subtracting matrices

Skillsheet

Rules for adding and subtracting matrices

1. Matrices are added by adding the elements that are in the same positions.
2. Matrices are subtracted by subtracting the elements that are in the same positions.
3. **Matrix addition and subtraction** can only be done if the two matrices have the *same order*.

Example 3 Adding and subtracting of matrices

Complete the following addition and subtraction of matrices.

a $\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 9 & -1 \end{bmatrix}$

b $\begin{bmatrix} 7 & 3 \\ 2 & 8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 9 \\ 3 & 7 \end{bmatrix}$

Solution

a **1** Write the addition.

$$\begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 9 & -1 \end{bmatrix}$$

2 Add the elements that are in the same positions.

$$= \begin{bmatrix} 2+9 & 4+8 \\ 5+9 & 1+(-1) \end{bmatrix}$$

3 Evaluate each element.

$$= \begin{bmatrix} 11 & 12 \\ 14 & 0 \end{bmatrix}$$

b **1** Write the subtraction.

$$\begin{bmatrix} 7 & 3 \\ 2 & 8 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 9 \\ 3 & 7 \end{bmatrix}$$

2 Subtract the elements that are in the same positions.

$$= \begin{bmatrix} 7-4 & 3-2 \\ 2-(-1) & 8-9 \\ 1-3 & 0-7 \end{bmatrix}$$

3 Evaluate each element.

$$= \begin{bmatrix} 3 & 1 \\ 3 & -1 \\ -2 & -7 \end{bmatrix}$$

▶ The zero matrix, 0

In a **zero matrix** every element is zero.

The following are examples of zero matrices.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Just as in arithmetic with ordinary numbers, adding or subtracting a zero matrix does not make any change to the original matrix. For example:

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Also, subtracting any matrix from itself gives a zero matrix. For example:

$$\begin{bmatrix} 9 & 4 & 8 \end{bmatrix} - \begin{bmatrix} 9 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

Exercise 8C

SF

Example 3

1 Calculate the sums or differences of the following matrices.

a $\begin{bmatrix} 4 & 3 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 6 & 1 \end{bmatrix}$

b $\begin{bmatrix} 8 & 6 \\ 9 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 0 \end{bmatrix}$

c $\begin{bmatrix} 3 & 5 \\ 7 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

d $\begin{bmatrix} 9 \\ 8 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

e $\begin{bmatrix} 8 & 6 \\ 2 & 9 \end{bmatrix} - \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$

f $\begin{bmatrix} 7 & 4 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 2 & -8 \end{bmatrix}$

g $\begin{bmatrix} 4 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 5 \end{bmatrix}$

h $\begin{bmatrix} 7 & -5 \end{bmatrix} - \begin{bmatrix} 7 & -5 \end{bmatrix}$

i $\begin{bmatrix} 4 & -3 \end{bmatrix} + \begin{bmatrix} -4 & 3 \end{bmatrix}$

j $\begin{bmatrix} 4 & -3 & 2 & -1 \end{bmatrix} - \begin{bmatrix} 6 & -5 & -1 & 8 \end{bmatrix}$

2 Consider the matrices below:

$$A = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 7 \\ 1 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 2 \\ 1 & 0 \\ 3 & -8 \end{bmatrix} \quad D = \begin{bmatrix} -3 & 5 \\ 4 & -2 \\ 1 & 7 \end{bmatrix} \quad E = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$$

Find, where possible:

a $A + B$ **b** $B + A$ **c** $A - B$ **d** $B - A$

e $B + E$ **f** $C + D$ **g** $B + C$ **h** $D - C$

SF

3 Two people shared the work of a telephone poll surveying voting intentions. The results for each person's survey are given in matrix form.

Sample 1:

$$\begin{array}{c} \\ \textit{Men} \\ \textit{Women} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Liberal} & \textit{Labor} & \textit{Democrat} & \textit{Green} \end{array} \\ \begin{bmatrix} 19 & 21 & 7 & 3 \\ 18 & 17 & 11 & 4 \end{bmatrix} \end{array}$$

Sample 2:

$$\begin{array}{c} \\ \textit{Men} \\ \textit{Women} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Liberal} & \textit{Labor} & \textit{Democrat} & \textit{Green} \end{array} \\ \begin{bmatrix} 24 & 21 & 3 & 2 \\ 19 & 20 & 6 & 5 \end{bmatrix} \end{array}$$

Write a matrix showing the overall result of the survey.

CF

4 The weights and heights of four people were recorded and then checked again one year later.

2004 results:

$$\begin{array}{c} \\ \textit{Weight}\text{ (kg)} \\ \textit{Height}\text{ (cm)} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Aida} & \textit{Bianca} & \textit{Chloe} & \textit{Donna} \end{array} \\ \begin{bmatrix} 32 & 44 & 59 & 56 \\ 145 & 155 & 160 & 164 \end{bmatrix} \end{array}$$

2005 results:

$$\begin{array}{c} \\ \textit{Weight}\text{ (kg)} \\ \textit{Height}\text{ (cm)} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Aida} & \textit{Bianca} & \textit{Chloe} & \textit{Donna} \end{array} \\ \begin{bmatrix} 38 & 52 & 57 & 63 \\ 150 & 163 & 167 & 170 \end{bmatrix} \end{array}$$

a Write the matrix that gives the changes in each person's weight and height after one year.

b Who gained the most weight?

c Which person had the greatest increase in height?

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8D Scalar multiplication

A *scalar* is just a number. Multiplying a matrix by a number is called **scalar multiplication**.

Multiplying a matrix by a scalar

Scalar multiplication is the process of multiplying a matrix by a number (a scalar).

In scalar multiplication each element is multiplied by that scalar (number).

The following is an example of scalar multiplication of a matrix.

$$5\begin{bmatrix}1 & 2\\2 & 0\end{bmatrix} = \begin{bmatrix}5\times 1 & 5\times 2\\5\times 2 & 5\times 0\end{bmatrix} = \begin{bmatrix}5 & 10\\10 & 0\end{bmatrix}$$

Example 4 Scalar multiplication

If $A = \begin{bmatrix}5 & 1\\-3 & 0\end{bmatrix}$, find $3A$.

Solution

1 If $A = \begin{bmatrix}5 & 1\\-3 & 0\end{bmatrix}$, then $3A = 3\begin{bmatrix}5 & 1\\-3 & 0\end{bmatrix}$.

$$3A = 3\begin{bmatrix}5 & 1\\-3 & 0\end{bmatrix}$$

2 Multiply each number in the matrix by 3.

$$= \begin{bmatrix}3\times 5 & 3\times 1\\3\times -3 & 3\times 0\end{bmatrix}$$

3 Evaluate each element.

$$= \begin{bmatrix}15 & 3\\-9 & 0\end{bmatrix}$$

Scalar multiplication has many practical applications. It is particularly useful in scaling up the elements of a matrix; for example, add the GST to the cost of the prices of all items in a shop by multiplying a matrix of prices by 1.1.

Example 5 Application of scalar multiplication

A gymnasium has enrolments in the courses shown in this matrix.

$$\begin{array}{c} \\ Men \\ Women \end{array}\begin{array}{c} \begin{array}{ccc} Body\ building & Aerobics & Fitness \end{array} \\ \begin{bmatrix} 70 & 20 & 80 \\ 10 & 50 & 60 \end{bmatrix} \end{array}$$

The gym has a membership drive and the enrolments in each course double. Show this in a matrix.

Solution

1 Each element in the matrix is multiplied by 2.

$$2 \times \begin{bmatrix} 70 & 20 & 80 \\ 10 & 50 & 60 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 70 & 2 \times 20 & 2 \times 80 \\ 2 \times 10 & 2 \times 50 & 2 \times 60 \end{bmatrix}$$

2 Evaluate each element.

$$= \begin{array}{c} \\ Men \\ Women \end{array}\begin{array}{c} \begin{array}{ccc} Body\ building & Aerobics & Fitness \end{array} \\ \begin{bmatrix} 140 & 40 & 160 \\ 20 & 100 & 120 \end{bmatrix} \end{array}$$

Scalar multiplication can also be used in conjunction with addition and subtraction of matrices.

Example 6 Scalar multiplication and subtraction of matrices

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find the matrix equal to $2A - 3B$.

Solution

1 Write $2A - 3B$ in expanded matrix form.

$$2A - 3B = 2 \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - 3 \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2 Multiply the elements in A by 2 and the elements in B by 3.

$$= \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$$

3 Subtract the elements in corresponding positions.

$$= \begin{bmatrix} 2 - 0 & 2 - 3 \\ 0 - 3 & 2 - 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$

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Exercise 8D

SF

1 Calculate the values of the following.

Example 4

a $2\begin{bmatrix} 7 & -1 \\ 4 & 9 \end{bmatrix}$ **b** $5\begin{bmatrix} 0 & -2 \\ 5 & 7 \end{bmatrix}$ **c** $-4\begin{bmatrix} 16 & -3 \\ 1.5 & 3.5 \end{bmatrix}$ **d** $1.5\begin{bmatrix} 1.5 & 0 \\ -2 & 5 \end{bmatrix}$

e $3\begin{bmatrix} 6 & 7 \end{bmatrix}$ **f** $6\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ **g** $\frac{1}{2}\begin{bmatrix} 4 & 6 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ **h** $-1\begin{bmatrix} 3 & 6 & -8 \end{bmatrix}$

Example 6

2 Consider the following matrices:

$$A = \begin{bmatrix} 3 & -4 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 6 \\ 1 & -4 \end{bmatrix} \quad C = \begin{bmatrix} -3 & 4 \\ -2 & -5 \end{bmatrix} \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Find the matrix required for:

a $3A$ **b** $2B + 4C$ **c** $5A - 2B$ **d** $2O$ **e** $3B + O$

3 Enter the matrices $A = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 0 & 5 \end{bmatrix}$ into your graphics calculator and use them to evaluate:

a $17A - 14B$ **b** $29B - 21A$ **c** $9A + 7B$ **d** $3(5A - 4B)$

CF

4 Consider the following matrices:

$$A = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}$$

Find the matrix for:

a $3A + 4B$ **b** $5C - 2D$ **c** $2(3A + 4B)$ **d** $3(5C - 2D)$

CU

Example 5

5 The expenses arising from costs and wages for each section of three stores, A, B and C, are shown in the Costs matrix. The Sales matrix shows the money from the sale of goods in each section of the three stores. Figures represent the nearest million dollars.

Costs:

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{ccc} \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \\ \left[\begin{array}{c} 12 \\ 11 \\ 15 \end{array}\right. & \begin{array}{c} 10 \\ 8 \\ 14 \end{array} & \left.\begin{array}{c} 15 \\ 17 \\ 7 \end{array}\right] \end{array}$$

Sales:

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{ccc} \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \\ \left[\begin{array}{c} 18 \\ 16 \\ 19 \end{array}\right. & \begin{array}{c} 12 \\ 9 \\ 13 \end{array} & \left.\begin{array}{c} 24 \\ 26 \\ 12 \end{array}\right] \end{array}$$

a Write a matrix showing the profits in each section of each store.

b If 30% tax must be paid on profits, show the amount of tax that must be paid by each section of each store. No tax needs to be paid for a section that has made a loss.

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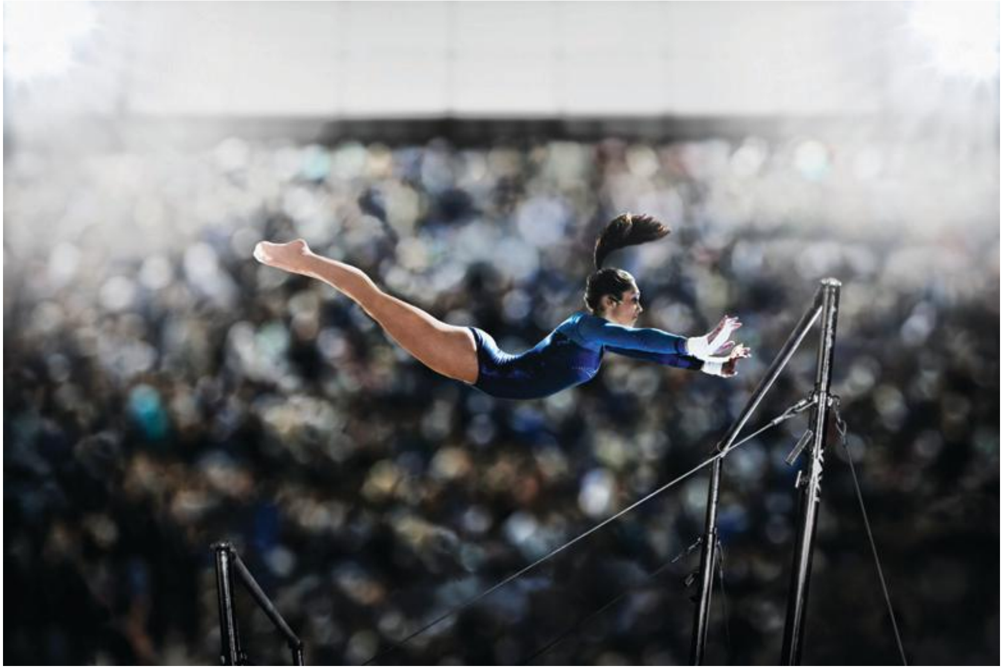
6 Zoe competed in the gymnastics rings and parallel bars events in a three-day gymnastics tournament. A win was recorded as 1 and a loss as 0. The three column matrices show the results for Saturday, Sunday and Monday.

$$\begin{array}{lccc} & \textit{Sat} & \textit{Sun} & \textit{Mon} \\ \textit{Gymnastics rings} & \left[\begin{array}{c}1\\0\end{array}\right] & \left[\begin{array}{c}1\\1\end{array}\right] & \left[\begin{array}{c}1\\1\end{array}\right] \\ \textit{Parallel bars} & & & \end{array}$$

a Give a 2 × 1 column matrix which records her total wins for each of the two types of events.

b Zoe received \$50 for each win. Give a 2 × 1 matrix that records her total prize money for each of the two types of events.



8E Matrix multiplication and power of a matrix

Matrix multiplication is the multiplication of a matrix by another matrix. It is not to be confused with scalar multiplication, which is the multiplication of a matrix by a number.

The matrix multiplication of matrices A and B can be written as $A \times B$ or just AB. Although it is called multiplication and the symbol × may be used, matrix multiplication is not the simple multiplication of numbers but a routine involving the sum of pairs of numbers that have been multiplied.

For example, the method of matrix multiplication can be demonstrated by using a practical example. The numbers of DVDs and computer games sold by Fatima and Gaia are recorded in matrix N. The selling prices of the DVDs and computer games are shown in matrix P.

$$N = \begin{array}{c} \\ Fatima \\ Gaia \end{array} \begin{array}{cc} DVDs & Games \\ \left[\begin{array}{c} 7 \\ 5 \end{array}\right. & \left.\begin{array}{c} 4 \\ 6 \end{array}\right] \end{array} \qquad P = \begin{array}{c} \\ DVDs \\ Games \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 20 \\ 30 \end{bmatrix} \end{array}$$

We want to make a matrix, S, that shows the value of the sales made by each person.

Fatima sold: **7** DVDs at **$20** + **4** Games at **$30**.

Gaia sold: **5** DVDs at **$20** + **6** Games at **$30**.

$$S = \begin{array}{c} \\ Fatima \\ Gaia \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 7 \times 20 + 4 \times 30 \\ 5 \times 20 + 6 \times 30 \end{bmatrix} \end{array}$$

The steps used in this example follow the routine for the matrix multiplication of $N \times P$.

As we move across the *first row* of matrix N we move down the *column* of matrix P, adding the products of the pairs of numbers as we go.

Then we move across the *second row* of matrix N and down the *column* of matrix P, adding the products of the pairs of numbers as we go.

$$N \times P$$

$$\begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 7 \times 20 + 4 \times 30 \\ \end{bmatrix}$$

$$\begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 7 \times 20 + 4 \times 30 \\ 5 \times 20 + 6 \times 30 \end{bmatrix}$$

$$= \begin{bmatrix} 140 + 120 \\ 100 + 180 \end{bmatrix}$$

$$= \begin{array}{c} \$ \\ \begin{bmatrix} 260 \\ 280 \end{bmatrix} \end{array} \begin{array}{c} \\ Fatima \\ Gaia \end{array}$$

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Rules for matrix multiplication

Because of the way the products are formed, the number of columns in the first matrix must equal the number of rows in the second matrix. Otherwise, we say that matrix multiplication is not defined, meaning it is not possible.

Matrix multiplication

For matrix multiplication to be defined:

Think of the orders as two railway carriages that must be the same where they meet.

order of 1st matrix order of 2nd matrix

$m \times n$ $n \times p$

must be the same

In our example of the sales of DVDs and computer games:

order of 1st matrix order of 2nd matrix

2×2 2×1

the same

Notice that the outside numbers give the order of the product matrix: the matrix made by multiplying the two matrices. In our case, the answer is a 2 × 1 matrix.

Order of the product matrix

The order of the product matrix is given by:

Think: when the ‘railway carriages’ meet, the result has an order given by the end numbers.

order of 1st matrix order of 2nd matrix

$m \times n$ $n \times p$

order of answer

$m \times p$

We will check that these two important rules hold in the examples that follow.

Methods of matrix multiplication

Some people like to think of the matrix multiplication of $A \times B$ using a *run and dive* description. This procedure can be very tedious and error prone, so we will only do simple cases by hand so that you understand the process.

Matrix multiplication of $A \times B$

The *run and dive* description of matrix multiplication is to add the products of the pairs made as you:

- *run* along the first row of A and *dive* down the first column of B
- repeat running along the first row of A and diving down the next column of B until all columns of B have been used
- now start running along the next row of A and repeat diving down each column of B, entering your results in a new row
- repeat this routine until all rows of A have been used.

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Example 7 Matrix multiplication

Consider the following matrices:

$$A = \begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 9 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 4 & 7 \end{bmatrix} \quad D = \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}$$

- Decide whether the matrix multiplication in each question below is defined.
- If matrix multiplication is defined, give the order of the answer matrix and then do the matrix multiplication.

a AB **b** BA **c** CD

Solution

a AB

1 Write the order of each matrix.

$$\begin{array}{cc} A & B \\ 3 \times 2 & 2 \times 1 \end{array}$$

2 The inside numbers are the same.

Matrix multiplication is defined for $A \times B$.

3 The outside numbers give the order of $A \times B$.

The order of the product AB is 3×1.

4 Move across the first row of A and down the column of B, adding the products of the pairs.

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ \\ \\ \end{bmatrix}$$

5 Move across the second row of A and down the column of B, adding the products of the pairs.

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ 4 \times 8 + 6 \times 9 \\ \\ \end{bmatrix}$$

6 Move across the third row of A and down the column of B, adding the products of the pairs.

$$\begin{bmatrix} 5 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 8 + 2 \times 9 \\ 4 \times 8 + 6 \times 9 \\ 1 \times 8 + 3 \times 9 \end{bmatrix}$$

7 Tidy up by doing some arithmetic.

$$= \begin{bmatrix} 40 + 18 \\ 32 + 54 \\ 8 + 27 \end{bmatrix}$$

8 Write your answer.

$$\text{So } A \times B = \begin{bmatrix} 58 \\ 86 \\ 35 \end{bmatrix}$$

b BA

1 Write the order of each matrix.

$$\begin{array}{cc} B & A \\ 2 \times 1 & 3 \times 2 \end{array}$$

2 Are the inside numbers the same? No.

Multiplication is not defined for $B \times A$.

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c For CD where $C = \begin{bmatrix} 2 & 4 & 7 \end{bmatrix}$ and $D = \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}$:

1 Write the order of each matrix.

$\begin{array}{cc} C & D \\ 1 \times 3 & 3 \times 1 \end{array}$

2 Are the inside numbers the same? Yes.

Multiplication is defined for $C \times D$.

3 The outside numbers give the order of $C \times D$.

The order of the product CD is 1×1.

4 Move across the row of C and down the column of D, adding the products of the pairs.

$$\begin{bmatrix} 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \times 8 + 4 \times 6 + 7 \times 5 \end{bmatrix}$$

5 Tidy up by doing some arithmetic.

$$= \begin{bmatrix} 16 + 24 + 35 \end{bmatrix}$$

6 Write your answer.

So $C \times D = \begin{bmatrix} 75 \end{bmatrix}$

In the previous example, $AB \neq BA$. Usually, when we reverse (*commute*) the order of the matrices in matrix multiplication, we get a different answer. This differs from ordinary arithmetic, where multiplication gives the same answer when the terms are commuted; for example, $3 \times 4 = 4 \times 3$.

Matrix multiplication

In general, matrix multiplication is not commutative: $AB \neq BA$

Spreadsheet

Spreadsheet activity 8E: Matrix operations

Activities for demonstration of matrix operations using graphics calculators are available in the teacher resources to use for demonstrations.

▶ Identity matrix

In ordinary arithmetic, the number 1 is called the *multiplicative identity element*. When a number is multiplied by 1, the answer is always *identical* to the original number. Is there a matrix that can multiply any matrix and give an answer identical to the original matrix? Consider the following example.

Example 8 The identity matrix

$$A = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

a Find AI.

b Find IA.

Solution

a AI

1 Write the order of each matrix. The inside numbers are the same, so matrix multiplication is defined. The outside numbers tell us that the answer is a 2×2 matrix.

$$\overset{A}{2 \times 2} \quad \overset{I}{2 \times 2}$$

2 Do the matrix multiplication by hand or using your calculator.

$$A \times I = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \times 1 + 2 \times 0 & 5 \times 0 + 2 \times 1 \\ 8 \times 1 + 3 \times 0 & 8 \times 0 + 3 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$$

b IA

1 Write the order of each matrix. The inside numbers are the same, so matrix multiplication is defined. The outside numbers tell us that the answer is a 2×2 matrix.

$$\overset{I}{2 \times 2} \quad \overset{A}{2 \times 2}$$

2 Do the matrix multiplication by hand or using your calculator.

$$I \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 5 + 0 \times 8 & 1 \times 2 + 0 \times 3 \\ 0 \times 5 + 1 \times 8 & 0 \times 2 + 1 \times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$$

Identity matrix for 2 × 2 matrices

The 2×2 identity matrix is $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and is the only matrix that has the property $AI = A = IA$ for any 2×2 matrix A.

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The **identity matrix**, I, also has the special property that it is *commutative* in matrix multiplication. When I is one of the matrices in the multiplication and the other is a square matrix of the same order, the answer is the same when the order of the matrices is commuted (reversed).

In Example 8: $AI = IA = \begin{bmatrix} 5 & 2 \\ 8 & 3 \end{bmatrix}$

Remember that matrix multiplication is not usually commutative.

The *identity matrix for any square matrix* is a square matrix of the same order with 1s along the *leading diagonal* (from the top left to the bottom right) and 0s in all the other positions.

$$[1] \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Power of a matrix

The **power of a matrix** has the same meaning as a power in arithmetic or algebra.

For example:

Arithmetic $7^3 = 7 \times 7 \times 7$

Algebra $x^3 = x \times x \times x$

Matrices $A^3 = A \times A \times A$

$$\begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}^3 = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

The power of a matrix, A^n

$$A^n = A \times A \times A \times \ldots \times A$$

A occurs n times in the expanded multiplication.

Only square matrices can be raised to a power of 2 or more.

Repeated multiplication involving higher powers or large matrices is time consuming and error prone. Technology such as matrix multiplication apps, spreadsheets or graphics calculators should be used.

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Example 9 Finding powers of a matrix

Consider matrix A. $A = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$

Without using technology find:

a A^2

b A^3

c Apply technology to matrix A to find A^3

Solution

a **1** Write the expanded form of A^2.

$$A^2 = A \times A = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$$

2 Do matrix multiplication.

$$= \begin{bmatrix} 9 & 0 \\ 8 & 1 \end{bmatrix}$$

b **1** Expand as $A^3 = A \times A^2$

$$A^3 = A \times A^2$$

2 Use A^2 from part **a**.

$$= \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 9 & 0 \\ 8 & 1 \end{bmatrix}$$

3 Multiply the matrices.

$$= \begin{bmatrix} 27 & 0 \\ 26 & 1 \end{bmatrix}$$

c **1** Enter the values into a matrix and store as matrix a.

$$\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \rightarrow a \qquad \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$$

2 Apply the power of 3 to matrix a.

$$a^3 \qquad \begin{bmatrix} 27 & 0 \\ 26 & 1 \end{bmatrix}$$

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Exercise 8E

Matrix multiplication

Example 7

1 Consider the following matrices: $A = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 3 \\ 0 & 8 \\ 2 & -5 \end{bmatrix}$

SF

- Decide whether the matrix multiplication in each question below is defined.
- If matrix multiplication is defined, give the order of the answer matrix and then perform the matrix multiplication.

a AB **b** BA **c** CB **d** BC
e AA **f** BB **g** AC **h** CA

2 Write the orders of each pair of matrices and decide if matrix multiplication is defined. If matrix multiplication is defined, find the answer.

a $\begin{bmatrix} 3 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ **c** $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

d $\begin{bmatrix} 8 & -2 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ **e** $\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ **f** $\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

3 Consider these matrices: $A = \begin{bmatrix} 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

Find:

a **i** $3A$ **ii** $5A$ **iii** $8A$ **iv** $3A + 5A$
b **i** $6B$ **ii** $6B + B$ **iii** $7B$
c **i** $2A$ **ii** $3B$ **iii** AB **iv** $2A \times 3B$ **v** $6AB$

4 Perform these matrix multiplications without using technology.

a $\begin{bmatrix} 4 & 2 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ **b** $\begin{bmatrix} 8 & 4 \\ 5 & 9 \end{bmatrix}\begin{bmatrix} 7 \\ 2 \end{bmatrix}$ **c** $\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 6 & 7 \end{bmatrix}$

d $\begin{bmatrix} 3 & 5 \\ 1 & 8 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -2 & -3 \end{bmatrix}$ **e** $\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **f** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 9 & 6 \\ 7 & 8 \end{bmatrix}$

g $\begin{bmatrix} 2 & 5 \\ 4 & 1 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \end{bmatrix}$ **h** $\begin{bmatrix} 7 & 4 \\ 1 & 0 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ **i** $\begin{bmatrix} 5 & 8 \end{bmatrix}\begin{bmatrix} 7 \\ 6 \end{bmatrix}$

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j $\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 9 \end{bmatrix}$ **k** $\begin{bmatrix} 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}$ **l** $\begin{bmatrix} 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$

m $\begin{bmatrix} 6 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ **n** $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \\ 8 \end{bmatrix}$ **o** $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$

5 Use technology to repeat the matrix multiplications in Question **4**.

6 $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

a Find AB. **b** Find BA. **c** Does $AB = BA$?

7 Use these matrices to find the required products.

$$C = \begin{bmatrix} 9 & 8 \\ 7 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 8 & 6 \\ 4 & 2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

a CD **b** CE **c** CF **d** DE **e** DF

8 Perform the following matrix multiplications using technology.

a $\begin{bmatrix} 6 & 8 & 12 \\ 14 & 17 & 11 \end{bmatrix}\begin{bmatrix} 26 & 9 & 21 & 6 \\ 8 & -7 & -4 & 9 \\ 13 & 10 & 5 & 26 \end{bmatrix}$ **b** $\begin{bmatrix} 15 & 9 & 23 & 72 \end{bmatrix}\begin{bmatrix} -6 \\ 22 \\ -8 \\ 19 \end{bmatrix}$

c $\begin{bmatrix} 16 \\ 10 \\ 24 \\ -18 \end{bmatrix}\begin{bmatrix} -31 & 47 & 61 & -14 \end{bmatrix}$ **d** $\begin{bmatrix} 8 & -7 & 9 \\ 6 & 11 & 14 \\ 3 & 21 & -5 \end{bmatrix}\begin{bmatrix} 8 & -19 & 24 \\ 33 & 16 & 19 \\ 4 & 0 & 13 \end{bmatrix}$

Identity matrices

Example 8

9 Which of these matrices is the 2×2 identity matrix?

A $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ **B** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **C** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

D $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **E** $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

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10 Find the required products.

a $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}7 & 3\\5 & 6\end{bmatrix}$ **b** $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}7 & 3\\5 & 6\end{bmatrix}$ **c** $\begin{bmatrix}4 & 8\\11 & -3\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

d $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}4 & 8\\11 & -3\end{bmatrix}$ **e** $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}3 & 1 & 4\\6 & 9 & 2\\8 & 0 & 7\end{bmatrix}$

Power of a matrix

11 Noting that $A^2 = A \times A$, $A^3 = A \times A \times A$, etc., calculate using technology:

i A^2 **ii** A^3 **iii** A^4

for each of the following matrices.

a $A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ **b** $A = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ **c** $A = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

d $A = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ **e** $A = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

Example 9 **12** Consider $A = \begin{bmatrix}4 & 7\\0 & 6\end{bmatrix}$

a Without using technology find:

i A^2 **ii** A^3

b Using technology find A^3.

13 Consider $B = \begin{bmatrix}2 & -1\\1 & 0\end{bmatrix}$

Without using technology find:

a B^2 **b** B^3 **c** B^4 **d** $B^2 \times B^2$

14 Investigating *The Curious Case of* A^0.

$$A = \begin{bmatrix}4 & 3\\1 & 2\end{bmatrix}$$

Technology can be used to find powers of any matrix A, such as A^2, A^3 and A^4.

a Use technology to find A^0.

b Find $A^0 \times A$.

c What is special about A^0?

d Make up another 2×2 matrix, B, and find B^0.

e Find $B^0 \times B$.

f What is a another name for A^0?

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8F Problem-solving and modelling with matrices

Data represented in matrix form can be multiplied to produce new useful information.

Example 10 Business application of matrices

Fatima and Gaia’s store has a special sales promotion. One free cinema ticket is given with each DVD purchased. Two cinema tickets are given with the purchase of each computer game.

The number of DVDs and games sold by Fatima and Gaia are given in matrix S.

The selling price of a DVD and a game, together with the number of free tickets is given by matrix P.

$$S = \begin{array}{r} \\ \textit{Fatima} \\ \textit{Gaia} \end{array}\begin{array}{c} \begin{array}{cc} \textit{DVDs} & \textit{Games} \end{array} \\ \begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix} \end{array} \qquad P = \begin{array}{r} \\ \textit{DVDs} \\ \textit{Games} \end{array}\begin{array}{c} \begin{array}{cc} \$ & \textit{Tickets} \end{array} \\ \begin{bmatrix} 20 & 1 \\ 30 & 2 \end{bmatrix} \end{array}$$

Find the matrix product $S \times P$ and interpret.

Solution

1 Complete the matrix multiplication $S \times P$.

$$\begin{array}{r} \\ \text{Fatima} \\ \text{Gaia} \end{array}\begin{array}{c} \begin{array}{cc} \text{DVDs} & \text{Games} \end{array} \\ \begin{bmatrix} 7 & 4 \\ 5 & 6 \end{bmatrix} \end{array} \times \begin{array}{r} \\ \text{DVDs} \\ \text{Games} \end{array}\begin{array}{c} \begin{array}{cc} \$ & \text{Tickets} \end{array} \\ \begin{bmatrix} 20 & 1 \\ 30 & 2 \end{bmatrix} \end{array}$$

$$= \begin{array}{r} \\ \text{Fatima} \\ \text{Gaia} \end{array}\begin{array}{c} \begin{array}{cc} \$ & \text{Tickets} \end{array} \\ \begin{bmatrix} 7 \times 20 + 4 \times 30 & 7 \times 1 + 4 \times 2 \\ 5 \times 20 + 6 \times 30 & 5 \times 1 + 6 \times 2 \end{bmatrix} \end{array}$$

$$= \begin{array}{r} \\ \text{Fatima} \\ \text{Gaia} \end{array}\begin{array}{c} \begin{array}{cc} \$ & \text{Tickets} \end{array} \\ \begin{bmatrix} 260 & 15 \\ 280 & 17 \end{bmatrix} \end{array}$$

2 Interpret the matrix.

Fatima had sales of $260 and gave out 15 tickets.

Gaia had sales of $280 and gave out 17 tickets.

▶ Properties of row and column matrices

Row and column matrices provide efficient ways of extracting information from data stored in large matrices. Matrices of a convenient size will be used to explore some of the surprising and useful properties of row and column matrices.

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Example 11 Using row and column matrices to extract information

Three rangers completed their monthly park surveys of feral animal sightings in matrix S.

$$S = \begin{array}{r} \\ \textit{Aaron} \\ \textit{Barra} \\ \textit{Chloe} \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Cats} & \textit{Dogs} & \textit{Foxes} & \textit{Rabbits} \end{array} \\ \begin{bmatrix} 27 & 9 & 34 & 59 \\ 18 & 15 & 10 & 89 \\ 35 & 6 & 46 & 29 \end{bmatrix} \end{array} \quad A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

a Evaluate $S \times B$.

b What information about matrix S is given in the product $S \times B$?

c Evaluate $A \times S$.

d What information about matrix S is given in the product $A \times S$?

Solution

a Matrix multiplication of a 3×4 and a 4×1 matrix produces a 3×1 matrix.

$$S \times B = \begin{bmatrix} 27 & 9 & 34 & 59 \\ 18 & 15 & 10 & 89 \\ 35 & 6 & 46 & 29 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 27 + 9 + 34 + 59 \\ 18 + 15 + 10 + 89 \\ 35 + 6 + 46 + 29 \end{bmatrix} = \begin{bmatrix} 129 \\ 132 \\ 116 \end{bmatrix}$$

b Look at the second last step in the working of $S \times B$.

Each row of SB gives the sum of the rows in S. Namely, the total sightings made by each ranger.

c Matrix multiplication of a 1×3 and a 3×4 matrix produces a 1×4 matrix.

$$A \times S = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 27 & 9 & 34 & 59 \\ 18 & 15 & 10 & 89 \\ 35 & 6 & 46 & 29 \end{bmatrix}$$

$$A \times S = \begin{bmatrix} 27 + 18 + 35 & 9 + 15 + 6 & 34 + 10 + 46 & 59 + 89 + 29 \end{bmatrix}$$

$$= \begin{bmatrix} 80 & 30 & 90 & 177 \end{bmatrix}$$

d In the second last step of part **c**, we see that each element is the sum of the sightings for each type of animal.

Each column of AS gives the sum of the columns in S, which gives the sum of the sightings of each type of animal.

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Exercise 8F

General applications

Example 10 **1** The first matrix below shows the number of milkshakes and sandwiches that Helen had for lunch. The number of kilojoules (kJ) present in each food is given in the second matrix.

$$\begin{array}{c|cc} & \text{Milkshakes} & \text{Sandwiches} \\ \hline \text{Helen} & 2 & 3 \end{array} \qquad \begin{array}{c|c} & kJ \\ \hline \text{Milkshakes} & 1400 \\ \text{Sandwiches} & 1000 \end{array}$$

Use a matrix product to calculate how many kilojoules Helen had for lunch.

SF

2 The first matrix shows the number of cars and bicycles owned by two families. The second matrix records the wheels and seats for cars and bicycles.

$$\begin{array}{c|cc} & \text{Cars} & \text{Bicycles} \\ \hline \text{Smith} & 2 & 3 \\ \text{Jones} & 1 & 4 \end{array} \qquad \begin{array}{c|cc} & \text{Wheels} & \text{Seats} \\ \hline \text{Car} & 4 & 5 \\ \text{Bicycle} & 2 & 1 \end{array}$$

Use a matrix product to find a matrix that gives the numbers of wheels and seats owned by each family.

CF

3 Eve played a game of darts. The parts of the dartboard that she hit during one game are recorded in matrix H. The bullseye is a small area in the centre of the dartboard. The points scored for hitting different regions of the dartboard are shown in matrix P.

$$H = \begin{array}{c|ccc} & \text{Bullseye} & \text{Inner region} & \text{Outer region} \\ \hline \text{Hits} & 2 & 13 & 5 \end{array}$$

$$P = \begin{array}{c|c} & \text{Points} \\ \hline \text{Bullseye} & 20 \\ \text{Inner region} & 5 \\ \text{Outer region} & 1 \end{array}$$

Use matrix multiplication to find a matrix giving her score for the game.

Business applications

4 On a Saturday morning Sand Café sold 18 quiches, 12 soups and 64 coffees. A quiche costs $5, soup costs $8 and a coffee costs $3.

- **a** Use a row matrix to record the number of each type of item sold.
- **b** Write the cost of each item in a column matrix.
- **c** Use matrix multiplication of the matrices from parts **a** and **b** to find the total value of the mornings sales.

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CF

5 Han's stall at the football made the sales shown in the table.

Tubs of chips	Pasties	Pies	Sausage rolls
90	84	112	73

The selling prices were: chips \$4, pastie \$5, pie \$5 and sausage roll \$3.

a Record the numbers of each product sold in a row matrix.

b Write the selling prices in a column matrix.

c Find the total value of the sales by using matrix multiplication of the row and column matrices found in parts **a** and **b**.

CU

6 Supermarkets sell eggs in boxes of 12, apples in bags of 8 and yoghurt tubs in sets of 4. This is represented by matrix A.

$$A = \begin{array}{r} \\ \text{Items per packet} \end{array} \begin{array}{c} \begin{matrix} \text{Eggs} & \text{Apples} & \text{Yoghurt} \end{matrix} \\ \begin{bmatrix} 12 & 8 & 4 \end{bmatrix} \end{array}$$

The cost for each type of packet is given by matrix B.

$$B = \begin{array}{r} \\ \text{Eggs} \\ \text{Apples} \\ \text{Yoghurt} \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 7 \\ 4 \\ 3 \end{bmatrix} \end{array}$$

The sales of each type of packet are given by matrix C as a column matrix and by matrix D as a row matrix.

$$C = \begin{array}{r} \\ \text{Eggs} \\ \text{Apples} \\ \text{Yoghurt} \end{array} \begin{array}{c} \text{Packets} \\ \begin{bmatrix} 100 \\ 50 \\ 30 \end{bmatrix} \end{array} \qquad D = \begin{array}{r} \\ \text{Packets} \end{array} \begin{array}{c} \begin{matrix} \text{Eggs} & \text{Apples} & \text{Yoghurt} \end{matrix} \\ \begin{bmatrix} 100 & 50 & 30 \end{bmatrix} \end{array}$$

Choose the appropriate matrices and use matrix multiplication to find:

a the total number of items sold (counting each egg, apple or yoghurt tub as an item)

b the total value of all sales.

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Using row and column matrices to extract information from a matrix

Example 11 **7** The number of study hours by three students over four days is shown in matrix H.

$$H = \begin{array}{r} \\ Issie \\ Jack \\ Kaiya \end{array}\begin{array}{c} \begin{array}{cccc} Mon & Tues & Wed & Thur \end{array} \\ \begin{bmatrix} 2 & 3 & 2 & 3 \\ 1 & 4 & 0 & 2 \\ 3 & 4 & 3 & 2 \end{bmatrix} \end{array}$$

Use matrix multiplication with a suitable row or column matrix to do the following.

- **a** Produce a matrix showing the total study hours for each student.
- **b** From the total study hours, find a matrix with the average hours of study for each student.
- **c** Obtain a matrix with the total number of hours studied on each night of the week.
- **d** From the total number of hours, find a matrix with the average number of hours studied each night, correct to 1 decimal place.

8 Matrix R records four students' results in five tests.

$$R = \begin{array}{r} \\ Ellie \\ Felix \\ George \\ Hannah \end{array}\begin{array}{c} \begin{array}{ccccc} T1 & T2 & T3 & T4 & T5 \end{array} \\ \begin{bmatrix} 87 & 91 & 94 & 86 & 88 \\ 93 & 76 & 89 & 62 & 95 \\ 73 & 61 & 58 & 54 & 83 \\ 66 & 79 & 83 & 90 & 91 \end{bmatrix} \end{array}$$

Choose an appropriate row or column matrix and use matrix multiplication to do the following.

- **a** Obtain a matrix with the sum of each student's results.
- **b** From these sums, give a matrix with each student's average test score.
- **c** Derive a matrix with the sum of the scores for each test.
- **d** From these sums, give a matrix with the average score on each test.

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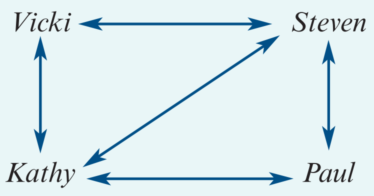
8G Communications and connections

Social networks, communication pathways and connections can be represented and analysed using matrix techniques.

Example 12 Applying matrices to social networks



The diagram shows the communications within a group of friends.

- A double-headed arrow connecting two names indicates that those two people communicate with each other.
- If there is no arrow directly connecting two people, they do not communicate.

a These links are called one-step connections because there is just one direct step in making contact with the other person. Record the social links in a matrix, N, using the first letter of each name to label the columns and rows. Explain how the matrix should be read.

b Explain why there is symmetry about the leading diagonal of the matrix.

c What information is given by the sum of a column or row?

d N^2 gives the number of two-step communications between people; that is, the number of ways one person can communicate with someone via another person. Find the matrix N^2, the square of matrix N.

e Use the matrix N^2 to find the number of two-step ways Kathy can communicate with Steven and write the connections.

f In the N^2 matrix there is a 3 where the S column meets the S row. This indicates that there are three two-step communications Steven can have with himself. Explain how this can be given a sensible interpretation.

Solution

a

$$N = \begin{array}{c c} & \begin{array}{cccc} V & S & K & P \end{array} \\ \begin{array}{c} V \\ S \\ K \\ P \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

For example, reading across row S to column K, a 1 indicates that Steven communicates with Kathy. The number 0 is used where there is no communication.

b The symmetry occurs because the communication is two way. For example, Vicki communicates with Steven and Steven communicates with Vicki.

c The sum of a column or row gives the total number of people that a given person can communicate with.

For example, Kathy can communicate with: $1 + 1 + 0 + 1 = 3$ people.

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d

$$N^2 = \begin{array}{c|cccc|} & V & S & K & P \\ \hline V & 2 & 1 & 1 & 2 \\ S & 1 & 3 & 2 & 1 \\ K & 1 & 2 & 3 & 1 \\ P & 2 & 1 & 1 & 2 \\ \hline \end{array}$$

e Reading across the K row to the S column the 2 indicates there are 2 two-step communications between Kathy and Steven.

These can be found in the arrows diagram.

Kathy ⟶ Vicki ⟶ Steven

Kathy ⟶ Paul ⟶ Steven

f There are three ways Steven can communicate with himself via another person.

Steven ⟶ Vicki ⟶ Steven

Steven ⟶ Kathy ⟶ Steven

Steven ⟶ Paul ⟶ Steven

For example, using the first case above, Steven might ring Vicki and ask her to ring him back later to remind him of an appointment.

The matrix N^3 would give the three-step communications between people. The number of ways of communicating with someone via two people.

The matrix methods of investigating communications can be applied to friendships, travel between towns and other types of two-way connections.

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Exercise 8G

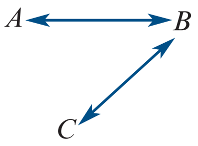
Note: The multiplication of large matrices may be done using technology.

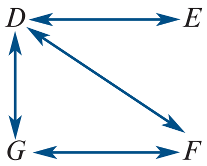
Example 12 **1** Assume that communications are a two-way process. So if A communicates with B then B communicates with A. The letters represent the names of people. Find the error in this communications matrix.

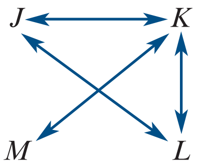
$$\begin{array}{c|cccc|} & A & B & C & D \\ A & 0 & 1 & 1 & 1 \\ B & 1 & 0 & 1 & 0 \\ C & 1 & 1 & 0 & 1 \\ D & 0 & 0 & 1 & 0 \end{array}$$

Where row B meets column A, the number 1 indicates that B communicates with A. The number 0 is used to show when there is no direct communication between two people.

2 Write the matrix for each communications diagram. Use the number 1 when direct communication between two people exists and 0 for no direct communication.

a 

b 

c 

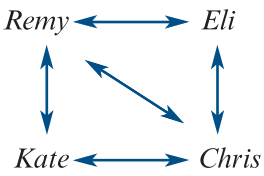
3 Road connections between towns are recorded in the matrices below. The letters represent towns. Where row C meets column B the number 1 indicates that there is a road directly connecting town C to town A. The number 0 is used to show when there is no road directly connecting two towns.
Draw a diagram corresponding to each matrix showing the roads connecting the towns.

a $$\begin{array}{c|ccc|} & A & B & C \\ A & 0 & 0 & 1 \\ B & 0 & 0 & 1 \\ C & 1 & 1 & 0 \end{array}$$

b $$\begin{array}{c|cccc|} & P & Q & R & S \\ P & 0 & 1 & 0 & 1 \\ Q & 1 & 0 & 1 & 0 \\ R & 0 & 1 & 0 & 1 \\ S & 1 & 0 & 1 & 0 \end{array}$$

c $$\begin{array}{c|cccc|} & T & U & V & W \\ T & 0 & 0 & 1 & 1 \\ U & 0 & 0 & 1 & 0 \\ V & 1 & 1 & 0 & 1 \\ W & 1 & 0 & 1 & 0 \end{array}$$

4 Communication connections between Chris, Eli, Kate and Remy are shown in the diagram.



a Write a matrix Q to represent the connections. Label the columns and rows in alphabetical order using the first letter of each name. Enter 1 to indicate that two people communicate directly or 0 if they do not.

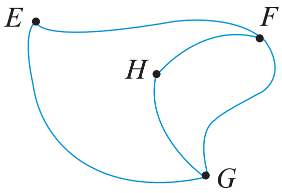
b What information is given by the sum of column R?

c **i** Find Q^2.

ii Using the matrix Q^2, find the total number of ways that Eli can communicate with a person via another person.

iii Write the chain of connections for each way that Eli can communicate with a person via another person.

5 Roads connecting the towns Easton, Fields, Hillsville and Gorges are shown in the diagram. The first letter of each town is used.



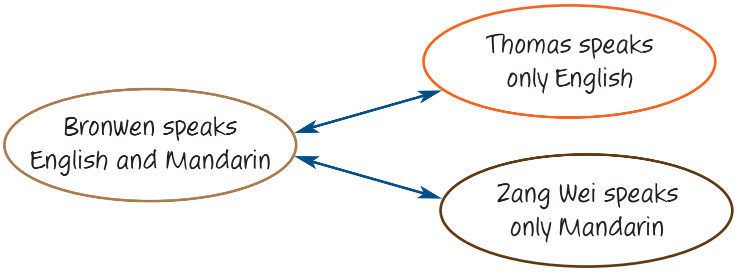
a Use matrix R to represent the road connections. Label the columns and rows in alphabetical order using the first letter of the name of each town. Write 1 when two towns are directly connected by a road and write 0 if they are not connected.

b What does the sum of column F reveal about Fields?

c **i** Find R^2.

ii How many ways are there to travel from Fields to a town via another town? List the possible ways of starting and ending at Fields.

6 The diagram shows the communications between three people. Bronwen speaks English and Mandarin. She communicates with Thomas, who only speaks English, and with Zang Wei, who only speaks Mandarin. The double-headed arrows are called one-step connections because they represent the one direct communication between two people. There is no one-step connection between Thomas and Zang Wei.



a Record the one-step connections in matrix C, using the first letter of each person's name, B, T and Z, to label the columns and rows.

b Explain why there is a symmetry about the leading diagonal of the matrix C.

c What information is given by the sum of a row or column?

d The matrix C^2 gives the number of two-step communications between people. A two-step communication is from one person to another via a third person. For example: $Z \rightarrow B \rightarrow T$ and $T \rightarrow B \rightarrow T$. In the second case, Thomas rang Bronwen and left a message for her to ring him back, which she did. List all the possible two-step communications.

e How can the matrix C^2 be used to check the total number of possible two-step connections?

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8H Further application and problem-solving tasks

Exercise 8H

1 Scalar multiplication occurs when a number multiplies a matrix.
For a 2 × 2 matrix it has the general form:

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

Suggest why the matrix $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ is called a *scalar matrix*.
Give examples to support your view.

Information can be stored in matrices with hundreds of columns and rows. Using conveniently sized matrices, we will explore methods that can be used for extracting information from huge matrix data banks.

2 The mobile phone bills of Anna, Boyd and Charlie for the four quarters of 2018 are recorded in matrix P. We will investigate the effect of multiplying by matrix E.

$$P = \begin{array}{r} \\ Anna \\ Boyd \\ Charlie \end{array}\begin{array}{c} \begin{array}{cccc} Q1 & Q2 & Q3 & Q4 \end{array} \\ \begin{bmatrix} 47 & 43 & 52 & 61 \\ 56 & 50 & 64 & 49 \\ 39 & 41 & 44 & 51 \end{bmatrix} \end{array} \qquad E = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

a Find $P \times E$ and comment on the result of that matrix multiplication.

b State the matrix F needed to extract the second quarter, $Q2$, costs.

A matrix is needed so it can multiply matrix P to extract the four quarterly costs on Charlie's phone bill.

c What will be the order of the matrix that displays Charlie's quarterly costs?

d State the order of matrix G that, when it multiplies a 3 × 4, gives a 1 × 4 matrix as the result? Should matrix G pre-multiply or post-multiply matrix P? Premultiply means it multiplies at the front (left) of matrix P; post-multiply means the matrix multiplies when written after (to the right of) matrix P.

e Suggest a suitable matrix G that will multiply matrix P and produce a matrix of Charlie's quarterly phone bills. Check that it works.

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Key ideas and chapter summary

Matrix	A **matrix** is a rectangular array of numbers set out in rows and columns within square brackets. The rows are horizontal; the columns are vertical.
Order of a matrix	The **order (size) of a matrix** is: number of rows × number of columns. The number of rows is always given first.
Elements of a matrix	The **elements of a matrix** are the numbers within it. The position of an element is given by its row and column in the matrix. Element $a_{i\,j}$ is in row i and column j. Row is always given first.
Connections	A matrix can be used to record various types of connections, such as social communications and roads directly connecting towns.
Equal matrices	Two matrices are equal when they have the same numbers in the same positions. To do this they need to have the same order (shape).
Adding matrices	Matrices of the same order can be *added* by adding numbers in the same positions.
Subtracting matrices	Matrices of the same order can be *subtracted* by subtracting numbers in the same positions.
Zero matrix, 0	A **zero matrix** is any matrix with zeroes in every position.
Scalar multiplication	**Scalar multiplication** is the multiplication of a matrix by *a number*.
Matrix multiplication	The process of multiplying a matrix by a matrix.
Power of a matrix, A^n	The matrix A is used n times in repeated multiplication.
Identity matrix, I	An *identity matrix* I behaves like the number 1 in arithmetic. Any matrix multiplied by I remains unchanged. For 2 × 2 matrices, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ where $AI = A = IA$ Matrix multiplication by the identity matrix is commutative.

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Skills check

Having completed this chapter you should be able to:

- state the order of a given matrix
- describe the location of an element within a matrix
- decide whether two matrices are equal
- add and subtract matrices
- perform scalar multiplication of a matrix
- identify a zero matrix
- decide whether it is possible to do matrix multiplication with two given matrices
- give the order of the matrix resulting from matrix multiplication
- perform matrix multiplication
- use technology to calculate the power of a matrix
- state the identity matrix for an $n \times n$ matrix and know its properties
- represent and solve communications.

Multiple-choice questions

SF

Use matrix F in Questions 1 and 2.

$$F = \begin{bmatrix} 4 & 8 & 6 \\ 5 & 1 & 7 \end{bmatrix}$$

1 The order of matrix F is:

A 6 **B** 2×3 **C** 3×2 **D** $2 + 3$ **E** $3 + 2$

2 The element $f_{2\,1}$ is:

A 3 **B** 2 **C** 8 **D** 1 **E** 5

3 The results of a survey of the number of electronic devices owned by the families of three students are shown in the matrix.

$$\begin{array}{r|ccc|} & \text{TVs} & \text{VCRs} & \text{PCs} \\ \hline \text{Caroline} & 4 & 3 & 2 \\ \text{Delia} & 1 & 0 & 5 \\ \text{Emir} & 2 & 1 & 3 \end{array}$$

The number of PCs owned by Emir's family is:

A 1 **B** 2 **C** 3 **D** 4 **E** 5

4 The matrix gives the numbers of roads directly connecting one town to another. The total number of roads directly connecting town E to other towns is:

$$\begin{array}{c} \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{ccc} D & E & F \end{array} \\ \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 0 \end{bmatrix} \end{array}$$

A 0 **B** 2 **C** 3 **D** 5 **E** 12

5 For these two matrices to be equal, the required value of x is:

$$\begin{bmatrix} 4 & 3x \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix}$$

A 2 **B** 3 **C** 4 **D** 6 **E** 18

Use matrices M and N in Questions 6 to 10.

$$M = \begin{bmatrix} 7 & 6 \\ 4 & 3 \end{bmatrix} \quad N = \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix}$$

6 The matrix $M + N$ is:

A $\begin{bmatrix} 12 & 8 \\ 5 & 3 \end{bmatrix}$ **B** $\begin{bmatrix} 12 & 4 \\ 4 & 3 \end{bmatrix}$ **C** $\begin{bmatrix} 12 & 4 \\ 4 & 0 \end{bmatrix}$ **D** $\begin{bmatrix} 12 & 4 \\ 5 & 3 \end{bmatrix}$ **E** $\begin{bmatrix} 12 & 4 \\ 5 & 0 \end{bmatrix}$

7 The matrix $M - N$ is:

A $\begin{bmatrix} 2 & 8 \\ 3 & 0 \end{bmatrix}$ **B** $\begin{bmatrix} 2 & 4 \\ 3 & 0 \end{bmatrix}$ **C** $\begin{bmatrix} 2 & 4 \\ 3 & 3 \end{bmatrix}$ **D** $\begin{bmatrix} 2 & 8 \\ 3 & 3 \end{bmatrix}$ **E** $\begin{bmatrix} 2 & 8 \\ 4 & 3 \end{bmatrix}$

8 The matrix $N - N$ is:

A 0 **B** $\begin{bmatrix} 0 \end{bmatrix}$ **C** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **D** $\begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix}$ **E** $\begin{bmatrix} -5 & 2 \\ -1 & 0 \end{bmatrix}$

9 The matrix $2N$ is:

A $\begin{bmatrix} 10 & -4 \\ 2 & 0 \end{bmatrix}$ **B** $\begin{bmatrix} 7 & 0 \\ 3 & 2 \end{bmatrix}$ **C** $\begin{bmatrix} 10 & -4 \\ 1 & 0 \end{bmatrix}$ **D** $\begin{bmatrix} 10 & -2 \\ 2 & 0 \end{bmatrix}$ **E** $\begin{bmatrix} 10 & -4 \\ 2 & 2 \end{bmatrix}$

10 The matrix $2M + N$ is:

A $\begin{bmatrix} 14 & 10 \\ 7 & 5 \end{bmatrix}$ **B** $\begin{bmatrix} 14 & 6 \\ 7 & 5 \end{bmatrix}$ **C** $\begin{bmatrix} 24 & 8 \\ 10 & 6 \end{bmatrix}$ **D** $\begin{bmatrix} 19 & 14 \\ 9 & 6 \end{bmatrix}$ **E** $\begin{bmatrix} 19 & 10 \\ 9 & 6 \end{bmatrix}$

Use the matrices P, Q, R, S in Questions 11 to 14.

$$P\begin{bmatrix}5 & 4 & 1\\7 & 6 & 8\end{bmatrix}\quad Q=\begin{bmatrix}2\\6\\0\end{bmatrix}\quad R=\begin{bmatrix}4 & 7\end{bmatrix}\quad S=\begin{bmatrix}1 & 3\\0 & 4\end{bmatrix}$$

11 Matrix multiplication is not defined for:

A PQ **B** SS **C** SP **D** PS **E** RS

12 The order of matrix QR is:

A 1 × 1 **B** 3 × 2 **C** 2 × 3 **D** 6 **E** 5

13 Which of the following matrix multiplications gives a 1 × 3 matrix?

A QQ **B** RQ **C** PR **D** QR **E** RP

14 The matrix multiplication PQ gives the matrix:

A $\begin{bmatrix}34\\50\end{bmatrix}$ **B** $\begin{bmatrix}10 & 24 & 0\\14 & 36 & 0\end{bmatrix}$ **C** $\begin{bmatrix}10 & 14\\24 & 0\\36 & 0\end{bmatrix}$ **D** $\begin{bmatrix}5 & 4 & 1\\7 & 6 & 8\\2 & 6 & 0\end{bmatrix}$ **E** $\begin{bmatrix}34 & 50\end{bmatrix}$

15 The 2 × 2 identity matrix is:

A $\begin{bmatrix}1\end{bmatrix}$ **B** $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ **C** $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ **D** $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ **E** $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

16 The matrix $A=\begin{bmatrix}4 & -3\\-1 & 2\end{bmatrix}$. Matrix A^2 is:

A $\begin{bmatrix}16 & -9\\-1 & -4\end{bmatrix}$ **B** $\begin{bmatrix}16 & 9\\1 & 4\end{bmatrix}$ **C** $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ **D** $\begin{bmatrix}19 & -18\\-6 & 7\end{bmatrix}$ **E** $\begin{bmatrix}19 & 18\\6 & 7\end{bmatrix}$

Short-answer questions

Use matrix A in Questions 1 to 4.

$$A=\begin{bmatrix}4 & 2 & 1 & 0\\3 & 4 & 7 & 9\end{bmatrix}$$

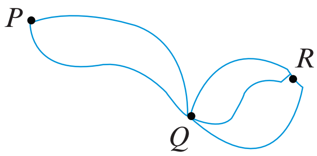
1 State the order of matrix A.

2 Identify the element $a_{1\,3}$.

3 If $C = [5 \quad 6]$, find CA.

4 If the order of a matrix B was 4×1, what would be the order of the matrix resulting from AB?

5 Roads are shown joining towns P, Q and R. Use a matrix to record the number of roads directly connecting one town to another town.



6 Consider the matrices below.

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 5 \\ 7 & 6 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find:

a $3A$ **b** $A + B$ **c** $B - A$ **d** $2A + B$
e $A - A$ **f** AB **g** BA **h** IB
i A^2 **j** AI **k** A^4

Extended-response questions

1 Farms A and B have their livestock numbers recorded in the matrix shown.

$$\begin{array}{c} \\ Farm\,A \\ Farm\,B \end{array} \begin{array}{c} \begin{array}{ccc} Cattle & Pigs & Sheep \end{array} \\ \begin{bmatrix} 420 & 50 & 100 \\ 300 & 40 & 220 \end{bmatrix} \end{array}$$

a How many pigs are on Farm B?

b What is the total number of sheep on the two farms?

c Which farm has the larger total number of livestock?

2 A bakery recorded the sales for Shop A and Shop B of cakes, pies and rolls in a Sales matrix, S. The prices were recorded in the Prices matrix, P.

$$S = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{ccc} Cakes & Pies & Rolls \end{array} \\ \begin{bmatrix} 12 & 25 & 18 \\ 15 & 21 & 16 \end{bmatrix} \end{array} \qquad P = \begin{array}{c} \\ Cakes \\ Pies \\ Rolls \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \end{array}$$

a How many pies were sold by Shop B?

b What is the selling price of pies?

c Calculate the matrix product SP.

d What information is contained in matrix SP?

e Which shop had the greater income from its sales? How much were its takings?

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3 Patsy and Geoff decided to participate in a charity fun run.

a Patsy plans to walk for 4 hours and jog for 1 hour. Geoff plans to walk for 3 hours and jog for 2 hours. Write out matrix A, filling in the missing information.

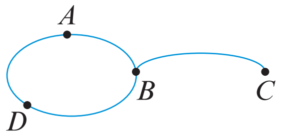
$$A = \begin{array}{ccc} & \textit{Hours walking} & \textit{Hours jogging} \\ \textit{Patsy} & \square & \square \\ \textit{Geoff} & \square & \square \end{array}$$

b Walking raises \$2 per hour and burns 1500 kJ/h (kilojoules per hour).
Jogging raises \$3 per hour and burns 2500 kJ/h.
Write out matrix B, filling in the missing information.

$$B = \begin{array}{ccc} & \$ & \textit{kJ} \\ \textit{Walking} & \square & \square \\ \textit{Jogging} & \square & \square \end{array}$$

c Use matrix multiplication to find a matrix that shows the money raised and the kilojoules burned by each person.

4 Roads connecting the towns Adz, Bez, Coz and Dzo are shown in the diagram. The first letter of each town is used.



a Use matrix R to record the road connections. Label the rows and columns in alphabetical order using the first letter of the name of each town. Use 1 to indicate that two towns are directly connected by a road and write 0 when they are not directly connected.

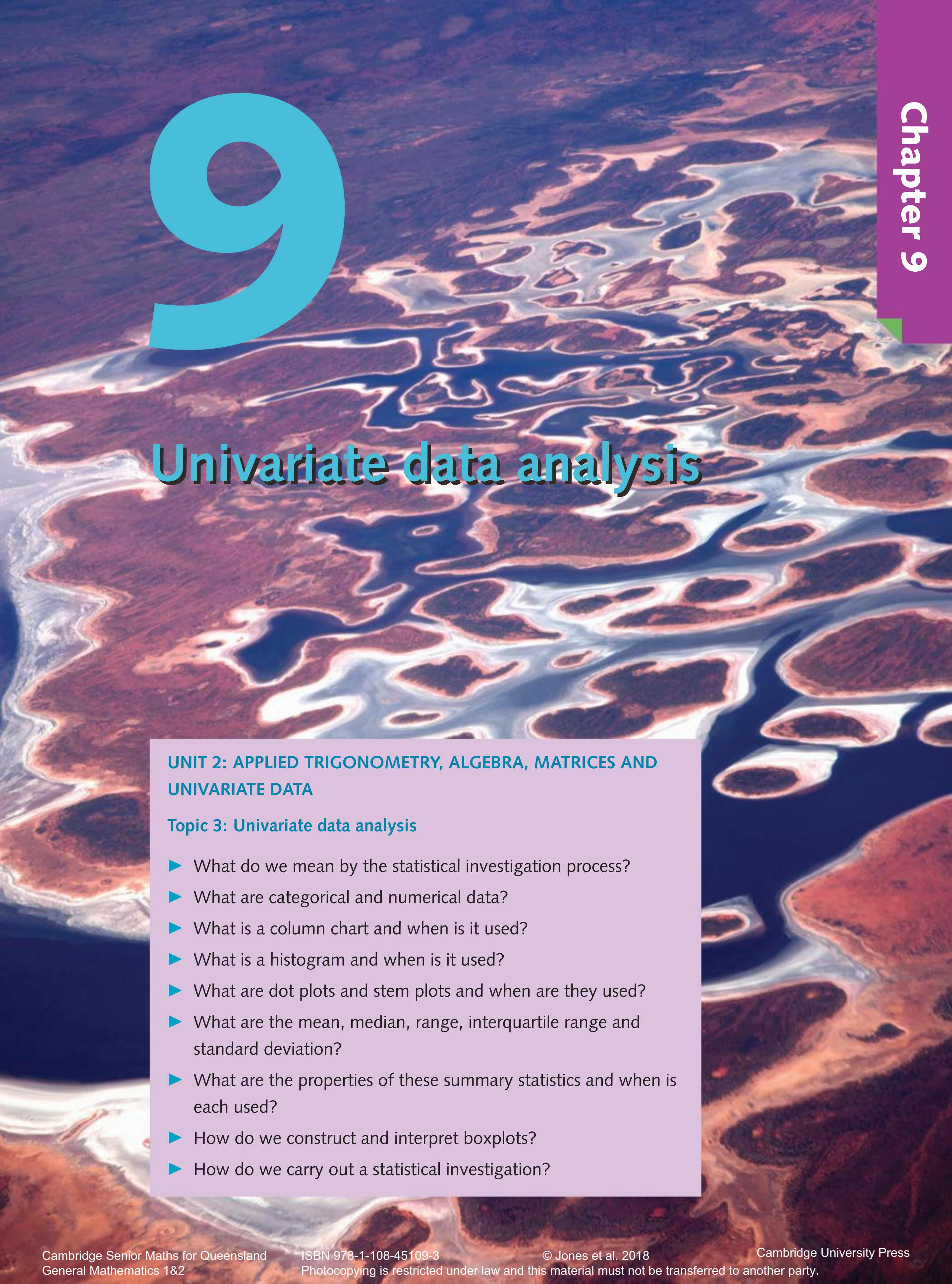
b What does the sum of column B tell us about the town Bez?

c **i** Find R^2.

ii How many ways are there to travel from Bez to a town via one other town? List the possible ways that start and end at Bez.

9

Univariate data analysis



UNIT 2: APPLIED TRIGONOMETRY, ALGEBRA, MATRICES AND UNIVARIATE DATA

Topic 3: Univariate data analysis

- What do we mean by the statistical investigation process?
- What are categorical and numerical data?
- What is a column chart and when is it used?
- What is a histogram and when is it used?
- What are dot plots and stem plots and when are they used?
- What are the mean, median, range, interquartile range and standard deviation?
- What are the properties of these summary statistics and when is each used?
- How do we construct and interpret boxplots?
- How do we carry out a statistical investigation?

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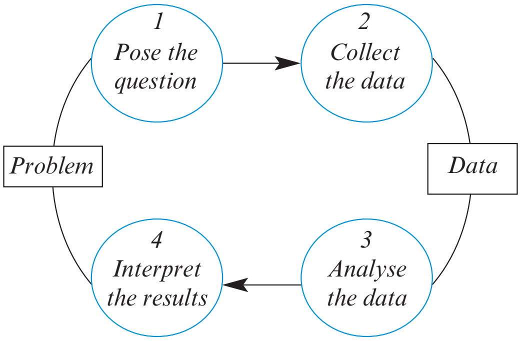
Introduction

Whether issues are of worldwide importance, such as global warming, or of national significance, such as our attitude to gun legislation, or of personal relevance, such as the time teenagers spend each week on social media, a consistent approach can be applied to better understand each situation. In each case, we can know more by gathering some relevant data, and analysing the data appropriately. The process we use for this is sometimes called the **statistical investigation process**, and it encompasses the following steps:

1. Pose the question – Decide what data would allow you to address the problem.
2. Data – Collect or obtain the data.
3. Analyse – Summarise and display the data to answer the question posed.
4. Conclusion – Interpret the results and communicate what has been learned.



Consider, for example, the problem of global warming. Scientists working on this issue have many decisions to make before they can start collecting data. Should they measure maximum temperature, minimum temperature or average temperature? Or should they measure the number of hours a day of sunlight, or the number of days without rain? Where should they take measurements, at how many locations, and how often? Posing a clear question will determine what data are required.

Once the question is in place, the data can either be collected directly by counting or measurement (called primary data), or obtained from another source if it has already been collected by someone else (called secondary data). Our scientists looking at global warming may have access to weather records from all over the world, so they could use this data if they had posed the questions 'Have maximum daily temperatures increased over the last fifty years?'

Once the data have been collected, there are many statistical techniques that can be used to make sense of the data set. In this chapter we will look at some of the techniques that are used when the data are collected from a single variable. Such data are called **univariate data**, and arise when we conduct a study that looks at only one variable. Suppose, for example, that we conducted a survey to estimate the average height of Year 11 students. As we would only be working with one variable (height), we would be working with univariate data.

The final step in the process is to interpret the results of the analyses and communicate clearly the answer to the question posed. This is also a focus of this chapter. By its end you will have developed all of the skills necessary to undertake a simple statistical investigation.

Univariate data

Univariate data is data collected from a single variable.

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9A Types of data

It is not enough to decide which variables to measure when carrying out a statistical investigation. It is also important to determine how to measure the variable. For example, if one is interested in measuring temperature then there are choices such as degrees Celsius, degrees Fahrenheit, kelvins, or maybe just a scale that says ‘cold, warm, hot’!

Consider the following situation. In completing a survey, students are asked, for example, to:

- indicate their sex by circling an ‘F’ for female or an ‘M’ for male on the form
- indicate their preferred coffee cup size when buying takeaway coffee as ‘small’, ‘medium’ or ‘large’
- write down the number of brothers they have
- measure and write their hand span in centimetres.

The information collected from four students is displayed in the table below.

Sex	Coffee size	Number of brothers	Hand span (in cm)
M	Small	0	23.6
F	Large	2	19.6
F	Small	1	20.2
M	Large	1	24.0

As the answers to each question in the survey will vary from student to student, each question defines a different **variable** namely: *sex*, *coffee size*, *number of brothers* and *hand span*. The values we collect about each of these variables are called **data**.

The data in the table fall into two broad types: *categorical* or *numerical*.

Categorical data

The data arising from the students’ responses to the first and second questions in the survey are called **categorical data** because the data values can be used to place the person into one of several groups or categories. However, the properties of the data generated by these two questions differ slightly.

- The question asking students to use an ‘M’ or ‘F’ to indicate their sex will prompt a response of either M or F. This identifies the respondent as either male or female, but tells us no more. We call this **nominal data** because it simply *names* or *nominates*.
- However, the question with responses ‘small’, ‘medium’ and ‘large’ that indicates the students’ preferred coffee size tells us two things. First, it names the coffee size, but it also enables us to order the students according to their preferred coffee sizes. We call this **ordinal data** because it enables us to both name and order their responses.

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Numerical data

The data arising from the responses to the third and fourth questions in the survey are called **numerical data** because they have values for which arithmetic operations such as adding and averaging make sense. However, the properties of the data generated by these questions differ slightly.

- The question asking students to write down the number of brothers they have will prompt whole number responses such as $0, 1, 2, \ldots$
  Because the data can only take particular numerical values it is called **discrete data**. Discrete data arises in situations where counting is involved. For this reason, discrete data is sometimes called count data.
- In response to the hand span question, students who wrote 24 cm could have an actual hand span of anywhere between 23.5 and 24.4 cm, depending on the accuracy of the measurement and how the student rounded their answer. This is called **continuous data**, because the variable we are measuring, in this case, *hand span*, can take any numerical value within a specified range.
  Continuous data are often generated when measurement is involved. For this reason, continuous data is sometimes called measurement data.

Types of variables

Categorical variables

Variables that generate categorical data are called **categorical variables** or, if we need a finer distinction, **nominal** or **ordinal** variables. For example, *sex* is a nominal variable, and *coffee size* is an ordinal variable.

Numerical variables

Variables that generate numerical data are called numerical variables or, if we need a finer distinction, discrete or continuous variables. For example, *number of brothers* is a **discrete variable**, while *hand span* is a continuous variable.

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Exercise 9A

Classifying data

SF

1 Classify the categorical data arising from people answering the following questions as either nominal or ordinal.

- **a** What is your favourite football team?
- **b** How often do you exercise? Choose one of ‘never’, ‘once a month’, ‘once a week’, ‘every day’.
- **c** Indicate how strongly you agree with ‘alcohol is the major cause of accidents’ by selecting one of ‘strongly agree’, ‘agree’, ‘disagree’, ‘strongly disagree’.
- **d** Which language will you study next year, ‘French’, ‘Chinese’, ‘Spanish’ or ‘none’?

2 Classify the data generated in each of the following as categorical or numerical.

- **a** Kindergarten pupils bring along their favourite toys, and the toys are grouped together under the headings ‘dolls’, ‘soft toys’, ‘games’, ‘cars’ and ‘other’.
- **b** The numbers of students on each of 20 school buses are counted.
- **c** A group of people each write down their favourite colour.
- **d** Each student in a class is weighed and the weight recorded in kilograms.
- **e** Students are weighed and then classified as ‘light’, ‘average’ or ‘heavy’.
- **f** People rate their enthusiasm for a certain rock group as ‘low’, ‘medium’ or ‘high’.

3 Classify the data generated in each of the following situations as nominal, ordinal or numerical (discrete or continuous).

- **a** The different brand names of instant soup sold by a supermarket were recorded.
- **b** A group of people were asked to indicate their attitude to capital punishment by selecting a number from 1 to 5, where 1 = strongly disagree, 2 = disagree, 3 = undecided, 4 = agree and 5 = strongly agree.
- **c** The number of computers per household was recorded during a census.

Classifying variables

4 Classify the numerical variables identified below (in italics) as discrete or continuous.

- **a** The *number of pages* in a book
- **b** The *price* paid to fill the tank of a car with petrol
- **c** The *volume* of petrol (in litres) used to fill the tank of a car
- **d** The *time* between the arrival of successive customers at an ATM
- **e** The *number of people* at a football match

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9B Displaying and describing categorical data distributions

To make sense of a data set, we first need to organise it into a more manageable form. For categorical data, frequency tables and column charts are used for this purpose.

Skillsheet

▶ The frequency table

Frequency

A **frequency table** is a listing of the values a variable takes in a data set, along with how often (frequently) each value occurs.

Frequency can be recorded as a:

- **frequency**: the number of times a value occurs
- **percentage frequency**: the percentage of times a value occurs, where:

$$\text{percentage frequency} = \frac{\text{count}}{\text{total}} \times 100\%$$

- frequency **distribution**: a listing of the values a variable takes, along with how frequently each of these values occurs.

Example 1 Constructing a frequency table for categorical data

Thirty children chose a sandwich, a salad or a pie for lunch, as follows:

sandwich, salad, salad, pie, sandwich, sandwich, salad, salad, pie, pie, pie, salad, pie, sandwich, salad, pie, salad, pie, sandwich, sandwich, pie, salad, salad, pie, pie, pie, salad, pie, sandwich, pie

Construct a table for the data showing both frequency and percentage frequency.

Solution

1 Set up a table as shown. The variable *lunch choice* has three categories: ‘sandwich’, ‘salad’ and ‘pie’.

2 Count the numbers of children choosing a sandwich, a salad or a pie. Record them in the ‘Number’ column.

3 Add the frequencies to find the total number.

	Frequency	
Lunch choice	Number	%
Sandwich	7	23.3
Salad	10	33.3
Pie	13	43.3
Total	30	99.9

4 Convert the frequencies into percentages and record them in the ‘%’ column. For example, percentage frequency for pies equals $\frac{13}{30} \times 100\% = 43.3\%$.

5 Total the percentages and record the total in the table. Note that the percentages add up to 99.9%, not 100%, because of rounding.

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Column charts

When there are a lot of data, a frequency table can be used to summarise the information, but we generally find that a graphical display is also useful. When the data are categorical, the appropriate display is a **column chart** (also known as a bar chart).

Column charts

In a column chart:

- frequency or percentage frequency is shown on the vertical axis
- the variable being displayed is plotted on the horizontal axis
- the height of the column gives the frequency (or percentage)
- the columns are drawn with gaps to indicate that each value is a separate category
- there is one column for each category.

Example 2 Constructing a column chart and percentage column chart from a frequency table

Use the frequency table for lunch choice from Example 1 to construct:

a a column chart

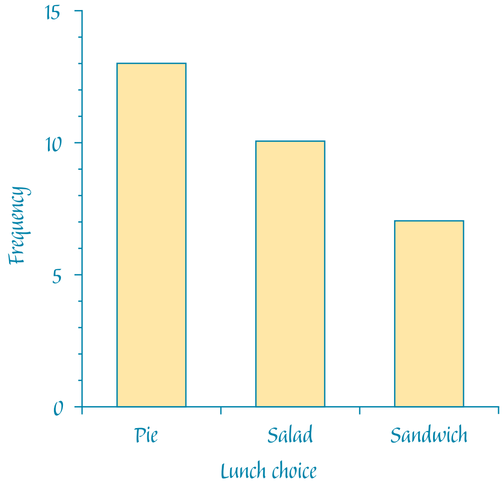
b a percentage column chart.

Solution

a **1** Label the horizontal axis with the variable name, 'Lunch choice'. Mark the scale off into three equal intervals and label them 'Pie', 'Salad' and 'Sandwich'.

2 Label the vertical axis 'Frequency'. Insert a scale allowing for the maximum frequency of 13. Up to 15 would be appropriate. Mark the scale in multiples of 5.

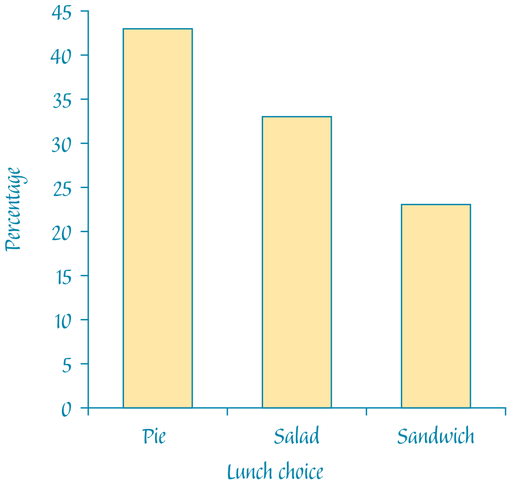
3 For each interval draw in a column as shown. Make the width of each column less than the width of the category intervals to show that the categories are quite separate. The height of each column is equal to the frequency.



Note: For nominal variables it is common, but not necessary, to list categories in decreasing order by frequency. This makes later interpretation easier.

b The lunch choice percentage frequency was: sandwich 23.3%, salad 33.3%, pie 43.3%.

To construct a percentage column chart of the lunch choice data, follow the procedure as for part **a** but label the vertical axis ‘Percentage’. Insert a scale allowing for a maximum percentage frequency up to 45%. Mark the vertical scale in intervals of 5%. The height of each column is equal to the percentage.



The mode or modal category

One of the features of a data set that is quickly revealed with a column chart is the **mode** or **modal category**. This is the most frequently occurring category. In a column chart, this is given by the category with the tallest column. For the column chart in Example 2, the modal category is clearly ‘pie’. That is, the most frequent or popular lunch choice was a pie.

When is the mode useful?

The mode is most useful when a single value or category in the frequency table occurs more often (frequently) than the others. Modes are of particular importance in popularity polls, when answering questions such as ‘Which is the most frequently watched TV station between the hours of 6 p.m. and 8 p.m.?’ or ‘When is a supermarket in peak demand?’

Exercise 9B

Constructing frequency tables

Example 1

SF

1 The *sex* of 15 people in a bus is as shown (F = female, M = male):

F M M M F M F F M M M F M M M

a Is the variable *sex* nominal or ordinal?

b Construct a frequency table for the data including frequency and percentage frequency.

2 The UK *shoe sizes* of 20 eighteen-year-old males are as shown:

8	9	9	10	8	8	7	9	8	9
10	12	8	10	7	8	8	7	11	11

a Is the variable *shoe size* nominal or ordinal?

b Construct a frequency table for the data including frequency and percentage frequency.

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Analysing frequency tables and constructing column charts

Example 2

SF

3 The table below shows the frequency distribution of the favourite type of fast food (*food type*) of a group of students.

a Complete the table.

b Is the variable *food type* nominal or ordinal?

c How many students preferred Chinese food?

d What percentage of students chose chicken as their favourite fast food?

e What was the favourite type of fast food for these students?

f Construct a column chart of the frequencies (number).

	Frequency	
Food type	Number	%
Hamburgers	23	33.3
Chicken	7	10.1
Fish and chips	6	
Chinese	7	10.1
Pizza	18	
Other	8	11.6
Total		99.9

CF

4 The following responses were received to a question regarding the return of capital punishment.

a Complete the table.

b Is the data used to generate this table nominal or ordinal?

c How many people said ‘Strongly agree’?

d What percentage of people said ‘Strongly disagree’?

e What was the most frequent response?

f Construct a column chart of the frequencies.

	Frequency	
Capital punishment	Number	%
Strongly agree	21	8.2
Agree	11	4.3
Don’t know	42	
Disagree		
Strongly disagree	129	50.4
Total	256	100.0

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5 A bookseller noted the types of books purchased on a particular day and recorded them in the table.

a Complete the table.

b Is the variable *type of book* nominal or ordinal?

c How many books purchased were classified as 'Fiction'?

d What percentage of books were classified as 'Children'?

e How many books were purchased in total?

f Construct a column chart of the percentage frequencies (%).

	Frequency	
Type of book	Number	%
Children	53	22.8
Fiction	89	
Cooking	42	18.1
Travel	15	
Other	33	14.2
Total	232	

6 A survey of the ways in which secondary school students preferred to spend their leisure time at home gave the results shown in the table.

a How many students were surveyed?

b Is the variable *leisure activity* nominal or ordinal?

c What percentage of students said that they preferred to spend their leisure time chatting with friends?

d What was the most popular way of spending their leisure time for these students?

e Construct a column chart of the percentage frequencies (%).

	Frequency	
Leisure activity	Number	%
Play video games	84	42
Read	26	13
Use social media	46	23
Chat with friends	24	12
Watch a movie	8	4
Other	12	6
Total	200	100

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9C Interpreting and describing frequency tables and column charts

As part of this subject, you will be expected to complete a statistical investigation. Under these circumstances, constructing a frequency table or a column chart is not an end in itself. It is merely a means to an end. The end is being able to understand something about the variables you are investigating that you didn't know before.

To complete the investigation, you will need to communicate this finding to others. To do this, you will need to know how to describe and interpret any patterns you observe in the context of your data investigation in a written report that is both systematic and concise. The purpose of this section is to help you develop such skills.

Some guidelines for describing the distribution of a categorical variable and communicating your findings

- Briefly summarise the context in which the data were collected, including the number of people (or things) involved in the study.
- If there is a clear modal category, make sure that it is mentioned.
- Include relevant counts or percentages in the report.
- If there are a lot of categories, it is not necessary to mention every category.
- Either counts or percentages can be used to describe the distribution.

These guidelines are illustrated in the following examples.

Example 3 Using a frequency table to describe the outcome of an investigation involving a categorical variable

A group of 30 children were offered a choice of a sandwich, a salad or a pie for lunch and their responses collected and summarised in the frequency table opposite.

Use the frequency table to report on the relative popularity of the three lunch choices, quoting appropriate frequencies to support your conclusions.

Lunch choice	Frequency
Sandwich	7
Salad	10
Pie	13
Total	30

Solution

Report

A group of 30 children were offered a choice of a sandwich, a salad or a pie for lunch. The most popular lunch choice was pie, chosen by 13 of the children. Ten of the children chose a salad. The least popular option was sandwich, chosen by only 7 of the children.

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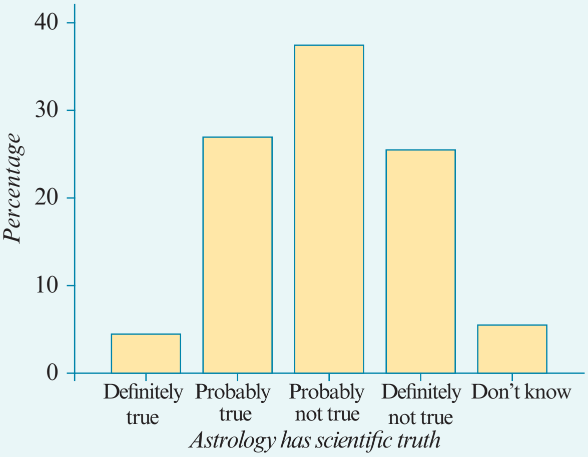
Example 4 Using a frequency table and a percentage column chart to describe the outcome of an investigation involving a categorical variable

A sample of 200 people were asked to comment on the statement ‘Astrology has scientific truth’ by selecting one of the options ‘definitely true’, ‘probably true’, ‘probably not true’, definitely not true’ or ‘don’t know’.

The data are summarised in the following frequency table and column chart. Note that the categories in the frequency table can be ordered in a definite order because the data are ordinal.

Astrology has scientific truth	Frequency	
	Number	%
Definitely true	9	4.5
Probably true	54	27.0
Probably not true	75	37.5
Definitely not true	51	25.5
Don’t know	11	5.5
Total	200	100.0



Write a report summarising the findings of this investigation, quoting appropriate percentages to support your conclusion.

Solution

Report
Two hundred people were asked to respond to the statement ‘Astrology has scientific truth’. The majority of respondents did not agree, with 37.5% responding that they believed that this statement was probably not true, and another 25.5% declaring that the statement was definitely not true. Over one quarter (27%) of the respondents thought that the statement was probably true, while only 4.5% of subjects thought that the statement was definitely true.

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Exercise 9C

Interpreting and describing frequency tables and column charts

SF

Example 3

1 A group of 69 students were asked to nominate their preferred type of fast food. The results are summarised in the percentage frequency table opposite. Use the information in the table to complete the report below by filling in the blanks.

Fast food type	%
Hamburgers	33.3
Chicken	10.1
Fish and chips	8.7
Chinese	10.1
Pizza	26.1
Other	11.6
Total	99.9

Report

A group of ☐ students were asked their favourite type of fast food. The most popular response was ☐ (33.3%), followed by pizza (☐). The rest of the group were almost evenly split between chicken, fish and chips, Chinese and other, all around 10%.

2 Two hundred and fifty-six people were asked whether they agreed that there should be a return to capital punishment in their state. Their responses are summarised in the table opposite. Use the information in the table to complete the report below by filling in the blanks.

Capital punishment	%
Strongly agree	8.2
Agree	4.3
Don't know	16.4
Disagree	20.7
Strongly disagree	50.4
Total	100.0

Report

A group of 256 people were asked whether they agreed that there should be a return to capital punishment in their state. The majority of these people ☐ (50.4%), followed by ☐ who disagreed. Levels of support for return to capital punishment were quite low, with only 4.3% agreeing and 8.2% strongly agreeing. The remaining ☐ said that they didn't know.

3 A group of 200 students were asked how they prefer to spend their leisure time.

The results are summarised in the frequency table.

Use the information in the table to write a brief report of the results of this investigation.

Leisure activity	%
Internet and digital games	42
Read	13
Listen to music	23
Watch TV or go to movies	12
Phone friends	4
Other	6
Total	100

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Example 4 **4** A group of 579 employees from a large company were asked to rate the importance of salary in determining how they felt about their job. Their responses are shown in the following frequency table and column chart.

Importance of salary	%
Very important	33.5
Important	56.8
Somewhat important	7.8
Not at all important	1.9
Total	100.0

Write a report describing how these employees rated the importance of salary in determining how they felt about their jobs.

9D Displaying and describing numerical data

Frequency tables can also be used to organise numerical data. For discrete numerical data, the process is the same as that for categorical data, as shown in the following example.

▶ Discrete data

Example 5 Constructing a frequency table for discrete numerical data

The number of brothers and sisters (siblings) reported by each of the 30 students in Year 11 are as follows:

2 3 4 0 3 2 3 0 4 1 0 0 1 2 3
0 2 1 1 4 5 3 2 5 6 1 1 1 0 2

Construct a frequency table for these data.

Solution

1. Find the maximum and the minimum values in the data set. Here the minimum is 0 and the maximum is 6.
2. Construct a table as shown, including all the values between the minimum and the maximum.
3. Count the number of 0s, 1s, 2s, etc. in the data set. For example, there are seven 1s. Record these values in the number column.
4. Add the frequencies to find the total.
5. Convert the frequencies to percentages, and record in the per cent (%) column.

   For example, percentage of 1s equals $\frac{7}{30} \times 100 = 23.3\%$.
6. Total the percentages and record.

	Frequency	
Number of siblings	Number	%
0	6	20.0
1	7	23.3
2	6	20.0
3	5	16.7
4	3	10.0
5	2	6.7
6	1	3.3
Total	30	100.0

▶ Grouping data

Some variables, such as the variable *number of children in a family*, can only take on a limited range of values. For these variables, it makes sense to list each of these values individually when forming a frequency distribution.

In other cases, the variable can take on a large range of values; for example, the variable *age* might take values from 0 to 100 or even more. Listing all possible ages would be tedious and would produce a large and unwieldy table. To solve this problem we **group the data** into a small number of convenient intervals.

These grouping intervals should be chosen according to the following principles:

- Every data value should be in an interval.
- The intervals should not overlap.
- There should be no gaps between the intervals.

The choice of intervals can vary but there are some guidelines.

- A division that results in about 5 to 15 groups is preferred.
- Choose an interval width that is easy for the reader to interpret, such as 10 units, 100 units or 1000 units (depending on the data).
- By convention, the beginning of the interval is given the appropriate exact value, rather than the end. As a result, intervals of 0–49, 50–99, 100–149 would be preferred over the intervals 1–50, 51–100, 101–150 etc.

Grouped discrete data

Example 6 Constructing a frequency table for a discrete numerical variable

A group of 20 people were asked to record how many cups of coffee they drank in a particular week, with the following results:

2	0	9	10	23	25	0	0	34	32
5	0	17	14	3	6	0	33	23	0

Construct a table of these data showing both frequency (count) and percentage frequency.

Solution

1 The minimum number of cups of coffee drunk is 0 and the maximum is 34. Intervals beginning at 0 and ending at 34 would ensure that all the data are included. Interval width of 5 will mean that there are 7 intervals. Note that the endpoints are within the interval, so that the interval 0 – 4 includes 5 values: 0, 1, 2, 3, 4.

2 Set up the table as shown.

3 Count the number of data values in each interval to complete the number column.

Cups of coffee	Frequency	
	Number	%
0–4	8	40
5–9	3	15
10–14	2	10
15–19	1	5
20–24	2	10
25–29	1	5
30–34	3	15
Total	20	100

4 Convert the frequencies into percentages and record in the per cent (%) column. For example, for the interval 5–9: % frequency $= \dfrac{3}{20} \times 100 = 15\%$

5 Total the percentages and record.

Grouped continuous data

Example 7 Constructing a frequency table for a continuous numerical variable

The following are the heights of the 41 players in a basketball club, in centimetres.

178.1	185.6	173.3	193.4	183.1	184.6	202.4	170.9	183.3
180.3	185.8	189.1	178.6	194.7	185.3	191.1	189.7	191.1
180.4	180.0	193.8	196.3	189.6	183.9	177.7	178.9	193.0
188.3	189.5	182.0	183.6	184.5	188.7	192.4	203.7	180.1
170.5	179.3	184.1	183.8	174.7				

Construct a frequency table of these data.

Solution

Heights	Frequency	
	Number	%
170.0–174.9	4	9.8
175.0–179.9	5	12.2
180.0–184.9	13	31.7
185.0–189.9	9	22.0
190.0–194.9	7	17.1
195.0–199.9	1	2.4
200.0–204.9	2	4.9
Total	41	100.1

1. Find the minimum and maximum heights, which are 170.5 cm and 203.7 cm. A minimum value of 170.0 and a maximum of 204.9 will ensure that all the data are included.
2. An interval width of 5 cm will mean that there are 7 intervals from 170.0 to 204.9, which is within the guidelines of 5–15 intervals.
3. Set up the table as shown. All values of the variable that are from 170.0 to 174.9 have been included in the first interval. The second interval includes values from 175.0 to 179.9, and so on for the rest of the table.
4. The number of data values in each interval is then counted to complete the number column of the table.
5. Convert the frequencies to percentages and record in the per cent (%) column. For example, for the interval 175.0–179.9: % frequency $= \frac{5}{41} \times 100 = 12.2\%$
6. Total the percentages and record.

The interval that has the highest frequency is called the **modal interval**. Here the modal interval is 180.0–184.9, as 13 players (31.7%) have heights that fall into this interval.

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Histograms

As with categorical data, we would like to construct a visual display of a frequency table for numerical data. The graphical display of a frequency table for a numerical variable is called a **histogram**. A histogram looks similar to a column chart but, because the data is numerical, there is a natural order to the plot and the column widths depend on the data values.

Histograms

In a histogram:

- frequency (number or percentage) is shown on the vertical axis
- the values of the variable being displayed are plotted on the horizontal axis
- each column corresponds to a data value, or a data interval if the data is grouped; alternatively, for ungrouped discrete data, the actual data value is located at the middle of the column
- the height of the column gives the frequency (number or percentage).

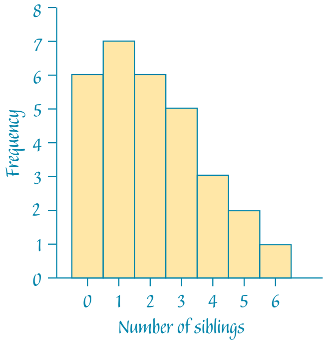
Example 8 Constructing a histogram for ungrouped discrete data

Construct a histogram for the data in the frequency table.

Siblings	Frequency
0	6
1	7
2	6
3	5
4	3
5	2
6	1
Total	30

Solution

1. Label the horizontal axis with the variable name ‘Number of siblings’. Mark in the scale in units, so that it includes all possible values.
2. Label the vertical axis ‘Frequency’. Insert a scale allowing for the maximum frequency of 7. Up to 8 would be appropriate. Mark the scale in units.
3. For each value for the variable draw in a column. The data is discrete, so make the width of each column 1, starting and ending halfway between data values.



For example, the column representing siblings starts at –0.5 and ends at 0.5 and the column representing 2 siblings starts at 1.5 and ends at 2.5. The height of each column is equal to the frequency.

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Example 9 Constructing a histogram for continuous data

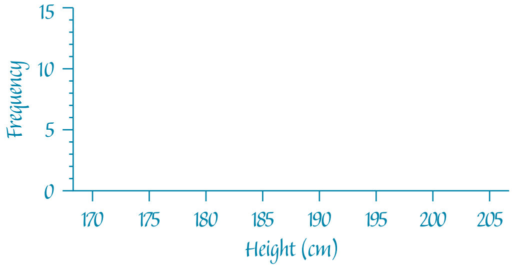
Construct a histogram for the data in the frequency table.

Height (cm)	Frequency
170.0–174.9	4
175.0–179.9	5
180.0–184.9	13
185.0–189.9	9
190.0–194.9	7
195.0–199.9	1
200.0–204.9	2
Total	41

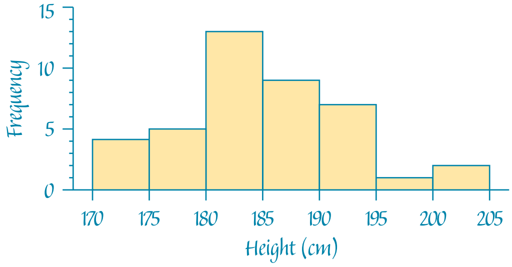
Solution

1 Label the horizontal axis with the variable name 'Height'. Mark in the scale using the beginning of each interval as the scale points; that is, 170, 175, …

2 Label the vertical axis 'Frequency'. Insert a scale allowing for the maximum frequency of 13. Up to 15 would be appropriate. Mark the scale in units.



3 For each interval draw in a column. Each column starts at the beginning of the interval and finishes at the beginning of the next interval. Make the height of each column equal to the frequency.



Desmos widget 9D: Constructing histograms

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Exercise 9D

Constructing frequency tables for numerical data

Example 5 **1** The number of magazines purchased in a month by 15 different people was as follows:

0 5 3 0 1 0 2 4 3 1 0 2 1 2 1

Construct a frequency table for the data, including both the frequency and percentage frequency.

Example 6 **2** The amount of money carried by 20 students is as follows:

$4.55 $1.45 $16.70 $0.60 $5.00 $12.30 $3.45 $23.60 $6.90 $4.35
$0.35 $2.90 $1.70 $3.50 $8.30 $3.50 $2.20 $4.30 $0.00 $11.50

Construct a frequency table for the data, including both the number and percentage in each category. Use intervals of $5, starting at $0.

Analysing frequency tables and constructing histograms

Example 7 **3** A group of 28 students were asked to draw a line that they estimated to be the same length as a 30 cm ruler. The results are shown in the frequency table below.

	Frequency	
Length of line (cm)	Number	%
28.0–28.9	1	3.6
29.0–29.9	2	7.1
30.0–30.9	8	28.6
31.0–31.9	9	32.1
32.0–32.9	7	25.0
33.0–33.9	1	3.6
Total	28	100.0

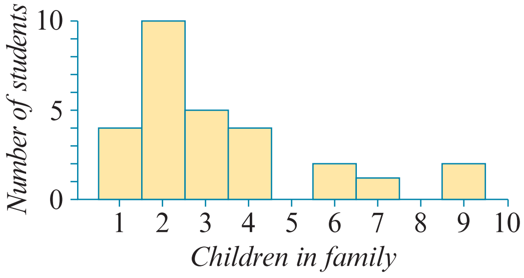
a How many students drew a line with a length:
  i from 29.0 to 29.9 cm?
  ii of less than 30 cm?
  iii of 32 cm or more?

b What percentage of students drew a line with a length:
  i from 31.0 to 31.9 cm?
  ii of less than 31 cm?
  iii of 30 cm or more?

c Use the table to construct a histogram using the counts.

Interpreting histograms

4 The number of children in the family for each student in a class is shown in the histogram.



a How many students are the only child in a family?

b What is the most common number of children in the family for students in this class?

c How many students come from families with 6 or more children?

d How many students are there in the class?

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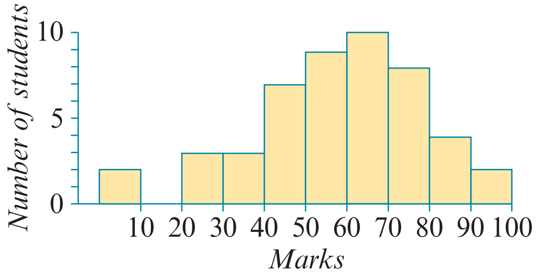
5 The histogram shown gives the scores on a general knowledge quiz for a class of Year 11 students.



a How many students scored from 10 to 19 marks?

b How many students attempted the quiz?

c What is the modal interval?

d If a mark of 50 or more is designated as a pass, how many students passed the quiz?

6 A student purchased 21 new textbooks from a schoolbook supplier. The prices (in dollars) are listed below:

41.65 34.95 32.80 27.95 32.50 53.99 63.99 17.80 13.50 18.99 42.98
38.50 59.95 13.20 18.90 57.15 24.55 21.95 77.60 65.99 14.50

a Construct a histogram with column width 10 and starting point 10.

b For this histogram:

- **i** what is the range of the third interval?
- **ii** what is the 'frequency' for the third interval?
- **iii** what is the modal interval?

CF

7 The maximum temperatures, in degrees Celsius, for a number of capital cities around the world on a particular day were:

17 26 36 32 17 12 32 2 16 15 18 25
30 23 33 33 17 23 28 36 45 17 19 37

a Construct a histogram with column width 2 and starting point 0.

b For this histogram:

- **i** what is the starting point of the second column?
- **ii** what is the 'frequency' for this interval?

c Redraw the histogram with a column width of 5 and a starting point of 0.

d For this histogram:

- **i** how many cities had maximum temperatures from 20°C to 25°C?
- **ii** what is the modal interval?

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9E Characteristics of distributions of numerical data: shape, location and spread

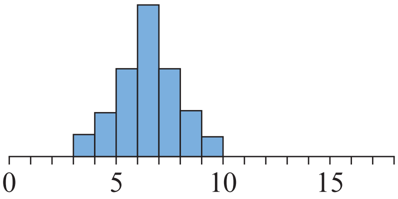
Distributions of numerical data are characterised by their shape and special features such as centre and spread.

▶ Shape of a distribution

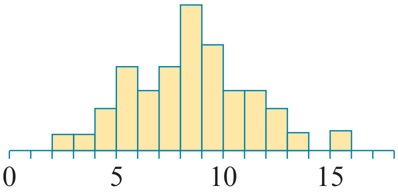
Symmetry and skew

A distribution is said to be **symmetric** if it forms a mirror image of itself when folded in the 'middle' along a vertical axis. Otherwise, the distribution is **skewed**.

Histogram A is symmetric, while histogram B shows a distribution that is approximately symmetric.



Histogram A



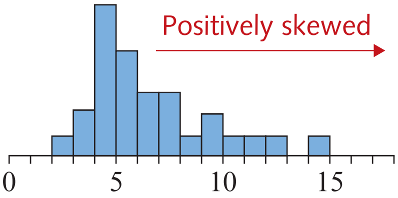
Histogram B

Positive and negative skew

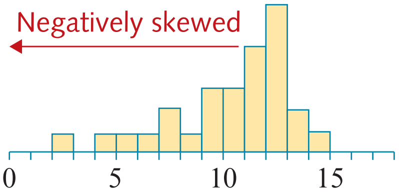
A histogram may be positively or negatively skewed.

- It is **positively skewed** if it has a short tail to the left and a long tail pointing to the right (because of the many relatively short columns towards the positive end of the distribution).
- It is **negatively skewed** if it has a short tail to the right and a long tail pointing to the left (because of the many relatively short columns towards the negative end of the distribution).

Histogram C is an example of a positively skewed distribution, and histogram D is an example of a negatively skewed distribution.



Histogram C



Histogram D

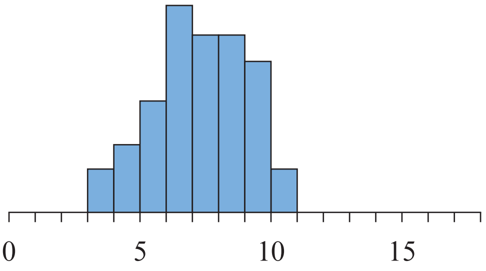
Knowing whether a distribution is skewed or symmetric is important, as this gives considerable information concerning the choice of appropriate summary statistics, as will be seen in the next section.

▶ Location and spread

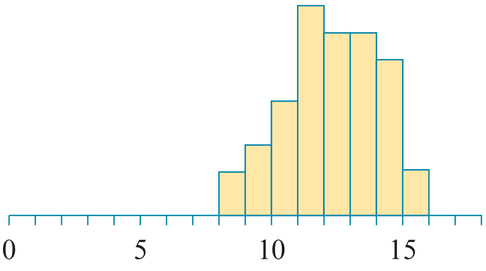
Comparing location

Two distributions are said to differ in **location** if the values of the data in one distribution are generally larger than the values of the data in the other distribution.

Consider, for example, the following histograms shown on the same scale. Histogram F is identical in shape and width to histogram E but moved horizontally several units to the right, indicating that these distributions differ in location.



Histogram E

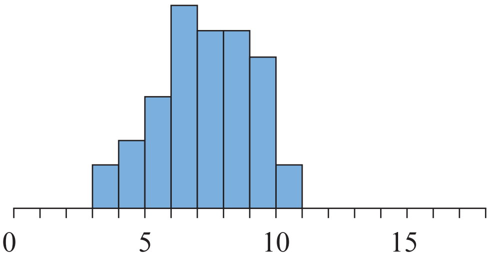


Histogram F

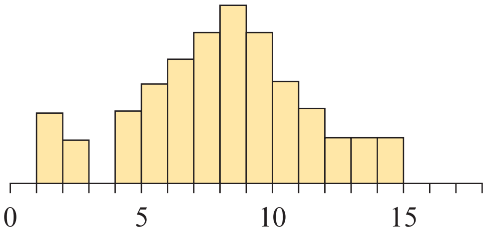
Comparing spread

Two distributions are said to differ in **spread** if the values of the data in one distribution tend to be more variable (spread out) than the values of the data in the other distribution.

Histograms G and H illustrate the difference in spread. Although both are centred at about the same place, histogram H is more spread out.



Histogram G



Histogram H

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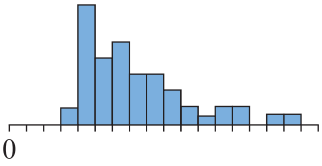
Exercise 9E

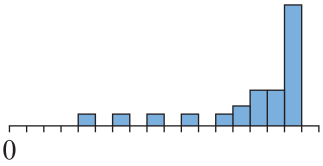
Describing shape using histograms

1 Describe the shape of each of the following histograms.

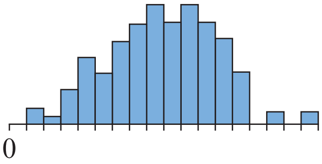
a



b

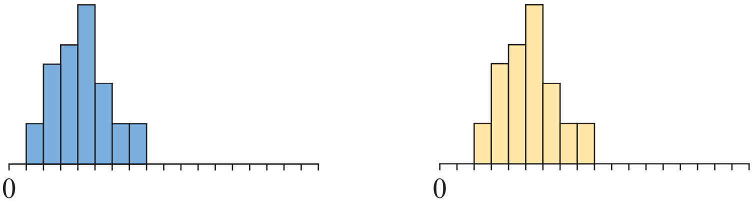


c

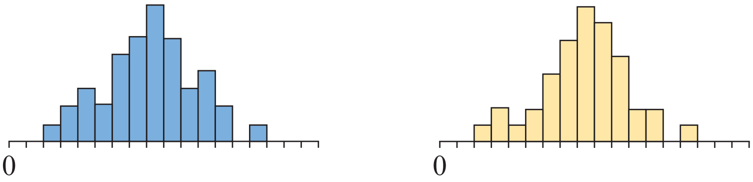


2 Do the following pairs of distributions differ in spread, location, both or neither? Assume that each pair of histograms is drawn on the same scale.

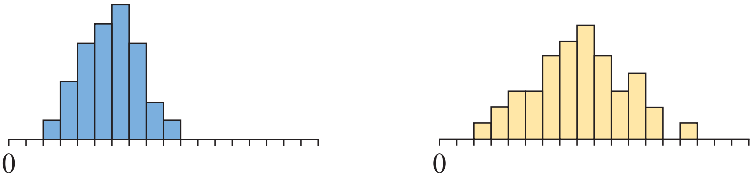
a



b



c



9F Dot plots and stem-and-leaf plots

Dot plots

The simplest display of numerical data is a **dot plot**.

Dot plot

A dot plot consists of a number line on which each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other.

Dot plots are a great way to display fairly small data sets in which the data takes a limited number of values.

Example 10 Constructing a dot plot

The number of hours worked by each of 10 students in their part-time jobs is as follows:

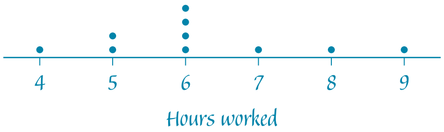
6 9 5 8 6 4 6 7 6 5

Construct a dot plot of these data.

Solution

1 Draw in a number line, scaled to include all data values. Label the line with the variable being displayed.

2 Plot each data value by marking in a dot above the corresponding value on the number as shown.



Stem-and-leaf plots

The **stem-and-leaf plot** or **stem plot** is another plot used for small data sets.

Example 11 Constructing a stem plot

The following is a set of marks obtained by a group of students on a test:

15 2 24 30 25 19 24 33 41 60 42 35 35
28 28 19 19 28 25 20 36 38 43 45 39

Display the data in the form of an ordered stem-and-leaf plot.

Solution

1 The data set has values in the units, tens, twenties, thirties, forties, fifties and sixties. Thus, appropriate stems are 0, 1, 2, 3, 4, 5 and 6. Write these down in ascending order, followed by a vertical line.

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2 Now attach the leaves. The first data value is 15. The stem is 1 and the leaf is 5. Opposite the 1 in the stem, write the number 5, as shown.

0	
1	5
2	
3	
4	
5	
6	

The second data value is 2. The stem is 0 and the leaf is 2. Opposite the 0 in the stem, write the number 2, as shown.

0	2
1	5
2	
3	
4	
5	
6	

Continue systematically working through the data, following the same procedure, until all points have been plotted. You will then have the *unordered* stem plot, as shown.

0	2
1	5 9 9 9
2	4 5 4 8 8 8 5 0
3	0 3 5 5 6 8 9
4	1 2 3 5
5	
6	0

3 Order the leaves in increasing value as they move away from the stem to give the *ordered* stem plot, as shown. Write the name of the variable being displayed (*Marks*) at the top of the plot and add a key (1|5 means 15 marks).

Marks 1|5 means 15 marks

0	2
1	5 9 9 9
2	0 4 4 5 5 8 8 8
3	0 3 5 5 6 8 9
4	1 2 3 5
5	
6	0

It can be seen from the preceding plot that the distribution is approximately symmetric, with one test score, 60, that seems to stand out from the rest. When a value sits away from the main body of the data, it is called an **outlier**.

▶ Choosing between plots

We now have three different plots that can be used to display numerical data: the histogram, the dot plot and the stem plot. They all allow us to make judgements concerning the important features of the distribution of the data, so how would we decide which one to use?

Although there are no hard and fast rules, the following guidelines are often used.

Plot	Used best when	How usually constructed
Dot plot	small data sets (say $n < 30$) discrete data	by hand or with technology when constructing histograms as well
Stem plot	small data sets (say $n < 50$)	by hand
Histogram	large data sets (say $n > 30$)	with technology

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Exercise 9F

Constructing and analysing dot plots

Example 10 **1** The number of children in each of 15 families is as follows:

0 7 2 2 2 4 1 3 3 2 2 2 0 0 1

a Construct a dot plot of the number of children.

b What is the mode of this distribution?

2 A group of 20 people were asked how many times in the last week they had shopped at a particular supermarket. Their responses were as follows:

0 1 1 0 0 6 0 1 2 2 3 4 0 0 1 1 2 3 2 0

a Construct a dot plot of this data.

b How many people did not shop at the supermarket in the last week?

3 The number of goals scored in an AFL game by each player on one team is as follows:

0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 2 2 3 6

a Construct a dot plot of the number of goals scored.

b What is the mode of this distribution?

c What is the shape of the distribution of goals scored?

4 In a study of the service offered at her café, Amanda counted the number of people waiting in the queue every 5 minutes from 12 noon until 1 p.m.

Time	12:00	12:05	12:10	12:15	12:20	12:25	12:30	12:35	12:40	12:45	12:50	12:55	1:00
Number	0	2	4	4	7	8	6	5	0	1	2	1	1

a Construct a dot plot of the number of people waiting in the queue.

b When does the peak demand at the café seem to be?

Constructing and analysing stem plots

Example 11 **5** The marks obtained by a group of students on an English examination are as follows:

92	65	35	89	79	32	38	46	26	43	83	79
50	28	84	97	69	39	93	75	58	49	44	59
78	64	23	17	35	94	83	23	66	46	61	52

a Construct a stem plot of the marks.

b How many students obtained 50 or more marks?

c What was the lowest mark?

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6 The stem plot on the right shows the ages, in years, of all the people attending a meeting.

Age (years) 1 | 2 represents 12 years

0 | 2 7
2 | 1 4 5 5 7 8 9
3 | 0 3 4 4 5 7 8 9
4 | 0 1 2 2 3 3 4 5 7 8 8 8
5 | 2 4 5 6 7 9
6 | 3 3 3 8
7 | 0

a How many people attended the meeting?

b What is the shape of the distribution of ages?

c How many of these people were less than 43 years old?

7 An investigator recorded the amount of time for which 24 similar batteries lasted in a toy. Her results (in hours) were:

26 40 30 24 27 31 21 27 20 30 33 22
4 26 17 19 46 34 37 28 25 31 41 33

a Draw a stem plot of these times.

b How many of the batteries lasted for more than 30 hours?

CF

8 The amount of time (in minutes) that a class of students spent on homework on one particular night were:

10 27 46 63 20 33 15 21 16 14 15
39 70 19 37 56 20 28 23 0 29 10

a Draw a stem plot of these times.

b How many students spent more than 60 minutes on homework?

c What is the shape of the distribution?

9 The prices of a selection of shoes at a discount outlet are as follows:

\$49 \$75 \$68 \$79 \$75 \$39 \$35 \$52 \$149 \$84
\$36 \$95 \$28 \$25 \$78 \$45 \$46 \$76 \$82

a Construct a stem plot of this data.

b What is the shape of the distribution?

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9G Summarising data

A statistic is any number computed from data. Certain special statistics are called **summary statistics**, because they numerically summarise important features of the data set. Of course, whenever any set of data is summarised into just one or two numbers, much information is lost. However, if a summary statistic is well chosen, it may reveal important information hidden in the data set.

For a single data distribution, the most commonly used summary statistics are either measures of centre or measures of spread.

Measures of centre

The mean

The most commonly used measure of the centre of a distribution of a numerical variable is the **mean**. The mean is calculated by summing the data values and then dividing by their number. The mean of a set of data is what many people call the 'average'.

The mean

$$\text{mean} = \frac{\text{sum of data values}}{\text{total number of data values}}$$

For example, consider the set of data: 1, 5, 2, 4

$$\text{Mean} = \frac{1+5+2+4}{4} = \frac{12}{4} = 3$$

Some notation

Because the rule for the mean is relatively simple, it is easy to write in words. However, later you will meet other rules for calculating statistical quantities that are extremely complicated and hard to write out in words. To overcome this problem, we generally use a shorthand notation that enables complex statistical formulas to be written out in a compact form.

In this notation we use:

- the Greek capital letter sigma, $\sum$, as a shorthand way of writing 'sum of'
- a lower case x to represent a data value
- a lower case x with a bar, $\bar{x}$ (pronounced 'x bar'), to represent the mean of the data values
- n to represent the total number of data values.

The rule for calculating the mean then becomes: $\bar{x} = \dfrac{\sum x}{n}$

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Example 12 Calculating the mean

The following data set shows the number of premierships won by each of the current AFL teams, until the end of 2017. Find the mean of the number of premierships won.

Team	Premierships won
Carlton	16
Essendon	16
Collingwood	15
Hawthorn	13
Melbourne	12
Fitzroy / Brisbane Lions	11
Richmond	11
Geelong	9
South Melbourne / Sydney Swans	5
North Melbourne	4
West Coast	3
Adelaide	2
Footscray / Western Bulldogs	2
Port Adelaide	1
St Kilda	1
Fremantle	0
Gold Coast	0
Greater Western Sydney	0

Solution

1 Write down the formula and the value of n.

$$\bar{x} = \frac{\sum x}{n} \qquad n = 18$$

2 Substitute into the formula and evaluate.

$$\bar{x} = \frac{16 + 16 + 15 + \ldots + 1 + 1 + 0 + 0 + 0}{18}$$

$$= \frac{121}{18}$$

3 We do not expect the mean to be a whole number, so give your answer to one decimal place.

$$= 6.7$$

4 Write the answer.

The mean number of premierships was 6.7.

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The median

Another useful measure of the centre of a distribution of a numerical variable is the middle value, or **median**. To find the value of the median, all the observations are listed in order and the middle one is the median.

For example, the median of the following data set is 6, as there are five observations on either side of this value when the data are listed in order.

median = 6
↓
2 3 4 5 5 6 7 7 8 8 11

When there is an even number of data values, the median is defined as the midpoint of the two middle values. For example, the median of the following data set is 6.5, as there are six observations on either side of this value when the data are listed in order.

median = 6.5
↓
2 3 4 5 5 6 7 7 8 8 11 11

Returning to the premiership data. As the data are already given in order, it only remains to determine the middle observation.

As there are 18 entries in the table there is no actual middle observation, so the median is chosen as the value halfway between the two middle observations, in this case the ninth and tenth (5 and 4).

$$\text{Median} = \frac{1}{2}(5 + 4) = 4.5$$

The interpretation here is that, of the teams in the AFL, half (or 50%) have won the premiership 5 or more times and half (or 50%) have won the premiership 4 or less times.

The following rule is useful for locating the median in the stem plot of a larger data set.

Determining the median

To determine the median of a distribution:

- arrange all the observations in ascending order according to size
- if n, the number of observations, is odd, then the median is the $\frac{n+1}{2}$th observation from the end of the list
- if n, the number of observations, is even, then the median is found by averaging the two middle observations in the list. That is, to find the median, the $\frac{n}{2}$ and the $\left(\frac{n}{2}+1\right)$th observations are added together and divided by 2.

Example 13 Determining the median

Find the median age of 23 people whose ages are displayed in this ordered stem plot.

Age (years)   1 | 2 represents 12 years

0	2 5
2	1 4 5 8
3	0 3 4 6
4	0 1 2 5 7
5	2 4 5 8
6	3 5 9 9

Solution

As the data are already given in order, it only remains to determine the middle observation.

1. Write down the number of observations. — $n = 23$
2. The median is located at the $\frac{n+1}{2}$th position. — Median is at the $\frac{23+1}{2}$ = 12th position
3. Evaluate and write the answer. — Thus the median age is 41 years.

Note: We can check to see whether we are correct by counting the number of data values either side of the median. They should be equal.

Comparing the mean and median

In Example 12 we found that the mean number of premierships won by the 18 AFL clubs was $\bar{x} = 6.7$. By contrast, we saw earlier that the median number of premierships won was 4.5.

These two measures are quite different and the interesting question is: Why are they different, and which is the better measure of centre in this situation?

To help us answer this question, consider a stem plot of these data values.

Premierships won

0	0 0 0 1 1 2 2 3 4
0	5 9
1	1 1 2 3
1	5 6 6

From the stem-and-leaf plot it can be seen that the distribution is positively skewed. This example illustrates a property of the mean. When the distribution is skewed or if there are one or two very extreme values, then the value of the mean may be far from the centre. The median is not so affected by unusual observations and always gives the middle value.

▶ Measures of spread

A measure of spread is calculated in order to judge the *variability* of a data set. That is, are most of the values clustered together, or are they rather spread out?

The range

The simplest measure of spread can be determined by considering the difference between the smallest and the largest observations. This is called the **range**.

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The range

The range (R) is the simplest measure of spread of a distribution.

The range is the difference between the largest and smallest values in the data set.

$$R = \text{largest data value} - \text{smallest data value}$$

Example 14 Finding the range

Consider the marks awarded to a group of students for two different tasks:

Task A

2 6 9 10 11 12 13 22 23 24 26 26 27 33 34
35 38 38 39 42 46 47 47 52 52 56 56 59 91 94

Task B

11 16 19 21 23 28 31 31 33 38 41 49 52 53 54
56 59 63 65 68 71 72 73 75 78 78 78 86 88 91

Find the range of each of these distributions.

Solution

For task A, the minimum mark is 2 and the maximum mark is 94.

Range for task A = 94 − 2 = 92

For task B, the minimum mark is 11 and the maximum mark is 91.

Range for task B = 91 − 11 = 80

The range for task A is greater than the range for task B. Is the range a useful summary statistic for comparing the spread of the two distributions? To help make this decision, consider the stem plots of the data sets:

	Task A
0	2 6 9
1	0 1 2 3
2	2 3 4 6 6 7
3	3 4 5 8 8 9
4	2 6 7 7
5	2 2 6 6 9
6	
7	
8	
9	1 4

	Task B
0	
1	1 6 9
2	1 3 8
3	1 1 3 8
4	1 9
5	2 3 4 6 9
6	3 5 8
7	1 2 3 5 8 8 8
8	6 8
9	1

From the stem-and-leaf plots of the data it appears that the spread of marks for the two tasks is not really described by the range. It is clear that the marks for task A are more concentrated than the marks for task B, except for the two unusual values for task A.

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Another measure of spread is needed, one that is not so influenced by these extreme values. The statistic we use for this task is the **interquartile range**.

The interquartile range

The interquartile range (IQR) gives the spread of the middle 50% of data values.

Determining the interquartile range

To find the interquartile range of a distribution, use these steps:

- Arrange all observations in order according to size.
- Divide the observations into two equal-sized groups, and if n is odd, omit the median from both groups.
- Locate Q_1, the *first quartile*, which is the median of the lower half of the observations, and Q_3, the *third quartile*, which is the median of the upper half of the observations.

The interquartile range IQR is then: IQR = $Q_3 - Q_1$

Definitions of the **quartiles** of a distribution sometimes differ slightly from the one given here. Using different definitions may result in slight differences in the values obtained, but these will be minimal and should not be considered a difficulty.

Example 15 Finding the interquartile range (IQR)

Find the interquartile ranges for tasks A and B in Example 14 and compare.

Solution

1 There are 30 values in total. This means that there are fifteen values in the lower 'half', and fifteen in the upper 'half'. The median of the lower half (Q_1) is the 8th value.	Task A Lower half: 2 6 9 10 11 12 13 ㉒ 23 24 26 26 27 33 34 $Q_1 = 22$
2 The median of the upper half (Q_3) is the 8th value.	Upper half: 35 38 38 39 42 46 47 ㊼ 52 52 56 56 59 91 94 $Q_3 = 47$
3 Determine the IQR.	$\text{IQR} = Q_3 - Q_1 = 47 - 22 = 25$
4 Repeat the process for task B.	Task B $Q_1 = 31$ $Q_3 = 73$ $\text{IQR} = Q_3 - Q_1 = 73 - 31 = 42$
5 Compare the IQR for task A to the IQR for task B.	The IQR shows the variability of task A marks is smaller than the variability of task B marks.

The interquartile range describes the range of the middle 50% of observations. It measures the spread of the data distribution around the median (M). Since the upper 25% and the lower 25% of the observations are discarded, the interquartile range is generally not affected by outliers in the data set, which makes it a reliable measure of spread.

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The standard deviation

The **standard deviation (s)**, measures the spread of a data distribution about the mean ($\bar{x}$).

The standard deviation

The standard deviation is defined to be:

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

where n is the number of data values (sample size) and $\bar{x}$ is the mean.

Although it is not easy to see from the formula, the standard deviation is an average of the squared deviations of each data value from the mean. We work with the *squared* deviations because the sum of the deviations around the mean will always be zero. For technical reasons we average by dividing by $n - 1$, not n. In practice this is not a problem, as dividing by $n - 1$ rather than n generally makes very little difference to the final value.

Normally, you will use your calculator to determine the value of a standard deviation. However, to understand what is involved when your calculator is doing the calculation, you should know how to calculate the standard deviation from the formula.

Example 16 Calculating the standard deviation

Use the formula:

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

to calculate the standard deviation of the data set: 2, 3, 4.

Solution

1 To calculate s, it is convenient to set up a table with columns for:
x, the data values
$(x - \bar{x})$, the deviations from the mean
$(x - \bar{x})^2$, the squared deviations.

	x	$(x-\bar{x})$	$(x-\bar{x})^2$
	2	−1	1
	3	0	0
	4	1	1
Sum	9	0	2

2 First find the mean and then complete the table as shown.

$$\bar{x} = \frac{\sum x}{n} = \frac{2+3+4}{3} = \frac{9}{3} = 3$$

3 Substitute the required values into the formula and evaluate.

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{2}{3-1}} = 1$$

▶ Using technology to calculate summary statistics

Scientific calculator activity 9G: Using a scientific calculator to calculate summary statistics

Spreadsheet **Spreadsheet activity 9G:** Using a spreadsheet to calculate summary statistics

Desmos widget 9G: Constructing summary statistics

Exercise 9G

Calculating the mean, median and IQR without using a calculator

SF

Example 12 **1** Find, without using a calculator, the mean and the median for each of these data sets.

a 2, 5, 7, 2, 9
b 4, 11, 3, 5, 6, 1
c 15, 25, 10, 20, 5
d 101, 105, 98, 96, 97, 109
e 1.2, 1.9, 2.3, 3.4, 7.8, 0.2

Example 14, 15 **2** Find, without using a calculator, the median, IQR and range of each of these ordered data sets.

a 2, 2, 5, 7, 9, 11, 12, 16, 23
b 1, 3, 3, 5, 6, 7, 9, 11, 12, 12
c 21, 23, 24, 25, 27, 27, 29, 31, 32, 33
d 101, 101, 105, 106, 107, 107, 108, 109
e 0.2, 0.9, 1.0, 1.1, 1.2, 1.2, 1.3, 1.9, 2.1, 2.2, 2.9

Example 13 **3** Without using a calculator, determine the median and IQR for the data displayed in the following stem plots.

a Monthly rainfall (mm)

4 | 8 9 9
5 | 0 2 7 7 8 9 9
6 | 0 7

b Battery time (hours)

0 | 4
1 | 7 9
2 | 0 1 2 4 5 6 6 7 7 8
3 | 0 0 1 1 3 3 4 7
4 | 0 1 6

Using a calculator to determine summary statistics

4 The following table gives the area, in hectares, of each of the suburbs of a city:

3.6	2.1	4.2	2.3	3.4	40.3	11.3	19.4	28.4	27.6	7.4	3.2	9.0

a Find the mean and the median areas, correct to one decimal place.
b Which is a better measure of centre for this data set? Explain your answer.

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5 The prices, in dollars, of apartments sold in a particular suburb during one month were:

$387 500 $329 500 $293 400 $600 000 $318 000 $368 000 $750 000
$333 500 $335 500 $340 000 $386 000 $340 000 $404 000 $322 000

a Find the mean and the median of the prices.

b Which is a better measure of centre of this data set? Explain your answer.

6 A manufacturer advertised that a can of soft drink contains 375 mL of liquid. A sample of 16 cans yielded the following contents:

357 375 366 360 371 363 351 369
358 382 367 372 360 375 356 371

Find the mean, standard deviation, median, IQR and range for the volume of drink in the cans. Give answers correct to one decimal place.

7 The serum cholesterol levels for a sample of 20 people are listed:

231 159 203 304 248 238 209 193 225 244
190 192 209 161 206 224 276 196 189 199

Find the mean, standard deviation, median, IQR and range of the serum cholesterol levels. Give answers correct to one decimal place.

8 Twenty babies were born at a local hospital on one weekend. Their birth weights are given in the stem plot.

Birth weight (kg) 3 | 6 represents 3.6 kg

2	1 5 7 9 9
3	1 3 3 4 4 5 6 7 7 9
4	1 2 2 3 5

Find the mean, standard deviation, median, IQR and range of the birth weights.

9 The results of a student's chemistry experiment are given:

7.3 8.3 5.9 7.4 6.2 7.4 5.8 6.0

Write all your answers correct to two decimal places.

a **i** Find the mean and the median of the results.

ii Find the IQR and the standard deviation of the results.

b Unfortunately, when the student was transcribing the results into his chemistry book, he made a small error and wrote:

7.3 8.3 5.9 7.4 6.2 7.4 5.8 60

i Find the mean and the median of these results.

ii Find the interquartile range and the standard deviation of these results.

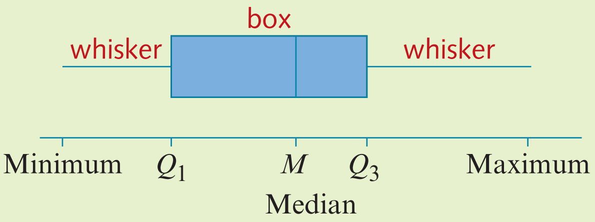
c Describe the effect the error had on the summary statistics in parts **a** and **b**.

9H Boxplots

Knowing the median and quartiles of a distribution means that quite a lot is known about the central region of the data set. If something is known about the tails of the distribution as well, then a good picture of the whole data set can be obtained. This can be achieved by knowing the **maximum** and **minimum** values of the data.

When we list the *median*, the *quartiles* and the *maximum* and *minimum* values of a data sets, we have what is known as a **five-number summary**. Its pictorial (graphical) representation is called a **boxplot** or a box-and-whisker plot.

Boxplots



- A boxplot is a graphical representation of a five-number summary.
- A box is used to represent the middle 50% of scores.
- The median is shown by a vertical line drawn within the box.
- Lines (whiskers) extend out from the lower and upper ends of the box to the smallest and largest data values of the data set respectively.

Example 17 Constructing a boxplot from a five-number summary

The following are the monthly rainfall figures for a year at the weather station.

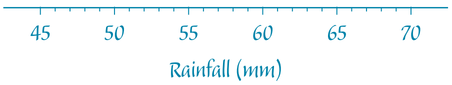
Month	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall (mm)	48	57	52	57	58	49	49	50	59	67	60	59

Construct a boxplot to display this data, given the five-number summary:

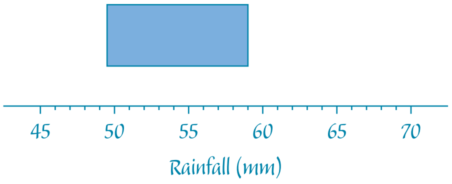
Min = 48, $Q_1 = 49.5$, $M = 57$, $Q_3 = 59$, Max = 67

Solution

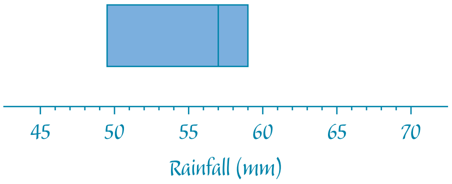
1 Draw a labelled and scaled number line that covers the full range of values.



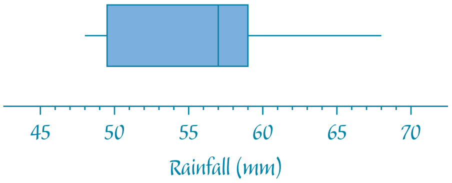
2 Draw a box starting at $Q_1 = 49.5$ and ending at $Q_3 = 59$.



3 Mark in the median value with a vertical line segment at $M = 57$.



4 Draw in the whiskers, lines joining the midpoint of the ends of the box, to the minimum and maximum values, 48 and 67 respectively.



Boxplots with outliers

An extension of the boxplot can also be used to identify possible outliers in a data set.

Outlier

An *outlier* is a data value that appears to be rather different from other observations.

Sometimes it is difficult to decide whether or not an observation is an outlier. For example, a boxplot might have one extremely long whisker. How might we explain this?

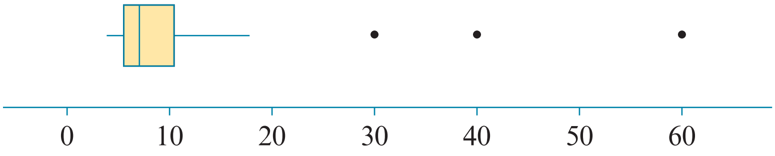
- The data distribution could be extremely skewed with lots of data values in its tail.
- Another explanation is that the long whisker hides one or more outliers.

By modifying the boxplots, we can decide which explanation is most likely.

Designating outliers

Any data point in a distribution that lies more than 1.5 interquartile ranges above the third quartile or more than 1.5 interquartile ranges below the first quartile could be an outlier.

These data values are plotted individually in the boxplot, and the whisker now ends at the largest or smallest data value that is not outside these limits. An example of a boxplot displaying outliers is shown below.



Upper and lower fences

When constructing a boxplot to display outliers, we must first determine the location of what we call the *upper and lower fences*. These are imaginary lines drawn one and a half the interquartile ranges (or box widths) to the left and right of the ends of the box (see next page). Data values outside these fences are classified as possible outliers and plotted separately.

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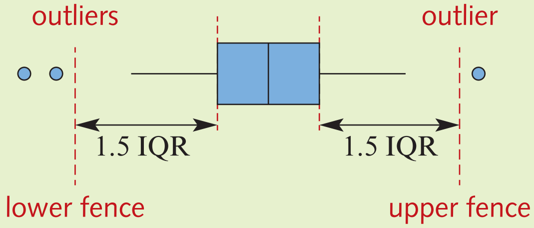
Using a boxplot to display possible outliers

In a boxplot, possible outliers are defined as those values that are:

- greater than $Q_3 + 1.5 \times \text{IQR}$ (upper fence)
- less than $Q_1 - 1.5 \times \text{IQR}$ (lower fence).



When drawing a boxplot, any observation identified as an outlier is indicated by a dot. The whiskers then end at the smallest and largest values that are not classified as outliers.

Example 18 Constructing a boxplot showing outliers

The number of hours that each of 33 students spent on a school project is shown below.

2	3	4	9	9	13	19	24	27	35	36
37	40	48	56	59	71	76	86	90	92	97
102	102	108	111	146	147	147	166	181	226	264

Construct a boxplot for this data set that can be used to identify possible outliers.

Solution

1 From the ordered list, state the minimum and maximum values. Find the median, the $\frac{1}{2}(33 + 1)$th = 17th value.

Min. = 2
Max. = 264
Median = 71

2 Determine Q_1 and Q_3. There are 33 values, so Q_1 is halfway between the 8th and 9th values and Q_3 is halfway between the 25th and 26th values.

$$\text{First quartile, } Q_1 = \frac{24 + 27}{2} = 25.5$$

$$\text{Third quartile, } Q_3 = \frac{108 + 111}{2} = 109.5$$

3 Determine the IQR.

$$\text{IQR} = Q_3 - Q_1 = 109.5 - 25.5 = 84$$

4 Determine the upper and lower fences.

$$\begin{aligned}\text{Lower fence} &= Q_1 - 1.5 \times \text{IQR} \\ &= 25.5 - 1.5 \times 84 \\ &= -100.5\end{aligned}$$

$$\begin{aligned}\text{Upper fence} &= Q_3 + 1.5 \times \text{IQR} \\ &= 109.5 + 1.5 \times 84 \\ &= 235.5\end{aligned}$$

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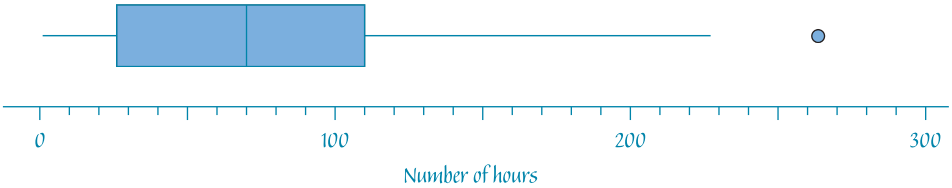
5 Locate any values outside the fences, and the values that lie just inside the limits (the whiskers will extend to these values).

There is one outlier 264.
The largest value that is not an outlier is 226.

6 The boxplot can now be constructed as shown below. The circle denotes the outlier.



There is one possible outlier, the student who spent 264 hours on the project.

Exercise 9H

Constructing a boxplot from a five-number summary

Example 17

SF

1 The heights (in centimetres) of a class of girls are listed below:

160	165	123	143	154	180	133	123	157	157
135	140	140	150	154	159	149	167	176	163
154	167	168	132	145	143	157	156		

The five-number summary for this data is:

Min = 123, $Q_1 = 141.5$, $M = 154$, $Q_3 = 161.5$, Max = 180

Use this five-number summary to construct a boxplot (there are no outliers).

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SF

2 The data shows how many weeks each of the singles in the Top 41 has been in the charts, in a particular week.

24	11	5	7	4	15	13	4	12	14	3	12	4	4
3	10	17	8	6	2	18	15	5	6	9	14	4	5
14	12	16	11	6	7	12	4	16	2	8	10	1	

The five-number summary for this data is:

Min = 1, $Q_1 = 4$, $M = 8$, $Q_3 = 13.5$, Max = 24

Use this five-number summary to construct a boxplot (there are no outliers).

Example 18

3 The amount of pocket money paid per week to a sample of Year 8 students is:

$5.00	$10.00	$12.00	$8.00	$7.50	$12.00	$15.00
$10.00	$10.00	$0.00	$5.00	$10.00	$20.00	$15.00
$26.00	$13.50	$15.00	$5.00	$15.00	$25.00	$16.00

The five-number summary for this data is:

Min = 0, $Q_1 = 7.75$, $M = 12$, $Q_3 = 15$, Max = 26

Use this five-number summary to construct a boxplot (there is one outlier).

Constructing boxplots from raw data

CF

4 The lengths of time, in years, that employees have been employed by a company are:

5	1	20	8	6	9	13	15	4	2
15	14	13	4	16	18	26	6	8	2
6	7	20	2	1	1	5	8		

Construct a boxplot of this data.

5 The times, in seconds, that 35 children took to tie up a shoelace are:

8	6	18	39	7	10	5	8	6	14	11	10
8	35	6	6	14	15	6	7	6	5	8	11
8	15	8	8	7	8	8	6	29	5	7	

Construct a boxplot of this data.

6 A researcher is interested in the number of books people borrow from a library. She selected a sample of 38 people and recorded the number of books each person had borrowed in the previous year. Here are her results:

7	28	0	2	38	18	0	0	4	0	0	5	13
2	13	1	1	14	1	8	27	0	52	4	11	0
0	12	28	15	10	1	0	2	0	1	11	0	

a Identify any possible outliers and write down their values.

b Construct a boxplot of the data, showing outliers.

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7 The following table gives the prices for houses sold in a particular suburb in one month (in thousands of dollars):

356	366	375	389	432
445	450	450	495	510
549	552	579	585	590
595	625	725	760	880
940	950	1017	1180	1625

a Identify any possible outliers and write down their values.

b Construct a boxplot of the data showing outliers.

8 The time taken, in seconds, for a group of children to complete a puzzle is:

8	6	18	39	7	10	5	8	6	14	11	5
10	8	60	6	6	14	15	6	7	6	5	7
8	11	8	15	8	8	7	8	8	6	29	

a Identify any possible outliers and write down their values.

b Construct a boxplot of the data showing outliers.

9 The percentage of people using the internet in 23 countries is given in the table:

Country	Internet users (%)	Country	Internet users (%)
Afghanistan	5.45	Italy	55.83
Argentina	55.80	Malaysia	65.80
Australia	79.00	Morocco	55.42
Brazil	48.56	New Zealand	82.00
Bulgaria	51.90	Saudi Arabia	54.00
China	42.30	Singapore	72.00
Colombia	48.98	Slovenia	68.35
Greece	55.07	South Africa	41.00
Hong Kong SAR, China	72.90	United Kingdom	87.48
		United States	79.30
Iceland	96.21	Venezuela	49.05
India	12.58	Vietnam	39.49

a Identify any possible outliers and write down their values.

b Construct a boxplot of the data, showing outliers.

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9I Comparing data for a numerical variable across two or more groups

It makes sense to compare the distributions of data sets when they are concerned with the *same* numerical variable, for example *height* measured for different groups of people, such as a basketball team and a gymnastic team.

For example, it would be useful to compare the distributions for each of the following:

- the maximum daily temperatures in Townsville in March and the maximum daily temperatures in Brisbane in March
- the test scores for a group of students who did not have a revision class and the test scores for a group of students who had a revision class.

For each of these examples we can actually identify *two variables*. One is a *numerical variable* and the other is a *categorical variable*.

For example:

- The variable *maximum daily temperature* is numerical but the variable *city*, which takes the values ‘Townsville’ or ‘Brisbane’, is categorical.
- The variable *test score* is numerical but the variable *attended a revision class*, which takes the values ‘yes’ or ‘no’, is categorical.

Thus, when we compare two data sets in this section, we will be actually investigating the relationship between two variables: a numerical variable and a categorical variable.

The outcome of these investigations will be a brief written report that compares the distribution of the numerical variable across two or more groups defined as categorical variables. The starting point for these investigations will be, as always, a graphical display of the data. To this end, you will meet and learn to interpret two new graphical displays: the **back-to-back stem plot** and the **parallel boxplot**.

Comparing distributions using back-to-back stem plots

A back-to-back stem plot differs from the stem plots you have already met in that it has a single stem with two sets of leaves, one for each of the two groups being compared. The leaves for one set of data are on one side of the stem, and the leaves for the second data set are on the other side.

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Example 19 Comparing distributions using back-to-back stem plots

The following back-to-back stem plot displays the distributions of life expectancies for males (in years) in several countries in the years 1970 and 2010.

In this situation, *life expectancy* is the numerical variable. *Year*, which takes the values 1970 and 2010, is the categorical variable.

Male life expectancy (in years)

1970		2010
8	3	
	4	
9 4 2	5	8
9 9 9 8 7 7 1 0	6	9
4	7	1 2 4 4 6 8 9 9
	8	0 0 0

5 | 8 = 58 years

1970	2010
M = 67 years	M = 76 years
IQR = 12.5 years	IQR = 8 years

Use the back-to-back stem plot and the summary statistics provided to compare these distributions in terms of centre and spread, and draw an appropriate conclusion.

Solution

1 Centre: Use the medians to compare centres.	The median life expectancy of males in 2010 (M = 76 years) was nine years higher than in 1970 (M = 67 years).
2 Spread: Use the IQRs to compare spreads.	The spread of life expectancies of males in 2010 (IQR = 8 years) was less than the spread in 1970 (IQR = 12.5).
3 Conclusion: Use the above observations to write a general conclusion.	In conclusion, the median life expectancy for these countries has increased over the last 40 years, and the variability in life expectancy between countries has decreased.

▶ Comparing distributions using parallel boxplots

Back-to-back stem plots can be used to compare the distribution of a numerical variable across two groups when the data sets are small. Parallel boxplots can also be used to compare distributions. Unlike back-to-back stem plots, boxplots can also be used when there are more than two groups.

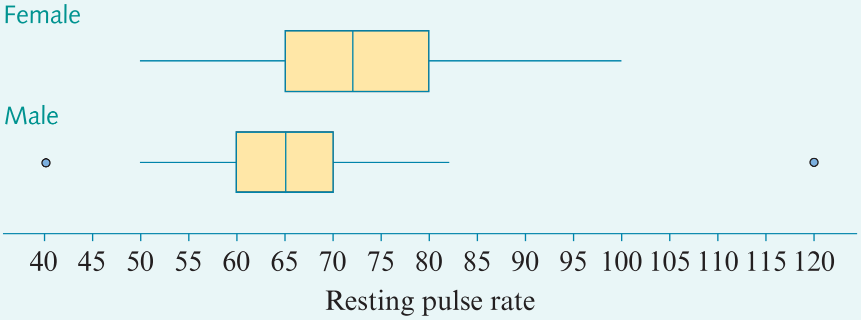
By drawing boxplots on the same axis, both the centre and spread for the distributions are readily identified and can be compared visually.

When comparing distributions of a numerical variable across two or more groups using parallel boxplots, the report should address the key features of:

- shape
- centre (the median)
- spread (the IQR)
- possible outliers.

Example 20 Comparing distributions across two groups using parallel boxplots

Use the following parallel boxplots to compare the pulse rates (in beats/minute) for a group of 70 male students and 90 female students.



Solution

1 Determine the shape for each distribution.

Both distributions are approximately symmetric.

2 Compare the medians by comparing the vertical line in the boxes.

The median for females is about 72, which is higher than that for males, which is about 65.

3 Compare the spread of the two distributions using the IQR, which can be read directly from the boxplot (the width of the boxes).

The IQR for females is 15, which is more than the IQR for males, which is 10.

4 Locate any outliers.

There are no outliers for the females and two for the males, one at 40 and one at 120.

5 Write the report comparing the distribution.

Report

The distributions of resting pulse rates for both male and female students were approximately symmetric. On average, the resting pulse rate for males is lower (median: male = 65, female = 72) and less variable than that for females (IQR: male = 10, female = 15). One male was found to have an extremely low pulse rate of 40, while another had a higher pulse rate than all other males and females.

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Exercise 9I

Comparing groups using back-to-back stem plots

Example 19

1 The stem plot displays the age distribution of ten females and ten males admitted to a regional hospital on the same day.

Females		Males
9	0	
5 0	1	3 6
7	2	1 4 5 6 7
7 1	3	4
3 0	4	0 7
0	5	
	6	
9	7	

4 | 0 = 40 years

a Calculate the median and the IQR for admission ages of the females and males in this sample.

b Write a report comparing these distributions.

2 The stem plot opposite displays the mark distribution of students from two different mathematics classes (Class A and Class B) who sat the test. The test was marked out of 100.

Marks %

Class B		Class A
3 2	1	9
	2	2
	3	9
	4	5 7 8
	5	5 8
9	6	5 8
6 4 3 3 2 2 1 0 0	7	1 6 7 9 9
8 8 4 4 3 2 1 1 0 0	8	0 1 2 2 5 5 9
8 1	9	1 9

7 | 1 = 71

a How many students in each class scored less than 50%?

b Determine the median and the IQR for the marks obtained by the students in each class.

c Write a report comparing these distributions.

3 The following table shows the number of nights spent away from home in the past year by a group of 21 Australian tourists and by a group of 21 Japanese tourists:

Australian

3	14	15	3	6	17	2
7	4	8	23	5	7	21
9	11	11	33	4	5	3

Japanese

14	3	14	7	22	5	15
26	28	12	22	29	23	17
32	5	9	23	6	44	19

a Construct a back-to-back stem-and-leaf plot of these data sets.

b Determine the median and IQR for each distribution.

c Write a report comparing the distributions of the number of nights spent away by Australian and Japanese tourists.

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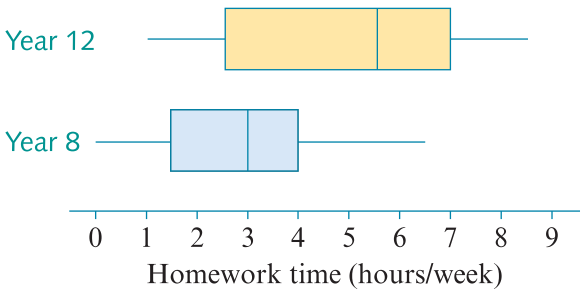
Comparing groups using parallel boxplots

Example 20

4 The boxplots below display the distributions of homework time (in hours/week) of a sample of Year 8 and a sample of Year 12 students.

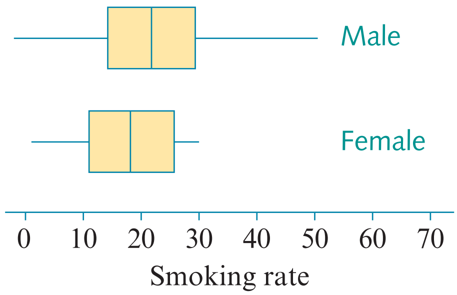


a Estimate the median and IQRs from the boxplots.

b Write a report comparing these distributions.

SF

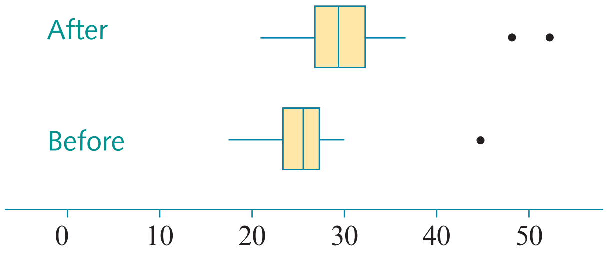
5 The boxplots below display the distribution of smoking rates (%) of males and females from several countries.



a Estimate the median and IQRs from the boxplots.

b Use the information in the boxplots to write a report comparing these distributions.

6 The boxplots below display the distributions of the number of sit-ups a person can do in one minute, both before and after a fitness course.



a Estimate the medians, IQRs and the values of any outliers from the boxplots.

b Write a report comparing these distributions.

CF

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7 To test the effect of alcohol on coordination, 20 randomly selected participants were timed to complete a task with both 0.00% blood alcohol and 0.05% blood alcohol. The times taken (in seconds) are shown in the accompanying table.

0.00% blood alcohol									
38	36	35	35	43	46	42	47	40	48
35	34	40	44	30	25	39	31	29	44

0.05% blood alcohol									
39	32	35	39	36	34	41	64	44	38
43	42	46	46	50	32	32	41	40	50

a Draw boxplots for each of the sets of scores on the same scale.

b Use the information in the boxplots to write a report comparing the distributions of the times taken to complete a task with 0.00% blood alcohol and 0.05% blood alcohol.

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9J Problem-solving using the statistical investigation process

The statistical investigation process is the principal means of problem-solving and modelling in statistics.

You will recall that the **statistical investigation process** encompasses the following steps:

- Pose the question – Decide what data would allow you to address the problem.
- Data – Collect or obtain the data.
- Analyse – Summarise and display the data to answer the question posed.
- Conclusion – Interpret the results and communicate what has been learned.

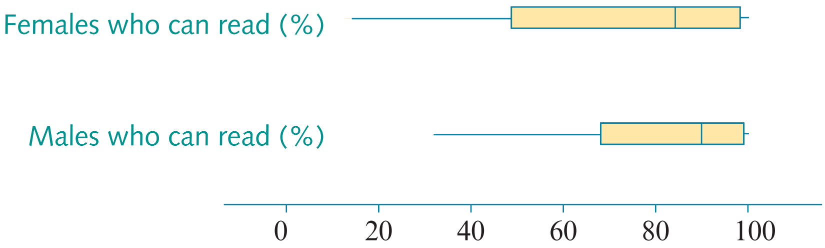
We can best demonstrate this statistical process through the following examples.

Example 21

Do females have the same access to education as males?

Solution

One measure that would allow us to investigate this problem is to look at adult literacy rates across various countries. Thus we can **pose the question** 'How do female literacy rates compare to those of males?'. This **data** can be accessed from the internet, and **analysed** using parallel boxplots of the literacy rates of males and females for 100 different countries:



Based on the analysis, the following report can be written to **answer the question**.

Report
The distribution of literacy rates is extremely negatively skewed for both males and females, with the distribution for males showing much less variation than the distribution for females. This can be seen from the spread of the middle 50% of literacy rates, which was from 67% to 99% for males (IQR = 32) compared to a spread from 48% to 98% for females (IQR = 50).

When comparing the literacy rates for males and females, it can be seen that the literacy rates for males were generally higher than the literacy rates for females. The average literacy rate for males across these countries was 90%, compared with the average literacy rate of 84.5% for females.

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Example 22

Researchers were interested in better understanding how students in Year 11 were spending their time. In particular, they were interested in the impact of ready access to the internet, which was now provided by a range of technologies such as smart phones, tablets and computers, on students' time allocation and whether this had changed over time.

Solution

To investigate this they **posed the question** 'How many hours each week do Year 11 students spend on the internet on any device?'.

Data was collected from a group of Year 11 students who were asked to keep a diary recording the time in hours that they spent on the internet during one week in 2015, and these data were collected again with another group of Year 11 students in 2017.

The data are given in the following table:

2015

5	8	7	18	6	22	11	31	10	4
6	10	15	11	23	5	14	14	0	8
14	14	5	10	5	10	16	3	23	5
7	9	12	14	40	20	15	10	10	10

2017

40	5	42	30	17	24	22	40	13	12
11	8	40	29	15	10	40	10	4	10
10	20	35	23	5	9	6	27	40	0
5	25	10	25	8	10	10	8	14	10

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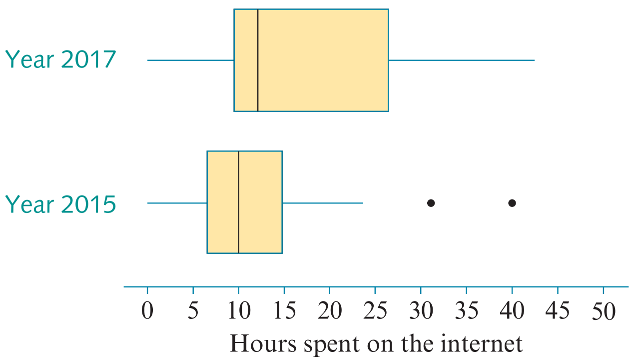
To **analyse the data** the researchers determined the following summary statistics:

	2015	2017
Minimum	0	0
Maximum	40	42
Q_1	6.5	9.5
Median	10	12.5
Q_3	14.5	26
Outliers	32, 40	None

and constructed the following parallel boxplots.



Based on these analyses we can now **answer the question** with the following report.

Report

A study was conducted to investigate the time spent by Year 11 students on the internet. A group of Year 11 students recorded the time they spent on the internet in one week in 2015, and these data were collected again with another group of Year 11 students in 2017. The distributions of time spent on the internet for both groups were found to be positively skewed. On average the time spent on the internet had increased between 2015 (median = 10 hours) and 2017 (median = 12.5 hours) and become much more variable (IQR: 2015= 8 hours, 2017 = 16.5 hours). Although there were two students whose use of the internet in 2015 was unusually high (32 hours and 40 hours respectively), these values were not unusual in 2017, when 25% of students spent more than 26 hours on the internet.

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Exercise 9J

CF

1 Use the following data to investigate and compare the mean number of hours per month that 40 Year 11 students spent watching television in 2015 and in 2017.

2015

30	1	7	27	10	12	30	0	1	2
6	3	10	0	4	10	2	5	40	10
7	4	5	8	24	4	0	5	11	18
25	1	21	10	20	8	5	3	20	2

2017

6	1	3	2	2	13	5	27	8	21
2	11	27	18	8	1	10	6	2	4
6	9	25	23	5	6	8	2	10	0
35	8	2	25	16	5	15	10	14	10

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2 To investigate the age of parents at the birth of their first child, a hospital recorded the ages of the mothers and fathers for the first 20 babies born in the hospital for each of the years 1970, 1990 and 2010.
The data are given below:

1970 Mother									
23	22	33	19	19	26	20	15	26	17
18	31	24	20	29	28	25	45	28	22
1970 Father									
29	15	39	29	22	35	32	26	37	29
25	31	20	34	28	22	33	25	34	46
1990 Mother									
28	14	38	28	21	34	31	25	36	28
24	30	19	33	27	21	32	24	33	45
1990 Father									
31	27	46	31	26	28	30	27	43	37
39	22	27	35	31	29	32	27	38	35
2010 Mother									
30	26	45	32	25	27	29	26	42	36
38	21	26	34	37	28	28	37	37	34
2010 Father									
37	31	39	36	21	34	34	23	17	37
23	33	31	32	24	39	45	30	35	34

Use the data to investigate:

a how the age of mothers has changed over time

b how the age of fathers has changed over time

c the relationship between the age of mothers and the age of fathers, and how this has changed over time.

3 Does Year 12 require more study time than Year 11? Carry out a statistical investigation to investigate this question. You will need to collect appropriate data to carry out this investigation.

4 Are Year 11 students the fittest in the school? Carry out a statistical investigation to investigate this question. You will need to collect appropriate data to carry out this investigation.

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Key ideas and chapter summary

Types of data	Data can be classified as **numerical** or **categorical**.
Frequency table	A **frequency table** is a listing of the values that a variable takes in a data set, along with how often (frequently) each value occurs. **Frequency** can be recorded as the number of times a value occurs or as a **percentage**, that is, the percentage of times a value occurs.
Categorical data	**Categorical data** arises when classifying or naming qualities or attributes. When the categories are naming the groups, the data is called **nominal**. When there is an inherent order in the categories, the data is called **ordinal**.
Column chart	A **column chart** is used to display the frequency distribution of a categorical variable.
Mode, modal category or class	The **mode** (or modal category) is the value of a variable (or the category) that occurs most frequently. The **modal interval**, for **grouped data,** is the interval that occurs most frequently.
Numerical data	**Numerical data** arises from measuring or counting some quantity. **Discrete** numerical data can only take particular values, usually whole numbers, and often arises from counting. **Continuous** numerical data describes numerical data that can take any value, sometimes in an interval, and often arises from measuring.
Histogram	A **histogram** is used to display the frequency distribution of a numerical variable. Histograms are suitable for medium to large-sized data sets.
Stem plot	A **stem plot** is a visual display of a numerical data set, an alternative display to the histogram. Stem plots are suitable for small to medium-sized data sets. Leading digits are shown as the stem and the final digit as the leaf.
Dot plot	A **dot plot** consists of a number line with each data point marked by a dot. Dot plots are suitable for small to medium-sized data sets.
Describing the distribution of a numerical variable	The **distribution of a numerical variable** can be described in terms of **shape** (**symmetric** or **skewed**: positive or negative), **centre** (the midpoint of the distribution) and **spread**.

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Summary statistics	**Summary statistics** are used to give numerical values to special features of a data distribution such as centre and spread.
Mean	The **mean** ($\bar{x}$) is a summary statistic that can be used to locate the centre of a symmetric distribution.
Range	The **range** ***(R)*** is the difference between the smallest and the largest data values. It is the simplest measure of spread. Range = largest value − smallest value
Standard deviation	The **standard deviation** (***s***) is a summary statistic that measures the spread of the data values around the mean.
Median	The **median** ***(M)*** is a summary statistic that can be used to locate the centre of a distribution. It is the midpoint of a distribution, such that 50% of the data values are less than this value and 50% are more. If the distribution is clearly skewed or there are outliers, the median is preferred to the mean as a measure of centre.
Quartiles	**Quartiles** are summary statistics that divide an ordered data set into four equal groups.
Interquartile range	The **interquartile range (IQR)** gives the spread of the middle 50% of data values in an ordered data set. If the distribution is highly skewed or there are outliers, the IQR is preferred to the standard deviation as a measure of spread.
Five-number summary	The median, the first quartile, the third quartile, along with the minimum and the maximum values in a data set, are known as a **five-number summary**.
Outliers	**Outliers** are data values that appear to stand out from the rest of the data set.
Boxplot	A **boxplot** is a visual display of a five-number summary with adjustments made to display outliers separately when they are present.

Skills check

Having completed this chapter you should be able to:

- differentiate between nominal, ordinal, discrete and continuous data
- interpret the information contained in a frequency table
- identify the mode from a frequency table and interpret it
- construct a column chart or histogram from a frequency table
- construct a histogram from raw data using a graphics calculator
- construct a dot plot and stem-and-leaf plot from raw data
- recognise symmetric, positively skewed and negatively skewed distributions
- identify potential outliers in a distribution from its histogram or stem plot
- locate the median and quartiles of a data set and hence calculate the IQR
- produce a five-number summary from a set of data
- construct a boxplot from a five-number summary
- construct a boxplot from raw data using a graphics calculator
- use a boxplot to identify key features of a data set such as centre and spread
- use the information in a back-to-back stem plot or a parallel boxplot to describe and compare distributions
- calculate the mean and standard deviation of a data set
- understand the difference between the mean and the median as measures of centre and be able to identify situations where it is more appropriate to use the median
- write a short paragraph comparing distributions in terms of centre, spread and outliers.

Multiple-choice questions

1 In a survey, a number of people were asked to indicate how much they exercised by selecting one of these options: ‘never’, ‘seldom’, ‘sometimes’ or ‘regularly’. The resulting variable was named *level of exercise*. The type of data generated is:

A variable **B** numerical **C** nominal **D** ordinal **E** metric

2 For which of the following variables is a column chart an appropriate display?

A Weight (kg) **B** Age (years)
C Distance between towns (km) **D** Hair colour
E Reaction time (seconds)

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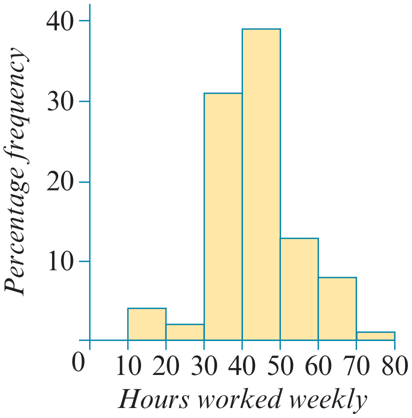
3 For which of the following variables is a histogram an appropriate display?

A Hair colour
B Sex (male, female)
C Distances between towns on a long road trip (km)
D Postcode
E Weight (underweight, average, overweight)

The following information relates to Questions 4 to 7.

The number of hours worked per week by employees in a large company is shown in this percentage frequency histogram.



4 The percentage of employees who work from 20 to fewer than 30 hours per week is closest to:

A 1% **B** 2% **C** 6%
D 10% **E** 33%

5 The percentage of employees who worked fewer than 30 hours per week is closest to:

A 2% **B** 3% **C** 4% **D** 6% **E** 30%

6 The modal interval for hours worked is:

A 10 to less than 20 **B** 20 to less than 30 **C** 30 to less than 40
D 40 to less than 50 **E** 50 to less than 60

7 The median number of hours worked is in the interval:

A 10 to less than 20 **B** 20 to less than 30 **C** 30 to less than 40
D 40 to less than 50 **E** 50 to less than 60

The following information relates to Questions 8 to 11.

A group of 18 employees of a company were asked to record the number of meetings they had attended in the last month.

1 1 2 3 4 5 5 6 7 9 10 12 14 14 16 22 23 44

8 The range of meetings is:

A 22 **B** 23 **C** 24 **D** 43 **E** 44

9 The median number of meetings is:

A 6 **B** 7 **C** 7.5 **D** 8 **E** 9

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10 The mean number of meetings is:

A 7 **B** 8 **C** 9 **D** 10 **E** 11

11 The interquartile range (IQR) of the number of meetings is:

A 0 **B** 4 **C** 9.5 **D** 10 **E** 14

12 The heights of six basketball players (in cm) are:

178.1 185.6 173.3 193.4 183.1 193.0

The mean and standard deviation are closest to:

A mean = 184.4; standard deviation = 8.0

B mean = 184.4; standard deviation = 7.3

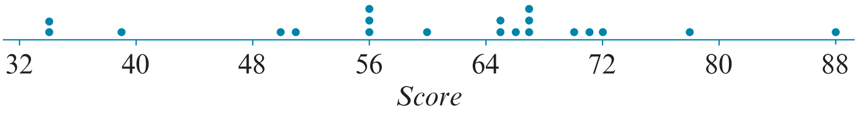
C mean = 182.5; standard deviation = 7.3

D mean = 182.5; standard deviation = 8.0

E mean = 183.1; standard deviation = 7.3

The following information relates to Questions 13 and 14.

The dot plot below gives the examination scores in mathematics for a group of 20 students.



13 The number of students who scored 56 on the examination is:

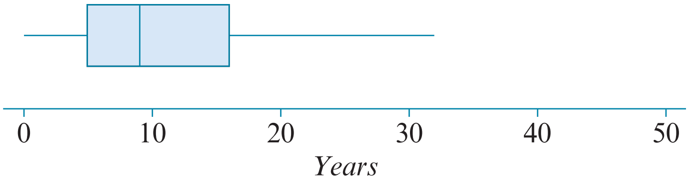
A 1 **B** 2 **C** 3 **D** 4 **E** 5

14 The percentage of students who scored between 40 and 80 on the exam is closest to:

A 60% **B** 70% **C** 80% **D** 90% **E** 100%

The following information relates to Questions 15 to 18.

The number of years for which a sample of people have lived at their current address is summarised in the boxplot.



15 The range is closest to:

A 10 **B** 15 **C** 20 **D** 25 **E** 30

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16 The median number of *years lived at this address* is closest to:

A 5 **B** 9 **C** 12 **D** 15 **E** 47

17 The interquartile range of the number of *years lived at this address* is closest to:

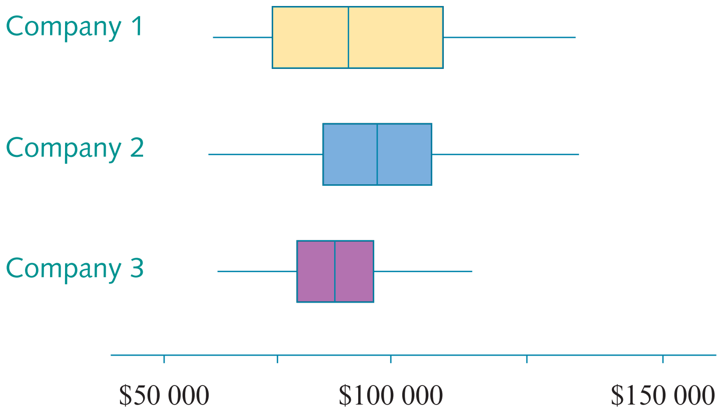
A 5 **B** 10 **C** 15 **D** 20 **E** 45

18 The percentage who have lived at this address for more than 15 years is closest to:

A 10% **B** 25% **C** 50% **D** 60% **E** 75%

The following information relates to Questions 19 to 21.

The amount paid per annum to the employees of each of three large companies is shown in the boxplots.



19 The company with the lowest median wage is:

A Company 1 **B** Company 2 **C** Company 3

D Company 1 and Company 2 **E** Company 2 and Company 3

20 The company with the largest general spread (IQR) in wages is:

A Company 1 **B** Company 2 **C** Company 3

D Company 1 and Company 2 **E** Company 2 and Company 3

21 Which of the following statements is *not* true?

A All workers in Company 3 earned less than $125 000 per year.

B More than half of workers in Company 2 earned less than $100 000 per year.

C 75% of workers in Company 2 earned less than the median wage in Company 3.

D More than half of the workers in Company 1 earned more than the median wage in Company 3.

E More than 25% of the workers in Company 1 earned more than the median wage at Company 2.

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Short-answer questions

SF

1 Classify the data that arises from the following situations as nominal, ordinal, discrete or continuous.

a The number of phone calls a hotel receptionist receives each day

b Interest in politics on a scale from 1 to 5, where 1 = very interested, 2 = quite interested, 3 = somewhat interested, 4 = not very interested, and 5 = uninterested

2 The column chart shows the percentage of working people in a certain town who are employed in private companies, work for the government or are self-employed.

a Is the data categorical or numerical?

b Approximately what percentage of the people are self-employed?

3 A researcher asked a group of people to record how many cigarettes they had smoked on a particular day. Here are her results:

0	0	9	10	23	25	0	0	34	32	0	0	30	0	4
5	0	17	14	3	6	0	33	23	0	32	13	21	22	6

Using class intervals of width 5, construct a histogram of this data.

4 A teacher recorded the time taken (in minutes) by each of a class of students to complete a test:

56	57	47	68	52	51	43	22	59	51	39
54	52	69	72	65	45	44	55	56	49	50

a Construct a dot plot of this data.

b Construct a stem-and-leaf plot of these times.

c Use this stem plot to find the median and quartiles for the time taken.

5 The weekly rentals, in dollars, for a group of people are given below:

285	185	210	215	320	680	280
265	300	210	270	190	245	315

Find the mean and standard deviation, the median and the IQR, and the range of the weekly rentals. Write your answers correct to two decimal places if they are not exact.

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SF

6 Geoff decided to record the time (in minutes) it takes him to complete his mail round each working day for four weeks. His data is recorded below:

170	189	201	183	168	182	161	166	167	173	182	167	188	211
164	176	161	187	180	201	147	188	186	176	174	193	185	183

Find the mean and standard deviation of the times for his mail round, correct to two decimal places.

7 A group of students recorded the number of SMS messages they sent in one 24-hour period. The following five-number summary was obtained from the data set.

Min = 0, $Q_1 = 3$, $M = 5$, $Q_3 = 12$, Max = 24

Use the summary to construct a boxplot of this data.

CF

8 The following data gives the number of students absent from a large secondary college on each of 36 randomly chosen school days:

7	22	12	15	21	16	23	23	17	23	8	16
7	3	21	30	13	2	7	12	18	14	14	0
15	16	13	21	10	16	11	4	3	0	31	44

a Construct a boxplot of this data.

b What was the median number of students absent each day during this period?

c On what percentage of days, correct to one decimal place, were more than 20 students absent?

Extended-response questions

SF

1 The divorce rates (in percentages) of 19 countries are:

27	18	14	25	28	6	32	44	53	0
26	8	14	5	15	32	6	19	9	

a Is the data categorical or numerical?

b Construct an ordered stem plot of divorce rates by hand.

c Construct a dot plot of divorce rates by hand.

d What shape is the distribution of the divorce rates?

e What percentage, correct to two decimal places, of the 19 countries have divorce rates greater than 30%?

f Calculate the mean and median of the distribution of divorce rates.

g Use your calculator to construct a histogram of the data with class intervals of width 10.

  i What is the shape of the histogram?

  ii How many of the 19 countries have divorce rates from 10% to less than 20%?

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2 TransLink has decided to improve its service on the Gold Coast line. Trains were timed on the run from Central to Varsity Lakes Station, and their times recorded over a period of six weeks at the same time each day.
The journey times are shown below (in minutes):

60	61	70	72	68	80	76	65	69	79	82
90	59	86	70	77	64	57	65	60	68	60
63	67	74	78	65	68	82	89	75	62	64
58	64	69	59	62	63	89	74	60		

a Use your calculator to construct a histogram of the times taken for the journey from Central to Varsity Lakes.

- **i** On how many days did the trip take 65–69 minutes?
- **ii** What shape is the histogram?
- **iii** What percentage of trains, correct to one decimal place, took less than 65 minutes to reach Varsity Lakes?

b Use your calculator to determine the following summary statistics for taken *journey times* (correct to two decimal places):

$$\bar{x},\ s,\ \text{Min},\ Q_1,\ M,\ Q_3,\ \text{Max}$$

c Use the summary statistics to complete the following report.

- **i** The mean time taken from Central to Varsity Lakes was ☐ minutes.
- **ii** 50% of the trains took more than ☐ minutes to travel from Central to Varsity Lakes.
- **iii** The range of travelling times was ☐ minutes, while the interquartile range was ☐ minutes.
- **iv** 25% of trains took more than ☐ minutes to travel to Varsity Lakes.
- **v** The standard deviation of travelling times was ☐ minutes.

d Summary statistics for the year before the improvements were made are:

$$\text{Min} = 55, \quad Q_1 = 65, \quad M = 70, \quad Q_3 = 89, \quad \text{Max} = 99$$

Construct boxplots for the last year and for the data from after the improvements.

e Use the information from the boxplots to write a report comparing the distribution of travelling times for the two transport corporations.

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10

Revision of Unit 2
Chapters 6–9

UNIT 2 APPLIED TRIGONOMETRY, ALGEBRA, MATRICES AND UNIVARIATE DATA

Topic 1: Applications of trigonometry

Topic 2: Algebra and matrices

Topic 3: Univariate data analysis

The revision exercises are arranged by chapter with these categories of questions:

- Simple familiar question types
- Complex familiar question types
- Complex unfamiliar question types
- Problem-solving and modelling questions
- Problem-solving and modelling investigations

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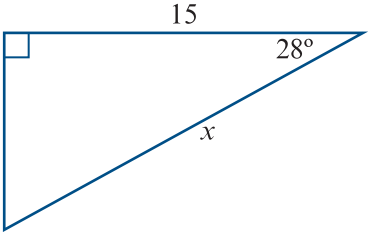
10A Revision of Chapter 6 Applied trigonometry

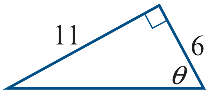
Multiple-choice questions – see Interactive Textbook

▶ Simple familiar questions

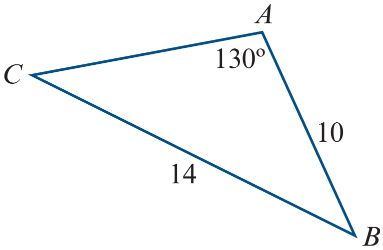
1 Find the unknown side x, correct to two decimal places.



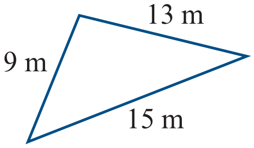
2 Find the angle θ in the right-angled triangle shown, correct to one decimal place.



3 Find angle C in the triangle shown, correct to one decimal place.



4 The boundary of a garden is shown in the diagram. Find the area to the nearest square metre.

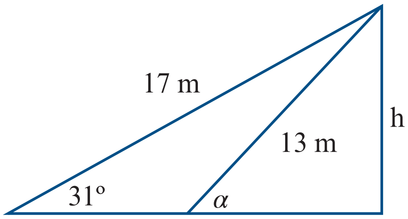


▶ Complex familiar questions

5 A rope 17 m long makes an angle of elevation of 31° with the top of a pole.

a Find the height, h, of the pole, correct to one decimal place.

b Find the angle of elevation, α, to the nearest degree, that a rope 13 m long would make with the top of the pole.



6 A surveyor left a road that runs north-south and walked on a true bearing of 220° for 5 km.

a **i** What is the shortest distance back to the road, correct to one decimal place?

ii State the true bearing of the shortest path back to the road.

b The surveyor decided to retrace her steps 5 km to return to her starting point. What was the true bearing of the direction she must travel?

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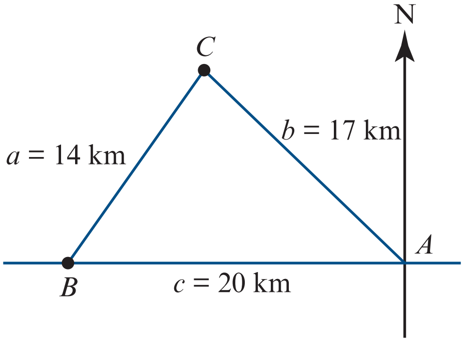
7 A ship left port A and sailed for 20 km west to port B. Another ship also left port A but sailed 17 km to port C. The distance between ports B and C is 14 km.



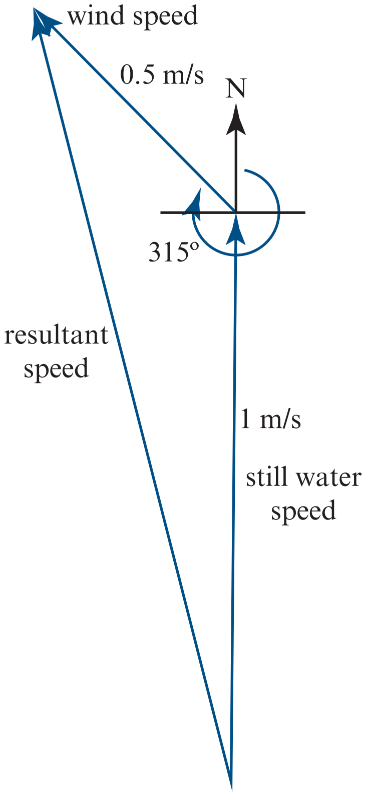
- **a** What was the angle between the directions of the two ships as they left port A? Give the angle correct to one decimal place.
- **b** Find the true bearing of port C from port A to one decimal place.
- **c** What is the true bearing of port C from port B?

► Complex unfamiliar questions

8 A vector can be used to represent the velocity of an object. The vector is drawn as an arrow. The length of the arrow is a measure of the speed of the object. The direction of the arrow gives the direction in which the object is travelling.

Lucy's boat travels in the still water of a sheltered river at a speed of 1 m/s. She points her boat north as it enters a lake, but the wind speed of 0.5 m/s true bearing 315° combines with the power of her boat to change her speed and direction.

The head to tail addition of the vectors representing her boat's still water speed and the wind's speed produce a resultant vector. The resultant vector shown in the diagram represents her resultant direction and speed.



- **a** Find the resultant speed of Lucy's boat, correct to one decimal place.
- **b** Find the true bearing of the resultant direction she travels, to the nearest degree.

Lucy's brother Jack also enters the lake pointing his boat north. He wants to achieve a resultant bearing of 350° to reach a town 3 km across the lake.

- **c** Given the wind speed is 0.5 m/s, what still water speed will his boat need to produce a resultant bearing of 350°? Give your answer correct to two decimal places.

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d Calculate his resultant speed, correct to two decimal places.

e Determine how long it will take him, to the nearest minute, to travel to the town.

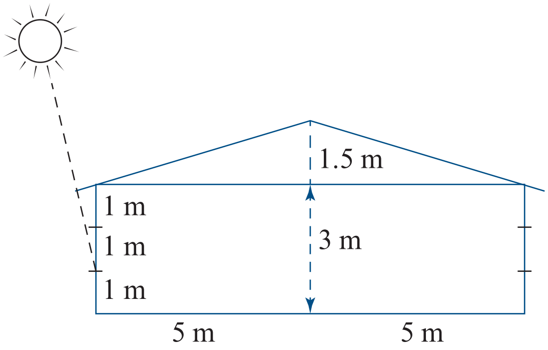
Note: Time taken = distance travelled ÷ speed.
Make sure all units used are in metres, metres per second and seconds.

9 A bushwalker left camp *A* and walked on a true bearing of 020° for 2 kilometres to arrive at camp *B*. From camp *B* she walked on a bearing of 280° until she reached camp *C*. At camp *C* she took a bearing of 150° and walked back to camp *A*. Find the total distance walked, correct to two decimal places.

▶ Problem-solving and modelling questions

The length of roof eaves to shade windows

10 A builder needs to determine the length of roof eaves to shade the windows of a house when the sun is at its highest and most intense. The roof eaves are the part of the roof that hangs out past the walls. The eaves provide some shade for the walls and windows. The windows are one metre above ground level and one metre in height.



a When the sun is at its most intense it makes an angle of 25° with the vertical. Enter this information into the house diagram.

b Find the sloping angle of the roof, correct to one decimal place.

c Find the angle the eaves makes with the house wall, correct to one decimal place.

d Find the required length of the eaves to the nearest centimetre.

e Let the angle the sun makes with the vertical be called θ. Derive a general rule for the required length of the eaves of this house in terms of θ.

f Use the general rule from part **e** to display a table of values for the length of eaves required for a selection of values of θ. Include $\theta = 25°$ to compare with your answer to part **d**.

▶ Problem-solving and modelling investigations

A paradoxical puzzle

11 The goal of this investigation is to explore the mathematics behind this puzzle and to use trigonometry to reveal the trick in the paradox. Check that the square and rectangle below each contain exactly the same four shapes. However, the area of the square is $8 \times 8 = 64$ square units and the area of the rectangle is $5 \times 13 = 65$ square units! Your goal is to discover the subtle trick involved.

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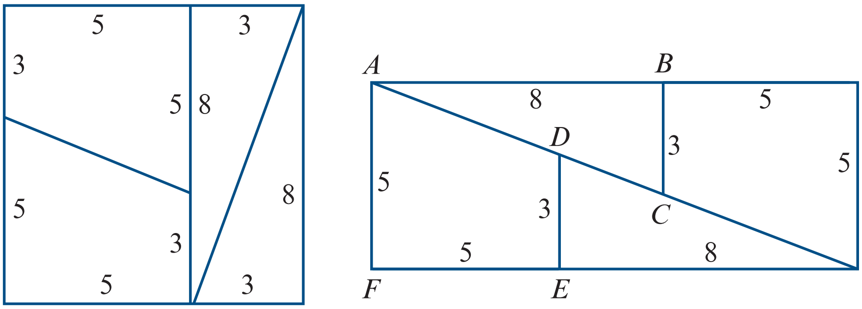

a Find angle *BAC* in triangle *ABC* of the rectangle diagram.

b Find angle *FAD* in parallelogram *ADEF* of the rectangle diagram.

c State the sum of angle *BAC* and angle *FAD*. If the two pieces fit perfectly together as part of a rectangle, what should be the sum of the two angles in parts **a** and **b**? Consider the opposite corner of the rectangle.

d Explain why the sum of the parts do not really form a rectangle with an area of 65 square units.

e Find another method to show the ‘rectangle’ is not truly a rectangle.

10B Revision of Chapter 7 Linear and non-linear relationships

Multiple-choice questions – see Interactive Textbook

▶ Simple familiar questions

1 Solve the following linear equations.

a $2a + 4 = 7a - 9$

b $5(4 - y) + 2(3y + 1) = 27$

c $2(0.9x + 4) = 8$

2 Transpose the following equations for the letter indicated in brackets.

a $A = P + PRT$ (R)

b $C = 2\pi r$ (r)

3 Two consecutive numbers sum to 49. What are the two numbers?

4 A taxi company charges a booking fee of \$5.50 plus \$3.20 per kilometre. How much will you pay if if you travel 19 km?

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▶ Complex familiar questions

5 The formula relating Celsius to Fahrenheit is:

$$F = \frac{9}{5}C + 32$$

- **a** Calculate the temperature in Fahrenheit when it is 38 degrees Celsius.
- **b** Rearrange the formula for C.
- **c** When is the temperature in Celsius equal to the temperature in Fahrenheit?

6 The local convenience store sells pies for \$4.95 each and tomato sauce packets for \$0.50 each.

- **a** Write a formula for the total cost, C, if x pies and y tomato sauce packets are purchased.
- **b** Katie bought 3 pies and 2 packets of tomato sauce. How much did it cost her?
- **c** If the total cost was \$26.25 for 5 pies and an unknown number of packets of tomato sauce, then how many packets of tomato sauce were purchased?

7 A plumber charges an upfront cost of \$90 plus \$40 for each hour, t.

- **a** Write an equation for the total cost C in terms of t.
- **b** If the total cost was \$230, for how long did the plumber work?

▶ Complex unfamiliar questions

8 Jacob currently has an average mark of 88% across his first four maths tests. What mark does he need to achieve on his fifth test, to increase his average to 90%?

9 Ally's piggy bank contains \$9.50 in 50 cent and \$1 coins. She has 13 coins in total. How many 50 cent coins and how many \$1 coins does she have?

▶ Problem-solving and modelling questions

10 A rock is thrown into the air from the ground. The height of the rock, h metres, at time t seconds after being thrown is given by the equation $h = 4t - t^2$.

- **a** Complete the following table of values.

t	0	1	2	3	4
h					

- **b** What is the maximum height that the rock reaches?
- **c** At what time does the rock reach its maximum height?
- **d** When is the rock exactly 3 metres above the ground?
- **e** Why are there two answers to part **d**?
- **f** What is the height of the rock at $t = 3.5$ seconds?
- **g** Find another time at which the rock is at the height found in part **f**. Consider the symmetry of the table.

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Problem-solving and modelling investigations

11 The area of a rectangle with length l metres and width w metres can be calculated using the formula $A = lw$.

a What is the area if $l = 6$ and $w = 4$?

Suppose the perimeter of a rectangle is 8 metres.

b Calculate the dimensions of the rectangle given that the length is 1 metre more than the width.

c Calculate the area of the rectangle if the length is 1 metre more than the width.

d Calculate the area of the rectangle if the width w is 1 metre.

e Calculate the area of the rectangle if:

  i $l = 1$ metres

  ii $l = 2$ metres

  iii $l = 3$ metres.

f Which value of l in part **e** gave the largest area, and do you think this is the maximum possible area for a rectangle with a perimeter of 8 m? How could you check this?

g What do you notice about the shape of the rectangle when the area is at a maximum?

10C Revision of Chapter 8 Matrices and matrix arithmetic

Multiple-choice questions – see Interactive Textbook

Simple familiar questions

1 The prices shown in the matrix must be increased by 10% to allow for GST. Display the final prices in a clearly labelled matrix.

$$\begin{array}{c|ccc|} & \textit{Shorts} & \textit{Tops} & \textit{Joggers} \\ \textit{Women} & 15 & 7 & 20 \\ \textit{Men} & 10 & 8 & 25 \end{array}$$

2 Find $2A - 3B$.

$$A = \begin{bmatrix} 7 & -4 \\ -8 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 5 & -3 \\ -5 & 2 \end{bmatrix}$$

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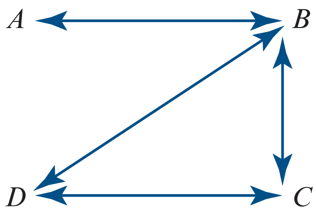
3 The hours Tim and Tam were jogging and swimming in the previous week are shown in matrix H. The kilojoules per hour, indicating the rate energy that was used for each activity, are listed in matrix K.

$$H = \begin{array}{c} \\ Tim \\ Tam \end{array} \begin{array}{c} \begin{array}{cc} \textit{Jogging} & \textit{Swimming} \end{array} \\ \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix} \end{array} \qquad K = \begin{array}{c} \\ Jogging \\ Swimming \end{array} \begin{array}{c} \textit{Kilojoules} \\ \begin{bmatrix} 2500 \\ 1600 \end{bmatrix} \end{array}$$

a To find the number of kilojoules each person used, should $H \times K$ or $K \times H$ be calculated?

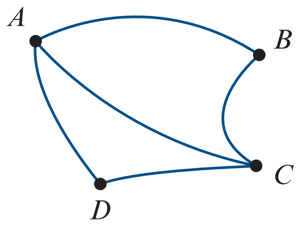
b Use the appropriate matrix product to obtain a matrix showing the total kilojoules each person used.

4 Write the matrix for the communication diagram. Use the number 1 when direct communication exists between two people and 0 for no direct communication.



Complex familiar questions

5 Bus routes connecting the towns Abel, Banach, Cantor and Dedekind are shown in the diagram. The first letter of each town is used.



a Use a matrix R to represent the bus routes. Label the columns and rows in alphabetical order using the first letter of each town's name. Write 1 when two towns are directly connected by a bus route and write 0 if they are not connected.

b What does the sum of column B reveal about the town of Banach?

c **i** Find R^2.

ii How many ways are there to travel from Banach to a town via another town? Include ways that start and end at Banach.

iii List the possible ways of part **ii**.

6 Three retail stores, X, Y and Z are preparing their statements for the previous financial year. The costs of wages and of purchasing stock for different departments of the three stores are shown in the Costs matrix. The values of sales are shown in the Sales matrix. Figures are given in millions of dollars.

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Costs **Sales**

$$\begin{array}{c|ccc|} & \textit{TVs} & \textit{Computers} & \textit{DVDs} \\ X & 8 & 11 & 4 \\ Y & 7 & 12 & 5 \\ Z & 6 & 10 & 3 \end{array} \qquad \begin{array}{c|ccc|} & \textit{TVs} & \textit{Computers} & \textit{DVDs} \\ X & 12 & 15 & 5 \\ Y & 8 & 14 & 6 \\ Z & 7 & 12 & 5 \end{array}$$

a Write a matrix showing the profit for each department of each store.

b Tax is calculated at 20% of the profit. Use a matrix to show the tax payable on sales for each department of each store.

c Show the after-tax profit in a matrix.

7 The day's sales at Sam's Café were 38 sandwiches, 23 cakes and 89 coffees. A sandwich costs \$7, a cake costs \$3 and coffee costs \$4.

a Record the number of each type of item in a row matrix.

b Write the cost of each item in a column matrix.

c Use matrix multiplication to find the total value of the day's sales.

▶ Complex unfamiliar questions

8 The results of three students in four exams are shown in matrix R.

$$R = \begin{array}{r|cccc|} & Ex1 & Ex2 & Ex3 & Ex4 \\ Abid & 88 & 79 & 91 & 75 \\ Boyd & 86 & 81 & 93 & 80 \\ Chaz & 82 & 74 & 79 & 83 \end{array} \qquad E = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

a **i** Suggest a matrix S that will multiply matrix R to give the sum of each student's results.

ii Should $S \times R$ or $R \times S$ be used in part **i**?

iii Use the appropriate matrix multiplication to display the sum of each student's results.

iv Calculate each student's average exam result.

b Find $R \times E$ and interpret the result.

c **i** State a matrix T that will multiply R to give the totals of how students scored in each separate exam.

ii Should $T \times R$ or $R \times T$ be used in part **i**?

iii Use matrix multiplication to find the total of the scores in each exam.

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9 A family owns 5 rabbits, 2 goats and 4 cavies.

a In one week each rabbit eats 4 serves of oats, 1 serve of wheat and 2 serves of carrots. Each goat eats 7 serves of oats, no wheat and 6 serves of carrots. Each cavy eats 2 serves of oats, 2 serves of wheat and 1 serve of carrots.

Display this information in a matrix E with Oats, Wheat and Carrots as column headings.

b **i** Write a matrix A that could multiply matrix E to give the total weekly consumption for each of oats, wheat and carrots.

ii Should $A \times E$ or $E \times A$ be used to obtain the result required in part **i**?

iii Use the appropriate matrix multiplication to obtain matrix T, the total weekly consumption for oats, wheat and carrots.

c Oats cost \$3 per serve, wheat costs \$2 per serve and carrots cost \$1 per serve.

i Show this information in a matrix C that could multiply matrix T to obtain the total weekly cost of feeding the animals.

ii Use the appropriate matrix multiplication of C and T to find the total weekly cost.

Problem-solving and modelling questions

Using row and column matrices to extract information

10 Matrix F records the number of pigs, goats and sheep on farms X and Y.

$$F = \begin{array}{cc} & \begin{array}{ccc} P & \;\;G & \;\;S \end{array} \\ \begin{array}{c} X \\ Y \end{array} & \begin{bmatrix} 20 & 30 & 50 \\ 15 & 20 & 60 \end{bmatrix} \end{array} \qquad A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

a Evaluate $F \times A$.

b Describe the information given by $F \times A$ and explain why the product of the two matrices gives that information.

c Evaluate $B \times F$.

d Describe the information given by $B \times F$ and explain why the product of the two matrices gives that information.

e Find a matrix C so that $F \times C$ evaluates to give a matrix with just the number of goats on each farm. Explain why $F \times C$ gives that information.

Problem-solving and modelling investigations

The surprising properties of a particular matrix

11 The goal of this investigation is to explore the powers of a given matrix and discover a powerful but simple rule for calculating the results. We will see if other matrices have similar properties.

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a Find the matrix resulting in each case for powers of $n = 1, 2, 3, 4$ and 5.

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^n$$

b Analyse the results of part **a** for each power used and express the answer in the form:

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^n = 2^p \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \text{ where } p \text{ is a whole number.}$$

c Express p in terms of n and write the equation below in terms of n.

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^n = 2^p \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

d Apply the rule found in part **c** to find:

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^{10}$$

Use technology to check that your answer is correct.

e Choose suitable powers to check if a rule similar to that found in part **c** applies to the matrices below:

i $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

ii $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

f Investigate if a rule exists for:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}^n, \text{ where } n \text{ is a power of the matrix.}$$

g Apply your rule to the case when $n = 4$ and demonstrate that your rule gives the correct answer.

10D Revision of Chapter 9 Univariate data analysis

Multiple-choice questions – see Interactive Textbook

▶ Simple familiar questions

1 Classify each of the following variables as nominal, ordinal, or numerical.

a The price of petrol per litre

b Eye colour

c Coffee size (small, medium, large, extra-large)

2 Find the median of this data set:

1 2 2 3 3 4 4 5 5 16

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3 The minimum daily temperatures (in degrees Celsius) in Brisbane each July day in a certain year were:

10.9	8.7	7.2	8.8	13.6	10.9	11.2	10.4	10.2	10.7	11.8
13.5	16.8	11.7	12.2	14.0	14.3	15.6	15.6	14.4	14.8	15.8
18.0	13.3	12.7	13.1	14.4	9.2	9.2	8.5	8.5		

a Construct a histogram with a column width of 1 and starting point of 7.

b Determine the percentage of days that July with a minimum temperature less than 10°C.

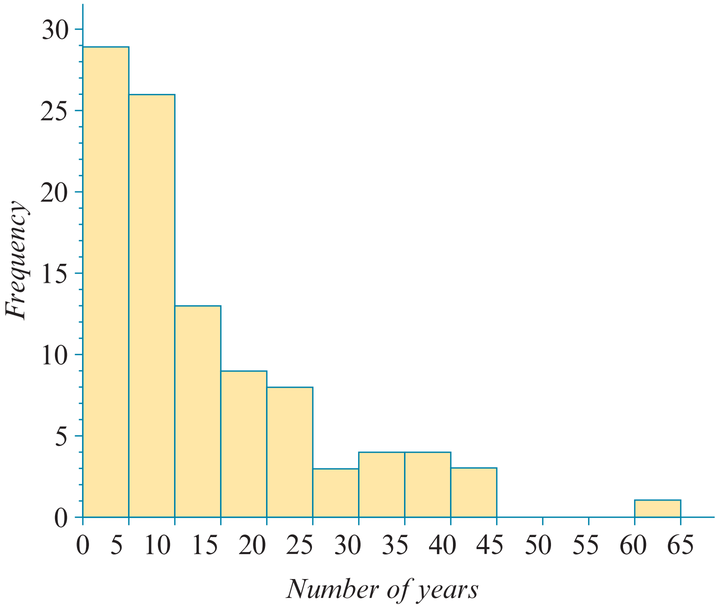
4 A group of 500 people recorded the number of hours per week they watch television, and the following summary statistics were determined:

Minimum	0
Q_1	5
Median	18
Q_3	35
Maximum	92

Use this information to construct a boxplot of this data.

▶ Complex familiar questions

5 A group of 100 people were asked how many years they have been doing their current type of work. Their responses are summarised in this histogram.



For this sample which would be the better measure of centre, the mean or the median? Explain your answer.

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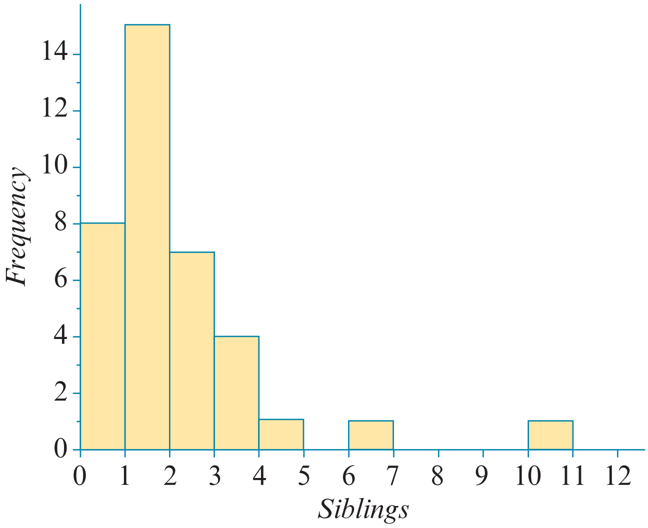
6 A group of 37 people were asked how many siblings they have. The data collected is summarised in the histogram below. Use the histogram to find the median number of siblings for this group.



7 For the following stemplot:

a Determine the values of the upper and lower fences.

b Give the value(s) of any outliers.

0	5 6 6 6 7 8
1	0 0 0 0 0 1 3
1	5 5 5 6 6 8
2	0 0 0 2 3
2	6 6 6 6 6 7
3	1 1 1 3 4
3	5 6 6 6 6 6 7
4	1
4	6
5	2
5	9 9
6	
6	
7	
7	
8	
8	5

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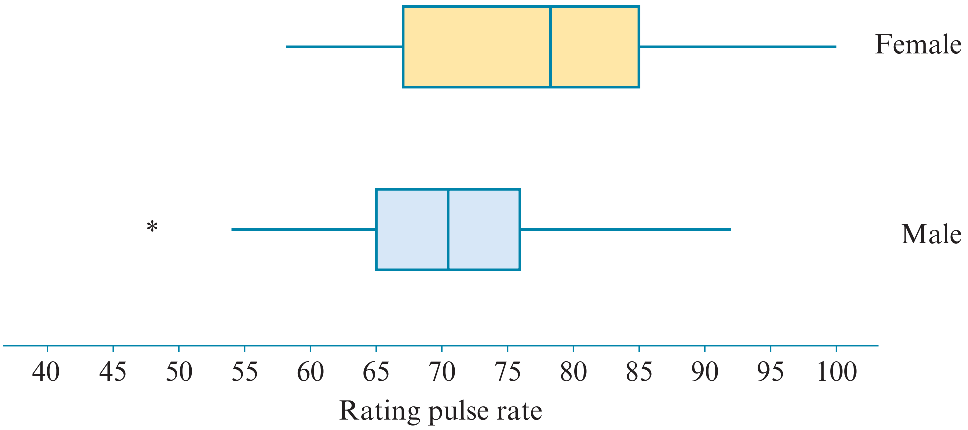
▶ Complex unfamiliar questions

8 Jack is carrying out a chemistry experiment. He repeats the experiment 10 times, and records the results. He calculates the mean of his results, and records this also. Unfortunately, before he gets to write up the experiment, he spills a cup of coffee on his results sheet, and one of the data values is no longer readable.

Find the missing data value if the mean $\bar{x} = 1.84$, and the 9 legible data values are as given below:

1.6 1.8 2.1 2.4 2.3 1.7 1.4 1.6 1.7

9 The parallel boxplots below display the resting pulse rates for a group of females and a group of males.



a Estimate the median, interquartile range and the values of any outliers for each boxplot.

b Write a report comparing these distributions.

▶ Problem-solving and modelling question

Comparing sporting teams

10 Select two teams from a sport of interest to you – football (any code), netball, basketball or cricket, for example. You need to be able to find data such as the age of the players, some physical characteristics such as height, and level of experience such as number of games played. For this information to be available, the players will probably have to be professional teams for which the information is available on the team website or on Wikipedia. Select two teams from that sport, one of which is successful (e.g. won the premiership) and the other of which is not (e.g. came bottom of the ladder).

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Can we find any differences between the two teams that might explain their differing levels of success?

a Let us start by looking at the age of the players.

  i Construct boxplots for age for each of the two teams.

  ii Complete the following table.

Characteristics of boxplot	Team 1	Team 2
Shape of the distribution		
Median		
Range		
Interquartile range		
Mean		
Standard deviation		
Outliers		

  iii Write a report describing and comparing the ages for each team.

b Investigate any other numerical variables which you think might help explain the difference between the teams. Some examples are physical characteristics (eg height of players) and experience (eg number of games played).

c Summarise you findings across all the variables you have considered. Do you have any advice for the coaches of each team?

▶ Problem-solving and modelling investigation

Time spent on social media

11 Investigate the amount of time male and female students spend on social media, using data related to students at your school.

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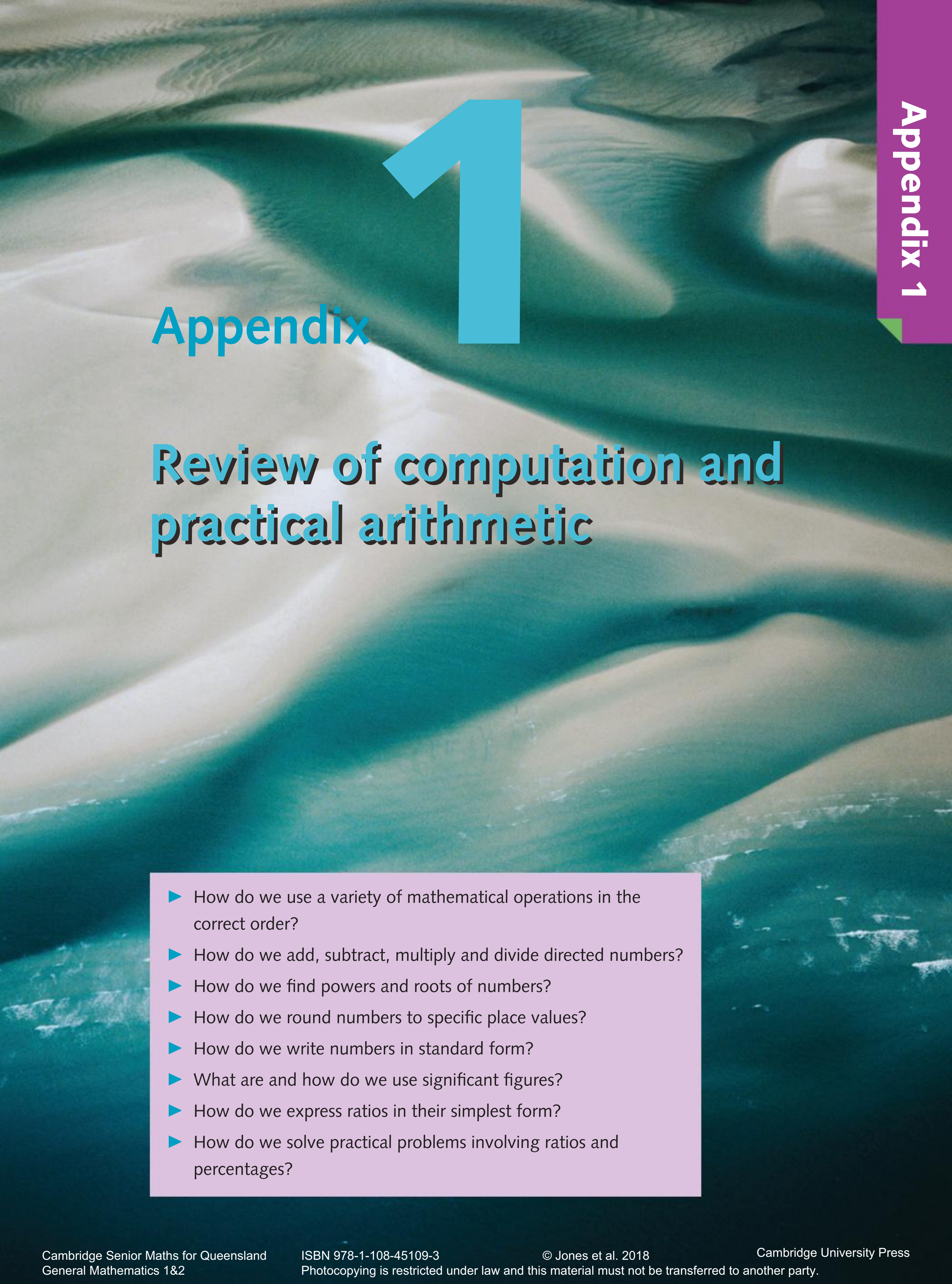

Appendix 1

Review of computation and practical arithmetic

- How do we use a variety of mathematical operations in the correct order?
- How do we add, subtract, multiply and divide directed numbers?
- How do we find powers and roots of numbers?
- How do we round numbers to specific place values?
- How do we write numbers in standard form?
- What are and how do we use significant figures?
- How do we express ratios in their simplest form?
- How do we solve practical problems involving ratios and percentages?

Introduction

This appendix revises basic methods of computation used in general mathematics. It will allow you to carry out the necessary numerical calculations for solving problems. We will begin with the fundamentals.

A1.1 Order of operations

Adding, subtracting, multiplying and dividing are some examples of operations that are used in mathematics. When carrying out a sequence of arithmetic operations, it is necessary to observe a definite sequence of rules. These rules, defining the **order of operations**, have been devised and standardised to avoid confusion.

Order of operation

The rules are to:

- always complete the operations in brackets first
- then carry out the division and multiplication operations (in order, from left to right)
- then carry out the addition and subtraction operations (in order, from left to right).

These rules can also be remembered by using **BODMAS**.

B	**B**rackets come first.
O	If a fraction **O**f a number is required or **O**rders (powers, square roots), you complete that next.
DM	**D**ivide and **M**ultiply, working left to right across the page.
AS	**A**dd and **S**ubtract, working left to right across the page.

A calculator with *algebraic logic* will carry out calculations in the correct order of operations. However, particular care must be taken with brackets.

Pronumeral

A number or **pronumeral** (letter) placed in front of a bracket means that you multiply everything in the bracket by that number or pronumeral.

$4(8)$ means $4 \times 8 = 32$

$5(x - 9) = 5x - 45$

$a(3a + 6) = 3a^2 + 6a$

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Example 1 Using correct order of operation

Evaluate the following.

a $3 + 6 \times 8$ **b** $(3 + 6) \times 8$ **c** $8 \div 2 - 2$ **d** $23 - (8 - 5)$

e $(4)3 - 2$ **f** $3 + 5(x - 1)$ **g** $(3 \times 8.5 - 4) - (4.1 + 5.4 \div 2)$

Solution

a
$$\begin{aligned} 3 + 6 \times 8 &= 3 + 48 \\ &= 51 \end{aligned}$$

b
$$\begin{aligned} (3 + 6) \times 8 &= 9 \times 8 \\ &= 72 \end{aligned}$$

c
$$\begin{aligned} 8 \div 2 - 2 &= 4 - 2 \\ &= 2 \end{aligned}$$

d
$$\begin{aligned} 23 - (8 - 5) &= 23 - 3 \\ &= 20 \end{aligned}$$

e
$$\begin{aligned} (4)3 - 2 &= 12 - 2 \\ &= 10 \end{aligned}$$

f
$$\begin{aligned} 3 + 5(x - 1) &= 3 + 5x - 5 \\ &= 5x - 2 \end{aligned}$$

g
$$\begin{aligned} (3 \times 8.5 - 4) - (4.1 + 5.4 \div 2) &= (25.5 - 4) - (4.1 + 2.7) \\ &= 21.5 - 6.8 \\ &= 14.7 \end{aligned}$$

Exercise A1.1

Example 1a–d

1 Evaluate the following, without using a calculator.

a $5 + 4 \times 8$ **b** $4 \times 3 - 7$

c $7 \times 6 - 4 + 4 \times 3$ **d** $15 \div 3 + 2$

e $3 + 12.6 \div 3$ **f** $4 \times (8 + 4)$

g $15 - 9 \div 2 + 4 \times (10 - 4)$ **h** $(3.7 + 5.3) \div 2$

i $8.6 - 3 \times 2 - 6 \div 3$ **j** $(3 \times 4 - 3) \div (2 - 3 \times 4)$

Example 1g

2 Use your calculator to find the answers to the following.

a $(8.23 - 4.5) + (3.6 + 5.2)$ **b** $(17 - 8.7) - (73 - 37.7)$

c $(6.2 + 33.17) \times (6.9 - 6.1)$ **d** $(3.2 + 0.5 \div 2.5) \div (8.6 - 1.3 \times 4)$

Example 1f–g

3 Evaluate the following.

a $9(3)$ **b** $2(x - 7)$ **c** $10(5 - y)$ **d** $w(8 - 2)$

e $k(k + 8)$ **f** $27(2) - 3(8)$ **g** $(5 - 3)x + 7(2)$ **h** $3(5) \times 2 - 8$

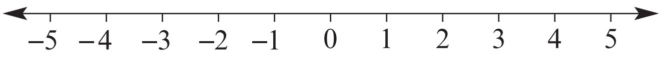
i $3(x + 1) - 8$ **j** $4 - 2(x + 3)$

A1.2 Directed numbers

Positive and negative numbers are **directed numbers** and can be shown on a number line.

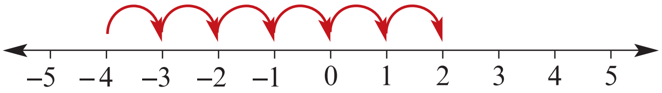
▶ Addition and subtraction

It is often useful to use a number line when adding directed numbers.



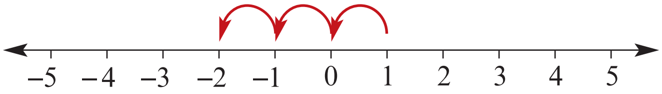
Adding a positive number means that you move to the right.

Example: $-4 + 6 = 2$



Adding a negative number means that you move to the left.

Example: $1 + (-3) = -2$



When subtracting a directed number, you add its opposite.

Example: $-2 - 3$ is the same as $-2 + (-3) = -5$

Example: $7 - (-9) = 7 + 9 = 16$

▶ Multiplication and division

Multiplying or dividing two numbers with the *same* sign gives a *positive* value.

Multiplying or dividing two numbers with *different* signs gives a *negative* value.

Multiplication and division with directed numbers

$+ \times + = +$ $+ \times - = -$ $- \times - = +$ $- \times + = -$

$+ \div + = +$ $+ \div - = -$ $- \div - = +$ $- \div + = -$

Example 2 Using directed numbers

Evaluate the following.

a $6 - 13$ **b** $(-5) - 11$ **c** $9 - (-7)$ **d** $(-10) - (-9)$

e $5 \times (-3)$ **f** $(-8) \times (-7)$ **g** $(-16) \div 4$ **h** $(-60) \div (-5)$

i $(-100) \div (-4) \div (-5)$ **j** $(-3)^2$

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Solution

a $6 - 13 = 6 + (-13)$
$= -7$

b $(-5) - 11 = (-5) + (-11)$
$= -16$

c $9 - (-7) = 9 + 7$
$= 16$

d $(-10) - (-9) = (-10) + 9$
$= -1$

e $5 \times (-3) = -15$

f $(-8) \times (-7) = 56$

g $(-16) \div 4 = -4$

h $(-60) \div (-5) = 12$

i $(-100) \div (-4) \div (-5) = 25 \div (-5)$
$= -5$

j $(-3)^2 = (-3) \times (-3)$
$= 9$

Exercise A1.2

Example 2a–d

1 Without using a calculator, find the answers to the following.

a $6 - 7$
b $-10 + 6$
c $-13 + (-3)$
d $-7 + 10$
e $-7 - 19$
f $(-18) - 7$
g $(-9) - 3$
h $4 - (-18)$
i $18 - (-4)$
j $15 - (-17)$
k $16 - (-12)$
l $(-3) - (-13)$
m $(-12) - (-6)$
n $(-21) - (-8)$

Example 2e–j

2 Without using a calculator, evaluate the following.

a $(-6) \times 2$
b $(-6)(-4)$
c $(-10) \div (-4)$
d $15 \div (-3)$
e $(5 + 2) \times 6 - 6$
f $-(-4) \times (-3)$
g $-7(-2 + 3)$
h $-4(-7 - (2)(4))$
i $-(3 - 2)$
j $-6 \times (-5 \times 2)$
k $-6(-4 + 3)$
l $-(-12 - 9) - 2$
m $-4 - 3$
n $-(-4 - 7(-6))$
o $(-5)(-5) + (-3)(-3)$
p $8^2 + 4(0.5)(8)(6)$

A1.3 Powers and roots

▶ Squares and square roots

When a number is multiplied by itself, we call this the *square* of the number.

$$4 \times 4 = 4^2 = 16$$

- 16 is called the *square* of 4 (or 4 squared).
- 4 is called the *square root* of 16.
- The square root of 16 can be written as $\sqrt{16} = 4$. ($\sqrt{\ }$ is the square root symbol)

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Cubes and cube roots

When a number is squared and then multiplied by itself again, we call this the *cube* of the number.

$$4 \times 4 \times 4 = 4^3 = 64$$

- 64 is called the *cube* of 4 (or 4 cubed).
- 4 is called the *cube root* of 64.
- The cube root of 64 can be written as $\sqrt[3]{64} = 4$. ($\sqrt[3]{\ }$ is the cube root symbol)

Other powers

When a number is multiplied by itself a number of times, the values obtained are called *powers* of the original number.

For example, $4 \times 4 \times 4 \times 4 \times 4 = 1024 = 4^5$, which is read as ‘4 to the *power* of 5’.

- 4 is the fifth root of 1024.
- $\sqrt[5]{1024}$ means the fifth root of 1024.
- Another way of writing $\sqrt{16}$ is $16^{\frac{1}{2}}$, which is read as ‘16 to the half’.
- Likewise, $8^{\frac{1}{3}}$, read as ‘8 to the third’, means $\sqrt[3]{8} = 2$.
- Powers and roots of numbers can be evaluated on the calculator by using the ˆ button.

Example 3 Finding the power or root of a number using a calculator

a Find 8^3.

b Find $8^{\frac{1}{3}}$.

Solution

a

```
8^3
                    512
```

b

```
8^(1/3)
                      2
```

Exercise A1.3

Example 3

1 Find the value of the following.

a 10^4 **b** 7^3 **c** $\sqrt{25}$ **d** $\sqrt[4]{16}$

e 2^6 **f** 12^4 **g** $9^{\frac{1}{2}}$ **h** $169^{\frac{1}{2}}$

i $1\,000\,000^{\frac{1}{2}}$ **j** $64^{\frac{1}{3}}$ **k** $32^{\frac{1}{5}}$

2 Find the value of the following.

a $\sqrt{10^2 + 24^2}$ **b** $\sqrt{39^2 - 36^2}$ **c** $\sqrt{12^2 + 35^2}$

d $\sqrt{(4+2)^2 - 11}$ **e** $10(3+5) - (\sqrt{9} - 2)$ **f** $\sqrt{(3+2)^2 - (5-2)^2}$

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A1.4 Approximations, decimal places and significant figures

Approximations occur when we are not able to give exact numerical values in mathematics. Some numbers are too long (e.g. 0.573 128 9 or 107 000 000 000) to work with and they are rounded to make calculations easier. Calculators are powerful tools and have made many tasks easier that previously took a considerable amount of time. Nevertheless, it is still important to understand the processes of **rounding** and estimation.

Some questions do not require an exact answer and a stated degree of accuracy is often sufficient. Some questions may only need an answer rounded to the nearest tenth, hundredth etc. Other questions may ask for an answer correct to two decimal places or to three significant figures.

▶ Rules for rounding

Rules for rounding

1. Look at the value of the digit to the right of the specified digit.
2. If the value is 5, 6, 7, 8 or 9, *round the digit up*.
3. If the value is 0, 1, 2, 3 or 4, *leave the digit unchanged*.

Example 4 Rounding to the nearest thousand

Round 34 867 to the nearest thousand.

Solution

1. Look at the first digit after the thousands. It is an 8.

   ⇓
   34 867

2. As it is 5 or more, increase the digit to its left by one. So the 4 becomes a 5. The digits to the right all become zero. Write your answer.

   35 000

Note: 34 867 is closer to 35 000 than to 34 000.

▶ Scientific notation (standard form)

When we work with very large or very small numbers, we often use **scientific notation**, also called *standard form*.

To write a number in scientific notation, we express it as a number between 1 and 10 multiplied by a power of 10.

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Scientific notation

Large numbers

$$\overbrace{249000000000} = 2.49 \times 100\,000\,000\,000$$

$$= 2.49 \times 10^{11}$$

The decimal point needs to be moved 11 places to the right to obtain the basic numeral.

Multiplying by $10^{\text{positive power}}$ gives the effect of moving the decimal point to the right to make the number larger.

Small numbers

$$\overbrace{0.000000002} = 2.0 \div 1\,000\,000\,000$$

$$= 2.0 \times 10^{-9}$$

The decimal point needs to be moved 9 places to the left to obtain the basic numeral.

Multiplying by $10^{\text{negative power}}$ gives the effect of moving the decimal point to the left to make the number smaller.

Example 5 Writing a number in scientific notation

Write the following numbers in scientific notation.

a 7 800 000

b 0.000 000 5

Solution

a **1** Write 7 800 000 as a number between 1 and 10 (7.8) and decide what to multiply it by to make 7 800 000.

$7\,800\,000 = 7.8 \times 1\,000\,000$

$7\ \overbrace{8\ 0\ 0\ 0\ 0\ 0}^{6 \text{ places}}$

2 Count the number of places the decimal point needs to move and whether it is to the left or right.

Decimal point needs to move 6 places to the right from 7.8 to make 7 800 000.

3 Write your answer.

$7\,800\,000 = 7.8 \times 10^6$

b **1** Write 0.000 000 5 as a number between 1 and 10 (5.0) and decide what to divide it by to make 0.000 000 5.

$0.000\,000\,5 = 5.0 \div 10\,000\,000$

$0.\ \overbrace{0\ 0\ 0\ 0\ 0\ 0\ 5}^{7 \text{ places}}$

2 Count the number of places the decimal point needs to move and whether it is to the left or right.

Decimal point needs to move 7 places to the left from 5.0 to make 0.000 000 5.

3 Write your answer.

$0.000\,000\,5 = 5.0 \times 10^{-7}$

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Example 6 Writing a scientific notation number as a basic numeral

Write the following scientific notation numbers as basic numerals.

a 3.576×10^7 **b** 7.9×10^{-5}

Solution

a **1** Multiplying 3.576 by 10^7 means that the decimal point needs to be moved 7 places to the right.

3.576×10^7

$3.\overbrace{5\;7\;6\;0\;0\;0\;0}^{\text{7 places}} \times 10^7$

2 Move the decimal place 7 places to the right and write your answer. Zeros will need to be added as placeholders.

$= 35\,760\,000$

b **1** Multiplying 7.9 by 10^{-5} means that the decimal point needs to be moved 5 places to the left.

7.9×10^{-5}

$\overbrace{0.\;0\;0\;0\;0\;7}^{\text{5 places}}\;9 \times 10^{-5}$

2 Move the decimal place 5 places to the left and write your answer.

$= 0.000\,079$

Significant figures

The first non-zero digit, reading from left to right in a number, is the first **significant figure**. It is easy to think of significant figures as all non-zero figures, except where the zero is between non-zero figures. The number of significant figures is shown in red below.

For example:

Number	Significant figures	Explanation
596.36	5	All numbers provide useful information.
5000	1	We do not know anything for certain about the hundreds, tens or units places. The zeros may be just placeholders or they may have been rounded off to give this value.
0.0057	2	Only the 5 and 7 tell us something. The zeros are placeholders.
0.00570	3	The last zero tells us that the measurement was made accurate to the last digit.
8.508	4	Any zeros between significant digits are significant.
0.00906	3	Any zeros between significant digits are significant.
560.0	4	The zero in the tenths place means that the measurement was made accurate to the tenths place. The first zero is between significant digits and is therefore significant.

Rules for significant figures

1. All non-zero digits are significant.
2. All zeros between significant digits are significant.
3. After a decimal point, all zeros to the right of non-zero digits are significant.

Example 7 Rounding to a certain number of significant figures

Round 93.738 095 to:

a two significant figures **b** one significant figure **c** five significant figures

Solution

a **1** Count the significant figures in 93.738 095.

There are eight significant figures.

2 For two significant figures, start counting two non-zero numbers from the left.

93.738 095

3 The next number (7) is 5 or more so we increase the previous number (3) by one (making it 4). Write your answer.

= 94 (two significant figures)

b **1** For one significant figure, count one non-zero number from the left.

93.738 095

2 The next number (3) is less than 5 so we leave the previous number (9) as it is and replace the 3 with 0 to make only one significant figure. Write your answer.

= 90 (one significant figure)

c **1** For five significant figures, start counting five non-zero numbers from the left.

93.738 095

2 The next number (0) is less than 5 so do not change the previous number (8). Write your answer.

= 93.738 (five significant figures)

Example 8 Rounding to a certain number of significant figures

Round 0.006 473 5 to:

a four significant figures **b** three significant figures **c** one significant figure

Solution

a **1** Count the significant figures.	There are five significant figures.
2 Count four non-zero numbers starting from the left.	0.006 473 5
3 The next number (5) is 5 or more. Increase the previous number (3) by one (4). Write your answer.	= 0.006 474 (four significant figures)
b **1** For three significant figures, count three non-zero numbers from the left.	0.006 473 5
2 The next number (3) is less than 5 so we leave the previous number (7) as it is. Write your answer.	= 0.006 47 (three significant figures)
c **1** For one significant figure, count one non-zero number from the left.	0.006 473 5
2 The next number (4) is less than 5 so do not change the previous number (6). Write your answer.	= 0.006 (one significant figure)

▶ Decimal places

23.798 is a decimal number with three digits after the decimal point. The first digit (7) after the decimal point is the first (or one) decimal place. Depending on the required accuracy we round to one decimal place, two decimal places, etc.

Example 9 Rounding correct to a number of decimal places

Round 94.738 295 to:

a two decimal places **b** one decimal place **c** five decimal places

Solution

a **1** For two decimal places, count two places after the decimal point and look at the next digit (8).	94.738 295
2 As 8 is 5 or more, increase the digit to the left of 8 by one. (3 becomes 4) Write your answer.	= 94.74 (to two decimal places)

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b	**1** For one decimal place, count one place after the decimal point and look at the next digit (3).	*94.738 295*
	2 As 3 is less than 5, the digit to the left of 3 remains unchanged. Write your answer.	*= 94.7 (to one decimal place)*
c	**1** For five decimal places, count five places after the decimal point and look at the next digit (5).	*94.738 295*
	2 As the next digit (5) is 5 or more, the digit to the left of 5 needs to be increased by one. As this is a 9, the next higher number is 10, so the previous digit also needs to change to the next higher number. Write your answer.	*= 94.738 30 (to five decimal places)*

Exercise A1.4

Example 4

1 Round to the nearest whole number.

a 87.15 **b** 605.99 **c** 2.5 **d** 33.63

Example 4

2 Round to the nearest hundred.

a 6827 **b** 46 770 **c** 79 999 **d** 313.4

Example 5

3 Write these numbers in scientific notation.

a 792 000 **b** 14 600 000 **c** 500 000 000 000 **d** 0.000 009 8

e 0.145 697 **f** 0.000 000 000 06 **g** 2 679 886 **h** 0.0087

4 Express the following approximate numbers using scientific notation.

a The mass of Earth is 6 000 000 000 000 000 000 000 000 kg.

b The circumference of Earth is 40 000 000 m.

c The diameter of an atom is 0.000 000 000 1 m.

d The radius of Earth's orbit around the Sun is 150 000 000 km.

Example 6

5 Write these scientific notation numbers as basic numerals.

a 5.3467×10^4 **b** 3.8×10^6 **c** 7.89×10^5 **d** 9.21×10^{-3}

e 1.03×10^{-7} **f** 2.907×10^6 **g** 3.8×10^{-12} **h** 2.1×10^{10}

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6 For each of the following numbers, state the number of significant figures.

a 89 156 **b** 608 765 **c** 900 000 000 000 **d** 0.709

e 0.10 **f** 0.006 **g** 450 000 **h** 0.008 007

Example 7, 8

7 Round the following to the number of significant figures indicated in the brackets.

a 4.8976 (2) **b** 0.078 74 (3)

c 1506.892 (5) **d** 5.523 (1)

8 Calculate the following and give your answer correct to the number of significant figures indicated in the brackets.

a $4.3968 \times 0.000\,743\,8$ (2) **b** $0.611\,35 \div 4.1119$ (5)

c $3.4572 \div 0.0109$ (3) **d** $50\,042 \times 0.0067$ (3)

Example 9

9 Use a calculator to find answers to the following. Give each answer correct to the number of decimal places indicated in the brackets.

a 3.185×0.49 (2) **b** $0.064 \div 2.536$ (3)

c 0.474×0.0693 (2) **d** $12.943 \div 6.876$ (4)

e $0.006\,749 \div 0.000\,382$ (3) **f** $38.374\,306 \times 0.007\,493$ (4)

10 Calculate the following, correct to two decimal places.

a $\sqrt{7^2 + 14^2}$ **b** $\sqrt{3.9^2 + 2.6^2}$ **c** $\sqrt{48.71^2 - 29^2}$

A1.5 Percentages

Per cent is an abbreviation of the Latin words *per centum*, which mean ‘by the hundred’. A **percentage** is a rate or a proportion expressed as part of one hundred. The symbol used to indicate percentage is %. Percentages can be expressed as common fractions or as decimals.

For example: 17% (17 per cent) means 17 parts out of every 100.

$$17\% = \frac{17}{100} = 0.17$$

Conversions

1 To convert a fraction or a decimal to a percentage, multiply by 100.

2 To convert a percentage to a decimal or a fraction, divide by 100.

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Example 10 Converting fractions to percentages

Express $\frac{36}{90}$ as a percentage.

Solution

1 Multiply the fraction $\frac{36}{90}$ by 100.

2 Evaluate and write your answer.

$$\frac{36}{90} \times 100$$
$$= 40\%$$

Example 11 Converting a decimal to a percentage

Express 0.75 as a percentage.

Solution

1 Multiply 0.75 by 100.

2 Evaluate and write your answer.

$$0.75 \times 100$$
$$= 75\%$$

Example 12 Converting a percentage to a fraction

Express 62% as a common fraction.

Solution

1 As 62% means 62 out of 100, this can be written as a fraction $\frac{62}{100}$.

$$62\% = \frac{62}{100}$$

2 Simplify the fraction by dividing both the numerator and the denominator by 2.

$$= \frac{62 \div 2}{100 \div 2}$$
$$= \frac{31}{50}$$

Example 13 Converting a percentage to a decimal

Express 72% as a decimal.

Solution

Write 72% as a fraction over 100 and express this as a decimal.

$$\frac{72}{100} = 0.72$$

▶ Finding a percentage of a quantity

To find a percentage *of* a number or a quantity, remember that in mathematics *of* means *multiply*.

Example 14 Finding a percentage of a quantity

Find 15% of \$140.

Solution

1 Write out the problem and rewrite 15% as a fraction out of 100.

$$15\% \text{ of } 140 = \frac{15}{100} \text{ of } 140$$

2 Change *of* to *multiply*.

$$= \frac{15}{100} \times 140$$

3 Perform the calculation and write your answer.

$$= 21$$

Comparing two quantities

One quantity or number may be expressed as a percentage of another quantity or number (both quantities must always be in the same units). Divide the quantity by what you are comparing it with and then multiply by 100 to convert it to a percentage.

Example 15 Expressing a quantity as a percentage of another quantity

There are 18 girls in a class of 25 students. What percentage of the class are girls?

Solution

1 Work out the fraction of girls in the class.

$$\text{Girls} = \frac{18}{25}$$

2 Convert the fraction to a percentage by multiplying by 100.

$$\frac{18}{25} \times 100$$

3 Evaluate and write your answer.

$$= 72$$

72% of the class are girls.

Example 16 Expressing a quantity as a percentage of another quantity with different units

Express 76 mm as a percentage of 40 cm.

Solution

1 First convert 40 centimetres to millimetres by multiplying by 10, as there are 10 millimetres in 1 centimetre.

$$40 \text{ cm} = 40 \times 10$$
$$= 400 \text{ mm}$$

2 Write 76 millimetres as a fraction of 400 millimetres.

$$\frac{76}{400}$$

3 Multiply by 100 to convert to a percentage.

$$\frac{76}{400} \times 100$$

4 Evaluate and write your answer.

$$= 19\%$$

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Exercise A1.5

Example 10, 11

1 Express the following as percentages.

a $\frac{1}{4}$ **b** $\frac{2}{5}$ **c** $\frac{3}{20}$ **d** $\frac{7}{10}$

e 0.19 **f** 0.79 **g** 2.15 **h** 39.57

i 0.073 **j** 1

Example 12, 13

2 Express the following as:

i common fractions, in their lowest terms

ii decimals.

a 25% **b** 50% **c** 75% **d** 68%

e 5.75% **f** 27.2% **g** 0.45% **h** 0.03%

i 0.0065% **j** 100%

Example 14

3 Find the following, correct to three significant figures.

a 15% of \$760 **b** 22% of \$500

c 17% of 150 m **d** $13\frac{1}{2}$% of \$10 000

e 2% of 79.34 cm **f** 19.6% of 13.46

g 0.46% of 35 € **h** 15.9% of \$28 740

i 22.4% of \$346 900 **j** 1.98% of \$1 000 000

Example 15

4 From a class of 35 students, 28 wanted to take part in a project. What percentage of the class wanted to take part?

5 A farmer lost 450 sheep out of a flock of 1200 during a drought. What percentage of the flock was lost?

6 In a laboratory test on 360 light globes, 16 globes were found to be defective. What percentage were satisfactory? Give your answer correct to one decimal place.

7 After three rounds of a competition, a basketball team had scored 300 points and 360 points had been scored against them. Express the points scored by the team as a percentage of the points scored against them. Give your answer correct to two decimal places.

8 In a school of 624 students, 125 are in Year 10. What percentage of the students are in Year 10? Give your answer to the nearest whole number.

Example 16

9 Express 75 cm as a percentage of 2 m.

10 In a population of $3\frac{1}{4}$ million people, 2 115 000 are under the age of 16. Calculate the percentage, to two decimal places, of the population who are under the age of 16.

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A1.6 Percentage increase and decrease

When increasing or decreasing a quantity by a given percentage, the percentage increase or decrease is always calculated as a percentage of the *original* quantity.

Example 17 Calculating the new price following a percentage increase

Sally's daily wage of \$175 is increased by 15%. Calculate her new daily wage.

Solution

Method 1

1 First find 15% of \$175 by rewriting 15% as a fraction out of 100 and changing *of* to multiply.	15% of 175
2 Perform the calculation and write your answer.	$= \frac{15}{100} \times 175$ $= 26.25$
3 As \$175 is to be increased by 15%, add \$26.25 to the original amount of \$175.	$175 + 26.25$ $= 201.25$
4 Write your answer in a sentence.	Sally's new daily wage is \$201.25.

Method 2

1 An increase of 15% means that the new amount will be the original amount (in other words, 100%) plus an extra 15%. Find 115% of 175.	115% of 175
2 Perform the calculation.	$= \frac{115}{100} \times 175$ $= 201.25$
3 Write your answer in a sentence.	Sally's new daily wage is \$201.25.

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Example 18 Calculating the new amount following a percentage decrease

A primary school's fun run distance of 2.75 km is decreased by 20% for students in years 2 to 4. Find the new distance.

Solution

Method 1

1 First find 20% of 2.75 by writing 20% as a fraction out of 100 and changing *of* to multiply (or use a calculator).

20% of 2.75

$= \frac{20}{100} \times 2.75$

2 Evaluate and write your answer.

$= 0.55$

3 As 2.75 km is to be decreased by 20%, subtract 0.55 km from the original 2.75 km.

$2.75 - 0.55$

$= 2.2$

4 Write your answer in a sentence.

The new distance is 2.2 km.

Method 2

1 A decrease of 20% means that the new amount will be the original amount (100%) minus 20%. Find 80% of 2.75.

80% of 2.75

$= \frac{80}{100} \times 2.75$

2 Perform the calculation.

$= 2.2$

3 Write your answer in a sentence.

The new distance is 2.2 km.

Example 19 Calculating a new price with a percentage discount

If a shop offers a discount of 15% on items in a sale, what would be the sale price of a pair of jeans originally priced at $95?

Solution

Method 1

1 Find 15% of 95.

15% of $95 = \frac{15}{100} \times 95$

$= 14.25$

2 As the jeans are discounted by 15%, this is a decrease, so we need to subtract the discounted price of $14.25 from the original price of $95.

$95 - 14.25$

$= 80.75$

3 Write your answer in a sentence.

The sale price would be $80.75.

Method 2

1 A discount of 15% means that the new amount is 85% of 95.

85% of 95

$= \frac{85}{100} \times 95$

2 Perform the calculation.

$= 80.75$

3 Write your answer in a sentence.

The sale price would be $80.75.

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Finding a percentage change

If we are given the original price and the new price of an item, we can find the percentage change. To find a percentage change, we compare the change (increase or decrease) with the original number.

Percentage change

$$\text{Percentage change} = \frac{\text{change}}{\text{original}} \times 100$$

Thus:

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original}} \times 100$$

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

Example 20 Calculating a percentage increase

A university increased its total size at the beginning of an academic year by 3000 students. If the previous number of students was 35 000, by what percentage, correct to two decimal places, did the student population increase?

Solution

1 To find the percentage increase, use the formula:

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

Substitute increase as 3000 and original as 35 000.

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

$$= \frac{3000}{35\,000} \times 100$$

2 Evaluate.

$$= 8.5714\ldots$$

3 Write your answer correct to two decimal places.

Student population increased by 8.57%.

Example 21 Calculating the percentage discount

Calculate the percentage discount obtained when a calculator with a normal price of $38 is sold for $32. Give the answer to the nearest whole per cent.

Solution

1 Find the amount of discount given by subtracting the new price, $32, from the original price, $38.

$$\text{Discount} = \$38 - \$32$$

$$= \$6$$

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2 To find the percentage discount, use the formula:

Percentage discount $= \dfrac{\text{discount}}{\text{original}} \times 100$

Substitute discount as 6 and original as 38 and evaluate.

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original}} \times 100 = \frac{6}{38} \times 100 = 15.7895\ldots$$

3 Write your answer to the nearest whole per cent.

The percentage discount is 16%.

Exercise A1.6

Example 19

1 A jewellery store has a promotion of 20% discount on all watches.

a How much discount will you get on a watch marked \$185?

b What is the sale price of the watch?

2 A store gave different savings discounts on a range of items in a sale. Copy and complete the following table.

	Normal price	% Discount	Saving	Sale price
a	\$89.99	5		
b	\$189.00	10		
c	\$499.00	15		
d	\$249.00	20		
e	\$79.95	22.5		
f	\$22.95	25		
g	\$599.00	27.5		
h	\$63.50	30		
i	\$1000.00	33		

3 In a particular shop the employees are given a $12\frac{1}{2}$% discount on any items they purchase. Calculate the actual price an employee would pay for each of the following.

a \$486 laptop

b \$799 HD LED television

c \$260 iPod

d \$750 digital camera

e \$246 digital video recorder

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4 A clothing store offers a 6% discount for cash sales. A customer who paid cash purchased the following items:

One pair of jeans $95.95
A leather belt at $29.95
Two jumpers at $45 each

a Calculate the total saving.

b What is the actual amount paid for the goods?

5 Which results in the larger sum of money, increasing $50 by 10% or decreasing $60 by 8%?

Example 17

6 The production of a particular model of car is increased from 14 000 by 6% over a 12-month period. What is the new production figure?

7 A new car is sold for $23 960 and three years later it is valued at $18 700. Calculate the percentage depreciation, correct to two decimal places.

8 A leading tyre manufacturer claims that a new tyre will average 12% more life than a previous tyre. The owner of a taxi fleet finds that the previous tyre averaged 24 000 km before replacement. How many kilometres should the new tyre average?

Example 21

9 Calculate the percentage discount for each of the following, to the nearest whole number.

	Normal price	Selling price	% Discount
a	$60.00	$52.00	
b	$250.00	$185.00	
c	$5000.00	$4700.00	
d	$3.80	$2.90	
e	$29.75	$24.50	
f	$12.95	$10.00	

10 A secondhand car advertised for sale at $13 990 was sold for $13 000. Calculate, correct to two decimal places, the percentage discount obtained by the purchaser.

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11 A sports shop advertised the following items in their end-of-year sale. Calculate the percentage discount for each of the items, to the nearest whole number.

		Normal price	Selling price	% Discount
a	Shoes	\$79.99	\$65.00	
b	12 pack of golf balls	\$29.99	\$19.99	
c	Exercise bike	\$1099.00	\$599.00	
d	Basket ball	\$49.99	\$39.99	
e	Sports socks	\$14.95	\$10.00	
f	Hockey stick	\$299.00	\$250.00	

12 Find the percentage increase that has been applied in each of the following.

a A book that is increased from \$20 to \$25

b An airfare that is increased from \$300 to \$420

c Accommodation costs that are increased from \$540 to \$580.50

A1.7 Ratio

Although you are likely to have studied ratio before, it is useful to review a few basic ideas here. It will also be useful for your study of similarity in Chapter 6 (Shape and measurement).

▶ What is a ratio?

Skillsheet

Ratios are used to numerically compare the values of two or more quantities.

A *ratio* can be written as **a : b** (read as 'a to b'). It can also be written as a fraction $\frac{a}{b}$.

The order of the numbers or numerals in a ratio is important. a : b is *not* the same as b : a.

Example 22 Expressing quantities as a ratio

In a Year 10 class of 26 students there are 14 girls and 12 boys. Express the number of girls to boys as a ratio.

Solution

As there are 14 girls and 12 boys, the ratio of girls to boys is 14 : 12.

Note: This could also be written as a fraction $\frac{14}{12}$.

Example 23 Expressing more than two quantities as a ratio

A survey of the same group of 26 students showed that 10 students walked to school, 11 came by public transport, and 5 were driven by their parents. Express as a ratio the number of students who walked to school to the number of students who came by public transport to the number of students who were driven to school.

Solution

The order of the numbers in a ratio is important.

10 students walked, 11 used public transport and 5 were driven so the ratio is 10 : 11 : 5.

▶ Expressing ratios in their simplest form

Ratios can be simplified by dividing through by a common factor or by multiplying each term as required.

Example 24 Simplifying ratios

Simplify the follow ratios.

a 15 : 20 **b** 0.4 : 1.7 **c** $\frac{3}{4} : \frac{5}{3}$

Solution

a **1** Divide both 15 and 20 by 5.

2 Evaluate and write your answer.

$$15 : 20$$
$$= \frac{15}{5} : \frac{20}{5}$$
$$= 3 : 4$$

b **1** Multiply both 0.4 and 1.7 by 10 to give whole numbers.

2 Evaluate and write your answer.

$$0.4 : 1.7$$
$$= 0.4 \times 10 : 1.7 \times 10$$
$$= 4 : 17$$

c Method 1

1 Multiply both fractions by 4.

$$\frac{3}{4} \times 4 : \frac{5}{3} \times 4$$
$$= 3 : \frac{20}{3}$$

2 Multiply both sides of the equation by 3.

$$= 3 \times 3 : \frac{20}{3} \times 3$$

3 Write your answer.

$$= 9 : 20$$

Method 2

1 Multiply both $\frac{3}{4}$ and $\frac{5}{3}$ by the lowest common multiple (LCM) of 3 and 4, which is 12, to eliminate fractions.

$$\frac{3}{4} : \frac{5}{3}$$
$$= \frac{3}{4} \times 12 : \frac{5}{3} \times 12$$

2 Evaluate and write your answer.

$$= 9 : 20$$

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In each of the previous examples, the ratios are equivalent and the information is unchanged. For example, the ratio:

12 : 8 is equivalent to the ratio 24 : 16 (multiply both 12 and 8 by 2)

and

12 : 8 is also equivalent to the ratio 3 : 2 (divide both 12 and 8 by 4).

Ratios

1. When ratios are written in terms of the smallest possible whole numbers, they are expressed in their *simplest form*.
2. The order of the figures in a ratio is important. 3 : 5 is *not* the same as 5 : 3.
3. Both parts of a ratio must be expressed in the same unit of measurement.

Example 25 Simplifying ratios with different units

Express 15 cm to 3 m as a ratio in its simplest form.

Solution

1. Write the ratio.
2. Convert 3 m to cm, by multiplying 3 m by 100, so that both parts of the ratio will be in the same units.
3. Simplify the ratio by dividing both 15 and 300 by 15.
4. Write your answer.

$$15 \text{ cm} : 3 \text{ m}$$

$$15 \text{ cm} : 3 \times 100 \text{ cm}$$

$$= 15 \text{ cm} : 300 \text{ cm}$$

$$= 15 : 300$$

$$= \frac{15}{15} : \frac{300}{15}$$

$$= 1 : 20$$

Example 26 Finding missing values in a ratio

Find the missing value for the equivalent ratios 3 : 7 = ☐ : 28.

Solution

1. Let the unknown value be *x* and write the ratios as fractions.
2. Solve for *x*.
3. Multiply both sides of equation by 28.
4. Evaluate and write your answer.

$$3 : 7 = x : 28$$

$$\frac{3}{7} = \frac{x}{28}$$

$$\frac{3}{7} \times 28 = \frac{x}{28} \times 28$$

$$x = 12$$

$$3 : 7 = 12 : 28$$

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Exercise A1.7

Example 22

1 A survey of a group of 50 year 11 students in a school showed that 35 of them have a part-time job and 15 do not. Express the number of students having a part-time job to those who do not as a ratio.

Example 22, 23

2 The table below shows the average life expectancy of some animals.

Animal	Life expectancy
Chimpanzee	40 years
Elephant	70 years
Horse	40 years
Kangaroo	9 years
Tortoise	120 years
Mouse	4 years
Whale	80 years

Find the ratios between the life expectancies of the following animals.

a Whale to horse
b Elephant to kangaroo
c Whale to tortoise
d Chimpanzee to mouse
e Horse to mouse to whale

Example 24

3 Express the following ratios in their simplest forms.

a 12 : 15
b 10 : 45
c 22 : 55 : 33
d 1.3 : 3.9
e 2.7 : 0.9
f $\frac{5}{3} : \frac{1}{4}$
g 18 : 8

Example 25

4 Express the following ratios in their simplest form after making sure that each quantity is expressed in the same units.

a 60 L to 25 L
b $2.50 to $50
c 75 cm to 2 m
d 5 kg to 600 g
e 15 mm to 50 cm to 3 m
f 1 km to 1 m to 1 cm
g 5.6 g to 91 g
h $30 to $6 to $1.20 to 60c

Example 26

5 For each of the following equivalent ratios find the missing value.

a 1 : 4 = ☐ : 20
b 15 : 8 = 135 : ☐
c 600 : 5 = ☐ : 1
d 2 : 5 = 2000 : ☐
e 3 : 7 = ☐ : 56

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6 Which of the following statements are true and which are false? For those that are false, suggest a correct replacement statement, if possible.

- **a** The ratio 4 : 3 is the same as 3 : 4.
- **b** The ratio 3 : 4 is equivalent to 20 : 15.
- **c** 9 : 45 is equivalent to 1 : 5.
- **d** The ratio 60 to 12 is equivalent to 15 to 3, which is the same as 4 to 1.
- **e** If the ratio of a father's age to his daughter's age is 7 : 1, then if the girl is 7 years old her father is 56.
- **f** If my weekly allowance is $\frac{5}{8}$ of that of my friend, then the ratio of my monthly allowance to the allowance of my friend is 20 : 32.

7 The following recipe is for Anzac biscuits.

Anzac biscuits (makes 25)

100 grams rolled oats

175 grams plain all-purpose flour, sifted

125 grams butter

2 tablespoons golden syrup

60 grams desiccated coconut

125 grams soft brown sugar

3 tablespoons boiling water

1 teaspoon bicarbonate of soda

- **a** What is the unsimplified ratio of rolled oats : coconut : flour : brown sugar : butter?
- **b** Simplify the ratio from part **a**.
- **c** You want to adapt the recipe to make 75 biscuits. What quantity of each ingredient do you need?

A1.8 Dividing quantities in given ratios

Example 27 Dividing quantities in given ratios

Calculate the number of students in each class if 60 students are divided into classes in the following ratios.

a 1 : 3 **b** 5 : 1 **c** 1 : 2 : 7

Solution

a **1** Add up the total number of parts. (Remember that a 1 : 3 ratio means that there is 1 part for every 3 parts).

The total number of parts is 1 + 3 = 4.

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	2	Divide the number of students (60) by the number of parts (4) to give the number of students in one group.	*60 ÷ 4 = 15* *One group of students will have 1 × 15 = 15 students.*
	3	Work out the number of students in the other group by multiplying the number of parts (3) by the number of students in one group (15).	*The other group will have 3 × 15 = 45 students.*
	4	Check this gives a total of 60 students and write your answer.	*15 + 45 = 60* *The two groups will have 15 and 45 students.*
b	**1**	Add up the total number of parts. (Remember that a 1 : 5 ratio means that there is 1 part for every 5 parts).	*The total number of parts is 1 + 5 = 6.*
	2	Divide the number of students (60) by the number of parts (6) to give the number of students in one group.	*60 ÷ 6 = 10* *One group of students will have 1 × 10 = 10 students.*
	3	Work out the number of students in the other group by multiplying the number of parts (5) by the number of students in one group (10).	*The other group will have 5 × 10 = 50 students.*
	4	Check this gives a total of 60 students and write your answer.	*10 + 50 = 60* *The two groups will have 10 and 50 students.*
c	**1**	To divide 60 students into classes in the ratio 1 : 2 : 7, first add up the total number of parts.	*The total number of parts is 1 + 2 + 7 = 10.*
	2	Divide the number of students (60) by the number of parts (10) to give the number of students in one group.	*60 ÷ 10 = 6* *One group of students will have 1 × 6 = 6 students.*
	3	Work out the number of students in the other two groups by multiplying the number of parts (2) and (7) by the number of students in one group (6).	*The other groups will have 2 × 6 = 12 students and 7 × 6 = 42 students.*
	4	Check that this gives a total of 60 students and write your answer.	*6 + 12 + 42 = 60* *The three groups will have 6, 12 and 42 students.*

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Exercise A1.8

Example 27a,b

1 If a 40 m length of rope is cut in the following ratios, what will be the lengths of the individual pieces of rope?

a 4 : 1 **b** 1 : 7 **c** 60 : 20 **d** 8 : 8

Example 27c

2 If a sum of $500 is to be shared among a group of people in the following ratios, how much would each person receive?

a 6 : 4 **b** 1 : 4 : 5 **c** 1 : 8 : 1 **d** 8 : 9 : 8
e 10 : 5 : 4 : 1

3 A basket contains bananas, mangos and pineapples in the ratio 10 : 1 : 4. If there are 20 pineapples in the basket, calculate:

a the number of bananas
b the number of mangos
c the total number of pieces of fruit in the basket.

4 7.5 litres of made-up cordial is required for a children's party. If the ratio of cordial to water is 1 : 4, calculate:

a the number of litres of cordial required
b the number of litres of water required.

5 The scale on a map is 1 : 20 000. If the measured distance on the map between two historical markers is 15 centimetres, what is the actual distance between the two markers in kilometres?

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Key ideas and chapter summary

Order of operation	The order of operations is important. Remember BODMAS or BOMDAS **B**rackets come first **O**f or **O**rders (powers, square roots) **D**ivision and **M**ultiplication come next, working from left to right then **A**ddition and **S**ubtraction, working from left to right
Directed numbers	Multiplying or dividing two numbers with the **same** sign gives a **positive** value. Multiplying or dividing two numbers with **different** signs gives a **negative** value.
Rounding	5.417 rounded to two decimal places is 5.42 (number after the 1 is 7 so round up).
Scientific notation	To write a number in scientific notation express it as a number between 1 and 10 multiplied by a power of 10.
Significant figures	All non-zero digits are significant. All zeros between significant digits are significant. After a decimal point, all zeros to the right of non-zero digits are significant.
Percentages	To convert a fraction or a decimal to a percentage, multiply by 100. To convert a percentage to a decimal or a fraction, divide by 100. $\text{Percentage change} = \dfrac{\text{change}}{\text{original quantity or price}} \times 100$
Ratios	The order of the figures in a ratio is important. 4 : 3 is not the same as 3 : 4. Ratios can be simplified. For example, 6 : 2 = 3 : 1

Skills check

Having completed this chapter you should be able to:

- use a variety of mathematical operations in the correct order
- add, subtract, multiply and divide directed numbers
- find powers and roots of numbers
- round numbers to specific place values
- write numbers in scientific notation (standard form)
- understand and use significant figures
- express ratios in their simplest form
- solve practical problems involving ratios and percentages.

Multiple-choice questions

1 Evaluate $4 + 7 \times 3$.

A 33 **B** 30 **C** 19 **D** 14 **E** 25

2 Evaluate $3 + (6 \div 3) - 2$.

A 3 **B** 6 **C** 1 **D** 9 **E** 8

3 $(8.7 - 4.9) \times (5.4 + 2.8)$ is equal to:

A 23.32 **B** 31.16 **C** −14.96 **D** 12.0 **E** −31.48

4 Evaluate $(-3) \times 4 \times 5$.

A 60 **B** 6 **C** −60 **D** 27 **E** 3

5 Evaluate $(-2) + 8$.

A 10 **B** 6 **C** −10 **D** −6 **E** 28

6 Evaluate $(-2) - (-3)$.

A −5 **B** 5 **C** 1 **D** −1 **E** 6

7 Evaluate $5 - (-9)$.

A −4 **B** 59 **C** 44 **D** −14 **E** 14

8 3.895 rounded to two decimal places is:

A 3.8 **B** 3.99 **C** 4.0 **D** 3.90 **E** 3.89

9 4679 rounded to the nearest hundred is:

A 5000 **B** 4600 **C** 4700 **D** 4670 **E** 4680

10 5.21×10^5 is the same as:

A 52 100 000 **B** 521 000 **C** 52 105 **D** 0.000 052 1 **E** 260.50

11 0.0048 written in scientific notation is:

A 48×10^{-4} **B** 48×10^{-3} **C** 4.8×10^{3} **D** 4.8×10^{-3} **E** 4.8×10^{-4}

12 28 037.2 rounded to two significant figures is:

A 28 000 **B** 20 000.2 **C** 20 007 **D** 7.2 **E** 28 000.2

13 0.030 69 rounded to two significant figures is:

A 0.03 **B** 0.000 69 **C** 0.0307 **D** 0.031 **E** 0.0306

14 10% as a fraction in its simplest form is:

A $\frac{1}{100}$ **B** $\frac{10}{100}$ **C** $\frac{1}{10}$ **D** 10 **E** $\frac{1.0}{100}$

15 56% as a fraction in its simplest form is:

A 0.56 **B** $\frac{56}{100}$ **C** $\frac{0.56}{100}$ **D** $\frac{5.6}{100}$ **E** $\frac{14}{25}$

16 15% of $1600 is equal to:

A $24 **B** $150 **C** $240 **D** $1840 **E** $24 000

17 An item with a cost price of $450 is marked up by 30%. Its selling price is:

A $585 **B** $135 **C** $480 **D** $1350 **E** $463.50

18 A box contains 5 green marbles, 7 blue marbles and 3 yellow marbles. The ratio of blue marbles to total marbles is:

A 7 : 5 : 3 **B** 7 : 8 **C** 7 : 15 **D** 5 : 7 : 3 **E** 5 : 7 : 3 : 15

19 $750 is divided in the ratio 1 : 3 : 2. The smallest share is:

A $250 **B** $125 **C** $375 **D** $750 **E** $150

20 In simplest ratio form, the ratio of 450 grams to 3 kilograms is:

A 3 : 20 **B** 450 : 3 **C** 9 : 60 **D** 150 : 1 **E** 15 : 100

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Short-answer questions

1 Evaluate the following.

a $3 + 2 \times 4$ **b** $25 \div (10 - 5) + 5$ **c** $14 - 21 \div 3$

d $(12 + 12) \div 12 + 12$ **e** $27 \div 3 \times 5 + 4$ **f** $4 \times (-2) + 3$

g $\frac{10 - 8}{2}$ **h** $\frac{4(3 + 12)}{2}$ **i** $\frac{-5 + 9}{2}$

2 Calculate the following and give your answer correct to two decimal places where appropriate.

a 5^3 **b** $\sqrt{64} - 5$ **c** $9^{\frac{1}{2}} + 9^{\frac{1}{2}}$ **d** $\sqrt{8}$

e $\sqrt{25 - 9}$ **f** $\sqrt{25} - 9$ **g** $\frac{6^3}{(10 \div 2)^2}$ **h** $\sqrt{6^2 + 10^2}$

3 Write each of the following in scientific notation.

a 2945 **b** 0.057 **c** 369 000 **d** 850.9

4 Write the basic numeral for each of the following.

a 7.5×10^3 **b** 1.07×10^{-3} **c** 4.56×10^{-1}

5 Write the following correct to the number of significant figures indicated in the brackets.

a 8.916 (2) **b** 0.0589 (2) **c** 809 (1)

6 Write the following correct to the number of decimal places indicated in the brackets.

a 7.145 (2) **b** 598.241 (1) **c** 4.0789 (3)

7 Express the following percentages as decimals.

a 75% **b** 40% **c** 27.5%

8 Express the following percentages as fractions, in their lowest terms.

a 10% **b** 20% **c** 22%

9 Evaluate the following.

a 30% of 80 **b** 15% of \$70 **c** $12\frac{1}{2}$% of \$106

10 A new LED smart television was valued at \$1038. During a sale it was discounted by 5%.

a What was the amount of discount?

b What was the sale price?

11 Tom's weekly wage of \$750 is increased by 15%. What is his new weekly wage?

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12 A 15-year-old girl working at a local bakery is paid $12.50 per hour. Her pay will increase to $15 per hour when she turns 16. What will be the percentage increase to her hourly pay (to the nearest per cent)?

13 A leather jacket is reduced from $516 to $278. Calculate the percentage discount (to the nearest per cent).

14 After dieting for three months, Melissa, who weighed 78 kg, lost 4 kg and Jody's weight dropped from 68 kg to 65 kg. Calculate the percentage weight loss, correct to two decimal places, for each girl.

15 True or false?

a The ratio 3 : 2 is the same as 2 : 3.

b 1 : 5 = 3 : 12

c 20 cm : 1 m is written as 20 : 1 in simplest form.

d 3 : 4 = 9 : 12

16 If a sum of $800 is to be shared among a group of people in the following ratios, how much would each person receive?

a 4 : 6 **b** 1 : 4 **c** 2 : 3 : 5 **d** 2 : 2 : 4

17 A recipe for pizza dough requires 3 parts wholemeal flour for each 4 parts of plain flour. How many cups of wholemeal flour are needed if 24 cups of plain flour are used?

18 The scale on a map is 1 : 1000. Find the actual distance (in metres) between two markers if the distance between the two markers on a map is:

a 2.7 cm **b** 140 mm

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Glossary

Note: for definitions and discussion of cognitive verbs used in the syllabus, See the Interactive Textbook.

A

Algebraic expression [p. 303] A mathematical relationship connecting two or more variables. It is also called a formula.

Angle of depression [p. 260] The angle between the horizontal and a direction *below* the horizontal.

Angle of elevation [p. 260] The angle between the horizontal and a direction *above* the horizontal.

Arc [p. 114] The part of a circle between two given points on the circle. The length of the arc of a circle is given by $s = r\left(\frac{\theta}{180}\right)\pi$, where r is the radius of the circle and θ is the angle in degrees subtended by the arc at the centre of the circle.

Area [p. 104] The area of a shape is a measure of the region enclosed by its boundaries, measured in square units.

Area formulas [p. 104] Formulas used to calculate the areas of regular shapes, including squares, rectangles, triangles, parallelograms, trapeziums, kites, rhombi and circles.

B

Back-to-back stem plot [p. 416] A type of stem plot used to compare two sets of data, with a single stem and two sets of leaves (one for each group).

Bearing [p. 265] *See* Three-figure bearing.

BODMAS [p. 452] An aid for remembering the order of operations: **B**rackets first; **O**rders (powers, square roots) and fractions **O**f numbers; **D**ivision and **M**ultiplication, working left to right; **A**ddition and **S**ubtraction, working left to right.

Boxplot [p. 410] A graphical representation of a five-number summary showing outliers if present. *See* Outliers.

C

Capacity [p. 120] The amount of substance that an object can hold.

Categorical data [p. 375] Data generated by a categorical variable. Even if numbers, for example, house numbers, categorical data *cannot* be used to perform meaningful numerical calculations.

Categorical variable [p. 376] Variables that are categories which are used to represent characteristics of individuals or objects, for example, place of birth, house number. Categorical variables come in types, nominal and ordinal.

Circumference [p. 112] The circumference of a circle is the length of its boundary. The circumference, C, of a circle with radius, r, is given by $C = 2\pi r$.

Column chart [p. 379] A statistical graph used to display the frequency distribution of categorical data, using vertical columns.

Column matrix [p. 335] A matrix with only one column.

Commutative [p. 352] Describes an operation or process that produces the same result when reversed.

Composite shape [p. 107] A shape that is made up of two or more basic shapes.

Compound interest [p. 65] Under compound interest, the interest paid on a loan or investment is credited or debited to the account at the end of each period. The interest earned for the next period is based on the principal plus previous interest earned. The amount of interest earned increases each year (it grows exponentially).

Compounding time period or **rests** [p. 68] The time period between compound interest calculations and payments; for example, a day, a month, or a year.

Connections [p. 338] A matrix can be used to record various types of connections, such as social communications and roads directly connecting towns.

Continuous data [p. 376] Measurements of a variable that can take any value (usually within a range) to any fraction or any number of decimal places.

Conversion of measurements [p. 153] The calculation required to change a value in one unit to the equivalent value in another unit.

Cosine ratio (cos θ) [p. 244] In right-angled triangles, the ratio of the side adjacent to a given angle (θ) to the hypotenuse.

Cosine rule [p. 285] In non-right-angled triangles, a rule used to find:

- the third side, given two sides and an included angle
- an angle, given three sides.

For triangle ABC and side a, the rule is:

$$a^2 = b^2 + c^2 - 2bc\ \cos A$$

$$\text{or } \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Similar rules exist for sides b and c.

D

Data [p. 375] Information collected about a variable.

Diameter [p. 112] The distance from one side of a circle (or sphere) to the opposite side through the centre; it is equal to twice the radius.

Directed numbers [p. 454] Positive and negative numbers.

Discount [p. 47] A reduction in the price of an item usually expressed as percentage decrease, but which may also be expressed as a fraction of the price or an amount subtracted from the price; also known as mark down.

Discrete data [p. 376] Data that is counted rather than measured and can take only specific numerical values, usually but not always whole numbers.

Discrete variable [p. 376] A numerical variable that represents a quantity that is determined by counting, for example, the number of people waiting in a queue is a discrete variable.

Distribution [p. 378] The pattern in a set of data values. It reflects how frequently different data values occur.

Dividend [p. 78] The financial return to shareholders on a share of a company. Dividends can be specified as the number of dollars each share receives or as a percentage of the current share price, called the dividend yield.

Dot plot [p. 397] A dot plot consists of a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other.

E

Elements of a matrix [p. 333] The numbers of symbols displayed in a matrix.

Exchange rate [p. 27] For a currency, the rate used to convert it into another currency.

F

Five-number summary [p. 410] A list of the five key points in a data distribution: the minimum value (Min), the first quartile (Q_1), the median (M), the third quartile (Q_3) and the maximum value (Max).

Formula [p. 303] A mathematical relationship connecting two or more variables, for example, $C = 5t + 20, P = 2L + 2W, A = \pi r^2$.

Frequency [p. 378] The number of times a value or a group of values occurs in a data set. Sometimes known as the count.

Frequency table [p. 378] A listing of the values that a variable takes in a data set along with how often (frequently) each value occurs. Frequency can also be recorded as a percentage.

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G

Gradient of a straight line [p. 178] *See* Slope of a straight line.

Gross salary [p. 2] The amount someone is paid for their employment before tax is deducted.

Grouped data [p. 388] Where there are many different data values, data is grouped in intervals such as 0–9, 10–19, …

GST [p. 84] GST (goods and services tax) is a tax, currently at the rate of 10%, that is added to most purchases of goods and services.

H

Heron's rule (Heron's formula) [p. 271] A rule for calculating the area of a triangle from its three sides.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a+b+c)$ and is called the semi-perimeter.

Histogram [p. 390] A statistical graph used to display the frequency distribution of a numerical variable; suitable for medium- to large-sized data sets.

Hypotenuse [p. 90] The longest side in a right-angled triangle.

I

Identity matrix (*I*) [p. 352] A square matrix with 1s along the leading diagonal and 0s in all other positions, that behaves like the number one in arithmetic,
represented by the symbol I. Any matrix multiplied by an identity matrix remains unchanged.

Inflation [p. 73] The tendency of prices to increase with time, resulting in the loss of purchasing power.

Intercept-slope form of the equation of a straight line [p. 182] A linear equation written in the form $y = a + bx$, where a and b are constants. In this equation, a represents the y-intercept and b represents the slope. For example, $y = 5 - 2x$ is the equation of a straight line with y-intercept of 5 and the slope of -2.

Interest [p. 54] An amount of money paid (earned) for borrowing (lending) money over a period of time. It may be calculated on a simple or compound basis.

Interest rate [p. 54] The rate at which interest is charged or paid. It is usually expressed as a percentage of the money owed or lent.

Interquartile range (IQR) [p. 406] A summary statistic that measures the spread of the middle 50% of values in a data distribution. It is defined as $\mathrm{IQR} = Q_3 - Q_1$.

L

Linear equation [p. 164] An equation that has a straight line as its graph. In linear equations, the unknown values are always to the power of 1, for example, $y = 2x - 3$, $y + 3 = 7$, $3x = 8$.

Location of distribution [p. 395] Two distributions are said to differ in location if the values of the data in one distribution are generally larger than the values of the data in the other distribution.

Loss (financial) [p. 236] *See* profit.

M

Mark down [p. 47] *See* discount.

Mark up [p. 47] An increase in the price of an item usually expressed as percentage increase, or the calculation of the price of an item as a fixed increase from its cost.

Matrix [p. 332] A rectangular array of numbers or symbols set out in rows and columns within square brackets. (Plural – matrices.)

Matrix addition [p. 340] Matrices are added by adding the elements that are in the same positions.

Matrix multiplication [p. 347] The process of multiplying a matrix by a matrix.

Matrix subtraction [p. 340] Matrices are subtracted by subtracting the elements that are in the same positions.

Maximum (Max) [p. 410] The largest value in a set of numerical data.

Mean (x) [p. 401] The balance point of a data distribution. The mean is given by $\bar{x} = \frac{\Sigma x}{n}$, where Σx is the sum of the data values and n is the number of data values.

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Median (*M*) [p. 403] The midpoint of an ordered data set that has been divided into two equal parts, each with 50% of the data values. It is equal to the middle value (for an odd number of data values) or the average of the two middle values (for an even number of data values). It is a measure of the centre of the distribution.

Metric system [p. 101] A decimal measuring system based on the metre and other standard units.

Minimum (Min) [p. 410] The smallest value in a set of numerical data.

Minimum monthly balance [p. 57] The lowest amount that a bank account contains in a given calendar month.

Modal category or interval [pp. 380, 389] The category or data interval that occurs most frequently in a data set.

Mode [p. 380] The most frequently occurring value in a data set. There may be more than one.

N

Negative slope [pp. 178, 180] A straight-line graph with a negative slope represents a decreasing y-value as the x-value increases. For the graph of a straight line with a negative slope, y decreases at a constant rate with respect to x.

Negatively skewed distribution [p. 394] A data distribution that has a long tail to the left. In negatively skewed distributions, the majority of data values fall to the right of the mean.

Nett salary [p. 2] The amount of pay left after tax has been deducted.

Network [p. 338] A set of points called vertices and connecting lines called edges, enclosing and surrounded by areas called faces.

Nominal data [p. 375] Type of categorical data, such as gender, in which categories are given names (‘nominations’) or labels rather than taking a numerical value.

Nominal variable [p. 376] A categorical variable that generates data values that can be used by name only, for example, eye colour: blue, green, brown.

Non-linear [p. 303] An equation or realationship with a graph that is *not* a straight line. In non-linear equations, the unknown values are not all to the power of 1, for example, $y = x^2 + 5, 3y^2 = 6, b^3 = 27$.

Numerical data [p. 376] Data obtained by measuring or counting some quantity. Numerical data can be discrete (for example, the *number* of people waiting in a queue) or continuous (for example, the *amount of time* people spent waiting in a queue).

O

Order of a matrix [p. 333] An indication of the size and shape of a matrix, written as $m \times n$, where m is the number of rows and n is the number of columns.

Order of operations [p. 452] The sequence in which arithmetical operations should be carried out. *See* BODMAS.

Ordinal data [p. 375] Type of categorical data, such as clothing size, in which categories are given labels that can be arranged in order, such as numbers or letters.

Ordinal variable [p. 376] A categorical variable that generates data values that can be used to both name and order, for example, house number.

Outliers [p. 398] Data values that appear to stand out from the main body of a data set. Using boxplots, possible outliers are defined as data values greater than $Q_3 + 1.5 \times \text{IQR}$ or less than $Q_1 - 1.5 \times \text{IQR}$.

P

Parallel boxplot [p. 416] A statistical graph in which two or more boxplots are drawn side by side. Used to compare distributions in terms of shape, centre and spread.

Percentage [p. 463] The number as a proportion of one hundred, indicated by the symbol %. For example, 12% means 12 per one hundred.

Percentage change [p. 48] The amount of the increase or decrease of a quantity expressed as a percentage of the original value.

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Percentage frequency [p. 378] Frequency of a value or group of values, expressed as a percentage of the total frequency.

Perimeter [p. 103] The distance around the edge of a two-dimensional shape.

Per annum [p. 69] For each year; an annual rate.

Piecewise linear graph [p. 212] A graph made up of two or more parts of different straight-line graphs, each representing different overlapping intervals on the x-axis. Sometimes called a line segment graph.

Positive slope [p. 178] A positive slope represents an increasing y-value with increase in x-value. For the graph of a straight line with a positive slope, y increases at a constant rate with respect to x.

Positively skewed distribution [p. 394] A data distribution that has a long tail to the right. In positively skewed distributions, the majority of data values fall to the left of the mean.

Power of matrix A^n [p. 352] The matrix A is used n time in repeated multiplication.

Price-to-earnings ratio [p. 77] A measure of the profit of a company, given by the *current share price/profit per share*. A lower value of the price-to-earnings ratio may indicate a better investment.

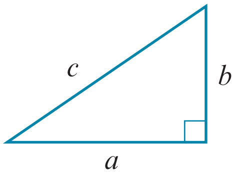
Principal (P) [p. 54] The initial amount borrowed, lent or invested.

Profit [p. 77] A financial gain, such as the amount left over when costs have been subtracted from revenue leaving a positive number. If the number left is negative, that is called a loss.

Pronumeral [p. 452] A symbol (usually a letter) that stands for a numerical quantity or variable.

Purchasing power [p. 73] A measure of how much a specific good or service money can buy at different times (due to inflation, for instance), or which different currencies can buy.

Pythagoras' theorem [p. 90] A rule for calculating the third side of a right-angled triangle given the length of the other two sides. In triangle ABC, the rule is: $c^2 = a^2 + b^2$, where c is the length of the hypotenuse.



Q

Quartiles (Q_1–Q_3) [p. 406] Summary statistics that divide an ordered data set into four equal-sized groups, each containing 25% of the scores.

R

Radius [p. 112] The distance from the centre of a circle (or sphere) to any point on its circumference (or surface); equal to half the diameter.

Range (R) [p. 404] The difference between the smallest (minimum) and the largest (maximum) values in a data set: a measure of spread.

Ratio [p. 472] A fraction, or two numbers in the form a:b, used to numerically compare the values of two or more quantities.

Relation [p. 175] The association between two variables that can be plotted as coordinates on a number plane, often described by an equation or formula, which can also be called a relationship.

Rest [p. 69] *See* compounding time period.

Right angle [p. 90] An angle equal to 90°.

Rise [p. 179] *See* Slope of a straight line.

Rounding [p. 457] Shortening a number by removing the last digit and adjusting the digit before it by the rules for rounding.

Row matrix [p. 335] A matrix with only one row.

Run [p. 179] *See* Slope of a straight line.

S

Scalar multiplication [p. 343] The multiplication of a matrix by a number.

Scale factor [p. 135] In similar figures, the ratio of the length of a side in one figure to the length of the corresponding side on the other figure.

Scale factor (areas) [p. 135] The scale factor, k^2, by which the area of a two-dimensional shape

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is scaled (increased or decreased) when its linear dimensions are scaled by a factor of k.

Scale factor (volumes) [p. 147] The scale factor, k^3, by which the volume of a solid shape is scaled (increased or decreased) when its linear dimensions are scaled by a factor of k.

Scientific notation [p. 457] A number expressed as a number between 1 and 10 and multiplied by a power of 10.

Shares [p. 84] A share is a unit of ownership of a company.

Significant figures [p. 459] The digits of a number that give its value to a required level of accuracy.

Similar figures [p. 135] Figures that have exactly the same shape but differ in size.

Similar triangles [p. 135] Different sized triangles in which the corresponding angles are equal. The ratios of the corresponding pairs of sides are always the same.

Simple interest [p. 54] Interest that is calculated for an agreed period and paid only on the original amount invested or borrowed. Also called flat-rate interest.

Simultaneous linear equations [p. 198] Two or more linear equations in two or more variables, for values that are common solutions to all equations. For example, $3x - y = 7$ and $x + y = 5$ are a pair of simultaneous linear equations in x and y, with the common solution $x = 3$ and $y = 2$.

Sine ratio (sin θ) [p. 244] In right-angled triangles, the ratio of the side opposite a given angle (θ) to the hypotenuse.

Sine rule [p. 276] In non-right-angled triangles, a rule used to find:

- an unknown side, given the angle opposite and another side and its opposite angle
- an unknown angle, given the side opposite and another side and its opposite angle.

For triangle ABC the rule is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Skewness [p. 394] Lack of symmetry in a data distribution. It may be positive or negative.

Slope of a straight line [p. 179] The ratio of the increase in the dependent variable (y) to the increase in the independent variable (x) in a linear equation. Also known as the gradient. The slope (or gradient) rise of a straight-line graph is defined to be:

$$\text{slope} = \frac{\text{rise}}{\text{run}}.$$

SOH–CAH–TOA [p. 244] A memory aid for remembering the trigonometric ratio rules.

Spread of a distribution [p. 395] A measure of the degree to which data values are clustered around some central point in the distribution. Measures of spread include the standard deviation (s), the interquartile range (IQR) and the range (R).

Square matrix [p. 335] A matrix with the same number of rows as columns.

Standard deviation (s) [p. 407] A summary statistic that measures the spread of the data values around the mean. It is given by

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}.$$

Stem plot (stem-and-leaf plot) [p. 397] A method for displaying data in which each observation is split into two parts: a ‘stem’ and a ‘leaf’. A stem plot is an alternative display to a histogram; suitable for small- to medium-sized data sets.

Step graph [p. 213] A step-graph consists of one or more non-overlapping horizontal line segments. They can be used to graphically represent situations where the value of one variable is constant for intervals of another variable.

Substitution method [p. 197] When solving simultaneous equations by substitution, the process is to substitute one variable from one equation into the other equation.

Summary statistics [p. 401] Statistics that give numerical values to special features of a data distribution, such as centre and spread. Summary statistics include the mean, median, range, standard deviation and IQR.

Surface area [p. 130] The total of the areas of each of the surfaces of a solid.

Symmetric distribution [p. 394] A data distribution in which the data values are evenly distributed around the mean. In a symmetric distribution, the mean and the median are equal.

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T

Tangent ratio (tan θ) [p. 244] In right-angled triangles, the ratio of the side opposite a given angle θ to the side adjacent to the angle.

Term [p. 164] One value in an algebraic expression.

Three-figure bearing [p. 265] An angular direction, measured clockwise from north and written with three digits, for example, 060°, 324°. Also called a true bearing.

Total surface area (TSA) [p. 131] The total surface area (TSA) of a solid is the sum of the surface areas of all of its faces.

Transposition [p. 321] The process of rearranging a formula in order to make a different pronumeral the subject of the equation.

Trigonometric ratios [p. 244] In right-angled triangles, the ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

True bearing [p. 265] *See* Three-figure bearing.

U

Undefined [p. 180] Has no meaning; has no value. The slope, or gradient, of a vertical line is undefined because $\frac{rise}{run}$ gives a zero denominator.

Unit cost [p. 24] The cost or price of a single item.

Univariate data [p. 374] Data that are collected from a single variable.

V

Variable [p. 375] A quantity that can have many different values in a given situation. Symbols such as x, y and z are commonly used to represent variables.

Volume [p. 154] The volume of a solid is a measure of the amount of space enclosed within it, measured in cubic units.

Volume formulas [p. 134] Formulas used to calculate the volumes of solids, including cubes, cuboids, prisms, pyramids, cylinders, cones and spheres.

Y

y-intercept [p. 182] The point at which a graph cuts the y-axis.

Z

Zero matrix (0) [p. 341] A matrix that behaves like zero in arithmetic, represented by the symbol O. Any matrix with zeros in every position is a zero matrix.

Zero slope [p. 180] A horizontal line has zero slope. The equation of this line has the form $y = c$ where c is any constant.

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Answers

Chapter 1

Exercise 1A

1 **a** $1782 **b** $3563 **c** $7721

2 **a** $1108, $2217 **b** $1509, $3019
c $2073, $4146 **d** $851, $1702

3 $1656

4 **a** $30 160 **b** $39 520
c $39 216 **d** $125 736

5 **a** $3000 **b** $78 000

6 $41 860

7 $81 120

8 Stephanie $49 348.00, Tahlia $45 852.00, Stephanie by $3496.00

9 Laura $32 110.00, Ebony $29 508.00; Laura by $2602.00

10 Tran $98 696.00, Jake $99 960.00; Difference $1264.00

11 **a** $892.50 **b** $943.50
c $1020.00 **d** $1071.00

12 $1130.50

13 **a** $444.00 **b** $351.50
c $858.40 **d** $511.71

14 **a** $15 787.20 **b** $31 720.00
c $39 062.40 **d** $24 731.20

15 **a** 40.50 h **b** $911.25

16 42 hours

17 9 hours

18 **a** $193.60 **b** $968.00
c $1936.00 **d** $50 336.00

19 Alyssa $332.10, Connor $320.00; Alyssa by $12.10

20 $635 481.60

21 $83 790.00

22 Computer application

23 Correct answer $1640.85 weekly pay; Isabelle's answer $1777.58
Calculations for weekly pay are incorrect as not every month has 4 weeks.

24 Lucy 18.75 hours, Ebony 16.67 hours

Exercise 1B

1 **a** $108.00 **b** $237.00 **c** $379.20
d $262.98

2 **a** $227.94 **b** $114.30 **c** $292.68
d $370.74

3 $1560.00 **4** $135.30 **5** $1023.40

6 **a** 45.5 hours **b** $1041.60

7 $9.80 **8** $16.00 **9** $1000.68

10 $1237.50

11 **a** $257.50 **b** $1287.50

12 $820.88 **13** $1761 **14** $3589.60

15 $2212 **16** $3720 **17** $451.05

18 $680.00

19 **a** $448.00 **b** $268.80 **c** 37.5%

20 5.89 hours

21 **a** $453.60 **b** 4 hours **c** $22.68
d $567.00

22 **b** $33.75 **d** $1288 **e** $5870

Exercise 1C

1 **a** $352.80 **b** $669.60 **c** $1368.80

2 **a** $9120.00 **b** $8400.00 **c** $5710.00
d $11 814.00

3 $1620.00

4 **a** $431.00 **b** $642.00 **c** $1640.00

5 $960.00

6 **a** 512.00 **b** $500.00

7 **a** \$1000, \$3250 **b** \$1000, \$4500
c \$1000, \$5750 **d** \$1000, \$7000

8 \$50 600.00

9 **a** \$3400.00 **b** \$4700.00 **c** \$5200.00

10 2%

11 \$460.00

12 **a** \$2250 **b** \$3870 **c** \$2880

13 \$500

14 \$5140

15 **a** \$1825
b Skirt: \$25/h, Shirt: \$23.33/h, Dress: \$23.57/h
c 40 skirts

16 \$4473.60

17 **a** \$13 096.32 **b** \$36 864.00
c \$15 712.35

18 **b** \$7794 **c** \$57 502.25

19 Computer application

Exercise 1D

1 **a** \$437.50 **b** \$11 375

2 **a** \$6858.80 **b** \$8801

3 **a** \$412.80 **b** \$10 732.80

4 **a** \$11 375 **b** \$11 375

5 **a** Youth **b** \$437.50
c Austudy **d** \$0

Exercise 1E

1 **a** \$15.60 **b** 84 seconds **c** \$885
d 10 km

2 7 red, 28 yellow

3 **a** 270 km **b** 225 km **c** 30 km
d 105 km **e** 330 km **f** 67.5 km

4 73 g cone for \$2.00

5 Brand A

6 **a** 550 km **b** 17 litres

7 51 eggs

Exercise 1F

1 **a** 506.98 EUR **b** \$3571.15 USD
c R\$511.62 BRL **d** \$2219.03 AUD
e \$3056.95 AUD **f** \$5855.04 AUD

2 17 245.20 BWP

3 328.32 EUR

Exercise 1G

1 **a** \$1536 **b** \$2550
c \$18 720 **d** \$23 400

2 \$13 473

3 **a** \$61 460.37 **b** \$59 769.77
c \$1690.60

4 **a**

Income		Expenses	
Job	\$74	Sport	\$24
Allow	\$30	Movies	\$22
		School	\$16
		Food	\$20
		Balance	\$22
	\$104		\$104

b \$22.00

5 Computer application

6 **a** \$526 **b** \$658 **c** \$77 total
d

Income		Expenses	
Job	\$1896	Mortgage	\$526
		Groceries	\$360
		Entertain	\$120
		Medical	\$18
		Car	\$160
		Electricity	\$20
		Telephone	\$14
		Rates	\$43
		Balance	\$635
	\$1896		\$1896

Exercise 1H

1 Answers will vary. See worked solutions for notes.

2 Answers will vary. See worked solutions for notes.

3 **a** **i** If the taxable income is equal to or less than \$18 200 then the tax payable is zero.
ii If the taxable income is equal to or more than \$18 201 and equal to or less than \$37 000 then the tax payable is 0.19 times the amount of taxable income over \$18 200.
iii If the taxable income is equal to or more than \$37 001 and equal to or less than \$87 000 then the tax payable is \$3572 plus 0.325 times the amount of taxable income over \$37 000.

iv If the taxable income is equal to or more than $87 001 and equal to or less than $180 000 the tax payable is .$19 822 plus 0.37 times the amount of taxable income over $87 000.

v If the taxable income is equal to or more than $180 001 the tax payable is .$54 232 plus 0.45 times the amount of taxable income over $180 000.

b **i** If Income ≤ $18 200, Tax = 0

ii If Income ≥ $18 201 and Income ≤ $37 000, Tax = 0.19* (Income–$18 200)

iii If Income ≥ $37 001 and Income ≤ $87 000, Tax = 0.325* (Income–$37 000)

iv If Income ≥ $87 001 and Income ≤ $180 000, Tax = 0.37* (Income–$87 000)

v If Income ≥ $180 000, Tax = 0.45* (Income–$180 000)

c Spreadsheet shows how to use IF functions to determine the bracket and apply the relevant formula.

4 **a** **i** She will earn $350 per week but has $100 in expenses, so $250 per week. 4800/250 = 19.2, so it will take at least 20 weeks to have $4800.

ii She will earn $500 per week but has $100 in expenses, so $400 per week. 4800/400 = 12, so it will take her at least 12 weeks to have $4800.

b The hourly rate is 100/4 = $25 per hour.

c A month has 4 weeks and Erin wishes to buy her goods in 3 months time (12 weeks).

i It takes 20 weeks for Erin to have enough working at the supermarket, so she will not be able to afford the goods in 3 months.

ii It takes 12 weeks for Erin to have enough delivering flyers, so she will have just enough. She will be able to afford the goods.

d She will not be able to afford the goods working at the supermarket. Her net earnings is $250 per week. In 3 months (12 weeks), she will have $3000. She is $4800 – $3000 = $1800 short.

e The hourly rate of pay for delivering flyers is the same as working at the supermarket. However, she would work less at the supermarket (14 hours per week), compared to delivering flyers (20 hours per week).

f Erin should deliver flyers around the local area, if she wishes to buy her goods in 3 months time.

5 Answers will vary. See worked solutions for notes.

Chapter 1 review

Multiple-choice questions

1 B **2** C **3** D **4** E

5 E **6** A **7** D **8** E

9 B **10** E **11** D **12** D

Short-answer questions

1 **a** $1855.20 **b** $38.65

2 $641.25

3 $5371.25

4 **a** $839.16 **b** $5634.36

5 **a** $4725.00 **b** $235 275.00

6 $4440

7 **a** $31 461.46 **b** $26 742.24

8 $1056.00

9 $5.35

10 **a** $1833.85 **b** 25.8%

11 $512

12 $2781.12

13 $1526

14 $27 103.50

15 **a** $264.67 **b** $185.14

16 **a** $105.76 **b** £66.57

Extended-response questions

1 **b** **i** $2085 **ii** $2220

c Rent $800, Phone $89.17, Food $285, Entertainment $356.67, Clothing $370, Petrol $160.83

d **i** $888.33 **ii** 4 months

e 3 months

2 **a** $269.51

b $70 072

c $57 572

d **i** 8 hours per week **ii** 11.4 hours

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Chapter 2

Exercise 2A

1 **a** 48.8% **b** 24.4% **c** 11.1% **d** 9.2% **e** 33.3% **f** 25%

2 **a** $2600 **b** $200 **c** $200 **d** $1.80 **e** $1865 **f** $20 000

3 **a** $86.40 **b** $180 **c** $0.58 **d** $73.08

4 **a** $291.20 **b** $626.40 **c** $68 **d** $6318

5 **a** $1865.50 **b** $11.14 **c** $27.72 **d** $10 282 **e** $847.70 **f** $2631.20

6 **a** 24% **b** 20%

7 **a** $60 **b** $50 **c** $71.43 **d** $88

8 **a** $13.30 **b** $62.22 **c** $104.50 **d** $4993.90

9 $212.75

10 $92.07

11 **a** $12.13 **b** $6.76 **c** $98.55 **d** $39.50

12 **a** $152.90 **b** $2945.80 **c** $10 835 **d** $1534.50

13 $2180.91

14 $3635.45

15 **a** $289.97 **b** $29.00

16 **a** $150 **b** 33.3%

17 **a** $20.00 **b** 20%

18 150%

Exercise 2B

1 **a** $80 **b** $300 **c** $600 **d** $384.38 **e** $4590 **f** $324.38 **g** $29.95 **h** $14.43 **i** $6243.75

2 **a**

	A	B
1		
2	*Simple interest*	
3	Year	Interest
4	1	$148.50
5	2	$297.00
6	3	$445.50
7	4	$594.00
8	5	$742.50
9	6	$891.00
10	7	$1,039.50
11	8	$1,188.00
12	9	$1,336.50
13	10	$1,485.00

b $742.50

3 **a** $600 **b** $932.10 **c** $1243.50 **d** $2411.25 **e** $2832

4 **a** $12 000 **b** $32 000

5 **a** $1950 **b** $11 950

6 $1243.50 **7** $9041.10

8 **a** $118.75 **b** $2750 **c** $2463.19 **d** $24 000 **e** $1983.63 **f** $13 617.92

9 **a** $4500 **b** $13.33

10 $3.12 **11** $1.88

12 **a** March $650.72, April $650.72, May $900.72
b $6.88

Exercise 2C

1 12% **2** 15% **3** 6.5 years

4 354 days **5** $1210 **6** $45 552

7 **a** $180 **b** $780 **c** 3 years **d** $1051.60 **e** 7% **f** $1335.15 **g** $4500 **h** $4650 **i** 5% **j** $3698.63 **k** 3 years **l** $220.50 **m** $1448.28 **n** $1500.78

8 4 years **9** 20 years

10 **a** $18 000 **b** $3000

Exercise 2D

1 **a** $4466.99 **b** $966.99

2 **a** $9523.42 **b** $2523.42

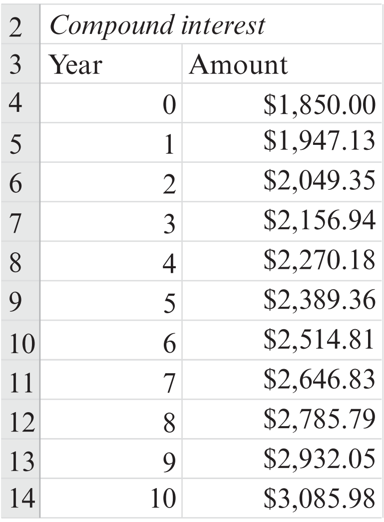
3 Difference: CI – SI = $ 202.61

4 **a** $1552.87 **b** $302.87

5 **a** $1338.23 **b** $338.23

6 Difference: CI – SI = $105.10

7 **a**



2	*Compound interest*	
3	Year	Amount
4	0	$1,850.00
5	1	$1,947.13
6	2	$2,049.35
7	3	$2,156.94
8	4	$2,270.18
9	5	$2,389.36
10	6	$2,514.81
11	7	$2,646.83
12	8	$2,785.79
13	9	$2,932.05
14	10	$3,085.98

b $420.18

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8 a

Years invested	Amount
0	$850.00
1	$962.63
2	$1,090.17
3	$1,234.62
4	$1,398.21
5	$1,583.47
6	$1,793.28
7	$2,030.89
8	$2,299.98

c $1583.47, $733.47

9 a

Years invested	Amount
0	$3,000.00
1	$3,169.50
2	$3,348.58
3	$3,537.77
4	$3,737.66
5	$3,948.83
6	$4,171.94
7	$4,407.66
8	$4,656.69
9	$4,919.79
10	$5,197.76

c $3737.66, $737.66

10 a 0.3% **b** $10 181.36
c $27 846.69

11 a 0.2% **b** $6482.97
c $1482.97

12 a i $18 696.04 **ii** $18 696.64
iii $18 697.13
b 60 cents

Exercise 2E

1 a $3.59 **b** $3.72

2 a $851.40 **b** $896.52

3 a $2.62 **b** $7.10

4 a $680 359.31 **b** $1 113 531.40

5 a $148 818.78 **b** $58 917.67

6 a $598.48 **b** $263.30 **c** $69.04

Exercise 2F

1 a 0.5% **b** $12 500

2 a 2000 **b** $50 000 **c** 1200

3 a A: 8.8, B: 10 **b** A

4 $400

5 a i $11 000 **ii** $12.10
b i $6, 600 **ii** $17.60

6 a $0.50 **b** $1.00
c 500 Alpha Oil, 250 Omega Mining
d $612.50

7 a $250 **b** 10.9%

8 a $1.25 **b** $2500

Exercise 2G

1 a $7200 **b** $50 256 **c** $14 256
d 16.5% p.a. **e** 13.5% p.a.

2 a 0.217%
b $52 665.82
c 24
d Advantage: higher interest rate.
Disadvantage: cannot take money out.
e $30 000
f $22 665.82

3 a $31 098.00
b Compounding periods = 52. Final amount is $31 099.29
c Compounding periods = 1. Final amount is $31 080.00.
d The more compounding periods, the higher the final amount.
e Interest is paid more frequently as the number of compounding periods increase. As interest is paid more frequently, the amount of interest received will be greater. Hence, the final amount is higher if the number of compounding periods is higher.

4 a $5715
b $57.15
c

■ Ingredients and supplies	$1400 pw
■ Cleaning accessories and costs	$20 pw
■ Maintenance	$16 pw
■ Gas and electricity	$40 pw
■ Rent	$200 pw
■ EFTPOS fees	$57.15 pw
■ Other expenses	$40

d $1773.15
e $25 500.
f *See table at top of next page.*
g $2273.15
h Yes. $3441.85.

	Amount still owing on loan	Interest calc	total	Loan Payment	Total of loan at end of week	Capital paid off
Week 1	$26,000.00	*Not Applicable*	$26,000.00	$500.00	$25,500.00	$500.00
Week 2	$25,500.00	*Not Applicable*	$25,500.00	$500.00	$25,500.00	$1,000.00
Week 3	$25,000.00	*Not Applicable*	$25,000.00	$500.00	$24,500.00	$1,500.00
Week 4	$24,500.00	$81.67	$24,581.67	$500.00	$24,081.67	$1,918.33
Week 5	$24,081.67	*Not Applicable*	$24,081.67	$500.00	$23,581.67	$2,418.33
Week 6	$23,581.67	*Not Applicable*	$23,581.67	$500.00	$23,081.67	$2,918.33
Week 7	$23,081.67	*Not Applicable*	$23,081.67	$500.00	$22,581.67	$3,418.33
Week 8	$22,581.67	$75.27	$22,656.94	$500.00	$22,156.94	$3,843.06
Week 9	$22,156.94	*Not Applicable*	$22,156.94	$500.00	$21,656.94	$4,343.06
Week 10	$21,656.94	*Not Applicable*	$21,656.94	$500.00	$21,156.94	$4,843.06

Chapter 2 review

Multiple-choice questions

1 B **2** B **3** A **4** D
5 A **6** C **7** E **8** C
9 E **10** A **11** B **12** D
13 A **14** E **15** B

Short-answer questions

1 a Rabbit Easter eggs **b** 0.7%

2 a $137.50
b Before discount: profit of 12.7%; after discount loss of 9.1%

3 a $36 000 **b** $11 000

4 a 0.2917% **b** $5527.04

5 $791.89

6 $14 859.47

7 a $1100 **b** 4.8%

Extended-response questions

1 a $612.50
b i $373.35 **ii** $23.35 **iii** $653.36
iv $6.67%

2 a, b

	5.3% SI	5.0% CI	
	Interest	Amount	Interest
Year			
0	$0.00	$3,000.00	$0.00
1	$159.00	$3,150.00	$150.00
2	$318.00	$3,307.50	$307.50
3	$477.00	$3,472.88	$472.88
4	$636.00	$3,646.52	$646.52
5	$795.00	$3,828.84	$828.84
6	$954.00	$4,020.29	$1,020.29
7	$1,113.00	$4,221.30	$1,221.30
8	$1,272.00	$4,432.37	$1,432.37
9	$1,431.00	$4,653.98	$1,653.98
10	$1,590.00	$4,886.68	$1,886.68

c i Plan A **ii** Plan B

3 a, b

	Part a	Part b	
Month	Amount	Amount after interest added	Amount after $100 withdrawn
0	$50,000.00	$50,000.00	
1	$50,208.33	$50,208.33	$50,108.33
2	$50,417.53	$50,317.12	$50,217.12
3	$50,627.61	$50,426.36	$50,326.36
4	$50,838.56	$50,536.05	$50,436.05
5	$51,050.38	$50,646.20	$50,546.20
6	$51,263.09	$50,756.81	$50,656.81
7	$51,476.69	$50,867.88	$50,767.88
8	$51,691.18	$50,979.41	$50,879.41
9	$51,906.56	$51,091.41	$50,991.41
10	$52,122.83	$51,203.87	$51,103.87
11	$52,340.01	$51,316.81	$51,216.81
12	$52,558.09	$51,430.21	$51,330.21

c $52 558.09
d $51 330.21
e $27.88 less

Chapter 3

Exercise 3A

1 a 4.9 cm **b** 83.1 cm **c** 24 mm
d 2.4 mm **e** 15.8 mm **f** 7.4 cm
g 6.4 cm **h** 141.4 mm **i** 15.4 m
2 2.9 m **3** 3.8 m **4** 5.3 m
5 48.88 km **6** 15 km **7** 12.81 km
8 20 cm **9** 9.4 m **10** 61.717 m
11 4.24 cm **12** 103 m

Exercise 3B

1 a 4.243 cm **b** 5.196 cm

2 a 10.77 cm **b** 11.87 cm **c** 6.40 cm

3 a 27.73 mm **b** 104.79 mm

4 9.54 cm

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5 **a** **i** 8.5 cm **ii** 9.1 cm
b **i** 10.6 cm **ii** 3.8 cm
6 17 cm **7** 13 cm **8** 25 cm
9 Yes it will fit **10** 8.02 m **11** 17.55 m

Exercise 3C

1 **a** 570 cm **b** 1587 m **c** 80 mm
d 6.7 m **e** 460 cm **f** 2.89 cm^2
g 52 000 cm^2 **h** 80 000 m^2 **i** 37 cm^2
j 6 000 000 mm^2 **k** 0.5 L
l 700 g **m** 2 300 000 mg
n 0.567 kL **o** 793.4 g **p** 500 mL
2 **a** 5.0×10^3 kg **b** 6.0×10^{-3} kg
c 2.71×10^{10} m^2 **d** 3.3×10^7 cm^3
e 4.87×10^{-4} km^2 **f** 2.8×10^{-2} L
g 6.0×10^5 cm **h** 1.125×10^{-3} kL
i 5.0×10^{-5} km^3 **j** 3.4×10^{-4} m^3
3 **a** 158 mm **b** 589.169 km
c 364.6 cm **d** 13.5 cm^2
4 7.86 m **5** 3000 kg
6 2 250 000 L **7** 31 trays
8 **a** **i** 60 cm **ii** 225 cm^2
b **i** 22.4 cm **ii** 26.1 cm^2
c **i** 312 cm **ii** 4056 cm^2
d **i** 44 cm **ii** 75 cm^2
9 **a** 56.2 cm^2 **b** 16.7 m^2
c 103.6 cm^2 **d** 73.8 cm^2
e 28 cm^2 **f** 35.9 cm^2
g 29.9 m^2 **h** 31.3 m^2
10 100 m^2 **11** 63 375 m^2
12 40 tiles **13** 4 L
14 **a** 5 cm^2 **b** 125 cm^2 **c** 40 cm^2
15 30.88 m^2
16 **a** 252 m^2 **b** 273 m^2

Exercise 3D

1 **a** **i** 31.4 cm **ii** 78.5 cm^2
b **i** 53.4 cm **ii** 227.0 cm^2
c **i** 49.6 mm **ii** 196.1 mm^2
d **i** 1.3 m **ii** 0.1 m^2
2 **a** **i** 25.71 cm **ii** 39.27 cm^2
b **i** 1061.98 mm **ii** 14 167.88 mm^2
c **i** 203.54 cm **ii** 2551.76 cm^2
d **i** 53.70 mm **ii** 150.80 mm^2
3 62.83 cm^2
4 **a** 343.1 cm^2 **b** 34.9 m^2
c 19.2 cm^2 **d** 177 377.4 mm^2
5 **a** 1051.33 m **b** 37 026.55 m^2
6 **a** 6 m **b** 3.4 m^2
7 1060 cm^2 **8** 30.91 m^2
9 8.73 cm **10** 8.19 m

Exercise 3E

1 **a** 125 cm^3 **b** 49 067.8 cm^3
c 3685.5 cm^3 **d** 3182.6 mm^3
e 29 250 cm^3 **f** 0.3 m^3
g 6756.2 cm^3 **h** 47.8 m^3
2 424 cm^3 **3** 516 cm^3 **4** 24 L
5 **a** 20 319.82 cm^3 **b** 20 L
6 228 cm^3

Exercise 3F

1 **a** 9500.18 cm^3 **b** 16.36 m^3
c 59.69 m^3 **d** 2356.19 mm^3
2 **a** 153.94 cm^3 **b** 705.84 m^3
c 102.98 cm^3 **d** 1482.53 cm^3
3 393 cm^3 **4** 7.87 m^3
5 0.02 L **6** 18 263.13 cm^3
7 2791 m^3
8 **a** 26.67 cm^3 **b** 420 m^3
c 24 m^3 **d** 68.64 cm^3
9 213.333 cm^3 **10** 1 694 000 m^3
11 **a** 335.6 cm^3 **b** 66.6 cm^3
12 3937.5 cm^3
13 **a** 523.60 mm^3 **b** 229.85 mm^3
c 7238.23 cm^3
14 **a** 179.59 cm^3 **b** 11 494.04 cm^3
c 33.51 cm^3
15 **a** 8578.64 cm^3 **b** 7679.12 cm^3
c 261.80 cm^3 **d** 4.09 m^3
16 44 899 mm^3 **17** 14 L

Exercise 3G

1 **a** 1180 cm^2 **b** 40 m^2
c 383.3 cm^2 **d** 531 cm^2
e 2107.8 cm^2 **f** 176.1 m^2
2 **a** 3053.63 cm^2 **b** 431.97 cm^2
c 277.59 m^2 **d** 7.37 m^2
e 242.53 cm^2 **f** 24.63 m^2
g 235.62 m^2 **h** 146.08 m^2
3 15 394 cm^2
4 **a** 1.08 m^2 **b** 6 m
5 0.23 m^2

Exercise 3H

1 **a** **i** $\frac{3}{1}$ or $k = 3$ **ii** $\frac{9}{1}$ or $k^2 = 9$
b **i** $\frac{2}{1}$ or $k = 2$ **ii** $\frac{4}{1}$ or $k^2 = 4$

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2 **a** Similar, $\frac{3}{1}$ or $k = 3$ **b** Similar, $\frac{2}{1}$ or $k = 2$
c Not similar
3 **a** Not similar **b** Similar, $\frac{4}{1}$ or $k = 4$
c Not similar **d** Similar $\frac{1}{3}$ or $k = \frac{1}{3}$
e Similar $\frac{3}{2}$ or $k = \frac{3}{2}$
4 $\frac{4}{1}$
5 **a** 3 cm **b** $\frac{9}{1}$
6 112 cm^2 **7** 864 cm^2
8 1.67 **9** **a** 36 km **b** 3 cm
10 14.4 cm
11 **a** 4.25 m **b** 50 cm
12 840 cm^2
13 **a** 15 m **b** 33.75 m^2

Exercise 3I

1 **a** SSS **b** AA **c** SAS or SSS or AA
2 **a** $x = 27$ cm, $y = 30$ cm
b $x = 26$ m, $y = 24$ m
3 **a** 28 cm, 35 cm **b** 119 cm
4 **a** AA **b** $\frac{1}{2}$ **c** 2 m
5 1.8 m **6** 72 cm^2 **7** 29.4 cm^2

Exercise 3J

1 27 times **2** **a** $\frac{4}{1}$ **b** $\frac{64}{1}$
3 $\frac{27}{1}$ **4** **a** 9 cm **b** $\frac{125}{1}$
5 **a** Scaled up **b** 27 **c** 3240 cm^3
6 **a** 6 cm **b** 27: 64
7 **a** 3 cm **b** Height = 12 cm, base = 16 cm
8 **a** 1 : 4 **b** 1 : 8

Exercise 3K

1 **a** 1.23 m^3 **b** 56 boards **c** 15 m^2
d 0.32 m^3 **e** 3.84 m^2
2 **a** 1.21 ha **b** 416.20 m
3 24 hectares
4 **a** 29.7 m^2 **b** 0.39 cm^2
5 **a** 14 cm **b** 56 cm, 44 cm
6 **a** 28.27 cm^2 **b** 4523.9 cm^3
c 4.524 L **d** 2827.4 cm^2
7 **a** 5 times **b** 32 768 000 cm^3
c 614 400 cm^2 **d** 7 times
8 4.8 m

Chapter 3 review

Multiple-choice questions

1 B **2** D **3** D **4** B **5** A
6 B **7** C **8** B **9** D **10** C
11 E **12** D **13** C **14** D **15** C
16 E

Short-answer questions

1 **a** 58 cm **b** 30 m
2 36 m **3** 68 cm
4 **a** 9.22 cm **b** 9 cm
5 **a** 140 cm^2 **b** 185 cm^2
6 37.5 cm^2
7 **a** 31.42 cm **b** 75.40 cm
8 **a** 78.54 cm^2 **b** 452.39 cm^2
9 **a** 373.85 cm^2 **b** 2.97 m^2 **c** 0.52 L
10 31 809 L
11 **a** 514 718 540 km^2 **b** 1.098×10^{12} km^3
12 **a** 376.99 cm^3 **b** 377 mL
13 6.4 m
14 **a** 30 m^2 **b** 15 m^2 **c** 5.83 m
d 69.97 m^2 **e** 421.94 m^2
15 33.32 m^3
16 4 **17** $\frac{1}{4}$
18 **a** 50.27 cm **b** 146.12 cm^2
19 Both equal 25.13 m

Extended-response questions

1 **a** 154.30 m^2 **b** 101.70 m
2 **a** 61.54 m **b** 140 m^2 **c** 120 m^3 **d** 128 m^2
3 13.33 m
4 **a** 15.07 m **b** 1.89 m^3
5 **a** $\frac{1.96}{1}$ or 1 : 1.96 **b** $\frac{2.744}{1}$ or 1 : 2.744
c 63 cm^3
6 2048 cm^3 **7** 18.71 cm
8 **a** 400 m
b 400 m, 406 m, 412 m, 419 m, 425 m, 431m
c Each starting point should be 6 m apart except for distance between 3rd and 4th runners which is 7 m.

Chapter 4

Exercise 4A

1 **a** $x = 9$ **b** $y = 15$ **c** $t = 5$ **d** $m = 6$
e $g = 6$ **f** $f = 19$ **g** $f = -3$ **h** $v = -5$
i $x = -1$ **j** $g = 1$ **k** $b = 5$ **l** $m = -2$

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m $y = 6$ **n** $e = 3$ **o** $h = -5$ **p** $a = -4$
q $t = -10$ **r** $s = -11$ **s** $k = 7$ **t** $n = 4$
u $a = 8$ **v** $b = 21$

2 **a** $x = 3$ **b** $g = 9$ **c** $n = 4$ **d** $x = -8$
e $j = -4$ **f** $m = 7$ **g** $f = 5.5$ **h** $x = 3.5$
i $y = 5$ **j** $s = -3$ **k** $b = -5$ **l** $d = -4.5$
m $r = 12$ **n** $q = 30$ **o** $x = 48$ **p** $t = -12$
q $h = 40$ **r** $m = 21$ **s** $a = 2$ **t** $f = -2$
u $a = 6$ **v** $y = 40$ **w** $r = 8$ **x** $x = 5$
y $m = 8$ **z** $x = 6.5$

3 **a** $y = 4$ **b** $x = 11$ **c** $g = 2$ **d** $x = 3$
e $x = 0.5$ **f** $m = 1.2$ **g** $a = 18$ **h** $r = 13$

4 **a** $x = 5$ **b** $a = 3$ **c** $b = 9$ **d** $y = 3$
e $x = 0$ **f** $c = -4$ **g** $f = -2$ **h** $y = -5$

5 **a** $a = \frac{11}{3}$ **b** $b = \frac{25}{4}$ **c** $w = \frac{5}{2}$ **d** $c = \frac{16}{7}$
e $y = \frac{23}{3}$ **f** $f = \frac{5}{2}$ **g** $h = \frac{13}{2}$ **h** $k = -\frac{4}{3}$
i $x = 1.84$ **j** $y = 2.5$ **k** $x = \frac{1}{2}$
l $x = \frac{1}{4}$ **m** $d = -10.5$ **n** $y = 4$
o $y = 0.6$

Exercise 4B

1 **a** $P = 27 + x$ **b** $P = 4x$
c $P = 2a + 2b$ **d** $P = 19 + y$

2 **a** $P = 22 + m$ **b** 8 cm

3 **a** $P = 4y$ **b** 13 cm

4 **a** $n + 7 = 15$ **b** 8

5 6 **6** 59

7 **a** $P = 4x + 12$ **b** 18 cm
c 24 cm, 18 cm

8 78 tickets **9** 25 invitations

10 Anne $750, Barry $250

11 30 km

12 **a** 47 min
b Ben 9.4 km, Amy 7.8 km

Exercise 4C

1 **a** $C = 0.5x + 0.2y$ **b** $16.50

2 **a** $C = 40x + 25y$ **b** $13 875

3 **a** $C = 1.6x + 1.4y$ **b** $141.20

4 **a** $C = 1.75x + 0.7y$ **b** $52.15

5 **a** $C = 3.5x + 5y$ **b** $312

6 **a** $C = 30x + 60y$ **b** $3480

7 **a** $N = x + y$ **b** $V = 0.5x + 0.2y$
c $37.90

8 5 balls

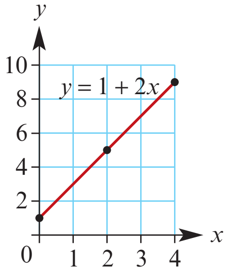
9 Maria is 21 years old, George is 16 years old.

10 667 cm

Exercise 4D

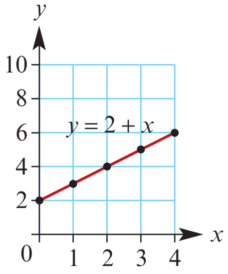
1 **a**

x	0	1	2	3	4
y	1	3	5	7	9



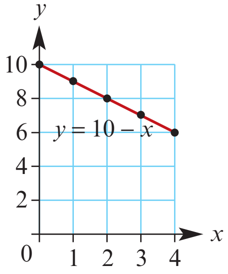
b

x	0	1	2	3	4
y	2	3	4	5	6



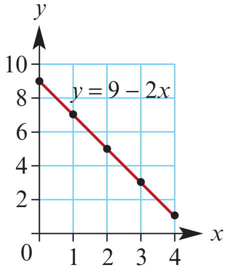
c

x	0	1	2	3	4
y	10	9	8	7	6



d

x	0	1	2	3	4
y	9	7	5	3	1



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2 a (0, 4), (2, 6), (3, 7), (5, 9)
b (0, 8), (1, 6), (2, 4), (3, 2)

Exercise 4E

1 *A* negative, *B* positive, *C* not defined, *D* zero

2 a $A\ \frac{-7}{3}$, $B\ \frac{7}{4}$, $C\ 1$
b $A\ 2$, $B\ -3$, $C\ 0$

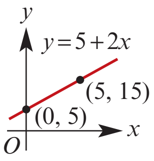
3 a 2 **b** −1 **c** 2 **d** 0.6
e 2 **f** −1

Exercise 4F

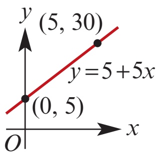
1 a y-intercept = 5, slope = 2
b y-intercept = 6, slope = −3
c y-intercept = 15, slope = −5
d y-intercept = 10, slope = −3
e y-intercept = 0, slope = 3
f y-intercept = −5, slope = −2
g y-intercept = 4, slope = 1
h y-intercept = 3, slope = 0.5
i y-intercept = −5, slope = 2
j y-intercept = 10, slope = 5
k y-intercept = 10, slope = −1
l y-intercept = 0, slope = 2
m y-intercept = 6, slope = −3
n y-intercept = −4, slope = 2
o y-intercept = −3, slope = 0.8
p y-intercept = −2, slope = 3

2 a $y = 2 + 5x$ **b** $y = 5 + 10x$
c $y = -2 + 4x$ **d** $y = 12 - 3x$
e $y = -2 - 5x$ **f** $y = 1.8 - 0.4x$
g $y = 2.9 - 2x$ **h** $y = -1.5 - 0.5x$

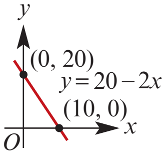
3 a



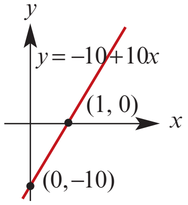
b



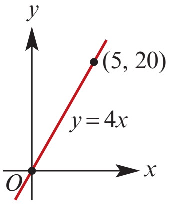
c



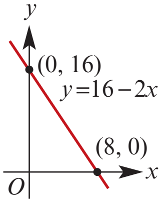
d



e



f



Exercise 4G

1 A: $y = 10 - 2.25x$ B: $y = 2 + 1.75x$
C: $y = x$

2 A: $y = 4 + 2x$ B: $y = 8 - 1.5x$
C: $y = 2 + 0.6x$

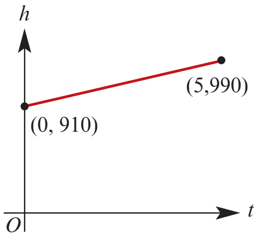
Exercise 4H

1 A: $y = 14.5 - 4.5x$ B: $y = -5 + 5x$
C: $y = -5 + 3x$

2 A: $y = 11.5 - 1.5x$ B: $y = -10 + 10x$
C: $y = 2 + 1.2x$

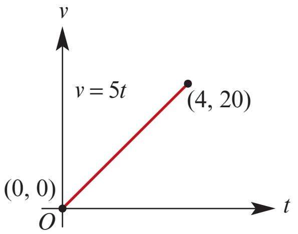
Exercise 4I-1

1 a $h = 910 + 16t$ for $0 \le t \le 5$
b



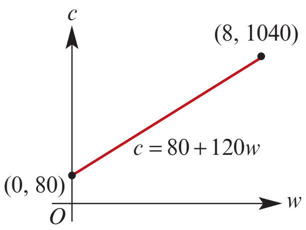
c 982 cm

2 a $V = 0 + 5t$ for $0 \le t \le 4$
b



c 16 litres

3 a $c = 80 + 120w$, for $0 \le w \le 8$
b



c $680

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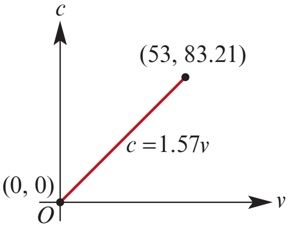
4 a $c = 1.57v$, for $0 \le v \le 53$

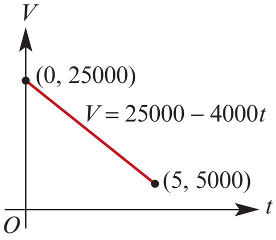
b



c \$83.21

5 a $V = 25\,000 - 4000t$, for $0 \le t \le 5$

b

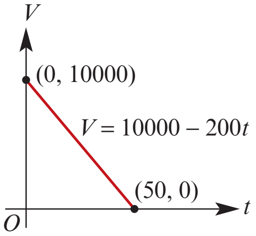


c \$14 600

6 a 50 days

b $V = 10\,000 - 200t$, for $0 \le t \le 50$

c



d 4000 litres

Exercise 4I-2

1 a \$10 **b** \$17.50 **c** $C = 10 + 0.075n$

d \$32.50 **e** \$0.075 (7.5 cents)

2 a 500 mL **b** 400 mL **c** 200 minutes

d $V = 500 - 2.5t$ **e** 212.5 mL

f 2.5 mL/min

3 a $F = 32 + 1.8C$ (or as more commonly written: $F = \frac{9}{5}C + 32$)

b i 122°F **ii** 302°F **iii** −40°F

c 1.8

Exercise 4J

1 a $x = 2, y = 1$ **b** $x = 2, y = 5$

c $x = 3, y = 4$ **d** $x = 9, y = 1$

e $x = 4, y = 3$ **f** $x = 1, y = 1$

Exercise 4K

1 a (−1, −1) **b** (3, −2)

c (1, 1) **d** (1, −1)

2 a (2, −4) **b** (−3, 2) **c** (1.5, 2.5)

d (2, 1) **e** (0, 6) **f** (7, 2)

g (0, 3) **h** (1, 5) **i** (0.4, −2.6)

j (7, 25) **k** (−8, −20) **l** (−3, 10)

m No intersection, lines are parallel

Exercise 4L

1 a $5t + 6p = 12.75$ and $7t + p = 12.30$

b Texta \$1.65, pencil \$0.75

2 Nails 1.5 kg, screws 1 kg

3 12 emus, 16 wombats

4 a $50p + 5m = 109$ and $75p + 5m = 146$

b Petrol \$1.48/L, motor oil \$7/L

5 a $6a + 10b = 7.10$ and $6a + 8b = 6.40$

b Banana 35c, orange 60c

6 6 cm, 12 cm **7** 22, 30 **8** 8, 27

9 Bruce 37, Michelle 33

10 Boy is 9, sister is 3

11 Chocolate thickshake \$5, fruit smoothie \$3

12 Mother 44, son 12

13 77 students

14 10 standard, 40 deluxe

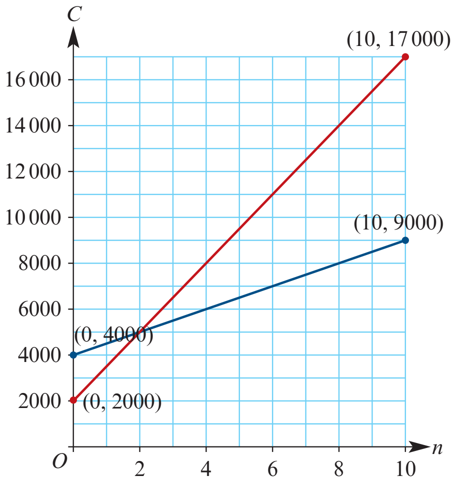
15 7542 L unleaded, 2458 L diesel

16 Width 24 m, length 36 m

17 28, 42, 35

18 a $C = 2000 + 1500n$

b and **d**

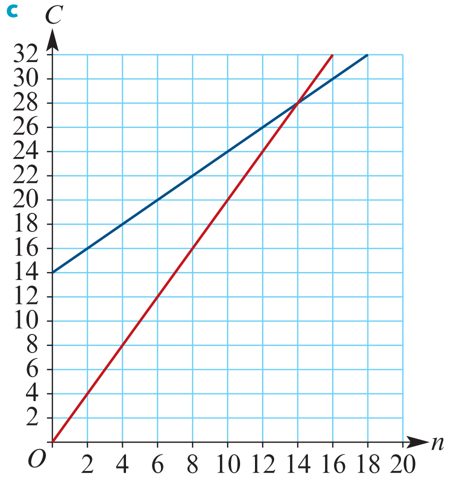


c $C = 4000 + 500n$ e 2 days

19 a Rabbitphone company \$24, Foxphone company \$20

a $C = 14 + n$ **b** $C = 2n$

c



d 14 minutes

3 **a** $V = 4800 - 200t$ **b** $V = 600 + 200t$

c 10.5 minutes

Exercise 4M

1 160 m^2, 260 m^2, 460 m^2

2 4 deluxe, 4 standard

3 **a** 31.90 L **b** 47.18 L **c** 176 cm **d** 40.61 L

e 45.65 L

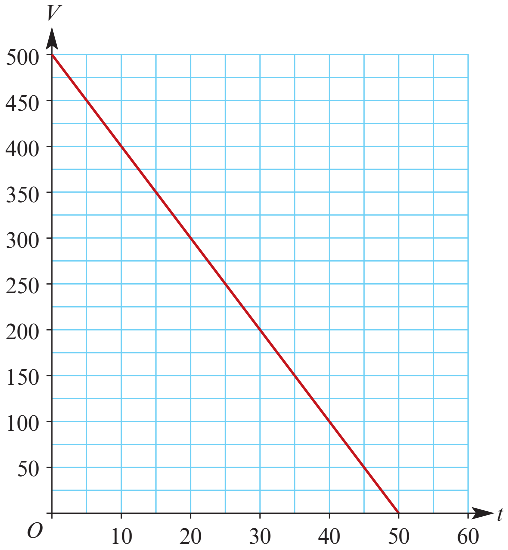
f

Weight	60	65	70	75	80
TBW	40.39	42.08	43.76	45.44	47.12

Weight	85	90	95	100
TBW	48.80	50.48	52.16	53.84

Weight	105	110	115	120
TBW	55.52	57.20	58.89	60.57

4 **a** 10

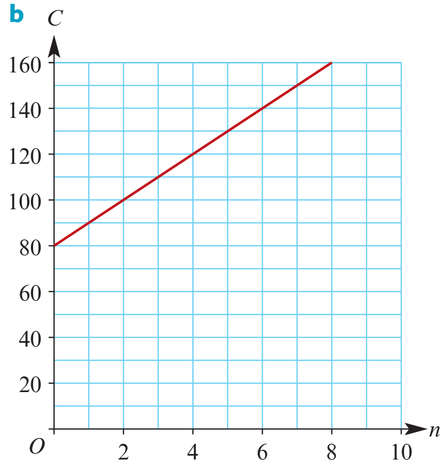


b 300 litres **c** 35 hours

d Yes, after 48 hours there are 20 litres left in the tank.

5 **a** $C = 80 + 10n$

b



c **i** \$130 **ii** 4 minutes

Exercise 4N

1 **a** **i** 5:00 **ii** 12:00 **iii** 9:00 **iv** 6:00

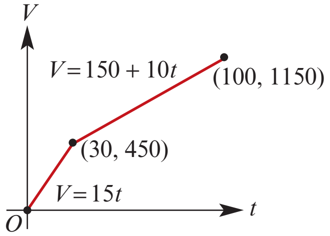
b 1hour

c 10:30

2 **a** **i** 300 L **ii** 450 L **iii** 750 L

iv 1150 L

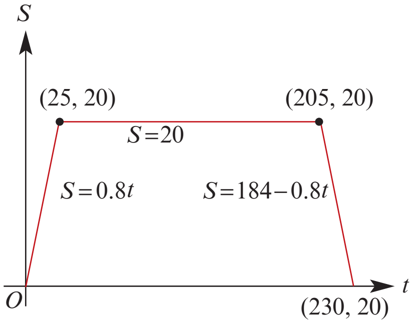
b



3 **a** **i** 8 m/s **ii** 20 m/s **iii** 20 m/s

iv 16 m/s

b



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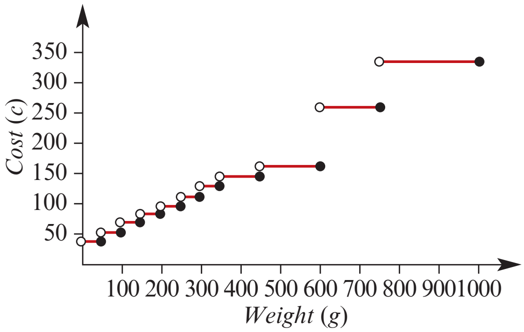

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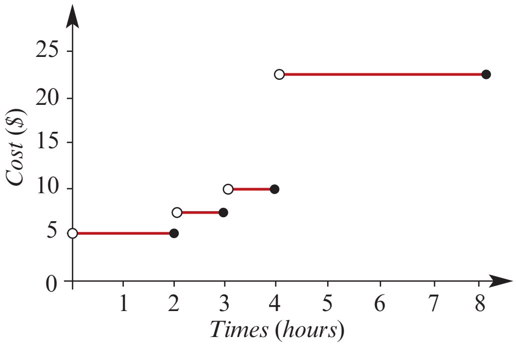

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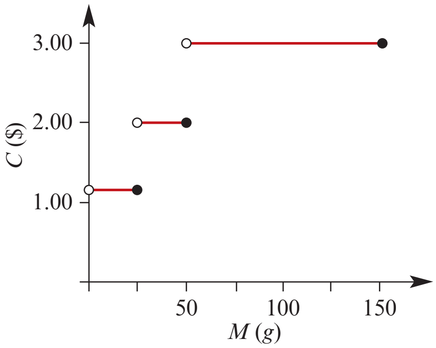
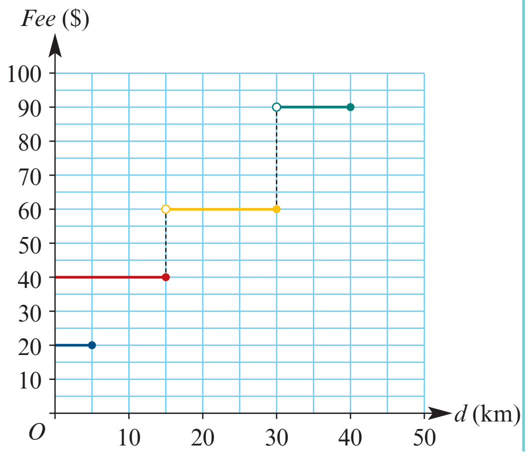


7 **a** Free **b** $3.00 **c** $5.00

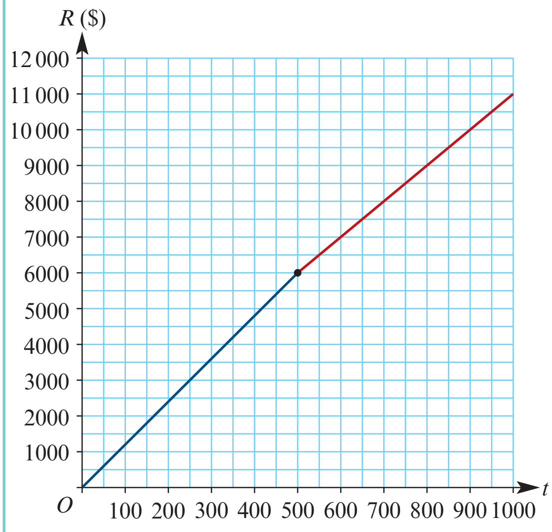
8 **a** $60 **b** 5 km

c



9 **a** **i** $4800 **ii** $9000

b



10 **a** $150 **b** $300

Chapter 4 review

Multiple-choice questions

1 B **2** D **3** B **4** D **5** C
6 E **7** D **8** A **9** B **10** A
11 D **12** C **13** C **14** E **15** C
16 B **17** E **18** C **19** B **20** B
21 A **22** C **23** D **24** D **25** D
26 A **27** D **28** C

Short-answer questions

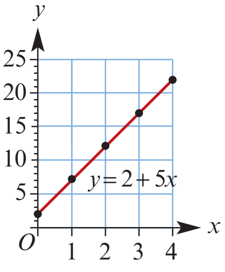
1 **a** $x = 10$ **b** $x = 11$ **c** $x = 8$ **d** $x = 6$
e $x = 1$ **f** $x = 7$ **g** $x = -6$ **h** $x = 11$
i $x = 3$ **j** $x = 3$ **k** $x = 15$ **l** $x = -24$

2 1

3 5

4 **a**

x	0	1	2	3	4
y	2	7	12	17	22



b

x	0	1	2	3	4
y	12	11	10	9	8

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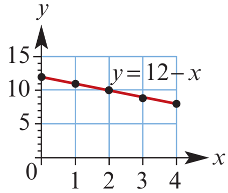
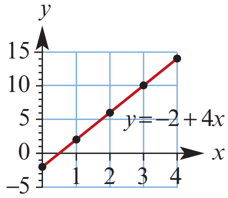


c

x	0	1	2	3	4
y	−2	2	6	10	14

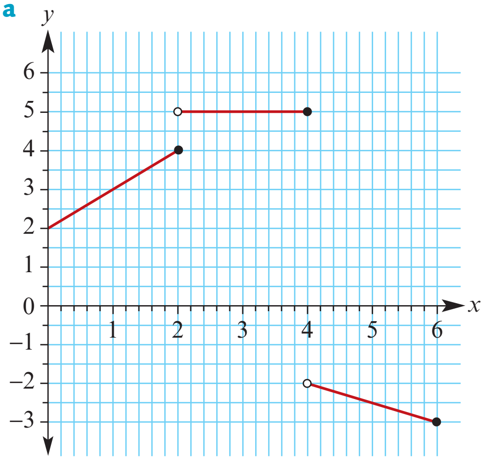


5 A −1.2, B 0.6

6 A 2.25, B −2.67

7 **a** \$755 **b** \$110

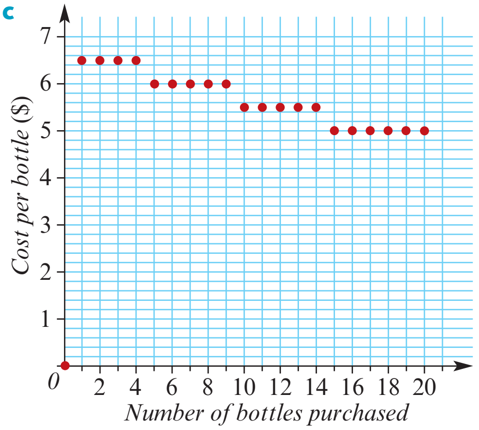
8 **a**



b $y = -\frac{5}{2}$

9 **a** \$5.00 **b** \$36.00

c



10 **a** (1, 3) **b** (4, 1) **c** (5, 1)

11 **a** $x = 2, y = 8$ **b** $x = 3, y = 2.5$
c $p = 5, q = -2$ **d** $p = 5, q = 2$
e $p = 2, q = 1$

12 $x = 200$

Extended-response questions

1 3 m

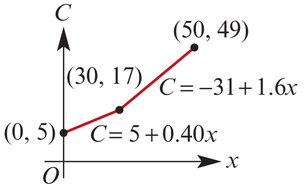
2 Indonesian 28; French 42; Japanese 35.

3 **a** \$200 000
b After 60 months (5 years)
c $V = 300\,000 - 5000t$
d \$120 000 **e** \$5000

4 **a** \$80 billion
b $A = 0.16N$ (with A in billions, N in thousands)
c \$96 billion **d** \$240 billion
e \$0.16 billion

5 **a** $3a + 5c = 73.5$; $2a + 3c = 46.5$
b \$12.00
c \$7.50

6 **a** **i** \$13 **ii** \$17 **iii** \$49
b **i** \$0.40 (40 cents) **ii** \$1.60
c



Chapter 5

Exercise 5A

5A Revision of Chapter 1 Consumer arithmetic: Personal finance

1 \$1056.40

2 **a** 34 hours **b** \$900

3 \$1149 **4** \$58.32

5 C, B, A

6 **a** NZ **b** \$160

7 **a** \$45 143, \$3761.92
b **i** \$188.08 **ii** \$2257
c 7 hours

8 **a** Eastern: \$1120.62, Centre: \$1063.80, Western: \$1176.00. He should go to Western.
b **i** \$2250.71 USD **ii** \$631.62 AUD
iii \$63.75 AUD

9 **a** \$1656.15 **b** \$1493.38
c \$51 270

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10 10 Answers will vary.

11 11 Answers will vary.

Exercise 5B

5B Revision of Chapter 2 Consumer arithmetic: Loans and investments

1 \$47.45 **2** \$800 **3** \$13 714

4 **a** \$5202 **b** \$202

5 **a** \$830 **b** \$7.2%

6 **a** \$11.73 **b** 32.47%

7 \$606

8 **a** \$8596.62

i \$8615.13

ii Better off to purchase immediately; after 3 years his investment is \$18.51 short of the purchase price.

9 **a** \$425.00

b \$434.88

c 5.12%

d 20 years

e 11.1%

10 **a** \$7320

b \$7370

c **i** 4.5 cents/cup **ii** 4.4 cents/cup

11 **a** \$1025 **b** \$1025

c \$409 525 **d** \$407 132

e \$398 076 **f** 168 months

g \$92 000 **h** Answers will vary.

12 Answers will vary.

Exercise 5C

5C Revision of Chapter 3 Shape and measurement

1 12 cm

2 6.75 cm

3 **a** $\frac{49\pi}{2}$ cm^2 **b** 49.7 cm^2

4 **a** 96 cm^3 **b** $\frac{200}{3} = 66.7$ cm^3 **c** $\frac{9}{2}\pi$ cm^3

5 **a** 192π cm^3 **b** 128π cm^2

6 **a** $\frac{10}{7} = \frac{7}{x}$ **b** $x = 4.9$

7 **a** 160 cm^2 **b** $k^2 = 4$ and $k^3 = 8$

c 800 cm^3

8 **a** $k = \frac{1}{5}$ **b** $k^3 = \frac{1}{125}$ **c** $\frac{1}{31}$ cm^3

9 **a** Extend the left and right sides of the trapezium so it forms a triangle.

b $\frac{8}{a} = \frac{x}{4+a}$ and $\frac{8}{a} = \frac{10}{a+7}$

c $x = \frac{64}{7}$

10 **a** $2r$ **b** $\sqrt{2}r$

c $2r + \sqrt{2}r + \sqrt{2}r = 2r(1 + \sqrt{2})$

d $r(\sqrt{2} + 2)$ **e** $\sqrt{2}$. Always true.

11 **a** $\frac{\varphi+1}{\varphi} = \frac{1}{\varphi-1}$ or $\frac{\varphi+1}{\varphi} = \frac{\varphi}{1}$ or $\frac{\varphi}{1} = \frac{1}{\varphi-1}$

b $\varphi = \frac{1 \pm \sqrt{5}}{2}$

c $\varphi = \frac{1-\sqrt{5}}{2}$ is invalid as it is negative. φ is a length, and lengths cannot be negative.

d φ is also known as the Golden Ratio. It is thought to give ideal proportions in architecture and art. Other answers are possible.

e 1.618

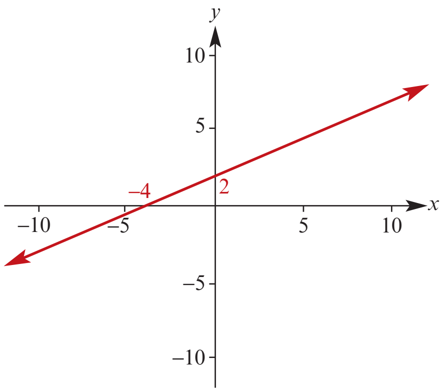
Exercise 5D

5D Revision of Chapter 4 Linear equations and their graphs

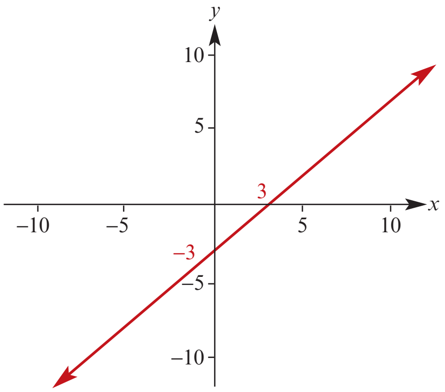
1 **a** $y = 7x - 3$ **b** $y = x + 3$

c $y = 2x + 1$

2 **a**



b



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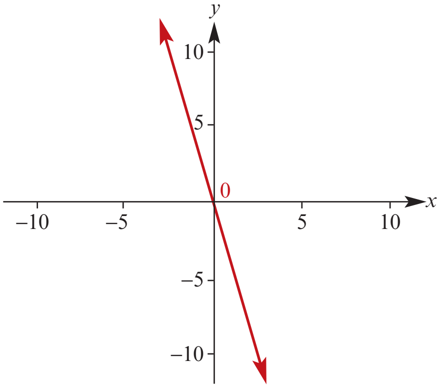
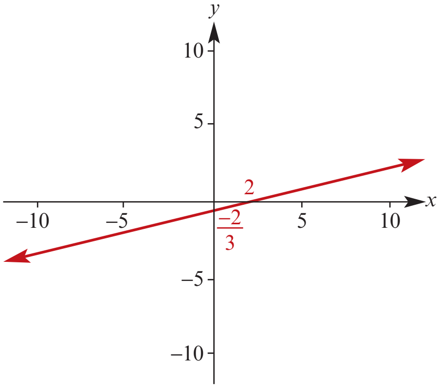
c



d



3 **a** $x = \frac{23}{7}$ and $y = \frac{1}{7}$ **b** $x = 3$ and $y = -4$

4 **a** y-int: 1 and slope: 4

b y-int: $-\frac{1}{2}$ and slope: $\frac{7}{4}$

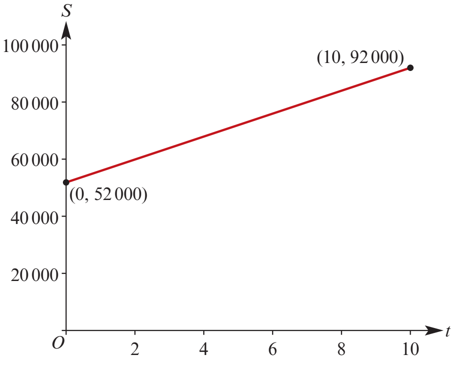
c y-int: 12 and slope: −3

d y-int: 3 and slope: −2

5 $\frac{9}{5} = 1.8$ degrees Fahrenheit

6 **a** $S = 52\,000 + 4000t$

b

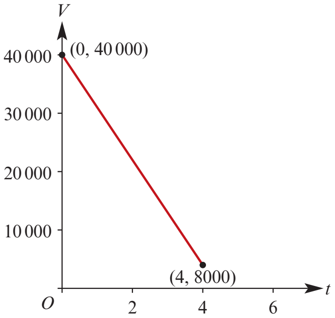


7 **a** Intersection at (1, 2)

b $x = 1$ and $y = 2$

8 **a** $V = 40\,000 - 8000t$

b



c $V = \$17\,600$

d $V = \$8000$

e $t = 5$ years

f Probably not valid. Cars usually don't depreciate by a constant amount every year. Rate of depreciation is highest in first few years, but slows down afterwards. The model assumes constant rate of depreciation which is not true.

9 **a** $T = 3 + 7n$

b $t = 7$ deliveries, driver earns \$52 (at least \$50)

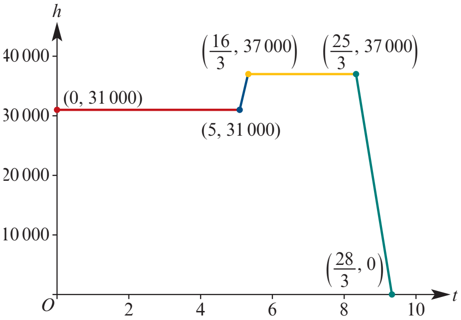
c $T_1 = 0.8 \times (3 + 7n)$

d $T_{net} = 0.8(3 + 7n) - 1 \times n = 4.6n + 2.4$

e \$4.6 per delivery.

10 **a** Starts at $t = 5$ hours and stops at $t = 5$ hours and 20 minutes.

b



c $5 \le t \le 5\frac{1}{3}$, $h = 18\,000x - 59\,000$ calculated using the points (5, 31 000) and $\left(5\frac{1}{3}, 37\,000\right)$

d $8\frac{1}{3} \le t \le 9\frac{1}{3}$, $h = -37\,000t + \frac{1\,036\,000}{3}$ calculated using the points $\left(8\frac{1}{3}, 37\,000\right)$ and $\left(9\frac{1}{3}, 0\right)$

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11 **a** Red: $y = 2$ Blue: $y = 2x$ Green:
$y = -2x + 20$
b and **c** Answers will vary.

Chapter 6

Exercise 6A

1 Answers are in order: hypotenuse, opposite, adjacent.
a 13, 5, 12 **b** 10, 6, 8
c 17, 8, 15 **d** 25, 24, 7
e 10, 8, 6 **f** 13, 12, 5

2 Answers are in order: $\sin\theta, \cos\theta, \tan\theta$.
a $\frac{5}{13}, \frac{12}{13}, \frac{5}{12}$ **b** $\frac{3}{5}, \frac{4}{5}, \frac{3}{4}$
c $\frac{8}{17}, \frac{15}{17}, \frac{8}{15}$ **d** $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$
e $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$ **f** $\frac{12}{13}, \frac{5}{13}, \frac{12}{5}$

3 **a** 0.4540 **b** 0.7314 **c** 1.8807 **d** 0.1908
e 0.2493 **f** 0.9877 **g** 0.9563 **h** 1.1106
i 0.9848 **j** 0.7638 **k** 5.7894 **l** 0.0750

Exercise 6B

1 **a** $\sin\theta$, 20.74 **b** $\cos\theta$, 20.76
c $\tan\theta$, 32.15 **d** $\cos\theta$, 8.24
e $\tan\theta$, 26.63 **f** $\sin\theta$, 7.55
g $\sin\theta$, 17.92 **h** $\tan\theta$, 15.59
i $\cos\theta$, 74.00 **j** $\tan\theta$, 17.44
k $\sin\theta$, 32.72 **l** $\sin\theta$, 37.28

2 **a** 78.05 **b** 25.67 **c** 8.58 **d** 54.99
e 21.32 **f** 11.59 **g** 30.67 **h** 25.38
i 63.00 **j** 62.13 **k** 4.41 **l** 15.59

3 **a** 12.8 **b** 28.3 **c** 38.5 **d** 79.4
e 16.2 **f** 15.0 **g** 14.8 **h** 37.7
i 59.6

Exercise 6C

1 **a** 28.8° **b** 51.1° **c** 40.9° **d** 30.0°
e 45.0° **f** 45.0° **g** 60.0° **h** 68.2°
i 33.0° **j** 73.0° **k** 17.0° **l** 30.0°
m 45.0° **n** 26.6° **o** 30.0° **p** 70.0°

2 **a** 32.2° **b** 59.3° **c** 28.3° **d** 55.8°
e 46.5° **f** 48.6° **g** 53.1° **h** 58.8°
i 22.6° **j** 53.1° **k** 46.3° **l** 22.6°
m 32.2° **n** 41.2° **o** 48.2°

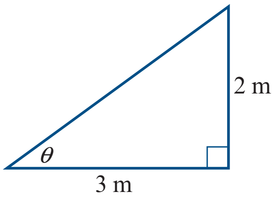
3 **a** 36.9° **b** 67.4° **c** 53.1° **d** 67.4°
e 28.1° **f** 43.6°

4 **a** 23° **b** 33° **c** 66°

Exercise 6D

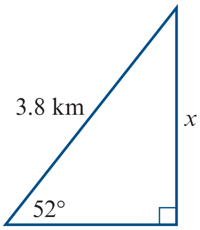
1 6.43 m **2** 21.0° **3** 10 m **4** 16 m

5 **a**



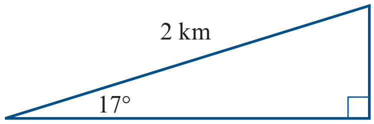
b 33.7°

6 **a**



b 3.0 km

7 **a**



b **i** Horizontal distance 1.91 km
ii Height 0.58 km

8 70.5° **9** 78.1 m **10** 5.77 m

Exercise 6E

1 413 m **2** 11 196 m **3** 33 m
4 164.8 m **5** 244 m **6** 14°
7 **a** 44.6 m **b** 36°
8 **a** 16.2 m **b** 62°
9 **a** 35 m **b** 64 m **c** 29 m
10 507 m

Exercise 6F

1 **a** 025° **b** 110° **c** 210° **d** 280°
2 **a** 25° **b** 7.61 km
3 **a** 236° **b** 056°
4 130°
5 **a** 4.2 km **b** 230°
6 **a** 10 km, 15 km **b** 5 km
c 8.7 km **d** 10 km
e 319°, 13.2 km
7 **a** 12.9 km **b** 15.3 km **c** 17.1 km
d 42° **e** 138°, 23.0 km

Exercise 6G

1 **a** 102 cm^2 **b** 40 cm^2 **c** 24 cm^2
d 52 cm^2 **e** 17.5 cm^2 **f** 6 cm^2

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2 **a** 25.7 cm^2 **b** 65.0 cm^2

c 26.0 cm^2 **d** 32.9 cm^2

e 130.5 cm^2 **f** 10.8 cm^2

3 **a** 36.0 km^2 **b** 9.8 m^2

c 23.5 cm^2 **d** 165.5 km^2

e 25.5 cm^2 **f** 27.7 cm^2

4 **a** iv **b** iii **c** i **d** ii

5 **a** 10 cm^2 **b** 23.8 cm^2 **c** 63.5 cm^2

d 47.3 m^2 **e** 30 m^2 **f** 30.1 m^2

g 100.9 km^2 **h** 21.2 km^2 **i** 6 km^2

6 224 cm^2 **7** 1124.8 cm^2

8 150.4 km^2 **9** 3500 cm^2

10 **a** 6 m^2 **b** 4.9 m^2 **c** 6.9 m^2

11 **a** 33.83 km^2 **b** 19.97 km^2

c 53.81 km^2

12 **a** 43.30 cm^2 **b** 259.81 cm^2

13 **a** **i** 12 km^2 **ii** 39 km^2 **iii** 21 km^2

b 29.6°

Exercise 6H

1 **a** $a = 15, b = 14, c = 13$

b $a = 19, b = 18, c = 21$

c $a = 31, b = 34, c = 48$

2 **a** $C = 50°$ **b** $A = 40°$ **c** $B = 105°$

3 **a** 5.94 **b** 12.08 **c** 45.11 **d** 86.8°

e 44.4° **f** 23.9°

4 **a** 41.0° **b** 53.7° **c** 47.2° **d** 50.3°

5 **a** 19.60 **b** 30.71 **c** 55.38 **d** 67.67

6 **a** 4.45 **b** 16.06 **c** 67.94 **d** 67.84

7 **a** $c = 10.16, B = 50.2°, C = 21.8°$

b $b = 7.63, B = 20.3°, C = 39.7°$

c $a = 52.22, c = 61.01, C = 37°$

d $b = 34.65, c = 34.23, C = 54°$

8 39.09 **9** 43.2° **10** 49.69

11 $a = 31.19, b = 36.56, A = 47°$

12 $A = 27.4°, C = 22.6°, c = 50.24$

13 $a = 154.54, b = 100.87, C = 20°$

14 2.66 km from A, 5.24 km from B

15 409.81 m

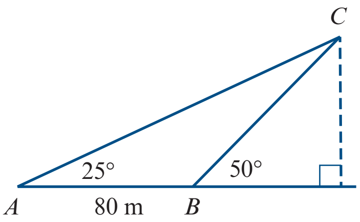
16 **a** 20.37 km from naval ship, 26.93 km from other ship

b 1.36 h (1 h 22 min)

17 **a** Airport A **b** 90.44 km

c Yes

18 **a**



b 130° **c** 25° **d** 145.01 m

e 61.28 m

Exercise 6I

1 **a** 36.72 **b** 47.62 **c** 12.00 **d** 14.55

e 29.95 **f** 11.39

2 17.41 **3** 27.09 **4** 51.51

5 **a** 33.6° **b** 88.0° **c** 110.7° **d** 91.8°

e 88.3° **f** 117.3°

6 50.5° **7** 63.2° **8** 40.9°

9 $B = 46.6°$ **10** $B = 73.2°$ **11** 33.6°

12 19.1 km **13** **a** 39.6° **b** 310°

14 **a** 60° **b** 42.51 km

15 5.26 km **16** 11.73 km

17 4.63 km **18** 45.83 m

Exercise 6J

Answers below are approximate, based upon measured quantities. Student answers may vary.

a **i** 103.2 km **ii** 178.4 km

b No **c** 1 h 35 min **d** \$354.92

e **i** 069° **ii** 226° **iii** 021°

f Fly 178.4 km on a bearing of 226°

g 23° **h** 92.64 km (using 23°)

i 3526 square km

Chapter 6 review

Multiple-choice questions

1 D	**2** C	**3** B	**4** E
5 B	**6** A	**7** A	**8** C
9 D	**10** B	**11** D	**12** B
13 E	**14** B	**15** D	**16** B
17 D	**18** C	**19** E	

Short-answer questions

1 35.87 cm **2** 117.79 cm **3** 4°

4 **a** 65, 72, 97 **b** $\frac{65}{97}$

5 14.02 cm **6** 76.3° **7** $A = 40.7°$

8 54.17 km **9** 760.7 cm^2 **10** 27.7 m^2

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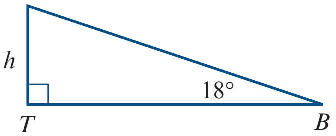
Extended-response questions

1 a 50.95 m **b** 112.23 m

c



Height of tree = 36.47 m

2 a 50°

b First group 3.68 km, second group 3.39 km

c 290°

3 a 110° **b** 81.26 km

4 a 44.4°, 57.1°, 78.5° **b** 14.70 m^2

c $426.21

Chapter 7

Exercise 7A

1 a $1400 **b** $1500 **c** $1425

2 380 km

3 a $10.50 **b** $14.40 **c** $30

4 a i $C = 157.08$ cm **ii** $A = 1963.50$ cm^2

b i $C = 18.85$ mm **ii** $A = 28.27$ mm^2

c i $C = 33.93$ cm **ii** $A = 91.61$ cm^2

d i $C = 45.24$ m **ii** $A = 162.86$ m^2

5 a $P = 14$ **b** $P = 46$ **c** $P = 23$

6 a $A = 4$ **b** $A = 14.25$

c $A = 16.74$

7 a 10°C **b** −17.8°C **c** 100°C

d 33.3°C

8 a $2400.00 **b** $180.00

c $375.00 **d** $2014.50

9 a i 15 points **ii** 37 points

iii 68 points

b Greenteam

10 Gold 23, Silver 22

11 Gold 30, Silver 27

12 6490 m/s

13 a i $V = 7420.70$ cm^3 **ii** $A = 1839.84$ cm^2

b i $V = 8181.23$ mm^3 **ii** $A = 1963.50$ mm^2

c i $V = 10.31$ m^3 **ii** $A = 22.90$ m^2

d i $V = 1047.39$ cm^3 **ii** $A = 498.76$ cm^2

14 1.6 amperes

15 71.4 km/h

16 a $\frac{28}{11}$ **b** $\frac{15}{4}$

17 a 13 **b** 23 **c** 101

18 a i 1 h 50 min **ii** 2 h 56 min

iii 3 h 6 min **iv** 2 h 38 min

b 5:15 p.m.

19 a 100.53 cm^2 **b** 123.76 cm^2

c 268.08 cm^3 **d** 16.76 cm^3

Exercise 7B

1

x	40	41	42	43	44	45
C ($)	86	88.15	90.3	92.45	94.6	96.75

x	46	47	48	49	50
C ($)	98.9	101.05	103.2	105.35	107.5

2

r	0	0.1	0.2	0.3	0.4	0.5
C	0	0.628	1.257	1.885	2.513	3.142

r	0.6	0.7	0.8	0.9	1.0
C	3.770	4.398	5.027	5.655	6.283

3

n	50	60	70	80	90	100
C ($)	49	50.8	52.6	54.4	56.2	58

n	110	120	130
C ($)	59.8	61.6	63.4

4

M (kg)	60	65	70	75	80	85	90
E (kJ)	650	695	740	785	830	875	920

M (kg)	95	100	105	110	115	120
E (kJ)	965	1010	1055	1100	1145	1190

5

n	3	4	5	6
S	180°	360°	540°	720°

n	7	8	9	10
S	900°	1080°	1260°	1440°

6

P		T			
		1	2	3	4
M	1	1	2	3	4
	2	2	4	6	8
	3	3	6	9	12
	4	4	8	12	16

7

H		Z			
		1	2	3	4
R	1	3	$\frac{3}{2}$	1	$\frac{3}{4}$
	2	4	2	$\frac{4}{3}$	1
	3	5	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$
	4	6	3	2	$\frac{6}{4} = \frac{3}{2}$

8 a $= 1.3 * A2 + 4$ **b** $43.00

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9 **a**

n	0	1	2	3	4	5
E ($)	680	740	800	860	920	980

n	6	7	8	9	10
E ($)	1040	1100	1160	1220	1280

b 6 cars

10

T (years)	1	2	3	4	5
I ($)	450	900	1350	1800	2250

T (years)	6	7	8	9	10
I ($)	2700	3150	3600	4050	4500

11

t (years)	5	10	15	20	25
A ($)	6535	8541	11 162	14 589	19 067

12

L	10	20	40	80	100	320	640
W	320	160	80	40	32	10	5

13 **a** $h = \dfrac{48}{b}$

b

b	2	4	8	12	16	24	48	96
h	24	12	6	4	3	2	1	0.5

14 **a**

C		p					
		0	2	4	6	8	10
t	0	0	0.4	0.8	1.2	1.6	2
	2	0.1	0.5	0.9	1.3	1.7	2.1
	4	0.2	0.6	1	1.4	1.8	2.2
	6	0.3	0.7	1.1	1.5	1.9	2.3
	8	0.4	0.8	1.2	1.6	2	2.4
	10	0.5	0.9	1.3	1.7	2.1	2.5

b $1.20

15 **a**

I		T				
		1	2	3	4	5
R	3	150	300	450	600	750
	3.2	160	320	480	640	800
	3.4	170	340	510	680	850
	3.6	180	360	540	720	900
	3.8	190	380	570	760	950
	4	200	400	600	800	1000

b $760.00

Exercise 7C

1 **a** $t = \dfrac{C - 1200}{50}$ **b** 5 hours

2 **a** $t = \dfrac{d}{v}$ **b** 2 h 45 min

3 **a** $K = \dfrac{F - 4}{1.3}$ **b** 22 km

4 6 cm **5** 13 cm **6** 15.2 cm

7 **a** $W = \dfrac{P - 2L}{2}$ **b** 26 m

8 **a** 5 years **b** $5400 **c** 3%

9 27 tries

10 **a** $R = \dfrac{240}{I}$ **b** 160 ohms

11 2 h 36 min **12** 360° F

Chapter 7 review

Multiple-choice questions

1 C **2** B **3** D **4** B
5 A **6** A **7** D **8** A
9 B

Short-answer questions

1 **a** $P = 40$ **b** $P = 130$

2 **a** $A = 30$ **b** $A = 54$

3 94.25 cm

4 424.12 cm^2

5 **a**

x	−20	−15	−10	−5	0
y	−716	−551	−386	−221	−56

x	5	10	15	20	25
y	109	274	439	604	769

b $x = 10$ **c** $x = -5$

6

k		g				
		−2	−1	0	1	2
h	−2	−6	−5	−4	−3	−2
	−1	−4	−3	−2	−1	0
	0	−2	−1	0	1	2
	1	0	1	2	3	4
	2	2	3	4	5	6

7 $r = \dfrac{S}{2\pi h}$, $r = 1.8$ cm

Extended-response questions

1 **a** $57 **b** 7 hours

2 **a**

n	60	70	80	90	100	110
C	55	60	65	70	75	80

n	120	130	140	150	160
C	85	90	95	100	105

b $105

3 **a**

P		c					
		0	1	2	3	4	5
a	0	0	42	84	126	168	210
	1	89	131	173	215	257	299
	2	178	220	262	304	346	388
	3	267	309	351	393	435	477
	4	356	398	440	482	524	566
	5	445	487	529	571	613	655

b $309

4 **a** $C = 80 + 45h$ **b** $215

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5 **a**

t	0	0.5	1	1.5	2	2.5	3	3.5	4
h	1	9.75	16	19.75	21	19.75	16	9.75	1

b 2 seconds

c 21 metres

d 1 second and 3 seconds

Chapter 8

Exercise 8A

1 **a** 3×4 **b** **i** 16 **ii** 3 **iii** 5
c 22 **d** 18

2 **a** **i** 2×3 **ii** 6, 7
b **i** 1×3 **ii** 2, 6
c **i** 3×2 **ii** −4, 5
d **i** 3×1 **ii** 9, 8
e **i** 2×2 **ii** 15, 12
f **i** 3×4 **ii** 20, 5

3 **a** B **b** D **c** E

4,5 **a** 9 **b** 2 **c** 3 **d** 10
e 8

6 **a** 32 students **b** 3×4
c 22 Year 11 students preferred football.

7 **a** $A\ 4 \times 3$, $B\ 2 \times 1$, $C\ 1 \times 2$, $D\ 2 \times 5$
b $a_{32} = 4$, $b_{21} = -5$, $c_{11} = 8$, $d_{24} = 7$

8 **a** **i** 75 ha **ii** 300 ha **iii** 200 ha
b 350 ha
c **i** Farm Y uses 0 ha for cattle.
ii Farm X uses 75 ha for sheep.
iii Farm X uses 150 ha for wheat.
d **i** f_{23} **ii** f_{12} **iii** f_{21}
e 2×3

Exercise 8B

1 **a** **i**
$$\begin{array}{c|cc} & A & B \\ \hline A & 0 & 3 \\ B & 3 & 0 \end{array}$$

ii
$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 2 & 0 \\ B & 2 & 0 & 1 \\ C & 0 & 1 & 0 \end{array}$$

iii
$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 2 & 0 \\ B & 2 & 0 & 2 \\ C & 0 & 2 & 0 \end{array}$$

iv
$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 1 \\ B & 1 & 0 & 2 \\ C & 1 & 2 & 0 \end{array}$$

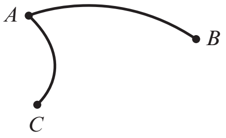
v
$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 2 & 1 \\ B & 2 & 0 & 1 \\ C & 1 & 1 & 0 \end{array}$$

vi
$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 2 \\ B & 1 & 0 & 2 \\ C & 2 & 2 & 0 \end{array}$$

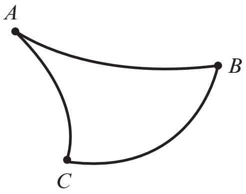
b The number of roads directly connected to B.

2 **a** Many answers are possible. Examples:

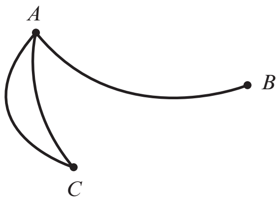
i



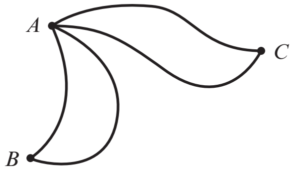
ii



iii



iv



b The number of roads directly connected to town A.

3 **a**
$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 1 \\ B & 1 & 0 & 1 & 1 \\ C & 0 & 1 & 0 & 0 \\ D & 1 & 1 & 0 & 0 \end{array}$$

b Compare the sums of the rows (or columns). The person with the highest total has met the most people.

c Person B

d Person C

Exercise 8C

1 **a** $\begin{bmatrix} 9 & 10 \\ 6 & 3 \end{bmatrix}$ **b** $\begin{bmatrix} 7 & 8 \\ 13 & 3 \end{bmatrix}$

c $\begin{bmatrix} 3 & 5 \\ 7 & 0 \end{bmatrix}$ **d** $\begin{bmatrix} 9 \\ 8 \end{bmatrix}$

e $\begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ **f** $\begin{bmatrix} 4 & -2 \\ 3 & 9 \end{bmatrix}$

g $\begin{bmatrix} 12 & 7 \end{bmatrix}$ **h** $\begin{bmatrix} 0 & 0 \end{bmatrix}$

i $\begin{bmatrix} 0 & 0 \end{bmatrix}$ **j** $\begin{bmatrix} -2 & 2 & 3 & -9 \end{bmatrix}$

2 a $\begin{bmatrix} 8 & 5 \\ 3 & 7 \end{bmatrix}$ **b** $\begin{bmatrix} 8 & 5 \\ 3 & 7 \end{bmatrix}$

c $\begin{bmatrix} -2 & -9 \\ 1 & 1 \end{bmatrix}$ **d** $\begin{bmatrix} 2 & 9 \\ -1 & -1 \end{bmatrix}$

e Not possible **f** $\begin{bmatrix} 3 & 7 \\ 5 & -2 \\ 4 & -1 \end{bmatrix}$

g Not possible **h** $\begin{bmatrix} -9 & 3 \\ 3 & -2 \\ -2 & 15 \end{bmatrix}$

3

$$\begin{array}{c|cccc|} & \textit{Liberal} & \textit{Labor} & \textit{Democrat} & \textit{Green} \\ \textit{Men} & 43 & 42 & 10 & 5 \\ \textit{Women} & 37 & 37 & 17 & 9 \end{array}$$

4 a

$$\begin{array}{c|cccc|} & \textit{Aida} & \textit{Bianca} & \textit{Chloe} & \textit{Donna} \\ \textit{Weight}\text{ (kg)} & 6 & 8 & -2 & 7 \\ \textit{Height}\text{ (cm)} & 5 & 8 & 7 & 6 \end{array}$$

b Bianca

c Bianca

Exercise 8D

1 a $\begin{bmatrix} 14 & -2 \\ 8 & 18 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & -10 \\ 25 & 35 \end{bmatrix}$

c $\begin{bmatrix} -64 & 12 \\ -6 & -14 \end{bmatrix}$ **d** $\begin{bmatrix} 2.25 & 0 \\ -3 & 7.5 \end{bmatrix}$

e $\begin{bmatrix} 18 & 21 \end{bmatrix}$ **f** $\begin{bmatrix} -12 \\ 30 \end{bmatrix}$

g $\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ **h** $\begin{bmatrix} -3 & -6 & 8 \end{bmatrix}$

2 a $\begin{bmatrix} 9 & -12 \\ 6 & 15 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & 28 \\ -6 & -28 \end{bmatrix}$

c $\begin{bmatrix} 1 & -32 \\ 8 & 33 \end{bmatrix}$ **d** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

e $\begin{bmatrix} 21 & 18 \\ 3 & -12 \end{bmatrix}$

3 a $\begin{bmatrix} 79 & -31 \\ 68 & -36 \end{bmatrix}$ **b** $\begin{bmatrix} -121 & 50 \\ -84 & 103 \end{bmatrix}$

c $\begin{bmatrix} 13 & -2 \\ 36 & 53 \end{bmatrix}$ **d** $\begin{bmatrix} 69 & -27 \\ 60 & -30 \end{bmatrix}$

4 a $\begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 5 & 3 & 3 \end{bmatrix}$ **c** $\begin{bmatrix} 6 \\ 14 \\ 8 \end{bmatrix}$

d $\begin{bmatrix} 0 & 15 & 9 & 9 \end{bmatrix}$

5 a

$$\begin{array}{c|ccc|} & \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \\ A & 6 & 2 & 9 \\ B & 5 & 1 & 9 \\ C & 4 & -1 & 5 \end{array}$$

b

$$\begin{array}{c|ccc|} & \textit{Clothing} & \textit{Furniture} & \textit{Electronics} \\ A & 1.8 & 0.6 & 2.7 \\ B & 1.5 & 0.3 & 2.7 \\ C & 1.2 & 0 & 1.5 \end{array}$$

6 a

$$\begin{array}{r|c|} & \textit{Wins} \\ \textit{Gymnastics rings} & 3 \\ \textit{Parallel bars} & 2 \end{array}$$

b

$$\begin{array}{r|c|} & \$ \\ \textit{Gymnastics rings} & 150 \\ \textit{Parallel bars} & 100 \end{array}$$

Exercise 8E

1 a Defined, 2 × 1, $\begin{bmatrix} 38 \\ 19 \end{bmatrix}$

b Not defined

c Defined, 3 × 1, $\begin{bmatrix} 17 \\ 32 \\ -10 \end{bmatrix}$

d Not defined

e Defined, 2 × 2, $\begin{bmatrix} 42 & 14 \\ 21 & 7 \end{bmatrix}$

f Not defined **g** Not defined

h Defined, 3 × 2, $\begin{bmatrix} 15 & 5 \\ 24 & 8 \\ -3 & -1 \end{bmatrix}$

2 a 1 × 2 and 2 × 1, [38]

b 1 × 2 and 3 × 1, not defined

c 1 × 3 and 3 × 1, [1]

d 1 × 3 and 2 × 1, not defined

e 1 × 4 and 4 × 1, [2]

f 1 × 4 and 3 × 1, not defined

3 a i $\begin{bmatrix} 6 & 9 \end{bmatrix}$ **ii** $\begin{bmatrix} 10 & 15 \end{bmatrix}$

iii $\begin{bmatrix} 16 & 24 \end{bmatrix}$ **iv** $\begin{bmatrix} 16 & 24 \end{bmatrix}$

b i $\begin{bmatrix} 30 \\ 24 \end{bmatrix}$ **ii** $\begin{bmatrix} 35 \\ 28 \end{bmatrix}$ **iii** $\begin{bmatrix} 35 \\ 28 \end{bmatrix}$

c i $\begin{bmatrix} 4 & 6 \end{bmatrix}$ **ii** $\begin{bmatrix} 15 \\ 12 \end{bmatrix}$ **iii** [22]

iv [132] **v** [132]

4,5 a $\begin{bmatrix} 22 \\ 33 \end{bmatrix}$ **b** $\begin{bmatrix} 64 \\ 53 \end{bmatrix}$ **c** $\begin{bmatrix} 0 & -8 \\ 4 & 2 \end{bmatrix}$

d $\begin{bmatrix} -4 & -3 \\ -14 & -20 \end{bmatrix}$ **e** $\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}$

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f $\begin{bmatrix}16 & 14\\16 & 14\end{bmatrix}$ **g** $\begin{bmatrix}31\\35\\21\end{bmatrix}$ **h** $\begin{bmatrix}11\\1\\7\end{bmatrix}$

i $\begin{bmatrix}83\end{bmatrix}$ **j** $\begin{bmatrix}21\end{bmatrix}$ **k** $\begin{bmatrix}8\end{bmatrix}$ **l** $\begin{bmatrix}4\end{bmatrix}$

m $\begin{bmatrix}30\end{bmatrix}$ **n** $\begin{bmatrix}36\end{bmatrix}$ **o** $\begin{bmatrix}3 & 3\end{bmatrix}$

6 a $\begin{bmatrix}19 & 22\\43 & 50\end{bmatrix}$ **b** $\begin{bmatrix}23 & 34\\31 & 46\end{bmatrix}$

c No

7 a $\begin{bmatrix}104 & 70\\80 & 54\end{bmatrix}$ **b** $\begin{bmatrix}9 & 8\\7 & 6\end{bmatrix}$

c $\begin{bmatrix}17 & 17\\13 & 13\end{bmatrix}$ **d** $\begin{bmatrix}8 & 6\\4 & 2\end{bmatrix}$

e $\begin{bmatrix}14 & 14\\6 & 6\end{bmatrix}$

8 a $\begin{bmatrix}376 & 118 & 154 & 420\\643 & 117 & 281 & 523\end{bmatrix}$ **b** $\begin{bmatrix}1292\end{bmatrix}$

c $\begin{bmatrix}-496 & 752 & 976 & -224\\-310 & 470 & 610 & -140\\-744 & 1128 & 1464 & -336\\558 & -846 & -1098 & 252\end{bmatrix}$

d $\begin{bmatrix}-131 & -264 & 176\\467 & 62 & 535\\697 & 279 & 406\end{bmatrix}$

9 D

10 a $\begin{bmatrix}7 & 3\\5 & 6\end{bmatrix}$ **b** $\begin{bmatrix}12 & 9\\12 & 9\end{bmatrix}$

c $\begin{bmatrix}4 & 8\\11 & -3\end{bmatrix}$ **d** $\begin{bmatrix}11 & -3\\4 & 8\end{bmatrix}$

e $\begin{bmatrix}3 & 1 & 4\\6 & 9 & 2\\8 & 0 & 7\end{bmatrix}$

11 a i $\begin{bmatrix}7 & 10\\15 & 22\end{bmatrix}$ **ii** $\begin{bmatrix}37 & 54\\81 & 118\end{bmatrix}$ **iii** $\begin{bmatrix}199 & 290\\435 & 634\end{bmatrix}$

b i $\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$ **ii** $\begin{bmatrix}4 & 4\\4 & 4\end{bmatrix}$ **iii** $\begin{bmatrix}8 & 8\\8 & 8\end{bmatrix}$

c i $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ **ii** $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ **iii** $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

d i $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ **ii** $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ **iii** $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

e i $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ **ii** $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ **iii** $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

12 a, i $\begin{bmatrix}16 & 70\\0 & 36\end{bmatrix}$ **ii, b** $\begin{bmatrix}64 & 532\\0 & 216\end{bmatrix}$

13 a $\begin{bmatrix}3 & -2\\2 & -1\end{bmatrix}$ **b** $\begin{bmatrix}4 & -3\\3 & -2\end{bmatrix}$

c, d $\begin{bmatrix}5 & -4\\4 & -3\end{bmatrix}$

14 a $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ **b** $\begin{bmatrix}4 & 3\\1 & 2\end{bmatrix}$

c When A is multiplied by A^0, the answer is A.

d $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ **e** $B^0 \times B = B$

f Identity matrix

Exercise 8F

1 5800 kJ

2
$$\begin{array}{cc} & \begin{matrix}\textit{Wheels} & \textit{Seats}\end{matrix}\\ \begin{matrix}\textit{Smith}\\ \textit{Jones}\end{matrix} & \begin{bmatrix}14 & 13\\12 & 9\end{bmatrix}\end{array}$$

3 $[\,110\,]$

4 a
$$\begin{array}{c}\begin{matrix}\textit{Quiche} & \textit{Soup} & \textit{Coffee}\end{matrix}\\ \begin{bmatrix}18 & 12 & 64\end{bmatrix}\end{array}$$

b
$$\begin{array}{cc} & \$\\ \begin{matrix}\textit{Quiche}\\ \textit{Soup}\\ \textit{Coffee}\end{matrix} & \begin{bmatrix}5\\8\\3\end{bmatrix}\end{array}$$

c $378

5 a
$$\begin{array}{c}\begin{matrix}\textit{Chips} & \textit{Pasties} & \textit{Pies} & \textit{Sausage rolls}\end{matrix}\\ \begin{bmatrix}90 & 84 & 112 & 73\end{bmatrix}\end{array}$$

b
$$\begin{array}{cc} & \$\\ \begin{matrix}\textit{Chips}\\ \textit{Pasties}\\ \textit{Pies}\\ \textit{Sausage rolls}\end{matrix} & \begin{bmatrix}4\\5\\5\\3\end{bmatrix}\end{array}$$

c $1559

6 a 1720 **b** $990

7 a
$$\begin{array}{cc} & \textit{Hours}\\ \begin{matrix}I\\ J\\ K\end{matrix} & \begin{bmatrix}10\\7\\12\end{bmatrix}\end{array}$$

b
$$\begin{array}{cc} & \textit{Av. Hours}\\ \begin{matrix}I\\ J\\ K\end{matrix} & \begin{bmatrix}2.5\\1.75\\3\end{bmatrix}\end{array}$$

c
$$\begin{array}{cc} & \begin{matrix}M & Tu & W & Th\end{matrix}\\ \textit{Hours} & \begin{bmatrix}6 & 11 & 5 & 7\end{bmatrix}\end{array}$$

d
$$\begin{array}{cc} & \begin{matrix}M & Tu & W & Th\end{matrix}\\ \textit{Av. Hours} & \begin{bmatrix}2 & 3.7 & 1.7 & 2.3\end{bmatrix}\end{array}$$

8 a
$$\begin{array}{cc} & \textit{Total score}\\ \begin{matrix}E\\ F\\ G\\ H\end{matrix} & \begin{bmatrix}446\\415\\329\\409\end{bmatrix}\end{array}$$

b
$$\begin{array}{cc} & \textit{Av. score}\\ \begin{matrix}E\\ F\\ G\\ H\end{matrix} & \begin{bmatrix}89.2\\83\\65.8\\81.8\end{bmatrix}\end{array}$$

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c
$$\begin{array}{cccccc} & T1 & T2 & T3 & T4 & T5 \\ \text{Test total} & 319 & 307 & 324 & 292 & 357 \end{array}$$

d
$$\begin{array}{cccccc} & T1 & T2 & T3 & T4 & T5 \\ \text{Test av.} & 79.75 & 76.75 & 81 & 73 & 89.25 \end{array}$$

Exercise 8G

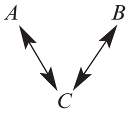
1 *A* communicates with *D*, yet *D* does not communicate with *A*.

2 a
$$\begin{array}{cccc} & A & B & C \\ A & 0 & 1 & 0 \\ B & 1 & 0 & 1 \\ C & 0 & 1 & 0 \end{array}$$

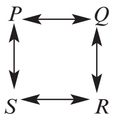
b
$$\begin{array}{ccccc} & D & E & F & G \\ D & 0 & 1 & 1 & 1 \\ E & 1 & 0 & 0 & 0 \\ F & 1 & 0 & 0 & 1 \\ G & 1 & 0 & 1 & 0 \end{array}$$

c
$$\begin{array}{ccccc} & J & K & L & M \\ J & 0 & 1 & 1 & 0 \\ K & 1 & 0 & 1 & 1 \\ L & 1 & 1 & 0 & 0 \\ M & 0 & 1 & 0 & 0 \end{array}$$

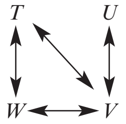
3 a



b



c



4 a
$$Q = \begin{array}{ccccc} & C & E & K & R \\ C & 0 & 1 & 1 & 1 \\ E & 1 & 0 & 0 & 1 \\ K & 1 & 0 & 0 & 1 \\ R & 1 & 1 & 1 & 0 \end{array}$$

b Remy communicates with 3 people.

c i
$$Q^2 = \begin{array}{ccccc} & C & E & K & R \\ C & 3 & 1 & 1 & 2 \\ E & 1 & 2 & 2 & 1 \\ K & 1 & 2 & 2 & 1 \\ R & 2 & 1 & 1 & 3 \end{array}$$

ii Add column *E* to get 6 ways.

iii $E \to R \to C$

$E \to R \to E$

$E \to C \to E$

$E \to R \to K$

$E \to C \to K$

$E \to C \to R$

5 a
$$R = \begin{array}{cccccc} & E & F & G & H & \\ E & 0 & 1 & 1 & 0 & E \\ F & 1 & 0 & 1 & 1 & F \\ G & 1 & 1 & 0 & 1 & G \\ H & 0 & 1 & 1 & 0 & H \end{array}$$

b Three roads directly connected to Fields.

c i
$$R^2 = \begin{array}{cccccc} & E & F & G & H & \\ E & 2 & 1 & 1 & 2 & E \\ F & 1 & 3 & 2 & 1 & F \\ G & 1 & 2 & 3 & 1 & G \\ H & 2 & 1 & 1 & 2 & H \end{array}$$

ii 7

iii $F \to G \to E$

$F \to E \to F$

$F \to G \to F$

$F \to H \to F$

$F \to E \to G$

$F \to H \to G$

$F \to G \to H$

6 a
$$C = \begin{array}{cccc} B & T & Z & \\ 0 & 1 & 1 & B \\ 1 & 0 & 0 & T \\ 1 & 0 & 0 & Z \end{array}$$

b The communications are two-way: Bronwen communicates with Thomas (first column, second row), Thomas communicates with Bronwen (second column, first row) and, so on. This gives symmetry.

c The sum of a row (or column) gives the total number of one-step connections for that person.

d $B \to T \to B$

$B \to Z \to B$

$T \to B \to T$

$T \to B \to Z$

$Z \to B \to T$

$Z \to B \to Z$

e The sum of all the numbers in matrix C^2 equals 6, which is the total number of two-step connections.

Exercise 8H

1 Multiplying by a scalar matrix has the same result as multiplying by a scalar.

2 a $\begin{bmatrix} 52 \\ 64 \\ 44 \end{bmatrix}$ Third quarter costs

b $F = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

c 1 × 4

d Pre-multiply with a 1 × 3 matrix.

e $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

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Chapter 8 review

Multiple-choice questions

1 B **2** E **3** C **4** D **5** A
6 D **7** D **8** C **9** A **10** E
11 D **12** B **13** E **14** A **15** D
16 D

Short-answer questions

1 2×4 **2** 1 **3** $\begin{bmatrix} 38 & 34 & 47 & 54 \end{bmatrix}$

4 2×1

5 $\begin{array}{c c} \begin{matrix} P & Q & R \end{matrix} & \\ \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix} & \begin{matrix} P \\ Q \\ R \end{matrix} \end{array}$

6 a $\begin{bmatrix} 9 & 3 \\ 12 & 6 \end{bmatrix}$ **b** $\begin{bmatrix} 3 & 6 \\ 11 & 8 \end{bmatrix}$

c $\begin{bmatrix} -3 & 4 \\ 3 & 4 \end{bmatrix}$ **d** $\begin{bmatrix} 6 & 7 \\ 15 & 10 \end{bmatrix}$

e $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **f** $\begin{bmatrix} 7 & 21 \\ 14 & 32 \end{bmatrix}$

g $\begin{bmatrix} 20 & 10 \\ 45 & 19 \end{bmatrix}$ **h** $\begin{bmatrix} 0 & 5 \\ 7 & 6 \end{bmatrix}$

i $\begin{bmatrix} 13 & 5 \\ 20 & 8 \end{bmatrix}$ **j** $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

k $\begin{bmatrix} 269 & 105 \\ 420 & 164 \end{bmatrix}$

Extended-response questions

1 a 40 pigs **b** 320 sheep **c** Farm *A*

2 a 21 pies **b** \$2 **c** $\begin{bmatrix} 104 \\ 103 \end{bmatrix}$

d Value of sales for each shop

e Shop *A*, \$104

3 a $\begin{array}{c c} & \begin{matrix} \textit{Hours walking} & \textit{Hours jogging} \end{matrix} \\ \begin{matrix} \textit{Patsy} \\ \textit{Geoff} \end{matrix} & \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} \end{array}$

b $\begin{array}{c c} & \begin{matrix} \$ & kJ \end{matrix} \\ \begin{matrix} \textit{Walking} \\ \textit{Jogging} \end{matrix} & \begin{bmatrix} 2 & 1500 \\ 3 & 2500 \end{bmatrix} \end{array}$

c $\begin{array}{c c} & \begin{matrix} \$ & kJ \end{matrix} \\ \begin{matrix} \textit{Patsy} \\ \textit{Geoff} \end{matrix} & \begin{bmatrix} 11 & 8500 \\ 12 & 9500 \end{bmatrix} \end{array}$

4 a $R = \begin{array}{c c} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$

b 3 roads connect to Bez.

c i $R^2 = \begin{array}{c c} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \end{array}$

ii 5 ways **iii** $B \to D \to A$
$B \to A \to B$
$B \to C \to B$
$B \to D \to B$
$B \to A \to D$

Chapter 9

Exercise 9A

1 a Nominal **b** Ordinal
c Ordinal **d** Nominal

2 a Categorical **b** Numerical
c Categorical **d** Numerical
e Categorical **f** Categorical

3 a Nominal **b** Ordinal
c Numerical (discrete)

4 a Discrete **b** Discrete
c Continuous **d** Continuous
e Discrete

Exercise 9B

1 a Nominal

b

	Frequency	
Sex	Number	%
Female	5	33.3
Male	10	66.7
Total	15	100.0

2 a Ordinal

b

	Frequency	
Shoe size	Number	%
7	3	15
8	7	35
9	4	20
10	3	15
11	2	10
12	1	5
Total	20	100

3 a 69; 8.7%, 26.1% **b** Nominal
c 7 students **d** 10.1%
e Hamburger

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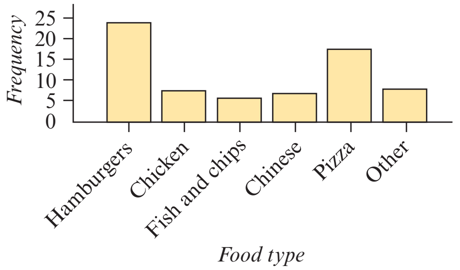
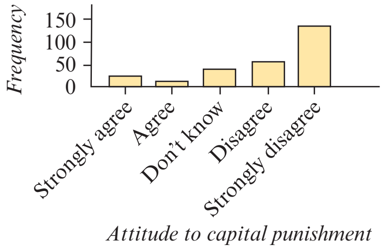
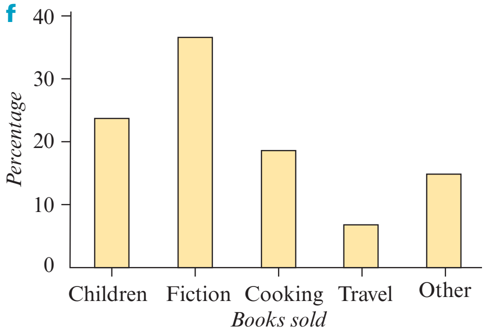
f



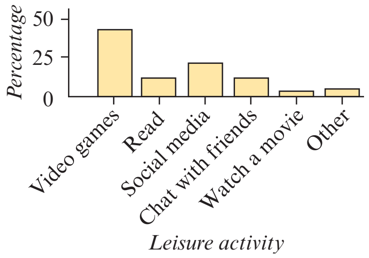
4 **a** 53; 16.4%, 20.7% **b** Ordinal
c 21 people **d** 50.4%
e Strongly disagree
f



5 **a** 38.4%, 6.5%, 100.0%
b Nominal **c** 89 books
d 22.8% **e** 232 books
f



6 **a** 200 students **b** Nominal
c 12% **d** Play video games
e



Exercise 9C

1 69, hamburgers, 26.1%

2 Strongly disagreed, 20.7%, 16.4%

3 A group of 200 students were asked how they prefer to spend their leisure time. The most popular response was using the internet and playing digital games (42%), followed by listening to music (23%), reading (13%), watching TV or going to a movie (12%) and chatting with phoning friends (4%). The remaining 6% said 'other'. For this group of students, spending time on the internet and digital games was clearly the most popular leisure time activity.

4 A group of 579 employees from a large company were asked about the importance to them of the salary that they earned in the job. The majority of employees said that it was important (56.8%), or very important (33.5%). Only a small number of employees said that it was somewhat important (7.8%), with even fewer saying that it was not at all important (1.9%). Salary was clearly important to almost all of the employees in this company.

Exercise 9D

1

Number of magazines	Frequency	
	Number	%
0	4	26.7
1	4	26.7
2	3	20.0
3	2	13.3
4	1	6.7
5	1	6.7
Total	15	100.0

2

Amount of money ($)	Frequency	
	Number	%
0.00–4.99	13	65
5.00–9.99	3	15
10.0 –14.99	2	10
15.00–19.99	1	5
20.00–24.99	1	5
Total	20	100

3 **a** **i** 2 students **ii** 3 students
iii 8 students
b **i** 32.1% **ii** 39.3% **iii** 89.3%

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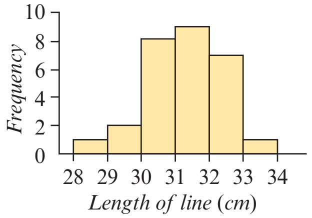
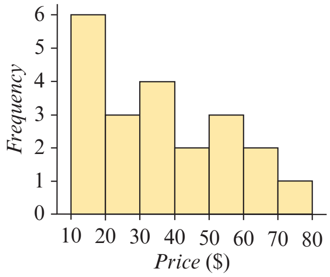
c



4 **a** 4 students **b** 2 children
c 5 students **d** 28 students

5 **a** 0 students **b** 48 students
c 60–69 marks **d** 33 students

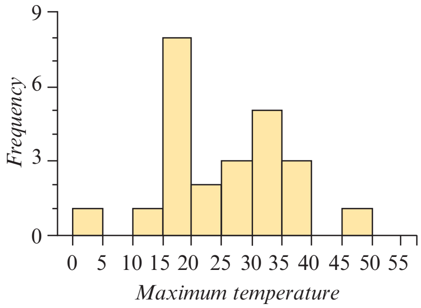
6 **a**



b **i** \$30 to < \$40 **ii** 4 books
iii \$10 to < \$20

7 **a** *See figure at bottom of this page.*
b **i** 2°C **ii** 1 city

c



d **i** 2 cities **ii** 15°C to < 20°C

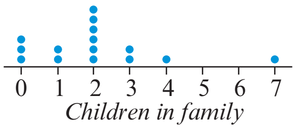
Exercise 9E

1 **a** Positively skewed **b** Negatively skewed
c Approximately symmetric

2 **a** Location **b** Neither **c** Both

Exercise 9F

1 **a**

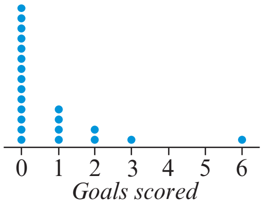


b 2 children

2 **a**

b 7 people

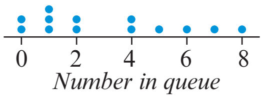
3 **a**



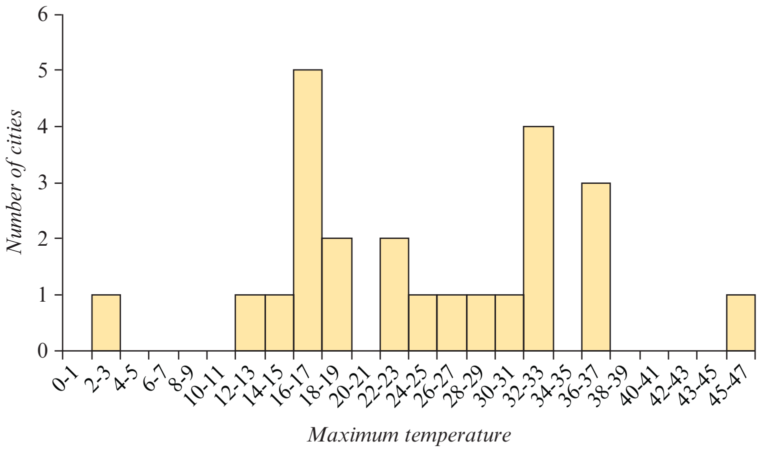
b 0 goals

c Positively skewed with a possible outlier, the player who kicked six goals.

4 **a**



b Around 12:25 p.m.



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5 a English marks

1	7
2	3 3 6 8
3	2 5 5 8 9
4	3 4 6 6 9
5	0 2 8 9
6	1 4 5 6 9
7	5 8 9 9
8	3 3 4 9
9	2 3 4 7

5 | 0 represents 50 marks

b 21 students **c** 17 marks

6 a 40 people

b Approximately symmetric

c 21 people

7 a Battery time (hours)

0	4
1	7 9
2	0 1 2 4 5 6 6 7 7 8
3	0 0 1 1 3 3 4 7
4	0 1 6

2 | 5 represents 25 hours

b 9 batteries

8 a

Homework time (minutes)

0	0
1	0 0 4 5 5 6 9
2	0 0 1 3 7 8 9
3	3 7 9
4	6
5	6
6	3
7	0

4 | 6 represents 46 minutes

b 2 students

c Positively skewed

9 a Price ($)

2	5 8
3	5 6 9
4	5 6 9
5	2
6	8
7	5 5 6 8 9
8	2 4
9	5
10	
11	
12	
13	
14	9
15	

16 | 4 represents $164
(truncated)

b Approximately symmetric with an outlier ($149)

Exercise 9G

1 a Mean = 5; median = 5

b Mean = 5; median = 4.5

c Mean = 15; median = 15

d Mean = 101; median = 99.5

e Mean = 2.8; median = 2.1

2 a $M = 9$, IQR = 10.5, $R = 21$

b $M = 6.5$, IQR = 8, $R = 11$

c $M = 27$, IQR = 7, $R = 12$

d $M = 106.5$, IQR = 4.5, $R = 8$

e $M = 1.2$, IQR = 1.1, $R = 2.7$

3 a $M = 57$ mm; IQR = 59 − 49.5 = 9.5 mm

b $M = 27.5$ hours; IQR = 33 − 23 = 10 hours

4 a $\bar{x} = 12.5$ ha, $M = 7.4$ ha

b The median, as it is typical of more suburbs. The median is not affected by the outlier.

5 a $\bar{x} = \$393\,386$, $M = \$340\,000$

b The median, as it is typical of more apartment prices.

6 $\bar{x} = 365.8$, $s = 8.4$, $M = 366.5$, IQR = 12.5, $R = 31$

7 $\bar{x} = 214.8$, $s = 35.4$, $M = 207.5$, IQR = 42, $R = 145$

8 $\bar{x} = 3.5$ kg, $s = 0.6$ kg, $M = 3.5$ kg, IQR = 1 kg, $R = 2.4$ kg

9 a i $\bar{x} = 6.79$, $M = 6.75$

ii IQR = 1.45, $s = 0.93$

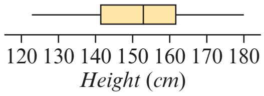
b i $\bar{x} = 13.54$, $M = 7.35$

ii IQR = 1.80, $s = 18.79$

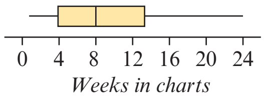
c The error does not affect the median or interquartile range very much. It doubles the mean and increases the standard deviation by a factor of 20.

Exercise 9H

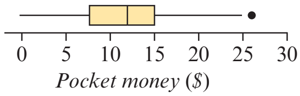
1



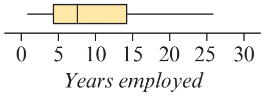
2



3



4



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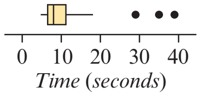

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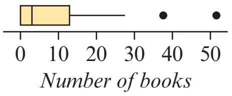
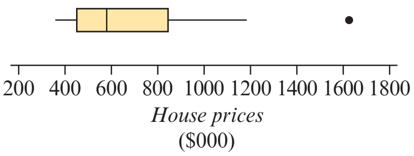
6 **a** There are two possible outliers: the people who borrowed 38 and 52 books respectively.

b



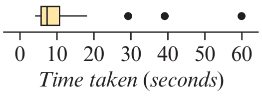
7 **a** There was one outlier: the house that sold for $1 625 000.

b



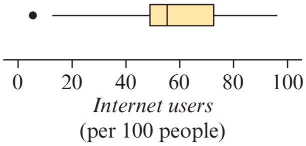
8 **a** There are three possible outliers: the three children who took 29, 39 and 60 seconds respectively to complete the puzzle.

b



9 **a** There is one possible outlier, Afghanistan, which recorded extremely low percentages of internet users (5.45%). At 12.52%, India is just above the outlier cutoff of 12.05%.

b



Exercise 9I

Note: The written reports should only be regarded as sample reports. There are many ways of writing the same thing.

1 **a** Females: M = 34 years, IQR = 28 years
Males: M = 25.5 years, IQR = 13 years

b Report: The distributions are both approximately symmetric. The median age of the females (M = 34 years) was higher than the median age of males (M = 25.5 years). The spread of ages of the females (IQR = 25 years) was greater than the spread of ages of the males (IQR = 13 years). In conclusion, the median age of the females admitted to the hospital on that day was higher than that for males. Their ages were also more variable.

2 **a** Class A: 6; Class B: 2

b Class A: M = 76.5 marks, IQR = 30.5 marks
Class B : M = 78 marks, IQR = 12 marks

c Report: Both distributions are negatively skewed. The median mark for Class A (M = 76.5) was lower than the median mark for Class B (M = 78). The spread of marks for Class A (IQR = 30.5) was greater than the spread of marks of Class B (IQR = 12). In conclusion, Class B had a higher median mark than Class A and their marks were less variable.

3 **a**

Japan		*Australia*
3	0	2 3 3 3 4 4
9 7 6 5 5	0	5 5 6 7 7 8 9
4 4 2	1	1 1 4
9 7 5	1	5 7
3 3 2 2	2	1 3
9 8 6	2	
2	3	3
	3	
4	4	

6 | 2 represents 26   1 | 5 represents 15

b Japanese: M = 17 days, IQR = 16.5 days
Australians: M = 7 days, IQR = 10.5 days

c Report: Both distributions are positively skewed. The median time spent away from home by the Japanese (M = 17 days) was much higher than the median time spent away from home by the Australians (M = 7 days). The spread in the time spent away from home by the Japanese (IQR = 16.5 days) was also greater than the time spent away from home by the Australians (IQR = 10.5). In conclusion, the median time spent away from home by the Japanese was longer than the Australians and the length of time they spent away from home more variable.

4 **a** Year 12: M = 5.5 hours, IQR = 4.5 hours
Year 8: M = 3 hours, IQR = 2.5 hours
(values can vary a little)

b Report: The distribution for Year 12 is negatively skewed and for Year 8 approximately symmetric. The median homework time for the Year 12 students (M = 5.5 hours/week) was higher than the median homework time for Year 8 students (M = 3 hours/week). The spread in the homework time for the Year 12 students (IQR = 4.5 hours/week) was also greater than for Year 8 students (IQR = 2.5 hours/week). In conclusion, the median homework time for the Year 12 students was higher that for Year

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8 students and the time they spent on homework was more variable.

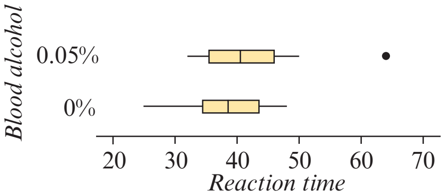
5 **a** Males: $M = 22\%$, IQR $= 15\%$
Females: $M = 19\%$, IQR $= 14\%$ (values can vary a little)

b Report: The distribution of smoking rates is slightly negatively skewed for females and approximately symmetric for males. The median smoking rate for males ($M = 22\%$) was higher than for females ($M = 19\%$). The spread in smoking rates for males (IQR $= 15\%$) was similar to that for females (IQR $= 14\%$). In conclusion, median smoking rates were higher for males than females but the variability in smoking rates was similar.

6 **a** Before: $M = 26$, IQR $= 4$, outlier $= 45$
After: $M = 30$, IQR $= 6$, outliers $= 48, 52$
(values can vary a little)

b Report: The distribution is negatively skewed before the course, and approximately symmetric after the course. The median number of sit ups before the fitness class ($M = 26$) was lower than that after the fitness class ($M = 30$). The spread in number of sit ups before the fitness class (IQR $= 6$) was less than after the fitness class (IQR $= 9$). There was one outlier before the fitness class, the person who did 45 sit ups. There were two outliers after the fitness class, the person who did 48 sit ups and the person who did 52 sit ups. In conclusion, the median number of sit ups increased after taking the fitness class and there was more variability in the number of sit ups that could be done.

7 **a**



b Report: The distribution is negatively skewed for the 0% blood alcohol group and positively skewed for the 0.05% alcohol group. The median time is slightly higher for the 0.05% blood alcohol group ($M = 40.5$) than for the 0% blood alcohol group ($M = 38.5$). The spread in time is also slightly higher for the 0.05% blood alcohol group (IQR $= 9.5$) than for 0% blood alcohol (IQR $= 9.0$). There was one outlier, the person with 0.05% blood alcohol who had a very long time of 64 seconds.
In conclusion, the median time was longer for the 0.05% blood alcohol group than for the 0% blood alcohol group but the variability in times was similar.

Exercise 9J

1 A study was conducted to investigate the time spent by Year 11 students watching television. A group of Year 11 students recorded the time they spent watching television per month in 2015, and these data were collected again with another group of Year 11 students in 2017. The distributions of time spent watching television for both groups were found to be positively skewed. Generally, the time spent watching television had increased only very slightly between 2015 (median = 7 hours) and 2017 (median = 8 hours) and variation has decreased a little (IQR: 2015 = 12 hours, 2017 = 11 hours). In 2015 there was one students whose hours watching television was unusually high (40 hours and another in 2017 who spent 35 hours watching television.

2 **a** Report: A study was conducted to investigate how the age of mothers at the birth of their first child has changed over the 40 year period 1970–2010. Data was collected from 20 mothers in each of 1970, 1990 and 2010. The distributions of these ages was approximately symmetric in 1970, slightly positively skewed in 1990 and slightly negatively skewed in 2010. The age of mothers at the birth of their first child has increased over this time period, with the median in 1970 of 23.5 years, in 1990 of 28 years, and in 2010 of 31 years. There is also an increase in variation in age over this time, with IQR increasing from 8.5 years in 1970 to 9.0 years in 1990 and 10.5 years in 2010. Interestingly, one mother who gave birth at age 45 years was considered an outlier in 1970, but this age would not have been particularly unusual in 1990 or 2010.

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b Report: A study was conducted to investigate how the age of fathers at the birth of their first child in has changed over the 40 year period 1970–2010. Data was collected from 20 fathers in each of 1970, 1990 and 2010. The distributions of these ages was approximately symmetric in 1970 and 1990 and positively skewed in 2010. The age of fathers at the birth of their first child has increased over this time period, with the median in 1970 of 29 years, in 1990 of 31 years, and in 2010 of 33.5 years. There has been little change in the variation in age over this time, with IQR of 9.0 years in 1970, 9.0 years in 1990 and 9.5 years in 2010.

c Report: A study was conducted to investigate the relationship between the age of mothers and the age of fathers, and how this has changed over the 40 year period 1970–2010. Data was collected from 20 mothers and 20 fathers in each of 1970, 1990 and 2010. On average the age of fathers at the birth of their first child is higher than the age of mothers at the birth of their first child, with fathers typically 5.5 years older in 1970, 3 years older in 1990 and 2.5 years older in 2010.

Chapter 9 review

Multiple-choice questions

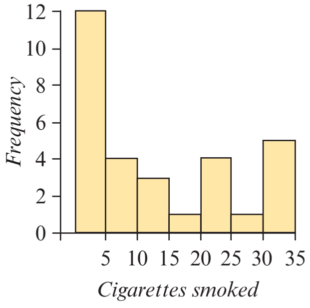
1 D	**2** D	**3** C	**4** B	**5** D
6 D	**7** D	**8** D	**9** D	**10** E
11 D	**12** A	**13** C	**14** C	**15** E
16 B	**17** B	**18** B	**19** C	**20** A
21 C				

Short-answer questions

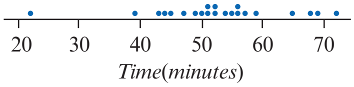
1 **a** Discrete **b** Ordinal

2 **a** Categorical **b** 7.5%

3



4 **a**



b Time (minutes)

2	2
3	9
4	3 4 5 7 9
5	0 1 1 2 2 4 5 6 6 7 9
6	5 8 9
7	2

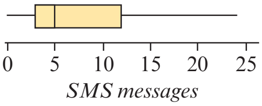
4 | 7 represents 47 minutes

c $M = 52$ minutes, $Q_1 = 47$ minutes, $Q_3 = 57$ minutes

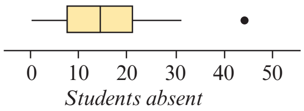
5 $\bar{x} = \$283.57$, $s = \$122.72$, $M = \$267.50$, IQR = \$90, $R = \$495$

6 $\bar{x} = 178.89$ minutes, $s = 13.99$ minutes

7



8 **a**



b 14.5 students **c** 27.8%

Extended-response questions

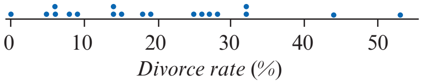
1 **a** Numerical

b Divorce rate (%)

0	0 5 6 6 8 9
1	4 4 5 8 9
2	5 6 7 8
3	2 2
4	4
5	3

3 | 2 represents 32%

c

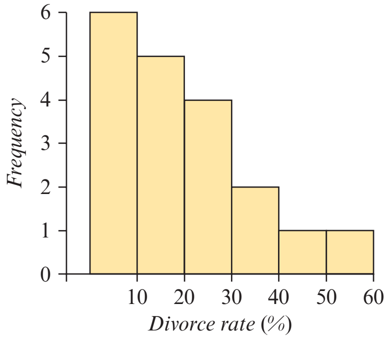


d Positively skewed

e 21.05%

f $\bar{x} = 20.05\%$, $M = 18\%$

g



i Positively skewed **ii** 5 countries

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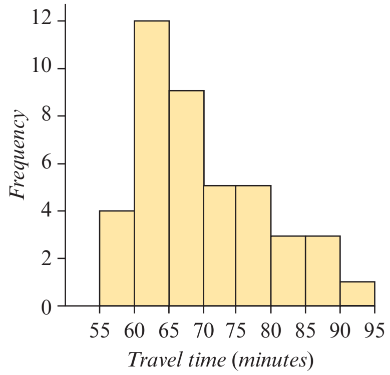
2 **a**



i 9 days **ii** Positively skewed

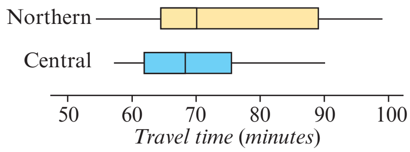
iii 38.1%

b $\bar{x} = 69.60$ minutes, $s = 9.26$ minutes, Min = 57 minutes, $Q_1 = 62$ minutes, $M = 68$ minutes, $Q_3 = 76$ minutes, Max = 90 minutes

c **i** 69.60 **ii** 68 **iii** 33, 14

iv 76 **v** 9.26

d



e The distribution of travel times is positively skewed for both corporations. The median travel times for Northern ($M = 70$ minutes) tend to be longer than the median travel times for Central ($M = 68$ minutes). The spread of times is also longer for Northern (IQR = 24 min) than for Central (IQR = 14 min). The median travel times and variability in travel times were both less for Central than for Northern.

Chapter 10

Exercise 10A

10A Revision of Chapter 6 Applied trigonometry

1 16.99

2 61.4°

3 33.2°

4 58 m^2

5 **a** 8.8 m **b** 42°

6 **a** **i** 3.2 km **ii** 090°

b 040°

7 **a** 43.5° **b** 313.5° **c** 33.2°

8 **a** 1.4 m/s **b** 345° **c** 1.65 m/s

d 2.04 m/s **e** 25 minutes

9 6.57 km

10 **a** Not required **b** 16.7° **c** 73.3°

d 85 cm **e** $\dfrac{2\sin\theta}{\sin(106.7° - \theta)}$

f

Angle θ	Length of eaves
15°	52 cm
20°	69 cm
25°	85 cm
30°	103 cm
35°	121 cm

11 **a** 20.56°

b 68.20°

c 88.76°, 90°

d The triangles on either side of the diagonal would have a gap between them if they were drawn correctly on a large scale.

e If the large triangle was truly right-angled, the tan ratio for it and the small triangle would be the same. However, $5 \div 13 \neq 3 \div 8$.

Exercise 10B

10B Revision of Chapter 7 Algebra: Linear and non-linear relationships

1 **a** $a = \dfrac{13}{5}$ **b** $y = 5$ **c** $x = 0$

2 **a** $R = \dfrac{A - P}{PT}$

b $r = \dfrac{C}{2\pi}$

3 24 and 25.

4 $66.30.

5 **a** 100.4 degrees Fahrenheit.

b $C = \dfrac{5}{9}(F - 32)$ **c** $C = F = -40$

6 **a** $C = 4.95x + 0.50y$ **b** $C = \$15.85$

c 3 packets of tomato sauce.

7 **a** $C = 90 + 40t$ **b** 3.5 hours

8 98%

9 7 × 50 cent coins and 6 × $1 coins.

10 **a**

t	0	1	2	3	4
h	0	3	4	3	0

b Maximum height is 4 m above ground.

c At time $t = 2$ seconds.

d $t = 1$ second and $t = 3$ second

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e One is when the rock is projected upwards, and the other is when it is falling towards the ground.

f $h = 1.75$ m

g $t = 0.5$ seconds

11 a $A = 24\text{ m}^2$

b $l = 2.5$ m and $w = 1.5$ m

c $A = \dfrac{15}{4} = 3.75\text{ m}^2$

d 3 m^2

e **i** 3 m^2 **ii** 4 m^2 **iii** 3 m^2

f When the rectangle is a square, $l = 2$ metres.

g The rectangle is a square when the area is at a maximum.

Exercise 10C

10C Revision of Chapter 8 Matrices and matrix arithmetic

1

$$\begin{array}{r|ccc|} & \textit{Shorts} & \textit{Tops} & \textit{Joggers} \\ \textit{Women} & 16.50 & 7.70 & 22 \\ \textit{Men} & 11 & 8.80 & 27.50 \end{array}$$

2 $\begin{bmatrix} -1 & 1 \\ -1 & -2 \end{bmatrix}$

3 a $H \times K$ **b**

$$\begin{array}{r|c|} & \textit{kJ} \\ \textit{Tim} & 14\,800 \\ \textit{Tam} & 15\,500 \end{array}$$

4

$$\begin{array}{c|cccc|} & A & B & C & D \\ A & 0 & 1 & 0 & 0 \\ B & 1 & 0 & 1 & 1 \\ C & 0 & 1 & 0 & 1 \\ D & 0 & 1 & 1 & 0 \end{array}$$

5 a

$$R = \begin{array}{c|cccc|} & A & B & C & D \\ A & 0 & 1 & 1 & 1 \\ B & 1 & 0 & 1 & 0 \\ C & 1 & 1 & 0 & 1 \\ D & 1 & 0 & 1 & 0 \end{array}$$

b There are two bus routes directly connected to Banach.

c **i**

$$R^2 = \begin{array}{c|cccc|} & A & B & C & D \\ A & 3 & 1 & 2 & 1 \\ B & 1 & 2 & 1 & 2 \\ C & 2 & 1 & 3 & 1 \\ D & 1 & 2 & 1 & 2 \end{array}$$

ii 6 ways

iii $B \rightarrow C \rightarrow A$

$B \rightarrow A \rightarrow B$

$B \rightarrow C \rightarrow B$

$B \rightarrow A \rightarrow C$

$B \rightarrow A \rightarrow D$

$B \rightarrow C \rightarrow D$

6 a

$$\begin{array}{c|ccc|} & \textit{TVs} & \textit{Computers} & \textit{DVDs} \\ X & 4 & 4 & 1 \\ Y & 1 & 2 & 1 \\ Z & 1 & 2 & 2 \end{array}$$

b

$$\begin{array}{c|ccc|} & \textit{TVs} & \textit{Computers} & \textit{DVDs} \\ X & 0.8 & 0.8 & 0.2 \\ Y & 0.2 & 0.4 & 0.2 \\ Z & 0.2 & 0.4 & 0.4 \end{array}$$

c

$$\begin{array}{c|ccc|} & \textit{TVs} & \textit{Computers} & \textit{DVDs} \\ X & 3.2 & 3.2 & 0.8 \\ Y & 0.8 & 1.6 & 0.8 \\ Z & 0.8 & 1.6 & 1.6 \end{array}$$

7 a

$$\begin{array}{r|ccc|} & \textit{Sandwiches} & \textit{Cakes} & \textit{Coffee} \\ \textit{Sold} & 38 & 23 & 89 \end{array}$$

b

$$\begin{array}{r|c|} & \$ \\ \textit{Sandwiches} & 7 \\ \textit{Cakes} & 3 \\ \textit{Coffee} & 4 \end{array}$$

c \$691

8 a **i** $S = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ **ii** $R \times S$

iii

$$\begin{array}{r|c|} \textit{Abid} & 333 \\ \textit{Bik} & 340 \\ \textit{Chaz} & 318 \end{array}$$

iv

$$\begin{array}{r|c|} \textit{Abid} & 83.25 \\ \textit{Bik} & 85 \\ \textit{Chaz} & 79.5 \end{array}$$

b

$$C = \begin{array}{r|c|} & \textit{Ex2} \\ \textit{Abid} & 79 \\ \textit{Bik} & 81 \\ \textit{Chaz} & 74 \end{array}$$; it gives students' Exam 2 results.

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c **i** $T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

ii $T \times R$

iii
$$T = \begin{array}{c} \begin{array}{cccc} Ex1 & Ex2 & Ex3 & Ex4 \end{array} \\ \begin{bmatrix} 256 & 234 & 263 & 238 \end{bmatrix} \end{array}$$

9 **a**
$$E = \begin{array}{c|ccc} & Oats & Wheat & Carrots \\ \hline Rabbit & 4 & 1 & 2 \\ Goat & 7 & 0 & 6 \\ Cavy & 2 & 2 & 1 \end{array}$$

b **i**
$$A = T\begin{array}{c} \begin{array}{ccc} Rabbits & Goats & Cavies \end{array} \\ \begin{bmatrix} 5 & 2 & 4 \end{bmatrix} \end{array}$$

ii $A \times E$

iii
$$T = \begin{array}{c} \begin{array}{ccc} Oats & Wheat & Carrots \end{array} \\ \begin{bmatrix} 42 & 13 & 26 \end{bmatrix} \end{array}$$

c **i**
$$C = \begin{array}{c|c} & \$ \\ \hline Oats & 3 \\ Wheat & 2 \\ Carrots & 1 \end{array}$$

ii $T \times C = \$[178]$

10 **a** $\begin{bmatrix} 100 \\ 95 \end{bmatrix}$

b The matrix produced by $F \times A$ gives the total number of animals on each farm. Each element in the first row of matrix F is multiplied by each element in column matrix A. The sum gives $20 \times 1 + 30 \times 1 + 50 \times 1 = 100$. This is the total of all animals on farm X.
Similarly, each element in the second row of matrix F is multiplied by each element in column matrix A.
The sum gives $15 \times 1 + 20 \times 1 + 60 \times 1 = 95$. This is the total of all animals on farm Y.

c $\begin{bmatrix} 35 & 50 & 110 \end{bmatrix}$

d The matrix produced by $B \times F$ gives the total number of each type of animal. Each element in the row matrix B is multiplied by each element in the first column of matrix F. The sum gives $1 \times 20 + 1 \times 15 = 35$. This is the total number of pigs. Similarly for the second and third columns of matrix F.
$1 \times 30 + 1 \times 20 = 50$. This is the total number of goats.
$1 \times 50 + 1 \times 60 = 110$. This is the total number of sheep.

e Use the matrix $C = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$F \times C = \begin{bmatrix} 20 & 30 & 50 \\ 15 & 20 & 60 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 20 \times 0 + 30 \times 1 + 50 \times 0 \\ 15 \times 0 + 20 \times 1 + 60 \times 0 \end{bmatrix}$$
$$= \begin{bmatrix} 30 \\ 20 \end{bmatrix}$$

Multiplying by zero has eliminated the unwanted values, while multiplying by 1 has retained the wanted values, which were the numbers of goats on each farm.

11 **a** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}, \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}, \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$

b $2^0\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, 2^1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, 2^2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, 2^3\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},$
$2^4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

c $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^n = 2^{n-1}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

d $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^{10} = 2^9\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 512 & 512 \\ 512 & 512 \end{bmatrix}$

e **i** No **ii** $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^n = 2^n\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

f $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}^n = 3^{n-1}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

g Self-correcting check.

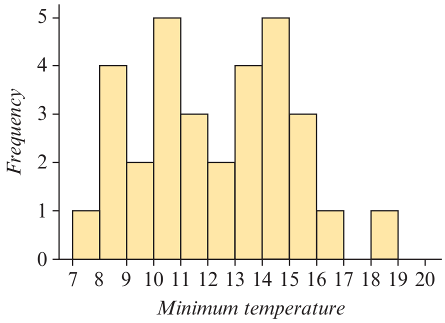
Exercise 10D

10D Revision of Chapter 9 Univariate data analysis

1 **a** Numerical **b** Nominal
c Ordinal

2 3.5

3 **a**



b 22.6%

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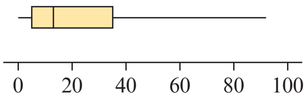

5 The better measure of centre would be the median. The distribution is positively skewed so the mean would be higher than the median, and would overstate the number of years people had been working in their current roles.

6 1

7 **a** Lower fence = −23.25, upper fence = 70.75
b Outlier = 85

8 1.8

9 **a** Male: Median = 70, IQR = 12, one outlier = 48
Female: Median = 78, IQR = 20, no outliers

b The distribution of resting pulse rates is approximately symmetric for both males and females in these samples. The median pulse rate for the females is higher than for males (around 78 beats/minute compared to 70 beats/minute) and the IQR is greater for females (around 20 beats/minute compared to 12 beats/minute). There is one male who has an unusually low pulse rate for this group at 48 beats/minute. In summary, for this group of people, the pulse rates were higher and more variable for females than males.

10 Answers will vary.

11 Answers will vary.

Appendix 1

Exercise A1.1

1 **a** 37 **b** 5 **c** 50 **d** 7
e 7.2 **f** 48 **g** 34.5 **h** 4.5
i 0.6 **j** −0.9

2 **a** 12.53 **b** −27 **c** 31.496 **d** 1

3 **a** 27 **b** $2x - 14$ **c** $50 - 10y$
d $6w$ **e** $k^2 + 8k$ **f** 30 **g** $2x + 14$
h 22 **i** $3x - 5$ **j** $-2 - 2x$

Exercise A1.2

1 **a** −1 **b** −4 **c** −16 **d** 3
e −26 **f** −25 **g** −12 **h** 22
i 22 **j** 32 **k** 28 **l** 10
m −6 **n** −13

2 **a** −12 **b** 24 **c** 2.5 **d** −5
e 36 **f** −12 **g** −7 **h** 60
i −1 **j** 60 **k** 6 **l** 19
m −7 **n** −38 **o** 34 **p** 160

Exercise A1.3

1 **a** 10 000 **b** 343 **c** 5 **d** 2
e 64 **f** 20 736 **g** 3 **h** 13
i 1000 **j** 4 **k** 2

2 **a** 26 **b** 15 **c** 37
d 5 **e** 79 **f** 4

Exercise A1.4

1 **a** 87 **b** 606 **c** 3 **d** 34

2 **a** 6800 **b** 46 800 **c** 80 000 **d** 300

3 **a** 7.92×10^{5} **b** 1.46×10^{7}
c 5.0×10^{11} **d** 9.8×10^{-6}
e $1.456\,97 \times 10^{-1}$ **f** 6.0×10^{-11}
g $2.679\,886 \times 10^{6}$ **h** 8.7×10^{-3}

4 **a** 6×10^{24}
b 4×10^{7}
c 1×10^{-10}
d 1.5×10^{8}

5 **a** 53 467 **b** 3 800 000
c 789 000 **d** 0.009 21
e 0.000 000 103 **f** 2 907 000
g 0.000 000 000 003 8 **h** 21 000 000 000

6 **a** 5 **b** 6 **c** 1 **d** 3
e 2 **f** 1 **g** 2 **h** 4

7 **a** 4.9 **b** 0.0787 **c** 1506.9 **d** 6

8 **a** 0.0033 **b** 0.148 68
c 317 **d** 335

9 **a** 1.56 **b** 0.025 **c** 0.03
d 1.8823 **e** 17.668 **f** 0.2875

10 **a** 15.65 **b** 4.69 **c** 39.14

Exercise A1.5

1 **a** 25% **b** 40% **c** 15% **d** 70%
e 19% **f** 79% **g** 215% **h** 3957%
i 7.3% **j** 100%

2 **a** **i** $\frac{1}{4}$ **ii** 0.25
b **i** $\frac{1}{2}$ **ii** 0.5
c **i** $\frac{3}{4}$ **ii** 0.75
d **i** $\frac{17}{25}$ **ii** 0.68

e **i** $\frac{23}{400}$ **ii** 0.0575
f **i** $\frac{34}{125}$ **ii** 0.272
g **i** $\frac{9}{2000}$ **ii** 0.0045
h **i** $\frac{3}{10\,000}$ **ii** 0.0003
i **i** $\frac{13}{200\,000}$ **ii** 0.000 065
j **i** 1 **ii** 1

3 **a** \$114 **b** \$110 **c** 25.5 m **d** \$1350
e 1.59 cm **f** 2.64 **g** €0.161 **h** \$4570
i \$77 700 **j** \$19 800

4 80% **5** 37.5%
6 95.6% **7** 83.33%
8 20% **9** 37.5%
10 65.08%

Exercise A1.6

1 **a** \$37 **b** \$148

2 **a** \$4.50; \$85.49 **b** \$18.90; \$170.10
c \$74.85; \$424.15 **d** \$49.80; \$199.20
e \$17.99; \$61.96 **f** \$5.74; \$17.21
g \$164.73; \$434.28 **h** \$19.05; \$44.45
i \$330; \$670

3 **a** \$425.25 **b** \$699.13 **c** \$227.50
d \$656.25 **e** \$215.25

4 **a** \$12.95 **b** \$202.95

5 Decreasing \$60 by 8%

6 14 840 cars **7** 21.95%

8 26 880 km

9 **a** 13% **b** 26% **c** 6%
d 24% **e** 18% **f** 23%

10 7.08%

11 **a** 19% **b** 33% **c** 45%
d 20% **e** 33% **f** 16%

12 **a** 25% **b** 40% **c** 7.5%

Exercise A1.7

1 35 : 15

2 **a** 80 : 40 **b** 70 : 9 **c** 80 : 120
d 40 : 4 **e** 40 : 4 : 80

3 **a** 4 : 5 **b** 2 : 9 **c** 2 : 5 : 3 **d** 1 : 3
e 3 : 1 **f** 20 : 3 **g** 9 : 4

4 **a** 12 : 5 **b** 1 : 20 **c** 3 : 8 **d** 25 : 3
e 3 : 100 : 600 **f** 100 000 : 100 : 1
g 4 : 65 **h** 50 : 10 : 2 : 1

5 **a** 5 **b** 72 **c** 120 **d** 5000 **e** 24

6 **a** False **b** False 3 : 4 = 15 : 20
c True **d** False 60 : 12 = 15 : 3 = 5 : 1
e False, the father would be 49. **f** True

7 **a** 100 : 60 : 175 : 125 : 125
b 20 : 12 : 35 : 25 : 25
c 300 g rolled oats, 180 g coconut, 525 g flour, 375 g brown sugar, 375 g butter, 9 tbsp water, 6 tbsp golden syrup, 3 tsp bicarb soda

Exercise A1.8

1 **a** 32 m and 8 m **b** 5 m and 35 m
c 30 m and 10 m **d** 20 m and 20 m

2 **a** \$300 and \$200 **b** \$50, \$200 and \$250
c \$50, \$400 and \$50 **d** \$160, \$180 and \$160
e \$250, \$125, \$100 and \$25

3 **a** 50 bananas **b** 5 mangos
c 75 pieces of fruit

4 **a** 1.5 litres **b** 6 litres

5 3 kilometres

Appendix 1 review

Multiple-choice questions

1 E	**2** A	**3** B	**4** C	**5** B
6 C	**7** E	**8** D	**9** C	**10** B
11 D	**12** A	**13** D	**14** C	**15** E
16 C	**17** A	**18** C	**19** B	**20** A

Short-answer questions

1 **a** 11 **b** 10 **c** 7 **d** 14
e 49 **f** −5 **g** 1 **h** 30
i 2

2 **a** 125 **b** 3 **c** 6 **d** 2.83
e 4 **f** −4 **g** 8.64 **h** 11.66

3 **a** 2.945×10^3 **b** 5.7×10^{-2}
c 3.69×10^5 **d** 8.509×10^2

4 **a** 7500 **b** 0.001 07 **c** 0.456

5 **a** 8.9 **b** 0.059 **c** 800

6 **a** 7.15 **b** 598.2 **c** 4.079

7 **a** 0.75 **b** 0.4 **c** 0.275

8 **a** $\frac{1}{10}$ **b** $\frac{1}{5}$ **c** $\frac{11}{50}$

9 **a** 24 **b** \$10.50 **c** \$13.25

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10 **a** \$51.90 **b** \$986.10

11 \$862.50

12 20%

13 46%

14 Melissa 5.13%, Jody 4.41%

15 **a** False **b** False **c** False **d** True

16 **a** \$320 and \$480 **b** \$160 and \$640
c \$160, \$240 and \$400
d \$200, \$200 and \$400

17 18 cups

18 **a** 27 m **b** 140 m

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