



OXFORD

**INSIGHT**  
**MATHEMATICS**  
**STANDARD 1**

**12**

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**ADDITIONAL RESOURCE**  
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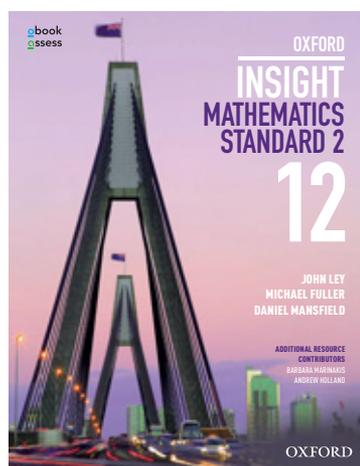
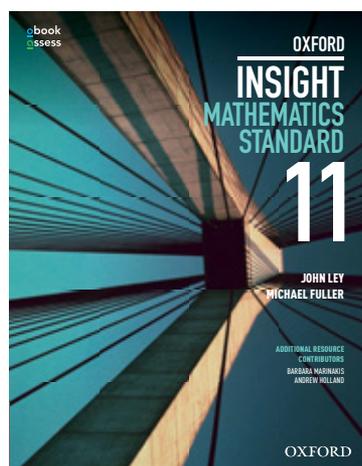
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# Using Oxford Insight Mathematics Standard 1 Year 12

New South Wales' most trusted Mathematics series has been updated for the new Mathematics Standard Stage 6 syllabus. The new edition includes comprehensive exercise sets, carefully graded exercises and worked examples embedded where students need them. Enhanced opportunities for support and extension, as well as consolidation and practice are offered in reviews, cumulative reviews, exam-style questions and integrated technology.

helpful resources are outlined at the beginning of each unit

visual mathematics glossary boosts understanding of key concepts

worked examples and visuals located next to the relevant exercise

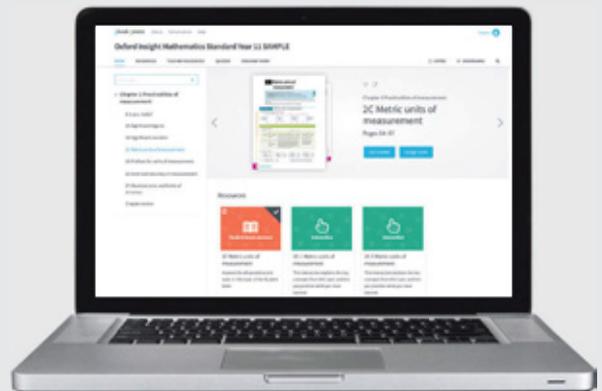
'Working Mathematically' syllabus components clearly signposted

## Student obook assess

Oxford Insight Mathematics Standard 1 Year 12 is supported by a range of engaging and relevant digital resources via obook assess.

Students receive:

- > a complete digital version of the Student book with notetaking and bookmarking functionality
- > targeted instructional videos by a team of Australia's most experienced Mathematics Standard teachers designed to help students prepare for assessment tasks and exams
- > interactive auto-correcting multiple-choice quizzes
- > access to teacher-assigned work including readings, homework, tests and assignments.



## Teacher obook assess

In addition to the student resources, teachers also receive:

- > detailed planning resources
- > printable (and editable) class tests with exam-style questions and answers
- > the ability to set up classes, set assignments, monitor progress and graph results, and to view all available content and resources in one place.

# Top tips for study success

## Tip 1 – read key documents

The first step to success is to gather all key documents and read them carefully.

- > Your most important tool is the syllabus. It sets out all of the information about the course, including what you are expected to learn and how you will be assessed. You can download a copy from the NESAs website.
- > Keep all documents from your teacher relating to assessment tasks and copies of any assessment advice (e.g. marking criteria or assessment rubrics). Understanding exactly what is required in an assessment task is crucial.

## Tip 2 – study regularly

If you're going to perform at your best, you need to allocate time for regular periods of study and revision. Studying regularly will help you to continually reinforce new concepts and avoid the stress of last-minute cramming. During your study you might:

- > summarise theory and key examples in your own words
- > focus on topics you find difficult and work through the relevant examples and questions
- > test your understanding with revision questions, practice papers and past exams.

## Tip 3 – manage your study time

When studying, it helps to put some practical strategies in place to stay on track. Try the following time management strategies.

- > **Create a study timetable** to set up periods of regular study and revision around your school and personal schedule.
- > **Use a diary, wall planner or calendar** to record the dates of upcoming assessment tasks, tests or exams and allow you to adequately prepare.
- > **Make lists** of daily, weekly or monthly goals. It helps to keep the bigger picture in mind and breaks big tasks down into smaller, more manageable tasks, so that you gain a sense of achievement.

## Tip 4 – take care of yourself

Looking after yourself during for HSC is important:

- > eat a balanced diet and stay hydrated – try to avoid too much caffeine and junk food
- > get enough sleep and regular exercise
- > make time for breaks from study – a walk to get some fresh air will help you reset before the next study session.

## Tip 5 – know the structure of exams

It's important for you to become familiar with the format of the exam and the types of questions that typically appear. In an exam you should also:

- > show your working when answering a question
  - even if a question is incorrect or left unfinished, you might still get some marks for your working
- > keep an eye on the clock to make sure you have enough time to answer every question
- > re-read questions so you know that you have provided a complete and accurate answer.

## Tip 6 – understand key terms

Assessment tasks will likely include key terms. These range in level of difficulty. Some, such as *solve* or *find*, are simple to understand and master. Others, such as *justify*, are more challenging and will take practice to master. Below is a list of common key terms and an explanation of what they mean.

TERM	DEFINITION
<b>analyse</b>	examine something complex by breaking it down into smaller parts and show how they relate to one another
<b>calculate</b>	work out an answer mathematically
<b>classify</b>	categorise into groups
<b>convert</b>	change to a different form without changing the value
<b>describe</b>	give a detailed account of the features
<b>evaluate</b>	determine the value
<b>explain</b>	make something clear by describing the relationships between different aspects and giving reasons
<b>express</b>	represent an answer as a number, figure, formula or symbol
<b>find</b>	determine the value or answer to a problem.
<b>identify</b>	determine and state clearly
<b>justify</b>	present an argument providing evidence
<b>solve</b>	work out the solution to a question

# 1

---

## Investments

The main mathematical ideas investigated are:

- ▶ making compound interest calculations using the formula
- ▶ making compound interest calculations using a compounded value table
- ▶ comparing different investment strategies
- ▶ calculating the price of goods following inflation
- ▶ calculating new salaries after increases in line with inflation
- ▶ calculating the appreciated value of items.

# ARE YOU READY?

- 1A** ▶ **1** What is the result of  $2000 \times 0.05 \times 3$ ?  
**A** 1200                      **B** 30 000  
**C** 2003.05                  **D** 300
- 1A** ▶ **2** What is 9.5% expressed as a decimal?  
**A** 0.95                        **B** 0.095  
**C** 9.5                          **D** 950
- 1A** ▶ **3** Given that  $a = 3$ ,  $b = 4$  and  $c = 8$ , what is the value of  $abc$ ?  
**A** 348                         **B** 15  
**C** 96                            **D** 3.48
- 1A** ▶ **4** Given that  $a = 2$ ,  $b = 5$  and  $c = 9$ , what is the value of  $a(b + c)$ ?  
**A** 214                         **B** 54  
**C** 28                            **D** 16
- 1A** ▶ **5** What is 20% of 970?  
**A** 19 400                      **B** 194  
**C** 1940                        **D** 19.4
- 1A** ▶ **6** What is 7.5% of \$11 300?  
**A** \$84 750                    **B** \$847.50  
**C** \$1506.67                  **D** \$1.51
- 1A** ▶ **7** How much interest is earned if \$1000 is put into a simple interest account paying 5% p.a. for 1 year?  
**A** \$5000                      **B** \$50  
**C** \$500                        **D** \$1050
- 1A** ▶ **8** \$2000 is put into a simple interest account paying 7% p.a. How much is in the account after 1 year?  
**A** \$2070                      **B** \$14 000  
**C** \$140                        **D** \$2140
- 1B** ▶ **9** How many days are there in 3 years (excluding leap years)?  
**A** 1095                        **B** 36  
**C** 1098                        **D** 156
- 1B** ▶ **10** Given that  $x = 150$  and  $y = 1.1$ , what is the value of  $xy^2$ ?  
**A** 181.5                        **B** 165  
**C** 123.97                      **D** 151.1
- 1E** ▶ **11** To the nearest cent, what is the value of  $\$35 \times 1.05^3$ ?  
**A** \$36.75                      **B** \$36.86  
**C** \$110.25                    **D** \$40.52

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1**                      **Support sheet 1A.1 Multiplying and dividing decimal numbers**  
**Q2**                      **Support sheet 1A.2 Converting percentages, fractions and decimals**  
**Q3–4**                   **Support sheet 1A.3 Substituting for pronumerals**  
**Q5–6**                   **Support sheet 1A.4 Percentage of a quantity**  
**Q7–8**                   **Support sheet 1A.5 Understanding the simple interest formula**  
**Q9**                      **Support sheet 1B.1 Converting units of time**  
**Q10**                    **Support sheet 1B.2 Evaluating algebraic expressions involving powers**  
**Q11**                    **Support sheet 1E.1 Finding cubes of numbers**

# 1A Comparing simple and compound interest investments

These resources are available on your obook assess:

- **Spreadsheet 1A:** Compare simple and compound interest investments
- **Worksheet 1A:** Practise your skills with extra problems for simple interest
- **assess quiz 1A:** Test your skills with an auto-correcting multiple-choice quiz

o  
i  
c  
i

## simple interest

interest that is calculated on the original principal for the lifetime of the investment or loan; also known as flat rate interest

**Simple interest** is a type of interest based on a fixed percentage of the original amount invested or borrowed, i.e. the principal. Simple interest can be calculated by using the following formula.

## Simple interest formula

$$I = Prn$$

where  $I$  = amount of interest in dollars

$P$  = the principal, the amount invested (or borrowed)

$r$  = interest rate per time period

$n$  = number of time periods.

!

## EXAMPLE 1A-1 Calculating simple interest on investments

Calculate the simple interest earned on these investments.

- \$5000 at 6.7% p.a. over 4 years
- \$2300 at 1.56% per month for 19 months
- \$3000 at 15% p.a. over 17 months

	Solve	Think/Apply
<b>a</b>	$P = \$5000, r = 0.067, n = 4$ $I = Prn$ $= 5000 \times 0.067 \times 4$ $= \$1340$	Convert the percentage interest rate to a decimal by dividing by 100. If needed, convert the interest rate to a rate for the specified time period. Substitute the values of $P$ , $r$ and $n$ into the formula $I = Prn$ . 
<b>b</b>	$P = \$2300, r = 0.0156, n = 19$ $I = Prn$ $= 2300 \times 0.0156 \times 19$ $= \$681.72$	
<b>c</b>	$P = \$3000$ $r = 15 \div 100 \div 12$ $= 0.0125$ $n = 17$ $I = Prn$ $= 3000 \times 0.0125 \times 17$ $= \$637.50$	

# EXERCISE 1A Comparing simple and compound interest investments

- Calculate the simple interest earned on these investments.
  - \$6000 at 5.8% p.a. over 3 years
  - \$3200 at 1.1% per month for 13 months
  - \$780 at 0.025% per day for 19 days
- Calculate the simple interest on the following investments.
 

<b>a</b> \$5600 at 13% p.a. for 16 months	<b>b</b> \$2900 at 15% p.a. for 23 days
<b>c</b> \$7890 at 18.6% p.a. for 11 months	<b>d</b> \$3540 at 12.8% p.a. for 53 days

## EXAMPLE 1A-2 Calculating the monthly repayment for simple interest investments

Dominic borrows \$2200 to buy a guitar. The simple interest rate is 9.75% p.a. and he takes the loan over 2 years.

- Find the interest on the loan.
- Find the total amount to be repaid.
- Find the monthly repayment.

	Solve/Think		Apply
<b>a</b>	$I = Prn$ $= 2200 \times \frac{9.75}{100} \times 2$ $= \$429$		The total amount to be repaid is the interest added to the principal. $\text{Monthly repayment} = \frac{\text{total to be repaid}}{\text{no. of months of the loan}}$
<b>b</b>	$\text{Total to be repaid} = 2200 + 429$ $= \$2629$		
<b>c</b>	$\text{Monthly repayment} = \frac{2629}{24}$ $= \$109.54$		

- Calculate the total amount to be repaid on a simple interest loan of:
 

<b>a</b> \$4500 at 13% p.a. over 3 years	<b>b</b> \$5750 at 0.9% per month over 15 months
<b>c</b> \$7100 at 0.031% per day over 19 days	<b>d</b> \$5290 at 14% p.a. over 17 months.
- Chad borrows \$14 300 to buy a car. The simple interest rate is 12.5% p.a. and he takes the loan over 3 years. Complete the following to find the:
  - interest on the loan =  $14\,300 \times \frac{\square}{100} \times \square$   
= \$\_\_\_\_\_
  - total to be repaid =  $14\,300 + \underline{\hspace{2cm}}$   
= \$\_\_\_\_\_
  - monthly repayment =  $\frac{\square}{\square}$   
= \$\_\_\_\_\_
- Monica borrows \$5800 to buy a bedroom suite. The simple interest rate is 8.6% p.a. and she takes the loan over 4 years.
 

<b>a</b> Find the interest on the loan.	<b>b</b> Find the total amount to be repaid.
<b>c</b> Find the monthly repayment.	

**compound interest**

interest that is calculated on the current balance of an investment, including the interest from the previous time period

For a **compound interest** investment, the interest earned at the end of each time period is added to the principal. This increases the principal that is used to calculate the interest for the next time period. Therefore, with compound interest you are earning interest on the interest you have previously earned.

**EXAMPLE 1A-3** Calculating the total value of an investment and compound interest earned

\$2000 is invested for 3 years at 7% p.a. interest compounded annually.

- a Find the amount the \$2000 will grow to after 3 years.
- b Find the amount of interest earned.

Solve			
<b>a</b>	<b>Year</b>	<b>Balance at start of year</b>	<b>Interest</b>
			<b>Balance at end of year</b>
	1	\$2000	$\frac{7}{100} \times 2000 = \$140$
			$2000 + 140 = \$2140$
	2	\$2140	$\frac{7}{100} \times 2140 = \$149.80$
			$2140 + 149.80 = \$2289.80$
	3	\$2289.80	$\frac{7}{100} \times 2289.80 = \$160.29$
			$2289.80 + 160.29 = \$2450.09$

The amount the \$2000 will grow to after 3 years is \$2450.09.

- b The amount of interest earned =  $2450.09 - 2000 = \$450.09$

	Think	Apply
<b>a</b>	Use $I = Prn$ with $P = 2000$ , $r = 0.07$ and $n = 1$ to find the interest for the first year of \$140. Add \$140 to \$2000 to get a new principal of \$2140, then calculate the interest on \$2140.	Use $I = Prn$ with $n = 1$ to calculate the interest each year. The principal each year is the previous principal plus the interest for that year. The interest earned is the total balance less the original principal.
<b>b</b>	Subtract \$2000 from the total balance.	

- 6 a Complete the table to determine the final value of \$2800 invested at 7% p.a. compound interest for 3 years.

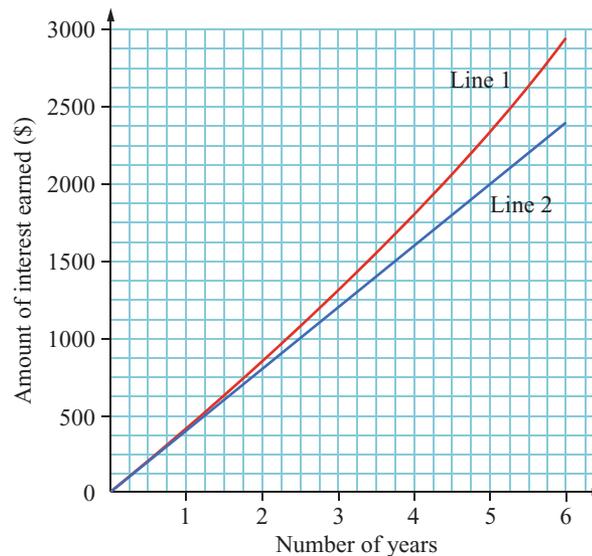
Year	Balance at start of year	Interest	Balance at end of year
1	\$2800	\$196	\$2996
2	\$2996	\$209.72	
3			

- b Calculate the total interest earned.  
Interest = \_\_\_\_\_ - 2800 = \$\_\_\_\_\_

- 7** Toby invested \$6500 for 4 years at 6.5% p.a. interest compounded annually.
- Using a table, find the value of Toby's investment after 4 years.
  - Find the amount of interest earned by Toby in the 4 years.
- 8** Given an annual interest rate (p.a.), we can calculate a monthly interest rate by dividing this quantity by 12. Calculate the monthly interest rates for the following interest rates.
- 12% p.a.
  - 6% p.a.
  - 9% p.a.
  - 7.2% p.a.
  - 6.6% p.a.
- 9** Julianne invested \$20000 in a saving account paying 8.4% p.a., with interest compounded monthly.
- What is the monthly interest rate for the savings account?
  - Using a table, find the value of Julianne's investment after 5 months.
  - Find the amount of interest earned by Julianne in 5 months.

The graph of the amount of interest earned through a simple interest investment is a straight line. The graph of the amount of interest earned through a compound interest investment is a curved line.

- 10** The following graph shows the amount of interest earned in both a simple interest investment and a compound interest investment when \$5000 is invested for 6 years at 8% p.a.
- Which line represents the simple interest investment and which line represents the compound interest investment?
  - What is the difference in the interest earned after 6 years?



- 11 a** Complete the table to determine the final value of \$980 invested at 3% p.a. compound interest for 4 years.

Year	Balance at start of year	Interest	Balance at end of year
1			
2			
3			
4			

- Calculate the total interest earned.
- Use digital technology to produce a graph of the value of the investment over 4 years.

**12** Adele decided to invest her savings of \$10350 for 5 years at 7.7% p.a. compound interest.

**a** Complete the table.

Year	Balance at start of year	Interest	Balance at end of year
1			
2			
3			
4			
5			

**b** If Adele intends to buy a car that is expected to be valued at \$14495 when her investment matures, will she have enough to buy the car? Explain.

**c** By how much is the investment over or under the value of the car?



**13** June receives a gift of \$5000 from her grandparents for her 21st birthday. She looks at different investment options, and wants to compare simple and compound interest investments at 5% p.a.

**a** Complete the following table to determine the final values of the two investments after 5 years.

Time	Simple interest at 5% p.a.	Compound interest at 5% p.a.
Start of first year	\$5000	\$5000
Start of second year	\$5250	\$5250
Start of third year		
Start of fourth year		
Start of fifth year		
Start of sixth year		

**b** What is the difference in the value of the investments after the 5 years?

**14** Leo wants to compare the potential value of two different investment opportunities. Bank A offers a simple interest rate of 6.2% p.a. and bank B offers a compound interest rate of 6.0% p.a., with the interest compounded monthly.

**a** If \$10000 is invested with each bank at the start of the year, which investment will have the higher balance at the end of the first year, and by how much?

**b** To receive the offered interest rates, the investments have to be made for a minimum of 2 years. Which investment will have the higher balance at the end of the second year, and by how much?

**c** Explain why the answers to parts **a** and **b** are different.

We can use a spreadsheet to generate tables for compound interest investments. Open a new spreadsheet and type in the column headings from the example below into cells A1 to D1. Then follow these instructions.

Enter in the amount of the principal (in dollars) into cell B2.

Type the formula  $=a*B2$  into cell C2, where  $a$  represents the interest rate per time period expressed as a decimal. **Fill down** to C5 and beyond.

Type the formula  $=B2+C2$  into cell D2 and **fill down** to D5 and beyond.

Type the formula  $=D2$  into cell B3 and **fill down** to B5 and beyond.

	A	B	C	D
1	Year	Balance at start of year (\$)	Interest (\$)	Balance at end of year (\$)
2	1			
3	2			
4	3			
5	...			

- 15 a** \$1000 is invested at 7% p.a. interest compounding annually. Use a spreadsheet to calculate the value of the investment at the end of each year for 10 years.
- b** Use digital technology to produce a graph showing the value of the investment over a period of 10 years.
- c** On the same set of axes as part **b**, draw a simple interest graph for the same time period, but with a rate of 9% p.a.
- d** Use the graph drawn in part **c** to determine the number of years it takes for the two investments to be the same total amount.
- 16 a** \$4000 is invested at 7.4% p.a. interest compounding annually. Use a spreadsheet to create a table showing the value of the investment at the end of each year for 10 years.
- b** Use digital technology to produce a graph showing the value of the investment over a period of 10 years.
- c** Find the time for the investment to be worth \$6500.
- d** On the same set of axes as part **b**, draw a straight line joining the point representing the initial value of the investment and the point representing the investment value after 10 years.
- e** Calculate the gradient of the straight line drawn for part **d** to help you determine the equivalent simple interest rate.
- 17** An amount of \$70 000 is to be invested for 9 years.
- a** Use a spreadsheet to determine the total interest earned on the investment if the initial amount is invested at:
- 9.2% p.a. simple interest
  - 9.2% p.a. compound interest with interest compounded annually.
- b** Explain why the final value is different for the two investment options.
- c** On the one set of axes, draw graphs to show how the value of the investment changes for each option over the 9 years.

# 1B The compound interest formula

These resources are available on your obook assess:

- **Video tutorial 1B:** Watch and listen to an explanation of Example 1B-1
- **Spreadsheet 1B:** Use the compound interest formula
- **Worksheet 1B:** Practise your skills with extra problems for compound interest
- **Investigation 1B:** Investigate another use of the compound interest formula
- **assess quiz 1B:** Test your skills with an auto-correcting multiple-choice quiz

In the financial world, the principal, or initial amount, is known as the present value of the investment. The amount to which the principal grows is known as the future value of the investment.

Compound interest can be calculated by using the following formula.

## Compound interest formula

$$FV = PV(1 + r)^n$$

where  $FV$  = the future value

$PV$  = the present value

$r$  = interest rate per compounding period

$n$  = number of compounding periods

### EXAMPLE 1B-1 Finding the interest earned for a compound interest investment

- a** Use the compound interest formula to calculate the future value of a fixed term investment of \$5000 over 5 years at 6.5% p.a. interest compounding yearly.
- b** Find the total interest earned.

	Solve	Think	Apply
<b>a</b>	$FV = PV(1 + r)^n$ $= 5000 \times (1 + 0.065)^5$ $= 5000 \times (1.065)^5$ $= \$6850.43$	$PV = 5000$ $r = 0.065$ , as the compounding period is annual, the interest rate is the annual rate. $n = 5$ , as the compounding period is annual, the number of time periods is the same as the number of years.	The interest rate and the time period must correspond. Substitute into the formula. The interest is calculated by subtracting the original investment amount (the present value) from the future value.
<b>b</b>	$\text{Interest} = 6850.43 - 5000$ $= \$1850.43$	Subtract the present value of \$5000 from the answer to part <b>a</b> .	

## EXERCISE 1B The compound interest formula

- 1 a** Using the compound interest formula, complete the following to calculate the future value when \$6500 is invested for 7 years at 4.2% p.a. interest compounding annually.
- $$PV = \underline{\hspace{2cm}} \qquad r = 4.2 \div \underline{\hspace{2cm}} = 0.\underline{\hspace{2cm}} \qquad n = \underline{\hspace{2cm}}$$
- $$FV = PV(1 + r)^n$$
- $$= \underline{\hspace{2cm}}(1 + \underline{\hspace{2cm}})^7$$
- $$= 6500(\underline{\hspace{2cm}})^{\square}$$
- $$= \underline{\hspace{2cm}}$$
- b** Complete the following to find the total interest earned.
- $$\text{Interest} = \underline{\hspace{2cm}} - 6500 = \underline{\hspace{2cm}}$$
- 2 a** Use the compound interest formula to calculate the future value of a fixed-term investment of \$4000 over 6 years at 7.5% p.a. interest compounding yearly.
- b** Find the total interest earned.
- 3 a** Use the compound interest formula to calculate the future value of a fixed-term investment of \$6453 over 3 years at 4.95% p.a. interest compounding yearly.
- b** Find the total interest earned.

### EXAMPLE 1B-2 Calculating the future value of a compound interest investment

Use the compound interest formula to calculate the future value of a fixed-term investment of \$3500 over 7 years at 6.2% p.a. interest compounding quarterly.

Solve	Think	Apply
$FV = PV(1 + r)^n$ $= 3500 \times (1 + 0.0155)^{28}$ $= 5000 \times (1.0155)^{28}$ $= \$5384.01$	<p>There are 4 quarters in a year, so <math>n = 7 \times 4 = 28</math> time periods.</p> <p>Quarterly interest rate is the annual rate divided by 4:  <math>r = 0.062 \div 4 = 0.0155</math></p> <p>Present value is \$3500.</p>	<p>Calculate to find the number of time periods, then find the interest rate for the required time period.</p> <p>Substitute into the compound interest formula. Usually, the time period is multiplied and the interest rate is divided by the same number.</p>

- 4** Using the compound interest formula, complete the following to calculate the future value of a fixed-term investment of \$1200 over 5 years at 8.4% p.a. interest compounding quarterly.
- $$PV = \underline{\hspace{2cm}} \qquad n = \underline{\hspace{2cm}} \times 4 = \underline{\hspace{2cm}} \qquad r = 0.084 \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$
- $$FV = PV(1 + r)^n$$
- $$= 1200(1 + \underline{\hspace{2cm}})^{\square}$$
- $$= 1200 \times (\underline{\hspace{2cm}})^{\square}$$
- $$= \underline{\hspace{2cm}}$$
- 5** Use the compound interest formula to calculate the future value of a fixed-term investment of \$950 over 3 years at 4.1% p.a. interest compounding quarterly.



- 9 Calculate the amount that must be invested at 5% p.a. interest compounding annually to have \$1600 at the end of 7 years.
- 10 Calculate the amount that must be invested at 11.2% p.a. interest compounding annually to have \$10000 at the end of 6 years.

**EXAMPLE 1B-4** Calculating the present value for a non-annual compounding period

Calculate the amount that must be invested at 7.5% p.a. interest compounding quarterly to have \$1700 at the end of 3 years.

Solve	Think	Apply
$PV = \frac{1700}{(1+r)^n}$ $= \frac{1700}{(1+0.01875)^{12}}$ $= \frac{1700}{(1.01875)^{12}}$ $= \$1360.31$ <p>\$1360.31 must be invested.</p>	<p>Quarterly interest rate  <math>= 0.075 \div 4 = 0.01875</math></p> <p>Number of quarters  <math>= 3 \times 4 = 12</math></p> <p><math>FV = 1700, r = 0.01875,</math>  <math>n = 12</math></p> <p>Substitute into the formula and solve.</p>	<p>Calculate to find the number of time periods, then divide to find the interest rate for the required time period.</p> <p>Substitute and solve.</p>

- 11 Complete the following to calculate the amount that must be invested at 8.5% p.a. interest compounding quarterly to have \$2300 at the end of 7 years.

Quarterly interest rate =  $0.085 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

Number of quarters =  $\underline{\hspace{1cm}} \times 4 \qquad n = \underline{\hspace{1cm}}$

$$PV = \frac{FV}{(1+r)^n}$$

$$= \frac{2300}{(1+\underline{\hspace{1cm}})^{\square}}$$

$$= \underline{2300}$$

$$= \$\underline{\hspace{1cm}}$$

- 12 Calculate the amount that must be invested at:
- a 4% p.a. interest compounding quarterly to have \$1540 at the end of 8 years
  - b 10.2% p.a. interest compounding quarterly to have \$10000 at the end of 4 years
  - c 9% p.a. interest compounding monthly to have \$3000 at the end of 3 years
  - d 4.5% p.a. interest compounding monthly to have \$950 at the end of 8 years
  - e 7.2% p.a. interest compounding six-monthly to have \$2000 at the end of 10 years
  - f 6.3% p.a. interest compounding six-monthly to have \$7500 at the end of 5.5 years.
- 13 How much does Paul need to invest at 4.95% to have \$2500 in 3 years' time, if the interest compounds monthly?
- 14 A company will need \$20000 to replace its computer system in 4 years' time. How much needs to be invested at 4.95% p.a. interest compounding quarterly to have this amount available?



- 19** A sum of \$5000 was invested for 4 years in a compound interest account paying 5.7% p.a. with interest compounded quarterly. How much would need to be invested to achieve the same amount of interest if the interest rate was 4.2% p.a. compounded monthly? Give your answer to the nearest dollar.
- 20** Use trial and error to determine the compound interest rate that will give a future value of \$5165.22 if \$4000 is invested for 4 years with interest compounded annually.
- 21** If the interest rate on a compound interest investment is doubled, will the interest earned also be doubled? Use an example to help you illustrate your answer.



We can enter the compound interest formula into a spreadsheet to determine the future value of an investment given the present value, the interest rate per annum, the number of compounding periods per annum, and the term of the investment.

Type in the headings in column A, and enter in the formula into cell B1, as shown.

	A	B
1	Future value (\$)	=B2*(1+(B3/(100*B4)))^(B4*B5)
2	Present value (\$)	
3	Interest rate (% p.a.)	
4	Compounding periods per annum	
5	Term of the investment (years)	

To help you, you might like to use the prepared spreadsheet file (Spreadsheet 1B) provided on your [obook assess](#).

- 22 a** Use a spreadsheet to calculate the future value of a \$5000 investment, invested for 3 years with the interest compounded annually, at the following compound interest rates.
- i** 4%      **ii** 6%      **iii** 8%      **iv** 10%
- b** Discuss how changing the interest rate of a compound interest investment affects the future value of the investment.
- 23 a** Use a spreadsheet to calculate the future value of a \$5000 investment, invested for 2 years at 5% p.a. at the following compounding periods.
- i** Annually  
**ii** Quarterly  
**iii** Monthly  
**iv** Daily
- b** Discuss how changing the compounding period of a compound interest investment affects the future value of the investment.



# 1C Investment strategies

These resources are available on your obook assess:

- **Video tutorial 1C:** Watch and listen to an explanation of Example 1C-1
- **o assess quiz 1C:** Test your skills with an auto-correcting multiple-choice quiz

o  
i  
o  
i

Comparing and contrasting different investment strategies can allow people to make smart financial decisions which will benefit them in the long run.

## EXAMPLE 1C-1 Choosing the best investment strategy

Justine wants to invest \$3000 for 3 years, but is struggling to decide where to invest her money. She narrows her choices down to three options.

Option A: A simple interest account at 9% p.a.

Option B: A compound interest account at 8.5% p.a., with interest compounded annually

Option C: A compound interest account at 8.3% p.a., with interest compounded daily

Determine which option will give Justine the highest future value on her investment.



Solve	Think	Apply
<p>Option A:</p> $I = Prn$ $= 3000 \times \frac{9}{100} \times 3$ $= \$810$ <p>Future value = <math>3000 + 810</math> = \$3810</p>	<p>Option A:</p> <p>Substitute <math>P = 3000</math>, <math>r = \frac{9}{100}</math> and <math>n = 3</math> into <math>I = Prn</math>.</p> <p>Add the interest to the principal to determine the future value.</p>	<p>Calculate the final value of each investment by using the simple interest and compound interest formulas.</p>
<p>Option B:</p> $FV = PV(1 + r)^n$ $= 3000 \times (1 + 0.085)^3$ $= 3000 \times (1.085)^3$ $= \$3831.87$	<p>Option B:</p> <p>Substitute <math>PV = 3000</math>, <math>r = 0.085</math> and <math>n = 3</math> into <math>FV = PV(1 + r)^n</math>.</p>	
<p>Option C:</p> $FV = PV(1 + r)^n$ $= 3000 \times \left(1 + \frac{0.083}{365}\right)^{1095}$ $= 3000 \times (1.000227\dots)^{1095}$ $= \$3848.12$	<p>Option C:</p> <p>Substitute <math>PV = 3000</math>, <math>r = \frac{0.083}{365}</math> and <math>n = 365 \times 3 = 1095</math> into <math>FV = PV(1 + r)^n</math>.</p>	
<p>Option C will give Justine the highest future value on her investment.</p>		

## EXERCISE 1C Investment strategies

- Sienna is looking to invest \$5500 for 3 years, but would like some help in picking the best investment option. Determine which option will give Sienna the highest future value on her investment.  
Option A: A simple interest account at 6.6% p.a.  
Option B: A compound interest account at 6.2% p.a., with interest compounded annually  
Option C: A compound interest account at 6% p.a., with interest compounded monthly
- Georgio has \$12 000 to invest and wants to invest it for 4 years. Which investment option should he choose?  
Option A: Compound interest at 4.7% p.a., with interest compounded annually  
Option B: Compound interest at 4.45% p.a., with interest compounded monthly  
Option C: Compound interest at 4.35% p.a., with interest compounded weekly
- Will a \$5000 simple interest investment earn more or less interest than a \$5000 compound interest investment if the interest rate is the same for both investments? Justify your answer.
- Given two compound interest investments of the same amount, with the same interest rate and different compounding periods, which investment will earn more interest? Justify your answer.
- Sam invested \$3000 for three years in a compound interest account paying 8.1% p.a. with interest compounded monthly. Sam's brother, Lee, also invested \$3000 for three years, but in a simple interest account paying 8.1% p.a. How much more interest did Sam earn over the period of the investment?

- Dijana's financial advisor suggested that Dijana put her \$20 000 inheritance for 5 years into a compound interest investment paying 7.8% p.a. with interest compounded weekly.
  - Calculate the future value of Dijana's investment after 5 years.
  - What equivalent simple interest rate p.a. will Dijana need to find to achieve the same return? Give your answer to two decimal places.
- Xavier's trust account matures on his 18th birthday. On that day it will be worth \$12 500. His mother persuades him to invest the full amount for at least 3 years and together they find three different investment opportunities.  
Option A: Compound interest at 7.55% p.a., with interest compounded monthly  
Option B: Simple interest at 7.4% p.a., with a bonus payment of \$1000 paid after 3 years  
Option C: Compound interest at 4.5% p.a., with interest compounded annually, plus a bonus 5% of the future value added after 3 years.
  - Calculate the potential future value of each of the investment options.
  - What is the difference between the options with the lowest and highest future values?
- Julio is helping his father out with his finances and they're looking for a place to invest his savings. They've narrowed down their choices to the following options.  
Option A: A compound interest account paying 5.6% p.a. compound interest, with interest compounded daily  
Option B: Investing in the stock market, with an expected long-term return of 9.8% p.a.  
Option C: Investing in the property market, with an expected long-term return of 9.5% p.a.
  - Julio's father is willing to invest \$50 000. If he invests his money for 5 years, determine the expected future value for each option.
  - Why might Julio advise his father to not invest in the option that has the highest expected future value?

# 1D Using a compounded value table

These resources are available on your obook assess:

- **assess quiz 1D:** Test your skills with an auto-correcting multiple-choice quiz



A compounded value table can be used to quickly and efficiently calculate the future value of a compound interest investment, given the number of time periods and the interest rate per time period.

Time period	Interest rate per time period									
	1%	2%	3%	4%	5%	6%	8%	9%	10%	15%
1	1.010	1.020	1.030	1.040	1.050	1.060	1.080	1.090	1.100	1.150
2	1.020	1.040	1.061	1.082	1.103	1.124	1.166	1.188	1.210	1.323
3	1.030	1.061	1.093	1.125	1.158	1.191	1.260	1.295	1.331	1.521
4	1.041	1.082	1.126	1.170	1.216	1.262	1.360	1.412	1.461	1.750
5	1.051	1.104	1.159	1.217	1.276	1.338	1.469	1.539	1.611	2.011
6	1.062	1.126	1.194	1.265	1.340	1.419	1.587	1.671	1.772	2.313
7	1.072	1.149	1.230	1.316	1.407	1.504	1.714	1.828	1.949	2.660
8	1.083	1.172	1.267	1.369	1.477	1.594	1.851	1.993	2.144	3.059

## EXAMPLE 1D-1 Using the table of compounded values to find the present value

Taryn needs to have \$5000 in 6 years' time. She can invest money at 4% p.a. compounded annually. What amount needs to be invested now? Use the compounded values in the table to help you answer the question.

Solve	Think	Apply
Amount to be invested $= 5000 \div 1.265$ $= \$3952.57$ Taryn needs to invest \$3952.57 now to have \$5000 in 6 years' time.	Find the value at the intersection of the 6th row and 4% column. This is 1.265. Divide 5000 by 1.265 to find the present value.	Find the value in the table at the intersection of the time and rate. Divide the future value given by this amount to find the present value.

## EXERCISE 1D Using a compounded value table

- 1** Use the interest table on the previous page to answer the following questions.
  - a** Complete the following to find the value of \$8000 invested at 5% compounded annually for 3 years.  
Amount =  $8000 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
  - b** Find the value of \$5000 invested at 10% compounded annually for 7 years.
  - c** Find the value of \$12 000 invested at 10% compounded annually for 6 years.
  - d** Complete the following to find the value of \$1120 invested at 1% per quarter compounded quarterly for 6 quarters. Amount =  $1120 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
  - e** Find the value of \$10 500 invested at 5% per quarter compounded quarterly for 2 quarters.
  - f** Find the value of \$1140 invested at 10% per quarter compounded quarterly for 5 quarters.
  - g** Find the value of \$1550 invested at 1% per month compounded monthly for 7 months.
  - h** Find the value of \$78 420 invested at 1% per month compounded monthly for 4 months.
  
- 2** Use the interest table on the previous page to answer the following questions.  
Find the future value of these investments.
  - a** \$5000 invested at 9% compounded annually for 4 years
  - b** \$3500 invested at 6% compounded annually for 8 years
  - c** \$7500 invested at 6% compounded annually for 7 years
  - d** \$1300 invested at 3% per quarter compounded quarterly for 7 quarters
  - e** \$6800 invested at 3% per quarter compounded quarterly for 3 quarters
  - f** \$2350 invested at 6% per quarter compounded quarterly for 2 years
  - g** \$890 invested at 12% per annum compounded monthly for 3 months
  - h** \$8431 invested at 9% per month compounded monthly for 5 months
  
- 3** Use the interest table on the previous page to answer the following questions.
  - a** Complete the following to find the amount that needs to be invested at 4% compounded annually to have \$6000 in 5 years' time. Amount to be invested =  $6000 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
  - b** What amount needs to be invested at 6% compounded annually to have \$3500 in 7 years' time?
  - c** What amount needs to be invested at 8% compounded annually to have \$3800 in 4 years' time?
  - d** Complete the following to find the amount that needs to be invested at 8% per quarter compounded quarterly for 8 quarters to have \$1500. Amount to be invested =  $\underline{\hspace{1cm}} \div 1.851 = \underline{\hspace{1cm}}$
  - e** What amount needs to be invested at 4% per quarter compounded quarterly for 5 quarters to have \$2900?
  - f** What amount needs to be invested at 2% per quarter compounded quarterly for 2 years to have \$2200?
  - g** What amount needs to be invested at 2% per month compounded monthly for 5 months to have \$1150?
  - h** What amount needs to be invested at 6% per month compounded monthly for 7 months to have \$5280?
  
- 4** Janet invested \$3300 in a compound interest account paying 8% p.a., with interest compounded half-yearly. Use the interest table on the previous page to determine the future value of her investment after 3.5 years.
  
- 5** A high-interest short-term loan charges 24% p.a., with interest compounded monthly. If \$1500 was borrowed for 7 months, use the interest table on the previous page to determine how much would need to be paid back.
  
- 6** Stig wants to buy a new sound system that costs \$2200. Use the interest table on the previous page to determine how much he would need to invest now, in a compound interest account paying 8% p.a. with interest compounding quarterly, if he wants to purchase it in 18 months' time?
  
- 7** Use the interest table on the previous page to find out how much more interest will be earned if \$5000 is invested for 6 years at 8% p.a. compounded annually compared to at 6% p.a. compounded annually.

# 1E Inflation and appreciated value

These resources are available on your obook access:

- **Video tutorial 1E:** Watch and listen to an explanation of Example 1E-1
- **assess quiz 1E:** Test your skills with an auto-correcting multiple-choice quiz

1010

## **inflation**

an increase in the price of goods and services over a period of time

## **appreciate**

a rise in the value or price of an item over a period of time

**Inflation** is the rise in the cost of consumer goods and services. It is usually expressed as a percentage rate. The inflation rate reflects the increase in the cost of items.

Some items, such as antiques, also **appreciate** in value, meaning that they gain in value each year.

We can calculate appreciated values on a year-by-year basis, or, if the rate of inflation is constant over the time period, we can use an adjusted version of the compound interest formula,  $FV = PV(1 + r)^n$ .

$$\text{Price after } n \text{ years} = \text{original price} \times \left(1 + \frac{\text{inflation rate}}{100}\right)^n$$

!

## EXAMPLE 1E-1 Calculating the price of an item after inflation

The price of petrol increases at the same rate as inflation. The inflation rate over 3 years is 4% p.a. If petrol is 139.50 c/L now, how much will it be in 3 years' time?



Solve	Think	Apply
Price after 1 year = $139.5 \times 1.04$ = 145.08 c/L Price after 2 years = $145.08 \times 1.04$ = 150.8832 c/L Price after 3 years = $150.8832 \times 1.04$ = 156.918528 = 156.92 c/L (2 decimal places)	Add 4% to 139.50 c by multiplying by 1.04. Repeat the multiplication three times.	If the rate of increase is constant (the same each year) apply the compound interest formula; otherwise, calculate the increase year by year.
Using the formula: Price after $n$ years = original price $\times \left(1 + \frac{\text{inflation rate}}{100}\right)^n$ = $139.50 \times \left(1 + \frac{4}{100}\right)^3$ = 156.92 c/L (2 decimal places)		





**EXAMPLE 1E-3** Calculating the annual rate of inflation

Three years ago a packet of crumpets cost \$2.12. It now costs \$2.36. What is the average annual rate of inflation of the cost of crumpets over the 3 years? Give your answer to one decimal place.

Solve	Think	Apply
$FV = PV \left(1 + \frac{r}{100}\right)^n$ $2.36 = 2.12 \left(1 + \frac{r}{100}\right)^3$ $\frac{2.36}{2.12} = \left(1 + \frac{r}{100}\right)^3$ $1.113\ 207\ 547 = \left(1 + \frac{r}{100}\right)^3$ $\sqrt[3]{1.113\ 207\ 547} = 1 + \frac{r}{100}$ $1.036\ 395\ 171 = 1 + \frac{r}{100}$ $0.036\ 395\ 171 = \frac{r}{100}, \text{ so } r \approx 3.6\%$ <p>The average annual inflation rate is 3.6%.</p>	It is possible to use the compound interest formula, as the rate of inflation is constant. Use $FV = 2.36$ as the future value, $PV = 2.12$ as the present value, and $n = 3$ .	As there is a constant increase in each time period, the compound interest formula is used.

- 14** Three years ago the price of chuck steak was \$8.99/kg. It now costs \$9.69/kg. Calculate the average yearly inflation rate, correct to two decimal places, that would produce this rise.
- 15** In 2013, the cost of a standard university lecture was \$152.13. In 2017 the price was \$176.42. What was the annual rate of inflation (assuming it is constant) correct to two decimal places?



- 16 a** For the Samurai sword in question 12, calculate the actual annual rate of increase (assuming it is constant) correct to two decimal places.
- b** For the rugby jumper in question 13, calculate the actual annual rate of increase (assuming it is constant) correct to two decimal places.
- 17 a** Use the internet to research how to calculate the average rate of inflation over a number of years. Present a report containing your findings to your class.
- b** If the rate of inflation was 3.0% one year and 4.0% the following year, would the average rate of inflation over the two years be 3.5%? Explain your answer.

# CHAPTER 1 REVIEW INVESTMENTS

You should be able to:

- ✓ calculate the future value of a compound interest investment using both tables and the compound interest formula, including for time periods other than years
- ✓ calculate the present value of a compound interest investment given the future value, the interest rate and the number of compounding periods
- ✓ use a compounded value table to calculate the future or present value of a compound interest investment
- ✓ determine the best investment strategy given a range of options
- ✓ calculate the effect of inflation on the cost of goods
- ✓ calculate the appreciated value of items.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

## REVIEW MULTIPLE-CHOICE QUESTIONS

- 1A** 1 What is the future value when \$3550 is invested at 5.8% p.a. compounded annually for 4 years?  
A \$4373.60                      B \$823.60                      C \$898.06                      D \$4448.06
- 1A** 2 What is the amount of interest earned when \$680 is invested for 5 years at 4.9% p.a. interest compounding annually?  
A \$863.75                      B \$183.75                      C \$166.60                      D \$846.60
- 1B** 3 What is the future value when \$1630 is invested at 4.3% p.a. compounded quarterly for 7 quarters?  
A \$2188.66                      B \$558.66                      C \$1756.68                      D \$126.68
- 1B** 4 What is the amount that must be invested at 4.8% p.a. interest compounding monthly to have \$4500 at the end of 3 years?  
A \$3897.61                      B \$3909.57                      C \$832.16                      D \$3899.84
- 1B** 5 How much more interest does an investment of \$5000 for 3 years at 6.8% compound interest earn if the interest is compounded monthly as compared to yearly?  
A \$6127.96                      B \$1127.96                      C \$40.44                      D \$37.03
- 1B** 6 Which of the following investment options provides the best return if \$8000 is invested for 3 years?  
Option A: Simple interest at 6.99% p.a.  
Option B: Compound interest at 6.77% p.a. with interest compounded annually  
Option C: Compound interest at 6.6% with interest compounded monthly  
Option D: Compound interest at 6.55% with interest compounded daily.  
A Option A                      B Option B                      C Option C                      D Option D

Use the following table to answer questions 7 and 8.

Periods (years)	Interest rate per period			
	1%	5%	10%	15%
4	1.041	1.216	1.461	1.750
5	1.051	1.276	1.611	2.011
6	1.062	1.340	1.772	2.313
7	1.072	1.407	1.949	2.660

- 1C** ▶ **7** What is the future value of \$3500 invested at 5% p.a. interest compounding annually for 5 years?  
**A** \$4256                      **B** \$4466                      **C** \$4690                      **D** \$4924.50
- 1C** ▶ **8** What is the future value of \$1300 invested at 20% p.a. interest compounding quarterly for 6 quarters?  
**A** \$1380.60                      **B** \$1742                      **C** \$3006.90                      **D** \$2303.60
- 1D** ▶ **9** A box of cereal costs \$3.78 today. The price increases at the same rate as inflation. The rate of inflation for the next 3 years is predicted to be constant at 6% p.a. What is the expected cost of the cereal in 3 years' time?  
**A** \$4.07                      **B** \$4.25                      **C** \$4.50                      **D** \$4.77
- 1D** ▶ **10** A signed Australia–Pakistan cricket bat used in Australia's win in the 2008 series was purchased for \$600. It increased in value by 3% p.a. What was the appreciated value in 2018, correct to the nearest dollar?  
**A** \$806                      **B** \$780                      **C** \$618                      **D** \$3600

## REVIEW SET 1

- a** Use the compound interest formula to calculate the future value of a fixed-term investment of \$6000 over 5 years at 4.7% p.a., with interest compounding yearly.

**b** Find the total interest earned.
- What amount must be invested at 7.5% p.a. interest compounding annually to have \$5000 at the end of 6 years?
- Calculate the future value when \$3220 is invested at 2% per month compounded monthly for 7 months.
- In three successive years the annual rate of inflation was 3.3%, 4.1% and 2.9%. How much would you expect to pay after these 3 years for a dozen eggs if they cost \$5.20 at the beginning of this period?
- Frank has the option of investing \$5000 for 3 years in either a simple interest account paying 7.3% p.a. or a compound interest account paying 6.9% p.a. with interest compounded monthly. Which option should he choose, and how much extra interest will this extra option earn?
- Jordan was earning \$47 000 a year. In two successive years her salary rose in line with the rate of inflation, which was 2.6% and 3.1% respectively. How much was her salary, to the nearest dollar, after the two pay rises?
- A business owner must repay a \$160 000 interest-free loan in full in 5 years' time. What is the single amount that he should invest now at 5.4% p.a. interest compounded monthly in order to be able to pay the debt?

## REVIEW SET 2

- 1 Use the compound interest formula to calculate the future value of a fixed-term investment of \$2700 over 8 years at 4.99% p.a. interest compounding quarterly.
- 2 Calculate the amount that must be invested at 3.8% p.a. interest compounding quarterly to have \$1350 at the end of 3 years.
- 3 Calculate the future value when \$7000 is invested at 1% per quarter compounding quarterly for 5 quarters.
- 4 In three successive years the annual rate of inflation was 2.8%, 3.1% and 3.8%. How much would you expect to pay after these 3 years for a box of chocolates that cost \$24.95 at the beginning of this period?
- 5 What's the difference in interest earned between an investment of \$5000 for 3 years earning 6.9% p.a. interest compounding monthly and an investment of \$4500 for 3 years earning 8.1% p.a. interest compounding monthly?
- 6 Priscilla started a new job earning \$75 000 as a business analyst. In the first three years of her job, her salary rose in line with the annual rate of inflation: 3.2%, 3.8% and 2.5%. What was her revised salary, to the nearest dollar, after three years?

## REVIEW SET 3

- 1
  - a Use the compound interest formula to calculate the future value of a fixed-term investment of \$2458 over 2 years at 6.95% p.a. interest compounding yearly.
  - b Find the total interest earned.
- 2 Calculate the amount when \$5200 is invested at 2% per quarter compounded quarterly for 7 quarters.
- 3 In three successive years the annual rate of inflation was 7.4%, 4.6% and 5.8%. How much would you expect to pay after these 3 years for a block of cheese that cost \$4.70 at the beginning of this period?
- 4 Calculate the future value of \$50 000 invested for 6 years at 4.9% p.a. interest compounded monthly.
- 5 What single amount must be invested now at 6.2% p.a. interest compounded quarterly in order to have \$30 000 in 5 years' time?
- 6 Katrina was earning \$50 000 before she received 4 successive pay rises of 3% each. How much was she earning after the pay rises, to the nearest dollar?
- 7 Dudi wanted to help advise his brother on where he could invest his \$20 000 inheritance for the next 3 years. He narrowed the choice down to 3 options:  
Option 1: A simple interest account paying 5.5% p.a. with a \$500 bonus after 3 years  
Option 2: A compound interest account paying 5.5% p.a. with interest compounding monthly  
Option 3: A compound interest account paying 5.4% p.a. with interest compounding weekly  
By performing appropriate mathematical calculations, determine which option would provide the highest return on investment.

## REVIEW SET 4

- 1 Use the compound interest formula to calculate the future value of a fixed-term investment of \$2950 over 4 years at 5.1% p.a. interest compounding quarterly.
- 2 Erin was earning \$88 000 before her pay rose by 3.5% and 3.6% in successive years. What was the increase in Erin's salary, correct to the nearest dollar, over the two-year period?
- 3 Calculate the amount that must be invested at 6% p.a. interest compounding annually to have \$1900 at the end of 5 years.
- 4 Calculate the amount that must be invested at 3.2% p.a. interest compounding quarterly to have \$10 000 at the end of 7 years.
- 5 In three successive years the annual rate of inflation was 5.8%, 4.5% and 3.1%. How much would you expect to pay after these 3 years for a hamburger that cost \$3.20 at the beginning of this period?
- 6 Calculate the extra interest earned on a \$30 000 investment invested for 5 years at 6% p.a. compound interest if interest is compounded daily as opposed to annually.

## REVIEW PRACTICE EXAMINATION QUESTION

- 1 a
  - i Convert 8% p.a. to a rate per quarter as a decimal. (1 mark)
  - ii How many quarters in 5 years? (1 mark)
  - iii Calculate the amount \$6250 grows to when invested at 8% p.a. compounding quarterly for 5 years. (2 marks)
  - iv How much interest is earned? (1 mark)
- b Stephen received a work bonus of \$7000 and decided to invest it for 5 years.
  - i What would the value of Stephen's investment be in 5 years if he placed the bonus into a simple interest account paying 7% p.a.? (2 marks)
  - ii What would the value of Stephen's investment be in 5 years if he placed the bonus into a compound interest account paying 6.6% p.a. with interest compounded monthly. (2 marks)
  - iii Into which account (i or ii) should Stephen place his investment, and how much more interest would he earn in this account than the other account? (2 marks)
- c Calculate the future value of \$1000 that is invested at 6% p.a. for 7 years. (1 mark)
- d Tournya needs \$6200 in 5 years' time. What amount does he need to invest now at 6% p.a. compounding annually? (1 mark)
- e Megan signed a contract to rent a house for \$1200 per month at the beginning of 2017. Her monthly rent increases each year by the amount of inflation for that year. If the inflation rate for 2017 was 4.5% p.a. and the inflation rate for 2018 was 5.1% p.a, find her rent at the start of 2019. (2 marks)

**TOTAL:**  
**15 marks**

# 2

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## Rates

The main mathematical ideas investigated are:

- ▶ using, simplifying and converting between units of rates
- ▶ using rates to solve practical problems (including determining costs and best buys)
- ▶ working with speed as a rate
- ▶ interpreting distance-time graphs and using them to solve problems relating to speed, distance and time
- ▶ performing calculations relating to fuel consumption rate
- ▶ measuring a person's heart rate and blood pressure over time and under different conditions
- ▶ interpreting charts, constructing graphs and performing a range of calculations in relation to heart rate and blood pressure.

# ARE YOU READY?

**2A ▶ 1** Which measurement is equivalent to 4.5 m?  
**A** 45 cm                      **B** 450 cm  
**C** 0.045 km                **D** 45 mm

**2A ▶ 2** Which measurement is equivalent to 260 L?  
**A** 0.26 kL                    **B** 26 kL  
**C** 26000 mL                **D** 0.026 ML

**2A ▶ 3** Which measurement is equivalent to 6 ha?  
**A** 600 m<sup>2</sup>                    **B** 6000 m<sup>2</sup>  
**C** 60000 m<sup>2</sup>                **D** 6 km<sup>2</sup>

**2A ▶ 4** Which measurement is equivalent to 850 g?  
**A** 0.85 kg                    **B** 8500 mg  
**C** 8.5 kg                      **D** 0.0085 t

**2A ▶ 5** Which measurement is equivalent to 2 h?  
**A** 120 s                        **B** 1200 s  
**C** 3600 s                      **D** 7200 s

**2A ▶ 6** What is the highest common factor of 45 and 72?  
**A** 3                              **B** 9  
**C** 18                            **D** 360

**2A ▶ 7** Which fraction is equivalent to  $\frac{1}{3}$ ?  
**A**  $\frac{5}{16}$                         **B**  $\frac{9}{18}$   
**C**  $\frac{2}{15}$                         **D**  $\frac{4}{12}$

**2A ▶ 8** What is 24 000 written in scientific notation?  
**A**  $24 \times 10^3$                 **B**  $2.4 \times 10^3$   
**C**  $2.4 \times 10^4$                 **D**  $2.4 \times 10^{-4}$

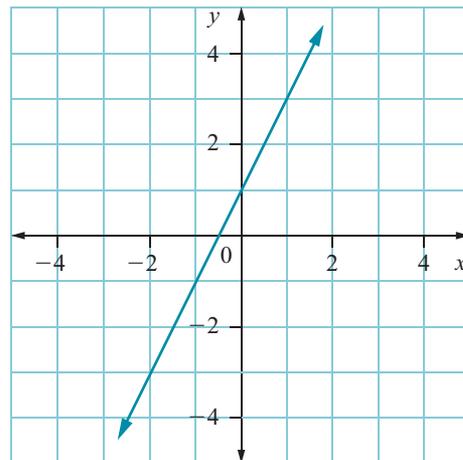
**2A ▶ 9** What is the result of  $40 \times 10^2 \times 10^5$  written in scientific notation?  
**A**  $40 \times 10^7$                 **B**  $40 \times 10^{10}$   
**C**  $4 \times 10^7$                     **D**  $4 \times 10^8$

**2B ▶ 10** A train travels at a constant speed for 2 hours and covers a distance of 170 km. What is the speed of the train?  
**A** 1.4 km/h                    **B** 85 km/h  
**C** 170 km/h                    **D** 340 km/h

**2B ▶ 11** Abbey jogs at a constant speed of 8 km/h for 45 minutes. How far has she jogged?  
**A** 16 km                        **B** 8 km  
**C** 6 km                         **D** 4 km

**2B ▶ 12** How long does it take a cyclist travelling at a constant speed of 20 km/h to cover a distance of 5 km?  
**A** 4 min                        **B** 15 min  
**C** 1 h                            **D** 4 h

**2B ▶ 13** What is the gradient of the line shown?



**A**  $\frac{1}{2}$                               **B** 2  
**C** 3                                **D** 4

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook](#) [\\_assess](#).

- Q1–5**      **Support sheet 2A.1 Converting units of measurement**
- Q6**        **Support sheet 2A.2 Finding the highest common factor**
- Q7**        **Support sheet 2A.3 Simplifying fractions**
- Q8–9**     **Support sheet 2A.4 Scientific notation**
- Q11–12**   **Support sheet 2B.1 Calculating speed, distance and time**
- Q13**      **Support sheet 2B.2 Finding the gradient of a line**

# 2A Rates

These resources are available on your obook assess:

- **Interactive 2A:** Explore key ideas for rates
- **Video tutorial 2A:** Watch and listen to an explanation of Examples 2A–3, 2A–4
- **Worksheet 2A:** Practise your skills with extra problems for writing rates
- **Investigation 2A.1:** Explore rates involved with antipasto platters
- **Investigation 2A.2:** Explore rates when producing different chemical solutions
- **assess quiz 2A:** Test your skills with an auto-correcting multiple-choice quiz

o  
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## rate

a comparison of one quantity with another; e.g. speed is a rate, as distance is compared to time

A **rate** is a comparison between quantities of different kinds.

The comparison is made by dividing one quantity by the other, in the required order. The rate is then expressed in the form ‘the first quantity per unit of the second quantity’.

!

We may compare distance travelled with petrol used, distance travelled in a given time, amount of fertiliser needed with area of land, amount of pay with time worked, etc.

## EXAMPLE 2A–1 Writing a rate to compare two quantities

Wendy types 600 words in 8 minutes. How many words per minute does she type?



Solve	Think	Apply
$\begin{aligned} \text{Rate} &= \frac{600 \text{ words}}{8 \text{ min}} \\ &= \frac{(600 \div 8) \text{ words}}{(8 \div 8) \text{ min}} \\ &= \frac{75 \text{ words}}{1 \text{ min}} \\ &= 75 \text{ words/min} \end{aligned}$	<p>This rate is comparing the number of words with time, in that order. So, divide the number of words by the time. Simplify until the rate is expressed as the number of words in 1 minute.</p> $\begin{aligned} \text{Rate} &= 600 \text{ words in } 8 \text{ min} \\ &= (600 \div 8) \text{ words in } 1 \text{ min} \\ &= 75 \text{ words in } 1 \text{ min} \\ &= 75 \text{ words/min} \end{aligned}$ <p>This is the number of words per unit of time.</p>	<p>Compare the quantities by dividing one quantity by the other. Simplify until the second quantity is 1 unit.</p>

# EXERCISE 2A Rates

UNDERSTANDING, FLUENCY AND COMMUNICATING

1 Complete the following to write the rate for each scenario.

**a** Jenny typed 300 words in 5 minutes. Find her typing rate per minute.

$$\begin{aligned} \text{Rate} &= \frac{\text{number of words}}{\text{time}} \\ &= \frac{\square \text{ words}}{\square \text{ min}} \\ &= \frac{\square \text{ words}}{1 \text{ min}} \\ &= \underline{\hspace{2cm}} \text{ words/min} \end{aligned}$$

**b** A 2.5 kg box of soap powder costs \$10.90. Find the cost per kg.

$$\begin{aligned} \text{Rate} &= \frac{\text{cost}}{\text{weight}} \\ &= \frac{\$ \square}{\square \text{ kg}} \\ &= \frac{\$ \square}{1 \text{ kg}} \\ &= \underline{\hspace{2cm}} \text{ \$/kg or } \$\underline{\hspace{2cm}}/\text{kg} \end{aligned}$$

**c** Vicki was paid \$82.50 for 6 hours of work. Calculate her rate of pay.

$$\begin{aligned} \text{Rate} &= \frac{\text{amount earned}}{\text{hours worked}} \\ &= \frac{\$ \square}{\square \text{ h}} \\ &= \frac{\$ \square}{1 \text{ h}} \\ &= \underline{\hspace{2cm}} \text{ \$/h or } \$\underline{\hspace{2cm}}/\text{h} \end{aligned}$$

**d** Nathan had to pay \$55.20 for 120 telephone calls. Calculate the cost per call.

$$\begin{aligned} \text{Rate} &= \frac{\text{total cost}}{\text{number of calls}} \\ &= \frac{\$ \square}{\square \text{ calls}} \\ &= \frac{\$ \square}{1 \text{ call}} \\ &= \underline{\hspace{2cm}} \text{ \$/call or } \$\underline{\hspace{2cm}}/\text{call} \end{aligned}$$

- 2 **a** The temperature rose  $14^\circ\text{C}$  in  $3\frac{1}{2}$  h. At what rate, in degrees per hour, did the temperature rise?  
**b** Jeremy spread 24 kg of fertiliser over an area of  $60 \text{ m}^2$ . Calculate the rate of application in  $\text{kg per m}^2$ .  
**c** Peggy drove 195 km in  $2\frac{1}{2}$  h at a constant speed. Calculate her speed in  $\text{km/h}$ .  
**d** Calculate the flow rate per minute when 119 L of water flows through a pipe in 35 min.

In some cases, it is more convenient or useful to express a rate using a different set of units. For example, a speed in kilometres per hour (km/h) can be converted to a speed in metres per second (m/s).

We need to convert the given units of each quantity in the rate to the required units. Note that we are still comparing the same quantities; for example, speed is a rate comparing distance and time.



## EXAMPLE 2A-2 Converting units for a rate comparing mass and area

Convert 8 t/ha to:

**a** kg/ha

**b**  $\text{kg/m}^2$

**c**  $\text{g/m}^2$

	Solve	Think	Apply
<b>a</b>	$\begin{aligned} 8 \text{ t/ha} &= \frac{8 \text{ t}}{1 \text{ ha}} \\ &= \frac{8000 \text{ kg}}{1 \text{ ha}} \\ &= 8000 \text{ kg/ha} \end{aligned}$	Convert the mass from tonnes to kilograms using $1 \text{ t} = 1000 \text{ kg}$ . There is no change to the area units.	Convert the given units to the required units and divide in the order given.

MEASUREMENT

	Solve	Think	Apply
<b>b</b>	$8 \text{ t/ha} = \frac{8 \text{ t}}{1 \text{ ha}}$ $= \frac{8000 \text{ kg}}{10\,000 \text{ m}^2}$ $= (8000 \div 10\,000) \text{ kg/m}^2$ $= 0.8 \text{ kg/m}^2$	Convert the mass from tonnes to kilograms (1 t = 1000 kg) and the area from hectares to square metres (1 ha = 10 000 m <sup>2</sup> ). Divide the mass by the area.	
<b>c</b>	$8 \text{ t/ha} = \frac{8 \text{ t}}{1 \text{ ha}}$ $= \frac{(8 \times 1000 \times 1000) \text{ g}}{10\,000 \text{ m}^2}$ $= (8\,000\,000 \div 10\,000) \text{ g/m}^2$ $= 800 \text{ g/m}^2$	Convert the mass from tonnes to grams (1 t = (1000 × 1000) g). Convert the area from hectares to square metres (1 ha = 10 000 m <sup>2</sup> ). Divide the mass by the area.	

### EXAMPLE 2A-3 Converting each quantity in a rate to smaller units

Convert 72 L/h to mL/s.

Solve	Think	Apply
$72 \text{ L/h} = \frac{72 \text{ L}}{1 \text{ h}}$ $= \frac{(72 \times 1000) \text{ mL}}{(60 \times 60) \text{ s}}$ $= (72\,000 \div 3600) \text{ mL/s}$ $= 20 \text{ mL/s}$	Convert the capacity from litres to millilitres (1 L = 1000 mL). Convert the time from hours to seconds (1 h = (60 × 60) s). Divide the capacity by the time.	Convert the given units to the required units and divide in the order given.

**3** Complete each of the following to convert:

**a** 5 t/ha to kg/ha

$$\frac{5 \text{ t}}{1 \text{ ha}} = \frac{\square \text{ kg}}{1 \text{ ha}}$$

$$= \underline{\hspace{2cm}} \text{ kg/ha}$$

**c** \$3.75/h to cents/min

$$\frac{\$3.75}{1 \text{ h}} = \frac{\square \text{ cents}}{\square \text{ min}}$$

$$= \underline{\hspace{2cm}} \text{ c/min}$$

**e** 15%/year to %/month

$$\frac{15\%}{1 \text{ year}} = \frac{15\%}{\square \text{ months}}$$

$$= \underline{\hspace{2cm}} \text{ %/month}$$

**b** 16 t/ha to kg/m<sup>2</sup>

$$\frac{16 \text{ t}}{1 \text{ ha}} = \frac{\square \text{ kg}}{\square \text{ m}^2}$$

$$= \underline{\hspace{2cm}} \text{ kg/m}^2$$

**d** \$1.80/m to cents/mm

$$\frac{\$1.80}{1 \text{ m}} = \frac{\square \text{ cents}}{\square \text{ mm}}$$

$$= \underline{\hspace{2cm}} \text{ c/mm}$$

**f** 2.4 kg/L to g/mL

$$\frac{2.4 \text{ kg}}{1 \text{ L}} = \frac{\square \text{ g}}{\square \text{ mL}}$$

$$= \underline{\hspace{2cm}} \text{ g/mL}$$

**4** Convert the following.

**a** 45 L/h to mL/s

**d** 27 L/h to mL/s

**b** \$12/kg to cents/g

**e** 7.2 kg/day to g/min

**c** 18 km/h to m/s

**f** 14 km/h to cm/s

### EXAMPLE 2A-4 Converting each quantity in a rate to larger units

Convert 5 m/s to km/h.

Solve	Think	Apply
$5 \text{ m/s} = \frac{5 \text{ m}}{1 \text{ s}}$ $= \frac{(5 \times 60 \times 60) \text{ m}}{1 \text{ h}}$ $= \frac{18\,000 \text{ m}}{1 \text{ h}}$ $= \frac{(18\,000 \div 1000) \text{ km}}{1 \text{ h}}$ $= \frac{18 \text{ km}}{1 \text{ h}}$ $= 18 \text{ km/h}$	<p>First find the number of metres travelled in 1 hour by multiplying the distance by the number of seconds in an hour.</p> $5 \text{ m in } 1 \text{ s} = (5 \times 60) \text{ m in } 1 \text{ min}$ $= (5 \times 60 \times 60) \text{ m in } 1 \text{ h}$ $= 18\,000 \text{ m in } 1 \text{ h}$ <p>Then convert the distance from metres to kilometres.</p> $18\,000 \text{ m in } 1 \text{ h}$ $= (18\,000 \div 1000) \text{ km in } 1 \text{ h}$	<p>Convert the speed from m/s to m/h by multiplying the distance by the number of seconds in an hour. Then convert the rate from m/h to km/h by dividing the distance by the number of metres in a kilometre.</p>

5 Complete each of the following to convert:

a 6 m/s to km/h

The number of seconds in 1 hour

$$= \underline{\quad} \times \underline{\quad}$$

$$= \underline{\quad}$$

$$\frac{6 \text{ m}}{1 \text{ s}} = \frac{(6 \times \square) \text{ m}}{1 \text{ h}}$$

$$= \frac{\square \text{ m}}{1 \text{ h}}$$

The number of metres in a kilometre =  $\underline{\quad}$

$$\frac{\square \text{ m}}{1 \text{ h}} = \frac{(\square \div \square) \text{ km}}{1 \text{ h}}$$

$$= \frac{\square \text{ km}}{1 \text{ h}}$$

So 6 m/s =  $\underline{\quad}$  km/h

b 3 cents/minute to \$/day

The number of minutes in 1 day

$$= 24 \times \underline{\quad}$$

$$= \underline{\quad}$$

$$\frac{3 \text{ cents}}{1 \text{ minute}} = \frac{(3 \times \square) \text{ cents}}{1 \text{ day}}$$

$$= \frac{\square \text{ cents}}{1 \text{ day}}$$

The number of cents in \$1 =  $\underline{\quad}$

So 3 cents/minute

=  $\underline{\quad}$  \$/day or \$ $\underline{\quad}$ /day

- |     |                                    |         |          |
|-----|------------------------------------|---------|----------|
| 6 a | Convert 20 m/s to:                 | i m/h   | ii km/h  |
| b   | Convert 5 g/mL to:                 | i g/L   | ii kg/L  |
| c   | Convert 0.8c/g to:                 | i c/kg  | ii \$/kg |
| d   | Convert 0.75 kg/m <sup>2</sup> to: | i kg/ha | ii t/ha  |
| e   | Convert 0.4 mL/s to:               | i mL/h  | ii L/h   |
| f   | Convert 0.8c/m to:                 | i c/km  | ii \$/km |

Rates can be used to make comparisons. For example, when comparing two or more purchase options to find the one that is the best value or the **best buy**, we calculate the unit price for each option.



**best buy**

found by comparing a number of purchase options to find the one that is the best value for money

**EXAMPLE 2A-5** Using unit prices to make comparisons

A box of 10 pens costs \$6.80. These pens can also be bought in a pack of 15 for \$9.60.

Which option provides better value or is the best buy?

Solve	Think	Apply
<p><i>Option 1:</i>                      Rate = \$6.80 for 10 pens                      = <math>(\\$6.80 \div 10)</math> for 1 pen                      = \$0.68/pen</p> <p><i>Option 2:</i>                      Rate = \$9.60 for 15 pens                      = <math>(\\$9.60 \div 15)</math> for 1 pen                      = \$0.64/pen</p> <p>The best buy is the pack of 15 pens.</p>	<p>For each option, the rate is comparing the cost with the number of pens. Divide the cost by the number of pens to find the unit cost in each case.</p> <p>The option with the lower cost per pen is the best buy.</p>	<p>Compare the unit price for each option; that is, find the rate for cost per one pen (cost/pen).</p>

- 7** At the supermarket, a carton of 12 eggs costs \$5.80. Eggs can also be bought at the Sunday market in a carton of 18 for \$8.95. Which option provides better value or is the best buy?
- 8** Which bottle of drink is the better value for money: the 2 L bottle of orange mineral water for \$3.50 or the 1.5 L bottle of cola for \$2.75?
- 9** Tyler bought a 400 g pack of tomatoes for \$3.60. Lisa bought 600 g of loose tomatoes for \$5.60.
- Calculate the amount paid by Tyler and Lisa for 100 g of tomatoes.
  - Who made the best buy? Explain.
  - Why does a store offer both options?
- 10** Which is the best buy: 700 g of mince steak for \$10.80 from the butcher or 1.2 kg of mince steak for \$16.50 from the supermarket?
- 11** A car travels at 60 km/h.
- How far will it travel in  $2\frac{1}{2}$  hours?
  - How long will it take to travel 225 km?
- 12** Fertiliser is to be spread at the rate of  $0.2 \text{ kg/m}^2$ .
- How much fertiliser would be needed for an area of  $600 \text{ m}^2$ ?
  - If the fertiliser is sold in 50 kg bags, how many bags are needed for an area of  $600 \text{ m}^2$ ?
  - What area could be fertilised with 1 t of fertiliser?
- 13** A patient in hospital is given an antibiotic solution intravenously at the rate of 80 mL/h.
- How much antibiotic solution will the patient receive in  $6\frac{1}{2}$  h?
  - How often would 600 mL containers of the antibiotic solution need to be changed?
  - If 1 mL of this solution contains 15 drops, calculate the rate at which the patient receives the antibiotic in drops/minute.



- 14** Anna is paid at the rate of \$23.60/h.
- How much would she be paid for working 15 h?
  - How long would she need to work to earn \$472?
  - Anna wants to save for a trip. If she works 28 h in week 1, and 31 h in the next week, how many more hours does she need to work to earn \$2000?
- 15** Tony charges a call-out fee of \$150 and \$68 per hour to service and repair an air-conditioning system. How much will the client be charged if it took 2.5 hours for Tony to complete the job?
- 16** Sonia charges \$26 per hour plus the cost of materials to paint rooms in a house. The painting job took 19 hours and she charged the homeowner \$790. What was the cost of the materials?
- 17** Compare the rates in each listed pair. Which one of the two rates is larger? Use calculations to justify your answer in each case.
- |                               |                              |
|-------------------------------|------------------------------|
| <b>a</b> 60 L/h or 900 mL/min | <b>b</b> 40 km/h or 15 m/s   |
| <b>c</b> 125 g/mL or 130 kg/L | <b>d</b> 18 km/h or 180 cm/s |
- 18** A cheetah can run at a speed of 93 km/h. Is this faster or slower than the top speed of 12.5 m/s recorded for Usain Bolt? Justify your answer.
- 19 a** While resting, Peter counted 25 heart beats in 20 seconds. Write his heart rate in beats/minute (or bpm).
- b** While exercising, Peter counted 20 heart beats in 10 seconds. Write his heart rate in beats/minute.
- c** Ten minutes after completing his exercise, Peter counted 20 heart beats in 15 seconds. Is his heart rate higher or lower than before he started exercising? Use calculations to justify your answer.
- 20** An old washing machine uses 115 L of water per load. A new washing machine uses 50 L per load. How much water is saved each year if two loads of washing are completed each week using the new machine?



- 21** The conversion rate for Australian dollars (A\$) to American dollars (US\$) is 0.95 A\$/US\$ (US\$1 = A\$0.95).
- How many US dollars would I receive for A\$2750?
  - How many Australian dollars would I receive for US\$1300?
  - Convert US\$1800 into A\$.
- 22** The average distance of the Earth from the Sun is  $1.49 \times 10^8$  km. Assume the Earth travels in a circular orbit around the Sun.
- Calculate (to three significant figures) the distance travelled by the Earth in one complete orbit. (*Hint: use  $C = 2\pi r$ .*)
  - It takes the Earth 1 year (365.25 days) to travel this distance. Using the answer from part **a**, find the average speed at which the Earth travels through space in:
    - km/h
    - km/s.

# 2B Distance-time graphs and speed

These resources are available on your obook assess:

- **Interactive 2B:** Explore distance-time graphs and calculate speed
- **Video tutorial 2B:** Watch and listen to an explanation of Example 2B-1
- **assess quiz 2B:** Test your skills with an auto-correcting multiple-choice quiz

01  
10

## distance-time graph

graph that shows the relationship between distance and time where time is shown on the horizontal axis and distance is shown on the vertical axis

We have seen that speed is a rate comparing distance and time.

When the speed is constant (stays the same), the distance travelled increases by the same amount in each equal time interval. For example, a constant speed of 60 km/h means that the distance increases by 60 km in each time interval of 1 hour.

When the speed is not constant, we can calculate the average speed over a time interval.

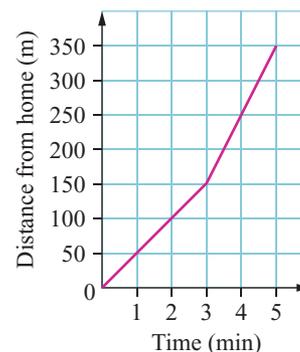
That is:  $\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$

A **distance-time graph** (also called a travel graph) shows the relationship between distance and time. We can use this graph to find speed, distance and time.

## EXERCISE 2B Distance-time graphs and speed

UNDERSTANDING, FLUENCY AND COMMUNICATING

- Consider the distance-time graph shown on the right which represents Liam's journey from his home.
  - How long did it take for Liam to be 100 m from home?
  - How long did it take for Liam to be 250 m from home?
  - How far had Liam travelled after:
    - 1 min?
    - 2 min?
    - 3 min?
  - What distance did Liam travel in the time interval from:
    - 0 min to 1 min?
    - 1 min to 2 min?
    - 2 min to 3 min?
  - At what speed did Liam travel in the time interval from:
    - 0 min to 1 min?
    - 1 min to 2 min?
    - 2 min to 3 min?
  - Liam travelled at a constant speed in the first 3 min. How is this represented in the distance-time graph? (*Hint*: what is the shape of the graph in the first 3 min?)
  - What distance did Liam travel in the time interval from:
    - 3 min to 4 min?
    - 4 min to 5 min?
  - At what speed did Liam travel in the time interval from:
    - 3 min to 4 min?
    - 4 min to 5 min?
  - Liam's speed between 3 min and 5 min was faster than his speed in the first 3 min. How is this represented in the distance-time graph?
- Use your answers from question 1 to complete this statement: The steeper the slope of a line in a distance-time graph, the \_\_\_\_\_ (higher/lower) the speed.



- 3** A train travelled at a constant speed of 80 km/h for 5 hours.
- a** Copy and complete this table of values for distance and time.

<b>Time (h)</b>	0	1	2	3	4	5
<b>Distance (km)</b>	0	80	160			

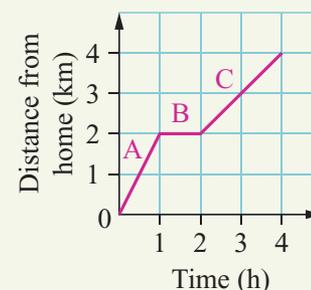
- b** Use the table of values to draw a distance-time graph for this journey. Show time along the horizontal axis and distance along the vertical axis.
- c** Find the gradient of the distance-time graph.
- d** Explain why the gradient of the graph is the same as the speed of the train.
- 4** A cyclist travelled at a constant speed of 50 km/h for 4 hours.
- a** Copy and complete this table of values for distance and time.

<b>Time (h)</b>	0	1	2	3	4
<b>Distance (km)</b>	0	50	100		

- b** Use the table of values to draw a distance-time graph for this journey.
- c** Find the gradient of the distance-time graph and compare it to the speed of the cyclist.

### EXAMPLE 2B-1 Interpreting a distance-time graph

This distance-time graph represents Lucy's journey from her home to the music store after stopping to visit her friend along the way.

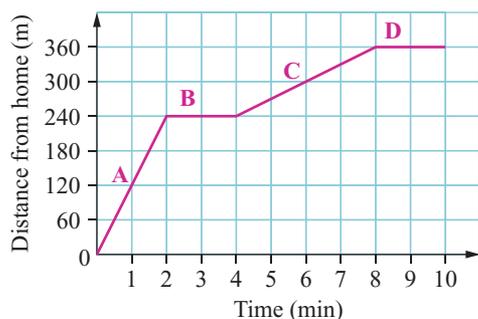


- a** Match the letters for each section of the graph (A, B and C) with the appropriate part of the description provided.
- b** How far from home was Lucy after each of these times?
- i** 0 hours      **ii** 1 hour      **iii** 2 hours
- iv** 3 hours      **v** 4 hours
- c** When did Lucy reach her friend's house?
- d** How long did she stay at her friend's house?
- e** How far did Lucy travel from her friend's house to the music store?
- f** In which section of the graph (A, B or C) did Lucy walk the fastest?
- g** What does the horizontal section on the graph indicate?

	<b>Solve</b>	<b>Think</b>	<b>Apply</b>
<b>a</b>	Section A: Lucy travels from home to her friend's house. Section B: Lucy visits her friend. Section C: Lucy travels from her friend's house to the music store.	Consider the order of the separate travel events in Lucy's journey.	Match the three sections of the graph with the description of Lucy's journey.
<b>b</b>	<b>i</b> After 0 hours, Lucy was 0 km from home. <b>ii</b> After 1 hour, Lucy was 2 km from home. <b>iii</b> After 2 hours, Lucy was 2 km from home. <b>iv</b> After 3 hours, Lucy was 3 km from home. <b>v</b> After 4 hours, Lucy was 4 km from home.	For each time value, read the corresponding distance from the graph.	Read the distance values from the graph for given time values.
<b>c</b>	Lucy reached her friend's house after 1 hour.	Section A represents Lucy's journey to her friend's house.	Read the time at the end of the time interval for section A.

	Solve	Think	Apply
<b>d</b>	Lucy stayed for one hour.	Section B represents the time Lucy spent at her friend's house.	Work out the time interval in the section where Lucy was not moving; that is, she was at her friend's house.
<b>e</b>	Lucy travelled 2 km.	Lucy travelled from her friend's house which is 2 km from her home to the music store which is 4 km from her home. This is a difference of 2 km.	Find the difference in the distance from home at the start and end of the appropriate section of the graph.
<b>f</b>	Lucy walked the fastest in section A.	Consider the slope of each section. The section with the steepest slope represents the fastest speed.	The steeper the line, the faster the speed.
<b>g</b>	Lucy was stationary, her distance from home had not changed.	There was no change in Lucy's distance from home so she was not moving.	Horizontal sections in a distance-time graph indicate that the distance has not changed and so the person is stationary.

**5** The distance-time graph below represents the first 10 minutes of Harry's journey from his home to the cinema after stopping to collect his friend along the way.



- Match the letters for each section of the graph (A, B, C and D) with the appropriate part of the description provided.
- How far from home was Harry after each of these times?
  - 0 min
  - 2 min
  - 4 min
  - 8 min
- When did Harry arrive at his friend's house?
- How long did he stay at his friend's house?
- How far did Harry travel from his friend's house to the cinema?
- In which section of the graph (A, B, C or D) did Harry walk the fastest?
- In which section/s of the graph was Harry stationary?



**EXAMPLE 2B-2** Calculating speed from a distance-time graph

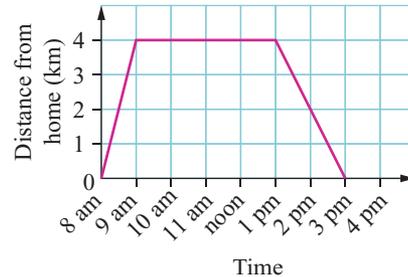
Consider the distance-time graph shown in Example 2B-1.

- a** Calculate Lucy's speed in:  
**i** section A                      **ii** section B                      **iii** section C.  
**b** What is the total distance travelled by Lucy in her 4-hour journey?  
**c** Calculate Lucy's average speed for her journey.

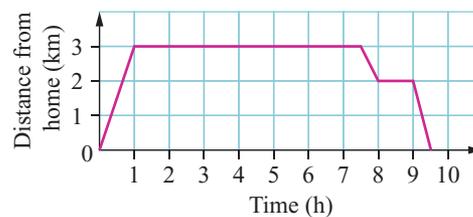
	Solve	Think	Apply
<b>a i</b>	In section A, Lucy walked 2 km in 1 hour. Speed = $\frac{2 \text{ km}}{1 \text{ h}}$ = 2 km/h or Gradient = $\frac{(2 - 0) \text{ km}}{(1 - 0) \text{ h}}$ = $\frac{2 \text{ km}}{1 \text{ h}}$ = 2 km/h	Calculate the speed by dividing the distance travelled in each section by the time taken. Read the distance and time values from the graph. Alternatively, calculate the gradient of the line for each section.	The gradient of a distance-time graph indicates the speed. The steeper the line, the faster the speed. A horizontal line has zero gradient and hence represents zero speed.
	<b>ii</b> In section B, Lucy walked 0 km in 1 hour. Speed = $\frac{0 \text{ km}}{1 \text{ h}}$ = 0 km/h or Gradient = $\frac{(2 - 2) \text{ km}}{(2 - 1) \text{ h}}$ = $\frac{0 \text{ km}}{1 \text{ h}}$ = 0 km/h		
	<b>iii</b> In section C, Lucy walked 2 km in 2 hours. Speed = $\frac{2 \text{ km}}{2 \text{ h}}$ = 1 km/h or Gradient = $\frac{(4 - 2) \text{ km}}{(4 - 2) \text{ h}}$ = $\frac{2 \text{ km}}{2 \text{ h}}$ = 1 km/h		
<b>b</b>	Lucy travelled a total distance of 4 km.	Lucy travelled away from home for the entire journey, starting at 0 km from home and finishing at 4 km from home.	Use the graph to find the total distance travelled.
<b>c</b>	Average speed = $\frac{4 \text{ km}}{4 \text{ h}}$ = 1 km/h	Lucy travelled at different speeds during her journey. Find the total distance travelled (4 km) and divide by the time taken (4 h).	Average speed is found by dividing the total distance travelled by the time taken.

- 6** Consider the distance-time graph in question 5.
- a** Calculate Harry's speed in:
    - i** section A
    - ii** section B
    - iii** section C
    - iv** section D.
  - b** What is the total distance travelled by Harry in his 10-minute journey?
  - c** Calculate Harry's average speed for his journey.

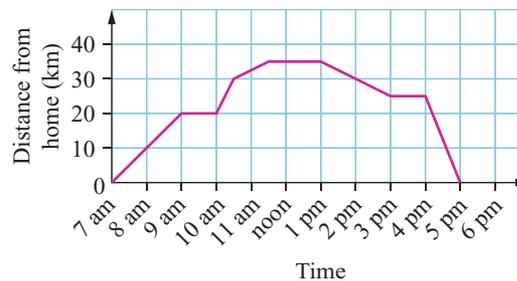
- 7** This travel graph represents Sam's journey to and from school.
- a** What is the distance between Sam's home and school?
  - b** How long was Sam at school?
  - c** Did he walk faster from home to school or from school to home? Explain your answer.
  - d** How far was Sam from home at 2 pm?
  - e** What was the total distance he walked to and from school?



- 8** Nadia walked from home to school. On her way home from school, she stopped at a sports centre for netball practice. This distance-time graph represents her journey.
- a** How far did Nadia walk from home to school?
  - b** When did she leave school to walk to the sports centre?
  - c** How long was her netball practice session?
  - d** In which time interval did Nadia travel the fastest? What was her speed?
  - e** In which time interval did Nadia travel at a constant speed of 2 km/h?
  - f** What time did Nadia arrive home if she left home at 7:45 am?
  - g** Interpret the graph by describing Nadia's journey.

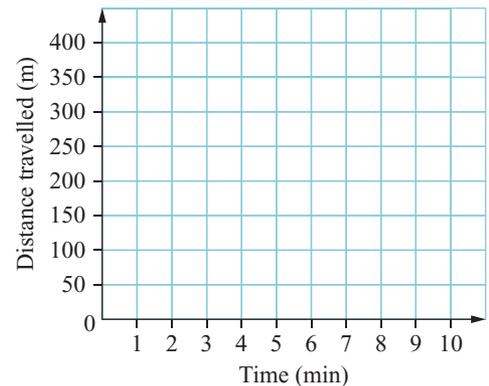


- 9** Leroy went for a bike ride. He left home at 7 am and arrived back at home at 5 pm. This graph represents his journey.
- a** How far did Leroy travel in the first two hours of this journey?
  - b** When did Leroy take his first rest break?
  - c** How far from home was he at noon?
  - d** At what time did he start to return home?
  - e** How much time did Leroy spend riding his bike on this journey?
  - f** In which time interval was Leroy travelling the fastest? What was his speed?
  - g** In which time interval was Leroy travelling at a constant speed of 20 km/h?
  - h** How fast was Leroy travelling at:
    - i** 8 am?
    - ii** 11 am?
    - iii** 2 pm?
  - i** What distance had Leroy travelled by 3 pm?
  - j** What was the total distance that Leroy rode on this journey?
  - k** What was his average speed for the entire journey?



**10** Matteo leaves home and walks at a constant speed for 100 m to the local bakery. This takes him 2 minutes. He spends another 2 minutes buying a loaf of bread, then leaves the bakery and walks at a constant speed for 4 minutes to his friend's house which is 120 m from the bakery. His friend is not there so he jogs at a constant speed for 2 minutes to his cousin's house which is 180 m away.

- a** Copy the set of axes shown on the right and draw a graph representing Matteo's journey. (*Note:* this graph is to show the distance travelled by Matteo and not Matteo's distance from home.)
- b** In which time interval did Matteo travel the fastest? How can you tell?
- c** When was Matteo stationary?
- d** Calculate Matteo's speed:
  - i** in the first two minutes
  - ii** in the last two minutes
  - iii** in the time interval from 4 to 8 minutes.
- e** Calculate Matteo's average speed for the entire journey.

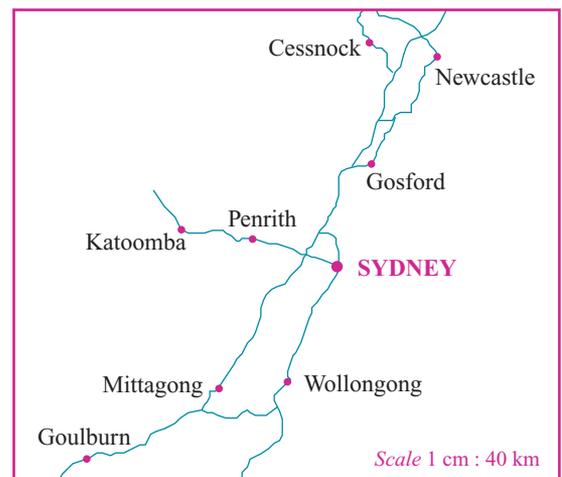


**11** Jenna cycles from home to the swimming pool at a constant speed of 4 km/h. It takes her 30 minutes. She remains at the pool for 4 hours and then returns home at a constant speed of 3 km/h.

- a** How far away is the swimming pool from Jenna's home?
- b** How long does it take Jenna to cycle home from the pool?
- c** Construct a distance-time graph representing Jenna's journey.
- d** If Jenna left home at 10:45 am, what time did she return back home?

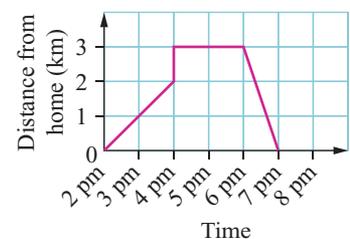
**12** The map on the right shows some routes out of Sydney. The scale indicates that a distance of 1 cm on the map represents an actual distance of 40 km.

- a** What is the actual distance (in a straight line) from Sydney to Katoomba?
- b** What is the actual distance from Goulburn to Gosford?
- c** Which town is about 90 km from Sydney?
- d** Which towns are about 40 km apart?
- e** Tina travels by car from Goulburn to Mittagong. The journey takes her 1 hour and 15 minutes. Using the straight-line distance between Goulburn and Mittagong as an estimate of the distance Tina travels, calculate her average speed for the journey.
- f** Estimate how long it would take Tina to drive from Mittagong to Gosford if her average speed is 72 km/h. Use the straight-line distance as an estimate of the distance Tina travels.



**13** Does this travel graph represent a journey that could really take place? Explain your answer.

**14** What assumption has been made in the way the distance-time graphs shown in this topic have been drawn? (*Hint:* can you realistically change from one constant speed to another without accelerating or decelerating?) Redraw one or more of the distance-time graphs shown in this exercise in a more realistic way.





	Solve	Think
<b>b</b>	Fuel consumption rate $= \frac{49 \text{ L}}{500 \text{ km}}$ $= 0.098 \text{ L/km}$	This rate is comparing the amount of petrol consumed with the distance travelled, in that order. Rate = 49 L for 500 km $= (49 \div 500) \text{ L for 1 km}$ $= 0.098 \text{ L/km}$ The car uses 0.098 L of petrol for every kilometre.
<b>c</b>	Fuel consumption rate $= \frac{49 \text{ L}}{500 \text{ km}}$ $= 0.098 \text{ L/km}$ $= (0.098 \times 100) \text{ L/100 km}$ $= 9.8 \text{ L/100 km}$ Or: Fuel consumption rate $= \frac{49 \text{ L}}{500 \text{ km}}$ $= \frac{(49 \div 5) \text{ L}}{100 \text{ km}}$ $= 9.8 \text{ L/100 km}$	As the rate in L/km is often small, fuel consumption is usually quoted as the amount of petrol used per 100 km. Rate = 0.098 L for 1 km $= (0.098 \times 100) \text{ L for 100 km}$ $= 9.8 \text{ L/100 km}$ Alternatively: Rate = 49 L for 500 km $= (49 \div 5) \text{ L for 100 km}$ $= 9.8 \text{ L/100 km}$
<b>Apply</b>		
Determine the order in which the two quantities are being compared and divide one by the other, in that order. Remember to write the appropriate units.		

## EXERCISE 2C Fuel consumption rate

- A car used 24 L of petrol on a trip of 280 km.
  - Calculate the number of kilometres the car travels per litre of petrol consumed.
  - Calculate the number of litres of petrol the car consumes per kilometre of travel.
  - Calculate the number of litres of petrol the car consumes per 100 km of travel.
- Calculate the fuel consumption rate for each trip in:
  - km/L
  - L/km
  - L/100 km
  - A car travels 450 km on 40 L of petrol.
  - A car travels 220 km on 24 L of petrol.
  - A motorcycle travels 192 km on 11 L of petrol.
- Calculate the fuel consumption rate in L/100 km for each trip.
  - A truck used 114 L of petrol on a trip of 600 km.
  - A car used 13.8 L of petrol on a trip of 120 km.
  - A motorcycle used 22 L of petrol on a trip of 345 km.

When the rate of fuel consumption is expressed in L/100 km, we can use the following formula to find the distance travelled in kilometres.

$$\text{Distance travelled (km)} = \frac{\text{amount of fuel (L)}}{\text{fuel consumption (L/100 km)}} \times 100$$



**EXAMPLE 2C-2** Calculating distance given the fuel consumption rate and fuel amount

How far can a Holden Astra travel on 42 L of petrol if its fuel consumption is 5.8 L/100 km?

Solve	Think	Apply
$\text{Distance} = \frac{42}{5.8} \times 100$ $\approx 724 \text{ km}$	The number of 'lots of 5.8 L' used is $\frac{42}{5.8}$ . The car travels 100 km for each 'lot of 5.8 L' used, so multiply the fraction $\frac{42}{5.8}$ by 100.	$\text{Distance travelled (km)} = \frac{\text{amount of fuel}}{\text{fuel consumption}} \times 100$ Fuel consumption is in L/100 km.

- 4 Complete the following to calculate how far a motorcycle can travel on 10 L of petrol if the fuel consumption is 5.6 L/100 km.

$$\text{Distance} = \frac{10}{\square} \times 100 \approx \text{ \_\_\_\_ km}$$

- 5 How far can a vehicle travel on:
- 30 L of petrol if its fuel consumption is 8.6 L/100 km?
  - 52 L of petrol if its fuel consumption is 9.3 L/100 km?
  - 98 L of petrol if its fuel consumption is 12.4 L/100 km?

When the rate of fuel consumption is expressed in L/100 km, we can use the following formula to find the amount of fuel used in litres.

$$\text{Amount of fuel used (L)} = \frac{\text{distance travelled (km)}}{100} \times \text{fuel consumption (L/100 km)}$$


**EXAMPLE 2C-3** Calculating the amount of fuel needed to travel a given distance

Calculate the amount of petrol used (to the nearest litre) by a Toyota Corolla on a trip of 480 km if its fuel consumption is 6.6 L/100 km.

Solve	Think	Apply
$\text{Amount of petrol used} = \frac{480}{100} \times 6.6$ $= 31.68$ $\approx 32 \text{ L}$	The number of 'lots of 100 km' travelled is $\frac{480}{100}$ . Each 'lot of 100 km' uses 6.6 L of petrol, so multiply the fraction $\frac{480}{100}$ by 6.6.	$\text{Amount of fuel used (L)} = \frac{\text{distance travelled}}{100} \times \text{fuel consumption}$ Fuel consumption is in L/100 km and distance travelled is in kilometres.

- 6 Complete the following to calculate the amount of fuel used (to the nearest litre) by a vehicle on a trip of 965 km, if the fuel consumption is 10.2 L/100 km.

$$\text{Amount of fuel used} = \frac{\square}{100} \times \square = \text{ \_\_\_\_ } \approx \text{ \_\_\_\_ L}$$

- 7 Calculate the amount of fuel used (to the nearest litre) by a vehicle on a trip of:
- 425 km, if the fuel consumption is 8.8 L/100 km
  - 670 km, if the fuel consumption is 11.2 L/100 km
  - 315 km, if the fuel consumption is 7.6 L/100 km.

**EXAMPLE 2C-4** Solving problems related to fuel consumption rate

A car uses 22 L of petrol on a trip of 250 km.

- a** Express the fuel consumption in L/100 km.
- b** Assuming the same rate of fuel consumption:
- i** how much fuel would the car use for a trip of 325 km?
- ii** how far could the car travel on a full tank of 50 L?
- c** Assuming the cost of petrol was 142.9c/L, what is the difference in cost of the fuel used for each of the two trips described in part **b**?

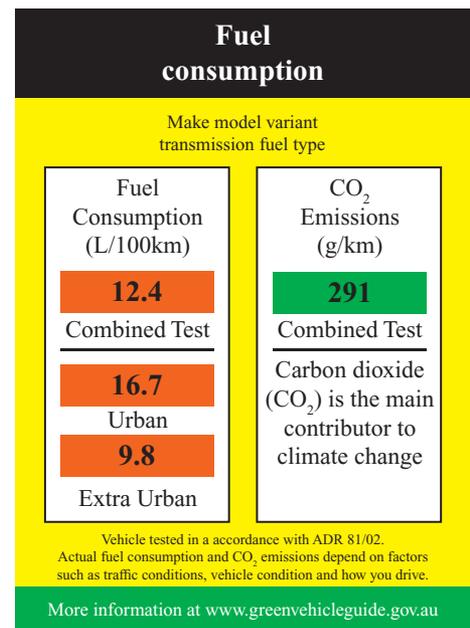
	Solve	Think	Apply
<b>a</b>	Fuel consumption rate $= \frac{22 \text{ L}}{250 \text{ km}}$ $= \frac{(22 \div 250) \text{ L}}{(250 \div 250) \text{ km}}$ $= \frac{0.088 \text{ L}}{1 \text{ km}}$ $= \frac{(0.088 \times 100) \text{ L}}{(1 \times 100) \text{ km}}$ $= \frac{8.8 \text{ L}}{100 \text{ km}}$ $= 8.8 \text{ L}/100 \text{ km}$	Write a fraction to compare the number of litres of fuel to the number of kilometres travelled. Multiply or divide the top and the bottom of the fraction by the same number until you obtain 100 km in the denominator.	One way to find fuel consumption rate in L/100 km is to first find the rate in L/km and then multiply the top and the bottom of the fraction by 100.
<b>b i</b>	Amount of petrol used $= \frac{325}{100} \times 8.8$ $= 3.25 \times 8.8$ $= 28.6 \text{ L}$	The number of 'lots of 100 km' travelled is $\frac{325}{100}$ ; that is, 3.25 lots of 100 km. Each 'lot of 100 km' uses 8.8 L of petrol, so multiply $\frac{325}{100}$ by 8.8.	Since fuel consumption rate is the number of litres used per 100 km, find the number of 'lots of 100 km' travelled and multiply by the fuel consumption rate.
<b>ii</b>	Distance $= \frac{50}{8.8} \times 100$ $= 568.181\dots$ $\approx 568.2 \text{ km}$	The car travels 100 km for each 8.8 L of petrol used. The number of 'lots of 8.8 L' used is $\frac{50}{8.8}$ , so multiply $\frac{50}{8.8}$ by 100.	Since fuel consumption rate is the number of litres used per 100 km, find the number of 'lots of 8.8 L' used and multiply by 100.
<b>c</b>	Cost of fuel for Trip 1 $= 142.9 \times 28.6$ $= 4086.94 \text{ cents}$ $= \$40.87$ Cost of fuel for Trip 2 $= 142.9 \times 50$ $= 7145 \text{ cents}$ $= \$71.45$ Difference in cost $= \$30.58$ Trip 2 costs \$30.58 more in fuel costs.	Cost of petrol is 142.9 cents or \$1.429 for each litre of fuel used, so multiply 142.9 cents or \$1.429 by the number of litres used. For Trip 1, use the number of litres calculated in part <b>b i</b> ; that is, 28.6 L. For Trip 2, use the number of litres given in part <b>b ii</b> ; that is, 50 L. Find the difference in fuel costs.	For each trip, multiply the cost per litre by the number of litres of fuel used. Think about whether the cost of the fuel in your calculation is in cents or dollars.

- 8** A car uses 30 L of petrol on a trip of 400 km.
- Express the fuel consumption in L/100 km.
  - Assuming the same rate of fuel consumption:
    - how much fuel would the car use for a trip of 500 km?
    - how far could the car travel on a full tank of 45 L?
  - Assuming the cost of petrol was 142.9c/L, what is the difference in cost of the fuel used for each of the two trips described in part **b**?
- 9** A courier averages 2800 km of city driving each month in a Ford Fiesta that has a fuel consumption of 6.8 L/100 km (city cycle). Calculate the cost of petrol used in a month in which the average price of unleaded petrol (ULP) is 139.9c/L.
- 10** In 2016 the average fuel consumption of Australian passenger vehicles was 10.6 L/100 km and the average yearly distance travelled was 13 716 km. If the average price of fuel was 135.9c/L, what was the average yearly fuel cost?
- 11** The table on the right shows the average fuel consumption rates for vehicles with engines containing a different number of cylinders.
- | Number of cylinders | Fuel consumption rate |
|---------------------|-----------------------|
| 1–3                 | 7.7 L/100 km          |
| 4                   | 10.0 L/100 km         |
| 6                   | 11.7 L/100 km         |
| 8 or more           | 12.9 L/100 km         |
- If the average price of petrol is 139.9c/L and the annual yearly distance travelled is 10 000 km, calculate the average yearly fuel cost for each type of vehicle.
  - What is the annual fuel cost saving of driving a 4-cylinder car compared to a 6-cylinder car?
  - An articulated truck has a fuel consumption of 56.3 L/100 km. Compare the annual fuel cost of the truck and an 8-cylinder car if the cost of diesel for the truck is 137.9c/L. Assume each vehicle is driven 10 000 km in a year.
- 12** A Mazda MX-5 uses 7.5 L/100 km of ULP and the diesel model of the same car uses 6.0 L/100 km of diesel fuel.
- Calculate the cost of driving the petrol model 800 km if ULP is 142.9c/L.
  - Calculate the cost of driving the diesel model 800 km if diesel fuel is 152.2c/L.
  - How much cheaper is the diesel option over this distance?
- 13** Eric's car uses fuel at the rate of 6.2 L/100 km for country driving and 7.8 L/100 km for city driving. On a trip, he drives 145 km in the country and 25 km in the city.
- Calculate the amount of fuel he used on this trip.
  - What is the average fuel consumption rate for this trip?
- 14** Liesel is comparing the costs of two different ways of travelling to her workplace. Her car uses one litre of fuel for every 10 km travelled. The cost of fuel is \$1.49/L and the distance from her home to her workplace is 38 km. The cost of travelling by bus is \$42.50 for 10 single trips. Which way of travelling is cheaper and by how much? Support your answer with calculations.



- 15** Tim owns a Holden Commodore that runs on ULP and has a fuel consumption of 8.9 L/100 km. When converted to run on liquid petroleum gas (LPG), the car's fuel consumption is 12.3 L/100 km. Tim averages 18 000 km per year.
- Calculate the annual cost for each type of fuel (assuming that the car only runs on one type of fuel for a year) if the average price of ULP is 139.9c/L and LPG is 71.9c/L.
  - How much does Tim save in fuel costs for the year by converting his car to LPG?
  - How much does he save per month?
  - The cost of converting the car to LPG is \$2600. How many months would Tim take to break even?
  - What distance would Tim travel before reaching the break-even point? (The break-even point is the point at which the cost of running the vehicle on each type of fuel is the same.)
- 16** Jasmine travels an average of 14 000 km per year and wants to buy a new car. The car she likes is available with a petrol motor or diesel motor. The petrol model has a fuel consumption of 10.8 L/100 km and the diesel model has a fuel consumption of 7.8 L/100 km.
- If Jasmine bought the car with the petrol engine, what would be her annual fuel cost if ULP is 139.9c/L?
    - If Jasmine bought the car with a diesel engine, what would be her annual fuel cost if diesel fuel is 137.2c/L?
    - How much per year would she save on fuel by buying the car with the diesel engine?
  - What is the average monthly saving with the diesel engine?
  - The diesel car costs \$1200 more to buy than the petrol car. How many months would it take for Jasmine to break even, if she buys the diesel car?
  - What distance would Jasmine travel before reaching her break-even point?

- 17** All new light vehicles sold in Australia are required to display a Fuel Consumption label on the front windscreen. The label indicates the vehicle's fuel consumption in litres of fuel per 100 kilometres (L/100 km) and its emissions of carbon dioxide in grams per kilometre (g/km). The label is designed to help Australian motorists make informed choices about the environmental impact of their new car and the cost of running their vehicle.
- Go to the Green Vehicle Guide website to find out why there are three fuel consumption rates given. A link is provided on your [obook assess](#). (*Hint*: locate 'Fuel consumption label' in the menu.)
  - Use the label shown to calculate the amount of fuel used to travel 200 km in the Combined test.
  - Calculate the difference in the cost of the fuel used to travel 200 km in Urban conditions compared to Extra Urban conditions when the cost of fuel is \$1.42/L.



- 18** Choose at least five different models of new vehicles and compare their fuel consumption rates and carbon dioxide emission rates. One website that you may find useful is Green Vehicle Guide. A link is provided on your [obook assess](#). Write a report that also includes a comparison of fuel consumption rates for different engine sizes and types of fuel used.

# 2D Heart rate

These resources are available on your [obook assess](#):

- **assess quiz 2D:** Test your skills with an auto-correcting multiple-choice quiz



## heart rate

the number of contractions of the heart per minute measured as beats per minute (bpm)

**Heart rate** is measured by the number of heart beats per minute (bpm). When awake, but relaxed, an 18-year-old male in a good state of health should have a resting heart rate of between 62 and 65 bpm, and an 18-year-old female in a good state of health should have a resting heart rate of between 66 and 69 bpm.

The maximum heart rate, which is the highest heart rate an individual can achieve without severe problems through exercise stress, depends on age. For both an 18-year-old female and for an 18-year-old male, maximum heart rate is about 200 bpm. However, these values can vary for individuals.

The table below shows a health rating for males in a number of age groups based on their resting heart rate (bpm).

Health rating	Age group for males					
	18–25	26–35	36–45	46–55	56–65	65+
Athlete	49–55	49–54	50–56	50–57	51–56	50–55
Excellent	56–61	55–61	57–62	58–63	57–61	56–61
Good	62–65	62–65	63–66	64–67	62–67	62–65
Above-average	66–69	66–70	67–70	68–71	68–71	66–69
Average	70–73	71–74	71–75	72–76	72–75	70–73
Below-average	74–81	75–81	76–82	77–83	76–81	74–79
Poor	82+	82+	83+	84+	82+	80+

The table below shows a health rating for females in a number of age groups based on their resting heart rate (bpm).

Health rating	Age group for females					
	18–25	26–35	36–45	46–55	56–65	65+
Athlete	54–60	54–59	54–59	54–60	54–59	54–59
Excellent	61–65	60–64	60–64	61–65	60–64	60–64
Good	66–69	65–68	65–69	66–69	65–68	65–68
Above-average	70–73	69–72	70–73	70–73	69–73	69–72
Average	74–78	73–76	74–78	74–77	74–77	73–76
Below-average	79–84	77–82	79–84	78–83	78–83	77–84
Poor	85+	83+	85+	84+	84+	84+

## EXERCISE 2D Heart rate

Use the tables on the opposite page to answer questions 1–6.

- 1** Find the resting heart rate range for:
- a** a 50-year-old male with a below-average health rating
  - b** a 34-year-old male with an excellent health rating
  - c** a 25-year-old male with a poor health rating
  - d** a 70-year-old male with a good health rating.
- 2** Find the health rating for each male with the given resting heart rate:
- a** a 44-year-old male, 58 bpm
  - b** a 51-year-old male, 70 bpm
  - c** a 18-year-old male, 80 bpm
  - d** a 58-year-old male, 75 bpm.
- 3**
- a** Find the minimum resting heart rate for a 35-year-old male in below-average health.
  - b** Find the minimum resting heart rate for a 35-year-old male athlete.
  - c** What is the difference in minimum heart rate for these two people?
- 4** Find the resting heart rate range for:
- a** a 34-year-old female with a below-average health rating
  - b** a 25-year-old female with an excellent health rating
  - c** a 50-year-old female with a poor health rating
  - d** a 71-year-old female with a good health rating.
- 5** Find the health rating for each female with the given resting heart rate:
- a** a 44-year-old female, 70 bpm
  - b** an 18-year-old female, 55 bpm
  - c** a 23-year-old female, 63 bpm
  - d** a 58-year-old female, 66 bpm.
- 6**
- a** Find the minimum resting heart rate for a 42-year-old female who has a below-average health rating.
  - b** Find the minimum resting heart rate for a 42-year-old female athlete.
  - c** What is the difference in minimum heart rate for the females in parts **a** and **b**?

### Maximum heart rate

There are many different formulae used to calculate maximum heart rate.

For both males and females, the simplest is  $220 - \text{age in years}$ .

### EXAMPLE 2D-1 Calculating the maximum heart rate

Use the formula  $220 - \text{age in years}$  to calculate the maximum heart rate for:

- a** an 18-year-old male
- b** a 35-year-old female.

	Solve	Think	Apply
<b>a</b>	202 bpm	$220 - 18 = 202$	Subtract age in years from 220 to calculate the maximum heart rate in beats per minute.
<b>b</b>	185 bpm	$220 - 35 = 185$	

- 7** Use the formula  $220 - \text{age in years}$  to calculate the maximum heart rate for:
- a** a 40-year-old male
  - b** a 53-year-old male
  - c** a 22-year-old male
  - d** a 21-year-old female
  - e** a 30-year-old female
  - f** a 64-year-old female.

### Target heart rate for exercise

One method for calculating the target heart rate for exercise is to use a range of values between 50% and 85% of a person's maximum heart rate.



### EXAMPLE 2D-2 Calculating the target heart rate range for exercise

Calculate the target heart rate range for exercise for:

**a** an 18-year-old male

**b** a 35-year-old female.

	Solve	Think	Apply
<b>a</b>	Maximum heart rate is 202 bpm. Lower bound = $0.5 \times 202 = 101$ Upper bound = $0.85 \times 202 \approx 172$ Target heart rate range for exercise is 101 to 172 bpm.	From Example 3B-1, the maximum heart rate is 202 bpm. Find 50% and 85% of 202. $50\% = \frac{50}{100} = 0.5$ $85\% = \frac{85}{100} = 0.85$	First calculate the maximum heart rate. Then find 50% and 85% of that amount. Exercise should be structured so that the heart rate stays within the target range for maximum benefit.
<b>b</b>	Maximum heart rate is 185 bpm Lower bound = $0.5 \times 185 = 93$ Upper bound = $0.85 \times 185 \approx 157$ Target heart rate range for exercise is 93 to 157 bpm.	From Example 3B-1, the maximum heart rate is 185 bpm. Find 50% and 85% of 185.	

- 8** Calculate the target heart rate range for a 30-year-old woman with a maximum heart rate of 190 bpm.
- 9** Calculate the maximum heart rate and the target heart rate range for:
- |                               |                               |                                |
|-------------------------------|-------------------------------|--------------------------------|
| <b>a</b> a 20-year-old male   | <b>b</b> a 43-year-old male   | <b>c</b> a 55-year-old male    |
| <b>d</b> a 19-year-old female | <b>e</b> a 28-year-old female | <b>f</b> a 58-year-old female. |

### Recovery rate number

One estimate of fitness is the recovery rate number. It is calculated by measuring the heart rate one minute after completing exercise, subtracting this value from the heart rate at the end of the exercise period and dividing the result by 10.

A recovery rate number from 2 to 2.9 is fair, from 3 to 3.9 is good, and from 4 to 6 is excellent.

A number over 6 is outstanding and a number below 2 is a sign of poor health or a lack of fitness.



### EXAMPLE 2D-3 Calculating the recovery rate number

Calculate the recovery rate number for a person whose heart rate at the end of exercising is 150 bpm and 90 bpm after one minute. State the level of fitness this indicates.

Solve	Think	Apply
Recovery rate number = $\frac{150 - 90}{10}$ $= 6$ This indicates an excellent level of fitness.	Subtract 90 from 150 then divide the result by 10.	The difference between heart rate at the end of exercise and one minute after exercise is divided by 10 to calculate the recovery rate number.

- 10** Calculate the recovery rate number for a person whose heart rate at the end of exercising is 125 bpm and 92 bpm one minute later. State the level of fitness this indicates.
- 11** Calculate the recovery rate number for each set of heart rate results and state the level of fitness this indicates.
- a** heart rate at the end of exercising is 165 bpm and 145 bpm one minute later
  - b** heart rate at the end of exercising is 155 bpm and 112 bpm one minute later
  - c** heart rate at the end of exercising is 135 bpm and 100 bpm one minute later
  - d** heart rate at the end of exercising is 118 bpm and 65 bpm one minute later

- 12 a** Complete this table for maximum heart rate.

Age (years)	10	30	50	70	90
Maximum heart rate (bpm)					

- b** Plot the points from the table and join them with a straight line or smooth curve to produce a graph representing this information.
  - c** Does the graph show a linear or non-linear relationship between age and maximum heart rate?
  - d** Use the graph to estimate the age of a person with maximum heart rate of 175 bpm.
- 13** A person trains for 20 minutes with their heart rate at 130 bpm. Calculate the number of heart beats in that time.
- 14** If each of the people in question 9 trained for 20 minutes at the lower level of their target heart range, calculate the total number of times their heart would beat in that time.
- 15** The average adult heart pumps about 70 mL of blood with each beat. Calculate the volume of blood pumped by the human heart beating for:
- a** 1 min at 65 bpm
  - b** 10 min at 120 bpm
  - c** 5 min at 95 bpm
  - d** 24 h at 70 bpm.
- 16** Have two or more students ride an exercise bike, if available, or jog on the spot for 6 minutes. Measure their heart rate every minute for the 6 minutes of exercise and then for the following 4 minutes.
- a** Graph the results.
  - b** Compare each person's initial resting heart rate with their heart rate 4 minutes after exercise is completed. What conclusions can you make?
  - c** Calculate the recovery rate numbers for each student. What does this indicate about the level of fitness of each student?
  - d** Repeat for other students, then graph and compare the results. From the graph, determine whether the relationships between heart rate and exercise time are linear. Explain your answer.

- 17** The formula  $(220 - \text{age in years})$  used to calculate the maximum heart rate is the formula attributed to Haskell and Fox. It is used in general circumstances and was not intended to be used in exercise physiology. Elite athletes and sports people need more specific information and further research gives other methods of calculation. Some of these methods are the study by Tanaka, Monahan and Seals; the Gulati formula for women and the Lund study.
- a** Research these formulae and any other formulae you find.
  - b** Calculate your maximum heart rate using at least three different formulae. Compare your answers and explain any differences according to the nature of the formula.
  - c** Find the average of all the maximum heart rates and compare this to the Haskell and Fox formula results. Explain which value you would use to determine your maximum heart rate.

### systolic blood pressure

the maximum pressure in the arteries as the heart pumps out blood with each beat, measured in millilitres of mercury (mmHg)

### diastolic blood pressure

the minimum pressure in the arteries as the heart relaxes before the next beat, measured in millilitres of mercury (mmHg)

# 2E Blood pressure

These resources are available on your obook assess:

- **Spreadsheet 2E:** Graph measurements for blood pressure
- **Assess quiz 2E:** Test your skills with an auto-correcting multiple-choice quiz

oigi

Blood pressure is the pressure of blood in the arteries as it is pumped around the body by the heart. Blood pressure changes to meet your body's needs. It is affected by various factors including emotional state, sleep, body position and exercise.

Blood pressure is recorded as two numbers. The first number, the **systolic blood pressure**, is the maximum pressure in the arteries as the heart pumps out blood with each beat. The second and smaller number, the **diastolic blood pressure**, is the minimum pressure in the arteries as the heart relaxes before the next beat. A person's blood pressure is usually expressed in terms of the systolic pressure over the diastolic pressure and is measured in millimetres of mercury (mmHg), for example 120/80 which is read as 120 over 80.

## EXAMPLE 2E-1 Using reference ranges for blood pressure

This table shows the reference ranges for blood pressure by age up to 18 years.

	Approximate age	Systolic (mmHg)	Diastolic (mmHg)
<b>Infants</b>	1 to 12 months	75–100	50–70
<b>Toddlers</b>	1 to 4 years	80–110	50–80
<b>Preschoolers</b>	3 to 5 years	80–110	50–80
<b>School age</b>	6 to 13 years	85–120	50–80
<b>Adolescents</b>	13 to 18 years	95–140	60–90

- a** Is a blood pressure reading of 125/80 within the normal range for an adolescent?  
**b** Is a blood pressure reading of 120/90 within the normal range for a toddler?

	Solve	Think	Apply
<b>a</b>	Yes	125 is within the range 95–140 and 80 is within the range 60–90.	Both the systolic and diastolic readings must be within the range for the reading to be normal for that age group.
<b>b</b>	No	120 is outside the range 80–110 and 90 is outside the range 50–80.	

## EXERCISE 2E Blood pressure

- Complete to determine if a blood pressure reading of 115/72 is within the normal range for an adolescent. 115 is \_\_\_ the range 95–\_\_\_ and 72 is within the range \_\_\_–\_\_\_ so it is/is not normal.
- Are the following blood pressure readings considered within the normal range?
 

<b>a</b> 80/60 for an infant	<b>b</b> 85/55 for a school-age child
<b>c</b> 100/50 for an adolescent	<b>d</b> 100/65 for a toddler
<b>e</b> 112/85 for an adolescent	<b>f</b> 80/55 for an infant

### EXAMPLE 2E-2 Identifying categories of blood pressure for adults

Category	Systolic (mmHg)	Diastolic (mmHg)
<b>Hypotension</b>	< 90	< 60
<b>Normal</b>	90–119	60–79
<b>Prehypertension</b>	120–139	80–89
<b>Stage 1 hypertension</b>	140–159	90–99
<b>Stage 2 hypertension</b>	160–179	100–109
<b>Hypertensive emergency</b>	≥ 180	≥ 110

What classification would be given to an adult with a blood pressure reading of:

- a** 115 over 75?      **b** 142 over 95?      **c** 82 over 51?      **d** 125 over 98?

	Solve	Think	Apply
<b>a</b>	Normal	115 is within the range 90–119 and 75 is within the range 60–79.	Check the range for the systolic and diastolic readings to find the classification. <i>Note:</i> if the readings are in different categories then use the category associated with the diastolic pressure, the second number.
<b>b</b>	Stage 1 hypertension	142 is within the range 140–159 and 95 is within the range 90–99.	
<b>c</b>	Hypotension	82 is less than 90 and 51 is less than 60.	
<b>d</b>	Stage 1 hypertension	125 is in the prehypertension range and 98 is in the stage 1 hypertension range, so the classification is stage 1 hypertension.	

- 3** Complete to find what classification would be given to an adult with a blood pressure reading of 125/85.

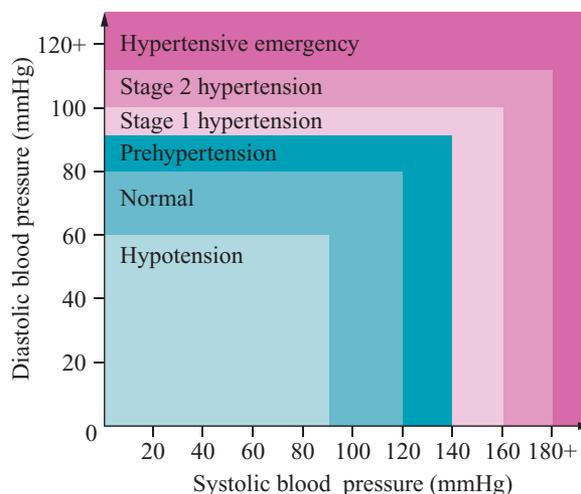
125 is in the range \_\_\_–139 and \_\_\_ is in the range 80–\_\_\_, so the classification is \_\_\_.

- 4** Determine the classification that would be given to an adult with each of the following blood pressure readings.

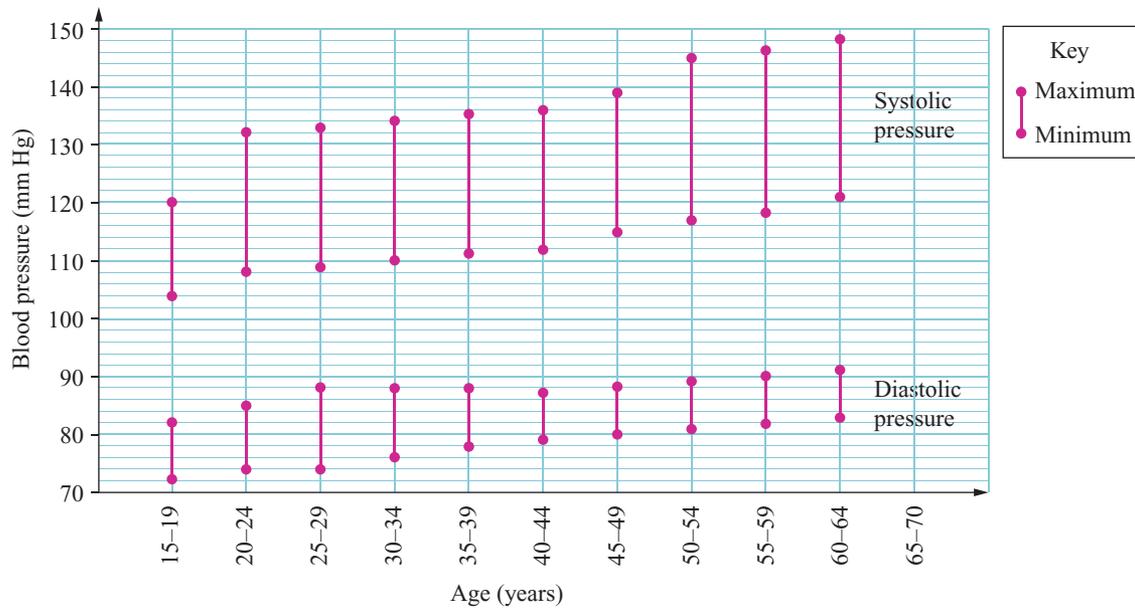
- a** 122 over 88      **b** 166 over 108  
**c** 95 over 75      **d** 118 over 81  
**e** 170 over 105      **f** 88 over 58

- 5** The graph at right shows the information from the table in Example 2E-2. Use the graph to classify an adult with each of the following blood pressure readings.

- a** 66 over 58  
**b** 165 over 105  
**c** 125 over 89  
**d** 115 over 63  
**e** 148 over 99  
**f** 190 over 125



**6** The bars on the graph below show the normal ranges of blood pressure for people of various ages.



- a** What is the normal range of blood pressure for a 46-year-old person?
- b** Lee, aged 52, had a blood pressure reading of 135/85. Is her reading within the normal range for her age?
- c** Tod, aged 27, had a blood pressure reading of 140/90. Is his reading within the normal range for his age?

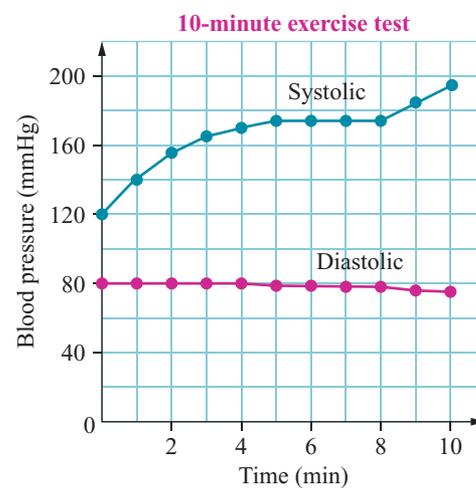
**7** Mike, aged 56, had a blood pressure reading of 180 over 110. A doctor prescribed Mike a medication to reduce his blood pressure. To check that the medication was being effective, the doctor measured Mike's blood pressure for 10 weeks and recorded the following results.

Week	1	2	3	4	5	6	7	8	9	10
Systolic pressure	180	165	144	148	140	136	137	136	135	135
Diastolic pressure	110	98	90	92	89	86	85	84	82	85

Based on the results in the table and the graph in question 6, from which week onwards was Mike's blood pressure consistently in the normal range?

**8** The graph shows a person's blood pressure during a 10-minute exercise test. Blood pressure measurements were taken every minute.

- a** What was the initial blood pressure reading?
- b** After some time the blood pressure readings level off. When is this? What is the reading?
- c** The exercise becomes more strenuous in the final 2 minutes. What is the increase in systolic pressure over the last 2 minutes?
- d** How does the diastolic pressure change during the course of the 10-minute exercise test?



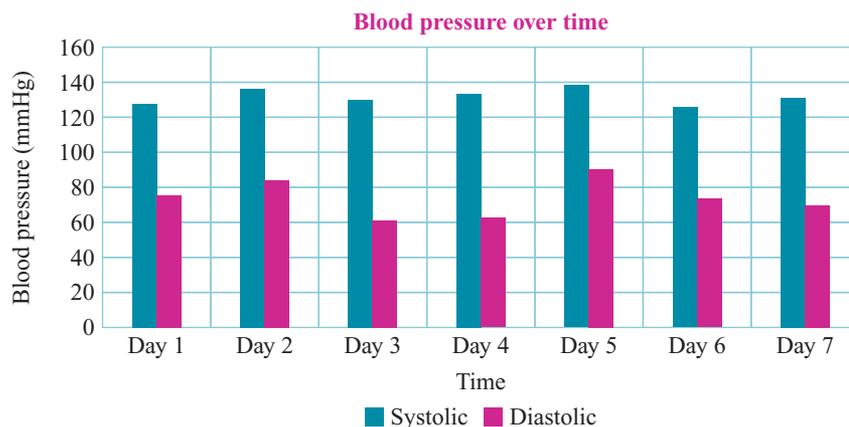
**9** Many people have high blood pressure which can negatively affect their health and damage their heart. Find out what can be done to lower blood pressure.

- 10** Use a blood pressure monitor to measure your blood pressure each day (preferably at the same time of day) for at least 7 days.
- Record your results in a table similar to the one shown below which contains sample data.
  - Produce a graph to display the results for systolic blood pressure and diastolic blood pressure on the same set of axes. You can use a spreadsheet by following these steps:

- Enter the data in three columns of the spreadsheet. See the sample data shown.
- Highlight the data in the three columns and from the **Insert** menu and **Charts**, select the graph type you wish to use. Click on the chart to bring up the Chart Elements menu to adjust the chart title and each axis title. You may like to use the prepared spreadsheet (Spreadsheet 2E) supplied on your [obook](#) [access](#).
- Experiment with different types of displays. Which type of graph is most appropriate for this data? Discuss your answer with a classmate.

Time	Systolic	Diastolic
Day 1	129	76
Day 2	137	85
Day 3	131	62
Day 4	134	63
Day 5	139	91
Day 6	126	74
Day 7	132	71

One type of graph (column) is shown below for the sample data in the table.



- 11** Repeat the task in question **10** but vary the conditions.
- Measure your blood pressure in the morning (preferably when you first get up) and late afternoon over a few days. Is your blood pressure different in the mornings compared to the afternoons?
  - Measure your blood pressure at regular times across one day. Can you see a trend in the data over time?
  - Measure your blood pressure before, during and after exercise. Comment on what the results show.
- 12** This investigation examines the changes in heart rate and blood pressure due to temperature. The heart rate is measured by taking the pulse rate on the left wrist and the blood pressure is measured using a blood pressure monitor attached to the upper right arm.
- Measure the heart rate and blood pressure of three groups of students while they are seated: one group with their right hand in a beaker of cold water ( $5^{\circ}\text{C}$ ), one group with their right hand in a beaker of hot water ( $42^{\circ}\text{C}$ ), and one group with their right hand in a beaker of water at room temperature ( $23^{\circ}\text{C}$ ).
  - Take three measurements of blood pressure and heart rate for each group of students: the first reading after initial exposure, the next after  $1\frac{1}{2}$  minutes of exposure, and the final after 3 minutes of exposure.
  - It is said that the average blood pressure for a person with their hand in the cold water would be higher than the average blood pressure of the person holding a hand in the hot water. Comment on this statement based on your measurements.

# CHAPTER 2 REVIEW RATES

You should be able to:

- ✓ calculate and simplify rates
- ✓ convert between units for rates
- ✓ use rates to make comparisons
- ✓ use rates to determine costs
- ✓ solve problems involving rates
- ✓ work with speed as a rate
- ✓ interpret distance-time graphs
- ✓ use distance-time graphs to solve problems related to speed, distance and time
- ✓ solve problems related to the fuel consumption rate of a motor vehicle
- ✓ measure heart rate and blood pressure over time under different conditions
- ✓ interpret charts, construct graphs and perform calculations related to heart rate
- ✓ interpret charts and construct graphs related to blood pressure.

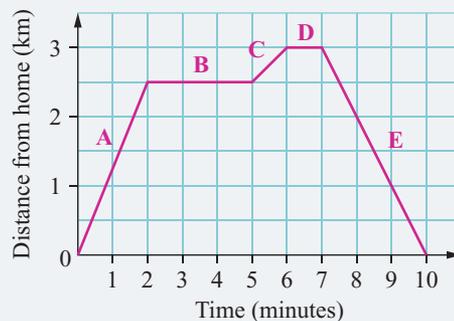
Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

## REVIEW MULTIPLE-CHOICE QUESTIONS

- 2A** 1 A garden hose can fill a 5 L bucket in 10 s. What is the rate of flow in litres per hour?  
A 180                                      B 1800                                      C 30                                      D 200
- 2A** 2 What speed is equivalent to 70 km/h?  
A 0.02 km/h                              B 1.2 m/s                                      C 19.4 m/s                                      D 1167 m/s
- 2A** 3 If the current exchange rate is 78 US cents for each A\$, how many Australian dollars would you need to exchange to get US\$1000?  
A 12.82                                      B 780                                      C 1000                                      D 1282.05

Tom's journey is represented in the distance-time graph. Use this graph to answer questions 4–7.

- 2B** 4 How far has Tom travelled after 5 minutes?  
A 1.5 km                                      B 2.5 km  
C 3 km                                      D 5 km
- 2B** 5 In which section of the graph is Tom travelling the fastest?  
A A                                      B B  
C C                                      D E
- 2B** 6 What is Tom's speed in section A of the graph?  
A 0 km/min                                      B 1.25 km/min  
C 2.5 km/min                                      D 5 km/min



- 2B** ▶ **7** In which section/s of the graph is Tom stationary?  
**A** A and C                      **B** A and E                      **C** B and D                      **D** B only
- 2C** ▶ **8** What is the fuel consumption rate of a car that travels 500 km on 70 L of petrol?  
**A** 0.14 L/100 km              **B** 7.14 L/100 km              **C** 14 L/100 km              **D** 70 L/100 km
- 2C** ▶ **9** How far can a motor vehicle travel on 40 L of petrol if its fuel consumption is 8.4 L/100 km?  
**A** 4.76 km                      **B** 21 km                      **C** 336 km                      **D** 476 km
- 2D** ▶ **10** A person's maximum heart rate can be calculated using the formula  $220 - \text{age in years}$ . What is the maximum heart rate for a 19-year-old male?  
**A** 239                      **B** 220                      **C** 201                      **D** 19
- 2D** ▶ **11** During exercise, a person's target heart rate is between 50% and 85% of their maximum heart rate. What is the target heart rate range for a 40-year-old woman with a maximum heart rate of 180?  
**A** 90–153                      **B** 140–220                      **C** 70–119                      **D** 0–180
- 2D** ▶ **12** The average adult heart pumps 70 mL of blood with each beat. What is the volume of blood pumped in 6 minutes at 115 bpm?  
**A** 9.85 mL                      **B** 630 mL                      **C** 1341 mL                      **D** 48 300 mL
- 2D** ▶ **13** A person's recovery rate number is calculated by subtracting the heart rate one minute after completing exercise from the heart rate at the end of exercising and dividing the result by 10. What is the recovery number for a person whose heart rate at the end of exercising is 153 bpm and 108 bpm 1 minute later?  
**A** 45                      **B** 4.5                      **C** 261                      **D** 2.61

Use this table to answer questions **14** and **15**.

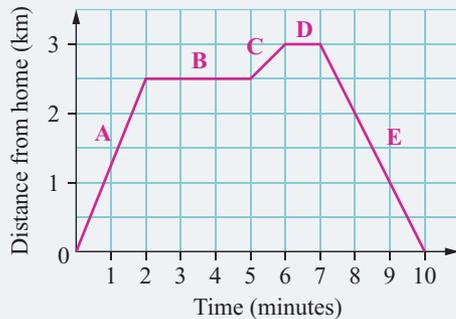
Category	Systolic (mmHg)	Diastolic (mmHg)
<b>Hypotension</b>	< 90	< 60
<b>Normal</b>	90–119	60–79
<b>Prehypertension</b>	120–139	80–89
<b>Stage 1 hypertension</b>	140–159	90–99
<b>Stage 2 hypertension</b>	160–179	100–109
<b>Hypertensive emergency</b>	$\geq 180$	$\geq 110$

- 2E** ▶ **14** An adult with a blood pressure reading of 115/75 would be given a classification of:  
**A** hypotension              **B** normal  
**C** prehypertension              **D** hypertension
- 2E** ▶ **15** An adult with a blood pressure reading of 135/85 would be given a classification of:  
**A** hypotension              **B** normal  
**C** prehypertension              **D** hypertension



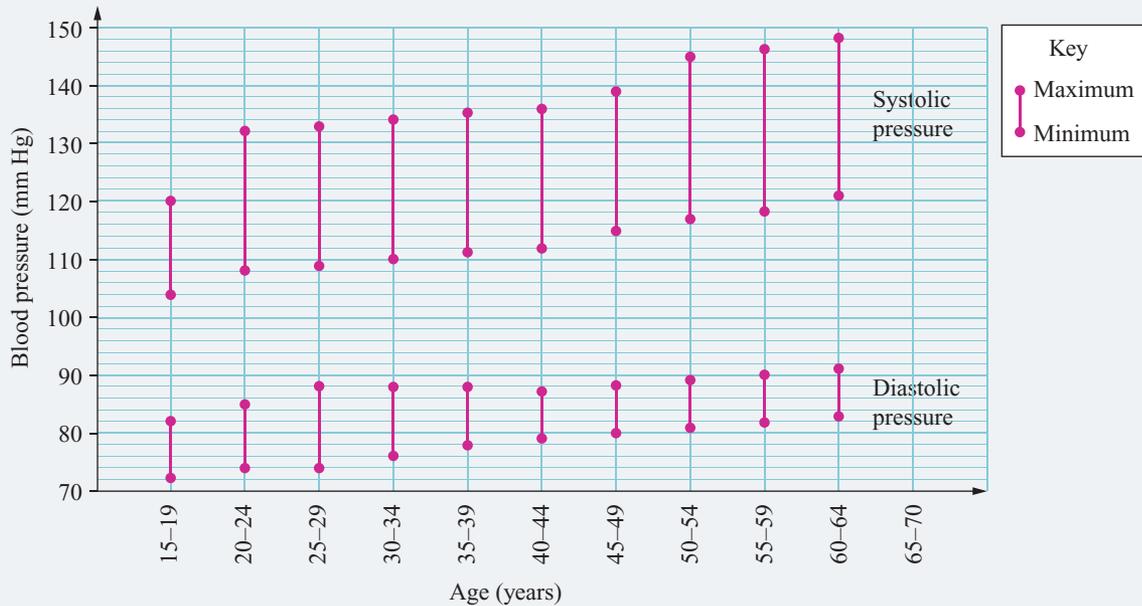
# REVIEW SET 1

- Convert these rates.
  - 12 t/ha to kg/m<sup>2</sup>
  - 9 m/s to km/h
- The cost of a pack of 8 pens is \$6.32 and the cost of a box of 5 pens is \$4.10. Which option is the best buy?
- Mark charges a call-out fee of \$50 and \$18 per hour as a cleaner. How much will the client be charged if it took 3.5 hours for Mark to complete the job?
- Katie drives her children from home to school and checks that they get to their classes on time. From the school, she heads to the post office to post a letter before returning home. This graph represents Katie's journey.



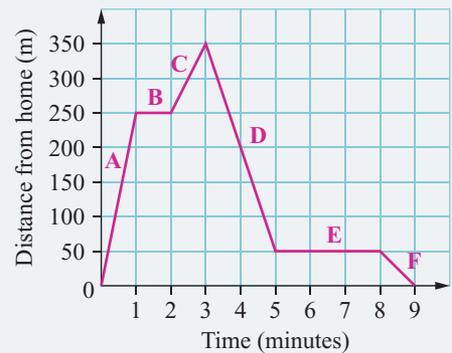
- Match the letters for each section of the graph (A, B, C, D and E) with the appropriate part of the description provided.
  - How far is the school from Katie's home?
  - How long did Katie spend at the school before leaving?
  - What is the distance between the school and the post office?
  - How long did Katie spend at the post office?
  - How long did it take Katie to travel from home to the school?
  - How long did Katie take to travel from the post office to her home?
  - What was Katie's speed in section E of the graph?
  - What was the total distance that she travelled?
  - What was Katie's average speed for her journey?
- How far can a vehicle travel on 38 L of fuel if its fuel consumption rate is 9.6 L/100 km?
  - A person's maximum heart rate can be calculated using the formula  $220 - \text{age in years}$ . Calculate the maximum heart rate for a 25-year-old female.
  - The average adult heart pumps about 70 mL of blood with each beat. Calculate the volume of blood pumped by a human heart beating for 12 minutes at 118 bpm.
  - Use the table shown on page 57 to determine the classification that would be given to an adult with a blood pressure reading of 135/88.

- 9 The bars on the graph below show the normal ranges of blood pressure for people of various ages.
- What is the normal range of blood pressure for an 18-year-old person?
  - Luke, aged 34, had a blood pressure reading of 140 over 90. Is his reading within the normal range for his age?



## REVIEW SET 2

- Sue buys 2.8 m of dress material for \$99.68. What is the cost per metre of the material?
- Water flows into a tank at the rate of 8 L/min.
  - How much water will flow into the tank in 3 h and 25 min?
  - If the tank has a capacity of 4.24 kL, how long will it take to fill the tank?
- Compare the rates in each listed pair. Which one of the two rates is larger? Use calculations to justify your answer in each case.
  - 70 cm/s or 45 m/min
  - 25 mg/mL or 24 g/L
- Jason leaves home and heads towards Tower Hill, jogging at a steady pace. He stops to catch his breath before continuing on to reach Tower Hill. Once there, he turns and heads back for home. Just before he arrives home, he stops to chat with his friend Gus for a while, before resuming his jog home. This graph shows his journey.
  - Match the letters for each section of the graph (A to F) with the appropriate part of the description.
  - In which section/s was Jason stationary?
  - Calculate Jason's speed in each section of the graph.
  - What was the total distance travelled by Jason over the 9 minutes?
  - What was his average speed over the 9 minutes?



- 5 A salesperson averages 5400 km of city driving each month in a Fiat 500 that uses 6.3 L/100 km. Calculate the monthly cost of petrol used by the salesperson when the average price of ULP is 139.9c/L.
- 6 During exercise, a person's target heart rate is between 50% and 85% of their maximum heart rate. Their maximum heart rate can be calculated using the formula  $220 - \text{age}$  in years. Calculate the target heart range for a 32-year-old female.

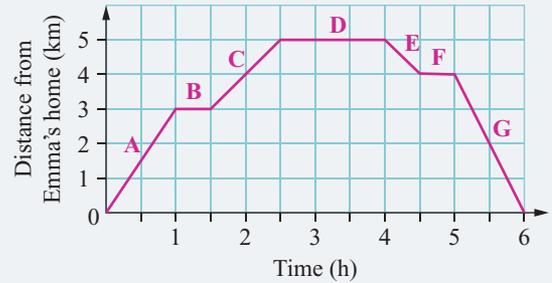


- 7 The average adult heart pumps about 70 mL of blood each beat. Calculate the volume of blood pumped by the human heart beating for 3 minutes at 127 bpm.
- 8 Use the table shown on page 57 to determine the classification that would be given to an adult with a blood pressure reading of 150/95.
- 9 Refer to the graph on page 59 which shows the normal ranges of blood pressure for people of various ages to answer the following questions.
  - a What is the normal range of blood pressure for a 43-year-old person?
  - b Tricia, aged 23, had a blood pressure reading of 130 over 80. Is her reading within the normal range for her age?

## REVIEW SET 3

- 1 Convert each of the following rates, giving the answer to two decimal places where necessary.
  - a 60 km/h to m/s
  - b 50 g/m<sup>2</sup> to kg/ha
- 2 At a market, Jess bought 8 apples for \$3.50 from one stall holder and Hunter bought 14 apples for \$6.50 from another. Who made the best buy?
- 3 Elliot is paid at the rate of \$27.50/h.
  - a How much would he be paid for working 38 h?
  - b How long would he need to work to earn \$660?

- 4** Emma walks to her friend's house and together they walk to the cinema. After the film, they go to a cafe before returning home. This graph represents Emma's journey.
- Match the letters for each section of the graph (A to G) with the appropriate part of the description provided.
  - How far did Emma walk to reach her friend's house?
  - How long did Emma spend at her friend's house?
  - What is the distance between her friend's house and the cinema?
  - How long did Emma and her friend spend at the cinema?
  - How far is the cafe from the cinema?
  - How long did Emma take to travel from the cafe to her home?
  - Calculate Emma's speed in each section of the graph.
  - What was the total distance that she travelled?
  - What was Emma's average speed for her journey?



- 5** Calculate the amount of fuel used by a car on a trip of 425 km, if the fuel consumption is 10.6 L/100 km.
- 6** A person's maximum heart rate can be calculated using the formula  $220 - \text{age in years}$ . Calculate the maximum heart rate for a 41-year-old male.
- 7** The average adult heart pumps about 70 mL of blood each beat. Calculate the volume of blood pumped by a human heart beating for 5 minutes at 151 bpm.
- 8** Use the table shown on page 57 to determine the classification that would be given to an adult with a blood pressure reading of 85/50.
- 9** Refer to the graph on page 59, which shows the normal ranges of blood pressure for people of various ages, to answer the following questions.
- What is the normal range of blood pressure for a 50-year-old person?
  - Anton, aged 61, had a blood pressure reading of 125 over 88. Is his reading within the normal range for his age?



## REVIEW SET 4

- The average rate of flow of a bathroom shower with a normal shower rose is 18 L/minute.
  - Nora has two 10-minute showers each day. Calculate the annual cost of Nora's showers, given the water usage charge is \$2.17/kL. (Use 1 year = 365 days.)
  - How much could Nora save each year by using a water-efficient shower rose that only uses 8.5 L/minute?
- Kate charges a travel fee of \$35 and \$24 per hour as a gardener. How much will the client be charged if it took 5 hours for Kate to complete the job?
- Compare the rates in each listed pair. Which one of the two rates is larger? Use calculations to justify your answer in each case.
 

<b>a</b> 20 m/s or 65 km/h	<b>b</b> 4 L/h or 70 mL/min
----------------------------	-----------------------------

- 4** Owen jogs from home to the football oval at a constant speed of 6 km/h. It takes him 30 minutes. He remains at the oval for 2 hours and then returns home at a constant speed of 4 km/h.
- How far away is the football oval from Owen's home?
  - How long does it take Owen to jog home from the oval?
  - Construct a distance-time graph representing Owen's journey.
  - If Owen left home at 6:30 pm, at what time did he return?

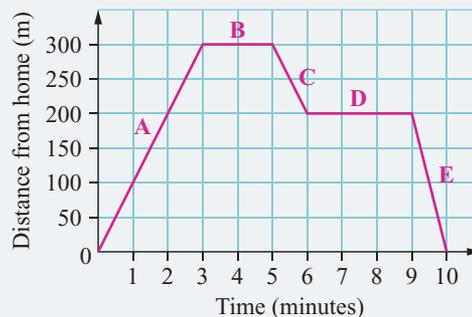


- 5** A car uses 32 L of petrol to travel 250 km.
- Calculate the petrol consumption in L/100 km.
  - At this rate of consumption, how much petrol (to the nearest litre) would be used to travel 650 km?
  - How far could the car travel on 56 L?
- 6** During exercise, a person's target heart rate is between 50% and 85% of their maximum heart rate. Their maximum heart rate can be calculated using the formula  $220 - \text{age in years}$ . Calculate the target heart rate range for a 23-year-old male.
- 7** The average adult heart pumps about 70 mL of blood each beat. Calculate the volume of blood pumped by the human heart beating for 7 minutes at 162 bpm.
- 8** Use the table shown on page 57 to determine the classification that would be given to an adult with a blood pressure reading of 190/122.
- 9** Refer to the graph on page 59 which shows the normal ranges of blood pressure for people of various ages to answer the following questions.
- What is the normal range of blood pressure for a 25-year-old person?
  - Liesel, aged 48, had a blood pressure reading of 145 over 95. Is her reading within the normal range for her age?

## REVIEW PRACTICE EXAMINATION QUESTION

- 1 a** A large park of area 1.2 ha is to be fertilised at the rate of 14 g/m<sup>2</sup>.
- Calculate, in kilograms, the amount of fertiliser needed. (2 marks)
  - If the fertiliser is only available in 25 kg bags, how many bags are needed? (1 mark)

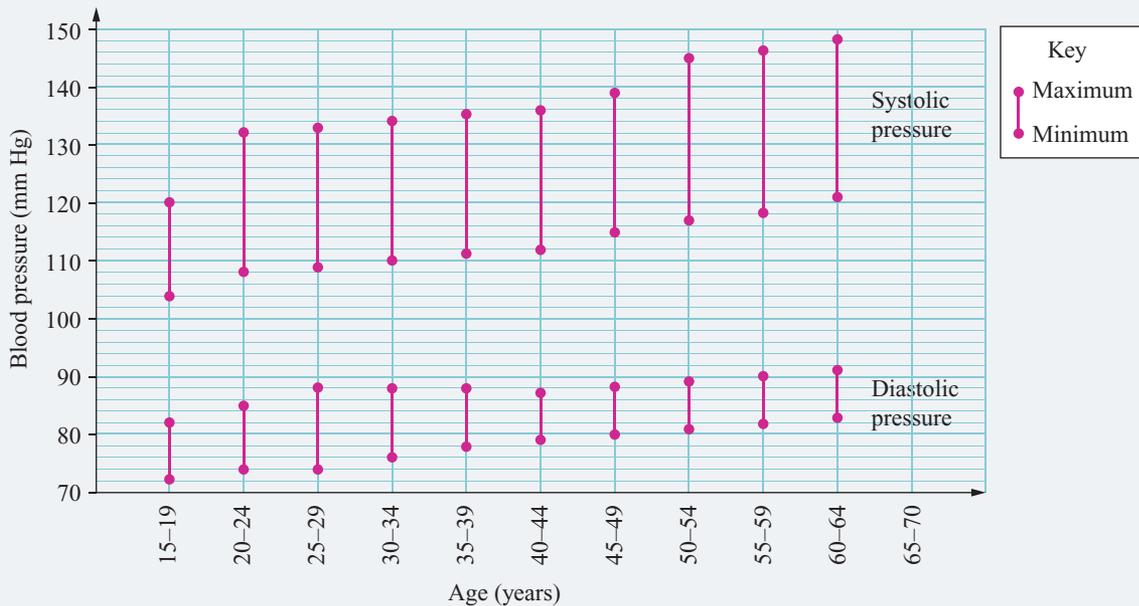
- b** The journey that is represented by this distance-time graph is described below.
- Daniel cycles from his home to the shops to buy a new cover for his tennis racquet. He then heads back home but stops to chat to a friend he sees on the way. Once he finishes chatting, he continues his journey home.



- Which section of the graph (A, B, C, D or E) represents Daniel's journey from the shops until he meets his friend? (1 mark)
- What was the total distance that Daniel rode in the 10 minutes? (1 mark)
- In which section/s was Daniel stationary? (1 mark)
- In which section was Daniel travelling the fastest? What was his speed? (1 mark)

- c How far can a car travel on 54 L of fuel if the fuel consumption is 7.8 L/100 km? (1 mark)
- d Bob is a 37-year-old male.
- A person's maximum heart rate can be calculated using the formula  $220 - \text{age in years}$ . Calculate Bob's maximum heart rate. (1 mark)
  - During exercise, a person's target heart rate is between 50% and 85% of their maximum heart rate. Calculate Bob's target heart range. (2 marks)
  - Bob trains for 18 minutes with his heart rate at 127 bpm. Calculate the total number of heart beats in that time. (1 mark)
- e Blood pressure is measured using two numbers: systolic pressure and diastolic pressure. A blood pressure measurement of 120 over 80 refers to a systolic pressure of 120 mmHg and a diastolic pressure of 80 mmHg.

The bars on the graph below show the normal ranges of blood pressure for people of various ages.



Julie, aged 38, had a blood pressure reading of 170 over 100. A doctor prescribed Julie a medication to reduce her blood pressure. To check that the medication was being effective, the doctor measured Julie's blood pressure for 10 weeks and recorded the following results.

Week	1	2	3	4	5	6	7	8	9	10
<b>Systolic pressure</b>	170	160	145	132	137	131	133	132	130	131
<b>Diastolic pressure</b>	100	95	90	86	87	89	84	85	83	84

- What is the normal range of blood pressure for a 38-year-old person? (1 mark)
- With reference to the data provided in the table, comment on the effectiveness of the medication during the 10-week period in returning Julie's blood pressure to the normal range. (2 marks)

**TOTAL:**  
15 marks

# 3

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## Scale drawings

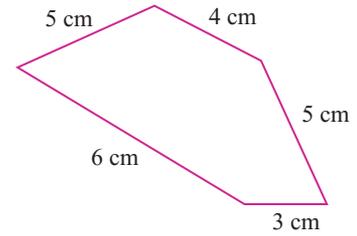
The main mathematical ideas investigated are:

- ▶ expressing ratios in simplest form
- ▶ finding ratios of two quantities
- ▶ dividing quantities in given ratios
- ▶ recognising properties of similar figures
- ▶ calculating the scale factor for similar figures
- ▶ using scale factors to find unknown side lengths of similar figures
- ▶ using scale drawings, building plans and maps
- ▶ solving practical problems using scale drawings and factors.

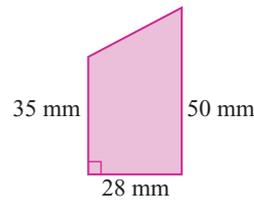
# ARE YOU READY?

- 3A** ▶ 1 Which measurement is equivalent to 4.5 m?  
**A** 45 cm                      **B** 450 cm  
**C** 0.045 km                **D** 45 mm
- 3A** ▶ 2 Which measurement is equivalent to 260 L?  
**A** 0.26 kL                    **B** 26 kL  
**C** 26 000 mL                **D** 0.026 ML
- 3A** ▶ 3 Which measurement is equivalent to 6 ha?  
**A** 600 m<sup>2</sup>                    **B** 6000 m<sup>2</sup>  
**C** 60 000 m<sup>2</sup>                **D** 6 km<sup>2</sup>
- 3A** ▶ 4 What is the highest common factor of 45 and 72?  
**A** 3                              **B** 9  
**C** 18                            **D** 360
- 3A** ▶ 5 Which fraction is equivalent to  $\frac{1}{3}$ ?  
**A**  $\frac{5}{16}$                           **B**  $\frac{9}{18}$   
**C**  $\frac{2}{15}$                           **D**  $\frac{4}{12}$
- 3A** ▶ 6 Which equation is equivalent to  $\frac{x}{2} = \frac{5}{7}$ ?  
**A**  $x = \frac{5}{7} \div 2$                 **B**  $x = \frac{5}{7} \times 2$   
**C**  $x = \frac{5}{7} + 2$                 **D**  $x = \frac{5}{7} - 2$
- 3A** ▶ 7 Which equation is equivalent to  $\frac{3}{y} = \frac{11}{2}$ ?  
**A**  $y = \frac{11}{2} \times 3$                 **B**  $y = \frac{11}{3} \times 2$   
**C**  $y = \frac{2}{11} \times 3$                 **D**  $y = \frac{2}{3} \times 11$

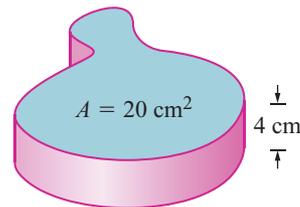
- 3D** ▶ 8 What is the perimeter of this shape?  
**A** 18 cm  
**B** 20 cm  
**C** 23 cm  
**D** 1800 cm



- 3D** ▶ 9 What is the area of a rectangle with length 8 m and breadth 3 m?  
**A** 11 m<sup>2</sup>                      **B** 22 m<sup>2</sup>  
**C** 24 m<sup>2</sup>                      **D** 48 m<sup>2</sup>
- 3D** ▶ 10 What is the area of this shape?  
**A**  $\frac{1}{2} \times 28 \times (35 + 50)$  mm<sup>2</sup>  
**B**  $(28 \times 50)$  mm<sup>2</sup>  
**C**  $\frac{1}{2} \times 50 \times (28 + 35)$  mm<sup>2</sup>  
**D**  $28 \times (35 + 50)$  mm<sup>2</sup>



- 3D** ▶ 11 What is the volume of this solid?  
**A** 5 cm<sup>3</sup>                      **B** 16 cm<sup>3</sup>  
**C** 24 cm<sup>3</sup>                      **D** 80 cm<sup>3</sup>



If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your obook assess.

- Q1–3**      **Support sheet 3A.1 Converting units of measurement**  
**Q4**        **Support sheet 3A.2 Finding the highest common factor**  
**Q5**        **Support sheet 3A.3 Simplifying fractions**  
**Q6–7**     **Support sheet 3A.4 Solving equations involving fractional terms**  
**Q8**        **Support sheet 3D.1 Finding perimeter**  
**Q9–10**   **Support sheet 3D.2 Calculating area**  
**Q11**      **Support sheet 3D.3 Calculating volume**

# 3A Ratios

These resources are available on your obook assess:

- **Interactive 3A.1:** Explore key ideas for understanding ratios
- **Interactive 3A.2:** Explore simplifying ratios and using equivalent ratios
- **Interactive 3A.3:** Explore key ideas for dividing quantities into ratios
- **Video tutorial 3A:** Watch and listen to an explanation of Examples 3A–4 and 3A–5
- **Worksheet 3A:** Practise your skills with extra problems on ratios
- **Investigation 3A:** Investigate the ratios of bicycle design
- **assess quiz 3A:** Test your skills with an auto-correcting multiple-choice quiz

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## ratio

comparison between quantities of the same kind, used as a measure of the relative sizes of the quantities

A **ratio** is a comparison between quantities of the same kind.

A ratio can be written using colon notation (:) or as a fraction.

A ratio does not have units and usually contains whole numbers.

The order of the quantities in a ratio is important.

!

Ratios are simplified by multiplying or dividing each term of the ratio by the same number. Unless otherwise specified, the terms should be left as whole numbers with no common factors and no units.

## EXAMPLE 3A–1 Writing comparisons as ratios

Tom's height is 169 cm and Laura's height is 165 cm.

- Write the ratio of Tom's height to Laura's height.
  - Write the ratio of Laura's height to Tom's height.
- b** Write each ratio from part **a** as a fraction.



	Solve	Think	Apply
<b>a i</b>	Tom's height to Laura's height = 169 cm to 165 cm = 169 : 165	The quantities compared are 169 cm and 165 cm. These are whole numbers and in the same unit, so write these quantities in the order given, separated by a colon, leaving out the units.	Check that the quantities are whole numbers and in the same unit. Write the quantities in the order given, separated by a colon or as a fraction, leaving out the units.  The order is important as $169 : 165 \neq 165 : 169$ .
<b>ii</b>	Laura's height to Tom's height = 165 cm to 169 cm = 165 : 169		
<b>b i</b>	$169 : 165 = \frac{169}{165}$	Write the first number in the ratio as the numerator of the fraction and the second number as the denominator.	
<b>ii</b>	$165 : 169 = \frac{165}{169}$		

# EXERCISE 3A Ratios

- 1 The table shows the number of each type of vehicle that passes the front of a school in an hour.  
Write the ratio of the numbers of:

- a cars to trucks
- b trucks to cars
- c motorcycles to cars
- d trucks to buses
- e cars to trucks to motorcycles
- f trucks to motorcycles to buses.

Type of vehicle	Number
Car	54
Truck	13
Motorcycle	7
Bus	4
Other	2

- 2 Write each ratio from parts a–d of question 1 as a fraction.

## EXAMPLE 3A-2 Writing comparisons as ratios in simplest form

Write each of these comparisons as a ratio in simplest form.

- a 24 to 18      b 16 to 12 to 20      c 1.5 to 2.7      d  $1\frac{3}{4}$  to  $\frac{2}{3}$       e 85 cm to 1.2 m

	Solve	Think	Apply
a	24 to 18 = 24 : 18 = 4 : 3	The two quantities are whole numbers, so they can be written as a ratio. The highest common factor (HCF) of 24 and 18 is 6, so divide each term by 6 to simplify the ratio. The fraction key on a calculator can be used when there are two parts to the ratio. Press: 24 $\frac{a}{b}$ 18 $\cdot$ SHIFT $\frac{a}{c}$ Display is 4   3	Ratios must contain whole numbers and can be simplified by multiplying or dividing each term by the same number. This produces an equivalent ratio.
b	16 to 12 to 20 = 16 : 12 : 20 = 4 : 3 : 5	The three quantities are whole numbers, so they can be written as a ratio. The HCF of 16, 12 and 20 is 4, so divide each term by 4.	
c	1.5 to 2.7 = 15 to 27 = 15 : 27 = 5 : 9	Ratios must contain whole numbers, so first multiply each term by 10. The HCF of 15 and 27 is 3, so divide each part by 3.	
d	$1\frac{3}{4}$ to $\frac{2}{3}$ = $\frac{7}{4}$ to $\frac{2}{3}$ = $\frac{7}{4} \times 12$ to $\frac{2}{3} \times 12$ = 21 to 8 = 21 : 8	Write the mixed number as an improper fraction. Since ratios must contain whole numbers, multiply each term by the lowest common denominator of 12. The ratio 21 : 8 cannot be simplified any further.	
e	85 cm to 1.2 m = 85 cm to 120 cm = 85 : 120 = 17 : 24	Write the terms so they have the same unit. Once the two quantities are expressed in the same unit and are whole numbers, they can be written as a ratio. The HCF of 85 and 120 is 5, so divide each term by 5.	

**3** Write each of these comparisons as a ratio in simplest form.

- |   |   |   |
|---|---|---|
| <b>a</b> 25 to 35                         | <b>b</b> 27 to 18                       | <b>c</b> 84 to 48                       |
| <b>d</b> 100 to 28 to 4                   | <b>e</b> 12 to 24 to 18                 | <b>f</b> 36 to 48 to 72                 |
| <b>g</b> 1.6 to 1.9                       | <b>h</b> 1.6 to 1.8                     | <b>i</b> 0.56 to 0.32                   |
| <b>j</b> 0.93 to 0.6                      | <b>k</b> 0.256 to 0.8                   | <b>l</b> $\frac{1}{2}$ to $\frac{1}{3}$ |
| <b>m</b> $1\frac{1}{2}$ to $2\frac{1}{2}$ | <b>n</b> $\frac{5}{8}$ to $\frac{2}{3}$ | <b>o</b> 25 cm to 1.1 m                 |
| <b>p</b> 2.2 kg to 850 g                  | <b>q</b> \$1.50 to 80 cents             | <b>r</b> $1\frac{1}{2}$ h to 40 min     |
| <b>s</b> 6 min to 2.1 h                   | <b>t</b> 600 mL to 1.5 L to 2.1 L       | <b>u</b> 40 g/L to 3 mg/mL to 2 mg/mL   |

Ratios generally contain whole numbers only. However, it can be useful to write them in the form  $n : 1$  or  $1 : n$  where  $n$  is not necessarily a whole number. This provides a direct comparison of how much bigger or smaller one quantity is than the other.



**EXAMPLE 3A-3** Expressing ratios in the form  $n : 1$  or  $1 : n$

- a** Express the ratio 25 : 10 in the form  $n : 1$ .  
**b** Express the ratio 24 : 16 in the form  $1 : n$ .

	Solve	Think	Apply
<b>a</b>	$25 : 10 = \frac{25}{10} : \frac{10}{10}$ $= 2.5 : 1$	To obtain 1 as the second number of the ratio and maintain an equivalent ratio, divide each number by 10. This means that the first number is 2.5 times the second.	To express the ratio in the form $n : 1$ , divide each term by the second number.
<b>b</b>	$24 : 16 = \frac{24}{24} : \frac{16}{24}$ $= 1 : \frac{2}{3} \text{ or } 1 : 0.\dot{6}$	To obtain 1 as the first number of the ratio and maintain an equivalent ratio, divide each number by 24. This means that the second number is $\frac{2}{3}$ (or $0.\dot{6}$ ) times the first.	To express the ratio in the form $1 : n$ , divide each term by the first number.

- 4** Express each of these ratios in the form  $n : 1$ .  
**a** 35 : 10                      **b** 72 : 40                      **c** 24 : 60                      **d** 72 : 80
- 5** Write each comparison as a ratio in the form  $n : 1$ . Explain the meaning of the answer.  
**a** 3.5 m to 70 cm                      **b** 1.2 kg to 800 g  
**c**  $2\frac{1}{2}$  cups to  $\frac{1}{2}$  cup                      **d** 0.04 ha to 500 m<sup>2</sup>
- 6** Express each of these ratios in the form  $1 : n$ .  
**a** 50 : 87                      **b** 40 : 90                      **c** 60 : 48                      **d** 125 : 8
- 7** Write each comparison as a ratio in the form  $1 : n$ . Explain the meaning of the answer.  
**a** 1 cm to 1 m                      **b** 1 mm to 1 m                      **c** 20 mL to 1 L                      **d** 50 mg to 15 g

**EXAMPLE 3A-4** Working with equivalent ratios (unknown quantity in numerator)

The ratio of the number of boys to girls in a school is 7 : 6. If there are 354 girls in the school, how many boys are there?

Solve	Think	Apply
number of boys : number of girls $= 7 : 6$ $\frac{\text{number of boys}}{\text{number of girls}} = \frac{7}{6}$ $\frac{\text{number of boys}}{354} = \frac{7}{6}$ number of boys $= \frac{7}{6} \times 354$ $= 413$ There are 413 boys.	Write the two equivalent ratios as fractions. number of boys : number of girls $= \frac{\text{number of boys}}{\text{number of girls}}$ $7 : 6 = \frac{7}{6}$ As the two equivalent ratios are equal, write as an equation. Solve the equation by multiplying both sides by 354.	Using fraction notation, write two equivalent ratios. (Remember the order of terms is important.) Treat this as an equation and solve.

- 8 The ratio of the number of boys to girls in a school is 9 : 8 and there are 312 girls at the school. Complete the following to find the number of boys.

$$\frac{\text{number of boys}}{\square} = \frac{9}{8}$$

$$\text{number of boys} = \frac{9}{8} \times \underline{\quad}$$

$$= \underline{\quad}$$

- 9 At an electrical store, the ratio of profit to sales is 2 : 7. Given that the annual sales for the year were \$145 600, what was the annual profit?
- 10 The ratio of Ben's net salary to the tax he pays is 10 : 3. Find his net salary, if he paid \$14 580 in tax for one year.

**EXAMPLE 3A-5** Working with equivalent ratios (unknown quantity in denominator)

The ratio of boys to girls in a school is 8 : 7. If there are 264 boys in the school, how many girls are there?

Solve	Think	Apply
$\frac{264}{\text{number of girls}} = \frac{8}{7}$ $\frac{\text{number of girls}}{264} = \frac{7}{8}$ number of girls $= \frac{7}{8} \times 264$ $= 231$ There are 231 girls.	Write two equivalent ratios. number of boys : number of girls = 8 : 7 Change the order so the number of girls is the first term in the ratio (and hence the numerator of the fraction). number of girls : number of boys = 7 : 8 As the two equivalent ratios are equal, write as an equation using fractions. Solve the equation by multiplying both sides by 264.	Using fraction notation, write two equivalent ratios. Change the order so that the unknown is the first term of the ratio and appears in the numerator of the fraction. Treat this as an equation and solve.

- 11** The ratio of males to females at a basketball match is 10:9. Complete the following to find the number of females, given that there are 470 males at the match:

$$\frac{\square}{\text{number of females}} = \frac{10}{9} \text{ so } \frac{\text{number of females}}{\square} = \frac{9}{10}$$

$$\text{number of females} = \frac{9}{10} \times \text{---} = \text{---}$$

- 12** The ratio of a daughter's height to that of her mother is 4:5. What is the mother's height, if her daughter is 132 cm tall?

- 13** The instructions on a pesticide product recommend a ratio of concentrated pesticide to water of 1:49.

- a** How many litres of water are required in a mixture with 2 L of concentrated pesticide?  
**b** How many millilitres of concentrated pesticide are required in a mixture with 10 L of water?

- 14** The instructions on a fertilising product recommend a ratio of concentrated fertiliser to water of 2:7.

- a** How many litres of water are required in a mixture with 10 L of concentrated fertiliser?  
**b** How much fertiliser mixture has been made?

- 15** A two-stroke fuel mixture is made by mixing oil and petrol in the ratio of 1:25.

- a** How many litres of petrol should be added to 750 mL of oil?  
**b** How many millilitres of oil should be added to 18 L of petrol?

- 16** A builder wants to make concrete in the ratio 1:2:3; that is, one part cement, two parts sand and three parts screenings.

- a** If the builder buys four bags of sand, how many bags of cement and screenings are needed?  
**b** If the builder only uses half a bag of cement, how many bags of sand and screenings are used?

### EXAMPLE 3A-6 Dividing a quantity in a given ratio

An inheritance of \$24 000 is to be divided between Sam and Jamie in the ratio 2:3. How much will each receive?

Solve	Think	Apply
The money needs to be divided into $2 + 3 = 5$ parts. Sam receives $\frac{2}{5} \times \$24\,000 = \$9600$ . Jamie receives $\frac{3}{5} \times \$24\,000 = \$14\,400$ .	For every \$2 that Sam receives, Jamie receives \$3. So Sam receives \$2 out of every \$5, or $\frac{2}{5}$ of the inheritance, and Jamie receives \$3 out of every \$5, or $\frac{3}{5}$ of the inheritance.	Add the terms of the ratio. This is the number of parts into which the quantity is to be divided. The appropriate fraction of the quantity can then be found.

- 17** If \$30 000 is to be divided between Hannah and Rachel in the ratio 5:3, how much will each girl receive?
- 18** If \$1500 is divided in the ratio 3:1, how much is the larger share?
- 19** Cordial and water are mixed in the ratio 1:8 to make a fruit drink. How much cordial and water would there be in a 180 mL glass of fruit drink?

- 20** A metal solder is made by combining lead and tin in the ratio 2 : 3. How much tin is needed to make 10 kg of the metal solder?
- 21** The masses of Alison, Vincent and Matthew are in the ratio 4 : 3 : 5. If their combined mass is 168 kg, find the mass of each person.
- 22** Statistics were gathered on the number of people who develop lung cancer according to their smoking habits. The results are shown in the table below.

Category	Number who develop lung cancer	Ratio (1 : <i>n</i> )
Males who have never smoked	10 in 760	1 : 76
Males who have smoked	25 in 300	
Males who currently smoke	20 in 90	
Females who have never smoked	3 in 471	
Females who have smoked	50 in 1150	
Females who currently smoke	15 in 132	

- a** Complete the table to express the risk, in the form 1 : *n*, of developing lung cancer.
- b** Which group of people is most likely to develop lung cancer?
- c** Which group of people is least likely to develop lung cancer?
- 23** A farmer planted lemon trees and orange trees in orchards in the ratio 2 : 3.
- a** In one orchard there are 60 orange trees. How many lemon trees were planted?
- b** In another orchard there are 18 lemon trees. How many orange trees were planted?
- c** How many trees did the farmer plant in total?
- d** What percentage of the planted trees are lemon trees? Explain how this relates to the ratio 2 : 3.
- 24** An average residential house construction will have labour costs of about 30% of the total project costs with materials making up the rest.
- a** Write the comparison of the cost of labour to the cost of materials as a ratio.
- b** If the labour costs for a project are \$87 000, what is the cost of materials?
- c** Calculate the labour costs for a house construction when the total cost of the project is \$420 000.
- 25** A spray solution of pesticide is made using 10 mL of concentrate per 5 L of water.
- a** Write the dilution ratio for concentrate to water in a spray solution.
- b** How much concentrate and how much water are needed to fill a 100 L spray tank with solution?
- 26** An investment fund has investments in property, shares and government bonds in the ratio 5 : 3 : 2.
- a** If the fund has a total of \$1.8 million invested, find the amount in each of these three investments.
- b** During the next year, the fund manager transfers \$160 000 from investments in government bonds to shares. Find the new ratio of investments in this fund.

- 27** Kristin and Daniel are to share \$100 between them in either the ratio 5 : 4 or 6 : 5. Which ratio should Kristin choose so as to obtain the larger payment? Explain your answer.
- 28** To create two distinct shades of pink paint, white and red paint is mixed in the ratios of 3 : 1 and 7 : 1. A 4 L can of each shade is purchased.
- a** Calculate the amount (in mL) of white and red paint required for each shade.
- b** The two cans are accidentally mixed together. What is the new ratio of white to red paint?

### similar figures

pair of figures that have the same shape but one is an enlargement of the other

### scale factor

indicates how many times larger or smaller one of the pair of similar figures is than the other. A scale factor larger than 1 produces an enlargement (image is larger) and a scale factor between 0 and 1 produces a reduction (image is smaller)

# 3B Similar figures and scale factors

These resources are available on your obook access:

- **Interactive 3B:** Explore key ideas for similar figures
- **Assess quiz 3B:** Test your skills with an auto-correcting multiple-choice quiz

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IGI

Two figures are defined as **similar figures** if one is an enlargement of the other. An enlargement factor is the same as a **scale factor**.

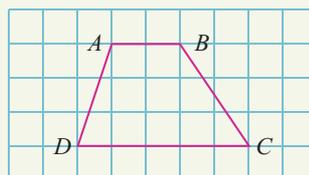
A pair of similar figures can be considered as an original figure and its image. The image is produced by enlarging (making larger) or reducing (making smaller) the original figure by a scale factor. Generally, it is assumed that the figure to the right of a pair of similar figures is the image. For example, a triangle  $ABC$  can be enlarged or reduced by a scale factor to produce triangle  $A'B'C'$ . The original figure  $ABC$  and its image  $A'B'C'$  are similar figures.

If the scale factor is greater than 1, the image is larger than the original figure.

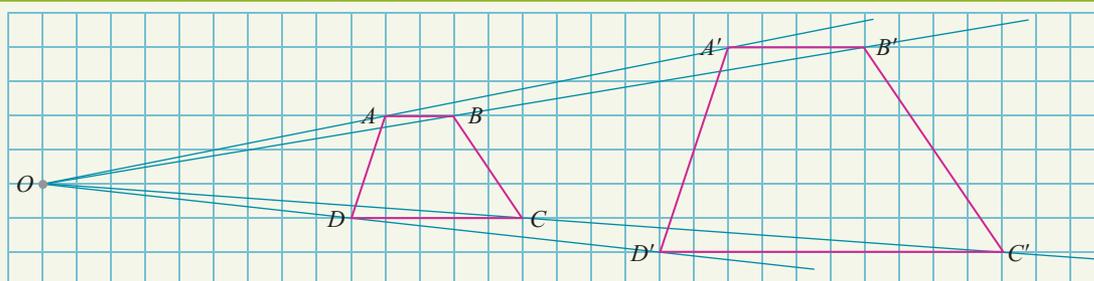
If the scale factor is between 0 and 1, the image is smaller than the original figure.

## EXAMPLE 3B-1 Enlarging a figure by a scale factor

On grid paper, enlarge this trapezium by a factor of 2.



### Solve



### Think

- Choose any convenient point  $O$  as the centre of enlargement.  $O$  may be inside or outside the trapezium.
- Draw lines from  $O$  through each of the vertices of the trapezium.
- Produce  $OA$  to  $OA'$  such that  $OA' = 2 \times OA$ .
- Produce  $OB$  to  $OB'$  such that  $OB' = 2 \times OB$ .
- Produce  $OC$  to  $OC'$  such that  $OC' = 2 \times OC$ .
- Produce  $OD$  to  $OD'$  such that  $OD' = 2 \times OD$ .
- Join  $A'B'C'D'$  to form the enlargement (image) of trapezium  $ABCD$ .

### Apply

To produce an enlargement, choose the centre of enlargement,  $O$ , and the enlargement (or scale) factor,  $k$ . The image of each point  $P$  is a point  $P'$  such that  $O, P$  and  $P'$  are in a straight line and  $OP' = k \times OP$ .  
*Note:* as  $k = 2$ , the image is an enlargement of the original figure.

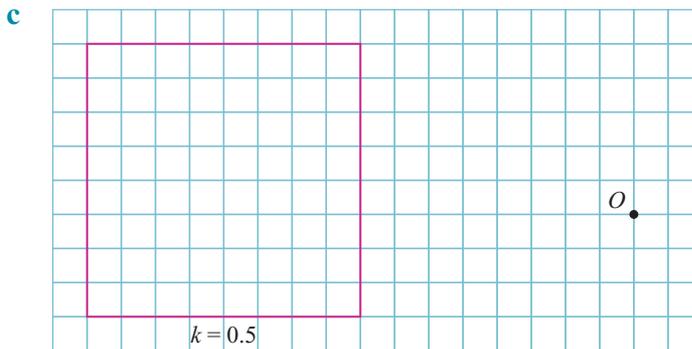
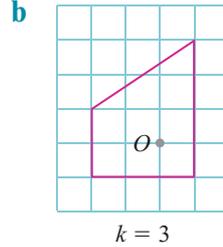
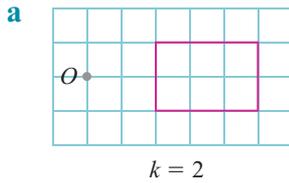
Note: trapezium  $A'B'C'D'$  and trapezium  $ABCD$  are similar figures.

Each angle and its image,  $\angle ABC$  and  $\angle A'B'C'$ ,  $\angle BCD$  and  $\angle B'C'D'$  etc., are called **corresponding angles**.

Each side and its image,  $AB$  and  $A'B'$ ,  $BC$  and  $B'C'$  etc., are called **corresponding sides**.

## EXERCISE 3B Similar figures and scale factors

- 1 On grid paper, copy and enlarge each figure using the centre of enlargement and scale factor ( $k$ ) given.



### corresponding angles

matching angles in a pair of similar figures, located in the same position when each figure is in the same orientation

### corresponding sides

matching sides in a pair of similar figures, located in the same position when each figure is in the same orientation

- 2 Look at the similar figures  $ABCD$  and  $A'B'C'D'$  in Example 3B-1.

a Use a protractor to measure and record the sizes of the angles of trapezium  $ABCD$ .

b Measure and record the sizes of the angles of trapezium  $A'B'C'D'$ .

c List the corresponding angle to these angles:

i  $\angle ABC$

ii  $\angle BCD$

iii  $\angle CDA$

iv  $\angle DAB$ .

d What do you notice about the corresponding angles in the two trapeziums?

e List the corresponding side to:

i  $AB$

ii  $BC$

iii  $CD$

iv  $DA$

f Complete the following statements by measurement and division.

i  $\frac{A'B'}{AB} = \underline{\hspace{2cm}}$

ii  $\frac{B'C'}{BC} = \underline{\hspace{2cm}}$

iii  $\frac{C'D'}{CD} = \underline{\hspace{2cm}}$

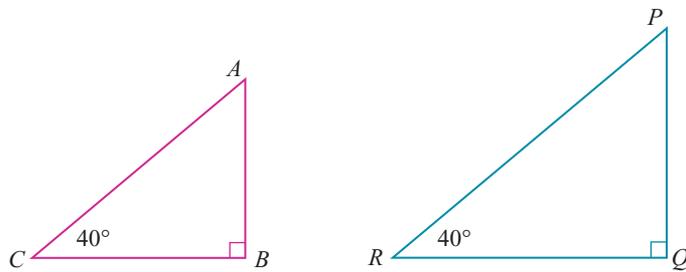
iv  $\frac{D'A'}{DA} = \underline{\hspace{2cm}}$

g What do you notice about the corresponding sides in the two trapeziums?

### Properties of similar figures

- The corresponding angles of similar figures are equal.
- The lengths of the corresponding sides of similar figures are in the same ratio (or proportion).
- The value of this ratio is equal to the scale factor  $k$ .
- For a pair of corresponding sides: scale factor  $k = \frac{\text{image length}}{\text{original length}}$ .
- When the scale factor is larger than 1, the image is larger than the original figure.
- When the scale factor is between 0 and 1, the image is smaller than the original figure.

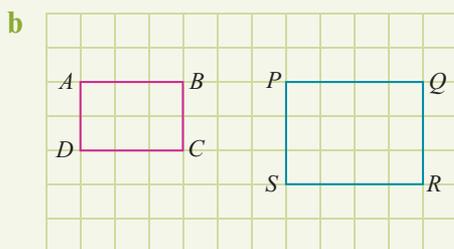
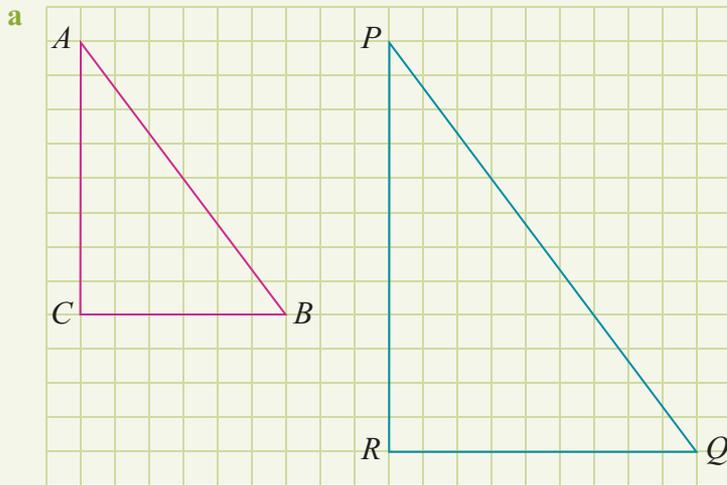
**3** Look at the following pair of triangles.



- a** Find the size of the remaining angle in each of these triangles.
- b** List the corresponding angle to:
  - i**  $\angle ACB$
  - ii**  $\angle CBA$
  - iii**  $\angle BAC$ .
- c** Are the corresponding angles equal?
- d** List the corresponding side to:
  - i**  $AB$
  - ii**  $AC$
  - iii**  $BC$ .
- e** Measure the lengths of the sides of the triangles and complete:  
 $\frac{PQ}{AB} = \text{---}$ ,  $\frac{PR}{AC} = \text{---}$ ,  $\frac{QR}{BC} = \text{---}$
- f** Are the ratios of the corresponding sides the same? What is this ratio?
- g** Are the triangles similar?

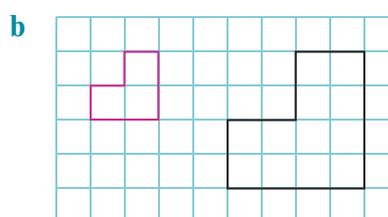
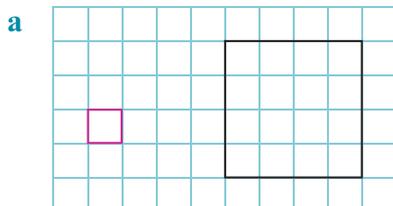
**EXAMPLE 3B-2** Determining whether two figures are similar

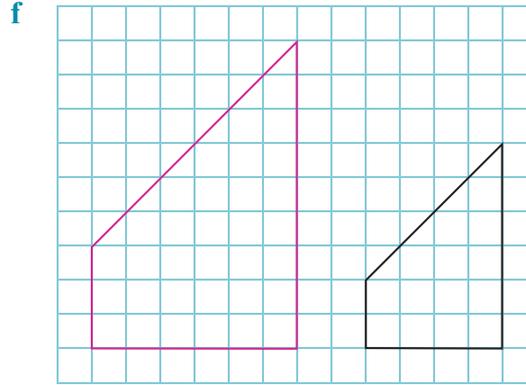
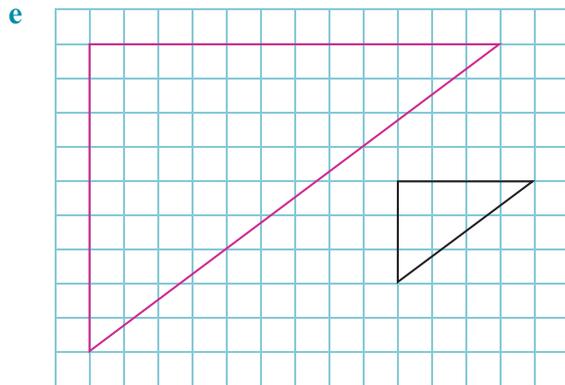
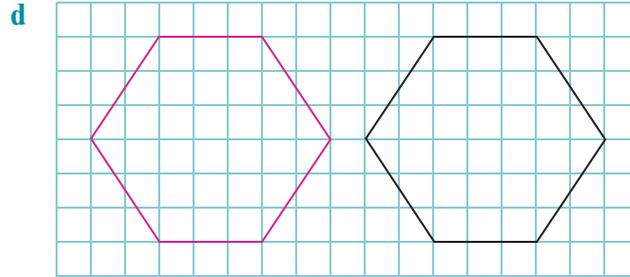
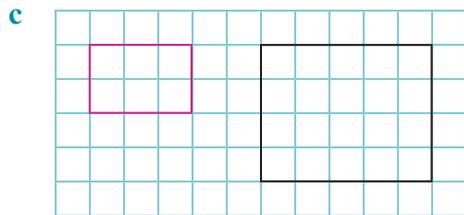
- i** Determine by measurement whether or not the following pairs of figures shown on grid paper are similar.
- ii** If they are similar, find the scale factor. The pink figure is the original figure in each case.



	Solve	Think	Apply
<b>a</b>	<p><b>i</b></p> $\angle ABC = \angle PQR (= 53^\circ)$ $\angle BCA = \angle QRP (= 90^\circ)$ $\angle CAB = \angle RPQ (= 37^\circ)$ $\frac{QR}{BC} = \frac{RP}{CA} = \frac{PQ}{AB} (= 1.5)$ The angles are equal and all pairs of corresponding sides are in proportion (in the same ratio). Hence, the figures are similar.	Measure all the angles. Measure the sides. Determine the ratio of the pairs of corresponding side lengths. Remember that the order of numbers in a ratio is important.	Two figures are similar if the corresponding angles are equal and the corresponding sides are in proportion.
	<p><b>ii</b></p> $k = \frac{QR}{BC}$ (or $\frac{RP}{CA}$ or $\frac{PQ}{AB}$ ) $= \frac{9}{6}$ $= 1.5$	If the figures are similar, the scale factor is the ratio of the lengths of any pair of corresponding sides. Choosing $QR$ and $BC$ , the ratio is $9:6$ or $\frac{9}{6}$ . This means that the length of each side of triangle $PQR$ is 1.5 times the length of each corresponding side of triangle $ABC$ . Alternatively, we can consider the ratio $\frac{BC}{QR}$ (or $\frac{CA}{RP}$ or $\frac{AB}{PQ}$ ) which is $\frac{2}{3}$ . This means that each side of triangle $ABC$ is $\frac{2}{3}$ times the length of each corresponding side of triangle $PQR$ .	Determine the value of the ratio of any pair of corresponding sides.
<b>b</b>	<p><b>i</b></p> $\angle ABC = \angle PQR (= 90^\circ)$ $\angle BCD = \angle QRS (= 90^\circ)$ $\angle CDA = \angle RSP (= 90^\circ)$ $\angle DAB = \angle SPQ (= 90^\circ)$ $\frac{PQ}{AB} = \frac{4}{3}, \frac{QR}{BC} = \frac{3}{2},$ $\frac{RS}{CD} = \frac{4}{3}, \frac{PS}{AD} = \frac{3}{2}$ The sides are not in proportion. Hence, the figures are not similar.	List the value of the corresponding angles. All the angles of both figures are equal ( $90^\circ$ ). Determine the ratio of the corresponding sides.	Two figures are similar if the corresponding angles are equal and the corresponding sides are in proportion.
	<p><b>ii</b></p> The figures are not similar, so there is no scale factor.		

- 4**
- i** Determine whether or not the following pairs of figures shown on grid paper are similar. Give reasons for your answer.
- ii** If they are similar, find the scale factor,  $k$ . The pink figure is the original figure in each case.

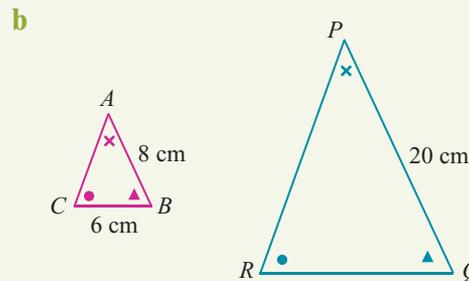
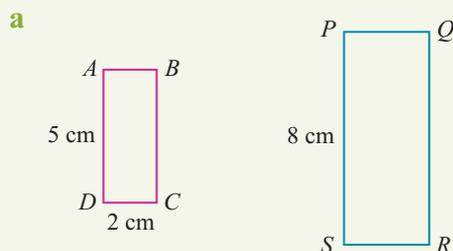




Note: figures that have a scale factor of 1 are called congruent figures.

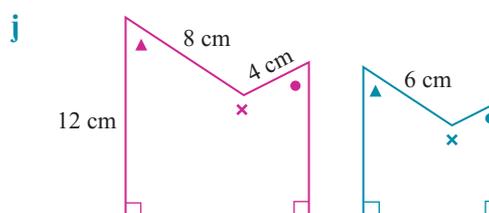
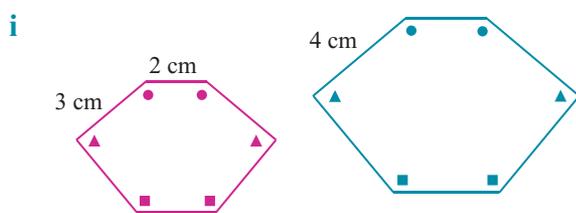
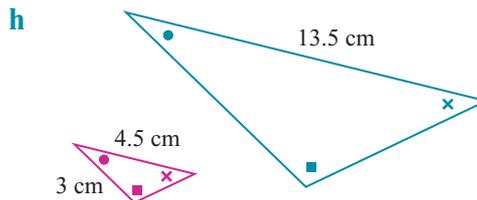
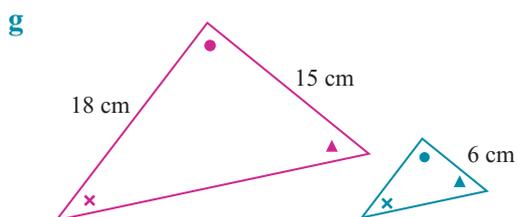
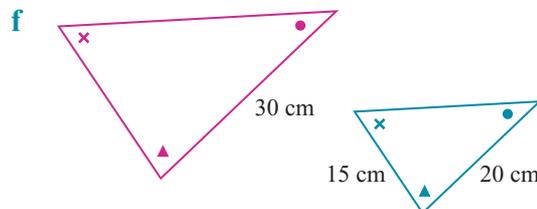
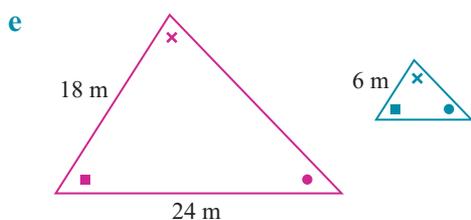
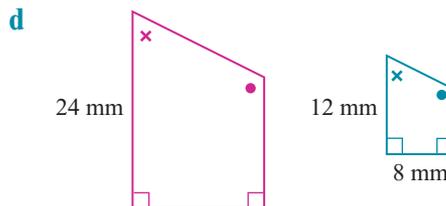
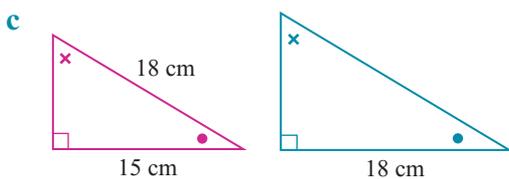
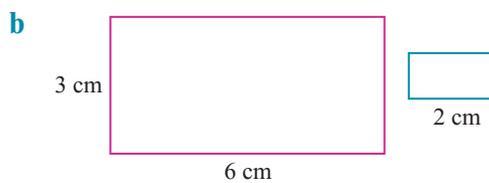
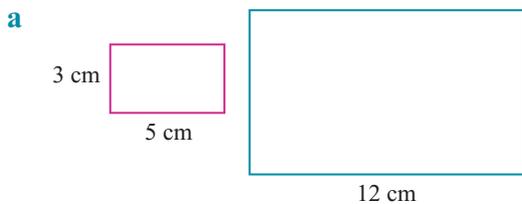
### EXAMPLE 3B-3 Finding the scale factor for similar figures

Given that the following pairs of figures are similar, find the scale factor. The pink figure is the original figure in each pair of similar figures.



Solve	Think	Apply
<p><b>a</b></p> $k = \frac{PS}{AD}$ $= \frac{8}{5}$ $= 1.6$	<p><math>PS</math> and <math>AD</math> are corresponding sides (<math>PS</math> is the enlargement of <math>AD</math>).</p> <p>Scale factor <math>k = \frac{\text{image length}}{\text{original length}} = \frac{PS}{AD}</math></p> <p>or the ratio of corresponding side lengths is <math>PS:AD = \frac{PS}{AD}</math>.</p>	<p>Identify a pair of corresponding sides with known lengths.</p> <p>The scale factor is the ratio of the lengths of this pair of corresponding sides.</p> <p>When <math>k &gt; 1</math>, the image produced is larger than the original figure.</p> <p>When <math>0 &lt; k &lt; 1</math>, the image produced is smaller than the original figure.</p>
<p><b>b</b></p> $k = \frac{PQ}{AB}$ $= \frac{20}{8}$ $= 2.5$	<p><math>PQ</math> and <math>AB</math> are corresponding sides.</p> <p>Scale factor <math>k = \frac{\text{image length}}{\text{original length}} = \frac{PQ}{AB}</math> or the ratio of corresponding side lengths is <math>PQ:AB = \frac{PQ}{AB}</math>.</p>	<p>Identify a pair of corresponding sides with known lengths.</p> <p>The scale factor is the ratio of the lengths of this pair of corresponding sides.</p> <p>When <math>k &gt; 1</math>, the image produced is larger than the original figure.</p> <p>When <math>0 &lt; k &lt; 1</math>, the image produced is smaller than the original figure.</p>

**5** Given that the following pairs of figures are similar, find the scale factor. The pink figure is the original figure in each pair of similar figures.

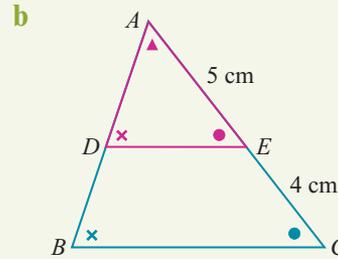
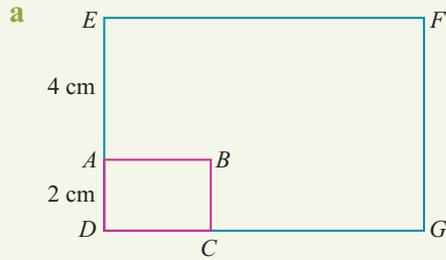


**6** Consider your answers to question 5.

- For which pairs of similar figures does the scale factor produce an enlargement (image is larger)?
- For which pairs of similar figures does the scale factor produce a reduction (image is smaller)?
- What do you notice about the size of the scale factor that produces an image that is larger than the original figure?
- What do you notice about the size of the scale factor that produces an image that is smaller than the original figure?

**EXAMPLE 3B-4** Finding the scale factor for similar figures when one is within the other

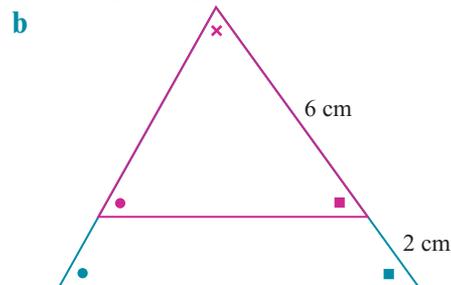
Determine the scale factor of the following pairs of similar figures. The pink figure is the original figure in each pair of similar figures.

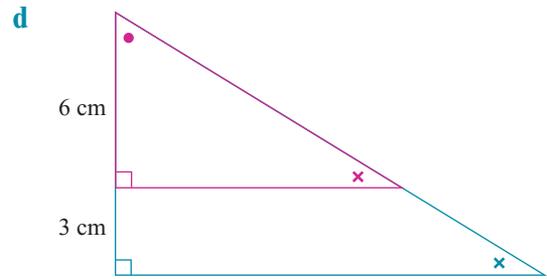
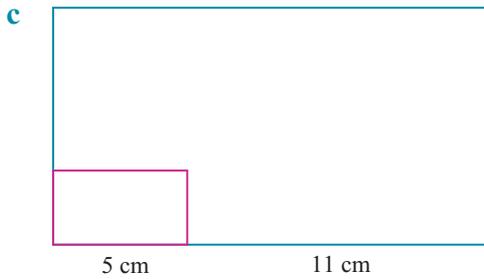


Rectangle  $EFGD$  is an enlargement of rectangle  $ABCD$ . Triangle  $ABC$  is an enlargement of triangle  $ADE$ .

Solve	Think	Apply
<p><b>a</b></p> $k = \frac{ED}{AD}$ $= \frac{6}{2}$ $= 3$	<p>Redraw the rectangles next to each other:</p> <p>Known corresponding side lengths are <math>AD = 2</math> cm and <math>ED = 6</math> cm. Scale factor <math>k = \frac{\text{image length}}{\text{original length}} = \frac{ED}{AD}</math>.</p>	<p>When one of the similar figures is within the other, separate them and draw one next to the other. Then identify a pair of corresponding sides and determine the scale factor using the ratio <math>\frac{\text{image length}}{\text{original length}}</math>.</p>
<p><b>b</b></p> $k = \frac{AC}{AE}$ $= \frac{9}{5}$ $= 1.8$	<p>Redraw the triangles next to each other:</p> <p>Known corresponding side lengths are <math>AE = 5</math> cm and <math>AC = 9</math> cm. Scale factor <math>k = \frac{\text{image length}}{\text{original length}} = \frac{AC}{AE}</math>.</p>	

**7** Determine the scale factor for each case. The pink figure is the original figure in each pair of similar figures.





PROBLEM SOLVING, REASONING AND JUSTIFICATION

- 8** A photograph 129 mm by 90 mm is enlarged to 172 mm by 120 mm. What is the enlargement factor?
- 9** There is a range of standard print sizes for photographs. Use the list on the right to compare the range of photos and find which are similar figures.

	Length	Width
<b>A</b>	15 cm	10 cm
<b>B</b>	17.5 cm	12.5 cm
<b>C</b>	20 cm	15 cm
<b>D</b>	25 cm	20 cm
<b>E</b>	30 cm	20 cm



- 10** Todd got a model plane for his birthday, which had a scale factor of  $\frac{1}{100}$ . What does this scale factor mean?
- 11** A photo is projected onto a screen. If the height of the photo is 10 cm and the height of its image on the screen is 1.2 m, what is the scale factor?
- 12** On a map, the length of a straight line between two landmarks represents the real distance between the landmarks. If the ratio of map length to real distance is 1 : 5000, what is the scale factor used to produce the map?

PRACTICAL TASKS

- 13** You will need an overhead projector, a transparency sheet, a black marker pen, a ruler and measuring tape.
- a** Draw a line 100 mm long on the transparency sheet and place it on the overhead projector. Focus the image on the wall. Measure the length of the image on the wall, in millimetres.
- b** Calculate the scale factor:  $k = \frac{\text{length of image}}{\text{length of object}} = \frac{\text{length of image}}{100} = \underline{\hspace{2cm}}$ .
- Note:* the 100-mm-long line is the object.
- c** Measure the distance of the object from the wall and record your results for parts **b** and **c** in a table.

Distance from wall						
Scale factor						

- d** Move the projector further away from the wall and focus the image.
- e** Measure the distance of the object from the wall and the length of the image. Calculate the scale factor and record the results in your table.
- f** Repeat parts **d** and **e** four times to complete your table.
- g** How is the scale factor related to the distance of the object from the wall?

## 3C

# Finding lengths of sides in similar figures

These resources are available on your obook assess:

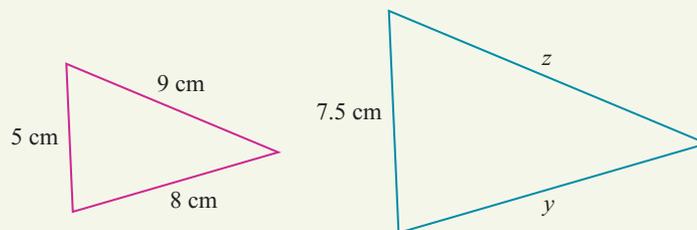
- **Interactive 3C:** Explore finding unknown side lengths in similar figures
- **Video tutorial 3C:** Watch and listen to an explanation of Example 3C-1
- **Worksheet 3C:** Practise your skills with extra problems on similar figures
- **Investigation 3C:** Calculate unknown heights using a mirror and similar triangles
- **Assess quiz 3C:** Test your skills with an auto-correcting multiple-choice quiz

10  
10  
10

The scale factor for a pair of similar figures indicates how much larger or smaller the lengths of the sides of the image will be, compared to the side lengths of the original figure. This scale factor can be used to help us calculate the length of an unknown side, if we know the length of the corresponding side.

## EXAMPLE 3C-1 Finding the length of unknown sides in the image

Given that these triangles are similar, find the length of the unknown sides labelled with a pronumeral. The pink figure is the original figure in the pair of similar triangles.



Solve	Think	Apply
<p><i>Method 1</i></p> $k = \frac{7.5}{5} = 1.5$ <p>So <math>y = 1.5 \times 8 = 12</math> cm and <math>z = 1.5 \times 9 = 13.5</math> cm</p> <p><i>Method 2</i></p> $\frac{y}{8} = \frac{7.5}{5}$ $y = \frac{7.5}{5} \times 8$ $= 12$ $\frac{z}{9} = \frac{7.5}{5}$ $z = \frac{7.5}{5} \times 9$ $= 13.5$ <p>So <math>y = 12</math> cm and <math>z = 13.5</math> cm.</p>	<p><i>Method 1</i></p> <p>Choose a pair of corresponding sides with known lengths (5 cm and 7.5 cm). Working from left (original figure) to right (image), the scale factor <math>k = \frac{7.5}{5} = 1.5</math>. So each side in the second triangle is 1.5 times the length of its corresponding side in the first triangle. Image length = 1.5 <math>\times</math> original length</p> <p><i>Method 2</i></p> <p>Identify the lengths of the corresponding sides in the two triangles: 7.5 cm and 5 cm, <math>y</math> and 8 cm, <math>z</math> and 9 cm. Take the ratio of each pair, in the same order (side length of second triangle : length of corresponding side of first triangle), and equate them to find the values of the pronumerals. (For similar triangles, the ratios of the lengths of the corresponding sides are equal.)</p>	<p><i>Method 1</i></p> <p>Find the scale factor using the known lengths of a pair of corresponding sides. Apply this scale factor to the side corresponding to the unknown side.</p> <p><i>Method 2</i></p> <p>Identify the pairs of corresponding sides. Take the ratio of each pair in the same order. Equate the ratios and solve to find the value of the pronumeral.</p>

To find the unknown length of side  $A'B'$ :  
 Length of  $A'B'$  = scale factor  $\times$  length of  $AB$   
 or image length = scale factor  $\times$  original length.

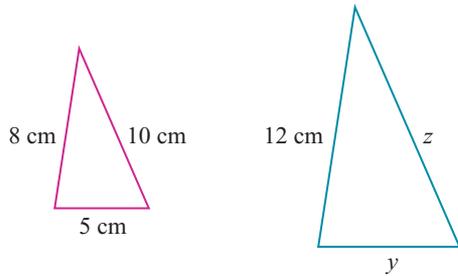


## EXERCISE 3C Finding lengths of sides in similar figures

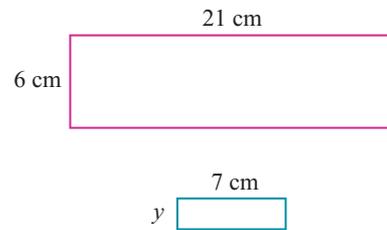
UNDERSTANDING, FLUENCY AND COMMUNICATING

- 1 For the following pairs of similar figures, find the length of the unknown sides labelled with a pronumeral. The pink figure is the original figure in each pair of similar figures.

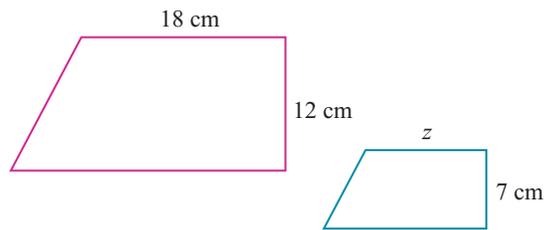
a



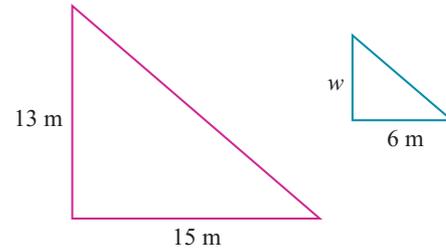
b



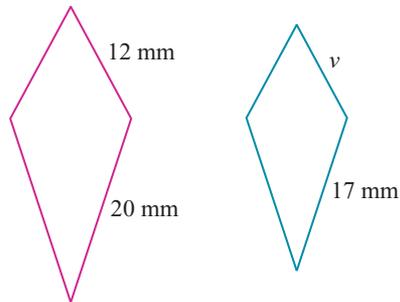
c



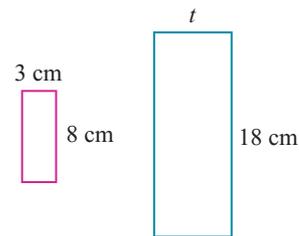
d



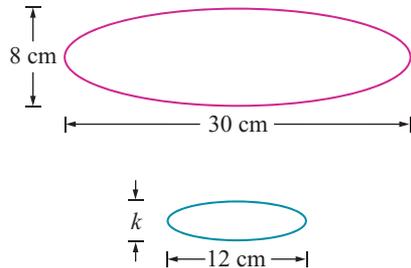
e



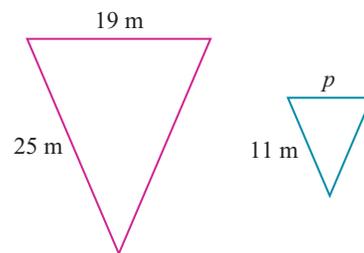
f



g

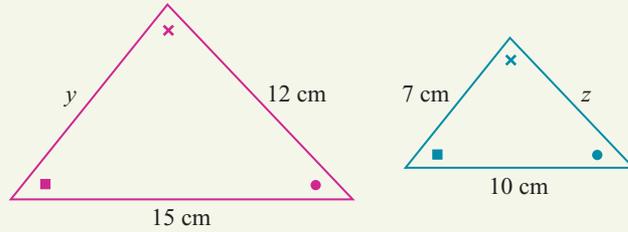


h



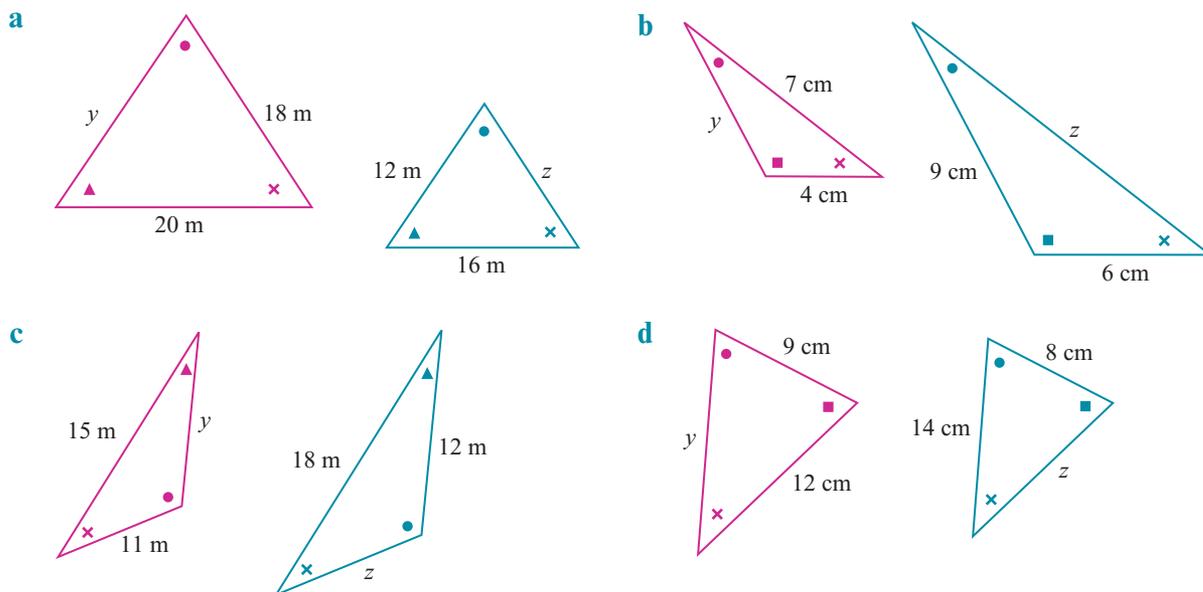
### EXAMPLE 3C-2 Finding the length of unknown sides in similar triangles

For the following pair of similar triangles, find the length of the unknown sides labelled with a pronumeral.



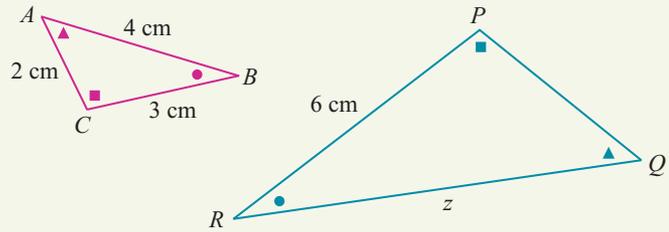
Solve	Think	Apply
<p><i>Method 1</i></p> <p>Scale factor <math>k = \frac{10}{15} = \frac{2}{3}</math></p> <p><math>z = \frac{2}{3} \times 12 = 8</math> cm</p> <p><math>7 = \frac{2}{3} \times y</math></p> <p><math>21 = 2y</math></p> <p><math>y = 10.5</math> cm</p> <p><i>Method 2</i></p> <p><math>\frac{z}{12} = \frac{10}{15}</math></p> <p><math>z = \frac{10}{15} \times 12</math></p> <p><math>= 8</math></p> <p><math>\frac{7}{y} = \frac{10}{15}</math></p> <p><math>7 = \frac{10}{15} \times y</math></p> <p><math>7 \times 15 = 10y</math></p> <p><math>10y = 105</math></p> <p><math>y = 10.5</math></p> <p>So <math>z = 8</math> cm and <math>y = 10.5</math> cm.</p>	<p><i>Method 1</i></p> <p>Scale factor <math>k = \frac{10}{15}</math> or <math>\frac{2}{3}</math>, so each side in the second triangle is <math>\frac{2}{3}</math> times the length of the corresponding side in the first triangle.</p> <p>Image length = <math>\frac{2}{3} \times</math> original length</p> <p><i>Method 2</i></p> <p>Identify the lengths of the corresponding sides in the two triangles: 10 cm and 15 cm, <math>z</math> and 9 cm, 7 cm and <math>y</math>.</p> <p>Take the ratio of each pair, in the same order (side length of second triangle : length of corresponding side in first triangle), and equate them to find the values of the pronumerals.</p> <p>To calculate <math>y</math> more easily, we could have formed the ratios in the opposite order (side length of first triangle : length of corresponding side in second triangle).</p> <p><math>\frac{y}{7} = \frac{15}{10}</math> or <math>y = \frac{15}{10} \times 7</math> or <math>y = 10.5</math></p>	<p><i>Method 1</i></p> <p>Find the scale factor using the known lengths of a pair of corresponding sides. Apply this scale factor to the side length in the second triangle corresponding to the side length in the first triangle.</p> <p><i>Method 2</i></p> <p>Identify the pairs of corresponding sides. Take the ratio of each pair in the same order. Equate the ratios and solve to find the value of the pronumeral.</p> <p>The most important point to remember is to form the ratios in exactly the same order before equating them.</p>

2 For the following pairs of similar triangles, find the length of the unknown sides labelled with a pronumeral.



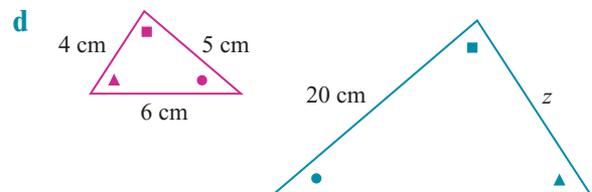
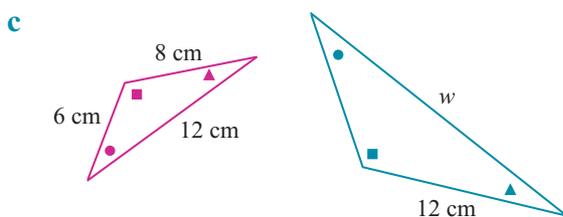
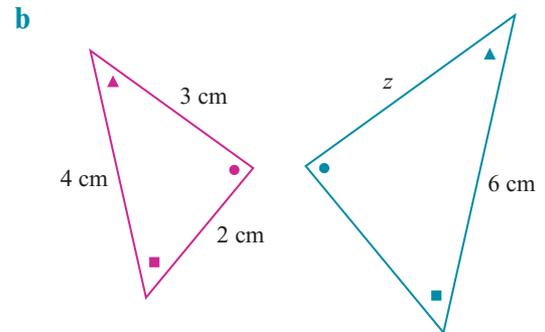
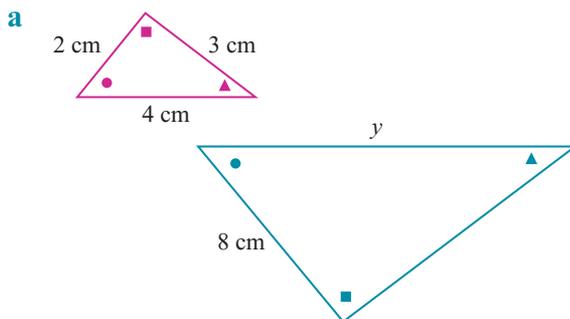
### EXAMPLE 3C-3 Finding the length of an unknown side for similar triangles in different orientation

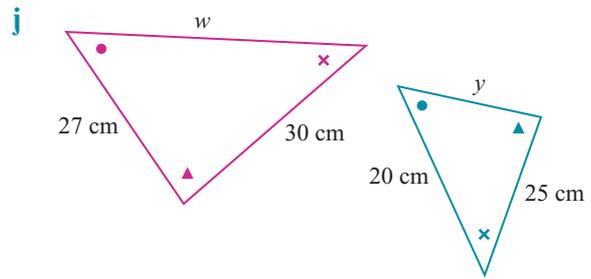
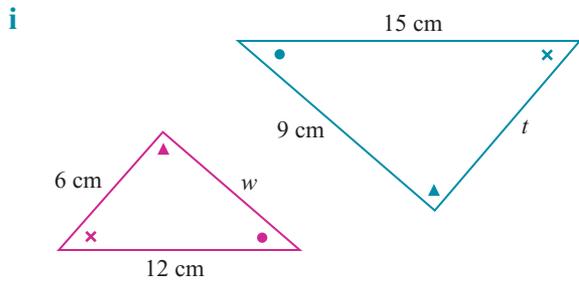
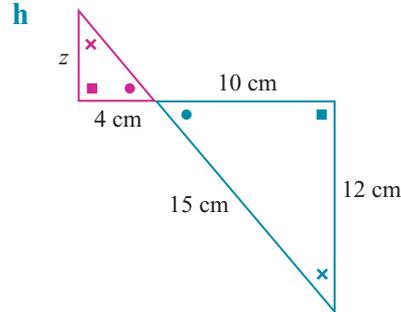
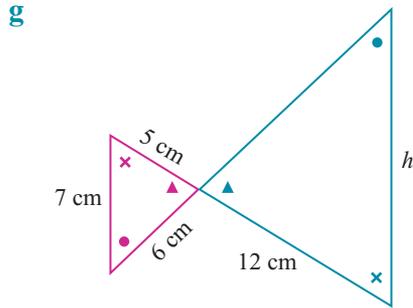
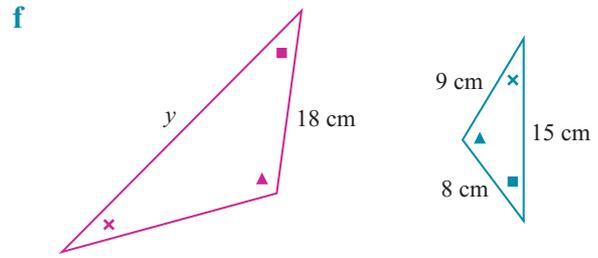
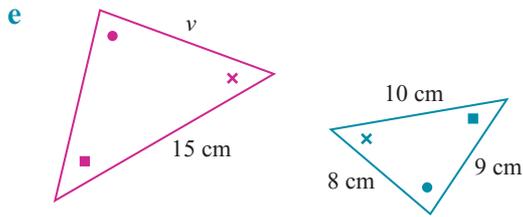
For the following pair of similar figures, find the length of the unknown side labelled with a pronumeral.



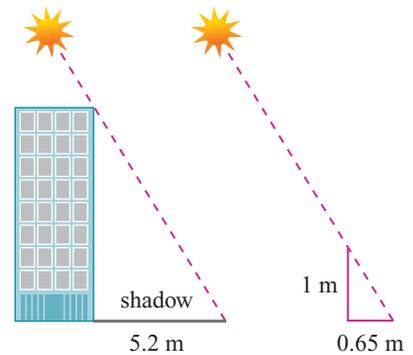
Solve	Think	Apply
<p><i>Method 1</i></p> <p>Scale factor:</p> $k = \frac{6}{3}$ $= 2$ $z = 2 \times 4$ $= 8 \text{ cm}$ <p><i>Method 2</i></p> $\frac{z}{4} = \frac{6}{3}$ $z = \frac{6}{3} \times 4$ $= 8 \text{ cm}$	<p><i>Method 1</i></p> <p>Working from left (original figure) to right (image), the scale factor is <math>\frac{6}{3}</math> or 2. So each side of the second triangle is two times the length of the corresponding side of the first triangle.</p> <p><i>Method 2</i></p> <p>The corresponding sides are opposite the pairs of equal angles, as shown in the diagram above. Hence <math>RP</math> and <math>BC</math>, <math>PQ</math> and <math>CA</math>, <math>QR</math> and <math>AB</math> are pairs of corresponding sides. Take the appropriate ratios of the corresponding sides:</p> $\frac{QR}{AB} = \frac{RP}{BC}$	<p><i>Method 1</i></p> <p>Identify pairs of corresponding sides using the pairs of equal angles. Find the scale factor using the known lengths of a pair of corresponding sides, taken in the appropriate order. Apply this scale factor to the side corresponding to the unknown side.</p> <p><i>Method 2</i></p> <p>Identify pairs of corresponding sides using the pairs of equal angles.</p>

3 For the following pairs of similar triangles, find the lengths of the unknown sides labelled with a pronumeral.





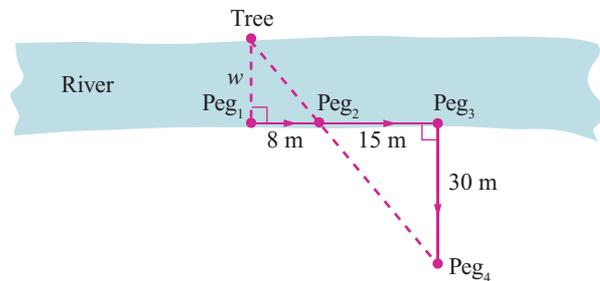
- 4** A photograph 15 cm long by 10 cm wide is enlarged so that the new length is 18 cm. What is the new width of the photograph?
- 5** A building casts a shadow 5.2 m long. At the same time, a metre rule casts a shadow 0.65 m long. Find the height of the building.



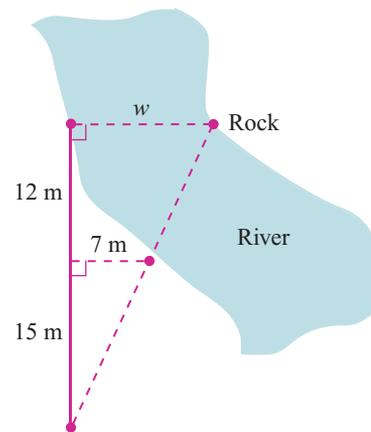
- 6** A man 1.8 m tall casts a shadow 3 m long. At the same time, his son's shadow is 2.5 m long.
- a** Draw a diagram showing a pair of similar triangles to illustrate this scenario.
- b** How tall is the son?

- 7** The shadow cast by a flagpole is 11 m long. At the same time, the shadow of a 30 cm ruler is 55 cm. Calculate the height of the flagpole.

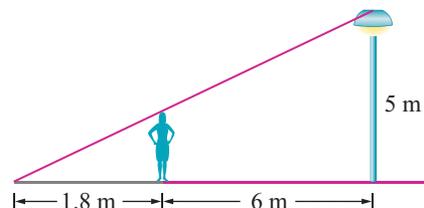
- 8** Jane wanted to find the width of a river without crossing it. She did this by placing four pegs and measuring the distances, as shown in the diagram on the right. Find the width of the river.



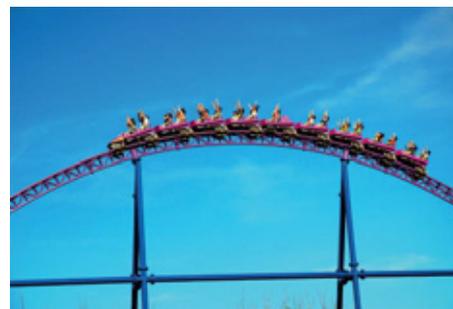
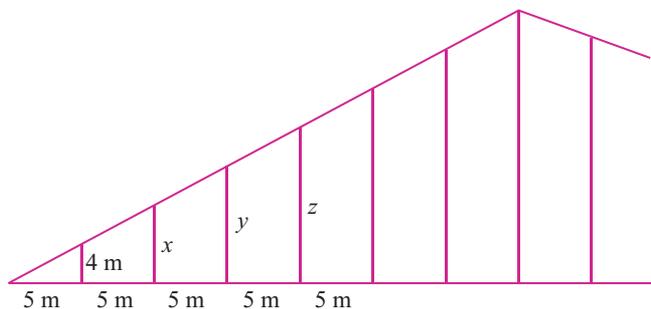
- 9 At another point on the river Jane made the measurements shown in this diagram. Calculate the width of the river,  $w$ , at this point.



- 10 A street lamp is 5 m high. When a girl stands 6 m from the base of the lamp her shadow is 1.8 m long. Calculate the height of the girl, to the nearest centimetre.



- 11 The supports on a straight section of a roller coaster ride in an amusement park are placed 5 m apart, as shown. If the length of the first support is 4 m, find the lengths of the next three supports.



- 12 Use the method applied in questions 5–7 to find the height of buildings, trees, the flagpole, etc. at your school.
- Measure the length of the shadow of the building, tree, flagpole, etc.
  - Measure the length of the shadow of a metre rule, at the same time.
  - Draw a diagram to illustrate the data.
  - Calculate the unknown height.
- 13 Use the method and a diagram similar to that shown in question 8 to find the width of the road at the front of your school, without crossing the road.
- Select an obvious feature on the opposite side of the road.
  - Place a peg or marker directly opposite this feature.
  - Walk (and measure) a convenient distance along the edge of the road and place another peg.
  - Walk (and measure) another convenient distance further along the road and place the third peg.
  - Walk at right angles to the road until the feature on the opposite side of the road is in line with the second peg (part c). Measure the distance walked to this point.
  - Draw a diagram and put all the measurements on it.
  - Use the properties of similar figures to calculate the width of the road.

# 3D Scale drawings

1010

These resources are available on your obook assess:

- **Interactive 3D:** Explore the key idea of scale
- **Video tutorial 3D:** Watch and listen to an explanation of Examples 3D-1 and 3D-2
- **Worksheet 3D:** Practise finding scale
- **Investigation 3D.1:** Using scale to calculate distances from a map
- **Investigation 3D.2:** Using ratios and scale to investigate travel by motorbike
- **assess quiz 3D:** Test your skills with an auto-correcting multiple-choice quiz

## scale drawing

a diagram that is similar to the original but is smaller (or larger) and in which the angles are the same and the lengths are in the same ratio as the original

## scale

the ratio of a length on a drawing to its real length expressed in the form  $1:n$  or  $\frac{1}{n}$

The aim of making a **scale drawing** is to draw a figure that is similar to the original; that is, the shape of the original is maintained by making the angles the same and the lengths in the same ratio as the original.

The scale factor of the length on the drawing to the real length is called the **scale** of the drawing.

$$\text{Scale} = \frac{\text{length on drawing}}{\text{real length}}$$

Scale may be expressed as a ratio such as  $1:1000$ , or as a fraction such as  $\frac{1}{1000}$ .

In general form, this can be written as  $1:n$  or  $\frac{1}{n}$ .

The scale can also be given in words, such as 1 cm to 10 m.

## EXAMPLE 3D-1 Finding the length on a scale drawing given the real length

Determine the length on a scale drawing that would represent a distance of 85 m on land, for these scales:

**a**  $1:10000$

**b**  $1:5000$

	Solve	Think	Apply
<b>a</b>	$\text{Scale} = 1:10000 = \frac{1}{10000}$ <p>So:</p> $\frac{\text{length on drawing}}{\text{real length}} = \frac{1}{10000}$ $\frac{\text{length on drawing}}{85 \text{ m}} = \frac{1}{10000}$ $\text{Length on drawing} = \frac{1}{10000} \times 85 \text{ m}$ $= 0.0085 \text{ m}$ $= 8.5 \text{ mm}$	<p>Express the ratio scale as a fraction and form an equation using</p> $\text{Scale} = \frac{\text{length on drawing}}{\text{real length}}$ <p>or <math>\frac{\text{length on drawing}}{\text{real length}} = \text{scale}</math>.</p> <p>Solve the equation by multiplying both sides by 85.</p>	<p>If the scale is <math>1:n</math>, then length on drawing or scaled length is <math>\frac{1}{n} \times \text{real length}</math>.</p>
<b>b</b>	$\text{Scale} = 1:5000 = \frac{1}{5000}$ <p>So</p> $\frac{\text{length on drawing}}{85 \text{ m}} = \frac{1}{5000}$ $\text{Length on drawing} = \frac{1}{5000} \times 85 \text{ m}$ $= 0.017 \text{ m}$ $= 17 \text{ mm}$		



**EXAMPLE 3D-3** Writing a scale as a ratio and a fraction

Express each of the scales below as:

**i** a ratio

**ii** a fraction.

**a** 1 cm to 5 m

**b** 1 mm to 25 m

	Solve	Think	Apply
<b>a i</b>	1 cm to 5 m = 1 cm to 500 cm = 1 : 500	Express both measurements in centimetres. As the two quantities are in the same unit, write as a ratio. Remember to leave out the units in the ratio.	Express both measurements in the same unit and simplify to either a ratio or a fraction.
	<b>ii</b> $1 : 500 = \frac{1}{500}$	Express the ratio scale as a fraction with the first number in the numerator and the second number in the denominator.	
<b>b i</b>	1 mm to 25 m = 1 mm to 25 000 mm = 1 : 25 000	Express both measurements in millimetres and write the quantities as a ratio.	
	<b>ii</b> $1 : 25\,000 = \frac{1}{25\,000}$	Express the ratio scale as a fraction.	

**3** Express each of the scales below as:

**i** a ratio

**ii** a fraction.

**a** 1 cm to 10 m

**b** 1 cm to 4 m

**c** 1 cm to 50 m

**d** 1 mm to 15 cm

**e** 1 mm to 5 m

**f** 1 mm to 20 m

**EXAMPLE 3D-4** Finding the scale given the length on the drawing and the real length

Find the scale on a drawing given that the length on the drawing is 32 cm and the real length is 1600 m.

Solve	Think	Apply
Length on drawing = 32 cm Real length = 1600 m = 160 000 cm Scale = $\frac{32 \text{ cm}}{160\,000 \text{ cm}}$ = $\frac{1}{5000}$ or 1 : 5000	Write each length in the same unit. Change 1600 m to cm by multiplying by 100. Substitute into $\text{Scale} = \frac{\text{length on drawing}}{\text{real length}}$ and use the fraction key on your calculator to simplify the fraction.	Express both measurements in the same unit and use $\text{Scale} = \frac{\text{length on drawing}}{\text{real length}}$ The scale can be expressed as a fraction or as a ratio.

**4** Find the scale on a drawing, given the following lengths.

**a** Length on the drawing is 52 cm and real length is 520 m.

**b** Length on the drawing is 52 cm and real length is 5200 m.

**c** Length on the drawing is 16 mm and real length is 64 m.

**d** Length on the drawing is 15 mm and real length is 7.5 m.

**e** Length on the drawing is 11.2 cm and real length is 56 km.

### EXAMPLE 3D-5 Finding a real length from a scale drawing

A scale drawing of a building is shown. Use your ruler and the scale given to find the real height of the building.



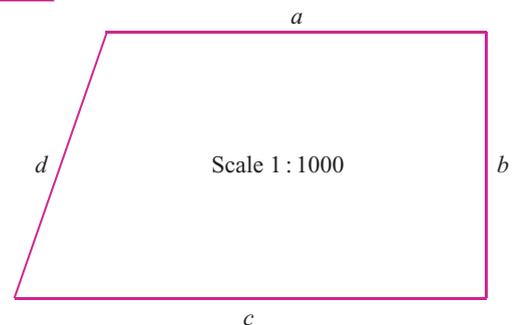
Scale is 1 : 400

Solve	Think	Apply
$\text{Scale} = 1 : 400 = \frac{1}{400}$ $\text{Height on drawing} = 40 \text{ mm}$ $\frac{\text{height on drawing}}{\text{real height}} = \text{scale}$ $\frac{40 \text{ mm}}{\text{real height}} = \frac{1}{400}$ $\frac{\text{real height}}{40 \text{ mm}} = \frac{400}{1}$ $\text{Real height} = 400 \times 40 \text{ mm}$ $= 16\,000 \text{ mm}$ $= 16 \text{ m}$	<p>Write the scale as a fraction and use a ruler to measure the height on the drawing. Form an equation using <math>\frac{\text{length on drawing}}{\text{real length}} = \text{scale}</math>.</p> <p>Take the reciprocal of both sides so the unknown height is in the numerator of the first fraction.</p> <p>Solve the equation by multiplying both sides by 40.</p>	<p>Measure the length on the scale drawing. If the scale is 1 : <math>n</math>, then real length = <math>n \times</math> scaled length.</p>

- 5 This diagram is a scale drawing of a rectangular field. Using your ruler and the scale given, determine the real length and breadth of the field.

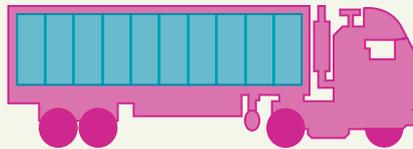


- 6 This is a scale drawing of a parking area. Use the scale given to find the real dimensions ( $a$ ,  $b$ ,  $c$ ,  $d$ ) of the parking area.



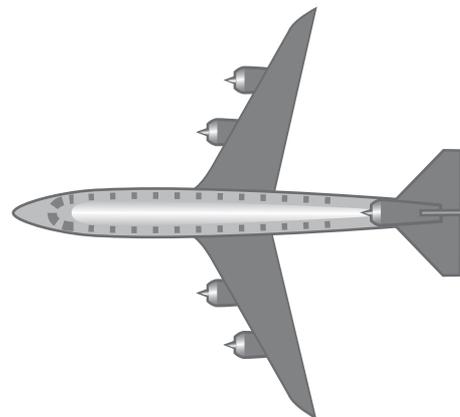
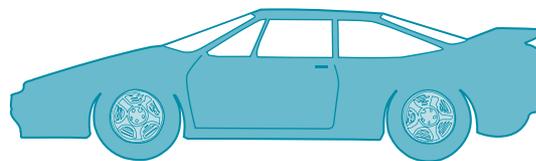
### EXAMPLE 3D-6 Finding the scale from a scale drawing

This is a scale drawing of a truck. The actual length of the truck is 5.4 m. What is the scale used in this drawing?

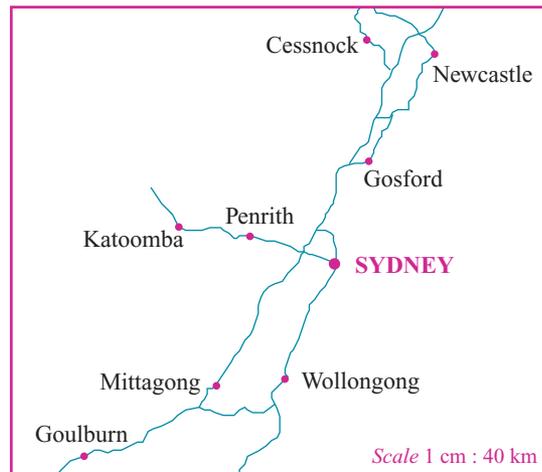


Solve	Think	Apply
Length on drawing = 54 mm Real length = 5.4 m = 5400 mm $\text{Scale} = \frac{\text{length on drawing}}{\text{real length}}$ $= \frac{54 \text{ mm}}{5400 \text{ mm}}$ $= \frac{1}{100} \text{ or } 1 : 100$	Use a ruler to measure the length of the truck in the scale drawing. Convert the real length to millimetres. Use the fraction key on a calculator to simplify the fraction.	Measure the length of the truck on the scale drawing. Express both measurements in the same unit and use $\text{Scale} = \frac{\text{length on drawing}}{\text{real length}}$

- 7** The diagram on the right is a scale drawing of a car.
- If the real length of the car is 5.6 m, what is the scale used in the drawing?
  - Calculate the real height of the car.
  - What is the actual diameter of the wheel?
- 8** The actual length of the plane shown in the scale drawing is 60 m.
- Find the scale used in the drawing.
  - Find the actual wingspan of the plane.



- 9** The map on the right shows some routes out of Sydney.
- What is the scale on the map?
  - What is the actual distance (in a straight line) from Sydney to Gosford?
  - What is the actual distance from Mittagong to Newcastle?
  - Which town is about 64 km from Sydney?
  - Which towns are about 280 km apart?

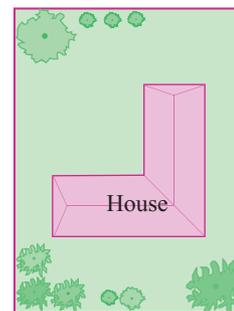


- 10** The image below shows a football field.
- Use the scale shown on the bottom right of the image to work out the length and width of the rectangular football field.
  - What is the area of the football field to the nearest square metre?



Scale 1 : 1750

- 11** A new house is situated within a block of land as shown in the site plan on the right.
- Use the scale to find the perimeter of the land.
  - To build a fence 1.8 m high along the boundary of the land, the owner received a quote of \$9100. How much is this per square metre of fencing?
  - What area of land does the house occupy?
  - The house was constructed on a concrete slab, which is 250 mm thick. Calculate the volume of concrete that was needed for the concrete slab. (*Hint: use  $V = A \times h$ .*)
  - To estimate the cost of painting the exterior walls of the house, which are 3 m high, the owner ignores any windows or doors. A 15 L can of paint costs \$225 and covers about 200 m<sup>2</sup> of wall. The owner wishes to apply two coats of paint.
    - Estimate how many cans of paint would be required.
    - Estimate the total cost of the paint.



Scale 1 : 1000



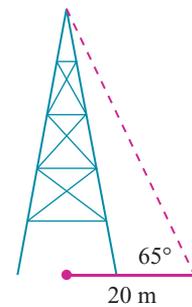
- f** As part of the landscaping, turf is to be laid at a cost of \$30 per square metre. What is the total cost of laying turf?
- g** The owner decides to have a square patio constructed between the two wings of the house using bluestone pavers instead of laying turf in that area. The pavers are 600 mm by 300 mm and cost about \$15 each (including installation).



- i** How many pavers would be needed for the patio?
  - ii** What is the total cost of paving the patio?
  - iii** How much extra would it cost to have the pavers instead of laying turf in this patio area?
- h** On one of the open sides of the patio, the owner wishes to have a brick wall built for a barbeque area. The wall is to be made of 15 rows of bricks and be two bricks thick. Each brick is 230 mm long and costs \$1.20. A bricklayer will charge \$1.50 per brick to build the wall. The owner has leftover materials to make the required concrete needed to be laid between the layers of bricks.
  - i** Estimate the number of bricks required for the brick wall. Explain how you worked this out.
  - ii** Estimate the cost of building the brick wall by calculating the total cost of the bricks and the bricklayer.

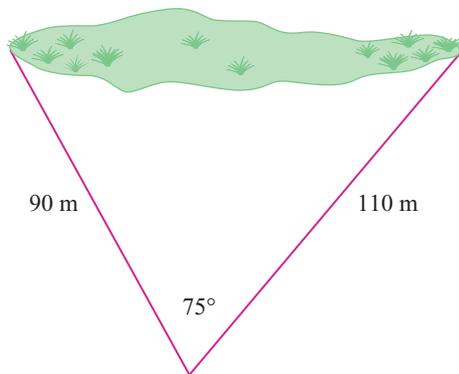
- 12** The angle of elevation to the top of a tower from a point on the ground 20 m from the base of the tower is  $65^\circ$ , as shown in the sketch.

- a** Make a scale drawing using a scale of 1 cm to 4 m.
- b** Find the actual height of the tower using the scale drawing.

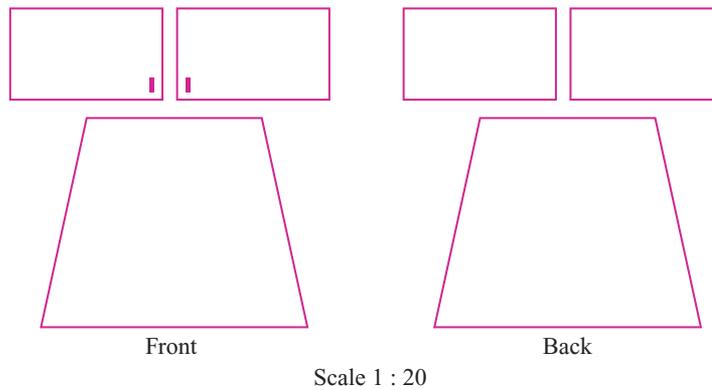


- 13** A surveyor marked this sketch below with measurements to find the distance across a swamp.

- a** Make a scale drawing of the figure using a scale of 1 : 1000.
- b** From the scale drawing, calculate the actual distance across the swamp.



- 14** Allie found a sewing pattern for a simple dress that is made up of six pieces. The pattern has been drawn smaller using a scale of 1 : 20.
- Use measurement and the scale to find the real dimensions of the pattern pieces.
  - Calculate the total area of the actual pattern pieces.
  - If Allie's chosen fabric is 90 cm wide, what is the minimum amount of fabric Allie needs? Is this a realistic amount of fabric? Explain.
  - Make a scale drawing showing how the pattern pieces could be laid out on the fabric.
  - Estimate how much fabric is needed to make the dress.
  - If the fabric costs \$14 per linear metre, what is the total cost of the fabric?
  - Allie finds a cheaper fabric of the same width which is only \$10 per linear metre but it has a repeat pattern. She estimates that she will need 20% more fabric to allow for pattern matching. Which will be the cheaper option?



- 15** Draw a floor plan of your classroom to scale. Remember to include the scale on your drawing.
- 16** Draw a map of a netball court (or other playing field) at your school to scale. Remember to include the scale on your drawing.
- 17** Design a novelty birthday cake that uses a number of cake pans of different sizes and shapes. Make a scale drawing of your design. These are some standard sizes of cake pans: round with 15 cm, 20 cm, 23 cm or 25 cm diameter; square with side length 20 cm, 23 cm or 25 cm; rectangular 28 cm by 18 cm or 30 cm by 20 cm.
- 18** To transport the cake you designed in question 17, a rectangular box is to be made of cardboard.
- Calculate the dimensions of the box and draw a quick sketch of its net with accompanying measurements.
  - Make a scale drawing of the net of the box. Remember to include the scale on your drawing.
  - To decorate the top of the box, a piece of ribbon is to be glued along the four edges and the two diagonals. Use your scale diagram to calculate the total length of ribbon needed.
- 19** Download a map or aerial photograph from the internet that contains a feature with known dimensions; for example, an Olympic swimming pool, an athletics field, or a playing field for a particular sport. Use this to estimate lengths or distances and to find the scale of the map or photograph.

# 3E Building plans

These resources are available on your obook assess:

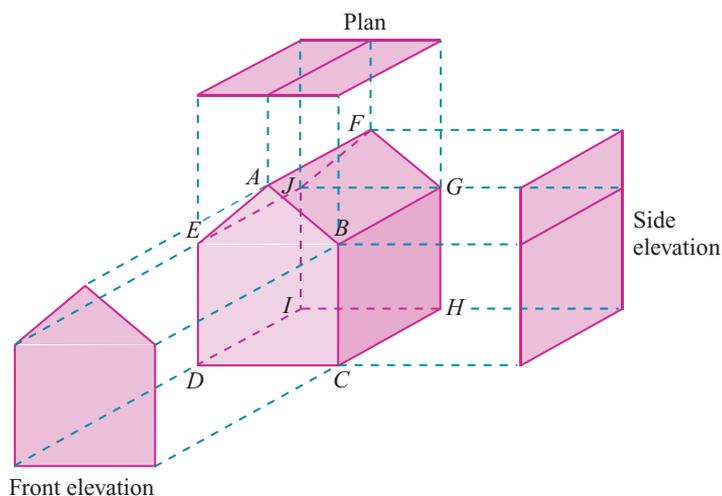
- **Investigation 3E:** Using scale to explore the plans for a new house
- **assess quiz 3E:** Test your skills with an auto-correcting multiple-choice quiz
- **website:** Building and Sustainability Index (BASIX)

Building plans are scale drawings of houses or other buildings. They include site plans, floor plans and side elevations, so that all construction details can be determined.

A site plan is a drawing of the block of land showing the position of the residence and any other buildings, or main features, on the block.

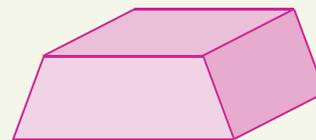
Plans and elevations are two-dimensional representations of a solid object. They are the views (what you can see) looking at the solid from various angles, usually from the top and sides.

The diagram above shows the plan (the view from directly above), the front elevation (the view from directly in front) and the side elevation (the view from one side) of a solid.



## EXAMPLE 3E-1 Drawing a plan, front elevation and side elevation

Draw the plan, front elevation and side elevation of the solid shown.



Solve	Think	Apply
<p>Plan:</p> <p>Front elevation:</p> <p>Side elevation:</p>	<p>The plan of a solid is the view seen from directly above it. The front elevation is the view from directly in front of it. The side elevation is the view from the side.</p>	<p>The plan and elevations are two-dimensional views of a three-dimensional object.</p>

# EXERCISE 3E Building plans

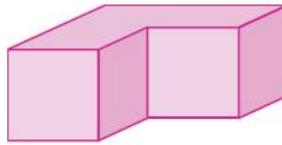
PROBLEM SOLVING, REASONING AND JUSTIFICATION

1 Draw the plan, front elevation and side elevation of the following solids.

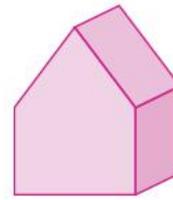
a



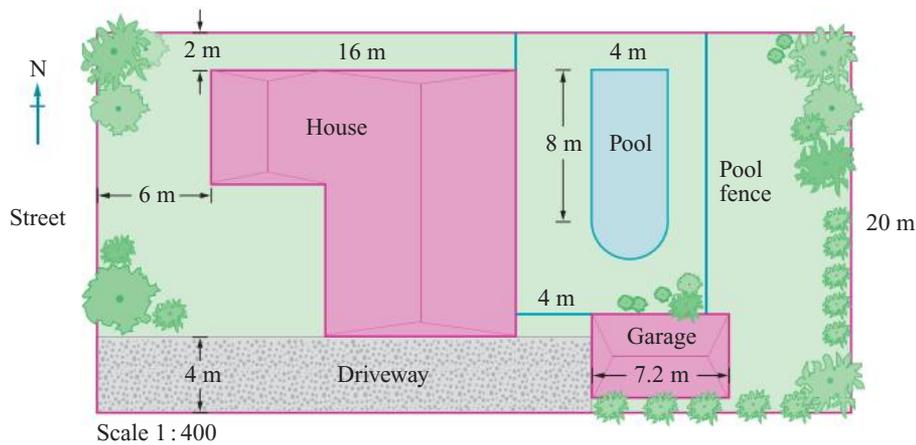
b



c



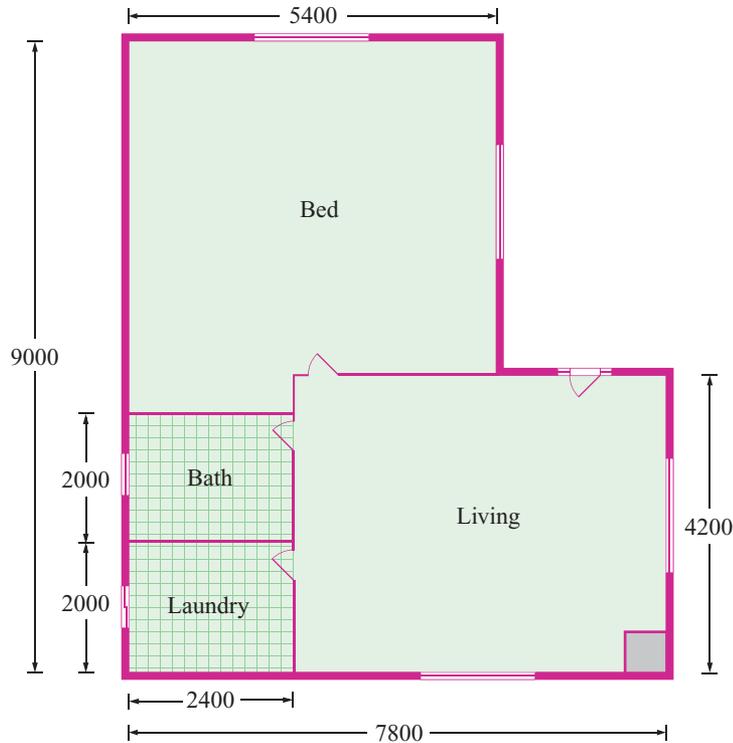
2 Answer the following questions using the information given on this site plan and the actual measurements on the scale drawing.



- a What are the dimensions of the block?
- b Find the length of the driveway to the garage.
- c Determine the distance between the house and the fence on:
  - i the northern boundary
  - ii the southern boundary.
- d How far is the house from the street?
- e How far is the garage wall from the southern boundary?
- f Calculate the area of the garage, to the nearest square metre.
- g Find the total length of the pool fences.
- h Calculate the area of land taken up by the pool, to the nearest square metre.
- i If the pool has an average depth of 1.2 m, calculate the volume of the pool. (*Hint*: remember that  $V = A \times h$ .)
- j Find the amount of water in the pool, to the nearest 1000 litres ( $1 \text{ m}^3 = 1 \text{ kL}$ ).
- k How many blocks of land of this size would fit into an area of 1 ha? ( $1 \text{ ha} = 10\,000 \text{ m}^2$ )



- 3** The diagram shows the plan of a small holiday house (all measurements are in millimetres). The owners wish to install air conditioning throughout the house, except in the bathroom and laundry.



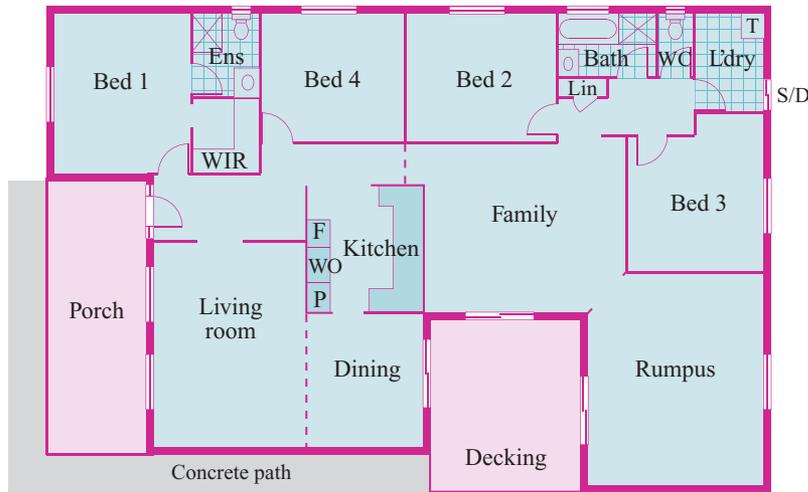
- a** Calculate the floor area of the house minus the bathroom and laundry.
- b** The power output of the air conditioning unit required is based on the volume of air to be conditioned.
- i** Calculate the volume of air to be air conditioned in this house, if the ceilings are 2400 mm high.  
(Hint: use  $V = A \times h$ .)
- ii** Use the table below to find the power output of the unit required.

<b>Volume (m<sup>3</sup>)</b>	50	60	70	80	90	100	110	120	130	140	150
<b>Power output required (kW)</b>	3.0	3.6	4.2	4.8	5.4	6.0	6.6	7.2	7.8	8.4	9.0

- c** Find the approximate power of the air conditioning unit required to air condition the living room of the house.
- d** Find the approximate power of the air conditioning unit required for the living room of the house, if the ceiling height is 2700 mm.

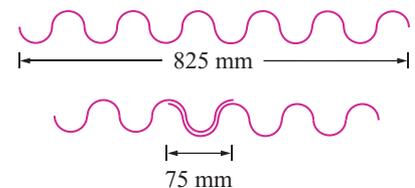


- 4 The floor plan below of a four-bedroom house is drawn to a scale of 1 : 200.
- a How many of each does the house have?  
 i windows (shown as )    ii hinged doors (shown as )    iii sliding doors (shown as )
- b Find the following symbols on the plan and give the meaning of each:  
 P, WO, F, WC, Ens, WIR, Lin, T



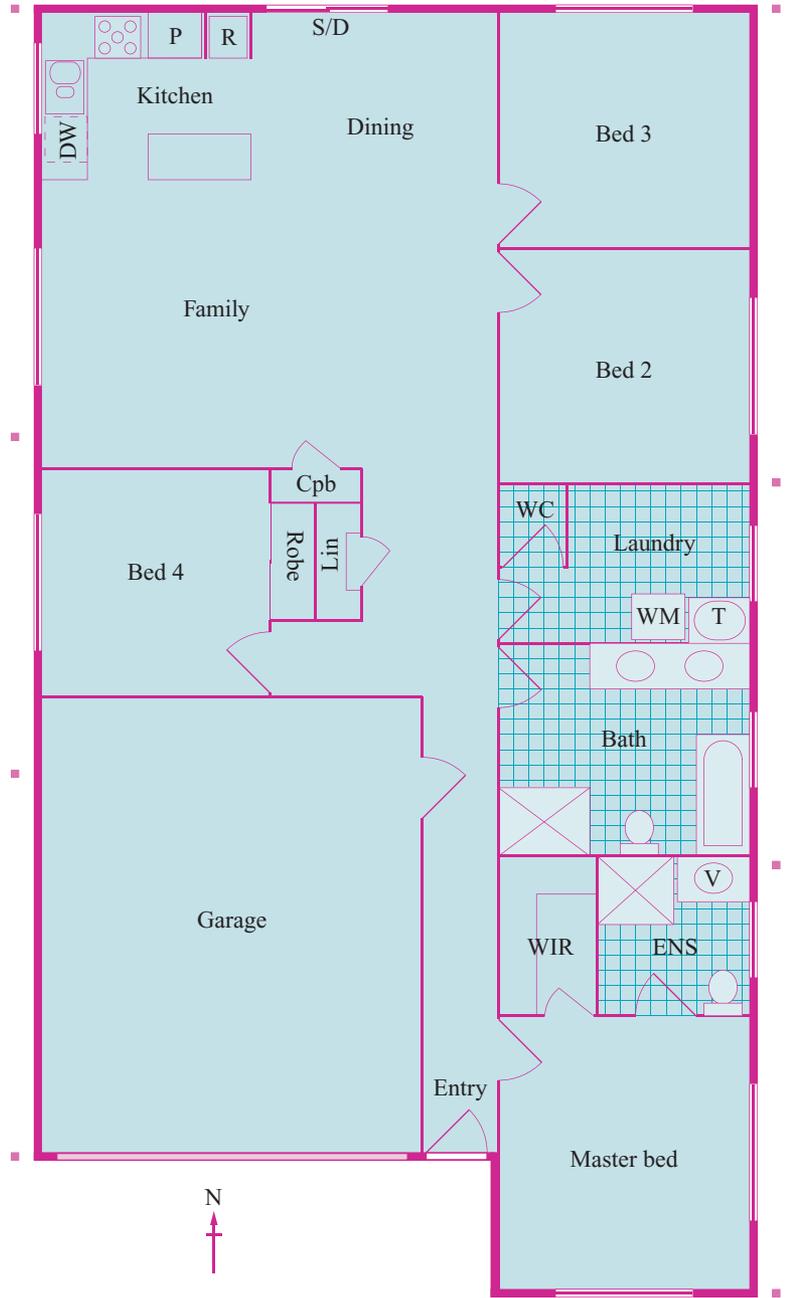
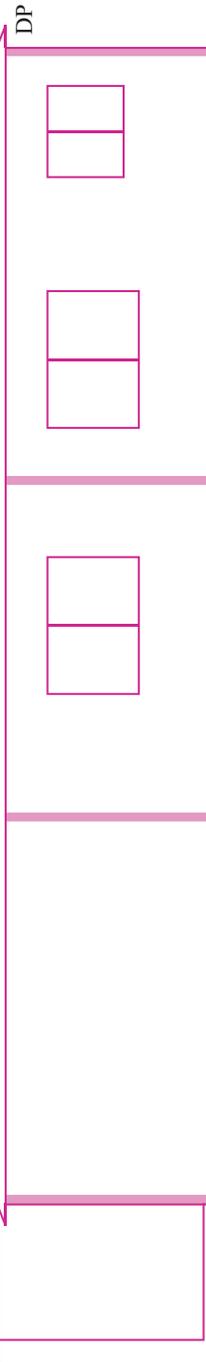
Scale 1 : 200

- c By measurement and calculation, find the dimensions of:  
 i the rumpus room (assume rectangular)    ii bedroom 2  
 iii the living room    iv the porch.
- d i What is the area of land occupied by the whole house, including the porch and decking? (Remember to include the thickness of the walls.)  
 ii A 'square' (100 square feet) is an old imperial unit used for measuring the area of a house. If 1 square  $\approx 9.29 \text{ m}^2$ , calculate the area of the house, including porch and decking, to the nearest square.
- e The porch is a reinforced concrete slab 100 mm thick. How much concrete, in cubic metres, was needed for the porch? (It is easier if you first convert all measurements to metres.)
- f The walls of bedroom 4 are to be repainted.  
 i Calculate the area of the four walls, including the window and door, given the ceiling height is 2.7 m.  
 ii Calculate the area to be painted given that the window height is 1.2 m and the door height is 2.1 m.  
 iii Find the amount of paint needed to apply two coats, if 1 L of paint covers  $16 \text{ m}^2$ .
- g i Consider the thickness of the internal walls and the external walls. Discuss why they are different.  
 ii Research and list the different ways in which the walls of a house could be constructed. How is this shown on a building plan?
- h i Find the dimensions of the laundry.  
 ii If the laundry floor is to be tiled with 250 mm by 250 mm square tiles, calculate the number of tiles needed.  
 iii The tiles come in boxes of 16. How many boxes must be bought?
- i The decking is to be covered with metal roofing that comes in 825 mm widths with the profile shown. How many of these sheets would be needed to cover the decking, if they are to be overlapped as shown? The sheets are placed so that the water runs to the side of the house.

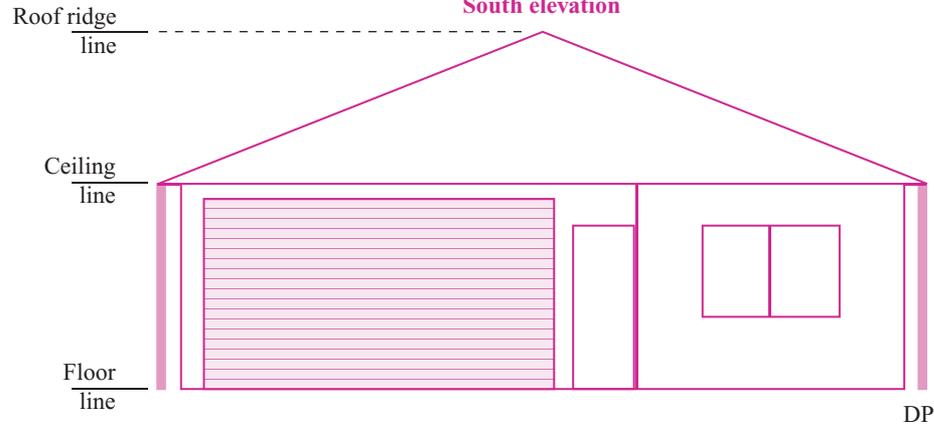




West elevation

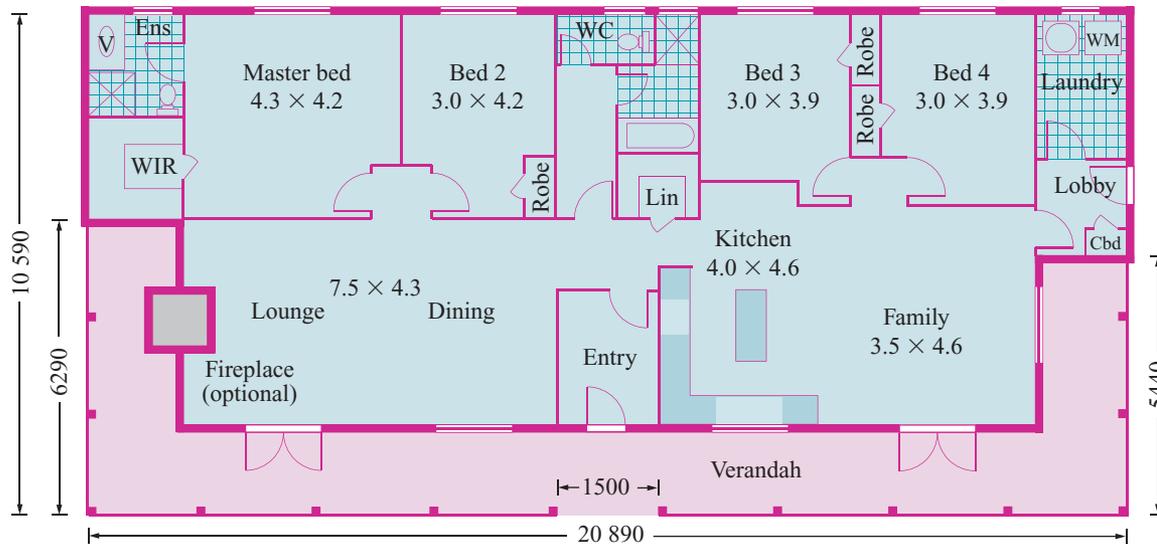


South elevation



Scale 1 : 100

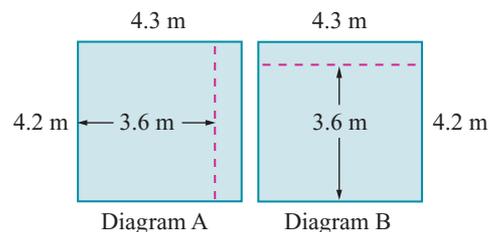
- 6 The following diagram shows the floor plan of a kit home. The external measurements are shown in millimetres and the internal measurements are shown in metres.



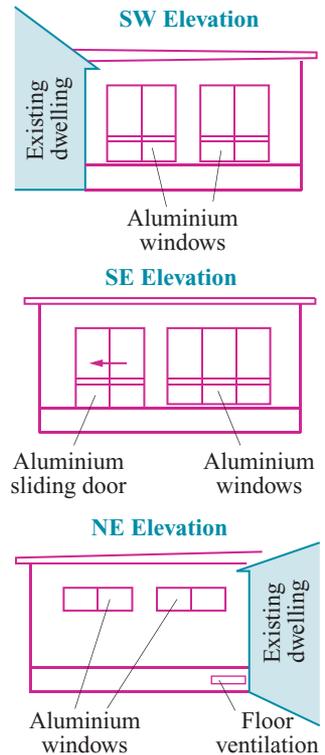
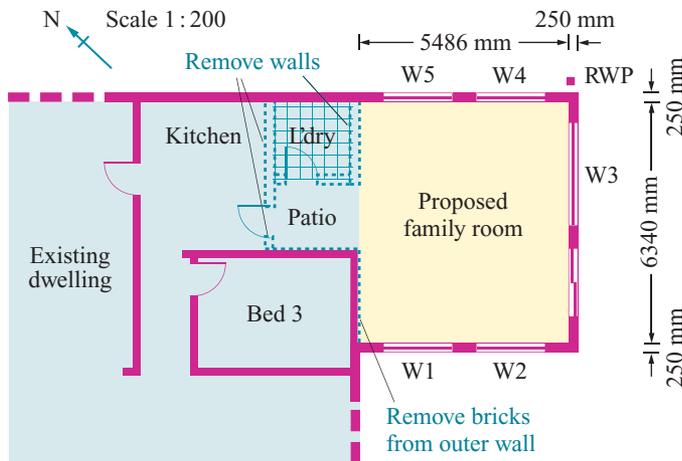
- Find the following symbols on the plan and give the meaning of each:  
WIR, Lin, Ens, WC, V, WM, Cbd
- How many hinged doors are shown on the plan? (Exclude doors to cupboards, wardrobes, etc.)
  - How many windows does the house have?
- How many vertical posts support the verandah roof?
- What is the length and width of the house (including the verandah)?
- Guttering is needed around the four sides of the house. Calculate the total length of guttering needed.
- Find the area of the house, including the verandah, to the nearest square metre.
- If built with weatherboard cladding, the house costs \$180 300 to complete. What is the cost per square metre of building the house (to the nearest dollar)?
- If constructed on a concrete slab 250 mm thick, what volume of concrete, in cubic metres, is needed?
- A double railing made of timber is built around the verandah, as shown. Calculate the number of metres of timber needed (leave a gap in front of the house entry and ignore the width of the posts).



- The master bedroom is to be carpeted using rolls that are 3.6 m wide.
  - The carpet is laid as in diagram A. How many metres would be needed, if there is to be only one join and the carpet pile must run in the same direction?
  - If it is to be laid as in diagram B, how many metres would be needed?
  - What features of the carpet or the house might affect which of the methods A and B is used?
- Find the cost of carpeting the master bedroom, if the carpet costs \$95 per linear metre. Use the best option from part j.



**7** The floor plan and elevations for a proposed extension to a house are shown.



- a What is the scale on the plan and elevations?
- b What are the internal dimensions of the proposed family room?
- c Find the thickness of the walls of the extension.
- d What length of existing wall indicated is to be removed?
- e Determine the dimensions of the new kitchen.
- f What length of guttering is needed across the rear of the family room?
- g What length of rainwater pipe is needed at the rear of the extension?
- h Find the height of the ceiling above the floor.
- i Use the plan and elevations to determine the dimensions of windows W1, W3 and W4.
- j What are the dimensions of the sliding door?
- k Calculate the area of the new room in square metres.
- l Calculate the area of the new kitchen in square metres.
- m How much would it cost to cover the floor of the new kitchen and family room with cork tiles that cost \$106/m<sup>2</sup> laid (to the nearest dollar)?

**8** To build homes that are energy-efficient, passive design features such as location, orientation, layout, window size, insulation and shading should be taken into consideration. Using materials that provide thermal mass to make the best use of absorbing heat energy at different times of the year is also beneficial. Investigate local council requirements for energy-efficient housing.

**9** The aim of the Building and Sustainability Index (BASIX) Certificate is to ensure the efficient use of water and energy in homes across New South Wales by including reduction targets in these areas for all development plans.

Imagine that you are about to build the house with the plans shown on the next two pages. Complete the following in preparation for undertaking a BASIX certificate for this development proposal.

- a Calculate the:
  - i site area
  - ii roof area
  - iii floor area.
  - iv ratio of floor area for which air conditioning applies to floor area for which air conditioning does not apply. (You will need to decide which rooms you will have air conditioned.)
- b Prepare a landscape design for the property and calculate the garden and lawn areas.
- c Assume that you will be installing a water tank and a swimming pool.
  - i Decide on the size of the tank and where it will be situated.
  - ii Calculate the volume of the swimming pool you select. (Keep the design simple.)
- d What are the thicknesses of the internal and external walls of the house?

- e What is the width of the eaves on each side of the house?
- f Investigate and calculate the amount of roof insulation required for this house.
- g Determine the orientation of the windows. Determine the breeze path and indicate it on the plan.

Now go to the NSW Government BASIX website. A link is provided on your obook assess. Use the plans given and your answers to the questions above to complete each section of the BASIX Certificate. You will need to modify your plans along the way until you have passed.

*Step 1:* On the home page, go to the login box and click on 'Register' and then 'next'. Select 'Education' as your user type and click on 'next'.

*Step 2:* Make up a username and password.

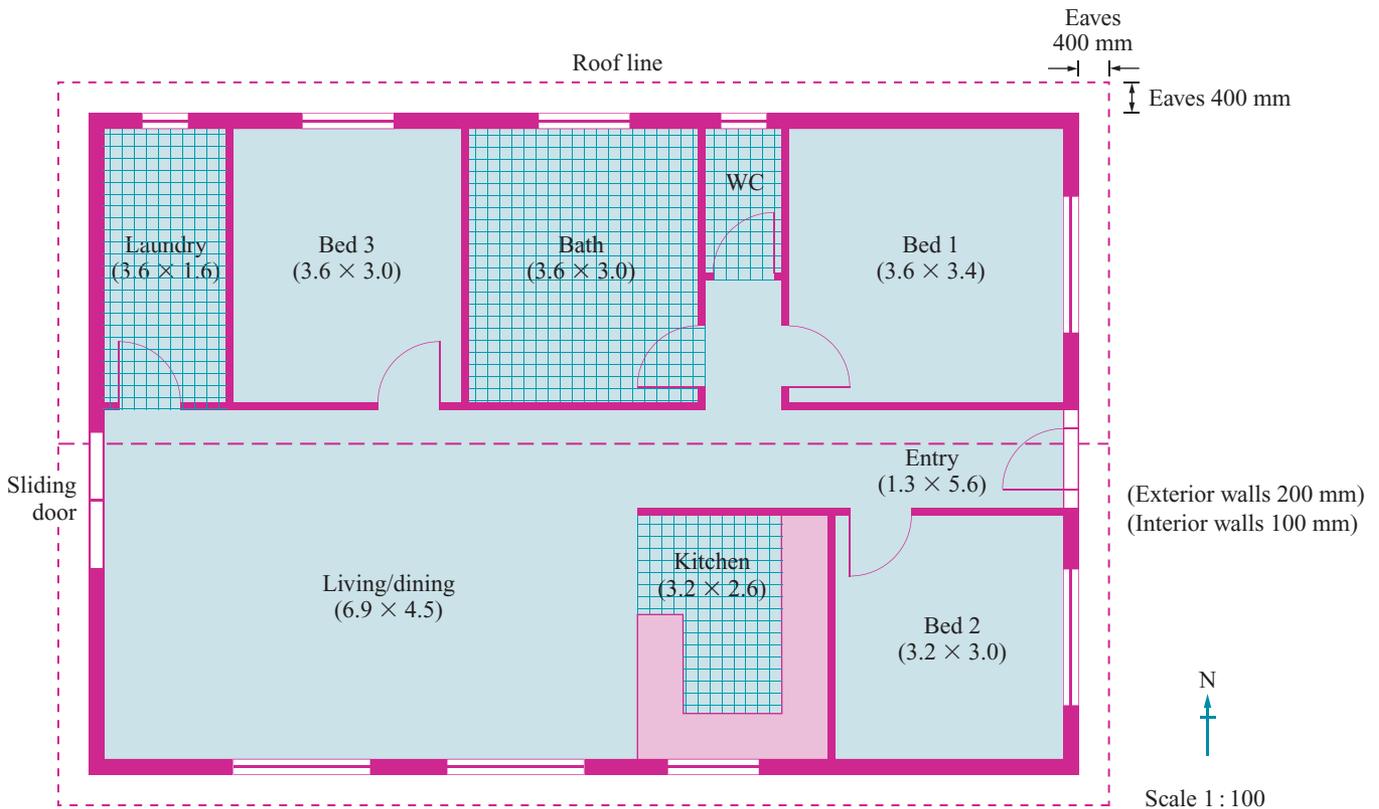
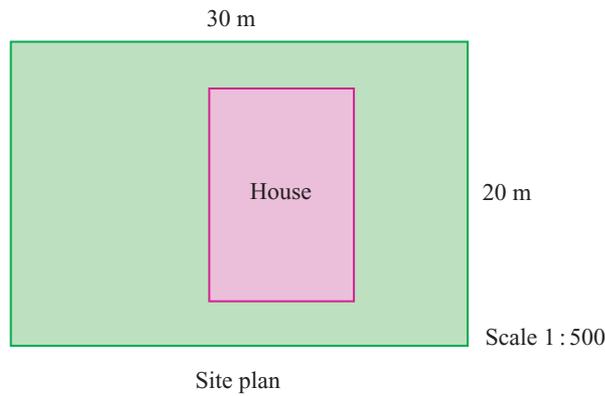
*Step 3:* Enter your name, school (institution) and email address. Click on 'next'. An email is then sent to you with your verification code.

*Step 4:* Return to the home page. In the login box enter your username and password. Tick '... terms and conditions' and enter the verification code sent to you by email. Submit.

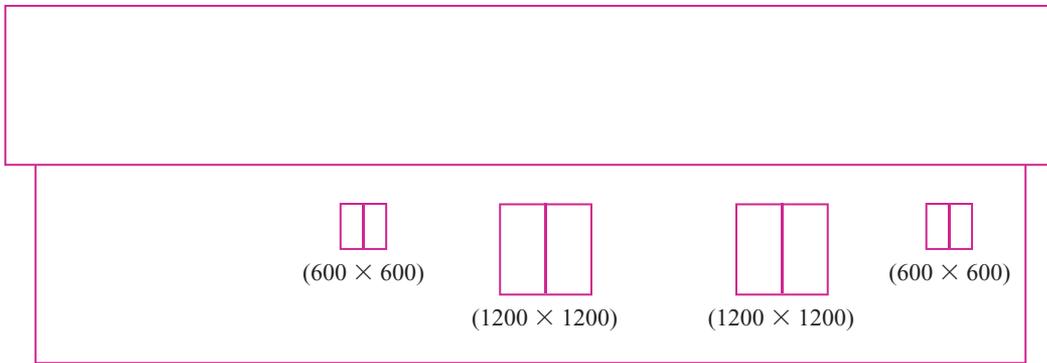
*Step 5:* Select 'New Dwellings' and enter.

*Step 6:* Click on 'Start a new project'.

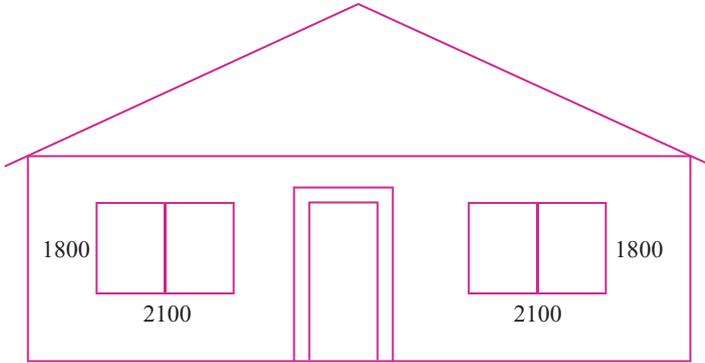
### Sample house plans



North elevation

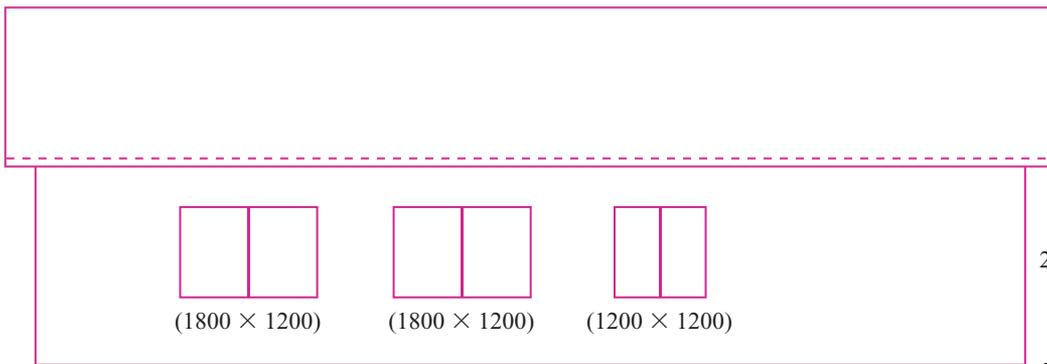


East elevation

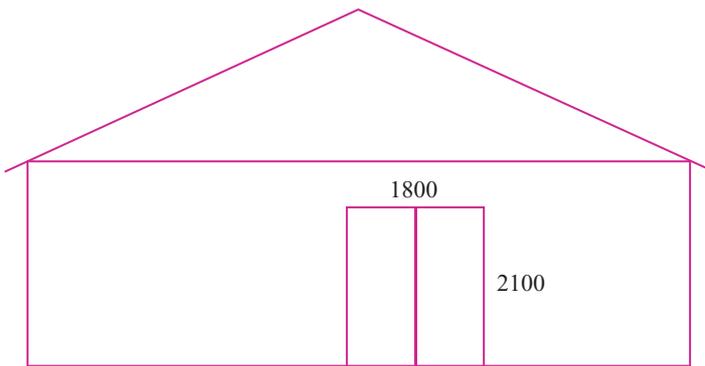


Scale 1 : 100

South elevation



West elevation



Scale 1 : 100

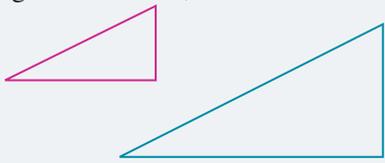
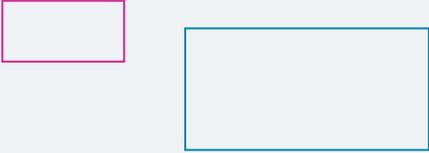
# CHAPTER 3 REVIEW SCALE DRAWINGS

You should be able to:

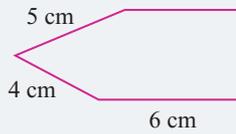
- ✓ find and simplify the ratio of two quantities
- ✓ divide a quantity in a given ratio
- ✓ determine whether or not two figures are similar
- ✓ find the scale factor
- ✓ calculate the unknown lengths of sides in similar figures
- ✓ interpret scale drawings, building plans and maps
- ✓ interpret common symbols and abbreviations on house plans
- ✓ use the scale to calculate actual dimensions on a plan or scale drawing
- ✓ estimate and compare quantities, materials and costs using actual measurements from scale drawings
- ✓ interpret plans and elevations to obtain the internal dimensions of rooms
- ✓ calculate perimeter, area and volume based on information on a map, plan or scale drawing.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

## REVIEW MULTIPLE-CHOICE QUESTIONS

- 3A** 1 The ratio of girls to boys in a school is 7:5. If there are 364 girls, how many boys are there?  
A 55                      B 77                      C 260                      D 275
- 3A** 2 When \$56 000 is divided in the ratio 5:3, how much is the larger amount?  
A \$21 000                      B \$28 000                      C \$33 600                      D \$35 000
- 3B** 3 For similar figures, which of the following statements are correct?  
a The corresponding angles are equal.  
b The corresponding angles are in proportion.  
c The corresponding sides are equal.  
d The corresponding sides are in proportion.  
A b and c                      B a and d                      C b and c                      D b and d
- 3B** 4 Using measurement, which of the following pairs of figures are similar?  
a                       b 
- A a and b                      B a only  
C b only                      D neither a nor b

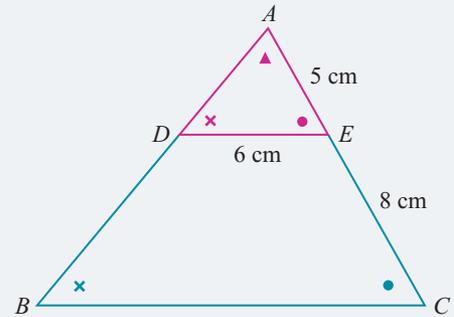
- 3B** 5 Given that the pair of figures shown are similar, what is the enlargement factor?



- A  $\frac{6}{5}$                       B  $\frac{6}{4}$                       C  $\frac{6}{6}$                       D  $\frac{5}{6}$

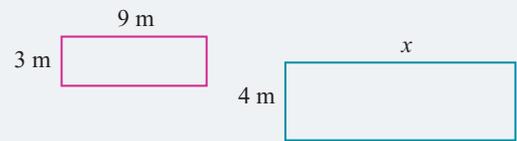
- 3B** 6 Triangle  $ADE$  has been enlarged to produce triangle  $ABC$ . What is the scale factor?

- A  $\frac{8}{5}$                       B  $\frac{13}{5}$   
C  $\frac{8}{6}$                       D  $\frac{13}{6}$



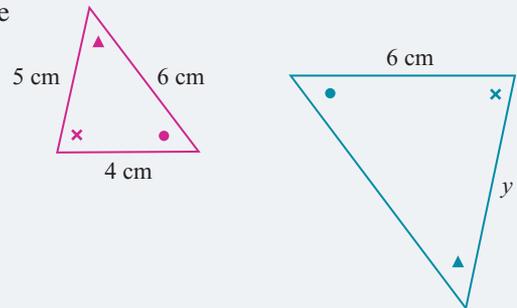
- 3C** 7 In this pair of similar figures, what is the length of the side labelled  $x$ ?

- A 27 m                      B 12 m  
C 9 m                      D 6.75 m



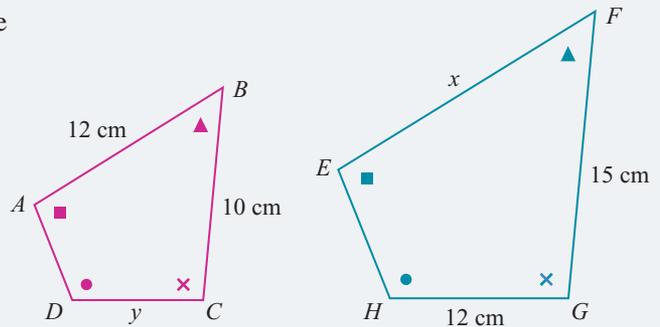
- 3C** 8 Which relationship describes a correct ratio of side lengths in these similar triangles?

- A  $\frac{y}{5} = \frac{6}{4}$   
B  $\frac{y}{4} = \frac{6}{5}$   
C  $\frac{y}{6} = \frac{6}{5}$   
D  $\frac{y}{6} = \frac{6}{4}$



- 3C** 9 What are the lengths of the sides labelled  $x$  and  $y$  in the similar figures on the right?

- A  $x = 15, y = 8$   
B  $x = 18, y = 8$   
C  $x = 18, y = 9.8$   
D  $x = 15, y = 9.8$



- 3C** 10 A photograph 16 cm long by 10 cm wide is enlarged so that the new length is 20 cm. What is the new width?

- A 32 cm                      B 14 cm                      C 12.5 cm                      D 8 cm

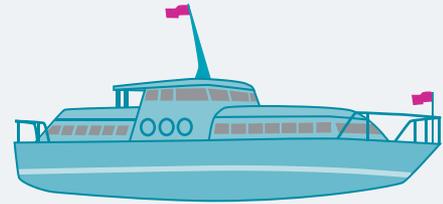
- 3D** 11 What length on a scale drawing would represent an actual length of 120 m, if a scale of 1 : 10000 is used?

- A 12 mm                      B 12 cm                      C 1.2 mm                      D 0.12 mm

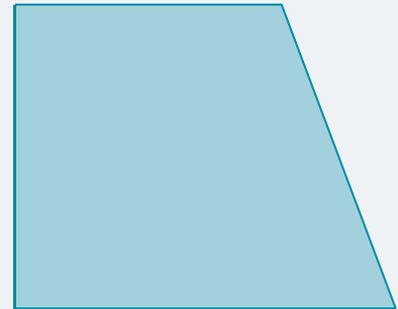
- 3D** 12 What is the scale 1 cm to 5 m equivalent to?

- A 1 : 5                      B 1 : 50                      C 1 : 500                      D 1 : 5000

- 3D** **13** The actual length of the boat shown in the scale drawing is 11.2 m.  
What scale has been used?
- A 1:112                      B 1:200  
C 1:500                      D 1:2000



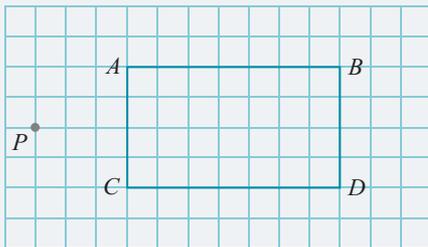
- 3E** **14** Using measurement and the scale given, what is the approximate perimeter of this site plan?
- A 134.4 m  
B 168 m  
C 1344 m  
D 1360 m



Scale 1 : 800

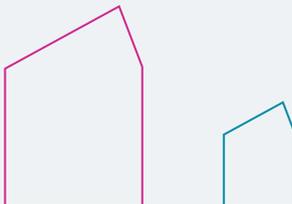
## REVIEW SET 1

- The ratio of boys to girls in a school is 8 : 9. If there are 256 boys, how many girls are there?
- Divide \$48 000 into two parts in the ratio 3 : 5.
- Copy the diagram and enlarge it using  $P$  as the centre of enlargement and the scale factor  $k = 2$ .

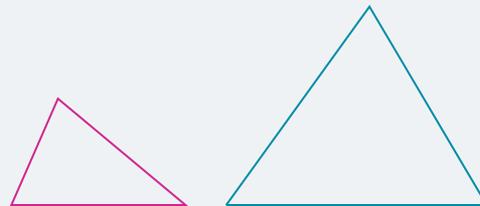


- Determine, by measurement, whether the following pairs of figures are similar.

**a**

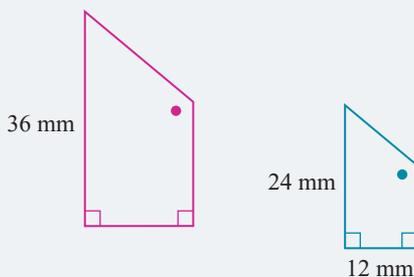


**b**

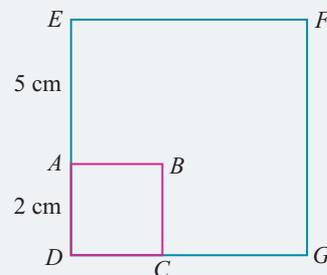


- Given that the following pairs of figures are similar, find the scale factor. The pink figure is the original figure in each pair.

**a**

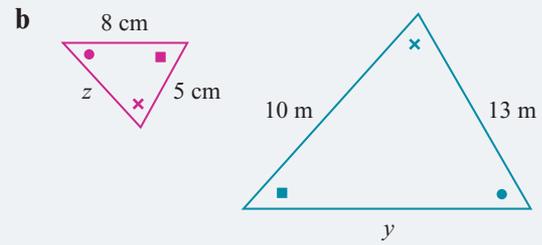
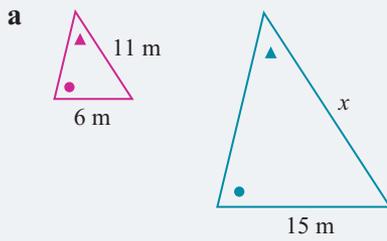


**b**

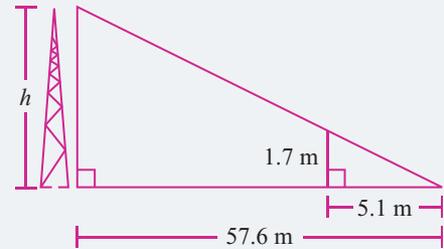


6 Consider the following pairs of similar figures.

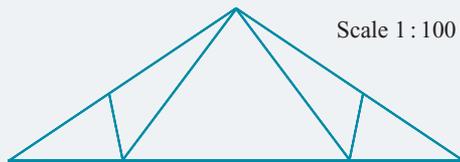
- i Find the scale factor, if the figure on the left is the original figure.
- ii What are the lengths of the unknown sides labelled with a pronumeral?



7 At a particular time during the day, a tower casts a shadow that is 57.6 m long. At the same time, a person who is 1.7 m tall casts a shadow 5.1 m long. What is the height of the tower?



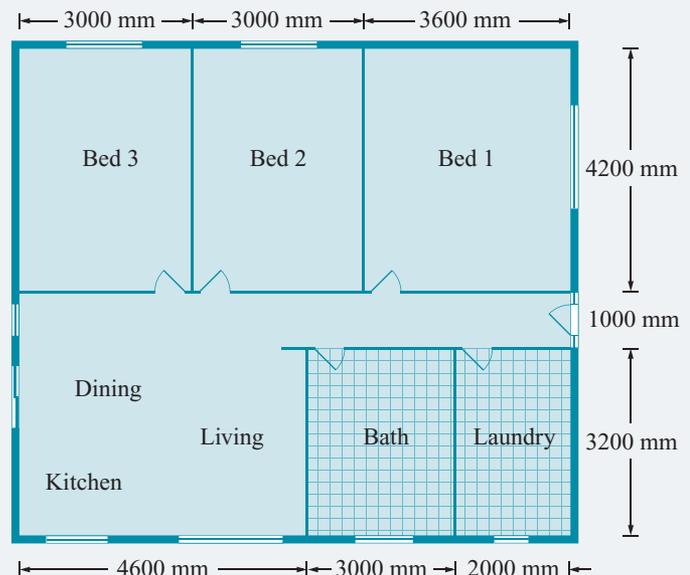
8 By measurement and calculation, find the total length of timber needed to construct the roof truss shown.



9 The floor plan of a house is shown below.

- a The bedrooms are to be carpeted so that the least amount of carpet is used.
  - i If carpet comes in rolls 3.6 m wide, how many metres of carpet will be needed?
  - ii If the carpet cost \$127 per linear metre, what is the cost of the carpet needed?
- b The floor of the bathroom and laundry are to be tiled with tiles 250 mm by 250 mm.
  - i Calculate the number of tiles needed.
  - ii Calculate the total cost, if the price is \$84/m<sup>2</sup> laid.
- c All the rooms in the house are to be air conditioned except the bathroom and laundry.
  - i Calculate the area to be air conditioned.
  - ii Calculate the volume of house to be air conditioned, if the ceilings are 2700 mm high.
  - iii Use the table below to find the power output of the air conditioning unit required.

Volume (m <sup>3</sup> )	Power output required (kW)
130	7.8
140	8.4
150	9.0
160	9.6
170	10.2
180	10.8



# REVIEW SET 2

1 Write each of these comparisons a ratio in simplest form.

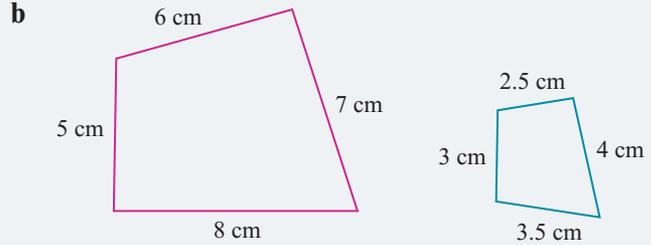
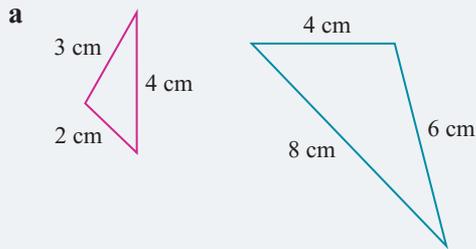
a 36 to 45

b  $1\frac{1}{4}$  to 2

c 1.5 to 2.4

2 On one day at the school canteen, the ratio of bread rolls to sandwiches sold was 3 : 2 and the ratio of sandwiches to pies sold was 1 : 4. If the canteen sold 72 pies on this day, how many bread rolls were sold?

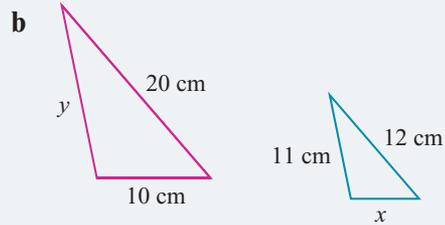
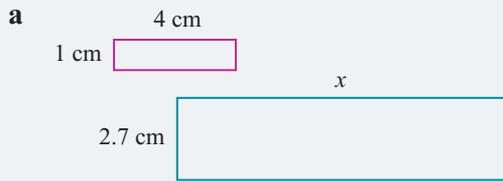
3 Determine which of the following pairs of figures are similar.



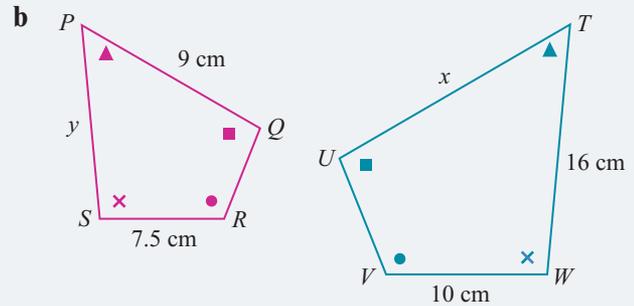
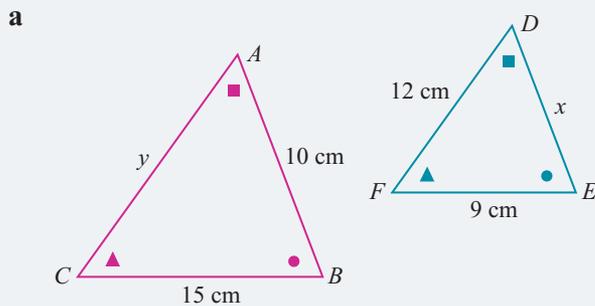
4 For the following pairs of similar figures, the pink figure is the original figure.

i Find the scale factor.

ii Find the length of the unknown sides labelled with a pronumeral.



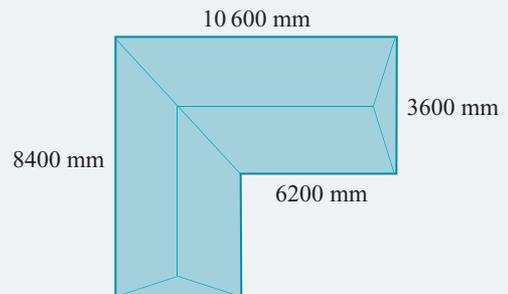
5 Find the lengths of the unknown sides labelled with pronumerals in the following pairs of similar figures.



6 A tree casts a shadow 5.6 m long. At the same time, a metre rule casts a shadow 1.6 m long. Calculate the height of the tree.

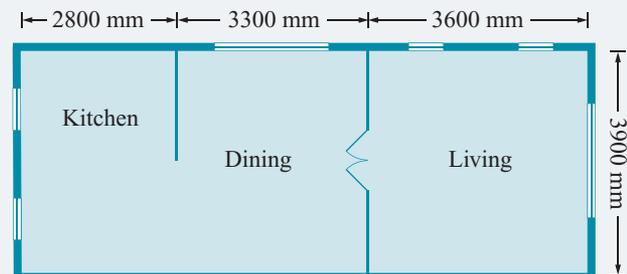
7 What is the real length represented by 3.6 cm on a scale drawing, if a scale of 1 : 10000 has been used?

8 Calculate the plan-view area of the roof of this house.



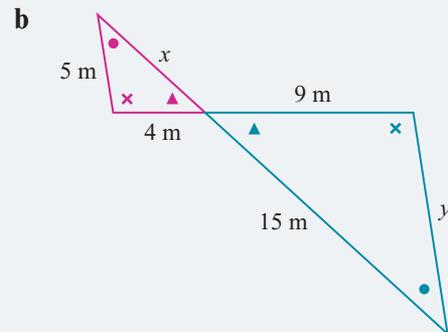
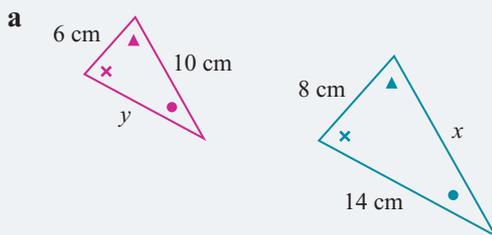
- 9 The floor plan of an extension to a house is shown below.
- The floors of the house are to be laid with porcelain tiles.
    - Calculate the area of the floor to be tiled.
    - Calculate the total cost, if the price is \$125/m<sup>2</sup> laid.
  - To estimate the cost of painting the ceilings and the internal walls of the extension, the owner ignores any windows or doors. The ceiling height is 2700 mm.
    - Estimate the area of wall and ceiling to be painted.
    - Find the amount of paint needed to apply two coats, if 1 L of paint covers 16 m<sup>2</sup>.
  - All the rooms in the house are to be air conditioned.
    - Calculate the area to be air conditioned.
    - Calculate the volume of house to be air conditioned.
    - Use the table below to find the power output of the air conditioning unit required.

Volume (m <sup>3</sup> )	Power output required (kW)
90	5.4
100	6.0
110	6.6
120	7.2
130	7.8
140	8.4

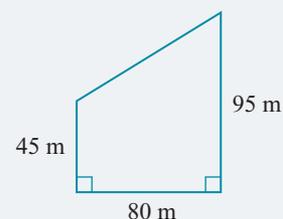


## REVIEW SET 3

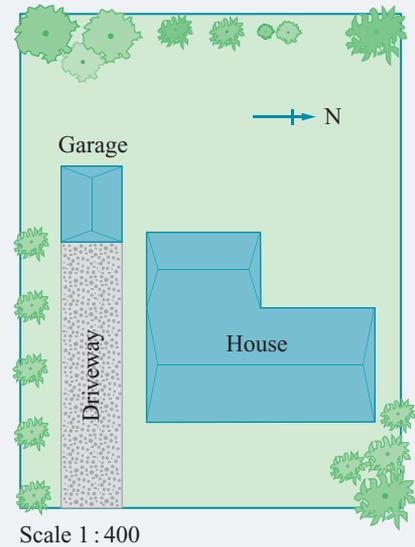
- An amount of \$15000 is divided between two people in the ratio 5 : 3. What is the value of the smaller share?
- The weight of meat decreases when it is cooked. If the ratio of the weight of raw meat to cooked meat is 5 : 4, calculate the weight of a 400 g piece of meat after it has been cooked.
- For the following pairs of similar figures, the pink triangle is the original figure.
  - What is the scale factor?
  - Find the lengths of the unknown sides labelled with a pronumeral.



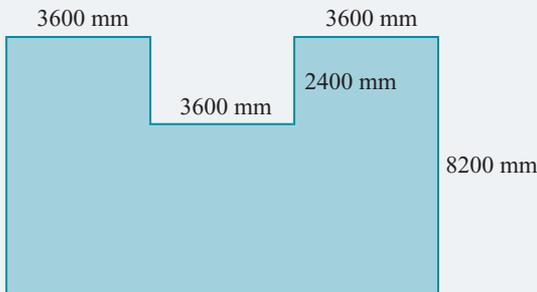
- Make a scale drawing of the field shown, using a scale of 1 : 1000.
  - What is the perimeter of the field?



- 5 Write the scale 2 cm to 5 m in the form 1 :  $n$ .
- 6 The site plan shown is drawn to scale.
- Find the dimensions of the garage.
  - Find the area of the driveway.
  - Find the actual dimensions of the house.
  - Find the area of the block of land.
  - If I walk from the front of the driveway to the garage, in which direction do I walk?

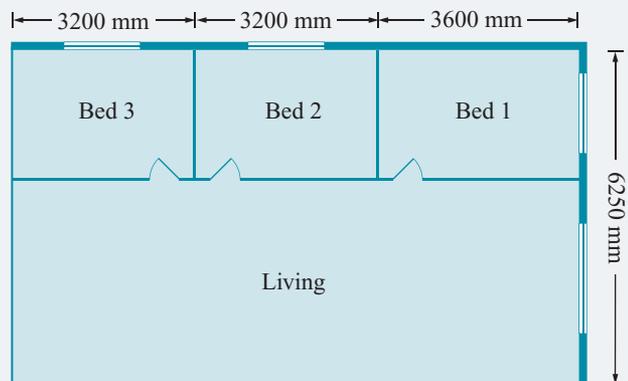


- 7 Calculate the plan-view area of the roof of this house.



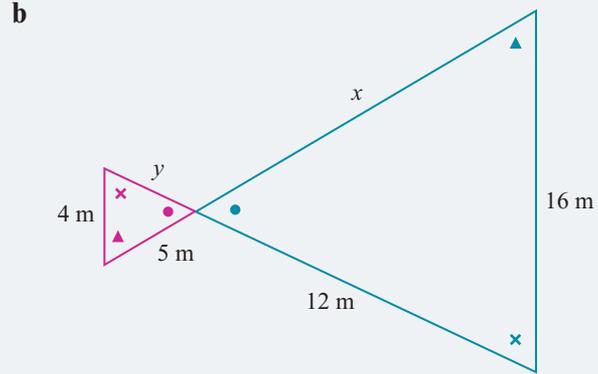
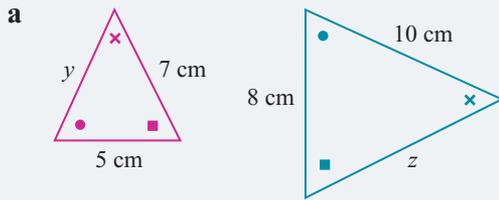
- 8 The floor plan of an extension to a house is shown below.
- The rooms are to be carpeted so that the least amount of carpet is used.
    - If carpet comes in rolls 3.6 m wide, estimate the number of metres of carpet that will be needed.
    - If the carpet costs \$145 per linear metre, what is the cost of the carpet needed?
  - To estimate the cost of painting the ceilings and the internal walls of the extension, the owner ignores any windows or doors. The ceiling height is 2400 mm and the width of each bedroom is 2300 mm.
    - Estimate the area of wall and ceiling to be painted.
    - Find the amount of paint needed to apply two coats, if 1 L of paint covers 14 m<sup>2</sup>.
  - All the rooms in the extension are to be air conditioned.
    - Calculate the area to be air conditioned.
    - Calculate the volume of house to be air conditioned, if the ceilings are 2400 mm high.
    - Use the table below to find the power output of the air conditioning unit required.

Volume (m <sup>3</sup> )	Power output required (kW)
140	8.4
150	9.0
160	9.6
170	10.2

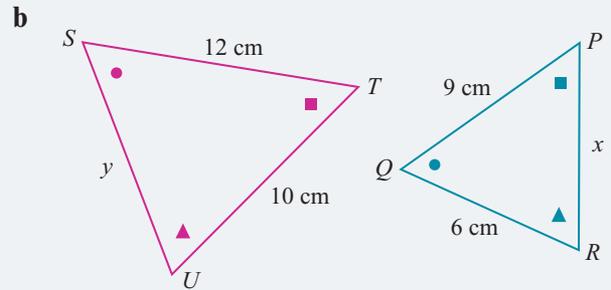
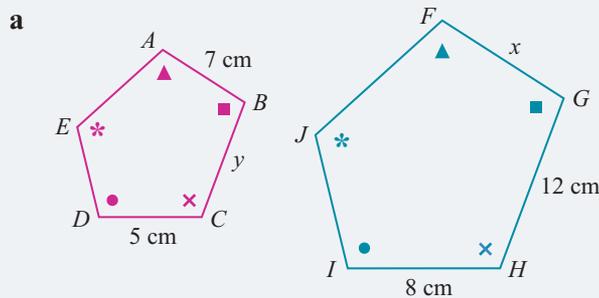


# REVIEW SET 4

- The profit on an investment is divided between Tony and Jen in the ratio 9 : 11. If the profit is \$12 000, calculate how much each person receives.
- For the following pairs of similar figures, the pink triangle is the original figure.
  - What is the scale factor?
  - Find the lengths of the unknown sides labelled with a pronumeral.



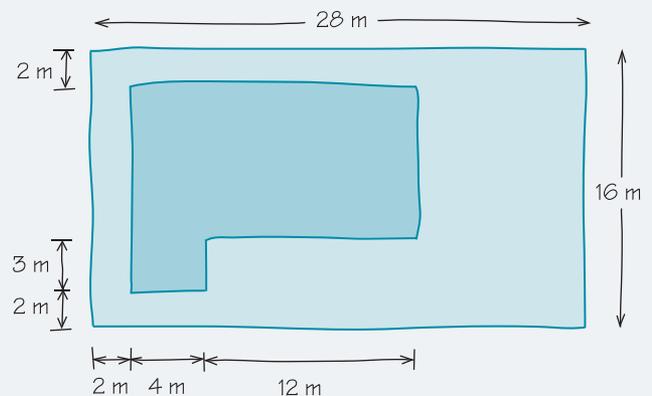
- Find the lengths of the unknown sides labelled with a pronumeral in the following pairs of similar figures.



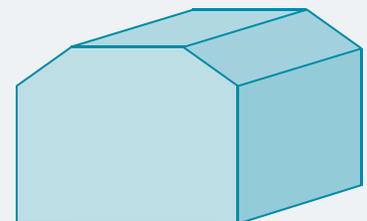
- A street lamp is 4 m high. When a man stands 5 m from the base of the lamp his shadow is 4.2 m long. Calculate the height of the man, to the nearest centimetre. (*Hint: draw a diagram to help visualise the situation.*)

- A rough sketch of the site plan of a house is shown.

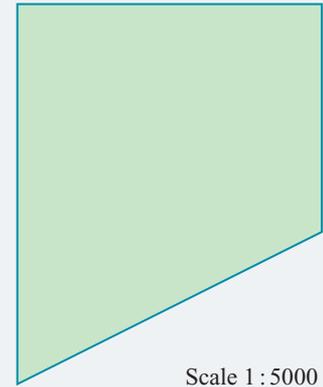
- Draw a scale diagram, using a scale of 1 cm to 2 m.
- What is the area of the block of land?
- What area of land is taken up by the house?
- What percentage of the block of land is occupied by the house?



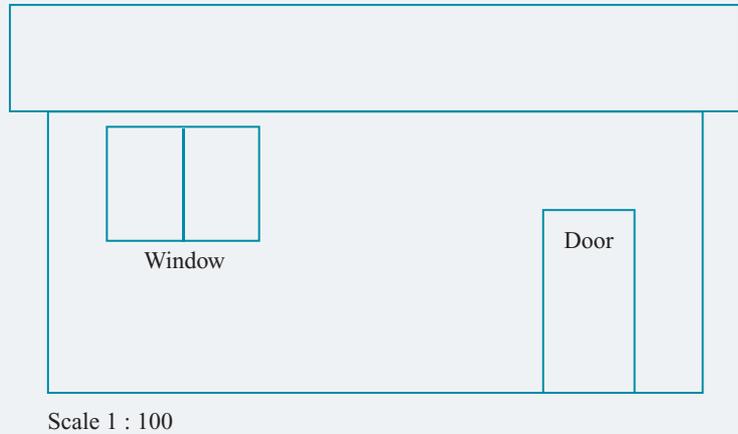
- Draw the plan, front elevation and side elevation of the solid shown.



- 7** Find the actual perimeter and area of the section of farmland shown in the scale diagram.

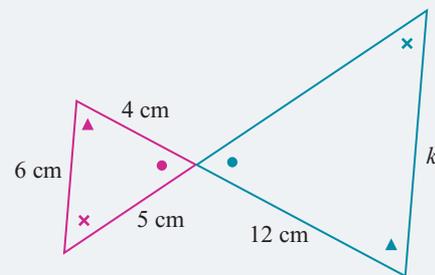


- 8** The diagram shows the north elevation of a house. The elevation is drawn to scale.
- Find the dimensions of the window.
  - Find the dimensions of the door.
  - Find the height of the top of the roof above the ground.
  - Find the length of the roof.
  - How far past the walls do each of the eaves extend?

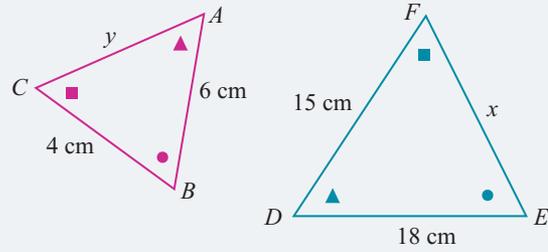


## REVIEW PRACTICE EXAMINATION QUESTION

- 1 a** A spray solution of fertiliser is made from using 10 mL of concentrate per 8 L of water.
- Write the dilution ratio for concentrate to water in the spray solution. (1 mark)
  - How many litres of water need to be added to 15 mL of concentrate to make a spray solution? (1 mark)
  - How many millilitres of concentrate need to be added to 200 L of water to make a spray solution? (1 mark)
- b** The profit of a company is divided between the three directors in the ratio 2 : 2 : 3. If the company makes a profit of \$630 000, calculate the largest share. (1 mark)
- c** Consider the pair of similar triangles shown. The pink triangle is the original figure.
- Determine the scale factor. (1 mark)
  - Find the length of the unknown side labelled  $k$ . (1 mark)



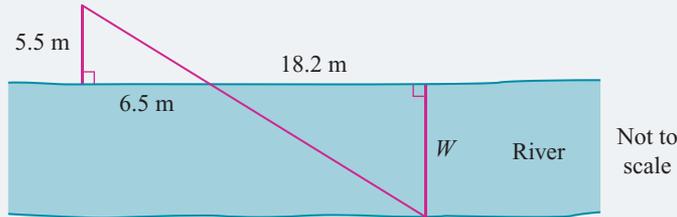
- d Find the lengths of the unknown sides labelled  $x$  and  $y$  in the pair of similar triangles shown.



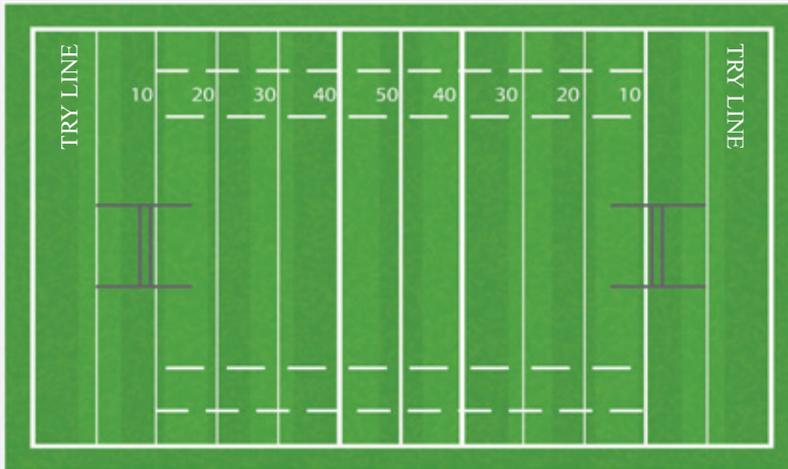
(2 marks)

- e The width ( $W$ ) of a river can be calculated using two similar triangles, as shown in the diagram. What is the width of the river?

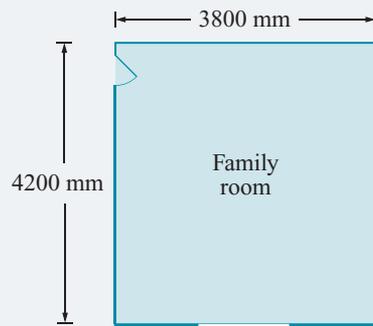
(2 marks)



- f The length of a rugby league field is 100 m between the try lines.



- i Calculate the scale used in the diagram. (1 mark)  
 ii Use the scale to calculate the width of the rugby field. (1 mark)  
 g The diagram shows the family room of a house. The three windows are 1800 mm by 1200 mm and the door is 2100 mm by 760 mm. The ceiling height is 2700 mm.



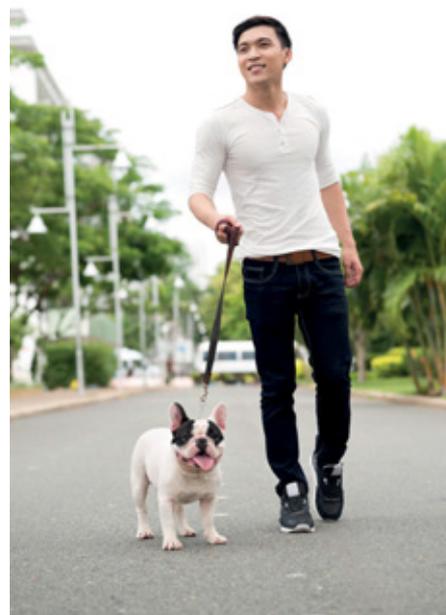
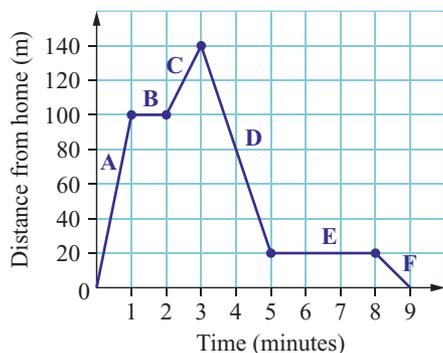
- The owner wants to paint the four walls and ceiling of this room. Calculate the area to be painted. (3 marks)

TOTAL: 15 marks

- 1 Aaron is going on a holiday. He borrows \$5200 over 4 years at a flat interest rate of 10.4% p.a.
  - a Find the simple interest paid.
  - b Find the total to be repaid.
- 2 Find the interest payable on a loan of \$7600 at a simple interest rate of 11.9% for these periods of time.
  - a  $3\frac{1}{2}$  years
  - b 21 months
- 3 Ginger borrows \$3800 to go on a cruise. She repays \$4730 over a period of 2 years. Calculate the simple interest rate.
- 4 Leo borrows \$15000 to buy a car. The simple interest rate is 11.3% and he takes the loan over 4 years.
  - a Find the interest on the loan.
  - b Find the total to be repaid.
  - c What is Leo's monthly payment?
- 5
  - a Use the compound interest formula to calculate the future value of a fixed-term investment of \$7580 over 3 years at 4.54% p.a. interest compounding yearly.
  - b Find the total interest earned.
- 6
  - a Use the compound interest formula to calculate the future value of a fixed-term investment of \$2300 over 5 years at 3.4% p.a. interest compounding quarterly.
  - b Calculate the amount that must be invested at 3.2% p.a. interest compounding annually to have \$10000 at the end of 9 years.
  - c Calculate the amount that must be invested at 5.5% p.a. interest compounding quarterly to have \$1040 at the end of 6 years.
  - d Calculate the amount to which \$2000 will grow if invested for 3 years at 6.6% p.a. interest compounding monthly.
  - e Calculate the amount that must be invested now at 3.8% p.a. interest compounding monthly in order to have \$10000 in 5 years' time.
- 7 In three successive years the annual rate of inflation was 2.8%, 2.5% and 2.1% respectively. How much would you expect to pay at the end of these 3 years for a book that cost \$23.99 at the start of this period?
- 8 A coin collection is purchased for \$2800. It appreciates in value by an average of 3.5% p.a. Find the appreciated value of the coin collection after:
  - a 10 years
  - b 20 years.
- 9 Convert the following rates.
  - a 24 t/ha to kg/m<sup>2</sup>
  - b 15 m/s to km/h
- 10 The cost of a pack of 5 candles is \$2.95 and the cost of a box of 24 candles is \$15.20. Which option is the better buy?
- 11 Kathy charges a call-out fee of \$40 and \$16 per hour as a gardener. How much will the client be charged if it took 4.5 hours for Kathy to complete the job?



12 This distance-time graph represents Isaac's journey.



- a How far from home was Isaac after:
  - i 1 min?
  - ii 2.5 min?
  - iii 4 min?
  - iv 7 min?
- b In which time interval/s was Isaac stationary?
- c What distance had Isaac travelled after 4 min?
- d In which section of the graph (A–F) was Isaac travelling the fastest? What was his speed?
- e What was Isaac's speed in section C of the graph?
- f What was Isaac's speed in section D of the graph?
- g What was the total distance travelled by Isaac over the 9 minutes?
- h What was his average speed over the 9 minutes?
- i Describe Isaac's journey.

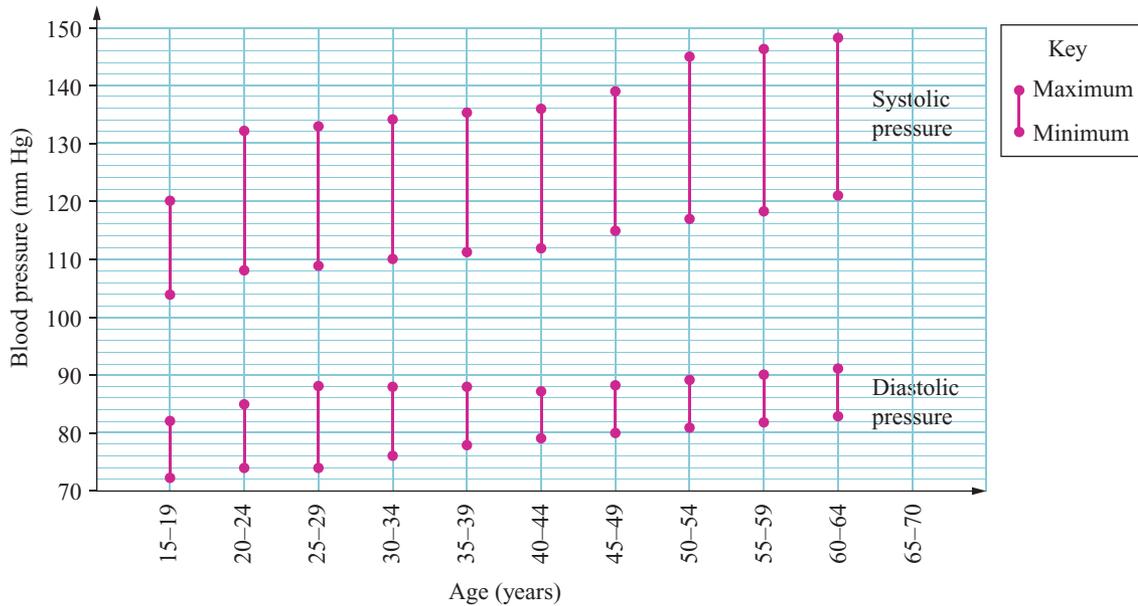
- 13 How far can a vehicle travel on 42 L of fuel if its fuel consumption rate is 10.1 L/100 km?
- 14 Calculate the fuel consumption rate in L/100 km for a car that used 35 L of petrol on a trip of 285 km.
- 15 A person's maximum heart rate can be calculated using the formula  $220 - \text{age in years}$ . Calculate the maximum heart rate for a 21-year-old male.
- 16 The average adult heart pumps about 70 mL of blood with each beat. Calculate the volume of blood pumped by a human heart beating for 15 minutes at 88 bpm.
- 17 The following table shows one type of blood pressure classification for adults.

Category	Systolic (mmHg)	Diastolic (mmHg)
Hypotension	< 90	< 60
Normal	90–119	60–79
Prehypertension	120–139	80–89
Stage 1 hypertension	140–159	90–99
Stage 2 hypertension	160–179	100–109
Hypertensive emergency	$\geq 180$	$\geq 110$

Use the table to determine the classification that would be given to an adult with each of the following blood pressure readings.

- a 125 over 86
- b 190 over 115
- c 105 over 75

- 18** The bars on the graph below show the normal ranges of blood pressure for people of various ages. Use the graph to answer the following.

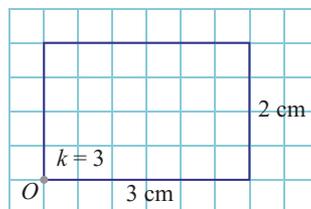


- a** What is the normal range of blood pressure for a 56-year-old person?
- b** Janelle, aged 27, had a blood pressure reading of 136 over 90. Is her reading within the normal range for her age?
- c** Colin, aged 48, had a blood pressure reading of 180 over 110. A doctor prescribed Colin a medication to reduce his blood pressure. To check that the medication was being effective, the doctor measured Colin's blood pressure for 10 weeks and recorded the following results.

Week	1	2	3	4	5	6	7	8	9	10
Systolic pressure	180	165	148	136	142	138	135	136	132	130
Diastolic pressure	110	100	95	85	88	86	84	85	82	83

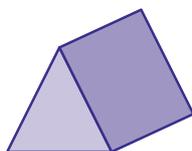
With reference to the data provided in the table, comment on the effectiveness of the medication during the 10-week period in returning Colin's blood pressure to the normal range.

- 19** The ratio of boys to girls in a school is 7 : 8. If there are 483 boys, how many girls are there?
- 20** Divide \$65 000 into two parts in the ratio 3 : 5.
- 21** Copy this figure on to grid paper and enlarge it using  $O$  as the centre of enlargement and a scale factor of 3.

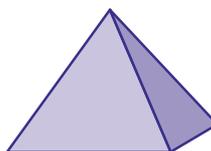


- 22** A photograph 16 cm long and 12 cm wide is enlarged so that the new length is 40 cm.
- Find the scale factor.
  - Find the new width of the enlarged photograph.
- 23** A tree casts a shadow 3.8 m long. At the same time a metre rule casts a shadow 0.73 m long. Calculate the height of the tree.
- 24 a** Using the scale 1 : 100 000, what real length would be represented by a length of 2.3 cm on a scale drawing?
- 25** Write the scale 4 cm to 5 m in the form 1 :  $n$ .
- 26** Draw the plan, front elevation and side elevation of the following solids.

**a**

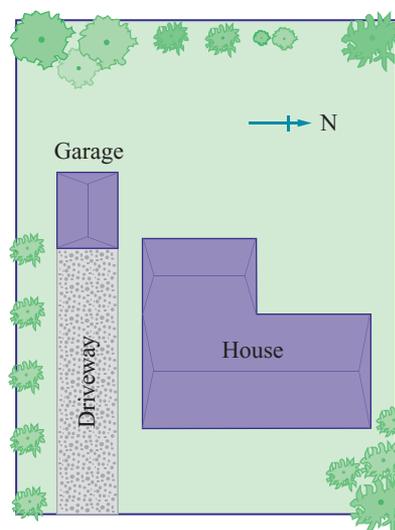


**b**



- 27** For the site plan shown, answer the following questions. The plan is drawn to scale.

- Find the dimensions of the garage.
- The boundary of the block of land is to be fenced.
  - Calculate the length of fencing required. (*Hint*: the entrance to the driveway is not to be fenced.)
  - If fencing costs \$55 per linear metre, calculate the cost of the fencing required.
- The driveway is to be laid with concrete.
  - Calculate the area of the driveway.
  - How much concrete is needed to cover the driveway to a depth of 10 cm?
- The house is to be air conditioned.
  - Estimate the area to be air conditioned by calculating the plan-view area of the roof of the house.
  - Calculate the volume of house to be air conditioned if the ceilings are 2400 mm high.
  - Use the table below to find the power output of the air conditioning unit required.



Volume (m <sup>3</sup> )	Power output required (kW)
210	12.6
220	13.2
230	13.8
240	14.4

- What percentage of the block of land is occupied by the house?

# 4

---

## Simultaneous linear equations

The main mathematical ideas in this chapter are:

- ▶ using and interpreting graphs of the form  $y = mx + c$
- ▶ modelling linear relationships
- ▶ working with linear models and their graphical representation
- ▶ identifying the solution to simultaneous linear equations from tables and graphs
- ▶ solving simultaneous linear equations graphically
- ▶ solving practical problems using simultaneous linear equations
- ▶ conducting a break-even analysis.

# ARE YOU READY?

- 4A** ▶ **1** What is the value of  $y$  in the equation  $y = 2x + 5$  when  $x = -1$ ?  
**A**  $-7$                       **B**  $-3$   
**C**  $3$                          **D**  $7$

- 4A** ▶ **2** Which table shows  $x$ - and  $y$ -values for  $y = 3x - 4$ ?

**A**

$x$	2	3	4
$y$	2	6	9

**B**

$x$	1	3	5
$y$	$-1$	5	11

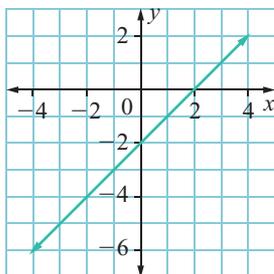
**C**

$x$	0	2	4
$y$	0	2	8

**D**

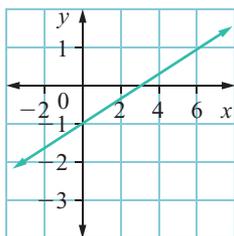
$x$	3	4	5
$y$	5	6	7

- 4A** ▶ **3** Which equation produces the following graph?



- A**  $y = x + 2$             **B**  $y = 2x - 4$   
**C**  $y = 2 - x$             **D**  $y = x - 2$

Use the following graph for questions **4**, **5** and **6**.

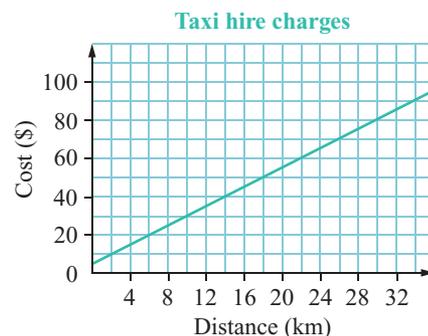


- 4A** ▶ **4** What is the gradient of this graph?  
**A**  $-1$                       **B**  $\frac{1}{3}$   
**C**  $\frac{2}{3}$                       **D**  $3$

- 4A** ▶ **5** What is the  $y$ -intercept of this graph?  
**A**  $2.5$                       **B**  $3$   
**C**  $-1$                       **D**  $-3$

- 4A** ▶ **6** What is the equation of this graph?  
**A**  $y = \frac{1}{3}x - 1$         **B**  $y = x + \frac{1}{3}$   
**C**  $y = \frac{1}{3}x + 1$         **D**  $y = 3x - 1$

Use the following graph for questions **7**, **8** and **9**.



- 4B** ▶ **7** What is the taxi hire charge for travelling 18 km?  
**A** \$30                      **B** \$40  
**C** \$50                      **D** \$60

- 4B** ▶ **8** How far can you travel by taxi for \$70?  
**A** 20 km                      **B** 22 km  
**C** 24 km                      **D** 26 km

- 4B** ▶ **9** Which equation best matches the graph of distance ( $d$ ) and cost ( $C$ )?  
**A**  $C = 2.5d + 5$         **B**  $C = 5d + 2.5$   
**C**  $d = 2.5C + 5$         **D**  $C = 2.5d - 5$

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook](#) [assess](#).

- Q1-2** Support sheet **4A.1** Substituting for pronumerals  
**Q3** Support sheet **4A.2** Plotting points from a table of values  
**Q4-6** Support sheet **4A.3** Identifying features of a linear graph  
**Q7-9** Support sheet **4B.1** Reading values from graphs

# 4A Graphs of the form

$$y = mx + c$$

## linear relationship

relationship between two variables where the coordinate points describing this relationship lie in a straight line when plotted on a number plane (or Cartesian plane)

These resources are available on your obook access:

- **Interactive 4A.1:** Explore key ideas for plotting linear graphs
- **Interactive 4A.2:** Explore gradient and intercepts of a linear graph
- **Interactive 4A.3:** Explore the drawing of graphs using gradient and  $y$ -intercept
- **Spreadsheet 4A.1:** Drawing linear graphs using a table of values
- **Spreadsheet 4A.2:** Drawing linear graphs using the gradient and  $y$ -intercept
- **assess quiz 4A:** Test your skills with an auto-correcting multiple-choice quiz

1010

In the Year 11 Mathematics Standard course, you used straight-line graphs to model practical situations and make predictions based on the model. Graphical representations are excellent models of practical situations because they provide a visual way to view trends.

This section will revise the generation of tables of values and the graphing of a **linear relationship**. You can also explore the use of technology to produce straight-line graphs.

### EXAMPLE 4A-1 Completing a table of values to graph a linear relationship

**a** Complete this table of values for the equation  $y = 5 - 2x$ .

$x$	-3	-2	-1	0	1	2	3
$y$							

**b** Plot each set of ordered pairs from the table for  $y = 5 - 2x$  on a number plane and draw a straight line through the points.

	Solve	Think	Apply																
<b>a</b>	<table border="1"> <tr> <td><math>x</math></td> <td>-3</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td><math>y</math></td> <td>11</td> <td>9</td> <td>7</td> <td>5</td> <td>3</td> <td>1</td> <td>-1</td> </tr> </table>	$x$	-3	-2	-1	0	1	2	3	$y$	11	9	7	5	3	1	-1	Substitute each $x$ -value into the equation $y = 5 - 2x$ to complete the table.	Substitute each $x$ -value into the linear relationship to find the matching $y$ -value.
$x$	-3	-2	-1	0	1	2	3												
$y$	11	9	7	5	3	1	-1												
<b>b</b>		Plot the points on a number plane and draw a straight line through them, extending as far as possible in both directions. Include an arrowhead at each end of the line to indicate that the line continues.	Plot the points on a number plane (also called a Cartesian plane) and join them with a straight line. The line extends beyond the plotted points. The graph of a linear relationship is always a straight line.																

# EXERCISE 4A Graphs of the form $y = mx + c$

- 1 Complete the following tables for the equations provided. Plot each set of ordered pairs on separate number planes and draw a straight line through the points.

a  $y = 2x + 1$

$x$	-2	-1	0	1	2
$y$					

b  $y = 2x + 3$

$x$	-4	-2	0	2	4
$y$					

c  $y = -3x - 4$

$x$	-4	-2	0	2	4
$y$					

d  $y = 4 - 5x$

$x$	-2	-1	0	1	2
$y$					

e  $y = 2x - 7$

$x$	-2	-1	0	1	2
$y$					

f  $y = 2 - 2x$

$x$	-2	-1	0	1	2
$y$					

- 2 By using a table of values such as the one shown below, draw graphs of the following linear relationships on separate number planes.

$x$	-3	0	3
$y$			

a  $y = x - 3$

b  $y = x + 2$

c  $y = 2x - 1$

d  $y = 1 - 3x$

e  $y = \frac{1}{2}x$

f  $y = \frac{2x + 3}{2}$

g  $y = \frac{2x}{3} - 6$

h  $y = 4 - 2x$

- 3 By using a table of values, draw the graph of  $y = \frac{x}{10} + 5$ . Explain your choice of  $x$ -values to use in the table and the scale to use on the axes of the number plane.

If the equation of a straight line is written in the form  $y = mx + c$ , the **gradient** of the line is  $m$  and the **y-intercept** is  $c$ .

*Note:*  $x$  is the **independent variable**, as any value may be used, and  $y$  is the **dependent variable**, as it depends on the value of  $x$ .

## gradient

steepness of a line (defined as  $m$ ) that can be measured using any two points on the line or interval using  $m = \frac{\text{rise}}{\text{run}}$

## y-intercept

point where a line crosses the  $y$ -axis of a number plane (or Cartesian plane)

## independent variable

variable whose outcomes are not due to those of another variable; usually represented on the horizontal axis of a graph; e.g. for the linear relationship  $y = 2x + 1$ , the independent variable is  $x$

## dependent variable

variable whose value depends on that of another; usually represented on the vertical axis of a graph; e.g. for the relationship  $y = 2x + 1$ , the dependent variable is  $y$  (as it depends on the value of  $x$ )

**EXAMPLE 4A-2** Writing the gradient and y-intercept of a line from its equation

Write the gradient and y-intercept of the straight lines with the following equations.

**a**  $y = 2x + 7$

**b**  $y = -3x + 2$

**c**  $y = \frac{2x}{3} - 5$

**d**  $y = -x + 3.6$

**e**  $y - \frac{x}{5} = \frac{4}{5}$

**f**  $4x + 2y = 1$

	Solve	Think/Apply
<b>a</b>	Gradient = 2, y-intercept = 7	When the equation is in the form $y = mx + c$ , the gradient is $m$ , which is the coefficient of $x$ (the number in front of $x$ ). The y-intercept is $c$ (the constant term). If the equation is not in the form $y = mx + c$ , it can be rearranged into this form. For part <b>e</b> , add $\frac{x}{5}$ to both sides of the equation. For part <b>f</b> , subtract $4x$ from both sides of the equation and then divide both sides by 2.
<b>b</b>	Gradient = -3, y-intercept = 2	
<b>c</b>	Gradient = $\frac{2}{3}$ , y-intercept = -5	
<b>d</b>	Gradient = -1, y-intercept = 3.6	
<b>e</b>	$y - \frac{x}{5} = \frac{4}{5}$ becomes $y = \frac{x}{5} + \frac{4}{5}$ . Gradient = $\frac{1}{5}$ , y-intercept = $\frac{4}{5}$	
<b>f</b>	$4x + 2y = 1$ becomes $y = -2x + \frac{1}{2}$ . Gradient = -2, y-intercept = $\frac{1}{2}$	

**4** Write the gradient and y-intercept of the straight lines with the following equations.

**a**  $y = 4x + 6$

**b**  $y = -2x + 9$

**c**  $y = \frac{3}{4}x - 8$

**d**  $y = x - 2$

**e**  $y = -x + 4$

**f**  $y = -1.2x - 2.8$

**g**  $y = 5.4x + 6.5$

**h**  $y = -\frac{3x}{8} + \frac{9}{2}$

**i**  $y = \frac{x}{2} + 7$

**j**  $y = \frac{x}{6} + \frac{5}{3}$

**k**  $y = -\frac{x}{3} - 2$

**l**  $y - \frac{x}{2} = 2$

**m**  $4x + y = 8$

**n**  $3x + 2y = -4$

**o**  $2x - y = 5$

**EXAMPLE 4A-3** Using the gradient and y-intercept to write the equation of a straight line

Write the equation of the straight line with:

**a** gradient 4 and y-intercept -3

**b** gradient -6 and y-intercept 1

**c** gradient  $\frac{4}{5}$  and y-intercept -2

**d** gradient 1 and y-intercept 0.9

**e** gradient  $\frac{1}{3}$  and y-intercept  $2\frac{1}{4}$ .

	Solve	Think	Apply
	The equation of a straight line is $y = mx + c$ , where $m$ is the gradient and $c$ is the y-intercept.	Substitute the values for $m$ and $c$ into $y = mx + c$ .	Write the equation in the form $y = mx + c$ , where $m$ is the gradient and $c$ is the y-intercept.
<b>a</b>	$y = 4x - 3$	$m = 4, c = -3$	
<b>b</b>	$y = -6x + 1$	$m = -6, c = 1$	
<b>c</b>	$y = \frac{4}{5}x - 2$	$m = \frac{4}{5}, c = -2$	
<b>d</b>	$y = x + 0.9$	$m = 1, c = 0.9$	
<b>e</b>	$y = \frac{1}{3}x + 2\frac{1}{4}$	$m = \frac{1}{3}, c = 2\frac{1}{4}$	

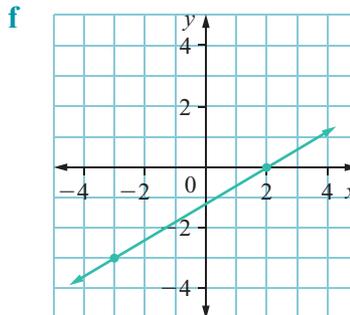
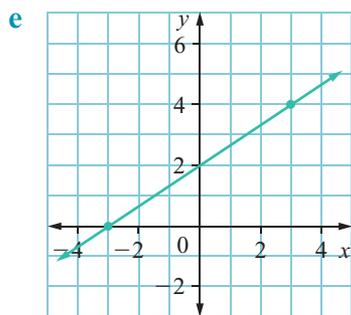
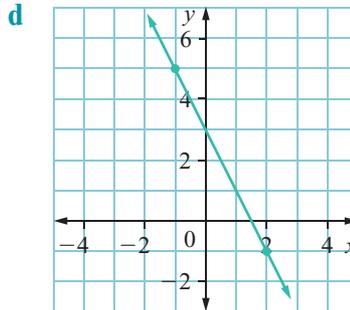
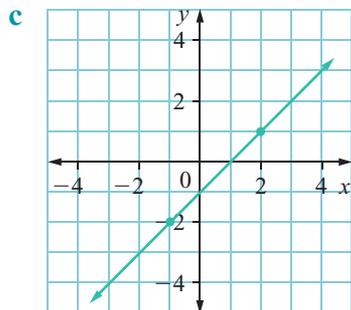
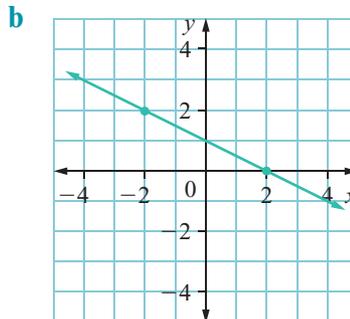
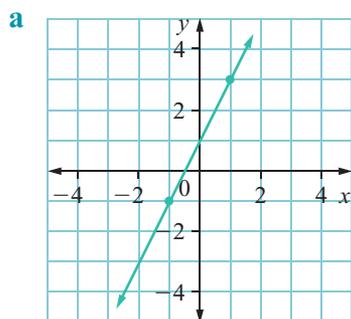


6 For the straight lines shown, find the:

i gradient

ii y-intercept

iii equation of the line.



**EXAMPLE 4A-5** Drawing a linear graph using the gradient and y-intercept

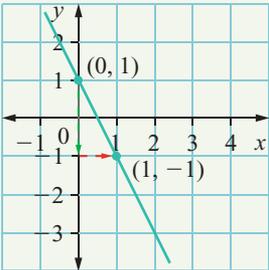
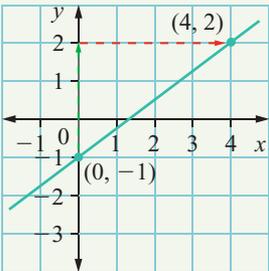
Draw the following straight lines without using a table of values.

a  $y = 3x - 2$

b  $y = -2x + 1$

c  $y = \frac{3x}{4} - 1$

	Solve	Think	Apply
a		<p>Gradient = 3, y-intercept = -2                      Start at the y-intercept (0, -2).                      As the gradient is <math>\frac{3}{1}</math>, move up 3 units                      then right 1 unit from (0, -2).                      This gives a second point (1, 1) on                      the line.                      Rule a line through these two points.</p>	<p>Find the gradient and                      y-intercept of the line.                      Use (0, c) as the first point                      on the line.                      Use Gradient = <math>\frac{\text{rise}}{\text{run}}</math> to find                      a second point on the line.                      Plot the two points and rule                      a line through them.</p>

	Solve	Think	Apply
<b>b</b>		<p>Gradient = <math>-2</math>, <math>y</math>-intercept = <math>1</math></p> <p>Start at the <math>y</math>-intercept <math>(0, 1)</math>.</p> <p>As the gradient is <math>-\frac{2}{1}</math>, move down 2 units then right 1 unit from <math>(0, 1)</math>.</p> <p>This gives a second point <math>(1, -1)</math> on the line.</p> <p>Rule a line through these two points.</p>	
<b>c</b>		<p>Gradient = <math>\frac{3}{4}</math>, <math>y</math>-intercept = <math>-1</math></p> <p>Start at the <math>y</math>-intercept <math>(0, -1)</math>.</p> <p>As the gradient is <math>\frac{3}{4}</math>, move up 3 units then right 4 units from <math>(0, -1)</math>.</p> <p>This gives a second point <math>(4, 2)</math> on the line.</p> <p>Rule a line through these two points.</p>	

**7** Draw each of the lines in question 4 a–f without using a table of values.

**8 a** Use graphing technology to draw the graphs of these equations on the same set of axes.

**i**  $y = 2x$

**ii**  $y = 2x + 3$

**iii**  $y = 2x - 2$

**iv**  $y = 2x + 1$

**b** What do you notice about the graphs?

**c** What is the effect of the constant term?

**9 a** Use graphing technology to draw the graphs of these equations on the same set of axes.

**i**  $y = x + 1$

**ii**  $y = 2x + 1$

**iii**  $y = 3x + 1$

**iv**  $y = \frac{1}{2}x + 1$

**b** What do you notice about the graphs?

**c** What is the effect of changing the coefficient of  $x$ ?

**10** Use graphing technology to draw the graphs with the given equations in questions 1, 2, 3 and 4 and find specific values. Check the accuracy of your answers from the hand-drawn graphs with those obtained using technology.

**11** Create a spreadsheet to produce the graph of a linear relationship from a table of values. Enter the values for  $x$  in the first column and values for  $y$  in the second column of the spreadsheet. Highlight the cells containing the  $x$ - and  $y$ -values, click **Insert** and **Charts**, and choose **Line graph**. Check some of your answers to questions 1 and 2 using your spreadsheet. You might like to use the prepared spreadsheet provided on your [obook assess](#).

**12** Create a spreadsheet to produce the graph of a linear relationship from its equation using the gradient and  $y$ -intercept. Check some of your answers to questions 1, 2, 3 and 4 using your spreadsheet. You might like to use the prepared spreadsheet provided on your [obook assess](#).

# 4B Linear models

## production cost

cost (or expense) of producing an item for sale, which can involve both a fixed cost and a variable cost that depends on the number of items produced

## revenue

amount of money (or income) received from the sale of items produced by a company, which can depend on the number of items sold

These resources are available on your obook assess:

- **Video tutorial 4B:** Watch and listen to an explanation of Example 4B-1
- **Investigation 4B.1:** Model the height of water flowing into a water tank
- **Investigation 4B.2:** Model the temperature of an icy-pole placed in a freezer
- **assess quiz 4B:** Test your skills with an auto-correcting multiple-choice quiz

1010

Linear relationships can be used to model practical situations. The linear equation used as a model will be of the form  $y = mx + c$ , where the gradient ( $m$ ) and y-intercept ( $c$ ) of the corresponding linear graph have practical meanings in each case. Usually the constant  $c$  is a fixed amount, and the gradient  $m$  is the rate of change. For example, a taxi fare can be calculated by adding a fixed cost to the charge per kilometre travelled.

In this section, we will also look at practical contexts involving **production cost** and **revenue**.

## EXAMPLE 4B-1 Drawing the graph of a cost model from its equation

The Magnificent Muffin Shop has a fixed cost of \$100 per day and a variable production cost of \$1.50 per muffin. The daily cost can be modelled using the equation  $C = 1.5n + 100$ .

- Explain each term in the equation.
- Draw the graph of  $C = 1.5n + 100$  for  $0 \leq n \leq 200$ .
- Use the graph to find the number of muffins produced for \$220.

	Solve	Think	Apply
a	1.5 is the cost per muffin, 100 is the fixed cost and $C$ is the total cost. All costs are in dollars. $n$ is the number of muffins made.	The gradient is the coefficient of $n$ , which is 1.5. The constant term $c$ is the fixed cost of 100.	The fixed cost is the intercept on the vertical axis. The variable cost relates to the gradient.
b		Use graphing technology or complete a table of values for $n$ and $C$ using $C = 1.5n + 100$ , and plot the points to produce the graph.	Draw the graph using technology or by completing a table of values and plotting the points. The gradient and y-intercept method of drawing a linear graph could also be used.
c	80 muffins are produced for \$220.	Draw a line across from the vertical axis at 220 until it meets the line. Draw down to the horizontal axis and read the value of $n$ .	Find values by drawing lines from one axis to the linear graph and then to the other axis.

# EXERCISE 4B Linear models

- 1** The Supertight Tie Company has a fixed cost of \$1200 per day and a variable production cost of \$3 per tie. The daily cost can be modelled using the equation  $C = 3n + 1200$ .

- a** Explain each term in the equation  $C = 3n + 1200$ .  
**b** Complete this table of values using the equation  $C = 3n + 1200$ .

$n$	0	50	100	150	200	250
$C$						

- c** Draw the graph of  $C = 3n + 1200$ .  
**d** Use your graph from part **c** to find the number of ties produced when the daily cost is \$1400.
- 2** The distance,  $d$  kilometres, travelled by a train in a time of  $t$  hours is  $d = 80t$ .

- a** Complete this table of values using the equation  $d = 80t$ .

$t$	1	2	3	4	5
$d$					

- b** Draw the graph of  $d = 80t$ .  
**c** Where does this graph cut the vertical axis? Why?  
**d** What is the gradient of this graph? What does this value represent in this case?  
**e** Use the graph to find how far away the train is after  $3\frac{1}{2}$  hours.  
**f** Use the graph to find when the train is 200 km away.

## EXAMPLE 4B-2 Drawing the graph of a revenue model from its equation

The Marvellous Muffin Shop sells its muffins for \$4 each. The sales revenue can be modelled using the equation  $R = 4n$ .

- a** Explain each term in the equation  $R = 4n$ .  
**b** Graph the straight line with equation  $R = 4n$  for  $0 \leq n \leq 100$ .  
**c** Use the graph to find the number of muffins that are sold if the revenue is \$240.

	Solve	Think	Apply
<b>a</b>	$R$ is the revenue, $n$ is the number of muffins sold. The gradient is 4, this is the price per muffin in dollars.	There is no fixed amount so the graph starts at the origin. The gradient is the price of each muffin.	If there are no sales then the revenue is zero, so the origin is a point on the graph.
<b>b</b>		Use graphing technology or complete a table of values for $n$ and $R$ using $R = 4n$ and plot the points to produce the graph.	Draw the graph using technology or by completing a table of values and plotting the points.

$n$	0	25	50	75	100
$R$	0	100	200	300	400

	Solve	Think	Apply
c	60 muffins must be sold for the revenue to be \$240.	Draw a line across from the vertical axis at 240 to the graph and then down to the $n$ -axis to read the value of $n$ .	Find values by drawing a line from one axis to the graph and then to the other axis.

**3** The Supertight Tie Company sells ties for \$15 each. The sales revenue may be modelled using the equation  $R = 15n$ .

- a** Explain each term in the equation  $R = 15n$ .  
**b** Complete this table of values for the equation  $R = 15n$ .

$n$	0	50	100	150	200	250
$R$						

- c** Draw the straight-line graph of the equation  $R = 15n$ .  
**d** Use the graph to find the number of ties sold when the revenue is \$1800.  
**e** What is the gradient of the straight line? What does this represent?

**4** Tina travels 200 km to her friend's home at an average speed of 80 km/h. The distance Tina is from her friend's home after travelling for  $t$  hours can be modelled using the linear equation  $D = 200 - 80t$ .

- a** Explain each term in the equation  $D = 200 - 80t$ .  
**b** Draw the straight-line graph of the equation  $D = 200 - 80t$  for  $t$  values up to 4 hours.  
**c** Use the graph to find how far Tina is from her friend's home after 1.5 hours.  
**d** Use the graph to find how long it takes for Tina to reach her friend's home.  
**e** What is the value of  $D$  when  $t$  is 4 hours? Explain why this linear model can only be used for particular values of  $t$  in this context.

**5** The Munchy Sandwich Shop has a monthly fixed expense of \$2000 and the cost of making each sandwich is \$1.20.

- a** Write an equation modelling this information. Use  $C$  for monthly cost in dollars and  $n$  for the number of sandwiches made.  
**b** Complete this table of values using your equation from part **a**.

$n$	0	500	1000	1500	2000	2500
$R$						

- c** Draw the straight-line graph for this linear model.  
**d** What is the value of the gradient? What does this value represent?  
**e** Use the graph to find the number of sandwiches made in a month if the cost is \$3500.

**6** The Tight Squeeze Fruit Juice Company has a weekly fixed cost of \$325. The cost to produce a cup of fruit juice is \$1.50.

- a** Write a linear equation modelling this information. Use  $C$  for the weekly cost in dollars and  $n$  for the number of cups of fruit juice.  
**b** Complete this table of values using your equation from part **a**.

$n$	0	200	400	600	800	1000
$C$						

- c** Draw the straight-line graph for this linear model.

- d** Where does the graph cut the vertical axis? What does this value represent?
- e** What is the gradient? What does this value represent?
- f** Use the graph to find the number of cups of juice made in a week if the cost is \$1000.

- 7** The Munchy Sandwich Shop sells sandwiches for \$3.50 each.
- a** Write an equation modelling this information. Use  $R$  for revenue in dollars and  $n$  for the number of sandwiches sold.
  - b** Complete this table of values using your equation from part **a**.

$n$	0	500	1000	1500	2000	2500
$C$						

- c** Draw a straight-line graph for this linear model.
  - d** Use the graph to find the number of sandwiches sold when the revenue is \$2800.
  - e** What is the gradient of this straight line? What does this represent?
- 8** The Tight Squeeze Fruit Juice Company sells juice for \$4.00 per cup.
- a** Write an equation modelling this information. Use  $R$  for revenue in dollars and  $n$  for the number of cups of juice sold.
  - b** Complete this table of values using your equation from part **a**.

$n$	0	200	400	600	800	1000
$R$						

- c** Draw the straight-line graph for this linear model.
  - d** Use the graph to find the number of cups of juice sold when the revenue is \$3000.
  - e** What is the gradient of this straight line? What does this represent?
- 9** The cost of hiring a taxi is \$6 flagfall and \$3.60 per kilometre travelled.
- a** Write a linear equation modelling this information to relate cost and distance travelled.
  - b** Use graphing technology or a table of values to draw the graph of this linear model for distances up to 20 km.
  - c** Use the graph to find the cost of travelling 10 km.
  - d** Use the graph to estimate how far you could travel for \$30.
- 10** On a building site, columns with a fixed cross-sectional area of  $0.78 \text{ m}^2$  are to be constructed by pouring concrete into moulds of different heights.
- a** Write a linear equation modelling this information to relate volume of concrete (in cubic metres) and height of a column (in metres).
  - b** Use graphing technology or a table of values to draw the graph of this linear model for height values up to 7 m.
  - c** Use the graph to estimate the volume of concrete needed to construct a column that is 4 m high.
  - d** Use the graph to estimate the height of a column that can be constructed using  $2.4 \text{ m}^3$  of concrete.

- 11** The conversion from temperature in degrees Celsius ( $C$ ) to temperature in degrees Fahrenheit ( $F$ ) is given by the equation  $F = \frac{9}{5}C + 32$ .
- a** Using graphing technology or by completing a table, draw the graph of this relationship for values of  $C$  between 0 and 100.
  - b** A 'rule of thumb' conversion is to double the temperature in degrees Celsius and add 30. Graph this model for values of  $C$  between 0 and 100.
  - c** Compare the two graphs. Describe the accuracy of the rule of thumb. Which would you use? When?

# 4C Identifying solutions to simultaneous linear equations

These resources are available on your [obook](#) [access](#):

- **assess quiz 4C:** Test your skills with an auto-correcting multiple-choice quiz



Consider the following tables of values for the linear equations given.

$$y = 2x + 7$$

<b>x</b>	-3	-2	-1	0	1	2	3
<b>y</b>	1	3	5	7	9	11	13

$$y = -4x - 5$$

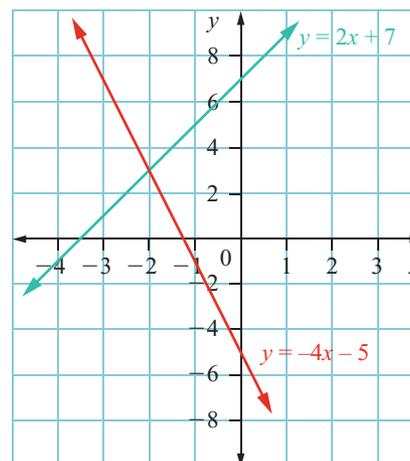
<b>x</b>	-3	-2	-1	0	1	2	3
<b>y</b>	7	3	-1	-5	-9	-13	-17

The equation  $y = 2x + 7$  has a solution of  $x = -3$  and  $y = 1$  because, when we substitute these values into the equation, the statement  $1 = 2 \times -3 + 7$  is true. In fact, this equation has an infinite number of solutions as  $x = -2$  and  $y = 3$ ,  $x = -1$  and  $y = 5$ ,  $x = 0$  and  $y = 7$ , etc. are also solutions. Similarly the equation  $y = -4x - 5$  has an infinite number of solutions as  $x = -3$  and  $y = 7$ ,  $x = -2$  and  $y = 3$ ,  $x = -1$  and  $y = -1$ , etc. are solutions.

We can see that  $x = -2$  and  $y = 3$  is a solution of *both* equations. We say that the linear equations  $y = 2x + 7$  and  $y = -4x - 5$  are solved simultaneously by the values  $x = -2$  and  $y = 3$ . Hence, to solve a pair of linear equations simultaneously, or solve a pair of **simultaneous linear equations**, means to find the values of the variables that make both equations true.

The graphs of the two linear equations can be drawn on the same number plane as shown on the right.

What are the coordinates of the **point of intersection** of the two lines? How does this relate to the solution of the simultaneous linear equations  $y = 2x + 7$  and  $y = -4x - 5$ ?



### simultaneous linear equations

two or more linear equations that use the same variables

### point of intersection

point where two or more lines intersect or cross over one another on a graph

## EXERCISE 4C Identifying solutions to simultaneous linear equations

- 1 Consider the tables given below to find the  $x$ -value and the  $y$ -value that is the solution to each pair of simultaneous linear equations.

**a**  $y = 2x$

<b>x</b>	-3	-2	-1	0	1	2	3
<b>y</b>	-6	-4	-2	0	2	4	6

$x + y = 6$

<b>x</b>	-3	-2	-1	0	1	2	3
<b>y</b>	9	8	7	6	5	4	3

**b**  $y = 2x - 1$

<b>x</b>	0	1	2	3	4
<b>y</b>	-1	1	3	5	7

**c**  $y = 3x - 5$

<b>x</b>	5	6	7	8	9
<b>y</b>	10	13	16	19	22

$x + y = 8$

<b>x</b>	0	1	2	3	4
<b>y</b>	8	7	6	5	4

$x + y = 23$

<b>x</b>	5	6	7	8	9
<b>y</b>	18	17	16	15	14

**EXAMPLE 4C-1** Checking the solution to simultaneous linear equations

 Check if each given  $x$ -value and  $y$ -value is the solution to the pair of simultaneous linear equations.

**a**  $x = 2, y = 4$

$y = 3x - 2$  and  $y = -2x + 8$

**b**  $x = 5, y = -3$

$y = -2x + 7$  and  $y = 2x - 11$

	Solve	Think	Apply
<b>a</b>	$y = 3x - 2$ When $x = 2$ and $y = 4$ , LHS = 4 and RHS = $3 \times 2 - 2 = 4$ So LHS = RHS $y = -2x + 8$ When $x = 2$ and $y = 4$ , LHS = 4 and RHS = $-2 \times 2 + 8 = 4$ So LHS = RHS As $x = 2, y = 4$ satisfies both equations, it is the solution.	Substitute $x = 2$ and $y = 4$ into the left-hand side (LHS) and right-hand side (RHS) of each equation. Check if the LHS and RHS are equal for each equation. They are equal (the values make both equations true statements). Hence $x = 2, y = 4$ is the solution to the simultaneous linear equations.	Substitute the given $x$ - and $y$ -values into each equation and check if they make both equations true statements. Decide if the given $x$ -value and $y$ -value is the solution to the simultaneous linear equations.
<b>b</b>	$y = -2x + 7$ When $x = 5$ and $y = -3$ , LHS = -3 and RHS = $-2 \times 5 + 7 = -3$ So LHS = RHS $y = 2x - 11$ When $x = 5$ and $y = -3$ , LHS = -3 and RHS = $2 \times 5 - 11 = -1$ So LHS $\neq$ RHS As $x = 5, y = -3$ does not satisfy both equations, this is not the solution.	Substitute $x = 5$ and $y = -3$ into the left-hand side (LHS) and right-hand side (RHS) of each equation. Check if the LHS and RHS are equal for each equation. They are equal for the first equation but not the second. Hence $x = 5, y = -3$ is not the solution to the simultaneous linear equations.	

**2** Check if each given  $x$ -value and  $y$ -value is the solution to the pair of simultaneous linear equations.

**a**  $x = 1, y = 5$

$y = x + 4$  and  $y = -2x + 7$

**c**  $x = 5, y = -2$

$y = x - 9$  and  $y = -2x + 7$

**e**  $x = 4, y = 3$

$y = 2x - 5$  and  $y = -x + 1$

**b**  $x = -2, y = 1$

$y = x + 3$  and  $y = -3x - 5$

**d**  $x = 3, y = -7$

$y = -2x - 1$  and  $y = -x - 4$

**f**  $x = -6, y = 0$

$y = x - 6$  and  $y = -x - 6$

To find the solution of a pair of simultaneous linear equations from a graph, find the point of intersection of the two lines. At the point of intersection, each line will simultaneously have the same  $x$ -coordinate and  $y$ -coordinate. The coordinates of the point of intersection provide the  $x$ -value and the  $y$ -value of the solution.

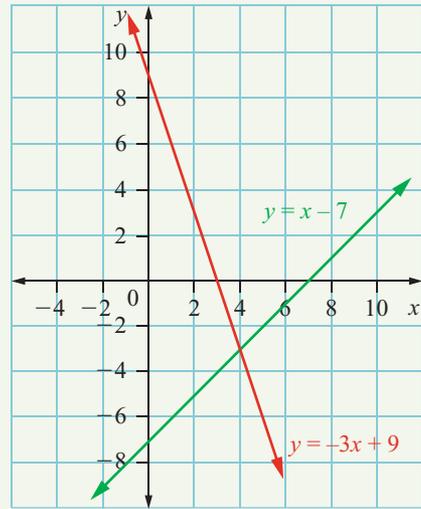


### EXAMPLE 4C-2 Identifying the solution to simultaneous linear equations from graphs

Consider the following pair of simultaneous linear equations and their graphs drawn on the same number plane.

$$y = x - 7 \text{ and } y = -3x + 9$$

- a Write the coordinates of the point of intersection of the two lines.
- b Use the coordinates of this point to write the solution to the simultaneous linear equations.



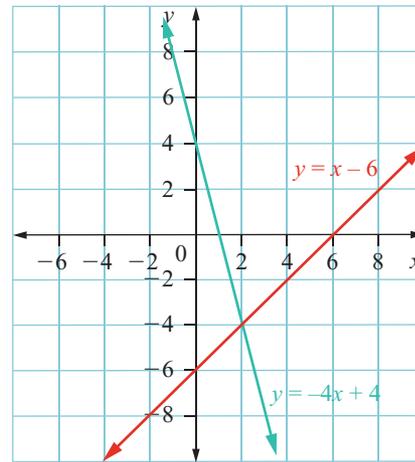
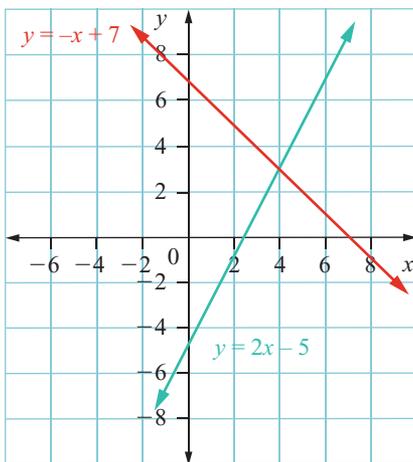
	Solve	Think	Apply
a	Point of intersection at (4, -3).	Identify the point where the two lines cross and write the x-value and the y-value of the point as coordinates.	Identify the point of intersection of the two lines and list the coordinates.
b	The solution to the simultaneous equations $y = x - 7$ and $y = -3x + 9$ is $x = 4, y = -3$ .	Use the coordinates of the point to write the solution.	Write the solution as the x-value and the y-value that makes both equations true.

3 Consider each pair of simultaneous linear equations and their graphs.

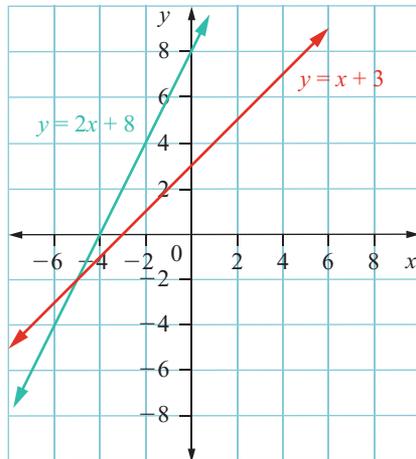
- i Write the coordinates of the point of intersection of the two lines.
- ii Use the coordinates of this point to write the solution to the simultaneous linear equations.

a  $y = 2x - 5$  and  $y = -x + 7$

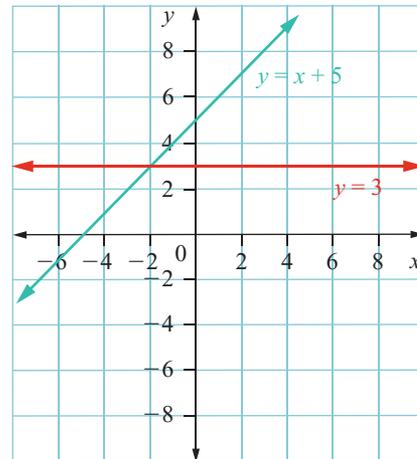
b  $y = -4x + 4$  and  $y = x - 6$



**c**  $y = 2x + 8$  and  $y = x + 3$

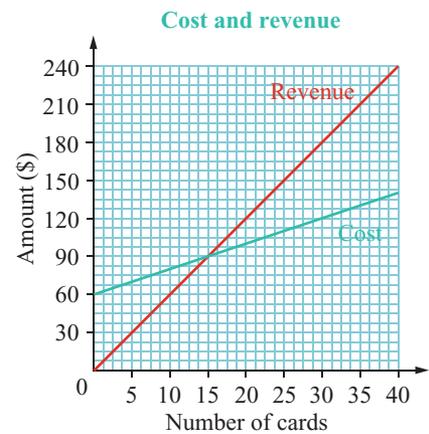


**d**  $y = x + 5$  and  $y = 3$

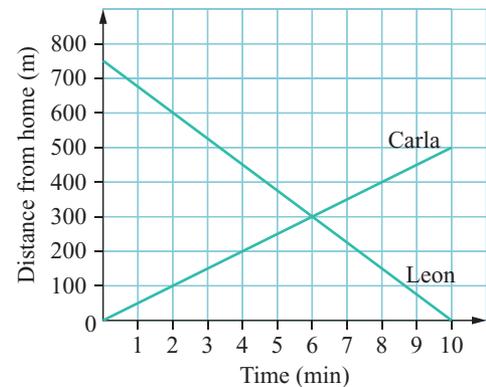


**4** Use substitution to check your solutions to the simultaneous linear equations in question 3.

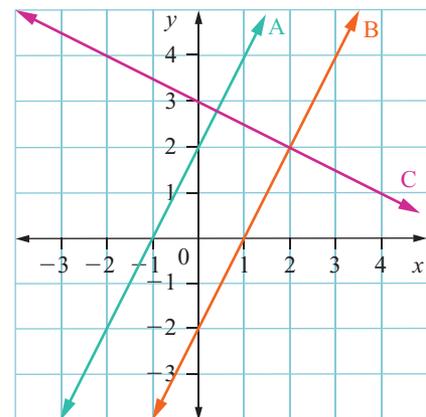
- 5** Suzy makes hand-painted cards. She invests \$60 in a set of paintbrushes and enough paint for 40 cards. The cost of each plain white card is \$2 and she sells the painted cards for \$6 each. The production cost and the revenue generated by the sale of the cards can be modelled with two linear equations and shown on a graph.
- How many cards does Suzy need to sell to recover all her costs?
  - How much profit does Suzy make if she sells 30 cards?
  - What is the maximum profit Suzy can make?



- 6** Carla walks from home along a road towards the town centre. Her brother Leon walks from the town centre towards home along the same road. They each start walking at the same time but at different speeds. This graph shows their journeys for the first 10 minutes.
- How far from home is Carla after she has walked for 2 minutes?
  - How far from home is Leon after he has walked for 2 minutes?
  - How far apart are they after walking for 4 minutes?
  - How long does it take for them to meet along the road?
  - How far has each travelled before they meet?



- 7** Consider the straight lines A, B and C shown on this graph.
- Write the coordinates of the point of intersection of lines B and C and hence give the solution to the simultaneous equations for these lines.
  - Use the graph to solve the equations for lines A and C simultaneously.
  - Can the equations for lines A and B be solved simultaneously? Explain why or why not.



# 4D Solving simultaneous linear equations graphically

These resources are available on your obook assess:

- **Video tutorial 4D:** Watch and listen to an explanation of Example 4D-1
- **Worksheet 4D:** Practise your skills with extra problems
- **Investigation 4D:** Solve simultaneous linear equations for train travel
- **assess quiz 4D:** Test your skills with an auto-correcting multiple-choice quiz

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In Topic 4C, the solution to a pair of simultaneous linear equations was identified from given tables or graphs. In this section you will use your skills in drawing linear graphs to solve simultaneous linear equations graphically. You can use technology or complete tables of values to help you produce the graphs.

If using tables of values, it is easier to have three sets of  $x$ - and  $y$ -values. Although only two points need to be plotted to produce a straight-line graph, using three (or four) points provides a check of the substitution calculations. If the plotted points do not lie in a straight line, we can easily see that an error has been made.

## EXAMPLE 4D-1 Drawing graphs from tables of values to solve simultaneous linear equations

- a** Complete the given tables below for the linear equations  $y = 2x + 1$  and  $y = 4 - x$ .

$$y = 2x + 1$$

$x$	-2	0	2
$y$			

$$y = 4 - x$$

$x$	-2	0	2
$y$			

- b** Plot the values from the tables to draw the graphs of  $y = 2x + 1$  and  $y = 4 - x$  on the same number plane.  
**c** Find the point of intersection of the two lines.  
**d** Write the solution to the simultaneous linear equations  $y = 2x + 1$  and  $y = 4 - x$ .

	Solve	Think/Apply																
<b>a</b>	$y = 2x + 1$ <table border="1"> <tr> <td><math>x</math></td> <td>-2</td> <td>0</td> <td>2</td> </tr> <tr> <td><math>y</math></td> <td>-3</td> <td>1</td> <td>5</td> </tr> </table> $y = 4 - x$ <table border="1"> <tr> <td><math>x</math></td> <td>-2</td> <td>0</td> <td>2</td> </tr> <tr> <td><math>y</math></td> <td>6</td> <td>4</td> <td>2</td> </tr> </table>	$x$	-2	0	2	$y$	-3	1	5	$x$	-2	0	2	$y$	6	4	2	<p>Complete the table of values for each equation by substituting each <math>x</math>-value into the equation to find the corresponding <math>y</math>-value.</p> <p>The coordinates of the point of intersection of the two straight lines give the solution of the simultaneous equations.</p>
$x$	-2	0	2															
$y$	-3	1	5															
$x$	-2	0	2															
$y$	6	4	2															

	Solve	Think/Apply
<b>b</b>		Use the tables of values to plot points for each equation on the same number plane and draw straight lines through them.
<b>c</b>	The point of intersection is (1, 3).	Identify the point where the two lines cross and write the coordinates of this point.
<b>d</b>	The solution is $x = 1, y = 3$ .	As the point of intersection lies on both lines, its coordinates satisfy both equations; that is, $x = 1$ and $y = 3$ is the solution of the simultaneous equations.

## EXERCISE 4D Solving simultaneous linear equations graphically

**1** Consider the following pairs of linear equations.

- Complete the given tables for the linear equations.
- Plot the values from the tables to draw the graphs of the equations on the same number plane.
- Find the point of intersection of the two lines.
- Write the solution to the simultaneous linear equations.

**a**  $y = x + 2$

$x$	-2	0	2
$y$			

$y = 2x - 1$

$x$	-2	0	2
$y$			

Draw a number plane showing  $x$ -values from  $-4$  to  $8$  and  $y$ -values from  $-6$  to  $8$ .

**b**  $y = 2x + 2$

$x$	0	2	4
$y$			

$y = 3x + 1$

$x$	0	2	4
$y$			

Draw a number plane showing  $x$ -values from  $-4$  to  $8$  and  $y$ -values from  $-2$  to  $14$ .

**c**  $y = 2x - 2$

$x$	-1	0	1
$y$			

$y = 4 - x$

$x$	-1	0	1
$y$			

Draw a number plane showing  $x$ -values from  $-2$  to  $5$  and  $y$ -values from  $-4$  to  $5$ .

**2** Solve each of the following pairs of simultaneous linear equations graphically using the approach used in question 1.

**a**  $y = 6 - x$  and  $y = 10 - 2x$

**b**  $y = x - 3$  and  $y = 1 - x$

**c**  $y = x - 1$  and  $y = 2x$

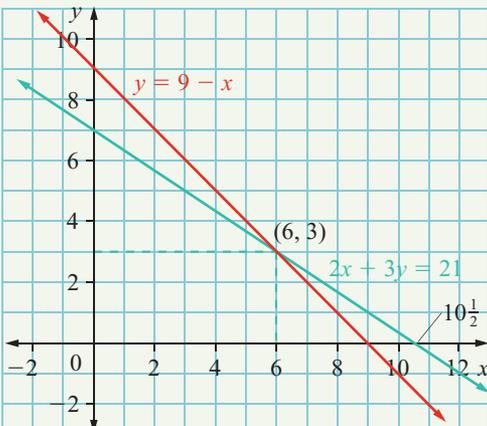
**d**  $y = 4 - x$  and  $y = 5 - 2x$

**e**  $y = 6 - x$  and  $y = 10 - 2x$

**f**  $y = x + 1$  and  $y = -\frac{2}{3}x + 2$

### EXAMPLE 4D-2 Solving simultaneous linear equations graphically

Use a graphical method to solve the simultaneous linear equations  $2x + 3y = 21$  and  $y = 9 - x$ .

Solve	Think																				
 <p>The point of intersection is <math>(6, 3)</math>, so the solution to the simultaneous linear equations <math>2x + 3y = 21</math> and <math>y = 9 - x</math> is <math>x = 6, y = 3</math>.</p>	<p>Draw the lines for the equations using graphing technology or by completing a table of values and plotting the points.</p> <p>The equation <math>2x + 3y = 21</math> can be rearranged to become <math>y = -\frac{2}{3}x + 7</math>. Choose <math>x</math>-values that make the substitution calculations easier.</p> <p>For <math>y = -\frac{2}{3}x + 7</math>:</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td style="text-align: center;"><math>x</math></td><td style="text-align: center;">0</td><td style="text-align: center;">3</td><td style="text-align: center;">6</td><td style="text-align: center;">9</td></tr> <tr><td style="text-align: center;"><math>y</math></td><td style="text-align: center;">7</td><td style="text-align: center;">5</td><td style="text-align: center;">3</td><td style="text-align: center;">1</td></tr> </table> <p>For <math>y = 9 - x</math>:</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td style="text-align: center;"><math>x</math></td><td style="text-align: center;">0</td><td style="text-align: center;">2</td><td style="text-align: center;">4</td><td style="text-align: center;">8</td></tr> <tr><td style="text-align: center;"><math>y</math></td><td style="text-align: center;">9</td><td style="text-align: center;">7</td><td style="text-align: center;">5</td><td style="text-align: center;">1</td></tr> </table> <p>Locate the coordinates of the point where the lines cross and write the solution.</p>	$x$	0	3	6	9	$y$	7	5	3	1	$x$	0	2	4	8	$y$	9	7	5	1
$x$	0	3	6	9																	
$y$	7	5	3	1																	
$x$	0	2	4	8																	
$y$	9	7	5	1																	

#### Apply

Produce the linear graphs using technology or by completing a table of values and plotting the points. The gradient and  $y$ -intercept method of drawing a linear graph could also be used. Find the point of intersection of the two lines and use the coordinates to write the solution to the simultaneous equations.

- 3** Use a graphical method to solve these pairs of simultaneous linear equations.
 

<b>a</b> $y = x - 3$ and $y = 1 - x$	<b>b</b> $x - y = 1$ and $y = 2x$
<b>c</b> $4x + 3y = 10$ and $x - 2y = -3$	<b>d</b> $3x + y = 5$ and $2x - 3y = 18$
<b>e</b> $y = 3x - 8$ and $3x + y = -2$	<b>f</b> $2x + y = 4$ and $2x - y = 8$
<b>g</b> $3x + y = -5$ and $2x + 3y = -8$	<b>h</b> $x - 3y = -9$ and $2x - 3y = -12$
<b>i</b> $x + y = 1$ and $3x + 2y = 1$	<b>j</b> $3x - y = 2$ and $x - y = -4$
  
- 4** Consider the linear equations  $y = 2x + 3$  and  $y = 8 - 3x$ .
  - a** Use graphing technology to draw the graphs of the two linear equations on the same set of axes.
  - b** Find the point of intersection of these lines.
  - c** Hence write the solution to the simultaneous linear equations  $y = 2x + 3$  and  $y = 8 - 3x$ .
  
- 5** Use graphing technology to solve the following pairs of simultaneous linear equations. Compare your answers with those obtained in question 2.
 

<b>a</b> $y = 6 - x$ and $y = 10 - 2x$	<b>b</b> $y = x - 3$ and $y = 1 - x$
<b>c</b> $y = x - 1$ and $y = 2x$	<b>d</b> $y = 4 - x$ and $y = 5 - 2x$
<b>e</b> $y = 6 - x$ and $y = 10 - 2x$	<b>f</b> $y = x + 1$ and $y = -\frac{2}{3}x + 2$

- 6** The cost (\$ $C$ ) of hiring a car from company A is \$50 per day; that is,  $C = 50d$ , where  $d$  is the number of days for which the car is hired. Company B charges a flat fee of \$60 plus \$40 per day; that is,  $C = 60 + 40d$ .
- Use graphing technology or complete tables to graph the equations on the same set of axes.
  - Solve the equations simultaneously using your graph. Explain the meaning of the solution.
  - After how many days is it cheaper to hire from company B?
- 7** Joanna is a salesperson who earns her income from commissions. Her supervisor offers her a choice of two methods of weekly payment.
- A straight commission of 10% of her sales:  $I = 0.1S$ , where  $I$  is income and  $S$  is sales in dollars
  - A retainer of \$300 plus 6% of sales:  $I = 300 + 0.06S$
- Use graphing technology or complete tables to graph each linear equation on the same set of axes.
  - Find the solution of the simultaneous linear equations. Explain the meaning of the solution.
  - What sales would Joanna have to achieve to earn more income using method **ii**?
- 8** A new car with a petrol engine uses 10.0 L/100 km. If the cost of petrol is \$1.50/L then the cost (\$ $C$ ) of fuel to drive the car is given by  $C = 0.15d$ , where  $d$  is the distance travelled. The same car with a diesel engine costs \$2000 more to purchase but only uses 6.0 L/100 km. The cost of diesel fuel is \$1.60/L. The cost of driving the diesel model of the car is given by  $C = 2000 + 0.096d$ .
- Use graphing technology or complete tables of values to graph each linear equation on the same set of axes.
  - Use the graph to compare the cost of driving each model for 10 000 km. Which is cheaper?
  - Use the graph to compare the cost of driving each model for 30 000 km. Which is cheaper?
  - How far do you have to travel before the cost of the two models is the same?
  - Explain when the diesel model would be cheaper to drive.
- 9** The perimeter of a printed rectangular photo is 50 cm.
- Use  $l$  for length and  $w$  for width to write a linear equation for the perimeter of the photo.
  - Use  $l$  and  $w$  to write a linear equation linking the length and width of the photo, given that the length is 5 cm longer than the width.
  - Solve the two equations simultaneously using a graphical method, and hence state the dimensions of the photo.
- 10** At the cinema, a large drink costs \$2 more than an ice cream. Liam buys five drinks and seven ice creams for \$70.
- Write two linear equations to model this information.
  - Solve the equations simultaneously using a graphical method to find the cost of each item.
- 11** Tom rides his bicycle from Town A to Town B at an average speed of 20 km/h. Julia walks from Town B to Town A along the same road at an average speed of 5 km/h. The distance along the road between the two towns is 15 km and they each start off at the same time.
- Write two linear equations to model the two journeys. In each case, relate the distance in kilometres *from Town A* to the time travelled in hours.
  - Solve the equations simultaneously using a graphical method to find the time at which they meet along the road between the two towns.
  - How far has each travelled before they meet?
- 12** Solve the following pairs of simultaneous linear equations graphically and explain your results.
- $y = 2x + 1$  and  $y = 2x - 3$
  - $x - 2y = 4$  and  $2x - 4y = 8$



# 4E Break-even analysis

## break-even point

point at which the revenue (or income) and the cost of production (or expenses) are equal; it can be identified as the point of intersection of the combined graphs of cost and revenue

These resources are available on your [obook assess](#):

- **Video tutorial 4E:** Watch and listen to an explanation of Example 4E-2
- **assess quiz 4E:** Test your skills with an auto-correcting multiple-choice quiz

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In Topic 4B, straight-line graphs were drawn to model both production cost and revenue. If we combine graphs for cost and revenue for the same business, we can find the **break-even point**. This is the point at which the graphs intersect and revenue equals cost. Sales exceeding the break-even number result in a profit for the business, and sales below the break-even number result in a loss.

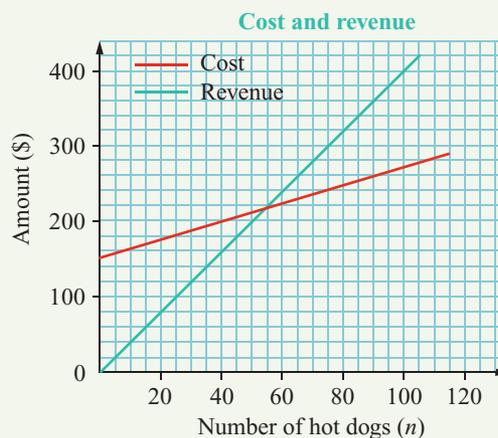
This section uses simultaneous linear equations to look at break-even points, and profits and losses.

## EXAMPLE 4E-1 Determining the break-even point from a given graph

Holly's Hot Dogs makes and sells hot dogs. There is a fixed cost per day of \$150 and each hot dog costs \$1.20 to produce. The hot dogs are sold for \$4 each.

The graphs of the production cost and the revenue made from selling hot dogs are drawn on the same set of axes.

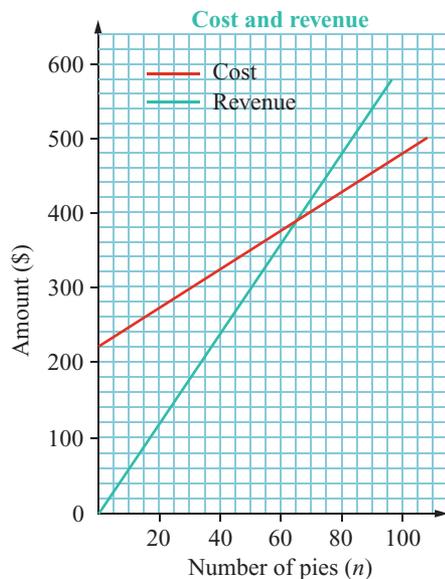
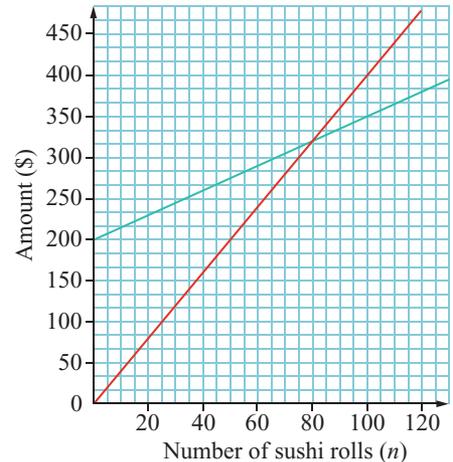
- How many hot dogs must be sold to break even?
- What is the revenue at the break-even point?
- Find the profit made when 90 hot dogs are sold.



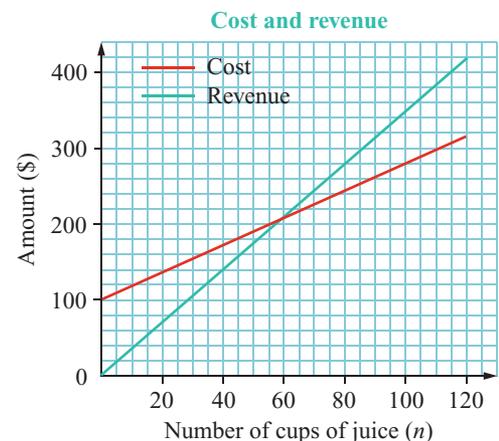
	Solve	Think	Apply
<b>a</b>	Approximately 54 hot dogs must be sold to break even.	To break even, the revenue has to equal the production cost. Read the value for $n$ at the point of intersection of the two lines. <i>Note:</i> in this case we need to estimate the value.	The break-even point occurs where the lines for revenue and cost intersect.
<b>b</b>	At the break-even point, revenue is about \$215.	Estimate the value for the revenue from the vertical axis (amount in dollars) when $n \approx 54$ .	Read the value for the revenue at the point of intersection of the two lines.
<b>c</b>	When $n = 90$ , revenue = \$360 cost = \$260 Profit = revenue - cost = $360 - 260$ = 100	Read the values from the vertical axis for the revenue line and the cost line when $n = 90$ . Subtract the cost from the revenue to find the profit.	Find the values for the revenue and cost by reading across to the vertical axis from each line for the given number of hot dogs.

## EXERCISE 4E Break-even analysis

- Suki's Sushi Bar makes and sells sushi rolls. There is a fixed cost per day of \$200 and each sushi roll costs \$1.50 to produce. The sushi rolls are sold for \$4 each. The graphs of production cost and revenue are drawn on the same set of axes.
  - How many sushi rolls must be sold to break even?
  - What is the revenue at the break-even point?
  - Subtract the cost from the revenue to find the profit made when 100 sushi rolls are sold.
- Peter's Pie Palace makes and sells gourmet pies. There is a fixed cost per day of \$220 and each pie costs \$2.60 to produce. The pies are sold for \$6 each.



- How many pies must be sold to break even?
  - What is the revenue at the break-even point?
  - Find the profit made when 90 pies are sold?
- Jenny's Juice Joint makes and sells fresh juices. There is a fixed cost per day of \$100 and each cup of juice costs \$1.80 to produce. The juices are sold for \$3.50 each. The graphs of production cost and revenue are drawn on the same set of axes.
    - How many cups of juice must be sold to break even?
    - What is the revenue at the break-even point?
    - Find the profit made by Jenny's Juice Joint when 110 cups of juice are sold.
    - Find the difference between the revenue and cost when 30 cups of juice are sold. Does Jenny make a profit? Explain your answer.



- 4** A plastics company has a maximum daily production of 700 items.
- a** There is an initial cost of \$3000 per day plus \$8 per item produced. This can be represented by the linear equation  $P = 8n + 3000$ , where  $P$  is the production cost in dollars and  $n$  is the number of items. Complete this table of values for the production cost.

Number of items ( $n$ )	0	100	300	500	700
Production cost ( $P$ , \$)					

- b** The selling price of each item is \$15. This can be represented by the linear equation  $R = 15n$ , where  $R$  is the revenue in dollars on the sale of  $n$  items. Complete this table of values for the revenue.

Number of items ( $n$ )	0	100	300	500	700
Revenue ( $R$ , \$)					

- c** Choose a suitable scale and graph these lines on the same set of axes:
- i** production cost versus number of items
  - ii** revenue versus number of items.
- d** Find the point of intersection of the two graphs. This is the break-even point.
- e** How many items must be sold each day to break even?
- f** What is the break-even revenue?

- 5** The Tin Lid factory produces sports caps. There is a fixed monthly cost of \$1800 and it costs \$3 to produce a cap. The caps are sold for \$8 each. The maximum monthly production of caps is 800.

- a** Use the equation  $C = 3n + 1800$ , where  $C$  is the production cost in dollars to produce  $n$  sports caps, to graph the cost of production.
- b** Use the equation  $R = 8n$ , where  $R$  is the revenue in dollars on the sale of  $n$  sports caps, to graph the revenue on the same set of axes as the cost equation in part **a**.
- c** Find the point of intersection of the two lines. This is the break-even point.
- d** How many caps must be sold in a month to break even?
- e** What is the break-even revenue?



- 6** The Supertight Tie Company has a fixed cost of \$1200 per day and a variable cost of \$3 per tie. This can be represented by the equation  $C = 3n + 1200$ , where  $C$  is the cost of producing  $n$  ties. The revenue of \$15 per tie sold can be represented by the equation  $R = 15n$ , where  $R$  is the revenue on the sale of  $n$  ties.

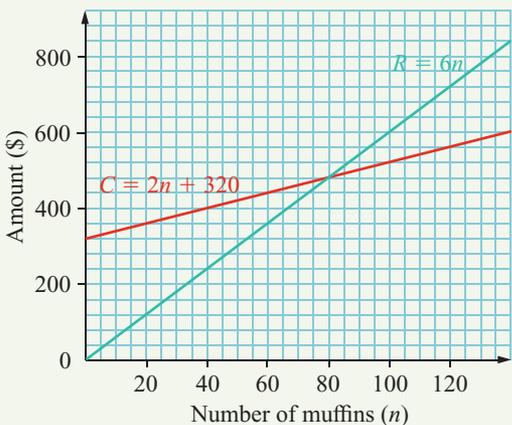
- a** Graph these two equations on the same set of axes for  $0 \leq n \leq 250$ .
- b** Find the break-even point.
- c** How many ties must be sold to break even?
- d** What is the break-even revenue?



**EXAMPLE 4E-2** Modelling revenue and cost for break-even analysis

The Majestic Muffin Shop has a fixed cost of \$320 per day and a variable production cost of \$2 per muffin. The selling price of each muffin is \$6. The maximum daily production is 120 muffins.

- Write an equation for the cost, \$ $C$ , of producing  $n$  muffins.
- Graph the linear equation for the production cost.
- Write an equation for the revenue, \$ $R$ , for the number of muffins sold,  $n$ . Draw the straight line for the revenue equation on the same set of axes as the cost equation in part **b**.
- Find the point of intersection.
- How many muffins must be sold to break even?
- What is the break-even revenue?
- On the graph drawn for parts **b** and **c**, shade the loss zone and the profit zone.

	Solve	Think	Apply										
<b>a</b>	The cost equation is $C = 2n + 320$ .	The fixed cost is \$320. The variable cost is $2n$ since the cost per muffin (and the gradient) is 2.	The vertical intercept is the fixed cost. The gradient relates to the variable cost.										
<b>b</b>		Draw the graph using graphing technology, or the gradient (2) and y-intercept (320) method, or by completing a table of values and plotting the points. <table border="1" data-bbox="872 1081 1209 1174"> <tbody> <tr> <td><math>n</math></td> <td>0</td> <td>40</td> <td>80</td> <td>120</td> </tr> <tr> <td><math>C</math></td> <td>320</td> <td>400</td> <td>480</td> <td>560</td> </tr> </tbody> </table>	$n$	0	40	80	120	$C$	320	400	480	560	The maximum number of muffins is 120 so this is the largest value for $n$ .
$n$	0	40	80	120									
$C$	320	400	480	560									
<b>c</b>	The revenue equation is $R = 6n$ . The graph is drawn on the same set of axes as $C = 2n + 320$ in part <b>b</b> .	Draw the graph using graphing technology, or the gradient (6) and y-intercept (0) method, or by completing a table of values and plotting the points on the same set of axes. <table border="1" data-bbox="872 1591 1209 1685"> <tbody> <tr> <td><math>n</math></td> <td>0</td> <td>40</td> <td>80</td> <td>120</td> </tr> <tr> <td><math>R</math></td> <td>0</td> <td>240</td> <td>480</td> <td>720</td> </tr> </tbody> </table>	$n$	0	40	80	120	$R$	0	240	480	720	There is no fixed cost, so the equation has no constant value. Both $C$ and $R$ are values in dollars and so can both be represented on the vertical axis, labelled as 'Amount (\$)'.
$n$	0	40	80	120									
$R$	0	240	480	720									
<b>d</b>	The point of intersection has the coordinates (80, 480).	Find the values at the point where the lines intersect.	The break-even point is when revenue equals cost. This is the point of intersection of the two lines.										

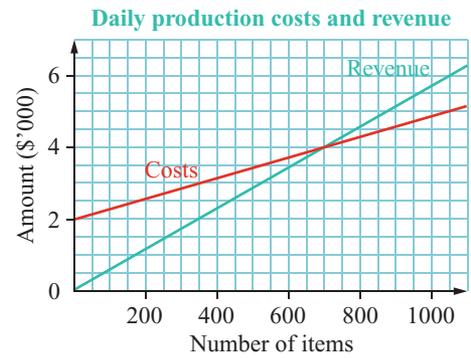
	Solve	Think	Apply
e	80 muffins must be sold to break even.	Read the value for $n$ for the point of intersection of the two lines.	
f	The break-even revenue is \$480.	Read the amount ( $C$ or $R$ ) in dollars for the point of intersection of the two lines.	
g	<p style="text-align: center;"><b>Cost and revenue</b></p>	<p>For the loss zone, shade the zone between the two lines to the left of the break-even point to indicate where the revenue is less than the cost for making <math>n</math> items.</p> <p>For the profit zone, shade the zone between the two lines to the right of the break-even point to indicate where the revenue is more than the cost for making <math>n</math> items.</p>	<p>The loss zone is the zone between the two lines where the revenue line is below the cost line. The profit zone is the zone between the two lines where the revenue line is above the cost line.</p>

- 7** The Munchy Sandwich Shop (from Exercise 4B) had a fixed expense of \$2000 per month and a cost of \$1.20 per sandwich. The sandwiches were sold for \$3.50 each.
- Write the linear equations that can be used to model the production cost and revenue for  $n$  sandwiches.
  - Graph straight lines modelling the cost and revenue for  $0 \leq n \leq 1000$ .
  - Find the break-even point.
  - How many sandwiches must be sold to break even?
  - What is the break-even revenue?
  - The maximum number of sandwiches that can be made in a month is 800. Explain the significance of this.
  - On the graph drawn for part **b**, shade the loss zone. Explain the significance of this.



- 8** The Full Flavour Juice Company has a weekly fixed cost of \$325 and a cost of \$1.50 per cup of juice. The cups of juice are sold for \$4.
- Graph straight lines modelling the production cost and revenue for up to 200 cups of juice.
  - Find the break-even point.
  - How many cups of juice must be sold to break even?
  - What is the break-even revenue?
  - The maximum number of cups of juice per week is 100. Is this a profitable business? Explain.
  - On the graph drawn for part **a**, shade the loss zone.
- 9** Consider the graphs you have drawn to model the cost and revenue for each company in questions **4**, **5** and **6**.
- Shade the loss zone for the graph drawn in question **4**.
  - Shade the profit zone for the graph drawn in question **5**.
  - Shade the loss zone for the graph drawn in question **6**.

- 10** The daily production costs and revenue from the sale of items is shown on the graph.
- Use the graph to determine the number of items that must be sold to break even.
  - How much profit or loss is made when:
    - 100 items are sold?
    - 1000 items are sold?
  - How many items need to be sold for the company to make a:
    - \$1000 profit?
    - \$1000 loss?

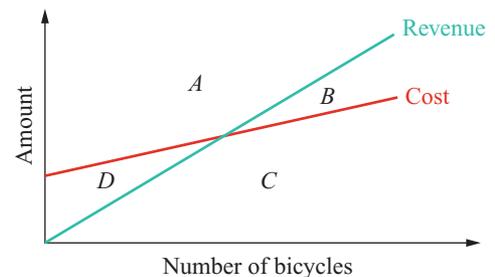


- 11** The cost of producing cotton bags is initially \$2000 plus \$2.50 per item. Each bag is sold for \$5.
- Model the production cost and the revenue with linear equations.
  - Use graphing technology or plot values from tables to draw the linear graphs for the production cost and revenue on the same set of axes, for up to 1200 bags.
  - How many bags need to be sold to break even?
  - How much profit or loss is made when:
    - 100 bags are sold?
    - 1000 bags are sold?
  - How many bags need to be sold for the company to make:
    - \$1000 profit?
    - \$1000 loss?

- 12** The production cost of printing books is initially \$8000 plus \$3000 per 1000 books printed. The books are sold for \$6.50 each.
- Model the production cost and the revenue with linear equations.
  - Use graphing technology or plot values from tables to draw the linear graphs for the production cost and revenue on the same set of axes.
  - How many books need to be sold to break even?
  - How much profit or loss is made when:
    - 1000 books are sold?
    - 2500 books are sold?
    - 3200 books are sold?
  - How many books need to be sold for the company to make:
    - a loss of \$3000?
    - a profit of \$2500?



- 13** A company manufactures bicycles. The company's revenue equation and cost equation are drawn on the same graph. Which region of the graph (*A*, *B*, *C* or *D*) is the profit zone?



# CHAPTER 4 REVIEW SIMULTANEOUS LINEAR EQUATIONS

You should be able to:

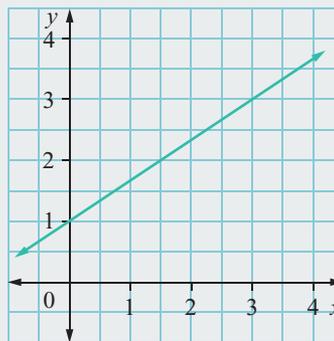
- ✓ complete tables, plot points and draw graphs of the form  $y = mx + c$
- ✓ model situations using linear equations
- ✓ generate tables and graph linear models
- ✓ use technology to graph linear models
- ✓ identify the solution from tables or graphs when solving linear equations simultaneously
- ✓ find the point of intersection between two straight-line graphs using technology
- ✓ solve simultaneous linear equations graphically and interpret the solution
- ✓ solve practical problems using simultaneous linear equations
- ✓ determine and interpret the break-even point of a simple business problem where cost and revenue are represented by linear equations.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

## REVIEW MULTIPLE-CHOICE QUESTIONS

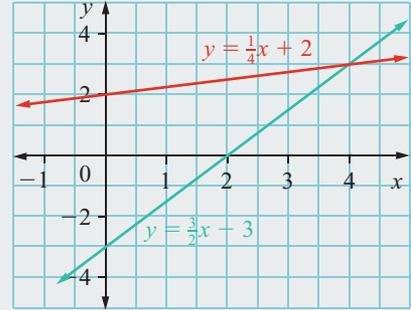
- 4A** 1 What are the gradient and  $y$ -intercept respectively of the line  $y = -2x + 3$ ?  
A 3 and  $-2$                       B  $-2$  and 3                      C  $-2$  and  $-3$                       D  $-3$  and  $-2$
- 4A** 2 What is the equation of the straight line with gradient of  $\frac{1}{2}$  and  $y$ -intercept  $-1$ ?  
A  $y = x - \frac{1}{2}$                       B  $y = -x - \frac{1}{2}$   
C  $y = \frac{1}{2}x - 1$                       D  $y = -\frac{1}{2}x + 1$

Use the graph on the right for questions 3 and 4.



- 4A** 3 What is the gradient of the line shown?  
A  $\frac{2}{3}$                       B  $\frac{3}{2}$   
C 1                      D 3
- 4A** 4 What is the equation of the line?  
A  $y = \frac{2}{3}x + 1$                       B  $y = \frac{3}{2}x + 1$   
C  $y = x + \frac{2}{3}$                       D  $y = x + \frac{3}{2}$
- 4A** 5 The Cactus Water Company purifies water. The weekly fixed cost is \$450 and the cost per bottle is \$2. Which equation models this information where  $C$  is the total cost and  $n$  is the number of bottles?  
A  $C = 2n + 450$                       C  $C = 450n - 2$   
B  $C = 450n + 2$                       D  $C + 450 = n$
- 4B** 6 The equation  $C = 3n + 150$  models the costs for a sandwich shop. What could the 150 represent?  
A number of sandwiches sold                      B the cost per sandwich  
C fixed daily cost                      D number of sandwiches made

- 4C** ▶ **7** The straight lines for equations  $y = \frac{3}{2}x - 3$  and  $y = \frac{1}{4}x + 2$  are shown on the graph. What is the solution when the two equations are solved simultaneously?



- A**  $x = -8, y = 0$   
**B**  $x = 0, y = -3$   
**C**  $x = 3, y = 4$   
**D**  $x = 4, y = 3$

- 4C** ▶ **8** Which pair of simultaneous equations has the solution  $x = -5, y = 3$ ?

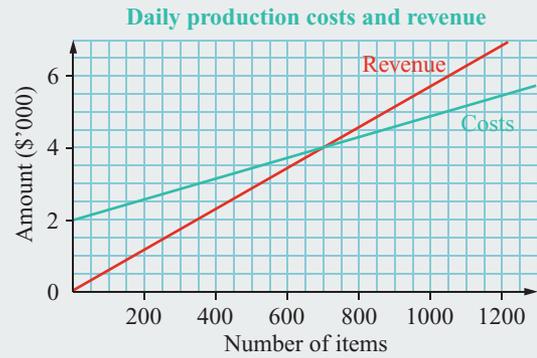
- A**  $y = x + 8$  and  $4x - 3y = 29$       **B**  $y = x + 8$  and  $3x - 2y = -21$   
**C**  $x + y = 2$  and  $5x + y = -22$       **D**  $y - x = 8$  and  $4x - 3y = 26$

- 4D** ▶ **9** What is the solution to the simultaneous equations  $y = 2x - 1$  and  $y = 14 - 3x$ ?

- A**  $x = 5, y = 3$       **B**  $x = 3, y = 5$   
**C**  $x = 2, y = -1$       **D**  $x = 14, y = -3$

- 4E** ▶ **10** From the graph, determine the number of items that need to be sold for the business to break even.

- A** 4  
**B** 4000  
**C** 700  
**D** 650



## REVIEW SET 1

- 1 a** Find the gradient and y-intercept of the line  $y = \frac{3}{8}x - 4$ .  
**b** Write the equation of the line with gradient of 0.08 and y-intercept 3.2.
- 2 a** Complete this table of values for  $y = 4x + 3$ .  
**b** Use the table of values to graph  $y = 4x + 3$  on a number plane.

$x$	-3	-2	-1	0	1	2	3
$y$							

- c** Use technology to produce a graph of  $y = 4x + 3$  and compare your answer with that for part **b**.

- 3** The distance,  $d$  kilometres, travelled by a train over time  $t$  hours is  $d = 70t$ .

- a** Complete this table of values for  $d = 70t$ .

$t$	1	2	3	4	5
$d$					

- b** Draw the graph of  $d = 70t$ .  
**c** How far away is the train after  $3\frac{1}{2}$  hours?  
**d** When is the train 200 km away?

- 4** Graphically solve the simultaneous equations  $y = \frac{1}{2}x + 1$  and  $y = \frac{3}{4}x$ .

- 5** Solve the equations  $y = -3x - 4$  and  $x + 2y = 2$  simultaneously using a graphical method.

- 6** The Happy Feet Shoe Company has a weekly fixed cost of \$3725 and a cost of \$7.50 per pair of shoes. The shoes are sold for \$45 a pair.
- Graph straight lines modelling the cost and revenue for up to 300 pairs of shoes on the same set of axes.
  - Find the break-even point.
  - How many pairs of shoes must be sold to break even?
  - What is the break-even revenue?
  - Shade the loss zone on the graph you produced in part **a**.

## REVIEW SET 2

- 1** By plotting the points (1, 3) and (5, 8), or otherwise, find the gradient of the line through them.

- 2 a** Complete this table of values for  $y = 3x + 2$ .

$x$	-3	-2	-1	0	1	2	3
$y$							

- Use the table of values to graph  $y = 3x + 2$  on a number plane.
- Use technology to produce a graph of  $y = 3x + 2$  and compare your answer with that for part **b**.

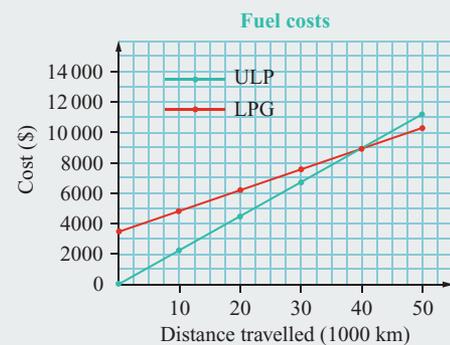
- 3** The Smart Tie Company has a fixed cost of \$1400 per day and a variable cost of \$4 per tie. The daily cost can be modelled using the straight-line equation  $C = 4n + 1400$ .

- Explain each term in the equation  $C = 4n + 1400$ .
- Complete this table of values using the equation  $C = 4n + 1400$ .

$n$	0	50	150	200	250
$C$					

- Draw the straight-line graph of  $C = 4n + 1400$  for  $0 \leq n \leq 250$ .
- From the graph, determine the number of ties produced when the cost is \$1500.

- 4** The fuel cost of driving a large 4WD when running on unleaded petrol (ULP) is shown in the graph on the right. When the vehicle is converted to run on liquid petroleum gas (LPG or autogas), the cost for travelling the same distance was calculated and drawn on the same set of axes. From the graph, estimate the distance travelled before the costs are the same.



- 5** Solve the following pairs of simultaneous linear equations using a graphical method.

- $y = 2x - 3$  and  $y = 6 - x$
- $2x + 3y = 0$  and  $3x - y = -11$

- 6** The Supersheer Stocking Company has a fixed cost of \$1500 per day and a variable cost of \$2 per pair of stockings. This can be modelled by the equation  $C = 2n + 1500$ , where  $C$  is the cost in dollars of producing  $n$  pairs of stockings. The revenue of \$12 per pair of stockings sold can be represented by the equation  $R = 12n$ , where  $R$  is the revenue in dollars on the sale of  $n$  pairs of stockings.

- Graph these two equations on the same set of axes for  $0 \leq n \leq 300$ .
- Find the break-even point.
- How many pairs of stockings must be sold to break even?
- What is the break-even revenue?

# REVIEW SET 3

- 1 a Complete this table of values for  $y = 5x - 2$ .

$x$	-3	-2	-1	0	1	2	3
$y$							

- b Use the table of values to graph  $y = 5x - 2$  on a number plane.  
 c Use technology to produce a graph of  $y = 5x - 2$  and compare your answer with that for part b.

- 2 The Silky Scarf Company sells scarves for \$18 each. Sales revenue may be modelled using the equation  $R = 18n$ .

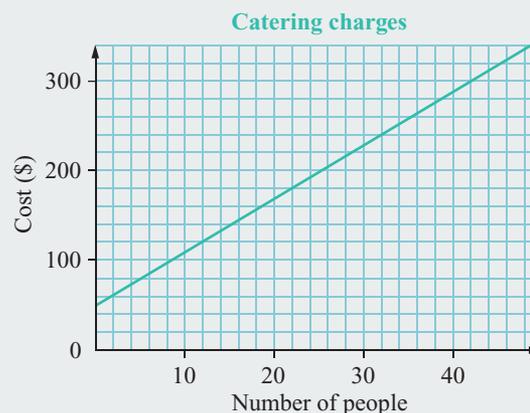
- a Explain each term in the equation  $R = 18n$ .  
 b Complete this table of values for  $R = 18n$ .

$n$	0	50	100	200	250
$R$					

- c Draw the straight-line graph of  $R = 18n$ .  
 d From the graph, find the number of scarves sold when the revenue is \$2070.

- 3 This graph models the cost for a company to cater for different numbers of people.

- a How much would it cost to cater for 35 people?  
 b How many people could be catered for \$300?  
 c Find the gradient. What is its meaning?  
 d Find the intercept on the vertical axis. What is its meaning?  
 e The model for another catering company is represented by the equation  $C = 8n$ , where  $C$  is the cost in dollars to cater for  $n$  people. Copy the graph on the right and draw the line for  $C = 8n$  on the same set of axes.  
 f For how many people is the cost of catering the same for each company? What is this cost?



- 4 Solve the equations  $x - 3y = 11$  and  $5x - 2y = 16$  simultaneously using a graphical method.

- 5 The cost (\$) of hiring a car from company A is given by  $C = 60d$ , where  $d$  is the number of days the car is hired. For company B, the cost of hire is  $C = 100 + 40d$ .

- a Draw the graphs of cost versus number of days for each company on the same set of axes for up to 8 days.  
 b Find the solution of the simultaneous equations  $C = 60d$  and  $C = 100 + 40d$ .  
 c After how many days is the cost the same for both companies?

- 6 The Stay Dry Company produces umbrellas. There is a fixed monthly cost of \$2800 and it costs \$7 to produce an umbrella. The umbrellas are sold for \$22 each. The maximum monthly production of umbrellas is 300.

- a Write two linear equations that can be used to model the production cost and revenue for  $n$  umbrellas.  
 b Graph straight lines modelling the cost and revenue for  $0 \leq n \leq 300$ .  
 c Find the point of intersection of the two lines. This is the break-even point.  
 d How many umbrellas must be sold in a month to break even?  
 e What is the break-even revenue?

# REVIEW SET 4

- 1 a Complete this table of values for  $y = 7 - 2x$ .

$x$	-3	-2	-1	0	1	2	3
$y$							

- b Use the table of values to graph  $y = 7 - 2x$  on a number plane.  
 c Use technology to produce a graph of  $y = 7 - 2x$  and compare your answer with that for part b.

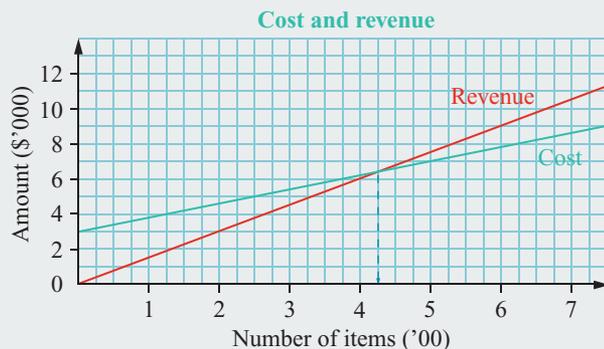
- 2 The Tropical Delight Fruit Juice Company has a weekly fixed cost of \$400. The cost per cup of juice is \$1.75.

- a Write an equation modelling this information. Use  $C$  for cost and  $n$  for the number of cups of juice.  
 b Complete this table of values using the cost equation from part a.

$n$	0	200	400	600	800	1000
$C$						

- c Draw the straight-line graph of your cost equation using the values from the completed table in part b.  
 d Use the graph to find the number of cups of juice made when the cost is \$1000.

- 3 A plastics company has an initial cost of \$3000 per day plus \$8 per item produced. The selling price of the items is \$15 each, and the maximum daily production is 1000 items. The revenue and cost models are represented in the graph on the right.



- a How many items need to be sold for the company to break even?  
 b What is the break-even revenue?

- 4 a Draw the graphs of  $y = 8 - x$  and  $2x + 3y = 23$  on the same number plane.

- b Find the point of intersection of the two lines.  
 c Hence write the solution to the simultaneous equations  $y = 8 - x$  and  $2x + 3y = 23$ .

- 5 The cost (\$ $C$ ) of electricity to run a home is \$200 per month. This may be written as  $C = 200m$ , where  $m$  is the number of months. For the same house, the cost of installing a solar system is \$4000, but the monthly cost of electricity decreases to \$150. The total cost of electricity using a solar system is then  $C = 4000 + 150m$ .

- a Use graphing technology or complete the following tables to graph each equation on the same number plane.

$$C = 200m$$

$m$	0	50	100
$C$			

$$C = 4000 + 150m$$

$m$	0	50	100
$C$			

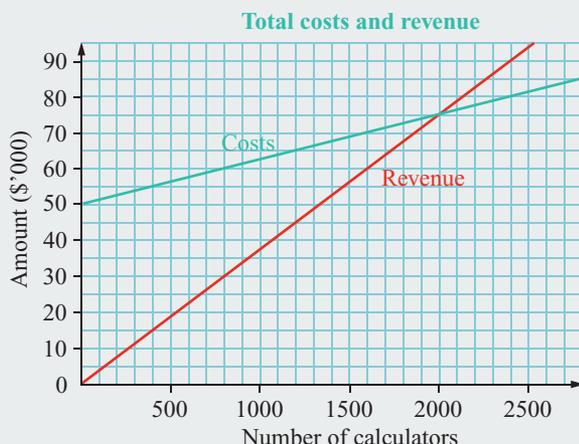
- b Find the simultaneous solution of the linear equations.  
 c After installing the solar system, how long would it take to start saving money?

# REVIEW PRACTICE EXAMINATION QUESTION

- 1 a i Complete this table and graph  $y = 3 - 2x$ . (2 marks)

$x$	-3	-2	-1	0	1	2	3
$y$							

- ii What is the gradient of the line? (1 mark)  
 iii What is the  $y$ -intercept of the line? (1 mark)
- b The graph shows the total costs and revenue for a manufacturer of calculators.



- i Find the total cost to the manufacturer of producing 1000 calculators. (1 mark)  
 ii Will the manufacturer make a profit when 1000 calculators are produced? Give reasons for your answers. (1 mark)  
 iii How many calculators must be produced for the manufacturer to break even? (1 mark)  
 iv How many calculators must be produced to make a profit of \$10 000? (1 mark)  
 v What is the initial set-up cost to the manufacturer? (1 mark)  
 vi What is the cost to produce each calculator, after the initial set-up costs? (1 mark)
- c The Creative Cake Company makes cupcakes. The cost of production is \$200 per day and the variable production cost is \$2.50 per cupcake. Cupcakes are sold for \$7.50 each. Maximum daily production is 100 cupcakes.
- i Write an equation for the cost  $\$C$  of producing  $n$  cupcakes. (1 mark)  
 ii Write an equation for the revenue  $\$R$  of selling  $n$  cupcakes. (1 mark)  
 iii Graph both equations for  $0 \leq n \leq 100$  on the same set of axes. (1 mark)  
 iv How many cupcakes must be sold to break even? (1 mark)  
 v If all 100 cupcakes are sold, how much profit is made? (1 mark)

**TOTAL:**  
15 marks

# 5

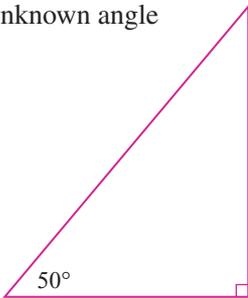
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## Right-angled triangles

The main mathematical ideas investigated are:

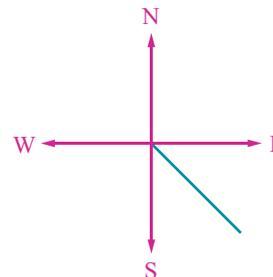
- ▶ using Pythagoras' theorem to solve practical problems
- ▶ calculating trigonometric ratios using a diagram
- ▶ using trigonometric ratios to calculate angle sizes to the nearest minute
- ▶ using trigonometric ratios to calculate side lengths
- ▶ using trigonometric ratios to solve practical problems
- ▶ solving problems involving compass and true bearings
- ▶ understanding different navigational methods
- ▶ solving problems involving angles of elevation and depression.

# ARE YOU READY?

- 5A** ▶ **1** What is the value of  $6.5^2$ ?  
**A** 42.25                      **B** 652  
**C** 13                              **D** 3.25
- 5A** ▶ **2** What is the value of  $\sqrt{36}$ ?  
**A**  $\pm 3$       **B**  $\pm 6$       **C**  $\pm 9$       **D**  $\pm 18$
- 5A** ▶ **3** If  $x^2 = 25$ , what are the possible values of  $x$ ?  
**A** 2.5 only                      **B** 5 only  
**C** -25 or 25                      **D** -5 or 5
- 5A** ▶ **4** If  $y^2 + 14 = 50$ , what are the possible values of  $y$ ?  
**A** 36 only                      **B** -6 or 6  
**C** -8 or 8                      **D** 64 only
- 5B** ▶ **5** What is the angle sum of a triangle?  
**A**  $45^\circ$                       **B**  $90^\circ$   
**C**  $180^\circ$                       **D**  $360^\circ$
- 5B** ▶ **6** What is the size of the unknown angle in this triangle?  
**A**  $30^\circ$   
**B**  $40^\circ$   
**C**  $50^\circ$   
**D**  $90^\circ$
- 
- 5D** ▶ **7** Which step can you follow to solve  $\frac{x}{12} = 0.75$ ?  
**A** Multiply both sides by 12  
**B** Multiply both sides by  $x$   
**C** Divide both sides by 12  
**D** Divide both sides by  $x$

- 5D** ▶ **8** What is the solution of  $\frac{x}{14} = 0.35$ ?  
**A** 4                              **B** 4.9  
**C** 40                              **D** 49

Use this diagram of a compass rose to answer questions 9–11.



- 5F** ▶ **9** What is the compass bearing shown in the diagram?  
**A** SE                              **B** NE  
**C** ES                              **D** SW
- 5F** ▶ **10** What is the true bearing (or three-figure bearing) shown in the diagram?  
**A**  $045^\circ$                       **B**  $090^\circ$   
**C**  $135^\circ$                       **D**  $225^\circ$
- 5F** ▶ **11** What is the true bearing that is directly opposite to that shown in the diagram?  
**A**  $045^\circ$                       **B**  $135^\circ$   
**C**  $225^\circ$                       **D**  $315^\circ$

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook](#) [assess](#).

- Q1–2** Support sheet 5A.1 Calculating squares and square roots  
**Q3–4** Support sheet 5A.2 Solving simple quadratic equations  
**Q5** Support sheet 5B.1 Angle sum of a triangle  
**Q6** Support sheet 5B.2 Finding unknown angles  
**Q7–8** Support sheet 5D.1 Solving linear equations  
**Q9–11** Support sheet 5F.1 Understanding bearings

# 5A Pythagoras' theorem

These resources are available on your obook assess:

- **Video tutorial 5A:** Watch and listen to an explanation of Example 5A–4
- **Interactive 5A.1:** Explore key ideas for Pythagoras' theorem
- **Interactive 5A.2:** Using Pythagoras' theorem to find the length of the hypotenuse
- **Interactive 5A.3:** Using Pythagoras' theorem to find the length of a shorter side
- **Worksheet 5A:** Practise your skills with Pythagoras' theorem
- **Investigation 5A:** Investigate Pythagoras' theorem with shapes other than squares on the sides of a right-angled triangle
- **assess quiz 5A:** Test your skills with an auto-correcting multiple-choice quiz

1010

## hypotenuse

the longest side in a right-angled triangle; it is opposite the right angle

In a right-angled triangle, the length of the **hypotenuse** squared is equal to the sum of the squares of the lengths of the other two sides.

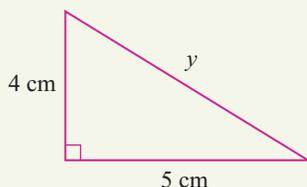
$$c^2 = a^2 + b^2$$

The hypotenuse is opposite the right angle. It is always the longest side in a right-angled triangle.

The value of this theorem, which was known to the ancient Greeks, is that if we know the lengths of any two sides of a right-angled triangle, we can calculate the length of the third side. Also, if we know the lengths of the three sides, we can determine whether or not the triangle is right-angled.

## EXAMPLE 5A–1 Finding the length of the hypotenuse

Find the length of the hypotenuse in the given triangle.

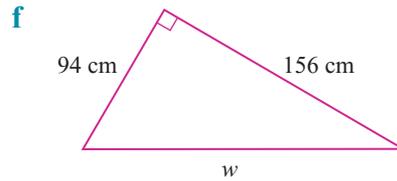
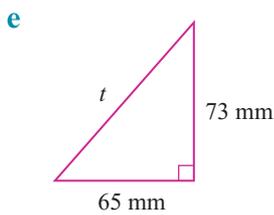
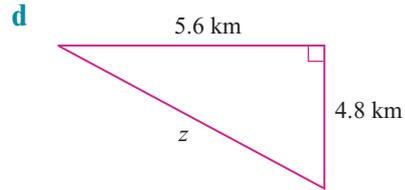
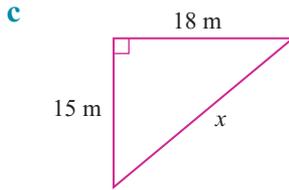
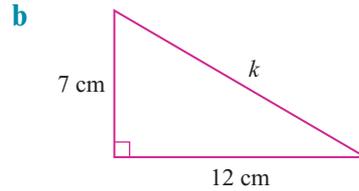
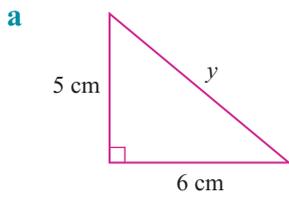


Solve	Think	Apply
$y^2 = 4^2 + 5^2$ $y^2 = 16 + 25$ $y^2 = 41$ $y = \sqrt{41}$ $y = 6.4$ (to one decimal place) The length of the hypotenuse is 6.4 cm.	The unknown side, $y$ , is the hypotenuse (as it is opposite the right angle). The other two sides are 4 cm and 5 cm.	Identify the hypotenuse and apply Pythagoras' theorem.

# EXERCISE 5A Pythagoras' theorem

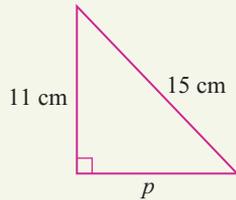
UNDERSTANDING, FLUENCY AND COMMUNICATING

1 Find the length of the hypotenuse, correct to one decimal place, in the following triangles.



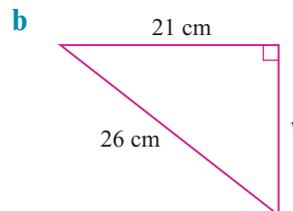
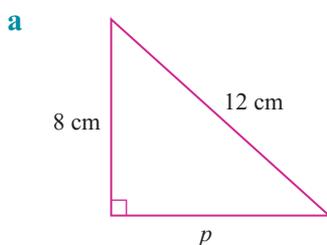
## EXAMPLE 5A-2 Finding the length of the unknown side

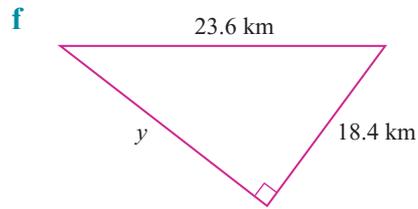
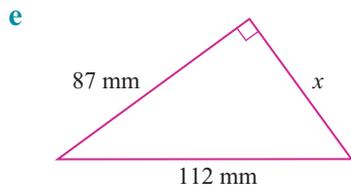
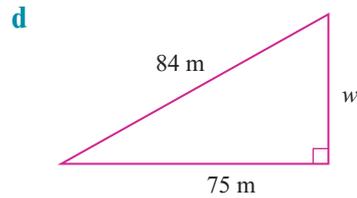
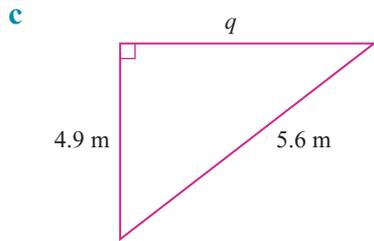
Find the length of the unknown side in the given triangle.



Solve	Think	Apply
$p^2 + 11^2 = 15^2$ $p^2 + 121 = 225$ $p^2 = 225 - 121$ $p^2 = 104$ $p = \sqrt{104}$ $p = 10.2$ (to one decimal place) The length of the unknown side is 10.2 cm.	The hypotenuse is 15 cm. The other two sides are $p$ cm and 11 cm. So $p^2 + 11^2 = 15^2$ . Solve the resulting equation by subtracting 121 from both sides and then taking the square root.	Identify the hypotenuse and apply Pythagoras' rule. Solve the resulting equation for the unknown.

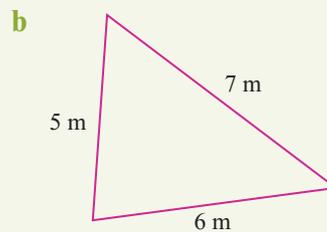
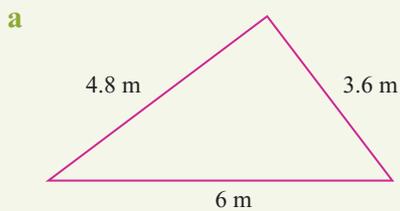
2 Find the length of the unknown side in these right-angled triangles. Give the answer correct to one decimal place.





**EXAMPLE 5A-3** Determining whether a triangle is right-angled

- i** Determine whether the following triangles are right-angled.  
**ii** If the triangle is right-angled, sketch the triangle and mark the right-angle on it.



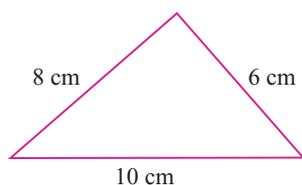
	Solve	Think	Apply
<b>a i</b>	$6^2 = 36$ $4.8^2 + 3.6^2 = 23.04 + 12.96$ $= 36$ $\therefore 6^2 = 4.8^2 + 3.6^2$ The triangle is right-angled.	Calculate the square of the longest side. Calculate the sum of the squares of the other two sides.	If the square of the longest side is equal to the sum of the squares of the other two sides, then the triangle is right-angled.
<b>ii</b>		The right-angle is opposite the longest side.	If the triangle is right-angled, then the right angle is opposite the longest side.
<b>b i</b>	$7^2 = 49$ $5^2 + 6^2 = 25 + 36$ $= 61$ $\therefore 7^2 \neq 5^2 + 6^2$ The triangle is not right-angled.	Calculate the square of the longest side. Calculate the sum of the squares of the other two sides.	If the square of the longest side is <i>not</i> equal to the sum of the squares of the other two sides, the triangle is not right-angled.

**3** Consider the following triangles.

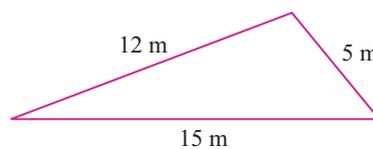
**i** State whether the triangle is right-angled.

**ii** If the triangle is right-angled, sketch the triangle and mark the right-angle on it.

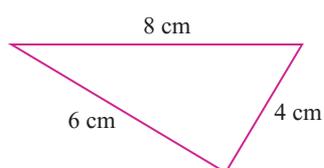
**a**



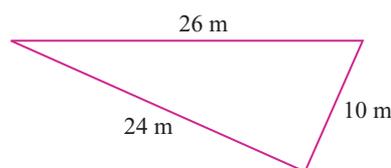
**b**



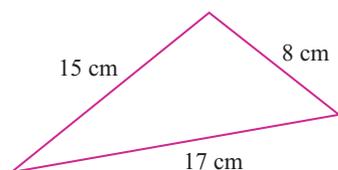
**c**



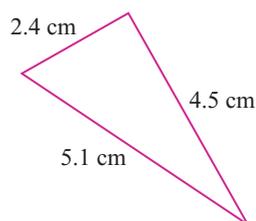
**d**



**e**



**f**



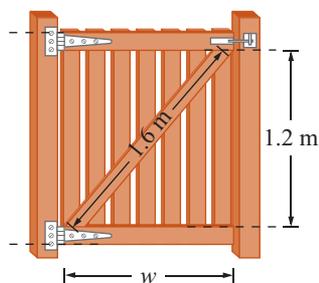
**4** A carpenter builds the frame for the wall of a house. The frame is 2700 mm by 3200 mm and the diagonal measures 4187 mm. Is the frame rectangular?

**5** A door frame is 2500 mm by 850 mm. A builder measures the diagonal to be 3000 mm. Is the door frame rectangular?

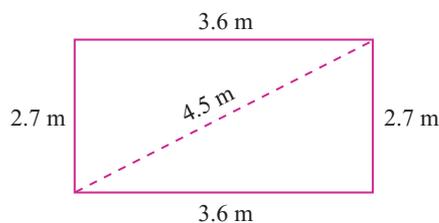
**6 a** Find the length of the diagonal of a 6 cm by 4 cm rectangle.

**b** Find the length of the diagonal of a 8 cm by 8 cm square.

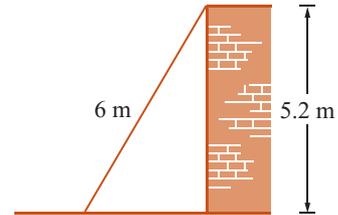
**7** A rectangular gate of height 1.2 m has a diagonal brace of length 1.6 m. Calculate the width of the gate.



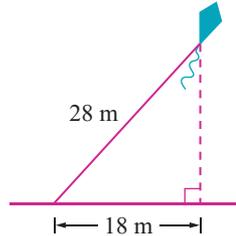
**8** Kylie measures out an area of ground to be concreted for the floor of a tool shed, as shown in the diagram. To check that the floor will be rectangular, she measures the diagonal and finds it to be 4.5 m. Is the area of ground rectangular?



- 9 A ladder 6 m long reaches 5.2 m up a wall. How far is the foot of the ladder from the base of the wall?



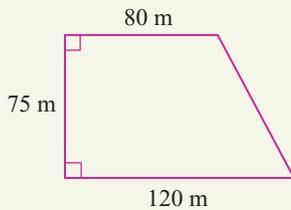
- 10 A kite is flying with 28 m of string let out. At this time, the horizontal distance to the kite is 18 m. Calculate the height of the kite above the ground.



- 11 A ship leaves Sydney Heads and travels 45 km due east. It then turns and travels 86 km due north. How far is the ship from its starting point?

### EXAMPLE 5A-4 Using Pythagoras' theorem to calculate the perimeter

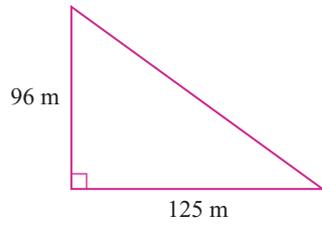
Find, to the nearest metre, the perimeter of the following shape.



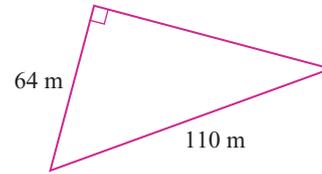
Solve	Think	Apply
$k^2 = 40^2 + 75^2$ $k^2 = 7225$ $k = \sqrt{7225}$ $k = 85 \text{ m}$ $\text{Perimeter} = 120 + 75 + 80 + 85$ $= 360 \text{ m}$	<p>Divide the figure into a rectangle and a right-angled triangle as shown. Let the length of the hypotenuse be <math>k</math> m. Using the fact that the opposite sides of a rectangle are equal, the other two sides of the triangle will be 75 m and 40 m.</p> <p>Hence, applying Pythagoras' theorem, <math>k^2 = 40^2 + 75^2</math>.</p>	<p>Divide the figure into a right-angled triangle and rectangle. Use the dimensions of the rectangle to determine two sides of the triangle and apply Pythagoras' rule to find the third side. The perimeter is the sum of the lengths of all the sides.</p>

**12** Find the perimeter of the following figures, to the nearest metre.

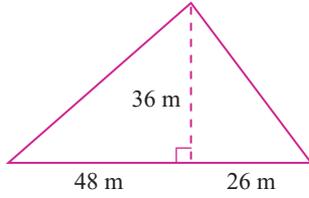
**a**



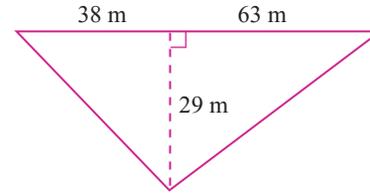
**b**



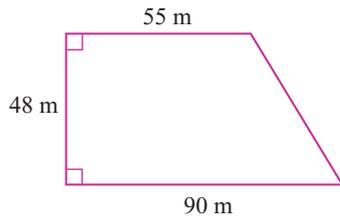
**c**



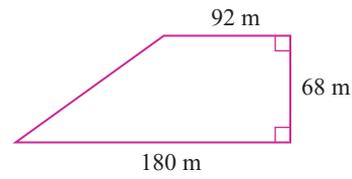
**d**



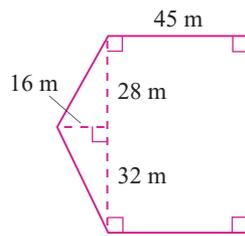
**e**



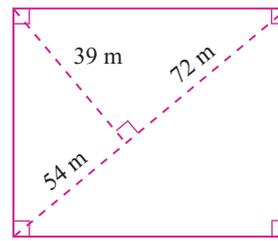
**f**



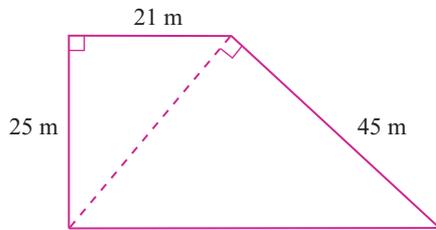
**g**



**h**

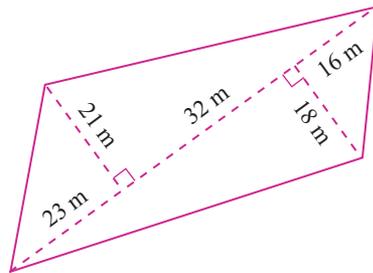


**i**

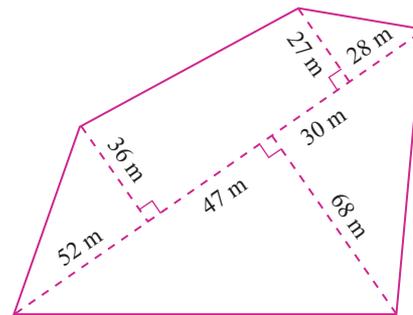


**13** A surveyor makes the following measurements of some fields. Calculate the perimeter of each field to the nearest metre.

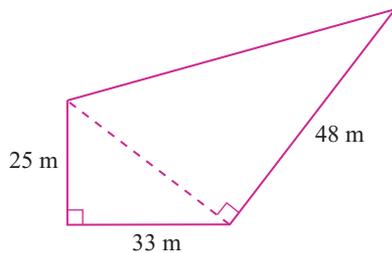
**a**



**b**



**c**



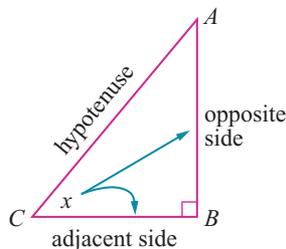
# 5B Trigonometric ratios

These resources are available on your obook assess:

- **Video tutorial 5B:** Watch and listen to an explanation of Example 5B–4
- **Interactive 5B:** Explore key ideas for understanding trigonometry
- **assess quiz 5B:** Test your skills with an auto-correcting multiple-choice quiz



In trigonometry, the sides of a triangle are given names in relation to the angles of the triangle.



In a right-angled triangle, the side opposite the right angle is called the hypotenuse.

The other sides are named relative to the angle marked  $x$ :

- $AB$  is the side that is opposite  $x$ . It is called the **opposite side**.
- $CB$  is the side that is adjacent to the angle. It is called the **adjacent side**.

From topic 3B we can deduce that:

### opposite side

the side opposite the given angle in a right-angled triangle

### adjacent side

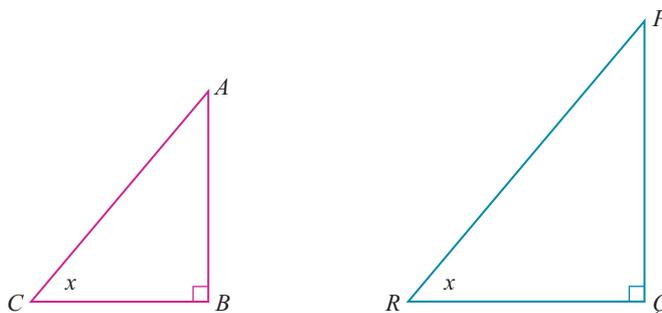
the side next to the given angle in a right-angled triangle that is not the hypotenuse

If three angles of one triangle are equal to three angles of another triangle, then the triangles are similar.



- 1 If two angles of one triangle equal two angles of another triangle, then the remaining angles in the triangles must be equal and the triangles are similar. (The angle sum of a triangle is  $180^\circ$ .)
- 2 The above result is only true for triangles. For example, if four angles of a quadrilateral are equal to four angles of another, the quadrilaterals are only similar if the corresponding sides are also in proportion.

Consider the two right-angled triangles shown below.



These triangles are similar because the three angles of triangle  $ABC$  are equal to the three angles of triangle  $PQR$ .

Thus, in the triangles shown  $\frac{PQ}{AB} = \frac{PR}{AC}$  (the corresponding sides of similar figures are in proportion).

This may be rearranged to  $\frac{PQ}{PR} = \frac{AB}{AC}$ .

So for angle  $x$ :

$$\frac{\text{length of opposite side of } \Delta PQR}{\text{length of hypotenuse of } \Delta PQR} = \frac{\text{length of opposite side of } \Delta ABC}{\text{length of hypotenuse of } \Delta ABC}$$

Thus, for any angle  $x$  in a right-angled triangle, the ratio  $\frac{\text{length of opposite side}}{\text{length of hypotenuse}}$  is constant.

This ratio is given a name. It is called the sine (abbreviated sin) of the angle  $x$ .

In a right-angled triangle, the **sine** of an angle  $x$  is the ratio of the length of the opposite side to the length of the hypotenuse. This may be written:

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}} \left( = \frac{O}{H} \right)$$

Similarly, in the triangles shown  $\frac{QR}{BC} = \frac{PR}{AC}$  (the corresponding sides of similar triangles are in proportion).

This may be arranged to  $\frac{QR}{PR} = \frac{BC}{AC}$ .

So for angle  $x$ :

$$\frac{\text{length of adjacent side of } \Delta PQR}{\text{length of hypotenuse of } \Delta PQR} = \frac{\text{length of adjacent side of } \Delta ABC}{\text{length of hypotenuse of } \Delta ABC}$$

Thus, for any angle  $x$  in a right-angled triangle, the ratio  $\frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$  is constant.

This ratio is given a name. It is called the cosine (abbreviated cos) of the angle  $x$ .

In a right-angled triangle, the **cosine** of an angle  $x$  is the ratio of the length of the adjacent side to the length of the hypotenuse. This may be written:

$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}} \left( = \frac{A}{H} \right)$$

Similarly, in the triangles shown  $\frac{PQ}{AB} = \frac{QR}{BC}$  (the corresponding sides of similar triangles are in proportion).

This may be arranged to  $\frac{PQ}{QR} = \frac{AB}{BC}$ .

So for angle  $x$ :

$$\frac{\text{length of opposite side of } \Delta PQR}{\text{length of adjacent of } \Delta PQR} = \frac{\text{length of opposite side of } \Delta ABC}{\text{length of adjacent side of } \Delta ABC}$$

Thus, for any angle  $x$  in a right-angled triangle, the ratio  $\frac{\text{length of opposite side}}{\text{length of adjacent side}}$  is constant.

This ratio is given a name. It is called the tangent (abbreviated tan) of the angle  $x$ .

#### sine

the trigonometric ratio calculated by dividing the length of the opposite side by the length of the hypotenuse in a right-angled triangle

#### cosine

the trigonometric ratio calculated by dividing the length of the adjacent side by the length of the hypotenuse in a right-angled triangle

#### tangent

the trigonometric ratio calculated by dividing the length of the opposite side by the length of the adjacent side in a right-angled triangle

In a right-angled triangle, the **tangent** of an angle  $x$  is the ratio of the length of the opposite side to the length of the adjacent side. This may be written:

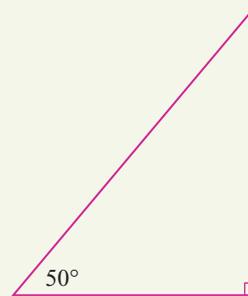
$$\tan x = \frac{\text{opposite}}{\text{adjacent}} \left( = \frac{O}{A} \right)$$



To help remember the above definitions, try using the following mnemonic: SOH CAH TOA.

### EXAMPLE 5B-1 Calculating trigonometric ratios by measuring side lengths

Calculate the value of  $\sin 50^\circ$ ,  $\cos 50^\circ$  and  $\tan 50^\circ$  in the given triangle.  
Give your answers correct to two decimal places.

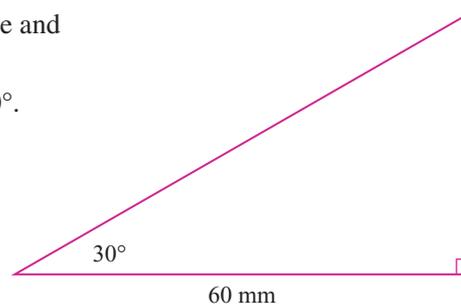


Solve	Think	Apply
$\sin 50^\circ = \frac{48}{63}$ $= 0.76$ $\cos 50^\circ = \frac{40}{63}$ $= 0.63$ $\tan 50^\circ = \frac{48}{40}$ $= 1.2$	By measurement: hypotenuse = 63 mm opposite = 48 mm adjacent = 40 mm	Measure the lengths of the sides and use these ratios: $\sin x = \frac{O}{H}$ $\cos x = \frac{A}{H}$ $\tan x = \frac{O}{A}$

## EXERCISE 5B Trigonometric ratios

UNDERSTANDING, FLUENCY AND COMMUNICATING

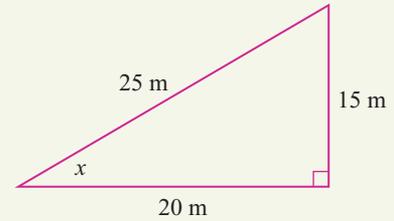
- 1 a** Measure accurately the length of the opposite side and the hypotenuse, for the  $30^\circ$  angle in this triangle.  
**b** Calculate the value of  $\sin 30^\circ$ ,  $\cos 30^\circ$  and  $\tan 30^\circ$ .



- 2 a** Draw any right-angled triangle with a  $65^\circ$  angle.  
**b** Measure accurately the length of the opposite side, the adjacent side and the hypotenuse, for the  $65^\circ$  angle.  
**c** Calculate the value of  $\sin 65^\circ$ ,  $\cos 65^\circ$  and  $\tan 65^\circ$ .
- 3** Repeat question **2** for a right-angled triangle with a  $25^\circ$  angle.

### EXAMPLE 5B-2 Calculating trigonometric ratios with given side lengths

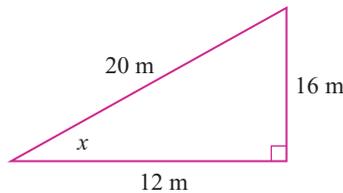
Calculate the value of  $\sin x$ ,  $\cos x$  and  $\tan x$  in the triangle shown.  
(It is not drawn to scale.)



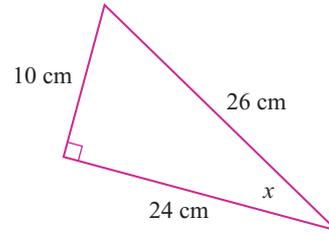
Solve	Think	Apply
$\sin x = \frac{15}{25}$ $= \frac{3}{5} \text{ or } 0.6$	The side opposite $x$ is 15 m and the hypotenuse is 25 m. $\therefore \sin x = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{15}{25}$	Identify the hypotenuse and then the opposite and adjacent sides for the given angle. Use the definitions to determine the sine, cosine and tangent ratios for the given angle.
$\cos x = \frac{20}{25}$ $= \frac{4}{5} \text{ or } 0.8$	The side adjacent to $x$ is 20 m and the hypotenuse is 25 m. $\therefore \cos x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{20}{25}$	
$\tan x = \frac{15}{20}$ $= \frac{3}{4} \text{ or } 0.75$	The opposite side is 15 m and the adjacent side is 20 m. $\therefore \tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{15}{20}$	

4 Calculate the value of  $\sin x$ ,  $\cos x$  and  $\tan x$  in the triangles below (not drawn to scale) to two decimal places.

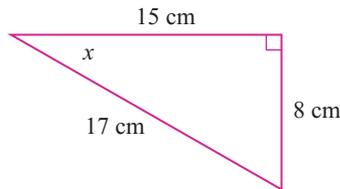
a



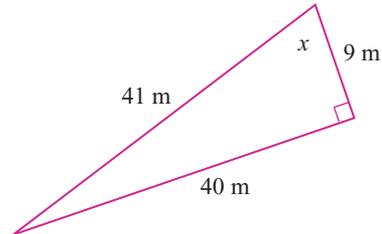
b



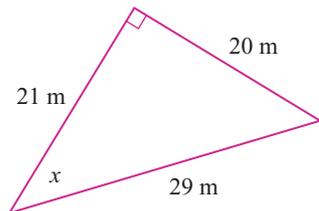
c



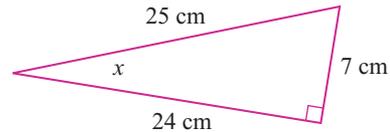
d



e



f



#### Using trigonometric ratios on a calculator

We have found the values of the sine, cosine and tangent of some angles by measurement and calculation. As this is quite time-consuming, and because the values of the ratios are so useful, they are usually calculated using a scientific calculator.

A minute is a fraction of a degree. If you are asked to express your answer in degrees and minutes, the answer will be more accurate than when given to the nearest degree.

1 degree = 60 minutes



### EXAMPLE 5B-3 Finding trigonometric values for angles in degrees and minutes

Use your calculator to find these values, correct to four decimal places.

- a**  $\sin 54^\circ$                       **b**  $\tan 68.3^\circ$                       **c**  $\cos 67^\circ 13'$                       **d**  $\sin 11^\circ 52'$

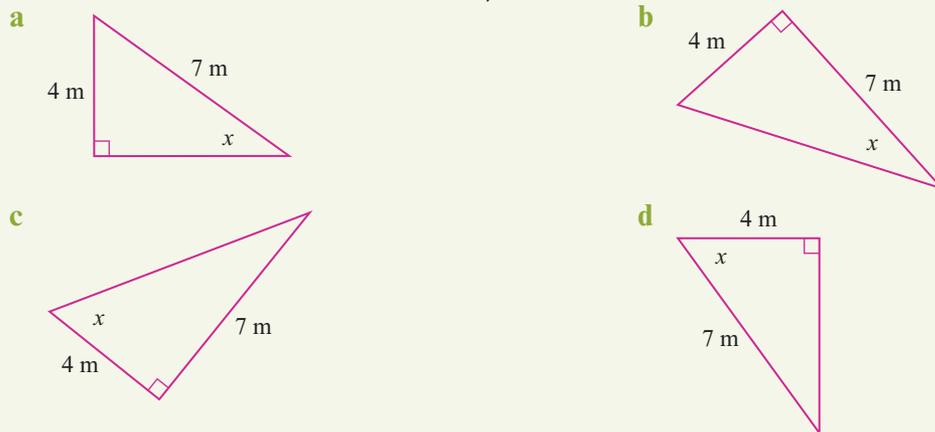
	Solve	Think	Apply
<b>a</b>	$\sin 54^\circ = 0.809\ 016\dots$ $\approx 0.8090$	One degree of angular measure can be divided into 60 smaller units called minutes: 1 degree = 60 minutes, written $1^\circ = 60'$ To find $\cos 67^\circ 13'$ on a CASIO calculator, press	Use the appropriate keys on your calculator to enter degrees and minutes.
<b>b</b>	$\tan 68.3^\circ \approx 2.5129$		
<b>c</b>	$\cos 67^\circ 13' \approx 0.3872$		
<b>d</b>	$\sin 11^\circ 52' \approx 0.2056$		

**5** Use your calculator to find these values, correct to four decimal places.

- a**  $\sin 47^\circ$                       **b**  $\cos 52^\circ$                       **c**  $\tan 66^\circ$                       **d**  $\sin 77.3^\circ$   
**e**  $\tan 13.7^\circ$                       **f**  $\cos 77.9^\circ$                       **g**  $\tan 74.9^\circ$                       **h**  $\sin 37.28^\circ$   
**i**  $\sin 43^\circ 45'$                       **j**  $\cos 64^\circ 3'$                       **k**  $\tan 34^\circ 26'$                       **l**  $\cos 60^\circ 55'$   
**m**  $\tan 71^\circ 41'$                       **n**  $\sin 60^\circ 27'$                       **o**  $\tan 55^\circ 21'$                       **p**  $\sin 79^\circ 34'$

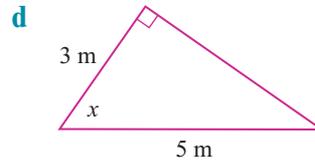
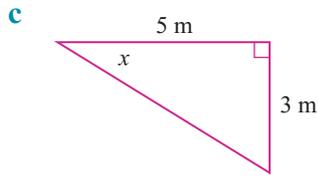
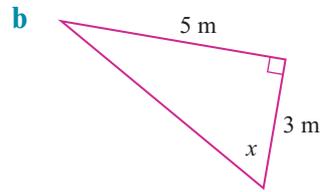
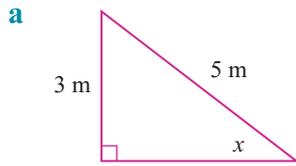
### EXAMPLE 5B-4 Identifying trigonometric ratios

For the following triangles, state whether  $\frac{4}{7} = \sin x$ ,  $\cos x$ ,  $\tan x$ , or none of these ratios.

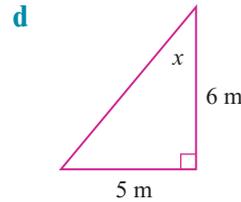
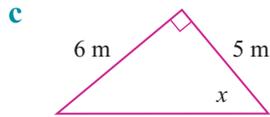
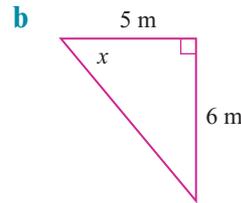
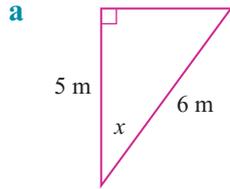


	Solve	Think	Apply
<b>a</b>	$\frac{4}{7} = \sin x$	$\frac{4}{7} = \frac{\text{opposite}}{\text{hypotenuse}} = \sin x$	Identify the given sides as opposite, adjacent or hypotenuse. Determine the trigonometric ratio that connects them in the correct order.  Remember SOH CAH TOA.
<b>b</b>	$\frac{4}{7} = \tan x$	$\frac{4}{7} = \frac{\text{opposite}}{\text{adjacent}} = \tan x$	
<b>c</b>	$\frac{4}{7} = \text{none of these}$	$\frac{4}{7} = \frac{\text{adjacent}}{\text{opposite}} = \text{none of these}$	
<b>d</b>	$\frac{4}{7} = \cos x$	$\frac{4}{7} = \frac{\text{adjacent}}{\text{hypotenuse}} = \cos x$	

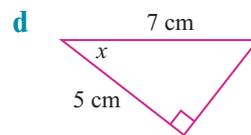
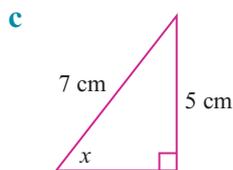
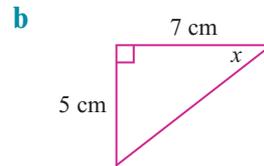
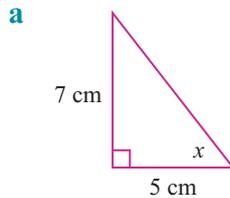
**6** For the following triangles, state whether  $\frac{3}{5}$  is  $\sin x$ ,  $\cos x$ ,  $\tan x$ , or none of these ratios.



**7** For the following triangles, state whether  $\frac{5}{6}$  is  $\sin x$ ,  $\cos x$ ,  $\tan x$ , or none of these ratios.



**8** For the following triangles, state where  $\frac{5}{7}$  is  $\sin x$ ,  $\cos x$ ,  $\tan x$ , or none of these ratios.



**9** Complete the following sentences.

- a** As the value of  $x$  increases from  $0^\circ$  to  $90^\circ$ , the value of  $\sin x$  \_\_\_\_\_.
- b** As the value of  $x$  increases from  $0^\circ$  to  $90^\circ$ , the value of  $\cos x$  \_\_\_\_\_.
- c** As the value of  $x$  increases from  $0^\circ$  to  $90^\circ$ , the value of  $\tan x$  \_\_\_\_\_.

**10** Explain why  $\tan x = \frac{\sin x}{\cos x}$ .

# 5C Using trigonometry to calculate angles

These resources are available on your obook assess:

- **Interactive 5C:** Explore key ideas for using trigonometry to find angles
- **assess quiz 5C:** Test your skills with an auto-correcting multiple-choice quiz

1010  
1010

Given the value of the trigonometric ratio, it is possible to find the angle, using a calculator.

## EXAMPLE 5C-1 Finding the angle size to the nearest degree

- a** Find the size of the angle  $x$ , if  $\sin x = 0.64$ .  
**b** Find the size of the angle  $x$ , if  $\tan x = \frac{5}{7}$ . Give your answer correct to the nearest degree.

	Solve	Think	Apply
<b>a</b>	$x = 40^\circ$ (to the nearest degree)	Possible steps on a CASIO calculator are  The calculator displays 39.7918... Round to the nearest whole number.	Determine the correct sequence of keys for your calculator.
<b>b</b>	$x = 36^\circ$ (to the nearest degree)	Possible steps on a CASIO calculator are  The calculator displays 35.537...	

## EXERCISE 5C Using trigonometry to calculate angles

UNDERSTANDING, FLUENCY AND COMMUNICATING

- 1** Find the size of the angle  $x$ , to the nearest degree, if:
- |                          |                          |                          |
|--------------------------|--------------------------|--------------------------|
| <b>a</b> $\sin x = 0.7$  | <b>b</b> $\cos x = 0.4$  | <b>c</b> $\tan x = 0.5$  |
| <b>d</b> $\sin x = 0.68$ | <b>e</b> $\cos x = 0.96$ | <b>f</b> $\tan x = 1.25$ |
| <b>g</b> $\cos x = 0.28$ | <b>h</b> $\tan x = 0.46$ | <b>i</b> $\sin x = 0.45$ |
| <b>j</b> $\cos x = 0.86$ | <b>k</b> $\sin x = 0.99$ | <b>l</b> $\tan x = 0.70$ |
- 2** Find, to the nearest degree, the size of the angle  $x$ , if:
- |                                   |                                   |                                   |
|-----------------------------------|-----------------------------------|-----------------------------------|
| <b>a</b> $\sin x = \frac{3}{4}$   | <b>b</b> $\cos x = \frac{5}{8}$   | <b>c</b> $\tan x = \frac{3}{7}$   |
| <b>d</b> $\cos x = \frac{8}{15}$  | <b>e</b> $\tan x = \frac{8}{3}$   | <b>f</b> $\sin x = \frac{5}{6}$   |
| <b>g</b> $\tan x = \frac{6}{11}$  | <b>h</b> $\cos x = \frac{4}{9}$   | <b>i</b> $\tan x = \frac{23}{48}$ |
| <b>j</b> $\sin x = \frac{56}{75}$ | <b>k</b> $\sin x = \frac{13}{21}$ | <b>l</b> $\cos x = \frac{13}{21}$ |
- 3** Find, to the nearest degree, the size of the angle  $\theta$ , if:
- |  |  |  |
|--|--|--|
| <b>a</b> $\sin \theta = \frac{2.5}{3.6}$   | <b>b</b> $\cos \theta = \frac{7.8}{10.7}$  | <b>c</b> $\tan \theta = \frac{12.9}{15.2}$ |
| <b>d</b> $\cos \theta = \frac{48.1}{69.4}$ | <b>e</b> $\tan \theta = \frac{65.5}{37.8}$ | <b>f</b> $\sin \theta = \frac{4.1}{9.7}$   |

A minute can be further divided into seconds.

1 minute = 60 seconds

When giving an answer to the nearest minute:

- if the number of seconds is  $< 30$  round down
- if the number of seconds is  $\geq 30$  round up.



### EXAMPLE 5C-2 Finding the angle size to the nearest minute

Use your calculator to find the size of angle  $\theta$ , correct to the nearest minute, when:

**a**  $\sin \theta = 0.7511$

**b**  $\cos \theta = 0.6725$

**c**  $\tan \theta = \frac{15.3}{11.2}$

**d**  $\sin \theta = \frac{6}{19}$

	Solve	Think	Apply
<b>a</b>	$\sin \theta = 0.7511$ $\theta \approx 48^\circ 41'$	These steps are appropriate for a CASIO calculator. Press <b>SHIFT</b> <b>sin</b> 0.7511 <b>=</b> 48.6857... The answer is in degrees. To change to degrees and minutes, press <b>° ' "</b> The display shows $48^\circ 41' 8.71''$ , this means the result is $48^\circ 41' 8.71''$ . If the number of seconds is $< 30$ round down, if the number of seconds is $\geq 30$ round up. Rounding gives $\theta \approx 48^\circ 41'$ .	Find the angle for a given trigonometric value using the inverse key on a calculator. Change the angle in degrees to degrees, minutes and seconds, and then round to the nearest minute.
<b>b</b>	$\cos \theta = 0.6725$ $\theta \approx 47^\circ 44'$	<b>SHIFT</b> <b>cos</b> 0.6725 = 47.7396... To change to degrees and minutes, press <b>° ' "</b> The display shows $47^\circ 44' 22.88''$ . Rounding gives $\theta \approx 47^\circ 44'$ .	
<b>c</b>	$\tan \theta = \frac{15.3}{11.2}$ $\theta \approx 53^\circ 48'$	<b>SHIFT</b> <b>tan</b> 15.3 <b>=</b> 11.2 <b>=</b> 53.7948...° <b>° ' "</b> gives $53^\circ 47' 41.63''$ which rounds to $53^\circ 48'$ .	
<b>d</b>	$\sin \theta = \frac{6}{19}$ $\theta \approx 18^\circ 25'$	<b>SHIFT</b> <b>sin</b> 6 <b>=</b> 19 <b>=</b> 18.4084...° <b>° ' "</b> gives $18^\circ 24' 30.53''$ which rounds to $18^\circ 25'$ .	

**4** Use your calculator to find the size of angle  $\theta$ , correct to the nearest minute.

**a**  $\sin \theta = 0.5789$

**b**  $\cos \theta = 0.1427$

**c**  $\tan \theta = 0.5499$

**d**  $\tan \theta = 0.4952$

**e**  $\sin \theta = 0.3367$

**f**  $\cos \theta = 0.5247$

**g**  $\cos \theta = 0.6723$

**h**  $\tan \theta = 0.4459$

**i**  $\sin \theta = 0.0496$

**5** Use your calculator to find the size of angle  $\theta$ , correct to the nearest minute.

**a**  $\sin \theta = \frac{12}{19}$

**b**  $\cos \theta = \frac{15}{22}$

**c**  $\tan \theta = \frac{3}{8}$

**d**  $\tan \theta = \frac{9}{7}$

**e**  $\sin \theta = \frac{7}{8}$

**f**  $\cos \theta = \frac{5}{11}$

**g**  $\cos \theta = \frac{27.6}{49.8}$

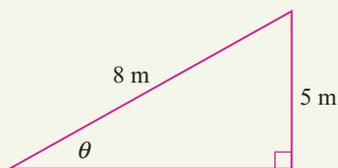
**h**  $\tan \theta = \frac{18.1}{12.5}$

**i**  $\sin \theta = \frac{6.2}{9.8}$

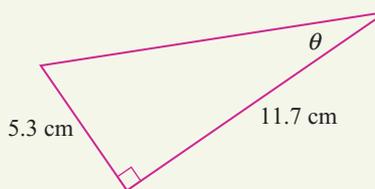
### EXAMPLE 5C-3 Finding the size of an unknown angle

Find the size of angle  $\theta$  in each of these triangles, to the nearest minute.

**a**



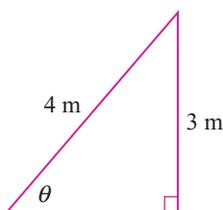
**b**



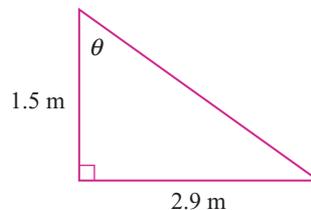
	Solve	Think	Apply
<b>a</b>	$\sin \theta = \frac{5}{8}$ $= 0.625$ $\theta = 38.682\dots^\circ$ $\approx 38^\circ 41'$	The known side lengths are on the opposite side and the hypotenuse. The trigonometric ratio connecting these sides is the sine ratio.	Identify the sides with the given side lengths and use the appropriate trigonometric ratio.
<b>b</b>	$\tan \theta = \frac{5.3}{11.7}$ $= 0.45299\dots$ $\theta = 24.370\dots^\circ$ $\approx 24^\circ 2' 2'$	The known side lengths are on the opposite side and the adjacent side. The trigonometric ratio connecting these sides is the tangent ratio.	Change angle in degrees to degrees, minutes and seconds, and then round to the nearest minute.

**6** For each of the following, find the size of angle  $\theta$ , to the nearest degree.

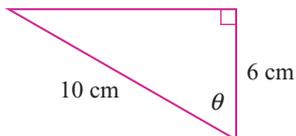
**a**



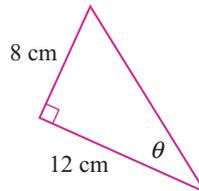
**b**



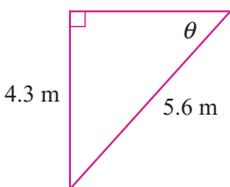
**c**



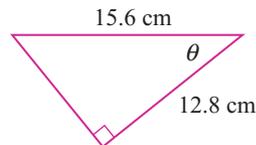
**d**



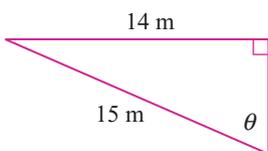
**e**



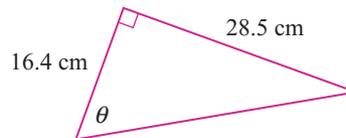
**f**



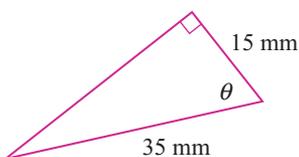
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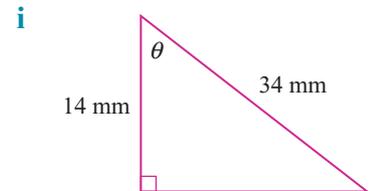
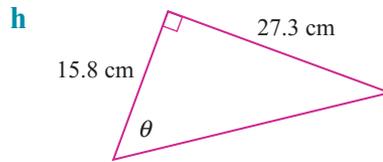
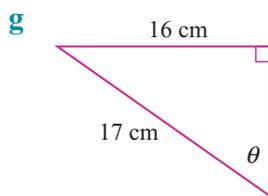
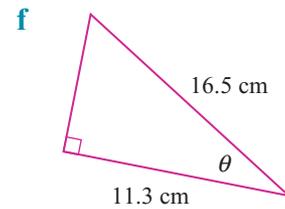
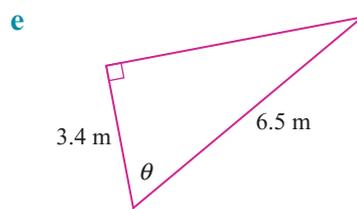
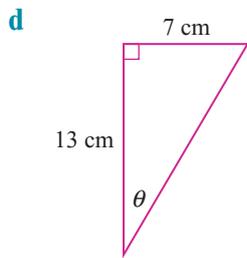
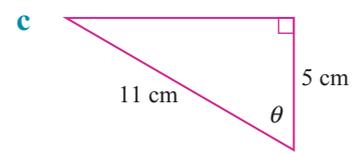
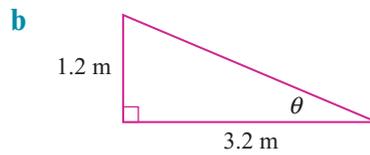
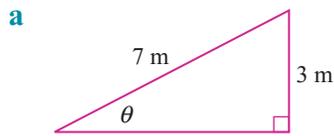
**h**



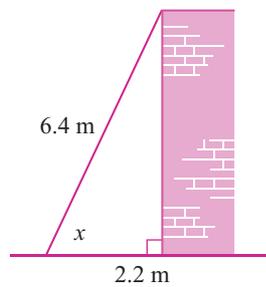
**i**



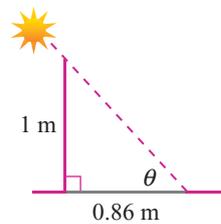
**7** Find the size of the angle  $\theta$  in each of these triangles, to the nearest minute.



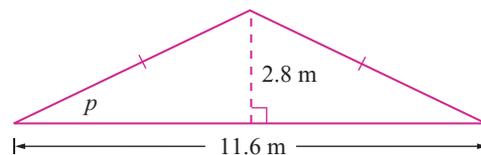
**8** The foot of a ladder 6.4 m long is placed 2.2 m from the base of a wall. Calculate the angle the ladder makes with the ground.



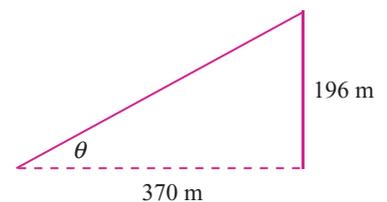
**9** A metre rule casts a shadow 0.86 m long. What angle do the sun's rays make with the ground at this time?



**10** The diagram shows the dimensions of the gable roof at the end of a house. Calculate the angle of pitch,  $p$ , of the roof.



**11** A ski slope falls 196 m over a run of 370 m. Find the angle the slope makes with the horizontal.



# 5D Using trigonometry to calculate side lengths

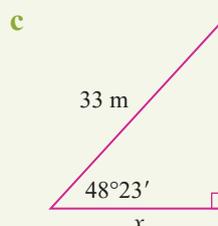
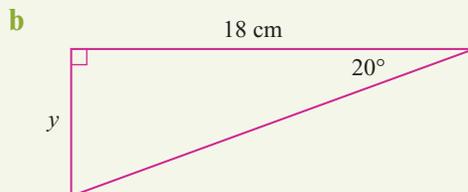
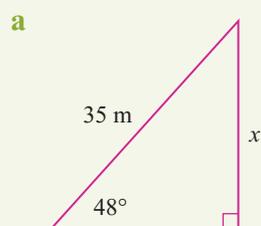
These resources are available on your obook assess:

- **Interactive 5D:** Explore key ideas for using trigonometry to find lengths
- **Worksheet 5D:** Practise your skills with extra problems for trigonometry
- **Investigation 5D:** Using trigonometry to find an approximate value for  $\pi$
- **assess quiz 5D:** Test your skills with an auto-correcting multiple-choice quiz

1010

## EXAMPLE 5D-1 Finding an unknown side length

For the following triangles, find the length of the unknown side correct to one decimal place.



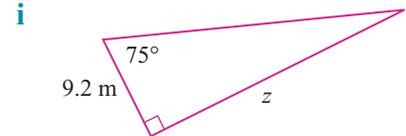
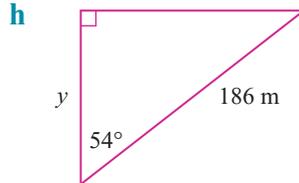
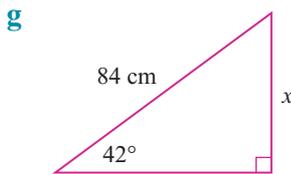
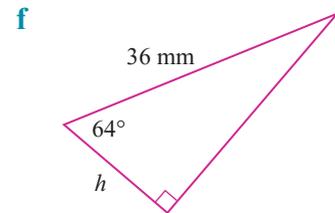
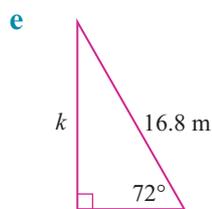
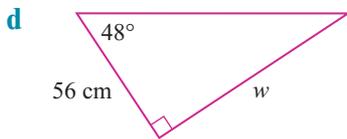
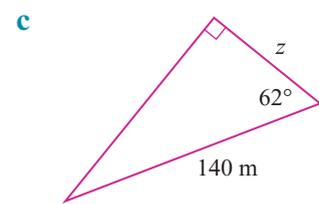
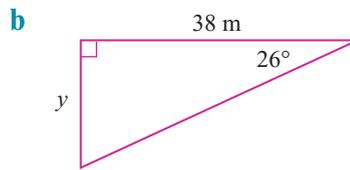
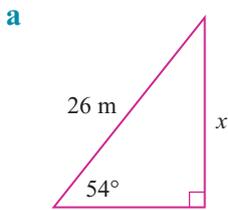
	Solve	Think	Apply
<b>a</b>	$\frac{x}{35} = \sin 48^\circ$ $x = 35 \times \sin 48^\circ$ $= 26.0 \text{ m}$ (to the nearest 0.1 m)	Relative to the given angle, the unknown side ( $x$ ) is the opposite side and the known side (35 m) is the hypotenuse. The trigonometric ratio that connects these two sides is the sine ratio. Hence $\frac{x}{35} = \frac{\text{opposite}}{\text{hypotenuse}} = \sin 48^\circ$ .	Identify the unknown and given sides as opposite, adjacent or hypotenuse and determine the trigonometric ratio that connects them. Solve the resulting equation.
<b>b</b>	$\frac{y}{18} = \tan 20^\circ$ $y = 18 \times \tan 20^\circ$ $= 6.6 \text{ cm}$ (to the nearest 0.1 cm)	Relative to the given angle, $y$ is the opposite side and 18 cm is the adjacent side. The trigonometric ratio that connects these sides is the tangent ratio. Hence $\frac{y}{18} = \frac{\text{opposite}}{\text{adjacent}} = \tan 20^\circ$ .	
<b>c</b>	$\frac{x}{33} = \cos 48^\circ 23'$ $x = 33 \times \cos 48^\circ 23'$ $= 21.9 \text{ m}$ (to the nearest 0.1 m)	Relative to the given angle, $x$ is the length of the adjacent side and 33 m is the length of the hypotenuse. The trigonometric ratio that connects these two sides is the cosine ratio. Hence $\frac{x}{33} = \frac{\text{adjacent}}{\text{hypotenuse}} = \cos 48^\circ 23'$ .	

# EXERCISE 5D

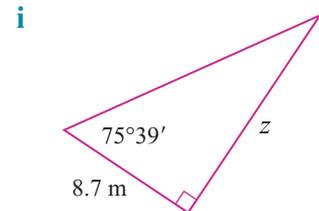
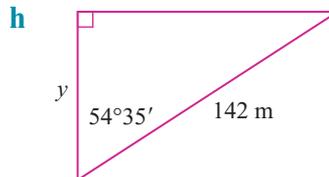
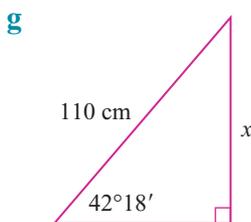
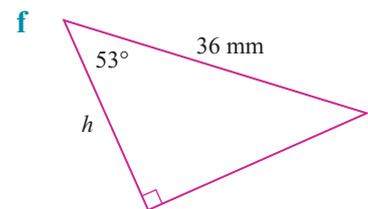
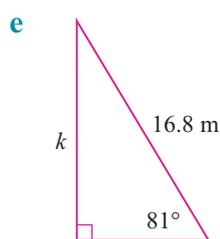
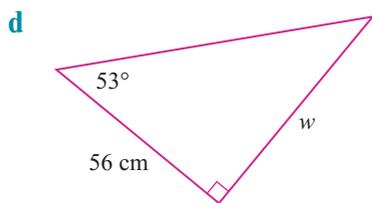
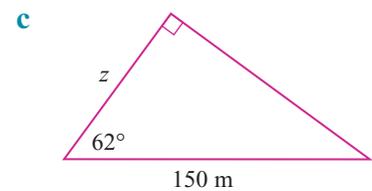
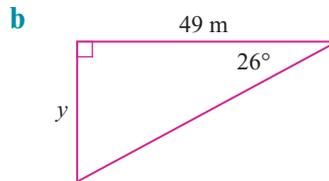
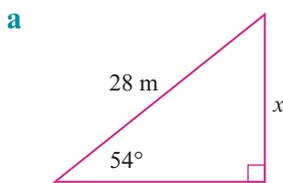
## Using trigonometry to calculate side lengths

UNDERSTANDING, FLUENCY AND COMMUNICATING

1 Find the length of the unknown side, correct to one decimal place.

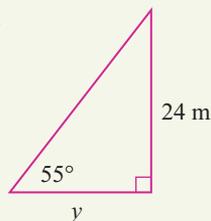


2 Find the unknown side length labelled with a pronumeral in each of these triangles, correct to one decimal place.



### EXAMPLE 5D-2 Using the other angle to find the side length

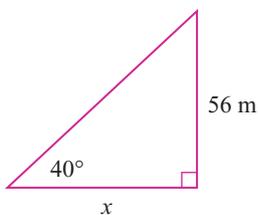
Find the length of the unknown side,  $y$ , correct to one decimal place.



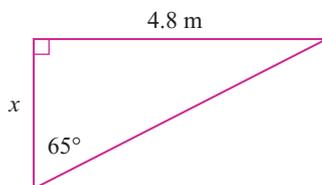
Solve	Think	Apply
$\frac{y}{24} = \tan 35^\circ$ $y = 24 \times \tan 35^\circ$ $= 16.8 \text{ m}$ (to one decimal place)	Relative to the $55^\circ$ angle, 24 m is the opposite side and $y$ is the adjacent side. Hence $\tan 55^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{24}{y}$ But it is easier to use the third angle. Third angle = $180^\circ - 90^\circ - 55^\circ = 35^\circ$ . Hence $\frac{y}{24} = \tan 35^\circ$ (unknown in numerator) $y = 24 \tan 35^\circ$	When using the tangent ratio, determine which angle must be used to ensure that the unknown is in the numerator.

**3** Find the length of the unknown side, correct to one decimal place.

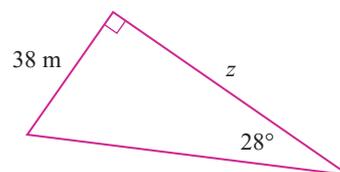
**a**



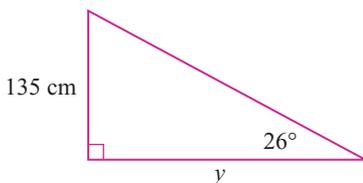
**b**



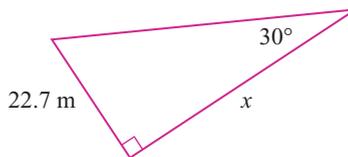
**c**



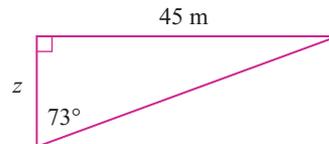
**d**



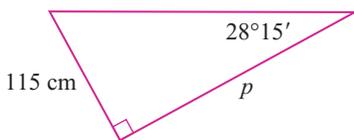
**e**



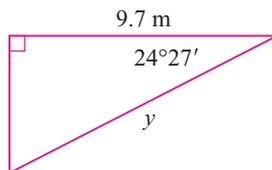
**f**



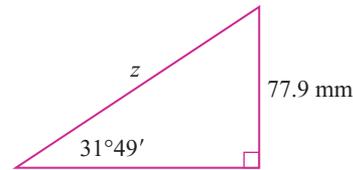
**g**



**h**



**i**



**4 a** Complete the following table.

$x$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$85^\circ$	$90^\circ$
<b>sin</b>		0.5		0.866		1
<b>cos</b>			0.707	0.5	0.087	
<b>tan</b>	0	0.577			11.430	undefined

**b** Describe what happens to the value of  $\sin x$  as  $x$  increases from  $0^\circ$  to  $90^\circ$ .

**c** Complete:  $\_\_\_ \leq \sin x \leq \_\_\_$ .

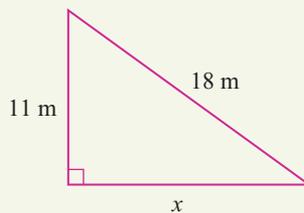
**d** Describe what happens to the value of  $\cos x$  as  $x$  increases from  $0^\circ$  to  $90^\circ$ .

- e Complete:  $\underline{\hspace{1cm}} \leq \cos x \leq \underline{\hspace{1cm}}$ .
- f Describe what happens to the value of  $\tan x$  as  $x$  increases from  $0^\circ$  to  $90^\circ$ .
- g Complete: For  $0^\circ \leq x \leq 45^\circ$ ,  $\underline{\hspace{1cm}} \leq \tan x \leq \underline{\hspace{1cm}}$ . For  $x > 45^\circ$ ,  $\tan x > \underline{\hspace{1cm}}$ .

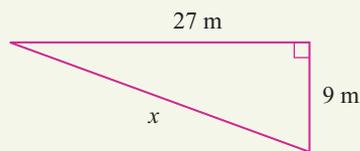
**EXAMPLE 5D-3** Determining reasonable values by considering the hypotenuse

By considering the measurements given in the diagram, determine whether the value of the unknown in the following triangles seems reasonable. (You do not need to calculate the value of the unknown.)

- a i  $x = 21$  m                      ii  $x = 14$  m



- b i  $x = 28$  m                      ii  $x = 26$  m

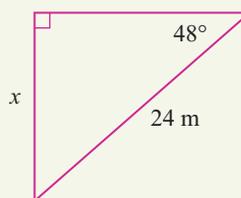


	Solve	Think	Apply
a i	No	The hypotenuse is the longest side of a right-angled triangle, hence the length of the unknown side must be less than 18 m. (Note that $x = 14$ m is possible, but not necessarily correct.)	Compare the answer with the length of the hypotenuse. The hypotenuse is the longest side of a right-angled triangle.
ii	Yes		
b i	Yes	As the unknown side is the hypotenuse its length must be greater than 27 m.	The hypotenuse is the longest side of a right-angled triangle.
ii	No		

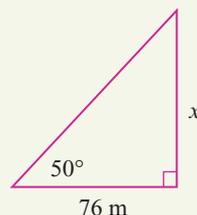
**EXAMPLE 5D-4** Determining reasonable values by considering trigonometric ratios

By considering the measurements given in the diagram, determine whether or not the value of the unknown in the following triangles seems reasonable. (You do not need to calculate the value of the unknown.)

- a i  $x = 178$  m                      ii  $x = 17.8$  m



- b i  $x = 62$  m                      ii  $x = 84$  m

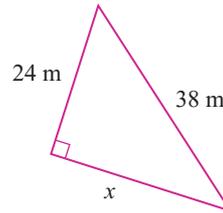


		Solve	Think	Apply
<b>a</b>	<b>i</b>	No	The length of the unknown side must be smaller than the length of the hypotenuse. $\frac{x}{24} = \sin 48^\circ$ $\therefore x = 24 \times \sin 48^\circ$ As $\sin 48^\circ < 1$ (from question 4), then $24 \times \sin 48^\circ < 24$ .	The hypotenuse is the longest side of a right-angled triangle. Or use $0 \leq \sin x \leq 1$ $0 \leq \cos x \leq 1$ For $0 \leq x \leq 45^\circ$ , $0 \leq \tan x \leq 1$ and for $x > 45^\circ$ , $\tan x > 1$ .
	<b>ii</b>	Yes		
<b>b</b>	<b>i</b>	No	$\frac{x}{76} = \tan 50^\circ$ $\therefore x = 76 \times \tan 50^\circ$ As $\tan 50^\circ > 1$ (from question 4), then $76 \times \tan 50^\circ > 76$ .	Use $0 \leq \sin x \leq 1$ $0 \leq \cos x \leq 1$ For $0 \leq x \leq 45^\circ$ , $0 \leq \tan x \leq 1$ and or $x > 45^\circ$ , $\tan x > 1$ .
	<b>ii</b>	Yes		

**5** By considering the measurements given in each diagram, determine whether the value of the unknown for the following triangles seems reasonable. The drawings are not to scale. (You do not need to calculate the value of the unknown.)

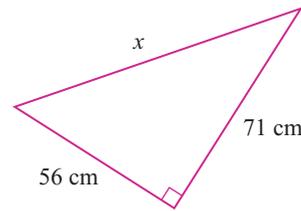
**a i**  $x = 45$  m

**ii**  $x = 29$  m



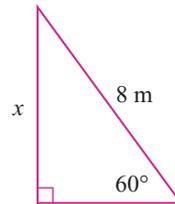
**b i**  $x = 90$  cm

**ii**  $x = 44$  cm



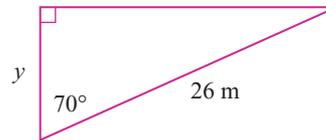
**c i**  $x = 9$  m

**ii**  $x = 7$  m



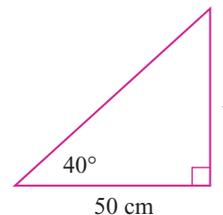
**d i**  $y = 76$  m

**ii**  $y = 9$  m



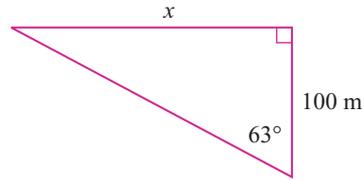
**e i**  $z = 60$  cm

**ii**  $z = 42$  cm



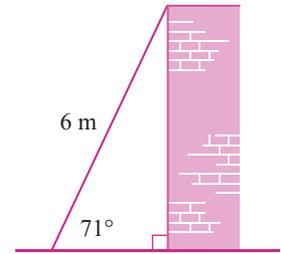
f i  $x = 196$  m

ii  $x = 51$  m

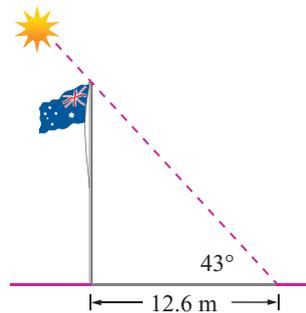


Unless stated otherwise, find all lengths correct to one decimal place and all angles to the nearest minute.

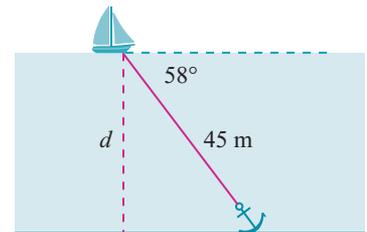
- 6 A ladder 6 m long makes an angle of  $71^\circ$  with the ground.  
 a How far does the ladder reach up the wall?  
 b How far is the foot of the ladder from the base of the wall?



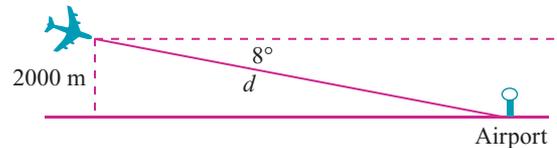
- 7 A flagpole casts a shadow 12.6 m long when the sun's rays make an angle of  $43^\circ$  with the ground. Find the height of the flagpole.



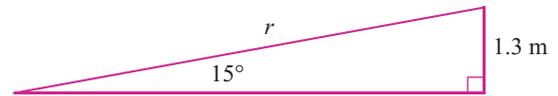
- 8 The anchor rope of a boat is 45 m long. When it is let out fully, it makes an angle of  $58^\circ$  with the surface of the water. Calculate the depth of the water at this point.



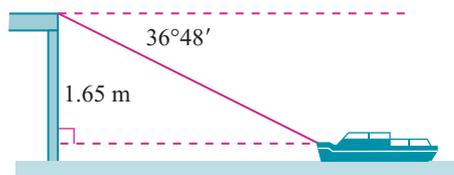
- 9 The pilot of a plane flying at an altitude of 2000 m notes that the angle from the horizontal line of sight to the base of the airport tower is  $8^\circ$ . How far, in a straight line to the nearest metre, is the plane from the tower?



- 10 A ramp, inclined at  $15^\circ$  to the ground, rises 1.3 m vertically to an entrance. How long is the ramp?



- 11 A boat is tied to a wharf that is 1.65 m above the boat, as shown in the diagram. The rope makes an angle of  $36^\circ 48'$  with the horizontal. How long is the rope to the nearest centimetre?



# 5E

# Angles of elevation and depression

### angle of elevation

angle between a horizontal line and the line of sight to an object above the horizontal

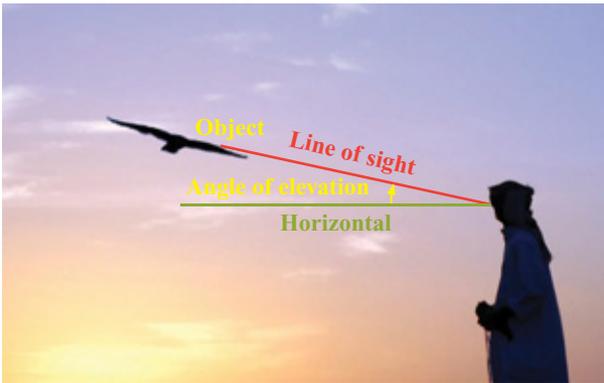
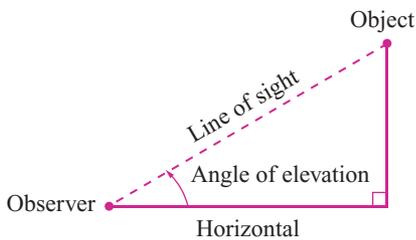
### angle of depression

angle between a horizontal line and the line of sight to an object below the horizontal

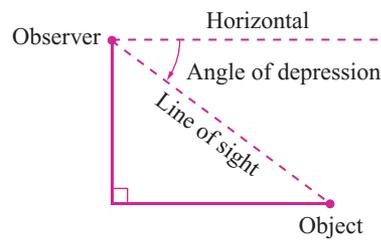
These resources are available on your obook assess:

- **Interactive 5E:** Explore key ideas for angles of elevation and depression
- **Investigation 5E:** Apply your skills to finding the height of an object
- **assess quiz 5E:** Test your skills with an auto-correcting multiple-choice quiz

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The **angle of elevation** of an object, from an observer, is the angle between the horizontal and the line of sight *up* to the object.

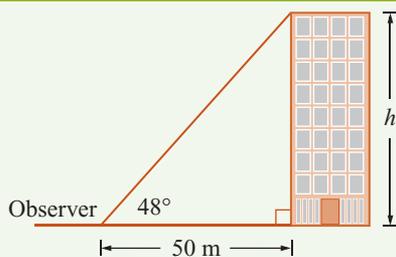


The **angle of depression** of an object, from an observer, is the angle between the horizontal and the line of sight *down* to the object.

## EXAMPLE 5E-1 Sketching an angle of elevation

The angle of elevation of the top of a building from a point 50 m from its base is  $48^\circ$ . Draw a diagram to illustrate this information.

### Solve

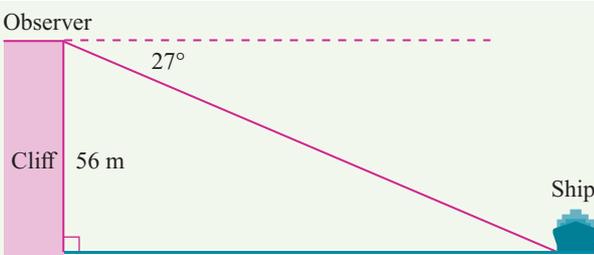


### Think/Apply

Draw a horizontal baseline, then draw a sketch of a building, labelling the height,  $h$ . Mark the position of the observer 50 m from the base of the building. From the observer, draw the line of sight to the top of the building. Mark the angle between the horizontal and the line of sight as  $48^\circ$ .

### EXAMPLE 5E-2 Sketching an angle of depression

The angle of depression from the top of a vertical cliff 56 m high to a ship at sea is  $27^\circ$ . Draw a diagram to illustrate this information.

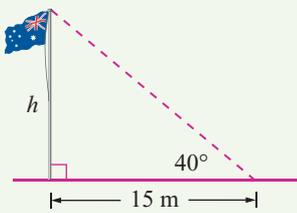
Solve	Think/Apply
	<p>Draw a horizontal line to represent sea level, then draw a sketch of a cliff at right angles to sea level, labelling the height 56 m. Mark the position of the ship at sea. Draw the horizontal through the observer at the top of the cliff. Draw the line of sight from the observer to the ship. Mark the angle between the horizontal and the line of sight as <math>27^\circ</math>.</p>

## EXERCISE 5E Angles of elevation and depression

- 1 Draw diagrams to illustrate the following information.
  - a The angle of elevation of the top of a building from a point 60 m from its base is  $65^\circ$ .
  - b The angle of depression from the top of a vertical cliff 36 m high to a ship at sea is  $15^\circ$ .
  - c The angle of elevation of the top of a tree from a point 20 m from its base is  $32^\circ$ .
  - d From the top of a 36 m tall building, the angle of depression to a car on the street below is  $54^\circ$ .
  - e A flagpole casts a shadow 15 m long when the angle of elevation of the sun is  $67^\circ$ .

### EXAMPLE 5E-3 Calculating the height using an angle of elevation

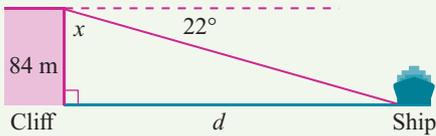
The angle of elevation of the top of a flagpole, as observed from a point 15 m from its base, is  $40^\circ$ . Find the height of the flagpole.

Solve	Think	Apply
<p>Draw a diagram. Let the height of the pole be <math>h</math>.</p>  <p>By trigonometry,  <math>\frac{h}{15} = \tan 40^\circ</math>  <math>\therefore h = 15 \times \tan 40^\circ</math>  <math>= 12.6 \text{ m (to one decimal place)}</math></p>	<p>For the <math>40^\circ</math> angle, <math>h</math> is the opposite side and 15 m is the adjacent side. The trigonometric ratio that connects these two sides is the tangent ratio. Hence:  <math>\frac{h}{15} = \frac{\text{opposite}}{\text{adjacent}} = \tan 40^\circ</math></p>	<p>Draw a diagram and mark the angle of elevation. Identify the unknown and given sides as opposite, adjacent or hypotenuse, and determine the trigonometric ratio that connects them. Solve the resulting equation.</p>

- 2 The angle of elevation of the top of a flagpole is  $42^\circ$ , when observed from a point 19 m from its base. Find the height of the flagpole.
- 3 From a point 30 m from the base of a building, the angle of elevation of the top of the building is  $65^\circ$ . How high is the building?
- 4 When the angle of elevation of the sun is  $38^\circ$ , a tree casts a shadow 23 m long. Find the height of the tree.

### EXAMPLE 5E-4 Finding the horizontal distance using an angle of depression

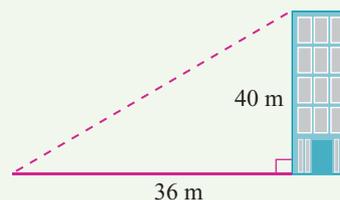
The angle of depression from the top of a cliff 84 m above sea level, to a ship, is  $22^\circ$ . Find the distance of the ship from the base of the cliff.

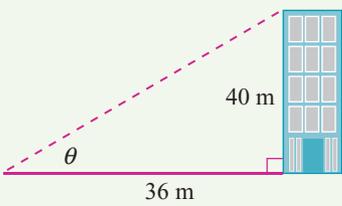
Solve	Think	Apply
<p>Draw a diagram and let the distance be <math>d</math>.</p>  <p>In the diagram, <math>x = 90^\circ - 22^\circ = 68^\circ</math>            By trigonometry: <math>\frac{d}{84} = \tan 68^\circ</math>  <math>\therefore d = 84 \times \tan 68^\circ</math>  <math>= 208 \text{ m (to nearest m)}</math></p>	<p><math>x + 22^\circ = 90^\circ</math>, as the cliff is vertical.            For the <math>68^\circ</math> angle, <math>d</math> is the opposite side and 84 m is the adjacent side. The trigonometric ratio that connects these two sides is the tangent ratio. Hence:  <math>\frac{d}{84} = \frac{\text{opposite}}{\text{adjacent}} = \tan 68^\circ</math></p>	<p>Draw a diagram. Mark the angle of depression.            Calculate the size of angle <math>x</math> in the triangle. Identify the sides as opposite, adjacent or hypotenuse, and determine the trigonometric ratio that connects them. Solve the resulting equation.</p>

- The angle of depression from the top of a cliff 76 m above sea level, to a ship is  $18^\circ$ . Find the distance of the ship from the base of the cliff, correct to the nearest metre.
- From the top of a building 96 m high, the angle of depression to a parked truck on the road below is  $33^\circ$ . How far, to the nearest metre, is the truck parked from the base of the building?
- From a plane approaching Sydney at a height of 3000 m above the ground, the angle of depression to the Harbour Bridge is  $15^\circ$ . Find the horizontal distance (i.e. the land distance) of the plane from the bridge. Give your answer to the nearest metre.

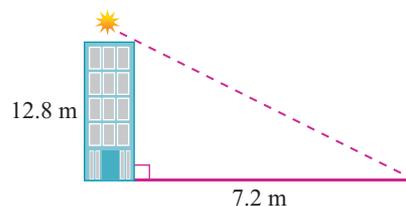
### EXAMPLE 5E-5 Using trigonometry to calculate the angle of elevation or depression

A building is 40 m high. Calculate the angle of elevation of the top of the building from a point that is 36 m from its base, correct to the nearest minute.

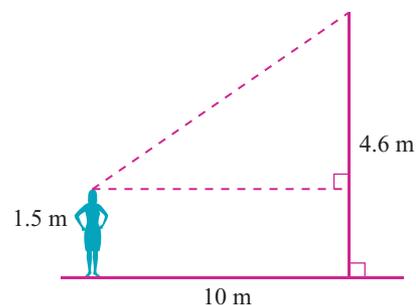


Solve	Think	Apply
 <p><math>\tan \theta = \frac{40}{36}</math>  <math>\theta \approx 48^\circ 1'</math></p>	<p>On the diagram, label the unknown angle as <math>\theta</math>.            For <math>\theta</math> in the right-angled triangle, 40 m is the length of the opposite side and 36 m is the length of the adjacent side.            Use <math>\tan \theta = \frac{\text{opposite}}{\text{adjacent}}</math>.</p>	<p>The angle of elevation of an object from an observer is the angle between the horizontal and the line of sight up to the object.</p>

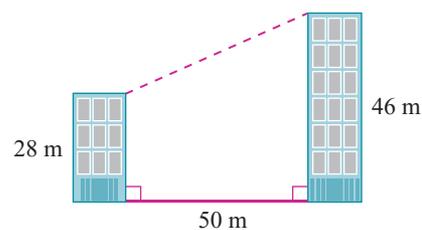
**8** A building 12.8 m high casts a shadow 7.2 m long. What is the angle of elevation of the Sun at this time?



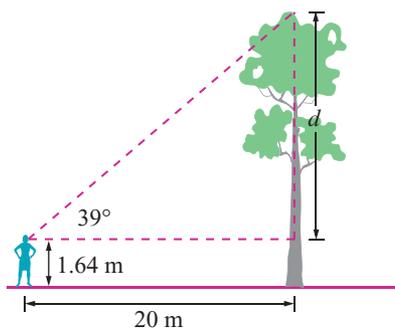
**9** A person 1.5 m tall is standing 10 m from the base of a statue that is 4.6 m high. Calculate the angle of elevation from the person to the top of the statue.



**10** Two buildings, 28 m and 46 m high respectively, are 50 m apart. Calculate the angle of elevation from the top of the shorter building to the top of the other.



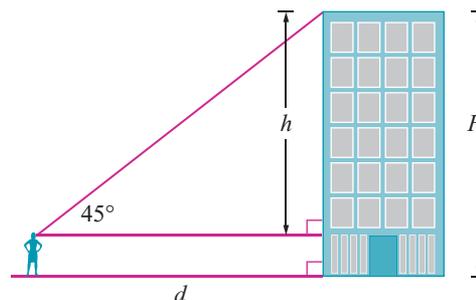
**11** From a point 20 m from the base of a tree, Sophia measures the angle of elevation of the top of the tree to be  $39^\circ$ . Her eye level is 1.64 m above the ground.



- a Calculate the distance  $d$  in the diagram.
- b Hence find the height of the tree.

**12** Using the method of question 11, find the height of some of the buildings, trees or the flagpole at your school.

- a Choose one of your school buildings that is on level ground.
- b Walk away from the base of the building and, by trial and error, find the point on the ground from which the angle of elevation to the top of the building is  $45^\circ$ .
- c Measure this distance, shown as  $d$  on the diagram.
- d Hence find the height  $h$  on the diagram.
- e Measure your height to eye level and calculate the height  $H$  of the building.



# 5F Bearings and navigational methods

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These resources are available on your obook assess:

- **Video tutorial 5F:** Watch and listen to an explanation of Example 5F-2
- **Interactive 5F:** Explore key ideas for compass bearings and true bearings
- **Worksheet 5F:** Practise your skills for compass bearings and true bearings
- **Investigation 5F:** Apply your skills with mapping your school grounds
- **Assess quiz 5F:** Test your skills with an auto-correcting multiple-choice quiz

## bearing

direction from one position to another, using the angles around a magnetic compass

## compass bearing

indicates direction as an angle measured from north or south towards east or west; e.g.  $N40^{\circ}W$  represents a bearing of  $40^{\circ}$  from north towards west and  $S75^{\circ}E$  represents a bearing of  $75^{\circ}$  from south towards east

## true bearing

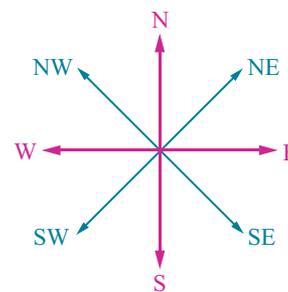
indicates direction as an angle measured from north in a clockwise direction and written with three digits; e.g. east is represented by  $090^{\circ}T$  and south-west is represented by  $225^{\circ}T$

Navigation involves determining position and direction. In the Mathematics Standard Year 11 course, you looked at describing position on Earth's surface with latitude and longitude. Modern navigational methods rely on receiving electronic information from satellites via receivers. The Global Positioning System (GPS) is currently the world's most used satellite navigation system.

Navigating a course for a ship or an aircraft also requires accurate directions. These directions are usually given in the form of a **bearing**. There are two types of bearings: a **compass bearing** and a **true bearing**.

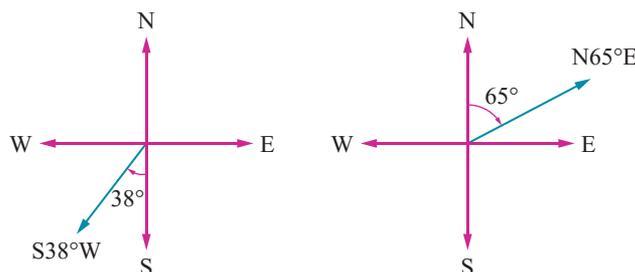
## Compass bearings

Compass bearings are stated using the compass rose. The compass rose has four major (or cardinal) points: north, east, south and west (N, E, S, W). The next four are in between these and are north-east, south-east, south-west and north-west (NE, SE, SW, NW). Note that north and south are written first, then east and west. This compass rose shows these eight compass points.



For directions between these eight compass points, we start with N or S, whichever is closer, and end with E or W. For example,  $S38^{\circ}W$  or  $N65^{\circ}E$ .

$S38^{\circ}W$  means face south, then turn  $38^{\circ}$  to west.  $N65^{\circ}E$  means face north, then turn  $65^{\circ}$  to east.

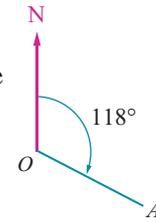
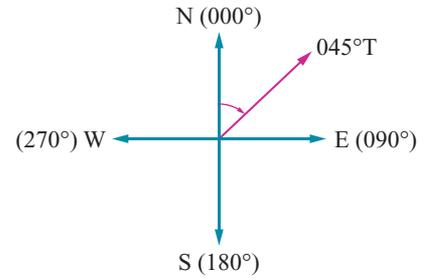


# True bearings

True bearings are measured as an angle of turn. The agreed convention is that the direction of travel is measured by a clockwise rotation from north and written using three digits.

For example, the diagram on the right shows us that the bearing of the direction north is  $000^\circ\text{T}$ , north-east is  $045^\circ\text{T}$ , east is  $090^\circ\text{T}$ , south is  $180^\circ\text{T}$  and west is  $270^\circ\text{T}$ . (*Note: the letter T indicates it is a true bearing.*)

In the diagram on the right, the bearing of  $A$  from  $O$  is the measure of the angle between the line  $OA$  and the line through  $O$  in the north direction. The bearing of  $A$  from  $O$  is written as  $118^\circ\text{T}$ .

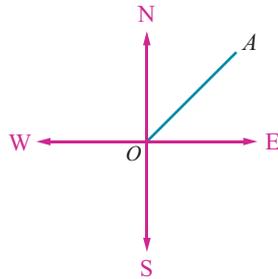


## EXERCISE 5F Bearings and navigational methods

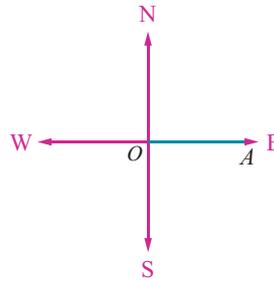
Unless stated otherwise, find all distances correct to one decimal place and all angles to the nearest degree.

1 Use one of the eight cardinal points to write the compass bearing of  $A$  from  $O$  for each diagram.

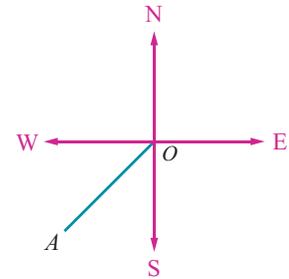
a



b

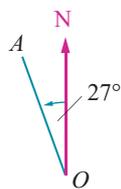


c

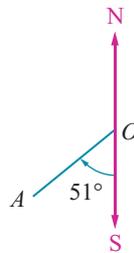


2 Write the compass bearing of  $A$  from  $O$  for each diagram.

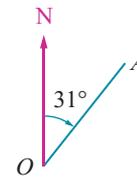
a



b

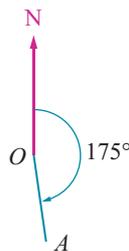


c

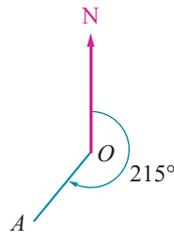


3 Write the true bearing of  $A$  from  $O$  for each diagram.

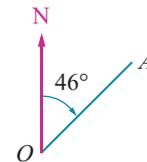
a



b



c



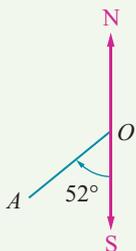
### EXAMPLE 5F-1 Writing compass bearings and true bearings

Write the bearing of  $A$  from  $O$  in the following diagrams as:

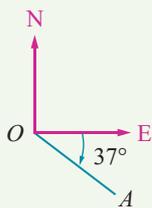
**i** a compass bearing

**ii** a true bearing.

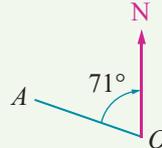
**a**

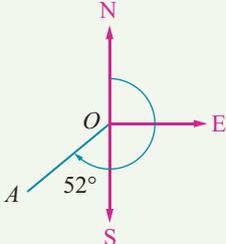
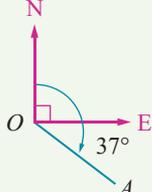
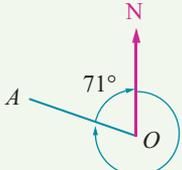


**b**



**c**



	Solve	Think
<b>a i</b>	Compass bearing of $A$ from $O$ is $S52^\circ W$ .	The direction from $O$ to $A$ is closer to south than north. Identify the angle between south and $OA$ . Write the bearing as the number of degrees from south towards west.
<b>ii</b>	$180^\circ + 52^\circ = 232^\circ$ True bearing of $A$ from $O$ is $232^\circ T$ .	The angle between north and south is $180^\circ$ . Add $180^\circ$ to $52^\circ$ to calculate the total number of degrees from north to $OA$ . 
<b>b i</b>	$90^\circ - 37^\circ = 53^\circ$ Compass bearing of $A$ from $O$ is $S53^\circ E$ .	The direction from $O$ to $A$ is closer to south than north. Calculate the angle between south and $OA$ . Write the bearing as the number of degrees from south towards east.
<b>ii</b>	$90^\circ + 37^\circ = 127^\circ$ True bearing of $A$ from $O$ is $127^\circ T$ .	The angle between north and east is $90^\circ$ . Add $90^\circ$ to $37^\circ$ to calculate the total number of degrees from north to $OA$ . 
<b>c i</b>	Compass bearing of $A$ from $O$ is $N71^\circ W$ .	The direction from $O$ to $A$ is closer to north than south. Identify the angle between north and $OA$ . Write the bearing as the number of degrees from north towards west.
<b>ii</b>	$360^\circ - 71^\circ = 289^\circ$ True bearing of $A$ from $O$ is $289^\circ T$ .	A full turn is $360^\circ$ . Subtract $71^\circ$ from $360^\circ$ to calculate the total number of degrees from north to $OA$ . 

#### Apply

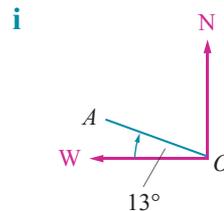
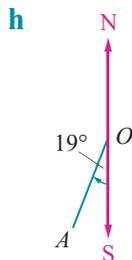
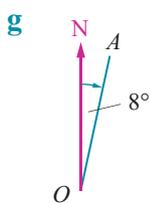
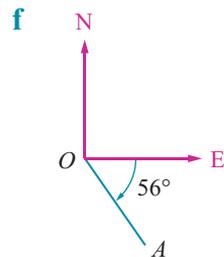
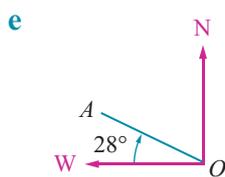
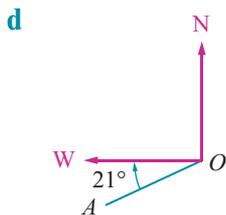
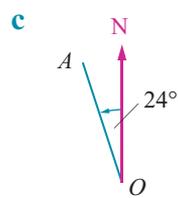
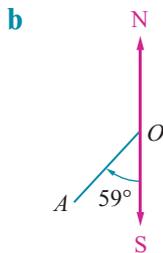
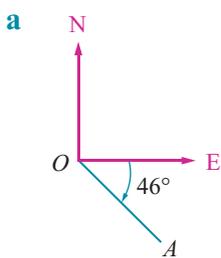
For the compass bearing, calculate the angle for  $OA$  from north or south, whichever is closer, towards east or west.

For the true bearing, calculate the amount of turn in a clockwise direction from north to  $OA$ .

4 For each diagram, write the bearing of  $A$  from  $O$  as:

i a compass bearing

ii a true bearing.



### EXAMPLE 5F-2 Finding the true bearing in the opposite direction

If the bearing of  $A$  from  $O$  is  $110^\circ\text{T}$ , what is the bearing of  $O$  from  $A$ ?

Solve	Think	Apply
<p> <math>\angle OAQ + \angle POA = 180^\circ</math>  <math>\angle OAQ + 110^\circ = 180^\circ</math>  <math>\angle OAQ = 70^\circ</math>                      Angle at <math>A</math>, clockwise from north to <math>AO</math>  <math>= 360^\circ - 70^\circ</math>  <math>= 290^\circ</math>                      Bearing of <math>O</math> from <math>A</math> is <math>290^\circ\text{T}</math>.                 </p>	<p>Draw a diagram showing a true bearing of <math>110^\circ</math> from <math>O</math> to <math>A</math>, labelling a point <math>P</math> on the north reference line. Construct another north reference line at <math>A</math> and label a point <math>Q</math>.</p> <p>Since <math>OP</math> is parallel to <math>AQ</math>, angles <math>OAQ</math> and <math>POA</math> are co-interior angles and add to <math>180^\circ</math>. Hence <math>\angle OAQ = 70^\circ</math>.</p> <p>Calculate the angle at <math>A</math>, clockwise from north to <math>AO</math>, to find the true bearing of <math>O</math> from <math>A</math>.</p>	<p>For parallel lines, co-interior angles add up to <math>180^\circ</math>. Angles at a point add up to <math>360^\circ</math>.</p>

5 Given these bearings of A from O, find the bearing of O from A.

a  $123^\circ\text{T}$

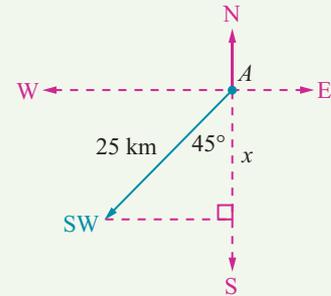
b  $047^\circ\text{T}$

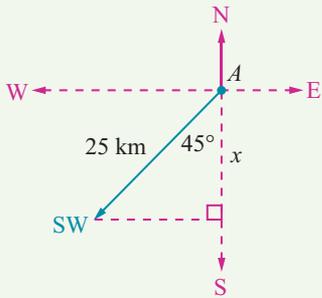
c  $231^\circ\text{T}$

d  $195^\circ\text{T}$

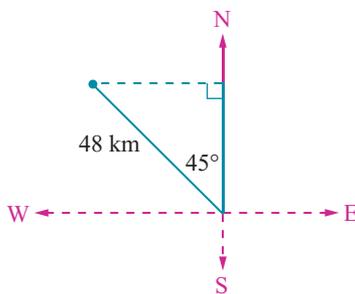
**EXAMPLE 5F-3** Using a compass bearing and trigonometry to calculate a distance

A ship sails for 25 km in a SW direction from a port A. Find the distance the ship is south of its starting point.

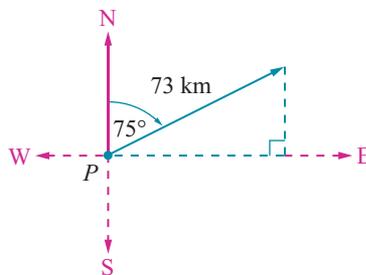


Solve	Think	Apply
 <p> <math>\cos 45^\circ = \frac{x}{25}</math>  <math>x = 25 \times \cos 45^\circ</math>  <math>= 17.677\dots</math>  <math>\approx 17.7 \text{ km}</math>                      The ship is about 17.7 km south of its starting position.                 </p>	<p>Form a right-angled triangle on the diagram. Label the side length representing the distance south with <math>x</math>. Label the angle of <math>45^\circ</math>.</p> <p>In the right-angled triangle, the length of the hypotenuse is 25 km and the unknown side length is on the adjacent side. Use the cosine ratio.</p>	<p>Form a right-angled triangle on the diagram so trigonometry can be used to find the unknown side length.</p>

6 A ship sails a distance of 48 km in a NW direction. Find how far north of its starting point it has travelled.

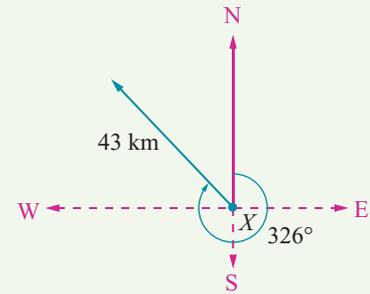


7 A ship sails 73 km from port P on a bearing of  $\text{N}75^\circ\text{E}$ . Find how far east the ship has sailed.



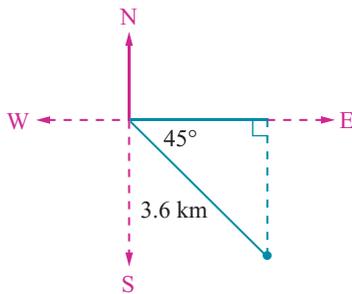
**EXAMPLE 5F-4** Using a true bearing and trigonometry to calculate a distance

A ship sails 43 km from port  $X$  on a true bearing of  $326^\circ$ . Find how far the ship is west of port  $X$ .

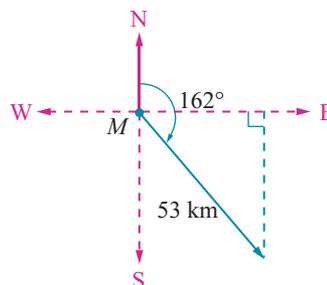


Solve	Think	Apply
<p> <math>\sin 34^\circ = \frac{x}{43}</math>  <math>x = 43 \times \sin 34^\circ</math>  <math>= 24.045\dots</math>  <math>\approx 24.0</math> km                      The ship is about 24.0 km west of port.                 </p>	<p>Form a right-angled triangle on the diagram. Label the side length representing the distance west with <math>x</math>. The angle inside the right-angled triangle is found by subtraction.</p> <p>Angle = <math>360^\circ - 326^\circ = 34^\circ</math></p> <p>The length of the hypotenuse is 43 km and the unknown side length is on the opposite side. Use the sine ratio.</p>	<p>Form a right-angled triangle on the diagram so trigonometry can be used to find the unknown side length.</p>

- 8** A cross-country skier skis 3.6 km on a true bearing of  $135^\circ$ . Find how far east she has travelled from her starting point.

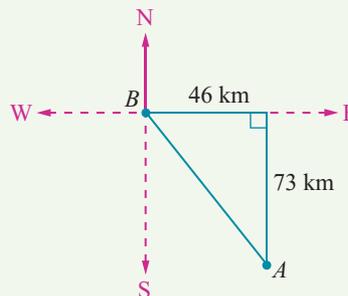


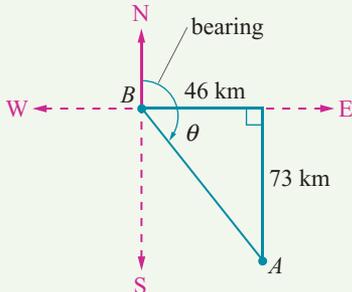
- 9** A ship sails 53 km from port  $M$  on a bearing of  $162^\circ$ . Find how far south the ship has sailed.



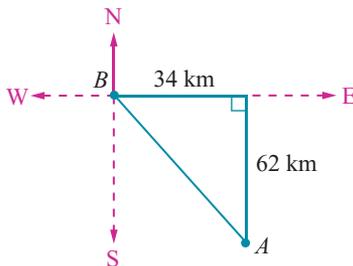
### EXAMPLE 5F-5 Using trigonometry to find a bearing

Town A is 46 km east and 73 km south of town B. Find the true bearing of A from B, to the nearest minute.

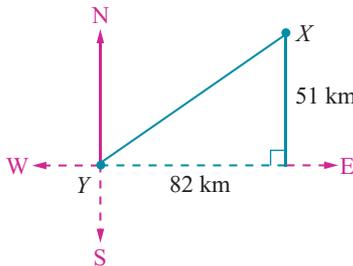


Solve	Think	Apply
 <p> <math>\tan \theta = \frac{73}{46}</math>  <math>\theta \approx 57^{\circ}47'</math>                      Bearing is  <math>90^{\circ} + 57^{\circ}47' = 147^{\circ}47'T.</math> </p>	<p>Indicate the angle to be found in the right-angled triangle, <math>\theta</math>, to help find the bearing. The opposite side to <math>\theta</math> has a length of 73 km and the adjacent side has a length of 46 km. Use the tangent ratio. Remember to add <math>90^{\circ}</math>, as the angle is between east and south.</p>	<p>Form a right-angled triangle and label the required angle to be found in the right-angled triangle. Use trigonometry to find the unknown angle and hence work out the bearing.</p>

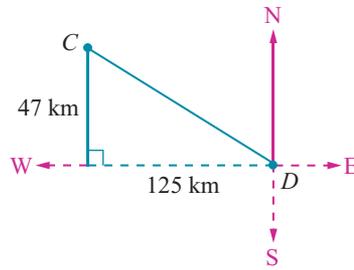
- 10** Town A is 34 km east and 62 km south of town B. Find the true bearing of A from B, to the nearest degree.



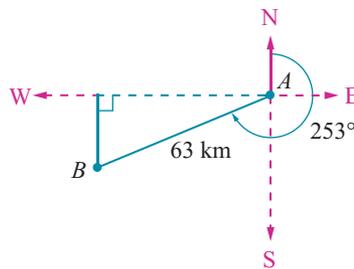
- 11** Town X is 82 km east and 51 km north of town Y. Find the true bearing of X from Y, to the nearest minute.



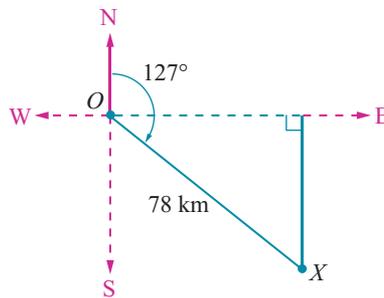
- 12** Town  $C$  is 125 km west and 47 km north of town  $D$ . Find the compass bearing of  $C$  from  $D$ .



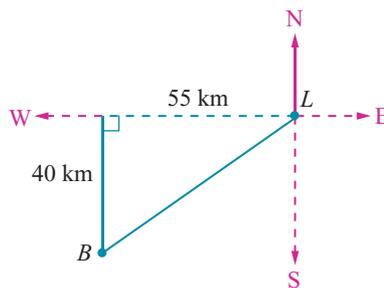
- 13** A ship sails 63 km from port  $A$  on a bearing of  $253^\circ\text{T}$ , to a buoy  $B$ . Find how far the ship is west and south of  $A$ .



- 14** A ship sails 78 km from port  $O$  on a bearing of  $127^\circ\text{T}$ . Find how far the ship is east and south of  $O$ .

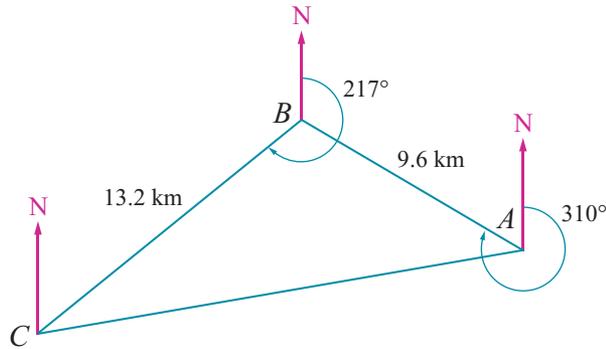


- 15** A ship sails from port  $L$  and travels 55 km west then 40 km south to a buoy  $B$ . Find the compass bearing of  $B$  from  $L$ .





- 20** A yacht sets out in a race with three legs as shown. The first leg is 9.6 km from  $A$  on a bearing of  $310^\circ\text{T}$  to  $B$ . The second leg from  $B$  to  $C$  is 13.2 km on a bearing of  $217^\circ\text{T}$ .



- a** How far is:
- i**  $B$  north of  $A$ ?
  - ii**  $C$  south of  $B$ ?
  - iii**  $C$  south of  $A$ ?
- b** How far is:
- i**  $B$  west of  $A$ ?
  - ii**  $C$  west of  $B$ ?
  - iii**  $C$  west of  $A$ ?
- c** Using the results from parts **a** and **b**, find:
- i** the length of the third leg from  $C$  to  $A$
  - ii** the true bearing from  $C$  to  $A$ , to the nearest minute.
- 21 a** Define *true north* and *magnetic north*. Explain the difference.
- b** Research the history of magnetic north and find the current location of magnetic north.
- c** Why is true north used on maps? How is magnetic north shown?
- 22** Find out more about modern navigational methods including the use of GPS.
- 23** Research and investigate the sport of orienteering. Write a short report on your findings.
- 24** Some cultures use different navigational methods. For example, since Aboriginal and Torres Strait Islander peoples did not have a written language, they committed their navigational information to memory in the form of songlines that could be passed to others. Songlines are oral maps of the landscape to enable trade routes to be followed over large distances. In many cases, oral maps of the landscape mirrored patterns of stars in the sky. These star maps were used to help memorise waypoints such as waterholes or turning places along the route.
- a** Find out more about songlines and how they are used as a navigational tool in the Aboriginal and Torres Strait Islander cultures.
  - b** Investigate how some modern Australian highways follow the path of Aboriginal songlines.
  - c** The concept of cardinal directions (north, south, east and west) is common among Aboriginal language groups in Australia. Find out more about this and how it can relate to sleeping position and burial traditions.
  - d** Investigate other navigational methods used by different cultures.



# CHAPTER 5 REVIEW RIGHT-ANGLED TRIANGLES

You should be able to:

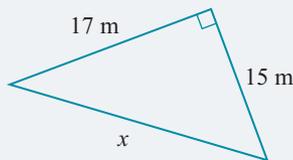
- ✓ use Pythagoras' theorem to find sides in right-angled triangles
- ✓ use Pythagoras' theorem to test if triangles are right-angled
- ✓ define the trigonometric ratios sine, cosine, tangent
- ✓ find angles of right-angled triangles using trigonometry
- ✓ find side lengths of right-angled triangles using trigonometry
- ✓ solve practical problems using trigonometry
- ✓ solve practical problems using angles of elevation and depression
- ✓ understand the difference between compass and true bearings
- ✓ use trigonometry with bearings to solve practical problems.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

## REVIEW MULTIPLE-CHOICE QUESTIONS

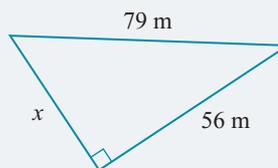
5A 1 The length of the unknown side in this triangle is:

- A 8 m
- B 23 m
- C 32 m
- D 514 m



5A 2 The length of the unknown side in this triangle is closest to:

- A 3105 m
- B 56 m
- C 97 m
- D 135 m

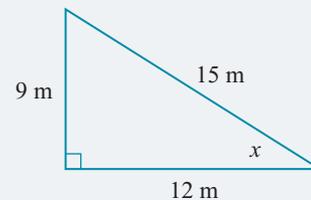


5B 3 The sine ratio is the ratio of:

- A  $\frac{\text{adjacent}}{\text{hypotenuse}}$
- B  $\frac{\text{opposite}}{\text{adjacent}}$
- C  $\frac{\text{adjacent}}{\text{opposite}}$
- D  $\frac{\text{opposite}}{\text{hypotenuse}}$

5B 4 In the triangle shown,  $\tan x =$

- A  $\frac{12}{9}$
- B  $\frac{9}{12}$
- C  $\frac{9}{15}$
- D  $\frac{12}{15}$

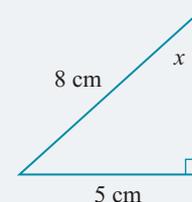


5B 5 Correct to two decimal places,  $\sin 58^\circ =$

- A 1.60
- B 0.53
- C 0.85
- D 1.18

5B 6 In this triangle,  $\frac{5}{8} =$

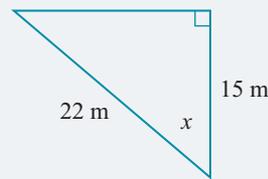
- A  $\sin x$
- B  $\cos x$
- C  $\tan x$
- D none of these



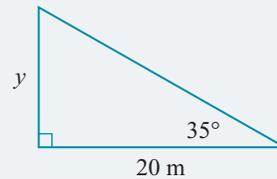
- 5C ➤ 7 If  $\cos x = \frac{3}{8}$ , then, to the nearest degree,  $x =$   
 A  $22^\circ$                       B  $68^\circ$

- C  $21^\circ$                       D  $69^\circ$

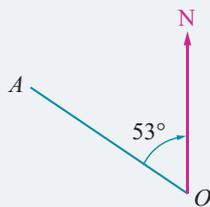
- 5C ➤ 8 In this triangle, to the nearest degree,  $x =$   
 A  $43^\circ$   
 B  $47^\circ$   
 C  $34^\circ$   
 D  $56^\circ$



- 5D ➤ 9 To the nearest 0.1 m, the unknown side  $y =$   
 A 11.5 m  
 B 16.4 m  
 C 14.0 m  
 D 24.4 m



Use this diagram to answer questions 10–12.



- 5E ➤ 10 What is the compass bearing of A from O in the diagram?

- A N53°E                      B N53°W                      C W53°N                      D N37°W

- 5E ➤ 11 What is the true bearing of A from O in the diagram?

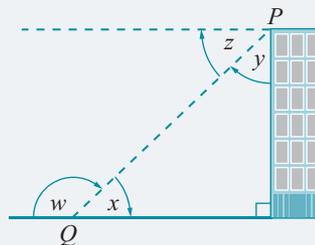
- A 053°T                      B 127°T                      C 307°T                      D 53°N

- 5E ➤ 12 A ship sails 57 km from O to A on the bearing shown in the diagram. How far north of its starting point is the ship?

- A 34.3 km                      B 45.5 km                      C 57.0 km                      D 75.6 km

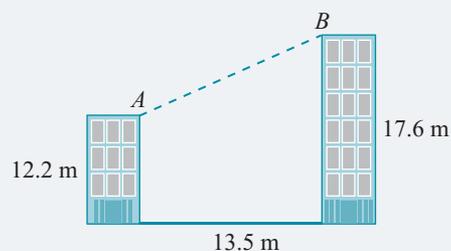
- 5F ➤ 13 Which angle is the angle of depression from P to Q?

- A  $w$                       B  $x$                       C  $y$                       D  $z$



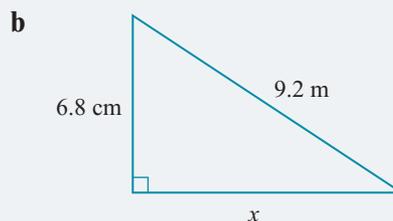
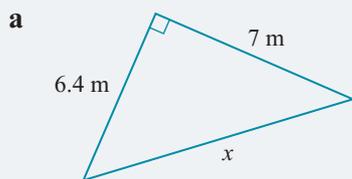
- 5F ➤ 14 What is the angle of elevation from A to B?

- A  $22^\circ$                       B  $24^\circ$   
 C  $53^\circ$                       D  $55^\circ$



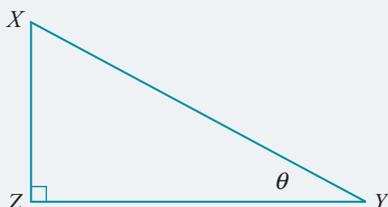
# REVIEW SET 1

1 Find the length of the unknown side in each triangle.



2 For the angle marked  $\theta$  in this triangle, name the:

- a opposite side                      b adjacent side.



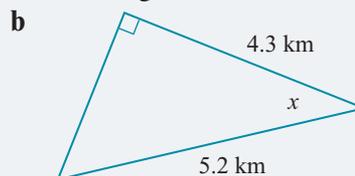
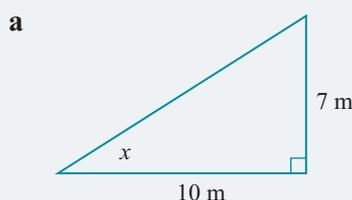
3 Find the following, correct to three decimal places:

- a  $\sin 56^\circ$                                       b  $\cos 18^\circ$                                       c  $\tan 75^\circ$

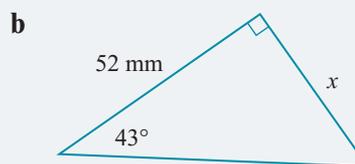
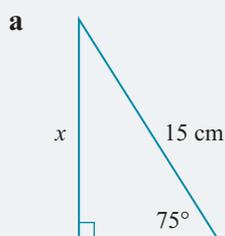
4 Find  $x$ , to the nearest degree, given:

- a  $\sin x = 0.314$                                       b  $\cos x = 0.426$                                       c  $\tan x = 1.12$

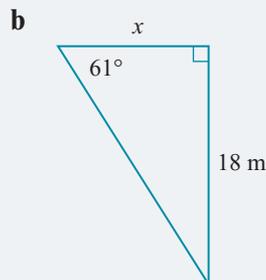
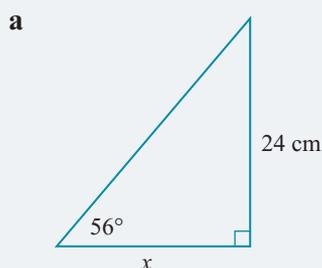
5 Find angle  $x$  in the triangles shown. Give the answer to the nearest degree.



6 Find the length of the unknown side in each triangle (correct to one decimal place).



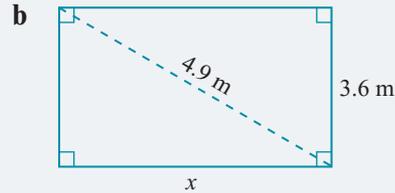
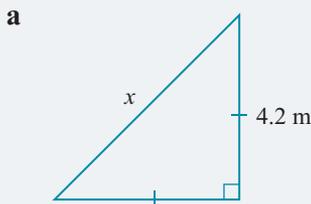
7 Find the length of the unknown side in each triangle (correct to one decimal place).



- 8 a A ship sails 78 km from port  $O$  on a bearing of  $148^\circ$ T. How far east has the ship sailed to the nearest km?  
 b Town  $A$  is 43 km east and 74 km south of town  $B$ . Find the compass bearing and the true bearing of  $A$  from  $B$ .

# REVIEW SET 2

1 Find the value of  $x$  in the following triangles.



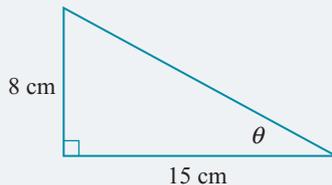
2 For this triangle,  $\frac{8}{15} =$

A  $\sin \theta$

B  $\cos \theta$

C  $\tan \theta$

D none of these



3 Find  $x$  to the nearest degree, given that:

**a**  $\sin x = 0.92$

**b**  $\cos x = 0.145$

**c**  $\tan x = 0.5$

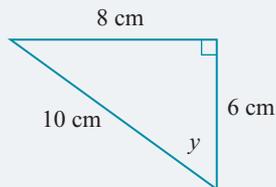
4 In this triangle,  $\cos y =$

A  $\frac{8}{10}$

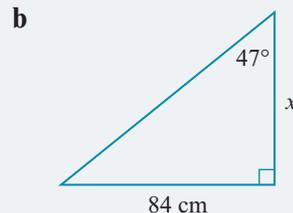
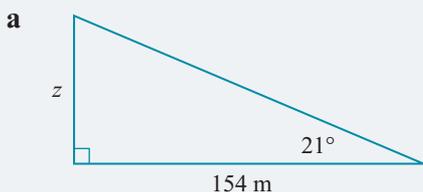
B  $\frac{6}{8}$

C  $\frac{6}{10}$

D  $\frac{8}{6}$



5 Find the length of the unknown side in the following triangles.



6 The angle of elevation of the top of a building, from a point 50 m from its base, is  $58^\circ$ . Find the height of the building.

7 **a** A ship sails 78 km from port on a bearing of  $236^\circ\text{T}$ . How far west has the ship sailed to the nearest metre?

**b** Town  $P$  is 48 km west and 94 km south of town  $Q$ . Find the compass bearing and the true bearing of  $P$  from  $Q$ .

# REVIEW SET 3

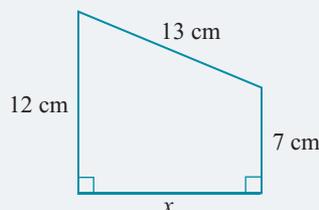
1 In the diagram shown,  $x =$

A 3 cm

B 12 cm

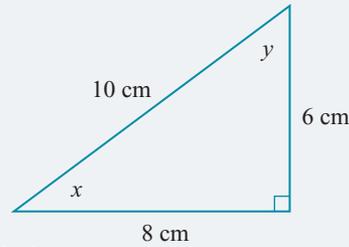
C 5 cm

D 1 cm



2 For the triangle shown, which statement is true?

- A  $\cos x < \sin x$
- B  $\sin x < \sin y$
- C  $\tan y < \tan x$
- D  $\cos y > \tan y$



3 Find the value of the following, correct to four decimal places.

- a  $\sin 54^\circ$
- b  $\cos 18^\circ$
- c  $\tan 76^\circ$

4 Find  $x$ , to the nearest degree, if:

- a  $\sin x = \frac{3}{5}$
- b  $\cos x = \frac{7}{8}$
- c  $\tan x = \frac{5}{12}$

5 A tree casts a shadow 5.5 m long when the angle of elevation of the sun is  $43^\circ$ . What is the height of the tree?

6 The foot of a 6 m long ladder is placed 2.5 m from the base of a wall. What angle does the ladder make with the ground, assuming the ground is horizontal?

7 From the top of a lighthouse 53.2 m above sea level, the angle of depression to a ship at sea is  $6^\circ$ . What is the distance of the ship from the base of the lighthouse?

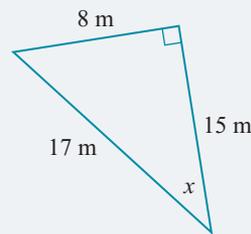
8 a A ship sails 236 km from port on a bearing of  $196^\circ$ T. How far south has the ship sailed to the nearest km?

b Town  $T$  is 93 km west and 73 km north of town  $S$ . Find the compass bearing and the true bearing of  $T$  from  $S$ .

## REVIEW SET 4

1 In the triangle shown, what is the ratio of:

- a  $\sin x$ ?
- b  $\cos x$ ?
- c  $\tan x$ ?

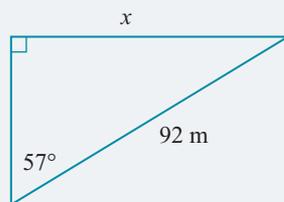


2 Find  $x$ , to the nearest degree, if:

- a  $\sin x = 0.512$
- b  $\cos x = \frac{7}{9}$
- c  $\tan x = \frac{1.2}{0.86}$

3 In the triangle shown,  $x =$

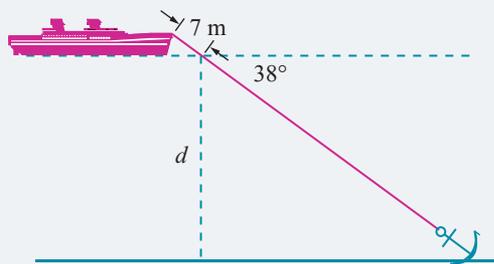
- A  $92 \sin 57^\circ$
- B  $92 \tan 57^\circ$
- C  $92 \cos 57^\circ$
- D  $\frac{92}{\tan 57^\circ}$



4 A ramp that is 5.6 m long rises 1.2 m vertically to an entrance. What angle does the ramp make with the ground, assuming the ground is horizontal?

5 A pilot flying at an altitude of 1500 m, notes that the angle of depression to a bridge is  $25^\circ$ . What is the horizontal distance (distance across land) of the aeroplane from the bridge?

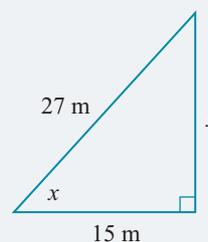
- 6** A boat drops its anchor. Its anchor rope is 72 m long. When the rope is inclined at an angle of  $38^\circ$  to sea level, 7 m of the rope is above the water. Calculate the depth of the water.



- 7 a** A ship sails 112 km from port on a bearing of  $153^\circ\text{T}$ . How far east has the ship sailed to the nearest km?  
**b** Town  $R$  is 125 km east and 87 km north of town  $S$ . Find the compass bearing and the true bearing of  $R$  from  $S$ .

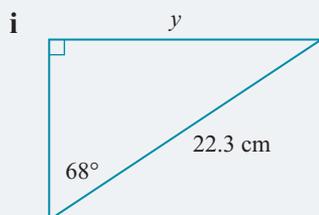
## REVIEW PRACTICE EXAMINATION QUESTION

- a i** Find the size of the unknown angle, to the nearest degree.  
**ii** Find the length of the unknown side, to one decimal place.

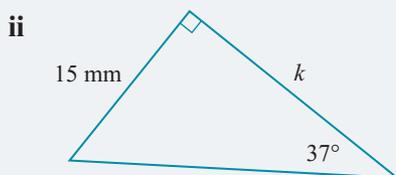


(2 marks)  
(2 marks)

- b** Calculate the size of the unknown sides in the following triangles, correct to one decimal place.



(2 marks)



(2 marks)

- c** From the top of a vertical cliff 48 m high, the angle of depression to a boat at sea is  $60^\circ$ .

- i** Draw a clear diagram to illustrate this information. (1 mark)  
**ii** Calculate the distance of the boat from the base of the cliff. (3 marks)

- d** After 3 hours of driving, a family is 185 km from home on a true bearing of  $217^\circ$ .

- i** State the compass bearing of their location from home. (1 mark)  
**ii** How far north or south are they from home, correct to the nearest km? (2 marks)

**TOTAL:**  
15 marks

# 6

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## Statistical analysis

The main mathematical ideas investigated are:

- ▶ identifying the target population for a statistical investigation
- ▶ avoiding bias in sampling methods and survey questions
- ▶ understanding the statistical investigation process
- ▶ analysing data collected from a statistical investigation
- ▶ comparing data across two or more groups
- ▶ illustrating bivariate data on a scatterplot
- ▶ identifying independent and dependent variables within bivariate datasets
- ▶ describing the correlation between bivariate datasets displayed in a scatterplot
- ▶ drawing a line of best fit by eye
- ▶ adding the trendline to a scatterplot using technology
- ▶ using interpolation and extrapolation to make predictions.

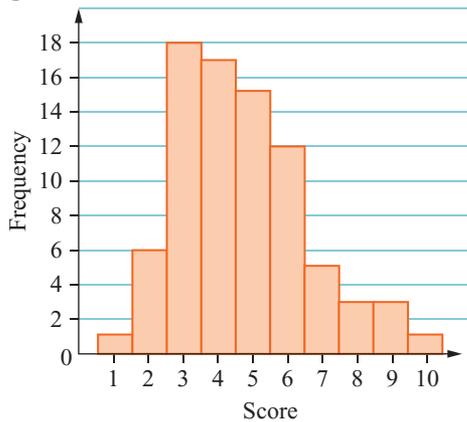
# ARE YOU READY?

Use the following dataset to answer questions 1 to 5.

4, 7, 9, 15, 8, 11, 7, 12, 13, 18, 7, 5, 14, 12, 8

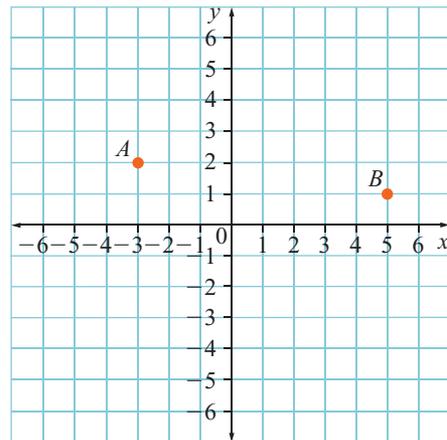
- 6B** 1 What is the mean of the dataset?  
**A** 7      **B** 9      **C** 10      **D** 11
- 6B** 2 What is the median of the dataset?  
**A** 7      **B** 9      **C** 10      **D** 11
- 6B** 3 What is the mode of the dataset?  
**A** 7      **B** 9      **C** 10      **D** 11
- 6B** 4 What is the range of the dataset?  
**A** 4      **B** 6      **C** 14      **D** 18
- 6B** 5 What is the interquartile range of the dataset?  
**A** 4      **B** 6      **C** 14      **D** 18

Use the frequency histogram below to answer questions 6 to 8.



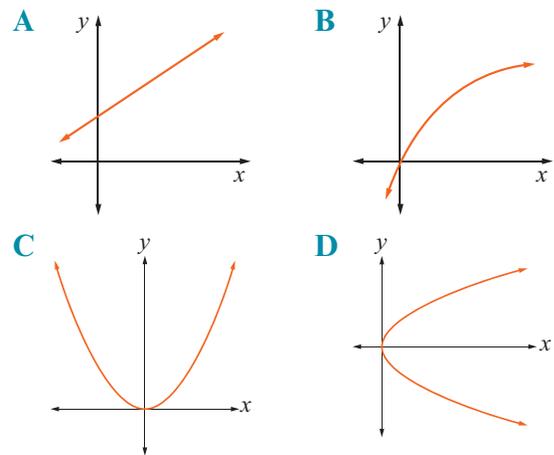
- 6B** 6 What is the mean of the dataset shown in the frequency histogram?  
**A** 3      **B** 4      **C** 4.7      **D** 9
- 6B** 7 What is the median of the dataset shown in the frequency histogram?  
**A** 3      **B** 4      **C** 4.7      **D** 9
- 6B** 8 What is the mode of the dataset shown in the frequency histogram?  
**A** 3      **B** 4      **C** 4.7      **D** 9

- 6A** 9 What are the coordinates of points *A* and *B* respectively in this graph?



- A** (2, 3) and (5, 1)      **B** (-2, 3) and (1, 5)  
**C** (2, -3) and (5, 1)      **D** (-3, 2) and (5, 1)

- 6A** 10 Which of the following graphs shows a linear relationship between the two variables?



- 6D** 11 Given the equation  $y = 3x - 5$ , when  $x = 3$ , what is the value of  $y$ ?  
**A** 28      **B** 4      **C** -4      **D** 3
- 6D** 12 Given the equation  $y = -2x + 7$ , when  $y = 11$ , what is the value of  $x$ ?  
**A** -15      **B** -9      **C** 2      **D** -2

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your obook assess.

- Q1-3** Support sheet 6B.1 Mean, median and mode for data in a list  
**Q4-5** Support sheet 6B.2 The range and the interquartile range  
**Q6-8** Support sheet 6B.3 Frequency histograms  
**Q9** Support sheet 6C.1 The Cartesian plane (four quadrants)  
**Q10** Support sheet 6C.2 Plotting relationships  
**Q11-12** Support sheet 6F.1 Substituting into and solving simple equations

# 6A Statistical investigations

These resources are available on your obook assess:

- **Video tutorial 6A:** Watch and listen to an explanation of Example 6A-1
- **Worksheet 6A:** Practise your skills with extra questions for understanding bias
- **assess quiz 6A:** Test your skills with an auto-correcting multiple-choice quiz

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## data

pieces of information that can be interpreted and analysed

Statistics refers to the science of collecting facts and information. The information is organised into a form that can easily be analysed and from which conclusions and predictions can accurately be made. The facts or pieces of information collected are called **data**. Data may be collected by counting, measuring and asking questions.

Statistics can be used to answer a wide range of real-life questions such as the following:

- Does Sydney receive more sunshine than Melbourne?
- Which drug is most effective in the treatment of the common cold?
- How does time on social media affect the concentration of teenagers?
- Does the location of your school have an impact on your HSC results?

The statistical investigation process can be used to answer these statistical questions. It involves 4 distinct steps:

- 1 Identifying your question and planning your investigation
- 2 Collecting the data
- 3 Representing and analysing the data
- 4 Interpreting and communicating the results

The results from a statistical investigation should only ever be used to interpret information about the population that is used in the study. For example, if you are investigating the hip to waist ratio in women, the conclusions you draw will probably not be valid for men.



Statistical questioning can be done by interview, either in person or by phone, or by questionnaire. The usefulness of the responses to the questions asked will depend on the relevance, clarity and structure of the questions. Questions should use simple language, be unambiguous so that there can be no doubt about their meaning, and they should be free from bias, that is, free from unfair influence.

Here are some examples of question types that could be used to start an investigation into school uniform, with a discussion of some of the advantages and disadvantages of each type.

Type of question	Example	Advantage/disadvantage
Free-response/open-ended	What do you like/dislike about the present school uniform?	This question would possibly collect a lot of information, but, because there would be many different responses, it might be difficult to organise and interpret the responses.
Yes or No	Would you like to change the present school uniform?	The responses to this question will indicate whether there is a desire to change the uniform, but will not indicate what change, if any, is wanted. (This of course could be done by asking further questions.)
True or False	The present school uniform is satisfactory. True or false?	As for the Yes and No questions, the responses will not indicate the changes wanted.
Tick the box	Which of the following items of school uniform would you like to see changed? <ul style="list-style-type: none"> <li>• shirt</li> <li>• trousers</li> <li>• shoes</li> <li>• pullover</li> </ul>	The responses to this question will give information about what particular changes are wanted.
Response scale	Circle the response that most closely agrees with your opinion of the present school uniform. <ol style="list-style-type: none"> <li><b>1</b> strongly like</li> <li><b>2</b> like</li> <li><b>3</b> no opinion</li> <li><b>4</b> dislike</li> <li><b>5</b> strongly dislike</li> </ol>	The responses to this question will give a clearer indication of the depth of feeling concerning a change, but will not indicate what change, if any, is wanted.

*Note:* when using response scales, some thought should be given to the number of choices. An even number of choices may force an opinion one way or another, in particular circumstances, but sometimes it may be desirable to allow a neutral choice by having an odd number of choices.

When writing questionnaires, the questions should be simple and straightforward so that there is only one interpretation possible. They should show respect for the privacy of individuals and should not involve any calculations. In general, the simpler the question and the more precise the response required, the more accurate the data will be. It is a good idea to trial your questions on a few people to check that the questions are achieving the purpose for which you are designing them.

**Ethics** must also be considered when conducting a statistical investigation. Be aware of any cultural sensitivities and act accordingly. Statistical investigations should be conducted objectively and honestly. You may not always find the outcome that you are either looking for, or expecting to find.

**ethics**  
moral principles that guide the way that we should act in a given situation

**EXAMPLE 6A-1** Identifying questions that can be answered using statistics

Which of these questions can be answered using statistics?

- a** What is the distance between Sydney and Newcastle?
- b** Do students who receive a high mark in Mathematics Standard earn more in their career than students who receive a low mark?
- c** Do teenagers eat more than adults?
- d** What is the average finishing time in the Hobart to Sydney Yacht Race?
- e** Which team scored the least number of tries in the NRL last season?

	Solve	Think	Apply
<b>a</b>	No	The answer to this question can be looked up online.	For a question to be answered using statistics it needs to either involve statistical calculations (such as calculating the mean), or a statistical investigation.  Questions that can be answered without statistical calculations, e.g. by looking up the information, do not need to be answered using statistics.
<b>b</b>	Yes	This question would be best answered by conducting a statistical investigation.	
<b>c</b>	Yes	This question would be best answered by conducting a statistical investigation.	
<b>d</b>	Yes	The answer to this question would involve calculating the mean or median of finishing times.	
<b>e</b>	No	The answer to this question can be looked up online.	

**EXERCISE 6A** Statistical investigations

- 1** Which of these questions can be answered using statistics?
  - a** How many more wickets did Shane Warne take in Test cricket than Glenn McGrath?
  - b** Are houses more expensive in Sydney or London?
  - c** Do tall people earn more than short people?
  - d** How hot was the warmest day in Broken Hill last year?
  - e** How tall is the tallest building in Sydney?
  - f** What is the average height of a skyscraper in Sydney?
  - g** How much does it cost to fly from the Central Coast to Perth?
  - h** Who is the most famous movie star in the world?



One of the first decisions to be made when collecting data is from whom, or what, the information is to be collected. The target **population** is the entire group about whom or from which information is sought. For instance, if we were interested in collecting information about the cleanliness of trains, the target population would be all commuters who use rail transport. If we wanted to determine the most popular colour for new cars in one particular year, the target population would be all new cars sold in that year.

There are two ways of collecting the information: a **census** or a **sample**.

**population**

all of the potential pieces of data that can be acquired

**census**

a collection of the entire population data

**sample**

a collection of a selection of the population data

A census involves collecting data about every individual in the whole population. The individuals may be people or objects. A census is detailed and accurate but is expensive, time consuming and often impractical.

A sample involves collecting data about a portion of the population. It is cheaper and quicker than a census but is not as detailed or as accurate. Conclusions drawn from samples always involve some degree of error.

- 2** What would be the target population if we wanted to:
- a** collect information about the pocket money given to school children?
  - b** collect information about the incomes of home owners?
  - c** investigate the success of a new treatment for the common cold?
  - d** investigate corruption in professional sports?



**EXAMPLE 6A-2** Identifying whether a census or sample should be used

Would a census or a sample be used to investigate:

- a** the length of time a portable hard drive will last?
- b** the causes of knee injuries in professional rugby league?
- c** the number of people who use Apple iPhones?

	Solve	Think	Apply
<b>a</b>	Sample	It is impractical to test every portable hard drive, as there would be none left for sale!	A sample collects data about a subset of the population.
<b>b</b>	Census	An accurate analysis of all knee injuries would be required.	A census collects information about every accident.
<b>c</b>	Sample	It would be very time consuming to interview the whole population to find out who uses, or does not use, iPhones.	A sample collects data about a subset of the population.

- 3** State whether a census or a sample would be used for each of these investigations.
- a** the most popular television channel
  - b** the number of pets in Australian families
  - c** the most popular pet of Australian students
  - d** the number of chocolate bars bought each week by Australian teenagers
  - e** the number of hours of exercise Australians get each week
  - f** the amount of rain in a week
  - g** the favourite holiday destinations of Australians
  - h** the favourite sport of students in a class
  - i** the number of lemons a lemon tree produces
  - j** the gender of lawyers
  - k** the population of Sydney suburbs
  - l** the weight of African elephants

The most common way of collecting information is by using a sample. The purpose of a sample is to provide an estimate of a particular characteristic of the whole population. Therefore, the challenge in selecting a sample is to make it as free from prejudice (**bias**) as possible and large enough to be representative of the whole population.

Bias occurs when one outcome is favoured above other outcomes. Bias can be either intentional or unintentional, and can also occur in the collection of data. Leading questions, which lead the respondent towards a particular response, can introduce bias into an investigation.

**bias**  
a prejudice in favour of one particular group or outcome

**EXAMPLE 6A-3** Identifying potential bias in sampling methods

Identify why the following sampling methods may result in a biased statistical investigation:

- a** Investigating the religious beliefs of Australians by surveying people at shopping malls on Sunday mornings
- b** Conducting a survey of the views of Australians about the current Prime Minister by interviewing people from only one electorate.

	Solve/Think	Think	Apply
<b>a</b>	Many religious people attend church on Sunday mornings, so these people will likely be under-represented at shopping malls at this time.		Identify the population that you are investigating. Does the proposed method take a random sample of that population, or are some people more likely to be included or excluded than others?
<b>b</b>	Political opinions vary across the country according to a range of factors (urban/rural, income, etc.), so one electorate will not provide a representative sample of the population.		

- 4** Identify why the following sampling methods may result in a biased statistical investigation.
- a** Conducting an investigation on the shopping habits of 17-year-olds by interviewing students from schools in inner-Sydney
  - b** Researching favourite types of music by interviewing people outside an Adele concert
  - c** Finding opinions about the new cycle lanes by interviewing members of the local cycling club
  - d** Looking at the state of the economy by interviewing investment bankers
  - e** Researching the favourite films of teenagers by conducting interviews at Comic-Con

- 5** Melissa wants to investigate the wages of university graduates. If Melissa only interviewed graduates with business degrees, why would her data not be suitable for estimating the wage of a 24-year-old science graduate?

**EXAMPLE 6A-4** Identifying potential bias in survey questions

Explain why the following survey questions may result in a biased statistical investigation:

- a** Do you agree that lowering the company tax rate will be good for society?
- b** Polls show that 90% of customers prefer the new Strawberry Dream frozen yoghurt range. What do you think?



	Solve/Think	Apply
<b>a</b>	By asking 'do you agree', the likelihood that the respondent will agree with the survey question is increased.	Leading questions lead the respondent to answer in a particular way. Unbiased questions should not lead the respondent in any way.
<b>b</b>	By explaining that the majority of people prefer the new frozen yoghurt range, this question is leading people to answer that they also prefer it.	

- 6** Explain why the following survey questions may result in a biased statistical investigation.
- a** Taking into account the budget deficit, should the government cut military expenditure?
  - b** You don't support the proposal for the new railway line, do you?
  - c** The Avengers was voted the best film of the 21st Century. What's your favourite film of the 21st Century?
  - d** I can't understand how the new road laws were passed. What is your opinion of them?
  - e** Soccer is the most played sport by Australian teenagers. What's your favourite sport to play?

- 7** Ryan was researching the health issues that staff at a large company had over the past 10 years. He kept his research in an unlocked spreadsheet, which detailed the names of the staff, as well as their personal information, such as their salary. Explain why there may be a privacy issue with Ryan's handling of the data.

- 8** Discuss any problems in collecting the data from question 3 with the class.

- 9** Use the internet to investigate ways that bias can be reduced or eliminated in both sampling methods and in the wording of survey questions.



# 6B Analysing and comparing data

These resources are available on your obook assess:

- **Worksheet 6B.1:** Practise your skills with extra questions for summary statistics
- **Worksheet 6B.2:** Practise your skills with extra questions for measures of centre and spread
- **assess quiz 6B:** Test your skills with an auto-correcting multiple-choice quiz

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When data from a statistical investigation has been collected, it can be analysed and conclusions can be drawn about the population. Conclusions from data collected from a census will be accurate for the whole population, whereas conclusions drawn from a sample will contain a degree of uncertainty.

## EXAMPLE 6B-1 Using summary statistics to analyse data

Justin collected information on the number of people living in each household in his street. The data he collected was as follows.

3, 4, 3, 6, 2, 3, 4, 4, 1, 4, 3, 2, 5, 4, 5, 2, 5, 3, 4, 1, 3, 6, 4, 3, 4, 4, 4, 6, 4, 2

- Rearrange this information into a frequency distribution table.
- What is the mean number of people living in each household?
- What is the modal number of people living in each household?
- What is the range of the number of people living in each household?



Solve			Think	Apply	
a	<b>Number of people</b>	<b>Tally</b>	<b>Frequency</b>	Work from left to right, use tally marks to indicate each data point. Add the tally marks and record the total in the frequency column.	The total of the tally marks for each category gives the frequency for that category. Check that the total of the frequency column is the same as the number of observations recorded.
	1		2		
	2		4		
	3		7		
	4		11		
	5		3		
	6		3		
		<b>Total</b>	<b>30</b>		

	Solve			Think	Apply
<b>b</b>	<b>Number of people (<math>x</math>)</b>	<b>Frequency (<math>f</math>)</b>	<b><math>f \times x</math></b>	Find the sub-totals of each category by using an $f \times x$ column. Then find the sum of this column, and divide by the sum of the frequency column.	Divide the sum of the $f \times x$ column by the sum of the frequency column to determine the mean.
	1	2	2		
	2	4	8		
	3	7	21		
	4	11	44		
	5	3	15		
	6	3	18		
		$\Sigma f = 30$	$\Sigma fx = 108$		
	$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ $= \frac{108}{30}$ $= 3.6$				
<b>c</b>	Mode = 4			From the frequency table, the category with the highest frequency is 4.	The mode is the score with the highest frequency.
<b>d</b>	Range = $6 - 1$ = 5			Subtract the lowest score from the highest score.	The range is the difference between the highest and lowest scores.

## Exercise 6B Analysing and comparing data

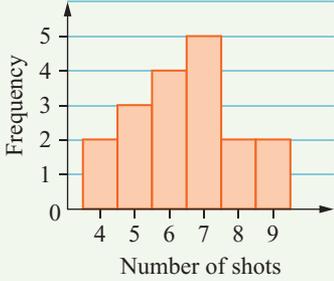
- 1** Francesca did a survey on the amount of pets students in her year at school have. The data she collected was as follows.  
0, 2, 4, 2, 0, 3, 1, 0, 0, 2, 1, 1, 5, 2, 2, 0, 1, 8, 0, 2, 3, 2, 1, 1, 0, 4, 2, 1, 3, 4
- Rearrange this information into a frequency distribution table.
  - What is the mean number of pets?
  - What is the modal number of pets?
  - What is the range of the number of pets?
- 2** Josh recorded the number of cars that went past his house in two-minute intervals over an hour.  
6, 6, 8, 9, 5, 7, 4, 6, 11, 9, 7, 7, 8, 5, 6, 12, 10, 6, 8, 10, 5, 4, 5, 8, 3, 9, 7, 6, 10, 9
- Rearrange this information into a frequency distribution table.
  - What is the mean number of cars passing Josh's house each two minutes?
  - What is the modal number of cars passing Josh's house each two minutes?
  - What is the range of the number of cars passing Josh's house each two minutes?

### EXAMPLE 6B-2 Analysing data from a frequency table

Jodie recorded the amount of shots she took in an 18-hole round of golf. The following frequency table details the scores she recorded.

Number of shots	Frequency
4	2
5	3
6	4
7	5
8	2
9	2
<b>Total</b>	<b>18</b>

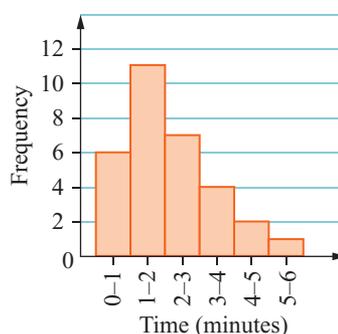
- Represent the data in a histogram.
- What is the median number of shots taken on a hole?
- What is the interquartile range for the number of shots taken on a hole?

	Solve	Think	Apply
<b>a</b>		<p>Choose a suitable scale. Put number of shots on the horizontal axis and frequency on the vertical axis. Draw the columns the same width, and centred above the temperatures they represent. The width of each column is the distance between successive scores on the horizontal axis. The height of each column is the frequency of each score.</p>	<p>A frequency histogram is a column graph with frequency on the vertical axis. The height of each column represents the frequency of each score.</p>
<b>b</b>	<p>As <math>n = 18</math>, the median is the average of the 9th and 10th ordered data values.</p> <p>The 9th data value is 6 and the 10th data value is 7.</p> $\text{So median} = \frac{6 + 7}{2}$ $= 6.5$	<p>There are 18 data points, so the median will be the <math>\frac{18 + 1}{2} = 9.5</math>th data value. This means that the median is the number midway between the 9th and 10th data values.</p> <p>The 9th value is 6 and the 10th value is 7, so the median is <math>\frac{6 + 7}{2} = 6.5</math>.</p>	<p>The median is the middle score in an ordered dataset.</p>
<b>c</b>	$\text{IQR} = Q_3 - Q_1$ $= 7 - 5$ $= 2$	<p>The median divides the data into two parts with 9 scores in each.</p> <p>The lower quartile (<math>Q_1</math>) is the median of the first 9 scores: the 5th score = 5.</p> <p>The upper quartile (<math>Q_3</math>) is the median of the second 9 scores: the 14th score = 7.</p> <p>The interquartile range is found by subtracting the first lower quartile from the upper quartile.</p> $\text{IQR} = Q_3 - Q_1$ $= 7 - 5$ $= 2$	<p>The median divides the data into two groups with equal numbers in each. The middle score of the lower group is the lower quartile (<math>Q_1</math>). The middle score of the upper group is the upper quartile (<math>Q_3</math>). The interquartile range (IQR) = <math>Q_3 - Q_1</math></p>

- 3** Stephen recorded the amount of shots he took in an 18-hole round of golf. The following frequency table details the scores he recorded.

Number of shots	Frequency
4	5
5	5
6	4
7	1
8	2
9	1
<b>Total</b>	<b>18</b>

- a** Represent the data in a histogram.  
**b** What is the median number of shots taken on a hole?  
**c** What is the interquartile range for the number of shots taken on a hole?  
**d** What was Stephen's score for his round of golf? *Hint*: find the sum of the data values.
- 4** Megan was training for a 10 km run, and recorded the time it took her to run 8 km over a period of 3 months. She rounded her results to the nearest minute. The results were as follows.  
 55, 54, 54, 53, 52, 50, 50, 49, 48, 48, 48, 47, 48, 47, 46, 46, 45, 44, 44
- a** Represent the data in a frequency table.  
**b** Represent the data in a histogram.  
**c** What is the range of the times of her runs?  
**d** What is the interquartile range of the times of her runs?
- 5** The following histogram shows the amount of time it took a restaurant to seat customers in the evening.



- a** What is the modal class for the time taken to seat customers?  
**b** What is the range for the time taken to seat customers?  
**c** What was the longest wait customers had to endure before being seated?

Statistical investigations will often produce datasets across more than one group, for example males and females, or for different sports teams. When the data has been collected, we can compare the data across these groups and draw statistical conclusions about similarities and differences between them.

**EXAMPLE 6B-3** Comparing data across two groups

Mr Strict wanted to compare the results that males and females in his class achieved in their latest Mathematics test. The ordered results were as follows:

Males: 33, 36, 40, 41, 48, 53, 53, 57, 62, 68, 69, 72, 75, 81, 88

Females: 42, 46, 49, 50, 55, 58, 63, 66, 66, 69, 73, 76, 78, 80, 82

- What is the mean of the male and female results?
- What is the median of the male and female results?
- Compare how the two different groups performed in the test.

	Solve	Think	Apply
<b>a</b>	Males: $\bar{x} = \frac{\sum x}{n}$ $= \frac{876}{15}$ $= 58.4$ Females: $\bar{x} = \frac{\sum x}{n}$ $= \frac{953}{15}$ $= 63.5 \text{ (to 1 decimal place)}$	For each group, divide the sum of the scores by the number of data points to calculate the mean.	Use the formula $\bar{x} = \frac{\sum x}{n}$ to calculate the mean for each group.
<b>b</b>	Males: 57 Females: 66	There are 15 data points, so the median will be the $\frac{15+1}{2} = 8$ th data value for each group.	The median is the middle score in an ordered dataset.
<b>c</b>	Even though a male achieved the highest individual score (88), overall females performed better in the test than males.	The females achieved the higher mean and median scores, although the highest individual score (88) was achieved by a male.	Consider the values calculated in parts <b>a</b> and <b>b</b> , as well as the highest and lowest value in each group.

- Mrs Kind wanted to compare the results that the males and females in her class achieved in their latest science test. The ordered results were as follows:

Males: 42, 46, 53, 55, 58, 60, 64, 66, 69, 69, 72, 75, 77, 81

Females: 39, 44, 48, 56, 62, 62, 64, 67, 72, 74, 74, 79, 83

  - What is the mean of the male and female results?
  - What is the median of the male and female results?
  - Compare how the two different groups performed in the test.
- A statistical investigation was undertaken to compare the income (in \$'000s) of full-time workers in their 20s and in their 50s. The following data was recorded:

20s: 48, 39, 80, 54, 62, 44, 37, 46, 112, 65, 49, 56, 42, 38, 50

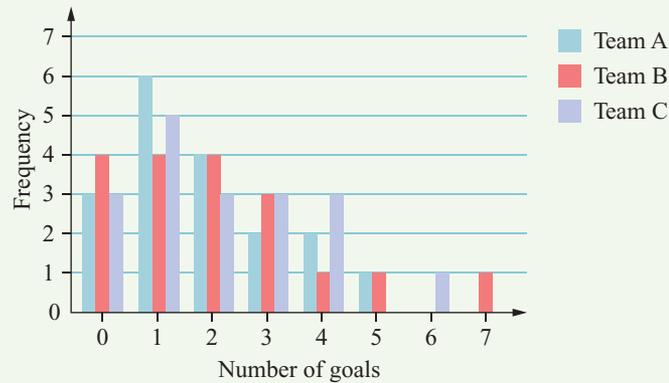
50s: 72, 55, 69, 125, 80, 48, 56, 105, 77, 91, 49, 63, 44, 84, 56

  - What is the mean of each group?
  - What is the range of each group?
  - What is the interquartile range of each group?
  - Compare the incomes of each group.

We can use data representations to help us to compare datasets containing two or more groups. Multiple bar graphs are bar graphs which split each category into two or more groups. Back-to-back histograms are usually displayed horizontally and share the same scale, which is placed in the centre of the two histograms.

### EXAMPLE 6B-4 Interpreting information from a multiple bar graph

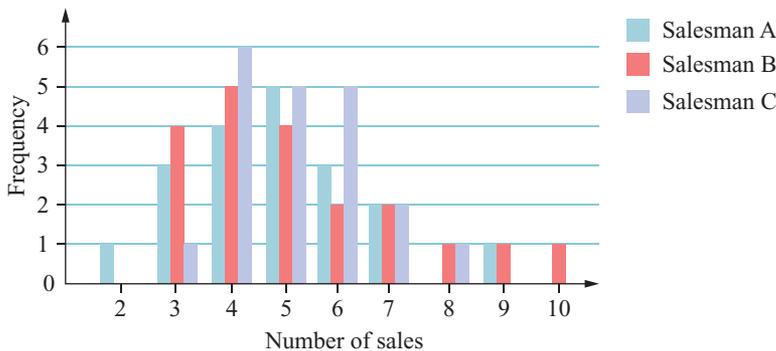
The following multiple bar graph shows the number of goals three soccer teams scored in games this season.



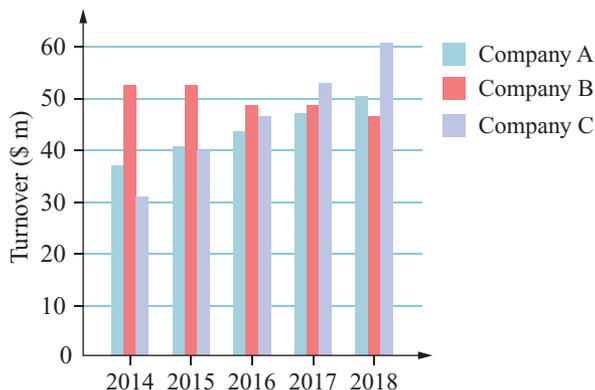
- Which team scored the most goals over the season?
- Which team scored the most goals in any game?
- Compare how the three teams performed in terms of scoring over the season.

	Solve	Think/Apply
<b>a</b>	<p>Team A:  <math>0 \times 3 + 1 \times 6 + 2 \times 4 + 3 \times 2 + 4 \times 2 + 5 \times 1</math>  <math>= 0 + 6 + 8 + 6 + 8 + 5</math>  <math>= 33</math></p> <p>Team B:  <math>0 \times 4 + 1 \times 4 + 2 \times 4 + 3 \times 3 + 4 \times 1 + 5 \times 1 + 7 \times 1</math>  <math>= 0 + 4 + 8 + 9 + 4 + 5 + 7</math>  <math>= 37</math></p> <p>Team C:  <math>0 \times 3 + 1 \times 5 + 2 \times 3 + 3 \times 3 + 4 \times 3 + 6 \times 1</math>  <math>= 0 + 5 + 6 + 9 + 12 + 6</math>  <math>= 38</math></p> <p>Team C scored the most goals over the season.</p>	<p>Calculate the total goals for each team by multiplying the number of goals in a game by the frequency, and finding the sum of these totals.</p>
<b>b</b>	<p>Team B, with 7 goals in one game</p>	<p>Identify the team which has a bar in the furthest right-hand category.</p>
<b>c</b>	<p>Teams B and C both scored a similar number of goals (38 and 37), with Team B scoring the highest number of goals in a game (7). Both teams performed significantly better than Team A, who only scored 33 goals in the season.</p>	<p>Consider the values calculated in parts <b>a</b> and <b>b</b>.</p>

- 8** The following multiple bar graph shows the number of cars three salesmen sold over a 20-day period.
- Which salesman sold the most cars over the 20-day period?
  - Which salesman sold the least cars on any day?
  - Compare how the three salesmen performed over the 20-day period.



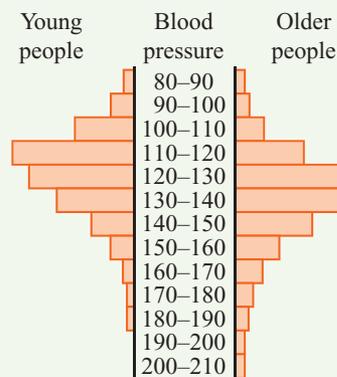
- 9** The following multiple bar graph shows the annual turnover of three companies over a 5-year period.
- Which company had the highest turnover in 2014?
  - Which company had the highest turnover in 2017?
  - Compare how the three companies performed over the 5-year period.



**EXAMPLE 6B-5** Interpreting information from a back-to-back histogram

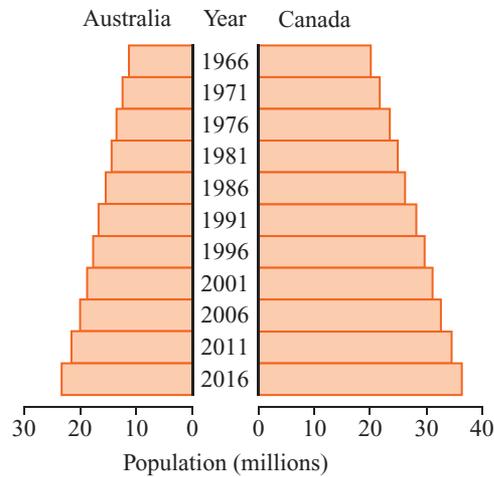
The following back-to-back histogram shows the systolic blood pressure of young people (aged 20–24) and older people (aged 60–64).

- Which age group had higher blood pressure overall?
- Which age group had the highest range of blood pressure?
- Compare the blood pressures of young and old people.



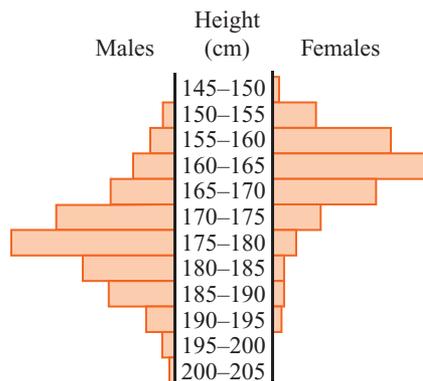
	Solve	Think/Apply
<b>a</b>	The older people (aged 60–64)	Look at the shape of the back-to-back histograms. The histogram with longer bars for the higher blood pressure categories is the older people.
<b>b</b>	The older people (aged 60–64)	Look at the bars at the top and bottom of the back-to-back histograms. The older people histogram has values in the first and last category, so has the highest range.
<b>c</b>	Overall younger people have lower blood pressure than older people, although there is a significant crossover in the histograms. Most young people have systolic blood pressure between 100 and 140, and most old people have systolic blood pressure between 110 and 150.	Look at the shape of the back-to-back histograms, and identify where the modal blood pressure is, as well as other high frequency categories.

**10** The following back-to-back histogram shows the populations of Australia and Canada from 1966 to 2016.



- What was the approximate population of Australia in 1976?
- What was the approximate population of Canada in 1991?
- Compare the population of the two countries over the 50-year period.

**11** The following back-to-back histogram shows the heights of Australian adult males and females.



- What is the modal height of Australian adult males?
- What is the modal height of Australian adult females?
- Compare the height of Australian adult males and females.

**12** Why do conclusions which are drawn from a sample contain a degree of uncertainty?

**13** Research the amount of tries that three rugby league teams scored in home and away games in the most recently completed NRL season. Draw a multiple bar graph to display your data, and write a paragraph to compare the difference between the amount of tries scored in home and away games.

**14** Complete a statistical investigation about the heights of Year 12 students in your school or college. Decide whether you will be conducting a census or a sample and collect the relevant data. Draw a back-to-back histogram to display your results, and write a conclusion about your findings.

# 6C Bivariate scatterplots

These resources are available on your obook assess:

- **Worksheet 6C:** Practice your skills with independent and dependent variables
- **assess quiz 6C:** Test your skills with an auto-correcting multiple-choice quiz

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The aim of many statistical investigations is to determine whether there is a relationship between the two variables being investigated. For instance, medical researchers might be interested in the relationship, if any, between the amount of a drug administered and the number of patients cured. A business enterprise might be interested in the relationship, if any, between the amount of money spent on advertising and the change in sales.

In this topic we will investigate a way of illustrating data so that a relationship, if it does exist, can be seen. A simple method of illustrating numerical data that relates two variables is to plot the data as ordered pairs on a number plane. The resulting diagram is known as a **scatterplot**.

In general, if the points are scattered at random over the grid, as in Graph A below, the variables are not mathematically related. If the points are scattered along a straight line, as in Graphs B and C, there may be a mathematical relationship between the two variables. The strength of this relationship, or how closely linked the variables are, is called **correlation**. Correlation will be investigated in more detail in topic 6D.

## scatterplot

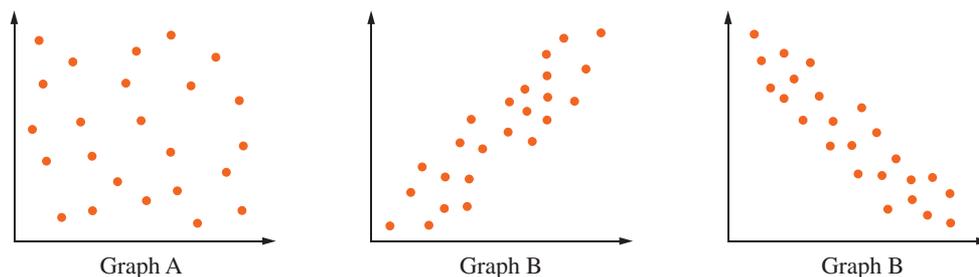
a diagram using Cartesian coordinates to display data values that relate two variables

## correlation

a measure of the extent to which two variables are related

## bivariate datasets

a dataset involving two different variables for each data point



When dealing with **bivariate datasets**, we are often working with an independent variable and a dependent variable. We can investigate how the dependent variable changes in response to the independent variable; the expected value of the dependent variable *depends* on the value of the independent variable.

When plotting a bivariate dataset with an independent and dependent variable, we put the independent variable on the horizontal axis and the dependent variable on the vertical axis.

If both variables in a bivariate dataset are independent, then either variable can be put on the horizontal and vertical axes.

## EXAMPLE 6C-1 Illustrating data on a scatterplot

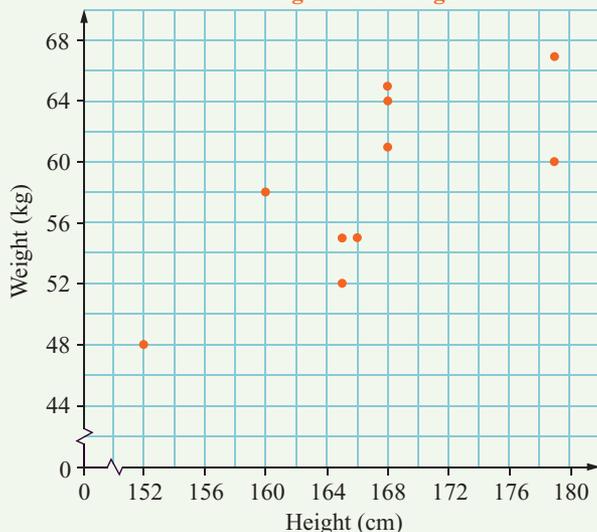
The heights and weights of 10 students were measured and the results shown in the table below.

Student	1	2	3	4	5	6	7	8	9	10
Height (cm)	179	165	160	179	152	168	168	165	168	166
Weight (kg)	60	55	58	67	48	64	61	52	65	55

Illustrate this data on a scatterplot and determine whether there is a possible relationship between the two variables.

### Solve

Weight versus height



From the distribution of points plotted, there appears to be a trend that as height increases so does weight. This might indicate a relationship between these two variables, but because of the scatter of the points, there does not appear to be a strong link. There would not appear to be a mathematical relationship that would allow the weight of a student to be accurately predicted from his or her height.

### Think/Apply

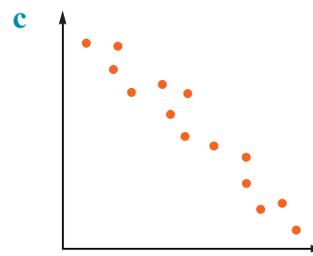
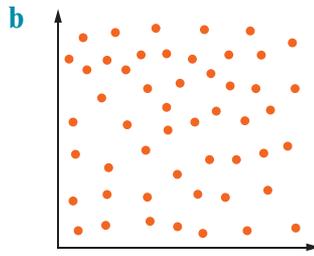
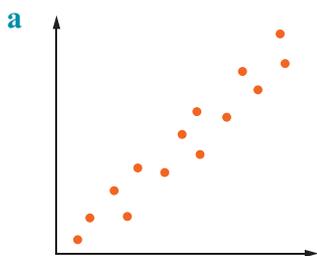
The data points are plotted as ordered pairs with height on the horizontal axis and weight on the vertical axis.

For student 1, go across to 179 on the horizontal axis then up to 60 on the vertical axis and plot the point. Repeat for the other students.

## EXERCISE 6C Bivariate scatterplots

UNDERSTANDING, FLUENCY AND COMMUNICATING

- 1 State whether or not there appears to be a linear relationship between the variables plotted on these scatterplots.



- 2 **i** Draw scatterplots for the data in the tables.  
**ii** Comment on any possible linear relationship between the variables.

**a**

$x$	40	50	60	70	80	90
$y$	42	47	51	59	62	72

**b**

$x$	10	15	20	25	30	35	40
$y$	12	17	19	24	29	31	37

**c**

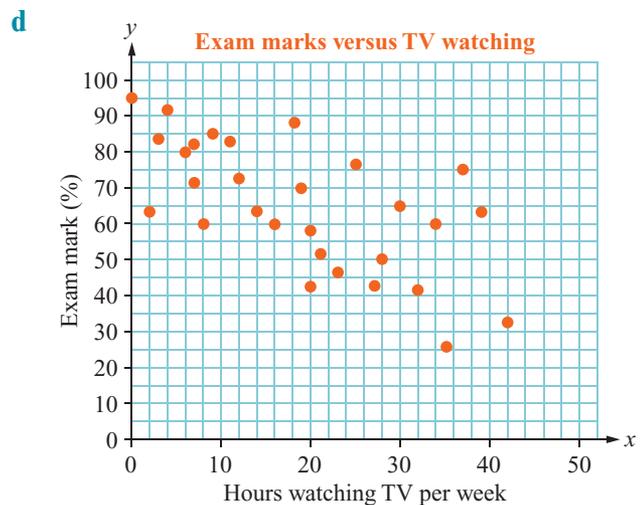
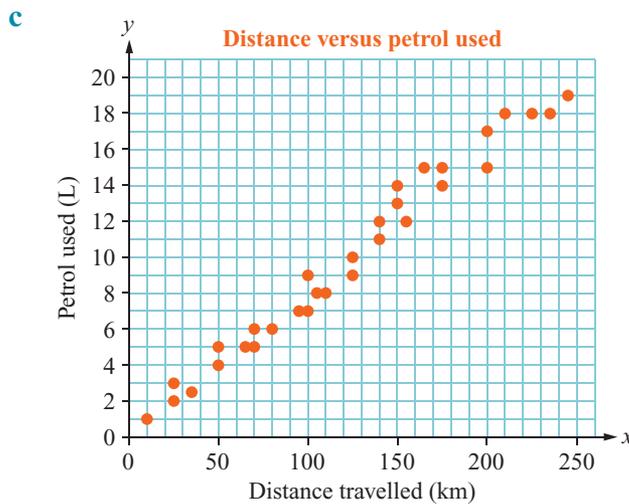
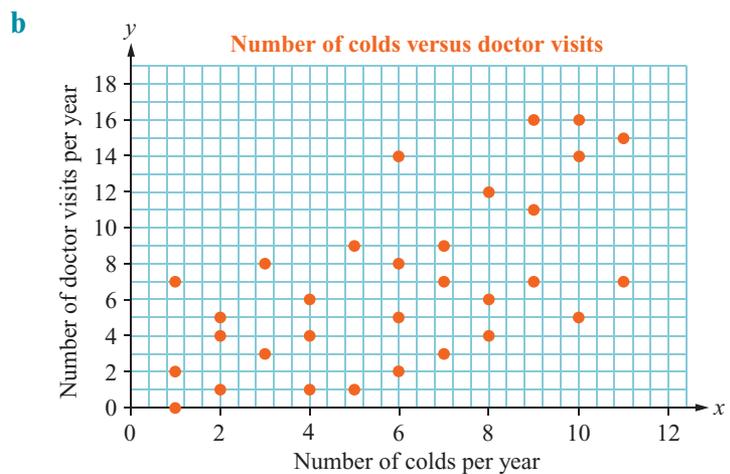
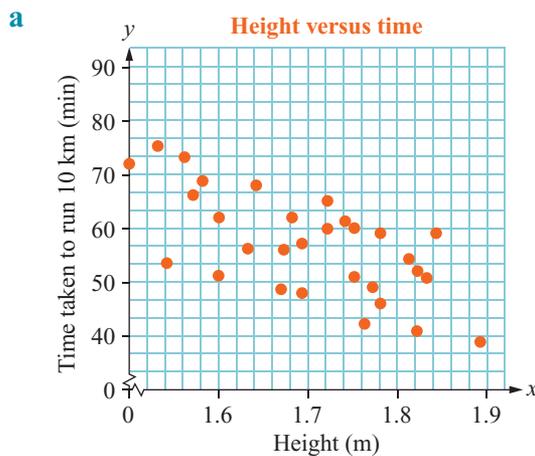
$x$	30	40	45	50	60	70	80	90
$y$	45	35	85	50	50	40	90	60

### EXAMPLE 6C-2 Identifying the independent and dependent variables

Cathy wants to investigate if there is a relationship between a person's height and their shoe size. Identify the independent variable and the dependent variable in this situation.

Solve	Think	Apply
Independent variable = height Dependent variable = shoe size	A person's shoe size is likely to vary depending on the height of the person, so shoe size is the dependent variable and height is the independent variable.	Identify which of the two variables is likely to change depending on the value of the other variable. This is the dependent variable.

3 Identify the independent variable and the dependent variable in these bivariate scatterplots.



- 4 Mika wants to investigate how the amount of sugar in a person's diet affects the size of their waistline. Identify the independent and dependent variables in this situation.
- 5 Jelena is looking into the relationship between an NBL player's height and the average number of points they score per game. Identify the independent and dependent variables in this situation.

An **outlier** is a data point that appears to be an anomaly in a dataset. When you identify an outlier it makes sense to interrogate it. Some outliers are genuine; for example, one student may score significantly higher in a mathematics test than all of the other students in their class. Other outliers may be the result of a recording or input error. Unless there is a strong reason to believe they are the result of an error, outliers should not be removed from a dataset.



**outlier**  
a data point that appears to be inconsistent with the other data points in a dataset

- 6 The results of a group of students in History and Geography tests are compared.

Student	1	2	3	4	5	6	7	8
History test ( <i>H</i> )	84	53	63	74	68	79	70	61
Geography test ( <i>G</i> )	80	62	75	88	70	54	82	70

- a Illustrate the given data on a scatterplot.  
 b Which student's results appear to be an outlier?  
 c Is there any reason to believe that this outlier may be the result of an error? Explain your answer.
- 7 In your own words explain the difference between an independent variable and a dependent variable.
- 8 Javi is using the Australian Bureau of Statistics website to write a report on the life expectancy of females in NSW over time. He collects the following data. The time is measured in years since 1971.

Year	1971	1976	1981	1986	1991	1996	2001	2006	2011
Time (years)	0	5	10	15	20	25	30	35	40
Life expectancy	74.4	76.2	78.3	78.8	80.2	81.2	82.6	83.7	84.2

- a Identify the independent and dependent variables in this situation.  
 b Draw a scatterplot for this data.  
 c Does there seem to be a linear relationship between the two variables?

A spreadsheet can be used to easily construct a scatterplot by following these steps.

- Step 1:* Put the data values into the spreadsheet, with the independent variable in the first column and the dependent variable in the second column.  
*Step 2:* Highlight all of the data values in the spreadsheet, including the two column headings.  
*Step 3:* From the Insert menu select X-Y Scatter Chart type.

- 9 The data in the table gives the life expectancy,  $L$ , of Australian males for the years 1881 to 2001. The time,  $T$ , is measured in years since 1881.

Year	1881	1901	1921	1947	1961	1981	2001
Time (years)	0	20	40	60	80	100	120
Life expectancy (years)	47.2	55.2	59.2	66.1	67.9	71.4	77.4

- a Draw a scatterplot comparing  $T$  and  $L$ .  
 b Does there seem to be a linear relationship between the two variables? If so, explain the relationship.

# 6D Correlation

These resources are available on your obook assess:

- **Video tutorial 6D:** Watch and listen to an explanation of Example 6D–1
- **Worksheet 6D:** Practise your skills with extra problems for scatterplots
- **assess quiz 6D:** Test your skills with an auto-correcting multiple-choice quiz

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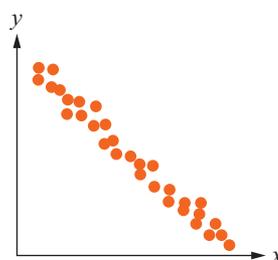
We can describe a bivariate dataset in terms of form, which means determining whether the relationship between the two variables is linear or non-linear.

If the relationship between the two variables is linear, then, in general, as the value of one variable increases, the value of the other variable will increase or decrease at a constant rate.

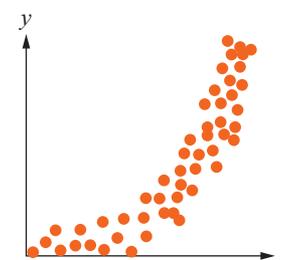
If a relationship between two variables is linear, then we can further describe it in terms of direction (positive and negative) and strength (strong, moderate or weak).

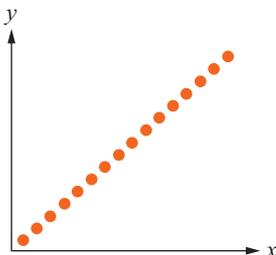
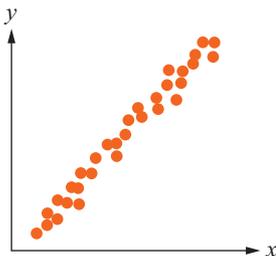
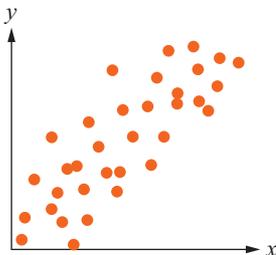
A positive relationship means that as one variable increases, the other variable increases, whereas a negative relationship means that as one variable increases, the other variable decreases.

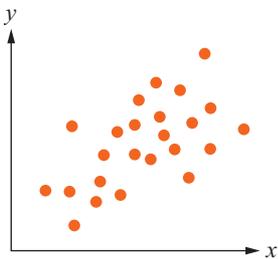
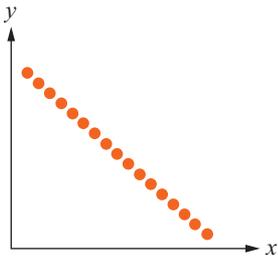
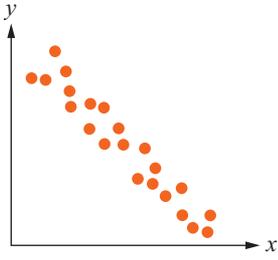
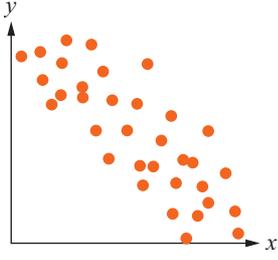
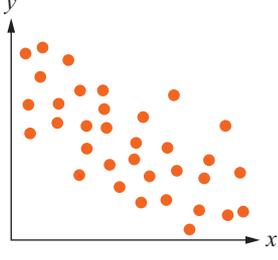
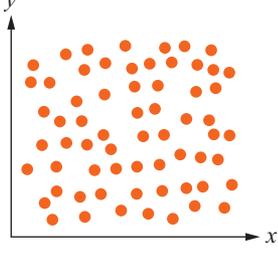
**Linear relationship**



**Non-linear relationship**

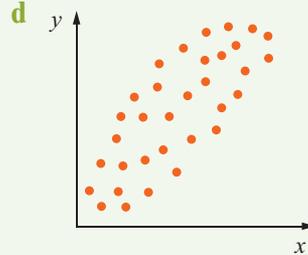
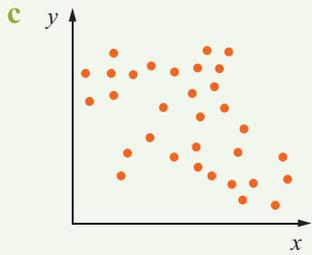
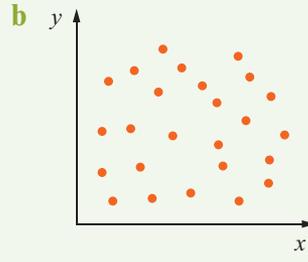
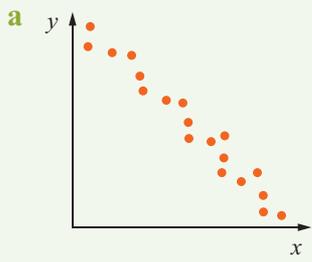


Type of correlation	Example scatterplot	Description
Perfect positive correlation		All of the points lie in a straight line with an upwards trend.
Strong positive correlation		The points approximate a straight line with an upwards trend.
Moderate positive correlation		The points are moving in an upwards trend, but are some way off a straight line.

Type of correlation	Example scatterplot	Description
Weak positive correlation		The points are moving in an upwards trend, but do not resemble a straight line.
Perfect negative correlation		All of the pointers lie with a downwards trend.
Strong negative correlation		The points approximate a straight line with a downwards trend.
Moderate negative correlation		The points are moving in a downwards trend, but are some way off a straight line.
Weak negative correlation		The points are moving in a downwards trend, but do not resemble a straight line.
No correlation		There is no noticeable positive or negative trend to the points.

### EXAMPLE 6D-1 Determining the strength of correlation via a scatterplot

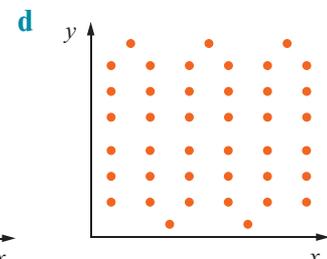
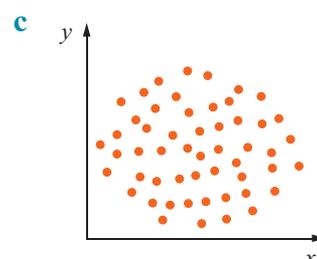
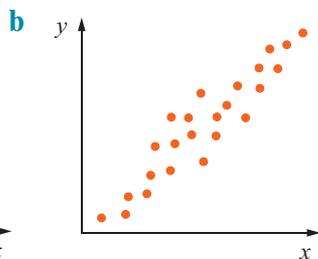
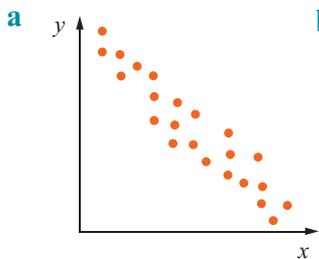
State whether the pairs of variables graphed below have perfect, strong, moderate, weak or no correlation.



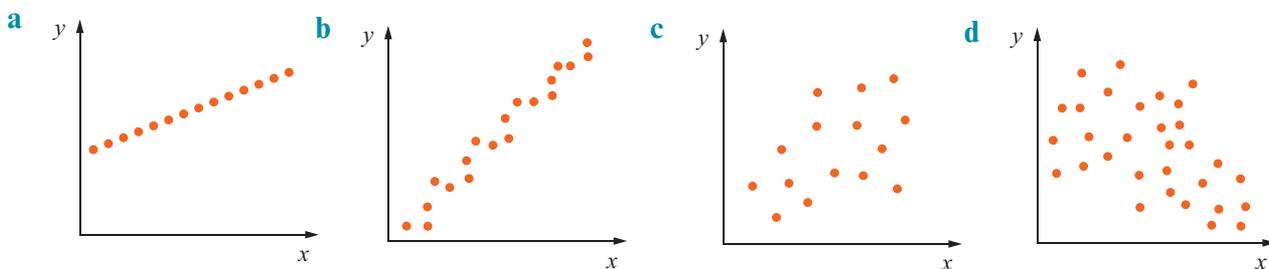
	Solve	Think	Apply
<b>a</b>	Strong correlation	All the points are marginally off a straight line, so this is a strong correlation.	For a perfect correlation the points will lie in a straight line.
<b>b</b>	No correlation	The points appear at random, so there is no correlation.	For a strong correlation the points will be marginally off appearing in a straight line.
<b>c</b>	Weak correlation	There is a general downwards trend, although the points are widely spread, so this is a weak correlation.	For a moderate correlation there will be an obvious trend in the dataset. For a weak correlation there will be a slight trend in the dataset.
<b>d</b>	Moderate correlation	There is a definite upwards trend with the points being slightly spread, so this is a moderate correlation.	For no correlation the points will appear at random.

## EXERCISE 6D Correlation

- 1 For each of these scatterplots, state whether the correlation is positive, negative or neither. Give reasons for your answers.



**2** State whether the pairs of variables graphed below have perfect, strong, moderate, weak or no correlation.



**3** Draw a scatterplot for two variables that have the following correlations.

- a** strong positive
- b** weak negative
- c** perfect negative
- d** no correlation
- e** perfect positive
- f** moderate negative

### EXAMPLE 6D-2 Interpreting the correlation

The members of a sporting club are using a new method of training to increase thigh circumference. Each fortnight, the thigh circumference of each member is measured. At the conclusion of the training period, there was a moderate positive correlation comparing thigh circumference to time.



**a** What does this mean?

	Solve/Think	Solve/Think	Apply
<b>a</b>	The new training increases thigh circumference.	The training method is working to increase thigh circumference.	A positive correlation means that as one variable increases the other increases. A negative correlation means that as one variable increases the other decreases.

**4** A gymnasium is using a new training program to increase bicep circumference. Each fortnight the bicep circumference of each person is measured. At the end of the program, the bicep measurements are compared with time. There is a moderate positive correlation. What does this mean? Explain your answer.

**5** Ten people are on a 6-week diet to lose weight. Their weight is recorded every week. There is a moderate negative correlation. Does this show that the diet works?



### EXAMPLE 6D-3 Discussing the expected correlation between variables

Discuss the expected strength of the relationship (correlation) between these variables.

- a speed and distance travelled
- b age and weight of a baby, up to 12 months of age
- c height and weight of 18-year-old girls
- d height of 18-year-old girls and mark in Mathematics in the HSC exam

	Solve/Think	Apply
a	As speed increases, there is a proportional increase in the distance travelled. This is an example of perfect positive correlation.	Sometimes both quantities increase or decrease but are unrelated; that is, there is no correlation.
b	As a baby's age increases so does its weight. However, this will happen at different rates for different babies; hence, this is an example of strong positive correlation.	
c	In general, taller girls weigh more than shorter girls; that is, larger heights are associated with larger weights and smaller heights are associated with smaller weights, but there are many exceptions. This is an example of weak positive correlation.	
d	There is no reason to suspect that there is any relationship between these two variables; that is, height will have no bearing on performance in the HSC or vice versa. This is an example of no correlation.	

6 Discuss the expected strength and direction of the relationship between the following variables.

- a the distance travelled and the cost for a taxi journey
- b the volume of water remaining in a tank and the time the tap is on
- c the number of police cars and the number of accidents on a highway
- d the height and shoe size of male adults
- e the age of cars and their price
- f the number of sunny days and the sales of umbrellas for a month
- g the speed of a car and the stopping distance
- h family income and the number of family pets
- i lengths of left arm and right arm of people
- j eyesight and age
- k hours spent studying and examination marks
- l smoking and lung cancer



A high degree of correlation between two variables does not necessarily imply that one causes the other.

A **cause and effect** relationship exists only when the change in one variable causes a direct change in another variable.

A **spurious correlation** is one in which two variables are correlated, but the correlation occurs either due to a third related variable, or due to **coincidence**.



#### cause and effect

the result of the change in one variable is directly responsible for the change in another variable

### EXAMPLE 6D-4 Identifying why a relationship between two variables exists

Comment on the following findings. Discuss whether a cause and effect relationship exists, or whether it is a case of spurious correlation.

- a** The heights and reading speeds of children were measured and a strong positive correlation was found.
- b** The number of televisions sold in Newcastle and the number of stray dogs in Wollongong were recorded over several years and a strong positive correlation was found between these variables.

	Solve/Think	Apply
<b>a</b>	Increases in height were associated with increases in reading speed. However, height does not affect reading speed and reading speed does not affect height. The strong correlation may be attributed to the fact that both variables are closely linked to a third variable, age. That is, as age increases, so do height and reading speed. This is an example of spurious correlation.	These are examples of spurious correlation. The strong correlation occurs because of the existence of a third related variable, or because both variables happen by chance to be increasing or decreasing at the same time.
<b>b</b>	Obviously an increase in the number of televisions sold in Newcastle does not cause an increase in the number of stray dogs in Wollongong, or vice versa. Both variables must have simply happened to be increasing over this period (i.e. coincidentally). This is an example of spurious correlation.	When variables are related such that one variable does cause an effect on the other (i.e. if one is changed, the other will change), we say that a causal relationship exists.

#### spurious correlation

two variables are mathematically related, but the relationship is not due to cause and effect

#### coincidence

two or more events occurring simultaneously through natural random variation

- 7** The following pairs of variables were measured and a moderate or strong positive correlation between them was found. Discuss whether a cause and effect relationship exists, or whether it is a case of spurious correlation.
  - a** the length of a person's left arm and right foot
  - b** company expenditure on advertising and sales
  - c** daily temperature and ice-cream consumption
  - d** the damage caused by a fire and the number of firemen who attend the fire
  - e** the number of people unemployed and the price of eggs
  - f** the height of parents and the height of adult offspring
  - g** the number of hotels and the number of churches in rural towns



- 8** Measure the height and foot length of 15–20 students from your school. Plot the data you collect as a scatterplot. Write a paragraph interpreting the correlation and any possible relationship between height and foot length.
- 9** Use the ABS website to write a paragraph on the average weekly earnings of Australian full-time workers over the past 5 years. Determine the correlation between time and earnings, and interpret this in your report.

# 6E Lines of best fit

These resources are available on your obook assess:

- **Spreadsheet 6E:** Draw a scatterplot with a trendline
- **Worksheet 6E:** Calculate the equation of the trendline using a calculator
- **assess quiz 6E:** Test your skills with an auto-correcting multiple-choice quiz

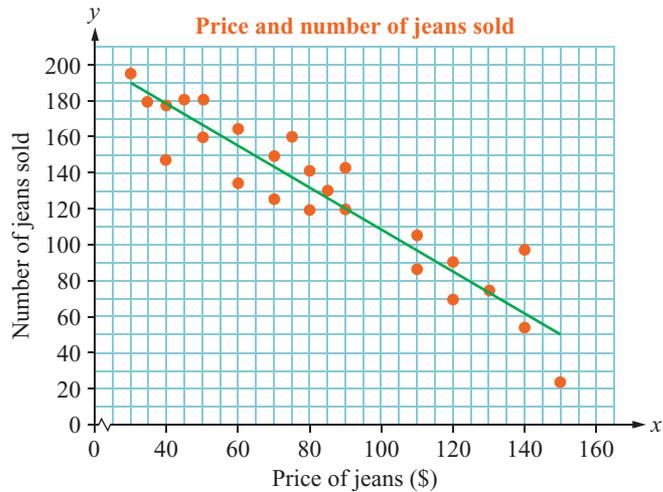
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**line of best fit**  
a straight line that provides a representation of all of the data points in a scatterplot; also known as the trendline

If a pair of variables appears to be related, as indicated by a linear pattern of dots on a scatterplot, then we can draw a straight line that fits the points. This line is known as the **line of best fit** or the trendline.

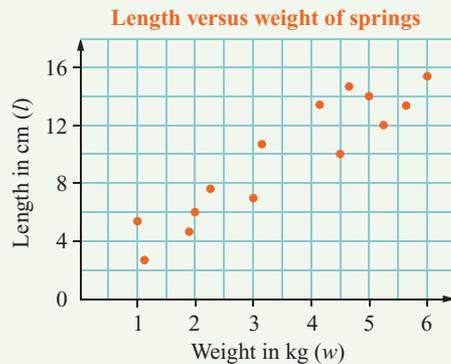
The simplest way to draw a line of best fit is by eye. The objective is to fit a line to the scatterplot so that the line is as close to as many data points as possible. An approximately equal number of

data points should lie above and below the line of best fit, with the distance of the points from the line being minimised. The line of best fit does not need to pass through any data points.



## EXAMPLE 6E-1 Drawing a line of best fit by eye

- a Draw a line of best fit on the scatterplot.

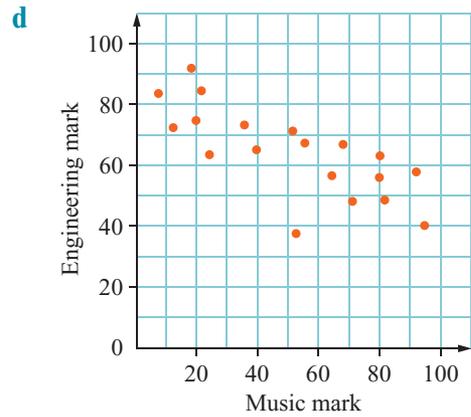
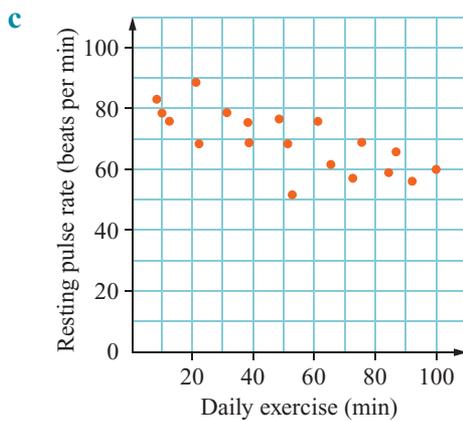
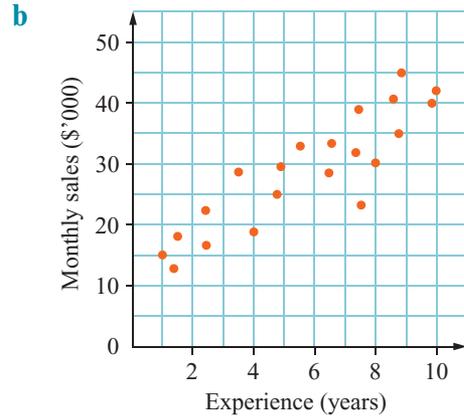
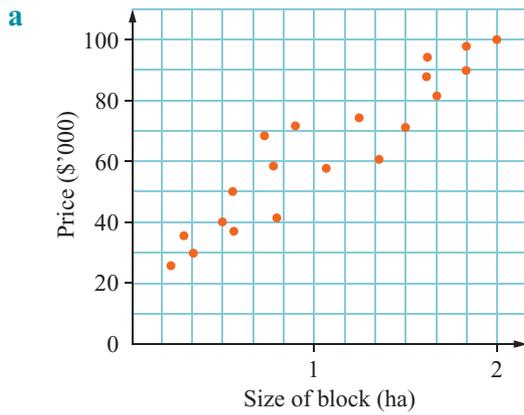


a

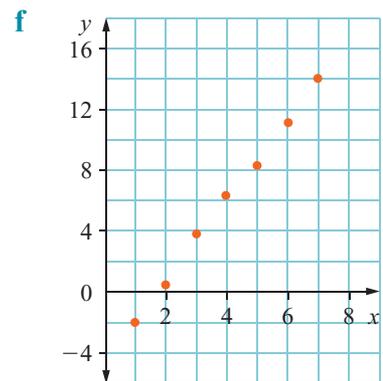
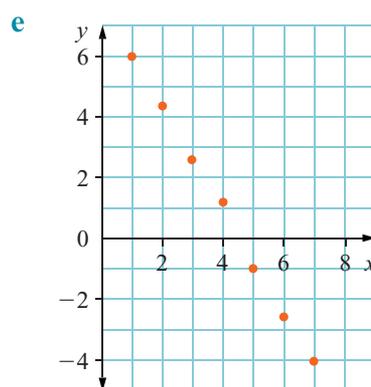
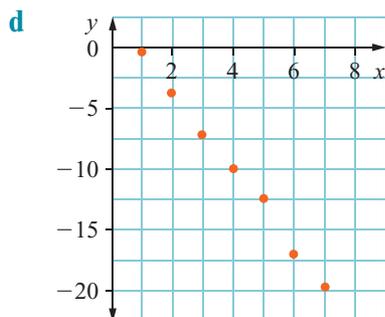
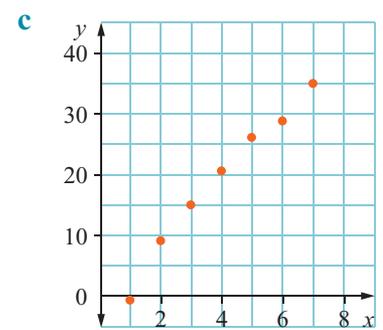
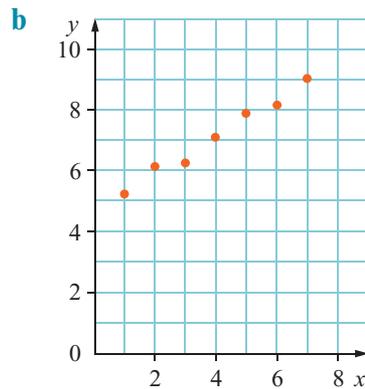
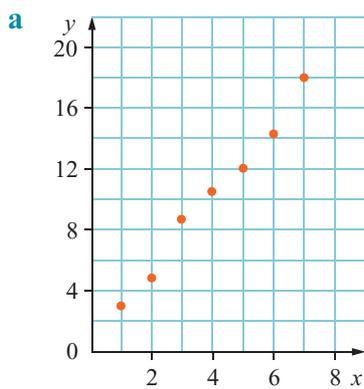
	Solve	Think	Apply
		<p>The line must have about the same number of dots above and below it.</p>	<p>The line need not pass through any of the points but must balance the points above and below it.</p>

# EXERCISE 6E Lines of best fit

1 Draw a line of best fit on each of the scatterplots below.



2 Draw the line of best fit for these bivariate datasets and describe the relationship in terms of direction (positive/negative) and strength (perfect/strong/moderate/weak/no correlation).



**3** Draw a scatterplot and line of best fit for the following data.

**a**

<i>x</i>	100	120	125	140	170	180	190	210	220	240
<i>y</i>	90	85	100	90	100	115	105	125	110	120

**b**

<i>x</i>	10	14	20	22	28	35	38	43	47
<i>y</i>	9	15	16	13	24	20	29	22	27

**c**

<i>x</i>	8	14	17	17	22	27	30	33
<i>y</i>	11	14	20	16	22	29	28	35

**d**

<i>x</i>	5	10	14	15	24	27	32	33
<i>y</i>	10	11	12	14	15	17	20	20

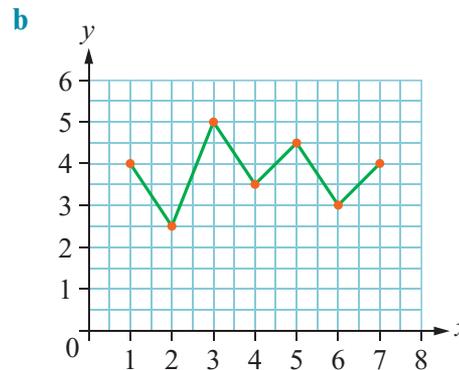
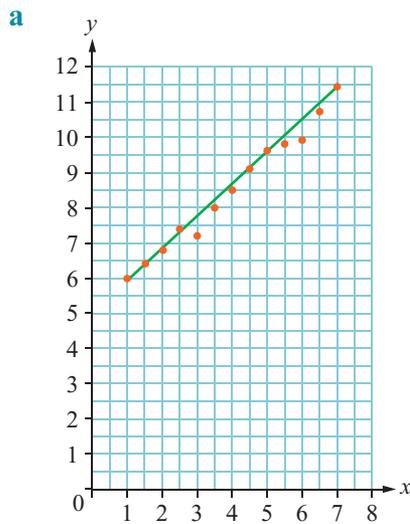
**e**

<i>x</i>	86	95	100	90	100	105	98	80	110	100	93
<i>y</i>	120	74	20	104	120	50	96	94	10	25	100

A mathematically determined line of best fit (or trendline) can be determined by using technology. The equation of the trendline will be in the form  $y = mx + c$ , where  $m$  is the value of the gradient of the line and  $c$  is the value of the vertical intercept (where the line crosses the vertical axis).  $y$  is the variable graphed on the vertical axis and  $x$  is the variable graphed on the horizontal axis.



**4** Explain why the lines depicted on the following scatterplots do not represent lines of best fit.



**EXAMPLE 6E-2** Interpreting the gradient and vertical intercept of the regression line

For a group of girls, the humerus length (elbow to shoulder) was measured and compared with height. The results are listed in this table.

<b>Humerus length (cm)</b>	37	35	40	31	35	33	31	40	34	39
<b>Height (cm)</b>	176	174	184	172	173	178	171	189	180	188

- Enter this data into a spreadsheet.
- Draw a scatterplot.
- Add the trendline and show the equation.

	Solve	Think	Apply
<b>a</b>	Enter the data into a spreadsheet.	Put the data into two columns.	Put a heading into the first row. Use the built-in formula.
<b>b</b> <b>c</b>		From the Insert menu select Scatterplot then first scatterplot type. From the Chart tools select Linear Trendline. Right click on the line and select Format Trendline from the drop-down menu. Check the Display equation on chart box and close.	

- 5** The data in the table gives the population,  $P$ , of Australia for the years 1931 to 2011. The time,  $T$ , is measured in years since 1931.

Year	1931	1941	1951	1961	1971	1981	1991	2001	2011
Time (years)	0	10	20	30	40	50	60	70	80
Population (millions)	6.53	7.11	8.42	10.55	13.07	14.92	17.28	19.27	22.34

- a** Enter this data into a spreadsheet.  
**b** Draw a scatterplot.  
**c** Add the trendline and show the equation.
- 6** The data in the table gives the percentage,  $P$ , of the Australian population born in the United Kingdom and Ireland for the years 1971 to 2011. The time,  $T$ , is measured in years since 1971.

Year	1971	1976	1981	1986	1991	1996	2001	2006	2011
Time (years)	0	5	10	15	20	25	30	35	40
Population born in the UK and Ireland (%)	8.53	8.22	7.72	7.18	6.91	6.33	5.79	5.48	5.43

- a** Illustrate the data ( $T$  vs  $P$ ) on a scatterplot using a spreadsheet.  
**b** Determine the equation of the trendline and add the line to your scatterplot.
- 7** The height of a plant  $n$  days after being planted is given in the following table.

Days ( $n$ )	3	6	9	12	15	18	21	24
Height ( $H$ cm)	4	6	9	11	13	16	18	20

- a** Illustrate the data ( $H$  vs  $n$ ) on a scatterplot using a spreadsheet.  
**b** Determine the equation of the trendline and add the line to your scatterplot.
- 8** Lines of best fit are usually only given if there appears to be a linear relationship between two variables. Why are lines of best fit less useful for non-linear relationships, e.g. exponential relationships?

# 6F Making predictions

These resources are available on your obook assess:

- **Video tutorial 6F:** Watch and listen to an explanation of Example 6F–2
- **Worksheet 6F:** Practise your skills with making predictions
- **assess quiz 6F:** Test your skills with an auto-correcting multiple-choice quiz

o  
i  
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When the line of best fit has been fitted to a dataset, we can use it to make predictions given a value of one of the two variables.

The strength of correlation determines the accuracy of the predictions made from the line of best fit for the data. The stronger the relationship between the variables, the more accurate predictions will be, and the weaker the relationship between the variables, the less accurate predictions will be.

The line of best fit will be more accurate for the population data when we are using a larger sample, therefore predictions made using a line of best fit are also more accurate when there are more data points in the dataset.

Lines of best fit can be extended beyond the range of points in a data set. **Interpolation** is when we make a prediction within the existing range of data points, and **extrapolation** is when we make a prediction outside the existing range of data points.

## interpolation

predicting where a new data point will lie within the existing range of data points

## extrapolation

predicting where a new data point will lie outside the existing range of data points

Interpolation is more accurate than extrapolation, as we cannot be sure that the behaviour of data points we observed will continue outside the range of values in the dataset.

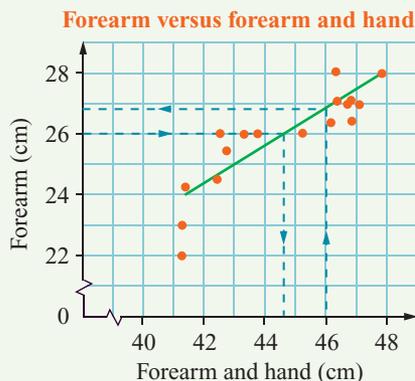
Extrapolation should not be used if the required data value lies far outside the range of the other data values.

!

## EXAMPLE 6F–1 Making predictions using a line of best fit

This scatterplot shows the forearm and hand against forearm only measurements for a group of students. Use the line of best fit to predict:

- the forearm measurement for a student with a forearm and hand measurement of 46 cm
- the forearm and hand measurement for a student with a forearm measurement of 26 cm.

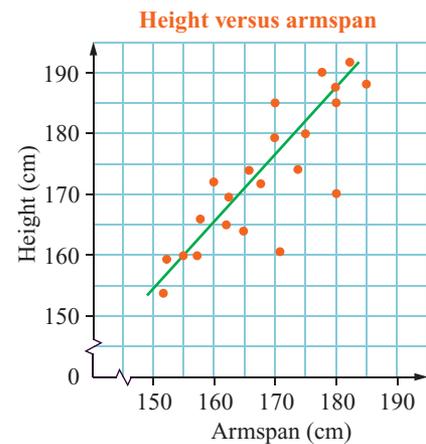


	Solve	Think	Apply
<b>a</b>	The forearm measurement is about 26.8 cm.	Draw a vertical line from 46 on the forearm and hand axis to meet the line. From this point of intersection, draw a horizontal line to meet the forearm axis. Read off the approximate forearm measurement.	The line of best fit will approximate the values. It is not a good indicator for values outside the range of the plotted points. The greater the number of points and the closeness of the points to the line, the better the line is as a predictor.
<b>b</b>	The forearm and hand measurement is about 44.6 cm.	Start at 26 on the forearm axis and reverse the process in part <b>a</b> .	

## EXERCISE 6F Interpolation and extrapolation

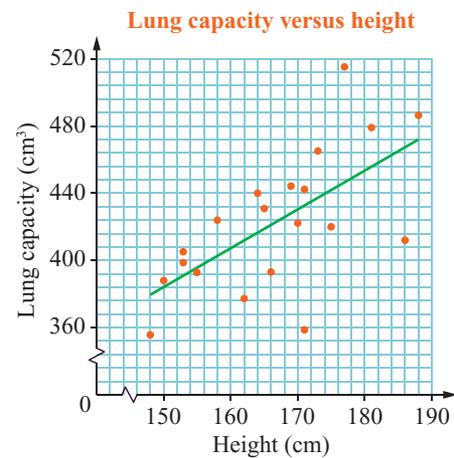
- 1** This scatterplot shows the heights and armspans of a group of students. A line of best fit has been drawn for these points. Use the line of best fit to predict:

- a** the heights of students with these armspans
- i** 160 cm                      **ii** 175 cm                      **iii** 180 cm
- b** the armspans of students with these heights
- i** 160 cm                      **ii** 175 cm                      **iii** 185 cm.



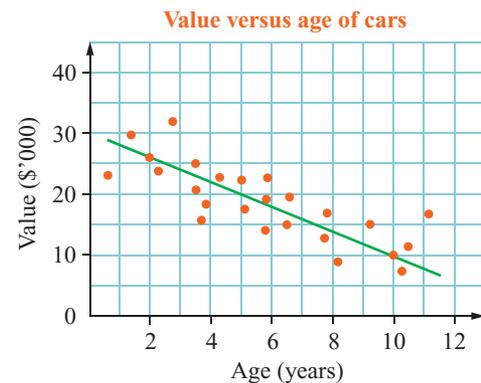
- 2** This scatterplot shows the height and lung capacity of a group of students. A line of best fit has been drawn for these points. Use the line of best fit to predict:

- a** the lung capacity of a student with a height of
- i** 160 cm                      **ii** 178 cm                      **iii** 172 cm
- b** the height of a student with a lung capacity of
- i** 400 cm<sup>3</sup>                      **ii** 430 cm<sup>3</sup>                      **iii** 450 cm<sup>3</sup>.



- 3** This scatterplot shows the age and value for a sample of cars of a particular model. A line of best fit has been drawn for these points. Use the line to predict:

- a** the value of a car of this model of age
- i** 2 years                      **ii** 5 years                      **iii** 10 years
- b** the age of a car of this model with a value of
- i** \$28 000                      **ii** \$18 000                      **iii** \$12 000.



### EXAMPLE 6F-2 Making predictions using the regression line equation

The equation of the trendline in Example 6F-1 is  $F = 0.6 \times A - 0.7$ , where  $F$  represents the forearm length and  $A$  represents the forearm and hand length. Use the equation to predict:

- a the forearm length of a student with a forearm and hand length of 45 cm
- b the forearm and hand length of a student with a forearm length of 25 cm.

	Solve	Think	Apply
a	$F = 0.6 \times 45 - 0.7$ $= 26.3$ cm	Substitute $A = 45$ into the equation and calculate $F$ .	Care must be taken if using the equation without viewing the data points.
b	$25 = 0.6 \times A - 0.7$ $25.7 = 0.6A$ $A = \frac{25.7}{0.6}$ $A = 42.8$ cm	Substitute $F = 25$ then solve the equation for $A$ .	

- 4 The equation of the trendline connecting height ( $H$ ) and armspan ( $A$ ), plotted on a scatterplot, is  $H = 1.2A - 36$ . Use the equation to predict:
- a the height of a student with an armspan length of
    - i 160 cm                      ii 170 cm                      iii 178 cm
  - b the armspan length of a student with a height of
    - i 160 cm                      ii 175 cm                      iii 183 cm.
- 5 The equation of the trendline connecting hip measurement ( $H$ ), in cm, and waist measurement ( $W$ ), in cm, is  $W = 0.7H - 2.1$ . Use the equation to predict:
- a the waist of a person with a hip measurement of
    - i 85 cm                      ii 96 cm                      iii 100 cm
  - b the hip measurement of a person with a waist of
    - i 60 cm                      ii 65 cm                      iii 71 cm.
- 6 The equation of the trendline connecting grape yield ( $G$ ), in tonnes, of a vineyard and the number of frosts ( $n$ ) during the growing season is  $G = -0.14 \times n + 5.6$ , when plotted on a scatterplot. Use the equation to predict:
- a the yield when there are
    - i 5 frosts                      ii 12 frosts                      iii 20 frosts
  - b the number of frosts given that the yield was
    - i 4.2 t                      ii 3.5 t                      iii 2.1 t.

### EXAMPLE 6F-3 Identifying interpolation and extrapolation

Ahn is doing an experiment measuring the time it takes a range of volunteers to complete a mental arithmetic task. He records the following results.

Age ( $n$ )	18	23	21	16	29	25	33	21	24
Time (s)	9.43	11.71	8.59	13.20	11.25	14.14	17.23	9.18	13.77

He calculates that the line of best fit for the data is  $s = 0.318n + 4.636$ .

- a Ahn wants to predict the completion time for a 45-year-old volunteer. Is this an example of interpolation or extrapolation?

- b** Ahn wants to predict the completion time for a 17-year-old volunteer. Is this an example of interpolation or extrapolation?

	Solve	Think	Apply
<b>a</b>	This is an example of extrapolation.	The range of values for the ages is from 16 to 29. Predicting values inside this range will be interpolation. Predicting values outside this range will be extrapolation.	If the data value we want to predict lies within the range of the other data values, we are interpolating. If it lies outside the range of the other data values, we are extrapolating.
<b>b</b>	This is an example of interpolation.		

- 7** Simone is a swimming coach. The time  $t$  it takes a number of her students to swim 25 m freestyle is recorded in the following table.

Age ( $n$ )	18	15	22	17	11	26	14	19	15
Time (s)	24	28	25	21	33	30	29	21	26

- a** If Simone wants to predict the swimming time for an 8-year-old, is this an example of interpolation or extrapolation?
- b** If Simone wants to predict the swimming time for a 30-year-old, is this an example of interpolation or extrapolation?



- 8** The results of a group of students on Mathematics and Science tests are compared.

Student	1	2	3	4	5	6	7	8	9	10
Maths test ( $M$ )	64	67	69	70	73	74	77	82	84	85
Science test ( $S$ )	68	73	68	75	78	73	77	84	86	89

- a** Use a spreadsheet to illustrate the data on a scatterplot, with  $M$  on the horizontal axis and  $s$  on the vertical axis.
- b** Add the trendline and show the equation of this line.
- c** Use the equation to predict the (average) score in Science of a student who scores 80 in Mathematics.
- d** Use the line of best fit equation to predict the (average) score in Mathematics of a student who scores 60 in Science.
- e** Is either part **c** or part **d** an example of extrapolation?

- 9 The population of a town over a period of 10 years is shown in the table. The time is measured in years from the start of 1990; that is,  $T = 1$  is the start of 1991,  $T = 2$  is the start of 1992, etc.

<b>Time (years)</b>	1	2	3	4	5	6	7	8	9	10
<b>Population</b>	3400	4100	4500	4900	5600	6100	6500	6900	7400	8000

- a Use a spreadsheet to illustrate the data on a scatterplot.  
 b Add the trendline and show the equation of this line.  
 c Use this equation to predict the population:  
     i after 4.5 years      ii after 7.5 years  
     iii after 12 years      iv at the start of 2007.  
 d Which of the answers in part c is/are the least reliable? Give reasons for your answer.  
 e Use this equation to estimate when the population:  
     i was 5000      ii will reach 10000.
- 10 April collects data on the shoe sizes of 100 female students from different year levels in her school. Explain why the line of best fit for this data would not be useful for predicting the shoe size of a 30-year-old woman.



- 11 May is investigating whether there is a link between the weight and systolic blood pressure of a number of patients at the hospital she works at. She collects the following information.

<b>Weight (kg)</b>	75	83	62	66	98	57	88	111	79	60
<b>Systolic blood pressure (mmHg)</b>	99	115	108	86	127	90	122	143	103	105

- a Use a spreadsheet to illustrate the data on a scatterplot.  
 b Add the trendline and show the equation of this line.  
 c Use the equation of the regression line to predict the systolic blood pressure of patients with the following weights:  
     i 115 kg      ii 140 kg  
 d Which of the predictions from part c is more reliable? Explain your answer.
- 12 June is looking to see whether there is a relationship between the number of goals and behinds in AFL football games. The table below is a sample of the data she collects.

<b>Goals (g)</b>	14	5	14	15	12	18	17	7	11	25
<b>Behinds (b)</b>	8	15	13	7	8	9	16	9	12	13

- a Use a spreadsheet to illustrate the data on a scatterplot, with  $g$  on the horizontal axis and  $b$  on the vertical axis.  
 b State the strength and direction of the correlation for this data.  
 c Add the trendline and show the equation of this line.  
 d Considering your answers to parts b and c, will the regression line be useful in making accurate predictions for this dataset? Explain your answer.

- 13** Explain why it will be more accurate to make predictions from a line of best fit that is calculated from many data points than from one that is calculated from just a few data points.
- 14** Bailey is doing a project about the price of inner-city apartments in Sydney. He collects the following data about apartment sizes (in  $\text{m}^2$ ) and their price (in \$'000s).

Size ( $\text{m}^2$ )	77	102	52	77	134	68	99	130	61	75
Price (\$'000s)	1180	1800	830	1120	3050	1270	1070	2600	660	2350

- a** Use a spreadsheet to illustrate the data on a scatterplot.
- b** Calculate the equation of the trendline for the data.
- c** Use the equation of the trendline to estimate the price of a  $90 \text{ m}^2$  inner-city apartment in Sydney. Give your answer to the nearest \$10 000.
- d** Bailey re-checks his data and discovers he made a mistake in inputting the size of the final apartment; it is actually  $125 \text{ m}^2$  instead of  $75 \text{ m}^2$ . Give the equation of the trendline for the corrected data.
- e** Recalculate the estimated price of the  $90 \text{ m}^2$  apartment from part **c**.
- f** Which factors do you think affect the price of apartments in inner-city Sydney?



- 15 a** Measure the height and handspan of the students in your class (separate results for males and females) and record the information in a table (or directly into a spreadsheet).
- b** Use a spreadsheet to illustrate the data on a scatterplot.
- c** Describe the correlation between these two variables in terms of direction (positive/negative) and strength (perfect/strong/moderate/weak). Is one variable a good predictor of the other?
- d** Add the trendline and show the equation of this line.
- e** Measure the heights of some students from another class and predict the handspans of these students, using the equation of the regression line.
- f** Find the handspans of these students by actual measurement and compare them with your predictions.
- g** Discuss the results in relation to your answer to part **c**.
- 16** Repeat the procedures in question **15** for these measurements.
- a** Height and shoe size
- b** Head circumference and height
- c** Length of femur (thigh bone) and height
- d** Length from hip to the ground and height
- 17** Use data from the ABS website to research the household income of Australian families over time. Write a report on your findings, discussing any potential correlation between household income and time.
- 18** Complete a statistical investigation about the amount of recycling Australian households produce in a week. Ask your classmates to record the amount of recycling their household produces in a week (in grams), and discuss if there is a correlation between this figure and the number of people living in a household.

# CHAPTER 6 REVIEW STATISTICAL ANALYSIS

You should be able to:

- ✓ identify which questions can be answered using statistics
- ✓ identify how and where bias can appear in both sample selection and data collection
- ✓ analyse numerical data collected from statistical investigations
- ✓ compare and contrast data from two or more groups
- ✓ create a scatterplot from a given dataset
- ✓ identify independent and dependent variables where appropriate
- ✓ describe the strength and direction of bivariate datasets
- ✓ draw a line of best fit for a bivariate dataset by eye
- ✓ determine the equation of the trendline using technology
- ✓ make predictions about a bivariate dataset by using either interpolation or extrapolation
- ✓ understand when predictions are accurate and when they are inaccurate.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

## REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 Which of the following questions can be answered without using statistics?
  - A What is the average time it takes Year 12 students to run 100 m?
  - B Does it rain more in Sydney or Brisbane?
  - C Which team has won the most NRL premierships?
  - D Do lawyers earn more than doctors?
- 2 For which of the following investigations would a census be used?
  - A The number of minutes walking Australians do each day
  - B The amount of recycling Australian businesses achieve
  - C The gender of journalists
  - D The favourite film of students in a class

Use the following dataset to answer questions 3–5.

7, 14, 12, 9, 12, 7, 8, 13, 11, 12, 8, 9, 12, 14, 17

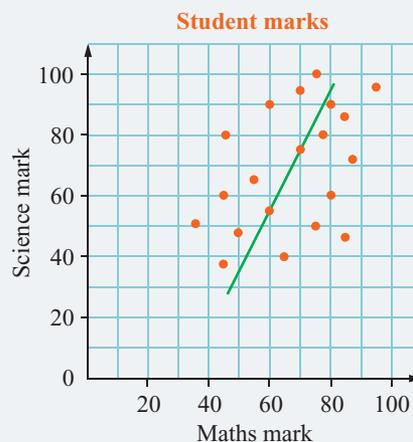
- 3 What is the mean of this dataset?

A 7	B 8	C 11	D 12
-----	-----	------	------
- 4 What is the median of this dataset?

A 7	B 8	C 11	D 12
-----	-----	------	------
- 5 What is the mode of this dataset?

A 7	B 8	C 11	D 12
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This graph is a scatterplot, with a line of best fit, of the Mathematics and Science marks for a group of students. Use the graph to answer questions 6–8.



**6** Using the trendline, what is the predicted Science mark for a student who scored 70 in Mathematics?

- A 60                      B 65                      C 70                      D 75

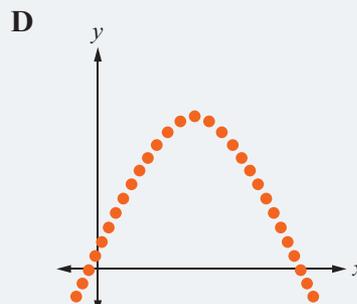
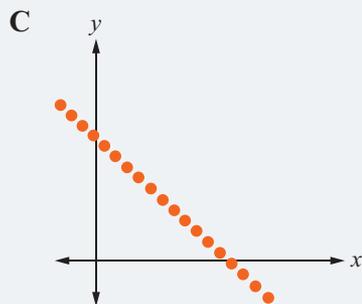
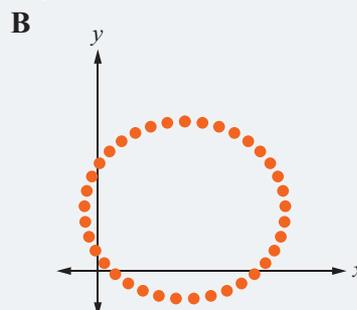
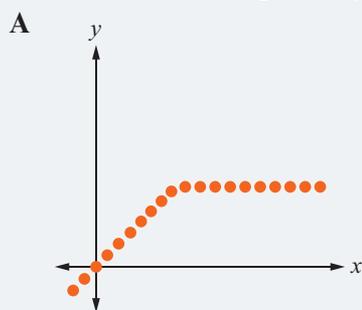
**7** Using the trendline, what is the predicted Mathematics mark for a student who scored 55 in Science?

- A 50                      B 55                      C 60                      D 65

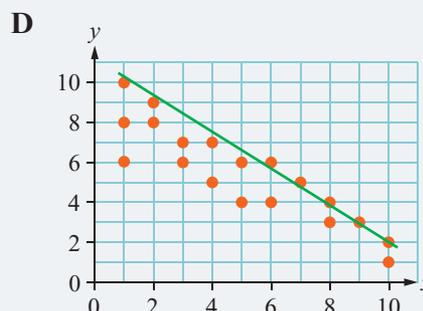
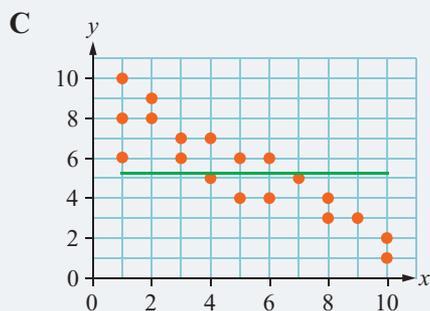
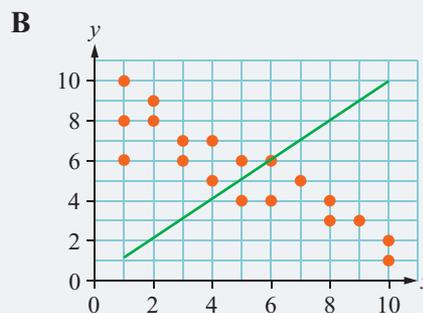
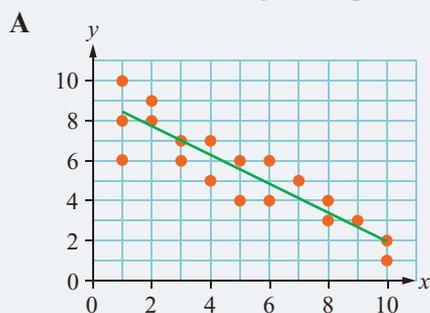
**8** At which score are the predicted values for Mathematics and Science the same?

- A 40                      B 50                      C 65                      D 75

**9** Which of the following scatterplots shows a linear relationship between the variables?



**10** Which of the following scatterplots correctly depicts a line of best fit?



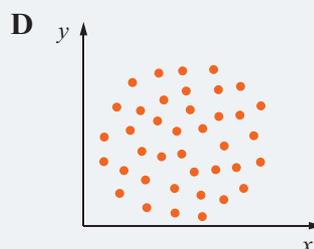
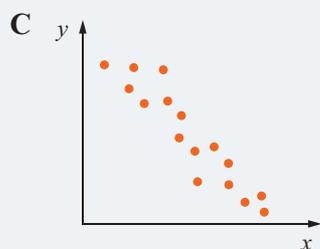
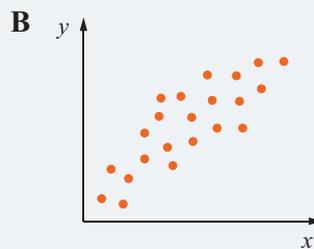
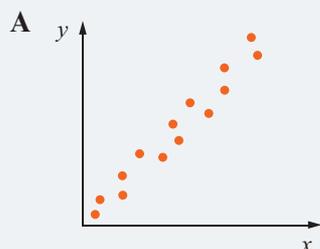
- 11** Given the following data collected about the time (in seconds) it took females and males to solve a logic problem, which of the following statements is not true?

Female: 48, 42, 58, 55, 33, 47, 66, 60, 39, 51, 59

Male: 58, 51, 45, 49, 31, 67, 47, 53, 36, 44, 65

- A The range for males is greater than it is for females.
- B The mean for males is greater than it is for females.
- C The interquartile range for females is greater than it is for males.
- D There is no mode for either males or females.

- 12** Which of the following scatterplots shows a strong negative correlation?



- 13** There is a strong degree of correlation between the lengths of the left and right feet of individuals. Which of the following best describes the association between the two variables?

- A cause and effect
- B spurious correlation
- C interpolation
- D extrapolation

Brooke is investigating the relationship between the number of handballs and kicks players make in AFL football. The line of best fit for her data is  $h = 0.85k + 2.5$ , where  $h$  represents the number of handballs and  $k$  represents the number of kicks. Use this equation to answer questions **14** and **15**.

- 14** If a player makes 15 kicks in a game, how many handballs would they expect to make (to the nearest whole number)?
- A 10
  - B 11
  - C 13
  - D 15
- 15** If a player makes 325 handballs in a season, how many kicks would they expect to make (to the nearest whole number)?
- A 274
  - B 278
  - C 379
  - D 385
- 16** The table shows the distance travelled, in 1000 km, versus the servicing costs, in \$1000, for a motor vehicle.

<b>Distance ('000 km)</b>	50	100	180	200	230	270	330	350	400
<b>Cost (\$'000)</b>	3.2	4.1	4.4	6	7.3	8.5	9.1	9.8	13.5

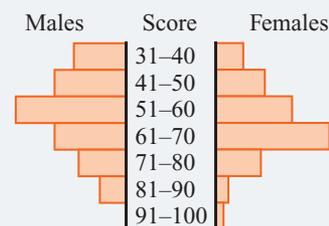
Which of the following is an example of extrapolation?

- A estimating the servicing costs for a motor vehicle that has travelled 385 000 km
- B estimating the distance travelled for a motor vehicle with servicing costs of \$5200
- C estimating the servicing costs for a motor vehicle that has travelled 20 000 km
- D estimating the distance travelled for a motor vehicle with servicing costs of \$10 500

# REVIEW SET 1

- Identify why the following sampling methods may result in a biased statistical investigation.
  - Interviewing people on the evening of a State of Origin match about what their favourite sport is.
  - Asking cinemagoers what their favourite film genre is straight after they have seen the latest Star Wars movie.
- The following dataset was collected from a statistical investigation:  
13, 14, 18, 12, 14, 16, 20, 13, 14, 19, 22, 13, 11, 16, 19
  - What is the mean?
  - What is the range?
  - What is the interquartile range?

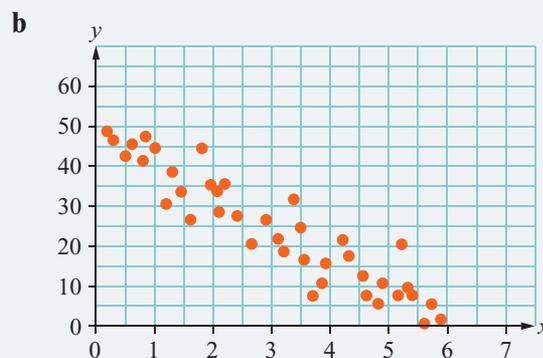
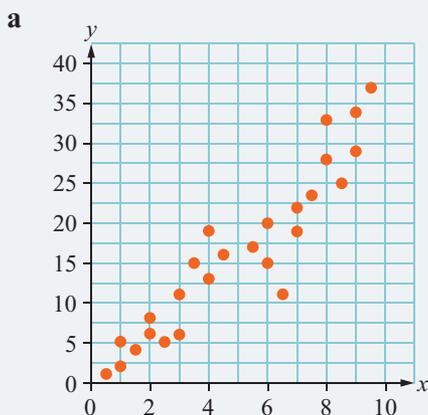
- The following back-to-back histogram shows the scores for males and females in a Mathematics test.



- What is the modal group for males?
  - What is the modal group for females?
  - Overall, which group performed better on the test? Justify your answer.
- Draw a scatterplot for the following table of data.

$x$	10	20	30	40	50	60	70	80
$y$	2.2	1.9	1.8	1.8	1.4	1.3	0.9	0.8

- Is the correlation between  $x$  and  $y$ :
    - perfect, strong, or weak?
    - positive or negative?
- Discuss the expected strength of the relationship between the following variables.
    - the size of a gold nugget and its value in dollars
    - the number of chickens on a farm and the number of eggs laid
    - the age of a tractor and its value in dollars
  - Draw a line of best fit on each of these scatterplots.



- The relationship between time spent training at a gym and the maximum weight that can be bench-pressed is given by the equation  $w = 35 + 12.5t$ , where  $w$  is the weight in kg and  $t$  is the time in years. Use the equation to predict the maximum bench-press weight of someone who has been training for:
  - 3 years
  - 7 years
  - 15 years



- 8** Julie is a marine biologist tracking the population of sea turtles on a Pacific island. Over 24 years she records the following data.

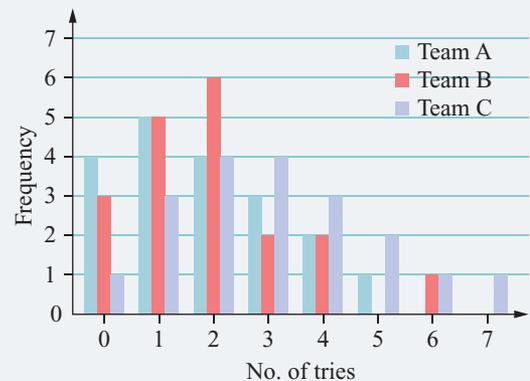
<b>Year</b>	1994	1997	2000	2003	2006	2009	2012	2015	2018
<b>Population</b>	3520	3314	3198	2805	2487	2108	1884	1539	1235

- a** Julie wants to know the year in which the population first fell to 2000. Is this an example of interpolation or extrapolation? Explain your answer.
- b** Julie wants to predict the population of sea turtles in 2020. Would this be an example of interpolation or extrapolation? Explain your answer.



## REVIEW SET 3

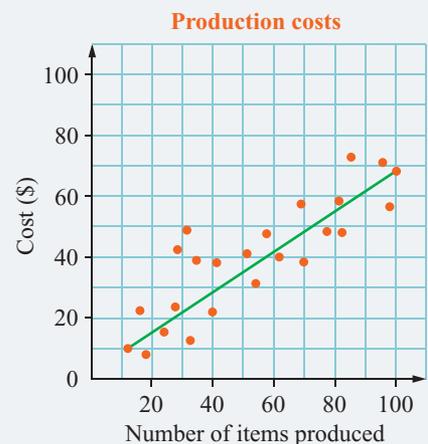
- 1** What would be the target population if we wanted to:
- collect information about the income of parents?
  - collect information about farm animals?
- 2** The multiple bar graph on the right shows the number of tries three rugby league teams scored in games this season.
- What team scored the most tries over the season?
  - What team had the most games without scoring a try?



- 3** Draw a scatterplot for the data in this table.

<b>x</b>	5	10	15	20	25	30	35	40	45	50	55
<b>y</b>	0	8	21	28	43	51	60	69	82	94	102

- 4** Use this line of best fit to predict:
- the cost when the number of items produced is 80
  - the number of items produced when the cost is \$35.
- 5** There was a moderate positive correlation between marks in an English exam and marks in a French exam. Explain what this means in the context of the data.



- 6** The following pairs of variables were measured and a strong correlation found. State whether it is a cause and effect relationship or a case of spurious correlation.
- the number of umbrellas sold and the number of swimming costumes sold
  - the number of storks nesting in chimneys and the birth rate
- 7** The equation of the regression line connecting the age of a tropical plant in days ( $d$ ) and height of the plant in cm ( $h$ ) is  $h = 1.45d - 3.5$ .
- Use the equation to predict the height of the plant after:
    - 8 days
    - 12 days
    - 20 days.
  - Does the vertical intercept make sense in the context of the data?
- 8** Discuss the expected strength of the relationship between the following variables.
- age of a computer and its processing power
  - height and size of a nose
  - experience in a job and salary

## REVIEW SET 4

- 1** Identify why the following sampling methods may result in a biased statistical investigation:
- researching bonuses in Australian businesses by interviewing a number of people who work in the finance sector
  - researching attitudes towards parental leave by speaking to 50 new parents

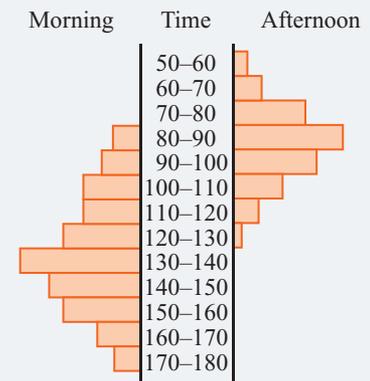
- 2** The following data was collected in a statistical investigation.

8, 13, 7, 10, 12, 8, 16, 11, 13, 11, 12, 15, 7, 10, 9

- What is the mean of the dataset?
- What is the median of the dataset?
- What is the interquartile range of the dataset?

- 3** The following back-to-back histogram shows the waiting time (in seconds) for customers at a coffee shop in the morning and in the afternoon.

- What is the modal group for the morning wait time?
- What is the modal group for the afternoon wait time?
- Overall, when would be the best time to get a coffee if you were in a rush? Justify your answer.



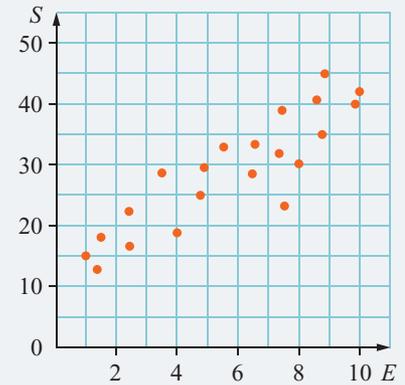
- 4** A transport company keeps a record of the annual maintenance costs of its fleet of semitrailers. The distance travelled (in thousands of kilometres), and cost (in thousands of dollars), is shown in the table below.

<b>Distance travelled ('000 km)</b>	50	100	180	200	230	270	330	350	400
<b>Cost (\$'000)</b>	2.3	2.7	3.3	3.5	3.7	4.1	4.5	4.7	5.1

- Plot the data on a scatterplot.
- Comment on the correlation between the variables (strong, moderate, weak, positive, negative).
- Determine the equation of the regression line that fits this data.
- Predict the annual maintenance cost for a semitrailer that has travelled 300 000 km. Give your answer to the nearest \$100.
- Using your answer for part **b**, comment on the accuracy of this prediction.

5 The scatterplot compares  $S$  and  $E$ .

- a Draw a line of best fit.
- b Find  $S$  when  $E = 7$ .
- c Find  $E$  when  $S = 40$ .



6 Karl is tracking the population of kangaroos in a rural area. He records the following data.

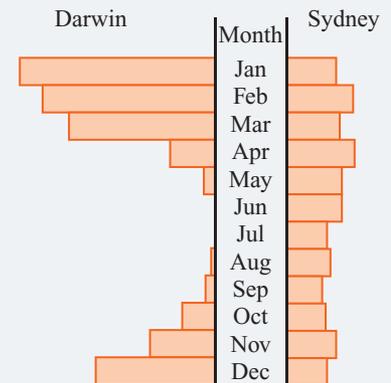
Year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Population	45 300	46 100	46 600	47 200	47 100	48 000	48 400	48 800	49 600

- a If Karl wanted to estimate the population of kangaroos in the area in 2004, would this be an example of interpolation or extrapolation?
- b Explain your answer to part a.

## REVIEW PRACTICE EXAMINATION QUESTION

1 a The following back-to-back histogram shows the average monthly rainfall for Sydney and Darwin each year.

- i What is the modal group for Darwin's monthly rainfall? (1 mark)
  - ii What is the modal group for Sydney's monthly rainfall? (1 mark)
  - iii How many months in a year does Sydney receive more rainfall than Darwin? (2 marks)
  - iv Overall, which city receives more rain in a year? Justify your answer. (2 marks)
  - v Which city has the more consistent rainfall throughout the year? Justify your answer. (2 marks)
- b i Draw a scatterplot for the data in this table (2 marks)



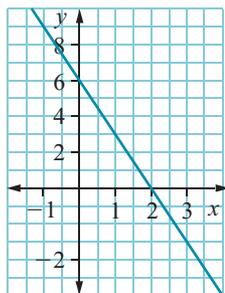
$T$	0	20	40	60	80	100	120	140
$H$	38	35	43	54	55	68	72	73

- ii Draw a line of best fit by eye and estimate the value of  $T$  when  $H = 50$ . (2 marks)
  - iii Would predicting the value of  $H$  when  $T = 160$  be an example of interpolation or extrapolation? (1 mark)
- c A scientist was investigating the relationship between the amount of medicine administered in a trial in mL ( $m$ ) and the number of bacteria in millions ( $b$ ).
- i Identify the independent and dependent variables in this situation. (1 mark)
- The line of best fit for the dataset is given by the equation  $b = -23m + 6325$ .
- ii What is the value of  $b$  when  $m = 50$ ? (1 mark)
  - iii What is the value of  $m$  when  $b = 4370$ ? (1 mark)

**TOTAL:**  
15 marks

# Chapters 4–6 CUMULATIVE REVIEW

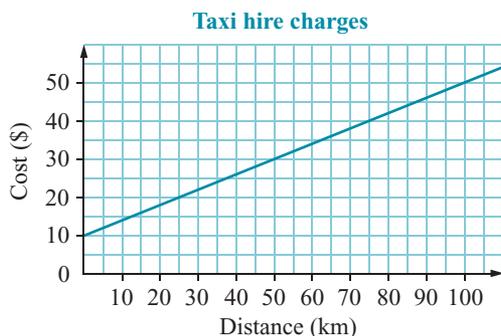
- 1 Find the gradient and  $y$ -intercept of this straight-line graph.



- 2 a Complete this table of values for  $y = 3x - 2$ .

$x$	-3	-2	-1	0	1	2	3
$y$							

- b Graph  $y = 3x - 2$  on a number plane.
- 3 Draw the straight-line graph  $y = -3 - 4x$ .
- 4 The graph below models taxi charges.
- a Find the cost of travelling 75 km.
- b How far can you travel for \$50?
- c Find the gradient. What is the meaning of the gradient in this context?
- d Find the intercept on the vertical axis. What is its meaning?



- 5 The Super Sock Company has fixed costs of \$800 per day and a variable cost of \$6 per pair of socks. The daily cost may be modelled using the straight-line equation  $C = 6n + 800$ .



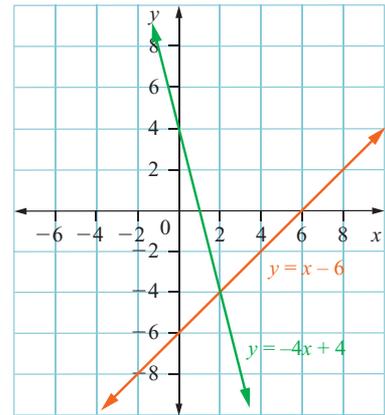
- a Explain each term in the formula  $C = 6n + 800$ .
- b Complete this table of values.

$n$	0	50	100	150	200	250
$C$						

- c** Draw the straight-line graph of  $C = 6n + 800$ .
- d** Use the graph to find the number of pairs of socks produced when the cost is \$1580.
- 6** A mobile telephone plan charges calls at 75 cents per minute.
- a** Write a linear equation to model this information. Relate the cost in dollars to the time in minutes.
- b** Draw the graph of time versus cost. Which is the dependent variable?
- c** Use the graph to find the cost of a call of 2.5 min.
- d** What is the maximum length of a call that costs \$2.50?

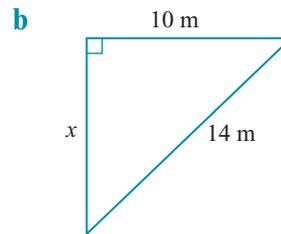
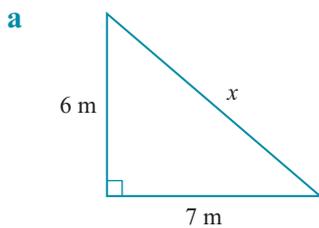


- 7** The graphs of the pair of simultaneous linear equations  $y = x - 6$  and  $y = -4x + 4$  are drawn on the same number plane.
- a** Write the coordinates of the point of intersection of the two lines.
- b** Use the coordinates of this point to write the solution to the simultaneous linear equations.
- 8** Solve each pair of simultaneous equations using a graphical method.
- a**  $y = 7x - 5$  and  $y = 5 - 3x$
- b**  $4x - 3y = 27$  and  $y = 1 - 2x$



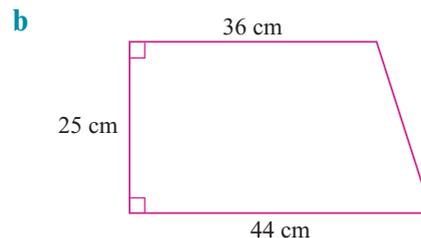
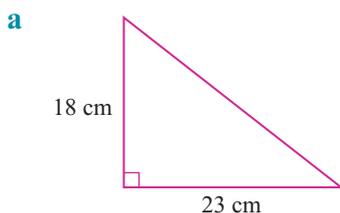
- 9** The Gossimer Stocking Company has fixed costs of \$900 per day and a variable cost of \$3 per pair of stockings,  $C = 3n + 900$ , and revenue of \$10 per pair sold,  $R = 10n$ , where  $n$  is the number of pairs of stockings.
- a** Graph these two equations on the same number plane.
- b** Find the break-even point.
- c** How many pairs of stockings must be sold to break even?
- d** What is the break-even income?

- 10** Find the value of  $x$  in the following triangles, correct to one decimal place.



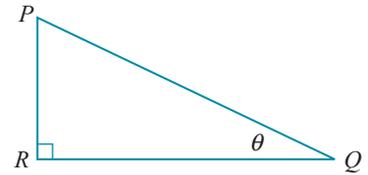
- 11 a** Find the length of the diagonal of a 8 cm by 3 cm rectangle.
- b** Find the length of the diagonal of a 10 cm by 10 cm square.

- 12** Find the perimeter of the following figures, to the nearest centimetre.



**13 a** For the angle marked  $\theta$  in this triangle, name the:

- i** opposite side
- ii** adjacent side
- iii** hypotenuse.



**b** Write these values as a ratio using the side lengths  $PQ$ ,  $PR$  and  $RQ$ .

- i**  $\sin \theta$
- ii**  $\cos \theta$
- iii**  $\tan \theta$

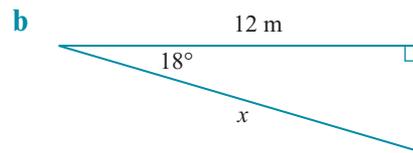
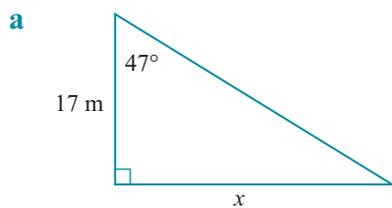
**14 a** Find these values correct to four decimal places.

- i**  $\sin 73.2^\circ$
- ii**  $\cos 71^\circ 15'$
- iii**  $\tan 18^\circ 46'$

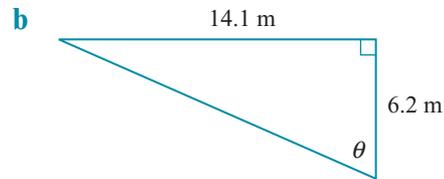
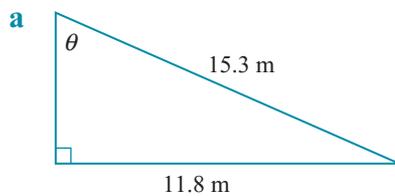
**b** Find  $\theta$  to the nearest minute, if:

- i**  $\sin \theta = 0.271$
- ii**  $\cos \theta = 0.389$
- iii**  $\tan \theta = 1.41$ .

**15** Find the length of the unknown side in these triangles.

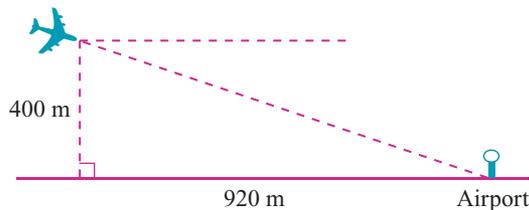


**16** Find the value of  $\theta$  to the nearest minute.



**17** The pilot of a plane flying at an altitude of 400 m observes an airport runway that is 920 m horizontally from the plane.

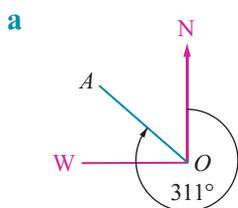
- a** What is the angle of depression from the plane to the runway, to the nearest minute?
- b** What is the actual distance, to the nearest metre, that the plane has to fly to reach the runway?



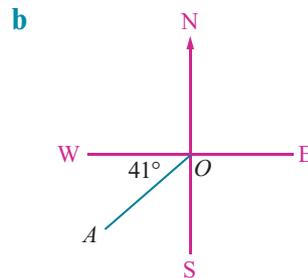
**18** When the angle of elevation of the sun is  $32^\circ$ , a statue casts a shadow 8.8 m long. Find the height of the statue.

19 Write the bearing of  $A$  shown by these diagrams as a:

i compass bearing



ii true bearing.



20 a A ship sails 143 km from port  $O$  on a bearing of  $194^\circ\text{T}$ . How far south has the ship sailed, to the nearest kilometre?

b Town  $P$  is 85 km west and 61 km north of town  $Q$ . Find the bearing of  $P$  from  $Q$  to the nearest minute.

21 What would be the target population if we wanted to:

a collect information about the weight of red kangaroos?

b investigate the price of package holidays online?

22 Lisa wants to investigate the most popular pet and she asks people she meets in the street the following survey question: Do you like cats or dogs? Explain why this survey question may result in a biased statistical investigation.



23 Given the following dataset:

8, 6, 9, 14, 11, 8, 10, 7, 11, 7, 17, 12, 8, 10, 15, 9, 8, 13, 7

a find the mean of the data

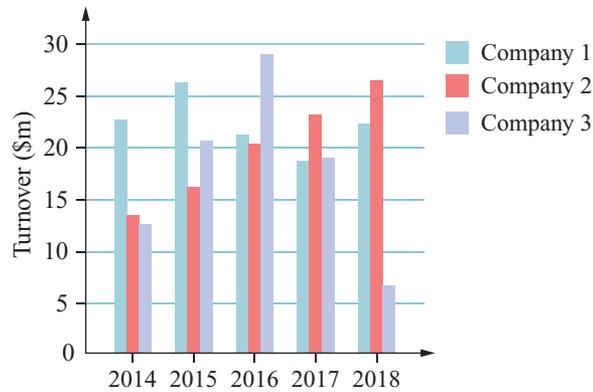
b find the median of the data

c find the mode of the data

d find the range of the data

e find the interquartile range of the data

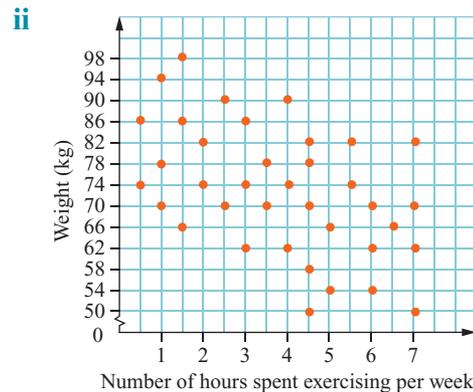
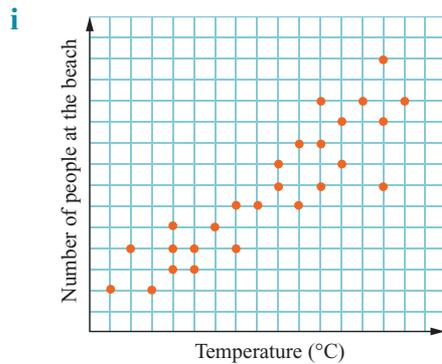
**24** The following multiple bar graph shows the annual turnover of three companies over a five-year period.



Match the following descriptions to the three companies.

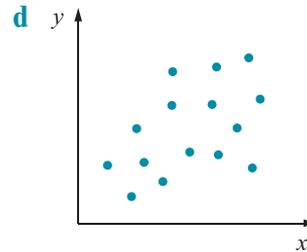
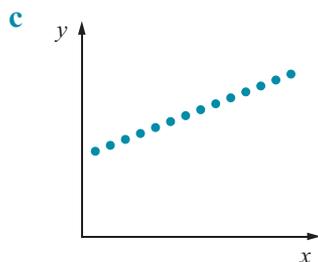
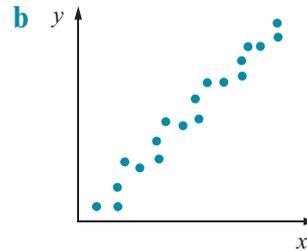
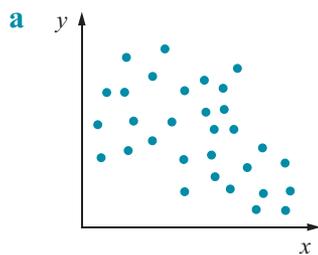
- a** This company's turnover rose quickly, before plummeting.
- b** This company's turnover steadily rose over the 5 years.
- c** This company's turnover fluctuated over the 5 years, ending up in a similar position to where they began.

**25** Consider the following bivariate scatterplots.



- a** Identify the independent variable and the dependent variable in each scatterplot.
- b** State whether or not there appears to be a linear relationship between the variables in each scatterplot.

**26** State whether the pairs of variables graphed below have perfect, strong, moderate, weak or no correlation.



**27** For each of the scatterplots in question 26, state whether the correlation is positive, negative or approximately zero.

**28 a** Enter the data shown in the table into a spreadsheet.

<b>x</b>	15	25	36	11	32	25	18	33	16	27	41	8	20	31	28
<b>y</b>	25	16	2	42	12	11	51	5	33	19	3	35	21	10	15

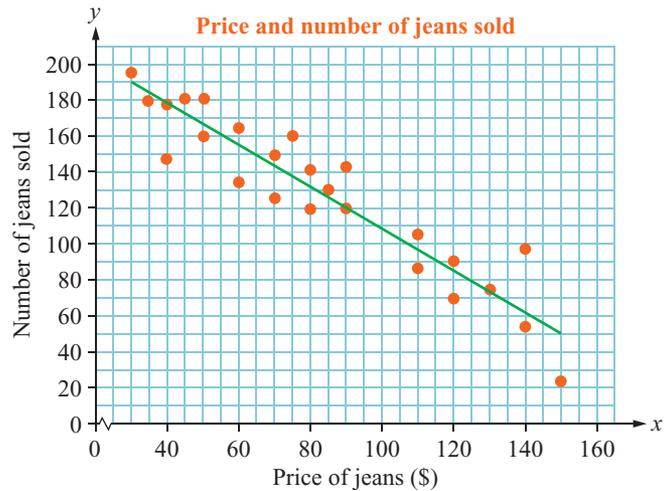
**b** Describe the trend shown in the scatterplot.

**29** The following pairs of variables were measured and a moderate or strong positive correlation between them was found. Discuss whether a cause and effect relationship exists, or whether it is a case of spurious correlation.

- a** the height of a person and the length of their left foot
- b** the number of people working at a supermarket and the price of milk
- c** daily temperature and cold drink consumption

**30** The graph shows a scatterplot with a line of best fit for the price and number of jeans sold. Use the line of best fit to predict:

- a** the number of jeans sold when the price of jeans is \$80
- b** the price of jeans when the number of jeans sold is 150.



**31** The Maths test score obtained by a student after studying for a number of hours is shown in this table.

<b>Hours of study</b>	2	1	4	5	2	3	3	2	4	1
<b>Maths test score</b>	67	55	87	81	70	75	59	60	93	62

- a** Use a spreadsheet to illustrate the data on a scatterplot.
- b** Add the trendline and show the equation of this line.
- c** Use this equation to predict a student's Maths test score:
  - i** after 3 hours of study
  - ii** after 4.5 hours of study
  - iii** after 30 minutes of study
  - iv** after 7 hours of study.
- d** Which of the answers in part c are the least reliable? Give reasons using your understanding of interpolation and extrapolation.

# 7

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## Depreciation and loans

The main mathematical ideas investigated are:

- ▶ constructing a depreciation table
- ▶ calculating the salvage value of an item using the declining-balance method of depreciation
- ▶ constructing reducing-balance loan tables
- ▶ calculating the amount owed on a reducing-balance loan at a given point in time
- ▶ calculating reducing-balance loan repayments
- ▶ understanding credit card statements
- ▶ calculating payments, charges and balances on credit cards

# ARE YOU READY?

- 7A ▶ 1** What is 18% of \$20 000?  
**A** \$360                      **B** \$1800  
**C** \$3600                      **D** \$16400
- 7A ▶ 2** What is  $0.8^2$ ?  
**A** 0.64                      **B** 0.82  
**C** 0.89                      **D** 64
- 7A ▶ 3** What is  $0.77 \times \$23\,215$  rounded to the nearest dollar?  
**A** \$17 880                      **B** \$17 875  
**C** \$17 876                      **D** \$17 870
- 7A ▶ 4** A washing machine purchased for \$1800 is depreciated by \$220 per year. What is the salvage value of the washing machine after 5 years?  
**A** \$700                              **B** \$2900  
**C** \$1580                              **D** \$920

	Column				
Row	1	2	3	4	5
<b>1</b>	15	33	29	70	38
<b>2</b>	43	81	16	47	69
<b>3</b>	22	75	36	41	92
<b>4</b>	18	64	51	68	35
<b>5</b>	49	12	77	56	79
<b>6</b>	85	63	30	48	52

Use the table in the previous column to answer questions **5** and **6**.

- 7B ▶ 5** What is the value in row 4 and column 2 of the table?  
**A** 68                              **B** 47  
**C** 64                              **D** 81
- 7B ▶ 6** In which row and column does the number 12 lie?  
**A** Row 2, column 3  
**B** Row 2, column 5  
**C** Row 3, column 2  
**D** Row 5, column 2
- 7C ▶ 7** How many days are there from 1 January to 5 February inclusive?  
**A** 34                              **B** 35  
**C** 36                              **D** 37
- 7C ▶ 8** Which of the following is greater: \$20 or 3% of \$600?  
**A** \$20                              **B** 3% of \$600  
**C** Same value                      **D** Unable to determine
- 7D ▶ 9** What is the value of  $245 + -100 + 77 + 153$ ?  
**A** 575                              **B** 375  
**C** -375                              **D** -575

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1**              **Support sheet 7A.1 Percentage of a quantity**  
**Q2**              **Support sheet 7A.2 Squares of decimal numbers**  
**Q3**              **Support sheet 7A.3 Rounding**  
**Q4**              **Support sheet 7A.4 Straight-line depreciation**  
**Q5–6**          **Support sheet 7B.1 Reading information from a table**  
**Q7**              **Support sheet 7C.1 Counting days**  
**Q8**              **Support sheet 7D.1 Comparing numbers**  
**Q9**              **Support sheet 7D.2 Adding and subtracting directed numbers**

# 7A Declining-balance method of depreciation

These resources are available on your obook assess:

- **Video tutorial 7A:** Watch and listen to an explanation of Example 7A-1
- **assess quiz 7A:** Test your skills with an auto-correcting multiple-choice quiz
- **Spreadsheet 7A:** Comparing declining-balance and straight-line depreciation

1010

## depreciation

a loss in the value of an item over time

## salvage value

the reduced value of an item after depreciation

The **depreciation** of an item is its loss in value due to age and usage. The value of an item after depreciation is called its **salvage value**, book value, scrap value or written-down value. Some items can be depreciated by businesses or workers to help reduce the tax they have to pay. The value by which an item has been depreciated can be used as a tax deduction against taxable income.

In the Year 11 Mathematics Standard course we calculated straight-line depreciation, which depreciates the value of an item by a constant amount each year. In this course we will look at the declining-balance method of depreciation, which depreciates the value of an item by a constant percentage each year.

### EXAMPLE 7A-1 Constructing a depreciation table

Construct a table to calculate the value of a \$20 000 car after 3 years if its rate of depreciation is 20% p.a.

Solve				Think	Apply
<b>Year</b>	<b>Value (\$)</b>	<b>Depreciation (\$)</b>	<b>Depreciated value (\$)</b>	Use 20% = 0.2.	Amount of depreciation each year = rate of depreciation × the value of the car at the beginning of the year. Subtract the amount of depreciation from the value at the beginning of the year.
1	20 000	4000	16 000	Depreciation year 1 = $0.2 \times 20\,000$ = \$4000	
2	16 000	3200	12 800	Value end of year 1 = $20\,000 - 4000$ = \$16 000	
3	12 800	2560	10 240	Depreciation year 2 = $0.2 \times 16\,000$ = \$3200	
				Value end of year 2 = $16\,000 - 3200$ = \$12 800	
				Depreciation year 3 = $0.2 \times 12\,800$ = \$2560	
				Value end of year 3 = $12\,800 - 2560$ = \$10 240	

# EXERCISE 7A Declining-balance method of depreciation

- 1 Complete the table to calculate the value of a \$24 900 car after 3 years if its rate of depreciation is 22% p.a.

Year	Value (\$)	Depreciation (\$)	Depreciated value (\$)
1	24 900	$0.22 \times 24\,900 = 5478$	19 422
2	19 422	$0.22 \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$	
3			

- 2 Complete the table to calculate the value after 4 years of office furniture costing \$34 800 if its rate of depreciation is 18% p.a.

Year	Value (\$)	Depreciation (\$)	Depreciated value (\$)
1	34 800	$\underline{\hspace{2cm}} \times 34\,800 = \underline{\hspace{2cm}}$	
2			
3			
4			

The formula for the declining-balance method of depreciation

$$S = V_0(1 - r)^n$$

where  $S$  = the salvage value of the asset

$V_0$  = the purchase price of the asset

$r$  = the percentage depreciation rate per time period, expressed as a decimal

$n$  = the total number of time periods.

You will notice that this formula is very similar to the compound interest formula, but with a minus sign before the  $r$ , as the value steadily decreases over time.



## EXAMPLE 7A-2 Calculating the salvage value and amount of depreciation

A new car is purchased for \$32 000. It depreciates in value at a rate of 22% per year.

- a Calculate the salvage value of the car after 3 years.  
 b By what amount has the car depreciated in value after 3 years?

	Solve	Think	Apply
a	$S = V_0(1 - r)^n$ $= 32\,000(1 - 0.22)^3$ $= \$15\,186 \text{ (to nearest \$)}$	Substitute $V_0 = 32\,000$ , $r = 22\% = 0.22$ and $n = 3$ into the formula.	Substitute the values of $V_0$ , $r$ and $n$ into the formula $S = V_0(1 - r)^n$ .
b	$\text{Depreciation} = 32\,000 - 15\,186$ $= \$16\,814$	Subtract the salvage value (\$15 186) from the original price (\$32 000).	The amount of depreciation is the change in the value of the car.

- 3** A new car is purchased for \$35 000. It depreciates in value at a rate of 24% per year. Complete the following to find:
- the salvage value of the car after 3 years  

$$S = V_0(1 - r)^n$$

$$= \text{___} (1 - 0.24)^{\square}$$

$$= \$\text{___} \text{ (to the nearest \$)}$$
  - the amount the car has depreciated in value after 3 years.  
 Change in value = 35 000 - \_\_\_  
 = \$\_\_\_
- 4** New office carpets are purchased for \$19 990. They depreciate in value at a rate of 28% per year.
- Calculate the salvage value of the carpets after 5 years.
  - By what amount have the carpets depreciated in value after 5 years?
- 5** Some manufacturing machinery is purchased for \$56 000. It depreciates in value at a rate of 35% per year.
- Calculate the salvage value of the machinery after 4 years.
  - By what amount has the machinery depreciated in value after 4 years?

### EXAMPLE 7A-3 Calculating the annual percentage rate of depreciation

An electronic security system depreciates in value from \$35 000 to \$22 000 in 2 years. Use the declining-balance formula to calculate the annual percentage rate of depreciation.

Solve	Think	Apply
$S = V_0(1 - r)^n$ $22\,000 = 35\,000(1 - r)^2$ $\frac{22\,000}{35\,000} = (1 - r)^2$ $\sqrt{\frac{22\,000}{35\,000}} = 1 - r$ $0.7928 = 1 - r$ $r = 1 - 0.7928\dots$ $= 0.2071\dots$ $\approx 0.21$ <p><math>\therefore</math> Rate of depreciation <math>\approx 21\%</math></p>	Substitute $S = 22\,000$ , $V_0 = 35\,000$ and $n = 2$ into the formula: $22\,000 = 35\,000(1 - r)^2$ Divide both sides by 35 000. Take the square root of both sides. Add $r$ to both sides. Subtract 0.7928 from both sides.	Substitute the values of $V_0$ , $r$ and $n$ into the formula $S = V_0(1 - r)^n$ and solve the resulting equation.

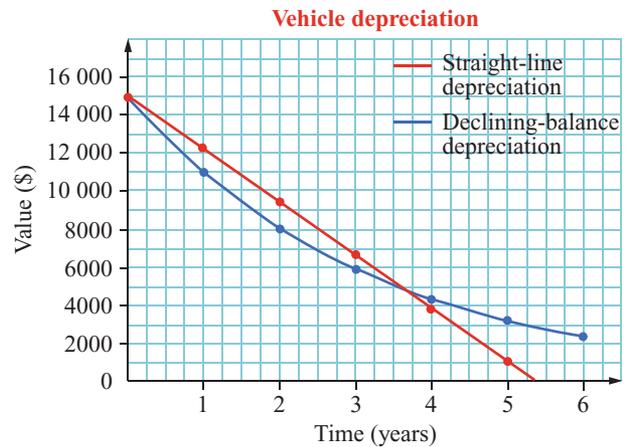
- 6** A computer server depreciates in value from \$29 000 to \$20 462 in 2 years. Complete the following to calculate the annual percentage rate of depreciation using the declining-balance formula:  $S = V_0(1 - r)^n$ .
- $$20\,462 = \text{___} (1 - r)^2$$
- $$\frac{20\,462}{\square} = (1 - r)^2$$
- $$\sqrt{\frac{20\,462}{\square}} = 1 - r$$
- $\therefore$  \_\_\_ = 1 -  $r$   
 $r = 1 - \text{___} \approx \text{___} \therefore$  Rate of depreciation  $\approx \text{___}\%$
- 7** An air conditioning system depreciates in value from \$36 000 to \$19 000 in 2 years. Use the declining-balance formula to calculate the annual percentage rate of depreciation.

- 8** Some office furniture depreciates in value from \$44 900 to \$32 440 in 2 years. Use the declining-balance formula to calculate the annual percentage rate of depreciation.
- 9** A plumbing fitting depreciates in value from \$15 000 to \$4500 in 3 years. Use the declining-balance formula to calculate the annual percentage rate of depreciation.
- 10** A network of office printers depreciates in value from \$68 000 to \$31 000 in 3 years. Use the declining-balance formula to calculate the annual percentage rate of depreciation.
- 11** The percentage rate of depreciation of some items is not always consistent from year to year. Georgina is able to depreciate the value of her car by 40% in the first year, 18% in the second year and 12% in the third year.
- If Georgina's car cost \$38 000, what is the salvage value of the car after three years?
  - What would be the equivalent annual percentage rate of depreciation? Give your answer to the nearest whole number.

**12** The graphs on the right shows the depreciation of a car using:

- the straight-line method
- the declining-balance method.

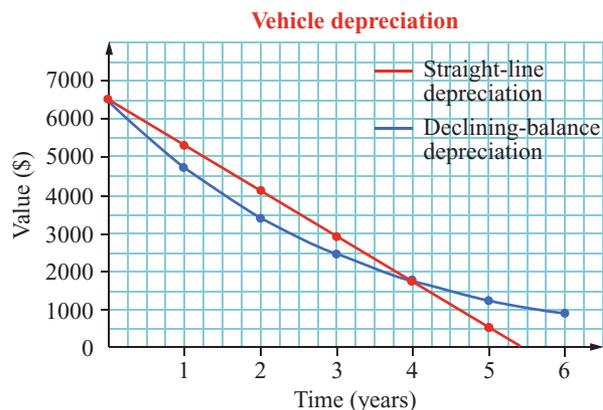
- What was the purchase price?
- What is the value of the car after 1 year using each method?
- When is the car worth \$8000, for each method?
- When is the salvage value the same for each type of depreciation? What is it?
- When is the greatest difference in book values? How much is it?
- For the straight-line method, what is the annual amount of depreciation?



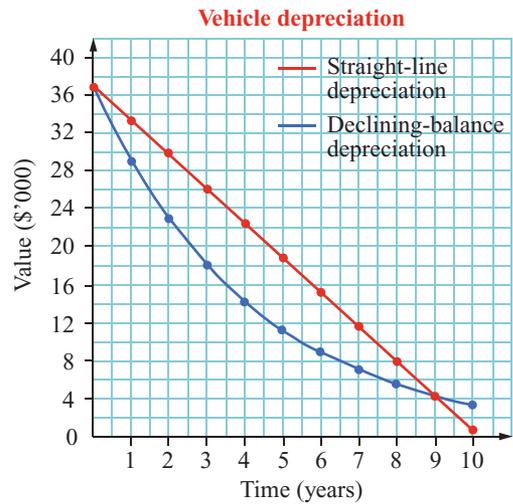
**13** The graphs on the following page show the depreciation of a scooter using:

- the straight-line method
- the declining-balance method.

- What was its purchase price?
- What is the value of the scooter after 2 years using each method?
- When is it worth \$4000 for each method?
- When is the salvage value the same for each type of depreciation? What is it?
- The scooter is scrapped after 5 years. What is its final written-down value for each method?
- For the straight-line method, what is the annual amount of depreciation?



- 14** The graphs on the right show the depreciation of a car using:
- i** the straight-line method
  - ii** the declining-balance method.
- a** What was the purchase price of the car?
  - b** What is the car's value after 2 years for each method?
  - c** When is the car worth half its original value for each method?
  - d** When is the salvage value the same for each type of depreciation? What is it?
  - e** For the straight-line method, what is the annual depreciation?
  - f** What is the annual rate of depreciation for the declining-balance method?



- 15** Peter purchases a computer for \$2300 which he uses only for work purposes. He is able to depreciate the value of the computer by 30% each year, and can use the depreciation amount as a tax deduction.
- a** What is the salvage value of Peter's computer after 1 year of use?
  - b** What amount can Peter reduce his taxable income by in the first year that he depreciates his computer?
  - c** What amount can Peter reduce his taxable income by in the second year that he depreciates his computer?
- 16** How much will the company that depreciated their manufacturing machinery in question 5 be able to reduce their taxable income by in the first year of this depreciation?

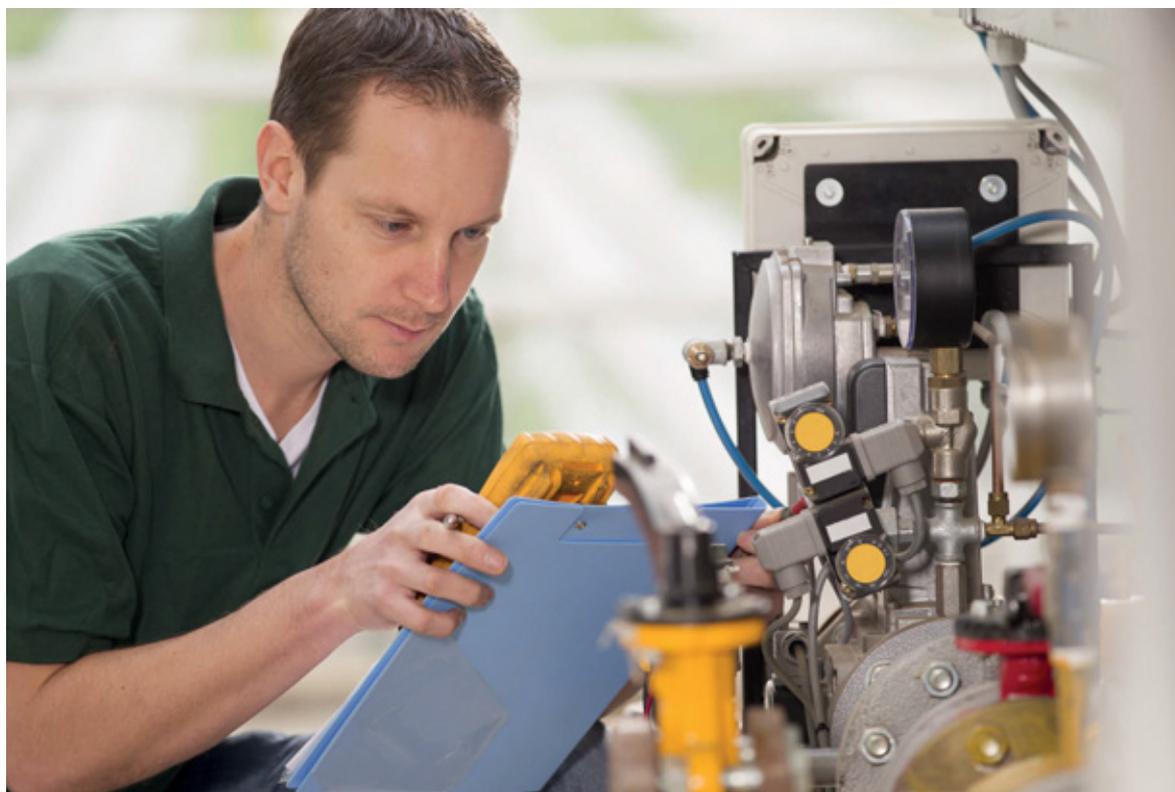
Spreadsheets can be used to create depreciation tables to model both declining-balance depreciation and straight-line depreciation.

Type in the headings in column A and row 5, and enter in the formulas into cells C6, D6, C7 and D7, as shown. The formulas in C7 and D7 can be filled down into the subsequent cells as required.

	A	B	C	D
1	Purchase cost			
2	Declining-balance percentage			
3	Straight-line depreciation amount			
4				
5		Year	Straight-line value (\$)	Declining-balance value (\$)
6		0	=B1	=B1
7		1	=C6-B\$3	=D6*(1-B\$2)
8		2		
9		3		
10		4		
11		5		

To help you, you may like to use the prepared Spreadsheet (Spreadsheet 7A) provided on your [obook](#) [assess](#).

- 17** A hot water system is purchased for \$19 900. The straight-line depreciation amount is \$3700 and the declining-balance percentage is 30%.
- Use a spreadsheet to create a depreciation table to model the depreciated value of the hot water system each year for 5 years using both methods of depreciation.
  - Draw a graph of the depreciation of the hot water system for each method, on the same set of axes.
  - From the table, when  $n = 2$ ,  $S = \underline{\hspace{2cm}}$  for the straight-line method and  $S = \underline{\hspace{2cm}}$  for the declining-balance method.
  - From the graphs, when  $n = 3.5$ ,  $S \approx \underline{\hspace{2cm}}$  for the straight-line method and  $S \approx \underline{\hspace{2cm}}$  for the declining-balance method.
  - From the graph, the straight line intersects the curve when  $n \approx \underline{\hspace{2cm}}$ .  
The values are the same after approximately  $\underline{\hspace{2cm}}$  years, or  $\underline{\hspace{2cm}}$  years and  $\underline{\hspace{2cm}}$  months.



- 18** A motorcycle used for courier work is purchased for \$11 350. The depreciation can be calculated as either \$2100 per year using the straight-line method, or 32% per year using the declining-balance method.
- Use a spreadsheet to create a depreciation table to model the depreciated value of the motorcycle each year for 5 years using both methods of depreciation.
  - Draw a graph of the depreciation of the motorcycle for each method, on the same set of axes.
  - Using the straight-line method, when is the salvage value less than that for the declining-balance method?
  - What is the salvage value of the motorcycle after  $3\frac{1}{2}$  years, using each method?
  - When is the motorcycle worth half its original value, under each method?
- 19** Use the internet to research the amount of depreciation of motor vehicles of different ages. Does the percentage of depreciation stay the same for the lifetime of the vehicle? Write a report on your findings.

# 7B Reducing-balance loans

These resources are available on your obook assess:

- **Spreadsheet 7B:** Create a reducing-balance loan table
- **Investigation 7B:** Investigate comparison interest rates
- **assess quiz 7B:** Test your skills with an auto-correcting multiple-choice quiz

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## reducing-balance loan

a loan for which the interest is calculated on the balance owing, which gradually reduces over time

A **reducing-balance loan** is a compound interest loan with periodic repayments. In this type of loan, the interest is calculated at the start of each repayment period. A home loan is an example of a reducing-balance loan, and for home loans the repayment period is often monthly, but may be fortnightly or weekly.

In a reducing-balance loan the repayment amount remains consistent throughout the lifetime of the loan, while the interest for each repayment period gradually reduces as the balance reduces. This means that early repayments on a reducing-balance loan pay a higher proportion of interest compared to latter repayments.

This topic examines various methods of calculating the balance owing and the monthly payments required. Usually, the information most frequently needed by borrowers concerns the amount that can be borrowed and the repayment. Most lending institutions have online calculators that can be used to find the monthly repayment, the amount of interest paid and the effects of changing the interest rate and making extra payments.

### EXAMPLE 7B-1 Looking up the monthly repayment on a loan

This table of home loan repayments was generated using a spreadsheet. The annual interest rate is 8.5% and the amount shown is the monthly repayment.

Years	\$160 000	\$200 000	\$240 000	\$280 000	\$320 000	\$360 000	\$400 000
5	\$3282.64	\$4103.30	\$4923.96	\$5744.62	\$6565.30	\$7385.96	\$8206.60
10	\$1983.78	\$2479.72	\$2975.66	\$3471.60	\$3967.54	\$4463.48	\$4959.44
15	\$1575.58	\$1969.48	\$2363.38	\$2757.28	\$3151.16	\$3545.06	\$3938.96
20	\$1388.52	\$1735.64	\$2082.78	\$2429.90	\$2777.04	\$3124.16	\$3471.28
25	\$1288.36	\$1630.46	\$1932.64	\$2254.64	\$2576.72	\$2898.82	\$3260.92
30	\$1230.26	\$1537.82	\$1845.40	\$2152.96	\$2460.52	\$2768.08	\$3075.64

Use the table to find the monthly repayment on a loan of \$240 000 over:

- a** 30 years                      **b** 25 years                      **c** 10 years.

	Solve	Think	Apply
<b>a</b>	\$1845.40	Look down the \$240 000 column and along the row showing the number of years.	Determine the appropriate column and row in the table.
<b>b</b>	\$1932.64		
<b>c</b>	\$2975.66		

## EXERCISE 7B Reducing-balance loans

Use the table in Example 7B–1 to answer questions 1–3.

- 1 Find the monthly repayment on a loan of \$320 000 over:
  - a 10 years
  - b 15 years
  - c 25 years
  - d 30 years.
  
- 2
  - a Find the monthly repayment on \$400 000 over 20 years.
  - b Find the monthly repayment on \$360 000 over 20 years.
  - c How much larger is the monthly repayment on \$400 000 than that on \$360 000 over 20 years?
  
- 3 From the amounts shown in the table, what is the maximum loan that can be taken if you can afford these monthly repayments?
 

a \$2000	b \$2300	c \$2480	d \$2600
----------	----------	----------	----------
  
- 4 Go to the website of a bank and use their online calculator to check the current results for questions 1 to 3.



### EXAMPLE 7B–2 Calculating the amount still owing on a loan

A home loan of \$250 000 is taken out at an interest rate of 7.75% p.a., reducible monthly, with a monthly repayment of \$1888. Use the table to calculate the amount still owing after 6 months.

Month	Principal at the start of the month	Monthly interest	Monthly repayment	Balance at end of month
1	\$250 000			
2				
3				
4				
5				
6				

Solve				
Month	Principal at the start of the month	Monthly interest	Monthly repayment	Balance at end of month
1	\$250 000	\$1614.58	\$1888	\$249 726.58
2	\$249 726.58	\$1612.82	\$1888	\$249 451.40
3	\$249 451.40	\$1611.04	\$1888	\$249 174.44
4	\$249 174.44	\$1609.25	\$1888	\$248 895.69
5	\$248 895.69	\$1607.45	\$1888	\$248 615.14
6	\$248 615.14	\$1605.64	\$1888	\$248 332.78

Think	Apply
<p>Monthly interest rate = <math>\frac{7.75}{12}</math>  <math>= 0.645\ 83\%</math></p> <p>Interest for 1st month <math>I = \frac{250\ 000 \times 0.645\ 83 \times 1}{100}</math>  <math>= \\$1614.58</math></p> <p>Hence:            Balance at end of 1st month  <math>= 250\ 000 + 1614.58 - 1888</math>  <math>= \\$249\ 726.58</math></p> <p>The first row can now be completed. The balance at the end of the first month becomes the principal for the second month.            Hence for the second month:</p> <p><math>I = \frac{249\ 726.58 \times 0.645\ 83 \times 1}{100}</math>  <math>= \\$1612.81</math></p> <p>Balance at end of 2nd month  <math>= 249\ 726.58 + 1612.81 - 1888</math>  <math>= \\$249\ 451.40</math></p> <p>The balance at the end of the second month becomes the principal for the third month, etc.</p>	<p>Calculate the balance at end of the month by adding the interest to the principal and then subtracting the repayment. This amount then becomes the principal for the next month.</p>

*Note:* in Example 7B–2 the amount of interest being paid each month is reducing because the balance is reducing each month. The total amount repaid over 6 months =  $6 \times 1888 = \$11\ 328$ .

The total interest paid =  $1614.58 + 1612.82 + \dots = \$9660.78$  and the principal has been reduced by  $11\ 328 - 9660.78 = \$1667.27$ .

If this was a simple interest loan, the total amount of interest paid would be  $6 \times 1614.58 = \$9687.48$ .

- 5** A home loan of \$420 000 is taken out at an interest rate of 7.2% p.a. with a monthly repayment of \$3306.83. Complete the table to calculate the amount still owing after 6 months.

Month	Principal at start of the month	Monthly interest	Monthly repayment	Balance at end of month
1	\$420 000	\$2520.00	\$3306.83	\$419 213.17
2				
3				
4				
5				
6				

- 6 A home loan of \$350 000 is taken out at an interest rate of 7.6% p.a. with a monthly repayment of \$2471.26. Complete the table to calculate the amount still owing after 6 months.

Month	Principal at start of the month	Monthly interest	Monthly repayment	Balance at end of month
1	\$350 000			
2				
3				
4				
5				
6				

### EXAMPLE 7B-3 Calculating the monthly repayment on a loan

This table shows the repayment per \$1000 borrowed on a monthly reducible loan. Find the monthly repayment on a loan of \$210 000 over:

- a 20 years at 7.75% p.a.      b 25 years at 8.25% p.a.

Term in years	7%	7.25%	7.5%	7.75%	8%	8.25%	8.5%
5	\$19.8012	\$19.9194	\$20.0379	\$20.1570	\$20.2765	\$20.3963	\$20.5164
10	\$11.6108	\$11.7401	\$11.8702	\$12.0011	\$12.1328	\$12.2653	\$12.3985
15	\$8.9883	\$9.1286	\$9.2701	\$9.4128	\$9.5566	\$9.7014	\$9.8474
20	\$7.7530	\$7.9036	\$8.0559	\$8.2095	\$8.3644	\$8.5207	\$8.6782
25	\$7.0678	\$7.2281	\$7.3899	\$7.5533	\$7.7182	\$7.8875	\$8.0522
30	\$6.6530	\$6.8218	\$6.9921	\$7.1641	\$7.3377	\$7.5127	\$7.6891

	Solve	Think	Apply
a	$\begin{aligned} \text{Repayment} &= 8.2095 \times 210 \\ &= \$1724.00 \end{aligned}$	From the table, the repayment on \$1000 at 7.75% over 20 years is \$8.2095.  Thousands borrowed $= \frac{210\,000}{1000} = 210$	Determine the repayment on \$1000 from the appropriate column and row in the table. Multiply this amount by the number of thousands being borrowed.
b	$\begin{aligned} \text{Repayment} &= 7.8875 \times 210 \\ &= \$1656.38 \end{aligned}$	Repayment on \$1000 over 25 years at 8.25% is \$7.8875.	

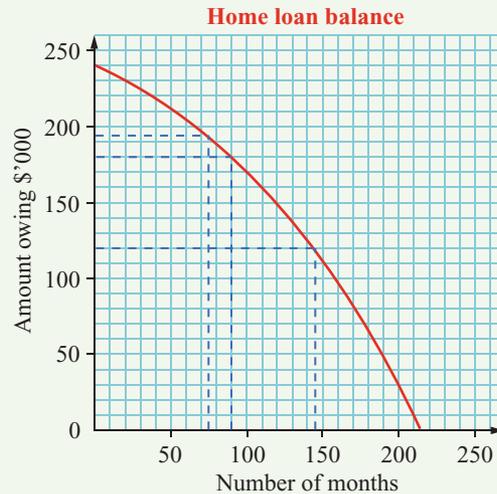
- 7 Use the table in Example 7B-3 to find the monthly repayment on each of these loans.

	Amount (\$)	Term (years)	Interest rate (%)
a	260 000	25	7.25
b	350 000	30	7
c	415 000	10	8.25
d	396 000	20	7.75
e	527 500	15	8
f	292 600	5	8.5

### EXAMPLE 7B-4 Interpreting information from a loan graph

The graph shows the amount outstanding after  $n$  months for a home loan of \$240 000 at 8.4% p.a. with a monthly repayment of \$2160.

- a How much is still owing after 75 months?
- b When is the amount owing equal to \$180 000?
- c When is the loan half paid?
- d How long does it take to repay the loan in full?



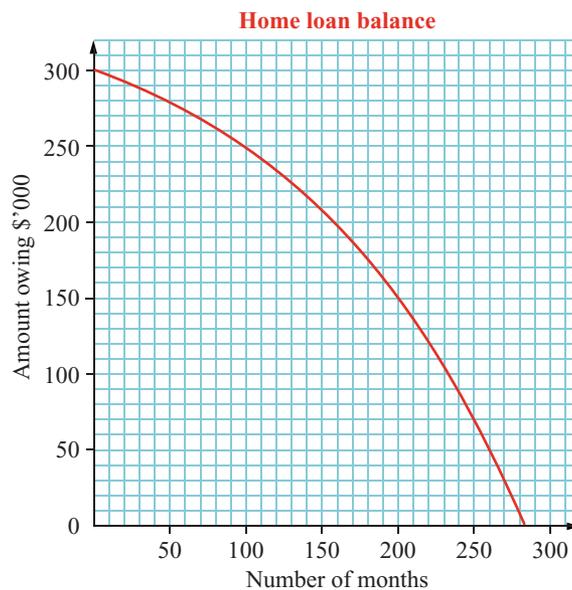
	Solve	Think	Apply
a	About \$195 000	From the graph, when: $n = 75$ months, $A \approx \$195\ 000$	Locate the appropriate point on the graph.
b	About 90 months	From the graph, when: $A = \$180\ 000$ , $n \approx 90$ months	
c	About 145 months	From the graph, when: $A = \frac{1}{2} \times 240\ 000 = \$120\ 000$ , $n \approx 145$	
d	About 215 months	From the graph, when: $A = \$0$ , $n \approx 215$ months	

8 Use the graph from Example 7B-4 to answer these questions.

- a How much is owing after 100 months?
- b How much is owing after 125 months?
- c When is the amount owing \$100 000?
- d When is the amount owing \$70 000?

9 Use this graph of the amount owing on a home loan on the right to answer the following questions.

- a How much was borrowed?
- b How much is still owing after 100 months?
- c How much is still owing after 200 months?
- d How much is still owing after  $12\frac{1}{2}$  years?
- e When is the amount owing \$200 000?
- f When is the amount owing \$50 000?
- g When is the loan half paid?
- h How long does it take to repay the loan in full?



- 10** The amount  $\$A_n$  owing on a home loan after  $n$  months is shown in the table below.

$n$	0	60	120	180	240	300
$A_n$	450 000	404 720	346 920	261 260	150 135	0

Plot the values in the table and draw a smooth curve through them to draw a graph of amount owing versus time for this loan. Use the graph to answer these questions.

- How much was borrowed?
  - How much is still owing after 100 months?
  - How much is still owing after 18 years?
  - When is the amount owing \$300 000?
  - When is the loan half paid?
  - How long does it take to repay the loan in full?
- 11** Here is a personal loan table showing the monthly repayments for amounts from \$5000 to \$15 000, over 1 to 3 years. The loan is monthly reducible with a rate of 10.45% p.a.

Years	Amount borrowed					
	\$5000	\$7000	\$9000	\$11 000	\$13 000	\$15 000
1	\$440.63	\$616.88	\$793.13	\$969.38	\$1145.63	\$1321.88
2	\$231.76	\$324.47	\$417.18	\$509.88	\$602.59	\$695.29
3	\$162.39	\$227.35	\$292.31	\$357.27	\$422.23	\$487.18

Find the monthly repayment on these personal loans.

- \$7000 over 3 years
  - \$11 000 over 2 years
  - \$13 000 over 1 year
  - \$15 000 over 3 years
- 12** The financial institution from Question 11 also has a higher interest rate for loans of more than 3 years. The table below shows the monthly repayments for personal loans for amounts from \$5000 to \$15 000, over 4 to 7 years. The loan interest is calculated on the monthly balance. The rate is 10.95% p.a. for loans of 4 to 5 years and 11.45% p.a. for loans of 6 to 7 years.

Years	Amount borrowed					
	\$5000	\$7000	\$9000	\$11 000	\$13 000	\$15 000
4	\$129.11	\$180.75	\$232.39	\$284.03	\$335.68	\$387.32
4.5	\$117.68	\$164.76	\$211.83	\$258.91	\$305.98	\$353.05
5	\$108.59	\$152.02	\$195.46	\$238.89	\$282.33	\$325.76
6	\$96.33	\$134.86	\$173.39	\$211.92	\$250.45	\$288.98
6.5	\$91.18	\$127.65	\$164.12	\$200.60	\$237.07	\$273.54
7	\$86.80	\$121.52	\$156.24	\$190.96	\$225.68	\$260.40

Find the monthly repayment on these loans.

- \$7000 over 4.5 years
- \$11 000 over 7 years
- \$13 000 over 5 years
- \$15 000 over 6.5 years

- 13** The table below shows the personal loan monthly repayments for loans over \$15 000. The interest rate increases as the time period increases. It is 9.95% p.a. for 1 to 3 years, 10.45% p.a. for 4 to 5 years and 10.95% p.a. for 6 to 7 years.

Years	Amount borrowed					
	\$15 000	\$16 000	\$17 000	\$18 000	\$19 000	\$20 000
1	\$1318.39	\$1406.28	\$1494.17	\$1582.07	\$1669.96	\$1757.85
1.5	\$900.51	\$960.54	\$1020.58	\$1080.61	\$1140.65	\$1200.68
2	\$691.83	\$737.95	\$784.07	\$830.19	\$876.32	\$922.44
2.5	\$566.82	\$604.61	\$642.40	\$680.19	\$717.98	\$755.76
3	\$483.66	\$515.90	\$548.14	\$580.39	\$612.63	\$644.87
4	\$383.69	\$409.27	\$434.85	\$460.43	\$486.01	\$511.58
4.5	\$349.38	\$372.67	\$395.96	\$419.25	\$442.54	\$465.83
5	\$322.04	\$343.51	\$364.98	\$386.44	\$407.91	\$429.38
6	\$285.13	\$304.14	\$323.14	\$342.15	\$361.16	\$380.17
6.5	\$269.64	\$287.61	\$305.59	\$323.56	\$341.54	\$359.51
7	\$256.44	\$273.54	\$290.63	\$307.73	\$324.83	\$341.92

- a** Find the monthly repayment on these loans.
- i** \$17 000 over 4.5 years      **ii** \$19 000 over 3 years
  - iii** \$20 000 over 5 years      **iv** \$16 000 over 2.5 years
  - v** \$18 000 over 6.5 years
- b** \$17 000 is borrowed over 4 years.
- i** Find the total amount repaid over the term of the loan.
  - ii** Find the amount of interest paid over the 4 years.
  - iii** Find the equivalent simple interest rate over the 4 years.

- 14** Carla and Ian take out a personal loan of \$15 000 to furnish their home. Interest is 14.4% p.a. monthly reducible. There is a loan establishment fee of \$390 and a monthly account-keeping fee of \$10 per month. Their monthly repayment is \$413.

Complete the following table to calculate how much they still owe after 6 months.



Month	Principal at start of the month	Interest	Account-keeping fee	Monthly repayment	Balance at end of month
1	$15\,000 + 390 = \$15\,390$	\$184.68	\$10	\$413	\$15 171.68
2	\$15 171.68				
3					
4					
5					
6					

- 15 a** How does making an additional lump-sum payment affect the time taken to repay a loan?  
**b** How does making an additional lump-sum payment affect the cost of a loan?

We can use a spreadsheet to model reducing-balance loans. Here we will create a spreadsheet in which you can modify the loan amount, the interest rate (p.a.) and the monthly repayment amount for a reducing-balance loan. Type in the headings in column A and row 5, and enter in the formulas into cells B6, C6, D6, E6 and B7, as shown.

These formulas can be filled down into the subsequent cells as required.

	A	B	C	D	E
<b>1</b>	Loan amount (\$)				
<b>2</b>	Interest rate (%p.a.)				
<b>3</b>	Monthly repayment (\$)				
<b>4</b>					
<b>5</b>	Month	Principal at start of the month (\$)	Monthly interest (\$)	Monthly repayment (\$)	Balance (\$)
<b>6</b>	1	=B1	=B6*(B\$2/1200)	=B\$3	=B6+C6-D6
<b>7</b>	2	=E6			

To help you, you might like to use the prepared spreadsheet (Spreadsheet 7B) provided on your [obook assess](#).

*Note:* the loan has been repaid in full when the balance in column E is either zero or negative.

- 16 a** Tanya takes out an \$8000 reducing-balance travel loan to pay for her dream holiday. The interest rate on the loan is 7.5% p.a. and she pays back the minimum monthly repayment of \$300. What is the term of the loan?  
**b** If the interest rate was 8% instead of 7.5%, how much longer would it take to pay back the loan?
- 17 a** Patrick takes out a \$25 000 reducing-balance loan to help pay for an extension to his home. The interest rate on the loan is 6.3% and the minimum monthly repayment amount is \$490. If Patrick only paid the minimum monthly repayment, how long would it take him to repay the loan?  
**b** Patrick wants to pay back the loan within 3 years. What is the minimum monthly amount he must pay back to achieve his goal? Round your answer up to the nearest \$10.  
**c** Patrick decides to pay back the amount calculated in part **b**, but before he starts repaying the loan the interest rate rises to 6.6%. Does this affect the time it will take him to pay back the loan?
- 18** Use a spreadsheet or an online calculator to model a \$10 000 reducing-balance loan with an interest rate of 6.6% p.a. and monthly repayments of \$310.  
**a** What is the term of the loan?  
**b** Find the total amount of interest paid.  
**c** If there is an interest rate rise to 7.5% p.a., how does this affect the term of the loan?  
**d** If the monthly repayments increased to \$400, how does this affect the term of the loan?  
**e** Are the findings from parts **c** and **d** applicable to any interest rate rise?  
**f** Are the findings from parts **c** and **d** applicable to any increase in the monthly repayment amount?

# 7C Credit card statements and fees

These resources are available on your obook assess:

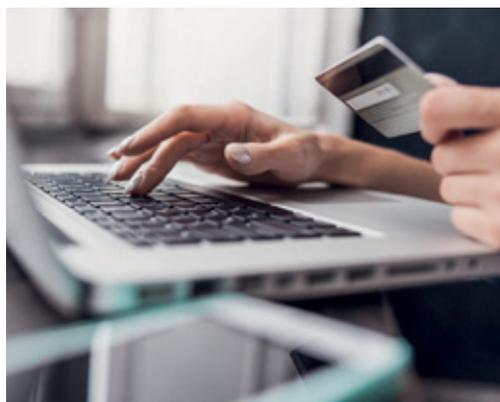
- **assess quiz 7C:** Test your skills with an auto-correcting multiple-choice quiz



## credit card

plastic card that allows the holder to purchase goods and services on credit

**Credit cards** are issued by banks and financial institutions as a convenient way for consumers to purchase goods and services from vendors. The vendor is paid by the bank and the bank recovers the money from the cardholder. Naturally, various fees and interest charges are applied by the banks for the use of this credit facility. The cardholder receives a monthly statement and must make a minimum payment of 2–5% of the balance owing. If the cardholder does not pay the account in full by the due date (for example, only the minimum payment is made), any amount outstanding will be carried over and interest charges will apply.



Credit cards usually have a higher rate of interest than most other consumer loans. Different rates of interest apply to purchases and cash advances for most cards. There is often an annual fee for the use of the card, and fees may be charged for exceeding the credit limit or making late payments. Many cards offer interest-free days under certain conditions.

Credit card statements are issued monthly, and detail expenditures made using the credit card, payments made against the credit card, the opening and closing balances, and other details.

## EXERCISE 7C Credit card statements and fees

RESEARCH

- 1 Investigate and make a list of the following current credit card fees: annual/monthly fee, cash advance fee, late payment fee, overlimit fee, card replacement fee, copy fee, payment dishonour fee, international transaction fee
- 2 Investigate the meaning of the following terms when used in credit card statements. Write the meaning of each term and make notes or give examples of any additional information that is useful to understand them. Annual percentage rate, available credit, cash advance, closing balance, credit limit, interest-free period/days, minimum amount due, opening balance, overdue amount/outstanding balance, overlimit amount, statement period/billing cycle. An example is shown below.

Term	Meaning	Notes
Cash advance	Cash withdrawn from a credit card account	This could include: <ul style="list-style-type: none"> <li>• withdrawing cash at an ATM</li> <li>• taking out cash when making a purchase at a store</li> <li>• using a credit card to gamble online or at a casino.</li> </ul>

- 3** Consider the credit card statement shown below.
- a**
    - i** What is the statement period?
    - ii** How many days is this?
  - b**
    - i** When is the minimum payment due?
    - ii** How many days is this from the start of the statement period?
  - c** What is the credit limit on this card?
  - d** What were the total debits for this period?
  - e**
    - i** What is the minimum payment due?
    - ii** What percentage is this of the closing balance?
  - f** What is the available credit?
  - g** Calculate the daily interest rate charges as a percentage and as a decimal for:
    - i** purchases
    - ii** cash advances.

CREDIT CARD STATEMENT			
		Statement begins	14 April 2018
		Statement ends	13 May 2018
		Account number	XXXX 1234 5678 9000
		Overdue amount due now	\$0
		Overlimit due now	\$0
		Payment due date	7 June 2018
		Minimum amount due	\$108.66
Opening balance	New charges	Payment received	Closing balance
\$2864.00	+\$958.00	-\$200.00	\$3622.00
Interest charged on purchases	Purchase rate 19.60%	Daily rate	
Interest charged on cash advances	Cash advance rate 21.40%	Daily rate	
Credit limit \$10 000	Available credit \$6378		

*Note:*

- As the closing balance for each month is paid in full by the due date, there are no interest charges for March and April.
- The minimum payment in April is \$10 since this is greater than 3% of \$223 (\$6.69).
- The due date for purchases made in March is 24 April, 55 days from 1 March (the start of the statement period).
- The number of interest-free days is up to 55. For example, the number of interest-free days for the purchase of furniture in March is 48 days (from 8 March to 24 April inclusive) and for the purchase of hardware is 39 days (from 17 March to 24 April inclusive).
- The interest-free period does not apply to cash advances (such as ATM withdrawals). These transactions attract interest from the day they appear on the statement.

March statement		
Date	Details	Amount (\$)
1 Mar	Opening balance	0
8 Mar	Furniture	1680
17 Mar	Hardware	67
Opening balance:		\$0
Closing balance:		\$1747
Minimum payment due: \$52.41		
Due date:		24 April

April statement		
Date	Details	Amount (\$)
11 April	Jeans	88
24 April	Payment	-1747
30 April	Sunglasses	135
Opening balance:		\$1747
Closing balance:		\$223
Minimum payment due: \$10		
Due date:		25 May

May statement		
Date	Details	Amount (\$)
9 May	Groceries	48
17 May	Electrical goods	136
25 May	Payment	-223
Opening balance:		\$223
Closing balance:		\$184
Minimum payment due: \$10		
Due date:		24 June

Questions 4–6 refer to the credit card shown in the statements above.

- 4** The minimum payment for this credit card is the greater of \$10 or 3% of the closing balance. Calculate the minimum payment due on these closing balances.
- a** \$96                      **b** \$390                      **c** \$1245                      **d** \$320
- 5** For this credit card, what would be the due date for purchases made in:
- a** June?                      **b** December?                      **c** February (not a leap year)?
- 6** How many interest-free days are available for the purchase of:
- a** jeans?                      **b** sunglasses?                      **c** groceries?                      **d** electrical goods?

- 7 Complete these statements, given that the statement period is from the first to the last day of the month, the minimum payment is the greater of \$10 or 4% of the closing balance, there are up to 55 interest-free days and the closing balance is paid in full on the due date.



September statement		
Date	Details	Amount (\$)
1 Sep	Opening balance	0
9 Sep	Clothes	80
18 Sep	Make-up	54
Opening balance:		\$0
Closing balance:		\$__
Minimum payment due:		\$__
Due date:		25 October

October statement		
Date	Details	Amount (\$)
5 Oct	Shoes	180
25 Oct	Payment	—
29 Oct	Television set	967
Opening balance:		\$__
Closing balance:		\$__
Minimum payment due:		\$__
Due date:		_____

November statement		
Date	Details	Amount (\$)
10 Nov	Groceries	48
16 Nov	DVDs	66
__ Nov	Payment	—
Opening balance:		\$__
Closing balance:		\$__
Minimum payment due:		\$__
Due date:		_____

# 7D Credit card balances

These resources are available on your obook assess:

- **Video tutorial 7D.1:** Watch and listen to an explanation of Example 7D-1
- **Video tutorial 7D.2:** Watch and listen to an explanation of Example 7D-2
- **Spreadsheet 7D:** Create a credit card statement
- **Assess quiz 7D:** Test your skills with an auto-correcting multiple-choice quiz

1010

The interest charged on credit cards is usually compounded on a daily basis. As such, a credit card is an example of a reducing-balance loan.

## EXAMPLE 7D-1 Calculating the balance on a credit card

Calculate the total amount due on an ATM cash withdrawal of \$400 using a credit card if the full amount is repaid after 15 days. The annual percentage rate (APR) for cash for the card is 21.5% and there is a fee of 1.5% of the cash advance amount.

Solve	Think	Apply
Daily interest rate $= \text{APR} \div 365$ $= 0.0589\dots\%$ $= 0.000589\dots$ $\text{Balance} = 400(1 + 0.000589\dots)^{15}$ $= \$403.55$ Cash advance fee $= \frac{1.5}{100} \times 400 = \$6$ Total amount to be repaid $= 403.55 + 6 = \$409.55$	Calculate the balance using $FV = PV(1 + r)^n$ . Calculate the cash advance fee.	Interest on a cash withdrawal is calculated daily from the date of the transaction, whether or not the card has an interest-free period. Daily interest rate $= \text{APR} \div 365$ days Balance $= \text{withdrawal} \times (1 + \text{daily interest rate})^{\text{number of days}}$ Total amount due $= \text{balance} + \text{cash advance fee}$

## EXERCISE 7D Credit card balances

UNDERSTANDING, FLUENCY AND COMMUNICATING

- Complete the following to calculate the total amount due on an ATM cash withdrawal of \$500 using a credit card if the full amount is repaid after 23 days. The annual percentage rate for cash for the card is 22.9% and there is a fee of 1.5% of the cash advance amount.  
 Daily interest rate  $= \underline{\hspace{1cm}}\% \div 365 = \underline{\hspace{1cm}}\% = \underline{\hspace{1cm}}$   
 Balance  $= \square \times (1 + \square)^\square = \$\underline{\hspace{1cm}}$   
 Cash advance fee  $= \frac{\square}{100} \times 500 = \$\underline{\hspace{1cm}}$   
 Total amount due  $= \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \$\underline{\hspace{1cm}}$



- Calculate the total amount due on an ATM cash withdrawal of \$450 using a credit card if the amount is repaid after 17 days. The annual percentage rate for cash is 20.9% and there is a fee of 1.5% of the cash advance amount.
- Calculate the total amount due on an over-the-counter cash withdrawal of \$150 using a credit card if the full amount is repaid after 21 days. The annual percentage rate for cash is 19.8% and the cash advance fee is the greater of \$2.50 or 1.5% of the cash advance amount.

When calculating the charges on a credit card payment, both the purchase date and the date of repayment must be included in the calculation.



### EXAMPLE 7D-2 Calculating a credit card balance after multiple purchases

Calculate the balance on this credit card at the end of April given that the annual percentage rate is 19.9%. Assume that there is no interest-free period on these purchases.

April statement		
Date	Details	Amount (\$)
1 Apr	Opening balance	37
7 Apr	Purchase	53
16 Apr	Purchase	29
23 Apr	Payment	-80

Solve	Think
$r = 19.9\% \div 365$ $= 0.0545\dots\%$ $= 0.000545\dots$ 1 to 6 April (6 days): $PV = 37, n = 6$ $FV = 37(1 + 0.000545\dots)^6$ $= \$37.12$ 7 to 15 April (9 days): $PV = 37.12 + 53 = 90.12, n = 9$ $FV = 90.12(1 + 0.000545\dots)^9$ $= \$90.56$ 16 to 22 April (7 days): $PV = 90.56 + 29 = 119.26, n = 7$ $FV = 119.56(1 + 0.000545\dots)^7$ $= \$120.02$ 23 to 30 April (8 days): $PV = 120.02 - 80 = 40.02, n = 8$ $FV = 40.02(1 + 0.000545\dots)^8$ $= \$40.19$ The balance at the end of April is \$40.19.	Split the month into time periods depending on when transactions are made. Calculate the balance at the end of each time period using, $FV = PV(1 + r)^n$ , where $FV$ is the balance at the end of the time period, $PV$ is the balance at the start of the time period, $n$ is the number of days in the time period, and $r$ is the daily rate expressed as a decimal.
Apply	
Future balance = previous balance $\times$ $(1 + \text{daily interest rate})^{\text{number of days}}$	

- 4 Calculate the balance at the end of May, given that the annual percentage rate is 18.6%. Assume that there is no interest-free period on these purchases.

May statement		
Date	Details	Amount (\$)
1 May	Opening balance	147
8 May	Purchase	88
20 May	Purchase	133
25 May	Payment	-100

- 5 Calculate the balance due at the start of September, given that the annual percentage rate is 18.8% and there is no interest-free period on these purchases.

September statement		
Date	Details	Amount (\$)
1 Sep	Opening balance	135
10 Sep	Purchase	59
19 Sep	Purchase	136
24 Sep	Payment	-100

If you have a credit card with an interest-free period, then these purchases do not appear on the balance, when calculating the interest on the balance, until the end of the interest-free period.

### EXAMPLE 7D-3 Calculating a credit card balance including interest-free period

Calculate the balance on this credit card at the end of August, given that the annual percentage rate is 17.9%. There is an interest-free period of 55 days for purchases and a 1.5% fee for cash advances (added on at the end of the month). Assume there is no interest-free period remaining on the opening balance.

August statement		
Date	Details	Amount (\$)
1 Aug	Opening balance	105
5 Aug	Purchase	99
11 Aug	Cash advance	100
21 Aug	Payment	-150

Solve	Think
$r = 17.9\% \div 365$ $= 0.0490\dots\%$ $= 0.000490\dots$ 1 to 4 August (4 days): $PV = 105, n = 4$ $FV = 105(1 + 0.000490\dots)^4$ $= \$105.21$ 5 to 10 August (6 days): $PV = 105.21, n = 6$ $FV = 105.21(1 + 0.000490\dots)^6$ $= \$105.52$ 11 to 20 August (10 days): $PV = 105.52 + 100 = 205.52, n = 10$ $FV = 205.52(1 + 0.000490\dots)^{10}$ $= \$206.53$ 21 to 31 August (11 days): $PV = 206.53 - 150 = 56.53, n = 11$ $FV = 56.53(1 + 0.000490\dots)^{11}$ $= \$56.84$ Cash advance fee = $\frac{1.5}{100} \times 100$ $= \$1.50$ Final balance = $56.84 + 99 + 1.50$ $= \$157.34$ The balance at the end of August is \$157.34.	Split the month into time periods depending on when transactions are made. Calculate the balance at the end of each time period using $FV = PV(1 + r)^n$ , where $FV$ is the balance at the end of the time period, $PV$ is the balance at the start of the time period (excluding interest-free purchases), $n$ is the number of days in the time period and $r$ is the daily rate expressed as a decimal. Add the cost of the interest-free purchases and the cash advance fee to the balance at the end of the month.

### Apply

Future balance = previous balance  $\times (1 + \text{daily interest rate})^{\text{number of days}}$

- 6** Calculate the balance at the end of December, given that the annual percentage rate is 21.6%. Note: there is an interest-free period of 55 days for purchases (the council rates charge counts as a purchase). Assume there is no interest-free period remaining on the opening balance.

December statement		
Date	Details	Amount (\$)
1 Dec	Opening balance	87
7 Dec	Council rates	488
14 Dec	Payment	-100
19 Dec	Purchase	43
24 Dec	Payment	-100

- 7** Calculate the balance on this credit card at the end of July. The interest rate on the credit is 22.9%, and there is an interest-free period of 55 days for purchases and a 1.5% fee for cash advances (added on at the end of the month). Assume there is no interest-free period remaining on the opening balance.

July statement		
Date	Details	Amount (\$)
1 July	Opening balance	637
4 July	Purchase	126
10 July	Cash advance	200
14 July	Payment	-100
23 July	Purchase	93
26 July	Payment	-150

- 8 a** Create a table to detail the following transactions for the month of November.  
 Opening balance = \$517  
 Purchases on 6 November of \$29, 13 November of \$134, 19 November of \$97 and 24 November of \$166  
 Cash advances on 12 November of \$200 and 25 November of \$150  
 Payments on 7 November of \$300 and 20 November of \$100
- b** Calculate the closing balance for November, given that the annual percentage rate is 19.4% p.a. for both purchases and cash withdrawals, and there is no interest-free period for purchases and a 1.5% fee for cash advances (added on at the end of the month).
- c** What is the total interest and fees for November?
- d** Determine the minimum payment due if it is the greater of \$25 or 4% of the closing balance.
- 9** In your own words, explain what is meant by an interest-free period on a credit card.
- 10** Use an internet calculator to find the time it would take to repay various credit card balances if you only make the minimum repayment each month. How much interest would be charged? How much can be saved by increasing your monthly repayment by \$5, \$10, etc.?
- 11** Use the internet to compare credit card interest rates with interest rates for other loan types. Why do you think the interest rates for credit cards are typically higher than for other loans? Write a report on your findings.

**12** Zoe was comparing two different credit cards.

- Card A: \$120 annual fee with an annual percentage rate of 15% p.a. for all purchases
- Card B: \$75 annual fee with an annual percentage rate of 17.9% p.a. for all purchases

Zoe's credit card transactions for November were as shown in the table.

November statement		
Date	Details	Amount (\$)
1 Nov	Opening balance	0
2 Nov	Purchase	115
5 Nov	Purchase	90
11 Nov	Payment	-65
12 Nov	Purchase	93
20 Nov	Payment	-100
25 Nov	Purchase	229

If there was no interest-free period on either card, which credit card would be more cost-efficient for Zoe?

*Note:* calculate the proportion of the annual card fee for one month.

**13** Gil was trying to choose between two different credit cards with the following costs.

- Card A: \$60 annual fee plus 24.9% p.a. for all purchases
- Card B: \$150 annual fee plus 16.9% p.a. for all purchases

For both cards there is no interest-free period for purchases and a 1.5% fee for cash advances (added on at the end of the month).

Gil's credit card activity for March was as follows.

March statement		
Date	Details	Amount (\$)
1 March	Opening balance	350
4 March	Cash advance	100
13 March	Purchase	49
17 March	Payment	-65
25 March	Purchase	180

Taking into account the proportion of the annual fee for March, which credit card saves Gil more money, and by how much?

- 14 a** Anita has a credit card balance of \$1650 at the end of April. The minimum payment due is the greater of \$50 or 3% of the closing balance. How much is the minimum payment?
- b** Anita only makes the minimum payment at the end of April. If the annual percentage rate is 21.9%, how much interest is charged on April's remaining balance in May?



We can use a spreadsheet to create a credit card statement. Here we will create a spreadsheet in which you can add different purchases, the number of days since the purchase and the interest rate (p.a.) for a credit card.

- Type in the headings in rows 1 and 2, and enter in the formulas into cells D3 and D4, as shown.
- The formula in cell D4 can be filled down into the subsequent cells as required.
- Enter in the interest rate into cell B1, the name of the items into column A (starting at A4), the amount of the items into column B (starting at B3) and the days into the month for that transaction into column C (starting at C4).
- Finish each statement with as 'End of month' item. For this item, in the 'Days into month' cell enter the days in the month + 1.

	A	B	C	D
1	Interest rate (% p.a.)			
2	Item	Amount (\$)	Time period	Balance
3	Opening balance		1	=B3
4				=D3*(1+B\$1/36500)^(C4-C3)+B4
5				

*Note:* this spreadsheet assumes that there is no interest-free period for the credit card.

To help you, you might like to use the prepared spreadsheet (Spreadsheet 1H) provided on your [obook assess](#).

- 15** Yani's credit card statement for January is shown in the table. His credit card has an interest rate of 21.9% p.a.

Use a spreadsheet to calculate the opening balance on Yani's credit card at the end of January. Assume that there is no interest-free period on his credit card.

Date	Item	Cost
1 January	Opening balance	\$224
6 January	Payment	-\$150
10 January	Purchase	\$89
22 January	Purchase	\$35
24 January	Purchase	\$112
30 January	Purchase	\$14

- 16** Jacinta's purchases are shown in the table.

Use a spreadsheet to calculate the difference in Jacinta's balance at the end of March if she makes these purchases on a credit card with an interest rate of 19.9% p.a. rather than using one with an interest rate of 27.9% p.a. Assume that there is no interest-free period on either card.

Date	Item	Cost
1 March	Groceries	\$122
4 March	School books	\$249
8 March	Groceries	\$135
13 March	Coffee machine	\$399
15 March	Groceries	\$148
19 March	Concert tickets	\$198
22 March	Groceries	\$110
29 March	Groceries	\$132



- 7B** ▶ **7** How will an additional lump-sum payment typically affect the term of a loan?  
**A** It will increase. **B** It will decrease.  
**C** It will be the same. **D** It's impossible to tell.
- 7C** ▶ **8** The minimum payment for a credit card is the greater of \$10 or 3% of the closing balance. If the closing balance is \$255, what is the minimum payment due?  
**A** \$7.65 **B** \$10 **C** \$76.50 **D** \$255
- 7C** ▶ **9** If the due date on a credit card is 50 days from the first day of the statement period, what would be the due date for purchases made in March?  
**A** 1 April **B** 18 April  
**C** 19 April **D** 20 April

Use the credit card statement on the right to answer questions **10** and **11**.

- 7D** ▶ **10** If the annual percentage rate is 19.6% and there is no interest-free period on purchases, what would the balance at the end of August be?  
**A** \$291.00 **B** \$293.63  
**C** \$297.75 **D** \$300.72
- 7D** ▶ **11** If there was an interest-free period of 40 days on purchases, what would the balance at the end of August be? Assume there is no interest-free period remaining on the opening balance.  
**A** \$291.00 **B** \$293.66  
**C** \$297.75 **D** \$300.72

August statement		
Date	Details	Amount (\$)
1 Aug	Opening balance	246
10 Aug	Water rates	395
16 Aug	Purchases	83
21 Aug	Payment	-500
27 Aug	Purchases	67

## REVIEW SET 1

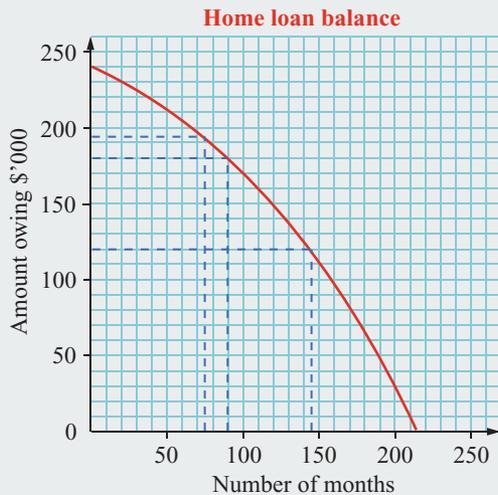
- A new tractor is purchased for \$63 000. It depreciates in value at a rate of 18% per year.
  - Calculate the salvage value of the tractor after 4 years.
  - By what amount has the tractor depreciated in value over the 4 years?
- A washing machine depreciates in value from \$1500 to \$1100 in 2 years. Use the declining-balance formula to calculate the annual percentage rate of depreciation. Give your answer to the nearest whole number.
- Use the following table to find the maximum loan that can be taken if you can afford the following monthly repayments.

Years	\$160 000	\$200 000	\$240 000	\$280 000	\$320 000	\$360 000	\$400 000
5	\$3282.64	\$4103.30	\$4923.96	\$5744.62	\$6565.30	\$7385.96	\$8206.60
10	\$1983.78	\$2479.72	\$2975.66	\$3471.60	\$3967.54	\$4463.48	\$4959.44
15	\$1575.58	\$1969.48	\$2363.38	\$2757.28	\$3151.16	\$3545.06	\$3938.96
20	\$1388.52	\$1735.64	\$2082.78	\$2429.90	\$2777.04	\$3124.16	\$3471.28
25	\$1288.36	\$1630.46	\$1932.64	\$2254.64	\$2576.72	\$2898.82	\$3260.92
30	\$1230.26	\$1537.82	\$1845.40	\$2152.96	\$2460.52	\$2768.08	\$3075.64

- a** \$1450 **b** \$1900 **c** \$2500 **d** \$2700



- 4 The graph below shows the amount outstanding after  $n$  months for a home loan of \$240 000 at 8.4% p.a. with a monthly repayment of \$2160. Use this graph to answer the following questions.
- How much is owing after 140 months?
  - When is the amount owing \$150 000?



- 5 The minimum payment on a credit card is the greater of \$20 or 3% of the closing balance. Calculate the minimum payment on these closing balances.
- \$700
  - \$1250
- 6 The following credit card statement is for the month of March.

March statement		
Date	Details	Amount (\$)
1 Mar	Opening balance	721
9 Mar	Purchase	356
19 Mar	Purchase	49
24 Mar	Payment	-600
27 Mar	Purchase	73

If the annual percentage rate is 21.6% and there is no interest-free period on purchases, calculate the balance at the end of March.

## REVIEW SET 3

- Some office furniture is purchased for \$15 000. It depreciates in value at a rate of 13% per year.
  - Calculate the salvage value of the office furniture after 3 years.
  - By what amount has the office furniture depreciated in value over the 3 years?
- A car depreciates in value from \$78 000 to \$41 000 in 3 years. Use the declining-balance formula to calculate the annual percentage rate of depreciation.
- Use the table in Review set 1 question 3 to find the monthly repayment on a loan of \$400 000 over:
  - 10 years
  - 30 years.



- 4** The graph in Review set 3 question 4 shows the amount owing on a home loan of \$300 000 against repayment periods. Use the graph to answer the following questions.
- How much is owing on the home loan after 90 months?
  - When is the amount owing \$120 000?

- 5** This credit card statement is for the month of June.

If the annual percentage rate is 22.1%, calculate the balance at the end of June.

- 6** If the due date on a credit card is 55 days from the first day of the statement period, what would be the due date for purchases made in December?

June statement		
Date	Details	Amount (\$)
1 Jun	Opening balance	589
8 Jun	Purchase	135
15 Jun	Purchase	68
21 Jun	Payment	-450
24 Jun	Purchase	33

## REVIEW PRACTICE EXAMINATION QUESTION

- 1 a** Kelly purchases new carpets for her apartment for \$4690. She depreciates the value of the carpets by 16% each year for 5 years.
- What is the salvage value of the carpets after 3 years? (2 marks)
  - What is the salvage value of the carpets after 5 years? (2 marks)
  - By what amount have the carpets depreciated in value after 5 years? (1 mark)

- b** A home loan of \$320 000 is taken out over 30 years at an interest rate of 8.5% p.a. monthly reducible. The monthly repayment is \$2460.52.

- i** Complete the next line of the table below to find the amount owing on the loan at the end of the second month. (2 marks)

Month	Principal at start of the month	Monthly interest	Monthly repayment	Balance at end of month
1	\$320 000	\$2266.67	\$2460.52	\$319 806.15
2				

- ii** What is the balance owing on the loan at the end of the sixth month? (2 marks)
- iii** By how much has the balance of the loan decreased after 6 months? (1 mark)
- c** Megan made the following purchases in December on her credit card.
- 8 December: \$69
  - 15 December: \$130
  - 23 December: \$335

Megan's credit card had a rate of 22.5% p.a., and the balance at the start of December was \$0.

- i** If Megan's credit card had an interest-free period for purchases of 55 days, what would be the balance on her credit card at the end of December? (1 mark)
- ii** If Megan's credit card had no interest-free period, what would be the balance on her credit card at the end of December? (3 marks)
- iii** How much interest would Megan's purchases in December accrue by the end of December if there was no interest-free period? (1 mark)

**TOTAL:**  
15 marks

# 8

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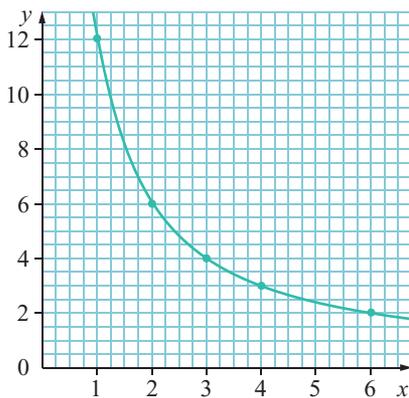
## Graphs of practical situations

The main mathematical ideas investigated are:

- ▶ working with linear and non-linear relationships and their graphical representation
- ▶ constructing graphs from tables of values
- ▶ sketching graphs from a description of a situation
- ▶ modelling linear relationships
- ▶ modelling exponential growth and decay
- ▶ using algebraic relationships to model physical phenomena
- ▶ determining the best model (linear or exponential) to approximate a graph by considering its shape
- ▶ recognising the limits of models.

# ARE YOU READY?

Use the following graph for questions 1 and 2.



**8A** ▶ 1 Which table of values matches the graph?

**A**

x	1	2	3	4	6
y	12	4	3	2.5	2

**B**

x	1	2	3	4	6
y	12	6	4	3	2

**C**

x	1	2	3	4	6
y	12	8	6	4	2

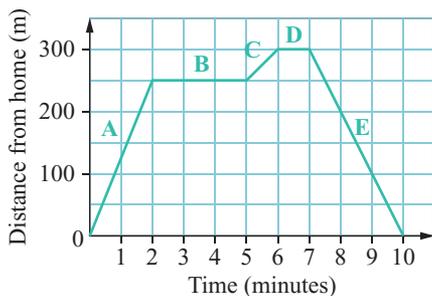
**D**

x	1	2	3	4	6
y	12	6	4	2.5	2

**8A** ▶ 2 What type of relationship is shown in the graph?

- A** linear                                      **B** quadratic  
**C** cubic                                        **D** inverse

Seth's journey is represented in the distance–time graph below. Use this graph to answer questions 3–5.



**8B** ▶ 3 In which section of the graph is Seth travelling the fastest?

- A** A            **B** B            **C** C            **D** E

**8B** ▶ 4 What is Seth's speed in section A of the graph?

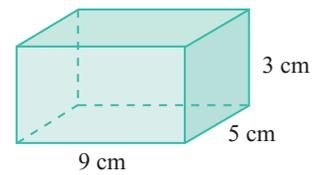
- A** 0 km/min                                      **B** 125 m/min  
**C** 250 m/min                                    **D** 500 m/min

**8B** ▶ 5 In which section/s of the graph is Seth stationary?

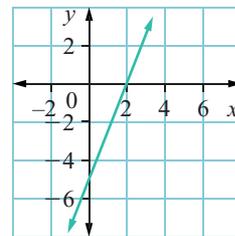
- A** A and C                                        **B** A and E  
**C** B and D                                        **D** B only

**8C** ▶ 6 What volume of water could this container hold?

- A** 15 cm<sup>3</sup>  
**B** 27 cm<sup>3</sup>  
**C** 135 cm<sup>3</sup>  
**D** 174 cm<sup>3</sup>



Use the following graph for questions 7 and 8.



**8D** ▶ 7 What is the gradient of this graph?

- A** -5            **B** 2            **C**  $\frac{5}{2}$             **D** 5

**8D** ▶ 8 What is the y-intercept of this graph?

- A** -5            **B** 2            **C**  $\frac{5}{2}$             **D** 5

**8E** ▶ 9 When  $x = 4$ , what is the value of  $3^x$ ?

- A** 7            **B** 12            **C** 64            **D** 81

**8E** ▶ 10 If  $y = 1000(1.05)^x$ , what is the approximate value of  $y$  when  $x = 12$ ?

- A** 1000    **B** 1796  
**C** 12 600    **D** 129 746

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook\\_ assess](#).

- Q1–2** Support sheet 8A.1 Plotting relationships from a table of values  
**Q3–5** Support sheet 8B.1 Interpreting distance–time graphs  
**Q6** Support sheet 8C.1 Calculating volume  
**Q7–8** Support sheet 8D.1 Identifying features of a linear graph  
**Q9–10** Support sheet 8E.1 Substitution in exponential relationships

## 8A

Constructing graphs  
from tables of values**linear  
relationship**

relationship between two variables in which the coordinate points describing this relationship lie in a straight line when plotted on a number plane (or Cartesian plane)

These resources are available on your obook assess:

- **Spreadsheet 8A.1:** Drawing graphs using a table of values
- **assess quiz 8A:** Test your skills with an auto-correcting multiple-choice quiz

10  
10

In the Mathematics Standard Year 11 course, straight-line graphs were used to model practical situations and make predictions based on the model. In real life, many situations cannot be modelled by a **linear relationship**; however, we can still plot graphs of these relationships by using a table of values. Graphical representations are excellent models of practical situations, because they provide a visual way to view trends.

**EXAMPLE 8A-1** Constructing a graph from a table of values

- a** For each table of values, construct the graph of the relationship by plotting the coordinate points on a number plane and joining them with a straight line or smooth curve.

**i**

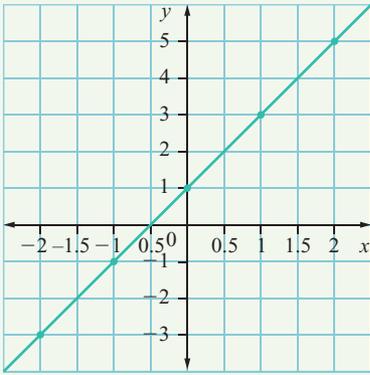
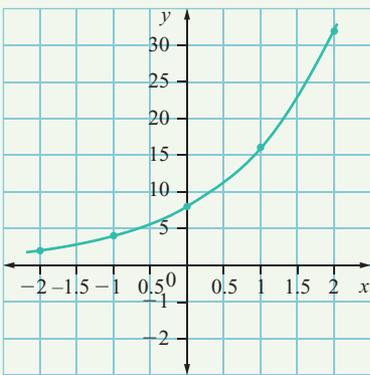
<b>x</b>	-2	-1	0	1	2
<b>y</b>	-3	-1	1	3	5

**ii**

<b>x</b>	-2	-1	0	1	2
<b>y</b>	2	4	8	16	32

- b** Does the graph show a linear relationship? Explain your answer.

**a i**

	Solve	Think	Apply
<b>a i</b>		<p>On a number plane, show a scale from <math>-2</math> to <math>2</math> along the <math>x</math>-axis and at least <math>-3</math> to <math>5</math> on the <math>y</math>-axis.</p> <p>Plot the five points on the number plane. As the points appear to lie in a straight line, rule a line through them.</p>	<p>Plot the coordinate points on a number plane after selecting the scale to be used along the axes from the values in the table.</p> <p>If the points appear to lie in a straight line, rule a straight line through the points.</p>
<b>ii</b>		<p>On a number plane, show a scale from <math>-2</math> to <math>2</math> along the <math>x</math>-axis and at least <math>-1</math> to <math>32</math> on the <math>y</math>-axis.</p> <p>Plot the five points on the number plane. As the points do not appear to lie in a straight line, draw a smooth curve through them.</p>	<p>If the points do not appear to lie in a straight line, draw a smooth curve through the points.</p>

	Solve	Think	Apply
<b>b i</b>	This is a linear relationship as the points lie in a straight line.	The points lie in a straight line so the relationship is linear.	Identify whether the points lie in a straight line.
<b>ii</b>	This is not a linear relationship as the points do not lie in a straight line.	The points do not lie in a straight line so the relationship is not linear.	

## EXERCISE 8A Constructing graphs from tables of values

- 1** For each table of values, construct the graph of the relationship by plotting the coordinate points on a number plane and joining them with a straight line or smooth curve.

**a**

<i>x</i>	-2	-1	0	1	2
<i>y</i>	-5	-2	1	4	7

**b**

<i>x</i>	-2	-1	0	1	2
<i>y</i>	6	3	2	3	6

**c**

<i>x</i>	-2	-1	0	1	2
<i>y</i>	1	2	4	8	16

**d**

<i>x</i>	-2	-1	0	1	2
<i>y</i>	-2	0	2	4	6

**e**

<i>x</i>	-2	-1	0	1	2
<i>y</i>	-8	-1	0	1	8

**f**

<i>x</i>	1	2	3	4	6
<i>y</i>	24	12	8	6	4

**g**

<i>x</i>	-2	-1	0	1	2
<i>y</i>	3	1	-1	-3	-5

**h**

<i>x</i>	0	1	2	3	4
<i>y</i>	80	40	20	10	5

- 2** Check your graphs from question 1 using graphing technology.
- 3** Which of the graphs drawn in question 1 show a linear relationship?
- 4** Consider each table of values in question 1. For each increase of 1 in  $x$ , how does  $y$  change? What do you notice about the difference between  $y$ -values for linear relationships when the difference between  $x$ -values is constant (the same)?
- 5** Use your results to question 4 to decide which of the following relationships are linear. You do not need to construct a graph.

**a**

<i>x</i>	1	2	3	4	5
<i>y</i>	0	3	8	15	24

**b**

<i>x</i>	1	2	3	4	5
<i>y</i>	-1	2	5	8	11

**c**

<i>x</i>	2	3	4	5	6
<i>y</i>	3	2	1	0	-1

**d**

<i>x</i>	4	5	6	7	8
<i>y</i>	1	3	9	27	81

**e**

<i>x</i>	0	1	2	3	4
<i>y</i>	0	-2	-8	-18	-32

**f**

<i>x</i>	-2	-1	0	1	2
<i>y</i>	-6	0	6	12	18



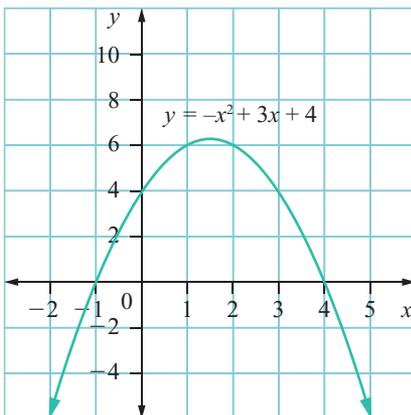
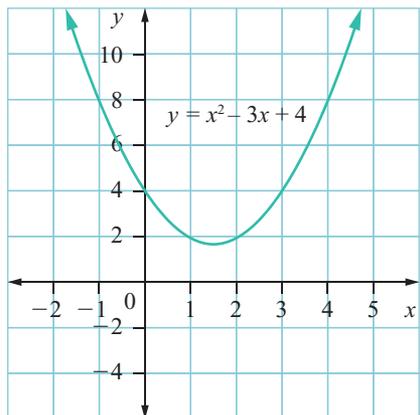
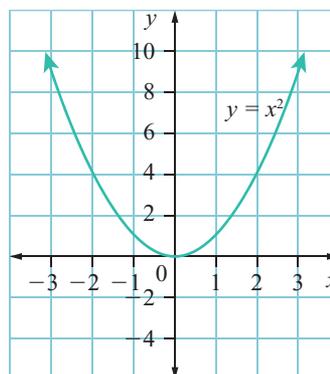
There are many non-linear relationships that can be graphed. Some of these are summarised below.

### Quadratic relationships

The parabola is the curved shape resulting from a quadratic relationship.

The simplest parabola has the equation  $y = x^2$ . It has a turning point at  $(0, 0)$ . The  $y$ -intercept is 0 and the  $x$ -intercept is 0.

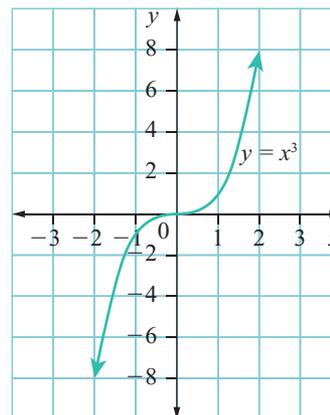
Other examples of a quadratic relationship are shown:



### Cubic relationships

The cubic graph is the curved shape resulting from a cubic relationship.

The simplest cubic graph has the equation  $y = x^3$ . It passes through  $(0, 0)$  so the  $y$ -intercept is 0 and the  $x$ -intercept is 0.

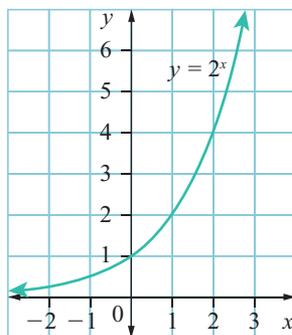


### Exponential relationships

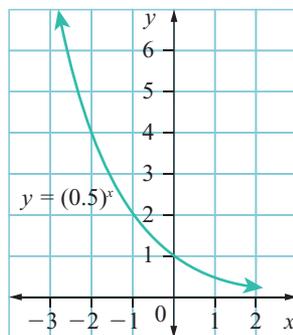
The exponential graph is the curved shape resulting from an exponential relationship.

The simplest exponential graph has the equation  $y = a^x$ , where  $a > 0$  and  $a \neq 1$ . It has a  $y$ -intercept of 1 and no  $x$ -intercepts.

When  $a > 1$ , the graph of  $y = a^x$  looks similar to this:



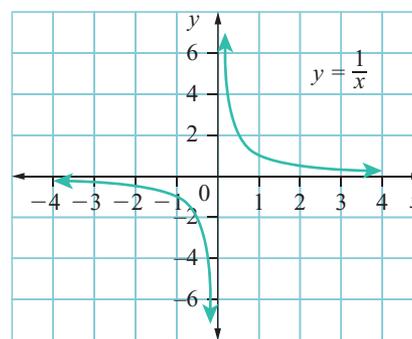
When  $0 < a < 1$ , the graph of  $y = a^x$  looks similar to this:



## Reciprocal or inverse relationships

The hyperbola is the curved shape resulting from an inverse relationship.

The simplest hyperbola has the equation  $y = \frac{1}{x}$ , where  $x \neq 0$ . It has two branches and no  $x$ -intercepts or  $y$ -intercepts.



- 6** Identify each of the non-linear relationships in question 1 as a quadratic, cubic, exponential or inverse relationship.
- 7** The distance,  $d$ , in kilometres, travelled by a train in time  $t$  hours is shown in the table below.

$t$	0	1	2	3	4
$d$	0	80	160	240	320

- Plot the values in the table by hand and join the points with a straight line or smooth curve to construct the graph of this relationship. Show  $t$  on the horizontal axis and  $d$  on the vertical axis.
  - Use graphing technology to produce the graph of this relationship by first entering the coordinate points.
  - Does the graph show a linear relationship?
  - For each increase of 1 in  $t$ , how does  $d$  change?
  - Predict the distance travelled by the train after 5 hours.
  - Extend your graph and predict the time it takes for the train to travel 500 km.
  - The common difference in  $d$  values you found in part **d** relates to the gradient of the line. Explain how.
- 8** The table below shows Rory's income versus the number of hours he worked.

<b>Time (h)</b>	1	2	3	4	5
<b>Income (\$)</b>	18	36	54	72	90

- Plot the values in the table by hand and join the points with a straight line or smooth curve to construct the graph of this relationship. Show time on the horizontal axis. Check your answer using graphing technology.
  - Does the graph show a linear relationship?
  - For each increase of 1 hour, how does Rory's income change? Is there a common difference in income values?
  - How does your answer to part **c** confirm that the relationship between income and number of hours worked is linear?
- 9** The table shows weight versus cost for buying tomatoes.

<b>Weight (kg)</b>	1	2	3	4	5	6
<b>Cost (\$)</b>	4.50	9.00	13.50	18.00	22.50	27.00

- Is this a linear relationship? How can you tell from the table?
- How much would it cost to buy 8 kg of tomatoes?
- Construct the graph of this relationship using the table of values or graphing technology. Extend the graph to check your answer to part **b**.



- 10** A ball is thrown into the air from a tower. Its height,  $h$ , in metres, after  $t$  seconds is shown in the table.

$t$	0	1	2	3	4	5
$h$	5	8	9	8	5	0

- Is the relationship between height of the ball and time linear? Explain your answer.
  - Construct the graph of this relationship using the table of values.
  - The relationship between  $t$  and  $h$  is quadratic. Explain how the features of the graph confirm this.
  - What was the initial height of the ball?
  - From the graph, estimate the height of the ball after 3.5 seconds.
  - What was the maximum height of the ball and when did this occur?
  - When did the ball hit the ground?
- 11** The distance taken for a car to stop on a dry road,  $d$ , in metres, without skidding when travelling at a given speed,  $s$ , in km/h, is shown in the table.

$s$	10	20	30	40	50	60
$d$	8	18	30	44	60	78



- Use graphing technology or plot the values in the table by hand to construct the graph of this relationship.
  - What type of relationship does the graph show?
  - What is the difference between the stopping distances in a school zone when travelling at a speed of 40 km/h and when travelling at a speed of 60 km/h?
  - Many residential areas have a 50 km/h speed limit. How much shorter is the stopping distance for a car travelling at 50 km/h than for a car travelling at 60 km/h? Does the 10 km/h reduction make a significant difference? Explain your answer.
  - Extend your graph to predict the stopping distance for a car travelling at 100 km/h.
- 12** Megan invested \$1000 at 10% p.a. compounded annually for 5 years. The table shows the amount,  $A$ , in dollars, of the investment after time,  $t$ , in years.

$t$	0	1	2	3	4	5
$A$	1000	1100	1210	1331	1464.10	1610.51

- Plot the values in the table by hand and join the points with a straight line or smooth curve to construct the graph of this relationship. Show  $t$  on the horizontal axis. Check your answer using graphing technology.
- Does the graph show a linear relationship?
- For each increase of 1 year, how does  $A$  change? Is there a common difference between the values of  $A$ ?
- Calculate the following ratios.
 

i $\frac{1100}{1000}$	ii $\frac{1210}{1100}$	iii $\frac{1331}{1210}$	iv $\frac{1464.10}{1331}$	v $\frac{1610.51}{1464.10}$
-----------------------	------------------------	-------------------------	---------------------------	-----------------------------
- What do you notice from your results for part **d**? This pattern (common ratio) indicates that the relationship is exponential.

- 13** The height,  $h$ , in millimetres, of a bean plant is measured over  $t$  days. The results are shown in the table.

$t$	0	1	2	3	4
$h$	200	300	450	675	1012.5

- Plot the values in the table by hand and join the points with a straight line or smooth curve to construct the graph of this relationship. Show  $t$  on the horizontal axis. Check your answer using graphing technology.
  - What type of relationship does the graph show? How can you check? (*Hint*: for each increase of 1 day for  $t$ , what is the ratio of  $h$  values?)
  - From the graph, estimate the height of the plant after 2.5 days.
  - From the graph, estimate how long it took for the plant to reach a height of 90 cm.
  - Extend your graph to predict the height of the plant after 6 days.
  - Could you continue to extend this graph to predict the height of the plant? Explain your answer.
- 14** The salvage value of a car,  $S$ , in dollars, decreases over time,  $t$ , in years as shown in the table.

$t$	0	1	2	3	4
$S$	32 000	24 000	18 000	13 500	10 125

- Use graphing technology or plot the values in the table by hand to construct the graph of this relationship.
  - What type of relationship does the graph show? (*Hint*: for each increase of 1 year for  $t$ , what is the ratio of  $S$  values?)
  - What does the intercept of the curve with the  $S$ -axis represent?
  - From the graph, estimate the salvage value of the car after 18 months.
  - From the graph, estimate how long it took for the car to be worth half of its initial value.
  - Extend your graph to predict the salvage value of the car after 6 years.
- 15** The average speed necessary to travel 240 km in a particular time, where  $t$  is the time in hours and  $s$  is the speed in km/h, is shown in the table.

$t$	2	3	4	6	8	12	24	48
$s$	120	80	60	40	30	20	10	5

- Plot the values in the table by hand and join the points with a straight line or smooth curve to construct the graph of this relationship. Show  $t$  on the horizontal axis. Check your answer using graphing technology.
- If the curve was extended in both directions, do you think it would cross the  $t$ -axis or the  $s$ -axis?
- What type of relationship does the graph show?
- From the graph, estimate the average speed required to complete the 240 km trip in 5 hours.
- From the graph, estimate how long it would take to complete the trip at 95 km/h.

- 16** Paul rents a guest house for \$720 per week. In order to save money, he has other people stay and divides the expense equally. The table shows the relationship between the cost per person,  $C$ , in dollars, and the number of people staying per week,  $n$ .

$n$	1	2	4	6	8
$C$	720	360	180	120	90

- Use graphing technology or the table of values to plot the data.
- Join the points with a straight line or smooth curve to show the trend of the data.
- What type of relationship does the graph show?
- From the graph, find the cost per person per week when three people are staying.
- From the graph, find the number of people staying in a week when the cost per person is \$144.

- 17** The volume of a cube,  $V$ , in cubic centimetres, with a side length,  $x$ , in centimetres, is shown in the table.

$x$	0	5	10	15	20	25
$V$	0	125	1000	3375	8000	15 625

- Use graphing technology or plot the values in the table by hand to construct the graph of this relationship.
- What type of relationship do you think the graph shows? Explain why the relationship is not linear, exponential or inverse. How can you decide whether the relationship is quadratic or cubic?
- From the graph, estimate the volume of a cube with side length 8 cm.
- From the graph, estimate the side length of a cube with volume 10 000 cm<sup>3</sup>.
- Extend your graph to predict the volume of a cube with side length 30 cm.

- 18** The distance a stone falls freely,  $d$ , in metres, after a period of time,  $t$ , in seconds, is shown in the table.

$t$	0	1	2	3	4	5
$d$	0	5	20	45	80	125

- Set up a number plane with a scale from 0 to 10 along the horizontal  $t$ -axis and a scale of 0 to 500 along the vertical  $d$ -axis. Use the table of values to construct the graph of this relationship for the first 5 seconds and then extend the curve to show the relationship for  $t$  values up to 10 seconds.
- From the graph, predict how far a stone falls down a well in 7 seconds.
- From the graph, estimate how long it would take for a stone to fall 400 m down a well.

- 19** The mass of algae in a pond varies under different conditions such as amount of sunshine or rainfall, temperature, etc. Data was recorded over 9 days, as shown in the table.

<b>Time (days)</b>	0	1	2	3	4	5	6	7	8	9
<b>Mass (g)</b>	3	6	10	8	13	16	14	18	21	19

- Set up a number plane with a scale from 0 to 12 along the horizontal time axis and a scale of 0 to 25 along the vertical mass axis. Use the table of values to construct the graph of this relationship for the first 9 days.
- Does the graph show a relationship you recognise? Even though the graph does not follow a linear, quadratic, cubic, exponential or inverse relationship, the trend shown by the curve may be used to estimate or predict values of mass or time.
- From the graph, estimate the mass of algae in the pond after  $4\frac{1}{2}$  days.
- From the graph, estimate when the mass of algae first reaches 8 g.
- Extend the graph to predict the mass of algae in the pond after 12 days. How reliable is this prediction? Explain your answer.

- 20** The times and heights of high tide and low tide measured at Fort Denison, Sydney, over four days are shown in the table below.

Day	Sunday				Monday			
<b>Time</b>	00:19	06:21	12:32	18:53	01:06	07:06	13:11	19:33
<b>Height (m)</b>	0.57	1.60	0.43	1.73	0.47	1.62	0.40	1.84

Day	Tuesday				Wednesday			
<b>Time</b>	01:53	07:54	13:52	20:16	02:42	08:44	14:36	21:01
<b>Height (m)</b>	0.38	1.62	0.39	1.93	0.32	1.60	0.41	1.99

One way to construct a graph of the relationship of height of tide over time is to first form a table showing the time,  $t$ , in hours, and height,  $h$ , in metres, from the start of Sunday.

- Copy and complete the table shown on the right. Show values of  $t$  to the nearest hour.
- Use graphing technology or plot the points by hand to construct a graph of the relationship between  $h$  and  $t$ . Use the table of values in part **a** and join the points with a smooth curve.
- Does the graph show a relationship you recognise? Can you use the trend shown by the curve to estimate or predict values of  $t$  or  $h$ ? Explain your answer.
- Estimate the height of the tide when  $t = 10$ h.
- Predict the height of the tide when  $t = 100$ h. What day and time is this?
- Predict when the first high tide will occur on Thursday. How high will it be?

$t$ (h)	$h$ (m)
0	0.57
6	1.60
13	0.43
19	1.73
25	0.47
31	1.62
37	
43	
...	...

- 21** The temperature at a ski resort,  $T$ , in  $^{\circ}\text{C}$ , was recorded every 2 hours starting at midnight. The results are shown in the table, where  $t$  is the number of hours after midnight.

$t$	0	2	4	6	8	10	12	14	16	18	20	22
$T$	-4	-6	-5	-3	-1	3	5	7	4	3	0	-2

- Construct the graph of this relationship using the table of values.
  - What was the temperature at 4 pm?
  - What was the lowest recorded temperature? When did this occur?
  - What was the highest recorded temperature? When did this occur?
  - From the graph, estimate the temperature at 2:30 pm.
  - From the graph, estimate for how long the temperature was above  $0^{\circ}\text{C}$ .
- 22** Olivia is riding on a Ferris wheel. The height,  $h$ , in metres, of her seat above the ground after time  $t$ , in minutes, is recorded in the table.

$t$	0	3	6	9	12	15	18
$h$	2	58	2	58	2	58	2

- Set up a number plane with a scale from 0 to 24 along the horizontal  $t$ -axis and a scale of 0 to 60 along the vertical  $h$ -axis. Plot the values from the table and join them with a smooth curve to construct the graph of this relationship for the first 18 minutes. Extend the curve to show the relationship for  $t$  values up to 24 minutes.
  - How long does it take for the Ferris wheel to complete one revolution?
  - How many revolutions does Olivia experience if the ride takes 24 minutes?
  - From the graph, estimate the height of Olivia's seat after:
    - 1 minute
    - 2 minutes
    - 10 minutes.
  - From the graph, estimate the time when Olivia's seat first reaches a height of 30 m.
  - Predict the height of Olivia's seat after 20 minutes.
  - Predict the next time at which her seat is at the height found in part **f**.
- 23** Create a spreadsheet to produce the graph of a relationship from a table of values. Enter the values for  $x$  in the first column and values for  $y$  in the second column of the spreadsheet. Highlight the cells containing the  $x$  and  $y$  values, click **Insert** and **Charts**, and then choose **Line graph**. Check some of the graphs you constructed in this exercise using your spreadsheet. You might like to use the prepared spreadsheet provided on your obook assess.

# 8B Sketching graphs related to speed, distance and time

These resources are available on your [ebook assess](#):

- **Video tutorial 8B:** Watch and listen to an explanation of Example 8B-1
- **assess quiz 8B:** Test your skills with an auto-correcting multiple-choice quiz

01  
19

## distance–time graphs

graphs that show the relationship between distance and time, where time is the independent variable shown on the horizontal axis and distance is the dependent variable shown on the vertical axis

## sketch

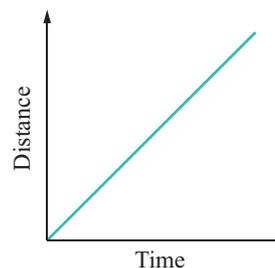
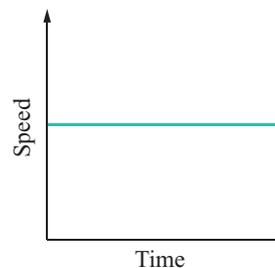
an approximate representation of a graph that indicates the general trend in the relationship between two variables without necessarily showing exact data points; includes labelled axes, intercepts and any other important relevant features, but generally there is no scale shown on the axes

In Chapter 2, we interpreted **distance–time graphs** and used them to solve problems related to speed, distance and time. A distance–time graph (or travel graph) shows the relationship between distance (as the dependent variable) and time (as the independent variable).

In this chapter, we will draw a **sketch** of a distance–time graph or a speed–time graph from the description of a situation. Compared to the actual graph, a sketch should be recognisably similar but does not need to be exact. We first need to identify the two quantities or variables in the relationship. These variables will be shown on the two axes of the number plane. As the independent variable increases, we indicate what happens to the dependent variable.

For example, if a car is travelling at a constant speed, the relationship between speed and time could be shown in the sketch graph at right. Notice that, as the time increases, the speed stays the same (remains constant).

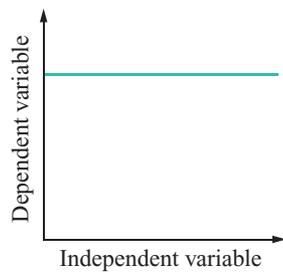
The relationship between distance and time for the same situation could be shown in the sketch graph below. As the time increases, the rate of change of the distance (speed) is the same. This is a linear relationship for which the gradient indicates the speed.



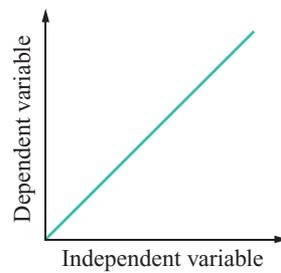
A sketch is used to indicate the relationship between two variables. The axes are labelled but no scale is shown. The main focus is on the shape of the graph, which indicates how the dependent variable changes as the independent variable increases. Here are four examples:



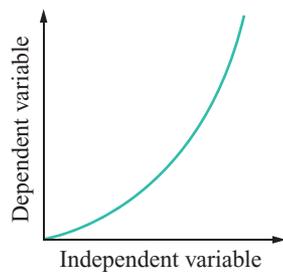
### Zero rate of change



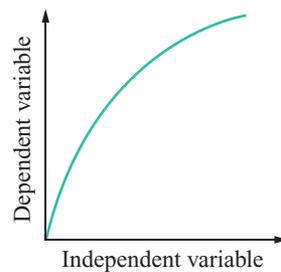
### Constant rate of change



### Increasing rate of change

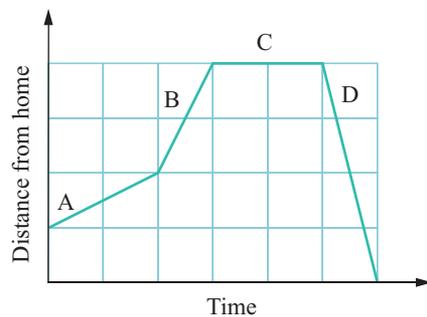


### Decreasing rate of change

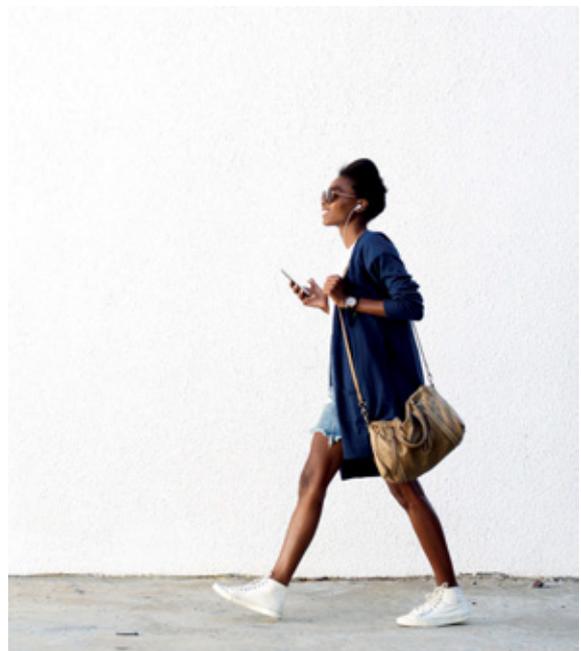


## EXERCISE 8B Sketching graphs related to speed, distance and time

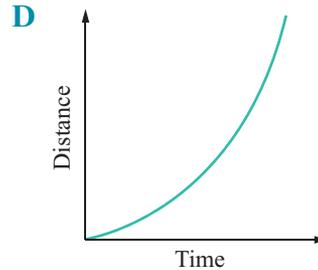
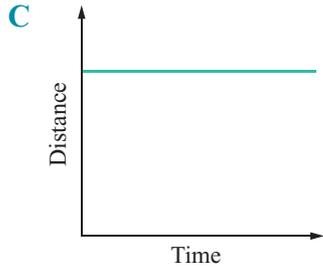
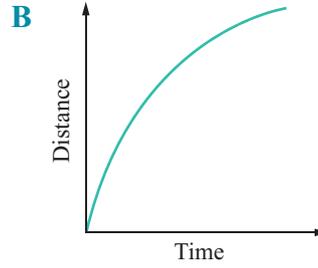
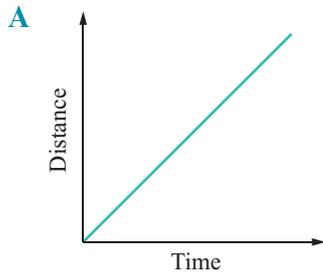
- 1 a Use the graph shown below of Alex's journey to answer true or false to each statement.



- i Section A shows that Alex is walking home at a constant speed.
  - ii Section B shows that Alex is walking at a constant but slower speed than in section A.
  - iii Section C shows that Alex is stationary.
  - iv Section D shows that Alex is walking at a constant but faster speed than in section B.
- b Write a description of Alex's journey.



**2** Alisa takes a journey in her car. Four sections of the journey are represented by the sketch graphs A, B, C and D.



Which graph represents the section of the journey where the car is:

- a** stationary (not moving)?
- b** moving at a constant speed?
- c** accelerating (moving at an increasing speed)?
- d** decelerating (moving at a decreasing speed)?

**EXAMPLE 8B-1** Sketching a distance–time graph

Tom walks from home at a constant speed to his friend’s house. They watch a movie and then Tom jogs home at a constant speed. From this description, sketch a graph showing Tom’s distance from home as time passes.

Solve	Think	Apply
	<p>Decide on the two variables to show on the graph: time on the horizontal axis and distance from home on the vertical axis.</p> <p>When Tom is walking at a constant speed, the distance from home increases at a constant rate. When he jogs home at a constant speed, the distance from home decreases at a constant rate but at a higher rate.</p>	<p>For a sketch, there is no need to show a scale on the axes. Label the axes with the two variables and draw a line or curve to indicate what happens to the distance as time passes.</p>

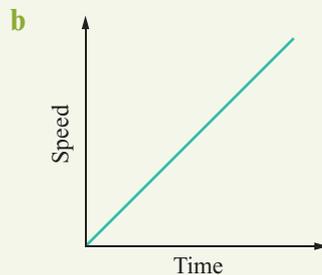
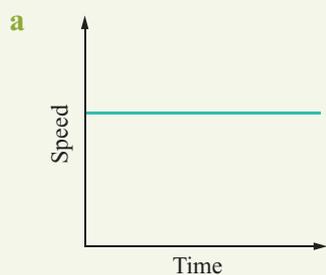
**3** Kate walks at a constant speed to collect her bicycle from her friend’s house. She then cycles at a constant speed to meet her sister at a cafe. They spend some time chatting over a cup of coffee. From this description, sketch a graph showing the distance Kate has travelled as time passes. (Assume no acceleration or deceleration occurs.)

- 4 Leo starts his parked car, drives on to the roadway and accelerates until he reaches the speed limit. He continues at a constant speed and then decelerates to park outside the supermarket. From this description, sketch a graph showing the distance Leo has travelled as time passes.
- 5 Hanna is driving her car at a constant speed. She sees a red traffic light ahead and decelerates to stop at the intersection. When the traffic light turns green, she accelerates to the speed limit and then maintains this speed. From this description, sketch a graph showing the distance Hanna has travelled as time passes.



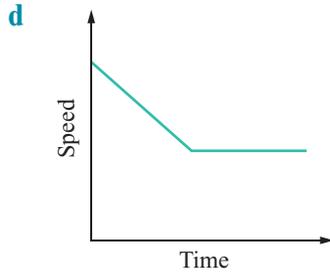
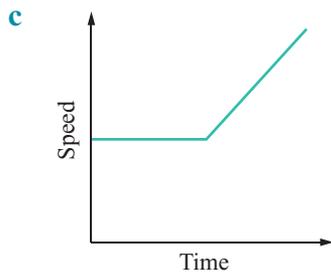
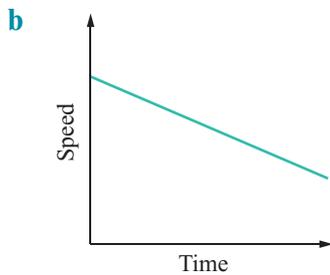
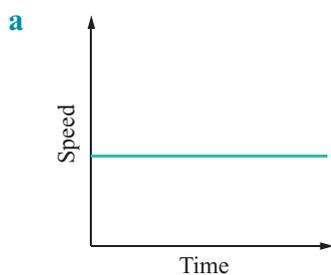
### EXAMPLE 8B-2 Interpreting a sketch of a speed-time graph

Explain what each of these sketch graphs represent.



	Solve	Think	Apply
<b>a</b>	Graph shows the relationship between speed and time. As time increases, speed remains constant (the same). This means that the distance travelled increases at a constant rate.	The horizontal line indicates that the speed is constant (not changing). Constant speed means that for each equal interval of time, the change in distance travelled is the same.	Consider what happens to the speed and distance travelled as time passes. Speed is a measure of how the distance changes as time passes. For constant speed, there is a constant increase in the distance travelled as time passes.
<b>b</b>	Graph shows the relationship between speed and time. As time increases, speed increases at a constant rate. This means that the distance travelled increases at an increasing rate. It is an example of acceleration.	The sloping line indicates that the speed is increasing at a constant rate. This means that for each equal interval of time, the speed increases by the same amount and the distance travelled increases at an increasing rate.	For increasing speed, there is an increasing change in the distance travelled as time passes.

6 Explain what each of the following sketch graphs represent.



7 For the distance–time graph in question 1, sketch the speed–time graph for:

- a** section A                      **b** section B
- c** section C                      **d** section D.

8 Sketch the matching speed–time graph for the situation described in question 4.

9 Sketch the matching speed–time graph for the situation described in question 5.

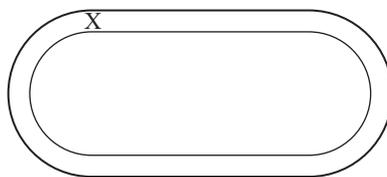


### EXAMPLE 8B–3 Sketching a distance–time graph from a speed–time graph

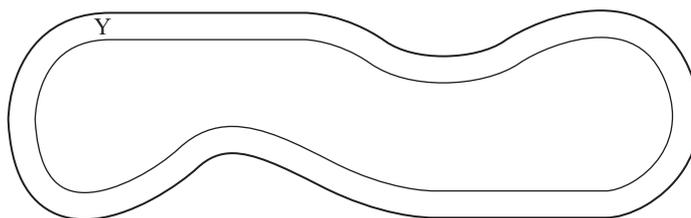
For each of the sketch graphs in Example 8B–2, draw a sketch of the matching distance–time graph.

	Solve	Think	Apply
<b>a</b>	<p>A distance-time graph with 'Distance' on the vertical axis and 'Time' on the horizontal axis. A straight line starts at the origin (0,0) and slopes upwards to the right, indicating constant speed.</p>	<p>Constant speed means the change in distance is constant as time passes. Sketch the graph as a straight line since the speed (gradient) is constant.</p>	<p>Consider how the constant speed affects the change in distance travelled as time passes.</p>
<b>b</b>	<p>A distance-time graph with 'Distance' on the vertical axis and 'Time' on the horizontal axis. A curve starts at the origin (0,0) and curves upwards, becoming steeper as time increases, indicating that speed is increasing.</p>	<p>As the speed is increasing at a constant rate, the distance is increasing at an increasing rate of change. The relationship is a curve since the speed (gradient) is increasing.</p>	<p>Consider how the increasing speed affects the change in distance travelled as time passes.</p>

- 10** For each of the sketch graphs in question 6, draw a sketch of the matching distance–time graph.
- 11** Hayden starts off from home on his bicycle. He accelerates to a certain speed and then maintains this speed along the bike track. Near the end of the bike track, he decelerates until he is stationary.
- Sketch a graph of speed versus time to represent this journey.
  - Sketch a graph of distance versus time to represent this journey.
- 12** Petra jogs along a straight, level track at a constant speed until she reaches a hill. She follows the track up a steep incline and continues over the hill and down the other side. Once the track levels out, she returns to jogging at a constant speed.
- Sketch a graph of speed versus time to represent this journey.
  - Sketch a graph of distance versus time to represent this journey.
- 13** Sketch a graph of speed versus time for a race car as it moves around the track shown in this diagram. The car starts from point X and moves in a clockwise direction around the track. (*Hint*: think about the speed of the car as it goes around curves in the track.)



- 14** Sketch a graph of speed versus time for a race car as it moves around the track shown in the diagram below. The car starts from point Y and moves in a clockwise direction around the track.



## 8C

# Sketching more graphs from a given description

These resources are available on your obook assess:

- **Video tutorial 8C:** Watch and listen to an explanation of Example 8C-1
- **assess quiz 8C:** Test your skills with an auto-correcting multiple-choice quiz

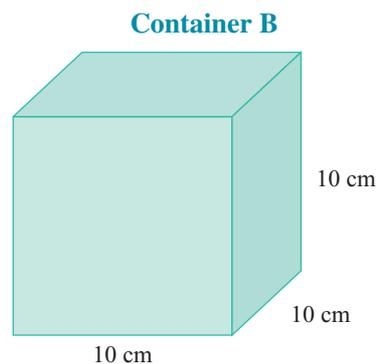
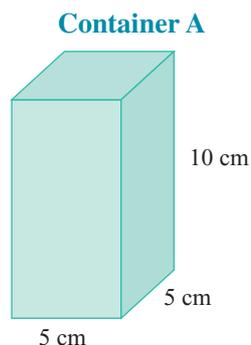
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As well as producing a sketch of a distance–time graph or speed–time graph, we can also draw a sketch from the description of other situations. Remember that, compared to the actual graph, a sketch should be recognisably similar but does not need to be exact. We first need to identify the two quantities or variables in the relationship. These variables will be shown on the two axes of the number plane. As the independent variable increases, we indicate what happens to the dependent variable.

## EXERCISE 8C

## Sketching more graphs from a given description

- 1 Consider these two containers which have the same height. Water is to be poured into each container at the same constant rate.



- a** Calculate the area of the base for:
- container A
  - container B.
- b** Explain how you can use the area of the base to calculate the volume of water, in  $\text{cm}^3$ , in each container if you know the depth of the water.
- c** Complete a table of values like the one shown to calculate the volume of water for different depths in:
- container A
  - container B.

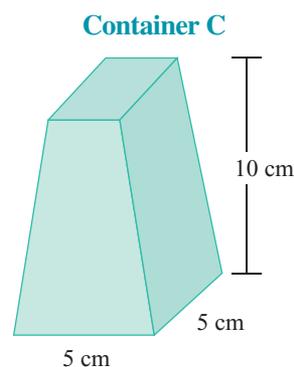
<b>Depth (cm)</b>	0	1	2	3	4	5
<b>Volume (<math>\text{cm}^3</math>)</b>	0					

- d** For each container, what is the increase in volume for an increase of 1 cm in depth? Is it a constant amount in each case?
- e** Explain why, in each case, the increase in volume is the same for each increase of 1 cm in depth. (*Hint:* how do the vertical sides of the containers affect the surface area of the water and hence the increase in volume?)

- f** On the same set of axes, construct graphs of volume versus depth for the two containers. What type of relationship does this show?
- g** Which graph has a steeper gradient? What does this mean?
- h** Explain what it means to pour water at a constant rate.
- i** Assuming water is poured into each container at the same constant rate, which container (A or B) would fill first? Explain your answer.

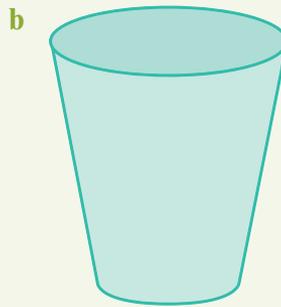
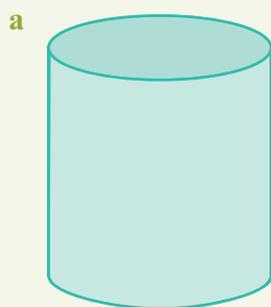
Now consider container C shown below. It will also have water poured into it at the same constant rate as for containers A and B.

- j** Would the increase in volume be the same for each increase of 1 cm in depth? Explain your answer.
- k** Does the surface area of the water in the container change when:
  - i** container A is filled?
  - ii** container B is filled?
  - iii** container C is filled?
- l** Which container (A, B or C) would fill first? Explain your answer.
- m** Summarise your findings. Describe how the shape of the container affects the rate at which it fills with water.



### EXAMPLE 8C-1 Sketching a graph for depth of water versus time

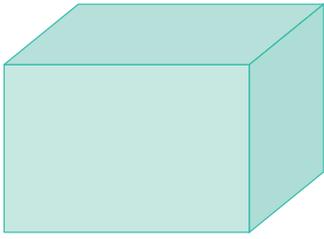
Water is poured into each of these containers at a constant rate. Sketch a graph for each container showing the depth of the water as time passes.



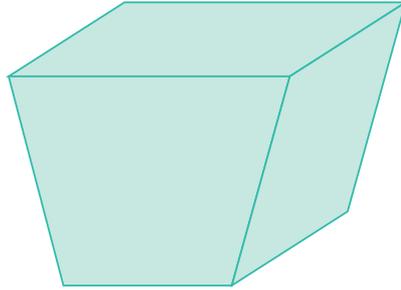
	Solve	Think	Apply
<b>a</b>		<p>Decide on the two variables to show on the graph: time on the horizontal axis and depth on the vertical axis. For a sketch, there is no need to show a scale on the axes. Draw a line or curve to indicate what happens to the depth as time passes.</p> <p>In part <b>a</b>, the area of the water surface stays the same as time passes so, for each identical time period, the change in depth is the same.</p>	<p>Since the water is poured at a constant rate, the depth is affected by the shape of the container.</p> <p>In part <b>a</b>, the container has vertical sides so the depth will change at a constant rate. The area of the water surface does not change.</p>
<b>b</b>		<p>In part <b>b</b>, the area of the water surface increases as time passes so, for each identical time period, the change in depth decreases.</p>	<p>In part <b>b</b>, the container has sloping sides so the area of the water surface increases as time passes.</p>

**2** Water is poured into each of these containers at a constant rate. Sketch a graph for each container that shows the depth of the water as time passes.

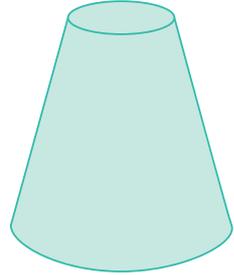
**a**



**b**

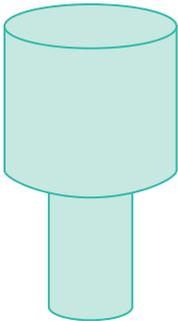


**c**

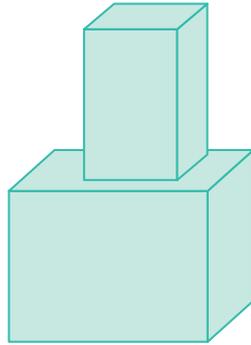


**3** These containers are made up of two sections. Water is poured into each container at a constant rate. Sketch a graph for each container that shows the depth of the water as time passes.

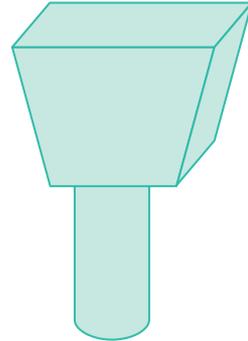
**a**



**b**

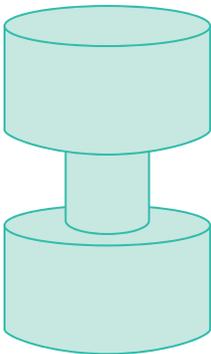


**c**

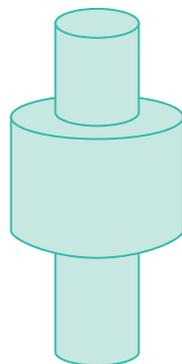


**4** These containers are made up of three sections. Water is poured into each container at a constant rate. Sketch a graph for each container that shows the depth of the water as time passes.

**a**



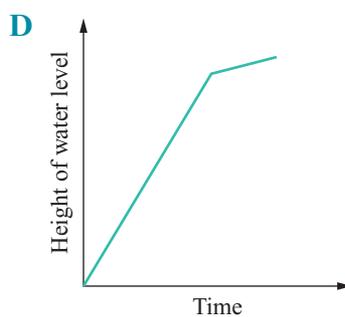
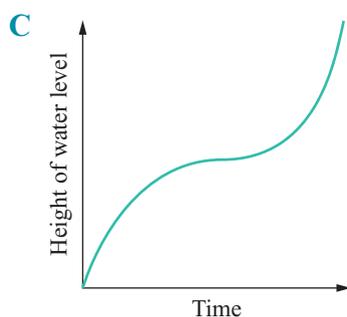
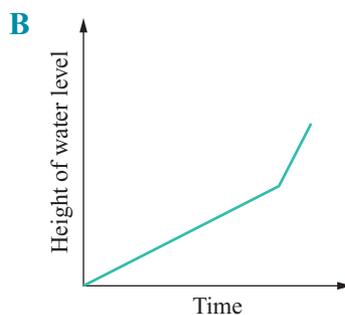
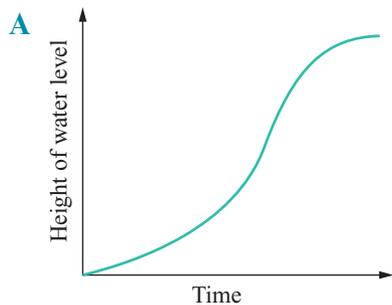
**b**



**c**



**5** Water is poured into this container at a constant rate. Which graph (A, B, C or D) best shows the height of the water level as time passes?



**6** Water is poured into each of these containers at a constant rate. Which of the graphs in question 5 best shows the height of the water level as time passes?

**a**



**b**



**7** Water is poured into each of these vases at a constant rate. Sketch a graph for each vase that shows the height of the water level as time passes.

**a**



**b**



**c**



**d**

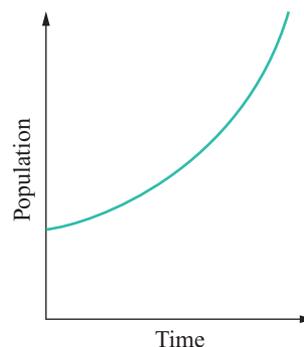


**EXAMPLE 8C-2** Sketching a graph from a description of a relationship

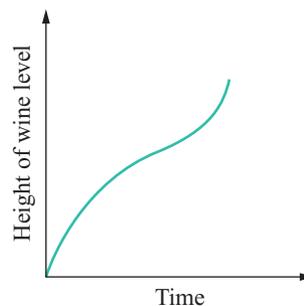
The number of bacteria in a culture doubles at regular time intervals. Sketch a graph that shows the relationship between the number of bacteria in the culture and time.

Solve	Think	Apply
	<p>On a set of axes, show time on the horizontal axis and number of bacteria on the vertical axis. The number of bacteria at the end of the next equal time period will be double the number of bacteria at the end of the current time period. This increase of 100% in each equal time period indicates an exponential relationship.</p>	<p>Label the axes with the two variables and draw a line or curve to indicate what happens to the number of bacteria as time passes. Look for a recognisable relationship if there is one.</p>

- 8 The mass of a newborn kitten increased by about 10% each day in its first two weeks. Sketch a graph that shows the relationship between the mass of the kitten and time.
- 9 The temperature of the atmosphere falls by 20% for each 5 km of height above sea level. Sketch a graph that shows the relationship between temperature of the atmosphere and height above sea level.
- 10 The price of bread increased at a constant rate over time. Sketch a graph that shows the relationship between the price of bread and time.
- 11 The area of a circle relates to the square of the radius of the circle. Sketch a graph that shows the relationship between area of a circle and its radius.
- 12 The time it takes to travel 200 km depends on the speed of the vehicle. Sketch a graph that shows the relationship between time and speed. (Show speed on the horizontal axis.)
- 13 Every person in a town is to receive an annual flu vaccination. The graph shows the population of the town over time.  
If the cost of each flu vaccination is assumed to be constant over time, sketch a graph that shows:
  - a the cost of each flu vaccination as time passes
  - b the total annual cost of vaccinating the population of the town as time passes.



- 14 Sparkling wine is poured into these three glasses at a constant rate. The sketch graph shows the height of the wine level over time for one of these glasses. Which glass (A, B or C) does the graph represent?



# 8D Graphs of linear models

These resources are available on your obook assess:

- **assess quiz 8D:** Test your skills with an auto-correcting multiple-choice quiz



As seen in Chapter 4, linear relationships can be used to model practical situations. The linear equation used as a model will be of the form  $y = mx + c$ , where the gradient ( $m$ ) and  $y$ -intercept ( $c$ ) of the corresponding linear graph have practical meanings in each case. In this section, we will first revise the skills needed to draw a linear relationship from its equation, identify the gradient and  $y$ -intercept, and then find the equation for a given linear graph.

A linear relationship that is used to model a practical situation can be shown as a graph or as an equation involving two variables.

The equation will be of the form  $y = mx + c$  where  $m$  is the gradient and  $c$  is the  $y$ -intercept of the line.

For practical situations, there may be restrictions on the values used for the two variables. For example, for a linear model involving time  $t$ , we know that  $t \geq 0$ .



## EXAMPLE 8D-1 Graphing a linear relationship from its equation

- Draw the graph of  $y = 2x - 1$ .
- Identify the gradient and  $y$ -intercept of the graph.

	Solve	Think/Apply																
a	<table border="1"> <tr> <td><math>x</math></td> <td>-3</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td><math>y</math></td> <td>-7</td> <td>-5</td> <td>-3</td> <td>-1</td> <td>1</td> <td>3</td> <td>5</td> </tr> </table> 	$x$	-3	-2	-1	0	1	2	3	$y$	-7	-5	-3	-1	1	3	5	<p>Draw the graph using technology or by completing a table of values and plotting the points. The gradient and <math>y</math>-intercept method of drawing a linear graph could also be used.</p>
$x$	-3	-2	-1	0	1	2	3											
$y$	-7	-5	-3	-1	1	3	5											
b	<p>From the graph:                      gradient = <math>\frac{3 - 1}{2 - 1} = 2</math>  <math>y</math>-intercept = <math>-1</math></p> <p>From the equation:  <math>y = 2x - 1</math> is of the form <math>y = mx + c</math>, where  <math>m = \text{gradient} = 2</math>  <math>c = y\text{-intercept} = -1</math></p>	<p>Using points (1, 1) and (2, 3) from the graph, we can identify a vertical rise of 2 for a horizontal run of 1 and substitute into gradient = <math>\frac{\text{rise}}{\text{run}}</math>.</p> <p>The <math>y</math>-intercept is the point where the line crosses the <math>y</math>-axis.</p> <p>Alternatively, we can identify <math>m</math> and <math>c</math> directly from the equation.</p>																

# EXERCISE 8D Graphs of linear models

UNDERSTANDING, FLUENCY AND COMMUNICATING

**1** For each of the following linear equations:

**i** draw the graph

**ii** identify the gradient and y-intercept of the graph.

**a**  $y = x + 3$

**b**  $y = x - 2$

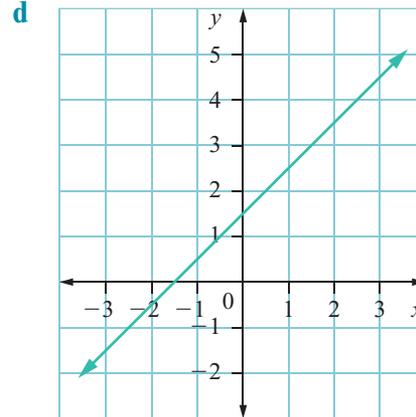
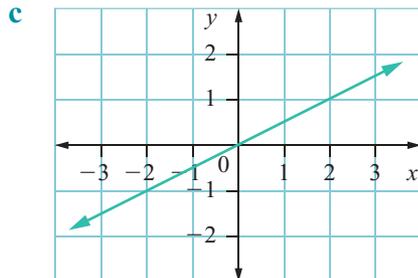
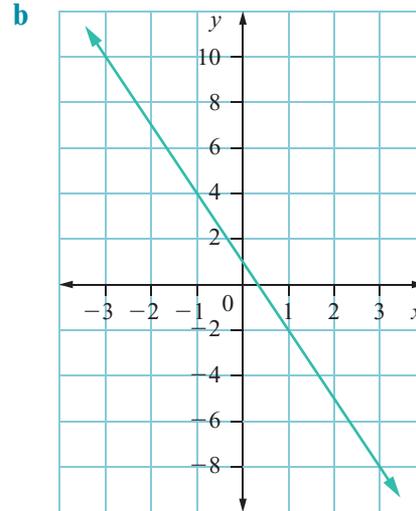
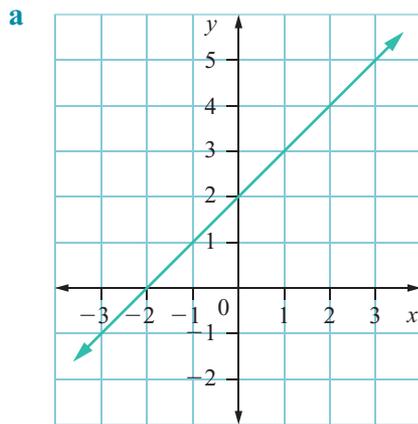
**c**  $y = 2x + 2$

**d**  $y = 4 - 2x$

**e**  $y = 3x - 6$

**f**  $y = 3 - 3x$

**2** Find the equation for each of these linear graphs by first identifying the gradient and y-intercept.



**3** Find the equation for each of these linear relationships by first constructing a graph.

**a**

<b>x</b>	-6	-4	-2	0	2
<b>y</b>	-2	0	2	4	6

**b**

<b>x</b>	-2	-1	0	1	2
<b>y</b>	-7	-4	-1	2	5

**c**

<b>x</b>	-4	-2	0	2	4
<b>y</b>	2	0	-2	-4	-6

**d**

<b>x</b>	-2	-1	0	1	2
<b>y</b>	5	3	1	-1	-3

**e**

<b>x</b>	-2	-1	0	1	2
<b>y</b>	-3	-1	1	3	5

**f**

<b>x</b>	-2	-1	0	1	2
<b>y</b>	8	5	2	-1	-4

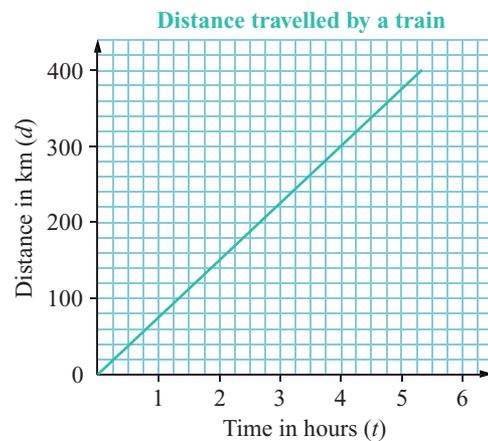
- 4** The amount of petrol used by a car travelling at constant speed can be modelled by the equation  $p = 0.1d$ , where  $p$  is the amount of petrol in litres and  $d$  is the distance in kilometres.
- How many litres of petrol are needed to travel 200 km at constant speed?
  - What distance could be travelled on 35 L of petrol?
  - Use graphing technology or a table of values to draw the graph of this linear model.
  - What is the gradient of the line? What does this represent?
  - Would this linear model be reliable for city driving? Explain your answer.

- 5** Catering charges can be modelled by the equation  $C = 60 + 15n$ , where  $C$  is the cost in dollars and  $n$  is the number of people.
- What is the cost to cater for 25 people?
  - How many people could be catered for \$300?
  - How many people could be catered for \$500?
  - Use graphing technology or a table of values to draw the graph of this linear model.
  - What is the gradient of the line? What does this represent?
  - What is the vertical intercept of the line? What does this represent?
  - Would this linear model work for calculating the catering cost for 1000 people? Explain your answer.

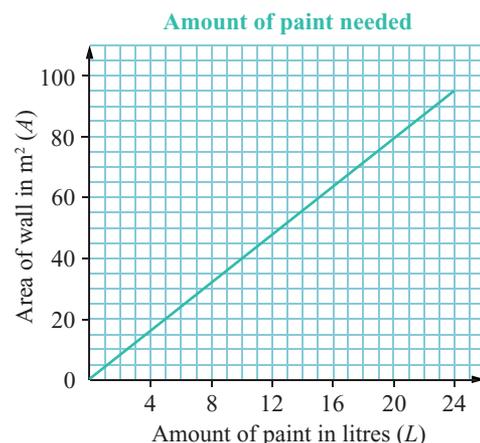


- 6** The average height,  $H$ , in centimetres, of a boy aged 6 to 11 years can be represented by a linear model with equation  $H = 5.6A + 81$ , where  $A$  is the age in years.
- What is the average height of a boy who is 6 years old?
  - What is the average height of a boy who is  $8\frac{1}{2}$  years old?
  - The gradient of the line is 5.6. What does this indicate about the heights of boys aged 6 to 11?
  - Give one reason why the equation is not suitable for predicting the heights of boys older than 11.
  - Use technology to draw the graph of this linear model for suitable values of  $A$ .

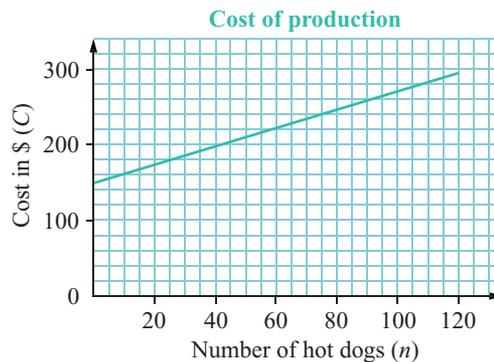
- 7** This graph models the distance,  $d$ , in kilometres, travelled by a train at a constant speed over time,  $t$ , in hours.
- From the graph, estimate how long the train will take to travel 150 km.
  - From the graph, estimate how far the train will travel in  $2\frac{1}{2}$  hours.
  - What is the gradient of this linear graph? What does it represent?
  - Write the equation for this linear model.
  - How far would the train travel in 8 hours?
  - What are the strengths and limitations of using this model?



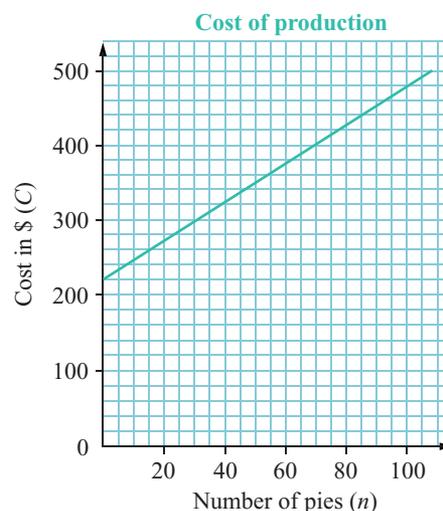
- 8** The graph shows the amount of paint,  $L$ , in litres, needed to paint an area of a wall,  $A$ , in  $m^2$ .
- From the graph, estimate the area of wall that can be painted using 20 L of paint.
  - From the graph, estimate the amount of paint required for a wall area of  $45 m^2$ .
  - What is the gradient of this graph? What does this represent?
  - Write the equation for this linear model.
  - What area of wall could be painted using 30 L of paint?
  - List one factor that could affect the accuracy of this model.



- 9** The cost of producing hot dogs is shown in this graph. The model represents the relationship between  $C$  and  $n$ , where  $C$  is the cost in dollars and  $n$  is the number of hot dogs.
- From the graph, estimate the cost of producing 70 hot dogs.
  - From the graph, estimate how many hot dogs are produced for \$250.
  - What is the gradient of this graph? What does this represent?
  - What is the vertical intercept of this graph? What does this represent?
  - Write the equation for this linear model.
  - What is the cost of producing 150 hot dogs?
  - What are the strengths and limitations of using this model?



- 10** The cost of producing gourmet pies is shown in this graph. The model represents the relationship between  $C$  and  $n$ , where  $C$  is the cost in dollars and  $n$  is the number of pies.
- From the graph, estimate the cost of producing 50 pies.
  - From the graph, estimate how many pies are produced for \$400.
  - What is the gradient of this graph? What does this represent?
  - What is the vertical intercept of this graph? What does this represent?
  - Write the equation for this linear model.
  - What is the cost of producing 1000 pies? Is this accurate? What limitations could be placed on this model?



- 11** The average weight,  $W$ , in kilograms, of a girl of age,  $A$ , in years, is shown in the table.

- Draw the graph of this linear model for values of  $A$  from 2 to 7 years.
- What is the gradient of the line? What does this indicate about the average weights of girls aged 2 to 7 years?
- If the line was extended to the vertical axis, what would be the vertical intercept? What would this represent?
- Write the equation for this linear model.
- What is the average weight of a girl who is  $3\frac{1}{2}$  years old?
- Explain why this model is not reliable for predicting the weights of girls younger than 2 and older than 7.

$A$	2	3	4	5	6	7
$W$	12	14	16	18	20	22

- 12** The volume of an inflated balloon increases as the temperature rises. There is a linear relationship between temperature,  $T$ , in  $^{\circ}\text{C}$ , and volume,  $V$ , in  $\text{cm}^3$ , if the pressure stays the same. Some data is shown in the table.

- Use the table to draw the graph of this linear relationship for values of  $T$  from 25 to 75.
- What is the gradient of the line? What does this indicate?
- If the line was extended to the vertical axis, what would be the vertical intercept? What would this represent?
- Write the equation for this linear model.
- What is the volume of the balloon when the temperature is  $52^{\circ}\text{C}$ ?
- Explain why this model is not reliable for predicting the volume of a balloon when the temperature is lower than  $25^{\circ}\text{C}$  or higher than  $75^{\circ}\text{C}$ .

$T$	25	35	45	55	65	75
$V$	600	620	640	660	680	700

**exponential growth**

relationship between two variables in which, as one variable increases, the other increases at an increasing rate of change; the graph curves upwards from left to right

**exponential decay**

relationship between two variables in which, as one variable increases, the other decreases at a decreasing rate of change; the graph curves downwards from left to right

**8E**

# Graphs of exponential models

These resources are available on your obook assess:

- **Video tutorial 8E:** Watch and listen to an explanation of Example 8E-1
- **Investigation 8E:** Explore the exponential growth of one grain of rice
- **assess quiz 8E:** Test your skills with an auto-correcting multiple-choice quiz

1010  
1010

Exponential relationships can also be used to model practical situations. Population growth, compound interest, declining-balance depreciation and radioactive decay are just a few examples of **exponential growth** or **exponential decay**.

In this section, we will focus on exponential models that can be represented by the general equation  $y = ka^x$ , where  $k > 0$ ,  $a > 0$  and  $a \neq 1$ . Recall that an exponential relationship is a non-linear relationship in which the independent variable is the exponent or power or index of a term in index form. For practical situations, there may be restrictions on the values used for the two variables. For example, for an exponential model involving time  $t$ , we know that  $t \geq 0$ .

## EXERCISE 8E Graphs of exponential models

UNDERSTANDING, FLUENCY AND COMMUNICATING

- 1 a** Complete this table of values for the equation  $y = 2^x$ .

$x$	-4	-3	-2	-1	0	1	2	3	4
$y$									

- b** Plot these points on a number plane and join with a smooth curve.  
**c** Use graphing technology to produce the curve for values of  $x$  from  $-4$  to  $4$ .  
**d** What happens to the value of  $y$  as the value of  $x$  increases? This is known as exponential growth.  
**e** Are there any negative values for  $y$ ? Explain.  
**f** What is the  $y$ -intercept of this exponential graph?  
**g** Are there any  $x$ -intercepts?

- 2 a** Complete this table of values for the equation  $y = (0.5)^x$ .

$x$	-4	-3	-2	-1	0	1	2	3	4
$y$									

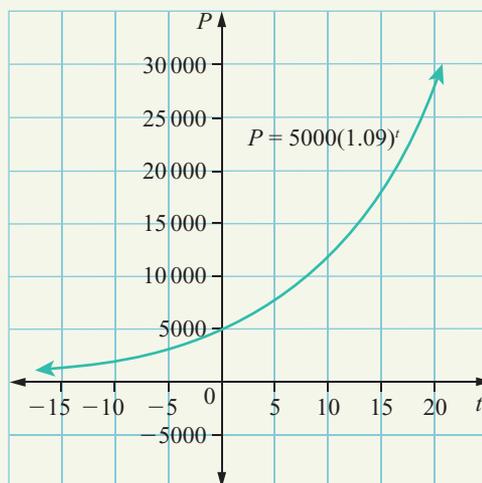
- b** Plot these points on a number plane and join with a smooth curve.  
**c** Use graphing technology to produce the curve for values of  $x$  from  $-4$  to  $4$ .  
**d** What happens to the value of  $y$  as the value of  $x$  increases? This is known as exponential decay.  
**e** Are there any negative values for  $y$ ? Explain.  
**f** What is the  $y$ -intercept of this exponential graph?  
**g** Are there any  $x$ -intercepts?

- 3** Use graphing technology or a table of values to draw the graph of each of the following exponential relationships. Adjust the interval of  $x$ -values you use so that you can see the shape of the graph clearly. Indicate the  $y$ -intercept for each graph.
- a**  $y = 1.4(3^x)$       **b**  $y = 3.5(0.6)^x$       **c**  $y = 200(1.02)^x$       **d**  $y = 700(0.95)^x$
- 4** Which of your graphs from question **3** show exponential growth and which show exponential decay?
- 5** How can you tell from the equation of an exponential relationship whether it is modelling exponential growth or exponential decay?

**EXAMPLE 8E-1** Interpreting the graph of an exponential model

The population of an island can be modelled using the equation  $P = 5000(1.09)^t$ , where  $P$  is the population and  $t$  is the time in years. The graph of the equation  $P = 5000(1.09)^t$  is drawn on the right.

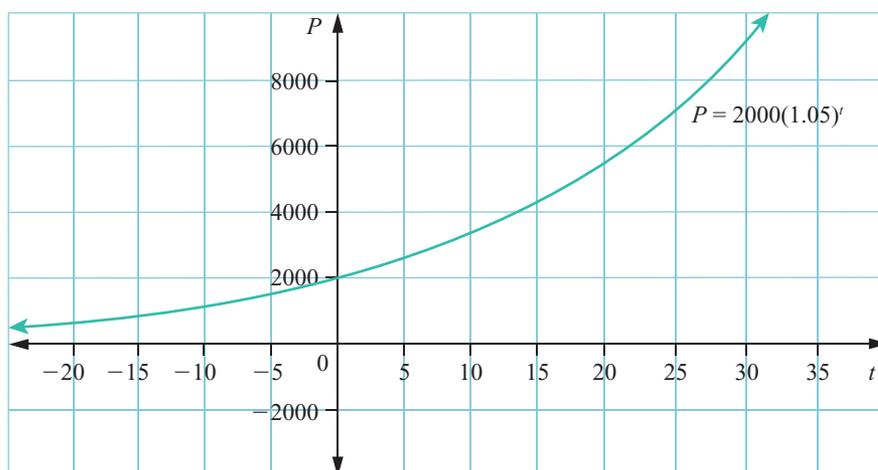
- a** Which part of the exponential curve applies to the population model in this case?
- b** Does the graph show exponential growth or decay?
- c** What was the initial population of the island?
- d** Use the graph to estimate the population after 12 years.
- e** How long did it take for the population to double?
- f** What are the strengths and limitations of using this model?



	Solve	Think	Apply
<b>a</b>	The curve matches the population model for $t \geq 0$ .	Time cannot be negative, so the values for $t$ must be greater than or equal to zero.	Identify the values of the variables that apply to the practical context.
<b>b</b>	Exponential growth	As $t$ increases, $P$ increases at an increasing rate.	The curve for exponential growth moves upwards at an increasing rate from left to right.
<b>c</b>	When $t = 0$ , $P = 5000$ . The initial population was 5000.	Find the population when $t = 0$ . Read the value for $P$ where the curve cuts the vertical axis (i.e. find the $P$ -intercept).	The initial population is when time is zero.
<b>d</b>	When $t = 12$ , $P \approx 14000$ . After 12 years, the population is about 14000.	Find the population when $t = 12$ . Move up from 12 on the horizontal axis to the curve and then move left to the vertical axis to read the value for $P$ .	Use the graph to read the value for the population when time is 12 years.
<b>e</b>	When the population is double the initial population, $P = 10000$ . When $P = 10000$ , $t \approx 8$ . The population has doubled after about 8 years.	Find the time when $P = 10000$ . Move across from 10000 on the vertical axis to the curve and then move down to the horizontal axis to read the value for $t$ .	Use the graph to read the value for the time when the population is 10000.

	Solve	Think	Apply
f	The model allows predictions of the future population of the island to be made, which can be useful in forward planning of the resources and amenities needed. However, the model may not be representative after some time if factors such as disease, immigration and change in birth rate affect the population of the island.	Describe the strength of the exponential model in allowing reasonably accurate predictions to be made. Limitations of the model may involve unexpected factors that affect the population and that have not been taken into consideration or that could not have been predicted.	Describe how the model is useful in making predictions (strength) and how it cannot always be representative of the situation (limitations).

- 6 The population of a mining town can be modelled using the equation  $P = 2000(1.05)^t$ , where  $P$  is the population and  $t$  is the time in years. The graph of the equation  $P = 2000(1.05)^t$  is drawn below.



- Which part of the exponential curve applies to the population model in this case?
  - Does the graph show exponential growth or decay? Explain.
  - What was the initial population of the town?
  - From the graph, estimate the population after 26 years.
  - How long did it take for the population to double?
- 7 The equation  $m = 3(1.5)^t$  can be used to model the mass of a bacterial culture, where  $m$  is the mass, in milligrams, after  $t$  days.

- a Complete this table of values for the equation  $m = 3(1.5)^t$  for values of  $t$  up to 4.

$t$	0	1	2	3	4
$m$					

- b Use graphing technology or the table of values to draw the exponential graph with equation  $m = 3(1.5)^t$  for values of  $t$  from 0 to 4, putting the values of  $m$  on the vertical axis and  $t$  on the horizontal axis.



- c What is the initial mass of the bacterial culture?
- d Estimate the mass of the bacterial culture after  $2\frac{1}{2}$  days.
- e After about how many days would the bacterial culture have doubled in mass?
- f Each day, the mass increases by a factor of 1.5. What is the daily growth rate of the mass of the bacterial culture? Write your answer as a percentage.
- g What are the strengths and limitations of using this model?

8 The equation  $m = 1.5(1.2)^x$  can be used to model the mass of a baby orang-utan up to 6 months of age, where  $m$  is the mass in kilograms and  $x$  is the age in months.

a Complete this table of values for the equation  $m = 1.5(1.2)^x$  for values of  $x$  up to 6.

$x$	0	1	2	3	4	5	6
$m$							



- b Use graphing technology or the table of values to draw the exponential graph with equation  $m = 1.5(1.2)^x$  for values of  $x$  from 0 to 6. Show the values of  $m$  on the vertical axis and  $x$  on the horizontal axis.
- c Estimate the mass of the orang-utan at age  $3\frac{1}{2}$  months.
- d At about what age would the orang-utan weigh 3.4 kg?
- e Each month, the mass increases by a factor of 1.2. What is the monthly growth rate of the baby orang-utan's mass? Write your answer as a percentage.
- f Why is this model only useful up to age 6 months? Explain your answer.

9 The population of an island is modelled using the equation  $P = 30\,000(1.02)^t$ , where  $P$  is the population and  $t$  is the time in years.

a Complete this table of values for the equation  $P = 30\,000(1.02)^t$ .

$t$	0	5	10	15	20	25
$P$						



- b Use graphing technology or the table of values to draw the graph of the population model for values of  $t$  from 0 to 25.
- c What does the intercept of the curve with the vertical axis represent?
- d Use the graph to estimate the population after 18 years.
- e Estimate the time taken for the population to reach 39 000.
- f Extend your graph to obtain an estimate of the time taken for the population to double.
- g What is the percentage increase in population each year? Explain how you can work this out from looking at the equation  $P = 30\,000(1.02)^t$ .

- 10** Radioactive decay is a physical phenomenon that can be modelled using an exponential relationship. A sample of radioactive material decays according to the equation  $m = 800(0.95)^t$ , where  $m$  is the mass of radioactive substance in grams and  $t$  is the number of years.

**a** Complete this table of values for  $m = 800(0.95)^t$ .

$t$	0	5	10	15	20	25
$m$						

- b** Use graphing technology or the table of values to draw the graph of  $m = 800(0.95)^t$  for values of  $t$  from 0 to 25.
- c** Explain how we can see that this model represents exponential decay.
- d** What was the initial mass of radioactive material?
- e** Use the graph to estimate the mass of radioactive substance after 23 years.
- f** Estimate the time taken for the mass of radioactive substance to reach 600 g.
- g** Extend your graph to obtain an estimate of the time taken for the mass of radioactive substance to reach 100 g.
- h** What is the percentage rate of decay each year? Explain how you can work this out from looking at the equation  $m = 800(0.95)^t$ .
- 11** Depreciation, appreciation and inflation may all be modelled using graphs. Depreciation is an example of exponential decay, and appreciation and inflation are examples of exponential growth.
- a** Draw the graph of  $y = 1000(1.08)^x$  using values of  $x$  from 0 to 10.
- b** Draw the graph of  $y = 1000(0.92)^x$  using values of  $x$  from 0 to 10.
- c** Which of these graphs would be used to model depreciation? Why?
- d** Which of these graphs would be used to model inflation? Why?
- e** What would be the difference in the value of two items after 8 years, if both were purchased for \$1000 and one appreciated at 8% p.a. and the other depreciated at 8% p.a.?

- 12** A new motorcycle is purchased for \$20 000 and depreciates in value at a rate of 25% per year. The salvage value of the motorcycle can be modelled using the equation  $S = 20000(0.75)^n$ , where  $S$  is the salvage value in dollars after  $n$  years.

- a** Use graphing technology or a table of values to draw the graph of this exponential model for values of  $n$  from 0 to 6.
- b** Does this graph represent exponential growth or decay? Explain your answer.
- c** What does the intercept of the curve with the vertical axis represent?
- d** Use the graph to find the salvage value of the motorcycle after 2 years.
- e** From the graph, estimate the number of years it takes for the salvage value of the motorcycle to be less than \$6000.



- 13** The formula for the declining-balance method of depreciation is  $S = V_0(1 - r)^n$ , where  $S$  is the salvage value of the asset in dollars,  $V_0$  is the purchase price of the asset in dollars,  $r$  is the depreciation rate per time period as a decimal, and  $n$  is the total number of time periods. Nick's work tools depreciate in value at the rate of 20% per year and can be represented by an exponential model.
- If Nick bought the tools for \$2500, show that the equation of this exponential model is  $S = 2500(0.8)^n$ .
  - Use technology or a table of values to draw the graph of  $S = 2500(0.8)^n$  for values of  $n$  from 0 to 10.
  - Use the graph to find the salvage value of Nick's tools after 3 years.
  - From the graph, estimate the number of years it takes for the salvage value to be less than \$500.

- 14** A new car is purchased for \$39 000. It depreciates in value at a rate of 28% per year.

- Write an equation to model the value of the car over time.
- Does this equation represent exponential growth or decay?
- Use technology to draw the graph of this exponential model.
- From the graph, find the salvage value of the car after 5 years.
- Estimate the number of years it takes for the car to be worth half of its original value.



- 15** Simone invests \$3000 at 10% p.a. interest compounded annually. After investing a principal,  $P$ , in dollars, for  $n$  years at an annual compound interest rate,  $r$  (as a decimal), the amount, in dollars, of the investment,  $A$ , can be represented by the formula  $A = P(1 + r)^n$ .

- Show that the model for this context can be represented by the equation  $A = 3000(1.1)^n$ .
- Use technology or a table of values to draw the graph of  $A = 3000(1.1)^n$  for values of  $n$  from 0 to 10.
- What does the intercept of the curve with the vertical axis represent?
- From the graph, find the amount of the investment after 4 years.
- From the graph, estimate the number of years it takes for the investment amount to be more than \$6000.
- Simone invests \$4000 instead of \$3000. Write an equation for this new investment model and hence compare the investment amount after 4 years.

- 16** A model for the population of Earth is given by  $P = 3.03(1.02)^t$ , where  $P$  is the population in billions and  $t$  is the time in years since 1960.

- Use graphing technology or a table of values to draw the graph of this population model for values of  $t$  from 0 to 50.
  - Use the graph to estimate the population after 15 years. This was the population in the year 1975. Check this against the actual population in 1975.
  - Use the model to estimate the population in the year 2001. Check this against the actual population.
  - Use the graph to estimate the year in which the population would reach:
 

<b>i</b> 4 billion	<b>ii</b> 5 billion	<b>iii</b> 7 billion	<b>iv</b> 10 billion.
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- You may need to extend your graph.
- What factors could influence the actual population?

# 8F Determining an appropriate model

These resources are available on your obook assess:

- **assess quiz 8F:** Test your skills with an auto-correcting multiple-choice quiz

01  
19  
GI

We have seen that observing a trend in a graph helps us to estimate data values and make predictions. Identifying the best model that represents the relationship between two variables is an advantage.

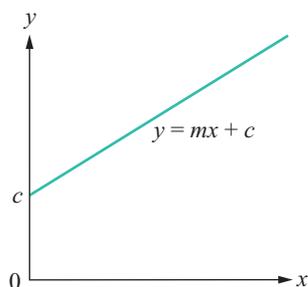
In this section, we will focus on determining whether a linear model or an exponential model best approximates the graph of a set of data by considering its shape.

Since we are modelling practical situations, we only consider positive values of the independent variable and zero; for example, for a relationship between  $x$  and  $y$  we only consider  $x \geq 0$ .

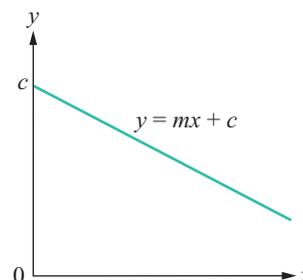
## Linear models

For a linear model, the equation is of the form  $y = mx + c$ , where  $m$  is the gradient and  $c$  is the  $y$ -intercept.

For an increasing linear relationship, the graph is a straight line with a positive gradient.



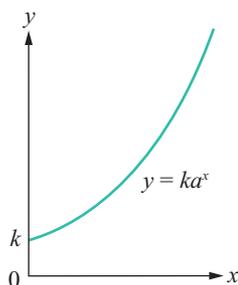
For a decreasing linear relationship, the graph is a straight line with a negative gradient.



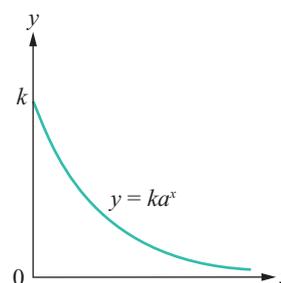
## Exponential models

For an exponential model, the simplest equation is of the form  $y = ka^x$ , where  $k > 0$ ,  $a > 0$  and  $a \neq 1$ .

For exponential growth,  $a > 1$  and the graph curves upwards from left to right at an increasing rate of change.



For exponential decay,  $0 < a < 1$  and the graph curves downwards from left to right at a decreasing rate of change.



### EXAMPLE 8F-1 Determining the best model

Consider the set of data shown in each table of values.

- i Plot the points on a number plane.
- ii Draw a straight line or exponential curve to best represent the trend of the data.
- iii State which model (linear or exponential) best approximates the data.

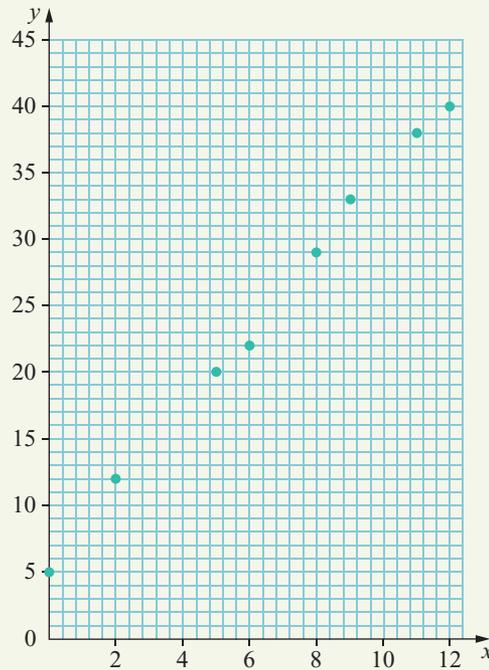
**a**

$x$	0	2	5	6	8	9	11	12
$y$	5	12	20	22	29	33	38	40

**b**

$x$	0	2	4	5	7	8	10	12
$y$	5	7	11	12	17	23	30	46

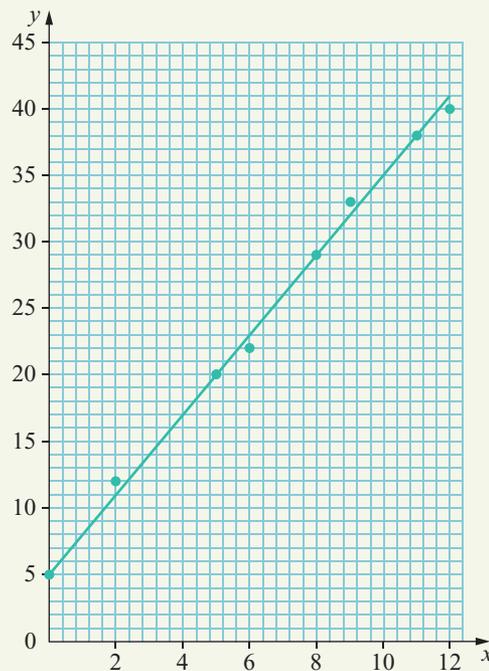
**a i**



**Think/Apply**

Draw a set of axes labelled  $x$  and  $y$ . Show a scale from 0 to 12 on the  $x$ -axis and a scale from 0 to at least 40 on the  $y$ -axis. Plot the coordinate points on the number plane.

**ii**



Decide whether a straight line or an exponential curve best suits the trend of the relationship shown by the plotted points.

In this case, the points appear to approximately form a straight line. Rule a straight line (line of best fit) that closely matches the points.

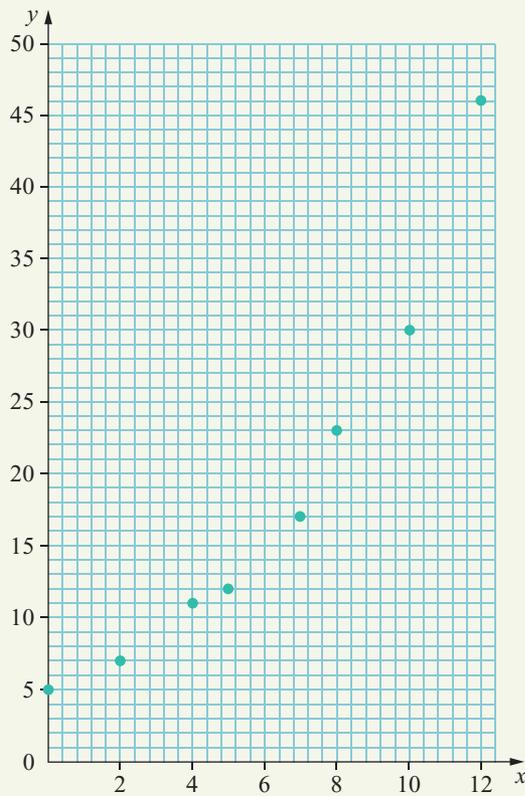
**iii**

A linear model best suits the data.

The data can be modelled by an increasing linear relationship.

b i

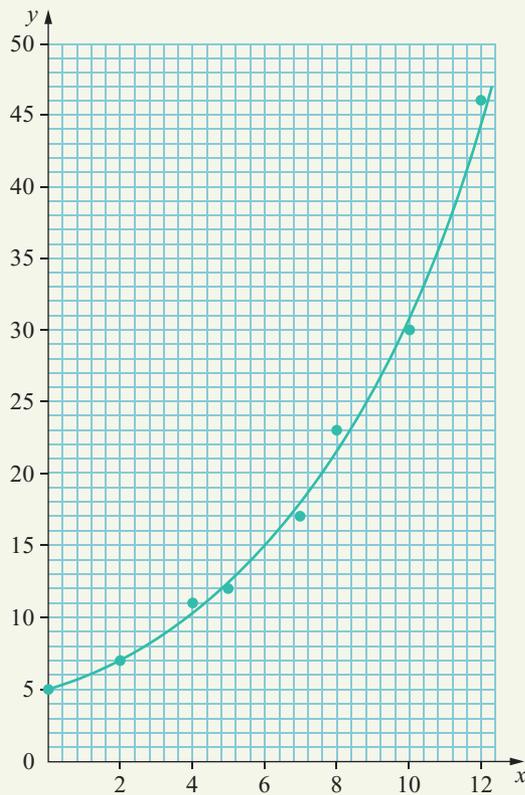
Solve



Think/Apply

Draw a set of axes labelled  $x$  and  $y$ . Show a scale from 0 to 12 on the  $x$ -axis and a scale from 0 to at least 46 on the  $y$ -axis. Plot the coordinate points on the number plane.

ii



Decide whether a straight line or an exponential curve best suits the trend of the relationship shown by the plotted points.

In this case, the points do not appear to form a straight line, so draw a curve that closely matches the points. *Note:* we could draw a straight line to approximate this data but it would not be the best model to use.

iii

An exponential model best suits the data.

The data can be modelled by an increasing exponential relationship that represents exponential growth.

# EXERCISE 8F Determining an appropriate model

- 1 Consider the set of data shown in each table of values.
  - i Plot the points on a number plane.
  - ii Draw a straight line or exponential curve to best represent the trend of the data.
  - iii State which model (linear or exponential) best approximates the data.

**a**

$x$	1	2	5	8	11	12	14	15
$y$	48	46	41	34	28	27	22	21

**b**

$x$	0	5	7	8	12	14	17	20
$y$	4	7	11	12	21	29	41	65

**c**

$x$	0	20	30	50	55	70	76	80
$y$	100	48	36	18	13	9	7	6

**d**

$x$	4	6	9	11	14	18	20	25
$y$	4	10	13	19	22	34	35	45

## EXAMPLE 8F-2 Determining the best model using a spreadsheet

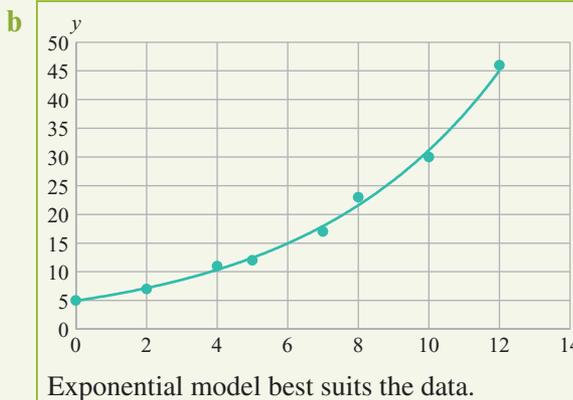
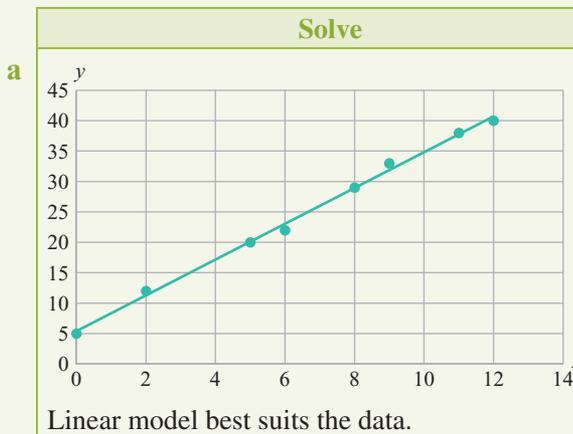
Consider each table of values in Example 8F-1. Use a spreadsheet to determine the best model (linear or exponential) that approximates the data.

**a**

$x$	0	2	5	6	8	9	11	12
$y$	5	12	20	22	29	33	38	40

**b**

$x$	0	2	4	5	7	8	10	12
$y$	5	7	11	12	17	23	30	46



**Think/Apply**

Enter the data from the table in two columns of the spreadsheet and highlight the data. From the **Insert** menu, select **Scatter** from the **Charts** option and then select the first scatterplot type. Right click on one of the data points in the plot and select **Add Trendline**. This opens the Trendline options. Select Linear or Exponential as appropriate. In this menu, you can also select **Display Equation on chart** to display the equation of the line or curve. You can also use the **Forecast** option to extend the line or curve.

- 2** Use a spreadsheet to determine the best model (linear or exponential) that approximates the dataset shown in each table.

**a**

<i>x</i>	10	12	16	24	25	28	29	32
<i>y</i>	14	17	25	55	65	110	134	158

**b**

<i>x</i>	2	6	7	9	10	12	15	20
<i>y</i>	56	145	182	235	247	305	360	510

- 3** Check your answers for question **1** using a spreadsheet. (*Note:* as a comparison, you might like to show both the linear and the exponential trendlines on the same set of axes for each dataset. To do this, click on the arrow to the right of the Trendlines option in the Chart Elements box and click on both Linear and Exponential.)

- 4** Tania's height was recorded from age 3 years to age 12 years. The data is shown in the table.

<b>Age (years)</b>	3	4	5	6	7	8	9	10	11	12
<b>Height (cm)</b>	94	102	108	118	121	126	133	137	144	150

- a** Plot the points for the dataset on a number plane. Show a scale from 0 to 13 on the horizontal axis for age and a scale from 0 to 160 on the vertical axis for height.
- b** Draw a straight line or exponential curve to best represent the trend of the data.
- c** State which model (linear or exponential) best approximates the data.
- d** From the trendline, estimate Tania's height when she was  $8\frac{1}{2}$  years old.
- e** Extend your trendline to predict Tania's height when she is 13 years old.
- f** Would you be able to use this model to predict Tania's height when she is 16 years old? Explain your answer.
- g** Find the equation for this model by calculating the gradient and extending your trendline to find the intercept with the vertical axis.
- h** Use the equation to calculate answers to parts **d** and **e**. How do they compare?
- 5** Mick invested an initial lump sum in an investment account that earns interest. At times, he withdraws small amounts from the account and then deposits small amounts back in. The amount in Mick's investment account has increased over time as shown in the table.

<b>Time (years)</b>	0	2	4	6	8	10	12	14	16
<b>Amount (\$'000)</b>	10.0	12.1	14.2	17.9	20.8	27.2	35.7	46.4	55.3

- a** What was the initial investment?
- b** Plot the points for the dataset on a number plane. Show a scale from 0 to 20 on the horizontal time axis and a scale from 0 to 100 on the vertical axis.
- c** Draw a straight line or exponential curve to best represent the trend of the data.
- d** State which model (linear or exponential) best approximates the data.
- e** From the trendline, estimate the amount in Mick's account after 9 years.
- f** From the trendline, estimate when the amount in Mick's account is double his initial investment.
- g** Extend your trendline to predict the amount in Mick's account after 20 years.

- 6** Consider the dataset in question 4.
- Use a spreadsheet to plot the points and draw the appropriate trendline.
  - Find the equation of the trendline. (*Hint*: right click on the trendline, select **Format Trendline** and check the box for **Display Equation on chart**.)
  - Use the displayed equation to calculate Tania's height when she was  $8\frac{1}{2}$  years old. How does this compare to the answer you found in part **d** of question 4?
  - Predict Tania's height when she is 13 years old using the Forecast tool. The Forecast tool extends the trendline by a nominated amount. (*Hint*: right click on the trendline, select **Format Trendline** and enter the required Forward period under Forecast.) How does this compare to the answer you found in part **e** of question 4?
- 7** Consider the dataset in question 5.
- Use a spreadsheet to plot the points and draw the appropriate trendline.
  - Does the trendline indicate exponential growth or exponential decay?
  - From the trendline, estimate the amount in Mick's account after 9 years. How does this compare to the answer you found in part **e** of question 5?
  - From the trendline, estimate when the amount in Mick's account is double his initial investment. How does this compare to the answer you found in part **f** of question 5?
  - Use the Forecast tool to extend the trendline and predict the amount Mick will have in his investment account after 20 years. How does this compare to the answer you found in part **g** of question 5?

- 8** Ivy bought a new car for \$32 000. The market value of her car changes over time. The data is shown in the table.

Time (years)	Value (\$'000)
0	32.0
1	24.2
2	18.1
3	12.9
4	10.8
5	7.3
6	5.1
7	4.9
8	3.7
9	2.3
10	1.8

- Plot the points for the dataset on a number plane, using technology or by hand.
- Draw a straight line or exponential curve to best represent the trend of the data.
- State which model (linear or exponential) best approximates the data.
- From the trendline, estimate the market value of Ivy's car after  $4\frac{1}{2}$  years.
- From the trendline, estimate when the market value of her car is \$15 000.
- Predict the market value of Ivy's car after 12 years.
- Does the relationship between market value and time indicate straight-line depreciation or declining-balance depreciation?

- 9** Consider the dataset in question 8.
- Use a spreadsheet to plot the points. Draw a linear trendline for the data.
  - From the linear trendline, estimate the market value of Ivy's car after  $4\frac{1}{2}$  years. How does this compare to the answer you found in part **d** of question 8?
  - From the linear trendline, estimate when the market value of her car is \$15 000. How does this compare to the answer you found in part **e** of question 8?
  - Extend the linear trendline to predict the market value of Ivy's car after 12 years. How does this compare to the answer you found in part **f** of question 8?
  - Explain what might happen if the best model is not chosen to represent a dataset.

# CHAPTER 8 REVIEW GRAPHS OF PRACTICAL SITUATIONS

You should be able to:

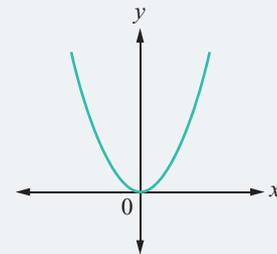
- ✓ construct the graphs of linear and non-linear relationships from tables of values
- ✓ use technology to graph linear and non-linear relationships
- ✓ distinguish between linear and non-linear relationships
- ✓ sketch the shape of a graph from a description of a situation, including distance–time and speed–time relationships, and height of water level in a container over time
- ✓ model linear relationships for practical problems
- ✓ recognise, draw and interpret exponential graphs
- ✓ model exponential relationships for practical problems
- ✓ understand the difference between exponential growth and exponential decay
- ✓ determine the best model (linear or exponential) to approximate a graph by considering its shape
- ✓ identify the strengths and limitations of linear and non-linear models in given practical contexts.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

## REVIEW MULTIPLE-CHOICE QUESTIONS

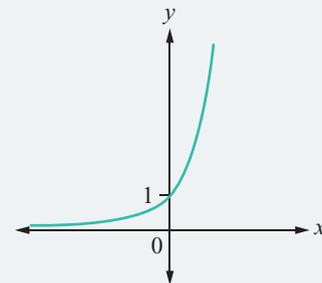
8A ➤ 1 What type of relationship is shown in the graph?

- A linear
- B quadratic
- C exponential
- D inverse



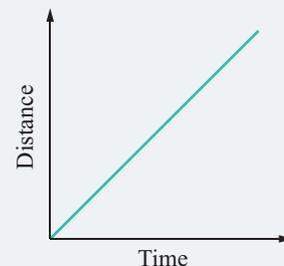
8A ➤ 2 What type of relationship is shown in the graph?

- A linear
- B quadratic
- C exponential
- D inverse

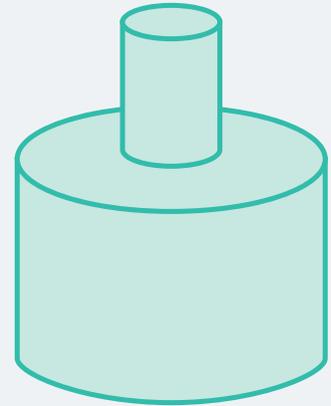
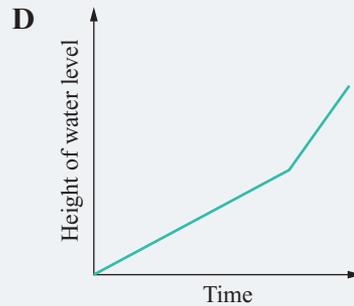
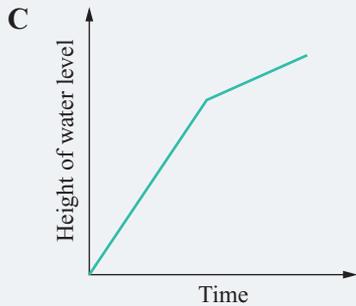
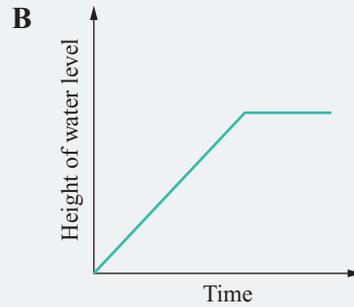
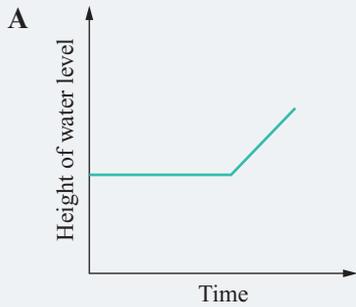


8B ➤ 3 Which description best matches the sketch graph of distance over time for Tina's journey?

- A Tina's speed is constant.
- B Tina's speed is zero.
- C Tina's speed is increasing.
- D Tina's speed is decreasing.



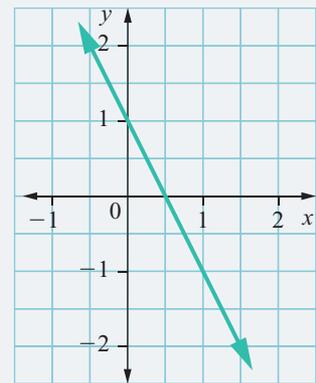
- 8B** ▶ **4** Water is poured into this container at a constant rate. Which sketch represents the height of the water level as time passes?



Use the graph shown on the right to answer questions **5** and **6**.

- 8B** ▶ **5** What is the gradient of the linear graph?

- A** 2                      **B** -2  
**C**  $\frac{1}{2}$                     **D**  $-\frac{1}{2}$



- 8B** ▶ **6** What is the equation of the linear graph?

- A**  $y = 2x + 1$             **B**  $y = -2x + \frac{1}{2}$   
**C**  $y = -\frac{1}{2}x + 1$         **D**  $y = 1 - 2x$

- 8E** ▶ **7** Which of the following equations could represent exponential decay?

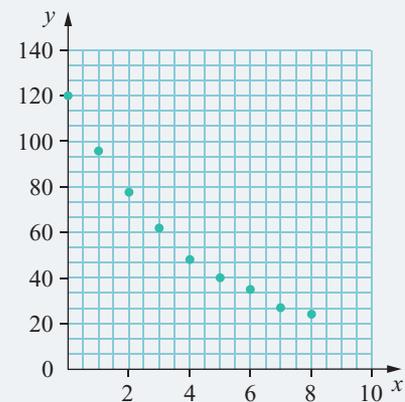
- A**  $y = (0.98)^x$             **B**  $y = (1.02)^x$             **C**  $y = \frac{6}{x}$                       **D**  $y = (0.5)x^2$

- 8E** ▶ **8** The population of a mining town can be modelled using the equation  $P = 1000(1.02)^t$ , where  $P$  is the population after  $t$  years. According to the model, what was the initial population of the town?

- A** 1.02                      **B** 500                      **C** 1000                      **D** 1020

- 8F** ▶ **9** Which relationship would best model the data shown in this graph?

- A** linear                      **B** quadratic  
**C** exponential              **D** inverse



# REVIEW SET 1

- 1 a For this table of values, construct the graph of the relationship by plotting the coordinate points on a number plane and joining them with a straight line or smooth curve.

$x$	0	1	2	3	4	5
$y$	3	2	7	18	35	58

- b Does the graph show a linear relationship? If not, what type is it?

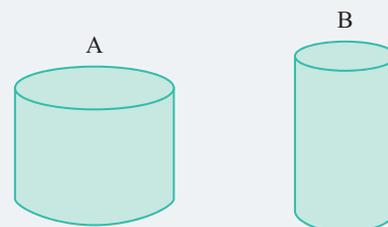
- 2 The table shows the cost per person,  $C$ , in dollars, to hire a bus if there are  $n$  people sharing the total cost.

$n$	1	2	4	6	7
$C$	420	210	105	70	60

- a Use graphing technology or the table of values to plot the coordinate points.  
 b Draw a curve through the points to indicate the trend in the data.  
 c What type of relationship is shown by the curve?  
 d From the graph, find the cost per person when 5 people hire the bus.  
 e From the graph, find the number of people hiring the bus if the cost per person is \$140.  
 f Give one limitation of the graph used to model this situation.

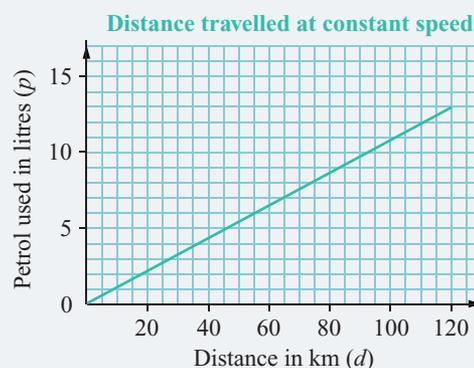
- 3 Liam cycles at a constant speed to his friend's house. He then walks at a constant speed with his bicycle and his friend to a cafe. They spend some time having lunch before he cycles home at a constant speed. From this description, sketch a graph representing Liam's distance from home over time. (Assume no acceleration or deceleration occurs.)

- 4 Water is poured at the same constant rate into each container. On the same set of axes, sketch a graph for each container, showing the height of the water level as time passes.



- 5 The graph shows the amount of petrol,  $p$ , in litres, used by a car travelling at a constant speed to cover a distance of  $d$  kilometres.

- a How much petrol does the car use to travel 100 km?  
 b How far does the car travel if it uses 5 L of petrol?  
 c What is the gradient of the line? What does this represent?  
 d Write the equation of the linear graph.  
 e How much petrol does the car use to travel 220 km?



- 6 The population of an island is modelled using the equation  $P = 40\,000(1.03)^t$ , where  $P$  is the population and  $t$  is the time in years.

- a What was the initial population of the island?  
 b Complete this table of values for the equation  $P = 40\,000(1.03)^t$ .

$t$	0	5	10	15	20	25
$P$						

- c Draw the population graph with  $t$  on the horizontal axis and  $P$  on the vertical axis.
- d Use the graph to estimate the population after 18 years.
- e Use the graph to estimate the time taken for the population to reach 59 000.
- f Extend your graph to obtain an estimate of the time taken for the population to reach 100 000.
- g What is the percentage increase in population each year? Explain how you worked this out from the equation.

- 7 Tristan bought a new motorbike for \$24 000. The salvage value of his motorbike changes over time. The data is shown in the table.

<b>Time (years)</b>	0	1	2	3	4	5	6	7
<b>Value (\$'000)</b>	24.0	18.5	14.9	11.3	8.9	7.1	5.2	4.3

- a Plot the points for the dataset on a number plane, using technology or by hand.
- b Draw a straight line or exponential curve to best represent the trend of the data.
- c State which model (linear or exponential) best approximates the data.
- d From the trendline, estimate the salvage value of Tristan's bike after  $2\frac{1}{2}$  years.
- e From the trendline, estimate when the salvage value of his bike is \$10 000.
- f Extend the trendline to predict the salvage value of Tristan's bike after 8 years.
- g Does the relationship between salvage value and time indicate straight-line depreciation or declining-balance depreciation?

## REVIEW SET 2

- 1 The height,  $h$ , in millimetres, of a plant is measured over  $t$  days. The results are shown in the table.

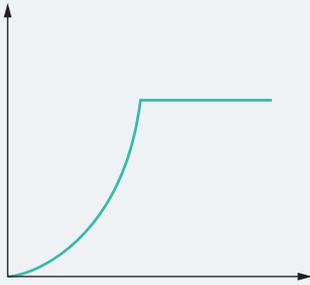
<b><math>t</math></b>	0	1	2	3	4
<b><math>h</math></b>	300	360	432	518	622

- a Plot the values in the table by hand and join the points with a straight line or smooth curve to construct the graph of this relationship. Show  $t$  on the horizontal axis. Check your answer using graphing technology.
  - b What type of relationship does the graph show?
  - c From the graph, estimate the height of the plant after 3.5 days.
  - d From the graph, estimate how long it took for the plant to reach a height of 40 cm.
  - e Extend your graph to predict the height of the plant after 5 days.
  - f Comment on the reliability of this model to predict the height of the plant after 5 days.
- 2 Helena is driving her car at a constant speed. She decelerates to stop at a school crossing. When the school crossing is clear, she accelerates to the speed limit and then maintains this speed.
- a From this description, sketch a graph representing speed versus time for Helena's journey.
  - b Sketch a graph representing distance versus time for Helena's journey.



**3** The value of a car falls by 25% each year. Sketch a graph that shows the value of a car over time.

**4** Write a description to match this sketch graph.



**5** The speed,  $s$ , in m/s, of a racing car over time,  $t$ , in seconds, is shown in the table.

$t$ (s)	1	2	3	4	5	6
$s$ (m/s)	25	30	35	40	45	50

- Draw the graph of this relationship for the values in the table.
- Does the graph show a linear relationship? How can you tell from the graph? How can you tell by looking at the table?
- What is the gradient of the line? What does this indicate?
- If the line was extended to the vertical axis, what would be the vertical intercept? What would this represent?
- Write the equation for this linear model.
- What is the speed of the racing car after 8 seconds?
- Can this equation be used to calculate the speed of the racing car for any value of  $t$ ? Explain your answer.



**6 a** Complete this table of values for the equation  $y = \frac{1}{8}(2)^x$ .

$x$	0	1	2	3	4	5	6	7	8
$y$									

- Plot these points on a number plane and join with a smooth curve.
- What type of curve has been drawn?
- What happens to the value of  $y$  as the value of  $x$  increases?

**7** The equation  $m = 2(1.8)^t$  can be used to model the mass of a bacterial culture, where  $m$  is the mass in milligrams after  $t$  days.

**a** Complete this table of values for the equation  $m = 2(1.8)^t$  for values of  $t$  up to 5.

$t$	0	1	2	3	4	5
$m$						

- Use graphing technology or the table of values to draw the exponential graph with equation  $m = 2(1.8)^t$  for values of  $t$  from 0 to 5. Show  $m$  on the vertical axis and  $t$  on the horizontal axis.
- What is the initial mass of the bacterial culture?
- From the graph, estimate the mass of the bacterial culture after  $3\frac{1}{2}$  days.

- e After about how many days would the bacterial culture have tripled in mass?
- f Each day, the mass increases by a factor of 1.8. What is the daily growth rate of mass of the bacterial culture? Write your answer as a percentage.
- g What are the strengths and limitations of using this model?

**8** The population of the urban area of Sydney from 1980 to 2018 is shown for selected years in the table on the right.

- a Plot points for the dataset on a number plane, using technology or by hand. Instead of using the actual year as the independent variable, use 0 for 1980, 5 for 1985, 10 for 1990, ... and 38 for 2018 to represent the number of years after 1980.
- b Draw a straight line or exponential curve to best represent the trend of the data.
- c State which model (linear or exponential) best approximates the data.
- d From the trendline, estimate the population in 2008.
- e Extend the trendline or find the equation for this model to predict what the population will be in 2025.
- f We have often seen that an increase in population over time is an example of exponential growth. What factors could affect the type of relationship we see here?

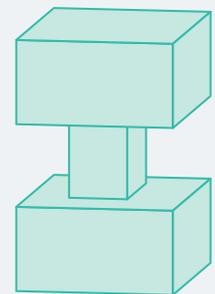
Year	Population (millions)
1980	3.3
1985	3.4
1990	3.6
1995	3.8
2000	4.1
2005	4.2
2010	4.4
2015	4.5
2018	4.6

## REVIEW SET 3

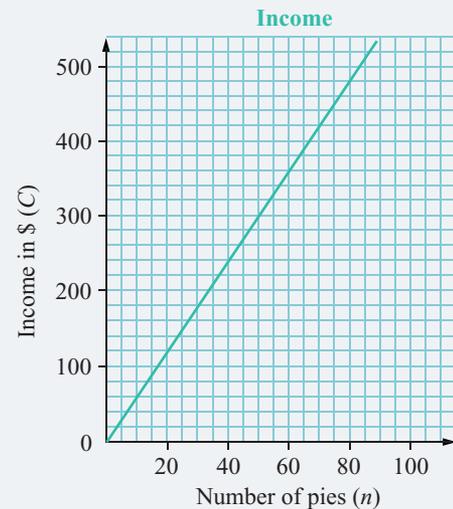
**1** The temperature at a desert holiday resort,  $T$ , in  $^{\circ}\text{C}$ , was recorded every 6 hours from midday on Friday. The results are shown in the table, where  $t$  is the number of hours after midday on Friday.

$t$	0	6	12	18	24	30	36	42	48	54
$T$	34	25	6	18	32	26	7	17	33	25

- a Construct the graph of this relationship using the table of values. Show a scale on the horizontal time axis from 0 to 60 and a scale from 0 to 36 on the vertical temperature axis.
  - b What was the temperature at 6 am on Saturday?
  - c From the graph, estimate the temperature at 8 pm on Saturday.
  - d From the graph, estimate the temperature at 9:30 am on Sunday.
  - e Extend the graph to predict the temperature at 10 pm on Sunday.
- 2** A stationary car accelerates quickly from an intersection and reaches a constant speed. Sketch a graph showing how its distance changes over time.
- 3** Water is poured into the container on the right at a constant rate. Sketch a graph that shows the height of the water level over time.
- 4** The circumference of a circle relates to the diameter of the circle. Sketch a graph that shows the relationship between circumference of a circle and its diameter.



- 5 The graph shows the income made from the sale of gourmet pies.
- What is the income when 70 pies are sold?
  - How many pies need to be sold to have an income of \$500?
  - What is the gradient of the line? What does this represent?
  - Write the equation of the linear graph.
  - What is the income when 100 pies are sold?



- 6 Radioactive decay can be modelled using the equation  $m = 500(0.92)^t$ , where  $m$  is the mass of radioactive substance in grams and  $t$  is the number of years.

- a Complete this table of values for the equation  $m = 500(0.92)^t$ .

$t$	0	5	10	15	20	25
$m$						

- Draw the graph of this model with  $t$  on the horizontal axis and  $m$  on the vertical axis.
  - What was the initial mass of radioactive material?
  - Use the graph to estimate the mass of radioactive substance after 23 years.
  - Estimate the time taken for the mass of radioactive substance to reach 400 g.
  - Extend your graph to obtain an estimate of the time taken for the mass of radioactive substance to reach 50 g.
  - What is the percentage rate of decay each year? Explain how you worked this out from the equation.
- 7 The annual number of road fatalities per 100 000 of population in Australia from 1997 to 2017 is shown for selected years in the table below.

Year	Road fatalities per 100 000 population
1997	9.54
1999	9.32
2001	8.95
2003	8.15
2005	7.98
2007	7.62
2009	6.78
2011	5.71
2013	5.16
2015	5.08
2017	4.98



- a Plot points for the dataset on a number plane, using technology or by hand. Instead of using the actual year as the independent variable, use 0 for 1997, 2 for 1999, 4 for 2001, ... and 20 for 2017 to represent the number of years after 1997.

- b** Draw a straight line or exponential curve to best represent the trend of the data.
- c** State which model (linear or exponential) best approximates the data.
- d** From the trendline, estimate the number of road fatalities per 100 000 of population in 2012.
- e** The actual recorded number in 2012 is 5.78. How does this compare to the number estimated from the model?
- f** Use this model to predict what the number of fatalities per 100 000 of population will be in 2025. Explain the method you used to answer this.
- g** What factors could affect the reliability of this model to predict the number of fatalities in the future?

## REVIEW SET 4

- 1** A skyrocket is launched into the air. The height of the rocket,  $h$ , in metres, can be modelled using the results in the table, where  $t$  is the time in seconds.

$t$	0	2	4	6	8	10	12
$h$	0	140	240	300	320	300	240

- a** Draw a graph modelling the height of the rocket. Show  $t$  on the horizontal axis and  $h$  on the vertical axis.
  - b** What is the maximum height of the rocket? When does this occur?
  - c** From the graph, estimate the height of the rocket after 5 seconds.
  - d** From the graph, estimate when the rocket first reaches a height of 200 m.
  - e** Predict the height of the rocket after 13 seconds.
- 2** Owen walks at a constant speed from home to the football oval. He watches a match and then jogs home at a constant speed. Sketch a graph that shows Owen's distance from home over time.
- 3** Water is poured at a constant rate into the container on the right. Sketch a graph that shows the height of the water level over time.
- 4** The time it takes to travel a certain distance depends on the speed of the vehicle. Sketch a graph that shows the relationship between time and speed.
- 5** The mass of a bacterial culture grows at an increasing rate. Sketch a graph showing how the mass of the bacterial culture changes over time.
- 6** A physics student measures the length of a spring,  $L$ , in centimetres, when different weights of mass  $m$ , in kilograms, are hung on it. The results are shown in the table.

$m$ (kg)	0.2	0.3	0.4	0.5	0.6	0.7
$L$ (cm)	24	28	32	36	40	44

- a** Draw the graph of this linear model for values of  $m$  from 0.2 to 0.7.
- b** What is the gradient of the line? What does this indicate?
- c** If the line was extended to the vertical axis, what would be the vertical intercept? What would this represent?
- d** Write the equation for this linear model.
- e** Find the length of the spring when a weight of 375 g is hung on it.
- f** The student hangs a 1 kg weight on the spring and finds that it stretches to 62 cm. What does this tell you about the suitability of the linear model for predicting the lengths of the spring for weights of 1 kg or more? Would it be reliable for weights between 0.7 kg and 1 kg? Explain your answer.



- 7** A new car is purchased for \$28 000 and depreciates in value at a rate of 24% per year. The salvage value of the car can be modelled using the equation  $S = 28\,000(0.76)^n$ , where  $S$  is the salvage value in dollars after  $n$  years.
- Use graphing technology or a table of values to draw the graph of this exponential model for values of  $n$  from 0 to 6.
  - Does this graph represent exponential growth or decay? Explain your answer.
  - What does the intercept of the curve with the vertical axis represent?
  - Use the graph to find the salvage value of the car after 3 years.
  - Use the graph to estimate the number of years it takes for the salvage value of the car to be less than \$10 000.
- 8** The mass of algae, in grams, in a laboratory water tank was recorded each day for 10 days. The results are shown in the table.

<b>Time (days)</b>	0	1	2	3	4	5	6	7	8	9	10
<b>Mass (g)</b>	5.0	7.1	9.8	13.7	20.2	25.9	38.6	54.7	71.5	105.2	141.6

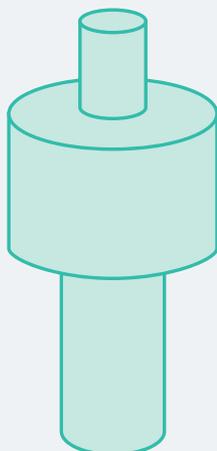
- Plot the points for the dataset on a number plane, using technology or by hand.
- Draw a straight line or exponential curve to best represent the trend of the data.
- State which model (linear or exponential) best approximates the data.
- From the trendline, estimate the mass of algae in the water tank after 60 hours.
- From the trendline, estimate when the mass of algae was 100 grams.
- Extend the trendline to predict what the mass of algae will be after 12 days.
- What factors could affect the reliability of this model to predict the mass of the algae in future weeks?

## REVIEW PRACTICE EXAMINATION QUESTION

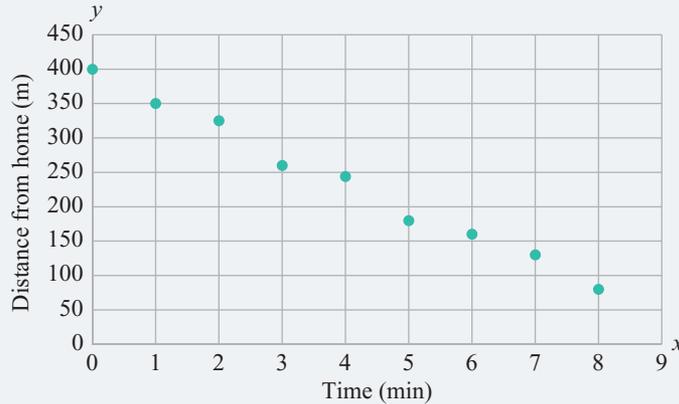
- 1 a** A ball is thrown into the air. The height of the ball,  $h$ , in metres, after  $t$  seconds is shown in the table.

<b><math>t</math></b>	0	1	2	3	4	5	6
<b><math>h</math></b>	1	16	25	28	25	16	1

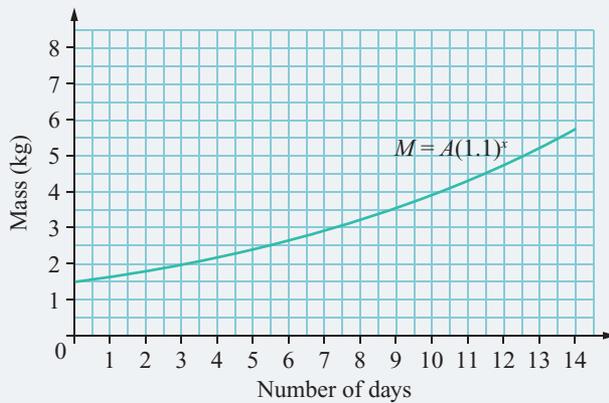
- Construct a graph of this relationship. (1 mark)
  - Does the graph show a linear relationship? (1 mark)
  - From the graph, find the maximum height of the ball and when this occurs. (1 mark)
  - From the graph, find when the ball first reaches a height of 20 m. (1 mark)
- b** Water is poured at a constant rate into the container below. Sketch a graph that shows the height of the water level over time. (2 marks)



- c** Rachel leaves home and jogs towards the park at a steady pace. She stops to remove a stone from her shoe before continuing on to the centre of the park. Once there, she turns and walks back home at a constant speed. Sketch a graph that shows her distance from home over time. (2 marks)
- d** Lily's distance from home was recorded each minute and plotted using a spreadsheet, as shown below.
- i** State which model (linear or exponential) would best approximate the data. (1 mark)
  - ii** Predict Lily's distance from home after 9 minutes. (1 mark)



- e** The mass  $M$  kg of a puppy at age  $x$  days is given by  $M = A(1.1)^x$ , where  $A$  is a constant. The graph of this equation is shown.



- i** Does this model show exponential growth or decay? Explain your answer. (1 mark)
- ii** What is the value of  $A$ ? (1 mark)
- iii** Estimate the mass of the puppy after 6 days. (1 mark)
- iv** Estimate the age of the puppy when it weighs 4.5 kg. (1 mark)
- v** What is the daily growth rate of the puppy's mass? Write your answer as a percentage. (1 mark)

**TOTAL:**  
15 marks

# 9

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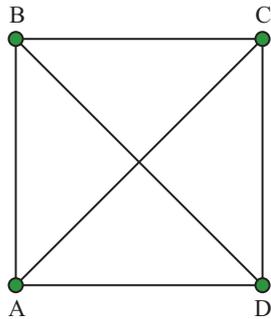
## Networks and paths

The main mathematical ideas in this chapter are:

- ▶ understanding the constituent parts of a network
- ▶ drawing network diagrams to represent information
- ▶ identifying walks, paths and cycles in a network
- ▶ finding trees and minimum spanning trees in a network
- ▶ using Kruskal's and Prim's algorithm to find a minimum spanning tree
- ▶ finding the shortest path between two vertices
- ▶ solving real-life problems using networks.

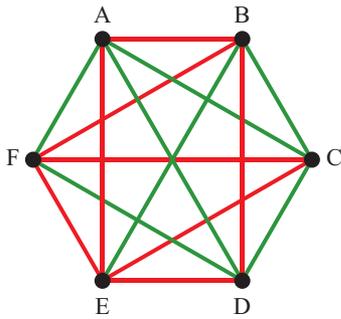
# ARE YOU READY?

- 9A ▶ 1** How many line segments (straight lines between two points) are shown in the network?



- A** 4                      **B** 6  
**C** 7                      **D** 8

Use the network below to answer questions **2** and **3**.



- 9A ▶ 2** Which of the following is a red triangle?  
**A** ADF                      **B** FCE  
**C** AED                      **D** ACD
- 9A ▶ 3** Which of the following is a green triangle?  
**A** ADB                      **B** FCE  
**C** AED                      **D** ACD

The following table contains the distance (in kilometres) between capital cities in mainland Australia. Use this table to answer questions **4** to **8**.

<b>Adelaide</b>						
2075	<b>Brisbane</b>					
1209	1267	<b>Canberra</b>				
3041	3435	4034	<b>Darwin</b>			
732	1813	651	3773	<b>Melbourne</b>		
2721	4434	3930	4037	3453	<b>Perth</b>	
1419	978	289	3971	876	3975	<b>Sydney</b>

- 9A ▶ 4** The distance between Brisbane and Melbourne is:  
**A** 651 km                      **B** 1813 km  
**C** 3435 km                      **D** 4434 km
- 9A ▶ 5** What is the total distance of a trip from Melbourne to Sydney, then Sydney to Canberra?  
**A** 876 km                      **B** 940 km  
**C** 1165 km                      **D** 1527 km
- 9A ▶ 6** What is the total distance of a trip from Melbourne to Canberra, then Canberra to Sydney?  
**A** 876 km                      **B** 940 km  
**C** 1165 km                      **D** 1527 km
- 9A ▶ 7** What is the smallest distance in the table?  
**A** 156 km                      **B** 289 km  
**C** 651 km                      **D** 4434 km
- 9A ▶ 8** A travelling salesman sets out from Melbourne to Brisbane. Along the way he must visit Sydney and Canberra. Which itinerary covers the shortest distance?  
**A** Melbourne, Sydney, Canberra, Brisbane  
**B** Melbourne, Canberra, Sydney, Brisbane  
**C** They cover the same distance  
**D** Impossible to tell

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook](#) [assess](#).

- Q1–3** Support sheet 9A.1 Lines and line segments  
**Q4–8** Support sheet 9A.2 Interpreting tables

# 9A

# Introduction to networks

These resources are available on your obook assess:

- **Video tutorial 9A:** Watch and listen to an explanation of Example 9A–3
- **Worksheet 9A:** Practise your skills with extra problems for networks
- **assess quiz 9A:** Test your skills with an auto-correcting multiple-choice quiz

### network

a collection of objects that are related to each other in some way

### vertex

a visual representation of some object, usually drawn as a point; sometimes called a node

### edge

a line between two vertices, which indicates a relationship between the vertices; sometimes called an arc

### loop

an edge that starts and ends at the same vertex

### degree

(of a vertex) the number of edges connected to a vertex

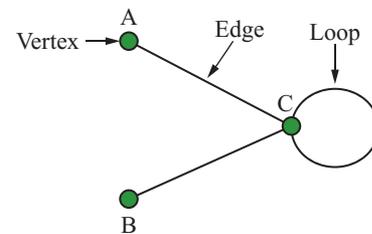
**Networks** deal with the idea of optimisation, which involves finding the *best possible* solution to a problem. Such as finding the *minimum* spanning tree, or the shortest path between two places.

Before we begin solving practical problems, we need to introduce some terminology.

A **vertex** is a point, and an **edge** is a line drawn between two vertices. An edge drawn from a vertex back to itself is called a **loop**.

The **degree** of a vertex is the number of edges connected to it (a loop adds 2 to the degree of a vertex).

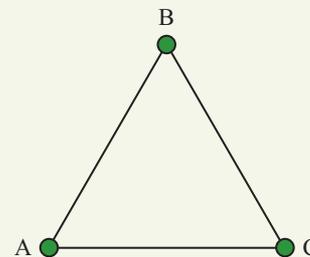
Together vertices and edges form a network.



## EXAMPLE 9A–1 Counting the number of vertices, edges and the degree of each vertex

For the network on the right, count:

- a the number of vertices
- b the number of edges
- c the degree of each vertex.



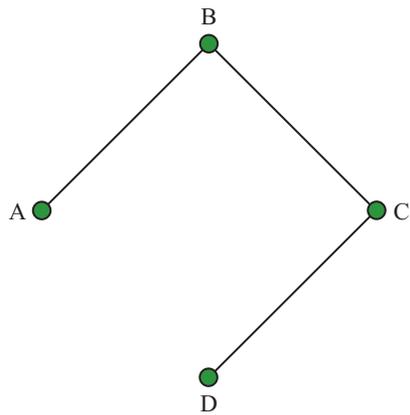
	Solve/Think	Apply
a	There are three vertices: A, B and C.	Vertices are the points in the network. They are denoted by a single letter. Edges are the lines joining two vertices together. They are denoted by the letters at either end of the edge, or by their own name. The degree of a vertex is the number of edges connected to it.
b	There are three edges: AB, BC and CA.	
c	Vertex A is connected to two edges: AB and CA. It has a degree of 2. Vertex B is connected to two edges: AB and BC. It has a degree of 2. Vertex C is connected to two edges: BC and CA. It has a degree of 2.	

# EXERCISE 9A Introduction to networks

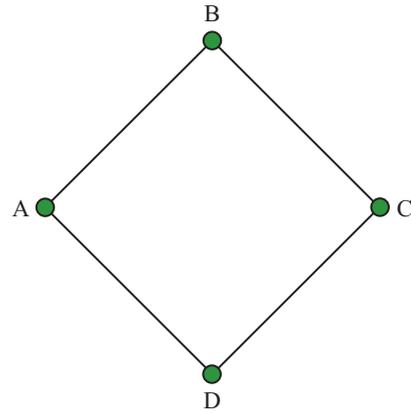
UNDERSTANDING, FLUENCY AND COMMUNICATING

1 Count the number of vertices in each network.

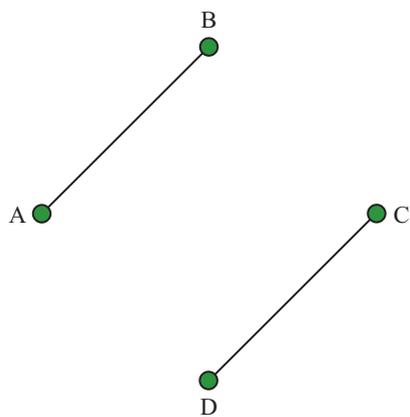
a



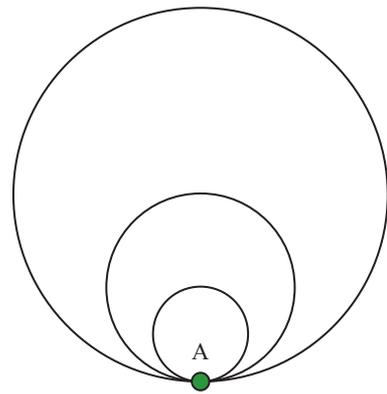
b



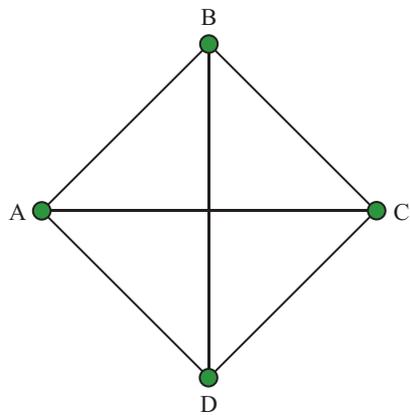
c



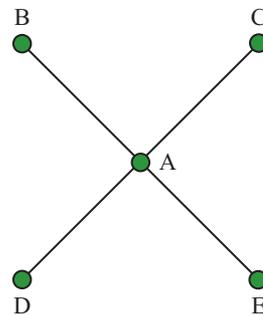
d



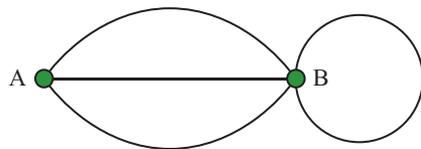
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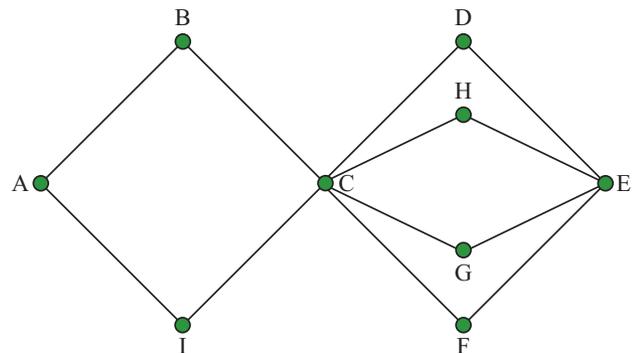
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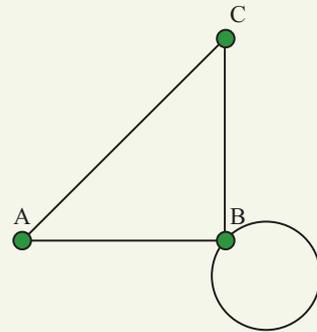
h



2 Count the number of edges in each network in question 1.

**EXAMPLE 9A-2** Determining the sum total of the vertex degrees of a network

- a Make a table of the vertex degrees for the network on the right.
- b How many edges are in this network?
- c What is the sum total of the vertex degrees?



Solve/Think				Apply								
a	<table border="1"> <tr> <th>Vertex</th> <td>A</td> <td>B</td> <td>C</td> </tr> <tr> <th>Degree</th> <td>2</td> <td>4</td> <td>2</td> </tr> </table>	Vertex	A	B	C	Degree	2	4	2	The loop contributes 2 to the vertex degree of B.		
Vertex	A	B	C									
Degree	2	4	2									
b	The number of edges is 4.											
c	The sum of vertex degrees is $2 + 4 + 2 = 8$ .											

Note that the sum of the vertex degrees (8) is twice the number of edges (4). This is the case for all networks.

- 3 i Make a table of the vertex degrees for each network in question 1.
- ii What is the sum total of the vertex degrees for each network? Verify that the sum total of the vertex degrees is always twice the number of edges.

**EXAMPLE 9A-3** Drawing a network from a table

Here is a list of people and their network of friends. Represent this information as a network.

Person	Ada	Ben	Carlos	Dalia	Eric
Friends	Ben, Carlos, Eric	Ada	Ada, Dalia	Carlos, Eric	Ada, Dalia

Solve	Think/Apply
	This information can be represented visually as a network by using a vertex to represent each person, and an edge to indicate that they are friends.

Note that this is just one of many ways to draw the information in the table. Another way would be to draw the vertex for Ben inside the loop.

- 4 Complete the table of the vertex degrees from Example 6A-3.

Vertex	Ada	Ben	Carlos	Dalia	Eric
Degree	3		2		

- 5 The following table represents airports and the destinations serviced by that airport. Draw a network that represents this information.

Airport	Albury (ABX)	Bathurst (BHS)	Coffs Harbour (CFS)	Dubbo City Regional (DBO)	Evans Head (EVH)
Destinations	BHS, DBO	ABX, CFS, DBO	BHS, EVH	ABX, BHS	CFS

6 a Draw a network that represents the following people and their friends.

<b>Person</b>	Uma	Vanessa	Wasim	Xavier	Yvonne	Zhang
<b>Friends</b>	Vanessa, Zhang	Wasim, Uma	Vanessa, Zhang, Yvonne, Xavier	Wasim, Yvonne	Wasim, Xavier	Uma, Wasim

b Complete the table of vertex degrees.

<b>Vertex</b>	Uma	Vanessa	Wasim	Xavier	Yvonne	Zhang
<b>Degree</b>						

- c Uma is telling Vanessa a story about what happened to a friend of a friend of hers. Use your answer to part a to find out who the story is about.
- d Yvonne is telling Wasim a story about what happened to a friend of a friend of hers. Use your answer to part a to find out who the story is *not* about.

**directed network**

a network in which the edges have a direction; the orientation of the edge is represented by an arrow

**weighted edge**

edges with a weight that represents a numerical quantity associated with the relationship between the vertices

A **directed network** is a network in which the edges are given a direction indicated by an arrow. A directed edge indicates a *one-sided* relationship between the vertices, such as the direction of a one-way street or the direction of the flow of water.

A **weighted edge** is an edge with a certain numerical value, such as the length of the street or the size of the flow of water in litres per second. In practice, the meaning of the edge direction and weight is determined by the context of the network.



**EXAMPLE 9A-4** Determining the travel time in a network with weighted edges

The Sydney CBD stations are connected by an underground rail network called the City Circle. Trains start at Central Station and travel in the City Circle clockwise (via Town Hall) or anticlockwise (via Museum). The following table shows the stations in the City Circle via the Town Hall line, and the time it takes to travel between them.

<b>Station</b>	Central	Town Hall	Wynyard	Circular Quay	St James	Museum
<b>Next stop</b>	Town Hall	Wynyard	Circular Quay	St James	Museum	Central
<b>Time (minutes)</b>	2	2	3	3	2	3

- a Represent the City Circle via Town Hall line as a directed network with weighted edges.
- b How long does it take to travel from Central to St James using the City Circle via the Town Hall line?

	Solve/Think	Apply
a		<p>Each station is a vertex. The connections between the stations are represented by directed edges and detailed with the time it takes to travel between them.</p>

Solve/Think	Apply
<p><b>b</b> It takes <math>2 + 2 + 3 + 3 = 10</math> minutes to travel from Central Station to St James.</p>	<p>Travel along this network is only permitted in the direction of the arrows, so although it would be shorter to travel from Central to St James via Museum, the direction of the edges in this network do not permit this route.</p>

**7** Using the network from Example 6A–4, find the time it takes to travel from Town Hall to Museum using the City Circle via the Town Hall line.

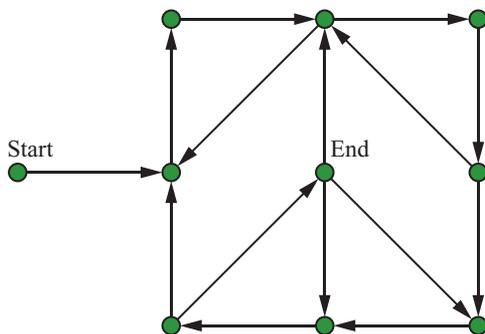
Vertices in a directed network have an indegree and an outdegree, instead of a degree. The **indegree** is the number of edges pointing towards the vertex, and the **outdegree** is the number of edges pointing away from the vertex.



**indegree**  
the number of edges pointing towards a vertex in a directed network

**outdegree**  
the number of edges pointing away from a vertex in a directed network

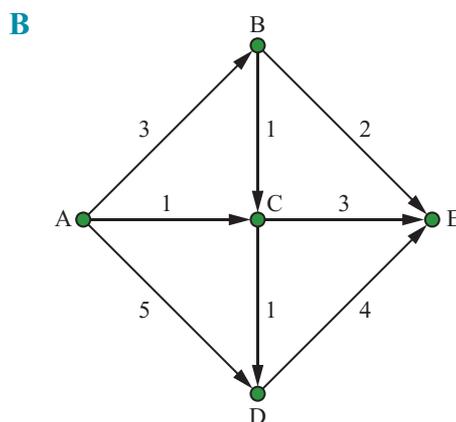
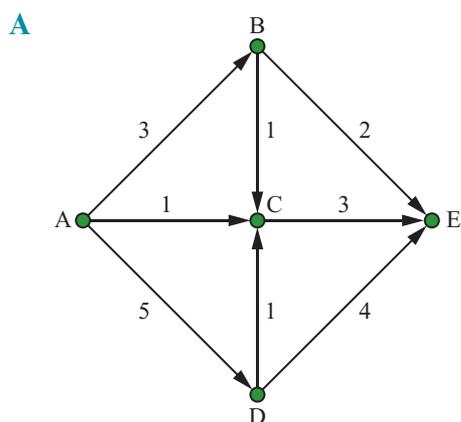
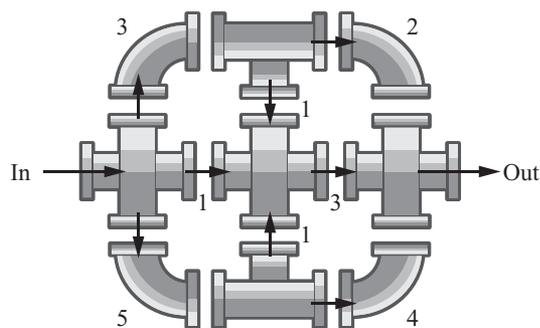
**8** Find your way to the centre of the labyrinth. You can only travel along edges in the direction of the arrows.

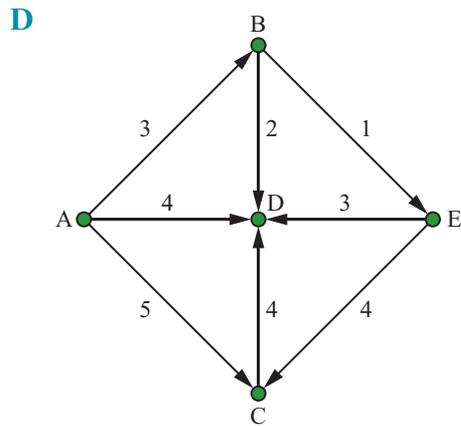
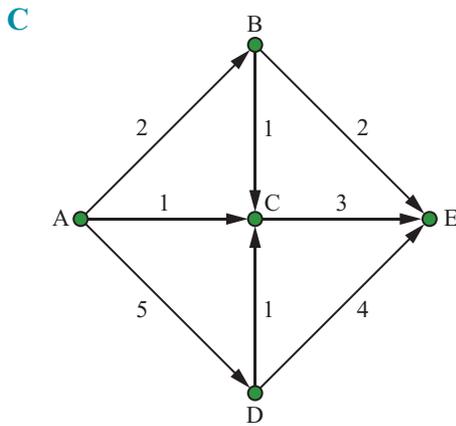


- How many vertices did you touch on your way (including the start and end)?
- How many edges did you travel along?
- What is the indegree of the End vertex?
- What is the outdegree of the End vertex?
- Is there more than one way from the Start to the End?

**9** A network of pipes flows from junction to junction as shown. The vertices A, B, C, D and E represent the intersection points of the pipes.

Which of the diagrams below shows the direction of the flow and the amount of the flow, in litres per second, as a directed and weighted network?



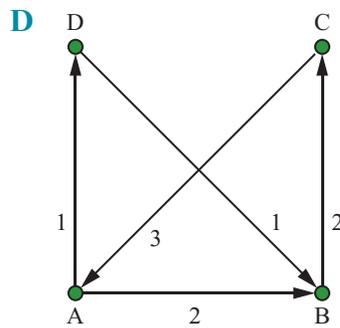
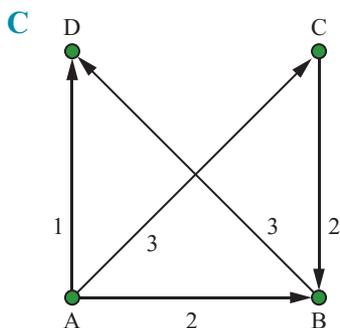
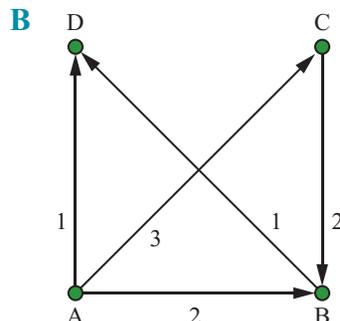
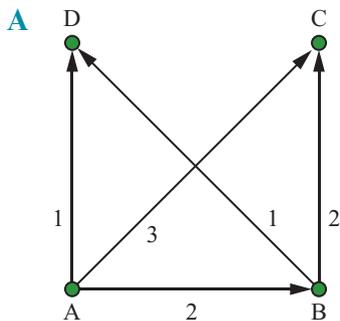


- 10** For the network in question 9, find the total flow, in litres per second that:
- a** flows out of vertex A
  - b** flows into vertex E
  - c** flows into vertex B
  - d** flows out of vertex C
  - e** flows out of vertex D.

- 11** Here is a table of vertices A, B, C and D, and the edge weights between these vertices.

From vertex	To vertex			
	A	B	C	D
A	–	2	3	1
B	–	–	–	3
C	–	2	–	–
D	–	–	–	–

Which of the following directed weighted networks represents this information?



**12** The table below shows the edge weights from question 9. Fill in the blank edge weights.

From vertex	To vertex				
	A	B	C	D	E
A	–	3			–
B	–	–	1	–	
C	–	–	–	–	3
D	–	–	1	–	
E	–	–	–	–	–

**13** Here is a table of vertices A, B, C and D, with the edge weights between these vertices. Draw a directed weighted network to represent this information.

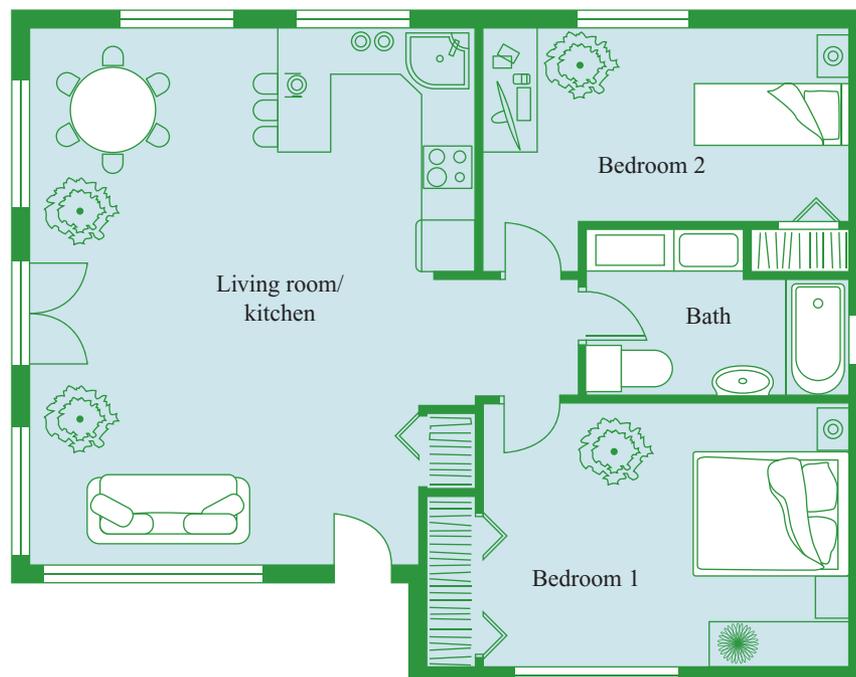
From vertex	To vertex			
	A	B	C	D
A	–	–	2	5
B	1	–	7	–
C	–	–	–	4
D	–	3	–	–

**14** Imagine that the networks in question 1 represent people at a party. The people are vertices, and an edge indicates that the two people have shaken hands (a loop means that the person shakes their other hand, and so counts for two handshakes). How many people shook an odd number of hands in each network? Verify that there is always an even number of such people.

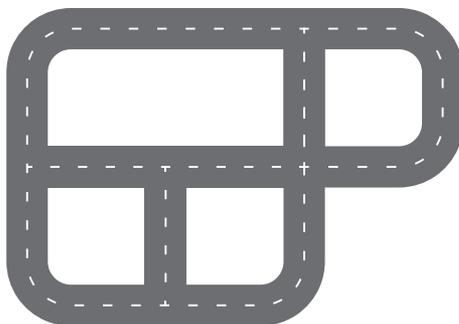
**15** A house has a printer, computer, TV and gaming console. All these devices must be connected to a single router via ethernet cables.

- Draw a network diagram to represent this information. Use vertex names P, C, T, G and R for the printer, computer, TV, gaming console and router respectively, and use edges to represent an ethernet connection between the devices.
- How many ethernet connections are required at R?

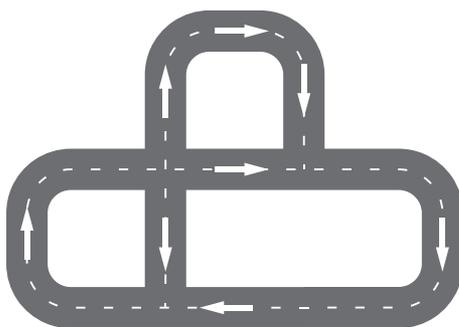
**16** Draw the following house plan as a network. Use vertices to represent the rooms and edges to represent the doorways that connect the rooms.



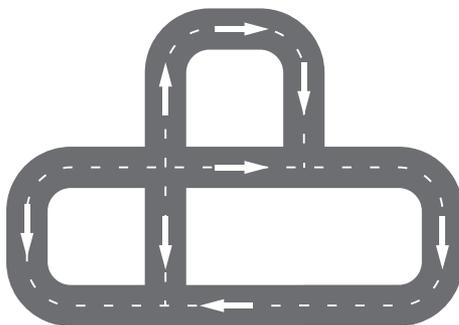
- 17** Redraw the roads below as a network using vertices to represent intersections, and edges to represent the span of road connecting two intersections.



- 18** Draw a directed network that represents the following street map. Represent the street intersections with vertices, and use directed edges to represent the span of road between intersections.

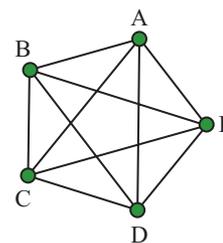


- 19** Here is the same road layout, except that the street orientation has changed. Represent this as a directed network, then use the notions of indegree and outdegree to describe the problem at each intersection.

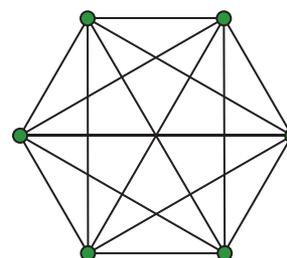


- 20** Use a red and blue pen to colour over all the edges in the network on the right. Try to do this without drawing a red triangle or a blue triangle.

*Hint:* first colour as many edges in red as you can without drawing a triangle, then do the same for blue.



- 21** **a** Colour all the edges of the network on the right in blue or red.  
**b** There will always be a red triangle or a blue triangle. See if you can find it.  
**c** Suppose that the vertices represent people. The blue edges connect people who have met before, and the red edges connect people who have not met before. You are hosting a dinner party and want to ensure that three people will all have met before, or three people will have never met before. What is the smallest number of people you should invite?  
 Hint: compare your answers to questions **20** and **21b**.



# 9B Paths and cycles

These resources are available on your obook assess:

- **Worksheet 9B:** Practise your skills with walks, paths and cycles
- **Assess quiz 9B:** Test your skills with an auto-correcting multiple-choice quiz



## walk

a sequence of vertices and the edges between them

## connected network

a network in which there is a walk between every pair of vertices

## disconnected network

a network that is not connected

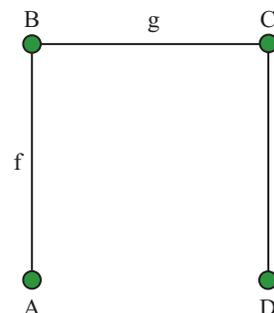
In this section we will explore walks through a network. A **walk** is a journey through a network that starts at a vertex and travels along the edges of the network, before ending at a vertex.

Trace your finger over the network on the right from vertex A to vertex D. The vertices and edges that you touched, in the order that you touched them, are:

A, f, B, g, C, h, D

This sequence of vertices and edges is a walk. It can be read as follows:

Starting from A, travel along edge f to vertex B, then along edge g to vertex C, then along edge h to vertex D. Vertex A is called the start vertex and vertex D is called the end vertex.



It is sometimes convenient to abbreviate the walk to just the vertices:

**ABCD**

The edges f, g and h are implicit in the walk that visits vertices A, B, C, D. This is convenient when the edges have no names, and we will use this style of abbreviation often in this chapter. Another style of abbreviation is to use only the edges:

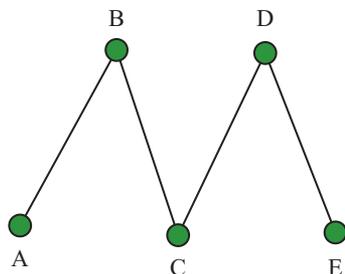
**fgh**

The vertices A, B, C and D are implicit in the walk with edges fgh. This is convenient when the edges do have names.

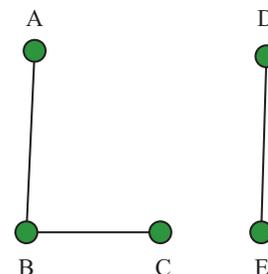
**Connected networks** are networks in which there is a walk between any pair of vertices.

If there is a pair of vertices with no walk between them the network is said to be

**disconnected.**



Connected network

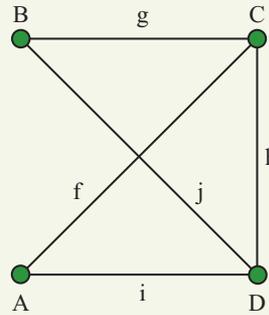


Disconnected network

### EXAMPLE 9B-1 Finding a walk in a network

Which of the following is a walk from A to B?

- A AB
- B A, f, C, g, B
- C A, f, C, h, D
- D B, j, D, i, A

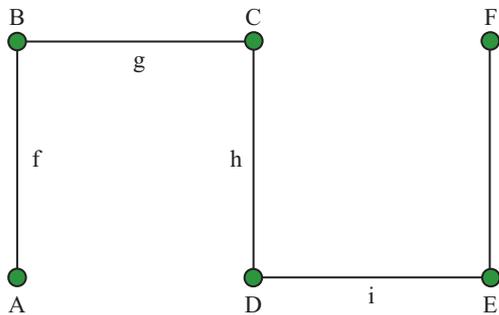


Solve/Think	Apply
Option B (A, f, C, g, B) is a walk from A to B.	Option A is not a walk since there is no edge between A and B. Option C is a walk from A to D. Option D is a walk from B to A.

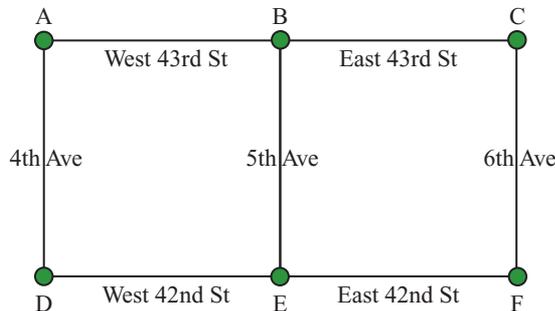
## EXERCISE 9B Paths and cycles

UNDERSTANDING, FLUENCY AND COMMUNICATING

- 1 Trace out a walk from A to F in the network below. Write down each vertex and edge that you visit in order.

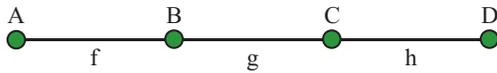


- 2 In the network from question 1, trace out the walk with edges  $ihg$ .
- a What is the start vertex?
  - b What is the end vertex?
- 3 The streets of Manhattan are divided into East and West by 5th Avenue. The following network is a map of Manhattan between 4th and 6th avenues and 43rd and 42nd streets.



You are given the following directions: Start at corner A. Follow West 43rd St to corner B. Follow 5th Avenue to Corner E. Follow East 42nd St to Corner F. Then follow 6th Avenue to corner C. Are these valid directions? Where do they lead?

- 4 a Consider the walk fgh in the network below. What is the sequence of vertices visited by this walk? What are the start and end vertices?



- b Consider the walk DCBA. What is the sequence of edges visited by this walk? What are the start and end vertices?
- c Consider the walk ABCDCBA. What is the sequence of edges visited by this walk? What are the start and end vertices?

**path**  
a walk that does not visit any vertex twice (or more)

**cycle**  
a walk that starts and ends at the same vertex, but does not visit any other vertices twice (or more)

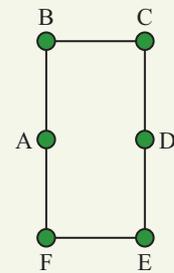
A **path** is a walk that doesn't visit any vertex more than once. A **cycle** is a walk with the same start and end vertex, which doesn't visit any other vertex more than once.



### EXAMPLE 9B-2 Identifying walks, paths and cycles

Identify the following as a walk, path, cycle, or other, giving reasons for your choice.

- a ABCDED
- b DEFA
- c ABCDEFA
- d AD



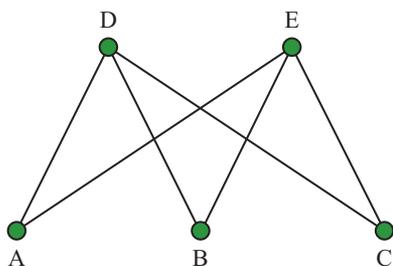
#### Solve/Think

- a This is a walk because it follows the edges of the network. It cannot be a path because it visits vertex D twice.
- b This is a path because it follows the edges of the network but does not visit any vertex more than once.
- c This is a cycle because it follows the edges of the network, and the start vertex and end vertex are the same.
- d There is no edge between A and D, hence this is something other than a walk, path or cycle.

#### Apply

In this example, the walks are referred to as a sequence of vertices. This is common practice when the edges do not have labels.

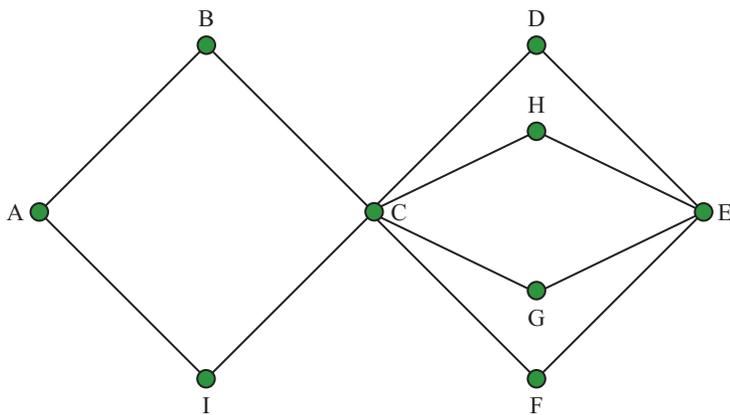
- 5 Which of the following is not a path from A to C?



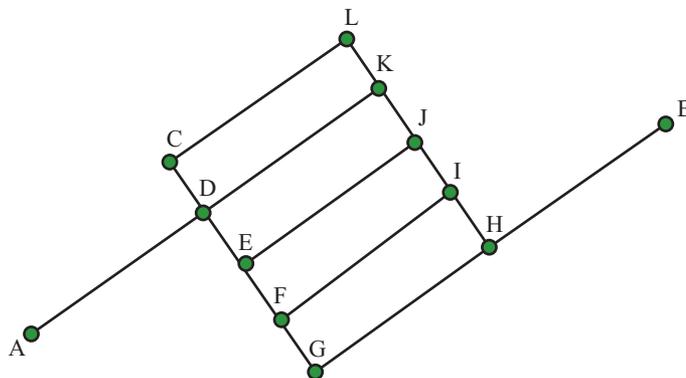
- A ABC
- B ADC
- C AEC
- D AEBDC

6 Which of the paths from question 5 visits every vertex?

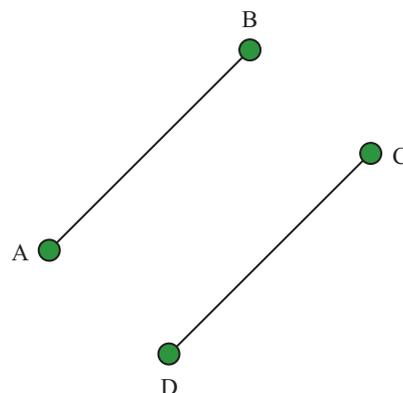
Use the network below to answer questions 7 and 8.



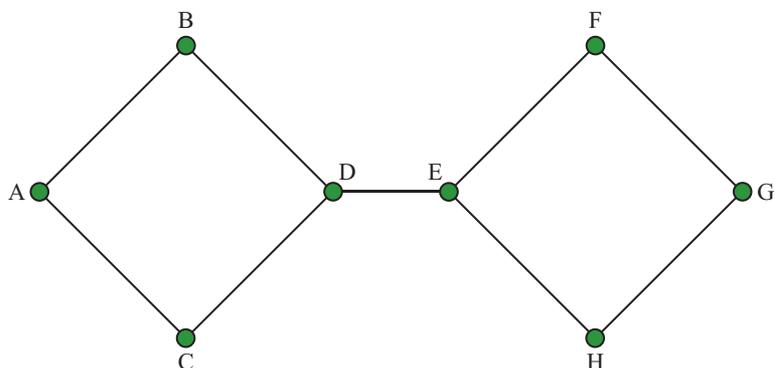
- 7
- Find a path from A to E.
  - Find a cycle that starts from vertex A.
  - Find a path from F to B that includes vertex D.
  - Find a cycle that starts from vertex H and includes vertex F.
- 8 Find a walk that visits every edge exactly once. Is your walk a cycle? Is your walk a path?
- 9 The following map shows a network of roads between Broken Hill High School (A) and the local park (B).
- If it takes one minute to walk down the short side of a block, and three minutes to walk down the long side, how long does it take to walk from A to B?
  - How many different routes from A to B take this amount of time?



- 10 The following network is not connected because there is no path between which vertices?
- |   |         |   |         |
|---|---------|---|---------|
| A | A and B | B | D and C |
| C | A and C | D | C and C |



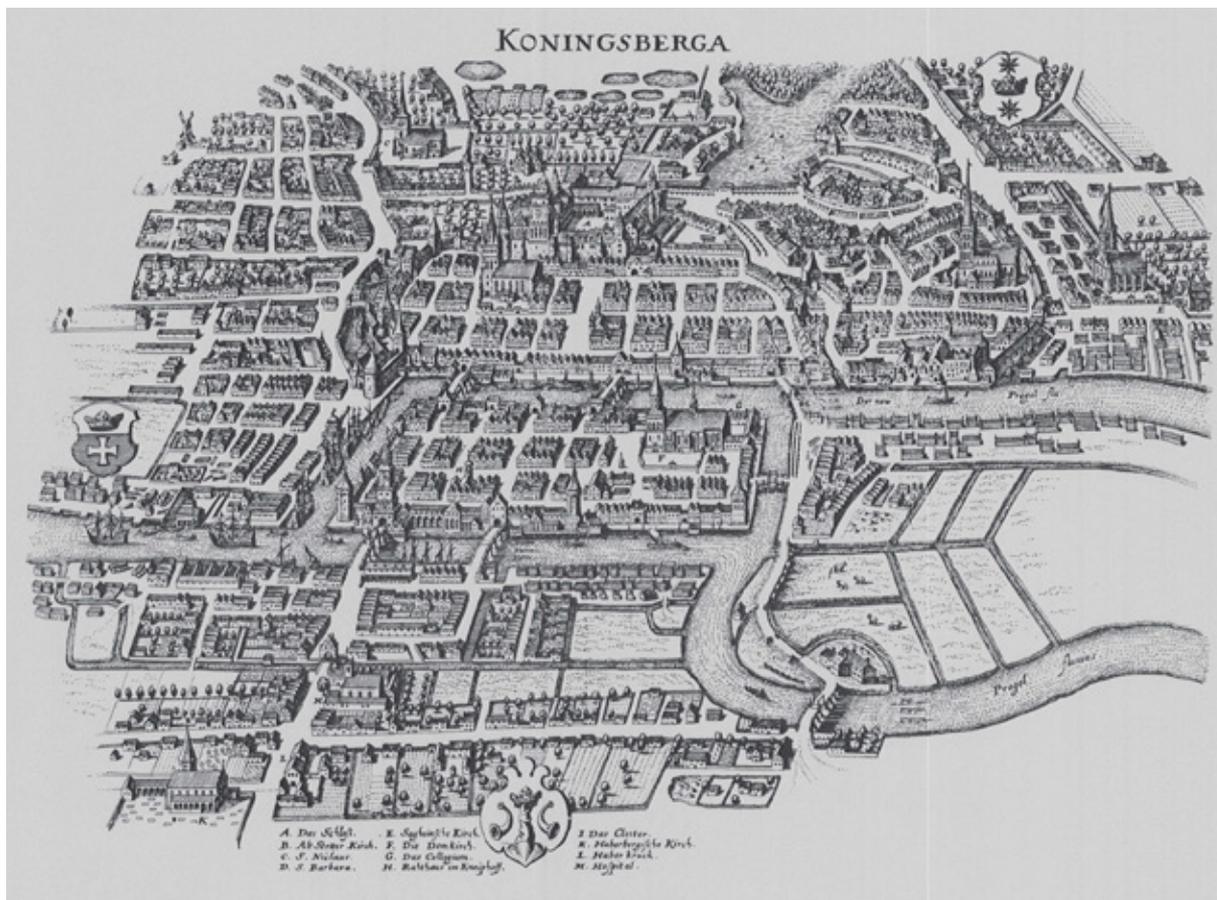
Use this network to answer questions 11 and 12.



- 11 Which single edge should be removed to disconnect the network?
- 12 Find a walk in the network that visits every edge exactly once.

Historically, the first network paths problem is known as the **Seven Bridges of Königsberg**.

The old city of Königsberg lies on a river connected by seven bridges, as shown below. The problem is to plan a walk through Königsberg that crosses every bridge exactly once.

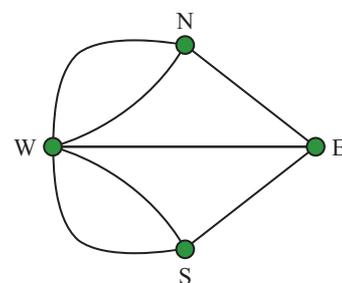


We can represent the city on each side of the river, and the two islands, by vertices. The bridges that connect the city can be represented by edges. This allows us to present a simplified version of the map that contains

only the relevant information. The question can then be rephrased as:  
Is there a walk that visits every edge exactly once?

The answer to the seven bridges of Königsberg problem is that there is no such walk. In 1736 Leonhard Euler solved the Königsberg problem, showing that it has only to do with the number of vertices with odd degree. If there are more than two vertices with odd degree, then no walk can visit every edge exactly once.

Let's look at the vertex degrees of this network.



Vertex	N	E	S	W
Degree	3	3	3	5

The network for this problem contains four vertices of odd degree, which is two too many! So it is not possible to find a walk that traverses each edge exactly once.

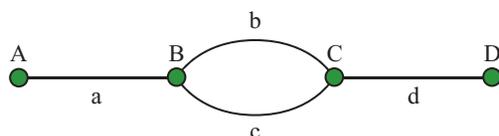
**To find a walk that traverses each edge exactly once in a network**

If there are exactly two vertices of odd degree, then these vertices must be the start and end.

If there are no vertices of odd degree, then the start and end vertex are the same.

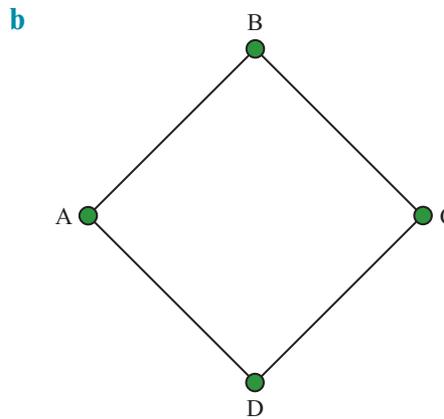
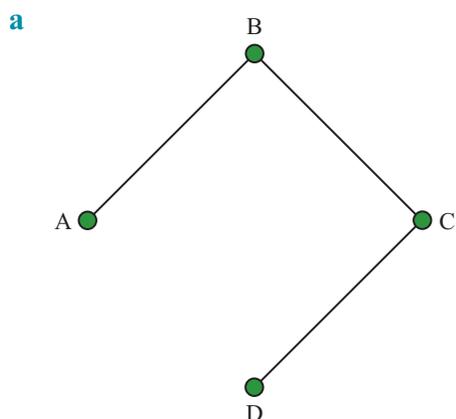


**13** There are two walks from A to D in the network below.

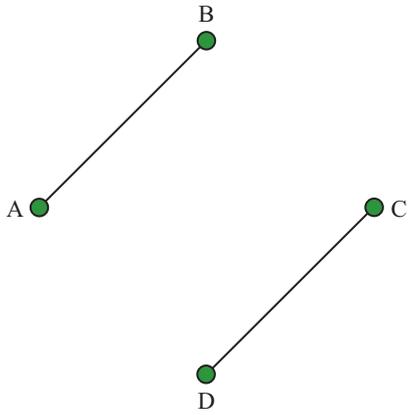


- Write down a walk from A to D as a sequence of vertices and edges.
- Write your answer to part **a** as a sequence of edges only.
- Write your answer to part **a** as a sequence of vertices only.
- Now find a different walk from A to D. Write down your walk as a sequence of vertices and edges.
- Write down your answer to part **d** as a sequence of edges only.
- Write down your answer to part **d** as a sequence of vertices only.
- Compare your answers to parts **c** and **f**. In this example, is it a good idea to abbreviate the walks to a sequence of vertices?

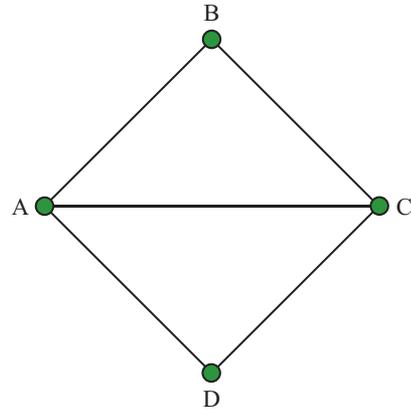
**14** For each of the networks below, first draw a table of vertex degrees and then decide if the network has a walk that traverses every edge exactly once. If such a walk exists then list the vertices of the walk.



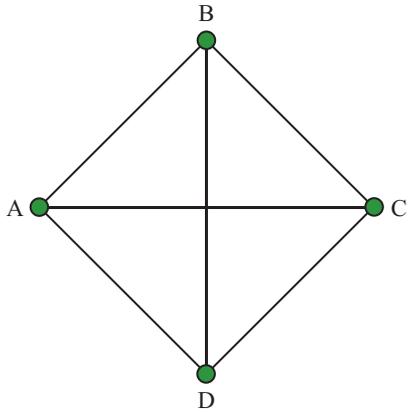
c



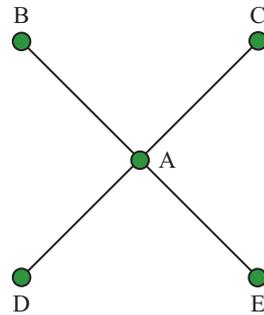
d



e

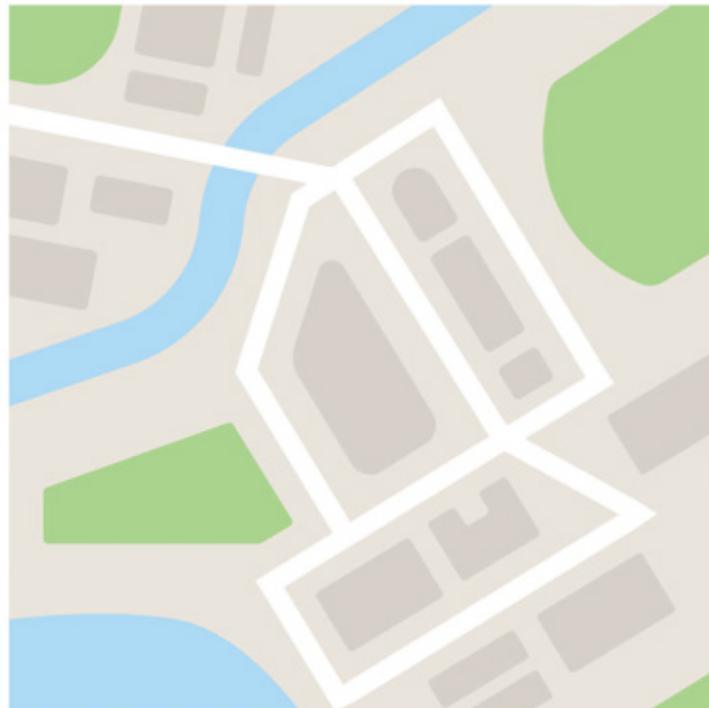


f

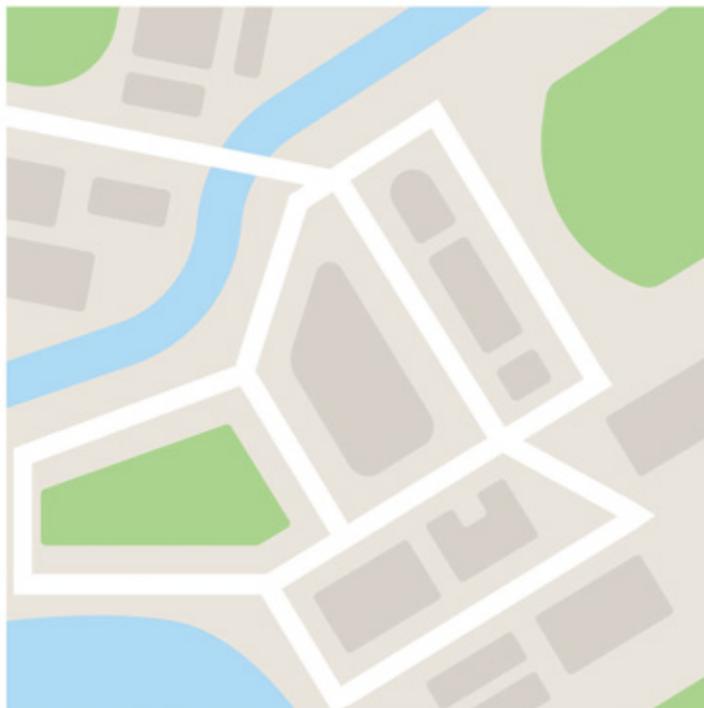


**15** For each of the networks in question 14 parts **a** to **d**, how many different paths can you find from A to D? Remember that a path cannot visit the same vertex twice.

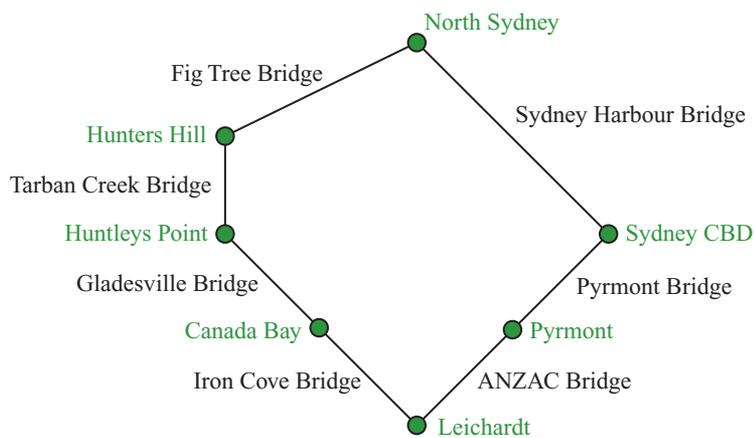
**16** Design a garbage truck route that visits every street in the map below exactly once, or explain why this task is impossible.



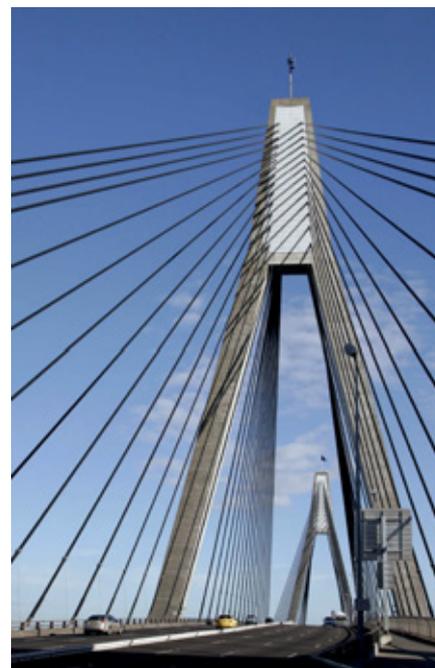
- 17** Design a garbage truck route that visits every street in the map below exactly once, or explain why this task is impossible.



- 18** In 2017 the ‘Seven Bridges Walk’ toured some of the scenic bridges of Sydney.



Count the vertex degrees, then state why it is possible to cross the seven bridges of Sydney without visiting the same bridge more than once.



# 9C Trees

**tree**

a connected network with no cycles

These resources are available on your obook assess:

- **Worksheet 9C:** Practise your skills with extra problems for trees and forests
- **assess quiz 9C:** Test your skills with an auto-correcting multiple-choice quiz

A **tree** is a special kind of network in which there is exactly one path between any two vertices. As such, a tree must be a connected network with no cycles.

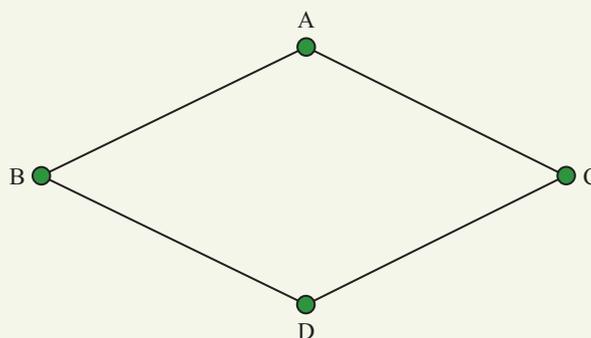
## EXAMPLE 9C-1 Identifying trees

Which of the following networks are trees?

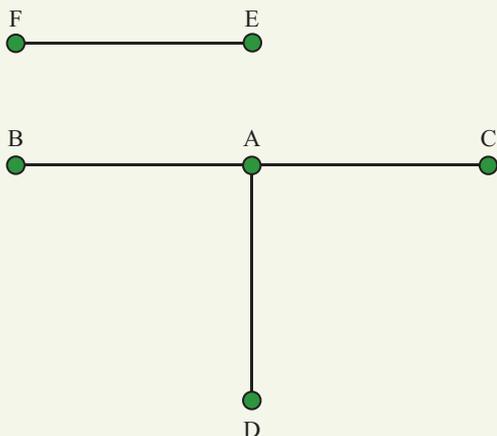
**a**



**b**



**c**



	Solve/Think	Apply
<b>a</b>	This is a tree because there is exactly one path from A to B: AB, and exactly one path from B to A: BA.	A more visual way to identify a tree is to use the alternative criteria: a tree is a connected network with no cycles. So, <b>a</b> is a tree because it has no cycles and is connected, <b>b</b> is not a tree because it has a cycle ABCDA, and <b>c</b> is not a tree because it is not connected.
<b>b</b>	This is not a tree because there are two paths from A to D: ABD and ACD.	
<b>c</b>	This is not a tree because there is no path from A to E.	

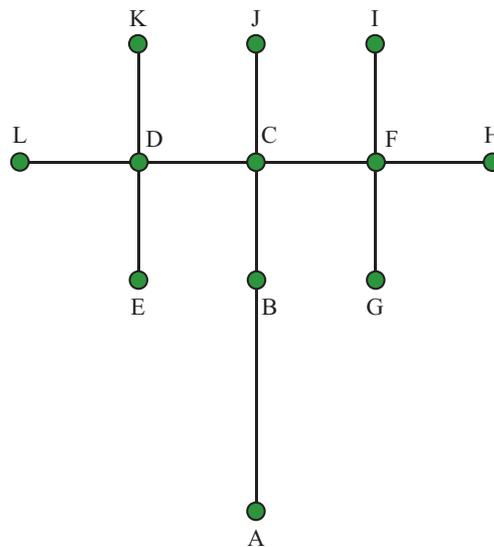
# EXERCISE 9C Trees

UNDERSTANDING, FLUENCY AND COMMUNICATING

- 1 The network given here is a tree, because there is exactly one path between any two vertices.

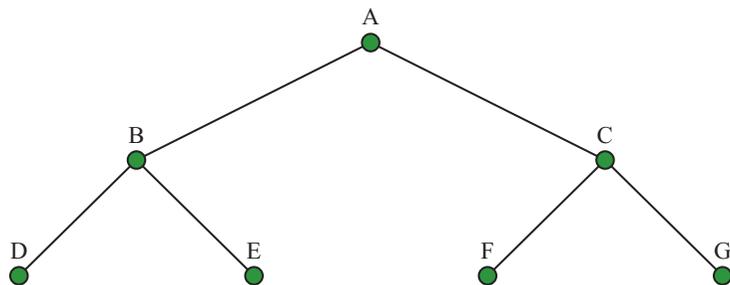
Find the unique path from:

- a A to L
- b L to E
- c B to H
- d G to K.

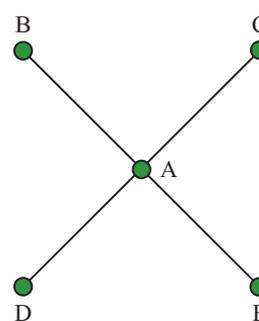


- 2 Which of the following networks are also trees?

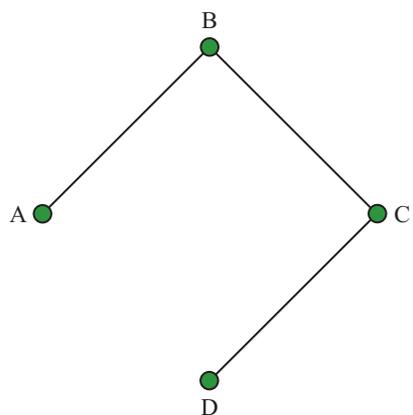
A



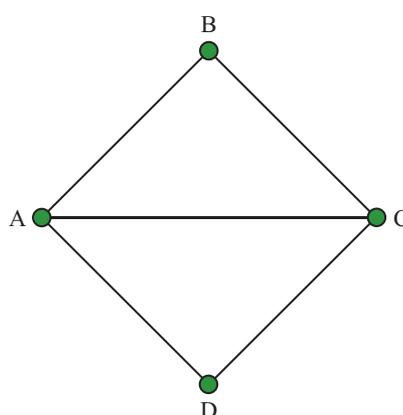
B



C



D



- 3 Count the number of edges and vertices in each of the trees from question 2.
- 4 Draw the tree on right as a network. Use vertices to represent the end of a branch (in network terminology, such a vertex is called a **leaf**), as well as the joins between branches. Draw edges to represent the branches. Is your network a tree?

**leaf**  
in a tree, any vertex of degree 1



For a connected network, a **spanning tree** is a tree that contains all the vertices of the original network, and some of the edges. It is helpful to think of a spanning tree as a tree that is carved out of a larger network. Every connected network has at least one spanning tree. A spanning tree can be constructed by following these three steps:

- Step 1:** Find a cycle.
- Step 2:** Remove any edge from that cycle.
- Step 3:** Repeat steps 1 and 2 until there are no more cycles.

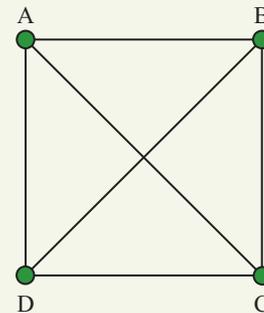
The remaining network will be a spanning tree. Note that a spanning tree always has one less edge than the number of vertices.



**spanning tree**  
a tree made from all of the vertices and some of the edges of the original network

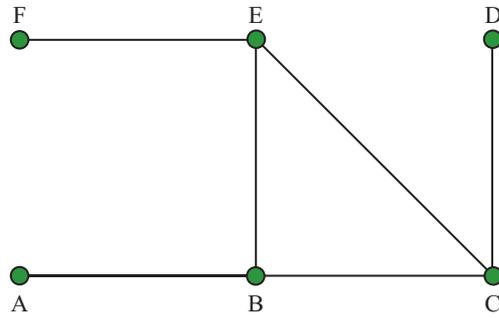
### EXAMPLE 9C-2 Constructing a spanning tree

Construct a spanning tree for the following network.



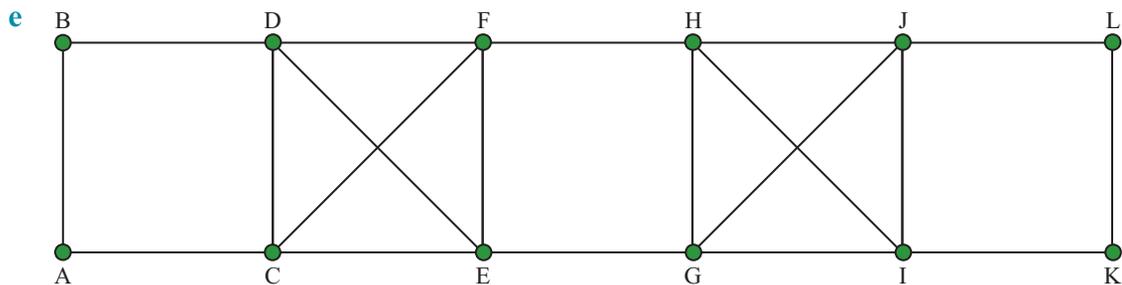
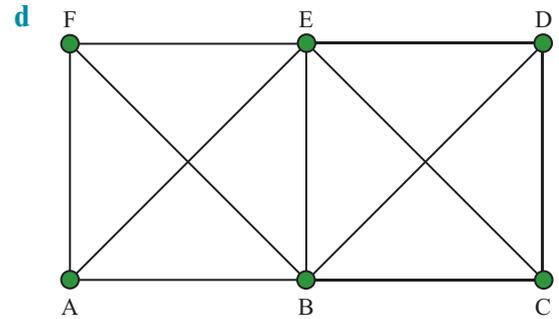
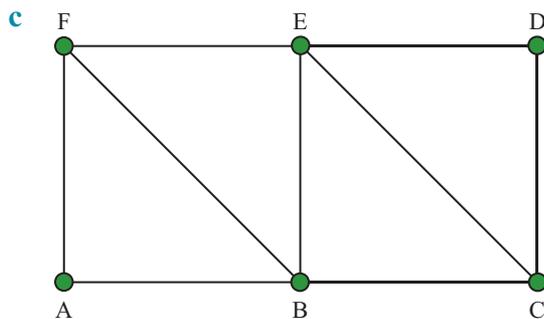
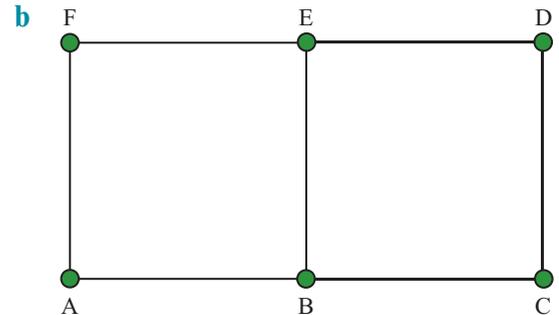
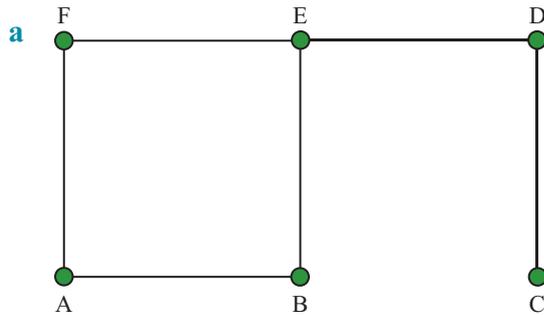
Step	Solve/Think	Apply
1.1	Identify that ABCA is a cycle.	
1.2	Remove any edge from the cycle. In this example we have chosen to remove AB.	
2.1	Identify that ACDA is a cycle.	
2.2	Remove any edge from the cycle. In this example we have chosen to remove AC.	
3.1	Identify that BCDB is a cycle.	
3.2	Remove any edge from the cycle. In this example we have chosen to remove BC.	
Stop	There are no more cycles.	

5 Which edge should be removed to transform this network into a tree?



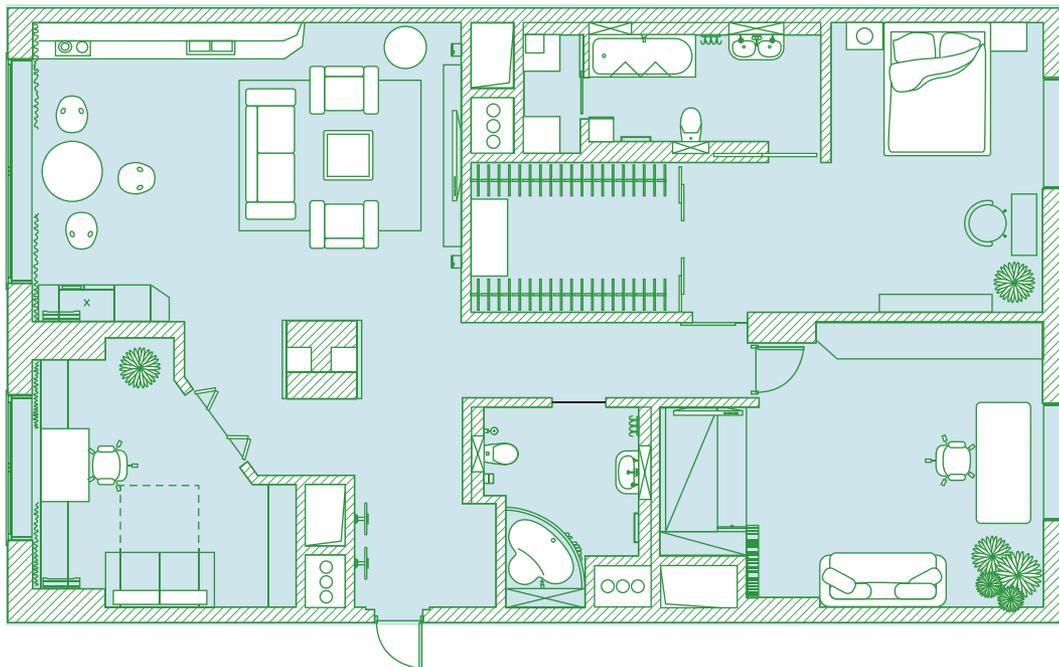
- A AB                      B BC                      C CD                      D EF

6 Find a spanning tree for the following networks.

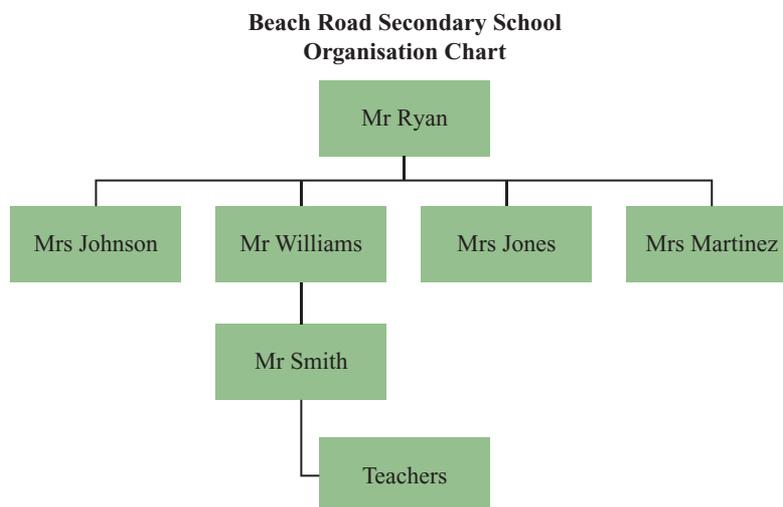


7 Find a different spanning tree to the one you found for question 6a to 6e (inclusive).

- 8 a** Draw a network that represents the following house plan. Use vertices to represent the rooms and edges to join rooms connected by a door.
- b** Is the network a tree? Does that make it a tree house?



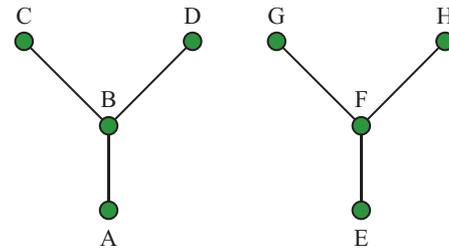
- 9** The following organisational chart shows the hierarchy within Beach Road Secondary School.
- a** Draw the organisational chart as a network.
- b** Is the network a tree?
- c** A teacher reports a serious incident to Mr Smith. How many steps will it take for this report to be handed up to Mr Ryan?



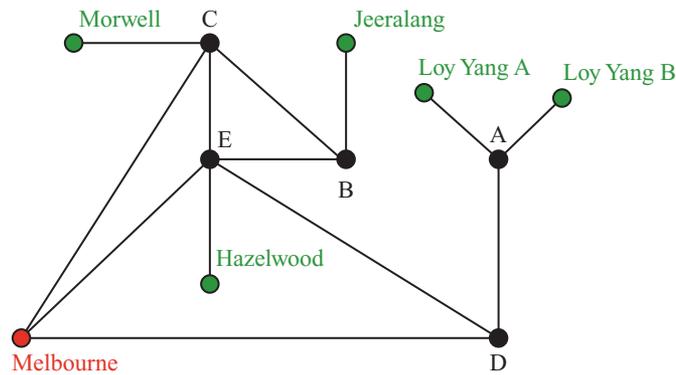
A forest is a network with no cycles. In other words, a forest is a network made up of separate trees. !

- 10** Which network in Example 9C–1 is a forest, but not a tree?

**11** Identify the two trees in the forest on the right. Show that the network is not connected by stating two vertices with no path between them, and then turn the forest into a tree by adding an edge between those vertices.

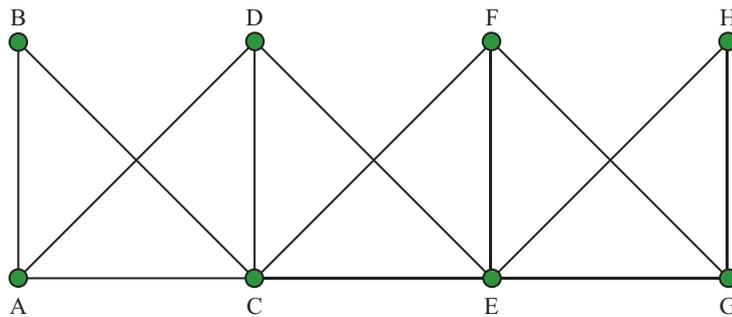


**12** The following network shows several power stations in the Latrobe Valley, Victoria. The power stations, represented by green vertices, are connected to terminals represented by black vertices. Power is transmitted to Melbourne across several transmission lines, represented by edges. Remove as many transmission lines as you can without disconnecting any power station from Melbourne.



**13** Suppose a spanning tree contains 10 vertices. How many edges are in the spanning tree?

**14** For the network shown, find a spanning tree that is also a path.



**15** For the network in question 14, find a spanning tree that is not a path.

**16** Can a single tree be a forest? Explain your thinking.

**17** Consider a network with five vertices and no edges. Is this a forest?

**18** Consider a network with five vertices and exactly one edge.

- a If the edge is a loop, is this a forest?
- b If the edge is not a loop, is this a forest?

**19** Working in pairs, take turns in removing a single edge from the network in question 14, without disconnecting the network. The winner is the first person to correctly identify when the network becomes a tree.

**20** Draw a network with at most 10 vertices, and challenge one of your classmates to a game of Network Jenga.

# 9D Minimum spanning trees

These resources are available on your [obook assess](#):

- **Video tutorial 9D:** Watch and listen to an explanation of Example 9D–2
- **Worksheet 9D:** Practise your skills with extra problems for minimum spanning trees
- **assess quiz 9D:** Test your skills with an auto-correcting multiple-choice quiz



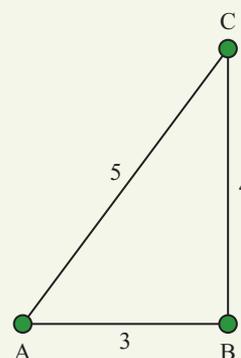
## minimum spanning tree

the spanning tree with the smallest total edge weight

For a network without weighted edges there is no notion of a ‘good’ or ‘bad’ spanning tree. But if the network is weighted, then the spanning trees also have weight, and some trees might be preferable to others. The spanning tree with the smallest total edge weight is called the **minimum spanning tree**.

### EXAMPLE 9D–1 Determining the minimum spanning tree by inspection

The network on the right shows the distances (in kilometres) between the three solar power plants A, B and C. The power plants must be connected by transmission lines. Design the shortest possible transmission network that connects power plants A, B and C.



#### Solve/Think

The power plants can be connected by a spanning tree. There are three possible spanning trees.

	Option 1	Option 2	Option 3
Spanning tree			
Total distance	$5 + 3 = 8$	$5 + 4 = 9$	$3 + 4 = 7$

The spanning tree in option 3 covers the least total distance, and hence it is the minimum spanning tree.

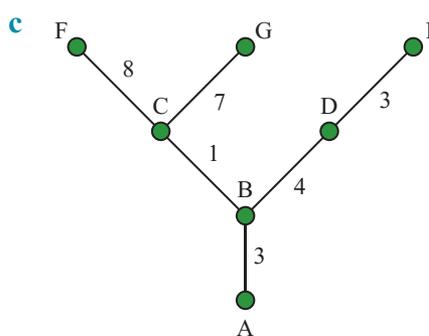
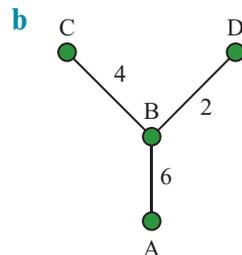
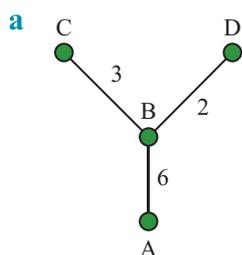
#### Apply

For a small network, we can list all possible spanning trees to determine the minimum spanning tree.

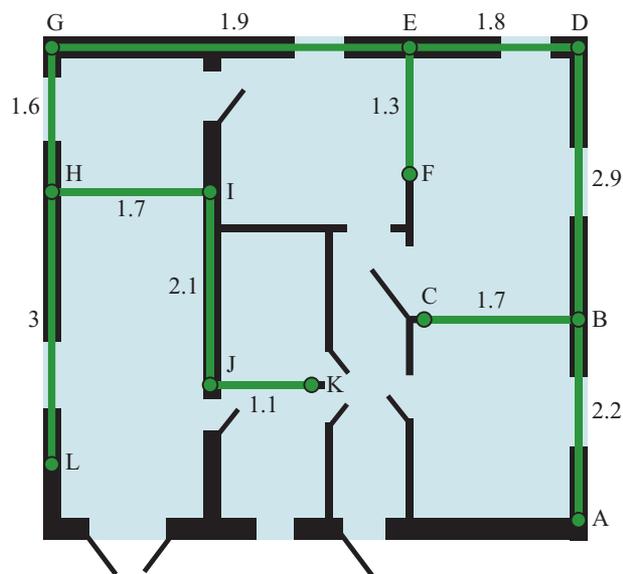
# EXERCISE 9D Minimum spanning trees

UNDERSTANDING, FLUENCY AND COMMUNICATING

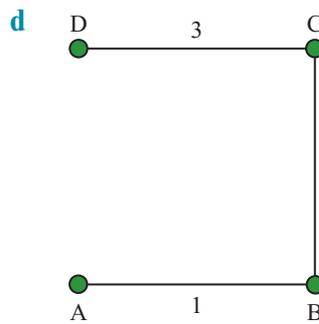
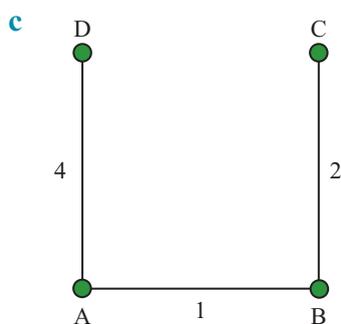
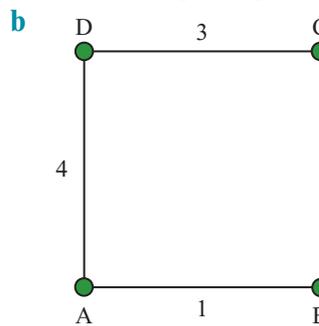
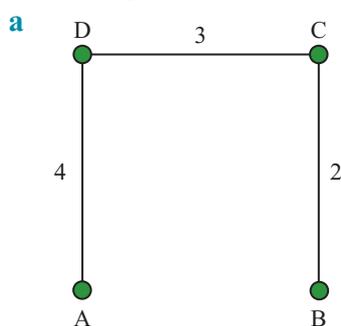
1 What is the total weight of the following trees?



2 The image on the right shows a house plan and the electrical cables that connect the electricity meter at A to the power outlets throughout the house. The length of wire is shown against each edge. What is the total length of wire required?



3 The four vertices A, B, C and D can be connected in four possible configurations. What is the total weight of each configuration below? Which configuration has the smallest total edge weight?



The strategy of finding all possible spanning trees and choosing the one with the least total edge weight becomes impractical for larger networks. Fortunately, there are two ways to find a minimum spanning tree that do not require listing every possible tree: Kruskal's algorithm and Prim's algorithm

**Kruskal's algorithm** was developed by Joseph Kruskal in 1956. Given a weighted network, a minimum spanning tree can be constructed as follows:

- 1 List all the edges of the network from smallest weight to largest.
- 2 Choose all of the vertices in the network, but none of the edges.
- 3 Proceed through your list of edges, starting with the edge of smallest weight. Add edges one by one, provided that your selection does not make a cycle.

If your original network has  $n$  vertices, then you can stop after selecting  $n-1$  edges. The outcome is guaranteed to be a spanning tree of minimum total edge weight.

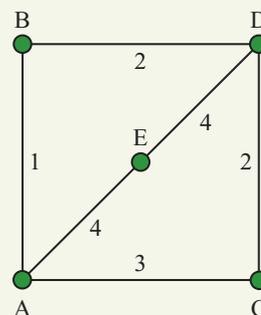
Kruskal's algorithm starts with a forest of vertices, and adds edges starting with the edge of smallest weight. At every step, the network is a forest. The final step is when the forest grows into a single minimum spanning tree.



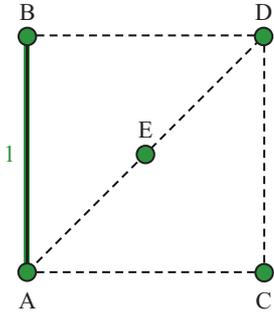
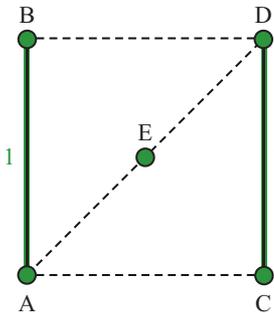
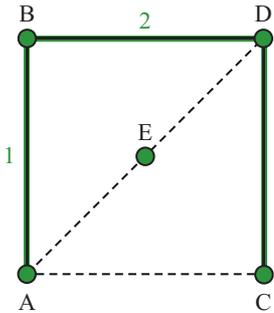
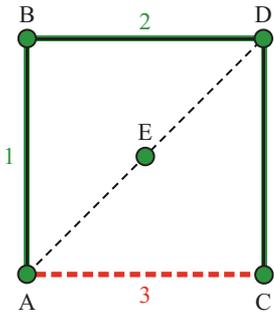
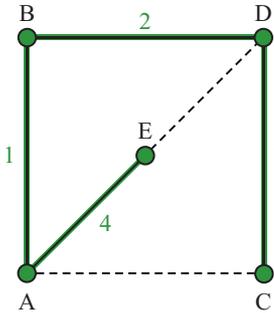
**Kruskal's algorithm**  
a method for finding the minimum spanning tree of a connected, weighted network by selecting the required edges one at a time

### EXAMPLE 9D-2 Using Kruskal's algorithm

Use Kruskal's algorithm to find the minimum spanning tree for the following weighted network.

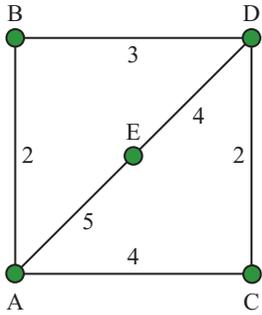


Step	Solve/Think														
<b>1</b>	<p>The list of edge weights is as follows.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <thead> <tr> <th style="width: 15%;">Edge</th> <th style="width: 15%;">AB</th> <th style="width: 15%;">DC</th> <th style="width: 15%;">BD</th> <th style="width: 15%;">CA</th> <th style="width: 15%;">AE</th> <th style="width: 15%;">ED</th> </tr> </thead> <tbody> <tr> <td style="background-color: #e0f0e0;"><b>Weight</b></td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td style="text-align: center;">2</td> <td style="text-align: center;">3</td> <td style="text-align: center;">4</td> <td style="text-align: center;">4</td> </tr> </tbody> </table> <p>Proceed through this list, adding edges that do not form a cycle. The network has 5 vertices, so we stop once <math>5 - 1 = 4</math> edges have been selected.</p>	Edge	AB	DC	BD	CA	AE	ED	<b>Weight</b>	1	2	2	3	4	4
Edge	AB	DC	BD	CA	AE	ED									
<b>Weight</b>	1	2	2	3	4	4									
<b>2</b>	<p>Choose all the vertices.</p> <div style="text-align: center; margin-top: 20px;"> </div>														

Step	Solve/Think	
3.1	Include the first edge AB.	
3.2	Include the second edge DC.	
3.3	Include the third edge BD.	
3.4	Skip the fourth edge CA, since ABDCA would be a cycle.	
3.5	Include the fifth edge AE.	
<b>Stop</b>	The selected vertices and edges form a minimum spanning tree.	

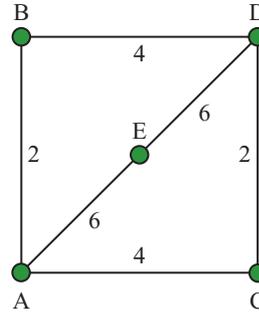
4 The edges from each network below are listed according to their edge weight. Fill in the remaining edge weights.

a



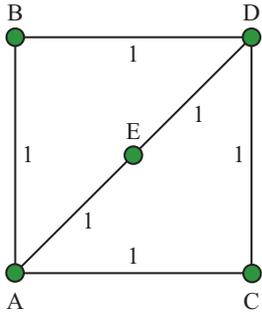
Edge	AB	DC	BD	DE	AC	AE
Weight	2		3			

b



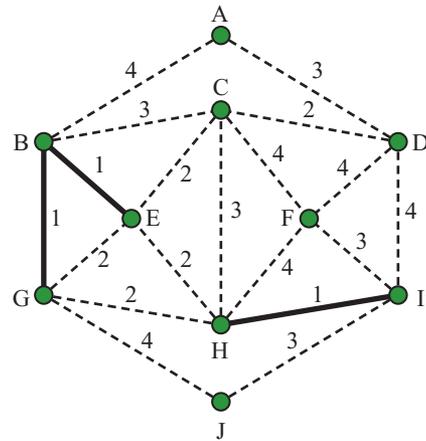
Edge	AB	DC	BD	AC	ED	AE
Weight		2				6

c



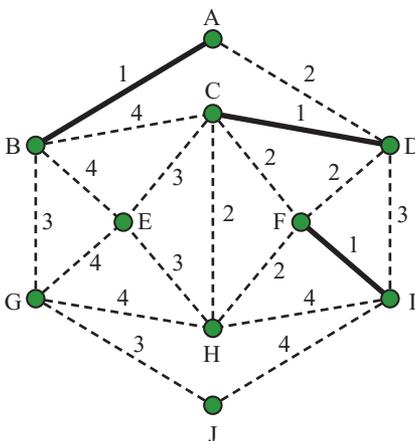
Edge						
Weight						

5 At every step of Kruskal's algorithm, the selected edges and vertices form a forest of trees. The first three steps of Kruskal's algorithm have been applied to the network on the right, resulting in all edges of weight 1 being added to the forest. Now proceed through all edges of weight 2, and add as many of them as possible to the forest (i.e. without creating a cycle). Then process all edges of weight 3, and then weight 4, until your forest grows into a single tree. The tree you created is a minimum spanning tree. Since this network has 10 vertices, your spanning tree should have 9 edges.

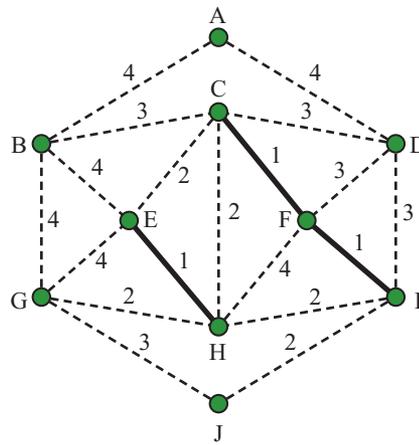


6 Repeat question 5 for the following networks.

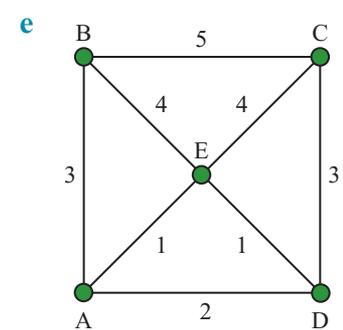
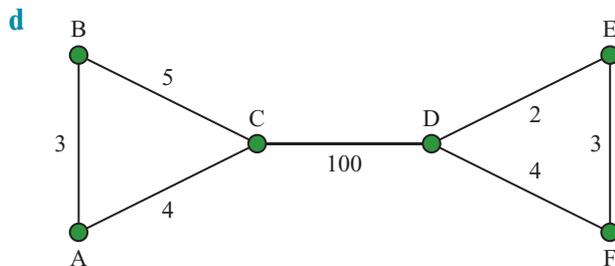
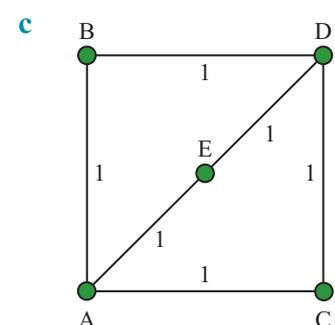
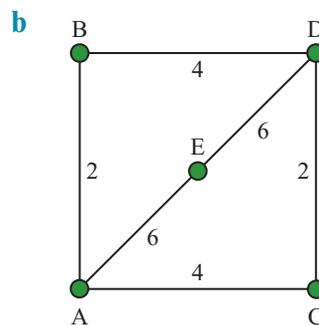
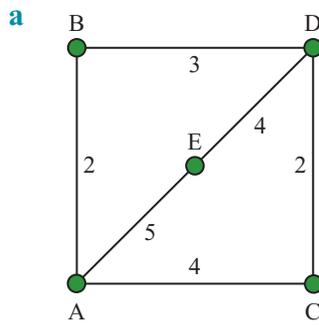
a



b



7 Use Kruskal's algorithm to find a minimum spanning tree for the following networks.



The second procedure for finding a minimum spanning tree is **Prim's algorithm**.

Starting with any vertex, Prim's algorithm selects the next 'best' edge and adds it to the tree without regard for the long-term consequences of that choice. Remarkably, this process still produces a minimum spanning tree.

The steps are as follows:

- 1 Choose any vertex as the start vertex. This is the start of the tree.
- 2 Of all the edges that join a vertex outside the tree to a vertex in the tree, choose the edge with *minimum weight* and add the vertex and the edge to the tree.
- 3 Repeat step 2 until the tree includes all vertices, which is when it becomes a spanning tree.

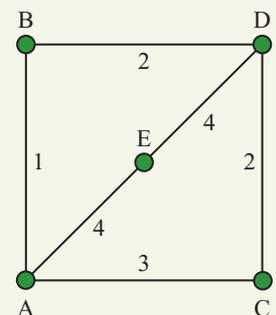
When there are many edges with the same minimum weight, any one of them can be chosen.

The selected edges and vertices form a tree at every step of Prim's algorithm. The algorithm continues until the tree includes every vertex. In contrast, Kruskal's algorithm contains every vertex right from the start. The two algorithms create minimum spanning trees, but in different ways.

**!** **Prim's algorithm**  
a method for finding the minimum spanning tree of a connected weighted network by iteratively adding the edge of smallest weight

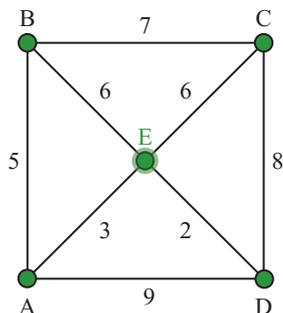
### EXAMPLE 9D-3 Using Prim's algorithm

Use Prim's algorithm to find the minimum spanning tree for the weighted network on the right.



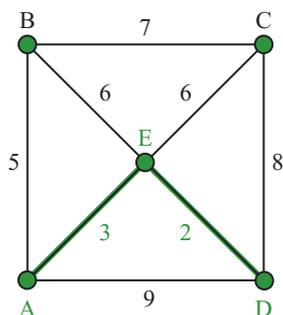
Step	Solve/Think											
<b>1</b>	Select any vertex as the start vertex. In this example vertex A is chosen.											
<b>2.1</b>	Edges AB, AC and AE will connect a new vertex to the tree. <table border="1" style="margin: 10px auto;"> <tr> <th>Edge</th> <th>AB</th> <th>AC</th> <th>AE</th> </tr> <tr> <th>Weight</th> <td>1</td> <td>3</td> <td>4</td> </tr> </table> Connect B to the tree via edge AB because it has the smallest edge weight.	Edge	AB	AC	AE	Weight	1	3	4			
Edge	AB	AC	AE									
Weight	1	3	4									
<b>2.2</b>	Edges BD, AC and AE will connect a new vertex to the tree. <table border="1" style="margin: 10px auto;"> <tr> <th>Edge</th> <th>BD</th> <th>AC</th> <th>AE</th> </tr> <tr> <th>Weight</th> <td>2</td> <td>3</td> <td>4</td> </tr> </table> Connect D to the tree via edge BD because it has the smallest edge weight.	Edge	BD	AC	AE	Weight	2	3	4			
Edge	BD	AC	AE									
Weight	2	3	4									
<b>2.3</b>	Edges AE, AC, DE and DC will connect a new vertex to the tree. <table border="1" style="margin: 10px auto;"> <tr> <th>Edge</th> <th>AE</th> <th>AC</th> <th>DE</th> <th>DC</th> </tr> <tr> <th>Weight</th> <td>4</td> <td>3</td> <td>4</td> <td>2</td> </tr> </table> Connect C to the tree via edge DC because it has the smallest edge weight.	Edge	AE	AC	DE	DC	Weight	4	3	4	2	
Edge	AE	AC	DE	DC								
Weight	4	3	4	2								
<b>2.4</b>	Edges AE and DE (but not AC) will connect a new vertex to the tree. <table border="1" style="margin: 10px auto;"> <tr> <th>Edge</th> <th>AE</th> <th>DE</th> </tr> <tr> <th>Weight</th> <td>4</td> <td>4</td> </tr> </table> As both edges have the minimum weight, we are free to choose either. In this example we choose DE. The tree now includes every vertex.	Edge	AE	DE	Weight	4	4					
Edge	AE	DE										
Weight	4	4										
<b>Stop</b>	The vertices and selected edges form a minimum spanning tree.											

- 8** The first step of Prim's algorithm has been applied to the network below. In this case vertex E was chosen.



Edge	EB	EC	ED	EA
Weight	6			

- a** Fill in the table of edges that connect a new vertex to the tree.
- b** Which of these edges has the smallest weight?
- 9** After a few more steps of Prim's algorithm, the selected tree has grown. Only vertices B and C remain to be connected.
- a** Complete the table of edges that connect these vertices to the tree.
- b** Which of these edges has smallest weight?

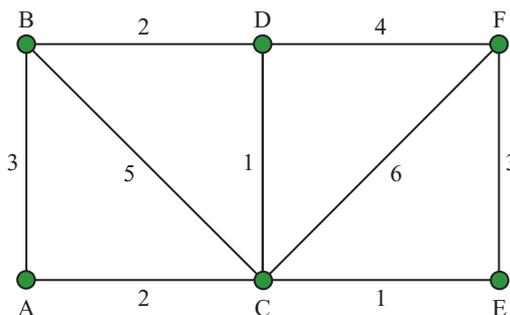


Edge	EB	EC	AB	CD
Weight				8

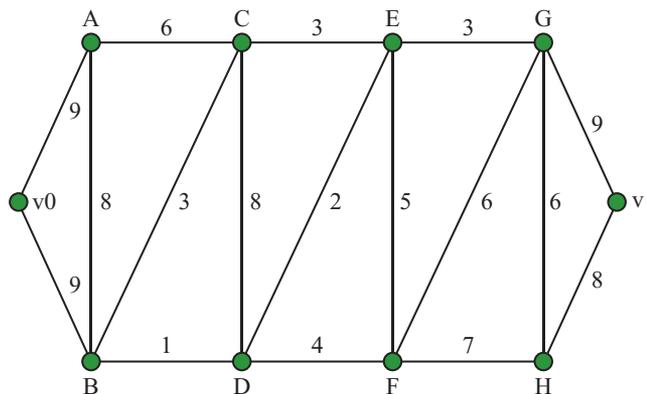
- 10** What is the minimum spanning tree for the network in question 8?

- 11** Repeat question 7 with Prim's algorithm instead of Kruskal's.

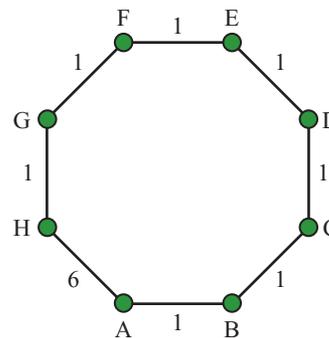
- 12** Find a minimum spanning tree for this network using any method.



- 13** Find a minimum spanning tree for this network.



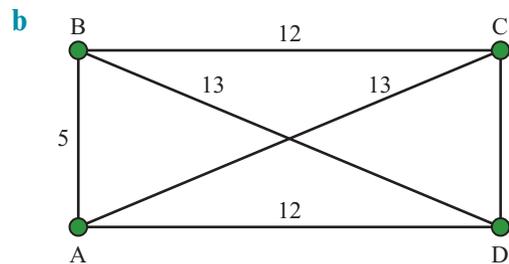
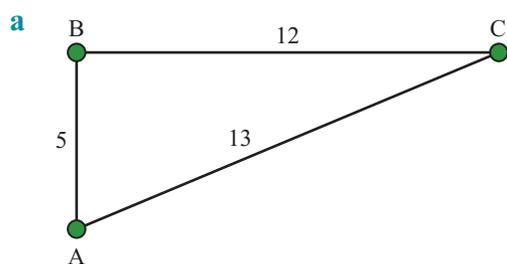
**14** A network of security cameras must be connected by wires. The cost of connecting the cameras is shown, in dollars, as the edge weight in the network on the right.



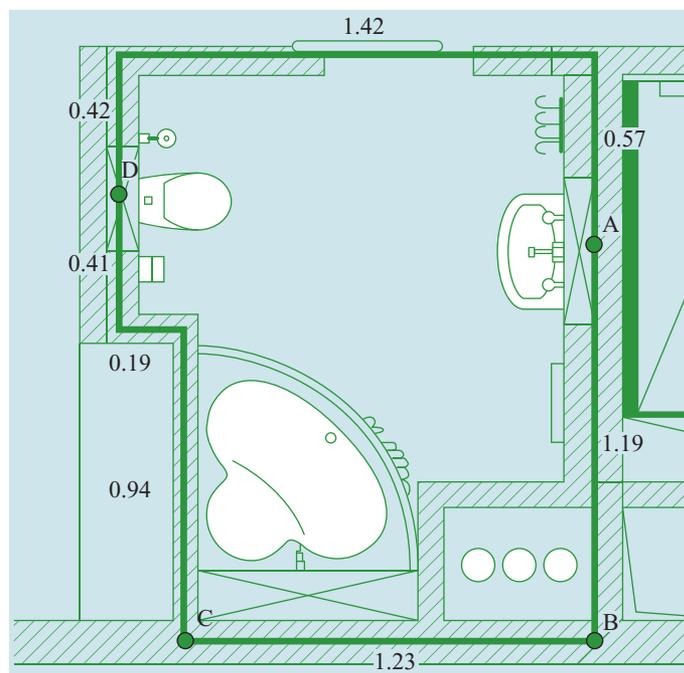
- What is the cheapest way to connect all eight cameras?
- What is the cheapest way to connect only cameras A and H?
- What is the cheapest way to connect only cameras A and H if the cost of wire between A and H is changed to 8?



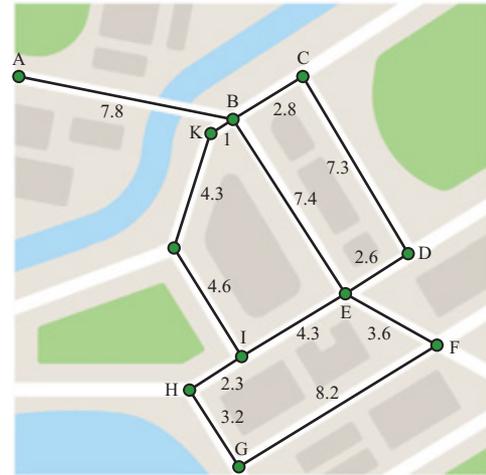
**15** Power plants and the distances between them are shown in the networks below. The power plants must be connected by transmission lines. Design the shortest possible transmission network in each scenario.



**16** This question will help you decide where to place the water pipes in the following floor plan for a bathroom, so that the sink (at vertex A), bath (at vertex C) and toilet (at vertex D) are connected to the water main (at vertex B). The distances along possible pipelines are shown. First represent this information as a network (i.e. add the distances to find the total distances between A and D, and D and C), and then use Kruskal's algorithm to find a spanning tree that minimises the total distance covered by the pipes.



- 17** The network on the right shows the location of fibre optic nodes that form part of a national high-speed internet network. The possible location of the fibre optic cable is given as edges, and the length of the cable (in kilometres) is specified as the edge weight. Design the most cost-effective possible way of connecting each node (i.e. vertex) with fibre optic cable. How many kilometres of cable does your network require?



- 18** The table below shows the distances between towns A, B, C and D in kilometres.

	A	B	C	D
A		3.3	5.8	2.5
B	3.3		4.4	1.5
C	5.8	4.4		3.6
D	2.5	1.5	3.6	

- Represent the information in the table as a network.
- What would be the most cost-effective way to connect the towns to each other by rail?
- If the cost per km of rail is \$5 million, how much would the most cost-effective railway cost?
- A local politician suggests rail lines along AB, BC and CD. How much would this railway cost?
- What is the difference in price between the most cost-effective railway and the local politician's suggested railway?



- 19** The following table shows the distances (in kilometres) between mainland capital cities in Australia.

Adelaide						
2075	Brisbane					
1209	1267	Canberra				
3041	3435	4034	Darwin			
732	1813	651	3773	Melbourne		
2721	4434	3930	4037	3453	Perth	
1419	978	289	3971	876	3975	Sydney

- What is the cheapest way to construct a high-speed rail network that connects the mainland capital cities to each other?
- How many kilometres of rail is required?
- Given the information in tabular form, would you choose to use Kruskal's or Prim's algorithm?

# 9E The shortest path

## shortest path

(in a network diagram) a path between two vertices in a network for which the sum of the weights of its edges is minimised

These resources are available on your obook assess:

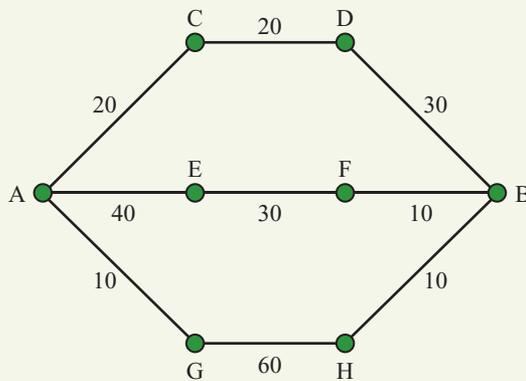
- **Video tutorial 9E:** Watch and listen to an explanation of Example 9E-2
- **Worksheet 9E:** Practise your skills with extra problems for the shortest path
- **assess quiz 9E:** Test your skills with an auto-correcting multiple-choice quiz

0101

In a weighted network, the **shortest path** between two vertices is the path with the smallest total edge weight.

### EXAMPLE 9E-1 Finding the shortest path by inspection

The network below shows cities connected by highways. The length of each highway is indicated by an edge weight. What is the shortest path between cities A and B?



#### Solve/Think

There are three paths from A to B.

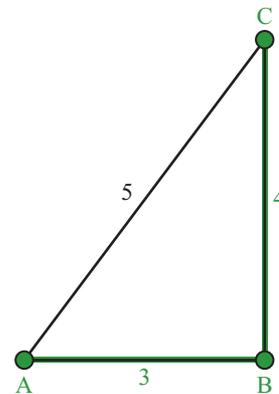
- 1 Path ACDB has a length of  $20 + 20 + 30 = 70$ .
- 2 Path AEFB has a length of  $40 + 30 + 10 = 80$ .
- 3 Path AGHB has a length of  $10 + 60 + 10 = 80$ .

The path with the shortest length is ACDB.

#### Apply

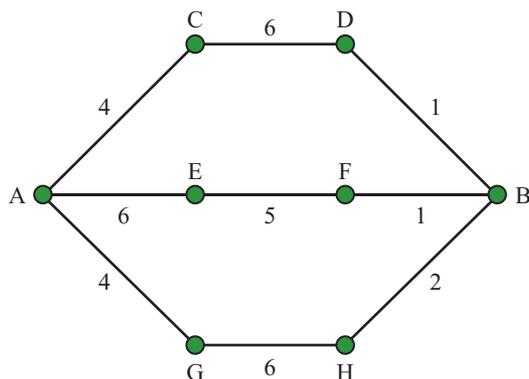
One way to find the shortest path is to calculate the length of every path, and then select the path with the shortest length. This method only works in practice when there is a small number of possible paths to consider.

The minimum spanning tree does not always provide the shortest path between two points. For example, in the network on the right the minimum spanning tree contains the path ABC (highlighted in green), whereas the shortest path between vertices A and C is AC.



# EXERCISE 9E The shortest path

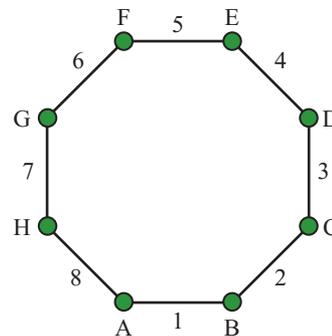
1 Consider the following network.



- a What is the shortest path from A to B?
- b What is the shortest path from F to G?

2 Consider the network on the right.

- a Which path from A to F is shorter: ABCDEF or AHGF?
- b Which path from A to G is shorter: ABCDEFG or AHG?
- c What is the minimum spanning tree for this network?



For larger networks it is difficult to consider all possible paths between two vertices. Fortunately, **Dijkstra's algorithm** gives us a method to construct a special spanning tree that contains all the shortest paths from a particular start vertex to every other vertex. Starting with any vertex, let's call it A, Dijkstra's algorithm grows a tree of shortest paths from A. The steps are as follows:

- 1 Make a tree that only contains the vertex A.
- 2 Make a list of all paths starting from A that extend the tree by exactly one edge. Choose the path that has the *minimum total weight* and add the vertex and edge to the tree.
- 3 Repeat step 2 until the tree includes all vertices.

This is very similar to Prim's algorithm, except in step 2 you select a new edge based on the total distance from A rather than the individual edge weight.

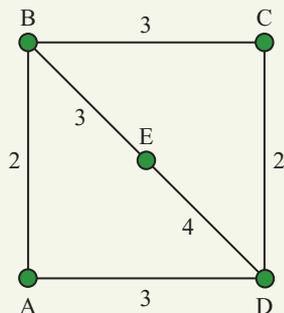


### Dijkstra's algorithm

a method for finding the shortest path between any two vertices of a connected weighted network

## EXAMPLE 9E-2 Using Dijkstra's algorithm

Use Dijkstra's algorithm to find the shortest path from A to E in the following network.



Step	Solve/Think											
<b>1</b>	Make a tree with only vertex A.											
<b>2.1</b>	<p>The tree contains only the vertex A. The list of all paths from A with exactly one vertex outside this tree is:</p> <table border="1" style="margin-left: 20px;"> <tr> <th style="background-color: #d9ead3;">Path</th> <td style="background-color: #d9ead3;">AB</td> <td style="background-color: #d9ead3;">AD</td> </tr> <tr> <th style="background-color: #d9ead3;">Total weight</th> <td style="background-color: #d9ead3;">2</td> <td style="background-color: #d9ead3;">3</td> </tr> </table> <p>Path AB has minimum total weight, so add it to the tree.</p>	Path	AB	AD	Total weight	2	3					
Path	AB	AD										
Total weight	2	3										
<b>2.2</b>	<p>The tree contains vertices A and B. The list of all paths starting at A with exactly one vertex outside this tree is:</p> <table border="1" style="margin-left: 20px;"> <tr> <th style="background-color: #d9ead3;">Path</th> <td style="background-color: #d9ead3;">AD</td> <td style="background-color: #d9ead3;">ABE</td> <td style="background-color: #d9ead3;">ABC</td> </tr> <tr> <th style="background-color: #d9ead3;">Total weight</th> <td style="background-color: #d9ead3;">3</td> <td style="background-color: #d9ead3;">5</td> <td style="background-color: #d9ead3;">5</td> </tr> </table> <p>Path AD has minimum total weight, so add it to the tree.</p>	Path	AD	ABE	ABC	Total weight	3	5	5			
Path	AD	ABE	ABC									
Total weight	3	5	5									
<b>2.3</b>	<p>The tree contains vertices A, B and D. The list of all paths from A with exactly one vertex outside this tree is:</p> <table border="1" style="margin-left: 20px;"> <tr> <th style="background-color: #d9ead3;">Path</th> <td style="background-color: #d9ead3;">ABE</td> <td style="background-color: #d9ead3;">ABC</td> <td style="background-color: #d9ead3;">ADE</td> <td style="background-color: #d9ead3;">ADC</td> </tr> <tr> <th style="background-color: #d9ead3;">Total weight</th> <td style="background-color: #d9ead3;">5</td> <td style="background-color: #d9ead3;">5</td> <td style="background-color: #d9ead3;">7</td> <td style="background-color: #d9ead3;">5</td> </tr> </table> <p>Paths ABE, ABC and ADC all have the same minimum total weight, so we are free to choose any one and add it to the tree. In this example we have chosen to add ABE.</p>	Path	ABE	ABC	ADE	ADC	Total weight	5	5	7	5	
Path	ABE	ABC	ADE	ADC								
Total weight	5	5	7	5								
<b>2.4</b>	<p>The tree contains vertices A, B, D and E. Only vertex C remains outside the tree. The paths from A with exactly one vertex outside the tree are:</p> <table border="1" style="margin-left: 20px;"> <tr> <th style="background-color: #d9ead3;">Path</th> <td style="background-color: #d9ead3;">ABC</td> <td style="background-color: #d9ead3;">ADC</td> </tr> <tr> <th style="background-color: #d9ead3;">Total weight</th> <td style="background-color: #d9ead3;">5</td> <td style="background-color: #d9ead3;">5</td> </tr> </table> <p>All paths have the same weight, so we are again free to choose any one and add it to the tree. In this example we have chosen to add ABC.</p>	Path	ABC	ADC	Total weight	5	5					
Path	ABC	ADC										
Total weight	5	5										
<b>2.5</b>	<p>The resulting tree is a spanning tree that contains the shortest path from A to any other vertex. In particular, the shortest path from A to E in this tree is ABE, which has total weight <math>2 + 3 = 5</math>.</p>											

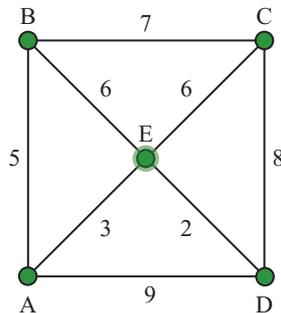
**Apply**

Follow the steps in Dijkstra's algorithm to create a spanning tree that contains all of the shortest paths in the network.

*Note:* we could have answered the question in step 2.3 when the path from A to E, ABE, was added to the tree of shortest paths.

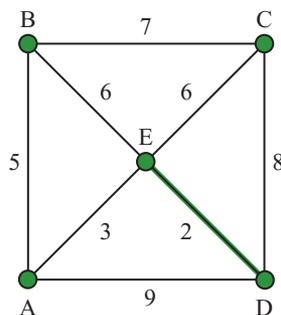
**3** Use Dijkstra's algorithm to find the shortest path from E to B in the network below.

**a** The first step of Dijkstra's algorithm is to create a tree with just the start vertex E. Below is a list of paths with exactly one vertex outside this tree. Calculate the total weight of each of these paths.



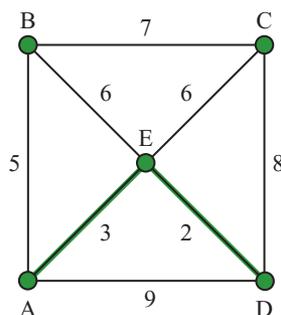
Path	EA	EB	EC	ED
Total weight	3			

**b** In the second step, the tree contains the vertices E and D, and the edge ED. Below is a list of all paths from E with exactly one vertex outside this tree. Calculate the total weight of each of these paths.



Path	EA	EB	EC	EDA	EDC
Total weight				11	

**c** In the third step, the tree contains vertices A, E and D, and the edges EA and ED. Below is a list of paths from E with exactly one vertex outside this tree. Calculate the total weight of each of these paths.

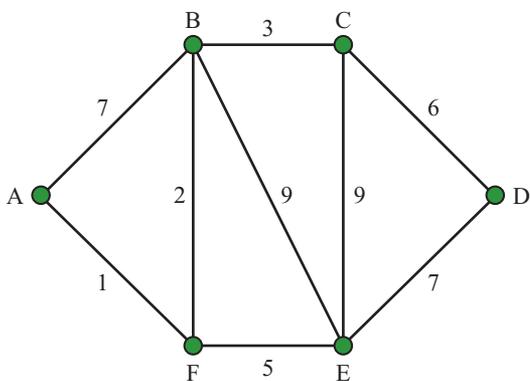


Path	EB	EC	EAB	EDC
Total weight			8	

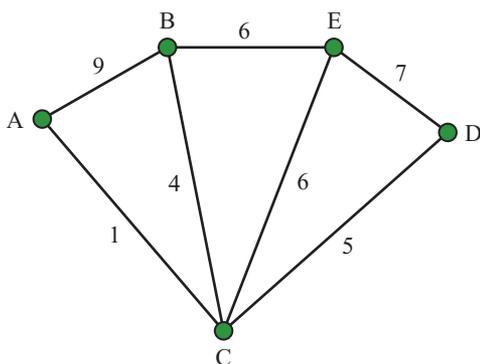
- i** Which of the paths in part **c** has the smallest total weight?
- ii** What is the minimum distance from E to B?

**4** What is the shortest path from B to E in the networks for Exercise 9D question 7?

5 What is the shortest path from A to D in the following network?

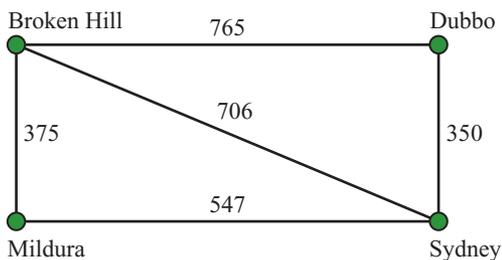


6 Find the shortest distance from A to E in the following network.

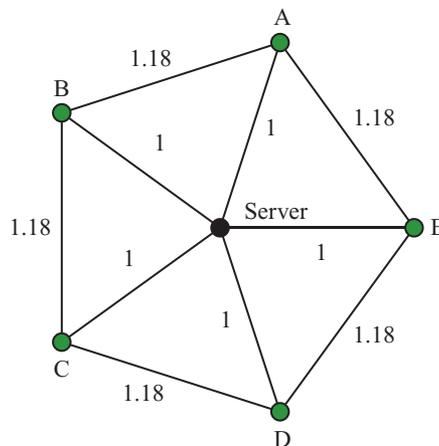


7 The cost of flying from Sydney to certain regional centres is shown below.

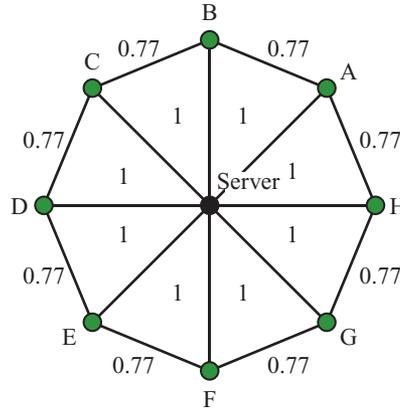
- a What is the cheapest route to Broken Hill?
- b Assuming the cheapest route is booked out, what is the second cheapest route?



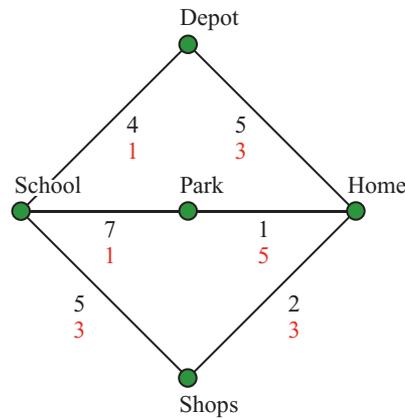
8 The network on the right shows the distances between the five computers A, B, C, D and E and a central server. Connect each computer to the server, so that the path from the server to the computer is as short as possible. Is your selected network also a minimum spanning tree?



- 9 Repeat question 8 for a network with eight computers. Is your selected network also a minimum spanning tree?

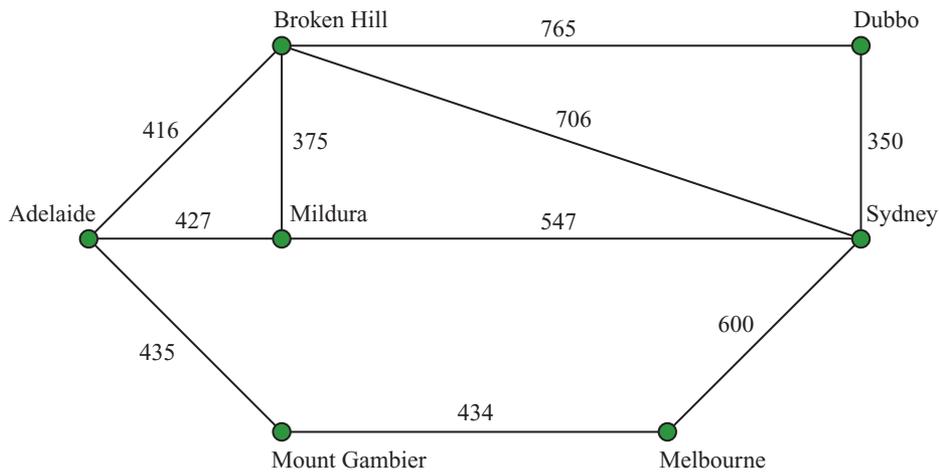


- 10 The following map shows the possible routes from school to home. The distance (in kilometres) is given by the black edge weight, and the level of danger is given by the red edge weight.



- a What is the shortest route from school to home?  
 b What is the least dangerous route from school to home?

- 11 Here is a larger map of regional flights, along with the cost of each route. What is the cheapest route in the network from Dubbo to Mount Gambier?



# CHAPTER 9 REVIEW NETWORK CONCEPTS

You should now be able to:

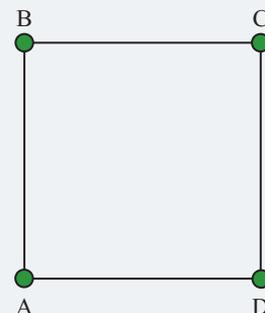
- ✓ convert a table of data into a network
- ✓ tabulate vertex degrees
- ✓ understand the difference between a walk, a path and a cycle
- ✓ determine if a network has a walk that visits every edge exactly once
- ✓ calculate the weight of a path in a network
- ✓ find a spanning tree
- ✓ find a minimum spanning tree using Kruskal's and Prim's algorithms
- ✓ find the shortest path between two vertices using Dijkstra's algorithm
- ✓ understand that a minimum spanning tree does not necessarily give the shortest path between two vertices.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

## REVIEW MULTIPLE-CHOICE QUESTIONS

Use the diagram on the right to answer questions 1–3.

- 6B** 1 Which of the following walks is a path?  
 A ABCB      B ABD      C DAB      D ABCDA
- 6B** 2 Which of the following walks is a cycle?  
 A ABAD      B ABCD      C ABDA      D ABCDA
- 6B** 3 Which of the following walks is not a path and not a cycle?  
 A AB      B ABA      C BCDAB      D ABAD
- 6C** 4 A spanning tree contains five edges. How many vertices are in the network?  
 A 3      B 4      C 5      D 6

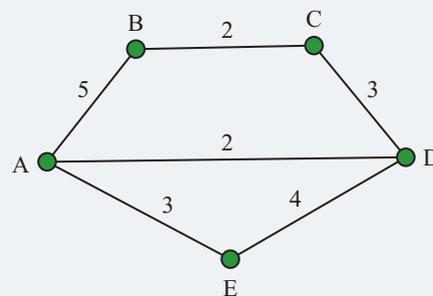


Use the diagram on the right to answer questions 5–9.

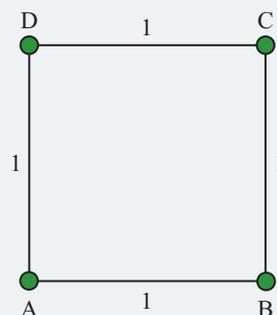
The diagram shows the possible paths (in km) for laying gas pipes between various locations.

Gas is to be supplied from one location. Any one of the locations can be the source of the supply.

- 6D** 5 What is the minimum total length of the pipes required to provide gas to all the locations?  
 A 10 km      B 19 km      C 12 km      D 11 km
- 6B** 6 What is the total weight of the cycle ADCBA?  
 A 7      B 9      C 10      D 12



- 6E** ▶ **7** The shortest path from B to E is:  
**A** BAE                      **B** BE                      **C** BCDE                      **D** BCDAE
- 6E** ▶ **8** The shortest path from A to C is:  
**A** AEDC                      **B** ABC                      **C** ADC                      **D** AC
- 6A** ▶ **9** How many vertices have odd degree?  
**A** none                      **B** one                      **C** two                      **D** more than two
- 6D** ▶ **10** How many different minimum spanning trees can you find for the following network?  
**A** one                      **B** two  
**C** three                      **D** four



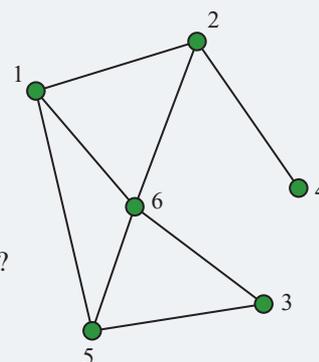
## REVIEW SET 1

Use the diagram on the right to answer questions 1–3.

- 1** Complete the table of vertex degrees for the network.

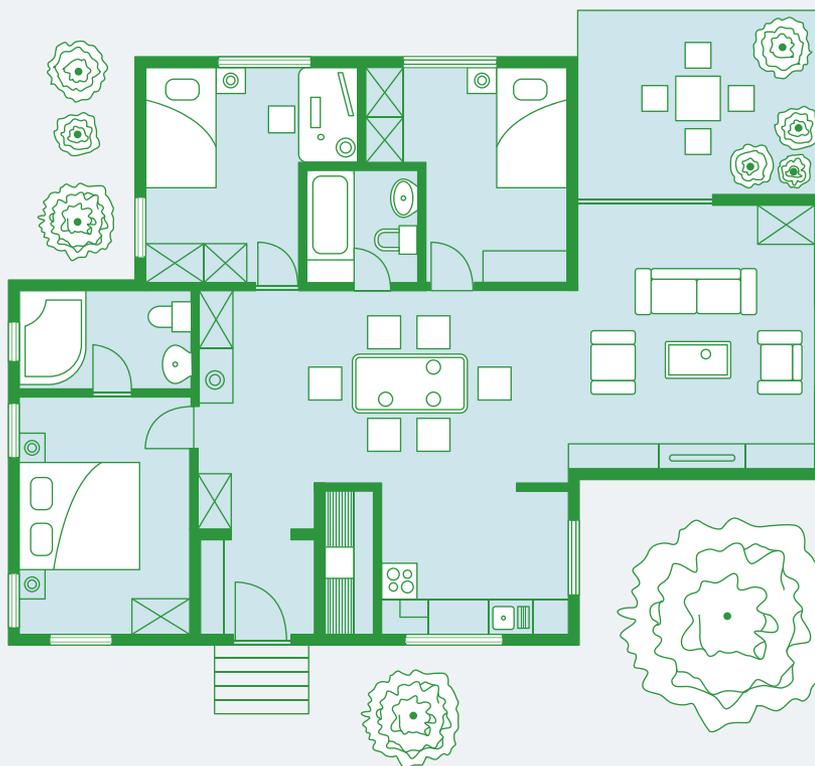
Vertex	1	2	3	4	5	6
Degree						

- 2** Is there a walk in the network that visits every edge exactly once? Why or why not?
- 3** Find a walk in the network that visits every vertex exactly once.



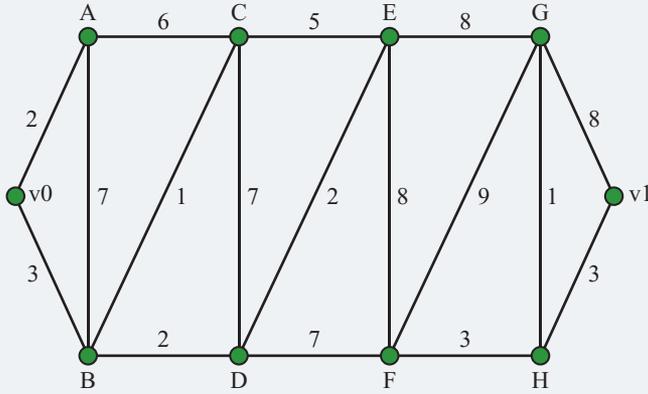
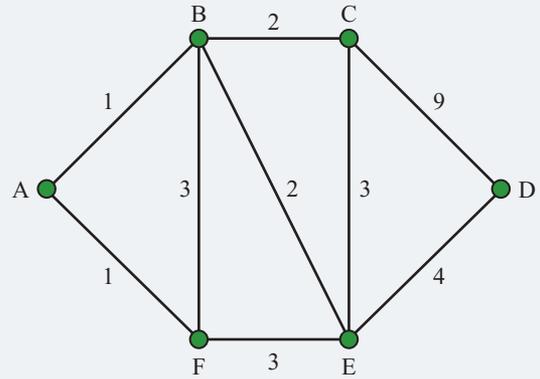
Use the diagram of the floor plan to answer questions 4–6.

- 4** Draw a network that represents the floor plan. Use vertices to represent rooms, and represent rooms connected by an internal door with an edge.
- 5** Is your network a tree?
- 6** Construct a table of vertex degrees for the network. Does there exist a walk through the house which passes through every internal door exactly once? Why or why not?



Use the diagram on the right to answer questions 7 and 8.

- 7 Use Prim's algorithm to find the minimum spanning tree for the network.
- 8 What is the total weight of the path AFBCD for the network?
- 9 Find the shortest path from v0 to v1.

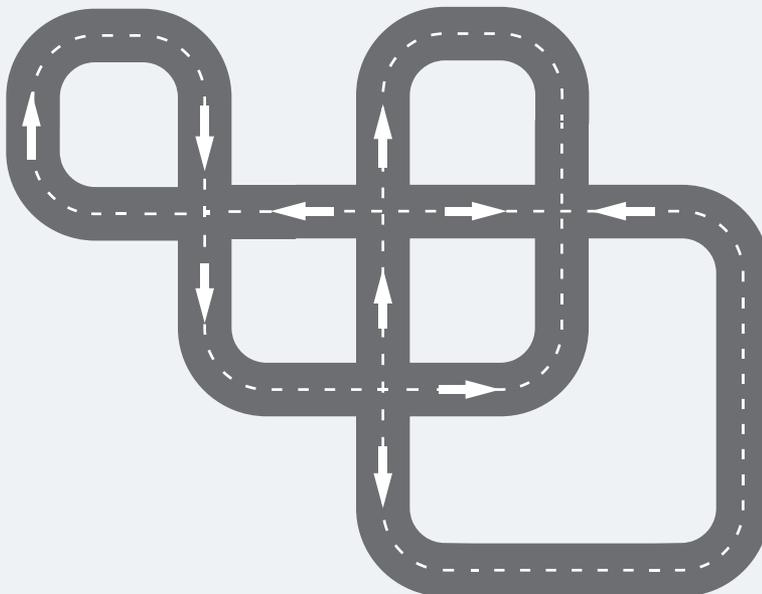


## REVIEW SET 2

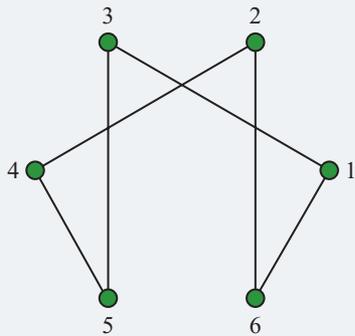
- 1 Represent the following table as a weighted network.

	A	B	C	D
A	–	2	8	9
B	2	–	7	4
C	8	7	–	1
D	9	4	1	–

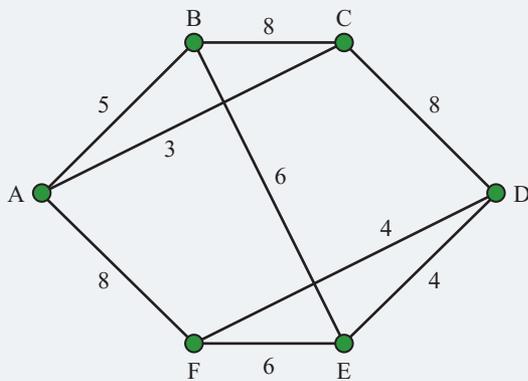
- 2 Draw this system of one way streets as a directed network.



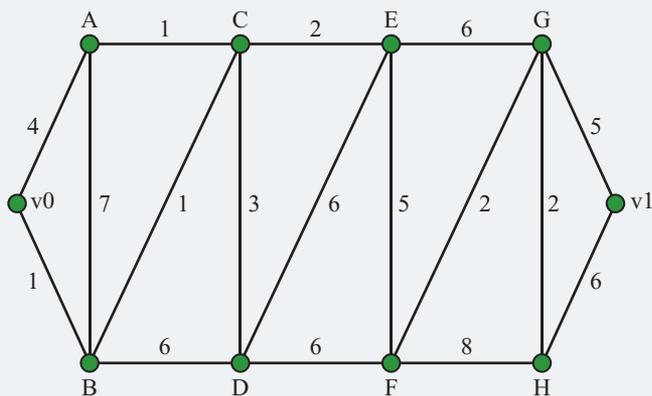
- 3** Using the concepts of indegree and outdegree, show that there is something wrong with the road design in question 2.
- 4** Find a cycle with six edges in the following network.



- 5** Transform the cycle in question 4 into a spanning tree by removing one or more edges.
- 6** Transform the spanning tree in question 5 into a forest of two trees by removing one or more edges.
- 7** Is there a walk in the network from question 4 that visits every edge exactly once?
- 8** Is there a walk in the network from your answer to question 5 that visits every edge exactly once?
- 9** Find a minimum spanning tree for the following network.

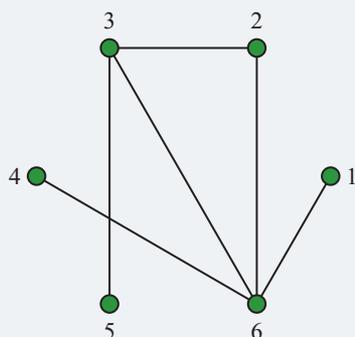


- 10** Find the shortest path from  $v_0$  to  $v_1$  for the following network.



# REVIEW SET 3

Use the diagram below to answer questions 1–2.

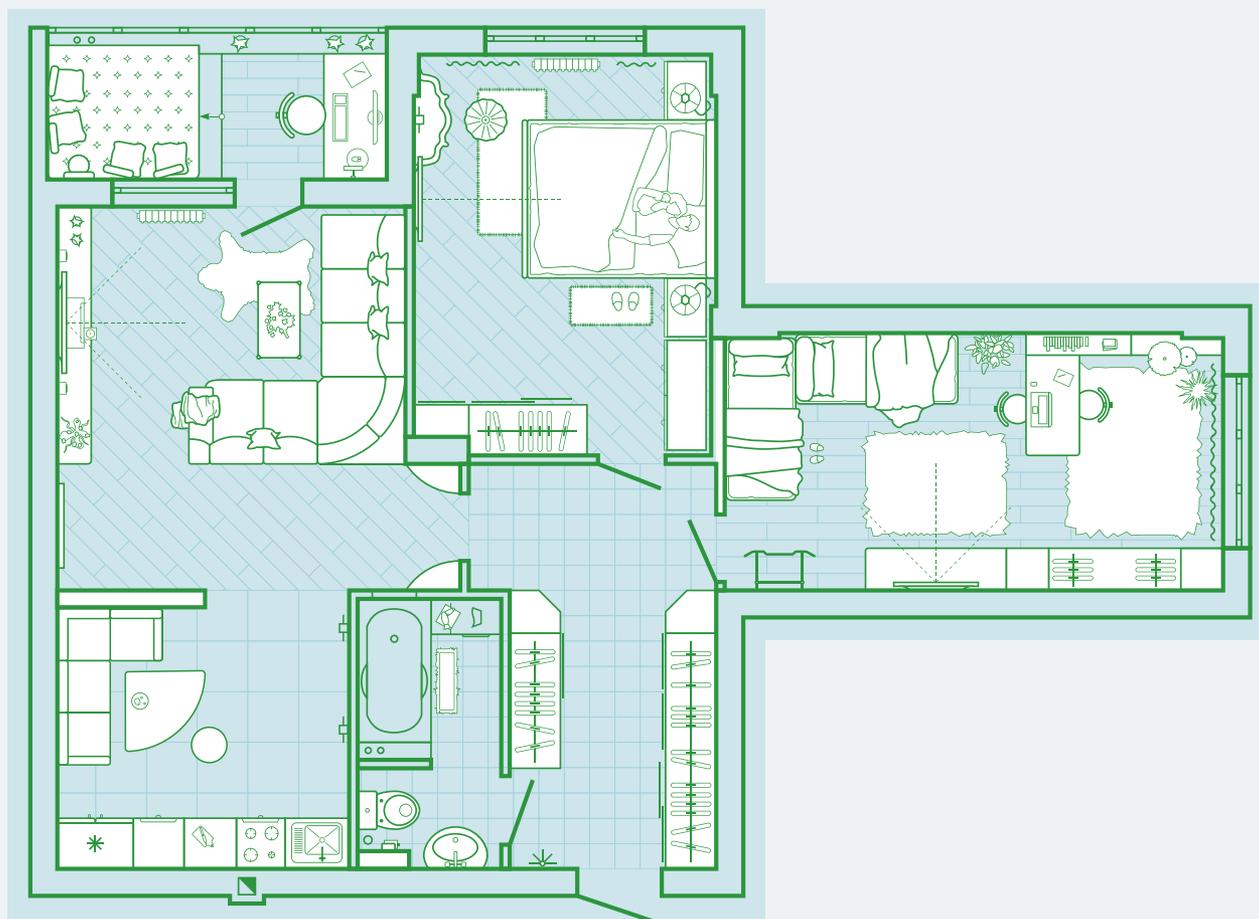


1 Complete the table of vertex degrees for the network.

Vertex	1	2	3	4	5	6
Degree						

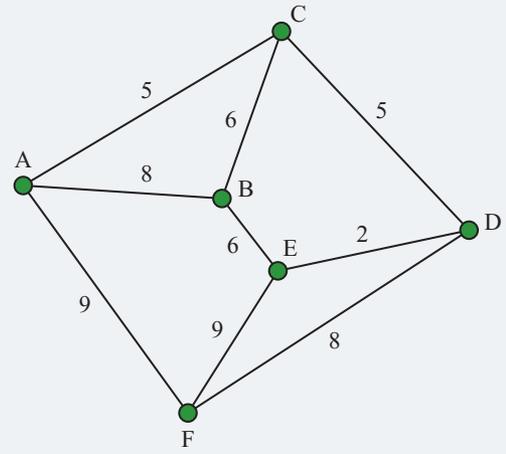
2 Is there a path in the network that visits every edge exactly once?

3 Draw a network that represents the following floor plan. Use vertices to represent rooms and represent rooms connected by a door with an edge.



4 Is the network from question 3 a tree?

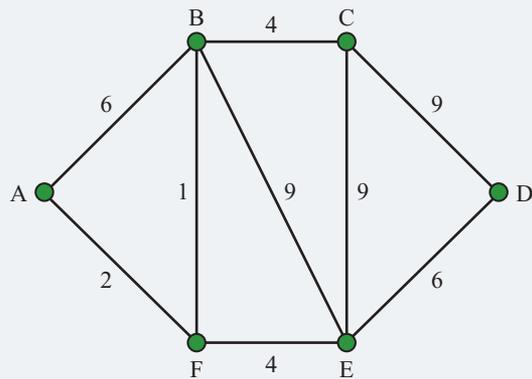
Use the diagram on the right to answer questions 5 and 6.



5 Find a minimum spanning tree for the network.

6 What is the weight of the walk ABEDC?

Use the diagram below to answer questions 7, 8 and 9.



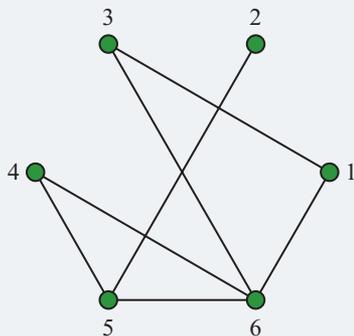
7 Find the shortest path from A to D in the network.

8 Find a walk that visits every edge of the network.

9 Use Kruskal's algorithm to find the minimum spanning tree for the network.

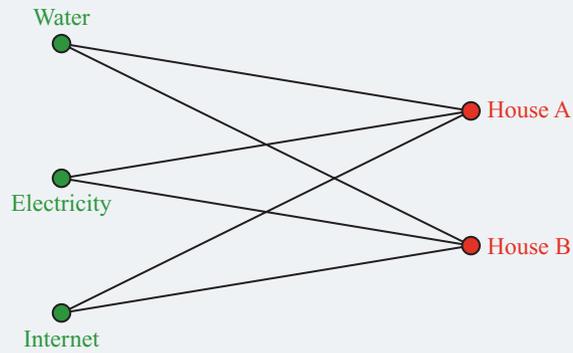
## REVIEW SET 4

1 Complete the table of vertex degrees for the following network.



Vertex	1	2	3	4	5	6
Degree						

2 Three essential services (water, electricity and internet) are to be connected to house A and house B.

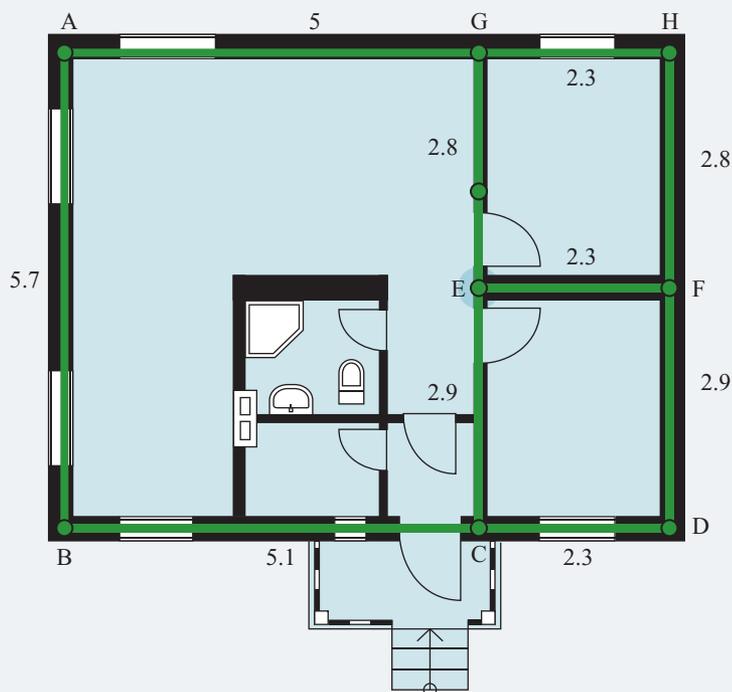


Re-draw this network so that the utility lines do not cross.

3 Find a walk through the network in question 2 that visits every edge exactly once.

Use the diagram below to answer questions 4–6.

The following house plan shows the possible locations of power cables.



4 What is the most cost-effective way to connect the points A, B, C, D, E and F to the power meter at G?

5 What is the weight of the path ABCDFEG for the network?

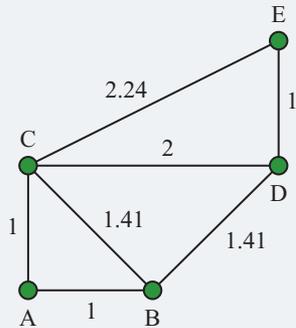
6 Use Prim's algorithm to find the minimum spanning tree for the network.



Gas is to be supplied from one location. Any one of the locations can be the source of the supply.

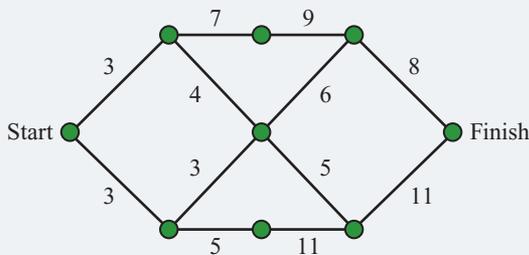
- i What is the minimum total length of the pipes required to provide gas to all the locations?  
**A** 32 km      **B** 33 km      **C** 29 km      **D** 40 km      (1 mark)
- ii How many different paths are there from New Town to Old Town?      (1 mark)
- iii What is the shortest path from Beachside to Hillside?      (1 mark)
- iv Find a walk that visits every edge exactly once.      (1 mark)

b This diagram shows the distance by road (in km) between towns.



- i What is the shortest path from town A to town E?      (1 mark)
- ii What roads should be resurfaced, to minimise the total length of resurfaced road while still connecting each town with newly resurfaced road?      (1 mark)

c The following network shows a maze. Distances are in metres.



- i What is the shortest path from the start of the maze to the finish?  
**A** 20 m      **B** 30 m      **C** 40 m      **D** 50 m      (1 mark)
- ii Why does this network not contain a walk that visits every edge exactly once?      (1 mark)
- iii Find a minimum spanning tree for this network.      (1 mark)

d Consider the road network below.

i Is the graph connected? (1 mark)

ii Which word best describes the road network? (1 mark)

A tree

B path

C cycle

D directed



e Consider the following town map as a network of intersections (vertices) connected by roads (edges).



i How many vertices are in the network? (1 mark)

ii What is the highest vertex degree? (1 mark)

iii Can you design a bus route that visits every street exactly once? (2 marks)

TOTAL:  
15 marks

# Chapters 7–9 CUMULATIVE REVIEW

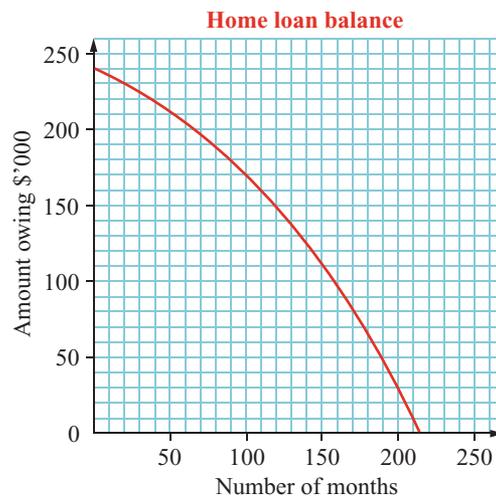
- A car is purchased for \$29 800. It depreciates in value at a rate of 28% per year.
  - Calculate the salvage value of the car after 3 years.
  - By what amount has the car depreciated in value after 3 years?
- A car depreciates in value from \$42 990 to \$25 820 in 2 years. Use the declining-balance formula to calculate the annual percentage rate of depreciation.
- Use the following table to find the monthly repayment on a reducing-balance loan of \$160 000 over:
  - 10 years
  - 15 years
  - 25 years
  - 30 years.

Years	\$160 000	\$200 000	\$240 000	\$280 000	\$320 000	\$360 000	\$400 000
5	\$3282.64	\$4103.30	\$4923.96	\$5744.62	\$6565.30	\$7385.96	\$8206.60
10	\$1983.78	\$2479.72	\$2975.66	\$3471.60	\$3967.54	\$4463.48	\$4959.44
15	\$1575.58	\$1969.48	\$2363.38	\$2757.28	\$3151.16	\$3545.06	\$3938.96
20	\$1388.52	\$1735.64	\$2082.78	\$2429.90	\$2777.04	\$3124.16	\$3471.28
25	\$1288.36	\$1630.46	\$1932.64	\$2254.64	\$2576.72	\$2898.82	\$3260.92
30	\$1230.26	\$1537.82	\$1845.40	\$2152.96	\$2460.52	\$2768.08	\$3075.64

- The following table shows the repayment per \$1000 borrowed on a monthly reducible loan. Find the monthly repayment on a loan of \$85 000 over:
  - 15 years at 7.25%
  - 10 years at 8.5%.

Term in years	7%	7.25%	7.5%	7.75%	8%	8.25%	8.5%
5	\$19.8012	\$19.9194	\$20.0379	\$20.1570	\$20.2765	\$20.3963	\$20.5164
10	\$11.6108	\$11.7401	\$11.8702	\$12.0011	\$12.1328	\$12.2653	\$12.3985
15	\$8.9883	\$9.1286	\$9.2701	\$9.4128	\$9.5566	\$9.7014	\$9.8474
20	\$7.7530	\$7.9036	\$8.0559	\$8.2095	\$8.3644	\$8.5207	\$8.6782
25	\$7.0678	\$7.2281	\$7.3899	\$7.5533	\$7.7182	\$7.8875	\$8.0522
30	\$6.6530	\$6.8218	\$6.9921	\$7.1641	\$7.3377	\$7.5127	\$7.6891

- The graph on the right shows the amount outstanding against repayment period for a home loan of \$240 000 at 8.4% p.a. with a repayment of \$2160 per month. Use the graph to find:
  - how much is owing after 100 months
  - when the amount owing is \$90 000.



6 Consider the following credit card statement.

CREDIT CARD STATEMENT			
Statement begins		14 June 2018	
Statement ends		13 July 2018	
Account number		XXXX 1234 5678 9000	
Overdue amount due now			\$0
Overlimit due now			\$0
Payment due date			7 Aug 2018
Minimum amount due			\$28.77
Opening balance	New charges	Payment received	Closing balance
\$920.00	+\$739	-\$700	\$959.00
Interest charged on purchases	Purchase rate 18.90%	Daily rate	
Interest charged on cash advances	Cash advance rate 22.90%	Daily rate	
Credit limit \$10 000	Available credit \$9041		

- a**
- i** When is the minimum payment due?
  - ii** How many days is this from the start of the statement period?
- b**
- i** What is the minimum payment due?
  - ii** What percentage is this of the closing balance?
- c** What is the available credit?
- d** Calculate the daily interest rate charges as a percentage and as a decimal for:
- i** purchases
  - ii** cash advances.
- 7** The minimum payment on a credit card is the greater of \$10 or 3% of the closing balance. Calculate the minimum payment on a closing balance of \$380.
- 8** Calculate the interest to be paid on a cash advance of \$2400 if the total amount is repaid after 19 days. The interest rate is 0.043 69% per day.
- 9** Calculate the balance at the end of September, given that the annual percentage rate is 19.9% and that there is no interest-free period on these purchases.

September statement		
Date	Details	Amount (\$)
1 Sep	Opening balance	85
7 Sep	Purchase	110
19 Sep	Purchase	229
24 Sep	Payment	250

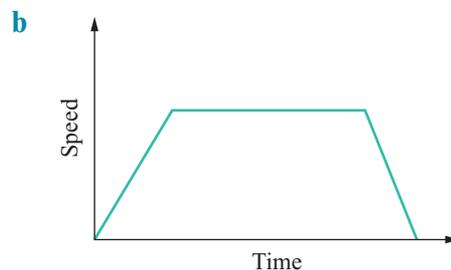
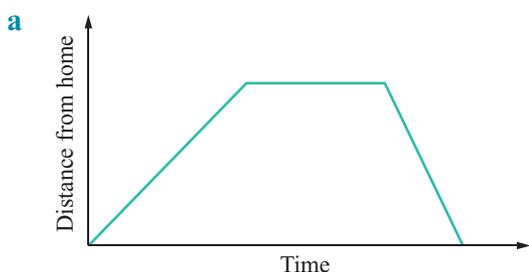
- 10 a** For this table of values, construct the graph of the relationship by plotting coordinate points on a number plane and joining them with a straight line or smooth curve.

$x$	-2	-1	0	1	2	3	4
$y$	5	0	-3	-4	-3	0	5

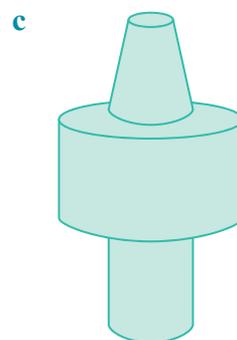
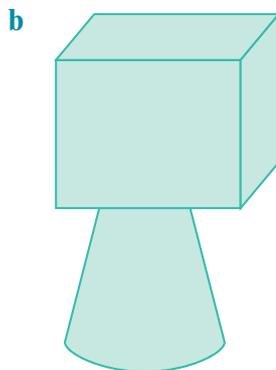
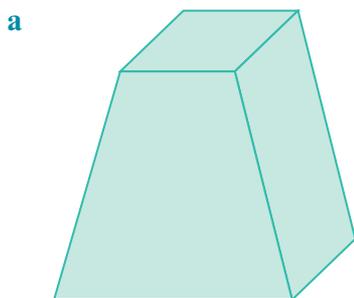
- b** Check your graph using graphing technology.  
**c** Does your graph show a linear relationship? If not, what type of relationship does it show?  
**d** From the graph, estimate the value of  $y$  when  $x$  is 2.5.
- 11** The average speed necessary to travel 480 km in a particular time, where  $t$  is the time in hours and  $s$  is the speed in km/h, is shown in the table.

$t$	4	6	8	12	16	24
$s$	120	80	60	40	30	20

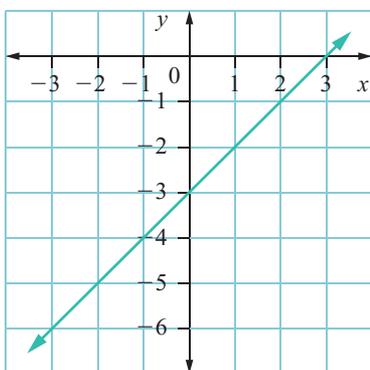
- a** Construct the graph of this relationship with  $t$  on the horizontal axis and  $s$  on the vertical axis.  
**b** What type of relationship does the graph show?  
**c** Use the graph to find the average speed required to complete the 480 km trip in 7 hours.  
**d** Use the graph to estimate the time it would take to complete the trip at an average speed of 45 km/h.
- 12** Sketch a graph showing distance as time passes to represent each description.
- a** Rolf is moving at a constant speed.  
**b** Leia is stationary.  
**c** Lawrence is decelerating (moving at a decreasing speed).  
**d** Tina is accelerating (moving at an increasing speed).
- 13** Explain what each of the following sketch graphs represents.



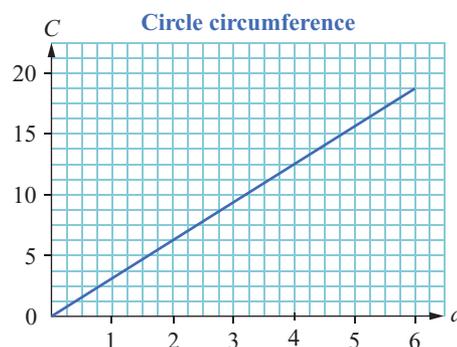
- 14** Water is poured into each of these containers at a constant rate. Sketch a graph for each container that shows the depth of the water as time passes.



- 15 Find the equation for this linear graph.



- 16 This graph models the circumference of a circle,  $C$ , given the diameter,  $d$ . Both the circumference and diameter are in centimetres.



- From the graph, estimate the circumference of a circle with a diameter of 5 cm.
- From the graph, estimate the diameter of a circle with a circumference of 11 cm.
- What is the gradient of this linear model?
- What is the vertical intercept of this linear model?
- Write the equation for this linear model.
- Use the equation to find the circumference of a circle with a diameter of 25 cm.

- 17 The Team Shirt Company has fixed costs of \$1400 per day and a variable cost of \$4 per T-shirt. The daily cost may be modelled using the straight-line equation  $C = 4n + 1400$  where  $C$  is the cost in dollars and  $n$  is the number of T-shirts.



- What is the cost to produce 180 T-shirts?
  - Use graphing technology or a table of values to draw the graph of this linear model for  $n$  values from 0 to 250.
  - What is the gradient of the line? What does this represent?
  - What is the vertical intercept of the line? What does this represent?
  - Use the graph to find the number of T-shirts produced if the cost is \$1900.
- 18 a Use graphing technology or a table of values to draw the graph of each of the following exponential relationships. Indicate the  $y$ -intercept for each graph.
- $y = 1.8(2^x)$
  - $y = 2.5(0.7)^x$
  - $y = 1000(1.03)^x$
- b Which of your graphs from part a show exponential growth and which show exponential decay?

**19** The population of an island is modelled using the equation  $P = 10000(1.05)^t$ , where  $P$  is the population and  $t$  is the time in years.

**a** Complete this table of values using the equation  $P = 10000(1.05)^t$ .

$t$	0	5	10	15	20
$P$					

**b** Use graphing technology or the table of values to draw the graph of the population model for values of  $t$  from 0 to 20.

**c** What does the intercept of the curve with the vertical axis represent?

**d** Use the graph to estimate the population after 12 years.

**e** Estimate the time taken for the population to reach 22000.

**f** Extend your graph to obtain an estimate of the time taken for the population to triple.

**g** What is the percentage increase in population each year? Explain how you can work this out from looking at the equation  $P = 10000(1.05)^t$ .

**20** The formula for the declining-balance method of depreciation is  $S = V_0(1 - r)^n$ , where  $S$  is the salvage value of the asset in dollars,  $V_0$  is the purchase price of the asset in dollars,  $r$  is the percentage interest rate per time period, as a decimal, and  $n$  is the total number of time periods. The value of Melita's work tools depreciates at the rate of 25% per year and can be represented by an exponential model.

**a** If Melita bought the tools for \$1200, show that the equation of this exponential model is  $S = 1200(0.75)^n$ .

**b** Use technology or a table of values to draw the graph of  $S = 1200(0.75)^n$  for values of  $n$  from 0 to 10.

**c** Use the graph to find the salvage value of Melita's tools after 3 years.

**d** Use the graph to estimate the number of years it takes for the salvage value to be less than \$300.

**e** Does this model represent exponential growth or exponential decay? Explain.

**21** The population of a regional town was recorded at the end of June each year from 2008 to 2018. The data is shown in the table.

**a** Plot the points for the dataset on a number plane, using technology or by hand. Use 0, 1, 2, ... for the number of years after 30 June 2008 as the independent variable.

**b** Draw a straight line or exponential curve to best represent the trend of the data.

**c** State which model (linear or exponential) best approximates the data.

**d** From the trendline, estimate the population at the end of December 2012.

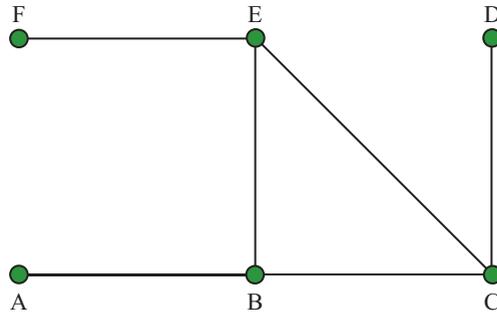
**e** From the trendline, estimate when the population was half of what was recorded in 2008.

**f** Predict what the recorded population will be in 2020. Explain the method you used to answer this.

**g** What factors could affect the reliability of this model to predict the population in the future?

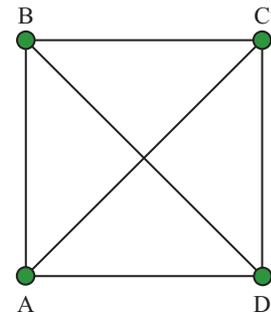
Year	Population
2008	2500
2009	2384
2010	2115
2011	2091
2012	1783
2013	1652
2014	1456
2015	1488
2016	1137
2017	945
2018	804

- 22** The network below shows vertices connected by edges. Fill out the table, which lists the vertices and the connections between them.



Vertex	A	B	C	D	E	F
Connected to	B	A, E, C				

- 23** Inspector Poirot has four suspects in his criminal investigation: A, B, C and D. Any one suspect may have committed the crime, or the suspects could be working together. Poirot indicates the possibility that two suspects are working together by drawing an edge between the corresponding vertices in the network shown.



Evidence emerges that suspects A and B are not working together, and that suspect D is completely innocent. What does Poirot's network of suspects look like with this new evidence?

- 24** The five boroughs of New York – Manhattan, the Bronx, Queens, Brooklyn and Staten Island – are connected by many bridges, as listed in the table. Queens and Brooklyn are connected by land, and so for this question are considered the same.

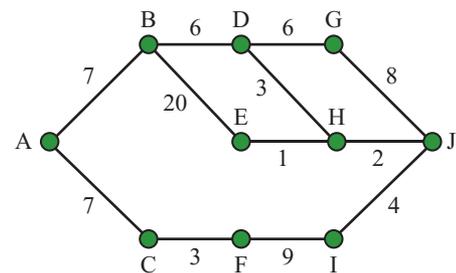
	Manhattan	The Bronx	Queens/Brooklyn	Staten Island
Manhattan	–	11	8	0
The Bronx		–	3	0
Queens/Brooklyn			–	1
Staten Island				–

Is it possible to cross every bridge in New York exactly once?

- 25** Find a spanning tree for the network in your answer to question 23.

Use the network on the right to answer questions 26–29.

- 26** Use Prim's algorithm to find a minimum spanning tree for this network.
- 27** Use Kruskal's algorithm to find a minimum spanning tree for this network.
- 28** Use Dijkstra's algorithm to find the shortest path between A and J in this network.
- 29** What is the shortest path between B and E in this network?



# ANSWERS

## CHAPTER 1 INVESTMENTS

### ARE YOU READY?

- 1 D 2 B 3 C 4 C 5 B 6 B 7 B  
8 D 9 A 10 A 11 D

### EXERCISE 1A COMPARING SIMPLE AND COMPOUND INTEREST INVESTMENTS

Note: answers may vary slightly due to rounding.

- 1 a \$1044 b \$457.60 c \$3.71  
2 a \$970.67 b \$27.41 c \$1345.25 d \$65.80  
3 a \$6255 b \$6526.25 c \$7141.82 d \$6339.18

4 a Interest on the loan =  $14\,300 \times \frac{12.5}{100} \times 3$   
= \$5362.50

b Total to be repaid =  $14\,300 + 5362.50$   
= \$19662.50

c Monthly repayment =  $\frac{19662.50}{36}$   
= \$546.18

- 5 a \$1995.20 b \$7795.20 c \$162.40

6 a

Year	Balance at start of year	Interest	Balance at end of year
1	\$2800	\$196	\$2996
2	\$2996	\$209.72	\$3205.72
3	\$3205.72	\$224.40	\$3430.12

b Interest =  $3403.12 - 2800$   
= \$603.12

- 7 a \$8362.03  
b \$1862.03

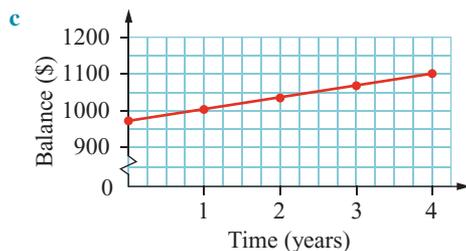
- 8 a 1% b 0.5% c 0.6% d 0.6% e 0.55%  
9 a 0.7% b \$20709.87 c \$709.87

- 10 a Line 1 = compound interest, line 2 = simple interest  
b About \$530 (\$534.37)

11 a

Year	Balance at start of year	Interest	Balance at end of year
1	\$980	\$29.40	\$1009.40
2	\$1009.40	\$30.28	\$1039.68
3	\$1039.68	\$31.19	\$1070.87
4	\$1070.87	\$32.13	\$1103.00

- b \$123.00



12 a

Year	Balance at start of year	Interest	Balance at end of year
1	\$10350	\$796.95	\$11146.95
2	\$11146.95	\$858.32	\$12005.27
3	\$12005.27	\$924.41	\$12929.68
4	\$12929.68	\$995.59	\$13925.27
5	\$13925.27	\$1072.25	\$14997.52

- b Yes, she will have \$14997.52 when her investment matures, which is enough to buy the car.  
c \$502.52

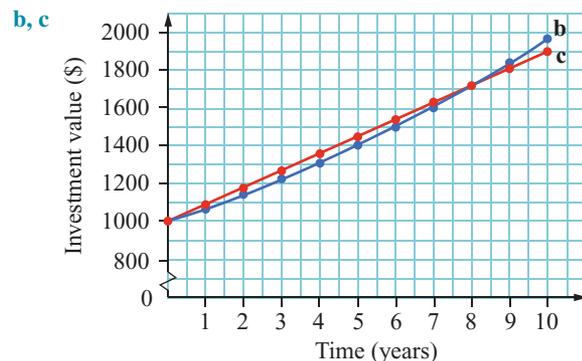
13 a

Time	Simple interest at 5% p.a.	Compound interest at 5% p.a.
Start of first year	\$5000	\$5000
Start of second year	\$5250	\$5250
Start of third year	\$5500	\$5512.50
Start of fourth year	\$5750	\$5788.13
Start of fifth year	\$6000	\$6077.54
Start of sixth year	\$6250	\$6381.42

- b \$131.42

- 14 a Bank A by \$3.22  
b Bank B by \$31.60  
c The balance for bank A increases by the same amount each year, whereas the balance for bank B increases by a greater amount each and every month.

- 15 a Year 1: \$1070, Year 2: \$1144.90, Year 3: \$1225.04,  
Year 4: \$1310.80, Year 5: \$1402.55, Year 6: \$1500.73,  
Year 7: \$1605.78, Year 8: \$1718.19, Year 9: \$1838.46,  
Year 10: \$1967.15

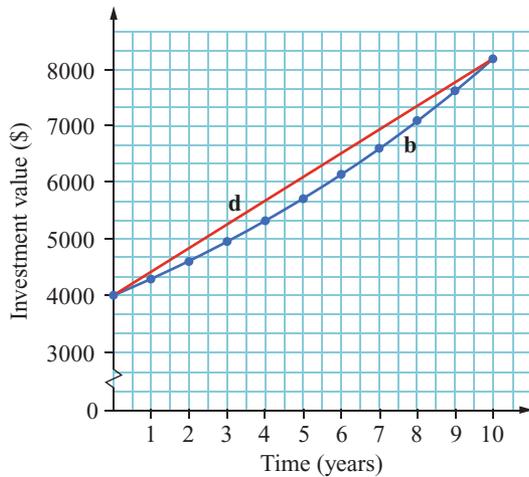


- d 8 years

16 a

Time (end of year)	Investment (\$)	Time (end of year)	Investment (\$)
1	\$4296.00	6	\$6138.83
2	\$4613.90	7	\$6593.10
3	\$4955.33	8	\$7080.99
4	\$5322.03	9	\$7604.99
5	\$5715.86	10	\$8167.76

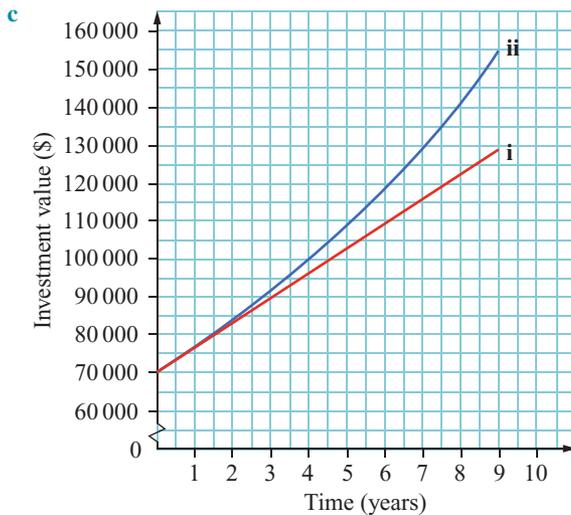
b, d



- c Just under 7 years  
e 10.4% p.a.

17 a i \$57960    ii \$84561.66

b The simple interest investment procures the same amount of interest each year, whereas the compound interest investment procures a higher amount of interest each year (when compared to the previous year).



### EXERCISE 1B THE COMPOUND INTEREST FORMULA

Note: answers may vary slightly due to rounding.

- 1 a  $PV = \$6500$      $r = 4.2 \div 100 = 0.042$      $n = 7$   

$$FV = PV(1 + r)^n$$

$$= 6500(1 + 0.042)^7$$

$$= 6500(1.042)^7$$

$$= \$8669.37$$
- b Interest =  $8669.37 - 6500 = \$2169.37$
- 2 a \$6173.21  
b \$2173.21
- 3 a \$7459.49  
b \$1006.49
- 4  $PV = 1200$      $n = 5 \times 4 = 20$      $r = 0.084 \div 4 = 0.021$   

$$FV = PV(1 + r)^n$$

$$= 1200(1 + 0.021)^{20}$$

$$= 1200(1.021)^{20}$$

$$= \$1818.43$$
- 5 \$1073.67

- 6 a \$836.93    b \$670.43    c \$13 245.75  
d \$894.66    e \$12573.94    f \$2566.36
- 7 Option B gives the best return. It returns \$1.59 more than option A and \$1.12 more than option C.
- 8  $FV = 6000$      $r = 0.07$      $n = 5$   

$$PV = \frac{FV}{(1 + r)^n}$$

$$= \frac{6000}{(1 + 0.07)^5}$$

$$= \frac{6000}{1.402...}$$

$$= \$4277.92$$
- 9 \$1137.09
- 10 \$5288.97
- 11 Quarterly interest rate =  $0.085 \div 4 = 0.02125$   
 Number of quarters =  $7 \times 4 = 28$   

$$PV = \frac{FV}{(1 + r)^n}$$

$$= \frac{2300}{(1 + 0.02125)^{28}}$$

$$= \frac{2300}{1.801...}$$

$$= \$1276.53$$
- 12 a \$1120.05    b \$6683.89    c \$2292.45  
d \$663.24    e \$985.90    f \$5332.12
- 13 \$2155.66
- 14 \$16 427.35
- 15 a 48    b 0.55%    c \$15 370.59
- 16 a \$62.87    b \$3.74
- 17 a \$6945.75    b \$7045.45    c \$7146.10
- 18 a \$29 591.12    b \$32 520.65    c \$35 740.19
- 19 \$5302.20
- 20 6.6%
- 21 The interest earned will only be doubled if there is only one compounding period. If there is more than one compounding period, then the doubled interest rate will earn more than double the interest of the lower interest rate.
- 22 a i \$5624.32    ii \$5955.08  
iii \$6298.56    iv \$6655
- b A higher interest rate will produce a higher future value of the investment.
- 23 a i \$5512.50    ii \$5522.43  
iii \$5524.71    iv \$5525.82
- b A shorter compounding period will produce a higher future value of the investment.

### EXERCISE 1C INVESTMENT STRATEGIES

- 1 Option A
- 2 Option A
- 3 Less, as a compound interest investment will earn interest on the interest earned during the life of the investment
- 4 The shorter compounding period will earn more interest, as the interest is compounded more frequently
- 5 \$93.08
- 6 a \$29 530.99  
b 9.53%
- 7 a Option A: \$15 666.41, Option B: \$16 275, Option C: \$14 977.81  
b \$1297.19
- 8 a Option A: \$66 155.07, Option B: \$79 796.11, Option C: \$78 711.94  
b The stock market is more volatile than the other two options, with the possibility of losing money as well as gaining money.

**EXERCISE 1D** USING A COMPOUNDED VALUE TABLE

- 1 a 1.158, \$9264    b \$9745    c \$21 264  
 d 1.062, \$1189.44    e \$11 581.50    f \$1836.54  
 g \$1661.60    h \$81 635.22
- 2 a \$7060    b \$5579    c \$11 280  
 d \$1599    e \$7432.40    f \$3745.90  
 g \$916.70    h \$12 975.31
- 3 a 1.217, \$4930.16    b \$2327.13  
 c \$2794.12    d \$1500, \$810.37  
 e \$2382.91    f \$1877.13  
 g \$1041.67    h \$3510.64
- 4 \$4342.80  
 5 \$1723.50  
 6 \$1953.82  
 7 \$840

**EXERCISE 1E** INFLATION AND APPRECIATED VALUE

- 1 Price after 4 years =  $12.95 \times \left(1 + \frac{3.8}{100}\right)^4 = \$15.03$
- 2 a \$2.99    b \$3.14    c \$3.30  
 d \$3.46    e \$3.64
- 3 a \$548.45    b \$912.87    c \$584.40  
 d \$155.22    e \$421.66
- 4 \$3.77  
 5 \$33.22  
 6 \$36.39  
 7 \$63 251  
 8 \$70 501
- 9 a \$3882.40    b \$6029.30    c \$22 581.60
- 10 a \$4057.45    b \$4703.70    c \$6321.40
- 11 \$59 211  
 12 \$1794.78
- 13 Under the expected value \$438.66
- 14 2.53%  
 15 3.77%
- 16 a 8.51%    b 4.56%
- 17 b No;  $1.03 \times 1.04 \neq 1.035^2$

**REVIEW** MULTIPLE-CHOICE QUESTIONS

- 1 D    2 B    3 C    4 B    5 D    6 C  
 7 B    8 D    9 C    10 A

**REVIEW** SET 1

- 1 a \$7548.92    b \$1548.92  
 2 \$3239.81  
 3 \$3698.77  
 4 \$5.75  
 5 The compound interest account earns \$51.27 extra  
 6 \$49 717  
 7 \$122 214.72

**REVIEW** SET 2

- 1 \$4014.78  
 2 \$1250.20  
 3 \$7357.07  
 4 \$27.45  
 5 \$86.86  
 6 \$82 350

**REVIEW** SET 3

- 1 a \$2811.53    b \$353.53  
 2 \$5973.17

- 3 \$5.59  
 4 \$67 049.05  
 5 \$22 055.79  
 6 \$56 275  
 7 Option 1

**REVIEW** SET 4

- 1 \$3612.92  
 2 \$6359  
 3 \$1419.79  
 4 \$8000.28  
 5 \$3.65  
 6 \$348.00

**REVIEW** PRACTICE EXAMINATION QUESTION

- 1 a i 0.02    ii 20    iii \$9287.17    iv \$3037.17  
 b i \$9450    ii \$9727.98    iii Account ii by \$277.98  
 c \$1503.63    d \$4633.00    e \$1317.95

**CHAPTER 2 RATES****ARE YOU READY?**

- 1 B    2 A    3 C    4 A    5 D    6 B    7 D  
 8 B    9 D    10 B    11 C    12 B    13 B

**EXERCISE 2A** UNDERSTANDING RATES

- 1 a  $\frac{300 \text{ words}}{5 \text{ min}} = 60 \text{ words/min}$   
 b  $\frac{\$10.90}{2.5 \text{ kg}} = 4.36 \text{ \$/kg or } \$4.36/\text{kg}$   
 c  $\frac{\$82.50}{6 \text{ h}} = 13.75 \text{ \$/h or } \$13.75/\text{h}$   
 d  $\frac{\$55.20}{120 \text{ calls}} = 0.46 \text{ \$/call or } \$0.46/\text{call}$
- 2 a 4 degrees/h    b 0.4 kg/m<sup>2</sup>    c 78 km/h    d 3.4 L/min
- 3 a  $\frac{5000 \text{ kg}}{1 \text{ ha}} = 5000 \text{ kg/ha}$   
 b  $\frac{16\,000 \text{ kg}}{10\,000 \text{ m}^2} = 1.6 \text{ kg/m}^2$   
 c  $\frac{375 \text{ cents}}{60 \text{ min}} = 6.25 \text{ c/min}$   
 d  $\frac{180 \text{ cents}}{1000 \text{ mm}} = 0.18 \text{ c/mm}$   
 e  $\frac{15\%}{12 \text{ months}} = 1.25\%/\text{month}$   
 f  $\frac{2400 \text{ g}}{1000 \text{ mL}} = 2.4 \text{ g/mL}$
- 4 a 12.5 mL/s    b 1.2 c/g    c 5 m/s  
 d 7.5 mL/s    e 5 g/min    f 388.9 cm/s
- 5 a  $60 \times 60 = 3600$ ,  $\frac{6 \text{ m}}{1 \text{ s}} = \frac{(6 \times 3600) \text{ m}}{1 \text{ h}} = \frac{21\,600 \text{ m}}{1 \text{ h}}$ ,  
 $1000$ ,  $\frac{21\,600 \text{ m}}{1 \text{ h}} = \frac{(21600 \div 1000) \text{ km}}{1 \text{ h}} = \frac{21.6 \text{ km}}{1 \text{ h}}$ ,  
 6 m/s = 21.6 km/h  
 b  $24 \times 60 = 1440$ ,  $\frac{3 \text{ cents}}{1 \text{ minute}} = \frac{(3 \times 1440) \text{ cents}}{1 \text{ day}}$   
 =  $\frac{4320 \text{ cents}}{1 \text{ day}}$ , 100, 43.2 \$/day or \$43.20/day
- 6 a i 72 000 m/h    ii 72 km/h  
 b i 5000 g/L    ii 5 kg/L  
 c i 800 c/kg    ii \$8/kg  
 d i 7500 kg/ha    ii 7.5 t/ha

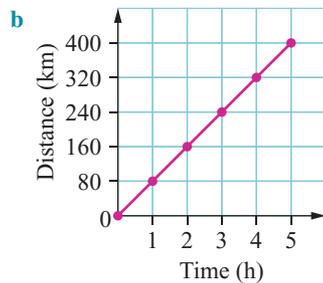
- e i 1440 mL/h    ii 1.44 L/h  
 f i 800 c/km    ii \$/km
- 7 Supermarket: \$0.48/egg, Sunday market: \$0.50/egg; cheaper to buy carton of 12 eggs at the supermarket
- 8 Mineral water: \$1.75/L, cola: \$1.83/L; 2-L bottle of orange mineral water is better value
- 9 a Tyler: \$0.90, Lisa: \$0.93  
 b Tyler paid less per 100 g so he made the best buy.  
 c Some customers may prefer to buy only what they need rather than a full pack.
- 10 Butcher: \$1.54/100 g, supermarket: \$1.38/100 g; supermarket offers the best buy
- 11 a 150 km    b 3.75 h
- 12 a 120 kg    b 3 bags    c 5000 m<sup>2</sup>
- 13 a 520 mL    b 7.5 h    c 20 drops/min
- 14 a \$354    b 20 h    c 26 h
- 15 \$320
- 16 \$296
- 17 a 60 L/h (as 60 L/h = 1000 mL/min)  
 b 15 m/s (as 40 km/h ≈ 11.1 m/s)  
 c 130 kg/L (as 130 kg/L = 130 g/mL)  
 d 18 km/h (as 18 km/h = 500 cm/s)
- 18 93 km/h is faster than 12.5 m/s as 93 km/h ≈ 25.8 m/s.
- 19 a 75 bpm    b 120 bpm  
 c After exercise, Peter's heart rate was 80 bpm which is higher than his resting heart rate of 75 bpm.
- 20 6760 L
- 21 a US\$2894.74    b A\$1235    c \$1710
- 22 a  $9.36 \times 10^8$  km  
 b i About 107 000 km/h    ii About 29.7 km/s

**EXERCISE 2B** DISTANCE-TIME GRAPHS AND SPEED

- 1 a 2 min    b 4 min  
 c i 50 m    ii 100 m    iii 150 m  
 d i 50 m    ii 50 m    iii 50 m/min  
 e i 50 m/min    ii 50 m/min    iii 50 m/min  
 f Constant speed is represented with a straight line in a distance-time graph.  
 g i 100 m    ii 100 m  
 h i 100 m/min    ii 100 m/min  
 i A faster speed is shown as a steeper straight line in a distance-time graph.
- 2 The steeper the slope of a line in a distance-time graph, the higher the speed.

3 a

Time (h)	0	1	2	3	4	5
Distance (km)	0	80	160	240	320	400

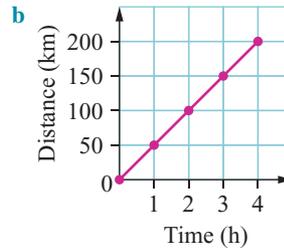


- c Gradient =  $\frac{(400 - 0) \text{ km}}{(5 - 0) \text{ h}} = 80 \text{ km/h}$
- d The gradient of a line is the vertical rise over the horizontal run. In this case the vertical rise is the change in distance and the horizontal run is the change in time. This is the same as the speed which is the distance

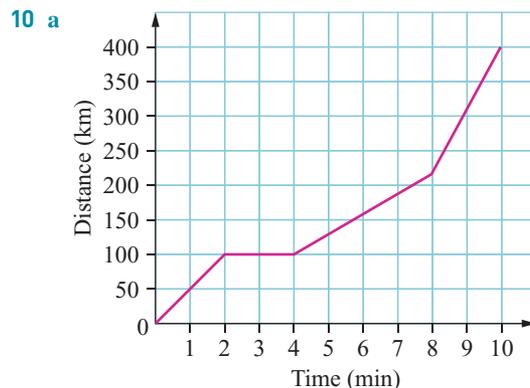
travelled (change in distance) over the time taken (change in time). Both are rates that are comparing the same quantities in a distance-time graph.

4 a

Time (h)	0	1	2	3	4
Distance (km)	0	50	100	150	200

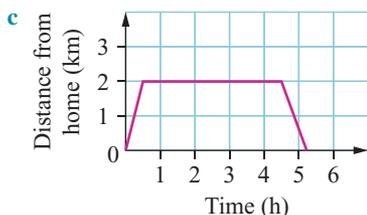


- c Gradient =  $\frac{(200 - 0) \text{ km}}{(4 - 0) \text{ h}} = 50 \text{ km/h}$ . This is the same as the given speed of 50 km/h.
- 5 a Section A: Harry travels to his friend's house, section B: Harry spends time at his friend's house, section C: Harry and his friend travel to the cinema, section D: Harry and his friend watch a film at the cinema.  
 b i 0 m    ii 240 m    iii 240 m    iv 360 m  
 c After 2 min    d 2 min    e 120 m  
 f Section A    g Sections B and D
- 6 a Section A: 120 m/min, section B: 0 m/min, section C: 30 m/min, section D: 0 m/min  
 b 360 m    c 36 m/min  
 7 a 4 km    b 4 h  
 c Sam walked faster from home to school. The line representing his travel from home to school is steeper than the line representing his travel from school to home. (Speed of 4 km/h compared to 2 km/h.)  
 d 2 km    e 8 km
- 8 a 3 km    b After 7.5 h    c 1 h  
 d Between 9 h and 9.5 h; 4 km/h  
 e Between 7.5 h and 8 h    f 5:15 pm  
 g Nadia left home at 7:45 am and walked to school at a constant speed of 3 km/h. She stayed at school for 6.5 h. Nadia left school at 3:15 pm and walked for 30 min to travel 1 km to the sports centre which was on her way home. After 1 hour of netball practice, she travelled at a constant speed of 4 km/h to return home at 5:15 pm.
- 9 a 20 km    b 9 am    c 35 km  
 d 1 pm    e 6.5 h  
 f Between 4 pm and 5 pm; 25 km/h  
 g Between 10 am and 10:30 am  
 h i 10 km/h    ii 5 km/h    iii 5 km/h  
 i 45 km    j 70 km    k 7 km/h



- b Between 8 min and 10 min; the line has the steepest slope  
 c Between 2 min and 4 min  
 d i 50 m/min ii 90 m/min iii 30 m/min  
 e 40 m/min

- 11 a 2 km b 40 min



- d 3:55 pm  
 12 a 84 km b 216 km  
 c Mittagong d Katoomba and Penrith  
 e 64 km/h f About 2 hours  
 13 No, you can't be in different places at the same time. This graph shows that you are at numerous distances from home at 4 pm.  
 14 To simplify the distance-time graphs, we have assumed that we can change from one constant speed to another instantaneously without accelerating or decelerating. Realistically, this change in speed should be shown as a curve between the sections of constant speed.

### EXERCISE 2C FUEL CONSUMPTION RATE

- 1 a 11.7 km/L b 0.086 L/km c 8.6 L/100 km  
 2 a i 11.25 km/L ii 0.089 L/km  
 iii 8.9 L/100 km  
 b i 9.2 km/L ii 0.109 L/km  
 iii 10.9 L/100 km  
 c i 17.5 km/L ii 0.057 L/km  
 iii 5.7 L/100 km  
 3 a 19L/100km b 11.25L/100km c 6.38L/100km  
 4 Distance =  $\frac{10}{5.6} \times 100 \approx 179$  km  
 5 a 349 km b 559 km c 790 km  
 6 Fuel used =  $\frac{965}{100} \times 10.2 = 98.43 \approx 98$  L  
 7 a 37 L b 75 L c 24 L  
 8 a 7.5 L/100 km  
 b i 37.5 L ii 600 km  
 c Trip costs \$10.72 more  
 9 \$266.37  
 10 \$1975.84  
 11 a 1–3 cylinders: \$1077.23, 4 cylinders: \$1399.00,  
 6 cylinders: \$1636.83, 8+ cylinders: \$1804.71  
 b \$237.83  
 c Annual fuel cost for truck is \$7763.77 which is more than 4 times as much as the cost for an 8-cylinder car.  
 12 a \$85.74 b \$73.06 c \$12.68  
 13 a 10.94 L b About 6.4 L/100 km  
 14 Cheaper to travel by bus; cost per one-way trip: bus \$4.25, car \$5.66; saving of about \$1.41 per one-way trip  
 15 a ULP: \$2241.20, LPG: \$1591.87  
 b \$649.33  
 c \$54.11  
 d Approx. 48 months  
 e About 72000 km  
 16 a i \$2115.29 ii \$1498.22 iii \$617.07  
 b \$51.42  
 c Approx. 23.3 months  
 d Approx. 27183 km

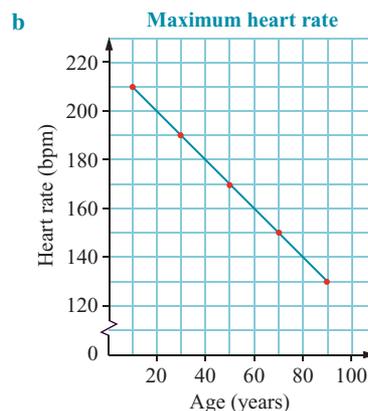
- 17 a The standardised test comprises two cycles: Urban cycle represents stop-start city driving, Extra Urban cycle is a high speed test to represent freeway or highway driving. The Combined test result is based on a weighting of the urban and extra urban figures according to the distance travelled in each cycle.  
 b 24.8 L c \$19.60

### EXERCISE 2D HEART RATE

- 1 a 77–83 bpm b 55–61 bpm  
 c 82+ bpm d 62–65 bpm  
 2 a Excellent b Above-average  
 c Below-average d Average  
 3 a 75 bpm b 49 bpm c 26 bpm  
 4 a 77–82 bpm b 61–65 bpm  
 c 84+ bpm d 65–68 bpm  
 5 a Above-average b Athlete  
 c Excellent d Good  
 6 a 79 bpm b 54 bpm c 25 bpm  
 7 a 180 bpm b 167 bpm c 198 bpm  
 d 199 bpm e 190 bpm f 156 bpm  
 8 95–162 bpm  
 9 a 200 bpm, 100–170 bpm b 177 bpm, 89–150 bpm  
 c 165 bpm, 83–140 bpm d 201 bpm, 101–171 bpm  
 e 192 bpm, 96–163 bpm f 162 bpm, 81–138 bpm  
 10 3.3, good  
 11 a 2, fair b 4.3, excellent c 3.5, good  
 d 5.3, excellent

12 a

Age (years)	10	30	50	70	90
Maximum heart rate (bpm)	210	190	170	150	130



- c Linear relationship  
 d 45  
 13 2600  
 14 a 2000 b 1780 c 1660  
 d 2020 e 1920 f 1620  
 15 a 4550 mL or 4.5 L b 84000 mL or 84 L  
 c 33250 mL or 33.25 L d 7056000 mL or 7056 L

### EXERCISE 2E BLOOD PRESSURE

- 1 115 is in the range 95–140 and 72 is within the range 60–90 so it is normal.  
 2 a Yes b Yes c No  
 d Yes e Yes f Yes  
 3 120, 85, 89, prehypertension



- e i Systolic 111–135 mmHg, diastolic 78–88 mmHg  
 ii Medication was effective in reducing Julie's blood pressure. From Week 2 onwards, both the systolic and diastolic readings were under the Week 1 readings. While a normal range was achieved in Week 4, the systolic reading was above the normal in Week 5 and the diastolic reading was above the normal in Week 6. A normal range was only consistently achieved from Week 7 onwards.

## CHAPTER 3 SCALE DRAWINGS

### ARE YOU READY?

- 1 B    2 A    3 C    4 B    5 D    6 B  
 7 C    8 C    9 C    10 A    11 D

### EXERCISE 3A RATIOS

- 1 a 54:13    b 13:54    c 7:54  
 d 13:4    e 54:13:7    f 13:7:4
- 2 a  $\frac{54}{13}$     b  $\frac{13}{54}$     c  $\frac{7}{54}$     d  $\frac{13}{4}$
- 3 a 5:7    b 3:2    c 7:4  
 d 25:7:1    e 2:4:3    f 3:4:6  
 g 16:19    h 8:9    i 7:4  
 j 31:20    k 8:25    l 3:2  
 m 3:5    n 15:16    o 5:22  
 p 44:17    q 15:8    r 9:4  
 s 1:21    t 2:5:7    u 40:3:2
- 4 a 3.5:1    b 1.8:1    c 0.4:1    d 0.9:1
- 5 a 5:1; 3.5 m is 5 times bigger than 70 cm.  
 b 1.5:1; 1.2 kg is  $1\frac{1}{2}$  times bigger than 800 g.  
 c 5:1;  $2\frac{1}{2}$  cups is 5 times bigger than  $\frac{1}{2}$  cup.  
 d 0.8:1; 0.04 ha is 0.8 times the size of 500 m<sup>2</sup>.
- 6 a 1:1.74    b 1:2.25    c 1:0.8    d 1:0.064
- 7 a 1:100; 1 cm is 100 times smaller than 1 m (or 1 m is 100 times bigger than 1 cm).  
 b 1:1000; 1 mm is 1000 times smaller than 1 m (or 1 m is 1000 times bigger than 1 mm).  
 c 1:50; 20 mL is 50 times smaller than 1 L (or 1 L is 50 times bigger than 20 mL).  
 d 1:300; 50 mg is 300 times smaller than 15 g (or 15 g is 300 times bigger than 50 mg).
- 8  $\frac{\text{number of boys}}{312} = \frac{9}{8}$ ; number of boys =  $\frac{9}{8} \times 312 = 351$
- 9 \$41 600
- 10 \$48 600
- 11  $\frac{470}{\text{number of females}} = \frac{10}{9}$  so  $\frac{\text{number of females}}{470} = \frac{9}{10}$   
 number of females =  $\frac{9}{10} \times 470 = 423$
- 12 165 cm
- 13 a 98 L    b 204 mL
- 14 a 35 L    b 45 L
- 15 a 18.75 L    b 720 mL
- 16 a 2 bags of cement and 6 bags of screenings  
 b 1 bag of sand and 1.5 bags of screenings
- 17 Hannah \$18 750, Rachel \$11 250
- 18 \$1125
- 19 20 mL, 160 mL
- 20 6 kg
- 21 Alison 56 kg, Vincent 42 kg, Matthew 70 kg

- 22 a Males who never smoked 1:76, males who have smoked 1:12, males who currently smoke 1:4.5, females who never smoked 1:157, females who have smoked 1:23, females who currently smoke 1:8.8  
 b Males who currently smoke  
 c Females who have never smoked
- 23 a 40 lemon trees    b 27 orange trees    c 145  
 d 40%; 2:3 is equivalent to 40:60 or 40% to 60%
- 24 a 3:7    b \$203 000    c \$126 000
- 25 a 1:500    b 200 mL of concentrate and 99.8 L of water
- 26 a Property \$900 000, shares \$540 000, bonds \$360 000  
 b 9:7:2
- 27 5:4, as this ratio gives her a larger fraction of \$100; that is  $\frac{5}{9}$  compared to  $\frac{6}{11}$  (\$55.56 compared to \$54.55)
- 28 a Dark pink: white 3000 mL, red 1000 mL; light pink: white 3500 mL, red 500 mL  
 b 13:3

### EXERCISE 3B SIMILAR FIGURES AND SCALE FACTORS

- 2 a  $\angle DAB = 108^\circ$ ,  $\angle ABC = 124^\circ$ ,  $\angle BCD = 56^\circ$ ,  $\angle CDA = 72^\circ$   
 b  $\angle D'A'B' = 108^\circ$ ,  $\angle A'B'C' = 124^\circ$ ,  $\angle B'C'D' = 56^\circ$ ,  $\angle C'D'A' = 72^\circ$   
 c i  $\angle A'B'C'$     ii  $\angle B'C'D'$   
 iii  $\angle C'D'A'$     iv  $\angle D'A'B'$   
 d The corresponding angles are equal.  
 e i  $A'B'$     ii  $B'C'$     iii  $C'D'$     iv  $D'A'$   
 f i  $\frac{A'B'}{AB} = 2$     ii  $\frac{B'C'}{BC} = 2$   
 iii  $\frac{C'D'}{CD} = 2$     iv  $\frac{D'A'}{DA} = 2$   
 g The corresponding sides are all in the same proportion.
- 3 a 50°  
 b i  $\angle PRQ$     ii  $\angle RQP$     iii  $\angle QPR$   
 c Yes  
 d i  $PQ$     ii  $PR$     iii  $QR$   
 e  $\frac{PQ}{AB} = 1.28$ ,  $\frac{PR}{AC} = 1.28$ ,  $\frac{QR}{BC} = 1.28$   
 f Yes, 1:1.28  
 g Yes
- 4 a i Similar: angles equal, sides in proportion  
 ii  $k = 4$   
 b i Similar: angles equal, sides in proportion  
 ii  $k = 2$   
 c i Not similar: sides not in proportion  
 d i Similar: figures are identical    ii  $k = 1$   
 e i Similar: angles equal, sides in proportion  
 ii  $k = \frac{1}{3}$   
 f i Similar: angles equal, sides in proportion  
 ii  $k = \frac{2}{3}$
- 5 a  $k = 2.4$     b  $k = \frac{1}{3}$  or 0.3    c  $k = 1.2$   
 d  $k = \frac{1}{2}$  or 0.5    e  $k = \frac{1}{3}$  or 0.3    f  $k = \frac{2}{3}$  or 0.6  
 g  $k = \frac{2}{5}$  or 0.4    h  $k = 3$     i  $k = \frac{4}{3}$  or 1.3  
 j  $k = \frac{3}{4}$  or 0.75
- 6 a Parts a, c, h, i  
 b Parts b, d, e, f, g, j  
 c A scale factor of more than 1 produces an enlargement.  
 d A scale factor between 0 and 1 produces a reduction.

- 7 a  $k = 5$     b  $k = \frac{4}{3}$  or  $1.\bar{3}$     c  $k = 3.2$     d  $k = 1.5$   
 8  $k = \frac{4}{3}$   
 9 A and E  
 10 The model plane is  $\frac{1}{100}$  the size of the real plane.  
 11  $k = 12$   
 12  $k = \frac{1}{5000}$

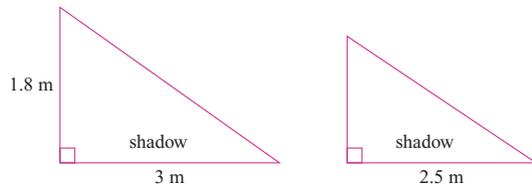
**EXERCISE 3C** FINDING LENGTHS OF SIDES IN SIMILAR FIGURES

- 1 a  $y = 7.5$  cm,  $z = 15$  cm    b  $y = 2$  cm  
 c  $z = 10.5$  cm    d  $w = 5.2$  m  
 e  $v = 10.2$  mm    f  $t = 6.75$  cm  
 g  $k = 3.2$  cm    h  $p = 8.36$  m  
 2 a  $y = 15$  m,  $z = 14.4$  m    b  $y = 6$  cm,  $z = 10.5$  cm  
 c  $y = 10$  m,  $z = 13.2$  m    d  $y = 15.75$  cm,  $z = 10.7$  cm  
 3 a  $y = 16$  cm    b  $z = 4.5$  cm  
 c  $w = 18$  cm    d  $z = 16$  cm  
 e  $v = 12$  cm    f  $y = 33.75$  cm  
 g  $h = 16.8$  cm    h  $z = 4.8$  cm  
 i  $w = 7.2$  cm,  $t = 7.5$  cm    j  $y = 22.5$  cm,  $w = 24$  cm

4 12 cm

5 8 m

6 a



- b 1.5 m  
 7 6 m  
 8 16 m  
 9 12.6 m  
 10 1.15 m  
 11  $x = 8$  m,  $y = 12$  m,  $z = 16$  m

**EXERCISE 3D** SCALE DRAWINGS

Note: answers may vary slightly due to measurement.

- 1 a i 9 mm    ii 18 mm    iii 90 mm  
 b i 6.5 mm    ii 13 mm    iii 65 mm  
 c i 16 mm    ii 32 mm    iii 160 mm  
 d i 18 mm    ii 36 mm    iii 180 mm  
 e i 24 mm    ii 48 mm    iii 240 mm  
 f i 30 mm    ii 60 mm    iii 300 mm  
 2 a i 8.5 m    ii 17 m    iii 340 m  
 b i 17.5 m    ii 35 m    iii 700 m  
 c i 21 m    ii 42 m    iii 840 m  
 d i 30 m    ii 60 m    iii 1200 m  
 e i 13 m    ii 26 m    iii 520 m  
 f i 4.5 m    ii 9 m    iii 180 m  
 3 a i 1:1000    ii  $\frac{1}{1000}$   
 b i 1:400    ii  $\frac{1}{400}$   
 c i 1:5000    ii  $\frac{1}{5000}$   
 d i 1:150    ii  $\frac{1}{150}$   
 e i 1:5000    ii  $\frac{1}{5000}$   
 f i 1:20000    ii  $\frac{1}{20000}$   
 4 a 1:1000    b 1:10000    c 1:4000  
 d 1:500    e 1:500000

- 5 Length = 250 m, breadth = 155 m  
 6  $a = 50$  m,  $b = 35$  m,  $c = 62$  m,  $d = 37$  m  
 7 a 1:80    b 1.6 m    c 0.88 m  
 8 a 1:1000    b 54 m  
 9 a 1:4000000    b 56 km    c 208 km  
 d Wollongong    e Goulburh to Newcastle  
 10 a Length 108.5 m, width 71.75 m  
 b 7785 m<sup>2</sup>  
 11 a 140 m    b \$36.11    c 256 m<sup>2</sup>    d 64 m<sup>3</sup>  
 e i 3    ii \$675  
 f \$28320 (944 m<sup>2</sup> × \$30)  
 g i 800    ii \$12000    iii \$7680  
 h i About 1560 bricks. Number of bricks for 12 m length =  $12 \div 0.23 \approx 52$ . For wall that is two bricks thick and 15 rows high,  $52 \times 2 \times 15 = 1560$ . This does not take into account that there will be concrete between each brick so it allows for wastage.  
 ii \$4212 (1560 × \$2.70)  
 12 b 43 m  
 13 b 123 m  
 14 a Rectangular panels: 40 cm × 24 cm  
 Trapezium: top length 46 cm, bottom length 70 cm, height 55 cm  
 b 10220 cm<sup>2</sup>  
 c 114 cm; this is not a realistic amount of fabric as the pieces will not fit together to take up all of the available fabric  
 d Answers may vary.  
 e Answers may vary. Possible answer is: 1.6 m length of fabric (or 160 cm × 90 cm = 14400 cm<sup>2</sup>)  
 f \$14 × 1.6 = \$22.40  
 g The new fabric (by \$22.40 – \$19.20 = \$3.20)

**EXERCISE 3E** BUILDING PLANS

Note: answers may vary slightly due to measurement.

- 1 a Plan    Front elevation    Side elevation  
 b Plan    Front elevation    Side elevation  
 c Plan    Front elevation    Side elevation  
 2 a 39.6 m by 20 m    b 26 m  
 c i 2 m    ii 4 m  
 d 6 m    e 0.8 m    f 32 m<sup>2</sup>    g 20.8 m  
 h 38 m<sup>2</sup>    i 46 m<sup>3</sup>    j 46000 L    k About 12.6  
 3 a 49.08 m<sup>2</sup>  
 b i Approx. 118 m<sup>3</sup>    ii 7.2 kW  
 c 3.6 kW    d 4.2 kW

- 4 a** i 12      ii 10      iii 4  
**b** P: pantry, WO: wall oven, F: fridge, WC: water closet (toilet), Ens: ensuite, WIR: walk in robe, Lin: linen cupboard, T: tub  
**c** i 5.6 m by 4.6 m      ii 4.0 m by 3.4 m  
     iii 5.4 m by 4.0 m      iv 7.2 m by 2.6 m  
**d** i 233 m<sup>2</sup>      ii 25 squares  
**e** 1.9 m<sup>3</sup>  
**f** i 38.9 m<sup>2</sup>      ii 35.5 m<sup>2</sup>      iii 4.4 L  
**h** i 2600 mm by 1800 mm  
     ii 75 tiles      iii 5 boxes  
**i** 6 sheets  
**j** i 4.0 m      ii 3.8 m      iii B  
**k** Bedroom 1  
     i 4.2 m      ii 4.2 m      iii Either  
     Bedroom 3  
     i 4.2 m      ii 4.2 m      iii Either  
     Bedroom 4  
     i 3.8 m      ii 3.6 m      iii B
- 5 a** 1: 100  
     i 150.7 m<sup>2</sup>      ii 37.7 m<sup>3</sup>  
**b** Cupboard, downpipe, dishwasher, ensuite, linen press, pantry, refrigerator, sliding door, tub, vanity, water closet, walk in robe, washing machine  
**c** i 3300 mm × 2800 mm      ii 3300 mm × 3100 mm  
     iii 2000 mm × 2100 mm      iv 6000 mm × 6000 mm  
**d** i 1800 mm × 1200 mm      ii 2.16 m<sup>2</sup>      iii 1.72 m<sup>2</sup>  
     iv 2700 mm      v 30.68 m<sup>2</sup>      vi 3 L  
**e** i 3300 mm × 2100 mm      ii 115  
**f** i 12.5 mm      ii \$1462.50  
**g** \$2736      h 53.24 m      i 1200 mm × 1000 mm  
**j** 4700 m      k 97.2 m<sup>3</sup>      l 8
- 6 a** WIR: walk in robe, Lin: linen cupboard, Ens: ensuite, WC: water closet (toilet), V: vanity, WM: washing machine, Cbd: cupboard  
**b** i 17 hinged doors      ii 11 windows  
**c** 14 vertical posts      d 20.89 m by 10.59 m  
**e** 62.96 m      f 221 m<sup>2</sup>      g \$816/m<sup>2</sup>  
**h** 55.3 m<sup>3</sup>      i 62.24 m  
**j** i 8.4 m      ii 8.6 m  
     iii Position of joins, cutting around central heating registers, boundary in rooms without carpet, or a one-way pattern on the carpet  
**k** \$798
- 7 a** 1: 200      b 6340 mm by 5486 mm  
**c** 250 mm      d 6.2 m  
**e** 5.6 m by 3.8 m      f 7640 mm (including eaves)  
**g** 3.4 m      h 2.8 m  
**i** W1: 1800 by 2000, W3: 2700 by 2000, W4: 1800 by 600  
**j** 1800 mm by 2100 mm  
**k** 34.8 m<sup>2</sup>      l 21.3 m<sup>2</sup>  
**m** \$5947
- 9 a** i 600 m<sup>2</sup>      ii about 131 m<sup>2</sup>      iii about 105 m<sup>2</sup>  
**d** Internal walls 100 mm, exterior walls 200 mm  
**e** 400 mm

### REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 C    2 D    3 B    4 A    5 B    6 B  
 7 B    8 A    9 B    10 C    11 A    12 C  
 13 B    14 A

### REVIEW SET 1

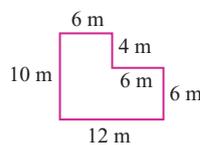
- 1 288 girls  
 2 \$18 000, \$30 000  
 4 a Similar      b Not similar  
 5 a  $\frac{2}{3}$       b  $\frac{2}{7}$   
 6 a i 2.5      ii  $x = 27.5$  m  
     b i 2      ii  $y = 16$  cm,  $z = 6.5$  cm  
 7 19.2 m  
 8 20 m ( $1 \times 6$  m,  $2 \times 3.6$  m,  $2 \times 2.5$  m,  $2 \times 0.9$  m)  
 9 a i 12.6 m      ii \$1600.20  
     b i 260      ii \$1365  
     c i 64.64 m<sup>2</sup>      ii 174.53 m<sup>3</sup>      iii 10.8 kW

### REVIEW SET 2

- 1 a 4:5      b 5:8      c 5:8  
 2 27  
 3 a Similar      b Not similar  
 4 a i 2.7      ii  $x = 10.8$  cm  
     b i 0.6      ii  $x = 6$  cm,  $y = 18\frac{1}{3}$  cm  
 5 a  $x = 6$ ,  $y = 20$       b  $x = 12$ ,  $y = 12$   
 6 3.5 m  
 7 360 m  
 8 59.28 m<sup>2</sup>  
 9 a i 37.83 m<sup>2</sup>      ii \$4728.75  
     b i About 143 m<sup>2</sup> ( $105.03$  m<sup>2</sup> +  $37.83$  m<sup>2</sup>)  
     ii About 18 L  
     c i 37.83 m<sup>2</sup>      ii 102.141 m<sup>3</sup>      iii 6.6 kW

### REVIEW SET 3

- 1 \$5625  
 2 320 g  
 3 a i  $\frac{4}{3}$       ii  $x = 13\frac{1}{3}$  cm,  $y = 10.5$  cm  
     b i  $\frac{9}{4}$       ii  $y = 11.25$  m,  $x = 6\frac{2}{3}$  m  
 4 b 314 m  
 5 1: 250  
 6 a 4 m by 3.2 m      b 44.8 m<sup>2</sup>



- d 520 m<sup>2</sup>      e West  
 7 79.92 m<sup>2</sup>  
 8 a i Least amount is 18.75 m ( $6.25$  m × 3) compared to 20 m ( $10$  m × 2)  
     ii \$2718.75  
     b i About 211 m<sup>2</sup> ( $148.08$  m<sup>2</sup> +  $62.5$  m<sup>2</sup>)  
     ii About 30 L  
     c i 62.5 m<sup>2</sup>      ii 150 m<sup>3</sup>      iii 9.0 kW

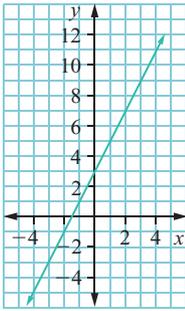
### REVIEW SET 4

- 1 Tony \$5400, Jen \$6600  
 2 a i 1.6 (or  $\frac{8}{5}$ )      ii  $z = 11.2$  cm,  $y = 6.25$  cm  
     b i 4      ii  $x = 20$  m,  $y = 3$  m  
 3 a  $x = 11.2$ ,  $y = 7.5$       b  $x = 7.5$ ,  $y = 8$   
 4 183 cm  
 5 b 448 m<sup>2</sup>      c 156 m<sup>2</sup>      d 34.8%



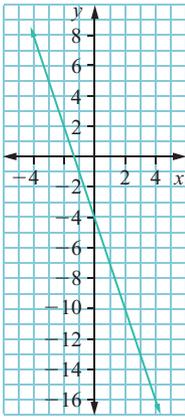
**b**  $y = 2x + 3$

$x$	-4	-2	0	2	4
$y$	-5	-1	3	7	11



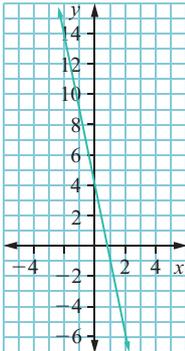
**c**  $y = -3x - 4$

$x$	-4	-2	0	2	4
$y$	8	2	-4	-10	-16



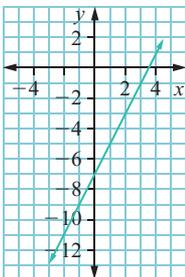
**d**  $y = 4 - 5x$

$x$	-2	-1	0	1	2
$y$	14	9	4	-1	-6



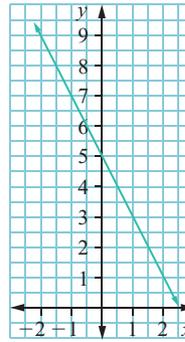
**e**  $y = 2x - 7$

$x$	-2	-1	0	1	2
$y$	-11	-9	-7	-5	-3



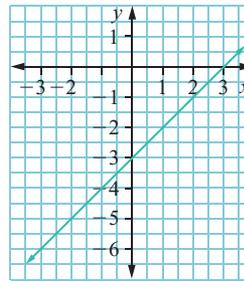
**f**  $y = 2 - 2x$

$x$	-2	-1	0	1	2
$y$	6	4	2	0	-2



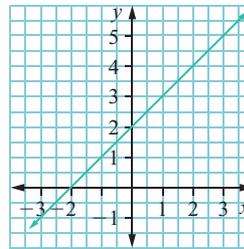
**2 a**  $y = x - 3$

$x$	-3	0	3
$y$	-6	-3	0



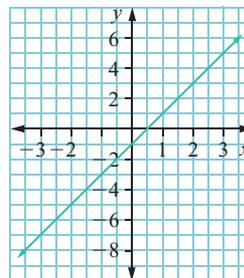
**b**  $y = x + 2$

$x$	-3	0	3
$y$	-1	2	5



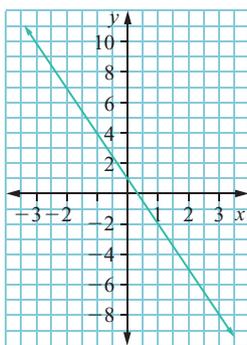
**c**  $y = 2x - 1$

$x$	-3	0	3
$y$	-7	-1	5



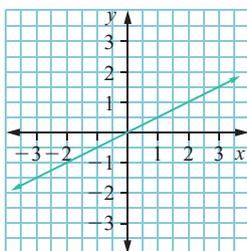
**d**  $y = 1 - 3x$

$x$	-3	0	3
$y$	10	1	-8



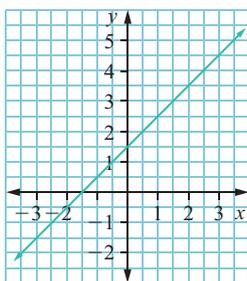
**e**  $y = \frac{1}{2}x$

$x$	-3	0	3
$y$	$-1\frac{1}{2}$	0	$1\frac{1}{2}$



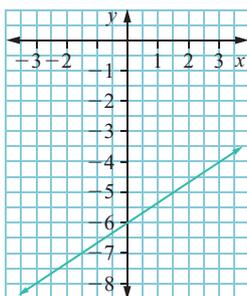
**f**  $y = \frac{2x + 3}{2}$

$x$	-3	0	3
$y$	$-\frac{3}{2}$	$\frac{3}{2}$	$\frac{9}{2}$



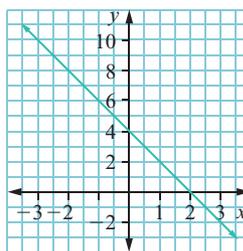
**g**  $y = \frac{2x}{3} - 6$

$x$	-3	0	3
$y$	-8	-6	-4



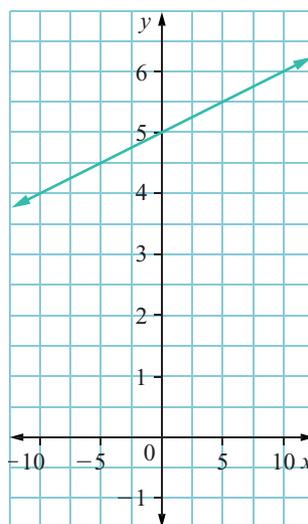
**h**  $y = 4 - 2x$

$x$	-3	0	3
$y$	10	4	-2



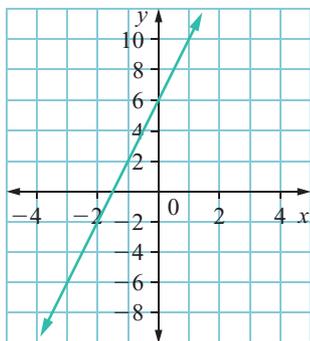
**3** One possible answer is:

$x$	-10	0	10
$y$	4	5	6

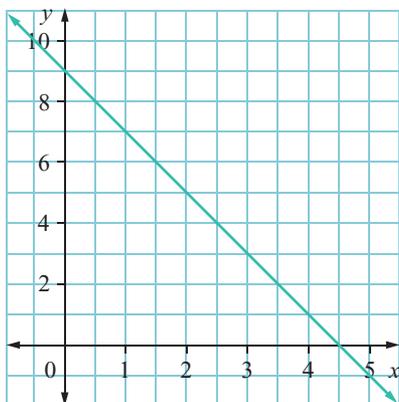


- 4** **a** 4, 6      **b** -2, 9      **c**  $\frac{3}{4}, -8$       **d** 1, -2  
**e** -1, 4      **f** -1.2, -2.8      **g** 5.4, 6.5      **h**  $-\frac{3}{8}, 4\frac{1}{2}$   
**i**  $\frac{1}{2}, 7$       **j**  $\frac{1}{6}, 1\frac{2}{3}$       **k**  $-\frac{1}{3}, -2$       **l**  $\frac{1}{2}, 2$   
**m** -4, 8      **n**  $-1\frac{1}{2}, -2$       **o** 2, -5
- 5** **a**  $y = 7x - 1$       **b**  $y = -2x + 5$   
**c**  $y = \frac{3}{7}x - 2$       **d**  $y = x + 4$   
**e**  $y = -x + 2.7$       **f**  $y = 0.2x + 3$   
**g**  $y = -1.4x - 0.3$       **h**  $y = \frac{5}{6}x + \frac{7}{12}$   
**i**  $y = -0.56x - 1.24$       **j**  $y = \frac{7}{5}x + 24$
- 6** **a** **i** 2      **ii** 1      **iii**  $y = 2x + 1$   
**b** **i**  $-\frac{1}{2}$       **ii** 1      **iii**  $y = -\frac{1}{2}x + 1$   
**c** **i** 1      **ii** -1      **iii**  $y = x - 1$   
**d** **i** -2      **ii** 3      **iii**  $y = -2x + 3$   
**e** **i**  $\frac{2}{3}$       **ii** 2      **iii**  $y = \frac{2}{3}x + 2$   
**f** **i** 0.6      **ii** -1.2      **iii**  $y = 0.6x - 1.2$

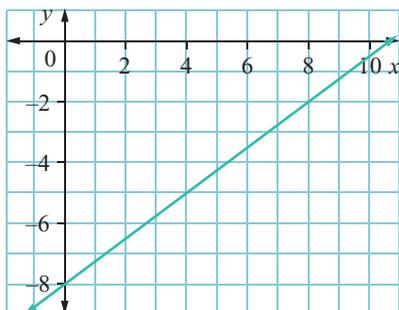
7 a  $y = 4x + 6$



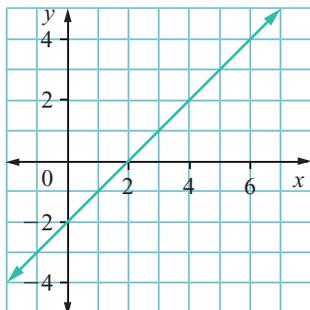
b  $y = -2x + 9$



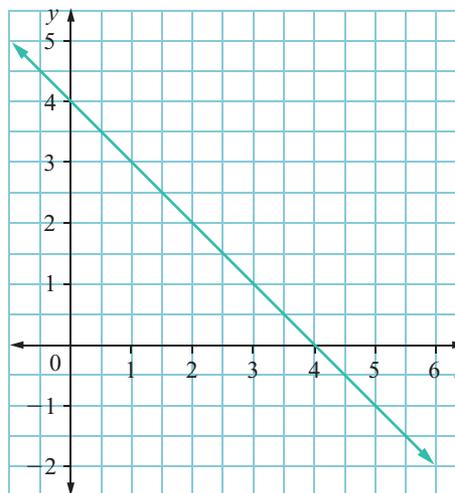
c  $y = \frac{3}{4}x - 8$



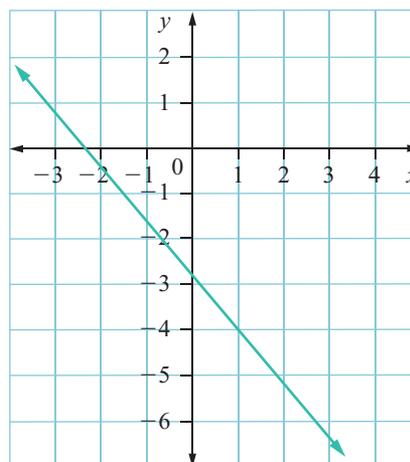
d  $y = x - 2$



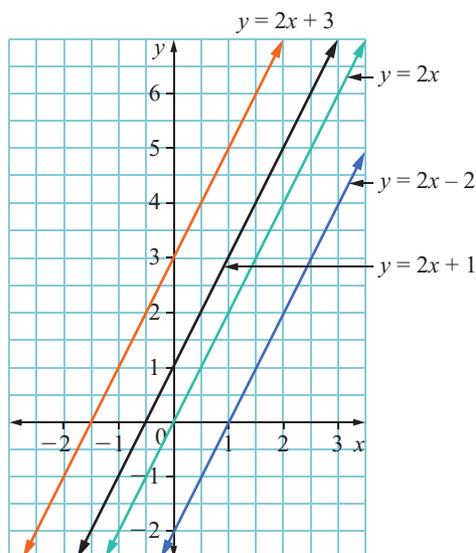
e  $y = -x + 4$



f  $y = -1.2x - 2.8$



8 a



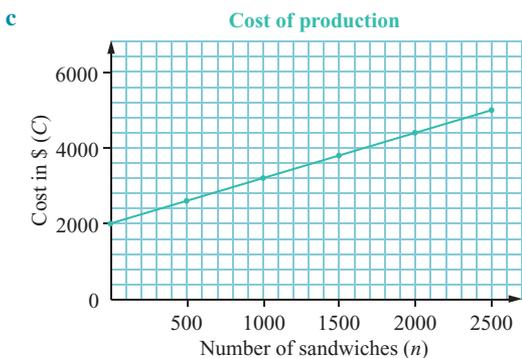
b The four lines are parallel; they have the same gradient but different y-intercepts.

c The constant term added to  $2x$  moves the line for  $y = 2x$  up or down. For example, adding 3 moves the line for  $y = 2x$  up 3 units. The constant term is also the y-intercept for each line.



5 a  $C = 1.2n + 2000$

<b>b</b>	$n$	0	500	1000	1500	2000	2500
	$C$	2000	2600	3200	3800	4400	5000

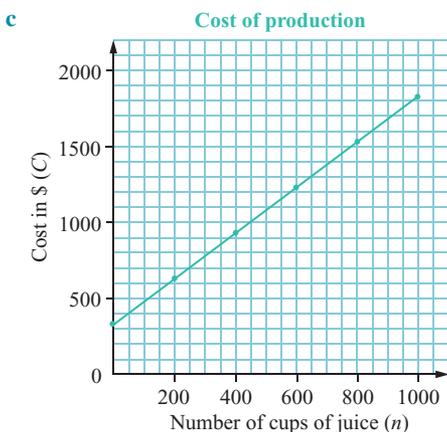


**d** Gradient is 1.2; it is the cost per sandwich (\$1.20).

**e** 1250 sandwiches

6 a  $C = 1.5n + 325$

<b>b</b>	$n$	0	200	400	600	800	1000
	$C$	325	625	925	1225	1525	1825



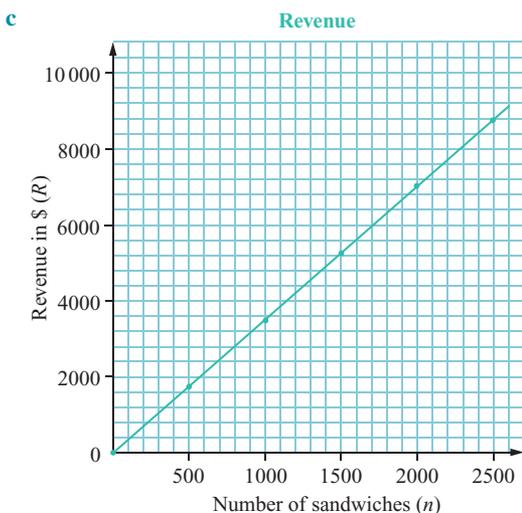
**d** y-intercept is 325, the weekly cost (\$325).

**e** Gradient is 1.5; it is the cost per cup of juice (\$1.50).

**f** 450 cups

7 a  $R = 3.5n$

<b>b</b>	$n$	0	500	1000	1500	2000	2500
	$R$	0	1750	3500	5250	7000	8750

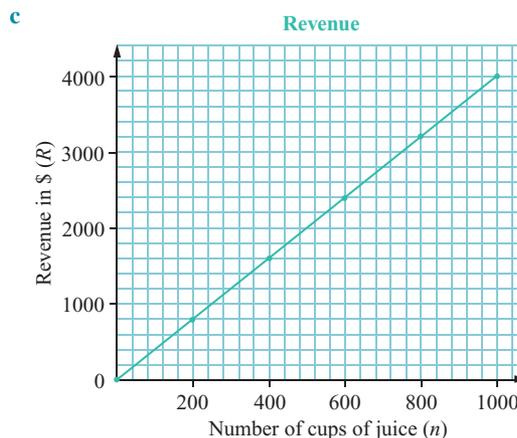


**d** 800 sandwiches

**e** Gradient is 3.5; it is the price of each sandwich (\$3.50).

8 a  $R = 4n$

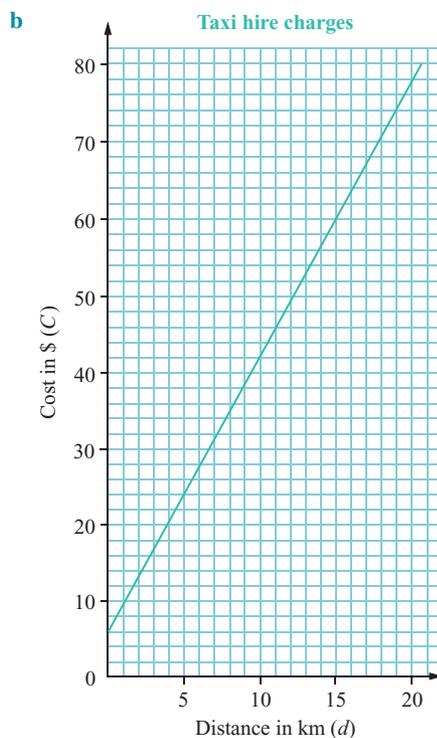
<b>b</b>	$n$	0	200	400	600	800	1000
	$R$	0	800	1600	2400	3200	4000



**d** 750 cups

**e** Gradient is 4; it is the price of each cup of juice (\$4).

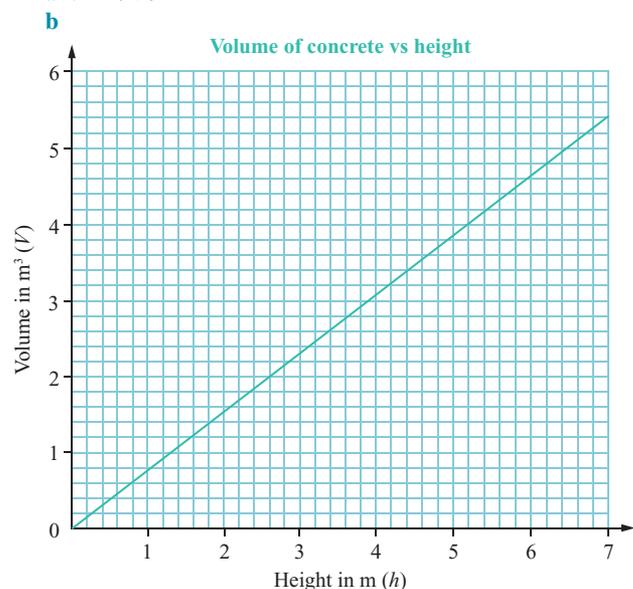
9 a  $C = 3.6d + 6$



**c** \$42

**d** About 7 km

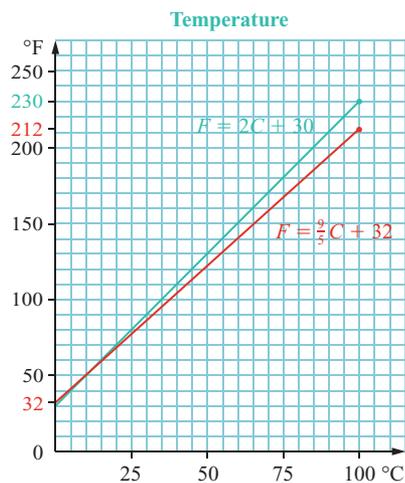
10 a  $V = 0.78h$



c About  $3 \text{ m}^3$

d About 3 m

11 a, b



c Rule of thumb is mostly a reasonable approximation for low temperatures.

**EXERCISE 4C** IDENTIFYING SOLUTIONS TO SIMULTANEOUS LINEAR EQUATIONS

- 1 a  $x = 2, y = 4$   
 b  $x = 3, y = 5$   
 c  $x = 7, y = 16$

- 2 a Yes      b Yes      c No  
 d Yes      e No      f No

- 3 a i (4, 3)      ii Solution is  $x = 4, y = 3$ .  
 b i (2, -4)      ii Solution is  $x = 2, y = -4$ .  
 c i (-5, -2)      ii Solution is  $x = -5, y = -2$ .  
 d i (-2, 3)      ii Solution is  $x = -2, y = 3$ .

- 5 a 15 cards  
 b \$60  
 c \$100  
 6 a 100 m  
 b 600 m  
 c 250 m  
 d 6 minutes  
 e Carla 300 m, Leon 450 m  
 7 a (2, 2), solution is  $x = 2, y = 2$ .  
 b Solution is  $x = 0.4, y = 2.8$ .  
 c No, lines A and B are parallel and do not intersect.

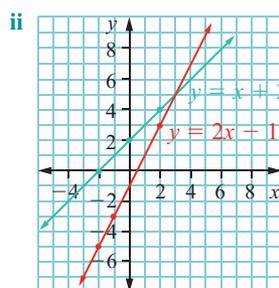
**EXERCISE 4D** SOLVING SIMULTANEOUS LINEAR EQUATIONS GRAPHICALLY

1 a i  $y = x + 2$

x	-2	0	2
y	0	2	4

$y = 2x - 1$

x	-2	0	2
y	-5	-1	3



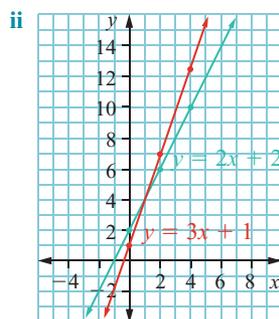
- iii (3, 5)  
 iv Solution is  $x = 3, y = 5$ .

b i  $y = 2x + 2$

x	0	2	4
y	2	6	10

$y = 3x + 1$

x	0	2	4
y	1	7	13

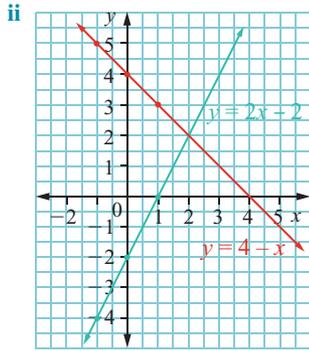


- iii (1, 4)  
 iv Solution is  $x = 1, y = 4$ .  
 c i  $y = 2x - 2$

x	-1	0	1
y	-4	-2	0

$x + y = 4$

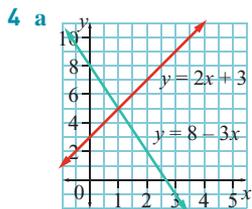
x	-1	0	1
y	5	4	3



iii (2, 2)

iv Solution is  $x = 2, y = 2$ .

- 2 a Solution is  $x = 4, y = 2$ .  
 b Solution is  $x = 2, y = -1$ .  
 c Solution is  $x = -1, y = -2$ .  
 d Solution is  $x = 1, y = 3$ .  
 e Solution is  $x = 4, y = 2$ .  
 f Solution is  $x = \frac{3}{5}$  or 0.6,  $y = 1\frac{3}{5}$  or 1.6.
- 3 a Solution is  $x = 2, y = -1$ .  
 b Solution is  $x = -1, y = -2$ .  
 c Solution is  $x = 1, y = 2$ .  
 d Solution is  $x = 3, y = -4$ .  
 e Solution is  $x = 1, y = -5$ .  
 f Solution is  $x = 3, y = -2$ .  
 g Solution is  $x = -1, y = -2$ .  
 h Solution is  $x = -3, y = 2$ .  
 i Solution is  $x = -1, y = 2$ .  
 j Solution is  $x = 3, y = 7$ .



b (1, 5)

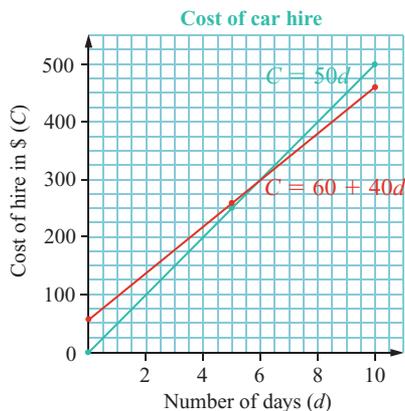
c Solution is  $x = 1, y = 5$ .

6 a  $C = 50d$

$d$	0	5	10
$C$	0	250	500

$$C = 60 + 40d$$

$d$	0	5	10
$C$	60	260	460



b  $d = 6, C = 300$ ; the cost of hiring a car for 6 days is the same for both companies, cost is \$300.

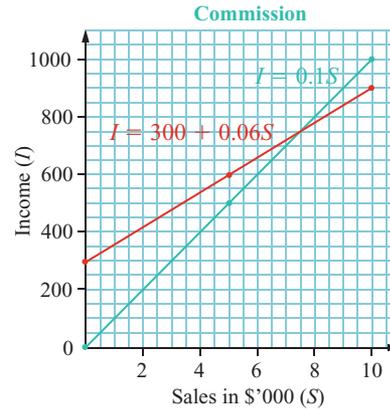
c 6 days

7 a  $I = 0.1S$

$S$	0	5000	10000
$I$	0	500	1000

$$I = 300 + 0.06S$$

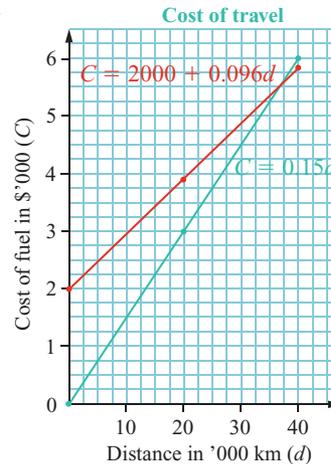
$S$	0	5000	10000
$I$	300	600	900



b  $S = 7500, I = 750$ ; when sales are \$7500, the two payment options give the same income of \$750

c Sales greater than \$7500.

8 a



b Petrol model \$1500, diesel model \$2960; petrol model is cheaper by \$1460.

c Petrol model \$4500, diesel model \$4880; petrol model is cheaper by \$380.

d About 37000 km

e Diesel model is cheaper when the distance travelled is more than 37000 km.

9 a  $2l + 2w = 50$

b  $l = w + 5$

c Solution is  $w = 10, l = 15$ . Dimensions of the photo are 15 cm by 10 cm.

10 a If  $d$  is the cost of a large drink and  $c$  is the cost of an ice cream,  $d = c + 2$  and  $5d + 7c = 70$ . All costs are in dollars.

b Solution is  $c = 5, d = 7$ ; large drink costs \$7 and an ice cream costs \$5.

- 11 a** If  $d$  is the distance in km from Town A after  $t$  hours,  $d = 20t$  (for Tom) and  $d = 15 - 5t$  (for Julia).  
**b** They meet after travelling 0.6 hours or 36 minutes.  
**c** Tom has travelled 12 km and Julia has travelled 3 km.
- 12 a** No solution as the two lines are parallel and do not intersect.  
**b** There is an infinite number of solutions, as the graphs of the two equations lie on top of one another; i.e. they are the same line.

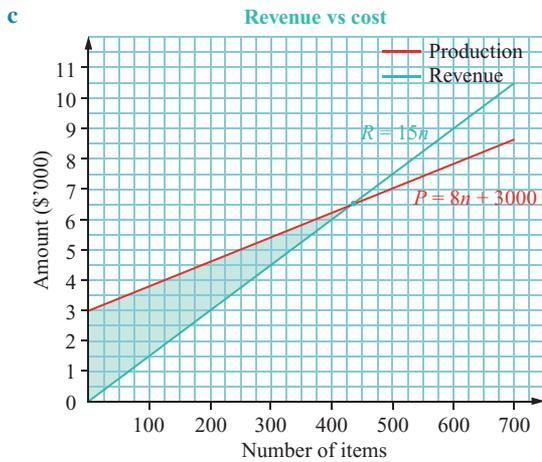
**EXERCISE 4E** BREAK-EVEN ANALYSIS

- 1 a** 80                      **b** \$320                      **c** \$50  
**2 a** About 65            **b** About \$390            **c** About \$85  
**3 a** About 60  
**b** About \$210  
**c** About \$85  
**d** Revenue – cost  $\approx -50$ . There is a loss of \$50. Jenny makes a loss not a profit.
- 4 a**  $P = 8n + 3000$

Number of items ( $n$ )	0	100	300	500	700
Production cost ( $P$ , \$)	3000	3800	5400	7000	8600

**b**  $R = 15n$

Number of items ( $n$ )	0	100	300	500	700
Revenue ( $R$ , \$)	0	1500	4500	7500	10500

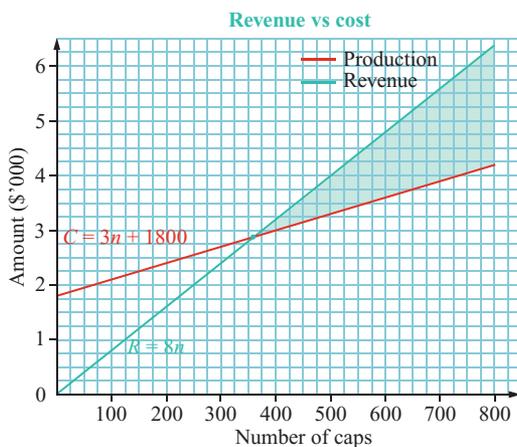


**d** About (430, 6500); actual coordinates are close to (428.57, 6428.57)

**e** About 430

**f** About \$6500

**5 a, b**

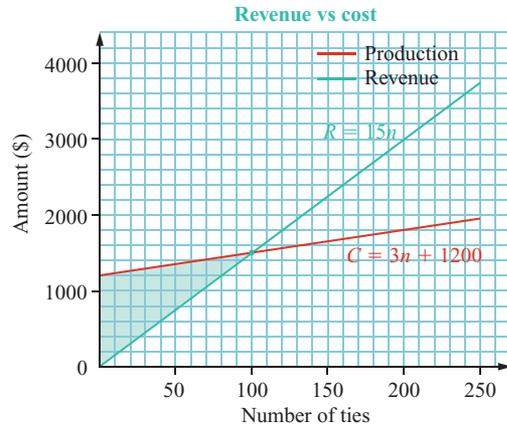


**c** About (360, 2900); actual coordinates are (360, 2880).

**d** 360

**e** About \$2900

**6 a**

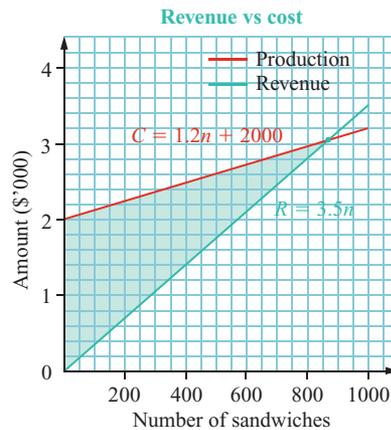


**b** (100, 1500)

**c** 100    **d** \$1500

**7 a**  $C = 1.2n + 2000$  and  $R = 3.5n$

**b**



**c** About (870, 3050); actual coordinates are close to (869.57, 3043.48).

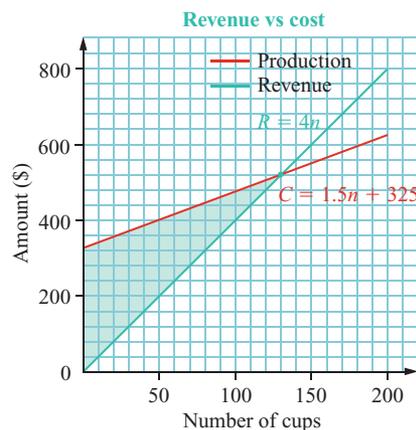
**d** About 870

**e** About \$3050

**f** Need to make 870 sandwiches to break even, therefore must always make a loss. Something needs to change to become profitable.

**g** See green shading on graph for part **b**. This indicates where a loss is made; i.e. The revenue is less than the cost for making  $n$  items.

**8 a**



**b** (130, 520)

**c** 130    **d** \$520

**e** Need to make 130 cups to break even, therefore operating at a loss. Need to increase price or production numbers or decrease costs to make a profit.

**f** See green shading on graph for part **a**.

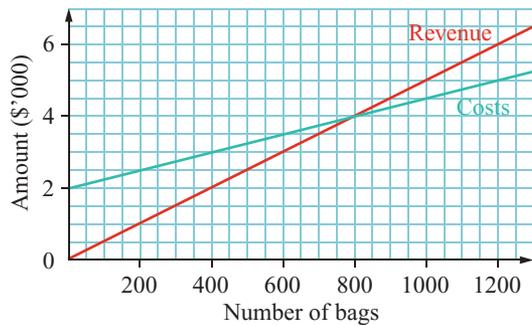
- 9 a See green shading on graph for question 4c.  
 b See green shading on graph for question 5a, b.  
 c See green shading on graph for question 6a.

10 a 700

- b i Loss of about \$1700      ii Profit of about \$800  
 c i 1050                              ii 350

11 a  $C = 2.5n + 2000$ ,  $R = 5n$  where  $C$  is the production cost in dollars and  $R$  is the revenue for  $n$  bags. Both  $C$  and  $R$  are in dollars.

b **Daily production costs and revenue**

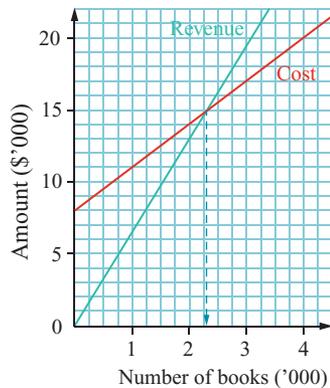


c 800 bags

- d i Loss of \$1750                      ii Profit of \$500  
 e i 1200 bags                              ii 400 bags

12 a  $C = 3n + 8000$ ,  $R = 6.5n$  where  $C$  is the production cost and  $R$  is the revenue for  $n$  books. Both  $C$  and  $R$  are in dollars.

b **Revenue vs cost**



c About 2300 books

- d i Loss of about \$4500              ii Profit of about \$750  
 iii Profit of about \$3200

e i Just under 1500 books ( $\approx 1430$ )  
 ii About 3000 books

13 Region B

**REVIEW** MULTIPLE-CHOICE QUESTIONS

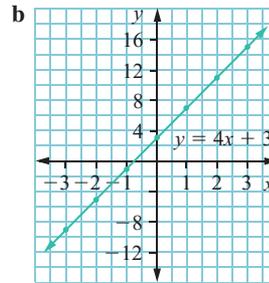
- 1 B      2 C      3 A      4 A      5 A  
 6 C      7 D      8 B      9 B      10 C

**REVIEW** SET 1

- 1 a  $m = \frac{3}{8}$ , y-intercept =  $-4$   
 b  $y = 0.08x + 3.2$

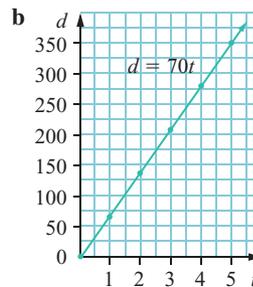
2 a

x	-3	-2	-1	0	1	2	3
y	-9	-5	-1	3	7	11	15



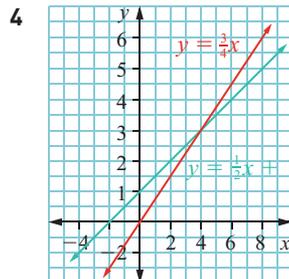
3 a

t	1	2	3	4	5
d	70	140	210	280	350



c Just under 250 km (245 km)

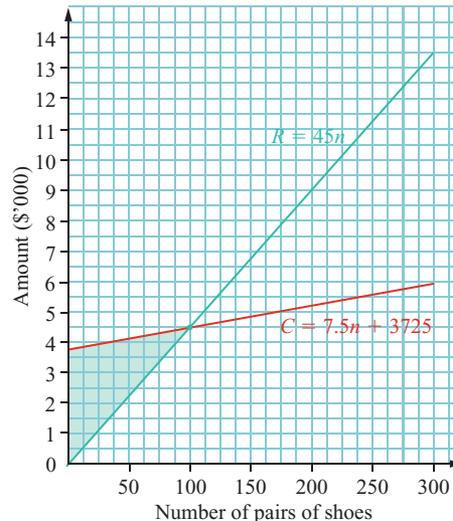
d About 2 h 50 min



Solution is  $x = 4$ ,  $y = 3$ .

5 Solution is  $x = -2$ ,  $y = 2$ .

6 a **Revenue vs cost**



b About (100, \$4500); actual coordinates are close to (99.3, 4470)

c About 100 pairs

d About \$4500

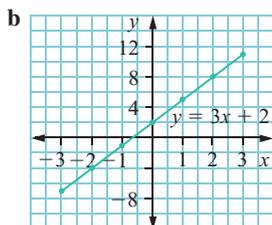
e See shading on graph in part a.

**REVIEW SET 2**

1 Gradient =  $\frac{5}{4}$

2 a

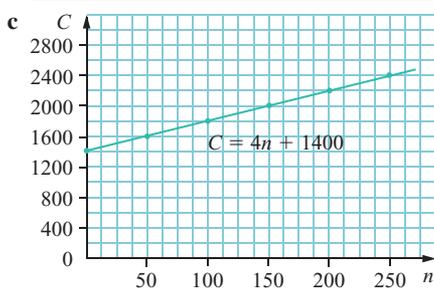
x	-3	-2	-1	0	1	2	3
y	-7	-4	-1	2	5	8	11



- 3 a  $C$  is the cost per day to make the ties.  
4 is the cost per tie.  
 $n$  is the number of ties made.  
1400 is the fixed cost.

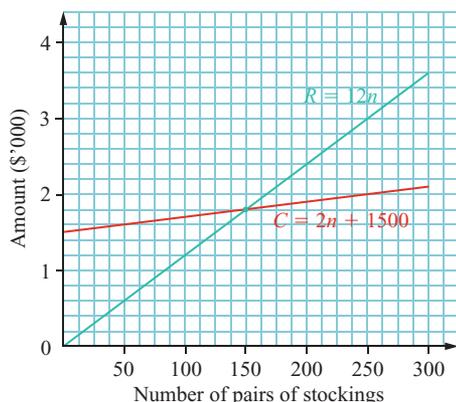
b

$n$	0	50	100	150	200	250
$C$	1400	1600	1800	2000	2200	2400



- d 25 ties  
4 40000 km  
5 a Solution is  $x = 3, y = 3$ .  
b Solution is  $x = -3, y = 2$ .

6 a **Revenue vs cost**

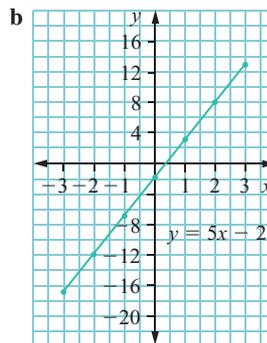


- b (150, 1800)    c 150 pairs    d \$1800

**REVIEW SET 3**

1 a

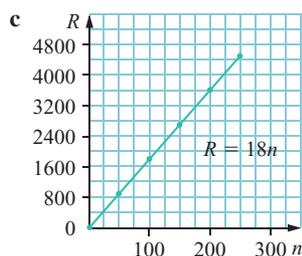
x	-3	-2	-1	0	1	2	3
y	-17	-12	-7	-2	3	8	13



- 2 a  $R$  is the revenue,  $n$  is the number of scarves sold and 18 is the price of each scarf.

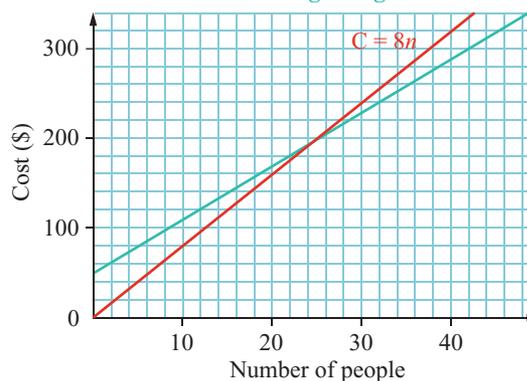
b

$n$	0	50	100	150	200	250
$R$	0	900	1800	2700	3600	4500



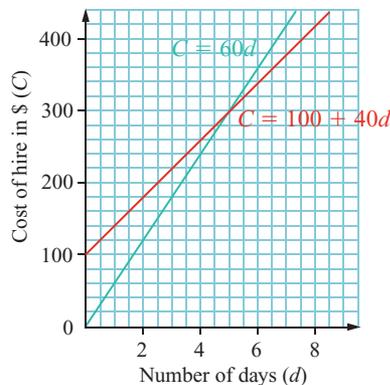
- d 115 scarves  
3 a About \$260    b 42  
c Gradient = 6, this is the cost per person.  
d \$50, this is the set up cost of \$50.  
e

**Catering charges**



- f 25 people at a cost of \$200  
4 Solution is  $x = 2, y = -3$ .

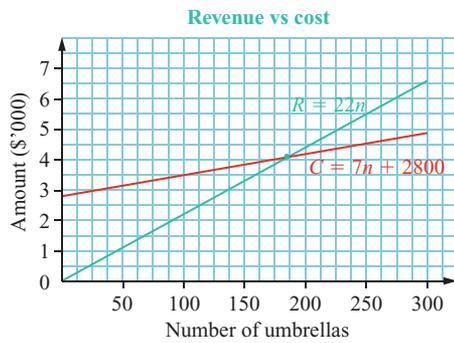
5 a **Cost of car hire**



- b  $d = 5, C = 300$     c 5 days

- 6 a  $C = 7n + 2800$  and  $R = 22n$ , where  $C$  is the production cost and  $R$  is the revenue for  $n$  umbrellas.

b



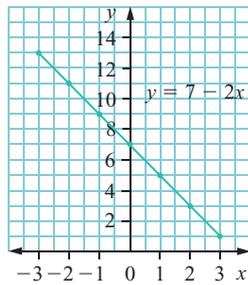
- c About (185, 4100); actual coordinates close to (186.67, 4106.67)  
d About 185 e About \$4100

### REVIEW SET 4

1 a

$x$	-3	-2	-1	0	1	2	3
$y$	13	11	9	7	5	3	1

b

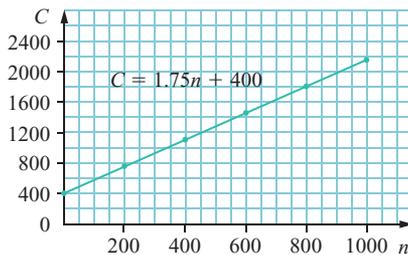


- 2 a  $C = 1.75n + 400$

b

$n$	0	200	400	600	800	1000
$C$	400	750	1100	1450	1800	2150

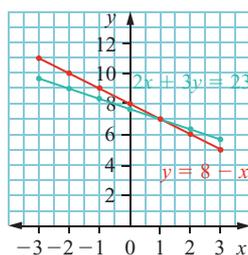
c



- d About 340 cups

- 3 a About 425; actual value close to 429  
b About \$6400; actual value close to \$6430

4 a



- b (1, 7)

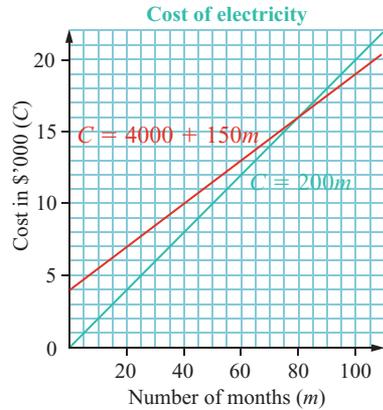
- c Solution is  $x = 1, y = 7$ .

- 5 a  $C = 200m$

$m$	0	50	100
$C$	0	10000	20000

$$C = 4000 + 150m$$

$m$	0	50	100
$C$	4000	11500	19000



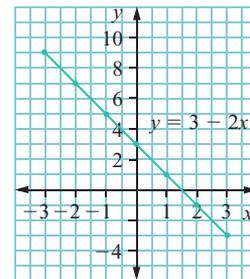
- b Solution is  $m = 80, C = 16000$ .

- c 80 months

### REVIEW PRACTICE EXAMINATION QUESTION

1 a i

$x$	-3	-2	-1	0	1	2	3
$y$	9	7	5	3	1	-1	-3



- ii Gradient is  $-2$ .

- iii  $y$ -intercept is 3.

- b i About \$63000

- ii No, revenue is less than cost.

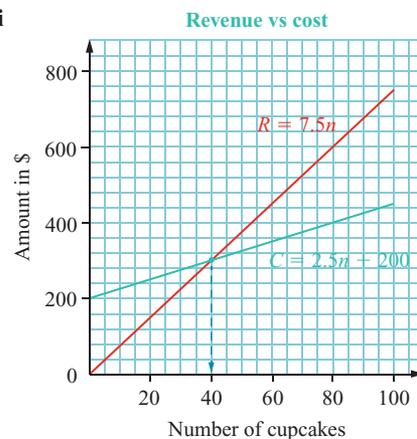
- iii 2000 iv 2400

- v \$50000 vi \$12.50

- c i  $C = 2.5n + 200$

- ii  $R = 7.5n$

iii



- iv 40

- v \$300

# CHAPTER 5 RIGHT-ANGLED TRIANGLES

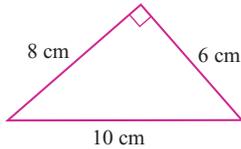
## ARE YOU READY?

- 1 A    2 B    3 D    4 B    5 C    6 B  
7 A    8 B    9 A    10 C    11 D

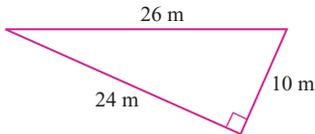
## EXERCISE 5A PYTHAGORAS' THEOREM

- 1 a 7.8 cm    b 13.9 cm    c 23.4 m  
d 7.4 km    e 97.7 mm    f 182.1 cm  
2 a 8.9 cm    b 15.3 cm    c 2.7 m  
d 37.8 m    e 70.5 mm    f 14.8 m  
3 a i Yes

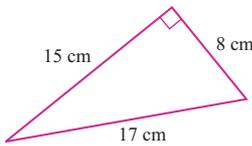
ii



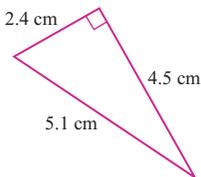
- b i No  
c i No  
d i Yes



- ii  
e i Yes



- f i Yes  
ii



- 4 Yes ( $4187^2 = 2700^2 + 3200^2$ )  
5 No ( $3000^2 \neq 2500^2 + 850^2$ )  
6 a 7.2 cm    b 11.3 cm  
7 1.1 m  
8 Yes  
9 3.0 m  
10 21.4 m  
11 97.1 km  
12 a 379 m    b 263 m    c 178 m  
d 218 m    e 252 m    f 451 m  
g 218 m    h 297 m    i 147 m  
13 a 165 m    b 389 m    c 169 m

## EXERCISE 5B TRIGONOMETRIC RATIOS

- 1 a Opposite = 35 mm, hypotenuse = 70 mm  
b  $\sin 30^\circ = 0.50$ ,  $\cos 30^\circ = 0.86$ ,  $\tan 30^\circ = 0.58$   
2 c  $\sin 65^\circ = 0.91$ ,  $\cos 65^\circ = 0.42$ ,  $\tan 65^\circ = 2.14$   
3 c  $\sin 25^\circ = 0.42$ ,  $\cos 25^\circ = 0.91$ ,  $\tan 25^\circ = 0.47$   
4 a  $\sin x = 0.80$ ,  $\cos x = 0.60$ ,  $\tan x = 1.33$   
b  $\sin x = 0.38$ ,  $\cos x = 0.92$ ,  $\tan x = 0.42$   
c  $\sin x = 0.47$ ,  $\cos x = 0.88$ ,  $\tan x = 0.53$   
d  $\sin x = 0.98$ ,  $\cos x = 0.22$ ,  $\tan x = 4.44$

- e  $\sin x = 0.69$ ,  $\cos x = 0.72$ ,  $\tan x = 0.95$   
f  $\sin x = 0.28$ ,  $\cos x = 0.96$ ,  $\tan x = 0.29$   
5 a 0.7314    b 0.6157    c 2.2460    d 0.9755  
e 0.2438    f 0.2096    g 3.7062    h 0.6057  
i 0.6915    j 0.4376    k 0.6856    l 0.4861  
m 3.0208    n 0.8699    o 1.4469    p 0.9835  
6 a  $\sin x$     b None    c  $\tan x$     d  $\cos x$   
7 a  $\cos x$     b None    c None    d  $\tan x$   
8 a None    b  $\tan x$     c  $\sin x$     d  $\cos x$   
9 a Increases    b Decreases    c Increases

- 10 Dividing both the numerator and denominator of the tangent ratio,  $\tan x = \frac{\text{opposite}}{\text{adjacent}}$ , by the hypotenuse gives  $\sin x$  on the numerator of the fraction and  $\cos x$  on the denominator of the fraction.

## EXERCISE 5C USING TRIGONOMETRY TO CALCULATE ANGLES

- 1 a  $44^\circ$     b  $66^\circ$     c  $27^\circ$     d  $43^\circ$   
e  $16^\circ$     f  $51^\circ$     g  $74^\circ$     h  $25^\circ$   
i  $27^\circ$     j  $31^\circ$     k  $82^\circ$     l  $35^\circ$   
2 a  $49^\circ$     b  $51^\circ$     c  $23^\circ$     d  $58^\circ$   
e  $69^\circ$     f  $56^\circ$     g  $29^\circ$     h  $64^\circ$   
i  $26^\circ$     j  $48^\circ$     k  $38^\circ$     l  $52^\circ$   
3 a  $44^\circ$     b  $43^\circ$     c  $40^\circ$     d  $46^\circ$   
e  $60^\circ$     f  $25^\circ$   
4 a  $35^\circ 22'$     b  $81^\circ 48'$     c  $28^\circ 48'$     d  $26^\circ 21'$   
e  $19^\circ 41'$     f  $58^\circ 21'$     g  $47^\circ 45'$     h  $24^\circ 2'$   
i  $2^\circ 51'$   
5 a  $39^\circ 10'$     b  $47^\circ 1'$     c  $20^\circ 33'$     d  $52^\circ 8'$   
e  $61^\circ 3'$     f  $62^\circ 58'$     g  $56^\circ 21'$     h  $55^\circ 22'$   
i  $39^\circ 15'$   
6 a  $49^\circ$     b  $63^\circ$     c  $53^\circ$     d  $34^\circ$   
e  $50^\circ$     f  $35^\circ$     g  $69^\circ$     h  $60^\circ$   
i  $65^\circ$   
7 a  $25^\circ 23'$     b  $20^\circ 33'$     c  $62^\circ 58'$     d  $28^\circ 18'$   
e  $58^\circ 28'$     f  $46^\circ 47'$     g  $70^\circ 15'$     h  $59^\circ 56'$   
i  $65^\circ 41'$   
8  $69^\circ 54'$   
9  $49^\circ 18'$   
10  $25^\circ 46'$   
11  $27^\circ 55'$

## EXERCISE 5D USING TRIGONOMETRY TO CALCULATE SIDE LENGTHS

- 1 a 21.0 m    b 18.5 m    c 65.7 m    d 62.2 cm  
e 16.0 m    f 15.8 mm    g 56.2 cm    h 109.3 m  
i 34.3 m  
2 a 22.7 m    b 23.9 m    c 70.4 m    d 74.3 cm  
e 16.6 m    f 21.7 mm    g 74.0 cm    h 82.3 m  
i 34.0 m  
3 a 66.7 m    b 2.2 m    c 71.5 m    d 276.8 cm  
e 39.3 m    f 13.8 m    g 214.0 cm    h 10.7 m  
i 147.8 mm

4 a

$x$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$85^\circ$	$90^\circ$
sin	0	0.5	0.707	0.866	0.996	1
cos	1	0.866	0.707	0.5	0.087	0
tan	0	0.577	1	1.732	11.430	undefined

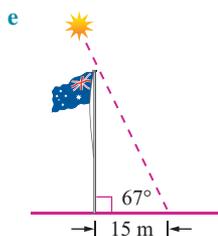
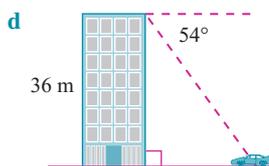
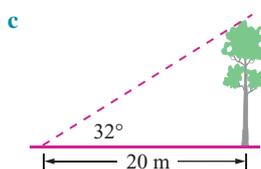
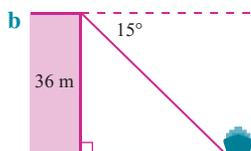
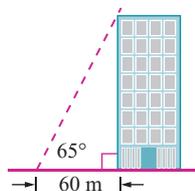
- b  $\sin x$  increases from 0 to 1    c  $0 \leq \sin x \leq 1$   
d  $\cos x$  decreases from 1 to 0    e  $0 \leq \cos x \leq 1$   
f  $\tan x$  increases indefinitely from 0  
g For  $0 \leq x \leq 45^\circ$ ,  $0 \leq \tan x \leq 1$ .  
For  $x > 45^\circ$ ,  $\tan x > 1$ .

- 5 a i No ii Yes  
 b i Yes ii No  
 c i No ii Yes  
 d i No ii Yes  
 e i No ii Yes  
 f i Yes ii No

- 6 a 5.7 m b 2.0 m  
 7 11.7 m  
 8 38.2 m  
 9 14 371 m  
 10 5.0 m  
 11 275 cm or 2.75 m

### EXERCISE 5E ANGLES OF ELEVATION AND DEPRESSION

1 a



- 2 17.1 m  
 3 64.3 m  
 4 18.0 m  
 5 234 m  
 6 148 m  
 7 11 196 m  
 8 60°39'  
 9 17°13'  
 10 19°48'  
 11 a 16.20 m  
 b 17.84 m

### EXERCISE 5F BEARINGS AND NAVIGATIONAL METHODS

- 1 a NE b E c SW  
 2 a N27°W b S51°W c N31°E

- 3 a 175°T b 215°T c 046°T  
 4 a i S44°E ii 136°T  
 b i S59°W ii 239°T  
 c i N24°W ii 336°T  
 d i S69°W ii 249°T  
 e i N62°W ii 298°T  
 f i S34°E ii 146°T  
 g i N8°E ii 008°T  
 h i S19°W ii 199°T  
 i i N77°W ii 283°T

- 5 a 303°T b 227°T c 051°T d 015°T  
 6 33.9 km  
 7 70.5 km  
 8 2.5 km  
 9 50.4 km  
 10 151°T  
 11 058°7'T  
 12 N69°W  
 13 60.2 km west, 18.4 km south  
 14 62.3 km east, 46.9 km south  
 15 S54°W  
 16 N53°W, 992 km  
 17 a 328°T b 148°T  
 18 a 360 km b 206 km c 2 h 28 min  
 19 a i 5.6 km ii 3.2 km iii 8.8 km  
 b i 2.1 km ii 3.8 km iii 5.9 km  
 c i 10.6 km ii 056°10'T  
 20 a i 6.2 km ii 10.5 km iii 4.3 km  
 b i 7.4 km ii 7.9 km iii 15.3 km  
 c i 15.9 km ii 074°18'T

### REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 B 2 B 3 D 4 B 5 C  
 6 A 7 B 8 B 9 C 10 B  
 11 C 12 A 13 D 14 A

### REVIEW SET 1

- 1 a 9.5 m b 6.2 m  
 2 a XZ b ZY  
 3 a 0.829 b 0.951 c 3.732  
 4 a 18° b 65° c 48°  
 5 a 35° b 34°  
 6 a 14.5 cm b 48.5 mm  
 7 a 16.2 cm b 10.0 m  
 8 a 41 km b S30°E, 150°T

### REVIEW SET 2

- 1 a 5.9 m b 3.3 m  
 2 C  
 3 a 67° b 82° c 27°  
 4 C  
 5 a 59.1 m b 78.3 cm  
 6 80 m  
 7 a 65 km b S27°W, 207°T

### REVIEW SET 3

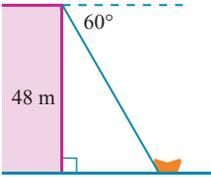
- 1 B  
 2 B  
 3 a 0.8090 b 0.9511 c 4.0108

- 4 a  $37^\circ$       b  $29^\circ$       c  $23^\circ$   
 5 5.1 m  
 6  $65^\circ$   
 7 506 m  
 8 a 227 km      b  $N52^\circ W, 308^\circ T$

**REVIEW** SET 4

- 1 a  $\sin x = \frac{8}{17}$       b  $\cos x = \frac{15}{17}$       c  $\tan x = \frac{8}{15}$   
 2 a  $31^\circ$       b  $39^\circ$       c  $54^\circ$   
 3 A  
 4  $12^\circ$   
 5 3217 m  
 6 40 m  
 7 a 51 km      b  $N55^\circ E, 055^\circ T$

**REVIEW** PRACTICE EXAMINATION QUESTION

- a i  $56^\circ$   
 ii 22.4 m  
 b i 20.7 cm      ii 19.9 mm  
 c i   
 ii 27.7 m  
 d i  $S37^\circ W$   
 ii 148 km south

**CHAPTER 6** STATISTICAL ANALYSIS**ARE YOU READY?**

- 1 C      2 B      3 A      4 C      5 C  
 6 C      7 B      8 A      9 D      10 A  
 11 D      12 B

**EXERCISE 6A** STATISTICAL INVESTIGATIONS

- 1 a No      b Yes      c Yes      d No  
 e No      f Yes      g Yes      h Yes  
 2 a School children  
 b Home owners  
 c Anyone who has used the new treatment  
 d All professional sports  
 3 a Sample  
 b Sample  
 c Sample  
 d Sample  
 e Sample  
 f Census  
 g Sample  
 h Census  
 i Census  
 j Sample  
 k Census  
 l Sample  
 4 a Students from schools in inner-Sydney are more likely to come from wealthy families.  
 b The favourite music of people outside an Adele concert is more likely to be Adele.

- c Members of a local cycling club are likely to be in favour of more cycle lanes.  
 d Investment bankers will tend to be optimistic about the state of the economy.  
 e Teenagers at Comic-Con will probably favour particular genres of film.  
 5 The jobs that graduates with business degrees will be applying for will be different from those that graduates with science degrees will apply for, so their expected wages are likely to be different.  
 6 a This question is leading people to say that military expenditure should be cut.  
 b The wording of this question suggests that it would be unusual to support the proposal for the new railway.  
 c By stating the film that won in a recent poll, people are more likely to also name this film when asked this question.  
 d The wording of this question suggests that the new road laws are bad.  
 e By stating that soccer is the most played sport, people are more likely to answer soccer when asked this question.  
 7 Ryan's spreadsheet could be easily accessed by someone within his company, or by a hacker. The spreadsheet is unlocked, so if someone obtains the file they will have access to the personal information of the staff.

**EXERCISE 6B** ANALYSING AND COMPARING DATA

1 a

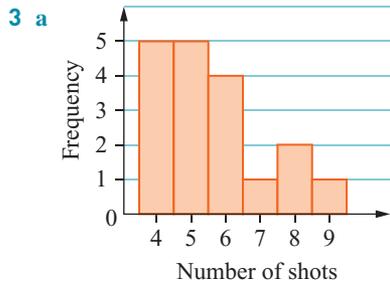
Number of pets	Frequency
0	7
1	7
2	8
3	3
4	3
5	1
6	0
7	0
8	1

- b 1.9  
 c 2  
 d 8

2 a

Number of cars	Frequency
3	1
4	2
5	4
6	6
7	4
8	4
9	4
10	3
11	1
12	1

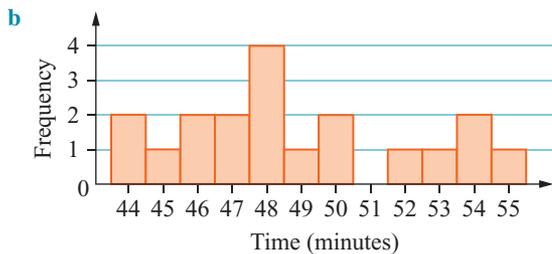
- b 7.2  
 c 6  
 d 9



- b** 5  
**c** 2  
**d** 101

**4 a**

Time (minutes)	Frequency
44	2
45	1
46	2
47	2
48	4
49	1
50	2
51	0
52	1
53	1
54	2
55	1
<b>Total</b>	<b>19</b>

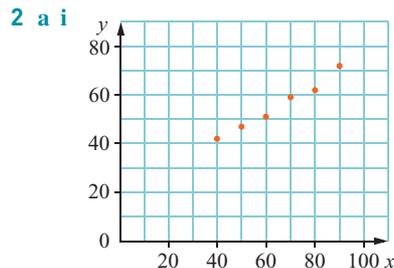


- c** 11 minutes  
**d** 6 minutes
- 5 a** 1–2 minutes  
**b** 6 minutes  
**c** 5–6 minutes
- 6 a** Male: 63.4, female: 63.4  
**b** Male: 65, female: 64  
**c** Males and females performed similarly in the test, within identical mean scores and almost identical median scores.
- 7 a** 20s: \$54 800, 50s: \$71 600  
**b** 20s: \$75 000, 50s: \$81 000  
**c** 20s: \$20 000, 50s: \$29 000  
**d** Most workers in their 50s receive a higher income than workers in their 20s, although there are a number of exceptions to this rule. Workers in their 50s have a greater spread (range and interquartile range) in their incomes than workers in their 20s.
- 8 a** Salesman B  
**b** Salesman A  
**c** Salesmen C and B both performed better than Salesman A over the 20-day period, selling a similar number of cars each (104 & 105). Salesman B was more consistent with his sales figures than the other two salesmen.

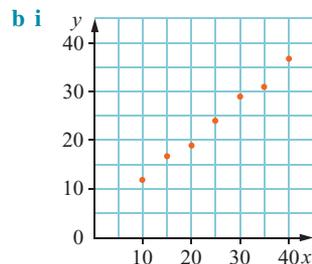
- 9 a** Company B  
**b** Company C  
**c** Companies A and C both steadily increased turnover over the 5-year period, although Company C's growth was much faster than company A's. Company B had the highest turnover in 2014, although its turnover decreased in subsequent years.
- 10 a** 14 million  
**b** 28 million  
**c** Both countries population steadily grew over the 50-year period, with Canada's population always being higher than Australia's.
- 11 a** 175–180 cm  
**b** 160–165 cm  
**c** Australian males tend to be taller than Australian females, although there is a significant crossover in height between males and females.
- 12** Samples only represent a certain proportion of the data, so any conclusions will always contain a degree of uncertainty. Larger samples will represent more of the data than smaller samples.

### EXERCISE 6C BIVARIATE SCATTERPLOTS

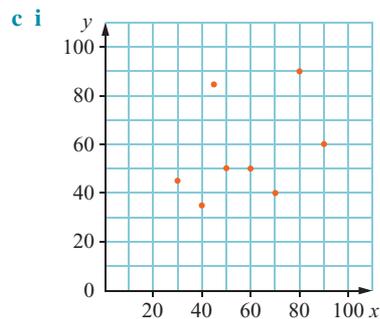
- 1 a** Yes      **b** No      **c** Yes



**ii** Linear relationship



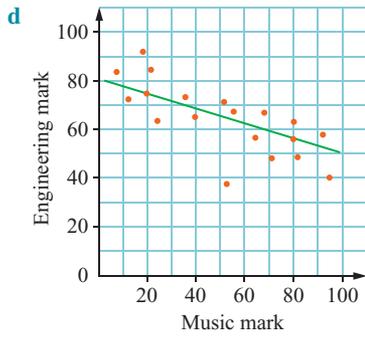
**ii** Linear relationship



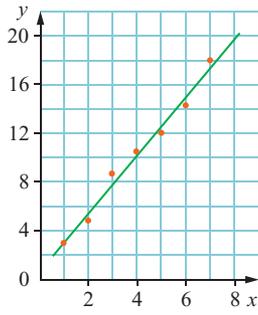
**ii** Non-linear relationship

- 3 a** Independent variable = height,  
 Dependent variable = time taken to run 10 km  
**b** Independent variable = number of colds per year,  
 Dependent variable = number of doctor visits per year

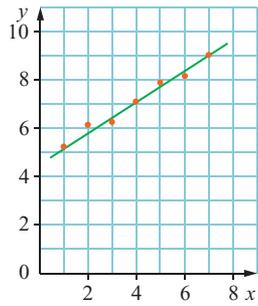




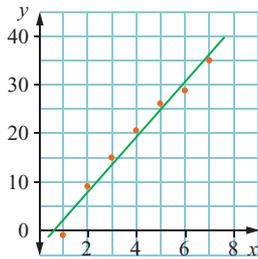
**2 a** Strong positive correlation



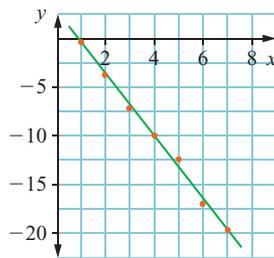
**b** Strong positive correlation



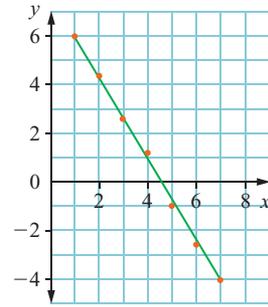
**c** Strong positive correlation



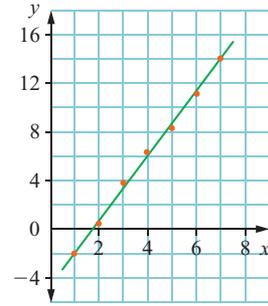
**d** Strong negative correlation



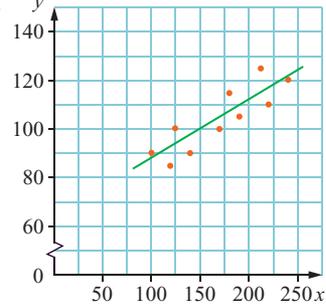
**e** Strong negative correlation



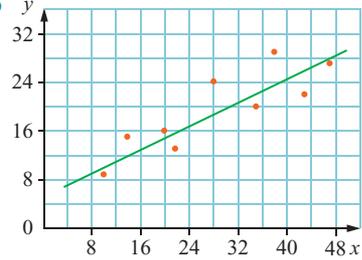
**f** Strong positive correlation



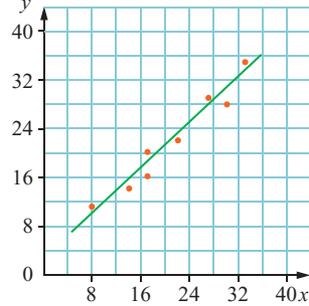
**3 a**

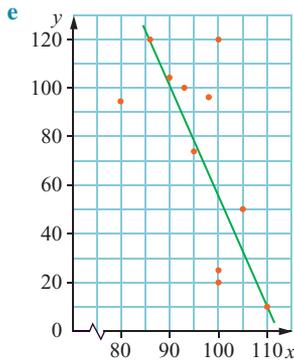
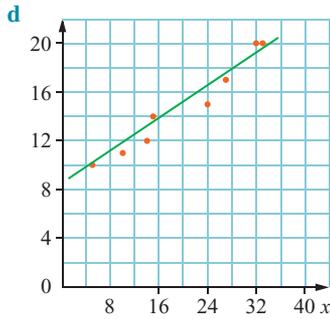


**b**



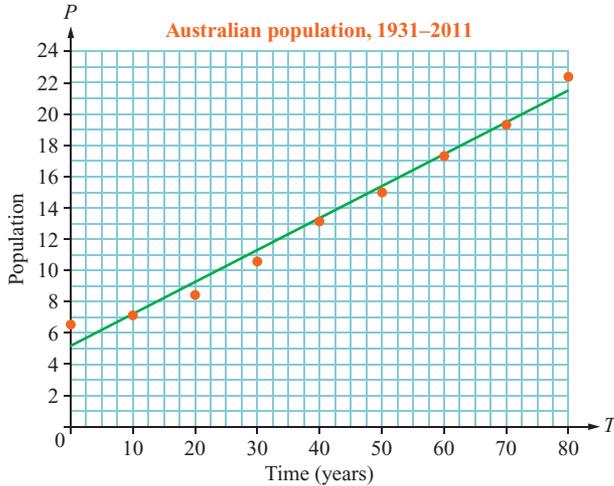
**c**





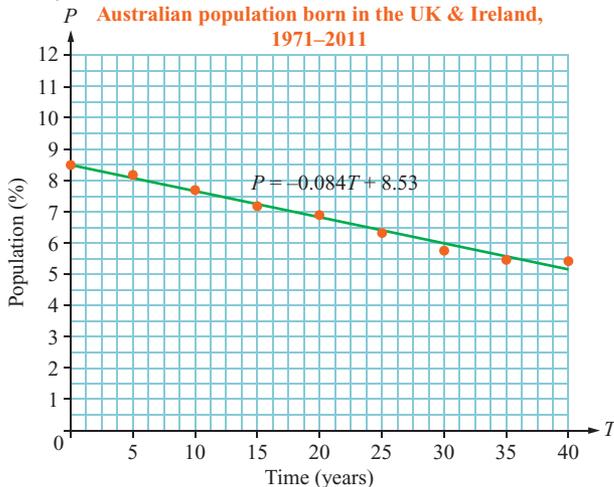
- 4 a** An approximately equal number of points should lie above and below the line of best fit.  
**b** The line of best fit is a straight line.

**5 a, b**

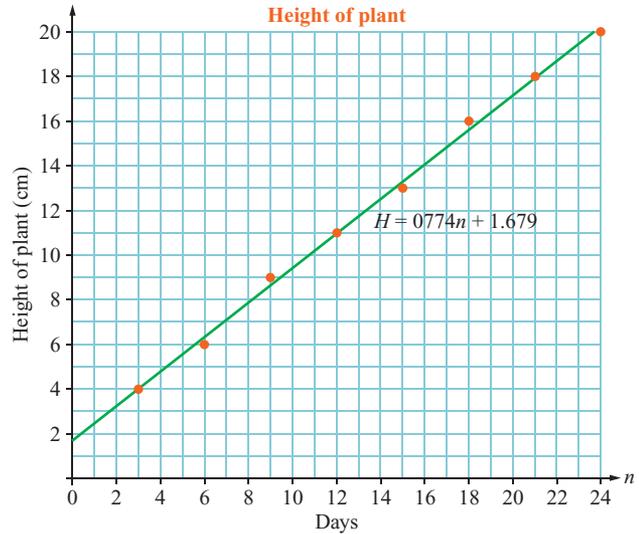


**c**  $P = 0.203T + 5.156$

**6 a, b**



**7 a, b**

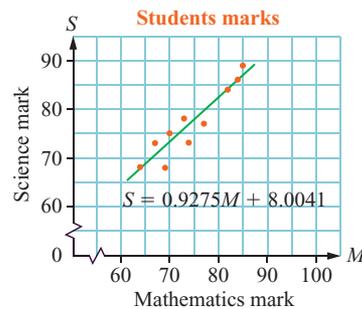


- 8** Lines of best fit are straight lines that represent the data values in a scatterplot, so they are less useful when the data does not appear in a straight line (i.e. is not linear).

**EXERCISE 6F MAKING PREDICTIONS**

- |                                  |                               |                                |
|----------------------------------|-------------------------------|--------------------------------|
| <b>1 a i</b> 165 cm              | <b>ii</b> 182 cm              | <b>iii</b> 187 cm              |
| <b>b i</b> 155 cm                | <b>ii</b> 168 cm              | <b>iii</b> 177 cm              |
| <b>2 a i</b> 408 cm <sup>3</sup> | <b>ii</b> 448 cm <sup>3</sup> | <b>iii</b> 435 cm <sup>3</sup> |
| <b>b i</b> 157 cm                | <b>ii</b> 170 cm              | <b>iii</b> 178 cm              |
| <b>3 a i</b> \$26 000            | <b>ii</b> \$20 000            | <b>iii</b> \$10 000            |
| <b>b i</b> 1 year                | <b>ii</b> 6 years             | <b>iii</b> 9 years             |
| <b>4 a i</b> 156 cm              | <b>ii</b> 168 cm              | <b>iii</b> 177.6 cm            |
| <b>b i</b> 163.3 cm              | <b>ii</b> 175.8 cm            | <b>iii</b> 182.5 cm            |
| <b>5 a i</b> 57.4 cm             | <b>ii</b> 65.1 cm             | <b>iii</b> 67.9 cm             |
| <b>b i</b> 88.7 cm               | <b>ii</b> 95.9 cm             | <b>iii</b> 104.4 cm            |
| <b>6 a i</b> 4.9 t               | <b>ii</b> 3.92 t              | <b>iii</b> 2.8 t               |
| <b>b i</b> 10 frosts             | <b>ii</b> 15                  | <b>iii</b> 25                  |
- 7 a** Extrapolation    **b** Extrapolation

**8 a, b**

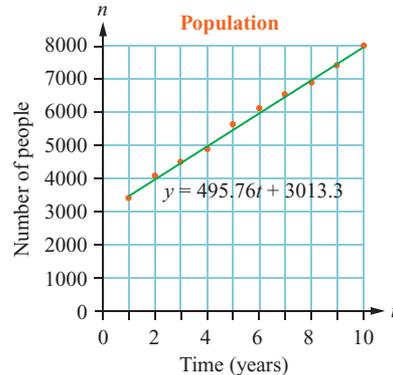


**c** 82

**d** 56

**e** Part **d** is an example of extrapolation.

**9 a, b**



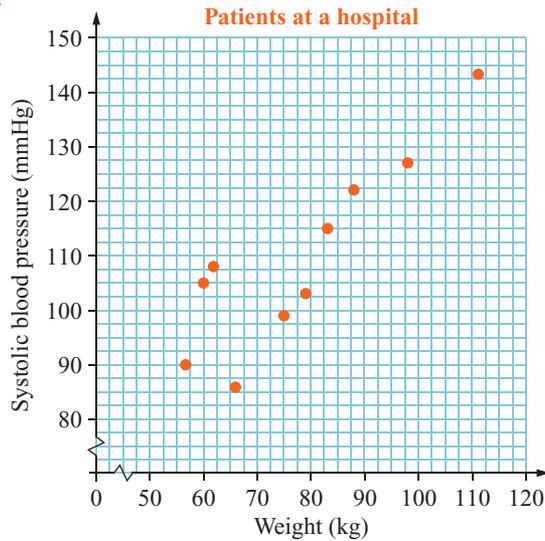
c i 5244    ii 6732    iii 8962    iv 11 441

d Answers **iii** and **iv** because they are based on values outside the measured range.

e i during 1994    ii during 2004

- 10 The range of ages in April's dataset will be the ages of school children, so a 30-year-old will be well outside the range of this dataset and the prediction would not be useful. In practical terms, we would expect shoe sizes to increase with age among school students, but to level out among adults.

11 a

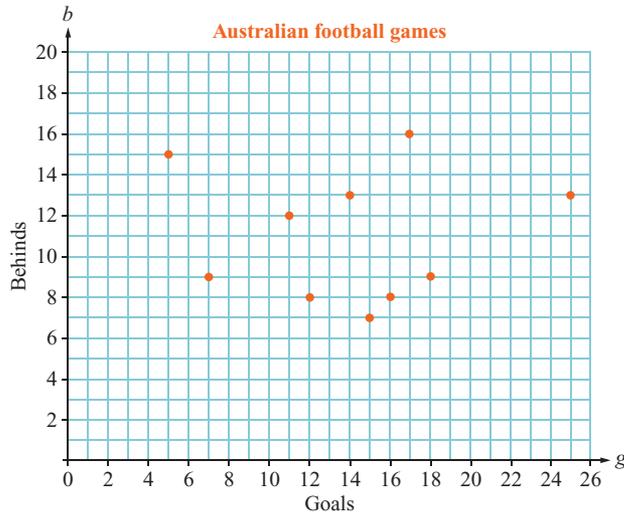


b  $y = 0.871x + 41.93$

c i 142 mmHg    ii 164 mmHg

d i as it is closer to the range of values in the dataset.

12 a



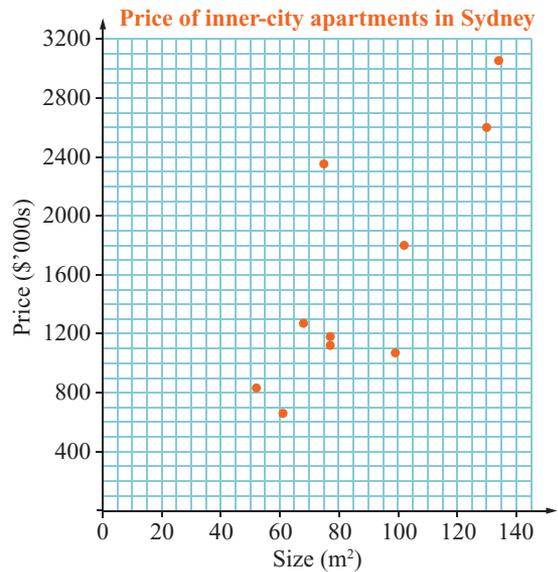
b No correlation

c  $b = 0.021g + 10.714$

d No; there is no correlation between the number of goals and the number of behinds scored, so predictions from the regression line will not be accurate.

- 13 A line of best fit calculated from many data points is going to be more representative of the population data; therefore, predictions made from this line of best fit will be more accurate.

14 a



b  $y = 23.538x - 466.589$

c \$1 650 000

d  $y = 25.320x - 749.132$

e \$1 530 000

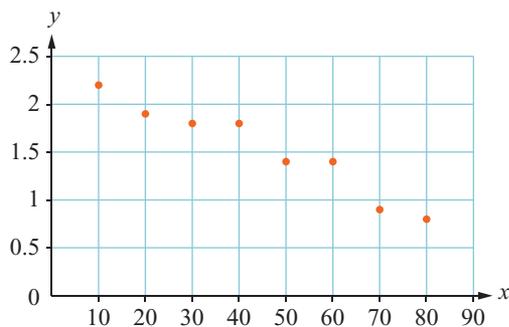
**REVIEW** MULTIPLE-CHOICE QUESTIONS

- 1 C    2 D    3 C    4 D    5 D    6 D  
 7 C    8 C    9 A    10 C    11 B    12 C  
 13 B    14 D    15 C    16 C

**REVIEW** SET 1

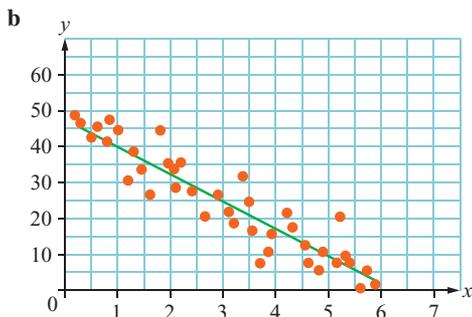
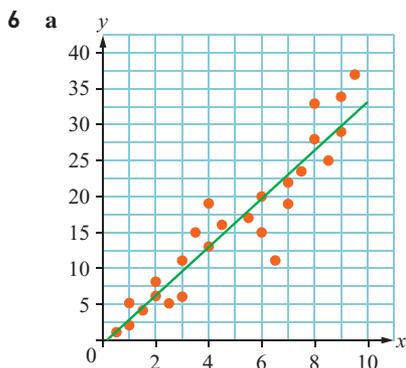
- 1 a On the evening of State of Origin people will be more likely to have rugby league on their mind, and the media will have been full of stories about rugby league for the past week, so they will be more likely to choose it as their favourite sport.  
 b People who have just seen the latest Star Wars movie are more likely to be fans of the sci-fi/action genres, so are going to be biased in choosing their favourite film genre.
- 2 a 15.6    b 11    c 6  
 3 a 51–60    b 61–70  
 c Overall, females performed better at the test. Fewer females achieved low scores, with females also having a higher model score.

4 a



b i Strong    ii Negative

- 5 a Strong positive correlation  
 b Strong positive correlation  
 c Strong negative correlation



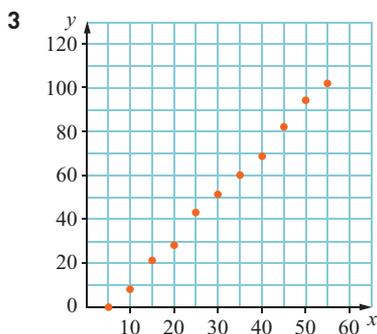
- 7 a 72.5 kg      b 122.5 kg      c 222.5 kg  
 8 It is not reasonable to expect the weight a person can bench press to continue to increase at a steady rate. It is more likely that their ability would level off. It is also possible that older people will start to lose some of their strength, and hence bench-pressing ability, due to other factors.  
 9 a 75              b 48

**REVIEW SET 2**

- 1 a The wording of this question suggests that the new building design is bad, thereby leading people to say that they also don't like it.  
 b Asking 'Do you agree ...', is more likely to lead people to say that they do agree than they don't agree.  
 2 a 2.5              b 9.3              c 0.7  
 3 a City A              b City B  
 4 a 272 minutes      b 252.5 minutes      c 220 minutes  
 5 238 hours  
 6 a 65              b 80  
 8 a Interpolation, as the population fell below 2000 within the range of the data  
 b Extrapolation, as the year 2020 is outside the range of the data

**REVIEW SET 3**

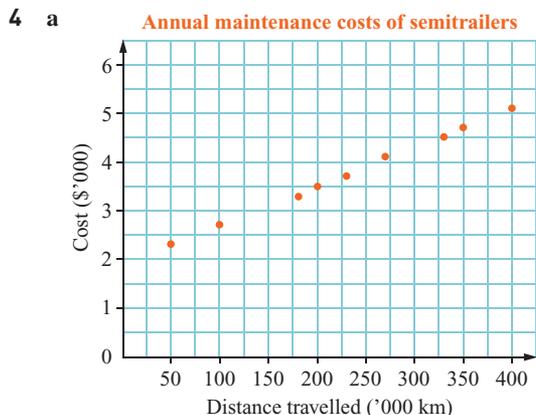
- 1 a Parents              b Farm animals  
 2 a Team C              b Team A



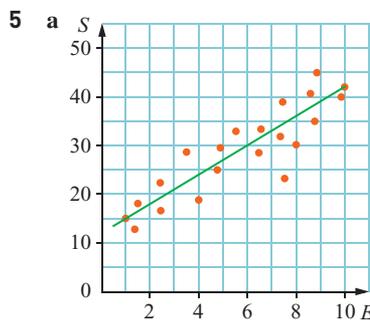
- 4 a \$55              b 50  
 5 The higher the mark a student receives in an English exam, the higher the mark they are likely to receive in a French exam.  
 6 a Spurious correlation  
 b Spurious correlation  
 7 a i 8.1 cm      ii 13.9 cm      iii 25.5 cm  
 b The vertical intercept does not make sense in the context of the data, as it suggests that the initial height of the plant is a negative value.  
 8 a Strong negative correlation  
 b No correlation  
 c Moderate positive correlation

**REVIEW SET 4**

- 1 a People who work in the finance sector often receive higher bonuses, which will lead to biased results when wanting to research bonuses in all Australian businesses.  
 b New parents are likely to have very favourable attitudes towards parental leave, which will lead to biased results in this research.  
 2 a 10.8              b 11              c 5  
 3 a 130–140              b 80–90  
 c When in a rush it would be better to get a coffee in the afternoon, as the wait time is usually shorter than in the morning.



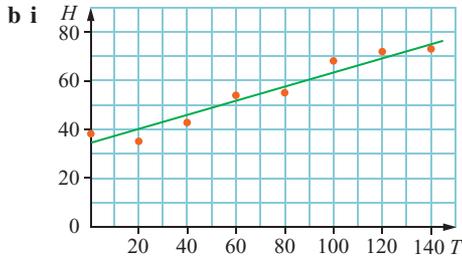
- b Strong positive correlation  
 c  $y = 0.008x + 1.891$   
 d \$4300  
 e This prediction should be accurate as it uses interpolation and there is a strong correlation between the variables.



- b 33              c 9.5  
 6 a Extrapolation  
 b 2004 is outside the range of the years given in the dataset.

**REVIEW** PRACTICE EXAMINATION QUESTION

- a i January  
 ii April  
 iii 7  
 iv Darwin; the total length of the bars is greater than for Sydney.  
 v Sydney; the range of the bars is smaller.



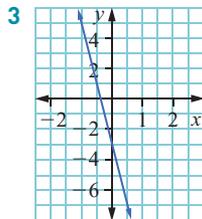
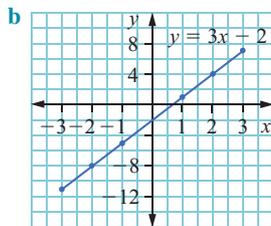
- ii 50  
 iii Extrapolation  
 c i Independent variable = amount of medicine administered,  
 Dependent variable = number of bacteria  
 ii 5175  
 iii 85

**CHAPTERS 4-6 CUMULATIVE REVIEW**

1 Gradient is  $-3$ , y-intercept is  $6$

2 a

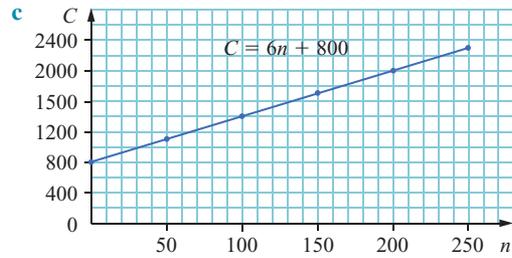
$x$	$-3$	$-2$	$-1$	$0$	$1$	$2$	$3$
$y$	$-11$	$-8$	$-5$	$-2$	$1$	$4$	$7$



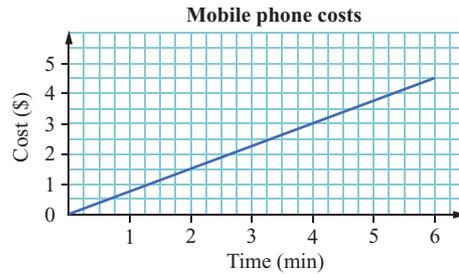
- 4 a \$40      b 100 km  
 c Gradient is  $0.4$ ; cost per km.  
 d Vertical intercept is \$10; flag fall.  
 5 a  $C$  = total cost,  $n$  is number of pairs of socks produced,  
 \$800 is fixed cost, \$6 is the cost per pair of socks.

b

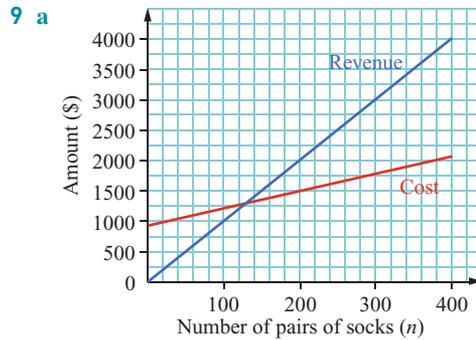
$n$	0	50	100	150	200	250
$C$	800	1100	1400	1700	2000	2300



- d About 130 pairs of socks (actually 130 pairs of socks)  
 6 a  $C = 0.75t$ , where  $C$  is the total cost in dollars after  $t$  minutes  
 b Dependent variable is cost.



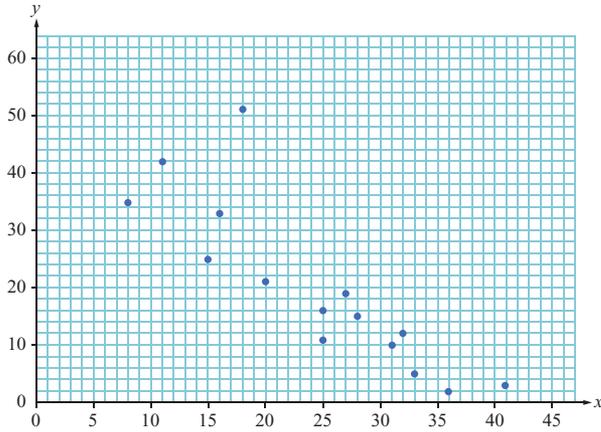
- c About \$1.90      d About  $3\frac{1}{3}$  min  
 7 a  $(2, -4)$       b Solution is  $x = 2, y = -4$ .  
 8 a Solution is  $x = 1, y = 2$ .  
 b Solution is  $x = 3, y = -5$ .



- b About  $(130, 1300)$   
 c About 130      d About \$1300  
 10 a 9.2 m      b 9.8 m  
 11 a 8.5 cm      b 14.1 cm  
 12 a 70 cm      b 131 cm  
 13 a i PR      ii RQ      iii PQ  
     b i  $\sin \theta = \frac{PR}{PQ}$     ii  $\cos \theta = \frac{RQ}{PQ}$     iii  $\tan \theta = \frac{PR}{PQ}$   
 14 a i 0.9573      ii 0.3214      iii 0.3398  
     b i  $15 \cdot 43'$     ii  $67 \cdot 6'$       iii  $54 \cdot 39'$   
 15 a 18.2 m      b 12.6 m  
 16 a  $50 \cdot 8'$       b  $66 \cdot 16'$   
 17 a  $23 \cdot 30'$       b 1003 m  
 18 5.5 m  
 19 a i N49°W      ii 311°T  
     b i S49°W      ii 229°T  
 20 a 139 km      b N54°20'W or 305°40'T  
 21 a Red kangaroos      b Online package holidays  
 22 People who have a dog as a pet regularly need to walk their dog, so will be more likely to be found walking outside, whereas most cat owners don't take their cats outside for walks. This question may target more dog owners than cat owners.  
 23 a 10    b 9    c 8    d 11    e 4  
 24 a Company 3    b Company 2    c Company 1

- 25 a i Independent variable is temperature and dependent variable is number of people at the beach.  
 ii Independent variable is number of hours spent exercising per week and dependent variable is weight.  
 b i Yes ii Yes  
 26 a Moderate correlation b Strong correlation  
 c Perfect correlation d Weak correlation  
 27 a Negative b Positive  
 c Positive d Positive

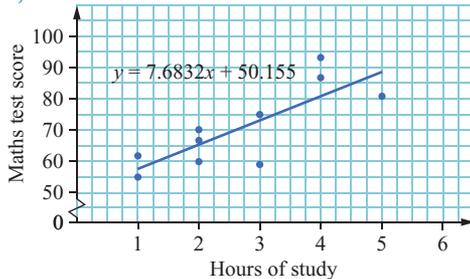
28 a



b Strong negative correlation

- 29 a Spurious correlation  
 b Spurious correlation  
 c Cause and effect relationship  
 30 a About 130 b About \$63

31 a,b



- c i 73 ii 85 iii 54 iv 104  
 d Parts iii and iv are the least reliable as the predicted results are outside the range of data; extrapolation was needed.

## CHAPTER 7 DEPRECIATION AND LOANS

### ARE YOU READY?

- 1 C 2 A 3 C 4 A 5 C  
 6 D 7 C 8 A 9 B

### EXERCISE 7A DECLINING-BALANCE METHOD OF DEPRECIATION

1

Year	Value (\$)	Depreciation (\$)	Depreciated value (\$)
1	24 900	$0.22 \times 24\,900 = 5478$	19 422
2	19 422	$0.22 \times 19\,422 = 4272.84$	15 149.16
3	15 149.16	$0.22 \times 15\,149.16 = 3332.82$	11 816.34

2

Year	Value (\$)	Depreciation (\$)	Depreciated value (\$)
1	34 800	$0.18 \times 34\,800 = 6264$	28 536
2	28 536	$0.18 \times 28\,536 = 5136.48$	23 399.52
3	23 399.52	4211.91	19 187.61
4	19 187.61	3453.77	15 733.84

- 3 a  $S = \$35\,000 (1 - 0.24)^3 = \$15\,364$   
 b Change in value =  $35\,000 - 15\,364 = \$19\,636$   
 4 a \$3867.90 b \$16 122.10  
 5 a \$9996.35 b \$46 003.65  
 6

$$20462 = 29000(1 - r)^2 \quad 20462 \div 29000 = (1 - r)^2$$

$$\sqrt{\quad} \quad 20462 \quad \div \quad 29000 \quad = 1 - r = 0.839\dots$$

$$r = 1 - 0.839\dots \approx 0.16$$

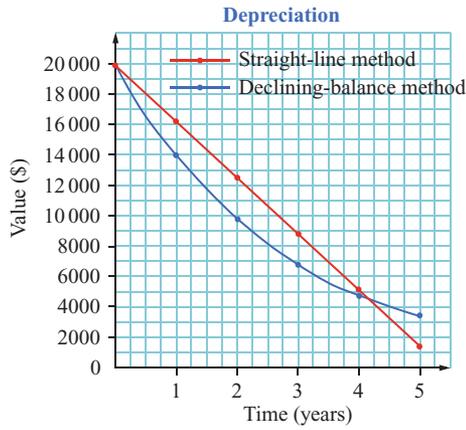
$\therefore$  Rate of depreciation = 16%

- 7 27%  
 8 15%  
 9 33%  
 10 23%  
 11 a \$16 452.48 b 24%  
 12 a \$15 000  
 b i \$12 200 ii \$11 000  
 c i  $2\frac{1}{2}$  years ii 2 years  
 d i 3.7 years ii \$4600  
 e After  $1\frac{1}{2}$  to 2 years  $\approx$  \$1500  
 f \$2800  
 13 a \$6500  
 b i \$4100 ii \$3400  
 c i 2.1 years ii  $1\frac{1}{2}$  years  
 d 4 years, \$1700  
 e i \$500 ii \$1250  
 f \$1200  
 14 a \$37 000  
 b i \$30 000 ii \$23 000  
 c i Just over 5 years ii Just less than 3 years  
 d 9 years, \$4500  
 e \$3600  
 f 21%  
 15 a \$1610 b \$690 c \$483  
 16 \$19 600

17 a

Year	Straight-line value (\$)	Declining-balance value (\$)
0	19 900	19 900
1	16 200	13 930
2	12 500	9 751
3	8 800	6 826
4	5 100	4 778
5	1 400	3 345

b

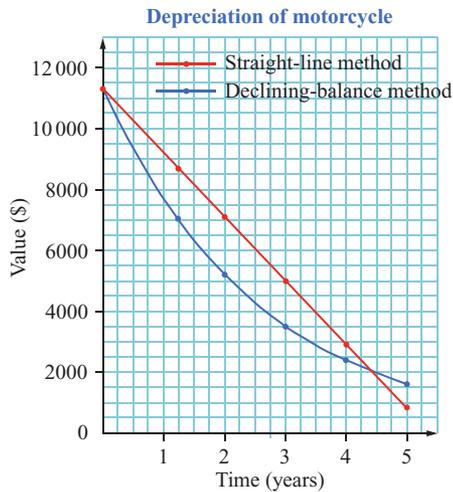


- c \$12 500, \$9751  
 d \$7000, \$5500  
 e  $\approx 4.2$  years  $\approx 4$  years 2 months

18 a

Year	Straight-line value (\$)	Declining-balance value (\$)
0	11 350	11 350
1	9 250	7 718
2	7 150	5 248
3	5 050	3 569
4	2 950	2 427
5	850	1 650

b



- c After about 4.4 years  
 d Straight-line: \$4000, Declining-balance: \$3000  
 e  $\approx 2.7$  years,  $\approx 1.8$  years

### EXERCISE 7B REDUCING-BALANCE LOANS

- 1 a \$3967.54      b \$3151.16  
 c \$2576.72      d \$2460.52  
 2 a \$3471.28      b \$3124.16  
 c \$347.12  
 3 a \$240 000      b \$280 000  
 c \$320 000      d \$320 000

5

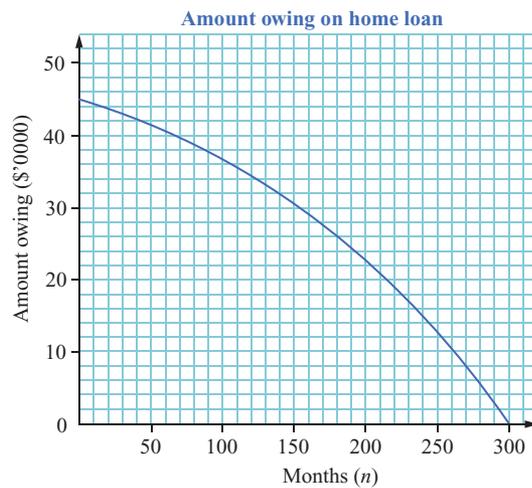
Month	Principal at the start of the month	Monthly interest	Monthly repayment	Balance at end of month
1	\$420 000	\$2520	\$3306.83	\$419 213.17
2	\$419 213.17	\$2515.28	\$3306.83	\$418 421.62
3	\$418 421.62	\$2510.53	\$3306.83	\$417 625.32
4	\$417 625.32	\$2505.75	\$3306.83	\$416 824.24
5	\$416 824.24	\$2500.94	\$3306.83	\$416 018.36
6	\$416 018.36	\$2496.11	\$3306.83	\$415 207.64

6

Month	Principal at the start of the month	Interest	Monthly repayment	Balance at end of month
1	\$350 000	\$2216.67	\$2471.26	\$349 745.41
2	\$349 745.41	\$2215.05	\$2471.26	\$349 489.20
3	\$349 489.20	\$2213.43	\$2471.26	\$349 231.37
4	\$349 231.37	\$2211.80	\$2471.26	\$348 971.91
5	\$348 971.91	\$2210.16	\$2471.26	\$348 710.81
6	\$348 710.81	\$2208.50	\$2471.26	\$348 448.05

- 7 a \$1879.31      b \$2328.55      c \$5090.10  
 d \$3250.96      e \$5041.11      f \$6003.10  
 8 a \$170 000      b \$140 000  
 c 160 months      d 180 months  
 9 a \$300 000      b \$250 000  
 c \$150 000      d \$210 000  
 e 160 months      f 260 months  
 g 200 months      h 285 months

10



- a \$450 000      b \$370 000  
 c \$200 000      d 150 months  
 e 200 months      f 300 months  
 11 a \$227.35      b \$509.88  
 c \$1145.63      d \$487.18  
 12 a \$164.76      b \$190.96  
 c \$282.33      d \$273.54  
 13 a i \$395.96      ii \$612.63      iii \$429.38  
 iv \$604.61      v \$323.56  
 b i \$20 872.80      ii \$3872.80      iii 5.70%  
 14

Month	Principal at start of the month	Monthly interest	Account-keeping fee	Monthly repayment	Balance at end of month
1	15 000 + 390 = \$15 390	\$184.68	\$10	\$413	\$15 171.68
2	\$15 171.68	\$182.06	\$10	\$413	\$14 950.74
3	\$14 950.74	\$179.41	\$10	\$413	\$14 727.15
4	\$14 727.15	\$176.73	\$10	\$413	\$14 500.88
5	\$14 500.88	\$174.01	\$10	\$413	\$14 271.89
6	\$14 271.89	\$171.26	\$10	\$413	\$14 040.15

- 15 a The time taken to repay the loan will reduce  
 b The cost of the loan will reduce
- 16 a 30 months      b Same time
- 17 a 60 months      b \$770  
 c No, he will still pay it back in 3 years
- 18 a 36 months (3 years)  
 b \$1038.76  
 c It increases to 37 months  
 d It reduces to 27 months  
 e An interest rate rise will either have no effect on the term of the loan, or will increase the term of the loan.  
 f An increase in the monthly repayment amount will either have no effect on the term of the loan, or will decrease the term of the loan.

### EXERCISE 7C CREDIT CARD STATEMENTS AND FEES

- 3 a i 14/04/18 to 13/05/18      ii 30 days  
 b i 7/06/18      ii 55 days  
 c \$10 000  
 d \$958  
 e i \$108.66      ii 3%  
 f \$6378  
 g i 0.0537% (to 3 s.f.), 0.000 537 (to 3 s.f.)  
 ii 0.0586% (to 3 s.f.), 0.000 586 (to 3 s.f.)
- 4 a \$10      b \$11.70      c \$37.35      d \$10
- 5 a 25 July      b 24 January      c 27 March
- 6 a 45      b 26      c 47      d 39

#### 7 September statement

Opening balance:	\$0
Closing balance:	\$134
Minimum payment due:	\$10
Due date:	25 Oct

#### October statement

Opening balance:	\$134
Closing balance:	\$1147
Minimum payment due:	\$45.88
Due date:	24 Nov

#### November statement

24 Nov	-1147
Opening balance:	\$1147
Closing balance:	\$114
Minimum payment due:	\$10
Due date:	25 Dec

### EXERCISE 7D CREDIT CARD BALANCES

- 1 Daily interest rate =  $22.9\% \div 365 = 0.062\ 74\%$   
 Balance =  $500 \times (1 + 0.06274)^{23}$   
 = \$507.22

$$\text{Interest} = \frac{500 \times 0.06274 \times 23}{100} = \$7.22$$

$$\text{Cash advance fee} = \frac{1.5}{100} \times 500 = \$7.50$$

$$\begin{aligned} \text{Total amount due} &= 507.22 + 7.50 \\ &= \$514.72 \end{aligned}$$

- 2 \$461.15  
 3 \$154.22  
 4 \$271.88  
 5 \$233.22  
 6 \$418.67  
 7 \$822.59

8 a

November statement		
Date	Details	Amount (\$)
1 Nov	Opening balance	517
6 Nov	Purchase	29
7 Nov	Payment	-300
12 Nov	Cash advance	200
13 Nov	Purchase	134
19 Nov	Purchase	97
20 Nov	Payment	-100
24 Nov	Purchase	166
25 Nov	Cash advance	150

- b \$907.55      c \$14.55      d \$36.30

- 12 Card B (by \$3.25)  
 13 Card A by \$4.20  
 14 a \$50      b \$30.03  
 15 \$327.79  
 16 \$6.26

### REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 A      2 C      3 B      4 C      5 B  
 6 D      7 B      8 B      9 C      10 C  
 11 A

### REVIEW SET 1

- 1 a \$28 483.67      b \$34 516.33  
 2 14%  
 3 a \$160 000      b \$240 000  
 c \$320 000      d \$320 000  
 4 a \$574.67      b \$552.13  
 5 a \$20.80      b \$10  
 6 a 26 July      b 25 November  
 7 \$826.95

### REVIEW SET 2

- 1 a \$14 298.13      b \$14 701.87  
 2 24%  
 3 a \$2429.90      b \$2777.04      c \$347.14  
 4 a \$125 000      b 120 months  
 5 a \$21      b \$37.50  
 6 \$614.97

### REVIEW SET 3

- 1 a \$9877.55      b \$5122.46  
 2 19%  
 3 a \$4959.44      b \$3075.64  
 4 a \$175 000      b 170 months  
 5 a \$13.80      b \$10

6 \$189.29

**REVIEW SET 4**

1

Year	Value (\$)	Depreciation (\$)	Depreciated value (\$)
1	22000	3300	18700
2	18700	2805	15895
3	15895	2384	13511
4	13511	2027	11484

2 26%

3 a \$2460.52      b \$2576.72

4 a \$255 000      b 220 months

5 \$385.76

6 24 January

**REVIEW PRACTICE EXAMINATION QUESTION**

1 a i \$2780      ii \$1961      iii \$2729

b i \$319 610.92      ii \$318 816.09      iii \$1183.91

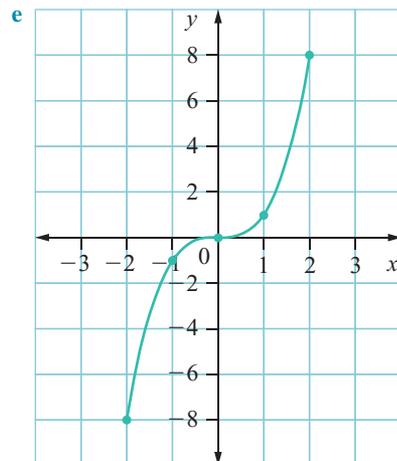
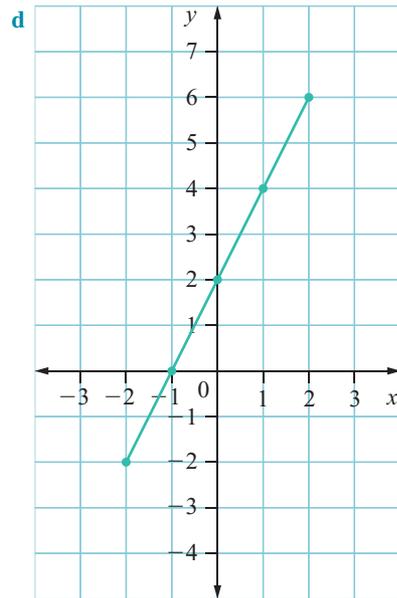
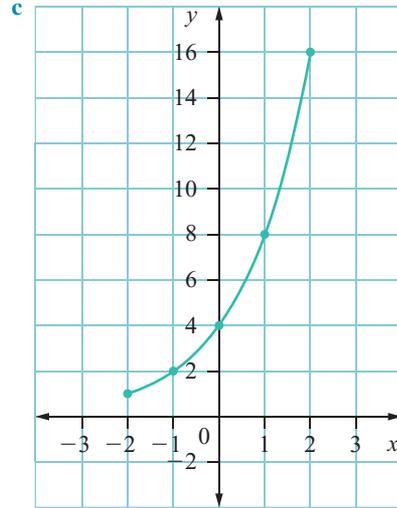
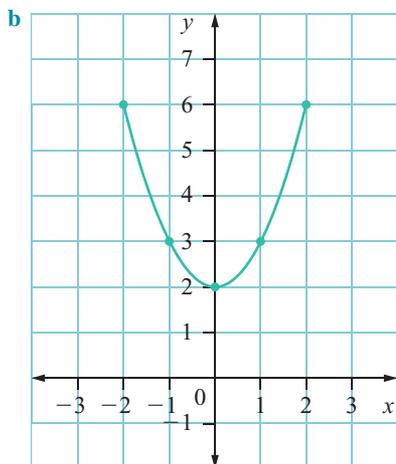
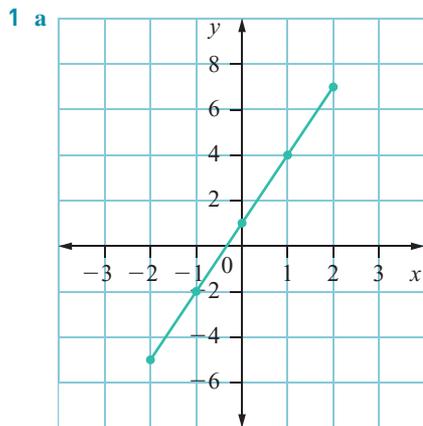
c i \$534      ii \$538.26      iii \$4.26

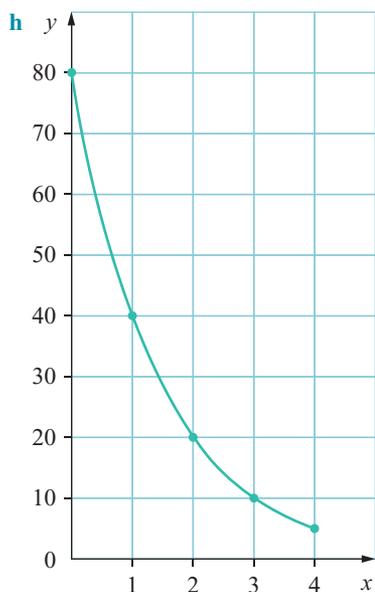
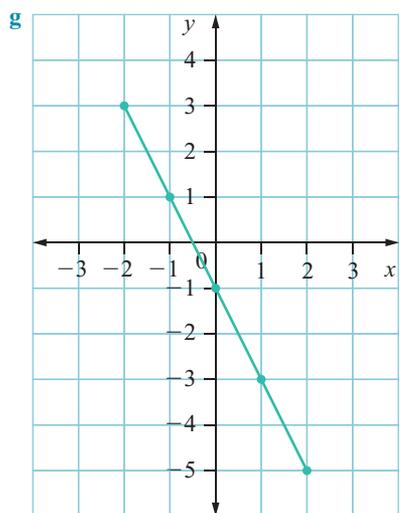
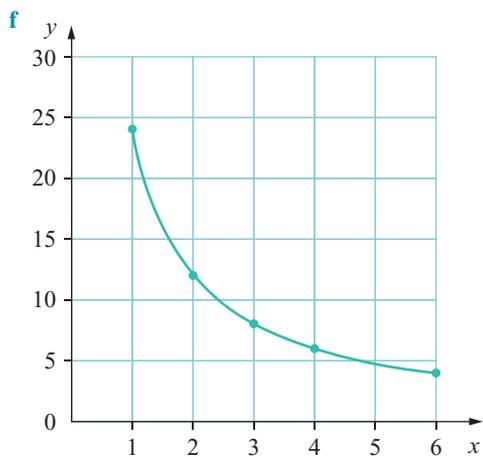
**CHAPTER 8 GRAPHS OF PRACTICAL SITUATIONS**

**ARE YOU READY?**

- 1 B      2 D      3 A      4 B  
 5 C      6 C      7 C      8 A  
 9 D      10 B

**EXERCISE 8A CONSTRUCTING GRAPHS FROM TABLES OF VALUES**

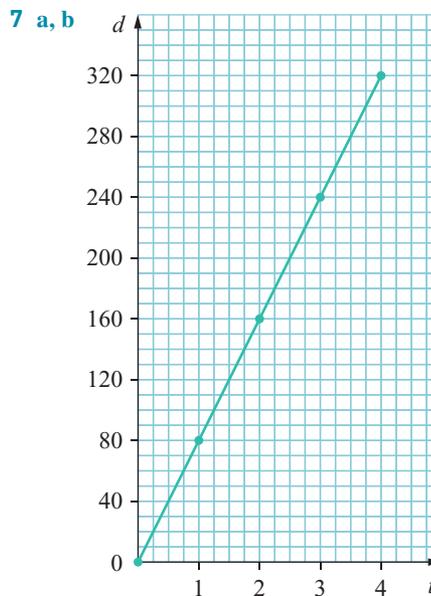




- 3** Linear relationships shown in parts **a**, **d** and **g**.  
**4 a** Difference in  $y$  is the same; common difference is 3.  
**b** Difference in  $y$  is not the same.  
**c** Difference in  $y$  is not the same.  
**d** Difference in  $y$  is the same; common difference is 2.  
**e** Difference in  $y$  is not the same.  
**f** Difference in  $y$  is not the same.  
**g** Difference in  $y$  is the same; common difference is  $-2$ .  
**h** Difference in  $y$  is not the same.

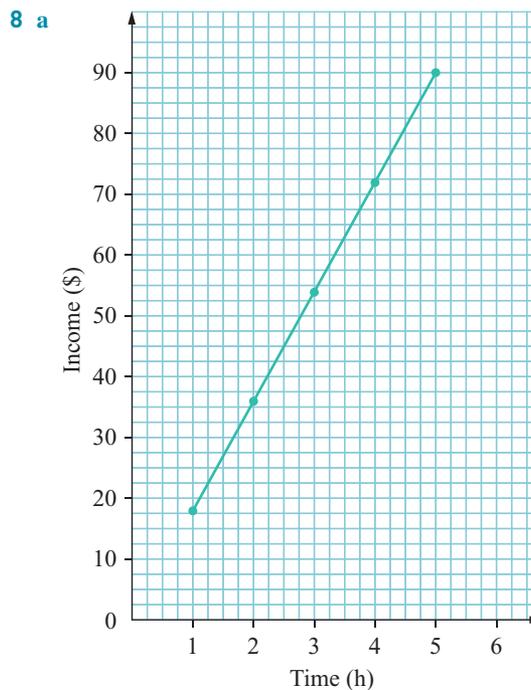
**5** Linear relationships shown in parts **b**, **c** and **f**.

- 6 a** Linear  
**b** Quadratic  
**c** Exponential  
**d** Linear  
**e** Cubic  
**f** Inverse  
**g** Linear  
**h** Exponential



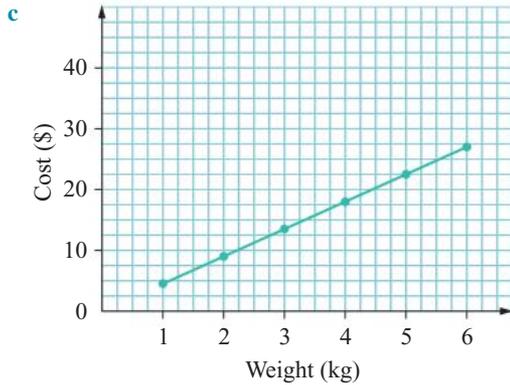
- c** Yes  
**d**  $d$  increases by 80  
**e** 400 km  
**f** About 6.3 hours (6.25 hours)  
**g** Using two consecutive points on the line,  

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{increase in } d}{\text{increase in } t} = \frac{80}{1} = 80.$$
 This shows that the common difference found in part **d** is the same as the gradient of the line.

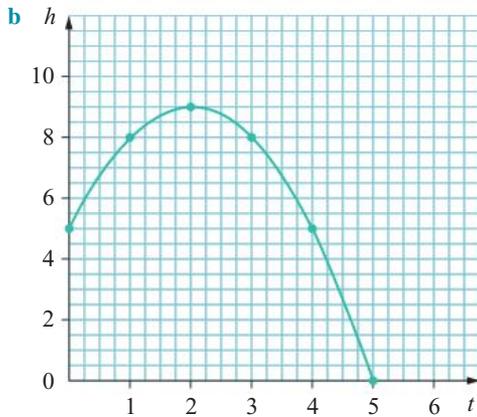


- b** Yes  
**c** Income increases by \$18, common difference is \$18  
**d** Common difference in income when the time increases by a constant amount indicates a linear relationship.

- 9 a** Yes, common difference of \$4.50  
**b** \$36.00

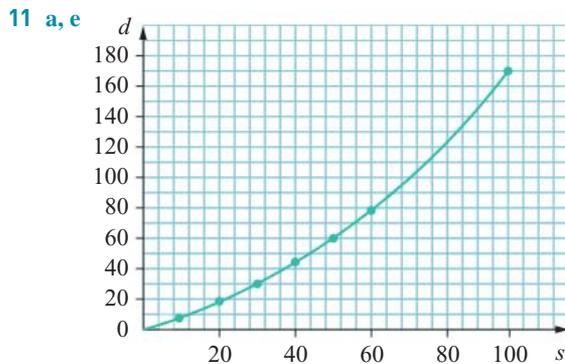


- 10 a** No, there is not a common difference between the  $h$  values as  $t$  increases by 1.

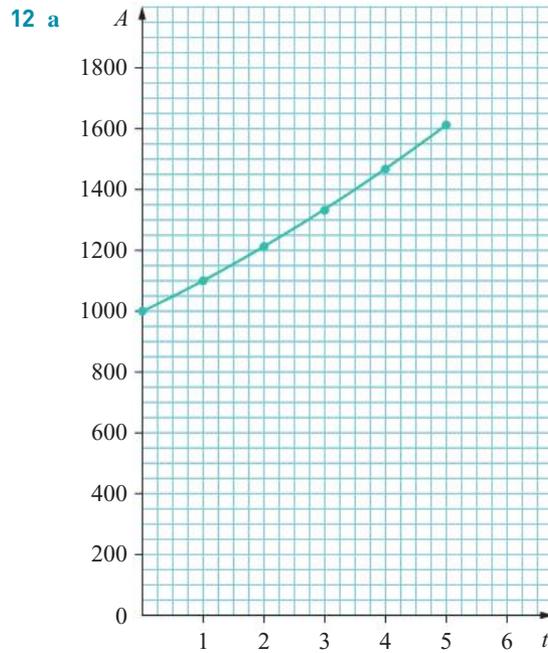


- c** Curve is symmetrical about an axis of symmetry and has a maximum turning point at (2, 9).

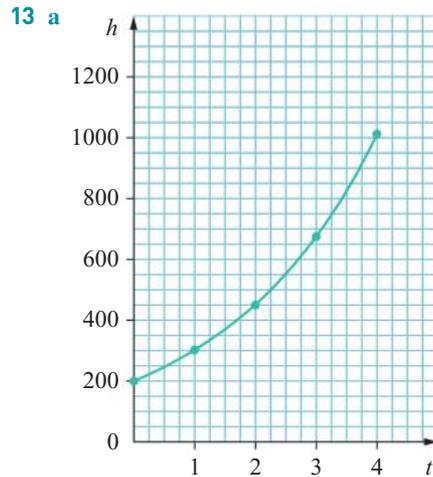
- d** 5 m  
**e** About 7 m (6.75 m)  
**f** Maximum height of 9 m after 2 seconds  
**g** 5 s



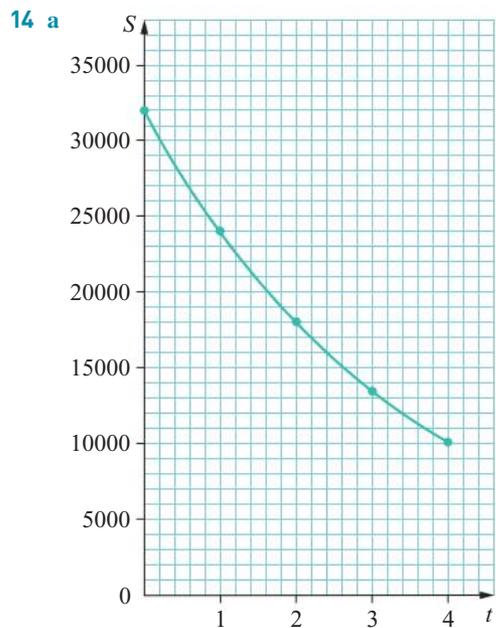
- b** Quadratic  
**c** 34 m  
**d** 18 m  
**e** 170 m



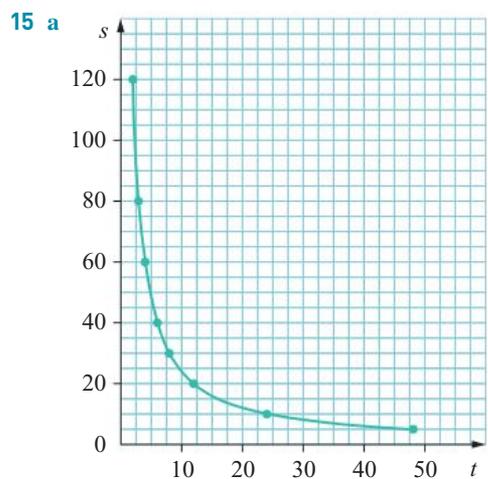
- b** No  
**c**  $1100 - 1000 = 100$ ,  $1210 - 1100 = 110$ ,  
 $1331 - 1210 = 121$ ,  $1464.10 - 1331 = 133.10$ ,  
 $1610.51 - 1464.10 = 146.41$ ; no common difference between values of  $A$   
**d i** 1.1    **ii** 1.1    **iii** 1.1    **iv** 1.1    **v** 1.1  
**e** There is a common ratio between the values of  $A$ .



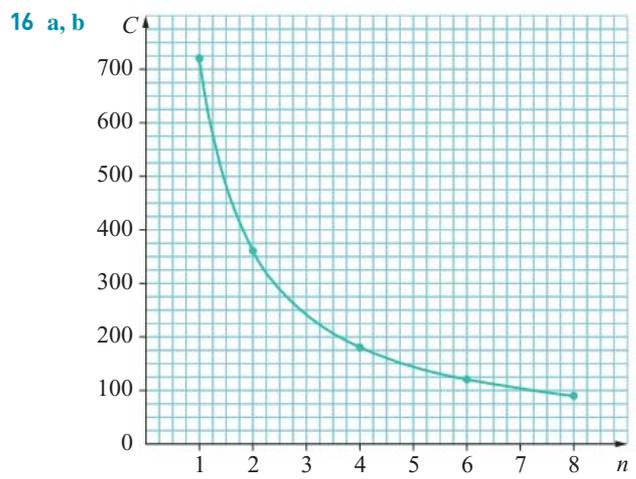
- b** Exponential; common ratio of 1.5  
**c** About 550 mm (551 mm)  
**d** About 3.7 days  
**e** About 2300 mm (2278 mm)  
**f** The height of the plant will not continue to increase indefinitely; it will only have a limited life span.



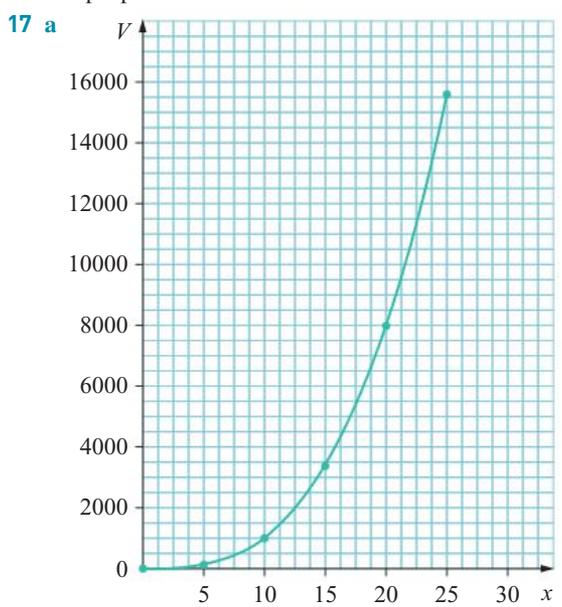
- b** Exponential; common ratio of 0.75
- c** Initial value of the car was \$32 000.
- d** About \$20 800 (\$20 784.61)
- e** About 2.4 years
- f** About \$5700 (\$5695.31)



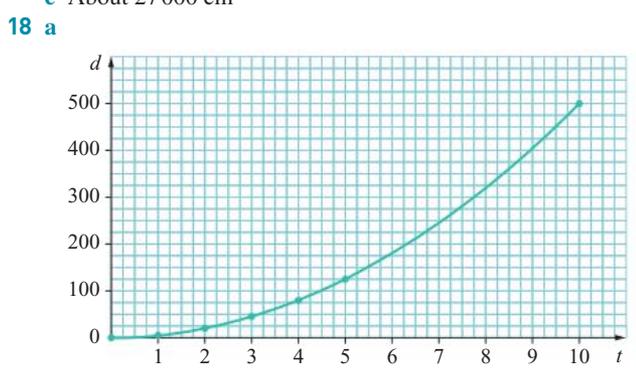
- b** The curve would not cross either axis.
- c** Inverse
- d** About 50 km/h (48 km/h)
- e** About 2.5 hours



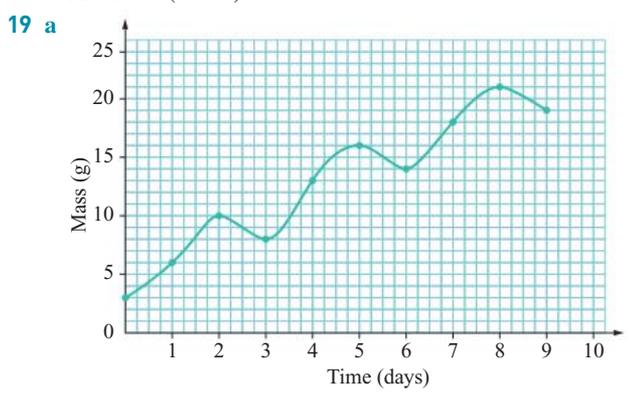
- c** Inverse
- d** \$240
- e** 5 people



- b** Cubic relationship; not linear as no common difference (and not a straight-line graph), not exponential as no common ratio, not inverse as  $V$  does not decrease as  $x$  increases, not quadratic as the difference between values of  $V$  increases at a higher rate as  $x$  increases.
- c** About 500 cm<sup>3</sup> (512 cm<sup>3</sup>)
- d** About 22 cm
- e** About 27 000 cm<sup>3</sup>



- b** About 250 m (245 m)
- c** About 9 s (8.94 s)

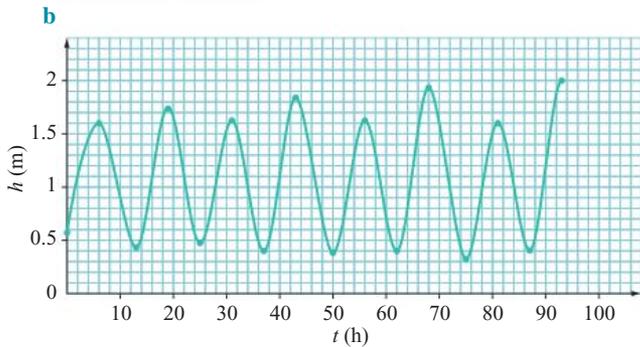


- b** No
- c** About 15 g
- d** About 1.5 days

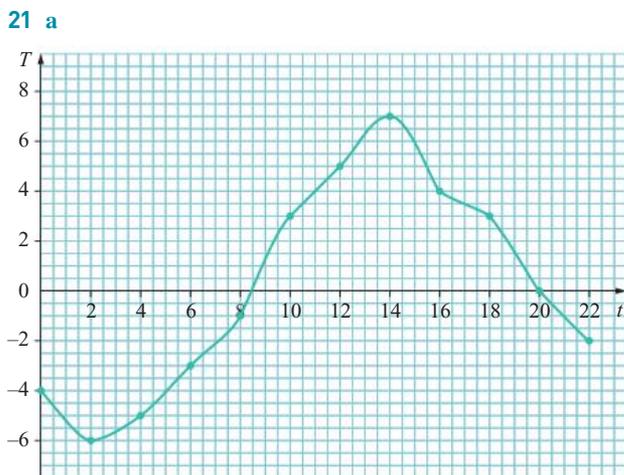
- e About 24 g; not reliable as this value is outside the range of the recorded data and there could be unknown factors that affect the mass of algae after 9 days.

20 a

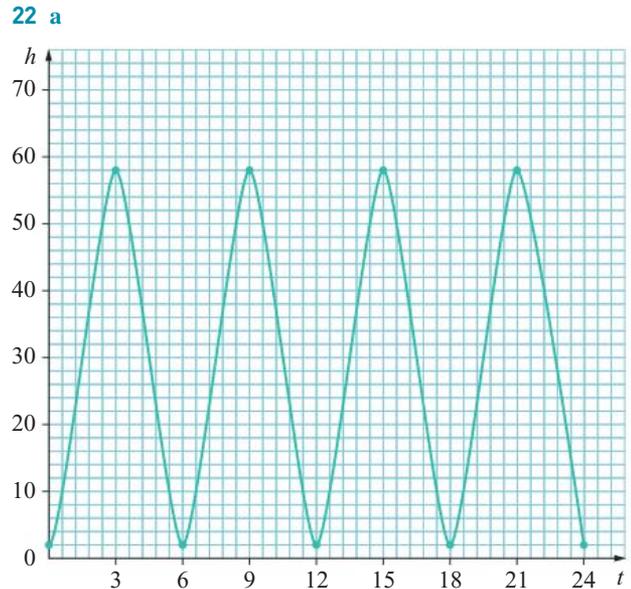
$t$ (h)	$h$ (m)
0	0.57
6	1.60
13	0.43
19	1.73
25	0.47
31	1.62
37	0.40
43	1.84
50	0.38
56	1.62
62	0.39
68	1.93
75	0.32
81	1.60
87	0.41
93	1.99



- c No recognisable relationship but can predict  $t$  or  $h$  values from the trend shown by the curve.  
 d About 0.9 m  
 e About 0.7 m; 4 am on Thursday  
 f About 1.6 m at approx. 10 am



- b  $4^{\circ}\text{C}$   
 c  $-6^{\circ}\text{C}$  at 2 am  
 d  $7^{\circ}\text{C}$  at 2 pm  
 e About  $6.5^{\circ}\text{C}$   
 f About 11.5 hours

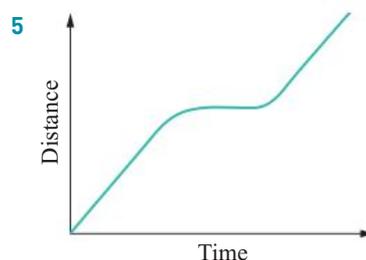
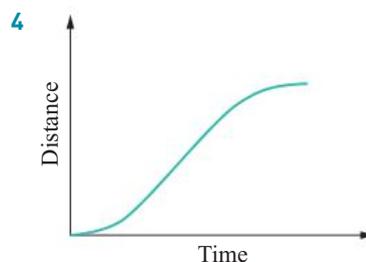
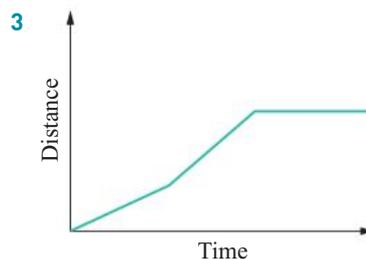


- b 6 minutes  
 c 4 revolutions  
 d i 16 m      ii 44 m      iii 44 m  
 e 1.5 minutes  
 f 44 m  
 g 22 minutes

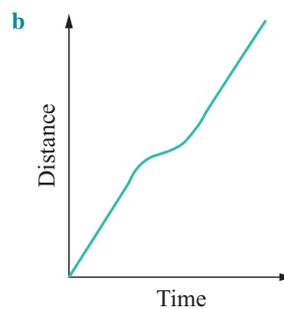
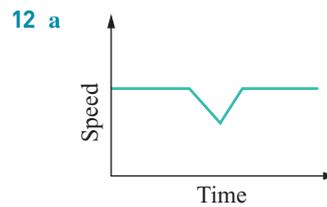
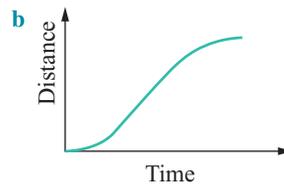
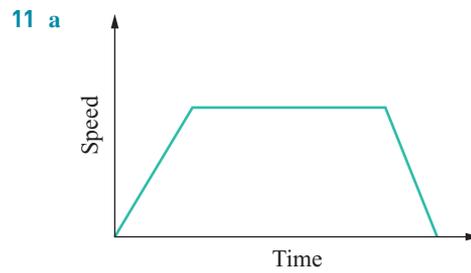
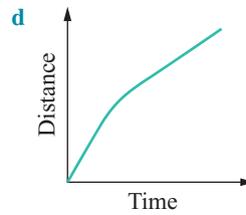
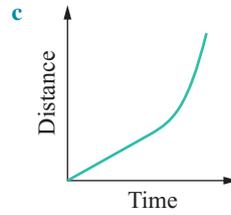
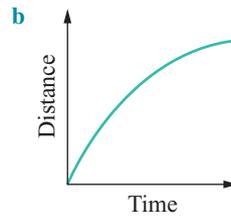
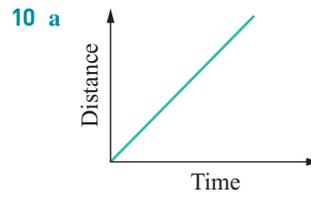
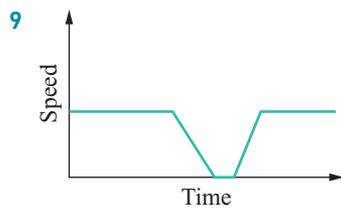
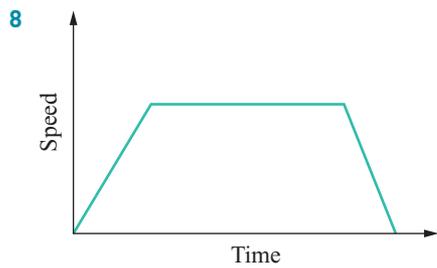
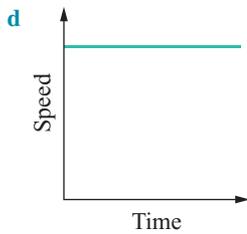
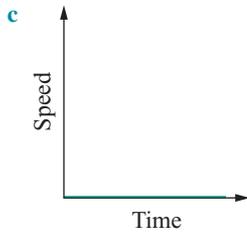
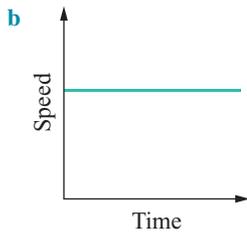
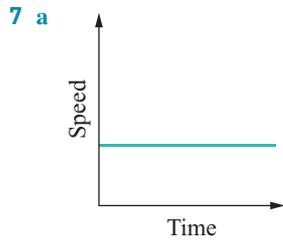
### EXERCISE 8B SKETCHING GRAPHS RELATED TO SPEED, DISTANCE AND TIME

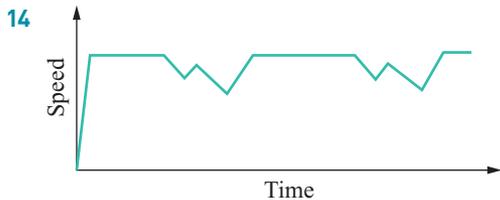
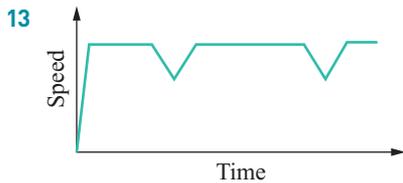
- 1 a i False      ii False      iii True      iv True  
 b Alex leaves home and walks at a constant speed (section A). He then walks at a faster speed (section B) before stopping (section C). He returns home at a constant speed that is faster than his earlier speeds (section D).

- 2 a C      b A      c D      d B



- 6 a** Speed is constant; distance travelled increases at a constant rate.  
**b** Speed is decreasing; distance travelled increases at a decreasing rate.  
**c** Speed is constant at first then increases; distance increases at a constant rate at first and then at an increasing rate.  
**d** Speed is decreasing at first then constant; distance increases at a decreasing rate at first and then at a constant rate.





**EXERCISE 8C** SKETCHING MORE GRAPHS FROM A GIVEN DESCRIPTION

- 1 a i  $25 \text{ cm}^2$       ii  $100 \text{ cm}^2$

b Volume = area of base  $\times$  depth

c i Container A

Depth (cm)	0	1	2	3	4	5
Volume ( $\text{cm}^3$ )	0	25	50	75	100	125

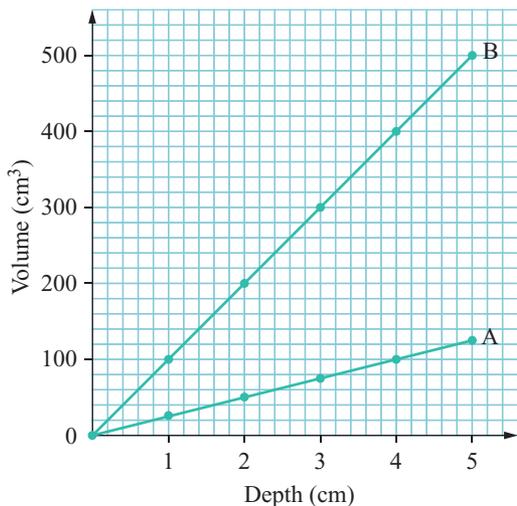
ii Container B

Depth (cm)	0	1	2	3	4	5
Volume ( $\text{cm}^3$ )	0	100	200	300	400	500

d Container A:  $25 \text{ cm}^3$ ; container B:  $100 \text{ cm}^3$

e The vertical sides mean that, for each increase of 1 cm in depth, the surface area of the water or 'area of base' remains the same, so the increase in volume is the same.

f Relationship is linear.



g The line for container B has the steeper gradient. This means that the volume of water is larger in container B for the same increase in depth.

h The same amount of water is poured in each equal-sized time interval.

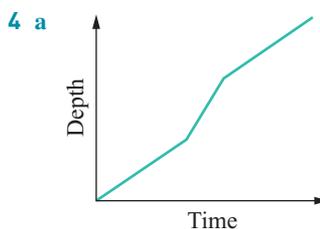
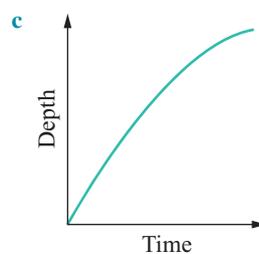
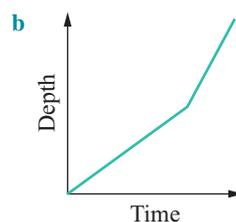
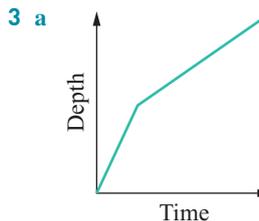
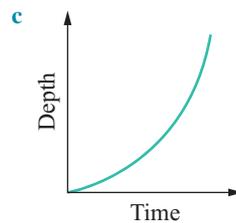
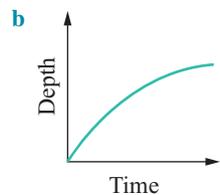
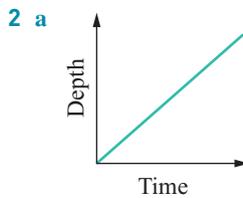
i Container A would fill first as it would take less time to fill a  $125 \text{ cm}^3$  container compared to a  $500 \text{ cm}^3$  container.

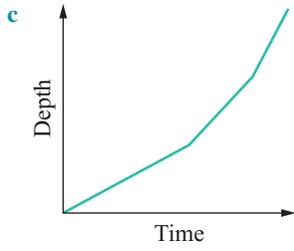
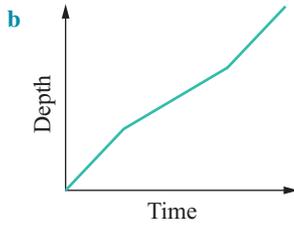
j No, as the surface area of the water decreases as the depth increases.

k i No      ii No      iii Yes

l Container C would fill first as its volume is less than  $125 \text{ cm}^3$ . Both containers A and C have the same base area of  $25 \text{ cm}^2$  but, as the depth increases, the surface area of water in container C decreases.

m If a container has vertical sides, the rate at which it fills is constant. If the surface area of the water increases as the depth increases, the rate at which it fills decreases. If the surface area of the water decreases as the depth increases, the rate at which it fills increases.

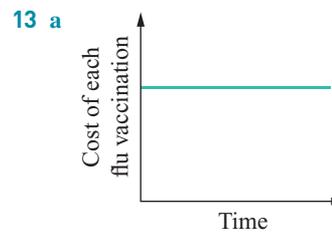
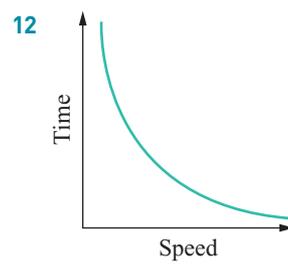
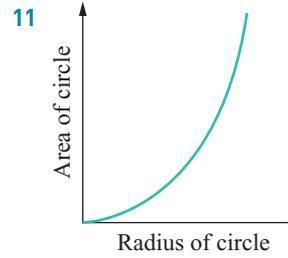
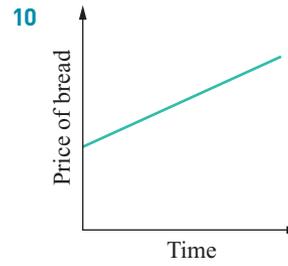
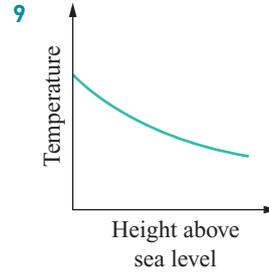
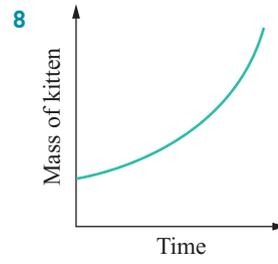
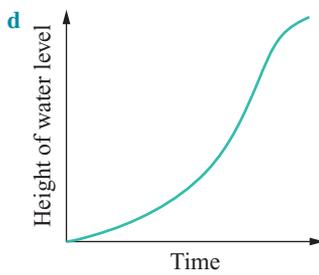
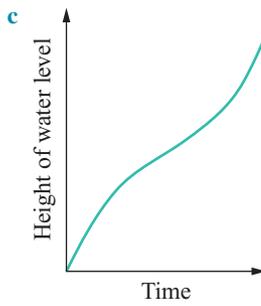
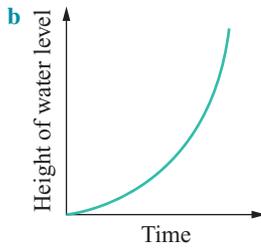
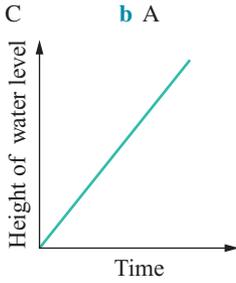


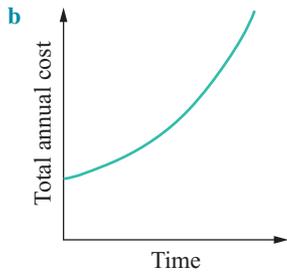


**5 B**

**6 a C**

**7 a**

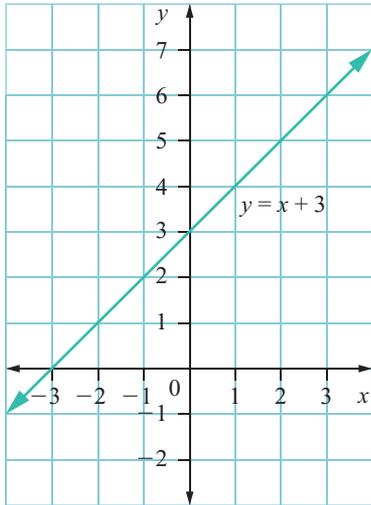




**14 B**

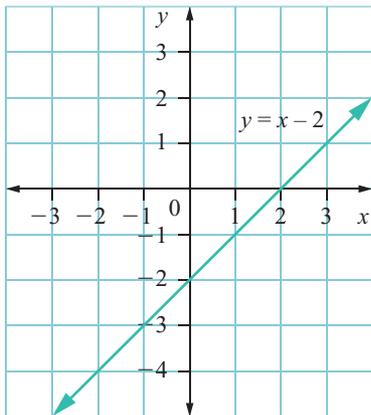
**EXERCISE 8D** GRAPHS OF LINEAR MODELS

**1 a i**



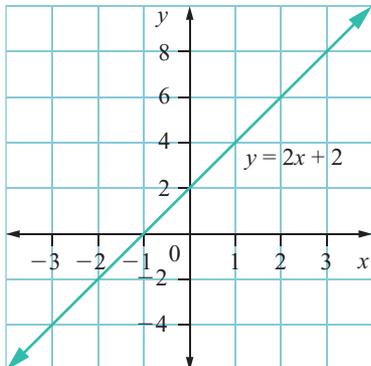
**ii** Gradient 1, y-intercept 3

**b i**



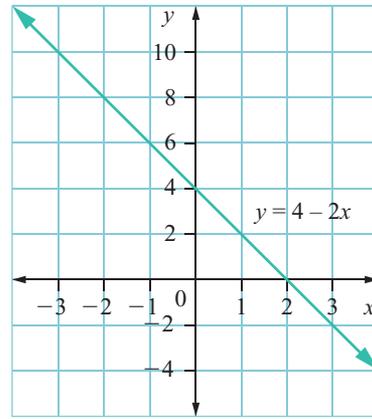
**ii** Gradient 1, y-intercept -2

**c i**



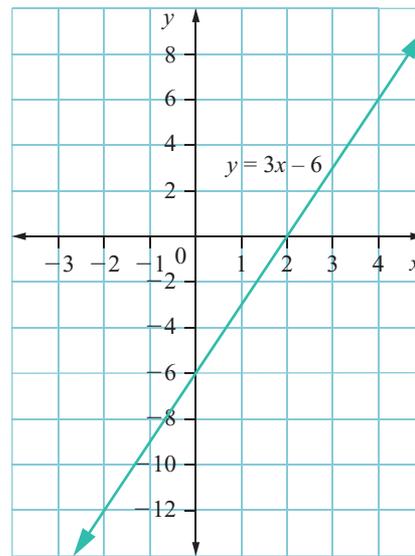
**ii** Gradient 2, y-intercept 2

**d i**



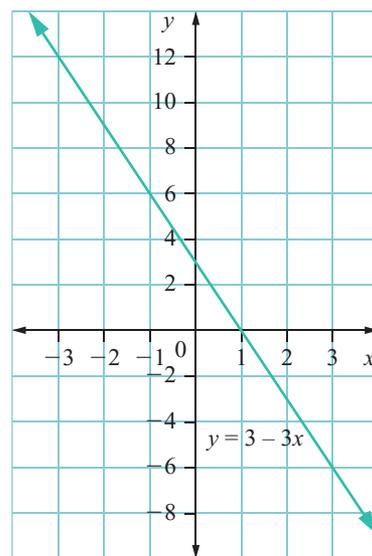
**ii** Gradient -2, y-intercept 4

**e i**



**ii** Gradient 3, y-intercept -6

**f i**

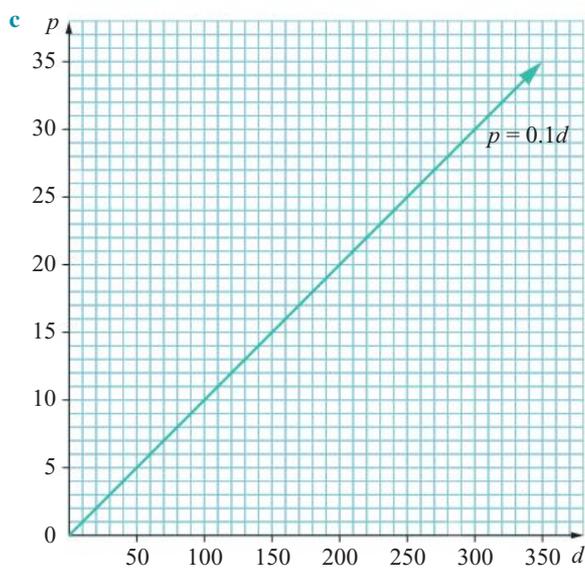


**ii** Gradient -3, y-intercept 3

- 2 a**  $y = x + 2$   
**b**  $y = -3x + 1$   
**c**  $y = \frac{1}{2}x$   
**d**  $y = x + 1.5$

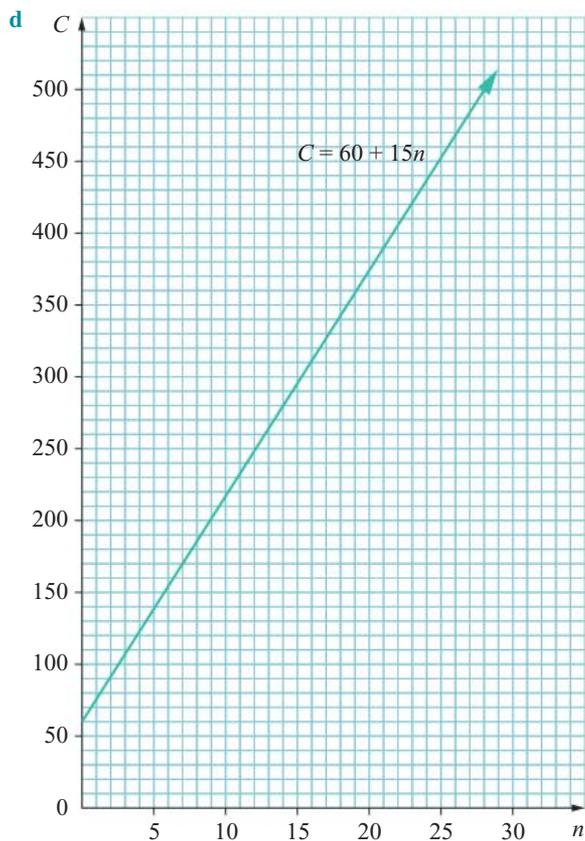
- 3 a  $y = x + 4$   
 b  $y = 3x - 1$   
 c  $y = -x - 2$   
 d  $y = -2x + 1$   
 e  $y = 2x + 1$   
 f  $y = -3x + 2$

- 4 a 20 L  
 b 350 km



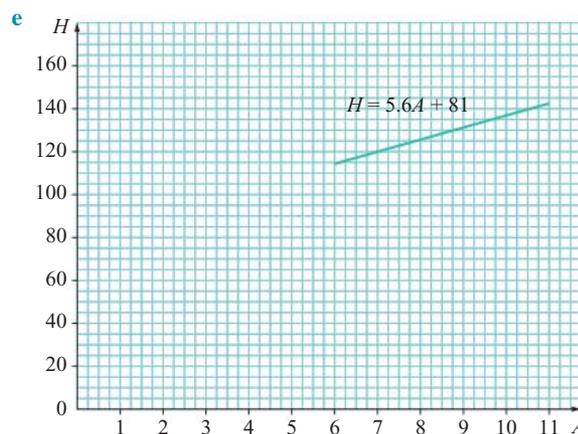
- d 0.1; It represents the number of litres of petrol consumed per km for the car travelling at constant speed (0.1 L/km).  
 e No, as speed would vary for city driving.

- 5 a \$435  
 b 16 people  
 c 29 people



- e 15; represents cost per person (\$15/person)  
 f 60; represents a fixed set-up cost (\$60)  
 g Probably not; the fixed set-up cost of \$60 may not be appropriate for large numbers of people.

- 6 a 114.6 cm  
 b 128.6 cm  
 c The average height of a boy aged 6 to 11 increases by 5.6 cm each year.  
 d During adolescence, boys may experience a growth spurt.

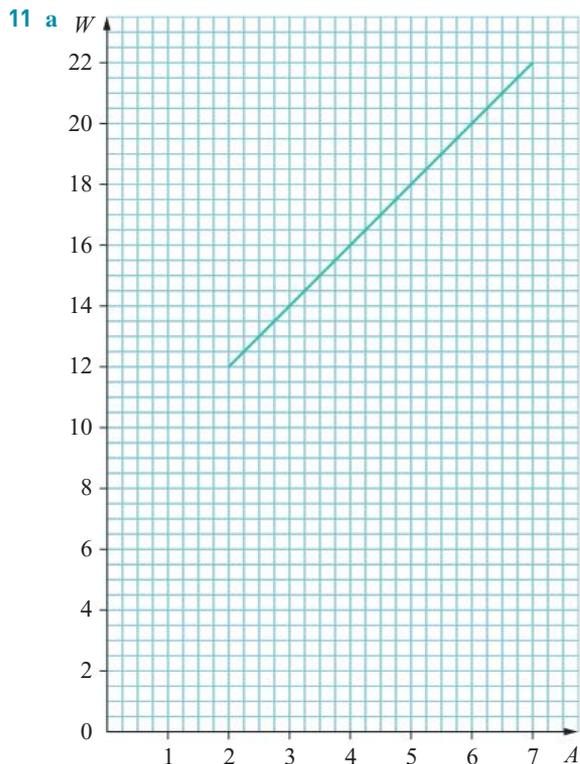


- 7 a 2 hours  
 b About 190 km (187.5 km)  
 c Gradient =  $\frac{300 - 0}{4 - 0} = 75$ ; represents distance travelled (in km) per hour (speed is 75 km/h)  
 d  $d = 75t$   
 e 600 km  
 f The strength of this model is that we can confidently predict the distance travelled using the equation or graph for values of  $t$  from 0 to 5. For values of  $t$  that are more than 5 (outside the range of recorded values shown in the graph), our predictions may not be reliable due to unknown factors.

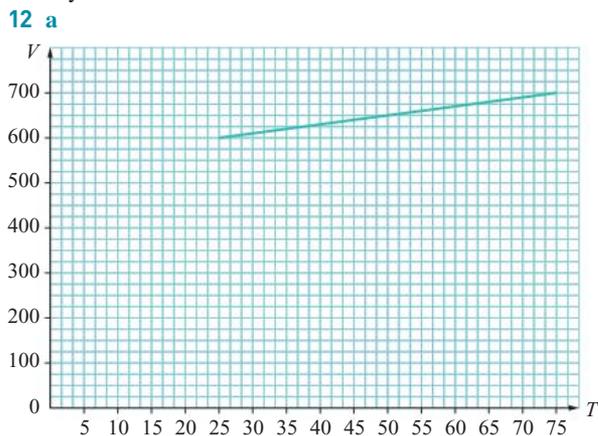
- 8 a 80 m<sup>2</sup>  
 b Just over 11 L (11.25 L)  
 c Gradient =  $\frac{60 - 0}{15 - 0} = 4$ ; it represents area of wall covered (in m<sup>2</sup>) with one litre of paint (4 m<sup>2</sup>/L).  
 d  $A = 4L$   
 e 120 m<sup>2</sup>  
 f Surface of wall could be porous and need more paint per m<sup>2</sup>; temperature could affect how much of the wall is covered by each litre of paint.

- 9 a About \$230  
 b About 85  
 c Gradient =  $\frac{240 - 150}{75 - 0} = 1.2$ ; represents cost per hot dog (\$1.20/hot dog)  
 d Vertical intercept = 150; represents a fixed set-up cost (\$150)  
 e  $C = 1.2n + 150$   
 f \$330  
 g The strength of this model is that we can confidently predict the cost of production using the equation or graph for values of  $n$  from 0 to 120. For values of  $n$  that are more than 120 (outside the range of recorded values shown in the graph), our predictions may not be reliable due to other factors such as increased set-up costs for large numbers of hot dogs.

- 10 a About \$350  
 b About 70  
 c Gradient =  $\frac{480 - 220}{100 - 0} = 2.6$ ; represents cost per pie (\$2.60/pie)  
 d Vertical intercept = 220; represents a fixed set-up cost (\$220)  
 e  $C = 2.6n + 220$   
 f \$2820; not accurate as there may be additional costs for large numbers of pies. Also the facilities may not be set up to produce 1000 pies. The model should be limited to a maximum number of pies that can be produced for the costs in the model.



- b Gradient =  $\frac{22 - 12}{7 - 2} = 2$ ; average weight of a girl aged 2 to 7 increases by 2 kg each year.  
 c Vertical intercept = 8; would represent average weight of new-born girl (8 kg)  
 d  $W = 2A + 8$   
 e 15 kg  
 f New-born baby girls have an average weight of about 3.3 kg (not 8 kg) and increase in weight at a higher rate in the first two years of their lives. After 7 years, girls start to grow taller and gain weight quicker than in the years 2 to 7.

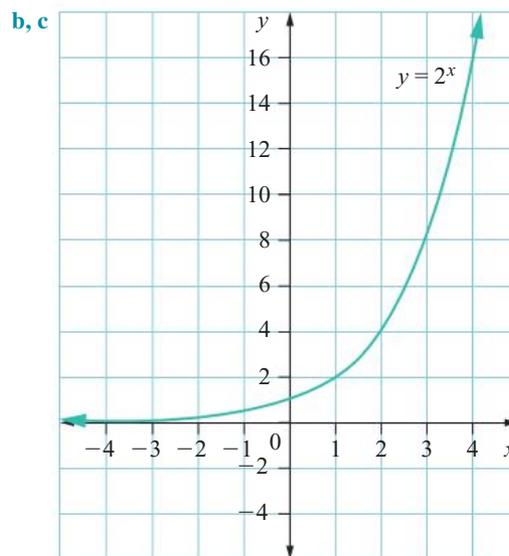


- b Gradient =  $\frac{700 - 600}{75 - 25} = 2$ ; volume of inflated balloon increases by 2 cm<sup>3</sup> for each increase of 1°C in temperature (2 cm<sup>3</sup>/°C).  
 c Vertical intercept = 550; would represent volume of balloon when temperature was 0°C (550 cm<sup>3</sup>).  
 d  $V = 2T + 550$   
 e 654 cm<sup>3</sup>  
 f Model is not reliable outside the range of recorded data. The rubber in the balloon may behave differently at extremely low or high temperatures and the balloon may burst once past its maximum volume.

### EXERCISE 8E GRAPHS OF EXPONENTIAL MODELS

1 a

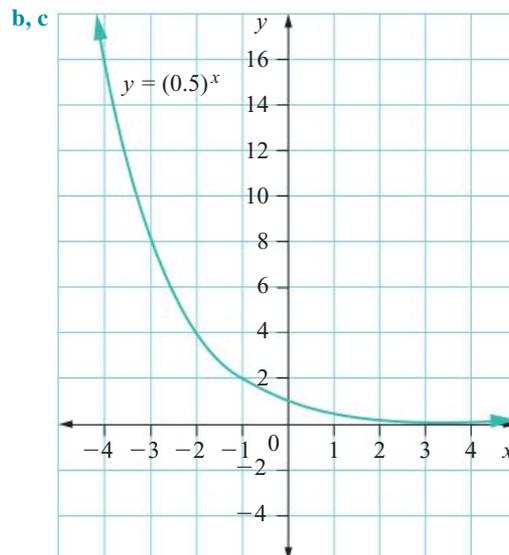
x	-4	-3	-2	-1	0	1	2	3	4
y	0.0625	0.125	0.25	0.5	1	2	4	8	16



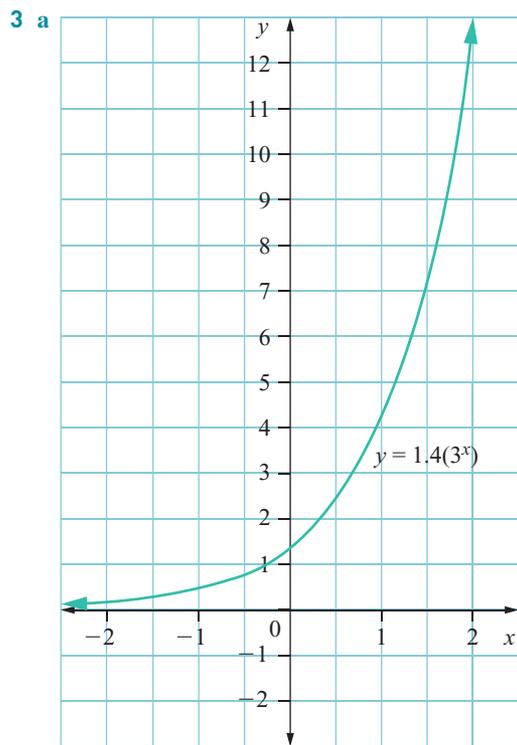
- d y increases at an increasing rate of change.  
 e No, all y-values are positive as 2 to any power always gives a positive result.  
 f 1  
 g No x-intercepts

2 a

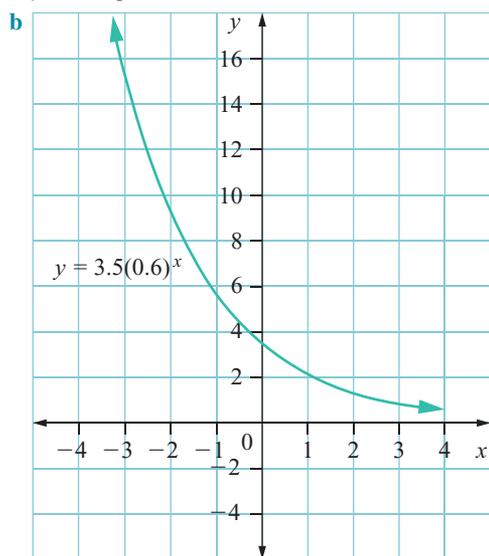
x	-4	-3	-2	-1	0	1	2	3	4
y	16	8	4	2	1	0.5	0.25	0.125	0.0625



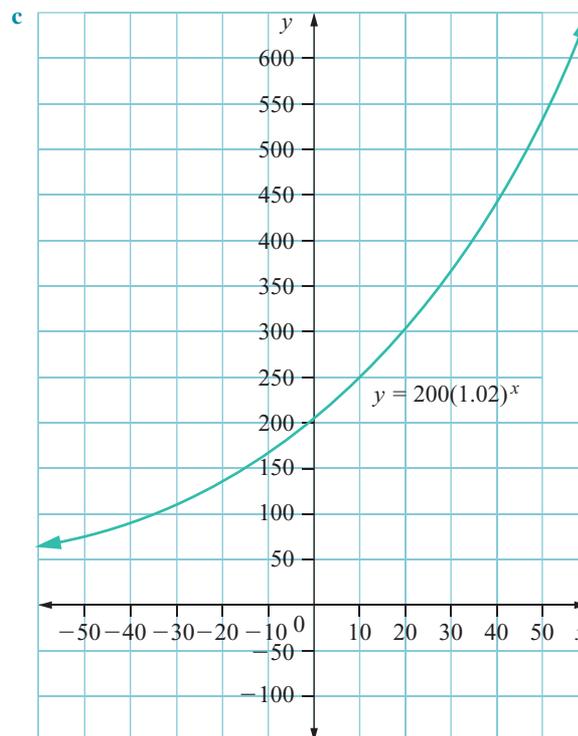
- d  $y$  decreases at a decreasing rate of change.
- e No, all  $y$ -values are positive as  $0.5$  to any power always gives a positive result.
- f 1
- g No  $x$ -intercepts



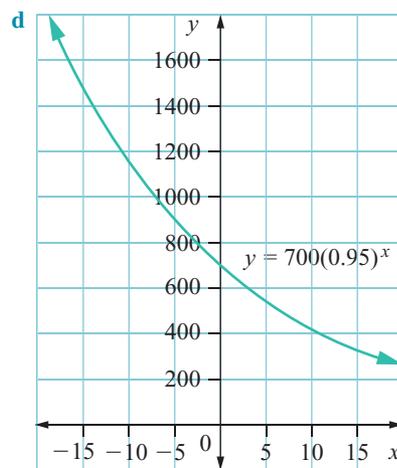
y-intercept at 1.4



y-intercept at 3.5



y-intercept at 200

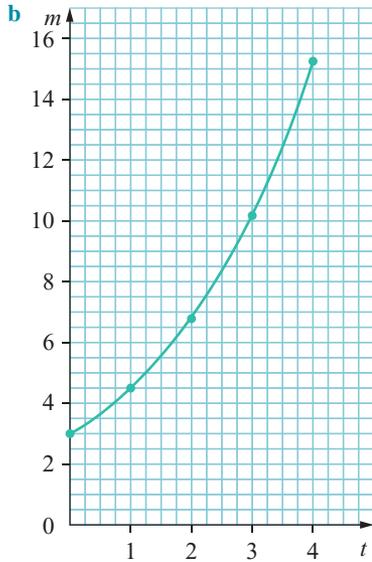


y-intercept at 700

- 4 Exponential growth: parts **a** and **c**; exponential decay: parts **b** and **d**
- 5 For equations of the form  $y = ka^x$ , exponential growth is when  $a > 1$  and exponential decay is when  $0 < a < 1$ .
- 6 **a** Population model matched for  $t \geq 0$ .  
**b** Exponential growth  
**c** 2000  
**d** About 7000 (7111)  
**e** About 14 years

**7 a**

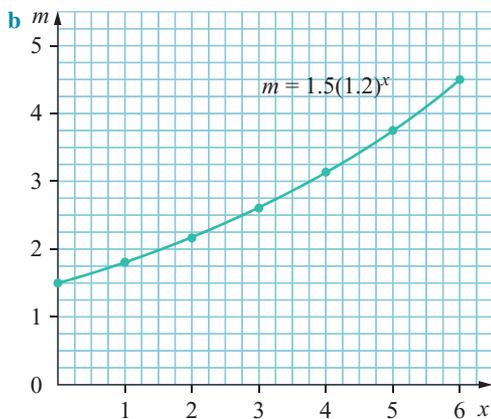
$t$	0	1	2	3	4
$m$	3	4.5	6.75	10.125	15.1875



- c** 3 mg  
**d** About 8 mg (8.267 mg)  
**e** About 1.5 days (1.7 days)  
**f** Mass increases by 50% each day.  
**g** The strength of this model is that we can predict the mass using the equation or graph for values of  $t$  from 0 to 4. For values of  $t$  that are more than 4 (outside the range of recorded values), our predictions may not be reliable due to unknown factors.

**8 a**

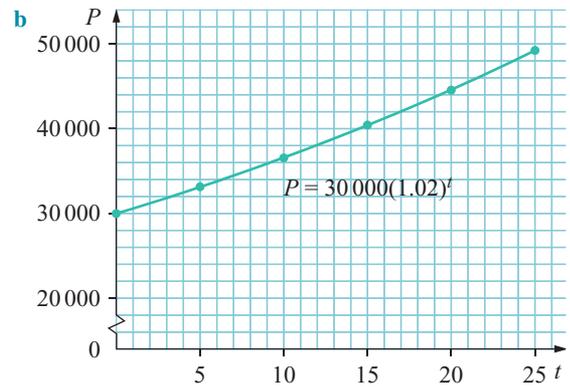
$x$	0	1	2	3	4	5	6
$m$	1.5	1.8	2.2	2.6	3.1	3.7	4.5



- c** About 2.8 kg (2.839 kg)  
**d** About 4.5 months  
**e** Mass increases by 20% each month.  
**f** Growth rate may change after 6 months.

**9 a**

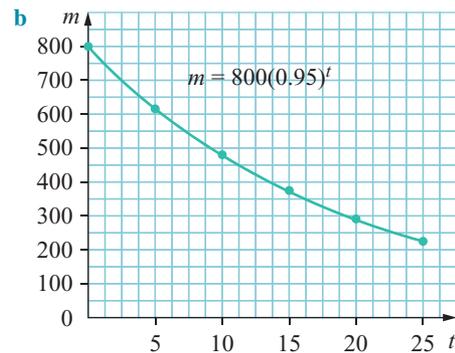
$t$	0	5	10	15	20	25
$P$	30000	33122	36570	40376	44578	49218



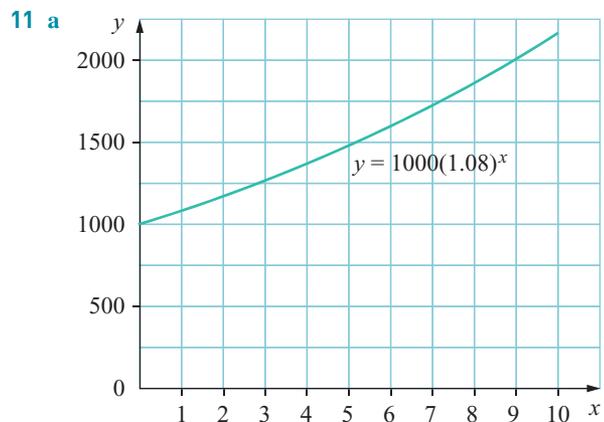
- c** Initial population  
**d** About 43000 (42 847)  
**e** About 13.2 years  
**f** 35 years  
**g** Population increases by 2% each year; can be seen from factor of 1.02 in equation which means that each year the population is 1.02 times or 102% of the previous year's population. This is an increase of 2% per year.

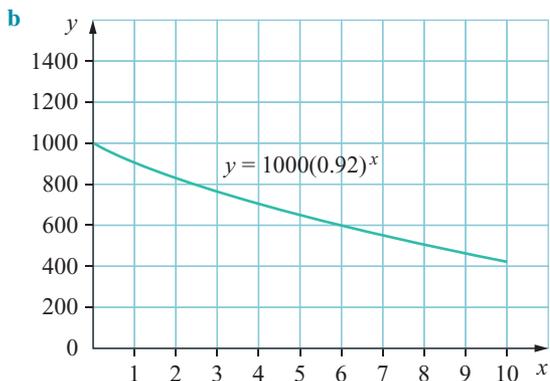
**10 a**

$t$	0	5	10	15	20	25
$m$	800	619	479	371	287	222

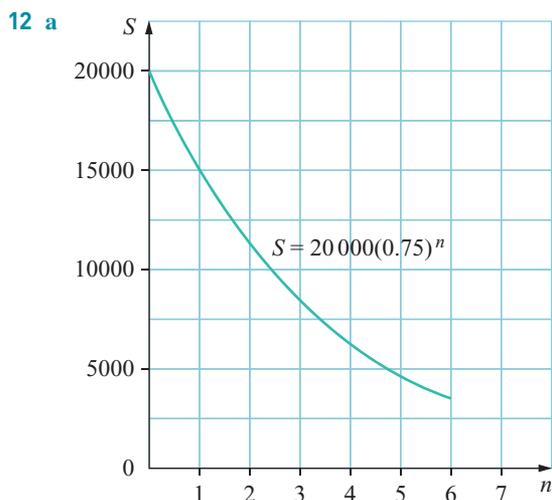


- c** As  $t$  increases,  $m$  decreases at a decreasing rate of change.  
**d** 800 g  
**e** About 250 g (246 g)  
**f** About 5.6 years  
**g** About 41 years  
**h** 5% per year, which can be seen from the factor of 0.95 in the equation; this means that each year's mass is 0.95 times or 95% of the mass for the previous year. This is a decrease of 5% per year.



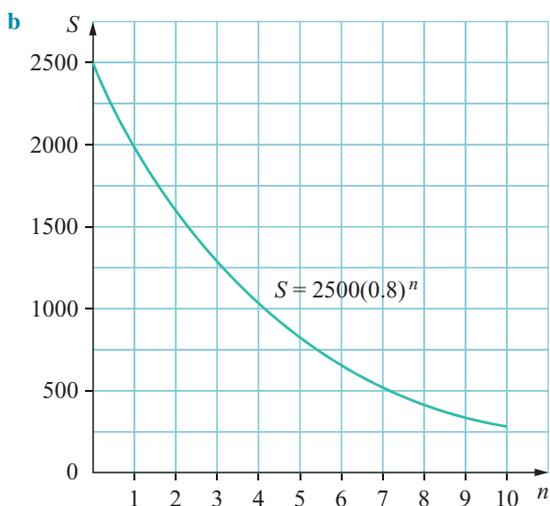


- c**  $y = 1000(0.92)^x$  as graph shows exponential decay.  
**d**  $y = 1000(1.08)^x$  as graph shows exponential growth  
**e** \$1337.71



- b** Exponential decay; graph decreases at a decreasing rate with time  
**c** Initial value (purchase price) of new motorcycle  
**d** \$11 250  
**e** About 4.2 years

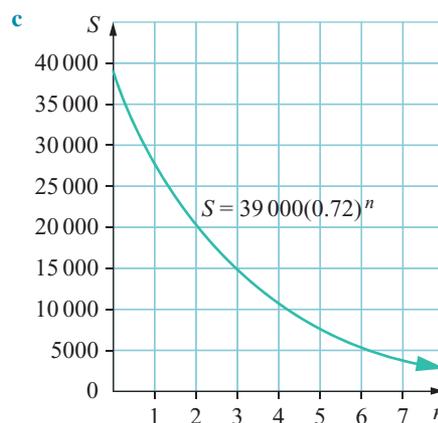
- 13 a**  $V_0 = \$2500$ ,  $r = 20\% = 0.2$  so  $S = V_0(1 - r)^n$  becomes  $S = 2500(1 - 0.2)^n$  or  $S = 2500(0.8)^n$



- c** About \$1300 (\$1280)  
**d** About 7.2 years

- 14 a**  $S = 39000(0.72)^n$ , where  $S$  is the salvage value in dollars after  $n$  years

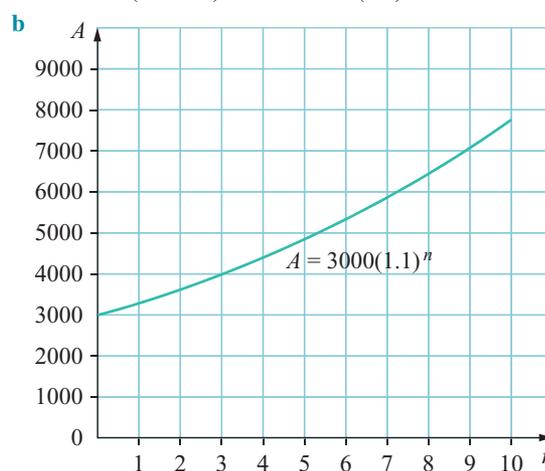
**b** Exponential decay



**d** About \$7500 (\$7546.18)

**e** About 2.1 years

- 15 a**  $P = 3000$ ,  $r = 10\% = 0.1$  so  $A = P(1 + r)^n$  becomes  $A = 3000(1 + 0.1)^n$  or  $A = 3000(1.1)^n$

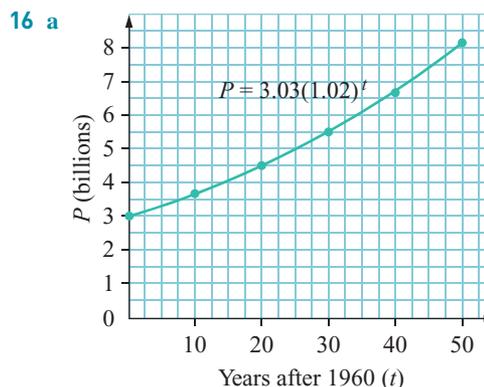


**c** Initial investment

**d** About \$4500 (\$4392.30)

**e** About 7.3 years

**f**  $A = 4000(1.1)^n$ . After 4 years, \$3000 investment becomes \$4392.30 while \$4000 investment becomes \$5856.40. Extra \$1000 investment yields an extra \$464.10.



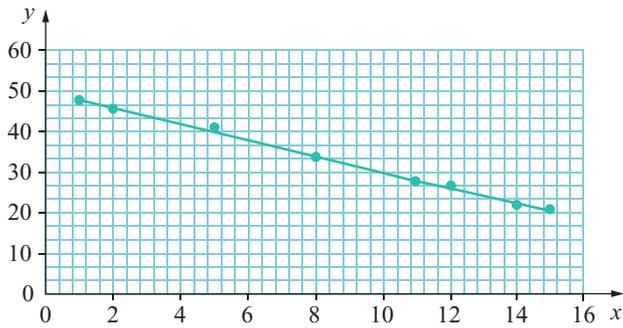
**b** About 4 billion (4.078 billion)

**c** About 6.8 billion (6.824 billion)

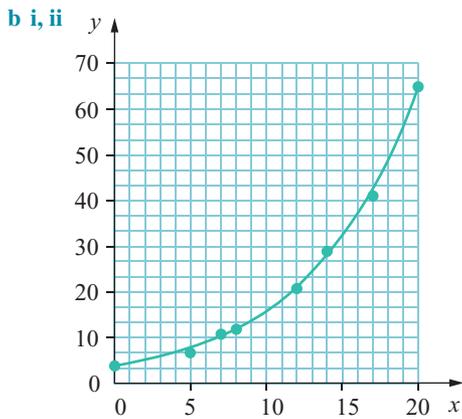
- d i**  $t = 14.03$  years; 1975      **ii**  $t = 25.3$  years; 1986
- iii**  $t = 42.3$  years; 2003      **iv**  $t = 60.3$  years; 2021
- e** Disease, war, health, etc.

**EXERCISE 8F** DETERMINING AN APPROPRIATE MODEL

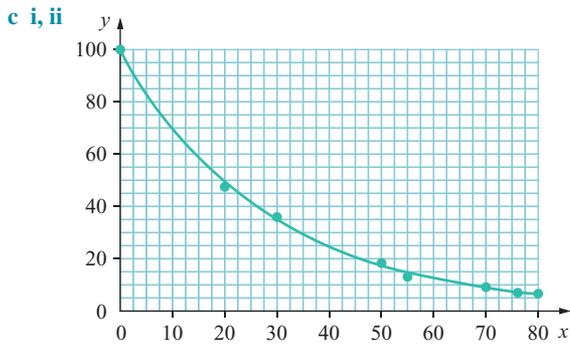
**1 a i, ii**



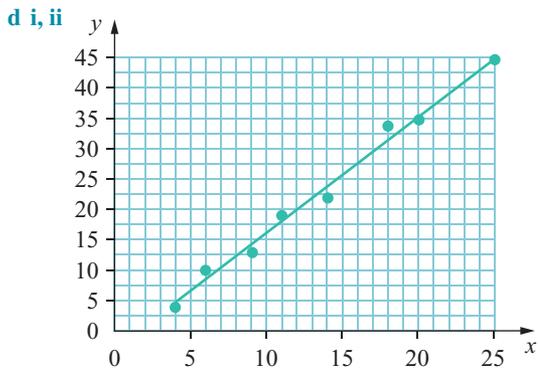
**iii** Linear



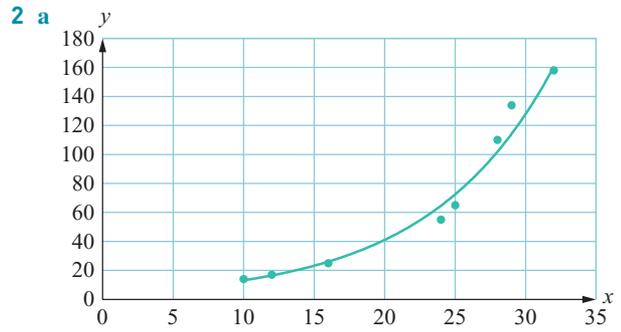
**iii** Exponential



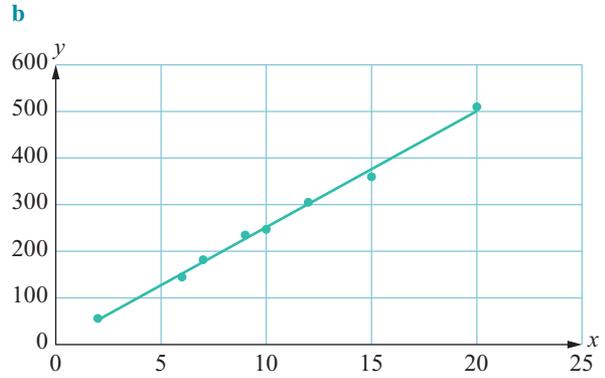
**iii** Exponential



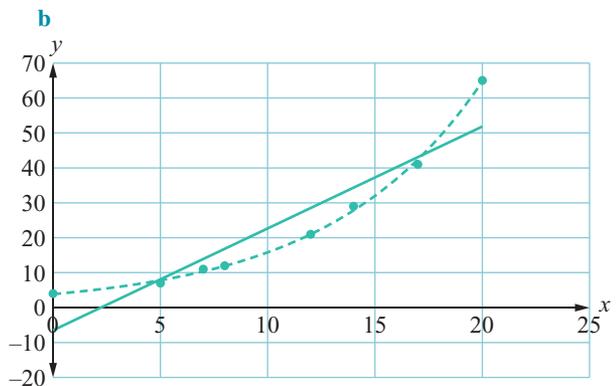
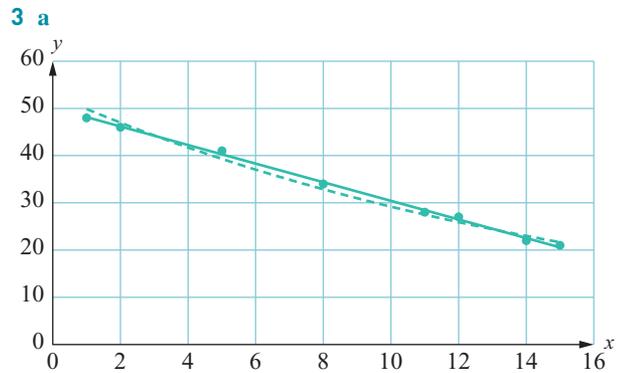
**iii** Linear

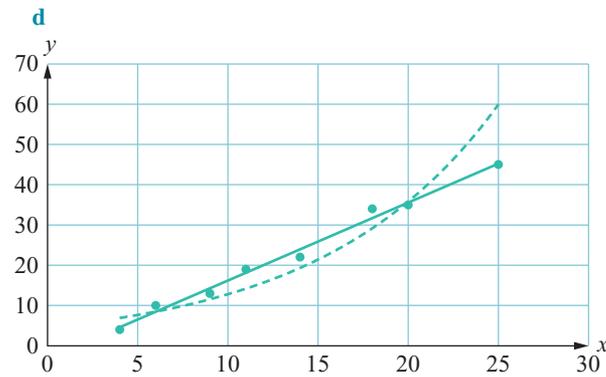
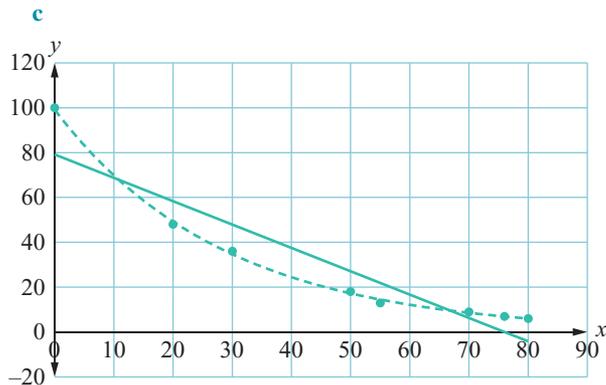


Exponential model is best.

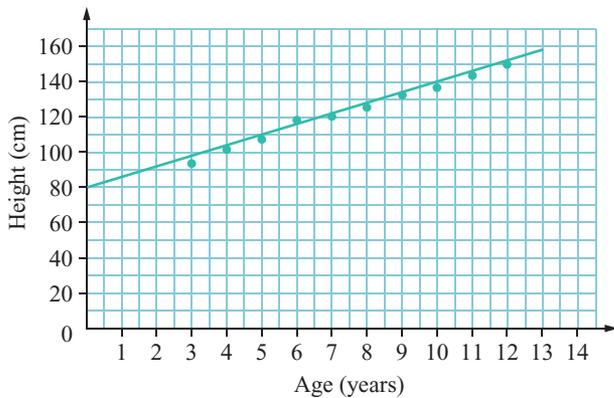


Linear model is best.



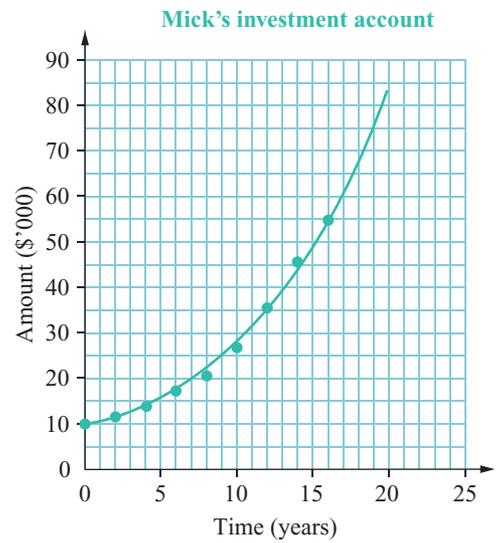


**4 a, b, e**

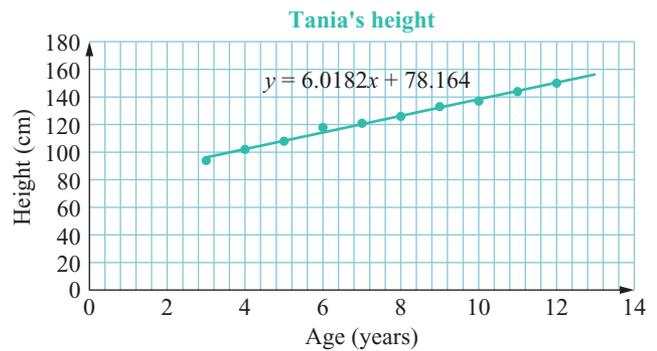


- c** Linear
- d** About 130 cm
- e** About 158 cm
- f** Model becomes less reliable when the age is outside the recorded values. Tania may have a growth spurt from early to mid-teens.
- g** Answers will vary. One possible answer is: gradient is about 6 and vertical intercept is about 80, so equation is  $h = 6a + 80$ , where  $h$  is Tania's height (in cm) and  $a$  is her age (in years).
- h** Answers will vary.  
When  $a = 8.5$ ,  $h = 6 \times 8.5 + 80 = 131$   
When  $a = 13$ ,  $h = 6 \times 13 + 80 = 158$

**5 a** \$10 000  
**b, c, g**



- d** Exponential
  - e** About \$25 000
  - f** About 7 years
  - g** About \$85 000
- 6 a, b**



- c** When  $a = 8.5$ ,  $h = 129.3187 \approx 129.3$
- d** When  $a = 13$ ,  $h = 156.4006 \approx 156.4$

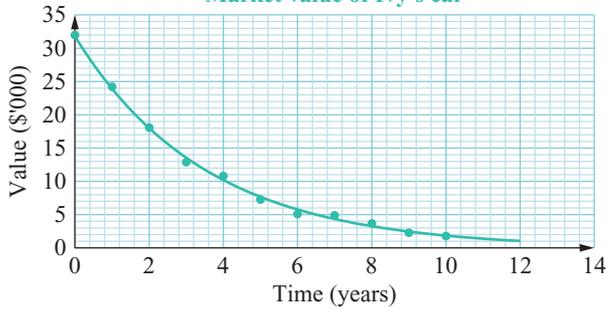
**7 a, e**



- b** Exponential growth
- c** About \$25 000 (\$25 323.34)
- d** About 6.8 years (6.8448 years)
- e** About \$85 000 (\$84 455.63)

8 a, b, f

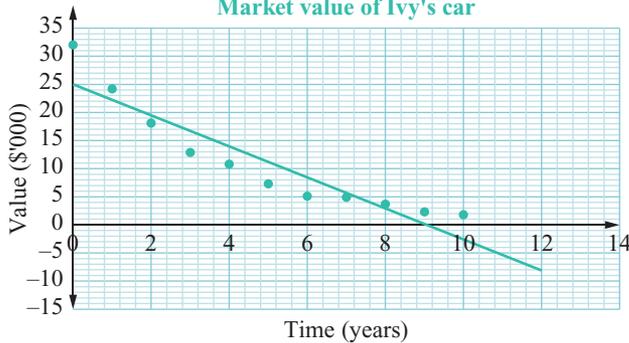
Market value of Ivy's car



- c Exponential
- d About \$9000 (\$8853)
- e About 2.5 years (2.64 years)
- f About \$1000 (\$1052)
- g Declining-balance depreciation

9 a

Market value of Ivy's car



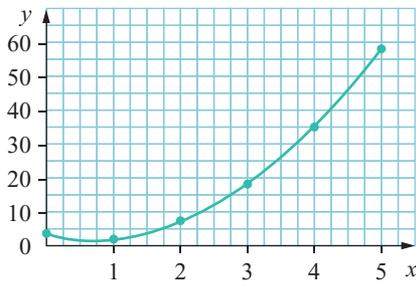
- b About \$12000 (\$12 570 using spreadsheet equation  $y = -2.7591x + 24.986$ ). Using the linear model gives a higher estimate for the market value, which does not seem reasonable compared to the data recorded for 4 and 5 years.
- c About 3.5 years (3.62 years using spreadsheet equation  $y = -2.7591x + 24.986$ ). Using the linear model, the car will be about one year older before it has a market value of \$15000. The linear trendline is above the recorded data points between 2 and 6 years and so the market value is overestimated.
- d No value as trendline shows a negative market value ( $-\$8123$  using spreadsheet equation  $y = -2.7591x + 24.986$ ). Using the exponential model, Ivy's car is predicted to still have some market value.
- e Not choosing the best model may produce estimated and predicted data that are not accurate or reliable.

REVIEW MULTIPLE CHOICE

- 1 B    2 C    3 A    4 D    5 B  
 6 D    7 A    8 C    9 C

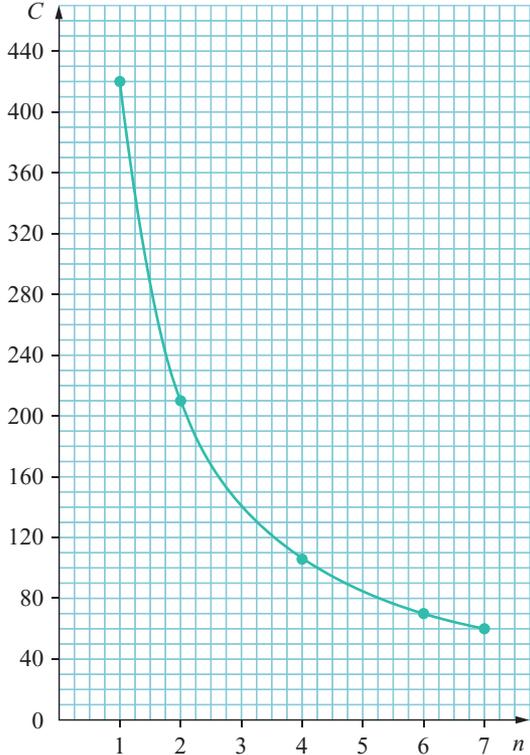
REVIEW SET 1

1 a



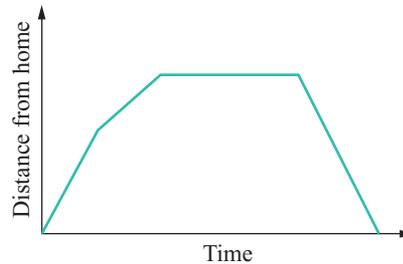
b No, it is a quadratic relationship.

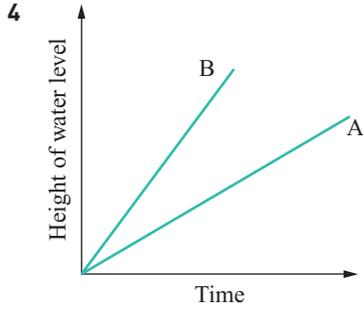
2 a, b



- c Inverse relationship
- d About \$85 (\$84)
- e 3 people
- f Cost ( $C$ ) cannot be zero or negative, or the number of people ( $n$ ) cannot be zero, or the number of people ( $n$ ) must be positive whole number values only. There is a limit to how many people can fit on the bus.

3



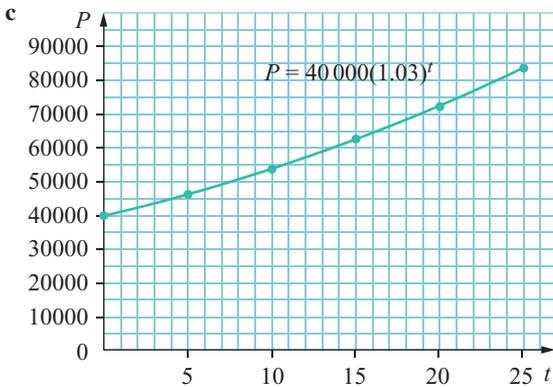


- 5 a 11 L  
 b 45 km  
 c Gradient =  $\frac{7}{65} \approx 0.1$ ; represents the fuel consumption rate in litres per kilometre (0.1 L/km)  
 d  $p = 0.1d$   
 e About 22 L

6 a 40 000

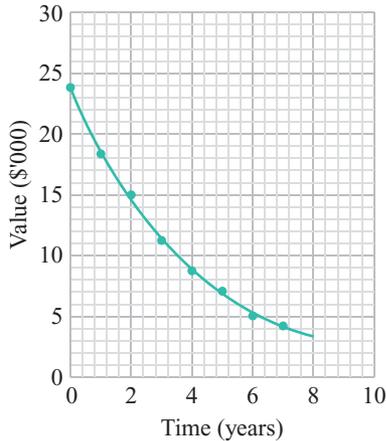
b

$t$	0	5	10	15	20	25
$P$	40 000	46 371	53 757	62 319	72 244	83 751



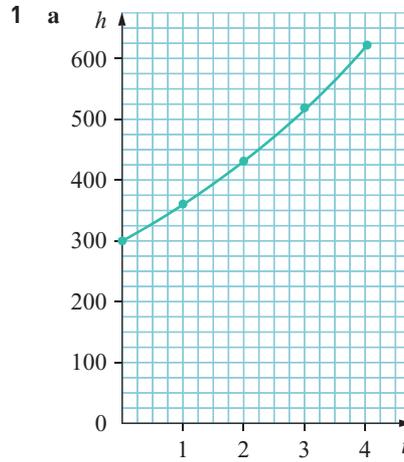
- d About 68 000 (68 097)  
 e About 13 years  
 f About 31 years  
 g 3% per year; this can be seen from the factor of 1.03 in the equation which means that each year's population is 1.03 times or 103% of the population of the previous year. This is an increase of 3% per year.

7 a, b, f **Value of Tristan's motorbike**

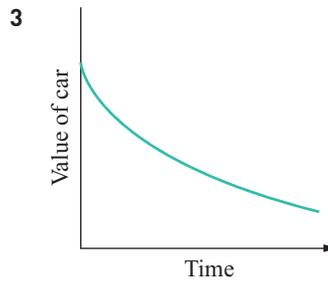
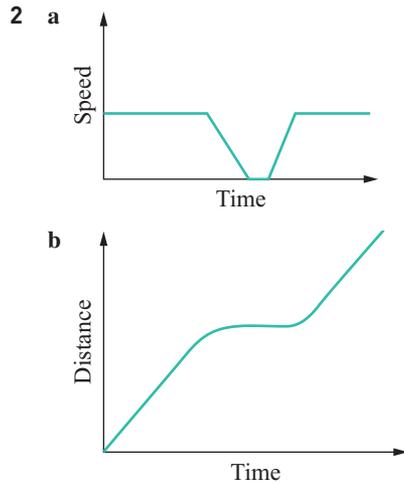


- c Exponential  
 d About \$13 000 (\$12 909)  
 e About 3.5 years (3.53 years)  
 f About \$3000 (\$3300)  
 g Declining-balance depreciation

**REVIEW SET 2**

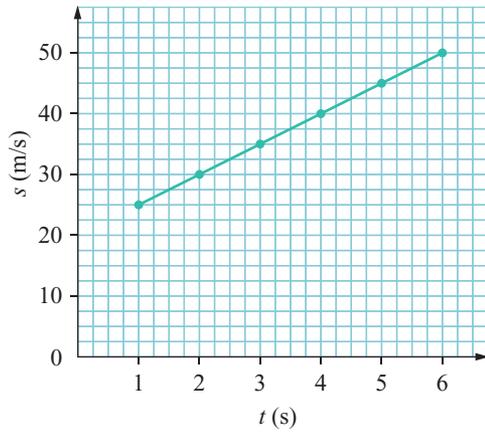


- b Exponential  
 c About 570 mm (567.88 mm)  
 d About 1.6 days (1.578 days)  
 e About 750 mm (746.5 mm)  
 f Model not reliable for values outside the range of the recorded data; there could be unknown factors that affect the height of the plant after 5 days.



- 4 Answers will vary. One possible answer is: The graph represents the distance travelled by a car over time. The car initially accelerates and then comes to a sudden stop and remains stationary. Another possible answer is: The graph represents the height of the water level in a vase over time. The vase has sides that taper inwards so that the depth increases at a faster rate. Once the water reaches the top of the vase, it overflows, so the height of the water level remains constant.

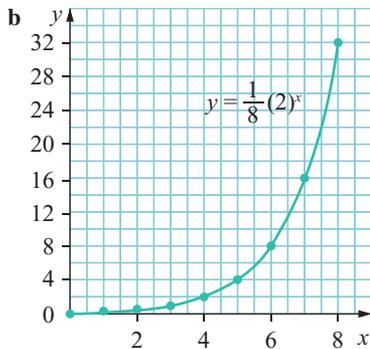
5 a



- b Yes, graph is a straight line and there is a common difference of 5 between the values of  $s$  in the table for each increase of 1 second.
- c 5; represents the increase in speed each second (acceleration of 5 m/s or 5 m/s<sup>2</sup>)
- d 20; represents the initial speed (20 m/s)
- e  $s = 5t + 20$
- f 60 m/s
- g No, there is a limit to the speed at which the racing car can travel.

6 a

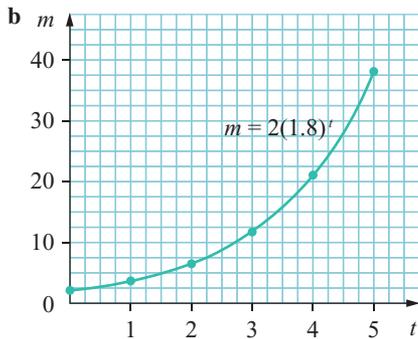
$x$	0	1	2	3	4	5	6	7	8
$y$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32



- c Exponential graph
- d  $y$  increases at an increasing rate.

7 a

$t$	0	1	2	3	4	5
$m$	2	3.6	6.48	11.664	20.9952	37.79136



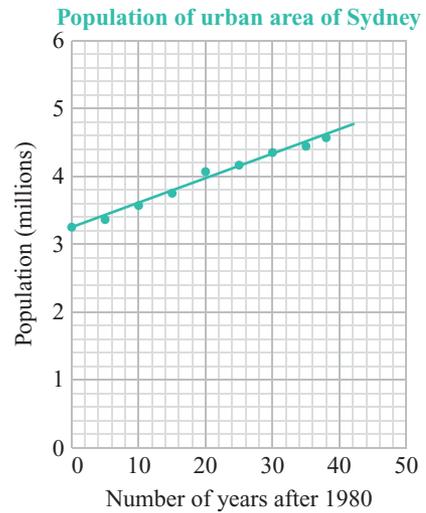
- c 2 mg
- d About 16 mg (15.65 mg)

e About 1.8 days (1.869 days)

f 80%

g The strength of this model is that we can predict the mass using the equation or graph for values of  $t$  from 0 to 5. For values of  $t$  that are more than 5 (outside the range of recorded values), our predictions may not be reliable due to unknown factors.

8 a, b, e



c Linear

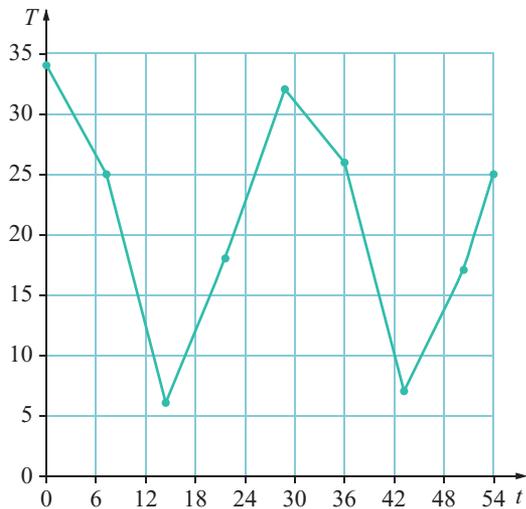
d About 4.3 million

e About 4.9 million (4 896 500 using equation of trendline  $y = 0.036x + 3.2765$ )

f Linear rather than exponential relationship seen due to factors such as decline in birth rate, people moving away from the area.

## REVIEW SET 3

1 a

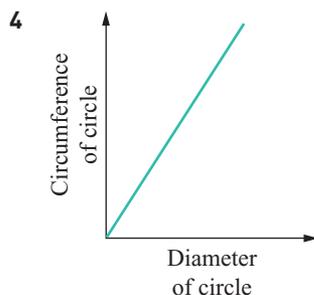
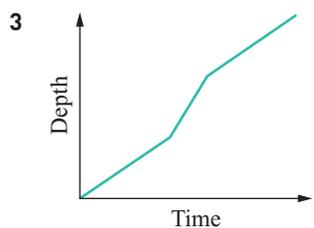
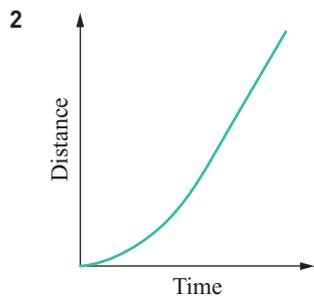


b 18°C

c About 15°C

d About 25°C

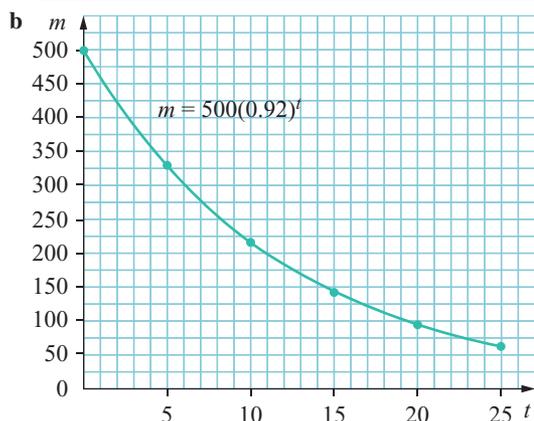
e About 19°C



- 5 a \$420  
 b About 83  
 c Gradient =  $\frac{420}{70} = 6$ ; represents the income made per pie (\$6/pie)  
 d  $C = 6n$   
 e \$600

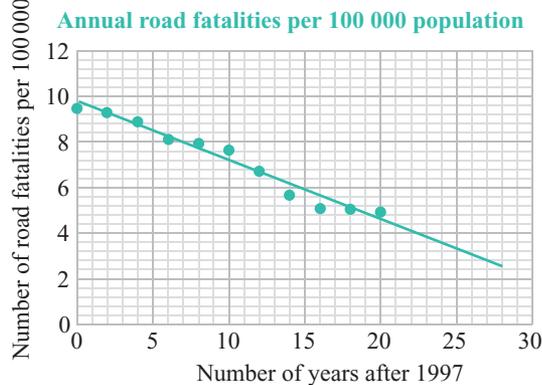
6 a

$t$	0	5	10	15	20	25
$m$	500	330	217	143	94	62



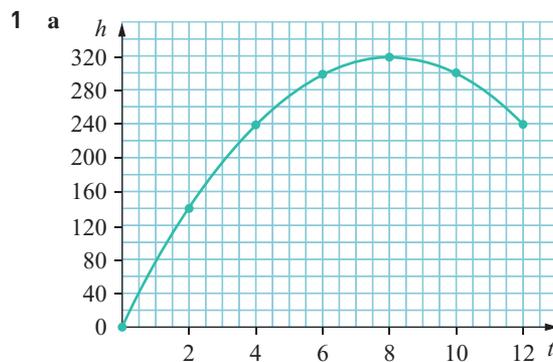
- c 500 g  
 d About 75 g (73.467 g)  
 e About 3 years (2.67 years)  
 f About 28 years (27.6 years)  
 g 8% per year; this can be seen from the factor of 0.92 in the equation, which means that the following year's mass is 0.92 times or 92% of the current year's mass. This is a decrease of 8% per year.

7 a, b

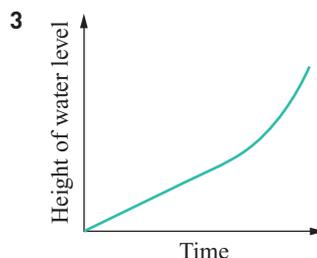
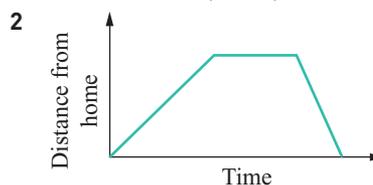


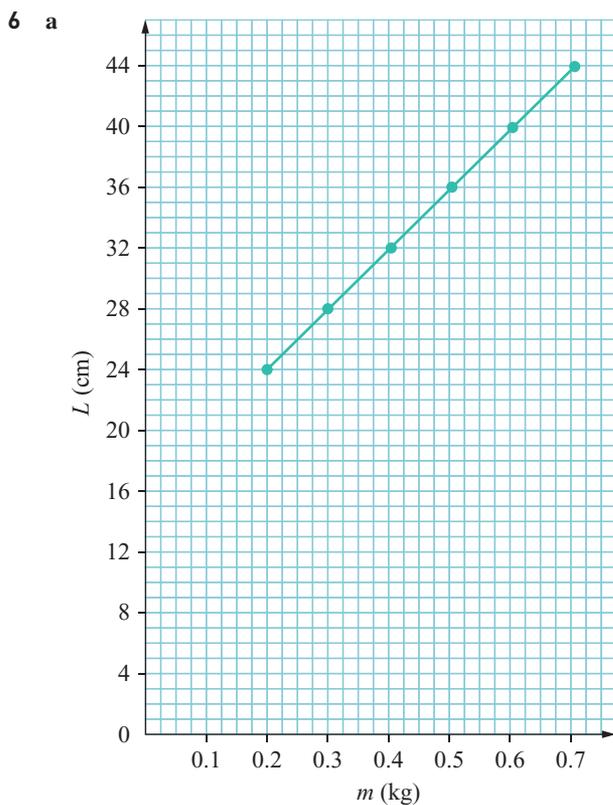
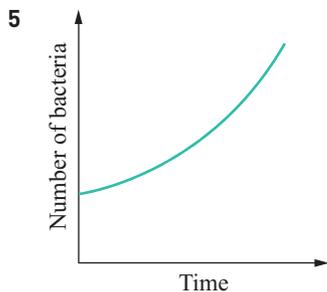
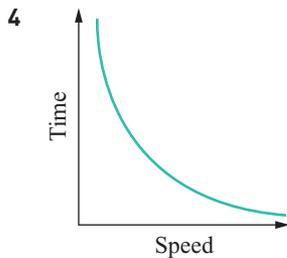
- c Linear  
 d About 6 per 100 000 (5.9068 per 100 000)  
 e Answer may vary. Actual recorded number is close to the model prediction or slightly lower.  
 f Extending the trendline gives a value of about 2.5 per 100 000; substituting 28 into equation of trendline ( $y = -0.26x + 9.8068$ ) gives 2.5268 per 100 000.  
 g Improved vehicle safety features, increased road safety education and better road infrastructure and conditions could lower the number of fatalities; changes to speed limits and higher numbers of vehicles on the road could increase the number of fatalities.

**REVIEW** SET 4



- b Maximum height is 320 m when  $t = 8$  s.  
 c About 270 m (275 m)  
 d Just over 3 s (3.1 s)  
 e About 200 m (195 m)

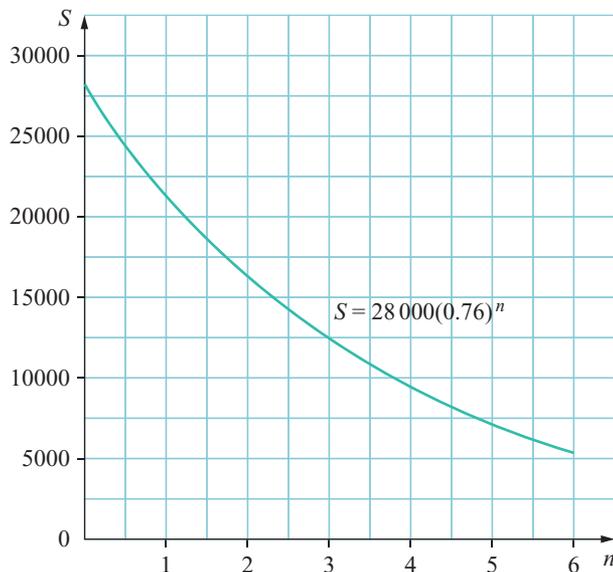




- b Gradient =  $\frac{4}{0.1} = 40$ ; represents the increase in length of the spring (in cm) for each additional kilogram of weight (40 cm/kg)  
 c Vertical intercept = 16; it represents the initial length of the spring before any weight is hung from it (16 cm)  
 d  $L = 40m + 16$   
 e 31 cm

f Model predicts a length of 56 cm, which is much lower than the actual result. So, for weights of 1 kg or more, the model is not suitable (the spring has lost its elasticity). Also, for weights between 0.7 kg and 1 kg, the model is not reliable since the values are outside the range of the recorded data.

7 a



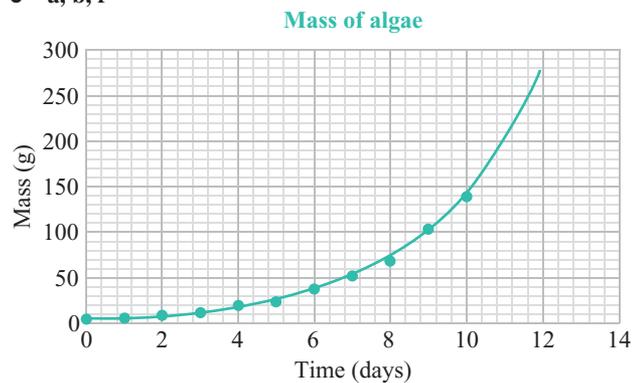
b Exponential decay; as  $n$  increases,  $S$  decreases at a decreasing rate

c Initial value (purchase price) of the new car

d About \$12 300 (\$12 291.33)

e After about 3.75 years

8 a, b, f



c Exponential

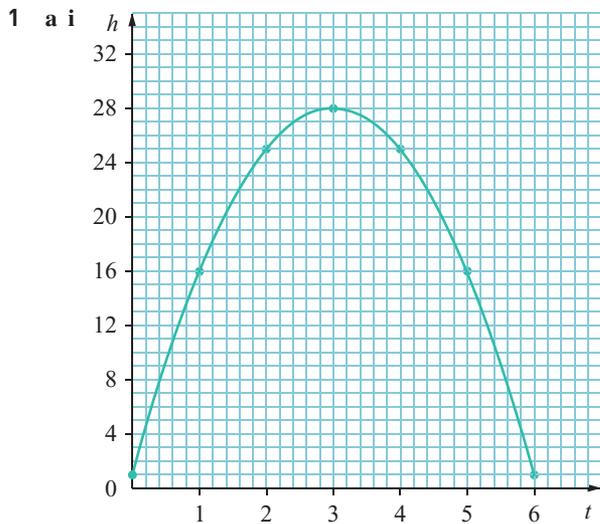
d About 12 g (11.688 g)

e About 9 days (8.902 days)

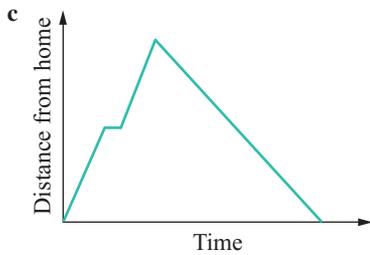
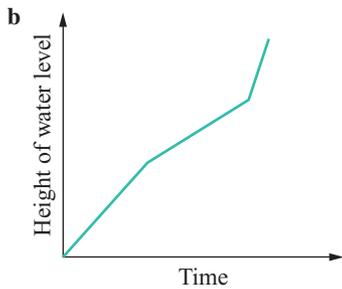
f About 280 g (282.562 g)

g Mass of algae could be affected by changes in the amount of light and the temperature of the water.

**REVIEW** PRACTICE EXAMINATION QUESTION



- ii No, it is not a straight line.  
 iii Maximum height is 28 m when  $t = 3$  s.  
 iv About 1.4 s (1.367 s)



- d i Linear  
 ii About 40 m (40.7 m)  
 e i Exponential growth; as  $x$  increases,  $M$  increases at an increasing rate.  
 ii 1.5  
 iii About 2.7 kg (2.657 kg)  
 iv About 11.5 days (11.527 days)  
 v 10%

**CHAPTER 9 NETWORK CONCEPTS**

**ARE YOU READY?**

- 1 B    2 B    3 D    4 B    5 C  
 6 B    7 B    8 B

**EXERCISE 9A** INTRODUCTION TO NETWORKS

- 1 a 4    b 4    c 4    d 1  
 e 4    f 5    g 2    h 9  
 2 a 3    b 4    c 2    d 3  
 e 6    f 4    g 4    h 12

3 a i

Vertex	A	B	C	D
Degree	1	2	2	1

ii 6

b i

Vertex	A	B	C	D
Degree	2	2	2	2

ii 8

c i

Vertex	A	B	C	D
Degree	1	1	1	1

ii 4

d i

Vertex	A
Degree	6

e i

Vertex	A	B	C	D
Degree	3	3	3	3

ii 12

f i

Vertex	A	B	C	D	E
Degree	4	1	1	1	1

ii 8

g i

Vertex	A	B
Degree	3	5

ii 8

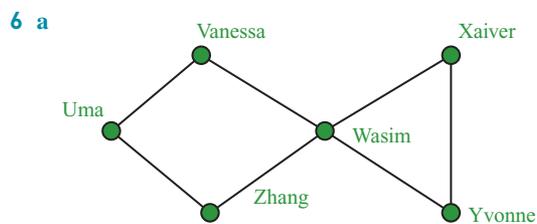
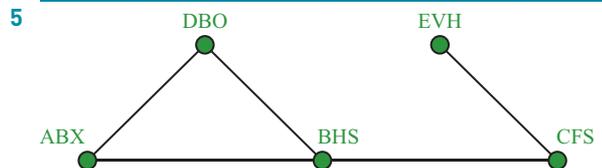
h i

Vertex	A	B	C	D	E	F	G	H	I
Degree	2	2	6	2	4	2	2	2	2

ii 24

4

Vertex	Ada	Ben	Carlos	Dalia	Eric
Degree	3	1	2	2	2



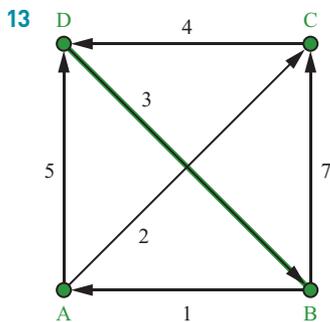
b

Vertex	Uma	Vanessa	Wasim	Xavier	Yvonne	Zhang
Degree	2	2	4	2	2	2

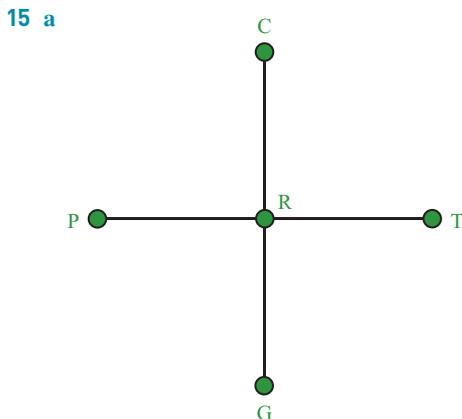
- c Wasim or Uma  
 d Uma  
 7 10 minutes

- 8 a 10 (including the start and end)  
 b 9  
 c 1  
 d 3  
 e Yes (look for a circle)
- 9 A
- 10 a 9 litres per second  
 b 8 litres per second  
 c 3 litres per second  
 d 4 litres per second  
 e 5 litres per second
- 11 C

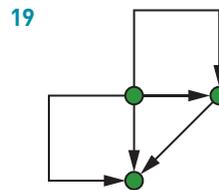
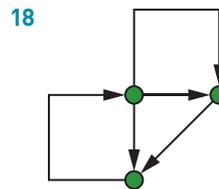
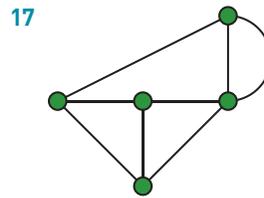
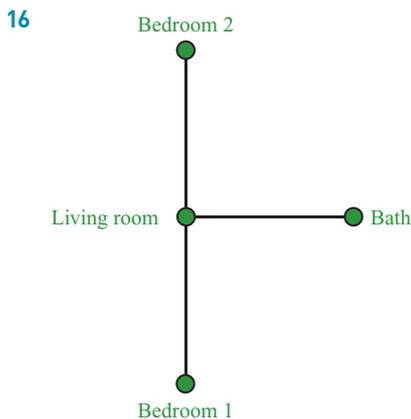
From vertex	To				
	A	B	C	D	E
A	-	3	1	5	-
B	-	-	1	-	2
C	-	-	-	-	3
D	-	-	1	-	4
E	-	-	-	-	-



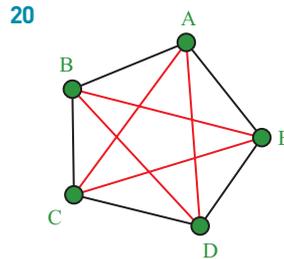
- 14 a 2      b 0      c 4      d 0  
 e 4      f 4      g 2      h 0



b 4



One vertex has zero indegree (cars cannot enter the intersection) and another has zero outdegree (cars cannot exit the intersection).



21 c 6

### EXERCISE 9B PATHS AND CYCLES

- 1 A, f, B, g, C, h, D, i, E, j, F
- 2 a E  
 b B
- 3 Corner C
- 4 a A, B, C, D. The start vertex is A and the end vertex is D.  
 b h, g, f. The start vertex is D and the end vertex is A.  
 c f, g, h, h, g, f. A is both the start and end vertex.
- 5 A
- 6 D
- 7 a ABCDE  
 b ABCIA  
 c FEDCB  
 d HCFEH
- 8 ABCDEFCHGECIA. This is neither a cycle nor a path.
- 9 a 12 minutes  
 b There are four paths that each take 12 minutes.
- 10 C
- 11 DE
- 12 DBACDEFGH
- 13 a A, a, B, b, C, d, D  
 b abd  
 c ABCD  
 d A, a, B, c, C, d, D  
 e acd  
 f ABCD  
 g No, because the walks are different but answers c and f are the same.

14 a

Vertex	A	B	C	D
Degree	1	2	2	1

This network has the walk ABCD.

b

Vertex	A	B	C	D
Degree	2	2	2	2

This network has the walk ABCDA.

c

Vertex	A	B	C	D
Degree	1	1	1	1

This network has no such walk because more than two vertices have an odd degree.

d

Vertex	A	B	C	D
Degree	3	2	3	2

This network has walk ABCADC.

e

Vertex	A	B	C	D
Degree	3	3	3	3

This network has no such walk because more than two vertices have an odd degree.

f

Vertex	A	B	C	D	E
Degree	4	1	1	1	1

This network has no such walk because more than two vertices have an odd degree.

- 15 a 1      b 2      c 0      d 3

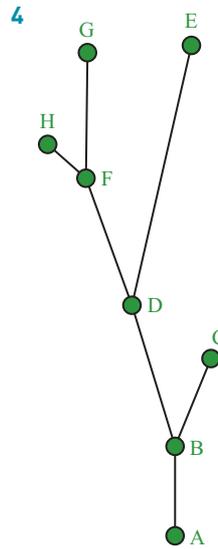


The garbage truck route is fgkjhi.

- 17 This task is not possible because there are more than two intersections with an odd degree.  
 18 There are zero vertices with odd degree so it is possible to cross every bridge exactly once.

**EXERCISE 9C** TREES

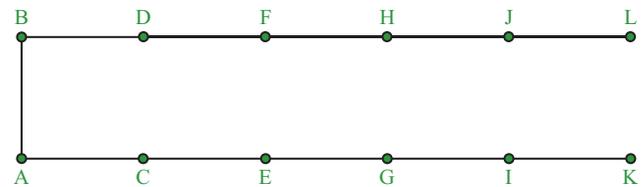
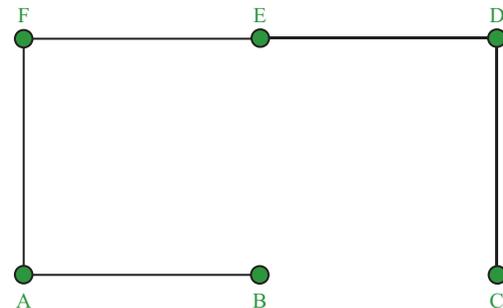
- 1 a ABCDL  
 b LDE  
 c BCFH  
 d GFCDK  
 2 A, B and C  
 3 a 7 vertices and 6 edges  
 b 5 vertices and 4 edges  
 c 4 vertices and 3 edges



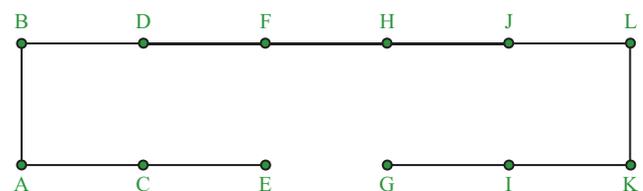
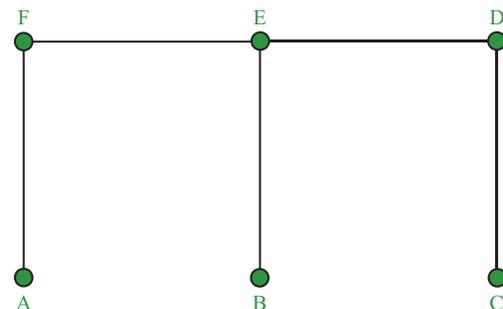
It is a tree.

- 5 B

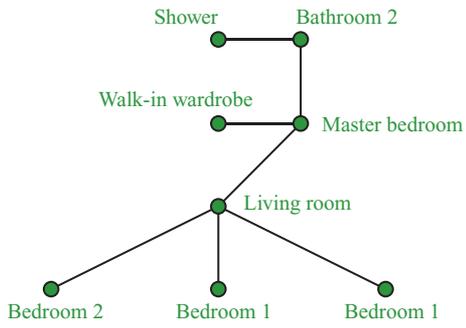
- 6 There are many correct answers. For a, b, c and d the same spanning tree can be chosen.



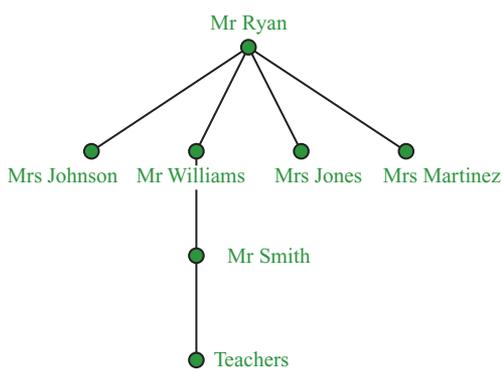
- 7 a, b, c, d



8 a



9 a



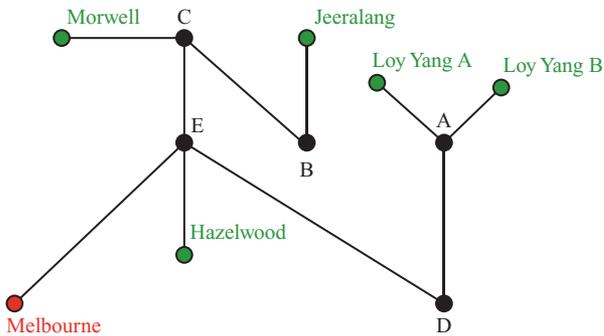
b It is a tree.

c 2

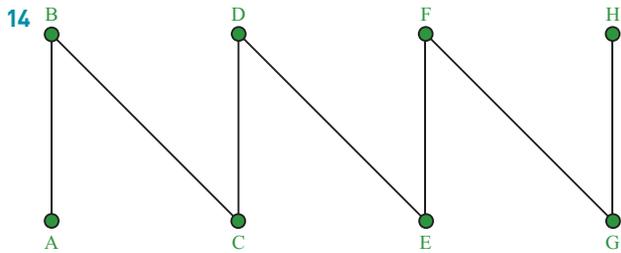
10 Part c

11 There is a left tree (with vertices A,B,C and D) and a right tree (with vertices E,F,G,H). Connecting D and G with an edge will turn the forest into a tree.

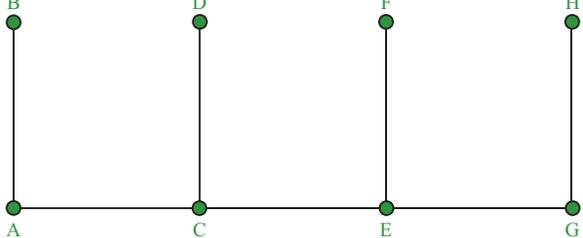
12 At most 3 edges can be removed.



13 9



15



16 Yes. Every tree is a forest, but not every forest is a tree.

17 Yes

18 a No                      b Yes

**EXERCISE 9D** MINIMUM SPANNING TREES

1 a 11                      b 12                      c 26

2 21.3 m

3 a 9

b 8

c 7

d 6 (smallest total edge weight)

4 a

Edge	AB	DC	BD	DE	AC	AE
Weight	2	2	3	4	4	5

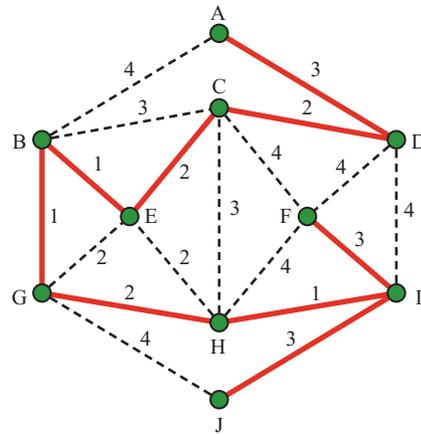
b

Edge	AB	DC	BD	AC	ED	AE
Weight	2	2	4	4	6	6

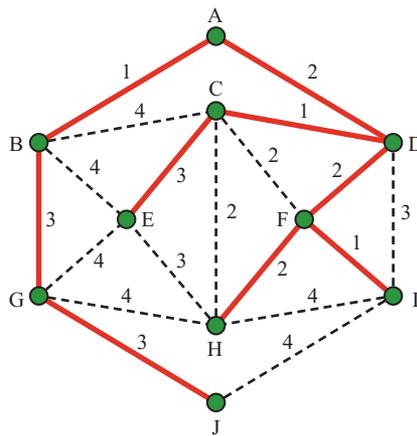
c

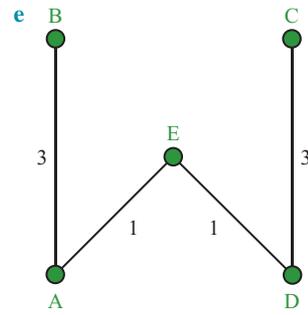
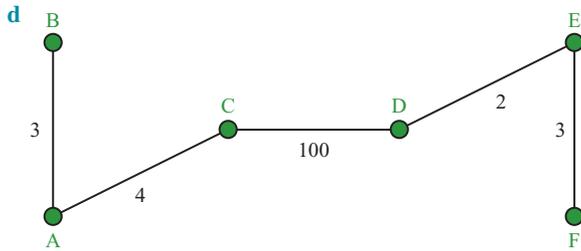
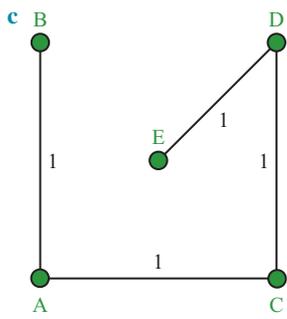
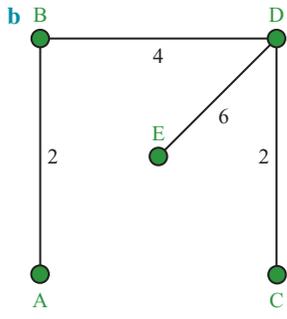
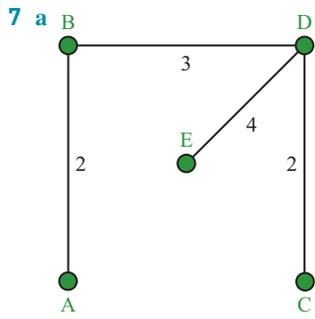
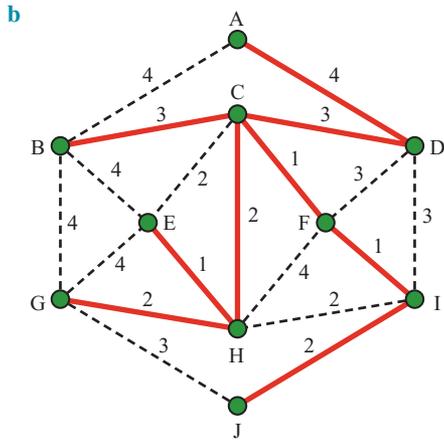
Edge	AB	AC	BD	CD	AE	ED
Weight	1	1	1	1	1	1

5



6 a





**8**

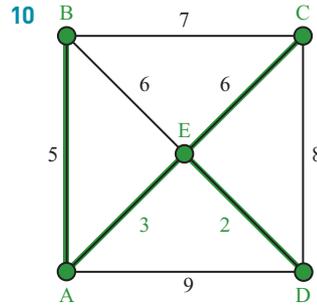
Edge	EB	EC	ED	EA
Weight	6	6	2	3

Edge ED has the smallest weight.

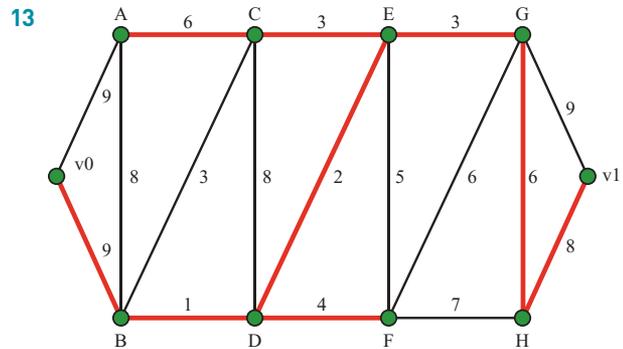
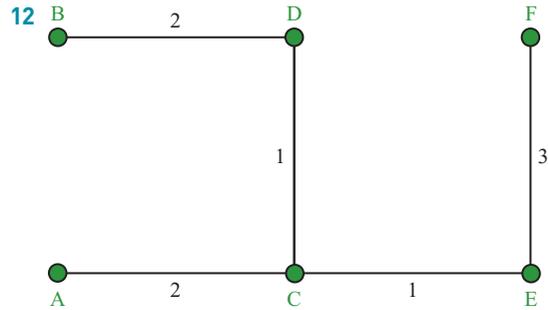
**9**

Edge	EB	EC	AB	CD
Weight	6	6	5	8

Edge AB has the smallest weight.

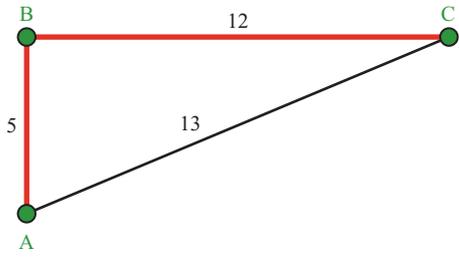


**11** See answer to question 7.

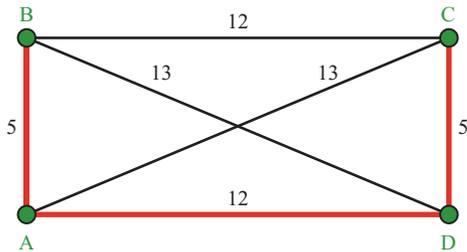


- 14** a \$7  
 b \$6  
 c \$7

15 a

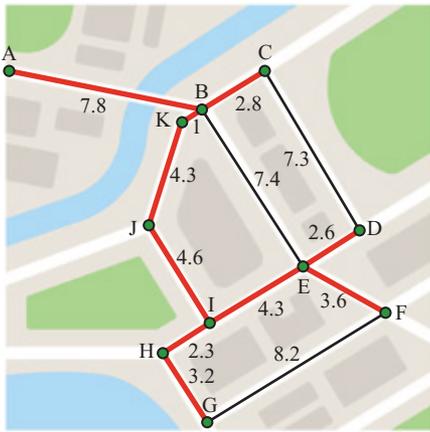


b



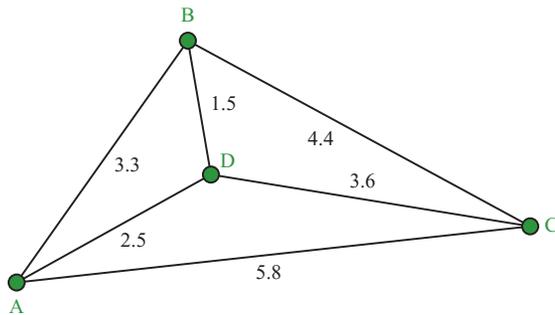
16 Connect AB, BC and CD.

17



36.5 km

18 a



b Connect rail lines along AD, BD and DC.

c \$38 million

d \$56.5 million

e \$18.5 million

19 a Perth to Adelaide, Adelaide to Melbourne, Melbourne to Canberra, Canberra to Sydney, Sydney to Brisbane, Adelaide to Darwin

b 8412 km

### EXERCISE 9E THE SHORTEST PATH

1 a 11

b 9

2 a ABCDEF

b AHG

c ABCDEFGH

3 a

Path	EA	EB	EC	ED
Total weight	3	6	6	2

b

Path	EA	EB	EC	EDA	EDC
Total weight	3	6	6	11	10

c

Path	EB	EC	EAB	EDC
Total weight	6	6	8	10

i EB and EC both have the smallest weight.

ii 6

4 a 7

b 8

c 2

d 107

e 4

5 12

6 7

7 a Direct flight (\$706)

b Via Mildura (\$922)

8 Yes

9 No

10 a Via the shops

b Via the depot

11 Dubbo, Sydney, Melbourne, Mount Gambier

### REVIEW MULTIPLE-CHOICE QUESTIONS

1 C

2 D

3 D

4 D

5 A

6 D

7 A

8 C

9 C

10 D

### REVIEW SET 1

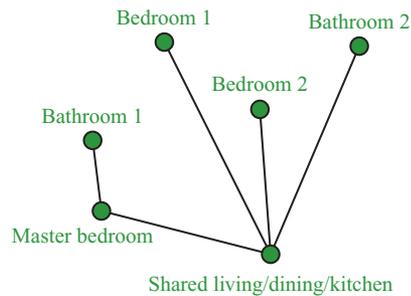
1

Vertex	1	2	3	4	5	6
Degree	3	3	2	1	3	4

2 No, there are more than two edges with odd degree.

3 4, 2, 1, 6, 5, 3

4



5 Yes

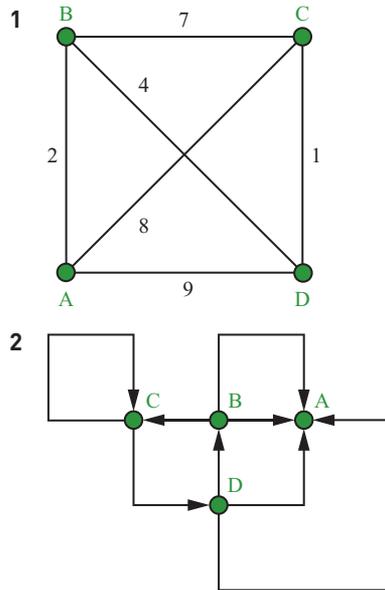
6

Vertex	Shared Living/Dining/Kitchen	Master Bedroom	Bathroom 1	Bedroom 1	Bathroom 2	Bedroom 2
Degree	4	2	1	1	1	1

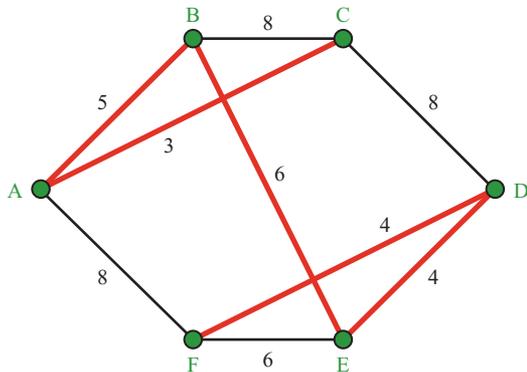
No, as there are more than two vertices with odd degree.

- 7 If you start with vertex A then Prim's algorithm chooses edges: AF, AB, BC, BE, ED.  
 8 15  
 9 The shortest path is v0, B, D, F, H, v1.

**REVIEW SET 2**



- 3 Vertex A has indegree 4 but outdegree 0, so cars cannot exit this intersection.  
 4 1, 3, 5, 4, 2, 6, 1  
 5 Remove any single edge.  
 6 Remove another edge (any one will do).  
 7 Yes  
 8 Yes  
 9

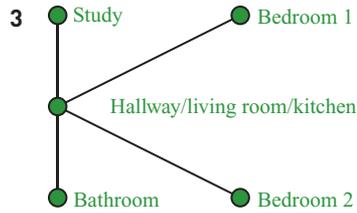


- 10 The shortest path is v0, B, C, E, G, v1.

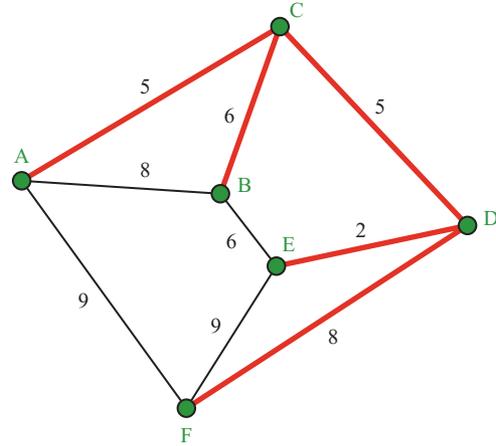
**REVIEW SET 3**

- 1 

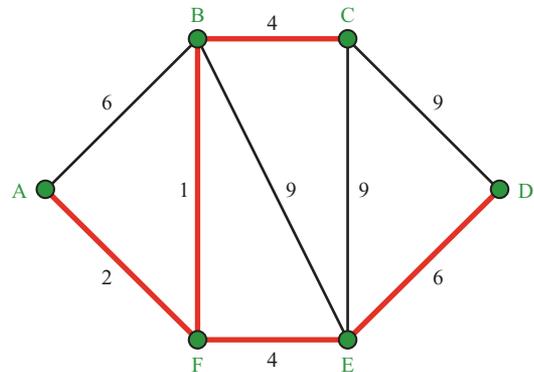
Vertex	1	2	3	4	5	6
Degree	1	2	3	1	1	4
- 2 No, there are more than two vertices with odd degree.



- 4 Yes  
 5



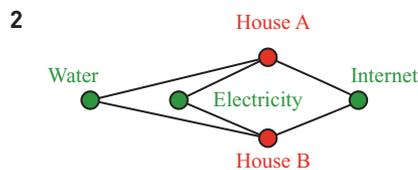
- 6 21  
 7 AFED  
 8 FBAFEBCEDC  
 9



**REVIEW SET 4**

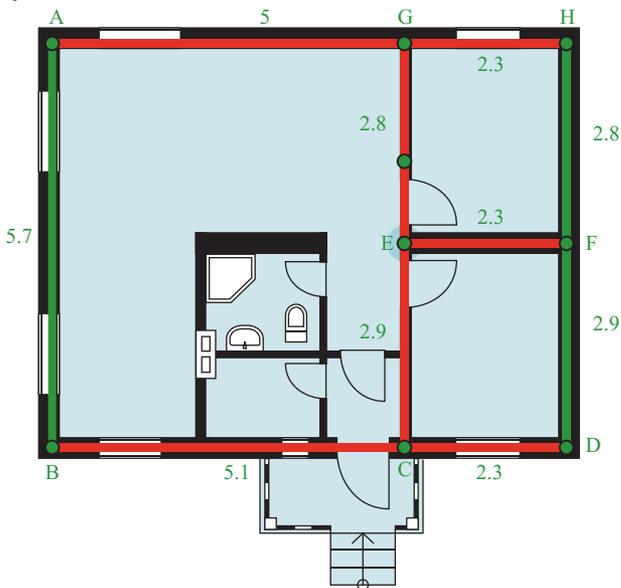
1 

Vertex	1	2	3	4	5	6
Degree	2	1	2	2	3	4



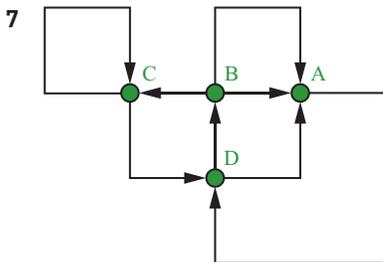
- 3 House A, Electricity, House B, Internet, House A, Water, House B

4



5 21.1

6 See answer to question 4



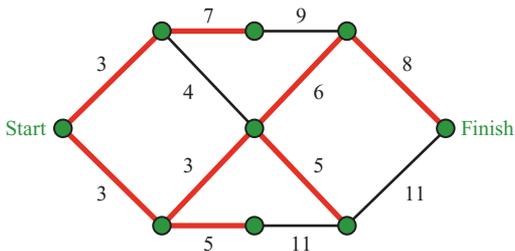
8  $v_0$ , B, C, E, F, H,  $v_1$

**REVIEW** PRACTICE EXAMINATION QUESTIONS

- a i A
- ii 4
- iii Either directly, or via New Town (both 20 km)
- iv Beachside, Hillside, Old Town, Beachside, New Town, Hillside

- b i ACE
- ii AC, AB, DE and BD

- c i A
- ii There are more than two vertices with odd degree.
- iii



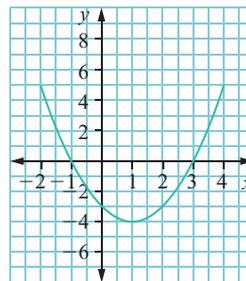
- d i Yes
- ii A
- e i 3
- ii 4
- iii Yes, since there are only two vertices with odd degree.

**CHAPTERS 7–9 CUMULATIVE REVIEW**

- 1 a \$11 122.79      b \$18 677.21
- 2 22.5%
- 3 a \$1983.78
- b \$1575.58
- c \$1288.36
- d \$1230.26
- 4 a \$775.93      b \$1053.87
- 5 a \$170 000      b About 165 months
- 6 a i 7/08/18      ii 55 days
- b i \$28.77      ii 3%
- c \$9041
- d i 0.0518% (to 3 s.f.), 0.000 518 (to 3 s.f.)
- ii 0.0627% (to 3 s.f.), 0.000 627 (to 3 s.f.)

- 7 \$11.40
- 8 \$19.92
- 9 \$177.40

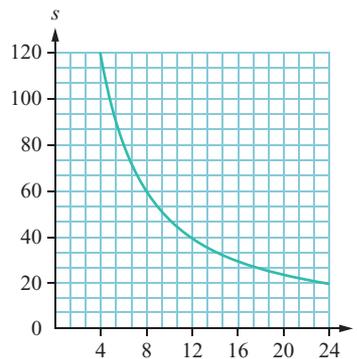
10 a, b



c Not a linear relationship as graph is not a straight line; relationship is quadratic.

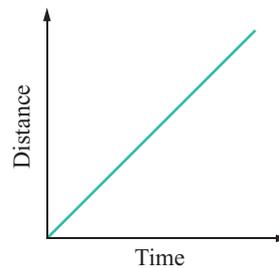
d About -2 (-1.75)

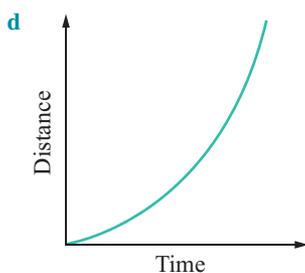
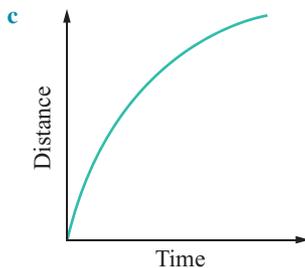
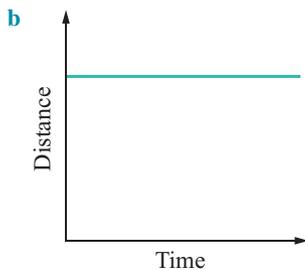
11 a



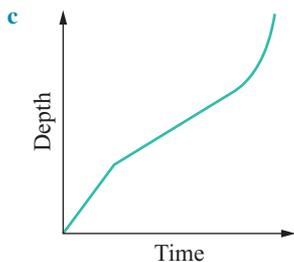
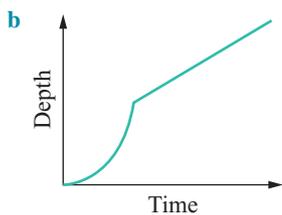
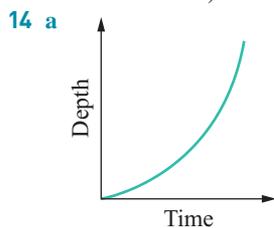
- b Inverse relationship
- c About 70 km/h (68.6 km/h)
- d About 10 h (10.7 h)

12 a



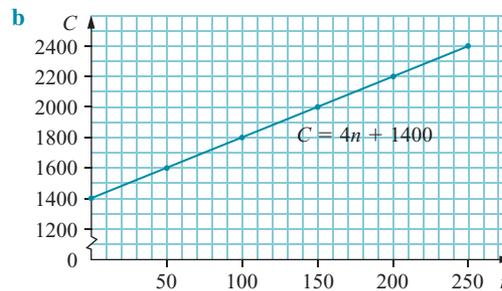


- 13 a** A person travels at a constant speed from home before stopping for a period of time. The person then travels home at a faster constant speed.
- b** A person accelerates from zero speed to a particular speed (speed is increasing at a constant rate) and then maintains this particular speed for a period of time. The person then decelerates to a stop (speed is decreasing at a constant rate).



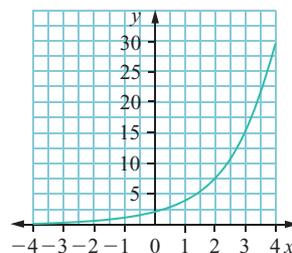
- 15**  $y = x - 3$
- 16 a** About 16 cm
- b** About 3.5 cm
- c** Gradient =  $\frac{7-0}{2.25-0} = 3.1$
- d** 0
- e**  $C = 3.1d$
- f** About 77.5 cm

- 17 a** \$2120



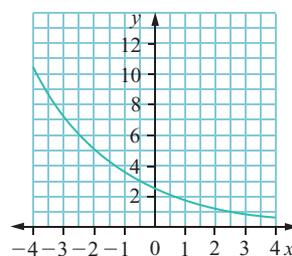
- c** 4; represents cost per T-shirt (\$4/T-shirt)
- d** 1400; represents a fixed set-up cost (\$1400)
- e** 125 T-shirts
- 18 a i** y-intercept is 1.8

$x$	-4	-3	-2	-1	0	1	2	3	4
$y$	0.1125	0.225	0.45	0.9	1.8	3.6	7.2	14.4	28.8



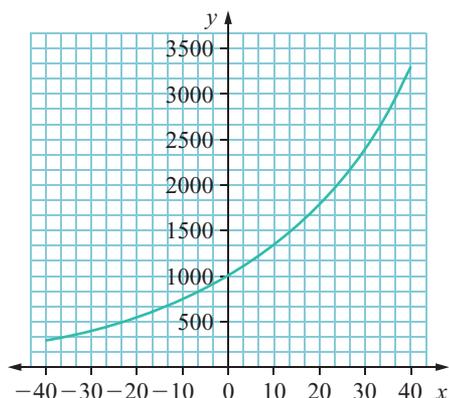
- ii** y-intercept is 2.5

$x$	-4	-3	-2	-1	0	1	2	3	4
$y$	10.41	7.29	5.10	3.57	2.5	1.75	1.23	0.86	0.60



- iii** y-intercept is 1000

$x$	-40	-30	-20	-10	0	10	20	30	40
$y$	307	412	554	744	1000	1344	1806	2427	3262

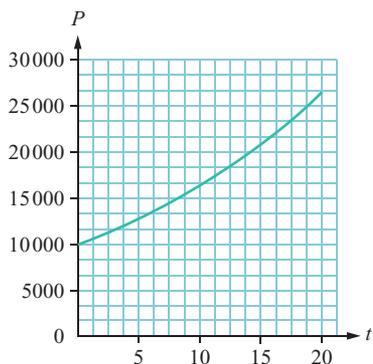


- b i** Exponential growth  
**ii** Exponential decay  
**iii** Exponential growth

**19 a**

$t$	0	5	10	15	20
$P$	10000	12763	16289	20789	26533

**b**

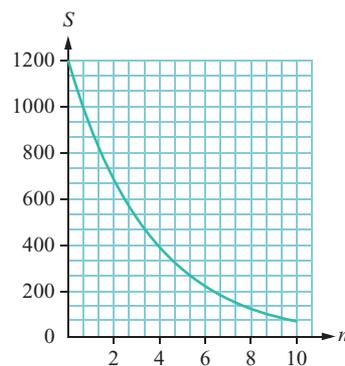


- c** The population when  $t = 0$   
**d** 18 000  
**e** 16 years  
**f**  $22\frac{1}{2}$  years  
**g** 5%; population increases by a factor of 1.05 each year which means that the population in the following year is 105% of the current year, an increase of 5%

- 20 a** Substituting  $V_0 = 1200$ ,  $r = 0.25$  (from 25%) into  $S = V_0(1 - r)^n$  gives  $S = 1200(1 - 0.25)^n$  or  $S = 1200(0.75)^n$ .

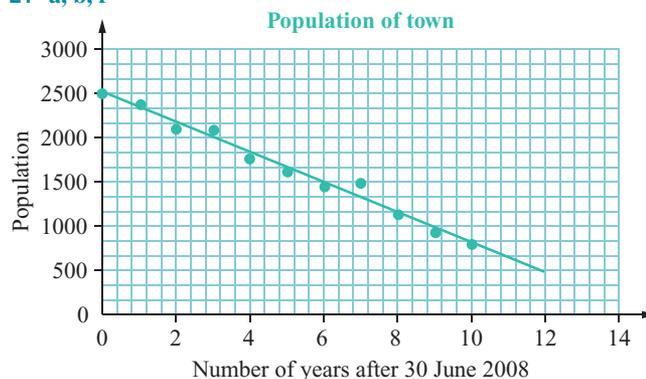
**b**

$n$	0	1	2	3	4	5	6	7	8	9	10
$S$	1200	900	675	506.25	379.69	284.77	213.57	160.18	120.14	90.10	67.58



- c** About \$500 (\$506.25)  
**d** About 4.8 years (4.82 years)  
**e** Exponential decay,  $S$  decreases as  $n$  increases

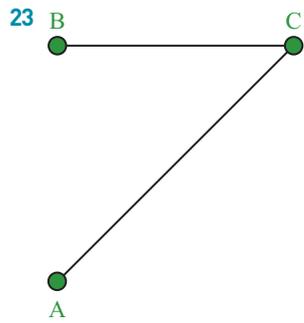
**21 a, b, f**



- c** Linear  
**d** About 1700 (1754)  
**e** Around the end of 2015 (7.5 years after 30 June 2008)  
**f** About 500 by extending and reading from trendline; 478 using spreadsheet equation  $y = -170.03x + 2518.8$ . The equation can also be found by using the gradient and vertical intercept of the trendline.  
**g** Changes to birth rate and death rate, changes to number of people relocating to a new town or moving into town

**22**

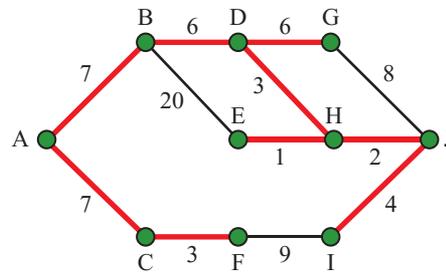
<b>Vertex</b>	A	B	C	D	E
<b>Connected to</b>	B	A, E, C	D, E, B	F, B, C	E



24 yes

25 Many possible answers, for example edges AB, BC and CD.

26, 27



28 ABDHJ

29 BDHE

# GLOSSARY

**adjacent side** side that is adjacent or next to the reference angle in a right-angled triangle (but is not the hypotenuse)

**angle of depression** angle between a horizontal line and the line of sight to an object below the horizontal

**angle of elevation** angle between a horizontal line and the line of sight to an object above the horizontal

**appreciate** a rise in the value or price of an item over a period of time

**bearing** direction from one position to another, using the angles around a magnetic compass

**best buy** found by comparing a number of purchase options to find the one that is the best value for money

**bias** a prejudice in favour of one particular group or outcome

**bivariate datasets** a dataset involving two different variables for each data point

**break-even point** point at which the revenue (or income) and the cost of production (or expenses) are equal; it can be identified as the point of intersection of the combined graphs of cost and revenue

**cause and effect** the result of the change in one variable is directly responsible for the change in another variable

**coincidence** two or more events occurring simultaneously through natural random variation

**compass bearing** indicates direction as an angle measured from north or south towards east or west; e.g.  $N40^\circ W$  represents a bearing of  $40^\circ$  from north towards west and  $S75^\circ E$  represents a bearing of  $75^\circ$  from south towards east

**corresponding angles** matching angles in a pair of similar figures, located in the same position when each figure is in the same orientation

**corresponding sides** matching sides in a pair of similar figures, located in the same position when each figure is in the same orientation

**census** a collection of the all of the entire population data

**compound interest** interest that is calculated on the current balance of an investment, including the interest from the previous time period

**connected network** a network in which there is a walk between every pair of vertices

**correlation** a measure of the extent to which two variables are related

**cosine** the trigonometric ratio calculated by dividing the length of the adjacent side by the length of the hypotenuse in a right-angled triangle

**credit card** plastic card that allows the holder to purchase goods and services on credit

**cycle** a walk that starts and ends in the same vertex, but does not visit any other vertices twice (or more)

**data** pieces of information that can be interpreted and analysed

**degree (of a vertex)** the number of edges connected to a vertex

**dependent variable** variable whose value depends on that of another; usually represented on the vertical axis of a graph; e.g. for the relationship  $y = 2x + 1$ , the dependent variable is  $y$  (as it depends on the value of  $x$ )

**depreciation** a loss in the value of an item over time

**Dijkstra's algorithm** a method for finding the shortest path between any two vertices of a connected weighted network

**diastolic blood pressure** the minimum pressure in the arteries as the heart relaxes before the next beat, measured in millilitres of mercury (mmHg)

**directed network** a network in which the edges have a direction; the orientation of the edge is represented by an arrow

**distance-time graph** graph that shows the relationship between distance and time where time is shown on the horizontal axis and distance is shown on the vertical axis

**disconnected network** a network that is not connected

**edge** a line between two vertices, which indicates a relationship between the vertices; sometimes called an arc

**ethics** moral principles that guide the way that we should act in a given situation

**exponential decay** relationship between two variables in which, as one variable increases, the other decreases at a decreasing rate of change; the graph curves downwards from left to right

**exponential growth** relationship between two variables in which, as one variable increases, the other increases at an increasing rate of change; the graph curves upwards from left to right

**extrapolation** predicting where a new data point will lie outside the existing range of data points

**fuel consumption rate** comparison between the two quantities fuel consumed and distance travelled; usually given as a rate in L/100 km

**gradient** steepness of a line (defined as  $m$ ) that can be measured using any two points on the line or interval using  $m = \frac{\text{rise}}{\text{run}}$

**heart rate** the number of contractions of the heart per minute measured as beats per minute (bpm)

**hypotenuse** the longest side in a right-angled triangle; it is opposite the right angle

**indegree** the number of edges pointing towards a vertex in a directed network

**independent variable** variable whose outcomes are not due to those of another variable; usually represented on the horizontal axis of a graph; e.g. for the linear relationship  $y = 2x + 1$ , the independent variable is  $x$

**inflation** an increase in the price of goods and services over a period of time

**interpolation** predicting where a new data point will lie within the existing range of data points

**Kruskal's algorithm** a method for finding the minimum spanning tree of a connected, weighted network by selecting the required edges one at a time

**leaf** in a tree, any vertex of degree 1

**line of best fit** a straight line that provides a representation of all of the data points in a scatterplot; also known as the trendline

**linear relationship** relationship between two variables where the coordinate points describing this relationship lie in a straight line when plotted on a number plane (or Cartesian plane)

**loop** an edge that starts and ends at the same vertex

**minimum spanning tree** the spanning tree with smallest total edge weight

**network** a collection of objects that are related to each other in some way

**opposite side** side that is opposite the reference angle in a right-angled triangle

**outdegree** the number of edges pointing away from a vertex in a directed network

**outlier** a data point that appears to be inconsistent with the other data points in a dataset

**path** a walk that does not visit any vertex twice (or more)

**point of intersection** point where two or more lines intersect or cross over one another on a graph

**population** all of the potential pieces of data that can be acquired

**Prim's algorithm** a method for finding the minimum spanning tree of a connected weighted network by iteratively adding the edge of smallest weight

**production cost** cost (or expense) of producing an item for sale, which can involve both a fixed cost and a variable cost that depends on the number of items produced

**rate** a comparison of one quantity with another; e.g. speed is a rate as distance is compared to time

**ratio** comparison between quantities of the same kind, used as a measure of the relative size of the quantities

**reducing-balance loan** a loan for which the interest is calculated on the balance owing, which gradually reduces over time

**revenue** amount of money (or income) received from the sale of items produced by a company, which can depend on the number of items sold

**salvage value** the reduced value of an item after depreciation

**sample** a collection of a selection of the population data

**scale drawing** a drawing of a diagram that is similar to the original but is smaller (or larger) in which the angles are the same and the lengths are in the same ratio as the original

**scale** the ratio of a length on a drawing to its real length expressed in the form  $1:n$  or  $\frac{1}{n}$

**scale factor** indicates how many times larger or smaller one of the pair of similar figures is than the other. A scale factor larger than 1 produces an enlargement (image is larger) and a scale factor between 0 and 1 produces a reduction (image is smaller)

**scatterplot** a diagram using Cartesian coordinates to display data values that relate two variables

**sketch** an approximate representation of a graph that indicates the general trend in the relationship between two variables without necessarily showing exact data points; includes labelled axes, intercepts and any other important relevant features, but generally there is no scale shown on the axes

**shortest path (in a network diagram)** a path between two vertices in a network for which the sum of the weights of its edges is minimised

**similar figures** pair of figures that have the same shape but one is an enlargement of the other

**simple interest** interest that is calculated on the original principal for the lifetime of the investment/loan; also known as flat rate interest

**simultaneous linear equations** two or more linear equations that use the same variables

**sine** the trigonometric ratio calculated by dividing the length of the opposite side by the length of the hypotenuse in a right-angled triangle

**spanning tree** a tree made from some of the edges and all of the vertices of the original network

**spurious correlation** two variables are mathematically related, but the relationship is not due to cause and effect

**systolic blood pressure** the maximum pressure in the arteries as the heart pumps out blood with each beat, measured in millilitres of mercury (mmHg)

**tangent** the trigonometric ratio calculated by dividing the length of the opposite side by the length of the adjacent side in a right-angled triangle

**tree** a connected network with no cycles

**true bearing** indicates direction as an angle measured from north in a clockwise direction and written with three digits; e.g. east is represented by  $090^\circ\text{T}$  and south-west is represented by  $225^\circ\text{T}$

**vertex** a visual representation of some object, usually drawn as a point; sometimes called a node

**walk** a sequence of vertices and the edges between them

**weighted edge** edges with a weight that represents a numerical quantity

associated with the relationship between the vertices

**y-intercept** point where a line crosses the y-axis of a number plane (or Cartesian plane)

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