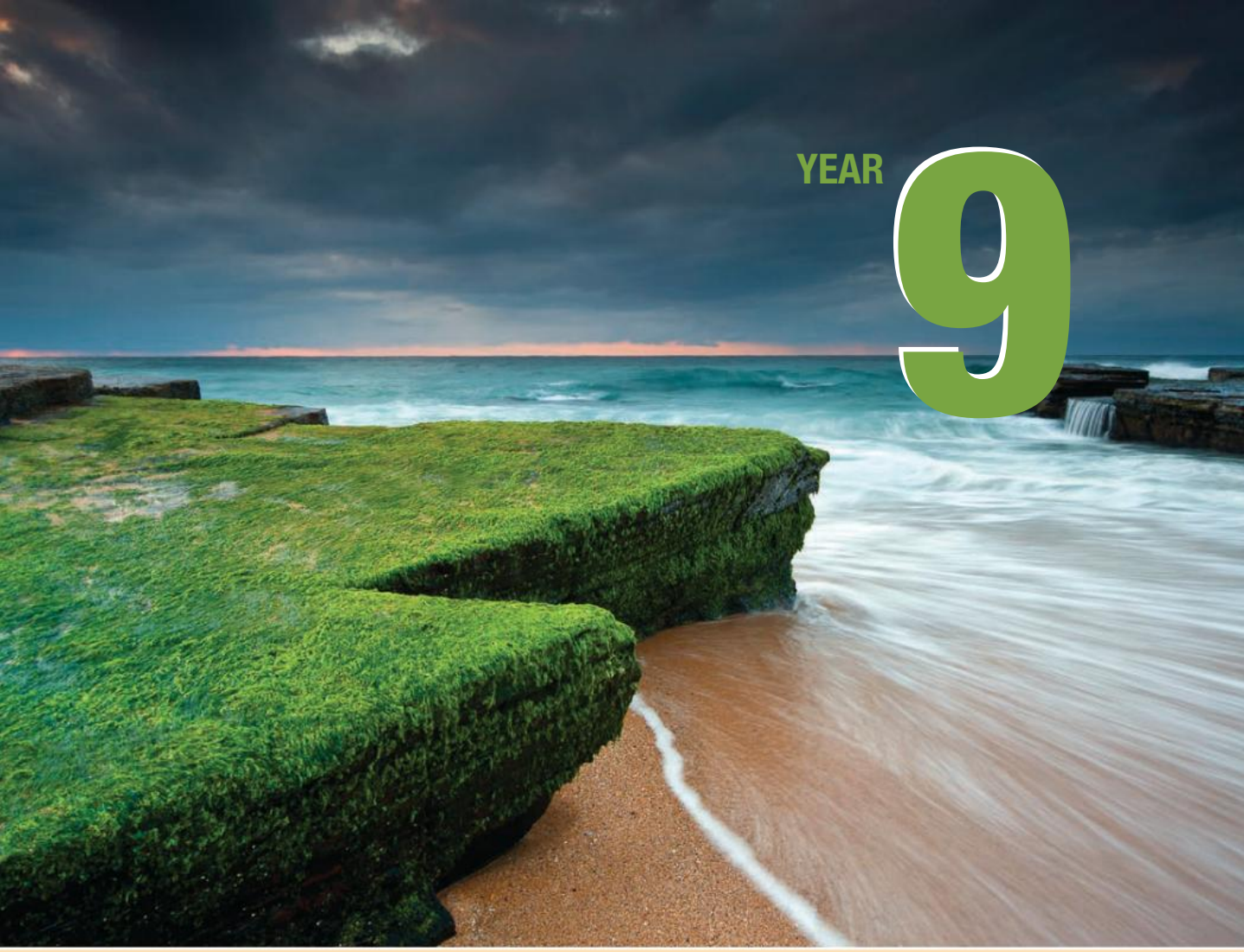

CambridgeMATHS NSW

STAGE 5.1 / 5.2

SECOND EDITION

STUART PALMER, KAREN McDAID
DAVID GREENWOOD, SARA WOOLLEY
JENNY GOODMAN, JENNIFER VAUGHAN

INCLUDES INTERACTIVE TEXTBOOK POWERED BY CAMBRIDGE HOTMATHS

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About the authors

Stuart Palmer was born and educated in NSW. He is a high school mathematics teacher with more than 25 years' experience teaching students from all walks of life in a variety of schools. He has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over NSW and beyond. He also works with pre-service teachers at The University of Sydney.

Karen McDaid has more that 17 years' experience teaching mathematics in several primary and secondary schools, and as a lecturer teaching mathematics to primary pre-service teachers at Western Sydney University. Karen was the Professional Teachers Association representative on the NSW Board Curriculum Committee during the development and consultation phases of both the Australian Curriculum and the NSW Mathematics K–10 Syllabus for the Australian Curriculum. She has been active on the executive committee of the Mathematical Association of NSW since 2003 and was the NSW Councillor on the board of the Australian Association of Mathematics Teachers from 2014 to 2018. Karen co-authored the CambridgeMATHS NSW GOLD books Years 7 to 10. Karen is currently Head of Mathematics K to 12 at Cluey Learning.

David Greenwood is the head of Mathematics at Trinity Grammar School in Melbourne and has 20 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 30 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics in Victoria from Years 7 to 12 since 2006 and has a keen interest in the creation of resources that cater for a wide range of ability levels.

Jenny Goodman has worked for over 20 years in comprehensive State and selective high schools in NSW and has a keen interest in teaching students of differing ability levels. She was awarded the Jones medal for education at Sydney University and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the *Spectrum* and *Spectrum Gold* series.

Jennifer Vaughan has taught secondary mathematics for over 30 years in NSW, WA, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.

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Introduction and guide to this book

The **second edition** of this popular resource features a new interactive digital platform powered by Cambridge HOTmaths, together with improvements and updates to the textbook, and additional online resources such as video demonstrations of all the worked examples, Desmos-based interactives, carefully chosen HOTmaths resources including widgets and walkthroughs, and worked solutions for all exercises, with access controlled by the teacher. The Interactive Textbook also includes the ability for students to complete textbook work, including full working-out online, where they can self-assess their own work and alert teachers to particularly difficult questions. Teachers can see all student work, the questions that students have 'red-flagged', as well as a range of reports. As with the first edition, the complete resource is structured on detailed teaching programs for teaching the NSW Syllabus, now found in the Online Teaching Suite.

The chapter and section structure has been retained, and remains based on a logical teaching and learning sequence for the syllabus topic concerned, so that chapter sections can be used as ready-prepared lessons. Exercises have questions graded by level of difficulty and are grouped according to the **Working Mathematically components** of the NSW Syllabus, with enrichment questions at the end. Working programs for three ability levels (Building Progressing and Mastering) have been subtly embedded inside the exercises to facilitate the management of differentiated learning and reporting on students' achievement (see page x for more information on the Working Programs). In the second edition, the *Understanding* and *Fluency* components have been combined, as have Problem-Solving and Reasoning. This has allowed us to better order questions according to difficulty and better reflect the interrelated nature of the Working Mathematically components, as described in the NSW Syllabus.

Topics are aligned exactly to the NSW Syllabus, as indicated at the start of each chapter and in the teaching program, except for topics marked as:

- REVISION — prerequisite knowledge
- EXTENSION — goes beyond the Syllabus
- FRINGE — topics treated in a way that lies at the edge of the Syllabus requirements, but which provide variety and stimulus.

See the Stage 5 books for their additional curriculum linkage.

The parallel ***CambridgeMATHS Gold*** series for Years 7–10 provides resources for students working at Stages 3, 4, and 5.1. The two series have a content structure designed to make the teaching of mixed ability classes smoother.

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Guide to the working programs

It is not expected that any student would do every question in an exercise. The print and online versions contain working programs that are subtly embedded in every exercise. For Stage 5.1 and 5.2, the suggested working programs provide two pathways through each book to allow differentiation for Building and Progressing students.

Each exercise is structured in subsections that match the Working Mathematically components, as well as Enrichment (Challenge).

The questions suggested for each pathway are listed in two columns at the top of each subsection:

- The left column (lighter-shaded colour) is the Building pathway
- The right column (darker-shaded colour) is the Progressing pathway

	Building	Progressing
UNDERSTANDING AND FLUENCY	1–3, 4, 5	3, 4–6
PROBLEM-SOLVING AND REASONING	7, 8, 11	8–12
ENRICHMENT	—	13

Gradients within exercises and question subgroups

The working programs make use of the gradients that have been seamlessly integrated into the exercises. A gradient runs through the overall structure of each exercise, where there is an increasing level of mathematical sophistication required in the Problem-solving and Reasoning group of questions than in the Understanding and Fluency group, and within each group the first few questions are easier than the last.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Building pathway will likely need more practice at Understanding and Fluency but should also attempt the easier Problem-Solving and Reasoning questions.

Choosing a pathway

There are a variety of ways of determining the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them if the chapter pre-tests can be used as a diagnostic tool.

For classes grouped according to ability, teachers may wish to set either the Building or Progressing pathway as the default setting for their entire class and then make individual alterations, depending on student need. For mixed-ability classes, teachers may wish to set both pathways within the one class, depending on previous performance and other factors.

The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1–4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e … or b, d, f, …)
- 2–4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- — : do not complete any of the questions in this section.

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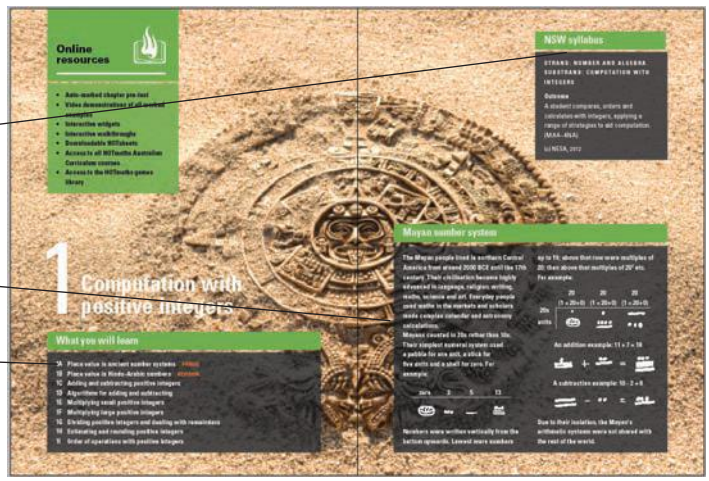
Guide to this book

Features:

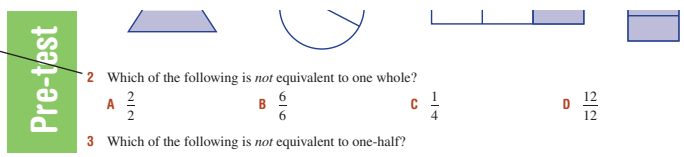
NSW Syllabus: strands, substrands and content outcomes for chapter (see teaching program for more detail)

Chapter introduction: use to set a context for students

What you will learn: an overview of chapter contents

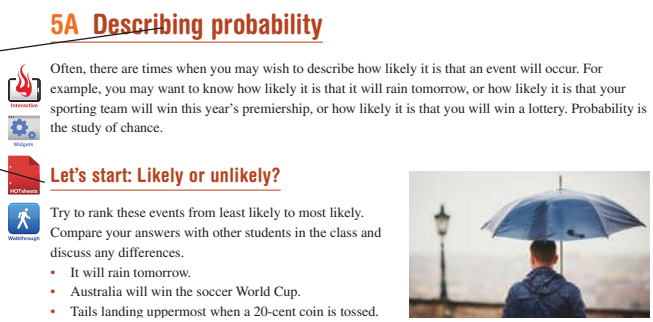


Pre-test: establishes prior knowledge (also available as an auto-marked quiz in the Interactive Textbook as well as a printable worksheet)

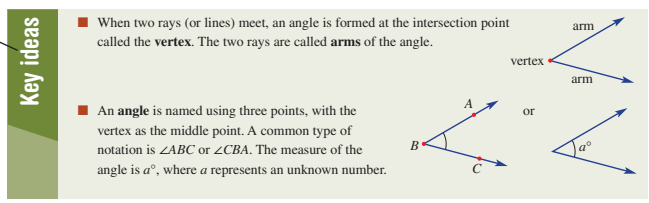


Topic introduction: use to relate the topic to mathematics in the wider world

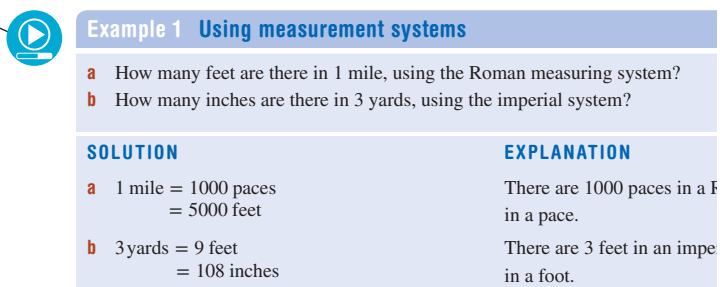
Let's start: an activity (which can often be done in groups) to start the lesson



Key ideas: summarises the knowledge and skills for the lesson



Examples: solutions with explanations and descriptive titles to aid searches. Video demonstrations of every example are included in the Interactive Textbook.



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Exercise questions categorised by the **working mathematically components** and **enrichment**

Example references link exercise questions to worked examples.

Investigations: inquiry-based activities

Puzzles and **challenges**

Exercise 10A FRINGE

UNDERSTANDING AND FLUENCY	1–8	4–9	5–9(½)

1 Complete these number sentences.
- **a** Roman system

Example 16 **4** Convert to the units shown in brackets.
- **a** 2 t (kg)
- **b** 70 kg (g)
- **c** 2.4 g (mg)
- **d** 2300 mg (g)
- **e** 4620 mg (g)
- **f** 21 600 kg (t)

PROBLEM-SOLVING AND REASONING	10–12, 18	12–14, 18	15–19

10 Arrange these measurements from smallest to largest.

ENRICHMENT	—	—	20

Very long and short lengths

20 When 1 metre is divided into 1 million parts, each part is called a **micrometre** (μm). At the other end of the spectrum, a **light year** is used to describe large distances in space.
- **a** State how many micrometres there are in:
  - **i** 1 m
  - **ii** 1 cm
  - **iii** 1 mm
  - **iv** 1 km

Investigation

The perfect billiard

When a billiard ball bounces o (with no side spin), we can ass at which it hits the wall (incon same as the angle at which it l (outgoing angle). This is simil reflects off a mirror.

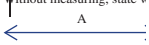
Single bounce

Use a ruler and protractor to d and then answer the questions.

- **a** Find the outgoing angle if:
  - **i** the incoming angle is 3
  - **ii** the centre angle is 104°
- **b** What geometrical reason d

Puzzles and challenges

1 Without measuring, state w

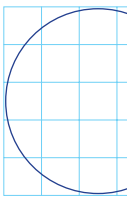


2 You have two sticks of leng

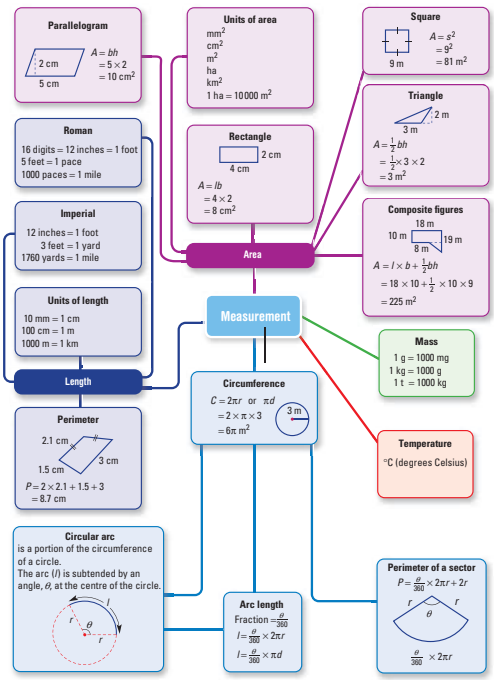
3 Count squares to estimate

- **a**

- **b**



Chapter summary: mind map of key concepts & interconnections



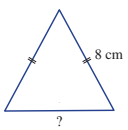
Chapter reviews with **multiple-choice, short-answer** and **extended-response questions**

Multiple-choice questions

1 Which of the following is a metric unit of capacity?
- **A** cm
- **B** pace
- **C** digit
- **D** yard
- **E** litre

2 Shonali buys 300 cm of wire that costs $2 per metre. How much does she pay for the wire?
- **A** $150
- **B** $600
- **C** $1.50
- **D** $3
- **E** $6

3 The triangle given has a perimeter of 20 cm. What is the missing base length?
- **A** 6 cm
- **B** 8 cm
- **C** 4 cm
- **D** 16 cm
- **E** 12 cm



4 The area of a rectangle with length 2 m and width 5 m is:
- **A** 10 m²
- **B** 5 m²
- **C** 5 m
- **D** 5 m³
- **E** 10 m

5 A triangle has base length 3.2 cm and height 4 cm. What is its area?
- **A** 25.6 cm²
- **B** 12.8 cm
- **C** 12.8 cm²
- **D** 6 cm
- **E** 6.4 cm²

Two **Semester reviews** per book

Semester review 1

Multiple-choice questions

1 Using numerals, thirty-five thousand, two hu
- **A** 350 260
- **B** 35 260
- **C** 3

2 The place value of 8 in 2 581 093 is:
- **A** 8 thousand
- **B** 80 thousand
- **C** 8

3 The remainder when 23 650 is divided by 4
- **A** 0
- **B** 4
- **C** 1

4 18 − 3 × 4 + 5 simplifies to:
- **A** 65
- **B** 135
- **C** 1

5 800 ÷ 5 × 4 is the same as:
- **A** 160 × 4
- **B** 800 ÷ 20
- **C** 8

Short-answer questions

1 Write the following numbers using words.
- **a** 1030
- **b** 13 000
- **d** 100 30
- **e** 100 300

Textbooks also include:
- Complete **answers**
- **Index**

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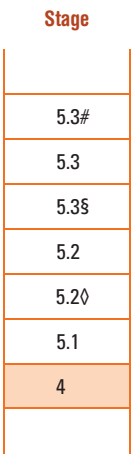
Stage Ladder icons

Shading on the ladder icons at the start of each section indicate the Stage covered by most of that section.

This key explains what each rung on the ladder icon means in practical terms. For more information see the teaching program and teacher resource package:



Stage	Past and present experience in Stages 4 and 5	Future direction for Stage 6 and beyond
5.3#	These are optional topics which contain challenging material for students who will complete all of Stage 5.3 during Years 9 and 10.	These topics are intended for students who are aiming to study Mathematics at the very highest level in Stage 6 and beyond.
5.3	Capable students who rapidly grasp new concepts should go beyond 5.2 and study at a more advanced level with these additional topics.	Students who have completed 5.1, 5.2 and 5.2 and 5.3 are generally well prepared for a calculus-based Stage 6 Mathematics course.
5.3§	These topics are recommended for students who will complete all the 5.1 and 5.2 content and have time to cover some additional material.	These topics are intended for students aiming to complete a calculus-based Mathematics course in Stage 6.
5.2	A typical student should be able to complete all the 5.1 and 5.2 material by the end of Year 10. If possible, students should also cover some 5.3 topics.	Students who have completed 5.1 and 5.2 without any 5.3 material typically find it difficult to complete a calculus-based Stage 6 Mathematics course.
5.2◊	These topics are recommended for students who will complete all the 5.1 content and have time to cover some additional material.	These topics are intended for students aiming to complete a non-calculus course in Stage 6, such as Mathematics Standard.
5.1	Stage 5.1 contains compulsory material for all students in Years 9 and 10. Some students will be able to complete these topics very quickly. Others may need additional time to master the basics.	Students who have completed 5.1 without any 5.2 or 5.3 material have very limited options in Stage 6 Mathematics.
4	Some students require revision and consolidation of Stage 4 material prior to tackling Stage 5 topics.	

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Overview of the digital resources

Interactive Textbook: general features

The **Interactive Textbook (ITB)** is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available separately. (A **downloadable PDF textbook** is also included for offline use). These are its features, including those enabled when the students' ITB accounts are linked to the teacher's **Online Teaching Suite (OTS)** account.

The features described below are illustrated in the screenshot below.

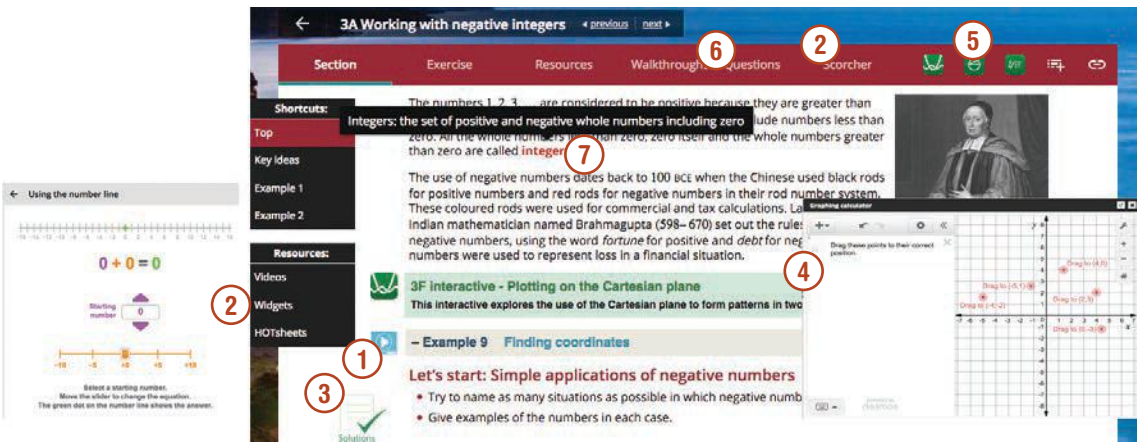
1. Every worked example is linked to a high-quality video demonstration, supporting both in-class learning and the 'flipped classroom'
2. Seamlessly blend with Cambridge HOTmaths, including hundreds of interactive widgets, walkthroughs and games and access to Scorcher
3. **Worked solutions** are included and can be enabled or disabled in the student accounts by the teacher
4. **Desmos interactives** based on embedded graphics calculator and geometry tool windows demonstrate key concepts and enable students to visualise the mathematics
5. The **Desmos scientific calculator** is also available for students to use (as well as the graphics calculator and geometry tools)
6. Auto-marked practice quizzes in each section with saved scores
7. **Definitions** pop up for key terms in the text, and access to the Hotmaths **dictionary**

Not shown but also included:

- Access to alternative HOTmaths lessons is included, including content from previous year levels.
- Auto-marked pre-tests and multiple-choice review questions in each chapter.

INTERACTIVE TEXTBOOK POWERED BY THE HOTmaths PLATFORM

Note: *HOTmaths platform features are updated regularly.*



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Interactive Textbook: Workspaces and self-assessment tools

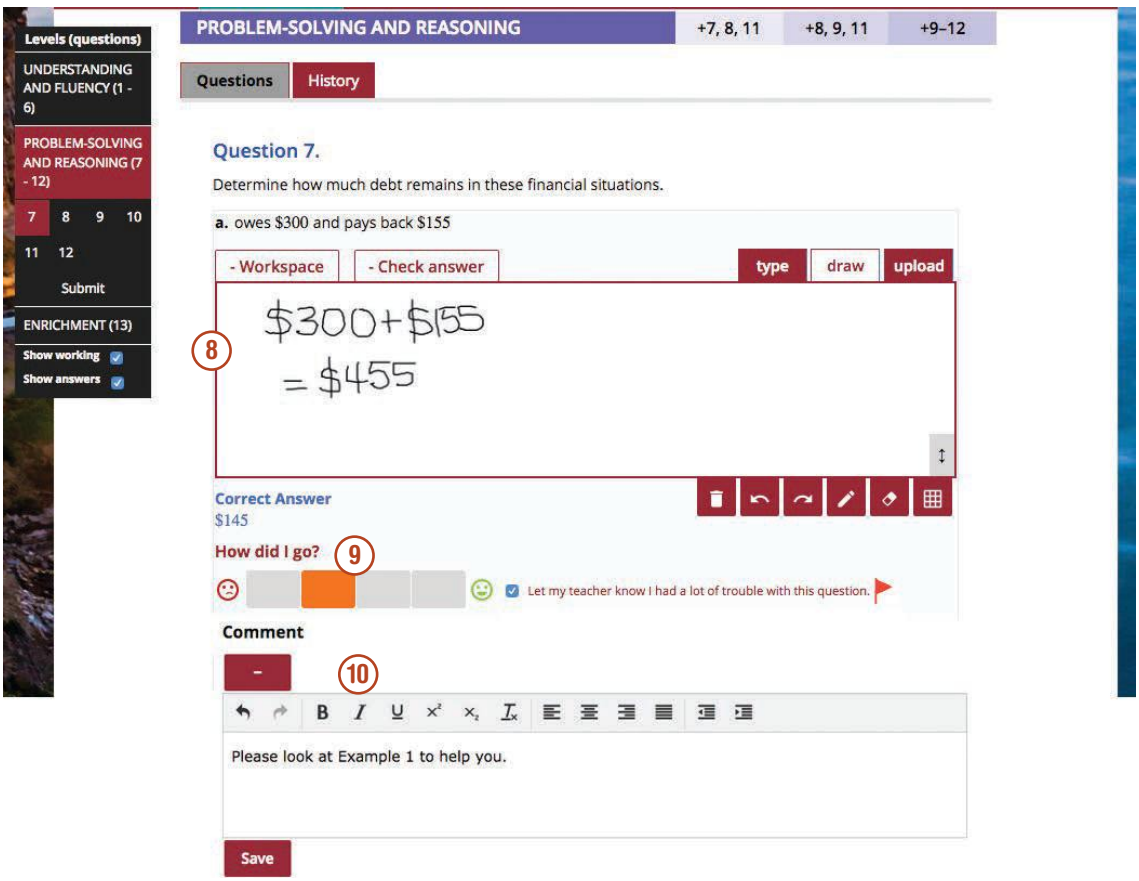
Almost every question in ***CambridgeMATHS NSW Second Edition*** can be completed and saved by students, including showing full working-out and students critically assessing their own work. This is done via the workspaces and self-assessment tools that are found below every question in the Interactive Textbook.

8 The new **Workspaces** enable students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploding images of writing or drawing.

9 The new **self-assessment tools** enable students to check answers including questions that have been red-flagged, and can rate their confidence level in their work, and alert teachers to questions the student has had particular trouble with. This self-assessment helps develop responsibility for learning and communicates progress and performance to the teacher.

10 Teachers can view the students' self-assessment individually or provide feedback. They can also view results by class.

WORKSPACES AND SELF-ASSESSMENT



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Downloadable PDF Textbook

The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.

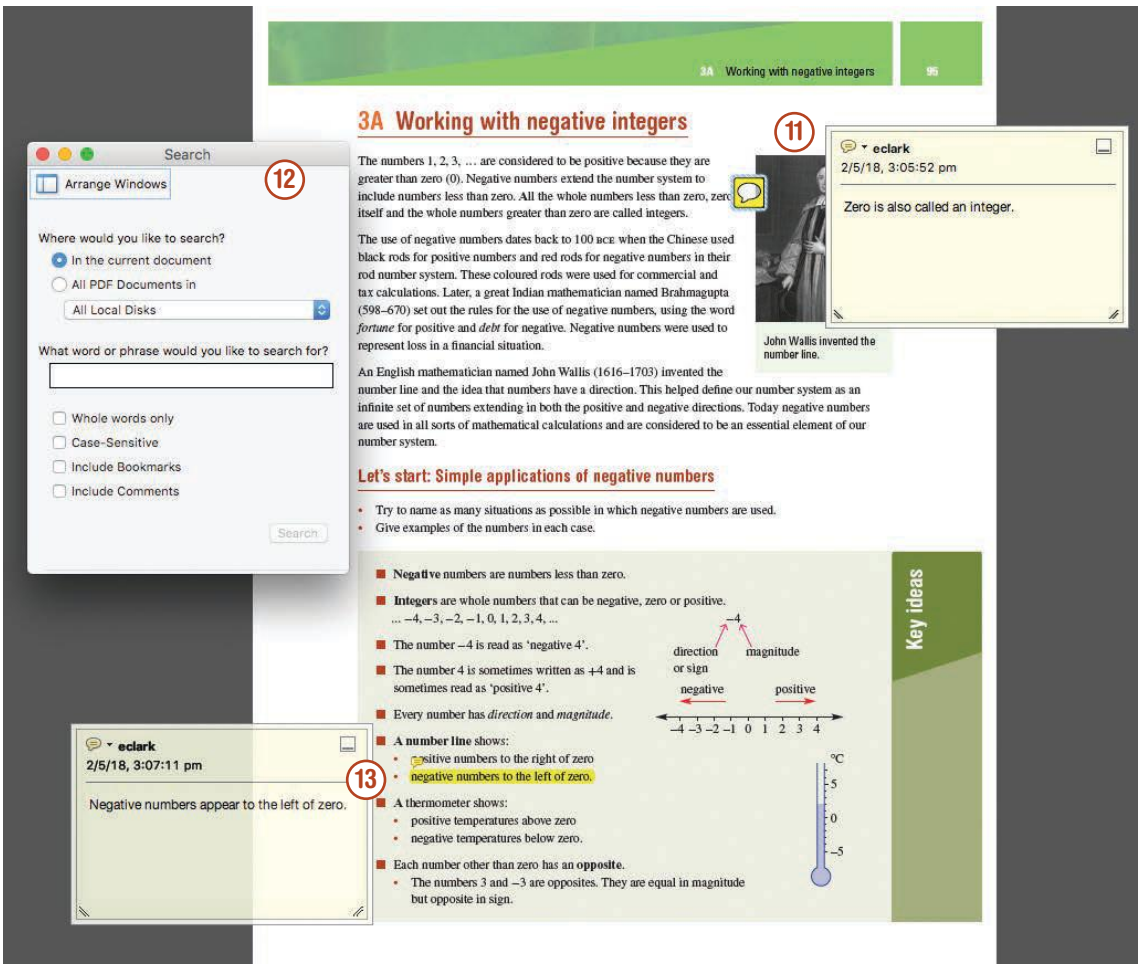
The features include:

11 PDF note-taking

12 PDF search features are enabled

13 highlighting functionality.

PDF TEXTBOOK



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Online Teaching Suite

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the assets and resources are in one place for easy access. The features include:

14 The HOTmaths learning management system with class and student analytics and reports, and communication tools

15 Teacher's view of a student's working and self-assessment, including multiple progress and completion reports viewable at both student and class level, as well as seeing the questions that a class has flagged as being difficult

16 A HOTmaths-style test generator

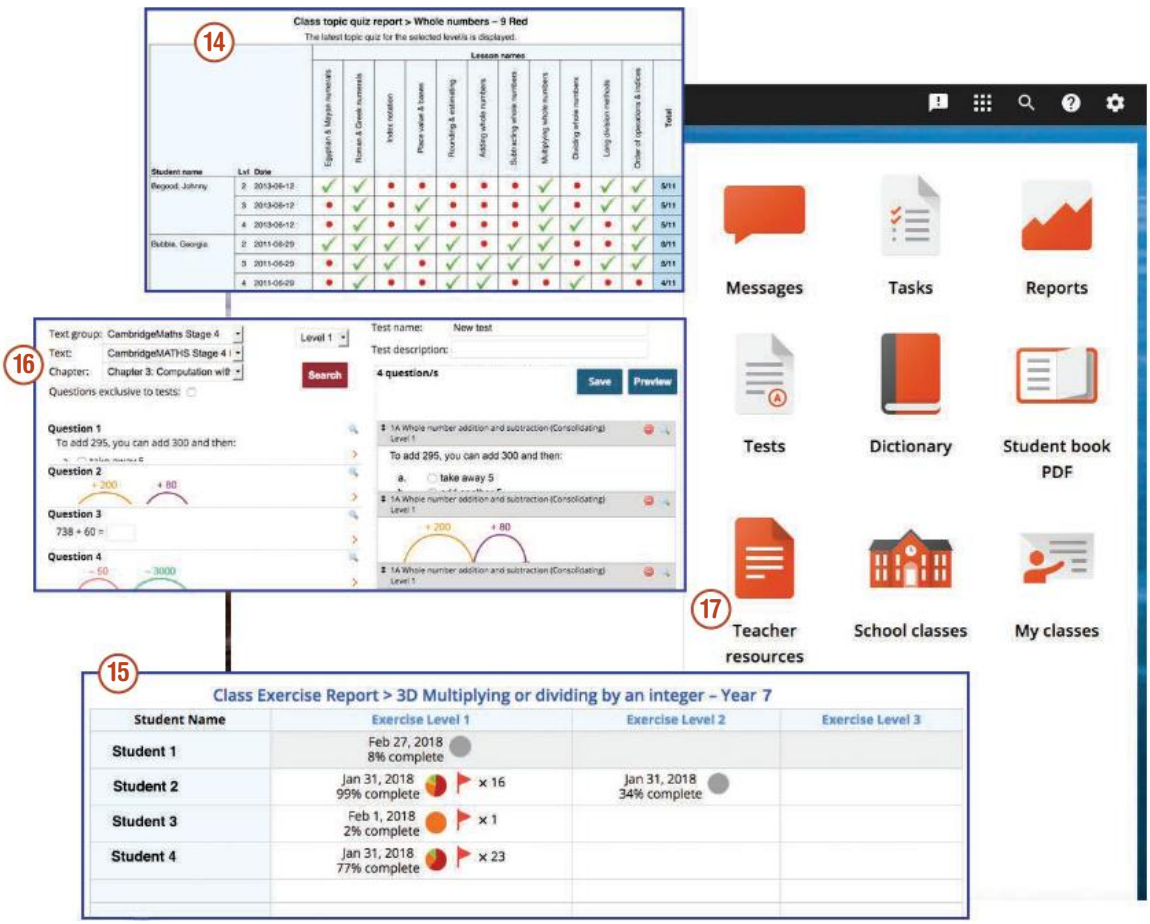
17 Chapter tests and worksheets

Not shown but also available:

- Editable teaching programs and curriculum grids.

ONLINE TEACHING SUITE POWERED BY THE HOTmaths PLATFORM

Note: *HOTmaths platform features are updated regularly.*



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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

1 Integers, decimals, fractions, ratios and rates

What you will learn

1A Adding and subtracting positive and negative integers REVISION
1B Multiplying and dividing positive and negative integers REVISION
1C Decimal places and significant figures
1D Rational numbers and irrational numbers REVISION
1E Adding and subtracting fractions REVISION
1F Multiplying and dividing fractions REVISION
1G Ratios REVISION
1H Rates and direct proportion

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NSW syllabus

STRANDS: NUMBER AND ALGEBRA; MEASUREMENT AND GEOMETRY

SUBSTRANDS: COMPUTATION WITH INTEGERS; FRACTIONS, DECIMALS AND PERCENTAGES; RATIOS AND RATES; NUMBERS OF ANY MAGNITUDE

Outcomes

A student compares, orders and calculates with integers, applying a range of strategies to aid computation.
(MA4–4NA)

A student operates with fractions, decimals and percentages.
(MA4–5NA)

A student operates with ratios and rates, and explores their graphical representation.
(MA4–7NA)

A student recognises direct and indirect proportion, and solves problems involving direct proportion.
(MA5.2–5NA)

A student interprets very small and vary large units of measurement, uses scientific notation, and rounds to significant figures.
(MA5.1–9MG)

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Mathematics and the origins of writing

Before there were symbols to represent numbers, like the Hindu-Arabic numeral '2' or Roman numeral 'V', people used objects to represent amounts of something. These objects were called tokens.

The Sumerians came up with the idea of using tokens of different shapes and sizes to keep track of goods bought and sold. Ten sheep, for example, could be represented by 10 tokens of a particular type.

As you can imagine, it would be unwieldy to carry around large amounts of individual tokens, so starting from around 3500 BCE the Sumerians made clay containers called *bullae* for the tokens. One bulla might contain 10 tokens of one type and 20 of another. The problem was: how could you tell what was in a bulla without breaking it open? The Sumerians marked the bullae with imprints of the tokens that were inside it.

Soon they realised that if the contents of the bulla were marked on the outside, then the tokens inside were not required. They began marking flat pieces of clay called tablets with tokens instead of round or barrel-shaped containers. After a while the Sumerians realised that they could *draw* the markings on the tablets with sharp instruments instead of using the tokens themselves to make imprints. It is thought that these markings evolved into cuneiform, the earliest known form of writing.

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Pre-test

1 Arrange the following mathematical terms under four headings: 'Addition', 'Subtraction', 'Multiplication' and 'Division'.

a Sum **b** Total **c** Less than **d** Lots of **e** Product
f Into **g** Take away **h** Difference **i** Add **j** Times
k Minus **l** More than **m** Quotient

2 Without using a calculator, find an answer to each of the following.

a 16 less 12 **b** 24 more than 8
c The difference between 12 and 8 **d** Increase 45 by 7
e The total of 40, 34 and 0 **f** 9 into 45
g The quotient of 7 and 63 **h** 480 shared between 12

3 Evaluate the following.

a $9 + 47$ **b** $135 - 35$ **c** $19 - 19$ **d** $56 + 89 - 12$
e 9×7 **f** $320 \div 4$ **g** 17×60 **h** $200 - 47 - 100$

4 Use a number line to find:

a $-5 - 7$ **b** $12 - 15$ **c** $-6 + 9$ **d** $-12 + 12$
e $16 - 17$ **f** $-4 + 3$ **g** $-7 + 4 + 3$ **h** $-4 - 4 - 4$

5 Copy and complete each of the following statements.

a $5 + 5 + 5 = \square \times 5$ **b** $-6 - 6 - 6 = \square \times (-6)$
c $9 - (+10) = 9 \ \square \ 10$ **d** $12 - (-2) = 12 \ \square \ 2$

6 The population of Australia in 2050 is projected to be 26 073 258. Round this number to the nearest:

a ten **b** hundred **c** thousand **d** million

7 Write down the place value of the 5 in each of the following numbers.

a 1256 **b** 345 **c** 5049 **d** 0.56 **e** 0.15 **f** 9.005

8 Arrange the numbers in each of the following sets in descending order (largest to smallest).

a 2.645, 2.654, 2.465 and 2.564 **b** 0.456, 0.564, 0.0456 and 0.654

9 Evaluate each of the following.

a $4.26 + 3.73$ **b** $3.12 + 6.99$ **c** $10.89 - 3.78$

10 Evaluate each of the following.

a 7×0.2 **b** 0.3×0.2 **c** 2.3×1.6
d 4.2×3.9 **e** $14.8 \div 4$ **f** $12.6 \div 0.07$

11 Evaluate each of the following.

a 0.345×100 **b** $3.74 \times 100\,000$ **c** $37.54 \div 1000$ **d** $3.7754 \div 100\,000$

12 Complete these equivalent fractions.

a $\frac{1}{2} = \frac{\square}{12}$ **b** $\frac{3}{4} = \frac{\square}{16}$ **c** $\frac{5}{6} = \frac{25}{\square}$ **d** $\frac{\square}{9} = \frac{2}{18}$

13 Find the lowest common denominator for these pairs of fractions.

a $\frac{1}{3}$ and $\frac{1}{5}$ **b** $\frac{1}{6}$ and $\frac{1}{4}$ **c** $\frac{1}{5}$ and $\frac{1}{10}$

14 Find:

a $\frac{3}{7} + \frac{2}{7}$ **b** $2 - \frac{3}{4}$ **c** $4 \div \frac{1}{2}$ **d** $\frac{3}{4} \times \frac{1}{2}$

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1A Adding and subtracting positive and negative integers REVISION

Interactive

Widgets

HOTsheets

Walkthrough

Integers are the set of positive and negative whole numbers, as well as zero.

Being able to work with whole numbers is very important, since whole numbers are used every day for counting, calculating measuring and ordering.

Numbers are used in marketplaces all around the world to describe prices, quantity and sometimes even quality.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Naming groups

Here are some groups of numbers. In groups of two or three, use the correct mathematical terms to describe each group. (Suggestions include: 'multiples of', 'factors of', 'integers', 'squares' and 'cubes'.)

- 2, 4, 6, 8, …
- 1, 4, 9, 16, …
- 1, 3, 5, 7, 9, …
- 1, 2, 3, 4, 5, 6, …
- –1, –2, –3, –4, –5, …
- 1, 8, 27, 64, …
- 1, 2, 3, 4, 6 and 12

Key ideas

- Integers are the positive and negative whole numbers, including zero.
  …, –3, –2, –1, 0, 1, 2, 3, …
- To add a negative number, you subtract its opposite: $a + (-b) = a - b$
  For example: $5 + (-7) = 5 - 7$
  $-6 + (-2) = -6 - 2$
- To subtract a negative number, you add its opposite: $a - (-b) = a + b$
  For example: $5 - (-7) = 5 + 7$
  $-6 - (-2) = -6 + 2$
- Adding or subtracting zero leaves a number unchanged.
  $a + 0 = a$ e.g. $5 + 0 = 5$
  $a - 0 = a$ e.g. $5 - 0 = 5$

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Exercise 1A REVISION

UNDERSTANDING AND FLUENCY | 1–4, 5–8(½) | 4, 5–8(½)

1 Match each of the following sentences to the correct expression on the right.

a The sum of 5 and 7	**i** $5 + (-7)$
b The total of negative 5 and 7	**ii** $5 - (-7)$
c The difference between negative 5 and 7	**iii** $5 + 7$
d The sum of 5 and negative 7	**iv** $-5 - 7$
e The difference between 5 and negative 7	**v** $-5 + 7$

2 Match each expression on the left to a number on the right.

a $10 + (-7)$	**i** -17
b $-10 - (-7)$	**ii** -3
c $10 - (-7)$	**iii** 0
d $-10 - 7$	**iv** 3
e $-10 + 7 + 3$	**v** 17

3 True or false?

a $9 - 7 = -2$ **b** $18 + 0 = 18$ **c** $-6 + 6 = 0$
d $9 + 8 = 8 + 9$ **e** $6 - (-4) = 6 + 4$ **f** $4 - (-2) = 4 - 2$

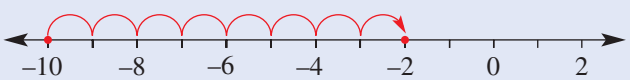
Example 1 Using a number line for addition and subtraction of integers

Use a number line to find:

a $-10 + 8$ **b** $-3 - 5$

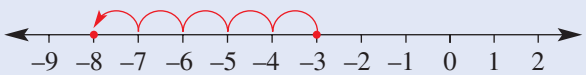
SOLUTION

a



$-10 + 8 = -2$

b



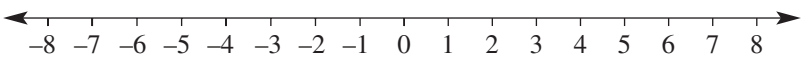
$-3 - 5 = -8$

EXPLANATION

Draw a number line showing -10 on it.
Start at -10 and count up (for addition) 8 places to finish at -2.

Draw a number line showing -3 on it.
Count down (for subtraction) 5 places to -8.

4 Use the number line below to help you simplify the following.



a $-6 + 4$ **b** $-6 + 8$ **c** $0 - 3$
d $-1 + 5$ **e** $6 - 10$ **f** $-2 - 3$
g $-1 - 1$ **h** $-3 + 3$ **i** $-3 - 3$

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5 Use a number line to evaluate the following.

a $9 - 6$ **b** $9 - 7$ **c** $9 - 8$
d $9 - 9$ **e** $9 - 10$ **f** $-9 - 0$
g $-9 + 8$ **h** $-9 - 8$ **i** $-6 - 4$
j $-3 + 12$ **k** $-8 + 6$ **l** $-1 + 12$
m $-12 + 5$ **n** $-10 - 8$ **o** $18 - 19$

On the number line, move right for addition and left for subtraction.

6 Evaluate the following.

a $4 + 8 - 7$ **b** $5 + 6 + 9$ **c** $-19 - 1$
d $-9 + 8 + 1$ **e** $-12 - 3 + 8$ **f** $8 + 3 - 5$
g $-12 - 12$ **h** $15 + 5 - 15$ **i** $-6 - 5 - 4$

Do in order from left to right.

Example 2 Adding a negative integer

Find $17 + (-12)$.

SOLUTION

$17 + (-12) = 17 - 12$
$= 5$

EXPLANATION

Adding a negative is the same as subtraction:
$17 + (-12) = 17 - 12$

7 Find the value of the following. Check your answers with a calculator.

a $9 + (-5)$ **b** $12 + (-16)$ **c** $3 + (-7)$
d $15 + (-24)$ **e** $-9 + (-23)$ **f** $-13 + (-25)$
g $-100 + (-89)$ **h** $56 + (-80)$ **i** $-9 + (-9)$
j $18 + (-18)$ **k** $-245 + (-560)$ **l** $98 + (-155)$
m $-89 + (-78)$ **n** $145 + (-3)$ **o** $-567 + (-237)$

$9 + (-5) = 9 - 5$
$-9 + (-23) = -9 - 23$

Example 3 Subtracting a negative integer

Find $-13 - (-9)$.

SOLUTION

$-13 - (-9) = -13 + 9$
$= -4$

EXPLANATION

Subtracting a negative is the same as addition:
$-13 - (-9) = -13 + 9$

8 Evaluate the following.

a $-5 - (-9)$ **b** $-8 - (-6)$ **c** $7 - (-6)$
d $2 - (-7)$ **e** $12 - (-12)$ **f** $-34 - (-34)$
g $-35 - (-7)$ **h** $-90 - (-9)$ **i** $90 - (-90)$
j $-68 - (-70)$ **k** $-90 - (-87)$ **l** $234 - (-6)$
m $670 - (-85)$ **n** $-6 - (-100)$ **o** $-230 - (-240)$

$-5 - (-9) = -5 + 9$

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PROBLEM-SOLVING AND REASONING

9, 10	9–12

9 Simplify each of these using the rules on the previous page. Check your answers with a calculator.

a $-9 - 9 - 9$
b $-23 - (-8) + 12$
c $50 - 46 - (-6)$
d $24 - (-8) + (-6)$
e $-20 - (-5) - (-15)$
f $-18 - (-6) + (-12)$
g $-90 - (-89) - 90$
h $125 - 35 - (-35)$
i $-90 + (-87) - 3$

Work from left to right.

10 Copy and complete each of the following statements.

a $\square + 6 = 8$
b $-6 + \square = 5$
c $12 - \square = 15$
d $-9 + \square = -10$
e $13 - \square = 20$
f $10 - \square = -1$

Try using a number line.

11 What must be added to each of the following to obtain a final result of zero?

a 8
b $-7 + 3$
c $-8 - 5$
d -124
e 19
f 0
g $12 - 8 + 18$
h -98
i $12 - (-14)$

12 The temperature inside a car was measured at 23°C. Fifteen minutes later it had risen by 18°C. The air conditioner was then switched on and the temperature fell by 22°C. What was the temperature inside the car at this time?

ENRICHMENT

–	13

Magic squares with integers

13 a Copy and complete this magic square. Each row, column and diagonal add to the same number.

−8		
	−2	−6
		4

What is the sum of the diagonal?

b Arrange these 9 integers into a 3 by 3 magic square. −13, −10, −7, −4, −1, 2, 5, 8 and 11

c Complete this 4 by 4 magic square.

5			
0	10		6
−1	4	2	8
		7	−4

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1B Multiplying and dividing positive and negative integers REVISION

Interactive

Widgets

HOTsheets

Walkthrough

Multiplication is a way of writing repeated addition. It is possible to develop rules for multiplying and dividing with negative integers.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Repeated addition

Write each of the following as a multiplication before finding the answer.

i $4 + 4 + 4$
ii $(-4) + (-4) + (-4)$
iii $5 + 5 + 5 + 5$
iv $(-5) + (-5) + (-5) + (-5)$
v $(-7) + (-7) + (-7) + (-7) + (-7)$
vi $(-8) + (-8)$

Use your results from above to answer these divisions.

i $12 \div 4$
ii $-12 \div (-4)$
iii $-12 \div 3$
iv $-20 \div 4$
v $-20 \div (-5)$
vi $20 \div 5$
vii $-35 \div (-7)$
viii $-35 \div 5$
ix $-16 \div (-8)$

What can you conclude about dividing a negative by a:

a positive number?
b negative number?

Key ideas

- When multiplying two numbers with the same sign, the result is a positive number.
  positive × positive = positive
  negative × negative = positive
  (If the signs are the same, the result is positive.)
  For example: $-5 \times (-4) = 20$

- When multiplying two numbers with different signs, the result is a negative number.
  positive × negative = negative
  negative × positive = negative
  For example: $-5 \times 4 = -20$

- The same rules apply for division:
  positive ÷ positive = positive
  negative ÷ negative = positive
  positive ÷ negative = negative
  negative ÷ positive = negative

- On your calculator, use brackets when finding powers of negative numbers.
  For example: $(-13)^2 = 169$ (not -169)

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Exercise 1B REVISION

UNDERSTANDING AND FLUENCY 1–3, 4–7(½) 3, 4–7(½)

1 Would these products be positive or negative?

a $2 \times (-8)$ **b** $78 \times (-1)$ **c** $56 \div (-2)$
d $-90 \div (-10)$ **e** 8×12 **f** $-18 \times (-9)$

2 Copy and complete this table.

×	−4	−3	−2	−1	0	1	2	3	4
−4									
−3									
−2									
−1									
0									
1									
2									
3									
4									

3 Complete the following statements.

a A negative number multiplied by a negative number equals a _____ number.
b A negative number multiplied by a positive number equals a _____ number.
c A negative number divided by a negative number equals a _____ number.
d A positive number divided by a negative number equals a _____ number.
e A negative number divided by a positive number equals a _____ number.

Example 4 Multiplying with negatives

Find the value of:

a $-6 \times (-7)$ **b** $8 \times (-12)$

SOLUTION

a $-6 \times (-7) = 42$

b $8 \times (-12) = -96$

EXPLANATION

The product of two negative numbers gives a positive answer.

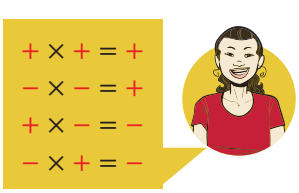
$- \times - = +$

The product of a positive number and a negative number results in a negative answer.

$+ \times - = -$

4 Find these products. Use a calculator to check your answers.

a -8×6 **b** -10×2 **c** -9×8
d $-9 \times (-8)$ **e** $-15 \times (-3)$ **f** $45 \times (-2)$
g $100 \times (-6)$ **h** $-9 \times (-9)$ **i** -12×12
j -13×11 **k** $11 \times (-13)$ **l** $34 \times (-1)$
m $-24 \times (-3)$ **n** $-18 \times (-5)$ **o** $36 \times (-3)$



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Example 5 Finding squares and cubes of negative numbers

Find the value of:

a $(-5)^2$ **b** $(-6)^3$

SOLUTION	EXPLANATION
a $(-5)^2 = 25$	$(-5)^2$ is the same as $-5 \times (-5)$. The product of two negative numbers gives a positive answer. $- \times - = +$
b $(-6)^3 = -216$	$(-6)^3 = -6 \times (-6) \times (-6)$ Multiply the first two numbers together: $-6 \times (-6) = 36$ Now: $36 \times (-6) = -216$

5 Find the answers to each of the following. Use a calculator to check your answers.

a $(-6)^2$ **b** $(-8)^2$ **c** $(-12)^2$
d $(-3)^2$ **e** $(-10)^2$ **f** $(-13)^2$
g $(-2)^3$ **h** $(-1)^3$ **i** $(-4)^3$

$(-6)^2 = -6 \times (-6)$
$(-3)^3 = -3 \times (-3) \times (-3)$

Example 6 Dividing with a negative number

Find the value of:

a $-24 \div 8$ **b** $-45 \div (-3)$

SOLUTION	EXPLANATION
a $-24 \div 8 = -3$	The division of two numbers with different signs gives a negative answer. $- \div + = -$
b $-45 \div (-3) = 15$	The division of two numbers with the same sign gives a positive answer. $- \div - = +$

6 Perform the following divisions. Use a calculator to check your answers.

a $18 \div (-2)$ **b** $-36 \div (-6)$ **c** $-100 \div (-10)$
d $-50 \div 5$ **e** $-6 \div (-2)$ **f** $-10 \div (-2)$
g $-50 \div (-25)$ **h** $16 \div (-16)$ **i** $24 \div (-3)$
j $-100 \div (-4)$ **k** $312 \div (-3)$ **l** $-45 \div (-9)$
m $185 \div (-5)$ **n** $-428 \div 2$ **o** $-156 \div (-12)$

$+ \div + = +$
$- \div - = +$
$+ \div - = -$
$- \div + = -$

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Example 7 Applying the order of operations

Find the value of $9 \times (-8) + (5 - 12)$.

SOLUTION	EXPLANATION
$9 \times (-8) + (5 - 12)$	Work out the value of the brackets first: $(5 - 12) = (-7)$
$= 9 \times (-8) + (-7)$	Multiplication is next: $9 \times (-8) = -72$
$= -72 + (-7)$	The addition of a negative is the same as subtraction:
$= -79$	$-72 + (-7) = -72 - 7$

7 Follow the order of operations and evaluate:

a $(7 - 3) \times 4$ **b** $(4 + 9) \times (-2)$
c $(6 - 2) \times (-3)$ **d** $-6 + 4 \times (-2)$
e $10 \div (-2) \times 4$ **f** $(-5 - 2) \times 4$
g $6 \times (-3) \times 4$ **h** $-21 \div 7 + (-5)$
i $-45 - 9 \times 5$ **j** $(4 - 6) \times (7 - 12)$
k $(15 - 9) \times (3 - 7)$ **l** $9 + 9 \times (-3)$
m $-(3 - 12)$ **n** $10 - 9 \times 4$
o $100 \div (-5) \times (-2)$ **p** $-4 \div (-7 + 5) \times (-3)$

The order of operations:
- brackets first, following the order of operations within them
- then × and ÷, working from left to right
- then + and –, working from left to right.

PROBLEM SOLVING AND REASONING 8, 9 8(½), 9–12

8 Copy and complete:

a $-5 \times \square = -35$ **b** $\square \div (-4) = 8$ **c** $-10 \times \square = 200$
d $17 \times \square = -68$ **e** $34 \div \square = -34$ **f** $-6 \times \square = -36$
g $\square \div 9 \times (-3) = 3$ **h** $15 \div \square \div 3 = -1$ **i** $-15 \times \square = 225$

9 The sum of two numbers is -3 and their product is -10. What are the two numbers?

10 Give the value of two different numbers that when squared each produce an answer of:

a 144 **b** 64 **c** 10 000

11 Explain why $-5 \times 4 \div (-3)$ produces a positive answer.

12 Decide whether each of the following would produce a positive or a negative answer.

a $-6 \times (-4) \times 5$ **b** $-12 \div 4 \times 9 \div (-3)$ **c** $-8 \div (-2) \times 5 \times (6 - 9)$
d $(-3)^4$ **e** $(-1)^{201}$ **f** $-(-2)^5$

ENRICHMENT – 13, 14

Substitutions involving negative integers

13 Evaluate the following by substituting $a = -2$, $b = 6$ and $c = -3$.

a $a^2 - b$ **b** $a - b^2$ **c** $2c + a$ **d** $b^2 - c^2$
e $a^3 + c^2$ **f** $3b + ac$ **g** $c - 2ab$ **h** $abc - (ac)^2$

14 Use a calculator to evaluate the following, where $x = -45$ and $y = -15$.

a $x + y$ **b** $x - y$ **c** xy **d** $x - 3y$
e $x^2 + y^2$ **f** $x^2 - y^2$ **g** $(x + y)^2$ **h** $3y^2$

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1C Decimal places and significant figures

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Walkthrough

Numbers with and without decimal places can be rounded. The time for a 100-metre sprint race, for example, might be 9.94 seconds, correct to two decimal places.

Another way of rounding numbers is to use significant figures. The number of cubic metres of gravel required for a road, for example, might be calculated as 3485 but is rounded to 3500. This number is written using two significant figures.

For road construction purposes, the volume of sand in these piles would only need to be known to two significant figures.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: In the middle

For each number given on the left, decide which of the two numbers on the right it is closest to.

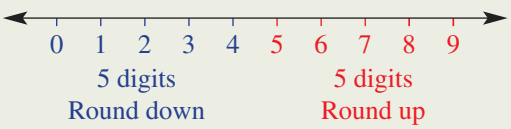
a	84	80 or 90
b	856	800 or 900
c	856	850 or 860
d	0.599	0.5 or 0.6
e	1.2099	1.2 or 1.3
f	1.2099	1.20 or 1.21
g	89 543	89 000 or 90 000
h	0.035	0.03 or 0.04

Key ideas

- **Rounding** involves approximating a decimal number using fewer digits.
- To round a number to a required number of decimal places:
  - Locate the digit in the required decimal place.
  - Round down (leave as is) if the next digit (**critical digit**) is 4 or less.
  - Round up (increase by 1) if the next digit is 5 or more.

Critical digit
The digit that determines whether you round the previous digit up or down

Critical digit



For example:
- To two decimal places, 1.543 rounds to 1.54 and 32.9283 rounds to 32.93.
- To one decimal place, 0.248 rounds to 0.2 and 0.253 rounds to 0.3.

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Key ideas

- The symbols ≑ and ≈ are used to mean 'approximately equal to'.
- To round a number to a required number of **significant figures**:
  - Locate the first *non-zero* digit, counting from left to right.
  - From this first significant digit, count out the number of digits, including zeros.
  - Stop at the required number of digits and round this last digit.
  - Replace any non-significant digits to the left of a decimal point with a zero.

Significant figure
A digit that indicates how accurate a number is

For example, these numbers are all rounded to **three** significant figures:

2.531 ≑ 2.54 0.002713 ≑ 0.00271 568810 ≑ 569000

Exercise 1C

UNDERSTANDING AND FLUENCY | 1–4, 5–6(½), 7, 8 | 4, 5–6(½), 7–9

1 How many decimal places do each of these numbers have?

a 0.46 **b** 1.467
c 0.7 **d** 0.08
e 146.1 **f** 47900
g 0.123 **h** 7000.6

2 How many significant figures do each of these numbers have?

a 0.46 **b** 1.467
c 0.7 **d** 0.08
e 146.1 **f** 479
g 0.123 **h** 4906

Count the digits starting from the first *non-zero* digit. These are the significant ones.

3 Answer each question.

a Is 44 closer to 40 or 50?
b Is 266 closer to 260 or 270?
c Is 7.89 closer to 7.8 or 7.9?
d Is 0.043 closer to 0.04 or 0.05?

4 Choose the correct answer if the first given number is rounded to three significant figures.

a 32124 is rounded to 321, 3210 or 32100.
b 431.92 is rounded to 431, 432 or 430.
c 5.8871 is rounded to 5.887, 5.88 or 5.89.
d 0.44322 is rounded to 0.44, 0.443 or 0.44302.
e 0.0019671 is rounded to 0.002, 0.00197 or 0.00196.

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Example 8 Rounding to a number of decimal places

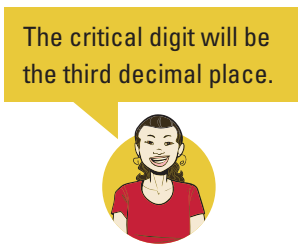
Round each of these to two decimal places.

a 256.1793 **b** 0.04459 **c** 4.8972

SOLUTION	EXPLANATION
a 256.1793 ≑ 256.18	256.1 7**9**3 The number after the second decimal place is 9, so round up (increase the 7 by 1).
b 0.044 59 ≑ 0.04	0.0 4**4** 59 The number after the second decimal place is 4, so round down.
c 4.8972 ≑ 4.90	4.8 9**7**2 The number after the second decimal place is 7, so round up. 89 rounds to 90.

5 Round each of the following numbers to two decimal places.

a 17.962 **b** 11.082 **c** 72.986
d 47.859 **e** 63.925 **f** 23.807
g 804.5272 **h** 500.5749 **i** 821.2749
j 5810.2539 **k** 1004.9981 **l** 2649.9974



Example 9 Rounding to a number of significant figures

Round each of these numbers to two significant figures.

a 2567 **b** 23 067.453 **c** 0.04059

SOLUTION	EXPLANATION
a 2567 ≑ 2600	**25**67 significant digits The next digit is 6, so round up. Replace last 2 digits with zeros to maintain place value of thousands.
b 23 067.453 ≑ 23 000	**23**067.453 significant digits The next digit is 0, so round down. Replace the remaining digits, in front of the decimal point, with zeros.
c 0.04059 ≑ 0.041	Locate the first non-zero digit: 0.0**4**059 significant digits The next digit is 5, so round up. No extra zeros are needed, as they are after the decimal point.

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6 Round each of these numbers to two significant figures.

a 2436	**b** 35 057.4	**c** 0.06049
d 34.024	**e** 107 892	**f** 0.00245
g 2.0745	**h** 0.7070	**i** 4706
j 59 134	**k** 0.4567	**l** 1.0631

Start at the first non-zero digit on the left.

7 Copy and complete this table.

	Number	Rounded to 2 decimal places	Rounded to 2 significant figures
a	1.4638		
b	0.0936		
c	23.7124		
d	0.00783		
e	100.465		

8 Round these numbers to the nearest whole number.

a 6.814 **b** 73.148 **c** 129.94 **d** 36 200.49

9 Round these numbers to one significant figure.

a 32 000	**b** 194.2
c 0.0492	**d** 0.0006413
e 4793	**f** 890
g 0.89	**h** 0.000304

48.06 = 50 to 1 significant figure.
4730 = 5000 to 1 significant figure.
0.638 = 0.6 to 1 significant figure.

PROBLEM SOLVING AND REASONING 10–12 12–15

Example 10 Estimating using significant figures

Estimate the answer to the following. Round each number in the problem to one significant figure. Use your calculator to check how reasonable your answer is.

$27 + 1329.5 \times 0.0064$

SOLUTION	EXPLANATION
$27 + 1329.5 \times 0.0064$	
$\approx 30 + 1000 \times 0.006$	Round each number to one significant figure and evaluate.
$= 30 + 6$	Recall multiplication occurs before the addition.
$= 36$	
The estimated answer is reasonable.	By calculator (to 1 d.p.): $27 + 1329.5 \times 0.0064 = 35.5$

10 Estimate the answers to the following by rounding each number in the problem to one significant figure. Check how reasonable your answer is with a calculator.

a $567 + 3126$	**b** $795 - 35.6$
c 97.8×42.2	**d** $965.98 + 5321 - 2763.2$
e $4.23 - 1.92 \times 1.827$	**f** $17.43 - 2.047 \times 8.165$
g $0.0704 + 0.0482$	**h** 0.023×0.98

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11 Copy and complete this table of common fractions.

Fraction form	Decimal form	Written to 1 decimal place	Written to 1 significant figure
$\frac{1}{3}$			
$\frac{2}{3}$			
$\frac{1}{9}$			
$\frac{3}{4}$			

12 An electronic timer records the time for a running relay between two teams, A and B. Team A's time is 54.283 seconds and team B's time is 53.791 seconds. What would be the difference in the times for teams A and B if the times were written down using:

a 1 decimal place?
b 4 significant figures?
c 2 significant figures?
d 1 significant figure?

13 A tonne (1000 kg) of soil is to be equally divided between 7 garden beds. How much soil does each garden bed get? Write your answer in tonnes rounded to the nearest kilogram.

14 A scientific experiment uses very small amounts of magnesium (0.0025 g) and potassium (0.0062 g). Why does it make sense to use two significant figures instead of two decimal places when recording numbers in a situation like this?

15 Should 2.14999 be rounded down or up if it is to be rounded to one decimal place? Give reasons.

ENRICHMENT – 16, 17

*n*th decimal place and π

16 $\frac{2}{11}$ can be written as 0.18181818 correct to 8 decimal places.

a Using the decimal pattern described, find the digit in the:
- **i** 20th decimal place
- **ii** 45th decimal place
- **iii** 1000th decimal place

b Express $\frac{1}{7}$ as a decimal correct to 13 decimal places.

c Using the decimal pattern from part **b**, find the digit in the:
- **i** 20th decimal place
- **ii** 45th decimal place
- **iii** 1000th decimal place

17 π is a decimal that is non-terminating and has no pattern.
$\pi = 3.141592653\ldots$

a How many decimal places can you remember?
b Quiz yourself and your classmates.
c Use the internet to find π correct to 100 decimal places.

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1D Rational numbers and irrational numbers REVISION

Interactive
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Walkthrough

Rational numbers are any numbers that can be written as a fraction in the form $\frac{a}{b}$, where $b \neq 0$.

Pythagoras and his followers discovered around 500 BC that not all numbers are rational. Pi(π) and $\sqrt{2}$ are examples of these numbers. When written as decimals, they do not terminate or repeat. We call them irrational numbers.

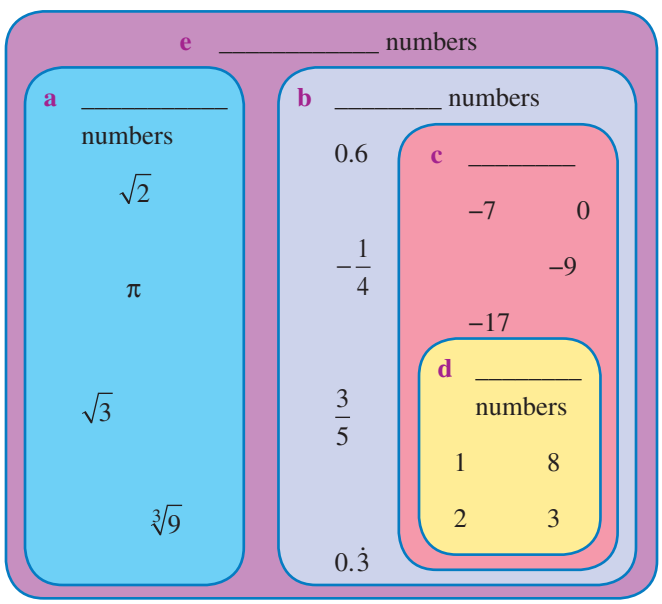
This is $\sqrt{2}$ to 100 decimal places:

1.41421356237309504880168872420969807856967187537694807317667973799073247846210703885038753432764157 27

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: The real number system

Copy the diagram below and insert these words correctly: rational, irrational, real, integers, counting.



Key ideas

Real numbers

Rational numbers (fraction) $\left(\text{e.g.}\ \frac{48}{6}, \frac{1}{3}, \frac{7}{1}\right)$

- Terminating decimals (e.g. 8, 1.2, −6.123)
- Recurring decimals (e.g. $0.\dot{5}$, $-1.\dot{4}$, $2.\overline{345}$)

Irrational numbers (e.g. $\sqrt{2}$, pi (π), phi(ϕ) – the golden ratio)

- Infinite non-recurring decimals (e.g. $\pi = 3.14159...$, $\phi = 1.618033...$, $\sqrt{2} = 1.414213...$)

Real number
Any positive or negative number, or zero

Rational number
A real number that can be expressed as a fraction

Irrational number
A real number that cannot be expressed as a fraction

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Key ideas

- An infinite or non-terminating decimal is one where the decimal places continue forever.
- Equivalent fractions have the same value. For example: $\frac{2}{3} = \frac{6}{9}$
- A fraction can be simplified by dividing the numerator and denominator by their highest common factor (HCF).
- If $\frac{a}{b}$ is a proper fraction, then $a < b$. For example: $\frac{2}{7}$
- If $\frac{a}{b}$ is an improper fraction, then $a > b$. For example: $\frac{10}{3}$
- A mixed numeral is written as a whole number plus a proper fraction. For example: $2\frac{3}{5}$
- Fractions can be compared using a common denominator. This should be the lowest common multiple of both denominators.
- A dot or bar is used to show a pattern in a recurring decimal number.

  For example: $\frac{1}{6} = 0.16666... = 0.1\dot{6}$, $\frac{3}{11} = 0.272727... = 0.\overline{27}$ or $0.\dot{2}\dot{7}$

Exercise 1D REVISION

UNDERSTANDING AND FLUENCY 1–3, 4(½), 5, 6, 7(½), 8 | 3, 4–8(½)

1 Are these true or false?

a $\sqrt{3}$ is an irrational number.

b $\sqrt{4}$ is an irrational number.

c 0.3 is a rational number.

An irrational number as a decimal is never-ending and has no pattern.

2 Which of the following can be written as a fraction?

a $0.\dot{6}$ **b** 1.4 **c** $\sqrt{9}$ **d** $\sqrt{5}$

3 Write each of these as a fraction in the form $\frac{a}{b}$.

a 0.7 **b** 0.3 **c** $1\frac{1}{2}$ **d** $2\frac{1}{3}$

Example 11 Identifying rational numbers

Choose the rational numbers in the following list.

$\sqrt{9}, \sqrt{2}, 0.6, \pi, -\frac{3}{4}$

SOLUTION

$\sqrt{9}$, 0.6 and $-\frac{3}{4}$ are rational numbers.

EXPLANATION

$\sqrt{9} = 3 = \frac{3}{1}$ $\therefore$ rational (a fraction)

$0.6 = \frac{6}{10}$ $\therefore$ rational

$-\frac{3}{4}$ is also rational.

π and $\sqrt{2}$ are infinite non-recurring decimals.

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4 Which of the following are rational numbers?

a 0.1 **b** 20% **c** $6\frac{1}{4}$

d $\frac{2}{5}$ **e** $\frac{2}{3}$ **f** 0.001

g $\sqrt{64}$ **h** $\sqrt{7}$ **i** 4^2

j π **k** $0.\dot{2}$ **l** $\sqrt{11}$

Rational numbers can be written as a fraction.

Example 12 Writing fractions as decimals

Write $3\frac{3}{8}$ as a decimal.

SOLUTION

$$\begin{array}{r} 0.\ 3\ 7\ 5 \\ 8\overline{)3.^{3}0^{6}0^{4}0} \end{array}$$

$3\frac{3}{8} = 3.375$

EXPLANATION

Find a decimal for $\frac{3}{8}$ by dividing 8 into 3 using the short division algorithm.

5 Write these fractions as decimals.

a $\frac{11}{4}$ **b** $\frac{7}{20}$ **c** $3\frac{2}{5}$ **d** $\frac{15}{8}$

e $2\frac{5}{8}$ **f** $3\frac{4}{5}$ **g** $\frac{37}{16}$ **h** $\frac{7}{28}$

Use short division.
$\frac{11}{4}$ means $11 \div 4$, i.e. $4\overline{)11}$.

Example 13 Recurring decimals from fractions

Write $\frac{5}{13}$ as a recurring decimal.

SOLUTION

$$\begin{array}{r} 0.\ 3\ \ 8\ 4\ 6\ 1\ 5\ 3 \\ 13\overline{)5.^{5}0^{11}0^{6}0^{8}0^{2}0^{7}0^{5}0} \end{array}$$

$\frac{5}{13} = 0.\overline{384615}$

EXPLANATION

Divide 13 into 5 and continue until the pattern repeats. Add a bar over the repeating pattern.

Writing $0.\dot{3}8461\dot{5}$ is an alternative.

6 Write these fractions as recurring decimals.

a $\frac{3}{11}$ **b** $\frac{7}{9}$ **c** $\frac{9}{7}$ **d** $\frac{5}{12}$

e $\frac{10}{9}$ **f** $3\frac{5}{6}$ **g** $7\frac{4}{15}$ **h** $\frac{29}{11}$

Check using your calculator.

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Example 14 Writing decimals as fractions

Write these decimals as fractions.

a 0.24 **b** 2.385

SOLUTION

a $0.24 = \frac{24}{100}$

$= \frac{6}{25}$

b $2.385 = \frac{2385}{1000}$ or $2\frac{385}{1000}$

$= \frac{477}{200}$ $= 2\frac{77}{200}$

$= 2\frac{77}{200}$

EXPLANATION

Write as a fraction using the smallest place value (hundredths) and then simplify using the HCF of 4.

The smallest place value is thousandths.

Simplify to an improper fraction or a mixed numeral.

7 Write these decimals as fractions.

a 0.35 **b** 0.06 **c** 3.7
d 0.56 **e** 1.07 **f** 0.075
g 3.32 **h** 7.375 **i** 2.005
j 10.044 **k** 6.45 **l** 2.101

Divide by 100 when there are two decimal places.

Example 15 Comparing fractions

Decide which is the larger fraction of the following.

$\frac{7}{12}$ or $\frac{8}{15}$

SOLUTION

LCM of 12 and 15 is 60.

$\frac{7}{12} = \frac{35}{60}$ and $\frac{8}{15} = \frac{32}{60}$

$\therefore \frac{7}{12} > \frac{8}{15}$

EXPLANATION

Find the lowest common multiple (LCM) of the two denominators (lowest common denominator).
Write each fraction as an equivalent fraction using the common denominator. Then compare numerators (i.e.35 > 32) to determine the larger fraction.

8 Decide which is the larger fraction in the following pairs.

a $\frac{3}{4}, \frac{5}{6}$ **b** $\frac{13}{20}, \frac{3}{5}$ **c** $\frac{7}{10}, \frac{8}{15}$

d $\frac{5}{12}, \frac{7}{18}$ **e** $\frac{7}{16}, \frac{5}{12}$ **f** $\frac{26}{35}, \frac{11}{14}$

g $\frac{7}{12}, \frac{19}{30}$ **h** $\frac{7}{18}, \frac{11}{27}$

First write both fractions using a common denominator.

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PROBLEM-SOLVING AND REASONING

9–11	10–13

9 Express the following quantities as simplified fractions.

a \$45 out of \$100

b 12 kg out of 80 kg

c 64 baskets out of 90 shots in basketball

d 115 mL out of 375 mL

10 These sets of fractions form a pattern. Find the next two fractions in the pattern.

a $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{\square}{\square}, \frac{\square}{\square}$

b $\frac{6}{5}, \frac{14}{15}, \frac{2}{3}, \frac{\square}{\square}, \frac{\square}{\square}$

c $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{\square}{\square}, \frac{\square}{\square}$

d $\frac{1}{2}, \frac{4}{7}, \frac{9}{14}, \frac{\square}{\square}, \frac{\square}{\square}$

11 The 'Weather forecast' website says there is a 0.45 chance that it will rain tomorrow. The 'Climate control' website says that the chance of rain is $\frac{14}{30}$. Which website gives the least chance that it will rain?

The chance of rain can be expressed as a decimal, a fraction or a percentage.

12 A jug has 400 mL of $\frac{1}{2}$ strength orange juice. The following amounts of full strength juice are added to the mix. Find a fraction to describe the strength of the orange drink after the full strength juice is added.

a 100 mL **b** 50 mL **c** 120 mL **d** 375 mL

13 If x is an integer, determine the values that x can take in the following.

a The fraction $\frac{x}{3}$ is a number between (and not including) 10 and 11.

b The fraction $\frac{x}{7}$ is a number between (and not including) 5 and 8.

c The fraction $\frac{34}{x}$ is a number between 6 and 10.

You may find more than one answer.

ENRICHMENT

–	14

Converting recurring decimals to fractions

14 Here are two examples of how to convert recurring decimals to fractions.

$$0.\dot{6} = 0.6666...$$
$$\text{Let } x = 0.6666.... \quad (1)$$
$$10x = 6.6666.... \quad (2)$$
$$(2) - (1)\ 9x = 6$$
$$x = \frac{6}{9} = \frac{2}{3}$$
$$\therefore 0.\dot{6} = \frac{2}{3}$$

$$1.\overline{27} = 1.272727...$$
$$\text{Let } x = 1.272727.... \quad (1)$$
$$100x = 127.2727.... \quad (2)$$
$$(2) - (1)\ 99x = 126$$
$$x = \frac{126}{99}$$
$$\therefore 1.\overline{27} = \frac{126}{99} = 1\frac{27}{99} = 1\frac{3}{11}$$

Convert these recurring decimals to fractions using the above method.

a $0.\dot{8}$ **b** $1.\dot{2}$ **c** $0.\overline{81}$ **d** $3.\overline{43}$

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1E Adding and subtracting fractions REVISION

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You will recall that fractions represent parts of a whole.

Fractions, like other numbers, can be added and subtracted. To add or subtract fractions they need to be the same type; that is, they both need to be eighths, quarters, thirds, tenths etc. They need to have a common denominator.

Let's start: Shade the fraction

Copy the diagrams and shade the fraction suggested by each of these additions and subtractions.

a $\frac{1}{4} + \frac{1}{4} =$

b $\frac{1}{4} + \frac{1}{2} =$

c $\frac{3}{4} - \frac{1}{2} =$

d $\frac{5}{8} + \frac{1}{4} =$

e $\frac{1}{2} + \frac{1}{3} =$

Key ideas

- Equivalent fractions are created by multiplying or dividing both the numerator and the denominator by the same factor.
  For example:

$$\frac{3}{4} \xrightarrow{\times 3} \frac{9}{12} \qquad \frac{10}{15} \xrightarrow{\div 5} \frac{2}{3}$$

- To add or subtract fractions, the denominators need to be the same.
  - If the denominators are the same, add or subtract the numerators, keeping the denominator unchanged.

    For example: $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$
  - If the denominators are different, find the lowest common denominator (LCD) using equivalent fractions. Then add or subtract the numerators, remembering to simplify your answer where possible.

    For example: $\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6}$ The LCD of halves and thirds is sixths.

    $= \frac{7}{6}$

    $= 1\frac{1}{6}$

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Exercise 1E REVISION

UNDERSTANDING AND FLUENCY 1–4, 5–7(½) 4, 5–7(½)

1 Copy and complete these equivalent fractions.

a $\frac{3}{4} = \frac{\square}{16}$ **b** $\frac{3}{5} = \frac{\square}{10}$ **c** $\frac{2}{3} = \frac{\square}{9}$

d $\frac{2}{3} = \frac{\square}{12}$ **e** $\frac{7}{10} = \frac{\square}{100}$ **f** $\frac{5}{6} = \frac{\square}{18}$

g $\frac{3}{7} = \frac{21}{\square}$ **h** $\frac{4}{5} = \frac{12}{\square}$ **i** $\frac{5}{7} = \frac{25}{\square}$

What you do to the numerator must also be done to the denominator, and vice versa.

2 True or false?

a $\frac{7}{9} + \frac{1}{9} = \frac{8}{9}$ **b** $\frac{6}{9} - \frac{1}{3} = \frac{5}{6}$

3 Copy and complete.

a $\frac{3}{10} + \frac{2}{5}$
$= \frac{3}{10} + \frac{\square}{10}$
$= \square$

b $\frac{1}{2} - \frac{3}{8}$
$= \frac{\square}{8} - \frac{3}{8}$
$= \frac{\square}{8}$

c $1\frac{1}{2} + \frac{1}{4}$
$= 1\frac{2}{\square} + \frac{1}{4}$
$= 1\frac{\square}{\square}$

4 What is the LCD needed for each of these additions?

a $\frac{1}{2}, \frac{1}{3}$ **b** $\frac{3}{7}, \frac{5}{9}$ **c** $\frac{2}{5}, \frac{1}{13}$ **d** $\frac{3}{10}, \frac{8}{15}$

e $\frac{1}{2}, \frac{1}{4}$ **f** $\frac{9}{11}, \frac{4}{33}$ **g** $\frac{5}{12}, \frac{7}{30}$ **h** $\frac{13}{29}, \frac{2}{3}$

The LCD is the lowest common multiple of both denominators.

Example 16 Adding and subtracting with common denominators

Find:

a $\frac{5}{11} + \frac{8}{11}$ **b** $1\frac{3}{8} - \frac{5}{8}$

SOLUTION	EXPLANATION
a $\frac{5}{11} + \frac{8}{11} = \frac{13}{11}$	Denominators are the same (elevenths). Keep the denominator and add the numerators.
$= 1\frac{2}{11}$	Write as a mixed numeral.
b $1\frac{3}{8} - \frac{5}{8} = \frac{11}{8} - \frac{5}{8}$	Write $1\frac{3}{8}$ as an improper fraction.
$= \frac{6}{8}$	Denominators are the same (eighths). Subtract the numerators.
$= \frac{3}{4}$	Simplify the answer by dividing by the HCF.

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5 Find:

a $\frac{2}{5}+\frac{1}{5}$ **b** $\frac{3}{8}+\frac{2}{8}$ **c** $\frac{9}{10}-\frac{2}{10}$

d $\frac{2}{7}+\frac{2}{7}$ **e** $\frac{13}{17}-\frac{8}{17}$ **f** $\frac{3}{5}-\frac{2}{5}$

g $\frac{11}{6}-\frac{1}{6}$ **h** $\frac{13}{8}-\frac{3}{8}$ **i** $\frac{4}{5}+\frac{3}{5}$

j $1\frac{1}{5}+\frac{3}{5}$ **k** $2\frac{2}{3}-\frac{1}{3}$ **l** $1\frac{1}{8}-\frac{5}{8}$

m $1\frac{1}{10}-\frac{7}{10}$ **n** $3\frac{1}{4}+1\frac{1}{4}$ **o** $2\frac{3}{10}-\frac{9}{10}$

Writing mixed numerals as improper fractions may help.

Example 17 Adding and subtracting with different denominators

Evaluate $\frac{1}{2}+\frac{3}{5}$.

SOLUTION

$\frac{1}{2}+\frac{3}{5}=\frac{5}{10}+\frac{6}{10}$

$=\frac{11}{10}$ or $1\frac{1}{10}$

EXPLANATION

The lowest common denominator of 2 and 5 is 10. Rewrite as equivalent fractions using a denominator of 10.

Add the numerators.

6 Evaluate the following.

a $\frac{1}{2}+\frac{1}{8}$ **b** $\frac{1}{2}-\frac{1}{6}$ **c** $\frac{3}{10}+\frac{1}{5}$

d $\frac{4}{3}-\frac{5}{6}$ **e** $\frac{3}{2}-\frac{3}{5}$ **f** $\frac{1}{3}+\frac{1}{4}$

g $\frac{3}{5}+\frac{1}{10}$ **h** $\frac{3}{4}-\frac{3}{5}$ **i** $\frac{5}{6}-\frac{3}{8}$

j $\frac{2}{3}+\frac{1}{2}$ **k** $\frac{5}{3}-\frac{2}{7}$ **l** $\frac{3}{8}+\frac{1}{3}$

m $\frac{2}{9}+\frac{1}{12}$ **n** $\frac{9}{21}+\frac{1}{7}$ **o** $\frac{27}{50}-\frac{7}{20}$

First rewrite with the LCD.

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Example 18 Adding and subtracting mixed numerals

Evaluate:

a $1\frac{2}{3} + 4\frac{5}{6}$

b $3\frac{2}{5} - 2\frac{3}{4}$

SOLUTION

a $1\frac{2}{3} + 4\frac{5}{6} = \frac{5}{3} + \frac{29}{6}$

$= \frac{10}{6} + \frac{29}{6}$

$= \frac{39}{6}$

$= \frac{13}{2}$ or $6\frac{1}{2}$

b $3\frac{2}{5} - 2\frac{3}{4} = \frac{17}{5} - \frac{11}{4}$

$= \frac{68}{20} - \frac{55}{20}$

$= \frac{13}{20}$

EXPLANATION

Change each mixed numeral to an improper fraction.
Remember that the lowest common denominator of 3 and 6 is 6. Change $\frac{5}{3}$ to an equivalent fraction with denominator 6, then add the numerators and simplify.

Convert to improper fractions, then rewrite as equivalent fractions with the same denominator.

Subtract the numerators.

7 Evaluate the following.

a $1\frac{1}{3} + 2\frac{1}{2}$ **b** $1\frac{1}{3} - \frac{3}{4}$ **c** $5\frac{1}{2} - 2\frac{7}{10}$

d $2\frac{1}{3} + \frac{1}{4}$ **e** $4\frac{1}{2} + 2\frac{1}{4}$ **f** $6\frac{1}{3} - 2\frac{1}{2}$

g $1\frac{3}{8} - 1\frac{1}{4}$ **h** $2\frac{3}{4} - 1\frac{1}{3}$ **i** $4\frac{1}{10} - 2\frac{2}{5}$

j $1\frac{7}{12} - \frac{2}{3}$ **k** $3\frac{1}{5} + 1\frac{1}{2}$ **l** $2\frac{5}{6} - 1\frac{1}{4}$

First rewrite using improper fractions.

PROBLEM-SOLVING AND REASONING 8–10 9–12

8 To remove impurities a mining company filters $3\frac{1}{2}$ tonnes of raw material. If $2\frac{5}{8}$ tonnes are removed, what quantity of material remains?

9 When a certain raw material is processed it produces $3\frac{1}{7}$ tonnes of mineral and $2\frac{3}{8}$ tonnes of waste. How many tonnes of raw material were processed?

The concentration (proportion) of the desired mineral within an ore body is vital information in the minerals industry.

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10 The ingredients in a punch recipe were $2\frac{1}{4}$ L of apple juice, $1\frac{1}{2}$ L of guava juice and $1\frac{1}{5}$ L of lemonade.

a How much punch is produced?

b If a cup holds 150 mL, how many cups can this punch serve?

11 Clarissa owns $\frac{3}{10}$ of a company, Sally owns another $\frac{1}{4}$ of the company and Keith owns $\frac{2}{5}$ of it. The bank owns the rest. How much of the company does the bank own?

12 Here is an example involving the subtraction of fractions where improper fractions are not used.

$$2\frac{1}{2} - 1\frac{1}{3} = 2\frac{3}{6} - 1\frac{2}{6}$$
$$= 1\frac{1}{6}$$

Try this technique on the following problem and explain the difficulty that you find.

$$2\frac{1}{3} - 1\frac{1}{2}$$

ENRICHMENT – 13

Which fractions do you choose?

13 The six fractions listed below are each used exactly once in the following questions.

a Arrange the six fractions correctly so that each equation produces the required answer. Use each fraction only once.

$\frac{1}{8}, \frac{5}{12}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

i $\square + \square = \frac{7}{8}$

ii $\square - \square = \frac{1}{4}$

iii $\square - \square = \frac{1}{3}$

b Which of the fractions in part **a** produce an answer of 2 when added?

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1F Multiplying and dividing fractions REVISION

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Rules must also be followed when multiplying and dividing fractions.

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5.1
4

Let's start: How much is left?

Tom, Sarah, Zara and Kimberly all share a strawberry tart.
How much is left for Kimberly to eat if:

- Tom eats half the tart?
- then Sarah eats half of what's left?
- then Zara eats a third of the remaining section?

Key ideas

- To multiply fractions (proper or improper), multiply the numerators together and multiply the denominators together.
  - In general: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

    For example: $\frac{2}{5} \times \frac{3}{7} = \frac{2 \times 3}{5 \times 7} = \frac{6}{35}$
- The word 'of' usually means multiply.

  For example: $\frac{1}{3}$ of $24 = \frac{1}{3} \times 24$
- Cancel any numerator with any denominator before multiplying.
- To find the reciprocal of a fraction, you simply invert it (that is, flip the proper or improper fraction upside down).
  - The reciprocal of 2 is $\frac{1}{2}$ since $2 = \frac{2}{1}$.
  - The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.
  - The reciprocal of $1\frac{1}{5}$ $\left(\text{given } 1\frac{1}{5} = \frac{6}{5}\right)$ is $\frac{5}{6}$.
- To divide a number by a fraction, multiply by its reciprocal.
  - In general: $\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c}$

    For example: $\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \times \frac{6}{5} = \frac{2}{\cancel{3}_1} \times \frac{\cancel{6}^2}{5} = \frac{4}{5}$

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Exercise 1F REVISION

UNDERSTANDING AND FLUENCY 1–4, 5–7(½) 4, 5–7(½)

1 Simplify the following fractions.

a $\frac{5}{10}$ **b** $\frac{10}{15}$ **c** $\frac{6}{12}$

d $\frac{12}{18}$ **e** $\frac{10}{20}$ **f** $\frac{15}{35}$

g $\frac{9}{27}$ **h** $\frac{12}{20}$ **i** $\frac{16}{24}$

Find the HCF of the numerator and denominator first.

2 Find the value of:

a $\frac{1}{2}$ of 24 **b** $\frac{3}{4}$ of 16 **c** $1\frac{1}{2}$ of 4

d $\frac{1}{3}$ of 12 **e** $\frac{9}{10}$ of 200 **f** $\frac{2}{5}$ of 30

3 Find the reciprocal of:

a $\frac{3}{4}$ **b** $\frac{1}{7}$ **c** $\frac{2}{3}$

d $\frac{5}{8}$ **e** 6 **f** 8

g $1\frac{1}{2}$ **h** $1\frac{2}{3}$ **i** $2\frac{3}{4}$

Whole numbers can be first written with a denominator of 1.

$6 = \frac{6}{1}, 8 = \frac{8}{1}$

4 True or false?

a $1\frac{3}{4} = \frac{7}{4}$ **b** $2\frac{1}{3} = 6$ **c** $\frac{2}{5} \times \frac{5}{2} = 1$

d $4 = \frac{4}{1}$ **e** $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ **f** $\frac{7}{10}$ of $20 = 14$

Example 19 Multiplying with proper fractions

Evaluate the following.

a $\frac{2}{3} \times \frac{5}{7}$ **b** $\frac{4}{5} \times \frac{25}{32}$

SOLUTION

a $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

b $\frac{4}{5} \times \frac{25}{32} = \frac{\cancel{4}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{25}^{5}}{\cancel{32}_{8}}$

$= \frac{5}{8}$

EXPLANATION

Multiply the numerators together.
Multiply the denominators together.

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7}$$

Cancel the common factors between the numerators and the denominators.

Multiply the numerators and then multiply the denominators.

$$\frac{1 \times 5}{1 \times 8}$$

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5 Evaluate the following.

a $\frac{3}{5} \times \frac{1}{4}$ **b** $\frac{2}{3} \times \frac{2}{5}$ **c** $\frac{3}{4} \times \frac{5}{7}$ **d** $\frac{1}{2} \times \frac{1}{3}$

e $\frac{1}{4} \times \frac{1}{5}$ **f** $\frac{1}{9} \times \frac{2}{3}$ **g** $\frac{5}{8} \times \frac{16}{17}$ **h** $\frac{1}{2} \times \frac{6}{7}$

i $\frac{2}{3} \times \frac{3}{5}$ **j** $\frac{8}{9} \times \frac{3}{4}$ **k** $\frac{4}{5} \times \frac{1}{2}$ **l** $\frac{5}{6} \times \frac{24}{25}$

m $\frac{6}{7} \times \frac{7}{8}$ **n** $\frac{7}{9} \times \frac{18}{21}$ **o** $\frac{8}{21} \times \frac{7}{12}$

Example 20 Multiplying with mixed numerals

Evaluate $1\frac{2}{3} \times 2\frac{1}{10}$.

SOLUTION

$$1\frac{2}{3} \times 2\frac{1}{10} = \frac{\cancel{5}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{21}^{7}}{\cancel{10}_{2}}$$

$$= \frac{7}{2} \text{ or } 3\frac{1}{2}$$

EXPLANATION

Rewrite as improper fractions.

Cancel common factors between numerators and denominators and then multiply numerators and denominators.

6 Evaluate the following. Check your answers using a calculator.

a $1\frac{1}{2} \times \frac{1}{3}$ **b** $1\frac{2}{3} \times \frac{3}{4}$ **c** $2\frac{1}{3} \times \frac{9}{10}$ **d** $1\frac{1}{2} \times 1\frac{1}{3}$ **e** $2\frac{1}{3} \times 1\frac{4}{5}$

f $\frac{3}{4} \times 1\frac{1}{7}$ **g** $2\frac{1}{2} \times 2\frac{1}{2}$ **h** $4\frac{1}{2} \times \frac{8}{9}$ **i** $1\frac{3}{4} \times 1\frac{2}{3}$ **j** $3\frac{3}{4} \times \frac{8}{5}$

Example 21 Dividing fractions

Evaluate the following.

a $\frac{4}{15} \div \frac{12}{25}$ **b** $1\frac{17}{18} \div 1\frac{1}{27}$

SOLUTION

a $$\frac{4}{15} \div \frac{12}{25} = \frac{4}{15} \times \frac{25}{12}$$

$$= \frac{\cancel{4}^{1}}{\cancel{15}_{3}} \times \frac{\cancel{25}^{5}}{\cancel{12}_{3}}$$

$$= \frac{5}{9}$$

EXPLANATION

To divide by $\frac{12}{25}$ we multiply by its reciprocal $\frac{25}{12}$.

Cancel common factors between numerators and denominators then multiply fractions.

b $$1\frac{17}{18} \div 1\frac{1}{27} = \frac{35}{18} \div \frac{28}{27}$$

$$= \frac{\cancel{35}^{5}}{\cancel{18}_{2}} \times \frac{\cancel{27}^{3}}{\cancel{28}_{4}}$$

$$= \frac{15}{8} \text{ or } 1\frac{7}{8}$$

Rewrite mixed numerals as improper fractions.

Multiply by the reciprocal of the second fraction.

The reciprocal of $\frac{28}{27}$ is $\frac{27}{28}$.

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7 Evaluate the following (recall that the reciprocal of 8 is $\frac{1}{8}$). Check your answers using a calculator.

a $\frac{4}{7} \div \frac{3}{5}$ **b** $\frac{3}{4} \div \frac{2}{3}$ **c** $\frac{5}{8} \div \frac{7}{9}$

d $\frac{3}{7} \div \frac{4}{9}$ **e** $\frac{3}{4} \div \frac{9}{16}$ **f** $\frac{4}{5} \div \frac{8}{15}$

g $\frac{8}{9} \div \frac{4}{27}$ **h** $\frac{15}{42} \div \frac{20}{49}$ **i** $15 \div \frac{5}{6}$

j $6 \div \frac{2}{3}$ **k** $12 \div \frac{3}{4}$ **l** $24 \div \frac{3}{8}$

m $\frac{4}{5} \div 8$ **n** $\frac{3}{4} \div 9$ **o** $\frac{8}{9} \div 6$

p $14 \div 4\frac{1}{5}$ **q** $6 \div 1\frac{1}{2}$ **r** $1\frac{1}{3} \div 8$

s $2\frac{1}{4} \div 1\frac{1}{2}$ **t** $4\frac{2}{3} \div 5\frac{1}{3}$

Multiply by the reciprocal of the second fraction.

Mixed numerals must become improper fractions first.

PROBLEM-SOLVING AND REASONING

8–10 | 9–12

8 Find the following amounts.

a $\frac{1}{2}$ of \$20 **b** $\frac{1}{2}$ of \$7 **c** $\frac{1}{2}$ of \$569

d $\frac{1}{3}$ of \$18 **e** $\frac{1}{10}$ of \$500 **f** $\frac{3}{4}$ of \$24

g $\frac{4}{5}$ of \$8000 **h** $\frac{5}{9}$ of \$27 **i** $\frac{9}{10}$ of \$1

The word 'of' means multiply. \$20 as a fraction is $\frac{20}{1}$

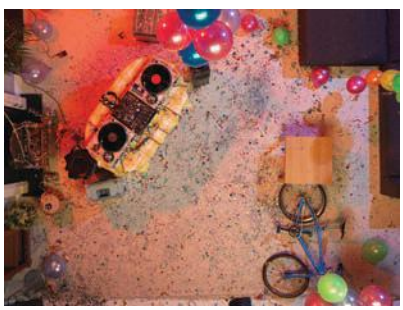
9 Evaluate these mixed-operation problems.

a $\frac{2}{3} \times \frac{1}{3} \div \frac{7}{9}$ **b** $\frac{4}{5} \times \frac{3}{5} \div \frac{9}{10}$ **c** $\frac{4}{9} \times \frac{6}{25} \div \frac{1}{150}$

10 In a $1\frac{1}{2}$ hour maths exam, $\frac{1}{6}$ of that time is allocated as reading time. How long is the reading time?

11 A car's fuel gauge shows that it has $\frac{1}{4}$ of a tank of petrol remaining. The petrol tank holds 64 litres of fuel when full. The car can travel 10 km on one litre. How many kilometres can you travel on the amount of petrol that remains in the tank?

12 Thomas, Ahn and Oscar agree to equally share the job of cleaning the house after a party. They estimate the job will take $4\frac{1}{2}$ hours to complete. How many minutes of work should they each contribute?



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ENRICHMENT – 13

Electronics and reciprocals

13 Reciprocals are used in the study of electronics. On the calculator, the x^{-1} can be used to find the reciprocal of a number.

a Use this button to find reciprocals of the following numbers.

i 2

ii 5

iii 6.2

iv 1.5

v -1

b If $\frac{1}{6.4} + \frac{1}{7.2} = A$, find the value of $\frac{1}{A}$ (answer to two decimal places).

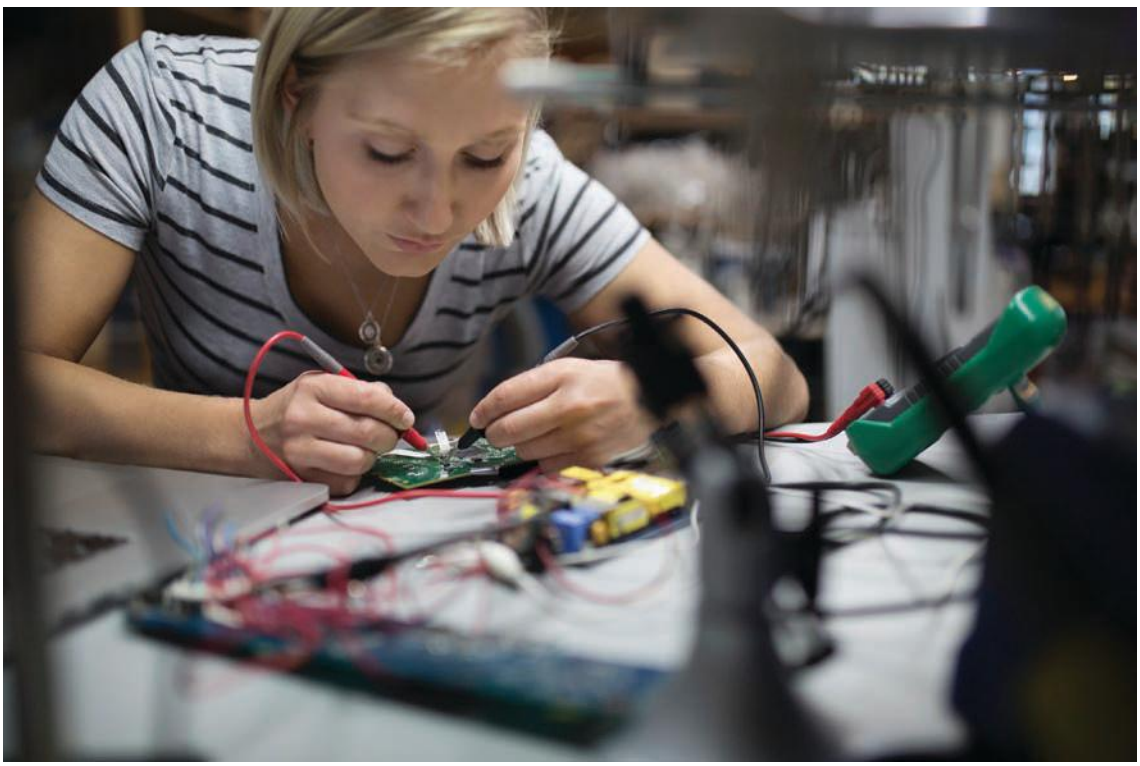
c If $\frac{1}{50} + \frac{1}{50} = B$, what is $\frac{1}{B}$?

The techniques in parts **b** and **c** are used to find the total resistance, measured in ohms, Ω, in electronics.

d Investigate, using your calculator, the types of positive numbers for which the reciprocal is bigger than the original number.

e What positive number is its own reciprocal?

f Are there any numbers that do not have a reciprocal?



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1G Ratios REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Fractions, ratios and rates are used to compare quantities. A lawnmower, for example, might require $\frac{1}{6}$ of a litre of oil to make a petrol mix of 2 parts oil to 25 parts petrol, which is an oil to petrol ratio of 2 to 25 or 2 : 25.

Two-stroke lawnmowers run on petrol and oil mixed in a certain ratio.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: The lottery win

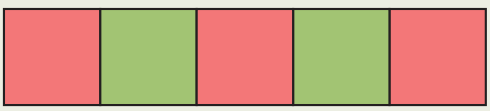
$100 000 is to be divided up for three lucky people into a ratio of 2 to 3 to 5 (2 : 3 : 5). Work out how the money is to be divided.

- Clearly write down your method and answer. There may be many different ways to solve this problem.
- Write down and discuss the methods suggested by other students in the class.

Key ideas

■ **Ratios** are used to compare quantities with the same units.
- The ratio of a is to b is written $a : b$.
- Ratios in simplest form use whole numbers that have no common factor.

In the diagram below the ratio can be written as:
green squares to red squares = 2 : 3



■ A ratio can be reduced to its simplest form by dividing by the highest common factor (HCF).

$$8 : 12 \xrightarrow{\div 4} 2 : 3 \qquad 18 : 6 \xrightarrow{\div 6} 3 : 1$$

■ The **unitary method** involves finding the value of one part of a total.
- Once the value of one part is found, then the value of several parts can be easily determined.

Ratio A way of comparing quantities of the same unit, separated by a colon

Unitary method A way of solving a problem by reducing one of the units to 1

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Exercise 1G REVISION

UNDERSTANDING AND FLUENCY 1–4, 5–6(½), 7, 8–9(½) 4, 5–9(½)

1 Write down the missing number.

a 2 : 5 = ☐ : 10 **b** 3 : 7 = ☐ : 28 **c** 5 : 8 = 15 : ☐ **d** 7 : 12 = 42 : ☐

e ☐ : 12 = 1 : 4 **f** 4 : ☐ = 16 : 36 **g** 8 : ☐ = 64 : 88 **h** ☐ : 4 = 12 : 16

2 Consider the ratio of boys to girls of 4 : 5.

a What is the total number of parts?

b What fraction of the total are boys?

c What fraction of the total are girls?

d If there were 18 students in total, how many of them are boys?

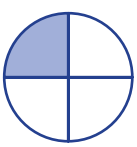
e If there were 18 students in total, how many of them are girls?

3 Draw up a table with the following headings.

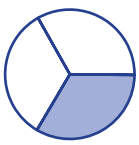
Diagram	Ratio of shaded parts to unshaded parts	Ratio of shaded parts to parts in the whole

Enter information about these diagrams to complete the table.

a



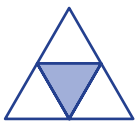
b



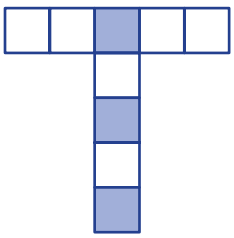
c

d

e



f



4 At a school presentation assembly, there were five times as many boys as adults and twice as many girls as boys. Write the ratio of:

a boys to adults

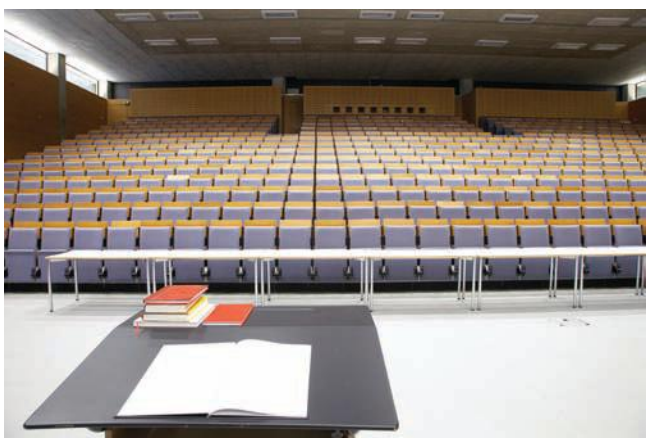
b boys to girls

c girls to boys

d girls to adults

e boys to the total number of people at the assembly

Remember that, with a ratio, the order matters.



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Example 22 Simplifying ratios using HCFs

Simplify the ratio 38 : 24.

SOLUTION	EXPLANATION
38 : 24 = 19 : 12	The HCF of 38 and 24 is 2 so divide both sides by 2.

5 Simplify these ratios.

a 6 : 30 **b** 8 : 20 **c** 40 : 50
d 2 : 20 **e** 15 : 18 **f** 9 : 27
g 18 : 6 **h** 24 : 36 **i** 52 : 39
j 144 : 36 **k** 48 : 96 **l** 30 : 10
m 2000 : 5600 **n** 3 : 6 : 12 **o** 15 : 30 : 10

Divide the numbers by their HCF.

Example 23 Simplifying ratios involving fractions and decimals

Simplify these ratios.

a 0.2 : 0.14 **b** $2\frac{1}{2} : 1\frac{1}{3}$

SOLUTION	EXPLANATION
a 0.2 : 0.14 = 20 : 14 = 10 : 7	Multiply by 100 to remove all the decimal places and simplify.
b $2\frac{1}{2} : 1\frac{1}{3} = \frac{5}{2} : \frac{4}{3}$	Write as improper fractions using the same denominator.
$= \frac{15}{6} : \frac{8}{6}$	
$= 15 : 8$	Multiply both sides by 6 to write as whole numbers.

6 Simplify these ratios.

a 0.1 : 0.2 **b** 0.3 : 4.1 **c** 0.5 : 3.2
d 0.3 : 0.9 **e** 0.7 : 3.5 **f** 0.4 : 0.12
g 8 : 0.2 **h** 15 : 0.01 **i** 1.6 : 0.56

7 Simplify the following.

a $\frac{1}{4} : \frac{1}{3}$ **b** $\frac{2}{3} : \frac{1}{4}$ **c** $1\frac{1}{2} : 3\frac{1}{3}$
d $2\frac{1}{4} : 1\frac{2}{5}$ **e** $\frac{3}{8} : 1\frac{3}{4}$ **f** $1\frac{5}{6} : 3\frac{1}{4}$

A simplified ratio must include only whole numbers (with no common factors).

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Example 24 Simplifying ratios involving units

Simplify the ratio of 560 metres to 2 kilometres.

SOLUTION	EXPLANATION
560 m : 2 km	Write using ratio notation.
= 560 m : 2000 m	Convert the units so that both are the same: 2 km = 2000 m
= 560 : 2000	Cancel the units and divide both numbers by their HCF. HCF of
= 7 : 25	560 and 2000 is 80.

8 Write each of the following as a ratio in simplest form.

a 80 c : \$8
b 90c : \$4.50
c 80 cm : 1.2 m
d 0.7 kg : 800 g
e 2.5 kg : 400 g
f 30 min : 2 hours
g 45 min : 3 hours
h 4 hours : 50 min
i 40 cm : 2 m : 50 cm
j 80 cm : 600 mm : 2 m
k 2.5 hours : 1.5 days
l 0.09 km : 300 m : 1.2 km

Convert to the same units first.

Example 25 Dividing into a given ratio

\$300 is to be divided into the ratio 2 : 3.
Find the value of the larger portion using the unitary method.

SOLUTION	EXPLANATION
Total number of parts is 2 + 3 = 5	Use the ratio 2 : 3 to get the total number of parts.
5 parts = \$300	
1 part = $\frac{1}{5}$ of \$300	Calculate the value of each part. (\$300 ÷ 5)
= \$60	
Larger portion = 3 × \$60	Calculate the value of 3 parts.
= \$180	

9 Divide:

a \$500 in the ratio of 1 : 4
b \$36 in the ratio of 4 : 5
c 88 kg in the ratio of 3 : 8
d \$96 in the ratio of 7 : 5
e \$500 in the ratio of 2 : 3
f 2000 g in the ratio of 3 : 5
g \$100 in the ratio of 7 : 3
h \$600 in the ratio of 1 : 1
i \$70 in the ratio of 2 : 7 : 1
j 420 g in the ratio of 8 : 2

First find the value of one part.

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PROBLEM SOLVING AND REASONING

10–12 | 11–14

10 420 g of flour is to be divided into a ratio of 7 : 3 for two different recipes. Find the smaller amount.

11 Kirsty manages a restaurant. Each day she buys watermelons and mangoes in the ratio of 3 : 2. How many watermelons did she buy if, on one day, the total number of watermelons and mangoes was 200?

12 If a prize of $6000 was divided among Georgia, Leanne and Maya in the ratio of 5 : 2 : 3, how much did each girl get?

13 The dilution ratio for a particular chemical with water is 2 : 3 (chemical to water). If you have 72 litres of chemical, how much water is needed to dilute the chemical?

14 Amy, Belinda, Candice and Diane invested money in the ratio of 2 : 3 : 1 : 4 in a publishing company. If the profit was shared according to their investment, and Amy's profit was $2400, find the profit each investor made.

ENRICHMENT

– | 15

Mixing drinks

15 Four jugs of cordial have a cordial to water ratio as shown and a given total volume.

Jug	Cordial to water ratio	Total volume
1	1 : 5	600 mL
2	2 : 7	900 mL
3	3 : 5	400 mL
4	2 : 9	330 mL

a How much cordial is in:
- **i** jug 1?
- **ii** jug 2?

b How much water is in:
- **i** jug 3?
- **ii** jug 4?

c Jugs 1 and 2 were mixed together to give 1500 mL of drink.
- **i** How much cordial is in the drink?
- **ii** Find the ratio of cordial to water in the drink.

d Find the ratio of cordial to water if the following jugs are mixed.
- **i** Jugs 1 and 3
- **ii** Jugs 2 and 3
- **iii** Jugs 2 and 4
- **iv** Jugs 3 and 4

e Which combination of two jugs gives the strongest cordial to water ratio?

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1H Rates and direct proportion

Interactive
Widgets
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Walkthrough

It is often necessary to compare two quantities with different units. When this occurs it is called a rate.

Rates are used to describe speed (m/s, km/h), pay, lap times in Formula One racing, and even the prices at the supermarket. Using rates is important in our everyday lives.

Lap times and speeds in Formula One racing are examples of rates.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Fastest animals on Earth

Working in pairs, arrange these animals from fastest to slowest. Then see if you can write their speeds in metres per second (m/s).

a Zebra 64.37 km/h

b Ostrich 70 km/h

c Giraffe 52 km/h

d African elephant 40.7 km/h

e Lion 80 km/h

f Cheetah 120 km/h

g Dog 72.4 km/h

h Galapagos tortoise 0.32 km/h

i Kangaroo 70.01 km/h

j Wildebeest 64.05 km/h

k Hyena 65 km/h

l Fox 32.03 km/h

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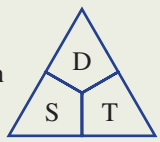
Key ideas

- A **rate** compares related quantities with different units.
  - The rate is usually written with one quantity compared to a single unit of the other quantity.
    For example: 50 km per 1 hour or 50 km/h.
- Rates can be used to determine best buys when purchasing products.
- Speed (S) = Distance (D) ÷ Time (T)
  For example: 80 km in 2 hours = 40 km/h



$S = \frac{D}{T}$, $D = S \times T$, $T = \frac{D}{S}$

- Two quantities are in **direct proportion** when they increase or decrease in the same ratio. For example:
  If the cost of petrol doubles from \$1.40 per litre to \$2.80 per litre, then the cost of filling your 60-litre petrol tank doubles from \$84 to \$168.

Rate
The number of units of one quantity for each single unit of another quantity

Direct proportion
The relationship between two quantities that increase or decrease at the same rate

Exercise 1H

UNDERSTANDING AND FLUENCY	1–4, 5(½), 6–9	4, 5(½), 6–9

1 Write the following as rates. For example, 40 km in 2 hours = 40 km/h.

a Tom travelled 60 km in 2 hours **b** \$84 in 2 hours **c** 600 km in 10 hours

d \$2 for $\frac{1}{2}$ kg **e** Mark earned \$80 in 5 hours

2 George earns \$720 for 8 hours work. What is his hourly rate of pay?

3 Find the cost of 1 kg if:

a 2 kg costs \$8 **b** 5 kg costs \$15 **c** 4 kg costs \$10

4 Callum's daily wage is directly proportional to the number of hours he works. If he works for 2 hours, he earns \$42. How much does he earn if the number of hours worked:

a doubles? **b** halves? **c** triples (×3)?

Example 26 Simplifying rates

Write these rates in simplest form.

a 120 km every 3 hours **b** 5000 revolutions in $2\frac{1}{2}$ minutes

SOLUTION	EXPLANATION
a 120 km per 3 hours $= \frac{120}{3}$ km/h	Divide by 3 to write the rate compared to 1 hour.
$= 40$ km/h	
b 5000 revolutions per $2\frac{1}{2}$ minutes	
$= 10\,000$ revolutions per 5 minutes	First multiply by 2 to remove the fraction.
$= \frac{10\,000}{5}$ rev/min	Then divide by 5 to write the rate using 1 minute.
$= 2000$ rev/min	

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5 Write these as rates in their simplest form.

a \$84 in 3 hours

b \$200 in 4 hours

c 12 kg every 2 minutes

d \$3.50 for $\frac{1}{2}$ kg

e 64 runs in 16 overs

f 623 points in 7 games

g 76 cm in 4 years

h 56 metres in 4 seconds

i 207 heart beats in $2\frac{1}{4}$ minutes

j 180 mL in 22.5 seconds

Work out each per one unit.

Example 27 Determining distance, speed and time

A family took 3 hours to complete the 210 km from their home to their holiday house.

a Find their average speed for the trip.

b If they increased their average speed to 90 km/h, how many hours and minutes would the trip have taken?

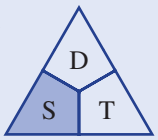
SOLUTION

a Average speed = distance ÷ time
= 210 km ÷ 3 hours
= 70 km/h

b Time = distance ÷ speed
= 210 km ÷ 90 km/h
$= 2\frac{1}{3}$ hours
= 2 hours 20 minutes

EXPLANATION

Use the DST triangle and cover the S to provide the formula for finding speed: $S = \frac{D}{T}$ or D ÷ T

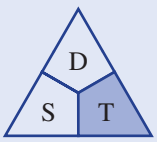


Substitute in your values and calculate the speed.

Remember to put units in your answer.

Use the DST triangle and cover the T to find the formula for time:

$T = \frac{D}{S}$ or D ÷ S



Substitute in your values and find the time:

$\frac{1}{3}$ hour $= \frac{1}{3} \times 60$ min = 20 minutes

6 Find the average speed (km/h) of a:

a car travelling 140 km in 2 hours

b bike travelling 60 km in 4 hours

c walker travelling 12 km in 3 hours

d horse galloping 2.7 km in 3 minutes ($\frac{1}{20}$ hour)

e truck travelling 760 km in 9.5 hours

7 Rick drives the 1040 km from Sydney to Melbourne to watch the Sydney Swans play the Western Bulldogs in the AFL. He averaged 80 km/h. How many hours did the trip take?

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Example 28 Finding best buys

Which is better value?
5 kg of potatoes for \$3.80 or 3 kg for \$2.20

SOLUTION	EXPLANATION
Price per kg:	
5 kg bag:	
1 kg costs \$3.80 ÷ 5 = \$0.76	Divide each price by the number of kilograms to find the price per kilogram.
3 kg bag:	
1 kg costs \$2.20 ÷ 3 = \$0.73	
∴ The 3 kg bag is better value.	Then compare. Choose the one that is the better value.

8 Determine the best buy in each of the following.

- **a** 2 kg of washing powder for \$11.70 or 3 kg for \$16.20
- **b** 1.5 kg of red delicious apples for \$4.80 or 2.2 kg of royal gala apples for \$7.92
- **c** 2.4 litres of orange juice for \$4.20 or 3 litres of orange juice for \$5.40
- **d** 0.7 GB of internet usage for \$14 or 1.5 GB for \$30.90 with different service providers

Example 29 Using direct proportion

The amount of fertiliser needed is directly proportional to the area of land being covered. A company claims that one 8 kg bag of its product covers 10 m^2.
How many kilograms would be needed to cover 25 m^2?

SOLUTION	EXPLANATION
8 kg per 10 m^2	Write the rate:
= 0.8 kg/m^2	8 kg/10 m^2
0.8 × 25	Divide by 10 to simplify the rate to kg per m^2.
= 20 kg are needed for 25 m^2	Multiply by 25 to find the kilograms needed for 25 m^2.

9 Consider the following.

- **a** The number of words Shute can type is directly proportional to the time he works. If Shute types 65 words a minute, how many words can he type in:
  - **i** 2 minutes? **ii** 5 minutes? **iii** 1 hour?
- **b** The number of grams of copper needed by a jeweller to make up his alloy is directly proportional to the number of grams of gold he uses. If he uses 3 grams of copper per 22 grams of gold, find the gold needed for:
  - **i** 6 grams of copper **ii** 30 grams of copper
- **c** Sally and Tom both pay the same proportion of their income in tax. When Sally earns \$25 000 she pays \$5000 in tax. How much tax does Tom pay if he earns four times what Sally does?

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PROBLEM-SOLVING AND REASONING

10–12 | 11–14

10 Hamish rides his bike at an average speed of 22 km/h. How far does he ride in:

a $2\frac{1}{2}$ hours? **b** $\frac{3}{4}$ hours? **c** 15 minutes?

11 Find the cost of 100 g of each product below, then decide which is the best buy.

a 300 g of coffee A at \$10.80 or 220 g of coffee B at \$8.58.

b 600 g of pasta A for \$7.50 or 250 g of pasta B for \$2.35

c 1.2 kg of cereal A for \$4.44 or 825 g of cereal B for \$3.30

12 A shearer sheared 80 sheep in 2 hours.

a Express this as a rate.

b If the shearer is able to continue at this rate for $5\frac{1}{2}$ hours, how many sheep can be sheared?

c How long would it take, at this rate, to shear all 1000 sheep on the property?

13 Light travels at approximately 300 000 km/s.

a Express this speed as km/h.

b How far does light travel in 2 minutes?

c How long does light from the Sun take to travel the 149 million kilometres to Earth?

14 Decide whether each of the following situations could reasonably be described as an example of direct proportion.

a An increase in the cost of petrol to the amount paid

b An increase in speed to the time taken

c The increase in food eaten to weight gained

d The increase in a toddler's height to their years of age

e The increase in heart rate to the time spent exercising

ENRICHMENT

– | 15

Practical rates, direct proportion

15 A cyclist's progress over a journey of 160 km is recorded. The ratio of time (in hours) to distance (in kilometres) is 1 : 20.

a Copy and complete the table showing the progress of the cyclist.

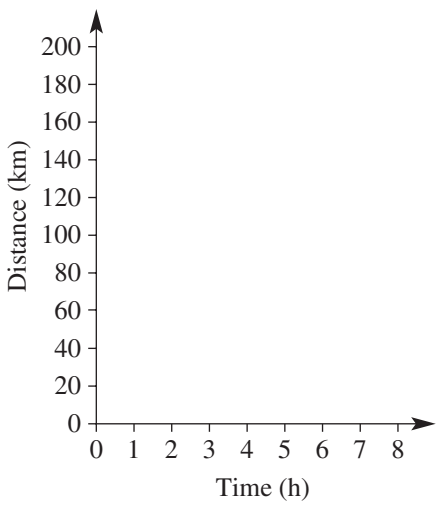
Time in hours	0	1	2	3	4	5	6	7	8
Distance in km	0	20							

b What is the cyclist's average speed?

c Draw a set of axes like the ones shown and record the information from the table onto the axes.

d What conclusions can you make about the shape of the graph? Why is this the case?

e If the cyclist doubles their speed, on the same set of axes plot a graph that shows the journey for this new speed.



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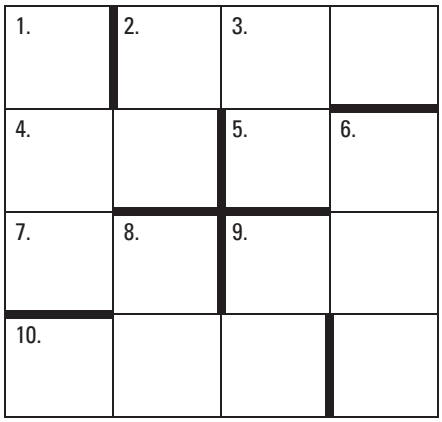
1 Can you arrange the first nine counting numbers with operations (+, –, ×, ÷) and brackets to give an answer of 100?

2 In how many ways can four girls and two boys be seated in a row if the two boys are to sit at the ends?

3 Complete the number cross below.



Clues

Across

2. The sum of 100, 150 and 3
4. The product of 5 and the first prime number after 6
5. Reverse the digits of the product of 13 and 5
7. One less than the square of 10
9. Half of 6 times 8
10. 2 to the power of 7

Down

1. A palindromic number
2. $5 \times 10 - 5 \times 5$
3. A multiple of 11
6. 639 written correct to 2 significant figures
8. The next even integer after 90
9. A multiple of 7

4 **a** Perfect numbers are positive integers that are equal to the sum of all their factors, excluding the number itself.

i Show that 6 is a perfect number.

ii There is one perfect number between 20 and 30. Find the number.

iii The next perfect number is 496. Show that 496 is a perfect number.

b Triangular numbers are the number of dots required to form triangles as shown in this table.

i Copy and complete this table.

Number of rows	1	2	3	4	5	6
Diagram	•	• • •	• • • • • •			
Number of dots (triangular number)	1	3				

ii Find the 7th and 8th triangular numbers.

c Fibonacci numbers are a sequence of numbers where each number is the sum of the two preceding numbers. The first two numbers in the sequence are 0 and 1.

i Write down the first 10 Fibonacci numbers.

ii If the Fibonacci numbers were to be extended in the negative direction, what would the first four negative Fibonacci numbers be?

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Chapter summary

Integers, decimals, fractions, ratios and rates

Decimals
Decimals are a way of writing fractions with denominators of 10, 100, 1000

$\frac{1}{10} = 0.1$, $\frac{1}{100} = 0.01$,

$\frac{1}{1000} = 0.001$

Rounding decimals
If the critical digit (the next one in the number) is 0, 1, 2, 3 or 4, round down. If it is 5, 6, 7, 8 or 9, round up.
4.763 = 4.8 (1 decimal place)
= 4.76 (2 decimal places)

Rational numbers
Rational numbers can be written as a fraction.
$3, -4, \frac{2}{3}, 0.\dot{3}, \sqrt{4}, 0.416$
are all rationals.
π is an example of an irrational number. It cannot be written as a fraction.

Ratios
Ratios compare two or more quantities with the same units in a definite order.
5 cents to \$1 = 5 : 100 = 1 : 20

You can simplify ratios by using the HCF.
24 : 36 = 2 : 3

Direct proportion
Two quantities are in direct proportion if when one quantity changes, the other quantity changes by the same factor. That is, they both halve or double etc.

Integers
Integers are the set of positive and negative whole numbers including zero.
$0, \pm1, \pm2, \pm3, \pm4, \ldots$

Working with negatives
- To add a negative number, you subtract.
  $a + (-b) = a - b$
- To subtract a negative number, you add.
  $a - (-b) = a + b$

× and ÷
- Two numbers of the same sign multiply or divide to give a positive answer. $-4 \times (-5) = 20$
  $-10 \div (-2) = 5$
- Two numbers with different signs multiply or divide to give a negative answer. $-4 \times 5 = -20$
  $10 \div (-2) = -5$

Fractions
- To add or subtract fractions, use the lowest common denominator.
- To multiply fractions (proper or improper), multiply the numerators and multiply the denominators after cancelling common factors.
  $\frac{8}{9} \times \frac{4}{7} = \frac{8 \times 4}{9 \times 7}$
- To divide by a fraction, multiply by the reciprocal.
  $\frac{3}{2} \div \frac{4}{5} = \frac{3}{2} \times \frac{5}{4}$

Reciprocals
The reciprocal of a proper or improper fraction is the fraction turned upside down (inverted).
The reciprocal of:

6 is $\frac{1}{6}$
$1\frac{1}{2}$ is $\frac{2}{3}$
$\frac{4}{5}$ is $\frac{5}{4}$ or $1\frac{1}{4}$

Rates
Rates compare two quantities with different units.
50 km in 2 hours = 25 km/h

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Multiple-choice questions

1 $\frac{2}{7}$ written as a decimal is:

A 0.29 **B** 0.286 **C** $0.28\dot{5}$ **D** $0.\overline{285714}$ **E** $0.28571\dot{4}$

2 3.0456 written to three significant figures is:

A 3.04 **B** 3.05 **C** 3.045 **D** 3.046 **E** 3.45

3 2.25 written as a fraction in simplest form is:

A $2\frac{1}{2}$ **B** $\frac{5}{4}$ **C** $\frac{9}{4}$ **D** $9\frac{1}{4}$ **E** $\frac{225}{100}$

4 $1\frac{1}{2} - \frac{5}{6}$ is equal to:

A $\frac{2}{3}$ **B** $\frac{5}{6}$ **C** $-\frac{1}{2}$ **D** $\frac{2}{6}$ **E** $\frac{1}{2}$

5 $\frac{2}{7} \times \frac{3}{4}$ is equivalent to:

A $\frac{8}{11}$ **B** $\frac{3}{7}$ **C** $\frac{5}{11}$ **D** $\frac{8}{12}$ **E** $\frac{3}{14}$

6 $\frac{3}{4} \div \frac{5}{6}$ is equivalent to:

A $\frac{5}{8}$ **B** 1 **C** 21 **D** $\frac{4}{5}$ **E** $\frac{9}{10}$

7 Simplifying the ratio 50 cm : 4 m gives:

A 50 : 4 **B** 8 : 1 **C** 25 : 2 **D** 1 : 8 **E** 5 : 40

8 0.28 as a fraction in its simplest form is:

A 0.28 **B** $\frac{28}{100}$ **C** $\frac{0.28}{100}$ **D** $\frac{2.8}{100}$ **E** $\frac{7}{25}$

9 Divide \$2.50 in the ratio of 4 : 1.

A 50 : 200 **B** 50c **C** \$2 : 50c **D** \$200 **E** \$2

10 A childcare centre requires the ratio of carers to children to be 1 : 5. How many carers are needed for 30 children?

A 5 **B** 6 **C** 150 **D** 30 **E** 36

11 Michael earns \$460 in 25 hours. His hourly rate of pay is:

A \$11 500/h **B** \$0.05/h **C** \$18.40/h **D** \$18.40/day **E** \$5/h

12 36 km in 40 minutes is the same as:

A 40 km/h **B** 36 km/h **C** 54 km/h **D** 90 km/h **E** 56 km/h

13 To mix a particular shade of pink, red paint is mixed in direct proportion to white paint. The ratio is 8 to 3. If 20 cans of red paint are used, how many cans of white paint will be needed to mix the correct shade of pink?

A 15 **B** 7.5 **C** 7 **D** 8 **E** 24

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Short-answer questions

1 Evaluate the following.

a $-4 - (-12)$ **b** $18 - 9 \times 7$ **c** $-90 \times (-3)$

d $(-4)^3$ **e** $-15 \div 5 \times (-2)$ **f** $(10 - 12) \times (7 - 11)$

2 Round these numbers to three significant figures.

a 21.483 **b** 29 130 **c** 0.15271 **d** 0.002414

3 Round these numbers to two decimal places.

a 14.976 83 **b** 0.7149871498… **c** 1.999

4 Write these fractions as decimals.

a $2\frac{1}{8}$ **b** $\frac{5}{6}$ **c** $\frac{13}{7}$

5 Write these decimals as fractions.

a 0.75 **b** 1.6 **c** 2.55

6 State the reciprocal of:

a $\frac{3}{4}$ **b** $\frac{1}{5}$ **c** 8 **d** $1\frac{2}{7}$

7 Simplify the following.

a $\frac{5}{6} - \frac{1}{3}$ **b** $1\frac{1}{2} + \frac{2}{3}$ **c** $\frac{13}{8} - \frac{4}{3}$

d $2\frac{1}{4} - 1\frac{1}{3}$ **e** $\frac{3}{8} \times \frac{4}{5}$ **f** $3\frac{1}{2} \times \frac{4}{7}$

g $5 \div \frac{4}{3}$ **h** $\frac{6}{7} \div 2$ **i** $3\frac{3}{4} \div 1\frac{2}{5}$

8 Simplify these ratios.

a 30 : 12 **b** 1.6 : 0.9 **c** $7\frac{1}{2} : 1\frac{2}{5}$

9 Divide 80 into the given ratio.

a 5 : 3 **b** 5 : 11 **c** 1 : 2 : 5

10 Dry dog food can be bought from store A for \$18 for 8 kg or from store B for \$14.19 for 5.5 kg.

a Determine the cost per kilogram at each store and state which is the best buy.

b Determine to the nearest whole number how many grams you get per dollar from each store.

11 Share \$660 in the ratio of 2 : 3 : 5.

12 Complete these rates.

a \$96 in 3 hours = \$ ____ /h

b 14 g in 2 minutes = ____ g/min

c 32 m in 10 seconds = ____ m/min

d \$7.40 in 30 minutes = \$ ____ /h

e 660 runs in 8 matches = ____ runs/match

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13 Write the following speeds in km/h.

a 780 km in 5 hours

b 90 km in $2\frac{1}{2}$ hours

14 Tom and Claire share their lottery win in the ratio of 3 : 2. If Tom got $9000, how much did Claire get?

15 In an electrical wire the resistance is directly proportional to the length of the wire. A length of 6 m has a resistance of 5 ohms. What is the length if the resistance is measured to be 30 ohms?

16 If a rectangle's perimeter is directly proportional to its sides, what is the effect on the perimeter of doubling the rectangle's length and width?

Extended-response questions

1 A class of 28 students has a ratio of boys to girls of 3 : 4.

a How many boys are in the class?

b Two boys leave and are replaced by two girls. How many boys and girls are now in the class?

c What is the new ratio of boys to girls?

2 The Tomslin family plans to drive the 856 km from Perth to Monkey Mia in Western Australia.

a At what speed should they travel, on average, if they hope to only take 10 hours?

b The family left home at 6 a.m. and arrived at Monkey Mia at 8 p.m. How long did the trip take?

c Calculate their average speed. Answer to one decimal place.

d How far would they have travelled at the speed in part **c** if they only drove for the original 10 hours as planned? Answer to the nearest km.

e The Tomslin's car has a fuel efficiency of 10.3 L per 100 km. How many litres of petrol is used on the drive to Monkey Mia?

f If the petrol is purchased at 149.7 cents per litre, what is the cost for the 856 km trip?

g The Tomslin's car has a 62 L fuel tank. After how many kilometres do they need to fill up? Answer to two decimal places.

h The Tomslin's decide to stop for petrol after they have completed 60% of the trip. What is the cost of their petrol?

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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

2 Financial mathematics

What you will learn

2A Percentages, fractions and decimals REVISION
2B Applying percentages REVISION
2C Percentage increase and decrease REVISION
2D Profits and discounts REVISION
2E Income
2F Taxation
2G Simple interest
2H Applications of simple interest

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NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRANDS: FRACTIONS, DECIMALS AND PERCENTAGES; FINANCIAL MATHEMATICS

Outcomes

A student operates with fractions, decimals and percentages.
(MA4–5NA)

A student solves financial problems involving purchasing goods.
(MA4–6NA)

A student solves financial problems involving earning, spending and investing money.
(MA5.1–4N)

© NESA, 2012

GFC: Global financial crisis

In 2007 the world's economy was rocked by what is considered the worst financial crisis since the Great Depression of the 1930s. Stock prices dropped and some of the world's most influential economies were in danger of collapsing.

Like the Great Depression, the GFC originated in the United States, yet the effects were felt around the world.

The GFC of 2007, and the five years following, was linked to the real estate market of the US. Low interest rates enticed people to borrow more, and banks issued more and more loans to potential homeowners, causing house prices to increase as demand for property increased. As with most trends, it could not continue indefinitely, so when people were not able to repay their loans as their interest rates increased, the real estate bubble burst, banks foreclosed and this put pressure on the economy. This impacted the global money markets and ordinary citizens around the world, as many other financial institutions had invested money in American real estate. Australia survived this period due to our strong mining industry and trade with Asia. Interest rates were lowered to stimulate growth in our economy and the housing market. It remains to be seen what the future brings as interest rates fluctuate over time.

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Pre-test

1 Simplify the following.

a $\frac{12}{100}$ **b** $\frac{20}{100}$ **c** $\frac{35}{100}$ **d** $\frac{75}{100}$ **e** $\frac{60}{100}$ **f** $\frac{50}{100}$

2 Multiply these decimals by 100.

a 0.99 **b** 0.58 **c** 0.9 **d** 1.22 **e** 0.08 **f** 1.5

3 Which is larger in each of the following pairs?

a $\frac{1}{2}$ or 55% **b** $\frac{3}{4}$ or 70% **c** 0.89 or 98%

4 Copy and complete.

a $61\% = \frac{\square}{100}$ **b** $9\% = \frac{\square}{100}$ **c** $37\% = \frac{\square}{100}$

d $121\% = \frac{\square}{100}$ **e** $1\% = \frac{\square}{100}$ **f** $75\% = \frac{\square}{100} = \frac{3}{\square}$

5 Copy and complete the following table.

Fraction	Decimal	Percentage
$\frac{1}{100}$		
	0.1	
	0.25	
		50%
$\frac{3}{4}$		

6 Write down 10% of these amounts.

a 100 g **b** 70 km **c** \$450 **d** \$8000 **e** \$5 **f** 90c

7 Find:

a 25% of \$400 **b** 75% of 80 m **c** 50% of \$3

d 10% of \$678 **e** 1% of 600 days **f** $\frac{1}{2}$% of 600 days

8 Complete the following.

a 1% of 60 = 60 divided by $\square$ **b** 10% of 50 = 50 divided by $\square$

c 5% of 100 = 100 divided by $\square$ **d** $33\frac{1}{3}$% of 963 = 963 divided by $\square$

e 25% of 88 = 88 divided by $\square$

9 Find $33\frac{1}{3}$% of 6300 km.

10 Which is larger: 56% of 78 or 87% of 65?

11 Copy and complete:

a 1 week = $\square$ days **b** 1 year = $\square$ days

c 1 year = $\square$ weeks **d** 1 year = $\square$ months

12 How many hours are there from:

a 5 a.m. to 7 p.m.? **b** 9 a.m. to 3 p.m.? **c** 8:30 a.m. to 9 p.m.?

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2A Percentages, fractions and decimals REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Percentages, fractions and decimals can be converted one into another. They express the same kind of quantities. We use percentages for many different things in our daily lives, especially in financial matters and profits. Some examples are loans, credit cards, discounts and profits.

We know from our previous work on percentages that they represent a fraction with a denominator of 100.

'Per cent' comes from the Latin term *per centum*, and means 'out of 100'.

Percentages are found all over the pages of the finance news.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Ordering with percentages

Ten different values are given in the box on the right.

$\frac{1}{2}$, 0.8, 0.05, 15%, $\frac{7}{20}$, 0.9, 9%, $\frac{3}{5}$, $\frac{1}{3}$, 0.3

In pairs, decide which of the numbers is the:

- smallest
- largest.

Write the 10 numbers in ascending order.

Key ideas

- There are six **conversions** involving fractions, decimals and percentages.

Conversion Changing (converting) a value into the equivalent value expressed in a different form

Conversion	Without a calculator	With a calculator
decimal to fraction $0.6 = \frac{6}{10} = \frac{3}{5}$	Use tenths, hundredths, etc. Simplify the fraction.	Type the decimal. Press =. Press the button that converts fractions to decimals.
fraction to decimal $\frac{13}{50} = \frac{26}{100} = 0.26$ $\frac{5}{8} = 5 \div 8 = 0.625$	Convert fraction to tenths or hundredths or divide numerator by denominator.	Type the fraction. Press =. Press the button that converts fractions to decimals.
decimal to percentage $0.24 \times 100 = 24$ $\therefore 0.24 = 24\%$	Multiply by 100. Add % symbol. (The decimal point appears to move two places to the right.)	Multiply by 100. Add % symbol.
percentage to decimal $24 \div 100 = 0.24$ $\therefore 24\% = 0.24$	Remove % symbol. Divide by 100. (The decimal point appears to move two places to the left.)	Remove % symbol. Divide by 100.

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Key ideas

Conversion	Without a calculator	With a calculator
fraction to percentage $\frac{13}{50} = \frac{26}{100} = 26\%$ $\frac{5}{8} = 5 \div 8 = 0.625$ $0.625 \times 100 = 62.5$ $\therefore \frac{5}{8} = 62.5\%$	Convert the fraction to hundredths. Add % symbol. Or divide numerator by denominator. Multiply by 100. Add % symbol.	Multiply by 100. Add % symbol.
percentage to fraction $60\% = \frac{60}{100} = \frac{3}{5}$	Remove % symbol. Write as 'hundredths'. Simplify fraction.	Remove % symbol. Divide by 100. Press the button that converts fractions to decimals.

- Expressing one quantity as a percentage of another (percentage composition):
  - Form a fraction, then convert to a percentage.

    For example: 4 goals from 10 attempts $= \frac{4}{10} = \frac{40}{100} = 40\%$

Exercise 2A REVISION

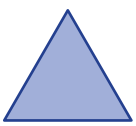
UNDERSTANDING AND FLUENCY 1–4, 5–9(½) 4, 5–9(½)

1 What percentage has been shaded?

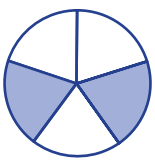
a

b

c



d



2 What percentage is not shaded?

a

b

3 Copy and complete this table of common percentages.

%	10%			50%			
Fraction			$\frac{1}{4}$			$\frac{1}{3}$	$\frac{2}{3}$
Decimal		0.2			0.75		

4 Scott scored 38 out of 50 on a maths quiz and Sarah scored 79% on the same test. Who scored the highest percentage?

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Example 1 Rewriting percentages and fractions

a Write $\frac{12}{25}$ as a percentage.

b Write 7.5% as a simple fraction.

SOLUTION

a $\frac{12}{25} = \frac{12}{25} \times 100$

$= 48\%$

b $7.5\% = \frac{7.5}{100}$

$= \frac{15}{200}$

$= \frac{3}{40}$

EXPLANATION

Multiply the fraction by 100 and write % symbol.

$\frac{12}{25} \times 100 = \frac{12}{\cancel{25}_{1}} \times \frac{\cancel{100}^{4}}{1}$

Write the percentage as a fraction, using a denominator of 100.

Multiply this fraction by $\frac{2}{2}$ so that we don't have a decimal in the fraction and it will be easier to simplify. $\frac{7.5}{100} \times \frac{2}{2} = \frac{15}{200}$

Simplify $\frac{15}{200} = \frac{3}{40}$ by dividing the numerator and denominator by their HCF (5)

5 Express the following fractions as percentages.

a $\frac{1}{5}$ **b** $\frac{4}{5}$ **c** $\frac{8}{10}$

d $\frac{3}{10}$ **e** $\frac{1}{4}$ **f** $\frac{1}{8}$

g $\frac{3}{4}$ **h** $\frac{12}{20}$ **i** $\frac{14}{25}$

j $\frac{7}{20}$ **k** $\frac{9}{100}$ **l** $\frac{3}{40}$

Multiply by 100

6 Express the following percentages as simplified fractions.

a 19% **b** 23% **c** 99%

d 5% **e** 22% **f** 45%

g 74% **h** 75% **i** 2.5%

j 17.25% **k** 1% **l** 125%

Divide by 100

Example 2 Converting between percentages and decimals

a Write 0.45 as a percentage.

b Write 25% as a decimal.

SOLUTION

a $0.45 = 0.45 \times 100$

$= 45\%$

b $25\% = 25 \div 100$

$= 0.25$

EXPLANATION

Multiply by 100. This moves the decimal point 2 places to the right.

Divide by 100. This moves the decimal point 2 places to the left.

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7 Express the following decimals as percentages.

a 0.78 **b** 0.95 **c** 0.65
d 0.48 **e** 0.75 **f** 1.42
g 0.07 **h** 0.3 **i** 0.03
j 1.04 **k** 0.12 **l** 0.1225

Move the decimal point two places to the right.

8 Express the following percentages as decimals.

a 12% **b** 83% **c** 57% **d** 88%
e 99% **f** 100% **g** 120% **h** 5%

Example 3 Writing a quantity as a percentage

Write 50c out of \$2.50 as a percentage.

SOLUTION

$$50\text{c out of \$2.50} = \frac{\cancel{50}^{1}}{\cancel{250}_{5}} \times 100$$
$$= 20\%$$

EXPLANATION

Convert to the same units (\$2.50 = 250c) and write as a fraction. Multiply by 100, cancelling first. 100 ÷ 5 = 20.

9 In each of the following cases, express the first quantity as a percentage of the second.

a 5 g out of 200 g **b** 40c out of \$4
c 10 km out of 200 km **d** 3 seconds out of 1 minute
e 200 m out of 1 km **f** 100 mL out of $\frac{1}{2}$L
g 200c out of \$1 **h** 45 marks out of a possible 60 marks

Write as a fraction, then multiply by 100.

PROBLEM-SOLVING AND REASONING 10–12 12–14

10 Copy and complete the table of the favourite summer sports of Year 9 students.

Sport	Number of students who chose sport	Fraction of the total	Percentage of the total
Swimming	44		
Golf	12		
Volleyball	58		
Cricket	36		
TOTAL			

11 Toni loses 31.5 cents in the dollar in tax. Express this as a percentage.

12 Bad weather stopped a cricket game for 35 minutes of a scheduled $3\frac{1}{2}$ hour match. What percentage of the scheduled time was lost?

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13 Joe lost 4 kg and now weighs 60 kg. What percentage of his original weight did he lose?

What was Joe's original weight?

14 A company claims that the apple pies it makes are 97% fat free. If the nutritional information on the side of the pack states that total fat is 7 grams of the 250-gram pie, is the claim correct?

ENRICHMENT – 15

Today's timeline

15 Copy and complete this table. In the first two columns, insert the times that you did each activity. Use that information to help you fill in the other columns.

a	Time you went to bed last night:	Time you woke up this morning:	Hours and minutes spent in bed:	Percentage of a day spent in bed:
b	Time you started breakfast today:	Time you finished breakfast today:	Minutes spent eating breakfast:	Percentage of the day you spent at breakfast:
c	Time school started today:	Time school finished today:	Hours and minutes spent at school:	Percentage of the day spent at school:
d	Time this maths lesson started:	Time this maths lesson will finish:	Minutes spent in the maths lesson:	Percentage of the day spent in the maths lesson:
e	Time school will finish today:	Time you will arrive home:	Minutes spent travelling home:	Percentage of the day spent travelling home:
f	Time you started your homework yesterday:	Time you finished your homework yesterday:	Minutes spent on homework:	Percentage of your day spent on homework:
g	Time you started watching TV or playing games yesterday:	Time you finished watching TV or playing games:	Minutes spent at this activity:	Percentage of the day spent at this activity:
h	Time you woke up today:	Time you will go to bed tonight:	Hours and minutes spent awake today:	Percentage of the day spent awake:

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2B Applying percentages REVISION

Interactive
Widgets
HOTsheets
Walkthrough

The media often quotes percentages in news stories and advertisements. For example:

- 90% of dentists prefer this toothbrush.
- There is a 45% chance of rain.
- A swing of 5% towards the Liberals is expected in the next election.

It is important to be able to find a percentage of a quantity or amount.

Australian Electoral Commission scrutineers count votes after an election. A swing to or away from a political party is described using percentages.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Today's newspaper challenge

In pairs, go through today's newspaper and find articles and advertisements that use percentages. Choose two and explain to the class how percentages are used in the items you choose.

Key ideas

- To find a percentage of a quantity (without a calculator):
  - Write the percentage as a fraction or decimal.
  - Multiply by the quantity.
- Calculator method:
  3% of 200 = 3 ÷ 100 × 200 = 6
- Mental strategies:

50%	25%	10%	5%	1%	75%
÷ 2	÷ 2 ÷ 2	÷ 10	÷ 10 ÷ 2	÷ 100	÷ 2 ÷ 2 × 3

- To find the original amount when given a percentage, you can use the unitary method or solve an equation.
  For example: 3% of an amount is 15. What is the original amount?

Unitary method

3% is 15

÷3 ÷3

1% is 5

×100 ×100

100% is 500

∴ The original amount is 500.

Equation method

3% of amount is 15.

$0.03 \times A = 15$

÷0.03 ÷0.03

$A = 500$

∴ The original amount is 500.

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Exercise 2B REVISION

UNDERSTANDING AND FLUENCY	1–3, 4–6(½)	3, 4–7(½)

1 Find these percentage amounts without using a calculator.

- **a** 10% of \$40
- **b** 5% of \$40
- **c** 25% of 8 kg
- **d** 75% of 8 kg
- **e** 1% of \$800
- **f** 2% of 140 seconds

2 If 1% of an amount is \$4, what is 100% of the amount?

3 True or false? 34% of 568 = 0.34 × 568

Example 4 Finding a percentage of a quantity

Find 15% of \$35.

SOLUTION

$$15\% \text{ of } \$35 = \frac{\cancel{15}^{3}}{\cancel{100}_{20}} \times \$35$$
$$= \$5.25$$

EXPLANATION

Write the percentage as a fraction out of 100 and multiply by \$35.
Or:
Write 0.15 × 35.
Note: 'of' means to 'multiply'.

4 Find the following amounts.

- **a** 10% of 20
- **b** 5% of 200
- **c** 20% of 40
- **d** 15% of 50
- **e** 8% of 720
- **f** 5% of 680
- **g** 15% of 8200
- **h** 70% of 60
- **i** 90% of 500
- **j** 75% of 44
- **k** 99% of 200
- **l** 3% of 50

$$10\% \text{ of } 20 = \frac{10}{100} \times 20$$

5 Find:

- **a** 10% of \$360
- **b** 50% of \$420
- **c** 75% of 64 kg
- **d** 12.5% of 240 km
- **e** 37.5% of 40 apples
- **f** 87.5% of 400 m
- **g** $33\frac{1}{3}$% of 750 people
- **h** $66\frac{2}{3}$% of 300 cars
- **i** $8\frac{3}{4}$% of \$560

These are all common percentages. Can you do them without a calculator?

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Example 5 Finding the original amount

Determine the original amount if 5% of the amount is \$45.

SOLUTION	EXPLANATION
5% of the amount = \$45 1% of the amount = \$9 100% of the amount = \$900 So the original amount is \$900	To use the unitary method, find the value of 1 part or 1% then multiply by 100 to find 100%.
Alternative method:	
5% of amount = 45 $0.05 \times A = 45$ $A = 45 \div 0.05$ $A = \$900$	Convert the information into an equation, letting A represent the amount. Solve the equation by dividing both sides by 0.05.

6 Determine the original amount if:

- **a** 10% of the amount is \$12
- **b** 6% of the amount is \$42
- **c** 3% of the amount is \$9
- **d** 40% of the amount is \$2.80
- **e** 90% of the amount is \$0.18
- **f** 6% of the amount is \$27
- **g** 12% of the amount is \$96
- **h** 15% of the amount is \$54

First find the value of 1%.

7 Determine the value of x in the following if:

- **a** 10% of x is \$54
- **b** 15% of x is \$90
- **c** 25% of x is \$127
- **d** 18% of x is \$225
- **e** 105% of x is \$126
- **f** 110% of x is \$44

x is the original amount.

PROBLEM-SOLVING AND REASONING 8–10 8, 10–14

8 Without a calculator, evaluate the following.

- **a** 10% of \$58
- **b** 5% of \$84
- **c** 1% of \$46
- **d** $2\frac{1}{2}$% of \$20
- **e** $33\frac{1}{3}$% of \$132
- **f** $66\frac{2}{3}$% of \$60

$33\frac{1}{3}\% = \frac{1}{3}$

9 If $\frac{1}{3}$ of 96 = 32, what is $66\frac{2}{3}$% of 96?

10 If 10% of \$800 is \$80, explain how you can use this result to find:

- **a** 1% of \$800
- **b** 5% of \$800
- **c** $2\frac{1}{2}$% of \$800

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11 About 80% of the mass of the human body is water. If Carla weighs 60 kg, how many kilograms of water make up her body weight?

12 In a class of 25 students, 40% have been to England. How many students have not been to England?

13 Explain why 10% of 24 = 24% of 10.

14 10% of 1 day is the same as x hours and y minutes. What is the value of x and y?

ENRICHMENT – 15

More than 100%

15 **a** Find 120% of 60.

b Determine the value of x if 165% of $x = 1.5$.

c Write 2.80 as a percentage.

d Write 325% as a fraction.

e \$2000 in a bank account increases to \$5000 over a period of time. By how much has the amount increased as a percentage?

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2C Percentage increase and decrease REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Percentages are often used to describe the amount by which a quantity has increased or decreased. The price of a car in the new year might be increased by 5%. On a \$70 000 car, this is a \$3500 increase. The price of a shirt might be marked down by 30%. If the shirt originally cost \$60, this provides an \$18 discount. It is important to note that the increase or decrease is calculated on the original amount.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: The quicker method

Two students, Nicky and Mila, consider the question: \$250 is increased by 15%. What is the final amount?

Nicky puts his solution on the board with two steps.

Step 1. 15% of \$250 $= 0.15 \times \$250$
$= \$37.50$

Step 2. Final amount $= \$250 + \$37.50$
$= \$287.50$

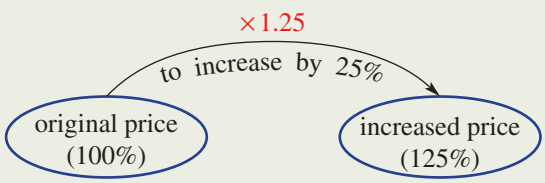
Mila says that the same problem can be solved with only one step using the number 1.15.

- Can you describe Mila's method? Write it down in words.
- What if the question was altered so that \$250 is decreased by 15%. How would Nicky's and Mila's methods work in this case?
- Which of the two methods do you prefer and why?

Key ideas

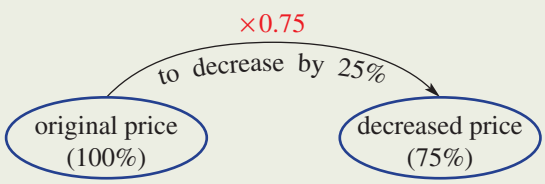
- To increase an amount by a given percentage:
  - add the percentage increase to 100%
  - multiply the amount by this new percentage.

  For example: To increase by 25%, multiply by (100% + 25%) = 125% = 1.25



- To decrease an amount by a given percentage:
  - subtract the percentage from 100%
  - multiply the amount by this new percentage.

  For example: To decrease by 25%, multiply by (100% − 25%) = 75% = 0.75



- To find a percentage change, use:

  $$\text{Percentage change} = \frac{\text{change in price}}{\text{original price}} \times 100\%$$

  For example: If a price changes from \$40 to \$50

  $$\text{Percentage change} = \frac{10}{40} \times 100\% = 25\%$$

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Exercise 2C REVISION

UNDERSTANDING AND FLUENCY	1–4, 5–6(½), 7	4, 5–6(½), 7, 8

1 Write the missing number.

- **a** To increase a number by 40%, multiply by _____.
- **b** To increase a number by 26%, multiply by _____.
- **c** To increase a number by _____, multiply by 1.6.
- **d** To increase a number by _____, multiply by 1.21.
- **e** To decrease a number by 20%, multiply by _____.
- **f** To decrease a number by 73%, multiply by _____.
- **g** To decrease a number by _____, multiply by 0.94.
- **h** To decrease a number by _____, multiply by 0.31.

2 The price of a watch increases from \$120 to \$150. What is the price increase?

3 A person's weight decreases from 108 kg to 96 kg. What is the weight decrease?

4
- **a** Write down 10% of 820 metres.
- **b** Increase 820 metres by 10%.
- **c** Decrease 820 metres by 10%.
- **d** Multiply 820 metres by 1.1. What do you notice?
- **e** Multiply 820 metres by 0.9. What do you notice?

Example 6 Increasing by a percentage

Increase \$70 by 15%.

SOLUTION	EXPLANATION
$100\% + 15\% = 115\%$	First add 15% to 100%.
$= 1.15$	
$\$70 \times 1.15 = \$80.50$	Multiply by 1.15 to give \$70 plus the increase in one step.

5 Complete the following.

- **a** Increase 56 by 10%.
- **b** Increase 980 by 20%.
- **c** Increase 100 by 12%.
- **d** Increase 890 by 5%.
- **e** Increase 180 by 15%.
- **f** Increase 450 by 20%.
- **g** Increase 8 by 50%.
- **h** Increase 98 by 100%.
- **i** Increase 30 by 5%.

Example 7 Decreasing by a percentage

Decrease \$5.20 by 40%.

SOLUTION	EXPLANATION
$100\% - 40\% = 60\%$	First subtract the 40% from 100% to find the percentage remaining.
$= 0.6$	
$\$5.20 \times 0.6 = \$3.12$	Multiply by $60\% = 0.6$ to get the result.

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6 Complete the following.

a Decrease 80 by 5%.
b Decrease 600 by 10%.
c Decrease 45 by 50%.
d Decrease 700 by 12%.
e Decrease 8000 by 8%.
f Decrease 450 by 25%.
g Decrease 68 by 75%.
h Decrease 9000 by 1%.
i Decrease 7000 by 100%.

Example 8 Finding a percentage change

a The price of a mobile phone increased from \$250 to \$280. Find the percentage increase.
b The population of a town decreases from 3220 to 2985. Find the percentage decrease and round to one decimal place.

SOLUTION	EXPLANATION
a Increase $= \$280 - \$250$ $= \$30$	First find the actual increase.
Percentage increase $= \frac{30}{250} \times \frac{100}{1}$ $= 12\%$	Divide the increase by the original amount and multiply by 100.
b Decrease $= 3220 - 2985$ $= 235$	First find the actual decrease.
Percentage decrease $= \frac{235}{3220} \times \frac{100}{1}$ $= 7.3\%$ (to 1 d.p.)	Divide the decrease by the original population and multiply by 100. Round as indicated.

7 Copy and complete the tables, showing percentage change. Round to one decimal place where necessary.

a

Original amount	New amount	Increase	Percentage change
40	60		
12	16		
100	125		
24	30		
88	100		
48	92		
200	250		

b

Original amount	New amount	Decrease	Percentage change
90	81		
100	78		
20	15		
24	18		
150	50		
9	8.3		
3	2.5		

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Example 9 Finding the original amount

After rain, the volume of water in a tank increased by 24% to 2200 L. How much water was in the tank before it rained? Round to the nearest litre.

SOLUTION	EXPLANATION
$100\% + 24\% = 124\%$	Write the total percentage.
$124\% = 2200$ L	The original volume is increased by 24% to give 2200 L.
$1\% = 17.74193...$	Divide by 124 to find 1%.
$100\% = 1774$ L (nearest litre)	Multiply by 100 to find the original amount.
Alternative method:	
$100\% + 24\% = 124\% = 1.24$	The volume has been multiplied by 1.24 to give 2200.
$V \times 1.24 = 2200$	Form an equation.
$V = 2200 \div 1.24$	Solve the equation by dividing both sides by 1.24.
$V = 1774$ L (nearest litre)	

8 Find the original amounts for each of the following if:

- **a** an increase of 10% on the cost of a can of cola drink increased it to \$3.30
- **b** an increase of 10% on the cost of a meal increased the cost to \$88
- **c** after an increase of 5%, the cost of a pair of running shoes came to \$210
- **d** a decrease of 30% made the cost of car insurance \$350
- **e** a decrease of 60% brought the price of a used car down to \$5000

PROBLEM-SOLVING AND REASONING	9–12	9, 11–16

9 The price of a computer was decreased by 15% in a sale. What is the sale price, if the original price was \$2100?

10 Plumbers on a salary of \$82 570 were given a $2\frac{1}{2}\%$ pay increase. Find their new annual salary.

11 A car manufacturer intends to increase sales by 14.7% next year. If the company sold 21 390 new cars this year, how many does it expect to sell next year?

12 The length of a bike sprint race is increased from 800 m to 1200 m. Find the percentage increase.

$$\% \text{ increase} = \frac{\text{increase}}{\text{original amount}} \times 100$$

13 The number of people on a bus decreased from 25 to 18 after one stop. Find the percentage decrease in the number of people on the bus.

14 After a price increase of 20%, the cost of entry to a museum rose to \$25.80. Find the original price.

15 The total price of an item including GST (at 10%) is \$120. How much GST is paid, to the nearest cent?

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16 An investor starts with \$1000.

- **a** After a bad day the initial investment is reduced by 10%. Find the balance at the end of the day.
- **b** The next day is better and the balance is increased by 10%. Find the balance at the end of the second day.
- **c** The initial amount decreased by 10% on the first day and increased by 10% on the second day. Explain why the balance on the second day didn't return to \$1000.

ENRICHMENT	–	17, 18

Repeated increase and decrease

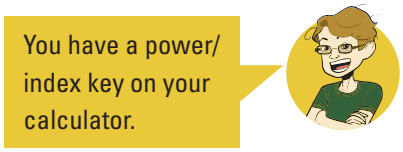
17 If the cost of a pair of shoes was increased twice, by 10% and then 15%, from an original price of \$80, then the final price would be:

$$\$80 \times 1.10 \times 1.15 = \$101.20$$

Use a similar technique to find the final price of these items. Round to the nearest cent.

- **a** Skis starting at \$450 and increasing by 20% and 10%
- **b** A computer starting at \$2750 and increasing by 6% and 11%
- **c** A Blu-ray player starting at \$280 and decreasing by 10% and 25%
- **d** A circular saw starting at \$119 and decreasing by 18% and 37%

18 If an amount is increased by the same percentage each time, powers can be used.



For example, 50 kg increased by 12% three times would increase to

$$\begin{aligned} &50 \text{ kg} \times 1.12 \times 1.12 \times 1.12 \\ &= 50 \text{ kg} \times (1.12)^3 \\ &= 70.25 \text{ kg (to 2 d.p.)} \end{aligned}$$

Use a similar technique to find the final value in these situations. Round to two decimal places.

- **a** The mass of a rat initially at 60 grams increases at a rate of 10% every month for 3 months.
- **b** The cost of a new car initially at \$80 000 increases by 5% every year for 4 years.
- **c** The value of an apartment initially at \$380 000 decreases by 4% per year for 3 years.
- **d** The length of a pencil initially at 16 cm decreases through being sharpened by 15% every week for 5 weeks.

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2D Profits and discounts REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Percentages are widely used in the world of finance. Profits, losses, commissions, discounts and taxation are often expressed and calculated using percentages.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Examples of percentages used in the media

Let's start: The best discount

Two book shops are selling the same book at a discounted price. The recommended retail price for the book is the same for both shops. Each shop has a sign near the book with the given details:

- Shop A. Discounted by 25%
- Shop B. Reduced by 20% then take a further 10% off that.

Which shop offers the bigger discount? Is the difference equal to 5% of the retail price?

Key ideas

- **Profit** is the amount of money made on a sale.
  Profit = selling price − cost price
- A **loss** is made when the selling price is less than the cost price.
  Loss = cost price − selling price
- **Mark-up** is the amount added to the cost price to produce the selling price.
  Selling price = cost price + mark-up
- The **percentage profit** or **percentage loss** can be found by dividing the profit or loss by the cost price and multiplying by 100:
  $$\% \text{ Profit/Loss} = \frac{\text{profit/loss}}{\text{cost price}} \times 100$$
- **Discount** is the amount by which an item is marked down.
  Discount = % discount × original price
  New price = original price − discount

Profit The amount of money made by selling something for more than its cost

Loss The amount of money lost by selling something for less than its cost

Discount An amount subtracted from a price

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Exercise 2D REVISION

UNDERSTANDING AND FLUENCY 1–7 4–8

1 Copy and complete the table of profits and losses.

Cost price ($)	7	18	24.80	7.30	460.95	3250
Selling price ($)	10	15.50	26.20	11.80	395	4430
Profit/Loss ($)						

2 Copy and complete the table of mark-ups.

Cost price ($)	30.95	99.95	199.95			18 000
Mark-up ($)	10	80	395.95	700	16 700	
Selling price ($)				1499.95	35 499	26 995

3 Copy and complete the table of discounts.

Original price ($)	100	49.95	29.95			2215
New price ($)	72	40.90	22.70	176	299.95	
Discount ($)				23	45.55	178

4 The following percentage discounts are given on the price of various products. State the percentage of the original price that each product is selling for.

a 10% off
b 20% off
c 15% off
d 8% off

Example 10 Calculating selling price from mark-up

An electrical store marks up all entertainment systems by 30%.
If the cost price of one entertainment system is $8000, what will be its selling price?

SOLUTION

Mark-up = 30% of $8000
= 0.3 × $8000
= $2400
∴ selling price = $8000 + $2400
= $10 400

EXPLANATION

Change percentage to a decimal or fraction and multiply by the cost price.

Selling price = cost price + mark-up
Alternatively, multiply by 1.3.

5 Copy and complete this table by calculating the mark-up amount and selling price of each item.

Item	Cost price	% mark-up	Mark-up amount	Selling price
Jeans	$60	28%		
Toaster	$40	80%		
Car	$22 000	45%		
Can of drink	$1.20	140%		
Loaf of bread	$1.80	85%		
Handbag	$80	70%		
Electronic tablet	$320	35%		

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Example 11 Finding the discount amount

An electrical store advertises a 15% discount on all equipment as a holiday special. Find the sale price on a projection system that has a marked price of \$18 000.

SOLUTION

Discount = 15% of \$18 000
= 0.15 × \$18 000
= \$2700

∴ The new price = \$18 000 − \$2700
= \$15 300

EXPLANATION

Change the percentage to a decimal and evaluate.

New price is original price minus discount. Alternatively, multiply by 0.85.

6 Copy and complete the table by writing in the missing values.

Item	Cost price	% discount	Discounted amount	Selling price
Camera	\$900	15%		
Car	\$24 000	20%		
Bike	\$600	25%		
Shoes	\$195	30%		
Blu-ray player	\$245	50%		
Electric razor	\$129	20%		
Lawnmower	\$880	5%		

Example 12 Determining profit

A manufacturer produces an item for \$400 and sells it for \$540.

a Determine the profit made.

b Express this profit as a percentage of the cost price.

SOLUTION

a Profit = \$540 − \$400
= \$140

b % Profit $= \frac{140}{400} \times 100$
= 35%

EXPLANATION

Profit = selling price − cost price

% Profit $= \frac{\text{profit}}{\text{cost price}} \times 100$

7 Find the missing values in these tables. Round to two decimal places where necessary.

a

Cost price (\$)	Selling price (\$)	Profit (\$)	Profit (%)
10	15		
24	30		
100	150		
250	255		
17.50	20		

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b

Cost price ($)	Selling price ($)	Loss ($)	Loss (%)
10	8		
16	12		
100	80		
34	19		
95	80.75		

Example 13 Calculating sale saving

A toy store discounts a toy by 10% in a sale. If the sale price was $10.80, what was the original price?

SOLUTION	EXPLANATION
$90\% = \$10.80$	The discount factor $= 100\% - 10\% = 90\%$.
$1\% = 10.80 \div 90$	Thus, $10.80 is 90% of the original price.
$1\% = 0.12$	Use the unitary method to find 1%.
$100\% = \$12$	Multiply by 100 to find the original amount.
The original price was $12.	Write the answer in words.
Alternative method:	
$100\% - 10\% = 90\% = 0.9$	The price given is 90% of the original price.
$0.9 \times P = \$10.80$	Form an equation.
$P = \$10.80 \div 0.9$	Solve the equation by dividing both sides by 0.9.
$P = \$12$	

8 Answer the following questions relating to finding the original price (cost price).

a Find the original price of a coffee mug that was discounted by 20% and sold for $4.40.

b Find the cost price of a pair of shoes that sold for $250 after a mark-up of 25%.

c Find the cost price after a discount of 10% was given on a surfboard that sold for $1350.

d Find the original price on a concert ticket for a major recording artist if it was marked up by 100% and sold for $250.

$4.40 is 80% of the original price.
$250 is 125% of the original price.

PROBLEM-SOLVING AND REASONING 9–11 9, 11–14

9 A manufacturer produces and sells items for the prices shown.

i Determine the profit made.

ii Express this profit as a percentage of the cost price.

a Cost price $10, selling price $12

b Cost price $20, selling price $25

c Cost price $120, selling price $136.80

d Cost price $1400, selling price $3850

10 It used to take 20 hours to fly to Los Angeles. It now takes 12 hours. What is the percentage decrease in travel time?

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11 Lenny marks up all computers in his store by 12.5%. If a computer cost him $890, what will be the selling price of the computer?

12 A used-car dealer purchases a vehicle for $13 000 and sells it for $18 500. Determine the percentage mark-up on the vehicle to one decimal place.

13 A refrigerator is discounted by 25%. If Paula pays $460 for it, what was the original price? Round to the nearest cent.

What percentage is $460 of the original price?

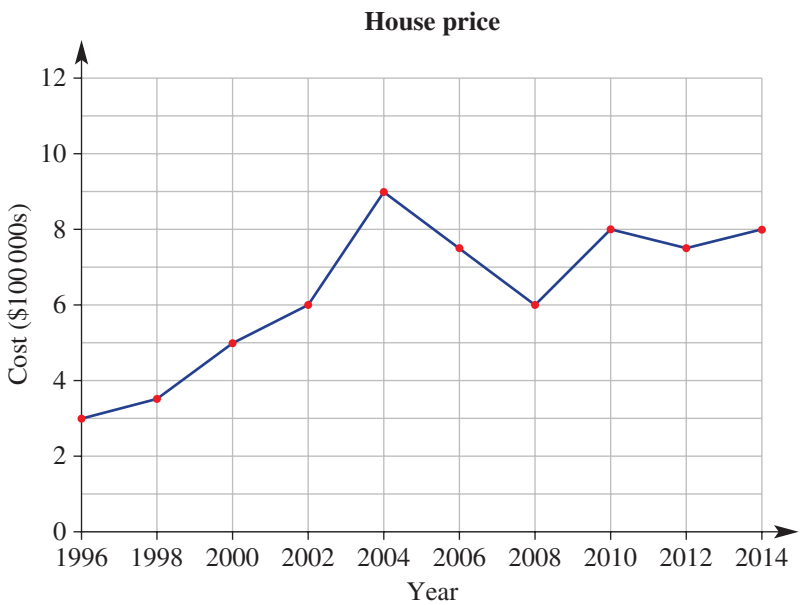
14 An electrical store buys a computer from the wholesaler for $500. The store marks up the computer by 80%.

- **a** What is the dollar amount of the mark-up?
- **b** What is the retail price of the computer after the store's mark-up?
- **c** The store offers the computer on sale for a discount of 15%. What is the price now?
- **d** If the 15% discount was calculated on the original $500 cost price, and then the computer was marked up 80% after that, would it make a difference to the sale price?

ENRICHMENT – 15

House value

15 The graph shows the changes in the value of a particular house in southern Sydney.



- **a** How much was the house worth in 1998?
- **b** Find the percentage increase, to one decimal place, in the value of the house from 1998 to:
  - **i** 2002
  - **ii** 2004
  - **iii** 2006
- **c** The owners sold the house in 2016 for one million dollars. What was the percentage increase in the two years from 2014 to 2016 for the house to be sold at that price?

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2E Income

Interactive
Widgets
HOTsheets
Walkthrough

There are many different ways of earning a living. You can be self-employed or work for someone else. Your income is usually related to the skills you have. It can be calculated and paid in different ways. You can earn, for example, a salary, a wage or even a commission.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Types of income

As a class, write down one example of a job that earns each of the following types of income.

- Salary
- Wage (i.e. hourly rate of pay)
- Overtime
- Commission

In the newspaper classifieds or online, find an advertisement for each of the income types above.

Key ideas

- Workers who earn a **wage** (for example, a casual waiter) are paid a fixed rate per hour. Hours outside the normal working hours (public holidays etc.) are paid at a higher rate called overtime. This can occur in a couple of common ways:
  - Time and a half: pay is 1.5 times the usual hourly rate
  - Double time: pay is twice the usual hourly rate
- Workers who earn a **salary** (for example, an engineer) are paid a fixed amount per year, say, $95 000. This is often paid monthly or fortnightly.
  - 12 months in a year and approximately 52 weeks in a year = 26 fortnights
- People who sell things may be paid a percentage of their sales. This is called a **commission**. They may also be paid a set amount called a **retainer**.

Wage Earnings paid to an employee based on an hourly rate

Salary A fixed agreed yearly amount that an employee earns

Commission Earnings of a salesperson based on a percentage of the value of goods or services sold

Exercise 2E

UNDERSTANDING AND FLUENCY	1–8	4–7, 8(½), 9

1 Tom earns $12.70 an hour. How much does he earn for:

a 2 hours of work? **b** 8 hours of work?

c 10 hours of work? **d** 38 hours of work?

2 Sela's hourly rate of pay is $24. Calculate her overtime rate at:

a time and a half **b** double time

3 William earns a salary of $136 875 each year. Approximately how much is this each:

a month?

b week?

c day?

1 year is:
- 12 months
- 52 week (approx.)
- 365 days.

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4 Find the commission earned on the given sales figures if the percentage commission is 20%.

a \$1000 **b** \$280
c \$4500 **d** \$725.50

Example 14 Comparing wages and salaries

Ken earns an annual salary of \$90 000 and works a 38-hour week. His wife Brooke works part-time in retail and earns \$61.80 per hour.

a Calculate how much Ken earns per week.
b Determine who has the higher hourly rate of pay.
c If Brooke works on average 18 hours per week, what is her yearly income?

SOLUTION	EXPLANATION
a Weekly rate = \$90 000 ÷ 52	\$90 000 pay in a year.
= \$1730.77	There are approximately 52 weeks in a year.
∴ Ken earns \$1730.77 per week.	Divide by 52 to find weekly wage.
b Brooke: \$61.80/h.	Ken works 38 hours in week.
Ken: \$1730.77 ÷ 38	Hourly rate = weekly rate ÷ number of hours.
= \$45.55/h	Round to the nearest cent.
∴ Brooke is paid more per hour.	Compare hourly rates.
c In one week: \$61.80 × 18	Weekly income = hourly rate × number of hours.
= \$1112.40	
Yearly income = \$1112.40 × 52	Multiply by 52 weeks to get yearly income.
= \$57 844.80	

5 Talib earns \$16 697.20 per year at a fast food restaurant.

a How much does he earn each week?
b Each week he works 38 hours. Calculate his hourly rate of pay.
c Find his fortnightly pay.
d Find his monthly pay.

6 Paul earns \$790 each week. How much does he earn each:

a year? **b** month? **c** hour, if he worked 40 hours each week?

7 Copy and complete the table.

Employee	Hourly rate	Hours worked	Income
Adam	\$10.40	8	
Betty	\$15.50	$8\frac{1}{2}$	
Ceanna	\$9.70	15	
David	\$24.30	38	
Edward	\$17.85	42	
Francis	\$30	27	
George	\$15.20	7.25	

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Example 15 Calculating overtime

Georgio works some weekends and late nights, and earns overtime for that work. His rate of pay is \$32 per hour.

a Calculate Georgio's time and a half rate of pay per hour.

b Calculate Georgio's double time rate of pay per hour.

c Calculate Georgio's weekly wage for a week where he works 18 hours at his normal rate, 2 hours at time and a half, and 1 hour at double time.

SOLUTION	EXPLANATION
a Time and a half $= \$32 \times 1.5$ $= \$48$	Time and a half is 1.5 times the hourly rate.
b Double time $= \$32 \times 2$ $= \$64$	Double time is 2 times the hourly rate.
c Wage $= \$32 \times 18 + \$48 \times 2 + \$64 \times 1$ $= \$736$	Find the sum of the: • normal hourly rate multiplied by the number of hours worked at the normal rate (18) • time and a half hourly rate multiplied by the number of hours worked at that rate (2) • double time hourly rate multiplied by the number of hours worked at that rate (1).

8 A job has a normal working hours pay rate of \$9.20 per hour. Calculate the pay, including overtime, from the following hours worked.

a 3 hours at the normal rate and 4 hours at time and a half

b 4 hours at the normal rate and 6 hours at time and a half

c 14 hours at the normal rate and 3 hours at double time

d 20 hours at the normal rate and 5 hours at double time

e 10 hours at the normal rate, 8 hours at time and a half and 3 hours at double time

f 34 hours at the normal rate, 4 hours at time and a half and 2 hours at double time

For time and a half: $\times 1.5$
For double time: $\times 2$

Example 16 Calculating commission

A saleswoman is paid a retainer of \$1500 per month. She also receives a commission of 5% on the value of goods she sells. If she sells goods worth \$5600 during the month, calculate her earnings for that month.

SOLUTION	EXPLANATION
Commission $= 5\%$ of \$5600 $= 0.05 \times \$5600$ $= \$280$	Calculate the commission on sales. Change the percentage to a decimal and evaluate.
Earnings $= \$1500 + \$280$ $= \$1780$	Earnings = retainer + commission

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9 Copy and complete the table.

Person	Weekly retainer	Rate of commission	Commission earned	Weekly wage
Adina	$0	12% on $7000		
Byron	$160	8% on $600		
Cindy	$300	5% on $680		
Deanne	$260	5% on $40 000		
Elizabeth	$500	8% on $5600		
Faruq	$900	2% on $110 000		
Gary	$1000	1.5% on $45 000		

PROBLEM-SOLVING AND REASONING

10–12 | 10, 12–15

10 Jim, a part-time gardener, earned $261 in a week. If he worked 12 hours during normal working hours and 4 hours overtime at time and a half, what was his hourly rate of pay?

4 hours at time and a half is equivalent to $4 \times 1.5 = 6$ hours at normal hourly rate.

11 Sally earned $329.40 in a week. She worked 10 hours during the week, 6 hours on Saturday at time and a half and 4 hours on Sunday at double time. What was her hourly rate of pay?

12 Amy works at Best Bookshop. During one week she sells books valued at $800. If she earns $450 per week plus 5% commission, how much does she earn in this week?

13 Jason works for a campervan company. If he sells $84 000 worth of campervans in a month, and he earns $650 per month plus 4% commission on sales, how much does he earn that month?

14 Stephen earns an hourly rate of $17.30 for the first 38 hours, time and a half for the next 3 hours and double time for each extra hour above that. Calculate his earnings if he works 44 hours in a week.

15 Calculate how many hours at the standard hourly rate the following working hours are the same as:

- **a** 3 hours and 2 hours at double time
- **b** 6 hours and 8 hours at time and a half
- **c** 15 hours and 12 hours at time and a half

ENRICHMENT

– | 16

Pay slips

16 Workers at a fast food restaurant are paid $7.21 per hour for working Monday to Friday up until 7 p.m. and time and a half after 7 p.m. They earn time and a half on Saturdays and double time on Sundays. They are given an unpaid 30-minute meal break for any shift over 5 hours.

Donna's shifts for the week are given below:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
–	4:30 p.m.–7 p.m.	4:30 p.m.–7:30 p.m.	5 p.m.–10 p.m.	5 p.m.–8 p.m.	10 a.m.–6:30 p.m.	10 a.m.–1:30 p.m.

- **a** How much can Donna expect to earn, to the nearest cent, if she works the hours she is rostered for?
- **b** The restaurant decides on a new workplace deal; it gets rid of all overtime and creates a flat hourly rate of $8.15 per hour. How much worse off will Donna be for the week above?
- **c** How many extra hours a week does Donna need to work during the week to make up the extra income? Answer in a whole number of hours.

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2F Taxation

Interactive
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Walkthrough

All wage and salary earners have some deductions taken out of their pay. Deductions usually include income tax. Tax is paid to the government. The government uses it to pay for roads, schools, hospitals, universities and services such as police.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Deductions from pay

What types of deductions can you and your class think of that might be taken out of someone's pay? Research the meaning of superannuation and the Medicare levy.

Key ideas

- **Gross income** = the total of all income
- **Net income** = gross income minus deductions such as tax, union fees and superannuation
- **Taxable income** = gross income minus worked-related expenses and donations to charity
- **Income tax** is paid to the government once a person's taxable income passes a set amount in the year.
- Income tax is calculated by the current tax table available through the Australian Tax Office (ATO).

  This table shows the **tax brackets** and tax rates for 2017/2018 from the ATO website.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$87 000	$3572 plus 32.5c for each $1 over $37 000
$87 001–$180 000	$19 822 plus 37c for each $1 over $87 000
$180 001 and over	$54 232 plus 45c for each $1 over $180 000

Gross income Total income before any deductions are made

Net income Income that remains after deductions have been made

Taxable income Gross income less certain items allowed by the tax office

Income tax An amount paid to the government by people earning an income

Tax bracket Each row in the table is called a tax bracket

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Exercise 2F

UNDERSTANDING AND FLUENCY 1–9 4–9

1 Copy and complete the table by inserting the net income amounts.

Gross income	Deductions	Net income
\$5600	\$450	\$5150
\$87 000	\$28 000	
\$50 000	\$6700	
\$890	\$76	
\$84 650	\$24 790	

2 Find Hoang's gross weekly income if he earns:
- \$1200 a week as a teacher
- \$60 an hour for tutoring, and he tutors for 3 hours a week
- \$25 in interest on his bank account per week.

3 John has a gross income of \$45 000 and a net income of \$20 000. How much did his deductions come to?

4 Wayne pays 31.5% of his gross income in tax. How much tax does he pay on a gross income of \$85 790?

Example 17 Calculating tax to find net income

Liam has an annual salary of \$52 800. His payslip each month shows deductions for taxation of \$968.

a Calculate Liam's net income each month.

b What percentage of Liam's monthly pay is being paid to the government by his employer for taxation?

c If the taxation rate for Liam's salary changes to 19% with the first \$18 200 now tax free, calculate Liam's net income for the year.

SOLUTION	EXPLANATION
a Monthly pay $= \$52\,800 \div 12$ $= \$4400$	Calculate gross income per month.
$\therefore$ net monthly income $= \$4400 - \$968$ $= \$3432$	Net income = gross income − taxation
b $\%\text{ tax} = \dfrac{968}{4400} \times 100$ $= 22\%$	Calculate what fraction \$968 is of the monthly income \$4400. Multiply by 100 to convert to a percentage.
c Salary for tax purposes $= \$52\,800 - 18\,200$ $= \$34\,600$	The first 18 200 is not taxed.
Tax amount $= 19\%$ of $\$34\,600$ $= 0.19 \times \$34\,600$ $= \$6574$	Calculate tax amount on \$34 600. Convert percentage to a decimal and evaluate.
$\therefore$ net income $= \$52\,800 - 6574$ $= \$46\,266$	Net income = gross income − tax amount.

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5 For each of the following find the:

- **i** annual net income
- **ii** percentage of gross income paid as tax. Round to one decimal place where necessary.

a Gross annual income = \$48 241, tax withdrawn = \$8206
b Gross annual income = \$67 487, tax withdrawn = \$13 581.20
c Gross monthly income = \$4041, tax withdrawn = \$606.15
d Gross monthly income = \$3219, tax withdrawn = \$714.62

6 Calculate the amount of tax to be paid using the following annual salaries and tax rates if the first \$6000 is tax free.

a salary = \$18 200, tax rate = 15%
b salary = \$44 300, tax rate = 21%
c salary = \$57 500, tax rate = 24.5%
d salary = \$84 200, tax rate = 30.4%

7 How much less tax will each person in Question **6** pay if the first \$18 200 is tax free?

8 Ed earns \$1400 per week and pays 27% of his annual income in tax.

a Calculate the amount of income tax that Ed pays in one year.
b Find Ed's annual net income.

Example 18 Using the tax table

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$37 000	19c for each \$1 over \$18 200
\$37 001–\$87 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$87 001–\$180 000	\$19 822 plus 37c for each \$1 over \$87 000
\$180 001 and over	\$54 232 plus 45c for each \$1 over \$180 000

Use the tax table above to find the income tax for an income of \$48 000.

SOLUTION

Tax = \$3572 + 0.325 × (\$48 000 − \$37 000)
= \$7147

EXPLANATION

Find the tax bracket in which the amount (\$48 000) lies (\$37 001 − \$87 000)
Write down the values in this bracket, remembering that 32.5c in the dollar is 32.5% or 0.325.
Subtract 37 000 from 48 000 to find the amount of income that the 32.5% applies to.
Use your calculator to find the answer.

9 Use the tax table in Example 18 to find the income tax payable on each of these incomes.

a \$6000
b \$10 000
c \$30 000
d \$50 000
e \$80 000
f \$129 000
g \$156 000
h \$200 000

Find the correct tax bracket first.

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PROBLEM-SOLVING AND REASONING 10, 12 11–13

10 Mel has a net annual income of $53 246 after 21% of her income is withdrawn for tax purposes. What was her gross income?

What percentage does $53246 represent of the gross income?

11 Fred earns $50 000 each year as a shop clerk and an extra $1200 each year for doing gardening on the weekends.

- **a** Calculate his annual gross salary.
- **b** Use his gross salary to find his income tax, using the tax table in Example 18.
- **c** What is his annual net income, after tax?
- **d** What is his approximate fortnightly net income, to the nearest cent, after tax?

12 William earns $450 a week. Using the tax table in Example 18, determine how much tax is deducted from his pay each week.

Use William's annual salary to find the annual tax amount first.

13 John pays $5000 in tax a year.

- **a** Which tax bracket does John fall into?
- **b** Work backwards from this amount to work out John's taxable income. Answer to the nearest dollar.

ENRICHMENT – 14–16

Other types of deductions and taxable income

The lower a person's income, the less tax they must pay. People therefore try to lower their taxable incomes by claiming allowable tax deductions.

Work-related expenses such as uniforms, stationery and job-related travel expenses are all examples of allowable tax deductions.

The income tax is therefore calculated on a person's taxable income.

$$\textit{Taxable income} = \textit{gross income} - \textit{allowable tax deductions}$$

Use the tax table from Example 18 to compare the following jobs.

14 An accountant earns $3120 a fortnight, with no allowable work-related expenses.

- **a** What is the accountant's taxable income?
- **b** How much tax does the accountant owe in a year?

15 A lawyer earns $3120 a fortnight with allowable work-related expenses totalling $2560 a year.

- **a** What is the lawyer's taxable income?
- **b** How much tax does the lawyer owe in a year?

16 How much less tax does the lawyer in Question **15** have compared to the accountant in Question **14** as a result of claiming $2560 in work-related expenses?

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2G Simple interest

Interactive

Widgets

HOTsheets

Walkthrough

Interest is charged when a person or institution borrows money. The interest is an extra amount that must be paid back, on top of the borrowed amount.

Interest is also earned when a person or institution invests money.

Simple or flat rate interest is usually charged or earned each year. It is calculated on the full amount borrowed or invested at the beginning of the loan.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Developing the rule

$5000 is invested in a bank and 5% simple interest is paid to the investor every year. In the table on the right, the amount of interest paid is shown for year 1, and the amount of total interest is shown for years 1 and 2.

- Copy and complete the table.
- How much interest would the investor earn in 10 years?

Year	Interest paid that year	Total interest
0	\$0	\$0
1	$\frac{5}{100} \times \$5000 = \$250$	$1 \times \$250 = \$250$
2		$2 \times \$250 = \$500$
3		
4		
5		

Key ideas

- To compute simple interest, we apply the formula:

  $I = PRN$

  I is the amount of simple interest (in \$)
  P is the **principal** amount; the money borrowed or invested (in \$)
  R is the interest rate per time period (expressed as a fraction or decimal)
  N is the number of time periods

- The total amount (\$$A$) equals the principal plus interest

  $A = P + I$

- p.a. means 'per annum' or 'per year'.

Principal An amount of money invested or borrowed

Exercise 2G

UNDERSTANDING AND FLUENCY 1–9 4–10

1 Complete these conversions.

a 1 year = ☐ months
b 36 months = ☐ years
c 18 months = ☐ years
d 52 months = ☐ years

2 \$12 000 is invested at 6% p.a. for 42 months.

a What is the principal amount?
b What is the interest rate?
c What is the time period in years?
d How much interest is earned each year?

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3 Use the rule $I = PRN$ to find the simple interest (I) earned in these financial situations.

a $P = \$10\,000$, $R = 10\%$, $N = 3$
b $P = \$6000$, $R = 12\%$, $N = 5$
c $P = \$5200$, $R = 4\%$, $N = 2$
d $P = \$3500$, $R = 6\%$, $N = 1.5$

I = interest
P = principal
R = interest rate, expressed as a decimal
N = time (years)

4 Jann earns \$560 p.a. in simple interest on an investment. How much would he earn on the investment in:

a 2 years? **b** 5 years? **c** 10 years?

Example 19 Using the simple interest formula

Calculate the simple interest earned if the principal is \$1000, the rate is 5% p.a. and the time is 3 years.

SOLUTION	EXPLANATION
$P = 1000$, $R = 5\%$, $N = 3$	List the information given.
$I = PRN$	Write the formula.
$= 1000 \times 0.05 \times 3$	$R = 5\% = 0.05$ as a decimal.
$= 150$	Substitute in the given values and evaluate.
Interest = \$150	

5 Find the simple interest earned on:

a \$5000 at 6% p.a. for 1 year
b \$5000 at 6% p.a. for 3 years
c \$8000 at 4% p.a. for 5 years
d \$15 000 at 3% p.a. for 7 years
e \$7250 at 5.5% p.a. for 3 years

Remember to write the rate as a decimal.

6 Wally invests \$15 000 at a rate of 6% p.a. for 3 years. Calculate the simple interest and the amount available at the end of 3 years.

Amount = principal + interest

Example 20 Using other time periods

Calculate the simple interest on \$7000 invested at $6\frac{1}{4}\%$ p.a. for 18 months.

SOLUTION	EXPLANATION
$P = 7000$, $R = 6\frac{1}{4}\% = 0.0625$	List the information given.
$N = 18\text{ months} = \frac{18}{12}$ or 1.5 years	Convert 18 months into years by dividing by 12.
$I = PRN$	Write the formula.
$I = 7000 \times 0.0625 \times 1.5$	Substitute in the values and evaluate.
$= 656.25$	
Interest = \$656.25	

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7 Calculate the simple interest earned on:

365 days = 1 year

- **a** \$500 at 7% p.a. for 18 months
- **b** \$1000 at 5% p.a. for 24 months
- **c** \$2000 at 4% p.a. for 6 months
- **d** Round to the nearest cent where necessary, \$4700 at $4\frac{1}{2}$% p.a. for 15 months
- **e** Round to the nearest cent where necessary, \$50 000 at 3.75% p.a. for 200 days

Example 21 Calculating the final balance

Allan and Rachel plan to invest some money for their child Kaylan. They invest \$4000 for 30 months in a bank that pays 4.5% p.a. Calculate the simple interest and the amount available at the end of the 30 months.

SOLUTION	EXPLANATION
$P = 4000, R = 4.5\% = 0.045, N = \frac{30}{12} = 2.5$	N is written in years since interest rate is per annum.
$I = PRN$ $= 4000 \times 0.045 \times 2.5$ $= 450$	Write the formula, substitute and evaluate.
Interest = \$450	
Total amount = \$4000 + \$450 = \$4450	Total amount = principal + interest

8 Annie invests \$22 000 at a rate of 4% p.a. for 27 months. Calculate the simple interest and the amount available at the end of 27 months.

9 Copy and complete the table.

	Principal, P(\$)	Annual interest rate, R	Time period, N	Interest, I	Final balance, $A = P + I$
a	7000	3%	4 years		
b	1500	7%	8 years		
c	40 000	2.5%	18 months		
d	70 000	$3\frac{1}{4}$%	2 years		
e	2000	4%	30 months		

10 A finance company charges 14% p.a. simple interest. If Lyn borrows \$2000 to be repaid over 2 years, calculate her total repayment.

PROBLEM SOLVING AND REASONING 11 11, 12

11 Markus borrows \$20 000 to buy a car. He is charged simple interest at 18% p.a. for a period of 5 years. What is the total amount that Markus will have paid at the end of the loan period?

He has to pay back the amount borrowed plus the interest.

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12 Wendy wins $5000 during a chess tournament. She wishes to invest her winnings, and has the two choices given below. Which one gives her the greater total at the end of the time?
Choice 1: 8.5% p.a. simple interest for 4 years
Choice 2: 8% p.a. simple interest for 54 months

ENRICHMENT – 13, 14

Simple interest tables and graphs

13 The table shows the amount of simple interest payable (in $, to the nearest $) on loans at a certain interest rate.

Amount of loan ($)	1 year	2 years	3 years	5 years	10 years
50	6	13	19	32	65
100	13	26	39	65	129
500	65	129	194	323	645
1000	129	258	387	645	1290
5000	645	1290	1935	3225	6450
10 000	1290	2580	3870	6450	12 900
50 000	6450	12 900	19 350	32 250	64 500
100 000	12 900	25 800	38 700	64 500	129 000

Use the table to find the interest payable on the following loans.

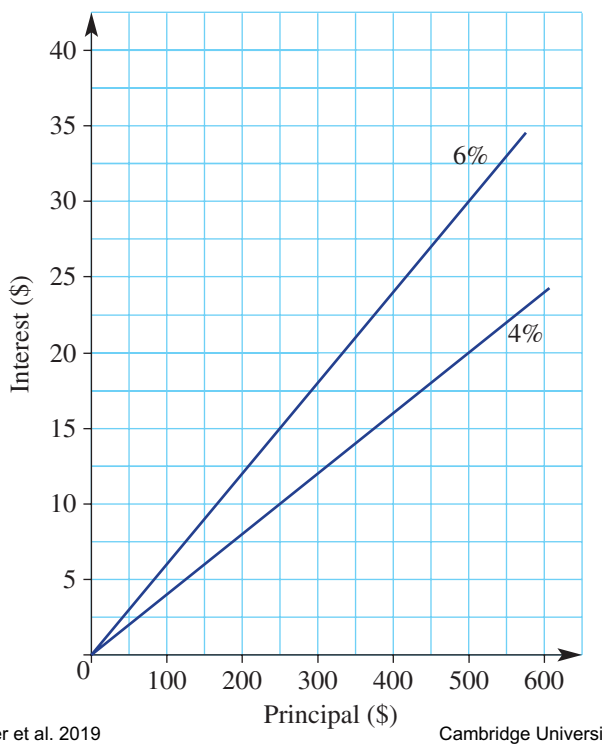
a $5000 for 1 year
b $500 for 3 years
c $100 for 10 years
d $150 for 1 year
e $85 500 for 5 years
f $9550 for 10 years
g $5000 for 4 years
h $50 000 for 9 years
i $100 000 for 10 years

14 The graph on the right shows the annual interest earned on investments for interests of 4% p.a. and 6% p.a. Use the graph to answer the following.

a Find the annual interest earned on an investment of:
- **i** $300 at 4% p.a.
- **ii** $520 at 6% p.a.
- **iii** $250 at 4% p.a.

b What investment would earn annual interest of (to the nearest $5):
- **i** $20 at 6% p.a.?
- **ii** $20 at 4% p.a.?
- **iii** $14 at 6% p.a.?

Annual interest earned at 4% and 6%



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2H Applications of simple interest

Interactive

Widgets

HOTsheets

Walkthrough

Decisions people make about where to borrow or invest money often relate to financial calculations.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Where do I invest?

Bank A: \$4000 at 5% p.a. for 8 years

Bank B: \$5000 at 8% p.a. for 4 years

Bank C: \$8000 at 4% p.a. for 5 years

Calculate the simple interest earned on each investment option. What do you notice?

Which bank would you choose and why?

Key ideas

- The simple interest formula, $I = PRN$, contains four variables:

  I is amount of interest

  P is principal (amount invested)

  R is interest rate, expressed as a decimal

  N is number of time periods

  If values are given for three of these, then they can be used to find the fourth.

- To repay a loan, you must repay the amount borrowed (the principal) and the interest.

- **Repayments** are the amount of money, usually the same amount each time period, used to repay a loan.

Repayment An amount paid to a bank or financial institution at regular intervals to repay a loan with interest included

Exercise 2H

UNDERSTANDING AND FLUENCY 1–9 4–9

1 If Phil earns \$10 in simple interest in one year, how long would it take him to earn:

a \$20? **b** \$40? **c** \$500? **d** \$25?

2 How many years does it take for \$100 to earn \$50 in interest if the simple interest rate is 10% p.a.?

3 A loan of \$4000 has interest of \$500 added to it. Calculate the size of each of the 24 repayments needed to repay the loan.

4 How much money do you need to invest to earn \$100 in interest in one year if the simple interest rate is 5%?

Require 5% of amount to be \$100.

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Example 22 Determining the investment period

Remy invests \$2500 at 8% p.a. simple interest, for a period of time, to produce \$50 interest. For how long did she invest the money?

SOLUTION	EXPLANATION
$I = 50,\ P = 2500,\ R = 8\% = 0.08$	List the information given.
$I = PRN$	Write the formula.
$50 = 2500 \times 0.08 \times N$	Substitute the known information and simplify.
$50 = 200 \times N$	
$200 \times N = 50$	Solve the remaining equation for N by dividing both sides by 200.
$N = \frac{50}{200}$	
$N = 0.25$	
Time $= 0.25$ years	Convert decimal time to months where appropriate.
$= 0.25 \times 12$ months	
$= 3$ months	

5 Alvi invests \$5000 at 8% p.a. simple interest and wants to earn \$1200 in interest. For how many years should Alvi invest his money?

6 Sam earns \$288 interest on his \$1600 investment. If the interest was calculated at 4% p.a., how many years did Sam invest the money for?

7 How many months will it take to earn \$600 interest at 5% p.a. on an investment of \$8000?

Example 23 Determining the interest rate

Bank East advertises \$450 interest a year on an investment of \$7500. Calculate the simple interest rate for this investment.

SOLUTION	EXPLANATION
$I = 450,\ P = 7500,\ N = 1,\ R = ?$	List the information given.
$I = PRN$	
$450 = 7500 \times R \times 1$	Write the formula with R the subject.
$7500 \times R = 450$	Substitute in the values and evaluate R.
$R = \frac{450}{7500}$	
$R = 0.06$	
$R = 6\%$	
Interest rate is 6% p.a.	Write R as a percentage by multiplying by 100.

8 Find the annual simple interest rate needed for each of the following situations.

a \$4000 earns \$500 in 2 years
b \$500 earns \$120 in 12 years
c \$18 000 earns \$3510 in 3 years
d \$950 earns \$470.25 in 9 years
e \$3000 earns \$945 in 18 months
f \$560 earns \$134.40 in 24 months
g \$1295 earns \$1554 in 10 years
h \$856 earns \$28.14 in 800 days

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Example 24 Calculating repayments

Deals 4 You offers a loan of \$24 000 at 16% p.a. simple interest if the loan is repaid in equal monthly instalments over 5 years.

a How much interest is charged on the loan?

b What is the total amount of the loan and the interest?

c Calculate the size of each repayment.

SOLUTION	EXPLANATION
a $I = PRN$ $P = 24\,000, R = 0.16, \text{N} = 5$ $\therefore I = 24\,000 \times 0.16 \times 5$ $= \$19\,200$	Write down the simple interest formula and list the information. Substitute in the values and evaluate.
b Total $= \$24\,000 + \$19\,200$ $= \$43\,200$	Total = principal + interest
c Repayments $= \dfrac{43200}{60}$ $= \$720$ Repayments come to \$720 a month.	Divide the total amount by the number of months ($5 \times 12 = 60$).

9 Copy and complete this table and find the monthly repayment for each loan.

Amount borrowed	Annual simple interest rate	Number of years	Interest	Total amount to be repaid	Monthly repayment
\$5000	21%	5			
\$14 000	15%	5			
\$10 000	6%	4			
\$55 000	8%	10			
\$250 000	7%	30			

PROBLEM-SOLVING AND REASONING 10, 11 11–13

10 Calculate the principal amount that earns \$500 simple interest over 3 years at a rate of 8% p.a. Round to the nearest cent.

11 Charlotte borrows \$9000 to buy a second-hand car. The loan must be repaid over 5 years at 12% p.a. simple interest. Calculate the:

a total amount to be repaid

b monthly repayment amount if the repayments are spread equally over the 5 years

12 If \$5000 grows to \$11 000 in 12 years, find the simple interest rate.

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13 An investor invests \$$P$ and wants to double this amount of money.

a How much interest must be earned to double this initial amount?

b What simple interest rate is required to double the initial amount in 8 years?

c If the simple interest rate is 5% p.a.:

i how many years will it take to double the investment?

ii how many years will it take to triple the investment amount?

iii how do the investment periods in parts **i** and **ii** compare?

Use the simple interest formula with \$$P$ and the interest of \$$P$.

ENRICHMENT – 14, 15

Compound interest

With simple interest, the principal and the interest earned each year remain the same for the period of the investment.

However, with compound interest, each time the interest is calculated it is added to the principal to give a new value. This means that the next time the interest is calculated on a larger amount.

In the following questions you will be asked to do repeated applications of simple interest to find the final compounded amount.

In compound interest, the balance grows faster as time passes.

14 \$500 is invested for 4 years at 10% p.a. interest compound annually.

a Copy and complete the following table to find the final value of the investment at the end of this time and the total.

Time (years)	Amount (A)	Interest (I)	New amount ($A + I$)
1	500	$500 \times 0.1 = 50$	$500 + 50 = 550$
2	550	$550 \times 0.1 =$	
3			
4			

b How much interest did the investment earn over the 4 years?

15 **a** Copy and complete the following table to find the final value of an investment of \$4500 compounded at 5% p.a. annually for 5 years.

Time (years)	Amount (A)	Interest (I)	New amount ($A + I$)
1	4500	225	4725
2	4725		
3			
4			
5			

b Type 4500×1.05^5 into your calculator. What do you notice about this answer?

c Can you explain how the answers to part **a** and part **b** relate?

d Use the formula $R = \dfrac{I}{PN}$ to find out what simple interest rate would be needed to create the same final amount over the 5 years. Answer correct to one decimal place.

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Puzzles and challenges

1 The answers to the clues are hidden in the wordfind. Can you find all 16 words?

S	A	G	R	O	S	S	W	H	K	L	O	M
A	P	E	R	C	E	N	T	A	G	E	C	O
L	A	R	C	O	M	M	I	S	S	I	O	N
A	S	E	R	V	I	N	T	E	R	E	S	T
R	T	Y	H	E	T	S	L	O	S	S	T	H
Y	E	F	O	R	T	N	I	G	H	T	I	L
W	S	M	O	T	A	X	A	T	I	O	N	Y
E	I	O	L	I	D	I	D	N	P	Y	I	E
K	M	N	Q	M	O	N	T	R	N	I	S	C
E	P	Y	U	E	D	I	S	C	O	U	N	T
M	L	R	E	P	A	Y	M	E	N	T	A	H
D	E	D	U	C	T	I	O	N	S	I	X	L

a A fixed annual income

b A percentage of the value of goods sold, which you earn as an income

c Working longer than normal working hours

d Money from your income given to the government

e Two weeks

f The total of all income

g Money taken from total pay

h Yearly

i 12 times a year

j Flat-rate interest

k Meaning 'out of 100'

l Money given to repay a loan

m Money earned on an investment

n An item offered for a sale price has had this happen

o The original price of an item

p Your incur this when you sell an item for less than you paid for it

2 Find out the four classical elements of the world by answering the following simple interest problems. Match the letter beside each question to its corresponding answer in each grid.

E = \$600 at 6% p.a. for 1 year	H = \$796 at 5% p.a. for 4 months
R = \$12 500 at $6\frac{1}{4}$% p.a. for 2 years	A = \$7000 at 5% p.a. for 3 years
I = \$1000 at 1% p.a. for 100 years	F = \$576.50 at 19% p.a. for 18 months
W = \$36 000 at 2% p.a. for 5 years	T = \$550 at 10% p.a. for 6 months

\$36	\$1050	\$1562.50	\$27.50	\$13.27

\$1050	\$1000	\$1562.50

\$164.30	\$1000	\$1562.50	\$36

\$3600	\$1050	\$27.50	\$36	\$1562.50

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Percentages and decimals

Percentage $\xrightarrow{\div 100}$ Decimal

Decimal $\xrightarrow{\times 100}$ Percentage

$49\% = 0.49$

Percentage composition

3 g out of 50 g

$= \frac{3}{50} \times 100$

$= 6\%$

Percentages and fractions

Percentage $\xrightarrow{\div 100}$ Fraction

Fraction $\xrightarrow{\times 100}$ Percentage

$19\% = \frac{19}{100}$

Percentage

Means 'out of 100'

$7\% = \frac{7}{100}$

Finding a percentage

17% of 30 $= \frac{17}{100} \times 30$

$= 0.17 \times 30$

$= 5.1$

Percentage increase and decrease

Increase	Decrease
20 by 6%	20 by 5%
$20 \times 1.06 = 21.2$	$20 \times 0.95 = 19$

Percentage change $= \frac{\text{change}}{\text{original}} \times 100$

Financial mathematics

Applications of percentages

Percentage profit/loss

$= \frac{\text{Profit/loss}}{\text{Cost price}} \times 100$

Mark-up and discount

Commission/tax

Income and tax

Employees can be paid:

wage: hourly rate with **overtime** at time and a half $= 1.5 \times$ hourly rate double time $= 2 \times$ hourly rate

salary: annual amount

commission: % of sales **net** income = **gross** income – deductions tax = money taken from pay and given to the government

Simple interest

$I = PRN$

where I = amount of interest

P = amount borrowed or invested

R = interest rate (as a fraction or decimal)

N = number of time periods

$I = PRN = \$2000 \times 0.05 \times 4$

$= \$400$

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Chapter review

Multiple-choice questions

1 2.8% as a decimal is:

A 2.8 **B** 0.28 **C** 0.028 **D** 0.0028 **E** 280

2 What percentage of \$2 is 50 cents?

A 4% **B** 40% **C** 25% **D** $2\frac{1}{2}\%$ **E** 400%

3 $12\frac{1}{2}\%$ as a simple fraction is:

A $\frac{12}{100}$ **B** $\frac{1}{8}$ **C** $\frac{3}{25}$ **D** 0.125 **E** 12.5

4 $33\frac{1}{3}\%$ of \$660 is the same as:

A $\$660 \div 2$ **B** $\$660 \times 0.3$ **C** $\$660 \times 0.03$ **D** $\$660 \div 3$ **E** $\$660 \div \frac{1}{3}$

5 15% of \$1600 is equal to:

A 24 **B** 150 **C** \$240 **D** \$24 **E** 240

6 If 110% of a number is 528, then the number is:

A 475.2 **B** 52.8 **C** 480 **D** 580.8 **E** 475

7 9670 increased by 12% becomes:

A 9682 **B** 9658 **C** 10830.4 **D** 1160.4 **E** 8509.6

8 Jane is paid a wage of \$7.80 per hour. She works 12 hours at this rate during a week, plus 4 hours on a public holiday when she gets paid at time and half. Her earnings in the week are:

A \$140.40 **B** \$124.80 **C** \$109.20 **D** \$156 **E** \$62.40

9 Simon earns a weekly retainer of \$370 and 12% commission of any sales he makes. If he makes \$2700 worth of sales in a particular week, he will earn:

A \$595 **B** \$652 **C** \$694 **D** \$738.40 **E** \$649.60

10 \$1200 is increased by 10% for two years with simple interest. The total balance at the end of the two years is:

A \$252 **B** \$1452 **C** \$1450 **D** \$240 **E** \$1440

Short-answer questions

1 Copy and complete the table on the right.

Decimal	Fraction	Percentage
0.6		
	$\frac{1}{3}$	
		$3\frac{1}{4}\%$
	$\frac{3}{4}$	
1.2		
		200%

2 Find:

a 25% of \$310

b 110% of 1.5

3 Determine the original amount if:

a 20% of the amount is 30

b 72% of the amount is 18

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4 a Increase 45 by 60%.
b Decrease 1.8 by 35%.
c Find the percentage change if $150 is reduced by $30.

5 The mass of a cat increased by 12% to 14 kg over a 12-month period. What was its previous mass?

6 A dining table at cost price $1800 is to be marked up by 15% before selling. What is its selling price?

7 Determine the discount given on a $15 000 car if it is discounted by 12%.

8 The cost price of an article is $150. If it is sold for $175:
a determine the profit made
b express the profit as a percentage of the cost price

9 Determine the hourly rate of pay for a person:
a with an annual salary of $36 062 working a 38-hour week
b who earns $429 working 18 hours at the hourly rate and 8 hours at time and a half.

10 Jo's monthly income is $5270. However, 20% of this is paid straight to the government in taxes. What is Jo's net yearly income?

11 Find the simple interest earned on $1500 at 7% p.a. for 5 years.

12 $600 simple interest is earned on an investment of $200 for 6 years. What is the interest rate per year?

Extended-response questions

1 Pauline buys a formal dress at cost price from her friend Tila. Pauline paid $420 for the dress, which is normally marked up by 55%.
a How much did she save?
b What is the normal selling price of the dress?
c If Tila gets a commission of 15%:
i how much commission did she get?
ii how much commission did Tila lose by selling the dress at cost price rather than the normal selling price?

2 Adam works a 38-hour week for a wage of $975.84.
a Calculate his hourly rate of pay.
Overtime is calculated at time and a half.
b What is Adam's overtime rate?
c How much does Adam earn for 4 hours of overtime work?
d How many hours of overtime did Adam work in a week if his wage for that week was $1226.22?
e If Adam usually works the amount of overtime in part **d** in the 52 weeks of the year he works, and he pays 27% of his pay in tax, what is his net annual income?
f If Adam invests 10% of his net income in an account earning 8% p.a. simple interest for 18 months, how much extra income will he have earned?

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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

3 Expressions and equations

What you will learn

3A Algebraic expressions REVISION
3B Adding and subtracting algebraic expressions REVISION
3C Multiplying and dividing algebraic expressions REVISION
3D Expanding algebraic expressions REVISION
3E Linear equations with pronumerals on one side REVISION
3F Solving linear equations involving fractions
3G Linear equations with brackets
3H Equations with pronumerals on both sides REVISION
3I Using linear equations to solve problems
3J Using formulas

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NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: ALGEBRAIC TECHNIQUES, EQUATIONS

Outcomes

A student generalises number properties to operate with algebraic expressions. (MA4–8NA)

A student uses algebraic techniques to solve simple linear and quadratic equations. (MA4–10NA)

A student solves linear and simple quadratic equations, linear inequalities and linear simultaneous equations, using analytical and graphical techniques. (MA5.2–8NA)

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Actuaries and risk assessment

Managing and assessing risk is an area studied in the field of actuarial science. Here trends can be examined and the risk to the client can be given a value.

Actuaries look at the changes in variables and the impact on individual scenarios before making recommendations to clients and companies.

Superannuation, which is compulsory in the Australian work place, requires actuaries to construct and apply mathematical models, looking at different input values and the projected outputs in each case. This is used to predict which investments are high in return yet low in risk. Algebra and linear/non-linear modelling are vital to the work done by actuaries in the superannuation industry.

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Pre-test

1 Write in simplified form.

a $3 \times x$ **b** $4 \times a \times b$ **c** $2 \times 5 \times x$ **d** $3 \times b \times 7$

2 If $x = 3$, find the value of:

a $x + 5$ **b** $10 - x$ **c** $4x$

d $\frac{18}{x}$ **e** $2x + 3$ **f** $2(x + 4)$

3 Write algebraic expressions for:

a 3 more than x

b the product of a and b

c 2 lots of y less 3

d the sum of x and 2, all divided by 3

4 Calculate:

a -6×3 **b** -7×4

c $-8 \times (-5)$ **d** $-1 \times (-3)$

e $-2 + 5$ **f** $-8 + 3$

g $7 - 11$ **h** $-2 - 6$

5 Simplify by collecting like terms.

a $2x + 5x - 4x$ **b** $7y + 5 - y$

c $4x - (-4x)$ **d** $3x + 12y - 3x + 5y$

e $3a + 4a^2 + 7a + 5a^2$ **f** $3xy - 4y + 2yx - 3y$

6 Simplify the following.

a $3 \times 2a$ **b** $7x \times (-3y)$

c $\frac{8b}{2}$ **d** $\frac{9mn}{6n}$

7 Expand the following.

a $2(x + 3)$ **b** $3(a - 5)$ **c** $4(2x + 1)$

8 Evaluate the following if $a = 3$ and $b = -2$.

a $2a - 5$ **b** $ab + 4$ **c** $\frac{9}{a} - b$ **d** $2a(b + 1)$

9 Which of the following equations is $x = 4$ a solution to?

a $2x + 3 = 9$ **b** $\frac{x}{2} + 3 = 5$ **c** $\frac{2x + 1}{3} = 3$ **d** $5 - 2x = -1$

10 Find the value of a that makes the following true.

a $a + 4 = 13$ **b** $a - 3 = 7$ **c** $2a + 1 = 9$ **d** $\frac{a - 1}{4} = 5$

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3A Algebraic expressions REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Algebra is central to the study of mathematics and is commonly used to solve problems in many everyday and more complex situations. Algebra involves working with unknown values in maths. Pronumerals are used to show these unknown values.

Variables representing unknown quantities are used in a wide range of jobs and occupations.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: What does the letter mean?

Consider the following situations and how they could be written as algebraic expressions.

- Fidel has 7 marbles and Jack has an unknown number of marbles, which we will call m. They then each get 4 more marbles.
  - **i** How many marbles does Fidel have now?
  - **ii** What mathematical operation ($+, -, \times, \div$) did you use to answer part **i**?
  - **iii** Using the same operation, how could you describe how many marbles Jack has now?
- Fidel can buy a chocolate bar for \$2. Jack can also buy a chocolate bar but he will pay an unknown amount for it. We'll call the amount Jack pays p (dollars).
  - **i** If Fidel buys 3 chocolate bars, how much will he pay?
  - **ii** If Fidel buys 10 chocolate bars, how much will he pay?
  - **iii** What mathematical operation did you use to answer parts **i** and **ii**?
  - **iv** Using the same operation, how could you write how much Jack will pay for 5 chocolate bars?
- Fidel earns \$10 an hour for raking leaves, while Jack earns \$$x$ per hour.
  - **i** How much does Fidel earn for two hours work?
  - **ii** Describe how much Jack would earn for two hours work.

Key ideas

In algebra, letters are used to represent numbers. These letters are called **pronumerals** or variables.

- In algebra, the multiplication and division signs are generally not shown.
  For example: $3 \times x$ is written as $3x$ and $x \div 4$ is written as $\frac{x}{4}$.
- An **expression** is a combination of numbers and pronumerals connected by the four operations $+, -, \times$ and $\div$. Brackets can also be used.
  For example: $5x + 4y - 1$ and $3(x + 2) - \frac{y}{5}$ are expressions.
- A **term** is a combination of numbers and variables connected with only multiplication and division. Terms are separated with the operations $+$ and $-$.
  For example: $5x + 7y$ is a two-term or binomial expression, $5x$ and $7y$.

Pronumeral A letter or symbol used to represent a number (also called a variable)

Expression A group of mathematical terms containing no equals sign

Term One of the components of an expression

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Key ideas

- **Coefficients** are the numbers being multiplied by pronumerals.
  For example: In $3x - 4y + \frac{z}{5}$,
  3 is the coefficient of x,
  -4 is the coefficient of y, and $\frac{1}{5}$ is the coefficient of z, since $\frac{z}{5} = \frac{1}{5} \times z$.
- **Constant terms** consist of a number only.
  For example: -2 in $x^2 + 4x - 2$ (The sign must be included.)
- Expressions can be evaluated by substituting a number for a pronumeral.
  For example: If $a = 2$,
  then $a + 6 = 2 + 6 = 8$.

3 is the *coefficient* of x. -2 is the *coefficient* of xy.

$3x + 7y - 2xy + 5$ ← 5 is a *constant term*.

This *expression* has 4 *terms*.

- Order of operations should be followed when evaluating expressions:
  1. Brackets
  2. Powers
  3. Multiplication and division, from left to right
  4. Addition and subtraction, from left to right

Coefficient A numeral placed before a pronumeral

Constant term The part of an expression without any pronumerals

Exercise 3A REVISION

UNDERSTANDING AND FLUENCY | 1–6, 7–9(½) | 4–6, 7–9(½)

1 Use the words from the list below to complete the following.

expression, coefficient, constant term, terms, pronumeral

- **a** $3x - 2y + 7$ is an example of an algebraic _____.
- **b** In $2a + b - 3$, -3 is the _____.
- **c** In $4m + 3$, m is called a _____.
- **d** In $3 - 2y + 4z$, 4 is the _____ of z.
- **e** In the expression $3s + 7t$, $3s$ and $7t$ are the _____.

2 Match an item in the left column with an item in the right column.

a	Product	**A**	Division
b	Sum	**B**	Subtraction
c	Difference	**C**	Multiplication
d	Quotient	**D**	The square of x
e	x^2	**E**	Addition

3 Write the following in simplified form.

a $7 \times y$ **b** $-2 \times x$ **c** $1 \times b$
d $a \times b$ **e** $y \div 2$ **f** $x \div y$

4 Substitute $x = 2$ into the following and evaluate.

a $x + 3$ **b** $10 - x$ **c** $7x$ **d** $\frac{8}{x}$

Substitute means to replace the pronumeral with the given numeral.

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Example 1 Identifying parts of algebraic expressions

Consider the expression $5a + 2ab + 7 - 4b$.

a How many terms are in the expression?

b What is the coefficient of:

i a?

ii b?

c What is the constant term?

SOLUTION	EXPLANATION
a There are 4 terms.	The terms in the expression are separated by + or –.
b i 5 is the coefficient of a.	$5a$ is $5 \times a$; 5 is the number being multiplied by a.
ii -4 is the coefficient of b.	$-4b$ is $-4 \times b$; the negative sign is included.
c 7 is the constant term.	7 is the number with no pronumeral part.

5 Complete each of the parts **i–iii** for the algebraic expressions **a–d**.

i How many terms are in the expression?

ii What is the coefficient of y?

iii What is the constant term?

a $5x + 2y + 3$

b $2x - 3y$

c $3xy + 7y - 4$

d $2x^2 - 1 + 4x + \frac{y}{2}$

Remember, the coefficient of a term includes the sign before it; in $3 - 2x$, -2 is the coefficient of x.

Example 2 Writing algebraic expressions for worded scenarios

Write an algebraic expression for the following.

a The number of tickets needed for 3 boys and r girls

b The cost in dollars of P pies at \$3 each

c The amount for each person if a \$300 prize is shared equally among m friends.

SOLUTION	EXPLANATION
a $3 + r$	3 tickets plus the number of girls, r.
b $3P$	2 pies would cost $\$3 \times 2 = \$6$. The cost is $3 \times$ the number of pies, so P pies costs $3 \times P$.
c $\frac{\$300}{m}$	\$300 shared between 3 people is \$300 divided into 3 parts, so \$300 divided into m parts is $\$300 \div m = \frac{\$300}{m}$.

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6 Write an algebraic expression for the following.

a The number of tickets required for:

- **i** 4 boys and r girls
- **ii** t boys and 2 girls
- **iii** x boys, y girls and z adults

b The cost in dollars of:

- **i** P pies at \$6 each
- **ii** 10 pies at \$$n$ each
- **iii** D drinks at \$2 each
- **iv** P pies at \$5 and D drinks at \$2

c The number of grams of lollies for one child if 500 g of lollies is shared equally among C children

For part **b i**:
1 pie costs \$6 × 1 = \$6
2 pies cost \$6 × 2 = \$12
5 pies cost \$6 × 5 = \$30
P pies cost . . .

Example 3 Converting words to expressions

Write an algebraic expression for the following.

a Five less than x
b Three more than twice x
c The sum of a and b is divided by 4
d The square of the sum of x and y

SOLUTION	EXPLANATION
a $x - 5$	5 below x is 5 subtracted from x.
b $2x + 3$	Twice x ($2 \times x$) plus 3
c $\dfrac{a+b}{4}$	The sum of a and b is done first ($a + b$) and the result is then divided by 4.
d $(x + y)^2$	The sum of x and y is done first and then the result is squared (recall that the square of a is $a^2 = a \times a$).

7 Write an algebraic expression for each of the following.

a The sum of 2 and x
b The sum of ab and y
c 5 less than x
d 7 subtracted from $2y$
e The product of x and 3
f Three times the value of p
g Four more than twice x
h The sum of x and y is divided by 5
i 10 less than the product of 4 and x
j 3 lots of x subtracted from 1
k The sum of 3 and y is divided by 2
l Half of 1 more than x
m The square of the sum of m and n
n The sum of the squares of m and n

Sum (+)
Product (×)
Twice (×2)

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Example 4 Substituting values into expressions

Evaluate these expressions if $a = 5$, $b = 6$ and $c = -3$.

a $3a + b$ **b** $2a - (b + c)$ **c** $a^2 - bc$

SOLUTION	EXPLANATION
a $3a + b = 3 \times 5 + 6$ $= 15 + 6$ $= 21$	Substitute the value 5 for a and the value 6 for b. Recall $3a = 3 \times a$.
b $2a - (b + c) = 2 \times 5 - (6 + (-3))$ $= 10 - 3$ $= 7$	Substitute the values for a, b and c. Then apply order of operations, with brackets evaluated first, before moving onto multiplication and division, then addition and subtraction. $6 + (-3) = 6 - 3 = 3$
c $a^2 - bc = (5)^2 - 6 \times (-3)$ $= 25 - (-18)$ $= 25 + 18$ $= 43$	Evaluate powers before the other operations: $5^2 = 5 \times 5$. A positive (6) times a negative (−3) gives a negative (−18). Subtracting a negative means to add.

8 Evaluate each of the following given $x = -4$

a $x - 9$
b $2x + 6$
c $5 - 3x$
d $(x + 2)^2$
e $x^2 - x$
f $-3x^2$
g $4x - 5x^3 + 9$
h $\dfrac{6 - 3x}{4}$
i $-x^2 - x - 1$

9 Evaluate these expressions if $a = 4$, $b = -3$ and $c = 8$.

a $3a + c$ **b** $ac - 7$ **c** $2c - (a + b)$
d $a^2 - 2c$ **e** $\dfrac{a + c}{2}$ **f** $b + 2(c - a)$
g $2c - ab$ **h** abc **i** $b - c + a^2$

When adding or subtracting a negative number:
$8 + (-3) = 8 - 3 = 5$
$8 - (-3) = 8 + 3 = 11$

PROBLEM SOLVING AND REASONING 10, 11 10–14

10 A rectangular garden bed is 12 m long and 5 m wide.

a Find the area of the garden bed.
b The length is increased by x m and the width is decreased by y m. Find the new length and width of the garden.
c One length and one width of the rectangular garden will be lined with paving. Write an expression for the total distance around the garden that will be paved.

Draw a diagram to help. Area of rectangle $= \ell \times b$

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11 Jelena earns $(10 + 8t)$ dollars for each shift that she works, where t is the number of hours worked in the shift.

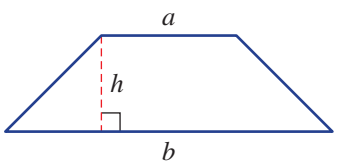
a How much does Jelena earn if $t = 2$?

b How much does Jelena earn from a 5-hour shift?

c Will she earn \$100 from a 10-hour shift?

d How many whole hours must she work to earn more than \$100?

12 The expression for the area of a trapezium is $\frac{1}{2}h(a + b)$, where a, b and h represent the lengths shown.



a Find the area of the trapezium with $a = 5$, $b = 7$ and $h = 3$.

b A trapezium has $h = 4$ and area 12. If a and b are whole numbers, what possible values can the variable a have?

13 Christina earns \$$A$ from selling 20 glasses of lemonade. Write an expression for the cost in dollars of:

a one glass of lemonade

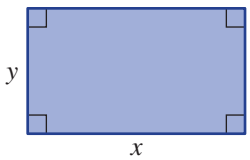
b three glasses of lemonade

c n glasses of lemonade

14 For each of these shapes, write an expression for the:

i perimeter **ii** area

a



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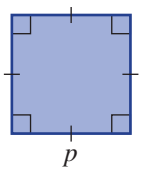

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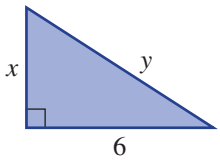

Perimeter is the distance around the outside of the shape.
Area of a rectangle is length × breadth.
Area of a triangle is $\frac{1}{2}$ × base × height.

ENRICHMENT – 15

Should there be brackets?

15 Brackets are used to ensure that the mathematics inside the brackets occurs first.

a By substituting values for x, decide whether the following are the same.

i $2(x + 1)$ and $2x + 1$ **ii** $3 + \frac{x}{2}$ and $\frac{3 + x}{2}$

b Write down two possible expressions that could describe each of the following.

i 3 lots of x plus 1

ii The sum of 5 and x divided by 3

iii Half of x plus y

c Write down a clear statement that describes the following expressions.

i $4(x + 2)$ **ii** $3 + \frac{x}{2}$ **iii** $\frac{1}{3}(m + n)$ **iv** $5x + 7$ **v** $\frac{x + y}{2}$ **vi** $\frac{1}{2}a + b$

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3B Adding and subtracting algebraic expressions REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Just as $2 + 2 + 2 + 2 = 4 \times 2$, so $x + x + x + x = 4 \times x$ or $4x$. We say that the expression $x + x + x + x$ is simplified to $4x$; i.e. 4 lots of x. Similarly, $3x + 5x$ is 3 lots of x plus 5 lots of x, which equals 8 lots of x, which equals $8x$, and $8x - 3x$, which equals $5x$.

All these expressions have like terms and can be simplified to an expression with a smaller number of terms by adding or subtracting their coefficients.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Two groups

Place the terms in the following four sets into groups of similar terms on either side of the dividing line. The first set has been started for you.

Set 1	Set 2	Set 3	Set 4
$3x$ $7y$ $2x$ $8x$	$4ab$ $2a$ $5ab$	2 $5y$ 7 1	$3ab$ $4a^2b$ $2ab$ $10ab$
$\cancel{4x}$ y $5x$ $2y$ $\cancel{3y}$	$7a$ ab $6ab$	$3y$ $2y$ 8 25	$5a^2b$ $12ab$ $9a^2b$
$4x$ | $3y$	|	|	|

- Describe how you classified each set into two groups.
- What do the terms in each group have in common?
- Could you use the above rules for grouping to combine any of the 8 groups together?

Key ideas

Like terms Two or more terms which contain the same pronumerals and indices

- $x \times 4$ and $4 \times x$ can be written as $4x$, which means '4 lots of x'.
- $x \times 1$ and $1 \times x$ and $1x$ can be written as x.
- $x \times 0$ and $0 \times x$ and $0x$ can be written as 0.
- **Like terms** contain the same pronumeral(s).
  For example: $5x$ and $7x$ are like terms.
  $3ab$ and $-8ab$ are like terms.
  - Since $a \times b = b \times a$, then ab and ba are also like terms.
- Pronumerals are usually written in alphabetical order. For example: xy rather than yx.
- Collect like terms to form a single term by adding or subtracting coefficients.
  For example: $4x + 7x = 11x$
  $10ab - 2ab = 8ab$
- Unlike terms do not contain the same pronumerals.
  For example: $5x$, x^2, xy, $2xy^2$ and 3 are all unlike terms.

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Exercise 3B REVISION

UNDERSTANDING AND FLUENCY 1–5, 6–7(½) 4, 5–7(½)

1 Use the list below to complete the following.

1, pronumeral, coefficients, like

a $4a$ and $7a$ are an example of _____ terms.
b To add like terms, we add the _____.
c Like terms have the same _____.
d The coefficient of y in $2x + y$ is _____.

2 Decide whether the following pairs of terms are like terms (Yes or No).

a $4ab$ and $3ab$ **b** $2x$ and $7xy$ **c** 5 and $4m$
d $3t$ and $-6tw$ **e** $7yz$ and yz **f** $2mn$ and $9nm$
g $5a$ and a **h** $3x^2y$ and $7xy^2$ **i** $4ab^2$ and $-3b^2a$

Like terms must have exactly the same pronumeral factors, including powers.

3 Complete the following.

a $2 - 5 =$ ____ **b** $3 - 4 =$ ____ **c** $1 - 6 =$ ____
d $-5 + 2 =$ ____ **e** $-8 + 3 =$ ____ **f** $-4 + 1 =$ ____
g $-2 + 5 =$ ____ **h** $-4 + 11 =$ ____ **i** $-1 + 4 =$ ____
j $-3 - 5 =$ ____ **k** $-1 - 4 =$ ____ **l** $-2 - 9 =$ ____

4 Which of the following represent the same expression as $w - x + y - z$?

a $w - z + y - x$ **b** $w - y + x - z$
c $y + w - z - x$ **d** $-x + w - z + y$
e $y - w - x + z$ **f** $y - z - x + w$

Make sure the correct pronumerals are being added or subtracted.

Example 5 Identifying like terms

Choose the pair(s) of like terms from the following sets.

a $3x, 3, 4xy, 5x, y$ **b** $4b, 4bc, b, 2abc, -7bc, 2bc^2$

SOLUTION	EXPLANATION
a $3x$ and $5x$ are like terms. The rest are not.	$3x$ and $5x$ contain the same pronumeral.
b $4b$ and b are like terms.	$4b$ and b have the same pronumeral.
$4bc$ and $-7bc$ are like terms.	$4bc$ and $-7bc$ have the same pronumeral. $2bc^2$ does not, because c is raised to the power of 2. All other terms do not have the same pronumeral.

5 Choose the pair(s) of like terms from the following sets.

a $4y, 5, 3xy, 2y, 7xy$ **b** $3x, 3y, 3, xy, 7x$
c $7ab, 2a, -3ab, b^2, 5a$ **d** $2a^2, 4a, 3ab, a, -3a^2$
e $2xy, 3x^2y, -5x, 7yx^2$ **f** $5ab^2, 3ab, 2b^2, 4ab^2, 7ba$

st and ts are like terms since $s \times t = t \times s$.

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Example 6 Collecting like terms

Simplify the following by collecting like terms.

a $4a + 7a$ **b** $3x + 4 - 2x$

SOLUTION	EXPLANATION
a $4a + 7a = 11a$	Since $4a$ and $7a$ are like terms, they can be simplified to one term by adding their coefficients. $4 + 7 = 11$.
b $3x + 4 - 2x = 3x - 2x + 4$ $= x + 4$	Collect like terms ($3x$ and $-2x$). The sign belongs to the term that follows. Combine their coefficients $3 - 2 = 1$. Recall that $1x$ is written as x.

6 Simplify the following by collecting like terms.

a $3a + 7a$ **b** $4n + 3n$ **c** $12y - 4y$
d $5x + 2x + 4x$ **e** $6ab - 2ab$ **f** $7mn + 2mn - mn$
g $4y + 3y + 8$ **h** $7x + 5 - 4x$ **i** $5m + 2 - m$
j $5ab + 3a + 7ab$ **k** $6xy + xy + 4y$ **l** $5bc - 4 - 2bc$

In order to add or subtract coefficients, the terms must be like terms.

Example 7 Combining like terms

Simplify the following by collecting like terms.

a $3x + 2y + 4x + 7y$ **b** $3xy + 4x + xy - 6x$ **c** $8ab^2 - 9ab - 5ab^2 + 3ba$

SOLUTION	EXPLANATION
a $3x + 2y + 4x + 7y = 3x + 4x + 2y + 7y$ $= 7x + 9y$	Collect the like terms ($3x$ and $4x$ and $2y$ and $7y$) and combine their coefficients ($3 + 4 = 7$ and $2 + 7 = 9$).
b $3xy + 4x + xy - 6x = 3xy + xy + 4x - 6x$ $= 4xy - 2x$	Collect the like terms ($3xy$ and xy and $4x$ and $-6x$). Combine their coefficients (recall $xy = 1xy$). $3 + 1 = 4$ and $4 - 6 = -2$.
c $8ab^2 - 9ab - 5ab^2 + 3ba$ $= 8ab^2 - 5ab^2 - 9ab + 3ab$ $= 3ab^2 - 6ab$	Collect like terms in ab^2 and ab. Remember that $ba = ab$ and the + or – sign belongs to the term that follows; i.e. $-5ab^2$. Combine coefficients: $8 - 5 = 3$ and $-9 + 3 = -6$.

7 Simplify the following by collecting like terms.

a $2a + 4b + 3a + 5b$ **b** $4x + 3y + 2x + 2y$ **c** $xy + 8x + 4xy - 4x$
d $6t + 5 - 2t + 1$ **e** $5x + 1 + 6x + 3$ **f** $3mn + 4 + 4nm - 5$
g $4ab + 2a + ab - 3a$ **h** $3st - 8ts + 2st + 3ts$ **i** $8ac - 6c + 2ac - 3c$
j $5xy^2 - 2xy^2$ **k** $8m^2n - 6nm^2 + m^2n$ **l** $2x^2y - 4xy + 5yx^2$
m $x^2 - 7x + 3x^2 + 4x$ **n** $a^2b - 4ab + 3a^2b + ba$ **o** $10pq^2 - 2qp - 3pq^2 - 6pq$

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PROBLEM SOLVING AND REASONING 8, 9 9(½), 10, 11

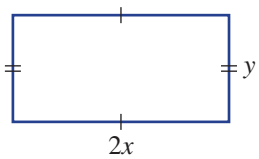
8 A farmer has x pigs and y chickens.

a Write an expression for the total number of heads.

b Write an expression for the total number of legs.

9 Write simplified expressions for the perimeter of the following shapes.

a



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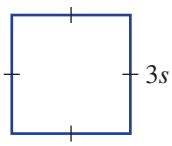

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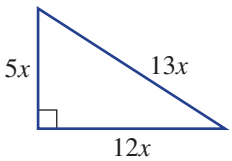

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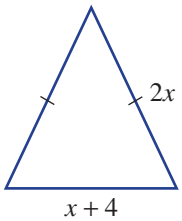

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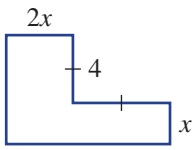

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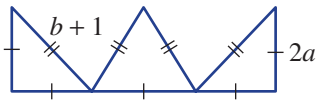
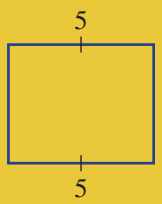


Sides with matching dash marks are equal in length.



10 A rectangular pool's length is three times its width, x metres. Write an expression for its perimeter.

11 Fill in the missing term to make each of the following correct.

a $11t + 3 - \square = 7t + 3$

b $\square + 3y - 2x = 7x + 3y$

c $3x + 4y - \square + 5y = x + 9y$

d $4a + 7b - \square - 2b = 5b - 3a$

e $3mn - \square + nm + 4n = 4mn - n$

f $2pq + 2p - \square - 5pq = 2p - 7pq$

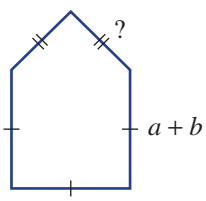
g $3x^2y - 11xy + 3yx^2 - \square = 6x^2y - 13xy$

h $4b^2 - 3b + 2b^2 + \square = 6b^2$

ENRICHMENT – 12

Around the edge

12 Consider the shape shown.



a If the unknown sides (?) have length $2b$, what is the perimeter in simplified form?

b If $a = 3$ and $b = 2$ and the perimeter is 23 units, what is the length of the unknown sides?

c If the perimeter is $(5a + 3b)$, what is the length of each unknown side?

d If the perimeter is $(7a + b)$, what is the length of each unknown side?

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3C Multiplying and dividing algebraic expressions REVISION

Interactive

Widgets

HOTsheets

Walkthrough

We have seen that $3x + 3x$ can be simplified to $6x$. $3x + 3x$ can also be written as $2 \times 3x$. Thus, $2 \times 3x = 6x$. So $4 \times 5a = 4 \times 5 \times a = 20a$. Any two or more terms can be simplified using multiplication; we do not need the terms to be like terms.

A single term such as $2 \times 5 \times x \div 10$ can also be simplified using multiplication and division, so $2 \times 5 \times x \div 10 = \frac{10x}{10} = x$, since $\frac{10}{10}$ simplifies to 1 by cancelling the common factor.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Are they equivalent?

These expressions can be separated into two groups. Group them so that the expressions in each group are equivalent.

$4x$ $\quad 2x - y$ $\quad 6x - x - x$ $\quad 10x - y - 8x$

$\frac{24x}{6}$ $\quad y + 2x - y + 2x$ $\quad 2 \times x - 1 \times y$ $\quad -y + 2x$

$\frac{1}{2} \times 8x$ $\quad 2 \times 2x$ $\quad \frac{8x^2}{2x}$ $\quad -1 \times y + \frac{2x^2}{x}$

Key ideas

- The symbols for multiplication ($\times$) and division ($\div$) are usually not shown in simplified algebraic terms.
  For example: $5 \times a \times b = 5ab$
  $$-7 \times x \div y^2 = -\frac{7x}{y^2}$$
- Order does not matter in multiplication; i.e. $2 \times 3 = 3 \times 2$, $a \times 2 \times b = 2 \times a \times b$.
- Algebraic terms are multiplied by multiplying coefficients with the pronumeral parts.
  For example: $4a \times 3b = 4 \times 3 \times a \times b = 12ab$
- The **factors** of a number are the set of numbers that divide evenly into it. For example, the factors of 6 are 1, 2, 3 and 6.

  Factor A whole number that will divide into another number without a remainder
- When dividing algebraic expressions, common factors can be cancelled. Cancel the highest common factor of the numerals and then the pronumerals.

  For example:
  $$\frac{{}_{1}\cancel{7}x}{{}_{2}\cancel{14}} = \frac{x}{2}$$
  $$\frac{{}_{2}\cancel{8}xy\cancel{{}_{1}}}{{}_{3}\cancel{12}y\cancel{{}_{1}}} = \frac{2x}{3}$$
  $$\frac{a^2b}{a} = \frac{{}_{1}\cancel{a} \times a \times b}{{}_{1}\cancel{a}} = ab$$

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Exercise 3C REVISION

UNDERSTANDING AND FLUENCY 1–4, 5–7(½) 4, 5–7(½)

1 Complete the simplified form of the following.

a $7 \times x =$ ____
b $a \times b =$ ____
c $3 \times 2 \times m =$ ____ m
d $8 \times 3 \times y = 24$____
e $6 \times 7 \times b =$ ____
f $4 \times x \times 5 =$ ____
g $a \times a = a$—
h $9 \times y \times y = 9$____

2 Fill in the missing word: *positive* or *negative*.

a A positive number multiplied by a negative number is _____.
b A negative number multiplied by a _____ number is positive.
c A negative number divided by a positive number is _____.

3 State the highest common factor of these pairs of numbers.

a 4 and 10
b 6 and 18
c 12 and 36
d 9 and 21
e 14 and 35
f 25 and 60

4 Without using a calculator, simplify each of the following fractions.

a $\frac{4}{6}$
b $\frac{20}{8}$
c $\frac{30}{10}$
d $\frac{8}{40}$
e $\frac{-4}{14}$
f $\frac{-5}{100}$
g $\frac{-24}{36}$
h $\frac{-72}{16}$

Simplify by cancelling the highest common factor of the numerator and denominator:
$\frac{6}{10} = \frac{2_1 \times 3}{2_1 \times 5} = \frac{3}{5}$

Example 8 Multiplying algebraic terms

Simplify the following.

a $3 \times 2b$
b $-2a \times 3b$

SOLUTION

a $3 \times 2b = 3 \times 2 \times b$
$= 6b$

b $-2a \times 3b = -2 \times 3 \times a \times b$
$= -6ab$

EXPLANATION

Multiply the coefficients.

Recall that multiplication can occur in any order; i.e. $2 \times 3 \times 4 = 2 \times 4 \times 3$ etc.
Multiply the coefficients and simplify.

5 Simplify the following.

a $5 \times 2m$
b $2 \times 6b$
c $3 \times 5p$
d $3x \times 2$
e $3p \times 6r$
f $4m \times 4n$
g $-2x \times 7y$
h $5m \times (-3n)$
i $-4c \times 3d$
j $2a \times 3b \times 5$
k $-4r \times 3 \times 2s$
l $5j \times (-4) \times 2k$
m $4m \times 5n \times (-3p)$
n $-8m \times (-5n)$
o $9pq \times -5$
p $8 \times (-w) \times 4 \times (-y)$

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Example 9 Dividing algebraic terms

Simplify the following.

a $\frac{4x}{8}$

b $10ab \div (15b)$

SOLUTION

a $\frac{{}_{1}\cancel{4}x}{{}_{2}\cancel{8}} = \frac{x}{2}$

b $10ab \div (15b) = \frac{10ab}{15b}$

$= \frac{{}^{2}\cancel{10} \times a \times \cancel{b}^{1}}{{}_{3}\cancel{15} \times \cancel{b}^{1}}$

$= \frac{2a}{3}$

EXPLANATION

Deal with numerals and promumerals separately, cancelling any common factors.
The highest common factor of 4 and 8 is 4.

Write division as a fraction first. Write numerator and denominator as a product and cancel the common factors 5 and b.

6 Simplify the following by cancelling.

a $\frac{8b}{2}$ **b** $\frac{2a}{6}$ **c** $\frac{4ab}{6}$

d $\frac{3mn}{6n}$ **e** $\frac{5xy}{20y}$ **f** $\frac{10st}{6t}$

g $\frac{3xy}{xy}$ **h** $\frac{27pq}{6p}$ **i** $2x \div 4$

j $12ab \div (2a)$ **k** $7mn \div (3n)$ **l** $8y \div (20xy)$

m $6p \div 3$ **n** $3p \div 6$ **o** $12pq \div (18p)$

p $\frac{12a - 3a}{6}$ **q** $\frac{2m + 5m}{m}$ **r** $\frac{2w \times 3x}{6x}$

Make sure they are in fraction form before cancelling; e.g. $2x \div 4 = \frac{2x}{4}$

Example 10 Multiplying and dividing with powers of 2

Simplify the following.

a $6x \times 5xy$

b $\frac{12a^2b}{3ab}$

SOLUTION

a $6x \times 5xy = 6 \times 5 \times x \times x \times y$

$= 30x^2y$

b $\frac{12a^2b}{3ab} = \frac{{}^{4}\cancel{12} \times {}^{1}\cancel{a} \times a \times \cancel{b}^{1}}{{}^{1}\cancel{3} \times \cancel{a}^{1} \times \cancel{b}^{1}}$

$= 4a$

EXPLANATION

Multiply the coefficients and the pronumerals. Recall that $x \times x$ is written as x^2 (x squared).

Write the numerator and denominator as products, with $a^2 = a \times a$. Cancel common factor (3) for numerals, then cancel common factors for pronumerals.

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7 Simplify the following.

a $4n \times 6n$

b $-3q \times q$

c $5s \times 2s$

d $7a \times 3ab$

e $5mn \times (-3n)$

f $-3gh \times (-6h)$

g $\dfrac{24ab^2}{8ab}$

h $\dfrac{25x^2y}{5xy}$

i $\dfrac{9m^2n}{18mn}$

j $\dfrac{2xy}{8xy^2}$

k $\dfrac{6a^2b}{10a}$

l $\dfrac{45p^2q^2}{15pq}$

Deal with numerals and pronumerals separately.

$\dfrac{\cancel{24}^{3}}{\cancel{8}_{1}}$ and $\dfrac{ab^2}{ab} = \dfrac{\cancel{a}_1 \times b \times \cancel{b}_1}{\cancel{a}_1 \times \cancel{b}_1} = b$

PROBLEM-SOLVING AND REASONING 8(½), 9 8(½), 10, 11(½)

8 Simplify the following by first writing in fraction form.

a $4 \times x \div y$

b $3 \times m \div 9$

c $4 \times a \div (2b)$

d $5x \times 4 \div (2y)$

e $6 \times 4mn \div (3m)$

f $8a \times 5b \div (8a)$

g $10m \times 4n \div (8mn)$

h $4x \times 3xy \div (2x)$

i $3pq \times p \div (6q)$

j $5ab^2 \times 4 \div (10b)$

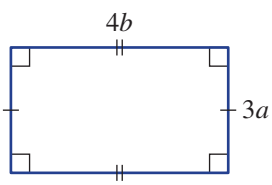
Apply one operation at a time from left to right in these; e.g.

$4 \times x \div y = 4x \div y$

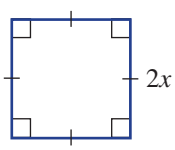
$= \dfrac{4x}{y}$

9 Write a simplified expression for the area of the following shapes.

a



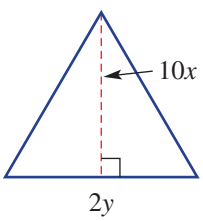
b



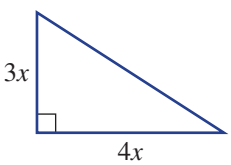
Area of a triangle

$= \dfrac{1}{2} \times$ base $\times$ height

c



d



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10 A bag of lollies contains x jelly beans. Jean buys 6 bags of lollies to share equally among her three grandchildren. Write a simplified expression for how many jelly beans each grandchild receives.

11 Fill in the missing term to make each of the following correct.

a $3m \times \square = 12mn$

b $4x \times \square = 28xy$

c $-2xy \times \square = -18xy^2$

d $6a \times \square = 30a^2b$

e $\dfrac{\square}{7} = 2x$

f $\dfrac{15}{\square} = \dfrac{3}{2y}$

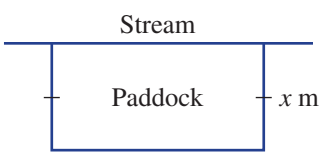
g $\dfrac{\square}{18ab} = \dfrac{2}{3b}$

h $\dfrac{\square}{4x} = 2xy$

ENRICHMENT – 12, 13

Rectangular paddocks

12 The length of a rectangular paddock is three times its width. Its width is x metres. It is bordered on one side by a stream.



a Write a simplified expression for the:

i area of the paddock

ii length of fencing needed for the three sides of the paddock

b If $x = 50$, use your answers to part **a** to give the:

i area of the paddock

ii length of fencing material required

13 The area of another rectangular paddock is given by the expression $4ab^2$. Find the width of the paddock if the length is:

a ab

b $2b$

c $4ab$

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3D Expanding algebraic expressions REVISION

Interactive
Widgets
HOTsheets
Walkthrough

We know from previous sections that when brackets are used in expressions we need to do the mathematics inside the brackets first.

For example, the expression $2(x + 3)$ means 'first add three to x, and then double the result'.

If $x = 10$, we get $2 \times (10 + 3) = 2 \times 13 = 26$.

Similarly, if we multiply each term inside the brackets separately by 2, the result is also 26: $(2 \times 10) + (2 \times 3) = 20 + 6 = 26$.

We can apply this to algebraic expressions, so that $2(x + 3)$ can also be written as $2 \times x + 2 \times 3 = 2x + 6$. This uses the distributive law for addition.

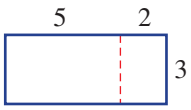
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Rectangular distributions

This rectangle has the dimensions as shown. Given that the area of a rectangle is length × breadth:

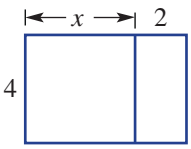
- describe two different ways to find the area of the rectangle
- fill in the boxes below to show the two methods used

$\square\,(\square + \square) = \square \times \square + \square \times \square$



This diagram also shows two joined rectangles with the given dimensions, with one unknown, x.

- Use your methods from above to find two different ways to write expressions for the combined area of the two rectangles.
- Complete the following: 4(_____) = _____ + _____



Examples of the distributive law can often be found in everyday tasks.

Key ideas

- The **distributive law** is used to expand and remove brackets.
  - A term on the outside of the brackets is multiplied by each term inside the brackets.

$$a(b + c) = ab + ac \quad or \quad a(b - c) = ab - ac$$

For example:

$$2(x + 4) = 2 \times x + 2 \times 4$$
$$= 2x + 8$$

  - If the number in front of the bracket is negative, the sign of each of the terms inside the brackets will change when expanded.
    For example: $-2(x - 3) = -2x + 6$ since $-2 \times x = -2x$ and $-2 \times (-3) = 6$

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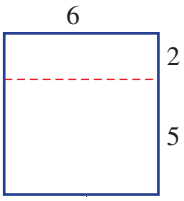
Exercise 3D REVISION

UNDERSTANDING AND FLUENCY 1–5, 6–8(½) 5, 6–9(½)

1 State a second way to find the area of these rectangles.

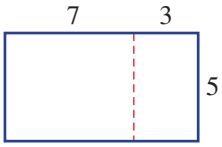
a



1 $A = 6 \times (5 + 2)$

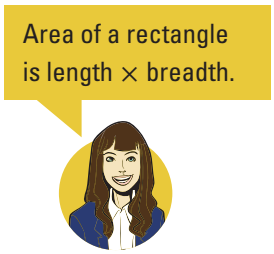
2 $A =$

b

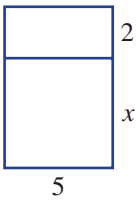


1 $A = 5 \times 3 + 5 \times 7$

2 $A =$



2 This diagram shows two joined rectangles with the given dimensions.



a Write an expression for the area of the:

i larger rectangle (x by 5)

ii smaller rectangle (2 by 5)

b Use your answers from part **a** to find the combined area of both rectangles.

c Write an expression for the total side length of the side involving x.

d Use your answer from part **c** to find the combined area of both rectangles. You will need brackets.

e Complete this statement: $5(x + 2) =$ ____ + ____

3 Substitute $x = 1, x = 3, x = -2$ and $x = 10$ into each expression and decide which two expressions could be equivalent.

a $2(x + 5)$ **b** $2x + 5$ **c** $2x + 10$

4 Complete the following.

a $3(x + 5) = __ \times x + __ \times 5$
$= __ + 15$

b $5(x + 1) = __ \times __ + 5 \times __$
$= __ + __$

c $2(x - 7) = 2 \times __ + 2 \times __$
$= __ - 14$

$a(b + c) = a \times b + a \times c$
$= ab + ac$

$a(b - c) = a \times b + a \times (-c)$
$= ab - ac$

5 **a** Substitute the value $x = 5$ into these expressions.

i $-2(x + 3)$

ii $-2x + 6$

b Do your answers from part **a** suggest that $-2(x + 3) = -2x + 6$? When expanded, what should $-2(x + 3)$ equal?

c Substitute the value $x = 10$ into these expressions.

i $-3(x - 1)$

ii $-3x - 3$

d Do your answers from part **c** suggest that $-3(x - 1) = -3x - 3$? When expanded, what should $-3(x - 1)$ equal?

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Example 11 Expanding simple expressions with brackets

Expand the following.

a $3(x + 4)$ **b** $5(x - 11)$

SOLUTION	EXPLANATION
a $3(x + 4) = 3x + 12$	The 3 on the outside is multiplied by each term inside the brackets; i.e. $3(x + 4) = 3 \times x + 3 \times 4$
b $5(x - 11) = 5x - 55$	$5 \times x = 5x$ and $5 \times (-11) = -55$

6 Expand the following.

a $2(x + 3)$
b $5(x + 12)$
c $2(x + 7)$
d $7(x + 9)$
e $3(x - 7)$
f $6(x - 11)$
g $5(x - 9)$
h $10(x - 1)$
i $3(2 + x)$
j $7(3 + x)$
k $4(7 - x)$
l $5(8 - x)$

Remember that the number in front of the brackets is multiplied by each term inside the brackets when expanding:
$2(x + 3) = \ldots$

Example 12 Expanding brackets with a negative number in front

Expand the following.

a $-2(x + 5)$ **b** $-3(x - 4)$

SOLUTION	EXPLANATION
a $-2(x + 5) = -2x - 10$	-2 is multiplied by each term inside the brackets. $-2 \times x = -2x$ and $-2 \times 5 = -10$
b $-3(x - 4) = -3x + 12$	Multiplying by a negative changes the sign of each term in the brackets. $-3 \times x = -3x$ and $-3 \times (-4) = +12$

7 Expand the following.

a $-3(x + 2)$ **b** $-2(x + 11)$
c $-5(x - 3)$ **d** $-6(x - 6)$
e $-4(2 - x)$ **f** $-13(3 + x)$
g $-8(9 + x)$ **h** $-300(1 - x)$

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Example 13 Expanding brackets and simplifying

Expand the following.

a $4(x + 3y)$

b $2x(4x - 3)$

SOLUTION

a $4(x + 3y) = 4 \times x + 4 \times 3y$
$= 4x + 12y$

b $2x(4x - 3) = 2x \times 4x + 2x \times (-3)$
$= 8x^2 - 6x$

EXPLANATION

Multiply each term inside the brackets by 4.
$4 \times x = 4x$ and $4 \times 3 \times y = 12y$.

Each term inside the brackets is multiplied by $2x$.
$2x \times 4x = 2 \times 4 \times x \times x = 8x^2$ and
$2 \times (-3) \times x = -6x$.

8 Expand the following.

a $2(a + b)$ **b** $5(a + 2b)$ **c** $3(2m + y)$
d $8(2x - 5)$ **e** $-3(4x + 5)$ **f** $4x(x - 2y)$
g $t(2t - 3)$ **h** $a(3a + 4)$ **i** $d(2d - 5)$
j $2b(3b - 5)$ **k** $2x(4x + 1)$ **l** $5y(1 - 3y)$

$x \times x$ is written as x^2.

Example 14 Simplifying by removing brackets

Expand the following and collect like terms.

a $4(x + 5) - 10$

b $2 - 3(x - 4)$

SOLUTION

a $4(x + 5) - 10 = 4x + 20 - 10$
$= 4x + 10$

b $2 - 3(x - 4) = 2 - (3x - 12)$
$= 2 - 3x + 12$
$= 14 - 3x$

EXPLANATION

Expand brackets first: $4(x + 5) = 4 \times x + 4 \times 5$.
Then collect like terms and simplify.

Expand brackets.
$3(x - 4) = 3x - 12$.
$-(3x - 12) = -1(3x - 12)$, so multiplying by negative 1 changes the sign of each term inside the brackets; i.e. $-1 \times 3x = -3x$ and $-1 \times (-12) = +12$.

9 Expand the following and collect like terms.

a $2(x + 4) + 3$ **b** $6(x + 3) + 4$ **c** $5(x + 2) - 4$
d $3(x + 4) - 2$ **e** $3 + 4(x - 2)$ **f** $7 + 2(x - 3)$
g $2 - 3(x - 2)$ **h** $1 - 5(x - 4)$ **i** $5 - 2(x + 3)$
j $12 - 3(x + 4)$ **k** $7 - (x + 4)$ **l** $4 - (3x - 2)$
m $6(2x - 1) + 5$ **n** $4(2x + 7) - 12$ **o** $2x(x - 3) + 2x$
p $m(2m - 3) - 5m$ **q** $12 + 3(3x - 5)$ **r** $5x - 6(x - 1)$
s $2x(x - 1) + x^2 + 5x$ **t** $2x + 3(7x - 2y)$ **u** $5x - (2x - 7) - 11$

Expand the brackets first, then collect like terms.

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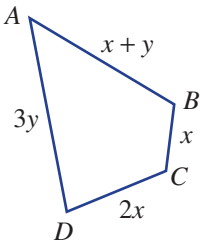
PROBLEM SOLVING AND REASONING 10, 11 | 10, 12–14

10 The diagram shows the route taken by a salesperson who travels from A to D via B and C.

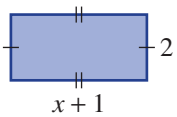


- **a** If the salesperson then returns directly to A, write an expression (in simplest form) for the total distance travelled.
- **b** If $y = x + 1$, write an expression for the total distance the salesperson travels in terms of x only. Simplify your expression.
- **c** When $y = x + 1$, how much would the distance have been reduced by (in terms of x) if the salesperson had travelled directly from A to D and straight back to A?

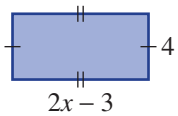
Subtract the distance from A to D and back again from the total distance.

11 Find the area of these basic shapes in expanded form.

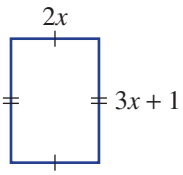
a



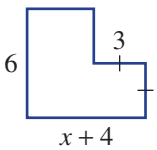
b



c



d



Remember to include brackets when multiplying length by breadth.

12 Murray has \$60 in savings. One Saturday, he does some odd jobs for a neighbour and earns \$$x$, which he adds to his savings. Murray's parents think he's doing a good job of saving money, so they decide to double his savings. Write an expanded expression for the amount of savings Murray has now.

13 Identify the errors in these expressions then write out the correct expansion.

- **a** $2(x + 6) = 2x + 6$
- **b** $x(x - 4) = 2x - 4x$
- **c** $-3(x + 4) = -3x + 12$
- **d** $-7(x - 7) = -7x - 49$
- **e** $5(x + 2) + 4 = 5x + 6$
- **f** $5 - 2(x - 7) = 5 - 2x - 14$
  $= -9 - 2x$

Watch out for the negative numbers.

14 Jill pays tax at 20c in the dollar for every dollar earned over \$10 000. Jill earns \$$x$ and $x > 10\,000$.

- **a** Write an expression for the amount that Jill pays tax on.
- **b** Write an expression for the amount Jill pays in tax. Expand your answer.

20c in the dollar is 0.2 of each dollar. In \$10, this is $0.2 \times 10 = \$2$.

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ENRICHMENT	–	15, 16

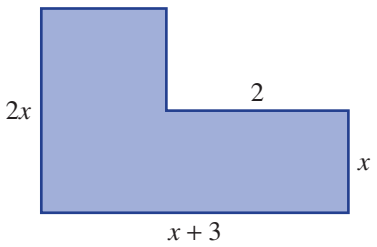
Pairs of brackets

15 Expand each pair of brackets first, then collect like terms.

a $2(x + 3) + 3(x + 2)$
b $2(x + 4) + 2(x - 1)$
c $3(2x + 4) + 5(x - 1)$
d $4(3x + 2) + 5(x - 3)$
e $-2(x + 2) + 3(x - 1)$
f $2(4x - 3) - 2(3x - 1)$
g $x(x + 2) + 2x(x + 4)$
h $3(x - 1) + x(x + 2)$
i $x(x + 2) - 4(x + 1)$

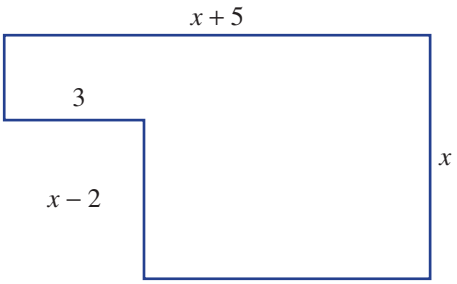
16 Use the technique in question **15** to find the area of these shapes in expanded form.

a

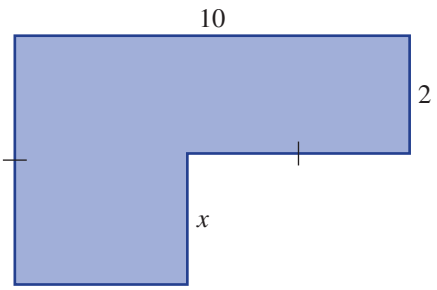


Split these shapes into rectangles and sum or subtract areas.

b



c



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3E Linear equations with pronumerals on one side

REVISION

Interactive
Widgets
HOTsheets
Walkthrough

A mathematical statement containing an equals sign, a left-hand side and a right-hand side is called an equation. $5 = 10 \div 2$, $3x = 9$, $x^2 + 1 = 10$ and $\frac{1}{x} = 4$ are examples of equations.

Linear equations can be written in the form $ax + b = c$, where the power of x is 1.

$4x - 1 = 6$, $3 = 2(x + 1)$ and $5x = 2x + 1$ are all linear equations.

Equations are solved by finding the value of the pronumeral that makes the equation true. This can be done by inspection for very simple linear equations (for example, if $3x = 15$, then $x = 5$, since $3 \times 5 = 15$). More complex linear equations can be solved through a series of steps where each step produces an equivalent equation.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Is $x = 4$ a solution?

State which of the following equations have $x = 4$ as the solution.

$5x = 20$	$2x - 1 = 5$	$14 = 10 - x$
$x + 9 = 13$	$14 = 3x + 2$	$-4x = 16$

- How did you find the equations for which $x = 4$ is a solution?
- The equations you have listed should be equivalent equations to $x = 4$. Starting from $x = 4$, describe what you would do to produce each equivalent equation; for example, 'add 1 to both sides'.

Key ideas

- An **equation** is a mathematical statement containing an equals sign. For example: $3x + 1 = 7$.
- **Equivalent equations** are created by:
  - adding a number to or subtracting a number from both sides of the equation

    $5x + 4 = 14$ $\quad -4$
    $5x = 10$

  - multiplying or dividing both sides of the equation by the same number (not including 0)

    $5x = 10$ $\quad \div 5$
    $x = 2$

  - swapping the left-hand side (LHS) with the right-hand side (RHS). For example: $13 = 2x + 1$ is equivalent to $2x + 1 = 13$.
- Linear equations can be solved by creating equivalent equations, then using opposite operations.

Equation A statement that two expressions have the same value

Equivalent equations Two or more equations that produce the same solution

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Key ideas

- The solution to an equation can be checked by substituting the solution into the original equation and checking that both sides are equal.

Equation: $5x + 4 = 14$
Possible solution: $x = 2$
LHS $= 5x + 4$ RHS $= 14$
$= 5 \times 2 + 4$
$= 14$
$\therefore x = 2$ *is* a solution

Exercise 3E REVISION

UNDERSTANDING AND FLUENCY 1–4, 5–6(½) 4, 5–7(½)

1 Which equations have a solution of $x = 3$?

a $x + 5 = 9$
b $x - 1 = 2$
c $4 - x = 1$
d $1 - x = -4$
e $2x + 4 = 10$
f $3x - 2 = 7$
g $2 = 7 - 2x$
h $-5 = 4 - 3x$
i $1 = -x + 2$

If $x = 3$ is a solution, the left-hand side must equal the value that is on the right-hand side of the = sign.

2 State the value of x that makes these equations true.

a $x + 4 = 10$ **b** $x - 3 = 5$
c $2 = x + 5$ **d** $2x = 8$
e $20 = 5x$ **f** $-3x = 9$

What value plus 4 equals 10 ?

3 Obtain an equivalent equation by completing the step given in brackets.

a $2x + 5 = 11$ (subtract 5 from both sides)
b $3x - 4 = 17$ (add 4 to both sides)
c $4x = 36$ (divide both sides by 4)

$2x + 5 = 11$
-5 -5
$2x = \ldots$

4 State the single operation that has been performed to obtain these pairs of equivalent equations; for example, 'add 3 to both sides'.

a $x + 9 = 11$
$x = 2$

b $2x - 1 = 5$
$2x = 6$

c $3x = 27$
$x = 9$

d $2x = 7$
$x = \frac{7}{2}$

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Example 15 Solving simple linear equations

Solve each of the following equations.

a $2x + 5 = 9$

b $11 = 3x - 7$

SOLUTION	EXPLANATION
a $2x + 5 = 9$	Subtract 5 from both sides to undo the +5.
$2x = 4$	Divide both sides by 2, since $2x = 2 \times x$ and $\frac{{}^{1}\cancel{2} \times x}{\cancel{2}^{1}} = x$.
$x = 2$	
Check:	
LHS $= 2x + 5$	Check your solution by substituting $x = 2$ into the original equation to check that the LHS equals the RHS.
$= 2 \times 2 + 5$	
$= 4 + 5$	
$= 9$	
RHS $= 9$	
b $11 = 3x - 7$	Swap LHS and RHS.
$3x - 7 = 11$	Add 7 to both sides to undo the -7, since $-7 + 7 = 0$.
$3x = 18$	$3x = 3 \times x$, so divide both sides by 3.
$x = 6$	
Check:	
LHS $= 11$	Check your solution using substitution.
RHS $= 3x - 7$	
$= 3 \times 6 - 7$	
$= 18 - 7$	
$= 11$	

5 Solve each of the following equations.
Check your answers.

a $2x + 7 = 15$
b $5a + 6 = 11$
c $16 = 3m + 4$
d $2x + 4 = 12$
e $2n + 13 = 17$
f $-7 = 3x + 5$
g $5b - 3 = 12$
h $3y - 2 = 19$
i $20 = 7a - 8$
j $4b - 7 = 25$
k $2x - 4 = -6$
l $-15 = 10y - 35$

Think carefully about each step to make sure you do produce an equivalent equation.

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Example 16 Solving equations with negative coefficients

Solve the following equations.

a $9 - 2x = 15$

b $7 - 4x = 10$

SOLUTION	EXPLANATION
a $9 - 2x = 15$ $-2x = 6$ $x = -3$	Subtract 9 from both sides, leaving $-2x$ on the LHS. Divide both sides by -2; recall that a positive number divided by a negative number is negative.
Check: LHS $= 9 - 2x$ RHS $= 15$ $= 9 - 2 \times (-3)$ $= 9 + 6$ $= 15$	Check your solution using substitution.
b $7 - 4x = 10$ $-4x = 3$ $x = -\frac{3}{4}$	Subtract 7 from both sides. Divide both sides by -4. Leave your answer in fraction form; i.e. $3 \div (-4) = -\frac{3}{4}$.
Check: LHS $= 7 - 4x$ RHS $= 10$ $= 7 - 4 \times \left(-\frac{3}{4}\right)$ $= 7 + 3$ $= 10$	Check your solution using substitution.

6 Solve each of the following equations.

a $12 - 2x = 18$
b $2 - 7x = 9$
c $15 - 5x = 25$
d $19 = 3 - 2x$
e $2 - 5x = -8$
f $24 - 7x = 10$
g $14 - 8x = -2$
h $-21 = 3 - 4x$

7 Solve these equations. Leave your answer in fraction form.

a $2x + 5 = 8$
b $5x - 1 = 2$
c $7 = 11x - 3$
d $6 - 2x = 5$
e $4 - 3x = 5$
f $4 = 8 - 5x$

If $5x = 3$, $x = \frac{3}{5}$ (divide both sides by 5). The answer can be left as a fraction.

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PROBLEM-SOLVING AND REASONING

8, 9 | 8, 10

8 For each of the following, write an equation and solve it to find the unknown value. Use x as the unknown value.

Form the algebraic expression (e.g. $x + 8$) and set it equal to the result to make the equation.

- **a** If 8 is added to a certain number, the result is 34.
- **b** Seven less than a certain number is 21.
- **c** I think of a number, double it and add 4. The result is 10.
- **d** I think of a number, triple it and subtract 4. The result is 11.
- **e** Four less than three times a number is 20.
- **f** I start with the number 25 and subtract twice a certain number. The result is 7.

9 Describe the error made in each of these incorrect solutions.

a
$$3x + 2 = 8$$
$$3x = 10$$
$$x = \frac{10}{3}$$

b
$$2x - 1 = 5$$
$$2x = 6$$
$$x = 12$$

c
$$5 - x = 12$$
$$x = 7$$

d
$$2x + 3 = 8$$
$$x + 3 = 4$$
$$x = 1$$

10 Solve these equations.

- **a** $-3 + 4x = 17$
- **b** $-5 - 2x = 13$
- **c** $-10 - 3x = -4$
- **d** $12 - 4x = 0$
- **e** $14 + 2x = -3$
- **f** $12 = 8m - 4$
- **g** $-10 = 2 - 4x$
- **h** $22 = 6x - 2$
- **i** $-3x - 9 = 0$
- **j** $-5m - 9 = 1$
- **k** $3m - 7 = 1$
- **l** $-9 - 7x = -1$
- **m** $6h - 5 = -7$
- **n** $7 - 3w = 4$
- **o** $-15 - 9d = 0$

ENRICHMENT

– | 11

Is the car speeding?

11 A car is travelling in zones where speed signs are in metres per second (m/s).

- **a** When it first sets out, its speed (s m/s) is given by $s = 2t$, where t is time in seconds. How long does it take for the car to reach a speed of 20 m/s (this is 72 km/h)?
- **b** The car starts to accelerate again, and its speed is now given by $s = 16 + 1.5t$. For how many seconds can it continue this acceleration before it reaches 28 m/s (about 100 km/h)?

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3F Solving linear equations involving fractions

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We saw in the previous section that to solve an equation such as $2x = 8$, we divide each side of the equation by 2. What happens if the equation involves a fraction? For the equation $\frac{1}{2}x = 2$, we could similarly divide both sides by $\frac{1}{2}$ to solve it. But we can also write the equation in a different way, to make it easier to solve. A half of x is equivalent to dividing x by 2, so $\frac{1}{2}x = 2$ can be written as $\frac{x}{2} = 2$. To solve this new equation, we can multiply both sides by 2, since $2 \times \frac{x}{2} = x$. This gives us $x = 4$. From this process, we can see that we could solve this equation by dividing both sides by $\frac{1}{2}$ *or* by multiplying both sides by 2.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

For more complex equations we'll also need to consider the order of operations.

Let's start: Fractions of x

Consider these problems involving fractions.

- If half of x is 8, what is x ?
- If a third of x is 5, what is x ?
- If three quarters of x is 30, what is x ?
  - State what you did to x to find its value each time.
  - Write an equation to represent each case.
  - Discuss how your method for finding the answer relates to the fraction in the equation.

Key ideas

- Pronumerals with a fraction coefficient can be expressed in fraction form.

  For example: 'One-third of a number' can be written as $x \div 3$ or $\frac{1}{3}x$ or $\frac{x}{3}$.
- Equations involving fractions can be solved by creating a series of simpler equivalent equations.
- At some stage, both sides of the equation will be multiplied by the number in the denominator of the fraction. The order of when this happens is important.

  For example:

$$\frac{x}{5} + 3 = 7$$

-3 $\qquad$ -3

$$\frac{x}{5} = 4$$

$\times 5$ $\qquad$ $\times 5$

$$x = 20$$

$$\frac{x+3}{5} = 7$$

$\times 5$ $\qquad$ $\times 5$

$$x + 3 = 35$$

-3 $\qquad$ -3

$$x = 32$$

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Key ideas

- It is always a good idea to check your solution using substitution.
  For example:
  Equation: $\frac{x}{5} + 3 = 7$
  Possible solution: $x = 20$
  Check:

$\text{LHS} = \frac{x}{5} + 3$	$\text{RHS} = 7$ ✓
$= \frac{20}{5} + 3$	
$= 7$ ✓	

Exercise 3F

UNDERSTANDING AND FLUENCY | 1–3, 4–5(½) | 3, 4–6(½)

1 Write down the value of x that is the solution to these equations. No written working is required.

a $\frac{x}{2} = 10$

b $\frac{x}{4} = 6$

c $\frac{x+1}{3} = 4$

d $\frac{x}{5} - 2 = 1$

2 Which one of the following is an equivalent equation to $\frac{x}{4} + 1 = 8$?

A $x + 1 = 32$

B $x = 36$

C $\frac{x}{4} = 7$

D $\frac{x+1}{4} = 2$

3 Match each equation in the first column with its equivalent equation in the second column and state the single operation that generates the equivalent equation (for example, multiply both sides by 5).

a $\frac{x}{3} - 1 = 2$ **i** $x + 2 = 3$

b $\frac{x}{4} = 2$ **ii** $x - 1 = 6$

c $\frac{x+2}{3} = 1$ **iii** $x = 8$

d $\frac{x-1}{2} = 3$ **iv** $\frac{x}{3} = 3$

Think about which step you would apply first to produce a simple equivalent equation.

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Example 17 Solving linear equations with fractional coefficients

Solve each of the following equations.

a $\frac{2x}{3} = 8$

b $\frac{x}{4} - 3 = 7$

SOLUTION	EXPLANATION
a $\frac{2x}{3} = 8$	
$2x = 24$ $x = 12$	Multiply both sides by 3 to remove the fraction on the left-hand side. Then divide both sides by 2.
Check:	
LHS $= \frac{2 \times 12}{3}$ RHS $= 8$ $= \frac{24}{3}$ $= 8$	Check your solution using substitution.
b $\frac{x}{4} - 3 = 7$	Add 3 to both sides first to undo the -3.
$\frac{x}{4} = 10$	Multiply both sides by 4.
$x = 40$	
Check:	
LHS $= \frac{40}{4} - 3$ RHS $= 7$ $= 10 - 3$ $= 7$	Check your solution using substitution.

4 Solve each of the following equations.

a $\frac{x}{5} = 3$

b $\frac{y}{2} = 7$

c $\frac{2y}{5} = 4$

d $6 = \frac{3x}{2}$

e $\frac{x}{4} + 3 = 5$

f $\frac{x}{2} + 4 = 5$

g $\frac{b}{3} + 5 = 9$

h $2 = 5 + \frac{t}{2}$

i $\frac{a}{3} + 4 = 2$

j $\frac{y}{5} - 4 = 2$

k $\frac{x}{3} - 7 = -2$

l $-1 = \frac{s}{2} - 3$

m $\frac{x}{4} - 5 = -2$

n $\frac{m}{4} - 2 = -3$

o $\frac{y}{5} + 4 = -3$

p $-2 = \frac{x}{3} + 5$

In part **e**, the first step is to subtract 3 from both sides.

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Example 18 Solving simple fractional equations

Solve the equation $\frac{x+4}{6} = 2$.

SOLUTION

$\frac{x+4}{6} = 2$

$x + 4 = 12$

$x = 8$

Check:

$\text{LHS} = \frac{8+4}{6}$ $\quad$ $\text{RHS} = 2$

$= \frac{12}{6}$

$= 2$

EXPLANATION

Multiply both sides by 6 first, since all of $(x + 4)$ is divided by 6. Subtract 4 from both sides and check the answer.

Check your solution using substitution.

5 Solve each of the following equations. Check your answers.

a $\frac{x+1}{3} = 4$ **b** $\frac{x+4}{2} = 5$ **c** $\frac{y+4}{3} = 2$

d $3 = \frac{b+6}{2}$ **e** $\frac{y-2}{3} = 6$ **f** $\frac{t-4}{5} = 7$

g $\frac{k-1}{7} = 8$ **h** $7 = \frac{x-7}{9}$ **i** $\frac{x+2}{4} = -2$

j $\frac{b+3}{7} = -3$ **k** $\frac{y-3}{5} = -5$ **l** $-7 = \frac{a-2}{2}$

Think carefully about the order of the steps here.

6 Solve this mixed selection of equations.

a $2t + 1 = 7$ **b** $\frac{m}{4} - 2 = 8$ **c** $9 = 5 - 2y$

d $\frac{x-4}{5} = 3$ **e** $3 + \frac{y}{6} = 2$ **f** $-8 = 4t - 5$

PROBLEM SOLVING AND REASONING 7, 8, 9 8(½), 10, 11(½)

7 By substituting into the left-hand side and the right-hand side of the equation, decide by writing True or False if the given value of x is the correct solution of the equation.

a $7x - 3 = x + 9$ $[x = 2]$

b $6 - x = x + 5$ $[x = 1]$

c $3x - 5 = x + 4$ $[x = 9]$

d $2 - 3x = 5 - x$ $[x = 5]$

e $10x + 15 = x - 9$ $[x = -2]$

f $5 - 3x = x + 9$ $[x = -1]$

g $9 - 5x = 1 + 3x$ $[x = 1]$

h $5x = 3x - 8$ $[x = 2]$

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8 For each of the following, write an equation and solve it to find the unknown value. Use x as the unknown value.

x halved is $\frac{x}{2}$.
A third of x is $\frac{x}{3}$.

- **a** If a certain number is divided by 3, the result is 12.
- **b** A certain number is doubled, then divided by 5. The result is 4.
- **c** I think of a number, halve it and subtract 4. The result is 10.
- **d** I think of a number, add 3 and divide this by 4. The result is 6.
- **e** A number is multiplied by 7 and the product is divided by 3. The final result is 8.
- **f** 5 is added to a third of a certain number. The result is 16.

9 A bag of chocolate eggs containing x eggs is shared equally between India and her two brothers. After India eats 2, she has 5 left. How many eggs were in the bag?

Start with an expression for the number of eggs India first receives.

10 Describe the error made in each of these incorrect solutions. Then write out the correct solution.

a
$$\frac{x+2}{3} = 7$$
$$\frac{x}{3} = 5$$
$$x = 15$$

b
$$\frac{x}{3} - 4 = 2$$
$$x - 4 = 6$$
$$x = 10$$

11 Solve these equations involving more than two steps.

Treat these just like a two-step equation, but add an extra step.

- **a** $\frac{2x}{3} - 1 = 7$
- **b** $\frac{3x}{4} - 2 = 7$
- **c** $1 = 3 + \frac{x}{2}$
- **d** $\frac{3b - 8}{2} = 5$
- **e** $\frac{2x + 2}{3} = 4$
- **f** $9 = \frac{7m - 8}{3}$
- **g** $\frac{5 - y}{3} = 2$
- **h** $\frac{4 - 2t}{6} = 3$
- **i** $5 = 1 - \frac{4x}{3}$

ENRICHMENT

–, 12, 13

Fractional coefficients

12 Solve these equations involving fractional coefficients.

- **a** $\frac{2}{3}x = 6$
- **b** $\frac{5}{6}x = 30$
- **c** $2 = -\frac{2}{7}x$
- **d** $\frac{3}{4}x - 1 = 5$
- **e** $\frac{7}{5}x - 3 = 4$
- **f** $7 = 1 - \frac{3}{5}x$

13 If $\frac{7}{5}x = 14$, does it matter whether you multiply by 5 first or divide by 7 first? Which method would be best here?

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3G Linear equations with brackets

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Just as we have seen expressions involving brackets, linear equations may also involve brackets. Brackets can be removed by expanding and then the remaining equation can be solved. Collecting like terms may also be required.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: To expand or not to expand?

The steps to solve two problems involving brackets are listed here in the incorrect order.

1 $3(x + 4) = 14$

$x = \frac{2}{3}$

$3x + 12 = 14$

$3x = 2$

2 $3(x + 2) = 12$

$x = 2$

$x + 2 = 4$

- Arrange the steps in the correct order, working from top to bottom.
- By considering all the steps in the correct order, explain what has happened in each step.
- Describe the two different approaches to solving the problems.
- Can you see why these methods have been used in each case?
- Apply method 1 to the second equation.

Key ideas

- Equations with brackets can be solved by first expanding the brackets and then solving the remaining equation.
  For example: $3(x + 1) = 2$ becomes $3x + 3 = 2$.
- Sometimes it is wise to swap the LHS and RHS.
  For example: $12 = 3(x + 1)$ becomes $3(x + 1) = 12$.
- You can check if a solution is correct by substituting the solution.
  For example:
  Equation: $2(x + 3) = 18$
  Possible solution: $x = 5$
  Check:

LHS $= 2(x + 3)$	RHS $= 18$ ✗
$= 2 \times (5 + 3)$	
$= 2 \times 8$	
$= 16$ ✗	

  $\therefore x = 5$ is *not* a solution because LHS $\neq$ RHS.

Exercise 3G

UNDERSTANDING AND FLUENCY | 1–4, 5–6(½) | 4, 5–7(½)

1 Complete the missing parts.

a $3(x + 7) = 3 \times \square + \square \times 7$
$= 3x + \square$

b $4(2x - 1) = \square \times 2x + 4 \times \square$
$= \square - 4$

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2 Expand the brackets.

a $4(x + 3)$ **b** $5(x + 2)$ **c** $7(x - 6)$
d $3(2x - 5)$ **e** $4(3x + 4)$ **f** $3(1 - 2x)$

3 Simplify by collecting like terms.

a $5x + 4 + 2x$
b $8x - 3 - 4x$
c $2x + 1 - 3x$
d $6 - 8x + x$

4 Expand these expressions and simplify.

a $3(x - 4) + x$ **b** $2(x + 1) + 2x$ **c** $5(x - 3) - 3x$
d $4(x + 2) - 6x$ **e** $2(x - 3) - 3x$ **f** $11 - 3(3 + x)$

Example 19 Solving equations with brackets

Solve $2(3x - 4) = 11$.

SOLUTION

$$2(3x - 4) = 11$$
$$6x - 8 = 11$$
$$6x = 19$$
$$x = \frac{19}{6} \text{ or } 3\frac{1}{6}$$

Check:

$$\text{LHS} = 2\left(3 \times \frac{19}{6} - 4\right) \qquad \text{RHS} = 11$$
$$= 2 \times \frac{11}{2}$$
$$= 11$$

EXPLANATION

Expand the brackets:

$2(3x - 4) = 2 \times 3x + 2 \times (-4)$.

Solve the remaining equation by adding 8 to both sides, then dividing both sides by 6.

Leave your answer in fraction form.

Check your solution using substitution.

5 Solve each of the following equations by first expanding the brackets.

a $2(x + 3) = 11$
b $5(a + 3) = 18$
c $3(m + 4) = 31$
d $5(y - 7) = 12$
e $4(p - 5) = 15$
f $2(k - 5) = 9$
g $4(5 - b) = 21$
h $13 = 2(1 - m)$
i $5(3 - x) = 19$
j $8 = 7(2a + 1)$
k $30 = 4(3x - 2)$
l $0 = 3(3n - 2)$
m $5(3 - 2x) = 16$
n $8 = 6(1 - 2y)$

Once you have expanded the brackets, the equation can be solved in the usual way.

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Example 20 Solving equations with brackets and collecting like terms

Solve $6(x + 3) - 4x = 32$.

SOLUTION

$$\begin{aligned} 6(x + 3) - 4x &= 32 \\ 6x + 18 - 4x &= 32 \\ 2x + 18 &= 32 \\ 2x &= 14 \\ x &= 7 \end{aligned}$$

Check:

$\text{LHS} = 6 \times 10 - 4 \times 7 \qquad \text{RHS} = 32$

$= 60 - 28$

$= 32$

EXPLANATION

Expand the brackets first, then collect any like terms: $6x - 4x = 2x$. Solve the remaining equation by subtracting 18 from both sides, and then divide both sides by 2.

Check your solution using substitution.

6 Expand and simplify, then solve each of the following equations.

- **a** $2(x + 4) + 2x = 12$
- **b** $2(x - 3) + 3x = 4$
- **c** $6(x + 3) - 2x = 26$
- **d** $5(x + 2) - 2x = 46$
- **e** $3(2x - 3) + x = 12$
- **f** $4(3x + 1) - 3x = 19$
- **g** $2(3x + 5) - 8x = 20$
- **h** $4(1 - x) - x = 9$

Expand the brackets and then combine like terms before solving.

7 Solve these equations.

- **a** $4(x - 1) + x - 1 = 0$
- **b** $3(x + 2) - 2 + 4x = 18$
- **c** $3(2x + 3) - 1 - 4x = 10$
- **d** $2(4x - 1) - 5x + 6 = 31$
- **e** $2(4x + 2) - 10x - 6 = 6$
- **f** $5(2x + 3) - 15x - 7 = 3$

PROBLEM-SOLVING AND REASONING 8, 9 8, 10, 11

8 Using x for the unknown number, write down an equation and then solve it to find the number. The first one has been started for you.

- **a** Three times 1 more than a number is 4. (i.e. $3(x + 1) = 4$)
- **b** Twice 2 less than a number is 19.
- **c** The product of 2 and 3 more than a number is 7.
- **d** The product of 3 and 4 less than a number is 8.
- **e** When 2 less than 3 lots of a number is doubled, the result is 5.
- **f** When 5 more than 2 lots of a number is tripled, the result is 10.

You will need to use brackets when setting up the expressions in these equations.

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9 Since Tara started her job, her original hourly wage ($\$x$) has been tripled, then decreased by \$6. Her pay is now to be doubled so that she earns \$18 an hour. What was her original hourly wage?

Tripling Tara's pay of $\$x$ is $3 \times x = 3x$.

10 Consider the equation $3(x - 2) = 9$.

- **a** Solve the equation by first dividing both sides by 3.
- **b** Solve the equation by first expanding the brackets.
- **c** Which of the above two methods is preferable and why?

11 Consider the equation $3(x - 2) = 7$.

- **a** Solve the equation by first dividing both sides by 3.
- **b** Solve the equation by first expanding the brackets.
- **c** Which of the above two methods is preferable and why?

You may recall this idea from the 'Let's start' activity at the beginning of the section.

ENRICHMENT – 12

Brackets or no brackets?

12 Can brackets be inserted into the following equations to make $x = 4$ the solution? If so, insert the brackets where required.

- **a** $6x - 2 = 10$
- **b** $8x - 12 = 12$
- **c** $4x + 1 = 20$
- **d** $2x - 4 + 3x + 1 = 13$
- **e** $3x + 2 + 2x - 5 = 12$
- **f** $2x - 3 - 3x - 1 = -4$

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3H Equations with pronumerals on both sides

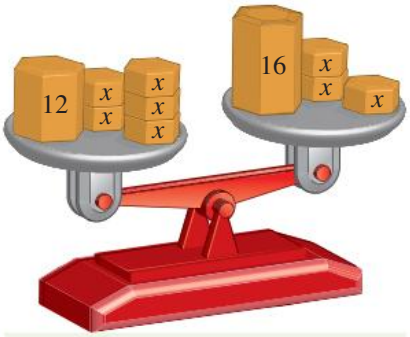
REVISION

Interactive Widgets HOTsheets Walkthrough

Some linear equations have pronumerals on both sides of the equation.

In the diagram:

- 12 could be subtracted from both sides.
- $3x$ could be subtracted from both sides.



This equation has numerals on both sides and pronumerals on both sides.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Which method? Same solution?

There are two methods to solve $5x + 12 = 3x + 16$.

Method 1

Start by subtracting $3x$ from both sides.

$$5x + 12 = 3x + 16$$
$$-3x \quad -3x$$
$$\square + 12 = 16$$
$$-12 \quad -12$$
$$\square = \square$$
$$\div 2 \quad \div 2$$
$$x = \square$$

Method 2

Start by subtracting 12 from both sides.

$$5x + 12 = 3x + 16$$
$$-12 \quad -12$$
$$5x = 3x + \square$$
$$-3x \quad -3x$$
$$\square = \square$$
$$\div 2 \quad \div 2$$
$$x = \square$$

Are both of these correct? Check by substitution.

Key ideas

- If an equation has pronumerals and numbers on both sides, add or subtract one of the terms. For example:

$$-2x \quad 6x + 4 = 2x - 3 \quad -2x$$
$$4x + 4 = -3$$

OR

$$-4 \quad 6x + 4 = 2x - 3 \quad -4$$
$$6x = 2x - 7$$

- At any time, the LHS (left-hand side) and RHS (right-hand side) can be swapped. For example:

$$2x + 3 = 5x - 2$$
$$5x - 2 = 2x + 3$$

Solving is easier if the larger coefficient is on the LHS.

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Exercise 3H REVISION

UNDERSTANDING AND FLUENCY 1–3, 4–6(½) 4–6(½)

1 In each of the following equations swap the LHS and RHS. Does this make the equation easier to solve?

a $5x = 2x + 3$
b $6x + 1 = 2x + 5$
c $3x - 5 = 7x + 3$
d $1 - 2x = 3x - 9$

The x coefficient is the numeral multiplied by the x.

2 State the step you would carry out to collect the x terms on the left-hand side; e.g. 'subtract $2x$ from both sides'.

a $7x = 3x + 4$
b $9x + 2 = 5x + 10$
c $3x - 1 = 3 - x$
d $4x + 2 = 8 - 2x$

For $3 - x$, adding x makes 3.

3 Show the next step only for the given equations and instructions.

a $3x = x + 6$ (subtract x from both sides)
b $4x - 3 = 2x + 1$ (subtract $2x$ from both sides)
c $2x = 1 - 3x$ (add $3x$ to both sides)
d $4 - x = 8 + 5x$ (add x to both sides)

Example 21 Solving equations with variables on both sides

Solve these equations with variables on each side.

a $4x = 2x + 10$ **b** $3x = 16 - 5x$

SOLUTION	EXPLANATION
a $4x = 2x + 10$ $2x = 10$ $x = 5$	Collect x terms on LHS (since the x coefficient is larger on that side) by subtracting $2x$ from both sides: $4x - 2x = 2x$. Solve by dividing both sides by 2.
Check: LHS $= 4 \times 5$ RHS $= 2 \times 5 + 10$ $= 20$ $= 20$	Check your solution using substitution.
b $3x = 16 - 5x$ $8x = 16$ $x = 2$	Collect x terms by adding $5x$ to both sides (collect on LHS since $3x$ coefficient is positive). Divide both side by 8.
Check: LHS $= 3 \times 2$ RHS $= 16 - 5 \times 2$ $= 6$ $= 6$	Check your solution using substitution.

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4 Solve these equations by first collecting the pronumeral on one side.

a $7x = 5x + 8$
b $8x = 3x + 15$
c $9x = 5x + 12$
d $10x = 9x + 7$
e $4x = x - 9$
f $6x = 2x - 8$
g $3x = 10 - 2x$
h $x = 12 - 2x$
i $x = 12 - x$
j $4x = 14 - 3x$

Remember: it is easier to collect on the side that will keep the x coefficient positive.

Example 22 Solving equations with pronumerals and numerals on both sides

Solve each of the following equations.

a $5x + 2 = 3x + 6$
b $3 - 2x = 5x - 4$

SOLUTION	EXPLANATION
a $5x + 2 = 3x + 6$ $2x + 2 = 6$ $2x = 4$ $\therefore x = 2$	Collect x terms on one side by subtracting $3x$ from both sides: $5x - 3x = 2x$. Collect on the side with the larger x term to keep it positive. Solve the remaining equation.
Check: LHS $= 5 \times 2 + 2$ RHS $= 3 \times 2 + 6$ $= 12$ $= 12$	Check your solution using substitution.
b $3 - 2x = 5x - 4$ $3 = 7x - 4$ $7 = 7x$ $1 = x$ $\therefore x = 1$	Add $2x$ to both sides to collect x terms on the side with the positive x coefficient. Solve the remaining equation by adding 4 to both sides and dividing both sides by 7. Swap LHS and RHS.
Alternative method:	
$3 - 2x = 5x - 4$	Swap LHS and RHS.
$5x - 4 = 3 - 2x$	Add $2x$ to both sides.
$7x - 4 = 3$	Add 4 to both sides.
$7x = 7$	Divide both sides by 7.
$x = 1$	Don't forget to check your solution using substitution.

5 Solve each of the following equations.

a $5x - 3 = 4x + 5$
b $9a + 3 = 8a + 6$
c $3m - 8 = 2m$
d $12x - 3 = 10x + 5$
e $8x + 4 = 12x - 16$
f $3x + 7 = 8x - 8$
g $2x + 6 = 9 - x$
h $3y + 6 = 14 - y$
i $5m - 18 = 15 - 6m$
j $4 - 7x = 2x - 23$
k $8 - 2b = 4b + 14$
l $3 - 4m = 3m + 24$

Collect the pronumerals on one side first.

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Example 23 Solving equations with brackets and pronumerals on both sides

Solve these equations involving brackets.

a $7(x + 2) = 3x + 2$

b $3(2x + 4) = 8(x + 1)$

SOLUTION

a
$$\begin{aligned} 7(x + 2) &= 3x + 2 \\ 7x + 14 &= 3x + 2 \\ 4x + 14 &= 2 \\ 4x &= -12 \\ \therefore x &= -3 \end{aligned}$$

b
$$\begin{aligned} 3(2x + 4) &= 8(x + 1) \\ 6x + 12 &= 8x + 8 \\ 12 &= 2x + 8 \\ 4 &= 2x \\ 2 &= x \\ \therefore x &= 2 \end{aligned}$$

EXPLANATION

Expand the brackets first, then subtract $3x$ from both sides to collect x terms.

Subtract 14 from both sides ($2 - 14 = -12$) and divide by 4. Recall that a negative number divided by a positive number is negative ($-12 \div 4 = -3$).

Expand the brackets on each side.

Subtract $6x$ from both sides. (Subtracting $6x$ keeps the x-coefficient positive, as alternatively subtracting $8x$ would end up with $-2x + 12 = 8$.)

Solve the equation and make x the subject.

6 Solve each of the following equations.

a $5(x - 2) = 2x + 11$
b $3(a + 1) = a + 13$
c $3(y + 4) = y + 16$
d $2(x + 5) = x - 4$
e $5b - 4 = 6(b + 2)$
f $2(4m - 5) = 4m + 2$
g $3(2a - 3) = 5(a + 2)$
h $4(x - 3) = 3(3x + 1)$
i $3(x - 2) = 5(x + 4)$
j $3(n - 2) = 4(n - 5)$
k $2(a + 5) = 2(2a + 3)$
l $4(x + 2) = 3(2x + 1)$

Expand the brackets before collecting the pronumeral on one side.

PROBLEM-SOLVING AND REASONING 7, 8 7, 9

7 Using x for the unknown number, write down an equation then solve it to find the number. The first one has been started for you.

a Twice a number is equal to 4 less than 3 times a number. ($2x = 3x - 4$)
b 4 more than 2 lots of a number is equal to 5 times the number.
c 10 more than 3 lots of a number is equivalent to 5 lots of the number.
d 2 more than 3 times the number is equivalent to 6 less than 5 times the number.
e 1 less than a doubled number is equivalent to 5 more than 3 lots of the number.
f 4 more than 2 lots of a number is equivalent to the number subtracted from 13.

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8 Mardy is x years old. He correctly tells his mother that she is 6 years older than 3 times his age. His mother replies that she is also 10 years younger than 5 times his age. How old is Mardy?

Build the equation step by step. You should get two expressions for the mother's age.

9 Consider the equation $3x + 1 = 5x - 7$.

a Solve the equation by first subtracting $3x$ from both sides.

b Solve the equation by first subtracting $5x$ from both sides.

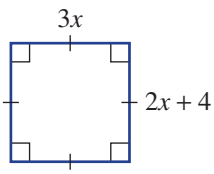
c Which method above do you prefer and why? Describe the differences.

ENRICHMENT – 10

Matching sides

10 Use the properties of these basic shapes to find their perimeters.

a



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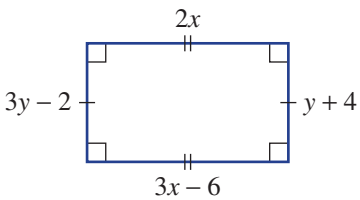
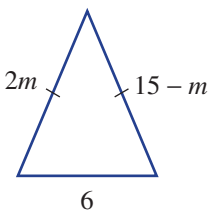
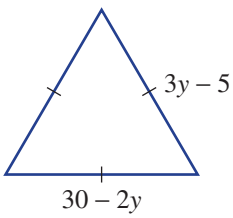


Find the value of each pronumeral first by solving an equation.

c



d



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3I Using linear equations to solve problems

Interactive
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Many types of problems can be solved by writing and solving linear equations. Often problems are expressed only in words. Reading and understanding the problem, defining a pronumeral and writing an equation become important steps in solving the problem.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Too much screen time?

Three friends, Rick, Kate and Sue, compare how much time they spend on their computers and smartphones in a week at home. Kate spends 3 times the amount of time that Rick spends. Sue spends 4 hours fewer than Kate. In total they spend 45 hours on their computers. How many hours did Rick spend?

- Underline each piece of important information in the question.
- Circle the item in the question that you are being asked to find.
- Let x hours be the number of hours spent by Rick.
- Write expressions for the number of hours spent by:
  - i Kate
  - ii Sue
- Add up the number of hours spent by Rick, Kate and Sue.
- Write an equation using all the information in the question.
- Solve the equation.
- Answer the question in the original problem.

Key ideas

- To solve a word problem using algebra:
  - Read the problem and find out what the question is asking you to find.
  - Define a pronumeral and write a statement such as: 'Let x be the number of …' The pronumeral is often what you have been asked to find.
  - Highlight the key pieces of information in the question.
  - Write an equation using your defined pronumeral.
  - Solve the equation.
  - Check that your solution makes sense and is reasonable.
  - Answer the question in words.

Exercise 3I

UNDERSTANDING AND FLUENCY 1–9 4–10

1 Match the written scenario **a–d** with its possible equation **i–iv**.

a 7 is 3 more than a number
b 2 less than a number is 10
c 16 is 1 more than 3 lots of a number
d half of a number is 3

i $3x + 1 = 16$
ii $\frac{a}{2} = 3$
iii $n + 3 = 7$
iv $y - 2 = 10$

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2 Arrange the following steps in order when solving a word problem using algebra.

- **i** Solve the equation.
- **ii** Read the problem.
- **iii** Answer the question in words.
- **iv** Set up an equation to represent the problem using the defined pronumeral.
- **v** Define a pronumeral to represent the unknown.
- **vi** Check that your solution is reasonable.

3 For each of the following examples, make x the unknown number and set up an equation to represent the problem. There is no need to solve the equation.

- **a** Three less than a certain number is equal to 5.
- **b** Seven is added to two lots of a number and the result is 9.
- **c** 2 more than a certain number is divided by 3. The result is 4.
- **d** 3 is added to a certain number, which has been halved. The result is 6.

4 The sum of the ages of Sam and his brother Bernard is 34. If Sam is 4 years older than Bernard, fill in the following to find their ages.

Start by underlining the key information.

Let x be the $\boxed{}$ of Bernard.

The age of Sam is $\boxed{}$.

The sum of their ages is 34.

$\therefore x + \boxed{} = 34$

$\boxed{} + 4 = 34$

$2x = \boxed{}$

$x = \boxed{}$

$\therefore$ Bernard is _____ years old and Sam is _____ years old.

Example 24 Turning a word problem into an equation

Five less than three times a certain number is 13. Write an equation and solve it to find the number.

SOLUTION	EXPLANATION
Let x be the number.	Define the unknown as a pronumeral. Interpret the wording bit by bit to construct the equation.
$3x - 5 = 13$	
$3x = 18$	3 times the number is $3x$, 5 less than this is $3x - 5$ and this equals 13.
$x = 6$	Solve the equation by adding 5 to both sides, then dividing both sides by 3.
Check:	
$3 \times 6 - 5 = 13$ ✓	Check that the solution is reasonable.
The number is 6.	Write the answer in words.

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5 Write an equation and solve it to find the unknown number in the following.

a 4 less than 2 times a number is 10.

b When a certain number is doubled, it results in a number that is 5 more than the original number.

c I think of a number, divide it by 3 and add 5. The result is 12.

d I think of a number, take away 2 and multiply the result by 4. This gives 24.

e 3 less than a certain number is 9 less than 4 times the number.

Example 25 Applying algebra to a word problem

A bicycle shop hires out bikes. It charges an initial fee of \$10 for hiring a bike and then a charge of \$8 per hour. Leah returns her bike and is charged \$42. For how many hours did she hire the bike?

SOLUTION	EXPLANATION
Let h be the number of hours of hire.	Define the unknown value as a pronumeral.
$10 + 8h = 42$	Write an equation from the information: the cost is \$10 plus \$8 per hour ($8 \times h$), which equals \$42.
$8h = 32$	
$\therefore h = 4$	Solve the equation for h.
Check:	
$\$10 + \$8 \times 4 = \$42$ ✓	Check that the answer is reasonable.
∴ Leah had the bike for 4 hours.	Answer the question in words.

6 Toby rented a car for a total cost of \$290. The rental company charged \$40 per day, plus a hiring fee of \$50.

a Define a pronumeral for the number of days Toby rented the car.

b Write an equation in terms of your pronumeral in part **a** to represent the problem.

c Solve the equation in part **b** to find the unknown value.

d For how many days did Toby rent the car?

7 A jeweller earns a weekly amount of \$200 plus \$10 per item she sells. If in one week she earned \$680, how many items did she sell?

What will you define your pronumeral as?

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Example 26 Solving word problems with equations

David and Usman made 254 runs between them in a cricket match. If Usman made 68 more runs than David, how many runs did each of them make?

SOLUTION

Let the number of runs for David be r.
Number of runs Usman made is $r + 68$.

$$r + (r + 68) = 254$$
$$2r + 68 = 254$$
$$2r = 186$$
$$r = 93$$

David made 93 runs and Usman made $93 + 68 = 161$ runs.

EXPLANATION

Define the unknown value as a pronumeral.
Write all other unknown values in terms of r.
Usman made 68 more runs than David: $r + 68$.
Write an equation: number of runs for David + number of runs for Usman = 254.
Collect like terms then solve the equation.
Answer the question in words and check that it is reasonable.

8 Leonie and Emma scored 28 goals between them in a netball match. Leonie scored 8 more goals than Emma.
- **a** Define a pronumeral for the number of goals scored by Emma.
- **b** Write the number of goals scored by Leonie in terms of the pronumeral in part **a**.
- **c** Write an equation in terms of your variable to represent the problem.
- **d** Solve the equation in part **c** to find the unknown value.
- **e** How many goals did each of them score?

9 A rectangle is four times as long as it is wide and its perimeter is 560 cm.

Draw a rectangle to help.

- **a** Define a pronumeral for the unknown width.
- **b** Write an expression for the length in terms of your pronumeral in part **a**.
- **c** Write an equation involving your pronumeral to represent the problem.
- **d** Solve the equation in part **c**.
- **e** What is the length and width of the rectangle?

10 A prize of \$1000 is divided between Adele and Benita so that Adele receives \$280 more than Benita. How much did each person receive?

PROBLEM-SOLVING AND REASONING 11, 12 | 11, 13–15

11 Andrew, Brenda and Cammi all work part-time at a supermarket. Cammi earns \$20 more than Andrew's weekly wage. Brenda earns \$30 less than twice Andrew's weekly wage. If their total combined wage is \$400, find how much each of these workers earns.

Remember to follow these steps.
1. Define a pronumeral for the unknown.
2. Set up an equation using the information from the question.
3. Solve the equation.
4. Answer the question in words and check that the answer is reasonable.

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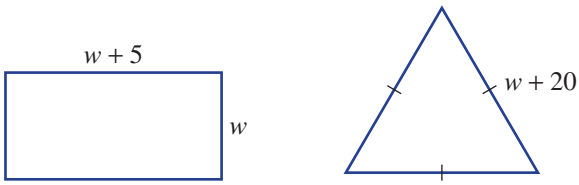
12 Ed walked a certain distance, and then ran twice as far as he walked. He then caught a bus for the last 2 km. If he travelled a total of 32 km, find how far Ed walked and ran.

13 Kate is three times as old as her son. If Kate is 30 years older than her son, what are their ages?

14 Two paddocks in the shapes shown below are to be fenced with wire. If the same total amount of wire is used for each paddock, what are the side lengths of each paddock, in metres?



Solve for w first.

15 Consecutive integers can be represented algebraically as $x, x + 1, x + 2$ etc.

a Find three consecutive numbers that add to 84.

b **i** Write three consecutive even numbers starting with x.

ii Find three consecutive even numbers that add to 18.

Make sure that you are still setting up an equation.

ENRICHMENT – 16

Profit and revenue

16 Tedco produces a teddy bear that sells for \$24. Each teddy bear costs the company \$8 to manufacture and there is an initial start-up cost of \$7200.

a Write a rule for the total cost, $\$T$, of producing x teddy bears.

b If the cost of a particular production run was \$9600, how many teddy bears were manufactured in that run?

c If x teddy bears are sold, write a rule for the revenue, $\$R$, received by the company.

d How many teddy bears were sold if the revenue was \$8400?

e If they want to make an annual profit of \$54000, how many teddy bears do they need to sell?

Revenue is the total amount of money collected.
Profit is the amount left over from the revenue after the costs have been taken out.
Profit = revenue − costs

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3J Using formulas

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A formula is an equation that relates two or more variables. You can find the value of one of the variables if you are given the value of all other unknowns. The following are some examples of formulas.

- $A = \pi r^2$ is the formula for finding the area, A, of a circle, given its radius, r.
- $V = lbh$ is the formula for finding the volume of a rectangular prism, given its length, l, breadth, b, and height, h.
- $F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.

We can find the volume of water in a fish tank if we know the tank's dimensions.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

A, V and F are said to be the subjects of the formulas given above.

Let's start: Common formulas

As a class group, try to list at least 10 formulas that you know.

- Write down the formulas and what they relate to.
- Try to describe what each variable in your formulas represents.
- What variable is the subject of each formula?

Key ideas

- The **subject** of a **formula** is a pronumeral that usually sits on its own on the LHS.
  For example: In $A = lb$, A is the subject of the formula.
- A pronumeral in a formula can be evaluated by substituting numbers for all other pronumerals.
- If the unknown is not the subject, a simple equation will need solving.
- A formula can be rearranged to make another pronumeral the subject.
- Similar steps are used to those applied when solving an equation.
  $A = lb$ can be rearranged to give $l = \frac{A}{b}$ with l now the subject.

Subject The pronumeral that is alone on one side of the equation

Formula A rule for finding the value of one quantity given the values of others

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Exercise 3J

UNDERSTANDING AND FLUENCY	1–5, 6(½)	4, 5–6(½)

1 Solve the following using inspection or guess-and-check.

a $8 = 2 \times w$

b $18 = \frac{1}{2} \times 6 \times h$

c $7 = \frac{d}{3}$

d $3s^2 = 27$

e $36 = 3 \times w \times 4$

f $8 = 2 \times (a + 1)$

Remember: multiplication can occur in any order
$2 \times x \times 4 = 2 \times 4 \times x$

2 State the letter that is the subject of these formulas.

a $A = \frac{1}{2}bh$

b $D = b^2 - 4ac$

c $M = \frac{a + b}{2}$

d $A = \pi r^2$

In $V = lbh$, V is the subject.

Example 27 Substituting values into formulas

Substitute the given values into the formula to work out the subject.

a $A = \frac{1}{2}h(a + b)$, when $a = 3$, $b = 7$ and $h = 5$

b $E = \frac{1}{2}mv^2$, when $m = 4$ and $v = 5$

SOLUTION	EXPLANATION
a $A = \frac{1}{2}h(a + b)$	
$A = \frac{1}{2} \times 5 \times (3 + 7)$	Substitute $a = 3$, $b = 7$ and $h = 5$. Work out the sum in the brackets first, then multiply out to find the value of A.
$= \frac{1}{2} \times 5 \times 10$	
$= 25$	
b $E = \frac{1}{2}mv^2$	
$E = \frac{1}{2} \times 4 \times 5^2$	Substitute $m = 4$ and $v = 5$. $5^2 = 5 \times 5 = 25$, then work out E by multiplying.
$= \frac{1}{2} \times 4 \times 25$	
$= 50$	

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3 Substitute the given values into each of the following formulas to work out the subject. Round to two decimal places where appropriate.

a $A = bh$, when $b = 3$ and $h = 7$

b $F = ma$, when $m = 4$ and $a = 6$

c $m = \dfrac{a + b}{4}$, when $a = 14$ and $b = -6$

d $t = \dfrac{d}{v}$, when $d = 18$ and $v = 3$

e $A = \pi r^2$, when $\pi = 3.14$ and $r = 12$

f $V = \dfrac{4}{3}\pi r^3$, when $\pi = 3.14$ and $r = 2$

g $c = \sqrt{a^2 + b^2}$, when $a = 12$ and $b = 22$

h $Q = \sqrt{2gh}$, when $g = 9.8$ and $h = 11.4$

i $I = \dfrac{MR^2}{2}$, when $M = 12.2$ and $R = 6.4$

j $x = ut + \dfrac{1}{2}at^2$, when $u = 0$, $t = 4$ and $a = 10$

$V = \dfrac{4}{3}\pi r^3$ is the formula for the volume of a sphere with radius r

4 Make x the subject in the following.

a $ax = 6$

b $\dfrac{x}{c} = 2$

c $bx - c = 5$

d $4xy = b$

Rearrange to make $x = \ldots$ Consider what you would do if the other pronumerals were numbers; i.e. if $2x = 6$, $x = \dfrac{6}{2}$.

Example 28 Finding the unknown value in a formula

Substitute the given values into each formula, then solve to find the unknown value.

a $S = \dfrac{d}{t}$ given $S = 15$ and $d = 60$

b $C = 2\pi r$ given $C = 30$ (answer to one decimal place)

SOLUTION	EXPLANATION
a $S = \dfrac{d}{t}$	Write the formula and substitute the given values of S and d.
$15 = \dfrac{60}{t}$	Solve for t by multiplying both sides by t.
$15t = 60$	Divide both sides by 15.
$t = 4$	
b $C = 2\pi r$	Substitute $C = 30$. Solve for r by dividing both sides by 2π in one step. $\dfrac{30}{2\pi} = 4.77465\ldots$
$30 = 2\pi r$	
$\dfrac{30}{2\pi} = r$	Round to one decimal place.
$r = 4.8$	Swap LHS and RHS.

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5 Substitute the given values into each of the following formulas. Solve the equations to find the unknown value. Round to two decimal places where appropriate.

a $m = \frac{F}{a}$, when $m = 12$ and $a = 3$

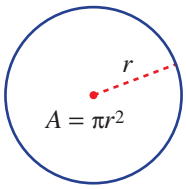
b $A = lb$, when $A = 30$ and $l = 6$

c $v = u + at$, when $v = 20$, $u = 5$ and $a = 2$

d $p = \frac{m}{v}$, when $p = 2$ and $m = 50$

e $A = \pi r^2$, when $A = 48$

f $A = \frac{1}{2}h(a + b)$, when $A = 64$, $b = 12$ and $h = 4$



Example 29 Transposing formulas

Transpose each of the following to make the red pronumeral the subject.

a $ax + by = c$ **b** $A = \pi r^2 \ (r > 0)$

SOLUTION	EXPLANATION
a $ax + by = c$ $by = c - ax$ $y = \frac{c - ax}{b}$	We need to get y on one side by itself. Follow steps as if you are solving the equation for y. Subtract ax from both sides. by is $b \times y$. Just as for $2y$ you would divide both sides by 2, divide both sides by b.
b $A = \pi r^2$ $\frac{A}{\pi} = r^2$ $r^2 = \frac{A}{\pi}$ $r = \sqrt{\frac{A}{\pi}}$	Divide both sides by π. Make r^2 the subject. Take the square root of both sides, since $\sqrt{r^2} = r$ if $r > 0$.

6 Transpose each of the following formulas to make the red pronumeral the subject.

a $C = \pi d$

b $a + bx = d$

c $p = m(x + n)$

d $I = \frac{Prt}{100}$

e $P = \frac{v^2}{R} \ (v > 0)$

f $A = 2\pi rh$

g $V = \pi r^2 h \ (r > 0)$

h $\sqrt{A} + B = 4C$

i $c^2 = a^2 + b^2 \ (a > 0)$

j $\sqrt{b + c} = a$

Follow similar steps as if you were solving to find the red pronumeral, such as $d = \ldots$

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PROBLEM-SOLVING AND REASONING | 7, 8 | 7, 9, 10

7 The formula $s = \frac{d}{t}$ gives the speed s km/h of a car that has travelled a distance of d km in t hours.

a Find the speed of a car that has travelled 400 km in 4.5 hours. Round to two decimal places.

b **i** Transpose the formula $s = \frac{d}{t}$ to make d the subject.

ii Find the distance covered if a car travels at 75 km/h for 3.8 hours.

8 The velocity, v m/s, of an object is described by the rule $v = u + at$, where u is the initial velocity in m/s, a is the acceleration in m/s^2 and t is the time in seconds.

a Find the velocity after 3 seconds if the initial velocity is 5 m/s and the acceleration is 10 m/s^2.

b Find the time taken for a body to reach a velocity of 20 m/s if its acceleration is 4 m/s^2 and its initial velocity is 12 m/s.

Write down the value of the known pronumerals first; i.e. $t = 3$, $u = 5$ etc.

Velocity is another word that relates to speed.

9 The volume of water (V litres) in a tank is given by $V = 4000 - 0.1t$, where t is the time in seconds after a tap is turned on.

a Over time, does the water volume increase or decrease according to the formula?

b Find the volume after 2 minutes. (Note: t is in seconds.)

c Find the time it takes for the volume to reach 1500 litres. Round to the nearest minute.

10 The formula $F = \frac{9}{5}C + 32$ converts degrees Celsius, C, to degrees Fahrenheit, F.

a Find what each of the following temperatures is in degrees Fahrenheit.

i 100°C

ii 38°C

b Calculate what each of the following temperatures is in degrees Celsius. Round to one decimal place where necessary.

i 14°F

ii 98°F

$\frac{9}{5}C$ can also be written as $\frac{9C}{5}$.

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ENRICHMENT – 11

Basketball formulas

11 The formula $T = 3x + 2y + f$ can be used to calculate the total number of points made in a basketball game where:

x = number of three-point goals
y = number of two-point goals
f = number of free throws made
T = total number of points

a Find the total number of points for a game where 12 three-point goals, 15 two-point goals and 7 free throws were made.

b Find the number of three-point goals made if the total number of points was 36, with 5 two-point goals made and 5 free throws made.

c The formula $V = \dfrac{\left(p + \dfrac{3r}{2} + 2a + \dfrac{3s}{2} + 2b\right) - (1.5t + 2f + m - o)}{g}$ can be used to calculate the value, V, of a basketball player where:

p = points earned
a = number of assists
b = number of blocks
f = number of personal fouls
o = number of offensive rebounds
r = number of rebounds
s = number of steals
t = number of turnovers
m = number of missed shots
g = number of games played

Calculate the value of a player with 350 points earned, 2 rebounds, 14 assists, 25 steals, 32 blocks, 28 turnovers, 14 personal fouls, 24 missed shots, 32 offensive rebounds and 10 games.

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Puzzles and challenges

1 In a magic square, all the rows, columns and main diagonals add to the same total. The total for this magic square is 15. Find the value of x, then complete the magic square.

$2x - 2$	$3x$	$x - 1$
x		

2 Simplify $2xy - 3x(y - 2x) - 8x^2y \div (2y) + 2x \times 6xy \div (4y)$.

3 Reveal the word below by solving each equation for x. The value of x gives the number of the letter in the alphabet; e.g. $x = 1$ would be A, $x = 2$ would be B etc.

$\frac{x-3}{4} = 5$	$6x = 4x + 30$	$2(x - 10) = 16$	$1 - 2x = -7$

4 Twelve years ago Eric's father was seven times as old as Eric was. If Eric's father is now 54 years old, how old is Eric now?

5 In a yacht race the second leg was half the length of the first leg. The third leg was two thirds of the length of the second leg. The last leg was twice the length of the second leg. If the total distance was 153 km, find the length of each leg.

6 A group of office workers had some prize money to distribute among themselves. When all but one took \$9 each, the last person only received \$5. When they all took \$8 each, there was \$12 left over. How much had they won?

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Expressions and equations

Expanding brackets

e.g. $2(x+3) = 2 \times x + 2 \times 3$
$= 2x + 6$
$-3(4-2x) = -3 \times 4 + (-3) \times (-2x)$
$= -12 + 6x$

Expressions

e.g. $\boxed{2x} - 3y + 5$

term, coefficient, constant

e.g. 2 more than 3 lots of m is $3m + 2$
3 less than x is $x - 3$

Substitution

The process of replacing a pronumeral with a given value
e.g. if $x = 3$, $2x + 4 = 2 \times (3) + 4$
$= 6 + 4$
$= 10$
e.g. if $x = 2$, $y = -4$,
then $xy = 2 \times (-4)$
$= -8$

Solving word problems

1 Define a pronumeral for the unknown.
2 Set up an equation.
3 Solve the equation.
4 Check the answer and write in words.

Addition/subtraction

Only like terms can be combined under addition or subtraction.
Like terms have the same pronumeral factors; i.e. $4xy$ and $2xy$.
e.g. $5a + 2 + 3a$
$= 5a + 3a + 2$
$= 8a + 2$
e.g. $3x + 2y - x + 4y$
$= 3x - x + 2y + 4y$
$= 2x + 6y$

Solving linear equations

The solution of a linear equation is the value that makes the equation true.
e.g. $2x + 5 = 9$
$2x = 4$ (subtract 5)
$x = 2$ (divide by 2)
e.g. $5(x-1) = 2x + 7$
$5x - 5 = 2x + 7$ (expand)
$3x - 5 = 7$ (subtract $2x$)
$3x = 12$ (add 5)
$x = 4$ (divide by 3)

Multiplication/division

In simplified form the multiplication sign ($\times$) and division sign ($\div$) are not shown.
That is, $2 \times x = 2x$ and $x \div 2 = \frac{x}{2}$.
Multiplying:
$2a \times 3b = 2 \times 3 \times a \times b$
$= 6ab$
Dividing:
$4x \div 2 = \frac{\cancel{4}^{2}x}{\cancel{2}_{1}} = 2x$
$\frac{6ab^2}{9b} = \frac{\cancel{6}^{2} \times a \times b \times \cancel{b}^{1}}{\cancel{9}_{3} \times \cancel{b}_{1}}$
$= \frac{2ab}{3}$

Equations involving fractions

e.g. $\frac{x}{3} + 1 = 5$
$\frac{x}{3} = 4$ (subtract 1)
$x = 12$ (multiply by 3)
e.g. $\frac{x-2}{4} = 1$
$x - 2 = 4$ (multiply by 4)
$x = 6$ (add 2)

Formulas

Some common formulas are
$A = lb$, $C = 2\pi r$
An unknown value can be found by substituting values for the other pronumerals. A formula can be transposed to make a different pronumeral the subject.
e.g. make r the subject in $A = \pi r^2$
$A = \pi r^2$
$\frac{A}{\pi} = r^2$
$\sqrt{\frac{A}{\pi}} = r$
$\therefore r = \sqrt{\frac{A}{\pi}}$ where $r > 0$

Chapter summary

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Chapter review

Multiple-choice questions

1 The algebraic expression that represents 2 less than 3 lots of n is:

A $3(n-2)$ **B** $2-3n$ **C** $3n-2$ **D** $3+n-2$ **E** n

2 The fully simplified form of $2ab \div (8a)$ is:

A $\frac{2ab}{8a}$ **B** $\frac{4}{b}$ **C** $\frac{b}{4}$ **D** $4b$ **E** $4a$

3 The simplified form of $6ab + 14a - 2ab + 3a$ is:

A $21ab$ **B** $4ab + 11a$ **C** $ab + 7a$ **D** $4ab + 17a$ **E** $4 + 17a$

4 The solution to the equation $2x - 3 = 7$ is:

A $x = 2$ **B** $x = -5$ **C** $x = 5$ **D** $x = 6.5$ **E** $x = 0.5$

5 The solution to $\frac{x}{3} - 1 = 4$ is:

A $x = 13$ **B** $x = 7$ **C** $x = 9$ **D** $x = 15$ **E** $x = \frac{5}{3}$

6 The expanded form of $3(2y - 1)$ is:

A $6y - 3$ **B** $5y - 1$ **C** $6y - 1$ **D** $6y + 2$ **E** $3y - 3$

7 An equivalent equation to $5x = 2x + 9$ is:

A $2x = 5$ **B** $3x = 9$ **C** $0 = 3x + 9$ **D** $7x = 9$ **E** $x = 9$

8 The solution to the equation $2(x - 3) = 7$ is:

A $x = 5$ **B** $x = \frac{13}{2}$ **C** $x = 2$ **D** $x = \frac{1}{2}$ **E** $x = \frac{7}{3}$

9 Eli is x years old. His sister is 2 years older. The sum of their ages is 22. A simplified equation to represent this is:

A $x + 2 = 22$ **B** $2x + 2 = 22$ **C** $2(x + 2) = 22$ **D** $x(x + 2) = 22$ **E** $2x = 22$

10 If $A = \frac{1}{2}bh$ with $A = 360$ and $h = 40$, then the value of b is:

A 90 **B** 4.5 **C** 20 **D** 18 **E** 9

Short-answer questions

1 Write algebraic expressions to represent the following.

a The sum of x and y
b The product of m and 7
c The cost of 3 movie tickets at m dollars each
d 3 less than n, all divided by 4

2 Evaluate the following if $x = 2$, $y = -1$ and $z = 5$.

a $xz + 1$ **b** $4x + y$ **c** $x - 2yz$ **d** $x(y + z)$ **e** $x^2 - 3z$

3 Simplify the following.

a $2 \times 4n$ **b** $3x \times 2y$ **c** $8a \div 2$ **d** $\frac{4x^2y}{12x}$

4 Simplify by collecting like terms.

a $2b + 4b + b$
b $6x + 3 - 2x$
c $4p - 3q - p + 5q$
d $3mn + 2m - 6mn + n$

5 Expand and simplify the following.

a $2(x + 7)$
b $-3(2x + 5)$
c $2x(3x - 4)$
d $-2a(5 - 4a)$
e $4(x + 2) + 5$
f $3(x - 2) - 1$

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6 Solve the following linear equations for x.

a $5x + 6 = 51$ **b** $7x - 4 = 10$ **c** $7 = 4 - x$ **d** $21 = 3 - 2x$

7 Solve these linear equations involving fractions.

a $\frac{2x}{5} = 4$ **b** $\frac{x}{2} - 1 = 3$ **c** $7 = \frac{x+2}{4}$ **d** $4 = \frac{1}{3}x + 2$

8 Write an equation to represent each of the following and then solve it for the pronumeral.

a A number, n, is doubled and increased by 3 to give 21.

b A number of lollies, l, is decreased by 5 and then shared equally among 3 friends so that they each get 7.

c 5 less than the result of Toni's age, x, divided by 4 is 0.

9 Solve the following linear equations by first expanding the brackets.

a $2(x + 4) = 18$ **b** $3(2x - 3) = 2$

10 Solve these equations with pronumerals on both sides.

a $8x = 2x + 24$ **b** $5x - 2 = 3x + 2$

c $3 - 4x = 7x - 8$ **d** $5(2x + 4) = 7x + 5$

11 Nick makes an initial bid of \$$x$ in an auction for a signed cricket bat. By the end of the auction he has paid \$550, which is \$30 more than twice his initial bid, x. Set up and solve an equation to determine Nick's initial bid.

12 Find the value of the unknown in each of the following formulas.

a $E = \sqrt{PR}$ when $P = 90$ and $R = 40$

b $v = u + at$ when $v = 20, u = 10, t = 2$

c $V = \frac{1}{3}Ah$ when $V = 20, A = 6$

13 Rearrange the following formulas to make the pronumeral in brackets the subject.

a $v^2 = u^2 + 2ax$ (x) **b** $P = RI^2, I > 0$ (I)

Extended-response questions

1 Julie hires a jumping castle for her daughter's birthday party. It costs \$60 for the set-up, plus \$25 for each hour that it is hired.

a What is the cost for:

i 1 hour of hire? **ii** 2 hours of hire?

b Julie is charged \$210 for the hire of the jumping castle.

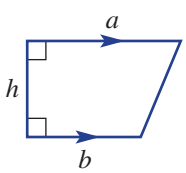
i Define a pronumeral to represent the number of hours for which Julie hired the castle.

ii Set up an equation using your pronumeral.

iii Solve the equation to find how many hours Julie hired the jumping castle for.

2 A new backyard deck is being designed in the shape of the trapezium shown.

The area of a trapezium is given by $A = \frac{1}{2}h(a + b)$.



Currently the dimensions are set so that $a = 12$ m and $b = 8$ m.

a Substitute the given values and rearrange the equation to make h the subject.

b Use your answer to part **a** to find the width of the deck (h) required to have a deck area of 100 m^2.

c It is decided to extend the width (h) to 12 m and increase the area to 150 m^2. If b is fixed at 8 m, find the value of a that gives the required area.

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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

4 Right-angled triangles

What you will learn

4A Exploring Pythagoras' theorem REVISION
4B Finding the length of the hypotenuse REVISION
4C Finding the lengths of the shorter sides REVISION
4D Using Pythagoras' theorem to solve two-dimensional problems REVISION
4E Introducing the trigonometric ratios
4F Finding unknown sides
4G Solving for the denominator
4H Finding unknown angles
4I Using trigonometry to solve problems

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NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY

SUBSTRAND: RIGHT-ANGLED TRIANGLES (PYTHAGORAS AND TRIGONOMETRY)

Outcomes

A student applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solves related problems. (MA4–16MG)

A student applies trigonometry, given diagrams, to solve problems, including problems involving angles of elevation and depression. (MA5.1–10MG)

A student applies trigonometry to solve problems, including problems involving bearings. (MA5.2–13MG)

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UAVs: unmanned aerial vehicles

UAVs, such as quadcopters, octocopters and drones, all use the same principles to navigate.

Before UAVs could be developed, the complex dynamics of their flight had to be represented mathematically. These equations were then coded to allow computer control. When in action, the flight controlling program uses data input from several miniature, electronic sensors including a GPS (for location), an accelerometer (for distance travelled) and a magnetometer (for direction of travel).

The GPS device receives time signals and position data from four of the 24 GPS satellites and calculates the UAV's distance from each satellite. Pythagoras' theorem and trigonometry is applied in three dimensions to compute the UAV's exact position on Earth, giving its latitude, longitude and altitude.

Using data input from the GPS, magnetometer and accelerometer, the UAV's computer program applies trigonometry to calculate the direction and distance to fly to a given destination. Using these same principles, a UAV can hold a position in windy conditions or automatically return to the start if its battery is running low or the control signal is broken.

UAVs can fly autonomously following a list of distances and bearings or GPS coordinates and also follow a variety of instructions such as where to photograph, turn, hover, orbit or land.

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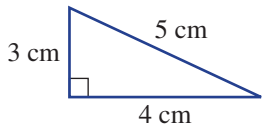
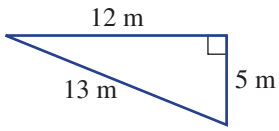
Pre-test

1 What is the length of the longest side given on these triangles?

a



b



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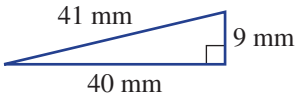


2 Calculate each of the following.

a 3^2 **b** 7^2 **c** $2^2 + 4^2$ **d** $9^2 - 6^2$

3 Round off each number correct to four decimal places.

a 0.45678 **b** 0.34569 **c** 0.04562 **d** 0.27997

4 Round off each number correct to two decimal places.

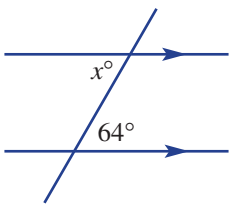
a 4.234 **b** 5.678 **c** 76.895 **d** 23.899

5 Use a calculator to calculate each of the following correct to two decimal places.

a $\sqrt{35}$ **b** $\sqrt{236}$ **c** $\sqrt{23.6}$ **d** $\sqrt{65.4}$

6 Find the value of x in these diagrams.

a



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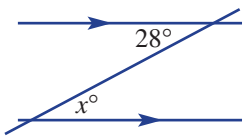

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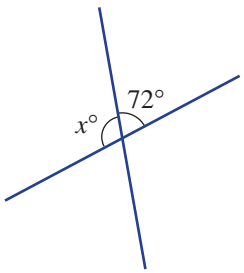

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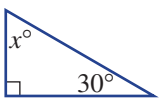

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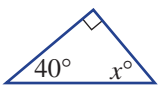

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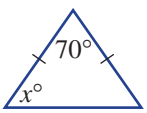

7 Solve each of the following equations to find the value of x. Multiply or divide both sides to start.

a $3x = 6$ **b** $4x = 12$ **c** $5x = 60$ **d** $8x = 48$

e $\frac{x}{3} = 4$ **f** $\frac{x}{5} = 6$ **g** $\frac{x}{7} = 4$ **h** $\frac{x}{13} = 14$

8 Solve each of the following equations to find the value of x correct to one decimal place. First multiply both sides by x, then divide to finish.

a $\frac{3}{x} = 5$ **b** $\frac{4}{x} = 7$ **c** $\frac{32}{x} = 15$ **d** $\frac{14}{x} = 27$

9 Given that x is a positive number, find its value in each of the following equations. Note, for example, that if $x^2 = 9$, $x = \sqrt{9} = 3$ (provided that $x > 0$).

a $x^2 = 4$ **b** $x^2 = 16$ **c** $x^2 = 3^2 + 4^2$ **d** $x^2 = 12^2 + 5^2$

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4A Exploring Pythagoras' theorem REVISION

Interactive
Widgets
HOTsheets
Walkthrough

The philosopher Pythagoras was born in Greece in the 6th century BC. He travelled to Egypt and Persia where he developed his ideas in mathematics and philosophy. His students and followers were called the Pythagoreans. They made many advances in mathematics.

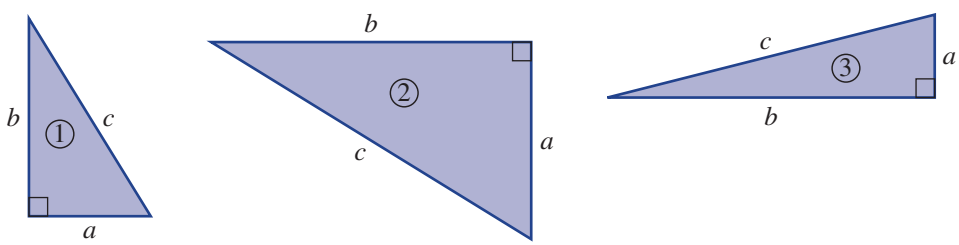
The Pythagoreans discovered the famous theorem, which is named after Pythagoras, and the existence of irrational numbers such as $\sqrt{2}$, which cannot be written down as a fraction or terminating decimal. The Pythagoreans called these numbers 'unutterable' numbers. Any member of the brotherhood who mentioned these numbers in public might be put to death.

A painting of the Pythagorean brotherhood in ancient Greece

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

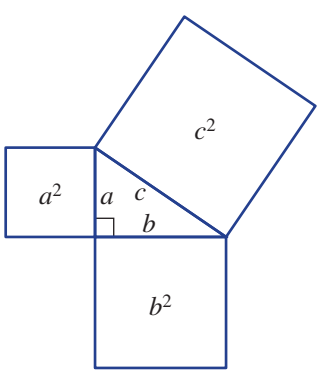
Let's start: Discovering Pythagoras' theorem

Use a ruler to measure the sides of these right-angled triangles to the nearest mm. Then copy and complete the table.



	a	b	c	a^2	b^2	c^2
①						
②						
③						

- Can you see any relationship between the numbers in the columns for a^2 and b^2 and the number in the column for c^2?
- Can you write down this relationship as an equation?
- Explain how you might use this relationship to calculate the value of c if it was unknown.
- Research how you can cut the two smaller squares (a^2 and b^2) to fit the pieces into the largest square (c^2).



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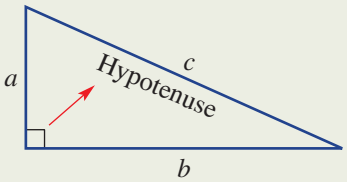
Key ideas

- The **hypotenuse**
  - It is the longest side of a right-angled triangle.
  - It is opposite the right angle.



Hypotenuse The longest side of a right-angled triangle

- **Pythagoras' theorem**
  - The square of the hypotenuse is the sum of the squares of the other two shorter sides.

    $a^2 + b^2 = c^2$ or $c^2 = a^2 + b^2$

Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides

- If you know the length of all three sides, you can decide whether or not the triangle is right-angled.

Exercise 4A REVISION

UNDERSTANDING AND FLUENCY 1–6 4, 5–6(½)

1 Calculate these squares and sums of squares.

a 3^2 **b** 5^2 **c** 12^2 **d** 1.5^2

e $2^2 + 4^2$ **f** $3^2 + 7^2$ **g** $6^2 + 11^2$ **h** $12^2 + 15^2$

2 Decide if these equations are true or false.

a $2^2 + 3^2 = 4^2$ **b** $6^2 + 8^2 = 10^2$ **c** $7^2 + 24^2 = 25^2$

d $5^2 - 3^2 = 4^2$ **e** $6^2 - 3^2 = 2^2$ **f** $10^2 - 5^2 = 5^2$

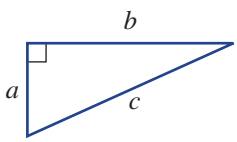
Does LHS = RHS?

3 Write the missing words in this sentence.

The _____ is the longest side of a right-angled _____.

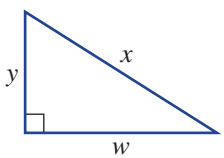
4 Which letter marks the length of the hypotenuse in these triangles?

a

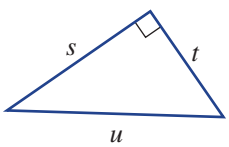


The hypotenuse is the longest side of a right-angled triangle.

b



c



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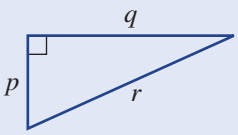
Example 1 Writing Pythagoras' theorem

Write down Pythagoras' theorem for this triangle.



SOLUTION

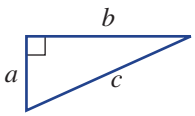
$p^2 + q^2 = r^2$ or $r^2 = p^2 + q^2$

EXPLANATION

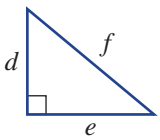
r is the length of the hypotenuse and p and q are the lengths of the two shorter sides.

5 Write down Pythagoras' theorem for these triangles.

a

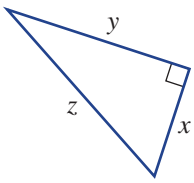


b



Replace the letters a, b and c in $a^2 + b^2 = c^2$ with the given letters.

c



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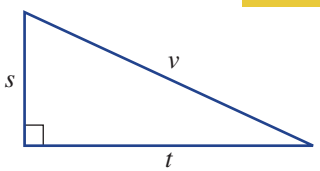

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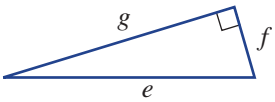

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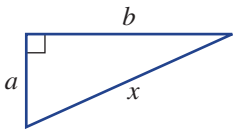
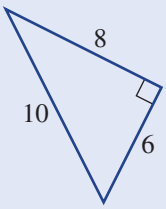


Example 2 Substituting with Pythagoras' theorem

Substitute the numbers on this triangle into $a^2 + b^2 = c^2$.



SOLUTION

$6^2 + 8^2 = 10^2$

EXPLANATION

10 is the length of the hypotenuse and 6 and 8 are the lengths of the two shorter sides.

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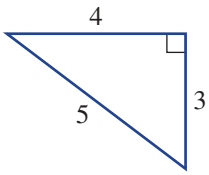
6 Substitute the numbers on these triangles into $a^2 + b^2 = c^2$.

a



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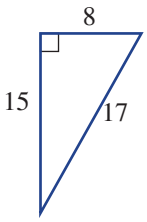

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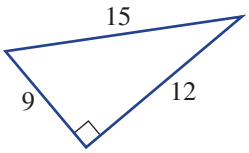

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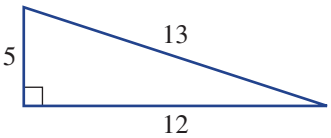

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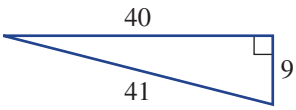

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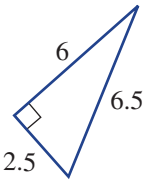
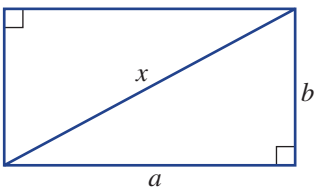


PROBLEM-SOLVING AND REASONING 7–9 | 7, 9–11

7 Write down Pythagoras' theorem using the letters given in these diagrams.

a



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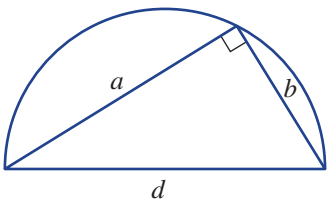

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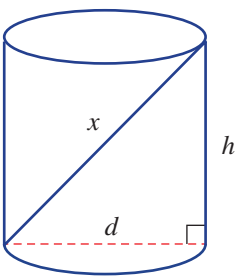

8 Copy and complete this table and answer the questions.

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3	4	5				
6	8	10				
8	15	17				

a Which two columns give equal results?

b What would be the value of c^2 if:

i $a^2 = 4$ and $b^2 = 9$?

ii $a^2 = 7$ and $b^2 = 13$?

c What would be the value of $a^2 + b^2$ if:

i $c^2 = 25$?

ii $c^2 = 110$?

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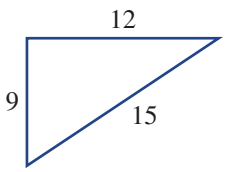
9 A cable connects the top of a 30 m mast to a point on the ground. The cable is 40 m long and connects to a point 20 m from the base of the mast.

a Using $c = 40$, decide if $a^2 + b^2 = c^2$.

b Do you think the triangle formed by the mast and the cable is right angled? Give a reason.

10 If $a^2 + b^2 = c^2$, we know that the triangle must have a right angle. Which of these triangles must have a right angle?

a



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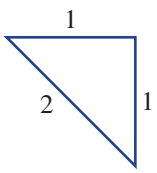
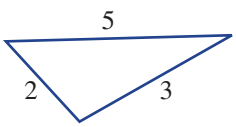


Substitute into $a^2 + b^2 = c^2$ to see if it's true.

c



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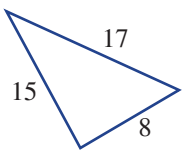

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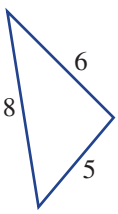

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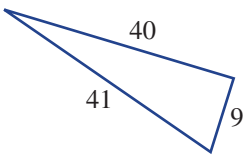

11 (3, 4, 5) and (5, 12, 13) are Pythagorean triads, since $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$. Find 10 more Pythagorean triads using whole numbers less than 100.

If (3, 4, 5) is a Pythagorean triad, then (6, 8, 10) is also a triad.

ENRICHMENT – 12

Pythagorean triads

12 Find the total number of Pythagorean triads with whole numbers of less than 100.

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4B Finding the length of the hypotenuse REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

From our knowledge of formulas and equations, we know that the value of one variable can be found if the values of all the other variables are known. With the help of Pythagoras' theorem, this means that an unknown side length of a right-angled triangle can be found if the other two sides are known. In this section we will find the length of the hypotenuse, given the two shorter sides.

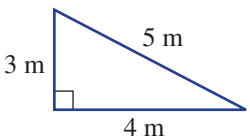
Let's start: Correct substitution

Here are three applications of Pythagoras' theorem for the given triangle.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 3^2 + 4^2 \\ &= 25 \\ \therefore c &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 4^2 + 3^2 \\ &= 25 \\ \therefore c &= \sqrt{25} \\ &= 5 \end{aligned}$$

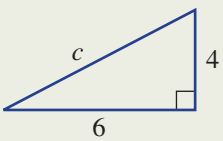
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 4^2 \\ &= 41 \\ \therefore c &= \sqrt{41} \end{aligned}$$



- Which set of working is incorrect? Give reasons.
- Why are the other two sets of working correct? Are they identical?

Key ideas

- Using Pythagoras' theorem to find the length of the hypotenuse:



• $c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$
• Substitute the shorter lengths for a and b.	$c^2 = 4^2 + 6^2$
• Find the value of $a^2 + b^2$.	$c^2 = 16 + 36$
• Take the square root.	$c^2 = 52$
• This is called the exact value.	$\therefore c = \sqrt{52}$
• Calculate the square root of 52 to find the value of c.	$c = 7.211\ldots$
• Round to one decimal place.	$c = 7.2$ (to 1 d.p.)

- Lengths can be expressed with exact values using **surds**. $\sqrt{2}$, $\sqrt{28}$ and $2\sqrt{3}$ are examples of surds.

Surd An irrational number expressed as a square root.

- When expressed as a decimal, a surd is an infinite non-recurring decimal with no pattern.

  For example: $\sqrt{2} = 1.4142135623\ldots$

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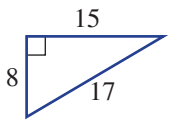
Exercise 4B REVISION

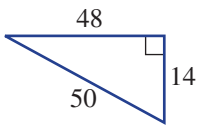
UNDERSTANDING AND FLUENCY	1–3, 4(½), 5, 6	3, 4–6(½)

1 State the value of the hypotenuse (c) in these right-angled triangles.

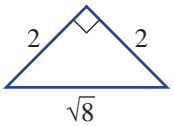
a



b

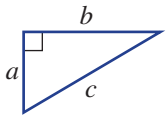


c



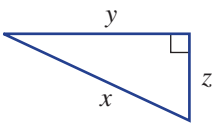
2 Write down Pythagoras' theorem using the given pronumerals for these right-angled triangles.

a

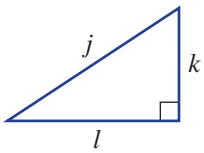


Write a statement such as $u^2 = s^2 + t^2$.

b



c



3 Evaluate the following, rounding to two decimal places in parts **g** and **h**.

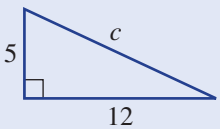
a 9^2 **b** 3.2^2 **c** $3^2 + 2^2$ **d** $9^2 + 5^2$

e $\sqrt{36}$ **f** $\sqrt{64 + 36}$ **g** $\sqrt{24}$ **h** $\sqrt{3^2 + 2^2}$

i $6.5^2 + 9.1^2$ **j** $10^2 + 12^2$ **k** $\sqrt{12^2 + 5^2}$ **l** $\sqrt{40^2 + 9^2}$

Example 3 Finding the length of the hypotenuse

Find the length of the hypotenuse in this right-angled triangle.



SOLUTION	EXPLANATION
$c^2 = a^2 + b^2$ $= 5^2 + 12^2$ $= 169$	Write the rule and substitute the lengths of the two shorter sides.
$\therefore c = \sqrt{169}$ $= 13$	If $c^2 = 169$ then $c = \sqrt{169} = 13$.

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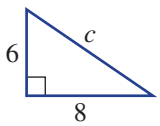
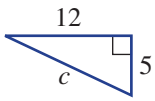
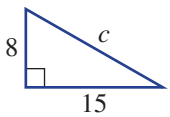
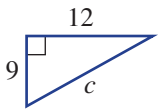
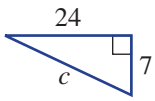
4 Find the length of the hypotenuse in each of the following right-angled triangles.

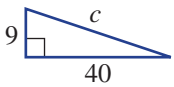
a


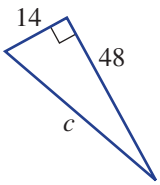
b


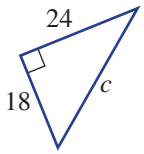
c


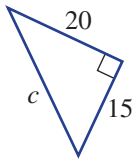
d


e


f


g


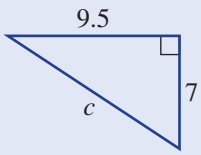
h


i


Substitute the two shorter sides for a and b in the equation $c^2 = a^2 + b^2$.

Example 4 Using rounding

Find the length of the hypotenuse in this right-angled triangle, rounding to two decimal places.



SOLUTION

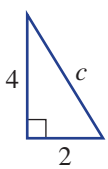
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 7^2 + 9.5^2 \\ &= 139.25 \\ \therefore\ c &= \sqrt{139.25} \\ &= 11.8004\ldots \\ &= 11.80 \text{ (to 2 d.p.)} \end{aligned}$$

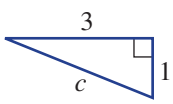
EXPLANATION

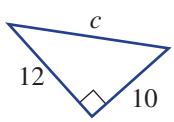
The order for a and b does not matter since $7^2 + 9.5^2 = 9.5^2 + 7^2$.

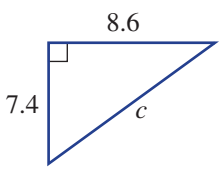
$\sqrt{139.25} = 11.8004\ldots$ and the third decimal place is zero, so round down.

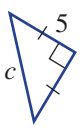
5 Find the length of the hypotenuse in each of these right-angled triangles, correct to two decimal places.

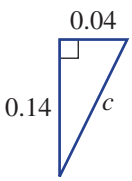
a


b


c


d


e


f


Use a calculator to find the value of c and round your answer to two decimal places.

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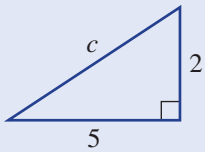
Example 5 Finding the length of the hypotenuse using exact values

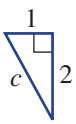
Find the length of the hypotenuse in this right-angled triangle, leaving your answer as an exact value.



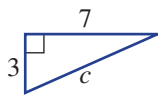
SOLUTION	EXPLANATION
$c^2 = a^2 + b^2$ $= 5^2 + 2^2$ $= 29$	Apply Pythagoras' theorem to find the value of c.
$\therefore\ c = \sqrt{29}$	Express the answer exactly using a surd, which is an exact value.

6 Find the length of the hypotenuse in these triangles, leaving your answer as an exact value.

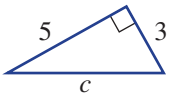
a



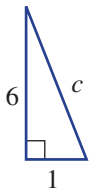
b



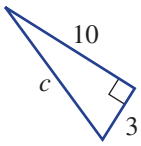
c



d



e



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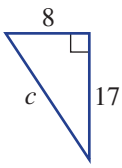
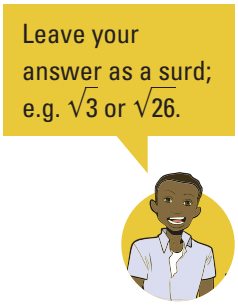
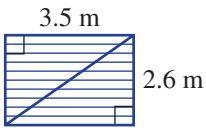


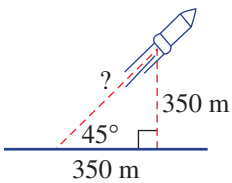


PROBLEM-SOLVING AND REASONING 7–10 10–13

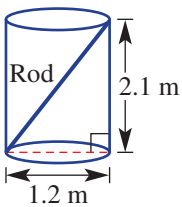
7 Find the length of the diagonal steel brace required to support a wall of length 3.5 m and height 2.6 m. Give your answer correct to one decimal place.



8 A miniature rocket blasts off at an angle of 45° and, after a few seconds, reaches a height of 350 m above the ground. At this point it has also covered a horizontal distance of 350 m. How far has the rocket travelled to the nearest metre?



9 Find the length of the longest rod, as shown, that will fit inside a cylinder of height 2.1 m and with circular end surface of 1.2 m diameter. Give your answer correct to one decimal place.



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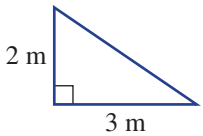
10 For each of these triangles, first calculate the length of the hypotenuse, then find the perimeter, correct to two decimal places.

a



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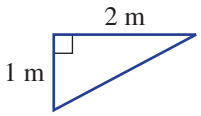
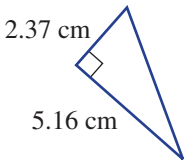
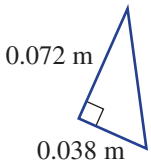


c



d



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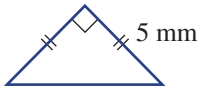

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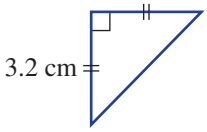
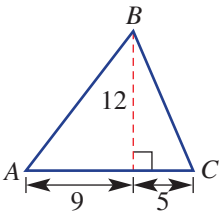


11 A helicopter hovers at a height of 150 m above the ground and is a horizontal distance of 200 m from a beacon on the ground. Find the direct distance of the helicopter from the beacon.

First draw a right-angled triangle.

12 Find the perimeter of this triangle.



You will need to find *AB* and *BC* first.

13 One way to check whether a four-sided figure is a rectangle is to make sure that both its diagonals are the same length. What should the length of the diagonals be if a rectangle has side lengths 3 m and 5 m? Answer to two decimal places.

ENRICHMENT – 14

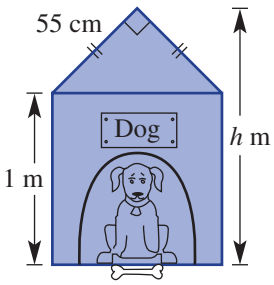
The dog kennel

14 A dog kennel has the dimensions shown in the diagram on the right. Give your answers to each of the following, correct to two decimal places.

a What is the width of the kennel?

b What is the total height, h m, of the kennel?

c If the sloping height of the roof was to be reduced from 55 cm to 50 cm, what difference would this make to the total height of the kennel? (Assume that the width is the same as in part **a**.)



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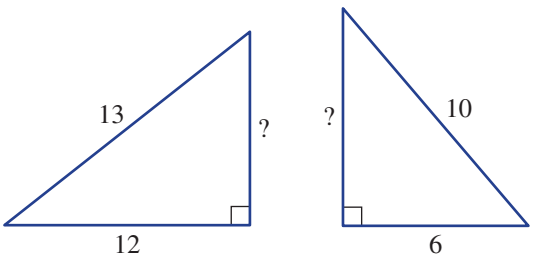
4C Finding the lengths of the shorter sides REVISION

Interactive
Widgets
HOTsheets
Walkthrough

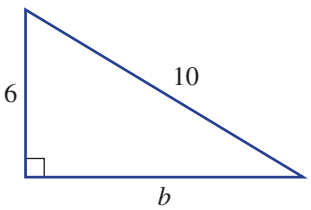
In a right-angled triangle, any two sides can be used to calculate the length of the third side.



Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: True or false?

Below are some mathematical statements relating to a right-angled triangle with hypotenuse 10 and the two shorter sides 6 and b.

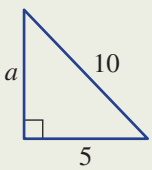


Some of these mathematical statements are true and some are false. Sort them into true and false groups.

$10^2 = 6^2 + b^2$	$b^2 + 10^2 = 6^2$	$b + 6 = 10$	$10^2 + 6^2 = b^2$
$b^2 = 6^2 + 10^2$	$6^2 + b^2 = 10^2$	$b^2 + 6^2 = 10^2$	$10^2 = b^2 + 6^2$

Key ideas

■ Using Pythagoras' theorem to find a shorter side:



- Use $a^2 + b^2 = c^2$. $\quad a^2 + b^2 = c$
- Substitute the known lengths for b and c. $\quad a^2 + 5^2 = 10^2$
- Solve the equation. $\quad a^2 + 25 = 100$

$$-25 \qquad -25$$

$$a^2 = 75$$

- This is called the exact value. $\quad \therefore\ a = \sqrt{75}$
- Calculate the square root to find the value of a. $\quad a = 8.66...$
- Round to one decimal place. $\quad a = 8.7$ (to 1 d.p.)

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Exercise 4C REVISION

UNDERSTANDING AND FLUENCY	1–5	3, 4–5(½)

1 Write the missing word or number.

a To solve for a in $a^2 + 9 = 25$, the first step would be to _____ 9 from both sides.

b To solve for b in $16 + b^2 = 49$, the first step would be to subtract _____ from both sides.

c If $a^2 + 25 = 36$, then $a^2 =$ _____.

2 Find the value of a or b in these equations. (Both a and b are positive numbers.)

a $a = \sqrt{196}$ **b** $a = \sqrt{121}$

c $a^2 = 144$ **d** $a^2 = 400$

e $b^2 + 9 = 25$ **f** $b^2 + 49 = 625$

g $36 + b^2 = 100$ **h** $15^2 + b^2 = 17^2$

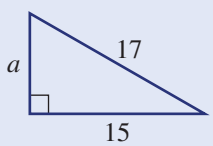
3 If $a^2 + 64 = 100$, decide if the following are true or false.

a $a^2 = 100 - 64$ **b** $64 = 100 + a^2$

c $a = 6$ **d** $a = 10$

Example 6 Finding a shorter side

Find the value of the pronumeral.



SOLUTION

$a^2 + 15^2 = 17^2$

$a^2 + 225 = 289$

$a^2 = 64$

$a = \sqrt{64}$

$a = 8$

EXPLANATION

Write the rule and substitute the known sides.

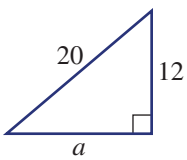
Square 15 and 17.

Subtract 225 from both sides.

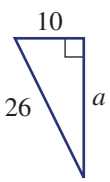
Take the square root of both sides.

4 Find the value of the pronumeral.

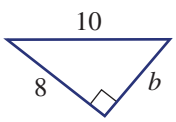
a



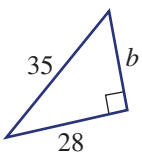
b



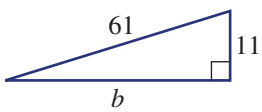
c



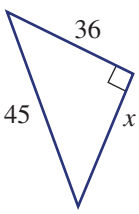
d



e



f



Substitute for a, b and c, then solve for the remaining pronumeral.

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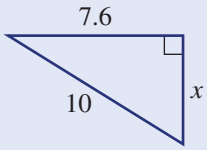
Example 7 Using rounding

Find the value of x, rounding to two decimal places.



SOLUTION

$x^2 + 7.6^2 = 10^2$

$x^2 + 57.76 = 100$

$x^2 = 42.24$

$\therefore\ x = \sqrt{42.24}$

$x = 6.499\ldots$

$x = 6.50$ (to 2 d.p.)

EXPLANATION

Write the rule and substitute $c = 10$ and $b = 7.6$.

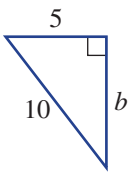
Subtract 57.76 from both sides.

Take the square root of both sides.

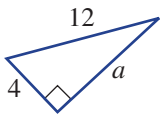
Round to two decimal places.

5 Find the value of the pronumeral. Express your answers correct to two decimal places.

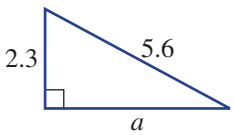
a



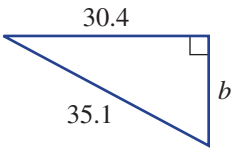
b



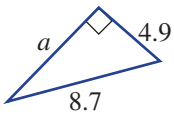
c



d



e



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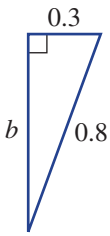
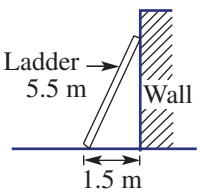


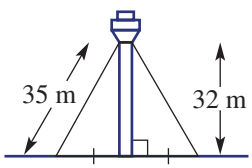
Use your calculator to get your answer, then round to two decimal places.

PROBLEM-SOLVING AND REASONING 6–8 6, 8–10

6 The base of a ladder leaning against a wall is 1.5 m from the base of the wall. The ladder is 5.5 m long. Find how high the top of the ladder is above the ground, correct to one decimal place.



7 A 32 m communication tower is supported by 35 m cables stretching from the top of the tower to a position at ground level. Find the distance from the base of the tower to the point where the cable reaches the ground, correct to one decimal place.



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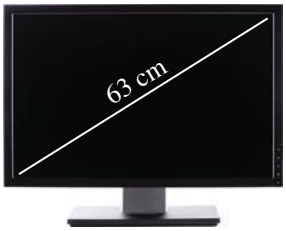
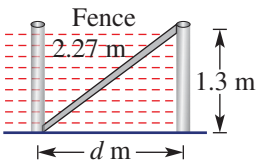
8 If a television has a screen size of 63 cm, it means that the diagonal length of the screen is 63 cm. If the vertical height of a 63 cm screen is 39 cm, find how wide the screen is to the nearest centimetre.

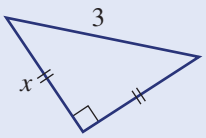


9 A 1.3 m vertical fence post is supported by a 2.27 m bar, as shown in the diagram on the right. Find the distance (d metres) from the base of the post to where the support enters the ground. Give your answer correct to two decimal places.



Example 8 Using exact values

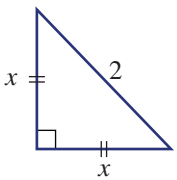
Find the value of x, giving your answer as a surd.



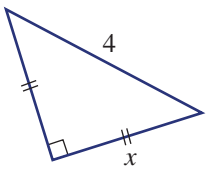
SOLUTION	EXPLANATION
$x^2 + x^2 = 3^2$	Two sides are of length x.
$2x^2 = 9$	Add like terms.
$x^2 = \frac{9}{2}$	Divide both sides by 2.
$\therefore\ x = \sqrt{\frac{9}{2}}$ or $\frac{3}{\sqrt{2}}$	Take the square root of both sides. To express as an exact answer, do not round.

10 Find the value of x as an exact answer.

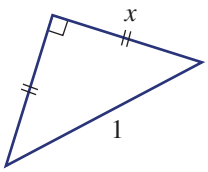
a



b



c

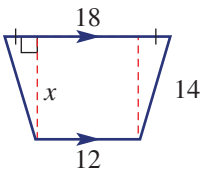


ENRICHMENT – 11

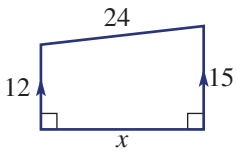
First find a missing length

11 Find the value of x. Give an exact answer each time.

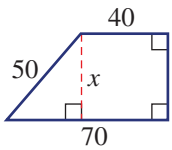
a



b



c



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4D Using Pythagoras' theorem to solve two-dimensional problems REVISION

Interactive
Widgets
HOTsheets
Walkthrough

To apply Pythagoras' theorem to solve a real problem, it can help to first draw a right-angled triangle. This makes it easier to identify the unknown side. As long as two sides of the right-angled triangle are known, the length of the third side can be found.

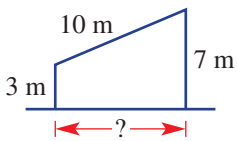
The length of each cable on the Anzac Bridge, Sydney, can be calculated using Pythagoras' theorem.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: But where is the right-angled triangle?

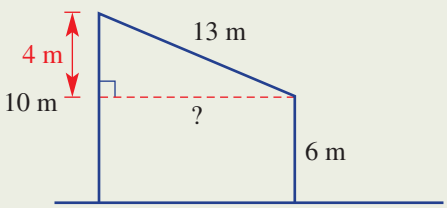
A shed with two walls of length 3 m and 7 m has a sloping 10 m roof. We need to determine the distance between the two walls.

- Can you identify a right-angled triangle?
- What two side lengths on the right triangle do you know?
- Show how Pythagoras' theorem can be used to find the unknown side.
- How would you put the answer in words?



Key ideas

- When applying Pythagoras' theorem:
  - Identify and draw right-angled triangles that may help to solve the problem.
  - Label the sides with their lengths or with a letter (pronumeral) if the length is unknown.
  - Use Pythagoras' theorem to solve for the unknown.
  - Solve the problem by making any further calculations and answering in words.
  - Check that the answer is reasonable (remembering that the hypotenuse is always the longest side).
  - Answer the question in a worded statement.



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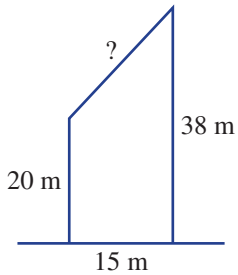
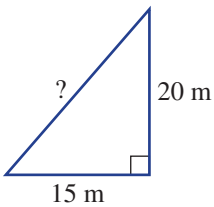
Exercise 4D REVISION

UNDERSTANDING AND FLUENCY | 1–6 | 2–6

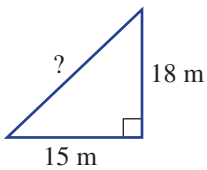
1 Two vertical posts are connected at the top by a wire. Which of the following triangles could be used to help find the cable length?



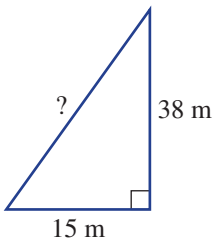
A



B



C



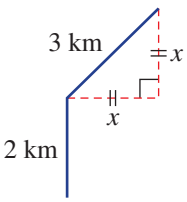
2 Match each problem (**a**, **b** or **c**) with both a diagram (**A**, **B** or **C**) and its solution (**I**, **II**, **III**).

a Two trees stand 20 m apart. They are 32 m and 47 m tall. What is the distance between the tops of the two trees?

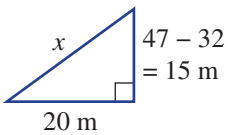
b A kite is flying with a kite string of length 35 m. Its horizontal distance from its anchor point is 17 m. How high is the kite flying?

c A man walks due north for 2 km then north-east for 3 km. How far north is he from his starting point?

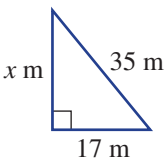
A



B



C



i The kite is flying at a height of 30.59 m.

ii The man has walked a total of 2 + 2.12 = 4.12 km north from his starting point.

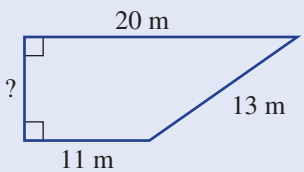
iii The distance between the top of the two trees is 25 m.

Example 9 Identifying right-angled triangles in a diagram

An area of lawn is in the shape of a trapezium, as shown.

a Draw a right-angled triangle to help find the unknown side length.

b Find the unknown side length correct to two decimal places.



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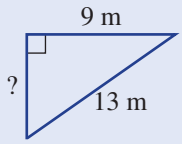
SOLUTION

a



b $a^2 + b^2 = c^2$

$a^2 + 9^2 = 13^2$

$a^2 + 81 = 169$

$a^2 = 88$

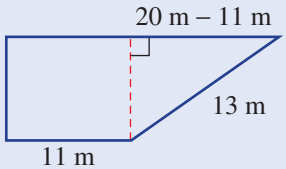
$\therefore a = \sqrt{88}$

$= 9.3808...$

The unknown length is 9.38 m (to 2 d.p.).

EXPLANATION

The top side of the triangle is 20 m − 11 m = 9 m.

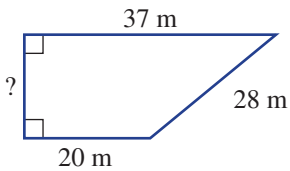


Use Pythagoras' theorem with the hypotenuse as 13 m. Solve for a and round as required.

3 A paved area is in the shape of a trapezium, as shown.

a Draw a right-angled triangle to help find the unknown side length.

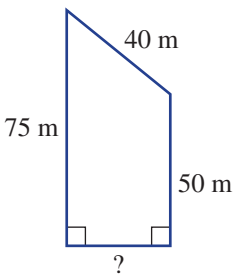
b Find the unknown side length correct to two decimal places.



4 The side of a sculpture is in the shape of a trapezium, as shown.

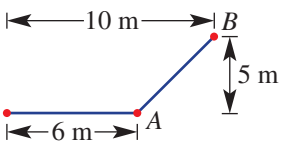
a Draw a right-angled triangle to help find the unknown side length.

b Find the unknown length correct to two decimal places.

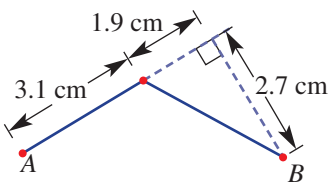


5 Find the direct distance between the points A and B in each of the following, correct to one decimal place.

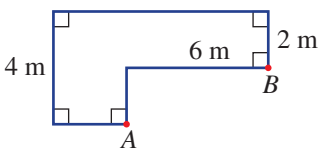
a



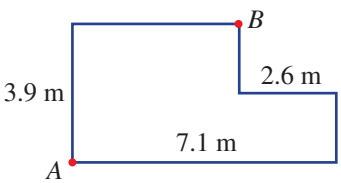
b



c



d



Draw in the right-angled triangle that will help to find the distance AB.

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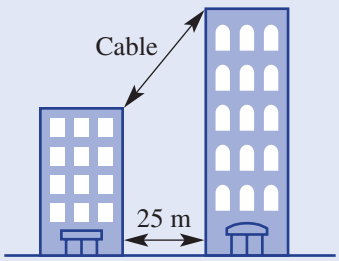
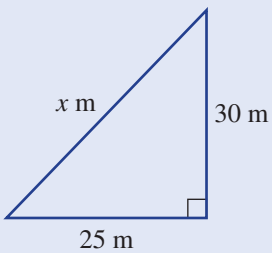
Example 10 Applying Pythagoras' theorem

Two skyscrapers are located 25 m apart and a cable links the tops of the two buildings. Find the length of the cable if the buildings are 50 m and 80 m in height. Give your answer correct to two decimal places.



SOLUTION

Let x m be the length of the cable.



$c^2 = a^2 + b^2$

$x^2 = 25^2 + 30^2$

$x^2 = 625 + 900$

$= 1525$

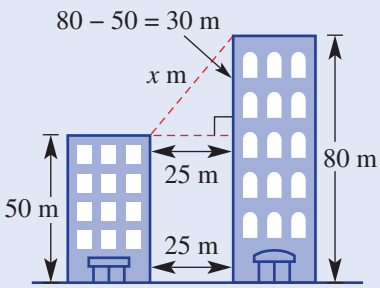
$\therefore\ x = \sqrt{1525}$

$= 39.05\ldots$

$\therefore$ The cable is 39.05 m long (to 2 d.p.).

EXPLANATION

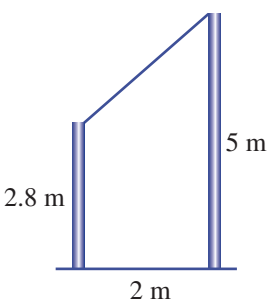
Draw a right-angled triangle and label the measurements and pronumerals.



Set up an equation using Pythagoras' theorem and solve for x.

Answer the question in words.

6 Two poles are located 2 m apart. A wire links the tops of the two poles. Find the length of the wire if the poles are 2.8 m and 5 m in height. Give your answer correct to one decimal place.



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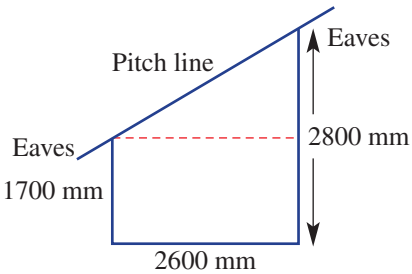
PROBLEM-SOLVING AND REASONING

7, 8	7, 9, 10

7 Two skyscrapers are located 25 m apart and a cable of length 62.3 m links the tops of the two buildings. If the taller building is 200 metres tall, what is the height of the shorter building? Give your answer correct to one decimal place.

First draw a right-angled triangle just like in Example 10. Then find the difference in their heights.

8 A garage is to be built with a skillion roof (a roof with a single slope). The measurements are given in the diagram. Calculate the pitch line length, to the nearest millimetre. Allow 500 mm for each of the eaves.



9 Two bushwalkers are standing on different sides of a mountain. According to their maps, one of them is at a height of 2120 m and the other is at a height of 1650 m. If the horizontal distance between them is 950 m, find the direct distance between the two bushwalkers. Give your answer correct to the nearest metre.

You will need to find the hypotenuse length of a right-angled triangle.

10 A 100 m radio mast is supported by six cables in two sets of three cables. They are anchored to the ground at an equal distance from the mast. The top set of three cables is attached at a point 20 m below the top of the mast. Each cable in the lower set of three cables is 60 m long and is attached at a height of 30 m above the ground. If all the cables have to be replaced, find the total length of cable required. Give your answer correct to two decimal places.

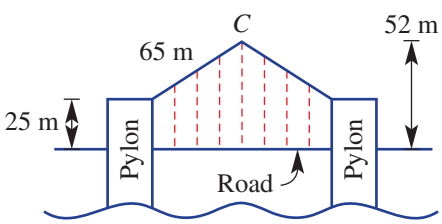
First draw a diagram of the cables as shown in the photo and add all the given information.

ENRICHMENT

–	11

The suspension bridge

11 A suspension bridge is built with two vertical pylons and two straight beams of equal length. They extend from the top of the pylons to meet at a point, C, above the centre of the bridge, as shown in the diagram on the right.



a Calculate the vertical height of the point C above the tops of the pylons.

b Calculate the distance between the pylons; that is, the length of the span of the bridge correct to one decimal place.

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4E Introducing the trigonometric ratios

Interactive
Widgets
HOTsheets
Walkthrough

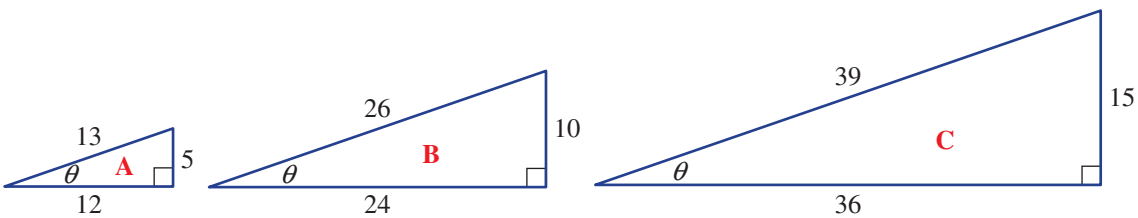
Trigonometry deals with the relationship between the side lengths and angles in triangles. Trigonometry dates back to the ancient Egyptian and Babylonian civilisations when trigonometry was used in the building of pyramids and in the study of astronomy. In the first century AD, Claudius Ptolemy developed tables of values linking the sides and angles of a triangle. The Greek letter theta, θ, is used to represent an unknown angle.

A basic form of trigonometry was used in the building of pyramids in ancient Egypt.

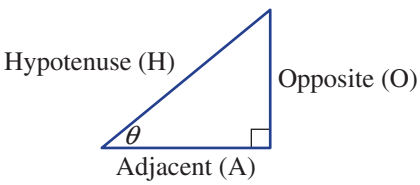
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: What is the sine ratio?

The triangles below are similar. They contain the same angles but triangles B and C are enlargements of triangle A.



Copy and complete the following:



- In triangle A, the ratio $\dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{\square}{\square}$ (in simplest form)
- In triangle B, the ratio $\dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{\square}{\square} = \dfrac{\square}{\square}$ (in simplest form)
- In triangle C, the ratio $\dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{\square}{\square} = \dfrac{\square}{\square}$ (in simplest form)

∴When a right-angled triangle is enlarged, the angles stay the same and the sine ratio stays the same.

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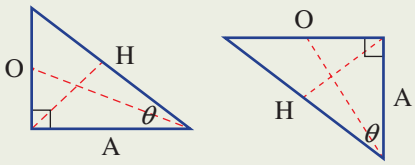
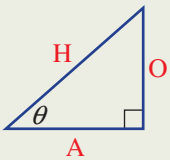
Key ideas

- In this topic the Greek letter θ (pronounced theta) is often used for the size of an unknown angle.
- If a right-angled triangle has another angle θ, then the:
  - longest side is called the hypotenuse (H); it is opposite the right angle
  - side opposite θ is called the **opposite** (O)
  - remaining side is called the **adjacent** (A).



- For a right-angled triangle with a given angle θ, the three ratios **sine** (sin), **cosine** (cos) and **tangent** (tan) are given by the:
  - sine ratio: $\sin\theta = \dfrac{\text{length of the opposite side}}{\text{length of the hypotenuse}}$
  - cosine ratio: $\cos\theta = \dfrac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$
  - tangent ratio: $\tan\theta = \dfrac{\text{length of the opposite side}}{\text{length of the adjacent side}}$
- For any right-angled triangle with the same angles, these ratios are always the same.



- Memory aid: The letters **SOHCAHTOA** are useful when trying to remember the three ratios.

SOH CAH TOA

$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

Opposite (side) In a right-angled triangle, the side opposite a particular angle

Adjacent (side) In a right-angled triangle, the side adjacent to (next to) a particular angle

Sine The ratio of the length of the opposite side to the length of the hypotenuse

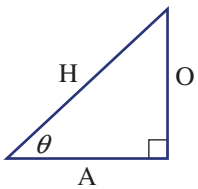
Cosine The ratio of the length of the adjacent side to the length of the hypotenuse

Tangent The ratio of the length of the opposite side to the length of the adjacent side

Exercise 4E

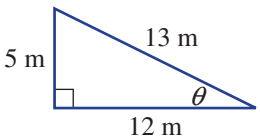
UNDERSTANDING AND FLUENCY	1–3, 4(½), 5, 6	3, 4(½), 5, 6

1 Write the missing word in these sentences.



a H stands for the word _____.
b O stands for the word _____.
c A stands for the word _____.
d $\sin\theta$ = _____ ÷ Hypotenuse.
e $\cos\theta$ = Adjacent ÷ _____.
f $\tan\theta$ = Opposite ÷ _____.

2 For this triangle, what is the length of the:



a hypotenuse?
b side opposite θ?
c adjacent to θ?

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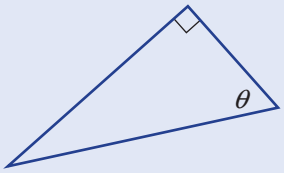
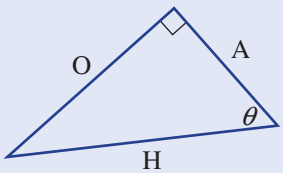
Example 11 Labelling the sides of triangles

Copy this triangle and label the sides as opposite to θ(O), adjacent to θ(A) or hypotenuse (H).

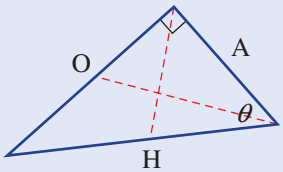


SOLUTION



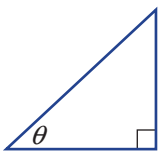
EXPLANATION

Draw the triangle and label the side opposite the right angle as hypotenuse (H), the side opposite the angle θ as opposite (O) and the remaining side next to the angle θ as adjacent (A).

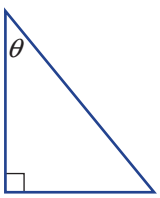


3 Copy each of these triangles and label the sides as opposite to θ(O), adjacent to θ(A) or hypotenuse (H).

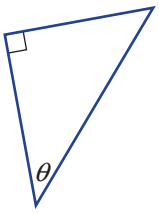
a



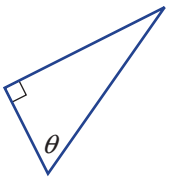
b



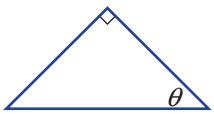
c



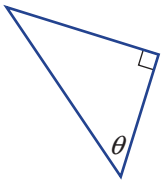
d



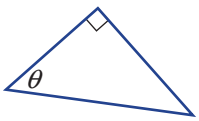
e



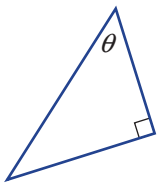
f



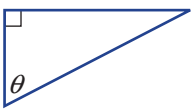
g



h



i



Alternatively, pencil the letters O, A and H onto this page.

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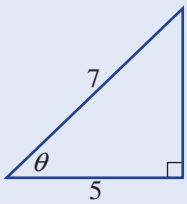
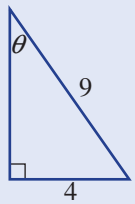
Example 12 Writing trigonometric ratios

Write a trigonometric ratio (in fraction form) for each of the following triangles.

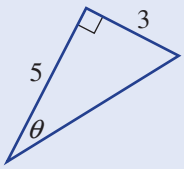
a



b

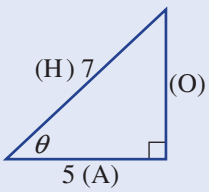


c



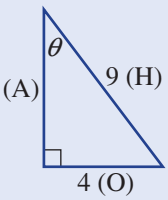
SOLUTION **EXPLANATION**

a $\cos\theta = \frac{A}{H} = \frac{5}{7}$



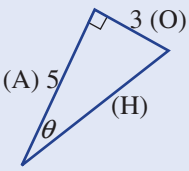
Side length 7 is opposite the right angle so it is the hypotenuse (H). Side length 5 is adjacent to angle θ so it is the adjacent (A).
Use $\cos\theta$ since we have A and H.

b $\sin\theta = \frac{O}{H} = \frac{4}{9}$



Side length 9 is opposite the right angle so it is the hypotenuse (H). Side length 4 is opposite angle θ so it is the opposite (O).
Use $\sin\theta$ since we have O and H.

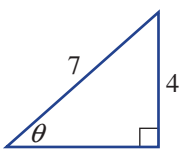
c $\tan\theta = \frac{O}{A} = \frac{3}{5}$



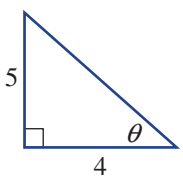
Side length 5 is the adjacent side to angle θ so it is the adjacent (A). Side length 3 is opposite angle θ so it is the opposite (O).
Use $\tan\theta$ since we have O and A.

4 Write a trigonometric ratio (in fraction form) for each of the following triangles and simplify where possible.

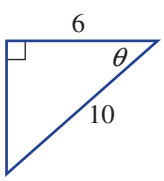
a



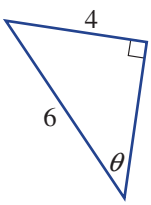
b



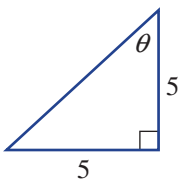
c



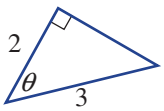
d



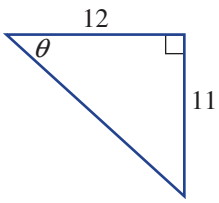
e



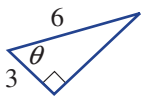
f



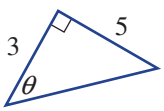
g



h



i



Use $\sin\theta$ if you have O and H, use $\cos\theta$ if you have A and H, and use $\tan\theta$ if you have O and A.

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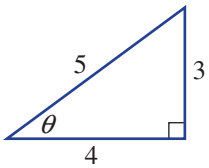
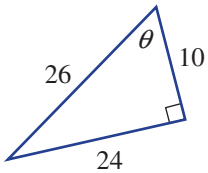
5 For each of these triangles, write a ratio (in simplified fraction form) for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

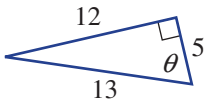
a



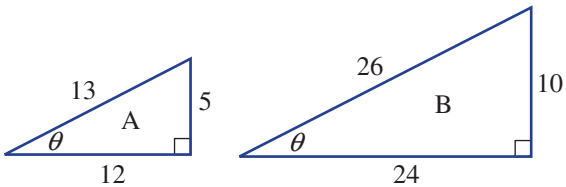
b



c



6 Here are two similar triangles, A and B.



a **i** Write the ratio $\sin\theta$ (as a fraction) for triangle A.
ii Write the ratio $\sin\theta$ (as a fraction) for triangle B.
iii What do you notice about your two answers from parts **a i** and **a ii**?

b **i** Write the ratio $\cos\theta$ (as a fraction) for triangle A.
ii Write the ratio $\cos\theta$ (as a fraction) for triangle B.
iii What do you notice about your two answers from parts **b i** and **b ii**?

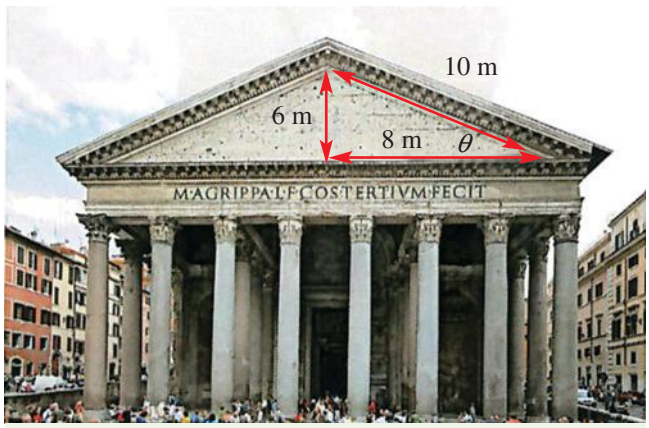
c **i** Write the ratio $\tan\theta$ (as a fraction) for triangle A.
ii Write the ratio $\tan\theta$ (as a fraction) for triangle B.
iii What do you notice about your two answers from parts **c i** and **c ii**?

Simplify your fractions from the larger triangle to help see the connection.

PROBLEM-SOLVING AND REASONING 7–9 | 7, 9–11

7 The facade of a Roman temple has the given measurements. Write down the simplified ratio for:

a $\sin\theta$
b $\cos\theta$
c $\tan\theta$



The Pantheon is a Roman temple that was built in 126 AD.

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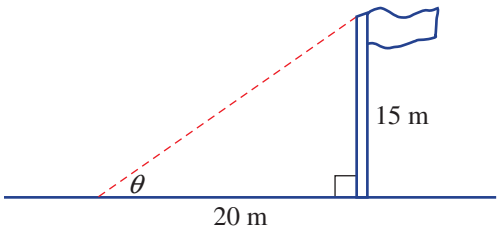
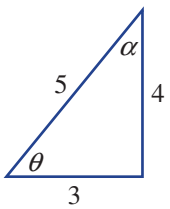
8 A vertical flag pole casts a shadow 20 m long. If the pole is 15 m high, find the ratio for $\tan\theta$.



Decide which side is the opposite to θ and which side is adjacent to θ.

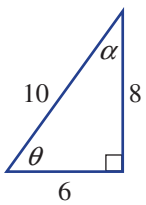
9 For the triangle shown, state the length of the side that corresponds to the:

a hypotenuse
b side opposite angle θ
c side opposite angle α
d side adjacent to angle θ
e side adjacent to angle α.



10 For the triangle shown on the right, write a ratio (in fraction form) for:

a $\sin\theta$
b $\sin\alpha$
c $\cos\theta$
d $\tan\alpha$
e $\cos\alpha$
f $\tan\theta$



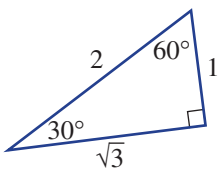
11 **a** Draw a right-angled triangle and mark one of the angles as θ. Mark in the length of the opposite side as 15 units and the length of the hypotenuse as 17 units.
b Using Pythagoras' theorem, find the length of the adjacent side.
c Determine the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

Use $a^2 + b^2 = c^2$ for part **b** with $c = 17$ and $b = 15$.

ENRICHMENT – 12, 13

The complementary connection

12 This triangle has angles 90°, 60° and 30° and side lengths 1, 2 and $\sqrt{3}$.



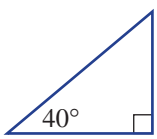
a Write a ratio for:

i $\sin 30°$ **ii** $\cos 30°$ **iii** $\tan 30°$
iv $\sin 60°$ **v** $\cos 60°$ **vi** $\tan 60°$

b What do you notice about the following pairs of ratios?

i $\cos 30°$ and $\sin 60°$
ii $\sin 30°$ and $\cos 60°$

13 **a** Measure all the side lengths of this triangle to the nearest millimetre.



b Use your measurements from part **a** to find an approximate ratio for:

i $\cos 40°$ **ii** $\sin 40°$ **iii** $\tan 40°$
iv $\sin 50°$ **v** $\tan 50°$ **vi** $\cos 50°$

c Do you notice anything about the trigonometric ratios for 40° and 50°?

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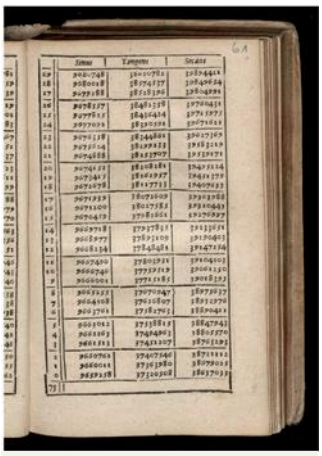
4F Finding unknown sides

Interactive
Widgets
HOTsheets
Walkthrough

Since ancient times, mathematicians have attempted to tabulate the three trigonometric ratios for varying angles. Here are the ratios for some angles in a right-angled triangle, correct to three decimal places.

Angle (θ)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
15°	0.259	0.966	0.268
30°	0.5	0.866	0.577
45°	0.707	0.707	1
60°	0.866	0.5	1.732
75°	0.966	0.259	3.732
90°	1	0	undefined



Trigonometric tables in a 400-year-old European book

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

We can now evaluate these values to a high degree of accuracy using calculators and they can be used to help solve problems involving triangles with unknown side lengths.

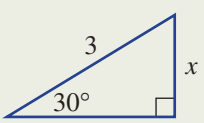
Let's start: Calculator start up

All scientific or CAS calculators can produce accurate values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

- Ensure that your calculator is in degree mode.
- Find the value of the following, correct to three decimal places:
  - i $\sin 50°$
  - ii $\cos 81.5°$
  - iii $\tan 36°15'$
- Use trial and error to find (to the nearest degree) an angle, θ, which satisfies these conditions:
  - i $\sin\theta = 0.454$
  - ii $\cos\theta = 0.588$
  - iii $\tan\theta = 9.514$

Key ideas

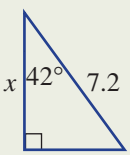
- If θ is in degrees, the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$ can be found accurately using a calculator in degree mode.
- If the angles and one side length of a right-angled triangle are known, then the other side lengths can be found using the $\sin\theta$, $\cos\theta$ or $\tan\theta$ ratios.



$\sin 30° = \dfrac{x}{3}$

$\therefore\ x = 3 \times \sin 30°$

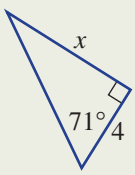
$= 1.5$



$\cos 42° = \dfrac{x}{7.2}$

$\therefore\ x = 7.2 \times \cos 42°$

$= 5.35$ (to 2 d.p.)



$\tan 71° = \dfrac{x}{4}$

$\therefore\ x = 4 \times \tan 71°$

$= 11.62$ (to 2 d.p.)

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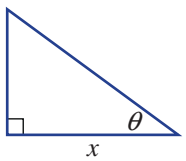
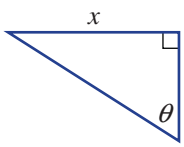
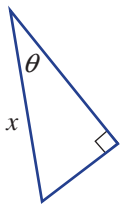
Exercise 4F

UNDERSTANDING AND FLUENCY | 1–3, 4–6(½) | 3, 4–6(½)

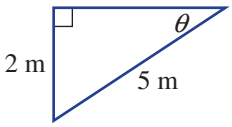
1 For the marked angle θ, decide if x represents the length of the opposite (O), adjacent (A) or hypotenuse (H) side.

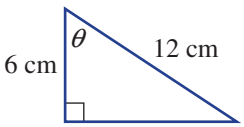
a


b


c


2 Which two sides (choose a pair from O, A and H) are given in these triangles?

a


b


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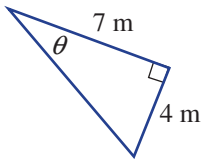
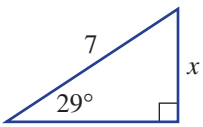
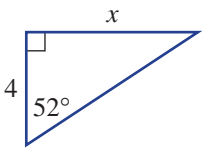


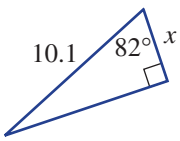
Choose either O, A or O, H or A, H.

3 Decide whether you would use $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$ or $\tan\theta = \frac{O}{A}$ to help find the value of x in these triangles.

Do not find the value of x; just state which ratio would be used.

a


b


c


Example 13 Using a calculator

Use a calculator to evaluate the following, correct to two decimal places.

a $\sin 50°$ **b** $\cos 16.1°$ **c** $\tan 77°18'$

SOLUTION	EXPLANATION
a $\sin 50° = 0.766044...$ $= 0.77$ (to 2 d.p.)	The 3rd decimal place is greater than 4, so round up.
b $\cos 16.1° = 0.9607...$ $= 0.96$ (to 2 d.p.)	The 3rd decimal place is less than 5, so round down.
c $\tan 77°18' = 4.4373...$ $= 4.44$ (to 2 d.p.)	Your calculator has a special button for degrees and minutes. One degree is equal to 60 minutes.

4 Use a calculator to evaluate the following correct to two decimal places.

a $\sin 20°$ **b** $\cos 37°$ **c** $\tan 64°$ **d** $\sin 47°$
e $\cos 84°$ **f** $\tan 14.1°$ **g** $\sin 27°24'$ **h** $\cos 76°12'$
i $\sin 81°$ **j** $\cos 56°10'$ **k** $\tan 34°15'$ **l** $\cos 24°12'$

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Example 14 Solving for x in the numerator of a trigonometric ratio

Find the value of x in the equation $\cos 20° = \frac{x}{3}$, correct to two decimal places.

SOLUTION

$$\cos 20° = \frac{x}{3}$$
$$x = 3 \times \cos 20°$$
$$= 2.8190\ldots$$
$$= 2.82 \text{ (to 2 d.p.)}$$

EXPLANATION

Multiply both sides of the equation by 3 and round as required.

5 Find the value of x correct to two decimal places.

a $\sin 50° = \frac{x}{4}$ **b** $\tan 81° = \frac{x}{3}$ **c** $\cos 33° = \frac{x}{6}$

d $\cos 75° = \frac{x}{3.5}$ **e** $\sin 24° = \frac{x}{4.2}$ **f** $\tan 42° = \frac{x}{10}$

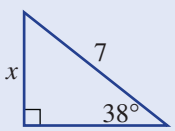
g $\frac{x}{7.1} = \tan 18.4°$ **h** $\frac{x}{5.3} = \sin 64°42'$ **i** $\frac{x}{12.6} = \cos 52°54'$

For part **a**, for example, multiply by 4 and use a calculator to evaluate $4 \times \sin 50°$.

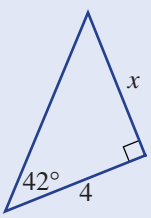
Example 15 Finding side lengths

For each triangle, find the value of x correct to two decimal places.

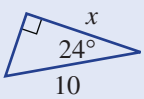
a



b



c



SOLUTION

a $\sin 38° = \frac{O}{H}$

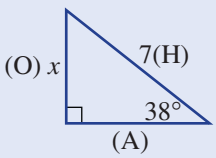
$$\sin 38° = \frac{x}{7}$$
$$x = 7 \sin 38°$$
$$= 4.3096\ldots$$
$$= 4.31 \text{ (to 2 d.p.)}$$

b $\tan 42° = \frac{O}{A}$

$$\tan 42° = \frac{x}{4}$$
$$x = 4 \tan 42°$$
$$= 3.6016\ldots$$
$$= 3.60 \text{ (to 2 d.p.)}$$

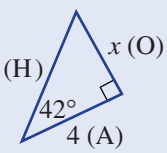
EXPLANATION

Since the opposite side (O) and the hypotenuse (H) are involved, the $\sin \theta$ ratio must be used. Use $\theta = 38°$.



Multiply both sides by 7 and evaluate using a calculator.

Since the opposite side (O) and the adjacent side (A) are involved, the $\tan \theta$ ratio must be used. Use $\theta = 42°$.



Multiply both sides by 4 and evaluate.

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c $\cos 24° = \dfrac{A}{H}$

$\cos 24° = \dfrac{x}{10}$

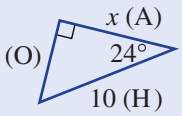
$x = 10 \cos 24°$

$= 9.135\ldots$

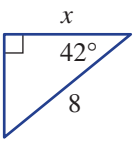
$= 9.14$ (to 2 d.p.)

Since the adjacent side (A) and the hypotenuse (H) are involved, the $\cos\theta$ ratio must be used.

Multiply both sides by 10.



6 Find the value of x correct to two decimal places.

a


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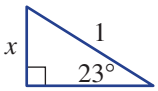

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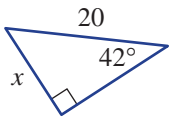

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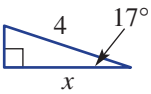

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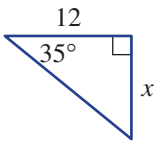

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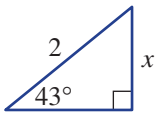

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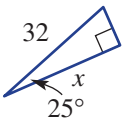

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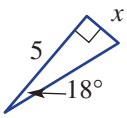

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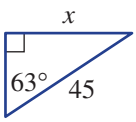

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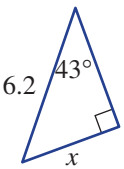

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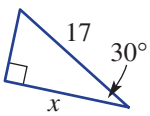

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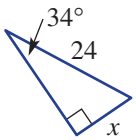

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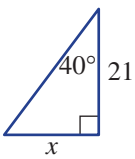

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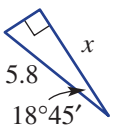

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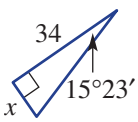
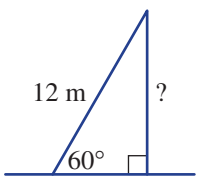


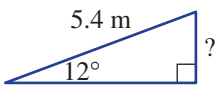
First decide which two sides you have, including the unknown (O, A or H), then choose the correct ratio ($\sin\theta$, $\cos\theta$ or $\tan\theta$).

PROBLEM-SOLVING AND REASONING 7–9 | 7, 9–11

7 A pole is supported by a 12 m cable that makes a 60° angle with the ground. How tall is the pole, correct to two decimal places?



8 Amy walks 5.4 m up a ramp that is inclined at 12° to the horizontal. How high (correct to two decimal places) is she above her starting point?



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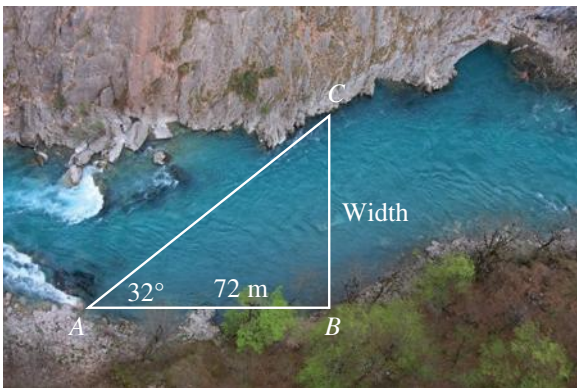
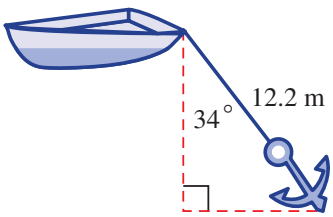
9 Kane wanted to measure the width of a river at a certain point. He placed two markers, A and B, 72 m apart along the bank. C is a point directly opposite marker B. Kane measured angle CAB to be 32°. Find the width of the river correct to two decimal places.



The 'width' is opposite (O) the 32° angle and the 72 m length is adjacent (A) to the 32° angle.

10 One end of a 12.2 m rope is tied to a boat. The other end is tied to an anchor, which is holding the boat steady in the water. If the anchor is making an angle of 34° with the vertical, how deep is the water? Give your answer correct to two decimal places.



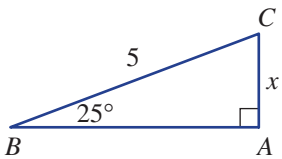
The depth length is adjacent (A) to the 34° angle.

11 For this right-angled triangle:

a find the angle $\angle C$

b calculate the value of x correct to three decimal places using the sine ratio

c calculate the value of x correct to three decimal places using the cosine ratio

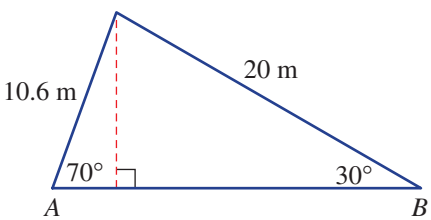


ENRICHMENT – 12

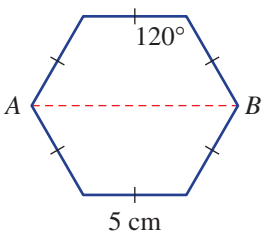
Combining multiple trigonometric calculations

12 Find the length AB in these diagrams. Round to two decimal places where necessary.

a



b



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4G Solving for the denominator

Interactive

Widgets

HOTsheets

Walkthrough

So far we have constructed trigonometric ratios using a pronumeral that has always appeared in the numerator. For example: $\sin 40° = \frac{x}{5}$

This makes it easy to solve for x where both sides of the equation can be multiplied by 5. If, however, the pronumeral appears in the denominator, there are a number of algebraic steps that can be taken to find the solution.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Solution steps

Three students attempt to solve $\sin 40° = \frac{5}{x}$ for x.

Nick says $x = 5 \times \sin 40°$.

Sharee says $x = \frac{5}{\sin 40°}$

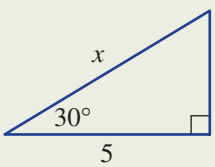
Dori says $x = \frac{1}{5} \times \sin 40°$.

- Which student has the correct solution?
- Can you show the algebraic steps that support the correct answer?

Key ideas

- If the unknown value of a trigonometric ratio is in the **denominator**, you need to rearrange the equation to make the pronumeral the subject.
- For example: for the triangle shown:
  - multiply both sides by x (removes the fraction) $\quad \cos 30° = \frac{5}{x}$
  - divide both sides by $\cos 30°$ ($\cos 30°$ is just a number). $\quad x \times \cos 30° = 5$, $\quad x = \frac{5}{\cos 30°}$

Denominator
The bottom part of a fraction

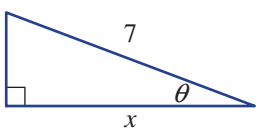


Exercise 4G

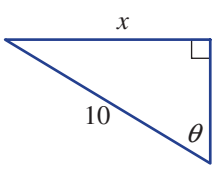
UNDERSTANDING AND FLUENCY | 1–3, 4–5(½) | 3, 4–5(½)

1 Which trigonometric ratio ($\sin\theta$, $\cos\theta$ or $\tan\theta$) would be used to find the value of x in these triangles?

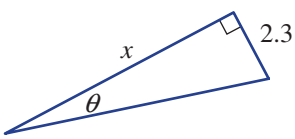
a



b



c



2 Solve these simple equations for x.

a $\frac{4}{x} = 2$ **b** $\frac{20}{x} = 4$ **c** $\frac{15}{x} = 5$ **d** $25 = \frac{100}{x}$

For part **a**, first multiply by x then divide by 2.

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3 Which of these equations has x in the denominator of the fraction?

a $\frac{x}{2} = \sin 40°$ **b** $\frac{3}{x} = \cos 70°$ **c** $\frac{7.1}{x} = \tan 34°$ **d** $\cos 47° = \frac{x}{5}$

Example 16 Solving for x in the denominator

Solve for x in the equation $\cos 35° = \frac{2}{x}$, correct to two decimal places.

SOLUTION	EXPLANATION
$\cos 35° = \frac{2}{x}$	Multiply both sides of the equation by x.
$x \cos 35° = 2$	Divide both sides of the equation by $\cos 35°$.
$x = \frac{2}{\cos 35°}$	
$= 2.4415\ldots$	Evaluate and round off to two decimal places.
$= 2.44$ (to 2 d.p.)	

4 Find the value of x correct to two decimal places.

a $\cos 43° = \frac{3}{x}$ **b** $\sin 36° = \frac{4}{x}$ **c** $\tan 9° = \frac{6}{x}$

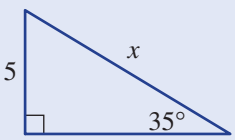
d $\tan 64° = \frac{2}{x}$ **e** $\cos 67° = \frac{5}{x}$ **f** $\sin 12° = \frac{3}{x}$

g $\sin 38.3° = \frac{5.9}{x}$ **h** $\frac{45}{x} = \tan 21°24'$ **i** $\frac{18.7}{x} = \cos 32°15'$

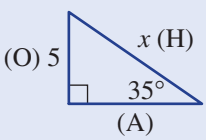
First multiply both sides by x.

Example 17 Finding side lengths

Find the value of x correct to two decimal places.



SOLUTION	EXPLANATION
$\sin 35° = \frac{O}{H}$	Since the opposite side (O) is given and we require the hypotenuse (H), use $\sin \theta$.
$\sin 35° = \frac{5}{x}$	
$x \sin 35° = 5$	Multiply both sides of the equation by x then divide both sides of the equation by $\sin 35°$.
$x = \frac{5}{\sin 35°}$	
$= 8.717\ldots$	Evaluate on a calculator and round off to two decimal places.
$= 8.72$ (to 2 d.p.)	



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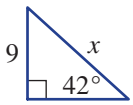
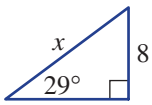
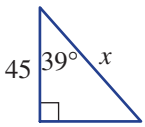
5 Find the value of x correct to decimal places, using the sine, cosine or tangent ratios.

a


b


c


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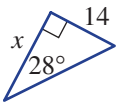

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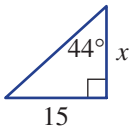

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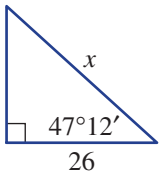
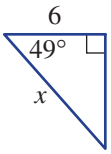


Are you going to use $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$ or $\tan\theta = \frac{O}{A}$?

g


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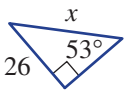

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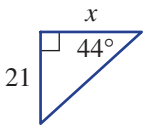

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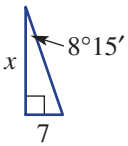

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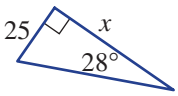

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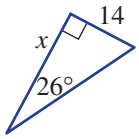
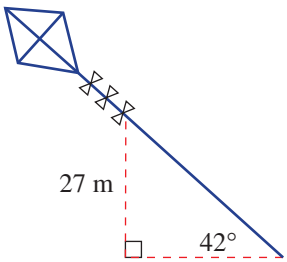


PROBLEM SOLVING AND REASONING

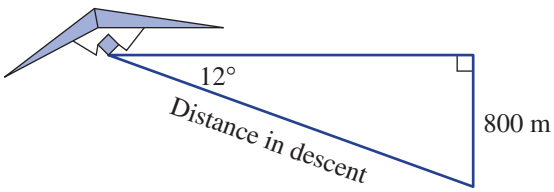
	6–8	6, 8–10

6 A kite is flying at a height of 27 m above the anchor point. If the string is inclined at 42° to the horizontal, find the length of the string correct to the nearest metre.



First decide if you are going to use $\sin\theta$ (SOH), $\cos\theta$ (CAH) or $\tan\theta$ (TOA).

7 A hang glider flying at a height of 800 m descends at an angle of 12° to the horizontal. How far (to the nearest metre) has it travelled in descending to the ground?



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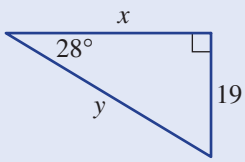
Example 18 Finding two values on one triangle

Find the value of x and y correct to two decimal places.



SOLUTION

$$\tan 28° = \frac{O}{A}$$

$$\tan 28° = \frac{19}{x}$$

$$x \tan 28° = 19$$

$$x = \frac{19}{\tan 28°}$$

$$= 35.73 \text{ (to 2 d.p.)}$$

$$\sin 28° = \frac{19}{y}$$

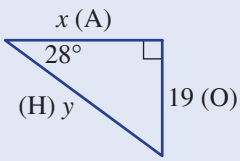
$$y \sin 28° = 19$$

$$y = \frac{19}{\sin 28°}$$

$$= 40.47 \text{ (to 2 d.p.)}$$

EXPLANATION

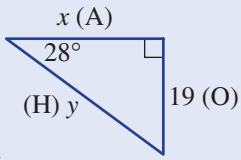
Since the opposite side (O) is given and the adjacent (A) is required, use $\tan\theta$.



Multiply both sides of the equation by x.

Divide both sides of the equation by $\tan 28°$ and round the answer to two decimal places.

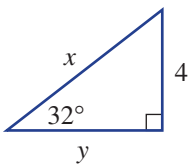
Use the opposite (19) as it is an exact length, so use $\sin\theta = \frac{O}{H}$.



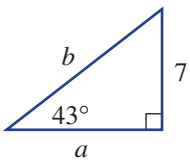
Alternatively, y can be found by Pythagoras' theorem.

8 Find the value of each pronumeral correct to one decimal place.

a



b



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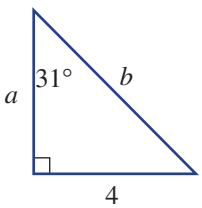

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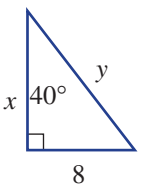

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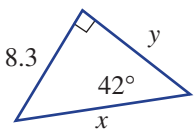

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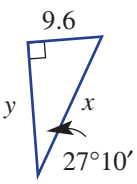

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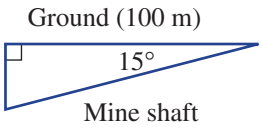
9 A mine shaft is dug at an angle of 15° to the horizontal. The horizontal length of the mine is 100 m. Answer the questions to the nearest metre.



a How far below ground level is the end of the shaft?

b How long is the mine shaft?

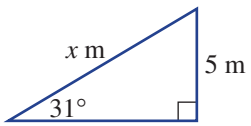
10 In calculating the value of x for the triangle below, correct to two decimal places, two students come up with these answers.

Student A: $x = \dfrac{5}{\sin 31°}$

$= \dfrac{5}{0.52} = 9.62$

Student B: $x = \dfrac{5}{\sin 31°}$

$= 9.71$



a Which of the two answers is more correct and why?

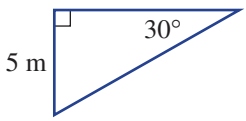
b What advice would you give to the student whose answer is not accurate?

ENRICHMENT – 11

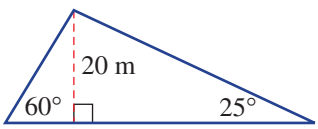
Trigonometry and perimeters

11 Find the perimeter of these triangles, correct to one decimal place. You will first need to find the lengths of all the sides.

a



b



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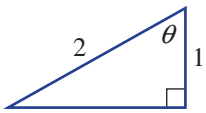
4H Finding unknown angles

Interactive
Widgets
HOTsheets
Walkthrough

Given two sides of a right-angled triangle, we can use trigonometry to calculate one of the angles.

By using one of the three trigonometric ratios, we can use inverse trigonometric functions to find a value of θ.

For example, if $\cos\theta = 0.5$, then $\theta = \cos^{-1}(0.5) = 60°$.



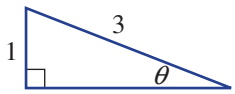
Learn how to do this on your calculator.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Trial and error can be slow

We know that for this triangle, $\sin\theta = \frac{1}{3}$.



- Guess the angle θ.
- For your guess, use a calculator to see if $\sin\theta = \frac{1}{3} (= 0.333\ldots)$ for your chosen value of θ.
- Update your guess and use your calculator to check once again.
- Repeat this trial-and-error process until you think you have the angle θ correct to three decimal places.
- Now use your calculator to evaluate $\sin^{-1}\left(\frac{1}{3}\right)$ and check your guess.

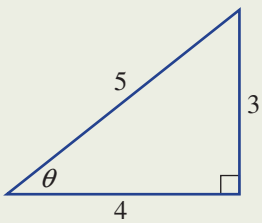
Key ideas

- Your calculator has functions for solving an equation such as $\sin\theta = \frac{3}{5}$.

  'sin⁻¹' is written above the 'sin' button.

  You must learn how to use this function.

- Use your calculator to check the following solutions:
  - $\sin\theta = \frac{3}{5}$ means $\theta = 37°$ (to nearest degree)
  - $\cos\theta = \frac{4}{5}$ means $\theta = 37°$ (to nearest degree)
  - $\tan\theta = \frac{3}{4}$ means $\theta = 37°$ (to nearest degree)



- When finding an unknown angle, you can express your answer in many different ways.

$\sin\theta = \frac{3}{5}$ or $\sin\theta = \frac{3}{5}$ or $\sin\theta = \frac{3}{5}$

$\theta = 37°$ (nearest degree) $\theta = 36.9°$ (1 d.p.) $\theta = 37°52'$ (nearest minute)

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Exercise 4H

UNDERSTANDING AND FLUENCY 1–3, 4–6(½) 3, 4–6(½)

1 Use a calculator to evaluate the following, rounding to two decimal places.

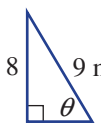
a $\sin^{-1}(0.2)$ **b** $\sin^{-1}(0.9)$ **c** $\cos^{-1}(0.75)$
d $\cos^{-1}(0.43)$ **e** $\tan^{-1}(0.5)$ **f** $\tan^{-1}(2.5)$

2 Evaluate each of the following to the nearest degree.

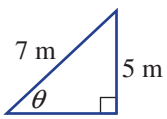
a $\sin^{-1}(0.7324)$ **b** $\cos^{-1}(0.9763)$ **c** $\tan^{-1}(0.3321)$
d $\tan^{-1}(1.235)$ **e** $\sin^{-1}(0.4126)$ **f** $\cos^{-1}(0.7462)$
g $\cos^{-1}(0.1971)$ **h** $\sin^{-1}(0.2247)$ **i** $\tan^{-1}(0.0541)$

3 Which trigonometric ratio should be used to solve for θ?

a



b



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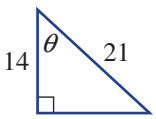

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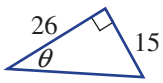

Choose $\sin\theta$ if using O and H, $\cos\theta$ if using A and H, and $\tan\theta$ if using O and A.

Example 19 Using inverse trigonometric ratios

Find the value of θ to the nearest degree if $\sin\theta = 0.3907$.

SOLUTION

$\sin\theta = 0.3907$
$\theta = 22.998\ldots$
$\theta = 23°$ (nearest degree)

EXPLANATION

Use the $\sin^{-1}$ key on your calculator.
Round off to the nearest whole number.

4 Find the value of θ to the nearest degree.

a $\sin\theta = 0.5$ **b** $\cos\theta = 0.5$ **c** $\tan\theta = 1$
d $\cos\theta = 0.8660$ **e** $\sin\theta = 0.7071$ **f** $\tan\theta = 0.5774$
g $\sin\theta = 1$ **h** $\tan\theta = 1.192$ **i** $\cos\theta = 0$
j $\cos\theta = 0.5736$ **k** $\cos\theta = 1$ **l** $\sin\theta = 0.9397$

Use the $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ functions on your calculator.

Example 20 Using inverse trigonometric ratios with fractions

Find the value of θ correct to two decimal places if $\tan\theta = \frac{1}{2}$.

SOLUTION

$\tan\theta = \frac{1}{2}$
$\theta = 26.56505\ldots$
$\theta = 26.57°$ (to 2 d.p.)

EXPLANATION

Use the $\tan^{-1}$ key on your calculator (i.e. $\tan^{-1}\left(\frac{1}{2}\right)$) and round the answer to two decimal places.

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5 Find the angle θ correct to two decimal places.

a $\sin\theta = \frac{4}{7}$ **b** $\sin\theta = \frac{1}{3}$ **c** $\sin\theta = \frac{9}{10}$

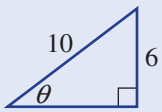
d $\cos\theta = \frac{1}{4}$ **e** $\cos\theta = \frac{4}{5}$ **f** $\cos\theta = \frac{7}{9}$

g $\tan\theta = \frac{3}{5}$ **h** $\tan\theta = \frac{8}{5}$ **i** $\tan\theta = 12$

For part **a**, type $\sin^{-1}\left(\frac{4}{7}\right)$ on your calculator.

Example 21 Finding an angle

Find the value of θ to the nearest degree.



SOLUTION

$\sin\theta = \frac{O}{H}$

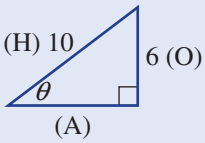
$\sin\theta = \frac{6}{10}$

$\theta = 36.869\ldots$

$\theta = 37°$ (nearest degree)

EXPLANATION

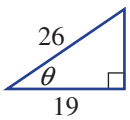
Since the opposite side (O) and the hypotenuse (H) are given, use $\sin\theta$.



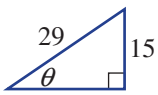
Use the $\sin^{-1}$ key on your calculator and round as required.

6 Find the value of θ to the nearest degree.

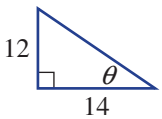
a



b



c



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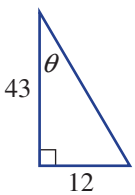

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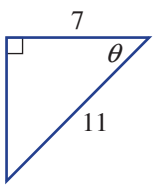

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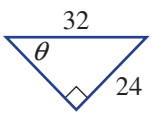

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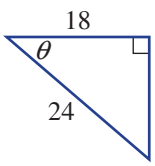

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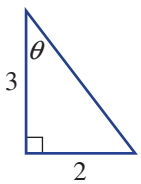

First set up an equation (like those in Question **5**), then use a calculator to find θ.

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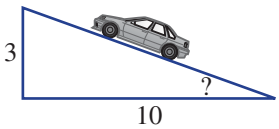
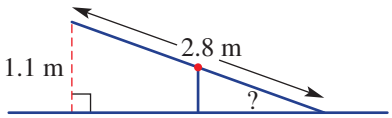
PROBLEM SOLVING AND REASONING

7–9	7, 9–12

7 A road rises at a grade of 3 in 10. Find the angle (to the nearest degree) the road makes with the horizontal.



8 When a 2.8 m long seesaw is at its maximum height, it is 1.1 m off the ground. What angle (correct to two decimal places) does the seesaw make with the ground?

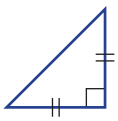


The 1.1 m length is opposite the angle and the 2.8 m length forms the hypotenuse.

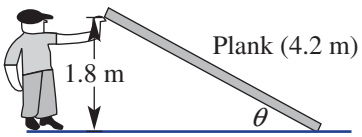
9 Consider this triangle.

a Calculate tan 45° on your calculator.

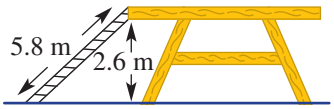
b Explain why tan 45° is exactly 1.



10 Adam holds one end of a plank of wood 1.8 m above the ground. The other end of the plank rests on the ground. The plank of wood is 4.2 m long. Find the angle that the plank makes with the ground (θ), correct to one decimal place.



11 A ladder has a length of 5.8 m. The ladder is resting against the top of a platform that is 2.6 m high. Find the angle the ladder makes with the ground, correct to one decimal place.



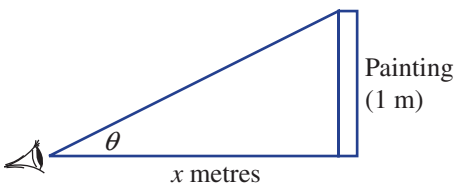
12 For what value of θ is $\sin\theta = \cos\theta$? Choose a value of θ between 0° and 90°.

ENRICHMENT

–	13

Viewing angle

13 Joe has trouble with his eyesight but every Sunday he goes to view his favourite painting at the gallery. His eye level is at the same level as the base of the painting and the painting is 1 metre tall.



Answer the following to the nearest degree for angles and to two decimal places for lengths.

a If $x = 3$, find the viewing angle θ.

b If $x = 2$, find the viewing angle θ.

c If Joe can stand no closer than 1 metre to the painting, what is Joe's largest viewing angle?

d When the viewing angle is 10°, Joe has trouble seeing the painting. How far is he from the painting at this viewing angle?

e What would be the largest viewing angle if Joe could go as close as he would like to the painting?

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4I Using trigonometry to solve problems

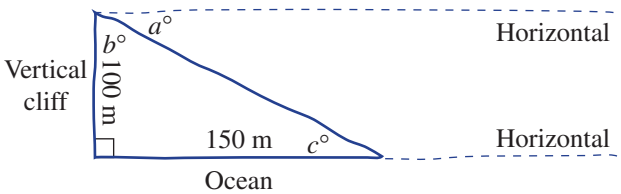
Interactive
Widgets
HOTsheets
Walkthrough

In many practical maths problems, trigonometry can be used to find unknown lengths and angles. Two special angles are called the angle of elevation and the angle of depression. These angles are measured from the horizontal.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Draw and measure

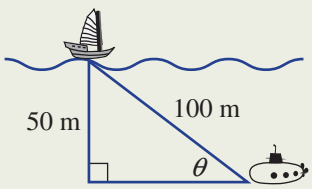
Use a scale of 1 mm represents 1 metre to draw a neat, accurate scale drawing of this diagram.



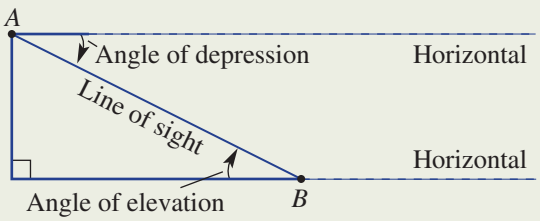
Use a protractor to estimate the values of a, b and c. Did a and c have the same value? Did you have the same values for a, b and c as other people in your class?

Key ideas

- To solve application problems involving trigonometry:
  - Draw a diagram and label the key information (e.g. a problem involving a submarine and a yacht).
  - Identify and draw the appropriate right-angled triangles separately.
  - Solve using trigonometry to find the missing measurements.
  - Check that your answer is reasonable.
  - Express your answer in words.



- **Angles of elevation** and **depression** are always measured from the horizontal.



Angle of elevation The angle of your line of sight from the horizontal when looking up at an object

Angle of depression The angle of your line of sight from the horizontal when looking down at an object

- The horizontal lines are parallel. Therefore, the angle of depression and the angle of elevation are equal alternate angles.

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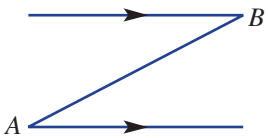
Exercise 4I

UNDERSTANDING AND FLUENCY	1–10	4–10

1 Draw this diagram and complete these tasks.



a Mark in the:

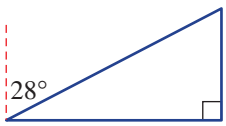
i angle of elevation (θ) of B from A

ii angle of depression (α) of A from B

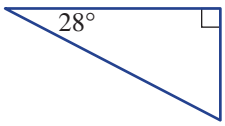
b Is $\theta = \alpha$ in your diagram? Why?

2 Choose the diagram (**A**, **B** or **C**) that matches this situation. A boy views a kite at an angle of elevation of 28°.

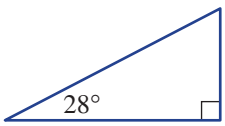
A



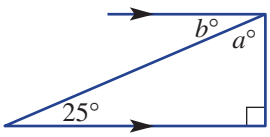
B



C



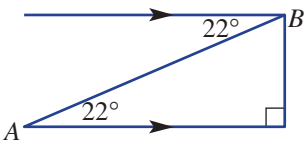
3 Find the values of the pronumerals in this diagram.



4 For this diagram what is the:

a angle of elevation of B from A?

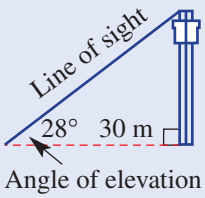
b angle of depression of A from B?



Why do you think the two angles are equal in this diagram?

Example 22 Using angles of elevation

The angle of elevation of the top of a tower from a point on the ground 30 m away from the base of the tower is 28°. Find the height of the tower to the nearest metre.



SOLUTION

Let the height of the tower be h m.

$$\tan 28° = \frac{\text{O}}{\text{A}}$$

$$= \frac{h}{30}$$

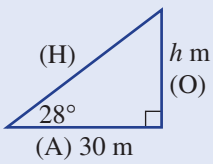
$$h = 30\tan 28°$$

$$= 15.95\ldots$$

The height is 16 m, to the nearest metre.

EXPLANATION

Since the opposite side (O) is required and the adjacent (A) is given, use $\tan\theta$.



Multiply both sides by 30 and evaluate, rounding to the nearest metre.

Write the answer in words.

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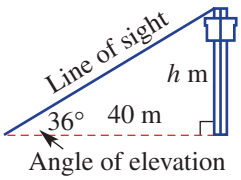
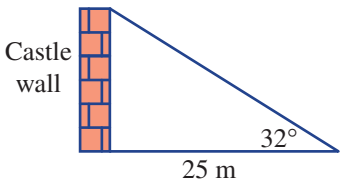
5 The angle of elevation of the top of a tower from a point on the ground 40 m from the base of the tower is 36°. Find the height of the tower to the nearest metre.



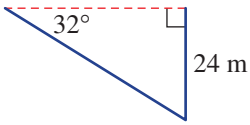
First draw a simple triangle and put on your side lengths and angle.

6 The angle of elevation of the top of a castle wall from a point on the ground 25 m from the base of the castle wall is 32°. Find the height of the castle wall to the nearest metre.



7 From a point on the ground, Emma measures the angle of elevation of an 80 m tower to be 27°. Find how far Emma is from the base of the tower, correct to the nearest metre.

8 From a pedestrian overpass, Chris spots a landmark at an angle of depression of 32°. How far away (to the nearest metre) is the landmark from the base of the 24 m high overpass?

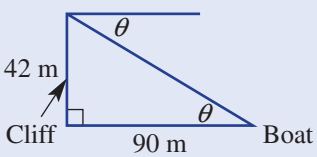


9 From a lookout tower, David spots a bushfire at an angle of depression of 25°. If the lookout tower is 42 m high, how far away (to the nearest metre) is the bushfire from the base of the tower?

Example 23 Finding an angle of depression

From the top of a vertical cliff, Andrea spots a boat out at sea. If the top of the cliff is 42 m above sea level and the boat is 90 m away from the base of the cliff, find Andrea's angle of depression to the boat to the nearest degree.

SOLUTION



$\tan\theta = \dfrac{O}{A}$

$\tan\theta = \dfrac{42}{90}$

$\theta = 25.0168\ldots$

The angle of depression is 25°, to the nearest degree.

EXPLANATION

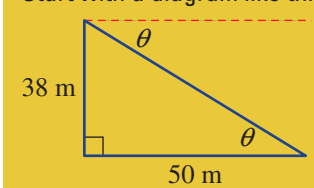
Draw a diagram and label all the given measurements.
Use alternate angles in parallel lines to mark θ inside the triangle.

Since the opposite (O) and adjacent sides (A) are given, use $\tan\theta$.

Use the $\tan^{-1}$ key on your calculator.
Round off to the nearest degree and express the answer in words.

10 From the top of a vertical cliff, Josh spots a swimmer out at sea. If the top of the cliff is 38 m above sea level and the swimmer is 50 m away from the base of the cliff, find the angle of depression from Josh to the swimmer, to the nearest degree.

Start with a diagram like this.



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PROBLEM SOLVING AND REASONING

11, 12	11, 13, 14

11 From a ship, a person is spotted floating in the sea 200 m away. If the viewing position on the ship is 20 m above sea level, find the angle of depression from the ship to the person in the sea. Give your answer to the nearest degree.

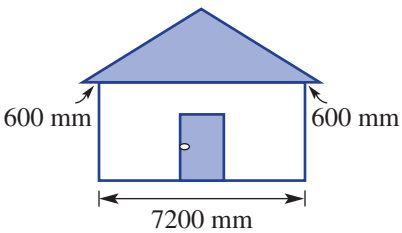
12 A power line is stretched from a pole to the top of a house. The house is 4.1 m high and the power pole is 6.2 m high. The horizontal distance between the house and the power pole is 12 m. Find the angle of elevation of the top of the power pole from the top of the house, to the nearest degree.

13 A road has a steady gradient of 1 in 10. It rises 1 m for every 10 m across.

a What angle does the road make with the horizontal? Give your answer to the nearest degree.

b A car starts from the bottom of the inclined road and drives 2 km along the road. How high, vertically, has the car climbed? Use your rounded answer from part **a** and give your answer correct to the nearest metre.

14 A house is to be built using the design shown on the right. The eaves are 600 mm and the house is 7200 mm wide, excluding the eaves. Calculate the length (to the nearest mm) of a sloping edge of the roof, which is pitched at 25° to the horizontal.

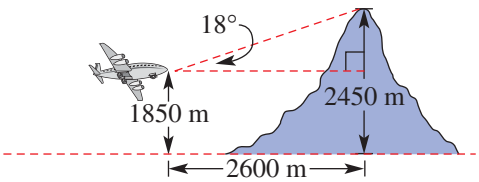


ENRICHMENT

–	15, 16

Plane trigonometry

15 A plane flying at 1850 m starts to climb at an angle of 18° to the horizontal when the pilot sees a mountain peak 2450 m high, 2600 m away from him in a horizontal direction. Will the pilot clear the mountain?



16 An aeroplane takes off and climbs at an angle of 20° to the horizontal, at 190 km/h along its flight path for 15 minutes.

a Find the:

i distance the aeroplane travels in 15 minutes

ii height the aeroplane reaches after 15 minutes, correct to two decimal places

b If the angle at which the plane climbs is twice the original angle but its speed is halved, will it reach a greater height after 15 minutes?

c If the plane's speed is doubled and its climbing angle is halved, will the plane reach a greater height after 15 minutes?

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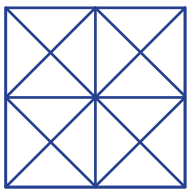
Puzzles and challenges

1 How many different right-angled triangles are there in this diagram?

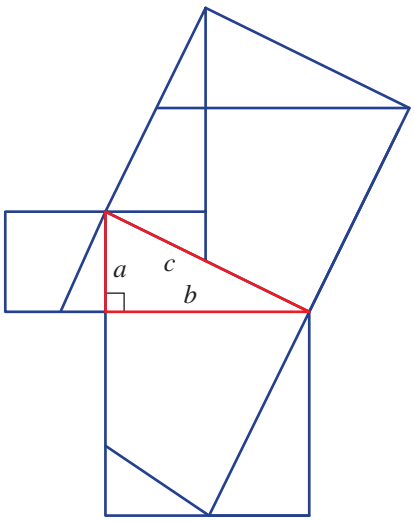


2 Look at this right-angled triangle and the squares drawn on each side. Each square is divided into smaller sections.

a Can you see how the parts of the two smaller squares would fit into the larger square?

b What is the area of each square if the side lengths of the right-angled triangle are a, b and c as marked?

c What do the answers to the above two questions suggest about the relationship between a, b and c?

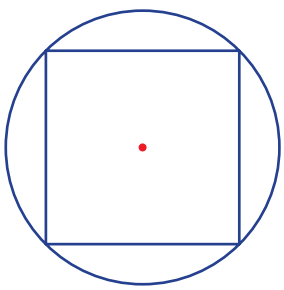


3 Imagine trying to cut the largest square from a circle of a certain size and calculating the side length of the square.

a If the circle has a diameter of 2 cm, can you find a good position to draw the diameter that also helps to form a right-angled triangle?

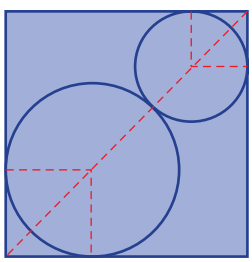
b Can you determine the side length of the largest square if the circle has a diameter of 2 cm?

c What percentage of the area of a circle does the largest square occupy?



4 Which is a better fit: a square peg in a round hole or a round peg in a square hole? Use area calculations and percentages to investigate.

5 Two circles of radius 10 cm and 15 cm respectively are placed inside a square. Find the perimeter of the square to the nearest centimetre. Hint: first find the diagonal length of the square using the diagram on the right.



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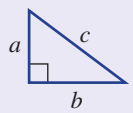
Chapter summary

Right-angled triangles

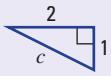
Pythagoras' theorem



$c^2 = a^2 + b^2$

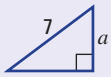
Finding the hypotenuse

$c^2 = 2^2 + 1^2$
$= 5$
$c = \sqrt{5}$
$= 2.24$ (to 2 d.p.)

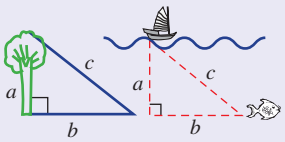


Shorter sides

$a^2 + 5^2 = 7^2$
$a^2 = 7^2 - 5^2$
$= 24$
$\therefore a = \sqrt{24}$ (exact value)



Applications

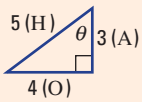


SOHCAHTOA

$\sin\theta = \frac{O}{H} = \frac{4}{5}$

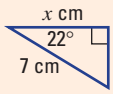
$\cos\theta = \frac{A}{H} = \frac{3}{5}$

$\tan\theta = \frac{O}{A} = \frac{4}{3}$



Finding side lengths

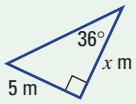
(x in the numerator)



$\cos 22° = \frac{x}{7}$
$x = 7 \times \cos 22°$
$= 6.49$ (2 d.p.)

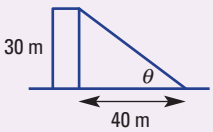
Finding side lengths

(x in the denominator)



$\tan 36° = \frac{5}{x}$
$x \times \tan 36° = 5$
$x = \frac{5}{\tan 36°}$
$= 6.88$ (2 d.p.)

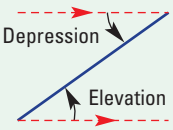
Applying trigonometry



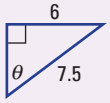
$\tan\theta = \frac{30}{40}$

$\theta = 36.87°$ (2 d.p.)

Elevation and depression



Finding angles



$\sin\theta = \frac{6}{7.5}$

$\theta = 53°$ (nearest degree)

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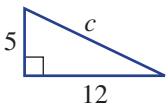
Chapter review

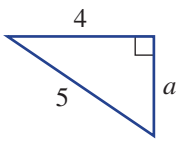
Multiple-choice questions

1 For the right-angled triangle shown, which is a correct statement?



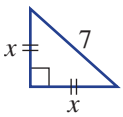
A $c^2 = 5^2 + 12^2$ **B** $c^2 = 5^2 - 12^2$ **C** $c^2 = 12^2 - 5^2$
D $c^2 = 5^2 \times 12^2$ **E** $(5 + 12)^2$

2 Which of the following is true for this triangle?



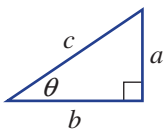
A $a = 5 - 4$ **B** $a^2 = 5^2 - 4^2$ **C** $a^2 = 5^2 + 4^2$
D $a = \sqrt{10}$ **E** $a^2 - 4^2 = 5^2$

3 For the right-angled triangle shown:



A $2x^2 = 49$ **B** $7x^2 = 2$ **C** $2x^2 = 7$
D $x^2 + 7^2 = x^2$ **E** $x^2 = \frac{2}{7}$

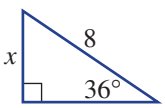
4 For the triangle shown:



A $\sin\theta = \frac{a}{b}$ **B** $\sin\theta = \frac{c}{a}$ **C** $\sin\theta = \frac{a}{c}$
D $\sin\theta = \frac{b}{c}$ **E** $\sin\theta = \frac{c}{b}$

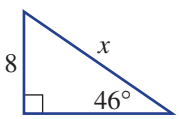
5 The value of cos 46°, correct to four decimal places, is:
A 0.7193 **B** 0.6947 **C** 0.594
D 0.6532 **E** 1.0355

6 In the diagram, the value of x, correct to two decimal places, is:



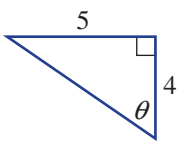
A 40 **B** 13.61 **C** 4.70
D 9.89 **E** 6.47

7 The value of x in the triangle is given by:



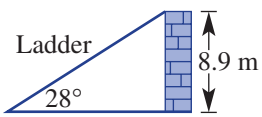
A $8\sin 46°$ **B** $8\cos 46°$ **C** $\frac{8}{\cos 46°}$
D $\frac{8}{\sin 46°}$ **E** $\frac{\cos 46°}{8}$

8 Which equation could be used to find the value of θ?



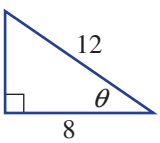
A $\theta = \tan\left(\frac{5}{4}\right)$ **B** $\theta = \tan\left(\frac{4}{5}\right)$ **C** $\tan\theta = \frac{5}{4}$
D $\sin\theta = \frac{4}{5}$ **E** $\tan\theta = \frac{4}{5}$

9 A ladder is inclined at an angle of 28° to the horizontal. If the ladder reaches 8.9 m up the wall, the length of the ladder correct to the nearest metre is:



A 19 m **B** 4 m **C** 2 m
D 10 m **E** 24 m

10 The value of θ in the diagram, correct to two decimal places, is:



A 0.73° **B** 48.19° **C** 41.81°
D 33.69° **E** 4.181°

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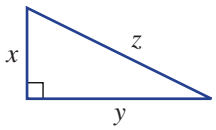
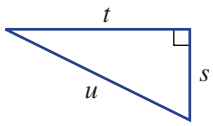
Short-answer questions

1 Write down Pythagoras' theorem for these triangles, using the given pronumerals.

a



b



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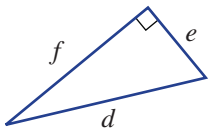
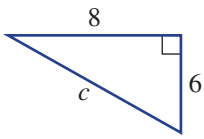


2 Find the length of the hypotenuse in these triangles. Round to two decimal places where appropriate.

a



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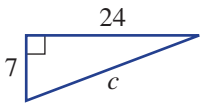

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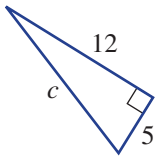

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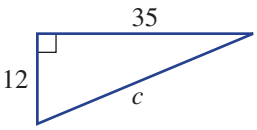

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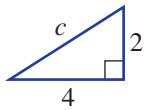

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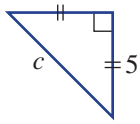
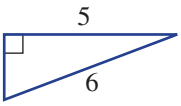
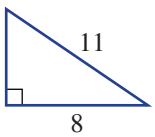


3 Find the length of the unknown side in these right-angled triangles. Round to two decimal places.

a



b



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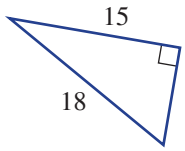
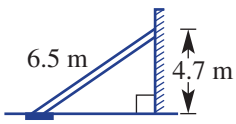


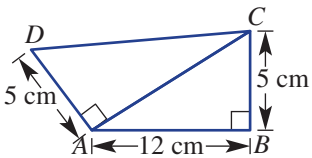
4 A steel support beam of length 6.5 m is connected to a wall at a height of 4.7 m from the ground. Find the distance (to decimal places) between the base of the building and the point where the beam is joined to the ground.



5 For this double triangle, find the length:

a AC

b CD (correct to two decimal places)



6 Two different cafés on opposite sides of an atrium in a shopping centre are respectively 10 m and 15 m above the ground floor. If the cafés are linked by a 20 m escalator, find the horizontal distance (to the nearest metre) across the atrium, between the two cafés.

7 Find the value of each of the following, correct to two decimal places.

a $\sin 40°$

b $\tan 66°$

c $\cos 44°$

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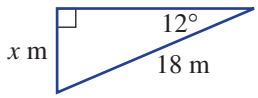
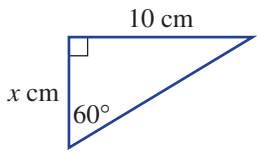
8 Which ratio $\left(\sin\theta = \frac{O}{H},\ \cos\theta = \frac{A}{H} \text{ or } \tan\theta = \frac{O}{A}\right)$ would be used to find the value of the unknown in these triangles?

a



b



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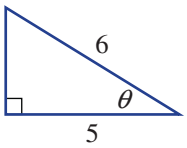
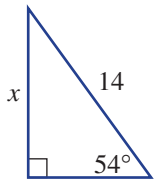


9 Find the value of each pronumeral, correct to two decimal places where necessary.

Use $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$ or $\tan\theta = \frac{O}{A}$.

a



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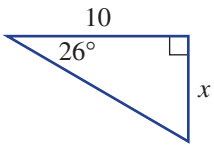

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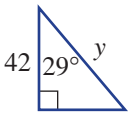

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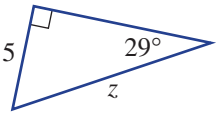

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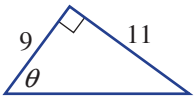

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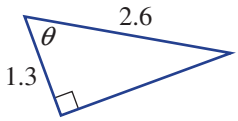
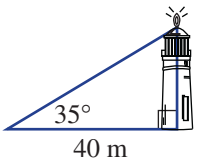
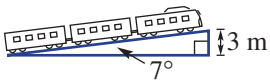


10 The angle of elevation of the top of a lighthouse from a point on the ground 40 m from its base is 35°. Find the height of the lighthouse, correct to two decimal places.



11 A train travels up a slope, making an angle of 7° with the horizontal. When the train is at a height of 3 m above its starting point, find the distance it has travelled up the slope, to the nearest metre.



12 From a point on the ground, Ahbed measures the angle of elevation of a 120 m tower to be 34°. How far from the base of the tower is Ahbed, correct to two decimal places?

13 From the roof of a skyscraper, Aisha spots a car at an angle of depression of 51° from the roof of the skyscraper. If the skyscraper is 78 m high, how far away is the car from the base of the skyscraper, correct to one decimal place?

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Extended-response questions

1 From the top of a 100 m cliff, Skevi sees a boat out at sea at an angle of depression of 12°.

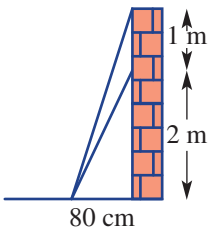
a Draw a simple diagram for this situation.

b Find how far out to sea the boat is to the nearest metre.

c A swimmer is 2 km away from the base of the cliff and in line with the boat. What is the angle of depression to the swimmer to the nearest degree?

d How far away is the boat from the swimmer, to the nearest metre?

2 An extension ladder is initially placed so that it reaches 2 metres up a wall. The foot of the ladder is 80 centimetres from the base of the wall.

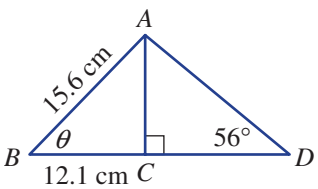


a Find the length of the ladder, to two decimal places, in its original position.

b Without the foot of the ladder being moved, the ladder is extended so that it reaches 1 m further up the wall. How far (to two decimal places) has the ladder been extended?

c The ladder is placed so that its foot is now 20 cm closer to the base of the wall. How far up the wall can the ladder length found in part **b** reach? Round to two decimal places.

3 Consider triangle *ABD*, where *AC* meets *BD* at 90°.



a Use Pythagoras' theorem to find the height *AC*, correct to 2 decimal places.

b Find θ, correct to the nearest degree.

c Find the length of

i *AD*

ii *CD*, each to 1 decimal place.

d Find the perimeter of triangle *ABD*.

e Calculate the area of triangle *ABD*.

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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

5 Linear relationships

What you will learn

5A Introducing linear relationships
5B The x-intercept and y-intercept
5C Graphing straight lines using intercepts EXTENSION
5D Lines with only one intercept
5E Gradient
5F Gradient and direct proportion
5G Gradient–intercept form
5H Finding the equation of a line using $y = mx + b$ EXTENSION
5I Midpoint and length of a line segment from diagrams
5J Linear modelling FRINGE
5K Introducing non-linear relationships

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NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRANDS: LINEAR RELATIONSHIPS, RATIOS AND RATES

Outcomes

A student creates and displays number patterns; graphs and analyses linear relationships; and performs transformations on the Cartesian plane. (MA4–11NA)

A student determines the midpoint, gradient and length of an interval, and graphs linear relationships. (MA5.1–6NA)

A student uses the gradient–intercept form to interpret and graph linear relationships. (MA5.2–9NA)

A student recognises direct and indirect proportion, and solves problems involving direct proportion. (MA5.2–5NA)

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Supply and demand

A business can use linear relations to find the most suitable selling price to make both profit and sales.

For example, consider the production and sales of a hatch-back car. If the selling price of this hatch-back car is too low there is no profit made. A high selling price brings profit, which enables the production of more cars. But of the customers who prefer this model, how many are willing to pay a higher price? Fewer sales also mean less profit. The selling price needs to be suitable for both the customer and the manufacturer.

Linear equations for supply (i.e. production) and demand (i.e. sales) relate P, the price of one car, and n, the number of cars. The supply graph has a positive slope since as P increases, n also increases. The demand graph has negative slope because as P increases, n decreases. The intersection point of these two lines occurs at the selling price when the number of cars produced equals the number that customers will buy at that price.

Applying mathematical techniques to achieve minimal waste and maximum profit are skills used by an operational research analyst. Work areas include manufacturing, finance, retail, the airline industry, computer services, healthcare, transportation, mining and the defence forces.

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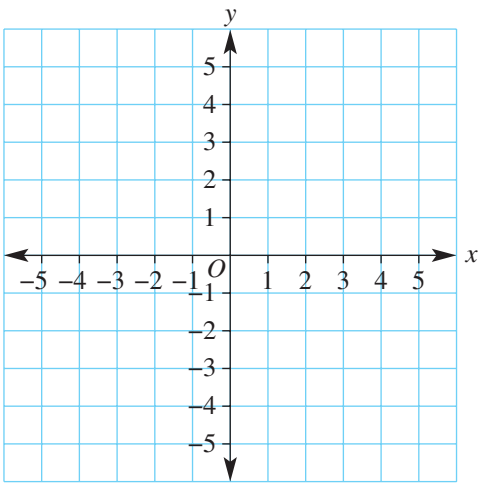
Pre-test

1 Plot and label the following points on a Cartesian plane (x-y axes).

a (3, 1) **b** (−2, 4) **c** (0, 0) **d** (5, −3)



2 Given $y = x - 2$, find the value of y when:

a $x = 3$ **b** $x = 1$ **c** $x = -1$ **d** $x = -2$

3 Solve the following equations for x.

a $3x = 15$ **b** $-2x = 4$ **c** $-x = 3$

d $0 = x - 4$ **e** $0 = 3x + 6$ **f** $0 = 2x - 8$

4 Find the value of y in each of the following when $x = 0$.

a $y = 2x + 3$ **b** $y = 3x - 4$ **c** $x + 2y = 8$ **d** $3x - 4y = 12$

5 Copy and complete the tables of values for these rules.

a $y = x + 3$

x	−2	−1	0	1	2
y					

b $y = 2x - 1$

x	−2	−1	0	1	2
y					

6 If $x = 2$ and $y = 1$, decide whether the following equations are true.

a $y = 2x + 1$ **b** $y = 3x - 5$ **c** $y = -2x + 5$ **d** $5x - 2y = 6$

7 Simplify:

a $\dfrac{4 + 10}{2}$ **b** $\dfrac{2 + 11}{2}$ **c** $\dfrac{-1 + 7}{2}$ **d** $\dfrac{-3 + (-7)}{2}$

8 Find the vertical distance between the following pairs of points.

a (3, 2) and (3, 7) **b** (−1, 1) and (−1, −3)

c (2, 3) and (2, −2) **d** (1, −4) and (1, −1)

9 Find the horizontal distance between the following pairs of points.

a (1, 3) and (5, 3) **b** (−1, 2) and (4, 2)

c (−2, −3) and (5, −3) **d** (−4, 1) and (−1, 1)

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5A Introducing linear relationships

Interactive
Widgets
HOTsheets
Walkthrough

If two variables are related in some way, we can use mathematical rules to more precisely describe this relationship. The most simple type of relationship is one that can be illustrated with a straight line graph. These are called linear relationships.

The volume of petrol in your car at a service bowser, for example, might initially be 10 L, then increase by 1.2 L per second after that. This is an example of a linear relationship between *volume* and *time* because the volume is increasing at a constant rate of 1.2 L/s.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

The amount of petrol in your car as you fill up at the bowser can be described by a linear relation.

Let's start: Which equation is linear?

Here are three equations and their tables of values. Copy and complete the tables of values and plot the points.

1 $y = 2^x - 1$

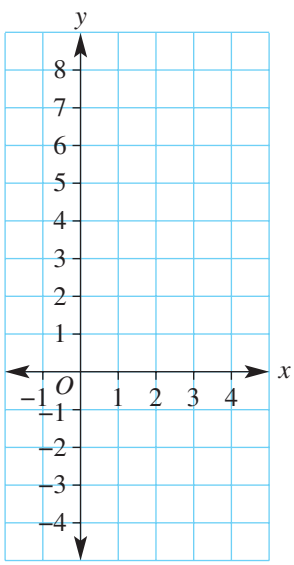
x	0	1	2	3
y				

2 $y = x^2 - 1$

x	0	1	2	3
y				

3 $y = 3x - 4$

x	0	1	2	3
y				



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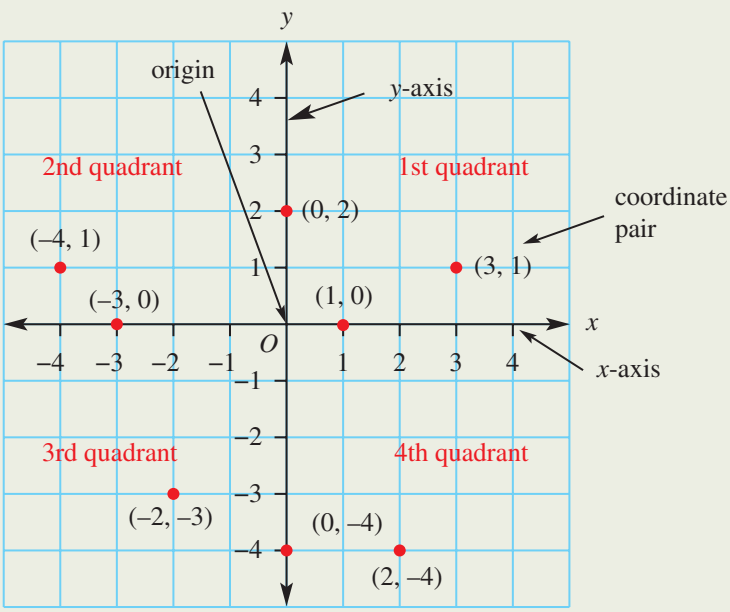
Key ideas

- Coordinate geometry provides a link between geometry and algebra.
- The **Cartesian plane** (or number plane) consists of two axes that divide the number plane into four quadrants.
  - The ***x*-axis** is the horizontal axis.
  - The ***y*-axis** is the vertical axis.
  - The x-axis and y-axis intersect at the origin (0, 0) at right angles.
  - A point is precisely positioned on a Cartesian plane using the *coordinate pair* (x, y). x describes the horizontal position of the point from the origin. y describes the vertical position of the point from the origin.



Cartesian plane A plane on which every point is related to a pair of numbers called coordinates

***x*-axis** The horizontal axis of the Cartesian plane

***y*-axis** The vertical axis of the Cartesian plane

Linear relationship A set of ordered pairs that give a straight line

- In the coordinate pair (2, −4), 2 is the x-coordinate and −4 is the y-coordinate. The point is 2 units to the right of the origin (horizontal direction) and 4 units below the origin (vertical direction).
- A **linear relationship** is a set of ordered pairs (x, y) that, when graphed, give a straight line.
- Linear relationships have rules that may be of the form:
  - $y = mx + b$, for example, $y = 2x + 1$
  - $ax + by = d$ or $ax + by + c = 0$, for example, $2x - 3y = 4$ or $2x - 3y - 4 = 0$
- Each point that is on the line fits the rule for the linear relation. For example, on the line with rule $y = 2x$, each y-coordinate will be two times the x-coordinate. So, (3, 6) will be on the line but (4, 10) will not.

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Exercise 5A

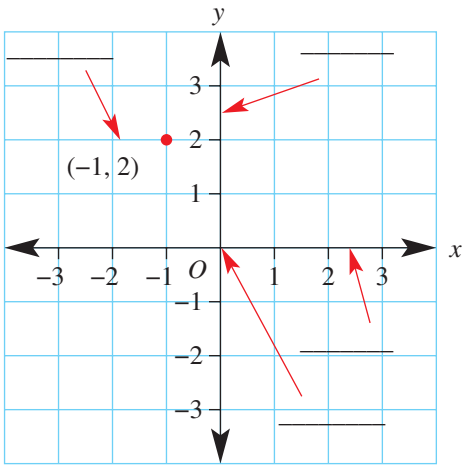
UNDERSTANDING AND FLUENCY	1–7	5, 6, 7(½)

1 **a** For the Cartesian plane shown, label the following parts.

- **i** x-axis
- **ii** origin
- **iii** y-axis
- **iv** coordinate pair

b For the point (3, −1):

- **i** 3 is the _____.
- **ii** −1 is the _____.
- **iii** From the origin, the point is 3 units to the _____ and 1 unit _____.



2 Which one of the following rules is linear?

A $y = x^2 + 4$ **B** $y = \frac{2}{x}$

C $y = 2x - 1$ **D** $y = 3^x$

Linear rules are often in the form $y = mx + b$ or $ax + by = d$.

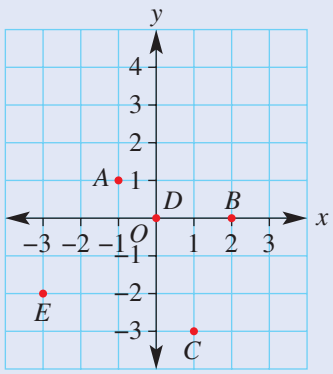
Example 1 Using the Cartesian plane

Using the grid shown:

a plot the points with coordinates:

- **i** (2, 4)
- **ii** (3, −1)
- **iii** (0, −2)
- **iv** (−2, 3)

b state the coordinates of the points marked A–E



Example is continued on the next page

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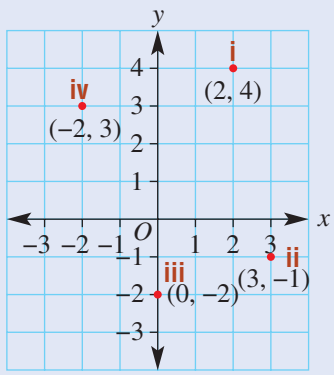
SOLUTION

a



b $A(-1, 1)$
$B(2, 0)$
$C(1, -3)$
$D(0, 0)$
$E(-3, -2)$

EXPLANATION

From the origin, start with the x-coordinate, then the y-coordinate.

i (2, 4) is 2 units right of the origin and 4 units above.

ii (3, −1) is 3 units right of the origin and 1 unit below.

iii For (0, −2), the x coordinate is 0, so there is no move left or right, so the point is 2 units below the origin.

iv (−2, 3) is 2 units left and 3 units above the origin.

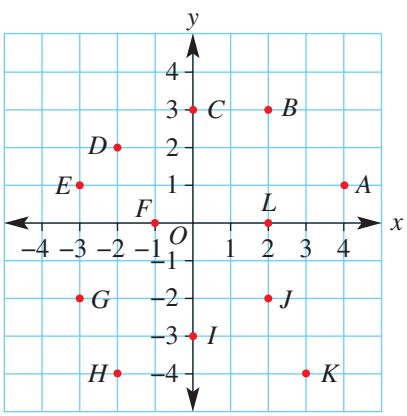
Each point is a coordinate pair (x, y) with the x-coordinate (horizontal direction) first, followed by the y-coordinate (vertical direction).
Start at the origin and move along the x-axis until you are above or below the point (this is the x-coordinate), then move up or down to the point (this is the y-coordinate).

3 Refer to the Cartesian plane shown.

a Draw this set of axes (without A–L) and plot the following points.

i (3, 1) **ii** (−4, −2)
iii (0, 2) **iv** (−1, 2)

b Give the coordinates of all the points A–L.



List the x-coordinate followed by the y-coordinate; e.g.(2, 4).

4 For the rule $y = 3x - 4$, find the value of y for these x values.

a $x = 2$ **b** $x = 1$
c $x = 0$ **d** $x = -1$

When $x = 2$,
$y = 3 \times 2 - 4 = \ldots$

5 Which pair of coordinates fits the rule $y = 3x$?

A (3, 1) **B** (3, 6) **C** (2, 6) **D** (−1, 3)

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Example 2 Filling in tables from rules

Complete the tables for the given rules.

a $y = 2x - 1$

x	−2	−1	0	1	2	3
y						

b $y = -3x + 2$

x	−2	−1	0	1	2
y					

SOLUTION

a

x	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5

b

x	−2	−1	0	1	2
y	8	5	2	−1	−4

EXPLANATION

Substitute each x value in the table into the rule $y = 2x - 1$ to find its corresponding y value.

For $x = -2$, $y = 2 \times (-2) - 1$
$= -4 - 1$
$= -5$

For $x = 1$, $y = 2 \times 1 - 1$
$= 2 - 1$
$= 1$

Substitute each x value into the rule $y = -3x + 2$.

Recall that negative × negative = positive.

For $x = -2$, $y = -3 \times (-2) + 2$
$= 6 + 2$
$= 8$

For $x = 0$, $y = -3 \times 0 + 2$
$= 0 + 2$
$= 2$

6 Copy and complete the tables for the given rules.

a $y = x + 2$

x	−2	−1	0	1	2	3
y						

b $y = 2x + 3$

x	−2	−1	0	1	2
y					

c $y = -2x$

x	−2	−1	0	1	2
y					

d $y = -3x - 1$

x	−2	−1	0	1	2	3
y						

A negative × positive is negative.
A negative × negative is positive.

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Example 3 Plotting points to graph straight lines

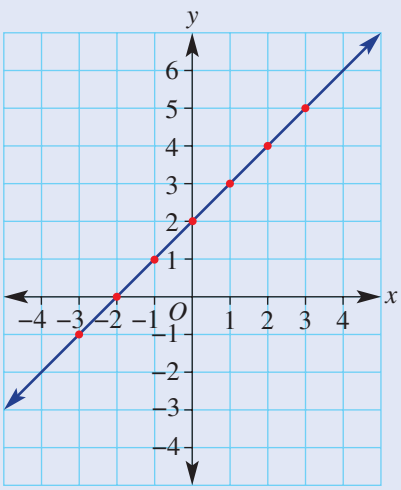
Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph for these linear relations.

a $y = x + 2$ **b** $y = -2x + 2$

SOLUTION

a

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5



EXPLANATION

Use $-3 \leqslant x \leqslant 3$ to construct a table of x values as instructed. Substitute each value of x into the rule $y = x + 2$.

The coordinates of the points are read from the table; i.e.(−3, −1), (−2, 0) etc.

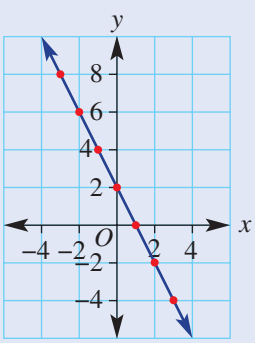
Draw a set of axes with an appropriate scale to show the points from the table.

Plot each point and join them to form a straight line.

Extend the line to show it continues in either direction.

b

x	−3	−2	−1	0	1	2	3
y	8	6	4	2	0	−2	−4



Use $-3 \leqslant x \leqslant 3$ as instructed. Substitute each value of x into the rule $y = -2x + 2$.

Plot each point and join them to form a straight line.

Extend the line beyond the plotted points.

7 Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph for these linear relationships.

a $y = x - 1$ **b** $y = x + 3$

c $y = 2x - 3$ **d** $y = -3x$

e $y = -2x - 1$ **f** $y = -x + 4$

g $y = 4 - x$ **h** $y = 2x - 4$

i $y = 3x$

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PROBLEM SOLVING AND REASONING	8–10	8, 10–12

Example 4 Deciding whether a point is on a line

Decide whether the point (3, 7) is on the line with the given rule.

a $y = 3x - 4$ **b** $y = 2x + 1$

SOLUTION	EXPLANATION
a $y = 3x - 4$	
Substitute $x = 3$	Find the value of y on the graph of the rule for $x = 3$.
$y = 3 \times 3 - 4$	
$= 5$ (not 7)	To fit the rule, the y value needs to be 5, not 7, so (3, 7) is not on the line.
$\therefore$ The point (3, 7) is not on the line	
b $y = 2x + 1$	
Substitute $x = 3$	By substituting $x = 3$ into the rule for the line, we find $y = 7$.
$y = 2 \times 3 + 1$	
$= 7$	
$\therefore$ The point (3, 7) is on the line	Since (3, 7) fits the rule, it is on the line.

8 Decide whether the point (2, 8) is on the line with the given rule.

a $y = 2x + 6$
b $y = 3x + 2$
c $y = -x + 6$

When $x = 2$, does $y = 8$?

9 Decide whether the point (−1, 4) is on the line with the given rule.

a $y = x + 5$
b $y = -2x + 2$
c $y = 4x$

10 Find a rule in the form $y = mx + b$ (e.g. $y = 2x - 1$) that matches these tables of values.

a

x	0	1	2	3	4
y	2	3	4	5	6

b

x	0	1	2	3
y	0	2	4	6

c

x	−2	−1	0	1	2	3
y	−3	−1	1	3	5	7

d

x	−2	−1	0	1	2	3
y	−7	−4	−1	2	5	8

e

x	−1	0	1	2	3	4
y	−3	−2	−1	0	1	2

Think about how you can find the y value from the x value. Is it double the x value each time or 3 more than the x value each time? etc.

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11 The table below shows the recorded height, y, of a seedling x days after it sprouts. Find a linear rule in the form $y = \ldots$ that matches the data in the table.

x (day)	1	2	3	4
y (height)	3	6	9	12

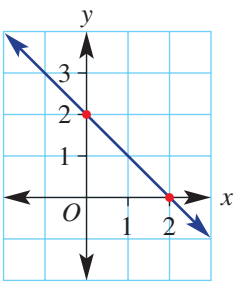
12 By considering the relationship between the coordinates, match these rules **i**, **ii**, **iii** and **iv** to the graphs **a**, **b**, **c** and **d**.

i $y = 2x + 1$ **ii** $y = -x - 1$

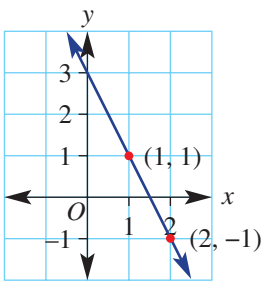
iii $y = -2x + 3$ **iv** $x + y = 2$

Each rule shows the relationship between the x- and y-coordinates for each point on the line. In **i**, each y-coordinate has to be 1 more than twice its x-coordinate.

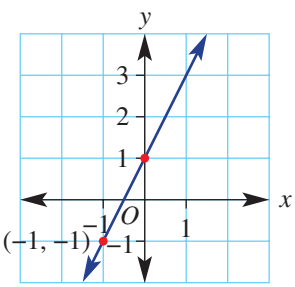
a



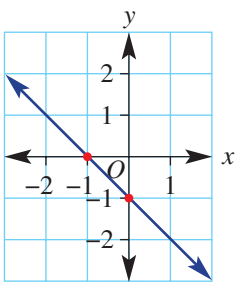
b



c



d



ENRICHMENT – 13

Tougher rule finding

13 Find the linear rule linking x and y in these tables.

a

x	−1	0	1	2	3
y	5	7	9	11	13

b

x	−2	−1	0	1	2
y	22	21	20	19	18

c

x	1	3	5	7	9
y	1	5	9	13	17

d

x	6	7	8	9	10
y	4	4.5	5	5.5	6

e

x	0	2	4	6	8
y	4	−2	−8	−14	−20

f

x	−5	−4	−3	−2	−1
y	−21	−16	−11	−6	−1

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5B The x-intercept and y-intercept

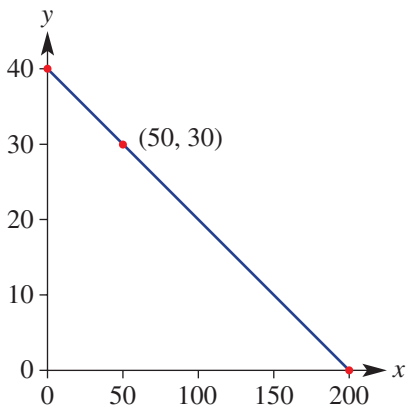
Interactive
Widgets
HOTsheets
Walkthrough

Each point on a line fits a rule for a linear relationship. When the linear relationship is a model for a real situation, each point gives a piece of information about that situation. Some points are very significant.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

For example, the following graph for the linear relationship $y = -0.2x + 40$ models how much petrol a car uses. The y-axis shows the amount of petrol in the tank and the x-axis shows the number of kilometres travelled. The coordinate pair (50, 30) shows that after travelling 50 km, 30 litres of petrol are left in the tank.



One important point shows us how much petrol was in the tank before the car has gone anywhere ($x = 0$). Using the linear relationship (substituting $x = 0$), we can see that this point is (0, 40) – this is where the graph cuts the y-axis. This is called the y-intercept.

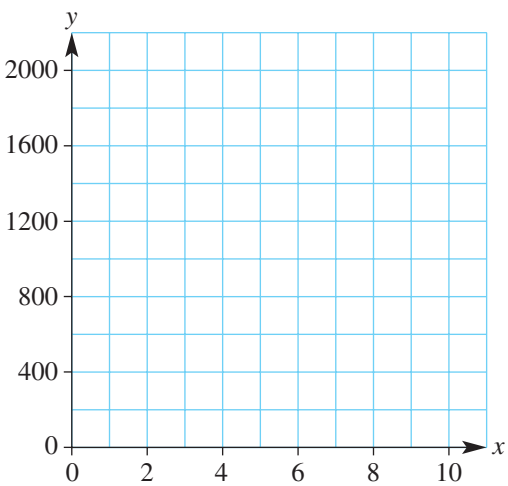
We also want to know when the car will run out of petrol ($y = 0$). Again using the linear relationship (substituting $y = 0$), we can see that this point is (200, 0) – this is where the graph cuts the x-axis. This is called the x-intercept.

Let's start: Are we there yet?

This table shows the distance (y metres) a student is from his home (at x minutes) when he rides his bike home from school.

x	0	2	4	6	8	10
y	2000	1600	1200	800	400	0

- Copy the axes shown and plot these points to form a line.
- How far is school from home?
- How long does it take to get home?
- Circle the above two points on the graph.
- Which point is the x-intercept? Which point is the y-intercept?
- If the rule for the table is $y = -200x + 2000$, how could you find the x- and y-intercepts without sketching the graph?



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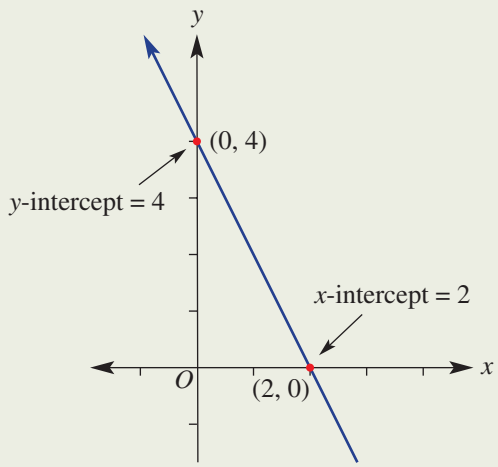
Key ideas

- The ***y*-intercept** is the y-value of the point where the line cuts (intersects) the y-axis.
- The ***x*-intercept** is the x-value of the point where the line cuts (intersects) the x-axis.

x-intercept

x	−2	−1	0	1	②	3
y	8	6	④	2	0	−2

y-intercept



- The rule of the linear relationship can be used to find the x- and y-intercepts.
  - To find the y-intercept, substitute $x = 0$ into the rule, since it lies on the y-axis.
  - To find the x-intercept, substitute $y = 0$ into the rule, since it lies on the x-axis.

Exercise 5B

UNDERSTANDING AND FLUENCY | 1–4, 5–7(½) | 4, 5–7(½)

1 Plot the points $A(3, 0)$ and $B(0, 2)$ on a Cartesian plane. Draw a line through A and B. What is the:

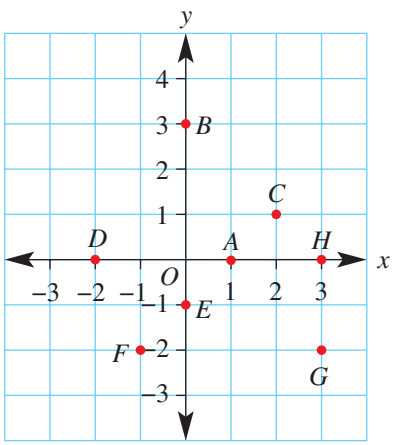
a x-intercept?

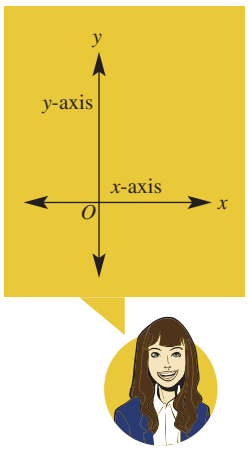
b y-intercept?

2 For the Cartesian plane shown, give the coordinates of each point that is on the:

a x-axis

b y-axis



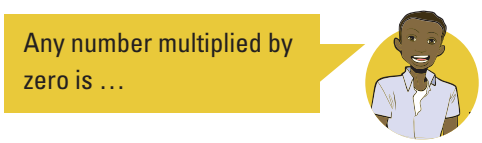


3 Simplify the following to find the value of y.

a $y = 2 \times 0 + 1$

b $y = 4 \times 0 - 2$

c $y = -2 \times 0 - 3$



4 Solve these simple equations.

a $4x = 8$ **b** $2y = 6$ **c** $-3y = 15$

d $x - 4 = 0$ **e** $2x - 6 = 0$ **f** $-3x + 9 = 0$

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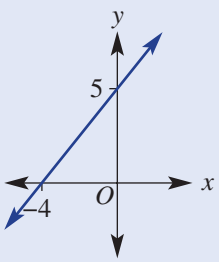
Example 5 Reading off the x-intercept and y-intercept

Read off the x-intercepts and y-intercepts from this table and graph.

a

x	−2	−1	0	1	2	3
y	12	9	6	3	0	−3

b



SOLUTION

a The x-intercept is 2.
The y-intercept is 6.

b The x-intercept is −4.
The y-intercept is 5.

EXPLANATION

(2, 0) is a point on the x-axis.
(0, 6) is a point on the y-axis.
The line cuts the horizontal axis at −4.
The line cuts the vertical axis at 5.

5 Read off the x- and y-intercepts from these tables and graphs.

a

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

b

x	−3	−2	−1	0	1	2	3
y	4	3	2	1	0	1	2

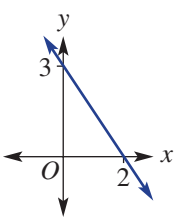
c

x	−1	0	1	2	3	4
y	10	8	6	4	2	0

d

x	−5	−4	−3	−2	−1	0
y	0	2	4	6	8	10

e



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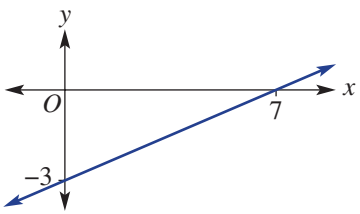

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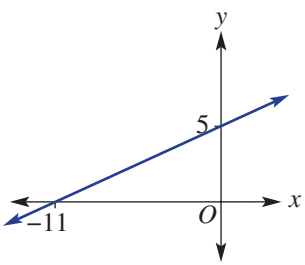

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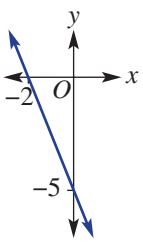

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Example 6 Finding the y-intercept

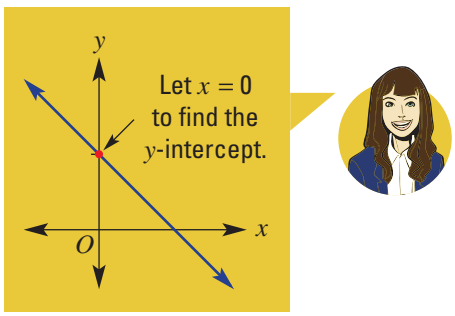
Find the y-intercept for each of these linear relations.

a $y = 2x - 1$ **b** $2x + 3y = 6$

SOLUTION	EXPLANATION
a $y = 2x - 1$	
y-intercept (let $x = 0$):	Substitute $x = 0$ into the rule to find the y value of the y-intercept.
$y = 2 \times 0 - 1$	Recall that anything multiplied by 0 is 0.
$= -1$	This line cuts the y-axis at $(0, -1)$.
$\therefore$ The y-intercept is -1.	
b $2x + 3y = 6$	
y-intercept (let $x = 0$):	Substitute $x = 0$ to find the y-intercept.
$2 \times 0 + 3y = 6$	Solve the equation $3y = 6$ by dividing both sides by 3.
$3y = 6$	This line cuts the y-axis at $(0, 2)$.
$y = 2$	
$\therefore$ The y-intercept is 2.	

6 Find the y-intercepts for these linear relations.

a $y = 2x + 5$ **b** $y = 3x + 1$
c $y = x - 7$ **d** $y = 2x - 3$
e $y = -4x + 2$ **f** $y = -5x - 4$
g $y = -x - 2$ **h** $y = -2x + 10$
i $2x + y = 11$ **j** $2x + 3y = 9$
k $x + 4y = 8$ **l** $-3x + 2y = 12$
m $4x - 2y = 6$ **n** $3x - 4y = 4$
o $-2x - y = 3$ **p** $-2x - 3y = -9$



Example 7 Finding the x-intercept

Find the x-intercepts for these linear relations.

a $y = 2x + 4$ **b** $3x - 2y = 12$

SOLUTION	EXPLANATION
a $y = 2x + 4$	
x-intercept (let $y = 0$):	Substitute $y = 0$ into the rule to find the x-intercept.
$0 = 2x + 4$	
$-4 = 2x$	Solve the equation by subtracting 4 from both sides.
$-2 = x$	Then divide both sides by 2.
$x = -2$	
$\therefore$ The x-intercept is -2.	This line cuts the x-axis at $(-2, 0)$.
b $3x - 2y = 12$	
x-intercept (let $y = 0$):	Substitute $y = 0$ to find the x-intercept.
$3x - 2 \times 0 = 12$	$2 \times 0 = 0$ and then solve the remaining equation for x.
$3x = 12$	
$x = 4$	
$\therefore$ The x-intercept is 4.	This line cuts the x-axis at $(4, 0)$.

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7 Find the x-intercept for these linear relationships.

a $y = x + 6$ **b** $y = x - 4$ **c** $y = 2x - 8$
d $y = 3x + 6$ **e** $y = -2x - 8$ **f** $y = -4x + 12$
g $x + 2y = 7$ **h** $2x - y = 4$ **i** $3x + 4y = 12$
j $4x - 6y = 24$ **k** $-2x + 3y = -2$ **l** $-5x - 7y = 15$

PROBLEM SOLVING AND REASONING

	8, 9	8, 10, 11

8 Find the x-intercept and y-intercept for each of these linear relationships involving fractions.

a $y = 2x - 1$ **b** $y = \frac{1}{2}x - 2$
c $y = \frac{1}{3}x + 4$ **d** $3x + 2y = 7$

Recall that another way of writing $\frac{1}{2}x$ is $\frac{x}{2}$.

9 The height, h, in metres, of a lift above ground after t seconds is given by $h = 100 - 5t$.
a What height does the lift start at ($t = 0$)?
b How long does it take for the lift to reach the ground ($h = 0$)?

10 Water is being drained from a fish tank. The amount of water in the tank, V litres, after t minutes is given by the rule $V = 80 - 4t$.
a How long will it be until the tank is empty?
b How much water was in the tank to begin with ($t = 0$)?

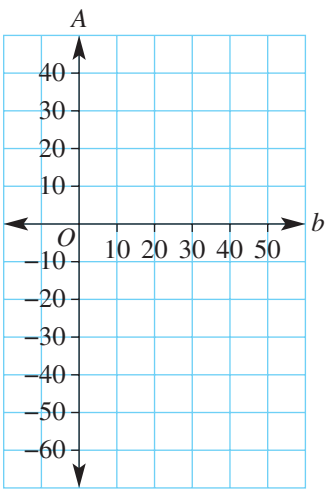
11 The line $y = 3x$ has both its x-intercept and y-intercept with coordinates (0, 0). Can you explain how this is the case, and what it means for the graph of $y = 3x$?

ENRICHMENT

	–	12

Making money

12 Ana makes badges to sell at the market. The rule for the amount of money, A dollars, she makes from the sale of b badges is given by $A = 2b - 60$.
a What is the value of A if she sells no badges ($b = 0$)?
b Can you explain your answer to part **a**?
c How many badges must she sell to cover her initial costs ($A = 0$)?
d What happens when she sells more badges than in your answer to part **c**?
e Use the information from your answers above to draw a graph for the rule on the axes shown. The A-axis is the y-axis and the b-axis is the x-axis.



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5C Graphing straight lines using intercepts EXTENSION

Interactive
Widgets
HOTsheets
Walkthrough

When linear equations are graphed, all the points lie in a straight line. This means that it is possible to graph a straight line using only two points. Two key values that help draw these graphs are the x-intercept and y-intercept. Once these are located, they can be joined in a straight line to give all the other points that lie on the line.

Computer software can be used to graph linear relationships.

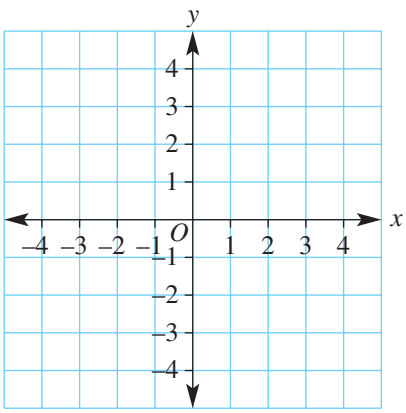
Straight-line graphs are frequently used in business to illustrate the relationship between two variables.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Two key values

Consider the relationship $y = 2x - 2$.

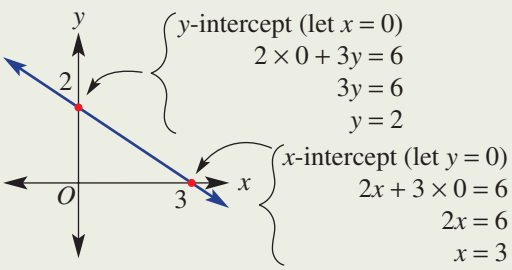
- Write a set of steps for another student, explaining to them how to find the y-intercept using the rule.
- Write another set of steps, this time explaining how to find the x-intercept using the rule.
- Now that you have these two points, describe how you would use them to sketch the graph of $y = 2x - 2$ on the axes shown, without drawing up a table of values.



Key ideas

- A straight line can be drawn using only two points.
- Two key values we can use to graph a straight line are the x-intercept and y-intercept.
- To graph a linear relationship using intercepts:
  - find the y-intercept by substituting $x = 0$ into the rule
  - find the x-intercept by substituting $y = 0$ into the rule
  - plot the two points on the axes
  - draw a straight line passing through the two intercepts.

Example: $2x + 3y = 6$



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Exercise 5C EXTENSION

UNDERSTANDING AND FLUENCY 1–5, 6–7(½) 3, 5, 6–7(½)

1 Insert the missing number.

a A minimum of _____ points are required to sketch a straight line.

b To find the y-intercept, substitute $x =$ _____.

c To find the x-intercept, substitute $y =$ _____.

2 For each of the given rules, complete a table like the one shown and plot points to draw a graph. Circle the x-intercept and y-intercept in each table.

x	−3	−2	−1	0	1	2	3
y							

a $y = x + 2$ **b** $y = \frac{1}{2}x - 1$ **c** $y = 3x$ **d** $y = -x + 3$

3 Plot each of the following pairs of points on a set of axes and join the points to form a straight line.

a $(3, 0)$ and $(0, -2)$

b $(0, 4)$ and $(2, 0)$

c $(-3, 0)$ and $(0, 6)$

(3, 0) is on the x-axis as it is 3 to the right of the origin and 0 up or down.

4 For these equations, find the y-intercept by letting $x = 0$.

a $x + y = 4$ **b** $x - y = 5$ **c** $2x + 3y = 9$ **d** $y = 2x - 4$

5 For these equations, find the x-intercept by letting $y = 0$.

a $x + 2y = 5$ **b** $2x - y = -4$ **c** $4x - 3y = 12$ **d** $y = 3x - 6$

Example 8 Sketching linear relations of the form $ax + by = c$ using intercepts

Sketch the graph of $2x + 3y = 6$, showing the x- and y-intercepts.

SOLUTION

$2x + 3y = 6$

y-intercept (let $x = 0$):

$2 \times 0 + 3y = 6$

$3y = 6$

$y = 2$

$\therefore$ The y-intercept is 2.

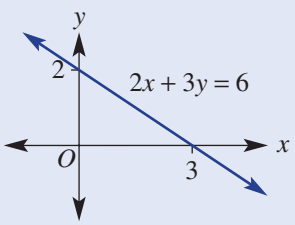
x-intercept (let $y = 0$):

$2x + 3 \times 0 = 6$

$2x = 6$

$x = 3$

$\therefore$ The x-intercept is 3.



EXPLANATION

Only two points are required to generate a straight line. For the y-intercept, substitute $x = 0$ into the rule and solve for y by dividing each side by 3.

State the y-intercept. Plot the point $(0, 2)$.

To find the x-intercept, substitute $y = 0$ into the rule and solve for x.

State the x-intercept. Plot the point $(3, 0)$.

Mark and label the intercepts on the axes and sketch the graph by joining the two intercepts. Continue the line past these points.

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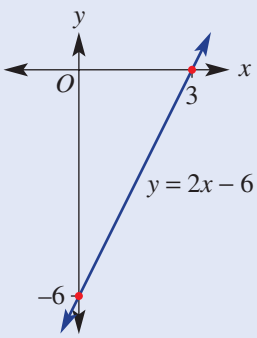
6 Sketch the graph of the following relations, by finding the x- and y-intercepts.

a $x + y = 2$
b $x + y = 5$
c $x - y = 3$
d $x - y = -2$
e $2x + y = 4$
f $3x - y = 9$
g $4x - 2y = 8$
h $3x + 2y = 6$
i $3x - 2y = 6$
j $y - 3x = 12$
k $-5y + 2x = -10$
l $-x + 7y = 21$

For the x-intercept, let $y = 0$.
For the y-intercept, let $x = 0$.

Example 9 Sketching linear relationships of the form $y = mx + b$ using intercepts

Sketch the graph of $y = 2x - 6$, showing the x- and y-intercepts.

SOLUTION	EXPLANATION
$y = 2x - 6$	
y-intercept (let $x = 0$):	Substitute $x = 0$ for the y-intercept.
$y = 2 \times 0 - 6$	Simplify to find the y-coordinate.
$y = -6$	
$\therefore$ The y-intercept is -6.	Plot the point $(0, -6)$.
x-intercept (let $y = 0$):	Substitute $y = 0$ for the x-intercept. Solve the remaining equation for x by adding 6 to both sides. Then divide both sides by 2.
$0 = 2x - 6$	
$6 = 2x$	
$x = 3$	
$\therefore$ The x-intercept is 3.	Plot the point $(3, 0)$.
	Mark in the two intercepts and join them to sketch the graph.

7 Sketch the graphs of the following relationships, showing the x- and y-intercepts.

a $y = 3x + 3$
b $y = 2x + 2$
c $y = x - 5$
d $y = -x - 6$
e $y = -2x - 2$
f $y = -3x - 6$
g $y = -2x + 4$
h $y = 2x - 3$
i $y = -x + 1$

To solve: $3x + 3 = 0$
$3x = -3$ (subtract 3)
$x = -1$ (divide by 3)

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PROBLEM-SOLVING AND REASONING

8, 9	8–10

8 The distance, d metres, of a remote-controlled car from an observation point after t seconds is given by the rule $d = 8 - 2t$.

a Find the distance from the observation point initially (at $t = 0$).

b After what time, t, is the distance, d, equal to 0 (substitute $d = 0$)?

c Sketch a graph of d versus t between the points from part **a** (the d-intercept) and part **b** (the t-intercept).

The axes will have d on the vertical axis and t on the horizontal axis.

9 Use your algebra and fraction skills to help sketch graphs for these relationships by finding x- and y-intercepts.

a $\frac{x}{2} + \frac{y}{3} = 1$

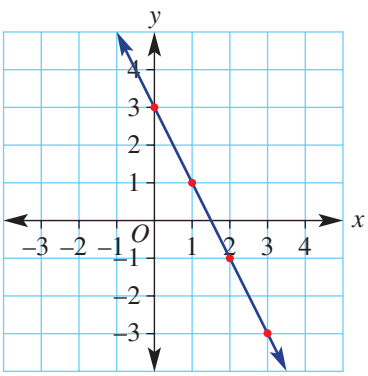
b $y = \frac{8 - x}{4}$

c $\frac{y}{2} = \frac{2 - 4x}{8}$

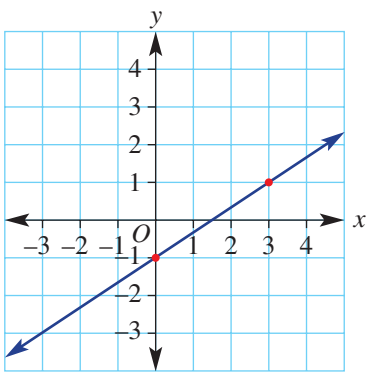
Recall: $\frac{0}{2} = 0$

10 Give reasons why the x-intercept on these graphs has the exact coordinates (1.5, 0).

a



b



ENRICHMENT

–	11

Points to rules

11 Write down the rules for the graphs with these axes intercepts. Write the rules in the form $ax + by = c$.

a x-intercept: 4; y-intercept: 4

b x-intercept: 2; y-intercept: 2

c x-intercept: 3; y-intercept: −3

d x-intercept: −1; y-intercept: 1

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5D Lines with only one intercept

Interactive

Widgets

HOTsheets

Walkthrough

Vertical and horizontal lines are special types of lines. You often see them in houses and construction sites.

When plotting a graph, a vertical line runs parallel to the y-axis. Since parallel lines never touch, the line will never cross the y-axis, and therefore has only one intercept, on the x-axis.

Horizontal lines run parallel to the x-axis and they therefore also have only one intercept, on the y-axis.

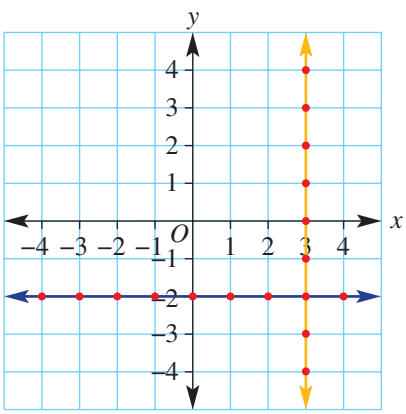
Lines that pass through the origin (0, 0) also have only one intercept, as the x- and y-intercepts are both 0.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: What rule satisfies all points?

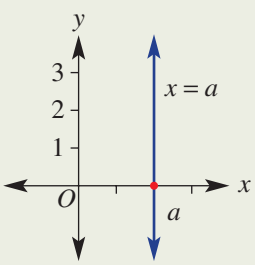
Here is one vertical (yellow) and one horizontal (blue) line.

- For the vertical line shown, write down the coordinates of all the points shown as dots.
- What is always true for each coordinate pair?
- Can you think of a simple equation that describes every point on the line?
- For the horizontal line shown, write down the coordinates of all the points shown as dots.
- What is always true for each coordinate pair?
- Can you think of a simple equation that describes every point on the line?

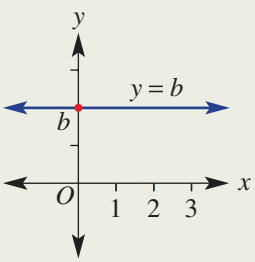


Key ideas

- Vertical line: $x = a$, for example $x = 2$, $x = -1$, $x = 0$
  - Parallel to the y-axis, the x-coordinate is the same for every point on the line.
  - The equation is of the form $x = a$, where a is a constant (fixed number).
  - The x-intercept is a.
  - There is no y-intercept.



- Horizontal line: $y = b$, for example $y = 2$, $y = -1$, $y = 0$
  - Parallel to the x-axis, the y-coordinate is the same for every point on the line.
  - The equation is of the form $y = b$, where b is a constant (fixed number).
  - The y-intercept is b.
  - There is no x-intercept.



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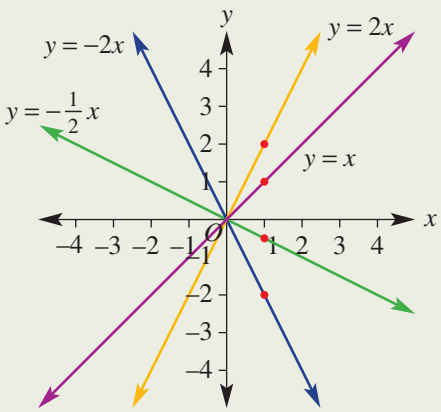
Key ideas

- Lines through the origin (0, 0): $y = mx$
  - The y-intercept is 0.
  - The x-intercept is 0.
  - Since the x- and y-intercepts give only one point, a second point is required. Substitute any x value into the rule to give a second point – use $x = 1$ for ease of calculation.



Exercise 5D

UNDERSTANDING AND FLUENCY | 1–6, 7–9(½) 10 | 6, 7–10(½)

1 Classify the following lines as either *horizontal* or *vertical*.

a $y = 3$ **b** $y = -1$
c $x = 2$ **d** $y = 0$
e $x = -5$

Will they be parallel to the x-axis or y-axis?

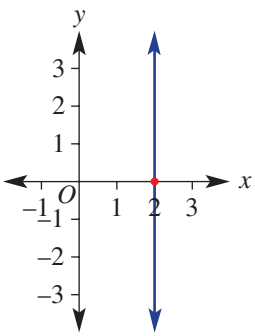
2 Select the features from the list (**i**–**v**) that apply to:

a horizontal lines
b vertical lines
  i Parallel to the y-axis
  ii Has a y-intercept
  iii Has an x-intercept
  iv Parallel to the x-axis
  v Always pass through the origin

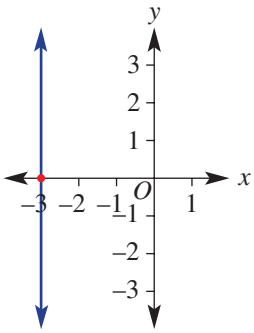
Draw a horizontal and vertical line to help.

3 What is the x-intercept for each of these graphs?

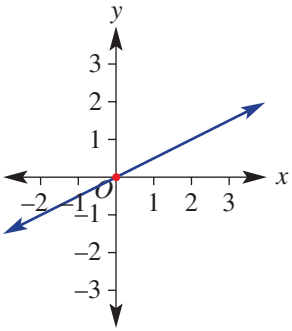
a



b



c



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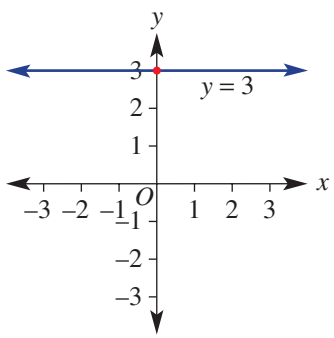
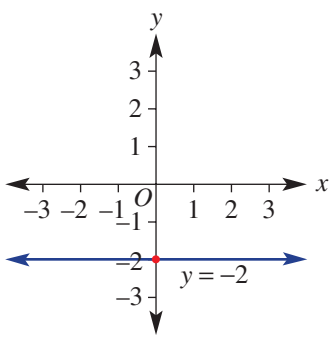
4 What is the y-coordinate of each point on these graphs?

a



b



5 List which of the following rules will have graphs that pass through (0, 0).

a $y = 4$ **b** $y = 2x$ **c** $y = 3x + 2$

d $x = 2$ **e** $y = -x$ **f** $y = \frac{1}{2}x$

Equations of the form $y = mx$ pass through the origin.

6 Find the value of y if $x = 1$, using these rules.

a $y = 5x$ **b** $y = \frac{1}{3}x$ **c** $y = -4x$ **d** $y = -0.1x$

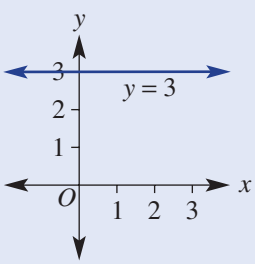
Example 10 Graphing vertical and horizontal lines

Sketch the graph of the following vertical and horizontal lines.

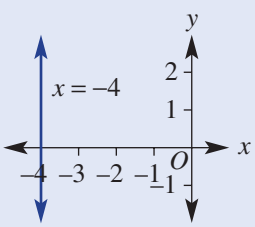
a $y = 3$ **b** $x = -4$ **c** $y = 0$

SOLUTION

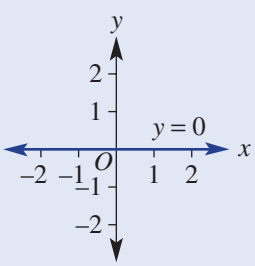
a



b



c



EXPLANATION

Each point on the line has a y-coordinate of 3, so the y-intercept is 3.
Sketch a horizontal line through all points where $y = 3$.

Each point on the line has an x-coordinate of -4, so the x-intercept is -4.
Sketch a vertical line through all points where $x = -4$.

Each point on the line has a y-coordinate of 0.
Sketch a horizontal line through all points where $y = 0$; this line is the x-axis.

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7 Sketch the graphs of the following vertical and horizontal lines.

a $x = 2$ **b** $x = 5$
c $y = 4$ **d** $y = 1$
e $x = -3$ **f** $x = -2$
g $y = -1$ **h** $y = -3$

Example 11 Sketching lines that pass through the origin

Sketch the graph of $y = 3x$.

SOLUTION

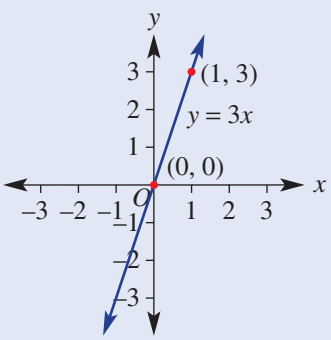
$y = 3x$

The x- and y-intercepts are 0.

Another point (let $x = 1$):

$$y = 3 \times (1)$$
$$y = 3$$

Another point is at $(1, 3)$.



EXPLANATION

The equation is of the form $y = mx$; i.e. when you substitute $x = 0$, $y = 3 \times 0 = 0$, giving $(0, 0)$ as both the x- and y-intercepts.

Two points are required to generate the straight line. Find another point by substituting $x = 1$. Any other x value could be used but the calculation is simplest for $x = 1$.

Plot and label both points and sketch the graph by joining the points in a straight line.

8 Sketch the graphs of the following linear relationships, which pass through the origin.

a $y = 2x$ **b** $y = 5x$ **c** $y = 4x$
d $y = x$ **e** $y = -4x$ **f** $y = -3x$
g $y = -2x$ **h** $y = -x$ **i** $y = \frac{1}{2}x$

Substitute $x = 1$ to obtain a second point.

9 Sketch the graphs of these special lines all on the same set of axes and label them with their equations.

a $x = -2$ **b** $y = -3$ **c** $y = 2$
d $x = 4$ **e** $y = 4x$ **f** $y = -\frac{1}{2}x$
g $y = -1.5x$ **h** $x = 0.5$ **i** $x = 0$
j $y = 0$ **k** $y = 2x$ **l** $y = 1.5x$

There are vertical lines, horizontal lines and lines through the origin.

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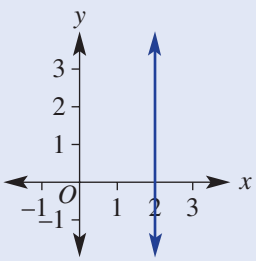
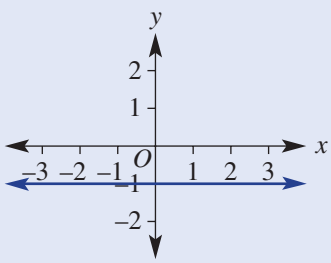
Example 12 Finding the equation of horizontal and vertical lines

Give the equations of the following vertical and horizontal lines.

a



b



SOLUTION

a $x = 2$

b $y = -1$

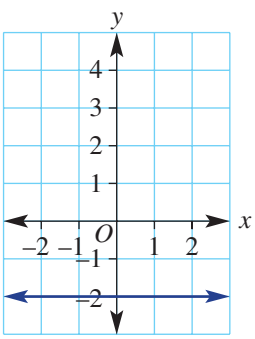
EXPLANATION

Line is vertical, with x-coordinate always 2.

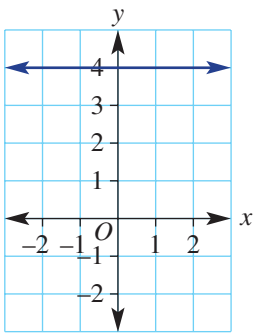
Line is horizontal, with y-coordinate always -1.

10 Give the equations of the following vertical and horizontal lines.

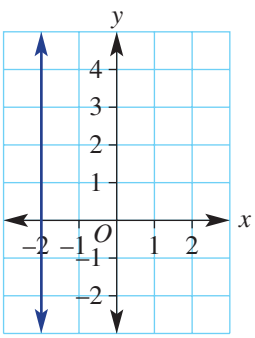
a



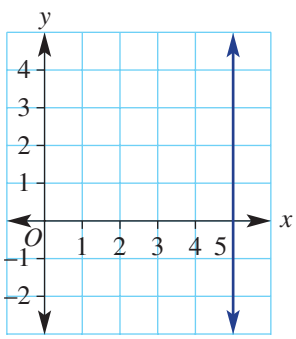
b



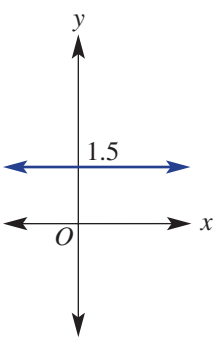
c



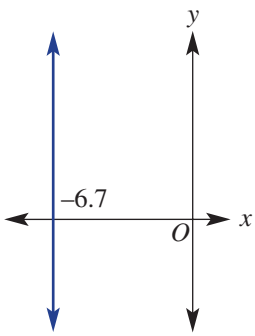
d



e



f



A horizontal line has the form $y = b$. A vertical line has the form $x = a$.

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PROBLEM-SOLVING AND REASONING 11, 12 11, 13

11 Find the equation of the straight line that is:

a parallel to the x-axis and passes through the point $(1, 3)$

b parallel to the y-axis and passes through the point $(5, 4)$

c parallel to the y-axis and passes through the point $(-2, 4)$

d parallel to the x-axis and passes through the point $(0, 0)$

First decide whether the line is horizontal or vertical, then use the point.

12 If, in a picture, the surface of the sea is represented by the x-axis, state the equation of the following paths.

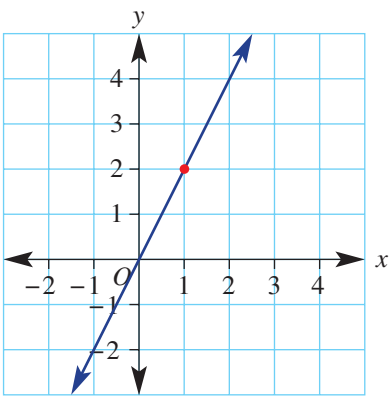
a A plane flies horizontally at 250 m above sea level. One unit is 1 metre.

b A submarine travels horizontally 45 m below sea level. One unit is 1 metre.

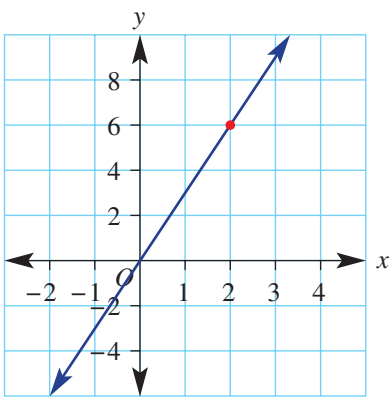
13 The rules of the following graphs are of the form $y = mx$. Use the points marked with a dot to find m and hence state the equations.

In a rule such as $y = 3x$, when $x = 1$, $y = 3 \times 1 = 3$.

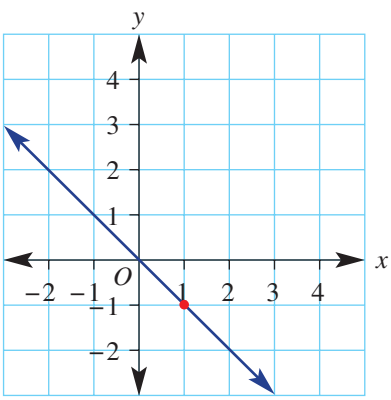
a



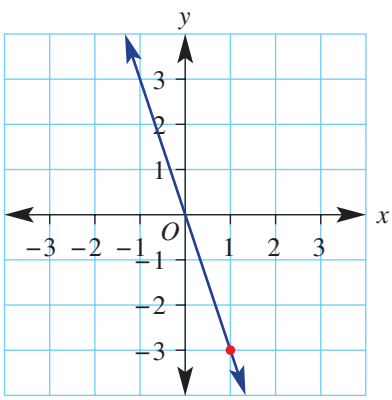
b



c



d



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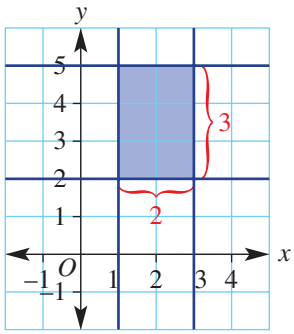
ENRICHMENT – 14, 15

Rectangular areas

The four lines $x = 1$, $x = 3$, $y = 2$, $y = 5$ drawn on the one set of axes form a rectangle, as shown (shaded). The area of this rectangle is length × breadth = 3 × 2 = 6 square units.



14 Find the areas of the rectangles contained within the following four lines.

a $x = 1$, $x = -2$, $y = -3$, $y = 2$

b $x = 0$, $x = 17$, $y = -5$, $y = -1$

15 The lines $x = -1$, $x = 3$ and $y = -2$ form three sides of a rectangle. Find the possible equation of the fourth line if the:

a area of the rectangle is:

i 12 square units

ii 8 square units

iii 22 square units

b perimeter of the rectangle is:

i 14 units

ii 26 units

iii 31 units

There may be multiple answers.

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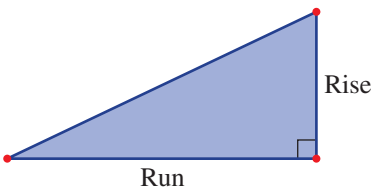
5E Gradient

Interactive
Widgets
HOTsheets
Walkthrough

The gradient of a line is a measure of its slope. It is a number that describes the steepness of a line. It is calculated by considering how far a line rises or falls between two points within a given horizontal distance. The horizontal distance between two points is called the *run*. The vertical distance is called the *rise*.



Gradients are used to describe the steepness of this rollercoaster track.

Stage

5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

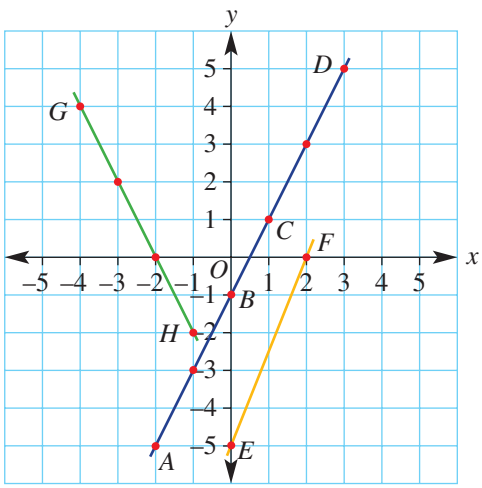
Let's start: Which line is the steepest?

The three lines here connect the points A, B, C, D, E, F, G and H.

- Calculate the rise and run (working from left to right) and also the fraction $\frac{\text{rise}}{\text{run}}$ for these segments.

  i AB **ii** BC **iii** BD
  iv EF **v** GH

- What do you notice about the fractions $\left(\frac{\text{rise}}{\text{run}}\right)$ for parts **i**, **ii** and **iii**?
- How does the $\frac{\text{rise}}{\text{run}}$ for EF compare with the $\frac{\text{rise}}{\text{run}}$ for parts **i**, **ii** and **iii**? Which of the two lines is the steepest?
- Your $\frac{\text{rise}}{\text{run}}$ for GH should be negative. Why is this the case?
- Discuss whether or not GH is steeper than AD.



Use computer software (dynamic geometry) to produce a set of axes and grid.

- Construct a line segment with endpoints on the grid. Show the coordinates of the endpoints.
- Calculate the rise (vertical distance between the endpoints) and the run (horizontal distance between the endpoints).
- Calculate the gradient as the *rise* divided by the *run*.
- Now drag the endpoints and explore the effect on the gradient.
- Can you drag the endpoints but retain the same gradient value? Explain why this is possible.
- Can you drag the endpoints so that the gradient is zero or undefined? Describe how this can be achieved.

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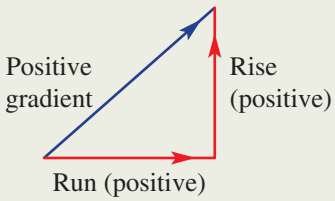
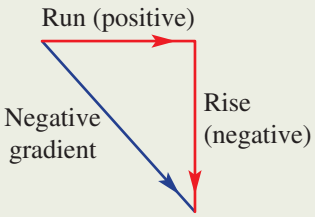
Key ideas

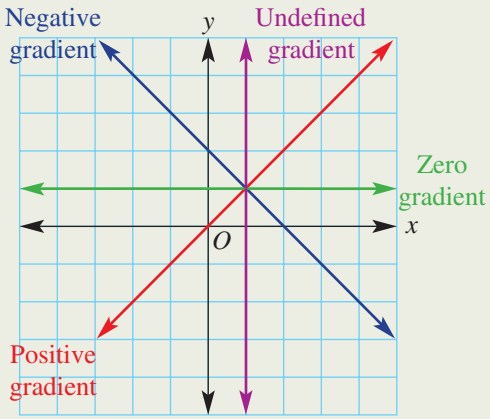
- The **gradient** of a line is a measure of its steepness or slope.
- The gradient of a line can be found using the formula $m = \frac{\text{rise}}{\text{run}}$.

Gradient The steepness of a straight line or interval





- We work from left to right, so the run is always positive.
- The gradient can be positive, negative, zero or undefined.
  - The gradient is positive if the graph increases from left to right.
  - The gradient is negative if the graph decreases from left to right.
  - The gradient is zero if the line is horizontal.
  - The gradient is undefined if the line is vertical.

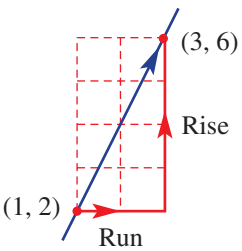


Exercise 5E

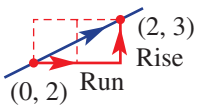
UNDERSTANDING AND FLUENCY 1–3, 4(½), 5, 6(½) | 3, 4(½), 5, 6(½)

1 For these lines, calculate the value for: **i** run **ii** rise

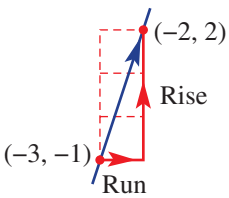
a



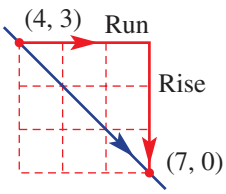
b



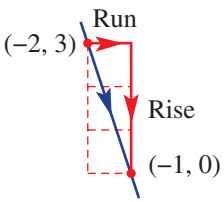
c



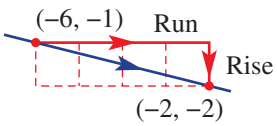
d



e



f



Remember that the rise is negative if the line slopes downward from left to right.

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2 Use the word 'positive', 'negative', 'zero' or 'undefined' to complete each sentence.

a The gradient of a horizontal line is _____.

b The gradient of the line joining (0, 3) with (5, 0) is _____.

c The gradient of the line joining (−6, 0) with (1, 1) is _____.

d The gradient of a vertical line is _____.

3 Simplify these fractions as much as possible.

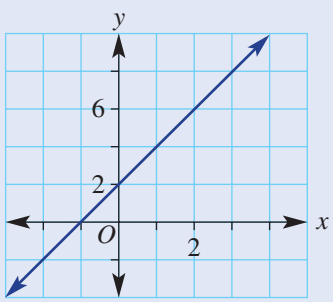
a $\frac{6}{3}$ **b** $\frac{-8}{2}$ **c** $\frac{4}{4}$

d $\frac{2}{4}$ **e** $\frac{-6}{4}$ **f** $\frac{-10}{6}$

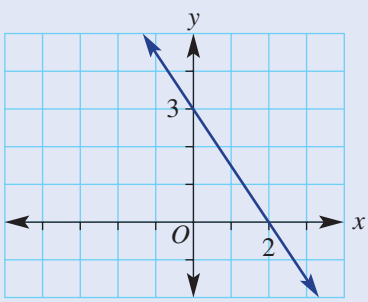
Example 13 Finding the gradient of a line

For each graph, state whether the gradient is positive, negative, zero or undefined, then find the gradient, where possible.

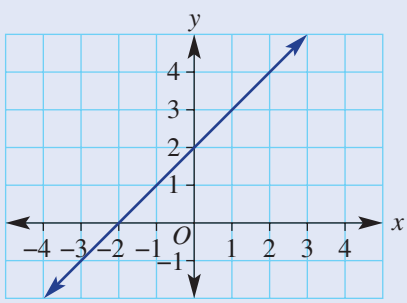
a



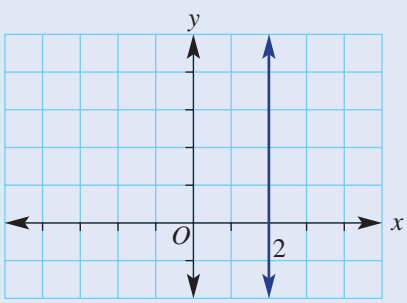
b



c

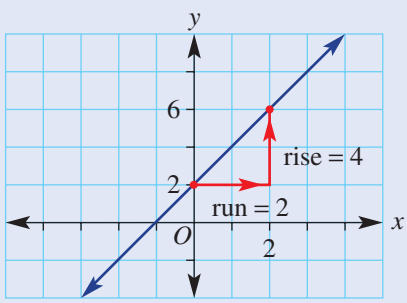


d



SOLUTION

a The gradient is positive.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{4}{2}$$
$$= 2$$

EXPLANATION

By inspection, the gradient is positive since the graph increases from left to right.

Select any two points and create a right-angled triangle between them to determine the rise and run.

Substitute rise = 4 and run = 2, and simplify.

Example is continued on the next page

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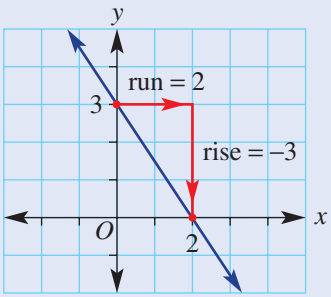
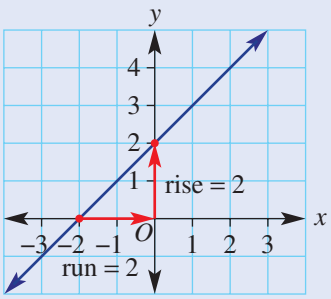
b The gradient is negative.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-3}{2} = -\frac{3}{2}$$

By inspection, the gradient is negative since y values decrease from left to right.
Create a right-angled triangle between two points.
Rise = −3 since it 'falls' 3 units, and run = 2.
It is best to leave the fraction as improper rather than a mixed numeral.

c The gradient is positive.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{2}{2} = 1$$

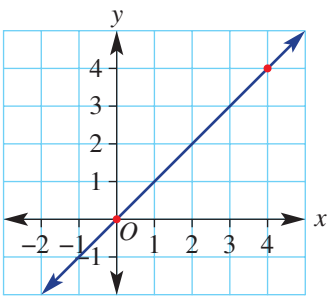
A rising graph indicates a positive gradient. For this triangle, rise = 2. The run is from left to right, from −2 to 0, so the run is +2.

d The gradient is undefined.

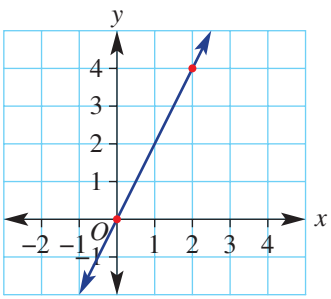
The line is vertical.

4 For each graph, state whether the gradient is positive, negative, zero or undefined, then find the gradient where possible.

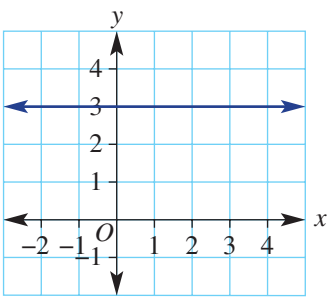
a



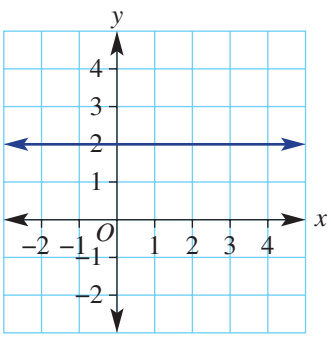
b



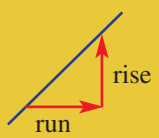
c



d



Create a right-angled triangle to find the rise and the run.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

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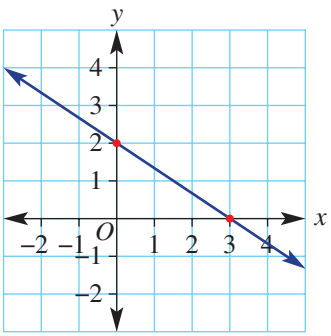
e



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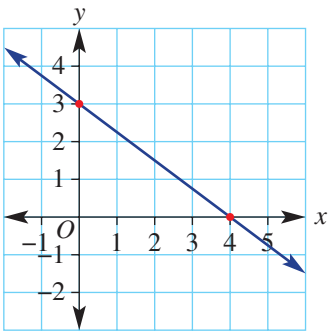
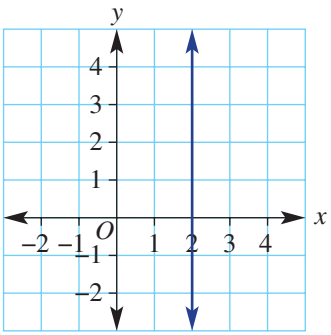
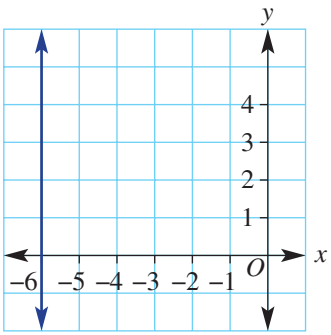


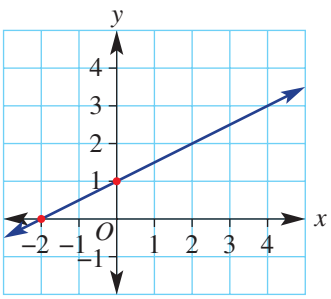
g



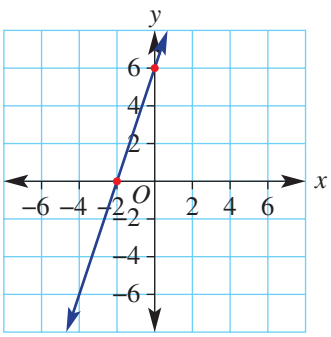
h



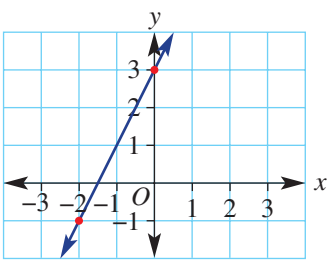
i



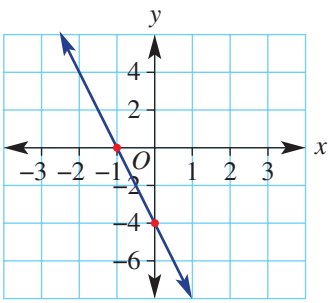
j



k



l



In part **k**, from −1 to 3 is a rise of 4.

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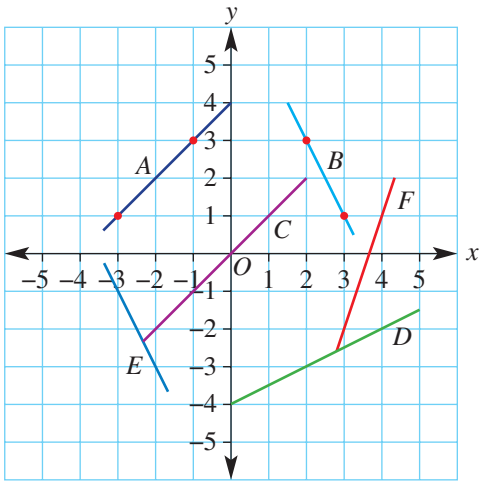
5 Find the gradient of each line A–F on this graph and grid.



Pick two known points on each line to find the gradient between them.

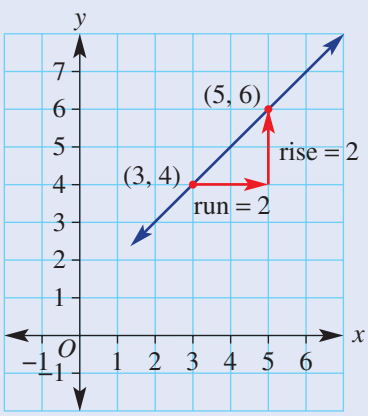
Example 14 Finding the gradient between two points

Use a graph to find the gradient (m) of the line joining the given points.

a $A(3, 4)$ and $B(5, 6)$ **b** $A(-3, 6)$ and $B(1, -3)$

SOLUTION

a

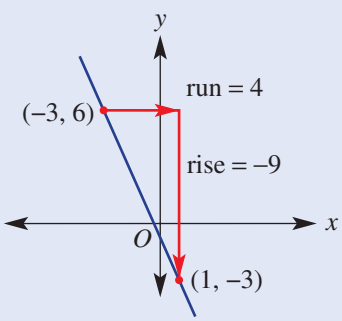


$$m = \frac{\text{rise}}{\text{run}} = \frac{2}{2} = 1$$

EXPLANATION

Plot the two points on a set of axes and construct a right-angled triangle between them. Observe from the graph that the gradient is positive. Calculate the gradient by finding the rise and run.

b



$$m = \frac{\text{rise}}{\text{run}} = \frac{-9}{4} \text{ or } -\frac{9}{4} \text{ or } -2.25$$

Plot points A and B and join in a line. Observe that the gradient is negative.
From left to right, -3 to 1, run $= 4$.
Rise from 6 to -3 is a 'fall' of 9 units, so rise $= -9$.
Note: leave the fraction as a simplified improper fraction, not a mixed numeral.

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6 Find the gradients of the lines joining the following pairs of points.

a $A(2, 3)$ and $B(3, 5)$
b $C(0, 8)$ and $D(2, 6)$
c $E(0, 4)$ and $F(2, 0)$
d $G(2, 1)$ and $H(5, 4)$
e $A(1, 5)$ and $B(2, 7)$
f $C(-2, 4)$ and $D(1, -2)$
g $E(-3, 4)$ and $F(2, -1)$
h $G(-1, 5)$ and $H(1, 6)$
i $A(-4, -2)$ and $B(-2, 1)$
j $C(1, 1)$ and $(3, -4)$
k $E(3, 2)$ and $F(0, 1)$
l $G(-1, 1)$ and $H(-3, -4)$

Plot the points first.

PROBLEM SOLVING AND REASONING 7, 8 7–10

7 Find the gradients, using $\frac{\text{rise}}{\text{run}}$, corresponding to the following slopes.

a A road falls 10 m for every 200 horizontal metres.
b A cliff rises 35 metres for every 2 metres horizontally.
c A plane descends 2 km for every 10 horizontal kilometres.
d A submarine ascends 150 m for every 20 horizontal metres.

The horizontal distance is the run.

8 A firecracker ascends with a gradient of 2. How far, horizontally, has the cracker travelled after rising 80 m?

9 Sally runs up a sand dune that has a gradient of $\frac{1}{3}$. How far, horizontally, has Sally moved after rising 30 m?

10 Find the missing number.

a The gradient joining the points $(0, 2)$ and $(1, ?)$ is 4.
b The gradient joining the points $(?, 5)$ and $(3, 9)$ is 2.
c The gradient joining the points $(-3, 4)$ and $(?, 1)$ is -1.
d The gradient joining the points $(-4, ?)$ and $(-2, -12)$ is -4.

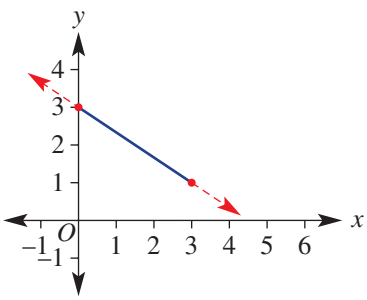
A gradient of $4 = \frac{4}{1}$; a rise of 4 units for every 1 unit to the right.

ENRICHMENT – 11

Where does it hit?

11 The line here has gradient $\frac{-2}{3}$, which means that it falls 2 units for every 3 units across. The y-intercept is 3.

a Use the gradient to find the y-coordinate on the line where:
  i $x = 6$
  ii $x = 9$
b What will be the coordinates of the x-intercept?
c What would be the x-intercept if the gradient was changed to:
  i $\frac{-1}{2}$?
  ii -2?
  iii $\frac{-5}{4}$?



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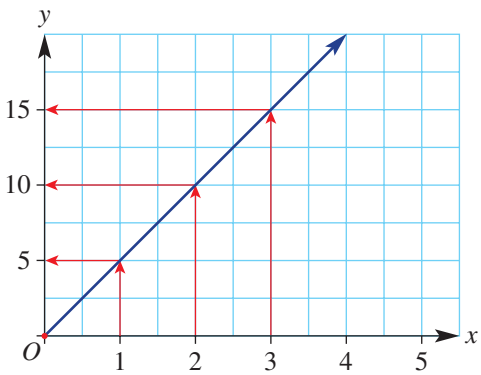
5F Gradient and direct proportion

Interactive
Widgets
HOTsheets
Walkthrough

The table below and graph opposite are showing that:

- x starts from O.
- y starts from O.
- Every time x increases by 1, y increases by 5.

x	0	1	2	3
y	0	5	10	15



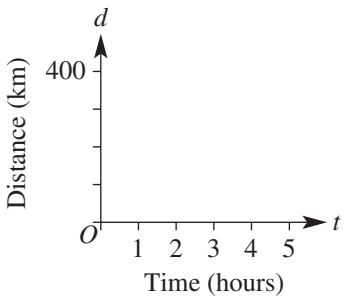
In situations like this we say 'y is in direct proportion to x' or 'y is directly proportional to x'.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Average speed

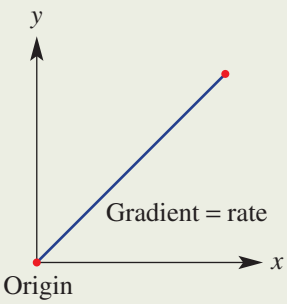
Over 5 hours, Sandy travels 400 km in a car.

- What is Sandy's average speed for the trip?
- Draw a graph of distance versus time for the journey, assuming a constant speed.
- Where does your graph intersect the axes and why?
- Find the gradient of your graph. What do you notice?
- Find a rule linking distance (d) and time (t).



Key ideas

- A **rate** is the change in one variable compared with another (often time). For example, 60 km/h (60 kilometres per hour) is a rate. It states a 60 km change in distance for each hour in time that passes.
- If two variables are **directly proportional**, the:
  - rate of change of one variable with respect to the other is constant
  - graph is a straight line passing through the origin
  - rule is of the form $y = mx$
  - gradient (m) of the graph equals the rate of change of the y variable with respect to the x variable.



Rate The number of units of one quantity for each single unit of another quantity

Direct proportion The relationship between two quantities that increase or decrease at the same rate

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Exercise 5F

UNDERSTANDING AND FLUENCY	1–7	5–8

1 Write 'Yes' or 'No' to state which of the following are rates.

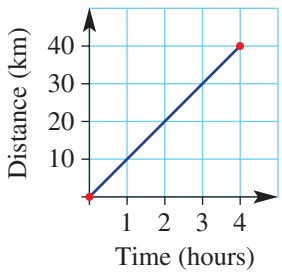
a \$120

b 12 cm/s

c 150 mL/min

d 80 km

2 This graph shows how far Caroline travels on her bike over 4 hours.



a State how far Caroline has travelled after:

i 1 hour

ii 2 hours

iii 3 hours

b Write down the speed of the bike in km/h (rate of change of distance over time).

c Find the gradient of the graph.

d What do you notice about your answers from parts **b** and **c**?

3 The rule linking the height of a plant over time is given by $h = 5t$, where h is in millimetres and t is in days.

a Find the height of the plant after 3 days ($t = 3$).

b Find the time for the plant to reach:

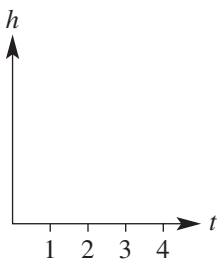
i 30 mm

ii 10 cm

c Copy and complete this table.

t	0	1	2	3	4
h					

d Copy and complete this graph.



Add an appropriate scale to the h-axis and plot and join the points from the table.

e Find the gradient of the graph.

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Example 15 Simplifying rates

Write the following rates in simplest form.

a 80 metres in 10 seconds

b 160 heartbeats in 2 minutes

SOLUTION	EXPLANATION
a $80 \div 10 = 8$ 8 m/s	Convert to metres per 1 second by dividing by 10.
b $160 \div 2 = 80$ 80 beats/min	Divide by 2 to get 80 heartbeats in 1 minute.

4 Write these rates in simplest form.

a 120 km in 2 hours

b 180 m in 20 seconds

c 400 L in 80 min

d \$30 for 20 litres

e \$900 in 30 hours

f 15°C in 5 min

g 70 grams in 30 seconds

h \$480 in 6 hours

i 12 mL in 4 minutes

j \$1.20 for 2 seconds

Simplest form is per 1 unit; e.g. 50 km in 1 hour, 50 km/h.

Example 16 Forming rules

Write a rule linking the variables A dollars and t hours if \$60 is earned in 5 hours of work.

SOLUTION	EXPLANATION
$60 \div 5 = 12$ Rate is \$12/h.	÷5 \$60 in 5 hours ÷5 \$12 in 1 hours
$A = 12 \times t$ $A = 12t$	The amount earned, A, is \$12 for each hour worked. t hours of work earns $12 \times t$ dollars.

5 Write down a rule linking the given variables.

a I travel 720 km in 12 hours. Use d for distance and t for time.

b A calf grows 12 cm in 6 months. Use g for growth height and t for time.

c The cost of petrol is \$120 for 80 litres. Use C for cost and n for the number of litres.

d The profit is \$10 000 for 500 tonnes. Use P for profit and t for the number of tonnes.

Establish the rate first.

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Example 17 Exploring direct proportion

Water is poured into an empty tank. It takes 3 hours to fill the tank with 6000 litres.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) using $0 \leqslant t \leqslant 3$.

c Find the:

- **i** gradient of your graph
- **ii** rule for V

d Use your rule to find the:

- **i** volume after 1.5 hours
- **ii** time to fill 5000 litres

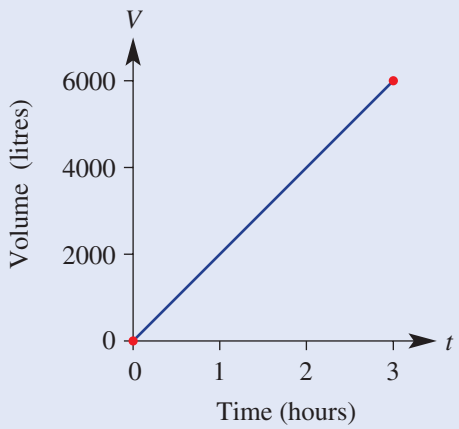
SOLUTION

a 6000 L in 3 hours = 2000 L/h

EXPLANATION

6000 L per 3 hours $\xrightarrow{\div 3}$ 2000 L per 1 hour

b



Draw axes with volume on the vertical axis and time on the horizontal axis, since volume depends on time.
Plot the two endpoints:
(0, 0), since the tank starts empty, and (3, 6000).
Join these points with a straight line.

c i Gradient $= \dfrac{6000}{3} = 2000$

ii $V = 2000\,t$

Using the endpoints, rise = 6000, run = 3.
Note that the gradient is the same as the rate.
2000 L are filled for each hour.

d i
$$\begin{aligned} V &= 2000\,t \\ &= 2000 \times 1.5 \\ &= 3000 \end{aligned}$$
$\therefore$ 3000 L after 1.5 hours

Substitute $t = 1.5$ into your rule.

ii
$$\begin{aligned} V &= 2000\,t \\ 5000 &= 2000\,t \\ 2.5 &= t \end{aligned}$$
$\therefore$ It takes $2\frac{1}{2}$ hours to fill 5000 L.

Substitute $V = 5000$ into the rule and solve for t by dividing each side by 2000.

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6 A 300-litre fish tank takes 3 hours to fill by hose.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) using $0 \leqslant t \leqslant 3$.

c Find the:

i gradient of your graph

ii rule for V

d Use your rule to find the:

i volume after 1.5 hours

ii time to fill 2000 litres

7 A solar powered car travels 100 km in 4 hours.

a What is the rate of change of distance over time (i.e. speed)?

b Draw a graph of distance (d km) vs time (t hours) using $0 \leqslant t \leqslant 4$.

c Find the:

i gradient of your graph

ii rule for d

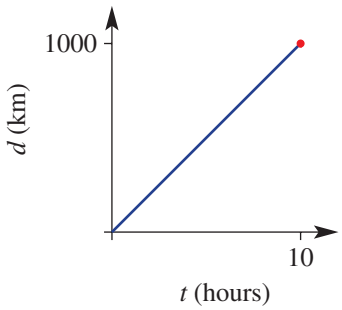
d Use your rule to find the:

i distance after 2.5 hours

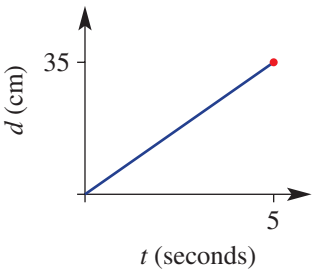
ii time to travel 40 km

8 Use the gradient to find the rate of change of distance over time (speed) for each of these graphs. Use the units given on each graph.

a



b



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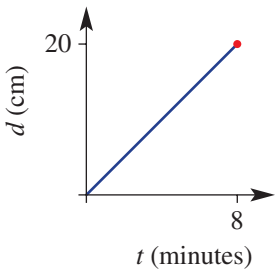
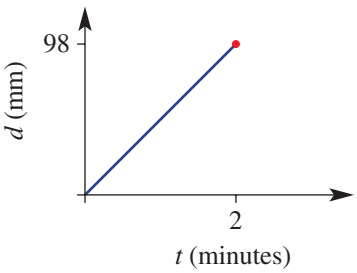


d



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PROBLEM SOLVING AND REASONING

	9, 10	9, 11, 12

9 Who is travelling the fastest?
- Mick runs 720 m in 2 minutes.
- Sally rides 550 m in 1 minute.
- Udhav jogs 2000 m in 5 minutes.

To compare rates, they will need to be in the same units.

10 Which animal is travelling the slowest?
- A leopard runs 400 m in 30 seconds.
- A jaguar runs 2700 m in 3 minutes.
- A panther runs 60 km in $1\frac{1}{3}$ hours.

11 A car's trip computer says that the fuel economy for a trip is 8.5 L per 100 km.
- **a** How many litres would be used for 120 km?
- **b** How many litres would be used for 850 km?
- **c** How many kilometres could be travelled if the capacity of the car's petrol tank was 68 L?

Consider how many litres are used in 1 kilometre.

12 The area of a particular rectangle is given by $A = lb$. If its length is fixed at 12 cm but its breadth, b cm, can vary:
- **a** write a rule for the area of this rectangle
- **b** draw a graph of A against b for $0 \leqslant b \leqslant 4$
- **c** decide if the area of this rectangle is directly proportional to its width. Explain.

Equations of the form $y = mx$ show that y is directly proportional to x.

ENRICHMENT

	–	13, 14

Rate challenge

13 Hose A can fill a bucket in 2 minutes and hose B can fill the same bucket in 4 minutes.
- **a** What fraction of a bucket does hose A fill in 1 minute?
- **b** What fraction of a bucket does hose B fill in 1 minute?
- **c** If both hoses were used at the same time, what fraction of a bucket could they fill in 1 minute?
- **d** If both hoses were used at the same time, how long would it take to fill the bucket?

14 Vonda can vacuum an office building in 2 hours and her husband Chris can vacuum the same office building in 3 hours. How long would it take to vacuum the office building if they both vacuumed at the same time?

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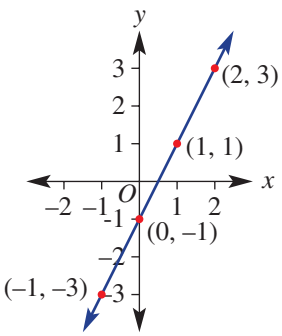
5G Gradient–intercept form

Interactive
Widgets
HOTsheets
Walkthrough

Shown here is the graph of the rule $y = 2x - 1$. It shows a gradient of 2 and a y-intercept of -1. The fact that these two numbers match numbers in the rule is no coincidence. This is why rules written in this form are called gradient–intercept form. Other examples of rules in this form include:

$$y = -5x + 2,\ y = \frac{1}{2}x - 0.5 \text{ and } y = \frac{x}{5} + 20.$$



Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: What's in common?

Sketch the following linear relationships on the same set of axes. Plot points or use the x- and y-intercepts.

a $y = 2x$ **b** $y = 2x + 2$ **c** $y = 2x - 1$

- What do these graphs have in common?
- Calculate the gradient of each line. What do you notice?
- How do the gradients relate to the rules?

Now, sketch the following linear relationships on the same set of axes. Plot points or use the x- and y-intercepts.

a $y = x + 1$ **b** $y = 2x + 1$ **c** $y = -\frac{1}{2}x + 1$

- What do these graphs have in common?
- How does the common feature relate to the rules?
- Can you form a rule for a linear relationship with a gradient of 3 and a y-intercept at 2?

Key ideas

m = gradient b = y-intercept

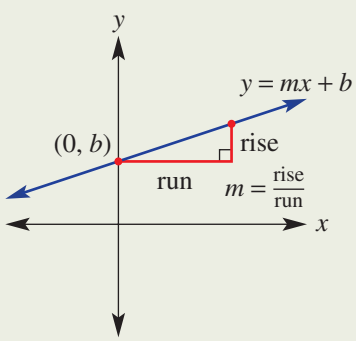
- $y = mx + b$ (or $y = mx + c$) is the **gradient–intercept form** of a straight line equation.

Gradient–intercept form The equation of a straight line, written with y as the subject of the equation

- If the y-intercept is zero, the equation becomes $y = mx$ and these graphs will pass through the origin.
- Any linear relationship can be rearranged to be written in gradient–intercept form by making y the subject.
- To sketch a graph using the gradient–intercept method, find the y-intercept and use the gradient to find a second point.

  For example: if $m = \frac{2}{5}$, move 5 across and 2 up from the y-intercept.

  For the gradient of $-2 = \frac{-2}{1}$ you would move 1 across and 2 down from the y-intercept.



- Graphs can also be drawn using computer software, such as Geogebra.

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Exercise 5G

UNDERSTANDING AND FLUENCY | 1–4, 5–8(½) | 4, 5–8(½)

1 In the gradient–intercept form $y = mx + b$:

a m is the _____.

b b is the _____.

c y is the _____ of the equation.

2 Match the given gradient and y-intercept in the first column with a rule in the left column.

a gradient = 2, y-intercept = 5

b gradient = 5, y-intercept = 2

c gradient = −2, y-intercept = 3

d gradient = −1, y-intercept = −2

i $y = 5x + 2$

ii $y = -2x + 3$

iii $y = 2x + 5$

iv $y = -x - 2$

3 **a** From the point (0, 2), give the coordinates of the point that is:

i 2 across and 4 up

ii 3 across and 1 down

iii 4 across and 6 down

b From the point (0, −1), give the coordinates of the point that is:

i 1 across and 2 up

ii 3 across and 4 down

Plot the point to help you visualise, if required.

4 Fill in the missing numbers.

a The gradient $\frac{5}{3}$ describes a run of _____ for a rise of _____.

b The gradient 4 describes a run of _____ for a rise of _____.

c The gradient $\frac{-1}{2}$ describes a run of _____ for a rise of _____.

Gradient = $\frac{\text{rise}}{\text{run}}$.

Example 18 Stating the gradient and y-intercept

State the gradient and the y-intercept for the graphs of the following relationships.

a $y = 2x + 1$

b $y = -3x$

SOLUTION	EXPLANATION
a $y = 2x + 1$ Gradient = 2 y-intercept = 1	The rule is given in gradient intercept form, $y = mx + b$. The gradient is the coefficient of x (the numeral multiplied by x). The constant term is the y-intercept.
b $y = -3x$ Gradient = −3 y-intercept = 0	The gradient is the coefficient of x including the negative sign. The constant term is not present so the y-intercept = 0.

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5 State the gradients and y-intercepts for the graphs of the following relationships.

a $y = 3x - 4$ **b** $y = -5x - 2$ **c** $y = -2x + 3$

d $y = \frac{1}{3}x + 4$ **e** $y = -4x$ **f** $y = 2x$

g $y = 2.3x$ **h** $y = -0.7x$

$y = mx + b$

gradient y-intercept

Example 19 Rearranging linear equations into the form $y = mx + b$

Rearrange these linear equations into the form $y = mx + b$.

a $2x + y = 7$ **b** $4x + 2y = 10$

SOLUTION

a $2x + y = 7$

$y = 7 - 2x$

$y = -2x + 7$

b $4x + 2y = 10$

$2y = -4x + 10$

$y = -2x + 5$

EXPLANATION

To have y by itself, subtract $2x$ from both sides.

$7 - 2x$ is the same as $-2x + 7$, which is in the form $mx + b$

Solve for y by first subtracting $4x$ from both sides.

Divide both sides of the equation by 2:

$\frac{-4x + 10}{2} = \frac{-4}{2}x + \frac{10}{2} = -2x + 5$

6 Rearrange these linear equations into the form $y = mx + b$.

a $3x + y = 4$ **b** $2x + y = 7$ **c** $y - 3x = 2$

d $y - 2x = 7$ **e** $5x - y = 3$ **f** $4x + 2y = 8$

g $3x + 2y = 6$ **h** $4x - 2y = 5$ **i** $5x - 3y = -6$

Follow steps as if you were solving an equation for y.

Example 20 Sketching linear graphs using the gradient and y-intercept

Find the value of the gradient and y-intercept for these relationships and sketch their graphs.

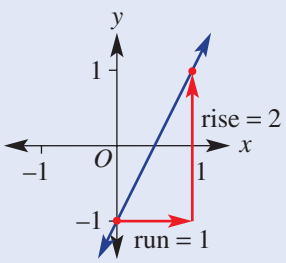
a $y = 2x - 1$ **b** $y = -\frac{3}{4}x + 2$

SOLUTION

a $y = 2x - 1$

y-intercept $= -1$

gradient $= 2 = \frac{2}{1}$



EXPLANATION

The rule is in gradient–intercept form so we can read off the gradient (coefficient of x) and the y-intercept (constant term).

Gradient $= \frac{2\text{ (rise)}}{1\text{ (run)}}$; for every 1 across, move 2 up.

Label the y-intercept at $(0, -1)$.

From $(0, -1)$ move 1 across (to the right) and 2 up to give a second point at $(1, 1)$.

Mark and join the points to form a line.

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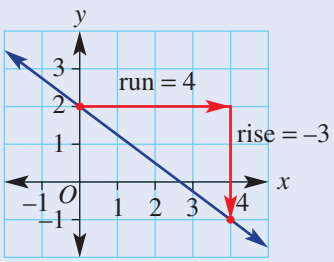
b $y = -\frac{3}{4}x + 2$

y-intercept $= 2$

Gradient $= \frac{-3}{4}$



Read off the y-intercept and gradient from the gradient–intercept form.
Rise $= -3$, run $= 4$.

Label the y-intercept $(0, 2)$ and from here move 4 across to the right (run) and 3 down (due to -3) to give a second point at $(4, -1)$.
Join points to form a line.

7 Find the gradients and y-intercepts for these relationships and sketch their graphs.

a $y = x - 2$ **b** $y = 2x + 1$ **c** $y = 3x + 2$

d $y = \frac{1}{2}x + 2$ **e** $y = -3x + 3$ **f** $y = \frac{3}{2}x + 1$

g $y = 2x$ **h** $y = -\frac{4}{3}x$

Plot the y-intercept first, then use the rise and run of the gradient.

Example 21 Rearranging equations to sketch using the gradient and y-intercept

Rearrange the equation $x + 2y = 6$ to find the gradient and y-intercept, and sketch its graph.

SOLUTION

$x + 2y = 6$

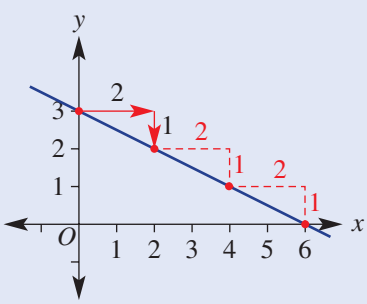
$2y = 6 - x$

$y = 3 - \frac{1}{2}x$

$= -\frac{1}{2}x + 3$

So y-intercept is 3.

Gradient $= -\frac{1}{2}$



EXPLANATION

Make y the subject by subtracting x from both sides and then dividing both sides by 2.

$\frac{6 - x}{2} = \frac{6}{2} - \frac{x}{2} = 3 - \frac{1}{2}x$

Rewrite in the form $y = mx + b$ to read off the gradient and y-intercept

Link the negative sign to the rise (-1) so the run is positive $(+2)$.

Mark the y intercept $(0, 3)$ then from this point move 2 right and 1 down to give a second point at $(2, 2)$.

Note that the x-intercept will be 6. If the gradient is $\frac{-1}{2}$ then a fall of 3 needs a run of 6.

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8 Rearrange these equations to find the gradients and y-intercepts and sketch their graphs.

a $x + y = 4$ **b** $y - x = 6$ **c** $4x + 2y = 6$
d $x + 2y = 8$ **e** $2x + 3y = 6$ **f** $4x + 3y = 12$
g $x - 2y = 4$ **h** $2x - 3y = 6$ **i** $3x - 4y = 12$
j $x + 4y = 0$ **k** $x - 5y = 0$ **l** $8x - 2y = 0$

Solve for y to make y the subject with equations in the form $y = mx + b$.

PROBLEM-SOLVING AND REASONING

9, 10 | 9, 11, 12

9 During a heavy rainstorm, a rain gauge is filling with water. The rule for the volume of water in the tank (V mL), t hours after the start of the storm, is given by $V = 4t + 2$.

a State the gradient and V-intercept of this relationship and sketch its graph.

b In this scenario, what do the gradient and the V-intercept represent?

Here, the V-intercept is like the y-intercept.

10 Vera is trying to convince her friend Li that $\frac{4x + 6}{2} = 2x + 6$. She offers to pay Li \$2 if she is wrong. Does Vera lose her money? Explain.

11 Which of these linear relationships have a gradient of 2 and y-intercept of -3?

a $y = 3 - 2x$ **b** $y = \frac{2x - 6}{2}$ **c** $y = \frac{4x - 6}{2}$
d $y = \frac{3 - 2x}{-1}$ **e** $2y = 4x - 3$ **f** $-2y = 6 - 4x$

Rewrite each in the form $y = mx + b$ and look for $y = 2x - 3$.

12 Jeremy says that the graph of the rule $y = 2(x + 1)$ has gradient 2 and y-intercept 1.

a Explain his error.

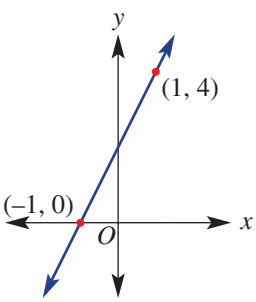
b What can be done to the rule to help show the y-intercept?

ENRICHMENT

– | 13

The missing y-intercept

13 This graph shows two points $(-1, 0)$ and $(1, 4)$, with a gradient of 2. By considering the gradient (2 up for every 1 across), the y-intercept can be calculated to be 2, so $y = 2x + 2$.



Use this approach to find the rules of the lines passing through these points.

a $(-1, 1)$ and $(1, 5)$
b $(-2, 4)$ and $(2, 0)$
c $(-2, -1)$ and $(3, 9)$
d $(-4, 1)$ and $(2, 4)$
e $(-1, 3)$ and $(1, 4)$

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5H Finding the equation of a line using $y = mx + b$ EXTENSION

Interactive
Widgets
HOTsheets
Walkthrough

When data points from an experiment are plotted, they may form a straight line. This shows a linear relationship between the two variables, such as time and growth, involved in the experiment.

By finding the equation of this line, we can develop a rule to relate the two variables. This rule can be used to make further predictions about the data, and could be applied to a larger scale experiment.

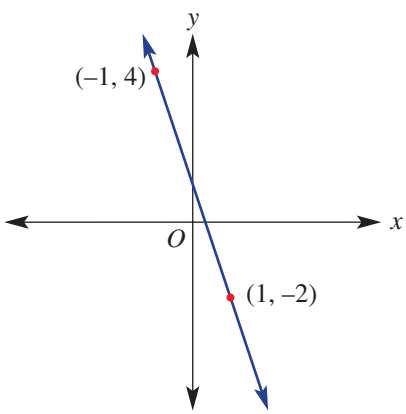
Using gradient–intercept form, the rule (or equation) of a line can be found by calculating the value of the gradient and the y-intercept.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: But we don't know the y-intercept!

A line with the rule $y = mx + b$ passes through two points $(-1, 4)$ and $(1, -2)$.

- Using the information given, is it possible to find the value of m? If so, calculate its value.
- The y-intercept is not given on the graph. Discuss what information could be used to find the value of the constant, b, in the rule. Is there more than one way you can find the y-intercept?
- Write the rule for the line.



Key ideas

- To find the equation of a line in gradient–intercept form, $y = mx + b$, you need to find the value of the:
  - gradient (m) using $m = \dfrac{\text{rise}}{\text{run}}$
  - constant (b), by observing the y-intercept or by substituting another point.
- All the points (x, y) on a line satisfy the equation of the line.
  For example: $(2, 5)$ is on the line with equation $y = 2x + 1$, since substituting $x = 2$ and $y = 5$ gives a true equation: $5 = 2 \times 2 + 1$.

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Exercise 5H EXTENSION

UNDERSTANDING AND FLUENCY	1–6, 7(½)	4, 5–7(½)

1 Fill in the missing words.
To find the equation of a line in the form $y = mx + b$, the _____ (m) is required as well as the _____ (b) or another _____.

2 Substitute the given values of m and b into $y = mx + b$ to write the rule.

a $m = 2, b = 5$ **b** $m = 4, b = -1$

c $m = -2, b = 5$ **d** $m = -1, b = -\frac{1}{2}$

3 Which one of the following lines does the point (3, 1) lie on?

A $y = 3x + 1$ **B** $y = x - 3$
C $y = -2x + 7$ **D** $y = 2x + 1$
E $y = -x - 2$

In which rule does $y = 1$ when $x = 3$?

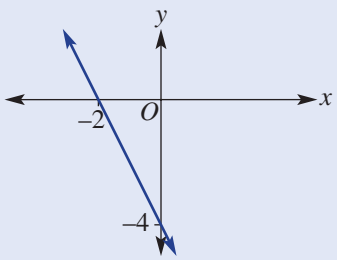
4 Substitute the point into the given rule and solve to find the value of b. For example, using (3, 4), substitute $x = 3$ and $y = 4$ into the rule.

a (3, 4), $y = x + b$ **b** (1, 5), $y = 2x + b$
c (−2, 3), $y = 3x + b$ **d** (−1, 6), $y = 4x + b$
e (3, −1), $y = -2x + b$ **f** (−2, 4), $y = -x + b$

In part **a**,
$4 = 3 + b$
$\therefore b = 1$

Example 22 Finding the equation of a line, given the y-intercept and another point

Determine the equation of the straight line shown here.



SOLUTION

$y = mx + b$

$m = \frac{\text{rise}}{\text{run}}$

$= \frac{-4}{2}$

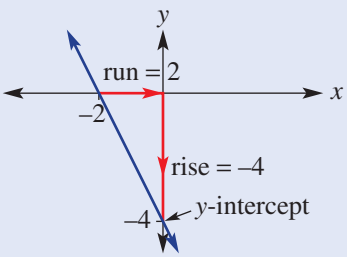
$= -2$

y-intercept $= -4$

$\therefore y = -2x - 4$

EXPLANATION

For the gradient–intercept form you need to find the gradient (m) and the y-intercept (b).



The y-intercept (−4) can be read from the graph.
In $y = mx + b$, substitute $m = -2$ and $b = -4$.

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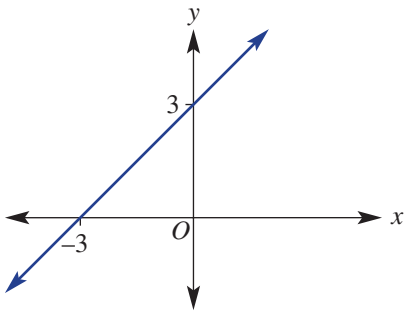
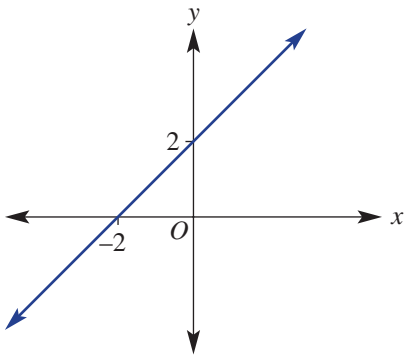
5 Determine the equations of the following straight lines.

Write in the form $y = mx + b$. You will need the gradient and the y-intercept.

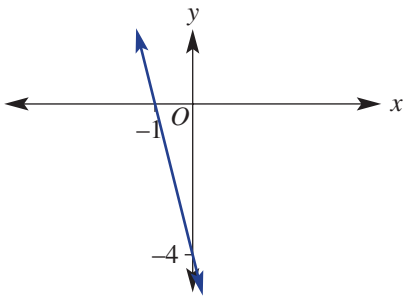
a



b



c



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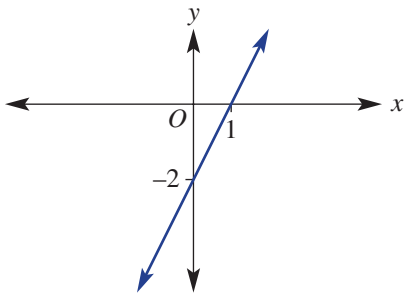

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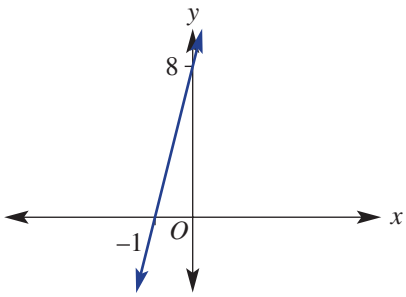

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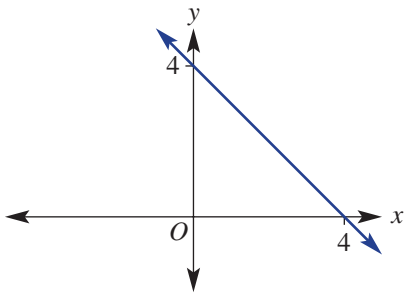
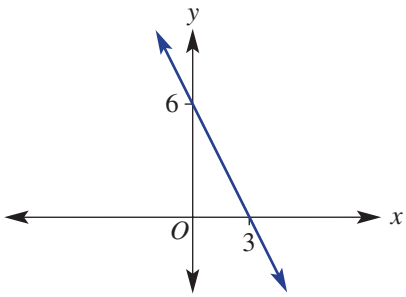


g



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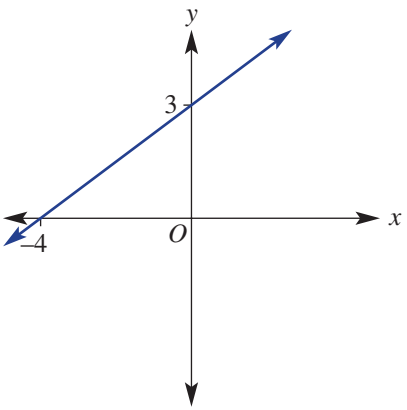
6 Find the equations of these straight lines that have fractional gradients. Simplify any fractions.

a



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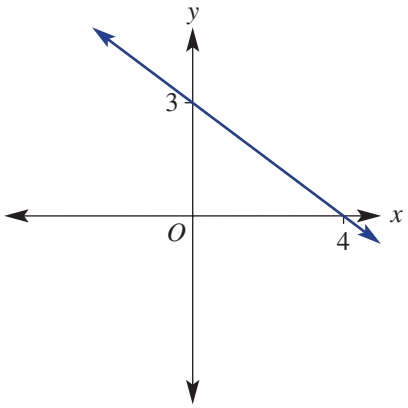

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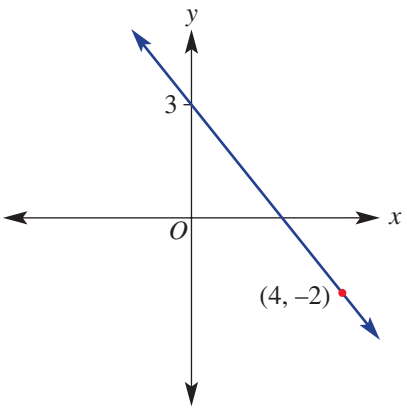

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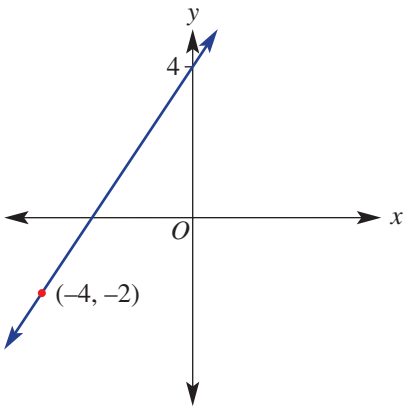
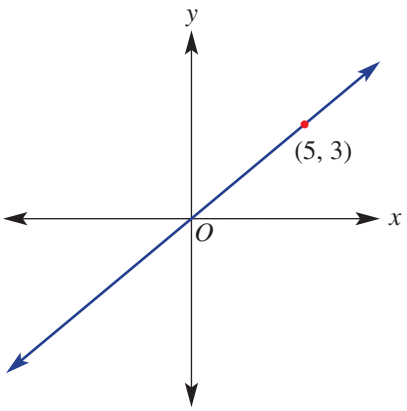
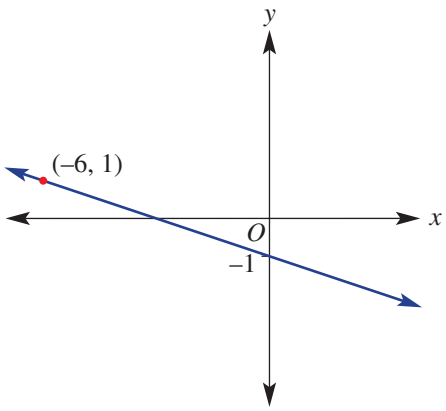


e



f



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Example 23 Finding the equation of a line, given the gradient and a point

Find the equation of the line that has a gradient, m, of 3 and passes through the point $(2, -1)$.

SOLUTION	EXPLANATION
$y = mx + b$	Substitute $m = 3$ into $y = mx + b$.
$y = 3x + b$	
$-1 = 3 \times 2 + b$	Since $(2, -1)$ is on the line, it must satisfy the equation $y = 3x + b$, hence substitute the point $(2, -1)$ where $x = 2$ and $y = -1$ to find b. Simplify and solve for b by subtracting 6 from both sides.
$-1 = 6 + b$	
$-7 = b$	
$\therefore y = 3x - 7$	Write the equation in the form $y = mx + b$.

7 Find the equation of the line that has a:

- **a** gradient of 3 and passes through the point $(1, 8)$
- **b** gradient of -2 and passes through the point $(2, -5)$
- **c** gradient of -3 and passes through the point $(2, 2)$
- **d** gradient of 1 and passes through the point $(1, -2)$
- **e** gradient of -3 and passes through the point $(-1, 6)$
- **f** gradient of 5 and passes through the point $(2, 9)$
- **g** gradient of -1 and passes through the point $(4, 4)$
- **h** gradient of -3 and passes through the point $(3, -3)$
- **i** gradient of -2 and passes through the point $(-1, 4)$
- **j** gradient of -4 and passes through the point $(-2, -1)$

In $y = mx + b$, insert the gradient value for m, then substitute the point for x and y to find b.

PROBLEM-SOLVING AND REASONING 8, 9 8, 10, 11

8 The coordinates $(0, 1)$ mark the take-off point 1 metre above the ground for a rocket constructed as part of a science class. The positive x direction is considered to be east. Find the equation of the rocket's path if it rises at a rate of 5 m vertically for every 1 m in an easterly direction.

First write the rule of the line.

9 A line has gradient -2 and y-intercept 5. Find its x-intercept.

10 Brad starts the year with \$80 in his bank account. He adds money into his account each week and does not take money out. After 4 weeks he has \$220 in his account. If the relationship is linear:

- **a** find a rule for the amount of money in the account after x weeks
- **b** use your rule to find when he will have \$500

Plot the linear information on a graph to help.

11 For the line connecting the following pairs of points find the:

- **i** gradient
- **ii** equation using $y = mx + b$

- **a** $(2, 6)$ and $(4, 10)$
- **b** $(1, 7)$ and $(3, -1)$
- **c** $(-3, 6)$ and $(5, -2)$
- **d** $(-4, -8)$ and $(1, -3)$

Compare with Example 23.

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ENRICHMENT – 12, 13

Two pieces of information make a linear rule

Assume that the relationships between the variables in these questions are linear.

12 Water is leaking from a tank. The volume of water in the tank is 100 L after 1 hour and 20 L after 5 hours.

a Find the equation of the line joining the points to find a rule. Use y for volume and x for time.

b Use the rule to state the amount of water that was in the tank to start with.

13 At 1 minute before midnight, the temperature inside a house was $-5°$C. The heater was switched on at this time. Three minutes after midnight, the temperature had reached $3°$C.

a Find the rule to represent this. (Plot the points on axes and find the equation of the line joining them. Use y for temperature and x for time.)

b Find the x-intercept and the y-intercept.

c Describe what the points in part **b** represent in a practical sense.

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5I Midpoint and length of a line segment from diagrams

Interactive
Widgets
HOTsheets
Walkthrough

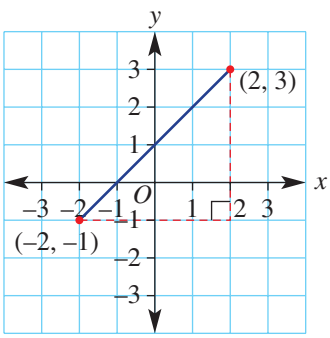
A line continues on forever, but a line segment (or line interval) has endpoints; this means it has a defined length and therefore must have a midpoint. Both the midpoint and length can be found by using the coordinates of the endpoints.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Choosing a method

This graph shows a line segment between the points at $(-2, -1)$ and $(2, 3)$.

- What is the horizontal distance between the two points?
- What is the vertical distance between the two points?
- Discuss and explain a method for finding the length of a line segment, using the right-angled triangle formed.
- What is the x-coordinate of the point halfway along the line segment?
- What is the y-coordinate of the point halfway along the line segment?
- Discuss and explain a method for finding the midpoint of a line segment.

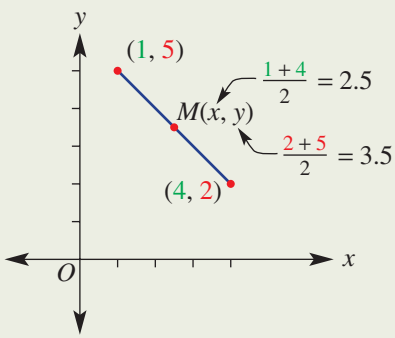


Using graphing software or dynamic geometry software, produce a line segment like the one shown above. Label the coordinates of the endpoints and the midpoint. Also find the length of the line segment. Now drag one or both of the endpoints to a new position.

- Describe how the coordinates of the midpoint relate to the coordinates of the endpoints. Is this true for all positions of the endpoints that you choose?
- Now use your software to calculate the vertical distance and the horizontal distance between the two endpoints. Then square these lengths. Describe these squared lengths compared to the square of the length of the line segment. Is this true for all positions of the endpoints that you choose?

Key ideas

- The **midpoint** (M) of a **line segment** is the halfway point between the two endpoints.
  - The x-coordinate is the average (mean) of the x-coordinates of the two endpoints.
  - The y-coordinate is the average (mean) of the y-coordinates of the two endpoints.
  - $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



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Key ideas

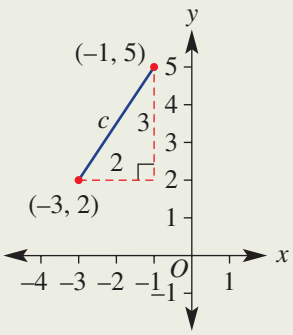
- The length of a line segment (or line interval) is found using Pythagoras' theorem.
  - The line segment is the hypotenuse (longest side) of a right-angled triangle.
  - The coordinates can be used to find the horizontal distance between them and the vertical distance.

Horizontal distance from -3 to -1 is 2
Vertical distance from 2 to 5 is 3
Pythagoras' theorem:

$$c^2 = 2^2 + 3^2$$
$$= 13$$
$$\therefore c = \sqrt{13}$$

Midpoint The point on a line segment that is an equal distance from each of the endpoints of the segment

Line segment A section of a straight line

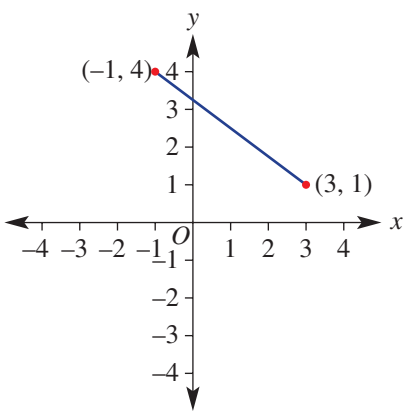


Exercise 5I

UNDERSTANDING AND FLUENCY 1–4, 5(½), 6, 7(½) | 4, 5(½), 6, 7(½)

1 Given the line segment shown, which are the coordinates of the midpoint of the segment?

A (2, 1.5)
B (1, 2.5)
C (2.5, 4.5)
D (0, 3.5)
E (–2, 2)



2 Find the number that is halfway between these pairs of numbers.

a 1, 7 **b** 5, 11 **c** –2, 4 **d** –6, 0
e 4, 7 **f** 0, 5 **g** –3, 0 **h** –4, –1

3 Evaluate c in the following, correct to two decimal places, given $c > 0$.

a $c^2 = 1^2 + 2^2$ **b** $c^2 = 5^2 + 7^2$ **c** $c^2 = 10^2 + 2^2$

If $c^2 = 7$,
$c = \sqrt{7}$

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4 **a** Find the horizontal distance between the points:
- **i** (1, 4) and (4, 6)
- **ii** (−2, 3) and (2, −1)

b Find the vertical distance between the points:
- **i** (−2, 3) and (0, 5)
- **ii** (2, 5) and (6, −2)

For horizontal distance, consider the x-coordinates. For vertical distance, consider the y-coordinates.

Example 24 Finding a midpoint

Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a (1, 0) and (5, 4) **b** (−3, −2) and (5, 3)

SOLUTION

a $x = \frac{1+5}{2}$

$= 3$

$y = \frac{0+4}{2}$

$= 2$

$\therefore M = (3, 2)$

b $x = \frac{-3+5}{2}$

$= \frac{2}{2}$

$= 1$

$y = \frac{-2+3}{2}$

$= \frac{1}{2}$

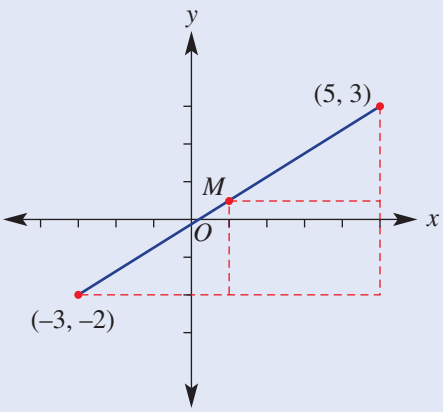
$= 0.5$

$\therefore M = (1, 0.5)$

EXPLANATION

Find the average (mean) of the x-coordinates and y-coordinates for both points.

By plotting the points to form the line segment, you can check that the coordinates you find for the midpoint appear to be the halfway point of the line segment.



5 Find the midpoints, $M(x, y)$, of the line segments joining these pairs of points.

a (0, 0) and (6, 6) **b** (0, 0) and (4, 4)
c (0, 2) and (2, 8) **d** (3, 0) and (5, 2)
e (−2, 0) and (0, 6) **f** (−4, −2) and (2, 0)
g (1, 3) and (2, 0) **h** (−1, 5) and (6, −1)
i (−3, 7) and (4, −1) **j** (−2, −4) and (−1, −1)
k (−7, −16) and (1, −1) **l** (−4, −3) and (5, −2)

Check that your coordinates for the midpoint appear to lie halfway along the line segment.

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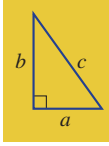
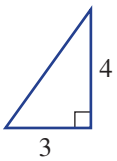
6 Use Pythagoras' theorem to find the length of the hypotenuse in these right-angled triangles. Round to one decimal place where required.

Pythagoras' theorem:

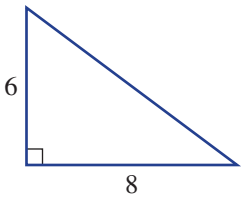


$c^2 = a^2 + b^2$

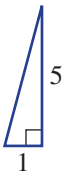
a



b



c



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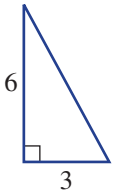
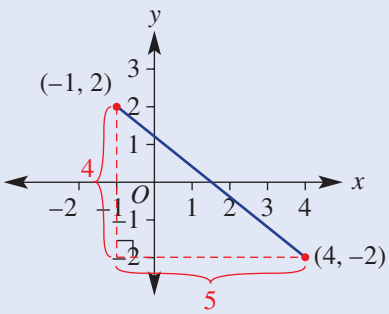


Example 25 Finding the length of a segment

Find the length of the segment joining $(-1, 2)$ and $(4, -2)$, correct to two decimal places.

SOLUTION



Horizontal length $= 5$

Vertical length $= 4$

$\therefore c^2 = 5^2 + 4^2$

$= 25 + 16$

$= 41$

$\therefore c = \sqrt{41}$

$\therefore$ length $= 6.40$ (to 2 d.p.)

EXPLANATION

Plot the points and form the line segment to visualise the problem.

Form the right-angled triangle.

Use the coordinates to find the lengths of the horizontal and vertical sides.

Horizontal length: from -1 to 4 is 5 units.
Vertical length: from 2 to -2 is 4 units.
Apply Pythagoras' theorem $c^2 = a^2 + b^2$.
From $c^2 = 41$, take the square root of both sides.

Round to two decimal places on a calculator as required.

7 Find the lengths of the segments joining these pairs of points, correct to two decimal places.

a $(1, 1)$ and $(2, 6)$
b $(1, 2)$ and $(3, 4)$
c $(0, 2)$ and $(5, 0)$
d $(-2, 0)$ and $(0, -4)$
e $(-1, 3)$ and $(2, 1)$
f $(-2, -2)$ and $(0, 0)$
g $(-1, 7)$ and $(3, -1)$
h $(-4, -1)$ and $(2, 3)$
i $(-3, -4)$ and $(3, -1)$

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PROBLEM SOLVING AND REASONING 8–10 8, 10–12

8 A fence line on the plans for a property shows fence posts at points with coordinates (−4, 8), (2, 3) and (8, −2). To reinforce the fence, two posts are to be placed halfway between the current posts. At what coordinates will these posts be placed?

9 Find the missing coordinates in this table if M is the midpoint of points A and B.

A	B	M
(4, 2)		(6, 1)
	(0, −1)	(−3, 2)
	(4, 4)	(−1, 6.5)

10 A circle has its centre at (2, 1). Find the coordinates of the endpoint of a diameter if the other endpoint has these coordinates.

a (7, 1) **b** (3, 6) **c** (−4, −1)

A diameter of a circle is shown: ⊘
It passes through the centre.

11 Find the perimeter of these shapes, correct to one decimal place.

a A triangle with vertices (−2, 0), (−2, 5) and (1, 3)

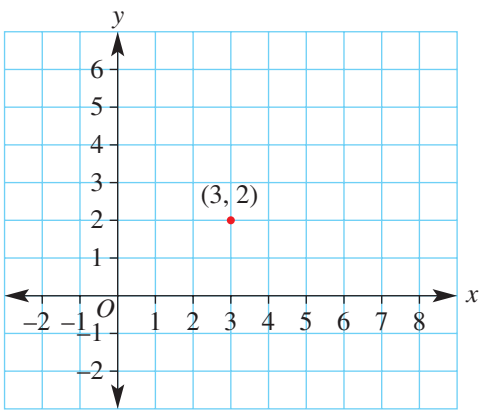
b A trapezium with vertices (−6, −2), (1, −2), (0, 4) and (−5, 4)

Start by plotting the points to form the line segments that make up the shape.

12 For the point (3, 2) shown:

a list four points that are 3 units from this point and add them to the axes

b describe the shape that would be formed by all the points that are 3 units from (3, 2)



ENRICHMENT – 13

Division by ratio

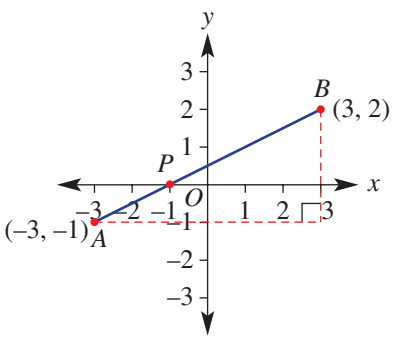
13 From left to right, this line segment shows the point $P(-1, 0)$, which divides the segment in the ratio 1 : 2.

a What fraction of the horizontal distance between the endpoints is P from A?

b What fraction of the vertical distance between the endpoints is P from A?

c Find the coordinates of point P on the segment AB if it divides the segment in the ratio:

i 2 : 1 **ii** 1 : 5 **iii** 5 : 1



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5J Linear modelling FRINGE

Interactive
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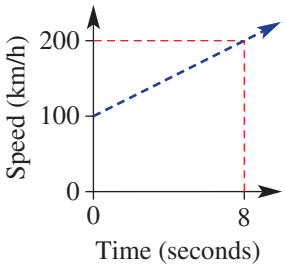
Walkthrough

If a relationship between two variables is linear, the graph will be a straight line. The equation linking the two variables can be written in gradient–intercept form. The process of using line graphs and rules for the relationship between two variables is called linear modelling. A test car, for example, increasing its speed from 100 km/h to 200 km/h in 8 seconds with constant acceleration, could be modelled by the rule $S = 12.5t + 100$. This rule could then be used to calculate the speed at different times in the test run.



Let's start: The test car

The above graph describes the speed of a racing car over an 8-second period.

- Explain how the rule $S = 12.5\,t + 100$ for the graph is found by describing what the 100 means and what the 12.5 means.
- Why might negative values of t not be considered for the graph?
- How could you accurately calculate the speed after 6.5 seconds?
- If the car continued to accelerate at the same rate, how could you accurately predict the car's speed after 13.2 seconds?

Key ideas

- Many situations can be modelled by using a linear rule or graph. The key elements of linear modelling include:
  - finding the rule linking the two variables
  - sketching a graph
  - using the graph or rule to predict or estimate the value of one variable, given the other
  - finding the rate of change of one variable with respect to the other variable. This is the same as finding the gradient.
- On a set of axes the **dependent variable** goes on the vertical axis and the **independent variable** goes on the horizontal axis.
  - If the variables are speed and time, then speed is on the vertical axis, since the speed reached depends on the time passed.
  - Time is usually on the horizontal axis.

Dependent variable The value that changes in response to changes made in the independent variable

Independent variable The value that is deliberately controlled when collecting data

Exercise 5J FRINGE

UNDERSTANDING AND FLUENCY | 1–7 | 4–8

1 **a** Insert the word *independent* or *dependent* in the following.

i The _____ variable goes on the vertical axis.

ii The _____ variable goes on the horizontal axis.

b Choose the dependent variable in the following cases.

i The volume of water, V, in a tank t minutes after turning on the hose

ii The height, h, of a tree t days after it was planted

iii The amount of money earned from the sale of m items

Which variable changes in response to changes in the other variable?

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2 A person gets paid \$50 plus \$20 per hour. Decide which rule describes the relationship between the pay, \$$P$, and number of hours, n.

A $P = 50 + n$ **B** $P = 50 + 20n$ **C** $P = 50n + 20$ **D** $P = 20 + 50$

3 The amount of money in a bank account is \$1000 and it is increasing by \$100 per month.

a Find the amount of money in the account after:

i 2 months **ii** 5 months **iii** 12 months

b Write a rule for the amount of money, A dollars, after n months.

4 If $d = 5t - 4$:

a find d if:

i $t = 10$ **ii** $t = 0$ **iii** $t = 2.5$

b find t if:

i $d = 6$ **ii** $d = 11$ **iii** $d = 27$

Example 26 Applying linear relationships

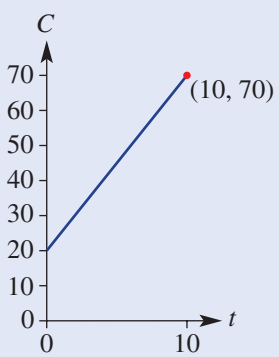
Netshare is an internet provider. It charges \$20 per month plus \$5 per hour of use.

a Write a rule for the total monthly cost, \$$C$, of using Netshare for t hours per month.

b Sketch the graph of C versus t using $0 \leqslant t \leqslant 10$.

c What is the total cost in a month when Netshare is used for 4 hours?

d If the monthly cost was \$50, for how many hours was Netshare used during the month?

SOLUTION	EXPLANATION
a $C = 20 + 5t$	A fixed amount of \$20 plus \$5 for each hour for t hours. This is $5 \times t$.
b C-intercept is20. At $t = 10$, $C = 20 + 5 \times 10 = 70$ $\therefore$ endpoint is $(10, 70)$	The cost, C, is the independent variable and so it goes on the vertical (y) axis. Let $t = 0$ to find the C-intercept. (This is the point on the vertical axis.) Letting $t = 10$ gives $C = 70$ and this gives the other endpoint.
	Sketch the graph using the points $(0, 20)$ and $(10, 70)$.
c $C = 20 + 5t$ $= 20 + 5 \times 4$ $= 40$ The cost is \$40.	Substitute $t = 4$ into the rule. Answer the question using the correct units.
d $C = 20 + 5t$ $50 = 20 + 5t$ $30 = 5t$ $\therefore t = 6$ Netshare was used for 6 hours.	Write the rule and substitute $C = 50$. Solve the resulting equation for t by subtracting 20 from both sides then dividing both sides by 5. Answer the question in words.

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5 A sales representative earns \$400 a week plus \$20 for each sale she makes.

a Write a rule that gives the total weekly wage, $\$W$, if the sales representative makes x sales.

b Draw a graph of W versus x using $0 \leqslant x \leqslant 40$.

c How much does the sales representative earn if, in a particular week, she makes 12 sales?

d If, in a particular week, the sales representative earns \$1000, how many sales did she make?

To sketch over $0 \leqslant x \leqslant 40$, find the endpoints; that is, the point where $x = 0$ and where $x = 40$.

6 A plumber charges a \$40 fee up-front per job, and \$50 for each hour he works.

a Find a linear equation for the total charge, $\$C$, for n hours of work.

b What will a 4-hour job cost?

c If the plumber works on a job for two days and averages 6 hours per day, what will be the total cost?

7 A catering company charges \$500 for the hire of a marquee (a giant tent), plus \$25 per guest.

a Write a rule for the cost, $\$C$, of hiring a marquee and catering for n guests.

b Draw a graph of C versus n for $0 \leqslant n \leqslant 100$.

c How much would a party catering for 40 guests cost?

d If a party cost \$2250, how many guests were catered for?

8 The cost, $\$C$, of recording a music CD is \$300, plus \$120 per hour of studio time.

a Write a rule for the cost, $\$C$, of recording a CD requiring t hours of studio time.

b Draw a graph of C versus t for $0 \leqslant t \leqslant 10$.

c How much does a recording using 6 hours of studio time cost?

d If a recording cost \$660 to make, for how long was the studio used?

PROBLEM SOLVING AND REASONING

9, 10 | 9, 11, 12

9 A petrol tank holds 66 litres of petrol. If it starts with 12 litres of petrol and the petrol pump fills it at 3 litres every 10 seconds, find:

a a linear equation for the amount of fuel (F litres) in the tank after t minutes

b how long it will take to fill the tank

Read units carefully.

10 A tank starts with 4000 L of water. Water from it is used at a rate of 20 L per minute.

a Write a rule for the volume, V litres, of water after t minutes.

b Calculate the volume after 1.5 hours.

c How long will it take for the tank to be emptied?

d How long will it take for the tank to have only 500 L?

If the water is being used, think about what should be happening to the volume of water in the tank.

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11 The rule for distance travelled, d km, over a given time, t hours, for a moving vehicle is given by $d = 50 + 80t$.

a What is the speed of the vehicle?

b If the speed was actually 70 km/h, how would this change the rule? Give the new rule.

12 The altitude, h metres, of a helicopter t seconds after it begins its descent is given by $h = 350 - 20t$.

a Use the rule to find or state:

i the rate at which the helicopter's altitude is decreasing

ii the helicopter's initial height ($t = 0$)

iii how long it will take for the helicopter to reach the ground

b If the rule was $h = 350 + 20t$, describe what the helicopter would be doing.

Altitude is the height above ground at which the helicopter is flying.

ENRICHMENT – 13

Where will they meet?

13 Car A heads out of town on the open highway travelling at an average speed of 65 km/h.

a Write a rule for the distance, d km, of the car from the town after t hours.

b Sketch a graph of distance (d) vs time (t) for $0 \leqslant t \leqslant 5$.

At the same time as car A leaves town, car B sets out on the highway from the opposite direction, travelling towards the town. The rule for the distance, d km, of car B from the town after t hours is given by $d = 350 - 75t$.

c How many kilometres from town does car B begin?

d How long will it take for car B to reach the town?

e Sketch the graph of the rule for car B on the same axes that you used in part **b**.

f Using your graph, find how many:

i hours into the journey the two cars will pass each other

ii kilometres out of town they will be when they pass each other

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5K Introducing non-linear relationships

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We have looked at linear relationships such as $j = 2x - 5$, and how they result in straight line graphs.

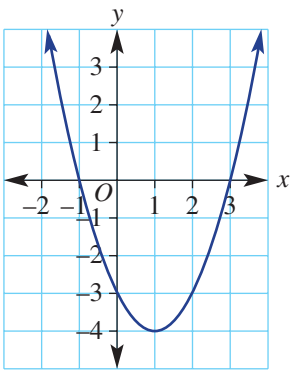
There are also many situations and rules that produce graphs that are non-linear. These rules involve terms such as x^2, 2^x and $\frac{1}{x}$.

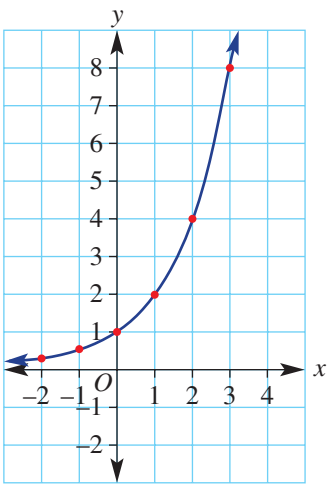
One common non-linear graph is the parabola. A parabola can model the path of a ball through the air (e.g. a soccer ball after it is kicked), an object thrown from a window or the arch of a bridge. Like a straight line, a parabola has some key features.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Finding features

Here are graphs of two types of non-linear relationships.





- With a partner, list what you consider to be the important features of each graph.
- Did your discussion include shape, intercepts and other key points?
- Relate the following list of words to one or both of the graphs.

  symmetry y-intercept x-intercept minimum
  parabola x-axis turning point positive
- Decide which rule below matches each graph.

  a $y = x^2 - 2x - 3$

  b $y = 2^x$

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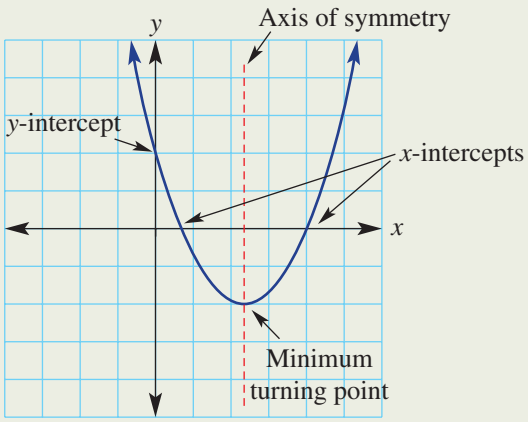
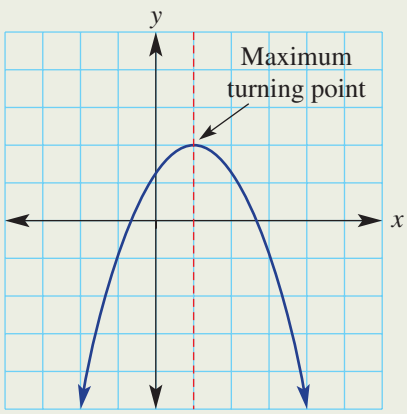
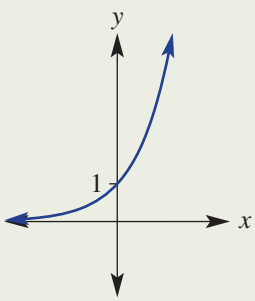
Key ideas

- The graph of a **quadratic relationship** is called a **parabola**.
  - The basic quadratic relationship is $y = x^2$ and has basic shape .
  - A quadratic relationship has a general equation $y = ax^2 + bx + c$, where a, b and c are constants, with $a \neq 0$.
- A parabola is symmetrical about a line called the axis of symmetry, and has a minimum or maximum **turning point**.





- The graph of an **exponential relationship** is called an exponential curve and it is always increasing or always decreasing.
  - The general exponential equation is $y = a^x$, $a > 0$. For $a > 1$



  - increasing curve
  - y-intercept is 1
  - the graph approaches the x-axis but does not cross it

Quadratic relationship An equation with a squared term but no term with a higher power

Parabola A graph of a quadratic relationship

Turning point The point at which a curve changes direction

Exponential relationship An equation with a term multiplied by itself one or more times

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Exercise 5K

UNDERSTANDING AND FLUENCY 1–5 3–6

1 Name two types of non-linear graphs.

2 Calculate:

a 2^2 **b** 3^2 **c** $(-2)^2$ **d** $(-1)^2$
e -2^2 **f** -1^2 **g** 0^2 **h** $-(-3)^2$

$3^2 = 3 \times 3$
$(-2)^2 = -2 \times (-2)$

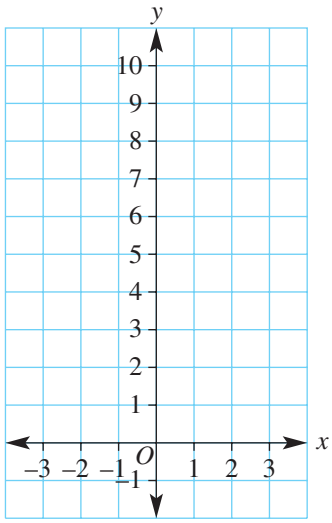
3 Calculate:

a 2^3 **b** 3^3 **c** 2^4 **d** 1^3 **e** 1^4

$2^3 = 2 \times 2 \times 2$
$2^4 = 2 \times 2 \times 2 \times 2$

4 Plot the points in this table on the axes to form the basic parabola $y = x^2$.

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9



Example 27 Plotting a parabola

Copy and complete the table of values shown and plot the points to sketch each of the following parabolas.

a $y = 2x^2$ **b** $y = -x^2 + 1$

x	−2	−1	0	1	2
y					

SOLUTION

a

x	−2	−1	0	1	2
y	8	2	0	2	8

EXPLANATION

Substitute each x value into the rule $y = 2x^2$.

For $x = 2$, $y = 2 \times 2^2$
$= 2 \times 4$
$= 8$

For $x = -1$, $y = 2 \times (-1)^2$ Recall $-1^2 = -1 \times (-1) = 1$
$= 2 \times 1$
$= 2$

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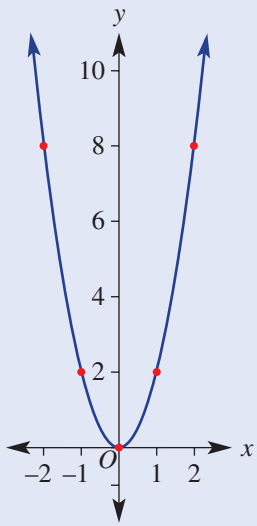
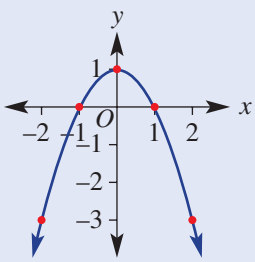


Plot points and join them to form a parabola.
Note that it has a minimum turning point.

b

x	–2	–1	0	1	2
y	–3	0	1	0	–3



Substitute each x value into the rule $y = -x^2 + 1$.

For $x = 2$, $y = -2^2 + 1$ $\quad 2^2 = 4$, so $-2^2 = -4$
$= -4 + 1$
$= -3$

For $x = -1$, $y = -(-1)^2 + 1$ $\quad (-1)^2 = -1 \times (-1) = 1$
$= -1 + 1$
$= 0$

Plot points and join them to form a parabola with a maximum turning point.

5 Copy and complete the table of values shown and plot the points to sketch the following parabolas.

x	–2	–1	0	1	2
y					

a $y = 3x^2$ **b** $y = x^2 + 2$ **c** $y = 2x^2 - 2$ **d** $y = x^2 + 2x + 1$
e $y = -2x^2$ **f** $y = -x^2 - 3$ **g** $y = -2x^2 + 8$ **h** $y = -x^2 + 2x - 2$

6 Plot the points from the tables below to form the exponential curves of the rules. Use an appropriate scale on your axes.

a $y = 2^x$

x	–2	–1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

b $y = 3^x$

x	–2	–1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

Look at the x values to choose a scale for the x-axis. Look at the y values to choose a scale for the y-axis. The scale must be even on each axis.

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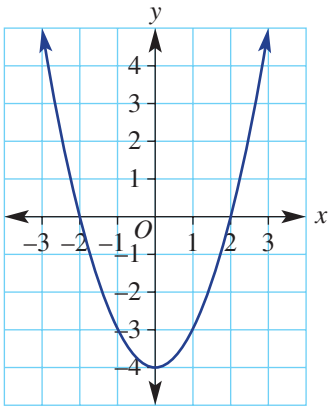
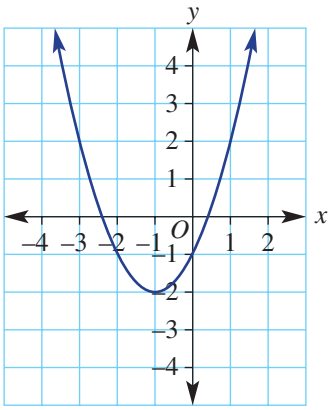
PROBLEM SOLVING AND REASONING 7, 8 7–9

7 Consider the four parabolas with rules shown below.

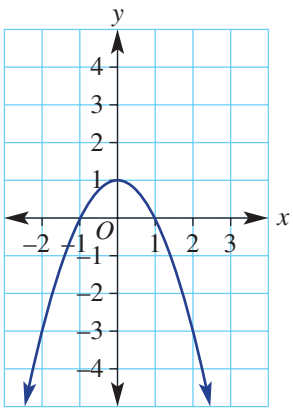
i $y = x^2 - 4$



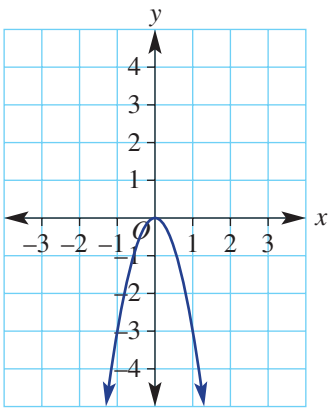
ii $y = x^2 + 2x - 1$



iii $y = -x^2 + 1$



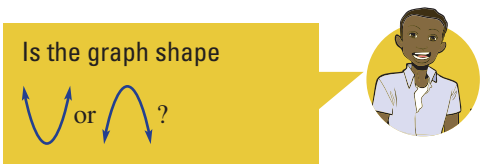
iv $y = -3x^2$



a For each graph, state whether it has a minimum turning point or a maximum turning point and give its coordinates.

b Given your answers to part **a**, what feature in the rule of your graph would be used to tell you if it has a maximum or minimum turning point?

c Draw in a line of symmetry on each graph.



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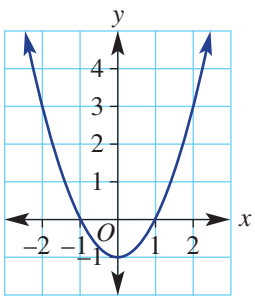
8 Match the following rules with the graphs.

a $y = 3x^2$ **b** $y = x^2 - 1$ **c** $y = 2^x$ **d** $y = 3^x$ **e** $y = -x^2$

i



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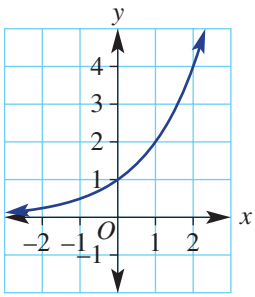

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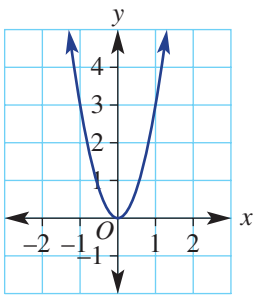
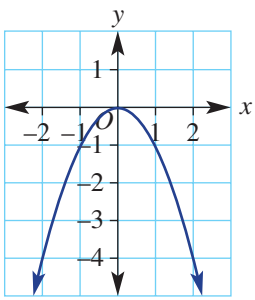


iv



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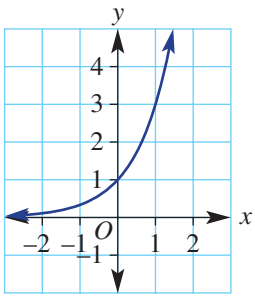
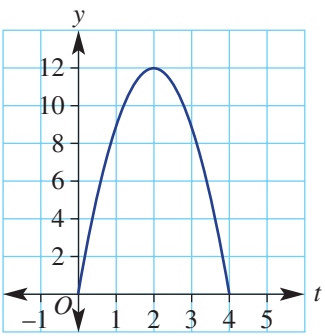


Substitute some x values into the rules to help you if needed.

9 The following graph shows the height of a cricket ball, y metres, as a function of time, t seconds.



a **i** At what times is the ball at a height of 9 m?

ii Why are there two different times?

b **i** At what time is the ball at its greatest height?

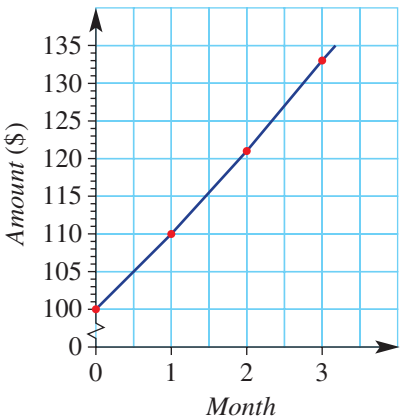
ii What is the greatest height the ball reaches?

iii After how many seconds does the ball hit the ground?

ENRICHMENT – 10

Increasing interest

10 This graph shows the amount of money in Yumi's bank account each month since the start of the year, as interest is added.



a What type of graph would you describe this as: linear, a parabola or exponential?

b How much was in the account at the start of the year?

c Use the graph to find how much was in the account after:

i 1 month **ii** 2 months **iii** 3 months

d Does the account increase by the same amount each month?

e Find the percentage increase in the account from:

i the start to month 1

ii month 1 to month 2

iii month 2 to month 3

f What can you say about the percentage increase from month to month?

g Find the approximate amount that will be in the account after month 4.

Remember to divide the increase by the original amount.

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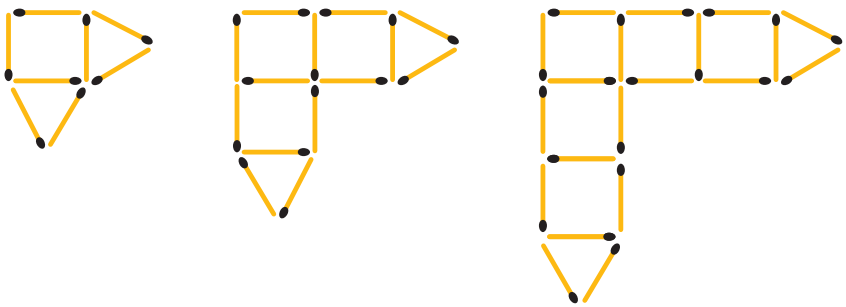
Puzzles and challenges

1 Matches are arranged by a student so that the first three diagrams in the pattern are:

How many matches are in the 50th diagram of the pattern?

2 The first three shapes in a pattern made with matches are:

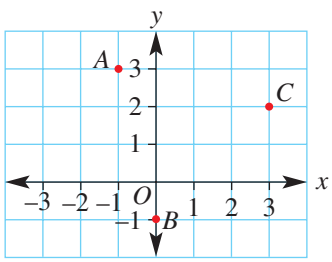


How many matches make up the 100th shape?

3 A tank with 520 L of water begins to leak at a rate of 2 L per day. At the same time, a second tank is being filled at a rate of 1 L per hour, starting at 0 L. How long does it take for the tanks to have the same volume?

4 The points $(-1, 4)$, $(4, 6)$, $(2, 7)$ and $(-3, 5)$ are the vertices of a parallelogram. Find the midpoints of its diagonals. What do you notice?

5 Prove that the triangle with vertices at the points $A(-1, 3)$, $B(0, -1)$ and $C(3, 2)$ is isosceles.



6 Bill takes 3 days to paint a house. Rashid takes 4 days to paint a house. Lucy takes 5 days to paint a house. How long would it take to paint a house (to the nearest hour) if all three of them worked together?

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Chapter summary

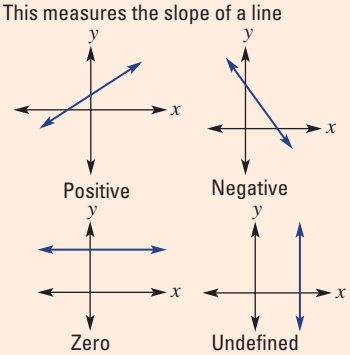
Linear relationships

Direct proportion

If two variables are directly proportional, their rule is of the form $y = mx$.
The gradient of the graph represents the rate of change of y with respect to x, e.g. for a car travelling at 60 km/h for t hours, distance, $d = 60t$.

Gradient

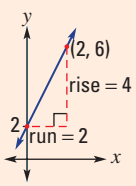
This measures the slope of a line



Gradient $m = \frac{\text{rise}}{\text{run}}$

$m = \frac{4}{2} = 2$

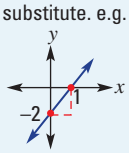
e.g.



Finding the equation of a line

Gradient–intercept form

($y = mx + b$) Require gradient and y-intercept or any other point to substitute. e.g.



$m = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$

y-intercept is –2

$\therefore\ y = 2x - 2$

Linear modelling

Define variables to represent the problem and write a rule relating the two variables.

Straight line

$y = mx + b$

gradient y-intercept

In the form $ax + by = c$, rearrange to $y = mx + b$ form to read off gradient and y-intercept

Midpoint and length of a line segment

Midpoint is halfway between segment endpoints.

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

e.g. midpoint of segment joining (–1, 2) and (3, 6)

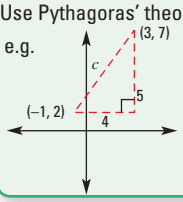
$x = \frac{-1 + 3}{2} = 1$

$y = \frac{2 + 6}{2} = 4$, i.e. (1, 4)

Length of segment

Use Pythagoras' theorem $c^2 = a^2 + b^2$

e.g.



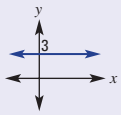
$c^2 = 4^2 + 5^2$

$c = \sqrt{41}$

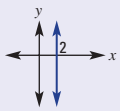
$c = 6.4$ (to 1 d.p.)

Lines with only one intercept

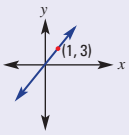
Horizontal line $y = b$
e.g. $y = 3$



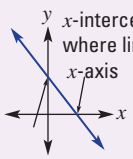
Vertical line $x = a$
e.g. $x = 2$



$y = mx$ passes through the origin, substitute $x = 1$ to find another point, e.g. $y = 3x$



x- and y-intercepts



e.g. for $2x + 3y = 6$

y-intercept ($x = 0$)
$2(0) + 3y = 6$
$3y = 6$
$y = 2$
y-intercept $= 2$

x-intercept ($y = 0$)
$2x + 3(0) = 6$
$2x = 6$
$x = 3$
x-intercept $= 3$

y-intercept where line cuts y-axis

x-intercept has y-coordinate 0.

y-intercept has x-coordinate 0.

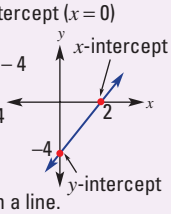
Sketching linear graphs

Sketch with two points. Often we find the x-intercept ($y = 0$) and y-intercept ($x = 0$)

e.g. $y = 2x - 4$

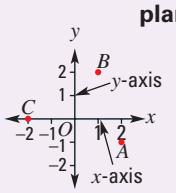
y-intercept: $x = 0$, $y = 2 \times 0 - 4$
$= -4$

x-intercept: $y = 0$, $0 = 2x - 4$
$2x = 4$
$x = 2$



Plot these points and join in a line.

Coordinates and the Cartesian plane

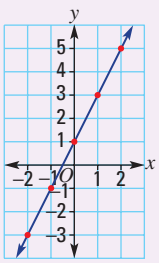


Each point on the plane is a coordinate pair (x, y).

(0, 0) is the origin.
A has coordinates (2, –1).
B has coordinates (1, 2).
C has coordinates (–2, 0).

A linear relationship is made up of points (x, y) that form a straight line when plotted, e.g. $y = 2x + 1$

x	–2	–1	0	1	2
y	–3	–1	1	3	5



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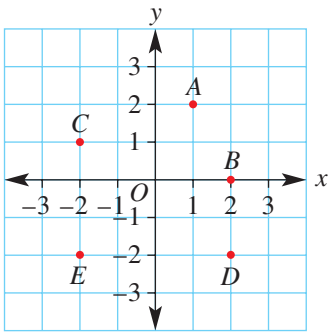
Multiple-choice questions

1 The point(s) on the Cartesian plane shown with an x-coordinate of 2 are:



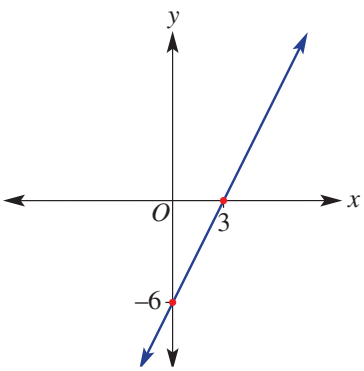
A B only **B** A only **C** C, D and E
D B and D **E** E and D

2 The coordinates of the x-intercept of the graph of $2x + 4y = 12$ are:

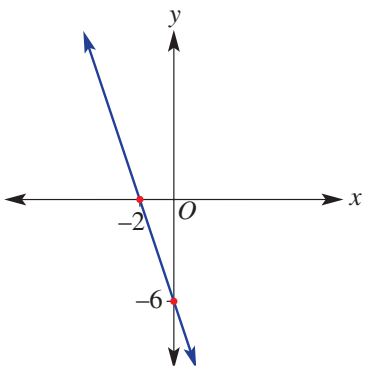
A (4, 0) **B** (0, 3) **C** (6, 0)
D (2, 0) **E** (0, 2)

3 The graph of $y = 3x - 6$ is represented by:

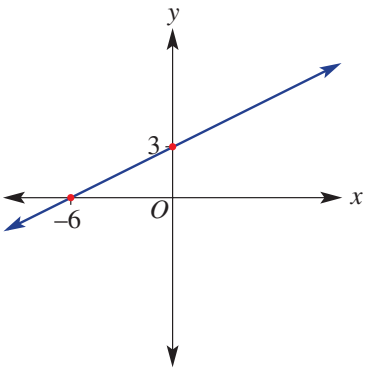
A



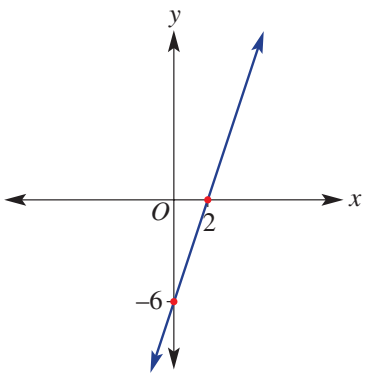
B



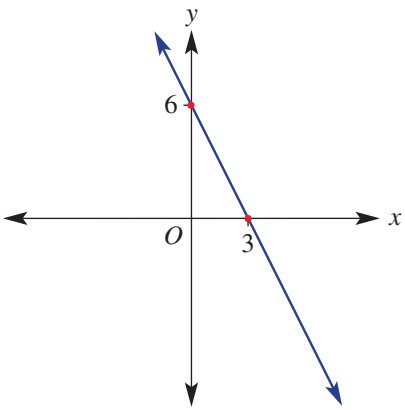
C



D



E



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4 The point that is not on the straight line $y = 2x - 1$ is:

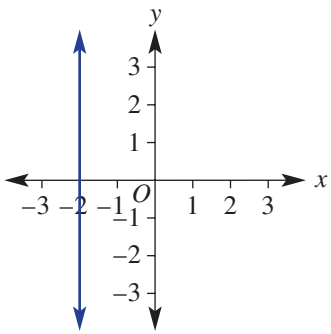
A (1, 1) **B** (0, −1) **C** (3, 5)
D (−2, 7) **E** (−1, −3)

5 The gradient of the line joining the points (1, −2) and (5, 6) is:

A 2 **B** 1 **C** 3
D −1 **E** $\frac{1}{3}$

6 The graph shown has equation:

A $y = x - 2$
B $y = -2x$
C $y = -2$
D $x = -2$
E $x + y = -2$

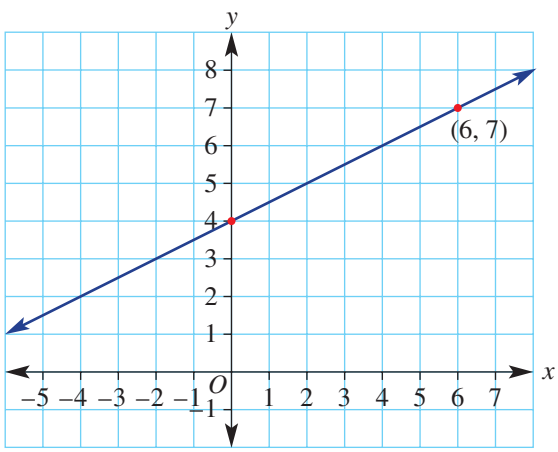


7 A straight line has a gradient of −2 and passes through the point (0, 5). Its equation is:

A $2y = -2x + 10$ **B** $y = -2x + 5$ **C** $y = 5x - 2$
D $y - 2x = 5$ **E** $y = -2(x - 5)$

8 The equation of the graph shown is:

A $y = x + 1$
B $y = \frac{1}{2}x + 4$
C $y = 2x + 1$
D $y = -2x - 1$
E $y = \frac{1}{2}x + 2$



9 The length of the line segment joining the points $A(1, 2)$ and $B(4, 6)$ is:

A 5 **B** $\frac{4}{3}$ **C** 7
D $\sqrt{5}$ **E** $\sqrt{2}$

10 A phone call costs a 20c connection fee plus 60c per minute. A rule to represent the cost, C cents, of a call lasting n minutes is:

A $C = 20n + 60$ **B** $n = 60C + 20$ **C** $C = 60n + 20$
D $n = 20C + 60$ **E** $20C + 60n = 1$

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Short-answer questions

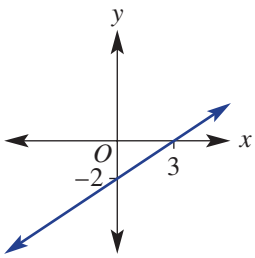
1 Using $-2 \leqslant x \leqslant 3$, construct a table of values and plot a graph for $y = 2x - 3$.

2 Read off the x- and y-intercepts from the table and graph.

a

x	−2	−1	0	1	2
y	0	2	4	6	8

b



3 Sketch the following linear graphs, labelling x- and y-intercepts.

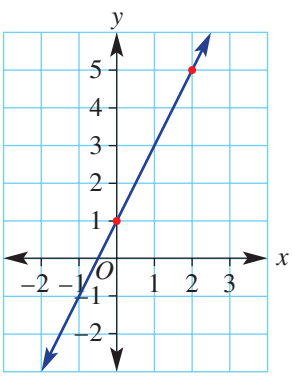
a $y = 2x - 4$ **b** $y = 3x + 9$ **c** $y = -2x + 5$ **d** $y = -x + 4$
e $2x + 4y = 8$ **f** $4x - 2y = 10$ **g** $2x - y = 7$ **h** $-3x + 6y = 12$

4 Sketch the following lines with one intercept and classify their gradient as positive, negative, zero or undefined.

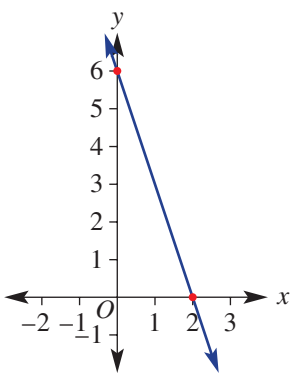
a $y = 3$ **b** $y = -2$ **c** $x = -4$
d $x = 5$ **e** $y = 3x$ **f** $y = -2x$

5 Find the gradients of the following lines.

a



b



6 Find the gradients of the lines passing through these points.

a (3, 1) and (5, 5) **b** (2, 5) and (4, 3)
c (−3, −2) and (1, 6) **d** (−2, 6) and (1, −4)

7 An empty swimming pool is being filled with water by a hose. It takes 4 hours to fill 8 000 L.

a What is the rate at which water is poured into the pool?
b Draw a graph of volume (V litres) vs time (t hours) for $0 \leqslant t \leqslant 4$.
c By finding the gradient of your graph, give the rule for V in terms of t.
d Use your rule to find the time to fill 5000 L.

8 For each of the following linear relationships, state the value of the gradient and the y-intercept and then sketch the graph using the gradient–intercept method.

a $y = 2x + 3$
b $y = -3x + 7$

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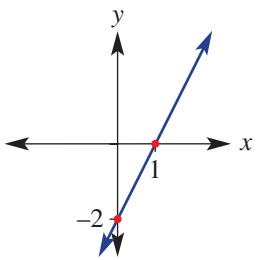
9 Rearrange the following equations into the form $y = mx + b$ and state the gradient and y-intercept.

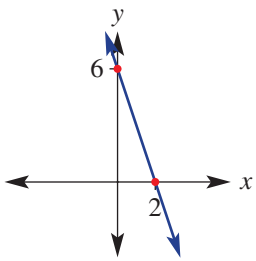
a $2x + y = 6$ **b** $x + 2y = 4$

10 Find the equations of these linear graphs in the form $y = mx + b$.

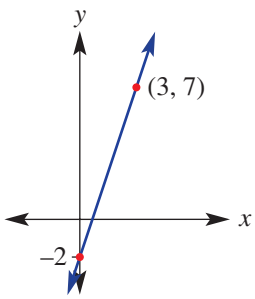
a



b



c



11 Give the equation of the straight line that has:

a gradient 3 and passes through the point (1, 4)

b gradient −2 and passes through the point (2, −1)

12 For the line segment joining the following pairs of points, find the:

i midpoint

ii length (to two decimal places)

a (2, 4) and (6, 8) **b** (5, 2) and (10, 7)

c (−2, 1) and (2, 7) **d** (−5, 7) and (−1, −2)

13 For the following relationships, construct a table of values for $-2 \leqslant x \leqslant 2$ and plot the points on a graph. Use your calculator to find 3^{-2} and 3^{-1}.

a $y = 2x^2$ **b** $y = 3^x$

Extended-response questions

1 Justin is hiring a clown for his son's birthday party. The clown charges a booking fee of \$100 as well as \$50 per hour at the party.

a Write a rule for the cost (C dollars) of hiring the clown for t hours.

b Sketch a graph of cost (C) vs time (t) for $0 \leqslant t \leqslant 5$.

c How much will it cost to hire the clown for 4 hours?

d At the end of the party, Justin writes a cheque for \$225. How long was the clown at the party for?

2 Luca runs a marathon at a constant speed. The distance, d km, he has remaining in the marathon t hours after the start is given by $d = 42 - 14t$.

a Find the t- and d-intercepts for the rule and sketch its graph between these points. (These are like the x- and y-intercepts respectively.)

b What is the distance of the marathon?

c How long does Luca take to run the marathon?

d Find the gradient of the graph.

e What speed is Luca running at?

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Semester review 1

Chapter 1: Integers, decimals, fractions, ratios and rates

Multiple-choice questions

1 $7 \times (-5) + 3 \times (-4)$ is equal to:

A 23 B −47 C 47 D −23 E 420

2 45.5 ÷ 8.5, when rounded to one significant figure, gives:

A 4.9 B 5.35 C 5.4 D 5 E 4.95

3 \$450 is divided in the ratio 4 : 5. The value of the smaller portion is:

A \$210 B \$250 C \$90 D \$200 E \$220

4 Con runs 4.5 km in 20 minutes. His speed is:

A 1.5 km/h B 1.5 km/min C 13.5 km/h D 90 km/h E 22.5 km/h

5 $\frac{7}{10000} + \frac{13}{100}$ is equal to:

A 0.713 B 0.0713 C 0.1307 D 0.83 E 7.13

Short-answer questions

1 Evaluate the following.

a $\frac{13}{10} - \frac{3}{10}$ b $\frac{3}{7} + \frac{1}{4}$ c $2\frac{1}{4} + \frac{1}{2}$ d $2\frac{1}{3} - 1\frac{5}{9}$

2 Evaluate:

a $\frac{9}{10} \times \frac{5}{12}$ b $3\frac{3}{4} \div 2\frac{1}{12}$

3 Round the following to three decimal places.

a 4.1255 b 21.00241 c 0.0096

4 Divide \$800 into the following ratios.

a 1 : 4 b 2 : 3 c 7 : 1

5 Write these rates and ratios in simplest form.

a Prize money is shared between 2 people in the ratio 60 : 36.

b Jodie travels 165 km in 3 hours.

c 30 mL of rain falls in $1\frac{1}{4}$ hours.

6 A car averages 68 km/h on a journey. How far will it travel at this speed for 2 hours and 40 minutes? Round to one decimal place.

7 The height of a tree is directly proportional to its width. A tree that is 2 metres tall has a width of 0.6 metres. How tall is a tree with width 1.2 metres?

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Extended-response question

Thomas walks, on average, 6 kilometres an hour. Phillip walks at an average speed of 8 km/h.

a How far does Thomas walk in 4 hours?

b Phillip starts at 8 a.m.

i If he stops at 9:45 a.m., how far has he walked?

ii What time will Phillip finish his walk if he walks twice as far as Thomas?

Chapter 2: Financial mathematics

Multiple-choice questions

1 $2\frac{1}{2}\%$ is the same as:

A 0.25 **B** $\frac{1}{40}$ **C** $\frac{25}{100}$ **D** 2.5 **E** $\frac{5}{2}$

2 If cost price is \$36 and selling price is \$28.80, the percentage loss is:

A 7.2% **B** 80% **C** 92.8% **D** 20% **E** 2%

3 The simple interest earned on \$660 invested at 5% p.a. for $1\frac{1}{2}$ years is:

A \$49.50 **B** \$40.50 **C** \$594 **D** \$33 **E** \$709.50

4 A book that costs \$27 is discounted by 15%. The new price is:

A \$20.25 **B** \$31.05 **C** \$4.05 **D** \$22.95 **E** \$25.20

5 Anna is paid a normal rate of \$12.10 per hour. In a week she works 6 hours at the normal rate, 2 hours at time and a half and 3 hours at double time. How much does she earn?

A \$181.50 **B** \$145.20 **C** \$193.60 **D** \$175.45 **E** \$163.35

Short-answer questions

1 Convert each of the following to a percentage.

a 0.6 **b** $\frac{5}{16}$ **c** 2 kg out of 20 kg **d** 75c out of \$3

2 Find:

a 10% of \$96 **b** 85% of \$900 **c** $5\frac{1}{2}\%$ of \$200

3 **a** Increase 80 m by 5%. **b** Decrease \$54 by 6%.

4 Julie spends 20% of her day reading a novel. How many hours and minutes did she spend reading?

5 Jamal earns a weekly retainer of \$400 plus 6% of the sales he makes. If he sells \$8200 worth of goods, how much will he earn for the week?

6 Sean invests \$800 at 6% p.a. for 18 months.

a Calculate his simple interest.

b What is the final balance on Sean's account after the interest has been paid?

7 How long does \$600 need to be invested at 8% p.a. simple interest to earn \$120 in interest?

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Extended-response question

Jill plans to trial a simple interest plan. Before investing her money she increases the amount in her account by 20% to \$21 000.

a What was the original amount in her account?

b She invests the \$21 000 for 4 years at a simple interest rate of 3% p.a. How much does she have in her account at the end of the four years?

c She continues with the plan in part **b** and after a certain number of years has obtained \$5670 interest. For how many years has she had the money invested?

d What percentage increase does this interest represent on her initial investment?

Chapter 3: Expressions and equations

Multiple-choice questions

1 The expression that represents 3 more than half a certain number, n, is:

A $\frac{1}{2}n + 1.5$ **B** $2n + 3$ **C** $\frac{3}{2}n$ **D** $\frac{n}{2} + 3$ **E** $\frac{n+3}{2}$

2 The simplified form of $5ab + 3a + 2ab - a$ is:

A $9ab$ **B** $10ab + 3a$ **C** $13ab - a$ **D** $7ab + 3$ **E** $7ab + 2a$

3 The expanded form of $-2(3m - 4)$ is:

A $-6m + 8$ **B** $-6m + 4$ **C** $-6m - 8$ **D** $-5m - 6$ **E** $5m + 8$

4 The solution to $\frac{d}{4} - 7 = 2$ is:

A $d = -20$ **B** $d = 15$ **C** $d = 36$ **D** $d = 1$ **E** $d = 30$

5 The formula $m = \sqrt{\frac{b-1}{a}}$, with $b = 9$ and $a = 2$, gives m equals:

A 3 **B** $\frac{3}{2}$ **C** 2 **D** $\sqrt{3}$ **E** $\frac{\sqrt{3}}{2}$

Short-answer questions

1 Simplify the following.

a $2x + 6y - 4x + y$ **b** $-3m \times 5n$ **c** $\frac{6xy}{18x}$

2 Solve the following equations.

a $3x + 7 = 25$ **b** $\frac{x-1}{4} = 2$

c $4(2m + 3) = 15$ **d** $5a - 8 = 3a - 2$

3 Noah receives m dollars pocket money per week. His older brother Jake gets 1 dollar more than twice Noah's amount. If Jake receives \$15:

a write an equation to represent the problem using the variable m

b solve the equation in part **a** to determine how much Noah receives each week

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4 The formula $S = \frac{n}{2}(a + l)$ gives the sum, S, of a sequence of n numbers with first term a and last term l.

a Find the sum of the sequence of 10 terms 2, 5, 8, …, 29.

b If a sequence of 8 terms has a sum of 88 and a first term equal to 4, use the formula to find the last term of this sequence.

Extended-response question

Chris and his brother Michael play basketball. In the last game, Chris scored 12 more points than Michael. Between them they scored 38 points. Let p be the number of points scored by Michael.

a Write an expression for the number of points scored by Chris.

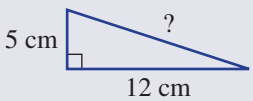
b Write an equation involving the unknown, p, to represent the problem.

c Solve the equation to find the number of points scored by Chris and Michael.

Chapter 4: Right-angled triangles

Multiple-choice questions

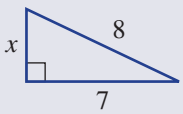
1 The length of the hypotenuse in this triangle is:



A 11 cm **B** 13 cm **C** 14.8 cm

D 169 cm **E** $\frac{1}{\sqrt{119}}$ cm

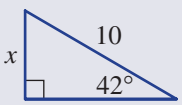
2 The value of x in the triangle shown is:



A 3.9 **B** $\sqrt{113}$ **C** $\sqrt{57}$

D $\sqrt{15}$ **E** 2.7

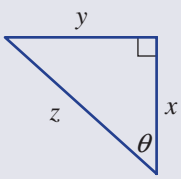
3 The value of x in the triangle shown is closest to:



A 6.7 **B** 7.4 **C** 14.9

D 13.5 **E** 9.0

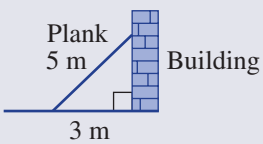
4 A correct ratio for this triangle is:



A $\tan\theta = \frac{x}{y}$ **B** $\cos\theta = \frac{y}{z}$ **C** $\tan\theta = \frac{y}{x}$

D $\sin\theta = \frac{y}{x}$ **E** $\cos\theta = \frac{x}{y}$

5 A 5-metre plank of wood is leaning up against a side of a building as shown. If the wood touches the ground 3 m from the base of the building, the angle the wood makes with the building is closest to:



A 36.9° **B** 59° **C** 53.1°

D 31° **E** 41.4°

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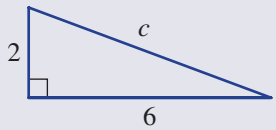
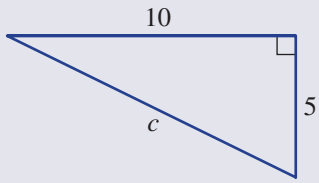
Short-answer questions

1 Find the value of each pronumeral, correct to one decimal place.

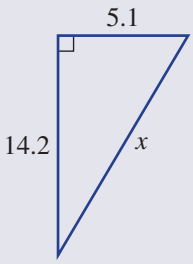
a



b



c



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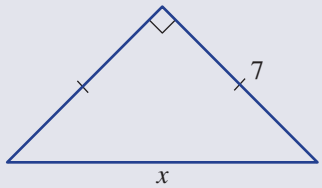

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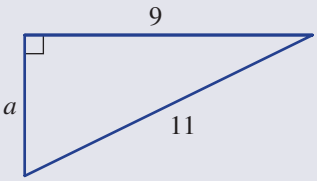

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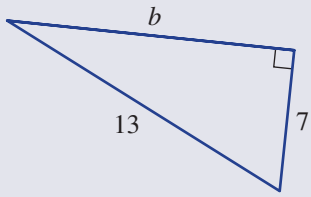
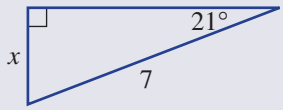


2 Find the value of each pronumeral, correct to one decimal place.

a



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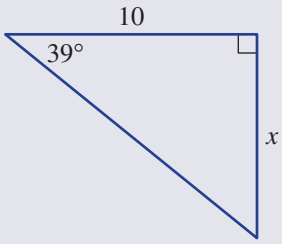

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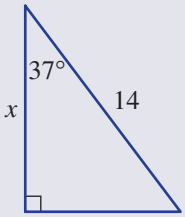

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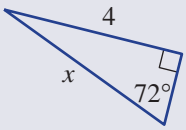

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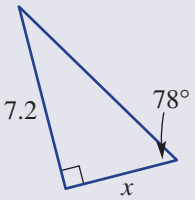

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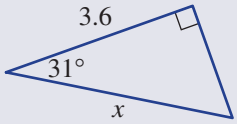

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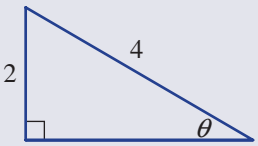

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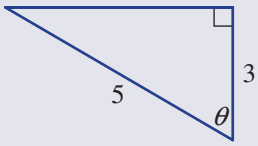

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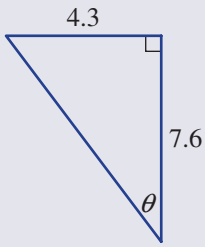

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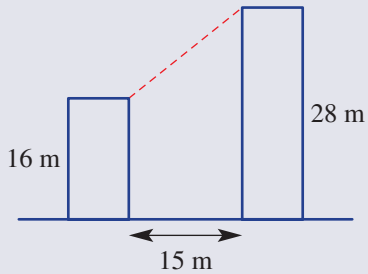
3 A wire is to be connected from the edge of the top of a 28 m high building to the edge of the top of a 16 m high building. The buildings are 15 m apart.



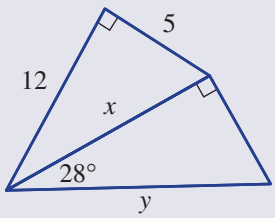
a What length of wire is required? Round to two decimal places.

b What is the angle of depression from the top of the taller building to the top of the smaller building? Round to one decimal place.

4 For the diagram below, use:

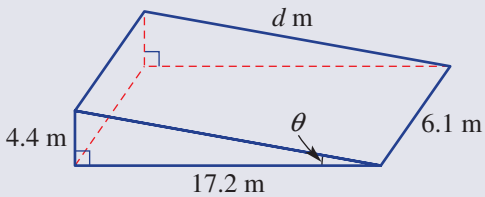
a Pythagoras' theorem to find the value of x

b trigonometry to find the value of y, correct to two decimal places.



Extended-response question

A skateboard ramp is constructed as shown.



a Calculate the distance d metres up the ramp, correct to two decimal places. Use Pythagoras' theorem.

b What is the angle of inclination (θ) between the ramp and the ground, correct to one decimal place?

c If the skateboarder rides from one corner of the ramp diagonally to the other corner, what distance would be travelled? Round to one decimal place.

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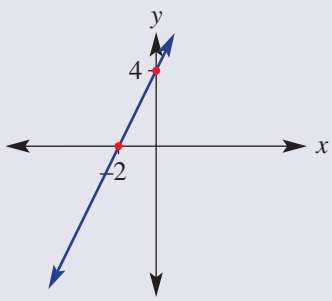
Chapter 5: Linear relationships

Multiple-choice questions

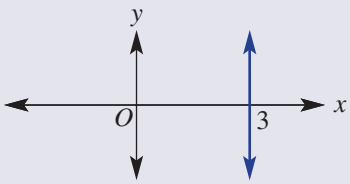
1 The points at which this line meets the x-axis and y-axis are:

A $(-2, 4)$ and $(4, -2)$
B $(0, 4)$ and $(0, -2)$
C $(-2, 0)$ and $(0, 4)$
D $(4, 0)$ and $(0, -2)$
E $(2, 0)$ and $(0, -4)$



2 The graph shown has equation:

A $y = 3x$ **B** $y = 3$ **C** $y = x + 3$
D $x = 3$ **E** $x + y = 3$



3 The line passing through the points $(-3, -1)$ and $(1, 1)$ has gradient:

A $\frac{1}{2}$ **B** -3 **C** 1 **D** 2 **E** -2

4 If the point $(-1, 3)$ is on the line $y = 2x + b$, the value of b is:

A 1 **B** 5 **C** -7 **D** -5 **E** -1

5 The midpoint and length to one decimal place of the line segment joining the points $(-2, 1)$ and $(4, 6)$ are:

A $(1, 3.5)$ and 7.8
B $(3, 5)$ and 5.4
C $(3, 3.5)$ and 6.1
D $(1, 3.5)$ and 3.3
E $(3, 3.5)$ and 3.6

Short-answer questions

1 Sketch the following linear graphs, labelling x- and y-intercepts.

a $y = 2x - 6$ **b** $3x + 4y = 24$ **c** $y = 4x$ **d** $y = -2$

2 Find the gradient of each of the following.

a The line passing through the points $(-1, 2)$ and $(2, 4)$
b The line passing through the points $(-2, 5)$ and $(1, -4)$
c The line with equation $y = -2x + 5$
d The line with equation $-4x + 3y = 9$ (rearrange to $y = mx + b$ first)

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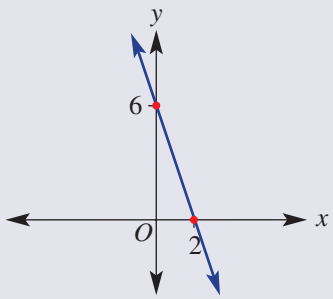
3 Give the equation of the following lines in gradient-intercept form.

a The line with the following graph



b The line with gradient 3 and passing through the point (2, 5)

Extended-response question

Doug works as a labourer. He is digging a trench and has 180 kg of soil to remove. He has taken 3 hours to remove 36 kg.

a What is the rate at which he is removing the soil?

b If he maintains this rate, write a rule for the amount of soil, S(kg), remaining after t hours.

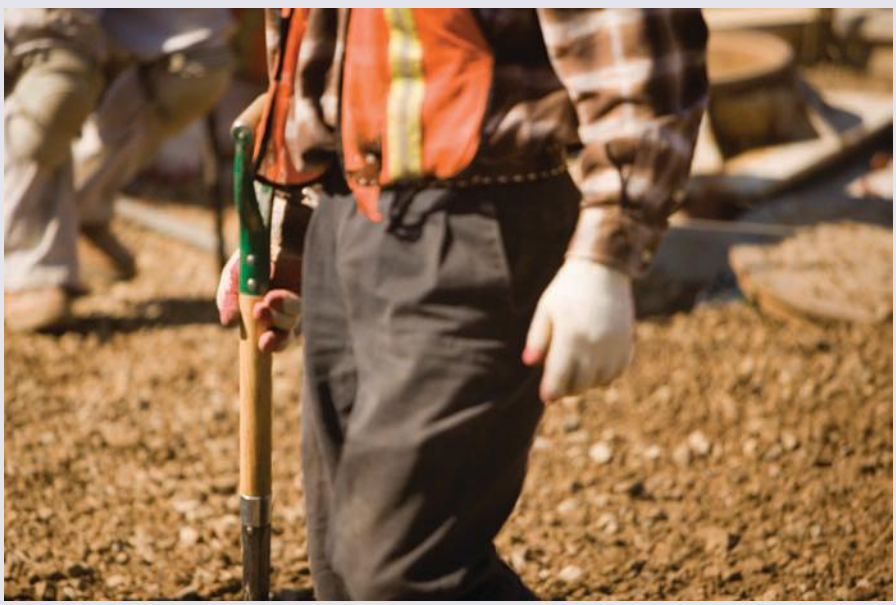
c Draw a graph of your rule.

d How long will it take to remove all of the soil?

e Doug is paid \$40 for the job plus \$25 per hour worked.

i Write a rule for his pay P dollars for working h hours.

ii How much will he be paid to remove all the soil?



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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

6 Length, area, surface area and volume

What you will learn

6A Length and perimeter REVISION
6B Circumference of circles and perimeter of sectors REVISION
6C Area of quadrilaterals and triangles REVISION
6D Area of circles REVISION
6E Perimeter and area of composite shapes
6F Surface area of prisms
6G Surface area of cylinders
6H Volume of prisms
6I Volume of cylinders

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NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY
SUBSTRANDS: AREA AND SURFACE AREA, VOLUME

Outcomes

A student uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area. (MA4–13MG)
A student calculates the areas of composite shapes, and the surface areas of rectangular and triangular prisms. (MA5.1–8MG)
A student calculates the surface areas of right prisms, cylinders and related composite solids. (MA5.2–11MG)
A student uses formulas to calculate the volumes of prisms and cylinders, and converts between units of volume. (MA4–14MG)

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Body surface area

Body surface area (BSA) is measured in square metres and is the total surface area of a person's body. It is used in medicine to calculate correct drug dose and the volume of fluids needed to be administered intravenously.

Calculating the surface area of prisms and pyramids is relatively easy, as we use the area formulas for each face, and then find the total. However, calculating the actual surface area of the irregular solid, such as a person, is a difficult task. To find the surface area of a person the following formula can be used.

$$\text{BSA (m}^2\text{)} = \sqrt{\frac{\text{body weight (kg)}}{\text{height (cm)}} \div 3600}$$

Note that the average BSA for a 12–13-year-old is 1.33 m² and 1.7 m² for the average adult.

Paediatricians need to know a child's BSA to use Clarke's formula to calculate the correct dose required by each patient. Clarke's formula states:

$$\text{child's dose} = \frac{\text{surface area of child (m}^2\text{)}}{\text{average surface area of adult (1.7 m}^2\text{)}} \times \text{adult dose}$$

Doctors and nurses need to be able to perform these calculations quickly and accurately in-order to save lives.

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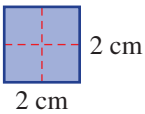
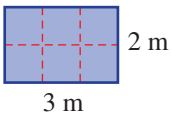
Pre-test

1 Count the number of squares in these shapes to find the areas.

a



b



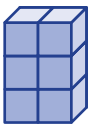
c

2 Find the distances (perimeters) around the shapes in Question **1**.

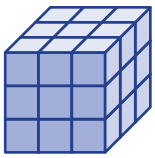
3 Count the number of cubes in these solids to find the volumes.

a

b

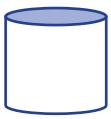


c



4 **a** Name this solid.

b What is the name of the shape (shaded) on top of the solid?

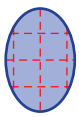


5 Calculate the following by moving the decimal point right (for ×) or left (for ÷).

a 2.3×10 **b** 0.048×1000 **c** $270 \div 100$

d $52\,134 \div 10\,000$ **e** $0.0005 \times 100\,000$ **f** $72\,160 \div 1000$

6 Estimate the area (number of squares) in this ellipse.



7 Convert the following to the units shown in the brackets.

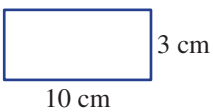
Remember: 1 km = 1000 m, 1 m = 100 cm and 1 cm = 10 mm.

a 3 cm (mm) **b** 20 m (cm) **c** 1.6 km (m) **d** 23 mm (cm)

e 3167 m (km) **f** 72 cm (m) **g** 20 000 m (km) **h** 0.03 km (m)

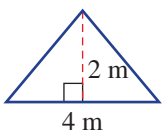
8 Find the area of these basic shapes, using the given formulas.

a



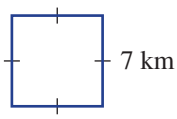
$A = l \times b$

b



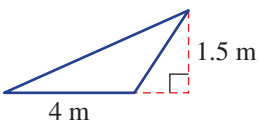
$A = \frac{1}{2}bh$

c



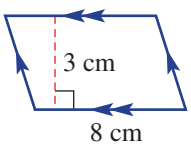
$A = s^2$

d



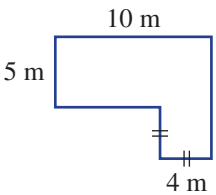
$A = \frac{1}{2}bh$

e



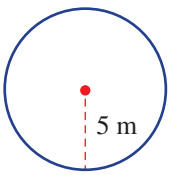
$A = b \times h$

f



$A = l \times b + l^2$

9 Find the circumference ($C = 2\pi r$) and area ($A = \pi r^2$) of this circle, rounding to two decimal places.



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6A Length and perimeter REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Measurement is the branch of mathematics that includes the consideration of length, area and volume. This includes units of measurement, perimeter, circumference, surface area, composite shapes and capacity.

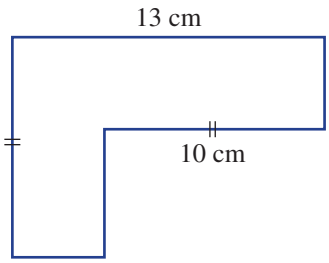
We use measurement when we design buildings, water our gardens, control satellites, fill our cars with petrol or participate in school athletics days.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Not enough information?

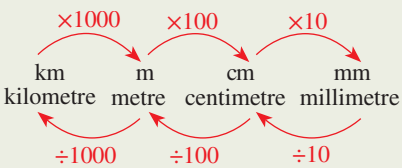
All the angles at each vertex in this shape are 90° and the two given lengths are 10 cm and 13 cm.

- Is there enough information to find the perimeter of the shape?
- If there is enough information, find the perimeter and discuss your method. If not, what information needs to be provided?



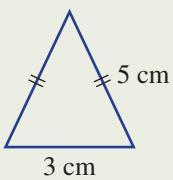
Key ideas

- To convert between metric units of length, multiply or divide by the appropriate power of 10.

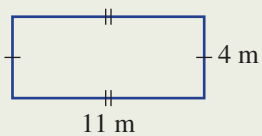


- **Perimeter** is the distance around a closed shape.
  - Sides with the same markings are of equal length.

Perimeter The total distance (length) around the outside of a figure



$P = 2 \times 5 + 3$
$= 13$ cm

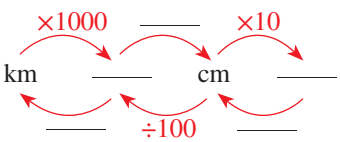


$P = 2 \times 11 + 2 \times 4$
$= 30$ m

Exercise 6A REVISION

UNDERSTANDING AND FLUENCY	1–4, 5(½)	4, 5(½), 6

1 Fill in the gaps on this flow chart.



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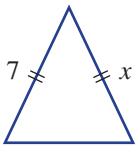
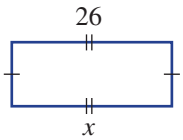
2 Write down the value of x in these diagrams.

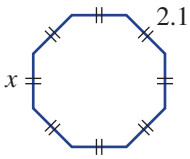
a



b



c



Example 1 Converting units of length

Convert to the units shown in the brackets.

a 5.41 cm (mm) **b** 3200 m (km)

SOLUTION	EXPLANATION
a 5.41 cm = 5.41 × 10 = 54.1 mm	1 cm = 10 mm and you are moving to a smaller unit, so multiply. Move the decimal point one place to the right.
b 3200 m = 3200 ÷ 1000 = 3.2 km	1 km = 1000 m and you are moving to a larger unit, so divide. Move the decimal point three places to the left.

3 Convert the following length measurements into the units given in the brackets.

a 5 cm (mm) **b** 41 cm (mm)
c 2.8 m (cm) **d** 0.4 m (cm)
e 4.6 km (m) **f** 0.9 km (m)
g 521 mm (cm) **h** 36 mm (cm)
i 240 cm (m) **j** 83.7 cm (m)
k 7000 m (km) **l** 2170 m (km)

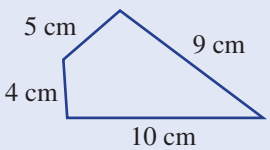
Multiply when changing to a smaller unit and divide when changing to a larger unit.

4 A steel beam is 8.25 m long and 22.5 mm wide. Write down the length and the width of the beam in centimetres.

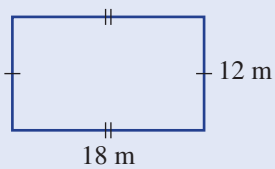
Example 2 Finding perimeters of simple shapes

Find the perimeter of each of the following shapes.

a



b



SOLUTION	EXPLANATION
a $P = 10 + 9 + 5 + 4$ $= 28$ cm	To find the perimeter, add all the side lengths together.
b $P = 2 \times 12 + 2 \times 18$ $= 24 + 36$ $= 60$ m	There are two lengths of 12 m and two lengths of 18 m.

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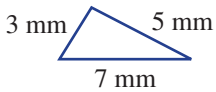
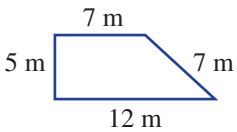
5 Find the perimeter of each of the following shapes.

a



b



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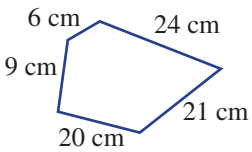
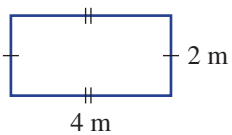
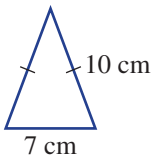


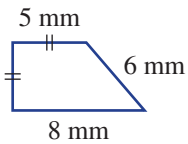
d



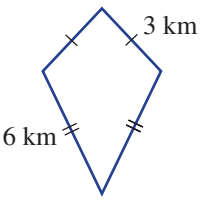
e



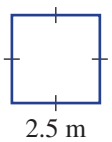
f



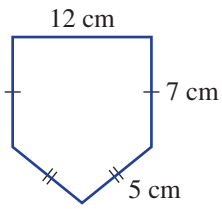
g



h

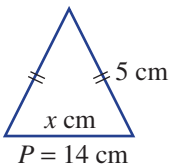


i



6 Find the unknown side length in these shapes with the given perimeters.

a



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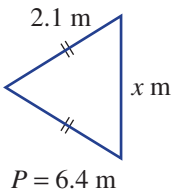
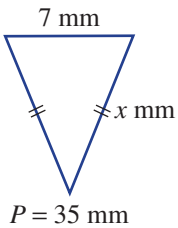


c

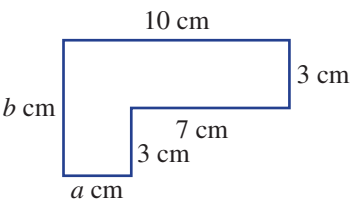


Use trial and error or an equation to find the value of x.

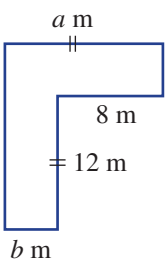
PROBLEM-SOLVING AND REASONING 7, 8 | 7, 8(½), 9–11

7 Write down the values of the pronumerals in these shapes. All angles are right angles.

a

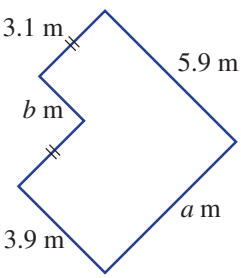


b



Look at the sides opposite the pronumerals.

c



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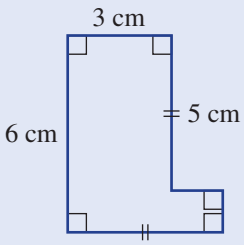
Example 3 Finding perimeters of composite shapes

Find the perimeter of this composite shape.

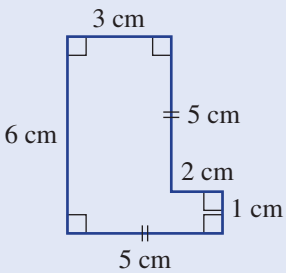


SOLUTION

$P = (2 \times 5) + 6 + 3 + 2 + 1$
$= 22$ cm

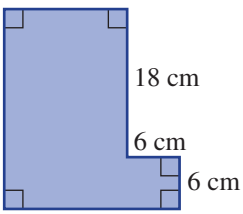
EXPLANATION

Missing sides are:
5 cm − 3 cm = 2 cm
6 cm − 5 cm = 1 cm
Alternatively,
$2 \times 6 + 2 \times 5 = 22$ cm

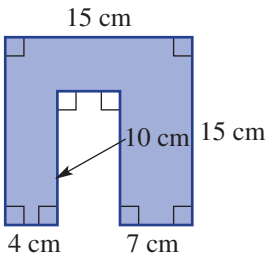


8 Find the perimeter of each of the following composite shapes.

a

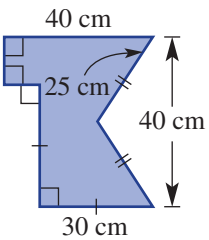


b

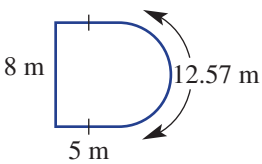


First label all the missing side lengths, then find the perimeter.

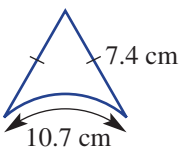
c



d



e



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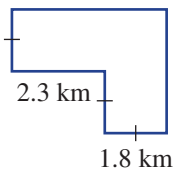

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9 A lion cage is made up of five straight fence sections. Three sections are 20 m in length and the other two sections are 15.5 m and 32.5 m. Find the perimeter of the cage.

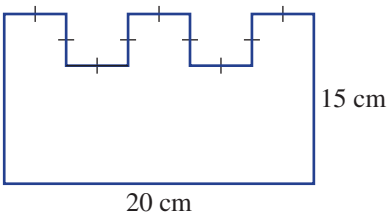
10 Convert the following measurements into the units given in the brackets.

a 8 m (mm) **b** 110 000 mm (m)
c 0.00001 km (cm) **d** 0.02 m (mm)
e 28 400 cm (km) **f** 62 743 000 mm (km)

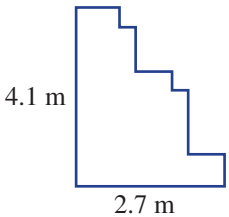
In part **a**, first convert to cm, then to mm.

11 Find the perimeters of these shapes. All angles are right angles.

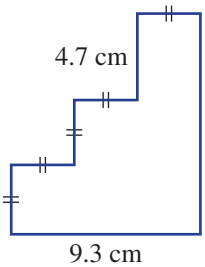
a



b



c

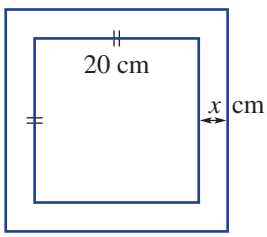


ENRICHMENT – 12, 13

Picture framing

12 A photo 12 cm wide and 20 cm long is surrounded with a picture frame 3 cm thick. Find the outside perimeter of the framed picture.

13 A square picture of side length 20 cm is inside a frame of width x cm.



a Find the perimeter of the framed picture if:
- **i** $x = 2$
- **ii** $x = 3$
- **iii** $x = 5$

b Write a rule for the perimeter, P, of the framed picture in terms of x.

c Use your rule to find the perimeter if $x = 3.7$.

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6B Circumference of circles and perimeter of sectors REVISION

Interactive
Widgets
HOTsheets
Walkthrough

For thousands of years, mathematicians have known about a special number that links a circle's diameter to its circumference. The Egyptians, Babylonians and Ancient Indians knew about this special number, and thought its value was about 3.15.

Today we call this number pi (π) and know it to be 3.14159, correct to five decimal places.

An exact value of pi cannot be written down as a decimal, as it has an infinite number of decimal places with no pattern.

Ancient Egyptian mathematicians knew that pi was about 3.15.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Working with string

Use a pair of compasses, a ruler and a piece of string for this activity.

The table below contains some approximate circle measurements.

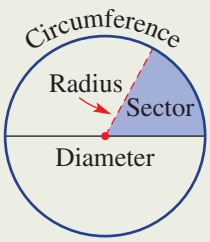
Circumference (C)	Diameter (d)	$C \div d$
18.8	6.0	
12.6	4.0	
…	…	
…	…	

- Copy the table, leaving two blank rows.
- Use your compass to draw a circle, then measure its diameter using a ruler and its circumference using your string (try to be as accurate as possible). Add this information to the table.
- Do this again, with a different-sized circle.
- Calculate $C \div d$ in the third column for each of the four circles. What do you notice?
- What does this say about how to calculate the circumference of a circle?

Key ideas

- Features of a circle:
  - **Diameter** (d) is the distance across the centre.
  - **Radius** (r) is the distance from the centre to the circumference. The radius is half of the diameter.
  - **Circumference** of a circle is $C = 2\pi r$ or $C = \pi d$.

    Use $\frac{22}{7}$ or 3.14 to approximate π, or use technology for more precise calculations.



Diameter A line passing through the centre of a circle with its endpoints on the circumference

Radius The distance from the centre of a circle to its outside edge

Circumference The curved boundary of a circle

- Special circle sectors:
  - A half circle is called a semicircle.
  - A quarter circle is called a quadrant.

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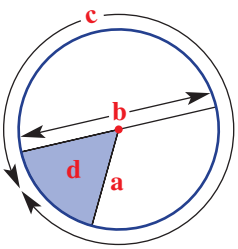
Exercise 6B REVISION

UNDERSTANDING AND FLUENCY 1–8 4–8

1 Name features **a–d** shown in this circle.



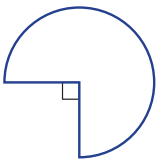
2 **a** What is the radius of a circle if its diameter is 5.6 cm?
b What is the diameter of a circle if its radius is 48 mm?

3 Write down the rule for the circumference of a circle using:
a r, the radius **b** d, the diameter

4 Determine the fraction of a circle shown in these sectors. Write the fraction in simplest form.

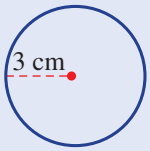
a

b

c


Example 4 Finding the circumference of a circle

Find the circumference of this circle, correct to two decimal places.



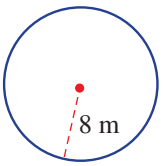
SOLUTION

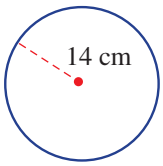
$C = 2\pi r$
$= 2 \times \pi \times 3$
$= 6\pi$
$= 18.85$ cm (to 2 d.p.)

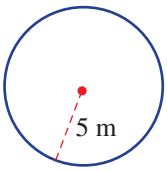
EXPLANATION

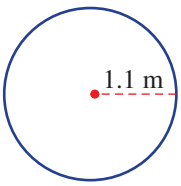
Use the formula $C = 2\pi r$ or $C = \pi d$ and substitute $r = 3$ (or $d = 6$).
6π would be the exact answer and 18.85 is the rounded answer.

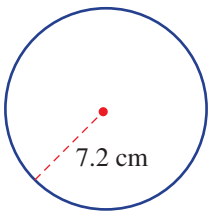
5 Find the circumferences of these circles, correct to two decimal places. Use a calculator for the value of π.

a


b


c


d


e


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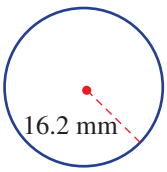

Write the rule $C = 2\pi r$, then substitute the radius length.

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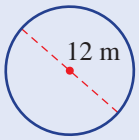
Example 5 Finding the circumference using the diameter

Find the circumference of this circle, correct to two decimal places.



SOLUTION

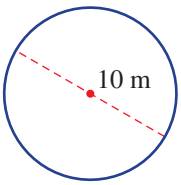
$C = \pi d$
$= \pi \times 12$
$= 37.70$ m (to 2 d.p.)

EXPLANATION

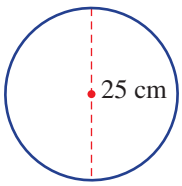
Write the formula. $C = \pi d$ is preferred since d is given. Substitute $d = 12$ and multiply by π. Use a calculator and round.
Note: 37.6991 rounds to 37.70 for two decimal places.

6 Find the circumferences of these circles, correct to two decimal places.

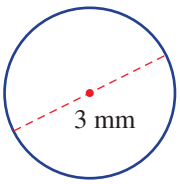
a



b

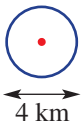


c

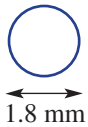


Write the rule $C = \pi d$ and substitute the diameter length.

d



e



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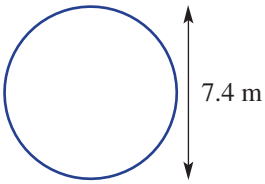
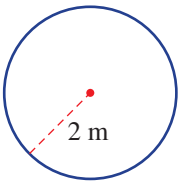


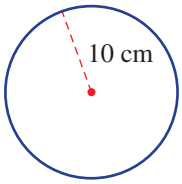
7 Find the circumferences of these circles without a calculator, using the given approximation of π.

a



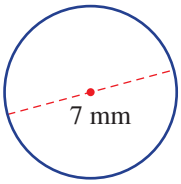
$\pi = 3.14$

b



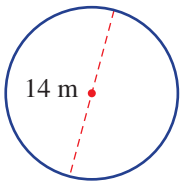
$\pi = 3.14$

c



$\pi = \frac{22}{7}$

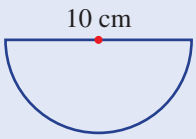
d



$\pi = \frac{22}{7}$

Example 6 Working with a semicircle

Find the perimeter of this semicircle, correct to two decimal places.



SOLUTION

$P = \frac{1}{2} \times \pi d + 10$

$= \frac{1}{2} \times \pi \times 10 + 10$

$= 25.71$ cm (to 2 d.p.)

EXPLANATION

The perimeter consists of half the circumference of a circle (with diameter 10 cm) plus the 10 cm diameter across the top.

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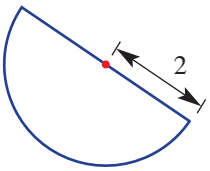
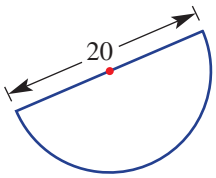
8 Find the perimeters of these sectors, correct to two decimal places.

a



b



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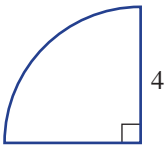

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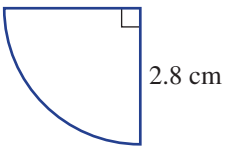

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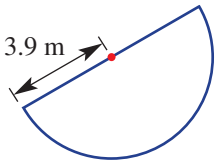
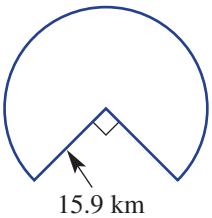


f



Decide what fraction of the circumference you want and don't forget to add the straight sides.

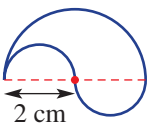
PROBLEM-SOLVING AND REASONING	9, 10	9–11

9 Find the distance around the outside of a circular pool of radius 4.5 m, correct to two decimal places.

10 Find the length of string required to surround the circular trunk of a tree that has a diameter of 1.3 m, correct to one decimal place.

11 Give the perimeters of these shapes, correct to two decimal places.

a



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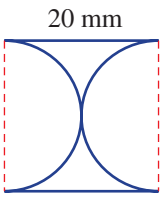
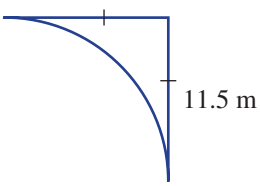


Two semicircles of the same size make a full circle.

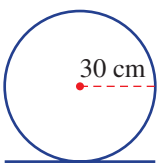
c



ENRICHMENT	–	12

The rolling wheel

12 A wheel of radius 30 cm is rolled in a straight line.



a Find the circumference of the wheel, correct to two decimal places.

b How far, correct to two decimal places, has the wheel rolled after completing:

i 2 rotations?

ii 10.5 rotations?

c Can you find how many rotations would be required to cover at least 1 km in length? Round to the nearest whole number.

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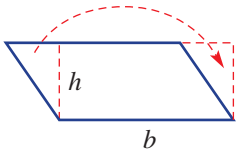
6C Area of quadrilaterals and triangles REVISION

Interactive
Widgets
HOTsheets
Walkthrough

For many common shapes, such as the parallelogram and trapezium, the rules for finding their area can be found using simple rectangles and triangles. The parallelogram, for example, can be seen as a 'pushed over' rectangle. Its area can be calculated in the same way as for a rectangle.

So A = length × breadth = $b \times h$



The Dockland Office Building in Hamburg has a cross-section that is in the shape of a parallelogram.

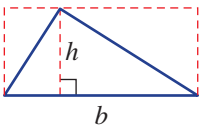
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Build the formulas

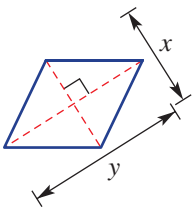
From the introduction, you can see how the rule for the area of a parallelogram is given by $A = b \times h$.

Use the given diagrams to explain the rules for the areas of each shape.

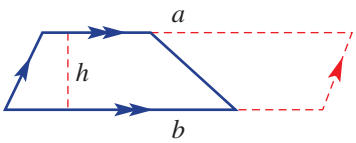
- Triangle: $A = \frac{1}{2}b \times h$



- Rhombus: $A = \frac{1}{2}xy$

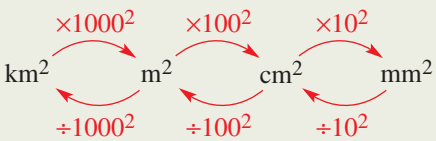


- Trapezium: $A = \frac{1}{2}h(a + b)$

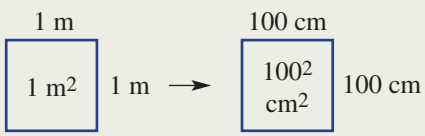


Key ideas

■ Conversion of area units



$10^2 = 10 \times 10 = 100$
$100^2 = 100 \times 100 = 10\,000$
$1000^2 = 1000 \times 1000 = 1\,000\,000$



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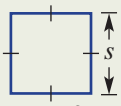
Key ideas

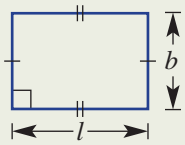
- The area of a two-dimensional shape is a measure of the space enclosed within its boundaries.

Square



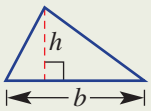
$A = s^2$

Rectangle



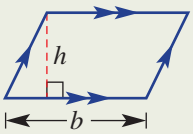
$A = lb$

Triangle



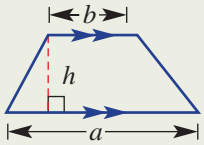
$A = \frac{1}{2}bh$

Parallelogram, rhombus



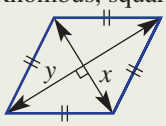
$A = bh$

Trapezium



$A = \frac{1}{2}h(a + b)$

Rhombus, square, kite



$A = \frac{1}{2}xy$

Exercise 6C REVISION

UNDERSTANDING AND FLUENCY	1–6	3, 4–6(½)

1 Write the missing number.

a $1\text{ cm}^2 = 10^2\text{ mm}^2 =$ _____ mm^2

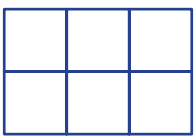
b $1\text{ m}^2 = 100^2\text{ cm}^2 =$ _____ cm^2

c $1\text{ km}^2 = 1\,000^2\text{ m}^2 =$ _____ m^2

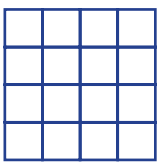
2 Count the number of squares to find the area of these shapes. Each square in each shape represents one square unit.

For parts **d** and **e**, note that each triangle is half of a rectangle.

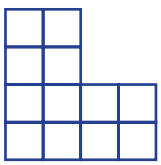
a



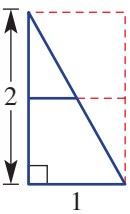
b



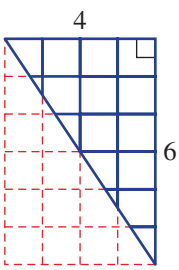
c



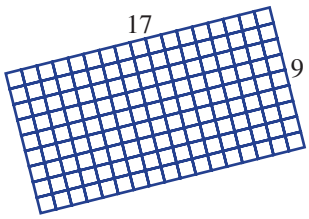
d



e



f



3 Name the shape that has the given area formula.

a $A = lb$

b $A = \frac{1}{2}xy$ (3 shapes)

c $A = \frac{1}{2}bh$

d $A = \frac{1}{2}h(a + b)$

e $A = bh$

f $A = s^2$

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Example 7 Converting units of area

Convert the following area measurements into the units given in the brackets.

a 859 mm^2 (cm^2)

b 2.37 m^2 (cm^2)

SOLUTION

a $859\ mm^2 = 859 \div 10^2\ cm^2$
$= 8.59\ cm^2$

b $2.37\ m^2 = 2.37 \times 100^2\ cm^2$
$= 23\,700\ cm^2$

EXPLANATION

cm^2 mm^2 859.
$\div 10^2 = 100$

$\times 100^2 = 10\,000$
m^2 cm^2 2.3700

4 Convert the following area measurements into the units given in the brackets.

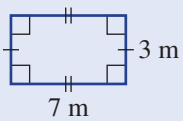
a 2 cm^2 (mm^2)
b 0.4 cm^2 (mm^2)
c 500 mm^2 (cm^2)
d 310 mm^2 (cm^2)
e 2.1 m^2 (cm^2)
f 0.2 m^2 (cm^2)
g 210 000 cm^2 (m^2)
h 3 700 cm^2 (m^2)
i 0.001 km^2 (m^2)
j 4.3 km^2 (m^2)
k 3 200 000 m^2 (km^2)
l 39 400 m^2 (km^2)

Multiply or divide by: $10^2 = 100$
$100^2 = 10\,000$
or $1\,000^2 = 1\,000\,000$.

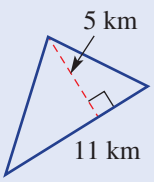
Example 8 Finding areas of rectangles, triangles and parallelograms

Find the area of each of the following plane figures.

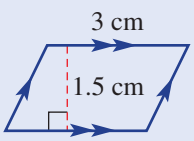
a



b



c



SOLUTION

a Area $= l \times b$
$= 7 \times 3$
$= 21\ m^2$

b Area $= \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 11 \times 5$
$= 27.5\ km^2$

c Area $= b \times h$
$= 3 \times 1.5$
$= 4.5\ cm^2$

EXPLANATION

Use the area formula for a rectangle.
Substitute $l = 7$ and $b = 3$.
Include the correct units.

Use the area formula for a triangle.

Substitute $b = 11$ and $h = 5$, where h is the height perpendicular (at 90°) to the base.

Use the area formula for a parallelogram.
Multiply the base length by the perpendicular height.

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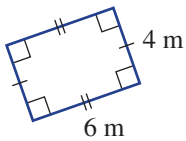
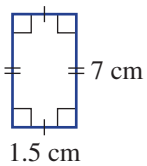
5 Find the area of each of the following plane figures.

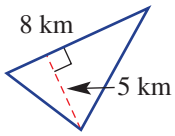
a



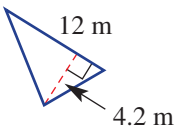
b



c



d



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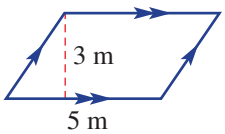

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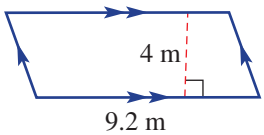
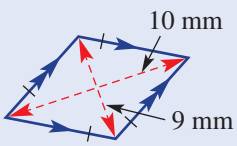


Choose from:
$A = l \times b$,
$A = \frac{1}{2}b \times h$ or
$A = b \times h$.

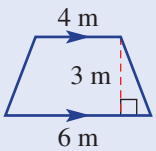
Example 9 Finding areas of rhombuses and trapeziums

Find the area of each of the following plane figures.

a



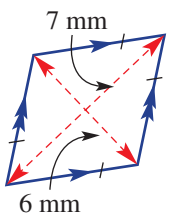
b



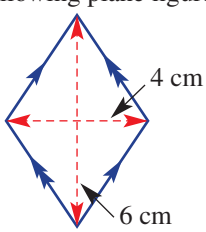
SOLUTION	EXPLANATION
a Area $= \frac{1}{2} \times x \times y$	Use the area formula for a rhombus.
$= \frac{1}{2} \times 10 \times 9$	x and y are the lengths of the diagonals.
$= 45 \text{ mm}^2$	$\frac{1}{2} \times 10 \times 9 = 5 \times 9 = 45$
b Area $= \frac{1}{2}h(a + b)$	Use the area formula for a trapezium.
$= \frac{1}{2} \times 3 \times (4 + 6)$	Substitute $a = 4$, $b = 6$ and $h = 3$.
$= 15 \text{ m}^2$	Include the correct units.

6 Find the area of each of the following plane figures.

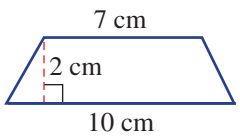
a



b

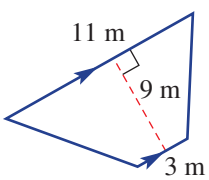


c

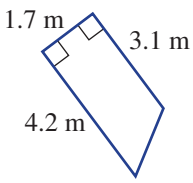


Choose from:
$A = \frac{1}{2}x \times y$ or
$A = \frac{1}{2}h(a + b)$

d



e



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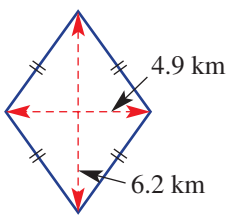

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PROBLEM-SOLVING AND REASONING

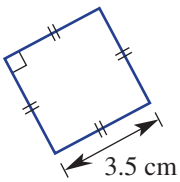
7, 8	7, 9–11

7 A piece of land has an area of half a square kilometre (0.5 km^2). How many square metres (m^2) is this?

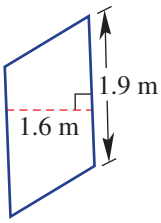
8 A rectangular park covers an area of 175 000 m^2. Give the area of the park in km^2.

9 Find the area of each of the following mixed-plane figures.

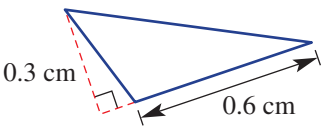
a



b

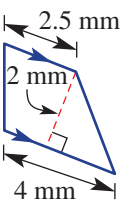


c



Choose from:
$A = s^2$, $A = b \times h$,
$A = \frac{1}{2}b \times h$ or
$A = \frac{1}{2}h(a + b)$.

d



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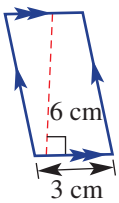

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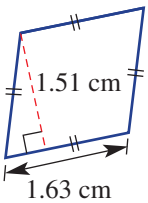


10 An old picture frame that was once square now leans to one side to form a rhombus. If the distances between pairs of opposite corners are 85 cm and 1.2 m, find the area inside the frame in m^2.

11 Convert the following measurements into the units given in the brackets.

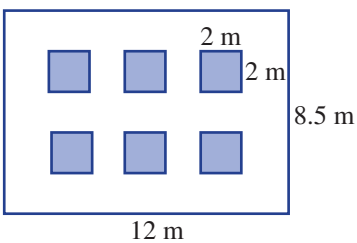
a 1.5 km^2 (cm^2) **b** 0.000005 m^2 (mm^2) **c** 75 000 mm^2 (m^2)

ENRICHMENT

–	12

Windows

12 Six square windows of side length 2 m are placed into a 12 m wide by 8.5 m high wall, as shown. The windows are positioned so that the vertical spacing between the windows and the wall edges are equal. The horizontal spacings are also equal.



a **i** Find the horizontal distance between the windows.

ii Find the vertical distance between the windows.

b Find the area of the wall, not including the window spaces.

c If the wall included 3 rows of 4 windows (instead of 2 rows of 3), would it be possible to space all the windows so that the horizontal and vertical spacings are the same? (Horizontal spacing doesn't have to be the same as vertical spacing.)

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6D Area of circles REVISION

Interactive

Widgets

HOTsheets

Walkthrough

We know that the circumference of a circle is connected to the diameter by the special number pi (π).

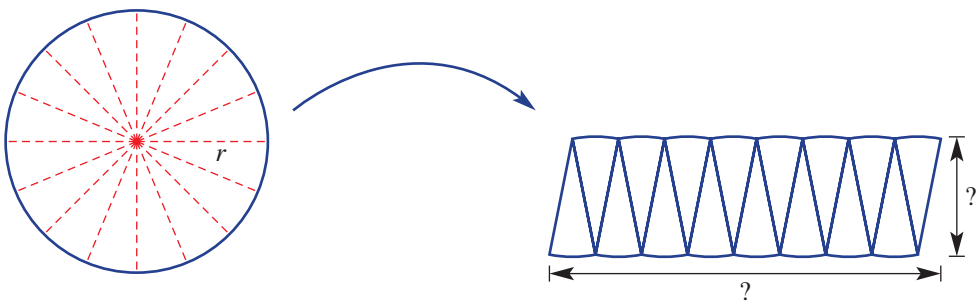
Similarly, the area of a circle is also connected to pi. This time it is the product of pi and the square of the radius that gives the area; so, $A = \pi r^2$.

The area of this circular stage is the product of pi and the square of its radius.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: How does a circle become a rectangle?

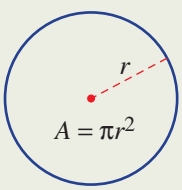
Consider a circle cut into small sectors, as shown, then rearranged to form a rectangular-style shape.



- Compared to the circle, what is the height of the rectangle close to?
- Compared to the circle, what is the length of the rectangle close to?
- What does this say about the area of the rectangle and hence the area of a circle?
- How could the rectangle be improved?

Key ideas

- The area of a circle is given by $A = \pi r^2$.
  - If the diameter is given, halve it to find the radius.



- The radius (r) is half the diameter (d).

$d = 10$ cm

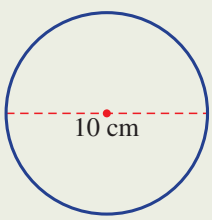
$\therefore r = 5$ cm

$\therefore A = \pi \times 5^2$

$\therefore$ Area $= 25\pi$ cm^2 (exact value)

$= 78.539\ldots$

$= 78.54$ cm^2 (to 2 d.p.)



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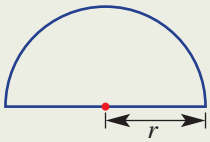
Key ideas

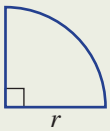
- A half circle is called a **semicircle**.

  $A = \frac{1}{2}\pi r^2$



- A quarter circle is called a **quadrant**.

  $A = \frac{1}{4}\pi r^2$



Semicircle Half a circle

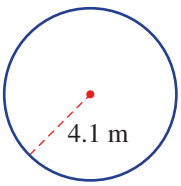
Quadrant A quarter of a circle

Exercise 6D REVISION

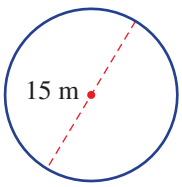
UNDERSTANDING AND FLUENCY 1–5 3–6

1 What is the radius of each of these circles?

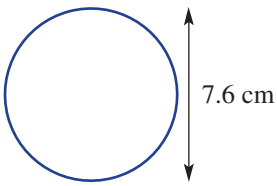
a



b



c



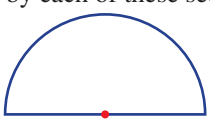
2 Which is the correct formula for the area of a circle?

A $A = \pi d^2$ **B** $A = \frac{1}{2}\pi r^2$ **C** $A = \pi r^2$ **D** $A = (\pi r)^2$

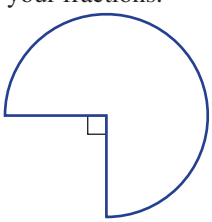
3 What fraction of a full circle is shown by each of these sectors? Simplify your fractions.

a

b

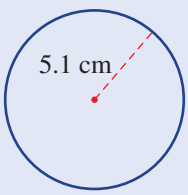


c



Example 10 Finding areas of circles

Find the area of this circle, correct to two decimal places.



SOLUTION

$A = \pi r^2$
$= \pi \times (5.1)^2$
$= 81.7128\ldots$
$= 81.71\ \text{cm}^2$ (to 2 d.p.)

EXPLANATION

Write the rule and substitute $r = 5.1$.

81.7128 rounds to 81.71 since the third decimal place is 2.

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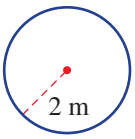
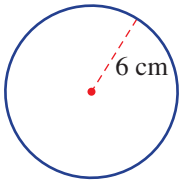
4 Find the area of each of these circles, correct to two decimal places.

a



b



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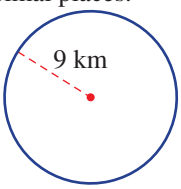

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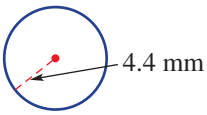

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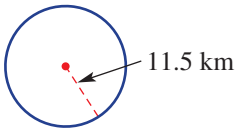

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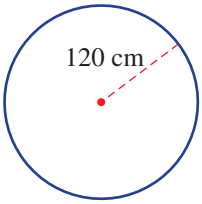
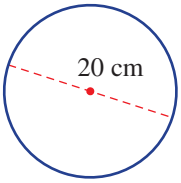


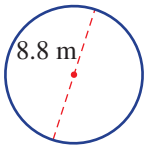
Substitute the radius into $A = \pi r^2$.

5 Find the area of each of these circles, correct to two decimal places.

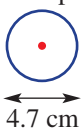
a



b



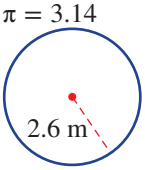
c



First work out the radius.

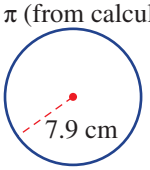
6 Find the area of each of these circles, using the given value for pi (π). Round to two decimal places.

a $\pi = 3.14$



b $\pi = \frac{22}{7}$

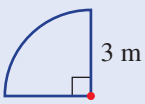
c π (from calculator)



PROBLEM-SOLVING AND REASONING	7–9	7, 9–11

Example 11 Finding areas of quadrants

Find the area of this quadrant, correct to two decimal places.



SOLUTION	EXPLANATION
$A = \frac{1}{4} \times \pi r^2$	A quadrant has one quarter of the area of a full circle.
$= \frac{1}{4} \times \pi(3)^2$	Substitute $r = 3$ and evaluate.
$= 7.0685\ldots$	
$= 7.07$ m^2 (to 2 d.p.)	

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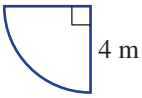
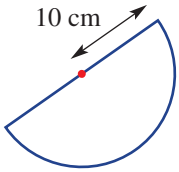
7 Find the areas of these sectors, correct to two decimal places.

a



b



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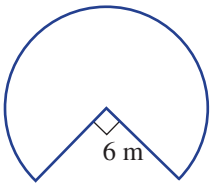


8 A pizza of radius 15 cm is divided into quarters. Find the area of each quarter correct to the nearest cm^2.

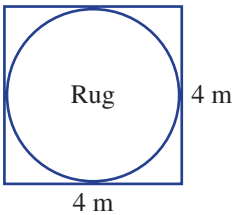
9 A circular rug touches the edges of a square room of side length 4 m.

a What is the radius of the rug?

b Find the area of the rug, correct to two decimal places.

c Find the area not covered by the rug, correct to two decimal places.

d Find the percentage of the floor that is not covered by the rug, correct to one decimal place.



$$\% \text{ area} = \frac{\text{rug area}}{\text{total area}} \times \frac{100}{1}$$

10 A pizza shop is considering increasing the diameter of its family pizza tray from 32 cm to 34 cm. Find the percentage increase in area, correct to two decimal places, from the 32 cm tray to the 34 cm tray.

11 You can rearrange $A = \pi r^2$ to give $r = \sqrt{\frac{A}{\pi}}$. Use this new rule to find the radius of a circle for these areas. Round to one decimal place where necessary.

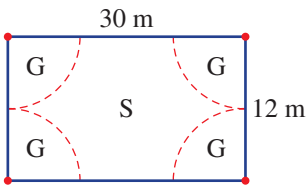
a 10 cm^2 **b** 117.8 m^2 **c** 4π km^2

ENRICHMENT – 12

Tennis lights

12 A tennis court area is lit by 4 corner lights. The area close to each light is considered to be good (G), while the remaining area is satisfactory (S).

What percentage of the area is 'good'? Round to the nearest per cent.



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6E Perimeter and area of composite shapes

Interactive
Widgets
HOTsheets
Walkthrough

Composite shapes are made from simple shapes. You can find the perimeters and areas of composite shapes by starting with simple shapes for which we have known formulas.

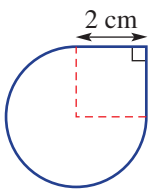
The area of this window can be worked out using the area of triangles and trapeziums.

The area of this window can be worked out using the area a semi-circle and rectangle.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: A fraction of a circle plus a square

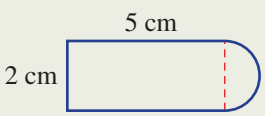
This diagram shows a fraction of a circle plus a square.



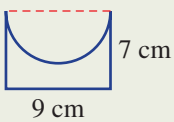
- How could you find the area and perimeter of the shape?
- See if you can write down a full solution for finding the area and perimeter of the shape.
- See if your teacher or another student can easily follow your solution.

Key ideas

- Composite shapes are made up of more than one basic shape.
- Addition and/or subtraction can be used to find areas and perimeters of composite shapes.
  - Use addition.



  - Use subtraction (for area).



- The layout of the relevant mathematical working needs to make sense so that the reader of your work understands each step.

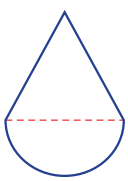
Exercise 6E

UNDERSTANDING AND FLUENCY 1–5 3, 4(½), 5, 6(½)

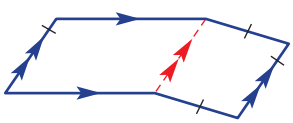
1 Name the two different shapes that make up each of these composite shapes; e.g. square and semicircle.

a

b



c



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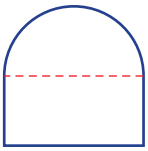
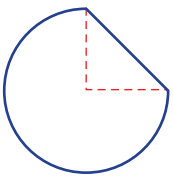
2 What fraction of a circle is used in each of these shapes?

a



b

c



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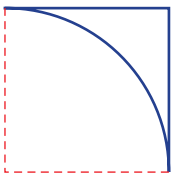
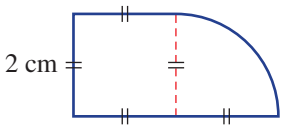


Choose from $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$.

For a full circle:
$A = \pi r^2$
$C = 2\pi r$

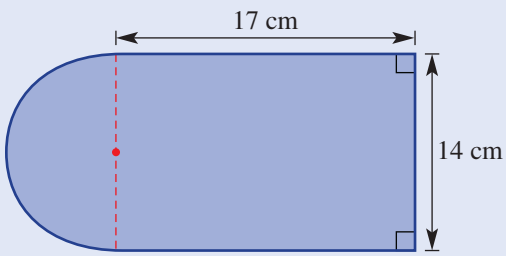
3 This composite shape includes a square and a quadrant ($\frac{1}{4}$ circle).



a Find the area of the square.
b Find the area of the quadrant, correct to two decimal places.
c Find the total area, correct to two decimal places.
d Find the perimeter of the entire shape, correct to two decimal places.

Example 12 Finding perimeters and areas of composite shapes

Find the perimeter and area of this composite shape, rounding answers to two decimal places.



SOLUTION

$P = 2 \times l + b + \frac{1}{2} \times 2\pi r$

$= 2 \times 17 + 14 + \frac{1}{2} \times 2\pi \times 7$

$= 69.99$ cm (to 2 d.p.)

Area (rectangle) $= l \times b$

$= 17 \times 14$

$= 238$ cm^2

Area (semicircle) $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 7^2$

$= 76.969\ldots$ cm^2

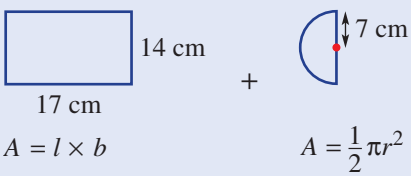
Total area $= 238 + 76.969\ldots$

$= 314.97$ cm^2 (to 2 d.p.)

EXPLANATION

3 straight sides ⊐ + semicircle arc ☾
Recall that $C = 2\pi r$.
Substitute $l = 17$, $b = 14$ and $r = 7$.
Calculate and round to two decimal places.

The total area consists of a rectangle plus a semicircle.



$A = l \times b$ $\qquad$ $A = \frac{1}{2}\pi r^2$

Round to two decimal places.

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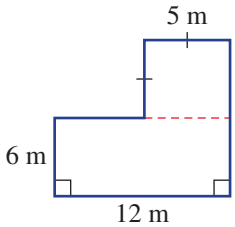
4 Find the perimeter and the area of each of these simple composite shapes, rounding answers to two decimal places where necessary.

First decide which two shapes you will be dealing with in each case.

a



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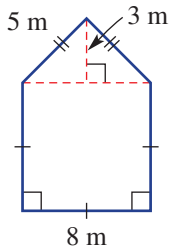

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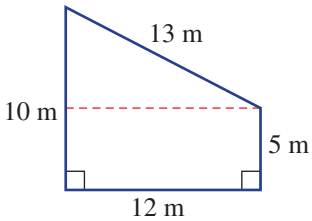

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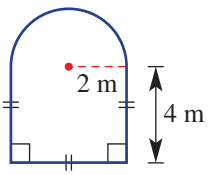

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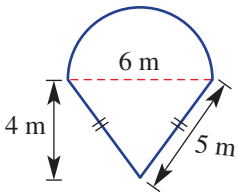

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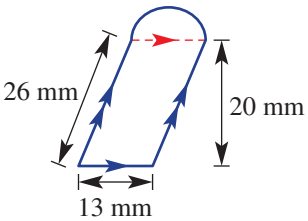
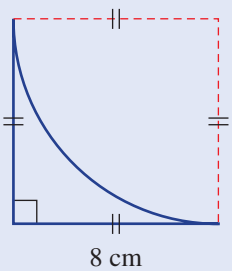


Example 13 Finding more perimeters and areas

This shape is a square with a quadrant (quarter circle) subtracted.
Find its perimeter and area, correct to two decimal places.



SOLUTION	EXPLANATION
$P = 2 \times s + \frac{1}{4} \times 2\pi r$	Includes 2 sides and a quarter of the circumference of a circle of radius 8 cm.
$= 2 \times 8 + \frac{1}{4} \times 2\pi \times 8$	
$= 28.57$ cm (to 2 d.p.)	
$A = s^2 - \frac{1}{4} \times \pi r^2$	Includes a square; subtract a quarter of the area of a circle of radius 8 cm.
$= 8^2 - \frac{1}{4} \times \pi \times 8^2$	
$= 13.73$ cm^2 (to 2 d.p.)	

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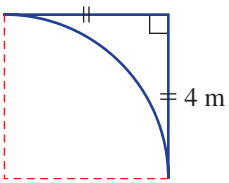
5 Find the perimeters and areas of these shapes, correct to two decimal places.

a



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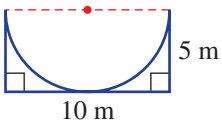

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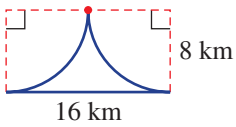
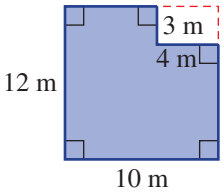


Use subtraction to find the area.

6 Find the area of the shaded part of each of these composite shapes.

Use subtraction in each case.

a



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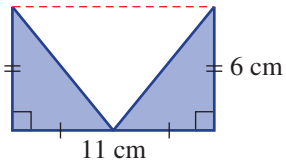

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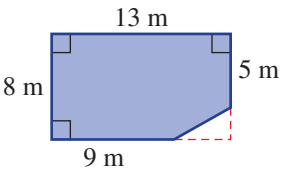

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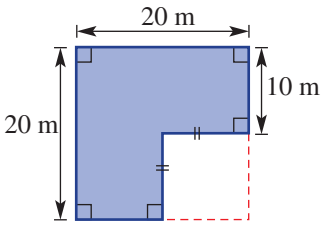

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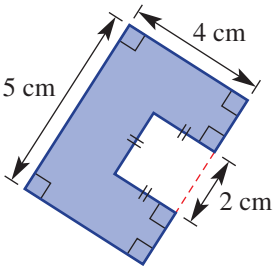

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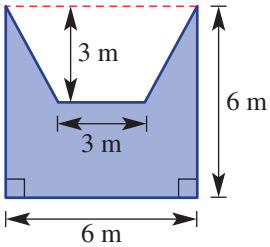

PROBLEM-SOLVING AND REASONING

	7, 8	7, 9, 10

7 An area of lawn is made up of a rectangle measuring 10 m by 15 m and a semicircle of radius 5 m. Find the total area of lawn, correct to two decimal places.

8 Twenty circular pieces of pastry, each of diameter 4 cm, are cut from a rectangular layer of pastry 20 cm long and 16 cm wide. What is the area, correct to two decimal places, of pastry remaining after the 20 pieces are removed?

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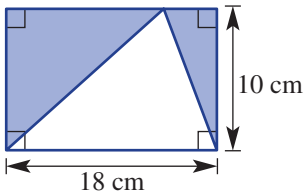
9 Find the area of the shaded region of each of the following shapes by subtracting the area of the clear shape from the total area. Round to two decimal places where necessary.

a



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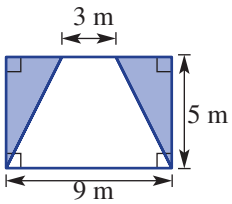

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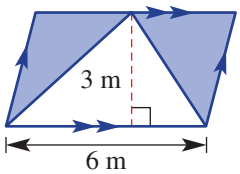

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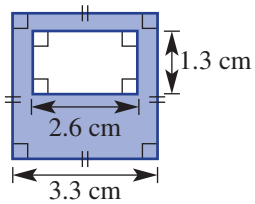

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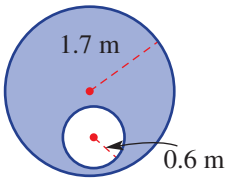

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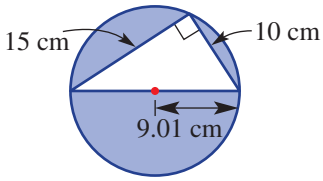

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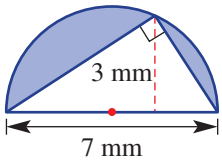

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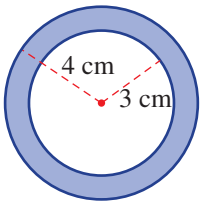

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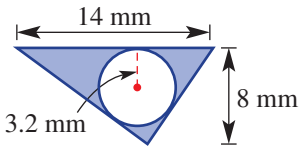
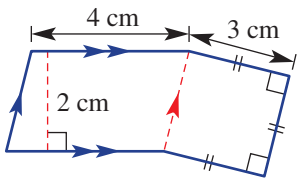


10 Find the area of each of the following composite shapes.

a



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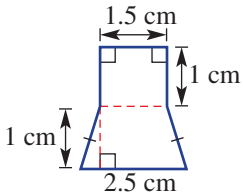

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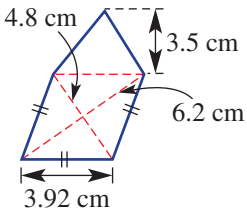
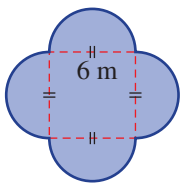


ENRICHMENT – 11

Circular challenges

11 Find the perimeter and the area of each of the following composite shapes, correct to two decimal places where necessary.

a



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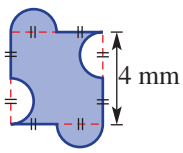

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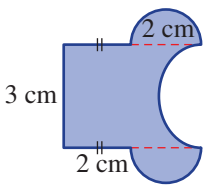

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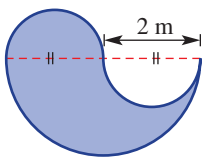

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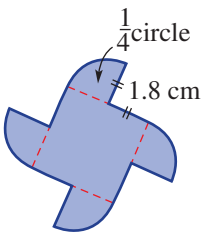

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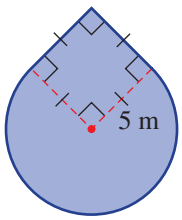

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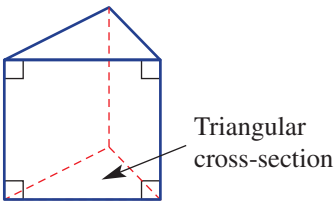
6F Surface area of prisms

Interactive
Widgets
HOTsheets
Walkthrough

Three-dimensional objects or solids have outside surfaces that together form the total surface area. Nets are very helpful for determining the number and shapes of the surfaces of a three-dimensional object.

For this section we will deal with right prisms. A right prism has a uniform cross-section with two identical ends and the remaining sides are rectangles.



Right triangular prism

The Flatiron Building in New York City is the shape of a triangular prism.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Drawing prisms

Prisms are named by the shape of their cross-section.

- Try to draw as many different right prisms as you can.
- Describe the different types of shapes that make up the surface of your solids.
- Which solids are the most difficult to draw and why?

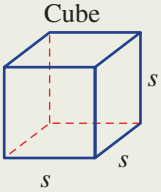
Key ideas

- The **surface area** of a solid is the sum of the areas of all the surfaces.
- A **net** is a two-dimensional illustration of all the surfaces of a solid.

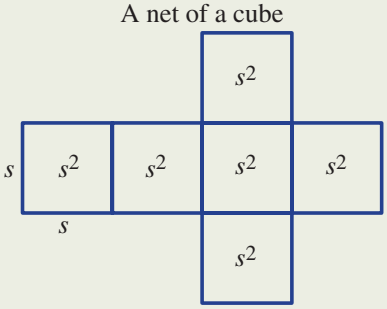
Surface area The number of square units needed to cover the outside of a solid

Net A diagram showing how the plane faces of a solid are joined to each other

Cube



A net of a cube



The surface area of a cube is given by $A = 6s^2$.

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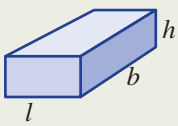
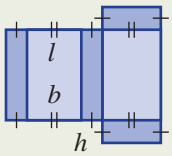
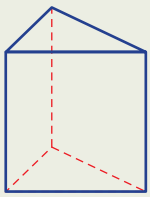
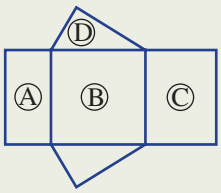
Key ideas

- A **right prism** is a solid with a uniform cross-section and with remaining sides as rectangles.
  - Prisms are named by the shape of their cross-section.
- The nets for a rectangular prism (cuboid) and triangular prism are shown here.

Right prism A solid with a uniform cross-section, and remaining sides are rectangles

Solid	Net	Surface area
Rectangular prism 		$A = 2(lb + bh + lh)$
Triangular prism 		A = area Ⓐ + area Ⓑ + area Ⓒ + 2 × area Ⓓ

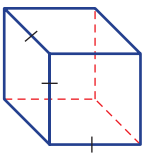
Exercise 6F

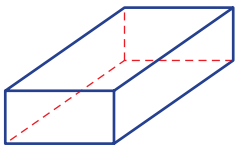
UNDERSTANDING AND FLUENCY 1–5 3, 4(½), 5, 6

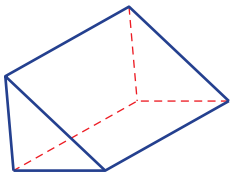
1 How many faces do the following solids have?

a rectangular prism **b** cube **c** triangular prism

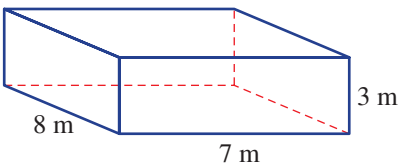
2 Draw a suitable net for these prisms and name each solid.

a


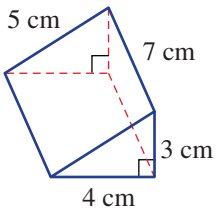
b


c


3 Copy and complete the working to find the surface area of these solids.

a


$A = 2 \times (__ \times __ + __ \times __ + __ \times __)$

b


$A = __ \times __ + __ \times __ + __ \times __ + 2 \times \left(\frac{1}{2} \times __ \times __\right)$

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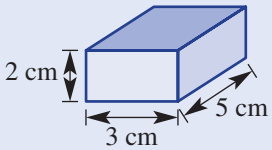
Example 14 Finding the surface area of a rectangular prism

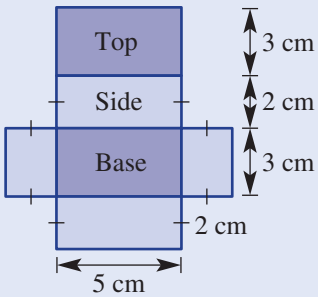
Find the surface area of this rectangular prism.



SOLUTION

$A = 2 \times (3 \times 5 + 2 \times 5 + 2 \times 3)$
$= 2 \times 31$
$= 62 \text{ cm}^2$

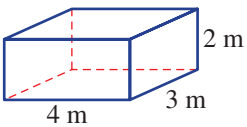
EXPLANATION



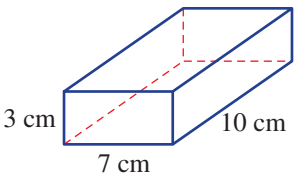
Top = 3×5
Side = 2×5
End = 2×3

4 Find the surface area of each of the following rectangular prisms. Draw a net of each solid to help you.

a

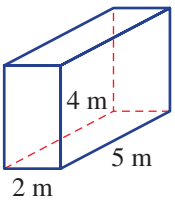


b

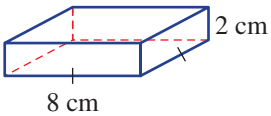


Add up all the pairs of rectangles using $A = l \times b$

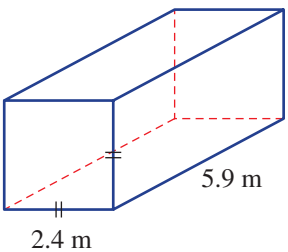
c



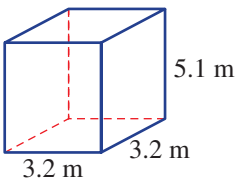
d



e



f



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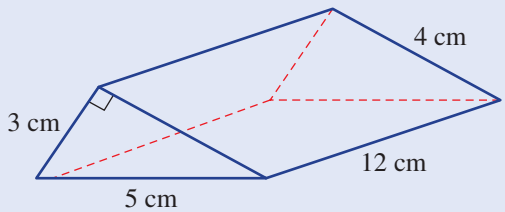
Example 15 Finding the surface area of a right triangular prism

Find the surface area of this triangular prism.



SOLUTION

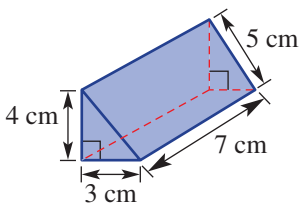
$$A = 12 \times 5 + 12 \times 4 + 12 \times 3 + 2 \times \left(\frac{1}{2} \times 4 \times 3\right)$$
$$= 60 + 48 + 36 + 12$$
$$= 156 \text{ cm}^2$$

EXPLANATION

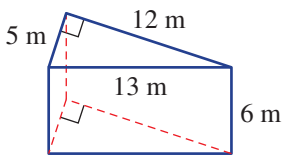
The surface is made up of three rectangles and two identical triangles. Use 3 cm and 4 cm for the base and height, respectively, of the triangles.

5 Find the surface area of each of the following triangular prisms.

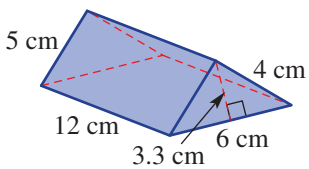
a



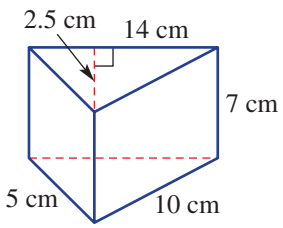
b



c



d



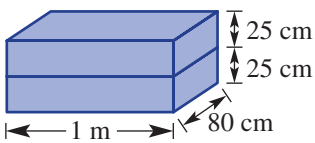
Each one consists of three rectangles and two triangles.

6 Find the surface area of a cube of side length 1 metre.

PROBLEM-SOLVING AND REASONING 7, 8 7, 9, 10

7 A rectangular box is to be covered in material. How much material is required to cover the entire box if it has the dimensions 1.3 m, 1.5 m and 1.9 m?

8 Two wooden boxes, both with dimensions 80 cm, 1 m and 25 cm, are placed on the ground, one on top of the other, as shown. The entire outside surface, including the underside of the bottom box, is then painted. Find the area of the painted surface in cm^2.



Only include the outside surfaces that you could paint.

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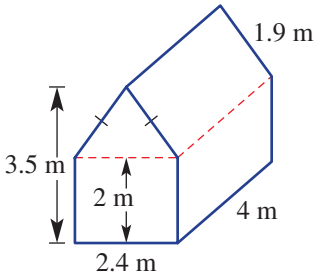
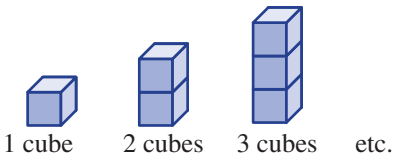
9 The four walls and roof of a barn (shown) are to be painted.

a Find the surface area of the barn, not including the floor.

b If 1 litre of paint covers 10 m^2, find how many litres are required to complete the job.



10 Cubes of side length one unit are stacked as shown.



a Complete this table.

Number of cubes (n)	1	2	3	4	5	6	7	8	9
Surface area (A)									

b Can you find the rule for the surface area (A) for n cubes stacked in this way? Write down the rule for A in terms of n.

c Use your rule to find the surface area if there are 100 cubes.

An example of a rule is $A = 2n + 5$, but this is not the rule for this question.

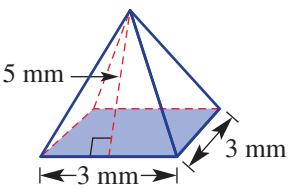
ENRICHMENT – 11

Pyramids

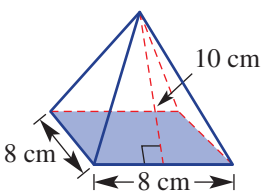
11 Pyramids consist of a base and a number of triangular faces. The surface area can be calculated by adding all the face areas, similar to that of prisms. Remember that the area of a triangle is given by $A = \frac{1}{2}bh$.

Find the surface area of each of these pyramids. Draw a net of each solid to help you.

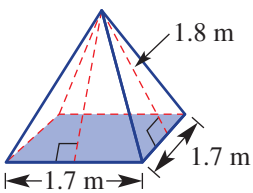
a



b



c



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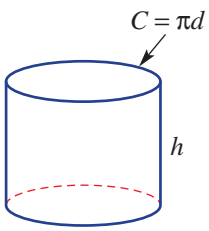
6G Surface area of cylinders

Interactive
Widgets
HOTsheets
Walkthrough

Like a rectangular prism, a cylinder has a uniform cross-section (a circle), but because it doesn't have rectangular sides, it isn't classified as a right prism.

A cylinder's total surface area can be found by considering the curved part as a rectangle with length $2\pi r$ (or πd) and height h.

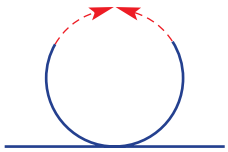


These pipes are hollow cylinders.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Curved area

- Roll a piece of paper to form the curved surface of a cylinder.
- Do not stick the ends together, so that you can allow the paper to return to a flat surface.
- What shape is the paper when lying flat on a table?
- When curved to form the cylinder, what do the sides of the rectangle represent on the cylinder? How does this help to find the surface area of a cylinder?

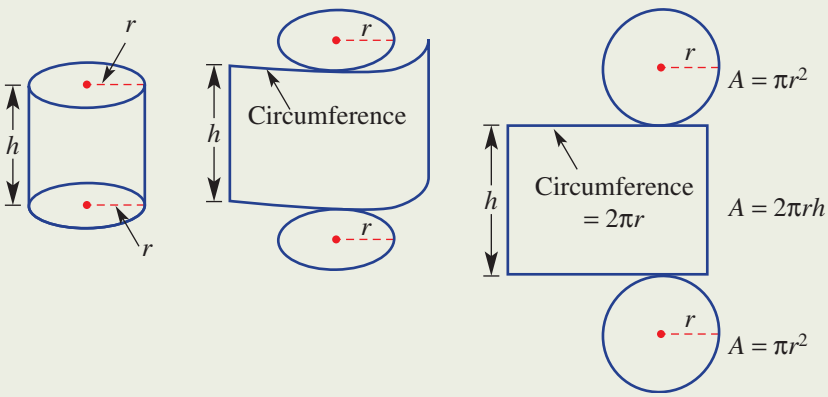


Key ideas

■ Surface area of a **cylinder**

$$A = 2 \text{ circles} + 1 \text{ rectangle}$$
$$= 2 \times \pi r^2 + 2\pi r \times h$$
$$= \underbrace{2\pi r^2}_{\text{2 circular ends}} + \underbrace{2\pi rh}_{\text{curved area}}$$

Cylinder A solid with two parallel, congruent circular faces connected by a curved surface



■ A cylindrical can with no top or bottom will have the curved surface only. The surface area of the outer surface is given by $A = 2\pi rh$.

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Exercise 6G

UNDERSTANDING AND FLUENCY | 1–6 | 4–7

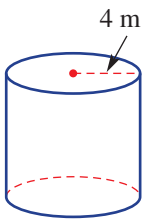
1 The formula for the surface area of a cylinder is $A = 2\pi r^2 + 2\pi rh$.

a Which part of the formula works out the area of the curved surface?

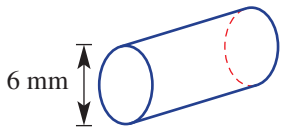
b Which part of the formula works out the area of the two ends?

2 Find the circumference of the circular end of each of these cylinders. Round to two decimal places.

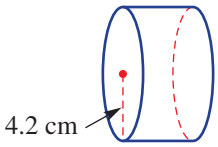
a



b



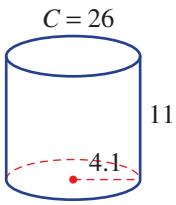
c



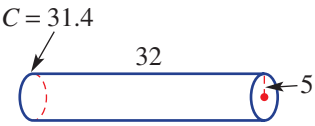
Use $C = \pi d$ or $C = 2\pi r$.

3 Draw a net suited to each of these cylinders. Label the sides using the given measurements.

a

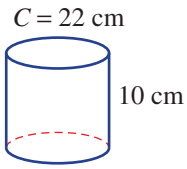


b

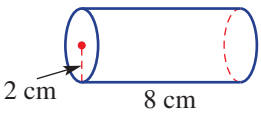


4 The curved surface of each of these cylinders is allowed to flatten out to form a rectangle. What would be the length and breadth of the rectangles? Round to two decimal places where necessary.

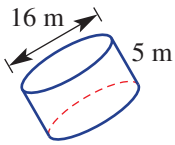
a



b



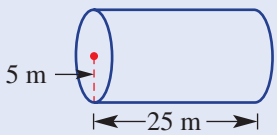
c



Only work out and write down the length and breadth.

Example 16 Finding the surface area of a cylinder

Find the surface area of this cylinder, rounding to two decimal places.



SOLUTION	EXPLANATION
$A = 2\pi r^2 + 2\pi rh$	In the diagram, $r = 5$ and $h = 25$.
$= 2 \times \pi \times 5^2 + 2 \times \pi \times 5 \times 25$	
$= 50\pi + 250\pi$	
$= 300\pi$	300π is the *exact* answer.
The surface area is 924.48 m^2 (to 2 d.p.).	Calculate and round off.

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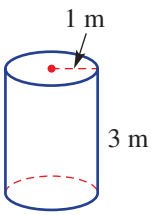
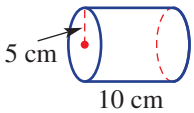
5 Find the surface area of each of these cylinders, rounding to two decimal places.

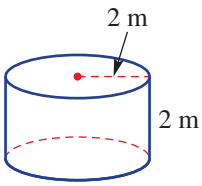
a



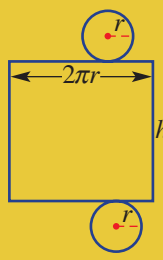
b



c

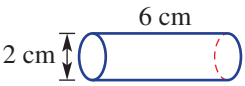


Draw a net to help:

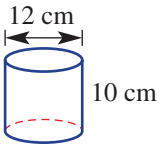


6 Find the surface area of each of these cylinders, rounding to one decimal place.

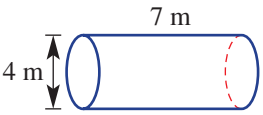
a



b



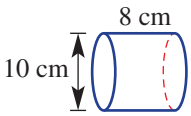
c



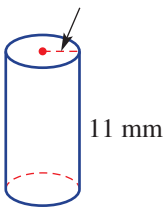
Remember that the radius of a circle is half the diameter.

7 Find the areas of the curved surfaces only for these cylinders, correct to two decimal places.

a



b



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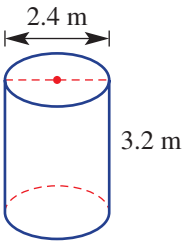


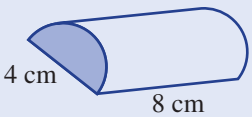
For the curved part only, use $A = 2\pi rh$

PROBLEM-SOLVING AND REASONING	8, 9	8–11

8 Find the surface area of a cylindrical plastic container of height 18 cm and with a circle of radius 3 cm at each end, correct to two decimal places.

Example 17 Finding surface areas of cylindrical portions

Find the surface area of this half-cylinder, rounding to two decimal places.



SOLUTION

Area (two semicircles) $= \pi r^2$

$= \pi \times 2^2$

$= 12.566\ldots \text{ cm}^2$

Area (rectangles) $= 8 \times 4 + 8 \times \frac{1}{2} \times (2\pi \times 2)$

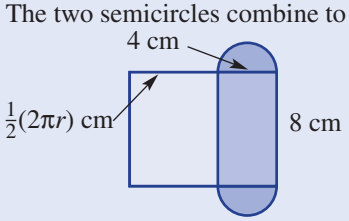
$= 82.265\ldots \text{ cm}^2$

$A = 12.566\ldots \text{ cm}^2 + 82.265\ldots \text{ cm}^2$

$= 94.83 \text{ cm}^2$ (to 2 d.p.)

EXPLANATION

The two semicircles combine to form a full circle.



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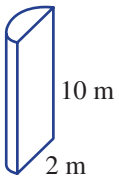
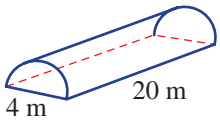
9 Find the surface area of each of these solids, rounding to two decimal places.

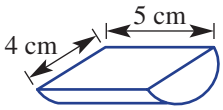
a



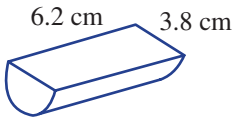
b



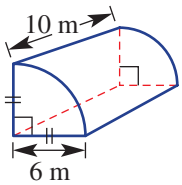
c



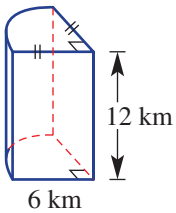
d



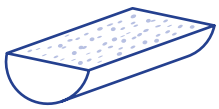
e



f



10 A water trough is in the shape of a half-cylinder. Its semicircular ends have diameter 40 cm and the trough length is 1 m. Find the outside surface area, in cm^2, of the curved surface plus the two semicircular ends, correct to two decimal places.

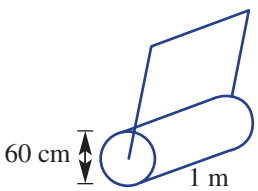


11 A solid cylinder cut in half gives half the volume but not half the surface area. Explain why.

ENRICHMENT – 12

Roller revolutions

12 A cylindrical roller is used to prepare a smooth flat surface for a bowling green. The entire rectangular bowling green is 30 m long and 15 m wide. The roller has a width of 1 m and diameter 60 cm.



$1\ m^2 = (100 \times 100)\ cm^2$

a Find the surface area of the curved part of the roller, in cm^2, correct to three decimal places.

b Find the area, in m^2 to two decimal places, of lawn that can be pressed after:

i 1 revolution

ii 20 revolutions

c Find the minimum number of complete revolutions required to press the entire bowling green area.

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6H Volume of prisms

Interactive
Widgets
HOTsheets
Walkthrough

We use volume to describe the amount of space inside a three-dimensional object. We use metric units, such as:

- cubic kilometres for the volume of water in the sea
- cubic metres for the volume of concrete poured at a building site
- cubic centimetres for the volume of space occupied by this book
- cubic millimetres for the volume of metal in a pin.

Units for capacity (millilitres, litres, kilolitres and mega-litres) are used for liquids and gases.

It is helpful to know the volume of concrete necessary for a given area.

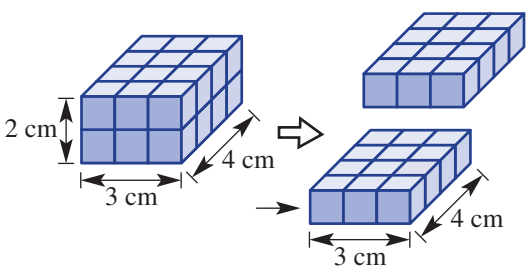
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Why length × breadth × height?

For most people, the first thing that comes to mind when dealing with volume is length × breadth × height. But this rule only applies to finding the volume of rectangular prisms.

Let's look at a rectangular prism split into two layers.

- How many cubes sit on one layer?
- What is the area of the base? What do you notice?
- What is the height and how many layers are there?
- Why is the volume rule given by $V = lbh$ in this case?



Key ideas

- Common metric units for **volume** include cubic kilometres (km^3), cubic metres (m^3), cubic centimetres (cm^3) and cubic millimetres (mm^3).

$$km^3 \xrightarrow{\times 1000^3} m^3 \xrightarrow{\times 100^3} cm^3 \xrightarrow{\times 10^3} mm^3$$
$$km^3 \xleftarrow{\div 1000^3} m^3 \xleftarrow{\div 100^3} cm^3 \xleftarrow{\div 10^3} mm^3$$

$1000^3 = 1\,000\,000\,000$
$100^3 = 1\,000\,000$
$10^3 = 1000$

Volume The amount of three-dimensional space inside an object

- For **capacity**, common units include:
  - megalitres (ML) 1ML = 1000 kL
  - kilolitres (kL) 1 kL = 1000 L
  - litres (L) 1L = 1000 mL
  - millilitres (mL)

$$ML \xrightarrow{\times 1000} kL \xrightarrow{\times 1000} L \xrightarrow{\times 1000} mL$$
$$ML \xleftarrow{\div 1000} kL \xleftarrow{\div 1000} L \xleftarrow{\div 1000} mL$$

Capacity The amount of liquid a container can hold

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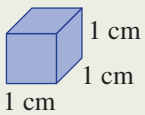
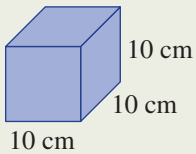
Key ideas

- A cubic centimetre holds 1 mL of liquid.



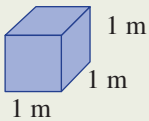
$\boxed{1\text{ cm}^3 \text{ holds } 1\text{ mL}}$

- This container holds 1 litre of liquid.



$\boxed{1000\text{ cm}^3 \text{ holds } 1\text{ L}}$

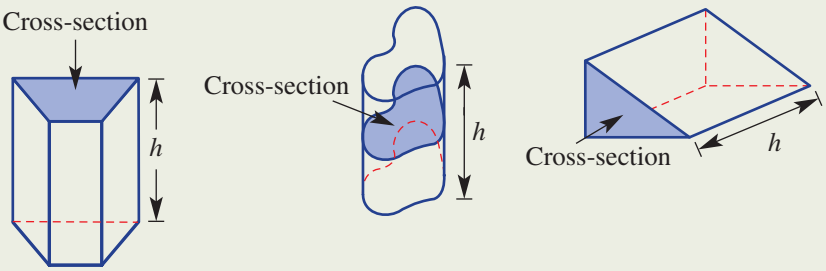
- This container holds 1000 litres or 1 kilolitre.



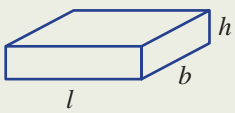
$\boxed{1\text{ m}^3 \text{ holds } 1000\text{ L or } 1\text{ kL}}$

- Volume of solids with a uniform **cross-section** is equal to area of cross-section (A) × height (h).
  $V = A \times h$

Cross-section
The plane figure formed when you slice a solid figure parallel to one of its surfaces



- The 'height' is the length of the edge that runs perpendicular to the cross-section in any solid.
- Volume of a rectangular prism:
  $V = l \times b \times h$

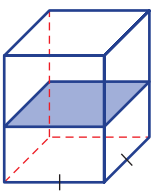


Exercise 6H

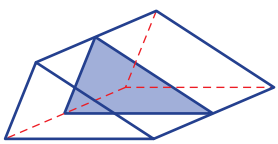
UNDERSTANDING AND FLUENCY	1–9	3, 4–5(½), 6, 7, 8(½), 9

1 What is the name given to the shape of the shaded cross-section of each of the following solids?

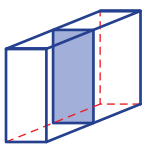
a



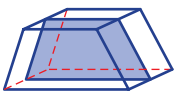
b



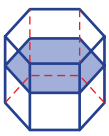
c



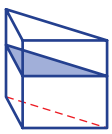
d



e



f



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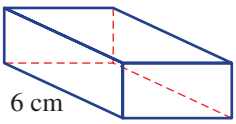
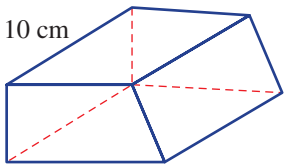
2 Draw the cross-sectional shape for each of these prisms and state the given 'height' (perpendicular to the cross-section).

a

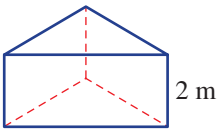


b

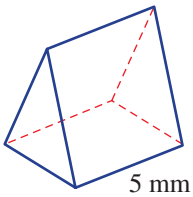


'Perpendicular' means 'at a right angle (90°)'.

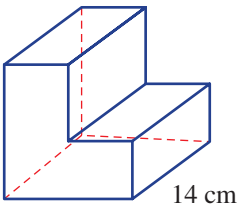
c



d



e



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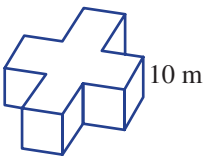

3 Write the missing number.

a The number of mm in 1 cm is _____.
b The number of mm^2 in 1 cm^2 is _____.
c The number of mm^3 in 1 cm^3 is _____.
d There are _____ cm^3 in 1 m^3.
e There are _____ m^3 in 1 km^3.
f There are _____ mL in 1 L.
g There are _____ L in 1 kL.
h There are _____ cm^3 in 1 mL.

Example 18 Converting units of volume

Convert the following volume measurements into the units given in the brackets.

a 2.5 m^3 (cm^3)
b 458 mm^3 (cm^3)

SOLUTION

a $2.5\,m^3 = 2.5 \times 100^3\,cm^3$
$= 2\,500\,000\,cm^3$

b $458\,mm^3 = 458 \div 10^3\,cm^3$
$= 0.458\,cm^3$

EXPLANATION

$\times 100^3 = 1\,000\,000$

m^3 cm^3 2.500000

cm^3 mm^3

$\div 10^3 = 1000$ 458.

4 Convert the following volume measurements into the units given in brackets.

a 3 cm^3 (mm^3)
b 0.3 cm^3 (mm^3)
c 2000 mm^3 (cm^3)
d 0.001 m^3 (cm^3)
e 8.7 m^3 (cm^3)
f 5900 cm^3 (m^3)
g 0.00001 km^3 (m^3)
h 21 700 m^3 (km^3)
i 430 000 cm^3 (m^3)

$1\,km^3 = 1000^3\,m^3$
$1\,m^3 = 100^3\,cm^3$
$1\,cm^3 = 10^3\,mm^3$

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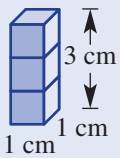
5 Convert these units of capacity to the units given in brackets.

a 3 L (mL)
b 0.2 kL (L)
c 3500 mL (L)
d 0.021 L (mL)
e 37 000 L (kL)
f 42 900 kL (ML)
g 2 cm^3 (mL)
h 2 L (cm^3)
i 1 m^3 (L)

1 ML = 1 000 kL
1 kL = 1 000 L
1 L = 1 000 mL

Example 19 Finding the volume of a rectangular prism

Find the volume of this rectangular prism.



SOLUTION

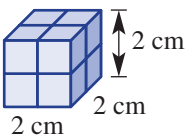
Volume = $l \times b \times h$
= $1 \times 1 \times 3$
= 3 cm^3

EXPLANATION

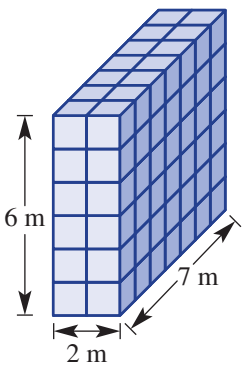
The solid is a rectangular prism.
Length = 1 cm, breadth = 1 cm and height = 3 cm

6 Find the volume of these three-dimensional rectangular prisms.

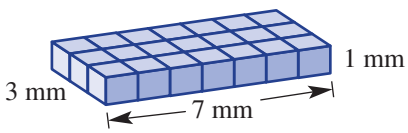
a



b

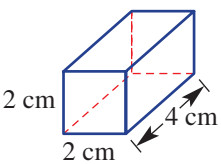


c



7 Find the volume of each of these rectangular prisms (cuboids).

a



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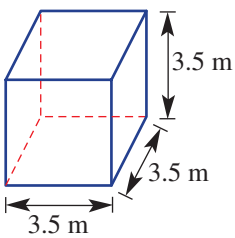
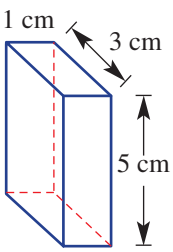


c



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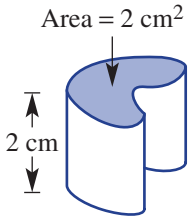
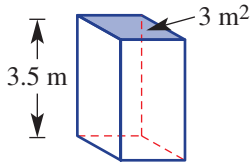
8 Find the volume of each of these three-dimensional objects. The cross-sectional area has been given.

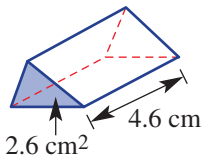
a



b

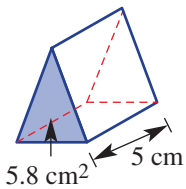


c

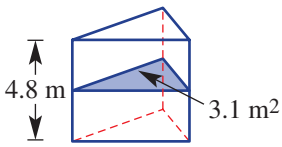


Simply use $V = A \times h$, since the area of the cross-section is given.

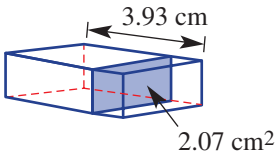
d



e

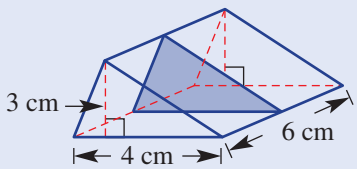


f



Example 20 Finding the volume of a triangular prism

Find the volume of this triangular prism.



SOLUTION

Area of cross-section $= \frac{1}{2} \times b \times h_1$

$= \frac{1}{2} \times 4 \times 3$

$= 6 \text{ cm}^2$

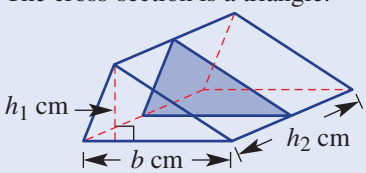
$V = A \times h_2$

$= 6 \times 6$

$= 36 \text{ cm}^3$

EXPLANATION

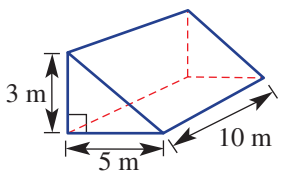
The cross-section is a triangle.



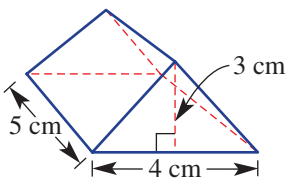
Multiply the area of the cross-section by the height.

9 Find the volumes of these prisms.

a

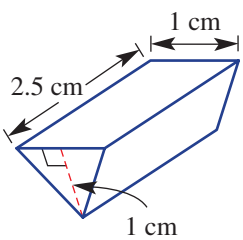


b



First find the area of the triangular cross-section.

c



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PROBLEM-SOLVING AND REASONING

	10, 11	10, 12, 13

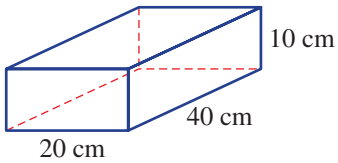
10 A brick is 10 cm wide, 20 cm long and 8 cm high. How much space would five of these bricks occupy?

11 25 L of water is poured into a rectangular fish tank that is 50 cm long, 20 cm wide and 20 cm high. Will it overflow?

There are 1000 cm^3 in 1L.

12 Find the volumes of these solids, converting your answers to litres.

a



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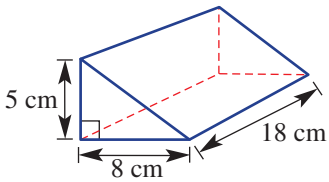

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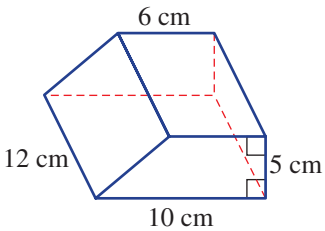
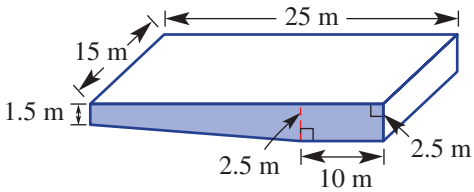


Area of a trapezium:
$A = \frac{1}{2}h(a + b)$

13 This diagram is a sketch of a new 25 m swimming pool to be installed in a school sports complex.

a Find the area of one side of the pool (shaded).

b Find the volume of the pool in litres.
Use $1\,m^3 = 1\,000$ L.



ENRICHMENT

	–	14

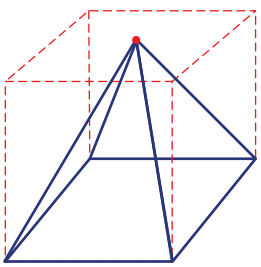
Volume of a pyramid

14 Someone tells you that the volume of a pyramid is half of the volume of a rectangular prism with the same base. Do you think this is true?

a Make an educated guess as to what fraction of the prism's volume is the pyramid's volume.

b Use the internet to find the actual answer to part **a**.

c Draw some pyramids and find their volume using the results from part **b**.



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6I Volume of cylinders

Interactive
Widgets
HOTsheets
Walkthrough

A cylinder is not a right prism because its sides are not rectangles. It does have a uniform cross-section (a circle), so its volume can be calculated in a similar way to that of a prism. Cylindrical objects are often used to store gases and liquids, so working out the volume of a cylinder is an important measurement calculation.

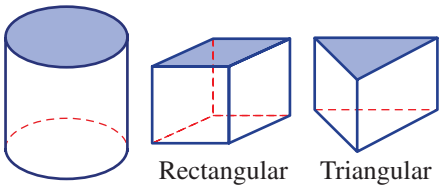
The volume of liquid in this tanker can be estimated using the volume formula for a cylinder.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: How is a cylinder like a prism?

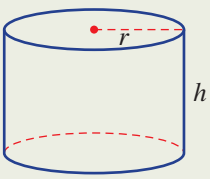
Here is a cylinder and two prisms.

- What do they all have in common?
- Do you think you can work out the volume of the cylinder in the same way as for a prism? Why?
- What do you think the formula would be for the volume of a cylinder?



Key ideas

- The volume of a cylinder is given by:
  $V = \pi r^2 \times h$ or $V = \pi r^2 h$
  where r is the radius of the circular ends
  h is the length or distance between the circular ends.

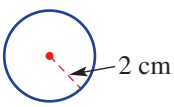


Exercise 6I

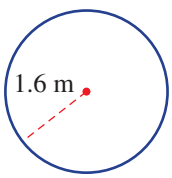
UNDERSTANDING AND FLUENCY 1–4 3, 4(½)

1 Find the area of each of these circles, correct to two decimal places.

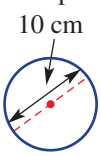
a



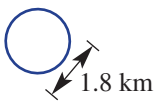
b



c



d



2 Convert the following into the units given in the brackets. Remember, 1 L = 1000 cm^3 and 1 m^3 = 1000 L.

a 2000 cm^3 (L) **b** 4.3 cm^3 (mL) **c** 3.7 L (cm^3)
d 1 m^3 (L) **e** 38 000 L (m^3) **f** 0.0002 m^3 (mL)

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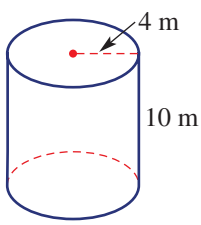
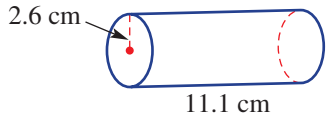
3 State the radius (r) and the height (h) of these cylinders.

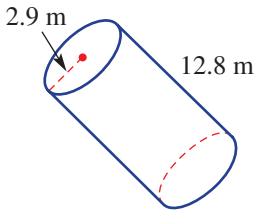
a



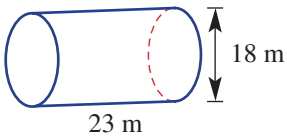
b



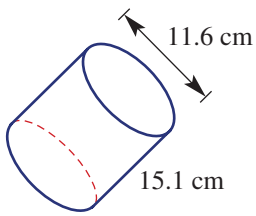
c



d



e



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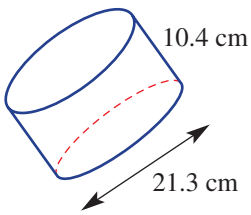
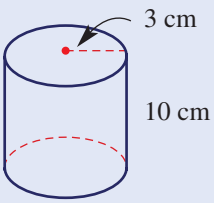


The height is perpendicular (at 90°) to the circular ends.

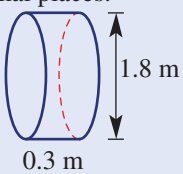
Example 21 Finding the volume of a cylinder

Find the volumes of these cylinders, correct to two decimal places.

a



b



SOLUTION

a $V = \pi r^2 h$
$= \pi \times (3)^2 \times 10$
$= 90\pi \text{ cm}^3$
$= 282.74 \text{ cm}^3$

b $V = \pi r^2 h$
$= \pi \times (0.9)^2 \times 0.3$
$= 0.7634\ldots$
$= 0.76 \text{ m}^3$ (to 2 d.p.)

EXPLANATION

πr^2 gives the area of the circular cross-section and h is the distance between the ends.
Substitute $r = 3$ and $h = 10$ into the rule.

The diameter is 1.8 m so $r = 0.9$.
The distance between the ends is 0.3 m, so $h = 0.3$.

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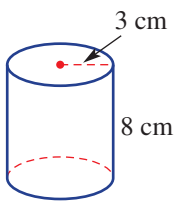
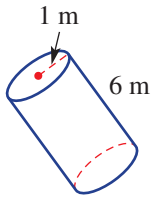
4 Find the volumes of these cylinders, correct to two decimal places.

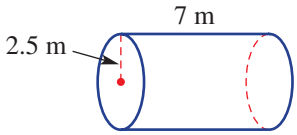
a



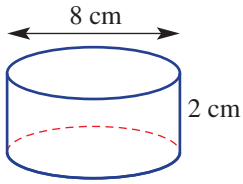
b



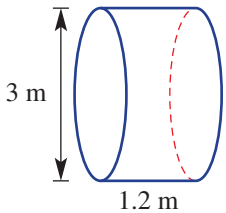
c



d



e



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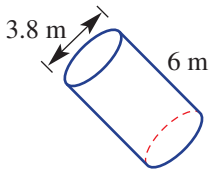


Substitute the values of r and h into $V = \pi r^2 h$. Divide the diameter by 2 to determine the radius when the diameter is given.

PROBLEM-SOLVING AND REASONING	5–7	5–10

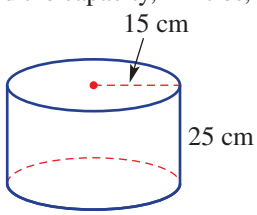
Example 22 Finding the capacity of a cylinder

Find the capacity, in litres, of a cylinder with radius 30 cm and height 90 cm. Round to the nearest litre.

SOLUTION	EXPLANATION
$V = \pi r^2 h$	
$= \pi \times (30)^2 \times 90$	Substitute $r = 30$ and $h = 90$.
$= 81\,000\pi$	
$= 254\,469.004\ldots$ cm^3	
$= 254\,469.004\ldots$ mL	There are 1000 cm^3 in 1 L, so divide by 1000 to convert to litres.
$= 254.469\ldots$ L	
$= 254$ L (nearest litre)	

5 Find the capacity, in litres, of each of these cylinders. Round to the nearest litre.

a



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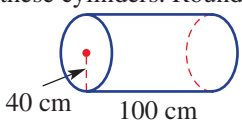

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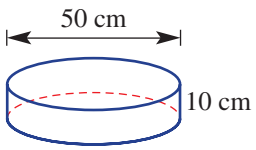

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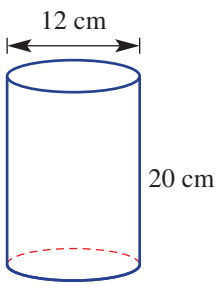

First work out the volume in cm^3 then use 1 L = 1000 cm^3 to find the capacity in litres.

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6 A cylindrical storage drum has a radius of 0.5 m and a height of 2 m.

a Find its volume, in m^3, correct to three decimal places.

b Find its volume, in L, correct to the nearest litre ($1\,m^3 = 1\,000$ L).

7 Which has a greater capacity: a 10 cm by 10 cm by 10 cm cube or a cylinder with radius 6 cm and height 10 cm?

8 A cylindrical water tank has a radius of 2 m and a height of 2 m.

a Find its capacity, in m^3, rounded to three decimal places.

b Find its capacity, in L, rounded to the nearest litre.

9 How many litres of gas can a tanker carry if its tank is cylindrical with a 2 m diameter and is 12 m in length? Round to the nearest litre.

$1\,m^3 = 1000$ L

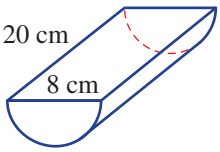
10 Draw a cylinder with its circumference equal to its height. Try to draw it to scale.

ENRICHMENT – 11

Cylinder portions

11 Find the volumes of these cylindrical portions, correct to two decimal places.

a



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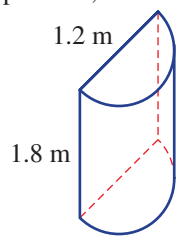
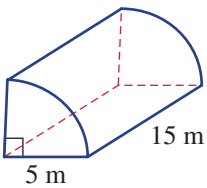


First determine what fraction of a cylinder is involved. In part **f**, subtract the 'missing' volume.

c



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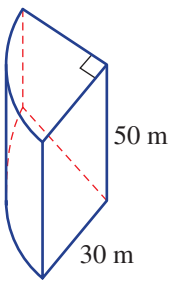

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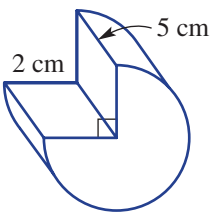
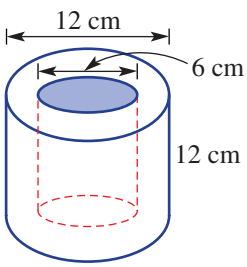


f



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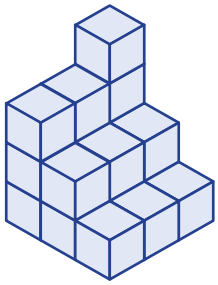
1 These towers are made by stacking cubes. How many cubes are needed in each case?

a



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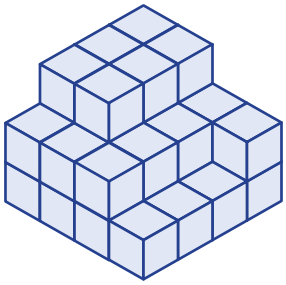
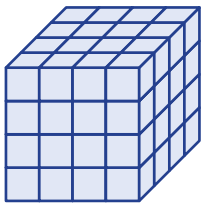
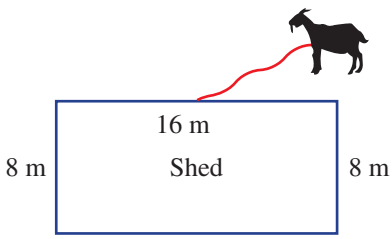


2 A large cube of side length 4 cm is painted and then cut into 64 single 1 cm cubes. How many 1 cm cubes are not painted on any face?

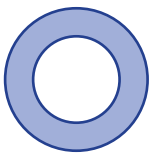


3 To the nearest metre, how far will a wheel of diameter 1 m travel after 100 revolutions?

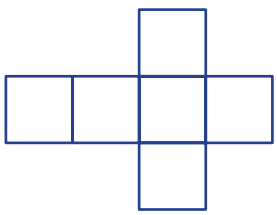
4 A goat is tethered to the centre of one side of a shed with a 10 m length of rope. In what area of grass can the goat graze?



5 A circle of radius 10 cm has a hole cut out of its centre to form a ring. Find the radius of the hole if the remaining area is 50% of the original area. Round to one decimal place.



6 Here is one net for a cube. How many different nets are possible? Do not count nets that can be rotated or reflected to give another net.



7 A factory has a flat roof that has an area of 100 m^2. All the water that collects on the roof is fed into a rainwater tank. If there is 1 mm of rainfall, how many litres of water go into the tank?

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Chapter summary

Length, area, surface area and volume

Length and perimeter

Circle circumference

$C = 2\pi r$ or πd

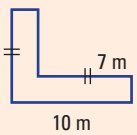
Units

$\times 1000$ $\times 100$ $\times 10$

km m cm mm

$\div 1000$ $\div 100$ $\div 10$

Perimeter



$P = 2 \times 10 + 2 \times 7$
$= 34$ m

Volume

Units

$\times 1000^3$ $\times 100^3$ $\times 10^3$

km^3 m^3 cm^3 mm^3

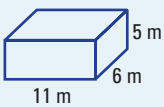
$\div 1000^3$ $\div 100^3$ $\div 10^3$

1 L = 1000 mL = 1000 cm^3

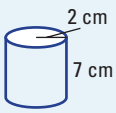
Prisms and cylinders

$V = Ah$
A = area of cross-section
h = height

$V = lbh$
$= 11 \times 6 \times 5$
$= 330$ m^3



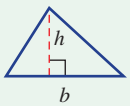
$V = \pi r^2 h$
$= \pi(2)^2 \times 7$
$= 87.96$ cm^3



Area

Triangle

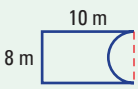
$A = \frac{1}{2}bh$



Quadrilaterals

Square $A = s^2$
Rectangle $A = lb$
Parallelogram $A = bh$
Trapezium $A = \frac{1}{2}h(a + b)$
Rhombus or kite $A = \frac{1}{2}xy$

Composite shapes



$A = lb - \frac{1}{2}\pi r^2$
$= 10 \times 8 - \frac{1}{2}\pi \times 4^2$
$= 54.87$ m^2

$P = 2 \times 10 + 8 + \frac{1}{2}2\pi(4)$
$= 40.57$ m

Units

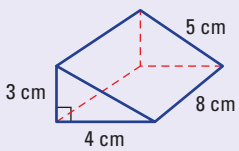
$\times 1000^2$ $\times 100^2$ $\times 10^2$

km^2 m^2 cm^2 mm^2

$\div 1000^2$ $\div 100^2$ $\div 10^2$

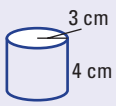
Surface area

Prisms



A = 2 identical triangles + 3 rectangles
$= 2 \times \frac{1}{2} \times 4 \times 3 + 8 \times 5$
$+ 8 \times 4 + 8 \times 3$
$= 108$ cm^2

Cylinder



$A = 2\pi r^2 + 2\pi rh$
$= 2\pi(3)^2 + 2\pi(3)(4)$
$= 131.95$ cm^2

Circle

$A = \pi r^2$
$A = \pi \times 6^2$
$= 113.10$ m^2

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Multiple-choice questions

1 If the area of a square field is 25 km^2, the length of one of its sides is:

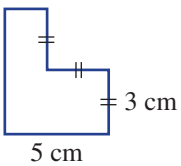
A 10 km **B** 5 km **C** 4 km **D** 50 km **E** 25 km

2 If 1 m^2 = 100^2 cm^2, 2.7 m^2 is the same as:

A 270 cm^2 **B** 0.0027 km^2 **C** 27 000 cm^2 **D** 2700 mm^2 **E** 27 cm^2

3 The perimeter of this shape is:

A 21 cm
B 14 cm
C 24 cm
D 20 cm
E 22 cm

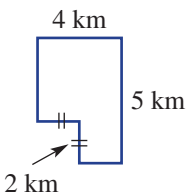


4 A parallelogram has area 10 m^2 and base 5 m. Using $A = bh$, its perpendicular height is:

A 50 m **B** 2 m **C** 50 m^2 **D** 2 m^2 **E** 0.5 m

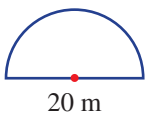
5 This composite shape could be considered as a rectangle with an area in the shape of a square removed. The shape's area is:

A 16 km^2
B 12 km^2
C 20 km^2
D 24 km^2
E 6 km^2



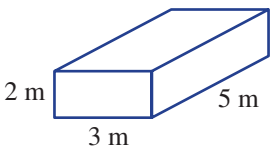
6 A semicircular goal area has diameter 20 m. Its perimeter, correct to the nearest metre, is:

A 41 m **B** 36 m **C** 83 m
D 51 m **E** 52 m



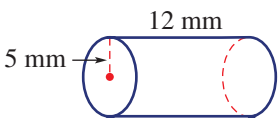
7 The surface area of the rectangular prism shown is:

A 30 m^2
B 62 m^2
C 31 m^2
D 60 m^2
E 100 m^2



8 The area of the *curved surface only* of a cylinder with radius 5 mm and height 12 mm is closest to:

A 534 mm^2 **B** 94 mm^2 **C** 188 mm^2
D 754 mm^2 **E** 377 mm^2



9 A prism's cross-sectional area is 100 m^2. If its volume is 6500 m^3, the prism's total height would be:

A 0.65 m **B** 650 000 m **C** 65 m **D** 6.5 m **E** 650 m

10 The volume of a cylinder with radius 3 cm and height 10 cm is closest to:

A 188 cm^2 **B** 251 cm **C** 141 cm^3 **D** 94 cm^3 **E** 283 cm^3

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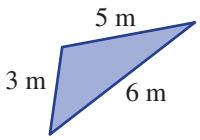
Short-answer questions

1 Convert the following measurements into the units given in brackets.

a 3.8 m (cm) **b** 1.27 km (m) **c** 48 mm (cm) **d** 273 mm^2 (cm^2)
e 5.2 m^2 (cm^2) **f** 0.01 m^3 (cm^3) **g** 53 100 mm^3 (cm^3) **h** 3100 mL (L)
i 0.043 L (mL) **j** 2.83 kL (L) **k** 4000 cm^3 (L) **l** 1 m^3 (L)

2 Find the perimeter of each of the following shapes.

a


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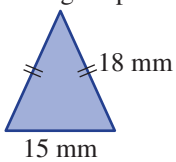
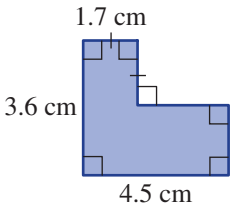
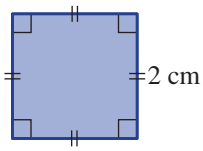


c


3 Find the area of each of the following plane figures.

a


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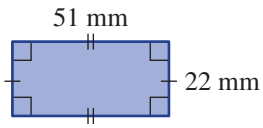

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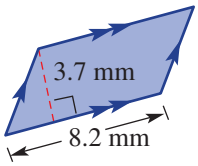

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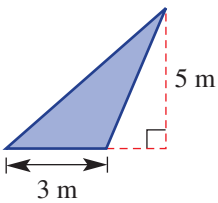

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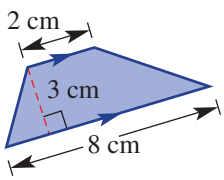

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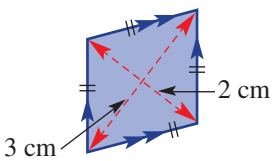
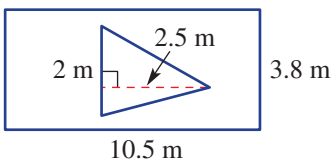
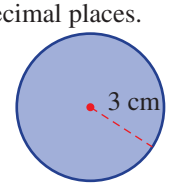


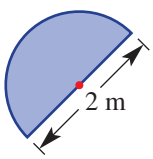
4 Inside a rectangular lawn area of length 10.5 m and breadth 3.8 m, a new garden bed is to be constructed. The garden bed is to be the shape of a triangle with base 2 m and height 2.5 m. Find the area of the:

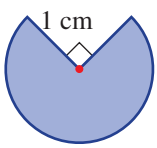
a garden bed
b lawn remaining around the garden bed



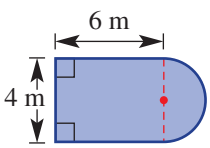
5 Find the area and circumference/perimeter of each of the following shapes, correct to two decimal places.

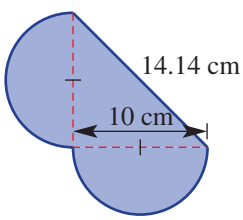
a


b


c


6 Find the perimeter and area of each of the following composite shapes, correct to two decimal places.

a


b


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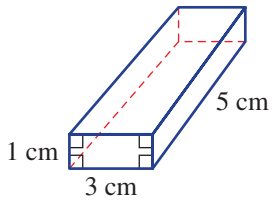
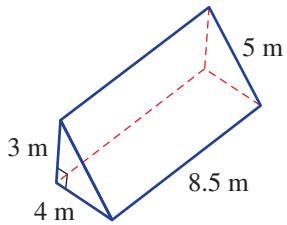
7 Find the surface area of each of the following solid objects.

a

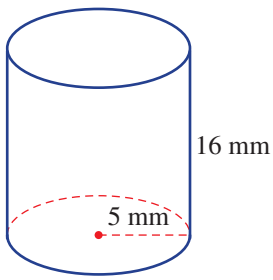


b

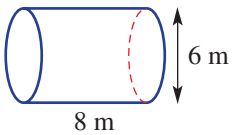


8 Find the surface area of each of these cylinders, correct to two decimal places.

a

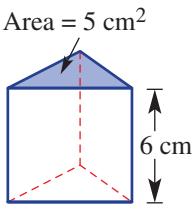


b

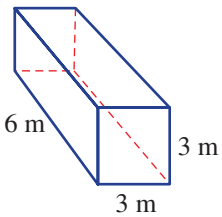


9 Find the volume of each of these solid objects, rounding to two decimal places where necessary.

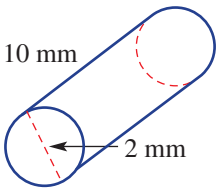
a



b

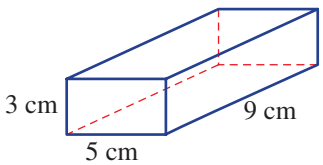


c



Extended-response questions

1 A kindergarten teacher collects some blocks of wood for a painting activity. Each block is a rectangular prism, as shown.



a Find the volume of each block.

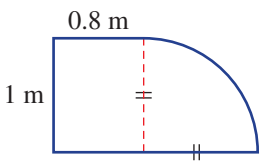
b Find the total outside area to be painted of each block.

c If the paint costs \$2.50 per 100 cm², find the cost of painting 10 blocks.

d Another wood block is a cylinder with radius 3 cm and height 9 cm. Compared to the rectangular block, does it have a greater:

i volume? **ii** surface area?

2 An office receives five new desks with a bench shape made up of a rectangle and quarter-circle as shown.



The edge of the bench is lined with a rubber strip at a cost of \$2.50 per metre.

a Find the length of the rubber edging strip for one desk, correct to two decimal places.

b Find the total cost of the rubber strip for the five desks. Round to the nearest dollar.

The manufacturer claims that the desk top area space is more than 1.5 m².

c Is the manufacturer's claim correct?

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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

7 Indices

What you will learn

7A Index notation
7B Index laws for multiplying and dividing
7C The zero index and power of a power
7D Index laws extended
7E Negative indices
7F Scientific notation
7G Scientific notation using significant figures

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NSW syllabus

STRANDS: NUMBER AND ALGEBRA, MEASUREMENT AND GEOMETRY
SUBSTRAND: INDICES

Outcomes

A student operates with positive-integer and zero indices of numerical bases.
(MA4–9NA)

A student operates with algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases.
(MA5.1–5NA)

A student applies index laws to operate with algebraic expressions involving integer indices.
(MA5.2–7NA)

A student interprets very small and very large units of measurement, uses scientific notation and rounds to significant figures.
(MA5.1–9MG)

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Deadly bacteria

Mycobacterium tuberculosis **is a bacteria that causes tuberculosis, a disease that is one of the most common causes of death in developing countries.**

Bacteria, such as *M. tuberculosis*, reproduce rapidly. Each bacteria cell divides in two within a given period of time. So, after n divisions, the number of cells, N, is given by $N = 2^n$, where n is the index in the rule. Rules involving indices help to model population growth of bacteria and other forms of growth and decay.

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Pre-test

1 Draw a factor tree for 36 and then write 36 as a product of prime factors.

2 **a** List the factors of 24.
b List the factors of 45.
c List the prime factors of 24.
d List the prime factors of 45.

3 Write each of the following in index form (as a power; e.g. 6^4).
a $7 \times 7 \times 7$
b $5 \times 5 \times 2 \times 2 \times 2$
c $(3 \times 3 \times 3 \times 3) \div (3 \times 3)$
d $(4 \times 4 \times 4) \div (4 \times 4)$

4 Write each of the following as 2 raised to a single power.
a $2^2 \times 2$ **b** $2^4 \times 2^2$ **c** $2^3 \times 2^2$

5 Evaluate the following.
a 5^2 **b** 10^2 **c** 1^4 **d** 5^1 **e** 4^0

6 Write each of the following in expanded form and evaluate.
a 3×2^2 **b** 4×3^2 **c** 5×2^3 **d** 3×10^4

7 Simplify the following by removing brackets.
a $(3^2)^2$ **b** $(2^4)^3$ **c** $(-3)^2$

8 Write each of these fractions as decimals.
a $\frac{3}{1000}$ **b** $\frac{4}{100}$ **c** $\frac{2}{1\,000\,000}$ **d** $\frac{8}{10\,000}$

9 Round the following to two decimal places.
a 3.732 **b** 24.6174 **c** 18.3654 **d** 4.3971

10 State the place value of the 4 in each of these numbers.
a 246 **b** 0.0043 **c** 4 320 000 **d** 32.48

11 State the number of significant figures in each of the following.
a 23.102 **b** 30.05 **c** 0.0012 **d** 49 501

12 Complete the following.
a $3.8 \times 10 =$ _____
b $2.31 \times 1000 =$ _____
c $17.2 \div 100 =$ _____
d $0.18 \div 100 =$ _____
e $3827 \div$ ____ $= 3.827$
f $6.49 \times$ ____ $= 64\,900$

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7A Index notation

Interactive
Widgets
HOTsheets
Walkthrough

When a product has the same number multiplied by itself over and over, index notation can be used to write a simpler expression. For example, $5 \times 5 \times 5$ can be written as 5^3 and $x \times x \times x \times x \times x$ can be written as x^5. The expression 5^3 is a power and we can say '5 to the power of 3'. The 5 is called the base and the 3 is the index, or exponent. Numbers written with indices are common in maths and can be applied to many types of problems.

Index notation is a way to carry out calculations such as how much mass is lost over time from ancient stone monuments.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Who has the most?

A person offers you one of two prizes, shown below.

- Which offer would you take?
- Try to calculate the final amount for prize B.
- How might you use index notation to help calculate the value of prize B?
- How can a calculator help to find the amount for prize B using the power button?

Key ideas

- When a number is multiplied by itself many times, that product can be written using **index form**.

  For example,

  Expanded form Index form

  $\overbrace{2 \times 2 \times 2 \times 2 \times 2}^{\text{Expanded form}} = \overbrace{2^5}^{\text{Index form}} = 32$

  base index basic numeral

  $x \times x \times x \times x = x^4$

  base index

Index form A way of writing numbers that are multiplied by themselves

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Key ideas

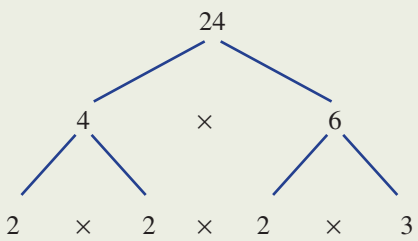
- The **base** is the factor in the product.
- The **index** is the number of times the factor (base number) appears.
  - 2^2 reads '2 to the power of 2' or '2 squared'.
  - 2^3 reads '2 to the power of 3' or '2 cubed'.
  - 2^5 reads '2 to the power of 5'.
- **Prime factorisation** involves writing a number as a product of its **prime factors**.
  - A prime number has only two factors: 1 and itself.

  Note that $a^1 = a$. For example: $5^1 = 5$
  - 3^2 does *not* mean $3 \times 2 = 6$.

Base A number or pronumeral that is being raised to a power

Index The number of times a factor is repeated under multiplication

Prime factor A prime number that divides exactly into another number



Prime factor form

$$24 = \overbrace{2 \times 2 \times 2 \times 3}$$
$$= \underbrace{2^3 \times 3}_{\text{Index form}}$$

Exercise 7A

UNDERSTANDING AND FLUENCY 1–5, 6–8(½), 9–11 | 5, 6–8(½), 9, 10–11(½)

1 **a** The product $3 \times 3 \times 3 \times 3$ is called the _____ or _____ form of 81.
b 3^4 is called the _____ form of 81.
c 3^4 reads '3 to the _____ of 4'.
d In 3^4 the special name for the 3 is the _____ number.
e In 3^4 the special name for the 4 is the _____ number.
f A _____ number has only two factors, itself and 1.
g Prime factorisation involves writing a number as a product of its _____ _____.

2 Use the information outlined in the key ideas to copy and complete the table below.

	Base	Index	Expanded form	Basic numeral
3^4				
5^2				
2^5				
$\left(\frac{2}{3}\right)^3$				
$(-3)^3$				

3 Write the number or pronumeral that is the base in these expressions.

a 3^7 **b** 6^4 **c** $(1.2)^5$ **d** $(-7)^3$
e $\left(\frac{2}{3}\right)^4$ **f** y^{10} **g** w^6 **h** t^2

4 Write the number that is the index in these expressions.

a 4^3 **b** 10^8 **c** $(-3)^7$ **d** $\left(\frac{1}{2}\right)^4$
e x^{11} **f** $(xy)^{13}$ **g** $\left(\frac{x}{2}\right)^9$ **h** $(1.3x)^2$

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5 Write the prime factors of these numbers.

a 6 **b** 15 **c** 30 **d** 77

Example 1 Writing in expanded form

Write the following in expanded form.

a 5^4 **b** a^3 **c** $(xy)^4$ **d** $2a^3b^2$

SOLUTION	EXPLANATION
a $5^4 = 5 \times 5 \times 5 \times 5$	Factor 5 appears four times.
b $a^3 = a \times a \times a$	Factor a appears three times.
c $(xy)^4 = xy \times xy \times xy \times xy$	Factor xy appears four times.
d $2a^3b^2 = 2 \times a \times a \times a \times b \times b$	Factor a appears three times and factor b appears twice. Factor 2 only appears once.

6 Write each of the following in expanded form.

a 4^3 **b** 7^4 **c** 3^5
d 5^3 **e** a^4 **f** b^3
g x^3 **h** $(xp)^6$ **i** $(5a)^4$
j $(3y)^3$ **k** $4x^2y^5$ **l** $(pq)^2$
m $-3s^3t^2$ **n** $6x^3y^5$ **o** $5(yz)^6$
p $4(ab)^3$

factor → a^5 ← number of times a factor appears

$a^5 = a \times a \times a \times a \times a$

Example 2 Expanding and evaluating

Write each of the following in expanded form and then evaluate.

a 5^3 **b** $(-2)^5$ **c** $\left(\frac{2}{5}\right)^3$

SOLUTION	EXPLANATION
a $5^3 = 5 \times 5 \times 5$ $= 125$	Write in expanded form with 5 appearing three times and evaluate.
b $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2)$ $= -32$	Write in expanded form with -2 appearing five times and evaluate.
c $\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$ $= \frac{8}{125}$	Write in expanded form. Evaluate by multiplying numerators and denominators.

7 Write each of the following in expanded form and then evaluate.

a 6^2 **b** 2^4 **c** 3^5 **d** 12^1
e $(-2)^3$ **f** $(-1)^7$ **g** $(-3)^4$ **h** $(-5)^2$
i $\left(\frac{2}{3}\right)^3$ **j** $\left(\frac{3}{4}\right)^2$ **k** $\left(\frac{1}{6}\right)^3$ **l** $\left(\frac{5}{2}\right)^2$
m $\left(\frac{2}{-3}\right)^3$ **n** $\left(\frac{-3}{4}\right)^4$ **o** $\left(\frac{-1}{4}\right)^2$ **p** $\left(\frac{5}{-2}\right)^5$

$\underbrace{-2 \times (-2)} \times (-2)$
$= 4 \times (-2)$
$= -8$

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Example 3 Writing in index form

Write each of the following in index form.

a $5 \times 5 \times 5 \times 5$ **b** $6 \times x \times x \times x \times x$ **c** $4 \times a \times 4 \times a \times 4 \times a$

SOLUTION	EXPLANATION
a $5 \times 5 \times 5 \times 5 = 5^4$	Factor 5 is repeated 4 times.
b $6 \times x \times x \times x \times x = 6x^4$	Factor x is repeated 4 times; 6 appears only once.
c $4 \times a \times 4 \times a \times 4 \times a$	Group the factors of 4 together and the factors of a together.
$= 4 \times 4 \times 4 \times a \times a \times a$	
$= 4^3a^3$	Write in index form.

8 Write each of the following in index form.

a $3 \times 3 \times 3$ **b** $8 \times 8 \times 8 \times 8 \times 8 \times 8$
c $y \times y$ **d** $3 \times x \times x \times x$
e $4 \times c \times c \times c \times c \times c$ **f** $5 \times 5 \times 5 \times d \times d$
g $x \times x \times y \times y \times y$ **h** $7 \times b \times 7 \times b \times 7$

The index or power is the number of appearances of a factor.

Example 4 Writing in index form with fractions

Write each of the following in index form.

a $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ **b** $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

SOLUTION	EXPLANATION
a $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \left(\frac{3}{4}\right)^3$	The fraction $\frac{3}{4}$ appears 3 times.
b $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \left(\frac{3}{7}\right)^2 \times \left(\frac{4}{5}\right)^3$	There are two groups of $\left(\frac{3}{7}\right)$ and three groups of $\left(\frac{4}{5}\right)$.

9 Write each of the following in index form.

a $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ **b** $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$
c $\frac{4}{7} \times \frac{4}{7} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$ **d** $\frac{7x}{9} \times \frac{7x}{9} \times \frac{y}{4} \times \frac{y}{4} \times \frac{y}{4}$

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Example 5 Writing in index form with a combination of pronumerals

Write each of the following in index form.

a $8 \times a \times a \times 8 \times b \times b \times a \times b$ **b** $3a \times 2m \times 3a \times 2m$ **c** $4am(4am)(4am)$

SOLUTION	EXPLANATION
a $8 \times a \times a \times 8 \times b \times b \times a \times b$	Group the numerals and like pronumerals and write in index form.
$= 8 \times 8 \times a \times a \times a \times b \times b \times b$	
$= 8^2a^3b^3$	$64a^3b^3$ and $64(ab)^3$ are alternative answers.
b $3a \times 2m \times 3a \times 2m$	Rearrange so like factors are grouped together and write in index form.
$= 2 \times 2 \times 3 \times 3 \times a \times a \times m \times m$	
$= 2^23^2a^2m^2$	$36a^2m^2$ and $36(am)^2$ are alternative answers.
c $4am(4am)(4am)$	Rearrange and write in index form.
$= 4 \times 4 \times 4 \times a \times a \times a \times m \times m \times m$	
$= 4^3a^3m^3$	$64a^3m^3$ and $64(am)^3$ are alternative answers.

10 Write each of the following in index form.

a $3 \times x \times y \times x \times 3 \times x \times 3 \times y$ **b** $3x \times 2y \times 3x \times 2y$

c $4d \times 2e \times 4d \times 2e$ **d** $6by(6by)(6y)$

e $3pq(3pq)(3pq)(3pq)$ **f** $7mn \times 7mn \times mn \times 7$

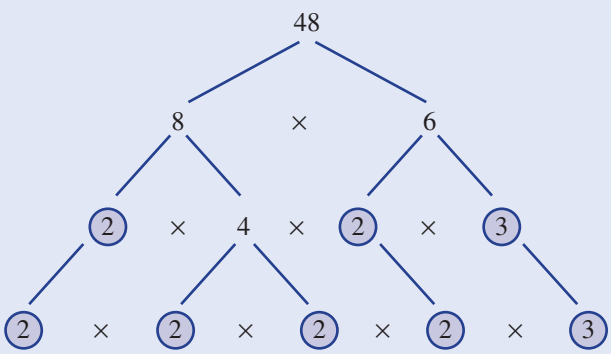
First rearrange the factors with numbers first, then form groups of like bases.

Example 6 Finding the prime factor form

Express 48 as a product of prime factors in index form. Prime numbers are divisible only by 1 and themselves.

SOLUTION

Tree diagram method



$48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

EXPLANATION

Choose a pair of factors of 48; for example, 8 and 6.
Choose a pair of factors of 8; i.e. 2 and 4.
Choose a pair of factors of 6; i.e. 2 and 3.
Continue this process until the factors are all prime numbers.

Write the prime factors of 48.
Express in index notation.

Example is continued on the next page

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Repeated division method

2	48	Start by dividing by a prime number.
2	24	$48 \div 2 = 24$
2	12	$24 \div 2 = 12$
2	6	$12 \div 2 = 6$
3	3	$6 \div 2 = 3$
	1	$3 \div 3 = 1$

$48 = 2 \times 2 \times 2 \times 2 \times 3$ Write prime factors in ascending order.

$= 2^4 \times 3$ Express in index notation.

11 Express each of the following as a product of prime factors in index form.

a 10 **b** 8 **c** 144

d 75 **e** 147 **f** 500

PROBLEM-SOLVING AND REASONING 12, 13 12–14

12 Copy and fill in the missing numbers or symbols.

a $3 \times 3 \times a \times a \times a = 3^{\square}a^{\square}$ **b** $\square \times \square \times k \times k \times k = 5^2k^{\square}$ **c** $\frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} = \left(\frac{2}{7}\right)^{\square}$

d $3p^2q = 3 \times \square \times \square \times q$ **e** $(abc)^1 = a\,\square\,b\,\square\,c$

Example 7 Evaluating expressions in index form by using substitution

If $a = 5$, $b = 2$ and $c = -5$, evaluate these expressions.

a $(ab)^2$ **b** $\left(\frac{a}{c}\right)^3$ **c** $\left(\frac{b}{c}\right)^2$

SOLUTION **EXPLANATION**

a $(ab)^2 = (5 \times 2)^2$ Replace a with 5 and b with 2. Include the $\times$ sign.

$= 10^2$ $5 \times 2 = 10$ is done first.

$= 10 \times 10$ Base of 10 repeated twice.

$= 100$

b $\left(\frac{a}{c}\right)^3 = \left(\frac{5}{-5}\right)^3$ Replace a with 5 and c with -5.

$= (-1)^3$ $5 \div (-5) = -1$

$= -1 \times (-1) \times (-1)$ $-1 \times (-1) = +1, \quad +1 \times (-1) = -1$

$= -1$

c $\left(\frac{b}{c}\right)^2 = \left(\frac{2}{-5}\right)^2$ Replace b with 2 and c with -5.

$= \frac{2}{-5} \times \frac{2}{-5}$ Write in expanded form.

$= \frac{4}{25}$ $2 \times 2 = 4, \quad -5 \times (-5) = 25$

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13 If $a = 3$, $b = 2$ and $c = -3$, evaluate these expressions.

a $(ab)^2$ **b** $(bc)^3$

c $\left(\frac{a}{c}\right)^4$ **d** $\left(\frac{b}{c}\right)^3$

e $(abc)^1$ **f** $c^2 + ab$

g ab^2c **h** c^2ab^3

$abc = a \times b \times c$

14 Find the missing number.

a $3^{\square} = 81$ **b** $2^{\square} = 256$

c $\square^3 = 125$ **d** $\square^5 = 32$

e $\square^3 = -64$ **f** $\square^7 = -128$

g $\square^3 = \frac{1}{8}$ **h** $\left(\frac{2}{3}\right)^{\square} = \frac{16}{81}$

For part **a**, ask 'How many times is 3 repeated so that the product is 81?'

ENRICHMENT – 15

Splitting cells

15 Certain bacteria cells split in 2 every 1 minute. New cells also continue splitting in the same way. So, after each minute, the number of bacteria cells has doubled.

a Copy and complete this table showing the number of bacteria after each minute for 10 minutes.

Time in minutes	Number of bacteria	Number in index form
0	1	2^0
1	$1 \times 2 = 2$	2^1
2	$2 \times 2 = 4$	2^2
3	$2 \times 2 \times 2 = 8$	2^3

b How long will it take for 1 cell to divide into:

i 4 cells? **ii** 16 cells? **iii** 64 cells?

c A single cell is set aside to divide for 24 minutes. Use index form to quickly find how many cells there will be after this time.

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7B Index laws for multiplying and dividing

Interactive
Widgets
HOTsheets
Walkthrough

When multiplying or dividing numbers with the same base, index laws can be used to simplify the expression.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Consider $5^{18} \times 5^{10}$:

Using expanded form: $5^{18} \times 5^{10} = \underbrace{\overbrace{5 \times 5 \times 5 \times \ldots \times 5}^{\text{18 factors of 5}} \times \overbrace{5 \times 5 \times \ldots \times 5}^{\text{10 factors of 5}}}_{\text{18 + 10 factors of 5}}$

$= 5^{18+10}$

$= 5^{28}$

So the total number of factors of 5 is $18 + 10 = 28$.

Also $5^{18} \div 5^{10} = \dfrac{\overbrace{5 \times 5 \times \ldots \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}^{\text{18 factors of 5}}}{\underbrace{\cancel{5} \times \ldots \times \cancel{5} \times \cancel{5}}_{\text{10 factors of 5}}}$

$= 5^{18-10}$

$= 5^{8}$

So the total number of factors of 5 is $18 - 10 = 8$.

Let's start: Discovering the index laws

Consider the two expressions $2^3 \times 2^5$ and $6^8 \div 6^6$.

Complete this working.

$2^3 \times 2^5 = 2 \times \square \times \square \times 2 \times \square \times \square \times \square \times \square$

$= 2^{\square}$

$6^8 \div 6^6 = \dfrac{6 \times \square \times \square \times \square \times \square \times \square \times \square \times \square}{6 \times \square \times \square \times \square \times \square \times \square}$

$= \dfrac{6 \times 6}{1}$

$= 6^{\square}$

- What do you notice about the given expression and the answer in each case? Can you express this as a rule or law in words?
- Repeat the type of working given above and test your laws on these expressions.

a $3^2 \times 3^7$ **b** $4^{11} \div 4^8$

Key ideas

- Index law for multiplication: $a^m \times a^n = a^{m+n}$
  - When multiplying terms with the same base, add the powers.
    For example: $7^3 \times 7^2 = 7^{3+2} = 7^5$
- Index law for division: $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$
  - When dividing terms with the same base, subtract the powers.
    For example: $8^5 \div 8^3 = \dfrac{8^5}{8^3} = 8^{5-3} = 8^2$

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Exercise 7B

UNDERSTANDING AND FLUENCY | 1–4, 5–8(½) | 4, 5–8(½)

1 Write the missing words.

a If you multiply two terms with the same _____, you _____ the powers.

b If you divide two terms with the same _____, you _____ the powers.

2 Copy and complete the following to give answers in index form.

a $3^2 \times 3^4 = 3 \times \square \times 3 \times \square \times \square \times \square$
$= 3^{\square}$

b $6^4 \times 6^3 = 6 \times \square \times \square \times \square \times 6 \times \square \times \square$
$= 6^{\square}$

c $k^3 \times k^2 = k \times \square \times \square \times \square \times \square$
$= k^{\square}$

d $m^4 \times m^3 = m \times \square \times \square \times \square \times \square \times \square \times \square$
$= m^{\square}$

The power shows how many times the factor is repeated.

3 Copy and complete the following to give answers in index form.

a $5^5 \div 5^3 = \dfrac{5 \times \square \times \square \times \square \times \square}{5 \times \square \times \square}$
$= 5^{\square}$

b $9^4 \div 9^2 = \dfrac{9 \times \square \times \square \times \square}{9 \times \square}$
$= 9^{\square}$

c $a^6 \div a^2 = \dfrac{a \times \square \times \square \times \square \times \square \times \square}{\square \times \square}$
$= a^{\square}$

d $\dfrac{n^6}{n^3} = \dfrac{n \times \square \times \square \times \square \times \square \times \square}{\square \times \square \times \square}$
$= n^{\square}$

Show cancelling:

for example: $\dfrac{5 \times 5^{\cancel{1}} \times 5^{\cancel{1}}}{{}^{\cancel{1}}5 \times 5^{\cancel{1}}}$

4 Copy and complete the following.

a $6^5 \times 6^7 = 6^{\square+\square} = 6^{\square}$

b $a^{13} \times a^2 = a^{\square+\square} = a^{\square}$

c $5^{12} \div 5^4 = 5^{\square-\square} = 5^{\square}$

d $\dfrac{m^{16}}{m^2} = m^{\square-\square} = m^{\square}$

When dividing, subtract the powers.

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Example 8 Multiplying and dividing with a common numerical base

Simplify the following, giving your answers in index form.

a $3^6 \times 3^4$ **b** $4^5 \times 4$ **c** $7^9 \div 7^5$ **d** $6^8 \div 6$

SOLUTION	EXPLANATION
a $3^6 \times 3^4 = 3^{6+4}$	$a^m \times a^n = a^{m+n}$
$= 3^{10}$	Add powers: $6 + 4 = 10$. The base 3 is unchanged.
b $4^5 \times 4 = 4^{5+1}$	$4 = 4^1$
$= 4^6$	Add powers: $5 + 1 = 6$. The base 4 is unchanged.
c $7^9 \div 7^5 = 7^{9-5}$	$a^m \div a^n = a^{m-n}$
$= 7^4$	Subtract powers: $9 - 5 = 4$. The base 7 is unchanged.
d $6^8 \div 6 = 6^{8-1}$	$6 = 6^1$
$= 6^7$	Subtract powers: $8 - 1 = 7$. The base 6 is unchanged.

5 Simplify the following, giving your answers in index form.

a $2^4 \times 2^3$ **b** $5^6 \times 5^3$ **c** $7^2 \times 7^4$ **d** $8^9 \times 8$
e $3^4 \times 3^4$ **f** $6^5 \times 6^9$ **g** $3^7 \div 3^4$ **h** $6^8 \div 6^3$
i $5^4 \div 5$ **j** $10^6 \div 10^5$ **k** $9^9 \div 9^6$ **l** $(-2)^5 \div (-2)^3$
m $10^3 \times 10^5$ **n** $(-5)^6 \times (-5)^2$ **o** $6^5 \times 6^{12} \div 6^8$ **p** $5^{12} \div 5^7 \times 5^2$
q $9^{11} \div 9$ **r** $3^5 \times 3^2 \times 3$ **s** $2 \times 2 \times 2^5$ **t** $(-3)^6 \times (-3)^2 \div (-3)$

Remember: $8 = 8^1$.

Example 9 Multiplying with non-numerical bases

Simplify each of the following using the index law for multiplication.

a $x^4 \times x^5 \times x^2$ **b** $x^3y^4 \times x^2y$

SOLUTION	EXPLANATION
a $x^4 \times x^5 \times x^2 = x^{4+5+2}$	Add the indices, since all terms have base x.
$= x^{11}$	
b $x^3y^4 \times x^2y = x^3x^2y^4y$	Regroup so like bases are together. Add the indices corresponding to each different base. Recall that $y = y^1$.
$= x^{3+2}y^{4+1}$	
$= x^5y^5$	

6 Simplify each of the following.

a $x^4 \times x^3$ **b** $a^6 \times a^3$
c $t^5 \times t^3$ **d** $y \times y^4$
e $d^2 \times d$ **f** $y^2 \times y \times y^4$
g $b \times b^5 \times b^2$ **h** $q^6 \times q^3 \times q^2$
i $a^2m^2 \times a^3m^2$ **j** $k^3p^2 \times k^2p$
k $x^2y^3 \times x^4y^5$ **l** $m^5e^3 \times m^2e$
m $a^7 \times a^2 \times a$ **n** $m^{12} \times m \times m^3$
o $pq^3 \times p^6q \times r^3$ **p** $-m^5n^6 \times m^8n^3$

Check that the bases are the same before adding the powers.

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Example 10 Using the index law for division

Simplify $x^{10} \div x^2$ using the index law for division.

SOLUTION

$$x^{10} \div x^2 = x^{10-2}$$
$$= x^8$$

EXPLANATION

Subtract the indices: $10 - 2 = 8$.
The base x is unchanged.

7 Simplify each of the following.

a $a^6 \div a^4$ **b** $x^5 \div x^2$ **c** $\dfrac{q^{12}}{q^2}$ **d** $\dfrac{d^7}{d^6}$

e $\dfrac{b^{10}}{b^5}$ **f** $\dfrac{d^9}{d^4}$ **g** $\dfrac{a^{14}}{a^7}$ **h** $\dfrac{y^{15}}{y^7}$

Recall:
$\dfrac{q^{12}}{q^2} = q^{12} \div q^2$

Example 11 Simplifying using index laws

Simplify each of the following using the index laws for multiplication or division.

a $3m^4 \times 2m^5$ **b** $12y^7 \div (4y^3)$ **c** $\dfrac{8a^6b^3}{12a^2b^2}$

SOLUTION

a $3m^4 \times 2m^5 = 3 \times 2 \times m^4 \times m^5$
$= 6 \times m^{4+5}$
$= 6m^9$

EXPLANATION

Regroup with numbers first, then like bases together. Multiply the numbers, then add the indices of the base m.

b $12y^7 \div 4y^3 = \dfrac{12y^7}{4y^3}$
$= 3y^{7-3}$
$= 3y^4$

$12 \div 4 = 3$
Subtract the indices.

c $\dfrac{8a^6b^3}{12a^2b^2} = \dfrac{8}{12} \times \dfrac{a^6}{a^2} \times \dfrac{b^3}{b^2}$
$= \dfrac{2}{3}a^4b$ or $\dfrac{2a^4b}{3}$

$\dfrac{8}{12} = \dfrac{2}{3}$ in simplest form.
Subtract the indices for each different base.
$6 - 2 = 4$, $3 - 2 = 1$.
Recall: $b = b^1$.

8 Simplify, using the index laws.

a $7x^3y^3 \times x^4y^2$ **b** $3x^7y^3 \times x^2y$ **c** $5x^3y^5 \times xy^4$

d $xy^4z \times 4xy$ **e** $3m^3 \times 5m^2$ **f** $4e^4f^2 \times 2e^2f^2$

g $5c^4d \times 4c^3d$ **h** $9yz^2 \times 2yz^5$ **i** $9m^3 \div (3m^2)$

j $14x^4 \div (2x)$ **k** $5y^4 \div y^2$ **l** $6a^6 \div (2a^5)$

m $\dfrac{36m^7}{12m^2}$ **n** $\dfrac{5w^2}{25w}$ **o** $\dfrac{4a^4}{20a^3}$

p $\dfrac{7x^5}{63x}$ **q** $\dfrac{16x^8y^6}{12x^2y^3}$ **r** $\dfrac{6s^6t^3}{14s^5t}$

s $\dfrac{8m^5n^4}{6m^4n^3}$ **t** $\dfrac{5x^2y}{xy}$

Rearrange first, and group numbers and like bases together.

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PROBLEM-SOLVING AND REASONING	9, 10	9, 11, 12

Example 12 Combining index laws

Simplify each of the following using the index laws.

a $x^2 \times x^3 \div x^4$

b $\dfrac{2a^3b \times 8a^2b^3}{4a^4b^2}$

SOLUTION

a $x^2 \times x^3 \div x^4 = x^5 \div x^4$
$= x$

b $\dfrac{2a^3b \times 8a^2b^3}{4a^4b^2} = \dfrac{2 \times 8 \times a^3a^2 \times bb^3}{4 \times a^4 \times b^2}$
$= \dfrac{16}{4} \times \dfrac{a^5}{a^4} \times \dfrac{b^4}{b^2}$
$= 4 \times a \times b^2$
$= 4ab^2$

EXPLANATION

Add the indices for $x^2 \times x^3$.
Subtract the indices for $x^5 \div x^4$.

Rearrange the question with numbers first, then like bases grouped together.
$2 \times 8 = 16$, $a^3a^2 = a^{3+2} = a^5$, $bb^3 = b^{1+3} = b^4$
$\dfrac{16}{4} = 4$, $\dfrac{a^5}{a^4} = a^{5-4} = a^1 = a$, $\dfrac{b^4}{b^2} = b^{4-2} = b^2$

9 Simplify each of the following.

a $b^5 \times b^2 \div b$
b $y^5 \times y^4 \div y^3$
c $c^4 \div c \times c^4$
d $x^4 \times x^2 \div x^5$
e $\dfrac{t^4 \times t^3}{t^6}$
f $\dfrac{p^2 \times p^7}{p^3}$
g $\dfrac{d^5 \times d^3}{d^2}$
h $\dfrac{x^9 \times x^2}{x}$
i $\dfrac{3x^3y^4 \times 8xy}{6x^2y^2}$
j $\dfrac{9b^4}{2g^3} \times \dfrac{4g^4}{3b^2}$

Write pronumerals in alphabetical order.

10 Simplify each of the following.

a $\dfrac{m^4}{n^2} \times \dfrac{m}{n^3}$
b $\dfrac{x}{y} \times \dfrac{x^3}{y}$
c $\dfrac{a^4}{b^3} \times \dfrac{b^6}{a}$
d $\dfrac{12a}{3c^3} \times \dfrac{6a^4}{4c^4}$
e $\dfrac{3f^2 \times 8f^7}{4f^3}$
f $\dfrac{4x^2b \times 9x^3b^2}{3xb}$

11 Write the missing number.

a $2^7 \times 2^{\square} = 2^{19}$
b $6^{\square} \times 6^3 = 6^{11}$
c $11^6 \div 11^{\square} = 11^3$
d $19^{\square} \div 19^2 = 19$
e $x^6 \times x^{\square} = x^7$
f $a^{\square} \times a^2 = a^{20}$
g $b^{13} \div b^{\square} = b$
h $y^{\square} \div y^9 = y^2$
i $\square \times x^2 \times 3x^4 = 12x^6$
j $15y^4 \div \left(\square\, y^3\right) = y$
k $\square\, a^9 \div (4a) = \dfrac{a^8}{2}$
l $13b^6 \div \left(\square\, b^5\right) = \dfrac{b}{3}$

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12 Evaluate without using a calculator.

a $7^7 \div 7^5$

b $10^6 \div 10^5$

c $13^{11} \div 13^9$

d $2^{20} \div 2^{17}$

e $101^5 \div 101^4$

f $200^{30} \div 200^{28}$

g $7 \times 31^{16} \div 31^{15}$

h $3 \times 50^{200} \div 50^{198}$

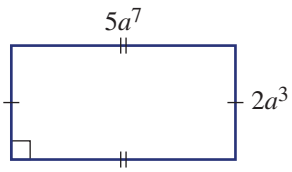
Simplify using index laws first.

ENRICHMENT – 13

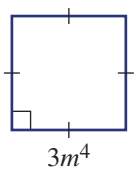
Areas and index notation

13 a Write the area of each of these shapes using index notation.

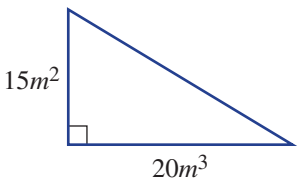
i



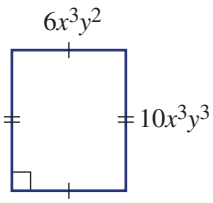
ii



iii

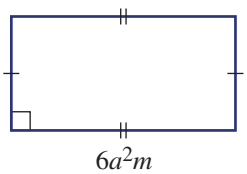


iv

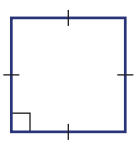


b Find the breadth of each of these shapes using index notation.

i Area $= 24a^3m^2$



ii Area $= 36x^2y^4$



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7C The zero index and power of a power

Interactive
Widgets
HOTsheets
Walkthrough

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Sometimes we find that expressions already written in index form are raised to another power, such as $(2^3)^4$ or $(a^2)^5$.

Consider $(a^2)^5$.

$$\begin{aligned}(a^2)^5 &= a^2 \times a^2 \times a^2 \times a^2 \times a^2 \\ &= \overbrace{a \times a} \times \overbrace{a \times a} \times \overbrace{a \times a} \times \overbrace{a \times a} \times \overbrace{a \times a} \\ &= a^{10}\end{aligned}$$

When we multiply a^2 by itself 5 times, there are a total of $5 \times 2 = 10$ factors of a. So if we multiply a^m by itself n times, there will be a total of $m \times n = mn$ factors of a.

The power of 0 has a special property.

Consider $\dfrac{a^3}{a^3}$.

Simplify using expanded form:

$$\begin{aligned}\frac{a^3}{a^3} &= \frac{\cancel{a^1} \times \cancel{a^1} \times \cancel{a^1}}{\cancel{a^1} \times \cancel{a^1} \times \cancel{a^1}} \\ &= 1\end{aligned}$$

Simplify by subtracting indices:

$$\begin{aligned}\frac{a^3}{a^3} &= a^{3-3} \\ &= a^0\end{aligned}$$

So $a^0 = 1$

Let's start: Power of a power and the zero power

Use the expanded form of 5^3 to simplify $(5^3)^2$ as shown.

$$\begin{aligned}(5^3)^2 &= 5 \times \square \times \square \times 5 \times \square \times \square \\ &= 5^{\square}\end{aligned}$$

- Repeat these steps to also simplify $(3^2)^4$ and $(x^4)^2$.
- What do you notice about the given expression and answer in each case? Can you express this as a law or rule in words?

Now copy and complete this table.

Index form	3^5	3^4	3^3	3^2	3^1	3^0
Basic numeral	243	81				

- What pattern do you notice in the basic numerals?
- What conclusion do you come to regarding 3^0?

Key ideas

- Index law for power of a power: $(a^m)^n = a^{m \times n} = a^{mn}$
  - When raising a term in index form to another power, retain the base and multiply the indices. For example: $(x^2)^3 = x^{2 \times 3} = x^6$.
  - A power outside brackets only applies to the expression inside those brackets. For example: $5(a^3)^2 = 5a^{3 \times 2} = 5a^6$.
- The zero power: $a^0 = 1$, where $a \neq 0$
  - Any term except 0 raised to the power of zero is 1. For example: $5^0 = 1$, $m^0 = 1$ and $(2a)^0 = 1$.

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Exercise 7C

UNDERSTANDING AND FLUENCY	1–4, 5–8(½)	4, 5–8(½)

1 Write the missing words or numbers in these sentences.

a When raising a term or numbers in index form to another power, _____ the indices.

b Any number (except 0) raised to the power 0 is equal to ___.

Indices means powers.

2 Copy and complete these tables.

a

Index form	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Basic numeral	64	32					

b

Index form	4^5	4^4	4^3	4^2	4^1	4^0
Basic numeral	1024	256				

3 Copy and complete this working.

a $(4^2)^3 = 4^2 \times 4^2 \times 4^2$

$= (4 \times \square) \times (4 \times \square) \times (4 \times \square)$

$= 4^{\square}$

b $(12^3)^2 = 12^{\square} \times 12^{\square}$

$= (12 \times \square \times \square) \times (12 \times \square \times \square)$

$= 12^{\square}$

c $(x^4)^2 = x^{\square} \times x^{\square}$

$= (x \times \square \times \square \times \square) \times (x \times \square \times \square \times \square)$

$= x^{\square}$

d $(a^2)^5 = a^{\square} \times a^{\square} \times a^{\square} \times a^{\square} \times a^{\square}$

$= (a \times \square) \times (a \times \square) \times (a \times \square) \times (a \times \square) \times (a \times \square)$

$= a^{\square}$

4 Find the value of each of the following.

a 6^0 **b** 21^0 **c** 2^0 **d** 1^0

e $(3.7)^0$ **f** 582^0 **g** $\left(\frac{3}{4}\right)^0$ **h** 2760^0

$a^0 = 1$

Example 13 Simplifying a power of a power

Simplify each of the following.

a $(x^5)^4$ **b** $3(y^5)^2$

SOLUTION	EXPLANATION
a $(x^5)^4 = x^{5\times4}$ $= x^{20}$	Keep x as the base and multiply the powers.
b $3(y^5)^2 = 3y^{5\times2}$ $= 3y^{10}$	Keep y and multiply the powers. The power of 2 is only applied inside the brackets. The 3 is unchanged.

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5 Simplify each of the following. Leave your answers in index form.

a $(y^6)^2$ **b** $(m^3)^6$ **c** $(x^2)^5$
d $(b^3)^4$ **e** $(3^2)^3$ **f** $(4^3)^5$
g $(3^5)^6$ **h** $(7^5)^2$ **i** $5(m^8)^2$
j $4(q^7)^4$ **k** $-3(c^2)^5$ **l** $2(j^4)^6$

Keep the base. Multiply the powers.

Example 14 Using the zero power

Apply the zero power rule to evaluate each of the following.

a $(-3)^0$ **b** $3(5x)^0$ **c** $2y^0 - (3y)^0$

SOLUTION	EXPLANATION
a $(-3)^0 = 1$	Any number raised to the power of 0 is 1.
b $3(5x)^0 = 3 \times 1$ $= 3$	Everything in the brackets is to the power of 0 so $(5x)^0$ is 1. The 3 is not to the power of 0.
c $2y^0 - (3y)^0 = 2 \times 1 - 1$ $= 2 - 1$ $= 1$	$2y^0$ has no brackets so the power applies to the y only, so $2y^0 = 2 \times y^0 = 2 \times 1$ while $(3y)^0 = 1$.

6 Evaluate each of the following.

a 5^0 **b** 9^0 **c** $(-6)^0$
d $(-3)^0$ **e** $(4)^0$ **f** $\left(\frac{3}{4}\right)^0$
g $\left(-\frac{1}{7}\right)^0$ **h** $(4y)^0$ **i** $5m^0$
j $-3p^0$ **k** $6x^0 - 2x^0$ **l** $-5n^0 - (8n)^0$
m $(3x^4)^0$ **n** $1^0 + 2^0 + 3^0$ **o** $3a^0 + (3a)^0$ **p** $100^0 - a^0$

Any number (except zero) raised to the power 0 equals 1.

Example 15 Combining index laws

Simplify $(x^2)^3 \times (x^3)^5$ by applying the various index laws.

SOLUTION	EXPLANATION
$(x^2)^3 \times (x^3)^5 = x^{2\times3} \times x^{3\times5}$ $= x^6 \times x^{15}$ $= x^{21}$	Use power of a power to remove brackets first by multiplying indices. Then use index law for multiplication to add indices.

7 Simplify each of the following by combining various index laws.

a $4 \times (4^3)^2$ **b** $(3^4)^2 \times 3$
c $x \times (x^0)^5$ **d** $y^5 \times (y^2)^4$
e $b^5 \times (b^3)^3$ **f** $(a^2)^3 \times a^4$
g $(d^3)^4 \times (d^2)^6$ **h** $(y^2)^6 \times y^4$
i $z^4 \times (z^3)^2 \times (z^5)^3$

First remove brackets by multiplying powers. Remember that $4 = 4^1$.

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Example 16 Combining index laws

Simplify $\frac{(m^3)^4}{m^7}$ by applying index laws.

SOLUTION

$$\frac{(m^3)^4}{m^7} = \frac{m^{3\times4}}{m^7}$$
$$= \frac{m^{12}}{m^7}$$
$$= m^5$$

EXPLANATION

Remove brackets by multiplying indices then simplify using index law for division.

$12 - 7 = 5$

8 Simplify each of the following.

a $\frac{(b^2)^5}{b^4}$ **b** $\frac{(x^4)^3}{x^7}$ **c** $\frac{(y^3)^3}{y^3}$

d $7^8 \div (7^3)^2$ **e** $(4^2)^3 \div 4^5$ **f** $(3^6)^3 \div (3^5)^2$

g $(m^3)^6 \div (m^2)^9$ **h** $(y^5)^3 \div (y^6)^2$ **i** $(h^{11})^2 \div (h^5)^4$

First remove brackets by multiplying the powers.

PROBLEM-SOLVING AND REASONING	9, 10	10, 11

Example 17 Combining index laws with the zero power

Simplify $\frac{4x^2 \times 3x^3}{6x^5}$ by applying index laws.

SOLUTION

$$\frac{4x^2 \times 3x^3}{6x^5} = \frac{4 \times 3 \times x^2 \times x^3}{6x^5}$$
$$= \frac{12x^5}{6x^5}$$
$$= 2x^0$$
$$= 2 \times 1$$
$$= 2$$

EXPLANATION

Regroup, then simplify the numerator first by multiplying numbers and adding indices of base x.
$12 \div 6 = 2$, and subtract indices: $5 - 5 = 0$
The zero power says $x^0 = 1$.

9 Simplify each of the following using various index laws.

a $\frac{3x^4 \times 6x^8}{9x^{12}}$ **b** $\frac{5x^5 \times 4x^6}{2x^{10}}$

c $\frac{24(x^4)^4}{8(x^4)^2}$ **d** $\frac{4(d^4)^3 \times (e^4)^2}{8(d^2)^5 \times e^7}$

e $\frac{6(m^3)^2(n^5)^3}{15(m^5)^0(n^2)^7}$ **f** $\frac{2(a^3)^4(b^2)^6}{16(a)^0(b^6)^2}$

Remove brackets, then rearrange with numbers first then like bases grouped together.

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10 If m and n are positive integers, in how many ways can $(a^m)^n = a^{16}$? Show each possibility.

11 Explain the error made in the following problems, then give the correct answer.

a $(a^4)^5 = a^9$

b $3(x^3)^2 = 9x^6$

c $(2x)^0 = 2$

ENRICHMENT – 12

Rabbits!

12 There are 100 rabbits on Mt Burrow at the start of the year 2000. The rule for the number of rabbits, N, after t years (from the start of the year 2000) is $N = 100 \times 2^t$.

a Find the number of rabbits at:

i $t = 2$

ii $t = 6$

iii $t = 0$

b Find the number of rabbits at the beginning of:

i 2003

ii 2007

iii 2010

c How many years will it take for the population to first rise to more than 500 000? Give a whole number of years.

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7D Index laws extended

Interactive

Widgets

HOTsheets

Walkthrough

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

It is common to find expressions such as $(2x)^3$ and $\left(\frac{x}{3}\right)^4$ in mathematical problems. These are different from most of the expressions in previous sections. They contain more than one single number or variable, connected by multiplication or division, raised to a power. These expressions can also be simplified using two index laws that remove the brackets.

Consider $(a \times b)^6$:

Using expanded form: $(a \times b)^6 = \overbrace{ab \times ab \times ab \times ab \times ab \times ab}^{\text{6 factors of } ab}$

$$= \overbrace{a \times a \times a \times a \times a \times a}^{\text{6 factors of } a} \times \overbrace{b \times b \times b \times b \times b \times b}^{\text{6 factors of } b}$$

$$= a^6 \times b^6$$

So this becomes a product of 6 factors of a and 6 factors of b.

Also, $\left(\frac{a}{b}\right)^6 = \overbrace{\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}}^{\text{6 factors of } \frac{a}{b}}$

$$= \frac{\overbrace{a \times a \times a \times a \times a \times a}^{\text{6 factors of } a}}{\underbrace{b \times b \times b \times b \times b \times b}_{\text{6 factors of } b}}$$

$$= \frac{a^6}{b^6}$$

A research scientist in a microbiology laboratory would use indices to express the number of microbes being studied.

So to remove the brackets, we can raise each of a and b to the power 6.

Let's start: Can you see a short-cut?

Use the expanded form of $(2x)^3$ and $\left(\frac{x}{3}\right)^4$ to help simplify the expressions.

$$(2x)^3 = 2x \times \square \times \square$$
$$= 2 \times 2 \times 2 \times \square \times \square \times \square$$
$$= 2^{\square} \times \square^{\square}$$

$$\left(\frac{x}{3}\right)^4 = \frac{x}{3} \times \square \times \square \times \square$$
$$= \frac{x \times \square \times \square \times \square}{3 \times \square \times \square \times \square}$$
$$= \frac{\square^{\square}}{\square^{\square}}$$

- Repeat these steps to also simplify the expressions $(3y)^4$ and $\left(\frac{x}{2}\right)^5$.
- What do you notice about the given expressions and the answer in each case? Can you express this as a rule or law in words?

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Key ideas

- Power of a product: $(a \times b)^m = (ab)^m = a^m b^m$
  - When multiplying two or more numbers or pronumerals raised to the power of m, raise each number or pronumeral in the brackets to the power of m.
    For example: $(2x)^2 = 2^2x^2 = 4x^2$
- Power of a fraction: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ and $b \neq 0$
  - When dividing two numbers or pronumerals raised to the power of m, raise each number or pronumeral in the brackets to the power of m.
    For example: $\left(\frac{y}{3}\right)^3 = \frac{y^3}{3^3} = \frac{y^3}{27}$

Exercise 7D

UNDERSTANDING AND FLUENCY	1–6, 7(½), 8	3–5, 6–9(½)

1 Copy and complete the following.

a $(a \times b)^m = a^m \times \square$

b $\left(\frac{a}{b}\right)^m = \frac{a^m}{\square}$

2 Copy and complete this working.

a $(5a)^3 = 5a \times \square \times \square$
$= 5 \times 5 \times 5 \times a \times \square \times \square$
$= 5^3 \times \square$

b $(ab)^4 = ab \times \square \times \square \times \square$
$= a \times \square \times \square \times \square \times b \times \square \times \square \times \square$
$= a^4 \times \square$

c $\left(\frac{x}{6}\right)^3 = \frac{x}{6} \times \square \times \square$
$= \frac{x \times \square \times \square}{6 \times \square \times \square}$
$= \frac{x^3}{\square}$

d $\left(\frac{a}{b}\right)^5 = \frac{a}{b} \times \square \times \square \times \square \times \square$
$= \frac{a \times \square \times \square \times \square \times \square}{b \times \square \times \square \times \square \times \square}$
$= \frac{a^5}{\square}$

3 Copy and complete these bracket expansions.

a $(xy)^5 = x^{\square}y^5$

b $(2m)^3 = 2^{\square}m^{\square}$

c $(abc)^4 = a^4b^{\square}c^{\square}$

d $\left(\frac{5}{k}\right)^3 = \frac{5^3}{k^{\square}}$

e $\left(\frac{a}{m}\right)^5 = \frac{a^{\square}}{m^{\square}}$

f $\left(\frac{2}{2}\right)^4 = \frac{2^{\square}}{3^{\square}} = \frac{16}{\square}$

Raise each value in the brackets to the power.

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4 Copy and complete these bracket expansions.

a $6(ab)^2 = 6a^{\square}b^2$

b $4(xy)^3 = \square x^{\square}y^{\square}$

c $7(2m)^2 = 7 \times 2^{\square}m^{\square}$
$= 7 \times \square \times m^2$
$= \square\, m^2$

d $4(3ab)^2 = \square \times 3^{\square}a^{\square}b^{\square}$
$= 4 \times \square\, a^{\square}b^{\square}$
$= \square\, a^{\square}b^{\square}$

The power outside the brackets only applies to the values inside the brackets.

5 Copy and complete these bracket expansions. Write each answer in index notation.

a $(2 \times 3^2)^4 = 2^4 \times (3^2)^{\square} = 2^4 \times 3^{2\times\square} = 2^4 \times 3^{\square}$

b $(5^2 \times 7^3)^4 = (5^2)^{\square} \times (7^3)^{\square} = 5^{2\times 4} \times 7^{3\times\square} = 5^{\square} \times 7^{\square}$

c $\left(\dfrac{11^2}{5}\right)^3 = \dfrac{(11^2)^{\square}}{5^3} = \dfrac{11^{\square}}{5^3} = \dfrac{11^{\square}}{5^3}$

d $\left(\dfrac{8^2}{3^5}\right)^4 = \dfrac{(8^2)^{\square}}{(3^5)^{\square}} = \dfrac{8^{2\times\square}}{3^{5\times\square}} = \dfrac{8^{\square}}{3^{\square}}$

Recall: $(3^2)^4 = 3^{2\times 4}$
$= 3^8$

Example 18 Applying a power to a product

Expand each of the following.

a $(3 \times 2^2)^3$ **b** $(9a^3)^2$

SOLUTION

a $(3 \times 2^2)^3 = 3^3 \times (2^2)^3$
$= 3^3 \times 2^6$

b $(9a^3)^2 = 9^2(a^3)^2$
$= 81a^{3\times 2}$
$= 81a^6$

EXPLANATION

Apply the power of 3 to each term in the brackets.
Raise (2^2) to the power of 3 by multiplying indices.
Raise 9 and (a^3) to the power of 2.
$9^2 = 9 \times 9 = 81$, $(a^3)^2 = a^{3\times 2} = a^6$

6 Expand each of the following. Write answers with index notation.

a $(5 \times 7^2)^3$ **b** $(3 \times 4^5)^2$
c $(2 \times 5^3)^4$ **d** $(2^2 \times 3^4)^2$
e $(5^3 \times 3^2)^4$ **f** $(8^2 \times 11^5)^5$

Follow the same steps as in **5a** and **b**.

7 Expand each of the following. Evaluate numbers raised to a power.

a $(2x)^3$ **b** $(5y)^2$ **c** $(4a)^3$
d $(3r)^2$ **e** $(3b)^4$ **f** $(7r)^3$
g $(2h^2)^4$ **h** $(5c^2)^4$ **i** $(3a^2)^3$
j $(7p^4)^2$ **k** $(5m^3)^2$ **l** $(3y^{10})^2$

The power outside the brackets must be applied to each value inside the brackets.

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Example 19 Applying a power to a fraction

Expand each of the following and write with index notation.

a $\left(\frac{2^2}{5}\right)^2$ **b** $\left(\frac{m^3}{5}\right)^2$

SOLUTION

a $\left(\frac{2^2}{5}\right)^2 = \frac{(2^2)^2}{5^2}$

$= \frac{2^4}{5^2}$

EXPLANATION

Raise each value in the brackets to the power of 2. Raise (2^2) to the power of 2. $(2^2)^2 = 2^{2\times2} = 2^4$.

b $\left(\frac{m^3}{5}\right)^2 = \frac{(m^3)^2}{5^2}$

$= \frac{m^{3\times2}}{5^2}$

$= \frac{m^6}{5^2}$

Raise (m^3) and 5 to the power of 2. $(m^3)^2 = m^{3\times2} = m^6$.

$\frac{m^6}{5^2}$ also equals $\frac{m^6}{25}$.

8 Expand each of the following. Write answers with index notation.

a $\left(\frac{3^2}{5}\right)^2$ **b** $\left(\frac{2^4}{7}\right)^2$

c $\left(\frac{9}{8^3}\right)^4$ **d** $\left(\frac{5^2}{12^3}\right)^2$

e $\left(\frac{3^4}{2^5}\right)^2$ **f** $\left(\frac{7^3}{4^2}\right)^5$

Follow the same steps as in **5c** and **d**.

9 Expand the following. Evaluate numbers raised to a power.

a $\left(\frac{p}{q}\right)^3$ **b** $\left(\frac{x}{y}\right)^4$

c $\left(\frac{4}{y}\right)^3$ **d** $\left(\frac{5}{p^2}\right)^4$

e $\left(\frac{2}{r^3}\right)^2$ **f** $\left(\frac{s^3}{7}\right)^2$

g $\left(\frac{7}{x^3}\right)^2$ **h** $\left(\frac{w^4}{10}\right)^2$

i $\left(\frac{a^2}{5}\right)^3$ **j** $\left(\frac{m^2}{2}\right)^4$

k $\left(\frac{2m}{n}\right)^5$ **l** $\left(\frac{2a^2}{3}\right)^3$

Recall that $4^3 = 4 \times 4 \times 4 = 64$.

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PROBLEM-SOLVING AND REASONING | 10, 11 | 10–12

Example 20 Applying the power of a product

Expand each of the following using the power of a product.

a $(-2x^3y)^4$ **b** $4(c^2d^3)^5$

SOLUTION

a $(-2x^3y)^4 = (-2)^4(x^3)^4y^4$
$= 16x^{12}y^4$

b $4(c^2d^3)^5 = 4(c^2)^5(d^3)^5$
$= 4c^{10}d^{15}$

EXPLANATION

Raise each value in the brackets to the power of 4. Evaluate $(-2)^4 = -2 \times (-2) \times (-2) \times (-2)$ and simplify $(x^3)^4$ using the power of a power rule.

Raise each value in the brackets to the power of 5. Note that the coefficient (4) is not raised to the power of 5 since it is not in the brackets. Simplify using index laws.

10 Expand each of the following using the power of a product.

a $(2x^3y^2)^5$ **b** $9(p^2q^4)^3$ **c** $2(x^3y)^2$
d $(8t^2u^9v^4)^0$ **e** $(-3w^3y)^3$ **f** $-4(p^4qr)^2$
g $(-5s^7t)^2$ **h** $-(-2x^4yz^3)^3$ **i** $(5a^4b^3)^4$
j $(-3m^7n)^2$ **k** $(7a^5b^4c)^2$ **l** $(-9m^5n)^2$

Powers only apply to values inside the brackets.

Example 21 Applying the power to a fraction

Expand $\left(\dfrac{-2a^2}{3bc^3}\right)^4$.

SOLUTION

$$\left(\frac{-2a^2}{3bc^3}\right)^4 = \frac{(-2)^4a^8}{3^4b^4c^{12}}$$
$$= \frac{16a^8}{81b^4c^{12}}$$

EXPLANATION

Raise each value in the brackets to the power of 4. $(a^2)^4 = a^8$ and $(c^3)^4 = c^{12}$.

Evaluate $(-2)^4$ and 3^4.

11 Expand the following. Evaluate numbers that are raised to a power.

a $\left(\dfrac{3n^3}{2m^4}\right)^3$ **b** $\left(\dfrac{-2r}{n}\right)^4$ **c** $\left(\dfrac{-3f}{2^3g^5}\right)^2$

d $\left(\dfrac{5w^4y}{2x^3}\right)^2$ **e** $\left(\dfrac{-3x}{2y^3g^5}\right)^2$ **f** $\left(\dfrac{3km^3}{4n^7}\right)^3$

g $-\left(\dfrac{-5w^4y}{2zx^3}\right)^2$ **h** $\left(-\dfrac{3x^2y^3}{2a^5b^3}\right)^2$

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Example 22 Using the various index laws

Simplify each of the following by applying various index laws.

a $a(-2a^2b)^3$

b $\left(\frac{x^2y^3}{c}\right)^3 \times \left(\frac{xc}{y}\right)^4$

SOLUTION

a $a(-2a^2b)^3 = a(-2)^3(a^2)^3b^3$
$= a \times -8a^6b^3$
$= -8a^7b^3$

EXPLANATION

Apply the power of 3 to the values inside the brackets.
$(-2)^3 = -2 \times (-2) \times (-2) = -8$
$(a^2)^3 = a^{2\times3} = a^6$
Combine powers with a common base:
$a \times a^6 = a^{1+6} = a^7$

b $\left(\frac{x^2y^3}{c}\right)^3 \times \left(\frac{xc}{y}\right)^4 = \frac{x^6y^9}{c^3} \times \frac{x^4c^4}{y^4}$
$= \frac{x^{10}y^9c^4}{c^3y^4}$
$= cx^{10}y^5$

Raise each value in the brackets to the power. Multiply the numerators, then divide.

Write the answer in alphabetical order.

12 Simplify each of the following by applying the various index laws.

a $a(3b)^2$ **b** $a(3b^2)^3$ **c** $-3(2a^3b^4)^2a^2$

d $2(3x^2y^3)^3$ **e** $(-4b^2c^5d)^3$ **f** $a(2a)^3$

g $a(3a^2)^2$ **h** $5a^3(-2a^4b)^3$ **i** $-5(-2m^3pt^2)^5$

j $\left(\frac{-3x^2y^0}{5a^5b^3}\right)^3$ **k** $\left(\frac{a^3b}{c}\right)^3 \times \left(\frac{ac^4}{b}\right)^2$ **l** $\left(\frac{x^2z}{y}\right)^4 \times \left(\frac{xy^2}{z}\right)^3$

Remove brackets. Combine numerators and divide for parts **j** to **l**.

ENRICHMENT – 13

Germinating seeds

13 The rule for the number of seeds germinating in a glasshouse over a two-week period is given by $N = \left(\frac{t}{2}\right)^3$, where N is the number of germinating seeds and t is the number of days.

a Find the number of germinating seeds after:
- **i** 4 days
- **ii** 10 days

b Expand to rewrite the rule without brackets.

c Use your rule in part **b** to find the number of seeds germinating after:
- **i** 6 days
- **ii** 4 days

d Find the number of days required to germinate:
- **i** 64 seeds
- **ii** 1 seed

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7E Negative indices

Interactive
Widgets
HOTsheets
Walkthrough

We know that $2^3 = 8$ and $2^0 = 1$. What about 2^{-1} or 2^{-6}? Numbers written in index form using negative indices also have meaning in mathematics.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

For example, simplify $a^2 \div a^5$.

Method 1: By subtracting the indices

$$\frac{a^2}{a^5} = a^{2-5}$$
$$= a^{-3}$$

Method 2: By cancelling

$$\frac{a^2}{a^5} = \frac{\cancel{a^1} \times \cancel{a^1}}{a \times a \times a \times \cancel{a_1} \times \cancel{a_1}}$$
$$= \frac{1}{a^3}$$

So we can see that $a^{-3} = \dfrac{1}{a^3}$

Let's start: Continuing the pattern

Explore the use of negative indices by copying and completing this table.

Index form	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Whole number or fraction	16	8					$\frac{1}{4} = \frac{1}{2^2}$	

÷2 ÷2 ÷2 ÷2 ÷2 ÷2 ÷2

- What do you notice about the numbers with negative indices in the top row and the fractions in the second row?
- Can you describe this connection in words?
- What might be another way of writing 2^{-7} or 5^{-4}?

Key ideas

- To express a negative index as a positive index, use the following rules:

  $a^{-m} = \dfrac{1}{a^m}$ and $a^m = \dfrac{1}{a^{-m}}$

  a raised to the power $-m$ is equal to the reciprocal of a raised to the power m. ($a \neq 0$)

  For example: $5^{-2} = \dfrac{1}{5^2}$, $\dfrac{1}{3^{-4}} = 3^4$.

  - Recall: the reciprocal of $\dfrac{a}{b} = \dfrac{b}{a}$.
- All the index laws also apply to negative indices.

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Exercise 7E

UNDERSTANDING AND FLUENCY	1–7	4, 5–8(½)

1 Write the following using positive indices. For example: $\frac{1}{8} = \frac{1}{2^3}$.

a $\frac{1}{4}$ **b** $\frac{1}{9}$

c $\frac{1}{125}$ **d** $\frac{1}{27}$

Write the denominator as a power of a prime number.

2 Copy and complete these tables.

a

Index form	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	81	27					$\frac{1}{9} = \frac{1}{3^2}$	

÷3 ÷3 ÷3 ÷3 ÷3 ÷3 ÷3

b

Index form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	10 000							$\frac{1}{1000} = \frac{1}{10^3}$

÷10 ÷10 ÷10 ÷10 ÷10 ÷10 ÷10

3 Copy and complete each of the following.

a $10^{-4} = \frac{1}{10^{\square}}$ **b** $3^{-2} = \frac{1}{3^{\square}}$ **c** $7^{-3} = \frac{1}{7^{\square}}$

d $8^{-6} = \frac{1}{8^{\square}}$ **e** $9^{-4} = \frac{1}{9^{\square}}$ **f** $5^{-4} = \frac{1}{5^{\square}}$

$a^{-m} = \frac{1}{a^m}$

So $10^{-4} = \frac{1}{10^4}$

4 Copy and complete each of the following.

a $\frac{1}{3^{-4}} = 3^{\square}$ **b** $\frac{1}{7^{-6}} = 7^{\square}$ **c** $\frac{1}{8^{-3}} = 8^{\square}$

d $\frac{1}{4^{-6}} = 4^{\square}$ **e** $\frac{1}{10^{-3}} = 10^{\square}$ **f** $\frac{1}{5^{-4}} = 5^{\square}$

$\frac{1}{a^{-m}} = a^m$

So $\frac{1}{3^{-4}} = 3^4$

Example 23 Using $a^{-m} = \frac{1}{a^m}$

Rewrite the following as fractions containing positive indices.

a 3^{-2} **b** 5×4^{-3}

SOLUTION	EXPLANATION
a $3^{-2} = \frac{1}{3^2}$	Use $a^{-m} = \frac{1}{a^m}$.
b $5 \times 4^{-3} = \frac{5}{1} \times \frac{1}{4^3}$	The 5 does not have a negative power so it remains unchanged. $4^{-3} = \frac{1}{4^3}$.
$= \frac{5}{4^3}$	Multiply numerators and denominators. $5 \times 1 = 5$, $1 \times 4^3 = 4^3$

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5 Write each of the following with positive powers.

a 5^{-2} **b** 7^{-4} **c** 8^{-3} **d** 3^{-5}
e 9^{-2} **f** 10^{-3} **g** 4^{-5} **h** 2^{-3}

Remember: $a^{-m} = \dfrac{1}{a^m}$

So $5^{-2} = \dfrac{1}{5^2}$

6 Rewrite the following with positive indices only.

a 3×2^{-4} **b** 5×4^{-3}
c 7×5^{-6} **d** 2×3^{-4}
e 4×3^{-5} **f** 9×5^{-2}
g 8×7^{-3} **h** 6×5^{-6}

$5 \times 2^{-3} = \dfrac{5}{2^3}$

Don't change

Example 24 Using $\dfrac{1}{a^{-m}} = a^m$

Rewrite the following with positive indices.

a $\dfrac{1}{2^{-3}}$ **b** $\dfrac{7}{3^{-2}}$

SOLUTION	EXPLANATION
a $\dfrac{1}{2^{-3}} = 2^3$	Use $\dfrac{1}{a^{-m}} = a^m$.
b $\dfrac{7}{3^{-2}} = 7 \times \dfrac{1}{3^{-2}}$	The 7 remains unchanged
$= 7 \times 3^2$	$\dfrac{1}{3^{-2}} = 3^2$

7 Write each of the following with positive powers.

a $\dfrac{1}{2^{-4}}$ **b** $\dfrac{1}{3^{-2}}$ **c** $\dfrac{1}{4^{-3}}$
d $\dfrac{1}{6^{-5}}$ **e** $\dfrac{1}{5^{-3}}$ **f** $\dfrac{1}{8^{-5}}$
g $\dfrac{1}{7^{-3}}$ **h** $\dfrac{1}{9^{-4}}$

Remember: $\dfrac{1}{a^{-m}} = a^m$

So $\dfrac{1}{2^{-4}} = 2^4$

8 Rewrite the following with positive indices only.

a $\dfrac{6}{4^{-3}}$ **b** $\dfrac{5}{8^{-2}}$
c $\dfrac{4}{7^{-5}}$ **d** $\dfrac{3}{2^{-5}}$
e $\dfrac{12}{5^{-4}}$ **f** $\dfrac{6}{10^{-3}}$
g $\dfrac{7}{9^{-4}}$ **h** $\dfrac{7}{10^{-6}}$

Don't change

$\dfrac{7}{3^{-4}} = 7 \times 3^4$

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PROBLEM-SOLVING AND REASONING	9, 10	9–12

Example 25 Changing to fractions

Rewrite 3×2^{-4} with a positive power and then as a fraction.

SOLUTION

$$3 \times 2^{-4} = 3 \times \frac{1}{2^4}$$
$$= \frac{3}{2^4}$$
$$= \frac{3}{16}$$

EXPLANATION

$$2^{-4} = \frac{1}{2^4}$$
$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

9 Rewrite each of these with a positive power and then as a fraction. Simplify where possible.

a 6×5^{-2}
b 2×3^{-2}
c 4×5^{-3}
d 6×7^{-1}
e 4×10^{-3}
f 2×10^{-1}
g 5×2^{-4}
h 4×5^{-1}

Remember: $5^1 = 5$.
So $3 \times 5^{-1} = 3 \times \frac{1}{5^1}$
$= \frac{3}{5}$

Example 26 Changing to fractions and decimals

Rewrite 5×10^{-3} with a positive power and then as a fraction and a decimal.

SOLUTION

$$5 \times 10^{-3} = 5 \times \frac{1}{10^3}$$
$$= \frac{5}{10^3}$$
$$= \frac{5}{1000} \text{ or } \frac{1}{200}$$
$$= 0.005$$

EXPLANATION

$$10^{-3} = \frac{1}{10^3}$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

5 thousandths = 0.005

10 Rewrite each of the following with a positive power and then as an unsimplified fraction and a decimal.

a 2×10^{-3}
b 5×10^{-2}
c 7×10^{-1}
d 3×10^{-4}
e 5×10^{-4}
f 8×10^{-5}
g 2×10^{-6}
h 4×10^{-8}

$\frac{1}{10} = 0.1$
$\frac{1}{100} = 0.01$
$\frac{1}{1000} = 0.001$
$\frac{1}{10\,000} = 0.0001$
$\frac{1}{100\,000} = 0.00001$

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Example 27 Evaluating without a calculator

Express using positive powers only, then evaluate without using a calculator.

a 3^{-4} **b** $\dfrac{5}{3^{-2}}$ **c** $\left(\dfrac{2}{3}\right)^{-4}$

SOLUTION	EXPLANATION
a $3^{-4} = \dfrac{1}{3^4}$ $= \dfrac{1}{81}$	Express 3^{-4} as a positive power. $3^4 = 3 \times 3 \times 3 \times 3 = 81$
b $\dfrac{5}{3^{-2}} = 5 \times \dfrac{1}{3^{-2}}$ $= 5 \times 3^2$ $= 5 \times 9$ $= 45$	$\dfrac{1}{3^{-2}} = 3^2$ $3^2 = 3 \times 3 = 9$
c $\left(\dfrac{2}{3}\right)^{-4} = \dfrac{2^{-4}}{3^{-4}}$	Apply the power to each numeral in the brackets.
$= 2^{-4} \times \dfrac{1}{3^{-4}}$	$\dfrac{a}{b} = a \times \dfrac{1}{b}$
$= \dfrac{1}{2^4} \times 3^4$	$2^{-4} = \dfrac{1}{2^4}$, $\dfrac{1}{3^{-4}} = 3^4$
$= \dfrac{3^4}{2^4}$	$3^4 = 3 \times 3 \times 3 \times 3 = 81$ $2^4 = 2 \times 2 \times 2 \times 2 = 16$
$= \dfrac{81}{16}$	Give the answer as an improper fraction.

11 Evaluate the following without the use of a calculator. Express the answers in simplified form.

a 5^{-1} **b** 3^{-2} **c** 4×10^{-2}

d -5×10^{-3} **e** $8 \times (2^2)^{-2}$ **f** $6^4 \times 6^{-6}$

g $8^{-7} \times (8^2)^3$ **h** $\dfrac{1}{8^{-1}}$ **i** $\dfrac{2}{2^{-3}}$

j $\dfrac{2^3}{2^{-3}}$ **k** $\left(\dfrac{3}{8}\right)^{-2}$ **l** $\left(\dfrac{4}{3}\right)^{-3}$

First write expressions using positive indices.

12 Describe the error made in these problems, then give the correct answers.

a $2x^{-2} = \dfrac{1}{2x^2}$ **b** $\dfrac{5}{a^4} = \dfrac{a^{-4}}{5}$ **c** $\dfrac{2}{(3b)^{-2}} = \dfrac{2b^2}{9}$

ENRICHMENT – 13

Kilograms and grams

13 **a** The mass of a small insect is 2^{-9} kg. How many grams is this? Round to two decimal places.

b The mass of an asteroid is 3^{20} kg. How many tonnes is this?

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7F Scientific notation

Interactive
Widgets
HOTsheets
Walkthrough

It is common in practical situations to work with very large or very small numbers. For example, the number of cubic metres of concrete used to build the Hoover Dam in the United States was 3 400 000 m^3. The mass of a molecule of water is 0.000000000000000000000000299 grams. Numbers like this can be written more efficiently using powers of 10 with positive or negative indices. This is called scientific notation. The number is written as a number between 1 and 10 that is multiplied by a power of 10. This notation is also used to show very large and very small time intervals.

At the time of contruction, the Hoover Dam was the largest concrete structure in the world.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Equal times

Work in pairs and help each other to complete this 'matching puzzle'.

For each time in the first column, find the equal time in the second column and in the third column. Write a list of the equal times from each column.

Column 1	Column 2	Column 3
a 6 thousand seconds	6 000 000 s	6×10^2 s
b 6 millionths of a second	0.006 s	6×10^3 s
c 6 hundred seconds	0.06 s	6×10^6 s
d 6 million seconds	6000 s	6×10^{-2} s
e 6 thousandths of a second	0.000006 s	6×10^{-3} s
f 6 hundredths of a second	600 s	6×10^{-6} s

Key ideas

- Numbers written in **scientific notation** are expressed in the form $a \times 10^m$ where a is a number from 1 to less than 10 and m is an integer.
  - To write numbers in scientific notation, place the decimal point after the first non-zero digit, then multiply by a power of 10.
- Large numbers written in scientific notation will use positive powers of 10.

  For example: 4 million years $= 4\,000\,000$ years
  $= 4 \times 1\,000\,000$ years
  $= 4 \times 10^6$ years
- Small numbers written in scientific notation will use negative powers of 10.

  For example: 5 thousandths of a metre $= 0.005$ metre
  $= \frac{5}{1000}$ metre
  $= \frac{5}{10^3}$ metre
  $= 5 \times 10^{-3}$ metre

Scientific notation A method used to express very large and very small numbers

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Exercise 7F

UNDERSTANDING AND FLUENCY	1–9	4, 5–9(½)

1 Copy and complete this table. The first row has been done for you.

Scientific notation	Power of 10 expanded	Basic numeral
5×10^3	5×1000	5000
3×10^4		
2×10^5		
7×10^2		
		70 000
		400 000
		6000
		2 000 000

The power of 10 is equal to the number of zeros.

2 Copy and complete this table. The first row has been done for you.

Scientific notation	Positive power	Fraction	Basic numeral
2×10^{-4}	$\frac{2}{10^4}$	$\frac{2}{10\,000}$	0.0002
3×10^{-2}			
5×10^{-3}			
7×10^{-6}			
			0.009
			0.08
			0.0006
			0.00004

3 Which of the numbers 1000, 10 000 or 100 000 completes each equation?

a $6.2 \times$ _____ $= 62\,000$ **b** $9.41 \times$ _____ $= 9410$

c $1.03 \times$ _____ $= 103\,000$ **d** $3.2 \div$ _____ $= 0.0032$

e $5.16 \div$ _____ $= 0.0000516$ **f** $1.09 \div$ _____ $= 0.000109$

The number of zeros tells you how many places to move the decimal point.

4 If these numbers were written in scientific notation, would positive or negative indices be used?

a 2000 **b** 0.0004 **c** 19 300 **d** 0.00101431

Example 28 Writing large numbers in scientific notation

Write 4 500 000 in scientific notation.

SOLUTION

$4\,500\,000 = 4.5 \times 10^6$

EXPLANATION

Place the decimal point after the first non-zero digit (4). Multiply by 10^6 since the decimal place has been moved 6 places to the left.

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5 Write the following in scientific notation.

a 40 000	**b** 230 000 000 000	**c** 16 000 000 000
d 7 200 000	**e** 3500	**f** 8 800 000
g 52 hundreds	**h** 3 million	**i** 21 thousands

Large numbers: use 10 to a positive power.

Example 29 Writing small numbers in scientific notation

Write 0.0000004 in scientific notation.

SOLUTION

$0.0000004 = 4 \times 10^{-7}$

EXPLANATION

The first non-zero digit is 4. Multiply by 10^{-7} since the decimal place has been moved 7 places to the right.

6 Write the following in scientific notation.

a 0.000003	**b** 0.0004	**c** 0.00876
d 0.00000000073	**e** 0.00003	**f** 0.000000000125
g 0.00000000809	**h** 0.000000024	**i** 0.0000345

Small numbers: use 10 to a negative power.

7 Write each of the following numbers in scientific notation.

a 6000	**b** 720 000	**c** 324.5
d 7869.03	**e** 8459.12	**f** 0.2
g 0.000328	**h** 0.00987	**i** 0.00001
j 460 100 000	**k** 17 467	**l** 128

Place the decimal point after the first non-zero digit.

Example 30 Writing basic numerals from positive powers

Express 9.34×10^6 as a basic numeral.

SOLUTION

$9.34 \times 10^6 = 9\,340\,000$

EXPLANATION

Move the decimal point 6 places to the right, so that the 9 increases in place value by 6 places from the units position to the millions position.

8 Express each of the following as a basic numeral.

a 5.7×10^4	**b** 3.6×10^6	**c** 4.3×10^8
d 3.21×10^7	**e** 4.23×10^5	**f** 9.04×10^{10}
g 1.97×10^8	**h** 7.09×10^2	**i** 6.357×10^5

Move the decimal point right to increase the place value of each digit.

Example 31 Writing basic numerals from negative powers

Express 4.71×10^{-6} as a basic numeral.

SOLUTION

$4.71 \times 10^{-6} = 0.00000471$

EXPLANATION

Move the decimal point 6 places to the left and insert zeros where necessary. The 4 decreases in place value by 6 places from the units position to the millionths position.

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9 Express each of the following as a basic numeral.

a 1.2×10^{-4} **b** 4.6×10^{-6} **c** 8×10^{-10}
d 3.52×10^{-5} **e** 3.678×10^{-1} **f** 1.23×10^{-7}
g 9×10^{-5} **h** 5×10^{-2} **i** 4×10^{-1}

Move the decimal point left to decrease the place value of each digit.

PROBLEM-SOLVING AND REASONING

10, 11 | 10, 12

10 Express each of the following approximate numbers in scientific notation.

a The mass of Earth is 6 000 000 000 000 000 000 000 000 kg.
b The circumference of Earth is 40 000 000 m.
c The diameter of a gold atom is 0.0000000001 m.
d The radius of Earth's orbit around the Sun is 150 000 000 km.
e The universal constant of gravitation is 0.0000000000667 N m^2/kg^2.
f The half-life of polonium-214 is 0.00015 seconds.
g Uranium-238 has a half-life of 4 500 000 000 years.

11 Express each of the following numbers as a basic numeral.

a Neptune is approximately 4.6×10^9 km from Earth.
b A population of bacteria contained 8×10^{12} organisms.
c Earth is approximately 3.84×10^5 km from the Moon.
d A fifty-cent coin is approximately 3.8×10^{-3} m thick.
e The diameter of the nucleus of an atom is approximately 1×10^{-14} m.
f The population of a city is 7.2×10^5.

12 Write the answers to each of these problems in scientific notation.

a Two planets are 2.8×10^8 km and 1.9×10^9 km from their closest sun. What is the difference between these two distances?
b Two particles weigh 2.43×10^{-2} g and 3.04×10^{-3} g. Find the difference in their masses.

ENRICHMENT

– | 13

Scientific notation with numbers larger than 10

13 The number 47×10^4 is not written in scientific notation, since 47 is not a number between 1 and 10. The following shows how to convert to scientific notation.

$$47 \times 10^4 = 4.7 \times 10 \times 10^4$$
$$= 4.7 \times 10^5$$

Write these numbers in scientific notation.

a 32×10^3 **b** 41×10^5
c 0.13×10^5 **d** 0.092×10^3
e 61×10^{-3} **f** 424×10^{-2}
g 0.02×10^{-3} **h** 0.0004×10^{-2}

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7G Scientific notation using significant figures

Interactive
Widgets
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Walkthrough

The number of digits used to record measurements depends on how accurately the measurements can be recorded. The volume of Earth, for example, has been calculated as 1 083 210 000 000 km^3. This could be written in scientific notation as 1.08321×10^{12}.
A more accurate calculation may include more non-zero digits in the last seven places.

The mass of a single oxygen molecule is known to be 0.000000000000000000000000053 g. This is written in scientific notation as 5.3×10^{-26}.

On many calculators you will notice that very large or very small numbers are automatically converted to scientific notation. Numbers can also be entered into a calculator using scientific notation.

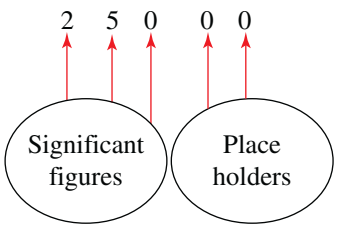
The accuracy of the measurement of the volume of the Earth depends in part of the number of significant figures.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: How many people in the ground?

The officials at a football match counted the people who entered the stadium, then rounded off to the nearest hundred. They announced: 'The attendance today is 25 000.'

- How many people were in the stadium?
- What is the smallest number of people?
- What is the largest number of people?



In this example, the officials are using 3 significant figures.

Key ideas

- **Significant figures** are counted from left to right, starting at the first non-zero digit.
  - Zeros on the end of a decimal are definitely significant.
  - Zeros on the end of a whole number might be significant.

  For example:

  0.0025 definitely has 2 significant figures.
  0.00250 definitely has 3 significant figures.
  0.205 definitely has 3 significant figures.
  6.0 definitely has 2 significant figures.
  66 definitely has 2 significant figures.
  60 might have 1 or 2 significant figures.
  66 000 might have 2, 3, 4 or 5 significant figures.
- When using scientific notation, the first significant figure sits to the left of the decimal point.

  For example:

  3.210×10^4 has four significant figures
- Calculators can be used to work with scientific notation.
  - Look for a $\boxed{\times 10^x}$ or $\boxed{\text{EXP}}$ button.
  - Use it to confirm that $(6 \times 10^9) \times (5 \times 10^9) = 3 \times 10^{19}$.

Significant figure A digit that indicates how accurate a number is

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Exercise 7G

UNDERSTANDING AND FLUENCY | 1–4, 5–9(½) | 4, 5–9(½)

1 **a** Round each of these numbers to the nearest hundred.
- **i** 267
- **ii** 32 740
- **iii** 18 350

b Round each of these numbers to the nearest tenth.
- **i** 0.063
- **ii** 0.1902
- **iii** 21.04

c Round each of these numbers to the nearest thousand.
- **i** 267 540
- **ii** 38 290
- **iii** 4 060 990

Rounding rules:
Locate the first digit to the right of the required digit.
- Round down (leave it as it is) for a 4 or less
- Round up (increase by 1) for a 5 or more

2 Which of these numbers definitely has two significant figures?
62 000, 30 500, 420, 0.0071, 0.0805, 201 000

Note that 204 has 3 significant figures but 240 might have 2 or 3 significant figures.

3 Copy and complete the tables, rounding each number to the given number of significant figures.

a 57 263

Significant figures	Rounded number
4	
3	57 300
2	
1	

b 4 170 162

Significant figures	Rounded number
5	
4	
3	4 170 000
2	
1	

57 263
6 is the 4th significant figure, so round to the nearest 10 for 4 significant figures.

c 0.0036612

Significant figures	Rounded number
4	
3	
2	
1	0.004

d 24.8706

Significant figures	Rounded number
5	
4	
3	
2	25
1	

24.8706
0 is the 5th significant figure so round to the nearest thousandth for 5 significant figures.

4 Are the following numbers written in scientific notation with three significant figures? (Yes or no)

a 4.21×10^4 **b** 32×10^{-3} **c** 1800×10^6
d 0.04×10^2 **e** 1.89×10^{-10} **f** 9.04×10^{-6}
g 5.56×10^{-14} **h** 0.213×10^2 **i** 26.1×10^{-2}

The number to the left of the decimal point must be from 1 to less than 10.

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Example 32 Stating the number of significant figures

State the number of significant figures given in these numbers.

a 401 **b** 0.005012 **c** 3.2×10^7 **d** 125 000

SOLUTION	EXPLANATION
a 3 significant figures	All the digits are significant.
b 4 significant figures	Start counting at the first non-zero digit (5).
c 2 significant figures	With scientific notation, the first significant figure is to the left of the decimal point.
d 3, 4, 5 or 6 significant figures	The zeros on the end of a whole number may or may not be significant.

5 State the number of significant figures given in these numbers.

a 27 200 **b** 1007 **c** 301 010
d 190 **e** 0.0183 **f** 0.20
g 0.706 **h** 0.00109 **i** 4.21×10^3
j 2.905×10^{-2} **k** 1.07×10^{-6} **l** 5.90×10^5

The zeros on the end of a whole number may or may not be significant.

Example 33 Writing numbers in scientific notation using significant figures

Write these numbers in scientific notation and using three significant figures.

a 2 183 000 **b** 0.0019482

SOLUTION	EXPLANATION
a $2\,183\,000 = 2.18 \times 10^6$	Put the decimal point after the first non-zero digit. The decimal point has moved 6 places so multiply by 10^6. Round the third significant figure down (i.e. leave it as is) since the following digit (3) is less than 5.
b $0.0019482 = 1.95 \times 10^{-3}$	Move the decimal point 3 places to the right and multiply by 10^{-3}. Round the third significant figure up to 5 since the following digit (8) is greater than 4.

6 Write these numbers in scientific notation and using three significant figures.

a 242 300 **b** 171 325 **c** 2829
d 3 247 000 **e** 0.00034276 **f** 0.006859
g 0.01463 **h** 0.001031 **i** 23.41
j 326.042 **k** 19.618 **l** 0.172046

7 Write each number in scientific notation, rounding to the number of significant figures given in the brackets.

a 47 760 (3) **b** 21 610 (2) **c** 4 833 160 (4)
d 37.16 (2) **e** 99.502 (3) **f** 0.014427 (4)
g 0.00201 (1) **h** 0.08516 (1) **i** 0.0001010 (1)

First round the number to the required number of significant figures.

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Example 34 Using a calculator with scientific notation

Use a calculator to evaluate $3.67 \times 10^5 \times 23.6 \times 10^4$. Leave your answer in scientific notation, correct to four significant figures.

SOLUTION

$3.67 \times 10^5 \times 23.6 \times 10^4$
$= 8.661 \times 10^{10}$

EXPLANATION

Use the $\boxed{\times 10^x}$ or $\boxed{\text{EXP}}$ button.
Write in scientific notation with four significant figures.

8 Use a calculator to evaluate each of the following. Leave your answers in scientific notation, correct to four significant figures.

a 4^{-6}

b 78^{-3}

c $(-7.3 \times 10^{-4})^{-5}$

d $\dfrac{3.185}{7 \times 10^4}$

e $2.13 \times 10^4 \times 9 \times 10^7$

f $5.671 \times 10^2 \times 3.518 \times 10^5$

g $9.419 \times 10^5 \times 4.08 \times 10^{-4}$

h $2.85 \times 10^{-9} \times 6.33 \times 10^{-3}$

i $12\,345^2$

j 87.14^8

k $\dfrac{1.83 \times 10^{26}}{4.5 \times 10^{22}}$

l $\dfrac{-4.7 \times 10^{-2} \times 6.18 \times 10^7}{3.2 \times 10^6}$

Example 35 Using a calculator with scientific notation with negative indices

Use a calculator to evaluate $7.6 \times 10^{-3} + \sqrt{2.4 \times 10^{-2}}$. Leave your answer in scientific notation, correct to four significant figures.

SOLUTION

$7.6 \times 10^{-3} + \sqrt{2.4 \times 10^{-2}} = 0.1625$
$= 1.625 \times 10^{-1}$

EXPLANATION

Use the $\boxed{\times 10^x}$ or $\boxed{\text{EXP}}$ button.
Write in scientific notation with a number between 1 and 10.

9 Use a calculator to evaluate each of the following. Leave your answers in scientific notation, correct to five significant figures.

a $\sqrt{8756}$

b $\sqrt{634 \times 7.56 \times 10^7}$

c $8.6 \times 10^5 + \sqrt{2.8 \times 10^{-2}}$

d $-8.9 \times 10^{-4} + \sqrt{7.6 \times 10^{-3}}$

e $\dfrac{5.12 \times 10^{21} - 5.23 \times 10^{20}}{2 \times 10^6}$

f $\dfrac{8.942 \times 10^{47} - 6.713 \times 10^{44}}{2.5 \times 10^{19}}$

g $\dfrac{2 \times 10^7 + 3 \times 10^8}{5}$

h $\dfrac{4 \times 10^8 + 7 \times 10^9}{6}$

i $\dfrac{6.8 \times 10^{-8} + 7.5 \times 10^{27}}{4.1 \times 10^{27}}$

j $\dfrac{2.84 \times 10^{-6} - 2.71 \times 10^{-9}}{5.14 \times 10^{-6} + 7 \times 10^{-8}}$

For fraction calculations, insert brackets so that the numerator and denominator are calculated before division.

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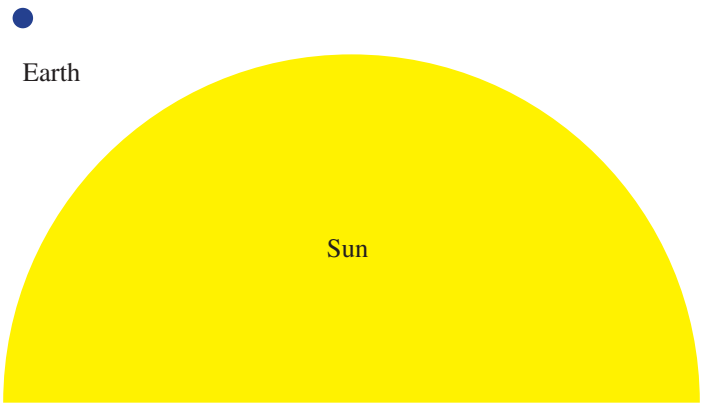
PROBLEM-SOLVING AND REASONING 10, 11 | 10, 12, 13

10 The mass of Earth is approximately 6 000 000 000 000 000 000 000 000 kg. The mass of the Sun is 330 000 times the mass of Earth. Find the mass of the Sun. Express your answer in scientific notation, correct to three significant figures.

11 The diameter of Earth is approximately 12 756 000 m. If the Sun's diameter is 109 times that of Earth, compute its diameter in kilometres. Express your answer in scientific notation, correct to three significant figures.



The size of the Sun and Earth compared (distance of Earth of Sun is not to scale)

12 Write these numbers from largest to smallest.
2.41×10^6, 24.2×10^5, 0.239×10^7, 2421×10^3, 0.02×10^8

First write each number in scientific notation.

13 The following output is common on a number of different calculators and computers. Write down the number that you think they represent.

a 4.26E6 **b** 9.1E-3 **c** 5.04EXP11
d 1.931EXP-1 **e** 2.1^{06} **f** 6.14^{-11}

ENRICHMENT – | 14

Combining bacteria

14 A flask of type A bacteria contains 5.4×10^{12} cells and a flask of type B bacteria contains 4.6×10^8 cells. The two types of bacteria are combined in the same flask.

a How many bacterial cells are there in the flask?

b If type A bacterial cells double every 8 hours, and type B bacterial cells triple every 8 hours, how many cells are in the flask after:

i 8 hours?

ii one day?

Express your answers to part **b** in scientific notation, correct to three significant figures.

Set up a table to show the number of each type of bacteria after every 8 hours.

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1 A population of bacteria doubles every 5 minutes. What is this type of growth called? Solve this puzzle to find the answer.

Write the basic numeral for each of the following. Write the letter corresponding to each answer in the boxes below to form a word.

T	$\frac{1}{10^3}$	**L**	2.15×10^3	**I**	$\frac{9^5}{9^3}$
A	5×10^{-2}	**O**	2^{-2}	**N**	2^4
P	$3^2 \times 2^2 - 3^0$	**X**	$5^2 + 2^4 - 3^2 \times 4$	**E**	$\frac{4^3}{8}$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
8	5	35	0.25	16	8	16	0.001	81	0.05	2150

2 Write the answer to these calculations without using a calculator:

a $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

b $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 4 \times 4 \times 4 \times 4 \times 4 \times 5 \times 5 \times 5 \times 5 \times 5$

3 Show that this expression is equal to 1.

$$\frac{1}{2} \times \frac{a^2b}{3am} \times \frac{6a^3}{b^2} \times \frac{10(ab)^2}{2bm^3} \times \frac{m^4}{5a^6}$$

4 Insert brackets in this expression to make the given answer correct.

$$4a \times 3m^3 \times \left(\frac{a}{6m}\right)^2 = 3a^3m$$

5 Nick and Maddy have a big bag of 10-cent coins. They decide to make 30 piles of coins, starting with one coin in the first pile, then 2 coins in the second pile, 4 coins in the third pile, 8 in the fourth pile and so on, continuing to double the number of coins for each new pile. Given that each 10-cent coin is 2 mm thick, find the heights of the 1st, 5th, 10th, 15th, 20th, 25th and 30th piles and the value in dollars of each of these piles.

6 Each day that a bushfire burns in a National Park, it has burned twice as much total area as the day before. If a National Park was totally burned out in 15 days, on which day was the park only half burned out?

7 It is thought that the game of chess was invented by an ancient Indian mathematician. The king was so pleased with the game that he offered the inventor any reward of his choice: rare jewels, bags of gold or even a large property.

To the king's surprise the Indian mathematician asked for some wheat! He asked for 1 grain for the first square of the chess board, 2 grains for the second square, 4 grains for the third square, 8 grains for the fourth square etc. continuing this way right up to the 64th square.

a If one grain of wheat weighs 2×10^{-8} tonnes, what weight of wheat would the mathematician have received for the 64th square? Answer in scientific notation, correct to three significant figures.

b How much money would this wheat be worth at the 2012 Australian price of \$275/tonne?

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Chapter summary

Indices

Index law of multiplication

$a^m \times a^n = a^{m+n}$

When multiplying terms with the same base, add indices.

e.g. $x^3 \times x^5 = x^{3+5}$
$= x^8$

Index law of division

$a^m \div a^n = a^{m-n}$

When dividing terms with the same base, subtract indices.

e.g. $\frac{x^7}{x^4} = x^{7-4}$
$= x^3$

Index form

a^m ← index

base

e.g. $2^3 = 2 \times 2 \times 2$
$3^4 = 3 \times 3 \times 3 \times 3$
$a \times b \times a \times b \times b = a^2b^3$

Power of a product

$(ab)^m = a^m b^m$

e.g. $(3x^2)^3 = 3^3 (x^2)^3$
$= 27x^6$

Power of a fraction

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

e.g. $\left(\frac{x}{2}\right)^4 = \frac{x^4}{2^4}$
$= \frac{x^4}{16}$

Zero index

$a^0 = 1$

Any number (except 0) to the power 0 equals 1.

e.g. $3x^0 = 3 \times 1$
$= 3$
$(3x)^0 = 1$

Power of a power

$(a^m)^n = a^{mn}$

To expand brackets, multiply indices.

e.g. $(x^2)^5 = x^{2\times5}$
$= x^{10}$

Negative indices

$a^{-m} = \frac{1}{a^m}$

e.g. $2^{-3} = \frac{1}{2^3}$

$2 \times 5^{-4} = \frac{2}{1} \times \frac{1}{5^4}$
$= \frac{2}{5^4}$

$\frac{1}{a^{-m}} = a^m$

e.g. $\frac{1}{2^{-3}} = 2^3$

$\frac{5}{7^{-4}} = 5 \times 7^4$

Scientific notation

In scientific notation, very large or very small numbers are written in the form $a \times 10^m$ where $1 \le a < 10$.

Large numbers will use positive powers of 10,

e.g. 2 350 000 kg $= 2.35 \times 10^6$ kg

Small numbers will use negative powers of 10,

e.g. 0.000000016 $= 1.6 \times 10^{-8}$

Significant figures

These are counted from left to right, starting at the first non-zero digit.

0.000217 has 3 significant figures.

1.07×10^4 has 3 significant figures.

2 403 000 might have 4, 5 or 6 significant figures.

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Multiple-choice questions

1 $3x^7 \times 4x^4$ is equivalent to:

A $12x^7$ **B** $12x^{28}$ **C** $7x^{11}$ **D** $12x^{11}$ **E** $7x^3$

2 $3(2y^2)^0$ simplifies to:

A 6 **B** 3 **C** $6y^2$ **D** $3y$ **E** 12

3 $(2x^2)^3$ expands to:

A $2x^5$ **B** $2x^6$ **C** $6x^6$ **D** $8x^5$ **E** $8x^6$

4 $\dfrac{x^6y^2}{4x^2y}$ simplifies to:

A $-4x^4y$ **B** $\dfrac{x^3y^2}{4}$ **C** $\dfrac{x^4y}{4}$ **D** $\dfrac{x^4y^2}{4}$ **E** $\dfrac{x^3y}{4}$

5 $\left(\dfrac{2a}{k}\right)^3$ is equal to:

A $\dfrac{2a^3}{k}$ **B** $\dfrac{2a^3}{k^3}$ **C** $\dfrac{6a^3}{k^3}$ **D** $\dfrac{8a^3}{k^3}$ **E** $\dfrac{6a}{3k}$

6 $\left(\dfrac{5^2}{7^3}\right)^4$ is equal to:

A $\dfrac{4 \times 5^2}{4 \times 7^3}$ **B** $\dfrac{5^8}{7^{12}}$ **C** $\dfrac{5^6}{7^3}$ **D** $\dfrac{5^8}{7^3}$ **E** $\dfrac{5^6}{7^7}$

7 $5(2am^3)^3$ is equal to:

A $30a^3m^9$ **B** $10a^3m^9$ **C** $40a^3m^6$ **D** $40a^3m^9$ **E** $10am^6$

8 4×7^{-3} expressed with positive indices is:

A 28^3 **B** $4^3 7^3$ **C** $\dfrac{4}{7^3}$ **D** $\dfrac{-4}{7^3}$ **E** $\dfrac{1}{4 \times 7^3}$

9 The mass of a shipping container is 2.32×10^4 kg. In expanded form this mass in kilograms is:

A 2 320 000 **B** 232 **C** 23 200 **D** 0.000232 **E** 2320

10 0.00032761 in scientific notation rounded to three significant figures is:

A 328×10^{-5} **B** 3.27×10^{-4} **C** 3.28×10^4 **D** 3.30×10^4 **E** 3.28×10^{-4}

Short-answer questions

1 Express each of the following in index form.

a $3 \times 3 \times 3 \times 3$ **b** $2 \times x \times x \times x \times y \times y$

c $3 \times a \times a \times a \times \dfrac{b}{a} \times b$ **d** $\dfrac{3}{5} \times \dfrac{3}{5} \times \dfrac{3}{5} \times \dfrac{1}{7} \times \dfrac{1}{7}$

2 Write each of the following as a product of prime factors in index form.

a 45 **b** 300

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3 Simplify using index laws.

a $x^3 \times x^7$ **b** $2a^3b \times 6a^2b^5c$ **c** $3m^2n \times 8m^5n^3$

d $a^{12} \div a^3$ **e** $x^5y^3 \div (x^2y)$ **f** $\dfrac{5a^8b^3}{10a^6b}$

4 Simplify:

a $(m^2)^3$ **b** $(3a^4)^2$ **c** $(-2a^2b)^5$

d $3a^0b$ **e** $2(3m)^0$ **f** $\left(\dfrac{a^2}{3}\right)^3$

5 Express each of the following with positive indices.

a 2^{-3} **b** 3×4^{-5} **c** 6×10^{-3}

d $\dfrac{1}{4^{-2}}$ **e** $\dfrac{6}{5^{-3}}$ **f** $\dfrac{2}{10^{-5}}$

6 Use index laws to simplify each of the following.

a $\dfrac{k^5 \times k^3}{k^2}$ **b** $\dfrac{9a^4}{2m^3} \times \dfrac{4m^4}{3a^2}$ **c** $(a^2)^3 \times 4a$

d $(h^{12})^3 \div (h^4)^5$ **e** $\dfrac{4(a^3)^2(b^4)^3}{20a^0(b^2)^5}$ **f** $5(3p^2q)^2$

g $(3a^7b)^3 \div \dfrac{ab^2}{3}$ **h** $(5m^{-2}n^8)^0 \times (3m^2n^3)^{-1}$ **i** $\dfrac{2a^{-2}b^4}{8a^5b^{-1}}$

7 Write each of the following numbers as a basic numeral and then arrange them in ascending order.

2.35, 0.007×10^2, 0.0012, 3.22×10^{-1}, 0.4, 35.4×10^{-3}.

8 Write each of the following numbers as a basic numeral.

a 3.24×10^2 **b** 1.725×10^5 **c** 2.753×10^{-1} **d** 1.49×10^{-3}

9 Write each of the following values in scientific notation using three significant figures.

a The population of Australia in 2010 was approximately 22 475 056.

b The area of the USA is 9 629 091 km^2.

c The time taken for light to travel 1 metre (in a vacuum) is 0.00000000333564 seconds.

d The wavelength of ultraviolet light is 0.000000294 m.

10 Write each of the following values using scientific notation in the units given in brackets.

Note: 1 tonne = 1000 kg.

a 25 years (hours) **b** 3 million years (months)

c 430 tonnes (kg) **d** 5 tonnes (grams)

Extended-response questions

1 Use a calculator to evaluate the following, giving your answer in scientific notation correct to two significant figures.

a $m_S \times m_E$ where m_S (mass of the Sun) $= 1.989 \times 10^{30}$ kg and m_E (mass of Earth) $= 5.98 \times 10^{24}$ kg

b The speed, v, in m/s of an object of mass $m = 2 \times 10^{-3}$ kg and kinetic energy $E = 1.88 \times 10^{-12}$ joules where $v = \sqrt{\dfrac{2E}{m}}$

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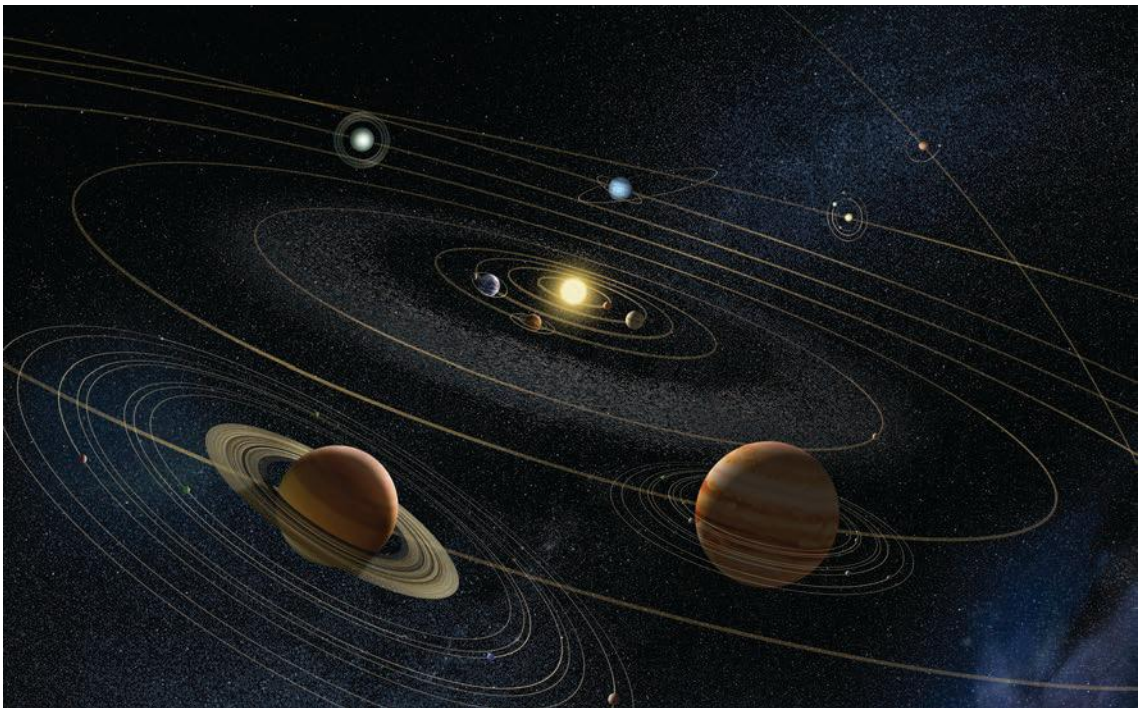
2 Use the table of Solar System data below to answer these questions. Write each answer in scientific notation rounded to three significant figures.

Approximate data for some planets in the Solar System			
Planet	**Distance from the Sun in millions of km**	**Length (in Earth days) of one revolution around the Sun**	**Mass compared to Earth**
Mercury	57.9	88.0	0.553
Venus	108.2	224.7	0.815
Earth	149.6	365.25	1.00
Mars	227.9	687.0	0.1074
Jupiter	778.3	4331	317 896
Saturn	1427	10 760	95 185

a Find the distance from the Sun, in kilometres, for:
- **i** Mercury
- **ii** Earth
- **iii** Jupiter
- **iv** Saturn

b How many Earth years will have passed for these planets to complete one full revolution around the Sun?
- **i** Mercury
- **ii** Mars
- **iii** Saturn

c The mass of Earth is approximately 5.98×10^{24} kg. Determine the mass of:
- **i** Venus
- **ii** Mars
- **iii** Jupiter



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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

8 Properties of geometrical figures

What you will learn

8A Angles and triangles REVISION
8B Parallel lines REVISION
8C Quadrilaterals REVISION
8D Polygons
8E Congruent triangles
8F Enlargement and similar figures
8G Similar triangles
8H Applying similar triangles

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NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY

SUBSTRAND: PROPERTIES OF GEOMETRICAL FIGURES

Outcomes

A student classifies, describes and uses the properties of triangles and quadrilaterals, and determines congruent triangles to find unknown side lengths and angles. (MA4–17MG)

A student describes and applies the properties of similar figures and scale drawings. (MA5.1–11MG)

A student calculates the angle sum of any polygon and uses minimum conditions to prove triangles are congruent or similar. (MA5.2–14MG)

A student identifies and uses angle relationships, including those related to transversals on sets of parallel lines. (MA4–18MG)

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Penrose stairs

Penrose stairs are a two-dimensional illusion that appears to show a three-dimensional, never-ending staircase. Such an illusion was featured in the 2010 film *Inception*. The Penrose stairs are a variation of the Penrose triangle, which shows three right angles joined in an impossible way to add up to 270°.

Dutch artist M.C. Escher (1898–1972) became famous for his drawings of impossible objects, including several based on the Penrose triangle.

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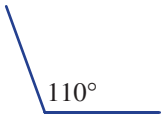
Pre-test

1 Name the following types of angles. Choose from acute, right, obtuse, straight, reflex or revolution.

a


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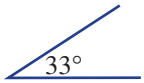
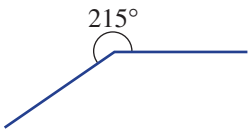
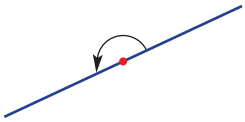
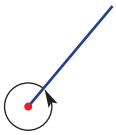


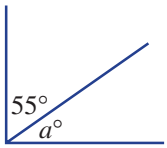
c


d

e


f


2 Find the value of a.

a


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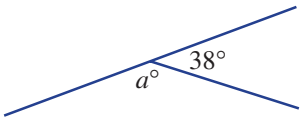

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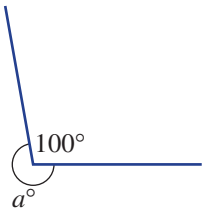
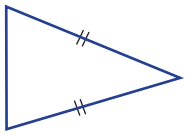
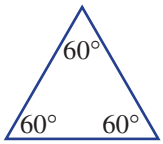
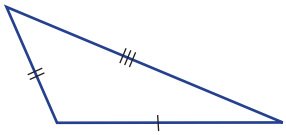


3 Identify the following triangles as equilateral, isosceles or scalene.

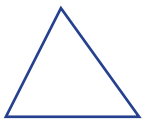
a


b


c


4 Identify the following as right, acute or obtuse triangles.

a

b


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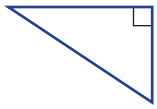
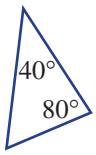
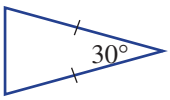
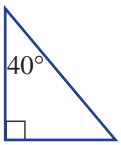


5 Find any missing angles in these triangles.

a


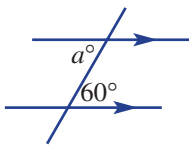
b


c


6 Of the six special quadrilaterals (square, rectangle, parallelogram, rhombus, kite and trapezium), which ones match the following descriptions?

a Two pairs of equal length sides **b** Two pairs of parallel sides **c** Equal length diagonals

7 Find the value of the pronumerals in each of the following.

a


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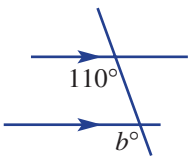
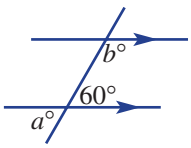
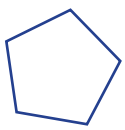


c


8 Name the following shapes.

a


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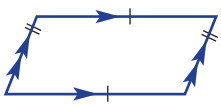

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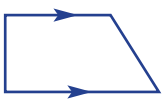

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8A Angles and triangles REVISION

Interactive
Widgets
HOTsheets
Walkthrough

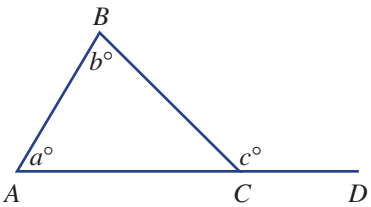
When looking at a building or structure, we see angles formed where two lines meet. We see triangles being used for their strength and rigidity. Lines meeting at a point and triangles have special properties and will be revised in this section.

Modern architecture using geometric shapes: the spire of the Arts Centre in Melbourne.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Exterior angle discovery

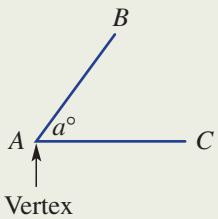
Here is a triangle with one side extended to form the exterior angle $\angle BCD$ ($c°$).



- If $a = 50$ and $b = 85$, find $\angle ACB$. Then find $\angle BCD$. What do you notice?
- Repeat for $a = 60$ and $b = 95$. What do you notice?
- What is the relationship between a, b and c? This is called the exterior angle theorem.

Key ideas

- When two **lines** or line segments meet at a point, an **angle** is formed.
  - The angle shown can be named $\angle A$ or $\angle BAC$ or $\angle CAB$.



Line A continuous set of points without breadth or thickness

Angle The difference in direction between two lines

- Angle types
  - Acute: between 0° and 90°
  - Right: 90°
  - Obtuse: between 90° and 180°
  - Straight: 180°
  - Reflex: between 180° and 360°
  - Revolution: 360°

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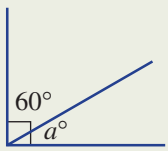
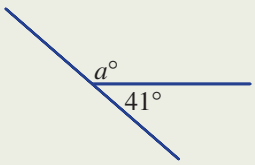
Key ideas

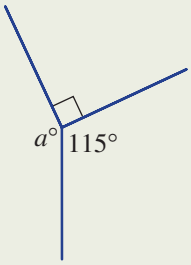
- Angles at a point
  - Angles in a right angle or adjacent complementry angles (sum to 90°)
  - Angles on a straight line or adjacent supplementry angles (sum to 180°)
  - Revolution (sum to 360°)
  - Vertically opposite angles (are equal)



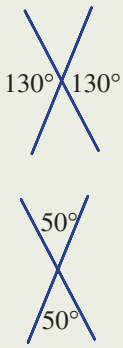
$a + 60 = 90$



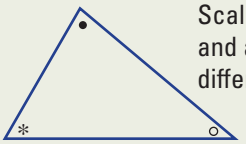
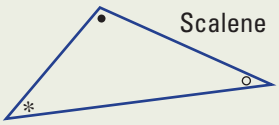
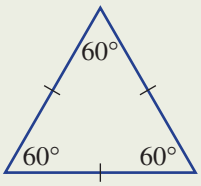
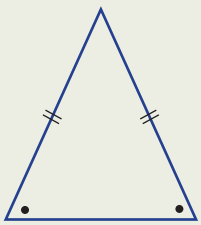
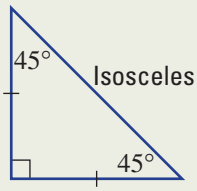
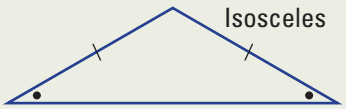
$a + 41 = 180$



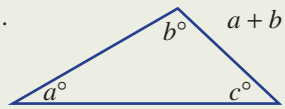
$a + 90 + 115 = 360$



- Types of triangles

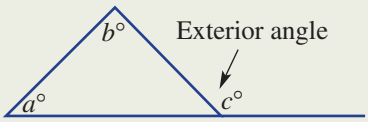
Acute-angled (all angles ∠90°)	Right-angled (includes a 90° angle)	Obtuse-angled (one angle > 90°)
 Scalene (all sides and angles are different sizes)	Scalene	 Scalene
 Equilateral (all angles 60° and all sides equal)  Isosceles (two angles equal and two sides equal)	 Isosceles	 Isosceles

- The sum of the angles in a triangle is 180°.



$a + b + c = 180$

- An **exterior angle** is formed by extending one side of a shape.
  - The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



$c = a + b$

Exterior angle The angle formed between the extended side and the adjacent side of a triangle

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Exercise 8A REVISION

UNDERSTANDING AND FLUENCY 1–5, 6(½) 3, 4–6(½)

1 Choose a word or number from the table to complete each sentence.

180°	revolution	acute	obtuse	180°
supplementary	180°	equal	90°	right

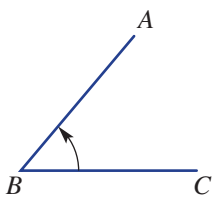
a A 90° angle is called a _____ angle.
b A _____ angle is called a straight angle.
c A 360° angle is called a _____.
d _____ angles are between 90° and 180°.
e _____ angles are between 0° and 90°.
f Reflex angles are between ____ and 360°.
g Complementary angles sum to _____.
h _____ angles sum to 180°.
i The three angles in a triangle sum to _____.
j Vertically opposite angles are _____.

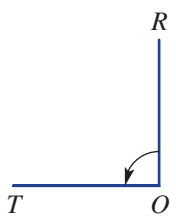
2 Match the triangles in the table with the descriptions below.

equilateral triangle	right-angled triangle	scalene triangle
obtuse-angled triangle	isosceles triangle	acute-angled triangle

a a pair of equal length sides
b one obtuse angle
c all angles 60°
d one pair of equal angles
e all angles acute
f all sides of different length
g one right angle

3 Estimate the size of each of the following angles and use your protractor to determine an accurate measurement.

a


b


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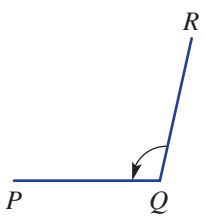
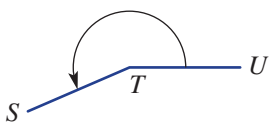


d


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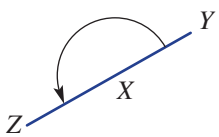

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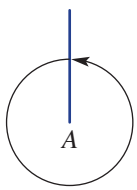

Example 1 Finding supplementary and complementary angles

For the angle 47°, determine the:
a supplementary angle
b complementary angle

SOLUTION	EXPLANATION
a $180° - 47° = 133°$	Supplementary angles sum to 180°.
b $90° - 47° = 43°$	Complementary angles sum to 90°.

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4 For each of the following angles determine the:

i supplementary angle

ii complementary angle

a 55° **b** 31° **c** 74° **d** 10°

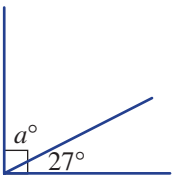
e 89° **f** 22° **g** 38° **h** 65°

i 47° **j** 77°

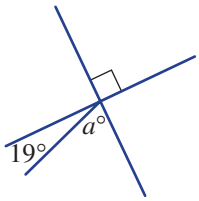
- Supplementary angles add to 180°.
- Complementary angles add to 90°.

5 Find the value of a in these diagrams.

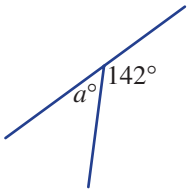
a



b

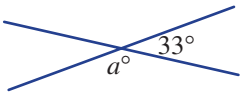


c

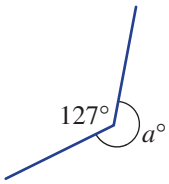


A straight angle is 180°.
A revolution is 360°.

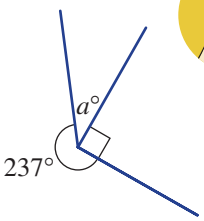
d



e



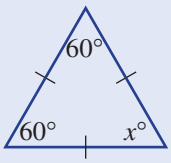
f



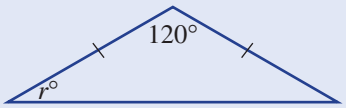
Example 2 Finding unknown angles in triangles

Name the types of triangles shown here and determine the values of the pronumerals.

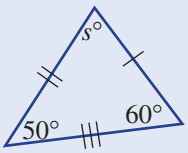
a



b



c



SOLUTION	EXPLANATION
a Equilateral triangle	All sides are equal, therefore all angles are equal.
$x = 60$	
b Obtuse isosceles triangle	One angle is more than 90° and two sides are equal.
$2r + 120 = 180$	Angles in a triangle add to 180° and since it is isosceles, the two
$2r = 60$	base angles are equal.
$r = 30$	Subtract 120° from both sides and then divide both sides by 2.
c Acute scalene triangle	All angles are less than 90° and all sides are of different length.
$s + 50 + 60 = 180$	Angles in a triangle add to 180°. Simplify and solve for s.
$s + 110 = 180$	
$s = 70$	

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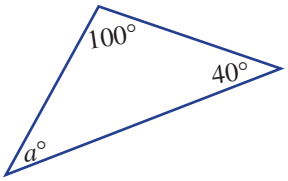
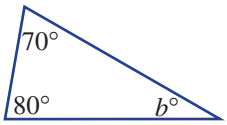
6 Name the types of triangles shown here. What are the values of the pronumerals?

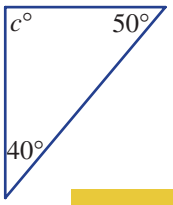
a



b

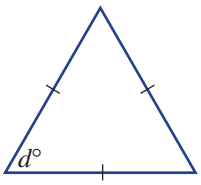


c

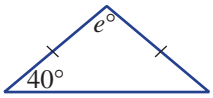


The angle sum of a triangle is 180°.

d



e



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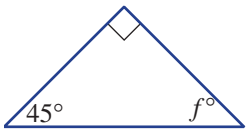
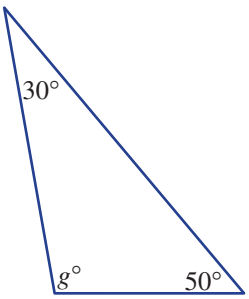


g



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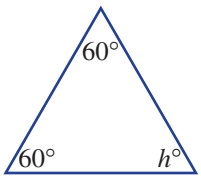

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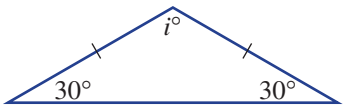

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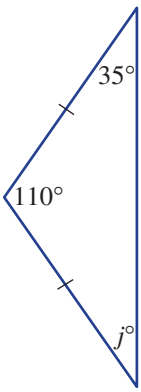

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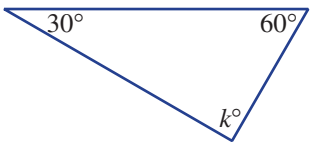

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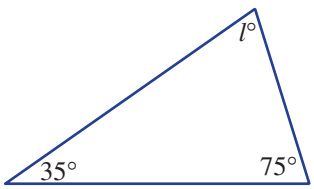


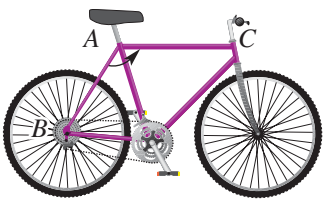
PROBLEM-SOLVING AND REASONING

	7, 8	7–9

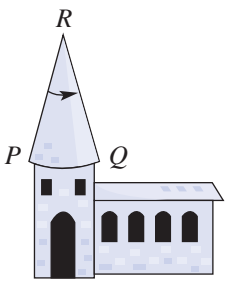
7 For each diagram:

- **i** name the angle shown (e.g. $\angle ABC$)
- **ii** state the type of angle given
- **iii** estimate the size of the angle
- **iv** measure the angle using a protractor

a



b



When naming an angle, put the letter at the vertex in the middle.

$\angle DEF$

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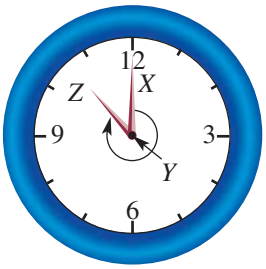
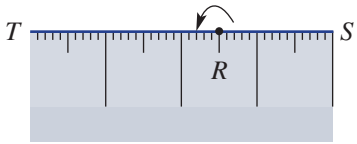
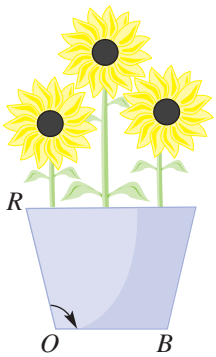
c



d



e

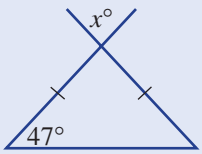


f

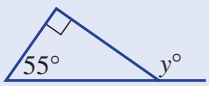
Example 3 Finding exterior angles

Find the value of each pronumeral. Give reasons for your answers.

a



b



SOLUTION

a $x + 47 + 47 = 180$ (angle sum of triangle)

$x + 94 = 180$

$x = 86$

b Let a be the unknown angle.

$a + 90 + 55 = 180$ (angle sum of triangle)

$a = 35$

$y + 35 = 180$ (angles on a straight line)

$y = 145$

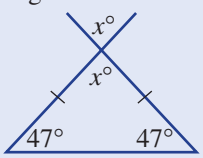
Alternative method:

$y = 90 + 55$ (exterior angle of triangle)

$= 145$

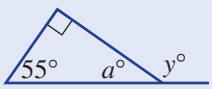
EXPLANATION

Note the isosceles triangle and vertically opposite angles and mark in those angles.



Angles in a triangle add to 180° and vertically opposite angles are equal. Simplify and solve for x.

Angles in a triangle sum to 180°.



Angles in a straight line are supplementary (sum to 180°).

Alternatively, use the exterior angle theorem, which says that the exterior angle is equal to the sum of the two opposite interior angles.

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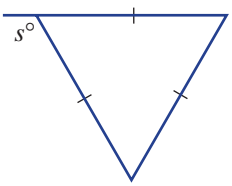
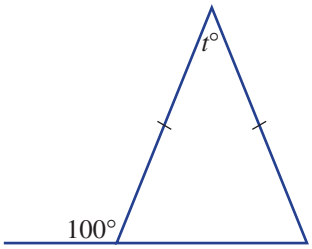
8 Find the value of each pronumeral.

a

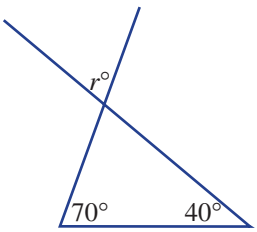


b

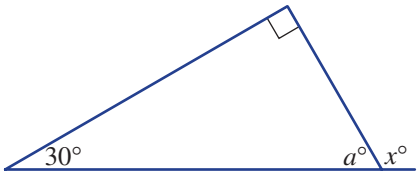


If you can't find the value of the pronumeral straight away, try finding one of the other missing angles.

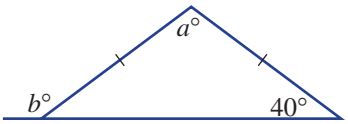
c



d



e



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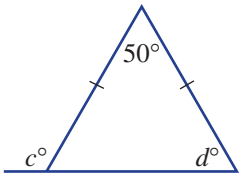
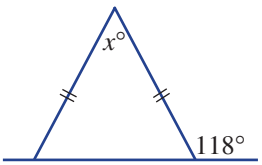


g



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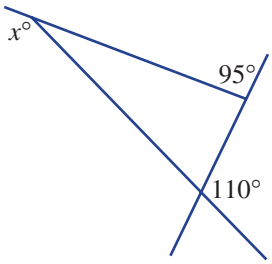
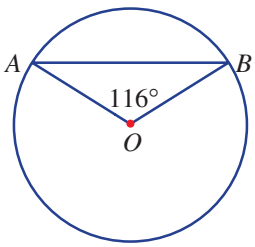


9 Explain why $\angle OAB$ is 32° in this circle if O is the centre of the circle.



ENRICHMENT – 10, 11

Analogue geometry

10 Calculate how many degrees the minute hand of a clock rotates in:

a 1 hour **b** $\frac{1}{4}$ of an hour **c** 10 minutes **d** 15 minutes

e 72 minutes **f** 1 minute **g** 2 hours **h** 1 day

11 Find the non-reflex angle between the hour and minute hands at these times. Remember to consider how the hour hand moves between each whole number. For example, at 9:30, the hour hand is halfway between the 9 and 10.

a 3 p.m. **b** 5 a.m. **c** 6:30 p.m.

d 11:30 p.m. **e** 3:45 a.m. **f** 1:20 a.m.

g 4:55 a.m. **h** 2:42 a.m. **i** 9:27 a.m.

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8B Parallel lines REVISION

Interactive
Widgets
HOTsheets
Walkthrough

When two lines are crossed by a third line, called a transversal, many special pairs of angles are formed. If the two original lines are parallel, then the pairs of angles are either equal or add up to 180°.

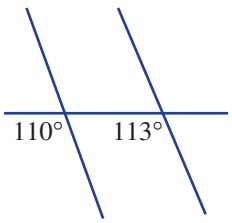
The steel rails of a railway track are parallel.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

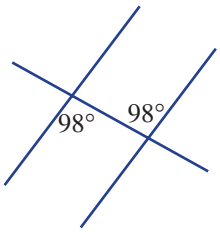
Let's start: Are they parallel?

Here are three diagrams that show a transversal crossing two other lines.

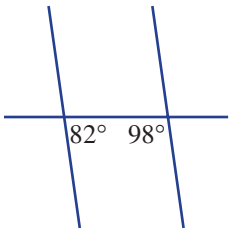
1



2



3

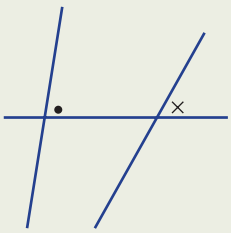
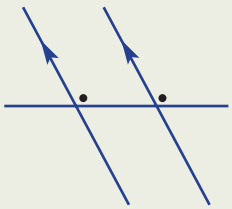
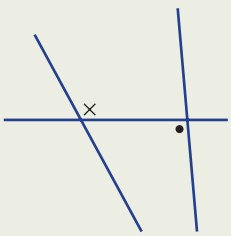
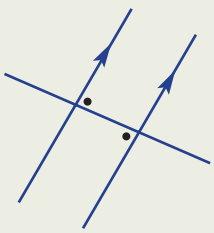
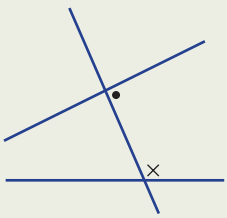
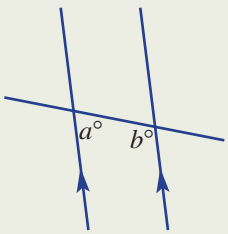


Decide whether each diagram contains a pair of parallel lines. Give reasons for your answer.

Key ideas

- Parallel lines never meet.
  - Arrows indicate that lines are parallel.
- A **transversal** is a line crossing two or more other lines.

Transversal A line that cuts two or more lines

	Non-parallel lines	Parallel lines
Corresponding angles • If lines are parallel, corresponding angles are equal.		
Alternate angles • If lines are parallel, alternate angles are equal.		
Co-interior angles • If lines are parallel, co-interior angles are supplementary (sum to 180°).		 $a + b = 180$

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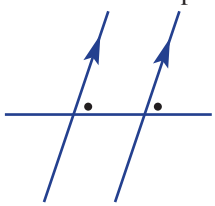
Exercise 8B REVISION

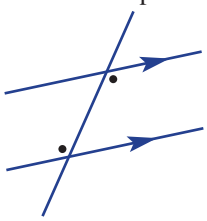
UNDERSTANDING AND FLUENCY	1, 2, 3–4(½), 5	2, 3–5(½)

1 Decide whether the pair of marked angles are corresponding, alternate or co-interior.

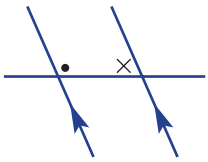
a



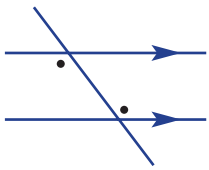
b



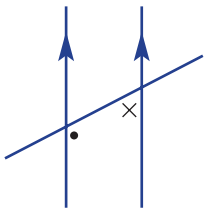
c



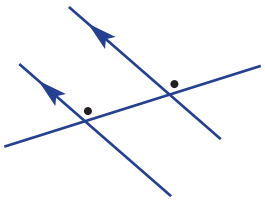
d



e



f



2 Use the word *equal* or *supplementary* to complete these sentences.

a If two lines are parallel, corresponding angles are _____.

b If two lines are parallel, alternate angles are _____.

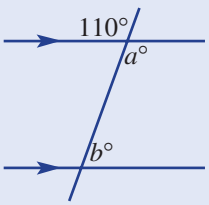
c If two lines are parallel, co-interior angles are _____.

Supplementary angles add to 180°.

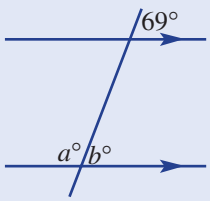
Example 4 Finding angles in parallel lines

Find the value of each of the pronumerals. Give reasons for your answers.

a



b



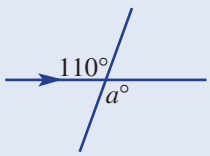
SOLUTION

a $a = 110$ (vertically opposite angles)

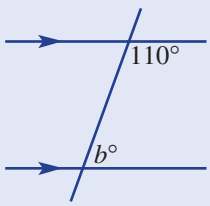
$b + 110 = 180$ (co-interior angles in parallel lines)

$b = 70$

EXPLANATION



Vertically opposite angles are equal.



Co-interior angles add to 180° in parallel lines.

Example is continued on the next page

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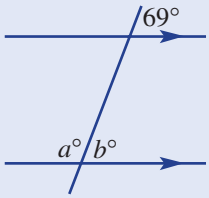
b $b = 69$ (corresponding angles in parallel lines)



Corresponding angles are equal in parallel lines.

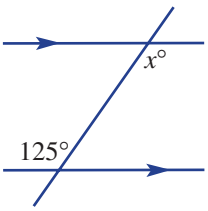
$a + 69 = 180$ (angles on a straight line)
$a = 111$

Angles on a straight line add to 180°, so $a + b = 180$.

3 Find the values of the pronumerals in these pairs of parallel lines. Give a reason for each answer.

For part **a**, $x = 125$.
Reason: Alternate angles in parallel lines.

a



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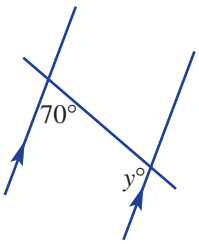

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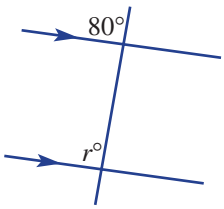

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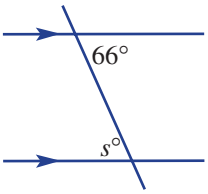

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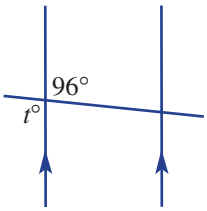

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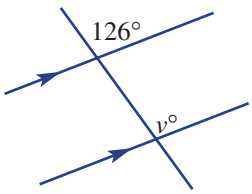

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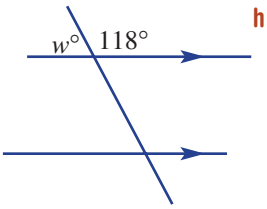

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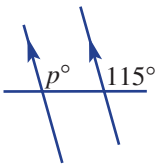

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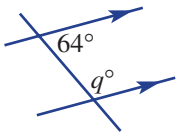
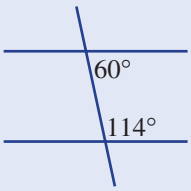


Example 5 Deciding whether lines are parallel

Decide whether each diagram contains a pair of parallel lines. Give reasons.

a



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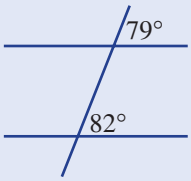

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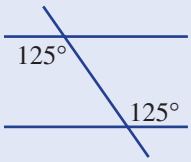

SOLUTION

a No. The two co-interior angles do not add to 180°.

b No. The two corresponding angles are not equal.

c Yes. The two alternate angles are equal.

EXPLANATION

$60° + 114° = 174° \neq 180°$

$79° \neq 82°$

If alternate angles are equal, then the lines are parallel.

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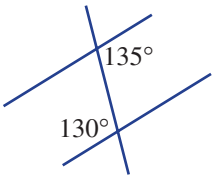
4 Decide if each diagram contains a pair of parallel lines. Give reasons.

a



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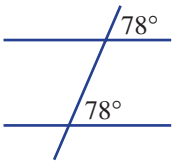

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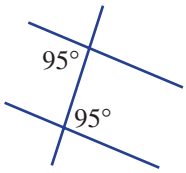

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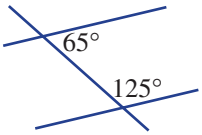

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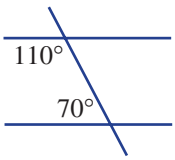

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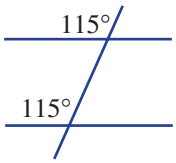

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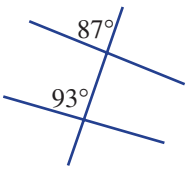

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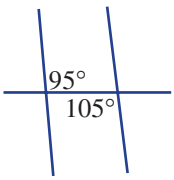

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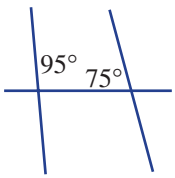
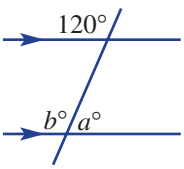


The reasons why lines could be parallel include:

1. Corresponding angles are equal.
2. Alternate angles are equal.
3. Co-interior angles add to 180° (are supplementary).

5 Find the value of each pronumeral. Give reasons for your answers.

a



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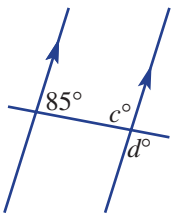

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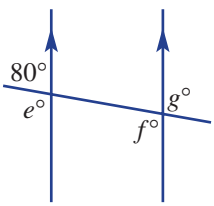

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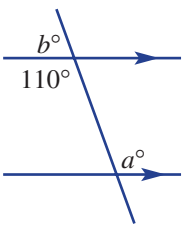

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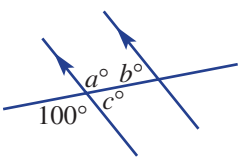

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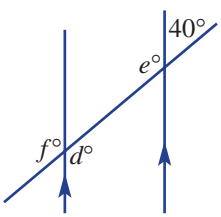

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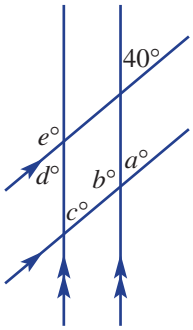
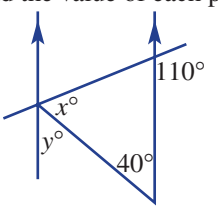
PROBLEM-SOLVING AND REASONING 6, 7 6, 8

6 Two pairs of straight train tracks meet at 40°, as shown on the right. Find thevalue of a, b, c, d and e.

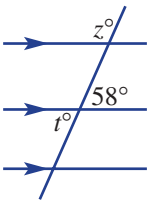


7 Find the value of each pronumeral.

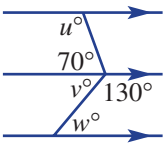
a



b



c



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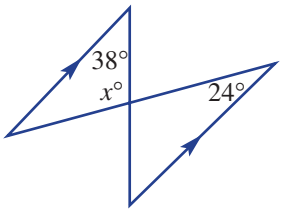

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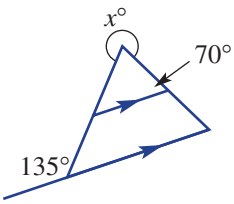

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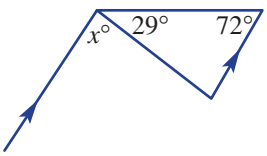
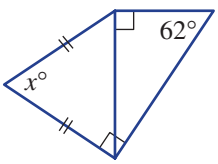


Look for a pair of angles inside parallel lines. Mark in all known missing angles to help find the value of the pronumeral.

8 Find the value of x.

a



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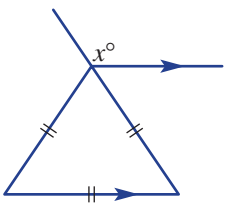

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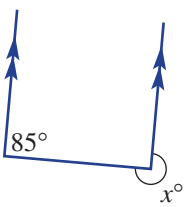

Use all the given information to mark in any missing angles.

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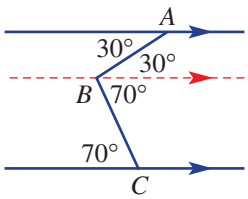
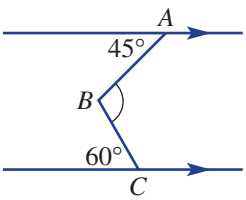
ENRICHMENT – 9

Add a new line to help

9 Sometimes you can add a third parallel line to a diagram to help you find an angle. For example, to find $\angle ABC$ in this diagram, you can draw a parallel line through B, then find the two alternate angles (30° and 70°). So $\angle BC = 30° + 70° = 100°$. Add a third parallel line to help find $\angle ABC$ in these diagrams.



a



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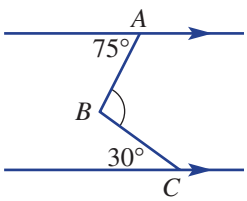

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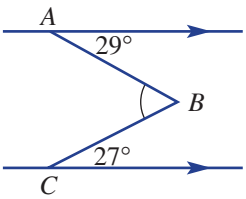

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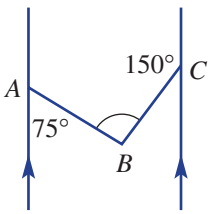

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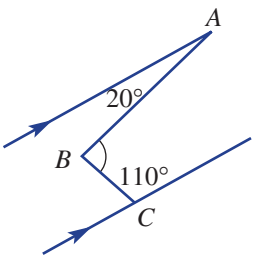

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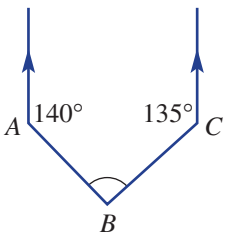

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8C Quadrilaterals REVISION

Interactive
Widgets
HOTsheets
Walkthrough

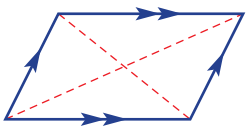
Quadrilaterals are polygons with four straight sides. They include six special shapes. Like triangles, they have a special angle sum. This sum is the same for all types of quadrilaterals.

Both the sign shapes are quadrilaterals.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: What is a parallelogram?

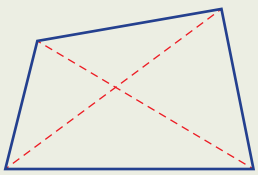
A parallelogram is a four-sided shape made from two pairs of parallel lines.



- What are some properties of a parallelogram?
- Do you think the following shapes are parallelograms?
  - Rectangle
  - Kite
  - Rhombus
  - Square
  - Trapezium
- Discuss the properties of each of the special quadrilaterals. Consider side lengths, the lengths of diagonals and the interior angles.

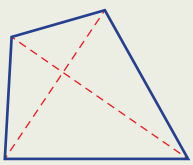
Key ideas

- Every quadrilateral has two diagonals.
  - In some quadrilaterals the diagonals bisect each other (i.e. cut each other in half).



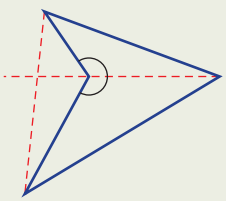
- **Quadrilaterals** can be convex or non-convex.
  - **Convex** quadrilaterals have all vertices pointing outwards.
  - **Non-convex** quadrilaterals have one vertex pointing inwards.
  - Both diagonals of convex quadrilaterals lie inside the figure.

Convex



All interior angles less than 180°

Non-convex



One reflex interior angle

Quadrilateral A two-dimensional shape with four sides of any length

Convex A polygon with all vertices pointing outward

Non-convex A polygon with at least one vertex pointing inwards

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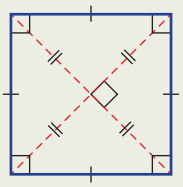
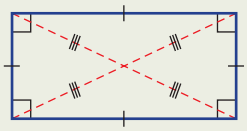
Key ideas

- Special quadrilaterals

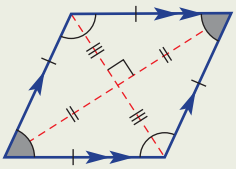
Square



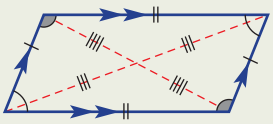
Rectangle



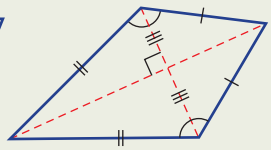
Rhombus



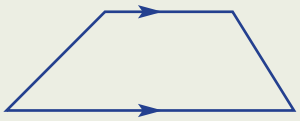
Parallelogram



Kite



Trapezium



Square A quadrilateral with all sides equal, both pairs of opposite sides parallel, and all angles are right angles

Rectangle A quadrilateral with both pairs of opposite sides equal and parallel and all angles are right angles

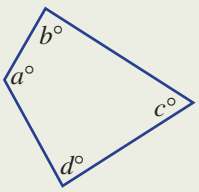
Rhombus A quadrilateral with both pairs of opposite sides parallel and all sides equal

Parallelogram A quadrilateral with both pairs of opposite sides parallel

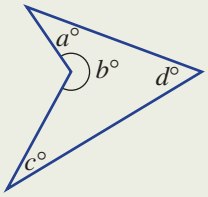
Kite A quadrilateral with two pairs of adjacent sides equal

Trapezium A quadrilateral with at least one pair parallel sides

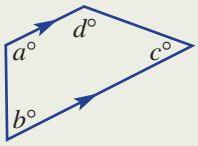
- The angle sum of any quadrilateral is 360°.



$$a + b + c + d = 360$$



- Quadrilaterals with parallel sides contain two pairs of cointerior angles.



$$c + d = 180$$
$$a + b = 180$$

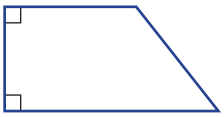
Exercise 8C REVISION

UNDERSTANDING AND FLUENCY	1–6	3, 4, 5–6(½)

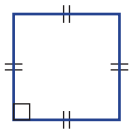
1 **a** Write down the names of the six special quadrilaterals.

b Based on the information in the diagrams below, choose the best name for each of these shapes.
Hint: All six have different names.

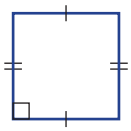
i



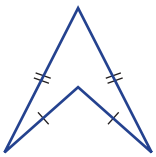
ii



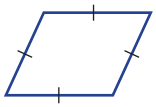
iii



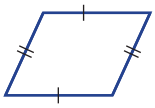
iv



v



vi



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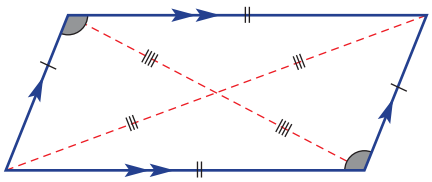
2 Using the diagram of a parallelogram given below, which of the following sentences are *definitely* true.

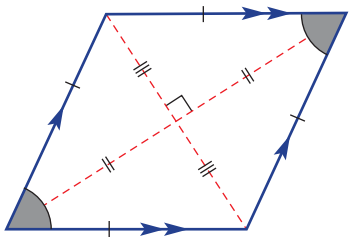
a All sides are equal in length.
b Opposite sides are equal.
c Opposite sides are parallel.
d All angles are 90°.
e Opposite angles are equal.
f The diagonals are equal in length.
g The diagonals bisect each other.
h The diagonals meet at right angles.

Parallelogram



3 Repeat Question **2**, using this diagram of a rhombus.

Rhombus



Example 6 Working with properties of quadrilaterals

Which of the six 'special' quadrilaterals *definitely* have these properties?

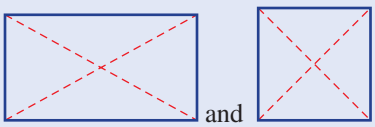
a Diagonals of equal length
b Diagonals intersecting at right angles

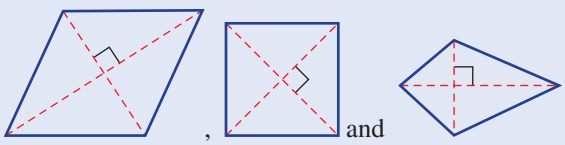
SOLUTION

a Rectangle and square

b Rhombus, square and kite

EXPLANATION





4 Which of the six 'special' quadrilaterals *definitely* have these properties?

a Opposite sides equal
b All interior angles 90°
c Diagonals of equal length
d Diagonals intersecting at right angles
e Exactly 1 pair of parallel sides
f Diagonals of different length

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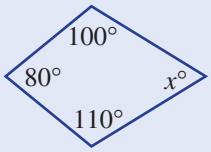
Example 7 Finding angles in quadrilaterals

Find the value of the pronumeral in this quadrilateral. Give a reason.



SOLUTION

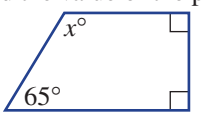
$x + 80 + 100 + 110 = 360$ (angle sum of a quadrilateral)
$x + 290 = 360$
$x = 70$

EXPLANATION

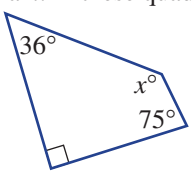
The angles in a quadrilateral add to 360°. Simplify and solve for x.

5 Find the value of the pronumeral x in these quadrilaterals.

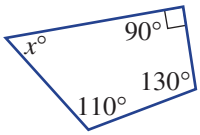
a



b

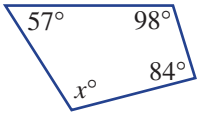


c

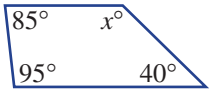


Use the angle sum of a quadrilateral, which is 360°.

d



e



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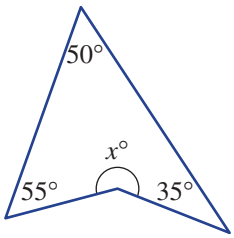
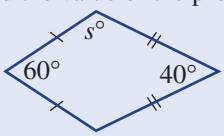


Example 8 Finding angles in special quadrilaterals

Find the value of the pronumeral in this kite and trapezium. Give a reason.

a



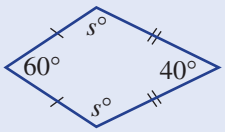
b

SOLUTION

a $2s + 60 + 40 = 360$ (angle sum of a quadrilateral)
$2s + 100 = 360$
$2s = 260$
$s = 130$

b $y + 125 = 180$ (co-interior angles in parallel lines)
$y = 55$

EXPLANATION



The angles in a quadrilateral add to 360° and the opposite angles (s°) are equal in a kite.

Co-interior angles inside parallel lines are supplementary (add to 180°).

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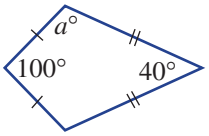
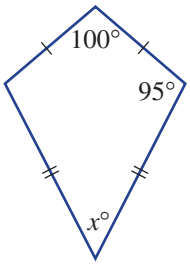
6 Find the value of the pronumerals.

a

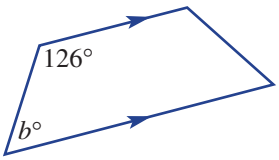


b



Remember: A kite has one pair of equal opposite angles. Co-interior angles inside parallel lines are supplementary (add to 180°).

c



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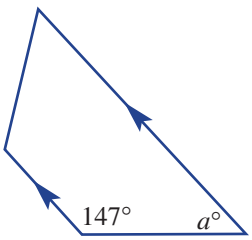
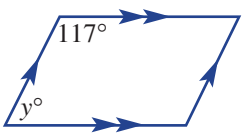


e



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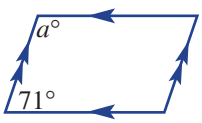


PROBLEM-SOLVING AND REASONING

	7, 8	7, 9, 10

7 Decide whether it is possible to draw a quadrilateral with:

a 1 interior reflex angle

b 2 interior reflex angles

c 3 acute angles

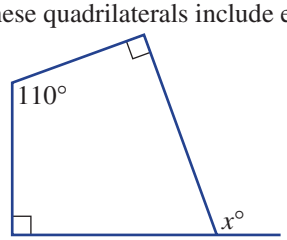
d 4 acute angles

8 A shopping centre's floor is a quadrilateral with one 120° angle and two 90° angles. What is the size of the fourth angle?

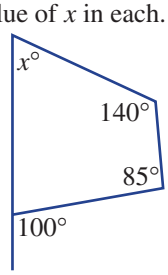
9 Explain why rectangles, squares and rhombuses are all parallelograms.

10 These quadrilaterals include exterior angles. Find the value of x in each.

a



b



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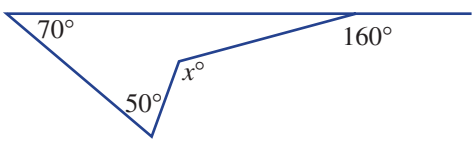

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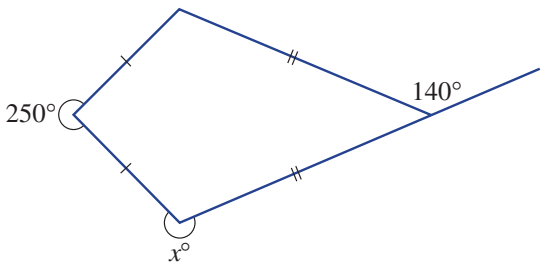
ENRICHMENT – 11

Quad x challenge

11 Find the value of x in these diagrams.

a



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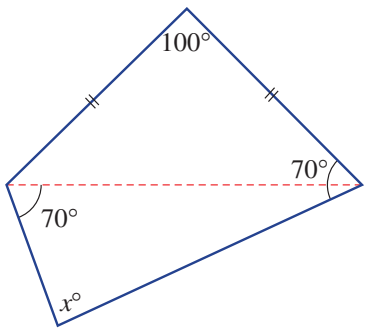

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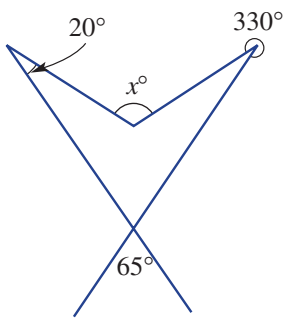
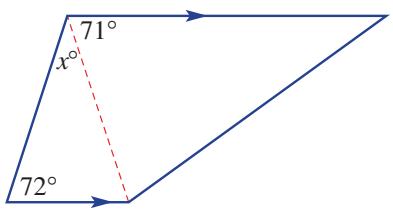


d



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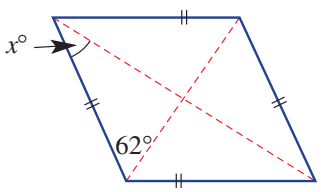

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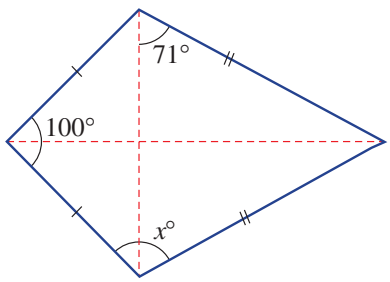

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8D Polygons

Interactive
Widgets
HOTsheets
Walkthrough

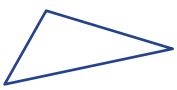
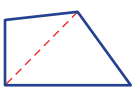
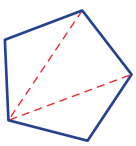
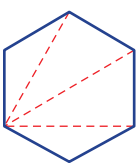
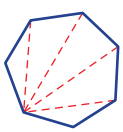
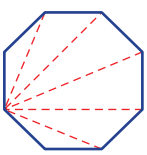
Closed two-dimensional shapes with straight sides are called polygons. They are classified by their number of sides. We have already seen triangles (3 sides) and quadrilaterals (4 sides), which have been classified further by their special properties.

This building in Singapore has a hexagonal structure over the windows.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Developing the rule

The following procedure uses the fact that the angle sum of a triangle is 180°. Copy and complete the table and try writing the general rule in the final row.

Shape	Number of sides	Number of triangles	Angle sum
Triangle 	3	1	$1 \times 180° = 180°$
Quadrilateral 	4	2	____ $\times$ 180° = ____
Pentagon 	5		
Hexagon 	6		
Heptagon 	7		
Octagon 	8		
n-sided polygon	n		(____) $\times$ 180°

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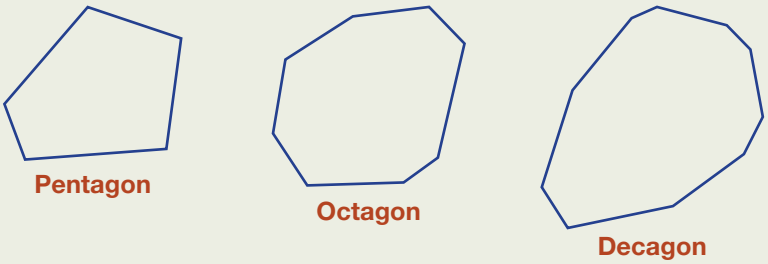
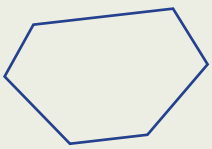
Key ideas

- **Polygons** are shapes with straight sides.



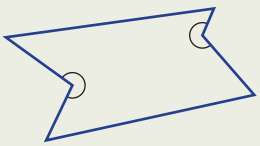
- Polygons can be convex or non-convex.
  - Convex polygons have all vertices pointing outwards.
  - Non-convex polygons have at least one vertex pointing inwards.

Convex



This means all interior angles are less than 180°.

Non-convex

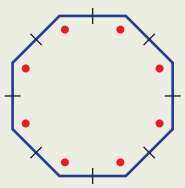


This means there is one or more reflex interior angles.

- Polygons are named according to their number of sides.

Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	**Hexagon**
7	**Heptagon**
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

- The angle sum, S, of a polygon with n sides is given by the rule $S = (n - 2) \times 180°$.
- The sum of the exterior angles of a convex polygon is 360°.
- A **regular polygon** has sides of equal length and equal interior angles.



A regular octagon

Polygon A plane figure bounded by line segments

Pentagon A five-sided polygon (with five angles)

Octagon An eight-sided polygon (with eight angles)

Decagon A ten-sided polygon

Hexagon A six-sided polygon (with six angles)

Heptagon A seven-sided polygon (with seven angles)

Regular polygon A polygon with all sides of equal length and all angles equal

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Exercise 8D

UNDERSTANDING AND FLUENCY 1–7 4, 5–7(½)

1 Write the missing word/rule from the table.

regular	non-convex	$(n - 2) \times 180°$	hexagan

a A _____ has 6 sides.
b A _____ polygon has equal sides and equal angles.
c A _____ polygon has at least one reflex angle.
d The rule for the angle sum of a polygon is $S =$ _____.

2 How many sides do these polygons have?

a pentagon **b** heptagon
c quadrilateral **d** undecagon
e nonagon **f** dodecagon

Refer to the key ideas for help.

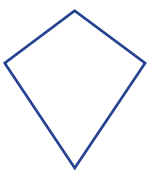
3 Use $S = (n - 2) \times 180°$ to find the angle sum, S, of these polygons.

a hexagon **b** octagon
c undecagon **d** heptagon

n is the number of sides of the polygon.

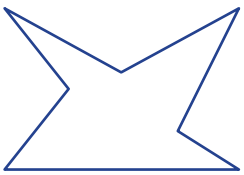
4 Name each of these shapes as convex or non-convex, and write their polygon name.

a



b

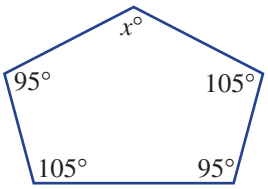
c



A convex polygon has all angles less than 180°.

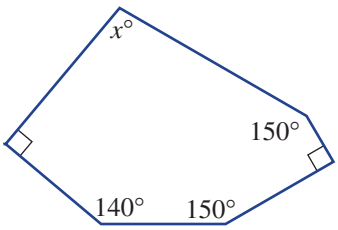
5 Find the value of x using the given angle sum.

a



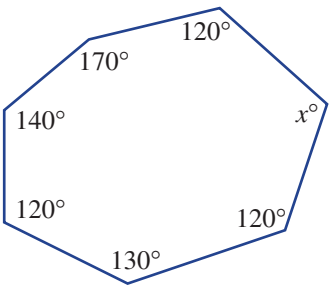
Angle sum = 540°

b



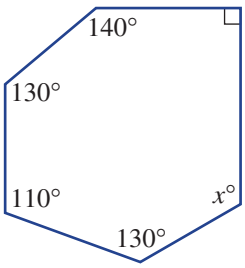
Angle sum = 720°

c



Angle sum = 900°

d



Angle sum = 720°

All the angles must sum to the given angle sum.

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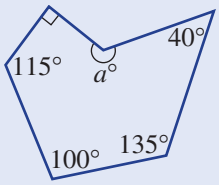
Example 9 Finding angles in polygons

Find the angle sum using $S = (n - 2) \times 180$, then find the value of a.



SOLUTION

$n = 6$ and $S = (n - 2) \times 180$
$= (6 - 2) \times 180$
$= 720$

$a + 90 + 115 + 100 + 135 + 40 = 720$
$a + 480 = 720$
$a = 240$

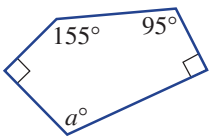
EXPLANATION

The shape is a hexagon with 6 sides so $n = 6$.
$4 \times 180 = 720$

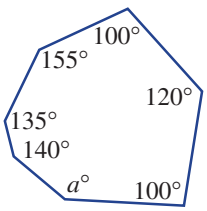
The sum of all angles is 720°. Simplify and solve for a.

6 For each polygon, find the angle sum using $S = (n - 2) \times 180$. Then find the value of a.

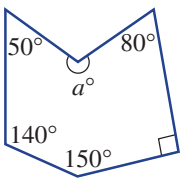
a



b

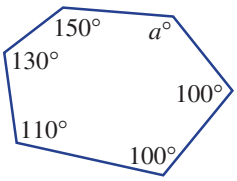


c

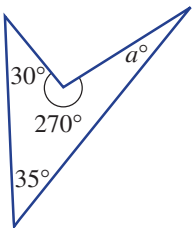


First find the angle sum using $s = (n - 2) \times 180$, where n is the number of sides.

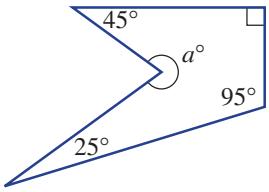
d



e

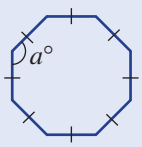


f



Example 10 Working with regular polygons

Find the size of an interior angle of a regular octagon.



SOLUTION

$n = 8$ and $S = (n - 2) \times 180$
$= (8 - 2) \times 180$
$= 1\,080$
$\therefore a = 1\,080 \div 8$
$= 135$

Alternative method:
The exterior angle sum is 360°.
The exterior angles are $360° \div 8 = 45°$
The interior angles are $180° - 45° = 135°$
$\therefore a = 135$

EXPLANATION

The regular octagon has 8 sides, so use $n = 8$.

Each interior angle in a regular polygon is equal, so divide by 8 to get a single angle.

This is true for *all* polygons.

Interior angle + exterior angle = 180°

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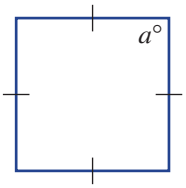
7 Find the size of an interior angle of these regular polygons. Round to two decimal places in part **e**.

a



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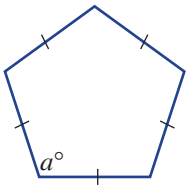

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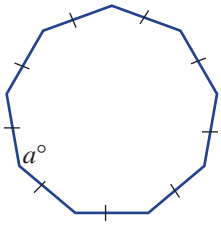

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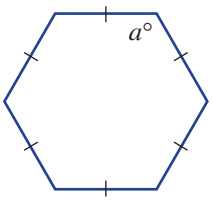

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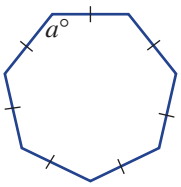

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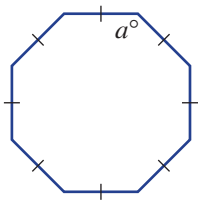


PROBLEM SOLVING AND REASONING	8, 9	8–10

8 Calculate the number of sides if a polygon has the given angle sum. Suggestion: Use the rule $S = (n - 2) \times 180$.

a 2520° **b** 4140° **c** 18 000°

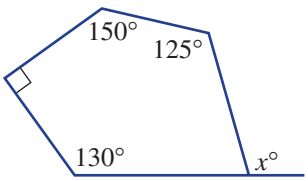
9 Decide whether the following are possible. If so, make a drawing.

a A hexagon with all angles equal but not all sides equal

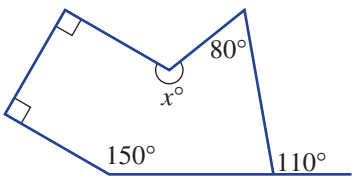
b A hexagon with all sides equal but not all angles equal

10 These polygons include exterior angles. Find the value of x in each.

a

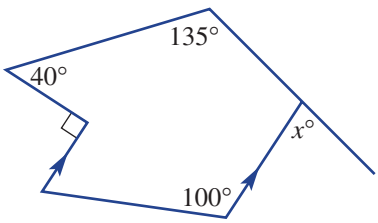


b



The interior angle sum will need to be calculated.

c



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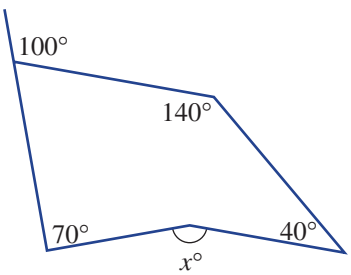

ENRICHMENT	–	11

Maximum reflex

11 Recall that a non-convex polygon has at least one reflex interior angle.

a Use drawings to decide whether the following are possible.

i A quadrilateral with 2 reflex angles **ii** A pentagon with 2 reflex angles

iii A hexagon with 4 reflex angles

b What is the maximum number of interior reflex angles possible for these polygons?

i Quadrilateral **ii** Pentagon **iii** Octagon

c Write a rule for the maximum number of interior reflex angles for a polygon with n sides.

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8E Congruent triangles

Interactive

Widgets

HOTsheets

Walkthrough

Congruent shapes have the same shape and size. Their matching angles are equal and their matching sides have the same length.

It is possible to determine whether or not two triangles are congruent by using four tests, which we call SSS, SAS, AAS and RHS.

The Petronas Twin Towers in Kuala Lumpur look congruent.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

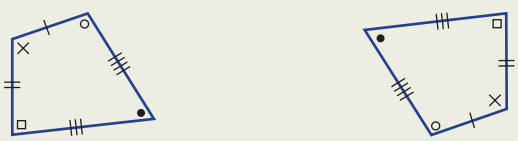
Let's start: Constructing congruent triangles

For this task you will need a ruler, pencil and protractor. (You might also use compasses.) Divide these constructions equally among the members of the class. Each group is to draw one of the following triangles, with the given properties.

1. Triangle ABC with $AB = 8$ cm, $AC = 5$ cm and $BC = 4$ cm
2. Triangle DEF with $DE = 7$ cm, $DF = 6$ cm and $\angle EDF = 40°$
3. Triangle GHI with $GH = 6$ cm, $\angle IGH = 50°$ and $\angle IHG = 50°$
4. Triangle JKL with $\angle JKL = 90°$, $JL = 5$ cm and $KL = 4$ cm
   - Compare all triangles with the vertices ABC. What do you notice? What does this say about two triangles that have three pairs of equal side lengths?
   - Compare all triangles with the vertices DEF. What do you notice? What does this say about two triangles that have two pairs of equal side lengths and the included angles equal?
   - Compare all triangles with the vertices GHI. What do you notice? What does this say about two triangles that have two equal corresponding angles and one corresponding equal length side?
   - Compare all triangles with the vertices JKL. What do you notice? What does this say about two triangles that have one right angle, the hypotenuse and one other corresponding equal length side?

Key ideas

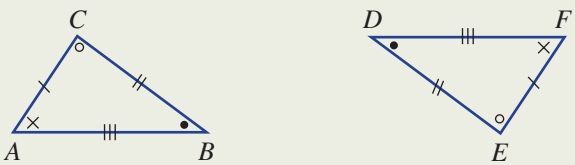
- **Congruent figures** have the same shape and size. They are identical.



Congruent figures Figures that are exactly the same size and shape

Matching sides Sides that are in the same position in two or more shapes

- If triangle ABC (ΔABC) is congruent to triangle DEF (ΔDEF), we write $\Delta ABC \equiv \Delta FDE$. This is called a congruence statement.



Matching sides	Matching angles
$AB = FD$	$\angle A = \angle F$
$BC = DE$	$\angle B = \angle D$
$AC = FE$	$\angle C = \angle E$

- **Matching sides** are opposite equal matching angles.

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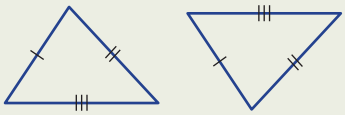
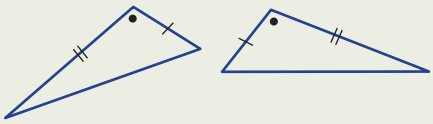
Key ideas

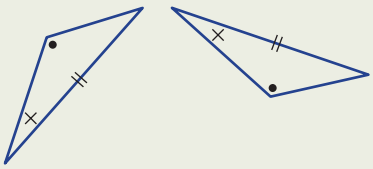
- Tests for triangle congruence:
  - Side, Side, Side (SSS)
    Three pairs of matching sides are equal.



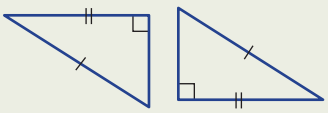
  - Side, Angle, Side (SAS)
    Two pairs of matching sides and the included angle are equal.



  - Angle, Angle, Side (AAS)
    Two angles and any pair of matching sides are equal.



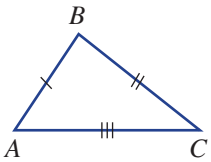
  - Right angle, Hypotenuse, Side (RHS)
    A right angle, the hypotenuse and one other pair of matching sides are equal.

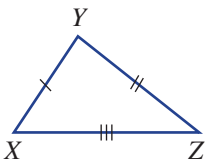


Exercise 8E

UNDERSTANDING AND FLUENCY 1–3, 4–5(½) 3, 4–5(½)

1 These two triangles are congruent.





a Name the vertex on ΔXYZ that matches:
- **i** point B
- **ii** point A
- **iii** point C

b Name the side on ΔXYZ that matches:
- **i** AB
- **ii** AC
- **iii** BC

c Name the angle in ΔABC that matches:
- **i** $\angle X$
- **ii** $\angle Y$
- **iii** $\angle Z$

2 Use the information in the Key Ideas to complete these sentences.
- **a** Congruent figures are exactly the same shape and _____.
- **b** If triangle ABC is congruent to triangle STU, then we write $\Delta ABC \equiv$ _____.
- **c** The short names of the four congruence tests for triangles are SSS, ____, ____ and _____.

3 Write a congruence statement if:
- **a** triangle ABC is congruent to triangle FGH
- **b** triangle DEF is congruent to triangle STU
- **c** triangle AMP is congruent to triangle CBD
- **d** triangle BMW is congruent to triangle SLK

Example: $\Delta DEF \equiv \Delta GHI$

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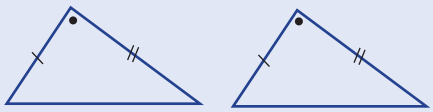
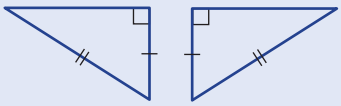
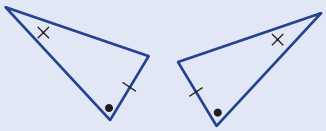
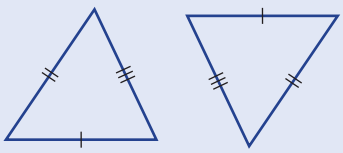
Example 11 Choosing a congruence test

Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

a 

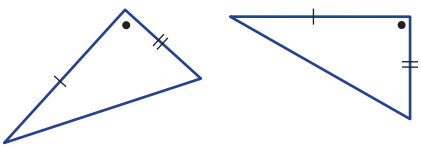
b 

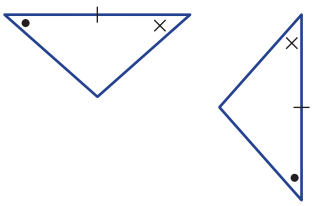
c 

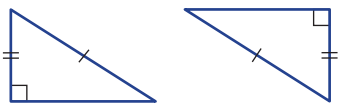
d 

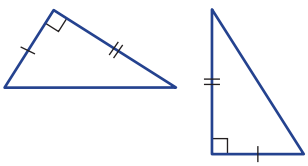
SOLUTION	EXPLANATION
a SAS	Two pairs of matching sides and the included angle are equal.
b RHS	A right angle, hypotenuse and one pair of matching sides are equal.
c AAS	Two angles and a pair of matching sides are equal.
d SSS	Three pairs of matching sides are equal.

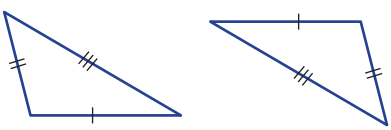
4 Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

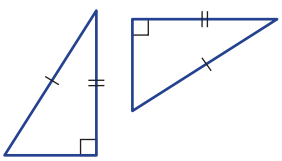
a 

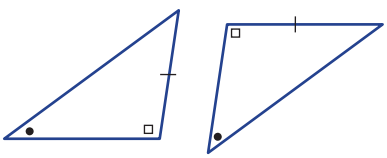
b 

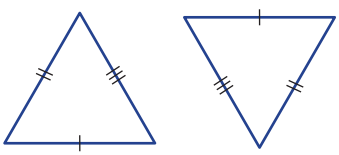
c 

d 

e 

f 

g 

h 

Choose SSS for three matching sides, SAS for two matching sides and the included angle, AAS for two equal angles and one matching side or RHS for a right angle, hypotenuse and one other matching side.

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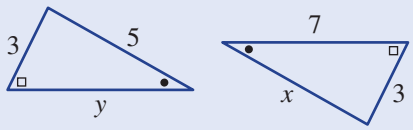
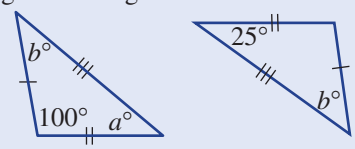
Example 12 Finding missing side lengths and angles using congruence

Find the values of the pronumerals in these pairs of congruent triangles.

a



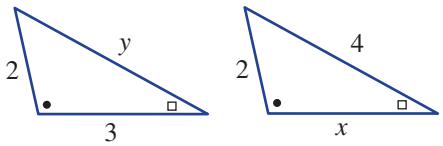
b



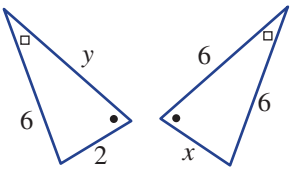
SOLUTION	EXPLANATION
a $x = 5$	The side of length x and the side of length 5 are in matching positions (opposite the □).
$y = 7$	The longest side on both triangles must be equal.
b $a = 25$	The angle marked $a°$ matches the 25° angle in the other triangle.
$b = 180 - 100 - 25$ $= 55$	The sum of three angles in a triangle is 180°.

5 Find the values of the pronumerals in these pairs of congruent triangles.

a



b



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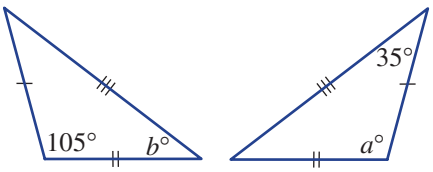

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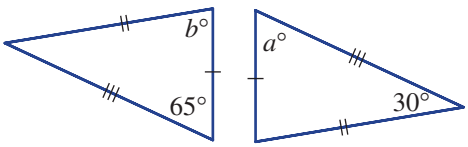

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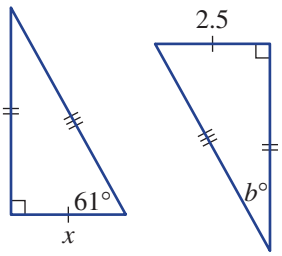

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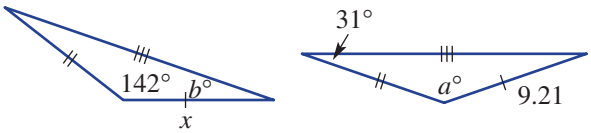

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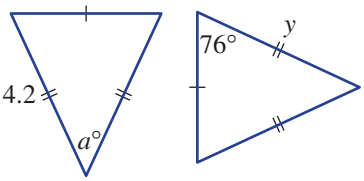

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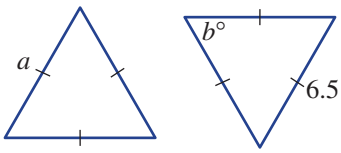

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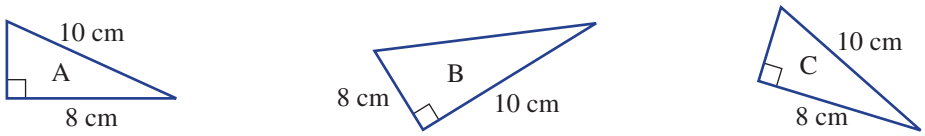
PROBLEM-SOLVING AND REASONING 6, 7 6, 8–10

6 Measurements were taken for three matching triangular tiles and are shown here.



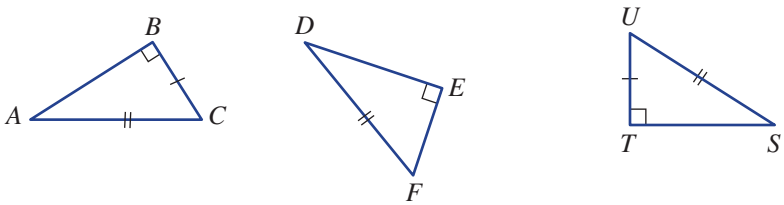
a Which two triangles are congruent?

b What reason (SSS, SAS, AAS, RHS) is used to explain their congruence?

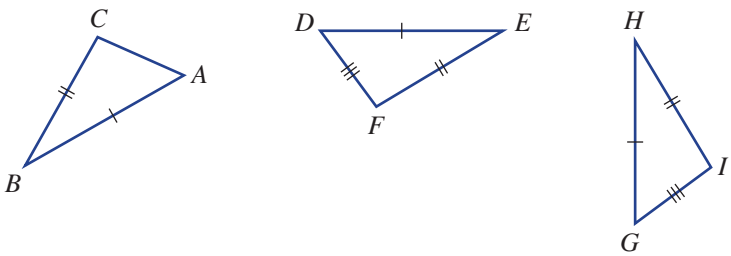
7 For each set of three triangles, choose the two that are congruent. Give a reason (SSS, SAS, AAS or RHS) and write a congruence statement (e.g. $\Delta ABC \equiv \Delta FGH$).

Write matching vertices in the same order.

a



b



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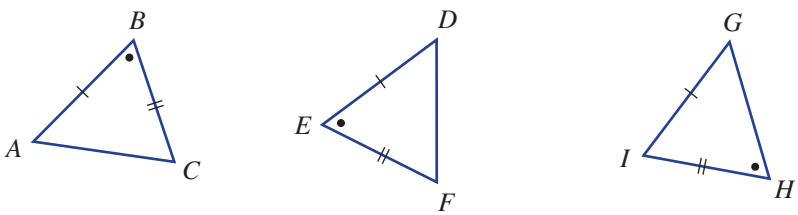

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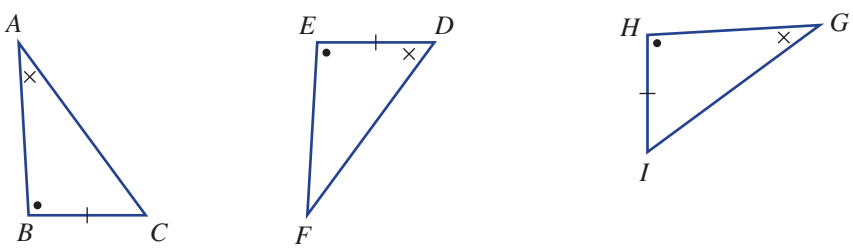

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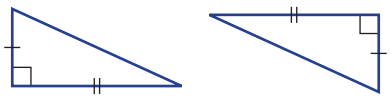
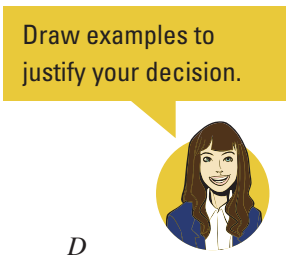
8 Explain why RHS is not the first reason you would use to explain why these two triangles are congruent.



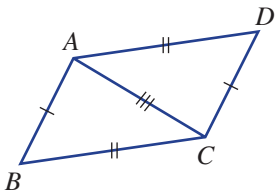
9 Are all triangles with three pairs of equal matching angles congruent? Explain why or why not.



10 *ABCD* is a parallelogram.

a Give the reason why $\Delta ABC \equiv \Delta CDA$.

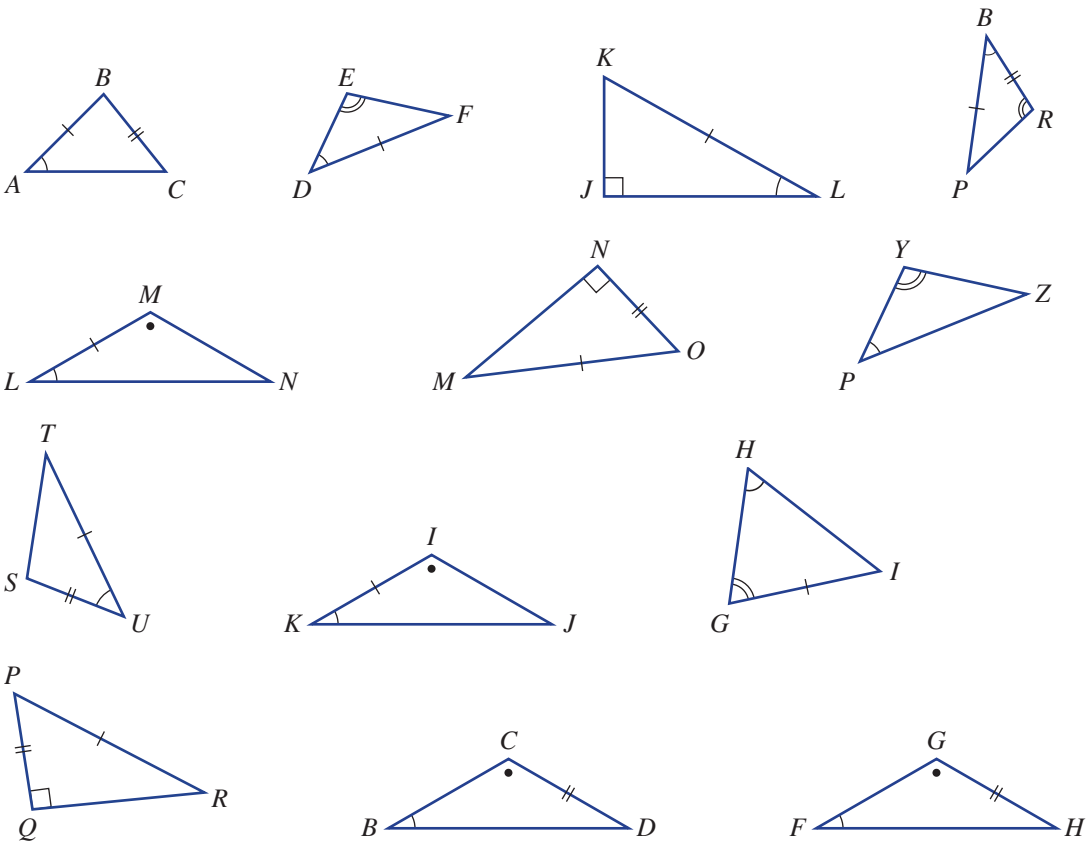
b What does this say about $\angle B$ and $\angle D$?



ENRICHMENT – 11

Fishing for congruence

11 Identify all pairs of congruent triangles from those below. Angles with the same mark are equal.



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8F Enlargement and similar figures

Interactive
Widgets
HOTsheets
Walkthrough

Whenever we look at a map, TV screen, microscope, model, movie screen or computer document we are looking at similar figures of other images.

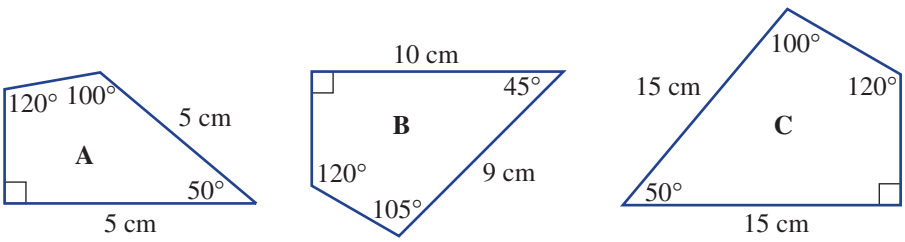
Similar figures have the same shape but not necessarily the same size. If two figures are similar, then one of them can be enlarged or reduced so that it is identical (congruent) to the other. If the scale factor is greater than 1, the image will be larger than the original. If the scale factor is between 0 and 1, the image will be smaller.

Architectural models are useful for communicating design ideas.

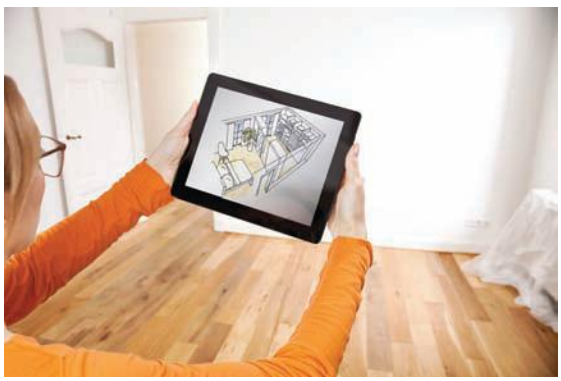
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Are they similar?

Here are three quadrilaterals.



- Which pair of quadrilaterals do you think are similar?
- Why do you think they are similar? Discuss your reasons.
- Give reasons why the other quadrilateral is not similar.



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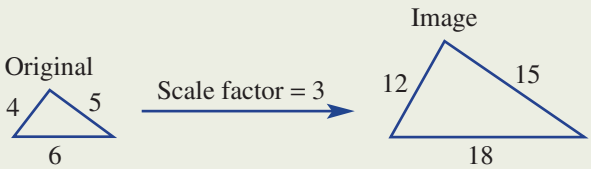
Key ideas

- **Enlargement** is a transformation that involves the increase or decrease in size of an object.
  - The 'shape' of the object is unchanged.
  - Enlargement is done by multiplying all dimensions of a shape by a **scale factor**.

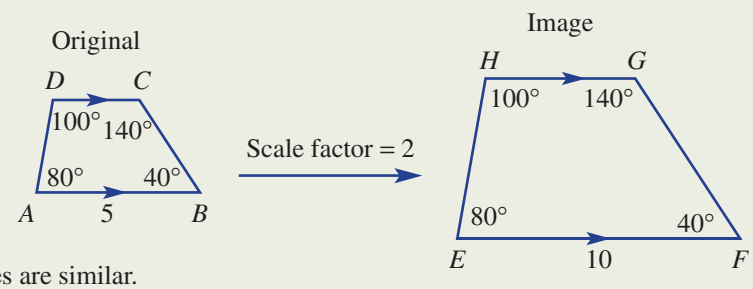


Enlargement A transformation that changes the size of a figure without changing its shape

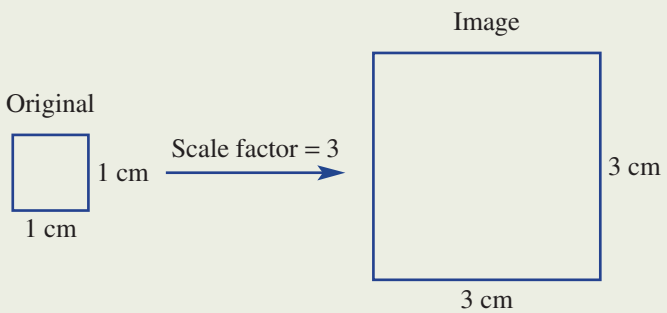
Scale factor The number by which you multiply each side length to enlarge or reduce the size of a shape

Similar figures Figures of the same shape but not the same size

- The scale factor $= \dfrac{\text{image length}}{\text{original length}}$
- If the scale factor is greater than 1, the image will be larger than the original. If the scale factor is between 0 and 1, the image will be smaller. If the scale factor is 1, the image will be congruent.
- Two figures are **similar** if one can be enlarged to be congruent to the other.
  - Matching angles are equal.
  - Pairs of matching sides are in the same proportion or ratio.
- The symbol ||| is used to describe similarity.
  For example: $ABCD \mathbin{|||} EFGH$
  - The letters are usually written in matching order.
  - Scale factor $= \dfrac{EF}{AB} = \dfrac{10}{5} = 2$, so all the sides of the image are twice the length of the original shape.



- All squares are similar.
- All circles are similar.
- All equilateral triangles are similar.



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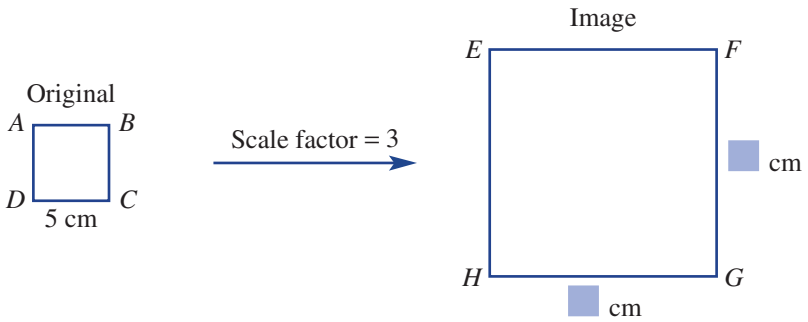
Exercise 8F

UNDERSTANDING AND FLUENCY	1–6	4, 5(½), 6, 7

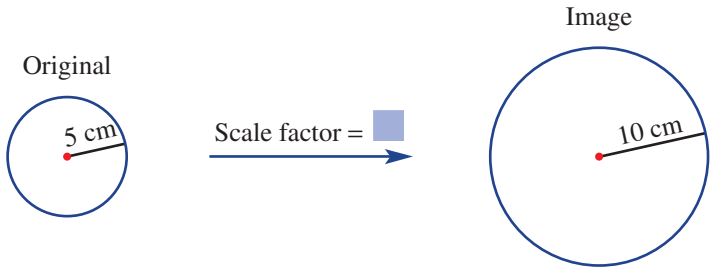
1 Write the missing information, represented by the grey square (■) in your exercise book.

Note: Diagrams are not to scale.

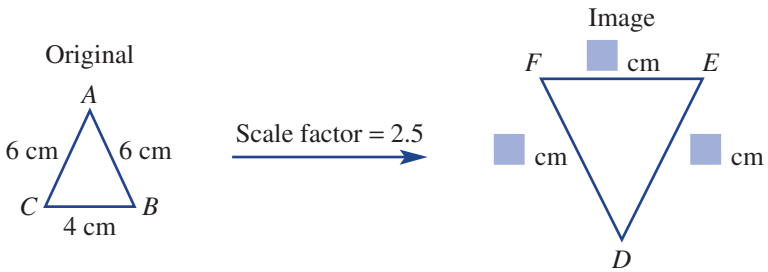
a *ABCD* is a square.



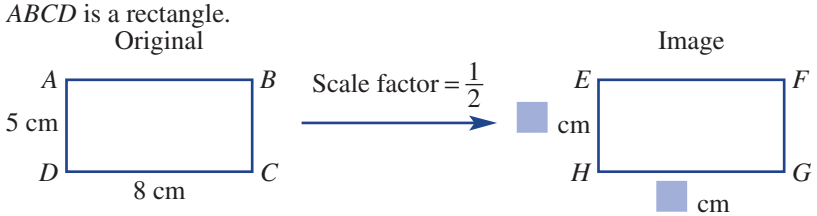
b



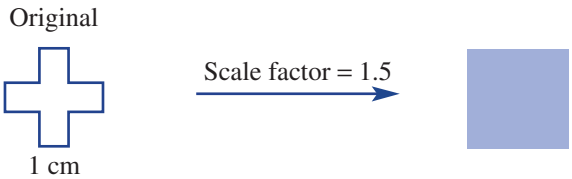
c



d *ABCD* is a rectangle.



e



All side are 1 cm.
All angles are 90°.

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2 The two figures below are similar.

a Name the angle in the larger figure that matches $\angle A$.

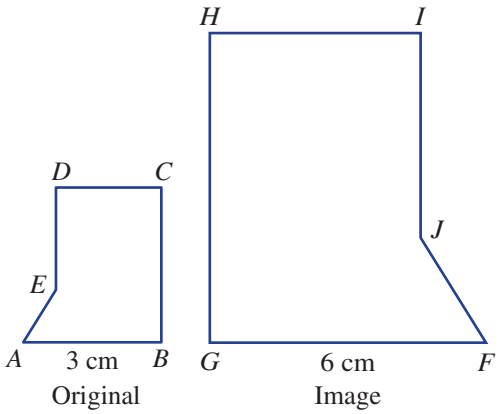
b Name the angle in the smaller figure that matches to $\angle I$.

c Name the side in the larger figure that matches to BC.

d Name the side in the smaller figure that matches to FJ.

e Use FG and AB to find the scale factor.

For part **e**, use Scale factor $= \dfrac{\text{image length}}{\text{original length}}$



3 This diagram shows ΔABC enlarged to give the image $\Delta A'B'C'$.

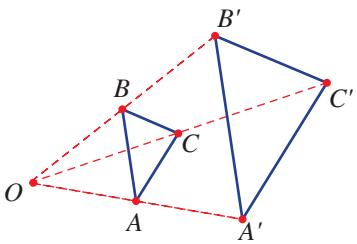
a Measure the lengths OA and OA'. What do you notice?

b Measure the lengths OB and OB'. What do you notice?

c Measure the lengths OC and OC'. What do you notice?

d What is the scale factor?

e Is $A'B'$ twice the length of AB? Measure to check.



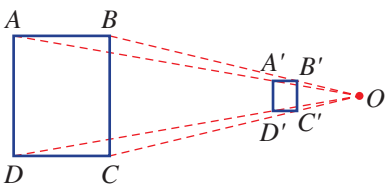
4 This diagram shows rectangle $ABCD$ enlarged (in this case reduced) to rectangle $A'B'C'D'$.

a Measure the lengths OA and OA'. What do you notice?

b Measure the lengths OD and OD'. What do you notice?

c What is the scale factor?

d Compare the lengths AD and $A'D'$. Is $A'D'$ one quarter of the length of AD?



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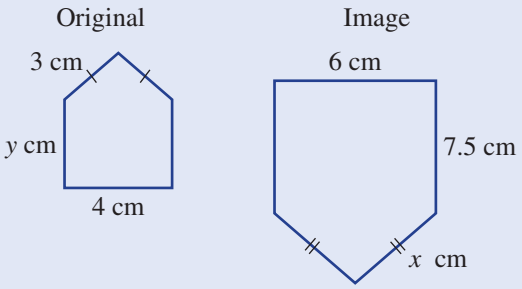
Example 13 Using the scale factor

These figures are similar.

a Find a scale factor.

b Find the value of x.

c Find the value of y.



SOLUTION

a Scale factor $= \frac{6}{4} = 1.5$

b $x = 3 \times 1.5$
$= 4.5$

c $y = 7.5 \div 1.5$
$= 5$

EXPLANATION

Choose two matching sides and use

$$\text{scale factor} = \frac{\text{image length}}{\text{original length}}$$

Multiply by the scale factor to get the length of the side on the larger image.

Divide by the scale factor to get the length of the side on the smaller image.

5 Each of the pairs of figures shown here are similar. For each pair find:

i a scale factor

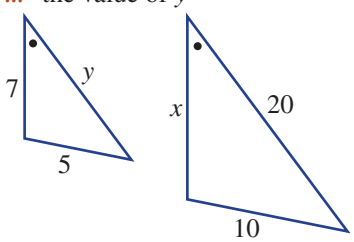
ii the value of x

iii the value of y

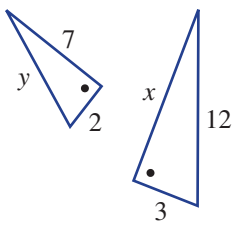
Scale factor $= \frac{\text{larger}}{\text{smaller}}$

Make sure you choose a matching pair of sides.

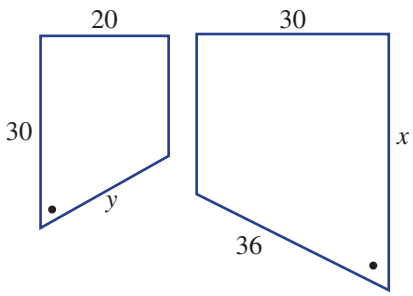
a



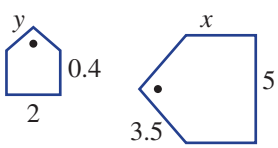
b



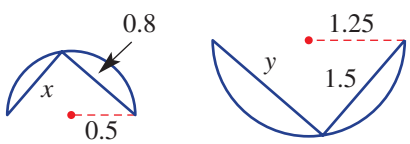
c



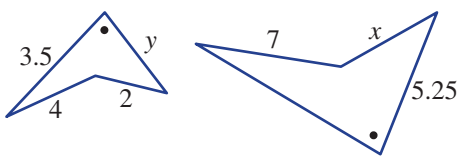
d



e



f



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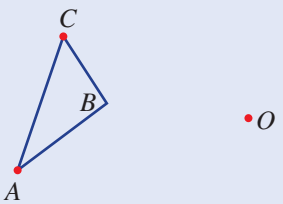
Example 14 Enlarging figures

Copy the given diagram using plenty of space, and use the given centre of enlargement (O) and these scale factors to enlarge ΔABC.

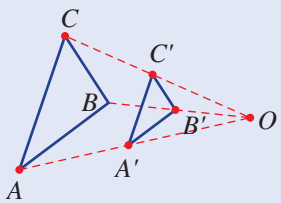
a Scale factor $\frac{1}{2}$

b Scale factor 3

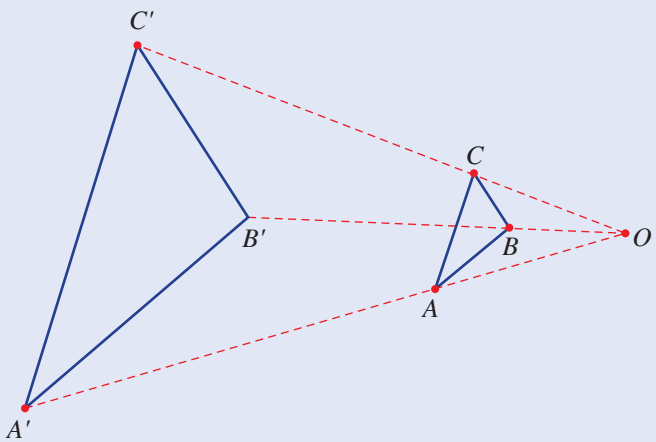


SOLUTION

a



b



EXPLANATION

Connect dashed lines between O and the vertices A, B and C.

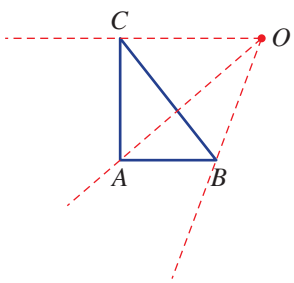
Since the scale factor is $\frac{1}{2}$, place A' so that OA' is half of OA. Repeat for B' and C'. Join vertices A', B' and C'.

Draw dashed lines from O through A, B and C. Place A' so that OA' is 3 times OA. Repeat for B' and C' and form $\Delta A'B'C'$.

6 Copy the given diagram, leaving plenty of space around it. Use the given centre of enlargement (O) and given scale factors to enlarge ΔABC.

a Scale factor $\frac{1}{3}$

b Scale factor 2



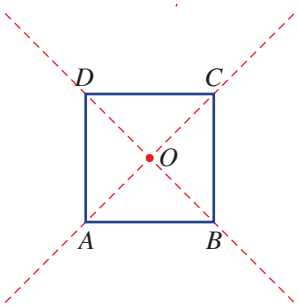
7 This diagram includes a square with centre O and vertices $ABCD$.

a Copy the diagram, leaving plenty of space around it.

b Enlarge square $ABCD$ by these scale factors and draw the image. Use O as the centre of enlargement.

i $\frac{1}{2}$

ii 1.5



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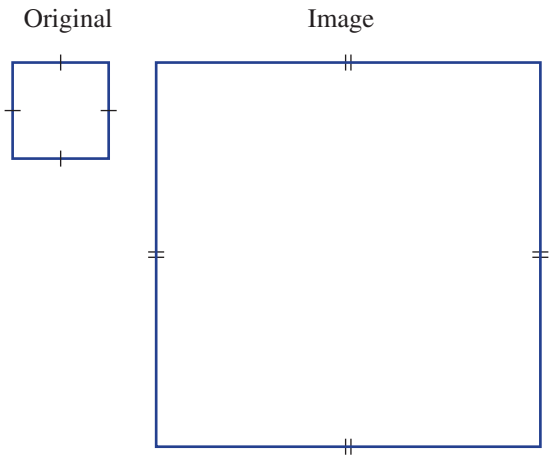
PROBLEM-SOLVING AND REASONING 8, 9 8, 10, 11

8 A square is enlarged by a scale factor of 4.

a Are the internal angles the same for both the original and the image?

b If the side length of the original square was 2 cm, what would be the side length of the image square?

c If the side length of the image square was 100 m, what would be the side length of the original square?



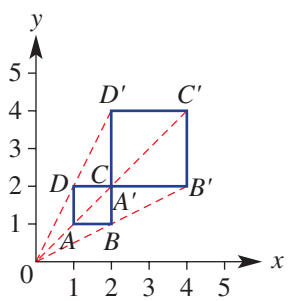
9 A photo of a sunflower is enlarged by a scale factor of 2. If the larger flower has a diameter of 7.4 cm, what is the diameter of the smaller flower?

10 These diagrams show a shape and its image after enlargement. For each part, find the:

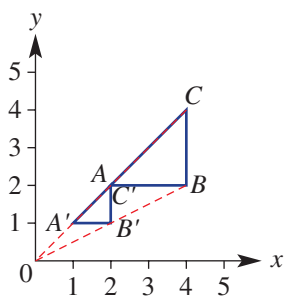
i scale factor

ii coordinates (x, y) of the centre of enlargement

a

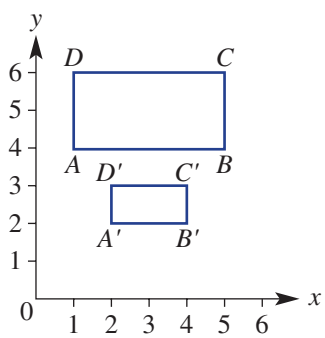


b

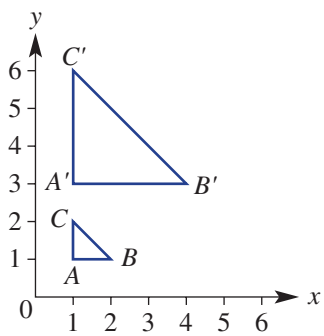


For parts **c** and **d**, first draw dashed red lines connecting A with A' etc.

c



d



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11 Explain why:

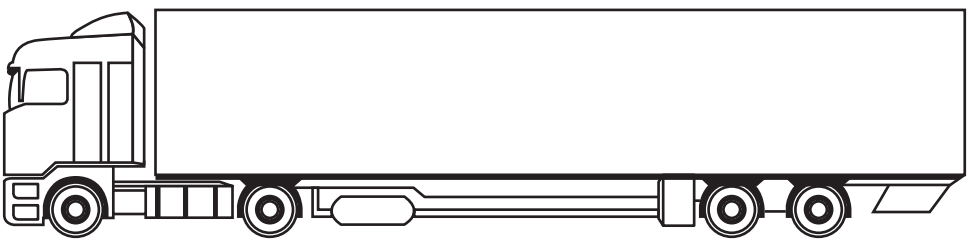
- **a** any two squares are similar
- **b** any two equilateral triangles are similar
- **c** any two rectangles are not necessarily similar
- **d** any two isosceles triangles are not necessarily similar

ENRICHMENT	–	12, 13

Actual lengths from drawings and maps

12 The container of this truck is 12.7 m long.

- **a** Measure the length of the container on the truck in the drawing.
- **b** Measure the height of the container on the truck in the drawing.
- **c** Estimate the actual height of the container on the truck.



13 A map has a scale ratio of 1 : 50000.

- **a** What length on the ground is represented by 2 cm on the map?
- **b** What length on the map is represented by 12 km on the ground?

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8G Similar triangles

Interactive

Widgets

HOTsheets

Walkthrough

As with congruent triangles, there are mathematical tests that can be used to see if two triangles are similar.

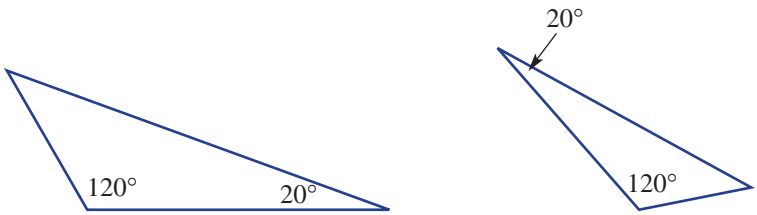
Once we know that two triangles are similar, we can work out any missing angles and lengths.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Do you need to know all six angles?

If all three angles are the same inside two triangles, then we know that the triangles are similar.

Here is a pair of triangles.



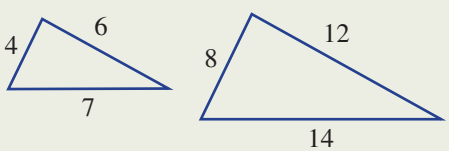
- Do you think they are similar?
- What is the unknown angle in each triangle?
- How many pairs of angles do you need to know?

Key ideas

- Two triangles are similar if:
  - matching angles are equal
  - matching sides are in proportion (the same ratio).
- The **similarity statement** for two similar triangles ΔABC and ΔDEF is:
  - $\Delta ABC \,|||\, \Delta DEF$

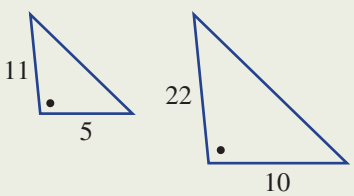
  Letters are usually written in matching order so $\angle A$ corresponds to $\angle D$ etc.
- **Tests for similar triangles.** (Not to be confused with the congruence tests for triangles.)
  - If the three sides of a triangle are proportional to the three sides of another triangle, then the two triangles are similar.

    $\frac{12}{6} = \frac{8}{4} = \frac{14}{7} = 2$ (i.e. scale factor is 2)



  - If two sides of a triangle are proportional to two sides of another triangle, and the included angles are equal, then the two triangles are similar.

    $\frac{22}{11} = \frac{10}{5} = 2$ (i.e. scale factor is 2)



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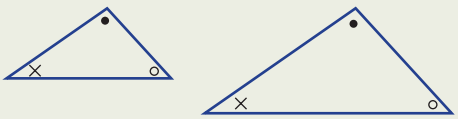
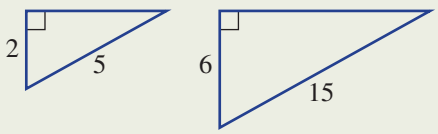
Key ideas

- If two angles of a triangle are equal to two angles of another triangle, then the two triangles are similar.
  (Note: if there are two equal pairs, then the third pair must be equal.)



- If the hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and a second side of another right-angled triangle, then the two triangles are similar.

  $\frac{15}{5} = \frac{6}{2} = 3$ (i.e. scale factor is 3)

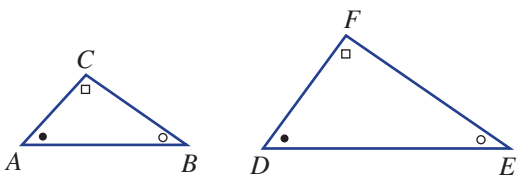


■ There are no abbreviations such as AAA in the NSW Syllabus for these tests of similarity.

Exercise 8G

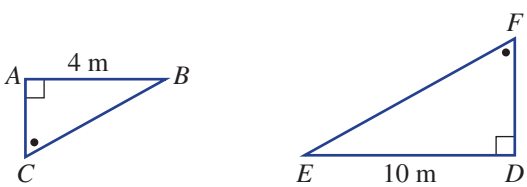
UNDERSTANDING AND FLUENCY	1, 2, 3(½), 4–6	2, 3–4(½), 5–7

1 These two triangles are similar.



a Which vertex on ΔDEF matches vertex B on ΔABC?
b Which vertex on ΔABC matches vertex F on ΔDEF?
c Which side on ΔDEF matches side AC on ΔABC?
d Which side on ΔABC matches side EF on ΔDEF?
e Which angle on ΔABC matches $\angle D$ on ΔDEF?
f Which angle on ΔDEF matches $\angle B$ on ΔABC?

2 What is the scale factor on this pair of similar triangles which enlarges ΔABC to ΔDEF?



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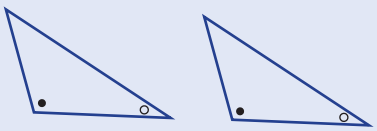
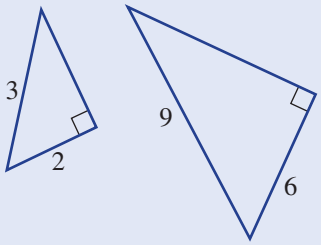
Example 15 Choosing a similarity test for triangles

Choose the similarity test that proves that these pairs of triangles are similar.

a



b



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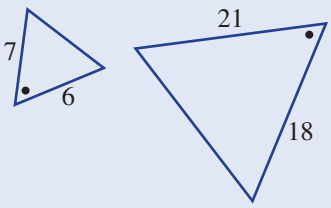

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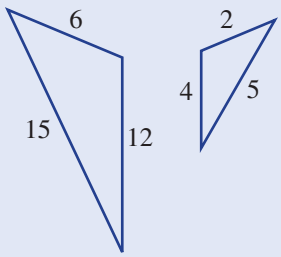

SOLUTION	EXPLANATION
a Two pairs of equal angles	If two pairs of angles are equal, then the third pair must also be equal.
b The hypotenuses and another pair of sides are in the same ratio.	Both are right-angled triangles and the hypotenuses and another pair of sides are in the same ratio $\left(\frac{9}{3} = \frac{6}{2} = 3\right)$.
c The sides adjacent to equal angles are in the same ratio.	Two pairs of matching sides are in the same ratio $\left(\frac{21}{7} = \frac{18}{6} = 3\right)$ and the included angles are equal.
d Three pairs of sides are in the same ratio.	Three pairs of matching sides are in the same ratio $\left(\frac{15}{5} = \frac{12}{4} = \frac{6}{2} = 3\right)$.

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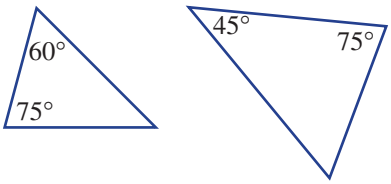
3 Explain why these pairs of triangles are similar.

a



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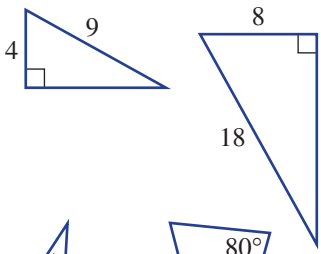

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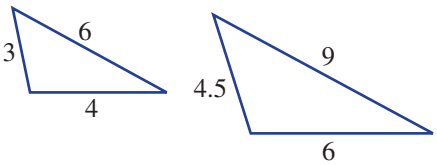

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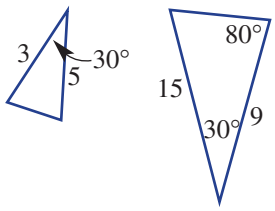

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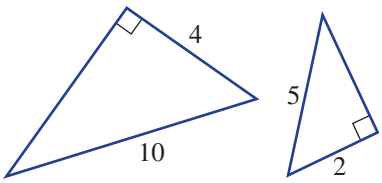
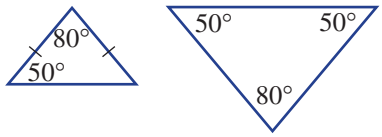
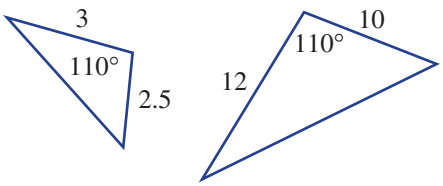


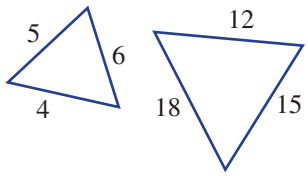
f



g

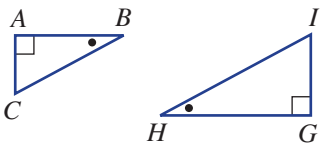


h



4 Write similarity statements for these pairs of similar triangles. Write letters in matching order.

a



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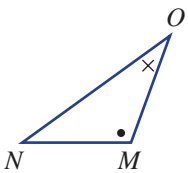
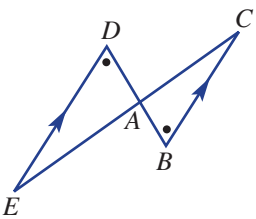
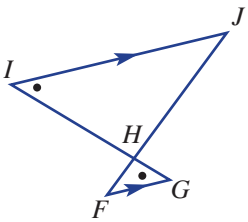


Here is an example of a similarity statement: $\Delta ABC \parallel\!| \Delta DEF$

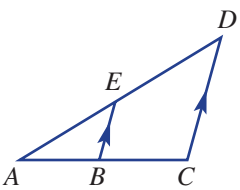
c



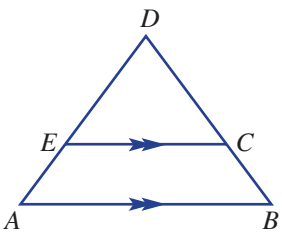
d



e



f



For parts **e** and **f**, use angles in parallel lines.

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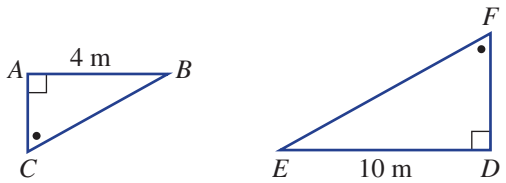
5 What is the scale factor on this pair of similar triangles that enlarges ΔABC to ΔDEF?



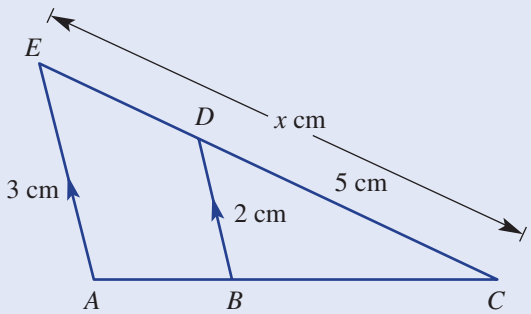
Example 16 Finding a missing length using similarity

For this pair of triangles:

a give a reason why the two triangles are similar

b find the scale factor

c find the value of x



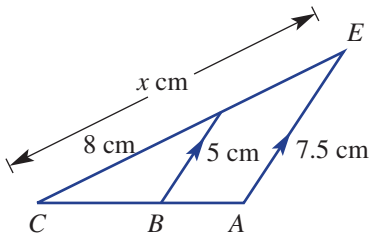
SOLUTION	EXPLANATION
a Two angles in ΔCDB are equal to two angles in ΔCEA.	$\angle EAC = \angle DBC$, since AE is parallel to BD and $\angle C$ is common to both triangles. (Also $\angle AEC = \angle BDC$, since AE is parallel to BC.)
b Scale factor $= \frac{3}{2} = 1.5$	$\frac{AE}{BD} = \frac{3}{2}$
c $x = 5 \times 1.5$ $= 7.5$	Multiply CD by the scale factor to find the length of the corresponding length CE.

6 For this pair of triangles:

a give a reason why the two triangles are similar

b find the scale factor

c find the value of x

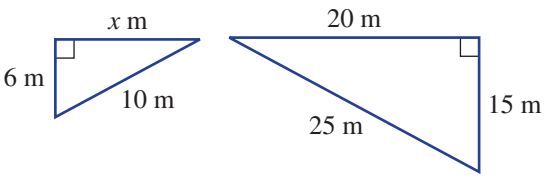


7 For this pair of triangles:

a give a reason why the two triangles are similar

b find the scale factor

c find the value of x



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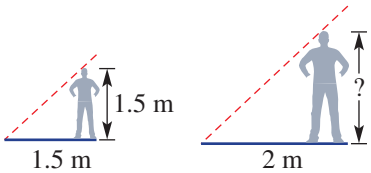
PROBLEM-SOLVING AND REASONING 8, 9 8, 10, 11

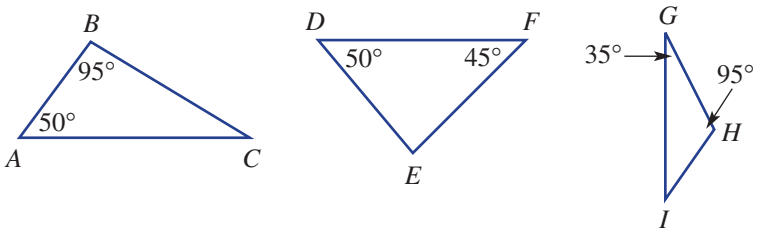
8 Two people create a shadow in the sun, as shown. What is the height of the taller person?



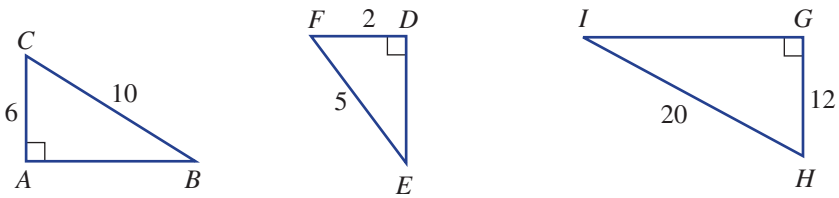
9 Name the triangle that is not similar to the other two in each group of three triangles.

Name a triangle like this: ΔABC or ΔSTU.

a



b



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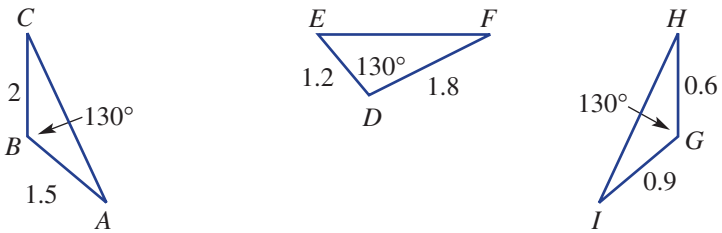

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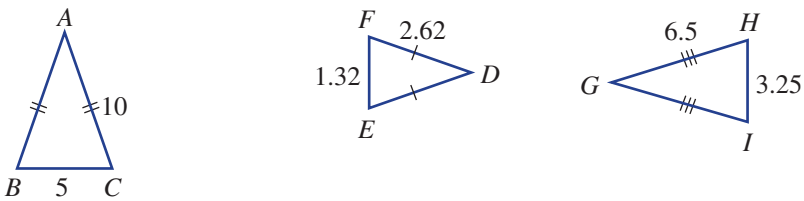


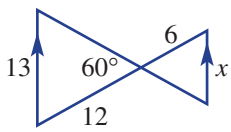
10 For each pair of similar triangles:

i give a reason why the two triangles are similar

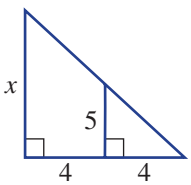
ii find the value of x

Consider angle properties in parallel lines.

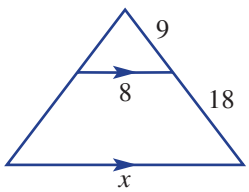
a



b



c



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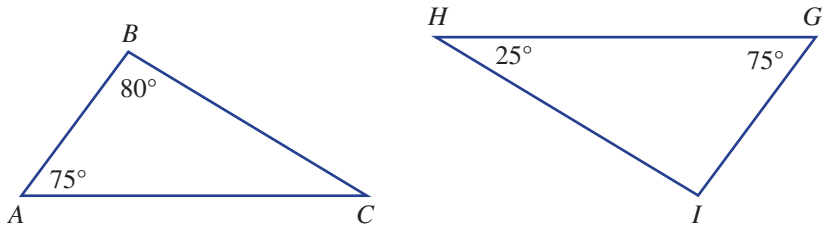
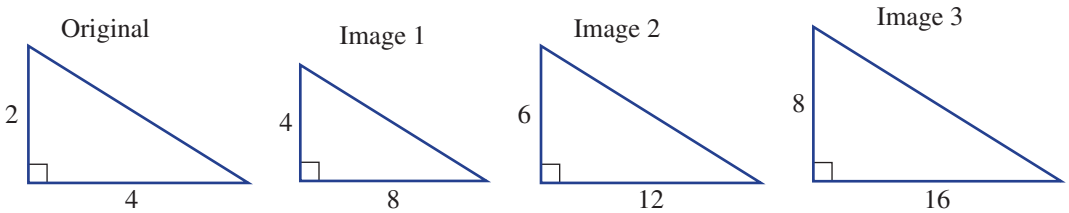
11 Give reasons why these two triangles are similar.



ENRICHMENT – 12

Area ratio

12 Consider these three similar triangles (not drawn to scale).



a Copy and complete this table, comparing each image to the original.

Triangle	Original	Image 1	Image 2	Image 3
Length scale factor	1	2		
Area	4	16		
Area scale factor	1			

b What do you notice about the area scale factor compared to the length scale factor?

c What would the area scale factor be if the length scale factor was:

i 10? **ii** 20? **iii** 100?

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8H Applying similar triangles

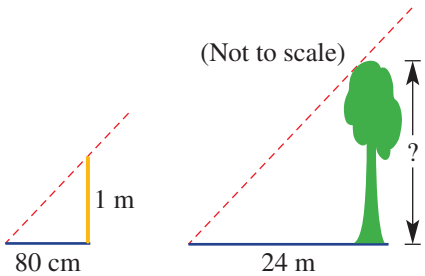
Interactive
Widgets
HOTsheets
Walkthrough

Similar triangles can be used in many mathematical and practical problems. If two triangles are proved to be similar, then the properties of similar triangles can be used to find missing lengths or unknown angles. The approximate height of a tall object, or the width of a projected image, can be found using similar triangles.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: How high is it?

A 1-metre vertical stick gives a shadow 80 cm long. At the same time, a tall tree has a shadow 24 m long.



- How can two pairs of equal angles be used to explain the two similar triangles?
- What is the scale factor?
- What is the height of the tree?

The height of the tallest building in the world, the Burj Khalifa in Dubai, can be verified using similar triangles.

Key ideas

- To apply similarity in practical problems involving triangles:
  - Prove that two triangles are similar by using one of the four tests.
  - Find a scale factor.
  - Find the value of any unknowns.

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Exercise 8H

UNDERSTANDING AND FLUENCY	1–6	3–7

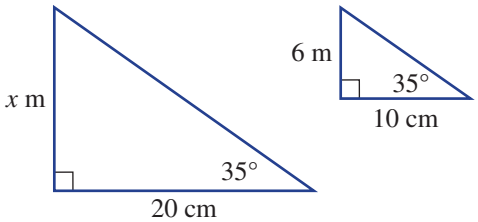
1 Consider the following triangles.

a How many pairs of equal matching angles are given?

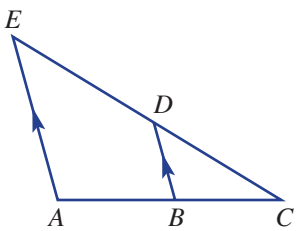
b Which test explains why they are similar?

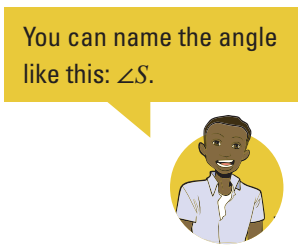
c What is the scale factor if the smaller triangle is enlarged to the size of the larger triangle?

d Find the value of x.



2 In this diagram, name the angle that is common to both ΔACE and ΔBCD.

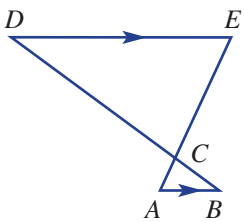




3 In this diagram name the:

a pair of vertically opposite angles

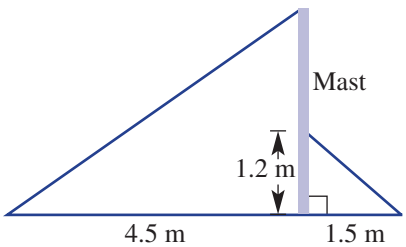
b two pairs of equal alternate angles



4 Two similar triangles are created by cables supporting a yacht's mast.

a Find a scale factor for the two triangles.

b Find the height of the mast.

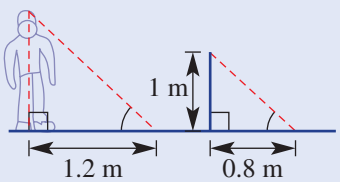


Example 17 Applying similarity

Chris' shadow is 1.2 m long while a 1 m vertical stick has a shadow 0.8 m long.

a Give a reason why the two triangles are similar.

b Determine Chris' height.



SOLUTION	EXPLANATION
a All angles are the same.	The Sun's rays will pass over Chris and the stick and hit the ground at approximately the same angle.
b Scale factor $= \frac{1.2}{0.8} = 1.5$	First find the scale factor.
$\therefore$ Chris' height $= 1 \times 1.5$ $= 1.5$ m	Multiply the height of the stick by the scale factor to find Chris' height.

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5 A tree's shadow is 20 m long while a 2 m vertical stick has a shadow 1 m long.

a Give a reason why the two triangles contained within the objects and their shadows are similar.

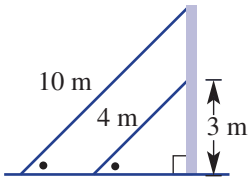
b Find the height of the tree.

Construct a diagram as in Example 17.

6 Two cables support a steel pole at the same angle as shown. The two cables are 4 m and 10 m in length while the shorter cable reaches 3 m up the pole.

a Give a reason why the two triangles are similar.

b Find the height of the pole.

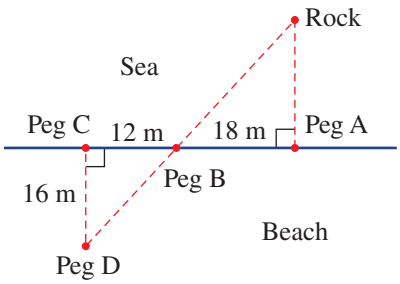


7 Ali is at the beach and decides to estimate how far an exposed rock is from the seashore. He places four pegs in the sand as shown, and measures the distance between them.

a Why do you think Ali has placed the four pegs in the way that is shown in the diagram?

b Are the two triangles similar? Which test could be used?

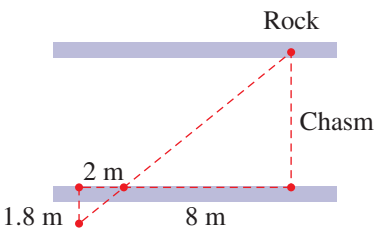
c How far is the rock from the beach?



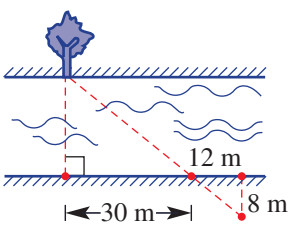
PROBLEM-SOLVING AND REASONING 8, 9 8, 10

8 A deep chasm has a large rock sitting on its side, as shown. Find the width of the chasm.

Don't forget to find the scale factor.



9 Joanne wishes to determine the width of the river shown without crossing it. She places four pegs as shown. Calculate the river's width.



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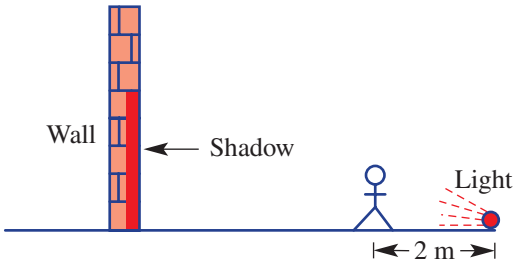
10 A person 1.8 m tall stands in front of a light that sits on the floor.

The person casts a shadow on the wall behind them.



a How tall will the shadow be if the distance between the wall and the light is:

i 4 m?

ii 10 m?

iii 3 m?

b How tall will the shadow be if the distance between the wall and the person is:

i 4 m?

ii 5 m?

c Find the distance from the wall to the person if the shadow is:

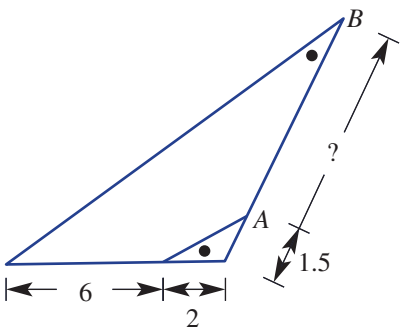
i 5.4 m

ii 7.2 m

ENRICHMENT – 11, 12

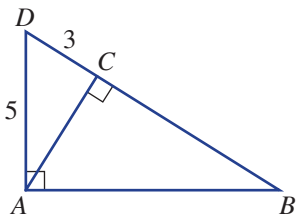
Visual challenge

11 Find the length AB in this diagram if the two triangles are similar.



Remember to line up corresponding sides for the scale factor.

12 In this diagram, AC is perpendicular (at right angles) to BD and ΔABD is right-angled.



a How many similar triangles are there?

b Find the lengths:

i BD

ii AC

iii AB

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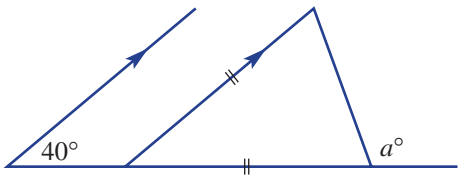
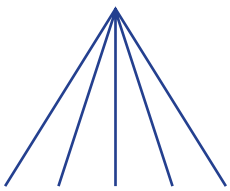
Puzzles and challenges

1 Use 12 match sticks to make 6 equilateral triangles.

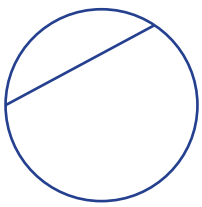
2 Find the value of a in the diagram on the below.



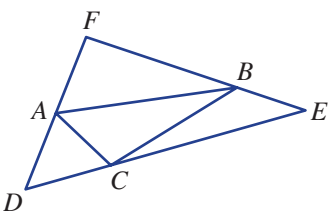
3 How many acute angles are there in this diagram?



4 A circle is divided using chords (one chord is shown here). What is the maximum number of regions that can be formed if the circle is divided with 4 chords?



5 Explore (using dynamic geometry software) where the points A, B and C should be on the sides of acute-angled ΔDEF so that the perimeter of ΔABC is a minimum.



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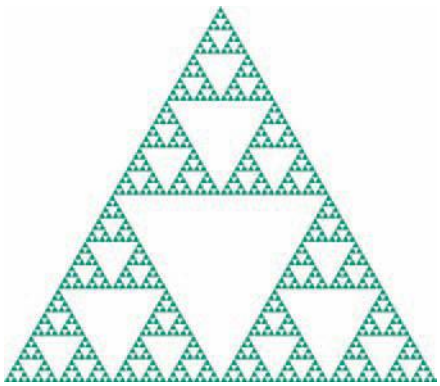
6 The Sierpinski triangle shown is a mathematically generated pattern. It is created by repeatedly enlarging triangles by a factor of $\frac{1}{2}$. The steps are:

- Start with an equilateral triangle, as in figure 1.
- Enlarge the triangle by a factor of $\frac{1}{2}$.
- Arrange three copies of the image, as in figure 2.
- Continue repeating steps 2 and 3 with each triangle.



a Make a large copy of figures 1–3, then draw the next two figures in the pattern.

Figure 1

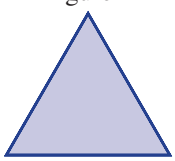


Figure 2

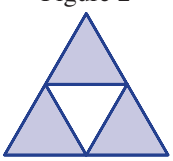
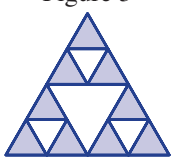


Figure 3



b If the original triangle (figure 1) had side length l, what is the side length of the smallest triangle in:

i figure 2?

ii figure 3?

iii figure 8 (assuming figure 8 is the 8th diagram in the pattern)?

c What fraction of the area is shaded in:

i figure 2?

ii figure 3?

iii figure 6 (assuming figure 6 is the 6th diagram in the pattern)?

d The Sierpinski triangle is one where the process of enlargement and copying is continued forever. What is the shaded area of a Sierpinski triangle?

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Chapter summary

Properties of geometrical figures

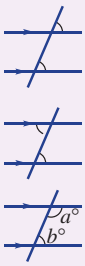
Angles

Acute $0° < \theta < 90°$
Right $90°$
Obtuse $90° < \theta < 180°$
Straight $180°$
Reflex $180° < \theta < 360°$
Revolution $360°$
Complementary angles sum to $90°$.
Supplementary angles sum to $180°$.
Vertically opposite angles are equal.

Parallel lines

If the lines are parallel:

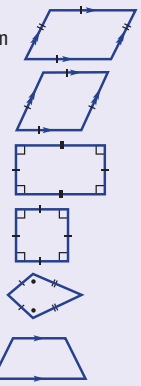
- corresponding angles are equal
- alternate angles are equal
- co-interior angles are supplementary $a + b = 180$



Quadrilaterals

Angle sum = $360°$

parallelogram
rhombus
rectangle
square
kite
trapezium



Congruent triangles

These are identical in shape and size. They may need to be rotated or reflected. We write $\Delta ABC \equiv \Delta DEF$

congruent to

Tests for congruence:
(SSS) – three pairs of matching sides equal
(SAS) – two pairs of matching sides and included angle are equal
(AAS) – two angles and any pair of matching sides are equal
(RHS) – a right angle, hypotenuse and one other pair of matching sides are equal

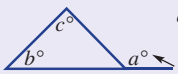
Polygons

The sum of the interior angles in a polygon with n sides is $S = (n - 2) \times 180$. Regular polygons have all sides and angles equal.

Triangles

Sum of angles is $180°$.
Exterior angle equals sum of two opposite interior angles

$a = b + c$



Types:
Acute angled – all angles $< 90°$
Obtuse angled – 1 angle $> 90°$
Right angled – 1 angle $90°$
Equilateral – all angles $60°$ – all sides are equal
Isosceles – 2 angles and 2 sides are equal
Scalene – all sides and angles are different sizes

Similar figures

These are the same shape but different size.

Scale factor = $\dfrac{\text{image length}}{\text{original length}}$

Similar triangles

Similar triangles have corresponding angles equal and corresponding sides are in the same ratio.
We say $\Delta ABC \,|||\, \Delta DEF$

Tests for similar triangles:

- three pairs of matching sides in the same ratio
- two pairs of matching sides in the same ratio and included angle equal
- all three pairs of matching angles are equal; two pairs is enough to prove this
- the hypotenuses of right-angled triangles and another pair of matching sides are in the same ratio

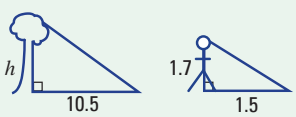
Applying similar triangles

In practical problems:

- look to identify and prove pairs of similar triangles
- find a scale factor
- use this to find the value of any unknowns.

e.g. The shadow cast by a tree is 10.5 m while a person 1.7 m tall has a 1.5 m shadow. How tall is the tree?

The triangles are similar.
Scale factor = $\dfrac{10.5}{1.5} = 7$
$\therefore h = 1.7 \times 7$
$= 11.9$ m
Tree is 11.9 m tall.



(Not to scale)

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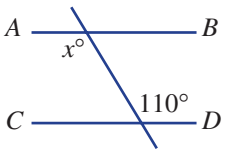
Multiple-choice questions

1 The angle that is supplementary to an angle of 55° is:

A 35° **B** 55° **C** 95°
D 125° **E** 305°

2 AB is parallel to CD. The value of x is:

A 110 **B** 70 **C** 20
D 130 **E** 120

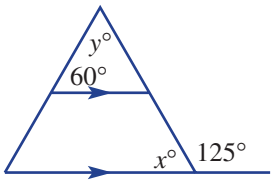


3 If two angles in a triangle are 30° and 80°, then the third angle is:

A 110° **B** 70° **C** 90°
D 80° **E** 30°

4 The values of x and y in the diagram are:

A $x = 35, y = 85$
B $x = 45, y = 45$
C $x = 50, y = 60$
D $x = 55, y = 65$
E $x = 65, y = 55$



5 Using $S = (n - 2) \times 180$, the sum of the interior angles in a hexagon is:

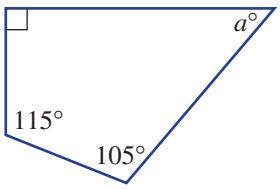
A 1078° **B** 360° **C** 720°
D 900° **E** 540°

6 The quadrilateral with all sides equal, two pairs of opposite parallel sides and no right angles is a:

A kite **B** trapezium **C** parallelogram
D rhombus **E** square

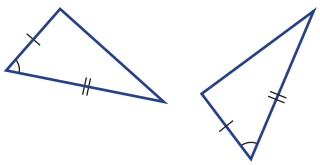
7 The value of a in the diagram is:

A 45
B 310
C 75
D 60
E 50



8 The abbreviated reason for congruence in the two triangles shown is:

A AA
B SAS
C SSS
D AAS
E RHS



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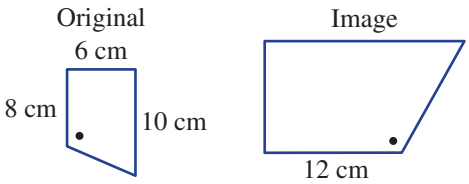
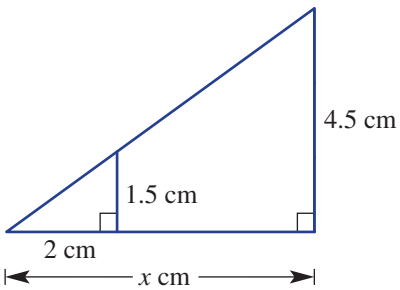
9 The scale factor in the two similar figures shown that enlarges the original figure to its image is:



A $\frac{2}{3}$
B 2
C 1.2
D 1.5
E 0.5

10 The value of x in the diagram is:

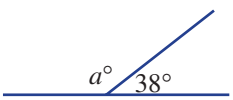


A 6
B 9
C 10
D 8
E 7.5

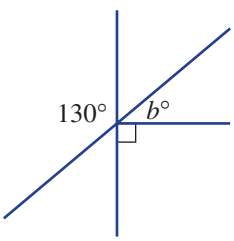
Short-answer questions

1 Find the value of the pronumerals.

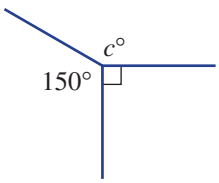
a



b

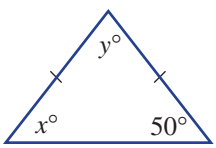


c

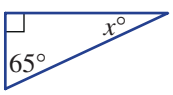


2 Name the following triangles and find the value of the pronumerals.

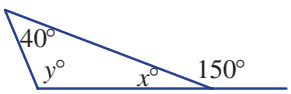
a



b

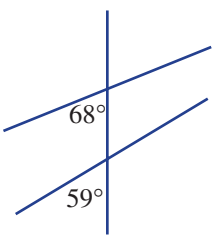


c

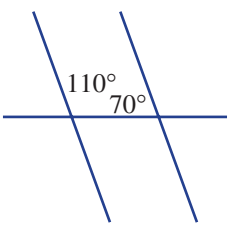


3 Decide whether the following contain a pair of parallel lines. Give a reason.

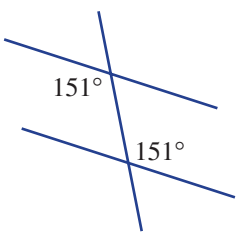
a



b



c



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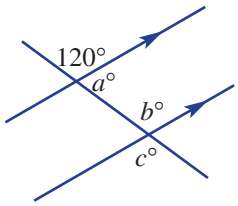
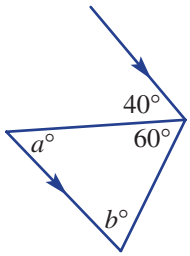
4 State the values of the pronumerals.

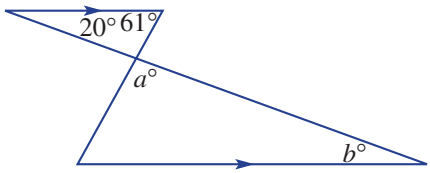
a



b



c



5 Which of the six 'special' quadrilaterals *definitely* have:

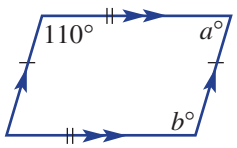
a diagonals of equal length?

b diagonals intersecting at right angles?

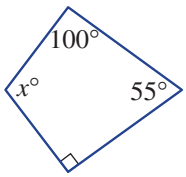
c equal opposite sides?

6 Find the value of each pronumeral in the following quadrilaterals.

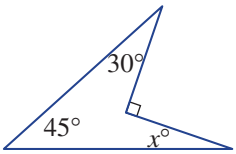
a



b

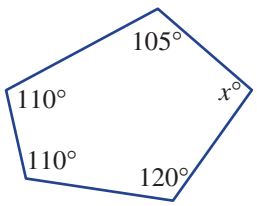


c



7 Find the value of the pronumerals in these polygons. Use $S = (n - 2) \times 180$ for the angle sum.

a



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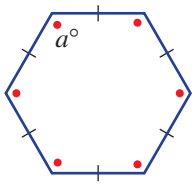

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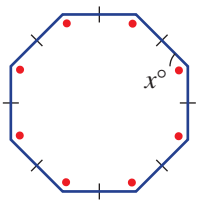
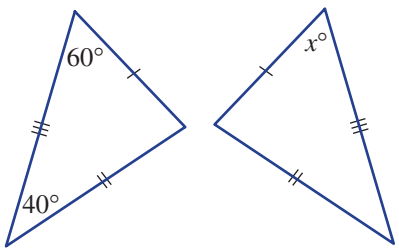


8 Determine whether each pair of triangles is congruent. If congruent, give the abbreviated reason (SSS, SAS, AAS or RHS) and state the value of any pronumerals.

a



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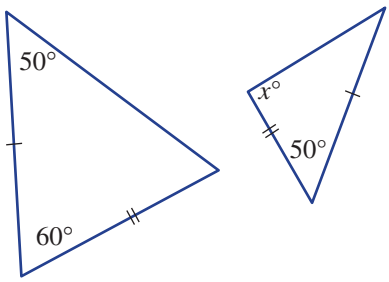

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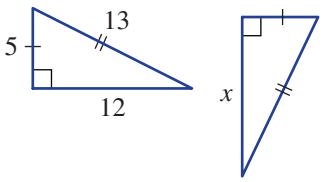
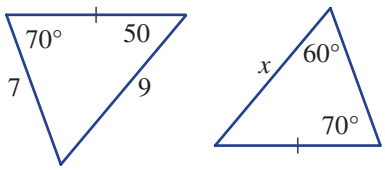


d



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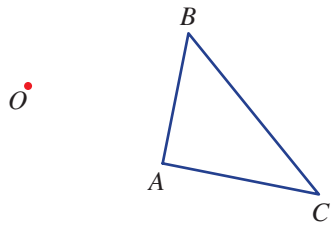
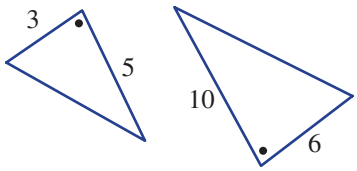
9 Copy the given diagram, allowing plenty of space. Using the centre of enlargement (O) and a scale factor of 3, enlarge ΔABC.



10 Determine whether the following pairs of triangles are similar, and state the similarity test that proves they are similar.

a



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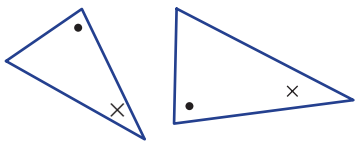

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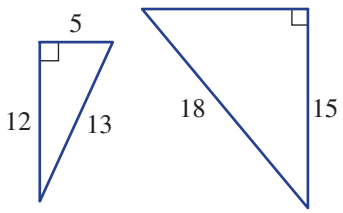

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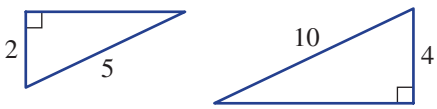
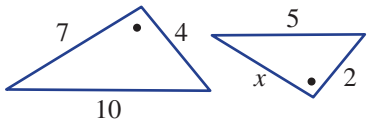


11 For the following pairs of similar triangles, find the value of x.

a



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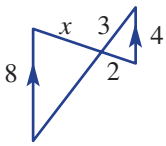

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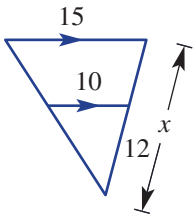
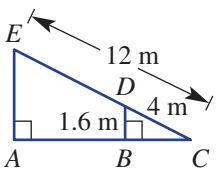


12 A conveyor belt loading luggage onto a plane is 12 m long. A vertical support 1.6 m high is placed under the belt. It is 4 m along the conveyor belt, as shown in the diagram.

a Find a scale factor for the two similar triangles.

b Find the height (AE) of the luggage door above the ground.



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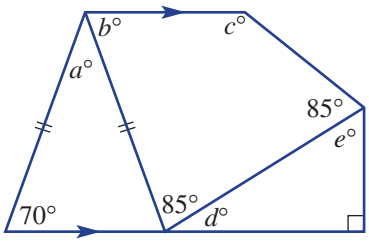
Extended-response questions

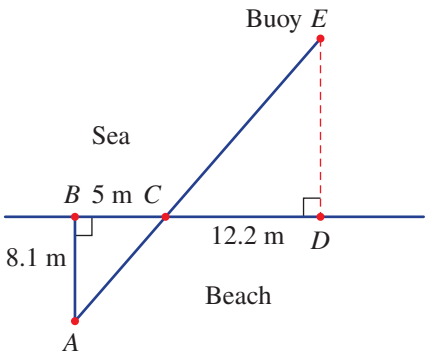
1 In this complicated diagram, there are two triangles and one quadrilateral.



a Name the types of triangles.

b Find the values of the pronumerals in alphabetical order. Give a reason at each step.

2 A buoy (E) is floating in the sea at some unknown distance from the beach, as shown. The points A, B, C and D are measured and marked out on the beach, as shown.



a Name the angle that is vertically opposite to $\angle ACB$.

b Explain, with reasons, why $\Delta ABC \parallel\!| \Delta EDC$.

c Find the distance from the buoy to the beach (ED) to one decimal place.

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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

9 Quadratic expressions and algebraic fractions

What you will learn

9A Reviewing algebra REVISION
9B Expanding binomial products
9C Expanding perfect squares EXTENSION
9D Difference of two squares EXTENSION
9E Factorising algebraic expressions
9F Simplifying algebraic fractions: multiplication and division
9G Simplifying algebraic fractions: addition and subtraction

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NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: ALGEBRAIC TECHNIQUES

Outcomes
A student generalises number properties to operate with algebraic expressions.
(MA4–8NA)
A student simplifies algebraic fractions, and expands and factorises quadratic expressions.
(MA5.2–6NA)

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Shopping with the distributive law

Suppose that you go to a shop that is having a 20%-off sale on all stock. You see three items with full retail price labels of \$38, \$42 and \$80.

To work out the total price of the items, you could reduce the cost of each item by 20%, then add them up to get the total. Another way to work out the cost would be to first add up all the retail prices, then take 20% off the total.

This is an application of the distributive law:
$ab + ac + ad = a(b + c + d)$.

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Pre-test

1 Write down the coefficient of x in these expressions.

a $4x - 1$ **b** $3y + 6x$ **c** $3xy - 2x$ **d** $-4x^2 - 7x$

2 What is the constant term in these expressions?

a $5 - 2x$ **b** $-16a + 2$ **c** $5xy - 2x + 1$ **d** $101x - 6$

3 Evaluate the following if $a = 2$ and $b = -5$.

a ab **b** $-3b + 1$ **c** $4 - b + a$ **d** $5a^2b$

e $\sqrt{a^2}$ **f** $2b^2 - a$ **g** $5 - b^2$ **h** $a^2b - 2b^2$

4 Expand the following using $a(b + c) = ab + ac$.

a $2(x + 3)$ **b** $3(a - 5)$ **c** $4x(3 - 2y)$ **d** $-3(2b - 1)$

5 Write down the highest common factor of:

a 4 and 6 **b** 12 and 18 **c** $2x$ and $4x$

d $3xy$ and $9y$ **e** $10x$ and $15x^2$ **f** $3a^2b$ and $4ab^2$

6 Factorise by taking out the highest common factor.

a $2a + 6$ **b** $3x + 12y$ **c** $5x^2 - 15x$ **d** $4m - 6mn$

7 Add or subtract these fractions. You will need a common denominator for parts **c** and **d**.

a $\frac{3}{7} + \frac{2}{7}$ **b** $\frac{4}{9} - \frac{5}{9}$ **c** $\frac{1}{2} + \frac{2}{3}$ **d** $\frac{3}{8} - \frac{1}{2}$

8 Multiply or divide these fractions.

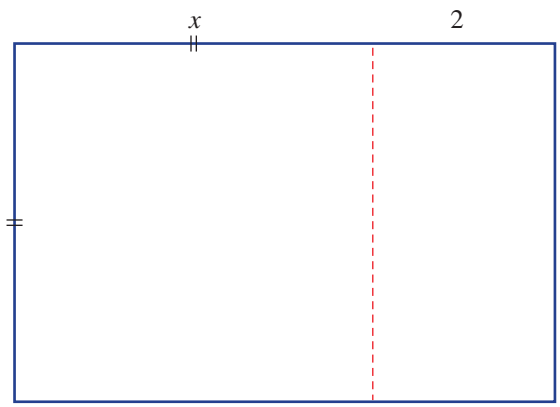
a $\frac{2}{3} \times \frac{4}{5}$ **b** $\frac{3}{4} \times \frac{2}{3}$ **c** $\frac{7}{14} \times \frac{2}{3}$ **d** $\frac{6}{11} \times \frac{22}{12}$

e $\frac{2}{3} \div \frac{1}{3}$ **f** $\frac{7}{8} \div \frac{14}{24}$ **g** $\frac{3}{2} \div \frac{4}{3}$ **h** $\frac{7}{9} \div \frac{4}{3}$

9 Expand and simplify the following.

a $3(x - 1) + 5$ **b** $4(1 - x) + 5x$ **c** $-2(5 + x) - x$

10 Write two expressions for the area of this rectangle, one with brackets and one without.



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9A Reviewing algebra REVISION

Interactive

Widgets

HOTsheets

Walkthrough

High levels of skill in algebra are required to solve more complex mathematical problems. These include adding, subtracting, multiplying and dividing algebraic expressions, as well as expanding and factorising expressions.

In this section we will review some basic concepts in algebra.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Vocab review

Consider the expression $3x^2 + 4xy - 2 - xy$.

- How many terms are there in the expression?
- Which letters have been used as pronumerals?
- What is the coefficient of x^2?
- What is the constant term?
- Are there any like terms?
- Can the expression be simplified? If so, how?
- What would be the value of the expression if $x = 2$ and $y = -1$?

Key ideas

- Terms: 3-term expression with pronumerals x and y

$$\overbrace{7y^2 + 2xy - 4}$$

Coefficient of y^2 is 7 **Constant term** is -4

- **Like terms** have the same pronumeral part.
  - They can be collected using addition and subtraction. For example: $5a - 7a = -2a$ and $3xy^2 + 2y^2x = 5xy^2$

- For multiplication and division, the symbols $\times$ and $\div$ are not usually shown.
  For example:
  $-3ab \times 2b = -6ab^2$

$$14a^2b \div (7ab) = \frac{14a^2b}{7ab} = 2a$$

- The **distributive law** is used to expand brackets.

  For example:

$$-3(x - 2) = -3x + 6$$

$$4x + 2(1 - x) = 4x + 2 - 2x$$
$$= 2x + 2$$

Coefficient A numeral placed before a pronumeral

Constant term The part of an equation or expression without any pronumerals

Like terms Terms with the same pronumerals and same powers

Distributive law Adding numbers in brackets, *then* multiplying the total, gives the same answer as multiplying each number in the brackets separately first, *then* adding the products

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Exercise 9A REVISION

UNDERSTANDING AND FLUENCY | 1–3, 4–6(½) | 3, 4–6(½)

1 Write expressions for each of the following.

1 movie pass is \$11.
2 movie passes are \$22.
x movie passes are …

- **a** The cost of:
  - **i** x movie passes at \$11 each
  - **ii** n apples at 50 cents each
- **b** The number of tickets purchased for:
  - **i** x adults and y children
  - **ii** b boys and g girls
- **c** The cost of hiring an electrician for n hours if the callout fee is \$50 and the cost of labour is \$60 per hour

2 Consider the expression $5x - 2xy - y^2 - 7$.

- **a** What is the coefficient of x?
- **b** What is the coefficient of xy?
- **c** What is the constant term?
- **d** Are there any like terms?
- **e** Evaluate the expression if $x = 1$ and $y = 2$.
- **f** Evaluate the expression if $x = 2$ and $y = -1$.

3 Write expressions for:

'Product' means multiply.

- **a** the sum of 5 and $2x$
- **b** 7 less than $4a$
- **c** 1 less than the square of y
- **d** the product of x and z
- **e** the sum of the squares of a and b
- **f** the square root of the number that is 8 more than x

Example 1 Collecting like terms

Collect like terms to simplify the following.

a $5y - x + 2y$ **b** $4a^2b + ab - ba^2$

SOLUTION	EXPLANATION
a $5y - x + 2y = 7y - x$	$5y$ and $2y$ are like terms; add coefficients.
b $4a^2b + ab - ba^2 = 3a^2b + ab$	$4a^2b$ and ba^2 (or a^2b) are like terms.

4 Simplify by collecting like terms.

Only collect 'like' terms. Remember that $ab = ba$ and that the + or − belongs only to the term that immediately follows.

- **a** $3a - 4a$
- **b** $4x + 7x$
- **c** $6ab + 2ab$
- **d** $5xy - 9xy$
- **e** $3a - 1 - 2a$
- **f** $7y + 2x - 11y$
- **g** $5x - y - 3y + x$
- **h** $ab + ba$
- **i** $a^2b - 5a^2b - 3$
- **j** $11xy - 14yx$
- **k** $10r^2a + 2ar^2$
- **l** $9st^2 - st - 8t^2s$

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Example 2 Multiplying and dividing terms

Simplify these expressions.

a $-4a \times 2ab$

b $21a^2b \div (3abc)$

SOLUTION

a $-4a \times 2ab = -8a^2b$

b $21a^2b \div (3abc) = \dfrac{21a^2b}{3abc}$

$= \dfrac{7a}{c}$

EXPLANATION

$-4 \times 2 = -8$ and $a \times a = a^2$

Write as a fraction, then cancel where possible.

Note: $\dfrac{a^2}{a} = \dfrac{a \times \cancel{a}^1}{\cancel{a}^1} = a$

5 Simplify the following.

a $5 \times 3x$ **b** $7a \times 2$ **c** $-4x \times 2y$

d $6a \times (-7a)$ **e** $-3ab \times b$ **f** $-6a^2b \times 2b$

g $4x \div 2$ **h** $7a \div a$ **i** $12a \div (4a)$

j $22a^2 \div (11a)$ **k** $40a \div (4a^2)$ **l** $100x^2y \div (25xy)$

m $\dfrac{7ab^2}{7b}$ **n** $\dfrac{48a^2bc}{16abc^2}$ **o** $\dfrac{12xy}{36xy^2}$

Example 3 Expanding brackets

Use the distributive law to expand and simplify:

a $-x(2 + x)$

b $4x - 2(x - 1)$

SOLUTION

a $-x(2 + x) = -2x - x^2$

b $4x - 2(x - 1) = 4x - 2x + 2$

$= 2x + 2$

EXPLANATION

$-x \times 2 = -2x$ and $-x \times x = -x^2$

First expand the brackets, then collect like terms.

Note: $-2 \times (-1) = 2$, not -2.

6 Use the distributive law to expand and simplify:

a $3(x + 2)$ **b** $2(4 + x)$ **c** $-3(x + 4)$

d $-6(x + 1)$ **e** $-2(x - 3)$ **f** $-x(x + 1)$

g $-3y(2 - 3y)$ **h** $-7a(a - b)$ **i** $3 + 2(x - 1)$

j $3x + 4(1 - x)$ **k** $3(x + 1) - 7x$ **l** $4(x + 2) - 2(x + 1)$

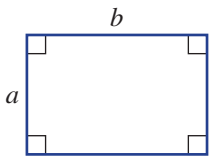
m $7(x - 3) - 3(x - 4)$ **n** $-4(1 - x) - 21x + 1$ **o** $-6(2 - x) - 7(4 - x)$

Remember: a negative times a negative is a positive.

PROBLEM-SOLVING AND REASONING 7, 8 7–11

7 For these shapes, write expressions for **i** perimeter and **ii** area.

a



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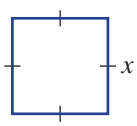

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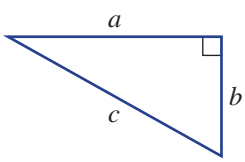

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8 Evaluate the following if $a = 3$, $b = -2$ and $c = -4$.

a $2a + b$ **b** abc **c** $2a \div (3b)$

d $-3c + b$ **e** $a^2 + b^2$ **f** $c^2 \div b$

g $\dfrac{a^2 + c^2}{2}$ **h** $\dfrac{a - c}{7}$ **i** $\dfrac{1}{a}(b + c)$

j $a^3 - b^3$ **k** $2b^3 - c$ **l** $\sqrt{a^2 + b^2}$

Substitute for the pronumerals and work out the answer.

9 You can hire a Ferrari for an upfront fee of \$100 plus \$80 per hour after that.

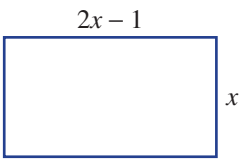
a What is the cost of hiring a Ferrari for:

i 2 hours? **ii** 9 hours?

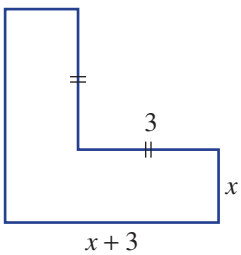
b Write an expression for the cost of hiring a Ferrari for n hours.

10 Find the area of these shapes in expanded form.

a



b



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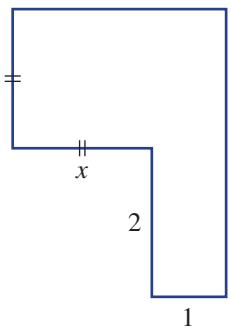

11 Identify the errors, then correct the calculations to find the answers.

a $-3(x - 1) = -3x - 3$

b $\dfrac{3ab}{6b} = 2a$

c $\dfrac{4a}{2a^2} = 2a$

d $3(x + 2) - 3(x - 1)$
$= 3x + 6 - 3x - 3$
$= 3$

ENRICHMENT – 12

Jake vs Lucas

12 Jake and Lucas operate separate computer consulting services.

- Jake charges \$60 per hour.
- Lucas charges \$40 callout plus \$50 per hour after that.

a What is the cost of hiring Jake for:

i 2 hours?

ii 10 hours?

b What is the cost of hiring Lucas for:

i 2 hours?

ii 10 hours?

c Write an expression for the cost of hiring Jake for n hours.

d Write an expression for the cost of hiring Lucas for n hours.

e After what time is the cost of hiring Jake or Lucas the same?

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9B Expanding binomial products

Interactive
Widgets
HOTsheets
Walkthrough

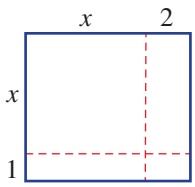
A binomial expression contains two terms, such as $x + 5$ or $x^2 + 3$. You will recall from the previous section that we looked at the product of a single term with a binomial expression; e.g. $2(x - 3)$ or $x(3x - 1)$. The product of two binomial expressions can also be expanded using the distributive law. This involves multiplying every term in one expression by every term in the other expression.

Expanding the product of two expressions can be applied to problems involving the expansion of rectangular areas, such as a farmer's paddock.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Rectangular expansions

If $(x + 1)$ and $(x + 2)$ are the side lengths of a rectangle as shown, the total area can be found as an expression in two different ways.



- Write an expression for the total area of the rectangle using length $= (x + 2)$ and breadth $= (x + 1)$.
- Now find the area of each of the four parts of the rectangle and combine to give an expression for the total area.
- Compare your two expressions above and complete this equation:

$$(x + 2)(____) = x^2 + ___ + ___.$$

- Can you explain a method for expanding the left-hand side to give the right-hand side?

Key ideas

- A binomial expression contains two terms.
  For example: $2x + 3$
- Expanding a **binomial product** uses the distributive law.

$$(a + b)(c + d) = a(c + d) + b(c + d)$$
$$= ac + ad + bc + bd$$

Binomial product The product of two binomial expressions

- Diagrammatically

$$(a + b)(c + d) = ac + ad + bc + bd$$

	a	b
c	ac	bc
d	ad	bd

For example:

$$(x + 1)(x + 5) = x^2 + 5x + x + 5$$
$$= x^2 + 6x + 5$$

	x	5
x	x^2	$5x$
1	$1x$	5

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Exercise 9B

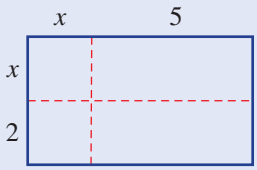
UNDERSTANDING AND FLUENCY	1–3, 4–5(½)	3, 4–5(½)

Example 4 Expanding with rectangles

The given diagram has the area $(x + 2)(x + 5)$.

a Write down an expression for the area of each of the four regions inside the rectangle.

b Copy and complete: $(x + 2)(x + 5) = ___ + 5x + ___ + ___$
$= ___ + ___ + ___$



SOLUTION

a x^2, $2x$, $5x$, 10

b $(x + 2)(x + 5)$
$= x^2 + 5x + 2x + 10$
$= x^2 + 7x + 10$

EXPLANATION

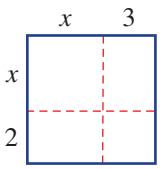
Multiply the length by the breadth for each rectangle within the larger rectangle.

Add all the region areas together and collect the like terms $5x$ and $2x$.

1 The given diagram shows the area $(x + 2)(x + 3)$.

a Write down an expression for the area of each of the four regions inside the rectangle.

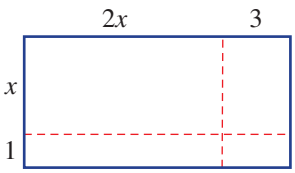
b Copy and complete: $(x + 2)(x + 3) = ___ + 3x + ___ + 6$
$= ___ + 5x + ___$



2 The given diagram shows the area $(2x + 3)(x + 1)$.

a Write down an expression for the area of each of the four regions inside the rectangle.

b Copy and complete: $(2x + 3)(____) = 2x^2 + ___ + 3x + ____$
$= ___ + ___ + ___$



3 Copy and complete these expansions.

a $(x + 1)(x + 5) = ___ + 5x + ___ + 5$
$= ___ + 6x + ___$

b $(x - 3)(x + 2) = ___ + ___ - 3x - ___$
$= ___ - x - ___$

c $(3x - 2)(7x + 2) = ___ + 6x - ___ - ___$
$= ___ - ___ - ___$

d $(4x - 1)(3x - 4) = ___ - ___ - 3x + ___$
$= ___ - 19x + ___$

The product is negative if there are opposite signs (+, –) or (–, +) and positive if they are the same sign (+, +) or (–, –).

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Example 5 Expanding binomial products

Expand $(x + 3)(x + 5)$.

SOLUTION

$$(x + 3)(x + 5) = x^2 + 5x + 3x + 15$$
$$= x^2 + 8x + 15$$

EXPLANATION

Use the distributive law to expand the brackets and then collect the like terms $5x$ and $3x$.

4 Expand the following.

a $(x + 2)(x + 5)$ **b** $(b + 3)(b + 4)$ **c** $(t + 8)(t + 7)$
d $(p + 6)(p + 6)$ **e** $(x + 9)(x + 6)$ **f** $(d + 15)(d + 4)$
g $(a + 1)(a + 7)$ **h** $(y + 10)(y + 2)$ **i** $(m + 4)(m + 12)$
j $(a + 9)(a + 1)$ **k** $(2 + a)(3 + a)$ **l** $(n + 6)(n + 6)$

First expand with the distributive law, then collect the two like terms.

Example 6 Expanding binomial products with subtraction signs

Expand the following.

a $(x - 4)(x + 7)$ **b** $(2x - 1)(x - 6)$ **c** $(5x - 2)(3x + 7)$

SOLUTION

a $(x - 4)(x + 7) = x^2 + 7x - 4x - 28$
$= x^2 + 3x - 28$

b $(2x - 1)(x - 6) = 2x^2 - 12x - x + 6$
$= 2x^2 - 13x + 6$

c $(5x - 2)(3x + 7) = 15x^2 + 35x - 6x - 14$
$= 15x^2 + 29x - 14$

EXPLANATION

After expanding to get the four terms, collect the like terms $7x$ and $-4x$.
Note: $x \times 7 = 7x$ and $-4 \times x = -4x$.
Remember: $2x \times x = 2x^2$ and $-1 \times (-6) = 6$.

Recall: $5x \times 3x = 5 \times 3 \times x \times x = 15x^2$.

5 Expand the following.

a $(x + 3)(x - 4)$ **b** $(x + 5)(x - 2)$ **c** $(x + 4)(x - 8)$
d $(x - 6)(x + 2)$ **e** $(x - 1)(x + 10)$ **f** $(x - 7)(x + 9)$
g $(x - 2)(x + 7)$ **h** $(x - 1)(x - 2)$ **i** $(x - 4)(x - 5)$
j $(4x + 3)(2x + 5)$ **k** $(3x + 2)(2x + 1)$ **l** $(3x + 1)(5x + 4)$
m $(2x - 3)(3x + 5)$ **n** $(8x - 3)(3x + 4)$ **o** $(3x - 2)(2x + 1)$
p $(5x + 2)(2x - 7)$ **q** $(2x + 3)(3x - 2)$ **r** $(4x + 1)(4x - 5)$
s $(3x - 2)(6x - 5)$ **t** $(5x - 2)(3x - 1)$ **u** $(7x - 3)(3x - 4)$

First expand with the distributive law, then collect the two like terms.

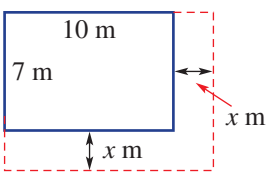
PROBLEM-SOLVING AND REASONING 6–8 6, 8–11

6 A 10 m by 7 m factory shed is expanded by x metres on two sides.

a Write expressions for the new:
i length of the shed **ii** breadth of the shed

b Expand the brackets to find an expression for the new total area.

c What would be the new area if $x = 2$?



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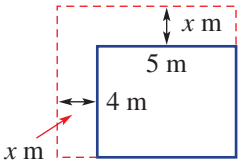
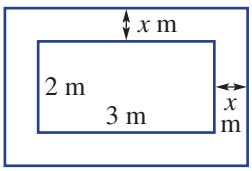
7 A room in a house with dimensions 4 m by 5 m is to be extended. Both the length and breadth are to be increased by x m.



First label the length and breadth of the new room.

- **a** Find an expanded expression for the area of the new room.
- **b** **i** If $x = 3$, find the area of the new room.
  - **ii** By how much has the area increased?

8 A rectangular trampoline of length 3 m and breadth 2 m is to be surrounded by padding of width x metres.



Don't forget to count the x on both sides.

- **a** Write expressions for the total:
  - **i** length of the trampoline and padding
  - **ii** breadth of the trampoline and padding
- **b** Find an expression for the total area by expanding brackets.
- **c** What would be the total area if $x = 1$?

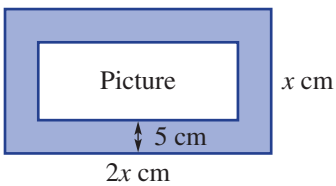
9 Write the missing terms in these expansions.

- **a** $(x + 2)(x + __) = x^2 + 5x + 6$
- **b** $(x + __)(x + 5) = x^2 + 7x + 10$
- **c** $(x + 1)(x + __) = x^2 + 7x + __$
- **d** $(x + __)(x + 9) = x^2 + 11x + __$
- **e** $(x + 3)(x - __) = x^2 + x - __$
- **f** $(x - 5)(x + __) = x^2 - 2x - __$

10 Expand these binomial products.

- **a** $(a + b)(a + c)$
- **b** $(a - b)(a + c)$
- **c** $(b - a)(a + c)$
- **d** $(2x + y)(x - 2y)$
- **e** $(2a + b)(a - b)$
- **f** $(3x - y)(2x + y)$

11 A picture frame with border 5 cm wide has a length that is twice the frame's total width, x cm



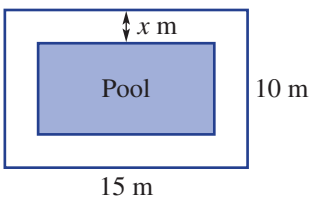
Length of picture $= 2x - 5 - 5 = 2x - 10$

- **a** Find an expression for the total area of the frame and picture.
- **b** Find an expression in expanded form for the area of the picture only.

ENRICHMENT – 12

Paving the pool

12 The outside edge of a path around a rectangular swimming pool is 15 m long and 10 m wide. The path is x metres wide.



- **a** Write expressions for the:
  - **i** length of the pool
  - **ii** breadth of the pool
- **b** Find an expression for the area of the pool in expanded form.
- **c** Find the area of the pool if $x = 2$.
- **d** What value of x makes the pool area 50 m^2? Use trial and error.

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9C Expanding perfect squares EXTENSION

Interactive
Widgets
HOTsheets
Walkthrough

A special type of binomial product involves perfect squares. $2^2 = 4$, $15^2 = 225$, x^2 and $(a + b)^2$ are all examples of perfect squares. To expand $(a + b)^2$ we multiply $(a + b)$ by $(a + b)$ and use the distributive law:

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

A similar result is obtained for the square of $(a - b)$:

$$\begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

Binomial products can be used to calculate the most efficient way to cut the shapes required for a fabrication out of a metal sheet.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Seeing the pattern

Using $(a + b)(c + d) = ac + ad + bc + bd$, expand and simplify the binomial products below.

$(x + 1)(x + 1) = x^2 + x + x + 1$
$\quad = $

$(x - 5)(x - 5) = $
$\quad = $

$(x + 3)(x + 3) = $
$\quad = $

$(x - 7)(x - 7) = $
$\quad = $

- Describe the patterns you see in the expansions above.
- Generalise your observations by completing the following expansions.

$(a + b)(a + b) = a^2 + __ + __ + __$
$\quad = a^2 + __ + __$

$(a - b)(a - b) = $
$\quad = $

Key ideas

- $3^2 = 9$, a^2, $(2y)^2$, $(x - 1)^2$ and $(3 - 2y)^2$ are all examples of **perfect squares**.
- Expanding perfect squares:

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

$$\begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

Perfect square
An algebraic expression formed by multiplying a binomial expression by itself

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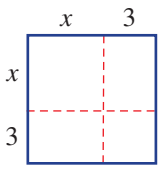
Exercise 9C EXTENSION

UNDERSTANDING AND FLUENCY 1–3, 4–5(½) 3, 4–6(½)

1 The side lengths of this square are $(x + 3)$ units.



a What are the areas of each of the four regions? Write expressions each time.

b Add up all the area expressions to find an expression for the total area.

c Complete the following: $(x + 3)(x + 3) = x^2 + 3x + __ + __$
$= __ + 6x + __$

2 Complete these expansions.

a $(x + 3)(x + 3) = x^2 + 3x + __ + __$
$= _____$

b $(x + 5)(x + 5) = x^2 + 5x + __ + __$
$= _____$

c $(x - 2)(x - 2) = x^2 - 2x - __ + __$
$= _____$

d $(x - 7)(x - 7) = x^2 - 7x - __ + __$
$= _____$

Expand using the distributive law:
$(a + b)(c + d) = ac + ad + bc + bd$

3 **a** Substitute the given value of b into $x^2 + 2bx + b^2$ and simplify.

i $b = 3$ **ii** $b = 11$

b Substitute the given value of b into $x^2 - 2bx + b^2$ and simplify.

i $b = 2$ **ii** $b = 9$

Example 7 Expanding perfect squares

Expand each of the following.

a $(x + 3)^2$ **b** $(x - 2)^2$

SOLUTION	EXPLANATION
a $(x + 3)^2 = (x + 3)(x + 3)$	Write in expanded form.
$= x^2 + 3x + 3x + 9$	Use the distributive law.
$= x^2 + 6x + 9$	Collect like terms.
Alternative solution:	
$(x + 3)^2 = x^2 + 2 \times x \times 3 + 3^2$	Expand using $(a + b)^2 = a^2 + 2ab + b^2$,
$= x^2 + 6x + 9$	where $a = x$ and $b = 3$.
b $(x - 2)^2 = (x - 2)(x - 2)$	Write in expanded form.
$= x^2 - 2x - 2x + 4$	Use the distributive law.
$= x^2 - 4x + 4$	Collect like terms.
Alternative solution:	
$(x - 2)^2 = x^2 - 2 \times x \times 2 + 2^2$	Expand using $(a - b)^2 = a^2 - 2ab + b^2$,
$= x^2 - 4x + 4$	where $a = x$ and $b = 2$.

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4 Expand each of the following perfect squares.

a $(x + 1)^2$ **b** $(x + 3)^2$ **c** $(x + 2)^2$ **d** $(x + 5)^2$
e $(x + 4)^2$ **f** $(x + 9)^2$ **g** $(x + 7)^2$ **h** $(x + 10)^2$
i $(x - 2)^2$ **j** $(x - 6)^2$ **k** $(x - 1)^2$ **l** $(x - 3)^2$
m $(x - 9)^2$ **n** $(x - 7)^2$ **o** $(x - 4)^2$ **p** $(x - 12)^2$

Recall:
$(a + b)^2 = (a + b)(a + b)$
$(a - b)^2 = (a - b)(a - b)$

Example 8 Expanding more perfect squares

Expand $(2x + 3)^2$.

SOLUTION	EXPLANATION
$(2x + 3)^2 = (2x + 3)(2x + 3)$	Write in expanded form.
$= 4x^2 + 6x + 6x + 9$	Use the distributive law.
$= 4x^2 + 12x + 9$	Collect like terms.
Alternative solution:	
$(2x + 3)^2 = (2x)^2 + 2 \times 2x \times 3 + 3^2$	Expand using $(a + b)^2 = a^2 + 2ab + b^2$ where
$= 4x^2 + 12x + 9$	$a = 2x$ and $b = 3$. Recall $(2x)^2 = 2x \times 2x = 4x^2$.

5 Expand each of the following perfect squares.

a $(2x + 1)^2$ **b** $(2x + 5)^2$ **c** $(3x + 2)^2$
d $(3x + 1)^2$ **e** $(5x + 2)^2$ **f** $(4x + 3)^2$
g $(7 + 2x)^2$ **h** $(5 + 3x)^2$ **i** $(2x - 3)^2$
j $(3x - 1)^2$ **k** $(4x - 5)^2$ **l** $(2x - 9)^2$

$(2x)^2 = 2x \times 2x$
$= 4x^2$

6 Expand each of the following perfect squares.

a $(3 - x)^2$ **b** $(5 - x)^2$ **c** $(1 - x)^2$
d $(6 - x)^2$ **e** $(11 - x)^2$ **f** $(4 - x)^2$
g $(7 - x)^2$ **h** $(12 - x)^2$ **i** $(8 - 2x)^2$
j $(2 - 3x)^2$ **k** $(9 - 2x)^2$ **l** $(10 - 4x)^2$

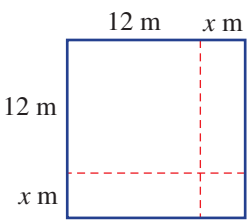
$-x \times (-x) = x^2$

PROBLEM SOLVING AND REASONING 7, 8 7, 9, 10

7 A farmer extends a 12 m square sheep pen on two sides by x metres.

a Write the expression for the side length of the new pen.
b Use your answer from part **a** to find the expression for the new pen area in expanded form.
c Use your result to find the new area if:
i $x = 2$
ii $x = 5$

An example of an expression is $10 - x$ or $x^2 - 20x + 100$.



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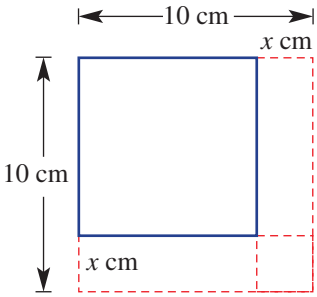
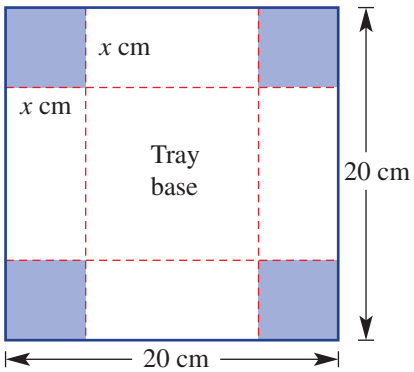
8 A child at preschool cuts off a strip of width x cm from two sides of a square piece of paper of side length 10 cm.



- **a** Write an expression for the new side length of the remaining paper.
- **b** Find the area of the new piece of paper in expanded form.
- **c** Use your result to find the new area if:
  - **i** $x = 2$
  - **ii** $x = 6$
- **d** What value of x makes the new area one quarter of the original?

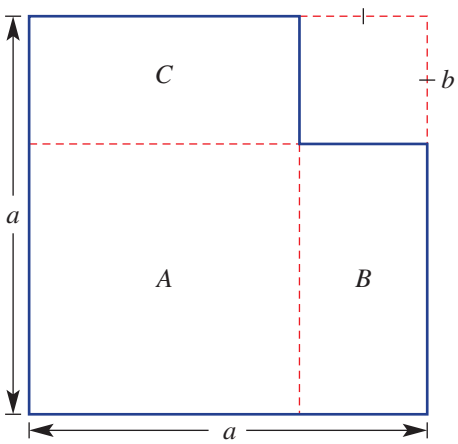
9 A square piece of tin of side length 20 cm has four squares of side length x cm removed from each corner. The sides are folded up to form a tray. The centre square forms the tray base.

- **a** Write an expression for the side length of the base of the tray.
- **b** Write an expression for the area of the base of the tray. Expand your answer.
- **c** Find the area of the tray base if $x = 3$.
- **d** Find the volume of the tray if $x = 3$.



10 A square of side length b is removed from a square of side length a.

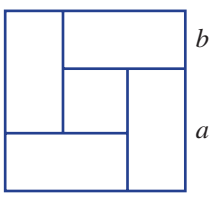
- **a** Using subtraction, write down an expression for the remaining area.
- **b** Write expressions for the area of these regions in expanded form.
  - **i** A **ii** B **iii** C
- **c** Add all the expressions from part **b** to see if you get your answer from part **a**.



ENRICHMENT – 11

Arranging tennis courts

11 Four tennis courts are arranged as shown, with a square storage area in the centre. Each court area has the same dimensions, $a \times b$.



- **a** Write an expression for the side length of the total area.
- **b** Write an expression for the area of the total area.
- **c** Write an expression for the side length of the inside storage area.
- **d** Write an expression for the area of the inside storage area.
- **e** Subtract your answer to part **d** from your answer to part **b** to find the area of the four courts.
- **f** Find the area of one court. Does your answer confirm that your answer to part **e** is correct?

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9D Difference of two squares EXTENSION

Interactive
Widgets
HOTsheets
Walkthrough

Another type of expansion deals with the product of the sum and difference of the same two terms. The result is called a 'difference of two squares':

$$(a + b)(a - b) = a^2 - ab + ba - b^2$$
$$= a^2 - b^2 \text{ (since } ab = ba \text{ the two middle terms cancel each other out.)}$$

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: How is 16 × 14 a difference of two squares?

Using the fact that $15^2 = 225$, follow these steps to show how 16 × 14 can be calculated mentally using a difference of two squares.

- $16 \times 14 = (15 + __) \times (15 - __)$ Rewrite.
  $= 15^2 - __ + __ - __$ Expand.
  $= __ - 1$ Simplify.
  $= __$ Evaluate.
- Now try this technique on 17 × 13 and 19 × 21.

Difference of two squares

$$(a + b)(a - b) = a^2 - ab + ba - b^2$$
$$= a^2 - b^2$$

$(a - b)(a + b)$ also expands to $a^2 - b^2$

The result is a difference of two squares.

Difference of two squares When one square term is subtracted from another

Key ideas

Exercise 9D EXTENSION

UNDERSTANDING AND FLUENCY 1–3, 4–5(½) 3, 4–5(½)

1 Why do the two middle terms in an expansion of $(x + a)(x - a)$ [e.g. $(x + 3)(x - 3) = x^2 - 3x + 3x - 9$] cancel out?

2 Decide whether each of the following shows a single perfect square or a difference of two squares.

a 4^2 **b** $7^2 - 3^2$ **c** $a^2 - b^2$ **d** x^2

3 Complete these expansions.

a $(x + 4)(x - 4) = x^2 - 4x + ____ - ____$
$= _____$

b $(x - 10)(x + 10) = x^2 + 10x - ____ - ____$
$= _____$

c $(2x - 1)(2x + 1) = 4x^2 + ____ - ____ - ____$
$= _____$

d $(3x + 4)(3x - 4) = 9x^2 - ____ + ____ - ____$
$= _____$

$(a + b)(a - b)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$

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Example 9 Forming a difference of two squares

Expand and simplify the following.

a $(x + 2)(x - 2)$ **b** $(x - 7)(x + 7)$

SOLUTION	EXPLANATION
a $(x + 2)(x - 2) = x^2 - \cancel{2x} + \cancel{2x} - 4$	Expand using the distributive law.
$= x^2 - 4$	$-2x + 2x = 0$.
Alternative solution:	
$(x + 2)(x - 2) = (x)^2 - (2)^2$	$(a + b)(a - b) = a^2 - b^2$. Here $a = x$ and $b = 2$.
$= x^2 - 4$	
b $(x - 7)(x + 7) = x^2 + \cancel{7x} - \cancel{7x} - 49$	Expand, then note that $7x - 7x = 0$.
$= x^2 - 49$	
Alternative solution:	
$(x - 7)(x + 7) = (x)^2 - (7)^2$	$(a - b)(a + b) = a^2 - b^2$, with $a = x$ and $b = 7$.
$= x^2 - 49$	

4 Expand and simplify the following to form a difference of two squares.

a $(x + 1)(x - 1)$ **b** $(x + 3)(x - 3)$
c $(x + 8)(x - 8)$ **d** $(x + 4)(x - 4)$
e $(x + 12)(x - 12)$ **f** $(x + 11)(x - 11)$
g $(x - 9)(x + 9)$ **h** $(x - 5)(x + 5)$
i $(x - 6)(x + 6)$ **j** $(5 - x)(5 + x)$
k $(2 - x)(2 + x)$ **l** $(7 - x)(7 + x)$

Recall:
$(a + b)(a - b) = a^2 - b^2$

Example 10 Forming more differences of two squares

Expand and simplify $(3x - 2y)(3x + 2y)$.

SOLUTION	EXPLANATION
$(3x - 2y)(3x + 2y) = 9x^2 + \cancel{6xy} - \cancel{6xy} - 4y^2$	Expand using the distributive law.
$= 9x^2 - 4y^2$	$6xy - 6xy = 0$.
Alternative solution:	
$(3x - 2y)(3x + 2y) = (3x)^2 - (2y)^2$	$(a + b)(a - b) = a^2 - b^2$, with $a = 3x$ and $b = 2y$
$= 9x^2 - 4y^2$	here. Recall that $(3x)^2 = 3x \times 3x$.

5 Expand and simplify the following.

a $(3x - 2)(3x + 2)$ **b** $(5x - 4)(5x + 4)$ **c** $(4x - 3)(4x + 3)$
d $(7x - 3y)(7x + 3y)$ **e** $(9x - 5y)(9x + 5y)$ **f** $(11x - y)(11x + y)$
g $(8x + 2y)(8x - 2y)$ **h** $(10x - 9y)(10x + 9y)$ **i** $(7x - 5y)(7x + 5y)$
j $(6x - 11y)(6x + 11y)$ **k** $(8x - 3y)(8x + 3y)$ **l** $(9x - 4y)(9x + 4y)$

$(3x^2) = 3x \times 3x$
$= 9x^2$

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PROBLEM SOLVING AND REASONING 6, 7 6, 8

6 To calculate 21×19, here is a method using the difference of two squares.

$$\begin{aligned} 21 \times 19 &= (20 + 1) \times (20 - 1) \\ &= 20^2 - 1^2 \\ &= 400 - 1 \\ &= 399 \end{aligned}$$

Use this technique to evaluate the following (mentally if you can).

a 31×29 (Note: $30^2 = 900$)
b 41×39 (Note: $40^2 = 1600$)
c 26×24 (Note: $25^2 = 625$)
d 51×49 (Note: $50^2 = 2500$)
e 22×18 (Use $20^2 = 400$)
f 23×17 (Use $20^2 = 400$)
g 35×25 (Use $30^2 = 900$)
h 54×46 (Use $50^2 = 2500$)

7 Lara is x years old and her two best friends are $(x - 2)$ and $(x + 2)$ years old.

a Write an expression for the:
  i square of Lara's age
  ii product of the ages of Lara's best friends

b Are the answers to parts **a i** and **ii** equal? If not, by how much do they differ?

Expand in part **a ii**.

8 A square of side length x has one side reduced by 1 unit and the other increased by 1 unit.

a Find an expanded expression for the area of the resulting rectangle.

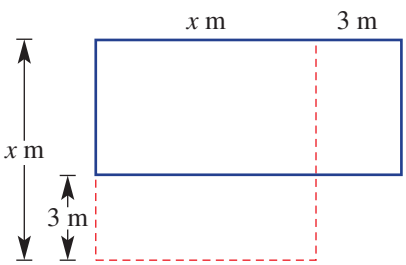
b Is the area of the original square the same as the area of the resulting rectangle? Explain why/why not.

Determine the expressions for the length and breadth of the rectangle first.

ENRICHMENT – 9

Classroom renovation

9 A square classroom is to be shortened on the south side by 3 m and extended on the east side by 3 m. The original side length of the classroom was x m.



a Write expressions for the:
  i original area of the classroom
  ii new length of the classroom
  iii new breadth of the classroom

b Expand to find an expression for the new area of the classroom.

c Is the new area the same as the original area? If not, by how much do they differ?

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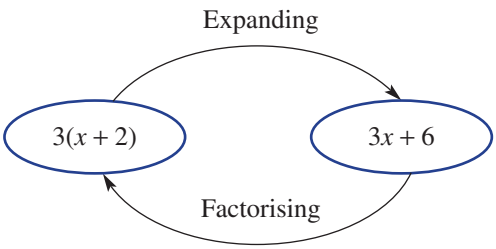
9E Using HCF to factorise algebraic expressions

Interactive
Widgets
HOTsheets
Walkthrough

The process of factorisation is a key step in the simplification of many algebraic expressions and in the solution of equations. It is the reverse process of expansion and involves writing an expression as a product of its factors.

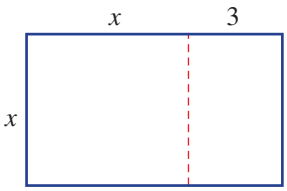


Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Factorised areas

Here is a rectangle of length $(x + 3)$ and breadth x.

- Write an expression for the total area using the given length and breadth.
- Write an expression for the total area by adding up the area of the two smaller regions.
- Are your two expressions equivalent? How could you work from your second expression (expanded) to the first expression (factorised)?



Key ideas

- When **factorising** expressions with common factors, take out the highest common factor (HCF). The HCF could be:
  - a number
    For example: $2x + 10 = 2(x + 5)$
  - a pronumeral
    For example: $x^2 + 5x = x(x + 5)$
  - the product of numbers and variables
    For example: $2x^2 + 10x = 2x(x + 5)$

HCF of $2x$ and 10

$2x + 10 = 2(x + 5)$

Expanded form Factorised form

- A factorised expression can be checked by using expansion.
  For example: $2x(x + 5) = 2x^2 + 10x$.

Factorise To write an expression as a product

Exercise 9E

UNDERSTANDING AND FLUENCY 1–3, 4–6(½) 3, 4–6(½)

1 Write down the highest common factor (HCF) of these pairs of numbers.

a 8, 12 **b** 10, 20 **c** 5, 60 **d** 24, 30
e 3, 5 **f** 100, 75 **g** 16, 24 **h** 36, 72

2 Write down the missing factor.

a $5 \times ___ = 5x$ **b** $7 \times ___ = 7x$ **c** $2a \times ___ = 2a^2$
d $5a \times ___ = 10a^2$ **e** $___ \times 3y = -6y^2$ **f** $___ \times 12x = -36x^2$
g $-3 \times ___ = 6x$ **h** $-2x \times ___ = 20x^2$ **i** $___ \times 7xy = -14x^2y$

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3 **a** Write down the missing factor in each part.

i $__(x^2 + 2x) = 6x^2 + 12x$

ii $__(2x + 4) = 6x^2 + 12x$

iii $__(x + 2) = 6x^2 + 12x$

b Which equation above uses the HCF of $6x^2$ and $12x$?

Expand to check.

Example 11 Finding the HCF

Determine the HCF of the following.

a $6a$ and $8ab$

b $3x^2$ and $6xy$

SOLUTION	EXPLANATION
a HCF of $6a$ and $8ab$ is $2a$.	HCF of 6 and 8 is 2. HCF of a and ab is a.
b HCF of $3x^2$ and $6xy$ is $3x$.	HCF of 3 and 6 is 3. HCF of x^2 and xy is x.

4 Determine the HCF of the following.

a $6x$ and $14xy$ **b** $12a$ and $18a$ **c** $10m$ and 4

d $12y$ and 8 **e** $15t$ and $6s$ **f** 15 and p

g $9x$ and $24xy$ **h** $6n$ and $21mn$ **i** $10y$ and $2y$

j $8x^2$ and $14x$ **k** $4x^2y$ and $18xy$ **l** $5ab^2$ and $15a^2b$

HCF stands for highest common factor.

Example 12 Factorising expressions

Factorise the following.

a $4x + 12$

b $40 - 16b$

SOLUTION	EXPLANATION
a $4x + 12 = 4(x + 3)$	4 is the HCF of $4x$ and 12. $4x \div 4 = x$ and $12 \div 4 = 3$. Check your answer by expansion.
b $40 - 16b = 8(5 - 2b)$	The HCF of 40 and $16b$ is 8. Place 8 in front of brackets and divide each term by 8.

5 Factorise the following.

a $7x + 7$ **b** $3x + 3$ **c** $4x - 4$ **d** $5x - 5$

e $4 + 8y$ **f** $10 + 5a$ **g** $3 - 9b$ **h** $6 - 2x$

i $12a + 3b$ **j** $6m + 6n$ **k** $10x - 8y$ **l** $4a - 20b$

m $x^2 + 2x$ **n** $a^2 - 4a$ **o** $y^2 - 7y$ **p** $x - x^2$

q $3p^2 + 3p$ **r** $8x - 8x^2$ **s** $4b^2 + 12b$ **t** $6y - 10y^2$

Always take out the highest common factor (HCF).

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Example 13 Using a negative number as a factor

Factorise $-8x^2 - 12x$.

SOLUTION

$-8x^2 - 12x = -4x(2x + 3)$

EXPLANATION

The HCF of the terms is $-4x$, including the common negative. Place the factor in front of the brackets and divide each term by $-4x$.
Note: $-8x^2 \div (-4x) = 2x$ and $-12x \div (-4x) = +3$.

6 Factorise the following by factoring out the negative sign.

a $-8x - 4$ **b** $-4x - 2$ **c** $-10x - 5y$
d $-7a - 14b$ **e** $-9x - 12$ **f** $-6y - 8$
g $-10x - 15y$ **h** $-4m - 20n$ **i** $-3x^2 - 18x$
j $-8x^2 - 12x$ **k** $-16y^2 - 6y$ **l** $-5a^2 - 10a$

Take the negative out of both terms.

PROBLEM-SOLVING AND REASONING 7, 8 7, 9–11

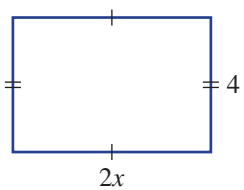
7 Write the missing number or expression.

a $3x + 9 = ___(x + 3)$
b $xy + x = x(___ + 1)$
c $a^2 - a = ___(a - 1)$
d $5xy + 10x = ___(y + 2)$
e $-7a - 14 = ___(a + 2)$
f $-24a^2 - 36a = ___(2a + 3)$

8 Write down the perimeter of these shapes in factorised form.

First find a simplified expression for the perimeter then factorise it.

a



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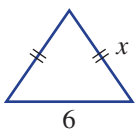

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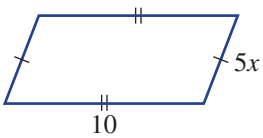

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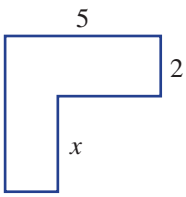

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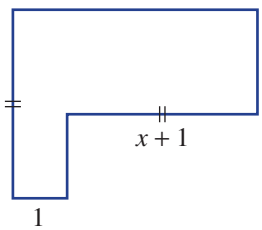

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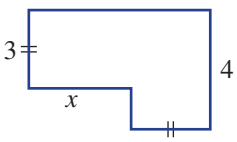

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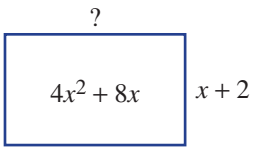
9 The expression for the area of a rectangle is $(4x^2 + 8x)$. Find an expression for its breadth if the length is $(x + 2)$.



10 The height, in metres, of a ball thrown in the air is given by $5t - t^2$, where t is the time in seconds.

a Write an expression for the ball's height in factorised form.

b Find the ball's height at these times:

i $t = 0$

ii $t = 2$

iii $t = 4$

c How long does it take for the ball's height to return to 0 metres? Use trial and error if required.

11 $7 \times 9 + 7 \times 3$ can be evaluated by first factorising to $7(9 + 3)$. This gives $7 \times 12 = 84$. Use a similar technique to evaluate the following.

a $9 \times 2 + 9 \times 5$

b $6 \times 3 + 6 \times 9$

c $-2 \times 4 - 2 \times 6$

d $-5 \times 8 - 5 \times 6$

e $23 \times 5 - 23 \times 2$

f $63 \times 11 - 63 \times 8$

ENRICHMENT – 12, 13

Further factorisation

12 Common factors can also be removed from expressions with more than two terms.
For example: $2x^2 + 6x + 10xy = 2x(x + 3 + 5y)$

Factorise these expressions by taking out the HCF.

a $3a^2 + 9a + 12$

b $5z^2 - 10z + zy$

c $x^2 - 2xy + x^2y$

d $4by - 2b + 6b^2$

e $-12xy - 8yz - 20xyz$

f $3ab + 4ab^2 + 6a^2b$

13 You can factorise some expressions by taking out a binomial factor.
For example: $3(x - 2) + x(x - 2) = (x - 2)(3 + x)$

In the example, $(x - 2)$ is the common factor.

Factorise the following by taking out a binomial common factor.

a $4(x + 3) + x(x + 3)$

b $3(x + 1) + x(x + 1)$

c $7(m - 3) + m(m - 3)$

d $x(x - 7) + 2(x - 7)$

e $8(a + 4) - a(a + 4)$

f $5(x + 1) - x(x + 1)$

g $y(y + 3) - 2(y + 3)$

h $a(x + 2) - x(x + 2)$

i $t(2t + 5) + 3(2t + 5)$

j $m(5m - 2) + 4(5m - 2)$

k $y(4y - 1) - (4y - 1)$

l $(7 - 3x) + x(7 - 3x)$

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9F Simplifying algebraic fractions: multiplication and division

Interactive

Widgets

HOTsheets

Walkthrough

Fractions such as $\frac{4}{6}$ and $\frac{24}{16}$ can be simplified by cancelling common factors.

This is also true of algebraic fractions such as $\frac{3x}{6}$, $\frac{5a^2}{10a}$ and $\frac{2x+4}{2}$.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Does $\frac{2x + \cancel{6}^{3}}{\cancel{2}^{1}} = 2x + 3$?

For the expression $\frac{2x+6}{2}$, a student attempts to cancel the 6 with the 2 in the denominator.

She writes $\frac{2x + \cancel{6}^{3}}{\cancel{2}^{1}} = 2x + 3$.

- Is this a correct method?
- Substitute $x = 1$ into the left-hand side and then the right-hand side. Are they equal?
- Can you show a correct method? Does it involve factorisation?

Key ideas

- Simplify algebraic fractions by cancelling common factors.

$$\frac{\cancel{3}^{1}x}{\cancel{6}^{2}} = \frac{x}{2}$$

$$\frac{5a^{\cancel{2}1}}{3\cancel{a}} = \frac{5a}{3}$$

$$\frac{2x+4}{2} = \frac{\cancel{2}^{1}(x+2)}{\cancel{2}^{1}} = x + 2$$

  - This is an incorrect cancellation: $\frac{2x + \cancel{4}^{2}}{\cancel{2}^{1}} = 2x + 2$.

- To multiply algebraic fractions, first cancel any numerator with any denominator.

$$\frac{\cancel{2}^{1}\cancel{x}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{15}^{3}}{\cancel{4}_{2}\cancel{x}_{1}y} = \frac{1 \times 3}{1 \times 2y} = \frac{3}{2y}$$

- To divide algebraic fractions, multiply by the reciprocal of the fraction following the division sign.
  - The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

$$\frac{3x}{4} \div \frac{9}{8y} = \frac{\cancel{3}^{1}x}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}y}{\cancel{9}^{3}} = \frac{2xy}{3}$$

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Exercise 9F

UNDERSTANDING AND FLUENCY | 1–3, 4–7(½) | 3, 4–7(½)

1 Cancel to simplify these fractions.

a $\frac{4}{2}$ **b** $\frac{4x}{2}$ **c** $\frac{4(x+2)}{2}$ **d** $\frac{4(x-1)}{2}$

2 Write down the reciprocal of these fractions.

a $\frac{2}{3}$ **b** $\frac{4}{3}$ **c** $\frac{7x}{2}$

d $\frac{5}{4a^2}$ **e** 7 **f** $6x$

Recall: $7 = \frac{7}{1}$ for part **e**.

3 Divide these simple fractions.

a $\frac{2}{3} \div \frac{3}{4}$ **b** $\frac{7}{8} \div \frac{3}{4}$ **c** $\frac{6}{7} \div \frac{4}{7}$

d $\frac{5}{14} \div \frac{10}{7}$ **e** $\frac{21}{4} \div \frac{7}{8}$ **f** $\frac{4}{9} \div \frac{4}{9}$

- Copy the first fraction.
- Change the ÷ to ×.
- Flip over the second fraction.

Example 14 Cancelling common factors

Simplify by cancelling common factors.

a $\frac{7x}{21}$ **b** $\frac{5(x+2)(x-1)}{x+2}$

SOLUTION	EXPLANATION
a $\frac{\cancel{7}^{1}x}{\cancel{21}_{3}} = \frac{x}{3}$	The HCF of 7 and 21 is 7.
b $\frac{5\cancel{(x+2)}^{1}(x-1)}{\cancel{x+2}^{1}} = 5(x-1)$	Treat $(x+2)$ as a common factor to both the numerator and denominator.

4 Simplify by cancelling common factors.

a $\frac{4x}{8}$ **b** $\frac{8x}{16}$ **c** $\frac{-4ab}{12}$ **d** $\frac{7a}{14b}$

e $\frac{4(x-1)}{4}$ **f** $\frac{-6(2x+1)}{3}$ **g** $\frac{-7x}{21x}$ **h** $\frac{4a^2}{40a}$

i $\frac{3(x+2)}{x+2}$ **j** $\frac{-2(x-7)}{x-7}$ **k** $\frac{5(x+3)}{10(x+3)}$ **l** $\frac{-16(x-14)}{24(x-14)}$

m $\frac{(x+2)(x-4)}{x+2}$ **n** $\frac{2(x-1)(x+3)}{x+3}$ **o** $\frac{-4(x+2)(x-5)}{2(x-5)}$ **p** $\frac{-7(x+1)(x-9)}{7(x+1)(x-9)}$

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Example 15 Simplifying by factorising

Simplify these fractions by factorising first.

a $\dfrac{2x+6}{2}$ **b** $\dfrac{-3x-9}{x+3}$

SOLUTION

a $\dfrac{2x+6}{2} = \dfrac{\cancel{2}^{1}(x+3)}{\cancel{2}^{1}}$

$= x+3$

b $\dfrac{-3x-9}{x+3} = \dfrac{-3\cancel{(x+3)}^{1}}{\cancel{x+3}^{1}}$

$= -3$

EXPLANATION

First factorise the numerator, then cancel the 2.

-3 is common to both $-3x$ and -9
$(x+3)$ can now be cancelled.

5 Simplify these fractions by factorising first.

a $\dfrac{4x+8}{4}$ **b** $\dfrac{2x-6}{2}$ **c** $\dfrac{3x-12}{3}$

d $\dfrac{3}{3x+3}$ **e** $\dfrac{-7}{14x+21}$ **f** $\dfrac{5a}{15a-10}$

g $\dfrac{2x+8}{x+4}$ **h** $\dfrac{7x+14}{x+2}$ **i** $\dfrac{-5x-15}{x+3}$

j $\dfrac{-4x-10}{2x+5}$ **k** $\dfrac{18+12x}{3+2x}$ **l** $\dfrac{-25-15x}{5+3x}$

First factorise the numerator or denominator, then cancel.

Example 16 Multiplying algebraic fractions

Simplify these products.

a $\dfrac{3x}{4} \times \dfrac{8}{9x}$

b $\dfrac{4(x+1)}{3} \times \dfrac{12}{x+1}$

SOLUTION

a $\dfrac{\cancel{3x}^{1}}{\cancel{4}^{1}} \times \dfrac{\cancel{8}^{2}}{\cancel{9x}^{3}} = \dfrac{1 \times 2}{1 \times 3}$

$= \dfrac{2}{3}$

b $\dfrac{4\cancel{(x+1)}^{1}}{\cancel{3}_{1}} \times \dfrac{\cancel{12}^{4}}{\cancel{x+1}_{1}} = \dfrac{4 \times 4}{1 \times 1}$

$= 16$

EXPLANATION

Cancel common factors from any numerator to any denominator, then multiply.

Cancel all common factors before multiplying.

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6 Simplify these products.

a $\frac{4x}{5} \times \frac{10}{8x}$ **b** $\frac{3x}{7} \times \frac{7}{6x}$

c $\frac{15a}{4} \times \frac{8}{25}$ **d** $\frac{16}{3a} \times \frac{9}{32}$

e $\frac{4y^2}{3} \times \frac{7x}{4y}$ **f** $\frac{8}{3x} \times \frac{5x^2}{16}$

g $\frac{2(x-1)}{3} \times \frac{6}{x-1}$ **h** $\frac{5(2x+1)}{4} \times \frac{4}{2x+1}$

i $\frac{3(x-7)}{7} \times \frac{21}{2(x-7)}$ **j** $\frac{x-3}{4} \times \frac{x+1}{x-3}$

k $\frac{3x+7}{x-2} \times \frac{x-2}{3x+7}$ **l** $\frac{5(x-2)}{3(x+7)} \times \frac{3(x+6)}{10(x-2)}$

Remember to cancel before multiplying.

Example 17 Dividing algebraic fractions

Simplify the following.

a $\frac{3x}{4} \div \frac{6}{7}$ **b** $\frac{5(x+1)}{6} \div \frac{2(x+1)}{9}$

SOLUTION

a $\frac{3x}{4} \div \frac{6}{7} = \frac{\cancel{3}^{1}x}{4} \times \frac{7}{\cancel{6}_{2}}$

$= \frac{7x}{8}$

b $\frac{5(x+1)}{6} \div \frac{2(x+1)}{9} = \frac{5\cancel{(x+1)}^{1}}{\cancel{6}_{2}} \times \frac{\cancel{9}^{3}}{2\cancel{(x+1)}^{1}}$

$= \frac{15}{4}$

EXPLANATION

Multiply by the reciprocal of the second fraction. Cancel before multiplying.

Invert the second fraction and multiply. Cancel before multiplying.

7 Simplify the following.

a $\frac{2x}{3} \div \frac{4}{7}$ **b** $\frac{5x}{4} \div \frac{15}{8}$

c $\frac{-3a}{4} \div \frac{2a}{7}$ **d** $\frac{-6a}{7} \div \frac{12}{7a}$

e $\frac{4(x+1)}{3} \div \frac{2(x+1)}{9}$ **f** $\frac{2(x-1)}{5} \div \frac{3(x-1)}{10}$

g $\frac{11(x+4)}{x+1} \div \frac{x+4}{x+1}$ **h** $\frac{-3(2x-1)}{2x+1} \div \frac{2(2x-1)}{2x+1}$ **i** $\frac{-7(4x+3)}{2x-7} \div \frac{14(4x+3)}{2x-4}$

Multiply by the reciprocal of the fraction after the division sign.

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PROBLEM-SOLVING AND REASONING

8, 9	8, 10

8 Find the error in these problems and then find the correct answer.

a $\dfrac{2x + \cancel{2}^{1}}{\cancel{2}^{1}} = 2x + 1$

b $\dfrac{-3x - 6}{3} = \dfrac{-3(x - 2)}{3}$
$= -(x - 2)$

c $\dfrac{2x}{3} \div \dfrac{4x}{9} = \dfrac{8x^2}{27}$

d $\dfrac{x + \cancel{2}^{1}}{\cancel{3}_{1}} \times \dfrac{\cancel{6}^{2}x}{\cancel{2}^{1}} = 2x^2 + 1$

9 Simplify the following.

a $\dfrac{2x + 4}{3} \times \dfrac{6}{x + 2}$

b $\dfrac{5x - 15}{7} \times \dfrac{7}{x - 3}$

c $\dfrac{-6x - 12}{x} \times \dfrac{3x}{x + 2}$

d $\dfrac{5x - 5}{2} \div \dfrac{x - 1}{6}$

e $\dfrac{-17x - 34}{x + 1} \div \dfrac{x + 2}{2x + 2}$

f $\dfrac{-7x + 14}{3x + 2} \div \dfrac{2 - x}{6x + 4}$

Factorise expressions before cancelling; e.g. $2x + 4$ can be written as $2(x + 2)$.

10 We know that $\dfrac{a^2}{a}$ cancels to $\dfrac{a}{1} = a$. Similarly, $\dfrac{(x + 1)^2}{x + 1} = \dfrac{(x + 1)\cancel{(x + 1)}^{1}}{\cancel{x + 1}^{1}} = x + 1$. Use this idea to cancel the following.

a $\dfrac{(x + 3)^2}{x + 3}$

b $\dfrac{(2x - 1)^2}{2x - 1}$

c $\dfrac{-5(x - 3)^2}{x - 3}$

d $\dfrac{-2(x - 4)}{6(x - 4)^2}$

ENRICHMENT

–	11

The negative 1 factor

11 Note that $3 - 2x = -1(2x - 3)$. Use this idea to cancel the following.

a $\dfrac{3 - 2x}{2x - 3}$

b $\dfrac{5 - 7x}{7x - 5}$

c $\dfrac{4(2 - x)}{3(x - 2)}$

d $\dfrac{-2(x - 5)}{5 - x}$

e $\dfrac{2 - x}{(x - 2)^2}$

f $\dfrac{16(x - 6)^2}{4(6 - x)}$

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9G Simplifying algebraic fractions: addition and subtraction

Interactive
Widgets
HOTsheets
Walkthrough

The process required for adding or subtracting algebraic fractions is similar to that used for fractions without variables.

For example, to simplify $\frac{2}{3} + \frac{4}{5}$, find the lowest common multiple of the denominators (15), and then express each fraction with this denominator. Adding the numerators completes the task.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Compare the working

Here is the working for the simplification of the sum of a pair of numerical fractions and the sum of a pair of algebraic fractions.

$$\frac{2}{5} + \frac{3}{4} = \frac{8}{20} + \frac{15}{20} = \frac{23}{20} \text{ or } 1\frac{3}{20}$$

$$\frac{2x}{5} + \frac{3x}{4} = \frac{8x}{20} + \frac{15x}{20} = \frac{23x}{20}$$

- What steps taken to simplify the algebraic fractions were the same as those used for the numerical fractions?
- Write down the steps required to add (or subtract) algebraic fractions.

Key ideas

- To add or subtract algebraic fractions:
  - determine the lowest common denominator (LCD)
  - express each fraction using the LCD
  - add or subtract the numerators
  - simplify if necessary.

Exercise 9G

UNDERSTANDING AND FLUENCY 1–5, 6–7(½) 5, 6–7(½)

1 Find the lowest common multiple of these pairs of numbers.

a 6 and 8
b 3 and 5
c 11 and 13
d 12 and 18

2 Simplify the following.

a $\frac{1}{2} + \frac{1}{3}$ **b** $\frac{3}{5} + \frac{1}{4}$ **c** $\frac{7}{8} + \frac{3}{4}$

d $\frac{9}{10} - \frac{1}{5}$ **e** $\frac{2}{3} - \frac{4}{5}$ **f** $\frac{11}{12} - \frac{5}{6}$

First find the LCD. Recall that the LCD of 8 and 4 is 8, not 32.

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3 Write equivalent fractions by stating the missing expression.

a $\frac{2x}{5} = \frac{\square}{10}$ **b** $\frac{7x}{3} = \frac{\square}{9}$ **c** $\frac{x+1}{4} = \frac{\square(x+1)}{12}$

d $\frac{3x+5}{11} = \frac{\square(3x+5)}{22}$ **e** $\frac{4}{x} = \frac{\square}{2x}$ **f** $\frac{30}{x+1} = \frac{\square}{3(x+1)}$

4 Copy and complete these simplifications.

a $\frac{x}{4} + \frac{2x}{3} = \frac{\square}{12} + \frac{\square}{12} = \frac{\square}{12}$

b $\frac{5x}{7} - \frac{2x}{5} = \frac{\square}{35} - \frac{\square}{35} = \frac{\square}{35}$

Remember to multiply the numerator by the same number by which you multiplied the denominator.

5 Write down the LCD for these pairs of fractions.

a $\frac{x}{3}, \frac{2x}{5}$ **b** $\frac{3x}{7}, \frac{x}{2}$ **c** $\frac{-5x}{4}, \frac{x}{8}$ **d** $\frac{2x}{3}, \frac{-5x}{6}$ **e** $\frac{7x}{10}, \frac{-3x}{5}$

Example 18 Adding and subtracting with a numeral in the denominator

Simplify:

a $\frac{7x}{3} + \frac{x}{6}$ **b** $\frac{x}{4} - \frac{2x}{5}$

SOLUTION	EXPLANATION
a $\frac{7x}{3} + \frac{x}{6} = \frac{14x}{6} + \frac{x}{6}$	Note that the LCD of 3 and 6 is 6, not $3 \times 6 = 18$.
$= \frac{15x}{6}$	For $\frac{7x}{3}$, multiply the numerator and denominator by 2.
$= \frac{5x}{2}$	$\frac{x}{6}$ already has a denominator of 6. $\frac{15}{6} = \frac{5}{2}$, given an HCF of 3.
b $\frac{x}{4} - \frac{2x}{5} = \frac{5x}{20} - \frac{8x}{20}$	Determine the LCD of 4 and 5; i.e. 20. Express each fraction as an equivalent fraction with a denominator of 20.
$= -\frac{3x}{20}$	$2x \times 4 = 8x$. Then subtract numerators.

6 Simplify the following.

a $\frac{x}{7} + \frac{x}{2}$ **b** $\frac{x}{3} + \frac{x}{15}$ **c** $\frac{x}{4} - \frac{x}{8}$ **d** $\frac{x}{9} + \frac{x}{5}$

e $\frac{y}{7} - \frac{y}{8}$ **f** $\frac{a}{2} + \frac{a}{11}$ **g** $\frac{b}{3} - \frac{b}{9}$ **h** $\frac{m}{3} - \frac{m}{6}$

i $\frac{m}{6} + \frac{3m}{4}$ **j** $\frac{a}{4} + \frac{2a}{7}$ **k** $\frac{2x}{5} + \frac{x}{10}$ **l** $\frac{p}{9} - \frac{3p}{7}$

m $\frac{b}{2} - \frac{7b}{9}$ **n** $\frac{9y}{8} + \frac{2y}{5}$ **o** $\frac{4x}{7} - \frac{x}{5}$ **p** $\frac{3x}{4} - \frac{x}{3}$

$\frac{x}{7}\frac{(\times 2)}{(\times 2)} = \frac{2x}{14}$

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Example 19 Adding more algebraic fractions

Simplify: $\frac{x+3}{2} + \frac{x-2}{5}$

SOLUTION

$$\frac{x+3}{2} + \frac{x-2}{5} = \frac{5(x+3)}{10} + \frac{2(x-2)}{10}$$
$$= \frac{5x+15+2x-4}{10}$$
$$= \frac{7x+11}{10}$$

EXPLANATION

The LCD of 2 and 5 is 10. Write as equivalent fractions with denominator 10.
Expand the brackets and simplify the numerator by adding and collecting like terms.

7 Simplify:

a $\frac{x+1}{2} + \frac{x+3}{5}$

b $\frac{x+3}{3} + \frac{x-4}{4}$

c $\frac{a-2}{7} + \frac{a-5}{8}$

d $\frac{y+4}{5} + \frac{y-3}{6}$

e $\frac{m-4}{8} + \frac{m+6}{5}$

f $\frac{x-2}{12} + \frac{x-3}{8}$

g $\frac{2b-3}{6} + \frac{b+2}{8}$

h $\frac{3x+8}{6} + \frac{2x-4}{3}$

i $\frac{2y-5}{7} + \frac{3y+2}{14}$

j $\frac{2t-1}{8} + \frac{t-2}{16}$

k $\frac{4-x}{3} + \frac{2-x}{7}$

l $\frac{2m-1}{4} + \frac{m-3}{6}$

Remember to include brackets when multiplying a numeral by these binomials numerators.

PROBLEM-SOLVING AND REASONING 8, 9 8, 10, 11

8 A student thinks that the LCD to use when simplifying $\frac{x}{2} + \frac{3x}{4}$ is 8.

a Complete the simplification using a common denominator of 8.

b Now complete the simplification using the actual LCD of 4.

c How does your working for parts **a** and **b** compare? Which method is preferable and why?

9 Find and describe the error in each set of working. Then give the correct answer.

a $\frac{4x}{5} - \frac{x}{3} = \frac{3x}{2}$

b $\frac{x+1}{5} + \frac{x}{2} = \frac{2x+1}{10} + \frac{5x}{10}$
$= \frac{7x+1}{10}$

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10 Simplify the following.

a $x + \frac{x}{2}$

b $x + \frac{x}{4}$

c $x + \frac{3x}{4}$

d $2x + 2x + \frac{x}{2}$

Recall that x can be written as $\frac{x}{1}$.

11 Simplify the following by finding the LCD.

a $\frac{2x}{5} - \frac{3x}{2} - \frac{x}{3}$

b $\frac{x}{4} - \frac{2x}{3} + \frac{5x}{6}$

c $\frac{5x}{8} - \frac{5x}{6} + \frac{3x}{4}$

d $\frac{x+1}{4} + \frac{2x-1}{3} - \frac{x}{5}$

e $\frac{2x-1}{3} - \frac{2x}{7} + \frac{x-3}{6}$

f $\frac{1-2x}{5} - \frac{3x}{8} + \frac{3x+1}{2}$

Choose a common denominator that works for all three fractions.

ENRICHMENT – 12

The missing algebraic fractions

12 Find the missing algebraic fraction. The fraction should be in simplest form.

a $\frac{x}{2} + \frac{\square}{\square} = \frac{5x}{6}$

b $\frac{\square}{\square} + \frac{x}{4} = \frac{3x}{8}$

c $\frac{2x}{5} + \frac{\square}{\square} = \frac{9x}{10}$

d $\frac{2x}{3} - \frac{\square}{\square} = \frac{7x}{15}$

e $\frac{\square}{\square} - \frac{x}{3} = \frac{5x}{9}$

f $\frac{2x}{3} - \frac{\square}{\square} = \frac{5x}{12}$

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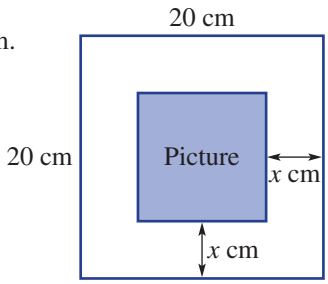
1 'You use me when dividing fractions.' Solve each expression, then match the answers to the letters to solve the riddle.

P	L	C	R	O	A	E	I
$-2x \times 3x$	$(x+1)(x-1)$	$\frac{2x-6}{2}$	$\frac{5x^2}{10x}$	$\frac{x}{2} - \frac{x}{3}$	$\frac{x+3}{2} \times \frac{6}{x+3}$	$\frac{2x}{3} \div \frac{4}{9}$	$7ab - 6ba$
☐	☐	☐	☐	☐	☐	☐	☐
$\frac{x}{2}$	$\frac{3x}{2}$	$x - 3$	ab	$-6x^2$	$\frac{x}{2}$	$\frac{x}{6}$	$x - 3$	3	$x^2 - 1$

2 A square framed photograph has side length 20 cm and frame width x cm.

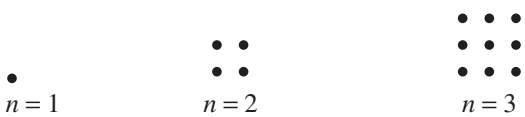
a Find an expanded expression for the area of the picture.

b What value of x makes the picture $\frac{1}{4}$ of the total area?



3 Write an expression that gives the total number of dots required for the nth diagram in each of these sequences.

a



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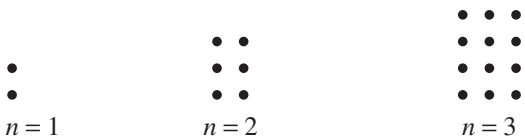

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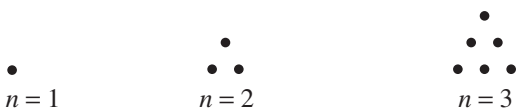

4 Simplify these algebraic fractions. Don't be fooled by the double negative.

a $\frac{x+1}{3} - \frac{x-2}{4}$

b $\frac{3x-4}{5} - \frac{4x-2}{3}$

5 **a** The difference between the squares of two consecutive numbers is 97.
What are the two numbers?

b The difference between the squares of two consecutive odd numbers is 136.
What are the two numbers?

c The difference between the squares of two consecutive multiples of 3 is 81.
What are the two numbers?

6 In a race over 4 km, Ryan ran at a constant speed. Sophie ran the first 2 km at a speed 1 km/h faster than Ryan. She ran the second 2 km at a speed 1 km/h slower than Ryan. Who won the race?

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Chapter summary

Quadratic expressions and algebraic fractions

Multiplying and dividing terms

$-3ab \times 2a = -6a^2b$

$15xy \div (3x^2y) = \dfrac{15xy}{3x^2y} = \dfrac{5}{x}$

Review

3-term expression

$\overbrace{7xy - 4yx + 6}$

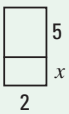
like terms (can collect) — constant term

The coefficient of x in $4 - 2x$ is -2.

Expansion

The process of removing brackets

e.g. $2(x + 5) = 2x + 10$



$(x + 3)(2x - 4) = 2x^2 - 4x + 6x - 12$
$= 2x^2 + 2x - 12$

Factorisation

The reverse process of expansion. Always remove the highest common factor first.

e.g. $2ab + 8b$ HCF $= 2b$
$= 2b(a + 4)$

e.g. $3x - 9 = 3(x - 3)$ HCF $= 3$

$-4x - 8 = -4(x + 2)$ HCF $= -4$

Difference of two squares

$(a - b)(a + b) = a^2 - b^2$

e.g. $(x - 5)(x + 5) = x^2 + \cancel{5x} - \cancel{5x} - 25$
$= x^2 - 25$

Simplifying by cancelling

Make sure in factorised form before cancelling.

e.g. $\dfrac{7(x + 1)}{2(x + 1)} = \dfrac{7}{2}$

e.g. $\dfrac{5x - 10}{5} = \dfrac{\cancel{5}^{1}(x - 2)}{\cancel{5}_{1}} = x - 2$

Perfect squares

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

e.g. $(2x + 3)^2 = (2x + 3)(2x + 3)$
$= 4x^2 + 6x + 6x + 9$
$= 4x^2 + 12x + 9$

Algebraic fractions

These involve algebraic expressions in the numerator and/or denominator.

e.g. $\dfrac{2x}{x - 7}, \dfrac{3}{5x}, \dfrac{7}{(x - 1)^2}$

Adding and subtracting

Must find lowest common denominator (LCD) before applying operation.

e.g. $\dfrac{2x}{3} - \dfrac{x}{4} = \dfrac{8x}{12} - \dfrac{3x}{12}$
$= \dfrac{5x}{12}$

e.g. $\dfrac{x - 1}{2} + \dfrac{x - 3}{3} = \dfrac{3(x - 1)}{6} + \dfrac{2(x - 3)}{6}$
$= \dfrac{3x - 3 + 2x - 6}{6}$
$= \dfrac{5x - 9}{6}$

Multiplying

Cancel any numerator with any denominator.

e.g. $\dfrac{\cancel{3x + 2}}{\cancel{5}_{1}} \times \dfrac{\cancel{15}^{3}}{2\cancel{(3x + 2)}_{1}}$
$= \dfrac{3}{2}$

Dividing

To divide, multiply by the reciprocal. Factorise all expressions, cancel and then multiply.

e.g. $\dfrac{3x}{4} \div \dfrac{9x}{8}$
$= \dfrac{\cancel{3x}^{1}}{\cancel{4}_{1}} \times \dfrac{\cancel{8}^{2}}{\cancel{9x}_{3}}$
$= \dfrac{2}{3}$

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Multiple-choice questions

1 The constant term in the expression $2x^2 - 3$ is:

A 3 **B** 2 **C** -2 **D** $2x^2$ **E** -3

2 The expanded form of $-2(x + 3)$ is:

A $2x + 6$ **B** $-2x - 3$ **C** $-2x - 6$ **D** $-2x + 6$ **E** $-2x + 3$

3 $x^2 - 2xy + 2yx$ is equal to:

A xy **B** x^2 **C** $x^2 - 4xy$ **D** 0 **E** $4xy$

4 $-3ab \times 4b$ is equal to:

A $-7ab^2$ **B** $-12ab^2$ **C** $-7a^2b$ **D** $-12a^2b$ **E** $12ab^2$

5 $(x - 3)(x - 4)$ expands to:

A 12 **B** $x^2 + 12$ **C** $x^2 - 7x + 12$ **D** $x^2 - x - 12$ **E** $x^2 + 7x - 12$

6 $(2x - 1)(x + 5)$ in expanded and simplified form is:

A $2x^2 + 9x - 5$

B $x^2 + 11x - 5$

C $4x^2 - 5$

D $3x^2 - 2x + 5$

E $2x^2 + 4x - 5$

7 $(3a + 2b)^2$ is equivalent to:

A $9a^2 + 6ab + 4b^2$

B $9a^2 + 4b^2$

C $3a^2 + 6ab + 2b^2$

D $3a^2 + 12ab + 2b^2$

E $9a^2 + 12ab + 4b^2$

8 $\frac{5x}{3} \div \frac{15x}{12}$ is equal to:

A $\frac{4}{3}$ **B** $\frac{4x}{3}$ **C** $\frac{4}{3x}$ **D** $\frac{25x^2}{12}$ **E** $\frac{25x}{12}$

9 $\frac{4x}{3} - \frac{2x}{9}$ is equal to:

A $\frac{10x}{27}$ **B** $\frac{30x}{9}$ **C** $\frac{2x}{9}$ **D** $\frac{2x}{3}$ **E** $\frac{10x}{9}$

10 $\frac{x + 2}{5} + \frac{2x - 1}{3}$ written as a single fraction is:

A $\frac{11x + 1}{15}$ **B** $\frac{11x + 9}{8}$ **C** $\frac{3x + 1}{8}$ **D** $\frac{11x + 7}{15}$ **E** $\frac{13x + 1}{15}$

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Short-answer questions

1 Simplify these expressions.

a $3x \times 2x$ **b** $-3ab \times 6b$ **c** $7ab \div (14a)$ **d** $-40x^2y \div (10xy)$

2 Expand the following.

a $3(x - 4)$ **b** $-2(x + 6)$ **c** $-x(x - 1)$ **d** $-4x(2x - 3)$

3 Collect like terms to simplify the following.

a $5x - 1 + 2x$ **b** $7ab - 2ba$ **c** $4a^2b - 7a^2b$ **d** $5x^2 - xy + 3x^2$

4 Expand the following binomial products.

a $(x - 3)(x + 4)$ **b** $(x - 7)(x - 2)$
c $(2x - 3)(3x + 2)$ **d** $(x - 1)(3x + 4)$

5 Expand the following.

a $(x + 3)^2$
b $(x - 4)^2$
c $(3x - 2)^2$
d $(x - 5)(x + 5)$
e $(7 - x)(7 + x)$
f $(11x - 4)(11x + 4)$

6 Write the following in fully factorised form by removing the highest common factor.

a $4a + 12b$
b $-3 - 9x$
c $x^2 + x$
d $6x - 9x^2$
e $-5x^2y - 10xy$
f $11a^2b - 33ab$

7 Simplify the following.

a $\dfrac{3(x - 1)}{6}$ **b** $\dfrac{(x + 1)(x - 3)}{x - 3}$ **c** $\dfrac{3x + 12}{3}$

d $\dfrac{-5x - 15}{10}$ **e** $\dfrac{2x - 16}{3x - 24}$ **f** $\dfrac{2(x + 2)(x - 1)}{x + 2}$

8 Simplify the following algebraic fractions by first factorising and cancelling where possible.

a $\dfrac{3}{2x} \times \dfrac{x}{6}$ **b** $\dfrac{7a^2}{x + 1} \times \dfrac{x + 1}{14a}$ **c** $\dfrac{x(x - 4)}{8(x + 1)} \times \dfrac{4(x + 1)}{x}$

d $\dfrac{3a}{5} \div \dfrac{9}{10}$ **e** $\dfrac{2(x - 1)}{3} \div \dfrac{x - 1}{6}$ **f** $\dfrac{2x}{5x + 20} \div \dfrac{x}{x + 4}$

9 Simplify the following by first finding the lowest common denominator.

a $\dfrac{x}{4} + \dfrac{2x}{3}$ **b** $\dfrac{5x}{6} - \dfrac{7x}{8}$ **c** $\dfrac{3a}{4} - \dfrac{a}{2}$

d $\dfrac{x}{2} + \dfrac{x - 1}{3}$ **e** $\dfrac{2x - 3}{4} + \dfrac{x + 1}{3}$ **f** $\dfrac{x - 1}{6} + \dfrac{x + 3}{8}$

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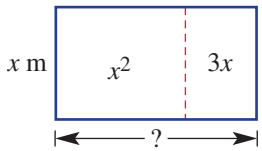
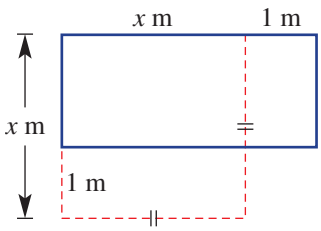
Extended-response questions

1 A pig pen for a small farm is being redesigned. It is originally a square of side length x m.



a In the planning, the breadth is initially kept as x m. The length is altered so that the area of the pen is $(x^2 + 3x)$ square metres. What is the new length? (Consider factorising.)

b Instead, it is decided that the original length will be increased by 1 metre and the original breadth will be decreased by 1 metre.



i What effect does this have on the perimeter of the pig pen compared with the original size?

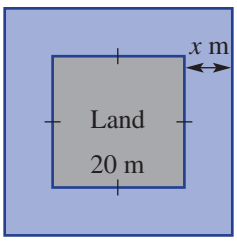
ii Work out an expression for the new area of the pig pen in expanded form. How does this compare to the original area?

c For the pig pen described in part **b** what:

i would be the area if $x = 2$?

ii value of x gives an area of 15 m^2? Use trial and error.

2 The security tower for a palace is on a small square piece of land 20 m by 20 m. It has a moat of width x metres the whole way around it, as shown.



a State the area of the piece of land.

b **i** Give expressions for the length and the breadth of the combined moat and land.

ii Find an expression, in expanded form, for the entire area occupied by the moat and the land.

c Use your answers to parts **a** and **b** to give an expression for the area occupied by the moat alone. Answer in factorised form.

d Use trial and error to find the value of x so that the area of the moat alone is 500 m^2.

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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

10 Probability and single variable data analysis

What you will learn

10A Probability review REVISION
10B Venn diagrams and two-way tables
10C Using arrays for two-step experiments
10D Using tree diagrams
10E Using relative frequencies to estimate probabilities
10F Mean, median, mode and range REVISION
10G Interpreting data from tables and graphs
10H Stem-and-leaf plots
10I Grouping data into classes FRINGE

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NSW syllabus

STRAND: STATISTICS AND PROBABILITY
SUBSTRANDS: PROBABILITY, SINGLE VARIABLE DATA ANALYSIS

Outcomes

A student represents probabilities of simple and compound events.
(MA4–21SP)
A student calculates relative frequencies to estimate probabilities of simple and compound events.
(MA5.1–13SP)
A student describes and calculates probabilities in multi-step chance experiments.
(MA5.2–17SP)
A student analyses single sets of data using measures of location, and range.
(MA4–20SP)
A student uses statistical displays to compare sets of data, and evaluates statistical claims made in the media.
(MA5.1–12SP)
A student uses quartiles and box plots to compare sets of data, and evaluates sources of data.
(MA5.2–15SP)
A student collects, represents and interprets single sets of data, using appropriate statistical displays.
(MA4–19SP)

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Once in 100 years

Water from burst pipes, storms or floods can cause extensive damage to a person's home and contents. Different insurers offer different types of cover and differing premiums based on the calculated risk of damage. The higher the risk, the higher the premiums, with some insurers offering cover for flood damage as an optional extra, while others do not offer cover at all.

Since the Brisbane floods of 2011, Australian regulators have introduced a standard definition of flood. All insurers have been made to adopt this definition into their policies since 19 June 2014. Flood damage, it should be noted, is different to storm damage or rainwater damage.

The Brisbane floods of 2011 have resulted in a 45% increase in some people's premiums. In New South Wales it is predicted that 13% of properties will suffer flood damage at some time, with 2% being of high risk.

Many people talk about the 'once in a hundred years' chance of a flood; this is misleading as it simply means that there is a 1% chance of this level of flood in any given year. Although unlikely, it is possible for such a flood to occur more than once in a 100 years. Brisbane has experienced severe flooding in 1893, 1974 and 2011.

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Pre-test

1 Write the following as decimals.

a $\frac{1}{10}$ **b** $\frac{2}{8}$ **c** 30% **d** 85% **e** 23.7%

2 Express the following in simplest form.

a $\frac{4}{8}$ **b** $\frac{7}{21}$ **c** $\frac{20}{30}$ **d** $\frac{100}{100}$ **e** $\frac{0}{4}$

f $\frac{12}{144}$ **g** $\frac{36}{72}$ **h** $\frac{36}{58}$ **i** $\frac{72}{108}$ **j** $\frac{2}{7}$

3 A six-sided die is rolled.

a List the possible outcomes.

b How many of the outcomes are:

i even? **ii** less than 3? **iii** less than or equal to 3?

iv at least 2? **v** not a 6? **vi** not odd?

4 One number is selected from the group of the first 10 positive integers $\{1, 2, \ldots, 10\}$. How many of the numbers are:

a odd?

b less than 8 ?

c greater than or equal to 5 ?

d no more than 7?

e prime?

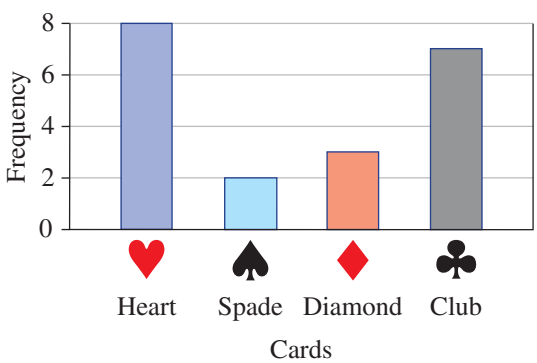
f not prime?

5 Several cards were randomly selected from a pack of playing cards. The suit of each card was noted, the card was replaced and the pack was shuffled. The frequency of each suit is shown in the column graph.

a How many times was a heart selected?

b How many times was a card selected in total?

c In what fraction of the trials was a diamond selected?



6 Consider this simple data set: 1, 2, 3, 5, 5, 7, 8, 10, 13.

a Find the mean (by finding the sum and dividing by 9).

b Find the median (the middle value).

c Find the mode (the most common value).

d Find the range (the difference between the highest and lowest value).

e If one of the numbers is randomly chosen from this data set, find the probability that it is a:

i 5

ii number that is not 5

iii number that is no more than 5

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10A Probability review REVISION

Interactive
Widgets
HOTsheets
Walkthrough

The mathematics used to calculate chance is called probability. We use it to compare the number of favourable outcomes to the total number of outcomes. This shows us how likely it is that the favourable event will occur. The probability of an event occurring is a number between 0 and 1. An impossible event has a probability of 0. An event that is certain to occur has a probability of 1.

We can use probability to find the likelihood of rolling a particular total score with two dice.

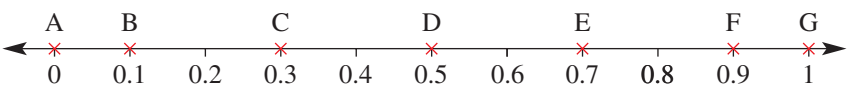
Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Events and probabilities

As a class group, write down and discuss at least three events that have the following chance of occurring.

- no chance
- very low chance
- medium to low chance
- even (50-50) chance
- medium to high chance
- very high chance
- certainty

For each decimal marked on the number line below, choose the probability description from the list above that best matches it.



Key ideas

Probability terminology

Terminology	Example	Definition
chance experiment	rolling a fair 6-sided die	A chance experiment is an activity which may produce a variety of different results that occur randomly. The example given is a single-step experiment.
trials	rolling a die 50 times	When an experiment is performed one or more times, each occurrence is called a trial. The example given indicates 50 trials of a single-step experiment.
outcome	rolling a 5	An outcome is one of the possible results of a chance experiment.
equally likely outcomes	rolling a 5 rolling a 6	Equally likely outcomes are two or more results that have the same chance of occurring.
sample space	{1, 2, 3, 4, 5, 6}	The sample space is the set of all possible outcomes of an experiment. It is usually written inside braces, as shown in the example.
event	rolling a 2 rolling an even number	An event is either one outcome or a collection of outcomes. It is a subset of the sample space.

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Key ideas

Terminology	Example	Definition
compound event	rolling an even number (e.g. 2 or 4 or 6)	A compound event is a collection of two or more outcomes from the sample space of a chance experiment.
mutually exclusive events	rolling a 5 rolling an even number	Two or more events are mutually exclusive if they share no outcomes.
non-mutually exclusive events	rolling a 5 rolling an odd number	Events are non-mutually exclusive if they share one or more outcomes. In the example given, the outcome 5 is shared.
complementary events	rolling a 2 or 3 rolling a 1, 4, 5, or 6	If *all* the outcomes in the sample space are divided into two events, they are complementary events.
complement	Rolling 2, 3, 4 or 5 is an event. Rolling a 1 or 6 is the complement.	If an experiment was performed and an event did not occur, then the complement definitely occurred.
favourable outcome(s)	In some games you must roll a 6 before you can start moving your pieces.	Outcomes are favourable when they are part of some desired event.
theoretical probability or likelihood or chance	The probability of rolling an odd number is written as: $P(\text{odd}) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$ Probabilities can be expressed as fractions, decimals and percentages.	Theoretical probability is the actual chance or likelihood that an event will occur when an experiment takes place. $P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ Probabilities range from 0 to 1 or 0% to 100%.
experimental probability	A die was rolled 600 times and showed a 5 on 99 occasions. The experimental probability of rolling a 5 on this die is: $P(5) = \frac{99}{600} = 0.165 = 16.5\%$	Sometimes it is difficult or impossible to calculate a theoretical probability, so an estimate can be found using a large number of trials. This is called the experimental probability. If the number of trials is large, then the experimental probability should be very close to the theoretical.
certain likely even chance unlikely impossible	rolling a number below 7 rolling a number below 6 rolling a 1, 2 or 3 rolling a 2 rolling a 7	The probability is 100% or 1. The probability is 50% or 0.5 or $\frac{1}{2}$. The probability is 0% or 0.
the sum of all probabilities in an experiment	$P(1) = \frac{1}{6}$ $P(2) = \frac{1}{6}$ $P(3) = \frac{1}{6}$ $P(4) = \frac{1}{6}$ $P(5)\frac{1}{6}$ $P(6)\frac{1}{6}$ $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$	The sum of the probablities of all the outcomes of a chance experiment is 1.
the sum of the probabilities of an event and its complement	$P(\text{rolling 1 or 6}) = \frac{2}{6}$ $P(\text{rolling 2, 3, 4 or 5}) = \frac{4}{6}$ $\frac{2}{6} + \frac{4}{6} + \frac{6}{6} = 1$	The sum of the probabilities of an event and its complement is 1. $P(\text{event}) + P(\text{complementary event}) = 1$

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Exercise 10A REVISION

UNDERSTANDING AND FLUENCY 1–9 4–9

1 Use the words in the table below to complete each statement.

even	impossible, certain	probability
complement	sample space	event

a The _____ _____ is the list of all possible outcomes in an experiment.
b An _____ is some of the outcomes from an experiment.
c P stands for _____.
d An _____ event has a probability of 0; an event that is _____ to occur has a probability of 1.
e An event with probability of 0.5 has an _____ or 50-50 chance of occurring.
f The _____ of an event, A, is the event where A does not occur.

2 Jim believes that there is a 1 in 4 chance that the flower on his prized rose will bloom tomorrow.
a Write the chance '1 in 4' as a:
i fraction
ii decimal
iii percentage
b Draw a number line from 0 to 1 and mark the level of chance described by Jim.

'1 in 4' means $\frac{1}{4}$.

3 Copy and complete this table.

Percentage ÷ 100 = decimal

	Percentage	Decimal	Fraction	Number line
a	50%	0.5	$\frac{1}{2}$	0 0.5 1
b	25%			
c			$\frac{3}{4}$	
d				0 0.2 0.5 1
e		0.6		
f			$\frac{17}{20}$	

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4 Ten people make the following estimates of the chance that they will get a salary bonus this year.

$0.7, \frac{2}{5}, 0.9, \frac{1}{3}$, 2 in 3, $\frac{3}{7}$ 1 in 4, $0.28, \frac{2}{9}, 0.15$

Order their estimates from lowest to highest.

First write each estimate as a decimal.

Example 1 Finding probabilities of events

This spinner has five equally divided sections.

a List the sample space using the given numbers.

b Find $P(3)$.

c Find $P(\text{not a } 3)$.

d Find $P(\text{a 3 or a 7})$.

e Find $P(\text{a number that is at least a 3})$.

SOLUTION	EXPLANATION
a $\{1, 2, 3, 3, 7\}$	Use braces, { }, and list all the possible outcomes in any order. Include 3 twice to represent its increased chance.
b $P(3) = \frac{2}{5}$ or 0.4	$P(3) = \frac{\text{number of sections labelled 3}}{\text{number of equal sections}}$
c $P(\text{not a } 3) = 1 - P(3)$	'Not a 3' is the complementary event of obtaining a 3.
$= 1 - \frac{2}{5}$ or $1 - 0.4$	
$= \frac{3}{5}$ or 0.6	Alternatively, count the number of sectors that are not 3.
d $P(\text{a 3 or a 7}) = \frac{2}{5} + \frac{1}{5}$	There are two 3 s and one 7 in the five sections.
$= \frac{3}{5}$	
e $P(\text{at least a 3}) = \frac{3}{5}$	Three of the sections have the numbers 3 or 7, which are at least 3; i.e. 3 or more.

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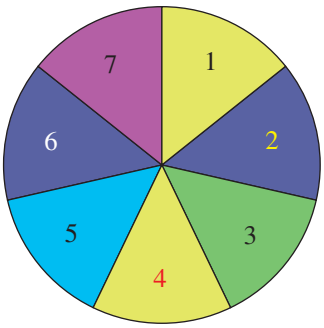
5 The spinners below have equally divided sections. Complete the following for each spinner. Write all probabilities as simplified fractions.

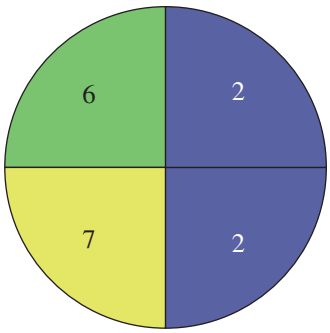
- **i** List the sample space using the given numbers.
- **ii** Find $P(2)$.
- **iii** Find $P(\text{not a } 2)$.
- **iv** Find $P(\text{a } 2 \text{ or a } 3)$.
- **v** Find $P(\text{a number which is at least a } 2)$.

P(event) means 'find the probability of the event'. 'At least 2' means '2 or more'.

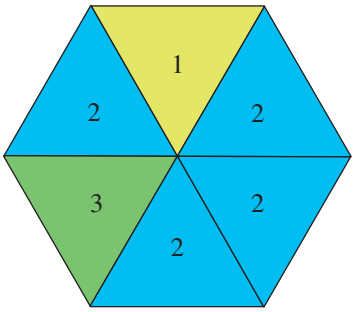
a



b



c



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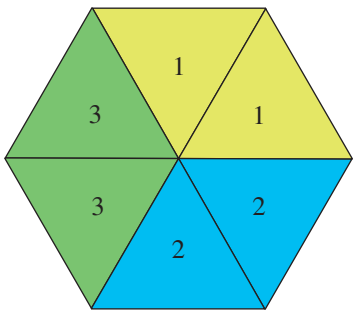
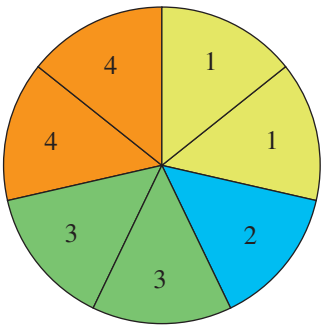
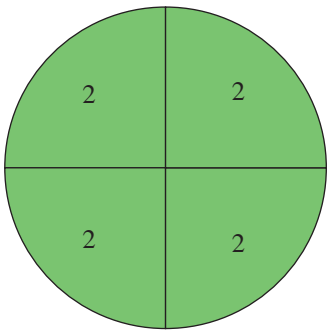


e



f



6 Find the probability of obtaining a blue ball if a ball is selected at random from a box that contains:

- **a** 4 blue balls and 4 red balls
- **b** 3 blue balls and 5 red balls
- **c** 1 blue ball, 3 red balls and 2 white balls
- **d** 8 blue balls, 15 black balls and 9 green balls
- **e** 15 blue balls only
- **f** 5 yellow balls and 2 green balls

$$P(\text{blue}) = \frac{\text{number of blue balls}}{\text{total number of balls}}$$

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7 Find the probability of *not* selecting a blue ball if a ball is selected at random from a box containing the balls described in question **6**, parts **a** to **f**, on the previous page.

$P(\text{not blue}) = 1 - P(\text{blue})$

8 If a swimming pool has eight lanes and each of eight swimmers has an equal chance of being placed in lane 1, find the probability that a particular swimmer:

a will swim in lane 1

b will not swim in lane 1

Example 2 Choosing letters from a word

A letter is randomly chosen from the word PROBABILITY. Find the following probabilities.

a $P(\text{L})$

b $P(\text{not L})$

c $P(\text{vowel})$

d $P(\text{consonant})$

e $P(\text{vowel or B})$

f $P(\text{vowel or consonant})$

SOLUTION	EXPLANATION
a $P(\text{L}) = \frac{1}{11}$	One of the 11 letters is an L.
b $P(\text{not L}) = 1 - \frac{1}{11}$ $= \frac{10}{11}$	The event 'not L' is the complement of the event 'selecting an L'. Complementary events sum to 1. $P(\text{not L}) = 1 - P(\text{L})$.
c $P(\text{vowel}) = \frac{4}{11}$	The vowels of the alphabet are A, E, I, O and U. There are 4 vowels in PROBABILITY: O, A and two letter Is.
d $P(\text{consonant}) = 1 - \frac{4}{11}$ $= \frac{7}{11}$	The events 'vowel' and 'consonant' are complementary. Alternatively, the other 7 letters are consonants; thus. $\frac{7}{11}$
e $P(\text{vowel or B}) = \frac{6}{11}$	There are 4 vowels and 2 letter Bs.
f $P(\text{vowel or consonant}) = 1$	This event includes all possible outcomes, since a letter is either a vowel or a consonant.

9 A letter is chosen at random from the word ALPHABET. Find the following probabilities.

a $P(\text{L})$

b $P(\text{A})$

c $P(\text{A or L})$

d $P(\text{vowel})$

e $P(\text{consonant})$

f $P(\text{vowel or consonant})$

g $P(\text{Z})$

h $P(\text{A or Z})$

i $P(\text{not A})$

j $P(\text{letter from the first half of the alphabet})$

Recall that vowels are A, E, I, O and U.

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PROBLEM SOLVING AND REASONING 10, 11 10, 12, 13

10 The school captain is to be chosen at random from four candidates. Two are girls (Hayley and Alisa) and two are boys (Rocco and Stuart).

a List the sample space.

b Find the probability that the school captain will be:

i Hayley

ii male

iii neither Stuart nor Alisa

The sample space is a list of the possible students.

11 A card is drawn at random from a pack of 52 playing cards. Find the probability that the selected card will be:

a the queen of diamonds

b an ace

c a red king

d a red card

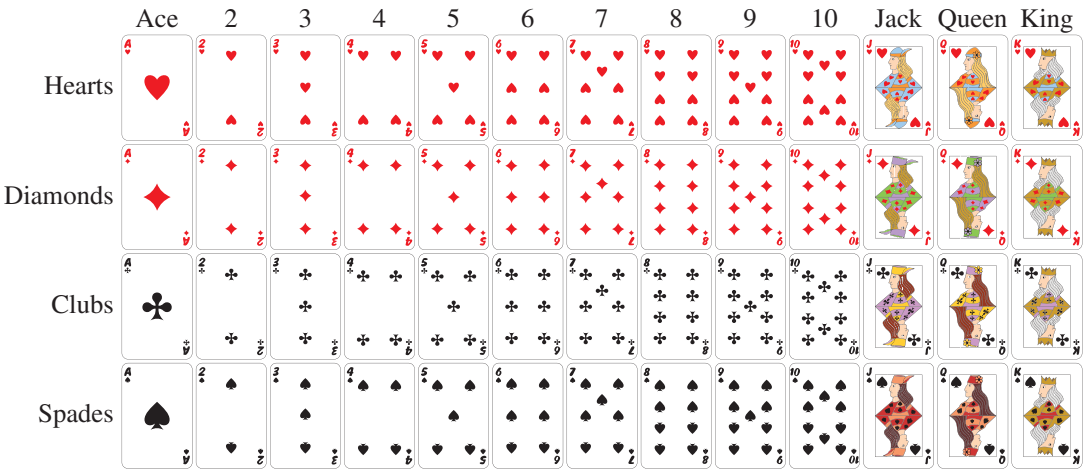
e a jack or a queen

f any card except a 2

g any card except a jack or a black queen

h not a black ace

A deck of 52 cards is shown below.



12 A six-sided die is rolled and the uppermost face is observed and recorded. Find the following probabilities.

a $P(6)$

b $P(3)$

c $P(\text{not } 3)$

d $P(1 \text{ or } 2)$

e $P(\text{number less than } 5)$

f $P(\text{even number or odd number})$

g $P(\text{square number})$

h $P(\text{not a prime number})$

i $P(\text{number greater than } 1)$

'Less than 5' means that 5 is not included.
1 is not prime.
Square numbers are $1^2 = 1$, $2^2 = 4$, $3^2 = 9$ etc.

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13 A letter is chosen at random from the word PROBABILITY. Find the probability that the letter will be:

- **a** a B
- **b** not a B
- **c** a vowel or a consonant
- **d** not a vowel
- **e** a consonant other than B
- **f** one of the first five letters in the alphabet
- **g** a letter from the word RABBIT
- **h** a letter that is not in the word RABBIT

ENRICHMENT – 14

Faulty CD player

14 A CD contains eight tracks. The time length for each track is shown in the table below.

Track	Time (minutes)
1	3
2	4
3	4
4	5
5	4
6	3
7	4
8	4

The CD is placed in a faulty CD player, which begins playing randomly at an unknown place somewhere on the CD, not necessarily at the beginning of a track.

- **a** Find the total number of minutes of music available on the CD.
- **b** Find the probability that the CD player will begin playing some where on track 1.
- **c** Find the probability that the CD player will begin somewhere on:
  - **i** track 2
  - **ii** track 3
  - **iii** a track that is 4 minutes long
  - **iv** track 4
  - **v** track 7 or 8
  - **vi** a track that is not 4 minutes long

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10B Venn diagrams and two-way tables

Interactive
Widgets
HOTsheets
Walkthrough

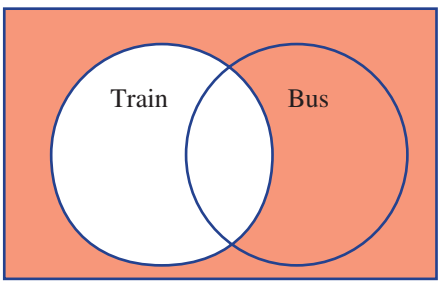
When the results of an experiment involve overlapping categories, it can be very helpful to organise the information into a Venn diagram or two-way table. Probabilities can easily be calculated from these types of diagrams.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Solving puzzles

Work with a partner to find the answer to each puzzle. For puzzles 1–4, draw some Venn diagrams, like the ones shown, to help you. For puzzle 5, use a two-way table.

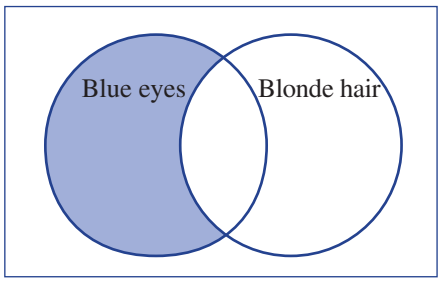
Puzzle 1



12 students travel to school by bus only and 10 students don't travel by either bus or train.

- How many students don't travel by train?

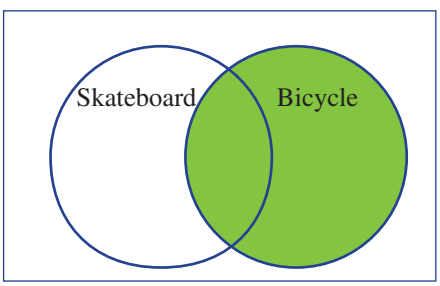
Puzzle 2



14 students in total have blue eyes, and of these students 5 have both blue eyes and blonde hair.

- How many have blue eyes but not blonde hair?

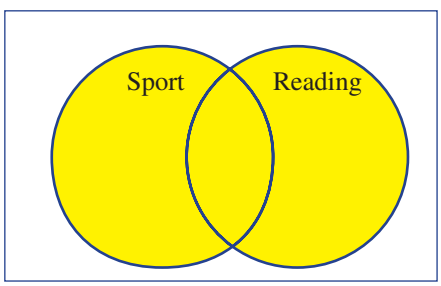
Puzzle 3



11 students own a bicycle only and 4 students own both a skateboard and a bicycle.

- How many students in total own a bicycle?

Puzzle 4



12 students in total like sport, 14 students in total like reading, and 9 students like both sport and reading.

- How many students altogether like either sport or reading or both?

Puzzle 5

A survey of 40 students found that a total of 22 play basketball, 9 play both volleyball and basketball and 6 do not play either basketball or volleyball.

- How many basketball players don't play volleyball?
- How many students in total don't play volleyball?
- How many students in total do play volleyball?

	Basketball	Not basketball	Total
Volleyball			
Not volleyball			
Total			

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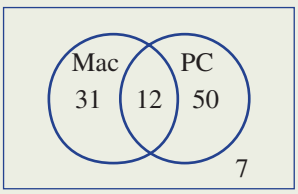
Key ideas

■ **Venn diagrams** and **two-way tables** help to organise outcomes into different categories.
This example shows the types of computers owned by 100 people.

Venn diagram



Two-way table

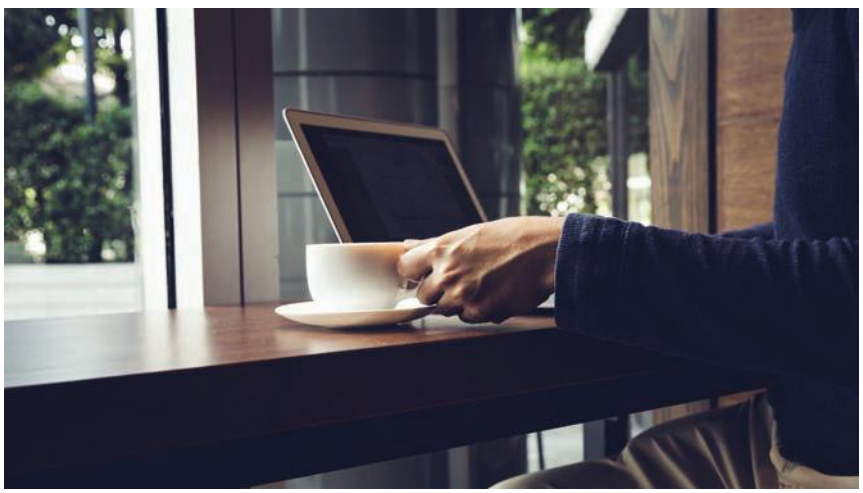
	Mac	No Mac	Total
PC	12	50	62
No PC	31	7	38
Total	43	57	100

These diagrams show, for example, that:

- 12 people own both a Mac and a PC
- 62 people own a PC
- 57 people do not own a Mac
- $P(\text{Mac}) = \frac{43}{100}$
- $P(\text{only Mac}) = \frac{31}{100}$
- $P(\text{Mac or PC}) = \frac{93}{100}$
- $P(\text{Mac and PC}) = \frac{12}{100} = \frac{3}{25}$

Venn diagram A diagram using overlapping circles to organise and show two or more sets of outcomes

Two-way table A table used to organise, display and compare two sets of outcomes



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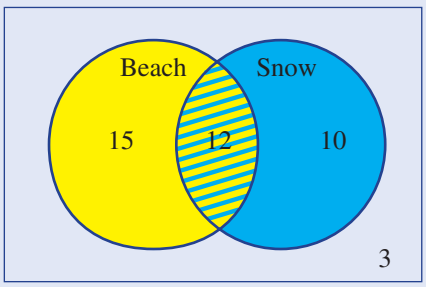
Exercise 10B

UNDERSTANDING AND FLUENCY | 1–8 | 4–9

Example 3 Understanding a Venn diagram

This Venn diagram shows the number of people who enjoy holidays at the beach and at the snow.



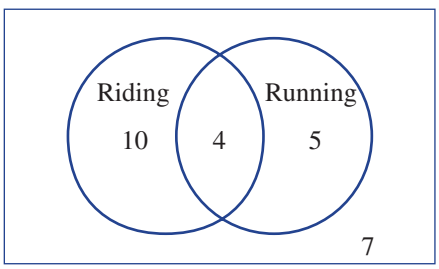
a How many people are represented in this Venn diagram?

b How many people enjoy holidays:
- **i** only at the beach?
- **ii** at the beach (in total)?
- **iii** both at the beach and at the snow?
- **iv** neither at the beach nor at the snow?

c How many people do not enjoy holidays at the:
- **i** beach?
- **ii** snow?

SOLUTION	EXPLANATION
a 40 in total	Add all the numbers in the Venn diagram. $15 + 12 + 10 + 3 = 40$
b i 15 enjoy the beach only	There are 15 people in the beach circle who do not appear in the overlapping section.
ii 27 enjoy the beach	Add the numbers in the beach circle. $15 + 12 = 27$
iii 12 enjoy the beach and the snow	12 people are in the overlapping section of the beach and snow circles.
iv 3 don't enjoy either	There are 3 people outside the circles.
c i 13 don't enjoy the beach	Add all numbers outside the beach circle. $10 + 3 = 13$
ii 18 don't enjoy the snow	Add all numbers outside the snow circle. $15 + 3 = 18$

1 This Venn diagram shows the number of people who enjoy riding and running.



a How many people in total are represented by this Venn diagram?

b How many people enjoy:
- **i** riding only?
- **ii** riding (in total)?
- **iii** running only?
- **iv** running (in total)?
- **v** both riding and running?
- **vi** neither riding nor running?
- **vii** riding or running?

c How many people do not enjoy:
- **i** riding?
- **ii** running?

'Riding only' does not include the overlapping section.

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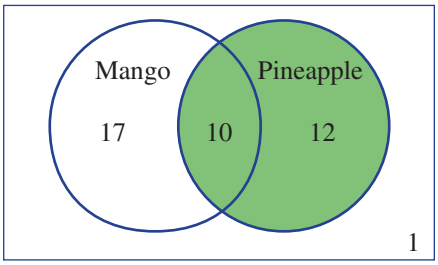
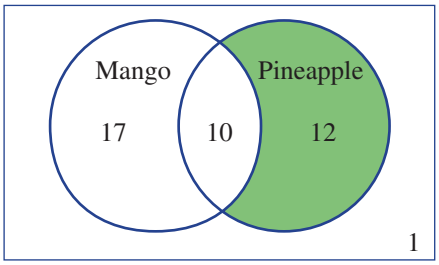
2 Each statement is about the shaded area in the Venn diagram. State the missing number in each statement.

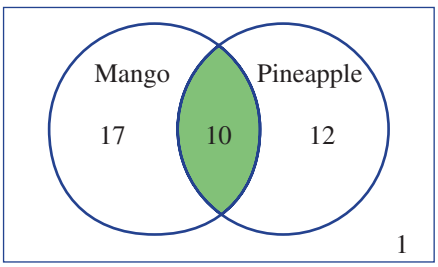
a _____ people in total like pineapple.



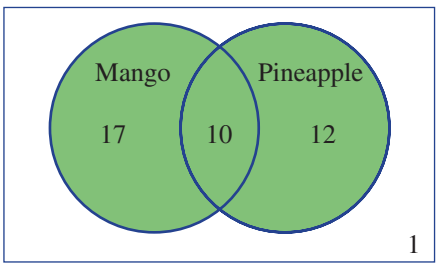
b _____ people like pineapple only.



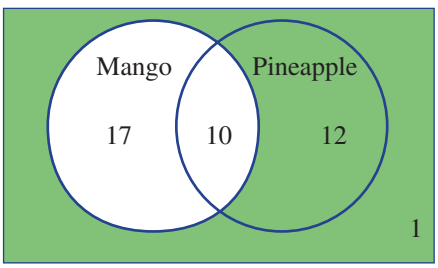
c _____ people like both pineapple and mango.



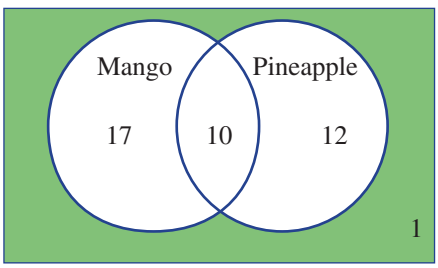
d _____ people like pineapple or mango or both.



e _____ people don't like mango.



f _____ person likes neither mango nor pineapple.



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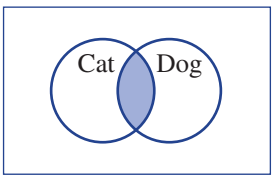
3 Match the diagrams **A–D** with the given description.

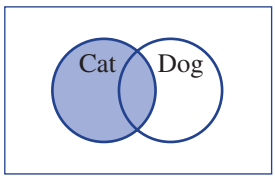
a Own a cat
b Own a cat only
c Own both a cat and a dog
d Own a cat or a dog or both

'Both a cat and a dog' is where the circles overlap.

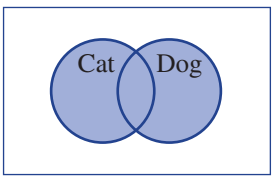
A



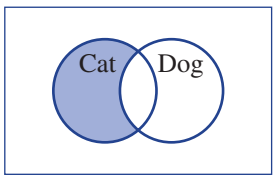
B



C



D



Example 4 Completing a two-way table

Copy and complete this two-way table.

	A	Not *A*	Total
B	7	6	
Not *B*		2	
Total	17		

SOLUTION

	A	Not *A*	Total
B	7	6	13
Not *B*	10	2	12
Total	17	8	25

EXPLANATION

Start with a row or column that has 2 numbers.
'*B* row: $7 + 6 = 13$
'*A*' column: $7 + 10 = 17$
'Not *B*' row: $10 + 2 = 12$
'Not *A*' column: $6 + 2 = 8$

Check that the sum of the row totals = the sum of the column totals.
Sum of column totals $= 17 + 8 = 25$
Sum of row totals $= 13 + 12 = 25$

4 Copy and complete these two-way tables.

a

	A	Not *A*	Total
B	7	8	
Not *B*		1	
Total	10		

b

	A	Not *A*	Total
B	2		7
Not *B*		4	
Total			20

Start with the column or row that has two numbers in it.

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Example 5 Constructing a Venn diagram

A survey of 30 people found that 21 like Australian Rules football and 12 like soccer. Also, 7 people like both football and soccer and 4 like neither football nor soccer.

a Construct a Venn diagram for the survey results.

b How many people:

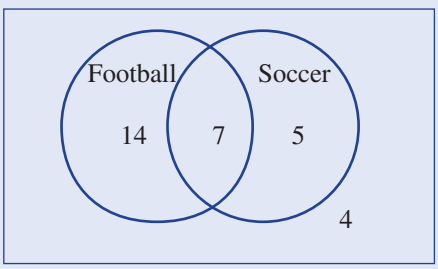
- **i** like football or soccer?
- **ii** do not like soccer?
- **iii** like only football?

c If one of the 30 people was randomly selected, find:

- **i** P(like football and soccer)
- **ii** P(like neither football nor soccer)
- **iii** P(like only soccer)

SOLUTION

a



EXPLANATION

Place the appropriate number in each category. Place the 7 in the overlap first, then ensure that:

- the total that like football is $21 (14 + 7 = 21)$.
- the total that like soccer is $12 (5 + 7 = 12)$.

The 4 people that like neither are placed outside the circles.

b **i** 26 like football or soccer or both

The total number of people who like football, soccer or both is $14 + 7 + 5 = 26$.

ii $30 - 12 = 18$ don't like soccer

12 like soccer, so 18 do not.

iii 14 like football but not soccer

21 like football but 7 of these also like soccer.

c **i** P(like football and soccer)

$= \frac{7}{30}$

7 out of 30 people like football and soccer.

ii P(like neither football nor soccer)

$= \frac{4}{30}$

$= \frac{2}{15}$

The 4 people who like neither football nor soccer sit outside both categories.

iii P(like soccer only)

$= \frac{5}{30}$

$= \frac{1}{6}$

5 people like soccer but not football.

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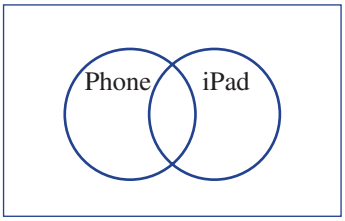
5 In a class of 30 students, 22 carried a phone and 9 carried an iPad. Three carried both a phone and an iPad and 2 students carried neither.



a Copy and complete this Venn diagram.

b How many people:

- **i** carried a phone or an iPad (includes carrying both)?
- **ii** do not carry an iPad?
- **iii** carry only an iPad?

c If one of the 30 people was selected at random, find the following probabilities.

- **i** P(carry a phone and an iPad)
- **ii** P(carry neither a phone nor an iPad)
- **iii** P(carry only a phone)

Start by writing a number in the overlapping section. '22 with a phone' is the total for the phone circle, including the overlap.

6 Match each diagram with the correct statement.

- **i** Not A
- **ii** A or B
- **iii** A and B
- **iv** B
- **v** B only
- **vi** Neither A nor B

'A or B' means 'either A or B or both'.
'A and B' means 'both A and B'.

Diagram P

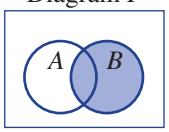


Diagram Q

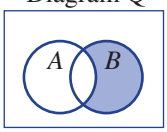


Diagram R

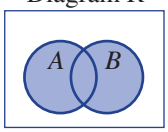


Diagram S

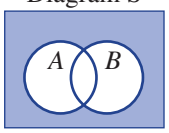


Diagram T

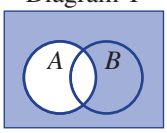
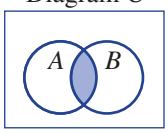


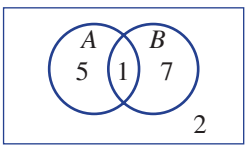
Diagram U



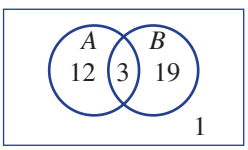
7 For each Venn diagram, find the following probabilities.

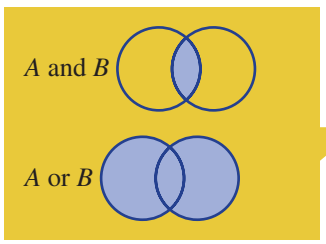
- **i** $P(A)$
- **ii** $P(A$ only$)$
- **iii** P(not B)
- **iv** $P(A$ and $B)$
- **v** $P(A$ or $B)$
- **vi** P(neither A nor B)

a



b





First calculate the total number in each sample.

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Example 6 Constructing a two-way table

At a car yard, 24 cars are tested for fuel economy. Eighteen of the cars run on petrol, 8 cars run on gas and 3 cars can run on both petrol and gas.

a Illustrate the situation using a two-way table.

b How many of the cars:

i do not run on gas? **ii** run on neither petrol nor gas?

c Find the probability that a randomly selected car:

i runs on gas **ii** runs on only gas **iii** runs on gas or petrol

SOLUTION

a

	Gas	Not gas	Total
Petrol	3	15	18
Not petrol	5	1	6
Total	8	16	24

b **i** 16

ii 1

c **i** $P(\text{gas}) = \frac{8}{24}$

$= \frac{1}{3}$

ii $P(\text{only gas})\ \frac{5}{24}$

iii $P(\text{gas or petrol}) = \frac{15 + 5 + 3}{24}$

$= \frac{23}{24}$

EXPLANATION

Set up a table as shown and enter the numbers (in black) from the given information.
Fill in the remaining numbers (in red), ensuring that each column and row adds to the correct total.

The total at the base of the 'Not gas' column is 16.
The number at the intersection of the 'Not gas' column and the 'Not petrol' row is 1.

Of the 24 cars, 8 cars run on gas.

Of the 8 cars that run on gas, 5 of them do not also run on petrol.
Of the 24 cars, some run on petrol only (15), some run on gas only (5) and some run on gas and petrol (3).

8 Of 50 desserts served at a restaurant one evening, 25 were served with ice-cream, 21 were served with cream and 5 were served with both cream and ice-cream.

25 is the total for the ice-cream row.

a Copy and complete this two-way table.

	Cream	Not cream	Total
Ice-cream			
Not ice-cream			
Total			

b How many of the desserts:

i did not have cream?

ii had neither cream nor ice-cream?

c Find the probability that a chosen dessert:

i had cream **ii** had only cream **iii** had cream or ice-cream

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9 Find the following probabilities using each of the given tables.
First copy and complete each two-way table.

i $P(A)$ **ii** $P(\text{not } A)$ **iii** $P(A \text{ and } B)$
iv $P(A \text{ or } B)$ **v** $P(B \text{ only})$ **vi** $P(\text{neither } A \text{ nor } B)$

'Neither A nor B' is the same as 'Not A and not B'.

a

	A	Not A	Total
B	3	1	
Not B	2		4
Total			

b

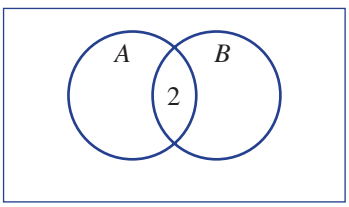
	A	Not A	Total
B		4	15
Not B	6		
Total			26

PROBLEM SOLVING AND REASONING 10–12 10, 12–14

10 For each two-way table, fill in the missing numbers then transfer the information to a Venn diagram.

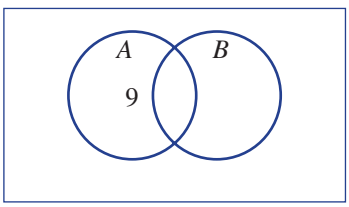
a

	A	Not A	Total
B	2		8
Not B			
Total		7	12



b

	A	Not A	Total
B		4	
Not B	9		13
Total	12		



11 In a group of 17 people, 13 rented their house, 6 rented a car and 3 did not rent either a car or their house.

14 people will be in the circles, but 13 + 6 = 19. How many must be in the overlap?

a Draw a Venn diagram, showing circles for 'Rents house' and 'Rents car'.
b How many people rented both a car and their house?
c Find the probability that one of the people in the group rented only a car.

12 One hundred citizens were surveyed regarding their use of water in their garden. Of these, 23 said that they used tank water, 48 said that they used tap water and 41 said that they did not use water on their garden at all.

a Copy and complete this two-way table.
b How many people used both tank and tap water?
c What is the probability that one of the people uses only tap water?
d What is the probability that one of the people uses tap water or tank water?

	Tank water	Not tank water	Total
Tap water			
Not tap water			
Total			

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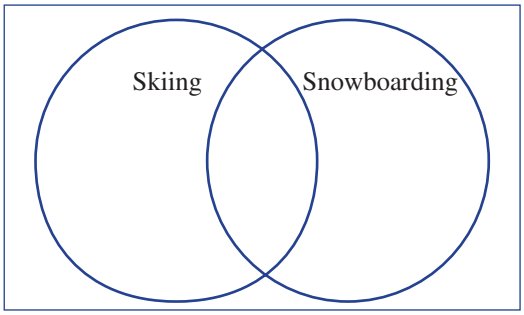
13 All members of a ski club enjoy skiing and/or snowboarding. Seven enjoy only snowboarding, 16 enjoy skiing and 4 enjoy both snowboarding and skiing.



- **a** Copy and complete this Venn diagram.
- **b** How many people are in the club?
- **c** If a person from the club is randomly selected, what is the probability of choosing a snowboarder?
- **d** If a person is randomly selected out of the group that likes skiing, what is the probability of choosing a snowboarder?
- **e** What is the probability of choosing a skier out of the group that likes snowboarding?

For part **d**, what fraction of skiers also like snowboarding?

14 Of a group of 30 cats, 24 like either tinned or dry food or both, 10 like only dry food and 5 like both tinned and dry food.

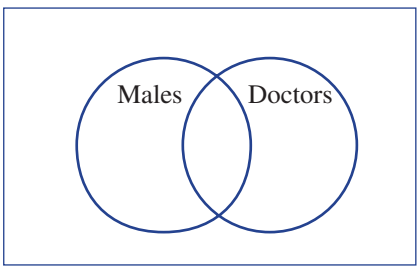
- **a** Find the probability that a selected cat likes only tinned food.
- **b** Out of the group of cats that eat dry food, what is the probability of selecting a cat that also likes tinned food?

Use a Venn diagram to record the information.

ENRICHMENT – 15

Numbers challenge

15 One hundred people were surveyed and it was found that 55 were males and 30 were doctors. The number of male doctors was 17. Copy and complete this Venn diagram and then determine the number of people in each question below.



- **a** The number who are neither male nor a doctor
- **b** The number who are not males
- **c** The number who are not doctors
- **d** The number who are male but not a doctor
- **e** The number who are a doctor but not male
- **f** The number who are female and a doctor
- **g** The number who are female or a doctor

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10C Using arrays for two-step experiments

Interactive
Widgets
HOTsheets
Walkthrough

Sometimes an experiment consists of two events, such as tossing a coin and then rolling a die. Or perhaps a card is pulled from a hat and then a spinner is spun. In this type of experiment, we can use arrays to list the sample space.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: The maths cup

This activity can be run in small groups or as a class.

1. Your teacher will draw the table below on the whiteboard. Each student should also have a copy.
2. Select one horse as your 'own' (choose a winner!).
3. Take turns with other students to roll 2 dice and state the sum of the uppermost faces of the dice.
4. The total of each roll refers to the horse number. When its number is rolled, that horse moves another 100 m towards the finish line. A cross is placed in the cell to show the move.
5. The winning horse is the first to reach the finish at 1000 m.
6. Keep rolling the dice until first, second and third places are decided.

	Horse	100 m	200 m	300 m	400 m	500 m	600 m	700 m	800 m	900 m	1000 m
1	SCRATCHED										
2	Greased Lightning										
3	Flying Eagle										
4	Quick Stix										
5	Break a Leg										
6	Slow and Steady										
7	The Donkey										
8	The Wombat										
9	Tooting Tortoise										
10	Ripper Racer										
11	Speedy Gonzo										
12	Pharoah										

Discussion questions

- Do you think that the horse that won your maths cup race would always win?
- In this game, do all the horses have the same chance of winning?
- Why is horse 1 scratched?
- When two dice are thrown, what are all the possible outcomes for the sum of the two uppermost faces?
- What are the horse numbers that are highly likely to win this race?
- What are the horse numbers that are unlikely to win this race?

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Key ideas

- A table called an array can be used to list the sample space of a two-step experiment.
- When listing outcomes, it is important to be consistent with the order for each outcome.
  For example: the outcome (heads, tails) is different from the outcome (tails, heads). So when two coins are tossed, the outcome of coin 1 is written first in each cell of the table.

		Coin 2	
		H	T
Coin 1	H	HH	HT
	T	TH	TT

  The sample space is shown in the body of the table; i.e. {HH, HT, TH, TT}.
- The probability is still given by:

$$P(\text{event}) = \frac{\text{number of outcomes where the event occurs}}{\text{total number of possible outcomes}}$$

  For example: When two coins are tossed, $P(\text{a tail and a head}) = \frac{2}{4} = \frac{1}{2}$
- Some experiments are conducted without replacement, which means that each individual trial outcome cannot be achieved again.
  For example: Two letters are chosen from the word CAT. 'Without replacement' means that the first letter chosen is removed and it is not possible to choose it again.

With replacement

		1st		
		C	A	T
2nd	C	(C, C)	(A, C)	(T, C)
	A	(C, A)	(A, A)	(T, A)
	T	(C, T)	(A, T)	(T, T)

9 outcomes

Without replacement

		1st		
		C	A	T
2nd	C	×	(A, C)	(T, C)
	A	(C, A)	×	(T, A)
	T	(C, T)	(A, T)	×

6 outcomes

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Exercise 10C

UNDERSTANDING AND FLUENCY 1–6 4–7

1 A letter is chosen from the word BYE and a letter is chosen from the word HI. Copy and complete this table to show the sample space.

	B	Y	E
H	HB		
I			

Write 'H' in each cell of the top row.
Write 'B' in each cell of the first column.

2 A coin and a four-sided die are tossed.

a Copy and complete this table to show the sample space.

	1	2	3	4
H				
T				

The sample space is all the outcomes in the central part of the table.

b How many outcomes are in the sample space?
c What is the probability of tossing an 'H3'?
d What is the probability of tossing a 'T2'?

3 A coin is flipped and then a spinner with numbers 1 to 5 is spun. The possible outcomes are listed in the table below.

	1	2	3	4	5
H	H1	H2	H3	H4	H5
T	T1	T2	T3	T4	T5

a How many outcomes are possible?
b List the four outcomes in which an even number is displayed on the spinner.
c Hence, state the probability that an even number is displayed.
d List the outcomes for which the coin shows tails and the spinner shows an odd number.
e What is $P(\text{T, odd number})$?

4 Two coins are tossed and the four possible outcomes are shown below.

		20-cent coin	
		H	T
50-cent coin	H	HH	HT
	T	TH	TT

The same face means either two heads or two tails.

a What is the probability that the 50-cent coin will be heads and the 20-cent coin will be tails?
b For which outcomes are the two coins displaying the same face?
c What is the probability of the two coins displaying the same face?

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Example 7 Using a table for multiple events

A spinner with the numbers 1, 2 and 3 is spun, and then a card is chosen at random from cards containing the letters A, T, H and S.

a Draw a table to list the sample space of this experiment.
b How many outcomes does the experiment have?
c Find the probability of the combination 2S.
d Find the probability of an odd number being spun and the letter H being chosen.

SOLUTION

a

	A	T	H	S
1	1A	1T	1H	1S
2	2A	2T	2H	2S
3	3A	3T	3H	3S

b There are 12 outcomes.

c $P(2, \text{S}) = \frac{1}{12}$

d $P(\text{odd, H}) = \frac{2}{12} = \frac{1}{6}$

EXPLANATION

The sample space of the spinner $\{1, 2, 3\}$ is put into the left column.
The sample space of the cards $\{\text{A, T, H, S}\}$ is put into the top row.

The table has $4 \times 3 = 12$ items in it.

All 12 outcomes are equally likely. Spinning 2 and choosing an S is one of the 12 outcomes.

Possible outcomes are 1H and 3H, so probability $= 2 \div 12$

5 A coin is flipped and then a regular die is rolled.

a Copy and complete this table to list the sample space for this experiment.

	1	2	3	4	5	6
H						
T						

b How many possible outcomes are there?
c Find the probability of the pair H3.
d Find the probability of 'heads' on the coin with an odd number on the die.

6 A letter is chosen from the word LINE and another is chosen from the word RIDE.

a Copy and complete this table to list the sample space.

	R	I	D	E
L				
I				
N				
E				

b How many possible outcomes are there?
c Find $P(\text{N, R})$; i.e. the probability that N is chosen from LINE and R is chosen from RIDE.
d Find $P(\text{L, D})$.
e Find the probability that two vowels are chosen.
f Find the probability that two consonants are chosen.
g Find the probability that the two letters chosen are the same.

Recall that the vowels are A, E, I, O and U.

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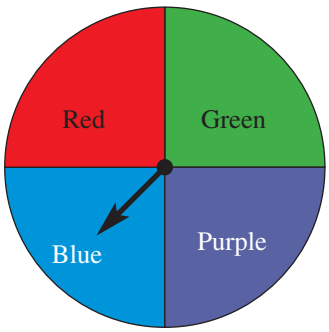
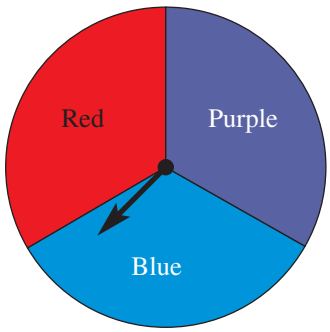
7 The spinners shown below are each spun.



Spinner 1



Spinner 2

a Copy and complete this table to list the sample space. Use R for red, P for purple and so on.

		Spinner 2		
		R	P	B
Spinner 1	R			
	P			
	G			
	B			

List the colour from spinner 1 first in each cell.

b Find the probability that spinner 1 will display red and spinner 2 will display blue.

c Find the probability that both spinners will display red.

d What is the probability that one of the spinners displays red and the other displays blue?

e What is the probability that both spinners display the same colour?

PROBLEM SOLVING AND REASONING 8, 9 8–11

8 Two dice are rolled for a board game. The numbers showing are then added together to get a number between 2 and 12.

a Draw a table to show the sample space and state the number of outcomes.

b Find the probability that the two dice add to 5.

c Find the probability that the two dice add to an even number.

d What is the most likely sum to occur?

e What are the two least likely sums to occur between 2 and 12 ?

Write the sum in each cell. For example, for (1, 1), write 2.

		Die 1					
		1	2	3	4	5	6
Die 2	1						
	2						
	3						
	4						
	5						
	6						

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9 In Rosemary's left pocket she has two orange marbles and one white marble. In her right pocket she has a yellow marble, a white marble and 3 blue marbles. She chooses a marble at random from each pocket.

The left-pocket outcomes are W, O, O.

- **a** Draw a table to describe the sample space.
- **b** Find the probability that she will choose an orange marble and a yellow marble.
- **c** What is the probability that she chooses a white marble and a yellow marble?
- **d** What is the probability that she chooses a white marble and an orange marble?
- **e** Find the probability that a white and a blue marble are selected.
- **f** What is the probability that the two marbles selected are the same colour?

Example 8 Finding the sample space for events without replacement

The letters T R E K are written on cards. A letter is chosen and not replaced so that this letter is now unavailable. Then a second letter is chosen.

- **a** In a table, list the outcomes from choosing two letters without replacement.
- **b** Find the probability that the two letters chosen are (E, R), in that order.
- **c** Find the probability of obtaining an outcome with an E in it.

SOLUTION

a

		1st choice			
		T	R	E	K
2nd choice	T	×	(R, T)	(E, T)	(K, T)
	R	(T, R)	×	(E, R)	(K, R)
	E	(T, E)	(R, E)	×	(K, E)
	K	(T, K)	(R, K)	(E, K)	×

EXPLANATION

List all the outcomes, writing the '1 st choice' letter first in each cell. Note that the same letter cannot be chosen twice.

b $P(\text{E, R}) = \frac{1}{12}$

Without replacement, there are 12 outcomes.

c $P(\text{includes E}) = \frac{6}{12} = \frac{1}{2}$

6 of the 12 outcomes contain an E.

10 Two letters are chosen from the word DOG *without replacement*.

- **a** Copy and complete the given table.

		1st		
		D	O	G
2nd	D	×	(O, D)	(G, D)
	O		×	
	G			×

'Without replacement' means the first letter chosen is removed, so it is not possible to choose it again.

- **b** Find the probability of obtaining the (G, D) outcome.
- **c** Find the probability of obtaining an outcome with an O in it.

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11 Two digits are selected *without replacement* from the set $\{1, 2, 3, 4\}$.

a Draw a table to show the sample space.

b Find:

i $P(1, 2)$

ii $P(4, 3)$

c Find the probability that:

i both numbers will be at least 3

ii the outcome will contain a 1 or a 4

iii the outcome will contain a 1 and a 4

iv the outcome will not contain a 3

Remember that doubles such as (1, 1), (2, 2) etc. are not allowed.

'At least 3' includes 3.

ENRICHMENT – 12

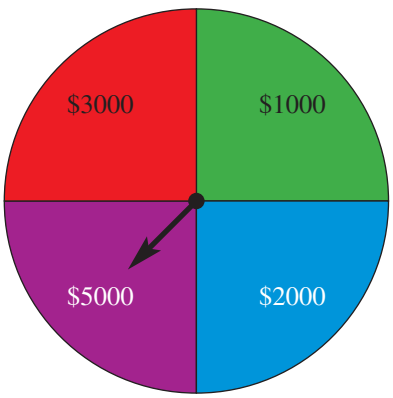
Game show

12 A wheel is spun during a game show to determine the prize money, and then a six-sided die is rolled. The prize money shown on the wheel is multiplied by the number on the die to give the total winnings.

a What is the probability that a contestant will win \$6000?

b What is the probability that a contestant will win more than \$11 000?

Draw a table to show all outcomes.
List the prize in brackets under each outcome.



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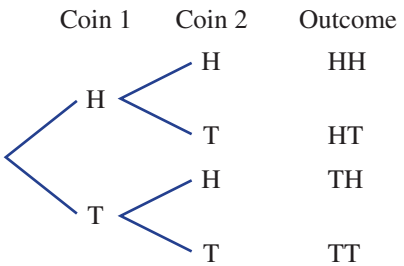
10D Using tree diagrams

Interactive
Widgets
HOTsheets
Walkthrough

When two coins are tossed, we can draw a table to list the sample space. However, if three coins were tossed, we would need a three-dimensional table to list all outcomes. Imagine trying to find probabilities when five coins are tossed!

Another tool that mathematicians use for probability is the tree diagram. The tree diagram shown here describes the four outcomes when two coins are tossed.



Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

It is important to be able to read a tree diagram correctly. The first row (HH) represents the outcome where the first coin tossed was heads and the second coin tossed was heads. The third row (TH) represents the outcome where the first coin was tails and the second was heads.

Let's start: Coin puzzle

- If two coins are tossed, rank these outcomes from most likely to least likely.
  - **a** Getting exactly two heads
  - **b** Getting exactly one head and exactly one tail
  - **c** Getting at least one coin showing tails
  - **d** Getting three tails.
- How might the order change if three coins are flipped? Compare your answers with other students.

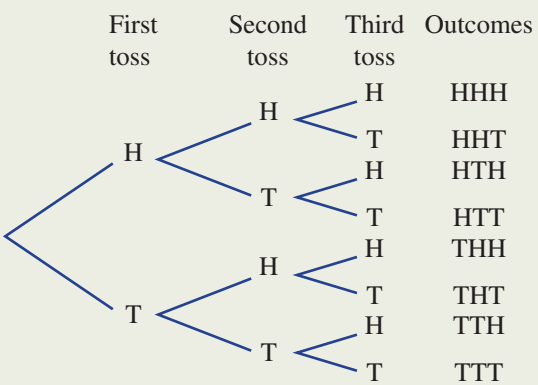
Key ideas

■ **Tree diagrams** are used to list the sample space for multistage experiments with two or more steps.

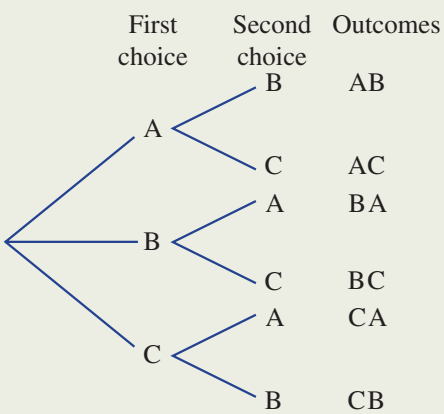
- The outcomes for each stage of the experiment are listed vertically and each stage is connected with branches. For example:

Tree diagram A diagram used to show all the possibilities when several different options are available

Tossing a coin 3 times (with replacement)



Selecting 2 letters from {A, B, C} (without replacement)



In these examples, each set of branches produces outcomes that are all equally likely. Tree diagrams can also be used where all outcomes are not equally likely.

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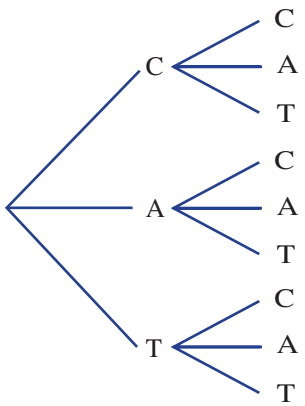
Exercise 10D

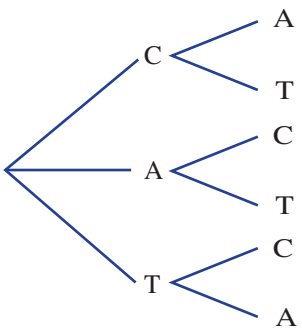
UNDERSTANDING AND FLUENCY	1–6	3–7

1 These two tree diagrams show the selection of two letters from the word CAT.

A



B



a Which tree diagram shows selection with replacement?

b Which tree diagram shows selection without replacement?

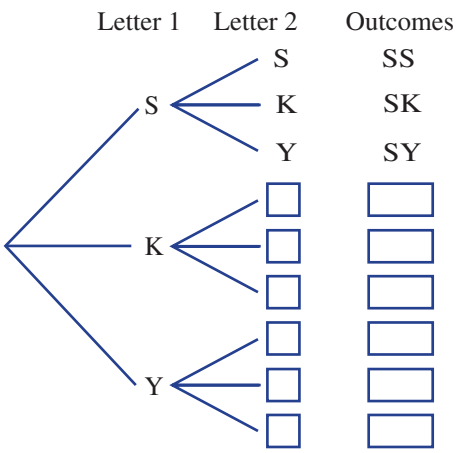
2 The letters S, K and Y are written on cards and placed in a hat. Two letters are randomly selected one at a time.

a Write the missing word or number from each statement about this event.

i 'With _____' means the first letter is returned to the hat before the second letter is selected.

ii When the first letter chosen is replaced, there are still _____ letters to choose from for the second letter.

b Copy and complete this tree diagram for selecting two letters *with replacement*.



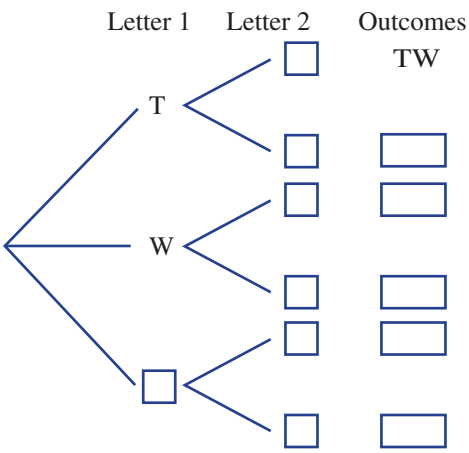
3 The letters T, W and O are written on cards and placed in a container. One letter is randomly selected and not returned to the container. A second letter is then selected.

a Write the missing word or number from each statement about this event.

i '_____ _____' means the first letter chosen is placed aside and the second letter is selected from the remaining letters.

ii Since the first letter chosen is not replaced, there will be only _____ letters to choose from for the second letter.

b Copy and complete this tree diagram for selecting two letters *without replacement*.



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Example 9 Constructing a tree diagram

An experiment involves tossing two coins.

a Complete a tree diagram to show all possible outcomes.

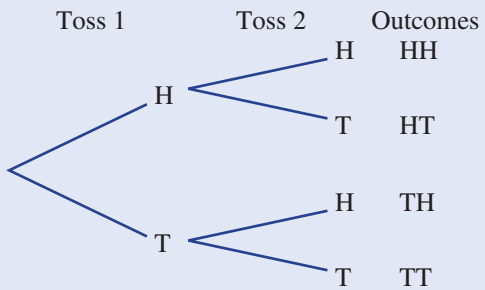
b What is the total number of outcomes?

c Find the probability of tossing:

i two tails **ii** one tail **iii** at least one head

SOLUTION

a



b The total number of outcomes is 4

c **i** $P(\text{TT}) = \frac{1}{4}$

ii $P(1\,\text{tail}) = \frac{2}{4} = \frac{1}{2}$

iii $P(\text{at least 1 head}) = \frac{3}{4}$

EXPLANATION

Tree diagram shows two coin tosses one after the other, resulting in $2 \times 2 = 4$ outcomes.

There are four possibilities in the outcomes column.

One out of the four outcomes is TT.

Two outcomes have one tail: {HT, TH}

Three outcomes have at least one head: {HH, HT, TH}

4 A coin is tossed twice.

a Complete this tree diagram to show all the possible outcomes.

b What is the total number of outcomes?

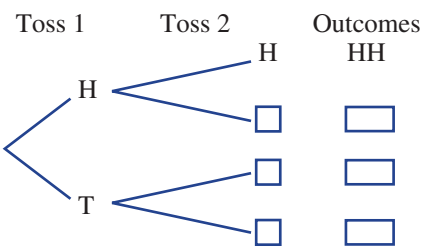
c Find the probability of obtaining:

i two heads

ii exactly one head

iii at least one head

iv at least one tail



'At least one' means one or two in this case.

5 A spinner with numbers 1, 2 and 3 is spun twice.

a Draw a tree diagram and list the outcomes.

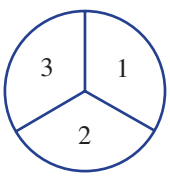
b Find $P(1 \text{ then } 1)$.

c Find $P(1 \text{ then } 2)$.

d Find $P(1 \text{ and } 2 \text{ spun in either order})$.

e Find $P(\text{both show the same number})$.

f Find $P(\text{numbers add to } 4)$.



Tree diagram headings:

First spin Second spin Outcomes

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Example 10 Constructing a tree diagram without replacement

Two people are selected without replacement from a group of three: Annabel (A), Brodie (B) and Chris (C).

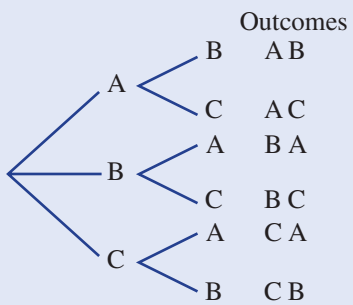
a List all the possible combinations for the selection using a tree diagram.

b Find the probability that the selection will contain:

- **i** Annabel and Brodie
- **ii** Chris
- **iii** Chris or Brodie

SOLUTION

a



EXPLANATION

On the first choice there are three options (A, B or C) but on the second choice there are only two remaining.

b **i** $P(\text{Annabel and Brodie}) = \frac{2}{6}$

$= \frac{1}{3}$

2 of the 6 outcomes contain Annabel and Brodie (A, B) and (B, A).

ii $P(\text{Chris}) = \frac{4}{6}$

$= \frac{2}{3}$

4 out of the 6 outcomes contain Chris.

iii $P(\text{Chris or Brodie}) = \frac{6}{6}$

$= 1$

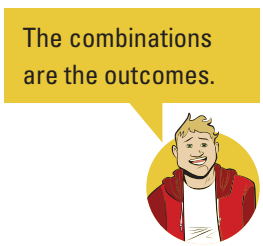
All of the outcomes contain at least one of Chris or Brodie.

6 Two people are selected without replacement from a group of three: Donna (D), Elle (E) and Fernando (F).

a List all the possible combinations for the selection using a tree diagram.

b Find the probability that the selection will contain:

- **i** Donna and Elle
- **ii** Fernando
- **iii** Fernando or Elle



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7 A drawer contains 2 red socks (R), 1 blue sock (B) and 1 yellow sock (Y) and two socks are selected at random without replacement.

a Complete this tree diagram.

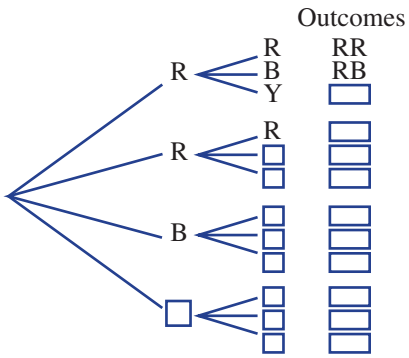
b Find the probability of obtaining:

i a red sock and a blue sock

ii two red socks

iii any pair of socks of the same colour

iv any pair of socks of different colour



PROBLEM SOLVING AND REASONING 8 8, 9

8 A coin is tossed three times.

a Draw a tree diagram and list the outcomes.

b Find P(3 tails).

c Find P(2 tails then 1 head).

d Find P(2 tails and 1 head, in any order).

e Which is more likely: getting exactly 3 tails or getting exactly 2 tails?

Use headings: 'Coin 1', 'Coin 2', 'Coin 3', 'Outcomes'

9 There are three bottles of white wine and two bottles of red wine on a shelf in a cellar. It is too dark to read the labels so two bottles are randomly selected, one at a time.

Find the probability that:

a two bottles of different colour wine are selected

b two bottles of the same colour wine are selected

c one or more bottles of red wine are selected

Draw a tree diagram using the 'without replacement' method.

ENRICHMENT – 10

Selecting matching clothes

10 A man randomly selects a tie from his collection of one green and two red ties. He selects a shirt from a collection of one red and two white. He then selects either a red hat or a black hat. Use a tree diagram to help find the probability that the man selects a tie, shirt and hat according to the following descriptions.

a A red tie, red shirt and black hat

b All three items red

c One item red

d Two items red

e At least two items red

f Green hat

g Green tie and a black hat

h Green tie or a black hat

i Not a red item

j Red tie or white shirt or black hat

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10E Using relative frequencies to estimate probabilities

Interactive

Widgets

HOTsheets

Walkthrough

The sample space is used to calculate theoretical probability. Experimental probability is calculated from the results of a survey, an experiment or a simulation. A larger number of trials make experimental probability calculations more accurate. Experimental probability is used to predict an expected number of results.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Newspaper theories

A tabloid newspaper reports that of 10 people interviewed in the street, 5 had a dose of the flu. At a similar time a medical student tested 100 people and found that 21 had the flu.

- What is the experimental probability of having the flu, according to the newspaper's survey?
- What is the experimental probability of having the flu, according to the medical student's results?
- Which of the two sets of results would be most reliable and why? Discuss the reasons.
- According to the results from the medical student, how many people would you expect to have the flu in a group of 1000, and why?

Key ideas

- The results of repeated trials of a probability experiment can be used to generate **relative frequencies**, which are sometimes called experimental probabilities.
- **Experimental probability** is calculated using the results of an experiment or survey.

$$\text{Experimental probability} = \frac{\text{number of times the outcomes occurs}}{\text{total number of trials in the experiment}}$$

- The **expected number of occurences** = probability × number of trials

$$E = P \times n$$

Relative frequency The proportion (or fraction) of times a particular outcome occurs, written as a fraction, a decimal or a percentage.

Experimental probability Probability based on measuring the outcomes of trials

Expected number of occurrences The expected number of favourable outcomes from an experiment

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Exercise 10E

UNDERSTANDING AND FLUENCY	1–6	2–7

1 Insert the word *experimental, theoretical* or *expected* to complete each statement.

a _____ probability is calculated from the results of an experiment or survey.

b _____ probability is calculated from the sample space of an event.

c When a coin is tossed, there are 2 possible equally likely outcomes. So the _____ probability of obtaining a head is $\frac{1}{2}$.

d Jayce tossed a coin 20 times and obtained a head 12 times. So the _____ probability of obtaining a head was $\frac{12}{20} = \frac{3}{5}$.

e A survey found that 3 out of every 5 students have a sister. Out of 1000 students, the _____ number of students with a sister would be $\frac{3}{5} \times 1000 = 600$.

2 Write the experimental probability from each set of results listed here.

a Out of 20 people surveyed, 18 preferred cereal for breakfast.

b 50 students were surveyed and it was found that 40 had pets.

c There were 25 boys and 20 girls at the school bus stop one afternoon. If one child is selected from those at the bus stop, state the experimental probability that the child is a boy.

In part **c**, first find the total number of students at the bus stop.

Example 11 Using experimental probability

Kris plays rugby and has a record of kicking a goal from a penalty kick 4 times out of every 7 attempts.

a State the experimental probability for Kris to achieve a penalty goal.

b Calculate the expected number of goals from 28 penalty kicks that Kris takes.

SOLUTION	EXPLANATION
a $P(\text{goal}) = \frac{4}{7}$	Experimentally, we know that Kris can kick 4 goals out of every 7 kicks.
b Expected number $= \frac{4}{7} \times 28 = 16$ goals	Expected number = probability × number of trials

3 Ashleigh has found that she can shoot a basketball through the hoop 4 times out of 10 from the '3-point' area.

a State the experimental probability for Ashleigh to shoot a basketball through the hoop from the 3-point area.

b Calculate the expected number of times the ball would go through the hoop from 100 shots Ashleigh makes from the 3-point area.

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4 The experimental probability of Jess hitting a bullseye on a dartboard is 0.05 (or $\frac{5}{100}$). How many bullseyes would you expect Jess to get if he threw the following number of darts?

a 100 darts
b 200 darts
c 1000 darts
d 80 darts

Expected number = probability × number of trials

5 A bus company surveyed a random selection of 90 people in one suburb. They found that 35 of these people regularly used a bus service from that suburb to the centre of the city.

a State the experimental probability for a person in that suburb to regularly use a bus service from that suburb to the city.
b If there are 2700 residents in that suburb, find the expected number who would regularly use a bus service from that suburb to the city.

Example 12 Finding the experimental probability

A box contains an unknown number of coloured balls and a ball is drawn from the box and then replaced. The procedure is carried out 100 times and the colour of the ball drawn is recorded each time. Twenty-five red balls were recorded.

a Find the experimental probability for selecting a red ball.
b Find the expected number of red balls if the box contained 500 balls in total.

SOLUTION	EXPLANATION
a $P(\text{red ball}) = \frac{25}{100}$	$P(\text{red ball}) = \frac{\text{number of red balls drawn}}{\text{total number of balls drawn}}$
$= 0.25$	There are 25 red balls and 100 balls in total.
b Expected number of red balls in 500	Expected number of occurrences
$= 0.25 \times 500$	= probability × number of trials
$= 125$	

6 A bag contains an unknown number of counters, and a counter is selected from the bag and then replaced. The procedure is carried out 100 times and the colour of the counter is recorded each time. Sixty of the counters drawn were blue.

Experimental probability $= \frac{\text{number of blue counters}}{\text{total number of counters selected}}$

a Find the experimental probability of selecting a blue counter.
b Find the expected number of blue counters if the bag contained:
i 100 counters **ii** 200 counters **iii** 600 counters

7 In an experiment involving 200 people chosen at random, 175 people said that they owned a home computer.

Expected number = probability × number of trials

a Calculate the experimental probability of choosing a person who owns a home computer.
b Find the expected number of people who would own a home computer in the following group sizes.
i 400 people **ii** 5000 people **iii** 40 people

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PROBLEM-SOLVING AND REASONING 8–10 8, 10–13

8 By calculating the experimental probability, estimate the chance that each of the following events will occur.

a Nat will walk to work today, given that she walked to work five times in the last 75 working days.

b Mike will win the next game of cards if, in the last 80 games, he has won 32.

c Brett will hit the bullseye on the dartboard with his next attempt if, in the last 120 attempts, he was successful 22 times.

9 This table shows the results of three different surveys of people in Perth about their use of public transport (PT).

Survey	Number who use PT	Survey size	Experimental probability
A	2	10	$\frac{2}{10} = 0.2$
B	5	20	_____
C	30	100	_____

A large survey size makes experimental probability calculations more accurate.

a What are the two missing numbers in the experimental probability list?

b Which survey should be used to estimate the probability that a person uses public transport and why?

10 The results of tossing a drawing pin and observing how many times the pin lands with the spike pointing up are shown in the table. Results are recorded at different stages of the experiment.

Number of throws	Frequency (spike up)	Experimental probability
1	1	1.00
5	2	0.40
10	5	0.50
20	9	0.45
50	18	0.36
100	41	0.41

Which experimental probability would you choose to best represent the probability that the pin will land spike up? Why?

11 A six-sided die is rolled 120 times. How many times would you expect the following events to occur?

a Rolling a 6

b Rolling a 1 or 2

c Rolling a number less than 4

d Rolling a number which is at least 5

Use the theoretical probability to calculate the expected numbers.

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12 The colour of cars driving along a highway was noted over a short period of time and summarised in this frequency table.

Colour	White	Silver	Blue	Green
Frequency	7	4	5	4

a How many cars had their colour recorded?

b Find the experimental probability that a car's colour is:

i blue

ii white

c If the colour of 100 cars was recorded, find the expected number of:

i blue cars

ii green cars

iii blue or green cars

13 A spinner is divided into three regions not necessarily of equal size. The regions are numbered 1, 2 and 3 and the spinner is spun 50 times. The table shows the results.

Number	1	2	3
Frequency	26	11	13

The angles of the spinner regions are multiples of 90°.

a Find the experimental probability of obtaining:

i a 1

ii at least a 2

iii a 1 or a 3

b Based on these results, how many 3s would you expect if the spinner is spun 300 times?

c In fact, the spinner is divided up using simple and common fractions. Draw and label how you think the spinner regions are divided up.

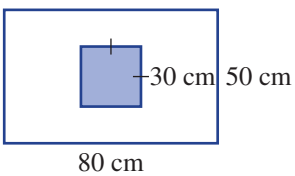
ENRICHMENT – 14

Different-shaped dartboards

14 One hundred darts are randomly thrown at the given dartboards. No darts miss the dartboard entirely. How many darts do you expect to hit the blue shaded region? Give reasons.

First find the proportion of the entire dartboard taken up by each blue region.

a



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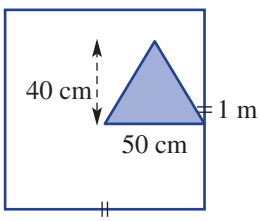

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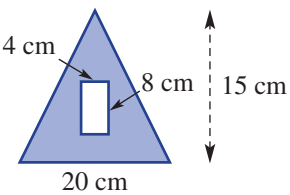

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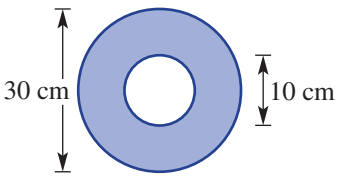

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10F Mean, median, mode and range REVISION

Interactive
Widgets
HOTsheets
Walkthrough

Statistics involves collecting and summarising data. It also involves drawing conclusions and making predictions. For example, the type and amount of product stocked on supermarket shelves is determined by the sales statistics and other measures such as average cost and price range.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

Let's start: Game purchase

Arathi purchases 7 computer games at a sale. 3 games cost \$20 each, 2 games cost \$30, 1 game costs \$50 and the last game cost \$200.

- Recall and discuss the meaning of the words *mean*, *median* and *mode*.
- Can you work out the mean, median or mode for the cost of Arathi's games?
- Which of the mean, median or mode gives the best 'average' for the cost of Arathi's games?
- Why is the mean greater than the median in this case?

Key ideas

- Statisticians use **summary statistics** to highlight important aspects of a data set. These are summarised below.
- Some summary statistics are called **measures of location**.
  - The two most commonly used measures of location are the **mean** and the **median**. These are also called '**measures of centre**' or '**measures of central tendency**'. Mean and median can only be applied to numerical data.
  - The **mean** is sometimes called the '**arithmetic mean**' or the '**average**'.
    The formula used for calculating the mean, $\bar{x}$, is:

$$\bar{x} = \frac{\text{sum of data values}}{\text{number of data values}}$$

For example, in the following data set

5	7	2	5	1

the mean is $\frac{5+7+2+5+1}{5} = 20 \div 5 = 4$

Mean ($\bar{x}$) The average value of data values

Median The middle score (or average of two middle scores) when a set of data is arranged in order

  - The **median** divides an ordered data set into two sets, each of which contain the same number of data values. It is often called the 'middle value'. The median is found by firstly ensuring the data values are in ascending order, then selecting the 'middle' value.

If the number of values is odd, simply choose the one in the middle.

2 2 3 | 4 | 6 9 9

Median = 4

If the number of values is even find the average of the two in the middle.

2 2 3 4 | 7 9 9 9

Median = $(4 + 7) \div 2 = 5.5$

  - The **mode** of a data set is the most frequently occurring data value. There can be more than one mode. When there are two modes, the data is said to be **bimodal**. The mode can be very useful for categorical data. It can also be used for numerical data, but it may not be an accurate measure of centre. For example, in the data set below the mode is 10, but it is the largest data value.

1	2	3	10	10

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Key ideas

- An **outlier** is a data value which is significantly greater than or less than the other data values in the data set. The inclusion of an outlier in a data set can inflate or deflate the mean and give a false impression of the 'average'.
  For example:

  In the data set 1, 2, 3, 4, 5, the mean is 3 and median is 3.

  If the data value 5 is changed to 20 it is now an outlier.

  In the data set 1, 2, 3, 4, 20, the median is still 3, but the mean is now 6, which is greater than four of the five data values. In this case the median is a better measure of centre than the mean.

Mode The most frequently occurring value in a set of data

Outlier Any value that is much larger or much smaller than the rest of the data in a set

Range The difference between the highest data value and the lowest data value in a set of data

■ Some summary statistics are called **measures of spread**

- The **range** is a measure of spread.
- The range of a data set is given by the formula:
  range = maximum data value − minimum data value
  For example, in the data set below the range is 7 − 1 = 6

5	7	2	5	1

maximum minimum

- Later, you will encounter some other measures of spread such as interquartile range and standard deviation.

Exercise 10F REVISION

UNDERSTANDING AND FLUENCY 1–5, 6(½), 7, 8 5, 6(½), 7–9

1 **a** To calculate the _____, you add up all the values and divide by the number of values.

b Find the mean of these data sets (round each answer to one decimal place).

i 3 6 8 10 12

ii 6 8 4 12 9 11 16 13

iii 9 12 1 5 7 10 5 7 13 15

iv 32 35 31 45

First find the sum of the scores, then divide by the number of scores.

Mean = $\frac{39}{5}$

2 **a** The mode is the most _____ value.

b State the mode of these data sets.

i 1 2 2 2 4 5 6

ii 1 4 8 8 9 10 10 10 12

iii 1 5 7 7 8 9 13

iv 21 23 25 23 21 25 27 25

3 **a** The median is the _____ value when the scores are listed in order.

b Find the median of these data sets.

i 1 3 5 6 8

ii 4 5 7 9 10 11

iii 5 6 1 2 8 9 3

iv 16 12 15 11 10 13 17 20

Order the data sets in iii and iv.

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4 **a** The _____ is the difference between the highest and lowest values.

b Find the range of these data sets.

i 3 6 7 12 15 24

ii 5 12 14 18 23 27

iii 22 24 26 27 30 35

iv 8 14 5 22 27 32 16

Range = highest value − lowest value

5 **a** An _____ is a score that is much larger or smaller than the rest of the data.

b For each of these data sets, if there is an outlier, state its value.

i 2 4 5 4 6 2 3 36

ii 21 25 3 27 28 24 29 30

iii 13 15 10 12 17 20 19 22

iv 24 21 100 23 25 27 30 25 25

Example 13 Finding measures of centre and the range

For the given data sets, find the:

i mean **ii** median **iii** mode **iv** range

a 5 2 4 10 6 1 2 9 6

b 17 13 26 15 9 10

SOLUTION	EXPLANATION
a i Mean $= \frac{45}{9}$ $= 5$	$5 + 2 + 4 + 10 + 6 + 1 + 2 + 9 + 6 = 45$ Find the sum of all the numbers and divide by the number of values.
ii 1 2 2 4 5 6 6 9 10 Median = 5	First, order the data. The median is the middle value.
iii Mode = 2 and 6	The data set is bimodal since there are two numbers with the highest frequency.
iv Range = 10 − 1 = 9	The range is the highest score minus the lowest score.
b i Mean $= \frac{90}{6}$ $= 15$	$17 + 13 + 26 + 15 + 9 + 10 = 90$ There are 6 values.
ii 9 10 13 15 17 26 14 Median $= \frac{13 + 15}{2}$ $= 14$	First, order the data. Since there are two values in the middle, find the average of them.
iii No mode	None of the values is repeated so there is no mode.
iv Range = 26 − 9 = 17	The highest score is 26 and the lowest score is 9.

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6 For the given data sets, find the:

i mean **ii** median
iii mode **iv** range

a 7 2 3 8 5 9 8
b 6 13 5 4 16 10 3 5 10
c 12 9 2 5 8 7 2 3
d 10 17 5 16 4 14
e 3.5 2.1 4.0 8.3 2.1
f 0.7 3 2.9 10.4 6 7.2 1.3 8.5
g 6 0 –3 8 2 –3 9 5
h 3 –7 2 3 –2 –3 4

The data must be in order to locate the middle value (median).

Recall that to subtract a negative, you add a positive. e.g. $7 - (-2) = 7 + 2 = 9$

7 These data sets include an outlier. Write down the outlier and then calculate the mean and the median. Include the outlier in your calculations.

a 5 7 7 8 12 33
b 2 40 42 45 47
c 1.3 1.1 1.0 1.7 1.5 1.6 –1.1 1.5
d –58 –60 –59 –4 –64

An outlier is much larger or smaller than the other scores.

8 Decide whether the following data sets are bimodal.

a 2 7 9 5 6 2 8 7 4
b 1 6 2 3 3 1 5 4 1 9
c 10 15 12 11 18 13 9 16 17
d 23 25 26 23 19 24 28 26 27

Bimodal means that there are two modes.

9 This ordered data set shows the number of fish Daniel caught in the 12 weekends that he went fishing during the year.

1 2 3 4 4 5 6 7 7 9 11 13

a Find the range.
b Find the median.
c Find the mean.
d Find the mode.

PROBLEM-SOLVING AND REASONING

10, 11 | 10, 12, 13

10 In three running races, Paula recorded the times 25.1 seconds, 24.8 seconds and 24.1 seconds.

a What is the mean time of the races? Round to two decimal places.
b Find the median time.

11 This is a data set of six house prices in Darwin.

\$324 000 \$289 000 \$431 000 \$295 000 \$385 000 \$1 700 000

a Which price would be considered the outlier?
b If the outlier was removed from the data set, by how much would the median change? (First work out the median for each case.)
c If the outlier was removed from the data set, by how much would the mean change, to the nearest dollar? (First work out the mean for each case.)

To find the median, list the prices in order, then find the middle value.

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Example 14 Finding a data value for a required mean

The hours a shop assistant spends cleaning the store in eight successive weeks are:
8, 9, 12, 10, 10, 8, 5, 10

a Calculate the mean for this set of data.

b Determine the score that needs to be added to this data to make the mean equal to 10.

SOLUTION	EXPLANATION
a Mean $= \frac{72}{8}$ $= 9$	$8 + 9 + 12 + 10 + 10 + 8 + 5 + 10 = 72$ Sum of the 8 data values is 72.
b Let a be the new score. Require $\frac{72 + a}{8 + 1} = 10$ $\frac{72 + a}{9} = 10$ $72 + a = 90$ $a = 18$ The new score would need to be 18.	$72 + a$ is the total of the new data and $8 + 1$ is the new total number of scores. Set this equal to the required mean of 10. Solve for a. 9 scores have a mean of 10, so the sum of the scores $= 9 \times 10 = 90$. Write the answer.

12 A netball player scored the following number of goals in her 10 most recent games:
15 14 16 14 15 12 16 17 16 15

a What is her mean score?

b What number of goals does she need to score in the next game for the mean of her scores to be 16?

13 Stevie obtained the following scores on her first five Maths tests: 92 89 94 82 93

a What is her mean test score?

b If there is one more test left to complete, and she wants to achieve an average of at least 85, what is the lowest score Stevie can obtain for her final test?

ENRICHMENT – 14

Aiming for an A

14 A school gives grades in Mathematics each semester according to this table. Raj has scored the following results for four topics this semester, and has one topic to go. 75 68 85 79

Average score	Grade
90–100	A+
80–	A
70–	B+
60–	B
50–	C+
0–	C

a What is Raj's mean score so far?

b What grade will Raj get for the semester if his fifth score is:

i 50? **ii** 68? **iii** 94?

c Find the maximum average score Raj can receive for the semester. Is it possible for him to get an A+?

d Find the least score that Raj needs in his fifth topic for him to receive an average of:

i B+ **ii** A

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10G Interpreting data from tables and graphs

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

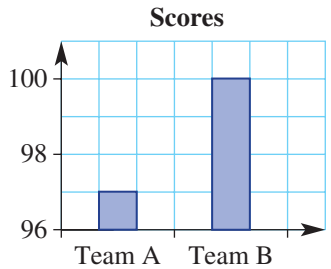
Interactive
Widgets
HOTsheets
Walkthrough

In our everyday lives, it is important to be able to understand many forms of information. Data that is presented in a table or a graph is much easier to interpret than a long list of data. The headings in a table add detail and graphs give a visual comparison between values or categories.

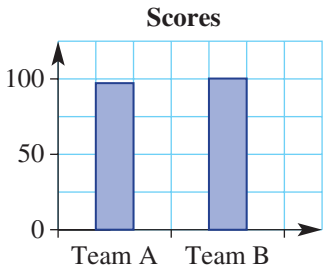
Let's start: An unfair comparison

Each pair of graphs below shows the same data. Identify which one of the pair is misleading and discuss the reasons why.

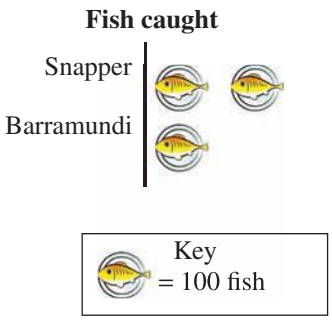
Graph A1



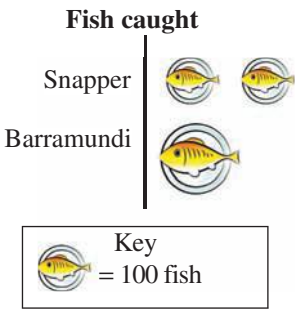
Graph A2



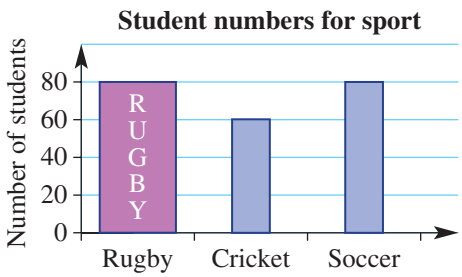
Graph B1



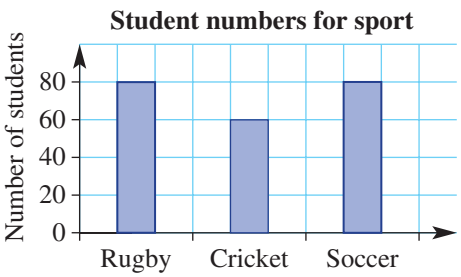
Graph B2



Graph C1



Graph C2



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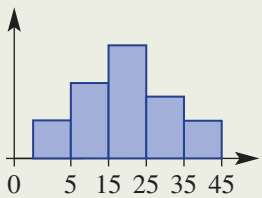
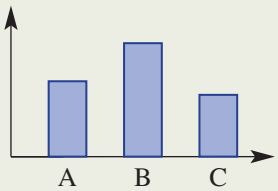
Key ideas

- Graphs can be presented in different forms. Some common examples are:

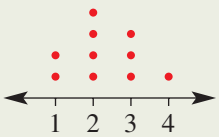
Histogram



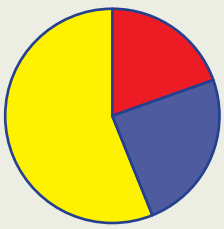
Column graph



Dot plot



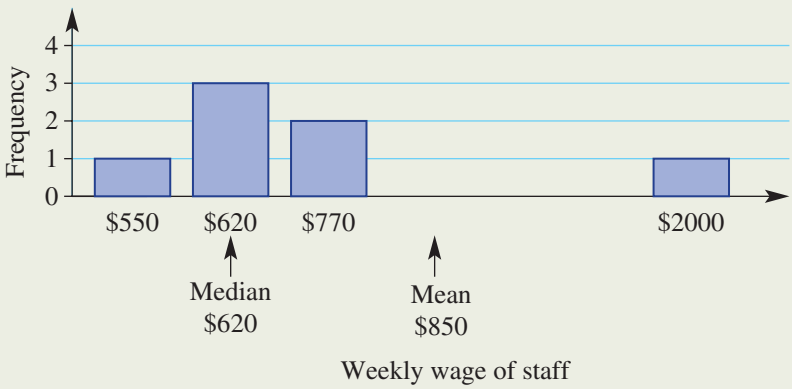
Pie chart



  - The titles, scales and column heights or sector angles tell us something about the data that is used to make the graphs.
- Misleading graphs give a false impression about data.
  - When only part of the scale is shown, the difference between results may be exaggerated.
  - If one column has a different size or colour, it may appear to have greater value than the other columns.
  - When pictograph symbols are not the same size, the larger symbol appears to have a higher value.
  - A measure of centre can be misleading if it is not a fair representation of the centre of the data values.

For example, for the data in the histogram below, the mean wage of $850 is actually larger than all other wages except the outlier of $2000. So the median (middle) wage of $620 is a better representation of the 'average' wage in this case.

Weekly wages for a small business



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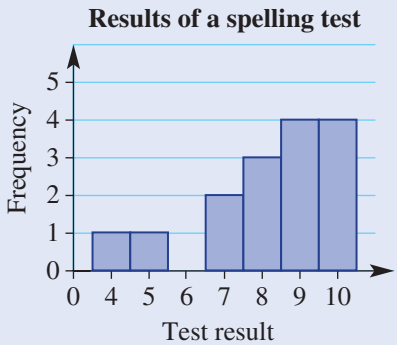
Exercise 10G

UNDERSTANDING AND FLUENCY	1–6	4–7

Example 15 Interpreting histograms

This histogram shows the results of a spelling test out of 10.

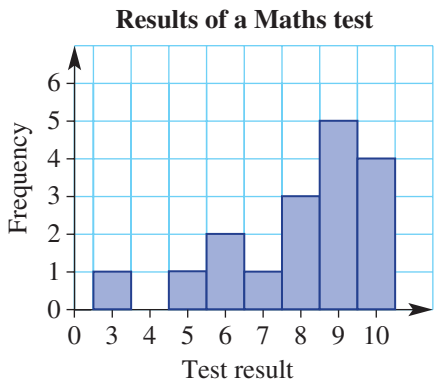
a How many results are shown in this histogram?
b List the results in ascending order.
c Calculate the mean.
d Calculate the median.
e What is the range of results from this test?
f Is this data skewed or symmetrical?
g What proportion of results is greater than or equal to 7?



SOLUTION	EXPLANATION
a 15	The frequency shows how many times each score occurred. Add the frequency values to find the total number of scores. $1 + 1 + 2 + 3 + 4 + 4 = 15$
b 4, 5, 7, 7, 8, 8, 8, 9, 9, 9, 9, 10, 10, 10, 10	1 lot of 4, 1 lot of 5, 2 lots of 7, 3 lots of 8, 4 lots of 9, 4 lots of 10.
c Mean $= \frac{123}{15}$ $= 8.2$	$4 + 5 + 7 + 7 + 8 + 8 + 8 + 9 + 9 + 9 + 9 + 10 + 10 + 10 + 10 = 123$
d Median = 9	The 8th score is the middle score.
e Range $= 10 - 4$ $= 6$	The range is the highest score minus the lowest score.
f Skewed	Results are bunched to the higher scores. The graph is not symmetrical.
g $\frac{13}{15}$	13 scores are greater than or equal to 7 out of a total of 15 scores.

1 This histogram shows the results of a Maths test out of 10.

a How many results are shown in this histogram?
b List the results in ascending order.
c Calculate the mean.
d Calculate the median.
e What is the range of results from this test?
f Is this data skewed or symmetrical?
g What proportion (fraction) of results is greater than or equal to 8?



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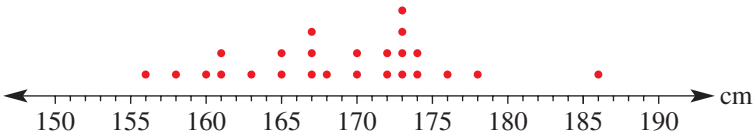
2 This dot plot shows the heights, in cm, of the students in a Year 9 class.

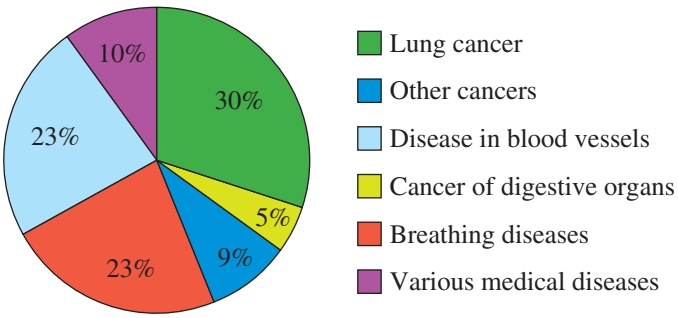


Each dot represents a student.

a How many students have their heights recorded on this dot plot?
b What is the range of heights?
c What is the mode of these heights?
d What is the median height for this class?
e What is the mean height for this class?
f What value is the outlier?

3 This pie chart shows the proportions of deaths in Australia from diseases caused by smoking.

Deaths from smoking-related diseases



Around 15 000 Australians in total die from smoking-related illness per year.

a What is the total percentage of smoking deaths caused by cancer?
b How many Australian smokers die of smoking-related cancer in a year?
c How many Australian smokers die of smoking-related cancer each day? Round to the whole number.
d How many Australian smokers die from various breathing diseases each day? Round to one decimal place.
e Calculate the sector angle for lung cancer.

Number of deaths $= \dfrac{\text{percentage}}{100} \times \text{total deaths}$

Sector angle = percentage × 360°

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4 The following table shows tide times and heights in December for Yamba, on the NSW north coast. The tide heights (in metres) shown are red for low tide and blue for high tide. The times are in 24-hour time.

1520 in 24-hour time is 3:20 p.m. in 12-hour time.

Saturday 17		*Sunday 18*		*Monday 19*		*Tuesday 20*		*Wednesday 21*	
Time	Height	Time	Height	Time	Height	Time	Height	Time	Height
0038	1.14	0145	1.18	0255	1.26	0401	1.37	0502	1.49
0619	0.45	0729	0.49	0847	0.50	1008	0.46	1123	0.38
1243	1.42	1340	1.34	1445	1.26	1556	1.21	1703	1.18
1925	0.29	2018	0.29	2115	0.28	2211	0.26	2307	0.22

a How high is the second high tide on Saturday 17 December?

b What time is the first low tide on Monday 19 December? Write your answer in both 24-hour time and 12-hour time.

c How much later in the morning is the low tide on Tuesday 20 December than the low tide on Monday 19 December?

d What is the difference in height between the two high tides on Wednesday 21 December?

e How long is it between the two high tides on Sunday 18 December?

5 The following is a table of life expectancy estimates for people in various countries.

Country	Life expectancy in years		
	Overall average	Male	Female
Japan	83	79	86
Switzerland	82	80	84
Australia	81	79	84
Malaysia	74	72	77
Vietnam	74	72	77
Indonesia	71	69	73
Papua New Guinea	57	55	60
South Africa	49	49	50
Mozambique	39	38	39

a How many years more do Australian males expect to live than Mozambiquan males?

b How many years more do Australian females expect to live than Papua New Guinean females?

c If the overall world average of life expectancy is 67 years, how far above the average is Japan's overall average life expectancy?

d Find the increase in life expectancy from Indonesian males to Australian males. Now calculate this increase as a percentage of Indonesian male life expectancy. Round to one decimal place.

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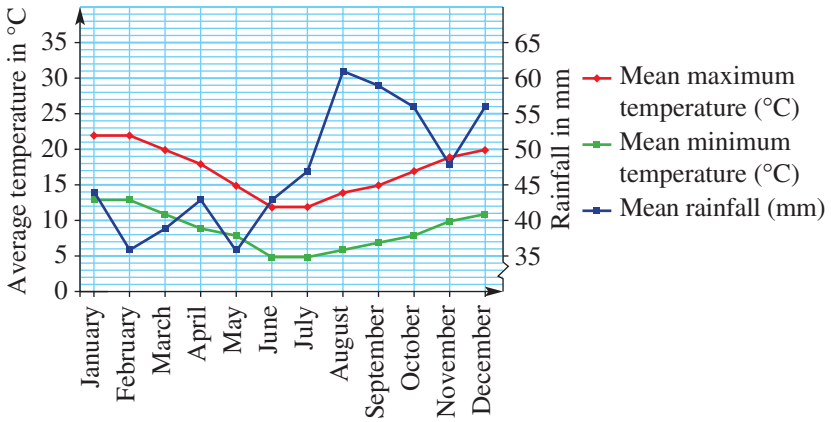
6 The line graphs below show the mean monthly minimum and maximum temperatures and the monthly rainfall in mm for Hobart.

The temperature values are read from the scale on the left and the rainfall values are read from the scale on the right. For example, in January the mean maximum temperature is 22°C and the rainfall is 44 mm.

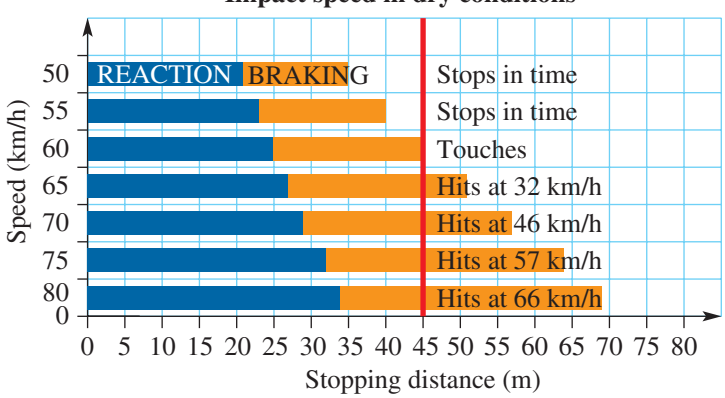
Hobart monthly temperature and rainfall



a What is the mean maximum temperature in April?
b What is the rainfall in April?
c What is the mean minimum temperature in July?
d What is the rainfall in July?
e During what months is the mean maximum temperature greater than 19°C?
f During what months is the rainfall less than 40 mm per month?
g Which are the wettest two months in Hobart?
h What is the temperature range in December?

7 The following graph shows stopping distances (reaction time and distance travelled while braking) for cars when driving at various speeds on a dry road.

Impact speed in dry conditions



Reaction time is shown by the blue bar, braking time is shown by the orange bar.

a The red line on this graph represents an object or person in front of the braking car. How far in front of the car is the object or person at the start of the driver's reaction time?
b How many metres are travelled during *reaction time* when driving at 60 km/h?
c How many metres are travelled during *braking time* when driving at 60 km/h?
d How much distance is needed, overall, to stop when driving at 80 km/h?
e By how much does the braking distance increase when driving at 80 km/h compared to 50 km/h?

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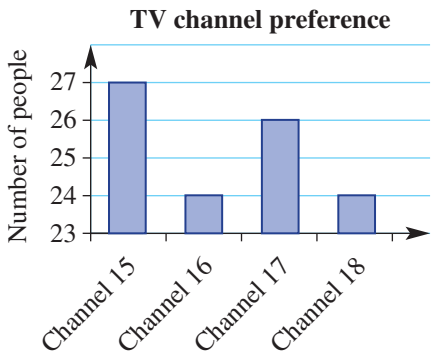
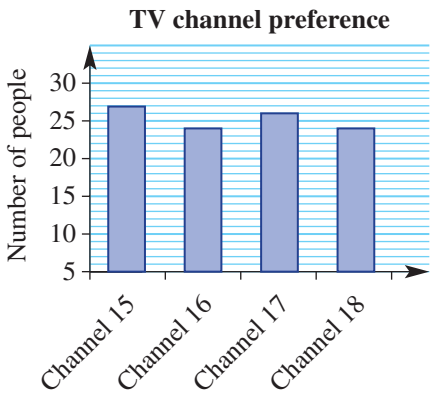
PROBLEM SOLVING AND REASONING 8, 9 8–11

8 Here are two column graphs, each showing the same results of a survey that asked people which TV channel they preferred.

Graph A

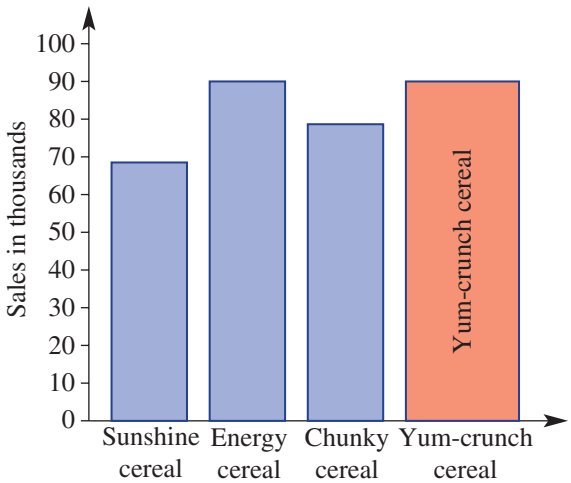


Graph B



a From graph A, write down how many viewers preferred each channel.
b Which graph could be titled 'Channel 15 is clearly most popular'?
c Which graph could be titled 'All TV channels have similar popularity'?
d What is the difference between the two graphs?
e Which graph is misleading, and why?

9 This column graph shows the number of sales for one month of four popular breakfast cereals. The height of each column represents the sales.



a List the breakfast cereals in order of sales.
b Did Yum-crunch have more sales than the other cereals?
c List three changes that have been made to the Yum-crunch column to make it look as though it has better sales than any of the other cereals.
d How should the columns be drawn so the graph is not misleading?

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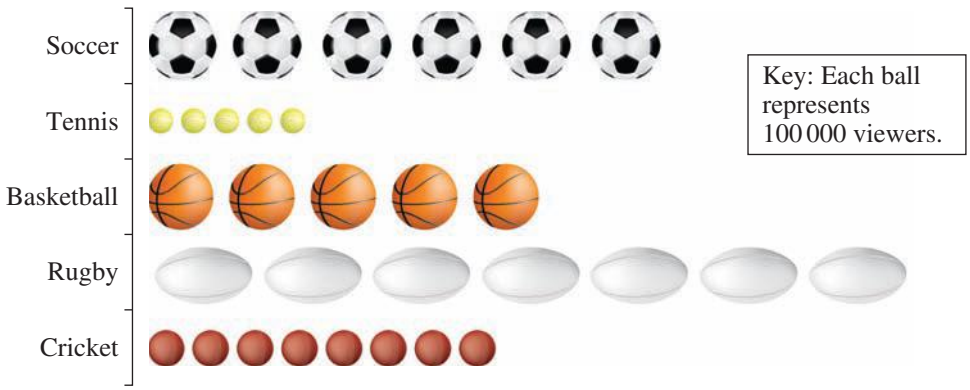
10 This pictograph shows the number of TV viewers for the final matches of various sports.

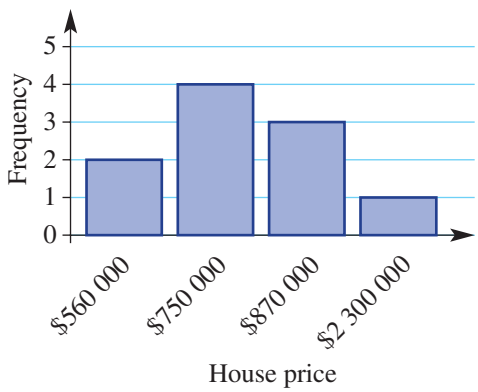
Size of TV audience for finals matches



a Which sport *appears* to have had the greatest TV audience?
b List the sports in order, according to their *length* on the pictograph.
c Using the key, determine the audience sizes for the rugby and soccer finals.
d Which sport actually had the largest TV audience? What was the size of its audience?
e List the sports in order according to the audience size calculated by using the key.
f In what way is this graph misleading?
g How should a pictograph be drawn so it is not misleading?

11 This histogram shows the price of some houses that were sold during one November in a Sydney suburb.

House sales in November



a How many house sales are shown in this histogram?
b List the price of each house in ascending order.
c Find the mean house price.
d Find the median house price.
e A newspaper headline read ‘House prices now average over $900 000’. Is this ‘average’ referring to the mean or median price?
f What proportion of house prices were less than the mean value?
g Do you think the mean or the median would better represent the ‘average’ house price? Give a reason for your answer.

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ENRICHMENT – 12

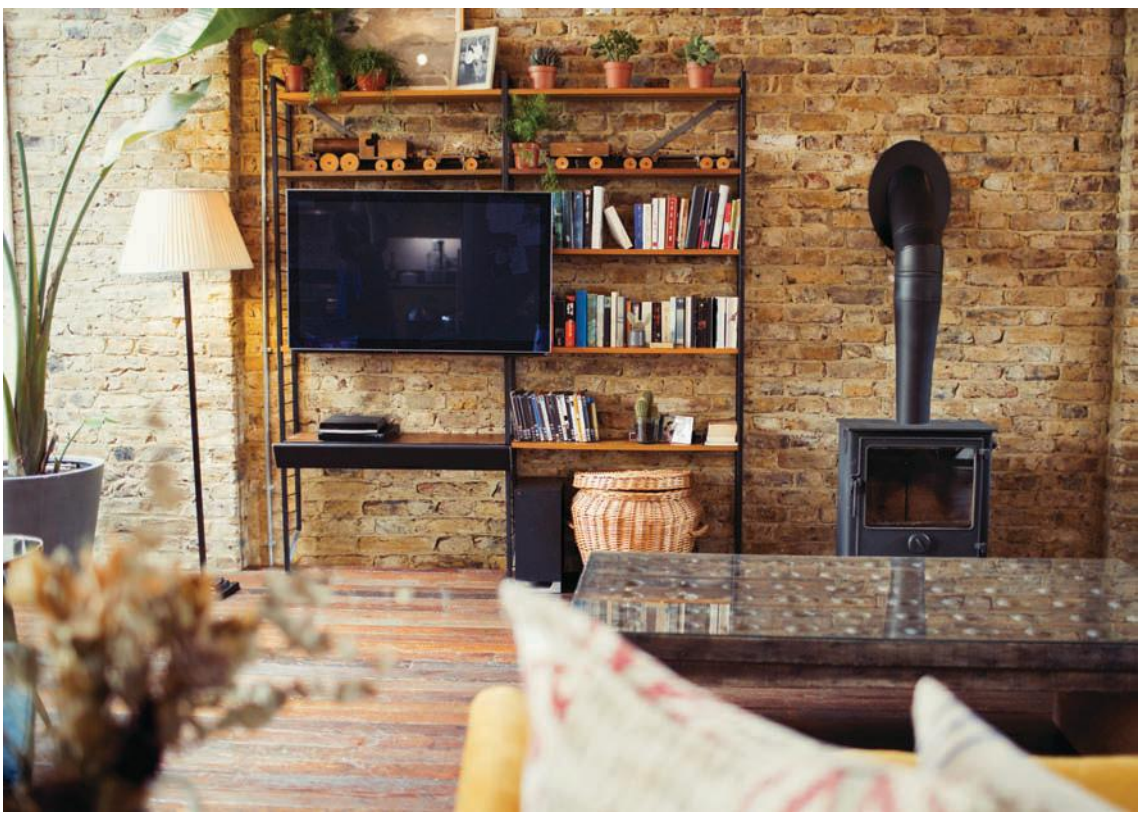
Viewing distance for TV

12 a Draw a line graph showing the minimum and maximum viewing distances for each TV screen size. Include a key.

TV screen size (length of diagonal in inches)	Maximum viewing distance (cm)	Minimum viewing distance (cm)
26	200	100
30	220	120
34	260	130
42	320	160
47	360	180
50	380	200
55	420	220
60	470	230
65	490	250

b If a lounge chair is 1.5 m in front of the TV, which sizes of TV would be suitable?

c If a lounge chair is 3 m in front of the TV, which sizes of TV would be suitable?



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10H Stem-and-leaf plots

Interactive
Widgets
HOTsheets
Walkthrough

Stem-and-leaf plots (or stem plots) are commonly used to display a single data set or two related data sets. They help to show how the data is distributed. They retain all the individual data elements so no detail is lost. The median and mode can be easily read from a stem-and-leaf plot because all the data sits in order.

Stage
5.3#
5.3
5.3§
5.2
5.2◊
5.1
4

For the data below, digits representing the stem give the tens, and digits representing the leaves give the units.

Let's start: Ships vs Chops

At a school, Ms Ships' class and Mr Chops' class sit the same exam. The scores are displayed using this back-to-back stem-and-leaf plot. Discuss the following.

- Which class had the most students?
- What were the lowest and highest scores from each class?
- What were the median scores from each class?
- Which class could be described as symmetrical and which as skewed?
- Which class had the better results?

Leaf Ms Ships' class	Stem	Leaf Mr Chops' class
3 1	5	0 1 1 3 5 7
8 8 7 5	6	2 3 5 5 7 9 9
6 4 4 2 1	7	8 9 9
7 4 3	8	0 3
6	9	1

7|8 means 78

Key ideas

- A **stem-and-leaf plot** is a way to display numerical data.
- Each number is split into a stem (the first digit or digits) and a leaf (the last digit).
  For example:

	Stem	Leaf
The number 7 is	0	7
The number 31 is	3	1
The number 152 is	15	2

- Leaves are aligned vertically, getting bigger as you move away from the stem.

Stem-and-leaf plot
A graph that lists numbers in order, grouped in rows

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Key ideas

- **Back-to-back stem-and-leaf plots** can be used to compare two sets of data. The stem is drawn in the middle, with the leaves on either side.

Scores for the last 30 AFL games

Winning scores		Losing scores
	7	4 5 8 8 9
1	8	0 0 3 3 6 7
8 7 5	9	1 2 3 6
8 4 4 1	10	3 9
9 5 0	11	1
1	12	

10 | 9 means 109

Lowest winning score was 81

Highest losing score was 111

Back-to-back stem-and-leaf plot A visual representation of two data sets

Symmetrical data A distribution of data which is symmetrical either side of the mean and the median

Skewed data A distribution of data which is sloped more to one side than the other. The mean and the median have different values

- **Symmetrical data** will produce a graph that is symmetrical about the centre. The 'winning scores' are symmetrical.

- **Skewed data** will produce a graph that lacks symmetry. The 'losing scores' are skewed.

Scores for the last 30 AFL games

Winning scores		Losing scores
	7	4 5 8 8 9
1	8	0 0 3 3 6 7
8 7 5	9	1 2 3 6
8 4 4 1	10	3 9
9 5 0	11	1
1	12	

10 | 9 means 109

'Shape' of the data is symmetrical

'Shape' of the data is skewed, not symmetrical

Exercise 10H

UNDERSTANDING AND FLUENCY 1–7 4–8

1 List the numbers in these stem-and-leaf plots, following the key.

a

Stem	Leaf
3	5 7
4	1 3 8

3|1 means 3.1

b

Stem	Leaf
5	2
6	0 1 7
7	3 5

6|3 means 63

The same stem goes with each leaf along each row.

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2 **a** Rewrite this stem-and-leaf plot with the leaves in order from smallest to largest.

Stem	Leaf
5	8 6 1 7
6	7 3 0 2
7	3 5 1 4

b If the stems are units and the leaves are tenths, write a possible key.

In part **b**, the key must include a decimal point.

3 This stem-and-leaf plot shows the number of minutes Alexis spoke on her phone for a number of calls.

Stem	Leaf
0	8
1	5 9
2	1 1 3 7
3	4 5

2|1 means 21 minutes

Count the leaves to find the number of scores.

a How many calls are represented by the stem and leaf plot?

b What is the length of the:

i shortest phone call?

ii longest phone call?

c What is the mode (the most common call time)?

d What is the median call time (middle value)?

4 This back-to-back stem-and-leaf plot shows the thickness of tyre tread on a selection of cars from the city and country.

City		Country
8 7 3 1 0 0 0	0	6 8 8 9
8 6 3 1 0	1	0 4 5 5 6 9
1	2	3 4 4

1|3 means 13 mm

06 is just written as 6.

a How many car tyres were tested altogether?

b What was the smallest tyre tread thickness in the:

i city?

ii country?

c What was the largest tyre tread thickness in the:

i city?

ii country?

d Find the median tyre tread thickness for tyres in the:

i city

ii country

e Is the distribution of tread thickness for city cars more symmetrical or skewed?

f Is the distribution of tread thickness for country cars more symmetrical or skewed?

Skewed distributions have only a few scores at one end.

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Example 16 Constructing and using a stem-and-leaf plot

Consider the following set of data.

0.3 2.5 4.1 3.7 2.0 3.3 4.8 3.3 4.6 0.1 4.1 7.5 1.4 2.4
5.7 2.3 3.4 3.0 2.3 4.1 6.3 1.0 5.8 4.4 0.1 6.8 5.2 1.0

a Organise the data into an ordered stem-and-leaf plot.
b Find the median.
c Find the mode.
d Describe the symmetry/skewness of the data.

SOLUTION

a

Stem	Leaf
0	1 1 3
1	0 0 4
2	0 3 3 4 5
3	0 3 3 4 7
4	1 1 1 4 6 8
5	2 7 8
6	3 8
7	5

3|4 means 3.4

b $\text{Median} = \dfrac{3.3 + 3.4}{2}$
$= 3.35$

c Mode is 4.1.

d Data is approximately symmetrical.

EXPLANATION

The minimum is 0.1 and the maximum is 7.5, so stems range from 0 to 7.
Place leaves in order from smallest to largest. Some numbers appear more than once; e.g. two instances of 0.1 means that the leaf 1 appears twice: 0.1, 0.1, …

There are 28 data values. The median is the average of the two middle values (the 14th and 15th values).

The most common value is 4.1.

The distribution of numbers is approximately symmetrical about the stem containing the median.

5 For each of the following data sets:
- **i** organise the data into an ordered stem-and-leaf plot
- **ii** find the median
- **iii** find the mode
- **iv** describe the symmetry/skewness of the data.

a 41 33 28 24 19 32 54 35 26 28 19 23 32 26 28

b 31 33 23 35 15 23 48 50 35 42 45 15 21 45
51 31 34 23 42 50 26 30 45 37 39

c 34.5 34.9 33.7 34.5 35.8 33.8 34.3 35.2 37.0 34.7
35.2 34.4 35.5 36.5 36.1 33.3 35.4 32.0 36.3 34.8

d 167 159 159 193 161 164 167 157 158 175 177 185
177 202 185 187 159 189 167 159 173 198 200

Draft version: Enter the leaves in the same order as in the question.
Final version: Re-write the stem-and-leaf plot with leaves in order. Remember to include the key.

In part **d**, use 16 as the stem for 167.

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6 The number of vacant rooms in a motel each week over a 20-week period is shown below.

12 8 11 10 21 12 6 11 12 16 14 22 5 15 20 6 17 8 14 9

a Draw a stem-and-leaf plot of this data.

b In how many weeks were there fewer than 12 vacant rooms?

c Find the median number of vacant rooms.

Example 17 Constructing back-to-back stem-and-leaf plots

A shop owner has two jeans shops. The daily sales in each shop over a 16-day period are monitored and recorded as follows.

Shop A

3 12 12 13 14 14 15 15 21 22 24 24 24 26 27 28

Shop B

4 6 6 7 7 8 9 9 10 12 13 14 14 16 17 27

a Draw a back-to-back stem-and-leaf plot.

b Compare and comment on differences between the sales made by the two shops.

SOLUTION

a

Shop A		Shop B
3	0	4 6 6 7 7 8 9 9
5 5 4 4 3 2 2	1	0 2 3 4 4 6 7
8 7 6 4 4 4 2 1	2	7

1|3 means 13

EXPLANATION

The data for each shop is already ordered. Stems are in intervals of 10. Record leaf digits for shop A on the left and shop B on the right, ordered from the middle to the outside.

b Shop A sales are generally between 12 and 28, with one low value of 3. Shop B sales are generally between 4 and 17, with one high value of 27. Shop A has a lot more high values than shop B. Shop B has more low values than shop A.

Look at both sides of the plot for the highest and lowest values and whether there are a few or many of the small and large numbers.

7 For each of the following sets of data:

i draw a back-to-back stem-and-leaf plot

ii state the smallest and largest value in each set and compare the numbers of small and large values in each set

a Set A: 46 32 40 43 45 47 53 54 40 54 33 48 39 43

Set B: 48 49 31 40 43 47 48 41 49 51 44 46 53 44

b Set A: 1 43 24 26 48 50 2 2 36 11 16 37 41 3 36
6 8 9 10 17 22 10 11 17 29 30 35 4 23 23

Set B: 9 18 19 19 20 21 23 24 27 28 31 37 37 38 39 39 39
40 41 41 43 44 44 45 47 50 50 51 53 53 54 54 55 56

c Set A: 0.7 0.8 1.4 8.8 9.1 2.6 3.2 0.3 1.7 1.9 2.5 4.1 4.3 3.3 3.4
3.6 3.9 3.9 4.7 1.6 0.4 5.3 5.7 2.1 2.3 1.9 5.2 6.1 6.2 8.3

Set B: 0.1 0.9 0.6 1.3 0.9 0.1 0.3 2.5 0.6 3.4 4.8 5.2 8.8 4.7 5.3
2.6 1.5 1.8 3.9 1.9 0.1 0.2 1.2 3.3 2.1 4.3 5.7 6.1 6.2 8.3

Order the leaves with the smallest on the inside and largest on the outside.
State whether each set has a few or many of the small numbers and large numbers.

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8 Draw a back-to-back stem-and-leaf plot for the final scores of St Kilda and Collingwood in the 24 AFL games given here.

St Kilda:	126	68	78	90	87	118
	88	125	111	117	82	82
	80	66	84	138	109	113
	122	80	94	83	106	68
Collingwood:	104	80	127	88	103	95
	78	118	89	82	103	115
	98	77	119	91	71	70
	63	89	103	97	72	68

a In what percentage of their games did each team score more than 100 points?

b Comment on the symmetry of the distribution of the scores for each team.

c Which team has scores that are more consistent? Which team has more higher scores?

Percentage = fraction × 100

PROBLEM SOLVING AND REASONING 9, 10 9, 11

9 This stem-and-leaf plot shows the time taken, in seconds, by Helena to run 100 m in her last 25 races.

a Find Helena's median time.

b What is the difference between the slowest and fastest time?

c If in her 26th race, her time was 14.8 seconds and this was added to the stem-and-leaf plot, would her median time change? If so, by how much?

Stem	Leaf
14	9
15	4 5 6 6 7 7 7 8 9
16	0 0 1 1 2 2 3 4 4 5 5 5 7 7
17	2

14|9 means 14, 9 seconds

10 Two brands of batteries were tested to determine their lifetime in hours. The data below shows the lifetime of 20 batteries of each brand.

Brand A: 7.3 8.2 8.4 8.5 8.7 8.8 8.9 9.0 9.1 9.2
9.3 9.4 9.4 9.5 9.5 9.6 9.7 9.8 9.9 9.9

Brand B: 7.2 7.3 7.4 7.5 7.6 7.8 7.9 7.9 8.0 8.1
8.3 9.0 9.1 9.2 9.3 9.4 9.5 9.6 9.8 9.8

The stems will be the 'units' value and each leaf the 'tenths' value.

a Draw a back-to-back stem-and-leaf plot for this data.

b How many batteries from each brand lasted more than 9 hours?

c Which brand shows the best performance?

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11 Find the median if all the data in each back-to-back stem-and-leaf plot was combined.

a

5	3	8 9
9 7 7 1	4	0 2 2 3 6 8
8 6 5 2 2	5	3 3 7 9
7 4 0	6	1 4

4|2 means 42

First combine each back-to-back plot into just one stem-and-leaf plot.

b

3	16	0 3 3 6 7 9
9 6 6 1	17	0 1 1 4 8 8
8 7 5 5 4 0	18	2 2 6 7
2	19	0 1

16|3 means 16.3

ENRICHMENT – 12

Birth weights and smoking

12 This back-to-back stem-and-leaf plot below shows the birth weight in kilograms of babies born to mothers who do or don't smoke.

Birth weight of babies

Smoking mothers		Non-smoking mothers
4 3 2 2	2	4
9 9 8 7 6 6 5 5	2*	8 9
4 3 2 1 1 1 0 0 0	3	0 0 1 2 2 3
6 5 5	3*	5 5 5 6 6 7 7 8
1	4	
	4*	5 5 6

2|4 means 2.4 kg

2*|5 means 2.5 kg

a What percentage of babies born to smoking mothers have a birth weight of less than 3 kg?

b What percentage of babies born to non-smoking mothers have a birth weight of less than 3 kg?

c Compare and comment on the differences between the birth weights of babies born to mothers who smoke and those born to mothers who don't smoke.

Do babies born to mothers who smoke weigh more or less than those born to mothers who don't smoke?

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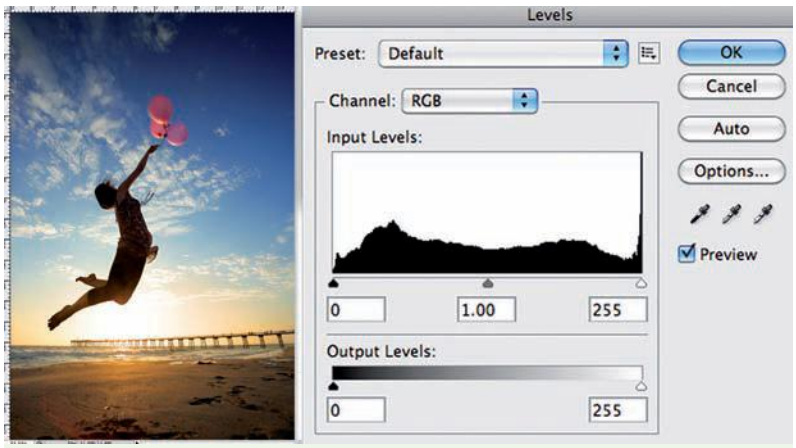
10I Grouping data into classes FRINGE

Interactive
Widgets
HOTsheets
Walkthrough

For some data, especially large sets, it makes sense to group the data and then record the frequency for each group to produce a frequency table. For numerical data, a graph generated from a frequency table gives a histogram. Like a stem-and-leaf plot, a histogram shows how the data is distributed across the full range of values. A histogram, for example, is used to display the level of exposure of the pixels in an image in digital photography. It uses many narrow columns to show how the light values are distributed across the scale from black to white.

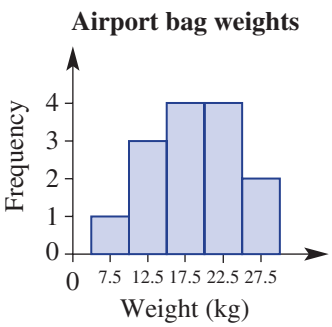


A luminance value histogram used in digital photography software

Let's start: Baggage check

This histogram shows the distribution of the weight of a number of bags checked at an airport.

- How many bags had a weight in the range 10–15 kg?
- How many bags were checked in total?
- Is it possible to determine the exact mean, median or mode of the weight of the bags by looking at the histogram? Discuss.
- Describe the distribution of checked bag weights for the given graph.



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Key ideas

- A **frequency table** shows the number of values within a set of categories or **class intervals**.
- Grouped numerical data can be illustrated using a **histogram**.
  - The height of a column corresponds to the frequency of values in that class interval.
  - There are usually no gaps between columns.
  - The scales are evenly spread with each bar spreading across the boundaries of the class interval.
  - A **percentage frequency histogram** shows the frequencies as percentages of the total.
- Like a stem-and-leaf plot, a histogram can show if the data is **skewed** or **symmetrical**.

Frequency table A table summarising data by showing all possible scores from lowest to highest in one column, and the frequency of each score in another column

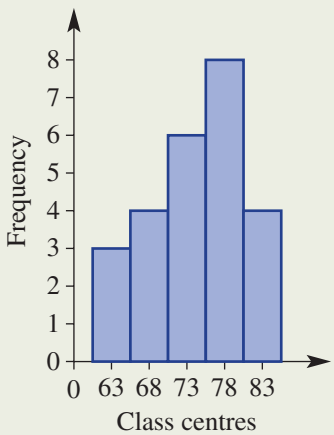
Class interval The name for a range of values that belong in a group of data

Histogram A special type of column graph with no gaps between the columns

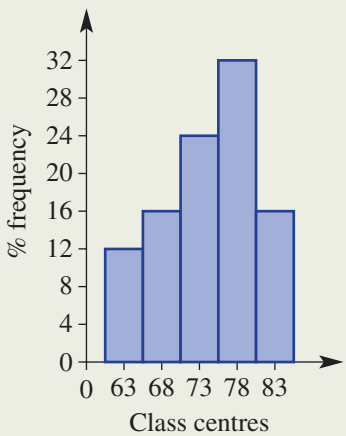
Frequency table

Class	Class centre	Frequency	Percentage frequency
61–65	63	3	12
66–70	68	4	16
71–75	73	6	24
76–80	78	8	32
81–85	83	4	16
	Total	25	100

Frequency histogram



Percentage frequency histogram



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Exercise 10I FRINGE

UNDERSTANDING AND FLUENCY	1–7	4–8

1 Write the missing word for each of these statements.

a A group of scores such as 10–15 kg is called a _____ _____.

b A _____ _____ shows the number of values in each class interval.

c A column graph that shows frequencies and scores is called a _____.

d There are _____ _____ between the columns in a histogram.

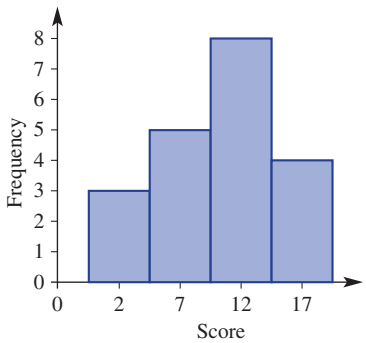
e A _____ _____ histogram shows each frequency as a percentage of the total number of scores.

2 Write the total number of scores in each of these displays.

a

Class interval	Class centre	Frequency
0–4	2	3
5–9	7	5
10–14	12	8
15–19	17	4

b

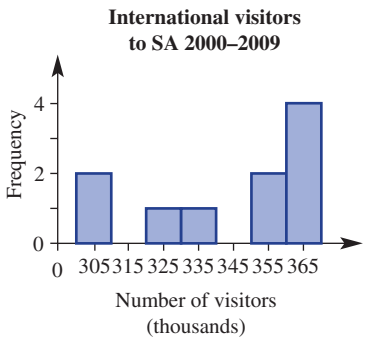


The sum of the frequencies equals the number of scores.

3 This frequency histogram shows the number of years for which the number of international visitors to South Australia was within a given range for the decade from 2000 to 2009.

a How many years in the decade were there fewer than 330 000 international visitors?

b Which range of visitor numbers had the highest frequency?



The frequency shows the number of years.

4 Some Year 9 students selected a sample of textbooks from the library and recorded the number of pages in each book. The frequency histogram on the right shows their results.

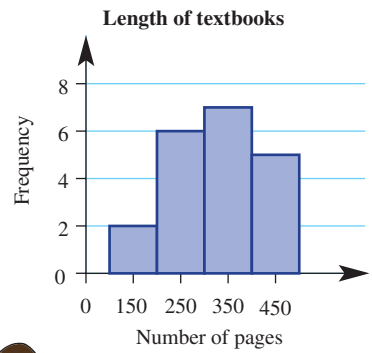
a How many textbooks had between 100 and 200 pages?

b How many textbooks were selected from the library?

c What percentage of textbooks had between:

i 200 and 300 pages?

ii 200 and 400 pages?



The frequency shows the number of textbooks.

$$\text{Percentage} = \frac{\text{frequency}}{\text{total}} \times 100$$

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Example 18 Constructing frequency tables and histograms

The data below shows the number of hamburgers sold each hour by a 24-hour fast-food store during a 50-hour period.

1	10	18	14	20	11	19	10	17	21
5	16	7	15	21	15	10	22	11	18
12	12	3	12	8	12	6	5	14	14
14	4	9	15	17	19	6	24	16	17
14	11	17	18	19	19	19	18	18	20

a Set up and complete a grouped frequency table, using class intervals 0–4, 5–9 etc. Include a percentage frequency column.

b Construct a frequency histogram.

c For how many hours did the fast-food store sell:

i fewer than 10 hamburgers?

ii at least 15 hamburgers?

SOLUTION

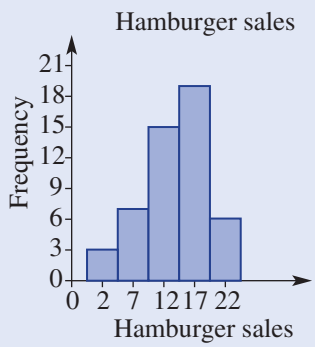
a

Class interval	Class centre	Frequency	Percentage frequency
0–4	2	3	6
5–9	7	7	14
10–14	12	15	30
15–19	17	19	38
20–24	22	6	12
Total		50	100

EXPLANATION

Create class intervals of 5 from 0 up to 25, since 24 is the maximum number. Record the number of data values in each interval in the frequency column. Convert to a percentage by dividing by the total (50) and multiplying by 100.

b



Create a frequency histogram with frequency on the vertical axis and the class intervals on the horizontal axis. The height of the column shows the frequency of that interval.

c i $3 + 7 = 10$ hours

ii $19 + 6 = 25$ hours

Fewer than 10 hamburgers covers the 0–4 and 5–9 intervals.

At least 15 hamburgers covers the 15–19 and 20–24 intervals.

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5 The data below shows the number of ice-creams sold from an ice-cream van over a 50-day period.

0	5	0	35	14	15	18	21	21	36	45	2	8
2	2	3	17	3	7	28	35	7	21	3	46	47
1	1	3	9	35	22	7	18	36	3	9	2	
11	37	37	45	11	12	14	17	22	1	2	2	

'Fewer than 20' doesn't include 20. 'At least 30' includes 30.

a Set up and complete a grouped frequency table using class intervals 0–9, 10–19, 20–29 etc. Include a percentage frequency column.

b Construct a frequency histogram.

c How many days did the ice-cream van sell:

i fewer than 20 ice-creams?

ii at least 30 ice-creams?

d What percentage of days were 20 or more ice-creams sold?

6 The data below shows the mark out of 100 on the Science exam for 60 Year 9 students.

50	67	68	89	82	81	50	50	89	52	60	82	52	60	87
89	71	73	75	83	86	50	52	71	80	95	87	87	87	74
60	60	61	63	63	65	82	86	96	88	50	94	87	64	64
72	71	72	88	86	89	69	71	80	89	92	89	89	60	83

The frequency for '50–59' will be the number of scores from 50–59.

a Set up and complete a grouped frequency table, using class intervals 50–59, 60–69, 70–79 etc. Include a percentage frequency column.

b Construct a frequency histogram.

c **i** How many marks were less than 70 out of 100?

ii What percentage of marks were at least 80 out of 100?

7 The number of goals kicked by a country footballer in each of his last 30 football matches is given below.

8	9	3	6	12	14	8	3	4	5	2	5	6	4	13
8	9	12	11	7	12	14	10	9	8	12	10	11	4	5

The frequency for '0–2' will be the number of scores that are 0, 1 or 2. Label the horizontal scale 1, 4, 7, 10 etc.

a Organise the data into a grouped frequency table using class intervals of 0–2, 3–5 etc.

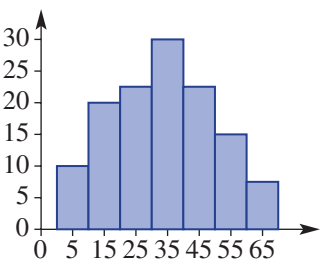
b Draw a frequency histogram for the data.

c In how many games did the player kick fewer than six goals?

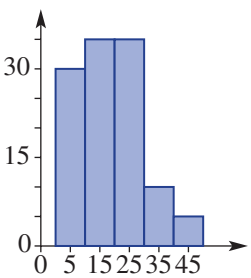
d In how many games did he kick more than 11 goals?

8 Which one of these histograms illustrates a symmetrical data set and which one shows a skewed data set?

a



b



Skewed data is bunched either below or above the middle.

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PROBLEM-SOLVING AND REASONING 9, 10 9, 11, 12

9 Write down the missing numbers in these frequency tables; i.e. find the values of the pronumerals.

a

Class interval	Class centre	Frequency	Percentage frequency
0–4	2	1	10
5–9	7	3	c
10–14	12	4	d
a–19	17	b	e
Total		10	f

b

Class interval	Class centre	Frequency	Percentage frequency
40–49	44.5	20	20
a–59	54.5	28	b
60–69	64.5	12	c
70–79	74.5	d	40
Total		100	e

10 The data below shows the length of overseas phone calls (in minutes) made by a particular household over a six-week period.

1.5	1	1.5	1	4.8	4	4	10.1	9.5	1	3
8	5.9	6	6.4	7	3.5	3.1	3.6	3	4.2	4.3
4	12.5	10.2	10.3	4.5	4.5	3.4	3.5	3.5	5	3.5
3.6	4.5	4.5	12	11	12	14	14	12	13	10.8
12.1	2.4	3.8	4.2	5.6	10.8	11.2	9.3	9.2	8.7	8.5

What percentage of phone calls were more than 3 minutes in length? Answer to one decimal place.

How many calls were 3 minutes or less?

$$\text{Percentage} = \frac{\text{frequency}}{\text{total}} \times 100$$

11 This percentage frequency histogram shows the heights of office towers in a city.

a What percentage of office towers have the following heights?

- **i** Between 50 m and 100 m
- **ii** Less than 150 m
- **iii** No more than 200 m
- **iv** At least 100 m
- **v** Between 100 m and 150 m or greater than 200 m

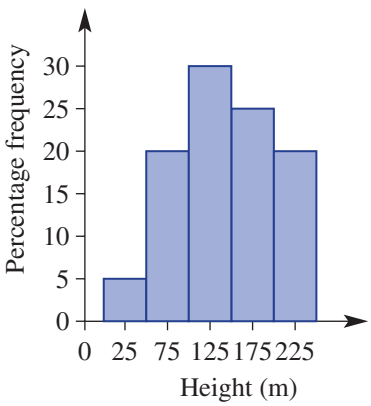
b If the city had 100 office towers, how many would have a height of:

- **i** between 100 m and 150 m?
- **ii** at least 150 m?

c If the city had 40 office towers, how many would have a height of:

- **i** between 0 m and 50 m?
- **ii** no more than 150 m?

In parts **b** and **c**, number of office towers = percentage × total



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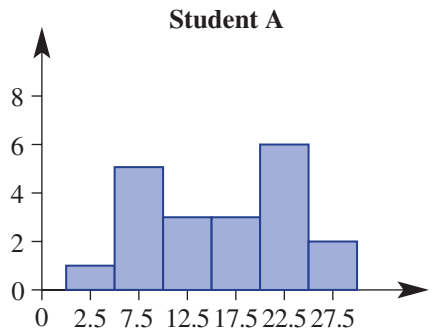
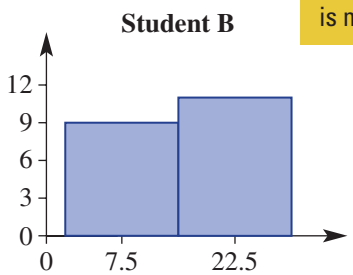
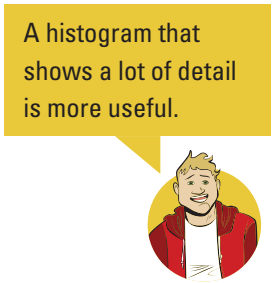
12 Two students show different histograms for the same set of data.







a Which histogram is more useful in helping to analyse the data?

b What would you advise student B to do differently when constructing the histogram?

ENRICHMENT – 13

The distribution of weekly waves

13 The data below shows the weekly wages (in dollars) of 50 people.

400 500 552 455 420 424 325 204 860 894 464 379 563 230
940 384 370 356 345 380 720 540 654 678 628 656 670 725
740 750 730 766 760 700 700 768 608 576 890 920 874 860
450 674 725 612 605 600 548 670

a What is the minimum weekly wage and the maximum weekly wage?

b **i** Organise the data into about 10 class intervals: 200–299, 300–399 etc.

ii Draw a frequency histogram for the data.

c **i** Organise the data into about four class intervals.

ii Draw a frequency histogram for the data.

d Discuss the shapes of the two graphs. Which graph represents the data better and why?

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Puzzles and challenges

1 One coin says to another coin: *How does my face appear?*

Answer these questions for each scenario, then match the letter in bold to the answers listed below to find the answer to the riddle.

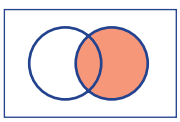
2 coins are tossed 12 times	**1 die is tossed 12 times**	**Choosing one letter from the word PUZZLES**
$P(\text{HH}) = \mathbf{D}$ $P(\text{HT or TH}) = \mathbf{I}$ Expected number of TT = **R** Expected number of one or more tails = **T**	$P(5) = \mathbf{A}$ $P(\text{greater than } 2) = \mathbf{N}$ Expected number of '6's = **S**	$P(\text{Z}) = \mathbf{M}$ $P(\text{consonant}) = \mathbf{O}$

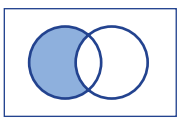
☐ ☐ ' ☐ ☐ ☐ ☐ ☐ ☐ ☐

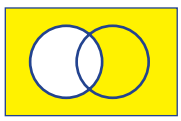
$\frac{1}{2}$ 9 2 3 $\frac{1}{6}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{5}{7}$ $\frac{2}{7}$

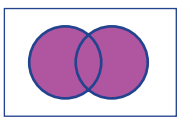
2 Match each description (**i**–**v**) with the most suitable diagram (**A**–**E**). Note: There is only one description for each diagram.

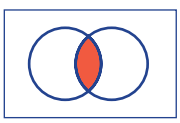
i Don't own an iPhone
ii Own an iPhone only
iii Own both an iPad and an iPhone
iv Own an iPad or an iPhone or both
v Own an iPad

A


B


C


D


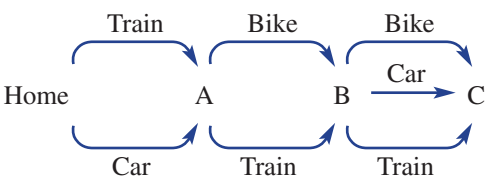
E


3 In a class of 28, each student owns a cat or a dog or both. If 18 students own cats and 16 students own dogs, how many students own both a cat and a dog?

4 Write down the set of five positive integers that has a mean of 5, a mode of 8 and a range of 6.

5 A 6-sided die and a 10-sided die are rolled simultaneously. What total sums have the highest chance of occurring? (A total sum means the sum of the two uppermost faces.)

6 Michael needs to deliver parcels to three places (A, B and C, in order) in the city. This diagram shows the different ways that he can travel.



a Draw a tree diagram showing all the possible combinations of transportation.
b What is the total number of possible outcomes?
c Find the probability that Michael will use a different method of transport each time.

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Probability and single variable data analysis

Two-step experiments

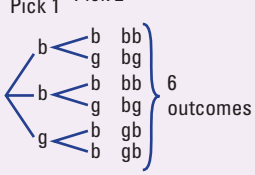
These can be represented by tables or tree diagrams. They involve more than one component and can occur with or without replacement.
e.g. a bag contains 2 blue counters and 1 green, 2 are selected at random.
(I) Table with replacement.

		Pick 1		
		b	b	g
Pick 2	b	bb	bb	gb
	b	bb	bb	gb
	g	bg	bg	gg

Sample space of 9 outcomes

$P(2 \text{ of same colour}) = \frac{5}{9}$

(II) Tree without replacement



$P(2 \text{ of same colour}) = \frac{2}{6} = \frac{1}{3}$

Probability review

The sample space is the list of all possible outcomes of an experiment.
For all possible outcomes:

$P(\text{event}) = \frac{\text{number of outcomes where event occurs}}{\text{total number of outcomes}}$

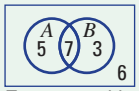
e.g. roll a normal six-sided die

$P(>4) = \frac{2}{6} = \frac{1}{3}$

$P(\text{not } A) = 1 - P(A)$

Venn diagrams and two-way tables

These organise data from two or more categories



Venn diagram

Two-way table

	A	Not A	Totals
B	7	3	10
Not B	5	6	11
Totals	12	9	21

i.e. 6 in neither category, 7 in both categories

$P(A) = \frac{12}{21} = \frac{4}{7}$ $P(A \text{ and } B) = \frac{7}{21} = \frac{1}{3}$

$P(B \text{ only}) = \frac{3}{21} = \frac{1}{7}$ $P(A \text{ or } B \text{ or both}) = \frac{15}{21} = \frac{5}{7}$

Stem-and-leaf plots

These display all the data values using a stem and a leaf. An ordered back-to-back stem-and-leaf plot.

Leaf	Stem	Leaf
9 8 7 2	1	0 3
7 4 3 3	2	2 2 4
5 2 1	3	3 6 7 8
7	4	4 5 9
0	5	0

skewed — symmetrical

3 | 5 means 35 ← key

Experimental probability

This is calculated from results of experiment or survey.

Experimental probability $= \frac{\text{number of times event occurs}}{\text{total number of trials}}$

Expected number of occurrences = probability × number of trials

Measure of spread

Range = maximum value − minimum value

Summarising data: Measures of centre

Mode is the most common value (there can be more than one). Two modes means data is bimodal.
Mean is the average of the data values.

Mean $(\bar{x}) = \frac{\text{sum of data values}}{\text{number of values}}$

Median is the middle value of data that is ordered.

Odd data set: 2 4 7 10 12 — Median

Even data set: 2 4 6 10 15 18 — $\frac{6+10}{2} = 8$

An outlier is a value that is not near the rest of the data.

Grouped data

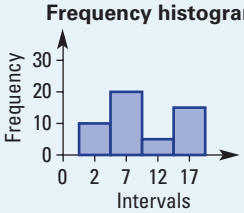
Data values can be grouped into class intervals, e.g. 0–4, 5–9, etc. and recorded in a frequency table.

The frequency or percentage frequency of each interval can be recorded in a histogram.

Frequency table

Class interval	Class centre	Frequency	Percentage frequency
0–4	2	10	20
5–9	7	20	40
10–14	12	5	10
15–19	17	15	30
Total		50	100

Frequency histogram



Chapter summary

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Chapter review

Multiple-choice questions

1 A letter is randomly chosen from the word XYLOPHONE. The probability that it is an O is:

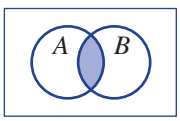
A $\frac{1}{8}$ **B** $\frac{2}{9}$ **C** $\frac{1}{4}$ **D** $\frac{1}{9}$ **E** $\frac{1}{3}$

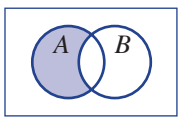
2 The values of x and y in the two-way table are:

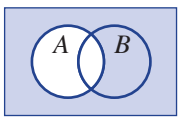
A $x = 12, y = 8$
B $x = 12, y = 11$
C $x = 16, y = 4$
D $x = 10, y = 1$
E $x = 14, y = 6$

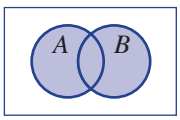
	A	Not A	Total
B		5	9
Not B	8	y	
Total	x		25

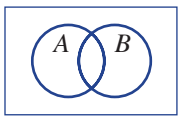
3 **i** Which shaded region represents both A and B?
ii Which shaded region represents A only?
iii Which shaded region represents A or B or both?
iv Which shaded region represents the complement of A?

A


B


C


D


E


4 A bag contains 2 green balls and 1 red ball. Two balls are randomly selected *without replacement*. Using a tree diagram, the probability of selecting one of each colour is:

A $\frac{1}{2}$ **B** $\frac{2}{3}$ **C** $\frac{5}{6}$ **D** $\frac{1}{3}$ **E** $\frac{3}{4}$

5 From rolling a biased die, a class finds an experimental probability of 0.3 of rolling a 5. From 500 rolls of the die, the expected number of 5s would be:

A 300 **B** 167 **C** 180 **D** 150 **E** 210

6 **a** The median of the data in this stem-and-leaf plot is:

A 74 **B** 71 **C** 86
D 65 **E** 70

b The range of the data in the stem-and-leaf plot is:

A 3 **B** 8 **C** 33
D 86 **E** 14

Stem	Leaf
5	3 5 8
6	1 4 7
7	0 2 4 7 9
8	2 6 6

7|4 means 74

7 If Jacob achieved scores of 12, 9, 7 and 12 on his last four language vocabulary tests, what score must he get on the fifth test to have a mean of 11?

A 16 **B** 14 **C** 11 **D** 13 **E** 15

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Short-answer questions

1 Determine the probability of each of the following.

a Rolling more than 2 on a normal six-sided die

b Selecting a vowel from the word EDUCATION

c Selecting a pink or white jelly bean from a packet containing 4 pink, 2 white and 4 black jelly beans

2 From a survey of 50 people, 30 have the newspaper delivered, 25 read it online, 10 do both and 5 do neither.

a Construct a Venn diagram for the survey results.

b How many people only read the newspaper online?

c If one of the 50 people were randomly selected, find:

i P(have paper delivered and read it online)

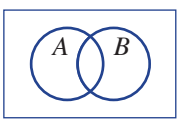
ii P(don't have it delivered)

iii P(only read it online)

3 **a** Copy and complete this two-way table.

	A	Not A	Total
B		16	
Not B	8		20
Total	17		

b Convert the information into a Venn diagram, as shown.



c Find the following:

i P(not B)

ii P (both A and B)

iii number of 'A only'

iv number in either A or B or both A and B

4 A spinner with equal areas of red, green and blue is spun and a four-sided die numbered 1 to 4 is rolled.

		Die			
		1	2	3	4
Spinner	red	(red, 1)	(red, 2)		
	green				
	blue				

a Complete a table like the one shown and state the number of outcomes in the sample space.

b Find the probability that the outcome:

i is red and an even number

ii is blue or green and a 4

iii does not involve blue

5 Libby randomly selects two coins from her pocket *without replacement*. Her pocket contains a \$1 coin and two 10-cent coins.

a List all the possible combinations using a tree diagram.

b If a chocolate bar costs \$1.10, find the probability that she can hand over the two coins to pay for it.

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6 A quality controller records the frequency of the types of chocolates from a sample of 120 off its production line.

Centre	Soft	Hard	Nut
Frequency	50	22	48

a What is the experimental probability of randomly selecting a nut centre?

b In a box of 24 chocolates, how many would be expected to have a non-soft centre?

7 Claudia records the number of emails she receives each weekday for two weeks as follows.

30 31 33 23 29 31 21 15 24 23

Find the:

a mean

b median

c mode

d range

8 Two mobile phone salespeople are both aiming to be promoted to assistant store manager. The best salesperson over a 15-week period will achieve the promotion. The number of mobile phones they sold each week is recorded below.

Employee 1: 21 34 40 38 46 36 23 51 35 25 39 19 35 53 45

Employee 2: 37 32 29 41 24 17 28 20 37 48 42 38 17 40 45

a Draw an ordered back-to-back stem-and-leaf plot for the data.

b For each employee, find the:

i median number of sales

ii mean number of sales

c By comparing the two sets of data, state, with reasons, who you think should get the promotion.

d Describe each employee's data as approximately symmetrical or skewed.

9 The data below represents the finish times, in minutes, of 25 competitors in a local car rally race.

134 147 162 164 145 159 151 143 136 155 163 157 168
171 152 128 144 161 158 136 178 152 167 154 161

a Record the above data in a frequency table in class intervals of 120–129, 130–139 etc. Include a percentage frequency column.

b Construct a frequency histogram.

c Determine the:

i number of competitors that finished in less than 140 minutes

ii percentage of competitors that finished between 130 and 160 minutes

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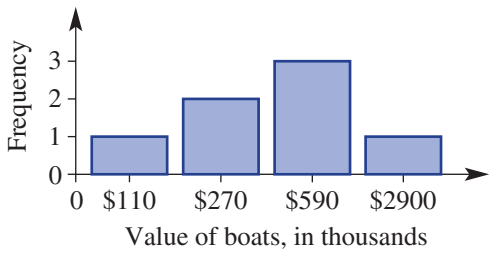
10 This column graph shows the value of some of the boats that were badly damaged in cyclone Yasi in North Queensland in 2011.

Value of boats damaged in cyclone



a Find the mean and median boat value.

b Do you think the mean or the median would better represent the 'average' of these boat values? Give a reason for your answer.

Extended-response questions

1 The local Sunday market has a number of fundraising activities.

		First spin				
		1	2	3	4	5
Second spin	1	(1, 1)	(2, 1)			
	2					
	3					
	4					
	5					

a For $1 you can spin a spinner numbered 1–5 twice. If you spin two even numbers, you receive $2 (your dollar back plus an extra dollar). If you spin two odd numbers, you receive your dollar back. For any other result you lose your dollar.

i Complete the table shown to list the sample space.

ii What is the probability of losing your dollar (i.e. spinning one odd and one even number)?

iii What is the probability of making a dollar profit (i.e. spinning two even numbers)?

iv In 50 attempts, how many times would you expect to lose your dollar?

v If you start with $100 and have 100 attempts, how much money would you expect to end up with?

b Forty-five people were surveyed as they walked through a market as to whether they bought a sausage and/or a drink from the sausage sizzle. Twenty-five people bought a sausage and 30 people bought a drink, with 15 buying both.

i Construct a Venn diagram to represent this information.

ii How many people bought neither a drink nor a sausage?

iii How many people bought a sausage only?

iv If a person was randomly selected from the 45, what is the probability they bought a drink but not a sausage?

v Find P(didn't buy a sausage).

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2 The delay time (in minutes) of the flight departure of the same evening flight of two rival airlines was recorded over 30 consecutive days. The data is shown below.

Airline A

2	11	6	14	18	1	7	4	12	14	9	2	13	4	19
13	17	3	52	24	19	12	14	0	7	13	18	1	23	8

Airline B

6	12	9	22	2	15	10	5	10	19	5	12	7	11	18
21	15	10	4	10	7	18	1	18	8	25	4	22	19	26

a Copy and complete this ordered back-to-back stem-and-leaf plot for the data. Use 2 lines for each stem, starting with '0' for leaves 0–4, and then '0*' for leaves 5–9 etc.

Airline A		Airline B
	0	
	0*	
	1	
	1*	
	2	
	2*	
	3	
	3*	
	4	
	4*	
	5	

1 | 2 means 12 1* | 5 means 15

b Does the data for airline A appear to have any outliers (numbers not near the majority of data elements)?

c By removing any outliers listed in part **b**, find the following for each airline, rounding to one decimal place where necessary.

i The median

ii The mean

d Airline A reports that half its flights for that month had a delay time of less than 10 minutes. Is this claim correct? Explain.

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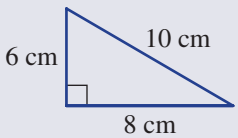
Chapter 6: Length, area, surface area and volume

Multiple-choice questions

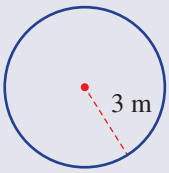
1 The perimeter and area of this triangle are:

A 48 cm, 24 m^2
B 68 cm, 80 cm^2
C 24 cm, 24 cm^2
D 24 cm, 12 cm^3
E 24 cm, 48 cm^2



2 The circumference and area for this circle, correct to two decimal places, are:

A 37.70 m, 18.85 m^2
B 9.42 m, 56.55 m^2
C 18.85 m, 28.27 m^2
D 18.85 m, 18.85 m^2
E 9.42 m, 28.27 m^2

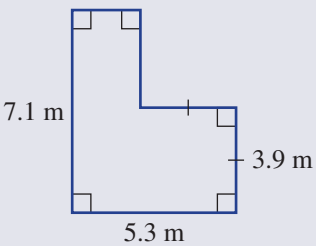


3 420 cm^2 is equal to:

A 4.2 m^2
B 0.42 m^2
C 42000 m^2
D 0.042 m^2
E 0.0042 m^2

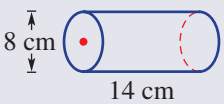
4 The perimeter and area of the figure shown are:

A 20.2 m, 22.42 m^2
B 24.8 m, 22.42 m^2
C 24.8 m, 25.15 m^2
D 20.2 m, 25.15 m^2
E 21.6 m, 24.63 m^2



5 The volume of the cylinder shown is closest to:

A 703.7 cm^3
B 351.9 cm^3
C 2814.9 cm^3
D 452.4 cm^3
E 1105.8 cm^3



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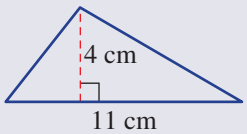
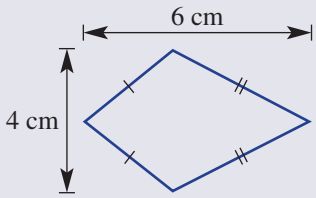
Short-answer questions

1 Find the areas of these basic shapes.

a



b



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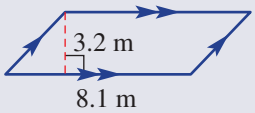
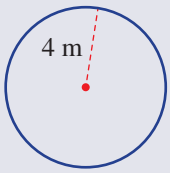
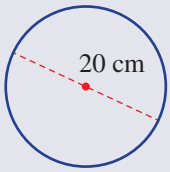


2 Find the circumferences and areas of these circles, correct to two decimal places.

a



b



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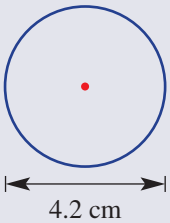
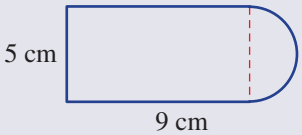
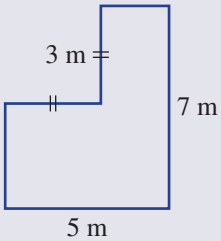


3 Find the area of each of the figures below. Round to two decimal places where necessary.

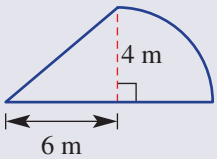
a



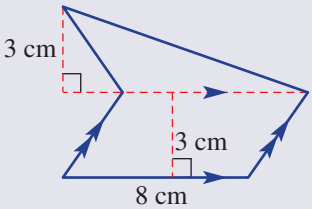
b



c

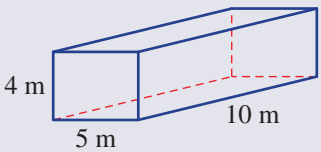


d

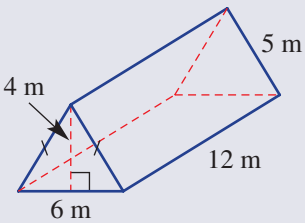


4 Find the surface areas of these solid objects. Round to two decimal places where necessary.

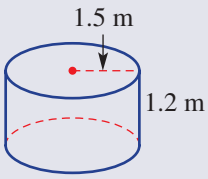
a



b



c



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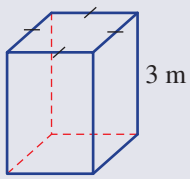
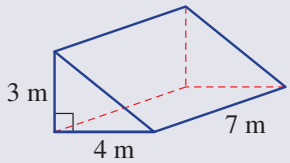
5 Find the volumes of these solids. Round to two decimal places for part **c**.

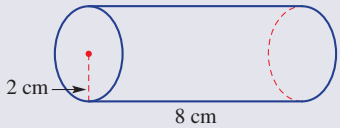
a



b

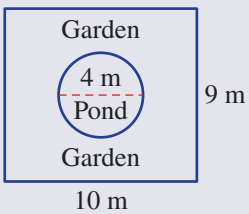


c



Extended-response question

A pond is to be built inside a rectangular garden, as shown.



a Find the area of the pond, correct to two decimal places.

b What is the area of the garden, not including the pond?

c The pond is to be surrounded by a low wall that costs \$50 per metre. What will it cost to build this wall? Round to the nearest dollar.

Chapter 7: Indices

Multiple-choice questions

1 $3a^2b^3 \times 4ab^2$ is equivalent to:

A $12a^2b^6$ **B** $7a^3b^5$ **C** $12a^3b^5$ **D** $12a^4b^5$ **E** $7a^2b^6$

2 $\left(\frac{2x}{5}\right)^3$ is equivalent to:

A $\frac{6x^3}{5}$ **B** $\frac{8x^3}{125}$ **C** $\frac{2x^3}{5}$ **D** $\frac{2x^4}{15}$ **E** $\frac{2x^3}{125}$

3 4^{-2} can be expressed as:

A $\frac{1}{4^{-2}}$ **B** $\frac{1}{8}$ **C** -16 **D** $\frac{1}{16}$ **E** -8

4 3×10^{-4} written with positive indices is:

A -3×10^4 **B** $\frac{1}{3 \times 10^4}$ **C** $\frac{-3}{10^4}$ **D** $\frac{1}{3 \times 10^{-4}}$ **E** $\frac{3}{10^4}$

5 0.00371 in scientific notation is:

A 0.371×10^{-3} **B** 3.7×10^{-2} **C** 3.71×10^{-3} **D** 3.71×10^3 **E** 371×10^3

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Short-answer questions

1 Use index laws to simplify the following.

a $5p^2q \times 3pq$ **b** $\dfrac{9a^6b^3}{18a^4b^2}$ **c** $(-3x^4y^2)^2 \times 6xy^2$ **d** $-3x^0 + (5x)^0$

2 Write each of the following using positive indices and then write as a basic numeral.

a $\dfrac{3}{4^{-2}}$ **b** 4×10^{-3}

3 Write these numbers as basic numerals.

a 2.4×10^3 **b** 1.08×10^6 **c** 7.1×10^{-3} **d** 2.06×10^{-5}

4 Write these numbers in scientific notation.

a 60 300 **b** 2 700 000 **c** 0.004 **d** 0.000703

5 Convert these numbers to the units given in brackets. Write your answer in scientific notation using three significant figures.

a 30.71 g (kg) **b** 4236 tonnes (kg)
c 3.4 hours (seconds) **d** 235 seconds (year)

Extended-response question

The average human body contains about 74 billion cells. (Note: 1 billion = 1 thousand million)

a Write this number of cells:
  i as a basic numeral
  ii in scientific notation

b If the population of a particular city is 2.521×10^6, how many human cells are there in the city? Give your answer in scientific notation, correct to three significant figures.

c If the average human weighs 64.5 kilograms, what is the average mass of one cell in grams? Give your answer in scientific notation, correct to three significant figures.

Chapter 8: Properties of geometrical figures

Multiple-choice questions

1 The supplementary angle to 55° is:

A 55° **B** 35° **C** 125° **D** 135° **E** 70°

2 A quadrilateral with all four sides equal and opposite sides parallel is best described as a:

A parallelogram **B** rhombus **C** rectangle
D trapezium **E** kite

3 The size of the interior angle in a regular pentagon with angle sum 540° is:

A 108° **B** 120° **C** 96° **D** 28° **E** 115°

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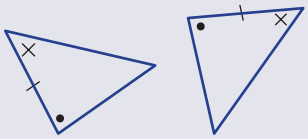
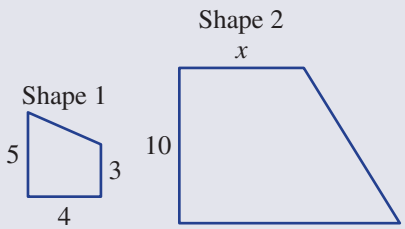
4 The test that proves congruence in these two triangles is:

A SAS
B RHS
C AAA
D SSS
E AAS



5 What is the scale factor that enlarges shape 1 to shape 2 in these similar figures, and what is the value of x?

A 2 and $x = 8$
B 2.5 and $x = 7.5$
C 3.33 and $x = 13.33$
D 2.5 and $x = 12.5$
E 2 and $x = 6$



Short-answer questions

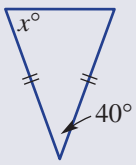
1 State the interior angle sums for these shapes.

a triangle **b** quadrilateral **c** pentagon
d heptagon **e** octagon **f** decagon

2 Find the value of each pronumeral in the following.

a

b



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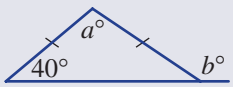

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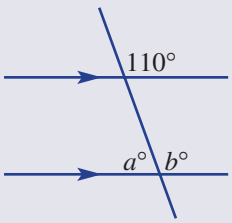

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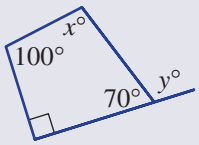

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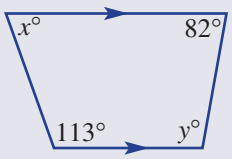

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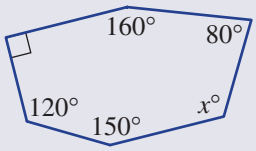

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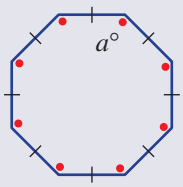

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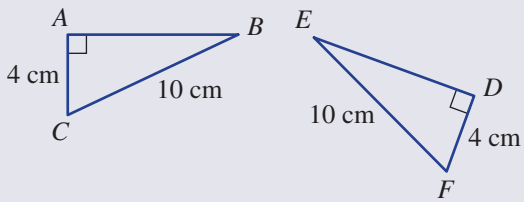
3 The two given triangles are similar.

a Are they congruent? Give a reason.

b Write a congruence statement.

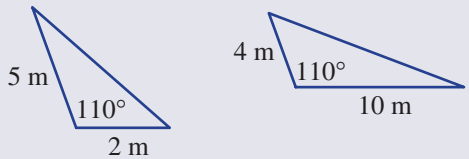
c Which side on ΔABC corresponds to side DE on ΔDEF?



4 For the given pair of triangles:

a give a reason why they are similar

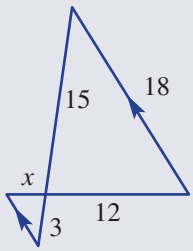
b calculate the scale factor



5 For this pair of triangles:

a give a reason why they are similar

b find the value of x



Extended-response question

A vertical wall is being supported by a piece of timber that touches the ground 10 metres from the base of the wall. A vertical metal support 4.5 m high is placed under the timber support 4 m from the wall.

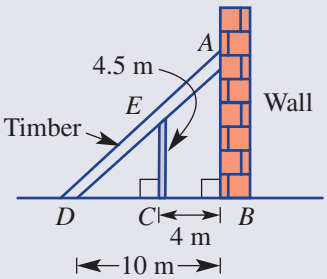
a Give a reason why $\Delta ABD \parallel\!| \Delta ECD$.

b What is the length DC?

c What is the scale factor for the two triangles formed by the timber and support?

d Find how far the timber reaches up the wall.

e How far above the ground is the point halfway along the timber support?



Chapter 9: Quadratic expressions and algebraic fractions

Multiple-choice questions

1 $-3(x - 4)$ is equal to:

A $-3x + 7$ **B** $-3x + 4$ **C** $-3x - 4$
D $-3x - 12$ **E** $-3x + 12$

2 The expanded form of $(x + 4)(3x - 2)$ is:

A $3x^2 + 12x - 8$ **B** $4x^2 + 14x - 8$ **C** $3x^2 + 12x - 10$
D $3x^2 - 8$ **E** $3x^2 + 10x - 8$

3 $(2x - 1)^2$ expands to:

A $4x^2 - 4x + 1$ **B** $2x^2 - 2x - 1$ **C** $2x^2 - 4x + 1$
D $4x^2 - 2x + 1$ **E** $4x^2 - 1$

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4 $\dfrac{2x-6}{2}$ simplifies to:

A $2x-3$ **B** $x-3$ **C** $2x-4$ **D** $x-6$ **E** $x-4$

5 $\dfrac{x+1}{3}+\dfrac{x}{4}$ simplifies to:

A $\dfrac{7x+1}{12}$ **B** $\dfrac{7x+4}{7}$ **C** $\dfrac{5x+4}{7}$ **D** $\dfrac{7x+4}{12}$ **E** $\dfrac{5x-3}{8}$

Short-answer questions

1 Expand these expressions.

a $-3(x+2)$ **b** $x(1-x)$ **c** $(x+1)(x-1)$ **d** $(x-4)^2$

e $(5x-2)(5x+2)$ **f** $(3x-4)^2$ **g** $(4x-1)(2x+3)$ **h** $5x-3(x-1)$

2 Simplify these expressions.

a $-3x \times 2yx$ **b** $7a \times 6ab^2$ **c** $4ab \div (8b)$

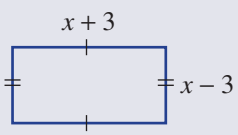
d $-6x^2y \div (xy^2)$ **e** $\dfrac{5x^2}{3} \times \dfrac{9}{10x}$ **f** $\dfrac{x-1}{2} \times \dfrac{3}{2(x-1)}$

g $\dfrac{4x-8}{x-2}$ **h** $\dfrac{-6x^2-x}{6x+1}$ **i** $\dfrac{4x}{3} \div \dfrac{8x}{9}$

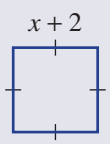
j $\dfrac{3(x-2)}{5a} \div \dfrac{x(x-2)}{15a^2}$

3 Find the areas of the following shapes in expanded form.

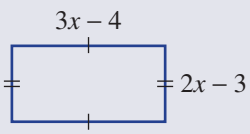
a



b



c



4 Factorise the following.

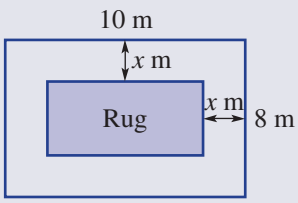
a $3x-12$ **b** $-7x-14$ **c** $5x^2+2x$ **d** $14x^2-21x$

5 Simplify these algebraic fractions.

a $\dfrac{x}{3}-\dfrac{x}{4}$ **b** $\dfrac{5x}{4}+\dfrac{7x}{5}$ **c** $\dfrac{x+1}{2}+\dfrac{x}{5}$ **d** $\dfrac{2x-1}{5}+\dfrac{x+2}{3}$

Extended-response question

A room that is 10 metres long and 8 metres wide has a rectangular rug in the middle of it that leaves a border, x metres wide, all the way around it as shown.



a Write expressions for the length and the breadth of the rug.

b Write an expression for the area of the rug in expanded form.

c What is the area of the rug when $x = 1$?

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Chapter 10: Probability and single variable data analysis

Multiple-choice questions

1 The probability of rolling a number less than five on a normal six-sided die is:

A $\frac{1}{3}$ **B** 4 **C** $\frac{1}{2}$ **D** $\frac{2}{3}$ **E** 3

2 From the two-way table, P(both A and B) is:

	A	Not A	Total
B		7	
Not B	5		
Total		11	20

A $\frac{1}{5}$ **B** 4 **C** $\frac{9}{20}$
D $\frac{1}{4}$ **E** 16

3 In a bag of 40 marbles, 28 are blue.

i The probability of selecting a blue marble is:

A 0.28 **B** 0.4 **C** 0.7 **D** 0.54 **E** 0.75

ii If a marble is selected 50 times *with replacement*, the number of blue marbles you would expect to select is:

A 28 **B** 38 **C** 70 **D** 35 **E** 40

4 The median, mean and range of the data set

12 3 1 6 10 1 5 18 11 15

are, respectively:

A 5.5, 8.2, 1–18 **B** 8, 8.2, 17 **C** 5.5, 8, 17
D 8, 8.2, 1–18 **E** 8, 74.5, 18

5 The median of the data set in the stem-and-leaf plot is:

A 55 **B** 67 **C** 7
D 69 **E** 4

Stem	Leaf
5	1 3 5 5 5 5
6	2 4 5 6 7 7 9
7	1 3 4 8 9
8	3 4
9	1 2 6

5 | 1 means 51

Short-answer questions

1 In a survey of 30 people, 18 people drink coffee during the day, 14 people drink tea and 8 people drink both. Let C be the set of people who drink coffee and T the set of people who drink tea.

a Construct a Venn diagram for the survey results.

b Find the number who:

i drink either coffee or tea or both

ii do not drink tea

c If one of the 30 people was randomly selected, find:

i P(drinks neither coffee nor tea)

ii P(C only)

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2 A coin and a four-sided die are tossed.

a Draw a table to show the sample space.

b How many outcomes are in the sample space?

c What is the probability of tossing a '*T*, 4'?

d What is the probability of tossing tails and an odd number?

3 A spinner has equal areas of the colours blue, orange and yellow. It is spun twice.

a Draw a tree diagram and list the outcomes.

b Find the probability of orange then blue.

c Find the probability that both outcomes are the same colour.

d Find the probability that blue and yellow show in any order.

4 Two ice-creams are randomly selected without replacement from a box containing one vanilla (V), two strawberry (S) and one chocolate (C) ice-creams.

a Draw a tree diagram to show each of the possible outcomes.

b What is the probability of selecting:

i a vanilla- and a strawberry-flavoured ice-cream?

ii two strawberry-flavoured ice-creams?

iii no vanilla-flavoured ice-creams?

5 The data below shows the number of aces served by a player in each of their grand slam tennis matches for the year.

15 22 11 17 25 25 12 31 26 18 32 11 25 32 13 10

a Construct a stem-and-leaf plot for the data.

b From the stem-and-leaf plot, find the mode and median number of aces.

c Is the data symmetrical or skewed?

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6 The frequency table shows the number of visitors, in intervals of 50, to a theme park each day in April.

a Copy and complete the frequency table shown. Round to one decimal place where necessary.

b Construct a frequency histogram.

c **i** On how many days were there fewer than 100 visitors?

ii What percentage of days had between 50 and 199 visitors?

Class interval	Class centre	Frequency	Percentage frequency (%)
0–49	24.5	2	
50–99	74.5	4	
100–149	124.5	5	
150–199	174.5	9	
200–249	224.5		
250–299	274.5	3	
	Total	30	

Extended-response question

A game at the school fair involves randomly selecting a green ball, and a red ball, each numbered 1, 2 or 3.

a List the outcomes in a table.

b What is the probability of getting an odd and an even number?

c Participants win $1 when they draw each ball showing the same number.

i What is the probability of winning $1?

ii If someone wins six times, how many games are they likely to have played?

d The ages of those playing the game in the first hour are recorded and are shown below.

12 16 7 24 28 9 11 17 18 18 37 9 40 16 32 42 14

i Approximately 50% of the participants are below what age?

ii If this data is used as a model for the 120 participants throughout the day, how many would be expected to be aged less than 30?

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Chapter 1

Pre-test

1 Addition: **a** Sum **b** Total **i** Add **l** More than
Subtraction: **c** Less than **g** Take away **h** Difference, **k** Minus
Multiplication: **d** Lots of **e** Product **j** Times
Division: **f** Into **m** Quotient

2 **a** 4 **b** 32 **c** 4 **d** 52 **e** 74 **f** 5 **g** 9 **h** 40

3 **a** 56 **b** 100 **c** 0 **d** 133 **e** 63 **f** 80 **g** 1020 **h** 53

4 **a** −12 **b** −3 **c** 3 **d** 0 **e** −1 **f** −1 **g** 0 **h** −12

5 **a** 3 **b** 3 **c** − **d** +

6 **a** 26 073 260 **b** 26 073 300 **c** 26 073 000 **d** 26 000 000

7 **a** 50 **b** 5 **c** 5000 **d** $\frac{5}{10}$ **e** $\frac{5}{100}$ **f** $\frac{5}{1000}$

8 **a** 2.654, 2.645, 2.564, 2.465
b 0.654, 0.564, 0.456, 0.0456

9 **a** 7.99 **b** 10.11 **c** 7.11

10 **a** 1.4 **b** 0.06 **c** 3.68 **d** 16.38 **e** 3.7 **f** 180

11 **a** 34.5 **b** 374 000 **c** 0.03754 **d** 0.000037754

12 **a** 6 **b** 12 **c** 30 **d** 1

13 **a** 15 **b** 12 **c** 10

14 **a** $\frac{5}{7}$ **b** $1\frac{1}{4}$ **c** 8 **d** $\frac{3}{8}$

Exercise 1A

1 **a** iii **b** v **c** iv **d** i **e** ii

2 **a** iv **b** ii **c** v **d** i **e** iii

3 **a** False **b** True **c** True **d** True **e** True **f** False

4 **a** −2 **b** 2 **c** −3 **d** 4 **e** −4 **f** −5 **g** −2 **h** 0 **i** −6

5 **a** 3 **b** 2 **c** 1 **d** 0 **e** −1 **f** −9 **g** −1 **h** −17 **i** −10 **j** 9 **k** −2 **l** 11 **m** −7 **n** −18 **o** −1

6 **a** 5 **b** 20 **c** −20 **d** 0 **e** −7 **f** 6 **g** −24 **h** 5 **i** −15

7 **a** 4 **b** −4 **c** −4 **d** −9 **e** −32 **f** −38 **g** −189 **h** −24 **i** −18 **j** 0 **k** −805 **l** −57 **m** −167 **n** 142 **o** −804

8 **a** 4 **b** −2 **c** 13 **d** 9 **e** 24 **f** 0 **g** −28 **h** −81 **i** 180 **j** 2 **k** −3 **l** 240 **m** 755 **n** 94 **o** 10

9 **a** −27 **b** −3 **c** 10 **d** 26 **e** 0 **f** −24 **g** −91 **h** 125 **i** −180

10 **a** 2 **b** 11 **c** −3 **d** −1 **e** −7 **f** 11

11 **a** −8 **b** 4 **c** 13 **d** 124 **e** −19 **f** 0 **g** −22 **h** 98 **i** −26

12 19°C

13 **a**

−8	6	−4
2	−2	−6
0	−10	4

b Example answer (other answers are possible):

2	−13	8
5	−1	−7
−10	11	−4

c

5	−2	7	3
0	10	−3	6
−1	4	2	8
9	1	7	−4

Exercise 1B

1 **a** negative **b** negative **c** negative **d** positive **e** positive **f** positive

2 **a**

×	−4	−3	−2	−1	0	1	2	3	4
−4	16	12	8	4	0	−4	−8	−12	−16
−3	12	9	6	3	0	−3	−6	−9	−12
−2	8	6	4	2	0	−2	−4	−6	−8
−1	4	3	2	1	0	−1	−2	−3	−4
0	0	0	0	0	0	0	0	0	0
1	−4	−3	−2	−1	0	1	2	3	4
2	−8	−6	−4	−2	0	2	4	6	8
3	−12	−9	−6	−3	0	3	6	9	12
4	−16	−12	−8	−4	0	4	8	12	16

3 **a** positive **b** negative **c** positive **d** negative **e** negative

4 **a** −48 **b** −20 **c** −72 **d** 72 **e** 45 **f** −90 **g** −600 **h** 81 **i** −144 **j** −143 **k** −143 **l** −34 **m** 72 **n** 90 **o** −108

5 **a** 36 **b** 64 **c** 144 **d** 9 **e** 100 **f** 169 **g** −8 **h** −1 **i** −64

6 **a** −9 **b** 6 **c** 10 **d** −10 **e** 3 **f** 5 **g** 2 **h** −1 **i** −8 **j** 25 **k** −104 **l** 5 **m** −37 **n** −214 **o** 13

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7 **a** 16 **b** −26 **c** −12 **d** −14 **e** −20
f −28 **g** −72 **h** −8 **i** −90 **j** 10
k −24 **l** −18 **m** 9 **n** −26 **o** 40
p −6

8 **a** 7 **b** −32 **c** −20 **d** −4 **e** −1
f 6 **g** −9 **h** −5 **i** −15

9 −5 and 2

10 **a** 12 and −12 **b** 8 and −8 **c** 100 and −100

11 Two negatives with × and ÷ produce a positive.

12 **a** positive **b** positive **c** negative
d positive **e** negative **f** positive

13 **a** −2 **b** −38 **c** −8 **d** 27
e 1 **f** 24 **g** 21 **h** 0

14 **a** −60 **b** −30 **c** 675 **d** 0
e 2250 **f** 1800 **g** 3600 **h** 675

Exercise 1C

1 **a** 2 **b** 3 **c** 1 **d** 2
e 1 **f** 0 **g** 3 **h** 1

2 **a** 2 **b** 4 **c** 1 **d** 1
e 4 **f** 3 **g** 3 **h** 4

3 **a** 40 **b** 270 **c** 7.9 **d** 0.04

4 **a** 32 100 **b** 432 **c** 5.89
d 0.443 **e** 0.00197

5 **a** 17.96 **b** 11.08 **c** 72.99 **d** 47.86
e 63.93 **f** 23.81 **g** 804.53 **h** 500.57
i 821.27 **j** 5810.25 **k** 1005.00 **l** 2650.00

6 **a** 2400 **b** 35 000 **c** 0.060 **d** 34
e 110 000 **f** 0.0025 **g** 2.1 **h** 0.71
i 4700 **j** 59 000 **k** 0.46 **l** 1.1

7 **a** 1.46, 1.5 **b** 0.09, 0.094 **c** 23.71, 24
d 0.01, 0.0078 **e** 100.47, 100

8 **a** 7 **b** 73 **c** 130 **d** 36 200

9 **a** 30 000 **b** 200 **c** 0.05 **d** 0.0006
e 5000 **f** 900 **g** 0.9 **h** 0.0003

10 **a** 3600, 3693 **b** 760, 759.4
c 4000, 4127.16 **d** 3000, 3523.78
e 0, 0.72216 **f** 1, 0.716245
g 0.12, 0.1186 **h** 0.02, 0.02254

11

$\frac{1}{3}$	$0.\dot{3}$	0.3	0.3
$\frac{2}{3}$	$0.\dot{6}$	0.7	0.7
$\frac{1}{9}$	$0.\dot{1}$	0.1	0.1
$\frac{3}{4}$	0.75	0.8	0.8

12 **a** A: 54.3, B: 53.8, 0.5 **b** A: 54.28, B: 53.79, 0.49
c A: 54, B: 54, 0 **d** A: 50, B: 50, 0

13 0.143 tonnes

14 Magnesium in this case would be zero if rounded to two decimal places rather than 2 significant figures.

15 2.14999 is closer to 2.1 correct to 1 decimal place ∴ round down.

16 **a** **i** 8 **ii** 1 **iii** 8
b 0.1428571428571
c **i** 4 **ii** 2 **iii** 8

17 Teacher to check

Exercise 1D

1 **a** True **b** False **c** True

2 **a** Yes **b** Yes **c** Yes **d** No

3 **a** $\frac{7}{10}$ **b** $\frac{3}{10}$ **c** $\frac{3}{2}$ **d** $\frac{7}{3}$

4 **a, b, c, d, e, f, g, i, k**

5 **a** 2.75 **b** 0.35 **c** 3.4 **d** 1.875
e 2.625 **f** 3.8 **g** 2.3125 **h** 0.25

6 **a** $0.\overline{27}$ **b** $0.\dot{7}$ **c** $1.\overline{285714}$ **d** $0.41\dot{6}$
e $1.\dot{1}$ **f** $3.8\dot{3}$ **g** $7.2\dot{6}$ **h** $2.\overline{63}$

7 **a** $\frac{7}{20}$ **b** $\frac{3}{50}$ **c** $3\frac{7}{10}$ **d** $\frac{14}{25}$
e $1\frac{7}{100}$ **f** $\frac{3}{40}$ **g** $3\frac{8}{25}$ **h** $7\frac{3}{8}$
i $2\frac{1}{200}$ **j** $10\frac{11}{250}$ **k** $6\frac{9}{20}$ **l** $2\frac{101}{1000}$

8 **a** $\frac{5}{6}$ **b** $\frac{13}{20}$ **c** $\frac{7}{10}$ **d** $\frac{5}{12}$
e $\frac{7}{16}$ **f** $\frac{11}{14}$ **g** $\frac{19}{30}$ **h** $\frac{11}{27}$

9 **a** $\frac{9}{20}$ **b** $\frac{3}{20}$ **c** $\frac{32}{45}$ **d** $\frac{23}{75}$

10 **a** $\frac{11}{6}, \frac{7}{3}$ **b** $\frac{2}{5}, \frac{2}{15}$ **c** $\frac{11}{12}, 1$ **d** $\frac{5}{7}, \frac{11}{14}$

11 Weather forecast website

12 **a** $\frac{3}{5}$ **b** $\frac{5}{9}$ **c** $\frac{8}{13}$ **d** $\frac{23}{31}$

13 **a** 31, 32 **b** 36, 37, 38, ... , 55 **c** 4, 5

14 **a** $\frac{8}{9}$ **b** $1\frac{2}{9}$ **c** $\frac{81}{99} = \frac{9}{11}$ **d** $3\frac{43}{99}$

Exercise 1E

1 **a** 12 **b** 6 **c** 6 **d** 8 **e** 70
f 15 **g** 49 **h** 15 **i** 35

2 **a** True **b** False

3 **a** $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$ **b** $\frac{4}{8} - \frac{3}{8} = \frac{1}{8}$ **c** $1\frac{2}{4} + \frac{1}{4} = 1\frac{3}{4}$

4 **a** 6 **b** 63 **c** 65 **d** 30
e 4 **f** 33 **g** 60 **h** 87

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5 **a** $\frac{3}{5}$ **b** $\frac{5}{8}$ **c** $\frac{7}{10}$ **d** $\frac{4}{7}$
e $\frac{5}{17}$ **f** $\frac{1}{5}$ **g** $\frac{10}{6} = 1\frac{2}{3}$ **h** $\frac{10}{8} = 1\frac{1}{4}$
i $\frac{7}{5} = 1\frac{2}{5}$ **j** $1\frac{4}{5}$ **k** $2\frac{1}{3}$ **l** $\frac{1}{2}$
m $\frac{2}{5}$ **n** $4\frac{1}{2}$ **o** $1\frac{2}{5}$

6 **a** $\frac{5}{8}$ **b** $\frac{1}{3}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$
e $\frac{9}{10}$ **f** $\frac{7}{12}$ **g** $\frac{7}{10}$ **h** $\frac{3}{20}$
i $\frac{11}{24}$ **j** $1\frac{1}{6}$ **k** $1\frac{8}{21}$ **l** $\frac{17}{24}$
m $\frac{11}{36}$ **n** $\frac{4}{7}$ **o** $\frac{19}{100}$

7 **a** $3\frac{5}{6}$ **b** $\frac{7}{12}$ **c** $2\frac{4}{5}$ **d** $2\frac{7}{12}$
e $6\frac{3}{4}$ **f** $3\frac{5}{6}$ **g** $\frac{1}{8}$ **h** $1\frac{5}{12}$
i $1\frac{7}{10}$ **j** $\frac{11}{12}$ **k** $4\frac{7}{10}$ **l** $1\frac{7}{12}$

8 $\frac{7}{8}$ tonnes

9 $5\frac{29}{56}$ tonnes

10 **a** $4\frac{19}{20}$ L **b** 33

11 $\frac{1}{20}$

12 $\frac{5}{6}$, problem is the use of negatives in the method, since $\frac{1}{3} < \frac{1}{2}$

13 **a** **i** $\frac{1}{8} + \frac{3}{4}$ **ii** $\frac{2}{3} - \frac{5}{12}$ **iii** $\frac{5}{6} - \frac{1}{2}$
b $\frac{1}{2} + \frac{2}{3} + \frac{5}{6} = 2$

Exercise 1F

1 **a** $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$ **e** $\frac{1}{2}$
f $\frac{3}{7}$ **g** $\frac{1}{3}$ **h** $\frac{3}{5}$ **i** $\frac{2}{3}$

2 **a** 12 **b** 12 **c** 6 **d** 4 **e** 180 **f** 12

3 **a** $\frac{4}{3}$ **b** 7 **c** $\frac{3}{2}$ **d** $\frac{8}{5} = 1\frac{3}{5}$ **e** $\frac{1}{6}$
f $\frac{1}{8}$ **g** $\frac{2}{3}$ **h** $\frac{3}{5}$ **i** $\frac{4}{11}$

4 **a** True **b** False **c** True
d True **e** True **f** True

5 **a** $\frac{3}{20}$ **b** $\frac{4}{15}$ **c** $\frac{15}{28}$ **d** $\frac{1}{6}$ **e** $\frac{1}{20}$
f $\frac{2}{27}$ **g** $\frac{10}{17}$ **h** $\frac{3}{7}$ **i** $\frac{2}{5}$ **j** $\frac{2}{3}$
k $\frac{2}{5}$ **l** $\frac{4}{5}$ **m** $\frac{3}{4}$ **n** $\frac{2}{3}$ **o** $\frac{2}{9}$

6 **a** $\frac{1}{2}$ **b** $1\frac{1}{4}$ **c** $2\frac{1}{10}$ **d** 2 **e** $4\frac{1}{5}$
f $\frac{6}{7}$ **g** $6\frac{1}{4}$ **h** 4 **i** $2\frac{11}{12}$ **j** 6

7 **a** $\frac{20}{21}$ **b** $1\frac{1}{8}$ **c** $\frac{45}{56}$ **d** $\frac{27}{28}$ **e** $1\frac{1}{3}$
f $1\frac{1}{2}$ **g** 6 **h** $\frac{7}{8}$ **i** 18 **j** 9
k 16 **l** 64 **m** $\frac{1}{10}$ **n** $\frac{1}{12}$ **o** $\frac{4}{27}$
p $3\frac{1}{3}$ **q** 4 **r** $\frac{1}{6}$ **s** $1\frac{1}{2}$ **t** $\frac{7}{8}$

8 **a** \$10 **b** \$3.50 **c** \$284.50 **d** \$6 **e** \$50
f \$18 **g** \$6400 **h** \$15 **i** 90c

9 **a** $\frac{2}{7}$ **b** $\frac{8}{15}$ **c** 16

10 $\frac{3}{12}$ hour = 15 minutes

11 160 km

12 90 minutes each

13 **a** **i** 0.5 **ii** 0.2 **iii** 0.16129 … **iv** $0.\dot{6}$ **v** −1
b 3.39 **c** 25
d Numbers greater than zero but less than one
e 1 **f** 0

Exercise 1G

1 **a** 4 **b** 12 **c** 24 **d** 72
e 3 **f** 9 **g** 11 **h** 3

2 **a** 9 **b** $\frac{4}{9}$ **c** $\frac{5}{9}$ **d** 8 **e** 10

3

a	1 : 3	1 : 4
b	1 : 2	1 : 3
c	2 : 3	2 : 5
d	1 : 2	1 : 3
e	1 : 3	1 : 4
f	1 : 2	1 : 3

4 **a** 5 : 1 **b** 1 : 2 **c** 2 : 1 **d** 10 : 1 **e** 5 : 16

5 **a** 1 : 5 **b** 2 : 5 **c** 4 : 5 **d** 1 : 10 **e** 5 : 6
f 1 : 3 **g** 3 : 1 **h** 2 : 3 **i** 4 : 3 **j** 4 : 1
k 1 : 2 **l** 3 : 1 **m** 5 : 14 **n** 1 : 2 : 4 **o** 3 : 6 : 2

6 **a** 1 : 2 **b** 3 : 41 **c** 5 : 32 **d** 1 : 3 **e** 1 : 5
f 10 : 3 **g** 40 : 1 **h** 1500 : 1 **i** 20 : 7

7 **a** 3 : 4 **b** 8 : 3 **c** 9 : 20
d 45 : 28 **e** 3 : 14 **f** 22 : 39

8 **a** 1 : 10 **b** 1 : 5 **c** 2 : 3 **d** 7 : 8
e 25 : 4 **f** 1 : 4 **g** 1 : 4 **h** 24 : 5
i 4 : 20 : 5 **j** 4 : 3 : 10 **k** 5 : 72 **l** 3 : 10 : 40

9 **a** \$100, \$400 **b** \$16, \$20
c 24 kg, 64 kg **d** \$56, \$40
e \$200, \$300 **f** 750 g, 1250 g
g \$70, \$30 **h** \$300, \$300
i \$14, \$49, \$7 **j** 336 g, 84 g

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10 126 g
11 120
12 \$3000, \$1200, \$1800 respectively
13 108 L
14 Amy: \$2400, Belinda: \$3600, Candice: \$1200, Diane: \$4800
15 a i 100 mL **ii** 200 mL
b i 250 mL **ii** 270 mL
c i 300 mL **ii** 1 : 4
d i 1 : 3 **ii** 7 : 19
iii 26 : 97 **iv** 21 : 52
e Jugs 3 and 4

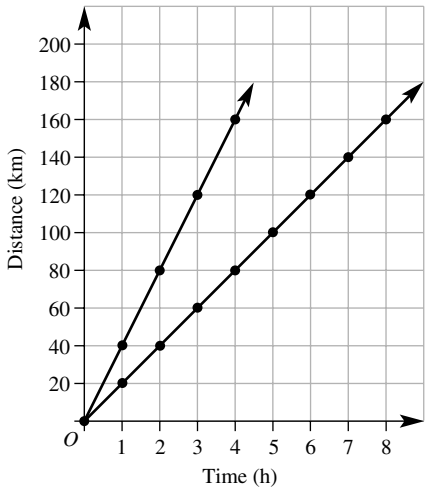
Exercise 1H

1 a 30 km/h **b** \$42/h **c** 60 km/h
d \$4/kg **e** \$16/h
2 \$90/h
3 a \$4 **b** \$3 **c** \$2.50
4 a \$84 **b** \$21 **c** \$126
5 a \$28/h **b** \$50/h **c** 6 kg/min
d \$7/kg **e** 4 runs/over **f** 89 pts/game
g 19 cm/year **h** 14 m/s
i 92 beats/min **j** 8 mL/s
6 a 70 km/h **b** 15 km/h **c** 4 km/h
d 54 km/h **e** 80 km/h
7 13 hours
8 a 3 kg deal **b** Red delicious
c 2.4 L **d** 0.7 GB
9 a i 130 words **ii** 325 words **iii** 3900 words
b i 44 g **ii** 220 g **c** \$20 000
10 a 55 km **b** $16\frac{1}{2}$km **c** $5\frac{1}{2}$km
11 a Coffee A\$3.60, coffee B\$3.90. Therefore, coffee A is the best buy.
b Pasta A \$1.25, pasta B \$0.94. Therefore, pasta B is the best buy.
c Cereal A \$0.37, cereal B \$0.40. Therefore, cereal A is the best buy.
12 a 40 sheep/h **b** 220 sheep **c** 25 hours
13 a 1 080 000 000 km/h
b 36 000 000 km
c $8\frac{5}{18}$min
14 a Yes **b** No **c** Yes **d** Yes **e** Yes
15 a

Time in hours	0	1	2	3	4	5	6	7	8
Distance in km	0	20	40	60	80	100	120	140	160

b 20 km/h

c, e



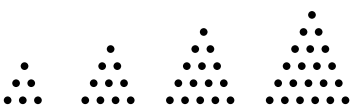
d Straight line, speed was constant

Puzzles and challenges

1 $1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9) = 100$
2 48 ways
3

1. **9**	2. **2**	3. **5**	**3**
4. **3**	**5**	5. **5**	6. **6**
7. **9**	8. **9**	9. **2**	**4**
10. **1**	**2**	**8**	**0**

4 a i $1 + 2 + 3 = 6$
ii 28 $(1 + 2 + 4 + 7 + 14 = 28)$
iii $1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496$
b i



ii 28, 36
c i 0, 1, 1, 2, 3, 5, 8, 13, 21, 34
ii −21, 13, −8, 5, −3, 2, −1, 1, 0, 1, … ∴ −21, −8, −3, −1

Multiple-choice questions

1 D **2** B **3** C **4** A **5** E
6 E **7** D **8** E **9** C **10** B
11 C **12** C **13** B

Short-answer questions

1 a 8 **b** −45 **c** 270 **d** −64 **e** 6 **f** 8
2 a 21.5 **b** 29 100 **c** 0.153 **d** 0.00241
3 a 14.98 **b** 0.71 **c** 2.00

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4 **a** 2.125 **b** $0.8\dot{3}$ **c** $1.\overline{857142}$

5 **a** $\frac{3}{4}$ **b** $1\frac{3}{5}$ **c** $2\frac{11}{20}$

6 **a** $\frac{4}{3}$ **b** 5 **c** $\frac{1}{8}$ **d** $\frac{7}{9}$

7 **a** $\frac{1}{2}$ **b** $2\frac{1}{6}$ **c** $\frac{7}{24}$ **d** $\frac{11}{12}$ **e** $\frac{3}{10}$
f 2 **g** $3\frac{3}{4}$ **h** $\frac{3}{7}$ **i** $2\frac{19}{28}$

8 **a** 5 : 2 **b** 16 : 9 **c** 75 : 14

9 **a** 50, 30 **b** 25, 55 **c** 10, 20, 50

10 **a** Store A \$2.25/kg, store B \$2.58/kg. Store A is the best buy.
b Store A 444 g/\$, store B 388 g/\$

11 \$132, \$198, \$330

12 **a** 32 **b** 7 **c** 192 **d** 14.80 **e** 82.5

13 **a** 156 km/h **b** 36 km/h

14 \$6000

15 36 m

16 Doubles the perimeter.

Extended-response questions

1 **a** 12 **b** 10 boys, 18 girls **c** 5:9

2 **a** 85.6 km/h **b** 14 hours
c 61.1 km/h **d** 611 km
e 88.168 L **f** \$131.99
g 601.94 km **h** \$79.19

Chapter 2

Pre-test

1 **a** $\frac{3}{25}$ **b** $\frac{1}{5}$ **c** $\frac{7}{20}$ **d** $\frac{3}{4}$ **e** $\frac{3}{5}$ **f** $\frac{1}{2}$

2 **a** 99 **b** 58 **c** 90 **d** 122 **e** 8 **f** 150

3 **a** 55% **b** $\frac{3}{4}$ **c** 98%

4 **a** 61 **b** 9 **c** 37 **d** 121 **e** 1 **f** 75, 4

5

Fraction	Decimal	Percentage
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%

6 **a** 10 g **b** 7 km **c** \$45
d \$800 **e** 50c **f** 9c

7 **a** \$100 **b** 60 m **c** \$1.50
d \$67.80 **e** 6 days **f** 3 days

8 **a** 100 **b** 10 **c** 20 **d** 3 **e** 4

9 2100 km

10 87% of 65

11 **a** 7 **b** 365 **c** 52 **d** 12

12 **a** 14 **b** 6 **c** 12.5

Exercise 2A

1 **a** 25% **b** 50% **c** 100% **d** 40%

2 **a** 75% **b** 70%

3

%	10%	20%	25%	50%	75%	$33\frac{1}{3}$%	$66\frac{2}{3}$%
Fraction	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$
Decimal	0.1	0.2	0.25	0.5	0.75	$0.\dot{3}$	$0.\dot{6}$

4 Sarah, $79\% > \frac{38}{50} = 76\%$

5 **a** 20% **b** 80% **c** 80% **d** 30%
e 25% **f** $12\frac{1}{2}$% **g** 75% **h** 60%
i 56% **j** 35% **k** 9% **l** $7\frac{1}{2}$%

6 **a** $\frac{19}{100}$ **b** $\frac{23}{100}$ **c** $\frac{99}{100}$ **d** $\frac{1}{20}$
e $\frac{11}{50}$ **f** $\frac{9}{20}$ **g** $\frac{37}{50}$ **h** $\frac{3}{4}$
i $\frac{1}{40}$ **j** $\frac{69}{400}$ **k** $\frac{1}{100}$ **l** $1\frac{1}{4}$

7 **a** 78% **b** 95% **c** 65% **d** 48%
e 75% **f** 142% **g** 7% **h** 30%
i 3% **j** 104% **k** 12% **l** 12.25%

8 **a** 0.12 **b** 0.83 **c** 0.57 **d** 0.88
e 0.99 **f** 1.0 **g** 1.2 **h** 0.05

9 **a** 2.5% **b** 10% **c** 5% **d** 5%
e 20% **f** 20% **g** 200% **h** 75%

10

Sport	Number of students who chose sport	Fraction of the total	Percentage of the total
Swimming	44	$\frac{22}{75}$	$29\frac{1}{3}$%
Golf	12	$\frac{2}{25}$	8%
Volleyball	58	$\frac{29}{75}$	$38\frac{2}{3}$%
Cricket	36	$\frac{6}{25}$	24%
TOTAL	150	1	100%

11 31.5% **12** $16\frac{2}{3}$% **13** $6\frac{1}{4}$%

14 Yes, 2.8% fat content

15 Answers will vary.

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Exercise 2B

1 **a** \$4 **b** \$2 **c** 2 kg
d 6 kg **e** \$8 **f** 2.8 seconds

2 \$400

3 True

4 **a** 2 **b** 10 **c** 8 **d** 7.5
e 57.6 **f** 34 **g** 1230 **h** 42
i 450 **j** 33 **k** 198 **l** 1.5

5 **a** \$36 **b** \$210 **c** 48 kg
d 30 km **e** 15 apples **f** 350 m
g 250 people **h** 200 cars **i** \$49

6 **a** \$120 **b** \$700 **c** \$300 **d** \$7
e \$0.20 **f** \$450 **g** \$800 **h** \$360

7 **a** \$540 **b** \$600 **c** \$508
d \$1250 **e** \$120 **f** \$40

8 **a** \$5.80 **b** \$4.20 **c** \$0.46
d \$0.50 **e** \$44 **f** \$40

9 64

10 **a** Divide by 10.
b Divide by 2.
c Divide by 2 and then 2 again (or just 4).

11 48 kg

12 15 students

13 $\frac{10}{100} \times 24 = \frac{24}{100} \times 10$ as $\frac{24 \times 10}{100}$, can multiply in any order

14 $x = 2$, $y = 24$

15 **a** 72 **b** $\frac{10}{11}$ **c** 280% **d** $3\frac{1}{4}$ **e** 150%

Exercise 2C

1 **a** 1.4 **b** 1.26 **c** 60% **d** 21%
e 0.8 **f** 0.27 **g** 6% **h** 69%

2 \$30

3 12 kg

4 **a** 82 m **b** 902 m **c** 738 m
d 902 m, same as increasing by 10%
e 738 m, same as decreasing by 10%

5 **a** 61.6 **b** 1176 **c** 112 **d** 934.5 **e** 207
f 540 **g** 12 **h** 196 **i** 31.50

6 **a** 76 **b** 540 **c** 22.5 **d** 616 **e** 7360
f 337.5 **g** 17 **h** 8910 **i** 0

7 **a**

Original amount	New amount	Increase	Percentage change
40	60	20	50%
12	16	4	$33\frac{1}{3}$%
100	125	25	25%
24	30	6	25%
88	100	12	13.6%
48	92	44	91.7%
200	250	50	25%

b

Original amount	New amount	Decrease	Percentage change
90	81	9	10%
100	78	22	22%
20	15	5	25%
24	18	6	25%
150	50	100	66.7%
9	8.3	0.7	7.8%
3	2.5	0.5	16.7%

8 **a** \$3 **b** \$80 **c** \$200
d \$500 **e** \$12 500

9 \$1785

10 \$84 634.25

11 24 534 cars

12 50%

13 28%

14 \$21.50

15 \$10.91

16 **a** \$900 **b** \$990
c As 10% of 1000 = 100 but 10% of 900 = 90

17 **a** \$594 **b** \$3235.65 **c** \$189 **d** \$61.48

18 **a** 79.86 g **b** \$97 240.50 **c** \$336 199.68 **d** 7.10 cm

Exercise 2D

1 \$3 profit, \$2.50 loss, \$1.40 profit, \$4.50 profit, \$65.95 loss, \$1180 profit

2 40.95, 179.95, 595.90, 799.95, 18 799, 8995

3 28, 9.05, 7.25, 199, 345.50, 2037

4 **a** 90% **b** 80% **c** 85% **d** 92%

5

Item	Cost price	% mark-up	Mark-up amount	Selling price
Jeans	\$60	28%	\$16.80	\$76.80
Toaster	\$40	80%	\$32	\$72
Car	\$22 000	45%	\$9900	\$31 900
Can of drink	\$1.20	140%	\$1.68	\$2.88
Loaf of bread	\$1.80	85%	\$1.53	\$3.33
Handbag	\$80	70%	\$56	\$136
Tablet	\$320	35%	\$112	\$432

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6

Item	Cost price	% discount	Discounted amount	Selling price
Camera	$900	15%	$135	$765
Car	$24 000	20%	$4800	$19 200
Bike	$600	25%	$150	$450
Shoes	$195	30%	$58.50	$136.50
Blu-ray player	$245	50%	$122.50	$122.50
Electric razor	$129	20%	$25.80	$103.20
Lawnmower	$880	5%	$44	$836

7 a

Cost price ($)	Selling price ($)	Profit ($)	Profit (%)
10	15	5	50%
24	30	6	25%
100	150	50	50%
250	255	5	2%
17.50	20	2.50	14.29%

b

Cost price ($)	Selling price ($)	Loss ($)	Loss (%)
10	8	2	20%
16	12	4	25%
100	80	20	20%
34	19	15	44.12%
95	80.75	14.25	15%

8 a $5.50 b $200 c $1500 d $125

9 a i $2 ii 20% b i $5 ii 25%
c i $16.80 ii 14% d i $2450 ii 175%

10 40%

11 $1001.25

12 42.3%

13 $613.33

14 a $400 b $900 c $765 d No

15 a $350 000
b i 71.4% ii 157.1% iii 114.3%
c 25%

Exercise 2E

1 a $25.40 b $101.60 c $127 d $482.60

2 a $36 b $48

3 a $11 406.25 b $2632.21 c $375

4 a $200 b $56 c $900 d $145.10

5 a $321.10 b $8.45 c $642.20 d $1391.43

6 a $41 080 b $3423.33 c $19.75

7

Employee	Hourly rate	Hours worked	Income
Adam	$10.40	8	$83.20
Betty	$15.50	$8\frac{1}{2}$	$131.75
Ceanna	$9.70	15	$145.50
David	$24.30	38	$923.40
Edward	$17.85	42	$749.70
Francis	$30	27	$810
George	$15.20	7.25	$110.20

8 a $82.80 b $119.60 c $184
d $276 e $257.60 f $404.80

9

Person	Weekly retainer	Rate of commission	Commission earned	Weekly wage
Adina	$0	12% on $7000	$840	$840
Byron	$160	8% on $600	$48	$208
Cindy	$300	5% on $680	$34	$334
Deanne	$260	5% on $40 000	$2000	$2260
Elizabeth	$500	8% on $5600	$448	$948
Faruq	$900	2% on $110 000	$2200	$3100
Gary	$1000	1.5% on $45 000	$675	$1675

10 $14.50 per hour

11 $12.20 per hour

12 $490

13 $4010

14 $839.05

15 a 7 b 18 c 33

16 a $250.55 b $46.80 c 6 hours

Exercise 2F

1

Gross income	Deductions	Net income
$5600	$450	$5150
$87 000	$28 000	$59 000
$50 000	$6700	$43 300
$890	$76	$814
$84 650	$24 790	$59 860

2 $1405

3 $25 000

4 $27 023.85

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5 **a** **i** \$40 035 **ii** 17.0%
b **i** \$53 905.80 **ii** 20.1%
c **i** \$41 218.20 **ii** 15%
d **i** \$30 052.56 **ii** 22.2%

6 **a** \$1830 **b** \$8043 **c** \$12 617.50 **d** \$23 772.80
7 **a** \$1830 **b** \$2562 **c** \$2989 **d** \$3708.80
8 **a** \$19 656 **b** \$53 144
9 **a** \$0 **b** \$0 **c** \$2242 **d** \$7797
e \$17 547 **f** \$35 362 **g** \$45 352 **h** \$63 232
10 \$67 400
11 **a** \$51 200 **b** \$8187 **c** \$43 013 **d** \$1654.35
12 \$19
13 **a** \$37 001–\$80 000 **b** \$41 394
14 **a** \$81 120 **b** \$17 911
15 **a** \$78 560 **b** \$17 079
16 \$832

Exercise 2G

1 **a** 12 **b** 3 **c** 1.5 **d** $4\frac{1}{3}$
2 **a** \$12 000 **b** 6% p.a.
c 3.5 years **d** \$720
3 **a** \$3000 **b** \$3600 **c** \$416 **d** \$315
4 **a** \$1120 **b** \$2800 **c** \$5600
5 **a** \$300 **b** \$900 **c** \$1600
d \$3150 **e** \$1196.25
6 \$2700, \$17 700
7 **a** \$52.50 **b** \$100 **c** \$40
d \$264.38 **e** \$1027.40
8 \$1980, \$23 980
9

a	\$840	\$7840
b	\$840	\$2340
c	\$1500	\$41 500
d	\$4550	\$74 550
e	\$200	\$2200

10 \$2560
11 \$38 000
12 Choice 2
13 **a** \$645 **b** \$194 **c** \$129
d \$19 **e** \$55 148 **f** \$12 320
g \$2580 **h** \$58 050 **i** \$129 000
14 **a** **i** \$12 **ii** \$31 **iii** \$10
b **i** \$335 **ii** \$500 **iii** \$235

Exercise 2H

1 **a** 2 years **b** 4 years **c** 50 years **d** $2\frac{1}{2}$ years
2 5 years
3 \$187.50
4 \$2000
5 3 years
6 $4\frac{1}{2}$ years
7 18 months
8 **a** 6.25% **b** 2% **c** 6.5% **d** 5.5%
e 21% **f** 12% **g** 12% **h** 1.5%
9

Interest	Total amount to be repaid	Monthly repayment
\$5250	\$10 250	\$170.83
\$10 500	\$24 500	\$408.33
\$2400	\$12 400	\$258.33
\$44 000	\$99 000	\$825
\$525 000	\$775 000	\$2152.78

10 \$2083.33
11 **a** \$14 400 **b** \$240
12 10%
13 **a** \$*P* **b** 12.5%
c **i** 20 years **ii** 40 years
iii The period in part **ii** is double the period in part **i**.
14 **a** Final amount = \$732.05 **b** \$232.05
15 **a** \$5743.27
b \$5743.27, same as final answer in part **a**.
c Increases the amount by 5% five times, giving the same result
d 5.5%

Puzzles and challenges

1

S	A	G	R	O	S	S	W	H	K	L	O	M
A	P	E	R	C	E	N	T	A	G	E	C	O
L	A	R	C	O	M	M	I	S	S	I	O	N
A	S	E	R	V	I	N	T	E	R	E	S	T
R	T	Y	H	E	T	S	L	O	S	S	T	H
Y	E	F	O	R	T	N	I	G	H	T	I	L
W	S	M	O	T	A	X	A	T	I	O	N	Y
E	I	O	L	I	D	I	D	N	P	Y	I	E
K	M	N	Q	M	O	N	T	R	N	I	S	C
E	P	Y	U	E	D	I	S	C	O	U	N	T
M	L	R	E	P	A	Y	M	E	N	T	A	H
D	E	D	U	C	T	I	O	N	S	I	X	L

2 EARTH, AIR, FIRE, WATER

Multiple-choice questions

1 C **2** C **3** B **4** D **5** C
6 C **7** C **8** A **9** C **10** E

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Short-answer questions

1

Decimal	Fraction	Percentage
0.6	$\frac{3}{5}$	60%
$0.\dot{3}$	$\frac{1}{3}$	$33\frac{1}{3}\%$
0.0325	$\frac{13}{400}$	$3\frac{1}{4}\%$
0.75	$\frac{3}{4}$	75%
1.2	$1\frac{1}{5}$	120%
2	2	200%

2 **a** \$77.50 **b** 1.65
3 **a** 150 **b** 25
4 **a** 72 **b** 1.17 **c** 20% **5** 12.5 kg
6 \$2070
7 \$1800
8 **a** \$25 **b** $16\frac{2}{3}\%$
9 **a** \$18.25 **b** \$14.30
10 \$50 592 **11** \$525 **12** 50%

Extended-response questions

1 **a** \$231 **b** \$651 **c** **i** \$63 **ii** \$34.65
2 **a** \$25.68 **b** \$38.52 **c** \$154.08 **d** 6.5
e \$46 547.31 **f** \$558.57

Chapter 3

Pre-test

1 **a** $3x$ **b** $4ab$ **c** $10x$ **d** $21b$
2 **a** 8 **b** 7 **c** 12 **d** 6 **e** 9 **f** 14
3 **a** $x + 3$ **b** ab **c** $2y - 3$ **d** $\frac{x+2}{3}$
4 **a** -18 **b** -28 **c** 40 **d** 3
e 3 **f** -5 **g** -4 **h** -8
5 **a** $3x$ **b** $6y + 5$ **c** $8x$
d $17y$ **e** $10a + 9a^2$ **f** $5xy - 7y$
6 **a** $6a$ **b** $-21xy$ **c** $4b$ **d** $\frac{3m}{2}$
7 **a** $2x + 6$ **b** $3a - 15$ **c** $8x + 4$
8 **a** 1 **b** -2 **c** 5 **d** -6
9 **b**, **c**
10 **a** 9 **b** 10 **c** 4 **d** 21

Exercise 3A

1 **a** expression **b** constant term **c** pronumeral
d coefficient **e** terms
2 **a** C **b** E **c** B **d** A **e** D
3 **a** $7y$ **b** $-2x$ **c** b
d ab **e** $\frac{y}{2}$ **f** $\frac{x}{y}$
4 **a** 5 **b** 8 **c** 14 **d** 4
5 **a** **i** 3 **ii** 2 **iii** 3
b **i** 2 **ii** -3 **iii** 0
c **i** 3 **ii** 7 **iii** -4
d **i** 4 **ii** $\frac{1}{2}$ **iii** -1
6 **a** **i** $4 + r$ **ii** $t + 2$ **iii** $x + y + z$
b **i** $6P$ **ii** $10n$ **iii** $2D$
iv $5P + 2D$
c $\frac{500}{C}$
7 **a** $2 + x$ **b** $ab + y$ **c** $x - 5$ **d** $2y - 7$
e $3x$ **f** $3p$ **g** $2x + 4$ **h** $\frac{x+y}{5}$
i $4x - 10$ **j** $1 - 3x$ **k** $\frac{3+y}{2}$ **l** $\frac{1}{2}(x + 1)$
m $(m + n)^2$ **n** $m^2 + n^2$
8 **a** -13 **b** -2 **c** 17
d 4 **e** 20 **f** -48
g 313 **h** $4\frac{1}{2}$ **i** -13
9 **a** 20 **b** 25 **c** 15
d 0 **e** 6 **f** 5
g 28 **h** -96 **i** 5
10 **a** 60 m^2
b Length $= 12 + x$, breadth $= 5 - y$
c $17 + x - y$
11 **a** \$26 **b** \$50 **c** No **d** 12 hours
12 **a** 18 square units **b** 1, 2, 3, 4, 5
13 **a** $\frac{A}{20}$ **b** $\frac{3A}{20}$ **c** $\frac{nA}{20}$
14 **a** **i** $P = 2x + 2y$ **ii** $A = xy$
b **i** $P = 4p$ **ii** $A = p^2$
c **i** $P = 6 + x + y$ **ii** $A = 3x$
15 **a** **i** No **ii** No
b **i** $3x + 1$ or $3(x + 1)$
ii $5 + \frac{x}{3}$ or $\frac{5+x}{3}$
iii $\frac{1}{2}(x + y)$ or $\frac{1}{2}x + y$
c Answers may vary.
i The sum of 2 and x is multiplied by 4.
ii 3 is added to a half of x.
iii A third of the sum of m and n.
iv 7 more than 5 lots of x.
v The sum of x and y is divided by 2.
vi The sum of b and a half of a

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Exercise 3B

1 **a** like **b** coefficients
c pronumeral **d** 1
2 **a** Yes **b** No **c** No
d No **e** Yes **f** Yes
g Yes **h** No **i** Yes
3 **a** -3 **b** -1 **c** -5 **d** -3 **e** -5 **f** -3
g 3 **h** 7 **i** 3 **j** -8 **k** -5 **l** -11
4 **a**, **c**, **d**, **f**
5 **a** $4y$ and $2y$, $3xy$ and $7xy$
b $3x$ and $7x$
c $7ab$ and $-3ab$, $2a$ and $5a$
d $2a^2$ and $-3a^2$, $4a$ and a
e $3x^2y$ and $7yx^2$
f $5ab^2$ and $4ab^2$, $3ab$ and $7ba$
6 **a** $10a$ **b** $7n$ **c** $8y$ **d** $11x$
e $4ab$ **f** $8mn$ **g** $7y + 8$ **h** $3x + 5$
i $4m + 2$ **j** $12ab + 3a$ **k** $7xy + 4y$ **l** $3bc - 4$
7 **a** $5a + 9b$ **b** $6x + 5y$ **c** $5xy + 4x$
d $4t + 6$ **e** $11x + 4$ **f** $7mn - 1$
g $5ab - a$ **h** 0 **i** $10ac - 9c$
j $3xy^2$ **k** $3m^2n$ **l** $7x^2y - 4xy$
m $4x^2 - 3x$ **n** $4a^2b - 3ab$ **o** $7pq^2 - 8pq$
8 **a** $x + y$ **b** $4x + 2y$
9 **a** $4x + 2y$ **b** $12s$ **c** $30x$
d $5x + 4$ **e** $6x + 16$ **f** $10a + 4b + 4$
10 $8x$ metres
11 **a** $4t$ **b** $9x$ **c** $2x$ **d** $7a$
e $5n$ **f** $4pq$ **g** $2xy$ **h** $3b$
12 **a** $3a + 7b$ **b** 4 units each
c a **d** $2a - b$

Exercise 3C

1 **a** $7x$ **b** ab **c** 6 **d** y
e $42b$ **f** $20x$ **g** 2 **h** y^2
2 **a** negative **b** negative **c** negative
3 **a** 2 **b** 6 **c** 12 **d** 3 **e** 7 **f** 5
4 **a** $\frac{2}{3}$ **b** $\frac{5}{2}$ **c** 3 **d** $\frac{1}{5}$
e $\frac{-2}{7}$ **f** $\frac{-1}{20}$ **g** $\frac{-2}{3}$ **h** $\frac{-9}{2}$
5 **a** $10m$ **b** $12b$ **c** $15p$ **d** $6x$
e $18pr$ **f** $16mn$ **g** $-14xy$ **h** $-15mn$
i $-12cd$ **j** $30ab$ **k** $-24rs$ **l** $-40jk$
m $-60mnp$ **n** $40mn$ **o** $-45pq$ **p** $32wy$
6 **a** $4b$ **b** $\frac{a}{3}$ **c** $\frac{2ab}{3}$ **d** $\frac{m}{2}$
e $\frac{x}{4}$ **f** $\frac{5s}{3}$ **g** 3 **h** $\frac{9q}{2}$
i $\frac{x}{2}$ **j** $6b$ **k** $\frac{7m}{3}$ **l** $\frac{2}{5x}$
m $2p$ **n** $\frac{p}{2}$ **o** $\frac{2q}{3}$ **p** $\frac{3a}{2}$
q 7 **r** w
7 **a** $24n^2$ **b** $-3q^2$ **c** $10s^2$ **d** $21a^2b$
e $-15mn^2$ **f** $18gh^2$ **g** $3b$ **h** $5x$
i $\frac{m}{2}$ **j** $\frac{1}{4y}$ **k** $\frac{3ab}{5}$ **l** $3pq$
8 **a** $\frac{4x}{y}$ **b** $\frac{m}{3}$ **c** $\frac{2a}{b}$ **d** $\frac{10x}{y}$
e $8n$ **f** $5b$ **g** 5 **h** $6xy$
i $\frac{p^2}{2}$ **j** $2ab$
9 **a** $12ab$ **b** $4x^2$ **c** $10xy$ **d** $6x^2$
10 $2x$
11 **a** $4n$ **b** $7y$ **c** $9y$ **d** $5ab$
e $14x$ **f** $10y$ **g** $12a$ **h** $8x^2y$
12 **a** **i** $3x^2$ m^2 **ii** $5x$ metres
b **i** 7500 m^2 **ii** 250 m
13 **a** $4b$ **b** $2ab$ **c** b

Exercise 3D

1 **a** $A = 6 \times 5 + 6 \times 2$ **b** $A = 5 \times (3 + 7)$
2 **a** **i** $5x$ **ii** 10
b $5x + 10$ **c** $x + 2$ **d** $5(x + 2)$ **e** $5x + 10$
3 **a** and **c** give the same values.
4 **a** $3(x + 5) = \underline{3} \times x + \underline{3} \times 5$
$= \underline{3x} + 15$
b $5(x + 1) = \underline{5} \times \underline{x} + 5 \times \underline{1}$
$= \underline{5x} + \underline{5}$
c $2(x - 7) = 2 \times \underline{x} + 2 \times (\underline{-7})$
$= \underline{2x} - 14$
5 **a** **i** -16 **ii** -4
b No, $-2x - 6$
c **i** -27 **ii** -33
d No, $-3x + 3$
6 **a** $2x + 6$ **b** $5x + 60$ **c** $2x + 14$ **d** $7x + 63$
e $3x - 21$ **f** $6x - 66$ **g** $5x - 45$ **h** $10x - 10$
i $6 + 3x$ **j** $21 + 7x$ **k** $28 - 4x$ **l** $40 - 5x$
7 **a** $-3x - 6$ **b** $-2x - 22$ **c** $-5x + 15$
d $-6x + 36$ **e** $-8 + 4x$ **f** $-39 - 13x$
g $-72 - 8x$ **h** $-300 + 300x$
8 **a** $2a + 2b$ **b** $5a + 10b$ **c** $6m + 3y$ **d** $16x - 40$
e $-12x - 15$ **f** $4x^2 - 8xy$ **g** $2t^2 - 3t$ **h** $3a^2 + 4a$
i $2d^2 - 5d$ **j** $6b^2 - 10b$ **k** $8x^2 + 2x$ **l** $5y - 15y^2$
9 **a** $2x + 11$ **b** $6x + 22$ **c** $5x + 6$
d $3x + 10$ **e** $4x - 5$ **f** $2x + 1$
g $8 - 3x$ **h** $21 - 5x$ **i** $-1 - 2x$
j $-3x$ **k** $3 - x$ **l** $6 - 3x$
m $12x - 1$ **n** $8x + 16$ **o** $2x^2 - 4x$
p $2m^2 - 8m$ **q** $9x - 3$ **r** $6 - x$
s $3x^2 + 3x$ **t** $23x - 6y$ **u** $3x - 4$
10 **a** $4x + 4y$ **b** $8x + 4$ **c** $2x - 2$
11 **a** $2x + 2$ **b** $8x - 12$ **c** $6x^2 + 2x$ **d** $6x + 15$

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12 $\$(2x + 120)$

13 a $2x + 12$ **b** $x^2 - 4x$ **c** $-3x - 12$ **d** $-7x + 49$
e $5x + 10 + 4 = 5x + 14$ **f** $5 - 2x + 14 = 19 - 2x$

14 a $\$(x - 10000)$
b $0.2(x - 10000) = \$(0.2x - 2000)$

15 a $5x + 12$ **b** $4x + 6$ **c** $11x + 7$
d $17x - 7$ **e** $x - 7$ **f** $2x - 4$
g $3x^2 + 10x$ **h** $5x - 3 + x^2$ **i** $x^2 - 2x - 4$

16 a $2x^2 + 4x$ **b** $x^2 + 2x + 6$ **c** $20 + 8x - x^2$

Exercise 3E

1 **b, c, e, f, h**

2 a 6 **b** 8 **c** -3 **d** 4 **e** 4 **f** -3

3 a $2x = 6$ **b** $3x = 21$ **c** $x = 9$

4 a subtract 9 from both sides
b add 1 to both sides
c divide both sides by 3
d divide both sides by 2

5 a $x = 4$ **b** $a = 1$ **c** $m = 4$ **d** $x = 4$
e $n = 2$ **f** $x = -4$ **g** $b = 3$ **h** $y = 7$
i $a = 4$ **j** $b = 8$ **k** $x = -1$ **l** $y = 2$

6 a $x = -3$ **b** $x = -1$ **c** $x = -2$ **d** $x = -8$
e $x = 2$ **f** $x = 2$ **g** $x = 2$ **h** $x = 6$

7 a $x = \frac{3}{2}$ **b** $x = \frac{3}{5}$ **c** $x = \frac{10}{11}$
d $x = \frac{1}{2}$ **e** $x = \frac{-1}{3}$ **f** $x = \frac{4}{5}$

8 a $x + 8 = 34, x = 26$ **b** $x - 7 = 21, x = 28$
c $2x + 4 = 10, x = 3$ **d** $3x - 4 = 11, x = 5$
e $3x - 4 = 20, x = 8$ **f** $25 - 2x = 7, x = 9$

9 a Added 2 to the right-hand side rather than subtracting 2.
b Multiplied the right-hand side by 2 in the last step rather than dividing by 2.
c The negative was dropped, $-x = 7$ so $x = -7$.
d Needed to subtract 3 from both sides before dividing both sides by 2.

10 a $x = 5$ **b** $x = -9$ **c** $x = -2$ **d** $x = 3$
e $x = -\frac{17}{2}$ **f** $m = 2$ **g** $x = 3$ **h** $x = -4$
i $x = -3$ **j** $m = -2$ **k** $m = \frac{8}{3}$ **l** $x = -\frac{8}{7}$
m $h = -\frac{1}{3}$ **n** $w = 1$ **o** $d = -\frac{5}{3}$

11 a 10 seconds **b** 8 seconds

Exercise 3F

1 a 20 **b** 24 **c** 11 **d** 15

2 C

3 a iv, add 1 to both sides
b iii, multiply both sides by 4
c i, multiply both sides by 3
d ii, multiply both sides by 2

4 a $x = 15$ **b** $y = 14$ **c** $y = 10$
d $x = 4$ **e** $x = 8$ **f** $x = 2$
g $b = 12$ **h** $t = -6$ **i** $a = -6$
j $y = 30$ **k** $x = 15$ **l** $s = 4$
m $x = 12$ **n** $m = -4$ **o** $y = -35$
p $x = -21$

5 a $x = 11$ **b** $x = 6$ **c** $y = 2$
d $b = 0$ **e** $y = 20$ **f** $t = 39$
g $k = 57$ **h** $x = 70$ **i** $x = -10$
j $b = -24$ **k** $y = -22$ **l** $a = -12$

6 a $t = 3$ **b** $m = 40$ **c** $y = -2$
d $x = 19$ **e** $y = -6$ **f** $t = \frac{-3}{4}$

7 a True **b** False **c** False **d** False
e False **f** True **g** True **h** False

8 a $\frac{x}{3} = 12, x = 36$ **b** $\frac{2x}{5} = 4, x = 10$
c $\frac{x}{2} - 4 = 10, x = 28$ **d** $\frac{x+3}{4} = 6, x = 21$
e $\frac{7x}{3} = 8, x = \frac{24}{7}$ **f** $\frac{x}{3} + 5 = 16, x = 33$

9 21

10 a Should have $\times 3$ before -2, $x + 2 = 21, x = 19$
b Should have $+4$ before $\times 3$, $\frac{x}{3} = 6, x = 18$

11 a $x = 12$ **b** $x = 12$ **c** $x = -4$
d $b = 6$ **e** $x = 5$ **f** $m = 5$
g $y = -1$ **h** $t = -7$ **i** $x = -3$

12 a $x = 9$ **b** $x = 36$ **c** $x = -7$
d $x = 8$ **e** $x = 5$ **f** $x = -10$

13 No, order does not matter for $\times$ or $\div$. By dividing by 7 first you deal with smaller numbers, since 7 is a factor of 14.

Exercise 3G

1 a $3(x + 7) = 3 \times \underline{x} + \underline{3} \times 7$
$= 3x + \underline{21}$
b $4(2x - 1) = \underline{4} \times 2x + 4 \times (-1)$
$= \underline{8x} - 4$

2 a $4x + 12$ **b** $5x + 10$ **c** $7x - 42$
d $6x - 15$ **e** $12x + 16$ **f** $3 - 6x$

3 a $7x + 4$ **b** $4x - 3$ **c** $-x + 1$ **d** $6 - 7x$

4 a $4x - 12$ **b** $4x + 2$ **c** $2x - 15$
d $-2x + 8$ **e** $-x - 6$ **f** $2 - 3x$

5 a $x = \frac{5}{2}$ or $2\frac{1}{2}$ **b** $a = \frac{3}{5}$ **c** $m = \frac{19}{3}$ or $6\frac{1}{3}$
d $y = \frac{47}{5}$ or $9\frac{2}{5}$ **e** $p = \frac{35}{4}$ or $8\frac{3}{4}$ **f** $k = \frac{19}{2}$ or $9\frac{1}{2}$
g $b = -\frac{1}{4}$ **h** $m = -\frac{11}{2}$ or $-5\frac{1}{2}$
i $x = -\frac{4}{5}$ **j** $a = \frac{1}{14}$ **k** $x = \frac{19}{6}$ or $3\frac{1}{6}$
l $n = \frac{2}{3}$ **m** $x = -\frac{1}{10}$ **n** $y = -\frac{1}{6}$

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6 **a** $x = 1$ **b** $x = 2$ **c** $x = 2$
d $x = 12$ **e** $x = 3$ **f** $x = 1\frac{2}{3}$
g $x = -5$ **h** $x = -1$

7 **a** $x = 1$ **b** $x = 2$ **c** $x = 1$
d $x = 9$ **e** $x = -4$ **f** $x = 1$

8 **a** $3(x + 1) = 4, x = \frac{1}{3}$ **b** $2(x - 2) = 19, x = 11\frac{1}{2}$
c $2(x + 3) = 7, x = \frac{1}{2}$ **d** $3(x - 4) = 8, x = 6\frac{2}{3}$
e $2(3x - 2) = 5, x = 1\frac{1}{2}$ **f** $3(2x + 5) = 10,\ x = \frac{-5}{6}$

9 \$5/hour

10 **a** $x = 5$ **b** $x = 5$
c Dividing both sides by 3 is faster because $9 \div 3$ is a whole number.

11 **a** $x = 4\frac{1}{3}$ **b** $x = 4\frac{1}{3}$
c Expanding the brackets is easier because $7 \div 3$ gives a fraction answer

12 **a** No **b** No
c Yes, $4(x + 1) = 20$
d Yes, $2(x - 4) + 3x + 1 = 13$
e Yes, $3x + 2 + 2(x - 5) = 12$
f Yes, $2x - 3 - 3(x - 1) = -4$

Exercise 3H

1 **a** $2x + 3 = 5x$ No **b** $2x + 5 = 6x + 1$ No
c $7x + 3 = 3x - 5$ Yes **d** $3x - 9 = 1 - 2x$ Yes

2 **a** Subtract $3x$ from both sides
b Subtract $5x$ from both sides
c Add x to both sides
d Add $2x$ to both sides

3 **a** $2x = 6$ **b** $2x - 3 = 1$
c $5x = 1$ **d** $4 = 8 + 6x$

4 **a** $x = 4$ **b** $x = 3$ **c** $x = 3$ **d** $x = 7$
e $x = -3$ **f** $x = -2$ **g** $x = 2$ **h** $x = 4$
i $x = 6$ **j** $x = 2$

5 **a** $x = 8$ **b** $a = 3$ **c** $m = 8$ **d** $x = 4$
e $x = 5$ **f** $x = 3$ **g** $x = 1$ **h** $y = 2$
i $m = 3$ **j** $x = 3$ **k** $b = -1$ **l** $m = -3$

6 **a** $x = 7$ **b** $a = 5$ **c** $y = 2$ **d** $x = -14$
e $b = -16$ **f** $m = 3$ **g** $a = 19$ **h** $x = -3$
i $x = -13$ **j** $n = 14$ **k** $a = 2$ **l** $x = 2.5$

7 **a** $2x = 3x - 4, x = 4$ **b** $2x + 4 = 5x, x = 1\frac{1}{3}$
c $3x + 10 = 5x, x = 5$ **d** $3x + 2 = 5x - 6, x = 4$
e $2x - 1 = 3x + 5, x = -6$ **f** $2x + 4 = 13 - x, x = 3$

8 Mardy is 8 years old.

9 **a** $x = 4$ **b** $x = 4$
c Method a: don't have to deal with negatives
Final step in method a: divide both sides by a positive number
Final step in method b: divide both sides by a negative number

10 **a** 48 units **b** 38 units **c** 26 units **d** 48 units

Exercise 3I

1 **a** iii **b** iv **c** i **d** ii

2 **ii, v, iv, i, vi, iii**

3 **a** $x - 3 = 5$ **b** $2x + 7 = 9$
c $\frac{x + 2}{3} = 4$ **d** $\frac{x}{2} + 3 = 6$

4 age, $x + 4$, $x + 4$, $2x$, 30, 15, Bernard is 15 and Sam is 19.

5 **a** 7 **b** 5 **c** 21 **d** 8 **e** 2

6 **a** Let d be the number of days the car is rented for.
b $50 + 40d = 290$ **c** $d = 6$
d The car was rented for 6 days.

7 48 items

8 **a** Let e be the number of goals for Emma.
b $e + 8$ **c** $e + e + 8 = 28$ **d** $e = 10$
e Emma scored 10 goals, Leonie scored 18 goals.

9 **a** Let b be the breadth in centimetres.
b Length $= 4b$
c $2b + 2(4b) = 560$
d $b = 56$
e Length = 224 cm, breadth = 56 cm

10 Benita \$360, Adele \$640

11 Andrew \$102.50, Brenda \$175, Cammi \$122.50

12 Walked 10 km, ran 20 km

13 15 and 45

14 Rectangle $l = 55$ m, $w = 50$ m. Triangle side = 70 m

15 **a** 27, 28, 29
b **i** $x, x + 2, x + 4$ **ii** 4, 6, 8

16 **a** $T = 8x + 7200$ **b** 300
c $R = 24x$ **d** $x = 350$ **e** 3825

Exercise 3J

1 **a** $w = 4$ **b** $h = 6$
c $d = 21$ **d** $s = 3$ (or $s = -3$)
e $w = 3$ **f** $a = 3$

2 **a** A **b** D **c** M **d** A

3 **a** 21 **b** 24 **c** 2 **d** 6
e 452.16 **f** 33.49 **g** 25.06
h 14.95 **i** 249.86 **j** 80

4 **a** $x = \frac{6}{a}$ **b** $x = 2c$ **c** $x = \frac{5 + c}{b}$ **d** $x = \frac{b}{4y}$

5 **a** $F = 36$ **b** $b = 5$ **c** $t = 7.5$
d $v = 25$ **e** $r = 3.91$ (or $r = -3.91$) **f** $a = 20$

6 **a** $d = \frac{C}{\pi}$ **b** $x = \frac{d - a}{b}$
c $n = \frac{p}{m} - x$ or $\frac{p - mx}{m}$ **d** $r = \frac{100I}{Pt}$
e $v = \sqrt{PR}$ **f** $h = \frac{A}{2\pi r}$
g $r = \sqrt{\frac{V}{\pi h}}$ **h** $A = (4C - B)^2$
i $a = \sqrt{c^2 - b^2}$ **j** $b = a^2 - c$

7 **a** 88.89 km/h **b** **i** $d = st$ **ii** 285 km

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8 **a** 35 m/s **b** 2 s
9 **a** Decrease **b** 3988 L **c** 417 min
10 **a** **i** 212°F **ii** 100.4°F
b **i** −10°C **ii** 36.7°C
11 **a** 73 **b** 7 **c** 42.05

Puzzles and challenges

1 $x = 3$

4	9	2
3	5	7
8	1	6

2 $5x^2 - xy$
3 WORD
4 Eric is 18 years old now.
5 1st leg = 54 km, 2nd leg = 27 km, 3rd leg = 18 km, 4th leg = 54 km
6 \$140

Multiple-choice questions

1 C **2** C **3** D **4** C **5** D
6 A **7** B **8** B **9** B **10** D

Short-answer questions

1 **a** $x + y$ **b** $7m$ **c** \$$3m$ **d** $\frac{n-3}{4}$
2 **a** 11 **b** 7 **c** 12 **d** 8 **e** −11
3 **a** $8n$ **b** $6xy$ **c** $4a$ **d** $\frac{xy}{3}$
4 **a** $7b$ **b** $4x + 3$ **c** $3p + 2q$ **d** $2m - 3mn + n$
5 **a** $2x + 14$ **b** $-6x - 15$ **c** $6x^2 - 8x$
d $-10a + 8a^2$ **e** $4x + 13$ **f** $3x - 7$
6 **a** $x = 9$ **b** $x = 2$ **c** $x = -3$ **d** $x = -9$
7 **a** $x = 10$ **b** $x = 8$ **c** $x = 26$ **d** $x = 6$
8 **a** $2n + 3 = 21, n = 9$ **b** $\frac{l-5}{3} = 7, l = 26$
c $\frac{x}{4} - 5 = 0, x = 20$
9 **a** $x = 5$ **b** $x = \frac{11}{6}$ or $1\frac{5}{6}$
10 **a** $x = 4$ **b** $x = 2$ **c** $x = 1$ **d** $x = -5$
11 \$260
12 **a** $E = 60$ **b** $a = 5$ **c** $h = 10$
13 **a** $x = \frac{v^2 - u^2}{2a}$ **b** $I = \sqrt{\frac{P}{R}}$

Extended-response questions

1 **a** **i** \$85 **ii** \$110
b **i** Let h be the number of hours of hire.
ii $60 + 25h = 210$ **iii** 6 hours
2 **a** $h = \frac{A}{10}$ **b** 10 m **c** 17 m

Chapter 4

Pre-test

1 **a** 5 cm **b** 13 m **c** 41 mm
2 **a** 9 **b** 49 **c** 20 **d** 45
3 **a** 0.4568 **b** 0.3457 **c** 0.0456 **d** 0.2800
4 **a** 4.23 **b** 5.68 **c** 76.90 **d** 23.90
5 **a** 5.92 **b** 15.36 **c** 4.86 **d** 8.09
6 **a** $x = 64$ **b** $x = 28$ **c** $x = 108$
d $x = 60$ **e** $x = 50$ **f** $x = 55$
7 **a** $x = 2$ **b** $x = 3$ **c** $x = 12$ **d** $x = 6$
e $x = 12$ **f** $x = 30$ **g** $x = 28$ **h** $x = 182$
8 **a** $x = 0.6$ **b** $x = 0.6$ **c** $x = 2.1$ **d** $x = 0.5$
9 **a** $x = 2$ **b** $x = 4$ **c** $x = 5$ **d** $x = 13$

Exercise 4A

1 **a** 9 **b** 25 **c** 144 **d** 2.25
e 20 **f** 58 **g** 157 **h** 369
2 **a** False **b** True **c** True
d True **e** False **f** False
3 hypotenuse, triangle
4 **a** c **b** x **c** u
5 **a** $a^2 + b^2 = c^2$ **b** $d^2 + e^2 = f^2$ **c** $x^2 + y^2 = z^2$
d $s^2 + t^2 = v^2$ **e** $f^2 + g^2 = e^2$ **f** $a^2 + b^2 = x^2$
6 **a** $3^2 + 4^2 = 5^2$ **b** $8^2 + 15^2 = 17^2$ **c** $9^2 + 12^2 = 15^2$
d $5^2 + 12^2 = 13^2$ **e** $9^2 + 40^2 = 41^2$ **f** $2.5^2 + 6^2 = 6.5^2$
7 **a** $a^2 + b^2 = x^2$ **b** $a^2 + b^2 = d^2$ **c** $d^2 + h^2 = x^2$
8

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3	4	5	9	16	25	25
6	8	10	36	64	100	100
8	15	17	64	225	289	289

a The last two columns
b **i** 13 **ii** 20
c **i** 25 **ii** 110
9 **a** No
b No, $a^2 + b^2 = c^2$ should be true for a right-angled triangle.
10 **a** Yes **b** No **c** No
d Yes **e** No **f** Yes
11/12 {(3, 4, 5), (6, 8, 10), (9, 12, 15), (12, 16, 20), (15, 20, 25), (18, 24, 30), (21, 28, 35), (24, 32, 40), (27, 36, 45), (30, 40, 50), (33, 44, 55), (36, 48, 60), (39, 52, 65), (42, 56, 70), (45, 60, 75), (48, 64, 80), (51, 68, 85), (54, 72, 90), (57, 76, 95)}, {(5, 12, 13), (10, 24, 26), (15, 36, 39), (20, 48, 52), (25, 60, 65), (30, 72, 78), (35, 84, 91)}, {(7, 24, 25), (14, 48, 50), (21, 72, 75)}, {(8, 15, 17), (16, 30, 34), (24, 45, 51), (32, 60, 68),

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(40, 75, 85)}, {(9, 40, 41), (18, 80, 82)}, {(11, 60, 61)}, {(20, 21, 29), (40, 42, 58), (60, 63, 87)}, {(12, 35, 37), (24, 70, 74)}, {(28, 45, 53)}, {(33, 56, 65)}, {(16, 63, 65)}, {(48, 55, 73)}, {(13, 84, 85)}, {(36, 77, 85)}, {(39, 80, 89)}, {(65, 72, 97)}; 50 Pythagorean triads

Exercise 4B

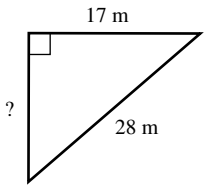
1 **a** 17 **b** 50 **c** $\sqrt{8}$
2 **a** $c^2 = a^2 + b^2$ **b** $x^2 = y^2 + z^2$ **c** $j^2 = k^2 + l^2$
3 **a** 81 **b** 10.24 **c** 13 **d** 106
e 6 **f** 10 **g** 4.90 **h** 3.61
i 125.06 **j** 244 **k** 13 **l** 41
4 **a** $c = 10$ **b** $c = 13$ **c** $c = 17$ **d** $c = 15$
e $c = 25$ **f** $c = 41$ **g** $c = 50$ **h** $c = 30$
i $c = 25$
5 **a** 4.47 **b** 3.16 **c** 15.62 **d** 11.35
e 7.07 **f** 0.15
6 **a** $\sqrt{5}$ **b** $\sqrt{58}$ **c** $\sqrt{34}$ **d** $\sqrt{37}$
e $\sqrt{109}$ **f** $\sqrt{353}$
7 4.4 m
8 495 m
9 2.4 m
10 **a** 8.61 m **b** 5.24 m **c** 13.21 cm
d 0.19 m **e** 17.07 mm **f** 10.93 cm
11 250 m
12 42 units
13 5.83 m
14 **a** 77.78 cm **b** 1.39 m
c Reduce by 8 cm

Exercise 4C

1 **a** subtract **b** 16 **c** 11
2 **a** 14 **b** 11 **c** 12 **d** 20
e 4 **f** 24 **g** 8 **h** 8
3 **a** True **b** False **c** True **d** False
4 **a** 16 **b** 24 **c** 6 **d** 21 **e** 60 **f** 27
5 **a** 8.66 **b** 11.31 **c** 5.11
d 17.55 **e** 7.19 **f** 0.74
6 5.3 m
7 14.2 m
8 49 cm
9 1.86 m
10 **a** $\sqrt{2}$ **b** $\sqrt{8}$ **c** $\sqrt{\frac{1}{2}}$
11 **a** $\sqrt{187}$ **b** $\sqrt{567}$ **c** 40

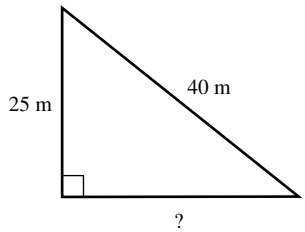
Exercise 4D

1 B
2 **a** B, iii **b** C, i **c** A, ii
3 **a**



b 22.25 m

4 **a**

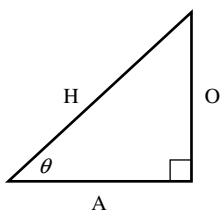


b 31.22 m

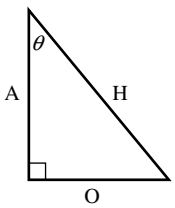
5 **a** 6.4 m **b** 5.7 cm **c** 6.3 m **d** 6.0 m
6 3.0 m
7 142.9 m
8 3823 mm
9 1060 m
10 466.18 m
11 **a** 27 m **b** 118.3 m

Exercise 4E

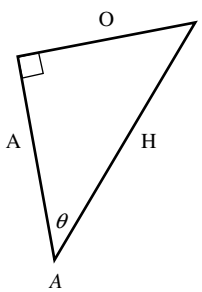
1 **a** hypotenuse **b** opposite **c** adjacent
d opposite **e** hypotenuse **f** adjacent
2 **a** 13 m **b** 5 m **c** 12 m
3 **a**



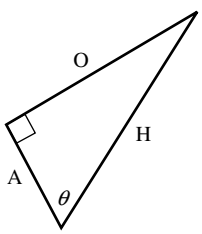
b



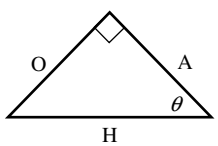
c



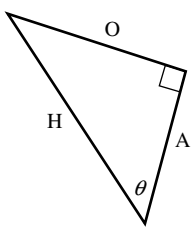
d



e



f



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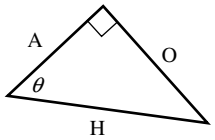
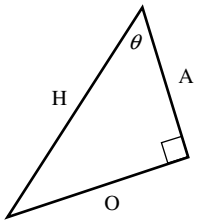
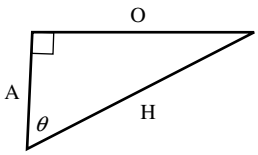
g



h



i



4 a $\sin\theta = \frac{4}{7}$ **b** $\tan\theta = \frac{5}{4}$ **c** $\cos\theta = \frac{3}{5}$

d $\sin\theta = \frac{2}{3}$ **e** $\tan\theta = 1$ **f** $\cos\theta = \frac{2}{3}$

g $\tan\theta = \frac{11}{12}$ **h** $\cos\theta = \frac{1}{2}$ **i** $\tan\theta = \frac{5}{3}$

5 a $\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

b $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$

c $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$

6 a i $\frac{5}{13}$ **ii** $\frac{5}{13}$

iii They are the same.

b i $\frac{12}{13}$ **ii** $\frac{12}{13}$

iii They are the same.

c i $\frac{5}{12}$ **ii** $\frac{5}{12}$

iii They are the same.

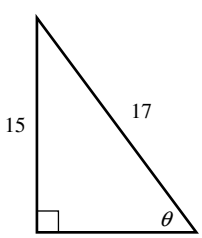
7 a $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** $\frac{3}{4}$

8 $\frac{3}{4}$

9 a 5 **b** 4 **c** 3 **d** 3 **e** 4

10 a $\frac{4}{5}$ **b** $\frac{3}{5}$ **c** $\frac{3}{5}$ **d** $\frac{3}{4}$ **e** $\frac{4}{5}$ **f** $\frac{4}{3}$

11 a



b 8

c $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$

12 a i $\frac{1}{2}$ **ii** $\frac{\sqrt{3}}{2}$ **iii** $\frac{1}{\sqrt{3}}$

iv $\frac{\sqrt{3}}{2}$ **v** $\frac{1}{2}$ **vi** $\sqrt{3}$

b i They are equal **ii** They are equal.

13 b i 0.766 **ii** 0.643 **iii** 0.839

iv 0.766 **v** 1.192 **vi** 0.643

c $\sin 40° = \cos 50°$, $\sin 50° = \cos 40°$, $\tan 50° = \frac{1}{\tan 40°}$

$\tan 40° = \frac{1}{\tan 50°}$

Exercise 4F

1 a A **b** O **c** H

2 a O, H **b** A, H **c** O, A

3 a sin **b** tan **c** cos

4 a 0.34 **b** 0.80 **c** 2.05 **d** 0.73
e 0.10 **f** 0.25 **g** 0.46 **h** 0.24
i 0.99 **j** 0.56 **k** 0.68 **l** 0.91

5 a 3.06 **b** 18.94 **c** 5.03
d 0.91 **e** 1.71 **f** 9.00
g 2.36 **h** 4.79 **i** 7.60

6 a 5.95 **b** 0.39 **c** 13.38 **d** 3.83 **e** 8.40
f 1.36 **g** 29.00 **h** 1.62 **i** 40.10 **j** 4.23
k 14.72 **l** 13.42 **m** 17.62 **n** 5.49 **o** 9.35

7 10.39 m

8 1.12 m

9 44.99 m

10 10.11 m

11 a 65° **b** 2.113 **c** 2.113

12 a 20.95 m **b** 10 cm

Exercise 4G

1 a $\cos\theta$ **b** $\sin\theta$ **c** $\tan\theta$

2 a $x = 2$ **b** $x = 5$ **c** $x = 3$ **d** $x = 4$

3 b, c

4 a 4.10 **b** 6.81 **c** 37.88
d 0.98 **e** 12.80 **f** 14.43
g 9.52 **h** 114.83 **i** 22.11

5 a 13.45 **b** 16.50 **c** 57.90 **d** 26.33
e 15.53 **f** 38.27 **g** 9.15 **h** 32.56
i 21.75 **j** 48.28 **k** 47.02 **l** 28.70

6 40 m

7 3848 m

8 a $x = 7.5, y = 6.4$ **b** $a = 7.5, b = 10.3$
c $a = 6.7, b = 7.8$ **d** $x = 9.5, y = 12.4$
e $x = 12.4, y = 9.2$ **f** $x = 21.0, y = 18.7$

9 a 27 m **b** 104 m

10 a Student B's answer is more correct because they did not use an approximation in their working out.

b Use your calculator and do not round sin 31° during working.

11 a 23.7 m **b** 124.9 m

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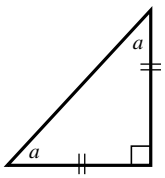
Exercise 4H

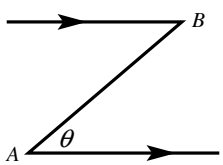
1 **a** 11.54 **b** 64.16 **c** 41.41
d 64.53 **e** 26.57 **f** 68.20
2 **a** 47° **b** 12° **c** 18°
d 51° **e** 24° **f** 42°
g 79° **h** 13° **i** 3°
3 **a** sin **b** sin
c cos **d** tan
4 **a** 30° **b** 60° **c** 45°
d 30° **e** 45° **f** 30°
g 90° **h** 50° **i** 90°
j 55° **k** 0° **l** 70°
5 **a** 34.85° **b** 19.47° **c** 64.16°
d 75.52° **e** 36.87° **f** 38.94°
g 30.96° **h** 57.99° **i** 85.24°
6 **a** 43° **b** 31° **c** 41°
d 16° **e** 55° **f** 50°
g 49° **h** 41° **i** 34°
7 17°
8 23.13°
9 **a** $\tan 45° = 1$



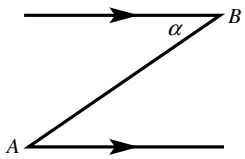
b This triangle is isoceles; therefore, 2 angles are equal
$a + a = 90$
$\therefore a = 45°$
$\tan 45° = \frac{O}{A}$
The O and A are equal
$\therefore \tan 45° = 1$
10 25.4°
11 26.6°
12 45°
13 **a** 18° **b** 27° **c** 45°
d 5.67 m **e** Up to 90°

Exercise 4I

1 **a** **i**



ii



b Yes, $\theta = \alpha$, alternate angles are equal on parallel lines
2 C
3 $a = 65$, $b = 25$
4 **a** 22° **b** 22°
5 29 m
6 16 m
7 157 m
8 38 m
9 90 m
10 37°
11 6°
12 10°
13 **a** 6° **b** 210 m
14 4634 mm
15 Yes, by 244.8 m
16 **a** **i** 47.5 km **ii** 16.25 km
b No **c** Yes

Puzzles and challenges

1 44
2 **b** a^2, b^2, c^2 **c** $a^2 + b^2 = c^2$
3 **a** A diagonal of the square **b** $\sqrt{2}$ cm
c 63.7%
4 Round peg, square hole
5 171 cm

Multiple-choice questions

1 A **2** B **3** A **4** C **5** B
6 C **7** D **8** C **9** A **10** B

Short-answer questions

1 **a** $x^2 + y^2 = z^2$ **b** $s^2 + t^2 = u^2$ **c** $e^2 + f^2 = d^2$
2 **a** 10 **b** 25 **c** 13
d 37 **e** 4.47 **f** 7.07
3 **a** 3.32 **b** 7.55 **c** 9.95
4 4.49 m
5 **a** 13 cm **b** 13.93 cm
6 19 m
7 **a** 0.64 **b** 2.25 **c** 0.72
8 **a** $\sin\theta$ **b** $\tan\theta$ **c** $\cos\theta$
9 **a** 11.33 **b** 4.88 **c** 48.02
d 10.31 **e** 50.71° **f** 60°
10 28.01 m
11 25 m
12 177.91 m
13 63.2 m

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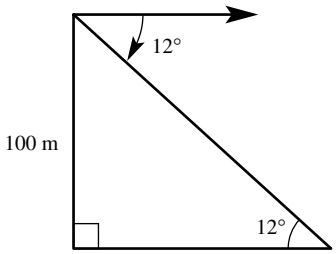
Extended-response questions

1 a



b 470 m **c** 3° **d** 1530 m

2 a 2.15 m **b** 0.95 m **c** 3.05 m

3 a 9.85 cm

b 39°

c i 11.9 cm

ii 6.6 cm

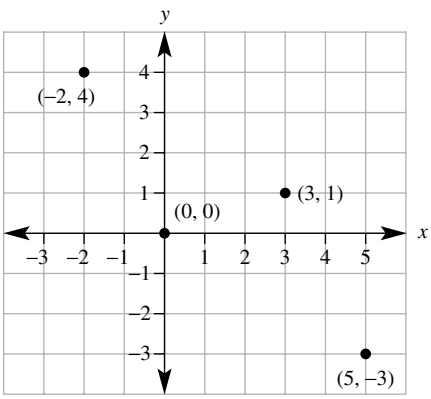
d 46.2 cm

e 92 cm^2 (nearest m^2)

Chapter 5

Pre-test

1



2 a 1 **b** −1 **c** −3 **d** −4

3 a $x = 5$ **b** $x = -2$ **c** $x = -3$

d $x = 4$ **e** $x = -2$ **f** $x = 4$

4 a 3 **b** −4 **c** 4 **d** −3

5 a

x	−2	−1	0	1	2
y	1	2	3	4	5

b

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

6 a False **b** True **c** True **d** False

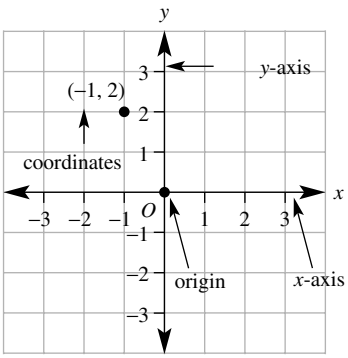
7 a 7 **b** 6.5 **c** 3 **d** −5

8 a 5 **b** 4 **c** 5 **d** 3

9 a 4 **b** 5 **c** 7 **d** 3

Exercise 5A

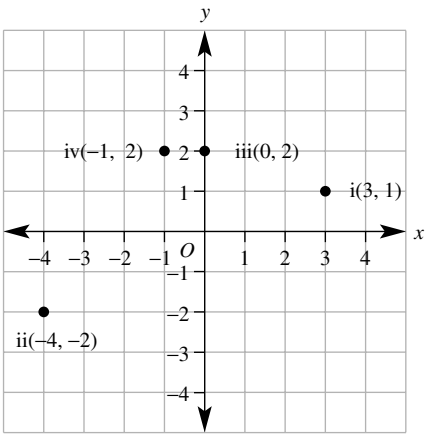
1 a



b i x-coordinate **ii** y-coordinate **iii** right, down

2 C

3 a



b $A(4, 1)$, $B(2, 3)$, $C(0, 3)$, $D(-2, 2)$, $E(-3, 1)$, $F(-1, 0)$, $G(-3, -2)$, $H(-2, -4)$, $I(0, -3)$, $J(2, -2)$, $K(3, -4)$, $L(2, 0)$

4 a 2 **b** −1 **c** −4 **d** −7

5 C

6 a

x	−2	−1	0	1	2	3
y	0	1	2	3	4	5

b

x	−2	−1	0	1	2
y	−1	1	3	5	7

c

x	−2	−1	0	1	2
y	4	2	0	−2	−4

d

x	−2	−1	0	1	2	3
y	5	2	−1	−4	−7	−10

7 a

x	−3	−2	−1	0	1	2	3
y	−4	−3	−2	−1	0	1	2

b

x	−3	−2	−1	0	1	2	3
y	0	1	2	3	4	5	6

c

x	−3	−2	−1	0	1	2	3
y	−9	−7	−5	−3	−1	1	3

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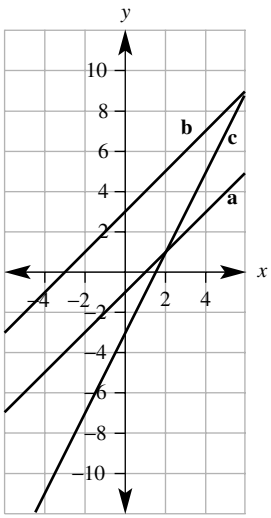
a $y = x - 1$ **b** $y = x + 3$ **c** $y = 2x - 3$



d

x	−3	−2	−1	0	1	2	3
y	9	6	3	0	−3	−6	−9

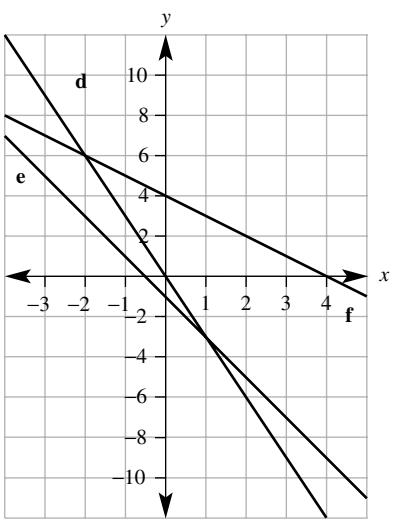
e

x	−3	−2	−1	0	1	2	3
y	5	3	1	−1	−3	−5	−7

f

x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1

d $y = -3x$ **e** $y = -2x - 1$ **f** $y = -x + 4$



g

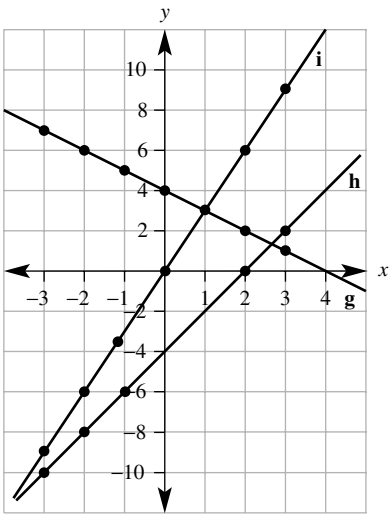
x	−3	−2	−1	0	1	2	3
y	7	6	5	4	3	2	1

h

x	−3	−2	−1	0	1	2	3
y	−10	−8	−6	−4	−2	0	2

i

x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9



8 **a** No **b** Yes **c** No

9 **a** Yes **b** Yes **c** No

10 **a** $y = x + 2$ **b** $y = 2x$ **c** $y = 2x + 1$
d $y = 3x - 1$ **e** $y = x - 2$

11 $y = 3x$

12 **a** iv **b** iii **c** i **d** ii

13 **a** $y = 2x + 7$ **b** $y = 20 - x$ **c** $y = 2x - 1$
d $y = \frac{1}{2}x + 1$ **e** $y = -3x + 4$ **f** $y = 5x + 4$

Exercise 5B

1 **a** 3 **b** 2

2 **a** A(1, 0), D(−2, 0), H(3, 0) **b** B(0, 3), E(0, −1)

3 **a** $y = 1$ **b** $y = -2$ **c** $y = -3$

4 **a** $x = 2$ **b** $y = 3$ **c** $y = -5$
d $x = 4$ **e** $x = 3$ **f** $x = 3$

5 **a** −2 and 2 **b** 1 and 1 **c** 4 and 8
d −5 and 10 **e** 2 and 3 **f** 7 and −3
g −11 and 5 **h** −2 and −5

6 **a** 5 **b** 1 **c** −7 **d** −3 **e** 2 **f** −4
g −2 **h** 10 **i** 11 **j** 3 **k** 2 **l** 6
m −3 **n** −1 **o** −3 **p** 3

7 **a** −6 **b** 4 **c** 4 **d** −2 **e** −4 **f** 3
g 7 **h** 2 **i** 4 **j** 6 **k** 1 **l** −3

8 **a** $\frac{1}{2}$ and −1 **b** 4 and −2
c 12 and 4 **d** $2\frac{1}{3}$ and $3\frac{1}{2}$

9 **a** 100 m **b** 20 s

10 **a** 20 minutes **b** 80 litres

11 When you substitute $x = 0$ into the rule, $y = 3 \times (0) = 0$.
It means the graph passes through the origin.

12 **a** $A = -60$
b She makes a loss of \$60.

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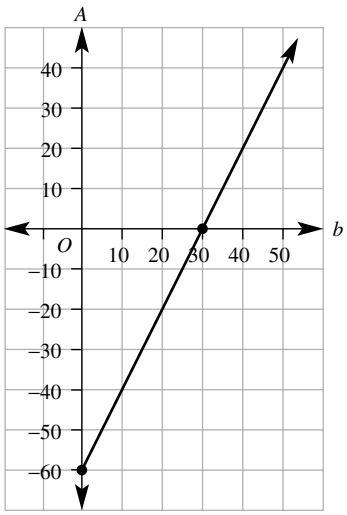
c 30 badges

d She then starts to make a profit from her stall.

e



Exercise 5C

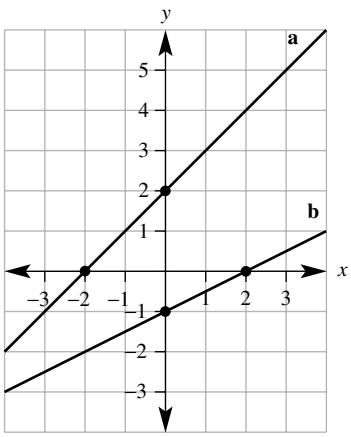
1 a 2 **b** 0 **c** 0

2 a $y = x + 2$

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

b $y = \frac{1}{2}x - 1$

x	−3	−2	−1	0	1	2	3
y	−2.5	−2	−1.5	−1	−0.5	0	0.5

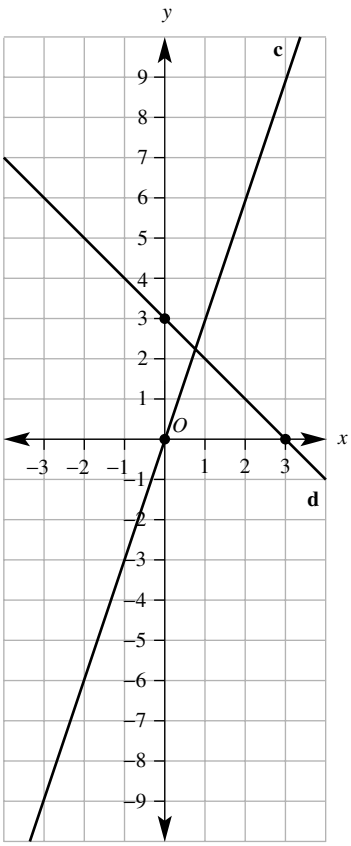


c $y = 3x$

x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9

d $y = -x + 3$

x	−3	−2	−1	0	1	2	3
y	6	5	4	3	2	1	0



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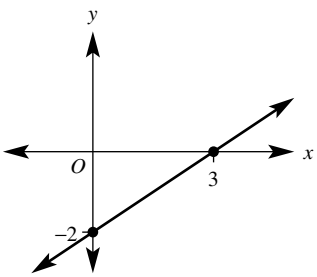

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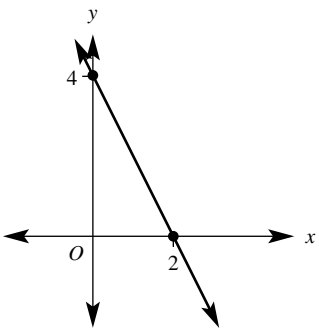

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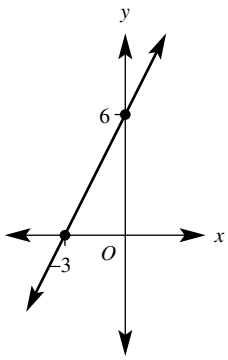
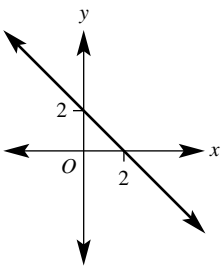


4 a 4 **b** −5 **c** 3 **d** −4

5 a 5 **b** −2 **c** 3 **d** 2

6 a



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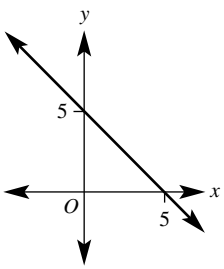

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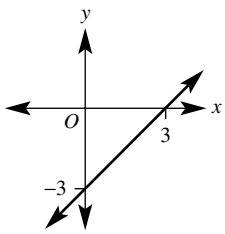

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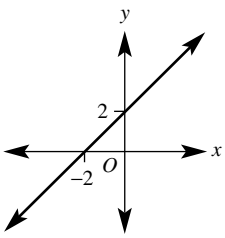

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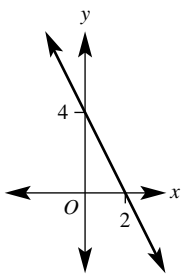

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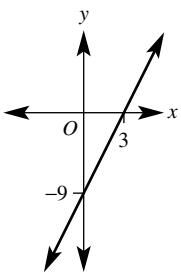

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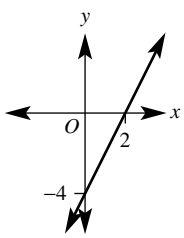

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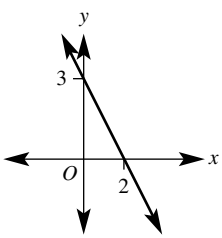

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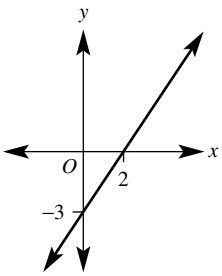

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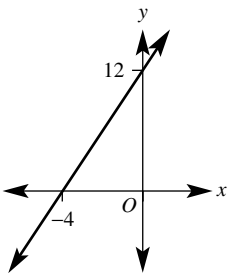

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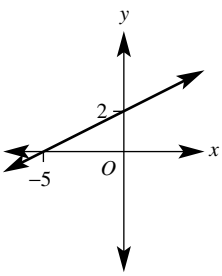

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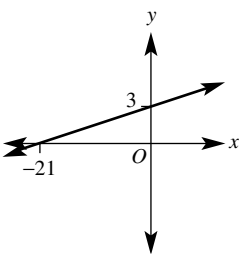

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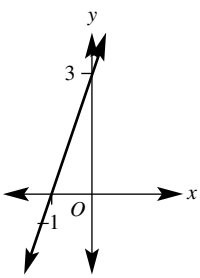

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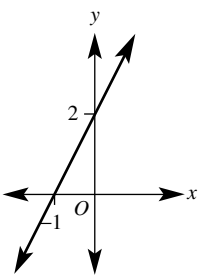

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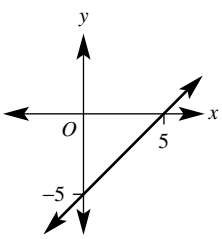

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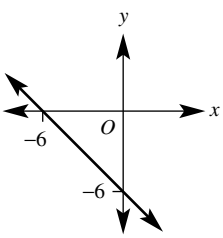

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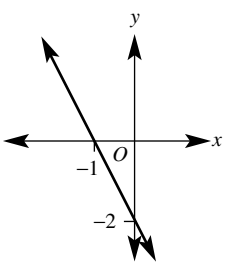

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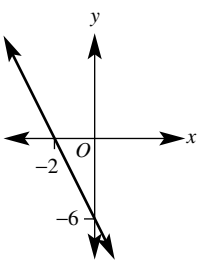

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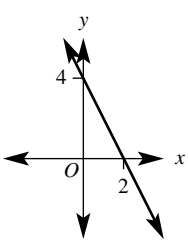

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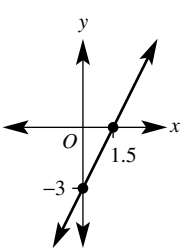

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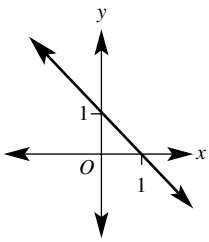
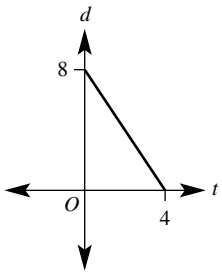


8 a 8 m **b** 4 seconds **c**



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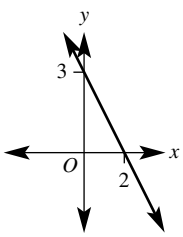

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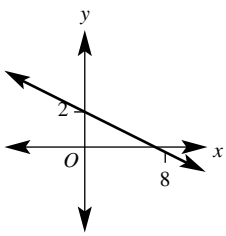

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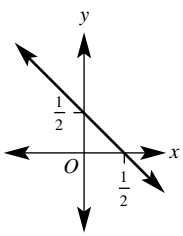


10 a For 0.5 across, graph moves 1 down.

b For 1.5 across, graph moves one up.

11 a $x + y = 4$ **b** $x + y = 2$

c $x - y = 3$ **d** $x - y = -1$

Exercise 5D

1 a Horizontal **b** Horizontal **c** Vertical

d Horizontal **e** Vertical

2 a ii, iv **b** i, iii

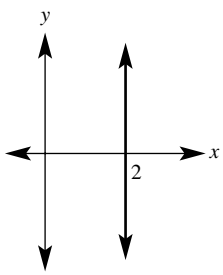
3 a 2 **b** −3 **c** 0

4 a 3 **b** −2

5 b, e, f

6 a 5 **b** $\frac{1}{3}$ **c** −4 **d** −0.1

7 a



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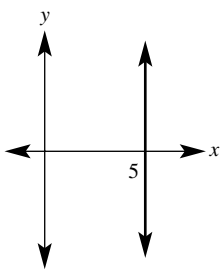

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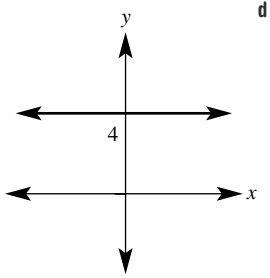

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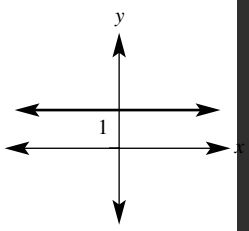

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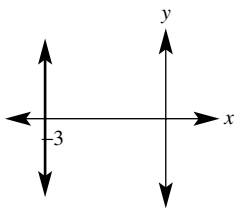

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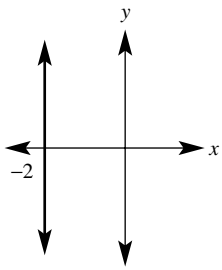

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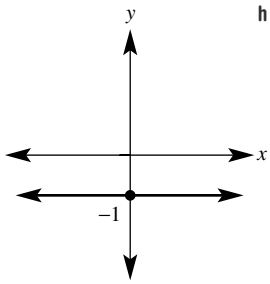

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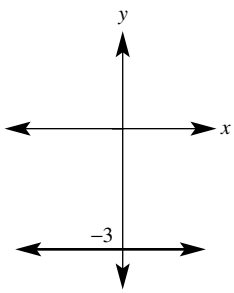

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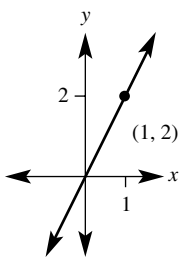

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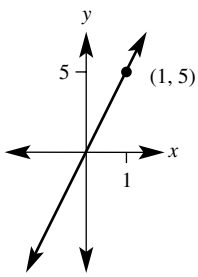

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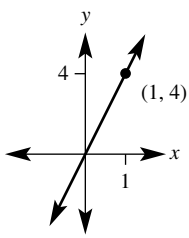

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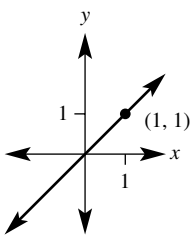

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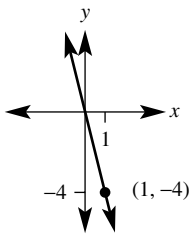
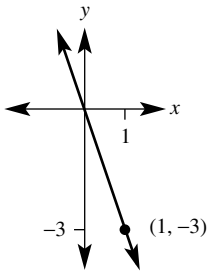
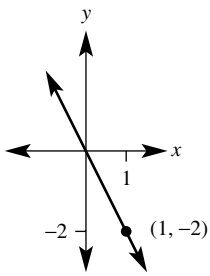
e



f



g



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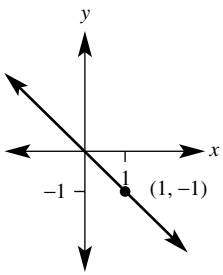
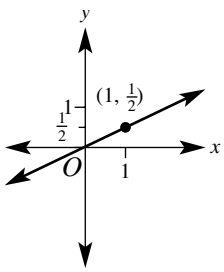


i



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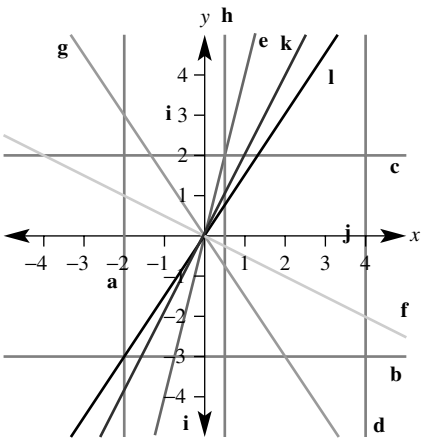


10 a $y = -2$ **b** $y = 4$ **c** $x = -2$
d $x = 5$ **e** $y = 1.5$ **f** $x = -6.7$

11 a $y = 3$ **b** $x = 5$ **c** $x = -2$ **d** $y = 0$

12 a $y = 250$ **b** $y = -45$

13 a $y = 2x$ **b** $y = 3x$ **c** $y = -x$ **d** $y = -3x$

14 a $A = 15$ square units **b** $A = 68$ square units

15 a i $y = 1$ or $y = -5$ **ii** $y = 0$ or $y = -4$
iii $y = 3\frac{1}{2}$ or $y = -7\frac{1}{2}$
b i $y = 1$ or $y = -5$ **ii** $y = 7$ or $y = -11$
iii $y = 9\frac{1}{2}$ or $y = -13\frac{1}{2}$

Exercise 5E

1 a i 2 **ii** 4 **b i** 2 **ii** 1
c i 1 **ii** 3 **d i** 3 **ii** −3
e i 1 **ii** −3 **f i** 4 **ii** −1

2 a zero **b** negative
c positive **d** undefined

3 a 2 **b** −4 **c** 1
d $\frac{1}{2}$ **e** $-\frac{3}{2}$ **f** $-\frac{5}{3}$

4 a Positive, 1 **b** Positive, 2 **c** Zero
d Zero **e** Negative, $\frac{-2}{3}$ **f** Negative $\frac{-3}{4}$
g Undefined **h** Undefined **i** Positive, $\frac{1}{2}$
j Positive, 3 **k** Positive, 2 **l** Negative, −4

5 *A* 1 *B* −2 *C* 1
D $\frac{1}{2}$ *E* −2 *F* 3

6 a 2 **b** −1 **c** −2 **d** 1
e 2 **f** −2 **g** −1 **h** $\frac{1}{2}$
i $\frac{3}{2}$ **j** $-\frac{5}{2}$ **k** $\frac{1}{3}$ **l** $\frac{5}{2}$

7 a $-\frac{1}{20}$ **b** 17.5 or $\frac{35}{2}$ **c** $-\frac{1}{5}$ **d** 7.5 or $\frac{15}{2}$

8 40 metres

9 90 metres

10 a 6 **b** 1 **c** 0 **d** −4

11 a i −1 **ii** −3 **b** (4.5, 0)
c i (6, 0) **ii** (1.5, 0) **iii** (2.4, 0)

Exercise 5F

1 a No **b** Yes **c** Yes **d** No

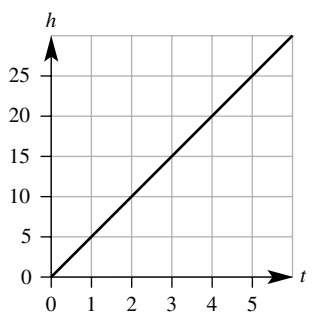
2 a i 10 km **ii** 20 km **iii** 30 km
b 10 km/h **c** 10 **d** They are the same.

3 a 15 mm **b i** 6 days **ii** 20 days

c

t	0	1	2	3	4
h	0	5	10	15	20

d



e 5

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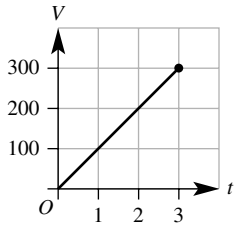
4 **a** 60 km/h **b** 9 m/s **c** 5 L/min **d** \$1.50/L
e \$30/h **f** 3°C/min **g** 140 g/min **h** \$80/h
i 3 mL/min **j** 60 c/s

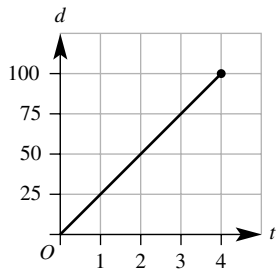
5 **a** $d = 60t$ **b** $g = 2t$ **c** $C = 1.5n$ **d** $P = 20t$

6 **a** 100 L/h
b



c **i** 100 **ii** $V = 100t$
d **i** 150 L **ii** 20 hours

7 **a** 25 km/h
b



c **i** 25 **ii** $d = 25t$
d **i** 62.5 km **ii** 1.6 hours

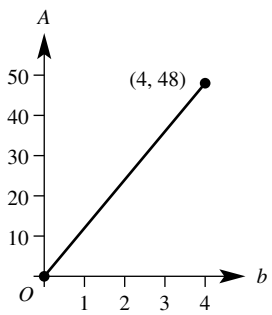
8 **a** 100 km/h **b** 7 cm/s
c 2.5 cm/min **d** 49 mm/min

9 Sally

10 The panther

11 **a** 10.2 L **b** 72.25 L **c** 800 km

12 **a** $A = 12b$
b



c Yes, the rule for the graph is in the form $y = mx$.

13 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$
d 1 min 20 s or $1\frac{1}{3}$ min

14 1.2 hours or 1 hour 12 minutes

Exercise 5G

1 **a** gradient **b** y-intercept **c** subject

2 **a** iii **b** i **c** ii **d** iv

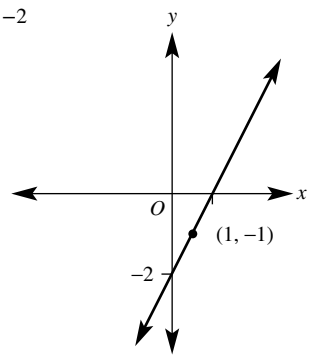
3 **a** **i** (2, 6) **ii** (3, 1) **iii** (4, −4)
b **i** (1, 1) **ii** (3, −5)

4 **a** 3, 5 **b** 1, 4 **c** 2, −1

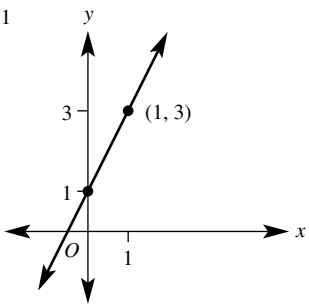
5 **a** gradient = 3, y-intercept = −4
b gradient = −5, y-intercept = −2
c gradient = −2, y-intercept = 3
d gradient = $\frac{1}{3}$, y-intercept = 4
e gradient = −4, y-intercept = 0
f gradient = 2, y-intercept = 0
g gradient = 2.3, y-intercept = 0
h gradient = −0.7, y-intercept = 0

6 **a** $y = -3x + 4$ **b** $y = -2x + 7$
c $y = 3x + 2$ **d** $y = 2x + 7$
e $y = 5x - 3$ **f** $y = -2x + 4$
g $y = \frac{-3}{2}x + 3$ **h** $y = 2x - \frac{5}{2}$
i $y = \frac{5}{3}x + 2$

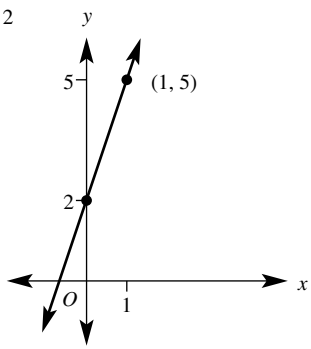
7 **a** $m = 1$, y-intercept = −2



b $m = 2$, y-intercept = 1



c $m = 3$, y-intercept = 2



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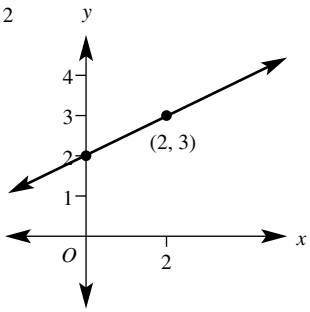
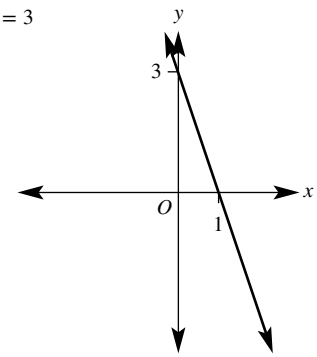
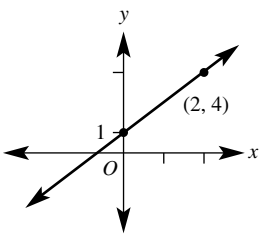
d $m = \frac{1}{2}$, y-intercept $= 2$



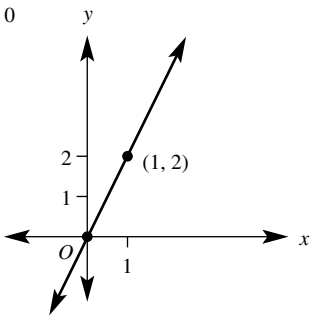
e $m = -3$, y-intercept $= 3$



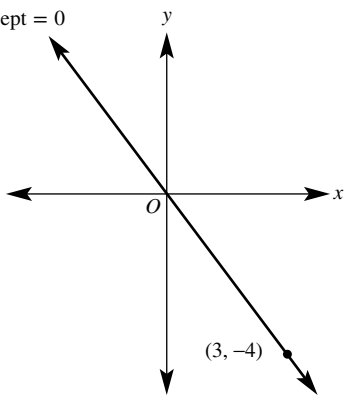
f $m = \frac{3}{2}$, y-intercept $= 1$



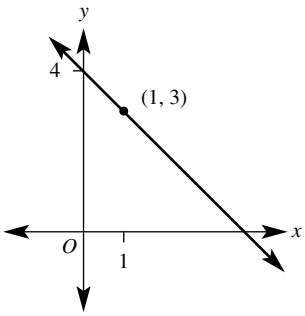
g $m = 2$, y-intercept $= 0$



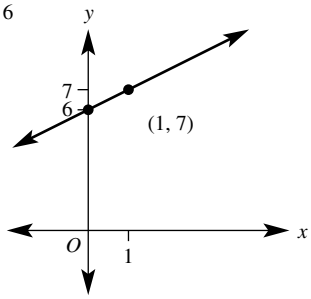
h $m = -\frac{4}{3}$, y-intercept $= 0$



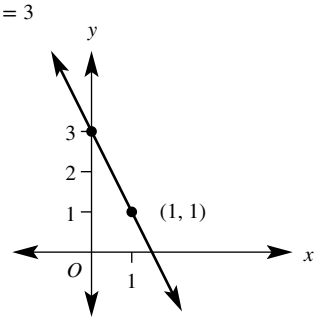
8 a $m = -1$, y-intercept $= 4$



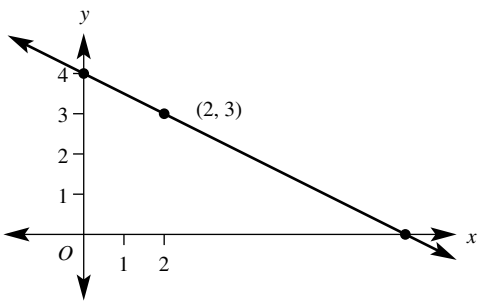
b $m = 1$, y-intercept $= 6$



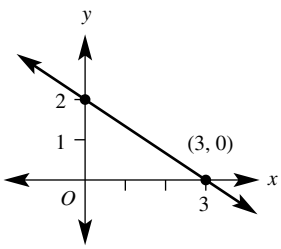
c $m = -2$, y-intercept $= 3$



d $m = \frac{-1}{2}$, y-intercept $= 4$



e $m = \frac{-2}{3}$, y-intercept $= 2$



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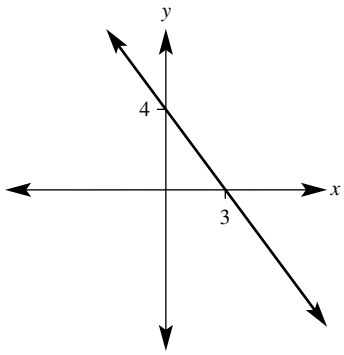
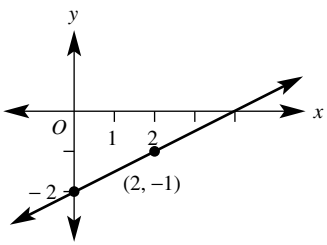
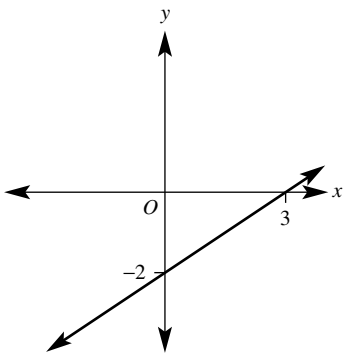
f $m = \frac{-4}{3}$, y-intercept $= 4$



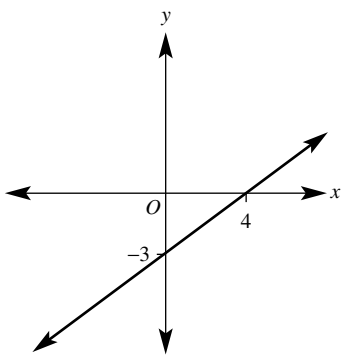
g $m = \frac{1}{2}$, y-intercept $= -2$



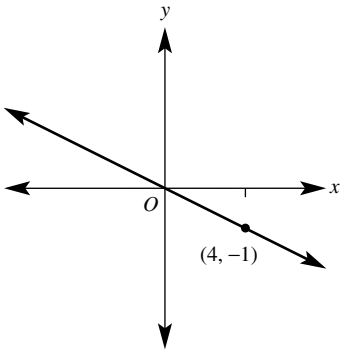
h $m = \frac{2}{3}$, y-intercept $= -2$



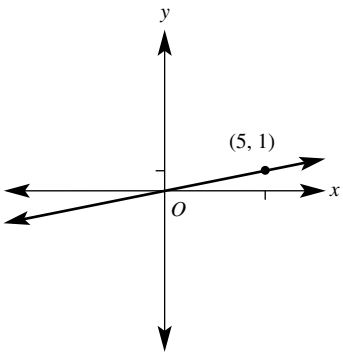
i $m = \frac{3}{4}$, y-intercept $= -3$



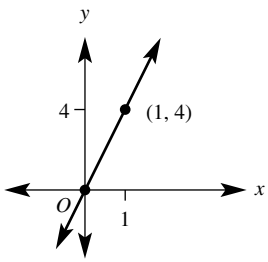
j $m = -\frac{1}{4}$, y-intercept $= 0$



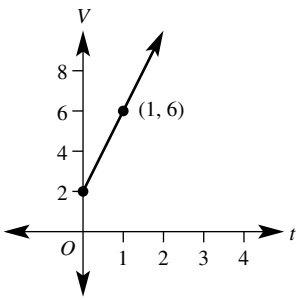
k $m = \frac{1}{5}$, y-intercept $= 0$,



l $m = 4$, y-intercept $= 0$



9 a gradient $= 4$, V-intercept $= 2$



b Gradient is the rate the water level is increasing (the rate of rain fall in mL/h), the V-intercept is the amount of water in the gauge before the storm

10 Yes she loses the bet. 6 needs to be divided by 2 also to give $2x + 3$.

11 c, d, f

12 a $y = 2x + 2$, y-intercept is 2

b Expand brackets

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13 a $y = 2x + 3$ **b** $y = -x + 2$
c $y = 2x + 3$ **d** $y = \frac{1}{2}x + 3$
e $y = \frac{1}{2}x + 3\frac{1}{2}$

Exercise 5H

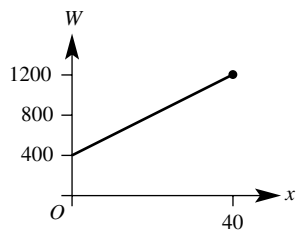
1 gradient, y-intercept, point
2 a $y = 2x + 5$ **b** $y = 4x - 1$
c $y = -2x + 5$ **d** $y = -x - \frac{1}{2}$
3 C
4 a 1 **b** 3 **c** 9 **d** 10 **e** 5 **f** 2
5 a $y = x + 3$ **b** $y = x + 2$ **c** $y = -4x - 4$
d $y = 2x - 2$ **e** $y = 8x + 8$ **f** $y = -x + 4$
g $y = -2x + 6$
6 a $y = \frac{3}{4}x + 3$ **b** $y = -\frac{3}{4}x + 3$ **c** $y = -\frac{5}{4}x + 3$
d $y = \frac{3}{2}x + 4$ **e** $y = \frac{3}{5}x$ **f** $y = -\frac{1}{3}x - 1$
7 a $y = 3x + 5$ **b** $y = -2x - 1$ **c** $y = -3x + 8$
d $y = x - 3$ **e** $y = -3x + 3$ **f** $y = 5x - 1$
g $y = -x + 8$ **h** $y = -3x + 6$ **i** $y = -2x + 2$
j $y = -4x - 9$
8 $y = 5x + 1$
9 2.5
10 a $y = 35x + 80$ **b** After 12 weeks
11 a i 2 **ii** $y = 2x + 2$
b i -4 **ii** $y = -4x + 11$
c i -1 **ii** $y = -x + 3$
d i 1 **ii** $y = x - 4$
12 a $y = -20x + 120$ **b** 120 L
13 a $y = 2x - 3$ **b** (1.5, 0), (0, −3)
c The x-intercept is the time when the temperature in the house reaches 0°C. The y-intercept is the temperature in the house at some moment in time. In this example, it is probably the temperature in the house at midnight.

Exercise 5I

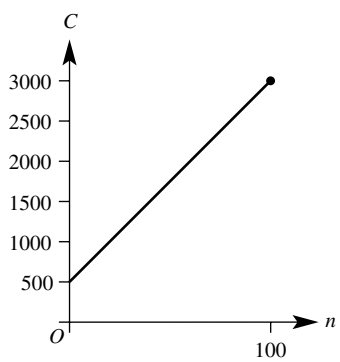
1 B
2 a 4 **b** 8 **c** 1 **d** −3
e 5.5 **f** 2.5 **g** −1.5 **h** −2.5
3 a 2.24 **b** 8.60 **c** 10.20
4 a i 3 **ii** 4 **b i** 2 **ii** 7
5 a (3, 3) **b** (2, 2) **c** (1, 5)
d (4, 1) **e** (−1, 3) **f** (−1, −1)
g (1.5, 1.5) **h** (2.5, 2) **i** (0.5, 3)
j (−1.5, −2.5) **k** (−3, −8.5) **l** (0.5, −2.5)
6 a 5 **b** 10 **c** 5.1 **d** 6.7
7 a 5.10 **b** 2.83 **c** 5.39
d 4.47 **e** 3.61 **f** 2.83
g 8.94 **h** 7.21 **i** 6.71
8 (−1, 5.5) and (5, 0.5)
9 $B(8, 0)$, $A(-6, 5)$, $A(-6, 9)$
10 a (−3, 1) **b** (1, −4) **c** (8, 3)
11 a 12.8 **b** 24.2
12 a (3, 5), (3, −1), (0, 2), (6, 2) are the obvious points.
b a circle
13 a $\frac{1}{3}$ **b** $\frac{1}{3}$
c i (1, 1) **ii** (−2, −0.5) **iii** (2, 1.5)

Exercise 5J

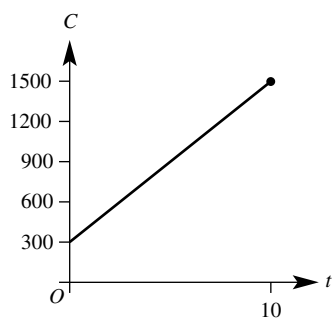
1 a i dependent **ii** independent
b i Volume **ii** Height
iii Amount of money
2 B
3 a i \$1200 **ii** \$1500 **iii** \$2200
b $A = 1000 + 100n$
4 a i 46 **ii** −4 **iii** 8.5
b i 2 **ii** 3 **iii** 6.2
5 a $W = 20x + 400$
b



c \$640 **d** 30
6 a $C = 50n + 40$ **b** \$240 **c** \$640
7 a $C = 25n + 500$
b



c \$1500 **d** 70
8 a $C = 120t + 300$
b



c \$1020 **d** 3 hours

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9 a $F = 18t + 12$ **b** 3 minutes

10 a $V = 4000 - 20t$ **b** 2200 L

c 200 minutes **d** 175 minutes

11 a 80 km/h

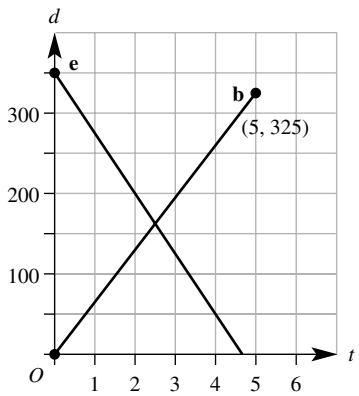
b Rate changes; i.e. new gradient = 70, $d = 50 + 70t$

12 a **i** 20 m/s **ii** 350 m **iii** 17.5 s

b Increasing altitude (i.e. rising) at rate of 20 m/s

13 a $d = 65t$

b and **e**



c 350 km

d 4 hours and 40 minutes ($4\frac{2}{3}$ hours)

f **i** 2.5 hours **ii** About 162.5 km

Exercise 5K

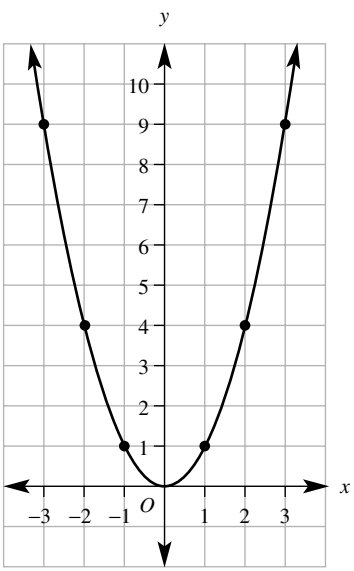
1 Parabola and exponential (circle, hyperbola etc. are others)

2 a 4 **b** 9 **c** 4 **d** 1

e −4 **f** −1 **g** 0 **h** −9

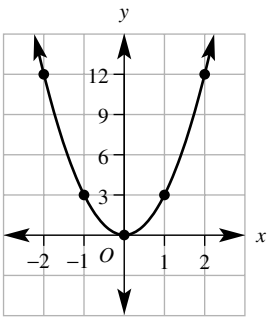
3 a 8 **b** 27 **c** 16 **d** 1 **e** 1

4



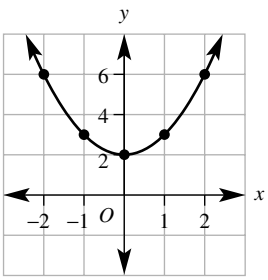
5 a

x	−2	−1	0	1	2
y	12	3	0	3	12



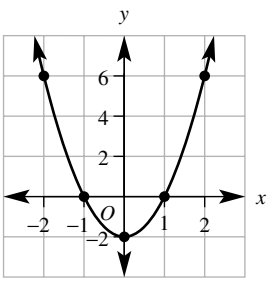
b

x	−2	−1	0	1	2
y	6	3	2	3	6



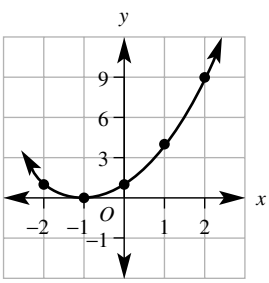
c

x	−2	−1	0	1	2
y	6	0	−2	0	6



d

x	−2	−1	0	1	2
y	1	0	1	4	9



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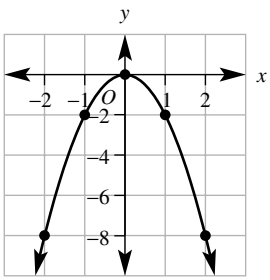
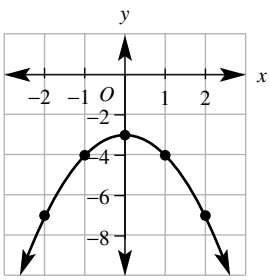
e

x	−2	−1	0	1	2
y	−8	−2	0	−2	−8



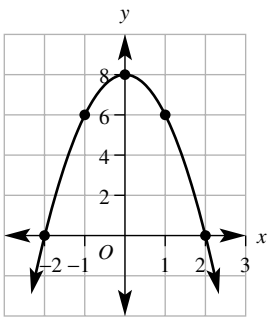
f

x	−2	−1	0	1	2
y	−7	−4	−3	−4	−7



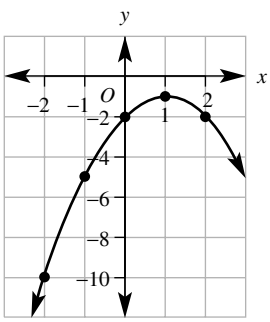
g

x	−2	−1	0	1	2
y	0	6	8	6	0

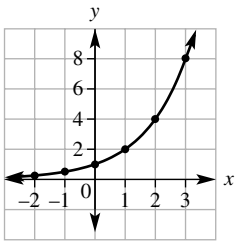


h

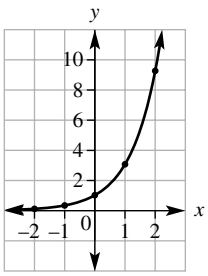
x	−2	−1	0	1	2
y	−10	−5	−2	−1	−2



6 a



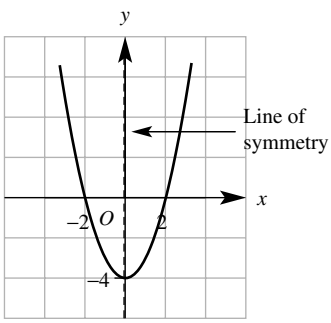
b



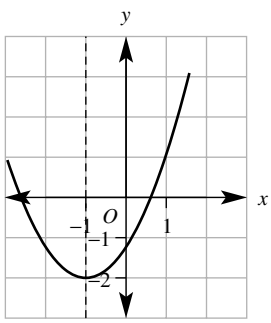
7 a i Minimum, (0, −4) **ii** Minimum, (−1, −2)
iii Maximum, (0, 1) **iv** Maximum (0, 0)

b The negative in front of x^2 creates a maximum turning point.

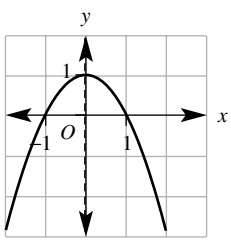
c i



ii



iii



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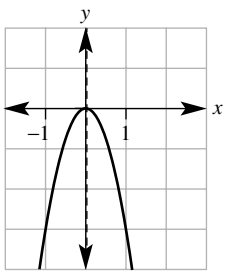
iv



8 a iii **b** i **c** ii **d** v **e** iv

9 a i 1 s, 3 s

ii One time is on the way up and the other is on the way down.

b i 2 s **ii** 12 m **iii** 4 s

10 a Exponential **b** \$100

c i \$110 **ii** \$121 **iii** \$133

d No

e i 10% **ii** 10% **iii** 10%

f It is constant. **g** \$146 (to the nearest dollar)

Puzzles and challenges

1 101 **2** 602 **3** 20 days

4 (0.5, 5.5), diagonals intersect at their midpoint

5 Length AB = length $AC = \sqrt{17}$

6 31 hours

Multiple-choice questions

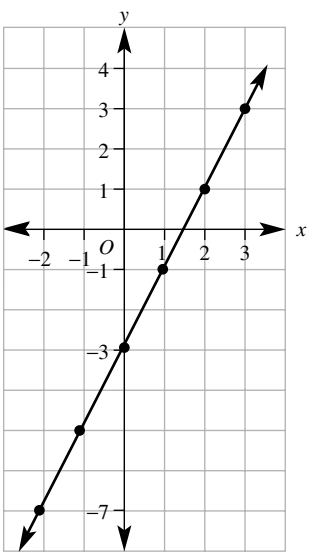
1 D **2** C **3** D **4** D **5** A

6 D **7** B **8** B **9** A **10** C

Short-answer questions

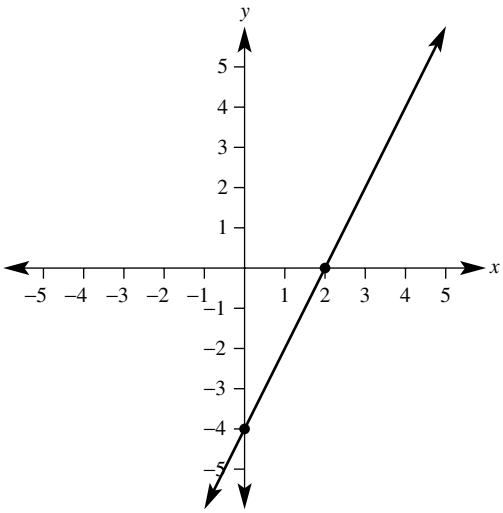
1

x	−2	−1	0	1	2	3
y	−7	−5	−3	−1	1	3

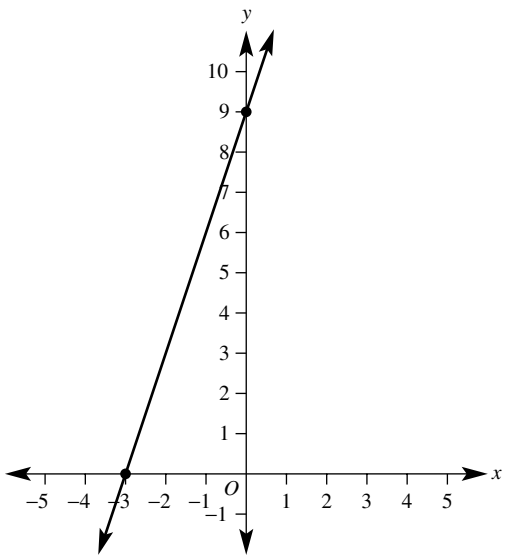


2 a −2 and 4 **b** 3 and −2

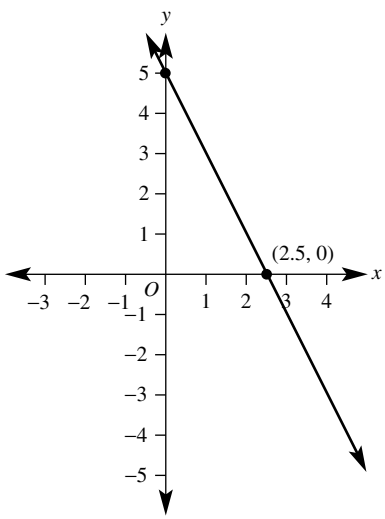
3 a



b



c



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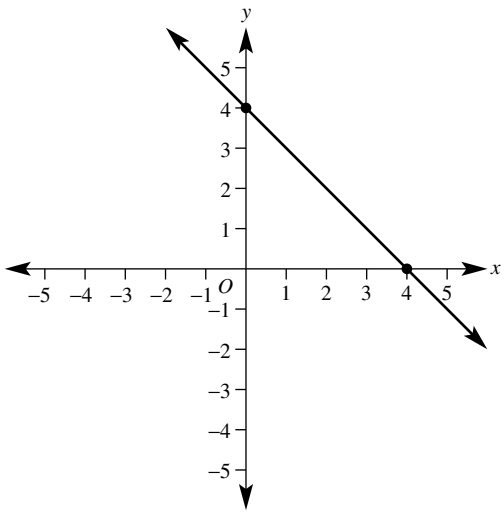
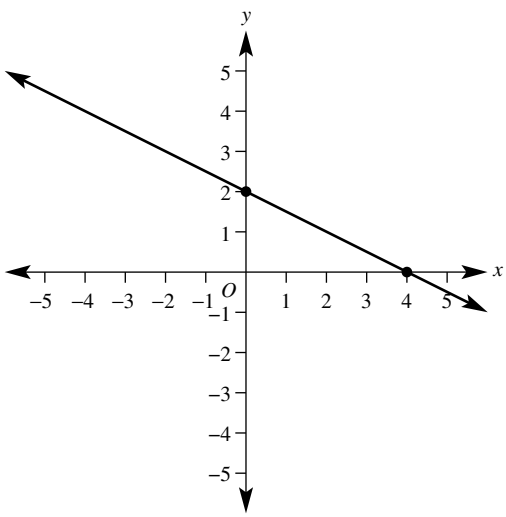
d



e



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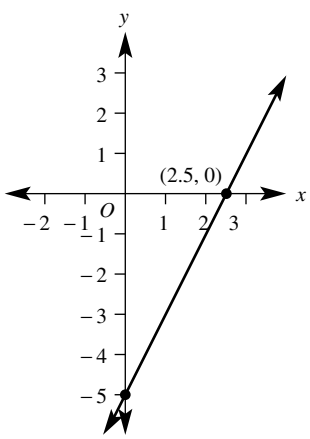
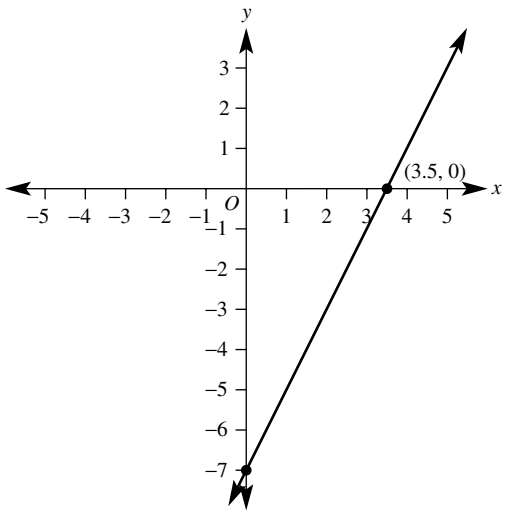


g



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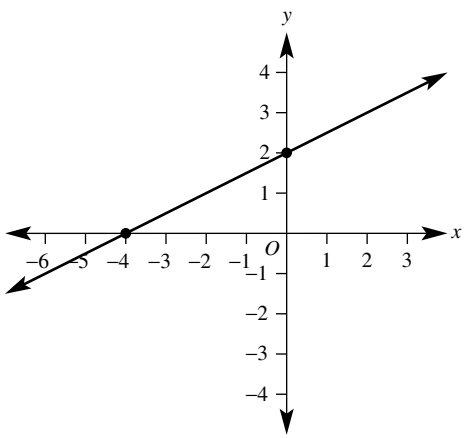
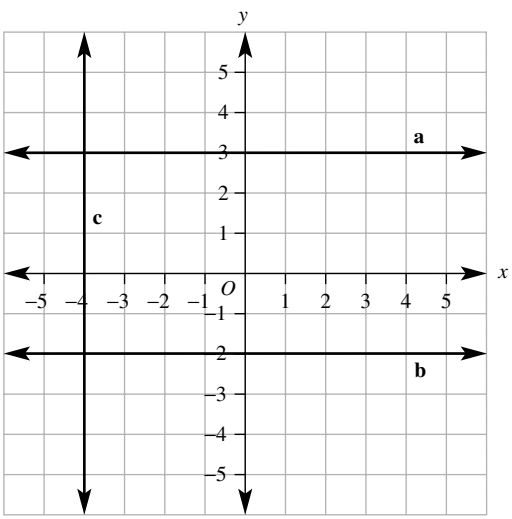


4 **a** Zero **b** Zero **c** Undefined



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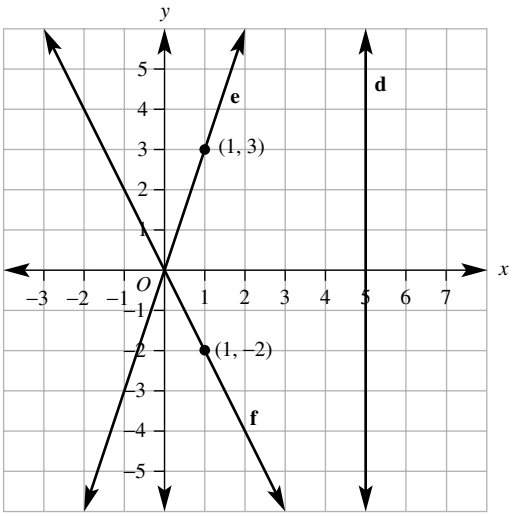
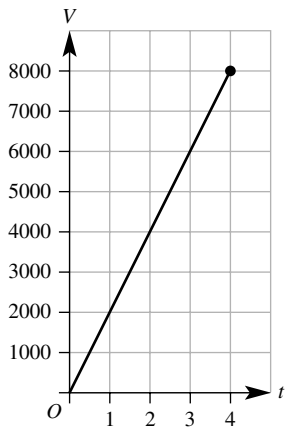
d Undefined **e** Positive **f** Negative



5 a 2 **b** −3

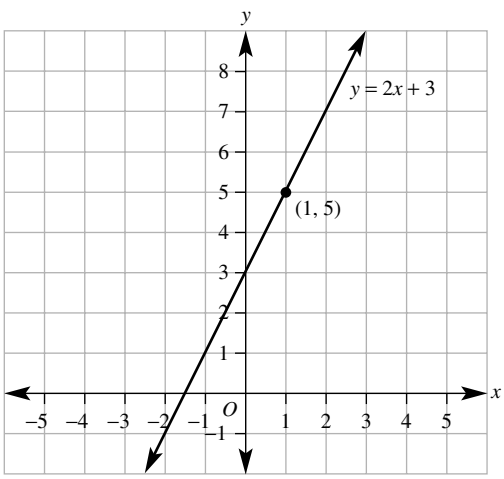
6 a 2 **b** −1 **c** 2 **d** $\frac{-10}{3}$

7 a 2000 L/h **b**

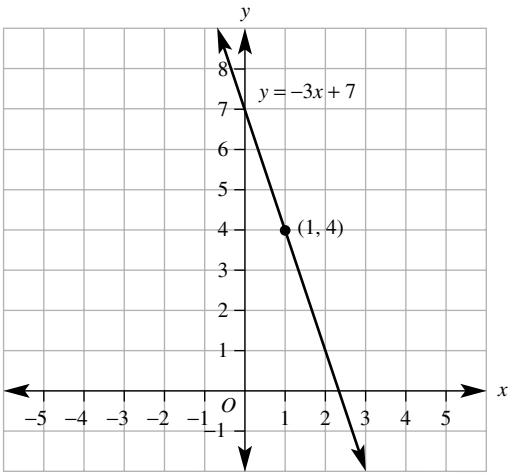


c $V = 2000t$ **d** 2.5 hours

8 a Gradient = 2, y-intercept = 3



b Gradient = −3, y-intercept = 7



9 a $y = -2x + 6$, gradient = −2, y-intercept = 6

b $y = -\frac{1}{2}x + 2$, gradient $= \frac{-1}{2}$, y-intercept = 2

10 a $y = 2x - 2$ **b** $y = -3x + 6$ **c** $y = 3x - 2$

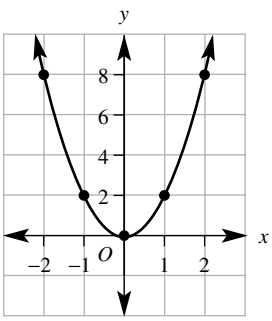
11 a $y = 3x + 1$ **b** $y = -2x + 3$

12 a $M(4, 6)$, 5.66 **b** $M(7.5, 4.5)$, 7.07

c $M(0, 4)$, 7.21 **d** $M(-3, 2.5)$, 9.85

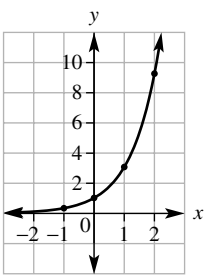
13 a

x	−2	−1	0	1	2
y	8	2	0	2	8



b

x	−2	−1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9



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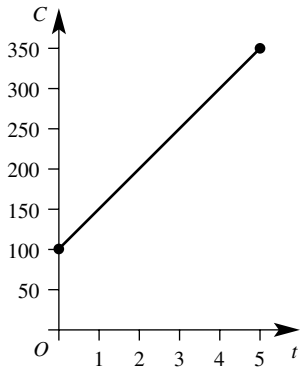
Extended-response questions

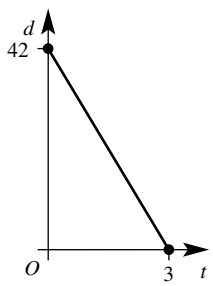
1 a $C = 50t + 100$

b



c \$300 **d** $2\frac{1}{2}$ hours

2 a



b 42 km **c** 3 hours **d** -14 **e** 14 km/h

Semester review 1

Integers, decimals, fractions, ratios and rates

Multiple-choice questions

1 B **2** D **3** D **4** C **5** C

Short-answer questions

1 a $\frac{10}{10} = 1$ **b** $\frac{19}{28}$ **c** $2\frac{3}{4}$ **d** $\frac{7}{9}$

2 a $\frac{3}{8}$ **b** $1\frac{4}{5}$

3 a 4.126 **b** 21.002 **c** 0.010

4 a \$160, \$640 **b** \$320, \$480 **c** \$700, \$100

5 a 5 : 3 **b** 55 km/h **c** 24 mL/h

6 181.3 km

7 4 m

Extended-response question

a 24 km

b i 14 km

ii 2 p.m.

Financial mathematics

Multiple-choice questions

1 B **2** D **3** A **4** D **5** A

Short-answer questions

1 a 60% **b** 31.25% **c** 10% **d** 25%

2 a \$9.60 **b** \$765 **c** \$11

3 a 84 m **b** \$50.76

4 4 h 48 min

5 \$892

6 a \$72 **b** \$872

7 $2\frac{1}{2}$ years

Extended-response question

a \$17 500 **b** \$23 520 **c** 9 years **d** 27%

Expressions and equations

Multiple-choice questions

1 D **2** E **3** A **4** C **5** C

Short-answer questions

1 a $-2x + 7y$ **b** $-15mn$ **c** $\frac{y}{3}$

2 a $x = 6$ **b** $x = 9$ **c** $m = \frac{3}{8}$ **d** $a = 3$

3 a $2m + 1 = 15$ **b** \$7

4 a 155 **b** 18

Extended-response question

a $p + 12$

b $p + (p + 12) = 38$; i.e. $2p + 12 = 38$

c Michael scored 13 points and Chris 25 points

Right-angled triangles

Multiple-choice questions

1 B **2** D **3** A **4** C **5** A

Short-answer questions

1 a 6.3 **b** 11.2 **c** 15.1

d 9.9 **e** 6.3 **f** 11.0

2 a 2.5 **b** 8.1 **c** 11.2

d 4.2 **e** 1.5 **f** 4.2

g 30° **h** 53.1° **i** 29.5°

3 a 19.21 m **b** 38.7°

4 a $x = 13$

b $y = 14.7$

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Extended-response question

a 17.75 m **b** 14.3° **c** 18.8 m

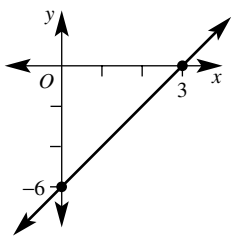
Linear relationships

Multiple-choice questions

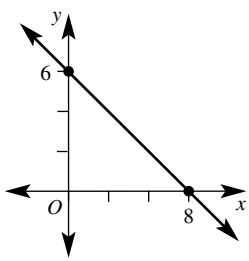
1 C **2** D **3** A **4** B **5** A

Short-answer questions

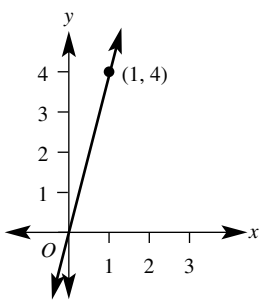
1 a



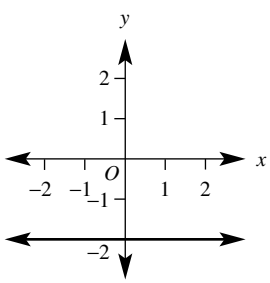
b



c



d



2 a $\frac{2}{3}$ **b** -3 **c** -2 **d** $\frac{4}{3}$

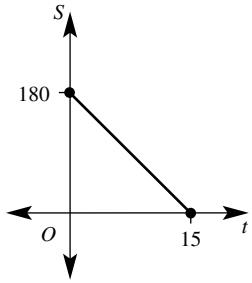
3 a $y = -3x + 6$ **b** $y = 3x - 1$

Extended-response question

a 12 kg/h

b $S = 180 - 12t$

c



d 15 hours

e i $P = 40 + 25h$ **ii** \$415

Chapter 6

Pre-test

1 a 4 cm² **b** 6 m² **c** 8 cm²

2 a 8 cm **b** 10 m **c** 18 cm

3 a 3 **b** 6 **c** 27

4 a Cylinder **b** Circle

5 a 23 **b** 48 **c** 2.7 **d** 5.2134 **e** 50 **f** 72.16

6 7

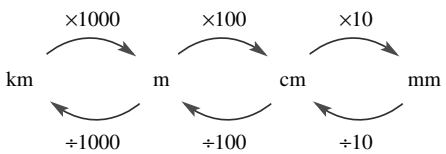
7 a 30 mm **b** 2000 cm **c** 1600 m **d** 2.3 cm **e** 3.167 km **f** 0.72 m **g** 20 km **h** 30 m

8 a 30 cm² **b** 4 m² **c** 49 km² **d** 3 m² **e** 24 cm² **f** 66 m²

9 $C = 31.42$ m, $A = 78.54$ m²

Exercise 6A

1



2 a 7 **b** 26 **c** 2.1

3 a 50 mm **b** 410 mm **c** 280 cm **d** 40 cm **e** 4600 m **f** 900 m **g** 52.1 cm **h** 3.6 cm **i** 2.4 m **j** 0.837 m **k** 7 km **l** 2.17 km

4 825 cm, 2.25 cm

5 a 15 mm **b** 31 m **c** 80 cm **d** 12 m **e** 27 cm **f** 24 mm **g** 18 km **h** 10 m **i** 36 cm

6 a $x = 4$ **b** $x = 2.2$ **c** $x = 14$

7 a $a = 3, b = 6$ **b** $a = 12, b = 4$ **c** $a = 6.2, b = 3.1$

8 a 90 cm **b** 80 cm **c** 170 cm **d** 30.57 m **e** 25.5 cm **f** 15.4 km

9 108 m

10 a 8000 mm **b** 110 m **c** 1 cm **d** 20 mm **e** 0.284 km **f** 62.743 km

11 a 86 cm **b** 13.6 m **c** 40.4 cm

12 88 cm

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13 a **i** 96 cm
ii 104 cm **iii** 120 cm
b $P = 4(20 + 2x)$ $\therefore P = 8x + 80$
c 109.6 cm

Exercise 6B

1 a Radius **b** Diameter
c Circumference **d** Sector
2 a 2.8 cm **b** 96 mm
3 a $C = 2\pi r$ **b** $C = \pi d$
4 a $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$
5 a 50.27 m **b** 87.96 cm **c** 31.42 m
d 6.91 m **e** 45.24 cm **f** 101.79 mm
6 a 31.42 m **b** 78.54 cm **c** 9.42 mm
d 12.57 km **e** 5.65 mm **f** 23.25 m
7 a 12.56 m **b** 62.8 cm **c** 22 mm **d** 44 m
8 a 10.28 **b** 51.42 **c** 14.28
d 10.00 cm **e** 20.05 m **f** 106.73 km
9 28.27 m
10 4.1 m
11 a 12.57 cm **b** 102.83 mm **c** 41.06 m
12 a 188.50 cm
b **i** 376.99 cm **ii** 1979.20 cm
c 531

Exercise 6C

1 a 100 **b** 10 000 **c** 1 000 000
2 a 6 **b** 16 **c** 12
d 1 **e** 12 **f** 153
3 a Rectangle **b** Rhombus, square, kite
c Triangle **d** Trapezium
e Parallelogram **f** Square
4 a 200 mm^2 **b** 40 mm^2 **c** 5 cm^2
d 3.1 cm^2 **e** 21 000 cm^2 **f** 2000 cm^2
g 21 m^2 **h** 0.37 m^2 **i** 1000 m^2
j 4 300 000 m^2 **k** 3.2 km^2 **l** 0.0394 km^2
5 a 24 m^2 **b** 10.5 cm^2 **c** 20 km^2
d 25.2 m^2 **e** 15 m^2 **f** 36.8 m^2
6 a 21 mm^2 **b** 12 cm^2 **c** 17 cm^2
d 63 m^2 **e** 6.205 m^2 **f** 15.19 km^2
7 500 000 m^2
8 0.175 km^2
9 a 12.25 cm^2 **b** 3.04 m^2 **c** 0.09 cm^2
d 6.5 mm^2 **e** 18 cm^2 **f** 2.4613 cm^2
10 0.51 m^2
11 a 1.5×10^{10}(15 000 000 000) cm^2
b 5 mm^2
c 0.075 m^2
12 a **i** 1.5 m **ii** 1.5 m
b 78 m^2 **c** Yes

Exercise 6D

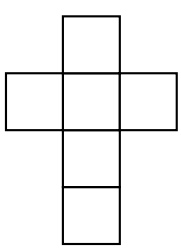
1 a 4.1 m **b** 7.5 m **c** 3.8 cm
2 C
3 a $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$
4 a 12.57 m^2 **b** 113.10 cm^2 **c** 254.47 km^2
d 60.82 mm^2 **e** 415.48 km^2 **f** 45 238.93 cm^2
5 a 314.16 cm^2 **b** 60.82 m^2 **c** 17.35 cm^2
6 a 21.23 m^2 **b** 216.51 km^2 **c** 196.07 cm^2
7 a 12.57 m^2 **b** 157.08 cm^2 **c** 84.82 m^2
8 177 cm^2
9 a 2 m **b** 12.57 m^2 **c** 3.43 m^2 **d** 21.5%
10 12.89%
11 a 1.8 cm **b** 6.1 m **c** 2 km
12 31%

Exercise 6E

1 a Semicircle and rectangle **b** Triangle and semicircle
c Rhombus and parallelogram
2 a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{1}{4}$
3 a 4 cm^2 **b** 3.14 cm^2 **c** 7.14 cm^2 **d** 11.14 cm
4 a 46 m, 97 m^2 **b** 34 m, 76 m^2
c 40 m, 90 m^2 **d** 18.28 m, 22.28 m^2
e 19.42 m, 26.14 m^2 **f** 85.42 mm, 326.37 mm^2
5 a $P = 14.28$ m, $A = 3.43$ m^2
b $P = 35.71$ m, $A = 10.73$ m^2
c $P = 41.13$ km, $A = 27.47$ km^2
6 a 108 m^2 **b** 33 cm^2 **c** 98 m^2
d 300 m^2 **e** 16 cm^2 **f** 22.5 m^2
7 189.27 m^2
8 68.67 cm^2
9 a 90 cm^2 **b** 15 m^2 **c** 9 m^2
d 7.51 cm^2 **e** 7.95 m^2 **f** 180.03 cm^2
g 8.74 mm^2 **h** 21.99 cm^2 **i** 23.83 mm^2
10 a 17 cm^2 **b** 3.5 cm^2 **c** 21.74 cm^2
11 a 37.70 m, 92.55 m^2 **b** 20.57 mm, 16 mm^2
c 18.00 cm, 11.61 cm^2 **d** 12.57 m, 6.28 m^2
e 25.71 cm, 23.14 cm^2 **f** 33.56 m, 83.90 m^2

Exercise 6F

1 a 6 **b** 6 **c** 5
2 a



Cube

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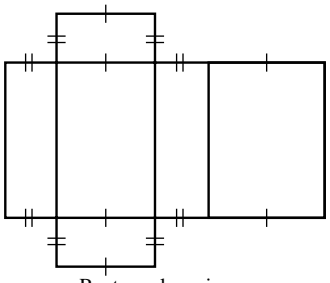
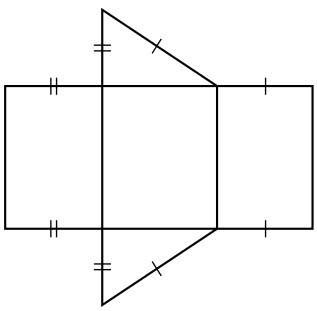
b



Rectangular prism

c



Triangular prism

3 a $A = 2 \times (8 \times 7 + 8 \times 3 + 7 \times 3)$
$= 202\ \text{m}^2$

b $A = 5 \times 7 + 4 \times 7 + 3 \times 7 + 2 \times \left(\frac{1}{2} \times 4 \times 3\right)$
$= 96\ \text{cm}^2$

4 a 52 m² **b** 242 cm² **c** 76 m²
d 192 cm² **e** 68.16 m² **f** 85.76 m²

5 a 96 cm² **b** 240 m² **c** 199.8 cm² **d** 238 cm²

6 6 m²

7 14.54 m²

8 34 000 cm²

9 a 44.4 m² **b** 4.44 L

10 a [6, 10, 14, 18, 22, 26, 30, 34, 38]
b $A = 4n + 2$
c 402

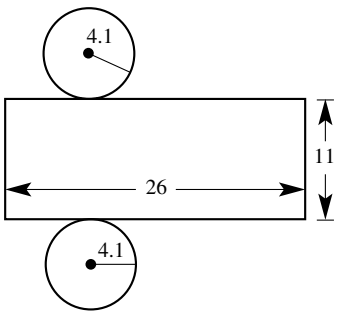
11 a 39 mm² **b** 224 cm² **c** 9.01 m²

Exercise 6G

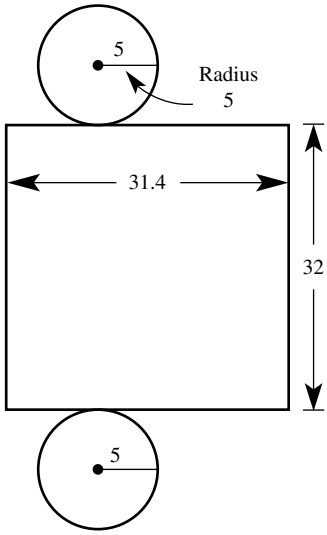
1 a $2\pi rh$ **b** $2\pi r^2$

2 a 25.13 m **b** 18.85 mm **c** 26.39 cm

3 a



b



4 a 22 cm by 10 cm **b** 12.57 cm by 8 cm
c 50.27 m by 5 m

5 a 25.13 m² **b** 471.24 cm² **c** 50.27 m²

6 a 44.0 cm² **b** 603.2 cm² **c** 113.1 m²

7 a 251.33 cm² **b** 207.35 mm² **c** 24.13 m²

8 395.84 cm²

9 a 54.56 m² **b** 218.23 m² **c** 63.98 cm²
d 71.91 cm² **e** 270.80 m² **f** 313.65 km²

10 7539.82 cm²

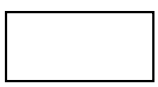
11 Half cylinder is more than half surface area because it includes new rectangular surface.

12 a 18 849.556 cm²
b i 1.88 m² **ii** 37.70 m² **c** 239

Exercise 6H

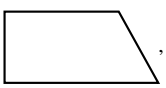
1 a Square **b** Triangle **c** Rectangle
d Trapezium **e** Hexagon **f** Triangle

2 a



, 6 cm

b

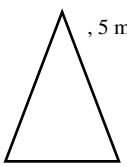


, 10 cm

c

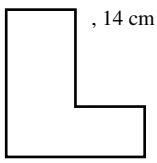
, 2 m

d



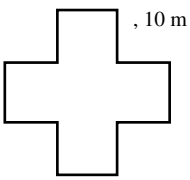
, 5 mm

e



, 14 cm

f



, 10 m

3 a 10 **b** 100 **c** 1000
d 1 000 000 **e** 1 000 000 000 **f** 1000
g 1000 **h** 1

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4 **a** 3000 mm^3 **b** 300 mm^3 **c** 2 cm^3
d 1000 cm^3 **e** 8 700 000 cm^3 **f** 0.0059 m^3
g 10 000 m^3 **h** 0.000 021 7 km^3 **i** 0.43 m^3
5 **a** 3000 mL **b** 200 L **c** 3.5 L
d 21 mL **e** 37 kL **f** 42.9 ML
g 2 mL **h** 2000 cm^3 **i** 1000 L
6 **a** 8 cm^3 **b** 84 m^3 **c** 21 mm^3
7 **a** 16 cm^3 **b** 42.875 m^3 **c** 15 cm^3
8 **a** 4 cm^3 **b** 10.5 m^3 **c** 11.96 cm^3
d 29 cm^3 **e** 14.88 m^3 **f** 8.1351 cm^3
9 **a** 75 m^3 **b** 30 cm^3 **c** 1.25 cm^3
10 8000 cm^3
11 Yes, the tank only holds 20 L.
12 **a** 8 L **b** 0.36 L **c** 0.48 L
13 **a** 55 m^2 **b** 825 000 L
14 **b** $\frac{1}{3}$

Exercise 6I

1 **a** 12.57 cm^2 **b** 8.04 m^2 **c** 78.54 cm^2 **d** 2.54 km^2
2 **a** 2 L **b** 4.3 mL **c** 3700 cm^3
d 1000 L **e** 38 m^3 **f** 200 mL
3 **a** $r = 4$ m, $h = 10$ m **b** $r = 2.6$ cm, $h = 11.1$ cm
c $r = 2.9$ m, $h = 12.8$ m **d** $r = 9$ m, $h = 23$ m
e $r = 5.8$ cm, $h = 15.1$ cm
f $r = 10.65$ cm, $h = 10.4$ cm
4 **a** 226.19 cm^3 **b** 18.85 m^3 **c** 137.44 m^3
d 100.53 cm^3 **e** 8.48 m^3 **f** 68.05 m^3
5 **a** 18 L **b** 503 L **c** 20 L **d** 2 L
6 **a** 1.571 m^3 **b** 1571 L
7 Cylinder by 131 cm^3
8 **a** 25.133 m^3 **b** 25 133 L
9 37 699 L
10 A number of answers. Require $h = 2\pi r$.
11 **a** 502.65 cm^3 **b** 1.02 m^3 **c** 294.52 m^3
d 35 342.92 m^3 **e** 47.12 cm^3 **f** 1017.88 cm^3

Puzzles and challenges

1 **a** 19 **b** 35
2 8 **3** 314 m **4** 163.4 m^2
5 7.1 cm **6** 11 **7** 100 L

Multiple-choice questions

1 B **2** C **3** E **4** B **5** A
6 D **7** B **8** E **9** C **10** E

Short-answer questions

1 **a** 380 cm **b** 1270 m **c** 4.8 cm
d 2.73 cm^2 **e** 52 000 cm^2 **f** 10 000 cm^3
g 53.1 cm^3 **h** 3.1 L **i** 43 mL
j 2830 L **k** 4 L **l** 1000 L
2 **a** 14 m **b** 51 mm **c** 16.2 cm
3 **a** 4 cm^2 **b** 1122 mm^2 **c** 30.34 mm^2
d 7.5 m^2 **e** 15 cm^2 **f** 3 cm^2
4 **a** 2.5 m^2 **b** 37.4 m^2
5 **a** $A = 28.27$ cm^2, $P = 18.85$ cm
b $A = 1.57$ m^2, $P = 5.14$ m
c $A = 2.36$ cm^2, $P = 6.71$ cm
6 **a** $P = 22.28$ m, $A = 30.28$ m^2
b $P = 45.56$ cm, $A = 128.54$ cm^2
7 **a** 46 cm^2 **b** 114 m^2
8 **a** 659.73 mm^2 **b** 207.35 m^2
9 **a** 30 cm^3 **b** 54 m^3 **c** 31.42 mm^3

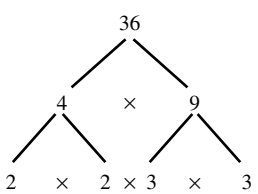
Extended-response questions

1 **a** 135 cm^3 **b** 174 cm^2 **c** \$43.50
d **i** Yes, by ~119 cm^3 **ii** Yes, by ~52 cm^2
2 **a** 5.17 m **b** \$65
c 1.59 m^2, claim is correct

Chapter 7

Pre-test

1



$36 = 2 \times 2 \times 3 \times 3$

2 **a** 24, 1, 12, 2, 8, 3, 6, 4 **b** 45, 1, 15, 3, 9, 5
c 2, 3 **d** 3, 5
3 **a** 7^3 **b** $5^2 2^3$ **c** 3^2 **d** 4^1 or 4
4 **a** 2^3 **b** 2^6 **c** 2^5
5 **a** 25 **b** 100 **c** 1 **d** 5 **e** 1
6 **a** 12 **b** 36 **c** 40 **d** 30 000
7 **a** 3^4 **b** 2^{12} **c** 9
8 **a** 0.003 **b** 0.04 **c** 0.000 002 **d** 0.0008
9 **a** 3.73 **b** 24.62 **c** 18.37 **d** 4.40
10 **a** tens **b** thousandths
c millions **d** tenths
11 **a** 5 **b** 4 **c** 2 **d** 5
12 **a** 38 **b** 2310 **c** 0.172 **d** 0.0018
e 1000 **f** 10 000

Exercise 7A

1 **a** expanded, factor **b** index
c power **d** base **e** index
f prime **g** prime factors

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2

	Base	Index	Expanded form	Basic numeral
3^4	3	4	$3 \times 3 \times 3 \times 3$	81
5^2	5	2	5×5	25
2^5	2	5	$2 \times 2 \times 2 \times 2 \times 2$	32
$\left(\frac{2}{3}\right)^3$	$\frac{2}{3}$	3	$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$	$\frac{8}{27}$
$(-3)^3$	-3	3	$(-3) \times (-3) \times (-3)$	-27

3 **a** 3 **b** 6 **c** 1.2 **d** -7
e $\frac{2}{3}$ **f** y **g** w **h** t

4 **a** 3 **b** 8 **c** 7 **d** 4
e 11 **f** 13 **g** 9 **h** 2

5 **a** 2, 3 **b** 3, 5 **c** 2, 3, 5 **d** 7, 11

6 **a** $4 \times 4 \times 4$ **b** $7 \times 7 \times 7 \times 7$
c $3 \times 3 \times 3 \times 3 \times 3$ **d** $5 \times 5 \times 5$
e $a \times a \times a \times a$ **f** $b \times b \times b$
g $x \times x \times x$
h $xp \times xp \times xp \times xp \times xp \times xp$
i $5a \times 5a \times 5a \times 5a$ **j** $3y \times 3y \times 3y$
k $4 \times x \times x \times y \times y \times y \times y \times y$
l $pq \times pq$ **m** $-3 \times s \times s \times s \times s \times t \times t$
n $6 \times x \times x \times x \times y \times y \times y \times y \times y$
o $5 \times yz \times yz \times yz \times yz \times yz \times yz$
p $4 \times ab \times ba \times ab$

7 **a** 36 **b** 16 **c** 243 **d** 12
e -8 **f** -1 **g** 81 **h** 25
i $\frac{8}{27}$ **j** $\frac{9}{16}$ **k** $\frac{1}{216}$ **l** $\frac{25}{4}$
m $-\frac{8}{27}$ **n** $\frac{81}{256}$ **o** $\frac{1}{16}$ **p** $-\frac{3125}{32}$

8 **a** 3^3 **b** 8^6 **c** y^2 **d** $3x^3$
e $4c^5$ **f** 5^3d^2 **g** x^2y^3 **h** 7^3b^2

9 **a** $\left(\frac{2}{3}\right)^4$ **b** $\left(\frac{3}{5}\right)^5$
c $\left(\frac{4}{7}\right)^2 \times \left(\frac{1}{5}\right)^4$ **d** $\left(\frac{7x}{9}\right)^2 \times \left(\frac{y}{4}\right)^3$

10 **a** $3^3x^3y^2$ **b** $(3x)^2(2y)^2$ or $3^2 2^2 x^2 y^2$
c $(4d)^2(2e)^2$ or $4^2 2^2 d^2 e^2$ **d** $6^3b^2y^3$
e $(3pq)^4$ or $3^4p^4q^4$ **f** $(7mn)^3$ or $7^3m^3n^3$

11 **a** 2×5 **b** 2^3 **c** $2^4 \times 3^2$
d 3×5^2 **e** 3×7^2 **f** $2^2 \times 5^3$

12 **a** $3 \times 3 \times a \times a \times a = 3^2a^3$ **b** $5 \times 5 \times k \times k \times k = 5^2k^3$
c $\frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} = \left(\frac{2}{7}\right)^3$ **d** $3p^2q = 3 \times p \times p \times q$
e $(abc)^1 = a \times b \times c$

13 **a** 36 **b** -216 **c** 1 **d** $-\frac{8}{27}$
e -18 **f** 15 **g** -36 **h** 216

14 **a** 4 **b** 8 **c** 5 **d** 2
e -4 **f** -2 **g** $\frac{1}{2}$ **h** 4

15 **a**

Time (min)	Number of bacteria	Number in index form
0	1	2^0
1	$1 \times 2 = 2$	2^1
2	$2 \times 2 = 4$	2^2
3	$2 \times 2 \times 2 = 8$	2^3
4	$2 \times 2 \times 2 \times 2 = 16$	2^4
5	$2 \times 2 \times 2 \times 2 \times 2 = 32$	2^5
6	$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$	2^6
7	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$	2^7
8	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$	2^8
9	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$	2^9
10	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$	2^{10}

b **i** 2 min **ii** 4 min **iii** 6 min
c $2^{24} = 16\,777\,216$ cells

Exercise 7B

1 **a** base, add **b** base, subtract

2 **a** $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$
b $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^7$
c $k \times k \times k \times k \times k = k^5$
d $m \times m \times m \times m \times m \times m \times m = m^7$

3 **a** $\frac{5 \times 5 \times \cancel{5}^1 \times \cancel{5}^1 \times \cancel{5}^1}{\cancel{5}^1 \times \cancel{5}^1 \times \cancel{5}^1} = 5^2$
b $\frac{9 \times 9 \times \cancel{9}^1 \times \cancel{9}^1}{\cancel{9}^1 \times \cancel{9}^1} = 9^2$
c $\frac{a \times a \times a \times a \times \cancel{a}^1 \times \cancel{a}^1}{\cancel{a}^1 \times \cancel{a}^1} = a^4$
d $\frac{n \times n \times n \times \cancel{n}^1 \times \cancel{n}^1 \times \cancel{n}^1}{\cancel{n}^1 \times \cancel{n}^1 \times \cancel{n}^1} = n^3$

4 **a** $6^{5+7} = 6^{12}$ **b** $a^{13+2} = a^{15}$
c $5^{12-4} = 5^8$ **d** $m^{16-2} = m^{14}$

5 **a** 2^7 **b** 5^9 **c** 7^6 **d** 8^{10}
e 3^8 **f** 6^{14} **g** 3^3 **h** 6^5
i 5^3 **j** 10 **k** 9^3 **l** $(-2)^2$
m 10^8 **n** $(-5)^8$ **o** 6^9 **p** 5^7
q 9^{10} **r** 3^8 **s** 2^7 **t** $(-3)^7$

6 **a** x^7 **b** a^9 **c** t^8 **d** y^5
e d^3 **f** y^7 **g** b^8 **h** q^{11}
i a^5m^4 **j** k^5p^3 **k** x^6y^8 **l** m^7e^4
m a^{10} **n** m^{16} **o** $p^7q^4r^3$ **p** $-m^{13}n^9$

7 **a** a^2 **b** x^3 **c** q^{10} **d** d
e b^5 **f** d^5 **g** a^7 **h** y^8

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8 **a** $7x^7y^5$ **b** $3x^9y^4$ **c** $5x^4y^9$ **d** $4x^2y^5z$
e $15m^5$ **f** $8e^6f^4$ **g** $20c^7d^2$ **h** $18y^2z^7$
i $3m$ **j** $7x^3$ **k** $5y^2$ **l** $3a$
m $3m^5$ **n** $\frac{w}{5}$ **o** $\frac{a}{5}$ **p** $\frac{x^4}{9}$
q $\frac{4x^6y^3}{3}$ **r** $\frac{3st^2}{7}$ **s** $\frac{4mn}{3}$ **t** $5x$

9 **a** b^6 **b** y^6 **c** c^7 **d** x
e t **f** p^6 **g** d^6 **h** x^{10}
i $4x^2y^3$ **j** $6b^2g$

10 **a** $\frac{m^5}{n^5}$ **b** $\frac{x^4}{y^2}$ **c** a^3b^3
d $\frac{6a^5}{c^7}$ **e** $6f^6$ **f** $12x^4b^2$

11 **a** 12 **b** 8 **c** 3
d 3 **e** 1 **f** 18
g 12 **h** 11 **i** 4
j 15 **k** 2 **l** 39

12 **a** $7^2 = 49$ **b** 10
c $13^2 = 169$ **d** $2^3 = 8$
e 101 **f** $200^2 = 40\,000$
g $7 \times 31 = 217$ **h** $3 \times 50^2 = 7500$

13 **a** **i** $10a^{10}$ **ii** $9m^8$ **iii** $150m^5$ **iv** $60x^6y^5$
b **i** $4am$ **ii** $6xy^2$

Exercise 7C

1 **a** multiply **b** 1

2 **a** 16, 8, 4, 2, 1 **b** 64, 16, 4, 1

3 **a** $4^2 \times 4^2 \times 4^2 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$
b $12^3 \times 12^3 = 12 \times 12 \times 12 \times 12 \times 12 \times 12 = 12^6$
c $x^4 \times x^4 = x \times x \times x \times x \times x \times x \times x \times x = x^8$
d $a^2 \times a^2 \times a^2 \times a^2 \times a^2 = a \times a \times a \times a \times a \times a$
$\times a \times a \times a \times a = a^{10}$

4 **a** 1 **b** 1 **c** 1 **d** 1
e 1 **f** 1 **g** 1 **h** 1

5 **a** y^{12} **b** m^{18} **c** x^{10} **d** b^{12}
e 3^6 **f** 4^{15} **g** 3^{30} **h** 7^{10}
i $5m^{16}$ **j** $4q^{28}$ **k** $-3c^{10}$ **l** $2j^{24}$

6 **a** 1 **b** 1 **c** 1 **d** 1
e 1 **f** 1 **g** 1 **h** 1
i 5 **j** -3 **k** 4 **l** -6
m 1 **n** 3 **o** 4 **p** 0

7 **a** 4^7 **b** 3^9 **c** x
d y^{13} **e** b^{14} **f** a^{10}
g d^{24} **h** y^{16} **i** z^{25}

8 **a** b^6 **b** x^5 **c** y^6 **d** 7^2 **e** 4
f 3^8 **g** 1 **h** y^3 **i** h^2

9 **a** 2 **b** $10x$ **c** $3x^8$ **d** $\frac{d^2e}{2}$ **e** $\frac{2m^6n}{5}$ **f** $\frac{a^{12}}{8}$

10 5 ways: $(a^{16})^1 = a^{16}$, $(a^1)^{16} = a^{16}$, $(a^2)^8 = a^{16}$, $(a^8)^2 = a^{16}$, $(a^4)^4 = a^{16}$

11 **a** 4×5 not $4 + 5$, a^{20}
b Power of 2 only applies to x^3, $3x^6$
c Power zero applies to whole bracket, 1

12 **a** **i** 400 **ii** 6400 **iii** 100
b **i** 800 **ii** 12 800 **iii** 102 400
c 13 years

Exercise 7D

1 **a** $a^m \times b^m$ **b** $\frac{a^m}{b^m}$

2 **a** $5a \times 5a \times 5a$
$= 5 \times 5 \times 5 \times a \times a \times a$
$= 5^3 \times a^3$
b $ab \times ab \times ab \times ab$
$= a \times a \times a \times a \times b \times b \times b \times b$
$= a^4 \times b^4$
c $\frac{x}{6} \times \frac{x}{6} \times \frac{x}{6}$
$= \frac{x \times x \times x}{6 \times 6 \times 6}$
$= \frac{x^3}{6^3}$
d $\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}$
$= \frac{a \times a \times a \times a \times a}{b \times b \times b \times b \times b}$
$= \frac{a^5}{b^5}$

3 **a** x^5y^5 **b** 2^3m^3 **c** $a^4b^4c^4$
d $\frac{5^3}{k^3}$ **e** $\frac{a^5}{m^5}$ **f** $\frac{2^4}{3^4} = \frac{16}{81}$

4 **a** $6a^2b^2$ **b** $4x^3y^3$
c $7 \times 2^2m^2 = 7 \times 4 \times m^2 = 28m^2$
d $4 \times 3^2a^2b^2 = 4 \times 9a^2b^2 = 36a^2b^2$

5 **a** $2^4 \times (3^2)^4 = 2^4 \times 3^{2\times4} = 2^4 \times 3^8$
b $(5^2)^4 \times (7^3)^4 = 5^{2\times4} \times 7^{3\times4} = 5^8 \times 7^{12}$
c $\frac{(11^2)^3}{5^3} = \frac{11^{2\times3}}{5^3} = \frac{11^6}{5^3}$
d $\frac{(8^2)^4}{(3^5)^4} = \frac{8^{2\times4}}{3^{5\times4}} = \frac{8^8}{3^{20}}$

6 **a** $5^3 \times 7^6$ **b** $3^2 \times 4^{10}$ **c** $2^4 \times 5^{12}$
d $2^4 \times 3^8$ **e** $5^{12} \times 3^8$ **f** $8^{10} \times 11^{25}$

7 **a** $8x^3$ **b** $25y^2$ **c** $64a^3$ **d** $9r^2$
e $81b^4$ **f** $343r^3$ **g** $16h^8$ **h** $625c^8$
i $27a^6$ **j** $49p^8$ **k** $25m^6$ **l** $9y^{20}$

8 **a** $\frac{3^4}{5^2}$ **b** $\frac{2^8}{7^2}$ **c** $\frac{9^4}{8^{12}}$ **d** $\frac{5^4}{12^6}$ **e** $\frac{3^8}{2^{10}}$ **f** $\frac{7^{15}}{4^{10}}$

9 **a** $\frac{p^3}{q^3}$ **b** $\frac{x^4}{y^4}$ **c** $\frac{64}{y^3}$ **d** $\frac{625}{p^8}$
e $\frac{4}{r^6}$ **f** $\frac{s^6}{49}$ **g** $\frac{49}{x^6}$ **h** $\frac{w^8}{100}$
i $\frac{a^6}{125}$ **j** $\frac{m^8}{16}$ **k** $\frac{32m^5}{n^5}$ **l** $\frac{8a^6}{27}$

10 **a** $32x^{15}y^{10}$ **b** $9p^6q^{12}$ **c** $2x^6y^2$ **d** 1
e $-27w^9y^3$ **f** $-4p^8q^2r^2$ **g** $25s^{14}t^2$ **h** $8x^{12}y^3z^9$
i $625a^{16}b^{12}$ **j** $9m^{14}n^2$ **k** $49a^{10}b^8c^2$ **l** $81m^{10}n^2$

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11 a $\frac{27n^9}{8m^{12}}$ **b** $\frac{16r^4}{n^4}$ **c** $\frac{9f^2}{64g^{10}}$ **d** $\frac{25w^8y^2}{4x^6}$

e $\frac{9x^2}{4y^6g^{10}}$ **f** $\frac{27k^3m^9}{64n^{21}}$ **g** $-\frac{25w^8y^2}{4z^2x^6}$ **h** $\frac{9x^4y^6}{4a^{10}b^6}$

12 a $9ab^2$ **b** $27ab^6$ **c** $-12a^8b^8$ **d** $54x^6y^9$

e $-64b^6c^{15}d^3$ **f** $8a^4$ **g** $9a^5$ **h** $-40a^{15}b^3$

i $160m^{15}p^5t^{10}$ **j** $\frac{-27x^6}{125a^{15}b^9}$ **k** $a^{11}bc^5$ **l** $x^{11}y^2z$

13 a i 8 **ii** 125 **b** $N = \frac{t^3}{8}$

c i 27 **ii** 8 **d i** 8 **ii** 2

Exercise 7E

1 a $\frac{1}{2^2}$ **b** $\frac{1}{3^2}$ **c** $\frac{1}{5^3}$ **d** $\frac{1}{3^3}$

2 a

Index form	3^4	3^3	3^2	3^1
Whole number or fraction	81	27	9	3

Index form	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	1	$\frac{1}{3}$	$\frac{1}{9} = \frac{1}{3^2}$	$\frac{1}{27} = \frac{1}{3^3}$

b

Index form	10^4	10^3	10^2	10^1
Whole number or fraction	10 000	1000	100	10

Index form	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	1	$\frac{1}{10}$	$\frac{1}{100} = \frac{1}{10^2}$	$\frac{1}{1000} = \frac{1}{10^3}$

3 a $10^{-4} = \frac{1}{10^4}$ **b** $3^{-2} = \frac{1}{3^2}$ **c** $7^{-3} = \frac{1}{7^3}$

d $8^{-6} = \frac{1}{8^6}$ **e** $9^{-4} = \frac{1}{9^4}$ **f** $5^{-4} = \frac{1}{5^4}$

4 a $\frac{1}{3^{-4}} = 3^4$ **b** $\frac{1}{7^{-6}} = 7^6$ **c** $\frac{1}{8^{-3}} = 8^3$

d $\frac{1}{4^{-6}} = 4^6$ **e** $\frac{1}{10^{-3}} = 10^3$ **f** $\frac{1}{5^{-4}} = 5^4$

5 a $\frac{1}{5^2}$ **b** $\frac{1}{7^4}$ **c** $\frac{1}{8^3}$ **d** $\frac{1}{3^5}$

e $\frac{1}{9^2}$ **f** $\frac{1}{10^3}$ **g** $\frac{1}{4^5}$ **h** $\frac{1}{2^3}$

6 a $\frac{3}{2^4}$ **b** $\frac{5}{4^3}$ **c** $\frac{7}{5^6}$ **d** $\frac{2}{3^4}$

e $\frac{4}{3^5}$ **f** $\frac{9}{5^2}$ **g** $\frac{8}{7^3}$ **h** $\frac{6}{5^6}$

7 a 2^4 **b** 3^2 **c** 4^3 **d** 6^5

e 5^3 **f** 8^5 **g** 7^3 **h** 9^4

8 a 6×4^3 **b** 5×8^2 **c** 4×7^5 **d** 3×2^5

e 12×5^4 **f** 6×10^3 **g** 7×9^4 **h** 7×10^6

9 a $\frac{6}{25}$ **b** $\frac{2}{9}$ **c** $\frac{4}{125}$ **d** $\frac{6}{7}$

e $\frac{1}{250}$ **f** $\frac{1}{5}$ **g** $\frac{5}{16}$ **h** $\frac{4}{5}$

10 a $\frac{2}{1000} = 0.002$ **b** $\frac{5}{100} = 0.05$

c $\frac{7}{10} = 0.7$ **d** $\frac{3}{10000} = 0.0003$

e $\frac{5}{10000} = 0.0005$ **f** $\frac{8}{100000} = 0.00008$

g $\frac{2}{1000000} = 0.000002$ **h** $\frac{4}{100000000} = 0.00000004$

11 a $\frac{1}{5}$ **b** $\frac{1}{9}$ **c** $\frac{1}{25}$ **d** $-\frac{1}{200}$ **e** $\frac{1}{2}$ **f** $\frac{1}{36}$

g $\frac{1}{8}$ **h** 8 **i** 16 **j** 64 **k** $\frac{64}{9}$ **l** $\frac{27}{64}$

12 a Negative power only applies to x, $\frac{2}{x^2}$

b $5 = 5^1$ has a positive power, $5a^{-4}$

c $\frac{2}{3^{-2}b^{-2}} = 2 \times 3^2 \times b^2 = 18b^2$

13 a 1.95 g **b** 3 486 784.401 tonnes

Exercise 7F

1

Scientific notation	Power of 10 expanded	Basic numeral
5×10^3	5×1000	5000
3×10^4	3×10000	30 000
2×10^5	2×100000	200 000
7×10^2	7×100	700
7×10^4	7×10000	70 000
4×10^5	4×100000	400 000
6×10^3	6×1000	6000
2×10^6	2×1000000	2 000 000

2

Scientific notation	Positive power	Fraction	Basic numeral
2×10^{-4}	$\frac{2}{10^4}$	$\frac{2}{10000}$	0.0002
3×10^{-2}	$\frac{3}{10^2}$	$\frac{3}{100}$	0.03
5×10^{-3}	$\frac{5}{10^3}$	$\frac{5}{1000}$	0.005
7×10^{-6}	$\frac{7}{10^6}$	$\frac{7}{1000000}$	0.000007
9×10^{-3}	$\frac{9}{10^3}$	$\frac{9}{1000}$	0.009
8×10^{-2}	$\frac{8}{10^2}$	$\frac{8}{100}$	0.08
6×10^{-4}	$\frac{6}{10^4}$	$\frac{6}{10000}$	0.0006
4×10^{-5}	$\frac{4}{10^5}$	$\frac{4}{100000}$	0.00004

3 a 10 000 **b** 1000 **c** 100 000

d 1000 **e** 100 000 **f** 10 000

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4 **a** positive **b** negative
c positive **d** negative

5 **a** 4×10^4 **b** 2.3×10^{11} **c** 1.6×10^{10}
d 7.2×10^6 **e** 3.5×10^3 **f** 8.8×10^6
g 5.2×10^3 **h** 3×10^6 **i** 2.1×10^4

6 **a** 3×10^{-6} **b** 4×10^{-4} **c** 8.76×10^{-3}
d 7.3×10^{-10} **e** 3×10^{-5} **f** 1.25×10^{-10}
g 8.09×10^{-9} **h** 2.4×10^{-8} **i** 3.45×10^{-5}

7 **a** 6×10^3 **b** 7.2×10^5 **c** 3.245×10^2
d 7.86903×10^3 **e** 8.45912×10^3 **f** 2×10^{-1}
g 3.28×10^{-4} **h** 9.87×10^{-3} **i** 1×10^{-5}
j 4.601×10^8 **k** 1.7467×10^4 **l** 1.28×10^2

8 **a** 57 000 **b** 3 600 000 **c** 430 000 000
d 32 100 000 **e** 423 000 **f** 90 400 000 000
g 197 000 000 **h** 709 **i** 635 700

9 **a** 0.00012 **b** 0.0000046
c 0.0000000008 **d** 0.0000352
e 0.3678 **f** 0.000000123
g 0.00009 **h** 0.05
i 0.4

10 **a** 6×10^{24} kg **b** 4×10^7 m
c 1×10^{-10} m **d** 1.5×10^8 km
e 6.67×10^{-11} Nm^2/kg^2 **f** 1.5×10^{-4} s
g 4.5×10^9 years

11 **a** 4 600 000 000 km
b 8 000 000 000 000 organisms
c 384 000 km
d 0.0038 m
e 0.00000000000001 m
f 720 000 people

12 **a** 1.62×10^9 km **b** 2.126×10^{-2} g

13 **a** 3.2×10^4 **b** 4.1×10^6 **c** 1.3×10^4
d 9.2×10^1 **e** 6.1×10^{-2} **f** 4.24
g 2×10^{-5} **h** 4×10^{-6}

Exercise 7G

1 **a** **i** 300 **ii** 32 700 **iii** 18 400
b **i** 0.1 **ii** 0.2 **iii** 21.0
c **i** 26 8000 **ii** 38 000 **iii** 4 061 000

2 0.0071

3 **a** 57 260, 57 300, 57 000, 60 000
b 4 170 200, 4 170 000, 4 170 000, 4 200 000, 4 000 000
c 0.003661, 0.00366, 0.0037, 0.004
d 24.871, 24.87, 24.9, 25, 20

4 **a** Yes **b** No **c** No
d No **e** Yes **f** Yes
g Yes **h** No **i** No

5 **a** 3, 4 or 5 **b** 4 **c** 5 or 6 **d** 2 or 3
e 3 **f** 2 **g** 3 **h** 3
i 3 **j** 4 **k** 3 **l** 3

6 **a** 2.42×10^5 **b** 1.71×10^5 **c** 2.83×10^3
d 3.25×10^6 **e** 3.43×10^{-4} **f** 6.86×10^{-3}
g 1.46×10^{-2} **h** 1.03×10^{-3} **i** 2.34×10^1
j 3.26×10^2 **k** 1.96×10^1 **l** 1.72×10^{-1}

7 **a** 4.78×10^4 **b** 2.2×10^4 **c** 4.833×10^6
d 3.7×10^1 **e** 9.95×10^1 **f** 1.443×10^{-2}
g 2×10^{-3} **h** 9×10^{-2} **i** 1×10^{-4}

8 **a** 2.441×10^{-4} **b** 2.107×10^{-6} **c** -4.824×10^{15}
d 4.550×10^{-5} **e** 1.917×10^{12} **f** 1.995×10^8
g 3.843×10^2 **h** 1.804×10^{-11} **i** 1.524×10^8
j 3.325×10^{15} **k** 4.067×10^3 **l** -9.077×10^{-1}

9 **a** 9.3574×10^1 **b** 2.1893×10^5 **c** 8.6000×10^5
d 8.6288×10^{-2} **e** 2.2985×10^{15} **f** 3.5741×10^{28}
g 6.4000×10^7 **h** 1.2333×10^9 **i** 1.8293
j 5.4459×10^{-1}

10 1.98×10^{30} kg

11 1.39×10^6 km

12 2421×10^3, 24.2×10^5, 2.41×10^6, 0.239×10^7, 0.02×10^8

13 **a** 4.26×10^6 **b** 9.1×10^{-3} **c** 5.04×10^{11}
d 1.931×10^{-1} **e** 2.1×10^6 **f** 6.14×10^{-11}

14 **a** 5.40046×10^{12}
b **i** 1.08×10^{13} **ii** 4.32×10^{13}

Puzzles and challenges

1 Exponential

2 **a** $10^8 = 100\,000\,000$
b $100^5 = 10\,000\,000\,000$

4 $4a \times (3m)^3 \times \left(\frac{a}{6m}\right)^2 = 3a^3m$

5

Pile	Height	Value
1	2 mm	\$0.10
5	3.2 cm	\$1.60
10	1.024 m	\$51.20
15	32.77 m	\$1638.40
20	1.05 km	\$52 428.80
25	33.55 km	\$1 677 721.60
30	1073.74 km	\$53 687 091.20

6 14th day

7 **a** 1.84×10^{11} tonnes
b \$50 729 000 000 000 or more than 50 trillion dollars

Multiple-choice questions

1 D **2** B **3** E **4** C **5** D
6 B **7** D **8** C **9** C **10** E

Short-answer questions

1 **a** 3^4 **b** $2x^3y^2$
c $3a^2b^2$ **d** $\left(\frac{3}{5}\right)^3 \times \left(\frac{1}{7}\right)^2$

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2 **a** $3^2 \times 5$ **b** $2^2 \times 3 \times 5^2$

3 **a** x^{10} **b** $12a^5b^6c$ **c** $24m^7n^4$ **d** a^9
e x^3y^2 **f** $\frac{a^2b^2}{2}$

4 **a** m^6 **b** $9a^8$ **c** $-32a^{10}b^5$
d $3b$ **e** 2 **f** $\frac{a^6}{27}$

5 **a** $\frac{1}{2^3}$ **b** $\frac{3}{4^5}$ **c** $\frac{6}{10^3}$
d 4^2 **e** 6×5^3 **f** 2×10^5

6 **a** k^6 **b** $6a^2m$ **c** $4a^7$
d h^{16} **e** $\frac{a^6b^2}{5}$ **f** $45p^4q^2$
g $81a^{20}b$ **h** $\frac{1}{3m^2n^3}$ **i** $\frac{b^5}{4a^7}$

7 $0.0012, 35.4 \times 10^{-3}, 3.22 \times 10^{-1}, 0.4, 0.007 \times 10^2, 2.35$

8 **a** 324 **b** 172 500
c 0.2753 **d** 0.00149

9 **a** 2.25×10^7 people **b** 9.63×10^6 km^2
c 3.34×10^{-9} seconds **d** 2.94×10^{-7} m

10 **a** 2.19×10^5 hours **b** 3.6×10^7 months
c 4.3×10^5 kg **d** 5×10^6 g

Extended-response questions

1 **a** 1.2×10^{55} kg^2 **b** 4.3×10^{-5} m/s

2 **a** **i** 5.79×10^7 km **ii** 1.50×10^8 km
iii 7.78×10^8 km **iv** 1.43×10^9 km
b **i** 2.41×10^{-1} **ii** 1.88 **iii** 2.95×10^1
c **i** 4.87×10^{24} kg **ii** 6.42×10^{23} kg **iii** 1.90×10^{30} kg

Chapter 8

Pre-test

1 **a** Obtuse **b** Acute **c** Reflex
d Right **e** Straight **f** Revolution

2 **a** 35 **b** 142 **c** 260

3 **a** Isosceles **b** Equilateral **c** Scalene

4 **a** Obtuse **b** Acute **c** Right

5 **a** 60° **b** 75° each **c** 50°

6 **a** Parallelogram, rectangle, kite
b Parallelogram, rectangle, square, rhombus
c Square, rectangle

7 **a** $a = 60$ **b** $b = 110$ **c** $a = 60, b = 120$

8 **a** Pentagon **b** Parallelogram **c** Trapezium

Exercise 8A

1 **a** right **b** 180°
c revolution **d** Obtuse
e Acute **f** 180°
g 90° **h** Supplementary
i 180° **j** equal

2 **a** Isosceles triangle **b** Obtuse-angled triangle
c Equilateral triangle **d** Isosceles triangle
e Acute-angled triangle **f** Scalene triangle
g Right-angled triangle

3 **a** 50° **b** 90° **c** 101°
d 202° **e** 180° **f** 360°

4 **a** **i** 125° **ii** 35°
b **i** 149° **ii** 59°
c **i** 106° **ii** 16°
d **i** 170° **ii** 80°
e **i** 91° **ii** 1°
f **i** 158° **ii** 68°
g **i** 142° **ii** 52°
h **i** 115° **ii** 25°
i **i** 133° **ii** 43°
j **i** 103° **ii** 13°

5 **a** $a = 63$ **b** $a = 71$ **c** $a = 38$
d $a = 147$ **e** $a = 233$ **f** $a = 33$

6 **a** Obtuse isosceles, $a = 40$
b Acute scalene, $b = 30$
c Right-angled scalene, $c = 90$
d Equilateral, $d = 60$
e Obtuse isosceles, $e = 100$
f Right-angled isosceles, $f = 45$
g Obtuse scalene, $g = 100$
h Equilateral, $h = 60$
i Obtuse isosceles, $i = 120$
j Obtuse isosceles, $j = 35$
k Right-angled scalene, $k = 90$
l Acute scalene, $l = 70$

7 **a** **i** $\angle BAC$ **ii** Obtuse **iv** 120°
b **i** $\angle PRQ$ **ii** Acute **iv** 30°
c **i** $\angle XYZ$ **ii** Reflex **iv** 310°
d **i** $\angle SRT$ **ii** Straight angle **iv** 180°
e **i** $\angle ROB$ **ii** Obtuse **iv** 103°
f **i** $\angle AOB$ **ii** Right **iv** 90°

8 **a** $s = 120$ **b** $t = 20$
c $r = 70$ **d** $a = 60, x = 120$
e $a = 100, b = 140$ **f** $c = 115, d = 65$
g $x = 56$ **h** $x = 155$

9 $AO = BO$ (equal radii)
$\therefore \angle BAO = \angle OBA$ (equal angles are opposite equal sides)
$\therefore \angle OAB = (180 - 116) \div 2 = 32°$

10 **a** 360° **b** 90° **c** 60° **d** 90°
e 432° **f** 6° **g** 720° **h** 8640°

11 **a** 90° **b** 150° **c** 15°
d 165° **e** 157.5° **f** 80°
g 177.5° **h** 171° **i** 121.5°

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Exercise 8B

1 **a** Corresponding **b** Alternate
c Co-interior **d** Alternate
e Co-interior **f** Corresponding
2 **a** equal **b** equal
c supplementary
3 **a** $x = 125$, alternate angles in ∥ lines
b $y = 110$, co-interior angles in ∥ lines
c $r = 80$, corresponding angles in ∥ lines
d $s = 66$, alternate angles in ∥ lines
e $t = 96$, vertically opposite
f $v = 126$, corresponding angles in ∥ lines
g $w = 62$, angles on a straight line
h $p = 115$, corresponding angles in ∥ lines
i $q = 116$, co-interior angles in ∥ lines
4 **a** No, alternate angles are not equal.
b Yes, corresponding angles are equal.
c Yes, alternate angles are equal.
d No, co-interior angles don't add to 180°.
e Yes, co-interior angles add to 180°.
f Yes, corresponding angles are equal.
g No, corresponding angles are not equal.
h No, alternate angles are not equal.
i No, co-interior angles do not add to 180°.
5 **a** $a = 60, b = 120$ **b** $c = 95, d = 95$
c $e = 100, f = 100, g = 100$ **d** $a = 110, b = 70$
e $a = 100, b = 80, c = 80$ **f** $e = 140, f = 140, d = 140$
6 $a = 40, b = 140, c = 40, d = 40, e = 140$
7 **a** $x = 70, y = 40$ **b** $t = 58, z = 122$
c $u = 110, v = 50, w = 50$ **d** $x = 118$
e $x = 295$ **f** $x = 79$
8 **a** 56 **b** 120 **c** 265
9 **a** 105° **b** 105° **c** 56°
d 105° **e** 90° **f** 85°

Exercise 8C

1 **a** Parallelogram, rhombus, rectangle, square, kite, trapezium
b **i** trapezium **ii** square
iii rectangle **iv** kite
v rhombus **vi** parallelogram
2 **b**, **c**, **e**, **g**
3 **a**, **b**, **c**, **e**, **g**, **h**
4 **a** Parallelogram, rectangle, rhombus, square
b Rectangle, square
c Square, rectangle
d Square, rhombus, kite
e Trapezium
f Parallelogram, rhombus, kite
5 **a** 115 **b** 159 **c** 30
d 121 **e** 140 **f** 220
6 **a** $a = 110$ **b** $x = 70$ **c** $b = 54$
d $a = 33$ **e** $y = 63$ **f** $a = 109$
7 **a** Yes **b** No **c** Yes **d** No
8 60°
9 A parallelogram has opposite sides parallel and equal; rectangles, squares and rhombi have these properties (and more) and are therefore all parallelograms.
10 **a** 110 **b** 55 **c** 140
11 **a** 255 **b** 80 **c** 115
d 37 **e** 28 **f** 111

Exercise 8D

1 **a** hexagon **b** regular
c non-convex **d** $(n - 2) \times 180°$
2 **a** 5 **b** 7 **c** 4 **d** 11 **e** 9 **f** 12
3 **a** 720° **b** 1080° **c** 1620° **d** 900°
4 **a** Convex quadrilateral
b Non-convex hexagon
c Non-convex heptagon
5 **a** 140 **b** 100 **c** 100 **d** 120
6 **a** 110 **b** 150 **c** 210
d 130 **e** 25 **f** 285
7 **a** 90° **b** 108° **c** 140°
d 120° **e** 128.57° **f** 135°
8 **a** 16 **b** 25 **c** 102
9 **a** Yes **b** Yes
10 **a** 135 **b** 240 **c** 85 **d** 150
11 **a** **i** No **ii** Yes **iii** No
b **i** One **ii** Two **iii** Five
c $(n - 3)$

Exercise 8E

1 **a** **i** Y **ii** X **iii** Z
b **i** XY **ii** XZ **iii** YZ
c **i** $\angle A$ **ii** $\angle B$ **iii** $\angle C$
2 **a** size **b** ΔSTU
c SAS, RHS, AAS
3 **a** $\Delta ABC \equiv \Delta FGH$ **b** $\Delta DEF \equiv \Delta STU$
c $\Delta AMP \equiv \Delta CBD$ **d** $\Delta BMW \equiv \Delta SLK$
4 **a** SAS **b** AAS **c** RHS **d** SAS
e SSS **f** RHS **g** AAS **h** SSS
5 **a** $x = 3, y = 4$ **b** $x = 2, y = 6$
c $a = 105, b = 40$ **d** $a = 65, b = 85$
e $x = 2.5, b = 29$ **f** $a = 142, x = 9.21, b = 7$
g $y = 4.2, a = 28$ **h** $a = 6.5, b = 60$
6 **a** A and C **b** RHS
7 **a** $\Delta ABC \equiv \Delta STU$, RHS **b** $\Delta DEF \equiv \Delta GHI$, SSS
c $\Delta ABC \equiv \Delta DEF$, SAS **d** $\Delta ABC \equiv \Delta GHI$, AAS
8 It is SAS; the hypotenuse is not given.
9 No – they can all be different sizes, one might have all sides 2 cm and another all sides 5 cm.

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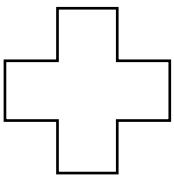
10 a SSS **b** Equal

11 $\Delta PBR \equiv \Delta FDE$
$\Delta LMN \equiv \Delta KIJ$
$\Delta FGH \equiv \Delta BCD$
$\Delta MNO \equiv \Delta RQP$
$\Delta PBR \equiv \Delta TUS$. Because $\Delta PBR \equiv \Delta TUS$ and $\Delta PBR \equiv \Delta FDE$, this also implies that $\Delta FDE \equiv \Delta TUS$, but we can't tell this by comparing the two directly.

Exercise 8F

1 a 15, 15 **b** 2
c 10, 15, 15 **d** 2.5, 4
e



1.5 cm

2 a $\angle F$ **b** $\angle D$ **c** GH **d** AE **e** 2

3 a OA' is double OA. **b** OB' is double OB.
c OC' is double OC. **d** 2
e Yes

4 a OA' is a quarter of OA. **b** OD' is a quarter of OD.
c $\frac{1}{4}$ **d** Yes

5 a i 2 **ii** 14 **iii** 10
b i $1\frac{1}{2}$ **ii** 10.5 **iii** 8
c i $1\frac{1}{2}$ **ii** 45 **iii** 24
d i 2.5 **ii** 1 **iii** 1.4
e i 2.5 **ii** 0.6 **iii** 2
f i 1.75 **ii** 3.5 **iii** 3

6 In student diagrams:
a $A'B'C'$ should have sides $\frac{1}{3}$ that of ABC
b $A'B'C'$ should have sides double that of ABC

7 b i $A'B'C'D'$ should have sides lengths $\frac{1}{2}$ that of $ABCD$.
ii $A'B'C'D'$ should have side lengths 1.5 times that of $ABCD$.

8 a Yes **b** 8 cm **c** 25 m

9 3.7 cm

10 a i 2 **ii** (0, 0)
b i $\frac{1}{2}$ **ii** (0, 0)
c i $\frac{1}{2}$ **ii** (3, 0)
d i 3 **ii** (1, 0)

11 a All angles of any square equal 90°, with only 1 side length.
b All angles in any equilateral triangle equal 60° with only 1 side length.
c The length and width might be multiplied by different numbers.
d Two isosceles triangles do not have to have the same size equal angles.

12 a 12.7 cm **b** 3 cm **c** 3 m

13 a 100 000 cm = 1 km **b** 24 cm

Exercise 8G

1 a E **b** C **c** DF **d** BC **e** $\angle A$ **f** $\angle E$

2 2.5

3 a Two pairs of equal angles.
b The hypotenuses and another pair of sides are in the same ratio.
c Three pairs of sides are in the same ratio.
d The sides adjacent to equal angles are in the same ratio.
e The hypotenuses and another pair of sides are in the same ratio.
f Two pairs of equal angles.
g The sides adjacent to equal angles are in the same ratio.
h Three pairs of sides are in the same ratio.

4 a $\Delta ABC \,|||\, \Delta GHI$ **b** $\Delta ABC \,|||\, \Delta MNO$
c $\Delta ABC \,|||\, \Delta ADE$ **d** $\Delta HFG \,|||\, \Delta HJI$
e $\Delta ADC \,|||\, \Delta AEB$ **f** $\Delta ABD \,|||\, \Delta ECD$

5 2.5

6 a Two pairs of equal angles
b 1.5 **c** 12

7 a The hypotenuses and another pair of sides are in the same ratio.
b 2.5 **c** 8

8 2 m

9 a ΔDEF **b** ΔDEF **c** ΔABC **d** ΔDEF

10 a i Two pairs of equal angles
ii 6.5
b i Two pairs of equal angles
ii 10
c i Two pairs of equal angles
ii 24

11 $\angle ACB = 25°$, AAA

12 a

Triangle	Original	Image 1	2	3
Length scale factor	1	2	3	4
Area	4	16	36	64
Area scale factor	1	4	9	16

b Area scale factor = (length scale factor)^2
c i 100 **ii** 400 **iii** 10 000

Exercise 8H

1 a two
b Two pairs of equal angles
c 2 **d** 12

2 $\angle C$

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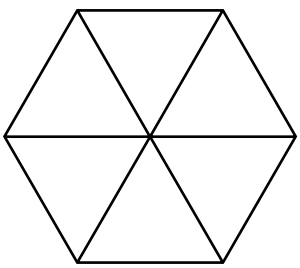
3 **a** $\angle ACB$ and $\angle ECD$
b $\angle BAC = \angle DEC$ and $\angle CBA = \angle CDE$
4 **a** 3 **b** 3.6 m
5 **a** Two pairs of equal angles
b 40 m
6 **a** Two pairs of equal angles
b 7.5 m
7 **a** To create two similar triangles
b Two pairs of equal angles
c 24 m
8 7.2 m
9 20 m
10 **a** **i** 3.6 m **ii** 9 m **iii** 2.7 m
b **i** 5.4 m **ii** 6.3 m
c **i** 4 m **ii** 6 m
11 $\frac{55}{6}$
12 **a** 3
b **i** $BD = \frac{25}{3}$ **ii** $AC = 4$ **iii** $AB = \frac{20}{3}$

Puzzles and games

1



2 110
3 10
4 11
5 Base of altitudes (an altitude is a line through a vertex that meets the opposite side at right angles)
6 **b** **i** $\frac{l}{2}$ **ii** $\frac{l}{4}$ **iii** $\frac{l}{128}$
c **i** $\frac{3}{4}$ **ii** $\frac{9}{16}$ **iii** $\frac{243}{1024}$
d Zero

Multiple-choice questions

1 D **2** A **3** B **4** D **5** C
6 D **7** E **8** B **9** D **10** A

Short-answer questions

1 **a** $a = 142$ **b** $b = 40$ **c** $c = 120$
2 **a** Acute-angled isosceles, $x = 50$, $y = 80$
b Right-angled, $x = 25$
c Obtuse-angled scalene, $x = 30$, $y = 110$
3 **a** No – corresponding angles are not equal.
b Yes – co-interior angles are supplementary.
c Yes – alternate angles are equal.
4 **a** $a = 60$, $b = 120$, $c = 120$
b $a = 40$, $b = 80$
c $a = 99$, $b = 20$
5 **a** Square, rectangle
b Square, rhombus, kite
c Rhombus, parallelogram, rectangle, square
6 **a** $a = 70$, $b = 110$
b $x = 115$ **c** $x = 15$
7 **a** $x = 95$ **b** $a = 120$ **c** $x = 135$
8 **a** SSS, $x = 60$ **b** Not congruent
c RHS, $x = 12$ **d** AAS, $x = 9$
9 For image $\Delta A'B'C'$, $OA' = 3OA$, $OB' = 3OB$, $OC' = 3OC$
10 **a** Yes. Two pairs of sides are in the same ratio and the included angles are equal.
b Yes. Two pairs of equal angles
c Not similar
d Yes. The hypotenuses and another pair of sides are in the same ratio.
11 **a** 3.5 **b** 4 **c** 18
12 **a** 3 **b** 4.8 m

Extended-response questions

1 **a** Isosceles, right-angled
b $a = 40$, angle sum in isosceles triangle
$b = 70$, angles in parallel lines
$c = 120$, angle sum of a quadrilateral
$d = 25$, angles on a straight line
$e = 65$, angle sum of a triangle
2 **a** $\angle ECD$
b $\angle ABC = \angle EDC$ (given 90°)
$\angle ACB = \angle ECD$ (vertically opposite)
$\therefore \Delta ABC \,|||\, \Delta EDC$ (two pairs of equal angles)
c 19.8 m

Chapter 9

Pre-test

1 **a** 4 **b** 6 **c** −2 **d** −7
2 **a** 5 **b** 2 **c** 1 **d** −6
3 **a** −10 **b** 16 **c** 11 **d** −100
e 2 **f** 48 **g** −20 **h** −70
4 **a** $2x + 6$ **b** $3a - 15$ **c** $12x - 8xy$ **d** $3 - 6b$
5 **a** 2 **b** 6 **c** $2x$
d $3y$ **e** $5x$ **f** ab
6 **a** $2(a + 3)$ **b** $3(x + 4y)$
c $5x(x - 3)$ **d** $2m(2 - 3n)$
7 **a** $\frac{5}{7}$ **b** $\frac{-1}{9}$ **c** $\frac{7}{6}$ **d** $\frac{-1}{8}$

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8 **a** $\frac{8}{15}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$ **d** 1
d 2 **e** $\frac{3}{2}$ **f** $\frac{9}{8}$ **g** $\frac{7}{12}$
9 **a** $3x + 2$ **b** $x + 4$ **c** $-10 - 3x$
10 $x(x + 2)$, $x^2 + 2x$

Exercise 9A

1 **a** **i** $\$11x$ **ii** $50n$ cents
b **i** $x + y$ **ii** $b + g$
c $\$(50 + 60n)$
2 **a** 5 **b** -2 **c** -7
d No **e** -10 **f** 6
3 **a** $5 + 2x$ **b** $4a - 7$ **c** $y^2 - 1$
d xz **e** $a^2 + b^2$ **f** $\sqrt{x + 8}$
4 **a** $-a$ **b** $11x$ **c** $8ab$ **d** $-4xy$
e $a - 1$ **f** $2x - 4y$ **g** $6x - 4y$ **h** $2ab$
i $-4a^2b - 3$ **j** $-3xy$ **k** $12ar^2$ **l** $st^2 - st$
5 **a** $15x$ **b** $14a$ **c** $-8xy$ **d** $-42a^2$
e $-3ab^2$ **f** $-12a^2b^2$ **g** $2x$ **h** 7
i 3 **j** $2a$ **k** $\frac{10}{a}$ **l** $4x$
m ab **n** $\frac{3a}{c}$ **o** $\frac{1}{3y}$
6 **a** $3x + 6$ **b** $8 + 2x$ **c** $-3x - 12$
d $-6x - 6$ **e** $-2x + 6$ **f** $-x^2 - x$
g $-6y + 9y^2$ **h** $-7a^2 + 7ab$ **i** $2x + 1$
j $4 - x$ **k** $3 - 4x$ **l** $2x + 6$
m $4x - 9$ **n** $-17x - 3$ **o** $13x - 40$
7 **a** **i** $2a + 2b$ or $2(a + b)$ **ii** ab
b **i** $4x$ **ii** x^2
c **i** $a + b + c$ **ii** $\frac{1}{2}ab$
8 **a** 4 **b** 24 **c** -1 **d** 10
e 13 **f** -8 **g** $\frac{25}{2}$ **h** 1
i -2 **j** 35 **k** -12 **l** $\sqrt{13}$
9 **a** **i** \$260 **ii** \$820 **b** $\$(100 + 80n)$
10 **a** $2x^2 - x$ **b** $x^2 + 6x$ **c** $x^2 + x + 2$
11 **a** $-3x + 3$ **b** $\frac{a}{2}$ **c** $\frac{2}{a}$ **d** 9
12 **a** **i** \$120 **ii** \$600
b **i** \$140 **ii** \$540
c $\$60n$ **d** $\$(40 + 50n)$ **e** 4 hours

Exercise 9B

1 **a** $x^2, 2x, 3x, 6$ **b** $x^2 + 3x + 2x + 6 = x^2 + 5x + 6$
2 **a** $2x^2, 2x, 3x, 3$
b $(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3$
$= 2x^2 + 5x + 3$
3 **a** $x^2 + 5x + x + 5 = x^2 + 6x + 5$
b $x^2 + 2x - 3x - 6 = x^2 - x - 6$
c $21x^2 + 6x - 14x - 4 = 21x^2 - 8x - 4$
d $12x^2 - 16x - 3x + 4 = 12x^2 - 19x + 4$
4 **a** $x^2 + 7x + 10$ **b** $b^2 + 7b + 12$ **c** $t^2 + 15t + 56$
d $p^2 + 12p + 36$ **e** $x^2 + 15x + 54$ **f** $d^2 + 19d + 60$
g $a^2 + 8a + 7$ **h** $y^2 + 12y + 20$ **i** $m^2 + 16m + 48$
j $a^2 + 10a + 9$ **k** $6 + 5a + a^2$ **l** $n^2 + 12n + 36$
5 **a** $x^2 - x - 12$ **b** $x^2 + 3x - 10$
c $x^2 - 4x - 32$ **d** $x^2 - 4x - 12$
e $x^2 + 9x - 10$ **f** $x^2 + 2x - 63$
g $x^2 + 5x - 14$ **h** $x^2 - 3x + 2$
i $x^2 - 9x + 20$ **j** $8x^2 + 26x + 15$
k $6x^2 + 7x + 2$ **l** $15x^2 + 17x + 4$
m $6x^2 + x - 15$ **n** $24x^2 + 23x - 12$
o $6x^2 - x - 2$ **p** $10x^2 - 31x - 14$
q $6x^2 + 5x - 6$ **r** $16x^2 - 16x - 5$
s $18x^2 - 27x + 10$ **t** $15x^2 - 11x + 2$
u $21x^2 - 37x + 12$
6 **a** **i** $(x + 10)$ m **ii** $(x + 7)$ m
b $(x^2 + 17x + 70)$ m^2 **c** 108 m^2
7 **a** $(x^2 + 9x + 20)$ m^2
b **i** 56 m^2 **ii** 36 m^2
8 **a** **i** $(2x + 3)$ m **ii** $(2x + 2)$ m
b $4x^2 + 10x + 6$ **c** 20 m^2
9 **a** 3 **b** 2 **c** 6, 6
d 2, 18 **e** 2, 6 **f** 3, 15
10 **a** $a^2 + ac + ab + bc$ **b** $a^2 + ac - ab - bc$
c $ab + bc - a^2 - ac$ **d** $2x^2 - 3xy - 2y^2$
e $2a^2 - ab - b^2$ **f** $6x^2 + xy - y^2$
11 **a** $2x^2$ **b** $2x^2 - 30x + 100$
12 **a** **i** $(15 - 2x)$ m **ii** $(10 - 2x)$ m
b $(150 - 50x + 4x^2)$ m^2 **c** 66 m^2
d $x = 2.5$

Exercise 9C

1 **a** $x^2, 3x, 3x, 9$ **b** $x^2 + 6x + 9$
c $(x + 3)(x + 3) = x^2 + 3x + 3x + 9$
$= x^2 + 6x + 9$
2 **a** $+3x + 9 = x^2 + 6x + 9$
b $+5x + 25 = x^2 + 10x + 25$
c $-2x + 4 = x^2 - 4x + 4$
d $-7x + 49 = x^2 - 14x + 49$
3 **a** **i** $x^2 + 6x + 9$ **ii** $x^2 + 22x + 121$
b **i** $x^2 - 4x + 4$ **ii** $x^2 - 18x + 81$
4 **a** $x^2 + 2x + 1$ **b** $x^2 + 6x + 9$
c $x^2 + 4x + 4$ **d** $x^2 + 10x + 25$
e $x^2 + 8x + 16$ **f** $x^2 + 18x + 81$
g $x^2 + 14x + 49$ **h** $x^2 + 20x + 100$
i $x^2 - 4x + 4$ **j** $x^2 - 12x + 36$
k $x^2 - 2x + 1$ **l** $x^2 - 6x + 9$
m $x^2 - 18x + 81$ **n** $x^2 - 14x + 49$
o $x^2 - 8x + 16$ **p** $x^2 - 24x + 144$
5 **a** $4x^2 + 4x + 1$ **b** $4x^2 + 20x + 25$
c $9x^2 + 12x + 4$ **d** $9x^2 + 6x + 1$
e $25x^2 + 20x + 4$ **f** $16x^2 + 24x + 9$

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g $49 + 28x + 4x^2$ h $25 + 30x + 9x^2$
i $4x^2 - 12x + 9$ j $9x^2 - 6x + 1$
k $16x^2 - 40x + 25$ l $4x^2 - 36x + 81$

6 a $9 - 6x + x^2$ b $25 - 10x + x^2$
c $1 - 2x + x^2$ d $36 - 12x + x^2$
e $121 - 22x + x^2$ f $16 - 8x + x^2$
g $49 - 14x + x^2$ h $144 - 24x + x^2$
i $64 - 32x + 4x^2$ j $4 - 12x + 9x^2$
k $81 - 36x + 4x^2$ l $100 - 80x + 16x^2$

7 a $(x + 12)$ m b $(x^2 + 24x + 144)$ m^2
c i 196 m^2 ii 289 m^2

8 a $(10 - x)$ cm b $(x^2 - 20x + 100)$ cm^2
c i 64 cm^2 ii 16 cm^2
d $x = 5$

9 a $(20 - 2x)$
b $(20 - 2x)(20 - 2x)$ cm^2 = $(400 - 80x + 4x^2)$ cm^2
c 196 cm^2 d 588 cm^3

10 a $a^2 - b^2$
b i $(a - b)^2 = a^2 - 2ab + b^2$ ii $b(a - b) = ab - b^2$
iii $b(a - b) = ab - b^2$
c Yes, $a^2 - 2ab + b^2 + ab - b^2 + ab - b^2 = a^2 - b^2$

11 a $a + b$
b $(a + b)(a + b) = a^2 + 2ab + b^2$
c $a - b$
d $(a - b)(a - b) = a^2 - 2ab + b^2$
e $4ab$
f ab so yes four courts area is $4ab$

Exercise 9D

1 Because $-3x + 3x = 0$

2 a Single b Difference
c Difference d Single

3 a $+4x - 16 = x^2 - 16$
b $-10x - 100 = x^2 - 100$
c $+2x - 2x - 1 = 4x^2 - 1$
d $-12x + 12x - 16 = 9x^2 - 16$

4 a $x^2 - 1$ b $x^2 - 9$ c $x^2 - 64$
d $x^2 - 16$ e $x^2 - 144$ f $x^2 - 121$
g $x^2 - 81$ h $x^2 - 25$ i $x^2 - 36$
j $25 - x^2$ k $4 - x^2$ l $49 - x^2$

5 a $9x^2 - 4$ b $25x^2 - 16$ c $16x^2 - 9$
d $49x^2 - 9y^2$ e $81x^2 - 25y^2$ f $121x^2 - y^2$
g $64x^2 - 4y^2$ h $100x^2 - 81y^2$ i $49x^2 - 25y^2$
j $36x^2 - 121y^2$ k $64x^2 - 9y^2$ l $81x^2 - 16y^2$

6 a 899 b 1599 c 624 d 2499
e 396 f 391 g 875 h 2484

7 a i x^2 ii $x^2 - 4$
b No, they differ by 4.

8 a $x^2 - 1$
b No, area of rectangle is 1 square unit less.

9 a i x^2 ii $x + 3$ iii $x - 3$
b $x^2 - 9$ c No, 9 m^2 less

Exercise 9E

1 a 4 b 10 c 5 d 6
e 1 f 25 g 8 h 36

2 a x b x c a d $2a$ e $-2y$
f $-3x$ g $-2x$ h $-10x$ i $-2x$

3 a i 6 ii $3x$ iii $6x$ b iii

4 a $2x$ b $6a$ c 2 d 4
e 3 f 1 g $3x$ h $3n$
i $2y$ j $2x$ k $2xy$ l $5ab$

5 a $7(x + 1)$ b $3(x + 1)$ c $4(x - 1)$
d $5(x - 1)$ e $4(1 + 2y)$ f $5(2 + a)$
g $3(1 - 3b)$ h $2(3 - x)$ i $3(4a + b)$
j $6(m + n)$ k $2(5x - 4y)$ l $4(a - 5b)$
m $x(x + 2)$ n $a(a - 4)$ o $y(y - 7)$
p $x(1 - x)$ q $3p(p + 1)$ r $8x(1 - x)$
s $4b(b + 3)$ t $2y(3 - 5y)$

6 a $-4(2x + 1)$ b $-2(2x + 1)$ c $-5(2x + y)$
d $-7(a + 2b)$ e $-3(3x + 4)$ f $-2(3y + 4)$
g $-5(2x + 3y)$ h $-4(m + 5n)$ i $-3x(x + 6)$
j $-4x(2x + 3)$ k $-2y(8y + 3)$ l $-5a(a + 2)$

7 a 3 b y c a
d $5x$ e -7 f $-12a$

8 a $4(x + 2)$ b $2(x + 3)$ c $10(x + 2)$
d $2(x + 7)$ e $2(2x + 3)$ f $2(x + 7)$

9 $4x$

10 a $t(5 - t)$
b i 0 m ii 6 m iii 4 m
c 5 seconds

11 a 63 b 72 c -20
d -70 e 69 f 189

12 a $3(a^2 + 3a + 4)$ b $z(5z - 10 + y)$
c $x(x - 2y + xy)$ d $2b(2y - 1 + 3b)$
e $-4y(3x + 2z + 5xz)$ f $ab(3 + 4b + 6a)$

13 a $(x + 3)(4 + x)$ b $(x + 1)(3 + x)$
c $(m - 3)(7 + m)$ d $(x - 7)(x + 2)$
e $(a + 4)(8 - a)$ f $(x + 1)(5 - x)$
g $(y + 3)(y - 2)$ h $(x + 2)(a - x)$
i $(2t + 5)(t + 3)$ j $(5m - 2)(m + 4)$
k $(4y - 1)(y - 1)$ l $(7 - 3x)(1 + x)$

Exercise 9F

1 a 2 b $2x$ c $2(x + 2)$ d $2(x - 1)$

2 a $\frac{3}{2}$ b $\frac{3}{4}$ c $\frac{2}{7x}$ d $\frac{4a^2}{5}$ e $\frac{1}{7}$ f $\frac{1}{6x}$

3 a $\frac{8}{9}$ b $\frac{7}{6}$ c $\frac{3}{2}$ d $\frac{1}{4}$ e 6 f 1

4 a $\frac{x}{2}$ b $\frac{x}{2}$ c $\frac{-ab}{3}$ d $\frac{a}{2b}$
e $x - 1$ f $-2(2x + 1)$ g $\frac{-1}{3}$ h $\frac{a}{10}$
i 3 j -2 k $\frac{1}{2}$ l $\frac{-2}{3}$
m $x - 4$ n $2(x - 1)$ o $-2(x + 2)$ p -1

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5 **a** $x + 2$ **b** $x - 3$ **c** $x - 4$ **d** $\frac{1}{x+1}$
e $\frac{-1}{2x+3}$ **f** $\frac{a}{3a-2}$ **g** 2 **h** 7
i -5 **j** -2 **k** 6 **l** -5

6 **a** 1 **b** $\frac{1}{2}$ **c** $\frac{6a}{5}$ **d** $\frac{3}{2a}$
e $\frac{7xy}{3}$ **f** $\frac{5x}{6}$ **g** 4 **h** 5
i $\frac{9}{2}$ **j** $\frac{x+1}{4}$ **k** 1 **l** $\frac{x+6}{2(x+7)}$

7 **a** $\frac{7x}{6}$ **b** $\frac{2x}{3}$ **c** $\frac{-21}{8}$
d $\frac{-a^2}{2}$ **e** 6 **f** $\frac{4}{3}$
g 11 **h** $\frac{-3}{2}$ **i** $\frac{-(x-2)}{2x-7}$

8 **a** $x + 1$ **b** $-(x + 2)$ **c** $\frac{3}{2}$ **d** $x(x + 2)$

9 **a** 4 **b** 5 **c** -18
d 15 **e** -34 **f** 14

10 **a** $x + 3$ **b** $2x - 1$ **c** $-5(x - 3)$ **d** $\frac{-1}{3(x-4)}$

11 **a** -1 **b** -1 **c** $\frac{-4}{3}$
d 2 **e** $\frac{-1}{x-2}$ **f** $-4(x - 6)$

Exercise 9G

1 **a** 24 **b** 15 **c** 143 **d** 36

2 **a** $\frac{5}{6}$ **b** $\frac{17}{20}$ **c** $\frac{13}{8}$
d $\frac{7}{10}$ **e** $-\frac{2}{15}$ **f** $\frac{1}{12}$

3 **a** $4x$ **b** $21x$ **c** 3
d 2 **e** 8 **f** 90

4 **a** $\frac{3x}{12} + \frac{8x}{12} = \frac{11x}{12}$ **b** $\frac{25x}{35} - \frac{14x}{35} = \frac{11x}{35}$

5 **a** 15 **b** 14 **c** 8 **d** 6 **e** 10

6 **a** $\frac{9x}{14}$ **b** $\frac{2x}{5}$ **c** $\frac{x}{8}$ **d** $\frac{14x}{45}$
e $\frac{y}{56}$ **f** $\frac{13a}{22}$ **g** $\frac{2b}{9}$ **h** $\frac{m}{6}$
i $\frac{11m}{12}$ **j** $\frac{15a}{28}$ **k** $\frac{x}{2}$ **l** $-\frac{20p}{63}$
m $-\frac{5b}{18}$ **n** $\frac{61y}{40}$ **o** $\frac{13x}{35}$ **p** $\frac{5x}{12}$

7 **a** $\frac{7x+11}{10}$ **b** $\frac{7x}{12}$ **c** $\frac{15a-51}{56}$
d $\frac{11y+9}{30}$ **e** $\frac{13m+28}{40}$ **f** $\frac{5x-13}{24}$
g $\frac{11b-6}{24}$ **h** $\frac{7x}{6}$ **i** $\frac{7y-8}{14}$
j $\frac{5t-4}{16}$ **k** $\frac{34-10x}{21}$ **l** $\frac{8m-9}{12}$

8 **a** $\frac{4x}{8} + \frac{6x}{8} = \frac{10x}{8} = \frac{5x}{4}$ **b** $\frac{2x}{4} + \frac{3x}{4} = \frac{5x}{4}$
c Using denominator 8 does not give an answer in simplified form and requires extra steps. Preferable to use actual LCD.

9 **a** Didn't make a common denominator and subtracted denominators, $\frac{7x}{15}$
b Didn't use brackets: $2(x + 1) = 2x + 2$, $\frac{7x+2}{10}$

10 **a** $\frac{3x}{2}$ **b** $\frac{5x}{4}$ **c** $\frac{7x}{4}$ **d** $\frac{9x}{2}$

11 **a** $-\frac{43x}{30}$ **b** $\frac{5x}{12}$ **c** $\frac{13x}{24}$ **d** $\frac{43x-5}{60}$
e $\frac{23x-35}{42}$ **f** $\frac{29x+28}{40}$

12 **a** $\frac{x}{3}$ **b** $\frac{x}{8}$ **c** $\frac{x}{2}$
d $\frac{x}{5}$ **e** $\frac{8x}{9}$ **f** $\frac{x}{4}$

Puzzles and games

1 RECIPROCAL

2 **a** $4x^2 - 80x + 400$ **b** $x = 5$

3 **a** n^2 **b** $n(n + 1)$ **c** $\frac{1}{2}n(n + 1)$

4 **a** $\frac{x+10}{12}$ **b** $\frac{-11x-2}{15}$

5 **a** 48, 49 **b** 33, 35 **c** 12, 15

6 Ryan

Multiple-choice questions

1 E **2** C **3** B **4** B **5** C
6 A **7** E **8** A **9** E **10** E

Short-answer questions

1 **a** $6x^2$ **b** $-18ab^2$ **c** $\frac{b}{2}$ **d** $-4x$

2 **a** $3x - 12$ **b** $-2x - 12$
c $-x^2 + x$ **d** $-8x^2 + 12x$

3 **a** $7x - 1$ **b** $5ab$
c $-3a^2b$ **d** $8x^2 - xy$

4 **a** $x^2 + x - 12$ **b** $x^2 - 9x + 14$
c $6x^2 - 5x - 6$ **d** $3x^2 + x - 4$

5 **a** $x^2 + 6x + 9$ **b** $x^2 - 8x + 16$ **c** $9x^2 - 12x + 4$
d $x^2 - 25$ **e** $49 - x^2$ **f** $121x^2 - 16$

6 **a** $4(a + 3b)$ **b** $-3(1 + 3x)$ **c** $x(x + 1)$
d $3x(2 - 3x)$ **e** $-5xy(x + 2)$ **f** $11ab(a - 3)$

7 **a** $\frac{x-1}{2}$ **b** $x + 1$ **c** $x + 4$
d $\frac{-(x+3)}{2}$ **e** $\frac{2}{3}$ **f** $2(x - 1)$

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8 **a** $\frac{1}{4}$ **b** $\frac{a}{2}$ **c** $\frac{x-4}{2}$
d $\frac{2a}{3}$ **e** 4 **f** $\frac{2}{5}$

9 **a** $\frac{11x}{12}$ **b** $\frac{-x}{24}$ **c** $\frac{a}{4}$
d $\frac{5x-2}{6}$ **e** $\frac{10x-5}{12}$ **f** $\frac{7x+5}{24}$

Extended-response questions

1 **a** $(x + 3)$ m
b **i** No change
ii $(x^2 - 1)$ m², 1 square metre less in area
c **i** 3 m² **ii** $x = 4$

2 **a** 400 m²
b **i** $L = W = (20 + 2x)$ m
ii $(4x^2 + 80x + 400)$ m²
c $4x(x + 20)$ m²
d $x = 5$

Chapter 10

Pre-test

1 **a** 0.1 **b** 0.25 **c** 0.3 **d** 0.85 **e** 0.237

2 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** 1
e 0 **f** $\frac{1}{12}$ **g** $\frac{1}{2}$ **h** $\frac{18}{29}$
i $\frac{2}{3}$ **j** $\frac{2}{7}$

3 **a** 1, 2, 3, 4, 5 or 6
b **i** 3 **ii** 2 **iii** 3
iv 5 **v** 5 **vi** 3

4 **a** 5 **b** 7 **c** 6 **d** 7 **e** 4 **f** 6

5 **a** 8 **b** 20 **c** $\frac{3}{20}$

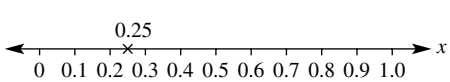
6 **a** 6 **b** 5 **c** 5 **d** 12
e **i** $\frac{2}{9}$ **ii** $\frac{7}{9}$ **iii** $\frac{5}{9}$

Exercise 10A

1 **a** sample space **b** event
c probability **d** impossible, certain
e even **f** complement

2 **a** **i** $\frac{1}{4}$
ii 0.25
iii 25%

b



3

	Percentage	Decimal	Fraction	Number line
a	50%	0.5	$\frac{1}{2}$	0 0.5 1 x
b	25%	0.25	$\frac{1}{4}$	0 0.25 0.5 0.75 1 x
c	75%	0.75	$\frac{3}{4}$	0 0.25 0.5 0.75 1 x
d	20%	0.2	$\frac{1}{5}$	0 0.2 0.4 0.6 0.8 1 x
e	60%	0.6	$\frac{3}{5}$	0 0.2 0.4 0.6 0.8 1 x
f	85%	0.85	$\frac{17}{20}$	0 0.85 1 x

4 0.15, $\frac{2}{9}$, 1 in 4, 0.28, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, 2 in 3, 0.7, 0.9

5 **a** **i** {1, 2, 3, 4, 5, 6, 7} **ii** $\frac{1}{7}$
iii $\frac{6}{7}$ **iv** $\frac{2}{7}$ **v** $\frac{6}{7}$
b **i** {2, 2, 6, 7} **ii** $\frac{1}{2}$
iii $\frac{1}{2}$ **iv** $\frac{1}{2}$ **v** 1
c **i** {1, 2, 2, 2, 2, 3} **ii** $\frac{2}{3}$
iii $\frac{1}{3}$ **iv** $\frac{5}{6}$ **v** $\frac{5}{6}$
d **i** {1, 1, 2, 2, 3, 3} **ii** $\frac{1}{3}$
iii $\frac{2}{3}$ **iv** $\frac{2}{3}$ **v** $\frac{2}{3}$
e **i** {1, 1, 2, 3, 3, 4, 4} **ii** $\frac{1}{7}$
iii $\frac{6}{7}$ **iv** $\frac{3}{7}$ **v** $\frac{5}{7}$
f **i** {2, 2, 2, 2} **ii** 1
iii 0 **iv** 1 **v** 1

6 **a** $\frac{1}{2}$ **b** $\frac{3}{8}$ **c** $\frac{1}{6}$ **d** $\frac{1}{4}$ **e** 1 **f** 0

7 **a** $\frac{1}{2}$ **b** $\frac{5}{8}$ **c** $\frac{5}{6}$ **d** $\frac{3}{4}$ **e** 0 **f** 1

8 **a** $\frac{1}{8}$ **b** $\frac{7}{8}$

9 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$ **d** $\frac{3}{8}$ **e** $\frac{5}{8}$
f 1 **g** 0 **h** $\frac{1}{4}$ **i** $\frac{3}{4}$ **j** $\frac{3}{4}$

10 **a** {H, A, R, S}
b **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$

11 **a** $\frac{1}{52}$ **b** $\frac{1}{13}$ **c** $\frac{1}{26}$
d $\frac{1}{2}$ **e** $\frac{2}{13}$ **f** $\frac{12}{13}$
g $\frac{23}{26}$ **h** $\frac{25}{26}$

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12 a $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$ **d** $\frac{1}{3}$ **e** $\frac{2}{3}$

f 1 **g** $\frac{1}{3}$ **h** $\frac{1}{2}$ **i** $\frac{5}{6}$

13 a $\frac{2}{11}$ **b** $\frac{9}{11}$ **c** 1 **d** $\frac{7}{11}$

e $\frac{5}{11}$ **f** $\frac{3}{11}$ **g** $\frac{7}{11}$ **h** $\frac{4}{11}$

14 a 31 minutes **b** $\frac{3}{31}$

c i $\frac{4}{31}$ **ii** $\frac{4}{31}$ **iii** $\frac{20}{31}$

iv $\frac{5}{31}$ **v** $\frac{8}{31}$ **vi** $\frac{11}{31}$

Exercise 10B

1 a 26

b i 10 **ii** 14 **iii** 5 **iv** 9

v 4 **vi** 7 **vii** 19

c i 12 **ii** 17

2 a 22 **b** 12 **c** 10

d 39 **e** 13 **f** 1

3 a B **b** D **c** A **d** C

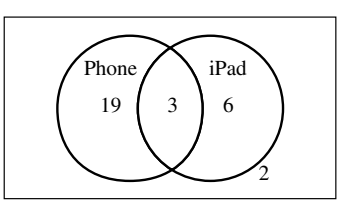
4 a

	A	Not *A*	Total
B	7	8	15
Not *B*	3	1	4
Total	10	9	19

b

	A	Not *A*	Total
B	2	5	7
Not *B*	9	4	13
Total	11	9	20

5 a



b i 28 **ii** 21 **iii** 6

c i $\frac{1}{10}$ **ii** $\frac{1}{15}$ **iii** $\frac{19}{30}$

6 i Not *A*: diagram T **ii** *A* or *B*: diagram R

iii *A* and *B*: diagram U **iv** *B*: diagram P

v *B* only: diagram Q **vi** Neither *A* nor *B*: diagram S

7 a i $\frac{2}{5}$ **ii** $\frac{1}{3}$ **iii** $\frac{7}{15}$

iv $\frac{1}{15}$ **v** $\frac{13}{15}$ **vi** $\frac{2}{15}$

b i $\frac{3}{7}$ **ii** $\frac{12}{35}$ **iii** $\frac{13}{35}$

iv $\frac{3}{35}$ **v** $\frac{34}{35}$ **vi** $\frac{1}{35}$

8 a

	Cream	Not cream	Total
Ice-cream	5	20	25
Not ice-cream	16	9	25
Total	21	29	50

b i 29 **ii** 9

c i $\frac{21}{50}$ **ii** $\frac{8}{25}$ **iii** $\frac{41}{50}$

9 a

	A	Not *A*	Total
B	3	1	4
Not *B*	2	2	4
Total	5	3	8

i $\frac{5}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{3}{8}$

iv $\frac{3}{4}$ **v** $\frac{1}{8}$ **vi** $\frac{1}{4}$

b

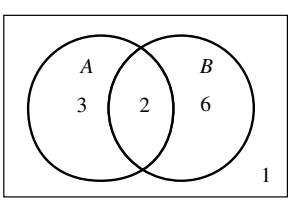
	A	Not *A*	Total
B	11	4	15
Not *B*	6	5	11
Total	17	9	26

i $\frac{17}{26}$ **ii** $\frac{9}{26}$ **iii** $\frac{11}{26}$

iv $\frac{21}{26}$ **v** $\frac{2}{13}$ **vi** $\frac{5}{26}$

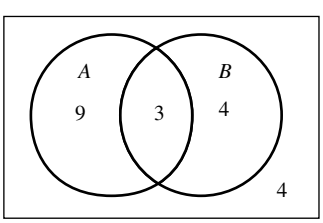
10 a

	A	Not *A*	Total
B	2	6	8
Not *B*	3	1	4
Total	5	7	12



b

	A	Not *A*	Total
B	3	4	7
Not *B*	9	4	13
Total	12	8	20



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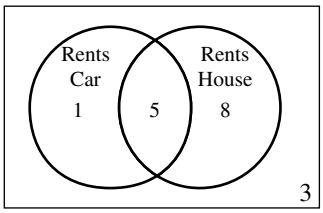
11 a



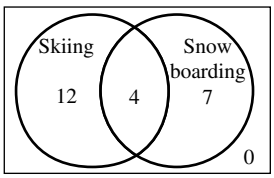
b 5 **c** $\frac{1}{17}$

12 a

	Tank water	Not tank water	Total
Tap water	12	36	48
Not tap water	11	41	52
Total	23	77	100

b 12 **c** $\frac{9}{25}$ **d** $\frac{59}{100}$

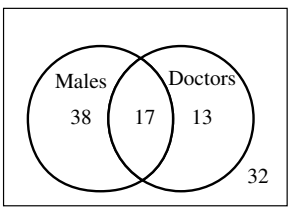
13 a



b 23 **c** $\frac{11}{23}$ **d** $\frac{4}{16} = \frac{1}{4}$ **e** $\frac{4}{11}$

14 a $\frac{3}{10}$ **b** $\frac{1}{3}$

15



a 32 **b** 45 **c** 70 **d** 38
e 13 **f** 13 **g** 62

Exercise 10C

1

	B	Y	E
H	HB	HY	HE
I	IB	IY	IE

2 a

	1	2	3	4
H	H1	H2	H3	H4
T	T1	T2	T3	T4

b 8 **c** $\frac{1}{8}$ **d** $\frac{1}{8}$

3 a 10 **b** H2, H4, T2, T4 **c** $\frac{2}{5}$
d T1, T3, T5 **e** $\frac{3}{10}$

4 a $\frac{1}{4}$ **b** HH, TT **c** $\frac{1}{2}$

5 a

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

b 12 **c** $\frac{1}{12}$ **d** $\frac{1}{4}$

6 a

	R	I	D	E
L	LR	LI	LD	LE
I	IR	II	ID	IE
N	NR	NI	ND	NE
E	ER	EI	ED	EE

b 16 **c** $\frac{1}{16}$ **d** $\frac{1}{16}$ **e** $\frac{1}{4}$ **f** $\frac{1}{4}$ **g** $\frac{1}{8}$

7 a

	R	P	B
R	RR	RP	RB
P	PR	PP	PB
G	GR	GP	GB
B	BR	BP	BB

b $\frac{1}{12}$ **c** $\frac{1}{12}$ **d** $\frac{1}{6}$ **e** $\frac{1}{4}$

8 a

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

36 outcomes

b $\frac{1}{9}$ **c** $\frac{1}{2}$ **d** 7 **e** 2 and 12

9 a

		Right				
		Y	W	B	B	B
Left	W	WY	WW	WB	WB	WB
	O	OY	OW	OB	OB	OB
	O	OY	OW	OB	OB	OB

b $\frac{2}{15}$ **c** $\frac{1}{15}$ **d** $\frac{2}{15}$ **e** $\frac{1}{5}$ **f** $\frac{1}{15}$

10 a

	D	O	G
D	X	(O, D)	(G, D)
O	(D, O)	X	(G, O)
G	(D, G)	(O, G)	X

b $\frac{1}{6}$ **c** $\frac{2}{3}$

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11 a

		1st			
		1	2	3	4
2nd	1	X	(2, 1)	(3, 1)	(4, 1)
	2	(1, 2)	X	(3, 2)	(4, 2)
	3	(1, 3)	(2, 3)	X	(4, 3)
	4	(1, 4)	(2, 4)	(3, 4)	X

b i $\frac{1}{12}$ **ii** $\frac{1}{12}$

c i $\frac{1}{6}$ **ii** $\frac{5}{6}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{2}$

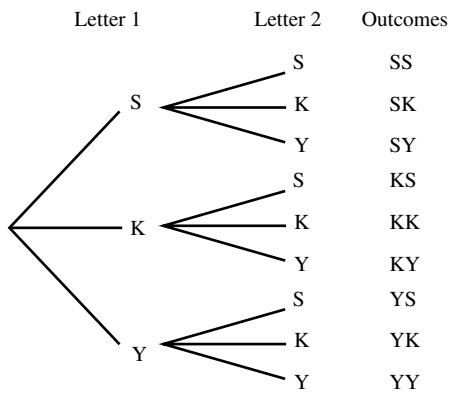
12 a $\frac{1}{8}$ **b** $\frac{1}{3}$

Exercise 10D

1 a A **b** B

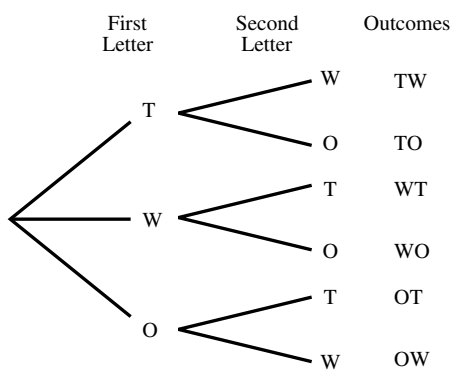
2 a i replacement **ii** 3

b

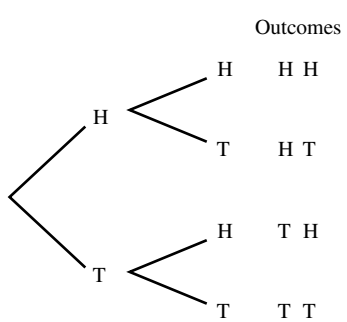


3 a i without replacement **ii** 2

b



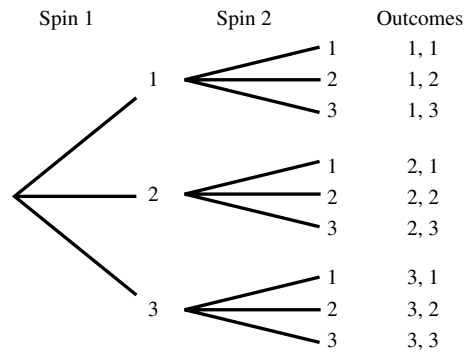
4 a



b 4

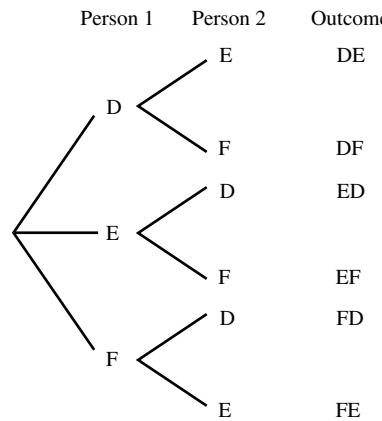
c i $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$ **iv** $\frac{3}{4}$

5 a



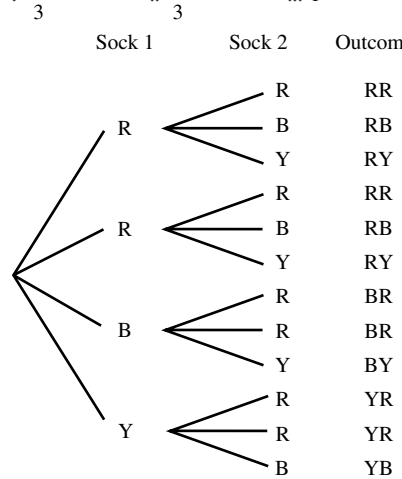
b $\frac{1}{9}$ **c** $\frac{1}{9}$ **d** $\frac{2}{9}$ **e** $\frac{1}{3}$ **f** $\frac{1}{3}$

6 a



b i $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** 1

7 a



b i $\frac{1}{3}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{6}$ **iv** $\frac{5}{6}$

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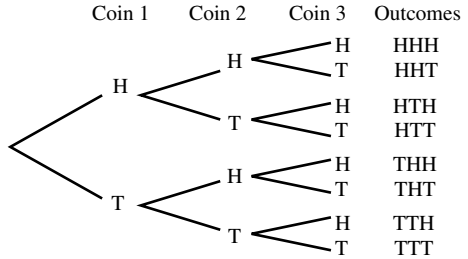
8 a



b $\frac{1}{8}$ **c** $\frac{1}{8}$ **d** $\frac{3}{8}$

e Getting exactly 2 tails

9 a $\frac{3}{5}$ **b** $\frac{2}{5}$ **c** $\frac{7}{10}$

10 a $\frac{1}{9}$ **b** $\frac{1}{9}$ **c** $\frac{7}{18}$ **d** $\frac{7}{18}$

e $\frac{1}{2}$ **f** 0 **g** $\frac{1}{6}$ **h** $\frac{2}{3}$

i $\frac{1}{9}$ **j** $\frac{17}{18}$

Exercise 10E

1 a experimental **b** theoretical **c** theoretical

d experimental **e** expected

2 a $\frac{9}{10}$ **b** $\frac{4}{5}$ **c** $\frac{5}{9}$

3 a $\frac{2}{5}$ **b** 40

4 a 5 **b** 10 **c** 50 **d** 4

5 a $\frac{35}{90} = \frac{7}{18}$

b 1050

6 a 0.6

b i 60 **ii** 120 **iii** 360

7 a $\frac{7}{8}$

b i 350 **ii** 4375 **iii** 35

8 a $\frac{1}{15}$ **b** $\frac{2}{5}$ **c** $\frac{11}{60}$

9 a B: $\frac{5}{20} = 0.25$ C: $\frac{30}{100} = 0.3$

b C because it is a larger sample size

10 0.41, from the 100 throws as the more times an experiment is carried out the closer the experimental probability becomes to the actual/theoretical probability

11 a 20 **b** 40 **c** 60 **d** 40

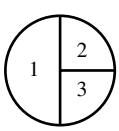
12 a 20

b i $\frac{1}{4}$ **ii** $\frac{7}{20}$ **c i** 25 **ii** 20 **iii** 45

13 a i 0.52 **ii** 0.48 **iii** 0.78

b 78

c $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$



14 a $\frac{\text{shaded area}}{\text{total area}} = 0.225 \therefore$ 100 shots ≈ 23

b $\frac{1}{10} \times 100 = 10$

c $\frac{150 - 32}{150} \times 100 \approx 79$

d $\frac{225\pi - 25\pi}{225\pi} \times 100 \approx 89$

Exercise 10F

1 a mean

b i 7.8 **ii** 9.9 **iii** 8.4 **iv** 35.8

2 a common (or frequent)

b i 2 **ii** 10 **iii** 7 **iv** 25

3 a middle

b i 5 **ii** 8 **iii** 5 **iv** 14

4 a range

b i 21 **ii** 22 **iii** 13 **iv** 27

5 a outlier

b i 36 **ii** 3 **iii** No outlier **iv** 100

6

	Mean	Median	Mode	Range
a	6	7	8	7
b	8	6	5, 10	13
c	6	6	2	10
d	11	12	none	13
e	4	3.5	2.1	6.2
f	5	4.5	none	9.7
g	3	3.5	−3	12
h	0	2	3	11

7 a Outlier = 33, mean = 12, median = 7.5

b Outlier = 2, mean = 35.2, median = 42

c Outlier = −1.1, mean = 1.075, median = 1.4

d Outlier = −4, mean = −49, median = −59

8 a Yes **b** No **c** No **d** Yes

9 a 12 **b** 5.5 **c** 6 **d** Bimodal; 4, 7

10 a 24.67 s **b** 24.8 s

11 a \$1 700 000

b (\$354 500 and \$324 000) drops \$30 500

c (\$570 667 and \$344 800) drops \$225 867

12 a 15 **b** 26

13 a 90 **b** 60

14 a 76.75

b i 71.4, B^+ **ii** 75, B^+ **iii** 80.2, A

c 81.4, he cannot get an A^+

d i 43 **ii** 93

Exercise 10G

1 a 17

b 3, 5, 6, 6, 7, 8, 8, 8, 9, 9, 9, 9, 9, 10, 10, 10, 10

c 8 **d** 9 **e** 7 **f** Skewed

g $\frac{12}{17}$

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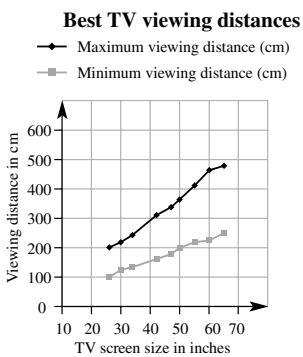
2 **a** 25 **b** 30 cm **c** 173 cm
d 170 cm **e** 168.88 cm **f** 186 cm
3 **a** 44% **b** 6600/year **c** 18/day
d 9.5/day **e** 108°
4 **a** 1.42 m **b** 0847, 8:47 a.m.
c 1 hour 21 minutes **d** 31 cm or 0.31 m
e 11 hours 55 minutes
5 **a** 41 years **b** 24 years
c 16 years **d** 10 years, 14.5%
6 **a** 18°C **b** 43 mm **c** 5°C **d** 47 mm
e January, February, March, December
f February, March, May
g August, September **h** 9°C
7 **a** 45 m **b** 25 m **c** 20 m **d** 69 m
e 35 m − 14 m = 21 m more
8 **a** Channel 15, 27; channel 16, 24; channel 17, 26; channel 18, 24
b Graph A **c** Graph B
d The scale on graph A starts at 23 but the scale on graph B starts at 5.
e Graph A is misleading as the scale expands the difference in column heights.
9 **a** Energy and Yum-crunch equal, Chunky, Sunshine
b No, it was equal to Energy
c The Yum-crunch column has wider dimensions, a different colour and the cereal name on the column.
d All columns should be the same width, same colour and either all should include the name or none include the name.
10 **a** Rugby
b Rugby, soccer, basketball, cricket, tennis
c Rugby: 700000; Soccer: 600000
d Cricket, 800000
e Cricket, rugby, soccer, basketball and tennis equal
f The length of each row is misleading because the ball sizes are not equal.
g Each picture should be an equal size.
11 **a** 10
b \$560000, \$560000, \$750000, \$750000, \$750000, \$750000, \$870000, \$870000, \$870000, \$2300000
c \$903000 **d** \$750000 **e** Mean
f 9 out of 10 house prices are less than the mean of \$903000.
g The median is a better measure of 'average' as it is the middle value. The mean is increased by one very large value.
12 **a**



b 26 inch, 30 inch, 34 inch
c 42 inch, 47 inch, 50 inch, 55 inch, 60 inch, 65 inch

Exercise 10H

1 **a** 3.5, 3.7, 4.1, 4.3, 4.8
b 52, 60, 61, 67, 73, 75
2 **a**

Stem	Leaf
5	1 6 7 8
6	0 2 3 7
7	1 3 4 5

b 6 | 4 means 6.4 (or other number)
3 **a** 9
b **i** 8 min **ii** 35 min **c** 21 min **d** 21 min
4 **a** 26
b **i** 0 mm **ii** 6 mm
c **i** 21 mm **ii** 24 mm
d **i** 8 mm **ii** 15 mm
e Skewed
f Symmetrical
5 **a** **i**

Stem	Leaf
1	9 9
2	3 4 6 6 8 8 8
3	2 2 3 5
4	1
5	4

2 | 3 means 23
ii 28 **iii** 28 **iv** Skewed
b **i**

Stem	Leaf
1	5 5
2	1 3 3 3 6
3	0 1 1 3 4 5 5 7 9
4	2 2 5 5 5 8
5	0 0 1

4 | 2 means 42
ii 35 **iii** 23, 45
iv Almost symmetrical
c **i**

Stem	Leaf
32	0
33	3 7 8
34	3 4 5 5 7 8 9
35	2 2 4 5 8
36	1 3 5
37	0

34 | 3 means 34.3
ii 34.85 **iii** 35.2, 34.5
iv Almost symmetrical

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d i

Stem	Leaf
15	7 8 9 9 9 9
16	1 4 7 7 7
17	3 5 7 7
18	5 5 7 9
19	3 8
20	0 2

17 | 7 means 177

ii 173 **iii** 159 **iv** Skewed

6 a

Stem	Leaf
0	5 6 6 8 8 9
1	0 1 1 2 2 2 4 4 5 6 7
2	0 1 2

1 | 2 means 12

b 9 **c** 12

7 a i

Set A		Set B
9 3 2	3	1
8 7 6 5 3 3 0 0	4	0 1 3 4 4 6 7 8 8 9 9
4 4 3	5	1 3

3 | 2 means 32

ii Set A has values spread between 32 and 54, whereas set B has most of its values between 40 and 53 with an outlier at 31.

b i

Set A		Set B
9 8 6 4 3 2 2 1	0	9
7 7 6 1 1 0 0	1	8 9 9
9 6 4 3 3 2	2	0 1 3 4 7 8
7 6 6 5 0	3	1 7 7 8 9 9 9
8 3 1	4	0 1 1 3 4 4 5 7
0	5	0 0 1 3 3 4 4 5 6

2 | 1 means 21

ii Set A has values between 1 and 50 with fewer large numbers, whereas set B has values between 9 and 56 with many large numbers.

c i

Set A		Set B
8 7 4 3	0	1 1 1 2 3 6 6 9 9
9 9 7 6 4	1	2 3 5 8 9
6 5 3 1	2	1 5 6
9 9 6 4 3 2	3	3 4 9
7 3 1	4	3 7 8
7 3 2	5	2 3 7
2 1	6	1 2
8 3	8	3 8
1	9	

4 | 1 means 4.1

ii Sets A and B are similar. Set A has values between 0.3 and 9.1 and set B has values between 0.1 and 8.8. Both sets have many small numbers and fewer large numbers.

8 a

Collingwood		St Kilda
8 3	6	6 8 8
8 7 2 1 0	7	8
9 9 8 2 0	8	0 0 2 2 3 4 7 8
8 7 5 1	9	0 4
4 3 3 3	10	6 9
9 8 5	11	1 3 7 8
7	12	2 5 6
	13	8

10 | 6 means 106

b Collingwood $33\frac{1}{3}$%, St Kilda $41\frac{2}{3}$%

c Collingwood is almost symmetrical data. St Kilda is not symmetrical.

d Collingwood results seem to be consistent. St Kilda has groups of similar scores and, while less consistent, has higher scores.

9 a 16.1 s **b** 2.3 s

c Yes, 0.05 s lower

10 a

Battery lifetime

Brand A		Brand B
3	7	2 3 4
	7	5 6 8 9 9
4 2	8	0 1 3
9 8 7 5	8	
4 4 3 2 1 0	9	0 1 2 3 4
9 9 8 7 6 5 5	9	5 6 8 8

8 | 3 represents 8.3 hours

b Brand A, 12; Brand B, 8

c Brand A consistently performs better than Brand B.

11 a 52 **b** 17.8

12 a 48% **b** 15%

c In general, birth weights of babies of mothers who smoke are lower.

Exercise 10I

1 a class interval **b** frequency table

c histogram **d** no spaces

e percentage frequency

2 a 20 **b** 20

3 a 3 **b** 360 000–370 000

4 a 2 **b** 20 **c i** 30% **ii** 65%

5 a

Class	Frequency	Percentage frequency
0–9	23	46
10–19	10	20
20–29	6	12
30–39	7	14
40–49	4	8
	50	100

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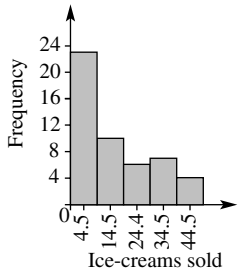
b

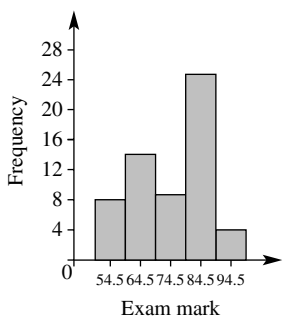


c **i** 33 **ii** 11 **d** 34%

6 a

Class	Frequency	Percentage frequency
50–59	8	$13\frac{1}{3}$
60–69	14	$23\frac{1}{3}$
70–79	9	15
80–89	25	$41\frac{2}{3}$
90–99	4	$6\frac{1}{3}$
	60	100

b

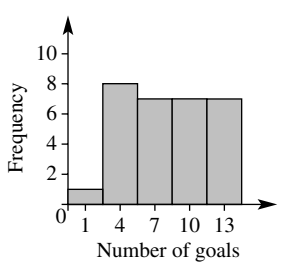


c **i** 22 **ii** $48\frac{1}{3}\%$

7 a

Number of goals	Frequency
0–2	1
3–5	8
6–8	7
9–11	7
12–14	7
	30

b



c 9 **d** 7

8 a Symmetrical data **b** Skewed data

9 a $a = 15$ $b = 2$ $c = 30$ $d = 40$
$e = 20$ $f = 100$

b $a = 50$ $b = 28$ $c = 12$ $d = 40$ $e = 100$

10 85.5%

11 a **i** 20% **ii** 55% **iii** 80%
iv 75% **v** 50%

b **i** 30 **ii** 45

c **i** 2 **ii** 22

12 a Student A

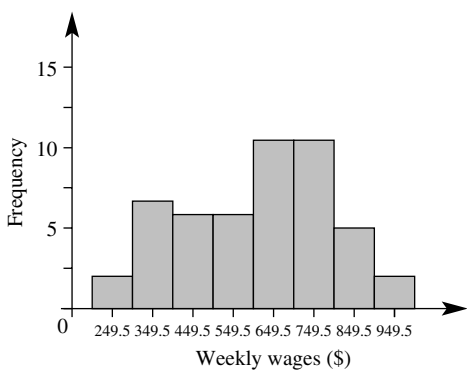
b Make the intervals for their groups of data smaller so that the graph conveys more information.

13 a minimum wage: \$204; maximum wage: \$940

b **i**

Weekly wages ($)	Frequency
200–299	2
300–399	7
400–499	6
500–599	6
600–699	11
700–799	11
800–899	5
900–999	2
Total	50

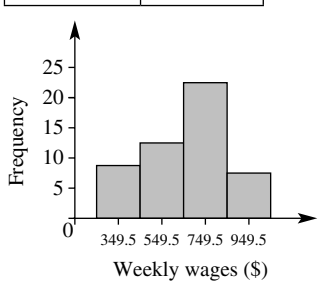
ii



c **i**

Weekly wages	Frequency
200–399	9
400–599	12
600–799	22
800–999	7
Total	50

ii



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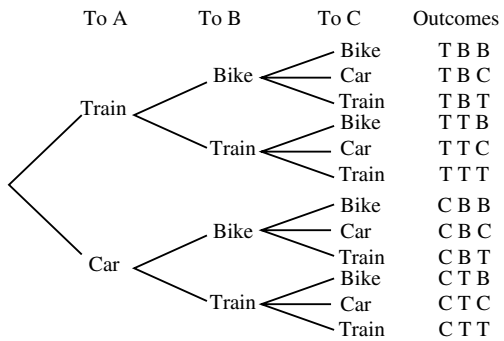
d More intervals shows greater detail. Since the first graph has each pair of intervals quite similar, these two graphs are quite similar.

Puzzles and challenges

1 IT'S RANDOM
2 i-C, ii-B, iii-E, iv-D, v-A
3 6 own both cat and a dog
4 2, 3, 4, 8, 8
5 7, 8, 9, 10 and 11
6 a



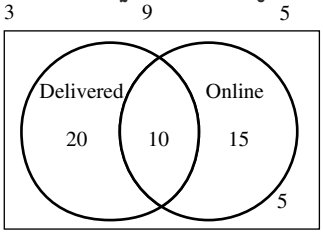
b 12 **c** $\frac{1}{4}$

Multiple-choice questions

1 B **2** A
3 i A **ii** B **iii** D **iv** C
4 B **5** D
6 a B **b** C
7 E

Short-answer questions

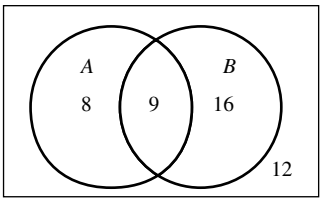
1 a $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{3}{5}$
2 a



b 15
c i $\frac{1}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{10}$
3 a

	A	Not *A*	Total
B	9	16	25
Not *B*	8	12	20
Total	17	28	45

b

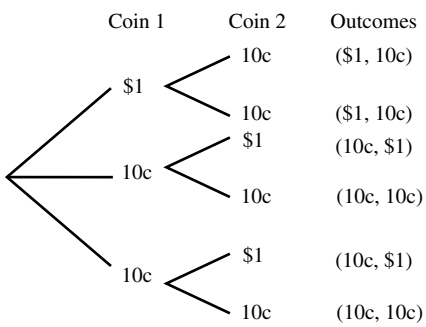


c i $\frac{4}{9}$ **ii** $\frac{1}{5}$ **iii** 8 **iv** 33

4 a 12 outcomes

	1	2	3	4
red	(red, 1)	(red, 2)	(red, 3)	(red, 4)
green	(green, 1)	(green, 2)	(green, 3)	(green, 4)
blue	(blue, 1)	(blue, 2)	(blue, 3)	(blue, 4)

b i $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{2}{3}$
5 a



b $\frac{2}{3}$
6 a $\frac{48}{120} = \frac{2}{5}$ **b** 14
7 a 26 **b** 26.5 **c** 23, 31 **d** 18
8 a

Employee 1		Employee 2
9	1	7 7
5 3 1	2	0 4 8 9
9 8 6 5 5 4	3	2 7 7 8
6 5 0	4	0 1 2 5 8
3 1	5	

2 | 4 means 24

b i Employee 1, 36; employee 2, 37
ii Employee 1, 36; employee 2, 33
c Employee 1, they have a higher mean and more sales at the high end.
d Employee 1 symmetrical, employee 2 skewed
9 a

Class interval	Frequency	Percentage frequency
120–129	1	4
130–139	3	12
140–149	4	16
150–159	8	32
160–169	7	28
170–179	2	8
Total	25	100

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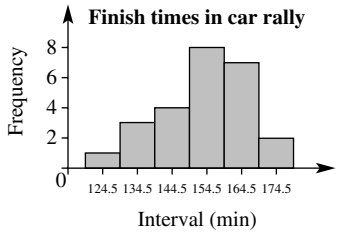
b



c i 4 **ii** 60%

10 a Mean = \$760 000
Median = \$590 000

b The median is better because the mean has been inflated by the outlier of \$2 900 000.

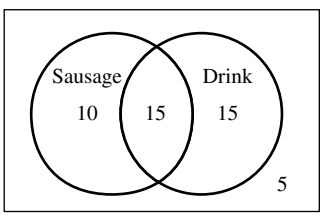
Extended-response questions

1 a i

		1st spin				
		1	2	3	4	5
2nd spin	1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)

ii $\frac{12}{25}$ **iii** $\frac{4}{25}$ **iv** 24 **v** \$68

b i



ii 5 **iii** 10 **iv** $\frac{1}{3}$ **v** $\frac{4}{9}$

2 a

Airline A		Airline B
4 4 3 2 2 1 1 0	0	1 2 4 4
9 8 7 7 6	0*	5 5 6 7 7 8 9
4 4 4 3 3 3 2 2 1	1	0 0 0 0 1 2 2
9 9 8 8 7	1*	5 5 8 8 8 9 9
4 3	2	1 2 2
	2*	5 6
2	5	

1 | 2 means 12 1* | 5 means 15

b Yes, 52 min

c i Airline A, 12
Airline B, 10.5

ii Airline A, 10.6
Airline B, 12.4

d No, the median time is 12 minutes, so half the flights have less than 12 minute delay.

Semester review 2

Length, area, surface area and volume

Multiple-choice questions

1 C **2** C **3** D **4** C **5** A

Short-answer questions

1 a 22 cm² **b** 12 cm² **c** 25.92 m²

2 a 25.13 m, 50.27 m² **b** 62.83 cm, 314.16 cm²
c 13.19 cm, 13.85 cm²

3 a 54.82 cm² **b** 26 m² **c** 24.57 m² **d** 36 cm²

4 a 220 m² **b** 216 m² **c** 25.45 m²

5 a 12 m³ **b** 42 m³ **c** 100.53 cm³

Extended-response question

a 12.57 m² **b** 77.43 m² **c** \$628

Indices

Multiple-choice questions

1 C **2** B **3** D **4** E **5** C

Short-answer questions

1 a $15p^3q^2$ **b** $\frac{a^2b}{2}$ **c** $54x^9y^6$ **d** -2

2 a $3 \times 4^2 = 48$ **b** $\frac{4}{10^3} = 0.004$

3 a 2400 **b** 1 080 000
c 0.0071 **d** 0.0000206

4 a 6.03×10^4 **b** 2.7×10^6
c 4×10^{-3} **d** 7.03×10^{-4}

5 a 3.07×10^{-2} kg **b** 4.24×10^6 kg
c 1.22×10^4 seconds **d** 7.45×10^{-6} years

Extended-response question

a i 74 000 000 000 **ii** 7.4×10^{10}

b 1.87×10^{17} **c** 8.72×10^{-7}

Properties of geometrical figures

Multiple-choice questions

1 C **2** B **3** A **4** E **5** B

Short-answer questions

1 a 180° **b** 360° **c** 540°
d 900° **e** 1 080° **f** 1 440°

2 a $x = 70$ **b** $x = 70$
c $a = 100$, $b = 140$ **d** $a = 70$, $b = 110$

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e $x = 100$, $y = 110$ **f** $x = 67$, $y = 98$
g $x = 120$ **h** $a = 135$

3 a Yes, RHS
b $\Delta ABC \equiv \Delta DEF$
c AB

4 a Two pairs of sides in same ratio and equal included angles
b 2

5 a Two pairs of equal angles
b 2.4

Extended-response question

a Two pairs of equal angles
b 6 m **c** $\frac{5}{3} = 1.\dot{6}$
d 7.5 m **e** 3.75 m

Quadratic expressions and algebraic fractions

Multiple-choice questions

1 E **2** E **3** A **4** B **5** D

Short-answer questions

1 a $-3x - 6$ **b** $x - x^2$
c $x^2 - 1$ **d** $x^2 - 8x + 16$
e $25x^2 - 4$ **f** $9x^2 - 24x + 16$
g $8x^2 + 10x - 3$ **h** $2x + 3$

2 a $-6x^2y$ **b** $42a^2b^2$ **c** $\frac{a}{2}$ **d** $\frac{-6x}{y}$
e $\frac{3x}{2}$ **f** $\frac{3}{4}$ **g** 4 **h** $-x$
i $\frac{3}{2}$ **j** $\frac{9a}{x}$

3 a $x^2 - 9$ **b** $x^2 + 4x + 4$
c $6x^2 - 17x + 12$

4 a $3(x - 4)$ **b** $-7(x + 2)$
c $x(5x + 2)$ **d** $7x(2x - 3)$

5 a $\frac{x}{12}$ **b** $\frac{53x}{20}$
c $\frac{7x + 5}{10}$ **d** $\frac{11x + 7}{15}$

Extended-response question

a $10 - 2x$ and $8 - 2x$
b $(10 - 2x)(8 - 2x) = 80 - 36x + 4x^2$
c 48 m^2

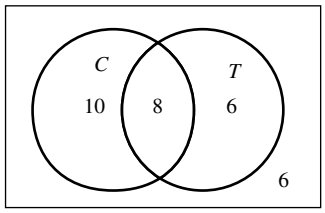
Probability and single variable data analysis

Multiple-choice questions

1 D **2** A **3 i** C **ii** D
4 B **5** B

Short-answer questions

1 a



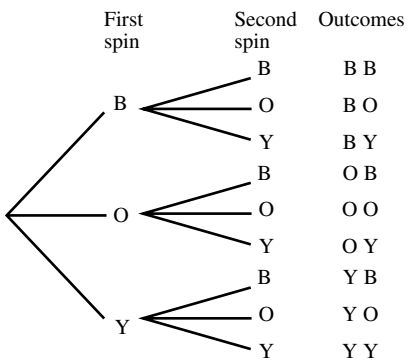
b i 24 **ii** 16
c i $\frac{1}{5}$ **ii** $\frac{1}{3}$

2 a

	1	2	3	4
H	H1	H2	H3	H4
T	T1	T2	T3	T4

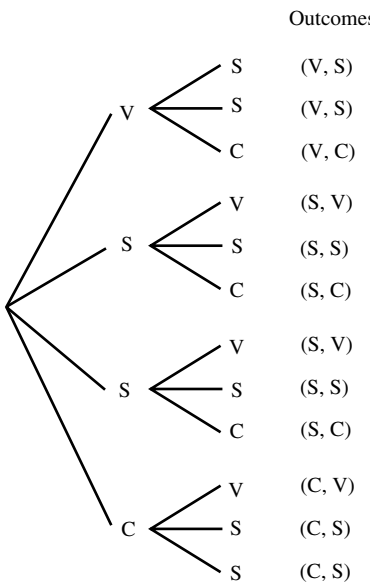
b 8 **c** $\frac{1}{8}$ **d** $\frac{1}{4}$

3 a



b $\frac{1}{9}$ **c** $\frac{1}{3}$ **d** $\frac{2}{9}$

4 a



b i $\frac{1}{3}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{2}$

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5 a

Stem	Leaf
1	0 1 1 2 3 5 7 8
2	2 5 5 5 6
3	1 2 2

1 | 3 means 13 aces

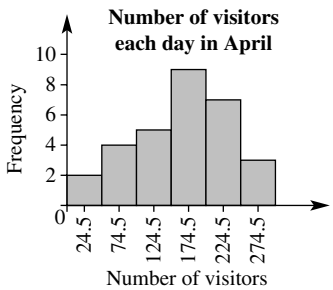
b Mode = 25, median = 20

c Skewed

6 a

Class interval	Class centre	Frequency	Relative frequency (%)
0–49	24.5	2	6.7
50–99	74.5	4	13.3
100–149	124.5	5	16.7
150–199	174.5	9	30
200–249	224.5	7	23.3
250–299	274.5	3	10
Total		30	100

b



c i 6 **ii** 60%

Extended-response question

a

		Red		
		1	2	3
Green	1	(1, 1)	(1, 2)	(1, 3)
	2	(2, 1)	(2, 2)	(2, 3)
	3	(3, 1)	(3, 2)	(3, 3)

b $\frac{4}{9}$

c i $\frac{1}{3}$ **ii** 18

d i 17 years **ii** 92

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