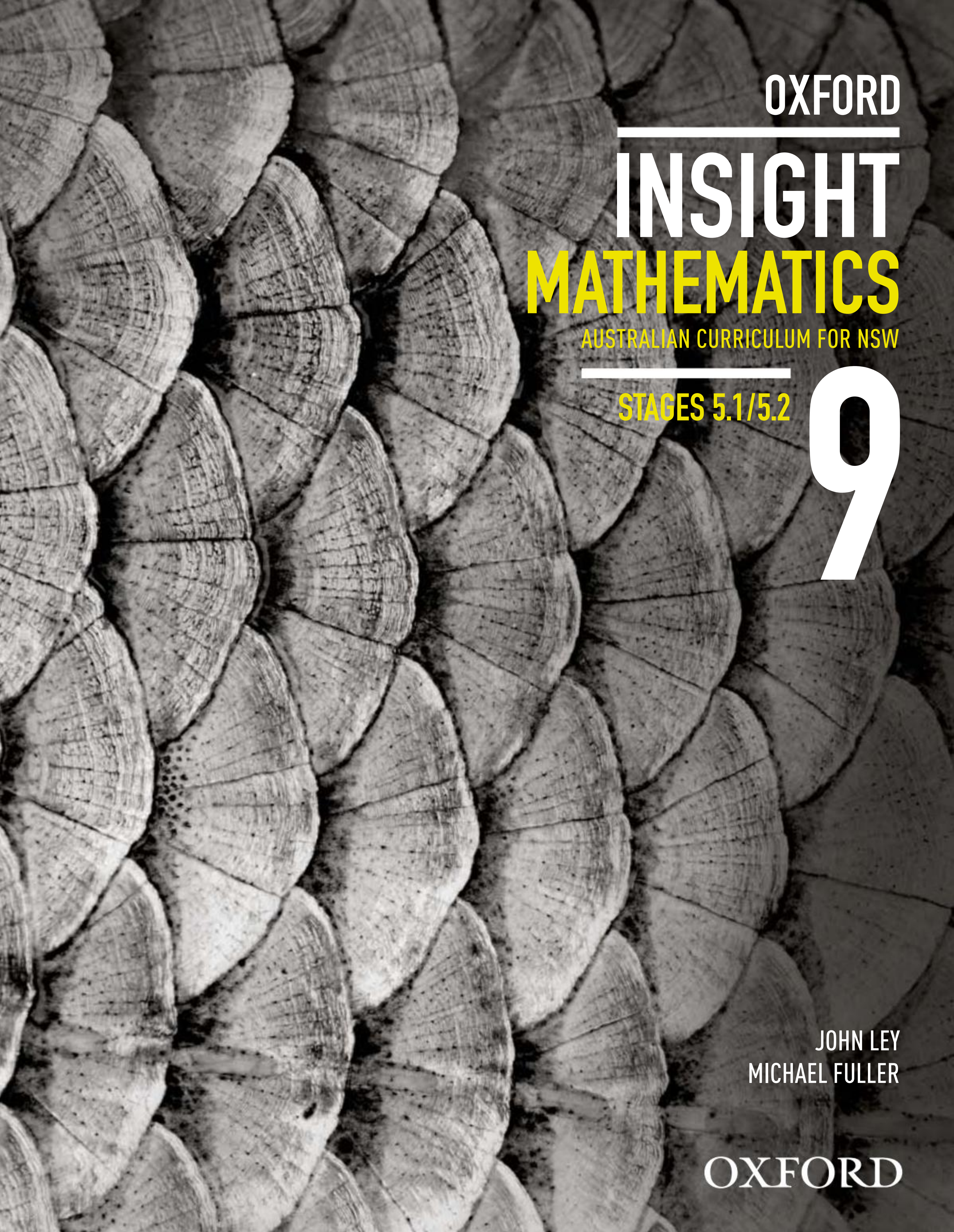

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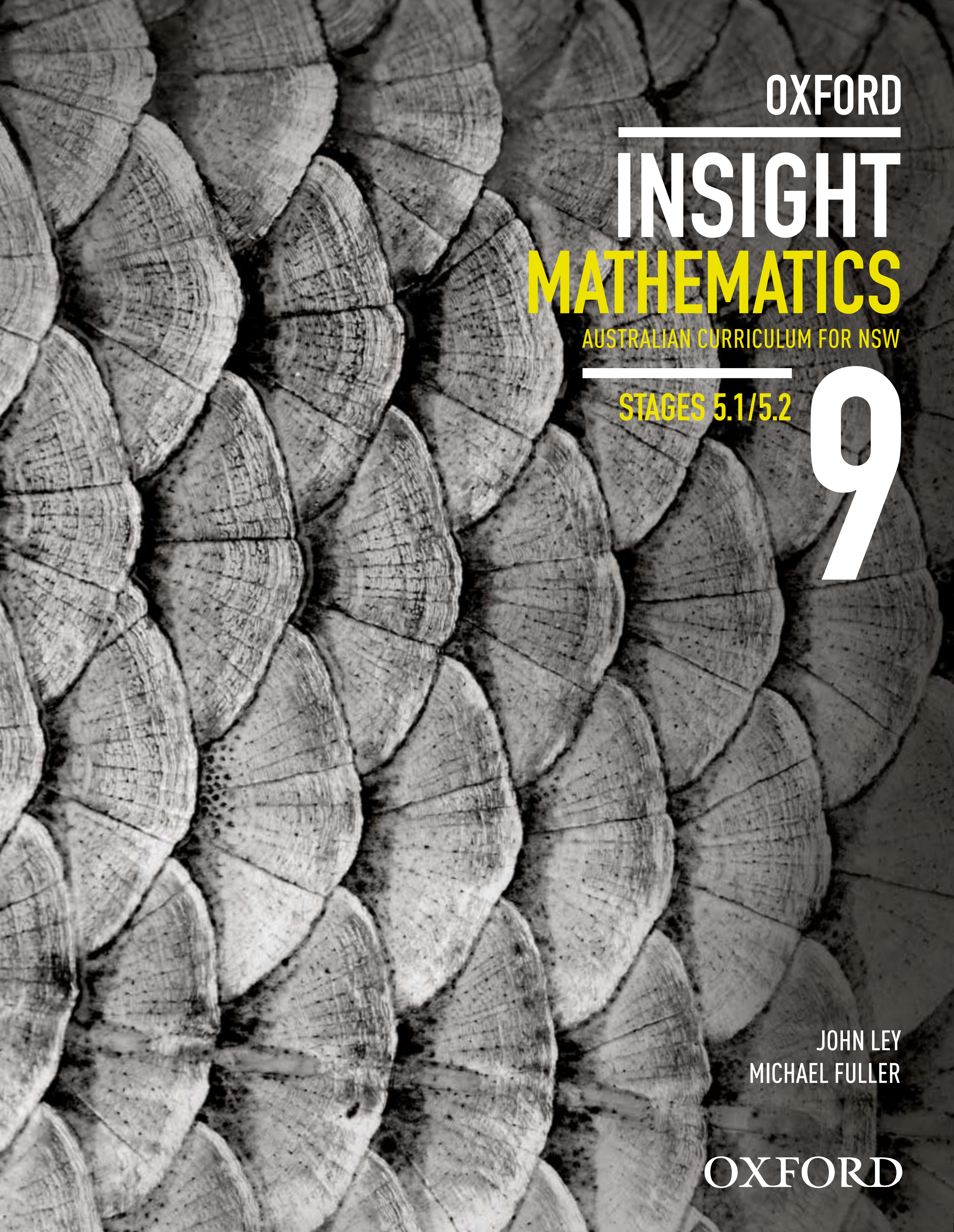

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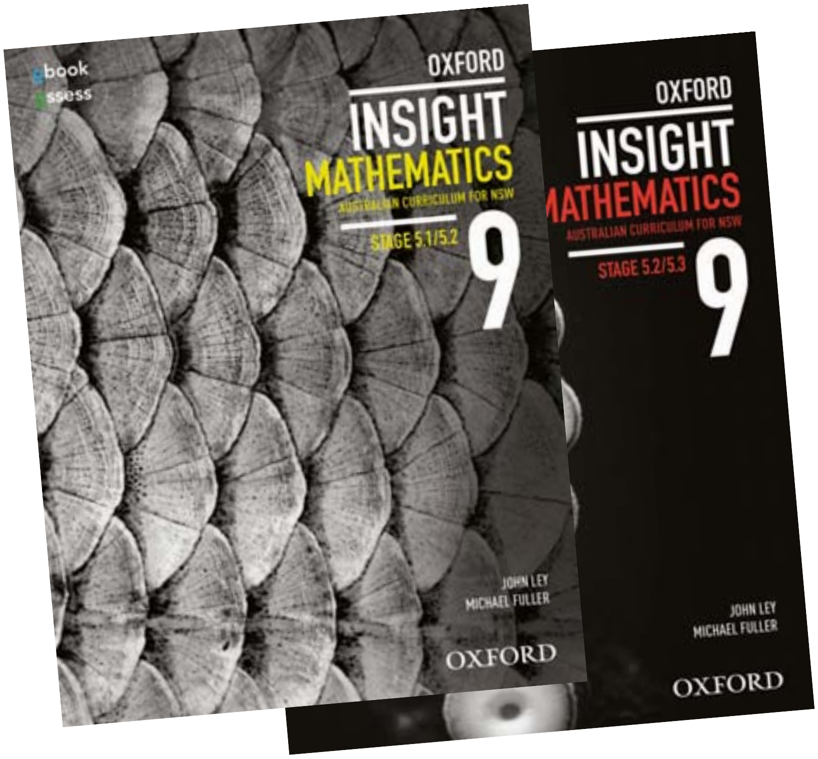

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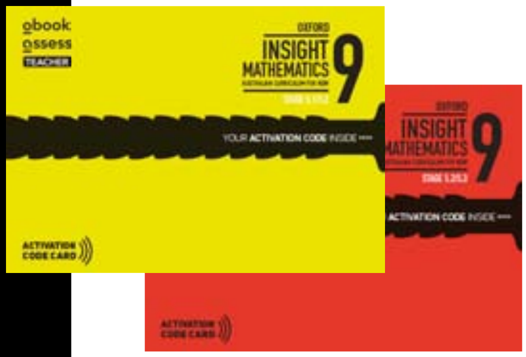

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SYLLABUS GRID

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2	Indices	MA5.1-1WM, MA5.1-3WM, MA5.1-5NA, MA5.2-1WM, MA5.2-3WM, MA5.2-6NA, MA5.2-7NA	5.1 N&A Indices, 5.2 N&A Indices, 5.2 N&A Algebraic techniques (part)	ACMNA209, ACMNA212, ACMNA213, ACMNA231
3	Collecting and analysing data	MA5.1-1WM, MA5.1-2WM, MA5.1-3WM, MA5.1-12SP, MA5.1-13SP	5.1 S&P Single variable data analysis	ACMSP228, ACMSP253, ACMSP282, ACMSP283
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6	Area and surface area	MA5.1-1WM, MA5.1-2WM, MA5.1-8MG	5.1 M&G Area and surface area	ACMMG216, ACMMG218
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CR 11–12 Cumulative review chapters 11–12				

1

Review of Year 8

This chapter reviews the Year 8 component of the mathematics syllabus and includes outcomes from Number and Algebra, Measurement and Geometry, and Statistics and Probability. You should be able to:

- complete data investigations
- calculate ratios and rates
- identify congruent figures including triangles, stating the conditions
- work with numbers and algebraic terms involving indices
- calculate the perimeter and area of plane and compound shapes
- calculate time using mixed units
- operate with fractions, decimals and percentages in worded problems
- calculate area and circumference of circles and surface area and volume of cylinders
- analyse sample data using mean, mode and median and make inferences
- use Pythagoras' theorem to perform calculations in right-angled triangles
- use algebraic techniques to simplify, expand and factorise simple algebraic expressions
- calculate volume and capacity
- solve linear equations and simple inequations
- solve probability problems involving simple events and use Venn diagrams
- graph and interpret linear relationships on the number plane.

A Data

Exercise 1A

1 Define the statistical term ‘sample’.

2 Would a census or a sample be used to investigate the number of people who use a particular brand of toothpaste? Why?

3 Describe the sample you would use if you wanted to gather support for improved skateboard facilities at your local park.

4 For the scores 11, 14, 15, 19, 19, 21, find the:

a mean **b** mode **c** median **d** range.

5 For the scores in this stem-and-leaf plot, find the:

a mean **b** mode

c median **d** range.

Stem	Leaf
2	7 8 8
3	0 0 1 2 3 4 5 6 6
4	1 2 4 4 4 6 8
5	3 5 7 8
6	2 3

6 The back-to-back stem-and-leaf plot compares the marks gained by classes A and B in their half-yearly Mathematics exam.

a Find the mean, mode, median and range for each class.

b Which class performed better? Explain your answer.

Class A Leaf	Stem	Class B Leaf
2 1	2	8 8
6 4 2 1	3	0 3 5 6
6 5 3 1 0	4	0 2 6 6 8
1 1 0	5	3 6 9
7	6	7

7 From a school of 800 students, a random sample of 50 students was selected. There were 13 left-handed students in the sample.

a What fraction of the sample was left-handed?

b Estimate how many students at the school were left-handed.

B Ratios and rates

Exercise 1B

1 Express the ratio 25 min : $1\frac{1}{4}$ h in simplest form.

2 Which of the following is equivalent to 3 : 5?

A 21 : 36 **B** 45 : 75

3 Find x if $\frac{4}{x} = \frac{7}{5}$.

4 Simplify the following ratios.

a 25 cm : 0.6 m **b** 360 m : 0.5 km

5 The ratio of teachers to students is 2 : 11. Calculate the number of students if there are 10 teachers.

6 A scalene triangle has side lengths in the ratio 2 : 5 : 4.

- **a** If the shortest side is 12.4 cm, find the lengths of the other two sides.
- **b** Calculate the perimeter of the triangle.

7 Ian jogs 3.5 km in 20 minutes. Express this as a rate of km/min.

8 Which is the better buy, A or B?

A 1.2 L of Fizz Whiz Cola at $1.05 **B** 2.5 L of Fizz Whiz Cola at $2.20

9 The scale of a model aeroplane is 1 : 120. If the wingspan of the model is 17 cm, calculate the actual wingspan of the real aeroplane.

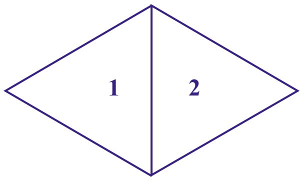
10 The actual height of a building is 825 m. If a model of the building is constructed using a scale of 1 : 1500, calculate the height of the model.

C Congruence

Exercise 1C

1 Which transformation(s) could have been used to produce each pair of congruent figures?

a



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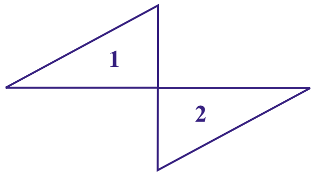
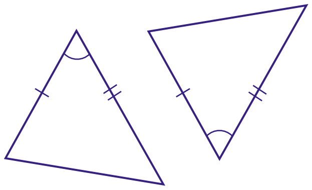


2 For each pair of triangles, state the congruency test used to show that the triangles are congruent.

a



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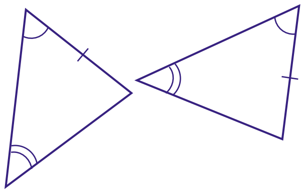

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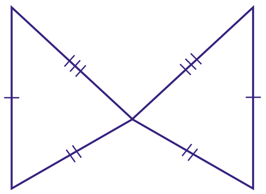

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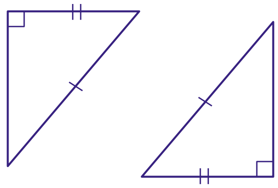
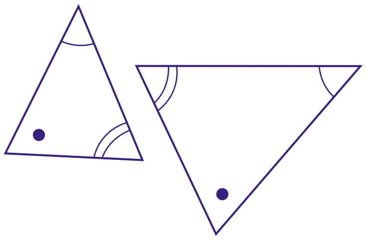


3 For each pair of triangles, state why the triangles are *not* congruent.

a



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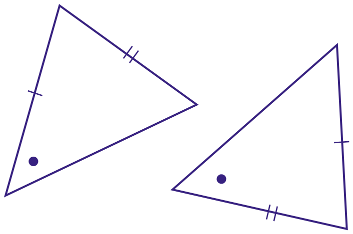

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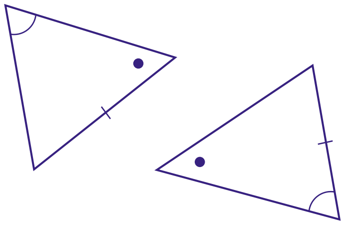
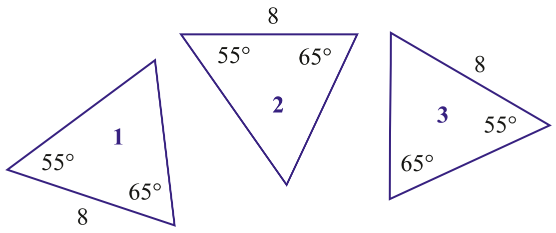


4 Which triangles in each group are congruent? Give a reason for your answer.

a



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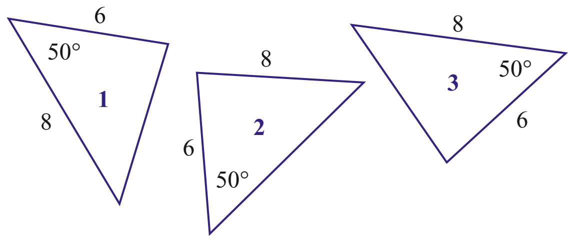
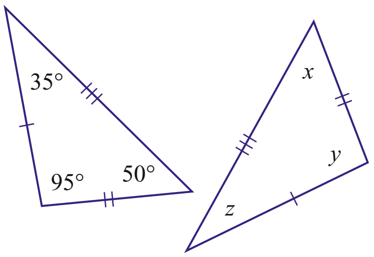


5 For each pair of triangles, state why the triangles are congruent. Hence find the values of the pronumerals.

a



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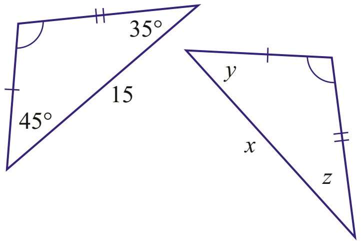

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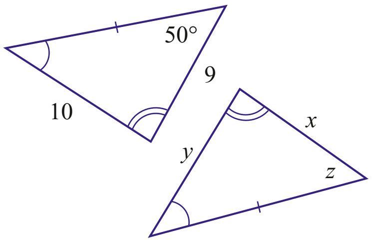


D Index laws

Exercise 1D

1 Write the following in index form.

a $7 \times 7 \times 7 \times 7 \times 7$ **b** $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$

c $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ **d** $10 \times 10 \times 10$

2 Write the base and index of each number.

a 3^8 **b** 5^2 **c** 8^4 **d** 6^0

3 Write the following in expanded form.

a 6^4 **b** 7^3 **c** 6^2 **d** 5^7

4 Evaluate:

a 2^9 **b** 3^6 **c** 5^4 **d** 7^3

5 Simplify, leaving your answers in index form.

a $3^{10} \times 3^3$ **b** $(7^2)^6$ **c** $4^{10} \div 4^5$ **d** $6^{12} \div 6$ **e** $(2^5)^4 \times 2^{10}$

6 Determine whether these calculations are true or false.

a $4^5 \times 2^6 = 8^{11}$ **b** $5^7 \div 5^3 = 1^4$ **c** $4^7 \times 3^4 = 12^{11}$ **d** $15^8 \div 3^2 = 5^6$

7 Evaluate:

a 6^2 **b** 6^1 **c** 6^0 **d** $(7^2)^0$

8 Simplify:

a $a \times a \times a$ **b** $6 \times r \times r \times r \times r$ **c** $x \times x \times y \times y \times y \times y$

9 Expand:

a t^2 **b** $5a^4$ **c** p^6 **d** $15e^5$

10 If $a = 3$, evaluate:

a a^2 **b** $4a^2$ **c** $(4a)^2$ **d** $4a^0$

E Perimeter and area

Exercise 1E

1 Estimate the width of your classroom.

2 Convert the following lengths to millimetres.

a 0.27 m

b 0.004 km

3 Convert the following lengths to centimetres.

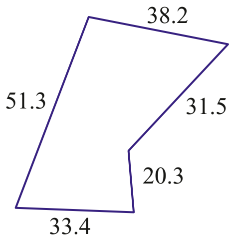
a 0.34 m

b 0.07 km

4 Calculate the perimeter of a rectangle with width 11.9 cm and length 26.3 cm.

5 Calculate the perimeter of each shape. All measurements are in centimetres.

a



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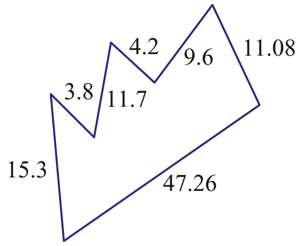
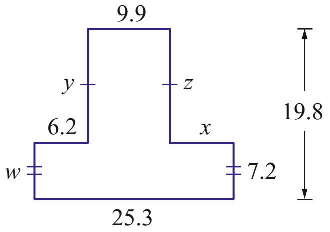
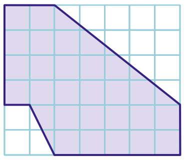


6 A regular octagon has a perimeter of 1012.16 cm. Calculate the length of each side.

7 Find the length of each side marked with a pronumeral, then calculate the perimeter. All measurements are in centimetres.

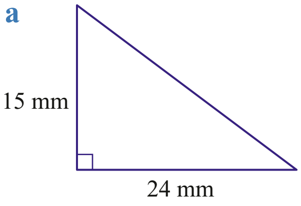


8 By counting squares, find the area of the shape.



9 Find the areas of the following shapes.

a



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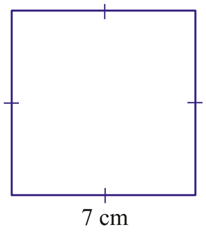

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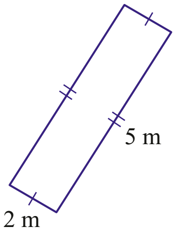

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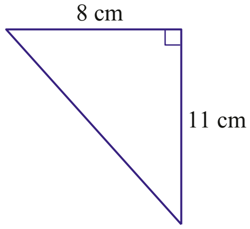
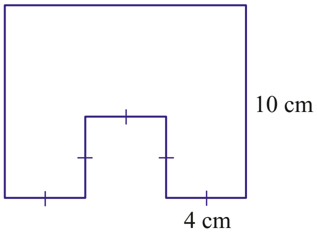


10 Find the areas of the following composite shapes.

a



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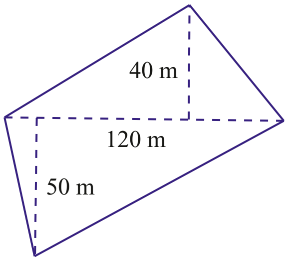

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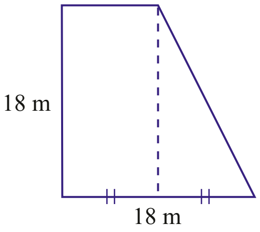
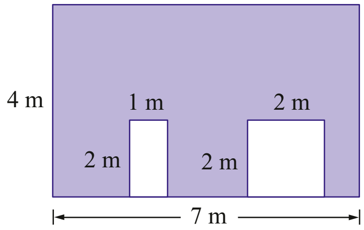


11 Find the shaded area in each shape.

a



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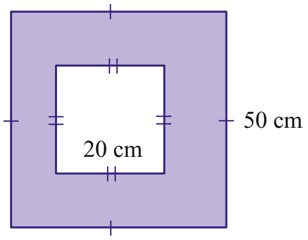


12 Complete these conversions.

a 5 cm = _____ mm
b 800 cm = _____ m
c 640 mm = _____ cm
d 11.6 m = _____ cm
e 43.8 cm = _____ mm
f 8400 cm = _____ m
g 8 cm^2 = _____ mm^2
h 7.2 m^2 = _____ cm^2
i 9000 mm^2 = _____ cm^2

F Time

Exercise 1F

1 How many hours in 2 days?

2 Complete the following conversions.

a 240 s = ____ min
b 300 min = ____ h

3 Convert 210 min to hours and minutes.

4 Calculate the following.

a 3 h 35 min + 5 h 48 min
b 3 h 21 min − 1 h 42 min

5 If Sergio caught the bus at 6:35 am, at what time did he arrive at work, given the bus trip took 42 min?

6 High tide is at 5:20 am and low tide is at 9:08 am. Calculate the time difference between high and low tide.

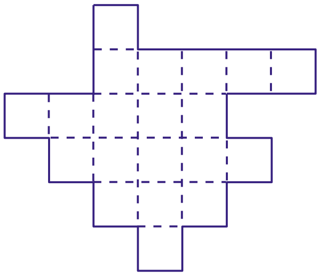
7 Convert $2\frac{1}{3}$ h to hours and minutes.

8 Round the digital clock display $\boxed{03:16:41}$ to the nearest minute. Express the time in hours and minutes.

G Fractions, decimals and percentages

Exercise 1G

1 Shade $\frac{7}{10}$ of this diagram.



2 In a class of 20 students, $\frac{1}{4}$ play soccer, $\frac{1}{5}$ play netball and the remainder play football. What fraction of the class plays football?

3 **a** Convert $\frac{146}{11}$ to a mixed numeral. **b** Convert $3\frac{5}{8}$ to an improper fraction.

4 **a** Complete: $\frac{155}{\square} = \frac{31}{20}$ **b** Simplify $\frac{175}{240}$.

5 Arrange in descending order: $\frac{4}{5}, \frac{8}{15}, \frac{2}{3}$

6 **a** State the reciprocal of $2\frac{2}{3}$. **b** Calculate $\frac{3}{8}$ of 592 kg.

7 Liam earns $600 per week. He banks $\frac{1}{5}$, spends $\frac{2}{3}$ on rent and food, and keeps the remaining money for personal use.

a How much does Liam bank each week?
b How much does he spend weekly on rent and food?
c What fraction of Liam's weekly wage is for personal use?
d How much is kept for personal use?

8 State the value of 2 in 4.0203.

9 Express $8 + \frac{3}{10} + \frac{7}{1000}$ as a decimal.

10 **a** Write 4.2 as a mixed numeral. **b** Write $3\frac{3}{8}$ as a decimal.

11 Express $\frac{1}{6}$ as a decimal correct to 2 decimal places.

12 **a** Round 3.854 44 to the nearest hundredth. **b** Round 3.5217 to the nearest whole number.

13 Simplify the following.

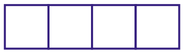
a $12.6 - 11.8 + 3.84$ **b** $15.5 \div 0.05 + 22.4$ **c** $16.2 \div 2 + 5.7 - 1.9$

14 Simplify the following.

a $(2.1 + 3) \times (11.9 - 5.9)$ **b** $(10.3 - 8.7) + (0.4 \times 9)$

15 **a** Ahmed earns $4.60 per hour. How much does he earn if he works for $10\frac{1}{2}$ hours?
b Sylvanna won $1 216 320 in a lottery. She decided to share it equally between eight people. How much did each person receive?

16 Shade 75% of this diagram.



17 Write 48 out of 100 as a percentage.

18 **a** Convert 37% to a fraction. **b** Convert 57% to a decimal.

19 Express the following as percentages.

a 3.8 **b** $\frac{5}{8}$

20 Convert to percentages and arrange in ascending order: $\frac{4}{5}$, 70%, 0.65

21 Convert:

a $\frac{27}{100}$ to a percentage **b** 15% to a simplified fraction **c** 425% to a decimal.

22 **a** Calculate 15% of $360. **b** Find 25% of 48 m.

23 Express 13 kg as a percentage of 52 kg.

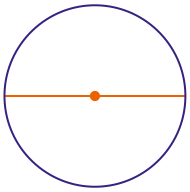
24 **a** Increase 100 by 30%. **b** Decrease 320 by 25%.

H Circles and cylinders

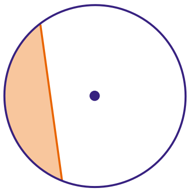
Exercise 1H

1 Name the features of each circle shown in orange.

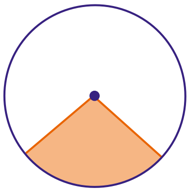
a



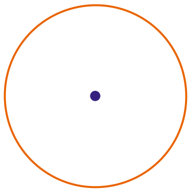
b



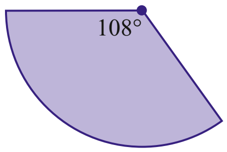
c



d



2 What fraction of a circle is represented by this sector?



3 Write the formula for the circumference of a circle when given the diameter.

4 Calculate the circumference of a circle with a diameter of 11.4 cm correct to 1 decimal place.

5 Write the formula for the circumference of a circle when given the radius.

6 Calculate the circumference of a circle with a radius of 6.8 cm correct to 2 decimal places.

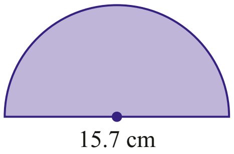
7 Write the formula for calculating the area of a circle when given the radius.

8 Calculate the area of a circle correct to 1 decimal place, given:

a radius = 7 cm

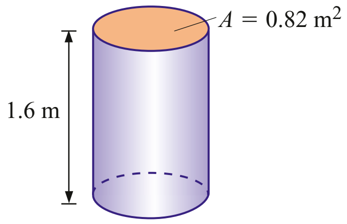
b diameter = 3.9 cm

9 Calculate the area of this shape correct to 1 decimal place.

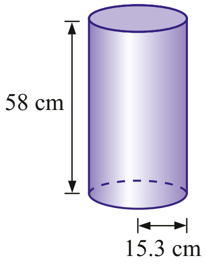


10 Calculate the volume of each cylinder.

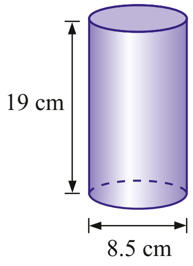
a



b



c



I Mean, mode, median and sampling

Exercise 1I

1 Write the outlier in each data set.

a 0, 71, 72, 72, 75

b 23, 24, 25, 25, 27, 28, 89

2 Consider the three data sets given.

A 8, 10, 11, 11, 13, 96 **B** 7, 7, 8, 9, 11, 12 **C** 4, 7, 8, 9, 9, 9, 9

In which data set(s) is the following measure *not* a central value?

a mean **b** mode **c** median

3 The sexes of 5 students chosen at random from Year 9 are female, male, male, female, male. For this data, find, where possible, the:

a mean **b** mode **c** median.

4 **a** The weights, in kilograms, of seven 1-year-old horses of the same breed were 420, 420, 430, 440, 460, 470, 650. For these weights, find the:

i mean **ii** mode **iii** median.

b Which measure would be the most appropriate to represent the weight of 1-year-old horses of this breed?

5 Five samples of 20 students were chosen from all students in a school. The students were asked to state the number of text messages they had sent the day before. The mean number of texts per day for each sample is shown in the table. Using the information given, what is the best estimate of the mean number of texts sent per day by students of this school? Give the answer to the nearest whole number.

Sample number	1	2	3	4	5
Mean number of texts	15.2	6.8	9.4	12.8	11.6

J Pythagoras' theorem

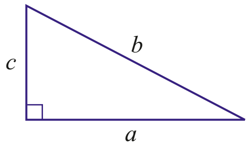
Exercise 1J

1 Consider the following triangles.

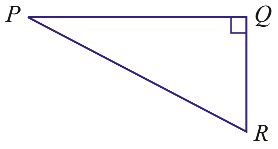
i Which side is the hypotenuse?

ii Write an expression for Pythagoras' theorem for the triangle.

a

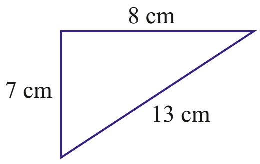


b

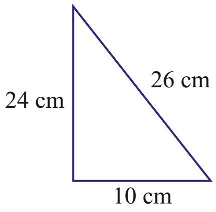


2 State whether each triangle is right-angled.

a



b

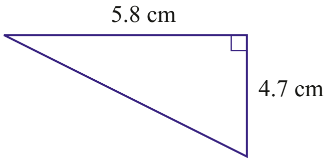


3 **a** Find the value of 9^2.

b Calculate the value of $\sqrt{70}$ correct to 1 decimal place.

4 Find the length of the hypotenuse in each triangle correct to 1 decimal place.

a



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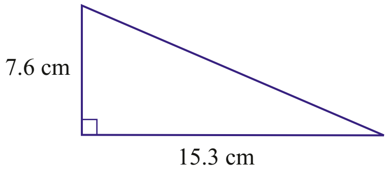
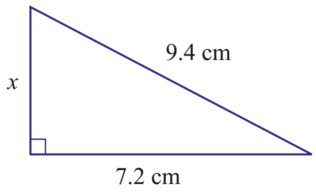


5 Find the length of the third side of each triangle correct to 1 decimal place.

a



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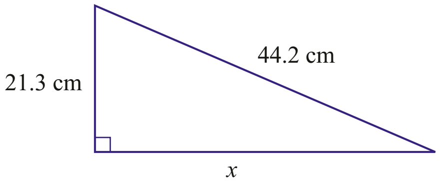
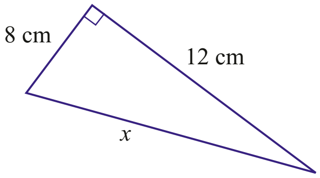
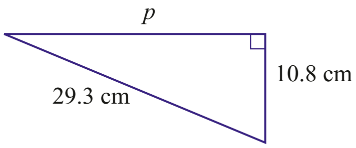


6 Find the value of the pronumeral in each of the following triangles correct to 1 decimal place.

a



b



7 Calculate the length of the diagonal of a square with side length 36 cm correct to 2 decimal places.

K Algebra

Exercise 1K

1 Simplify:

a $9x + 5x$ **b** $7y - y$ **c** $3a^2 + 4a^2$ **d** $9ac - 3ca$

2 Simplify:

a $5 \times 12n$ **b** $-5 \times 2a$ **c** $8m \times 3$ **d** $-5p \times -7$

3 Simplify:

a $10a \div 2$ **b** $12m \div -3$ **c** $\frac{abc}{a}$ **d** $\frac{12m}{3}$

4 Simplify:

a $4wx + 2y - 5xw - 5y$ **b** $6m + 2m - 8m$

5 Simplify:

a $\frac{4a}{7} + \frac{a}{7}$ **b** $\frac{a}{3} - \frac{a}{5}$ **c** $\frac{q}{5} \times q$ **d** $6p \div 2p$

6 Expand:

a $a(a - n)$ **b** $mn(2n - 5)$ **c** $4p(3p + 2)$ **d** $-2p(4y - 2w)$

7 Expand and simplify:

a $3(5a + 3) - 4(8 - 4a)$ **b** $3x(y - 4) + 4y(5x - 2)$

8 Factorise:

a $mn^2 + mn$ **b** $pq - aq$ **c** $4p - 12d$ **d** $25f - 15$

9 Factorise each by taking out a negative factor.

a $-3k + 9$ **b** $-4p - 12d$

10 Define the following terms.

a pronumeral

b coefficient

11 If $Q = 7$ and $p = -4$, evaluate:

a $4Q + p$

b $\dfrac{Qp}{8}$

c $6p - 5Q$

d $3(Q - p) + 7p - 8Q$

12 Write an algebraic expression for each.

a The product of six and d plus twenty-three

b The difference between x and seven multiplied by three and the result divided by eight

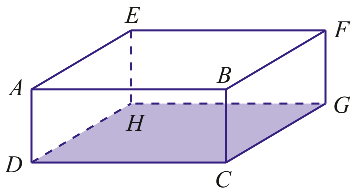
L Volume and capacity

Exercise 1L

1 **a** If $ABFE$ is the top face of the rectangular prism, name the bottom face.

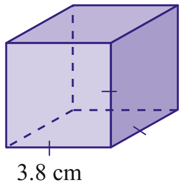
b Name the front and back faces.

c Name the two side faces.



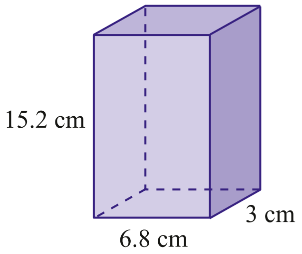
2 **a** Draw a net of the cube shown.

b Use the net to calculate the total surface area of the cube.

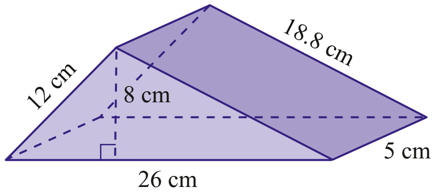


3 Calculate the surface area of each prism.

a

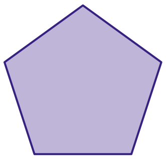


b



4 Construct prisms with the following cross-sections.

a



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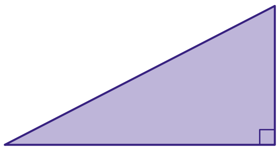
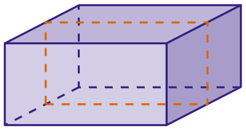
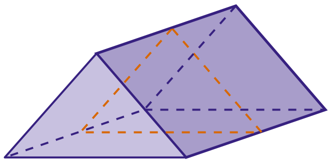


5 Draw the cross-section of each prism if it is cut along the orange dotted line shown.

a

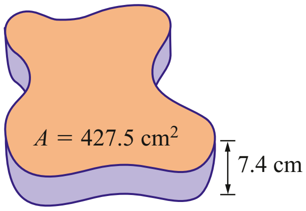


b



6 Calculate the volume of each solid.

a



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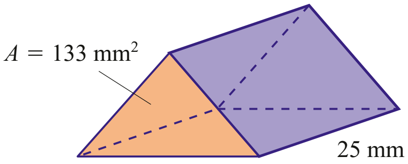
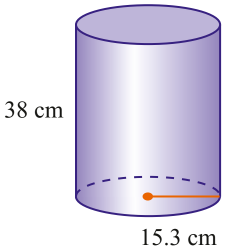
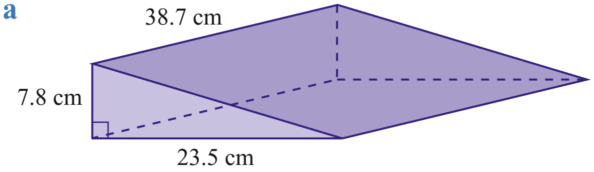


7 Calculate the volume of this cylinder to the nearest cm^3.

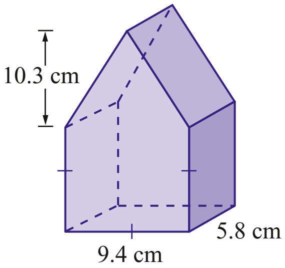


8 Calculate the volume of each solid.

a



b



9 Complete the following capacity conversions.

a $1\text{ cm}^3 = \_\_\_\_\text{ mm}^3$

b $1\text{ kL} = \_\_\_\_\text{ mL}$

c $1\text{ kL} = \_\_\_\_\text{ cm}^3$

d $1\text{ m}^3 = \_\_\_\_\text{ L}$

e $1\text{ m}^3 = \_\_\_\_\text{ kL}$

f $5.3\text{ kL} = \_\_\_\_\text{ cm}^3$

M Equations and inequations

Exercise 1M

1 Show each step required to get from the expression $4x + 12$ back to x.

2 Solve the following equations.

a $x + 11 = 17$

b $x + 9 = -6$

c $4x = 36$

d $-9x = 63$

e $3y + 18 = 29$

f $5 - 4p = -47$

g $4d + 8 = 3d - 12$

h $18 + 7c = 32 - 3c$

i $3(m + 6) = 2(m - 1)$

j $8(q - 5) = -3(10 + 3q)$

3 Solve the following equations.

a $\frac{4p}{5} = 6$

b $\frac{3x + 12}{7} = 12$

4 Is the given value for the pronumeral a solution to the equation?

a $5d + 12 = 28;\quad d = 3$

b $\frac{x}{5} + 7 = 24;\quad x = 3\frac{2}{5}$

5 Write an equation and solve this problem.

The sum of a certain number and 23 is 114. What is the number?

6 Solve the following inequations.

a $x + 9 \geqslant -3$

b $\frac{m}{7} < 4$

N Probability and Venn diagrams

Exercise 1N

1 A hat contains 1 red, 1 blue, 1 green and 1 yellow ticket. One ticket is chosen.
- **a** List the sample space.
- **b** What is the probability of selecting the red ticket?

2 Ten cards with the numbers 1 to 10 written on them are shuffled and one card is chosen.
- **a** List the sample space.
- **b** What is the probability that the card selected has 7 written on it?

3 Complete this table.

	Fraction	Decimal	Percentage
a		0.7	
b			25%
c	$\frac{5}{8}$		

4 A bag contains 4 green, 9 red and 7 blue marbles. One marble is selected at random.
- **a** How many marbles are in the bag?
- **b** How many marbles are red?
- **c** What is the probability of selecting a red marble?

5 One card is selected at random from a normal deck of 52 cards. What is:
- **a** P(diamond)?
- **b** P(red card)?
- **c** P(king)?

6
- **a** Write a statement describing a probability of 0.
- **b** Estimate a percentage probability for the phrase ‘even chance’.
- **c** Write a phrase to describe a probability of about 85%.

7 A die with the numbers 1–6 is rolled once. Describe an event that would be:
- **a** certain
- **b** impossible
- **c** of even chance.

8 A spinner has 5 equal-sized sectors coloured green, yellow, orange, brown and white. It is spun once. What is the probability of getting:
- **a** white?
- **b** any colour except white?
- **c** yellow or orange?
- **d** any colour except yellow or orange?

9
- **a** In a group of 29 girls, 15 play netball, 11 play oztag and 8 play both. Draw a Venn diagram to show this.
- **b** How many girls:
  - **i** play netball but not oztag?
  - **ii** play oztag but not netball?
  - **iii** play netball or oztag or both?
  - **iv** play netball or oztag but not both?
  - **v** play neither netball or oztag?

0 Coordinate geometry and straight lines

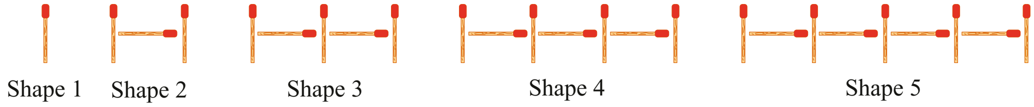
Exercise 10

1 Plot these points on a number plane: $A(0, -3)$, $B(-2, -3)$, $C(3, -4)$, $D(-3, 2)$, $E(2, 5)$

2 **a** Plot the points $A(-3, 6)$, $B(3, 6)$ and $C(3, 0)$.

b If $ABCD$ is a square, find the coordinates of point D.

3 **a** Use this pattern of matches to complete the table.



Shape number	1	2	3	4	5
Number of matches					

b Write a rule describing the number of matches required to make each shape.

c Using x to represent the shape number and y to represent the number of matches, write a set of coordinate points describing this information.

d Graph these points on a number plane.

e Mark in the next two points and write their coordinates.

4 Bulk ‘minute steak’ for barbecues is sold for $7.50 per kilogram with a minimum purchase of 2 kg. The following table shows weight versus cost for various quantities of minute steak.

Weight (kg)	2	4	6	10	20
Cost ($)	15	30	45	75	150

a Using x to represent the number of kilograms and y to represent the cost in dollars, write a set of coordinate points describing this information.

b Graph these points on a number plane and draw a straight line through them.

c Use the graph to find how much 16 kg of minute steak would cost.

d Use the graph to find how much minute steak could be purchased for $90.

5 Complete the table and draw the graph of $y = 2x - 3$.

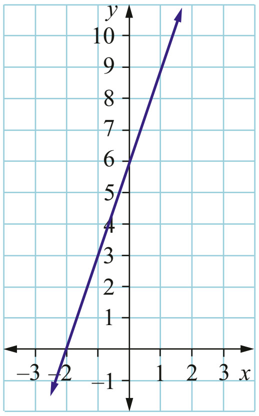
x	-2	-1	0	1	2
y		-5			1

6 The graph on the right shows a straight line.

a Use the graph to complete this table of values.

x	-2	-1	0	1
y				

b Write the rule describing this straight line.
The rule is of the form $y = \square x \pm \triangle$.



2

Indices

This chapter deals with indices and the distributive law.

After completing this chapter you should be able to:

- simplify algebraic products and quotients using the index laws
- simplify expressions involving the zero index
- evaluate numerical expressions involving negative (integral) indices
- simplify algebraic expressions involving negative (integral) indices
- apply the index laws to expressions with negative indices
- apply the distributive law to the expansion of algebraic expressions.

NSW Syllabus references: 5.1 N&A Indices, 5.2 N&A Indices, 5.2 N&A Algebraic techniques (part)
Outcomes: MA5.1-1WM, MA5.1-3WM, MA5.1-5NA, MA5.2-1WM, MA5.2-3WM, MA5.2-6NA, MA5.2-7NA
NUMBER & ALGEBRA – ACMNA209, ACMNA212, ACMNA213, ACMNA231

A The index laws

The **index laws** for numbers were established in Year 7:

$$3^{2}$$

2 ← **Index, power or exponent**

3 ← **Base**

*The plural of **index** is **indices**.*

1 When multiplying numbers with the same base, add the indices. For example: $3^6 \times 3^4 = 3^{6+4} = 3^{10}$

2 When dividing numbers with the same base, subtract the indices. For example: $3^6 \div 3^4 = 3^{6-4} = 3^2$

3 When raising a power of a number to a higher power, multiply the indices.
For example: $(3^6)^4 = 3^{6 \times 4} = 3^{24}$

*The **power** of a number is how many of that number are multiplied together.*

If we use letters to represent numbers, the rules can be generalised:

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$

EXAMPLE 1

Show by writing in expanded form that $m^4 \times m^3 = m^7$.

***Expand** means to spread out. Here it means by writing with multiplication signs.*

Solve	Think	Apply
$m^4 \times m^3 = m^7$	$m^4 \times m^3 = (m \times m \times m \times m) \times (m \times m \times m)$ $= m \times m \times m \times m \times m \times m \times m$ $= m^7$	Expand each term then write the answer in index form.

Exercise 2A

1 Complete the following.

a $m^3 \times m^4 = (m \times m \times m) \times (\underline{\qquad\quad})$

$= m \times m \times m \times \underline{\qquad\quad}$

$= \underline{\quad}$

b $m^5 \div m^3 = \dfrac{m \times m \times m \times m \times m}{\boxed{}}$

$= \dfrac{\boxed{}}{1}$

$= \underline{\quad}$

c $(m^2)^3 = (m \times m) \times (m \times m) \times (m \times m)$

$= \underline{\qquad\qquad\qquad\qquad\qquad}$

$= \underline{\quad}$

2 Show by writing in expanded form that:

a $m^2 \times m^4 = m^6$

b $m^6 \div m^2 = m^4$

c $(m^2)^4 = m^8$

EXAMPLE 2

a Use a calculator to evaluate these expressions when $a = 3$.

i $a^4 \times a^3$ **ii** a^7

b Does the value of $a^4 \times a^3$ equal the value of a^7?

		Solve/Think	Apply
a	**i**	$a^4 \times a^3 = 3^4 \times 3^3$ $= 81 \times 27$ $= 2187$	Substitute the value of the variable into each expression and evaluate using a calculator.
	ii	$a^7 = 3^7$ $= 2187$	
b		Yes, $a^4 \times a^3 = a^7$.	Compare the numerical answers.

3 **a** Use a calculator to complete the following when $a = 2$.

i $a^5 \times a^4 = 2^5 \times 2^4$
$= 32 \times$ ___
$=$ ___

ii $a^9 = 2^9$
$=$ ___

b Does the value of $a^5 \times a^4$ equal the value of a^9?

4 **a** Use a calculator to evaluate the following expressions when $m = 5$.

i $m^8 \div m^2$ **ii** m^6

b Does the value of $m^8 \div m^2$ equal the value of m^6?

5 **a** Use a calculator to evaluate the following expressions when $n = 3$.

i $(n^4)^2$ **ii** n^8

b Does the value of $(n^4)^2$ equal the value of n^8?

EXAMPLE 3

Use the index laws to simplify the following.

a $y^7 \times y^3$ **b** $y^{18} \div y^{17}$

c $(b^5)^{32}$

***Index** comes from the Latin word 'indicare': to point, disclose, show; as in using your index finger.*

	Solve	Think	Apply
a	$y^7 \times y^3 = y^{10}$	$y^7 \times y^3 = y^{7+3}$ $= y^{10}$	When multiplying powers with the same base, add the indices.
b	$y^{18} \div y^{17} = y^1 = y$	$y^{18} \div y^{17} = y^{18-17}$ $= y^1 = y$	When dividing powers with the same base, subtract the indices.
c	$(b^5)^3 = b^{15}$	$(b^5)^3 = b^{5 \times 3}$ $= b^{15}$	When raising a power of a number to a higher power, multiply the indices.

6 Complete the following using the index laws.

a $n^3 \times n^5 = n^{\square + \square}$
$= n^{\square}$

b $m^7 \div m^3 = m^{\square - \square}$
$= m^{\square}$

c $(k^2)^5 = k^{\square \times \square}$
$= k^{\square}$

7 Use the index laws to simplify the following.

a $m^3 \times m^6$ **b** $q^8 \times q^7$ **c** $t^{10} \times t^9$ **d** $b^{15} \times b \times b^4$ **e** $v \times v^5 \times v^7$

8 Use the index laws to simplify the following.

a $a^{12} \div a^{10}$ **b** $x^{15} \div x^5$ **c** $w^8 \div w^2$ **d** $b^6 \div b^5$ **e** $z^{20} \div z^{19}$

9 Use the index laws to simplify the following.

a $(b^4)^2$ **b** $(h^5)^3$ **c** $(k^8)^2$ **d** $(z^{10})^6$ **e** $(n^2)^4$

10 Use the index laws to simplify the following.

a $m^4 \times m^2$ **b** $x^9 \div x^6$ **c** $(b^4)^6$ **d** $m^3 \times m^6 \times m^4$ **e** $(v^7)^{10}$

f $n^8 \div n^7$ **g** $b^8 \div b$ **h** $(y^5)^5$ **i** $t^{10} \times t^{20} \times t$ **j** $a^{12} \div a^6$

EXAMPLE 4

Explain why the index laws cannot be used to simplify the following.

a $p^3 \times q^4$ **b** $m^6 \div n^4$

	Solve/Think	Apply
a	$p^3 \times q^4 = p \times p \times p \times q \times q \times q \times q$ $= p^3q^4$ As the bases are not the same, we cannot simplify further.	The index laws can only be used if the bases are the same.
b	$m^6 \div n^4 = \dfrac{m \times m \times m \times m \times m \times m}{n \times n \times n \times n}$ $= \dfrac{m^6}{n^4}$ Again, as the bases are not the same, we cannot simplify further.	

11 Explain why the index laws cannot be used to simplify the following.

a $k^5 \times m^3$ **b** $x^9 \div y^6$

12 Determine whether these statements are true or false.
If they are false, rewrite the answer to make them true.

a $b^4 \times b^3 = b^7$ **b** $m^5 \times m^2 = m^{10}$

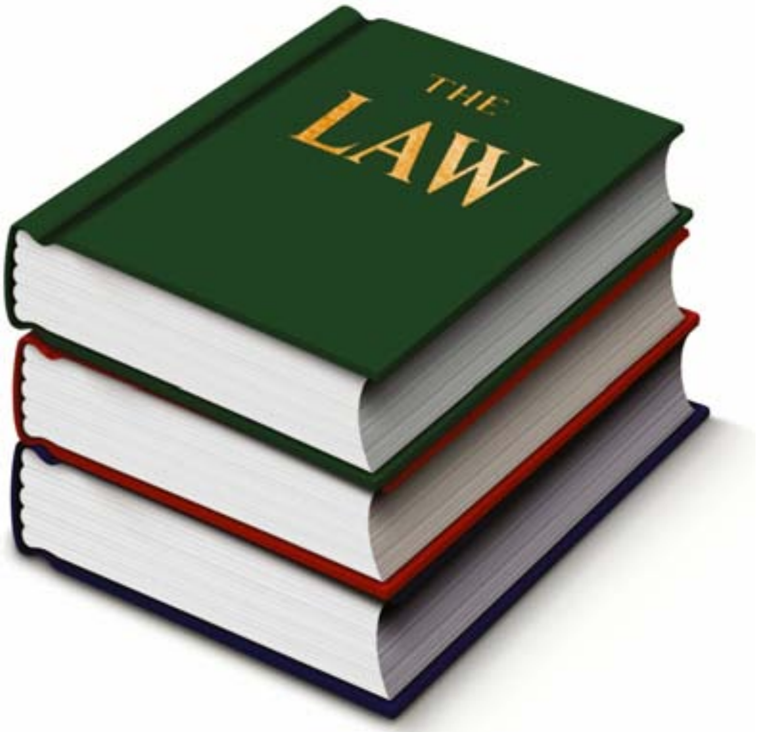
c $p^4 \times p^5 = p^{20}$ **d** $e^6 \times e^{10} = e^{16}$

e $a^4 \times b^5 = ab^9$ **f** $z^{10} \div z^2 = z^8$

g $p^{12} \div p^3 = p^4$ **h** $t^8 \div t^7 = t$

i $w^{15} \div w^3 = w^5$ **j** $\dfrac{p^6}{q^2} = \dfrac{p^4}{q}$

k $(b^7)^2 = b^{14}$ **l** $(n^{10})^3 = n^{13}$



B Applying the index laws

EXAMPLE 1

Simplify the following.

a $\dfrac{p^5 \times p^6}{p^8}$ **b** $\dfrac{(a^5)^4}{a^3 \times a^2}$

	Solve	Think	Apply
a	$\dfrac{p^5 \times p^6}{p^8} = p^3$	$\dfrac{p^5 \times p^6}{p^8} = \dfrac{p^{5+6}}{p^8}$ $= \dfrac{p^{11}}{p^8}$ $= p^{11-8} = p^3$	When multiplying powers with the same base, add the indices. When dividing, subtract the indices.
b	$\dfrac{(a^5)^4}{a^3 \times a^2} = \dfrac{a^{20}}{a^5}$ $= a^{15}$	$\dfrac{(a^5)^4}{a^3 \times a^2} = \dfrac{a^{5 \times 4}}{a^{3+2}}$ $= \dfrac{a^{20}}{a^5}$ $= a^{20-5} = a^{15}$	When raising a power to a higher power, multiply the indices.

Exercise 2B

1 Complete to simplify the following.

a $\dfrac{m^7 \times m^5}{m^8} = \dfrac{m^{\square}}{m^{\square}}$
$= m^{\square}$

b $\dfrac{(a^4)^6}{a^5 \times a^4} = \dfrac{a^{\square}}{a^{\square}}$
$= a^{\square}$

2 Simplify the following.

a $\dfrac{x^5 \times x^7}{x^6}$ **b** $\dfrac{w^3 \times w^{10}}{w^8}$ **c** $\dfrac{m^8 \times m^4}{m^{10}}$ **d** $\dfrac{k^{10} \times k^6}{k^8 \times k^5}$ **e** $\dfrac{a^7 \times a^6}{a^8 \times a^2}$

f $\dfrac{y^9 \times y^{11}}{y^{10} \times y^8}$ **g** $\dfrac{z^{16} \times z^2}{z^{10} \times z^7}$ **h** $\dfrac{x^{14}}{x^3 \times x^4}$ **i** $\dfrac{k^{30}}{k^{16} \times k^5}$ **j** $(m^2)^3 \times m^5$

k $(a^4)^5 \times (a^3)^4$ **l** $\dfrac{(t^5)^6}{t^{10}}$ **m** $\dfrac{(y^5)^5}{y^{20}}$ **n** $\dfrac{a^{16} \times a^6 \times a^4}{a^{12} \times a^8 \times a}$ **o** $\dfrac{b^{10} \times b^{20} \times b^{30}}{(b^4)^5}$

EXAMPLE 2

Simplify the following.

a $5m^4 \times 3m^6$ **b** $2k^7 \times 4k^3 \times 3k^5$

	Solve	Think	Apply
a	$5m^4 \times 3m^6 = 15m^{10}$	$5m^4 \times 3m^6 = 5 \times 3 \times m^4 \times m^6$ $= 15 \times m^{4+6} = 15m^{10}$	Multiply the numerical coefficients and use the index laws to multiply the pronumerals.
b	$2k^7 \times 4k^3 \times 3k^5 = 24k^{15}$	$2k^7 \times 4k^3 \times 3k^5 = 2 \times 4 \times 3 \times k^7 \times k^3 \times k^5$ $= 24 \times k^{7+3+5} = 24k^{15}$	

3 Complete the following.

$5t^7 \times 6t^4 = 5 \times 6 \times$ ___ $\times$ ___

$= 30 \times t^{\square} =$ ___

4 Simplify the following.

a $4m^5 \times 3m^7$　**b** $5p^4 \times 2p^6$　**c** $3t^8 \times 6t^4$

d $10a^{12} \times 7a^4$　**e** $4w^9 \times 6w^{10}$　**f** $5b^3 \times 6b^2 \times b^4$

g $3z^6 \times 4z^8 \times 2z^3$　**h** $2q^5 \times 5q^7 \times 8q^6$　**i** $d^4 \times 6d^6 \times 3d^8$

EXAMPLE 3

Simplify the following.

a $\dfrac{12m^8}{3m^6}$　**b** $\dfrac{20a^{10}}{16a^4}$

	Solve	Think	Apply
a	$\dfrac{12m^8}{3m^6} = 4m^2$	$\dfrac{12m^8}{3m^6} = \dfrac{12 \times m^8}{3 \times m^6}$ $= \dfrac{12}{3} \times \dfrac{m^8}{m^6}$ $= 4 \times m^2$ $= 4m^2$	Divide the numerical coefficients and use the index laws to divide the pronumerals. Be careful not to mix up the numerators and denominators.
b	$\dfrac{20a^{10}}{16a^4} = \dfrac{5a^6}{4}$	$\dfrac{20a^{10}}{16a^4} = \dfrac{20 \times a^{10}}{16 \times a^4}$ $= \dfrac{20}{16} \times \dfrac{a^{10}}{a^4}$ $= \dfrac{5}{4} \times a^6$ $= \dfrac{5a^6}{4}$	

5 Complete to simplify the following.

$$\frac{10a^7}{6a^4} = \frac{10 \times \square}{6 \times \square}$$
$$= \frac{10}{6} \times \frac{\square}{\square}$$
$$= \frac{5}{3} \times \square = \frac{\square}{3}$$

6 Simplify the following.

a $\dfrac{6m^7}{3m^2}$　**b** $\dfrac{10a^{12}}{5a^7}$

c $\dfrac{12w^{10}}{4w^8}$　**d** $\dfrac{8z^{12}}{6z^8}$

e $\dfrac{16k^9}{12k^3}$　**f** $\dfrac{9e^{10}}{6e^6}$

g $\dfrac{2m^8}{6m^3}$　**h** $\dfrac{6a^{15}}{12a^{10}}$

i $\dfrac{9t^{13}}{12t^6}$　**j** $\dfrac{15b^{11}}{20b^6}$

EXAMPLE 4

Simplify the following.

a $\frac{a^3}{a^7}$

b $\frac{10w^2}{8w^4}$

	Solve	Think	Apply
a	$\frac{a^3}{a^7} = \frac{1}{a^4}$	$\frac{a^3}{a^7} = \frac{{}^1\cancel{a} \times {}^1\cancel{a} \times {}^1\cancel{a}}{{}^1\cancel{a} \times {}^1\cancel{a} \times {}^1\cancel{a} \times a \times a \times a \times a}$ $= \frac{1}{a^4}$ Or $\frac{a^3}{a^7} = \frac{a^3 \div a^3}{a^7 \div a^3}$ $= \frac{1}{a^{7-3}} = \frac{1}{a^4}$	Write in expanded form and cancel or divide both the numerator and the denominator by the numerator.
b	$\frac{10w^2}{8w^4} = \frac{5}{4w^2}$	$\frac{10w^2}{8w^4} = \frac{10}{8} \times \frac{w^2}{w^4}$ $= \frac{5}{4} \times \frac{{}^1\cancel{w} \times {}^1\cancel{w}}{{}^1\cancel{w} \times {}^1\cancel{w} \times w \times w}$ $= \frac{5}{4} \times \frac{1}{w^2}$ $= \frac{5}{4w^2}$ Or $\frac{10w^2}{8w^4} = \frac{10}{8} \times \frac{w^2 \div w^2}{w^4 \div w^2}$ $= \frac{5}{4} \times \frac{1}{w^2} = \frac{5}{4w^2}$	Divide the numerical coefficients. Write the pronumerals in expanded form and cancel, or divide both the numerator and the denominator by the numerator.

7 Complete the following.

a $\frac{a^2}{a^5} = \frac{a \times a}{a \times a \times a \times a \times a}$

$= \frac{1}{\square}$

b $\frac{12y^4}{8y^5} = \frac{12}{8} \times \frac{y \times y \times y \times y}{y \times y \times y \times y \times y}$

$= \frac{\square}{\square} \times \frac{1}{\square}$

$=$ ____

8 Simplify the following.

a $\frac{m^4}{m^7}$ **b** $\frac{3k^2}{k^6}$ **c** $\frac{4p^3}{2p^5}$ **d** $\frac{5y^3}{15y^7}$ **e** $\frac{8z^2}{6z^7}$ **f** $\frac{12x^3}{18x^{10}}$

EXAMPLE 5

Simplify $(2a^3)^5$.

Solve	Think	Apply
$(2a^3)^5 = 32a^{15}$	$(2a^3)^5 = 2a^3 \times 2a^3 \times 2a^3 \times 2a^3 \times 2a^3$ $= 2 \times 2 \times 2 \times 2 \times 2 \times a^3 \times a^3 \times a^3 \times a^3 \times a^3$ $= 2^5 \times (a^3)^5$ $= 32 \times a^{15}$ $= 32a^{15}$	Expand, separate the numerical coefficients and the pronumerals and simplify using the index laws.

9 Complete to simplify the following.

$$\begin{aligned}(4b^2)^3 &= 4b^2 \times \_\_\_ \times \_\_\_\\ &= 4 \times 4 \times 4 \times \_\_\_ \times \_\_\_ \times \_\_\_\\ &= 4^3 \times (\_\_\_)^3\\ &= \_\_\_ \times b^{\square}\\ &= \_\_\_\end{aligned}$$

10 Simplify the following.

a $(3a^4)^3$ **b** $(2m^3)^6$ **c** $(7p^5)^2$ **d** $(10k^2)^4$

e $(5t^{11})^3$ **f** $(x^3y^2)^5$ **g** $(m^4n^6)^3$ **h** $(p^7q^3)^4$

i $(a^4b^{10})^2$ **j** $(2x^5y^2)^3$ **k** $(3x^7y^4)^3$ **l** $(5p^2q^4)^3$

EXAMPLE 6

Simplify the following.

a $5m^2n^4 \times 3m^5n^8$ **b** $\dfrac{12x^{10}y^8}{8x^6y^2}$

	Solve	Think	Apply
a	$5m^2n^4 \times 3m^5n^8$ $= 15m^7n^{12}$	$5m^2n^4 \times 3m^5n^8 = 5 \times 3 \times m^2 \times m^5 \times n^4 \times n^8$ $= 15 \times m^7 \times n^{12}$ $= 15m^7n^{12}$	Separate the terms and group together the numerical coefficients and the like terms. Use the index laws to simplify, where appropriate.
b	$\dfrac{12x^{10}y^8}{8x^6y^2} = \dfrac{3x^4y^6}{2}$	$\dfrac{12x^{10}y^8}{8x^6y^2} = \dfrac{12}{8} \times \dfrac{x^{10}}{x^6} \times \dfrac{y^8}{y^2}$ $= \dfrac{3}{2} \times x^4 \times y^6$ $= \dfrac{3x^4y^6}{2}$	

11 Complete to simplify the following.

a
$$\begin{aligned}4p^2q^3 \times 5p^3q &= 4 \times \_\_\_ \times \_\_\_ \times 5 \times \_\_\_ \times \_\_\_\\ &= 4 \times 5 \times p^2 \times p^3 \times \_\_\_ \times \_\_\_\\ &= \_\_\_ \times p^{\square} \times q^{\square}\\ &= \_\_\_\end{aligned}$$

b
$$\begin{aligned}\frac{15x^7y^5}{18x^4y^3} &= \frac{15}{18} \times \frac{x^7}{\square} \times \frac{\square}{\square}\\ &= \frac{5}{6} \times \_\_\_ \times \_\_\_\\ &= \_\_\_\end{aligned}$$

Remember to add indices when multiplying and subtract them when dividing.

12 Simplify the following.

a $4a^3b^2 \times 2a^5b^3$ **b** $5m^6n^7 \times 2m^4n$ **c** $3p^5q^8 \times 4p^6q^7$ **d** $10x^4y^3 \times 3x^6y$

e $2w^{10}z^{12} \times 6w^4z^5$ **f** $5a^2b^3c^4 \times 7ab^3c^2$ **g** $\dfrac{6a^5b^6}{4a^3b^2}$ **h** $\dfrac{15x^{10}y^9}{5x^6y^2}$

i $\dfrac{2k^7m^{12}}{10k^3m^6}$ **j** $\dfrac{9a^{11}b^6}{12a^8b}$ **k** $\dfrac{12m^7n^8}{15m^6n^8}$ **l** $\dfrac{5a^2b^3c^4}{7ab^3c^6}$

C The zero index

Any number raised to the power zero is equal to 1.

In general, **$a^0 = 1$**

$$1^0 = 5^0 = 127^0 = a^0 = 1$$

EXAMPLE 1

a Use the index laws to simplify $a^5 \div a^5$.
b Hence show that $a^0 = 1$.

	Solve	Think	Apply
a	$a^5 \div a^5 = a^0$	$a^5 \div a^5 = a^{5-5}$ $= a^0$	Use the index laws to divide one term by the other.
b	$a^5 \div a^5 = 1$ Hence $a^0 = 1$.	Any number divided by itself = 1.	Hence any number raised to the power zero is equal to 1.

Exercise 2C

1 **a** Use the index laws to simplify $a^4 \div a^4$.
b Hence show that $a^0 = 1$.

2 **a** Use the index laws to simplify $k^7 \div k^7$.
b Hence show that $k^0 = 1$.

EXAMPLE 2

Evaluate the following.

a x^0 **b** $(3x)^0$ **c** $3x^0$

	Solve	Think/Apply
a	$x^0 = 1$	Any number raised to the power zero is equal to 1.
b	$(3x)^0 = 1$	$3x = 3 \times x$ is a number. Any number raised to the power zero is equal to 1.
c	$3x^0 = 3$	$3x^0 = 3 \times x^0$ $= 3 \times 1$ $= 3$ The 3 is not to the power zero; only the x is to the zero power.

3 Evaluate the following.

a y^0 **b** $(3y)^0$ **c** $3y^0$ **d** $4k^0$ **e** $9t^0$
f $(6z)^0$ **g** $(10m)^0$ **h** $10m^0$ **i** $8b^0$ **j** $(7q)^0$
k $3m^0 + 1$ **l** $9e^0 - 3$ **m** $6p^0 + 7$ **n** $3a^0 + 2b^0$ **o** $6x^0 - 4y^0$

D Negative indices

EXAMPLE 1

Complete the table to find the meaning of 3^{-1}, 3^{-2}, 3^{-3}.

3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
27	9	3				

Solve	Think/Apply
3^3, 3^2, 3^1, 3^0, 3^{-1}, 3^{-2}, 3^{-3} 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9} = \frac{1}{3^2}$, $\frac{1}{27} = \frac{1}{3^3}$	Each number in the second row can be found by multiplying the number before it by $\frac{1}{3}$. Multiplying a number by $\frac{1}{3}$ is the same as dividing it by 3.

3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
27	9	3	1	$\frac{1}{3}$	$\frac{1}{9} = \frac{1}{3^2}$	$\frac{1}{27} = \frac{1}{3^3}$

Exercise 2D

1 Multiply the numbers in the second row by $\frac{1}{2}$ to complete the table. Hence find the meaning of 2^{-1}, 2^{-2}, 2^{-3}.

2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
8	$8 \times \frac{1}{2} = \underline{}$	$\underline{} \times \frac{1}{2} = \underline{}$				

Hence $2^{-1} = \frac{1}{\square} = \frac{1}{2^{\square}}$ $2^{-2} = \frac{1}{\square} = \frac{1}{2^{\square}}$ $2^{-3} = \frac{1}{\square} = \frac{1}{2^{\square}}$

2 Multiply the numbers in the second row by $\frac{1}{10}$ to complete the table and find the meaning of 10^{-1}, 10^{-2}, 10^{-3}.

10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
1000						

Hence $10^{-1} = \frac{1}{\square} = \frac{1}{10^{\square}}$ $10^{-2} = \frac{1}{\square} = \frac{1}{10^{\square}}$ $10^{-3} = \frac{1}{\square} = \frac{1}{10^{\square}}$

EXAMPLE 2

a Use the index laws to simplify $3^4 \div 3^6$.

b Write in expanded form and show that $3^4 \div 3^6 = \frac{1}{3^2}$.

c Hence show that $3^{-2} = \frac{1}{3^2}$.

	Solve/Think	Apply
a	$3^4 \div 3^6 = 3^{4-6} = 3^{-2}$	Simplify using the index laws and by writing in expanded form and cancelling. In general: $3^{-n} = \frac{1}{3^n}$
b	$3^4 \div 3^6 = \frac{{}^{1}\cancel{3} \times {}^{1}\cancel{3} \times {}^{1}\cancel{3} \times {}^{1}\cancel{3}}{{}^{1}\cancel{3} \times {}^{1}\cancel{3} \times {}^{1}\cancel{3} \times {}^{1}\cancel{3} \times 3 \times 3} = \frac{1}{3^2}$	
c	From parts **a** and **b**, $3^{-2} = \frac{1}{3^2}$.	

3 Complete the following.

Using the index laws to simplify, $2^2 \div 2^5 = 2^{\square - \square} = 2^{\square}$.

Writing in expanded form, $2^2 \div 2^5 = \dfrac{2 \times 2}{\square} = \dfrac{1}{2^{\square}}$.

Hence, $2^{-3} =$ ____.

4 **a** Use the index laws to simplify $5^3 \div 5^7$.

b By writing in expanded form, show that $5^3 \div 5^7 = \dfrac{1}{5^4}$.

c Hence show that $5^{-4} = \dfrac{1}{5^4}$.

5 Write the following with positive indices.

a 3^{-1} **b** 4^{-3} **c** 2^{-5} **d** 8^{-2} **e** 5^{-4}

f 12^{-1} **g** 9^{-2} **h** 6^{-1} **i** 7^{-3} **j** 3^{-6}

k 2^{-8} **l** 5^{-1} **m** 10^{-5} **n** 5^{-10} **o** 4^{-15}

EXAMPLE 3

Write the following as simplified fractions or mixed numerals.

a 5^{-2} **b** 3^{-5}

	Solve	Think	Apply
a	$5^{-2} = \dfrac{1}{25}$	$5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$	Write with a positive index then evaluate using a calculator if necessary.
b	$3^{-5} = \dfrac{1}{243}$	$3^{-5} = \dfrac{1}{3^5} = \dfrac{1}{243}$	

6 Complete the following.

a $2^{-3} = \dfrac{1}{2^{\square}} = \dfrac{1}{\square}$ **b** $5^{-1} = \dfrac{1}{5^{\square}} = \dfrac{1}{\square}$ **c** $2^{-4} = \dfrac{1}{2^{\square}} = \dfrac{1}{\square}$

7 Write the following as simplified fractions or mixed numerals.

a 3^{-2} **b** 2^{-5} **c** 4^{-3} **d** 5^{-4} **e** 2^{-10}

f 6^{-3} **g** 9^{-2} **h** 3^{-4} **i** 5^{-5} **j** 2^{-9}

k 7^{-3} **l** 4^{-4} **m** 3^{-6} **n** 10^{-4} **o** 9^{-2}

EXAMPLE 4

Write the following with negative indices.

a $\dfrac{1}{3}$ **b** $\dfrac{1}{3^2}$ **c** $\dfrac{1}{3^8}$

	Solve/Think	Apply
a	$\dfrac{1}{3} = \dfrac{1}{3^1} = 3^{-1}$	$3^{-n} = \dfrac{1}{3^n}$ is equivalent to $\dfrac{1}{3^n} = 3^{-n}$.
b	$\dfrac{1}{3^2} = 3^{-2}$	
c	$\dfrac{1}{3^8} = 3^{-8}$	

8 Write the following with negative indices.

a $\frac{1}{2}$

b $\frac{1}{2^2}$

c $\frac{1}{2^8}$

d $\frac{1}{2^5}$

e $\frac{1}{2^3}$

f $\frac{1}{5}$

g $\frac{1}{7^2}$

h $\frac{1}{4^3}$

i $\frac{1}{3^4}$

j $\frac{1}{5^6}$

k $\frac{1}{3^{10}}$

l $\frac{1}{6}$

m $\frac{1}{7^5}$

n $\frac{1}{4^9}$

o $\frac{1}{10}$

EXAMPLE 5

Write $\frac{1}{5^{-3}}$ with a positive index.

Solve/Think	Apply
$\frac{1}{5^{-3}} = \frac{1}{\frac{1}{5^3}}$ $= 1 \times \frac{5^3}{1} = 5^3$ Or $\frac{1}{5^{-3}} = \frac{5^0}{5^{-3}}$ $= 5^{0-(-3)} = 5^3$	Write 5^{-3} with a positive index and divide the fractions. Or write 1 as 5^0 and divide using the index laws. *To divide by a fraction, invert the fraction (turn it upside down) and multiply.*

9 Complete the following.

$\frac{1}{7^{-2}} = \frac{1}{\frac{1}{\square}} = 1 \times \frac{\square}{1} =$ ___ or $\frac{1}{7^{-2}} = \frac{7^0}{7^{-2}} = 7^{\square - (\square)} =$ ___

10 Write the following with positive indices.

a $\frac{1}{3^{-4}}$

b $\frac{1}{2^{-7}}$

c $\frac{1}{7^{-2}}$

d $\frac{1}{6^{-1}}$

e $\frac{1}{4^{-5}}$

EXAMPLE 6

Evaluate $\left(\frac{3}{7}\right)^{-1}$.

Solve/Think	Apply
$\left(\frac{3}{7}\right)^{-1} = \frac{1}{\frac{3}{7}}$ $= 1 \times \frac{7}{3} = \frac{7}{3}$ or $2\frac{1}{3}$	Write $\left(\frac{3}{7}\right)^{-1}$ with a positive index and divide the fractions.

11 Complete the following.

$\left(\frac{5}{8}\right)^{-1} = \frac{1}{\frac{\square}{\square}} = 1 \times \frac{\square}{\square} =$ ____

12 Evaluate the following.

a $\left(\frac{2}{3}\right)^{-1}$

b $\left(\frac{3}{4}\right)^{-1}$

c $\left(\frac{7}{8}\right)^{-1}$

d $\left(\frac{1}{5}\right)^{-1}$

e $\left(\frac{1}{10}\right)^{-1}$

f $\left(1\frac{1}{2}\right)^{-1}$

g $\left(2\frac{3}{4}\right)^{-1}$

13 Using the results of questions **11** and **12**, simplify $\left(\frac{a}{b}\right)^{-1}$.

E Negative indices with variables

A **negative index** means to **invert** the number (turn it upside-down).

In general, $a^{-n} = \frac{1}{a^n}$

EXAMPLE 1

a Use the index laws to simplify $\frac{a^4}{a^5}$.

b Expand and simplify $\frac{a^4}{a^5}$.

c Hence show that $a^{-1} = \frac{1}{a}$.

	Solve/Think	Apply
a	$\frac{a^4}{a^5} = a^{4-5} = a^{-1}$	Simplify using the index laws and by writing in expanded form and cancelling.
b	$\frac{a^4}{a^5} = \frac{{}^1\cancel{a}\times{}^1\cancel{a}\times{}^1\cancel{a}\times{}^1\cancel{a}}{{}^1\cancel{a}\times{}^1\cancel{a}\times{}^1\cancel{a}\times{}^1\cancel{a}\times a} = \frac{1}{a}$	
c	Since $\frac{a^4}{a^5} = a^{-1}$ and $\frac{a^4}{a^5} = \frac{1}{a}$, then $a^{-1} = \frac{1}{a}$.	

Exercise 2E

1 Complete the following.
Using the index laws to simplify, $\frac{a^3}{a^4} = a^{\square - \square} =$ ____.
Expanding and simplifying, $\frac{a^3}{a^4} = \frac{a \times a \times a}{\boxed{}} =$ ____. Hence, $a^{-1} =$ ____.

2 **a** Use the index laws to simplify $\frac{a^3}{a^5}$.
b Expand and simplify $\frac{a^3}{a^5}$.
c Hence show that $a^{-2} = \frac{1}{a^2}$.

3 **a** Use the index laws to simplify $\frac{a^2}{a^5}$.
b Expand and simplify $\frac{a^2}{a^5}$.
c Hence show that $a^{-3} = \frac{1}{a^3}$.

4 **a** Use the index laws to simplify $\frac{a^2}{a^6}$.
b Expand and simplify $\frac{a^2}{a^6}$.
c Hence show that $a^{-4} = \frac{1}{a^4}$.

5 **a** Use the index laws to simplify $\frac{a}{a^6}$.
b Expand and simplify $\frac{a}{a^6}$.
c Hence show that $a^{-5} = \frac{1}{a^5}$.

6 Complete: From questions **1** to **5** it can be seen that, in general, $a^{-n} =$ ____.

EXAMPLE 2

Write the following with positive indices.

a k^{-9} **b** m^{-15}

	Solve/Think	Apply
a	$k^{-9} = \frac{1}{k^9}$	In general $a^{-n} = \frac{1}{a^n}$.
b	$m^{-15} = \frac{1}{m^{15}}$	

7 Write the following with positive indices.

a y^{-2} **b** k^{-1} **c** m^{-3} **d** x^{-6} **e** t^{-10}

EXAMPLE 3

Write the following with negative indices.

a $\frac{1}{a^5}$ **b** $\frac{1}{y^7}$

	Solve/Think	Apply
a	$\frac{1}{a^5} = a^{-5}$	$a^{-n} = \frac{1}{a^n}$ is equivalent to $\frac{1}{a^n} = a^{-n}$.
b	$\frac{1}{y^7} = y^{-7}$	

8 Write the following with negative indices.

a $\frac{1}{a^8}$ **b** $\frac{1}{k^2}$ **c** $\frac{1}{x^{11}}$ **d** $\frac{1}{n^{14}}$ **e** $\frac{1}{z^{20}}$

EXAMPLE 4

Write the following with positive indices.

a $3m^{-2}$ **b** $(3m)^{-2}$

	Solve	Think	Apply
a	$3m^{-2} = \frac{3}{m^2}$	$3m^{-2} = 3 \times m^{-2}$ $= \frac{3}{1} \times \frac{1}{m^2}$ $= \frac{3}{m^2}$	First express the term as a product, then write it with a positive index.
b	$(3m)^{-2} = \frac{1}{9m^2}$	$(3m)^{-2} = \frac{1}{(3m)^2}$ $= \frac{1}{9m^2}$	First write the term with a positive index, then simplify.

9 Write the following with positive indices.

a $3k^{-1}$ **b** $(3k)^{-1}$ **c** $2y^{-5}$ **d** $(2y)^{-5}$ **e** $3t^{-4}$ **f** $(3t)^{-4}$

EXAMPLE 5

Write the following with negative indices.

a $\frac{1}{m^4}$ **b** $\frac{3}{m^4}$ **c** $\frac{1}{3m^4}$

	Solve/Think	Apply
a	$\frac{1}{m^4} = m^{-4}$	Write as a product, then use $\frac{1}{a^n} = a^{-n}$.
b	$\frac{3}{m^4} = 3 \times \frac{1}{m^4}$ $= 3 \times m^{-4}$ $= 3m^{-4}$	
c	$\frac{1}{3m^4} = \frac{1}{3} \times \frac{1}{m^4}$ $= \frac{1}{3} \times m^{-4}$ $= \frac{1}{3}m^{-4}$ or $\frac{m^{-4}}{3}$	

10 Write the following with negative indices.

a $\frac{1}{k^3} = k^{\square}$

b $\frac{2}{k^3} = \underline{\quad\quad} \times \frac{1}{k^3}$
$= \underline{\quad\quad} \times k^{\square}$
$= \underline{\quad\quad}$

c $\frac{1}{2k^3} = \underline{\quad\quad} \times \frac{1}{k^3}$
$= \underline{\quad\quad} \times k^{\square}$
$= \underline{\quad\quad}$ or $\underline{\quad\quad}$

11 Write the following with negative indices.

a **i** $\frac{1}{p^5}$ **ii** $\frac{7}{p^5}$ **iii** $\frac{1}{7p^5}$

b **i** $\frac{1}{m^{10}}$ **ii** $\frac{6}{m^{10}}$ **iii** $\frac{1}{6m^{10}}$

c **i** $\frac{1}{y^7}$ **ii** $\frac{4}{y^7}$ **iii** $\frac{1}{4y^7}$

F Further use of the index laws

EXAMPLE 1

Use the index laws to simplify the following.

a $m^{-6} \times m^2$ **b** $q^{-2} \div q^{-7}$ **c** $(x^{-3})^5$

	Solve	Think	Apply
a	$m^{-6} \times m^2 = m^{-4}$	$m^{-6} \times m^2 = m^{-6+2} = m^{-4}$	When multiplying add the indices.
b	$q^{-2} \div q^{-7} = q^5$	$q^{-2} \div q^{-7} = q^{-2-(-7)}$ $= q^{-2+7} = q^5$	When dividing subtract the indices.
c	$(x^{-3})^5 = x^{-15}$	$(x^{-3})^5 = x^{-3 \times 5}$ $= x^{-15}$	When raising a power to another power, multiply the indices.

Exercise 2F

1 Use the index laws to simplify the following.

a $a^{-5} \times a^{-2}$ **b** $y^{-3} \times y^{7}$ **c** $e^{5} \times e^{-7}$ **d** $n^{4} \times n^{-3}$ **e** $b^{-6} \div b^{2}$

f $w^{3} \div w^{-2}$ **g** $z^{-2} \div z^{-4}$ **h** $k^{-6} \div k^{-2}$ **i** $(y^{-2})^{4}$ **j** $(t^{5})^{-4}$

EXAMPLE 2

Simplify the following.

a $5m^{-3} \times 6m^{7}$ **b** $4y^{7} \div 5y^{-2}$ **c** $(5y^{-2})^{3}$

	Solve	Think	Apply
a	$5m^{-3} \times 6m^{7} = 30m^{4}$	$5m^{-3} \times 6m^{7} = 5 \times 6 \times m^{-3} \times m^{7}$ $= 30 \times m^{-3+7}$ $= 30 \times m^{4}$ $= 30m^{4}$	Separate the terms and group together the numerical coefficients and the like terms. Use the index laws to simplify, where appropriate.
b	$4y^{7} \div 5y^{-2} = \frac{4}{5}y^{9}$ or $\frac{4y^{9}}{5}$	$4y^{7} \div 5y^{-2} = \frac{4y^{7}}{5y^{-2}}$ $= \frac{4}{5} \times \frac{y^{7}}{y^{-2}}$ $= \frac{4}{5} \times y^{7-(-2)}$ $= \frac{4}{5} \times y^{9}$ $= \frac{4}{5}y^{9}$ or $\frac{4y^{9}}{5}$	
c	$(5y^{-2})^{3} = 125y^{-6}$	$(5y^{-2})^{3} = 5y^{-2} \times 5y^{-2} \times 5y^{-2}$ $= 5^{3} \times (y^{-2})^{3}$ $= 125y^{-6}$	

2 Complete the following.

a
$$\begin{aligned} 3a^{5} \times 4a^{-2} &= 3 \times 4 \times a^{5} \times a^{-2} \\ &= 12 \times a^{\square + (\square)} \\ &= \_\_\_ \times a^{\square} \\ &= \_\_\_ \end{aligned}$$

b
$$\begin{aligned} 3x^{2} \div 4x^{-3} &= \frac{3x^{2}}{4x^{-3}} \\ &= \frac{3}{4} \times \frac{x^{2}}{x^{-3}} \\ &= \_\_\_ \times x^{\square - (\square)} \\ &= \_\_\_ \times x^{\square} \\ &= \_\_\_ \text{ or } \_\_\_ \end{aligned}$$

c
$$\begin{aligned} (7m^{-3})^{2} &= 7m^{-3} \times 7m^{-3} \\ &= 7 \times 7 \times m^{-3} \times m^{-3} \\ &= \_\_\_ \times (m^{\square})^{\square} \\ &= \_\_\_ \times m^{\square} \\ &= \_\_\_ \end{aligned}$$

3 Simplify the following.

a $10a^{5} \times 9a^{-3}$ **b** $6b^{-5} \times 3b^{-2}$ **c** $3v^{-6} \times 2v^{2}$ **d** $8y^{5} \div 2y^{-1}$

e $6p^{-4} \div 2p^{2}$ **f** $3k^{-4} \div 8k^{-2}$ **g** $(5z^{-4})^{3}$ **h** $(2m^{-3})^{5}$

i $(3w^{-6})^{2}$ **j** $4n^{-3} \times 3n^{-4} \div 6n^{-5}$ **k** $(3x)^{-2}$ **l** $(5m^{2})^{-3}$

EXAMPLE 3

State whether the following are true or false.

a $m^3 \div m^5 = m^2$

b $3y^0 = 1$

c $6k^4 \div 2k^4 = 3$

d $2p^{-3} = \dfrac{1}{2p^3}$

e $x^{-3} = -3x$

	Solve	Think	Apply
a	Statement is false.	$m^3 \div m^5 = m^{3-5}$ $= m^{-2}$	Use the index laws, the results for the zero index and negative indices.
b	Statement is false.	$3y^0 = 3 \times y^0$ $= 3 \times 1 = 3$	
c	Statement is true.	$6k^4 \div 2k^4 = \dfrac{6}{2} \times \dfrac{k^4}{k^4}$ $= 3 \times k^{4-4}$ $= 3 \times k^0$ $= 3 \times 1 = 3$	
d	Statement is false.	$2p^{-3} = 2 \times p^{-3}$ $= 2 \times \dfrac{1}{p^3}$ $= \dfrac{2}{p^3}$	
e	Statement is false.	$x^{-3} = \dfrac{1}{x^3}$ $-3x = -3 \times x$	

4 State whether the following are true or false.

a $6m^0 = 1$

b $a^4 \div a^7 = a^3$

c $8t^9 \div 2t^9 = 4$

d $3c^{-2} = \dfrac{1}{3c^2}$

e $4k^0 = 4$

f $b \div b^6 = b^5$

g $5x^6 \div x^6 = 5x$

h $4y^{-3} = \dfrac{4}{y^3}$

i $(2p^{-1})^3 = \dfrac{8}{p^3}$

j $x^{-2} = -2x$

k $x^{-4} = \dfrac{-4}{x}$

l $(2x^{-1})^{-2} = \dfrac{x^2}{4}$

EXAMPLE 4

By substituting $a = 5$, show that $a^{-2} \neq -2a$.

Solve	Think	Apply
If $a = 5$, $a^{-2} = 5^{-2} = \dfrac{1}{25}$ and $-2a = -10$ Hence $a^{-2} \neq -2a$.	$5^{-2} = \dfrac{1}{5^2}$ and $-2a = -2 \times a$	Evaluate each expression by substituting the value of the variable.

5 By substituting $a = 3$, show the following.

a $a^2 \neq 2a$

b $a^3 \neq 3a$

c $a^{-2} \neq \dfrac{-2}{a}$

d $a^{-3} \neq \dfrac{-3}{a}$

e $a^2 \times a \neq a^2 + a$

f $a^2 + a^2 \neq a^4$

g $a^2 - a^2 \neq a^0$

h $5a^2 \times 3a \neq 5a^2 + 3a$

G Removing grouping symbols

EXAMPLE 1

Expand the following.

a $3(x + 5)$ **b** $4(3y - z)$

Expand *means to write the expression without the grouping symbols.*

	Solve	Think	Apply
a	$3(x + 5) = 3x + 15$	$3(x + 5)$ $= (x + 5) + (x + 5) + (x + 5)$ $= x + x + x + 5 + 5 + 5$ $= 3 \times x + 3 \times 5$ $= 3x + 15$	Use $a \times (b + c)$ $= a \times b + a \times c$.
b	$4(3y - z) = 12y - 4z$	$4(3y - z)$ $= (3y - z) + (3y - z) + (3y - z) + (3y - z)$ $= 3y + 3y + 3y + 3y - z - z - z - z$ $= 4 \times 3y + 4 \times (-z)$ $= 12y + (-4z)$ $= 12y - 4z$	*Adding $-4z$ is the same as subtracting $4z$.*

From Example 1 we can see that, in general:

$$a \times (b + c) = a \times b + a \times c$$

To remove grouping symbols, multiply each term inside them by the term (number and/or pronumeral) at the front. This is known as the **distributive law**.

EXAMPLE 2

Use the distributive law to expand the following.

a $5(2y + 3)$ **b** $7(3y - 4w)$

	Solve	Think	Apply
a	$5(2y + 3) = 10y + 15$	$5(2y + 3) = 5 \times 2y + 5 \times 3$ $= 10y + 15$	Use $a(b + c) = a \times b + a \times c$.
b	$7(3y - 4w) = 21y - 28w$	$7(3y - 4w) = 7 \times 3y + 7 \times (-4w)$ $= 21y + (-28w)$	

Exercise 2G

1 Complete to expand the following.

a $4(2x + 5) = \_\_\_ \times 2x + \_\_\_ \times 5$
$= \_\_\_ + \_\_\_$

b $3(4a - 2b) = 3 \times \_\_\_ + 3 \times (-2b)$
$= \_\_\_ + (\_\_\_)$
$= \_\_\_ - \_\_\_$

2 Use the distributive law to expand the following.

a $3(2w + 5)$
b $6(3z - 2)$
c $5(4a + 3b)$
d $2(4x - 3y)$
e $10(z^2 + 6)$
f $7(ab - 2a^2)$
g $4(m^2 + n^2)$
h $2(m^3 - 3mn)$
i $5(4b + 2a + 3)$
j $3(5x - 3y - 2z)$
k $13(2 - x)$
l $5(3a + b - 7c)$

EXAMPLE 3

Expand the following.

a $3w(2y + 4z)$
b $2a(3a - 4b)$
c $4m^2(m^3 + 2m^5)$

	Solve	Think	Apply
a	$3w(2y + 4z) = 6wy + 12wz$	$3w(2y + 4z) = 3w \times 2y + 3w \times 4z$ $= 6wy + 12wz$	Use $a(b + c)$ $= a \times b + a \times c$ and the index laws.
b	$2a(3a - 4b) = 6a^2 - 8ab$	$2a(3a - 4b) = 2a \times 3a - 2a \times 4b$ $= 6a^2 - 8ab$	
c	$4m^2(m^3 + 2m^5) = 4m^5 + 8m^7$	$4m^2(m^3 + 2m^5) = 4m^2 \times m^3 + 4m^2 \times 2m^5$ $= 4m^5 + 8m^7$	

3 Expand the following.

a $3a(2b + 4c)$
b $4x(3x - 2y)$
c $10k(6k - 4m)$
d $m(m^2 + 2)$
e $6x(2y - 5x^2)$
f $3k^2(2k^2 + 5)$
g $a^3(5a^2 - 2)$
h $2p^5(p^2 + 3p^3)$

EXAMPLE 4

Expand the following.

a $-3(2w + 5)$
b $-2(4a - 3b)$
c $-(4m + 3n)$

	Solve	Think	Apply
a	$-3(2w + 5) = -6w - 15$	$-3(2w + 5)$ $= -3 \times 2w + (-3) \times 5$ $= -6w + (-15)$ $= -6w - 15$	Use $a(b + c) = a \times b + a \times c$.
b	$-2(4a - 3b) = -8a + 6b$	$-2(4a - 3b)$ $= -2 \times 4a + (-2) \times (-3b)$ $= -8a + 6b$	
c	$-(4m + 3n) = -4m - 3n$	$-(4m + 3n)$ $= -1 \times 4m + (-1) \times 3n$ $= -4m + (-3n)$ $= -4m - 3n$	

4 Complete the following.

a $-2(3x + 7) = -2 \times$ ___ $+ (-2) \times$ ___
$=$ ___ $+$ (___)
$=$ ___ $-$ ___

b $-5(6a - 3b) = -5 \times$ ___ $+ (-5) \times$ (___)
$=$ ___ $+$ ___

5 Expand the following.

a $-2(y + 3)$ **b** $-5(a + 2)$ **c** $-3(w + 4)$ **d** $-4(m - 7)$ **e** $-(t + 3)$

f $-(b + 6)$ **g** $-3(2k + 5)$ **h** $-2(4m - 5)$ **i** $-(7w + 3)$ **j** $-(4x - 1)$

EXAMPLE 5

Expand the following.

a $-3a(5a^2 + 2ab)$ **b** $-n^3(2n^4 - 5n^2p)$

	Solve	Think	Apply
a	$-3a(5a^2 + 2ab)$ $= -15a^3 - 6a^2b$	$-3a(5a^2 + 2ab)$ $= -3a \times 5a^2 + (-3a) \times 2ab$ $= -15a^3 + (-6a^2b)$ $= -15a^3 - 6a^2b$	Use $a(b + c) = a \times b + a \times c$ with the index laws.
b	$-n^3(2n^4 - 5n^2p)$ $= -2n^7 + 5n^5p$	$-n^3(2n^4 - 5n^2p)$ $= -n^3 \times 2n^4 + (-n^3) \times (-5n^2p)$ $= -2n^7 + 5n^5p$	

6 Complete the following.

a $-4x^2(3x^3 + 2xy)$

$= -4x^2 \times \_\_\_ + (-4x^2) \times \_\_\_\_$

$= \_\_\_ + (\_\_\_)$

$= \_\_\_ - \_\_\_$

b $-a^2(5a^2 - 4ab)$

$= -a^2 \times \_\_\_ + (-a^2) \times (\_\_\_)$

$= \_\_\_ + \_\_\_$

7 Expand the following.

a $-2a(3a^2 + 2ab)$ **b** $-4x(2x^2 - 3xy)$ **c** $-3p^2(3p^2 + 4pq)$ **d** $-y^3(4y^2 - 3xy)$

e $-3m^4(2m^2 + 5mn)$ **f** $-y^2(y^3 - 4)$ **g** $-5x^2(2x^3 - 3xy)$ **h** $-t(mt^2 + t)$

EXAMPLE 6

Expand and simplify by collecting like terms.

a $3(a + 2) + 7$ **b** $3 + 2(3n - 5)$

__Like terms__ are terms that have the same power(s) of the same base(s). For example, x^2 and $3x^2$ are like terms, but x^2 and $3x$ are not.

	Solve	Think	Apply
a	$3(a + 2) + 7$ $= 3a + 6 + 7$ $= 3a + 13$	$3(a + 2) + 7 = 3 \times a + 3 \times 2 + 7$ $= 3a + 6 + 7$ $= 3a + 13$	Use $a(b + c)$ $= a \times b + a \times c.$
b	$3 + 2(3n - 5)$ $= 3 + 6n - 10$ $= 6n - 7$	$3 + 2(3n - 5) = 3 + 2 \times 3n - 2 \times (-5)$ $= 3 + 6n - (-10)$ $= 6n + 3 - 10$	Do the multiplication using the distributive law before the addition.

Remember to remove grouping symbols first.

8 Complete the following.

a $4(y + 5) - 3 = 4 \times \_\_\_ + 4 \times \_\_\_ - 3$

$= \_\_\_ + \_\_\_ - 3$

$= \_\_\_ + \_\_\_$

b $11 + 3(2x - 7) = 11 + 3 \times \_\_\_ + 3 \times (\_\_\_)$

$= 11 + \_\_\_ - \_\_\_$

$= \_\_\_ + 11 - \_\_\_$

$= \_\_\_ - \_\_\_$

9 Expand and simplify the following.

a $4(a + 3) + 6$ **b** $2(3b - 12) + 12$ **c** $3(4w + 2) - 7$

d $5(2y - 3) - 2$ **e** $6(3z - 1) + 4$ **f** $10 + 2(4x + 3)$

g $12 + 2(3b - 5)$ **h** $13 + 4(y + 5)$ **i** $4 + 3(2w - 4)$

EXAMPLE 7

Expand and simplify by collecting like terms.

a $5 - 2(4y - 3)$ **b** $-4(3x + 1) - 6$

	Solve	Think	Apply
a	$5 - 2(4y - 3) = 5 - 8y + 6$ $= -8y + 11$	$5 - 2(4y - 3)$ $= 5 - [2(4y - 3)]$ $= 5 - (2 \times 4y - 2 \times 3)$ $= 5 - (8y - 6)$ $= 5 - 8y -(-6)$ $= 5 - 8y + 6$ $= -8y + 11$	Do the multiplication using the distributive law before the subtraction.
b	$-4(3x + 1) - 6 = -12x - 4 - 6$ $= -12x - 10$	$-4(3x + 1) - 6$ $= -4 \times 3x + (-4) \times 1 - 6$ $= -12x - 4 - 6$ $= -12x - 10$	Use $a(b + c)$ $= a \times b + a \times c.$

10 Complete the following.

a $9 - 3(y - 2) = 9 - 3 \times y + (\_\_\_) \times (\_\_\_)$

$= \_\_\_ - 3y + \_\_\_$

$= -3y + \_\_\_ + \_\_\_$

$= \_\_\_ + \_\_\_$

b $-5(3x + 2) + 8 = -5 \times 3x + (-5) \times 2 + 8$

$= -15x - \_\_\_ + \_\_\_$

$= \_\_\_ - \_\_\_$

11 Expand and simplify the following.

a $12 - 2(a + 5)$ **b** $8 - 3(y - 2)$ **c** $9 - 4(b + 3)$

d $7 - 2(v - 6)$ **e** $20 - 3(2w + 5)$ **f** $2 - 5(3t - 4)$

g $4 - 3(5x + 2)$ **h** $10 - 2(3k - 1)$ **i** $5 - 3(3 + 4z)$

EXAMPLE 8

Expand and simplify the following.

a $3(x + 2) + 2(x - 4)$ **b** $4(2m - 3) + 3(m - 2)$

	Solve/Think	Apply
a	$3(x + 2) + 2(x - 4) = 3x + 6 + 2x - 8$ $= 3x + 2x + 6 - 8$ $= 5x - 2$	Expand using the distributive law and then collect like terms.
b	$4(2m - 3) + 3(m - 2) = 8m - 12 + 3m - 6$ $= 8m + 3m - 12 - 6$ $= 11m - 18$	

12 Complete the following.

a $3(4k - 2) + 5(k + 3) = 12k - \_\_\_ + 5k + \_\_\_$
$= 12k + 5k - \_\_\_ + \_\_\_$
$= 17k + \_\_\_$

b $4(3y - 5) + 2(5y - 1) = 12y - \_\_\_ + \_\_\_ - 2$
$= 12y + \_\_\_ - \_\_\_ - 2$
$= \_\_\_ - \_\_\_$

13 Expand and simplify the following.

a $5(2k + 3) + 3(k - 2)$
b $2(6m + 7) + 3(m - 1)$
c $4(2p - 1) + 2(3p + 5)$
d $3(3a + 2) + 4(a - 3)$
e $2(5x - 3) + 5(3x - 1)$
f $3(4y - 2) + (2y + 7)$
g $(6v - 1) + 3(2v - 5)$
h $4(3x + 2y) + 2(5x - 3y)$
i $7(2a - 3b) + 3(3a + 4b)$

EXAMPLE 9

Expand and simplify the following.

a $2(3p + 4q) - 4(2p - 3q)$
b $3(4m - 1) - (m + 4)$

	Solve/Think	Apply
a	$2(3p + 4q) - 4(2p - 3q) = 6p + 8q - 8p + 12q$ $= 6p - 8p + 8q + 12q$ $= -2p + 20q$	Expand using the distributive law and then collect like terms.
b	$3(4m - 1) - (m + 4) = 12m - 3 - m - 4$ $= 12m - m - 3 - 4$ $= 11m - 7$	

14 Complete the following.

a $3(2x + 4y) - 2(4x - 5y) = 6x + \_\_\_ - 8x + \_\_\_$
$= 6x - 8x + \_\_\_ + \_\_\_$
$= -2x + \_\_\_$

b $2(x - 5) - (x + 4) = 2x - \_\_\_ - x - \_\_\_$
$= 2x - x - \_\_\_ - \_\_\_$
$= x - \_\_\_$

15 Expand and simplify the following.

a $3(2k + 5) - 2(k + 3)$
b $5(w + 4) - 3(w - 2)$
c $2(6t + 1) - 3(t + 4)$
d $3(5z - 1) - (2z + 5)$
e $2(a + 5) - 4(a - 1)$
f $5(d - 3) - 3(2d + 1)$
g $4(3x + y) - (2x - 7y)$
h $3(2a - 3b) - (2a + 3b)$
i $2(q - 5) - 4(q - 5)$

Language in mathematics

1 Write the following in words.

a 3^5 **b** x^2 **c** 7^0 **d** x^{-3}

2 Write expressions for:

a seven squared **b** y cubed

c 6 to the power 5 **d** m to the fourth

3 Replace the missing vowels to make words that mean the same as 'power'.

a _nd_x **b** _xp_n_nt

4 When writing numbers in index notation, what name is given to:

a the number that is being repeated? **b** how many of this number are multiplied together?

5 Explain the following.

a $m^3 \times m^2 \neq m^6$ **b** $(x^3)^2 \neq x^5$ **c** $y^0 = 1$

d $a^{-2} \neq -2a$ **e** $3(k + 5) \neq 3k + 5$ **f** $5 + 4(n + 2) \neq 9(n + 2)$

6 Explain the difference between $3p^0$ and $(3p)^0$.

Terms

base
grouping
notation
distributive
index law
power
evaluate
indices
simplify
expand
invert
symbol
exponent
like terms
variable
expression
negative
zero

Check your skills

1 Which of the following does *not* simplify to a^8?

A $a^{16} \div a^8$ **B** $(a^2)^4$ **C** $a^7 \times a$ **D** $a^{16} \div a^2$

2 $5m^3n^2 \times 4m^4n^3 =$

A $20m^{12}n^{16}$ **B** $20m^7n^5$ **C** $9m^{12}n^6$ **D** $9m^7n^5$

3 $\dfrac{4a^2b^2 \times 2a^5b^{10}}{6a^4b^8} =$

A $\dfrac{4a^6b^{12}}{3}$ **B** $7a^6b^{12}$ **C** $\dfrac{4a^3b^4}{3}$ **D** $7a^3b^4$

4 $(3m^7)^3 =$

A $3m^{21}$ **B** $3m^{10}$ **C** $27m^{21}$ **D** $27m^{10}$

5 $\dfrac{x^5}{4x^8} =$

A $\dfrac{4}{x^3}$ **B** $\dfrac{1}{4x^3}$ **C** $4x^3$ **D** $\dfrac{x^3}{4}$

6 $5m^0 - 1 =$

A -1 **B** 0 **C** 4 **D** 5

7 $(5x)^0 =$

A 0 **B** 1 **C** 5 **D** $5x$

8 3^{-4} is the same as:

A $\frac{1}{4^3}$ **B** $\frac{1}{3^4}$ **C** -12 **D** $\frac{3}{4}$

9 Which of the following statements is *not* correct?

A $\left(1\frac{1}{2}\right)^{-1} = \frac{2}{3}$ **B** $\frac{1}{2^{-3}} = 2^3$ **C** $5p^{-2} = \frac{5}{p^2}$ **D** $2^5 \div 2^{-3} = 1^8$

10 $(5y)^{-3}$ simplifies to:

A $\frac{y^3}{125}$ **B** $\frac{125}{y^3}$ **C** $\frac{1}{125y^3}$ **D** $\frac{5}{y^3}$

11 $5p^3q^{-7} \times 2p^{-2}q^{-3} =$

A $7pq^{-10}$ **B** $10pq^{-10}$ **C** $7p^5q^{-4}$ **D** $10p^{-6}q^{21}$

12 $12k^{-2}m^3 \div 4k^{-5}m^2 =$

A $3k^{-7}m^5$ **B** $\frac{k^3}{3m}$ **C** $\frac{k^3m}{3}$ **D** $3k^3m$

13 $-5(4 - 3t) =$

A $-20 - 15t$ **B** $-20 + 15t$ **C** $-20 - 8t$ **D** $-20 + 8t$

14 $3x^2(7x - 2y) =$

A $21x^3 - 6x^2y$ **B** $21x^3 + 6x^2y$ **C** $10x^3 - 5x^2y$ **D** $21x^3 + 5x^2y$

15 $7 - 3(2x - 5) =$

A $8x - 20$ **B** $12 - 6x$ **C** $2 - 6x$ **D** $22 - 6x$

16 $3(2a + 3) - 4(a - 2) =$

A $2a + 17$ **B** $2a + 1$ **C** $10a + 17$ **D** $10a + 1$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2–5	6, 7	8	9, 10	11, 12	13–16
Section	A	B	C	D	E	F	G

2A Review set

1 Simplify:

a $y^{10} \times y^7$ **b** $k^{11} \div k^5$ **c** $(p^7)^2$ **d** $\frac{t^7 \times t^8}{t^3 \times t^4}$

e $(5m^4)^3$ **f** $3a^5b^3 \times 2ab^6$ **g** $\frac{5m^{10}n^8}{10m^3n \times m^5n^2}$

2 Simplify, writing the answers with positive indices.

a $\frac{a^4}{a^9}$ **b** $\frac{2a^4}{a^9}$ **c** $\frac{a^4}{2a^9}$ **d** $\frac{6a^4}{15a^9}$

3 Evaluate:

a v^0 **b** $5v^0$ **c** $(5v)^0$ **d** $2v^0 + 1$

4 Evaluate:

a $\frac{1}{2^{-3}}$ **b** $\left(\frac{3}{4}\right)^{-1}$ **c** $\left(\frac{9}{4}\right)^{-1}$

5 Write the following with positive indices.

a z^{-3} **b** $2z^{-3}$ **c** $(2z)^{-3}$

6 Write the following with negative indices.

a $\frac{1}{k^7}$ **b** $\frac{2}{k^7}$ **c** $\frac{1}{2k^7}$

7 Simplify:

a $y^{-3} \times y^5$ **b** $e^6 \div e^{-2}$ **c** $(n^{-4})^5$ **d** $6b^{-2} \times 3b^7$ **e** $4k^{-5} \div 2k^{-3}$

8 State whether the following are true or false.

a $4q^0 = 4$ **b** $a^5 \div a^7 = a^2$ **c** $6m^5 \div 3m^5 = 2m$

d $4b^{-2} = \frac{4}{b^2}$ **e** $n^2 \times n = n^2 + n$

9 Expand:

a $5(2v - 4w)$ **b** $a^3(2a^2 + 4a)$ **c** $-3(4x + 5)$

10 Expand and simplify:

a $4(m - 2) + 3(2m + 5)$ **b** $3(3a - b) - (2a - b)$ **c** $x(2 - x) - 2(x - 2)$

2B Review set

1 Simplify:

a $m^{14} \times m^6$ **b** $t^{25} \div t^5$ **c** $(z^6)^4$ **d** $\frac{(b^5)^6}{b^7 \times b^4}$

e $(2m^7)^4$ **f** $4p^3q^7 \times 5p^4q$ **g** $\frac{6a^7b^4 \times 3a^3b^5}{24a^9b}$

2 Simplify, writing the answers with positive indices.

a $\frac{m^7}{m^{10}}$ **b** $\frac{3m^7}{m^{10}}$ **c** $\frac{m^7}{3m^{10}}$ **d** $\frac{10m^7}{15m^{10}}$

3 Evaluate:

a s^0 **b** $4s^0$ **c** $(4s)^0$ **d** $4s^0 - 1$

4 Evaluate:

a $\frac{1}{3^{-2}}$ **b** $\left(1\frac{1}{4}\right)^{-1}$ **c** $\left(\frac{6}{11}\right)^{-1}$

5 Write the meaning of the following.

a e^{-4} **b** $3e^{-4}$ **c** $(3e)^{-4}$

6 Write the following with negative indices.

a $\frac{1}{p^4}$ **b** $\frac{5}{p^4}$ **c** $\frac{1}{5p^4}$

7 Simplify:

a $k^{-6} \times k^2$ **b** $m^{-4} \div m^{-1}$ **c** $(n^3)^{-5}$ **d** $5n^{-3} \times 4n^8$ **e** $2a^{-5} \div 4a^{-8}$

8 State whether the following are true or false.

a $3w^0 = 1$ **b** $b^6 \div b^9 = b^{-3}$ **c** $a^7 \div a^7 = a$ **d** $2t^{-2} = \frac{1}{2t^2}$ **e** $p^{-2} = -2p$

9 Expand:

a $-10(4p + 3)$ **b** $m^2(3m^5 - m^3)$ **c** $-a(2a - 5)$

10 Expand and simplify:

a $3(2q + 6) - 2(q - 7)$ **b** $2a(2a - 5) - (3a + 1)$ **c** $m(6 - t) + t(m - 6)$

2C Review set

1 Simplify:

a $p^6 \times p^8$ **b** $y^{16} \div y^8$ **c** $(t^5)^6$ **d** $\frac{c^{12} \times c^8}{(c^4)^5}$

e $(3v^7)^3$ **f** $3x^8y^9 \times 6x^2y^5$ **g** $\frac{5m^5n^3 \times 3m^{10}n^4}{2m^6n^2 \times 5m^7n^4}$

2 Simplify, writing the answers with positive indices.

a $\frac{k^4}{k^6}$ **b** $\frac{8k^4}{k^6}$ **c** $\frac{k^4}{8k^6}$ **d** $\frac{6k^4}{8k^6}$

3 Evaluate:

a a^0 **b** $7a^0$ **c** $(7a)^0$ **d** $7a^0 + 5$

4 Evaluate:

a $\frac{1}{3^{-3}}$ **b** $\left(\frac{7}{8}\right)^{-1}$ **c** $\left(1\frac{1}{5}\right)^{-1}$

5 Write the following with positive indices.

a b^{-5} **b** $3b^{-5}$ **c** $(3b)^{-5}$

6 Write the following with negative indices.

a $\frac{1}{t^3}$ **b** $\frac{2}{t^3}$ **c** $\frac{1}{2t^3}$

7 Simplify:

a $d^{-5} \times d^{-3}$ **b** $n^{-2} \div n^{-3}$ **c** $(k^{-2})^{-3}$ **d** $5a^{-6} \times 3a^3$ **e** $6m^3 \div 9m^{-4}$

8 State whether the following are true or false.

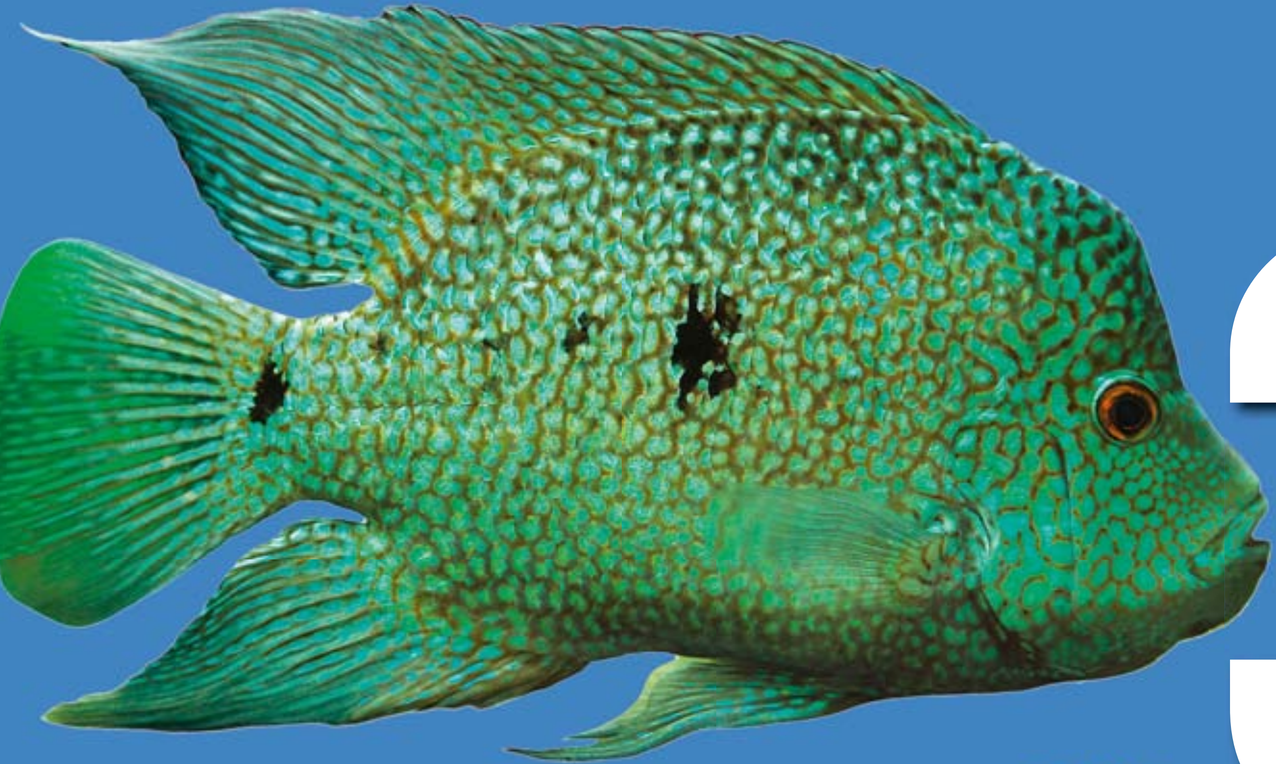
a $7h^0 = 7$ **b** $a \div a^5 = a^{-5}$ **c** $2p^5 \div 6p^5 = \frac{1}{3}p$ **d** $3s^{-4} = \frac{3}{s^4}$ **e** $n^2 + n^2 = n^4$

9 Expand:

a $4(5w + 2x)$ **b** $-k^4(2k^3 - 4k)$ **c** $-(4s - 7)$

10 Expand and simplify:

a $2(5t - 1) + 3(2 - 3t)$ **b** $10(x - 2y) - 5(2x - y)$ **c** $3(m + 2) - 8(3m - 2)$



3

Collecting and analysing data

This chapter deals with single variable data analysis.

After completing this chapter you should be able to:

- identify everyday questions and issues involving numerical and categorical variables
- describe data using the terms 'symmetric', 'skewed' and 'bimodal'
- construct back-to-back stem-and-leaf plots and histograms
- construct side-by-side histograms and dot plots
- compare data displays using the mean, median and range
- evaluate statistical reports in the media.

NSW Syllabus references: 5.1 S&P Single variable data analysis

Outcomes: MA5.1-1WM, MA5.1-2WM, MA5.1-3WM, MA5.1-12SP, MA5.1-13SP

STATISTICS & PROBABILITY – ACMSP228, ACMSP253, ACMSP282, ACMSP283

A Investigating data

Exercise 3A

Table 1 shows the monthly and annual rainfall for Sydney (Observatory Hill) from 2002 to 2011. Measurements are to the nearest millimetre.

Table 1: Rainfall for Sydney (mm)

Year	J	F	M	A	M	J	J	A	S	O	N	D	Annual
2002	98	348	45	68	93	28	24	20	22	6	32	75	860
2003	14	59	132	192	349	76	58	43	6	103	109	60	1200
2004	51	129	101	33	8	39	44	153	60	234	67	76	995
2005	68	125	154	33	48	79	63	2	51	43	125	25	816
2006	121	51	40	10	40	177	140	86	192	17	45	74	994
2007	45	108	65	180	10	511	67	152	41	27	170	123	1499
2008	57	258	63	147	3	127	90	44	99	67	73	54	1083
2009	25	128	61	153	126	130	53	6	16	180	13	67	956
2010	36	239	51	30	168	147	115	27	42	85	130	83	1154
2011	54	18	192	206	136	94	282	52	72	37	148	78	1369

1 In this time period, which year had the:

a highest annual rainfall? **b** lowest annual rainfall?

2 How much rain fell in:

a January 2006? **b** May 2007? **c** November 2011?

3 Which month had the highest rainfall in:

a 2004? **b** 2010?

4 Which month had the lowest rainfall in:

a 2003? **b** 2007?

5 Which year had the wettest:

a January? **b** June?
c December?

6 Which year had the driest:

a February? **b** May?
c November?

7 Considering winter to be the months June, July and August, which year had the:

a wettest winter?
b driest winter?

Table 2 summarises the rainfall statistics for Sydney for all records kept from 1859 to 2011.

Table 2: Rainfall statistics for Sydney (mm)

	J	F	M	A	M	J	J	A	S	O	N	D	Annual
Mean	102	118	130	127	121	131	98	80	69	77	84	78	1215
Lowest	6	3	8	1	3	4	2	0	2	1	2	3	583
Median	79	94	99	95	91	100	75	55	53	56	67	60	1160
Highest	387	631	521	622	585	643	336	483	356	285	517	402	2194

8 Consider the annual rainfall statistics for Sydney shown in Table 2. What is the annual:

a mean? **b** median?

9 **a** For the month of September, what is the:

i mean rainfall? **ii** median rainfall?

b Suggest why the 2000 Olympic Games were held in Sydney in the month of September.

10 On average, which month of the year is the:

a wettest? **b** driest?

11 **a** What is the least amount of rain that has fallen in any month?

b In which month did this occur?

12 **a** What is the greatest amount of rain that has fallen in any month?

b In which month did this occur?

13 Which month has the smallest difference between the mean and the median rainfall?

Use tables 1 and 2 to answer questions **14** and **15**.

14 For the period 2002–2011, in which years was the annual rainfall in Sydney greater than the long-term mean rainfall?

15 For the period 2002–2011, what percentage of years had rainfall that was less than the long-term median rainfall?

Table 3 shows the country of birth of settler arrivals in Australia for the year July 2010 to June 2011.

16 How many settlers arrived from:

a United Kingdom? **b** Vietnam? **c** Iraq?

17 The total number of settler arrivals in Australia, from more than 200 countries, was approximately 127 640 from July 2010 to June 2011. Determine the percentage of arrivals who came from:

a New Zealand **b** China **c** India

d South Africa **e** Sri Lanka.

18 How many more arrivals came from:

a New Zealand than from Iraq?

b the United Kingdom than from Sri Lanka?

Table 3: Country of birth of settler arrivals in Australia (July 2010 to June 2011)

Country of birth	Number of arrivals
New Zealand	25 772
China	14 611
United Kingdom	10 944
India	10 566
Philippines	5 048
South Africa	4 752
Vietnam	3 339
Sri Lanka	3 225
Iraq	2 988

Table 4 shows the top 15 countries of birth of Australian residents in 2006 and 2010 (excluding Australian born).

Table 4: Country of birth of Australian residents

Country of birth	Estimated population 2006	Estimated population 2010
United Kingdom	1 153 000	1 193 000
New Zealand	477 000	544 000
China	203 000	380 000
India	154 000	341 000
Italy	220 000	216 000
Vietnam	180 000	211 000
Philippines	136 000	177 000
South Africa	119 000	156 000
Malaysia	104 000	136 000
Germany	115 000	129 000
Greece	126 000	127 000
South Korea	49 000	100 000
Sri Lanka	71 000	92 000
Lebanon	87 000	90 000
Hong Kong	76 000	90 000

19 How many people who were resident Australian in 2010 were born in:

a New Zealand? **b** the Philippines? **c** South Africa? **d** Vietnam? **e** Greece?

20 What was the country of birth of the highest proportion of residents in:

a 2010? **b** 2006?

21 What was the country of birth of the lowest proportion of residents in:

a 2010? **b** 2006?

22 There was a decrease in the resident population from 2006 to 2010 of people born in which country?

23 In 2010 approximately 5 994 000 (≈27%) of the Australian population were born outside Australia. What percentage of those born outside Australia came from:

a China? **b** India? **c** South Korea? **d** Sri Lanka? **e** Lebanon?

24 In 2010 the resident population of Australia was approximately 22 370 000. What percentage of all Australian residents were born in:

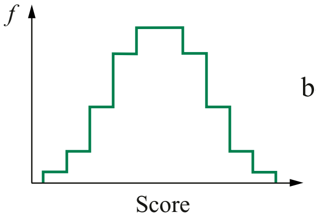
a China? **b** India? **c** South Korea? **d** Sri Lanka? **e** Lebanon?

25 From 2006 to 2010, what was the percentage increase in the resident population of people born in:

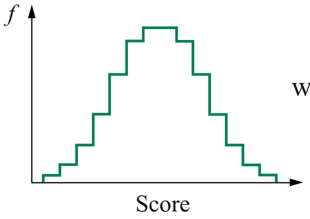
a China? **b** India? **c** South Korea? **d** Sri Lanka? **e** Lebanon?

B The shape of displays

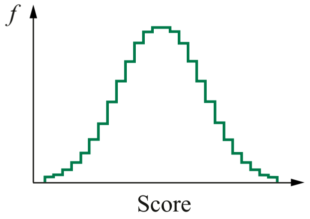
As the number of scores in a sample increases, the overall shape of the frequency histogram changes.



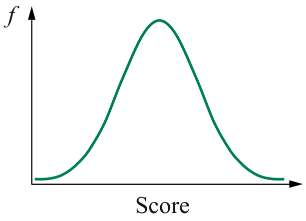
becomes



which becomes



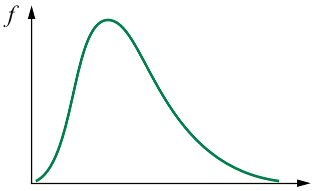
The general shape of a distribution can provide information about the scores. Here is the graph of a symmetric distribution, also referred to as a bell-shaped curve or a normal distribution.



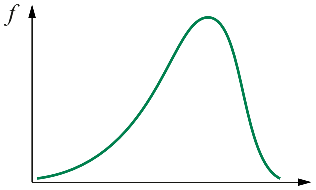
If the distribution is not symmetric then it is said to be **skewed**.

A distribution is **positively skewed** if most of the data is on the left-hand side of the distribution. The data has a ‘tail’ to the right as shown in the diagram on the left below.

A distribution is **negatively skewed** if most of the data is on the right-hand side of the distribution. The data has a ‘tail’ to the left as in the diagram on the right below.

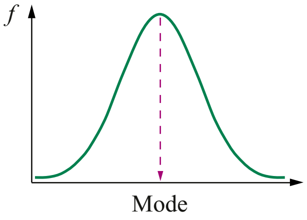


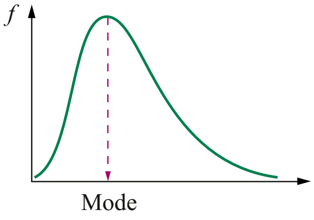
Positively skewed

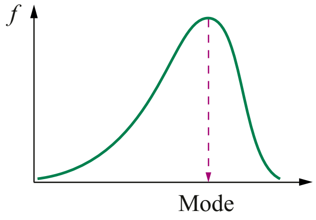


Negatively skewed

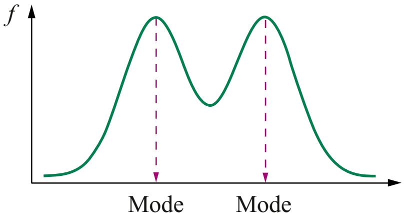
The mode is the score with the highest frequency. The mode for the normal and skewed distributions above are shown below.

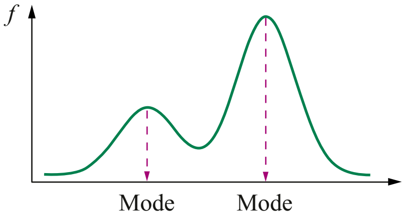






Some distributions have two modes. This is called a **bimodal** distribution. As long as the distribution has two distinct humps, not necessarily with the same frequency (height), then it is said to be bimodal. Two examples are shown below.

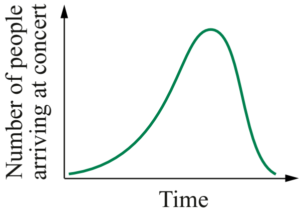




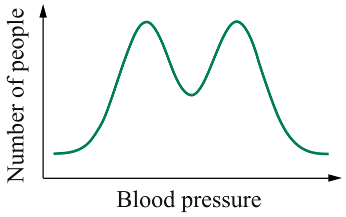
Exercise 3B

1 Describe the shape of the following distributions as symmetric, positively skewed, negatively skewed or bimodal.

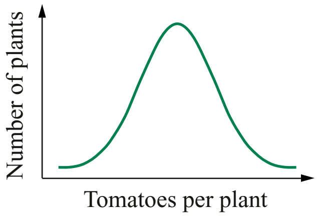
a



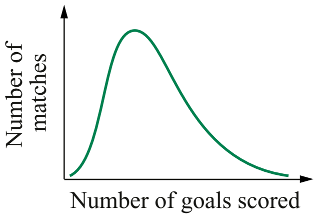
b



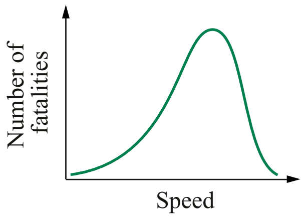
c



d



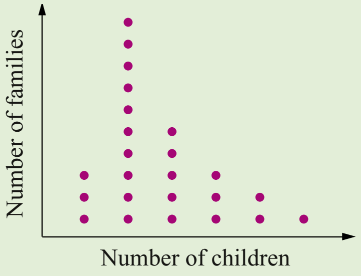
e



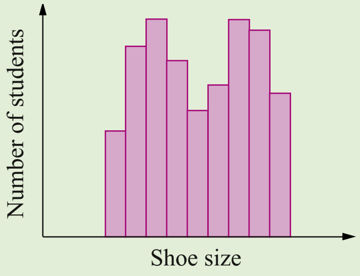
EXAMPLE 1

Describe the shape of each distribution as symmetric, positively skewed, negatively skewed or bimodal.

a

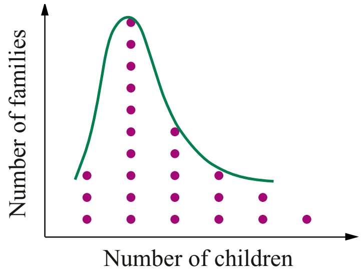
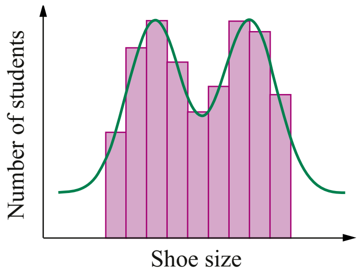


b

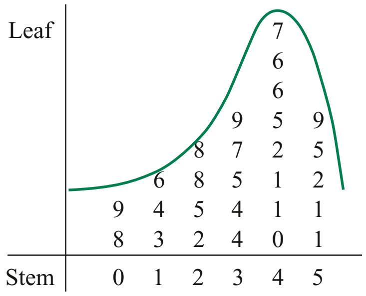


c **Marks on test**

Stem	**Leaf**
0	8 9
1	3 4 6
2	2 5 8 8
3	4 4 5 7 9
4	0 1 1 2 5 6 6 7
5	1 1 2 5 9

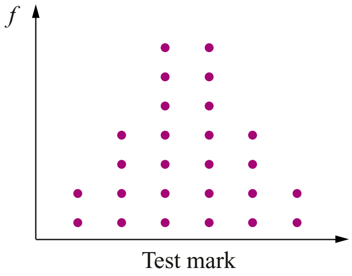
	Solve	Think	Apply
a	Positively skewed  	Draw a smooth curve to fit the tops of the columns. The tail is to the right.	Draw a smooth curve through the tops of the columns. Determine from the shape if the distribution is symmetric, positively skewed, negatively skewed or bimodal.
b	Bimodal  	Draw a smooth curve to fit the tops of the columns. The curve has two humps.	

EXAMPLE 1 CONTINUED

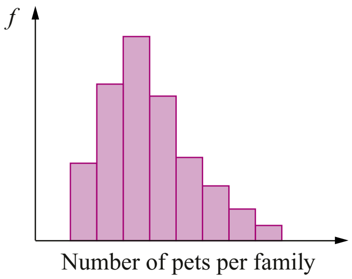
	Solve	Think	Apply
c	Negatively skewed 	Turn the stem-and-leaf plot on its side and draw a smooth curve to fit the tops of the columns. The tail is to the left.	Turn the stem-and-leaf plot on its side so that the stems are in increasing order, from left to right. Draw a smooth curve through the tops of the columns. Determine from the shape of the distribution if it is symmetric, positively skewed, negatively skewed or bimodal.

2 Describe the shape of each distribution as symmetric, positively skewed, negatively skewed or bimodal.

a



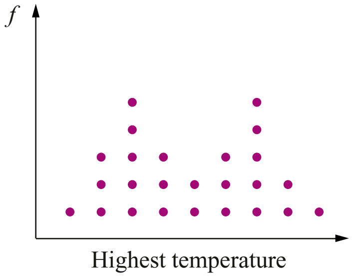
b



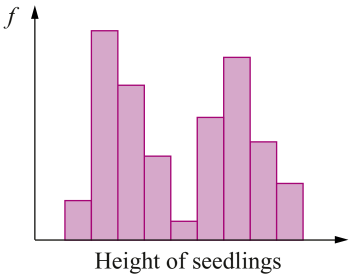
c **Marks on test**

Stem	Leaf
1	5
2	3 4
3	5 5 7
4	2 3 3 9
5	1 4 4 6 8 9
6	0 1 3

d



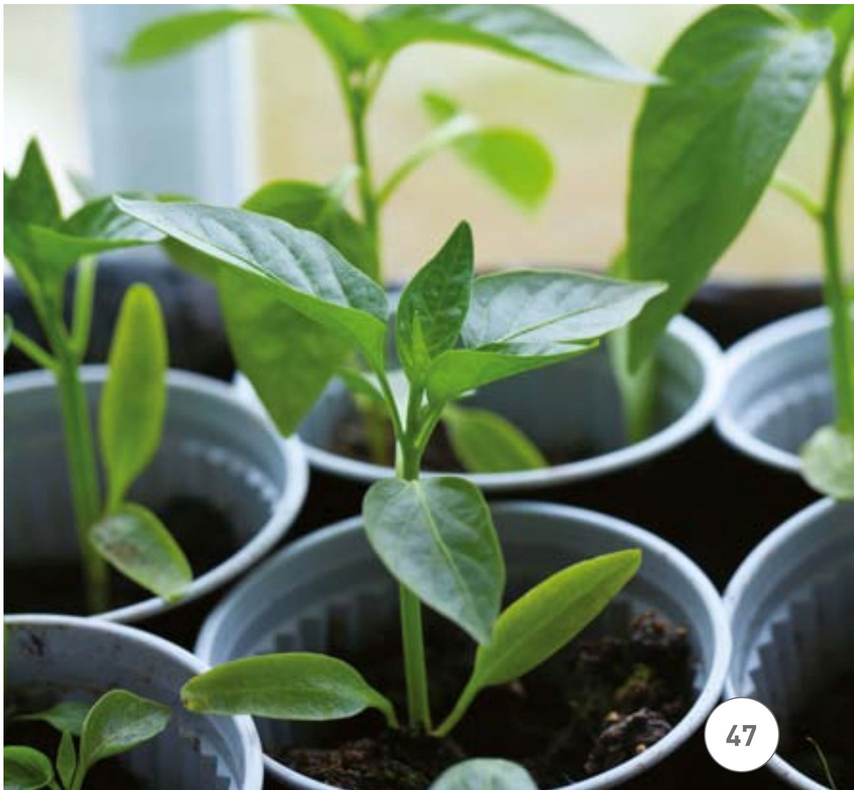
e



3 **a** Construct a frequency histogram and polygon for the data in the frequency distribution table below.

b Describe the shape of the distribution as positively skewed, negatively skewed or bimodal.

Score	Frequency
6	2
7	2
8	3
9	5
10	8
11	6

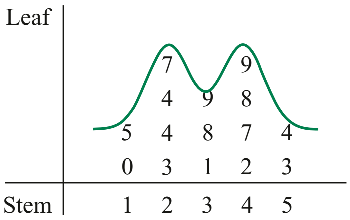
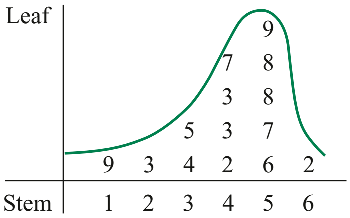


EXAMPLE 2

Describe the shape of both data sets in the back-to-back stem-and-leaf plot as symmetric, positively skewed, negatively skewed or bimodal.

Scores on topic test

Females	Stem	Males
9	1	0 5
3	2	3 4 4 7
5 4	3	1 8 9
7 3 3 2	4	2 7 8 9
9 8 8 7 6	4	3 4
4 2	6	

Solve		Think	Apply
Male scores Bimodal		For males, turn the plot on its side and draw a curve to fit the columns. For females, flip the data over the stem and turn it on its side. Draw a curve to fit the columns.	Turn each stem-and-leaf plot on its side so that the stems are in increasing order, from left to right. Draw a smooth curve through the tops of the columns and describe the shape of the distribution.
Female scores Negatively skewed			

4 Describe the shape of each data set in the following distributions as symmetric, positively skewed, negatively skewed or bimodal.

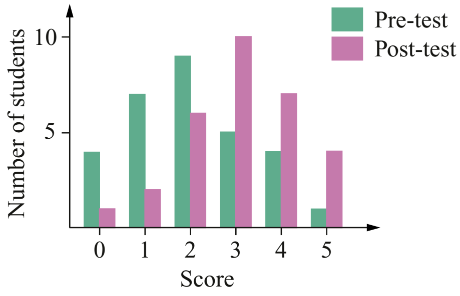
a **Scores on Science test**

Females	Stem	Males
9	0	7 7 8
4 3	1	2 3 5 7 9 9 9
7 1	2	0 1 1
9 6 4 0	3	5 7
9 7 5 5 2 2	4	6
3 1 1	5	1

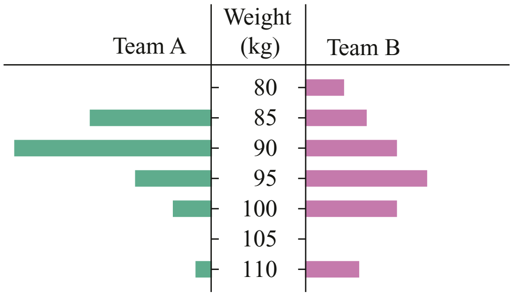
b **Scores on Mathematics test**

9 Blue	Stem	9 Red
9 9 8	4	7
8 7 2 1 1	5	3 4 7 8
4 3 2	6	1 5
5 4	7	0 5 5 6 8 9
3 0	8	3 4
1	9	2

5 **a** This distribution shows the scores for a group of students on a mathematics topic pre-test and post-test. This is called a **side-by-side** histogram. Describe the data distribution.



b This distribution shows the weight (in kg) of players in two Rugby League teams. This is called a **back-to-back** histogram. Describe the data distribution for each team.



C Comparing like sets of numerical data

EXAMPLE 1

The times taken for the students from two classes to travel to and from school are given below.

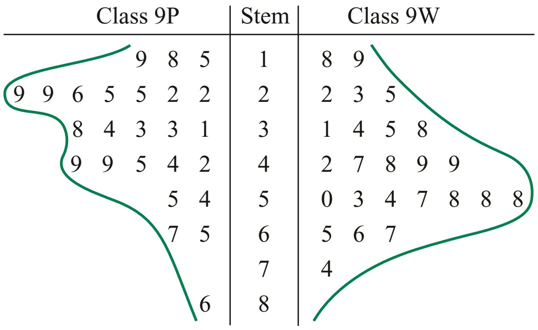
Class 9P: 19 49 25 25 22 55 26 38 54 22 33 44 15
86 31 18 33 67 34 42 49 29 45 65 29

Class 9W: 22 34 48 18 58 67 74 66 53 31 57 25 58
49 35 47 50 65 54 49 38 23 58 19 42

a Draw a back-to-back stem-and-leaf plot for this data.

b Comment on the shape of each distribution.

c Use the mean, median and range to compare the data.

	Solve	Think	Apply
a			Look for the tail to determine the shape of the distribution. The greater the mean and median, the longer the travel times. The greater the range, the greater the spread of times. The mean and range are greatly influenced by outliers, so it is often useful to ignore them.
b	The distribution of times for Class 9P is positively skewed; that is, there are more shorter than longer times. The distribution of times for Class 9W is negatively skewed; that is, there are more longer than shorter times.	For Class 9P the tail is to the right. For Class 9W the tail is to the left.	
c	For Class 9P: Mean $= \frac{955}{25} = 38.2$ Median $= 33$, Range $= 71$ For Class 9W: Mean $= \frac{1140}{25} = 45.6$ Median $= 49$, Range $= 56$ *Comment:* As expected from the distribution shape, the mean and median of Class 9W are greater than those of Class 9P. In general, the students of Class 9W taker longer than the students of Class 9P to get to and from school. The range of times for Class 9P is much larger than for Class 9W, indicating a greater spread of times for Class 9P. But, if we ignore the outlier, 86, the range of Class 9P becomes 52, which is similar to that of Class 9W.	Mean $= \frac{\text{sum of scores}}{\text{number of scores}}$ Median $=$ middle score Range $=$ highest score $-$ lowest score	

Class 9P	Stem	Class 9W
9 8 5	1	8 9
9 9 6 5 5 2 2	2	2 3 5
8 4 3 3 1	3	1 4 5 8
9 9 5 4 2	4	2 7 8 9 9
5 4	5	0 3 4 7 8 8 8
7 5	6	5 6 7
	7	4
6	8	

Exercise 3C

1 Two groups of Year 9 students were asked to unscramble a seven-letter word. Their times, in seconds, are shown below.

Group 1: 11 16 39 23 51 24 31 4 29 16 27 40 13 23 30 29 6 22 34 38 13
Group 2: 12 27 46 17 26 32 18 15 21 41 37 36 23 8 25 43 34 7 36 12 7

a Draw a back-to-back stem-and-leaf plot for this data.

b Comment on the shape of each distribution.

c Use the mean, median and range to compare the data.

2 The scores for a class of 16 students on two tests are given below.

Test 1: 22 42 34 30 19 39 46 41 38 35 47 39 24 45 27 32
Test 2: 13 18 21 6 40 16 26 24 35 12 20 26 31 13 15 19

a Draw a back-to-back stem-and-leaf plot for this data.

b Comment on the shape of each distribution.

c Use the mean, median and range to compare the data.

d Suggest a possible reason for the skewness.

3 **a** The table shows the January mean daily maximum temperatures for Sydney and Melbourne over 20 years. Draw a back-to-back stem-and-leaf plot for this data using stems 22 23 24 25 26 27 28 29.

b Comment on the shape of each distribution.

c Use the mean, median and range to compare the data.

Year	Sydney	Melbourne
1	26.7	26.9
2	25.4	28.6
3	26.2	23.9
4	25.0	25.0
5	27.2	25.3
6	26.2	24.0
7	26.9	28.0
8	26.9	24.3
9	26.3	29.5
10	26.4	27.4
11	26.4	24.2
12	25.4	23.9
13	26.8	23.8
14	25.9	23.1
15	27.0	24.4
16	26.8	28.1
17	25.4	25.7
18	25.5	26.0
19	28.9	26.0
20	25.3	22.4

EXAMPLE 2

The scores of two groups of university students on a mechanical aptitude test are given at right.

a Draw a back-to-back histogram and a side-by-side histogram for this data.

b Comment on the shape of each distribution.

c Compare the mean, median and range for each distribution.

d Suggest a possible reason for the differences in these distributions.

1: Business students

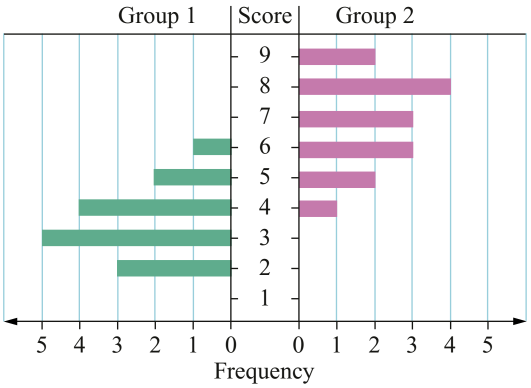
Score	Frequency
2	3
3	5
4	4
5	2
6	1

2: Engineering students

Score	Frequency
4	1
5	2
6	3
7	3
8	4
9	2

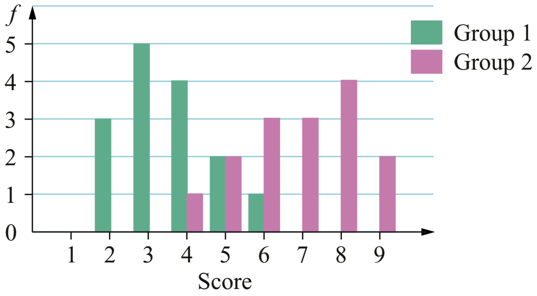
Solve/Think | **Apply**

a For the back-to-back histogram, put columns for each group on the opposite sides of the score axis.



Back-to-back histograms are drawn on opposite sides of the score axis. The axis can be horizontal or vertical.

For the side-by-side histogram, draw columns next to each other on the same horizontal axis.



Side-by-side histograms are drawn separately next to each other using the same scale on both axes or combined on the same score axis. Use shading to distinguish groups.

b Group 1 is positively skewed. (There are more lower than higher scores.)
The group 1 tail is to the right.
Group 2 is negatively skewed. (There are more higher than lower scores.)
The group 2 tail is to the left.

c For group 1: Mean $= \frac{53}{15} = 3.5$

Median $= 3$

Range $= 4$

For group 2: Mean $= \frac{103}{15} = 6.9$

Median $= 7$

Range $= 5$

Comment: The mean and median for group 2 are higher than for group 1. Overall, group 2 students have scored much higher marks than group 1 students on the test.

d The engineering students would be expected to perform better, as shown by the negatively skewed distribution of their scores. The business students would not be expected to score well, as shown by the positively skewed distribution.

4 The scores of female and male students on a class English test are shown in the tables.

Females

Score	Frequency
4	1
5	1
6	1
7	6
8	4
9	2

Males

Score	Frequency
3	2
4	4
5	2
6	2
7	4
8	1

a For this data, draw:

- **i** a back-to-back histogram
- **ii** a side-by-side histogram.

b Describe the shape of each distribution.

c Compare the mean, median and range for each distribution.

5 The 25 students in a Year 9 class were given a test in term 1 and a test in term 2. The results are given below.

Term 1: 8 9 10 10 10 9 8 7 9 8 10 9 10
9 8 9 3 4 9 9 8 5 5 8 9
Term 2: 2 3 1 0 3 2 5 7 8 9 8 7 6
3 4 4 2 3 4 4 0 1 1 2 3

a Complete the following tables and draw a back-to-back histogram for this data.

Term 1

Mark	Frequency
3	
4	
5	
6	
7	
8	
9	
10	

Term 2

Mark	Frequency
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

b Describe the shape of each distribution.

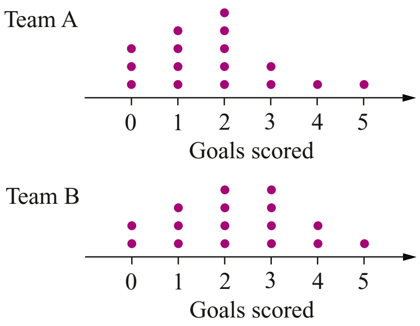
c Suggest a possible reason for the skewness.

d Use the mean, median and range to compare the data.

6 The parallel dot plot shows the number of goals scored by two soccer teams in a 16 match competition.

a Describe the shape of each distribution.

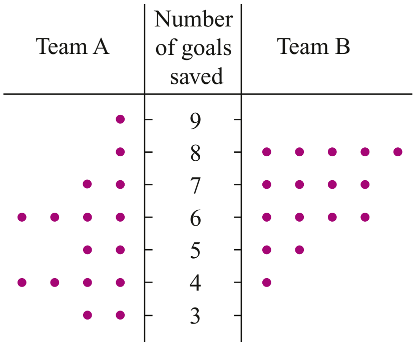
b Use the mean, median and range to compare the data.



7 This back-to-back dot plot shows the number of saves made by goalkeepers for the two teams in question **6**.

a Comment on the shape of each distribution.

b Use the mean, median and range to compare the data.



D Statistical claims in the media

Exercise 3D

1 Write a short evaluation or analysis of the following statistical statements. Is there anything misleading about each of them?

a Four out of five dentists recommend Britex toothpaste.

b A newspaper reported that ‘The Citico company has dismissed 10% of its workforce’.

c Twenty people at a netball game were surveyed and 70% said that they were going to vote for Mrs Brown at the coming state election. (Mrs Brown supports spending more money on sport facilities.)

d Of the two major political parties, one claimed that unemployment was rising and the other claimed that employment was rising. Could both parties be correct?

2 Consider the following two questions. What sort of responses are they likely to get?

‘Do you think that the taxes we pay should be used to pay people who stay at home and don’t work?’

‘Do you think that the government should give financial assistance to people who can’t find employment?’

Investigation 1 Statistical claims made in the media

Collect various reports and statistical claims made in the media.

1 Investigate and interpret advertising that quotes various statistics.

2 Analyse graphical displays to determine features that may have been manipulated to cause a misleading interpretation, or which support a particular point of view.

3 Critically review claims linked to data displays.

4 Consider the reliability of conclusions from statistical investigations, including factors that may have masked the result, the accuracy of measurements taken and whether the results can be generalised in other situations.

Language in mathematics

1 Describe in words what is meant by a data distribution that is:

a symmetric **b** positively skewed **c** negatively skewed **d** bimodal

2 Read the following article about Hanna Neumann. Answer the questions in complete sentences.

a Where and when was Hanna Neumann born?

b What were her main areas of interest?

c For what is she remembered?

d Construct a timeline of her life.

Hanna Neumann (1914–1971)

Hanna Neumann was born Hanna von Coemmerer in Berlin. She was the daughter of a historian with teaching qualifications, who was killed in World War 1. As a result, her family was very poor and from the age of 13 she helped support them by tutoring younger children. Hanna became an extremely capable student and commenced studies at the University of Berlin in 1932. Her main area of study was mathematics, but she also had an interest in physics, history and religion.

Here she met her husband Bernhard, who left for England, in 1933, after deciding that living in Germany under the Nazis had become too dangerous. Hanna secretly travelled to England in 1934 to become engaged and then returned to continue her study in Berlin. Eventually, in 1938 after completing her studies and working as a research student at Gottingen University, she travelled to England and married Bernhard.

Hanna worked extremely hard and was recognised as being an excellent teacher. In 1948 she started opening her house in the evenings for others to come and discuss mathematics. She continued to be involved in teaching and studying mathematics and, in 1964, joined Bernhard at the Australian National University in Canberra.

In her position at the university, Hanna was recognised as having an enormous capacity for work and a great concern for those she taught. She held a number of administrative positions within professional mathematical bodies and travelled giving lectures. Unfortunately, at the age of 57, on a lecture tour in Canada, she became ill suddenly and died. She is remembered not only for her mathematical ability, but also for the willingness with which she was prepared to devote her time to teaching and her students.

Terms

back-to-back
bimodal
distribution
histogram
mean
median
mode
negatively skewed
parallel
parallel dot plot
positively skewed
range
side-by-side
stem-and-leaf plot
symmetric

Check your skills

Use the table on the next page showing the top 10 countries of residence of visitors to Australia for 2010 and 2011 to answer questions **1** to **3**.

1 The country from which most visitors came to Australia in 2011 was:

A New Zealand **B** China **C** Japan **D** United Kingdom

2 From which country was the increase in visitors from 2010 to 2011 greatest?

A New Zealand **B** China **C** Japan **D** United Kingdom

3 What percentage of visitors came from Malaysia in 2010?

A 23.6% **B** 5.6% **C** 25.6% **D** 3.6%

Top 10 countries of residence of visitors to Australia for 2010 and 2011

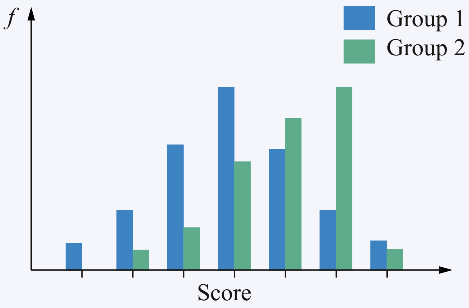
Country of residence	Visitors 2010 ('000)	Visitors 2011 ('000)
United Kingdom	646.7	608.3
USA	472.2	456.2
Malaysia	236.9	241.2
Singapore	308.0	318.5
China	453.8	542.0
Hong Kong	163.9	166.3
Japan	398.1	332.7
Korea	214.0	198.0
New Zealand	1161.8	1172.7
Germany	160.1	153.9

4 The distribution shown by the dot plot is:

A positively skewed
B negatively skewed
C bimodal
D symmetric

5 Which of the following statements about the shape of the distributions in this side-by-side histogram is correct?

A Group 1 is symmetric and group 2 is positively skewed.
B Group 1 is negatively skewed and group 2 is negatively skewed.
C Group 1 is symmetric and group 2 is negatively skewed.
D Group 1 is negatively skewed and group 2 is positively skewed.



Questions **6** to **10** refer to the back-to-back stem-and-leaf plot showing runs scored by Michael and Ricky.

Runs scored

Michael	Stem	Ricky
	0	0
9 5 2	1	9
9 8 7 7 6	2	3 8
6 5 2	3	7 8 8 9 9
2 1	4	1 3 6
	5	2

6 Ricky's lowest score is:

A 0
B 9
C 19
D 91

7 Michael's highest score is:

A 50
B 42
C 24
D 91

8 Michael's median score is:

A 82
B 29
C 28
D 27

9 The mean of Ricky's scores (to 1 decimal place) is:

A 5.6
B 6.1
C 34.1
D 36.9

10 Which of the following statements is true?

A Ricky's distribution of scores is slightly positively skewed.
B Michael's distribution of scores is slightly negatively skewed.
C The range of Ricky's scores is greater than the range of Michael's scores.
D All of the above.

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4, 5	6–10
Section	A	B	C

3A Review set

1 The table compares the population density of Australia with that of the 10 most densely populated countries in the world (with a population over 10 million).

Country	Population	Land area (km^2)	Density (population/km^2)
Australia	22 719 000	7 691 951	2.95
Bangladesh	152 518 000	147 750	1034
Taiwan	22 955 000	36 190	634
South Korea	48 456 000	99 538	487
Rwanda	10 718 000	26 338	407
England	53 013 000	130 440	406
Netherlands	16 760 000	41 526	404
India	1 210 193 000	3 287 263	368
Belgium	11 007 000	30 528	361
Japan	127 960 000	377 944	339
Sri Lanka	20 653 000	65 610	315

a Which country has the greatest:

i population? **ii** land area? **iii** population density?

b Which country has the smallest:

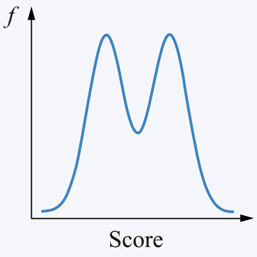
i population? **ii** land area? **iii** population density?

c Which countries have a population less than Australia's population?

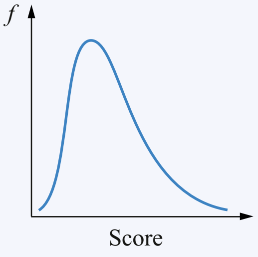
d Why is Australia's population density so much smaller than that of the other countries listed here?

2 Describe the shape of these distributions as symmetric, positively skewed, negatively skewed or bimodal.

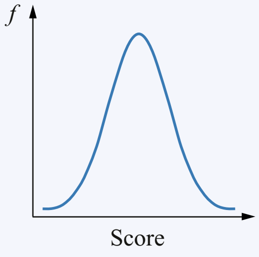
a



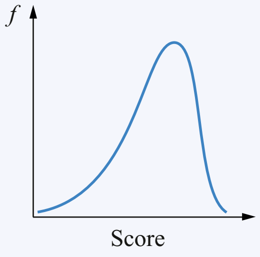
b



c



d



3 The heights, in centimetres, of the boys and girls in a Year 9 class are shown in the back-to-back stem-and-leaf plot.

a Comment on the shape of each distribution.

b Compare the mean, median and range of the heights.

Heights recorded

Boys	Stem	Girls
	14	9
6 4	15	2 6 7
9 5 5 3 2	16	0 1 2 3 4 7 8 9
9 8 7 5 4 1 0	17	1 2 3
3	18	

3B Review set

1 The table shows estimates of the population density of Australia's states and territories.

State or territory	Land area (km^2)	Population	Density (population/km^2)	% of population in capital city
ACT	2 358	344 200	146.0	99.6%
NSW	800 642	6 967 200	8.7	63%
Vic	227 416	5 297 600	23.3	71%
Qld	1 730 648	4 279 400	2.5	46%
SA	983 482	1 601 800	1.6	73.5%
WA	2 529 875	2 163 200	0.9	73.4%
Tas	68 401	498 200	7.3	41%
NT	1 349 129	219 900	0.2	54%

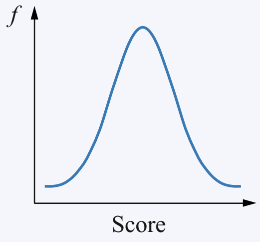
a Which state or territory has the greatest:
  i land area?
  ii population?
  iii population density?
  iv percentage of population in its capital city?

b Which state or territory has the smallest:
  i land area?
  ii population?
  iii population density?
  iv percentage of population in its capital city?

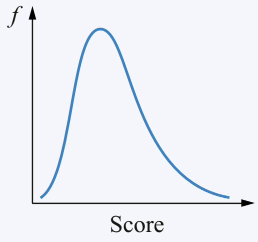
c Why does the ACT have such a high population density compared with the other states and territories?

2 State whether the shape of the following distributions are symmetric, positively skewed, negatively skewed or bimodal.

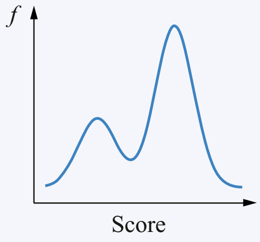
a



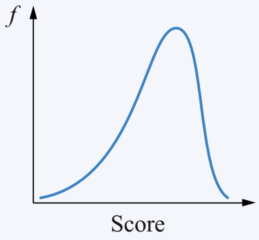
b



c



d



3 The numbers of goals per match scored by two soccer teams are shown in the tables below.

Team A

Number of goals	Number of matches
0	4
1	7
2	3
3	0
4	1

Team B

Number of goals	Number of matches
0	2
1	4
2	5
3	3
4	1

a Display this information in:
  i a back-to-back histogram
  ii a parallel dot plot.

b Comment on the shape of each distribution.

c Compare the mean, median and range for these two distributions.

d Which team do you think performed better?

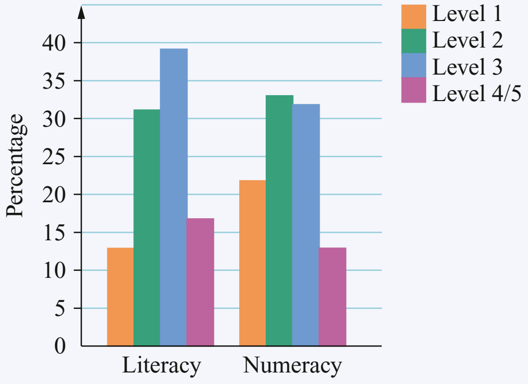
3C Review set

1 The following graphs show the results of a survey of the skill levels in literacy and numeracy of 15–74 year olds in Australia in 2011–12. Level 1 is the lowest.

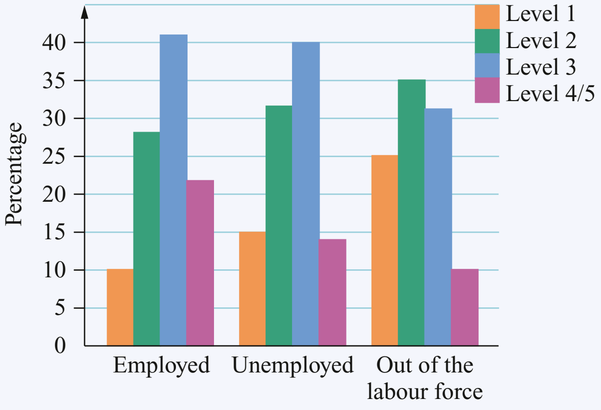
a What proportion of those surveyed was assessed at level 2 or less in:
  - **i** literacy?
  - **ii** numeracy?

b What proportion was assessed at level 4/5 in:
  - **i** literacy?
  - **ii** numeracy?

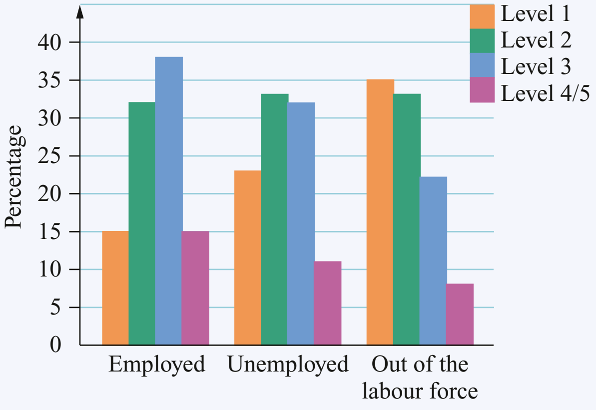
Proportion of Australians at each skill level in literacy and numeracy



Proportion at each literacy level by labour force status



Proportion at each numeracy level by labour force status



c What proportion was in level 3 literacy and:
  - **i** employed? **ii** unemployed? **iii** out of the labour force?

d Which group had the highest proportion of people assessed at level 1 for literacy?

e What proportion was in level 3 numeracy and:
  - **i** employed? **ii** unemployed? **iii** out of the labour force?

f Which group had the highest proportion of people assessed at level 1 for numeracy?

2 Draw a neat sketch of a:

a histogram of a positively skewed distribution

b dot plot of a negatively skewed distribution

c stem-and-leaf plot of a symmetric distribution

d frequency curve with bimodal distribution.

3 A Year 9 class was given a Geography test. The results are given for girls and boys.

Girls: 12 24 26 41 38 9 17 15 20 36 29 28 32 28 9
Boys: 43 31 26 32 32 17 51 49 18 24 49 35 48 46 40

a Draw a back-to-back stem-and-leaf plot for this data.

b Comment on the shape of each distribution.

c Use the mean, median and range to compare the data.

4

Numbers of any magnitude

This chapter deals with scientific notation, estimation, and the units and accuracy of measurements.

After completing this chapter you should be able to:

- express numbers in scientific notation
- interpret the meaning of prefixes used for units of measurement
- use appropriate units of time for very large or very small time intervals
- use the language of estimation appropriately
- describe the limits of accuracy of measuring instruments
- use the appropriate level of accuracy for calculations involving measurements.

NSW Syllabus references: 5.1 N&A Indices, 5.1 M&G Numbers of any magnitude
Outcomes: MA5.1-1WM, MA5.1-2WM, MA5.1-3WM, MA5.1-5NA, MA5.1-9MG
NUMBER & ALGEBRA – ACMNA210, ACMMG219

A Scientific notation

The distance of Mars from the Sun is approximately 229 000 000 km. The diameter of the hydrogen atom is 0.000 000 000 025 4 m. Scientists invented a more convenient method of writing very large and very small numbers. It is called **scientific notation** or **standard notation**. A number that is written in scientific notation, such as these, is written as the product of a number between 1 and 10 and a power of 10.

EXAMPLE 1

State whether or not the following numbers are written in scientific notation.

a 6.7×10^8 **b** 23×10^5 **c** 3.65×1000 **d** 2.96×10^{-7} **e** 480 000

	Solve	Think	Apply
a	Yes	The first number (6.7) is between 1 and 10 and it is multiplied by a power of 10 (10^8).	A number is written in scientific notation if it is expressed as the product of a number between 1 and 10 and a power of 10.
b	No	The first number (23) is not between 1 and 10.	
c	No	The second number (1000) is not expressed as a power of 10.	
d	Yes	The first number (2.96) is between 1 and 10 and it is multiplied by a power of 10 (10^{-7}).	
e	No	The number is not written as a product.	

Exercise 4A

1 State whether or not the following numbers are written in scientific notation.

a 5.9×10^6 **b** 34×10^8 **c** $8.97 \times 10\,000$ **d** 5.03×10^{-9} **e** 28 000

f 7×10^{-15} **g** 0.85×10^4 **h** 4.2×100 **i** 163 000 000 **j** 2.006×10^{68}

2 Complete this table.

10^0	10^1	10^2		10^4	10^5	
1	10		1000			1 000 000

EXAMPLE 2

Write the following numbers in scientific notation.

a 5 000 000 **b** 40 000

	Solve/Think	Apply
a	$5\,000\,000 = 5 \times 1\,000\,000$ $= 5 \times 10^6$ using the table in question **2** above.	Write the number as the product of a number between 1 and 10 and a multiple of 10. Then express the multiple of 10 as a power of 10.
b	$40\,000 = 4 \times 10\,000$ $= 4 \times 10^4$	

3 Complete to write the following numbers in scientific notation.

a $7000 = 7 \times$ ___
$= 7 \times 10^{\square}$

b $600\,000 = 6 \times$ ___
$= 6 \times 10^{\square}$

4 Write the following numbers in scientific notation.

a 3 000 000 **b** 70 000 **c** 8000 **d** 600 000 **e** 500

EXAMPLE 3

Write the following numbers in scientific notation.

a 5300 **b** 284 000

	Solve/Think	Apply
a	$5300 = 5.3 \times 1000$ $= 5.3 \times 10^3$	Write the number as the product of a number between 1 and 10 and a multiple of 10. Then express the multiple of 10 as a power of 10.
b	$284\,000 = 2.84 \times 100\,000$ $= 2.84 \times 10^5$	

5 Complete to write the following numbers in scientific notation.

a $67\,000 = 6.7 \times$ ___
$= 6.7 \times 10^{\square}$

b $8\,420\,000 =$ ___ $\times 1\,000\,000$
$=$ ___ $\times 10^{\square}$

6 Write the following numbers in scientific notation.

a 4800 **b** 392 000 **c** 64 000 **d** 2 180 000 **e** 760

EXAMPLE 4

Write the following numbers as ordinary decimal numbers.

a 6×10^5 **b** 3.94×10^6

	Solve/Think	Apply
a	$6 \times 10^5 = 6 \times 100\,000$ $= 600\,000$	Express the power of 10 as a multiple of 10 and perform the multiplication.
b	$3.94 \times 10^6 = 3.94 \times 1\,000\,000$ $= 3\,940\,000$	

7 Complete to write the following numbers as ordinary decimal numbers.

a $5 \times 10^4 = 5 \times$ ___
$=$ ___

b $4.93 \times 10^6 = 4.93 \times$ ___
$=$ ___

8 Write the following numbers as ordinary decimal numbers.

a 3×10^4 **b** 7×10^3 **c** 9×10^6 **d** 4×10^5
e 8×10^2 **f** 4.6×10^5 **g** 6.71×10^3 **h** 3.9×10^6
i 8.36×10^4 **j** 5.2×10^5

9 Complete this table.

0.1	0.01		0.0001		0.000 001
$\frac{1}{10}$		$\frac{1}{1000}$		$\frac{1}{100\,000}$	

EXAMPLE 5

Write the following numbers in scientific notation.

a 0.004 **b** 0.000 009

	Solve/Think	Apply
a	$0.004 = 4 \times 0.001$ $= 4 \times \frac{1}{1000}$ $= 4 \times \frac{1}{10^3}$ $= 4 \times 10^{-3}$	Write the number as the product of a number between 1 and 10 and a decimal fraction. Express the decimal fraction as a power of 10.
b	$0.000\,009 = 9 \times 0.000\,001$ $= 9 \times \frac{1}{1\,000\,000}$ $= 9 \times \frac{1}{10^6}$ $= 9 \times 10^{-6}$	

10 Complete to write the following numbers in scientific notation.

a $0.0007 = 7 \times 0.\_\_\_$

$= 7 \times \frac{1}{10^{\square}} = 7 \times 10^{\square}$

b $0.06 = 6 \times \_\_\_$

$= 6 \times \frac{1}{10^{\square}} = 6 \times 10^{\square}$

11 Write the following numbers in scientific notation.

a 0.003 **b** 0.000 007 **c** 0.0005 **d** 0.000 02 **e** 0.09

EXAMPLE 6

Write as ordinary decimal numbers.

a 5×10^{-2} **b** 7×10^{-6}

	Solve/Think	Apply
a	$5 \times 10^{-2} = 5 \times \frac{1}{10^2}$ $= 5 \times 0.01$ $= 0.05$	Express the power of 10 as a decimal fraction and perform the multiplication.
b	$7 \times 10^{-6} = 7 \times \frac{1}{10^6}$ $= 7 \times 0.000\,001$ $= 0.000\,007$	

12 Complete to write the following numbers as ordinary decimal numbers.

a $4 \times 10^{-3} = 4 \times \frac{1}{10^{\square}}$

$= 4 \times \frac{1}{\square}$

$= 4 \times 0.\_\_\_ = \_\_\_$

b $3 \times 10^{-2} = 3 \times \frac{1}{10^{\square}}$

$= 3 \times \frac{1}{\square}$

$= 3 \times 0.\_\_\_ = \_\_\_$

13 Write the following as ordinary decimal numbers.

a 6×10^{-2} **b** 3×10^{-6} **c** 2×10^{-3} **d** 5×10^{-4} **e** 9×10^{-6}

EXAMPLE 7

Explain the difference between:

a 2×10^4 and 2^4 **b** 2×10^{-4} and 2^{-4}

	Solve/Think	Apply
a	$2 \times 10^4 = 2 \times 10\,000$ $= 20\,000$ $2^4 = 2 \times 2 \times 2 \times 2$ $= 16$	Evaluate each numerical expression to show the difference.
b	$2 \times 10^{-4} = 2 \times \frac{1}{10^4} = 2 \times \frac{1}{10\,000}$ $= 2 \times 0.0001 = 0.0002$ $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ $= 0.0625$	

14 Explain the difference between:

a 3×10^4 and 3^4 **b** 5×10^{-2} and 5^{-2} **c** 2×10^3 and 2^3

d 2×10^{-5} and 2^{-5} **e** 4×10^6 and 4^6 **f** 5×10^6 and 5^6

EXAMPLE 8

Write 246 000 in scientific notation.

Solve	Think	Apply
$246\,000 = 2.46 \times 10^5$	A quick method for writing in scientific notation is: *Step 1:* Move the decimal point so that it is after the first non-zero digit of the number. This produces a number between 1 and 10. In this case it is 2.460 00. *Step 2:* Count the number of places back to the original position of the decimal point in the number. 2.460 00 Number of places = 5 to the right = +5 *Step 3:* Write the number using the number of places moved for the power of 10. $246\,000 = 2.46 \times 10^5$	Move the decimal point so that it is after the first non-zero digit. This produces a number between 1 and 10. Count the number of places (left or right) back to the original position of the decimal point in the number. This becomes the power of 10.

15 Complete the following to write 3 970 000 in scientific notation.

Step 1: Move the decimal point so that it is after the first non-zero digit of the number. We get _____.

Step 2: Count the number of places back to the original position of the decimal point in the number.

Number of places = ___ to the _____ = ___

Step 3: Write the number using the number of places moved for the power of 10.

3 970 000 = ___ $\times 10^{\square}$.

16 Write the following numbers in scientific notation.

a 372 000 **b** 54 000 **c** 2 980 000 **d** 3400 **e** 609 000

f 87 500 **g** 7 698 000 **h** 361 000 000 **i** 8000 **j** 56 000 000

EXAMPLE 9

Write 0.000 71 in scientific notation.

Solve	Think	Apply
0.000 71 = 7.1×10^{-4}	*Step 1:* Move the decimal point so that it is positioned between the first and second digits of the number. In this case we get 7.1. *Step 2:* Count the number of places back to the original position of the decimal point in the number. 00007.1 Number of places = 4 to the left = −4 *Step 3:* Write the number using the number of places moved for the power of 10.	Move the decimal point so that it is after the first non-zero digit. This produces a number between 1 and 10. Count the number of places (left or right) back to the original position of the decimal point in the number. This becomes the power of 10.

17 Write the following numbers in scientific notation:

a 0.000 57 **b** 0.000 078 **c** 0.0061 **d** 0.000 002 96 **e** 0.000 801

f 0.000 000 5 **g** 0.004 39 **h** 0.000 002 8 **i** 0.000 09 **j** 0.000 000 004 9

EXAMPLE 10

Write 6.48×10^6 as an ordinary number.

Solve	Think	Apply
6.48×10^6 = 6 480 000	Reverse the process of Example 8. As the power of 10 is +6, move the decimal point 6 places to the right. 6.480 000 Hence 6.48×10^6 = 6 480 000.	The power of 10 indicates how many places to move the decimal point (left or right).

18 Complete to write 5.72×10^5 as an ordinary number.

Move the decimal point ___ places to the _____.

5.72×10^5 = ___

19 Write the following numbers as ordinary numbers.

a 7.32×10^6
b 5.2×10^4
c 5.67×10^5
d 3.8×10^3
e 9.27×10^7
f 6.914×10^4
g 3.275×10^6
h 7×10^5
i 2×10^8
j 3.08×10^5

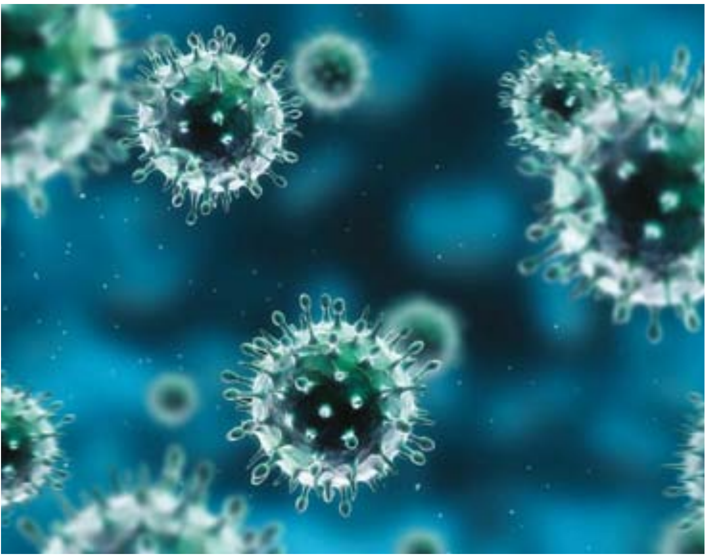
EXAMPLE 11

Write 3.51×10^{-6} as an ordinary number.

Solve	Think	Apply
$3.51 \times 10^{-6} = 0.000\,003\,51$	Reverse the process of Example 9. As the power of 10 is −6, move the decimal point 6 places to the left. 0000003.51 Hence $3.51 \times 10^{-6} = 0.000\,003\,51$.	The power of 10 indicates how many places to move the decimal point (left or right).

20 Write the following as ordinary numbers.

a 3.98×10^{-6}
b 5.3×10^{-4}
c 7.09×10^{-5}
d 8.8×10^{-3}
e 5.9×10^{-6}
f 3.07×10^{-7}
g 6×10^{-4}
h 3×10^{-6}
i 2.71×10^{-5}
j 3.6×10^{-10}



21 Express the following numbers in scientific notation.

a The number of hairs on a person's head is approximately 129 000.

b The distance from Earth to the Sun is 152 000 000 km.

c The diameter of a hydrogen atom is 0.000 000 002 54 cm.

d The size of the influenza virus is 0.000 000 26 m.

e The average speed of Earth around the Sun is 107 000 km/h.

22 Express the following as ordinary numbers.

a The distance of Mars from Earth is 7.83×10^7 km.

b The population of China is approximately 1.4×10^9.

c A human brain cell is 2.8×10^{-5} m long.

d A microsecond is equivalent to 2.8×10^{-10} h.

e The number of cells in the human body is approximately 10^{13}.

B Calculations using scientific notation

EXAMPLE 1

Use the index laws to calculate the following (leave the answers in scientific notation).

a $(3 \times 10^{15}) \times (6 \times 10^{-7})$ **b** $(8 \times 10^{-4}) \div (2 \times 10^{6})$ **c** $(5 \times 10^{7})^3$

	Solve/Think	Apply
a	$(3 \times 10^{15}) \times (6 \times 10^{-7}) = (3 \times 6) \times (10^{15} \times 10^{-7})$ $= 18 \times 10^{15 + (-7)}$ $= 18 \times 10^{8}$ $= 1.8 \times 10^{1} \times 10^{8}$ $= 1.8 \times 10^{9}$	First multiply the numbers, then multiply the powers of 10 using the index laws and express the answer in scientific notation.
b	$(8 \times 10^{-4}) \div (2 \times 10^{6}) = (8 \div 2) \times (10^{-4} \div 10^{6})$ $= 4 \times 10^{-4-6}$ $= 4 \times 10^{-10}$	First divide the numbers, then divide the powers of 10 using the index laws and express the answer in scientific notation.
c	$(5 \times 10^{7})^3 = 5^3 \times (10^{7})^3$ $= 125 \times 10^{21}$ $= 1.25 \times 10^{2} \times 10^{21}$ $= 1.25 \times 10^{23}$	Use the index laws to raise the number to the given power and express the answer in scientific notation.

Exercise 4B

1 Complete the following using the index laws.

a $(4 \times 10^{7}) \times (3 \times 10^{8})$
$= (4 \times 3) \times (\_\_\_ \times \_\_\_)$
$= 12 \times 10^{\square}$
$= 1.2 \times 10^{\square} \times 10^{\square}$
$= 1.2 \times 10^{\square}$

b $(6 \times 10^{3}) \div (4 \times 10^{7})$
$= (6 \div 4) \times (10^{3} \div 10^{7})$
$= 1.5 \times \_\_\_$

c $(2 \times 10^{-4})^5 = 2^5 \times (10^{-4})^5$
$= 32 \times 10^{\square}$
$= 3.2 \times 10^{\square} \times 10^{\square}$
$= 3.2 \times \_\_\_$

2 Use the index laws to calculate the following (leave the answer in scientific notation).

a $(3 \times 10^{8}) \times (4 \times 10^{6})$
b $(8 \times 10^{12}) \times (3 \times 10^{9})$
c $(7 \times 10^{15}) \times (6 \times 10^{-7})$
d $(2 \times 10^{-8}) \times (3 \times 10^{-7})$
e $(5 \times 10^{-9}) \times (4 \times 10^{20})$
f $(9 \times 10^{16}) \div (3 \times 10^{6})$
g $(6 \times 10^{-4}) \div (2 \times 10^{6})$
h $(8 \times 10^{4}) \div (4 \times 10^{16})$
i $(2 \times 10^{5})^3$
j $(7 \times 10^{9})^2$
k $(3 \times 10^{-6})^3$
l $(8 \times 10^{-10})^2$
m $(5 \times 10^{-8}) \times (7 \times 10^{-5})$
n $(3 \times 10^{-6}) \div (4 \times 10^{-2})$

EXAMPLE 2

Use the index laws to approximate the following.

a $(5.31 \times 10^7) \times (6.8 \times 10^9)$ **b** $(7.6 \times 10^{-4}) \div (5.1 \times 10^{-8})$

	Solve	Think	Apply
a	$(5.31 \times 10^7) \times (6.8 \times 10^9)$ $\approx (5 \times 7) \times (10^7 \times 10^9)$ $= 35 \times 10^{16}$ $= 3.5 \times 10^1 \times 10^{16}$ $= 3.5 \times 10^{17}$	5.31 is approximately equal to 5 and 6.8 is approximately equal to 7, then multiply as in Example 1.	Approximate the numbers and then multiply or divide to give the order of magnitude of the answer.
b	$(7.6 \times 10^{-4}) \div (5.1 \times 10^{-8})$ $\approx (8 \div 5) \times (10^{-4} \div 10^{-8})$ $= 1.6 \times 10^4$	7.6 is approximately equal to 8 and 5.1 is approximately equal to 5, then divide as in Example 1.	

3 Use the index laws to approximate the following.

a $(4.9 \times 10^7) \times (7.1 \times 10^5)$ **b** $(2.14 \times 10^{-4}) \times (8.78 \times 10^{-5})$

c $(5.69 \times 10^{10}) \div (4.23 \times 10^5)$ **d** $(9.06 \times 10^{-4}) \div (4.91 \times 10^{-6})$

EXAMPLE 3

Use your calculator to evaluate the following. Leave the answers in scientific notation.

a $(5.3 \times 10^8) \times (7.2 \times 10^{11})$ **b** $(4.8 \times 10^{-15}) \div (1.6 \times 10^7)$

c $(3 \times 10^7)^4$ **d** $\sqrt{5.6 \times 10^{24}}$

	Solve	Think	Apply
a	$(5.3 \times 10^8) \times (7.2 \times 10^{11})$ $= 3.816 \times 10^{20}$	5.3 EXP 8 × 7.2 EXP 11 = The calculator display shows: $\boxed{3.816 \times 10^{20}}$ or $\boxed{3.816^{20}}$ It means 3.816×10^{20} not 3.816 to the power of 20. So $(5.3 \times 10^8) \times (7.2 \times 10^{11}) = 3.816 \times 10^{20}$	Use the appropriate keys on your calculator to enter the numbers and calculate the answers. The EXP button acts as a grouping symbol.
b	$(4.8 \times 10^{-15}) \div (1.6 \times 10^7)$ $= 3 \times 10^{-22}$	4.8 EXP − 15 ÷ 1.6 EXP 7 = This displays $\boxed{3 \times 10^{-22}}$ or $\boxed{3^{-22}}$ So $(4.8 \times 10^{-15}) \div (1.6 \times 10^7) = 3 \times 10^{-22}$	
c	$(3 \times 10^7)^4 = 8.1 \times 10^{29}$	3 EXP 7 x^y 4 = This displays $\boxed{8.1 \times 10^{29}}$ or $\boxed{8.1^{29}}$ So $(3 \times 10^7)^4 = 8.1 \times 10^{29}$	
d	$\sqrt{5.6 \times 10^{24}} = 2.4 \times 10^{12}$	√ 5.76 EXP 24 = This displays $\boxed{2.4 \times 10^{12}}$ or $\boxed{2.4^{12}}$ So $\sqrt{5.6 \times 10^{24}} = 2.4 \times 10^{12}$	

4 Use your calculator to evaluate the following. Leave the answers in scientific notation.

a $(4.8 \times 10^9) \times (3.2 \times 10^{10})$
b $(2.7 \times 10^6) \times (9 \times 10^{12})$
c $(3.6 \times 10^{13}) \times (2.5 \times 10^{-5})$
d $(1.8 \times 10^{-8}) \times (1.5 \times 10^{-10})$
e $(1.2 \times 10^{14}) \div (1.5 \times 10^7)$
f $(3.6 \times 10^{-12}) \div (4.8 \times 10^6)$
g $(8 \times 10^{12}) \div (3.2 \times 10^{-9})$
h $(5.6 \times 10^{-18}) \div (4 \times 10^{-6})$
i $(2 \times 10^8)^4$
j $(5.2 \times 10^{-6})^2$
k $(6 \times 10^{12})^3$
l $(5 \times 10^{-8})^5$
m $\sqrt{6.25 \times 10^{18}}$
n $\sqrt{1.369 \times 10^{-23}}$
o $\sqrt[3]{2.7 \times 10^{19}}$

5 **a** Calculate $(3.6 \times 10^8) - (4.9 \times 10^7)$. Is the answer positive or negative?
b Is (3.6×10^8) bigger or smaller than (4.9×10^7)?

6 **a** Calculate $(7.2 \times 10^5) - (2.6 \times 10^8)$. Is the answer positive or negative?
b Is (7.2×10^5) bigger or smaller than (2.6×10^8)?

7 **a** Calculate $(4.9 \times 10^{-4}) - (5.3 \times 10^{-5})$. Is the answer positive or negative?
b Is (4.9×10^{-4}) bigger or smaller than (5.3×10^{-5})?

EXAMPLE 4

Write each pair of numbers in order from smaller to larger.

a $2.5 \times 10^{16}, 7.8 \times 10^{14}$
b $1.9 \times 10^{-8}, 4.3 \times 10^{-10}$
c $4.8 \times 10^6, 7.8 \times 10^6$

	Solve	Think	Apply
a	$7.8 \times 10^{14} < 2.5 \times 10^{16}$	Compare the powers of 10; $14 < 16$, so $7.8 \times 10^{14} < 2.5 \times 10^{16}$. *Check:* $7.8 \times 10^{14} - 2.5 \times 10^{16}$ $= -2.422 \times 10^{16} < 0$	To compare numbers written in scientific notation, first compare the powers of 10. The number with the smaller power of 10 is the smaller number. If the powers of 10 are the same, compare the numbers by which the powers of 10 are multiplied.
b	$4.3 \times 10^{-10} < 1.9 \times 10^{-8}$	Compare the powers of 10; $-10 < -8$, so $4.3 \times 10^{-10} < 1.9 \times 10^{-8}$. *Check:* $4.3 \times 10^{-10} - 1.9 \times 10^{-8}$ $= -1.857 \times 10^{-8} < 0$	
c	$4.8 \times 10^6 < 7.8 \times 10^6$	The powers of 10 are the same. Compare the first numbers. $4.8 < 7.8$, so $4.8 \times 10^6 < 7.8 \times 10^6$. *Check:* $4.8 \times 10^6 - 7.8 \times 10^6$ $= -3 \times 10^6 < 0$	

8 Complete the following steps to order the numbers from smaller to larger.

a $6.5 \times 10^{-8}, 7.9 \times 10^{-10}$
Compare the powers of 10: ___ < ___, so ___ < ___.

b $2.09 \times 10^6, 5.3 \times 10^6$
Compare the powers of 10: ___ = ___
Compare the first numbers: ___ < ___, so ___ < ___.

9 Write the following numbers in increasing order (from smallest to largest).

a $7.2 \times 10^{15}, 4.6 \times 10^{14}$
b $4.5 \times 10^{16}, 3.4 \times 10^{18}$
c $9.6 \times 10^{-12}, 6.8 \times 10^{-9}$
d $3.8 \times 10^{-6}, 7.8 \times 10^{-8}$
e $2.5 \times 10^{-4}, 7.1 \times 10^5$
f $2.9 \times 10^{16}, 3 \times 10^{16}$
g $8.5 \times 10^{-10}, 6.4 \times 10^{-10}$
h $5.9 \times 10^{16}, 8.1 \times 10^{14}, 2.8 \times 10^{15}$
i $5 \times 10^{-6}, 3.9 \times 10^{-5}, 8.9 \times 10^{-8}$
j $6.3 \times 10^6, 7.8 \times 10^{-5}, 8.3 \times 10^{-3}$

10 Light travels at a speed of 3×10^5 km/s. How far will it travel in 1 h?

11 The star Alpha Centauri is 4.1×10^{13} km from Earth. The distance from Earth to the star Altair is 1.5×10^{14} km. Which star is closer to Earth?

12 The diameter of the hydrogen atom is 2.54×10^{-9} cm. If 1 million hydrogen atoms could be placed next to each other in a straight line, how long would the line be?

13 The average speed of Earth around the Sun is approximately 10^5 km/h. How many days would it take Earth to travel 9.6×10^8 km?

14 Light travels at 3×10^5 km/s and sound travels at 330 m/s. A timekeeper stands at the end of a straight 100 m running track. After the starter fires the starting gun, how long does it take:

- **a** the sight of the smoke to reach the timekeeper?
- **b** the sound of the gun to reach the timekeeper?

15 The area of Australia is approximately 7.7×10^{12} m^2. If the population of Australia in 2026 is expected to be 27 million people, how much land will there be for each person?

C Units of measurement

Below is a table of **prefixes** commonly used to write multiples of units of measurement in words. They are used to avoid writing very large or very small numerical values.

SI stands for Système Internationale, the international system of units.

Multiplying factor	SI prefix	Scientific notation
1 000 000 000 000 000 000 000 000	yotta (Y)	10^{24}
1 000 000 000 000 000 000 000	zetta (Z)	10^{21}
1 000 000 000 000 000 000	exa (E)	10^{18}
1 000 000 000 000 000	peta (P)	10^{15}
1 000 000 000 000	tera (T)	10^{12}
1 000 000 000	giga (G)	10^{9}
1 000 000	mega (M)	10^{6}
1 000	kilo (k)	10^{3}
0.001	milli (m)	10^{-3}
0.000 001	micro (μ)	10^{-6}
0.000 000 001	nano (n)	10^{-9}
0.000 000 000 001	pico (p)	10^{-12}
0.000 000 000 000 001	femto (f)	10^{-15}
0.000 000 000 000 000 001	atto (a)	10^{-18}
0.000 000 000 000 000 000 001	zepto (z)	10^{-21}
0.000 000 000 000 000 000 000 001	yocto (y)	10^{-24}

EXAMPLE 1

Convert the following to metres.

a 5.3 megametres

b 46 nanometres

	Solve	Think	Apply
a	$5.3\text{ Mm} = 5.3 \times 10^6\text{ m}$ (or 5 300 000 m)	From the table, $1\text{ Mm} = 10^6\text{ m}$.	Use the appropriate multiplying factor from the table.
b	$46\text{ nm} = 46 \times 10^{-9}\text{ m}$ $= (4.6 \times 10^1) \times 10^{-9}\text{ m}$ $= 4.6 \times 10^{-8}\text{ m}$ (or 0.000 000 046 m)	From the table, $1\text{ nm} = 10^{-9}\text{ m}$.	

Exercise 4C

1 Complete to convert 7.2 kilolitres and 84 microlitres to litres.

a 7.2 kL = 7.2 × ___ L (or ___ L)

b 84 µL = 84 × __ L = (8.4 × ___) × ___ L
= ___ × ___ L (or ___ L)

2 **a** Convert the following to grams.

i 7 gigagrams

ii 38 nanograms

b Convert the following to joules.

i 3.1 megajoules

ii 124 millijoules

3 **a** Convert the following to watts.

i 4 terawatts

ii 45 microwatts

b Convert the following to grams.

i 9 kilograms

ii 2 gigagrams

EXAMPLE 2

a Convert 12 000 metres to:

i kilometres

ii megametres.

b Convert 0.0006 metres to:

i millimetres

ii micrometres.

		Solve	Think	Apply
a	**i**	$12\,000\text{ m} = 12\,000 \div 10^3\text{ km}$ $= (1.2 \times 10^4) \div 10^3\text{ km}$ $= 1.2 \times 10^1\text{ km} = 12\text{ km}$	$1\text{ km} = 10^3\text{ m}$ How many lots of 10^3 m in 12 000 m?	To convert metres to kilometres or megametres, divide by the number of metres in a kilometre or in a megametre.
	ii	$12\,000\text{ m} = 12\,000 \div 10^6\text{ Mm}$ $= (1.2 \times 10^4) \div 10^6\text{ Mm}$ $= 1.2 \times 10^{-2}\text{ Mm} = 0.012\text{ Mm}$	$1\text{ Mm} = 10^6\text{ m}$ How many lots of 10^6 m in 12 000 m?	
b	**i**	$0.0006\text{ m} = 6 \times 10^{-4} \div 10^{-3}\text{ mm}$ $= (6 \times 10^{-4}) \times 10^3\text{ mm}$ $= 0.6\text{ mm}$	$1\text{ mm} = 10^{-3}\text{ m}$ How many lots of 10^{-3} m in 0.0006 m?	To convert metres to millimetres or micrometres, divide by the number of metres in a millimetre or in a micrometre.
	ii	$0.0006\text{ m} = 6 \times 10^{-4} \div 10^{-6}\text{ µm}$ $= (6 \times 10^{-4}) \times 10^6\text{ µm}$ $= 600\text{ µm}$	$1\text{ µm} = 10^{-6}\text{ m}$ How many lots of 10^{-6} m in 0.0006 m?	

4 Complete the following to convert 26 000 grams to:

a kilograms

26 000 g = 26 000 ÷ ___ kg
= (2.6 × ___) ÷ ___ kg
= ___ kg

b megagrams

26 000 g = (2.6 × 10^4) ÷ ___ Mg
= 2.6 × $10^{\square}$ Mg
= ___ Mg

5 Complete the following to convert 0.000 075 grams to:

a milligrams

0.000 075 g = 0.000 075 ÷ ___ mg
= 0.000 075 × ___ mg
= ___ mg

b nanograms

0.000 075 g = 0.000 075 ÷ ___ ng
= 0.000 075 × ___ ng
= ___ ng

6 a Convert 67 000 000 litres to:

i kilolitres **ii** gigalitres.

b Convert 0.000 000 82 litres to:

i microlitres **ii** picolitres.

7 a Convert 640 000 000 metres to:

i megametres **ii** gigametres.

b Convert 0.000 000 9 metres to:

i centimetres **ii** nanometres.

8 Convert 560 000 joules to:

a kilojoules **b** megajoules.

EXAMPLE 3

a How many kilometres are there in 2.4 gigametres?
b How many picometres are there in 7 millimetres?

Answer in scientific notation. !

	Solve	Think	Apply
a	2.4 Gm = (2.4 × 10^9) ÷ 10^3 km = 2.4 × 10^6 km	1 Gm = 10^9 m and 1 km = 10^3 m How many lots of 10^3 m are there in 2.4 × 10^9 m?	Convert both measurements to the base unit (metres) and divide one by the other, in the appropriate order.
b	7 mm = (7 × 10^{-3}) ÷ 10^{-12} pm = 7 × 10^9 pm	1 mm = 10^{-3} m and 1 pm = 10^{-12} m How many lots of 10^{-12} m are there in 7 × 10^{-3} m?	

9 Complete the following to find how many megametres there are in 1.8 terametres.

1 Tm = ___ m and 1 Mm = ___ m
Hence 1.8 Tm = 1.8 × ___ ÷ ___ Mm = ___ Mm

10 How many:

a kilometres in 3.6 megametres? **b** nanowatts in 5.2 milliwatts? **c** megalitres in 7.4 teralitres?
d picolitres in 8.9 nanolitres? **e** megagrams in 4.8 petagrams? **f** kilojoules in 1.8 gigajoules?

11 Convert the following.

a 7 milliwatts into microwatts **b** 2 kilograms to milligrams **c** 5 millilitres to microlitres
d 1.8 kilojoules to millijoules **e** 17 centimetres to micrometres **f** 300 megalitres to teralitres

12 The average distance from Earth to the Sun is 149.5 gigametres. Convert this measurement to:

a terametres

b kilometres.

13 The mass of Earth is approximately 597 yottagrams. Convert this to:

a kilograms

b zettagrams.

14 The diameter of an electron is approximately 5.6 femtometres. Convert this to:

a millimetres

b nanometres.

15 The standard unit of measurement for digital information is the kilobyte.

1 kilobyte (kB or KB) = 1024 (= 2^{10}) bytes

a Using this definition, 1 megabyte (MB) = 1024^2 bytes and 1 gigabyte (GB) = 1024^3 bytes. Convert:

- **i** 1 MB to kB
- **ii** 1 GB to MB
- **iii** 1 GB to kB

b Given that 1 terabyte (TB) = 1024^4 bytes, convert 1 TB to:

- **i** kB
- **ii** MB
- **iii** GB

c Convert the following to bytes (as a power of 2).

- **i** 1 MB
- **ii** 1 GB
- **iii** 1 TB

Investigation 1 Kilobytes

As 1024 ≈ 1000, 1 kilobyte is regarded as 1000 bytes in some parts of the information and technology industry.

Research and write a short report on the history and use of the base 2 definition and the base 10 definition of a kilobyte.

D Measurement of time

The basic unit of time in the international system of units is the **second** (s). Smaller units can be described using the prefixes in the table in Section 4C above. The most common larger units used are shown in the table.

1 minute (min)	60 seconds
1 hour (h)	60 minutes
1 day (d)	24 hours
1 week	7 days
1 fortnight	2 weeks
1 year	365 days (a leap year has 366 days) 52 weeks and 1 day 12 months
1 decade	10 years
1 century	100 years
1 millennium	1000 years

EXAMPLE 1

Convert the following.

a 3.6 seconds to milliseconds

b 5900 picoseconds to nanoseconds

	Solve	Think	Apply
a	$3.6 \text{ s} = 3.6 \div 10^{-3} \text{ ms}$ $= 3.6 \times 10^{3} \text{ ms (or 3600 ms)}$	$1 \text{ ms} = 10^{-3} \text{ s}$ How many lots of 10^{-3} s in 3.6 s?	Convert both measurements to the base unit (seconds) and divide one by the other, in the appropriate order.
b	$5900 \text{ ps} = 5900 \times 10^{-12} \div 10^{-9} \text{ ns}$ $= (5.9 \times 10^{-9}) \div 10^{-9} \text{ ns}$ $= 5.9 \times 10^{0} \text{ ns} = 5.9 \text{ ns}$	$1 \text{ ps} = 10^{-12}$ s and $1 \text{ ns} = 10^{-9}$ s. How many lots of 10^{-9} s in 5900×10^{-12} ps?	

Exercise 4D

1 Convert the following times.

a 2.9 seconds to milliseconds

b 730 microseconds to milliseconds

c 7400 picoseconds to nanoseconds

d 3.7 nanoseconds to femtoseconds

e 14 500 attoseconds to femtoseconds

f 684 nanoseconds to milliseconds

EXAMPLE 2

Convert the following.

a 1 day to seconds

b 360 000 years to millennia

	Solve	Think	Apply
a	$1 \text{ day} = 24 \times 60 \times 60 \text{ s}$ $= 86\,400 \text{ s}$	1 day = 24 hours 1 hour = 60 minutes 1 minute = 60 seconds	Convert 1 day into hours, hours into minutes and minutes into seconds.
b	$360\,000 \text{ years} = 360\,000 \div 1000 \text{ millennia}$ $= 360 \text{ millennia}$	1 millennium = 1000 years	Divide the number of years by the number of years in 1 millennium.

2 Complete the following time conversions.

a 3 weeks to hours

3 weeks $= 3 \times 7 \times 24$ h

$= \_\_\_$ h

b 3888 minutes to days

3888 min $= 3888 \div \_\_\_$ h

$= 64.8$ h

$= \_\_\_ \div \_\_\_$ days

$= \_\_\_$ days

Alternatively 3888 min $= 3888 \div (\_\_\_ \times 24)$ days

$= 2.7$ days

3 Convert the following.

a 5 weeks to seconds

b 1 fortnight to minutes

c 3 centuries to months

d 5400 years to centuries

e 756 years to decades

f 250 years to weeks

4 The age of Earth is thought to be approximately 143 petaseconds. How many years is this?

5 A honey bee takes 5 milliseconds to flap its wings once. How many times would it flap its wings if it was in flight for 10 seconds?

E Approximations

Consider these situations.

1 \$1600 is to be shared between seven people and each share deposited in a bank account.
\$1600 ÷ 7 = \$228.571 428 …
It is not possible to deposit this exact amount in a bank account, since the smallest unit of money we can deposit is a cent. Each share is closer to \$228.57 than to \$228.58, so we would deposit \$228.57 into each person's account.

2 A piece of timber 2600 mm long has to be cut into three equal lengths: 2600 mm ÷ 3 = 866.666 … mm
It is not possible to cut a piece of timber exactly this long, since the smallest unit of measurement we are likely to have on a tape measure is a millimetre. This length is closer to 867 mm than 866 mm, so we would measure and cut each piece of timber to be 867 mm.

In both of these examples we have approximated the result of a calculation to make the answer meaningful.

This process of approximating numbers is also called ***rounding***.

EXAMPLE 1

Write down the value of the digit 7 in each number.

a 273.6 **b** 1407.2 **c** 86.457 **d** 2764

	Solve/Think	Apply
a	7 tens or 7×10 (= 70)	The value of the digit depends on its place in the number.
b	7 units or 7×1 (= 7)	
c	7 thousandths or $7 \times \frac{1}{1000}\ (= \frac{7}{1000})$	
d	7 hundreds or 7×100 (= 700)	

Exercise 4E

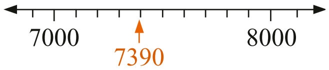
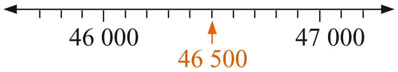
1 Write the value of the digit 6 in each number.

a 465.9	**b** 2346.1	**c** 3698	**d** 6284	**e** 16 382 000
f 12.836	**g** 5.698	**h** 30.562	**i** 20 600	**j** 0.0006

EXAMPLE 2

Round each number to the nearest thousand.

a 7390 **b** 24 830 **c** 46 500

	Solve	Think/Apply	Apply
a	7390 ≈ 7000	‘Round 7390 to the nearest thousand’ means: is 7390 closer to 7000 or 8000? Draw part of a number line showing thousands. Use the digit in the hundreds column (3) to find the approximate position of 7390. We can see that it is closer to 7000.  So 7390 ≈ 7000, to the nearest thousand.	Locate the position of the number on a number line and determine to which ‘thousand’ it is closer. *The symbol ≈ means ‘approximately equal to’.*
b	24 830 ≈ 25 000	‘Round 24 830 to the nearest thousand’ means: is 24 830 closer to 24 000 or 25 000? Draw part of a number line showing thousands. Use the digit in the hundreds column (8) to find the approximate position of 24 830. We can see that it is closer to 25 000.  So 24 830 ≈ 25 000, to the nearest thousand.	
c	46 500 ≈ 47 000	‘Round 46 500 to the nearest thousand’ means: is 46 500 closer to 46 000 or 47 000? Draw part of a number line showing thousands. Use the digit in the hundreds column (5) to find the position of 46 500. We know that it is exactly in the middle of 46 000 and 47 000. **By convention**, we round to 47 000.  So 46 500 ≈ 47 000, to the nearest thousand.	*‘By convention’ means everyone agrees to do it this way.*

To round a number to the nearest thousand, first locate the digit in the thousands column.

- If the digit to the right of the thousands column is smaller than 5, retain the thousands digit and replace the digits to the right of it by zeros. This is called **rounding down**.
- If the digit to the right of the thousands column is bigger than 5, increase the thousands digit by 1 and replace the digits to the right of it by zeros. This is called **rounding up**.
- By convention, if the digit to the right of the thousands column is equal to 5, round up.

EXAMPLE 3

Round each number to the nearest 1000.

a 874 296 **b** 28 741 **c** 2520

'To the nearest 1000' is known as the ***level of accuracy*** *of the answer.* ❗

	Solve	Think	Apply
a	874 296 ≈ 874 000	The digit in the thousands column is 4. The digit to the right of it is 2, which is smaller than 5. So we replace all the digits to the right of 4 with zeros. 874 296 ≈ 874 000, to the nearest 1000.	If the digit to the right of the thousands column is smaller than 5 then round down. If the digit to the right of the thousands column is bigger than or equal to 5 then round up.
b	28 741 ≈ 29 000	The digit in the thousands column is 8. The digit to the right of it is 7, which is bigger than 5. So we increase the thousands digit by 1 and replace all the digits to the right of 9 with zeros. 28 741 ≈ 29 000, to the nearest 1000.	
c	2520 ≈ 3000	The digit in the thousands column is 2. The digit to the right of it is 5. So we increase the thousands digit by 1 and replace the digits to the right of it by zeros. 2520 ≈ 3000, to the nearest 1000.	

2 Complete the following to round each number to the nearest 1000.

a 536 428: The digit in the thousands column is ___. The digit to the right of it is ___.
Hence, to the nearest thousand 536 428 ≈ ___.

b 57 856: The digit in the thousands column is ___. The digit to the right of it is ___.
Hence, to the nearest thousand 57 856 ≈ ___.

c 7517: The digit in the thousands column is ___. The digit to the right of it is ___.
Hence, to the nearest thousand 7517 ≈ ___.

3 Round each number to the nearest thousand.

a 36 800 **b** 83 500 **c** 524 100 **d** 8299
e 18 560 **f** 623 490 **g** 180 524 **h** 6287
i 2999 **j** 400 721 **k** 760 **l** 390

EXAMPLE 4

Round each number.

a 2380 to the nearest 100 **b** 7862 to the nearest 10 **c** 28.5 to the nearest whole number

	Solve	Think	Apply
a	2380 ≈ 2400	The digit in the hundreds column is 3. The digit to the right of it is 8, which is bigger than 5. So round up. 2380 ≈ 2400, to the nearest 100.	Locate the digit in the column to which the number is to be rounded. If the digit to the right of it is bigger than or equal to 5 then round up.

EXAMPLE 4 CONTINUED

	Solve	Think	Apply
b	7862 ≈ 7860	The digit in the tens column is 6. The digit to the right of it is 2, which is smaller than 5. So round down and leave it as 6. 7862 ≈ 7860, to the nearest 10.	If the digit to the right of it is smaller than 5 then round down, thus leaving it the same.
c	28.5 ≈ 29	The digit in the units column is 8. The digit to the right of it is 5. So round up. 28.5 ≈ 29, to the nearest whole number.	If the digit to the right of it is equal to 5 then round up.

4 Complete the following to round each number.

a 4690 to the nearest 100: The digit in the hundreds column is ___. The digit to the right of it is ___.
Hence, to the nearest hundred 4690 ≈ ___.

b 5125 to the nearest 10: The digit in the tens column is___. The digit to the right of it is ___.
Hence, to the nearest ten 5125 ≈ ___.

c 148.3 to the nearest whole number:
The digit in the units column is ___. The digit to the right of it is ___.
Hence, to the nearest whole number 148.3 ≈ ___.

5 Round each number to the nearest 100.

a 5360 **b** 16 829 **c** 20 421 **d** 849 **e** 369 **f** 240 230
g 3075 **h** 9628 **i** 450 **j** 147 **k** 86 **l** 29

6 Round each number to the nearest 10.

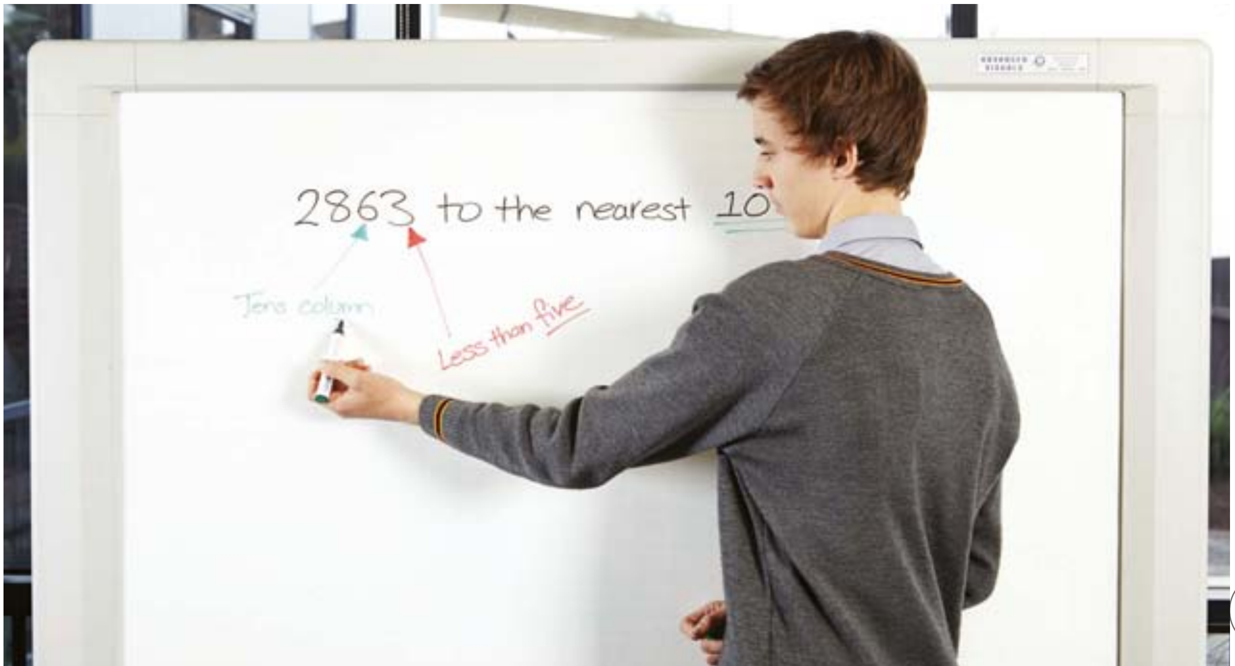
a 674 **b** 2368 **c** 825 **d** 1056 **e** 73 **f** 28
g 306 **h** 20 056 **i** 409 **j** 1251 **k** 8 **l** 4

7 Round each number to the nearest whole number.

a 16.7 **b** 25.3 **c** 81.5 **d** 236.67 **e** 583.1 **f** 265.07
g 20.9 **h** 106.28 **i** 300.7 **j** 55.5 **k** 0.71 **l** 0.36

8 Round each number below to the nearest:

i 1000 **ii** 100 **iii** 10 **iv** 1
a 46 783.5 **b** 28 456.7 **c** 39 165.5 **d** 8462.3 **e** 182 678.5



EXAMPLE 5

Round 12.3815 to:

a 1 decimal place **b** 2 decimal places **c** 3 decimal places.

	Solve	Think	Apply
a	12.3815 ≈ 12.4	The digit in the first column after the decimal point is 3. The digit to the right of it is 8, which is bigger than 5. So we round up the 3 and delete any digits to the right of it. 12.3815 ≈ 12.4 correct to 1 decimal place.	Locate the digit in the column to which the number is to be rounded. If the digit to the right of it is smaller than 5 then round down. If the digit to the right of it is bigger than or equal to 5 then round up.
b	12.3815 ≈ 12.38	The digit in the second column after the decimal point is 8. The digit to the right of it is 1, which is smaller than 5. So we round down to 8 and delete any digits to the right of it. 12.3815 ≈ 12.38 correct to 2 decimal places.	
c	12.3815 ≈ 12.382	The digit in the third column after the decimal point is 1. The digit to the right of it is 5. So we round up the 1 and delete any digits to the right of it. 12.3815 ≈ 12.382 correct to 3 decimal places.	

9 Complete the following.

a Round 63.5205 to 1 decimal place.
The digit in the first column after the decimal point is ___.
The digit to the right of it is ___.
Hence, to 1 decimal place 63.5205 ≈ ___.

b Round 63.5205 to 2 decimal places.
The digit in the second column after the decimal point is ___.
The digit to the right of it is ___.
Hence, to 2 decimal places 63.5205 ≈ ___.

c Round 63.5205 to 3 decimal places.
The digit in the third column after the decimal point is ___.
The digit to the right of it is ___.
Hence, to 3 decimal places 63.5205 ≈ ____.

10 Round 38.2683 to:

a 1 d.p. **b** 2 d.p. **c** 3 d.p.

1 decimal place can be abbreviated to 1 d.p.

11 Round each number below to:

i 1 d.p. **ii** 2 d.p. **iii** 3 d.p.

a 8.4382 **b** 6.5839 **c** 0.8625 **d** 0.1864 **e** 18.5555
f 21.6029 **g** 4.0611 **h** 5.0437 **i** 7.0069 **j** 3.0002

EXAMPLE 6

a Round 2.497 to 2 decimal places.
b Round 19.96 to 1 decimal place.

When rounding up to a zero, the last zero must be shown in order to indicate the level of accuracy of the approximation.

	Solve	Think	Apply
a	$2.497 \approx 2.50$	The digit in the second column after the decimal point is 9. The digit to the right of it is 7, which is bigger than 5. So round up the 9 and delete any digits to the right of it. $2.497 \approx 2.50$ correct to 2 decimal places.	When rounding up a 9, a zero must take its place.
b	$19.96 \approx 20.0$	The digit in the first column after the decimal point is 9. The digit to the right of it is 6, which is bigger than 5. So round up the 9 and delete any digits to the right of it. $19.96 \approx 20.0$ correct to 1 decimal place.	

12 Round each number to the given number of decimal places.

a 3.598 to 2 d.p.	**b** 49.96 to 1 d.p.	**c** 2.6895 to 3 d.p.	**d** 12.997 to 2 d.p.
e 0.996 to 2 d.p.	**f** 4.8997 to 3 d.p.	**g** 99.98 to 1 d.p.	**h** 69.995 to 2 d.p.

EXAMPLE 7

When a number was rounded to the nearest 10, the answer was 60.
a What is the smallest the number could have been?
b What is the largest the number could have been? Discuss.
c Write a mathematical statement that shows the range of possible numbers.

	Solve	Think	Apply
a	55	55 is halfway between 50 and 60, but by convention it is rounded up to 60. This is the smallest the number could have been.	If the answer is N when a number is rounded to the nearest k, the smallest the number could have been is $N - \frac{k}{2}$, and the largest the number could have been is less than $N + \frac{k}{2}$. $N - \frac{k}{2} \leqslant \text{number} < N + \frac{k}{2}$
b	<65	We cannot specify the largest number, but we do know that it has to be less than 65 (because 65 would be rounded up to 70).	
c	$55 \leqslant \text{number} < 65$	The number could be equal to 55, or between 55 and 65. This is written as $55 \leqslant \text{number} < 65$.	

13 When a number was rounded to the nearest 10, the answer was 80.
a What is the smallest the number could have been?
b What is the largest the number could have been?
c Write a mathematical statement that shows the range of possible numbers.

14 When a number was rounded to the nearest 100, the answer was 400.

- **a** What is the smallest the number could have been?
- **b** What is the largest the number could have been?
- **c** Write a mathematical statement that shows the range of possible numbers.

15 Consider parts **a** to **d** below.

- **i** What is the smallest the number could have been?
- **ii** What is the largest the number could have been?
- **iii** Write a mathematical statement that shows the range of possible numbers.

- **a** When the number was rounded to the nearest 1000, the answer was 28 000.
- **b** When the number was rounded to the nearest 1 (whole number), the answer was 43.
- **c** When the number was rounded to 1 decimal place, the answer was 5.7.
- **d** When the number was rounded to 2 decimal places, the answer was 6.32.

16 Emily was measured to be 163 cm tall, to the nearest centimetre. Within what range of values does her actual height lie?

17 The weight of a can of fruit was measured as 420 g, to the nearest 10 g. Within what range does the actual weight of the can lie?

18 The time taken for Ken to complete the 100 m sprint at the athletics carnival was 12.4 s, to the nearest tenth of a second. Within what range does his actual time lie?

F Significant figures

A method that combines the rounding techniques shown in Section 4E of this chapter involves the use of **significant figures**.

All digits that are not a zero are significant figures. The first significant figure in a number is the first digit that is not a zero (reading from left to right). Zeros between non-zero digits are significant. Zeros at the end of a number may or may not be significant.

EXAMPLE 1

Write down the first significant figure in each of these numbers.

a 3790 **b** 4.0625 **c** 0.002 86

	Solve/Think	Apply
a	The first digit that is not a zero is the 3.	The first significant figure in a number is the first non-zero digit.
b	The first digit that is not a zero is the 4.	
c	The first digit that is not a zero is the 2.	

Exercise 4F

1 Write down the first significant figure in each of the following numbers.

a 2876 **b** 5 069 836 **c** 1.0035 **d** 0.0791 **e** 0.000 802

EXAMPLE 2

Round 63.750 91 correct to the following number of significant figures.

a 1 **b** 2 **c** 3 **d** 4 **e** 5

	Solve	Think	Apply
a	63.750 91 ≈ 60	The first significant figure is 6, which is in the tens column. So we round to the nearest 10. 63.750 91 ≈ 60 correct to 1 significant figure.	If rounding to *n* significant figures, find the *n*th significant figure and determine the place value of the digits in this column (hundreds, tens, 2 decimal places). Round to this place value. The standard abbreviation for writing significant figures is s.f.
b	63.750 91 ≈ 64	The second significant figure is 3, which is in the units column. So we round to the nearest 1 (whole number). 63.750 91 ≈ 64 correct to 2 significant figures.	
c	63.750 91 ≈ 63.8	The third significant figure is 7, which is in the first place after the decimal point. So we round to 1 decimal place. 63.750 91 ≈ 63.8 correct to 3 significant figures.	
d	63.750 91 ≈ 63.75	The fourth significant figure is 5, which is in the second place after the decimal point. So we round to 2 decimal places. 63.750 91 ≈ 63.75 correct to 4 significant figures.	
e	63.750 91 ≈ 63.751	The fifth significant figure is 0, which is in the third place after the decimal point. So we round to 3 decimal places. 63.750 91 ≈ 63.751 correct to 5 correct significant figures.	

2 Complete the following to round 28.470 58 correct to the given level of accuracy.

Significant figures may be abbreviated to s.f.

- **a** The first significant figure is the ___, which is in the ___ column. So we round to the nearest ___.
  28.470 58 ≈ ___ correct to 1 s.f.
- **b** The second significant figure is the ___, which is in the ___ column. So we round to the nearest ___.
  28.470 58 ≈ ____ correct to 2 s.f.
- **c** The third significant figure is the ___, which is in the ___ place after the decimal point.
  So we round to ___.
  28.470 58 ≈ ___ correct to 3 s.f.
- **d** The fourth significant figure is the ___, which is in the ___ place after the decimal point.
  So we round to ___.
  28.470 58 ≈ ___ correct to 4 s.f.
- **e** The fifth significant figure is the ___, which is in the ___ place after the decimal point.
  So we round to ___.
  28.470 58 ≈ ___ correct to 5 s.f.

3 Round each number below to:

i 1 s.f. **ii** 2 s.f. **iii** 3 s.f.

a 428.3 **b** 6238 **c** 7.819 **d** 0.5273 **e** 53 689
f 725 600 **g** 0.039 26 **h** 0.005 072 **i** 6103 **j** 2005

EXAMPLE 3

Write each of the following correct to 3 significant figures.

a 249 700 **b** 629.51 **c** 0.001 896 **d** 6.998

	Solve	Think	Apply
a	249 700 ≈ 250 000	The third significant figure is 9 in the 1000s column. So we round to the nearest 1000. 249 700 ≈ 250 000 correct to 3 s.f.	Find the *n*th significant figure and determine the place value of the digit in this column (hundreds, tens, 2 decimal places). Round to this place value.
b	629.51 ≈ 630	The third significant figure is 9 in the units column. So we round to the nearest whole number. 629.51 ≈ 630 correct to 3 s.f.	
c	0.001 896 ≈ 0.001 90	The third significant figure is 9 in the fifth place after the decimal point. So we round to 5 decimal places. 0.001 896 ≈ 0.001 90 correct to 3 s.f.	*In parts **c** and **d** the zeros at the end are there to indicate the level of accuracy of the answer.*
d	6.998 ≈ 7.00	The third significant figure is 9 in the second place after the decimal point. So we round to 2 decimal places. 6.998 ≈ 7.00 correct to 3 s.f.	

4 Write each of the following correct to 3 significant figures.

a 369 800 **b** 239.6 **c** 0.005 798 **d** 8.997 **e** 299 700
f 499.7 **g** 0.039 98 **h** 0.299 9 **i** 0.001 999 **j** 999 900

EXAMPLE 4

When a number was rounded to 2 significant figures, the answer was:

a 430 **b** 3.7

i What is the smallest the number could have been?

ii What is the largest the number could have been?

iii Write a mathematical statement that shows the range of possible numbers.

	Solve	Think	Apply
a i	425	The second significant figure is in the tens column, hence the number has been rounded to the nearest 10. Although 425 is halfway between 420 and 430, it is rounded up to 430. This is the smallest the number could have been.	Find the place value of the *n*th significant figure. This indicates how the number has been rounded (to the nearest 100, 10, …, 2 decimal places). Complete as for Example 7 in Section 4E.
ii	<435	We cannot specify the largest number, but we know that it has to be less than 435, as 435 would be rounded up to 440.	
iii	$425 \leqslant$ number < 435	The number could be equal to 425 or between 425 and 435.	
b i	3.65	The second significant figure is in the first column after the decimal point, hence the number has been rounded to 1 decimal place. Although 3.65 is halfway between 3.6 and 3.7, it is rounded up to 3.7. This is the smallest the number could have been.	
ii	<3.75	We cannot specify the largest number, but we do know that it has to be less than 3.75, as 3.75 would be rounded up to 3.8.	
iii	$3.65 \leqslant$ number < 3.75	The number could be equal to 3.65 or between 3.65 and 3.75.	

5 When a number was rounded to 2 significant figures the answer was:

a 560 **b** 8.2 **c** 48 **d** 0.72 **e** 37 000 **f** 0.084

i What is the smallest the number could have been?

ii What is the largest the number could have been?

iii Write a mathematical statement that shows the range of possible numbers.

6 When a number was rounded to 3 significant figures the answer was:

a 483 **b** 3.86 **c** 14 500 **d** 0.128 **e** 56.9 **f** 3210

Write a mathematical statement that shows the range of possible numbers in each case.

7 Write a mathematical statement that shows the range of possible numbers if each of the following numbers was rounded to the given number of significant figures.

a 2 s.f. the answer is 300 **b** 2 s.f. the answer is 3000

c 3 s.f. the answer is 6000 **d** 3 s.f. the answer is 24 000

e 3 s.f. the answer is 500 000 **f** 2 s.f. the answer is 0.80

EXAMPLE 5

State the number of significant figures in each of the following numbers.

a 294 **b** 0.3 **c** 4.20 **d** 0.0017 **e** 56 000

	Solve	Think	Apply
a	3	There are 3 digits in the number 294.	For decimal numbers, zeros in front of the first non-zero digit are not significant, zeros after the first non-zero digit are significant. For integers (whole numbers), zeros on the end of the number may or may not be significant.
b	1	The first significant figure in 0.3 is the first non-zero digit. Hence the first zero is not significant.	
c	3	The zero on the end of this number indicates it has been rounded to 2 decimal places. Hence the zero in 4.20 is significant.	
d	2	The first significant figure in 0.0017 is the first non-zero digit. Hence the first three zeros are not significant.	
e	Cannot tell precisely.	The zeros on the end may or may not be significant. 56 300 rounded to the nearest 1000 ≈ 56 000. 55 970 rounded to the nearest 100 ≈ 56 000. 56 003 rounded to the nearest 10 ≈ 56 000. 55 999.6 rounded to the nearest whole number ≈ 56 000. Hence there could be 2, 3, 4 or 5 significant figures.	

8 How many significant figures are there in each of the following numbers?

a 38 **b** 0.49 **c** 2896 **d** 0.075 **e** 0.40 **f** 1.800
g 0.0053 **h** 0.060 **i** 400 **j** 7000 **k** 23 000 **l** 8 000 000

G Calculations and rounding numbers

Exercise 4G

1 **a** Calculate 12 853 + 678 + 15 692. Round the answer to the nearest 100.
b Round the following to the nearest 100.
i 12 853 **ii** 678 **iii** 15 692
c Find the sum of the three answers in part **b**.
d Compare the answers to parts **a** and **c**. Comment.

2 **a** Calculate 8085 − 2834 to the nearest 10.
b Round the following to the nearest 10:
i 8085 **ii** 2834
c Find the difference between the answers in part **b**.
d Compare the answers to parts **a** and **c**. Comment.

3 **a** Multiply 2.341 by 8 and round the answer to 1 decimal place.
b Write 2.341 correct to 1 decimal place.
c Multiply the answer to part **b** by 8.
d Compare the answers to parts **a** and **c**. Comment.

4 **a** Calculate 4.83×29.68 and give the answer correct to 1 decimal place.
b Round each of the numbers in part **a** to 1 decimal place and then perform the multiplication, giving the answer correct to 1 decimal place.
c Compare the answers to parts **a** and **b**. Comment.
d Truncate each number in part **a** correct to 1 decimal place and then perform the multiplication, giving the answer correct to 1 decimal place.
e Compare the answers to parts **a** and **d**. Comment.

Truncate to 1 decimal place means to cut off all digits after the first decimal place (without rounding).

5 **a** Calculate $32.683 + 16.87 + 25.619$.
b Round the answer to part **a** to 2 significant figures.
c Round the following to 2 significant figures.
 i 32.683 **ii** 16.87 **iii** 25.619
d Find the sum of the three answers in part **c** correct to 2 significant figures.
e Compare the answers to parts **b** and **d**. Comment.

6 **a** Multiply 3.5 by 17 and round the answer to 1 significant figure.
b Write 3.5 and 17 correct to 1 significant figure.
c Multiply the numbers in part **b** and give the answer correct to 1 significant figure.
d Compare the answers to parts **a** and **c**. Comment.

7 **a** Calculate $16.358 \div 7.499$, giving the answer correct to 3 significant figures.
b Find the following correct to 3 significant figures.
 i 16.358 **ii** 7.499
c Divide the answers in **b** and give the answer correct to 3 significant figures.
d Compare the answers to parts **a** and **c**. Comment.

8 **a** Multiply 32.86 by 128.57, giving the answer correct to 3 significant figures.
b Round each of the numbers in part **a** to 3 significant figures and then perform the multiplication, giving the answer correct to 3 significant figures.
c Compare the answers to **a** and **b**. Comment.

9 **a** Calculate $145 \div 784$, giving the answer correct to 2 significant figures.
b Round each of the numbers in part **a** to 2 significant figures and then perform the division, giving the answer correct to 2 significant figures.
c Compare the answers to **a** and **b**. Comment.

10 **a** Find $\sqrt{11}$ correct to 2 significant figures.
b Multiply the answer in part **a** by itself (square the answer). Is the result 11?
c Repeat parts **a** and **b** using:
 i 3 s.f. **ii** 4 s.f. **iii** 5 s.f. **iv** 6 s.f.

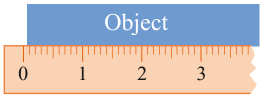
It should be clear that rounding numbers during a calculation often affects the accuracy of the result. So never round before the last step of a calculation.

11 The value of π is 3.141 592 65 correct to 9 significant figures. Calculate and comment on the accuracy of the following approximations for π. To how many significant figures is the approximation accurate?
a $\frac{22}{7}$ (Round the value of $\frac{22}{7}$ to 1, 2, 3, … significant figures and compare with the value of π.)
b $\frac{355}{113}$ **c** $\left(\frac{553}{312}\right)^2$ **d** $\sqrt[3]{31}$ **e** $\sqrt{9.87}$

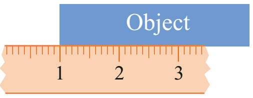
H Error in measurement

When physically measuring a quantity there are several sources of possible error and uncertainty.

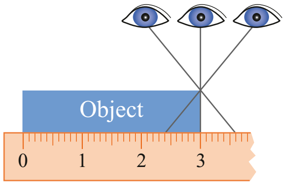
- Errors occur if the zero on the scale of the measuring instrument does not coincide with the end of the object or with the pointer on the measuring instrument.



- An error occurs if the end of the measuring instrument has been damaged. In this case start measuring from the 1, say, instead of 0.



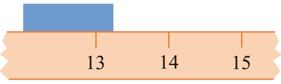
- Parallax error occurs if your eye is not directly above the scale on the measuring instrument.
- Calibration error can occur if the scale is not accurately marked on the measuring instrument.
- There is always an error due to the limit of reading the measuring instrument.
- Repeating a measurement a number of times and averaging the values can reduce the effect of most errors.



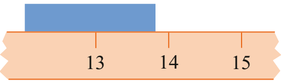
Investigation 2 Accuracy of measures

1 The diagrams below show several steel rods being measured with a ruler. Write down the length of each rod, using the scale given on the ruler.

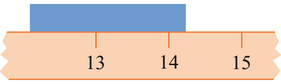
a



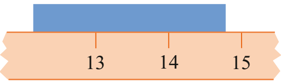
b



c



d



2 The length of a rod is measured using the ruler above, and the measurement is recorded as 14 cm.

a Would this be the exact length of the rod?

b Between what values would the actual length lie?

c What is the greatest possible error in stating that the length is 14 cm?

d How could we find a more accurate value of the length of the rod?

The rod in Investigation 2 question **2** has been measured to the nearest centimetre because this is the smallest unit on the ruler. Thus the length is closer to 14 cm than to 13 cm or 15 cm. The greatest possible error is 0.5 cm. This is half of the smallest scale (centimetres) on the ruler.

The smallest the true measurement can be is 14 cm − 0.5 cm = 13.5 cm. We cannot specify exactly what the largest the true measurement can be, but it must be less than 14 cm + 0.5 cm = 14.5 cm (14.5 would be rounded to 15), so 13.5 cm $\leqslant$ true length $<$ 14.5 cm. The lengths 13.5 cm and 14.5 cm are called the **limits of accuracy** of the measurement.

To obtain a more accurate measurement, we would need to use a more accurate ruler; that is, a ruler with smaller units on it. The smallest unit on a measuring instrument is called the **limit of reading** of the instrument.

The greatest possible error in measuring a quantity (sometimes called the **absolute error**) is equal to half the limit of reading.

EXAMPLE 1

For each of the following measurements, find:

- **i** the smallest unit of measurement (the limit of reading)
- **ii** the greatest possible error.

a 18 cm **b** 254 g **c** 2.4 kg **d** 12.6 s

		Solve	Think	Apply
a	**i**	Limit of reading = 1 cm	The measurement has been made to the nearest centimetre, so the smallest unit of measurement is 1 cm.	The limit of reading is the smallest unit on the measuring instrument. The greatest possible error is equal to half the limit of reading.
	ii	Greatest possible error = 0.5 cm	Greatest possible error $= \frac{1}{2} \times 1$ cm $= 0.5$ cm	
b	**i**	Limit of reading = 1 g	The measurement has been made to the nearest gram, so the smallest unit of measurement is 1 g.	
	ii	Greatest possible error = 0.5 g	Greatest possible error $= \frac{1}{2} \times 1$ g $= 0.5$ g	
c	**i**	Limit of reading = 0.1 kg	The measurement has been made to the nearest 0.1 (tenth) of a kilogram, so the smallest unit of measurement is 0.1 kg.	
	ii	Greatest possible error = 0.05 kg	Greatest possible error $= \frac{1}{2} \times 0.1$ kg $= 0.05$ kg	
d	**i**	Limit of reading = 0.1 s	The measurement has been made to the nearest 0.1 of a second, so the smallest unit of measurement is 0.1 s.	
	ii	Greatest possible error = 0.05 s	Greatest possible error $= \frac{1}{2} \times 0.1$ s $= 0.05$ s	

Exercise 4H

1 For each of the following measurements find:

i the smallest unit of measurement (the limit of reading)

ii the greatest possible error (absolute error).

a 16 cm **b** 286 g **c** 38 m **d** 14 min **e** 16 L

f 3.6 kg **g** 15.3 s **h** 2.8 m **i** 8.1 L **j** 3.76 m

EXAMPLE 2

For each of the measurements below:

i Find the limit of reading of the measuring instrument.

ii Find the greatest possible error.

iii Find the limits of accuracy.

iv Write a mathematical statement showing the range of values within which the true measurement lies.

a 16 s **b** 5.7 kg **c** 9.38 m

	Solve/Think	**Apply**
a i	This measurement of time has been made to the nearest second, so the smallest unit on the measuring instrument is seconds. Limit of reading $= 1$ s	The limits of accuracy $=$ measurement $\pm$ greatest possible error. Hence: measurement $-$ greatest possible error $\leqslant$ true measurement $<$ measurement $+$ greatest possible error.
ii	Greatest possible error $= \frac{1}{2} \times 1 \text{ s} = 0.5 \text{ s}$	
iii	Limits of accuracy $= 16 \pm 0.5$ s $= 15.5$ s, 16.5 s	
iv	$15.5 \text{ s} \leqslant \text{true time} < 16.5 \text{ s}$	
b i	This measurement of mass has been made to the nearest 0.1 of a kilogram, so the smallest unit on the measuring instrument is 0.1 kg. Limit of reading $= 0.1$ kg	
ii	Greatest possible error $= \frac{1}{2} \times 0.1 \text{ kg} = 0.05 \text{ kg}$	
iii	Limits of accuracy $= 5.7 \pm 0.05$ kg $= 5.65$ kg, 5.75 kg	
iv	$5.65 \text{ kg} \leqslant \text{true mass} < 5.75 \text{ kg}$	
c i	This measurement of length has been made to the nearest 0.01 of a metre, so the smallest unit on the measuring instrument 0.01 m. Limit of reading $= 0.01$ m	
ii	Greatest possible error $= \frac{1}{2} \times 0.01 \text{ m} = 0.005 \text{ m}$	
iii	Limits of accuracy $= 9.38 \pm 0.005$ m $= 9.375$ m, 9.385 m	
iv	$9.375 \text{ m} \leqslant \text{true length} < 9.385 \text{ m}$	

2 Complete the following for a measurement of 18 kg.

- **a** This measurement of mass has been made to the nearest ___, so the smallest unit on the measuring instrument is ___. Limit of reading = ___
- **b** The greatest possible error = $\frac{1}{2}$ × ___ = ___
- **c** Limits of accuracy = 18 ± ___
  = ___, ___
- **d** ___ ≤ true mass < ___

3 Complete the following for a measurement of 9.4 m.

- **a** This measurement of length has been made to the nearest ___, so the smallest unit on the measuring instrument is ___. Limit of reading = ___
- **b** The greatest possible error = $\frac{1}{2}$ × ___ = ___
- **c** Limits of accuracy = 9.4 ± ___
  = ___, ___
- **d** ____ ≤ true length < ___

4 For each of the following measurements find:

- **i** the limit of reading
- **ii** the greatest possible error (absolute error)
- **iii** the limits of accuracy
- **iv** the range of values of the true measurement.

- **a** 12 mm
- **b** 348 g
- **c** 375 mL
- **d** 8.2 km
- **e** 18.4 s
- **f** 4.9 kg
- **g** 2.37 m
- **h** 5.81 L

EXAMPLE 3

The mass of a car was given as 2300 kg to the nearest 100 kg.

- **a** Find the limit of reading.
- **b** Find the greatest possible error (absolute error) in the measurement.
- **c** Find the limits of accuracy.
- **d** State the range of values of the true measurement.

	Solve/Think	Apply
a	The smallest unit used for this measurement is given as 100 kg. Limit of reading = 100 kg	The level of accuracy of the measurement (in this case, 'to the nearest 100 kg') is the limit of reading of the measurement.
b	Absolute error = $\frac{1}{2} \times 100$ kg = 50 kg	
c	Limits of accuracy = 2300 ± 50 kg = 2250 kg, 2350 kg	
d	2250 kg ≤ true mass < 2350 kg	

5 For each of the following measurements find:

- **i** the smallest unit of measurement
- **ii** the absolute error in the measurement
- **iii** the limits of accuracy
- **iv** the range of values of the true measurement.

a The mass of a can of soup is 420 g, to the nearest 20 g.

b The capacity of a drink bottle is 370 mL, to the nearest 10 mL.

c The crowd at a cricket match was 38 000, to the nearest 1000.

d The time taken for a plane flight was $6\frac{1}{2}$ hours, to the nearest $\frac{1}{2}$ hour.

EXAMPLE 4

a If 17 m of rope is to be cut into 3 equal pieces, calculate the length of each piece correct to:

- **i** 1 s.f.
- **ii** 3 s.f.
- **iii** 4 s.f.

b Which unit of length would be appropriate to achieve each of the levels of accuracy in part **a**?

	Solve/Think	Apply
a i	17 m ÷ 3 = 5.6666… m Correct to 1 significant figure, 5.666… m ≈ 6 m	Perform the numerical calculation and round it to the required number of significant figures. Convert the number of significant figures to its equivalent ‘to the nearest ____’ and hence choose the appropriate unit of measurement to achieve this level of accuracy.
ii	Correct to 3 significant figures, 5.666… m ≈ 5.67 m	
iii	Correct to 4 significant figures, 5.666… m ≈ 5.667 m	
b i	Correct to 1 significant figures is equivalent to ‘to the nearest metre’. Thus an appropriate unit of length to use to achieve this level of accuracy would be ‘the metre’ (a measuring instrument with metres marked on it).	
ii	Correct to 3 significant figures is equivalent to ‘to the nearest centimetre’. An appropriate unit of length to use would be ‘the centimetre’ (a measuring instrument with centimetres marked on it).	
iii	Correct to 4 significant figures is equivalent to ‘to the nearest millimetre’. An appropriate unit of length to use would be ‘the millimetre’ (a measuring instrument with millimetres marked on it).	

6 **a** A 50 m length of curtain material is to be cut into 7 equal pieces. Complete the following to calculate the length of each piece correct to the number of significant figures given.

50 m ÷ 7 = ___ m

- **i** Correct to 1 significant figure: ___ m = ___ m
- **ii** Correct to 3 significant figures: ___ m = ___ m
- **iii** Correct to 4 significant figures: ___ m = ___ m

b Complete the following to find which unit of length would be appropriate to achieve each of the levels of accuracy in part **a**.

- **i** Correct to 1 significant figure is equivalent to ‘to the nearest ___’.
  Thus an appropriate unit of length to use to achieve this level of accuracy would be ‘the ___’.
- **ii** Correct to 3 significant figures is equivalent to ‘to the nearest ___’.
  Thus an appropriate unit of length to use would be ‘the ___’.
- **iii** Correct to 4 significant figures is equivalent to ‘to the nearest ___’.
  Thus an appropriate unit of length to use would be ‘the ___’.

7 **a** 400 g of flour is to be divided into 3 equal amounts. Complete the following to calculate the mass of each amount correct to the number of significant figures given.

400 g ÷ 3 = ___ g

- **i** Correct to 1 significant figure: ___ g = ___ g
- **ii** Correct to 2 significant figures: ___ g = ___ g
- **iii** Correct to 3 significant figures: ___ g = ___ g

b Complete the following to find which unit of mass would be appropriate to achieve each of the levels of accuracy in part **a**.

- **i** Correct to 1 significant figure is equivalent to ‘to the nearest ___’.
  Hence an appropriate unit of mass to use to achieve this level of accuracy would be ‘the ___’.
- **ii** Correct to 2 significant figures is equivalent to ‘to the nearest ___’.
  Hence an appropriate unit of mass to use would be ‘the ___’.
- **iii** Correct to 3 significant figures is equivalent to ‘to the nearest ___’.
  Hence an appropriate unit of mass to use would be ‘the ___’.

The number of significant figures indicates the level of accuracy of the measurement.

EXAMPLE 5

Explain the difference between measurements of 5.62 m and 5.620 m.

Solve	Think	Apply
5.62 m has been measured to the nearest centimetre and 5.620 m has been measured to the nearest millimetre. The second measurement is more accurate.	5.62 m has 3 significant figures. The last significant figure (2) is in the hundredths (of a metre) column. This indicates that the measurement has been made to the nearest hundredth of a metre: the nearest centimetre. 5.620 m has 4 significant figures. The last significant figure (0) is in the thousandths (of a metre) column. This indicates that the measurement has been made to the nearest thousandth of a metre (millimetre). The second measurement, 5.620 m, is more accurate.	Determine the number of significant figures and hence the level of accuracy for each measurement.

8 Complete the following to explain the difference between measurements of 8.76 kg and 8.760 kg.

8.76 kg has ____ significant figures.

The last significant figure is in the ___ (of a kilogram) column.

Thus the measurement has been made to the nearest ___ of a kilogram; that is, to the nearest ___ g.

8.760 kg has ___ significant figures.

The last significant figure is in the ___ (of a kilogram) column.

Thus the measurement has been made to the nearest ___ of a kilogram; that is, to the nearest ___ g.

The ____ measurement is more accurate.

9 Explain the difference between measurements of:

a 3.64 m and 3.640 m
b 5.8 kg and 5.80 kg
c 12 s and 12.0 s
d 36 cm and 36.0 cm
e 23.8 s and 23.80 s
f 7.29 km and 7.290 km
g 1.5 t and 1.50 t
h 5.83 L and 5.830 L

I Calculations involving measurements

As there will always be some degree of error in a numerical value found by measurement (measurements are never exact), it follows that the results of any calculations using measurements will also contain some degree of error.

EXAMPLE 1

The length and breadth of a rectangle were measured to be 8 cm and 6 cm respectively, to the nearest centimetre.

a Calculate the perimeter of the rectangle using these measurements.
b Write down the greatest possible error in each of these measurements.
c Hence find the limits of accuracy of the length and breadth.
d Calculate the lower and upper limits of the true perimeter.
e Find the maximum error in the answer calculated in part **a**.

	Solve/Think	Apply
a	The perimeter of the rectangle is 8 + 8 + 6 + 6 = 28 cm.	Calculate the lower and upper limits of each measurement. Use these to calculate the lower and upper limits of the perimeter. Determine the maximum error between the perimeters, using the given measurements and the lower and upper limits of the perimeter.
b	The greatest possible error of each measurement is 0.5 cm.	
c	The limits of accuracy of the length are 7.5 cm and 8.5 cm. The limits of accuracy of the breadth are 5.5 cm and 6.5 cm.	
d	Lower limit of perimeter is 7.5 + 7.5 + 5.5 + 5.5 = 26 cm. Upper limit of perimeter is 8.5 + 8.5 + 6.5 + 6.5 = 30 cm.	
e	The maximum error in the calculation of the perimeter is 30 − 28 (or 28 − 26) = 2 cm.	

Exercise 4I

1 The length and breadth of a rectangle were measured to be 9 cm and 5 cm respectively, to the nearest centimetre. Complete the following.

a The perimeter of the rectangle using these measurements is ___.

b The greatest possible error of each measurement is ___ cm.

c The limits of accuracy of the length are ___ cm and ___ cm and the limits of accuracy of the breadth are ___ cm and ___ cm.

d Lower limit of the perimeter = ___ + ___ + ___ + ___ cm = ___ cm

Upper limit of the perimeter = ___ + ___ + ___ + ___ cm = ___ cm

e The maximum error in the calculation of the perimeter = ___ − ___ cm = ___ cm

2 The masses of two bags of sand were measured and found to be 47 kg and 52 kg, to the nearest kg. Complete the following.

a The total mass of the two bags using these measurements = ___ + ___ kg = ___ kg

b The greatest possible error of each measurement = ____ kg

c The limits of accuracy of each mass are ___ kg and ___ kg, ___ kg and ___ kg

d Lower limit of total mass = ___ + ___

Upper limit of total mass = ___ + ___

e The maximum error in the calculation of the total mass = ___ − ___ kg = ___ kg

EXAMPLE 2

The length and breadth of a rectangle were measured to be 8 cm and 6 cm respectively, to the nearest centimetre.

a Calculate the area of the rectangle using these measurements.

b Write down the greatest possible error in each of these measurements.

c Hence find the limits of accuracy of the length and breadth.

d Calculate the lower and upper limits of the true area.

e Find the maximum error in the answer calculated in part **a**.

	Solve/Think	Apply
a	The area of the rectangle is 8×6 cm^2 = 48 cm^2.	Calculate the lower and upper limits of each measurement. Use these to calculate the lower and upper limits of the area. Determine the maximum error for the area using the given measurements and the lower and upper limits of the area.
b	The greatest possible error of each measurement is 0.5 cm.	
c	The limits of accuracy of the length are 7.5 cm and 8.5 cm. The limits of accuracy of the breadth are 5.5 cm and 6.5 cm.	
d	Lower limit of area is $7.5 \times 5.5 = 41.25$ cm^2. Upper limit of area is $8.5 \times 6.5 = 55.25$ cm^2.	
e	$48 - 41.25 = 6.75$ cm^2 $55.25 - 48 = 7.25$ cm^2 The maximum error in the area calculation is 7.25 cm^2.	

3 The length and breadth of a rectangular room were measured to be 5 m and 3 m respectively, to the nearest metre. Complete the following.

a The area of the rectangle using these measurements is ___ × ___ m^2 = ___ m^2.

b The greatest possible error of each measurement is ___ m.

c The limits of accuracy of the length are ___ m and ___ m.
The limits of accuracy of the breadth are ___ m and ___ m.

d Lower limit of area = ___ × ___ m^2 = ___ m^2
Upper limit of area = ___ × ___ m^2 = ___ m^2

e ___ − ___ m^2 = ___ m^2
___ − ___ m^2 = ___ m^2
The maximum error in the calculation of the area is ___ m^2.

EXAMPLE 3

The length and breadth of a rectangle were measured to be 12.6 cm and 6.4 cm respectively. Give a reasonable estimate of the area of the rectangle, using these measurements.

Solve	Think	Apply
Area = 12.6 × 6.4 cm^2 = 80.64 cm^2 = 81 cm^2 to 2 s.f.	From previous examples we know that errors in the measurement of length and breadth accumulate to produce an error in the calculation of area. It is common in calculations involving multiplication and division of measurements to round the answer to the least number of significant figures in the data. The length (12.6 cm) has 3 significant figures and the breadth (6.4 cm) has 2 significant figures. The breadth has the least number of significant figures, thus we round the area calculation to 2 significant figures. Area = 80.64 cm^2 = 81 cm^2 to 2 s.f.	Perform the numerical calculation and round the answer to the least number of significant figures in the given measurements.

4 The length and breadth of a rectangular block of land were measured to be 135 m and 42 m respectively. Complete the following to find a reasonable estimate of the area of this block, using these measurements.

The length (135 m) has ___ significant figures and the breadth (42 m) has ___ significant figures.

The least number of significant figures in the data is ___.

Thus we round the calculation of area to ___ significant figures.

Area = ___ × ___ m^2
= ___ m^2
= ___ m^2 to ___ s.f.

5 The mass and volume of a block of lead were measured to be 2.894 kg and 0.255 m^3 respectively. Complete the following to give a reasonable estimate of the density of lead, using these measurements.

The mass (2.894 kg) has ___ significant figures and the volume (0.255 m^3) has ___ significant figures.

The least number of significant figures in the data is ___.

Thus we round the calculation of density to ___ significant figures.

$$\begin{aligned}\text{Density} &= \frac{\text{mass}}{\text{volume}}\\ &= \frac{2.894\text{ kg}}{0.255\text{ m}^3}\\ &= \underline{\quad}\text{ kg/m}^3\\ &= \underline{\quad}\text{ kg/m}^3\text{ to }\underline{\quad}\text{ s.f.}\end{aligned}$$

EXAMPLE 4

Assuming the numerical values given are rounded measurements, calculate the answers to the appropriate number of significant figures.

a 4.6 × 2158 **b** 21.6 ÷ 0.845

	Solve	Think	Apply
a	$4.6 \times 2158 = 9926.8$ $= 9900$ to 2 s.f.	4.6 has 2 significant figures and 2158 has 4 significant figures. The least number of significant figures is 2. Thus round the calculation to 2 significant figures. $4.6 \times 2158 = 9926.8$ $= 9900$ to 2 s.f.	Perform the numerical calculation, then round the answer to the least number of significant figures in the given measurements.
b	$21.6 \div 0.845 = 25.562\,13\ldots$ $= 25.6$ to 3 s.f.	21.6 has 3 significant figures and 0.845 has 3 significant figures. The least number of significant figures is 3. Thus round the calculation to 3 significant figures. $21.6 \div 0.845 = 25.562\,13\ldots$ $= 25.6$ to 3 s.f.	

6 Assuming the numerical values given are rounded measurements, complete the following to calculate the answers to the appropriate number of significant figures.

a 48.9 × 3.156

48.9 has ___ significant figures and 3.156 has ___ significant figures.

The least number of significant figures in the data is ___.

Thus round the calculation to ___ significant figures.

$48.9 \times 3.156 = \underline{\quad}$

$= \underline{\quad}$ to ___ s.f.

b 6784 ÷ 9.5

6784 has ___ significant figures and 9.5 has ___ significant figures.

The least number of significant figures in the data is ___.

Thus round the calculation to ___ significant figures.

$6784 \div 9.5 = \underline{\quad}$

$= \underline{\quad}$ to ___ s.f.

EXAMPLE 5

Assuming the numerical values given are rounded measurements, calculate the answers to the appropriate degree of accuracy.

a 25.8 + 19.53 **b** 46 − 21.57

	Solve	Think	Apply
a	25.8 + 19.53 = 45.33 = 45.3 (1 decimal place)	It is common in calculations involving addition and subtraction of measurements to round the answers to the least decimal place accuracy given in the data. 25.8 is accurate to 1 decimal place and 19.53 is accurate to 2 decimal places. The least decimal place accuracy given is to 1 decimal place from 25.8. Thus round to 1 decimal place. 25.8 + 19.53 = 45.33 = 45.3 (1 decimal place)	Perform the numerical calculation and round the answer to the least decimal place accuracy in the given measurements.
b	46 − 21.57 = 24.43 = 24 (nearest whole number)	46 is accurate to the nearest whole number, 21.57 is accurate to 2 decimal places. The least decimal place accuracy given in the data is to the nearest whole number. Thus round to the calculation to the nearest whole number. 46 − 21.57 = 24.43 = 24 (nearest whole number)	

7 Assuming the numerical values given are rounded measurements, complete the following to calculate the answers to the appropriate degree of accuracy.

a 12.365 + 14.92

12.365 is accurate to ___ decimal places and 14.92 is accurate to ___ decimal places.

The least decimal place accuracy given in the data is 'correct to ____'.

Thus round the calculation to ___.

12.365 + 14.92 = ___

= ___ correct to ____

b 86.89 − 57

86.89 is accurate to ___ decimal places and 57 is accurate to ____.

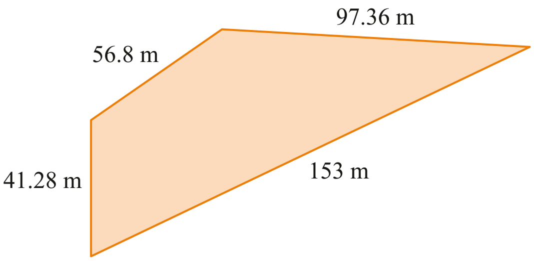
The least decimal place accuracy given in the data is 'to ____'.

Thus round the calculation to ___.

86.89 − 57 = ___

= ___ to ____

8 The sides of an irregular field were measured to be 56.8 m, 97.36 m, 153 m and 41.28 m. Calculate the perimeter of the field to an appropriate level of accuracy.



Language in mathematics

1 What is scientific notation?

2 Explain why measurements are never exact.

3 For a measurement, explain what is meant by these terms:

a limit of reading **b** greatest possible error **c** limits of accuracy

4 Explain the difference between measurements of 9 seconds and 9.0 seconds.

5 Using the table of prefixes in Section 4C, write the following measurements with the correct unit abbreviation.

a 7 milliseconds **b** 4.6 gigajoules **c** 3.5 terametres
d 2.4 nanograms **e** 6 kilometres **f** 5.2 megalitres

6 Three of the words in the following list are spelt incorrectly. Find these words and write the correct spelling.

significent dijit equivalent trunckate figure

7 Explain how to round 0.0358 to 2 significant figures.

Terms

accuracy affect appropriate approximation calculation convert decimal delete
digit effect estimate equivalent figure identify prefix product
retain round scientific notation significant figures terminate truncate

Check your skills

1 Which of the following numbers is written in scientific notation?

A 53×10^{18} **B** $9.2 \times 100\,000$ **C** 300 000 **D** 3.7×10^{-45}

2 When written in scientific notation, 23 000 000 becomes:

A 2.3×10^{6} **B** 2.3×10^{7} **C** 2.3×10^{8} **D** 2.3×10^{9}

3 When written as an ordinary number, 5.1×10^{-6} is:

A 0.000 51 **B** 0.000 051 **C** 0.000 005 1 **D** 0.000 000 51

4 $(8.6 \times 10^{18}) \times (2.5 \times 10^{14}) =$

A 2.15×10^{32} **B** 2.15×10^{33} **C** 2.15×10^{252} **D** 2.15×10^{253}

5 $(3.6 \times 10^{7}) \div (4.5 \times 10^{-8}) =$

A 8×10^{-14} **B** 8×10^{-15} **C** 8×10^{14} **D** 8×10^{15}

6 When the numbers 2.9×10^{-7}, 5.2×10^{-9}, 3.8×10^{-9} are written in order from smallest to largest, the answer is:

A 2.9×10^{-7}, 3.8×10^{-9}, 5.2×10^{-9} **B** 5.2×10^{-9}, 3.8×10^{-9}, 2.9×10^{-7}
C 2.9×10^{-7}, 5.2×10^{-9}, 3.8×10^{-9} **D** 3.8×10^{-9}, 5.2×10^{-9}, 2.9×10^{-7}

7 1.8 gigametres is equivalent to:

A 1800 kilometres **B** 18 000 kilometres **C** 180 000 kilometres **D** 1 800 000 kilometres

8 The dinosaurs became extinct approximately 63 million years ago. How many millennia is this?

A 63 000 **B** 6300 **C** 630 **D** 63

9 The value of the digit 3 in the number 156.832 is:
A 30 **B** 3 **C** $\frac{3}{10}$ **D** $\frac{3}{100}$

10 When rounded to the nearest thousand, 23 629 is:
A 23 **B** 24 **C** 23 000 **D** 24 000

11 When 7982 is rounded to the nearest hundred, the answer is:
A 7900 **B** 7990 **C** 7980 **D** 8000

12 Which of the following numbers is not equal to 36.5, when rounded to 1 decimal place?
A 36.48 **B** 36.54 **C** 36.55 **D** 36.50

13 When a number is rounded to the nearest 10, the answer is 70. The smallest the number could be is:
A 69.99 **B** 69 **C** 65.01 **D** 65

14 The first significant figure in the number 0.005 064 is:
A 0 **B** 5 **C** 6 **D** 4

15 Which of the following numbers is not equal to 4600, when rounded to 2 significant figures?
A 4639 **B** 4608 **C** 4550 **D** 4650

16 When rounded to 3 significant figures, 4.5976 is:
A 4.597 **B** 4.598 **C** 4.59 **D** 4.60

17 Which of the following numbers does not have 2 significant figures?
A 7.29 **B** 5.0 **C** 36 **D** 0.034

18 Rounding numbers before the last step of a calculation:
A never affects the accuracy of the answer **B** often affects the accuracy of the answer
C always affects the accuracy of the answer **D** makes the answer bigger than it should be

19 The mass of a bag of flour is 637 g, to the nearest gram. The limit of reading of the measuring instrument is:
A 0.5 g **B** 1 g **C** 5 g **D** 7 g

20 The greatest possible error in the measurement in question **19** is:
A 0.5 g **B** 1 g **C** 5 g **D** 7 g

21 The limits of accuracy of the measurement in question **19** are:
A 636.5 g, 637.5 g **B** 636 g, 638 g **C** 632 g, 642 g **D** 630 g, 640 g

22 The time taken to win the 100 m sprint at a school athletics carnival was 12.4 s, to the nearest 0.1 of a second. Within what range does the true time lie?
A 12.3 s $\leqslant$ true time $<$ 12.5 s **B** 12.35 s $\leqslant$ true time $<$ 12.45 s
C 12.0 s $\leqslant$ true time $<$ 13.0 s **D** 12 s $\leqslant$ true time $<$ 13 s

23 The length and breadth of a rectangle were measured to be 49.6 m and 5.3 m respectively. A reasonable estimate of the area of this rectangle using these measurements is:
A 300 m^2 **B** 260 m^2 **C** 263 m^2 **D** 262.88 m^2

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4, 6	7	8	9–13	14–17	18	19–22	23
Section	A	B	C	D	E	F	G	H	I

4A Review set

1 Write the following numbers in scientific notation.

a 17 000 000 000 **b** 0.000 000 35

2 Write the following as ordinary numbers.

a 2.86×10^5 **b** 3.06×10^{-4}

3 Explain why $3 \times 10^4 \neq 3^4$.

4 Use your calculator to find the value of the following (leave the answer in scientific notation).

a $(3 \times 10^{-8}) \times (5.3 \times 10^{-6})$ **b** $(4.5 \times 10^{-9}) \div (5 \times 10^{-16})$

c $(7.3 \times 10^{15})^2$ **d** $\sqrt[3]{1.25 \times 10^{-7}}$

5 Write the following numbers in order from smallest to largest.

a 2.94×10^{15}, 2.94×10^{16} **b** 6.5×10^{-14}, 1.4×10^{-10}

6 Convert the following to grams. Write the answers in scientific notation.

a 6.4 gigagrams **b** 6.4 nanograms

7 Convert 37 000 metres to:

a kilometres **b** megametres

8 How many nanowatts in 4.2 milliwatts?

9 Convert 1 megasecond to days.

10 Write the value of the digit 5 in each of the following numbers.

a 253.6 **b** 1405.2 **c** 86.457 **d** 2564

11 Round the following.

a 2470 to the nearest hundred **b** 7926 to the nearest ten

c 33.5 to the nearest whole number **d** 49 900 to the nearest thousand

12 Round 8.4625 to:

a 1 decimal place **b** 2 decimal places **c** 3 decimal places

13 Round the following.

a 3.497 to 2 decimal places **b** 19.98 to 1 decimal place

14 When a number was rounded to the nearest 10, the answer was 40.

a What is the smallest the number could have been?

b What is the largest the number could have been? Discuss.

c Write a mathematical statement that shows the range of possible numbers.

15 State the first significant figure in each of the following numbers.

a 3790 **b** 4.0625 **c** 0.002 86

16 Round 48.350 92 to:

a 1 s.f. **b** 2 s.f. **c** 3 s.f. **d** 4 s.f. **e** 5 s.f.

17 How many significant figures are there in each of the following numbers?

a 795 **b** 0.6 **c** 8.20 **d** 0.0032 **e** 28 000

18 The time taken for a girl to complete a task in an aptitude test was measured to be 17 s.

- **a** Find the limit of reading of the measuring instrument used.
- **b** Determine the greatest possible error in the measurement.
- **c** What are the limits of accuracy of the measurement?
- **d** Write a mathematical statement that shows the range of values within which the true time lies.

19 Explain the difference between measurements of 3.65 m and 3.650 m.

20 The length and breadth of a rectangle were measured to be 9 cm and 7 cm respectively, to the nearest centimetre.

- **a** Calculate the perimeter of the rectangle using these measurements.
- **b** Write down the greatest possible error in each of these measurements.
- **c** Hence find the limits of accuracy of the length and the breadth.
- **d** Calculate the lower and upper limits of the true perimeter.
- **e** Find the maximum error in the answer calculated in part **a**.

21 The length and breadth of a rectangle were measured to be 11.4 cm and 6.3 cm respectively. Give a reasonable estimate of the area of this rectangle using the level of accuracy of the measurements.

4B Review set

1 Write the following numbers in scientific notation.

- **a** 46 000
- **b** 0.0003

2 Write the following as ordinary numbers.

- **a** 4×10^{7}
- **b** 1.8×10^{-6}

3 Explain why $4 \times 10^{6} \neq 4^{6}$.

4 Use your calculator to find the value of the following (leave answer in scientific notation).

- **a** $(3.9 \times 10^{13}) \times (4 \times 10^{-5})$
- **b** $(8 \times 10^{10}) \div (1.6 \times 10^{-5})$
- **c** $(3 \times 10^{-6})^{5}$
- **d** $\sqrt{1.96 \times 10^{-10}}$

5 Write the following numbers in order from smallest to largest.

- **a** 4.1×10^{9}, 5×10^{9}
- **b** 4.5×10^{-11}, 3.1×10^{-15}

6 Convert the following to joules. Write the answers in scientific notation.

- **a** 5.6 megajoules
- **b** 86 millijoules

7 Convert 0.0007 metres to:

- **a** millimetres
- **b** nanometres

8 How many kilograms in 1.4 gigagrams?

9 Convert 1 hour to kiloseconds.

10 Write the value of the digit 6 in each of the following numbers.

- **a** 253.6
- **b** 1607.2
- **c** 83.456
- **d** 2564

11 Round the following.

- **a** 13 560 to the nearest hundred
- **b** 4063 to the nearest ten
- **c** 147.55 to the nearest whole number
- **d** 99 900 to the nearest thousand

12 Round 1.5607 to:

a 1 decimal place **b** 2 decimal places **c** 3 decimal places

13 Round the following.

a 2.695 to 2 decimal places **b** 39.96 to 1 decimal place

14 Susan's height was measured to be 164 cm to the nearest centimetre. Within what range of values does her actual height lie?

15 Write down the first significant figure in each of the following numbers.

a 24 560 **b** 15.0715 **c** 0.005 09

16 Write these numbers correct to 3 significant figures.

a 365 400 **b** 539.53 **c** 0.002 397 **d** 1.998

17 How many significant figures are there in each of the following numbers?

a 37 **b** 1.3 **c** 17.90 **d** 0.0008 **e** 4000

18 The mass of a bag of sand was measured to be 15.4 kg.

a What was the limit of reading of the measuring instrument used?
b Write down the greatest possible error in the measurement.
c Hence find the limits of accuracy of the measurement.
d Write a mathematical statement that shows the range of values within which the true mass lies.

19 Explain the difference between measurements of 1.54 kg and 1.540 kg.

20 The length and breadth of a rectangle were measured to be 8 cm and 5 cm, to the nearest centimetre.

a Calculate the perimeter of the rectangle using these measurements.
b Write down the greatest possible error in each of these measurements.
c Hence find the limits of accuracy of the length and the breadth.
d Calculate the lower and upper limits of the true perimeter.
e Find the maximum error in the answer calculated in part **a**.

21 Over a measured time interval of 15 s, a bicycle travelled a distance of 112.5 m. Give a reasonable estimate of the average speed of the bicycle using the level of accuracy of the measurements.

4C Review set

1 Write the following numbers in scientific notation.

a 23 000 000 **b** 0.000 05

2 Write the following as ordinary numbers.

a 9.8×10^4 **b** 3.7×10^{-5}

3 Explain why $7 \times 10^5 \neq 7^5$.

4 Use your calculator to find the value of the following (leave answer in scientific notation).

a $(3.4 \times 10^4) \times (3.5 \times 10^9)$ **b** $(5.6 \times 10^{10}) \div (1.4 \times 10^5)$

c $(3 \times 10^9)^5$ **d** $\sqrt{2.25 \times 10^{12}}$

5 Write the following numbers in increasing order.

a 3.8×10^{15}, 4.6×10^{13} **b** 7.7×10^{-16}, 3.1×10^{-12}

6 Convert the following to metres. Write the answers in scientific notation.

a 36 terametres **b** 45 micrometres

7 Convert 0.000 000 91 grams to:

a milligrams **b** nanograms

8 How many picolitres in 5.9 nanolitres?

9 Convert 22.8 millennia into years.

10 Write the value of the digit 8 in each of the following numbers.

a 318.6 **b** 36.8 **c** 23.487 **d** 8567

11 Round:

a 13 827 to the nearest hundred **b** 765 to the nearest ten

c 24.09 to the nearest whole number **d** 89 600 to the nearest thousand

12 Round 13.0652 to:

a 1 decimal place **b** 2 decimal places **c** 3 decimal places

13 Round the following.

a 4.196 to 2 decimal places **b** 20.95 to 1 decimal place

14 Write the first significant figure in each of the following numbers.

a 3790 **b** 4.0625 **c** 0.002 86

15 Round 17.6308 to:

a 1 s.f. **b** 2 s.f. **c** 3 s.f. **d** 4 s.f. **e** 5 s.f.

16 A number was rounded to 2 significant figures and the answer was 430.

a What is the smallest the number could have been?

b What is the largest the number could have been? Discuss.

c Write a mathematical statement that shows the range of possible numbers.

17 How many significant figures are there in each of the following numbers?

a 795 **b** 0.6 **c** 8.20 **d** 0.0032 **e** 28 000

18 The length of a piece of timber was measured to be 4.38 m.

a What was the limit of reading of the measuring instrument used?

b What is greatest possible error in the measurement?

c Write the limits of accuracy of the measurement.

d Write a mathematical statement that shows the range of values within which the true length lies.

19 Explain the difference between measurements of 6 cm and 6.0 cm.

20 A rectangle has a length of 6.8 cm and a breadth of 5.2 cm, measured to the nearest 0.1 of a cm.

a Calculate the perimeter of the rectangle using these measurements.

b What is the greatest possible error in each of these measurements?

c Hence find the limits of accuracy of the length and the breadth.

d Calculate the lower and upper limits of the true perimeter.

e Find the maximum error in the answer calculated in part **a**.

21 A man's height was measured as 1.88 m and mass as 85 kg. Calculate his body mass index, BMI, using the formula $\text{BMI} = \frac{\text{mass}}{\text{height}^2}$. Give a reasonable estimate of BMI using the level of accuracy of the measurements.

Chapters 2–4 Cumulative review

1 **a** Write the following in index form.

i $6 \times 6 \times 6 \times 6 \times 6 \times 6$ **ii** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

b Write the base and index of these numbers.

i 7^8 **ii** 4^{-11}

c Write in expanded form.

i 4^6 **ii** 3^7

d Use a calculator to evaluate the following.

i 5^4 **ii** 2^{11}

e Simplify the following, leaving the answer in index form.

i $5^{10} \times 5^{10}$ **ii** $(3^7)^{12}$ **iii** $5^{15} \div 5^{12}$ **iv** $4^{25} \div 4$ **v** $3^{28} \times (3^5)^4$

f Write true or false.

i $2^6 \times 2^5 = 4^{11}$ **ii** $7^8 \div 7^4 = 1^4$

g Write the meaning of:

i 7^{-5} **ii** 24^{-1}

h Evaluate:

i 3^{-4} **ii** 9^{-2} **iii** 4^{-1} **iv** 16^0

2 **a** Complete:

i $\dfrac{3h}{5} = \dfrac{\square}{25}$ **ii** $\dfrac{4xy}{3} = \dfrac{\square}{15}$

b Reduce to simplest form.

i $\dfrac{6w}{24}$ **ii** $\dfrac{9mn}{7n}$

c Simplify:

i $\dfrac{m^5}{m^6}$ **ii** $\dfrac{p^7}{p^{10}}$ **iii** $\dfrac{r^3}{r^5}$ **iv** $\dfrac{t^5}{t^2}$

d Simplify:

i $p^4 \times p^8$ **ii** $y^{16} \div y^4$ **iii** $(t^5)^7$

iv $\dfrac{c^{12} \times c^8}{(c^2)^5}$ **v** $(3v^5)^3$ **vi** $3x^8y^9 \times 7x^2y^6$

e Evaluate:

i c^0 **ii** $7c^0$ **iii** $(7c)^0$ **iv** $7\mathrm{c}^0 + 5$

f Write the meaning of:

i c^{-5} **ii** $4c^{-5}$ **iii** $(4c)^{-5}$

g Simplify:

i $d^{-5} \times d^{-2}$ **ii** $n^{-2} \div n^{-5}$ **iii** $(k^{-2})^{-5}$

iv $5a^{-9} \times 3a^3$ **v** $6m^3 \div 9m^{-2}$ **vi** $12n^{-3} \div 8n^{-2}$

h State whether the following are true or false.

i $5h^0 = 5$ **ii** $a \div a^5 = a^{-5}$ **iii** $2p^4 \div 6p^4 = \frac{1}{3}p$

iv $4s^{-4} = \dfrac{4}{s^4}$ **v** $n^2 + n^2 = n^4$ **vi** $3^3 \times 2^3 = 6^3$

3 **a** Expand:

i $7(5w + 2x)$ **ii** $-k^4(2k^3 - 3k)$ **iii** $-(4s - 8)$

b Expand and simplify:

i $4(5t - 1) + 3(2 - 3t)$ **ii** $10(x - 2y) - 4(2x - y)$

4 **a** The results of a test for a Year 9 class are given for girls and boys.

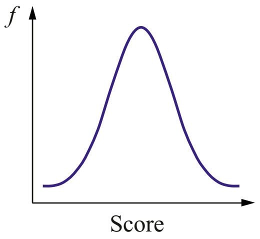
Girls: 27, 39, 41, 56, 53, 24, 32, 30, 35, 51, 44, 43, 47, 43, 24

Boys: 58, 46, 41, 47, 47, 32, 66, 64, 33, 39, 54, 49, 63, 61, 55

- **i** Draw back-to-back stem-and-leaf plots for this data.
- **ii** Comment on the shape of each distribution.
- **iii** Use the mean, median and mode to compare the data.

b Describe the shape of each distribution.

i



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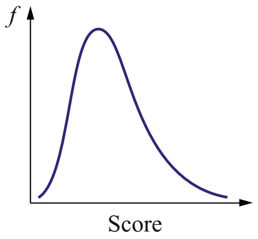

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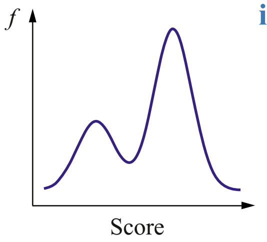

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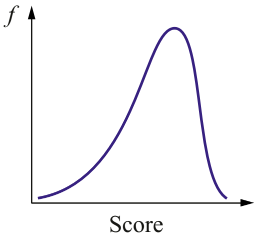


5 **a** Write the following numbers in scientific notation.

- **i** 325 000 000
- **ii** 0.000 074

b Write the following as ordinary numbers.

- **i** 6.21×10^7
- **ii** 3×10^{-5}

c Explain why $4 \times 10^6 \neq 4^6$.

d Use your calculator to find the value of the following (leave the answers in scientific notation).

- **i** $(6.5 \times 10^{-6}) \times (3 \times 10^{30})$
- **ii** $(3.4 \times 10^{15}) \div (7 \times 10^{-8})$
- **iii** $(8 \times 10^{18})^4$
- **iv** $\sqrt[3]{3.43 \times 10^{17}}$

e Write the following numbers in order from smallest to largest:

7.6×10^{-10}, 6.7×10^{-10}, 2.4×10^{-11}, 7.5×10^{-16}

f When a number was rounded to 2 significant figures, the answer was 520.

- **i** What is the smallest the number could have been?
- **ii** What is the largest the number could have been?
- **iii** Write a mathematical statement that shows the range of possible numbers.

g Explain the difference between measurements of 7 cm and 7.0 cm.

5

Financial mathematics

This chapter deals with solving financial mathematics problems involving earning and spending money, simple interest and loans.

After completing this chapter you should be able to:

- calculate weekly, fortnightly, monthly and yearly earnings for various types of income
- calculate net income after considering common deductions
- calculate tax payable
- calculate simple interest using the formula
- apply the simple interest formula to problems involving investing money
- calculate and compare the cost of purchasing goods by different means.

NSW Syllabus references: 5.1 N&A Financial mathematics
Outcomes: MA5.1-1WM, MA5.1-2WM, MA5.1-3WM, MA5.1-4NA
NUMBER & ALGEBRA – ACMNA211

Investigation 1 Earning an income

There are a number of different ways in which people are paid for providing their labour, knowledge, skills and services. People who work for themselves charge a fee, but most people work for an employer.

By research and discussion, complete the table below, which shows the ways in which people are paid when they work for an employer.

Method of payment	Description	Examples of occupations	Advantages/ disadvantages
Salary	A fixed amount per year, usually paid weekly or fortnightly		
Wages	An hourly rate for an agreed number of hours per week, usually paid weekly or fortnightly		
Commission	A percentage of the value of goods or services sold. Sometimes a low wage, called a **retainer**, is paid in addition to this		
Piecework	A fixed amount for each item produced or completed		
Fee	A fixed amount for a service provided		

A Salaries and wages

Employees who are paid a salary or wages may be permanent or casual. **Permanent** employees have security of employment through a workplace agreement and receive benefits such as sick leave and holiday leave. They may be employed **full-time** or **part-time**. Casual employees will be discussed in Section 5C.

3 days a week is part-time. !

EXAMPLE 1

Georgina works for 3 days a week and earns a salary of \$670.85 per week. How much does she earn per:

a fortnight? **b** year? **c** month? **d** quarter?

	Solve	Think	Apply
a	Fortnightly = \$670.85 × 2 = \$1341.70	There are 2 weeks in a fortnight. Multiply the weekly salary by 2.	There are 52 weeks in a year. If there were 4 weeks in a month then a year would be 48 weeks. Monthly pay is averaged over 12 months. There are 3 months in a quarter, so the quarterly salary could also be found by multiplying the monthly salary by 3.
b	Yearly = \$670.85 × 52 = \$34 884.20	There are 52 weeks in a year. Multiply the weekly salary by 52.	
c	Monthly = $\frac{\$34\ 884.20}{12}$ = \$2907.02 to the nearest cent	There are 12 months in a year. Divide the yearly salary by 12. One month is *not* 4 weeks.	
d	Quarterly = \$670.85 × 13 = \$8721.05	One quarter of 52 is 13. Multiply the weekly salary by 13.	

Exercise 5A

1 Jim works for 4 days a week and earns a salary of \$842.66 per week. Complete the following to find how much he earns for these periods to the nearest cent.

a Fortnightly = ____ × 2 = \$___

b Yearly = \$842.66 × ___ = \$___

c Monthly = ____ ÷ 12 = \$___

d Quarterly = \$___ × 13 = \$___

2 Neil works for 3 days a week and earns a salary of \$728.56 per week. How much does he earn per:

a fortnight? **b** year? **c** month? **d** quarter?

3 Convert the following weekly salaries into the equivalent salary per:

i fortnight **ii** year **iii** month **iv** quarter.

a \$914 **b** \$790 **c** \$1025.60 **d** \$984.60 **e** \$1378.94

EXAMPLE 2

Harry works full-time and earns a salary of \$68 600 p.a.
How much does he earn per:

a week? **b** fortnight? **c** month? **d** quarter?

p.a. is short for per annum, which means per year.

	Solve	Think	Apply
a	Weekly $= \dfrac{\$68\ 600}{52}$ $= \$1319.23$ to the nearest cent	There are 52 weeks in a year. Divide the yearly salary by 52.	Yearly amounts allow conversions to be straightforward. If unsure about converting, calculate the yearly amount first.
b	Fortnightly $= \dfrac{\$68\ 600}{26}$ $= \$2638.46$ to the nearest cent	There are 26 fortnights in a year Divide the yearly salary by 26.	
c	Monthly $= \dfrac{\$68\ 600}{12}$ $= \$5716.67$ to the nearest cent	There are 12 months in a year. Divide the yearly salary by 12.	
d	Quarterly $= \dfrac{\$68\ 600}{4}$ $= \$17\ 150$	There are 4 quarters in a year. Divide the yearly salary by 4.	

4 Helene works full-time and earns a salary of \$74 250 p.a. Complete the following to find how much she earns for these periods to the nearest cent.

a Weekly $= \dfrac{\$\square}{52} = \$$___

b Fortnightly $= \dfrac{\$74\ 250}{\square} = \$$___

c Monthly $= \dfrac{\$\square}{12} = \$$___

d Quarterly $= \dfrac{\$\square}{4} = \$$___

5 Tara works full-time and earns a salary of \$64 800 p.a. How much does she earn per:

a week? **b** fortnight? **c** month? **d** quarter?

6 Convert the following yearly salaries to the equivalent salary per:

i week **ii** fortnight **iii** month **iv** quarter.

a \$52 400 **b** \$36 600 **c** \$95 370 **d** \$76 280 **e** \$82 900

7 Convert the annual salaries shown in the advertisements below to the equivalent:

i weekly salary **ii** fortnightly salary **iii** monthly salary.

a

Fashion

Girl's Surfwear Designer
\$80K
Exciting position for the
right person. Ph 9444 222

b

Foreman \$110K
Experienced foreman required
for city project. Ph 9333 000

c

Cleaner/Housekeeper
\$40K Rare opportunity
to work in fine home.
Ph 9666 000

\$40K is a short way of indicating \$40 000.

EXAMPLE 3

Bruno is employed full-time and earns a salary of \$4600 per month. What is his equivalent weekly salary?

Solve	Think	Apply
Yearly salary $= \$4600 \times 12$ $= \$55\ 200$ Weekly salary $= \dfrac{\$55\ 200}{52}$ $= \$1061.54$ to the nearest cent	Find the yearly salary first by multiplying \$4600 by 12. Then find the weekly salary by dividing by 52.	Do not divide monthly by 4. Most months are 4 weeks plus 2 or 3 extra days, so always convert to yearly.

8 Jessinta is employed full-time and earns \$7320 per month. Complete the following to find her equivalent weekly salary to the nearest cent.

Yearly salary $= \$\_\_\_ \times 12 = \$\_\_\_$

Weekly salary $= \dfrac{\$\square}{52} = \$\_\_\_$

9 Samantha is employed full-time and earns \$6900 per month. What is her equivalent weekly salary?

10 Convert the following monthly salaries to the equivalent weekly salaries.

a \$4200 **b** \$2890 **c** \$5635 **d** \$7000 **e** \$3599

11 Scott earns \$68 840 p.a., Lisa earns \$1350 per week, Paula earns \$5700 per month and Tranh earns \$17 050 per quarter. Who earns the most?

EXAMPLE 4

Ella works a 35-hour week as a waitress and is paid \$23.86 per hour. What is her weekly wage?

Solve	Think	Apply
Weekly wage $= \$23.86 \times 35$ $= \$835.10$	Find the pay per week by multiplying the hourly rate of \$23.86 by 35 hours.	Many positions are paid by the hour.

12 Robert works a 35-hour week and is paid \$23.65 per hour. Complete the following to find his weekly wages.

Weekly wage $= \$\_\_\_\_ \times 35 = \$\_\_\_\_$

13 Dean works a 35-hour week and is paid \$21.70 per hour. What are his weekly wages?

14 Calculate the weekly wages for a person who works a 35-hour week and is paid:

a \$18.90/h **b** \$22.30/h **c** \$26.48/h **d** \$53.67/h **e** \$84.50/h

EXAMPLE 5

Yoshi earns \$992.50 for working a 25-hour week as a security guard.
What is his hourly rate of pay?

Less than 30 hours a week is part-time. !

Solve	Think	Apply
Hourly rate $= \frac{\$992.50}{25}$ $= \$39.70$	Find the hourly rate by dividing the weekly wage of \$992.50 by 25 hours.	The hourly rate is useful to compare pay rates for different jobs.

15 Patrice earns \$520.30 for working a 22-hour week. Complete the following to find her hourly rate of pay.

Hourly rate $= \frac{\$\square}{22} = \$\_\_\_$

16 Lauren earns \$715 for working a 25-hour week. What is her hourly rate of pay?

17 Calculate the hourly rate of pay for Phil who works a 25-hour week and is paid the following weekly wages:

a \$710 **b** \$877.50 **c** \$605 **d** \$485 **e** \$447.50

EXAMPLE 6

Sophie works a 38-hour week and is paid \$28.75 per hour. How much does she earn in a:

a week? **b** fortnight? **c** year? **d** average month?

	Solve	Think	Apply
a	Weekly wages $= \$28.75 \times 38$ $= \$1092.50$	Find the weekly wage by multiplying the hourly rate of \$28.75 by 38 hours.	Always convert to a weekly or annual wage to find the amount over other time periods.
b	Fortnightly wages $= \$1092.50 \times 2$ $= \$2185$	Find the fortnightly wage by multiplying the weekly wage by 2.	
c	Yearly wages $= \$1092.50 \times 52$ $= \$56\,810$	Find the yearly wage by multiplying the weekly wage by 52.	
d	Monthly wages $= \frac{\$56\,810}{12}$ $= \$4734.17$ to the nearest cent	There are 12 months in a year, so divide the yearly wage by 12.	*Remember: 1 month is not equal to 4 weeks.* !

18 Ashton earns \$19.65 per hour and works a 31-hour week. Complete the following to find his:

a Weekly wages $= \$\_\_\_ \times 31 = \$\_\_\_$

b Fortnightly wages $= \$\_\_\_ \times 2 = \$\_\_\_$

c Yearly wages $= \$\_\_\_ \times \_\_\_ = \$\_\_\_$

d Monthly wages $= \frac{\$\square}{12} = \$\_\_\_$

19 Trevor earns $17.20 per hour and works a 36-hour week. How much does he earn in a:

a week? **b** fortnight?
c year? **d** average month?

20 If a person is paid the following hourly rates for a 38-hour week, how much do they earn in a:

i week? **ii** fortnight?
iii year? **iv** month?

a $43/h **b** $27.60/h
c $52.90/h **d** $18.45/h
e $75.30/h **f** $23.55/h

B Additional payments

Overtime

Employees who earn wages are expected to work a certain number of hours each day, or each week, as negotiated in their workplace agreement. Overtime is paid to people who work additional hours and is usually paid at a higher rate. The most common rates of overtime payment are time-and-a-half and double-time.

- **Time-and-a-half:** the employee is paid at $1\frac{1}{2}$ times the normal hourly rate of pay. For example, if the normal rate of pay is $26/h, the employee would be paid ($26 × $1\frac{1}{2}$ =) $39/h.
- **Double-time:** the employee is paid double the normal rate of pay. For example, if the normal rate of pay is $26/h, the employee would be paid ($26 × 2 =) $52/h.

EXAMPLE 1

Ben normally works a 35-hour week as a kitchen hand and is paid $18.90 per hour. Calculate his total wages for a week in which he works an additional 5 hours overtime at time-and-a-half.

Solve	Think	Apply
Normal pay = $18.90 × 35 = $661.50 Overtime = ($18.90 × 1.5) × 5 = $141.75 Total wages = $661.50 + $141.75 = $803.25	Multiply the hourly rate of $18.90 by 35 hours to get the weekly wages. Multiply the hourly rate of $18.90 by the overtime loading of 1.5 to get the time-and-a-half hourly rate. Multiply the 5 hours overtime by this rate. Add $661.50 and $141.75 to find the total wages.	To calculate the overtime payment, multiply the overtime hours or the rate by 1.5, but not both.

Exercise 5B

1 Agneszka normally works a 28-hour week and is paid $22.36 per hour. Complete the following to calculate her total wages for a week in which she works an additional 3 hours overtime at time-and-a-half.

Normal pay = $___ × 28 = $___
Overtime = ($___ × 1.5) × ___ = $___
Total wages = $___ + $___
= $___

2 Jenny normally works a 35-hour week and is paid $23.40 per hour. Calculate her total wages for a week in which she works an additional 4 hours overtime at time-and-a-half.

3 Con normally works a 22-hour week and is paid $36.50 per hour. Calculate his total wages for a week in which he works an additional 7 hours overtime at time-and-a-half.

4 Rebecca normally works a 36-hour week and is paid $17.20 per hour. Calculate her total wages for a week in which she works an additional 3 hours overtime at time-and-a-half.

5 Carla normally works a 30-hour week and is paid $27.95 per hour. Calculate her total wages for a week in which she works an additional 6 hours overtime at time-and-a-half.

6 Tim is paid $18.60 per hour for a normal 35-hour week and time-and-a-half for any extra hours worked. How much would he earn for a week in which he worked 40 hours?

EXAMPLE 2

Rina normally works a 35-hour week as an analyst and is paid $36.15 per hour. Calculate her total wages for a week in which she works an additional 5 hours at time-and-a-half and 3 hours at double-time.

Solve	Think	Apply
Normal pay = $36.15 × 35 = $1265.25 Overtime = ($36.15 × 1.5) × 5 + ($36.15 × 2) × 3 = $488.03 to the nearest cent Total wages = $1265.25 + $488.03 = $1753.28	Multiply the hourly rate of $36.15 by 35 hours to get the weekly wages. Multiply the hourly rate of $36.15 by 1.5 and by 2 to get the overtime rates. Multiply 5 hours overtime at the time-and-a-half rate and 3 hours by the double-time rate. Add $1265.25 and $488.03 to find the total wages.	Calculate each amount of overtime separately and make sure that this is added to the normal weekly wage.

7 Zhong normally works a 30-hour week as a paper folder and is paid $29.60 per hour. Complete the following to calculate his total wages for a week in which he works an additional 7 hours at time-and-a-half and 5 hours at double-time.

Normal pay = $___ × 30 = $___
Overtime = ($29.60 × ___) × 7 + ($___ × 2) × ___ = $___
Total wages = $___ + $___
= $___

8 Jarrod normally works a 35-hour week as a baker and is paid \$31.20 per hour. Calculate his total wages for a week in which he works an additional 5 hours at time-and-a-half and 3 hours at double-time.

9 Martha normally works a 30-hour week and is paid \$28.60 per hour. Calculate her total wages for a week in which she works an additional 8 hours at time-and-a-half and 5 hours at double-time.

10 Dana normally works a 38-hour week and is paid \$36.15 per hour. Calculate her total wages for a week in which she works an additional 4 hours at time-and-a-half and 2 hours at double-time.

11 Erin is paid \$24.70 per hour. She is paid the normal rate for the first 7 hours worked each day, time-and-a-half for the next 2 hours and double-time for any other time. Calculate her total wages for a day on which she worked these hours:

a 8 **b** 9 **c** 10

12 Rob is paid \$26.30 per hour. He is paid the normal rate for the first 6 hours worked each day, time-and-a-half for the next 2 hours and double-time for any other time. Calculate his total wages for a day on which he worked these hours:

a 8 **b** 9 **c** 10

Weekend, night and public holiday loadings

Employees who earn wages may be paid higher rates for working on weekends, at night or on public holidays. These are often calculated as time-and-a-half and double-time.

EXAMPLE 3

Melissa is paid \$21.60 per hour on weekdays. On weekends she is paid time-and-a-half.

a Find her weekend rate.

b Find her wage for 8 hours during the week and 6 hours on the weekend.

	Solve	Think	Apply
a	Weekend rate = \$21.60 × 1.5 = \$32.40	Multiply the hourly rate of \$21.60 by 1.5 to get the time-and-a-half rate.	Sometimes weekend work is paid as overtime, but not always.
b	Weekday wage = \$21.60 × 8 = \$172.80 Weekend wage = \$32.40 × 6 = \$194.40 Total wages = \$172.80 + \$194.40 = \$367.20	Multiply the weekday rate of \$21.60 by 8 hours. Multiply time-and-a-half rate of \$32.40 by 6 hours. Add \$172.80 and \$194.40 to find the total wages.	Calculate the normal pay and the overtime. Add them both.

13 Rakesh is paid \$31.68 per hour for work on weekdays and time-and-a-half for weekend work. Complete the following to find:

a his weekend rate

Weekend rate = \$___ × 1.5 = \$___

b his total wage for working 17 hours during the week and 5 hours on the weekend.

Weekday wage = \$___ × ___ = \$___

Weekend wage = \$___ × 5 = \$___

Total wages = \$___ + \$___

= \$___

14 Matthew is paid \$27.28 per hour for work on weekdays and time-and-a-half for weekend work.

a Find his weekend rate per hour.

b Find his pay for 15 hours during the week and 7 hours on the weekend.

15 Maddie works as a waitress and is paid \$19.40 per hour for work on weekdays and time-and-a-half for night work.

a Find her night rate per hour.

b Find her pay for a week in which she works 8 hours during the day and 14 hours at night.

16 Christopher is paid \$31.08 per hour for work on weekdays and double-time for weekend work.

Double-time means that the normal rate is multiplied by 2.

a Find his weekend rate per hour.

b Find his pay for a week in which he works 13 hours during the week and 11 hours on the weekend.

17 Holly is paid \$23.57 per hour for work on weekdays and double-time for night work.

a Find her night rate per hour.

b Find her pay for a week in which she works 20 hours during the day and 17 hours at night.

18 Maree is paid \$41.20 per hour for work on weekdays, time-and-a-half for Saturdays and double-time for Sundays and public holidays.

a Find her Saturday rate per hour.

b Find her Sunday and public holiday rate per hour.

c Find her pay for a week in which she works 12 hours during the week, 4 hours on Saturday, 5 hours on Sunday and 5 hours on the public-holiday Monday.

Bonuses

A **bonus** is an extra payment made to employees, often as a reward, for example, for meeting deadlines, exceeding profit targets, or producing a high quality of work.

EXAMPLE 4

Paul works for a builder and earns a salary of \$66 000 per year. At the end of the year, the builder decides to pay Paul a bonus equal to one month's salary. Calculate Paul's bonus.

Solve	Think	Apply
$\text{Bonus} = \frac{\$66\ 000}{12}$ $= \$5500$	Divide the yearly salary by 12 months to get the bonus amount.	When calculating monthly salary, always work from the annual salary.

19 Seveta works as a clerk and earns a salary of $74 760 per year. At the end of the year her employer pays her a bonus of one month's salary. Complete the following to calculate Seveta's bonus.

$$\text{Bonus} = \frac{\$\square}{12} = \$\_\_\_$$

20 Jenni works as a secretary and earns a salary of $58 600 per year. At the end of the year her employer pays her a bonus of one month's salary. Calculate Jenni's bonus.

21 Abdul is paid $23.50 per hour and works a normal 35-hour week. At the end of the year his employer pays him a bonus of 5% of his yearly wages. Calculate Abdul's bonus.

22 A company made a profit of $194 000 for the year. The owner decided to share 60% of the profit between her 80 employees as a bonus. Calculate the bonus paid to each employee.

23 For completing a project ahead of schedule, each member of the project team was given a bonus of 3% of the after-tax profit made. Calculate the bonus paid to each team member if the after-tax profit was $120 000.

Holiday loading

Holiday loading (leave loading) is an extra payment given to permanent employees when they take their annual recreation leave. It is usually calculated as 17.5% of 4 weeks normal salary or wages.

EXAMPLE 5

Tanya earns $1215 per week as a librarian. She is entitled to 4 weeks' annual recreation leave and receives an additional holiday loading of 17.5%. Calculate Tanya's:

a holiday loading

b total pay for this holiday period.

	Solve	Think	Apply
a	4 weeks pay = $1215 × 4 = $4860 Holiday loading = 17.5% of $4860 = 0.175 × $4860 = $850.50	Calculate 4 weeks pay by multiplying $1215 by 4. Find $17\frac{1}{2}$% of $4860.	Holiday loading is an extra amount and is added to the normal 4 weeks pay. It is usually a percentage of 4 weeks pay.
b	Total pay = normal pay + holiday loading = $4860 + $850.50 = $5710.50	Add the 4 weeks pay and the holiday loading to get the total holiday pay.	

24 Maddison earns $884 per week. She is entitled to 4 weeks annual leave and receives an additional holiday loading of 17.5%. Complete the following to find:

a Maddison's holiday loading

4 weeks pay = $___ × 4 = $___
Holiday loading = 17.5% of $___
= 0.175 × $___ = $___

b Maddison's total pay for this holiday period.

Total pay = normal pay + holiday loading
= $___ + $___ = $___

25 Kylie earns $760 per week. She is entitled to 4 weeks annual leave and receives an additional holiday loading of 17.5%. Calculate Kylie's:

a holiday loading **b** total pay for this holiday period.

26 Vinh works a normal 35-hour week and is paid $17.90 per hour. He is entitled to 4 weeks' annual leave and receives an additional holiday loading of 17.5%. Calculate Vinh's:

a holiday loading **b** total pay for this holiday period.

27 Sunny earns $2460 per fortnight as a receptionist. She is entitled to 4 weeks annual leave and receives an additional holiday loading of 17.5%. Calculate Sunny's:

a holiday loading **b** total pay for this holiday period.

28 Wesley earns $43 940 per year. He is entitled to 4 weeks annual leave and receives an additional holiday loading of 17.5%. Calculate Wesley's:

a holiday loading **b** total pay for this holiday period.

Investigation 2 Calculating total pay

Here is a spreadsheet to calculate the total pay for the employees of a factory.

	A	B	C	D	E	F
1	Employee	Rate ($/h)	Normal time (h)	Overtime (h) time-and-a-half	Overtime (h) double-time	Total pay
2	Brody	24.72	36	8	4	
3	Chloe	18.94	36	6	1	
4	Alan	23.65	28	4	2	
5	Gillian	26.36	35	5	3	
6	Natasha	33.56	30			
7	Yani	19.43	40	1	1	

1 Copy the spreadsheet.

2 In cell F2 type the formula =(C2+D2*1.5+E2*2)*B2.

3 To find the total pay for the other employees:

- Highlight cells F2 to F7.
- Go to Edit.
- Select Fill Down.

4 Add some more employees. Enter their pay rates and the numbers of hours worked. Calculate their total pays.

C Casual employment

Casual workers are paid for the number of hours worked and may be employed full-time or part-time. The hourly rate is usually higher than for permanent workers because they are not entitled to benefits such as holiday leave and sick leave, and do not have security of employment through a workplace agreement. They may be paid special rates for working on weekends and public holidays.

EXAMPLE 1

The table below shows part of an award agreement for tradespersons in July 2013.

Tradesperson	Permanent $ per hour	Casual $ per hour
Bricklayer	$20.89	$25.07
Carpenter	$20.55	$24.66
Painter	$20.04	$24.04
Signwriter	$20.04	$24.04
Roof tiler	$20.22	$24.27

a Tom is a permanently employed bricklayer who works a 35-hour week. Calculate his normal weekly wages at award rates.

b Bob is a bricklayer who is employed at casual award rates for 35 hours a week. Calculate his weekly wage.

c How much more than Tom does Bob earn?

	Solve	Think	Apply
a	Tom's wages = $20.89 × 35 = $731.15	Multiply the hourly rate of $20.89 for a permanent bricklayer by 35 hours.	Although casual employees earn more per hour worked than permanent employees, they do not receive sick leave or holiday pay, and are not paid for public holidays.
b	Bob's wages = $25.07 × 35 = $877.45	Multiply the hourly rate of $25.07 for a casual bricklayer by 35 hours.	
c	$877.45 − $731.15 = $146.30 Bob earns $146.30 more than Tom.	Subtract the wages. Bob's wage is $146.30 more than Tom's wage.	

Exercise 5C

Use the table in Example 1 to answer questions **1** to **5**.

1 Joseph is a painter employed as a casual for 21 hours in one week. Complete to find how much he earns.
Joseph's wages = $___ × 21 = $___

2 Grant is a carpenter employed as a casual for 24 hours one week. How much does he earn?

3 Emma is a signwriter who does casual work. How much does she earn in a week in which she works the following hours?
Monday 3 hours, Tuesday 4 hours, Wednesday 3 hours, Friday 5 hours

4 **a** Matt is a permanently employed painter and works a 35-hour week. Calculate his normal weekly wages.
b During a busy period another painter is employed to work with him as a casual for the 35 hours. How much extra does the casual earn for the week's work?

5 Jack is a permanently employed roof tiler. Due to extra demand two casuals are employed to help him for 7 hours on each of 3 days. What is the wages bill for the two casuals?

EXAMPLE 2

The table below shows the pay rates in $ per hour for casuals in a restaurant.

	Monday–Friday	Saturday	Sunday
Kitchen hand	$20.40	$24.48	$29.38
Waiter	$21.10	$25.32	$30.38
Grill cook	$21.70	$26.04	$31.25
Chef	$25.40	$30.48	$36.58

Calculate the wages of a casual waiter who works a total of 8 hours from Monday to Friday, and then 6 hours on Saturday and 4 hours on Sunday.

Solve	Think	Apply
Wages $= \$21.10 \times 8$ $+ \$25.32 \times 6$ $+ \$30.38 \times 4$ $= \$442.24$	Multiply $21.10 per hour by 8 for weekday work. Multiply $25.32 per hour by 6 for Saturday work. Multiply $30.38 per hour by 4 for Sunday work. Add to find the total wages.	Make sure that the hours worked are multiplied by the correct rate.

Use the table in Example 2 to answer questions **6** to **10**.

6 Complete the following to calculate the wages of a casual chef who works 11 hours total from Monday to Friday, 3 hours on Saturday and 5 hours on Sunday.

Wages = $___ × 11 + $___ × 3 + $___ × ___
= $___

7 Calculate the wages of a casual waiter who works 9 hours total from Monday to Friday, 5 hours on Saturday and 6 hours on Sunday.

8 Emily is employed as a casual kitchen hand for 3 hours on each day Monday to Saturday. Calculate her wages.

9 Trent is a chef who works as a casual on Saturday for 6 hours and Sunday for 6 hours. Calculate his wages.

10 Calculate the wages of a casual grill cook who works the following hours.

M	T	W	T	F	S	S
–	3		3	4	6	3

The table below shows the casual Restaurant Award rates for Grade 1 juniors in $ per hour. Use the table to answer questions **11** to **13**.

Age (years)	Monday–Friday	Saturday	Sunday
17	$16.32	$19.59	$22.85
18	$18.36	$22.03	$25.70
19	$20.40	$24.48	$28.56
20	$22.44	$26.93	$31.42

11 Jenny is 19 years old and does casual work as a waitress. Calculate her wages for a week in which she works 12 hours total from Monday and Friday, 5 hours on Saturday plus 3 hours on Sunday.

12 **a** Ben is 18 years old and does casual work in a coffee shop. How much does he earn for working 4 hours on Saturday and 6 hours on Sunday?

b Lara is 20 years old. How much would she earn for working the same hours as Ben?

13 Sarah and Ella work in a café. Sarah is 17 years old and Ella is 18 years old. In one particular week they both work the same shifts, as shown below. How much more than Sarah does Ella earn for this week?

M	T	W	T	F	S	S
3	3	–	4	4	5	3

D Piecework

Sometimes an employee is paid for the number of items (pieces) produced or completed. This method of earning money is known as **piecework**.

EXAMPLE 1

Peta works at home sewing children's tops. She is paid $3.20 for each top she produces. How much does she earn if she produces 120 tops?

Solve	Think	Apply
Income = 120 × $3.20 = $384	Multiply the number of tops sewn by the rate of $3.20 per top.	The more items sewn, the greater the amount earned.

Exercise 5D

1 Trinh has a job assembling jewellery boxes. He receives $1.27 for each box he assembles. Complete the following to find how much he earns if he assembles 370 boxes.

Income = ___ × $1.27 = $___

2 Terry has a job assembling door locks. He receives 48 cents for each lock he assembles. How much does he earn if he assembles 450 locks?

3 Kerry is paid \$0.83 per item for ironing shirts in a factory. How much does she earn when she irons 240 shirts?

4 Patricia earns \$1.03 for each dress she finishes in a clothing factory. If, on average, she can finish 20 dresses per hour and she works a 35-hour week, what are her average weekly earnings?

5 Joe works for a men's hairdresser and is paid \$15 for each haircut. If he averages 26 haircuts per day for 6 days, how much does he earn?

6 Wayne works part-time for Sparkler Lighting Co. assembling lamps. He is paid the following daily piecework rates:

• for the first 50 lamps	\$1.45/lamp
• for each lamp over 50 and up to 70	\$1.60/lamp
• for each lamp over 70	\$1.90/lamp

The table shows the number of lamps he assembled in a particular week. Calculate Wayne's earnings for the week.

	Number $\leqslant 50$	Number > 50 and $\leqslant 70$	Number > 70
Monday	50	5	
Tuesday	48		
Wednesday	50	12	
Thursday	50	20	6
Friday	50	2	

E Commission

Employees such as sales people and real-estate agents are paid a percentage of the value of their sales. They are paid a **commission**. They may also be paid a retainer (a fixed weekly rate).

EXAMPLE 1

Georgia works as a salesperson and is paid a commission of 6% of the value of her sales. If Georgia sells \$12 000 worth of goods one week, what is her commission?

Solve	Think	Apply
Commission $= 6\%$ of \$12 000 $= 0.06 \times \$12\,000$ $= \$720$	Georgia sold \$12 000 worth of goods. As her commission is 6% of the sale, find 6% of \$12 000.	Commission is usually a percentage of the value of sales. Calculate the percentage of the amount sold to find the commission.

Exercise 5E

1 Reggi works as a salesperson and is paid a commission of 4% of the value of her sales. Complete the following to find Reggi's commission for a week in which she sells $6800 worth of goods.
Commission = ___% of $6800 = 0.04 × $___ = $___

2 Phillipa works as a salesperson and is paid a commission of 5% of the value of her sales. If Phillipa sells $16 000 worth of goods one week, what is her commission?

3 David works as a salesperson and is paid a commission of 8% of the value of his sales. If David sells $14 800 worth of goods one week, what is his commission?

4 Alex sells cars and is paid a commission of 2% of the value of his sales. What is the commission for a week in which Alex sells two cars costing $32 000 each?

5 A real-estate agent charges a commission of 1.5% of the value of any house he sells. Calculate how much he will earn if he sells a house for:
a $660 000 **b** $320 000 **c** $980 000

6 Joanne has a part-time job selling cosmetics. She is paid a commission of 18% of all sales. Calculate how much she earns in one month if her sales are:
a $9000 **b** $5400 **c** $2300

7 Daina is a stockbroker. She receives a commission of 2.5% of the selling price of any shares she sells. What commission would she earn for selling shares worth:
a $15 000? **b** $27 000? **c** $243 000?

The commission earned for buying or selling shares is called ***brokerage****.*

EXAMPLE 2

Steve sells clothing. He earns a commission of 19.5% of all weekly sales in excess of $5000. How much commission does he earn on sales of:
a $4800 **b** $8650?

'In excess of' means 'more than'.

	Solve	Think	Apply
a	Steve's sales are less than $5000, he earns no commission.	Commission is only applied if Steve's sales are more than $5000.	Check carefully for any excess, as commission is not earned until the excess is reached.
b	Excess = $8650 − $5000 = $3650 Commission = 0.195 × $3650 = $711.75	Commission is applied to sales over $5000. As $8650 − $5000 = $3650, find 19.5% of $3650.	

8 Steve sells clothing. He earns a commission of 18.5% of all weekly sales in excess of $2000. Complete the following to find how much commission he earns on sales of $5800.
Excess = $___ − $2000 = $___
Commission = 0.___ × $___ = $___

9 Fiona sells cleaning equipment. She earns a commission of 17.5% of all weekly sales in excess of $10 000. How much commission does she earn on weekly sales of:

a $8000? **b** $12 000? **c** $24 000?

EXAMPLE 3

Carol sells phone plans. She is paid the following rates of commission:

- 4.5% of the first $20 000 worth of sales
- 6.5% of any sales above $20 000.

Calculate how much she earns in a week in which her sales are:

a $16 000 **b** $24 000

	Solve	Think	Apply
a	Commission = 4.5% of $16 000 = 0.045 × $16 000 = $720	Carol sold $16 000 worth of plans. As her commission is 4.5% on sales less than $20 000, find 4.5% of $16 000.	The lower ratio applies to all of the first $20 000. If sales were more than $20 000, the extra sales amount earns commission at the higher rate.
b	Commission = 4.5% of $20 000 + 6.5% of ($24 000 − $20 000) = 0.045 × $20 000 + 0.065 × $4000 = $1160	Carol sold $24 000 worth of plans. As her commission is 4.5% on sales up to $20 000, find 4.5% of $20 000. Her commission is 6.5% on sales over $20 000, so find 6.5% of $4000. Add.	

10 Allyra sells phone plans. She is paid the following rates of commission:

- 4.5% of the first $8000 worth of sales
- 6.2% of any sales above $8000.

Complete the following to calculate how much she earns in a week in which her sales are $12 000.

Commission = ___% of $\_\_\_ + 6.2% of ($___ − $8000)

= 0.___ × $___ + 0.062 × $___ = $___

11 Harry sells internet plans. He is paid the following rates of commission:

- 2% of the first $20 000 worth of sales
- 3.5% of any sales above $20 000.

Calculate how much he earns in a week in which his sales are:

a $13 000 **b** $19 990 **c** $20 000 **d** $25 000 **e** $38 000

'Rate' means the percentage rate of commission. !

12 Zane has a job selling advertising. He is paid the following commission on his weekly sales:

- 1% for the first $50 000 worth of sales
- 2% of the next $30 000 worth of sales
- 6% of the value of any remaining sales

How much would Zane earn in a week in which his sales were:

a $46 000? **b** $50 000? **c** $65 000? **d** $80 000? **e** $92 000?

13 Marie is a real-estate agent. She charges the following commission for selling home units:

- 3% for the first $300 000 of the selling price of the unit
- 2% for the next $100 000
- 1.5% for the remainder of the selling price.

Calculate how much Marie would earn for selling a unit for:

a $290 000 **b** $350 000 **c** $380 000 **d** $455 000 **e** $550 000

EXAMPLE 4

Chad sells washing machines. He is paid a fixed wage of $400 per week plus a commission of 3% of sales. How much does he earn in a week in which his sales are $5480?

The fixed part of Chad's wages ($400) is called a retainer.

Solve	Think	Apply
Commission = 0.03 × $5480 = $164.40 Weekly earnings = retainer + commission = $400 + $164.40 = $564.40	Calculate the commission on Chad's weekly sales by finding 3% of $5480. Add the retainer of $400 and the commission of $164.40 to find Chad's weekly earnings.	Even if he does not make any sales, Chad will receive his fixed income of $400 per week.

14 Ibraham sells washing machines. He is paid a fixed wage of $270 per week plus a commission of 4% of sales. Complete the following to find how much he earns in a week in which his sales are $6240.

Commission = 0.04 × $___ = $___

Weekly earnings = retainer + commission

= $___ + $___

= $___

15 Therese sells printers for computers. She is paid a retainer of $250 per week plus a commission of 4% of her sales. How much does she earn in a week in which she sells printers to the value of $14 970?

16 Michael works for a bookseller. He is paid a retainer of $280 per week plus a commission of 2% of sales. How much does he earn in a week in which his sales are:

a $7650? **b** $3000? **c** $12 000? **d** $4700? **e** $8260?

17 Complete the following table to show the weekly earnings of the sales team for a pharmaceutical company.

Employee	Retainer	Rate	Sales	Commission	Weekly earnings
R. Roberts	$200	12%	$4 200		
H. Low	$150	15%	$8 600		
J. Thum	$100	16%	$10 450		
K. Trau	nil	18%	$12 900		
G. Flood	nil	19%	$15 360		

18 Jacqueline works as a sales representative for a hardware company. She is paid a retainer of $550 per week plus a 7% commission on any sales in excess of $6000. How much does she earn in a week if her sales are:

a $4500? **b** $6400? **c** $7200? **d** $8430? **e** $20 960?

19 Hassan gets a job as a salesperson with a phone company. He is offered two methods of weekly payment:

A retainer of $480 plus a commission of 3%, or

B no retainer and a commission of 15%.

a How much would Hassan earn, using each method, if his weekly sales were:

i $0? **ii** $3000? **iii** $4000? **iv** $5000? **v** $10 000?

b Which method of payment would you advise Hassan to choose? Give reasons.

F Taxable income

Taxable income is calculated using total income and allowable deductions. Total income includes income from all sources throughout the year. It may include wages and salaries, bonuses, interest earned, commissions and allowances. The total income may be subject to deductions that reduce the amount used to calculate the tax payable. Tax deductions may include tools used for work, safety equipment, self-education expenses, union fees, donations over $2, car travel expenses, uniforms and their cleaning, and tax agent fees. Which deductions are allowable will depend on the profession.

$$\text{Taxable income} = \text{total income} - \text{allowable deductions}$$

An extra tax that may be payable is the Medicare levy. Medicare is the public hospital medical system available without charge for Australians. The Medicare levy is currently calculated at 1.5% of taxable income. This is added to the amount of tax payable. In 2013, a full Medicare levy is payable if a person's taxable income exceeds $22 828. For incomes between $19 404 and $22 828, a reduced levy is payable, and for incomes of less than $19 404 no levy is payable. These thresholds are higher in some circumstances. People receiving family benefits have them added to their taxable income.

For taxation purposes, income is usually calculated in whole dollars, so always round down to the nearest whole dollar. (It does not follow the usual rules for rounding.)

EXAMPLE 1

Amita uses this information to calculate her taxable income: wages $35 980, interest $569 (joint account), bonus $200, cost of uniforms $140, tax agent fee $80, use of car $298. Calculate her taxable income.

Solve	Think	Apply
Income = $35 980 + ($569 ÷ 2) + $200 = $36 464.50 = $36 464 Deductions = $140 + $80 + $298 = $518 Taxable income = $36 464 − $518 = $35 946	Interest on a joint account is shared between both people. Add all income items to get $36 464. Add all deductions to get $518. Subtract deductions from total income to find the taxable income of $35 946.	Determine all income amounts and all deductions. Subtract to find the taxable income.

Exercise 5F

1 Annie uses this information to calculate her taxable income: wages $45 320, interest $665 (joint account), bonus $800, cost of uniforms $340, tax agent fee $60, use of car $45. Complete the following.

Income = ___ + (___ ÷ 2) + $800 = ___ = ___

Deductions = ___ + $60 + ___ = ___

Taxable income = ___ − ___ = ___

2 Use the following information to calculate Gary's taxable income: wages $42 330, interest $2355 (joint account), cost of uniforms $300, cleaning of uniforms $156, tax agent fee $100, use of car $3220.

3 Asha uses the following information to calculate her taxable income: wages $29 555, interest $243, tax agent fee $70, use of car $452, donations to charity $120. Calculate Asha's taxable income.

4 Paul uses the following information to calculate his taxable income: wages $53 022, interest $3659, bonus $1000, tax agent fee $120, use of car $1256, donations to charity $500. Calculate Paul's taxable income.

G Calculating tax

The calculation of income tax is done using a tax table. The following table shows the tax rates for 2013–14.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001–$180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

EXAMPLE 1

Use the tax table to calculate the tax payable on these incomes.

a $53 251 **b** $26 784 **c** $105 631

	Solve	Think	Apply
a	$53 251 is in the third row of the table. Excess = $53 251 − $37 000 = $16 251 Tax = $3572 + 0.325 × $16 251 = $8853.58	Subtract $37 000 to find the excess. Tax is $3572 plus 0.325 × excess.	Locate the taxable income range in the tax table. Use that row of the tax table if tax is payable. Subtract to find the ‘over’ amount (the excess) and multiply it by the rate in dollars. Add the first amount in the tax row, if appropriate.
b	$26 784 is in the second row of the table. Excess = $26 784 − $18 200 = $8584 Tax = 0.19 × $8584 = $1630.96	Subtract $18 200 to find the excess. Tax is 0.19 × excess.	
c	$105 631 is in the fourth row of the table. Excess = $105 631 − $80 000 = $25 631 Tax = $17 547 + 0.37 × $25 631 = $27 030.47	Subtract $80 000 to find the excess. Tax is $17 547 plus 0.37 × excess.	

Exercise 5G

1 Complete to find the tax payable on $47 953.

$47 953 is in the ___ row of the tax table.

Excess = ___ − $37 000 = ___

Tax = ___ + 0.325 × ___ = ___

2 What is the tax payable on these taxable incomes?

a $86 564 **b** $48 911 **c** $8100 **d** $31 000

3 Mrs Jones has an annual salary of $33 700 and receives an income of $896 from other sources.

a Calculate Mrs Jones’ total annual income.

b Determine the amount of tax Mrs Jones will have to pay on her total income, using the tax table at the beginning of this section.

4 Joshua earns $47 500 per year.

a Calculate the amount of tax he must pay.

b If Joshua has already paid $9084 in tax (PAYG – pay as you go) throughout the year, calculate the amount of money he receives in the form of a tax refund.

5 Sam has a few odd jobs. In total, he earns $22 500 per year. He receives a tax deduction of $2600.

a Calculate his taxable income.

b Calculate the amount of tax he must pay on his taxable income.

EXAMPLE 2

Sarah works as a secretary and receives a yearly salary of $34 479 plus an income of $1950 per year from baby-sitting. Her total tax deductions are $1570. During the year she paid tax instalments amounting to $3265.

a Calculate her total income.

b Calculate her taxable income.

c Determine the tax payable on her taxable income.

d Find her tax refund or balance payable.

	Solve	Think	Apply
a	Total income = $34 479 + $1950 = $36 429	Add Sarah's salary and baby-sitting income.	Calculate the taxable income by subtracting all tax deductions from total income. Use the tax table to calculate the tax payable. Compare the tax paid with the tax payable to determine the refund or balance payable.
b	Taxable income = $36 429 − $1570 = $34 859	Taxable income = total income − tax deductions	
c	Tax = 19 cents per $ in excess of $18 200 = 0.19 × ($34 859 − $18 200) = $3165.21 Tax payable on $34 859 is $3165.21.	Tax payable on $34 859 is in row 2 of the tax table.	
d	Tax refund or balance payable = $3265 − $3165.21 = $99.79 This is a refund of $99.79.	Sarah has already paid $3265. She only needed to pay $3165.21. She has paid too much and receives a refund.	

6 Brittany earns $86 452 as a company manager and $5203 as a part-time singer. She has tax deductions of $2612 and throughout the year pays a total of $20 867.40 in tax instalments. Complete the following to find:

a Total income = $86 452 + ___ = ___

b Taxable income = ___ − ___ = ___

c Tax payable = ___ + 0.37 × (___ − $80 000) = ___

d Tax refund or balance payable = ___ − $20 867.40 = ___

7 Mr Benton has a yearly salary of $29 760. He receives a salary of $2500 from his hobby of wood carving. His tax deductions amount to $1090 and throughout the year he has paid PAYG instalments of $3215.90.

a Calculate his total income.

b Calculate his taxable income.

c Determine the tax payable on his taxable income.

d Find his tax refund or balance payable.

8 Celine earns $32 056 per annum as an environmentalist and $8159 as a part-time analyst. She has tax deductions of $4903 and has paid $5123.90 in tax instalments.

a Calculate her total income.

b Calculate her taxable income.

c Determine the tax payable on her taxable income.

d Find the tax refund or balance payable.

9 Ricky earns \$84 365 as a cabaret artist and \$4620 from his share portfolio. He has tax deductions of \$13 884 and throughout the year pays a total of \$15 988 in tax instalments.

a Calculate his total income.

b Calculate his taxable income.

c Determine the tax payable on his taxable income.

d Find the tax refund or balance payable.

H Simple interest

When you invest money in a financial institution, such as a bank, the bank pays for the use of the money. This payment by the bank is called **interest** and is calculated as a percentage of the amount invested. Similarly, when you borrow money a charge is made for the use of the bank's money. This charge also is called **interest** and is calculated as a percentage of the amount borrowed. If the interest is calculated as a fixed percentage of the original amount invested (or borrowed), it is called **simple interest**.

EXAMPLE 1

Calculate the simple interest received when \$8000 is invested for 3 years at 4.5% p.a.

Solve	Think	Apply
Interest for 1 year = 4.5% of \$8000 = 0.045 × \$8000 = \$360 Interest for 3 years = \$360 × 3 = \$1080	Find the simple interest of 4.5% on \$8000 for 1 year and multiply it by 3 to find the interest over 3 years.	$\text{Interest} = \frac{\text{rate (p.a.)}}{100}$ × amount invested × number of years

If \$$P$ is invested for T years at r% p.a., the simple interest, I, can be found using the formula:

$$I = PRT$$

where P is the **principal**, R is the **interest rate** p.a. expressed as a decimal $\left(R = \frac{r}{100}\right)$ and T is the time in years.

EXAMPLE 2

Use the simple interest formula to calculate the simple interest earned on an investment of \$10 800 at 3.9% p.a. for 5 years.

Solve	Think	Apply
$I = PRT$ $= \$10\,800 \times 0.039 \times 5$ $= \$2106$	Use the formula $I = PRT$ where P is \$10 800, r is 3.9% so R is $\frac{3.9}{100} = 0.039$ and T is 5 years.	Substitute the values into the formula $I = PRT$, remembering that R is always a decimal not a percentage.

Exercise 5H

1 Complete the following using the simple interest formula to calculate the simple interest earned on an investment of \$8450 at 4.7% p.a. for 3 years.

$I = PRT$

$= \$\_\_\_ \times 0.\_\_\_ \times 3 = \$\_\_\_$

2 Calculate the simple interest received when \$7000 is invested for 2 years at 5% p.a.

3 Calculate the simple interest received when \$12 000 is invested for 4 years at 3% p.a.

4 Complete the following table.

Principal	Annual interest rate	Time invested (years)	Simple interest
\$5 800	7%	4	
\$15 000	3.5%	3	
\$24 000	4.5%	5	
\$6500	5%	6	
\$18 000	2.8%	2	
\$9 300	3.4%	4	
\$6 000	3%	3	

EXAMPLE 3

Calculate the amount to which \$7000 will grow in 3 years if invested at 6.5% p.a. simple interest.

Solve	Think	Apply
Interest $= \$7000 \times 0.065 \times 3$ $= \$1365$ Amount after 3 years $= \$7000 + \$1365$ $= \$8365$	Use the simple interest formula $I = PRT$ to calculate the interest over 3 years. Add the principal (\$7000) to the interest to find the total amount.	Convert the percentage interest rate to a decimal by dividing by 100.

5 Complete to calculate the amount to which \$5500 will grow in 4 years if invested at 4.5% p.a. simple interest.

Interest = \$___ × 0.045 × ___ = \$___

Amount after 4 years = \$5500 + \$___ = \$___

6 Calculate the amount to which \$9000 will grow in 3 years if invested at 6.5% p.a. simple interest.

7 Calculate the amount to which \$20 000 will grow in 5 years if invested at 4% p.a. simple interest.

8 If I invest \$13 500 at 7.4% p.a. simple interest, how much will I have in 4 years time?

EXAMPLE 4

Calculate the simple interest earned on \$6000 at 8% p.a. for 16 months.

Solve	Think	Apply
Interest $= \$6000 \times 0.08 \times \frac{16}{12}$ $= \$640$	Number of years the money is invested $= \frac{16}{12}$	Convert months to years by dividing by 12.

9 Complete the following to calculate the simple interest on \$4500 at 3% p.a. for 17 months.

Interest = \$___ × 0.0___ × $\frac{17}{12}$ = \$___

10 Calculate the simple interest earned on each of these investments.

a \$5000 at 9% p.a. for 18 months

b \$7000 at 8% p.a. for 15 months

c \$12 500 at 10% p.a. for 9 months

d \$3800 at 12% p.a. for 27 months

e \$24 000 at 7.8% p.a. for 45 months

f \$8600 at 9.6% p.a. for 6 months

EXAMPLE 5

Rene invested \$4700 at 6% p.a. simple interest. How long did it take to earn \$1128 in interest?

Solve	Think	Apply
Interest for 1 year $= 0.06 \times \$4700$ $= \$282$ Number of years invested $= \dfrac{\$1128}{\$282}$ $= 4$ Rene invested his money for 4 years.	Find the interest earned for 1 year: \$282. Divide the total interest by \$282 to get 4 years.	Calculate the annual interest. Divide by this amount to obtain the number of years.

11 Renash invested \$2300 at 4% p.a. simple interest. Complete the following to find how long it would take to earn \$276 in interest.

Interest for 1 year $= 0.04 \times \$\_\_\_ = \$\_\_\_$

Number of years invested $= \dfrac{\$276}{\$\square} = \_\_\_$ years

12 Harry invested \$13 000 at 6% p.a. simple interest. How long would it take to earn \$4680 in interest?

13 Joy invested \$2800 at 3.5% p.a. simple interest. How long would it take to earn \$490 in interest?

EXAMPLE 6

Colin invested \$4000 for 5 years and earned \$700 in interest. What was the annual rate of simple interest?

Solve	Think	Apply
Interest for 1 year $= \$700 \div 5$ $= \$140$ Annual interest rate $= \dfrac{\$140}{\$4000} \times 100\%$ $= 3.5\%$	Divide the interest earned by the number of years. Divide the annual interest by the principal and multiply by 100 to find the interest rate as a percentage.	When calculating interest rates always convert the amount of interest to amount per year.

14 Ivan invested \$3000 for 4 years and earned \$504 in interest. Complete the following to find the annual rate of simple interest.

Interest for 1 year $= \$504 \div \_\_\_ = \$\_\_\_$

Annual interest rate $= \dfrac{\$\square}{\$2300} \times 100\%$

$= \_\_\_\%$

15 Kim invested \$6000 for 5 years and earned \$2100 in interest. What was the annual rate of simple interest?

16 Lauren invested \$17 000 for 4 years, earning \$3128 in interest. What was the annual rate of simple interest?

I Purchasing goods by cash

When paying cash to purchase goods, the cost is rounded to the nearest 5 cents.

EXAMPLE 1

How much would you actually pay in cash to purchase goods that cost:

a 87 cents (87c)? **b** \$2.43? **c** \$2.99?

	Solve	Think	Apply
a	You pay 85 cents.	Rounding to the nearest 5 cents, you would pay 85 cents (85c).	78, 79, 80, 81 and 82 round to 80 cents, while 83, 84, 85, 86 and 87 round to 85 cents.
b	You pay \$2.45.	Rounding to the nearest 5 cents, you would pay \$2.45.	
c	You pay \$3.00.	Rounding to the nearest 5 cents, you would pay \$3.00.	

Exercise 5I

1 How much would you actually pay in cash to purchase goods that cost these amounts?

a 76c **b** \$5.28 **c** \$2.79 **d** \$7.31
e \$3.97 **f** \$8.52 **g** \$16.23 **h** \$21.99
i \$54.85 **j** \$39.14 **k** \$17.36 **l** \$69.98

2 Calculate the change given in these situations.

a \$10 is offered to pay for goods worth \$5.83
b \$10 is offered to pay for goods worth \$4.99
c \$15 is offered to pay for goods worth \$10.22
d \$20 is offered to pay for goods worth \$12.84
e \$18.50 is offered to pay for goods worth \$18.36
f \$7 is offered to pay for goods worth \$6.01
g \$40 is offered to pay for goods worth \$28.57
h \$50 is offered to pay for goods worth \$48.19

EXAMPLE 2

A computer store offers a discount of 12% for cash purchases. Find the cash price of an iPad marked at \$799.

Solve	Think	Apply
Discount = 12% of \$799 = 0.12 × \$799 = \$95.88 Cash price = \$799 − \$95.88 OR Cash price = 88% of \$799 = 0.88 × \$799 = \$703.12 The cash price is \$703.10 to nearest 5 cents.	Find 12% of \$799 to get the discount. Subtract this from the marked price to find the cash price. OR Find 88% (100% − 12%) of \$799 to get the cash price. Rounding to the nearest 5 cents, the cash price is \$703.10.	Subtracting the percentage discount from 100% allows the direct calculation of the price.

3 A computer store offers a discount of 11% for cash purchases. Complete to find the cash price of a laptop marked at $799.90.

Cash price = 89% of $___

= 0.___ × $___ = $___

The cash price is $___ to the nearest 5 cents.

4 An electrical store offers a discount of 12% for cash purchases. Find the cash price of a sound system marked at $479.

5 A builders' hardware store offers a discount of 6% for cash purchases. Find the cash price for goods worth:

a $147 **b** $463 **c** $224

d $180.56 **e** $68.99

6 List some advantages and disadvantages of using cash to purchase goods.

J Using credit cards

A **credit card** is a convenient method for purchasing and paying for goods. You can pay for the goods later, you don't need to carry large amounts of cash, you can take advantage of sales, and a monthly statement of purchases is provided.

The financial institution issuing the card charges an annual fee. If the balance owing at the end of each month is paid within the interest-free period (which varies from 0 to 55 days), no further costs are involved. However, if any balance is owing after the interest-free period has finished, there is an initial charge equal to 1 month's interest on the balance outstanding, and in addition an interest charge calculated daily from the end of the interest-free period. A minimum payment must be made each month.

You can also obtain cash advances up to a certain limit. In this case, interest is charged daily from the time the cash is withdrawn. Cash advances are usually charged a higher interest rate than purchases.

Exercise 5J

Use the credit card statement on the facing page to answer questions **1** to **3**.

1 What is the:

a period the statement covers? **b** annual interest rate for purchases?

c annual interest rate for cash advances? **d** daily interest rate for purchases?

e credit limit? **f** available credit?

g date by which payment must be made? **h** minimum payment that must be made?

2 **a** Calculate Opening balance + New charges − Payments/Refunds. Is this the Closing balance?

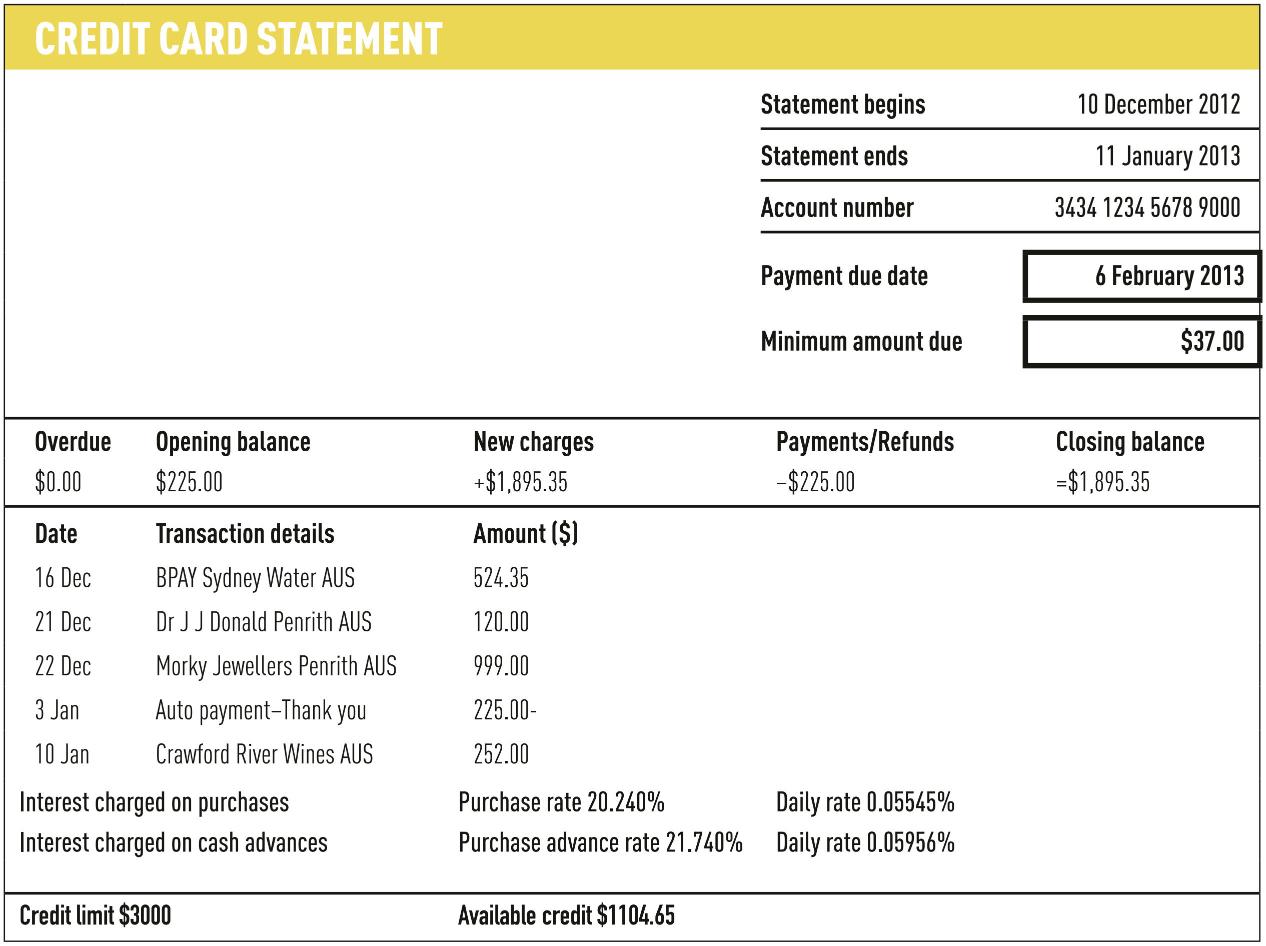
b Explain why the Closing balance is the same as the New charges.

c This card has a 30-day interest-free period. Explain why no interest has been charged on this statement.

3 **a** What percentage is the minimum repayment due of the closing balance?

b If only the minimum amount due is paid, on what amount will interest be charged on the *next* statement?

4 List some advantages and disadvantages of using credit cards to purchase goods.



CREDIT CARD STATEMENT

Statement begins	10 December 2012
Statement ends	11 January 2013
Account number	3434 1234 5678 9000
Payment due date	6 February 2013
Minimum amount due	$37.00

Overdue	Opening balance	New charges	Payments/Refunds	Closing balance
$0.00	$225.00	+$1,895.35	−$225.00	=$1,895.35

Date	Transaction details	Amount ($)
16 Dec	BPAY Sydney Water AUS	524.35
21 Dec	Dr J J Donald Penrith AUS	120.00
22 Dec	Morky Jewellers Penrith AUS	999.00
3 Jan	Auto payment–Thank you	225.00-
10 Jan	Crawford River Wines AUS	252.00

Interest charged on purchases	Purchase rate 20.240%	Daily rate 0.05545%
Interest charged on cash advances	Purchase advance rate 21.740%	Daily rate 0.05956%

Credit limit $3000 **Available credit $1104.65**

K Lay-by

Some retail stores allow customers to purchase goods by a method called **lay-by**. Under a lay-by agreement, a deposit is paid and the goods are put aside. The remainder of the cost price must be paid off within a given period, often 3 months. The customer cannot collect the goods until the balance is completely paid, but no interest is charged.

EXAMPLE 1

Nick decides to lay-by a tool set costing $849 and pays a deposit of $100. Over the next 3 months he makes payments of $150, $85, $90, $160, $120 and $70. How much more does he have to pay to be able to collect the tool set?

Solve	Think	Apply
Total amount paid = \$100 + \$150 + \$85 + \$90 + \$160 + \$120 + \$70 = \$775 Balance = \$849 − \$775 = \$74 Nick has \$74 to pay before he can collect the tool set.	Add all the payments and subtract from the balance owed.	With lay-by, the goods stay at the store until they are completely paid for.

Exercise 5K

1 Enrique decides to lay-by a laptop costing \$735 and pays a deposit of \$120. Over the next 3 months he makes payments of \$130, \$75, \$110, \$190 and \$75. Complete the following to determine how much more he has to pay to be able to collect the laptop.

Total amount paid = \$120 + \$130 + \$___ + \$___ + \$___ + \$75 = \$___

Balance = \$___ − \$___ = \$___

2 Ben decides to lay-by an electric saw costing \$569 and pays a deposit of \$120. Over the next 3 months he makes payments of \$60, \$45, \$90, \$70, \$70 and \$80. How much more does he have to pay to be able to collect the saw?

3 Zoe lay-bys a bicycle costing \$225 for her son's birthday and pays a deposit of \$60. Each week for the next 5 weeks she makes a payment of \$25. How much more does she have to pay to be able to collect the bicycle?

EXAMPLE 2

Isabella lay-bys a dress costing \$224 and pays a deposit of \$50. She wants to collect the dress in approximately 3 months to wear to a wedding. If she pays off the balance by making 3 equal monthly instalments, calculate:

a the balance to be paid

b the amount of each monthly instalment.

	Solve	Think	Apply
a	Balance $= \$224 - \$50$ $= \$174$	Subtract the \$50 deposit from the balance to be paid.	Always subtract the deposit before calculating payments.
b	Monthly instalment $= \dfrac{\$174}{3}$ $= \$58$	She owed \$174, which is divided by the 3 monthly instalments.	

4 Danielle lay-bys a onesie costing \$184 and pays a deposit of \$40. She wants to collect the onesie in approximately 3 months to wear to a party. If she pays off the balance by making 3 equal monthly instalments, calculate the following.

a Balance to be paid = \$___ − \$40 = \$___

b Amount of each monthly instalment = $\dfrac{\$\square}{3}$ = \$___

5 Martin lay-bys a suit costing \$485. He pays a deposit of \$80 and pays off the balance by making 4 equal monthly payments. Calculate:

a the balance to be paid

b the amount of each monthly payment.

6 Yvonne lay-bys a gas barbecue costing \$778. She is required to pay a 10% deposit and to pay the balance in 12 equal weekly instalments. Calculate:

a the deposit

b the balance to be paid

c the amount of each weekly instalment.

7 Josh lay-bys a Blu-ray player costing \$456. He is required to pay a 15% deposit and to pay the balance in 6 equal fortnightly instalments. Calculate:

- **a** the deposit
- **b** the balance to be paid
- **c** the amount of each fortnightly instalment.

8 List some advantages and disadvantages of using the lay-by method to purchase goods.

L Buying on terms

*Buying on terms is sometimes called **hire-purchase**.* !

When an item is bought **on terms**, a deposit is paid and the item is received immediately. The balance of the price is borrowed and this balance plus simple interest is repaid in equal instalments over a fixed period.

EXAMPLE 1

A camping set costing \$998 can be bought on terms for \$99 deposit and 24 monthly instalments of \$55.

- **a** Calculate the total cost of buying the camping set on terms.
- **b** How much would you save by paying cash?

	Solve	Think	Apply
a	Total cost = \$99 + 24 × \$55 = \$1419	Calculate all amounts paid: the deposit plus all monthly payments.	Make sure that the deposit is added to the amount paid.
b	Amount saved by paying cash = \$1419 − \$998 = \$421	Deduct the cost of purchase for cash from the cost of paying on terms.	

Exercise 5L

1 A snow skiing set costing \$742 can be bought on terms for \$75 deposit and 24 monthly instalments of \$34. Complete the following.

- **a** Total cost of buying the snow skiing set on terms = \$75 + 24 × \$___ = \$___
- **b** Amount saved by paying cash = \$___ − \$742 = \$___

2 A surfboard costing \$699 can be bought on terms for \$79 deposit and 24 monthly instalments of \$33.

- **a** Calculate the total cost of buying the surfboard on terms.
- **b** How much would you save by paying cash?

3 A laptop computer costing $2298 can be bought on terms for $229 deposit and 18 monthly repayments of $135.60.

- **a** Calculate the total cost of buying the computer on terms.
- **b** How much would you save by paying cash?

4 A home theatre system costing $1598 can be bought on the following terms: 10% deposit and 48 weekly instalments of $37.15.

- **a** Calculate the total cost of buying the system on terms.
- **b** How much would you save by paying cash?

5 A sound system costing $879 can be bought on the following terms: 15% deposit and 26 fortnightly repayments of $39.98.

- **a** Calculate the total cost of buying the sound system on terms.
- **b** How much would you save by paying cash?

EXAMPLE 2

A computer costing $1498 can be bought on terms for $300 deposit and 36 monthly repayments of $46.50.

- **a** Calculate the total cost of buying the computer on terms.
- **b** Find the total amount of interest charged.
- **c** Calculate the amount of interest paid annually.
- **d** What was the amount of money borrowed?
- **e** Calculate the annual rate of interest charged.

	Solve	Think	Apply
a	Total cost $= \$300 + 36 \times \$46.50$ $= \$1974$	Add the deposit to all the payments.	Pay particular attention to the deposit. It is the amount of money paid initially. It is always added to amounts you paid and subtracted from the amount owing.
b	Total interest $= \$1974 - \$1498$ $= \$476$	The amount paid in excess of the purchase price is the interest.	
c	Annual interest $= \dfrac{\text{total interest}}{\text{number of years}}$ $= \dfrac{\$476.00}{3}$ $= \$158.67$	Convert the total interest to amount of interest per year by dividing by the number of years.	
d	Amount borrowed $=$ balance owing after paying the deposit $= \$1498 - \$300$ $= \$1198$	The $300 deposit is subtracted from the balance owing to find the amount borrowed.	
e	Annual interest rate $= \dfrac{\text{annual interest}}{\text{amount borrowed}} \times 100\%$ $= \dfrac{\$158.67}{\$1198} \times 100\%$ $= 13.2\%$	Divide $158.67 by the amount borrowed and multiply by 100 to convert the rate to a percentage.	

6 A computer costing $1230 can be bought on terms for $150 deposit and 36 monthly repayments of $37.59. Complete the following.

a Total cost of buying the computer on terms = $___ + 36 × $37.59 = $___

b Total interest charged = $___ − $1230 = $___

c Amount of interest paid annually $= \dfrac{\text{total interest}}{\text{number of years}} = \dfrac{\$\square}{3} = \$$___

d Amount of money borrowed = balance owing after paying the deposit

= $1230 − $___ = $___

e Annual rate of interest charged $= \dfrac{\text{annual interest}}{\text{amount borrowed}} \times 100\%$

$= \dfrac{\$\square}{\$1080} \times 100\% =$ ___%

7 A $1499 digital SLR camera can be bought on terms for $200 deposit and 24 monthly repayments of $74.69.

a Calculate the total cost of buying the camera on terms.

b Find the total amount of interest charged.

c Calculate the amount of interest paid annually.

d What was the amount of money borrowed?

e Calculate the annual rate of interest charged.

8 A trailer costing $1890 can be bought on terms for $100 deposit and 36 monthly repayments of $66.50.

a Calculate the total cost of buying the trailer on terms.

b Find the total amount of interest charged.

c Calculate the amount of interest paid annually.

d What was the amount of money borrowed?

e Calculate the annual rate of interest charged.

9 A lounge suite was advertised for $5990 or $500 deposit and 48 monthly repayments of $187.58.

a Calculate the total cost of buying the lounge suite on terms.

b Find the total amount of interest charged.

c Calculate the amount of interest paid annually.

d What was the amount of money borrowed?

e Calculate the annual rate of interest charged.

EXAMPLE 3

A large screen TV set can be bought for $2998 cash or on the following terms: deposit $299, with the balance to be repaid over 2 years in 24 equal monthly repayments. Simple interest is charged on the balance at 12% p.a. The TV is bought on terms.

a Calculate the balance owing after the deposit is paid.

b Calculate the interest charged on the balance owing.

c What is the monthly repayment?

	Solve	Think	Apply
a	Balance owing = $2998 − $299 = $2699	Subtract the deposit.	Always subtract the deposit before calculating the interest. Add the interest and the principal. To check, multiply the payments by the time period and make sure that they exceed the balance owing.
b	Interest = $2699 × 0.12 × 2 = $647.76	Calculate the total interest.	
c	Balance owing + interest = $2699 + $647.76 = $3346.76 Monthly repayment $= \dfrac{\$3346.76}{24}$ = $139.45 to nearest cent	Add the balance owing and the interest. Divide by the number of months to find the repayment.	

10 A surround sound system can be bought for $998 cash or on the following terms: deposit $99, with the balance to be repaid over 2 years in 24 equal monthly repayments. Simple interest is charged at 9% p.a. on the balance. The surround sound system is bought on terms.

a Calculate the balance owing after the deposit is paid.

Balance owing = $998 − $____ = $____

b Calculate the interest charged on the balance owing.

Interest = $899 × 0.__ × ____ = $____

c What is the monthly repayment?

Balance owing + interest = $____ + $____ = $1060.82

Monthly repayment $= \frac{\$\square}{24}$ = $____ to nearest cent

11 Peter buys a second-hand car advertised for $9600 on the following terms: deposit $2000, the balance to be repaid over 2 years in equal monthly repayments. Simple interest is charged at 12% p.a.

a Calculate the balance owing.

b Calculate the interest charged on the balance owing.

c What is the monthly repayment?

12 Angela buys a motorbike advertised for $12 900 on the following terms: deposit $3000, the balance to be repaid over 3 years in equal monthly repayments. Simple interest is charged at 9% p.a.

a Calculate the balance owing.

b Calculate the interest charged on the balance owing.

c What is the monthly repayment?

13 Adrienne buys a washing machine advertised for $499 on the following terms: deposit 10% and the balance repaid over 2 years in equal monthly repayments. Simple interest is charged at 15% p.a.

a Calculate the deposit.

b Calculate the balance owing.

c Find the interest charged on the balance owing.

d What is the monthly repayment?

14 Robin buys a new car advertised for $19 900 on the following terms: deposit 15%, the balance to be repaid over 4 years in equal monthly repayments. Simple interest is charged at 11.9% p.a.

a Calculate the deposit.

b Calculate the balance owing.

c Find the interest charged on the balance owing.

d What is the monthly repayment?

15 List some advantages and disadvantages of purchasing goods on terms.

Investigation 3 Monthly repayments

The spreadsheet below calculates the monthly repayment for items bought on terms.

	A	B	C	D	E	F	G	H
1	Item	Cash price ($)	Deposit ($)	Interest rate (% p.a.)	Repayment period (years)	Balance owing ($)	Interest on balance ($)	Monthly repayment ($)
2	Gaming computer	2998	298	12.0	2			
3	LED TV	1899	189	15.0	2			
4	Furniture	4672	250	11.6	3			
5	Ducted air conditioning	7659	1000	14.2	4			
6	Refrigerator and freezer	3628	628	9.9	3			

1 Copy the spreadsheet.

2 In cell F2 type the formula: =B2−C2. This is the balance owing after the deposit is paid.

3 In cell G2 type the formula: =F2*D2/100*E2. This is the amount of interest charged on the balance.

4 In cell H2 type the formula: =(F2+G2)/(E2*12). This is the monthly repayment.

5 To find these values for the other items:

- Highlight cells F2 to H6.
- Go to Edit.
- Select Fill Down.

6 The gaming computer is to be paid off over 3 years instead of 2.

a What would be the monthly repayment?

b How much more interest would be paid?

(*Hint:* Change the repayment period to 3 and use the arrow key to move right.)

7 Try changing the repayment period and/or the interest rate for the other items to investigate the effect on the monthly repayment and the amount of interest paid.

8 Check the advertised monthly repayments for several items advertised in newspapers and magazines. If payments for the same items differ, investigate for hidden charges.

M Deferred payment

Many advertisements make statements such as ‘No repayments for 12 months’ or ‘Pay nothing until next June’. Under these arrangements the goods may be taken immediately the finance contract is approved and no payment needs to be made for the agreed period of time.

This type of financial arrangement is known as a **deferred payment** scheme.

EXAMPLE 1

Michael sees a skiing outfit advertised as shown. When Michael approaches the retailer, he is given the conditions shown for the deferred payment scheme.

a Calculate the total amount Michael would have to pay for the skiing outfit under this scheme.

b Calculate the monthly instalments.

c How much more than the advertised price would Michael pay under this scheme?

$2498

NO DEPOSIT
NO REPAYMENTS
FOR 12 MONTHS

Conditions:
1 Pay nothing for 12 months.
2 Balance plus interest to be repaid in equal monthly instalments over the 2 years following the repayment-free period.
3 Simple interest of 16% p.a. is charged for the 3-year period of the agreement.
4 Establishment fee of $110.
5 Account service fee of $2.95 per month for the 3-year period of the agreement.

	Solve	Think	Apply
a	$\text{Interest} = \$2498 \times 0.16 \times 3$ $= \$1199.04$ Total cost $= \$2498 + \$1199.04 + \$110 + \$2.95 \times 36$ $= \$3913.24$	Michael would pay interest on \$2498 at 16% p.a. for 3 years. Total cost = price + interest + establishment fee + account service fee	Read the conditions carefully. There are usually many costs, so make sure all costs are added into the calculation.
b	$\text{Monthly instalment} = \frac{\$3913.24}{24}$ $= \$163.05$ to the nearest cent	Michael would pay this amount by 24 equal instalments.	
c	Total cost − advertised price $= \$3913.24 - \$2498$ $= \$1415.24$ Michael would pay \$1415.24 more.	Subtract the advertised price to find how much more Michael pays.	

Exercise 5M

1 A computer is advertised as shown on the right.

a Calculate the total amount you would pay for the computer under this scheme.

b Calculate the monthly instalments.

c How much more than the advertised price does the computer cost under this scheme?

NO DEPOSIT
NO REPAYMENTS
FOR 12 MONTHS
(Conditions apply.)

$1999

Conditions: 1 Pay nothing for 12 months.
2 Balance plus interest to be repaid in equal monthly instalments over the 2 years following the repayment-free period.
3 Simple interest of 16% p.a. is charged for the 3-year period of the agreement.
4 Establishment fee of $110.
5 Account service fee of $2.95 per month for the 3-year period of the agreement.

$8599

No Deposit
No Repayments for
12 months (Conditions apply.)

Conditions:
1 Pay nothing for 12 months.
2 Balance plus interest to be repaid in equal monthly instalments over the 3 years following the repayment-free period.
3 Simple interest of 15% p.a. is charged for the 4-year period of the agreement.
4 Establishment fee of $125.
5 Account service fee of $2.55 per month for the 4-year period of the agreement.

2 A second-hand car is advertised as shown on the left.

a Calculate the total amount you would pay for the car under this scheme.

b Calculate the monthly instalments.

c How much more than the advertised price does the car cost under this scheme?

3 A sofa bed is advertised for $1598 with no deposit and no repayments for 6 months. The conditions of the agreement are:

- Pay nothing for 6 months.
- Balance plus interest to be repaid in equal monthly instalments over the 12 months following the repayment-free period.
- Simple interest of 1.5% per month is charged for the 18 months of the agreement.
- An establishment fee of $135.
- An account service fee of $2.85 per month for the 18-month period of the agreement.

a Calculate the total amount you would pay for the sofa under this scheme.
b Calculate the monthly instalments.
c How much more than the advertised price does the sofa cost under this scheme?

4 List some advantages and disadvantages of purchasing goods by deferred payment.

N Personal loans

To purchase expensive items, some people prefer to organise a **personal loan** through a bank, credit union or other financial institution. Personal loans often cost less than other forms of payment, such as buying on terms or deferred payment schemes, because of lower interest rates and lower fees. Banks have tables from which the monthly repayments can be determined. For example, the table below shows the monthly repayments of principal plus interest on each $1000 borrowed for various interest rates.

Amount per $1000 borrowed

Loan term (months)	Annual interest rate								
	10.0%	10.5%	11.0%	11.5%	12.0%	12.5%	13.0%	13.5%	14.0%
12	87.9159	88.1486	88.3817	88.6151	88.8488	89.0829	89.3173	89.5520	89.7871
18	60.0571	60.2876	60.5185	60.7500	60.9820	61.2146	61.4476	61.6811	61.9152
24	46.1449	46.3760	46.6078	46.8403	47.0735	47.3073	47.5418	47.7770	48.0129
30	37.8114	38.0443	38.2781	38.5127	38.7481	38.9844	39.2215	39.4595	39.6984
36	32.2672	32.5204	32.7387	32.9760	33.2143	33.4536	33.6940	33.9353	34.1776
42	28.3168	28.5547	28.7939	29.0342	29.2756	29.5183	29.7621	30.0071	30.2532
48	25.3626	25.6034	25.8455	26.0890	26.3338	26.5800	26.8275	27.0763	27.3265
54	23.0724	23.3162	23.5615	23.8083	24.0566	24.3064	24.5577	24.8104	25.0647
60	21.2470	21.4939	21.7424	21.9926	22.2444	22.4979	22.7531	23.0098	23.2683

EXAMPLE 1

Use the table above to calculate the monthly repayments on a loan of $8300 for 4 years at 13%.

Solve	Think	Apply
Monthly repayments for $8300 = $26.8275 × 8.3 = $222.67 to the nearest cent	From the table, the monthly repayment on $1000 borrowed for 4 years at 13% is $26.8275. Multiply $26.8275 by 8.3, the number of $1000s.	Tables sometimes give per dollar or per $100 only. Always multiply by the amount.

Exercise 5N

1 Using the table on page 139, complete the following to calculate the monthly repayments on a loan of $7000 for 2 years at 11%.

Monthly repayments for $7000 = $___ × 7
= $___ to the nearest cent

2 Use the table on page 139 to calculate the monthly repayments on a loan of $9000 for 3 years at 12%.

3 Use the table on page 139 to calculate the monthly repayments on these loans:

a $85 000 for $2\frac{1}{2}$ years at 10.5%
b $67 000 for 5 years at 11%
c $14 600 for $3\frac{1}{2}$ years at 13.5%
d $16 000 for 54 months at 14%
e $12 450 for 42 months at 11.5%
f $32 000 for 4 years at 13%

EXAMPLE 2

a Use the table on page 139 to calculate the monthly repayments on a loan of $6900 for 3 years at 12.5%.
b Calculate the total cost of the loan if there is a loan application fee of $180.

	Solve	Think	Apply
a	Monthly repayment = $33.4536 × 6.9 = $230.83 to the nearest cent	From the table, the repayment is $33.4536 per month. Multiply by 6.9, the number of $1000s.	Multiply the value from the table and add any other costs.
b	Total cost of loan = $230.83 × 36 + $180 = $8489.88	Add the fee of $180.	

4 Complete the following using the table on page 139.

a Calculate the monthly repayments on a loan of $4700 for 2.5 years at 11.5%.
Monthly repayment = $___ × 4.__ = $___ to the nearest cent

b Calculate the total cost of the loan if there is a loan application fee of $220.
Total cost of loan = $___ × 30 + $___ = $___

5 **a** Use the table on page 139 to calculate the monthly repayments on a loan of $7000 for 3 years at 11%.
b Calculate the total cost of the loan if there is a loan application fee of $180.

6 **a** Use the table on page 139 to calculate the monthly repayments on a loan of $15 500 for 4 years at 13.5%.
b Calculate the total cost of the loan if there is a loan application fee of $250.

7 Calculate the total cost of each of these loans. (Calculate the monthly repayment first.)

a $5000 for 3 years at 12%, loan application fee $300
b $12 000 for 4 years at 14%, loan application fee $200
c $8500 for 2 years at 10.5%, loan application fee $260
d $9400 for 60 months at 11.5%, loan application fee $210
e $18 000 for 42 months at 10%, loan application fee $190

8 In question **7**, by how much does the total cost of each loan exceed the amount borrowed?

9 List some advantages and disadvantages of purchasing goods by personal loan.

Language in mathematics

1 List four different ways in which people are paid for providing their labour or services.

2 Explain the meaning of these terms:
a overtime **b** bonus **c** holiday loading

3 Use the following terms in one sentence:
gross income, deductions, net earnings

4 What is the difference in meaning between the words ‘principal’ and ‘principle’?

5 The words ‘credit’, ‘deposit’ and ‘balance’ have a mathematical meaning as well as other meanings in ordinary English. Use a dictionary to find both meanings of each word.

6 Complete the following words from this chapter by replacing the vowels.
a f_rtn_ghtly **b** r_t_ _n_r **c** tax_bl_ **d** d_sc_ _nt

7 Three of the words in the following list have been spelt incorrectly. Rewrite them with the correct spelling:
peacework, purchase, cash, survice, loan, invesment

Terms

balance
bonus
buying on terms
cash
casual
commission
compare
credit card
deductions
deferred payment
deposit
discount
double-time
excluding
expenses
fee
financial
flat interest
fortnight
full-time
gross income
holiday loading
hire-purchase
including
income
instalment
investment
labour
lay-by
net income
personal loan
net earnings
overtime
part-time
permanent
piecework
principal
purchase
repayment
retainer
salary
service
simple interest
take-home pay
taxable income
time-and-a-half
wages

Check your skills

1 Samantha earns a salary of $980.40 per week. This is equivalent to a yearly salary of:
A $47 059.20 **B** $49 020 **C** $50 980.80 **D** $1961.20

2 A salary of $66 456 p.a. is *not* equivalent to:
A $1278 per week **B** $5112 per fortnight **C** $5538 per month **D** $16 614 per quarter

3 Garry works part-time and earns a salary of $684 per week. This is equivalent to a monthly salary of:
A $2736 **B** $2941.20 **C** $2964 **D** $3078

4 Sally is a junior and works a 36-hour week at a pay rate of $16.80 per hour. Her total wages for a week in which she works an additional 5 hours at time-and-a-half and 3 hours at double-time is:
A $831.60 **B** $739.20 **C** $806.40 **D** $873.60

5 Bianca earns $560 per week. She is entitled to 4 weeks annual leave and receives an additional holiday loading of 17.5%. Her total holiday pay for 4 weeks is:
A $2240 **B** $392 **C** $2338 **D** $2632

6 The table shows hourly pay in $ for a waiter employed as a casual.

Monday–Friday	Saturday	Sunday
20.92	23.65	26.38

The wage of a casual waiter who works 10 hours total from Monday to Friday, 4 hours on Saturday and 5 hours on Sunday is:

A $435.70 **B** $397.48 **C** $449.35 **D** $501.22

7 David is paid $1.37 for each tree that he plants. If he can plant an average of 18 trees per hour and he works a 36-hour week, his average weekly earnings are:

A $24.66 **B** $49.32 **C** $887.76 **D** $1479.52

8 Tony is a real-estate agent. He charges the following commission for selling home units: 3% of the first $250 000 and 1.2% for the remainder of the selling price. His commission for selling a home unit for $310 000 is:

A $3720 **B** $8220 **C** $9300 **D** $13 020

9 Stephen earns $853 per week. The deductions from his salary each week are tax $135, union fees $6 and private health insurance $11.80. His net pay for the week is:

A $718 **B** $712 **C** $706.20 **D** $700.20

10 The tax on $100 666 is $17 550 plus 37 cents per $ in excess of $80 000. What is the tax on $100 666?

A $7646.42 **B** $17 550 **C** $25 196.42 **D** $37 246.42

11 The simple interest on $3480 at 5.5% p.a. for 4 years is:

A $7656 **B** $191.40 **C** $765.60 **D** $4245.60

12 Michelle invested $5000 for 3 years and earned $825 in interest. The annual rate of interest was:

A 5.5% **B** 16.5% **C** 33.3% **D** 3.33%

13 A camera store offers a discount of 12% for paying cash. The cash payment for a camera marked as $459 is:

A $55.08 **B** $55.10 **C** $403.92 **D** $403.90

14 Bev's credit card statement showed an opening balance of $1735.80, payment of $500, purchases totalling $270 and interest of $41.35. If the minimum payment required is 2% of the closing balance, what is the minimum payment?

A $50.94 **B** $30.94 **C** $34.72 **D** $38.49

15 Goods can be purchased by paying a deposit and then paying the balance off over a short period of time. No interest is charged but the goods cannot be taken until full payment has been made. This method is called:

A time payment **B** hire-purchase **C** deferred payment **D** lay-by

16 A refrigerator costing $1395 can be bought on terms for $295 deposit and 24 monthly instalments of $61. The total cost of buying the refrigerator on terms would be:

A $2859 **B** $1759 **C** $1464 **D** $2564

17 An LCD television set costing $1089 is bought on the following terms: deposit $289 and the balance to be repaid over 2 years in equal monthly instalments. Simple interest is charged at 13% p.a. The monthly repayment will be:

A $42 **B** $57.17 **C** $37.67 **D** $51.27

18 A home gym is advertised as shown. The total amount you would have to pay for the home gym under this scheme is:

A \$1598 **B** \$1853.68
C \$2365.04 **D** \$2109.36

$1598

NO DEPOSIT
NO REPAYMENTS
FOR 12 MONTHS

Conditions:
1 Pay nothing for 12 months.
2 Balance plus interest to be repaid in equal monthly instalments over the 2 years following the repayment-free period.
3 Simple interest of 16% p.a. is charged for the 3-year period of the agreement.

19 Using the table in Section 5N, the monthly repayment on a loan of \$24 000 over 2 years at 11.5% is, to the nearest cent:

A \$38.51 **B** \$924.30
C \$696.82 **D** \$1124.17

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4, 5	6	7	8	9	10	11, 12	13	14	15	16, 17	18	19
Section	A	B	C	D	E	F	G	H	I	J	K	L	M	N

5A Review set

1 David works part time and earns \$463.90 per week. How much does he earn per:

a fortnight? **b** year? **c** month? **d** quarter?

2 Convert a salary of \$56 000 p.a. to the equivalent salary per:

a week **b** fortnight **c** month **d** quarter.

3 Alice is 17 and works a 35-hour week. If she is paid \$18.70 per hour, how much does she earn for a week in which she works an additional 4 hours at time-and-a-half and 3 hours at double-time?

4 Travis earns \$560 per week. He is entitled to 4 weeks annual leave and receives a holiday loading of 17.5%. Calculate his total pay for this holiday period.

5 Sam works as a casual in a fruit shop. He gets paid \$21.60 for any hours worked from Monday to Friday, \$22.90 per hour for Saturdays and \$23.60 per hour for Sundays. Calculate how much he earns for a week in which he works 6 hours total from Monday to Friday, 5 hours on Saturday and 4 hours on Sunday.

6 Nerida earns \$1.98 for each dress she finishes in a clothing factory. If, on average, she can finish 12 dresses per hour and she works 8 hours per day for 4 days, calculate her average weekly earnings.

7 Cass sells computers. She is paid a retainer of \$220 per week plus a commission of 2% of sales. How much does she earn in a week in which her sales are \$12 800?

8 Jack's gross weekly income is \$768 per week. The deductions from his salary each week are tax \$106, union fees \$8.40 and private health insurance \$13.76. Calculate his net weekly earnings.

9 Zara earns $62 552 p.a., has tax deductions of $8550 and pays $10 044.44 in tax instalments. Find her:
a taxable income **b** tax payable **c** refund or balance owing.

10 Calculate the simple interest on $3600 that is invested at 9% p.a. for:
a 4 years **b** 20 months.

11 A sports goods store offers a discount of 16% for cash purchases. Find the cash price of a pair of running shoes marked at $179.

12 List two advantages and two disadvantages of using a credit card to purchase goods.

13 Melanie lay-bys a swing set costing $524 by paying a deposit of $150. Over the next 3 months she makes payments of $100, $120 and $85. How much more does she have to pay in order to collect the swing set?

14 A car advertised for $10 999 can be bought on the following terms: deposit $3000, with the balance to be repaid over 4 years by 48 equal monthly repayments. Simple interest is charged on the balance at 12% p.a.
a Calculate the balance owing.
b Calculate the interest charged on the balance owing.
c What is the monthly repayment?

15 A carbon-fibre bicycle is advertised as shown at right.
a Calculate the total amount you would pay for the bicycle under this scheme.
b Calculate the monthly instalments.
c How much more than the advertised price would you pay under this scheme?

$3999

No Deposit
No Repayments for 12 months
(Conditions apply.)

Conditions:
1 Pay nothing for 12 months.
2 Balance plus interest to be repaid in equal monthly instalments over the 2 years following the repayment-free period.
3 Simple interest of 15% p.a. is charged for the 3-year period of the agreement.
4 Establishment fee of $110.

16 Use the table in Section 5N to calculate the monthly repayments on a loan of $7800 for $3\frac{1}{2}$ years at 12.5% p.a.

5B Review set

1 Dan works part-time and earns $368.54 per week. How much does he earn per:
a fortnight? **b** year? **c** month? **d** quarter?

2 Convert a salary of $45 600 p.a. to the equivalent salary per:
a week **b** fortnight **c** month **d** quarter.

3 Olivia works a 36-hour week and is paid $21.36 per hour. How much does she earn for a week in which she works an additional 6 hours at time-and-a-half and 2 hours at double-time?

4 Terry earns $680 per week. He is entitled to 4 weeks annual leave and receives a holiday loading of 17.5%. Calculate his total pay for this holiday period.

5 Dennis works as a casual in a coffee shop. He gets paid $20.90 for any hours worked from Monday to Friday, $23.64 per hour for Saturdays and $24.28 for Sundays. Calculate how much he earns for a week in which he works 10 hours total from Monday to Friday, 4 hours on Saturday and 6 hours on Sunday.

6 Joanne sews buttons on shirts in a clothing factory. She is paid $0.38 per shirt. Calculate her income for a week in which she completed the following numbers of shirts:
Mon 165, Tues 189, Wed 212, Thurs 194, Fri 176

7 Benita sells whitegoods. She is paid a retainer of $260 per week plus a commission of 1.5% of sales. How much does she earn in a week in which her sales are $22 400?

8 John's gross weekly income is $683 per week. He has the following deductions from his salary each week: tax $80, union fees $6.78, private health insurance $21.20 and savings $50. Calculate his weekly take-home pay.

9 Shona earns $38 996 p.a. as a part-time director and $7560 as a casual assessor. She has tax deductions of $3561. Calculate her tax payable.

10 Calculate the simple interest on $18 000 that is invested at 6% p.a. for:
a 3 years
b 15 months

11 An electrical goods store offers a discount of 14% for cash purchases. Find the cash price of a blender marked at $89.

12 In one month, Marcia pays $215 off her credit card, then uses the card for purchases of $188 and also draws $50 in cash. Her next statement shows an opening balance of $870 and closing balance of $918.36. How much interest was she charged?

13 List the advantages and disadvantages of using lay-by to purchase goods.

14 A washing machine and dryer combination costing $1655 can be bought on the following terms: deposit $200, the balance to be repaid over 2 years by 24 equal monthly repayments. Simple interest is charged on the balance at 15% p.a.
a Calculate the balance owing.
b Calculate the interest charged on the balance owing.
c What is the monthly payment?

15 List the advantages and disadvantages of using a deferred payment option to purchase goods.

16 Use the table in Section 5N to calculate the monthly repayments on a loan of $12 000 for 5 years at 10.5% p.a.

5C Review set

1 Convert a salary of $36 000 p.a. to the equivalent salary per:
a week
b fortnight
c month
d quarter.

2 Convert a part-time wage of $365 per week to the equivalent wage per:
a fortnight
b year
c month
d quarter.

3 Alice works a 38-hour week and is paid $19.20 per hour. How much does she earn for a week in which she works an additional 3 hours at time-and-a-half and 1 hour at double time?

4 Peta earns $538 per week. At the end of the year her employer pays her a bonus of 5% of her annual earnings. Calculate Peta's bonus.

5 James works as a casual in a bar. He is paid $21.20 for any hours worked from Monday to Friday, $27.68 per hour for Saturdays and $29.32 for Sundays. Calculate how much he earns for a week in which he works 12 hours total from Monday to Friday, 6 hours on Saturday and 6 hours on Sunday.

6 Isabella earns $1.21 for each part she builds in a factory that produces electrical appliances. If, on average, she can finish 15 parts per hour and she works 6 hours per day for 5 days, what are her average weekly earnings?

7 Cameron sells real-estate. He charges the following commission for selling home units:

- 3% of the first $400 000
- 2% of the next $100 000
- 1% of the remainder of the selling price.

Calculate how much Cameron would earn for selling a home unit for:

a $345 000 **b** $585 000 **c** $820 000

8 Phil's gross weekly income is $895. The deductions from his salary each week are tax $150, union fees $4.81 and private health insurance $18.26. He also deposits $100 a week into a special savings account and has $10 per week donated directly to a charity. Calculate his take-home pay each week.

9 Gloria earns $52 231 p.a., has deductions of $3698 and pays $8978.40 in tax instalments. Calculate her tax refund.

10 Calculate the simple interest on $13 000 that is invested at 6% p.a. for:

a 5 years **b** 21 months.

11 List the advantages and disadvantages of using cash to purchase goods.

12 Harrison pays the minimum payment off his credit card each month. Is he using his credit card wisely? Explain your answer.

13 Sylvie lay-bys a sewing machine costing $465 by paying a deposit of 10%. Over the next 4 weeks she makes repayments totalling $320. How much more does she have to pay in order to collect the machine?

14 A car costing $12 000 can be bought on the following terms: deposit $2000, the balance to be repaid over 3 years by 36 equal monthly repayments. Simple interest is charged on the balance at 8% p.a.

a Calculate the balance owing.
b Calculate the interest charged on the balance owing.
c What is the monthly repayment?

15 A set of golf clubs is advertised for $3650 with the following deferred payment scheme:

- Pay nothing for 12 months.
- Balance plus interest to be repaid in equal monthly instalments over the 3 years following the repayment-free period.
- Simple interest of 14% is charged for the 4-year period of the agreement.
- Establishment fee of $75.

a Calculate the total amount you would pay for the golf clubs under this scheme.
b Calculate the monthly instalments.
c How much would you save by buying the clubs on lay-by?

16 Use the table in Section 5N to calculate the monthly repayments on a loan of $24 000 for 4 years at 13% p.a.

6

Area and surface area

This chapter deals with calculating the areas of composite shapes and the surface areas of right prisms.

After completing this chapter you should be able to:

- use the formula to find the area of a trapezium, rhombus and kite
- calculate the areas of composite shapes by dissection into triangles, quadrilaterals and sectors of circles
- calculate the area of an annulus
- solve problems involving quadrilaterals and composite shapes
- identify the faces and edge lengths of rectangular and triangular prisms
- visualise and name a common solid, given its net
- sketch the nets of right prisms
- find the surface areas of right rectangular and triangular prisms using nets and Pythagoras' theorem
- solve problems involving the surface areas of right rectangular and triangular prisms.

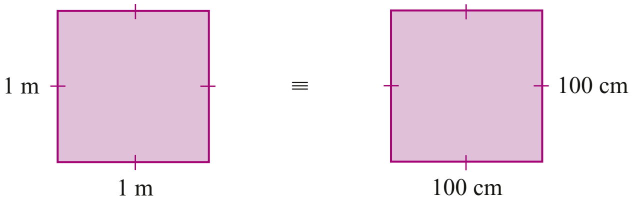
NSW syllabus references: 5.1 M&G Area and surface area
Outcomes: MA5.1-1WM, MA5.1-2WM, MA5.1-8MG
MEASUREMENT & GEOMETRY – ACMMG216, ACMMG218

A Review of area

Area conversions

$A = 1 \times 1\ \text{cm}^2$ or $A = 10 \times 10\ \text{mm}^2$

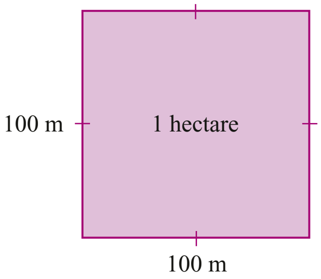
$\mathbf{1\ cm^2 = 100\ mm^2}$



$A = 1 \times 1\ \text{m}^2$ or $A = 100 \times 100\ \text{cm}^2$

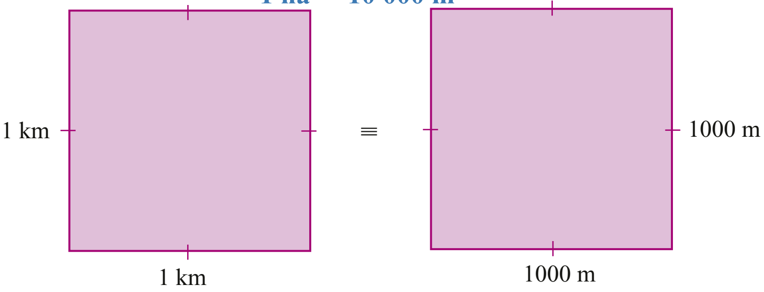
$\mathbf{1\ m^2 = 10\ 000\ cm^2}$

A hectare is the area of a square of side length 100 m.



$A = 100 \times 100\ \text{m}^2$

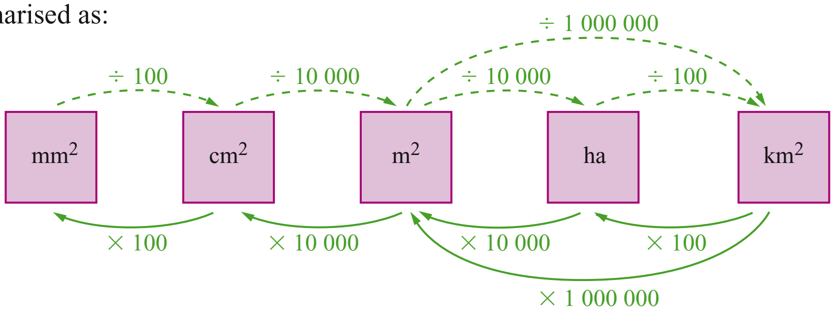
$\mathbf{1\ ha = 10\ 000\ m^2}$



$A = 1 \times 1\ \text{km}^2$ or $A = 1000 \times 1000\ \text{m}^2$

$\mathbf{1\ km^2 = 1\ 000\ 000\ m^2}$

This may be summarised as:



EXAMPLE 1

Convert the following measurements.

a 25 cm² to mm² **b** 2000 cm² to m² **c** 4.3 ha to m² **d** 5 600 000 m² to km²

	Solve	Think	Apply
a	$25 \text{ cm}^2 = 25 \times 100$ $= 2500 \text{ mm}^2$	Multiply by 100.	Use the conversion diagram to multiply or divide, as appropriate.
b	$2000 \text{ cm}^2 = \frac{2000}{10\,000}$ $= 0.2 \text{ m}^2$	Divide by 10 000.	
c	$4.3 \text{ ha} = 4.3 \times 10\,000$ $= 43\,000 \text{ m}^2$	Multiply by 10 000.	
d	$5\,600\,000 \text{ m}^2 = \frac{5\,600\,000}{1\,000\,000}$ $= 5.6 \text{ km}^2$	Divide by 1 000 000.	

Exercise 6A

1 Convert the following areas.

a 4 cm² to mm² **b** 31 m² to cm² **c** 32 km² to m² **d** 40 000 cm² to m²

e 7.3 ha to m² **f** 42 000 m² to ha **g** 15 cm² to mm² **h** 32 000 cm² to m²

i 3280 mm² to cm² **j** 235 000 m² to km² **k** 36.5 ha to m² **l** 780 m² to ha

EXAMPLE 2

Convert the following measurements.

a 3.8 m² to mm² **b** 0.5 km² to cm²

c 25 000 000 cm² to ha **d** 4 km² to ha

	Solve/Think	Apply
a	$3.8 \text{ m}^2 = 3.8 \times 10\,000 \text{ cm}^2 = 3.8 \times 10\,000 \times 100$ $= 3\,800\,000 \text{ mm}^2$	Use the conversion factors twice, as appropriate, to change to the required unit.
b	$0.5 \text{ km}^2 = 0.5 \times 1\,000\,000 \text{ m}^2 = 0.5 \times 1\,000\,000 \times 10\,000$ $= 5\,000\,000\,000 \text{ cm}^2$	
c	$25\,000\,000 \text{ cm}^2 = \frac{25\,000\,000}{10\,000} \text{ m}^2$ $= \frac{25\,000\,000}{10\,000 \times 10\,000}$ $= 0.25 \text{ ha}$	
d	$4 \text{ km}^2 = 4 \times 1\,000\,000 \text{ m}^2$ $= \frac{4 \times 1\,000\,000}{10\,000}$ $= 400 \text{ ha}$	

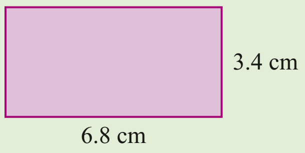
2 Convert the following.

a 7 000 000 mm² to m²
b 5.3 m² to mm²
c 3 600 000 cm² to ha
d 1 km² to ha
e 2.3 km² to ha
f 0.0042 ha to cm²
g 0.002 m² to mm²
h 6.3 ha to km²

Areas of rectangles, triangles, parallelograms and circles

EXAMPLE 3

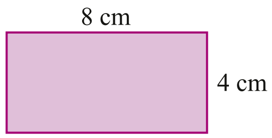
Find the area of this rectangle.



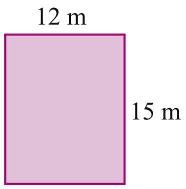
Solve/Think	Apply
$A = lb$ $= 6.8 \times 3.4 = 23.12$ cm²	Area of a rectangle $= l \times b$

3 Find the area of each rectangle.

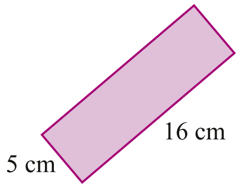
a



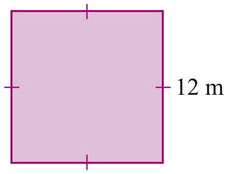
b



c



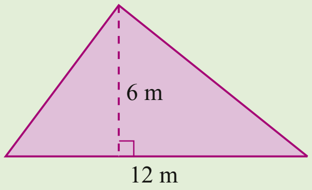
d



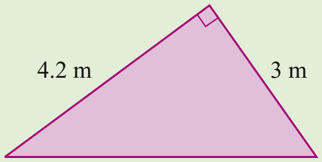
EXAMPLE 4

Find the area of each triangle.

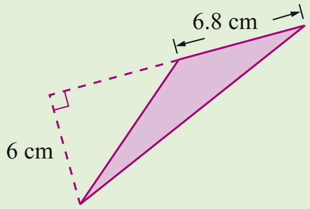
a



b



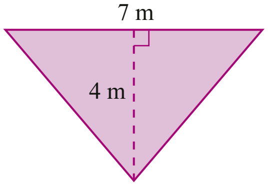
c



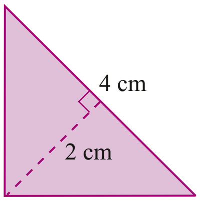
	Solve/Think	Apply
a	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 12 \times 6$ m² $= 36$ m²	Area of a triangle $= \frac{1}{2} \times b \times h$ The base and height must be perpendicular to each other. In an obtuse-angled triangle, the base or height may be ‘outside’ the triangle.
b	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 4.2 \times 3$ m² $= 6.3$ m²	
c	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 6.8 \times 6$ cm² $= 20.4$ cm²	

4 Find the area of each triangle.

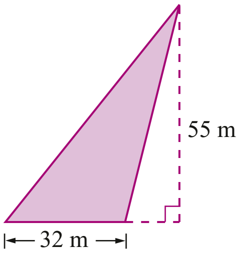
a



b



c



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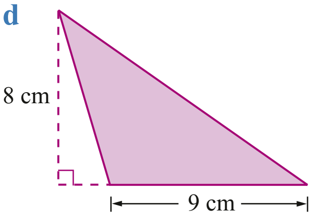
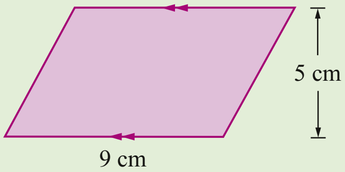


EXAMPLE 5

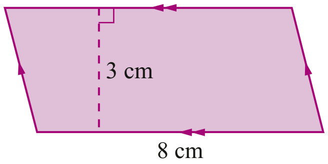
Find the area of this parallelogram.



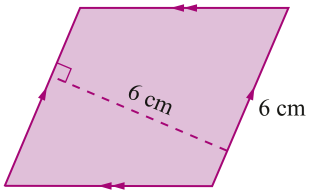
Solve/Think	Apply
$A = bh$ $= 9 \times 5 = 45 \text{ cm}^2$	Area of a parallelogram $= b \times h$ where b is the length of the base and h is the perpendicular height.

5 Find the area of each parallelogram.

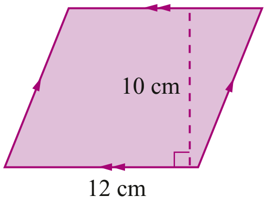
a



b



c



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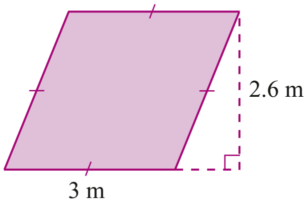
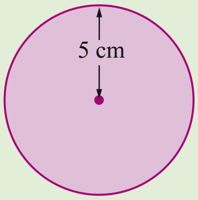


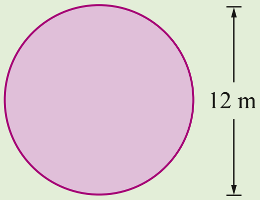
EXAMPLE 6

Find the area of each circle correct to 1 decimal place.

a



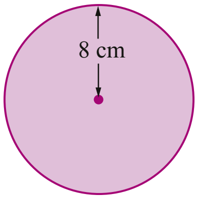
b



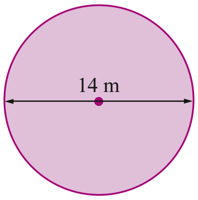
	Solve/Think	Apply
a	$A = \pi r^2 = \pi \times 5^2$ $= 25\pi \approx 78.5 \text{ cm}^2$	Area of a circle $= \pi r^2$
b	$A = \pi r^2 = \pi \times 6^2$ $= 36\pi \approx 113.1 \text{ m}^2$	

6 Find the area of each circle correct to 2 decimal places.

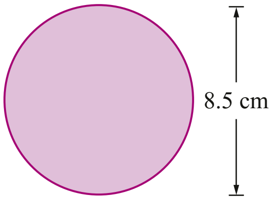
a



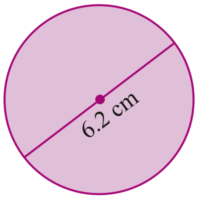
b



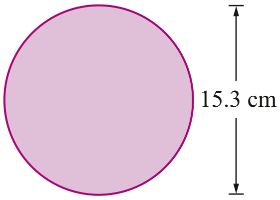
c



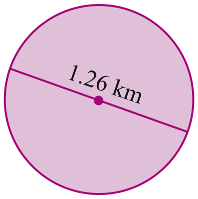
d



e



f



Investigation 1 Formulas for area

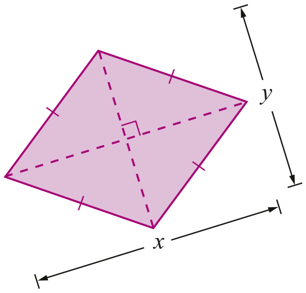
You know that the area of a triangle is: $A = \frac{1}{2}bh$

Use this formula to find expressions for the area of a rhombus, kite and trapezium.

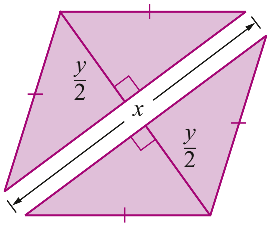
1 Rhombus

Use these diagrams to find expressions for the area of a rhombus with diagonals x and y units in length.

a

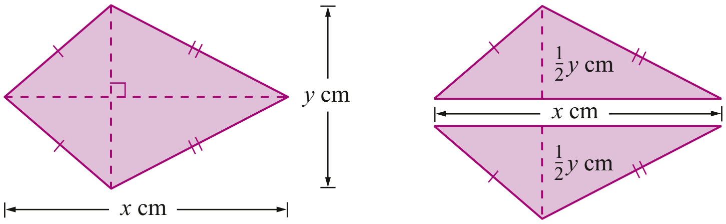


b



2 Kite

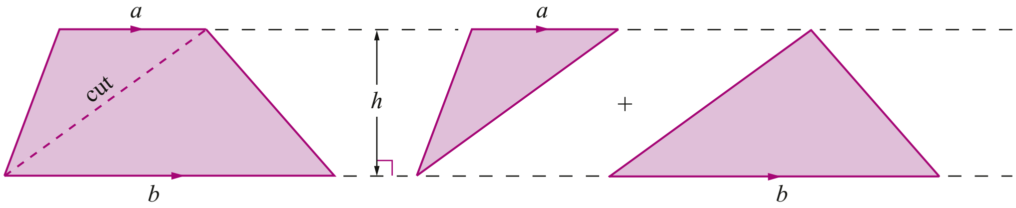
The formula for the area of a kite is the same as that for a rhombus. Compare this derivation with your expressions from question **1**.



$$\begin{aligned} A &= \tfrac{1}{2} \times x \times (\tfrac{1}{2}y) + \tfrac{1}{2} \times x \times (\tfrac{1}{2}y) \\ &= \tfrac{1}{4}xy + \tfrac{1}{4}xy \\ &= \tfrac{1}{2}xy \end{aligned}$$

3 Trapezium

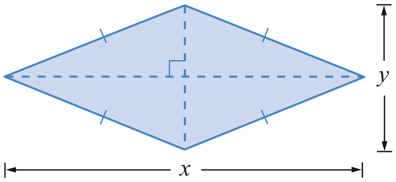
Use these diagrams to find an expression for the area of a trapezium.



B Areas of special quadrilaterals

From Investigation 1, we have developed the following formulas.

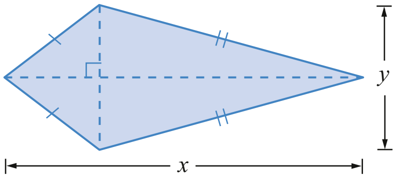
Rhombus



$A = \frac{1}{2}xy$

$A = \frac{1}{2} \times$ product of the lengths of the diagonals

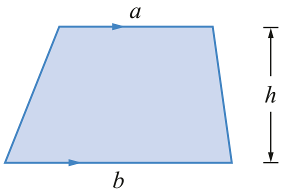
Kite



$A = \frac{1}{2}xy$

$A = \frac{1}{2} \times$ product of the lengths of the diagonals

Trapezium



$A = \frac{1}{2}ah + \frac{1}{2}bh$

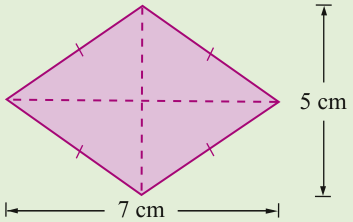
$= \frac{1}{2}h(a + b)$ or $A = \left(\frac{a + b}{2}\right)h$

$A =$ product of half the height and the sum of the lengths of the parallel sides
or product of the height and the average of the lengths of the parallel sides

Note: The height is the perpendicular distance between the two parallel sides. Sometimes it is a side but usually it is not.

EXAMPLE 1

Find the area of a rhombus with diagonals of length 5 cm and 7 cm.



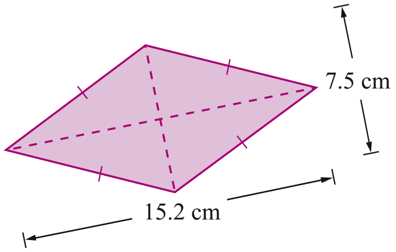
Solve/Think	Apply
$A = \frac{1}{2}xy$ $= \frac{1}{2} \times 5 \times 7 = 17.5\text{ cm}^2$	Area of a rhombus $= \frac{1}{2}xy$ where x and y are the lengths of the diagonals.

Exercise 6B

1 Complete the following to find the area of this rhombus.
The lengths of the diagonals are ___ and ___.

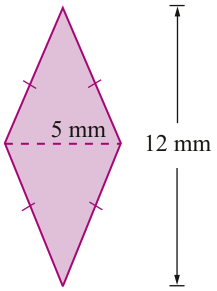
$A = \frac{1}{2} \times$ ___ $\times$ ___

$=$ ___ cm^2

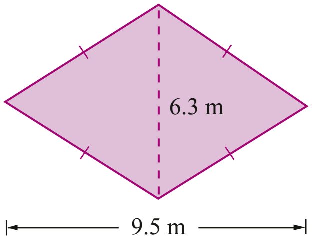


2 Find the area of each rhombus.

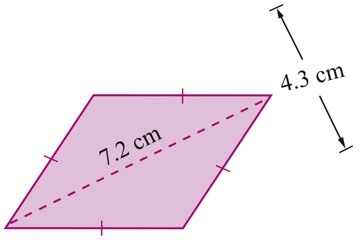
a



b

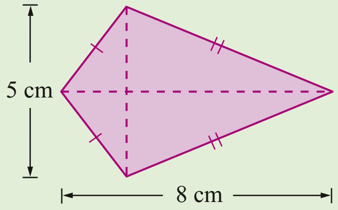


c



EXAMPLE 2

Find the area of this kite.



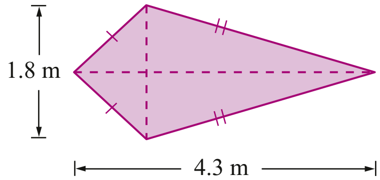
Solve/Think	Apply
$A = \frac{1}{2}xy$ $= \frac{1}{2} \times 5 \times 8 = 20\text{ cm}^2$	Area of a kite $= \frac{1}{2}xy$ where x and y are the lengths of the diagonals.

3 Complete the following to find the area of this kite.

The lengths of the diagonals are ___ and ___.

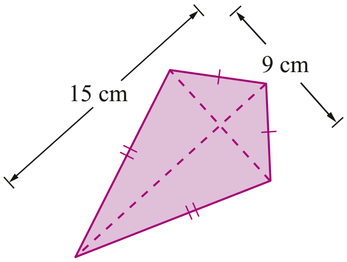
$A = \frac{1}{2} \times$ ___ $\times$ ___

$=$ ___ m^2

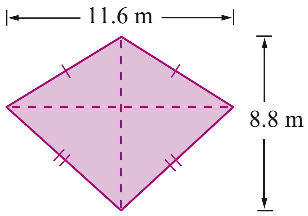


4 Find the area of each kite.

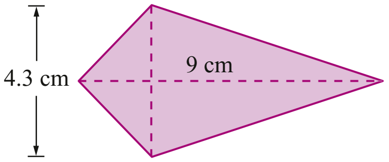
a



b

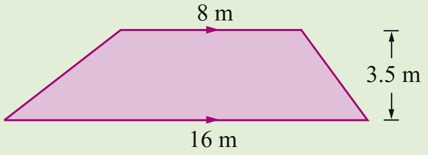


c



EXAMPLE 3

Find the area of this trapezium.



Solve/Think	Apply
First identify the height, then use the formula. $A = \frac{1}{2}h(a + b)$ $= \frac{1}{2} \times 3.5(8 + 16) = 42\text{ m}^2$	Area of a trapezium $= \frac{1}{2}h(a + b)$ or $\frac{h(a + b)}{2}$

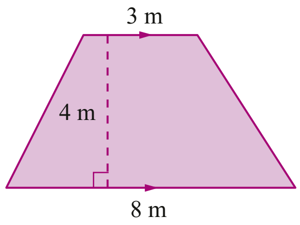
5 Complete the following to find the area of this trapezium.

The lengths of the parallel sides are ___ and ___.

The height is ___.

$A = \frac{1}{2}h(a + b)$

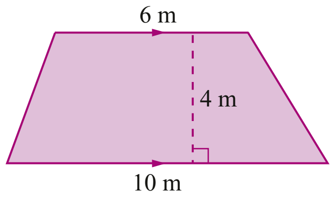
$= \frac{1}{2} \times \_\_\_ \times (\_\_\_ + \_\_\_) = \_\_\_\text{ m}^2$



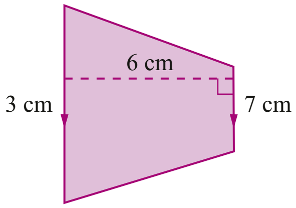
The height is perpendicular to the parallel sides. !

6 Find the area of each trapezium. Identify the height first.

a



b



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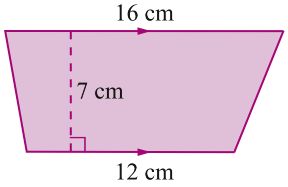
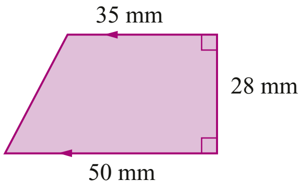
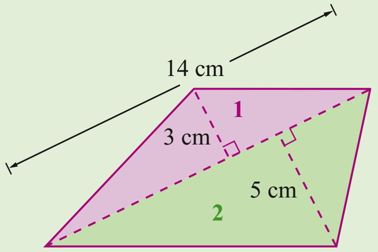


d



EXAMPLE 4

Find the area of this quadrilateral.



Solve/Think	Apply
$A =$ area of triangle 1 + area of triangle 2 $= \frac{1}{2} \times 14 \times 3 + \frac{1}{2} \times 14 \times 5$ $= 21 + 35 = 56\text{ cm}^2$	Divide the quadrilateral into 2 triangles.

7 Complete the following to find the area of the quadrilateral.

For triangle 1:

Base (b) = ___ cm

Height (h) = ___ cm

$A = \frac{1}{2} \times$ ___ $\times$ ___

= ___ cm^2

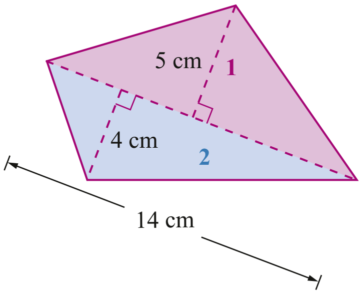
For triangle 2:

b = ___ cm

h = ___ cm

A = ___ $\times$ ___ $\times$ ___

= ___ cm^2



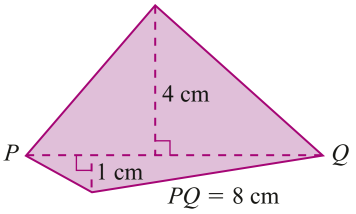
Area of quadrilateral = area of triangle 1 + area of triangle 2

= ___ + ___

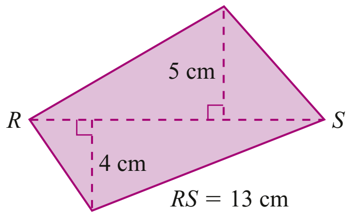
= ___ cm^2

8 Find the area of the each quadrilateral.

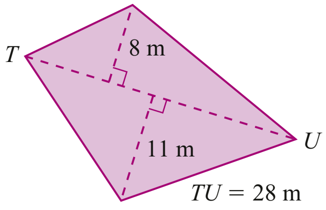
a



b

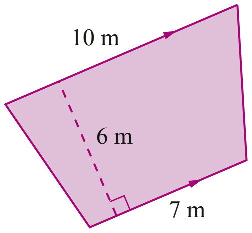


c

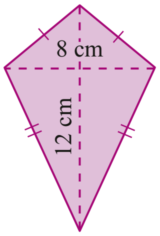


9 Use the correct formula to find the area of each quadrilateral.

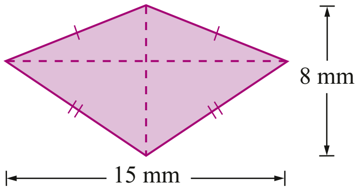
a



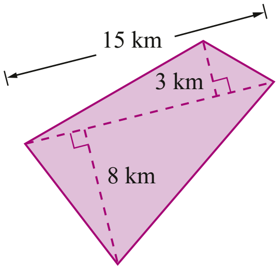
b



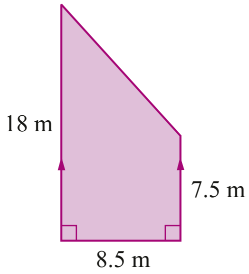
c



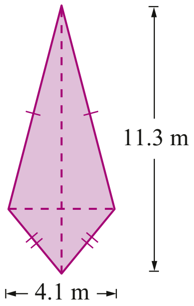
d



e



f

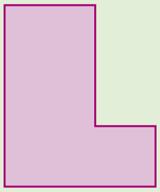


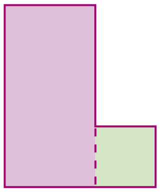
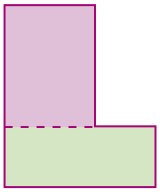
C Composite shapes

A **composite** shape is one that can be divided into simpler shapes.

EXAMPLE 1

Divide this shape into 2 rectangles (in two ways).

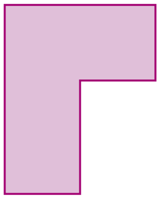


Solve/Think	Apply
 	Form rectangles using the edges of the shape as the sides of the rectangles.

Exercise 6C

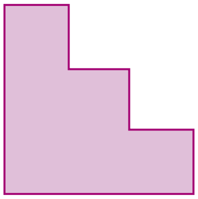
1 Divide the following composite shapes into the simpler shapes specified.

a



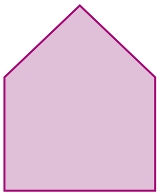
2 rectangles (in two ways)

b



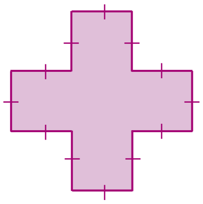
3 rectangles (in five ways)

c



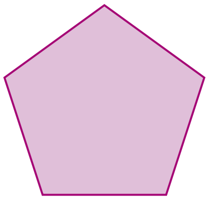
1 rectangle and 1 triangle

d



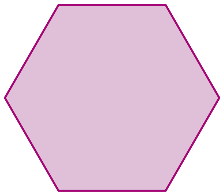
i 1 rectangle and 2 squares
ii 5 squares

e



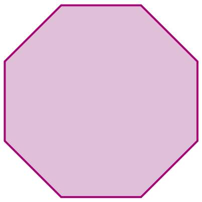
1 trapezium and 1 triangle

f



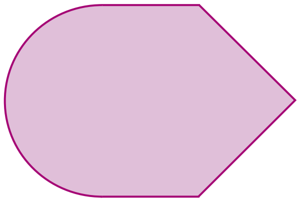
i 2 trapeziums
ii 1 rectangle and 2 triangles
iii 6 triangles

g



1 rectangle and 2 trapeziums

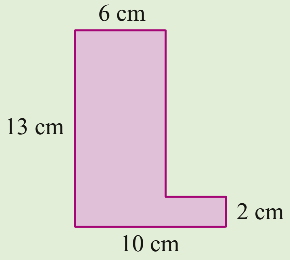
h

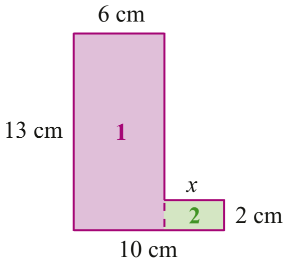
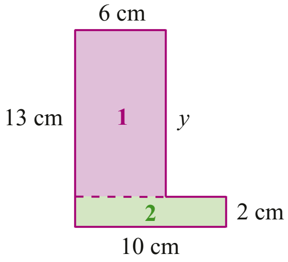


1 rectangle, 1 triangle and 1 semicircle

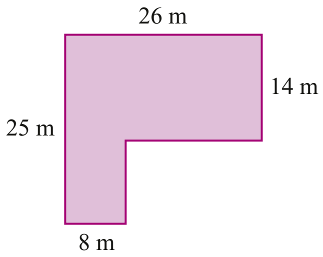
EXAMPLE 2

Find the area of this composite shape.

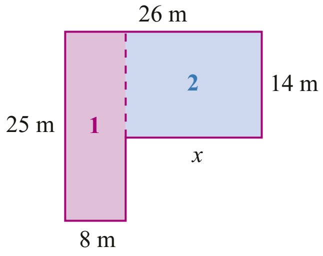


Solve/Think	Apply
Divide the shape into 2 rectangles, as shown. *For rectangle 1:* $A_1 = 13 \times 6 = 78\ \text{cm}^2$ *For rectangle 2:* $x = 10 - 6 = 4$ cm $A_2 = 4 \times 2 = 8\ \text{cm}^2$ Area of shape $= A_1 + A_2$ $= 78 + 8$ $= 86\ \text{cm}^2$  Or divide the shape as shown. *For rectangle 1:* $y = 13 - 2 = 11$ cm $A_1 = 11 \times 6 = 66\ \text{cm}^2$ *For rectangle 2:* $A_2 = 10 \times 2 = 20\ \text{cm}^2$ Area of shape $= A_1 + A_2$ $= 66 + 20$ $= 86\ \text{cm}^2$ 	Divide the shape into rectangles and use $A = lb$. Add or subtract to find the missing side lengths.

2 Complete the following to find the area of this shape.



Divide the shape into 2 rectangles, as shown.



For rectangle 1:

$A_1 =$ ___ $\times$ ___ $=$ ___ m^2

For rectangle 2:

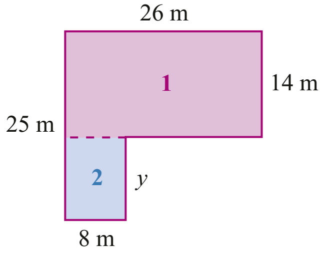
$x =$ ___ $-$ ___ $=$ ___ m

$A_2 =$ ___ $\times$ ___ $=$ ___ m^2

Area of shape = ___ + ___

= ___ m^2

Or divide the shape as shown.



For rectangle 1:

$A_1 =$ ___ $\times$ ___ $=$ ___ m^2

For rectangle 2:

$y =$ ___ $-$ ___ $=$ ___ m

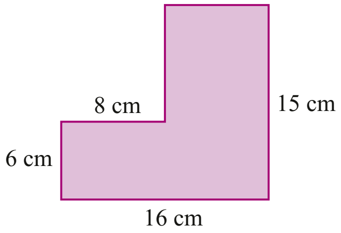
$A_2 =$ ___ $\times$ ___ $=$ ___ m^2

Area of shape = ___ + ___

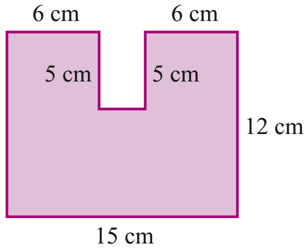
= ___ m^2

3 Find the areas of the following shapes.

a



b



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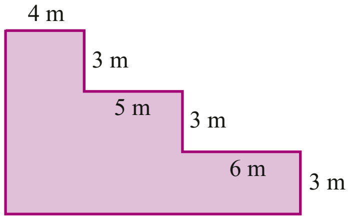

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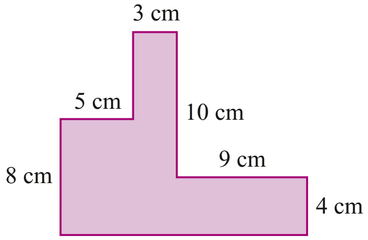
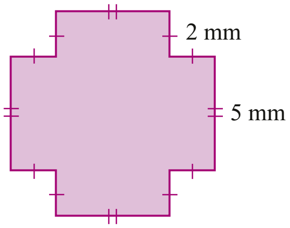


e



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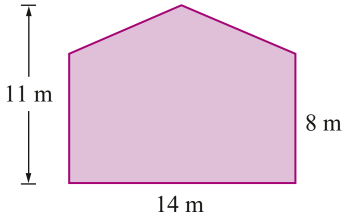
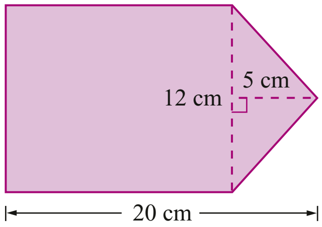


g



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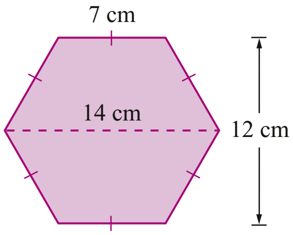
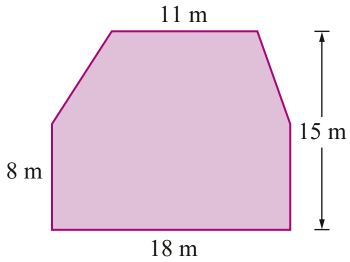
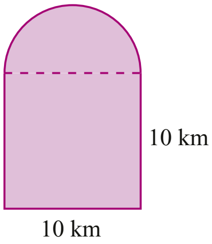


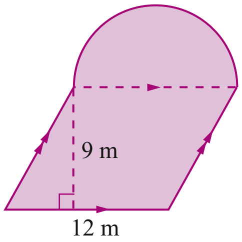
i



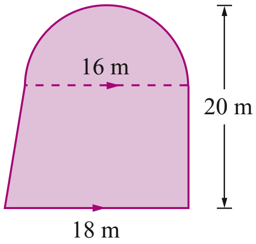
j



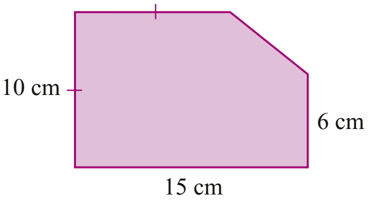
k



l



m



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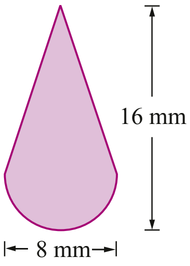
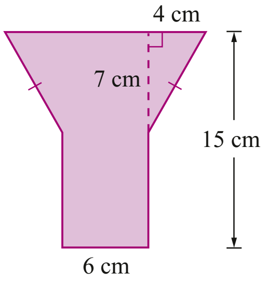
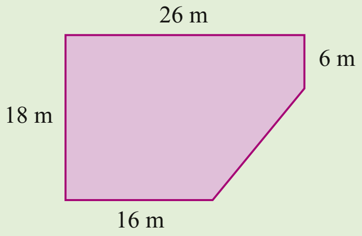


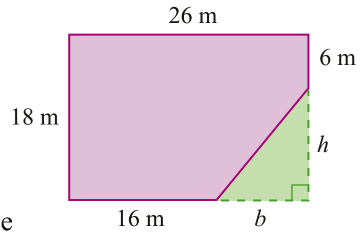
o



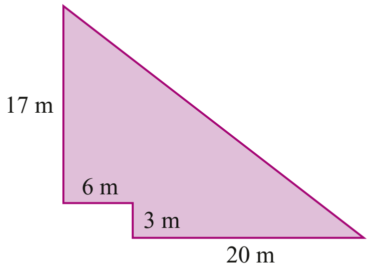
EXAMPLE 3

Find the area of this shape by subtraction.



Solve/Think	Apply
Complete a large rectangle as shown. *For the rectangle:* $A = 26 \times 18 = 468$ m^2 *For the triangle:* $b = 26 - 16 = 10$ m $h = 18 - 6 = 12$ m $A = \frac{1}{2} \times 10 \times 12 = 60$ m^2 Area of shape = area of rectangle − area of triangle $= 468 - 60 = 408$ m^2  	Complete the larger shape by extending the sides. Subtract the area of the smaller shape from the area of the larger shape.

4 Complete the following to find the area of this shape by subtraction.



Complete a triangle as shown.

For the triangle:

$b = \_\_\_ + \_\_\_ = \_\_\_$ m

$h = \_\_\_ + \_\_\_ = \_\_\_$ m

$A = \frac{1}{2} \times \_\_\_ \times \_\_\_ = \_\_\_$ m^2

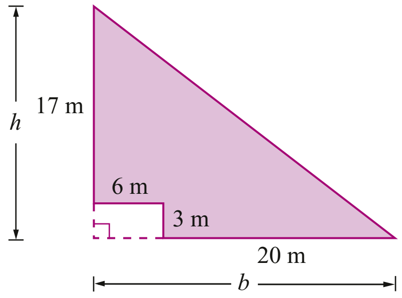
For the rectangle:

$A = \_\_\_ \times \_\_\_$

$= \_\_\_$ m^2

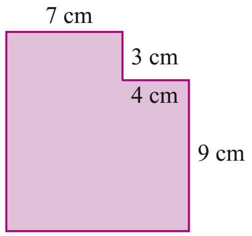
Area of shape = area of triangle − area of rectangle

$= \_\_\_ - \_\_\_ = \_\_\_$ m^2

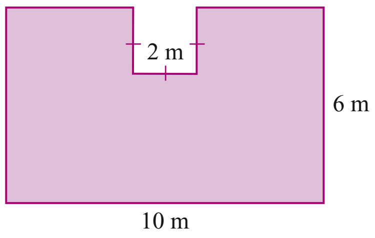


5 Find the areas of the following shapes by subtraction.

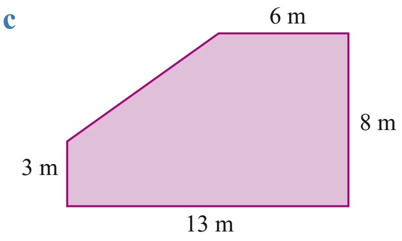
a



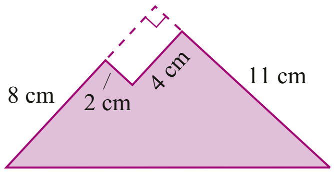
b



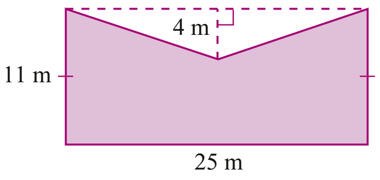
c



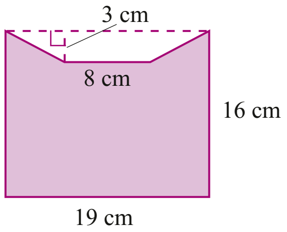
d



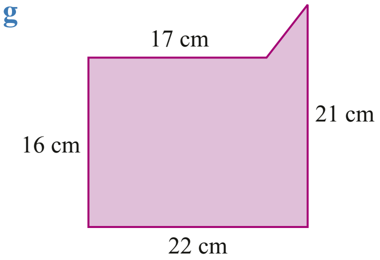
e



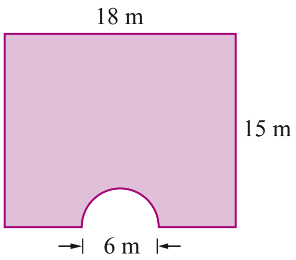
f



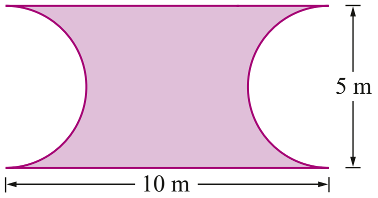
g



h

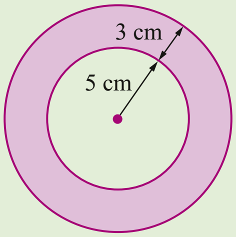


i



EXAMPLE 4

Find the area of this annulus.



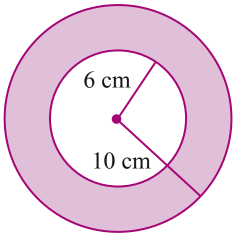
The region between two circles that have the same centre is called an annulus.

Solve/Think	Apply
Radius of large circle $= 5 + 3 = 8$ cm A = area of large circle − area of small circle $= \pi \times 8^2 - \pi \times 5^2$ $\approx 122.5\text{ cm}^2$ (1 decimal place).	Area of an annulus = area of large circle − area of small circle

6 Complete the following to find the area of this annulus.

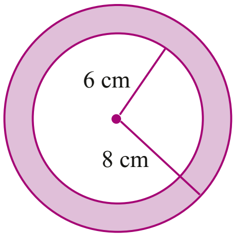
A = area of large circle − area of small circle

$= \pi \times \_\_\_ - \pi \times \_\_\_ \approx \_\_\_\text{ cm}^2$ (1 decimal place)

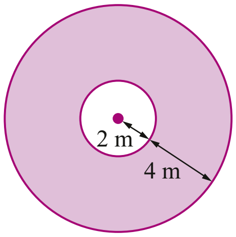


7 Find the shaded area of each shape correct to 1 decimal place.

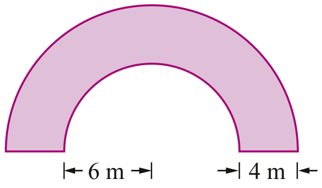
a



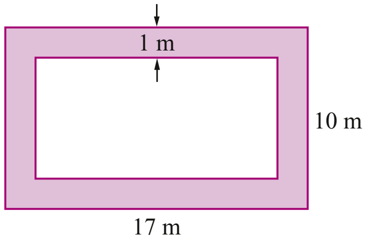
b



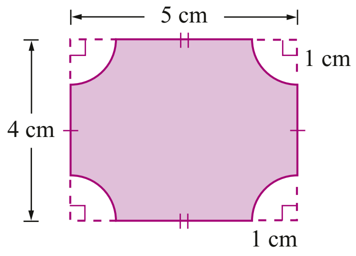
c



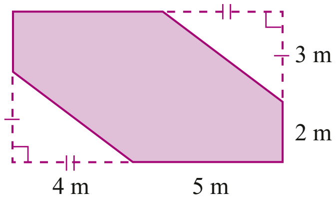
d



e



f



D Area applications

This section involves practical problems using area.

Exercise 6D

1 Find the cost of paving a rectangular patio 4.8 m long by 7.3 m wide, if paving costs $92.95 per square metre.

2 The diagram shows the floor plan of a conference room.

a Calculate the area of the conference room.

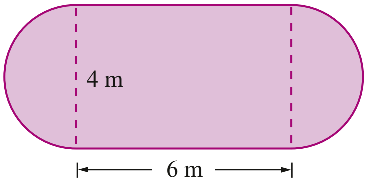
b Calculate the cost of tiling the floor of the conference room if the tiling costs $62.80 per square metre.

Scale 1 cm : 5 m

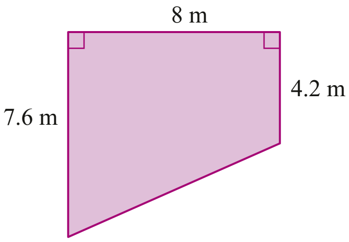
3 A family room is in the shape of a rectangle with a semicircle at each end, as shown in the diagram.

a Calculate the area of the family room.

b The cost to supply and lay tiles is $120 per square metre. Find the total cost of tiling the family room to the nearest dollar.



4 A concreter charges $59 per square metre. Calculate the cost of concreting the area shown.



5 A triangular hobby farm is situated along a main road.

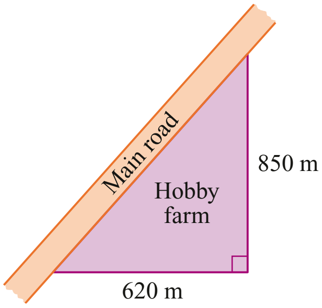
a Calculate the area (in m^2) of the farm.

b What is the area in hectares of this farm?

c If $\frac{7}{10}$ of the area is used for growing crops, how many hectares is this?

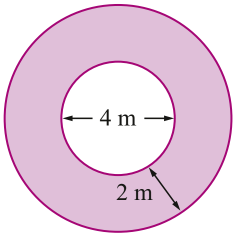
d If $\frac{3}{10}$ of the area is used for pasture and the homestead, how many hectares is this?

e Calculate the value of the hobby farm if each square metre is valued at $7.20.

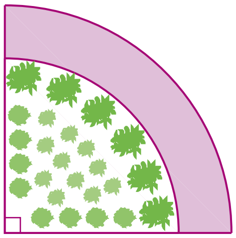


1 ha = 10 000 m². !

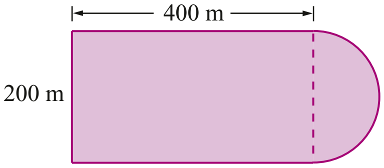
6 A path 2 m wide is placed around a circular pond of diameter 4 m. Find the area of the path to the nearest whole number.



7 A garden bed is in the shape of a quadrant of a circle of radius 3.5 m. A path 1 m wide is to be built around the curved boundary only. Find the area of the path, to the nearest whole number.



8 A farmer wants to spread fertiliser on a paddock at the rate of 200 kg of superphosphate per hectare. What weight of superphosphate is required for this paddock? Give your answer in tonnes.



E Areas involving sectors of a circle

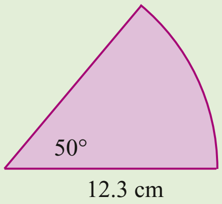
You can work out the area of a sector of a circle by comparing its angle with the angle of a full circle, 360°.

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{sector angle}}{360}$$

Hence $$A\text{ (of sector)} = \frac{\text{sector angle}}{360} \times \text{area of circle}$$

EXAMPLE 1

Find the area of this sector.



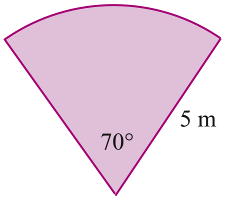
Solve/ Think	Apply
$A = \frac{\text{sector angle}}{360} \times \text{area of circle}$ $= \frac{50}{360} \times \pi \times 12.3^2 \approx 66.0\text{ cm}^2$ (1 decimal place)	Area of a sector $= \frac{\text{sector angle}}{360} \times$ area of circle

Exercise 6E

1 Complete the following to find the area of this sector.

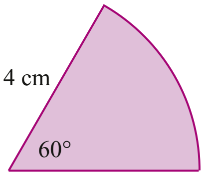
$$\text{Area of sector} = \frac{\square}{360} \times \pi \times \_\_\_$$

$$\approx \_\_\_ \text{ m}^2 \text{ (1 decimal place)}$$

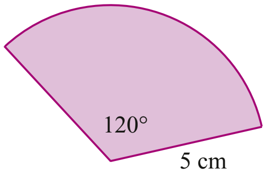


2 Calculate the area of each sector correct to 1 decimal place.

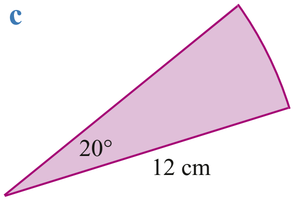
a



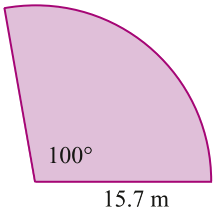
b



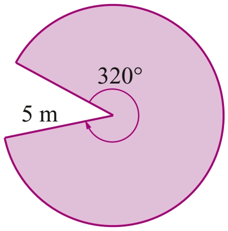
c



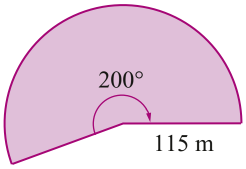
d



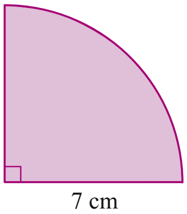
e



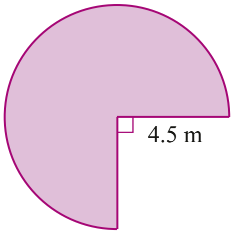
f



g

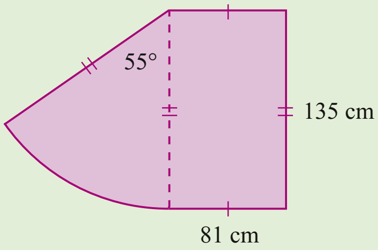


h



EXAMPLE 2

Calculate the area of this composite shape.



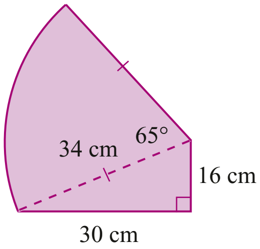
Solve/Think	Apply
A = area of sector + area of rectangle $= \frac{55}{360} \times \pi r^2 + lb$ $= \frac{55}{360} \times \pi \times 135^2 + 81 \times 135$ $\approx 19\,682.4 \text{ cm}^2$ (1 decimal place)	Divide the shape into simpler shapes and calculate their areas. Find the sum of the areas of the simpler shapes.

3 Complete the following to find the area of this composite shape.

A = area of sector + area of triangle

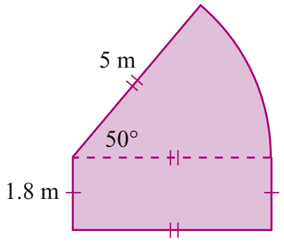
$$= \frac{\square}{360} \times \pi \times \_\_\_ + \_\_\_$$

$$\approx \_\_\_ \text{ cm}^2$$

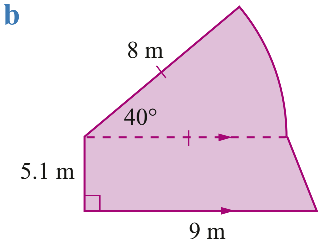


4 Calculate the area of each composite shape correct to 1 decimal place.

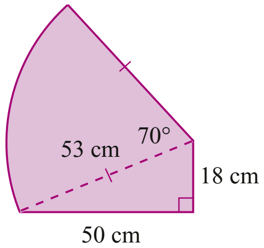
a



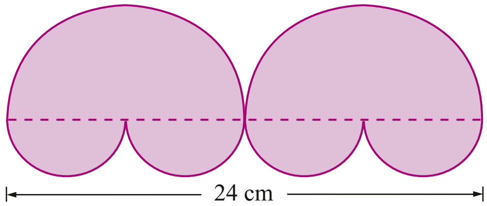
b



c



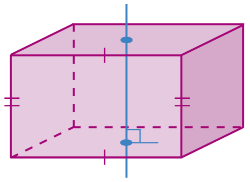
5 Calculate the area of this shape made of semicircles correct to 1 decimal place.



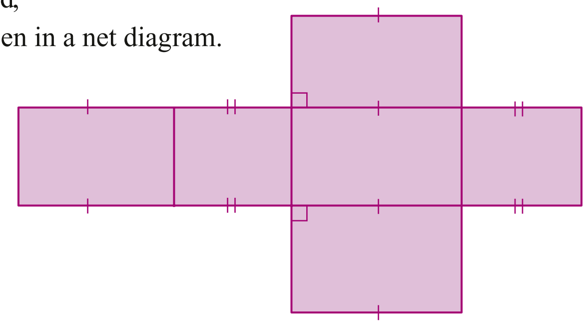
F Surface areas of prisms

Right and oblique prisms

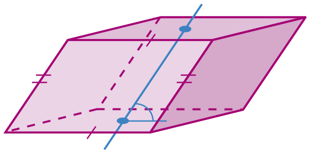
The **axis** of a prism is an imaginary line joining the centres of the identical bases or ends.
In a **right prism** the axis is perpendicular to the base or end, and the bases or ends are joined by rectangles, as can be seen in a net diagram.



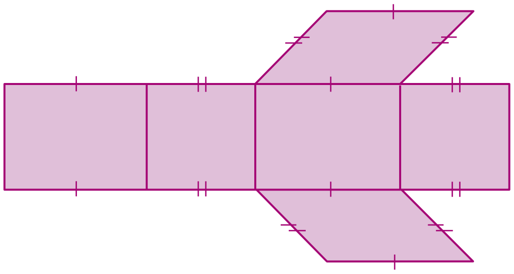
Right rectangular prism



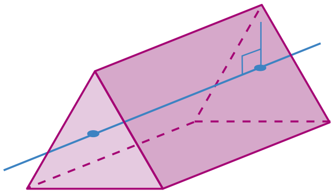
In an **oblique prism** the axis is *not* perpendicular to the base or end and the bases or ends are joined by parallelograms and rectangles (or just parallelograms).



Oblique rectangular prism



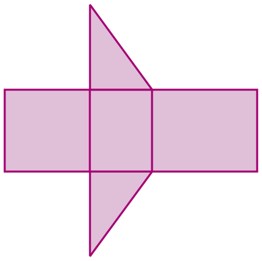
Pyramids and cylinders similarly can be described as right or oblique.
Note: The bases or ends of a prism may be at the sides, in which case the axis is not vertical.



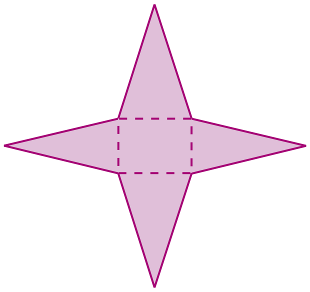
Exercise 6F

1 **i** Match each net in parts **a** to **e** with one of the solids shown in parts **A** to **E** below.
ii Name each solid.

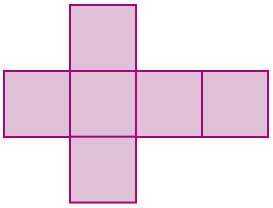
a



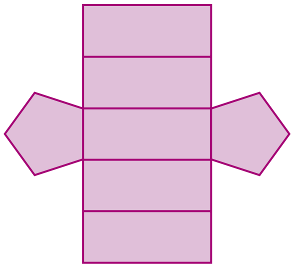
b



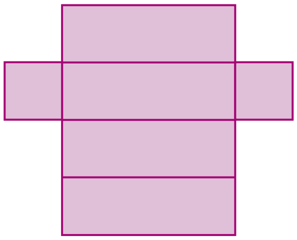
c



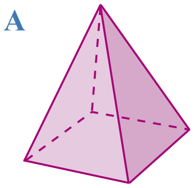
d



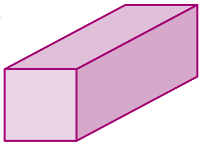
e



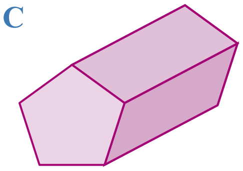
A



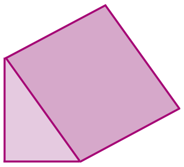
B



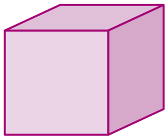
C



D



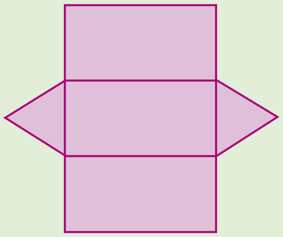
E



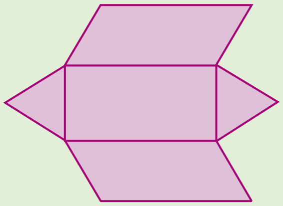
EXAMPLE 1

Determine from the net given whether each triangular prism is right or oblique.

a



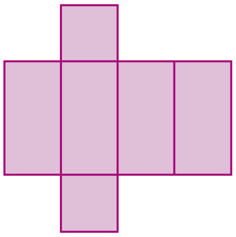
b



	Solve/Think	Apply
a	Identical ends are joined by rectangles, so this is a right triangular prism.	In a right prism the ends are joined by rectangles. In an oblique prism the ends are joined by parallelograms and rectangles (or just parallelograms).
b	Identical ends are joined by parallelograms and a rectangle, so this is an oblique triangular prism.	

2 State whether each net is for a right or oblique prism.

a



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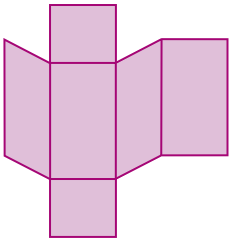
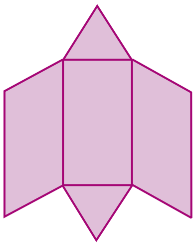
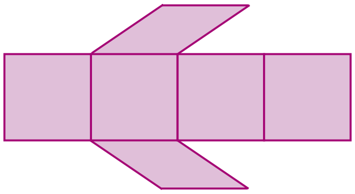


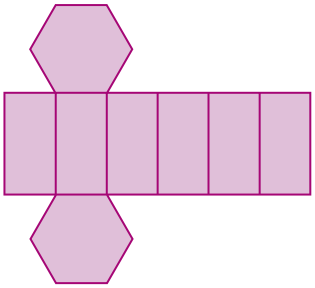
c



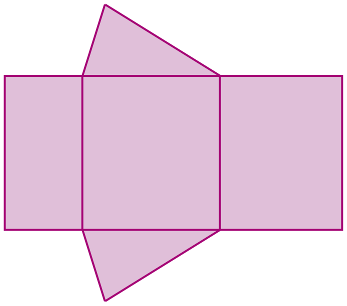
d



e

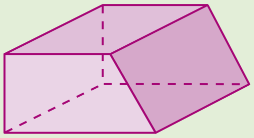


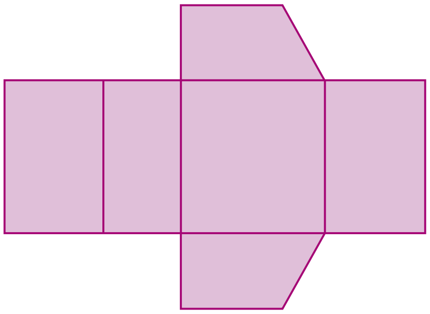
f



EXAMPLE 2

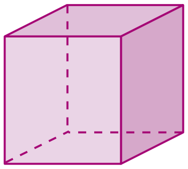
Draw a net for the solid shown and name the solid.



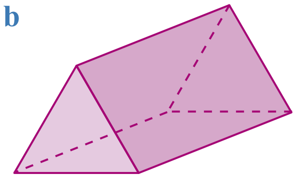
Solve/Think	Apply
The ends are identical trapeziums joined by rectangles, so the solid is a right trapezoidal prism. 	A prism has identical ends joined by rectangles. The prism is named according to the shape of the identical ends.

3 Draw a net for each solid and name the solid.

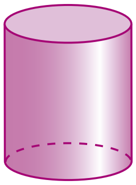
a



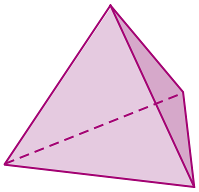
b



c



d



Surface areas of right rectangular and triangular prisms

The **surface area** of a solid is the sum of the areas of all its individual faces. Follow these steps to determine the surface area of a solid:

Step 1: Identify all its faces.

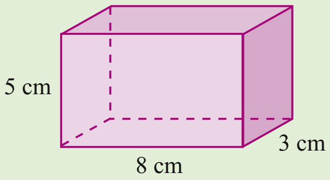
Step 2: Calculate the area of each face.

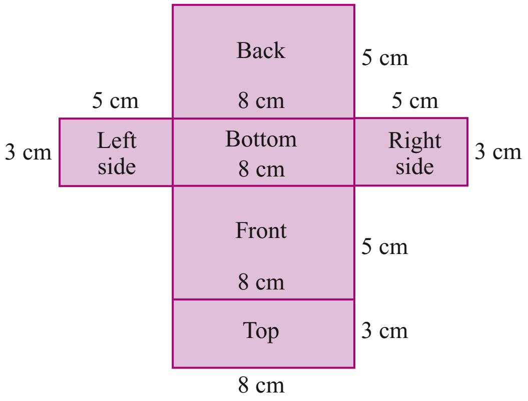
Step 3: Add the areas of all the individual faces.

EXAMPLE 3

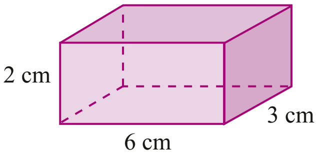
a Draw a net for this rectangular prism, showing the lengths of its edges.

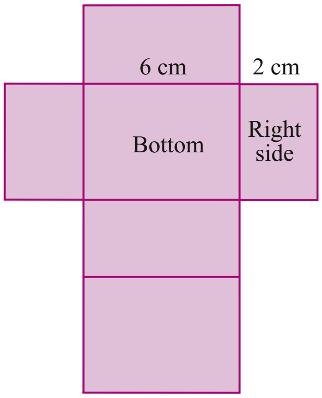
b Calculate the surface area of the prism.



	Solve/Think	Apply
a		Draw the net, identifying the faces, and transfer the edge lengths from the solid to the net. Calculate the area of each face and sum these areas.
b	$SA = (\text{bottom} + \text{top}) + (\text{front} + \text{back}) + (\text{left side} + \text{right side})$ $= (8 \times 3) \times 2 + (8 \times 5) \times 2 + (5 \times 3) \times 2$ $= 158\text{ cm}^2$	

4 **a** Complete the net of this rectangular prism by identifying all faces and marking its edge lengths.





b Calculate the surface area of the prism.

$SA = (\text{bottom} + \_\_\_) + (\text{front} + \_\_\_) + (\text{left side} + \_\_\_)$

$= (\_\_\_ \times \_\_\_) \times 2 + (\_\_\_ \times \_\_\_) \times 2 + (\_\_\_ \times \_\_\_) \times 2$

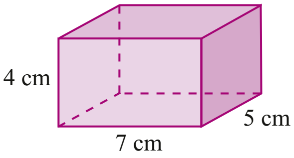
$= \_\_\_\text{ cm}^2$

5 For each of the following rectangular prisms:

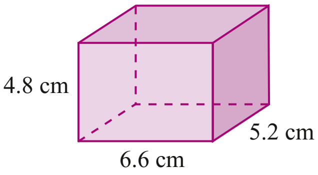
i Draw a net of the prism and mark its edge lengths.

ii Calculate its surface area.

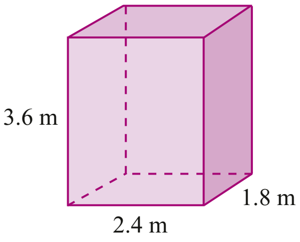
a



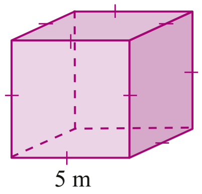
b



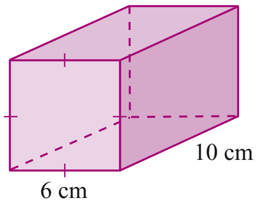
c



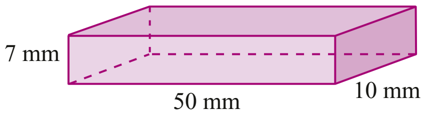
d



e

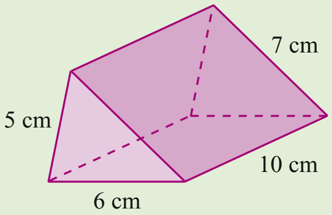


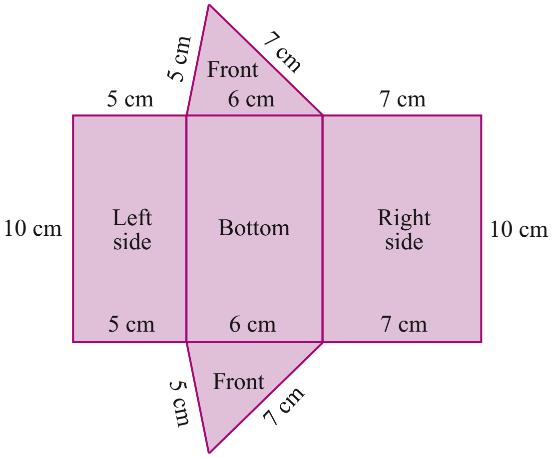
f



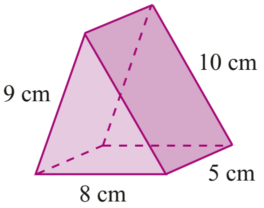
EXAMPLE 4

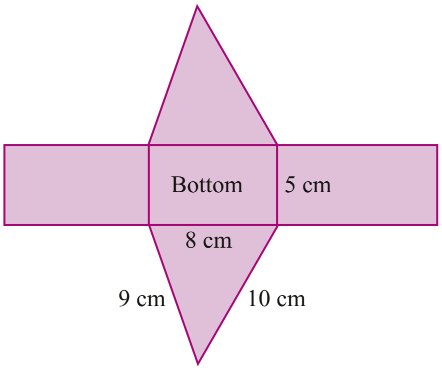
Draw a net for this triangular prism, marking its faces and edge lengths.



Solve/Think	Apply
	Draw the net identifying the faces and transfer the edge lengths from the solid to the net.

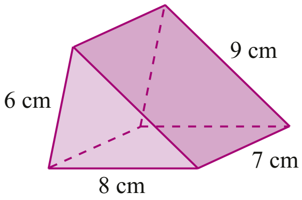
6 Complete the net for this triangular prism by marking its faces and edge lengths.



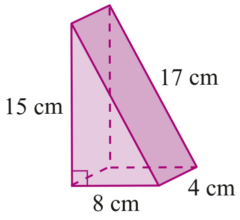


7 Draw a net for each of the following triangular prisms, marking its edge lengths.

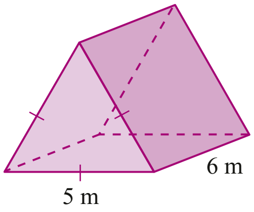
a



b



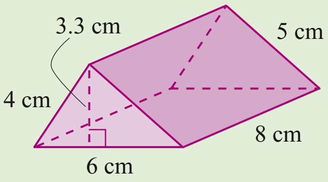
c

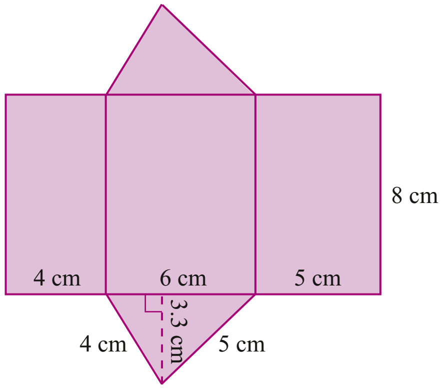


EXAMPLE 5

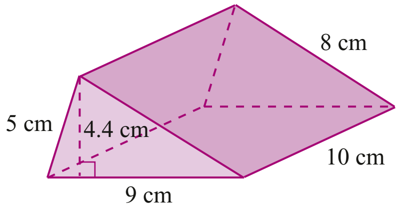
a Draw a net of this triangular prism, marking its edge lengths.

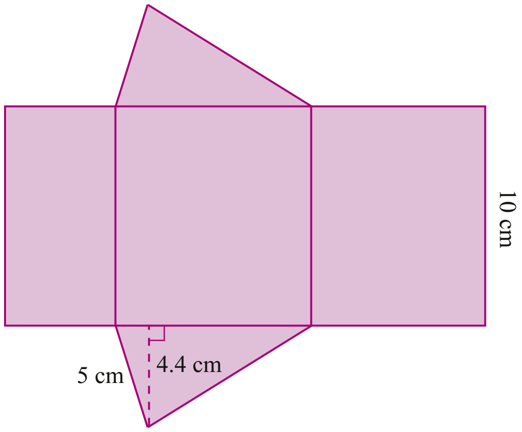
b Calculate the surface area of the prism.



	Solve/Think	Apply
a		Draw the net. Identify the faces and transfer the edge lengths from the solid to the net. Calculate the area of each face and sum these areas.
b	SA = area of 2 triangles + area of 3 rectangles $= (\frac{1}{2} \times 6 \times 3.3) \times 2 + 8 \times 4 + 8 \times 6 + 8 \times 5$ $= 139.8\text{ cm}^2$	

8 **a** Complete the net for this triangular prism by marking its edge lengths.





b Complete the following to calculate the surface area of the prism.

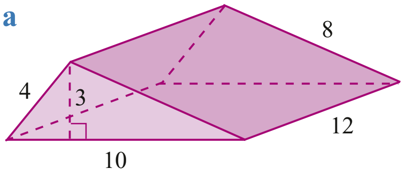
SA = area of 2 ___ + area of 3 ___

= (___ × ___ × ___) × ___ + ___ × ___ + ___ × ___ + ___ × ___

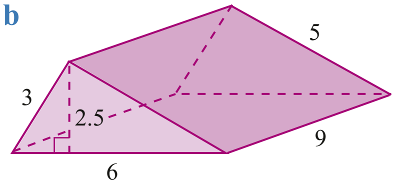
= ___

9 Calculate the surface area of each of the following triangular prisms. All measurements are in centimetres.

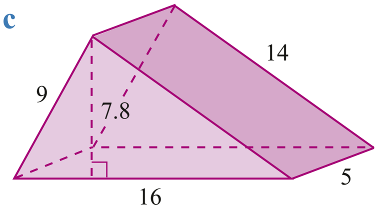
a



b



c

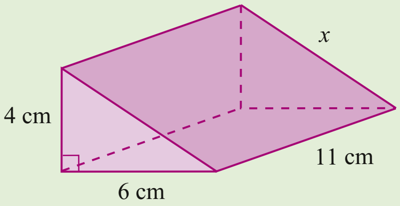


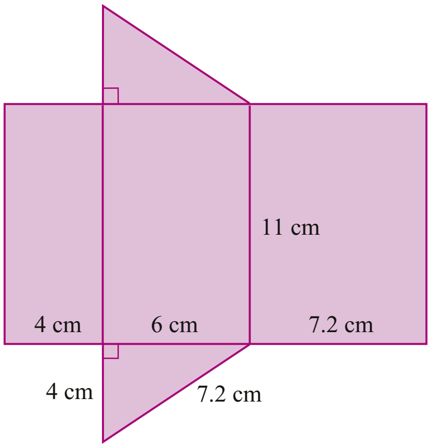
EXAMPLE 6

a Calculate the length of the unknown edge of this triangular prism.

b Draw a net of the prism.

c Calculate its surface area.



	Solve/Think	Apply
a	By Pythagoras' theorem: $x^2 = 4^2 + 6^2 = 52$ $\therefore x = \sqrt{52} \approx 7.2$ cm (1 decimal place)	Calculate the unknown edge using Pythagoras' theorem. Draw the net and calculate the surface area as before.
b		
c	$SA = (\frac{1}{2} \times 6 \times 4) \times 2 + 11 \times 4 + 11 \times 6 + 11 \times 7.2$ $= 213.2 \text{ cm}^2$	

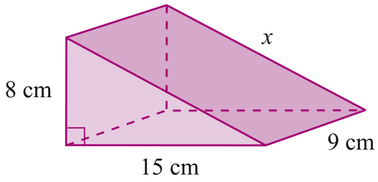
10 **a** Complete the following to find the length x.

$x^2 =$ ___ + ___

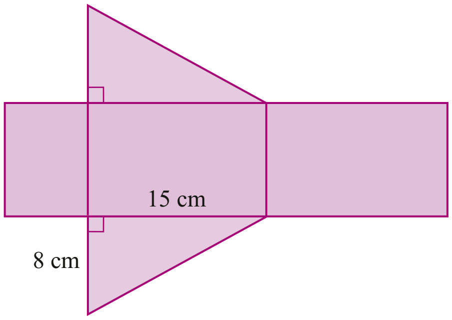
$=$ ___

$\therefore x =$ ___

$=$ ___ cm (1 decimal place if necessary)



b Use the result of part **a** to complete the net for this prism.



c Complete the following to find the surface area.

$SA = ($___ $\times$ ___ $\times$ ___$) \times$ ___ $+$ ___ $\times$ ___

$+$ ___ $\times$ ___ $+$ ___ $\times$ ___

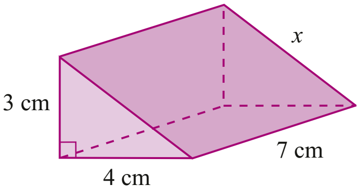
$=$ ___

11 For each triangular prism:

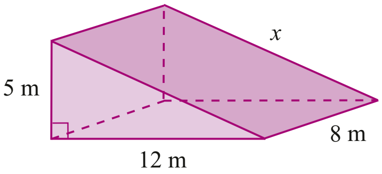
i Find x.

ii Calculate the surface area.

a



b



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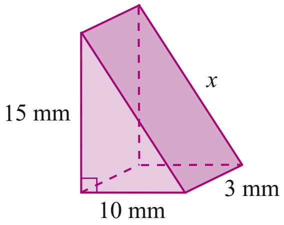
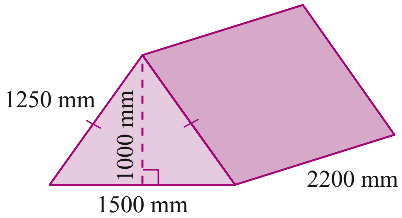


G Problems involving surface area

Exercise 6G

1 A timber door is 200 cm high, 80 cm wide and 3.5 cm thick. Calculate the surface area of the door.

2 A floorless tent is to be made with dimensions as shown. How much material is needed to make the tent? (Ignore seam allowances and overlaps.)

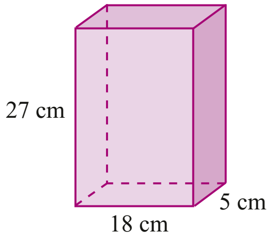


3 Ben is making 5 cm wooden cubes for a children's kindergarten. He intends to glue coloured paper to each face of the cubes. How much coloured paper would he need for 40 cubes?

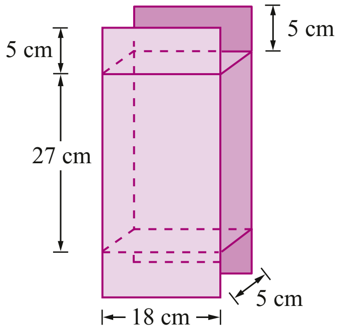
4 Jenny's bedroom is 4 m long, 3 m wide and 2.7 m high. It has a door that measures 2.1 m × 0.85 m and a window that is 1.8 m × 1.4 m. Calculate the cost of putting two coats of paint on the walls and ceiling if the paint costs $1.47/$m^2$.

5 In a craft class, a 20 cm × 15 cm × 8 cm cardboard box is to be covered inside and out, including the lid, with cotton fabric. How much cotton fabric is needed? (Ignore the thickness of the cardboard.)

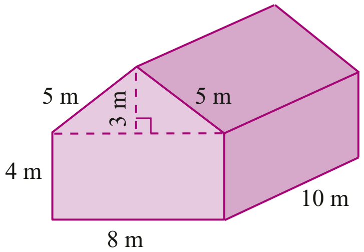
6 **a** How much cardboard is needed to make a 27 cm × 18 cm × 5 cm cardboard box?



b In order to make a 27 cm × 18 cm × 5 cm cardboard box for cereal, the box has to have a double flap at the top and the bottom, as shown. How much cardboard is needed to make the cereal box?



7 A farmer's shed with dimensions as shown is to be cleaned with a high-pressure water cleaner. How long will it take to clean the outside walls and roof if the pressure cleaner can clean 1 m^2 every $1\frac{1}{2}$ min?



Language in mathematics

Johann Kepler (1571–1630)

Johann Kepler was born in the German town of Wurttemberg. As a child he was small and suffered from ill health, but he was recognised as being intelligent. He was given a scholarship to attend the University of Tubingen, where he studied first for the Lutheran ministry and then science. He studied under a master in astronomy who believed in, and taught, the Copernican theory that Earth rotated around its own axis and around the Sun. Kepler taught mathematics in Graz from 1594.

In 1600 he went to Prague and became assistant to Tycho Brahe, an important astronomer. After Brahe's death, Kepler succeeded him as astronomer and mathematician to the emperor. Kepler had access to Brahe's extensive records of observations and calculations.

Kepler believed in the Copernican theory, and became one of the founders of modern astronomy. He developed three fundamental laws of planetary motion, now known as Kepler's Laws, in 1609. These proposed, among other things, that the Sun was at the centre of our planetary system, and that the orbits of the planets were elliptical rather than circular. Sixty years later these laws helped Newton to develop his Universal Law of Gravitation.

Kepler also suggested that tides are caused by the Moon's gravitational pull on the seas. He produced tables giving the positions of the Sun, Moon and planets, which were used for about 100 years. In 1611 he proposed an improved refracting telescope, and later he suggested a reflecting telescope that was developed by Newton.

1 **a** How old was Kepler when he died?
b When and where did Kepler teach mathematics?
c Describe the development of Kepler's ideas concerning planetary motion.
d Research Kepler's three laws.
e For how long were Kepler's tables of positions of the Sun, Moon and planets used?
f How are tides formed?

2 Insert the vowels in these glossary terms.
a c__rcl__
b q__ __dr__l__t__r__l
c c__mp__s__t__
d rh__mb__s
e k__t__
f s__ct__r
g tr__p__z__ __m
h tr__ __ngl__
i __bl__q__ __

3 Rearrange these words to form a sentence.
a a circle a semicircle A half is of.
b a is of quarter quadrant A circle a.
c may way than Composite more in areas one be found.

4 Use every third letter to find the sentence.
W D T R F H T G E H Y A U J R N H E G B A V F O E D F S W A A Z R D F H H J O L P M O
E B Q A U Z D S F Y O I J R B W A Q A K C G I H J T I I E O P I L L S G F H D E A S K L A X
F V B T H Q H S O E Y A P E F R H K O I P D N M U A E C S D T C G O H N F B E T W X H
A U E I O D A G I B H A J K G N H O D S N W E A D F L T Y S

Terms

area
circle
composite
diameter
formula
kite
oblique
parallelogram
prism
quadrant
quadrilateral
radius
rhombus
right
sector
semicircle
trapezium
triangle

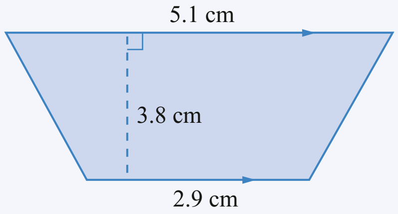
Check your skills

1 The formula $A = \frac{1}{2}xy$ could be used to find the area of a:

A parallelogram **B** trapezium **C** rhombus **D** all of these

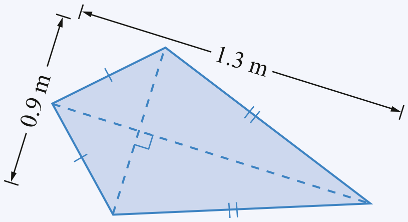
2 The area of this trapezium is:

A 15.2 cm² **B** 30.4 cm²
C 56.202 cm² **D** 28.101 cm²



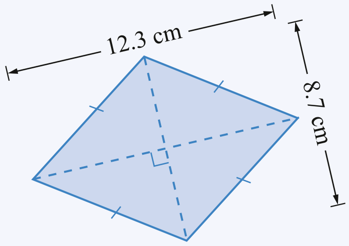
3 The area of this kite is:

A 1.17 m² **B** 2.34 m²
C 0.585 m² **D** 2.2 m²



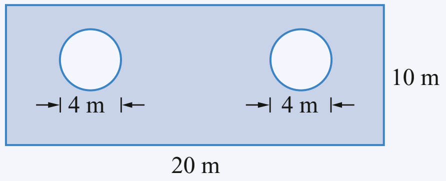
4 The area of this rhombus is:

A 107.01 cm² **B** 13.37625 cm²
C 26.7525 cm² **D** 53.505 cm²



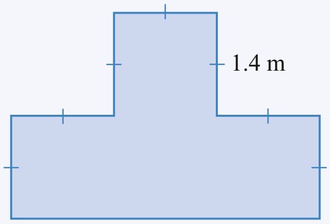
5 The shaded area is closest to:

A 174.9 m² **B** 99.5 m²
C 225.1cm² **D** 300.5 m²



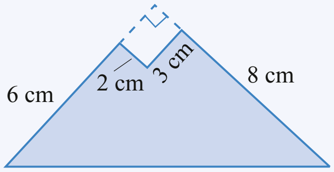
6 The area of this shape is:

A 7.84 m² **B** 14 m²
C 5.88 m² **D** 9.8 m²



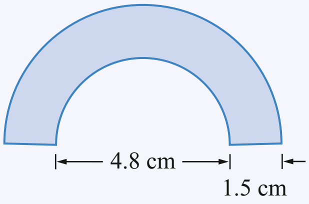
7 The area of this shaded shape is:

A 39 cm² **B** 30 cm²
C 24 cm² **D** 18 cm²



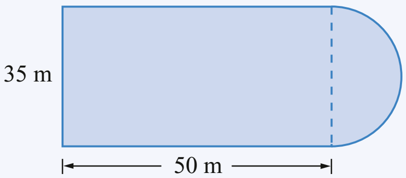
8 The shaded area is closest to:

A 25.16 cm² **B** 65.3 cm²
C 29.7 cm² **D** 14.8 cm²



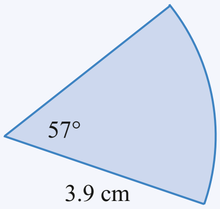
9 A farmer fertilises a paddock shaped as shown. The fertiliser is spread at the rate of 4.5 kg/m². The amount of fertiliser the farmer needs is closest to:

A 10 t **B** 7.875 t
C 12.2 t **D** 2.231 t



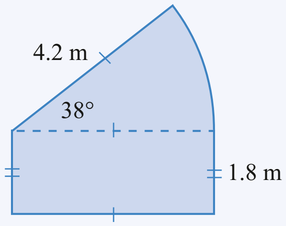
10 The area of this sector is closest to:

A 3.87 cm² **B** 7.6 cm²
C 15.6 cm² **D** 222.3 cm²



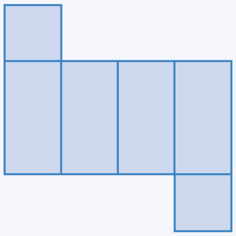
11 The area of this shape is closest to:

A 5.85 m² **B** 9.02 m²
C 13.41 m² **D** 14.8 m²

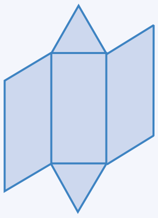


12 Which of the following is a net of an oblique prism?

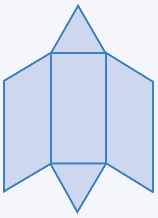
A



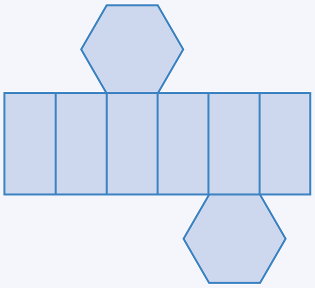
B



C

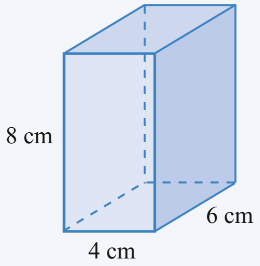


D



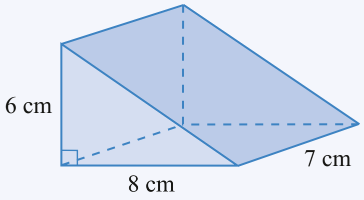
13 The surface area of this prism is:

A 104 cm² **B** 184 cm²
C 192 cm² **D** 208 cm²



14 The surface area of this prism is:

A 264 cm² **B** 234 cm²
C 216 cm² **D** 192 cm²



15 A lidded wooden box, 15 cm × 8.5 cm × 6 cm, is to be lacquered inside and out with two coats of lacquer. Ignoring the thickness of the wood, the total area to be lacquered is:

A 537 cm² **B** 2148 cm² **C** 1074 cm² **D** 2685 cm²

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–4	5–8	9	10, 11	12–14	15
Section	B	C	D	E	F	G

6A Review set

1 Complete the following.

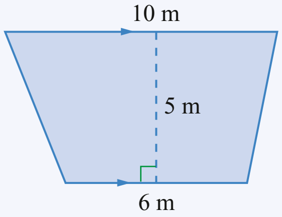
a 85 cm^2 = ___ mm^2 **b** 15 000 m^2 = ___ ha **c** 3.5 km^2 = ___ m^2

2 Write the formula for the area of each shape.

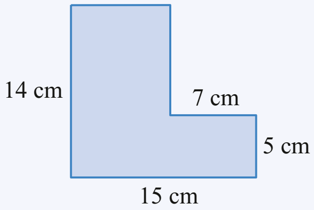
a a triangle **b** a circle **c** a square

3 Find the area of each shape.

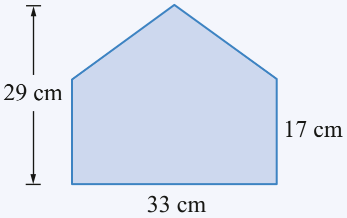
a



b

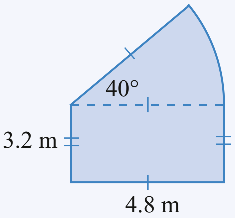


c

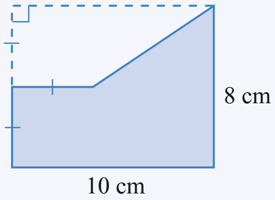


4 Find the shaded area correct to 1 decimal place.

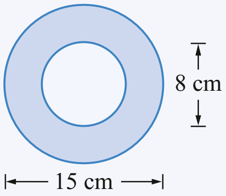
a



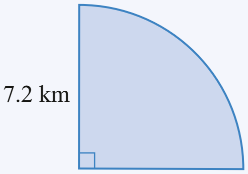
b



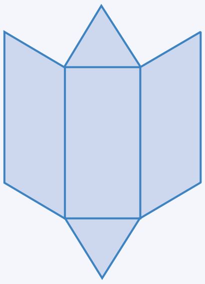
c



5 A river delta is shaped roughly like a quadrant, as shown. Calculate the population of the delta if 225 people per square kilometre live there.

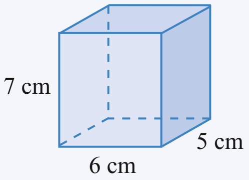


6 Name the solid whose net is shown.

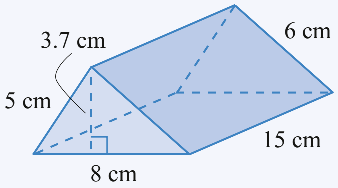


7 Calculate the surface area of each prism.

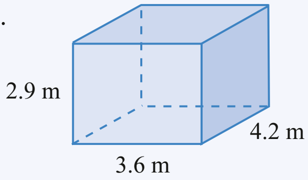
a



b



8 A closed metal tank, with the dimensions shown, is to be constructed. What area of metal is required for the tank?



6B Review set

1 Complete the following.

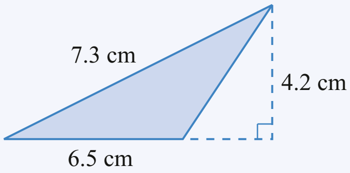
a 4.28 ha = ___ m^2 **b** 3 cm^2 = ___ mm^2 **c** 4300 cm^2 = ___ m^2

2 Write the formula for the area of each shape.

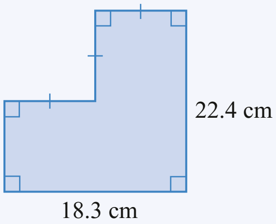
a a trapezium **b** a kite **c** a parallelogram

3 Find the area of each shape.

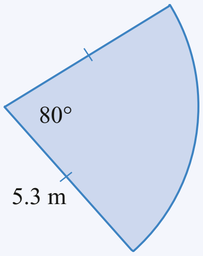
a



b

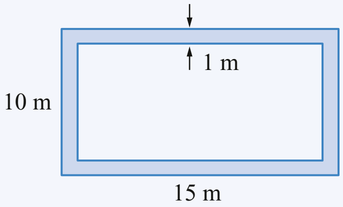


c

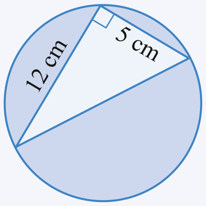


4 Find the shaded area correct to 1 decimal place.

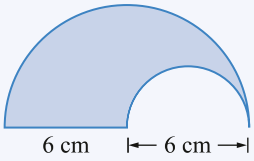
a



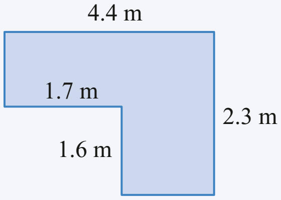
b



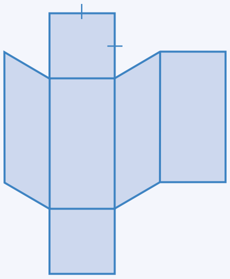
c



5 Deborah's family room is shown opposite. Calculate the cost of carpet-tiling the room if the carpet tiles costs $119.80 per square metre.

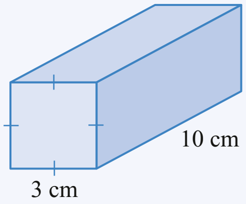


6 Name the solid whose net is shown.

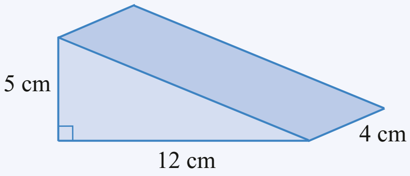


7 Calculate the surface area of each prism.

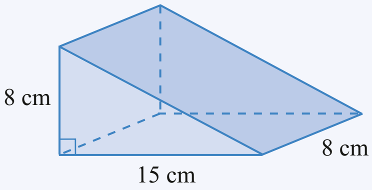
a



b



8 A door wedge shaped as shown is to be painted with two layers of paint. What total area is to be painted?



6C Review set

1 Complete the following.

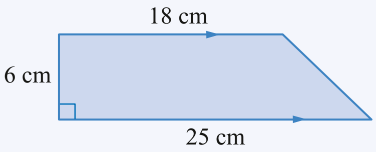
a 2.1 km^2 = ___ ha **b** 65 000 m^2 = ___ km^2 **c** 0.8 m^2 = ___ mm^2

2 Write the formula for the area of each shape.

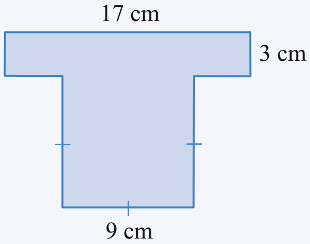
a a rectangle **b** a rhombus **c** a sector

3 Find the area of each shape.

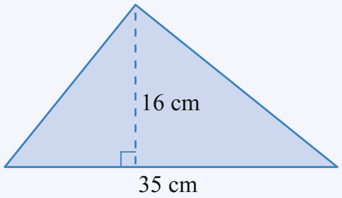
a



b

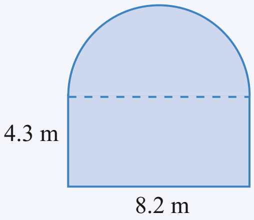


c

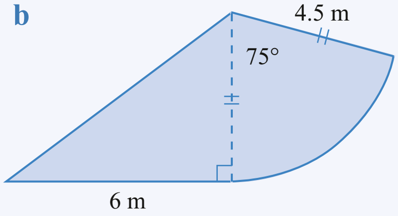


4 Find the area of each shape correct to 1 decimal place.

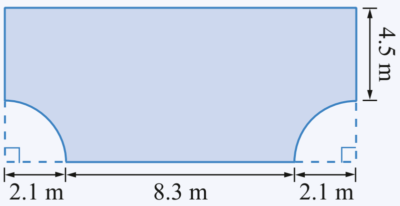
a



b



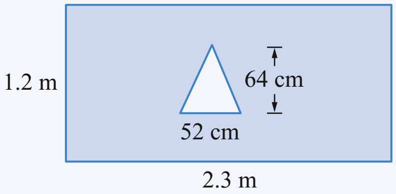
c



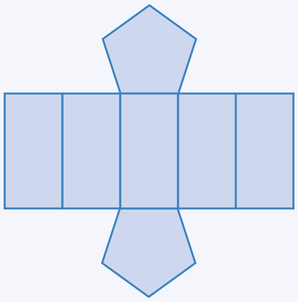
5 Karl wishes to cut a triangle from a rectangular piece of wood, as shown.

a Calculate the area of the triangle if the base is 52 cm and the height is 64 cm.

b Calculate the area remaining after the triangle is removed.

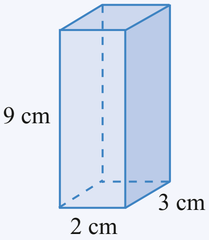


6 Name the solid whose net is shown.

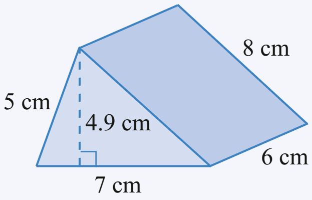


7 Calculate the surface area of each prism.

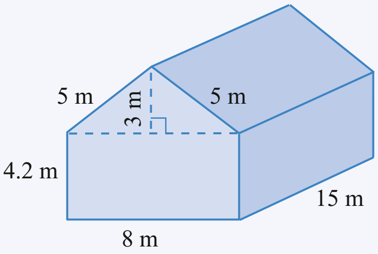
a



b



8 The army shed shown is to be painted in camouflage colours. What area is to be camouflaged?



7

Probability

This chapter deals with relative frequency and probabilities involving Venn diagrams and two-way tables.

After completing this chapter you should be able to:

- calculate relative frequencies to estimate probabilities of events
- calculate probabilities from data in Venn diagrams and two-way tables.

NSW Syllabus references: 5.1 S&P Probability
Outcomes: MA5.1-1WM, MA5.1-2WM, MA5.1-3WM, MA5.1-13SP
STATISTICS & PROBABILITY – ACMSP226

A Relative frequency and probability

If a six-sided die is rolled 100 times and a 6 comes up 13 times, the relative frequency of the 6 is $\frac{13}{100}$.

$$\textbf{Relative frequency} = \frac{\textbf{frequency of an event}}{\textbf{total number of trials}}$$

Note: Number of trials = number of times the experiment is repeated.

The relative frequency of an event can be used to predict the probability of an event occurring. When used in this way, relative frequency is often referred to as experimental probability. It is based on the assumption that the number of times that an event has occurred in the past is an indication of how often it will occur in the future.

Investigation 1 Probability experiments

1 a In pairs, roll a die 100 times.

b Record the results in a table like the one on the right.

c What is the relative frequency of getting a 6?

d What is the theoretical probability of getting a 6?

e Compare the relative frequency as an estimate of probability with the theoretical probability. Are they approximately the same? You will recall that:

$$\text{Theoretical probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Result	Tally	Frequency
1		
2		
3		
4		
5		
6		

f Combine the results of all groups in a table.

	Group 1	Group 2	Group 3	Group 4	Group 5	Group …
Relative frequency of getting a 6	$\frac{\square}{100}$	$\frac{\square}{100}$	$\frac{\square}{100}$	$\frac{\square}{100}$	$\frac{\square}{100}$	…

i Combine the results of the first five groups and compare the relative frequency of getting a 6 with the theoretical probability.

ii Combine the results of the first 10 groups and compare the relative frequency of getting a 6 with the theoretical probability.

iii Comment on the results of parts **i** and **ii**.

iv What do you think would happen if you rolled the die 1 000 000 times?

2 a In pairs, toss a coin 100 times.

b Record the results in a table like the one shown below.

Result	Tally	Frequency
H		
T		

c What is the relative frequency of getting a tail?

d What is the theoretical probability of getting a tail?

e Compare the experimental probability of getting a tail with the theoretical probability. Are they approximately the same?

f Combine the results of all groups in a table.

	Group 1	**Group 2**	**Group 3**	**Group 4**	**Group 5**	**Group …**
Relative frequency of getting a tail	$\frac{\square}{100}$	$\frac{\square}{100}$	$\frac{\square}{100}$	$\frac{\square}{100}$	$\frac{\square}{100}$	…

- **i** Combine the results of the first five groups and compare the experimental probability of getting a tail with the theoretical value.
- **ii** Combine the results of the first 10 groups and compare the experimental probability of getting a tail with the theoretical value.
- **iii** Comment on the results of parts **i** and **ii**.
- **iv** What do you think would happen if you tossed the coin 1 000 000 times?

3 **a** In pairs, select a card at random from a normal playing pack and record its suit. Shuffle the cards and repeat 100 times.

b Record the results in a table like the one below.

Suit	**Diamonds**	**Hearts**	**Spades**	**Clubs**
Frequency				

c What is the relative frequency of getting a heart?

d What is the theoretical probability of getting a heart?

e Compare the experimental probability with the theoretical probability. Are they approximately the same?

f Combine the results of all groups in a table.

	Group 1	**Group 2**	**Group 3**	**Group 4**	**Group 5**	**Group …**
Relative frequency of getting a heart	$\frac{\square}{100}$	$\frac{\square}{100}$	$\frac{\square}{100}$	$\frac{\square}{100}$	$\frac{\square}{100}$	…

- **i** Combine the results of the first five groups and compare the experimental probability of getting a heart with the theoretical value.
- **ii** Combine the results of the first 10 groups and compare the experimental probability of getting a heart with the theoretical value.
- **iii** Comment on the results of parts **i** and **ii**.
- **iv** What do you think would happen if you repeated the experiment 1 000 000 times?

From Investigation 1, you will have recognised that, as the number of trials increases, the relative frequency gets closer to the theoretical probability. (It is said that the probability estimates become more stable as the number of trials increases.) The theoretical probability can be thought of as being the likelihood of an event occurring under ideal conditions. Hence the probability of an event can be estimated from experimental data using the relative frequency of the event, as long as the number of trials is big enough

EXAMPLE 1

Consider the following statements.

i Five Year 9 students were interviewed. Four of these students indicated that Mathematics was their favourite subject.

ii A Year 9 class of 30 students was surveyed. Nineteen of these students indicated that Mathematics was their favourite subject.

iii A random sample of 100 Year 9 students was surveyed. Fifty-seven indicated that Mathematics was their favourite subject.

a Write down the relative frequency of the event 'Mathematics is my favourite subject' for each of the groups above.

b Which survey is the most reliable in estimating the probability that Mathematics is the favourite subject of a Year 9 student chosen at random?

	Solve	Think	Apply
a i	$\frac{4}{5}$ or 80%	Relative frequency $= \frac{\text{frequency of event}}{\text{total number of trials}}$	As the number of trials increases, the relative frequency gets closer to the theoretical probability; that is, the greater the number of trials the better will be the estimate of probability.
ii	$\frac{19}{30}$ or 63.3%		
iii	$\frac{57}{100}$ or 57%		
b	Survey **iii**		

Exercise 7A

1 Consider the following statements.

i A survey of 6 students indicated that 3 travelled to school by bus.
ii A survey of 20 students indicated that 11 travelled to school by bus.
iii A survey of 100 students indicated that 43 travelled to school by bus.

a Write the relative frequency of the event 'a student travels to school by bus' for each of the groups above.

b Which survey most reliably estimates the probability that a student chosen at random travels to school by bus?

2 The table shows the results of three groups of Year 9 students surveyed as to whether they approved of the school uniform.

	Approved	Did not approve
Group 1	4	6
Group 2	11	19
Group 3	34	66

a Write down the relative frequency of the event 'approve of the school uniform' for each group.

b Which survey is the most reliable in estimating the probability that a Year 9 student, chosen at random, approves of the school uniform?

EXAMPLE 2

A cylindrical can is tossed 150 times and the number of times it landed on its side and on an end is shown in the table.

a Write down the relative frequency of each outcome.

b Estimate the probability that in a future toss of the can it will land on its:

i end **ii** side.

Outcome	Frequency
End	36
Side	114

	Solve	Think	Apply
a	End: $\frac{36}{150}$ or 24% Side: $\frac{114}{150}$ or 76%	Relative frequency $= \frac{\text{frequency of event}}{\text{total number of trials}}$	The probability of an event can be estimated from experimental data using the relative frequency of the event.
b i	24%	Probability of 'land on end' ≈ relative frequency of 'land on end'	
ii	76%	Probability of 'land on side' ≈ relative frequency of 'land on side'	

3 A drawing pin is dropped 500 times and the number of times it landed 'point up' and 'point down' is shown in the table.

a Write down the relative frequency of each outcome.

b Estimate the probability that in a future drop of the drawing pin it will land:

i point up **ii** point down.

Outcome	Frequency
Point up	169
Point down	331

4 A box contains red, blue and green marbles. A marble is selected at random from this box, its colour noted and the marble replaced. This experiment is repeated 200 times. The results are shown in the table. Estimate the probability that a marble drawn at random from this box is:

a red **b** blue

c green **d** red or blue.

Colour	Frequency
Red	61
Blue	107
Green	32

5 A bag contains black and white tickets. Two tickets are drawn simultaneously from the bag and their colour noted and the tickets replaced. This is repeated 500 times. The results are shown in the table. Estimate the probability that two tickets drawn simultaneously from the bag are:

a the same colour **b** different colours.

Outcome	Frequency
Same colour	231
Different colours	269

6 Two coins were tossed simultaneously 1000 times and the results are shown in the table. When two coins are tossed, estimate the probability of getting:

a 2 heads **b** a head and a tail

c 2 tails **d** 2 heads or 2 tails.

Result	Frequency
2 heads	229
1 head and 1 tail	487
2 tails	284

7 Two dice were rolled 800 times and the sum of the numbers on the upper faces was noted. The sum of 7 occurred 158 times. Estimate the probability of getting a total of 7 when two dice are rolled.

8 A spinner with 6 equally sized sectors was spun 300 times and the results are shown in the table.

Result	1	2	3	4	5	6
Frequency	48	57	54	49	47	45

When this spinner is spun, estimate the probability of getting a:

a 2 **b** 6 **c** 4 or 5 **d** at least 4

9 **a** Design a spinner with 4 colours, red, blue, green and yellow, that with sufficient trials will produce a relative frequency of 25% for each outcome.

b Design a spinner with 2 colours, red and blue, that with sufficient trials will produce a relative frequency of 25% for red and 75% for blue.

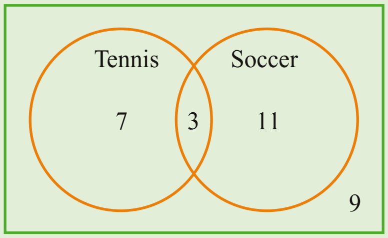
c Design a spinner with 3 colours, red, blue and green, that with sufficient trials will produce a relative frequency of 25% for the outcome red, 25% for blue and 50% for green.

B Venn diagrams and two-way tables

EXAMPLE 1

The Venn diagram represents the number of students in a class of 30 who play tennis and soccer. Calculate the probability that a student chosen at random from this class:

a plays tennis
b does not play tennis
c plays tennis but not soccer
d plays neither tennis nor soccer
e plays both tennis and soccer
f plays tennis or soccer or both
g plays tennis or soccer but not both.



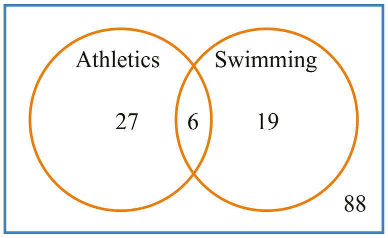
	Solve	Think	Apply
a	$P(\text{plays tennis}) = \frac{10}{30} = \frac{1}{3}$	Number of students who play tennis $=$ number inside the tennis circle $= 7 + 3 = 10$	Find the number of students in the appropriate region of the Venn diagram and calculate the probability.
b	$P(\text{does not play tennis}) = \frac{20}{30} = \frac{2}{3}$	Number of students who do not play tennis $=$ number outside the tennis circle $= 11 + 9 = 20$	
c	$P(\text{plays tennis but not soccer}) = \frac{7}{30}$	Number of students who play tennis but not soccer $=$ number inside the tennis circle but not inside the soccer circle $= 7$	
d	$P(\text{plays neither tennis nor soccer}) = \frac{9}{30} = \frac{3}{10}$	Number of students who play neither tennis nor soccer $=$ number outside both circles $= 9$	

EXAMPLE 1 CONTINUED

	Solve	Think
e	P(plays both tennis and soccer) $= \frac{3}{30} = \frac{1}{10}$	Number of students who play both tennis and soccer = number in the intersection of the circles = 3
f	P(plays tennis or soccer or both) $= \frac{21}{30} = \frac{7}{10}$	Number of students who play tennis or soccer or both = number inside the circles = 7 + 3 + 11 = 21
g	P(plays tennis or soccer but not both) $= \frac{18}{30} = \frac{3}{5}$	Number of students who play tennis or soccer but not both = number inside the two circles, but excluding the intersection = 7 + 11 = 18

Exercise 7B

1 The Venn diagram represents the number of students in Year 9 who were chosen for the school athletics and swimming teams. A student is chosen at random from Year 9. Complete the following.



a Number of students in Year 9 = 27 + ___ + ___ + 88 = ___

b Number of students chosen for the athletics team

= number of students inside the athletics circle = ___ + ___ = ___

P(student was chosen for the athletics team) $= \frac{\square}{\square}$

c Number of students not chosen for athletics team = number of students outside the athletics circle

= ___ + ___ = ___

P(student was not chosen for the athletics team) $= \frac{\square}{\square}$

d Number of students chosen for the athletics team but not the swimming team

= number of students inside the athletics circle but not inside the swimming circle = ___

P(student was chosen for the athletics team but not the swimming team) $= \frac{\square}{\square}$

e Number of students chosen for the swimming team = number of students inside the swimming circle

= ___ + ___ = ___

P(student was chosen for the swimming team) $= \frac{\square}{\square} = \frac{\square}{\square}$

f Number of students not chosen for the swimming team
= number of students outside the swimming circle = ___ + ___ = ___

P(student was not chosen for the swimming team) = $\frac{\square}{\square} = \frac{\square}{\square}$

g Number of students chosen for the swimming team but not the athletics team
= number of students inside the swimming circle but not inside the athletics circle = ___

P(student was chosen for the swimming team but not the athletics team) = $\frac{\square}{\square}$

h Number of students chosen for neither team = number of students outside both circles = ___

P(student was chosen for neither team) = $\frac{\square}{\square} = \frac{\square}{\square}$

i Number of students chosen for both teams = number of students in the intersection of both circles = ___

P(student was chosen for the both teams) = $\frac{\square}{\square} = \frac{\square}{\square}$

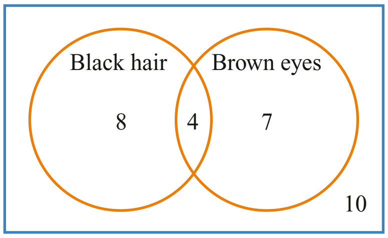
j Number of students who were chosen for either the athletics team or the swimming team or both
= number of students inside both circles = ___ + ___ + ___ = ___

P(student was chosen for either team or both) = $\frac{\square}{\square} = \frac{\square}{\square}$

k Number of students who were chosen for either the athletics team or the swimming team but not both
= number of students inside both circles but excluding the intersection = ___ + ___ = ___

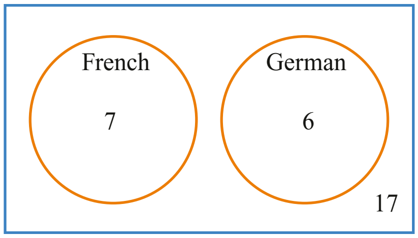
P(student was chosen for either team but not both) = $\frac{\square}{\square} = \frac{\square}{\square}$

2 The Venn diagram represents the number of students in a class who have black hair and brown eyes. What is the probability that a student chosen at random from this class:



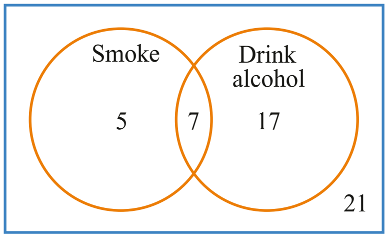
a has black hair?
b does not have black hair?
c has black hair but not brown eyes?
d has brown eyes?
e does not have brown eyes?
f has brown eyes but not black hair?
g does not have black hair or brown eyes?
h has both black hair and brown eyes?
i has black hair or brown eyes or both?
j has black hair or brown eyes but not both?

3 The Venn diagram shows the number of students in a class who study French and German. Calculate the probability that a student chosen at random from this class:



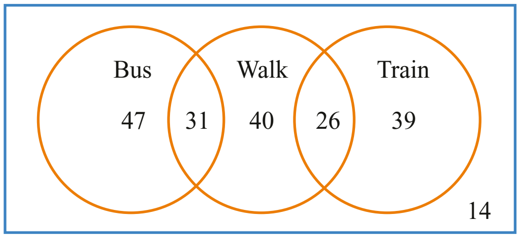
a studies French
b does not study French
c studies German
d does not study German
e does not study either language
f studies French or German
g studies both French and German.

4 The Venn diagram shows the numbers in a group of males who smoke cigarettes and drink alcohol. What is the probability that a male chosen at random from this group:



a smokes but does not drink?
b drinks but does not smoke?
c neither smokes nor drinks?
d both smokes and drinks?
e smokes or drinks or both?
f smokes or drinks but not both?

5 The Venn diagram represents the ways in which students in Year 9 travel to school.

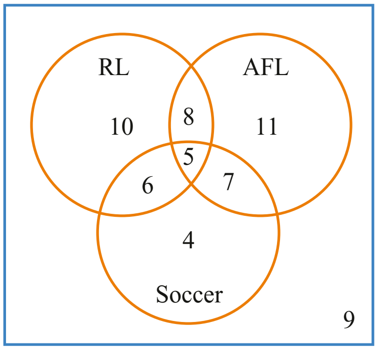


What is the probability that a student chosen at random from Year 9:

a travels by train only?
b travels by train and walks?
c travels by bus and walks?
d travels by train and bus?
e travels by bus or train?
f does not walk, travel by bus or travel by train?
g walks or travels by bus or travels by train or any two of these?
h does not travel by bus?
i does not travel by bus or train?

6 The Venn diagram represents the results of a survey of which sports people liked to watch. Calculate the probability that a person chosen at random from this group:

a does not like rugby league
b likes rugby league but not soccer or AFL
c likes soccer and AFL but not rugby league
d does not like any of these sports
e likes all three sports
f likes rugby league or AFL or both
g likes AFL or soccer but not both?



EXAMPLE 2

The two-way table shows the data collected from a survey of Year 9 students.

	Left-handed	Right-handed	
Male	2	14	16
Female	1	13	14
	3	27	30

What is the probability that a student chosen at random from this class is:

a female?

b left-handed?

c not left-handed?

d male and right-handed?

e male or right-handed or both?

f male or right-handed but not both?

g neither male nor right-handed?

h right-handed but not male?

	Solve	Think	Apply
a	$P(\text{female}) = \frac{14}{30} = \frac{7}{15}$	Number of females $= 14$	Find the number in the appropriate row and column and calculate the probability.
b	$P(\text{left-handed}) = \frac{3}{30} = \frac{1}{10}$	Number of left-handed students $= 3$	
c	$P(\text{not left-handed}) = \frac{27}{30} = \frac{9}{10}$	Number not left-handed $=$ number right-handed students $= 27$	
d	$P(\text{male and right-handed}) = \frac{14}{30} = \frac{7}{15}$	Number who are male and right-handed $= 14$	
e	$P(\text{male or right-handed or both}) = \frac{29}{30}$	Number who are male or right-handed or both $= 2 + 14 + 13 = 29$	
f	$P(\text{male or right-handed but not both}) = \frac{15}{30} = \frac{1}{2}$	Number who are male or right-handed but not both $= 2 + 13 = 15$	
g	$P(\text{neither male nor right-handed}) = \frac{1}{30}$	Number who are neither male nor right-handed $=$ number who are female and left-handed $= 1$	
h	$P(\text{right-handed but not male}) = \frac{13}{30}$	Number who are right-handed but not male $=$ number of right-handed females $= 13$	

7 The information in the table was collected from a group of athletes. Calculate the probability that an athlete chosen at random from this group is:

	Heavy	Light	
Tall	8	9	17
Short	3	10	13
	11	19	30

a tall

b short

c heavy

d light

e short and light

f short and heavy

g tall and light

h tall and heavy

i tall or light or both

j tall or light but not both

k short or heavy or both

l short or heavy but not both

m neither short nor heavy

n neither tall nor light

o heavy but not short

p light but not tall.

8 The table shows the results of a survey of a group of students.

	Born in Australia	Born overseas	
Male	87	29	116
Female	98	16	114
	185	45	230

Calculate the probability that a student chosen at random from this group:

a is male

b is female

c was born overseas

d was born in Australia

e is male and was born overseas

f is female and was born in Australia

g is male or was born overseas or both

h is male or was born overseas but not both

i is neither male nor born overseas

j is neither female nor born in Australia

k was born in Australia but is not male

l is female but was not born in Australia.

9 The data in the table was collected from a survey of people attending a fitness class.

	Male	Female	
Overweight	17	10	27
Not overweight	31	42	73
	48	52	100

Calculate the probability that a person chosen at random from this group is:

a female and overweight

b male but not overweight

c male or overweight or both

d male or overweight but not both

e male and overweight

f female but not overweight

g female or overweight or both

h female or overweight but not both

i neither overweight nor female.

10 Students were surveyed about whether they played soccer or netball. The results are shown in the table.

	Soccer	Not soccer	
Netball	10	6	16
Not netball	5	7	12
	15	13	28

Calculate the probability that a student chosen at random from this group plays:

a soccer

b soccer and netball

c soccer or netball or both

d neither soccer nor netball

e soccer but not netball

f netball but not soccer.

Language in mathematics

When your number's up – on average

How long will you live? This is a tantalising question throughout our lives. For those planning finances for retirement, it assumes special significance as they attempt to allocate financial resources to last for the remainder of their days. For the individual there is no way of determining a precise answer, but for the male and female populations there are satisfactory ways of establishing accurate life expectancy.

The results are of fundamental importance for organisations such as insurance companies, when they calculate payments for commonly used retirement products. In addition, these results can be used by individuals to determine accurately their chances of living to or beyond any specified age.

% chance of reaching a designated age

		Designated age (males)									
		55	60	65	70	75	80	85	90	95	100
Current age	55	100	92	80	64	45	14	12	4	1	0
	60		100	87	69	49	29	13	4	1	0
	65			100	80	57	33	15	4	1	0
	70				100	71	42	19	6	1	0
	75					100	59	26	8	1	0
	80						100	44	13	3	0
	85							100	30	6	1
	90								100	19	2
	95									100	12
	100										100

The means for considering life expectancy in the future are essentially based on the assumption that, in a population, patterns for deaths of women and men established in the immediate past are unlikely to change substantially in the immediate future. So past records of death in mortality tables can be used to predict future patterns with a high degree of accuracy.

Examples of figures extracted from a particular mortality table for males are given above. It gives the probability, in percentage terms, of a male who has attained a certain age living to another designated age.

For example, a man who is currently 65 years of age has an 80% chance of reaching 70, a 57% chance of reaching 75, a 33% chance of reaching 80 and so on across the line in the table. Alternatively, from a random sample of 100 males aged 65 alive today, 80 could be expected to reach 70 years of age, 57 to reach 75 years, 33 to reach 80 years and so on. For organisations such as insurance companies that deal with large groups of retirees, mortality tables provide invaluable information for calculating such products as life annuities. By dealing with sufficiently large groups of people, insurance companies can remove some of the uncertainty associated with planning finances for retirement.

The following questions relate to this article.

1 If a man is currently 55 years old, what is the chance that he will reach the age of:

a 65 years? **b** 75 years? **c** 85 years? **d** 95 years?

2 The table tells us that a 55-year-old man has a 0% chance of reaching 100. But a 0% chance means an impossibility. Discuss.

3 If 1000 60-year-old males were selected at random, how many of them would you expect to reach the age of:

a 65 years? **b** 70 years? **c** 75 years? **d** 80 years?

Word maze

The maze contains the following words, not necessarily in the order given. Movement in any direction, except diagonally, is possible. Work your way through the maze and find these words:

coins chance dice event experiment
expectation likelihood outcome probability
sample space spinner tree diagram

	N	T	E	X	P	E	R	E	S	out →
	E	N	E	M	I	R	E	X	N	I
	V	T	L	I	K	E	N	P	C	O
	E	E	C	H	I	L	N	E	N	O
in →		D	I	O	S	P	I	C	T	I
	E	L	D	O	M	A	R	G	A	T
	S	P	S	E	O	U	T	A	I	D
	P	M	A	C	H	C	C	O	M	E
	A	P	R	N	A	Y	T	I	E	E
	C	E	O	B	A	B	I	L	T	R

Terms

chance complementary events conditional probability dependent event
experiment independent lattice diagram multistage event outcome
probability tree sample space sampling with replacement sampling without replacement
tree diagram two-way table Venn diagram

Check your skills

1 A 6-sided die is rolled 50 times and the results are shown in the table.

Result	1	2	3	4	5	6
Frequency	7	9	12	8	10	4

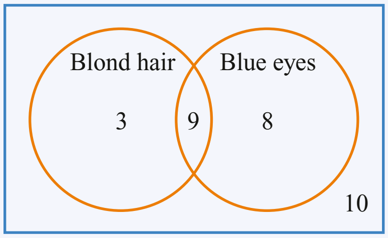
The relative frequency of the result 4 is:

A 8 **B** $\frac{8}{50}$ **C** 6 **D** $\frac{6}{21}$

2 The Venn diagram represents the number of students in a class who have blond hair and blue eyes. The probability that a student chosen at random from this class has both blond hair and blue eyes is:

A $\frac{2}{3}$ **B** $\frac{11}{30}$

C $\frac{6}{15}$ **D** $\frac{3}{10}$



3 The table shows the number of males and females in a class who were born in Australia and overseas. The probability of choosing a male who was born overseas from this class is:

	Born in Australia	Born overseas	
Male	9	5	14
Female	11	2	13
	20	7	27

A $\frac{5}{27}$ **B** $\frac{5}{7}$

C $\frac{5}{14}$ **D** $\frac{9}{27}$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2, 3
Section	A	B

7A Review set

1 One hundred batteries were selected at random from a production line and tested. The results are shown in the table.

a What is the relative frequency of the result 20–29 hours?

b Estimate the probability that a battery chosen at random from this manufacturer will last less than 10 hours.

Battery life (h)	Frequency
<10	3
10–19	21
20–29	60
30–49	15
40–49	1

2 The Venn diagram represents the number of students in a class who have black hair and brown eyes. What is the probability that a student chosen at random from this class will have:

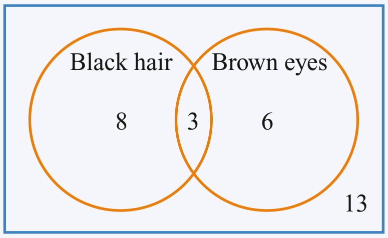
a black hair and brown eyes?

b black hair or brown eyes but not both?

c black hair or brown eyes or both?

d neither black hair nor brown eyes?

e black hair but not brown eyes?



3 The table shows the number of students in a class who study history and art.

	Study art	Do not study art	
Study history	7	11	18
Do not study history	6	5	11
	13	16	29

Find the probability that a student chosen at random from this class will study:

a history or art or both

b history or art but not both

c art and history

d neither history nor art

e art but not history.

7B Review set

1 Three groups of Year 9 students were surveyed as to whether they liked the food from the school canteen. The results are shown in the table.

	Like	Do not like
Group 1	6	4
Group 2	17	13
Group 3	55	45

a Write the relative frequency of the event ‘approve of the food from the canteen’ for each group.

b Which survey is the most reliable in estimating the probability that a student from Year 9, chosen at random, approves of the food from the school canteen?

2 The Venn diagram represents the number of students in a class who study physics and chemistry. What is the probability that a student chosen from this class studies:

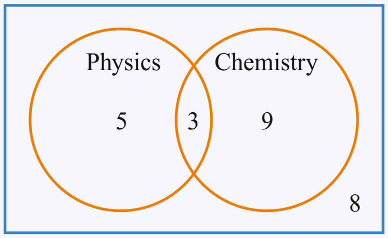
a both physics and chemistry?

b physics or chemistry but not both?

c physics or chemistry or both?

d neither physics nor chemistry?

e physics but not chemistry?



3 The table shows the results of a survey of Year 9 students.

	Male	Female	
Overweight	17	10	27
Not overweight	31	42	73
	48	52	100

Find the probability that a student chosen at random will be:

a overweight

b male and overweight

c female and not overweight

d female or overweight or both

e female or overweight but not both

f neither female nor overweight

g overweight but not male.

7C Review set

1 A card was selected from a normal playing pack and its value noted. This was repeated 150 times. (An ace was counted as 1 and the picture cards as 10.) The results are shown in the table.

Value	Frequency
<6	49
$\geqslant 6$	101

a What is the relative frequency of the result <6?

b Estimate the probability that the value of the card chosen is $\geqslant 6$.

2 The Venn diagram represents the method used by students in a class to travel to school. What is the probability that a student travels by:

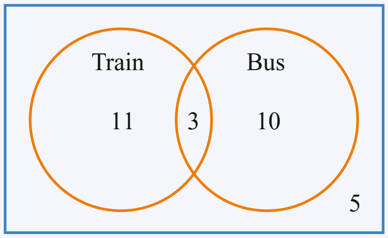
a both train and bus?

b train or bus but not both?

c train or bus or both?

d neither train nor bus?

e train but not bus?



3 The table shows the results of a Year 9 class in the term tests.

	Passed Mathematics	Did not pass Mathematics	
Passed English	24	3	27
Did not pass English	2	1	3
	26	4	30

Find the probability that a student chosen at random:

a passed Mathematics

b passed Mathematics and English

c passed Mathematics or English or both

d passed Mathematics or English but not both

e failed both subjects

f passed Mathematics but not English

g passed English but not Mathematics.

7D Review set

1 A drawing pin and a coin are tossed 100 times. The results are shown in the table.

a What is the relative frequency of the event pin up and tails?

b Estimate the probability that in another toss of the pin and the coin the result will be pin down and heads?

c How could we obtain a better estimate for the probability in part **b**?

Event	Frequency
Pin up and H	13
Pin up and T	17
Pin down and H	32
Pin down and T	38

2 The Venn diagram represents data collected from a group of people at a cafe about whether they drank tea or coffee. What is the probability that a person chosen at random from this group drinks:

a coffee?

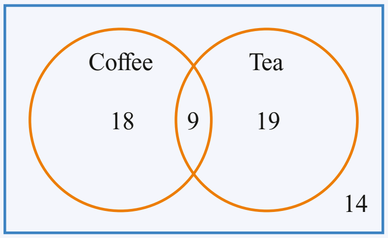
b coffee but not tea?

c both tea and coffee?

d coffee or tea or both?

e coffee or tea but not both?

f neither coffee nor tea?



3 The table shows the results of a survey of the newspaper read by people while travelling to work on the train.

	Read the *Herald*	Do not read the *Herald*	
Read the *Telegraph*	2	19	21
Do not read the *Telegraph*	15	4	19
	17	23	40

Find the probability that a person chosen at random from this group reads:

a the *Herald*

b the *Herald* but not the *Telegraph*

c the *Telegraph* but not the *Herald*

d both the *Herald* and the *Telegraph*

e the *Herald* or the *Telegraph* or both

f the *Herald* or the *Telegraph* but not both

g neither the *Herald* nor the *Telegraph*.

Chapters 5–7 Cumulative review

1 **a** Convert a salary of $67 000 p.a. to the equivalent salary per:

i week **ii** fortnight **iii** month.

b Holly earns $506 per week. How much is this per month?

c Aaron works a 38-hour week and is paid $17.10 per hour. How much does he earn for a week in which he works an additional 5 hours at time-and-a-half and 4 hours at double-time?

d Bert earns $820 per week. He is entitled to 4 weeks annual leave and receives an additional holiday loading of 17.5%. Calculate his total pay for this holiday period.

e Chloe earns $0.83 for each part she builds in a factory that produces electrical appliances. Given that she can finish 14 parts per hour and she works 7 hours per day for 5 days, calculate her average weekly earnings.

f Krystina sells mobile phone plans. She is paid a retainer of $240 per week plus a commission of 5% of sales. How much does she earn in a week in which her sales are $8200?

g Julie works as a casual in a cafe. She is paid $12.55 for any hours worked from Monday to Friday, $15.85 per hour for Saturdays and $17.98 for Sundays. Calculate how much she earns for a week in which she works 8 hours between Monday and Friday, 6 hours on Saturday and 5 hours on Sunday.

h Matt's gross weekly income is $795 per week. The deductions from his salary each week are tax $101, superannuation $32.81 and health insurance $28.26. He also deposits $200 per week into a special savings account and has $10 per week donated directly to a charity. Calculate his take-home pay each week.

i Calculate the simple interest on $12 600 invested at 7% p.a. for:

i 4 years **ii** 16 months.

j A store offers a discount of 14% for cash purchases. Find the cash price of a pair of sunglasses marked at $249.

k A car costing $20 999 can be bought on the following terms: deposit $7000, the balance to be repaid over 4 years by 48 equal monthly repayments. Simple interest is charged on the balance at 12% p.a.

- **i** Calculate the balance owing.
- **ii** Calculate the interest charged on the balance owing.
- **iii** What is the monthly repayment?

l A computer is advertised as shown.

- **i** Calculate the total amount you would have to pay for the computer under this scheme.
- **ii** Calculate the monthly instalments.

$3599

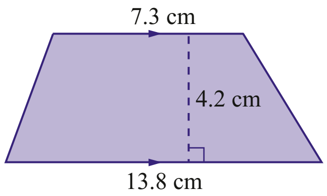
NO DEPOSIT
NO REPAYMENTS
FOR 12 MONTHS
(Conditions apply.)

Conditions:
1 Pay nothing for 12 months.
2 Balance plus interest to be repaid in equal monthly instalments over the 2 years following the repayment-free period.
3 Simple interest of 15% p.a. is charged for the 3-year period of the agreement.
4 Establishment fee of $110.

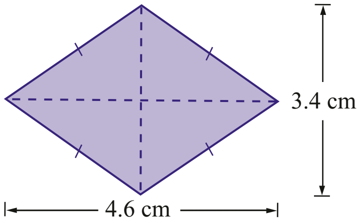
2 **a** For which quadrilateral is the formula $A = \frac{1}{2}h(a + b)$ used to find the area?

b Find the areas of these quadrilaterals.

i



ii



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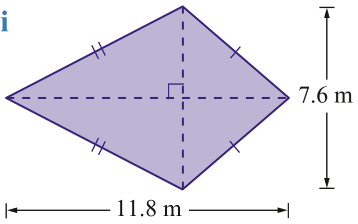
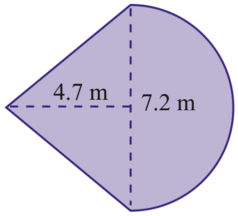


c Find the areas of these shapes.

i



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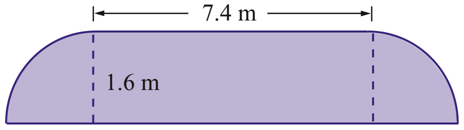
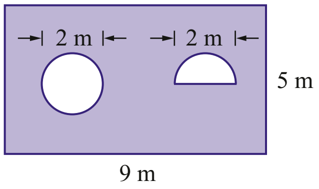


d Find the shaded areas of these figures.

i



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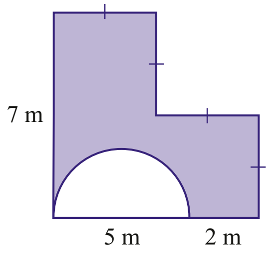


3 **a** A ball is selected at random from a bag containing 7 red, 5 blue and 2 orange balls. Determine the probability of selecting:

i a red ball
ii a blue ball
iii a blue ball or an orange ball
iv a non-red ball
v a non-black ball
vi a green ball.

b Two dice are thrown. Find the probability of obtaining:

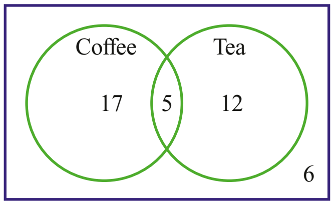
i a 3 and a 5
ii a 2 and a 3
iii a double 2
iv any double.

c This table represents data collected from 100 Year 9 students.

	Short	**Tall**	
Glasses	33	27	60
No glasses	30	10	40
	63	37	100

i Find the probability of selecting a tall person who wears glasses.
ii Find the probability of selecting a short person who does not wear glasses.
iii What type of person has a $\frac{1}{10}$ chance of being selected?

d The Venn diagram shows the number of students surveyed who drank tea or coffee. What is the probability that a person chosen at random from this group drinks:



i coffee?
ii coffee but not tea?
iii both tea and coffee?
iv coffee but not tea or both?

8

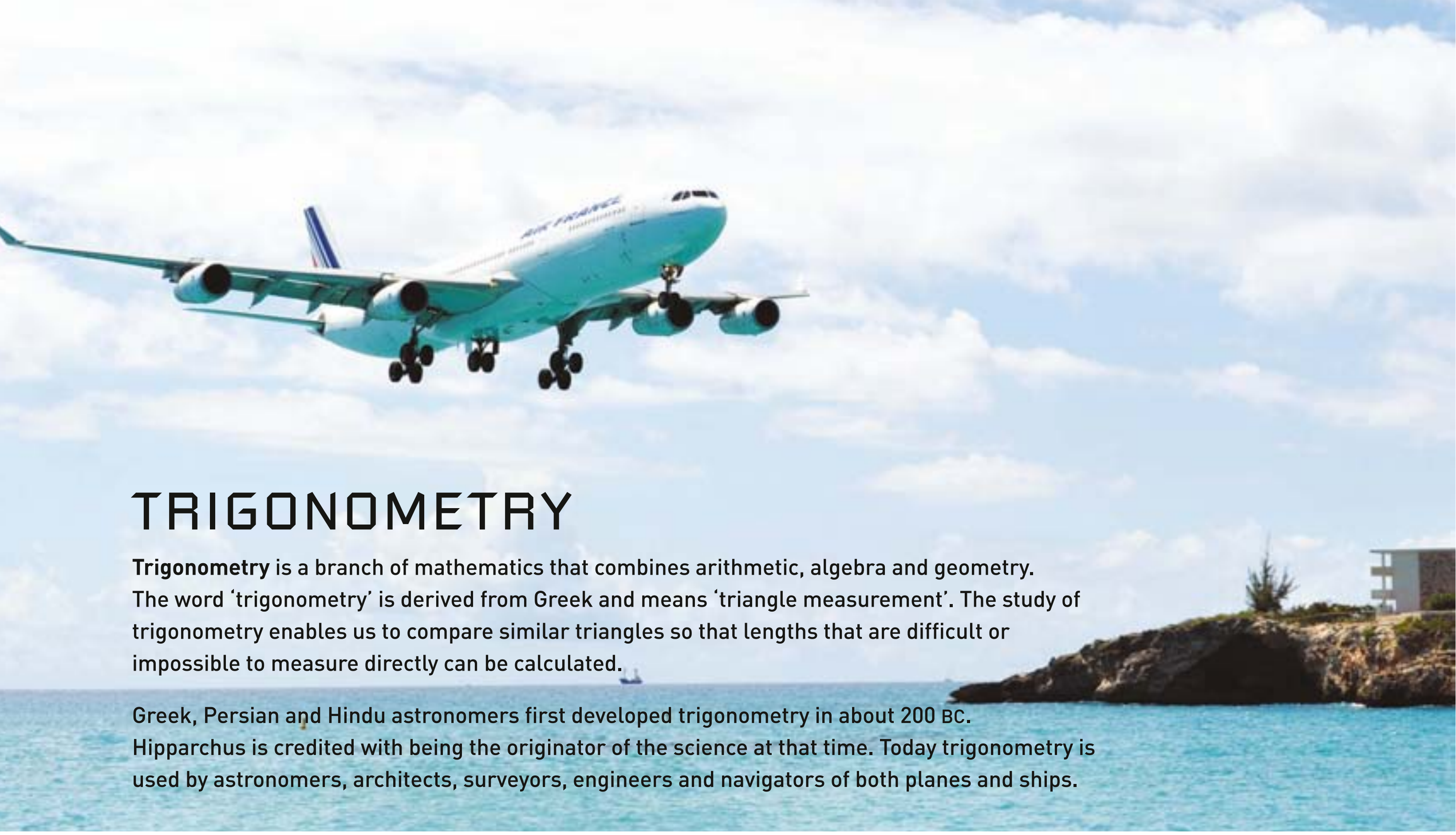
Right-angled trigonometry

This chapter deals with the solution of right-angled triangles.

After completing this chapter you should be able to:

- use Pythagoras' rule
- identify and label sides of a right-angled triangle
- define sine, cosine and tangent ratios
- use a calculator to find trigonometric ratios and angles
- use trigonometry to find unknown sides in right-angled triangles
- use trigonometry to find unknown angles in right-angled triangles
- solve worded trigonometric problems.

NSW syllabus references: 5.1 M&G Right-angled triangles (trigonometry)
Outcomes: MA5.1-1WM, MA5.1-2WM, MA5.1-3WM, MA5.1-10MG
MEASUREMENT & GEOMETRY – ACMMG222, ACMMG223, ACMMG224, ACMMG245



TRIGONOMETRY

Trigonometry is a branch of mathematics that combines arithmetic, algebra and geometry. The word 'trigonometry' is derived from Greek and means 'triangle measurement'. The study of trigonometry enables us to compare similar triangles so that lengths that are difficult or impossible to measure directly can be calculated.

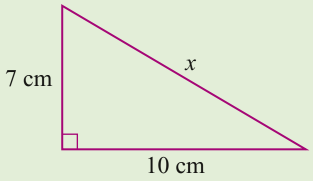
Greek, Persian and Hindu astronomers first developed trigonometry in about 200 BC. Hipparchus is credited with being the originator of the science at that time. Today trigonometry is used by astronomers, architects, surveyors, engineers and navigators of both planes and ships.

A Review of Pythagoras' theorem

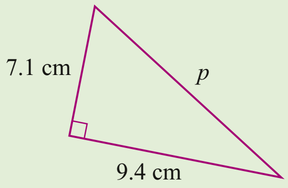
EXAMPLE 1

Use Pythagoras' theorem to find the length of the hypotenuse in each triangle.

a



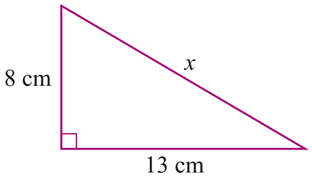
b



	Solve	Think	Apply
a	$c^2 = a^2 + b^2$ $x^2 = 7^2 + 10^2$ $= 149$ $x = \sqrt{149} = 12.2\ldots$ ≈ 12 cm	The two shorter sides are 7 cm and 10 cm. So $x^2 = 7^2 + 10^2$. Solve the equation then find the square root of 149. The hypotenuse is 12 cm to the nearest whole number.	Identify the hypotenuse and apply Pythagoras' rule. Solve the resulting equation for the unknown. Give the answer to the same number of decimal places as used in the question. Finding the hypotenuse involves addition.
b	$c^2 = a^2 + b^2$ $p^2 = 7.1^2 + 9.4^2$ $= 138.77$ $p = \sqrt{138.77} = 11.78\ldots$ ≈ 11.8 cm	The two shorter sides are 7.1 cm and 9.4 cm. So $x^2 = 7.1^2 + 9.4^2$. Solve the equation then find the square root of 138.77. The hypotenuse is 11.8 cm correct to 1 decimal place.	

Exercise 8A

1 Use Pythagoras' theorem to find the length of the hypotenuse in this triangle.



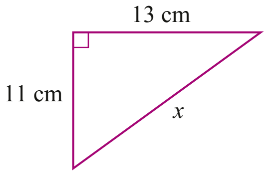
$c^2 = a^2 + b^2$

$x^2 = \_\_\_^2 + \_\_\_^2 = \_\_\_$

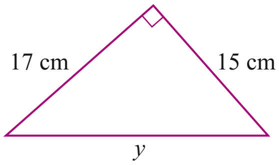
$x = \sqrt{233} \approx$ ___ cm correct to 1 decimal place

2 Find the hypotenuse of each triangle to the nearest whole number.

a



b



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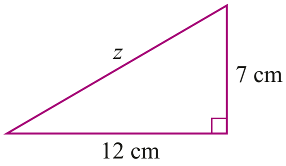
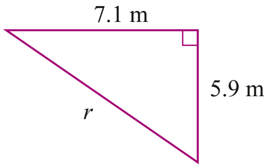


3 Find the hypotenuse of each triangle correct to 1 decimal place.

a



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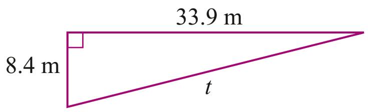

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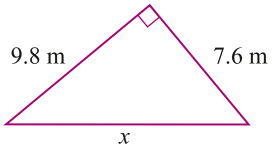
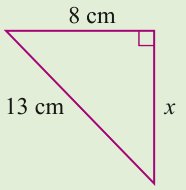


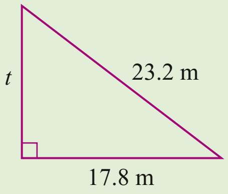
EXAMPLE 2

Calculate the value of the pronumeral and hence find the length of the unknown side in each triangle.

a

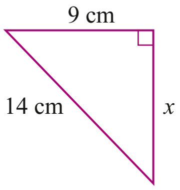


b



	Solve	Think	Apply
a	$a^2 = c^2 - b^2$ $x^2 = 13^2 - 8^2$ $= 105$ $x = \sqrt{105}$ ≈ 10 cm	The hypotenuse is 13 cm. The other two sides are x cm and 8 cm. $x^2 + 8^2 = 13^2$ so $x^2 = 13^2 - 8^2$. Solve the equation then find the square root of 105. The length of the unknown side is 10 cm to the nearest whole number.	Identify the hypotenuse and apply Pythagoras' rule. Solve the resulting equation for the unknown. Answer to the same number of decimal places as used in the question. Finding one of the shorter sides involves subtraction.
b	$a^2 = c^2 - b^2$ $t^2 = 23.2^2 - 17.8^2$ $= 221.4$ $t = \sqrt{221.4}$ ≈ 14.9 m	The hypotenuse is 23.2 m. The other two sides are t m and 17.8 m. So $t^2 = 23.2^2 - 17.8^2$. Solve the equation then find the square root of 221.4. The length of the unknown side is 14.9 cm correct to 1 decimal place.	

4 Calculate the value of the pronumeral and hence find the length of the unknown side in this triangle.



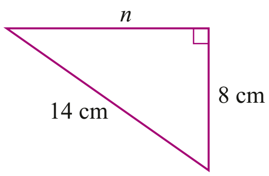
$c^2 = a^2 + b^2$

$x^2 = \_\_\_^2 - 9^2 = \_\_\_$

$x = \sqrt{\_\_\_} \approx$ ___ cm correct to 1 decimal place

5 Calculate the length of the unknown side in each triangle.

a



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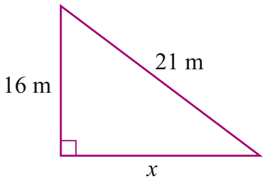

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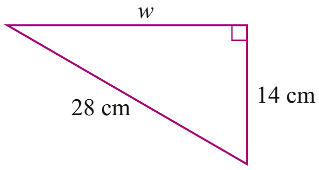

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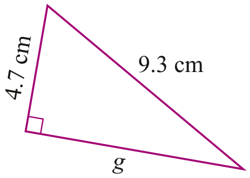
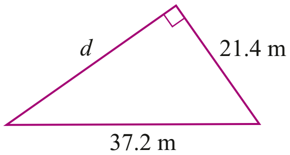
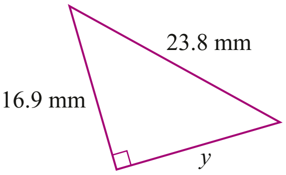


e



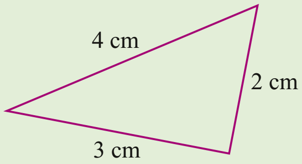
f



EXAMPLE 3

Determine if the following triangles are right-angled.

a



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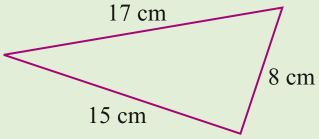


	Solve	Think	Apply
a	$c^2 = 4^2 = 16$ $a^2 + b^2 = 3^2 + 2^2$ $= 13$ $c^2 \neq a^2 + b^2$ Triangle is not right-angled.	The longest side is 4 cm. Calculate the square of the longest side. Calculate the sum of the squares of the other two sides. 16 ≠ 13, so the triangle is not right-angled.	In a right-angled triangle, the longest side is the hypotenuse. If the square of the longest side is equal to the sum of the squares of the other two sides, the triangle is right-angled. If the square of the longest side is *not* equal to the sum of the squares of the other two sides, the triangle is not right-angled.
b	$c^2 = 17^2 = 289$ $a^2 + b^2 = 15^2 + 8^2$ $= 289$ $c^2 = a^2 + b^2$ Triangle is right-angled.	The longest side is 17 cm. Calculate the square of the longest side. Calculate the sum of the squares of the other two sides. 289 = 289, so the triangle is right-angled.	

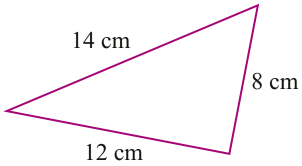
6 Determine if this triangle is right-angled.

$c^2 = 14^2 = \_\_\_$

$a^2 + b^2 = 8^2 + \_\_\_^2 = \_\_\_$

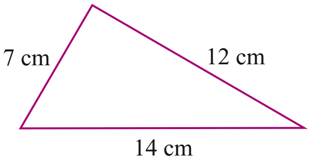
$c^2 \_\_\_ a^2 + b^2$

Triangle ___ right-angled.



7 Determine if following triangles are right-angled.

a



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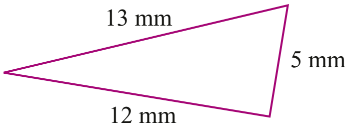

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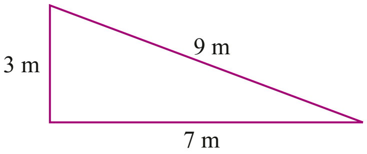

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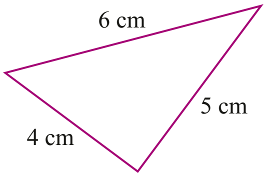

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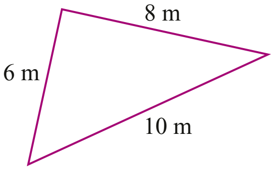

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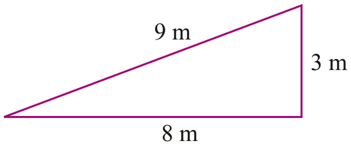
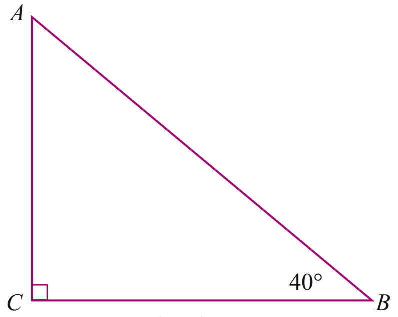
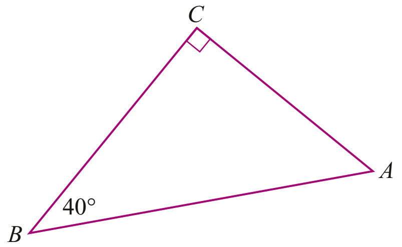


Investigation 1 Ratios of sides

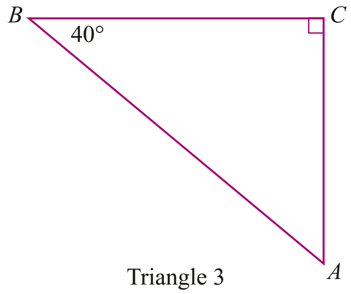
1 Here are three right-angled triangles. They have the same angles.



Triangle 1



Triangle 2



Triangle 3

a Measure the lengths of all sides to the nearest millimetre.

b Complete this table.

Triangle	BC	AC	AB	$\frac{AC}{BC}$	$\frac{AC}{AB}$	$\frac{BC}{AB}$
1						
2						
3						

c Compare your answers in the last three columns. What do you notice?

2 **a** Draw a triangle ABC with base AC of length 6 cm, angle A of 30° and angle C of 90°.

b Draw a second triangle ABC with AC of length 10 cm and angles A and C as before.

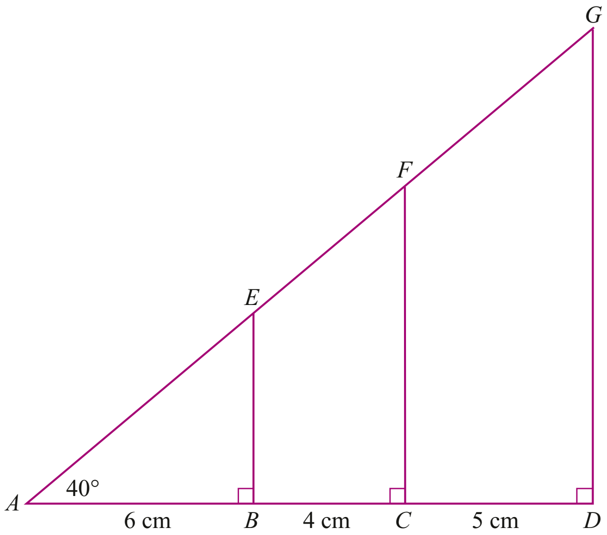
c Draw a third triangle ABC with AC of a length of your choosing and again angles A and C as before.

d Clearly, the triangles have the same angles. Measure the unknown sides and complete this table.

Triangle	AC	BC	AB	$\frac{BC}{AC}$	$\frac{BC}{AB}$	$\frac{AC}{AB}$
a	6 cm					
b	10 cm					
c						

e What do you notice about the answers in the last two columns?

3 **a** Draw this diagram to scale.



b Measure the unknown sides to the nearest millimetre, and complete this table.

△*ABE*	$AB =$	$EB =$	$AE =$	$\frac{EB}{AB} =$	$\frac{EB}{AE} =$	$\frac{AB}{AE} =$
△*ACF*	$AC =$	$FC =$	$AF =$	$\frac{FC}{AC} =$	$\frac{FC}{AF} =$	$\frac{AC}{AF} =$
△*ADG*	$AD =$	$GD =$	$AG =$	$\frac{GD}{AD} =$	$\frac{GD}{AG} =$	$\frac{AD}{AG} =$

c What do you notice about the answers in the last three columns?

4 Write a paragraph describing your results in this investigation.

B Defining trigonometric ratios

Naming sides

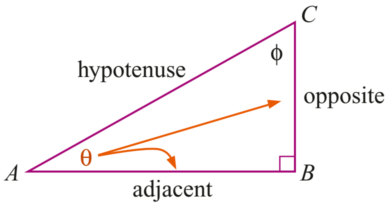
Recall that, for convenience, we give special names to the sides of a right-angled triangle. The side opposite the right angle is the **hypotenuse**, as used in Section 8A. It is the longest side.

Theta (θ) and phi (ϕ) are Greek letters often used for unknown angles.

In this triangle the hypotenuse is *AC*.

- The side *CB* is **opposite** angle *A* (marked θ).
- The side *AB* is **adjacent** or next to angle *A* (marked θ).

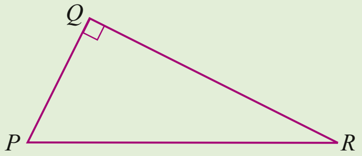
If we look at angle *C* (marked ϕ), *AB* is now the opposite side, and *CB* is the adjacent side.



EXAMPLE 1

In this triangle, name the:

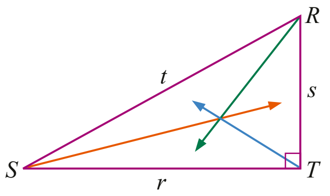
a hypotenuse
b side opposite angle *P*
c side adjacent to angle *P*
d side opposite angle *R*
e side adjacent to angle *R*.



	Solve	Think	Apply
a	The hypotenuse is *PR*.	The hypotenuse is side *PR* opposite the right angle.	Identify the hypotenuse and then the opposite and adjacent sides. The opposite and adjacent sides are relative to the chosen angle.
b	*QR* is the side opposite angle *P*.	The side opposite angle *P* is *QR*.	
c	*PQ* is the side adjacent to angle *P*.	The adjacent side is next to angle *P* but is not the hypotenuse.	
d	*PQ* is the side opposite angle *R*.	The side opposite angle *R* is *PQ*.	
e	*QR* is the side adjacent to angle *R*.	The adjacent side is next to angle *R* but is not the hypotenuse.	

Remember how to name sides.

Label the side opposite $\angle T$ as t, $\angle R$ as r, and $\angle S$ as s.

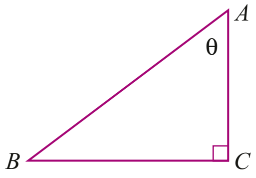


Exercise 8B

1 For each triangle below, name the:

- **i** hypotenuse
- **ii** side opposite the angle marked θ
- **iii** side adjacent to the angle marked θ.

a



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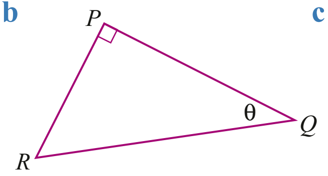

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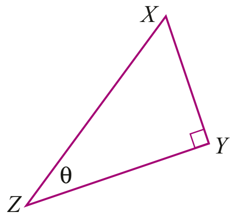

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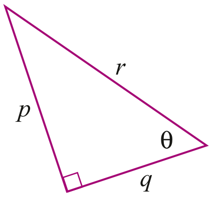

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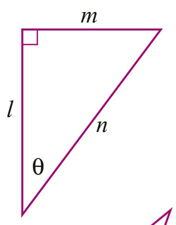

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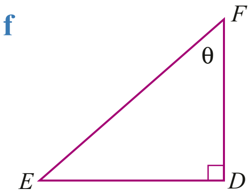

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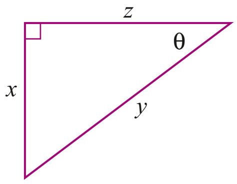

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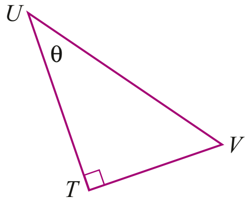

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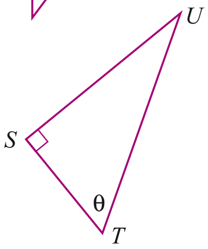

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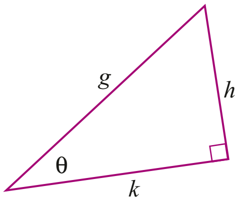
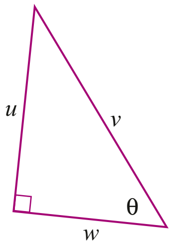


k



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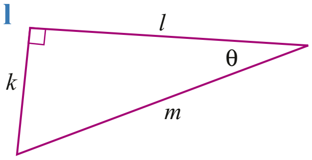
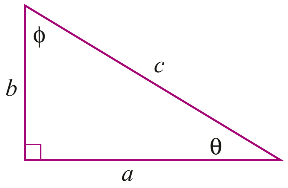


2 For the triangle shown, name the side:

a opposite the angle marked θ **b** opposite the angle marked ϕ

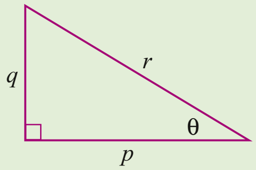
c adjacent to the angle marked θ **d** adjacent to the angle marked ϕ.



EXAMPLE 2

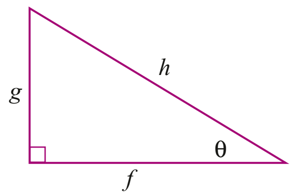
Using the given triangle, write expressions to complete the table for θ.

$\frac{\text{opposite}}{\text{adjacent}}$	$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$



<table>
<tr><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr><td><table><tr><th>$\frac{\text{opposite}}{\text{adjacent}}$</th><th>$\frac{\text{opposite}}{\text{hypotenuse}}$</th><th>$\frac{\text{adjacent}}{\text{hypotenuse}}$</th></tr><tr><td>$\frac{q}{p}$</td><td>$\frac{q}{r}$</td><td>$\frac{p}{r}$</td></tr></table></td><td>The hypotenuse is r, the side opposite the angle marked θ is q, and the side adjacent to θ is p.</td><td>The opposite and the adjacent sides are relative to the non-right angle chosen.</td></tr>
</table>

3 Using the given triangle, write expressions to complete the table for θ.



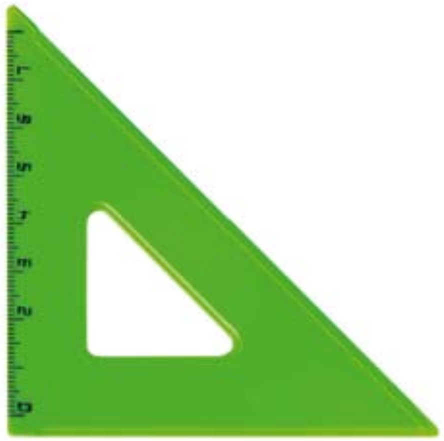
$\frac{\text{opposite}}{\text{adjacent}}$	$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$
$\frac{\square}{f}$	$\frac{g}{\square}$	$\frac{f}{\square}$

4 Complete this table for θ for each of the triangles in question **1**.

$\frac{\text{opposite}}{\text{adjacent}}$	$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$

The trigonometric ratios

The ratios from Example 2 are given names.



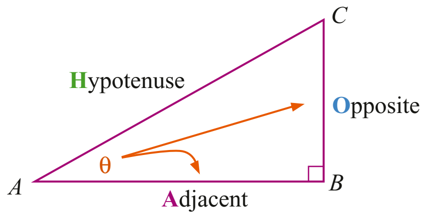
- The ratio $\frac{\text{opposite}}{\text{adjacent}}$ is the **tangent** of the angle marked θ.
  This is written as $\textbf{tan } \theta = \frac{\textbf{opposite}}{\textbf{adjacent}}$.
- The ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ is the **sine** of the angle marked θ.
  This is written as $\textbf{sin } \theta = \frac{\textbf{opposite}}{\textbf{hypotenuse}}$.
- The ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ is the **cosine** of the angle marked θ.
  This is written as $\textbf{cos } \theta = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$.

The trigonometric ratios can be remembered using a mnemonic: SOH CAH TOA.

SOH $\quad \text{Sin } \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

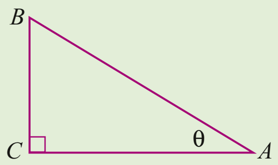
CAH $\quad \text{Cos } \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA $\quad \text{Tan } \theta = \frac{\text{Opposite}}{\text{Adjacent}}$



EXAMPLE 3

In triangle *ABC*, find expressions for tan θ, cos θ, and sin θ.



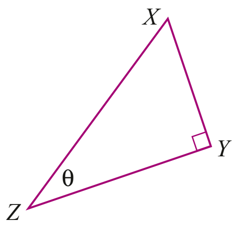
Solve	Think	Apply
$\tan\theta = \frac{BC}{AC}$ $\sin\theta = \frac{BC}{AB}$ $\cos\theta = \frac{AC}{AB}$	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	Locate the hypotenuse opposite the right angle. Identify the opposite and adjacent sides relative to the chosen angle.

5 In triangle *XYZ*, find expressions for tan θ, cos θ and sin θ.

$\tan\theta = \frac{\square}{YZ}$

$\sin\theta = \frac{XY}{\square}$

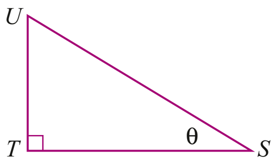
$\cos\theta = \frac{\square}{\square}$



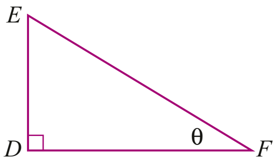
6 For each triangle, find an expression for:

i tan θ **ii** sin θ **iii** cos θ

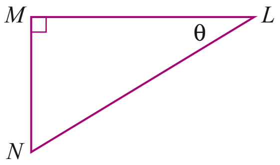
a



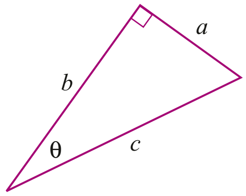
b



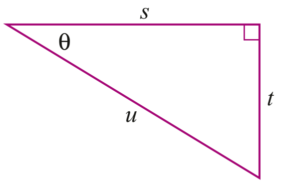
c



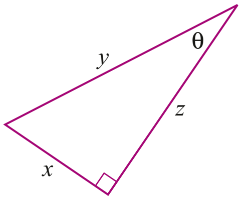
d



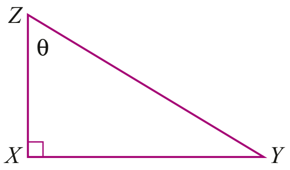
e



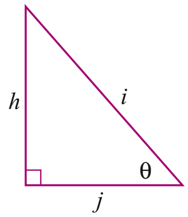
f



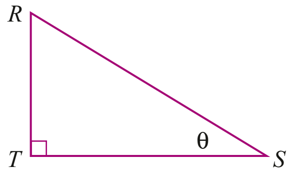
g



h



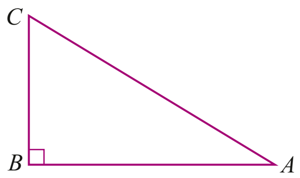
i



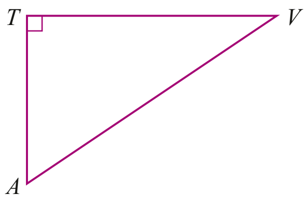
7 For each triangle, find an expression for:

i sin *A* **ii** cos *A* **iii** tan *A*

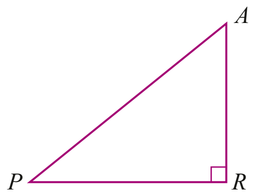
a



b



c



Investigation 2 Comparing ratios

You can find trigonometric ratios by measurement, as in Investigation 1, or by using the tan, sin and cos keys on a calculator.

With DAL calculators you enter the trig. ratio first. With some calculators you enter the angle first.

1 Use a calculator to find and verify these trigonometric ratios. Make sure it is in **degree mode** first.

a tan 25° ≈ 0.4663 tan 25 =

b sin 25° ≈ 0.4226 sin 25 =

c cos 25° ≈ 0.9063 cos 25 =

d tan 62° ≈ 1.8807 tan 62 =

e sin 62° ≈ 0.8829 sin 62 =

f cos 62° ≈ 0.4695 cos 62 =

2 Use your calculator to complete the table, giving values correct to 4 decimal places.

θ	**sin θ**	**cos θ**	**tan θ**	$\dfrac{\sin\theta}{\cos\theta}$	$\dfrac{\cos\theta}{\sin\theta}$
0°					
10°					
20°					
30°					
40°					
50°					
60°					
70°					
80°					
90°					

a What do you notice about the answers in the tan θ column compared with the $\dfrac{\sin\theta}{\cos\theta}$ column?

b Based on your answer to part **a**, complete: $\dfrac{\sin\theta}{\cos\theta} = \_\_\_\_$.

c From the table above, between which values do the following trigonometric ratios lie for angles from 0° to 90°?

i sin θ **ii** cos θ **iii** tan θ

C Trigonometric ratios of acute angles

This section introduces calculations involving angles. All angles are measured in degrees.

If two sides of a right-angled triangle are known, then one of the trigonometric ratios can be used to find the missing angles.

EXAMPLE 1

Find the following correct to 4 decimal places.

a cos 84° **b** sin 68° **c** tan 15°

	Solve	Think	Apply
a	cos 84° ≈ 0.1045	cos 84 =	Make sure your calculator is in degree mode.
b	sin 68° ≈ 0.9272	sin 68 =	
c	tan 15° ≈ 0.2679	tan 15 =	

Exercise 8C

1 Find the following correct to 4 decimal places.

a sin 28°
b cos 26°
c tan 11°
d cos 67°
e cos 46°
f tan 74°
g tan 77°
h sin 42°
i cos 63°
j cos 27°
k tan 35°
l sin 9°

EXAMPLE 2

Evaluate the following correct to 4 decimal places.

a 6 cos 27° **b** 7 sin 84° **c** 9.2 tan 54°

	Solve	Think	Apply
a	6 cos 27° ≈ 5.3460	6 × cos 27 =	Make sure that the calculator is in degrees mode.
b	7 sin 84° ≈ 6.9617	7 × sin 84 =	
c	9.2 tan 54° ≈ 12.6627	9.2 × tan 54 =	

2 Evaluate the following correct to 4 decimal places.

a 7 cos 29°
b 3 tan 18°
c 22 sin 55°
d 18 sin 56°
e 9.7 tan 72°
f 7.3 cos 39°
g 5 sin 63°
h 7.3 cos 58°
i 11.1 tan 27°
j 53 cos 21°
k 4.1 tan 17°
l 6.3 cos 6°

Using trigonometric ratios to find angles

You can work backwards on a calculator to find an angle from one of the trigonometric ratios, by using one of the key combinations SHIFT tan or SHIFT sin or SHIFT cos. These may appear on your calculator display as **tan**$^{-1}$ or **sin**$^{-1}$ or **cos**$^{-1}$.

For example, if $\sin\theta = 0.4369$
then $\theta = \sin^{-1} 0.4369$
where $\sin^{-1} 0.4369$ means 'the angle whose sine is 0.4369'.

Similarly, $\cos^{-1}$ means 'the angle whose cosine is' and $\tan^{-1}$ means 'the angle whose tangent is'.

EXAMPLE 3

Find θ to the nearest degree.

a $\sin\theta = 0.6314$ **b** $\tan\theta = 3.6$ **c** $\cos\theta = 0.8$

	Solve	Think	Apply
a	$\sin\theta = 0.6314$ $\theta = 39.153\ldots$ $\approx 39°$	SHIFT sin 0.6314 =	Make sure your calculator is in degree mode. Ensure that SHIFT is pressed before the trigonometric ratios so that the answer is an angle. Round the answers as normal.
b	$\tan\theta = 3.6$ $\theta = 74.475\ldots$ $\approx 74°$	SHIFT tan 3.6 =	
c	$\cos\theta = 0.8$ $\theta = 36.869\ldots$ $\approx 37°$	SHIFT cos 0.8 =	

3 Write these calculator displays as angles correct to the nearest degree.

a 31.278… **b** 64.773… **c** 47.135…
d 16.887… **e** 14.334… **f** 17.819…
g 28.931… **h** 43.22… **i** 84.373…
j 33.052… **k** 26.807… **l** 0.938…

4 Find θ to the nearest degree.

a $\sin\theta = 0.3625$ **b** $\cos\theta = 0.1445$ **c** $\tan\theta = 2.1351$
d $\cos\theta = 0.6731$ **e** $\tan\theta = 4.1371$ **f** $\sin\theta = 0.1113$
g $\tan\theta = 0.0371$ **h** $\sin\theta = 0.5512$ **i** $\cos\theta = 0.0314$
j $\sin\theta = 0.0027$ **k** $\tan\theta = 23.7215$ **l** $\cos\theta = 0.9811$
m $\cos\theta = 0.6614$ **n** $\sin\theta = 0.6262$ **o** $\tan\theta = 0.2222$

EXAMPLE 4

Find θ to the nearest degree.

a $\sin\theta = \frac{5}{9}$ **b** $\cos\theta = \frac{6}{13}$ **c** $\tan\theta = \frac{18}{7}$

	Solve	Think	Apply
a	$\sin\theta = \frac{5}{9}$ $\theta = 33.74...$ $\approx 34°$	SHIFT sin (5 ÷ 9) =	Make sure that the calculator is in degree mode. Press SHIFT first to obtain an angle. Put the fraction in brackets before pressing = .
b	$\cos\theta = \frac{6}{13}$ $\theta = 62.51...$ $\approx 63°$	SHIFT cos (6 ÷ 13) =	
c	$\tan\theta = \frac{18}{7}$ $\theta = 68.74\ldots$ $\approx 69°$	SHIFT tan (18 ÷ 7) =	

5 Find θ to the nearest degree.

a $\tan\theta = \frac{14}{3}$ **b** $\cos\theta = \frac{3}{11}$ **c** $\sin\theta = \frac{11}{18}$ **d** $\sin\theta = \frac{4}{29}$

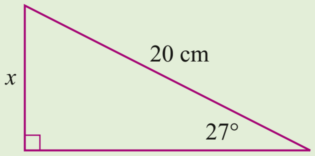
e $\tan\theta = \frac{6}{7}$ **f** $\cos\theta = \frac{14}{17}$ **g** $\sin\theta = \frac{0.013}{0.214}$ **h** $\cos\theta = \frac{6.2}{15}$

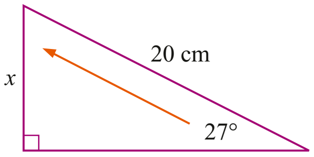
i $\tan\theta = \frac{11.27}{15}$ **j** $\cos\theta = \frac{1}{3}$ **k** $\sin\theta = \frac{3}{4}$ **l** $\tan\theta = \frac{4}{3}$

D Using trigonometry to find sides

EXAMPLE 1

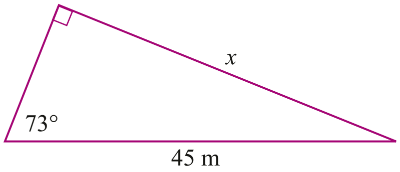
Use the sine ratio to find the value of x correct to 1 decimal place.



Solve	Think	Apply
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin 27° = \frac{x}{20}$ $\therefore x = 20 \sin 27°$ ≈ 9.1 cm	 20 × sin 27 =	x is opposite the given angle. The sine ratio is used when the opposite side and hypotenuse are the sides given. When finding the opposite side, multiply the hypotenuse by the sine of the angle.

Exercise 8D

1 Complete the following using the sine ratio to find the value of x correct to 1 decimal place.



$$\sin\theta = \frac{\underline{\hspace{1.5cm}}}{\text{hypotenuse}}$$

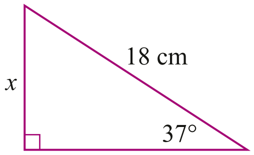
$$\sin \underline{\hspace{0.6cm}}° = \frac{x}{\square}$$

$$\therefore x = \underline{\hspace{0.6cm}} \sin 73°$$

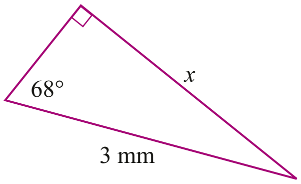
$$= \underline{\hspace{0.6cm}} = \underline{\hspace{0.6cm}} \text{ m (1 decimal place)}$$

2 Use the sine ratio to find the value of x correct to 1 decimal place.

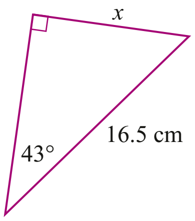
a



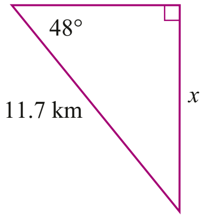
b



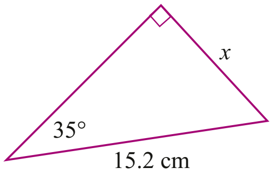
c



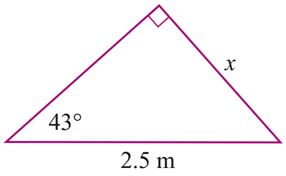
d



e

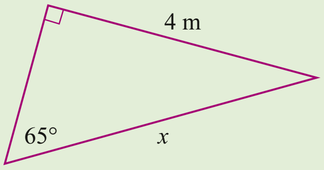


f



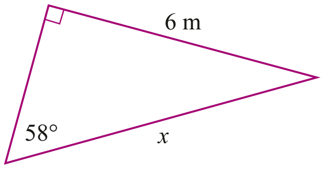
EXAMPLE 2

Use the sine ratio to find the length of the hypotenuse correct to 1 decimal place.



Solve	Think	Apply
$\sin 65° = \frac{4}{x}$ $x \sin 65° = 4$ $\therefore x = \frac{4}{\sin 65°}$ $= 4.413\ldots$ ≈ 4.4 m	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ x is the hypotenuse. 4 ÷ sin 65 =	When finding the hypotenuse, divide the opposite side by the sine of the angle.

3 Complete the following using the sine ratio to find the value of x correct to 1 decimal place.



$$\sin\theta = \frac{\text{opposite}}{\underline{\hspace{1.5cm}}}$$

$$\sin \underline{\hspace{0.6cm}}° = \frac{6}{\square}$$

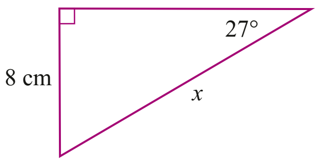
$$\therefore x = \frac{\square}{\sin \underline{\hspace{0.6cm}}°}$$

$$= \underline{\hspace{0.6cm}}$$

$$= \underline{\hspace{0.6cm}} \text{ m (1 decimal place)}$$

4 Use the sine ratio to find the length of the hypotenuse correct to 1 decimal place.

a



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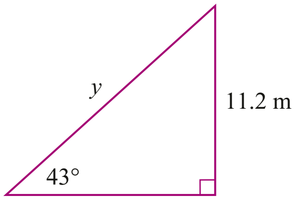

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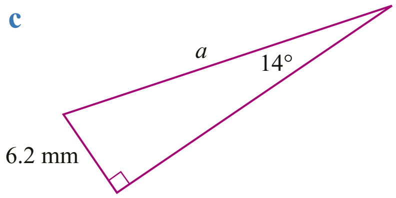

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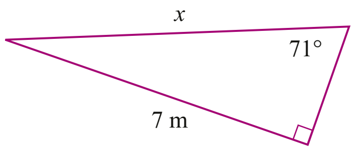
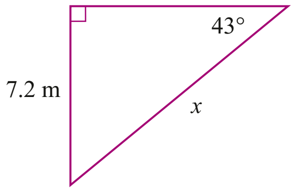


e



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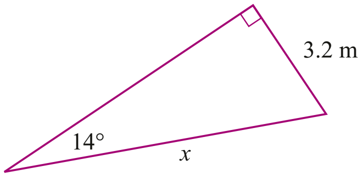
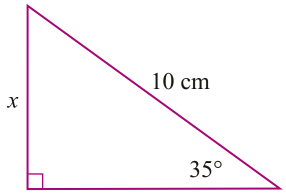


5 Find the unknown sides correct to 1 decimal place.

a



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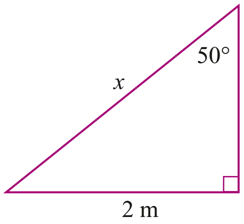

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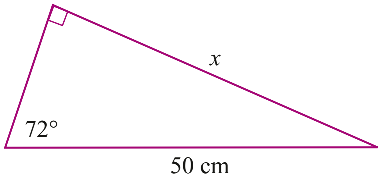
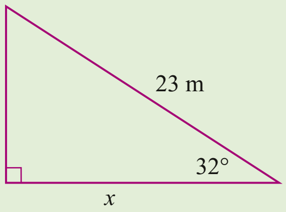
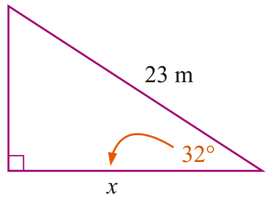


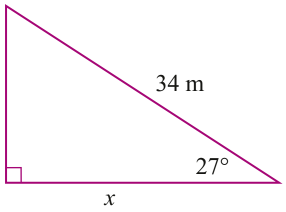
EXAMPLE 3

Use the cosine ratio to find the value of x correct to 1 decimal place.



Solve	Think	Apply
$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos 32° = \frac{x}{23}$ $\therefore x = 23 \cos 32°$ $= 19.505\ldots$ ≈ 19.5 m	x is adjacent to the given angle.  23 × cos 32 =	The cosine ratio is used when the adjacent side and hypotenuse are the sides given. As with sine, multiply when finding the adjacent side and divide by the cosine of the angle when finding the hypotenuse.

6 Use the cosine ratio to find the value of x correct to 1 decimal place.



$\cos\theta = \frac{\text{adjacent}}{\_\_\_\_}$

$\cos \_\_\_ = \frac{\square}{34}$

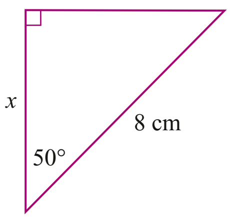
$\therefore x = \_\_\_ \cos \_\_\_°$

$= \_\_\_$

$= \_\_\_$ m (1 decimal place)

7 Use the cosine ratio to find the value of x correct to 1 decimal place.

a



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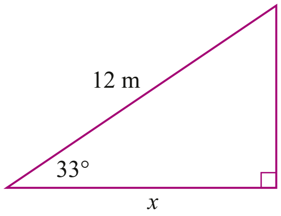

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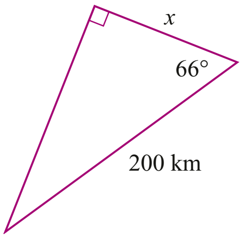

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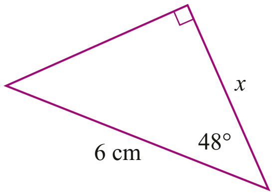

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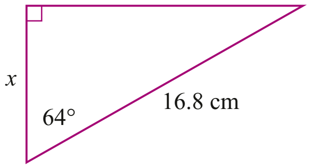

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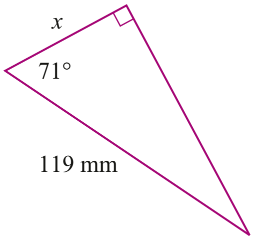
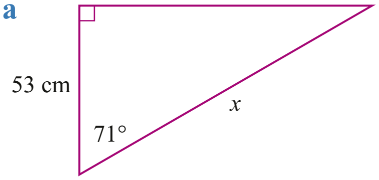


8 Use the cosine ratio to find the length of the hypotenuse correct to 1 decimal place.

a



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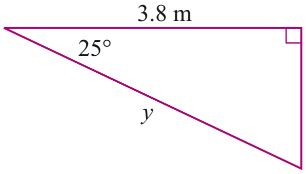

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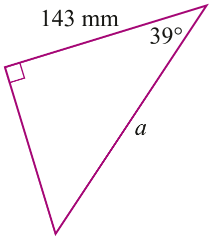

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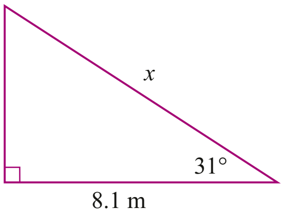
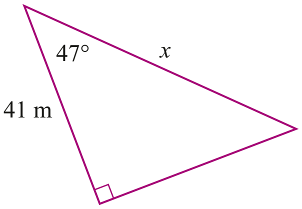


e



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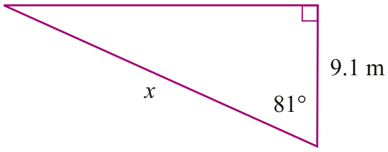
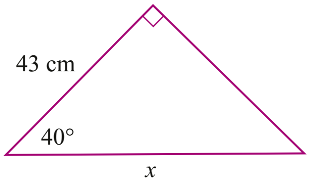


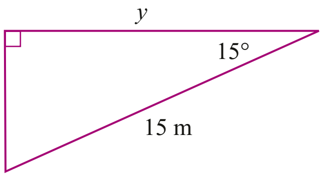
Remember to divide to find the hypotenuse.

9 Find the unknown sides correct to 1 decimal place.

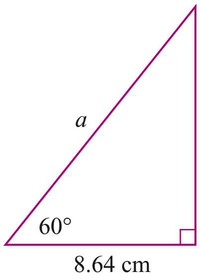
a



b

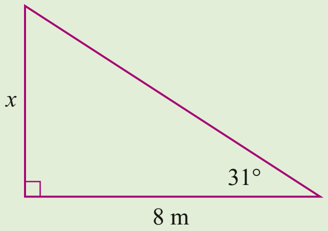


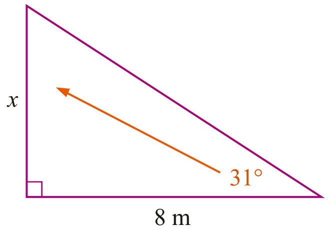
c



EXAMPLE 4

Use the tangent ratio to find the value of x correct to 1 decimal place.



Solve	Think	Apply
$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\tan 31° = \frac{x}{8}$ $\therefore x = 8\tan 31°$ $= 4.8069\ldots$ ≈ 4.8 m	x is opposite the given angle.  8 × tan 31 =	The tangent ratio is used when the hypotenuse is not given. Identify the opposite and adjacent sides. When finding the opposite side, multiply the side and the tangent of the angle. When finding the adjacent side, divide the opposite side by the tangent of the angle.

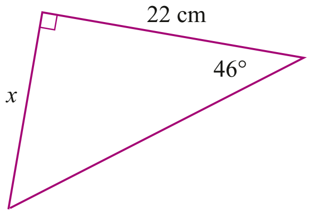
10 Use the tangent ratio to find the value of x correct to 1 decimal place.

$\tan\theta = \frac{\underline{\qquad}}{\text{adjacent}}$

$\tan 46° = \frac{x}{\square}$

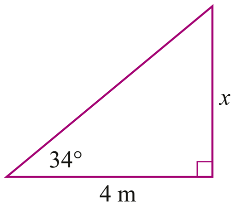
$\therefore x = \underline{\quad}\ \tan 46°$

$= \underline{\quad}$ m (1 decimal place)

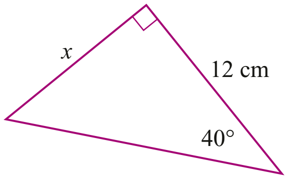


11 Use the tangent ratio to find the value of x correct to 1 decimal place.

a



b



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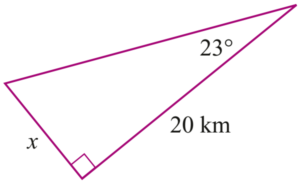

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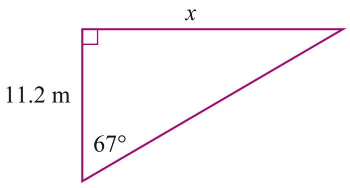
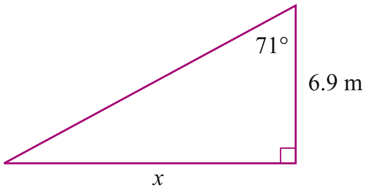


e



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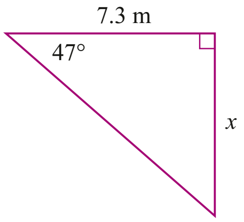
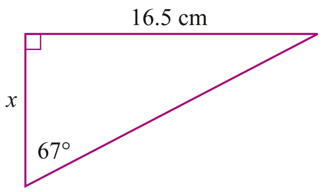


12 Use the tangent ratio to find the value of x correct to 1 decimal place.

a



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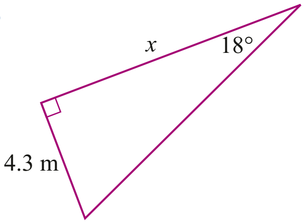

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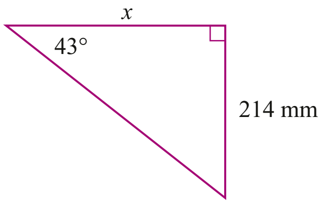
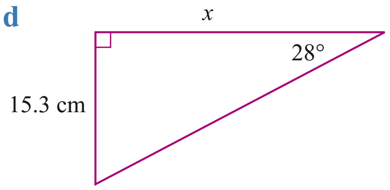


Remember to divide to find the adjacent side.

d



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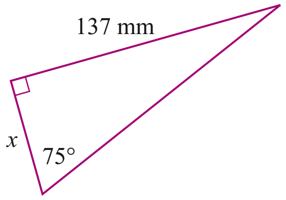

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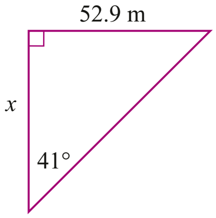
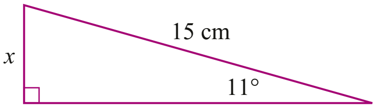


13 Use the sine, cosine or tangent ratios to find each unknown side correct to 1 decimal place.

a



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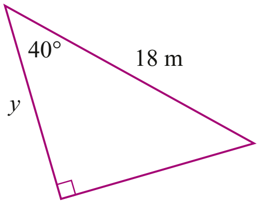

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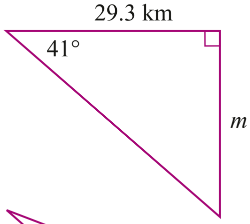

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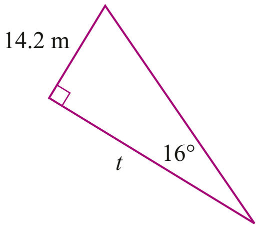

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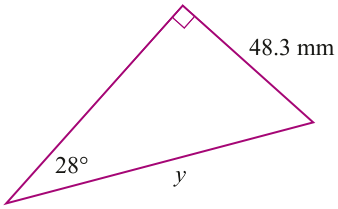

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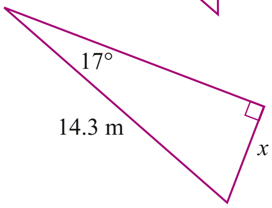

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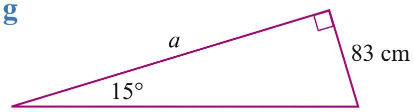

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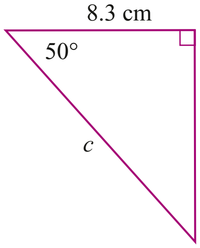
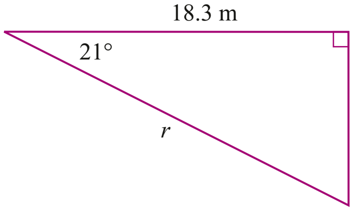


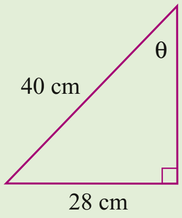
i

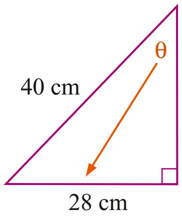


E Using trigonometry to find angles

EXAMPLE 1

Use the sine ratio to find θ to the nearest degree.



Solve	Think	Apply
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $= \dfrac{28}{40}$ $\therefore \theta = 44.4270\ldots$ $= 44°$	The side opposite θ and the hypotenuse are given.  SHIFT sin (28 ÷ 40) =	Press SHIFT before sin to obtain an angle.

Exercise 8E

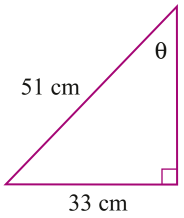
1 Use the sine ratio to find θ to the nearest degree.

$$\sin\theta = \frac{\text{opposite}}{\underline{\hspace{2em}}}$$

$$= \frac{\square}{51}$$

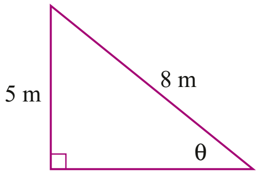
$$\therefore \theta = \underline{\hspace{2em}}\ldots$$

$$= \underline{\hspace{2em}}^\circ$$



2 Using the sine ratio, find θ to the nearest degree.

a



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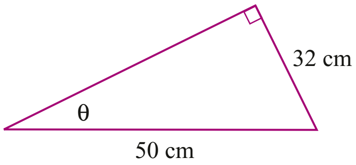

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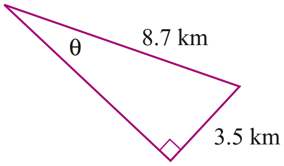

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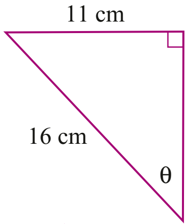

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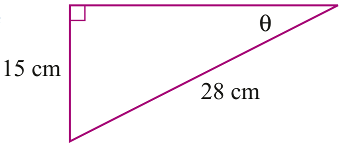

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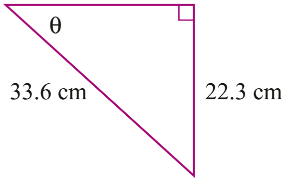

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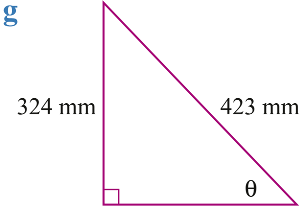

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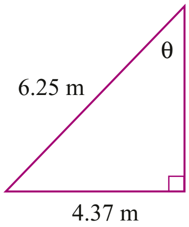

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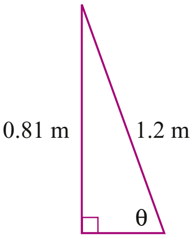
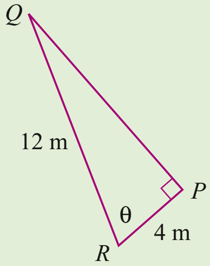
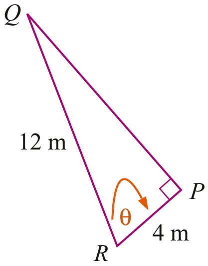


EXAMPLE 2

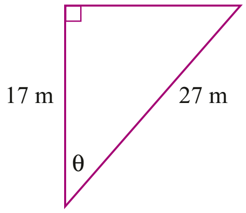
Use the cosine ratio to find θ correct to the nearest degree.



Solve	Think	Apply
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $= \dfrac{4}{12}$ $\therefore \theta = 70.5288\ldots$ $= 71^\circ$	The side adjacent to θ and the hypotenuse are given.  SHIFT cos (4 ÷ 12) =	Press SHIFT before cos to obtain an angle.

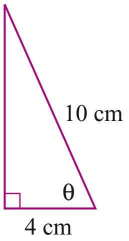
3 Use the cosine ratio to find θ to the nearest degree.

$$\cos \theta = \frac{\text{adjacent}}{\_\_\_\_\_\_}$$
$$= \frac{\square}{27}$$
$$\therefore \theta = \_\_\_\ldots$$
$$= \_\_\_^\circ$$

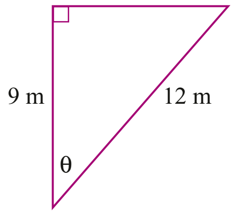


4 Using the cosine ratio, find θ to the nearest degree.

a



b



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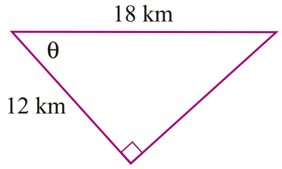
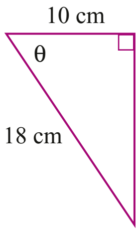
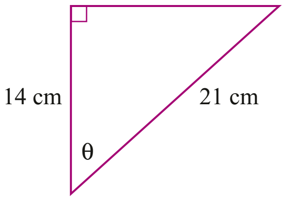


d



e



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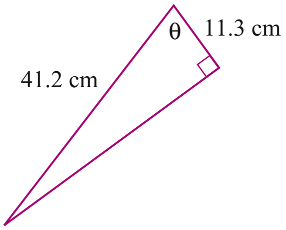

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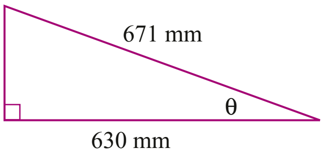

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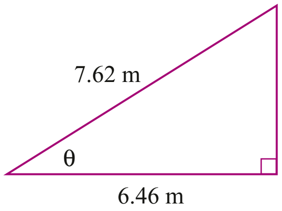
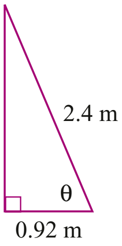
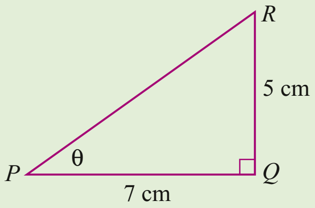


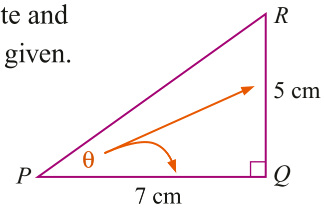
i



EXAMPLE 3

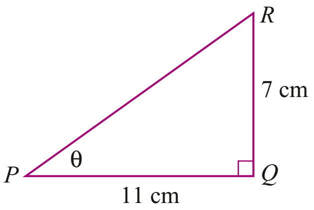
Use the tangent ratio to find θ to the nearest degree.



Solve	Think	Apply
$\tan \theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $= \dfrac{5}{7}$ $\therefore \theta = 35.5374\ldots$ $= 36^\circ$	The sides opposite and adjacent to θ are given.  SHIFT tan (5 ÷ 7) =	Press SHIFT before tan to obtain an angle.

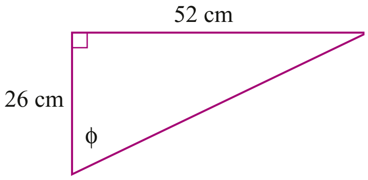
5 Using the tangent ratio, find θ to the nearest degree.

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\underline{\qquad}} \\ &= \frac{7}{\square} \\ \therefore \theta &= \underline{\qquad}\ldots \\ &= \underline{\qquad}^\circ \end{aligned}$$



6 Using the tangent ratio, find ϕ to the nearest degree.

a



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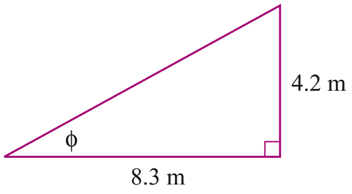

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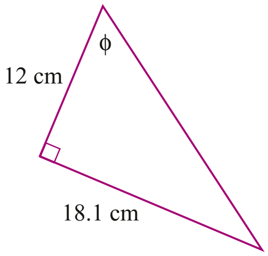

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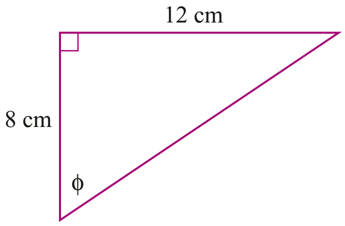

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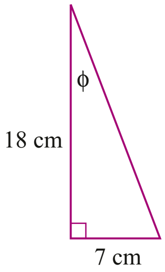

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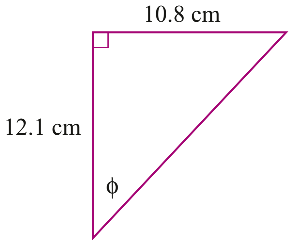

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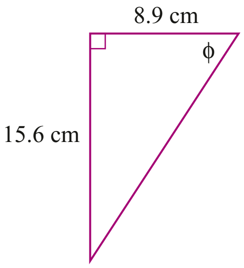

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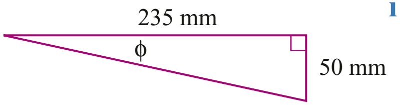

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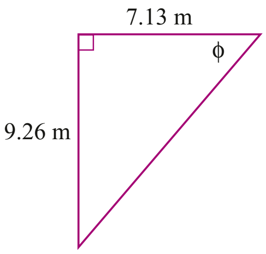
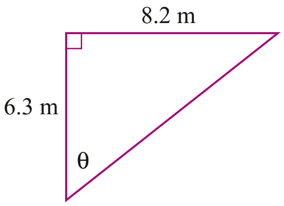
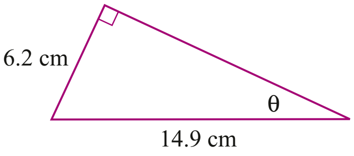


7 Use the sine, cosine or tangent ratios to find each unknown angle to the nearest degree.

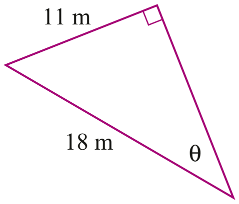
a



b



c



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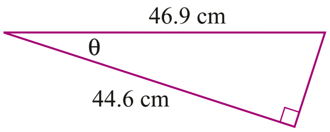
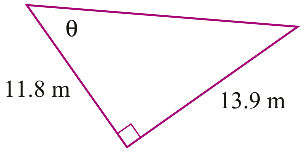


e



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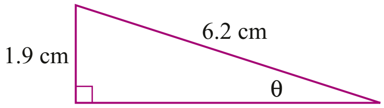

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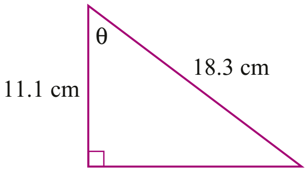

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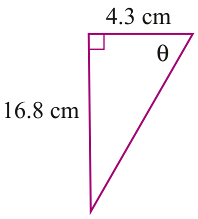
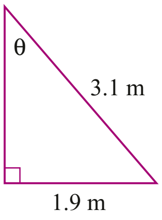


i



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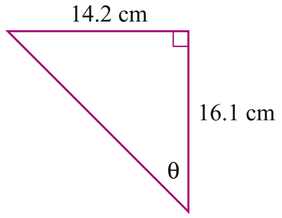
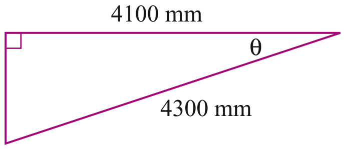
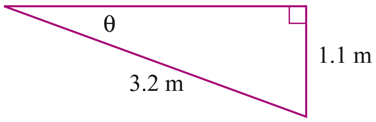


k

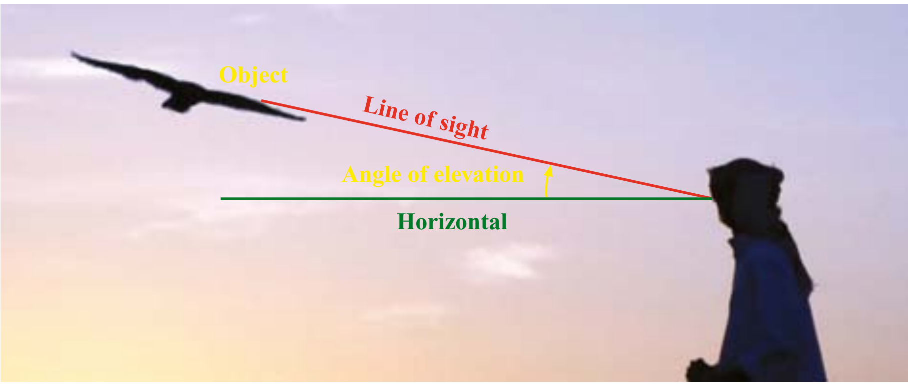


l

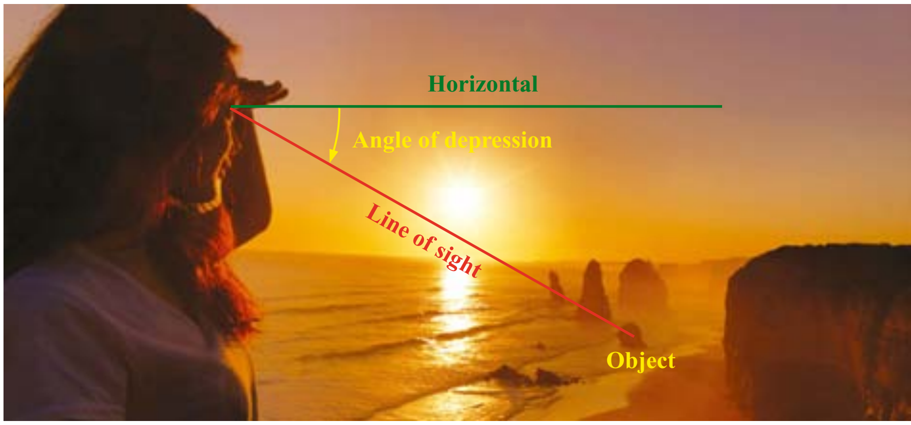


F Worded problems using trigonometry

The **angle of elevation** of an object from an observer is the angle between the horizontal and the line of sight *up* to the object.

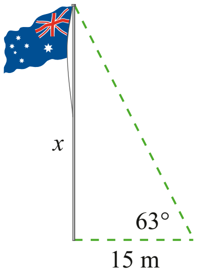


The **angle of depression** of an object from an observer is the angle between the horizontal and the line of sight *down* to the object.



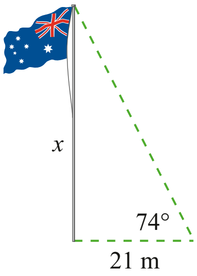
EXAMPLE 1

The angle of elevation of the top of a flagpole from the ground, as observed from a point 15 m from its base, is 63°. Draw a diagram and find the height of the flagpole.

Solve	Think	Apply
$\tan 63° = \frac{x}{15}$ $x = 15 \tan 63°$ ≈ 29.4 m The flagpole is about 29 m high.		'Elevation' means looking upwards. The angle is at ground level.

Exercise 8F

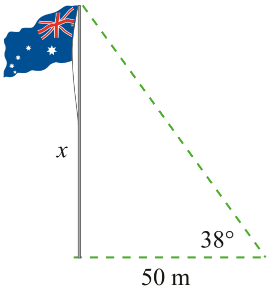
1 The angle of elevation of the top of a flagpole from the ground, as observed from a point 21 m from its base, is 74°. Find the height of the flagpole.



$$\begin{aligned} \tan \_\_\_^\circ &= \frac{x}{\square} \\ x &= 21 \tan \_\_\_^\circ \\ &= \_\_\_\ldots \\ &\approx \_\_\_ \text{ m} \end{aligned}$$

The flagpole is about ___ m high.

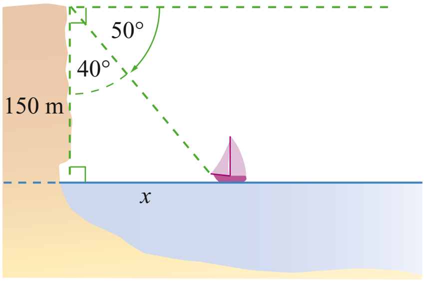
2 The angle of elevation of the top of a flagpole from the ground, as observed from a point 50 m from its base, is 38°. Find the height of the flagpole.



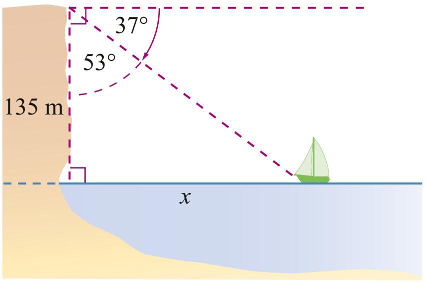
EXAMPLE 2

The angle of depression from the top of a vertical cliff, 150 m above sea level, to a boat below is 50°. Draw a diagram and find the distance of the boat from the base of the cliff.

We assume that the angle between the ground (or sea) and a building (or cliff) is always 90°.

Solve	Think	Apply
$\tan 40^\circ = \frac{x}{150}$ $x = 150 \tan 40^\circ$ ≈ 125.86 m The boat is about 126 m out from the base of the cliff.		'Depression' means looking downwards. Either subtract from 90° to find the angle in the triangle, or use parallel line properties to label the angle at the bottom as equal to the angle of depression.

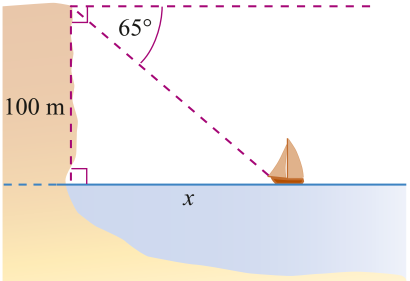
3 The angle of depression from the top of a vertical cliff , 135 m above sea level, to a boat below is 37°. Complete the following to find the distance of the boat from the base of the cliff.



$$\begin{aligned} \tan \_\_\_^\circ &= \frac{x}{\square} \\ x &= \_\_\_ \tan 53^\circ \\ &= \_\_\_\ldots \\ &\approx \_\_\_ \text{ m} \end{aligned}$$

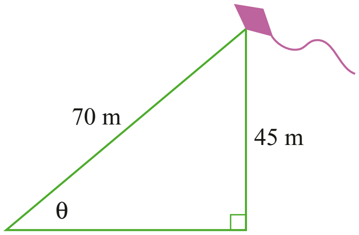
The boat is about ___ m out from the base of the cliff.

4 The angle of depression from the top of a cliff, 100 m above sea level, to a boat is 65°. Find the distance of the boat from the base of the cliff.



EXAMPLE 3

A kite is flying at a height of 45 m above the ground at the end of a string of length 70 m. Find, to the nearest degree, the angle of elevation from the ground to the string.

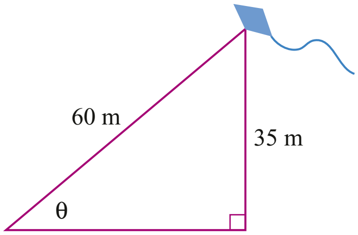
Solve	Think	Apply
$\sin\theta = \frac{45}{70}$ $\therefore \theta = 40.005\ldots°$ $\approx 40°$ The angle of elevation is 40°.	 SHIFT sin (45 ÷ 70) =	Determine the sides required and select the correct ratio.

5 A kite is flying at a height of 35 m above the ground at the end of a string of length 60 m. Find, to the nearest degree, the angle of elevation from the ground to the string.

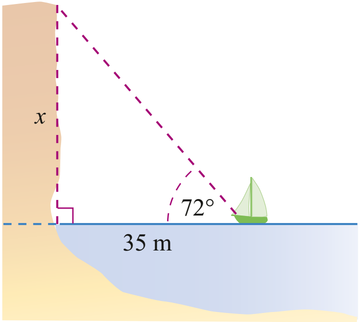
$\sin\theta = \frac{\square}{60}$

$\therefore \theta =$ ___... = ___°

The angle of elevation is ___°.

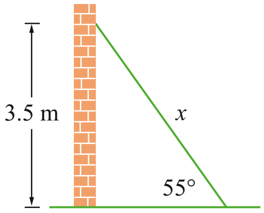


6 The angle of elevation of a vertical cliff from a point 35 m from its base is 72°. Find the height of the cliff to the nearest metre.



EXAMPLE 4

A ladder leaning against a vertical wall reaches 3.5 m up the wall and makes an angle of 55° with the ground. Determine the length of the ladder.

Solve	Think	Apply
$\sin 55^\circ = \frac{3.5}{x}$ $\therefore x \sin 55^\circ = 3.5$ $x = \frac{3.5}{\sin 55^\circ}$ $= 4.2727\ldots$ ≈ 4.3 m The ladder is 4.3 m long.	Use the opposite side and hypotenuse.  3.5 ÷ sin 55 =	Identify the sides required and select the correct ratio.

7 A ladder leaning against a vertical wall reaches 2.8 m up the wall and makes an angle of 67° with the ground. Determine the length of the ladder.

$\sin \_\_\_^\circ = \frac{\square}{x}$

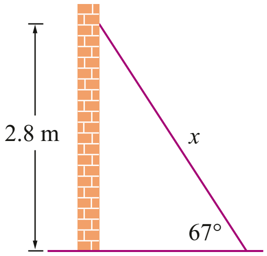
$\therefore x \sin \_\_\_^\circ = \_\_\_$

$x = \frac{\square}{\sin 67^\circ}$

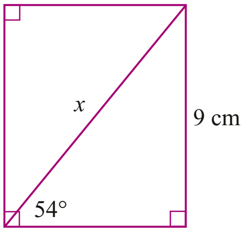
$= \_\_\_\ldots$

$\approx \_\_\_$ m

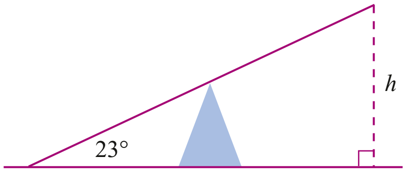
The ladder is ___ m long.



8 A rectangle has a longer side of 9 cm. The angle between the diagonal and the shorter side is 54°. Find the length of the diagonal.



9 A seesaw is 6.3 m long. When one end is resting on the ground it makes an angle of 23° with the ground. Find the height of the other end above ground level.



Language in mathematics

1 Add vowels to complete these words.

a _pp_s_t_

b hyp_t_n_s_

c c_s_n_

d t_ng_nt

e s_n_

f _dj_c_nt

2 Rearrange the words to form a sentence. The first word has a capital letter.

a measured degrees are Angles in.

b tangent all cosine ratios and Sine are.

c opposite hypotenuse The angle right is the.

d side Cosine the the divided adjacent hypotenuse by is.

3 Use every third letter to reveal a sentence about trigonometry.

A A T E R H F G E H H O N B R V F I A W G E E I S S N D R O T Y F H J T O P H M A E A S
W W E O R E R F R D Y G C U J O S E S F G I N J N O I E P O I O I S A S F X C R F G O R E
M W Q T Z C H V G E N H P K O H L I R J Y A H T S D E E V P C A Q O S W M D E P R F L
T H E U J M I K E O P N N G T R D O G A F J K S W E I G Y N T H E D E S T Y O A S T L O
H I U E U I C Y U O Y T S R T I T E N E R E W Q O S D F F T A A S N D E A R G N H J G K I
L O L E E F E D F Q E F U S D A C F L V G S B H T D W H A S E D G S J N I T V N U R E B
H O P Q F J L N O O I C V N I G E H J T K L Y A M D P L E B Z G F N R C K E O G E D B S
O X M U Y I T R N E W U Q A S Z S T E R H F V E B G A Y H N J U G I K L G F E

4 Investigate the origins of the terms ‘sine’, ‘cosine’, ‘tangent’ and ‘trigonometry’. Write a report.

Andrey Nikolayevich Kolmogorov (1903–1987)

Andrey Nikolayevich Kolmogorov was born in 1903 in Tambov, Russia. At the age of 17 he enrolled in Moscow State University, where his initial interest was ancient Russian arts; an interest he maintained throughout his life.

He began his first productive mathematical research in 1921 with research on trigonometric series and operations on sets. In the following years he made considerable contributions to the areas of differentiation, integration and measurable sets. He continued to expand his fields of interest to include mathematical logic.

In 1925 he graduated, was appointed a research associate and began to work in the field of probability theory. He later used this work to study the motion of the planets and the turbulent flow of air from a jet engine.

He was appointed a professor of the university in 1931, and subsequently a director of the Institute of Mathematics, but he continued to work in the field of stochastic (chance) processes and probability theory. He extended this theory to incorporate Markov processes, and related the theory to physics and to the areas of Brownian motion and diffusion. He also developed two systems of equations that describe Markov processes.

Throughout his research, Kolmogorov was surrounded by young mathematicians who wished to learn. In his later years he became interested in the mathematical education of schoolchildren and was appointed chairman of the Commission for Mathematical Education in the USSR. He was recognised as the twentieth century’s most influential Soviet mathematician, both in his own country and abroad. He received a number of prizes and was elected a member of numerous scientific academies, as well being awarded honorary doctorates from Paris, Stockholm and Warsaw universities.

5 **a** How old was Kolmogorov when he died?

b What did he research after graduating?

c What kind of processes are stochastic processes?

d What did he develop to describe Markov processes?

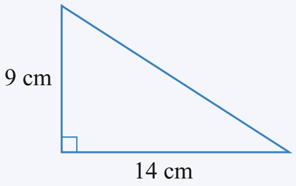
e What interested Kolmogorov in his later years?

Terms

acute
adjacent
angle of depression
angle of elevation
cosine (cos)
$\cos^{-1}$
degree
hypotenuse
north
opposite
phi
ratio
right-angled
sine (sin)
$\sin^{-1}$
tangent (tan)
$\tan^{-1}$
theta
triangle
trigonometry

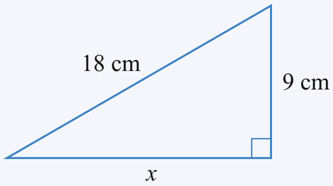
Check your skills

1 The length of the hypotenuse in this triangle is closest to:



A 115 cm
B 277 cm
C 16.6 cm
D 10.7 cm

2 The length of the side marked x in this triangle is closest to:

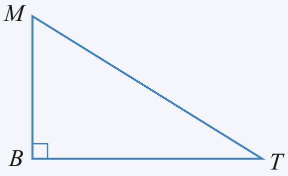


A 20.12 cm
B 15.6 cm
C 9 cm
D 27 cm

3 Which of the triangles with sides listed is *not* right-angled?

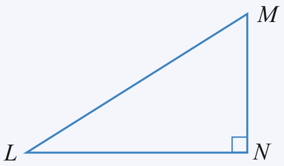
A 9 cm, 12 cm, 15 cm
B 5 cm, 12 cm, 13 cm
C 9 cm, 12 cm, 18 cm
D 3 cm, 4 cm, 5 cm

4 The side opposite angle B in this triangle is:



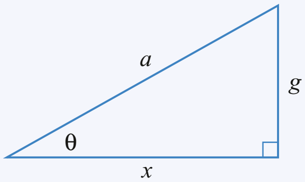
A MB
B BT
C TB
D the hypotenuse

5 The hypotenuse in this triangle is:



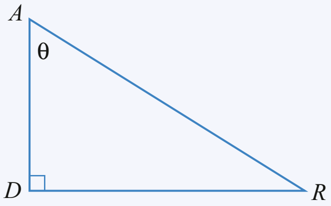
A LM
B LN
C MN
D m

6 The expression for $\sin \theta$ in this triangle is:



A $\frac{a}{g}$
B $\frac{g}{a}$
C $\frac{a}{x}$
D $\frac{g}{x}$

7 The expression for $\tan \theta$ in this triangle is:



A $\frac{AD}{AR}$
B $\frac{DR}{AD}$
C $\frac{AR}{AD}$
D $\frac{DR}{AR}$

8 The value of $\cos 28°$ is closest to:

A 28
B 0.8829
C 0.1392
D 0.1736

9 The value of θ in the expression $\tan \theta = 4.29$ is closest to:

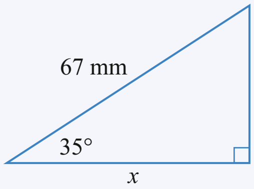
A 0.07°
B 0.075°
C 76°
D 77°

10 The value of angle A in the expression $\sin A = \frac{0.56}{1.8}$ is closest to:

A 0.311° **B** 0.0098 **C** 72° **D** 18°

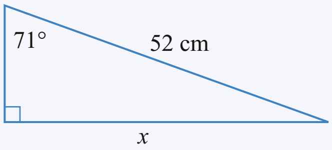
11 The value of x in this triangle is closest to:

A 82 mm **B** 55 mm
C 39 mm **D** 47 mm



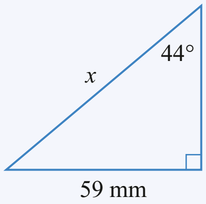
12 The value of x in this triangle is closest to:

A 151 cm **B** 160 cm
C 49 cm **D** 55 cm



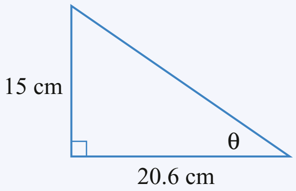
13 The value of x in this triangle is closest to:

A 82 mm **B** 61 mm
C 84 mm **D** 85 mm



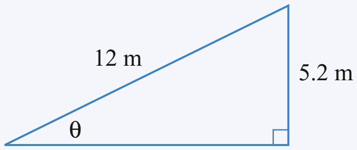
14 The value of θ in the triangle is closest to:

A 36° **B** 55°
C 46° **D** 43°



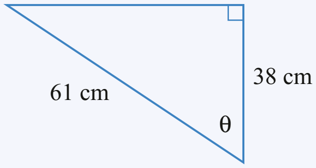
15 The value of θ in the triangle is closest to:

A 23° **B** 66°
C 26° **D** 64°



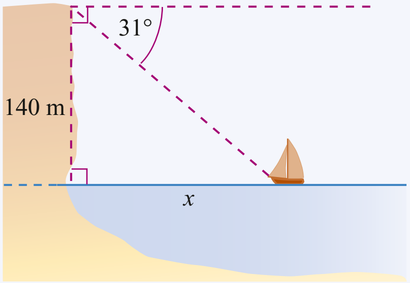
16 The value of θ in the triangle is closest to:

A 38° **B** 51°
C 47° **D** 58°



17 The angle of depression from the top of a cliff 140 m high to a boat at sea is 31°. The distance of the boat from the base of the cliff is closest to:

A 84 m **B** 72 m
C 272 m **D** 233 m



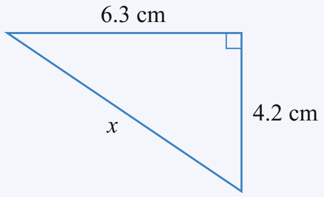
If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–5	6–7	8–10	11–13	14–16	17
Section	A	B	C	D	E	F

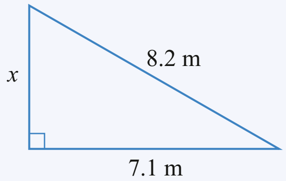
8A Review set

1 Find x correct to 1 decimal place.

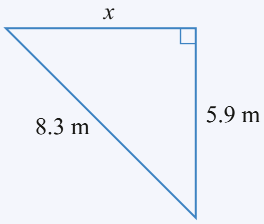
a



b



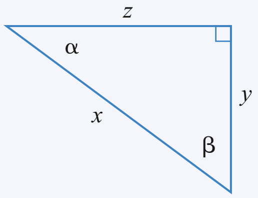
c



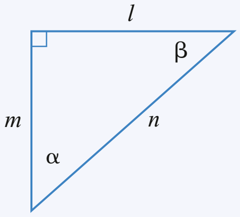
2 Write down expressions for sin α, cos α, tan α, and sin β, cos β, tan β in each of the following.

Alpha (α) and beta (β) are Greek letters often used for unknown angles.

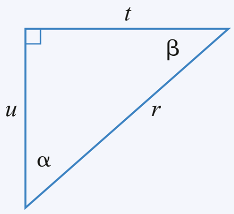
a



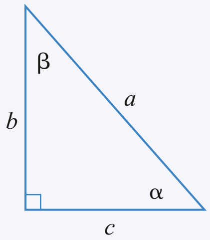
b



c

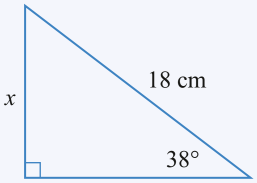


d

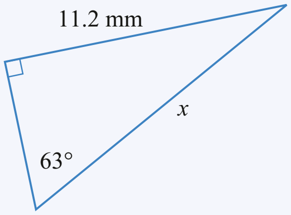


3 Find the length of the side marked x correct to 1 decimal place.

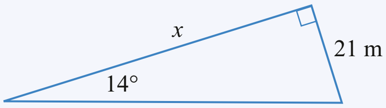
a



b

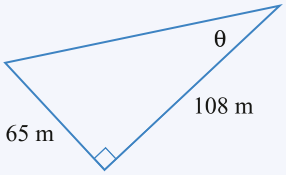


c

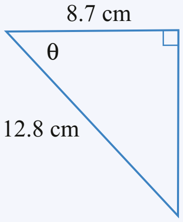


4 Find the value of θ correct to the nearest degree.

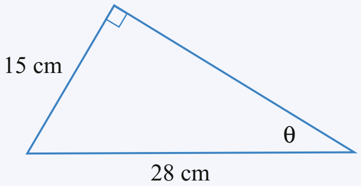
a



b

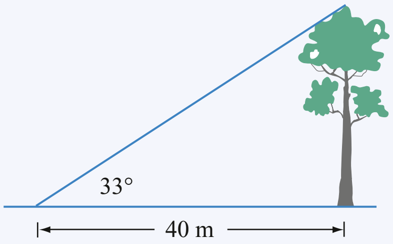


c

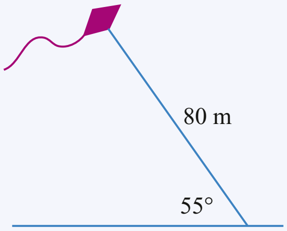


5 Solve the following problems.

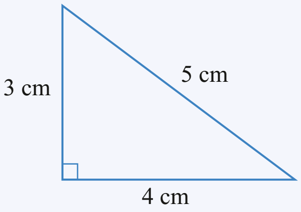
a The shadow of a tree is 40 m in length and the angle of elevation from the end of the shadow to the top of the tree is 33°. Find the height of the tree to the nearest one-tenth of a metre.



b A kite string is pinned to the ground. The string is 80 m long and makes an angle of 55° with the ground. How high is the kite above the ground? Give the answer to the nearest metre.



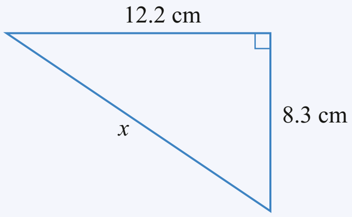
c A triangle has sides 3 cm, 4 cm and 5 cm. Find the angles of the triangle to the nearest degree.



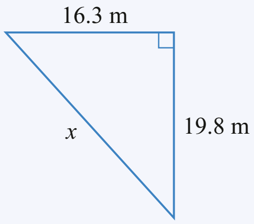
8B Review set

1 Find x correct to 1 decimal place.

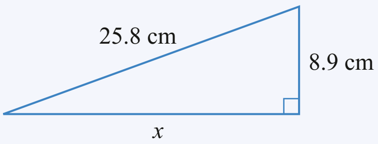
a



b

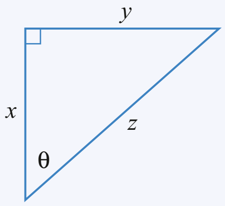


c

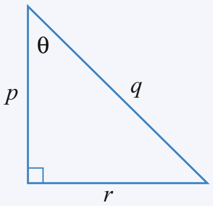


2 Write down expressions for sin θ, cos θ and tan θ in each of the following.

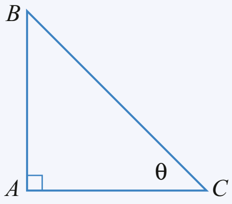
a



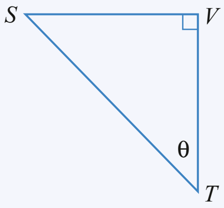
b



c

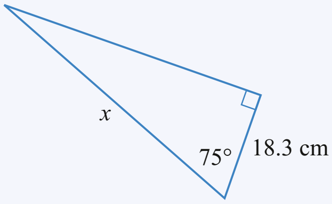


d

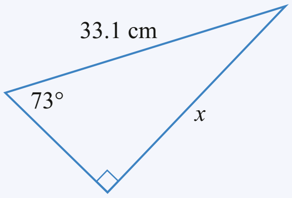


3 Find the length of the side marked x correct to 1 decimal place.

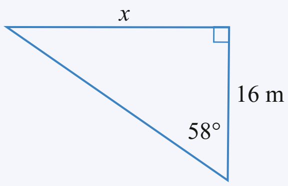
a



b

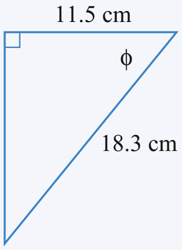


c

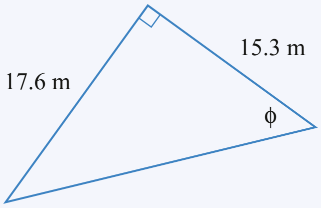


4 Find the value of ϕ to the nearest degree.

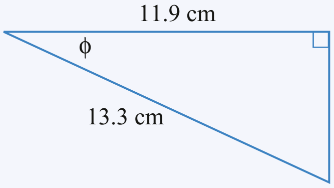
a



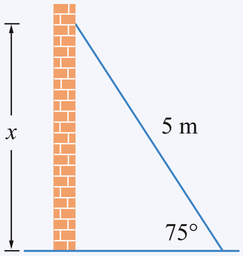
b



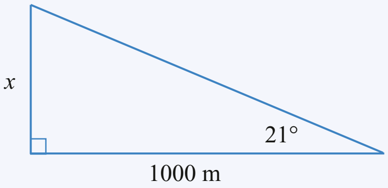
c



5 **a** A ladder is 5 m long and makes an angle of 75° with the ground. How far up the wall does it reach (to the nearest 10 cm)?



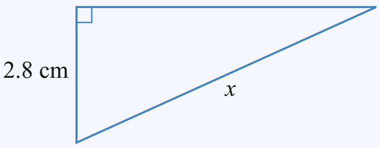
b An aeroplane takes off at a constant angle of 21°. When it has flown a horizontal distance of 1000 m, what is its altitude to the nearest metre?



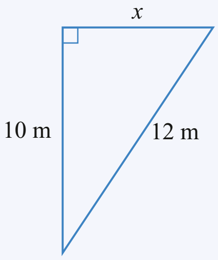
8C Review set

1 Find x correct to 1 decimal place.

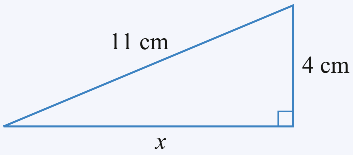
a



b

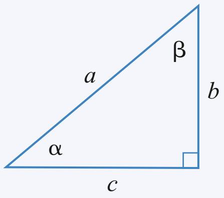


c

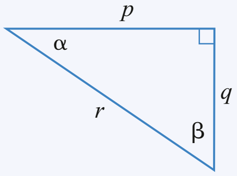


2 Write down expressions for sin α, cos α, tan α and sin β, cos β and tan β in each of the following.

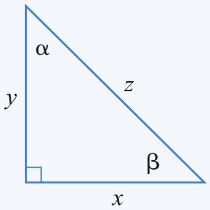
a



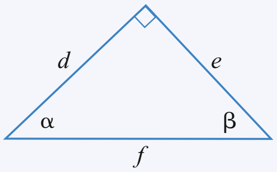
b



c

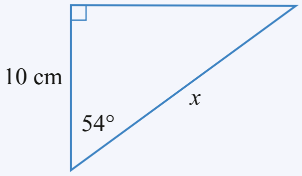


d

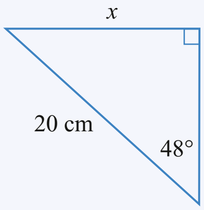


3 Find the length of the side marked x correct to 1 decimal place.

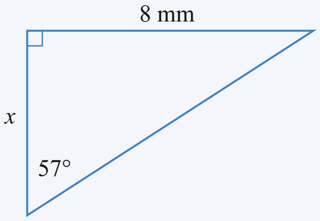
a



b

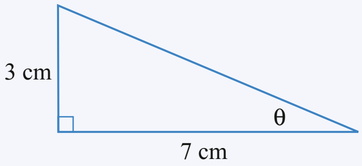


c

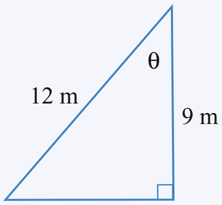


4 Find θ to the nearest degree.

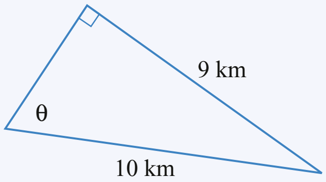
a



b

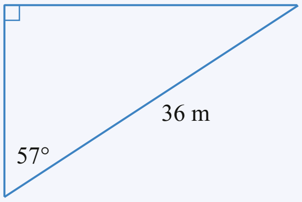


c

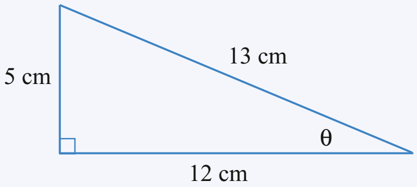


5 Solve these problems using trigonometry.

a Find all the sides and angles in the given triangle.



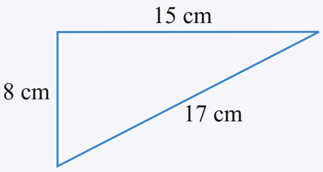
b Find the value of θ in the triangle at right in three different ways.



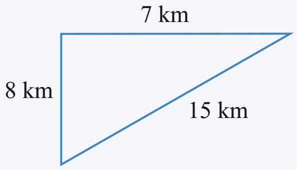
8D Review set

1 Determine whether the following are right-angled triangles.

a



b



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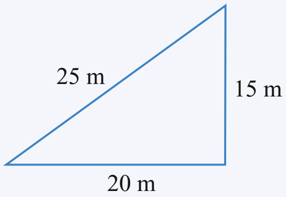
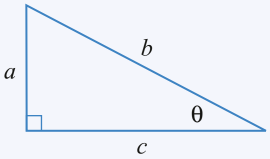
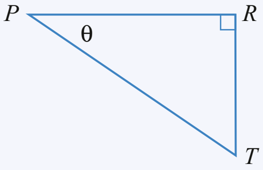


2 Write expressions for sin θ, cos θ and tan θ in the following diagrams.

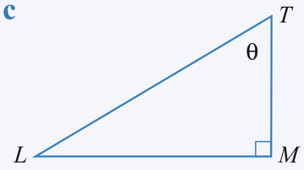
a



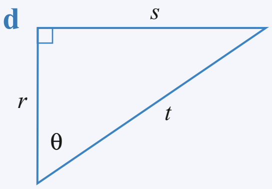
b



c

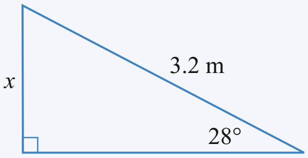


d

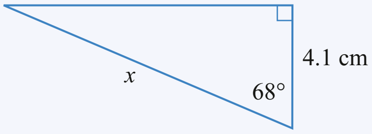


3 Find the length of the side marked x correct to 1 decimal place.

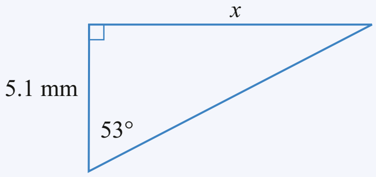
a



b

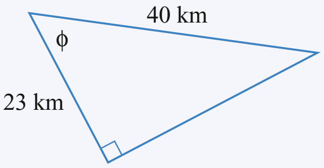


c

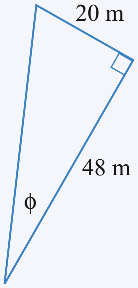


4 Find the value of ϕ correct to the nearest degree.

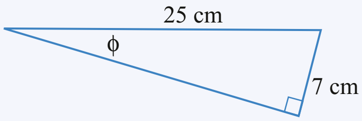
a



b

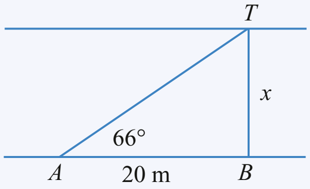


c

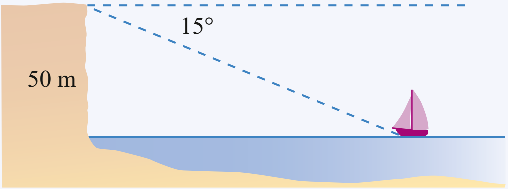


5 Solve the following problems.

a To measure the width of a river a surveyor finds a point B directly opposite a landmark T, such as a tree, on the bank on the other side of the river. He then moves 20 m along the bank at right angles to BT to a point A. With a theodolite he measures $\angle BAT$ as 66°. Calculate the width of the river to the nearest metre.



b From the top of a vertical cliff 50 m high, the angle of depression to a boat straight out to sea is 15°. How far is the boat from the foot of the cliff, to the nearest metre?



9

Similarity

This chapter deals with describing and applying the properties of similar figures and scale drawings.

After completing this chapter you should be able to:

- describe similar figures using the enlargement transformation
- match the sides and angles of similar figures
- describe the angle and side properties of similar figures
- determine the scale factor for pairs of similar polygons and circles
- calculate the dimensions of similar figures using the scale factor
- calculate unknown sides in similar triangles using a proportion statement
- construct, interpret and use scale drawings
- apply the scale factor to find unknown lengths in practical situations.

NSW Syllabus references: 5.1 M&G Properties of geometrical figures
Outcomes: MA5.1-1WM, MA5.1-2WM, MA5.1-3WM, MA5.1-11MG
MEASUREMENT & GEOMETRY – ACMMG220 (part), ACMMG221

A The enlargement transformation

Two figures are **similar** if an enlargement of one is **congruent** to the other. This may also be stated as:

Two figures are similar if one is an enlargement (or reduction) of the other.

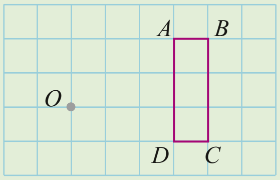
The enlargement factor is also called the scale factor.

EXAMPLE 1

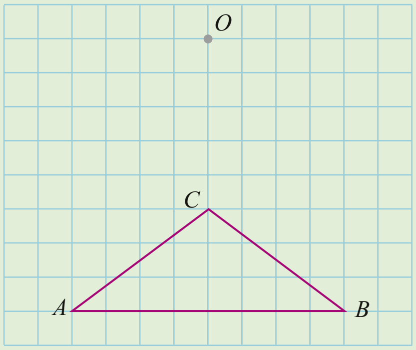
i Enlarge each of the figures below using the enlargement factor and centre of enlargement given. Label the vertices of the enlarged figure.

ii Name the pairs of **corresponding** (matching) sides in the similar figures.

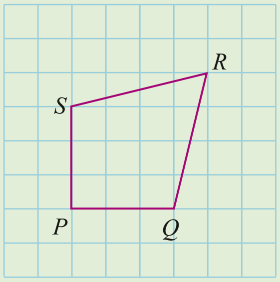
a Use enlargement factor $k = 3$ and O as centre of enlargement.

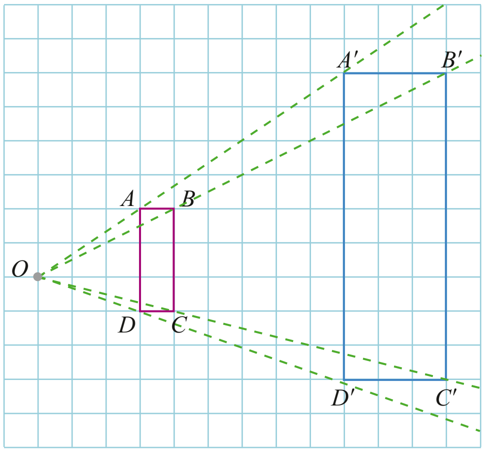


b Use $k = \frac{1}{2}$ and O as centre of enlargement.

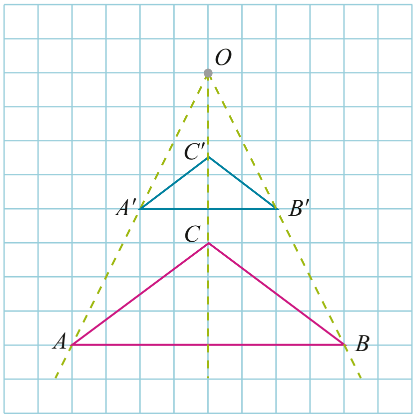
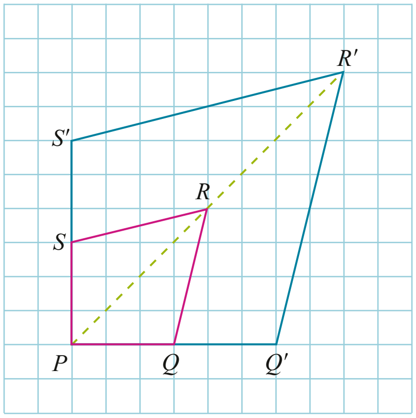


c Use $k = 2$ and P as centre of enlargement.



	Solve	Think	Apply
a i		Draw rays from O through each of the vertices of the rectangle. Produce OA to OA' such that $OA' = 3 \times OA$. Produce OB to OB' such that $OB' = 3 \times OB$. Produce OC to OC' such that $OC' = 3 \times OC$. Produce OD to OD' such that $OD' = 3 \times OD$. Join A', B', C' and D' to form the enlargement of rectangle $ABCD$ by a factor of 3.	To produce an enlargement, choose the centre of enlargement, O, and enlargement factor, k. The image of each point P is a point P' such that O, P and P' are in a straight line and $OP' = k \times OP$.
ii	$A'B'$ and AB, $B'C'$ and BC, $C'A'$ and CA, and $D'A'$ and DA	$A'B'$ is the image of AB, $B'C'$ is the image of BC, $C'D'$ is the image of CD and $D'A'$ is the image of DA.	

EXAMPLE 1 CONTINUED

	Solve	Think
b i		Draw rays from O through each of the vertices of the triangle. Locate A' on OA such that $OA' = \frac{1}{2} \times OA$. Locate B' on OB such that $OB' = \frac{1}{2} \times OB$. Locate C' on OC such that $OC' = \frac{1}{2} \times OC$. Join A', B' and C' to form the enlargement of $\triangle ABC$ by a factor of $\frac{1}{2}$.
ii	$A'B'$ and AB, $B'C'$ and BC, $C'A'$, and CA	$A'B'$ is the image of AB, $B'C'$ is the image of BC and $C'A'$ is the image of CA.
c i		Draw rays from P through each of the vertices of the quadrilateral. Produce PS to PS′ such that $PS' = 2 \times PS$. Produce PR to PR' such that $PR' = 2 \times PR$. Produce PQ to PQ' such that $PQ' = 2 \times PQ$. Join P, S', R' and Q' to form the enlargement of quadrilateral $PSQR$ with an enlargement factor of 2.
ii	PS' and PS, $S'R'$ and SR, $R'Q'$ and RQ, and $Q'P$ and QP	PS' is the image of PS, $S'R'$ is the image of SR, $R'Q'$ is the image of RQ, $Q'P$ is the image of QP.

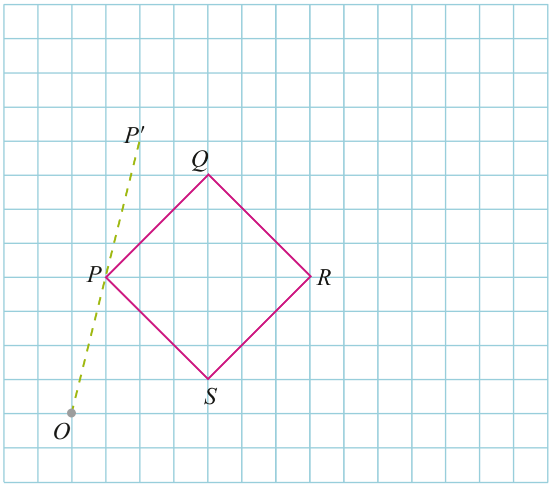
Exercise 9A

1 **a** Complete the following enlargement of $PQRS$ using O as the centre of enlargement and $k = 2$.

$OP' = 2 \times OP$

b The image of $PQRS$ is ___.

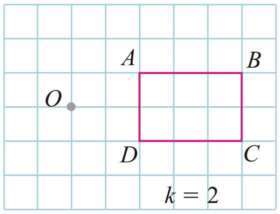
c The pairs of corresponding sides are $P'Q'$ and PQ, ___ and ___, ___ and ___, ___ and ___.



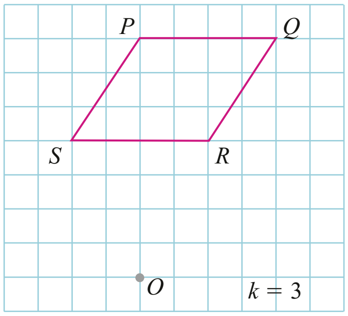
2 **i** Enlarge each of the figures below using the enlargement factor given and O as the centre of enlargement. Label the vertices of the enlarged figure.

ii Name the pairs of corresponding sides in the similar figures.

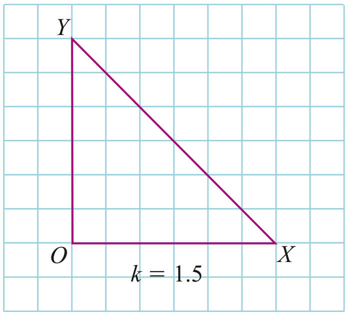
a



b



c

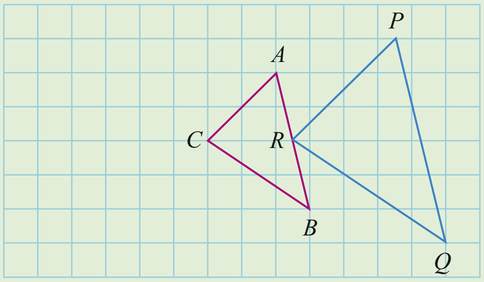


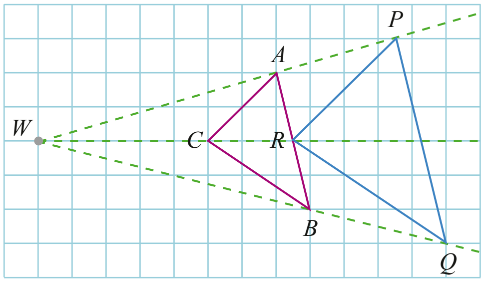
EXAMPLE 2

PQR and ABC are similar triangles; triangle PQR has been formed by enlarging triangle ABC.

a Find the centre of enlargement.

b Find the enlargement factor.



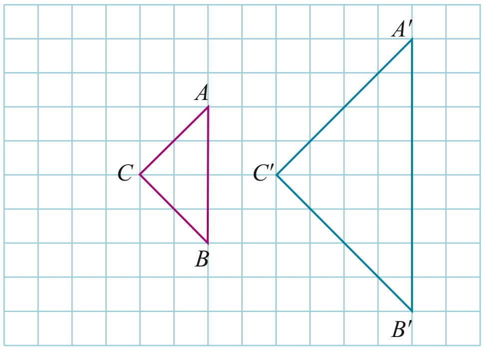
	Solve	Think	Apply
a		Draw (any two of) the rays PA, RC and QB. The point of intersection of these rays, W, is the centre of enlargement.	Draw a ray through each point and its image. The point of intersection of the rays is the centre of enlargement.
b	$k = \frac{7.5}{5} = 1.5$	$WR = 7.5$ units $WC = 5$ units $k = \frac{WR}{WC}\left(\text{or } \frac{WP}{WA} \text{ or } \frac{WQ}{WB}\right)$	$k = \frac{OP'}{OP}$ where O is the centre of enlargement and P' is the image of P.

3 Copy the following pairs of similar figures onto paper and find:

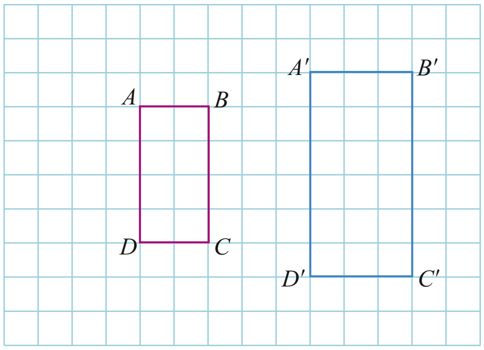
i the centre of enlargement

ii the enlargement factor.

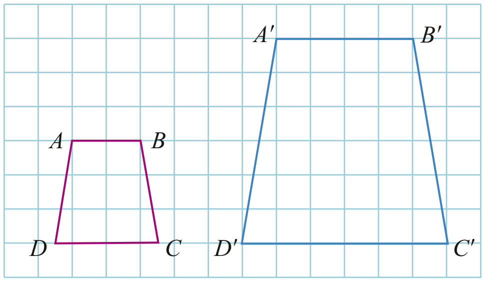
a



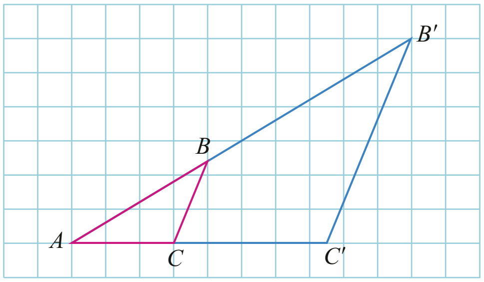
b



c



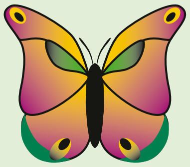
d

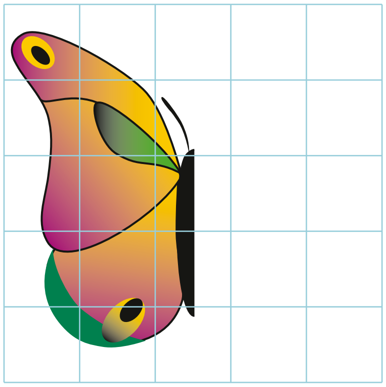
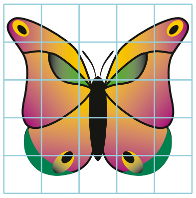


Another method of enlarging figures, especially those that are drawings rather than geometrical shapes, is shown in Example 3.

EXAMPLE 3

Enlarge the given picture to twice its original size.



Solve	Think	Apply
The enlarged picture is started below. 	Draw a 5 mm by 5 mm grid over the original picture.  To enlarge the picture to twice the original size (a scale factor of 2), copy each part of the picture in the 5 mm grid squares onto 10 mm grid squares.	Draw a square grid over the original picture. If the scale factor is *k*, copy the picture onto a larger square grid whose dimensions are *k* times those of the original.

4 **a** Copy and complete the enlarged picture in Example 3.

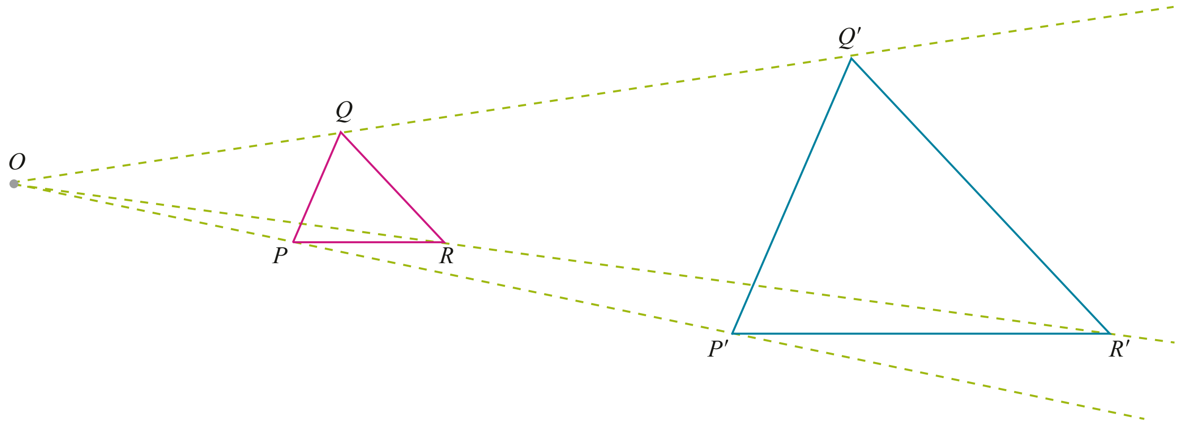
b Enlarge this picture using a scale factor of $\frac{1}{2}$ by using a $2\frac{1}{2}$ mm by $2\frac{1}{2}$ mm grid. (This is of course a reduction in size).

Investigation 1 Similar figures

Find examples of similar figures embedded in designs and art from different cultures and historical periods. Make a display to show and discuss with your class.

Investigation 2 Enlargements and scale factors

1 The triangle PQR has been enlarged to triangle $P'Q'R'$ from the centre of enlargement, O.



a Measure all sides to the nearest mm and all angles to the nearest degree and complete this table.

$\triangle PQR$	$\triangle P'Q'R'$	$\triangle PQR$	$\triangle P'Q'R'$
$PQ =$ ___	$P'Q' =$ ___	$\angle PQR =$ ___	$\angle P'Q'R' =$ ___
$QR =$ ___	$Q'R' =$ ___	$\angle QRP =$ ___	$\angle Q'R'P' =$ ___
$PR =$ ___	$P'R' =$ ___	$\angle RPQ =$ ___	$\angle R'P'Q' =$ ___

b What do you notice about the angles in each triangle?

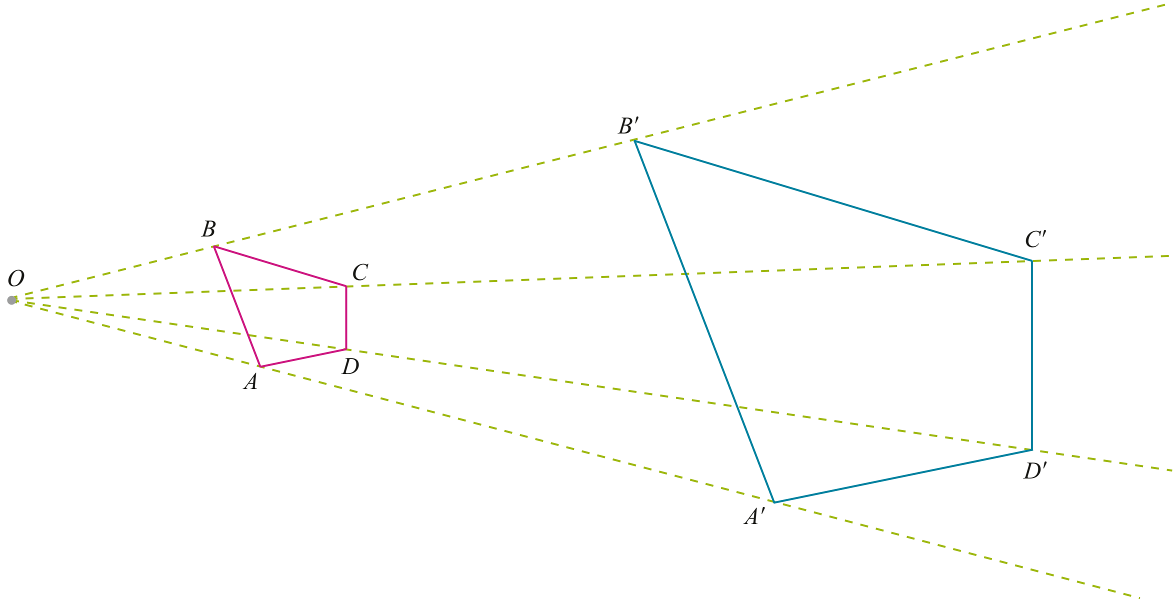
c Complete the following.

$$\frac{P'Q'}{PQ} = \_\_\_ \qquad \frac{Q'R'}{QR} = \_\_\_ \qquad \frac{P'R'}{PR} = \_\_\_$$

d What do you notice about the ratio of the pairs of matching sides in these similar figures?

e What is the scale factor in enlarging PQR to $P'Q'R'$?

2 The figure $ABCD$ has been enlarged to the figure $A'B'C'D'$, from the point O.



a Measure all the sides to the nearest mm and all angles to the nearest degree and complete this table.

ABCD	***A′B′C′D′***	***ABCD***	***A′B′C′D′***
$AB =$ ___	$A'B' =$___	$\angle ABC =$ ___	$\angle A'B'C' =$ ___
$BC =$ ___	$B'C' =$	$\angle BCD =$ ___	$\angle B'C'D' =$ ___
$CD =$ ___	$C'D' =$	$\angle CDA =$ ___	$\angle C'D'A' =$ ___
$AD =$ ___	$A'D' =$ ___	$\angle DAB =$ ___	$\angle D'A'B' =$ ___

b What do you notice about the angles in the quadrilaterals?

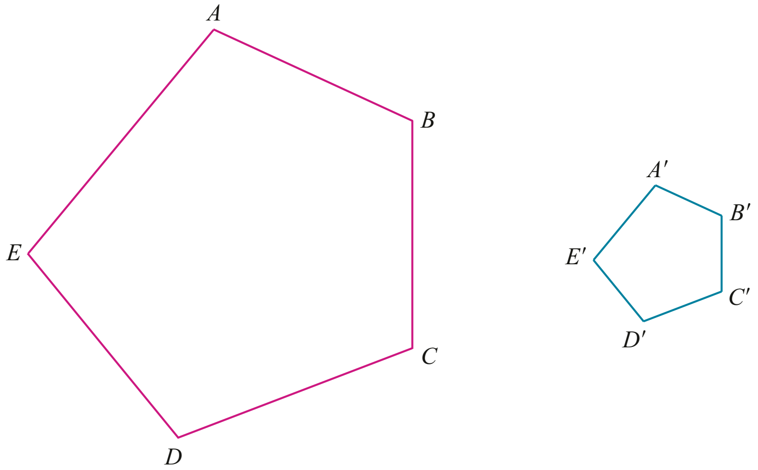
c Complete the following.

$\frac{A'B'}{AB} =$ ___ $\frac{B'C'}{BC} =$ ___ $\frac{C'D'}{CD} =$ ___ $\frac{A'D'}{AD} =$

d What do you notice about the ratio of the pairs of matching sides in these similar figures?

e What is the scale factor in enlarging $ABCD$ to $A'B'C'D'$?

3 Look at the figures below.



a Measure all the sides to the nearest mm and all angles to the nearest degree and complete this table.

ABCD	***A′B′C′D′***	***ABCD***	***A′B′C′D′***
$AB =$ ___	$A'B' =$___	$\angle ABC =$ ___	$\angle A'B'C' =$ ___
$BC =$ ___	$B'C' =$	$\angle BCD =$ ___	$\angle B'C'D' =$ ___
$CD =$ ___	$C'D' =$	$\angle CDE =$ ___	$\angle C'D'E' =$ ___
$DE =$ ___	$D'E' =$	$\angle DEA =$ ___	$\angle D'E'A' =$ ___
$EA =$ ___	$E'A' =$ ___	$\angle EAB =$ ___	$\angle E'A'B' =$ ___

b What do you notice about the angles in each pentagon?

c Complete the following.

$\frac{A'B'}{AB} =$ ___ $\frac{B'C'}{BC} =$ ___ $\frac{C'D'}{CD} =$ ___ $\frac{D'E'}{DE} =$ ___ $\frac{E'A'}{EA} =$

d What do you notice about the ratio of the pairs of matching sides in these similar figures?

e What is the scale factor reducing $ABCDE$ to $A'B'C'D'E'$?

B Properties of similar figures

From Investigation 2, you will have discovered that if two figures are similar:

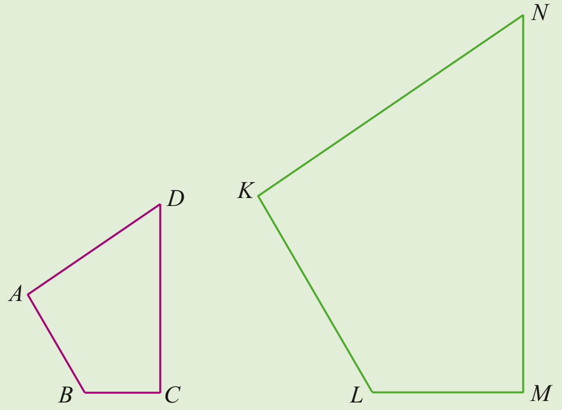
- The matching angles are equal and the lengths of the matching sides are in the same ratio. (The figures have the same shape but not necessarily the same size.)
- The ratio of the lengths of the matching sides is equal to the enlargement factor.

If the lengths of the matching sides are in the same ratio, we say that the sides are in **proportion**.

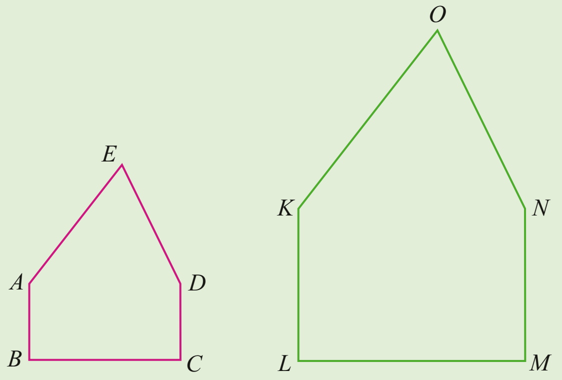
EXAMPLE 1

By measuring the angles and the lengths of the sides, determine whether or not the following figures are similar. If they are similar, state the enlargement factor.

a



b



	Solve/Think	Apply
a	In *ABCD*, $\angle A = 94°$, $\angle B = 120°$, $\angle C = 90°$, $\angle D = 56°$ and $AB = 1.5$ cm, $BC = 1$ cm, $CD = 2.5$ cm, $DA = 2.1$ cm. In *KLMN*, $\angle K = 94°$, $\angle L = 120°$, $\angle M = 90°$, $\angle N = 56°$ and $KL = 3$ cm, $LM = 2$ cm, $MN = 5$ cm, $NK = 4.2$ cm. The figures have four pairs of equal angles and the matching sides are in proportion: $\frac{KL}{AB} = \frac{LM}{BC} = \frac{MN}{CD} = \frac{NK}{DA} = 2$ Hence the figures are similar. The enlargement factor is 2.	Two figures are similar if the matching angles are equal and the matching sides are in the same ratio.
b	In *ABCDE*, $\angle A = 142°$, $\angle B = 90°$, $\angle C = 90°$, $\angle D = 154°$, $\angle E = 64°$ and $AB = 1$ cm, $BC = 2$ cm, $CD = 1$ cm, $DE = 1.7$ cm, $EA = 2$ cm. In *KLMNO*, $\angle K = 142°$, $\angle L = 90°$, $\angle M = 90°$, $\angle N = 154°$, $\angle O = 64°$ and $KL = 2$ cm, $LM = 3$ cm, $MN = 2$ cm, $NO = 2.6$ cm, $OK = 3$ cm. The figures have five pairs of equal angles but the matching sides are not in proportion: $\frac{KL}{AB} \neq \frac{LM}{BC}$, hence the figures are not similar.	

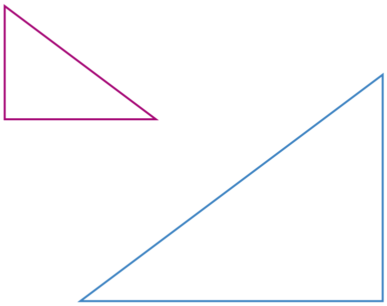
It is important to recall that similar figures have the same shape but not necessarily the same size.

Congruent figures have the same shape and are exactly the same size; that is, the enlargement factor $k = 1$.

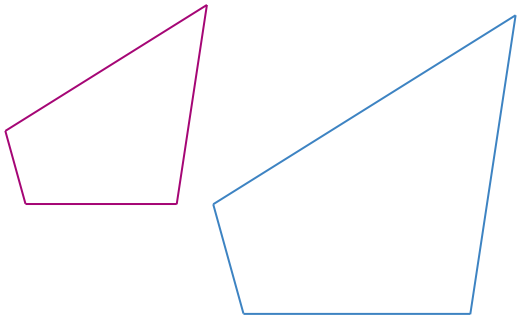
Exercise 9B

1 By measuring the angles and the lengths of the sides, determine whether or not the following pairs of figures are similar. If they are similar, state the enlargement factor.

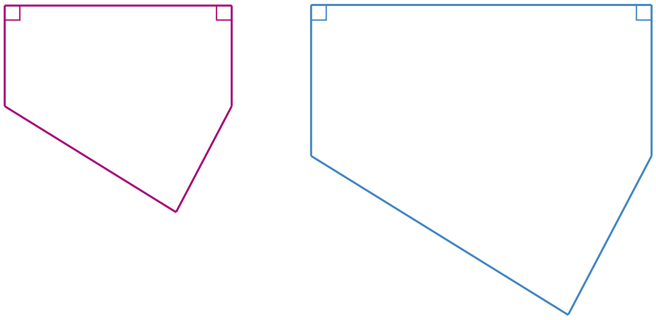
a



b

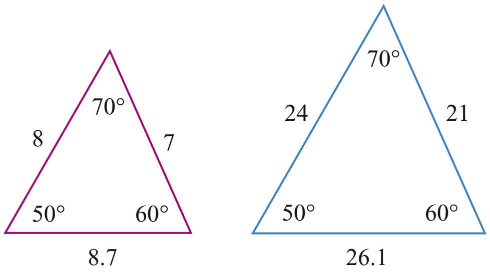


c

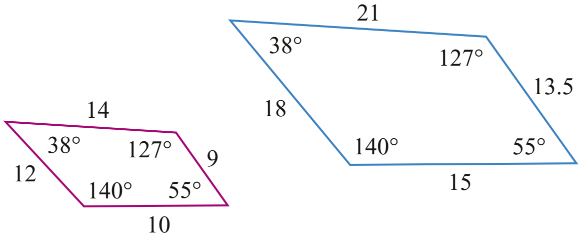


2 Use the information given to determine whether or not the following pairs of figures are similar. If they are similar, state the enlargement factor. (Diagrams are not drawn to scale. All lengths are in centimetres.)

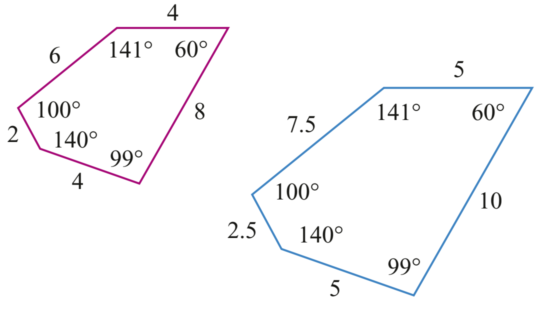
a



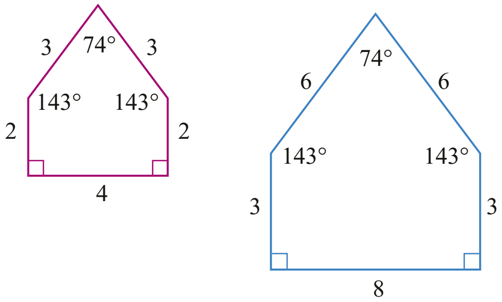
b



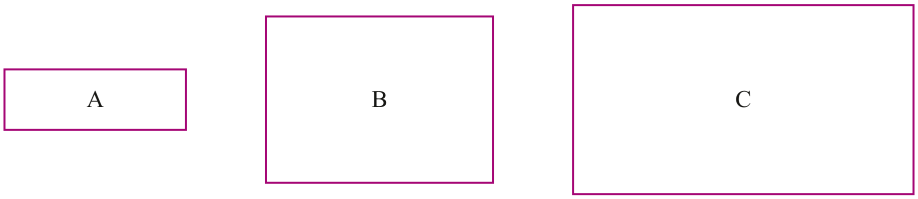
c



d



3 **a** Here are three rectangles. Are all rectangles equiangular? Give reasons for your answer.



b Complete the table below for the rectangles on the previous page.

	Rectangle A	Rectangle B	Rectangle C
Length			
Width			

c Complete the following.

$\frac{\text{length A}}{\text{length B}} =$ ___ $\frac{\text{length B}}{\text{length C}} =$ ___ $\frac{\text{length A}}{\text{length C}} =$ ___

$\frac{\text{width A}}{\text{width B}} =$ ___ $\frac{\text{width B}}{\text{width C}} =$ ___ $\frac{\text{width A}}{\text{width C}} =$ ___

d Are any of these rectangles similar? Explain.

e Comment on the statement ‘All rectangles are similar’.

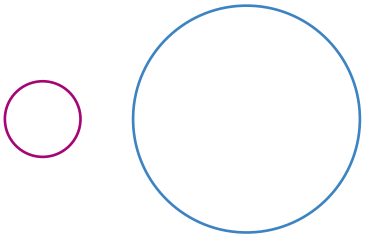
4 Determine whether the following statements are true or false.

a Any two circles are similar.
b Any two equilateral triangles are similar.
c Any two isosceles triangles are similar.
d Any two squares are similar.
e Any two scalene triangles are similar.

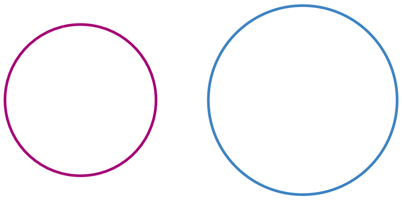
5 When are two similar figures congruent? Explain your answer.

6 What is the scale factor for each pair of circles below.

a



b

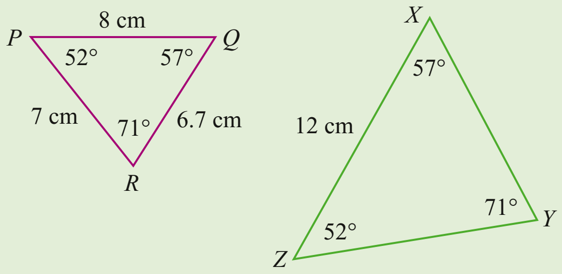


C Finding sides in similar figures

EXAMPLE 1

a Name the matching (corresponding) sides in these similar triangles.

b Hence find the enlargement factor.



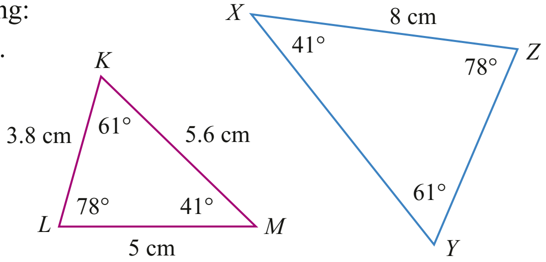
	Solve	Think	Apply
a	PQ and ZX, QR and XY, RP and YZ	The matching sides are opposite the matching angles.	The matching sides are opposite the matching angles and k equals the ratio of the lengths of the matching sides.
b	$k = \frac{12}{8} = 1.5$	$k = \frac{ZX}{PQ}$ or $\frac{XY}{QR}$ or $\frac{YZ}{RP}$	

Exercise 9C

1 For the pair of similar triangles shown, complete the following:

a KM and XY, ML and ___, KL and ___ are matching sides.

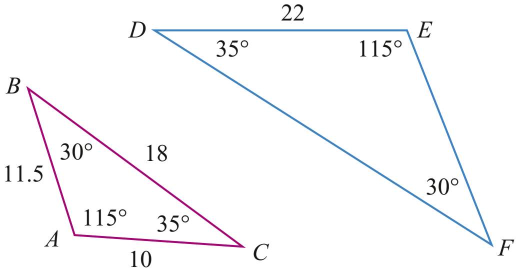
b $k = \frac{XZ}{\square} = \frac{\square}{\square} = $ ___



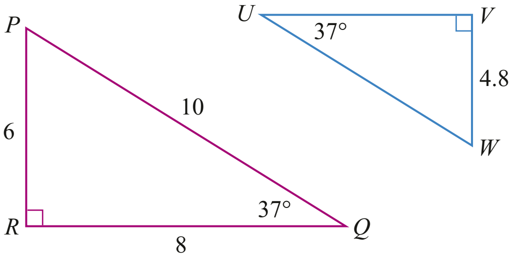
2 **i** Name the matching sides in the following pairs of similar triangles. (Diagrams are not drawn to scale. All lengths are in centimetres.)

ii Hence find the enlargement factor.

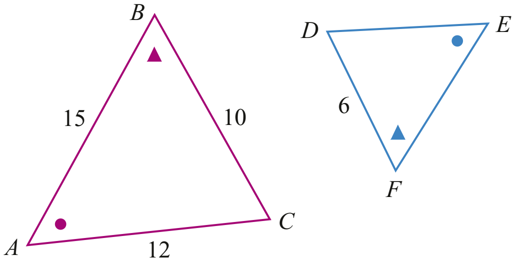
a



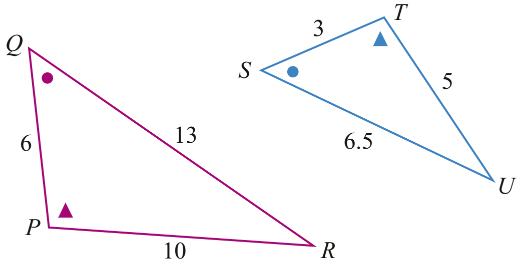
b



c

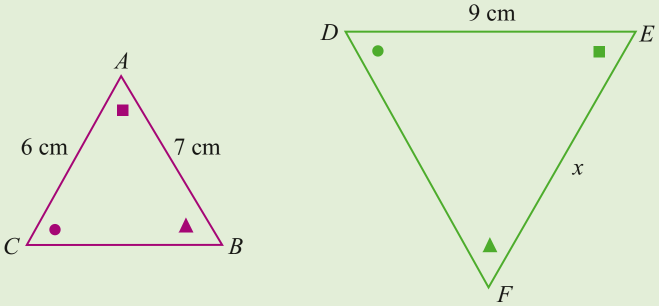


d



EXAMPLE 2

Find the length of the unknown side in these similar triangles.

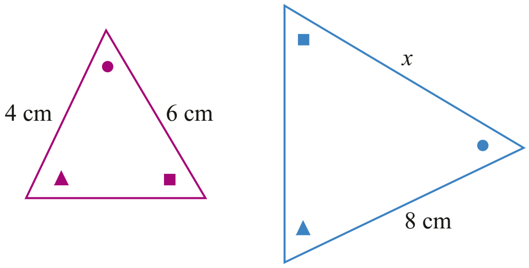


Solve	Think	Apply
$k = \frac{9}{6} = 1.5$ Hence: $x = 1.5 \times 7$ $= 10.5$ cm	As DE and AC are matching sides, $k = \frac{DE}{AC}$. As FE and BA are matching sides, $FE = k \times BA$.	Use a pair of matching sides of known length to find the scale factor, k. Each side of the second triangle is k times its matching side in the first triangle.

3 For the pair of similar triangles shown, complete the following to find the enlargement factor and the length of the unknown side.

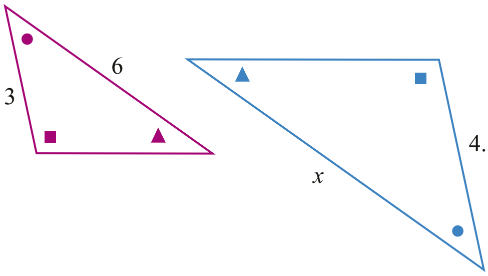
$$k = \frac{8}{\square} = \square$$

$$x = \_\_\_ \times 6 = \_\_\_ \text{ cm}$$

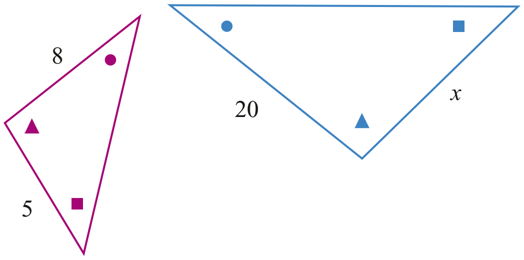


4 Find the length of the unknown side in the following pairs of similar triangles. (Diagrams are not drawn to scale. All lengths are in centimetres.)

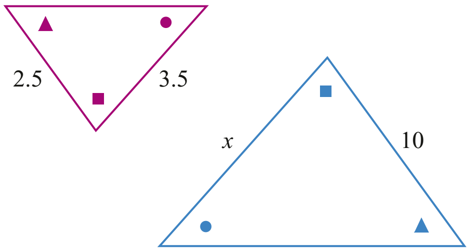
a



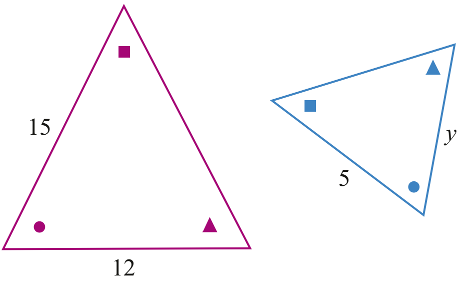
b



c

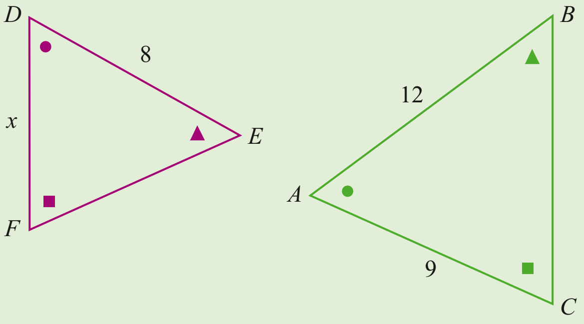


d



EXAMPLE 3

Find the length of the unknown side in the similar triangles shown. (All lengths are in centimetres.)

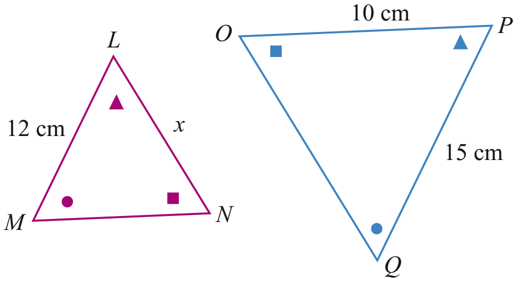


Solve	Think	Apply
$k = \frac{8}{12} = \frac{2}{3}$ Hence: $x = \frac{2}{3} \times 9$ $= 6$ cm	As we are to find a side in the first triangle, we need the enlargement factor from $\triangle ABC$ to $\triangle DEF$. DE and AB are matching sides hence $k = \frac{DE}{AB}$. As DF and AC are matching sides, $DF = k \times AC$.	Use a pair of matching sides of known length to find the scale factor, k. Each side of the first triangle is then k times its matching side in the second triangle.

5 For the pair of similar triangles shown, complete:

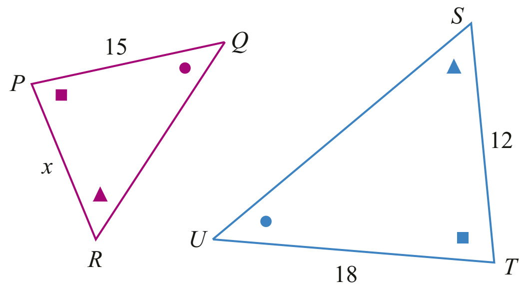
$$k = \frac{12}{\square} = \frac{\square}{\square} = \text{ or } \_\_\_$$

$$x = \frac{\square}{\square} \times 10 = \_\_\_ \text{ cm}$$

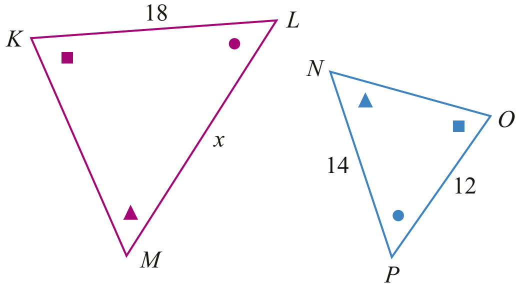


6 Find the length of the unknown side in the similar triangles shown. (All lengths are in centimetres.)

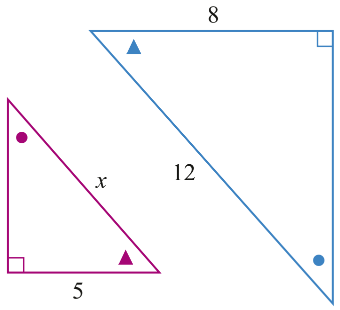
a



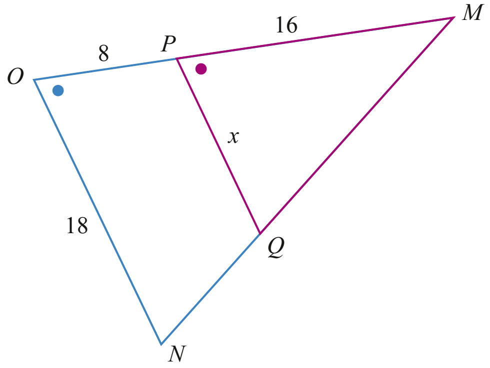
b



c

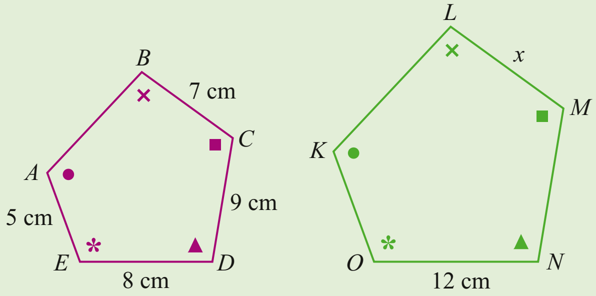


d



EXAMPLE 4

Find the length of the unknown side, given that the figures are similar. (Figures are not drawn to scale.)



Solve	Think	Apply
$k = \frac{12}{8} = 1.5$ Hence: $x = 1.5 \times 7$ $= 10.5$ cm	*ON* and *ED* are matching sides, hence $k = \frac{ON}{ED}$. As *LM* and *BC* are matching sides, $LM = k \times BC$.	Use a pair of matching sides of known length to find the scale factor, *k*. Each side of the second figure is then *k* times its matching side in the first figure.

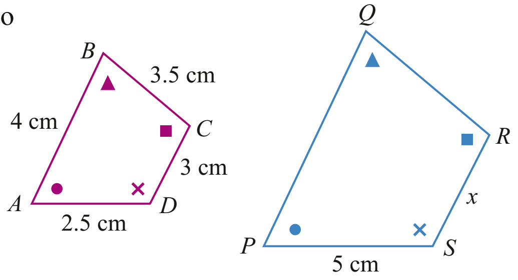
7 For the pair of similar triangles shown, complete the following to find the enlargement factor and the length of the unknown side.

PS and ___ are matching sides.

$k = \frac{5}{\square} =$ ___

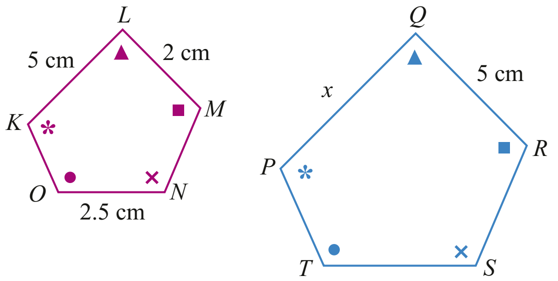
RS and ___ are matching sides.

$x =$ ___ $\times 3 =$ ___ cm

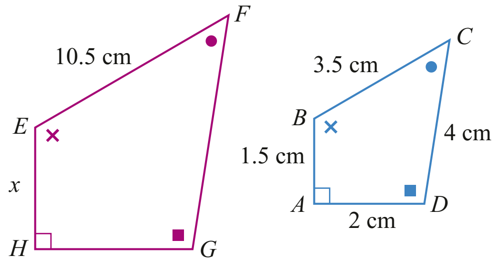


8 Find the length of the unknown side in the following pairs of similar figures.

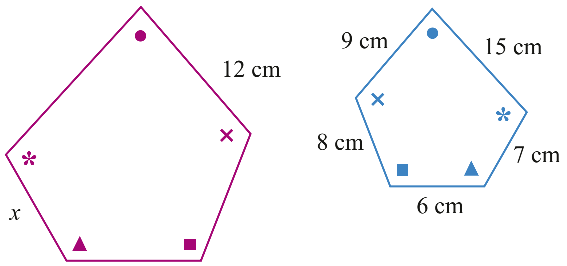
a



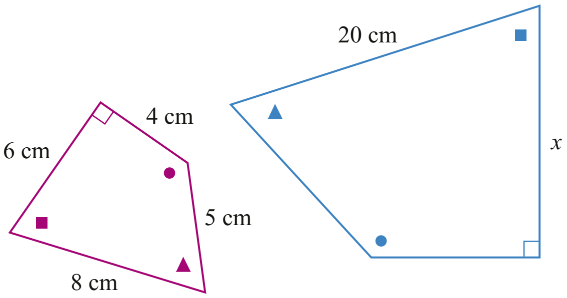
b



c

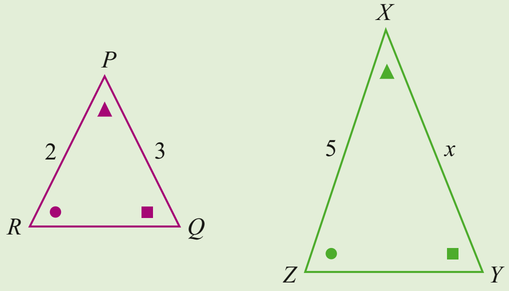


d



EXAMPLE 5

Given that the triangles are similar, find the length of the unknown side. (All lengths are in centimetres.)



This question can be answered by finding and using the enlargement factor as in the questions above. Here is an alternative method for solving these types of questions.

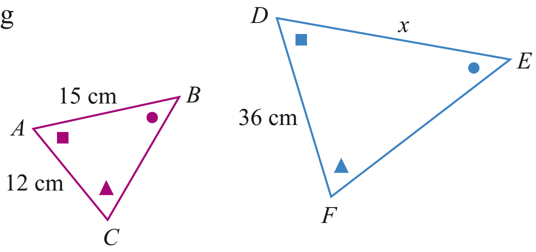
Solve	Think	Apply
$\frac{x}{3} = \frac{5}{2}$ $x = 3 \times \frac{5}{2} = 7.5$ cm	$\frac{XY}{PQ} = \frac{XZ}{PR} \left(= \frac{ZY}{RQ} = k\right)$	As the triangles are similar, the matching side lengths are in the same ratio.

9 For the pair of similar triangles shown, complete the following to find the length of the unknown side.

ED and ___, and *DF* and ___ are matching sides.

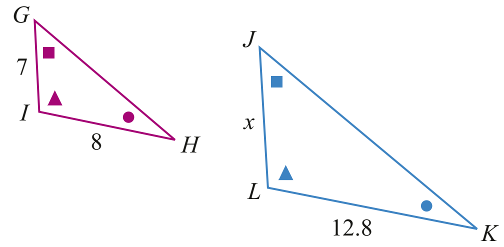
$\frac{x}{\square} = \frac{36}{\square}$

$x =$ ___ $\times \frac{36}{\square} =$ ___ cm

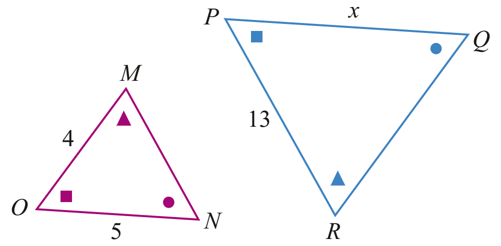


10 Use the method of Example 5 to find the length of the unknown side in the following pairs of similar triangles. (Diagrams are not drawn to scale. All lengths are in centimetres.)

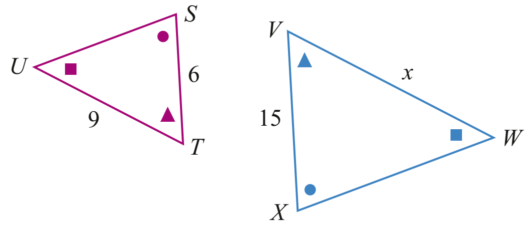
a



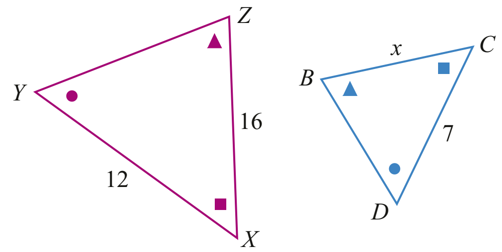
b



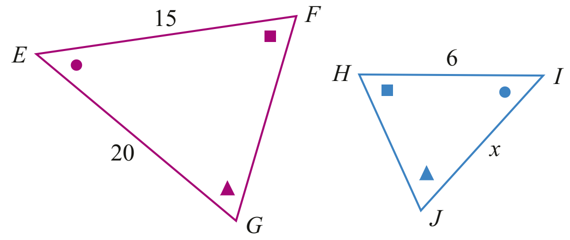
c



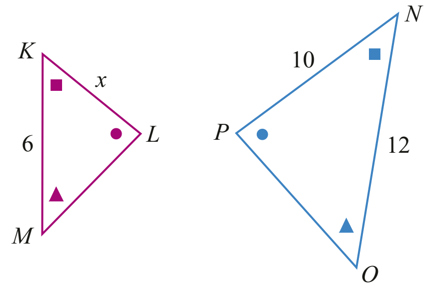
d



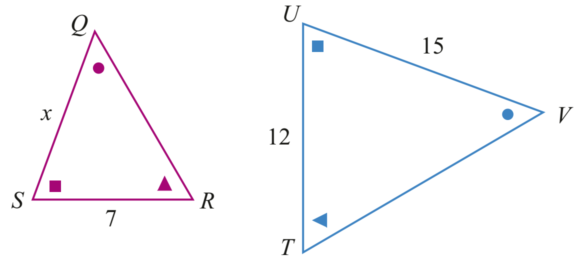
e



f

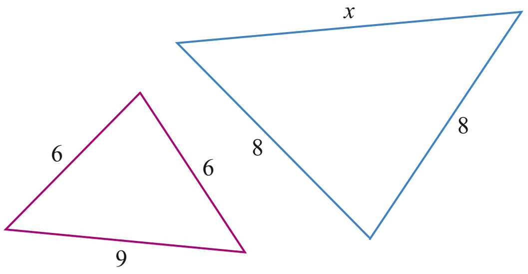


g

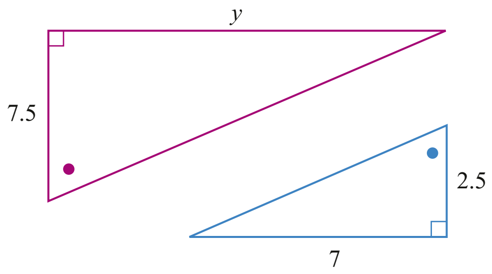


11 These triangles are similar. Find the value of the pronumerals. (All lengths are in centimetres.)

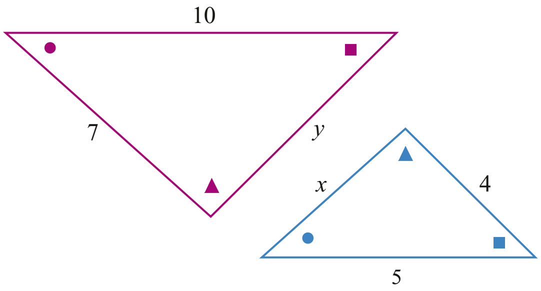
a



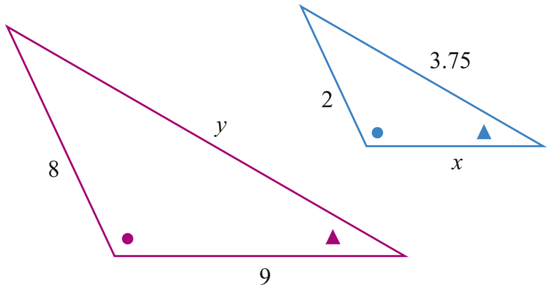
b



c



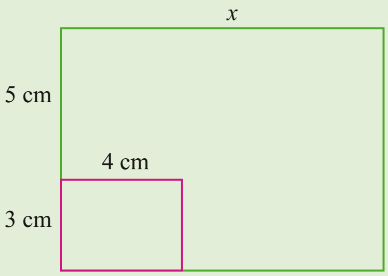
d



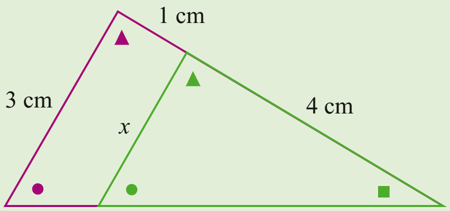
EXAMPLE 6

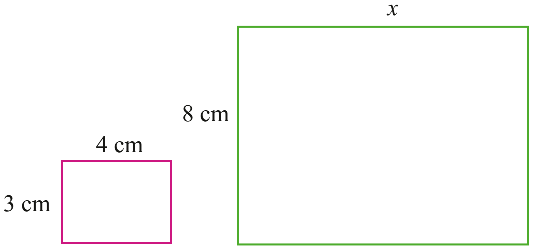
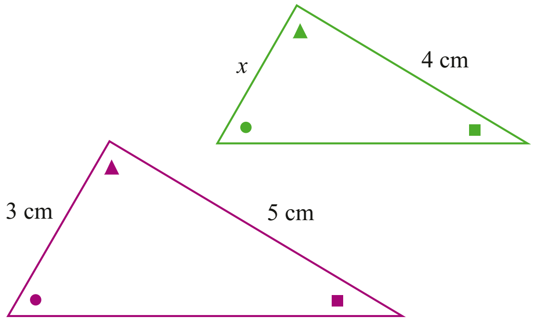
Find the lengths of the unknown sides in these pairs of similar figures.

a

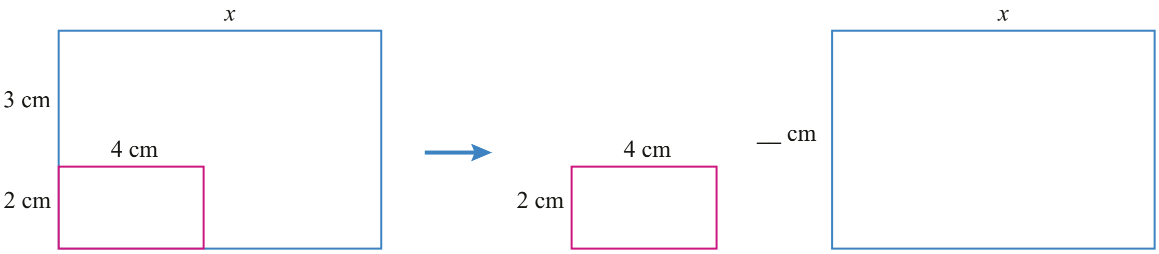


b



	Solve	Think	Apply
a	$k = \frac{8}{3}$ or $\frac{x}{4} = \frac{8}{3}$ $x = \frac{8}{3} \times 4$ $x = 4 \times \frac{8}{3}$ $= 10\frac{2}{3}$ $= 10\frac{2}{3}$ $x = 10\frac{2}{3}$ cm	Separate the similar rectangles. 	Separate into similar figures, find k and use it to find the unknown side.
b	$k = \frac{4}{5}$ or $\frac{x}{3} = \frac{4}{5}$ $x = \frac{4}{5} \times 3$ $x = 3 \times \frac{4}{5}$ $= 2.4$ $= 2.4$ $x = 2.4$ cm	Separate the similar triangles. 	

12 Complete the following to find the length of the unknown side in the pair of similar rectangles.



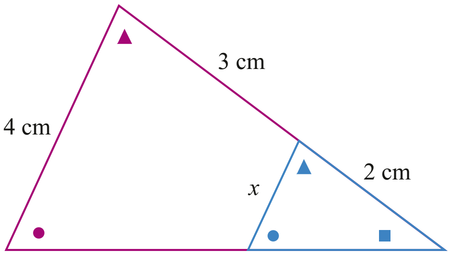
$$\frac{x}{4} = \frac{\square}{2}$$

$$x = 4 \times \frac{\square}{2}$$

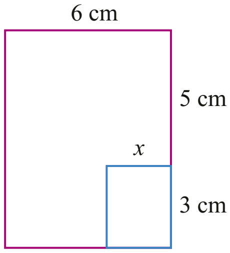
$$= \underline{\quad\quad} \text{ cm}$$

13 Find the lengths of the unknown sides in the following pairs of similar figures.

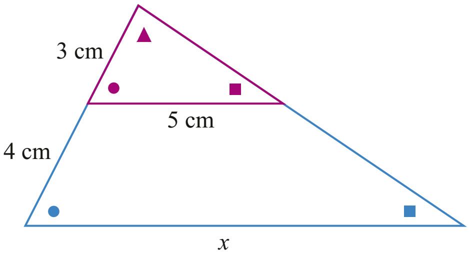
a



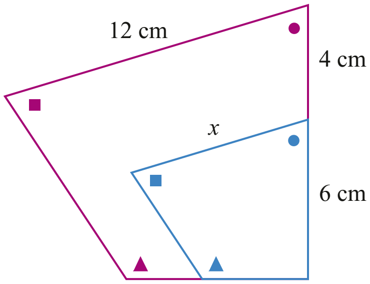
b



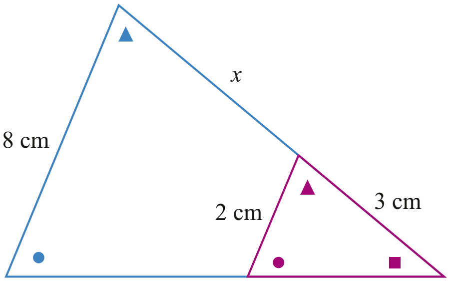
c



d



e

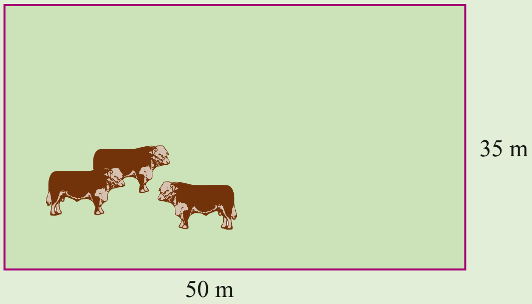


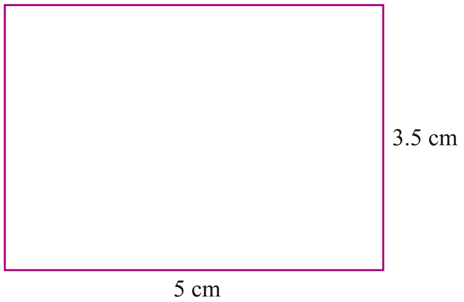
D Applications of similar figures

Scale drawings

EXAMPLE 1

The measurements of a rectangular cattle yard are shown on the diagram. Make a scale drawing of the paddock using a scale of 1 : 1000.

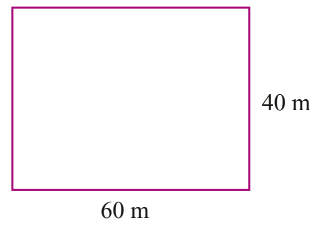


Solve	Think	Apply
Scaled length $= \frac{1}{1000} \times 50$ m $= 5$ cm Scaled breadth $= \frac{1}{1000} \times 35$ m $= 3.5$ cm  	Scale $= \frac{\text{length on drawing}}{\text{real length}}$ $= \frac{1}{1000}$ $\frac{\text{length on drawing}}{50 \text{ m}} = \frac{1}{1000}$ Length on drawing $= \frac{1}{1000} \times 50$ m $= \frac{1}{1000} \times 50 \times 100$ cm $= 5$ cm $\frac{\text{breadth on drawing}}{35 \text{ m}} = \frac{1}{1000}$ Breadth on drawing $= \frac{1}{1000} \times 35$ m $= \frac{1}{1000} \times 35 \times 100$ cm $= 3.5$ cm	If the scale is 1 : n, the length on the drawing or scaled length $= \frac{1}{n} \times$ real length.

Exercise 9D

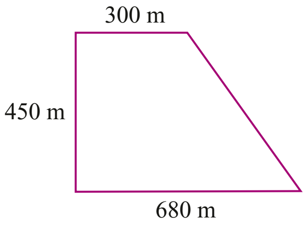
1 Make scale drawings of the following shapes using the scale given.

a



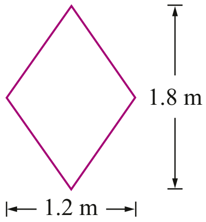
Scale 1 : 1000

b



Scale 1 : 10 000

c



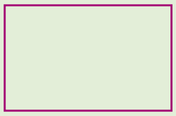
Scale 1 : 50

2 The length of a boat is 11.2 m. What would be its length on a scale drawing with a scale of 1 : 200?

3 The distance from Sydney to Melbourne by air is 710 km. What would this distance be on a map of Australia with a scale of 1: 10 000 000?

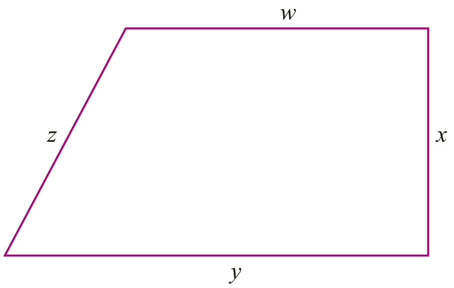
EXAMPLE 2

This scale drawing of a paddock was made by a surveyor using a scale of 1 : 10 000. What are the real dimensions of the paddock?



Solve	Think	Apply
By measurement: scaled length = 2.2 cm scaled breadth = 1.4 cm Real length = 10 000 × 2.2 cm = 22 000 cm = 220 m Real breadth = 10 000 × 1.4 cm = 14 000 cm = 140 m	$\frac{\text{length on drawing}}{\text{real length}} = \frac{1}{10\,000}$ $\frac{2.2 \text{ cm}}{\text{real length}} = \frac{1}{10\,000}$ Take the reciprocal of both sides: $\frac{\text{real length}}{2.2 \text{ cm}} = \frac{10\,000}{1}$ Real length = 10 000 × 2.2 cm $\frac{\text{breadth on drawing}}{\text{real breadth}} = \frac{1}{10\,000}$ $\frac{1.4 \text{ cm}}{\text{real breadth}} = \frac{1}{10\,000}$ Take the reciprocal of both sides: $\frac{\text{real breadth}}{1.4 \text{ cm}} = \frac{10\,000}{1}$ Real breadth = 10 000 × 1.4 cm	If the scale factor is 1: n, real length = n × scaled length.

4 The scale drawing of a field was made by a surveyor using a scale of 1 : 10 000. Find the real dimensions w, x, y, z of this field.

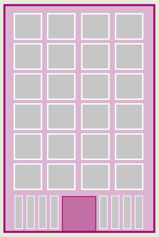


EXAMPLE 3

The real height of a building shown in the scale diagram is 18 m.

a Calculate the scale used.

b What is the width of the building.

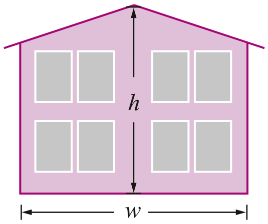


	Solve	Think	Apply
a	By measurement: scaled height = 30 mm Scale factor $= \frac{30}{18\,000}$ $= \frac{1}{600}$ Scale used is 1 : 600	Real height = 18 m = 18 000 mm. Scale factor $= \frac{\text{scaled height}}{\text{real height}}$	Measure the scaled length. Express both measurements in the same units. Use scale $= \frac{\text{scaled height}}{\text{real height}}$.
b	Scaled width = 20 mm Real width = 600 × 20 mm = 12 000 mm = 12 m	Measure the width on the diagram and multiply by the scale.	

5 The real height of the house shown in the scale diagram is 10 m.

a Calculate the scale used.

b What is the real width of the house?



6 The real height of the Eiffel Tower shown on the right is 324 m.

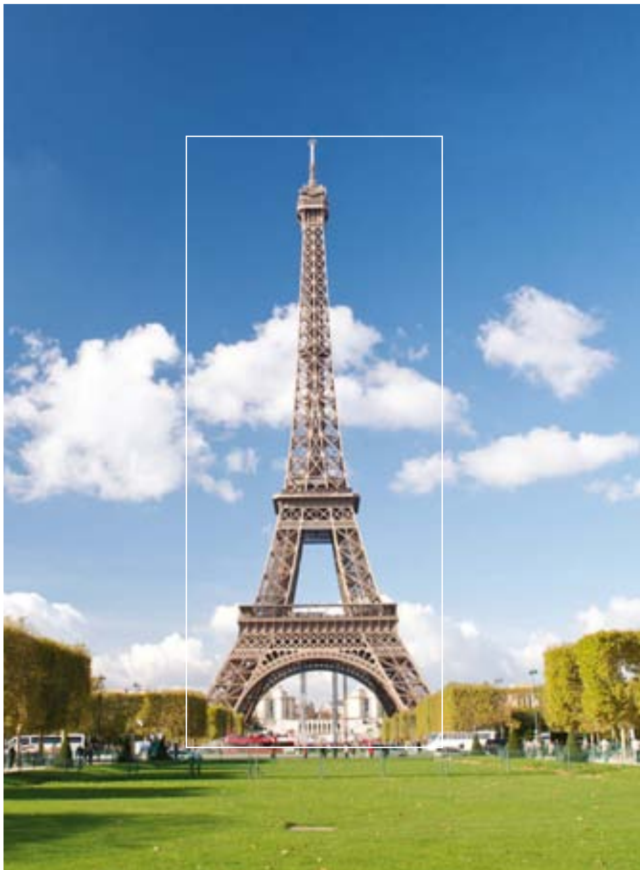
a Calculate the scale used.

b What is the real width of the Eiffel Tower?

7 The real width of the Sydney Opera House shown in the photograph below is 185 m.

a Calculate the scale used.

b What is the real height of the Sydney Opera House?



Investigation 3 Methods of producing scale drawings

Research the different methods that are used for producing scale drawings, including the use of digital technologies.

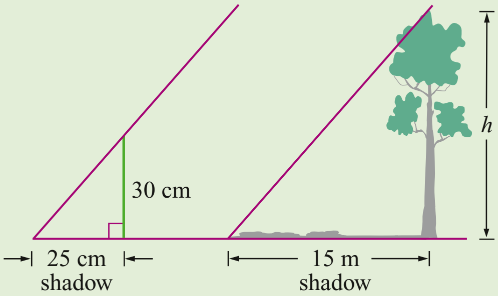
Investigation 4 Uses of scale drawings

Find examples in the media and other key learning areas of the uses of scales in photography, plans and drawings.

Practical problems involving similar triangles

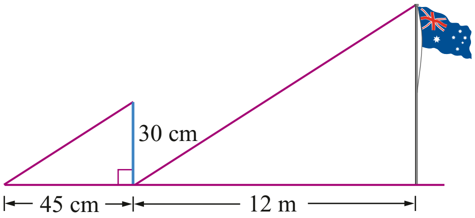
EXAMPLE 4

When a 30 cm ruler is stood vertically on the ground, it casts a 25 cm shadow. At the same time, a tree casts a shadow of length 15 m. How high is the tree?

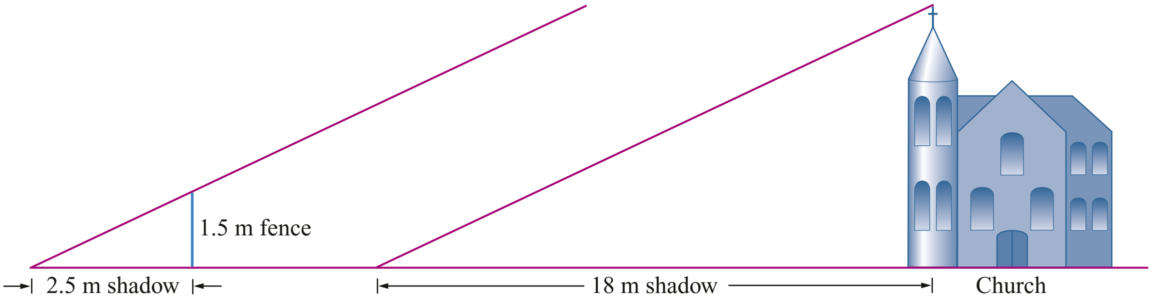


Solve	Think	Apply
$\frac{h}{30} = \frac{1500}{25}$ $h = 30 \times \frac{1500}{25}$ $= 1800$ The height of the tree is 1800 cm or 18 m.	The triangles are similar, hence $\frac{\text{height of tree}}{\text{height of ruler}} = \frac{\text{shadow of tree}}{\text{shadow of ruler}}$	Express all measurements in the same units. The triangles are similar, so the matching side lengths are in the same ratio.

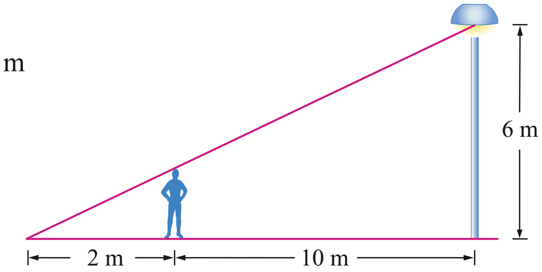
8 On a bright sunny day the shadow case by a flagpole is 12 m long. At the same time the shadow cast by a 30 cm ruler is 45 cm long. Find the height of the flagpole.



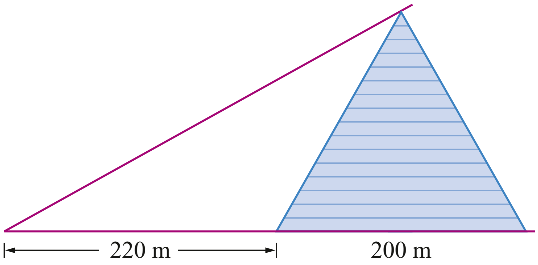
9 A picket fence surrounding a church is 1.5 m high. At a certain time of the day the shadow of the fence was 2.5 m long and the shadow of the church's steeple was 18 m long. How high is the steeple?



10 A light pole has a globe that is 6 m above the ground. How tall is a boy who casts a 2 m long shadow when standing 10 m from the base of the pole?



11 An Egyptian pyramid casts a 220 m shadow from its base at the same instant that the shadow cast by a metre rule is 3.2 m long. How high is the pyramid?



Language in mathematics

1 Match each word or phrase with its meaning.

- **a** corresponding
- **b** orientate
- **c** perpendicular
- **d** in proportion

- **A** at right angles to
- **B** in the same ratio
- **C** matching
- **D** point the same way

2 In your own words, write the meaning of these terms.

a similar figures **b** equiangular

3 Complete the sentence:

The enlargement factor is also called the ________ factor.

4 Explain how to find the matching sides in similar triangles.

5 Explain why any two equilateral triangles or any two squares are similar. When are they congruent?

6 Explain, with diagrams, why not all rectangles are similar.

7 Use a dictionary to explain the difference in meaning between:

a alternative and alternate **b** perpendicular and vertical

Terms

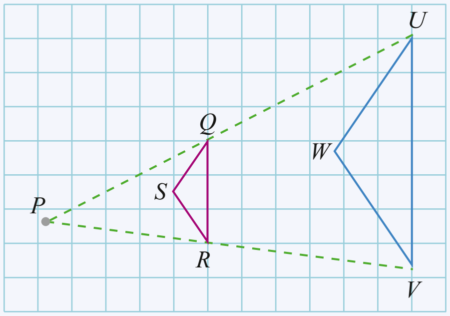
congruent
corresponding
enlargement
equiangular
equilateral
isosceles
matching
original
perpendicular
proportion
similar
symbol
transformation
vertices

Check your skills

1 The triangle *QRS* has been enlarged to *UVW* using *P* as the centre of enlargement. $PQ = 24$ mm and $QU = 30$ mm. The enlargement factor is:

A 1.25 **B** 2.25

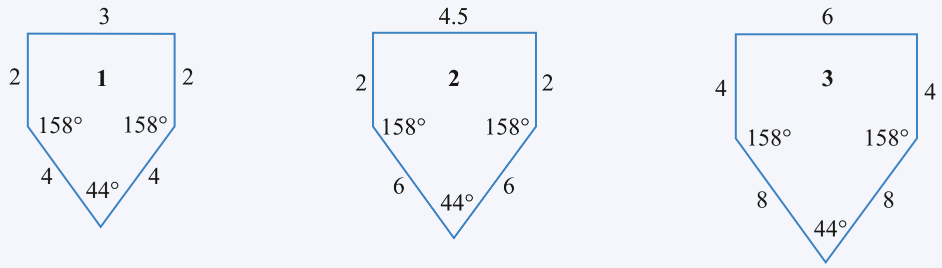
C $\frac{4}{5}$ **D** $\frac{4}{9}$



2 Which of the following statements is always true?

- **A** If two figures are similar then the matching angles are equal and the lengths of the matching sides are equal.
- **B** If two figures are similar then the matching angles are equal and the lengths of the matching sides are in the same ratio.
- **C** If two figures are similar then the matching angles are in the same ratio (but not equal) and the lengths of the matching sides are equal.
- **D** If two figures are similar then the matching angles are in the same ratio (but not equal) and the lengths of the matching sides are in the same ratio to each other.

3 Which of these figures are similar?



A 1 and 2

B 2 and 3

C 1 and 3

D 1, 2 and 3

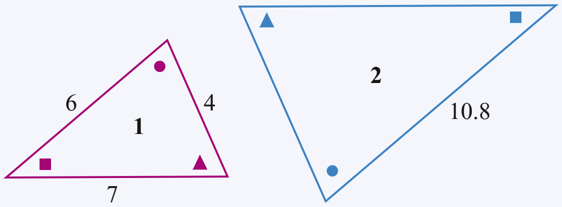
4 These triangles are similar. What is the enlargement factor (from 1 to 2)?

A 1.5

B 1.8

C 2.7

D 16.8



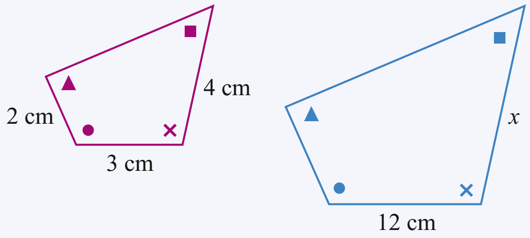
5 What is the value of the pronumeral in these similar figures?

A 24 cm

B 16 cm

C 12 cm

D 9.6 cm



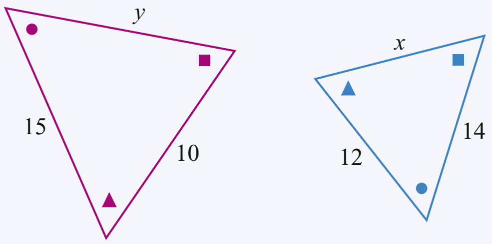
6 The values of the pronumerals in these similar triangles are:

A $x = 8, y = 11.2$

B $x = 8, y = 17.5$

C $x = 12.5, y = 11.2$

D $x = 12.5, y = 17.5$



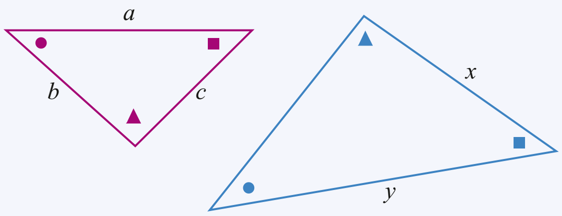
7 Which of the following statements about these similar triangles is correct.

A $\frac{x}{a} = \frac{y}{b}$

B $\frac{x}{b} = \frac{y}{a}$

C $\frac{x}{c} = \frac{y}{a}$

D $\frac{x}{c} = \frac{y}{b}$



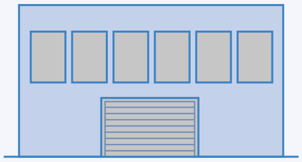
8 The real height of the warehouse shown in this scale diagram is 12 m. Its real length is:

A 21 m

B 120 m

C 210 m

D 60 m



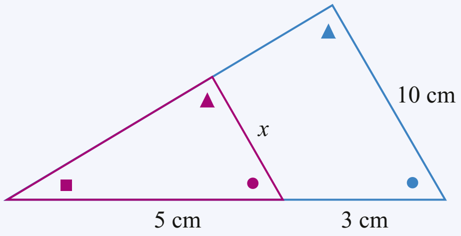
9 Given that the triangles are similar, the value of x in the diagram is:

A 3.75 cm

B 6 cm

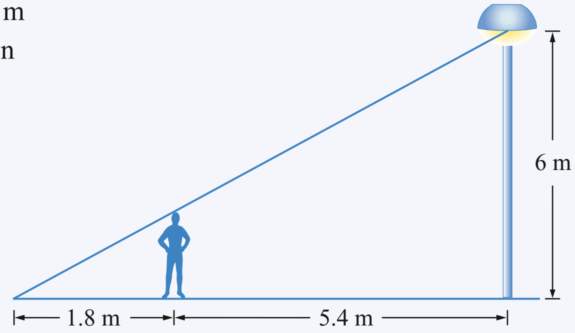
C $16\frac{2}{3}$ cm

D 6.25 cm



10 A street light is 6 m above the ground. A boy standing 5.4 m from the base of the light casts a shadow 1.8 m long. Given that the triangles are similar, the height of the boy is:

A 2.4 m **B** 2 m
C 1.62 m **D** 1.5 m



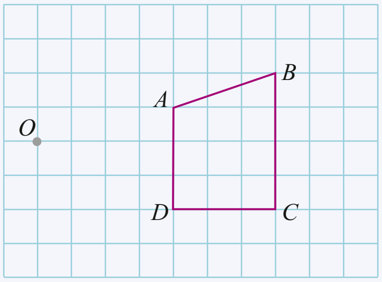
If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2, 3	4–7	8–10
Section	A	B	C	D

9A Review set

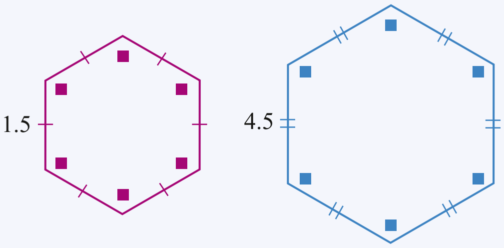
1 **a** Enlarge this figure using an enlargement factor of 2 and O as the centre of enlargement.

b Label the vertices of the enlarged figure and name the pairs of corresponding sides in the similar figures.

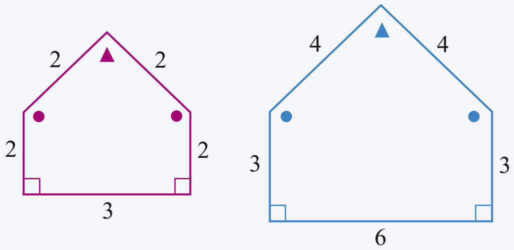


2 Use the information given to determine whether or not the following pairs of figures are similar. If they are similar, state the enlargement factor.

a



b



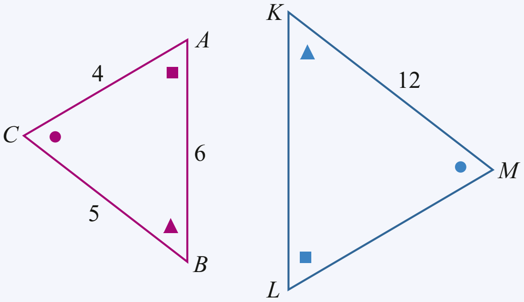
3 Determine whether the following statements are true or false.

a Any two equilateral triangles are similar.

b Any two isosceles triangles are similar.

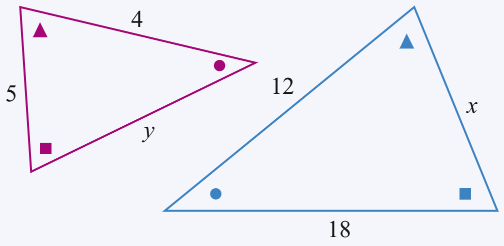
4 **a** Name the matching (corresponding) sides in these similar triangles.

b Hence find the enlargement factor.

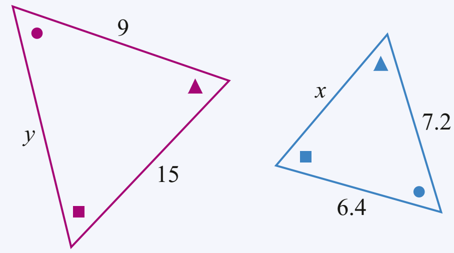


5 Find the enlargement factor and hence the length of the unknown sides in these similar triangles.

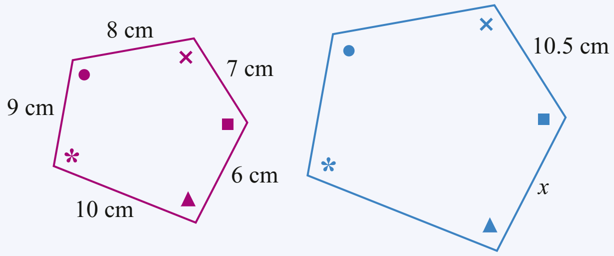
a



b

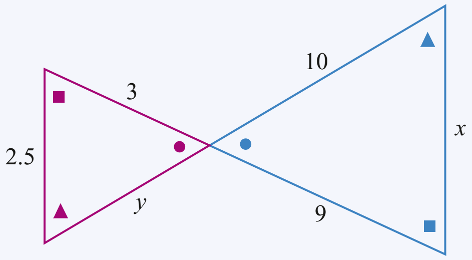


6 In the similar figures shown, find the enlargement factor and hence the length of the unknown side.

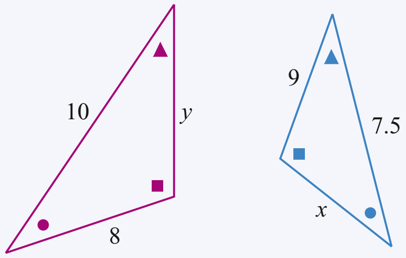


7 Use the ratios of matching sides to find the length of the unknown sides in these pairs of similar triangles.

a

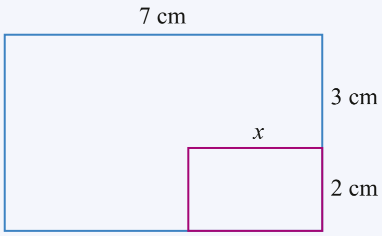


b

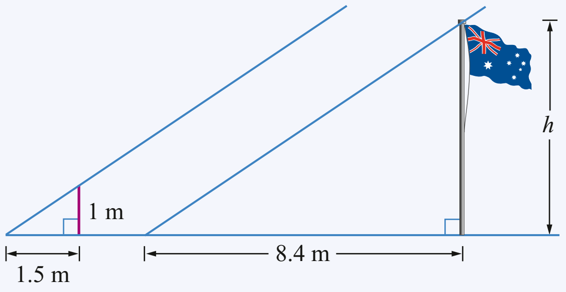


8 The length of a block of land is 49 m. What would be its length on a scale drawing with a scale of 1 : 500?

9 Find the value of x in this diagram, given that the rectangles are similar.



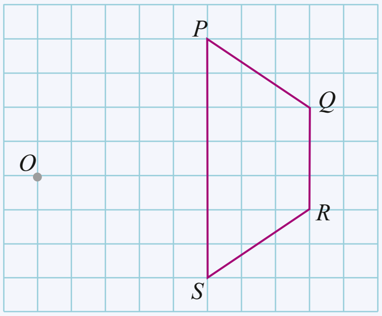
10 A vertical metre rule casts a shadow 1.5 m long. At the same time a flagpole casts a shadow 8.4 m long. How high is the flagpole, given that the triangles are similar?



9B Review set

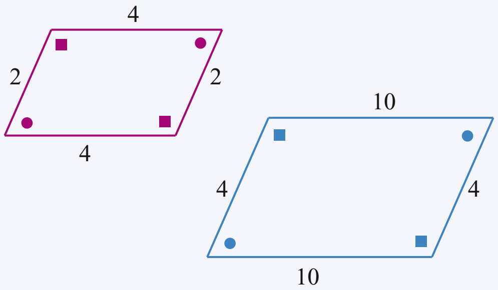
1 **a** Enlarge this figure using an enlargement factor of 2.5 and O as the centre of enlargement.

b Label the vertices of the enlarged figure and name the pairs of corresponding sides in the similar figures.

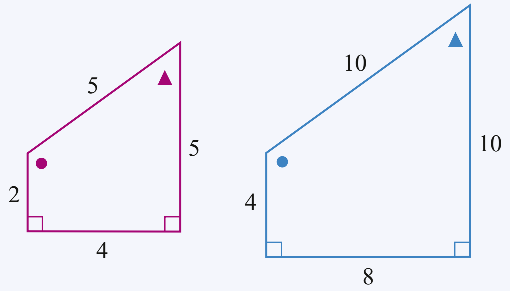


2 Use the information given to determine whether or not the following pairs of figures are similar. If they are similar, state the enlargement factor.

a



b

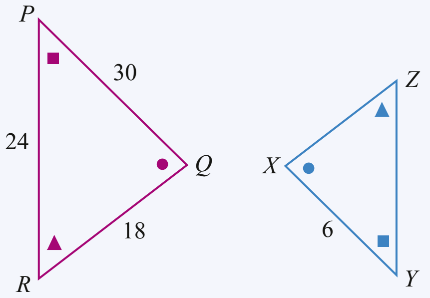


3 Determine whether the following statements are true or false.

a Any two squares are similar. **b** Any two rectangles are similar. **c** Any two circles are similar.

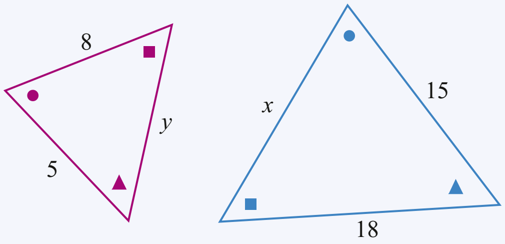
4 **a** Name the matching (corresponding) sides in these similar triangles.

b Hence find the enlargement factor.

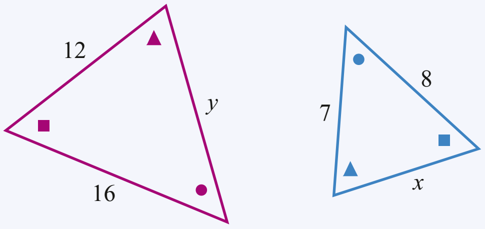


5 Find the enlargement factor and hence the length of the unknown sides in these pairs of similar triangles.

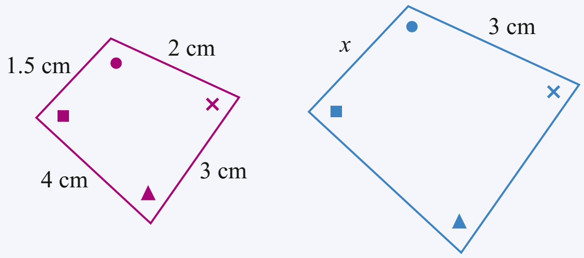
a



b

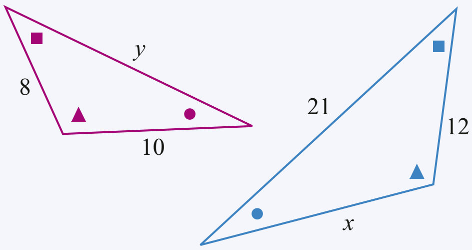


6 For these similar figures, find the enlargement factor and hence the length of the unknown side.

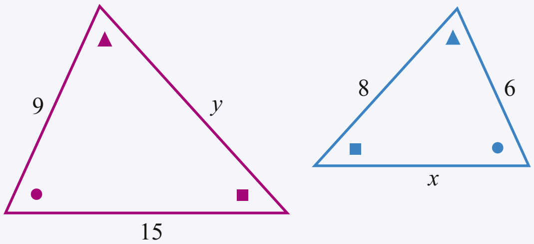


7 Use the ratios of matching sides to find the length of the unknown sides in these pairs of similar triangles.

a

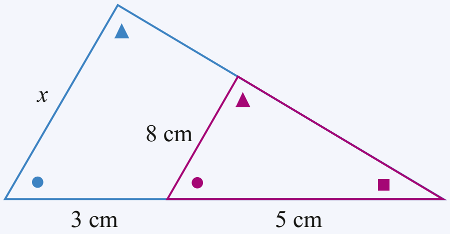


b

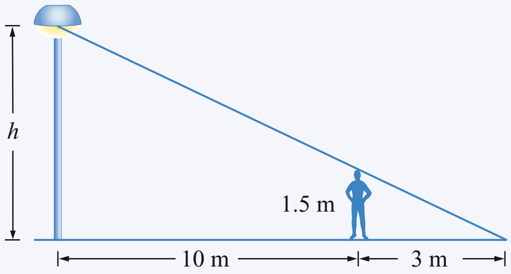


8 This scale drawing of a rectangular paddock was made by a surveyor using a scale of 1 : 10 000. What are the real dimensions of the paddock?

9 Find x in the diagram, given that the triangles are similar.



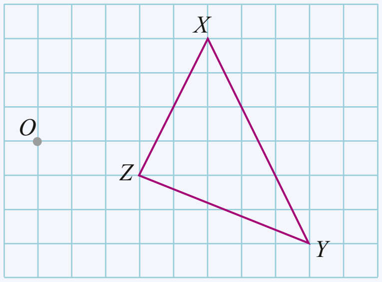
10 A boy, 1.5 m tall, stands 10 m from a lamp post and casts a shadow 3 m long. How high is the lamp post, given that the triangles are similar?



9C Review set

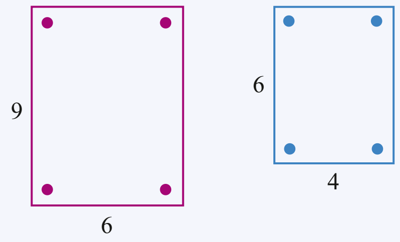
1 **a** Enlarge this figure using a scale factor of $\frac{1}{2}$ and O as the centre of enlargement.

b Label the vertices of the enlarged figure and name the pairs of corresponding sides in the similar figures.

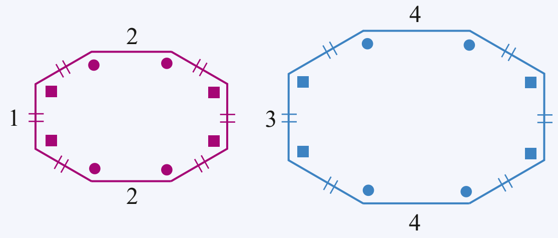


2 Use the information given to determine whether or not the following pairs of figures are similar. If they are similar, state the enlargement factor.

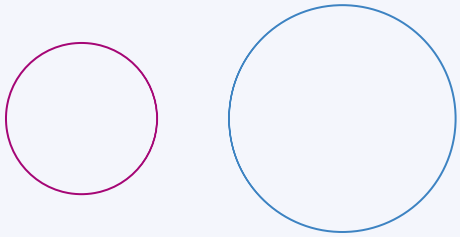
a



b

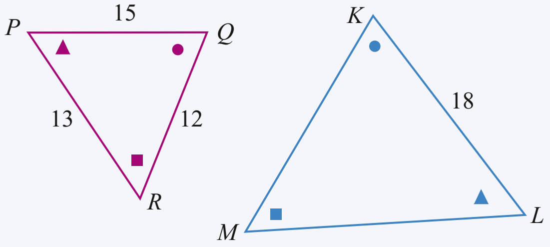


3 Determine the scale factor for this pair of circles.



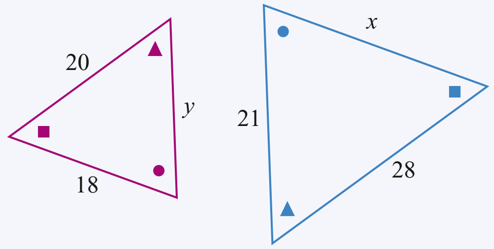
4 **a** Name the matching (corresponding) sides in these similar triangles.

b Hence find the enlargement factor.

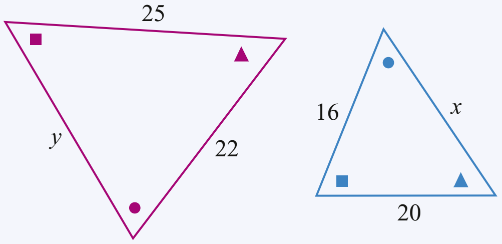


5 Find the enlargement factor and hence the length of the unknown sides in the similar triangles shown.

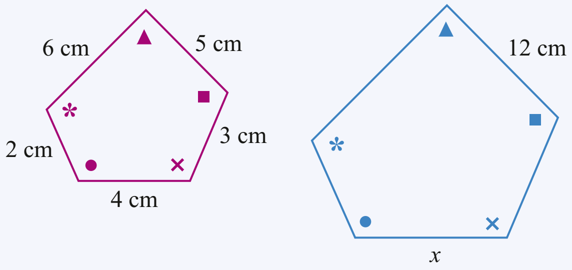
a



b

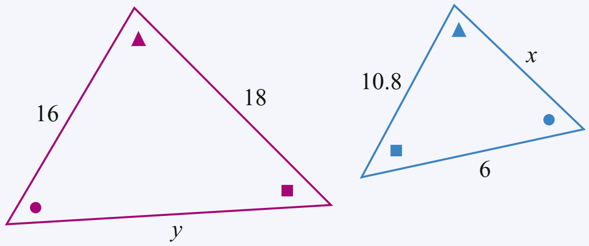


6 For these similar figures, find the enlargement factor and hence the length of the unknown side.

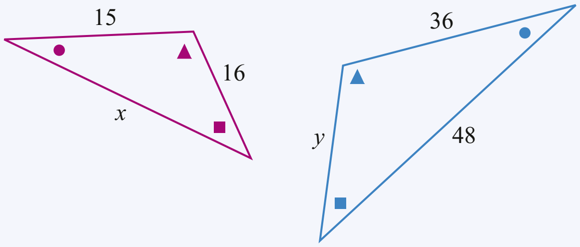


7 Use the ratios of matching sides to find the length of the unknown sides in these pairs of similar triangles.

a

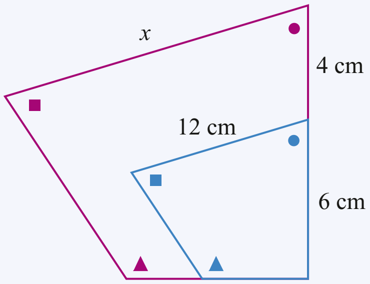


b

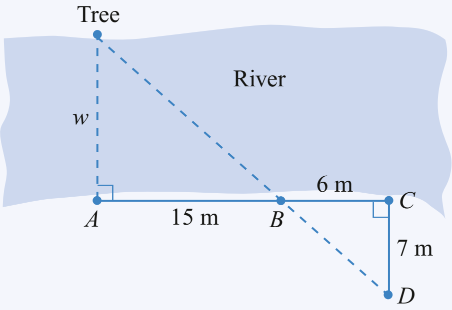


8 The distance by air from Sydney to Brisbane is 730 km. What would this distance be on a map with a scale of 1 : 10 000 000?

9 For this pair of similar figures, find the scale factor and hence the length of the unknown side.



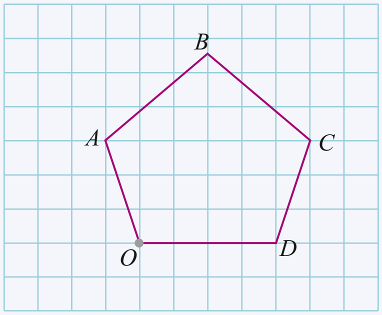
10 To determine the width of a river, a surveyor places pegs *A*, *B*, *C* and *D* along one bank, as shown. Calculate the width of the river, given that the triangles are similar.



9D Review set

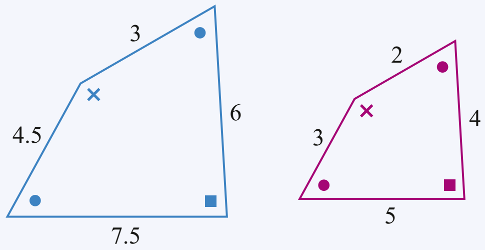
1 **a** Enlarge this figure using an enlargement factor of 1.5 and *O* as the centre of enlargement.

b Label the vertices of the enlarged figure and name the pairs of corresponding sides in the similar figures.

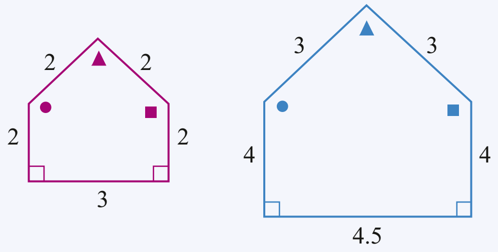


2 Use the information given to determine whether or not the following pairs of figures are similar. If they are similar, state the enlargement factor.

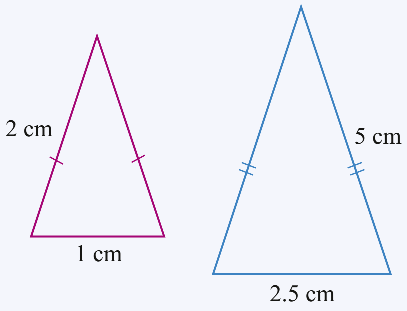
a



b

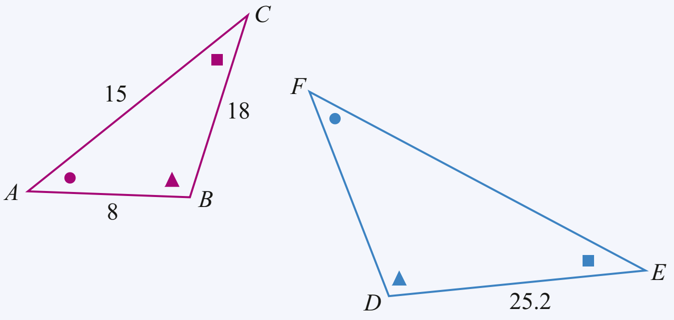


3 What extra condition would make the two isosceles triangles similar?



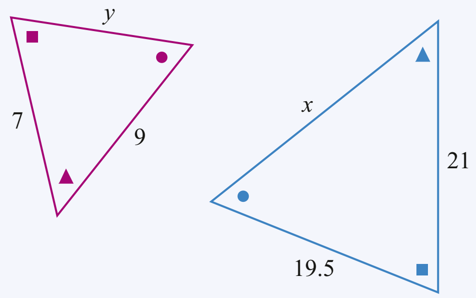
4 **a** Name the matching (corresponding) sides in these similar triangles.

b Hence find the enlargement factor.

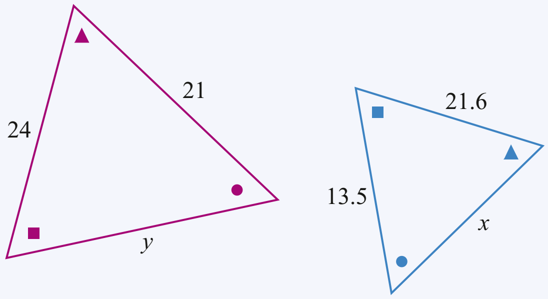


5 Find the enlargement factor and hence the length of the unknown sides in these similar triangles.

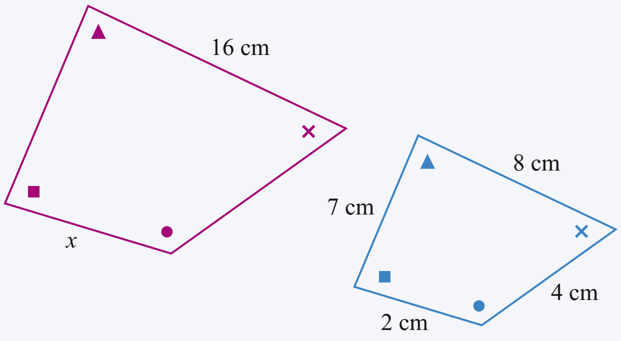
a



b

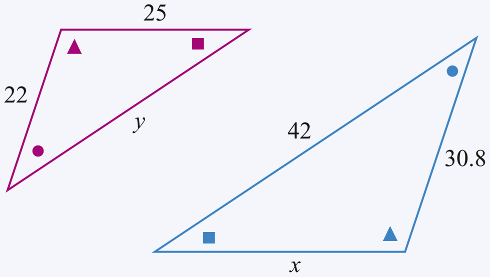


6 For these similar figures, find the enlargement factor and hence the length of the unknown side.

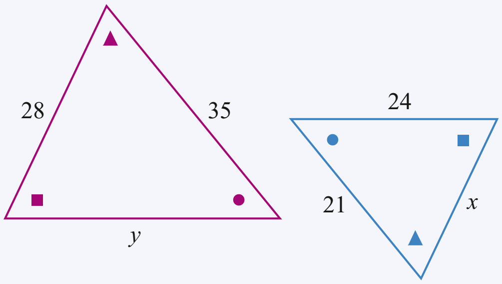


7 Use the ratios of matching sides to find the length of the unknown sides in these pairs of similar triangles.

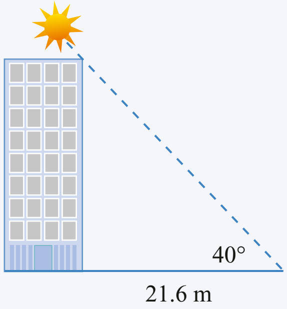
a



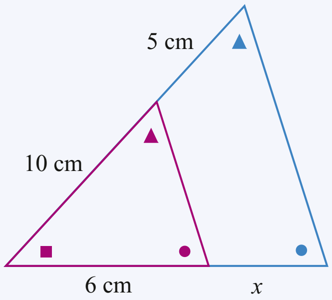
b



8 The length of the shadow cast by a building is 21.6 m when the angle of elevation of the Sun is 40°, as shown in the diagram. Make a scale drawing using a scale of 1 : 600 and determine the height of the building.



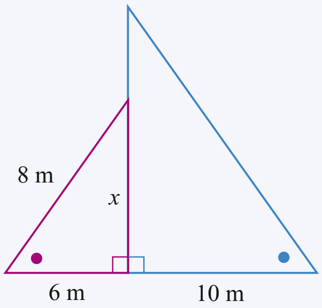
9 Find the value of x in the diagram, given that the triangles are similar.



10 A telecommunications tower has two cables attached on opposite sides of it. Each cable makes the same angle with the ground. The smaller cable, which is 8 m long, is attached to the ground 6 m from the base of the tower. The other cable is attached 10 m from its base. The triangles are similar.

a Calculate the height, x, at which the smaller cable is attached to the tower.

b Hence calculate the height of the tower.



10

Linear and non-linear relationships

This chapter deals with operations on the number plane.

After completing this chapter you should be able to:

- determine the midpoint of an interval from a diagram
- use Pythagoras' theorem to find the length of a line interval joining two points
- find the gradient of an interval using a diagram
- determine whether a line has positive or negative gradient
- find the gradient of a line using a right-angled triangle
- draw graphs of horizontal and vertical lines
- graph a variety of linear relationships
- graph simple non-linear relationships.

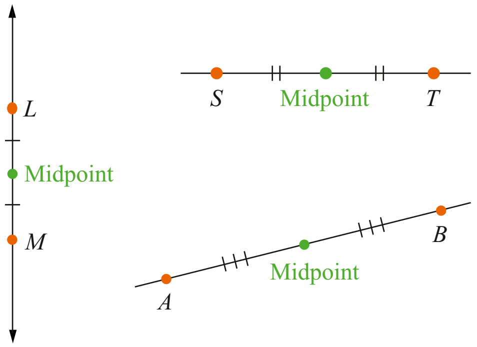
NSW Syllabus references: 5.1 N&A Linear relationships, 5.1 N&A Non-linear relationships

Outcomes: MA5.1-1WM, MA5.1-3WM, MA5.1-6NA, MA5.1-7NA

NUMBER & ALGEBRA – ACMNA214, ACMNA215, ACMNA239, ACMNA294, ACMNA296

A Midpoint

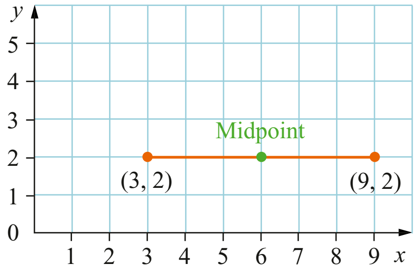
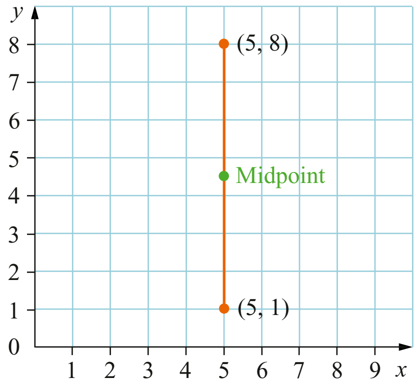
The **midpoint** of an interval is the point halfway between the endpoints of the interval. It is the average of the x-coordinates and the average of the y-coordinates.



EXAMPLE 1

Plot each pair of points, join them with a straight line and find the coordinates of the midpoint.

a (3, 2) and (9, 2) **b** (5, 1) and (5, 8)

	Solve	Think	Apply
a	 The midpoint coordinates are (6, 2).	The length of this horizontal line interval is $9 - 3 = 6$ units, so the midpoint is 3 units from each end. $3 + 3 = 6$ or $9 - 3 = 6$ Both points and the midpoint have a y-coordinate of 2. The coordinates of the midpoint are (6, 2).	Horizontal lines and their midpoint all have the same y-coordinate. Vertical lines and their midpoint all have the same x-coordinate.
b	 The midpoint coordinates are $(5, 4\frac{1}{2})$.	The length of this vertical line interval is $8 - 1 = 7$ units, so the midpoint is $3\frac{1}{2}$ units from each end. $1 + 3\frac{1}{2} = 4\frac{1}{2}$ or $8 - 3\frac{1}{2} = 4\frac{1}{2}$ Both points and the midpoint have an x-coordinate of 5. The coordinates of the midpoint are $(5, 4\frac{1}{2})$.	

Exercise 10A

1 Plot the following pairs of points, join them with a horizontal line and find the coordinates of the midpoint.

a (1, 4) and (9, 4) **b** (2, 3) and (12, 3) **c** (3, 6) and (7, 6)

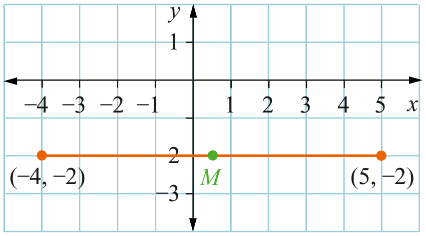
2 Plot the following pairs of points, join them with a vertical line and find the coordinates of the midpoint.

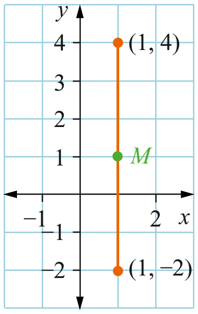
a (2, 1) and (2, 11) **b** (5, 3) and (5, 7) **c** (3, 4) and (3, 8)

EXAMPLE 2

Plot each pair of points, join them with a horizontal or vertical line, and find the coordinates of the midpoint.

a (−4, −2) and (5, −2) **b** (1, −2) and (1, 4)

<table>
<tr><th></th><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr><td>a</td><td>

The midpoint coordinate is $(\frac{1}{2}, -2)$.</td><td>The length of the interval is $5 - (-4) = 9$ units.
The midpoint is $\frac{9}{2} = 4\frac{1}{2}$ units from each end.
$-4 + 4\frac{1}{2} = \frac{1}{2}$ or $5 - 4\frac{1}{2} = \frac{1}{2}$
$\therefore$ The midpoint is $(\frac{1}{2}, -2)$.</td><td rowspan="2">Only one length needs to be calculated for horizontal or vertical lines.</td></tr>
<tr><td>b</td><td>

The midpoint coordinate is (1, 1).</td><td>The length of the interval is $4 - (-2) = 6$ units.
The midpoint is $\frac{6}{2} = 3$ units from each end.
$-2 + 3 = 1$ or $4 - 3 = 1$
$\therefore$ The midpoint is (1, 1).</td></tr>
</table>

3 Plot each pair of points, join them with a horizontal or vertical line, and find the coordinates of the midpoint.

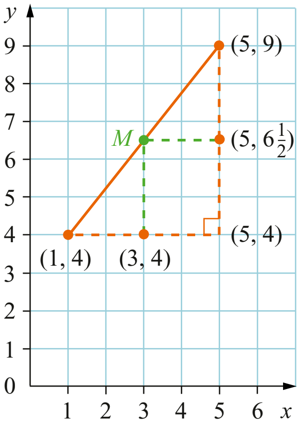
a (1, 2) and (1, 4) **b** (−2, 1) and (−2, 5) **c** (3, −1) and (3, 3)
d (5, 1) and (3, 1) **e** (5, 2) and (−3, 2) **f** (−4, −3) and (−4, 5)
g (0, 0) and (0, 9) **h** (0, 0) and (3, 0) **i** (7, 14) and (−3, 14)

EXAMPLE 3

*An **oblique** line is neither vertical nor horizontal.*

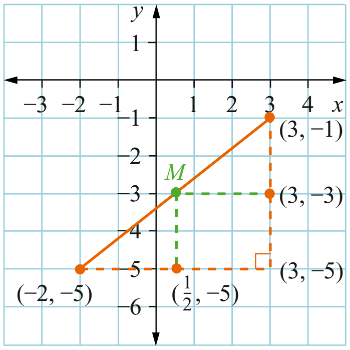
Plot each pair of points, join them with an oblique line, and find the coordinates of the midpoint.

a (1, 4) and (5, 9) **b** (−2, −5) and (3, −1)

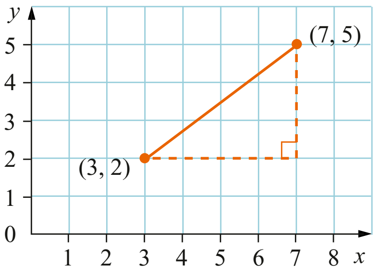
<table>
<tr><th></th><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr><td>a</td><td>

Midpoint of the line interval is $(3, 6\frac{1}{2})$.</td><td><i>Step 1:</i> Plot the points and join with an oblique straight line.
<i>Step 2:</i> Draw vertical and horizontal lines to make a right-angled triangle.
<i>Step 3:</i> Write the coordinates of the third vertex: (5, 4).
<i>Step 4:</i> Find the midpoints of the horizontal and vertical intervals: (3, 4) and $(5, 6\frac{1}{2})$.
<i>Step 5:</i> Join these points to the oblique line interval.</td><td>An oblique line can be formed into a triangle.
The vertical and horizontal lines give the midpoint.</td></tr>
</table>

EXAMPLE 3 CONTINUED

	Solve	Think	Apply
b	 Midpoint of the line interval is ($\frac{1}{2}$, −3).	Use the same steps as in part **a**. The third vertex is (3, −5). Half width $= \frac{3-(-2)}{2}$ $= \frac{5}{2} = 2\frac{1}{2}$ units *x*-coordinate $= -2 + 2\frac{1}{2} = \frac{1}{2}$ or $3 - 2\frac{1}{2} = \frac{1}{2}$ Half height $= \frac{-1-(-5)}{2}$ $= \frac{4}{2} = 2$ units *y*-coordinate $= -5 + 2 = -3$ or $-1 - 2 = -3$	The width and height are found by subtracting the *x*-coordinates and the *y*-coordinates. If a negative length results, reverse the sign and make it positive.

4 The points (3, 2) and (7, 5) are joined with an oblique line as shown.
Complete the triangle to find the coordinates of the midpoint.
The coordinates of the third vertex on the triangle are (___, ___).
The midpoint of the horizontal interval is (___, 2).
The midpoint of the vertical interval is (___, $3\frac{1}{2}$).
Join these points to the oblique line interval.
The midpoint is (5, ___).

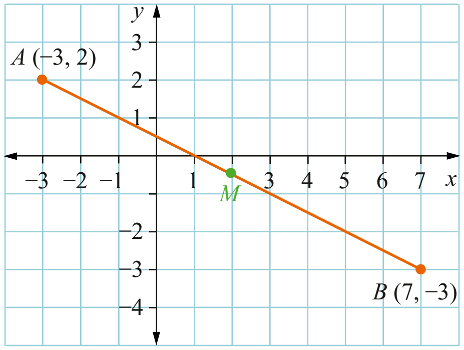


5 Plot each pair of points, join them with an oblique line and find the coordinates of the midpoint.

a (5, 2) and (1, 4)
b (2, 0) and (0, 8)
c (3, −1) and (1, −5)
d (−2, 5) and (2, −5)
e (2, −1) and (−1, 3)
f (5, 7) and (−3, −1)
g (−2, 3) and (−5, 1)
h (−4, −4) and (−1, 1)
i (−2, −3) and (2, 3)

EXAMPLE 4

Find the midpoint of the line segment joining points *A*(−3, 2) and *B*(7, −3).

Solve	Think	Apply
 Midpoint of the line interval is (2, $-\frac{1}{2}$).	Draw the line segment *AB*. As the midpoint *M* is halfway between *A* and *B*, the *x*-coordinate of *M* will be halfway between the *x*-coordinates of *A* and *B*. *x*-coordinate of *M* $= \frac{-3+7}{2}$ $= \frac{4}{2} = 2$ Similarly, the *y*-coordinate of *M* will be halfway between the *y*-coordinates of *A* and *B*. *y*-coordinate of *M* $= \frac{2+(-3)}{2} = -\frac{1}{2}$	Averaging the two *x*-coordinates and the two *y*-coordinates will give the coordinates of the midpoint. This is more efficient than the previous method.

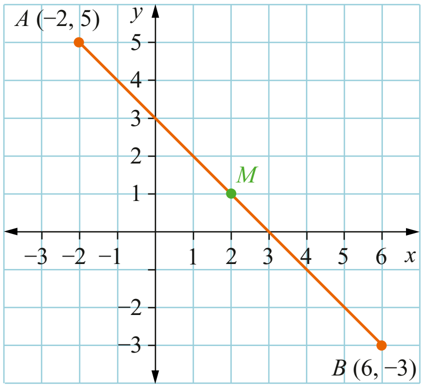
6 Complete the following to find the midpoint M of the line segment joining points $A(-2, 5)$ and $B(6, -3)$. The x-coordinate of M will be ___ between the x-coordinates of ___ and B.

x-coordinate of $M = \frac{-2 + \square}{2} =$ ___

The y-coordinate of M will be halfway between the y-___ of A and ___.

y-coordinate of $M = \frac{\square + (-3)}{2} =$ ___

Midpoint of the line interval is (___, ___).



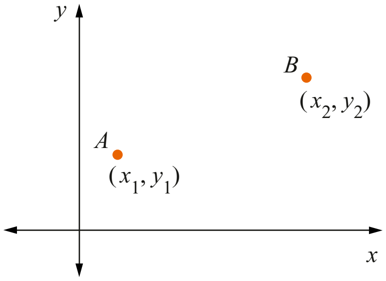
7 Find the midpoint of the line segment joining each pair of points.

a $(5, -8)$ and $(3, -3)$
b $(-2, -2)$ and $(6, -3)$
c $(0, 6)$ and $(6, 0)$
d $(15, 27)$ and $(17, 3)$
e $(51, -12)$ and $(-36, 11)$
f $(0, 0)$ and $(-7, -11)$
g $(2a, 4b)$ and $(4a, 6b)$
h $(5p, 3q)$ and $(-3p, 5q)$
i $(-4r, 7s)$ and $(4r, -7s)$

Extension

8 This diagram shows the points $A(x_1, y_1)$ and $B(x_2, y_2)$.

a Copy the diagram and join the points A and B.
b Complete a right-angled triangle.
c Show that the coordinates of the third vertex are (x_2, y_1).
d Average the x-coordinates and the y-coordinates to find the midpoint of the horizontal and vertical sides.
e Show that midpoint of AB is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
This is the midpoint formula.



EXAMPLE 5

Use the midpoint formula to find the midpoint of the line joining points $(-3, 2)$ and $(6, -5)$.

Solve	Think	Apply
Substitute into the formula: $\left(\frac{-3 + 6}{2}, \frac{2 + -5}{2}\right) = \left(\frac{3}{2}, -\frac{3}{2}\right)$ $= (1\frac{1}{2}, -1\frac{1}{2})$ The midpoint is $(1\frac{1}{2}, -1\frac{1}{2})$.	The midpoint formula is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. Label the points (x_1, y_1) and (x_2, y_2); that is, $(-3, 2)$ and $(6, -5)$.	The midpoint formula averages the coordinates. Choose any point as (x_1, y_1); the other is (x_2, y_2).

9 Complete the following using the midpoint formula to find the midpoint of the line joining the points $(-4, 7)$ and $(2, -1)$.

The midpoint formula is $\left(\frac{x_1 + x_2}{\square}, \frac{y_1 + y_2}{\square}\right)$.

$x_1 =$ ___, $x_2 = 2$
$y_1 =$ ___, $y_2 = -1$

Substitute into the formula:

$\left(\frac{-4 + \square}{2}, \frac{7 + \square}{2}\right) = \left(\frac{\square}{2}, \frac{\square}{2}\right) =$ (___, ___)

The midpoint is (___, ___).

10 Use the midpoint formula to find the midpoint of the line joining these points.

a (1, 2) and (5, 8) **b** (−3, 5) and (7, −2)

c $(-5\frac{1}{2}, 3\frac{1}{2})$ and (6, −2) **d** (−4, −5) and (1, 3)

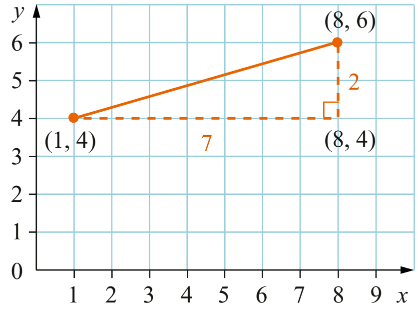
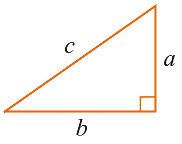
11 Explain why the midpoint formula is correct.

B Distance between two points

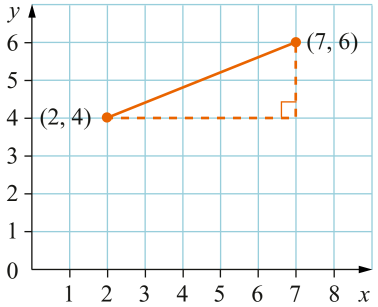
Pythagoras' rule can be used to find the length of the line interval joining two points.

EXAMPLE 1

Plot points (1, 4) and (8, 6) on a number plane and find the distance between them.

Solve	Think	Apply
 Distance between the points is 7.28 units (2 decimal places).	*Step 1:* Plot the points and draw lines to make a right-angled triangle. *Step 2:* Find the lengths of the sides. Vertical = 6 − 4 = 2 units Horizontal = 8 − 1 = 7 units *Step 3:* Use Pythagoras' theorem: $c^2 = a^2 + b^2$ $= 2^2 + 7^2$ $= 53$ $c = \sqrt{53}$ ≈ 7.28 (2 decimal places) 	Form the right-angled triangle in the same way as the midpoint section (Example 3). Use Pythagoras' theorem to find the hypotenuse.

Exercise 10B



1 The points (2, 4) and (7, 6) are joined with an oblique line as shown. Complete the following to find the distance between them.
The coordinates of the third vertex are (___, 4).
The lengths of the sides are:
Vertical = ___ − 4 = ___ units
Horizontal = 7 − ___ = ___ units
By Pythagoras' theorem:
$c^2 = a^2 + b^2$
$c^2 = \_\_\_^2 + 2^2 = \_\_\_$
$c = \sqrt{\_\_\_} \approx \_\_\_$ cm (2 decimal places)
Distance between the points is ___ units (2 decimal places).

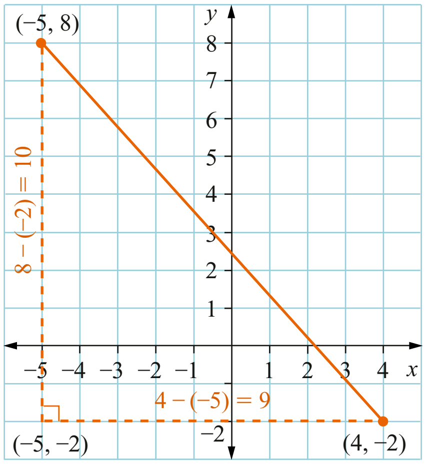
2 Plot each pair of points and find the distance between them correct to 2 decimal places.

a (2, 3) and (5, 7) **b** (5, 3) and (8, 6) **c** (8, 7) and (3, 3)

d (1, 9) and (7, 2) **e** (2, 8) and (7, 5) **f** (0, 0) and (5, 7)

EXAMPLE 2

Find the distance between the points $(-5, 8)$ and $(4, -2)$.

Solve	Think	Apply
 Distance is 13.45 units (2 decimal places).	*Step 1:* Plot the points and draw in a right-angled triangle. *Step 2:* Find the lengths of the horizontal and vertical sides. *Step 3:* Use Pythagoras' theorem: $c^2 = a^2 + b^2$ $= 10^2 + 9^2$ $= 181$ $c = \sqrt{181}$ $= 13.45$ (2 decimal places) Distance between the points is 13.45 units (2 decimal places).	When finding side lengths that cross the axis, take care with the negative values. All distances are positive quantities.

3 Complete the following to find the distance between the points $(-4, 7)$ and $(3, -4)$.

Vertical length $= 7 - \_\_\_ = 11$ units

Horizontal length $= (-4) - \_\_\_ = \_\_\_ = \_\_\_$ units

(distances must be positive)

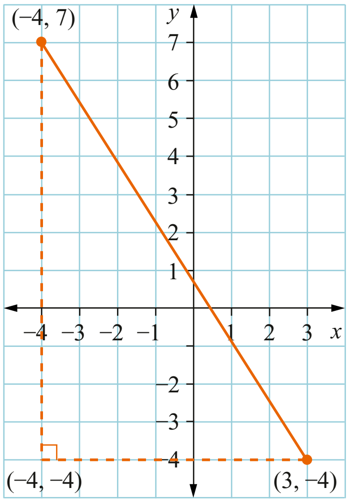
By Pythagoras' theorem:

$c^2 = a^2 + b^2$

$c^2 = 11^2 + \_\_\_^2 = \_\_\_$

$c = \sqrt{\_\_\_} = \_\_\_$ cm (2 decimal places)

Distance between the points is ___ units (2 decimal places).



4 Find the distance between each pair of points correct to 2 decimal places.

a $(-5, 3)$ and $(6, 2)$

b $(-2, -5)$ and $(3, 7)$

c $(-4, -5)$ and $(5, -1)$

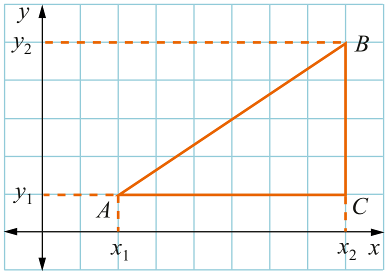
d $(-7, 0)$ and $(5, -4)$

e $(-8, -3)$ and $(0, 0)$

f $(4, 0)$ and $(0, -3)$

Investigation 1 Distance formula (extension)

The purpose of this investigation is to develop a formula to find the distance between the two points $A(x_1, y_1)$ and $B(x_2, y_2)$.



1 Use the graph to find an expression involving y_2 and y_1 for the length BC.

2 Use the graph to find an expression involving x_2 and x_1 for the length AC.

3 Use Pythagoras' rule to find an expression for d, the length AB.

4 Confirm that the distance formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

5 Check the formula by showing that the distance between $(-1, 4)$ and $(5, 2)$ is $\sqrt{40}$ units; that is, using $(-1, 4)$ for (x_1, y_1) and $(5, 2)$ for (x_2, y_2).

6 Show that the choice for (x_1, y_1) and (x_2, y_2) does not affect the outcome by using $(5, 2)$ for (x_1, y_1) and $(-1, 4)$ for (x_2, y_2) to show the distance is $\sqrt{40}$ units.

7 Use the distance formula to find the distance between the following pairs of points.

a $(4, 5)$ and $(-6, 3)$ **b** $(-5, -3)$ and $(2, -7)$

c $(0, 0)$ and $(14, 4)$ **d** $(-11, 13)$ and $(2, -5)$

8 Lianna uses the formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ to find the distance between two points.

a Show that this formula works for the points $(-5, -3)$ and $(2, -7)$.

b Expand the expression $(x_1 - x_2)^2 + (y_1 - y_2)^2$.

c Expand the expression $(x_2 - x_1)^2 + (y_2 - y_1)^2$.

d Explain why both formulas work.

9 Complete the following. In the distance formula:

a d stands for ____ **b** x_1 is ____ **c** x_2 is ____ **d** y_1 is ____ **e** y_2 is ____

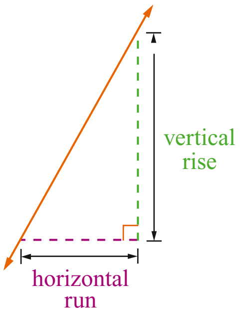
C Slope (gradient)

We use the word **slope** or **gradient** when talking about the degree of steepness of a line or a line segment. Horizontal lines have zero slope.

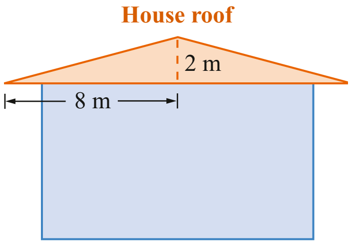
This line is very steep. It must therefore have a large slope.
To compare the slopes of different lines, we use the ratio of **vertical rise** to **horizontal run**.

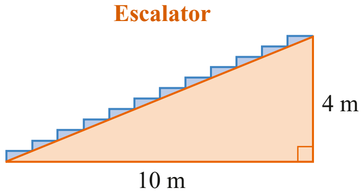
$$\textbf{Slope} = \frac{\textbf{vertical rise}}{\textbf{horizontal run}}$$

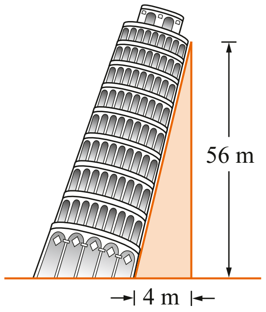
For a horizontal line, the vertical rise is 0, so the slope is 0.



The following illustrations indicate slopes of varying amounts.





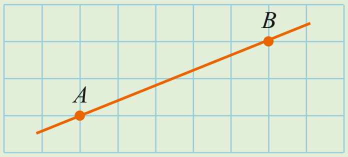


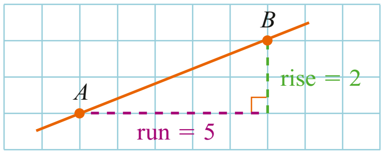
$\text{slope} = \frac{2}{8} = \frac{1}{4}$ $\text{slope} = \frac{4}{10} = \frac{2}{5}$ $\text{slope} = \frac{56}{4} = 14$

When line segments are drawn on graph paper, we can determine the slope of the line segments by drawing horizontal and vertical lines to complete a right-angled triangle.

EXAMPLE 1

Find the slope of line *AB*.



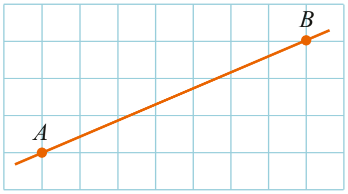
Solve	Think	Apply
 Slope of $AB = \frac{\text{vertical rise}}{\text{horizontal run}} = \frac{2}{5}$	Complete a right-angled triangle using *AB* as the hypotenuse. Then vertical rise = 2 and horizontal run = 5, so slope = $\frac{2}{5}$.	Complete a right-angled triangle. Determine the vertical rise and horizontal run to calculate the gradient. Gradient $= \frac{\text{rise}}{\text{run}}$

Exercise 10C

1 Complete the following to find the slope of line *AB*. First make a right-angled triangle using *AB* as the hypotenuse.

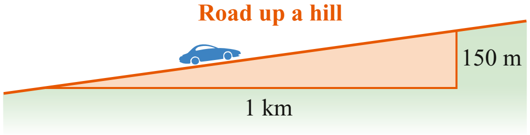
Vertical rise = ___ Horizontal run = ___

Slope of line $AB = \frac{\square}{\square}$

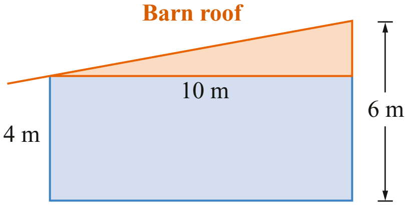


2 Find these slopes.

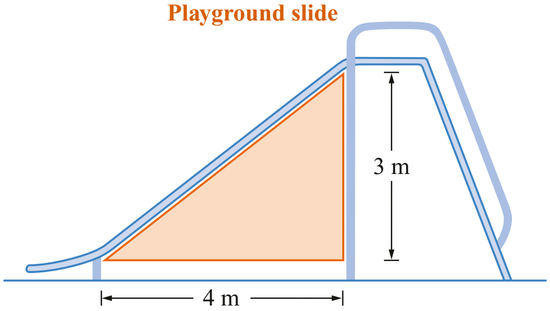
a Road up a hill



b Barn roof

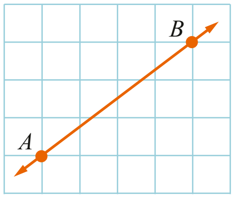


c Playground slide

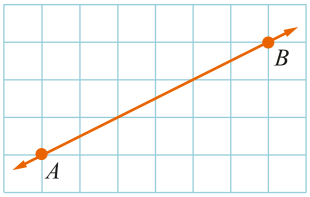


3 In each diagram, draw a right-angled triangle and find the gradient using: Gradient $= \frac{\text{vertical rise}}{\text{horizontal run}}$

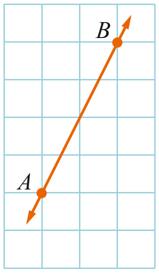
a



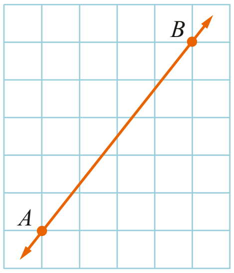
b



c

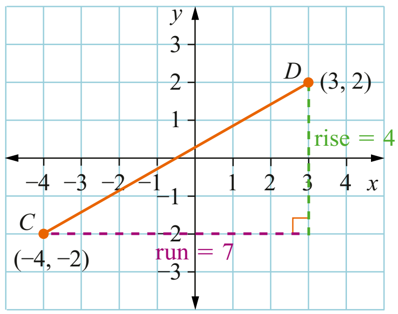


d



EXAMPLE 2

Find the gradient of the line passing through points $C(-4, -2)$ and $D(3, 2)$.

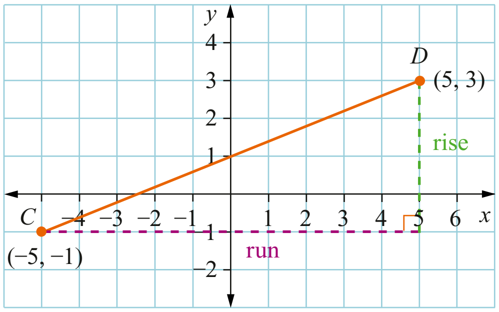
Solve	Think	Apply
 Gradient $= \frac{\text{rise}}{\text{run}} = \frac{4}{7}$	Complete a right-angled triangle using CD as the hypotenuse. Then vertical rise = 4 and horizontal run = 7, so gradient $= \frac{4}{7}$.	Complete a right-angled triangle using CD as the hypotenuse. Determine the vertical rise and horizontal run from C to D and calculate the gradient. Gradient $= \frac{\text{rise}}{\text{run}}$

4 Complete the following to find the gradient of the line passing through points $C(-5, -1)$ and $D(5, 3)$.

Vertical rise = ___

Horizontal run = 10

Gradient $= \frac{\square}{\square} = \frac{\square}{5}$



5 Find the gradient of the line passing through each pair of points.

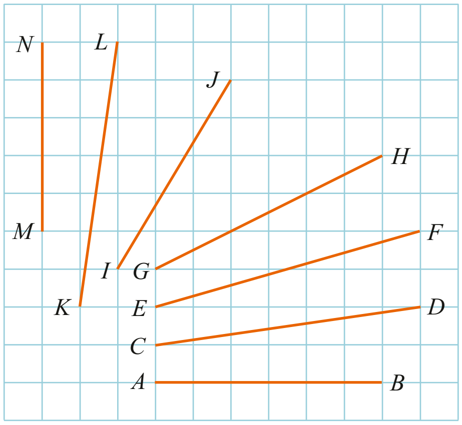
a $C(-5, -2)$ and $D(4, 5)$ **b** $A(-3, -1)$ and $B(5, 2)$

c $C(-5, 3)$ and $P(7, 7)$ **d** $M(1, -5)$ and $N(2, 6)$

Investigation 2 Varying the slope

1 Complete the table.

Line segment	*x*-run	*y*-rise	Slope
AB			
CD			
EF			
GH			
IJ			
KL			
MN			



2 Complete the following.

a The slope of a horizontal line is ___.

b The slope of a vertical line is ___.

c As the line segments become steeper, their slopes ___.

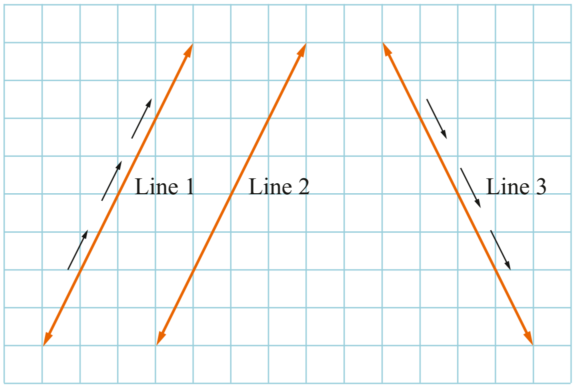
D Positive and negative gradients

In the diagram, lines 1 and 2 are parallel, and have the same slope of 2.

Line 3 is not parallel to lines 1 and 2, yet it has the same degree of steepness.

We say that lines 1 and 2 are **forwards sloping**, whereas line 3 is **backwards sloping**.

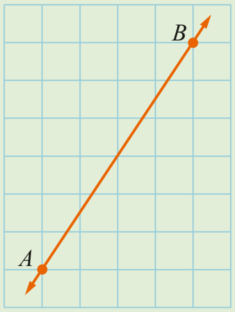
As we go from *left to right*, on line 1 we are going *uphill* and the slope (gradient) is **positive**, whereas on line 3 we are going *downhill* and the slope (gradient) is **negative**.



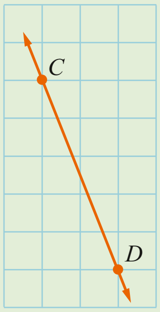
EXAMPLE 1

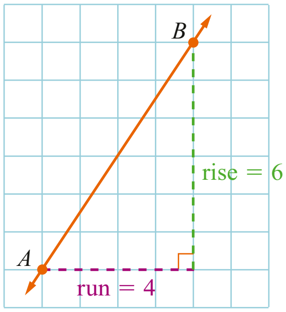
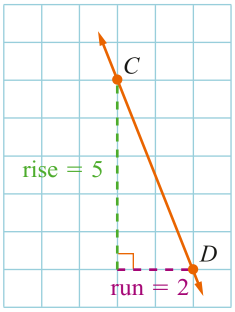
Find the slope of each line.

a



b

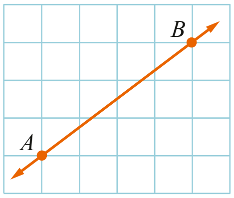


	Solve	Think	Apply
a	The slope of *AB* is positive (uphill). Slope $AB = \frac{\text{rise}}{\text{run}}$ $= +\frac{6}{4}$ $= +1\frac{1}{2}$	Draw in a right-angled triangle and find the rise and run.  	First determine whether the slope is positive or negative. For downhill slopes, the ‘rise’ is a ‘drop’, so the slope is a negative value.
b	The slope of *CD* is negative (downhill). Slope $CD = \frac{\text{rise}}{\text{run}}$ $= -\frac{5}{2}$ $= -2\frac{1}{2}$	Draw in a right-angled triangle and find the rise and run.  	

Exercise 10D

1 Complete the following to find the slope of each line.

a

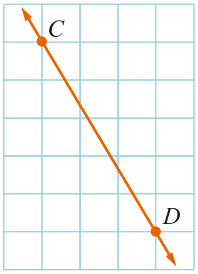


The slope of AB is ___.

$$\text{Slope of } AB = \frac{\text{rise}}{\text{run}}$$

$$= \frac{\square}{\square}$$

b

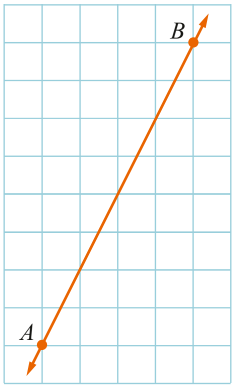


The slope of CD is ___.

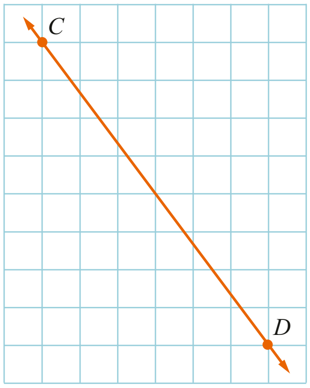
$$\text{Slope of } CD = \frac{\text{rise}}{\text{run}} = \frac{\square}{\square} = \_\_\_$$

2 Determine whether the slope is positive or negative and then find the gradient.

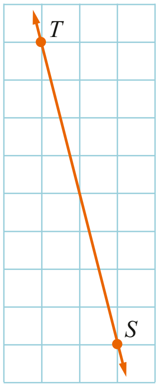
a



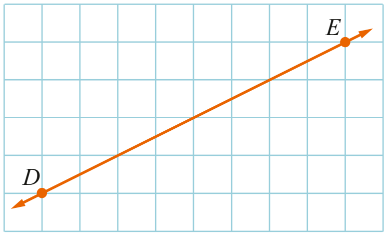
b



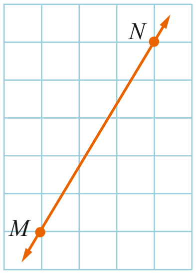
c



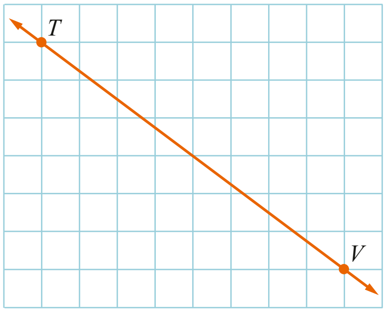
d



e

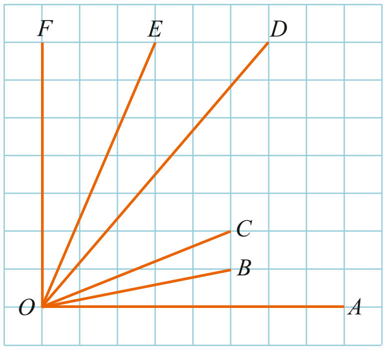


f



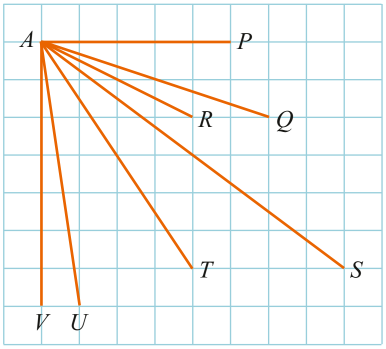
3 Find the gradient of each line.

a OA **b** OB
c OC **d** OD
e OE **f** OF

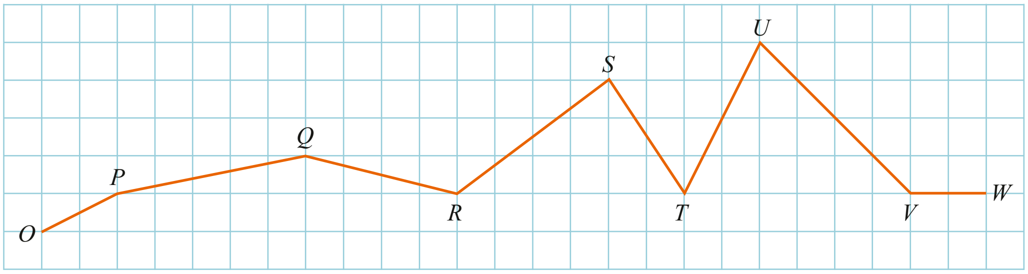


4 Find the gradient of each line.

a AP **b** AQ **c** AR
d AS **e** AT **f** AU
g AV



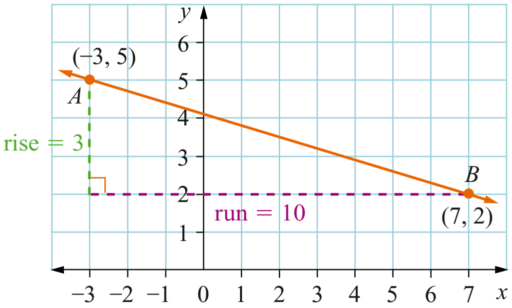
5 Imagine you are walking across the countryside from O to W (from left to right).



- **a** When are you going uphill?
- **b** When are you going downhill?
- **c** Where is the steepest positive slope?
- **d** Where is the steepest negative slope?
- **e** Where is the slope 0?
- **f** Where is the slope not zero but least?

EXAMPLE 2

Plot points $A(-3, 5)$ and $B(7, 2)$ and find the gradient of the line passing through them.

Solve	Think	Apply
 $\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= -\frac{3}{10}$	From the right-angled triangle, the slope is downhill, so the rise is -3 and the run is 10.	Plot the points, and draw a right-angled triangle. Find the rise and run. The gradient is negative (downhill).

6 Plot points $A(-2, 7)$ and $B(5, 1)$ on a number plane. Draw a right-angled triangle, then complete the following to find the gradient of the line passing through them.

The slope is ____.

The rise is ____ and the run is ____.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{\square}{\square}$$

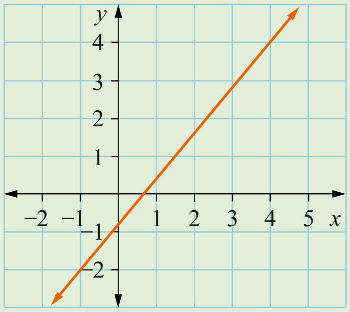
7 Plot each pair of points and find the gradient of the line passing through them.

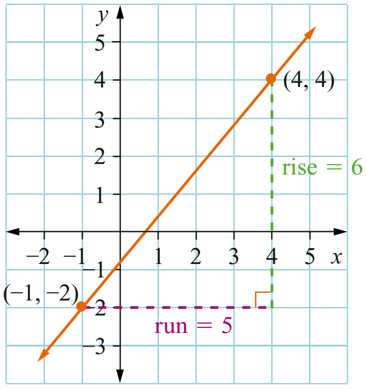
a $A(-4, 6)$ and $B(7, 2)$ **b** $C(-4, -1)$ and $D(5, 3)$ **c** $P(1, 3)$ and $Q(-4, -1)$

d $R(0, 0)$ and $S(5, 3)$ **e** $M(5, 3)$ and $N(-5, 2)$ **f** $S(-3, -2)$ and $T(4, -6)$

EXAMPLE 3

Find the gradient of this line.



Solve	Think	Apply
 $\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= +\frac{6}{5}$	Choose any two points on the line, say $(-1, -2)$ and $(4, 4)$. Draw in a right-angled triangle. The gradient is positive (uphill). The rise is 6 and the run is 5.	A straight line has the same gradient for its entire length. Choose any two points to calculate the gradient.

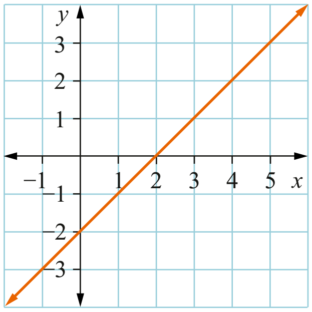
8 Complete the following to find the gradient of this line.

Choose any two points on the line, say $(0, -2)$ and $(5, 3)$, and draw a right-angled triangle.

The gradient is positive (uphill).

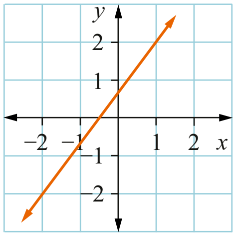
The rise is ___ and the run is ___.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{\square}{\square} = \_\_\_$$

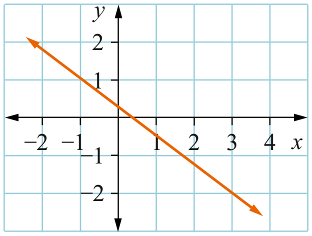


9 By choosing two points on each line, find the gradients.

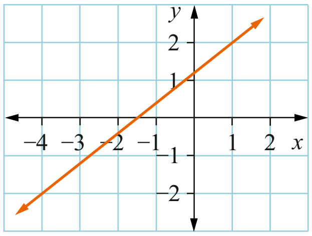
a



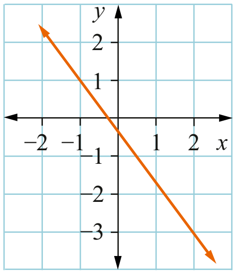
b



c

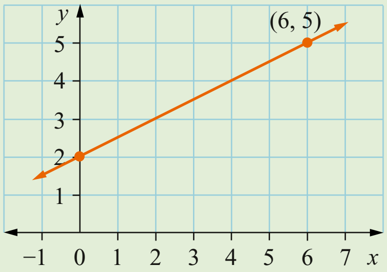


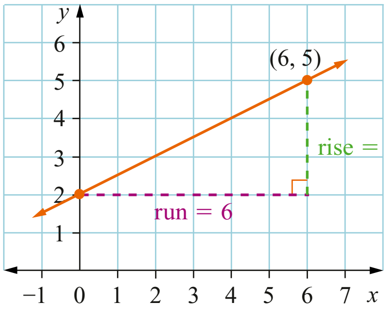
d



EXAMPLE 4

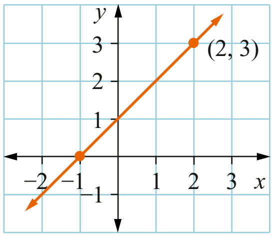
Find the gradient of the given line.



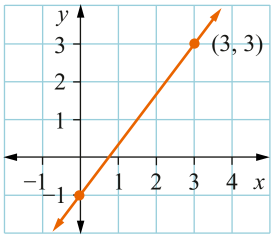
Solve	Think	Apply
 $\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= +\frac{3}{6} = +\frac{1}{2}$	The gradient is positive (uphill). The rise is 3 and the run is 6.	Draw in a right-angled triangle. Find the rise and run.

10 Find the gradients of these lines.

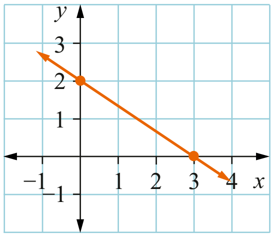
a



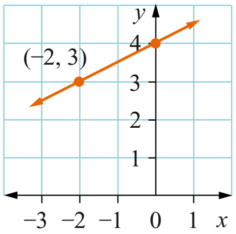
b



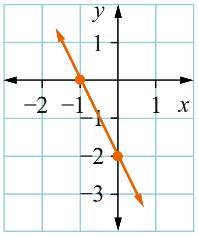
c



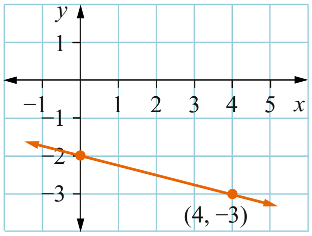
d



e

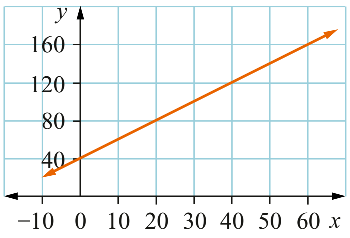


f

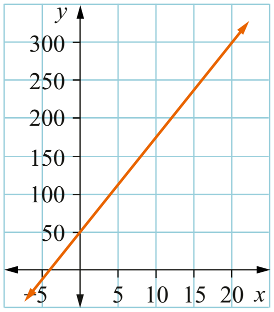


11 Find the gradient of each line.

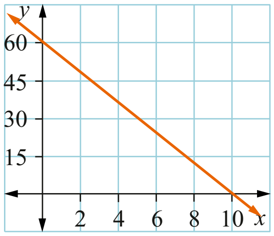
a



b

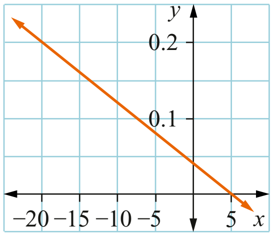


c

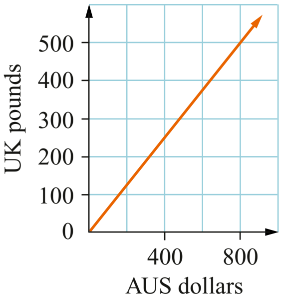


Be careful as the scales are not the same. **!**

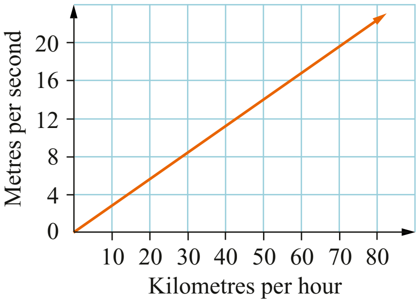
d



e



f



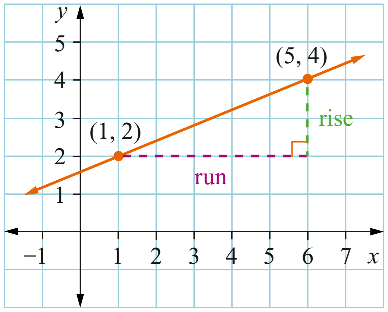
Investigation 3 Formula for gradient (extension)

The gradient has been found by drawing a right-angled triangle and finding the vertical rise and horizontal run.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

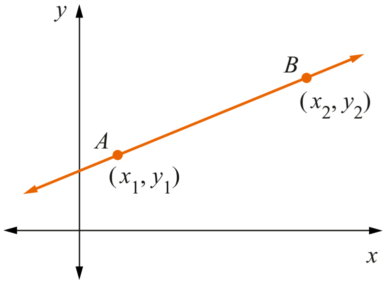
1 **a** Find values for the vertical rise and horizontal run as shown in the triangle on this graph.

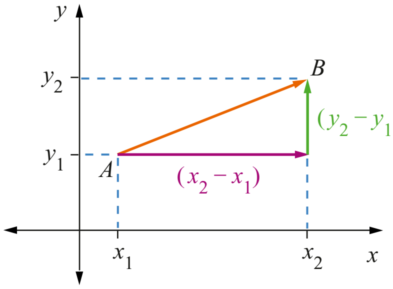
b Calculate the gradient.



2 **a** Copy this diagram.

b Draw in the triangle as shown on the right-hand diagram.





c If A is (x_1, y_1) and B is (x_2, y_2) then from the diagram:

- rise $= y_2 - y_1$
- run $= x_2 - x_1$
- the vertical rise from A to B is $y_2 - y_1$ (the difference between the y-coordinates)
- the horizontal run from A to B is $x_2 - x_1$ (the difference between the x-coordinates).

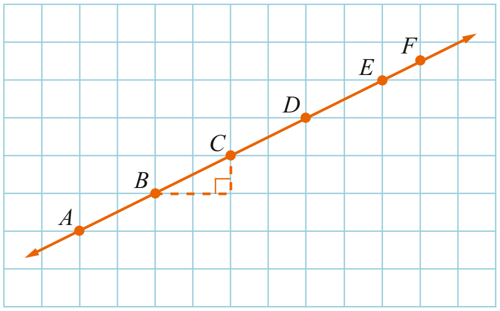
d The symbol for gradient is m. Complete the following.

$$m = \frac{y_2 - \square}{\square - \square}$$

Investigation 4 The slope of a line

1 Complete the table.

Line segment	x-run	y-rise	$\frac{y\text{-rise}}{x\text{-run}}$
BC	2	1	$\frac{1}{2}$
DE			
AC			
BE			
AE			
AF			



2 State, in sentence form, any conclusions you can draw from the graph and table.

Investigation 5 Relating gradient and the tangent ratio (extension)

1 Plot points $A(1, 2)$ and $B(5, 9)$.

2 Draw a right-angled triangle and write the lengths of the horizontal and vertical sides.

3 Find the gradient of AB.

4 Label the angle at A as θ.

5 With respect to θ, label the sides as opposite, adjacent and hypotenuse.

6 Write an expression for $\tan \theta$.

7 Compare $\tan \theta$ and the gradient.

8 Explain the result from question **7**.

9 Calculate the size of the angle that the line makes with the x-axis.

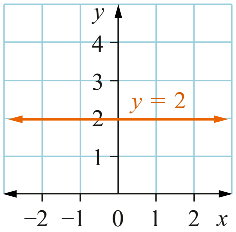
10 Calculate the angles for the gradients of the line joining the points in Exercise 10D question **5**.

11 Copy and complete the following.
The gradient of a line is equal to ___ θ, where θ is the angle made by the line and the ___ axis.

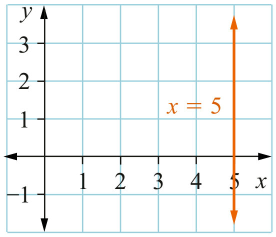
E Sketching lines parallel to the axes

In Section 10A we found the midpoints of horizontal and vertical lines.

In Example 1 part **a**, the points (3, 2) and (9, 2) were joined to give a midpoint of (6, 2). All three points have a y-coordinate of 2, so the equation of this horizontal line must be $y = 2$.



In Example 1 part **b**, the points (5, 1) and (5, 8) were joined by a vertical line, and the midpoint was found to be $(5, 4\frac{1}{2})$. All three points have an x-coordinate of 5, so the equation of this vertical line is $x = 5$.



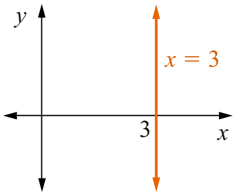
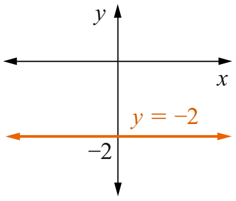
- Horizontal lines are of the form $y = a$, where a is a positive or negative number.
- Vertical lines are of the form $x = b$, where b is a positive or negative number.

EXAMPLE 1

Sketch the following lines.

a $x = 3$ **b** $y = -2$

A sketch shows only the axes and essential information.

	Solve	Think	Apply
a		$x = 3$ is a vertical line. All x-values are 3, whatever the value of y.	A line of the form $x = b$, is always vertical.
b		$y = -2$ is a horizontal line. All y-values are -2, whatever the value of x.	A line of the form $y = a$, is always horizontal.

Exercise 10E

1 **a** Sketch the vertical line $x = 4$.
b Sketch the horizontal line $y = -1$.

2 Sketch these horizontal and vertical lines.

a $y = 3$ **b** $x = 1$ **c** $x = -2$ **d** $y = -4$
e $x = 5$ **f** $y = 8$ **g** $y = -3$ **h** $x = 7$

3 **a** List the equations from question **2** that represent horizontal lines.
b Write the coordinates of the point at which each of these lines cuts the y-axis.

4 **a** List the equations from question **2** that represent vertical lines.
b Write the coordinates of the point at which each of these lines cuts the x-axis.

5 **a** Sketch these horizontal lines on the same number plane: $y = -2,\ y = -1,\ y = 1,\ y = 2$
b The x-axis is a horizontal line. The equation of the x-axis is $y =$ _____.
c Explain your answer to part **b**.

6 **a** Sketch these vertical lines on the same number plane: $x = -2,\ x = -1,\ x = 1,\ x = 2$
b Write the equation of the y-axis.
c Explain your answer to part **b**.

F Graphing linear relationships

Consider all points (x, y) in which $y = 2x$. We can write down any number of **ordered pairs** that satisfy this rule. For example, (1, 2), (2, 4), (3, 6), (−2, −4), (0.7, 1.4).

To visualise all the points that satisfy the equation $y = 2x$, we draw a graph of some points near the origin, O. A table of values is useful.

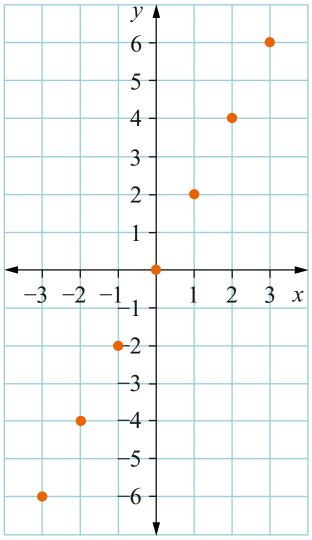
A table of values for $y = 2x$ is shown below.

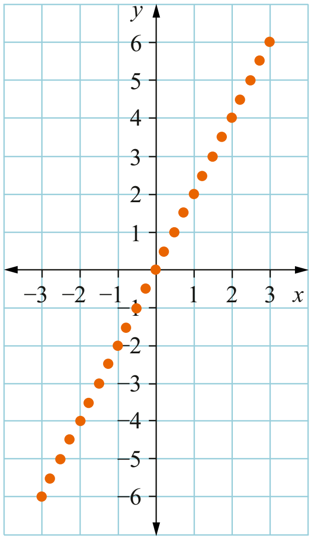
x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

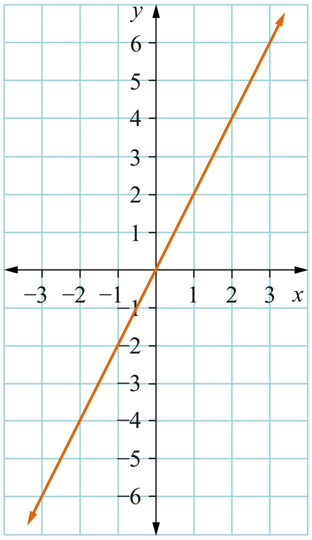
Graphing these seven points only, we get Graph 1.

However, not just the integers satisfy the rule $y = 2x$. Graph 2 is the plot of all points from $x = -3$ to $x = 3$, with x-values 0.25 units apart.

In fact, if we imagine ordered pairs for which the x-values are the complete set of real numbers, we would obtain the complete line as in Graph 3.


Graph 1


Graph 2


Graph 3

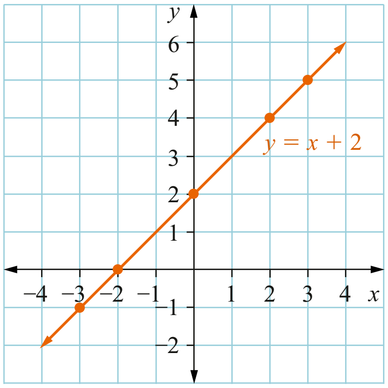
EXAMPLE 1

Draw the graph of the lines with these equations.

a $y = x + 2$ **b** $y = -2x$ **c** $y = \frac{1}{2}x - 1$

Solve | **Think/Apply**

a



Step 1: Construct a table of values. Choose any x-values you like.

$y = x + 2$

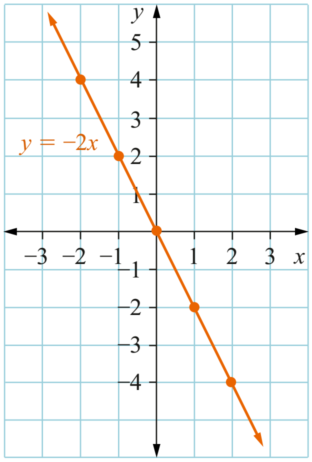
x	−3	−2	0	2	3
y	−1	0	2	4	5

Step 2: Plot the ordered pairs from the table of values on a number plane.

Step 3: Join the plotted points with a straight line. Extend the line past the plotted points and put an arrow on each end to show that it continues in both directions.

Step 4: Write the equation of the line.

b

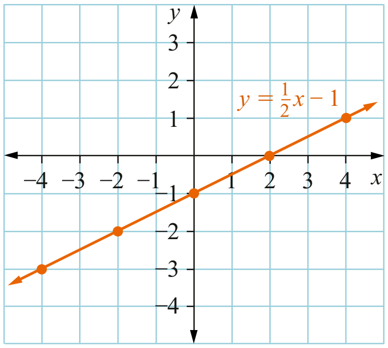


Proceed as in part **a**.

$y = -2x$

x	−2	−1	0	1	2
y	4	2	0	−2	−4

c



Proceed as in part **a**.

$y = \frac{1}{2}x - 1$

x	−4	−2	0	2	4
y	−3	−2	−1	0	1

Exercise 10F

1 Complete the table of ordered pairs for $y = 2x - 1$.

$y = 2(0) - 1 =$ ___

$y =$ ___$(2) - 1 =$ ___

$y =$ ___(___) $-$ ___ $=$ ___

Plot the points and draw the graph.

x	−1	0	1	2	3
y	−3		1		

2 Complete the table of ordered pairs for each equation given.

a $y = x$

x	1	2	3	4	5
y					

b $y = x - 2$

x	−1	0	1	2	3
y					

c $y = -x$

x	−2	−1	0	1	2
y					

d $y = 5 - x$

x	−2	−1	0	1	2
y					

e $y = 2x + 1$

x	−2	−1	0	1	2
y					

f $y = -\frac{1}{2}x$

x	−2	−1	0	1	2
y					

g $y = 8 - 2x$

x	−2	−1	0	1	2
y					

h $y = 1 - 3x$

x	−2	−1	0	1	2
y					

3 **i** Complete each table of values below using the rule provided.

ii Plot each set of ordered pairs and draw a straight line through the points.

a $y = x + 1$

x	−2	−1	0	1	2
y					

b $y = 2x + 3$

x	−2	−1	0	1	2
y					

c $y = -2x - 1$

x	−2	−1	0	1	2
y					

d $y = 4 - x$

x	−2	−1	0	1	2
y					

e $y = 2x - 2$

x	−2	−1	0	1	2
y					

f $y = 3 - 2x$

x	−2	−1	0	1	2
y					

4 Consider the straight lines in questions **2** and **3**.

a List the equations with a positive gradient.

b List the equations with a negative gradient.

c What is the difference between these groups of equations?

d Without drawing the graphs, state whether each of these equations has a positive or a negative gradient.

i $y = 2x - 1$ **ii** $y = -3x + 4$ **iii** $y = 5 - 7x$

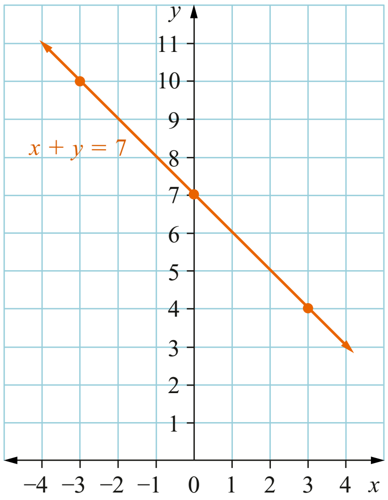
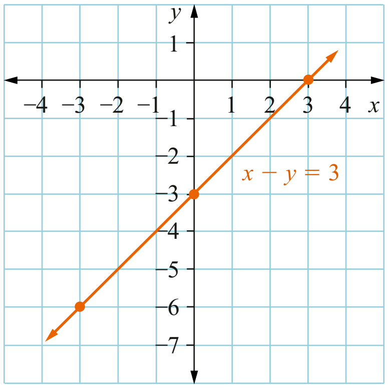
iv $y = 3 + 2x$ **v** $y = 7x - 1$ **vi** $y = -5x + 2$

EXAMPLE 2

By using this table of values, draw the graphs of:

a $x + y = 7$ **b** $x - y = 3$

x	-3	0	3
y			

<table>
<tr><th></th><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr><td>a</td><td>
</td><td>$x + y = 7$
<table><tr><td>x</td><td>-3</td><td>0</td><td>3</td></tr><tr><td>y</td><td>10</td><td>7</td><td>4</td></tr></table>When $x = -3$:
$-3 + y = 7$
$y = 7 + 3 = 10$
When $x = 0$:
$0 + y = 7$
$y = 7$
When $x = 3$:
$3 + y = 7$
$y = 7 - 3 = 4$</td><td rowspan="2">The table of values can be found using any x-values, with corresponding y-values calculated according to the equation of the line.
While two points define a line, a third point gives a check.</td></tr>
<tr><td>b</td><td>
</td><td>$x - y = 3$
<table><tr><td>x</td><td>-3</td><td>0</td><td>3</td></tr><tr><td>y</td><td>-6</td><td>-3</td><td>0</td></tr></table>When $x = -3$:
$-3 - y = 3$
$-y = 3 + 3 = 6$
$y = -6$
When $x = 0$:
$0 - y = 3$
$y = -3$
When $x = 3$:
$3 - y = 3$
$-y = 3 - 3 = 0$
$y = 0$</td></tr>
</table>

5 Complete the table of values for the equation $x + y = 5$.
For each x-value find the corresponding y-value.

x	-3	0	3
y			

When $x = -3$: $-3 + y =$ ___

$y =$ ___ $+ 3 =$ ___

When $x = 0$: ___ $+ y = 5$

$y =$ ___

When $x = 3$: $3 +$ ___ $=$ ___

$y = 5 -$ ___ $=$ ___

Plot the points and draw the graph.

6 Complete a table of values for each equation, then draw each graph on a separate number plane.

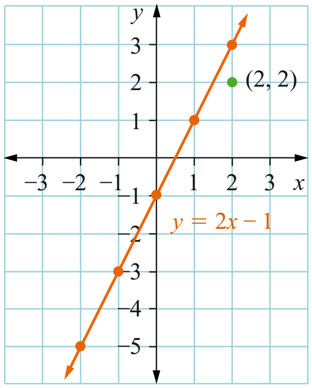
x	-3	0	3
y			

a $y = x - 4$
b $y = x + 4$
c $y = 2x$
d $y = 1 - x$
e $y = \frac{1}{2}x$
f $y = \frac{2x + 3}{4}$
g $x + y = -3$
h $x + y = 1$
i $x - y = 8$
j $x - y = 6$
k $y = -4 + x$
l $x + y = 8$
m $3y - 2x = -12$
n $y = \frac{3x}{2} - 2$
o $2x - 3y = 6$
p $5x + 3y = 15$

EXAMPLE 3

Does point (2, 2) lie on line $y = 2x - 1$?

Solve



The point (2, 2) does not lie on the line.

Think/Apply

Step 1: Construct a table of values for $y = 2x - 1$.

x	-2	-1	0	1	2
y	-5	-3	-1	1	3

Step 2: Plot these points and draw the line $y = 2x - 1$.
Step 3: Plot the point (2, 2). Check if the point lies on the line.

7 Using the graphs you drew in question **6**, answer the following questions.

a Does point $(1, -3)$ lie on line $y = x - 4$?
b Does point $(-1, 2)$ lie on line $y = x + 4$?
c Does point $(0, 2)$ lie on line $y = 2x$?
d Does point $(3, -2)$ lie on line $y = 1 - x$?
e Does point $(4, 1)$ lie on line $y = \frac{1}{2}x$?
f Does point $(0, 2)$ lie on line $y = \frac{2x + 3}{4}$?
g Does point $(-2, -1)$ lie on line $x + y = -3$?
h Does point $(2, -1)$ lie on line $x + y = 1$?
i Does point $(6, -2)$ lie on line $x - y = 8$?
j Does point $(2, -4)$ lie on line $x - y = 6$?

8 Match each table of values to its equation below.

a

x	-4	-2	0	2	4
y	-2	-1	0	1	2

b

x	0	1	2	3	4
y	0	2	4	6	8

c

x	-4	-2	0	1	3
y	-2	0	2	3	5

d

x	-2	0	1	2	3
y	4	2	1	0	-1

e

x	-3	-2	-1	0	1
y	3	2	1	0	-1

A $y = 2x$
B $y = -x$
C $y = \frac{1}{2}x$
D $y = x + 2$
E $y = 2 - x$

EXAMPLE 4

You can check whether a point lies on a line without drawing the line.
By substituting the x-value of point (13, 25) and finding the corresponding y-value, decide whether this point lies on line $y = 2x - 1$.

Solve	Think	Apply
(13, 25) $y = 2x - 1$ When $x = 13$, $y = 2(13) - 1$ $= 26 - 1 = 25$ As the y-values are equal, the point lies on the line.	Substitute $x = 13$ into the equation. If the y-value is 25, the point lies on the line.	Substitute the point into the equation. If it satisfies the equation, the point lies on the line.

9 Complete the following to decide whether the point (3, 5) lies on line $y = 3x - 4$.
Substitute the x-value of point (3, 5) to find the corresponding y-value.
When $x =$ ___, $y = 3($___$) - 4$
$=$ ___ $-$ ___ $=$ ___
As the y-values are ____, the point ____ lie on the line.

10 By substituting the x-value of the point given in brackets and finding the corresponding y-value, decide whether the point lies on the given line.

a $y = 4 - x$ (2, 2)
b $y = x + 4$ (1, 5)
c $y = 2x$ (3, 8)
d $y = 1 - x$ (5, 4)
e $y = \frac{1}{2}x$ (3, 6)
f $y = 2x + 3$ (−2, −1)
g $y = 2x - 3$ (−4, −11)
h $y = 3 - 2x$ (−5, 4)
i $y = 3x - 2$ (10, 28)
j $3x - 2y = 8$ (2, −2)

11 Find five points that lie on each line.
a $y = x + 3$
b $2x + y = 5$
c $3x - 2y = 6$

Investigation 6 Graphics calculator

1 These instructions are for a Casio fx-9860G AU series.

a Select GRAPH from the MAIN MENU.

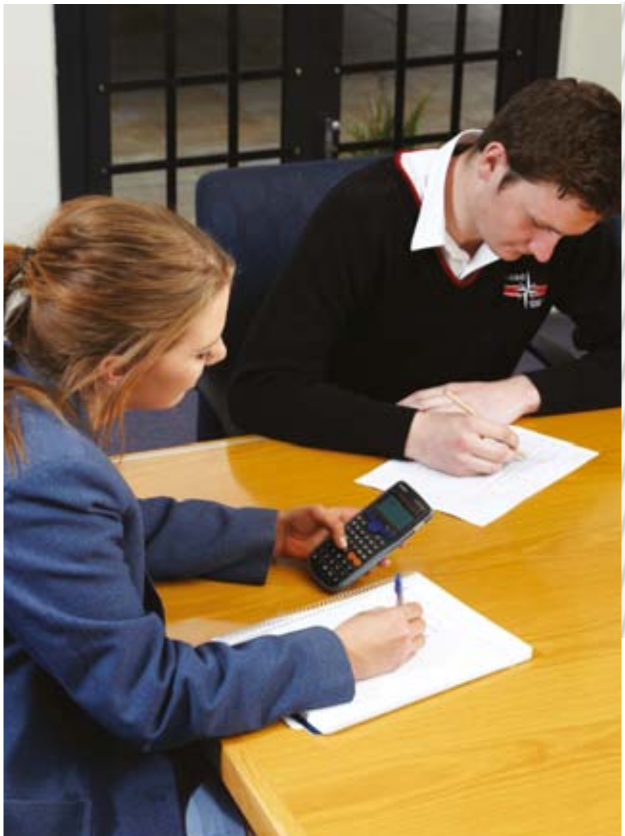
b Enter the equation $y = 2x + 3$ by pressing
2 × X,θ,T + 3 then EXE.

c Press F6 to DRAW.

d The graph should appear on the screen. If the scale of the axes needs adjustment, press V-WINDOW F3 and adjust as needed. Press EXIT to return.

e Press F2 F1 to delete any graphs after choosing the equations.

f To have more than one graph on the screen at a time, omit the instruction in part **e**.



NUMBER & ALGEBRA

2 **a** Graph $y = 2x + 1$, $y = 2x - 1$ and $y = 2x + 3$ on the same screen.
b What observation can you make?

3 **a** Graph $y = -3x - 1$, $y = -3x$ and $y = -3x + 2$ on the same screen.
b Comment on these graphs.

4 **a** Graph $y = x + 2$, $y = x$ and $y = x - 5$ on the same screen.
b Comment on these graphs.

5 **a** Graph $y = 3x - 1$, $y = 2x - 1$ and $y = -4x - 1$ on the same screen.
b What comment can be made?

6 **a** Draw the graphs of $y = x^2$, $y = x^2 + 1$ and $y = x^2 - 2$ on the same screen.
b Comment on these graphs.

7 **a** Graph $y = -x^2$, $y = -x^2 + 1$, $y = -x^2 - 1$ and $y = -x^2 + 3$ on the same screen.
b Comment on the shape of the graphs and the effect of adding and subtracting a constant.

8 **a** Graph $y = 2^x$, $y = 3^x$ and $y = 5^x$ on the same set of axes.
b Comment on the similarities in these graphs.

9 Draw the graphs from Exercise 10F and make up some of your own.

G Non-linear relationships

Straight lines are one type of relationship that can be graphed.
There are many relationships that, when graphed, are not straight lines.
This section examines some of these non-linear relationships.

Graphics calculators are an excellent tool for this section. !

- **Parabola** is the name given to the graph relating y to x^2.
  The simplest parabola is $y = x^2$.
- An **exponential** graph has x as a power. An example is $y = 2^x$.
- A **circle** graph relates x^2 and y^2. All circles of the form $x^2 + y^2 = r^2$ have centre (0, 0) and radius $= r$.

Exercise 10G

1 **a** Complete this table of values for $y = x^2$.

x	-4	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	4
y											

b Plot these points and draw a smooth curve through them.

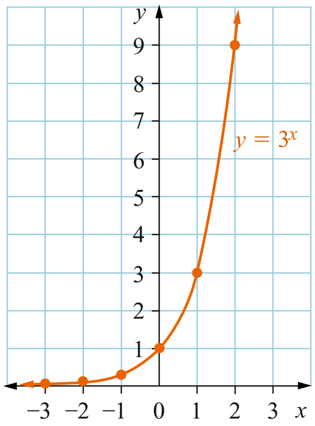
2 By using a table with the same values as in question **1**, or a graphics calculator, sketch each set of parabolas on the same number plane.
a $y = x^2$, $y = 2x^2$, $y = \frac{1}{2}x^2$, $y = -x^2$, $y = -2x^2$, $y = -\frac{1}{2}x^2$
b $y = x^2$, $y = 3x^2$, $y = 4x^2$, $y = \frac{1}{4}x^2$
c $y = x^2$, $y = x^2 + 1$, $y = x^2 + 2$, $y = x^2 - 1$, $y = -x^2$, $y = -x^2 + 1$, $y = -x^2 - 1$, $y = -x^2 + 3$

3 Write some observations about each set of parabolas in question **2**.

EXAMPLE 1

Complete a table of x values for $-3, -2, -1, 0, 1, 2$ and 3. Draw a graph of $y = 3^x$.

Solve



Think

Use the x^y key to find the y-values.

x	-3	-2	-1	0	1	2	3
y	0.04	0.1	0.3	1	3	9	27

Plot the graph.

Apply

Draw a smooth curve through the plotted points. Extend the graph past the endpoints.

4 The table for the graph of $y = 2^x$ is given.
Plot the points and draw the graph of $y = 2^x$.

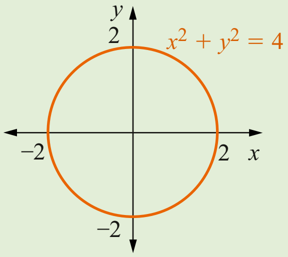
x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

5 Using a table similar to that in question **4** or a graphics calculator, graph or sketch these exponentials on the same number plane: $y = 2^x$, $y = 3^x$ and $y = 4^x$. What do you notice about all three graphs?

EXAMPLE 2

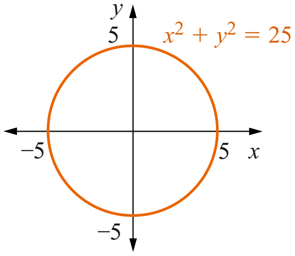
The circle $x^2 + y^2 = 4$ is sketched.

a Where is the centre of the circle?
b Where does it cut the x- and y-axes?
c What is the radius of the circle?



	Solve	Think/Apply
a	The centre of the circle is (0, 0).	All points on the circle are equidistant from the centre. If the centre is the origin, the equal intercepts occur on both axes. The radius is the distance from the centre to the circumference.
b	The circle cuts the x-axis at ± 2 and the y-axis at ± 2.	
c	The radius of the circle is $\sqrt{4} = 2$ units.	

6 The circle $x^2 + y^2 = 25$ is sketched. Complete the following.
The centre of the circle is (___, 0).
The circle cuts the x-axis at ± 5 and the y-axis at ____.
The radius of the circle is $\sqrt{}$ = ___ units.



7 Sketch the graphs of these circles, and state the centre and radius of each.

a $x^2 + y^2 = 1$
b $x^2 + y^2 = 9$
c $x^2 + y^2 = 16$
d $x^2 + y^2 = 6$

Language in mathematics

Pierre de Fermat (1601–1665)

Pierre de Fermat was born in Beaumont-de-Lomagne in France, near the border with Spain. He studied Latin and Greek literature, ancient science, mathematics and modern languages at the University of Toulouse, but his main purpose was to study law.

In 1629, Fermat studied the work of Apollonius, a geometer of ancient Greece, and discovered for himself that **loci** or sets of points could be studied using coordinates and algebra. His work *Introduction to Loci* was not published for another 50 years, then, together with *La Geometrie* by René Descartes (1596–1650), it formed the basis of Cartesian geometry.

In 1631 Fermat received his degree in law. He was later awarded the status of a minor nobleman, and in 1648 became King's Councillor.

Fermat was a man of great integrity who worked hard. He remained aloof from matters outside his own jurisdiction, and pursued his great interest in mathematics. He worked with Blaise Pascal (1623–1662) on the theory of probability and the principles of permutations and combinations. He worked on a variety of equations and curves and the Archimedean spiral. In 1657 he wrote *Concerning the Comparison of Curved and Straight Lines*, which was published during his lifetime.

Fermat died in 1665. He was acknowledged master of mathematics in France at the time, but his fame would have been greater if he had published more of his work while he was alive. He became known as the founder of the modern theory of numbers.

In mid-1993, one of the most famous unsolved problems in mathematics, **Fermat's Last Theorem**, was solved by Andrew Wiles of Princeton University (USA). Wiles made the final breakthrough after 350 years of searching by many famous mathematicians, both amateur and professional. Wiles is a former student and collaborator of Australian mathematician John Coates.

Fermat's Last Theorem is a simple assertion that he wrote in the margin of a mathematics book, but never proved, although he claimed he could. The theorem is:

> 'The equation $x^n + y^n = z^n$, when the exponent n is greater than 2, has no solutions in positive integers.'

Wiles's work established a whole new mathematical theory, proposed and developed over the last 60 years by the finest mathematical minds of the twentieth century.

1 Read the article about Pierre de Fermat and answer these questions.
- **a** For how long did Fermat live?
- **b** List four of Fermat's achievements.
- **c** How many publications did Fermat have in his lifetime?
- **d** Why was Fermat not as famous as he could have been?
- **e** What was the only article published by Fermat in his lifetime?

2 Complete these glossary words by inserting the vowels.

a v__rt__c__l

b h__r__z__nt__l

c gr__d__ __nt

d __nt__rv__l

e __bl__q__ __

f sl__p__

g l__n__ __r

h m__dp__ __nt

3 Rearrange these sentences, the first word has a capital letter.
- **a** The equation y-axis $x = 0$ has the.
- **b** has The x-axis $y = 0$ equation the.
- **c** positive uphill gradient slope An is a.
- **d** in is a midpoint middle The the of line.
- **e** a the If gradient is goes downhill negative line.
- **f** interval Pythagoras' found The is using an length of rule.

4 Use every third letter to reveal the message.

A W I E R N G H C U J O I K O O P R L K D N G I S E N R T A A S T X C E V F G B G E N H
O M J M N H E W E T R T R G F Y D C A E S N E D I F G N J K T O P E L K R J H V N B A G
F L C D J E S O W A I S D N F G S H J T K M W N B O V C P A S O D F I G H N B G T V F S
C D A D E N D S D S E H W S A Q A S A S L A S E A D N F G G H J T K L H I F A D F M G B
I H J D K M P D E O S C I F J N M Y T I N A X I N K E D S J S M W L Z F O G H P J K E

Terms

circle
coordinates
decimal place
endpoint
exponential
gradient
halfway
horizontal run
interval
linear
midpoint
negative
non-linear
number plane
oblique
ordered pair
parabola
positive
Pythagoras
relationship
right-angled triangle
sketch
slope
tangent
vertical rise

Check your skills

1 The midpoint of the line joining the points (3, 5) and (9, 5) is:
A (12, 5) **B** (6, 5) **C** (6, 10) **D** (6, 0)

2 The midpoint of the line joining the points (2, −4) and (2, 10) is:
A (2, 3) **B** (2, 6) **C** (0, 6) **D** (4, 6)

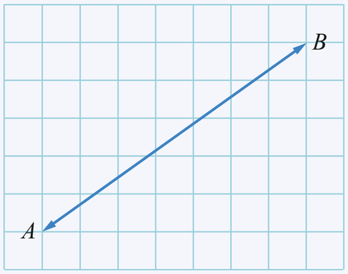
3 The midpoint of the line joining the points (−1, 3) and (9, 1) is:
A (8, 2) **B** (5, 2) **C** (8, 4) **D** (4, 2)

4 The midpoint of the line joining the points (−6, 3) and (5, −7) is:
A (−1, −4) **B** $(-\frac{1}{2}, -2)$ **C** $(\frac{1}{2}, 2)$ **D** $(-5\frac{1}{2}, 5)$

5 The distance between points (5, 3) and (0, 10) is:
A $\sqrt{12}$ units **B** $\sqrt{144}$ units **C** $\sqrt{24}$ units **D** $\sqrt{74}$ units

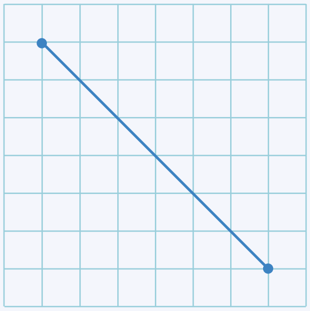
6 The distance between points (−5, 8) and (6, −5) is:
A $\sqrt{290}$ units **B** $\sqrt{48}$ units **C** $\sqrt{170}$ units **D** $\sqrt{2}$ units

7 The slope of *AB* is:
A $\frac{7}{5}$ **B** $-\frac{7}{5}$
C $\frac{5}{7}$ **D** $-\frac{5}{7}$



8 The gradient of the line passing through *A*(−7, −1) and *B*(3, 6) is:
A $\frac{10}{7}$ **B** $-\frac{10}{7}$
C $\frac{7}{10}$ **D** $-\frac{7}{10}$

9 The gradient of the line interval shown at right is:
A 6 **B** −6
C 1 **D** −1



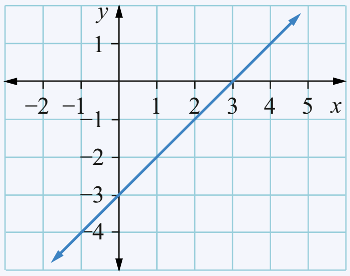
10 The gradient of the line joining points $A(-5, 7)$ and $B(3, 5)$ is:

A $\frac{1}{4}$ **B** $-\frac{1}{4}$ **C** 4 **D** -4

11 The gradient of the line shown at right is:

A $+1$ **B** -1

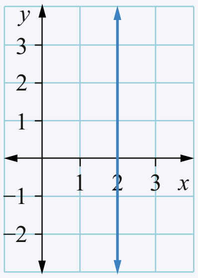
C $+5$ **D** -5



12 The equation of this line is:

A $y = 2$ **B** $x = 2$

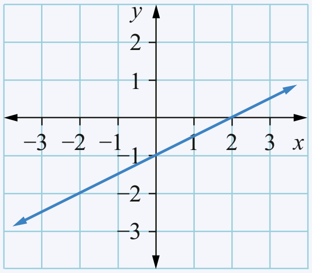
C $y = 2x$ **D** $x = 2y$



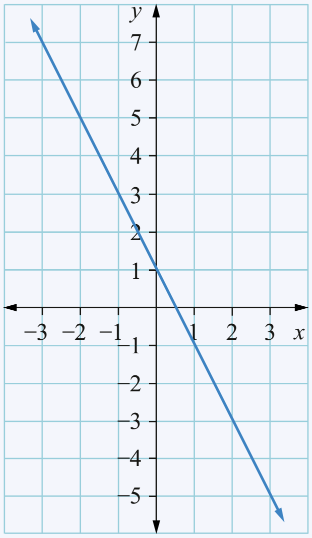
13 By completing this table of values, the graph of $y = 2x + 1$ is:

x	-3	0	3
y			

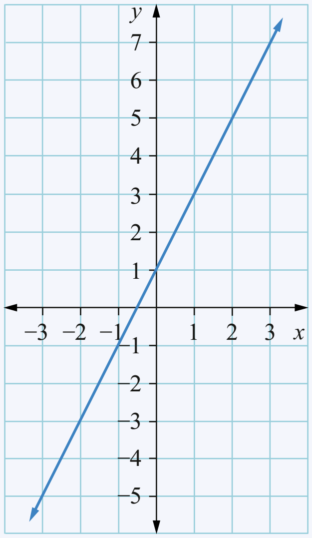
A



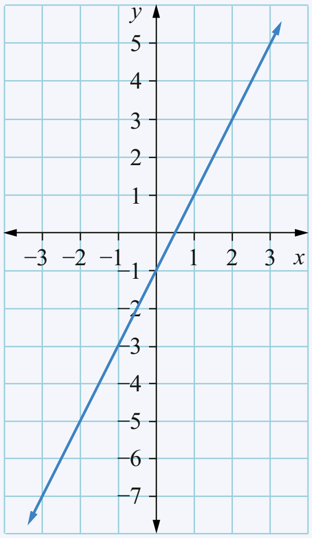
B



C

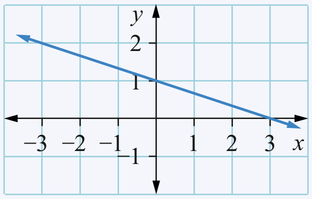


D



14 Complete this table of values for each equation.

x	-3	0	3
y			



The equation of the graph is:

A $y = 3x - 1$ **B** $y = -3x + 1$

C $y = -\frac{1}{3}x + 1$ **D** $y = \frac{1}{3}x - 1$

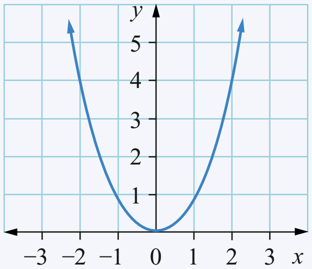
15 A line containing point $(-4, 9)$ is:

A $y = 3x - 8$ **B** $y = -3x - 3$ **C** $y = 4x + 14$ **D** $y = -4x - 11$

16 The equation of this graph is:

A $y = 3^x$ **B** $y = 2^x$

C $y = x^2$ **D** $y = x^2 + 1$



If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–4	5, 6	7, 8	9–11	12	13–15	16
Section	A	B	C	D	E	F	G

10A Review set

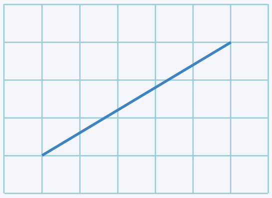
1 By plotting the points, find the midpoint of the line joining:

a $(4, 3)$ and $(10, 3)$ **b** $(2, 5)$ and $(2, 9)$ **c** $(-4, 3)$ and $(10, -1)$

2 Using Pythagoras' theorem, find the distance between each pair of points.

a $(4, 3)$ and $(10, 7)$ **b** $(-2, 5)$ and $(2, 3)$

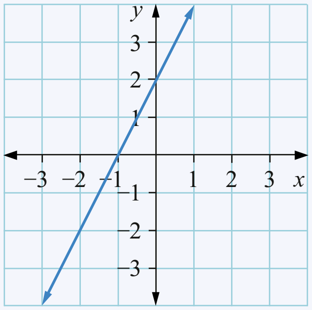
3 Draw a right-angled triangle to find the gradient of the line shown at right.



4 Find the gradient of the line passing through each pair of points.

a $(-5, -2)$ and $(6, 3)$ **b** $(-3, 6)$ and $(7, 1)$

5 Find the gradient of the line shown at right.



6 Sketch these lines.

a $y = 3$　　**b** $x = -4$

7 Draw a graph of each line by plotting points at $x = -3$, $x = 0$ and $x = 3$.

a $y = x + 3$　　**b** $y = 4 - 3x$

c $x + y = 5$　　**d** $x - y = 2$

8 Does point $(4, -3)$ lie on the line $y = 2x - 11$? Explain your answer.

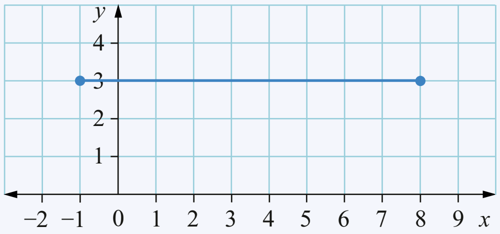
9 Complete this table for the relation $y = x^2 - 3$ and draw a graph.

x	-3	-2	-1	0	1	2	3
y							

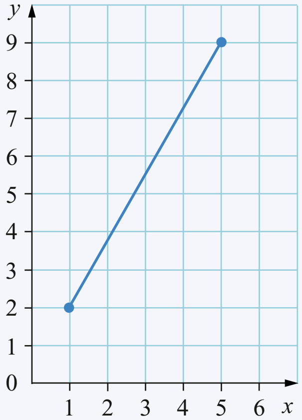
10B Review set

1 Find the midpoint of each line interval.

a



b



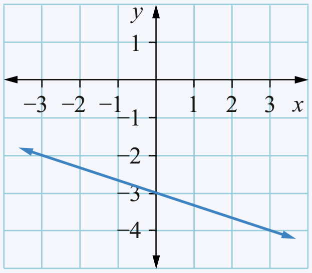
2 **a** Find the distance between the two points in question **1** part **b**.

b Find the distance between points $(1, 8)$ and $(5, 1)$.

3 **a** Find the gradient of the line in question **1** part **b**.

b Find the gradient of the line passing through points $(-4, -2)$ and $(2, 3)$.

4 Find the gradient of the line shown at right.



5 Draw a graph of each of these lines by plotting points at $x = -3$, $x = 0$ and $x = 3$.

a $y = x + 4$　　**b** $y = 2 - 3x$

c $x + y = -8$　　**d** $x - y = 0$

6 Does point $(-1, -3)$ lie on the line $x - y = 4$? Explain your answer.

7 Complete this table for the relation $y = 2^x$ and draw a graph.

x	−2	−1	0	1	2	3
y						

8 A circle has centre at (0, 0) and radius 5 units. Sketch the circle and write its equation.

10C Review set

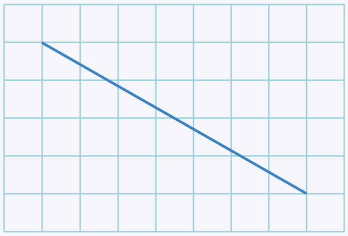
1 By plotting the points, find the midpoint of the line joining:

a (5, 3) and (11, 3) **b** (3, 1) and (8, 4) **c** (−5, 3) and (10, −4)

2 Use Pythagoras' theorem to find the distance between each pair of points.

a (4, 3) and (9, 8) **b** (−2, 7) and (3, 3)

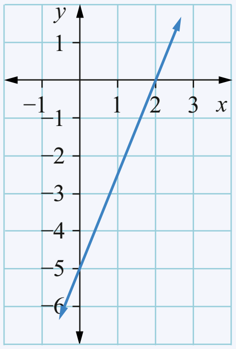
3 Draw a right-angled triangle to find the gradient of the line shown at right.



4 Find the gradient of the line passing through each pair of points.

a (−7, −2) and (6, 3) **b** (−3, 6) and (8, 3)

5 Find the gradient of the line shown at right.



6 Sketch these lines.

a $y = -5$ **b** $x = 2$

7 Draw a graph of each line by plotting points at $x = -3$, $x = 0$ and $x = 3$.

a $y = 5 - 2x$ **b** $y = 2x + 1$

c $x + y = -1$ **d** $x - y = 6$

8 Does point (2, 5) lie on the line $y = -3x - 1$? Explain your answer.

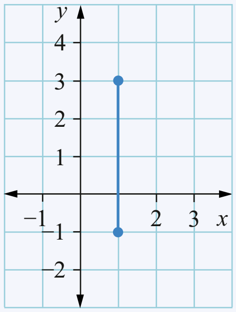
9 Complete this table for the relation $y = x^2 - 5$ and draw a graph.

x	−3	−2	−1	0	1	2	3
y							

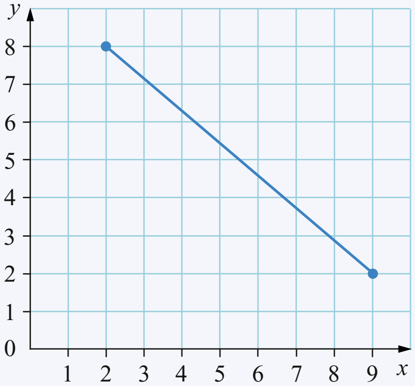
10D Review set

1 Find the midpoint of each line interval.

a



b



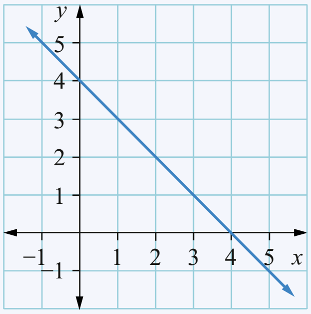
2 **a** Find the distance between the two points in question **1** part **b**.

b Find the distance between points (3, 8) and (5, 4).

3 **a** Find the gradient of the line in question **1** part **b**.

b Find the gradient of the line passing through points (−7, −2) and (2, 1).

4 Find the gradient of the line shown at right.



5 Draw a graph of each line by plotting points at $x = -3$, $x = 0$ and $x = 3$.

a $y = 3 - x$

b $y = 2x - 3$

c $2x + y = 1$

d $x - y = 3$

6 Does point (−1, 5) lie on the line $x + y = 4$? Explain your answer.

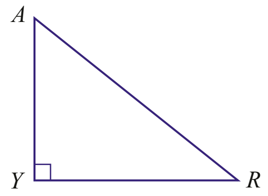
7 Complete this table for the relation $y = 3^x$ and draw a graph.

x	−2	−1	0	1	2	3
y						

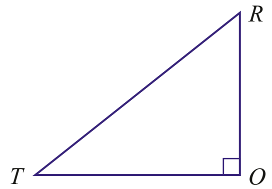
8 A circle has centre at (0, 0) and radius $\sqrt{5}$ units. Sketch the circle and write its equation.

Chapters 8–10 Cumulative review

1 **a** Name each of the sides in this triangle in two different ways.

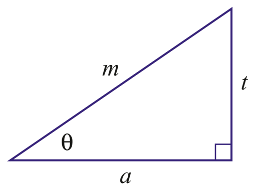


b Write the name of the hypotenuse of this triangle in two different ways.



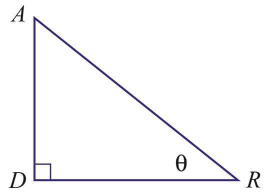
c Use this triangle to write expressions for:

i sin θ **ii** cos θ **iii** tan θ



d Write expressions in two different ways for:

i tan θ **ii** sin θ **iii** cos θ



e Find these values correct to 4 decimal places.

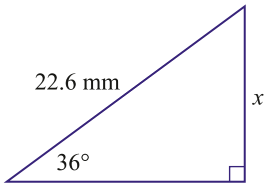
i cos 53° **ii** sin 71° **iii** tan 27°

f Find the value of θ in these expressions to the nearest degree.

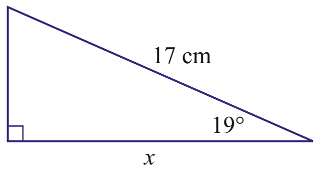
i $\tan\theta = 3.466$ **ii** $\sin\theta = 0.5473$ **iii** $\cos\theta = 0.3396$ **iv** $\tan\theta = 11.525$

g Find the value of x correct to 1 decimal place.

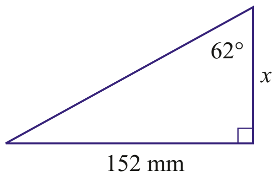
i



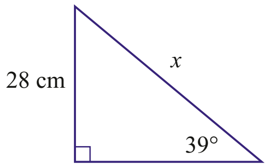
ii



iii

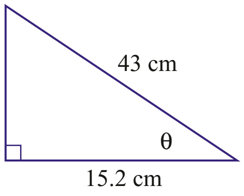


iv

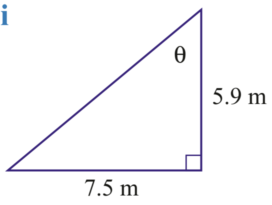


h Find the value of θ to the nearest degree.

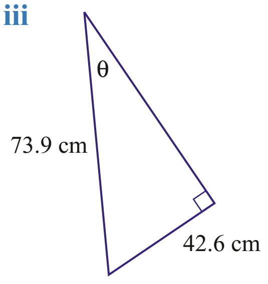
i



ii



iii



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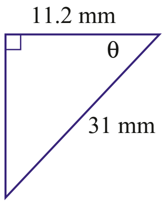
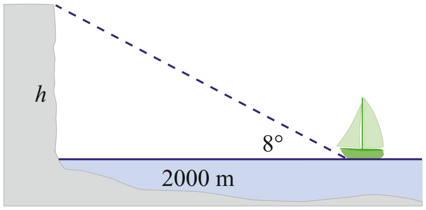
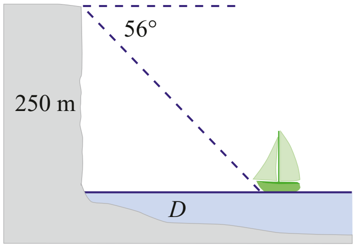


i The diagram shows that the angle of elevation of the top of a cliff from a boat 2000 m out to sea is 8°. Find the height of the cliff above sea level.

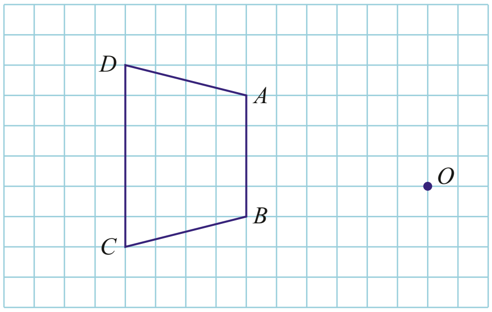


j The angle of depression from the top of a cliff 250 m above sea level to a boat is 56°. Find the distance of the boat from the cliff.

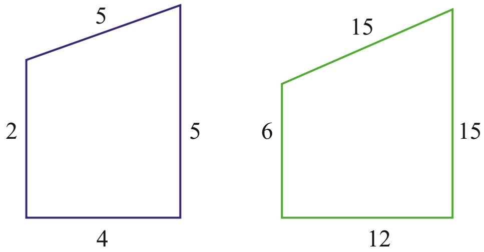


2 **a** **i** Enlarge this figure using an enlargement factor of 2 and *O* as the centre of enlargement.

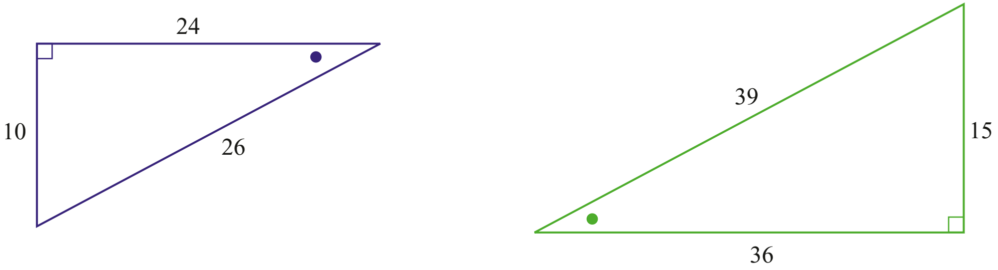
ii Label the vertices of the enlarged figure and name the pairs of corresponding sides in the similar figures.



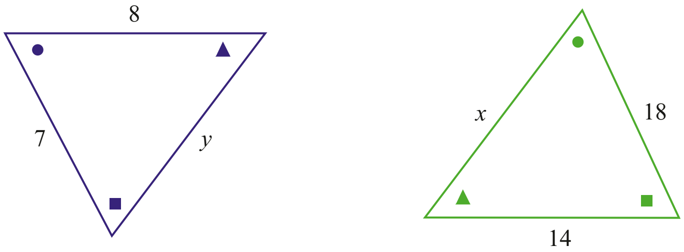
b Use the information given to determine whether or not these two figures are similar. If they are similar, state the enlargement factor.



c Determine whether or not these triangles are similar.

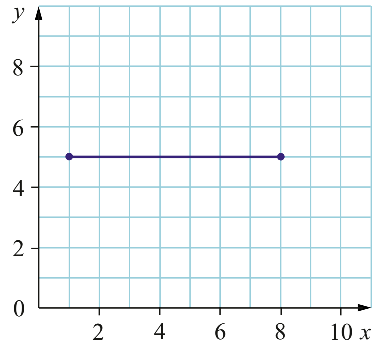


d Find the enlargement factor and hence the length of the unknown sides in these similar triangles.

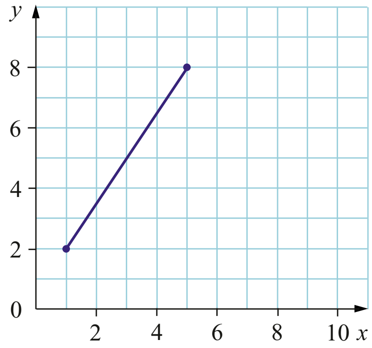


3 **a** Find the midpoint of these line intervals.

i



ii



b Plot these pairs of points, join them with a straight line and find the coordinates of their midpoint.

i (2, 3) and (10, 3)

ii (2, 4) and (2, 9)

iii (5, 1) and (8, 4)

iv (−5, 1) and (10, −4)

c **i** Find the distance between the two points in question **3** part **a ii**.

ii Use a diagram to find the distance between the points (2, 8) and (5, 1).

d Using Pythagoras' theorem, find the distance between these pairs of points:

i (4, 3) and (9, 8)

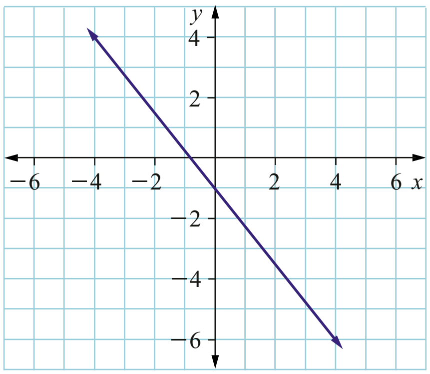
ii (−2, 7) and (3, 3)

e **i** Find the gradient of the line in question **3** part **a ii**.

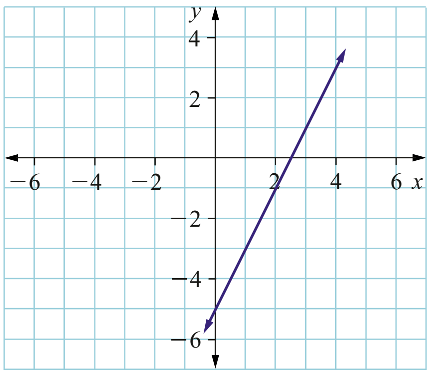
ii Find the gradient of the line passing through the points (−7, −2) and (2, 4).

f Find the gradient of each line.

i



ii



g Complete this table and draw a sketch of each line.

x	−3	0	3
y			

i $y = x + 2$

ii $y = 5 - 3x$

iii $x + y = -3$

iv $2x - y = 0$

h Does point (−1, −3) lie on the line $x - y = 2$? Explain your answer.

i **i** Complete this table for the relation $y = 3^x$.

x	−2	−1	0	1	2
y					

ii Draw a neat sketch of the relation $y = 3^x$.

j Draw neat sketches of these relations.

i $y = x^2$

ii $y = x^2 - 1$

11

Proportion and rates

This chapter deals with solving problems involving direct proportion.

After completing this chapter you should be able to:

- convert between units for rates
- identify and recognise everyday examples of direct and inverse proportion
- recognise direct and inverse proportion from graphs
- interpret and use conversion graphs
- use the equation $y = kx$ to model direct linear proportion
- graph equations representing direct linear proportion.

NSW Syllabus references: 5.2 N&A Ratios and rates
Outcomes: MA5.2-1WM, MA5.2-2WM, MA5.2-5NA
NUMBER & ALGEBRA – ACMNA208

A Rates

A **rate** is a comparison between quantities of different kinds. The comparison is made by dividing one quantity by the other, in the required order. The rate is then expressed in the form ‘the first quantity per unit of the second quantity’.

EXAMPLE 1

A car travelled 240 km in 3 hours. Calculate its rate of travel (speed).

Solve/Think	Apply
Rate $= \dfrac{240\text{ km}}{3\text{ h}} = 80$ km/h	Divide distance by time.

Exercise 11A

1 **a** A truck travelled 300 km in 5 hours. Calculate its average speed.

b Peter typed 480 words in 8 min. How many words per minute does he type?

c A farmer spread 16 t of fertiliser over 5 ha. Calculate the rate of application in t/ha.

d A 4 kg box of soap powder costs \$12.64. What is the cost per kg?

e Natasha spent \$54 on 120 phone calls. What is the average cost per call?

f A car used 16 L of petrol to travel 200 km. Calculate the rate at which petrol was used in L/km.

g Mr Lee’s water bill was \$148.50 for 55 kL of water. What is the cost per kilolitre?

h If 1.92 L of water drips from a tap in 8 hours, calculate the rate at which water leaks from the tap in litres per hour.

EXAMPLE 2

Convert 108 L/h to:

a L/min **b** mL/min **c** mL/s

	Solve	Think	Apply
a	108 L/h $= \dfrac{108\text{ L}}{1 \times 60\text{ min}}$ $= 1.8$ L/min	Change hours to minutes and divide.	Convert each quantity to the required unit and divide the first by the second.
b	108 L/h $= \dfrac{108 \times 1000\text{ mL}}{60\text{ min}}$ $= 1800$ mL/min	Change litres to millilitres, change hours to minutes and divide.	
c	108 L/h $= \dfrac{108 \times 1000\text{ mL}}{1 \times 60 \times 60\text{ s}}$ $= 30$ mL/s	Change litres to millilitres, change hours to seconds and divide.	

2 Complete to convert the following.

a 72 L/h to mL/s

$$\frac{72\text{ L}}{1\text{ h}} = \frac{\square \times \square\text{ mL}}{1 \times \square \times \square\text{ s}} = \_\_\_\text{ mL/s}$$

b \$7/m to c/mm

$$\frac{\$7}{1\text{ m}} = \frac{7 \times \square\text{c}}{1 \times \square\text{ mm}} = \_\_\_\text{ c/mm}$$

3 Convert the following.

a \$5.40/h to c/min
b \$8/m to c/cm
c 8 t/ha to kg/m^2
d 48.6 L/h to mL/s
e 9% /year to %/month
f \$15/kg to c/g
g 60 km/h to m/s
h 15 kg/day to g/h
i 1.8 kg/L to g/mL

EXAMPLE 3

Convert the following.

a 8 g/mL to kg/L
b 12 m/s to km/h

	Solve	Think	Apply
a	$8\text{ g/mL} = \dfrac{8 \div 1000\text{ kg}}{1 \div 1000\text{ L}}$ $= \dfrac{8}{1000}\text{ kg} \div \dfrac{1}{1000}\text{ L}$ $= 8\text{ kg/L}$	Divide 8 g by 1000 to convert to kilograms. Divide 1 mL by 1000 to convert to litres.	Divide each quantity by the appropriate factor to convert to the required unit, then use a calculator to evaluate.
b	$12\text{ m/s} = \dfrac{12 \div 1000\text{ km}}{1 \div 3600\text{ h}}$ $= \dfrac{12}{1000}\text{ km} \div \dfrac{1}{3600}\text{ h}$ $= 43.2\text{ km/h}$	Divide 12 m by 1000 to convert to kilometres. Divide 1 s by 60 × 60 = 3600 to convert to hours.	

4 Complete to convert the following.

a 7 g/mL to kg/L

$$\frac{7\text{ g}}{1\text{ mL}} = \frac{\square \div \square\text{ kg}}{1 \div \square\text{ L}}$$
$$= \frac{\square}{1000}\text{ kg} \div \frac{\square}{1000}\text{ L}$$
$$= \_\_\_\text{ kg/L}$$

b 8 m/s to km/h

$$\frac{8\text{ m}}{1\text{ s}} = \frac{\square \div \square\text{ km}}{1 \div \square\text{ h}}$$
$$\frac{8\text{ m}}{1\text{ s}} = \frac{\square}{1000}\text{ km} \div \frac{\square}{1000}\text{ h}$$
$$= \_\_\_\text{ km/h}$$

5 Convert the following.

a 6 g/mL to kg/L
b 15 m/s to km/h
c 12 c/min to \$/h
d 5 mL/s to L/h
e 9 c/cm to \$/m
f 0.3 kg/m^2 to t/ha
g 4.5 c/g to \$/kg
h 28 m/s to km/h

B Direct and inverse proportion

Two quantities are **directly proportional** if an increase in one of them causes a proportional increase in the other (or if a decrease in one causes a proportional decrease in the other). For example, if one quantity is doubled the other is also doubled, if one quantity is increased by a factor of 3 (multiplied by 3) then the other is increased by a factor of 3, if one quantity is halved the other is halved, and so on. The number of hours worked and the amount earned are directly proportional because, if the number of hours worked is doubled, the amount earned is doubled (or if the number of hours worked is halved, the amount earned is halved).

Two quantities are **inversely proportional** if an increase in one of them causes a proportional decrease in the other (or if a decrease in one causes a proportional increase in the other). For example, if one quantity is doubled the other is halved, if one quantity is increased by a factor of 3 the other is decreased by a factor of one-third, if one quantity is halved the other is doubled, and so on.

Consider the number of painters and the time it takes to paint a room. These are inversely proportional because, if the number of painters is doubled, the time taken is halved (or if the number of painters is halved the time taken is doubled), assuming each painter works at the same rate.

EXAMPLE 1

Determine whether the following variables are in direct proportion, inverse proportion or unrelated.

a the cost of tomatoes and the weight of tomatoes

b the average speed of a car and the time taken to travel from Sydney to Melbourne

c a person's weight and the number of pets owned

	Solve	Think	Apply
a	Direct proportion	If the weight of tomatoes is doubled the cost is also doubled. Hence these quantities are in direct proportion.	Use the doubling of one quantity as a test. If, when one quantity is doubled, the other is doubled, the quantities are in direct proportion. If, when one quantity is doubled, the other is halved, the quantities are in inverse proportion.
b	Inverse proportion	If the average speed of the car is doubled, the time taken for the journey is halved. Hence these quantities are inversely proportional.	
c	Unrelated	If the number of pets owned is doubled, there will be no change in a person's weight. Hence these quantities are unrelated.	

Exercise 11B

1 Determine whether the following variables are in direct proportion, inverse proportion or unrelated.

a the weight of potatoes and the cost of potatoes

b the side length of an equilateral triangle and the perimeter of the triangle

c the number of people digging and the time taken to dig a hole

d the height of a girl and her performance in mathematics

e the length of a rectangle of area 20 cm^2 and the breadth of the rectangle

f the rate of flow of water from a tap and the time taken to fill a tank

g the number of unemployed people and the price of eggs

h the number of people sharing a lottery and the amount each person receives

i the number of nights in a motel and the cost of the stay

j the speed of a car and the distance travelled

k the amount of money borrowed and the amount of interest charged

l the rate at which a secretary types and the time it takes to type a letter

C Graphs involving direct and inverse proportion

EXAMPLE 1

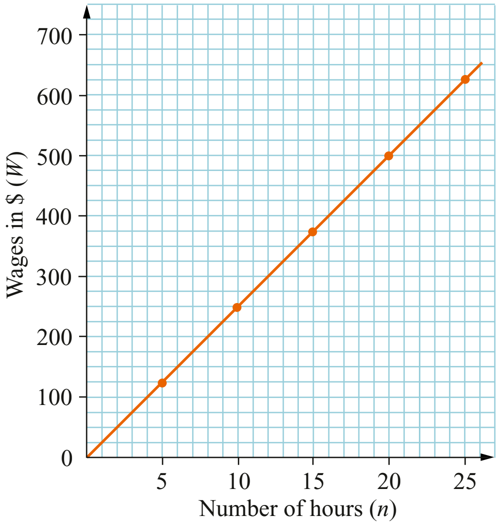
A waiter earns $25 per hour.

a Complete the following table and draw a graph of the waiter's wages compared with the number of hours worked.

Hours worked (n)	0	5	10	15	20	25
Wages in \$ (W)						

b Are the quantities n and W in direct proportion, inverse proportion or neither?

c Describe the features of the graph.

<table>
<tr><th></th><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr><td>a</td><td>
<table>
<tr><td>n</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr>
<tr><td>W</td><td>0</td><td>125</td><td>250</td><td>375</td><td>500</td><td>625</td></tr>
</table>
Wages for hours worked


</td><td>Complete the table. Plot the points (0, 0), (5, 125), (10, 250) etc. from the table and draw a straight line through them.</td><td rowspan="3">Plot the points from the table on a number plane and draw a straight line through them.
Use the ‘doubling test’ to determine if the quantities are in proportion.
Describe the features of the graph.</td></tr>
<tr><td>b</td><td>The quantities n and W are in direct proportion.</td><td>If n is doubled, W is doubled.</td></tr>
<tr><td>c</td><td colspan="2">The graph is a straight line that passes through the origin and has a positive gradient (it slopes up from left to right).</td></tr>
</table>

EXAMPLE 2

A cyclist has a distance of 40 km to ride.

a Complete the following table and draw a graph of the time the cyclist takes to ride this distance compared with her speed. (Join the points by a smooth curve.)

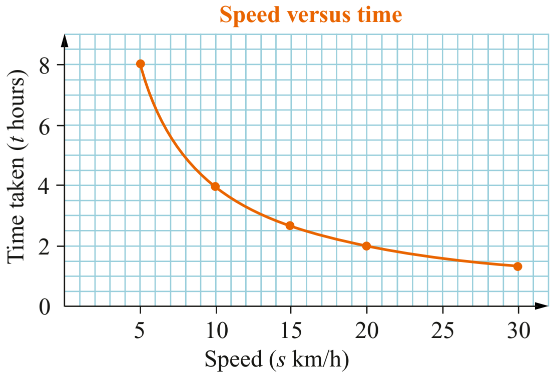
Speed (*s* km/h)	5	10	15	20	25	30
Time (*t* hours)						

b **i** Is there a point on the curve where $s = 0$?

ii Is there a point on the curve where $t = 0$?

c Are the quantities *s* and *t* in direct proportion, inverse proportion or neither?

d Describe the features of the graph.

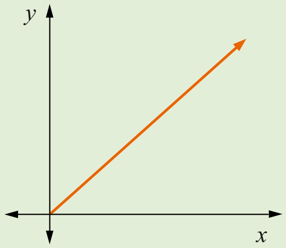
<table>
<tr><th></th><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr><td>a</td><td>
<table>
<tr><td><i>s</i> km/h</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td><td>30</td></tr>
<tr><td><i>t</i> hours</td><td>8</td><td>4</td><td>$2\frac{2}{3}$</td><td>2</td><td>1.6</td><td>$1\frac{1}{3}$</td></tr>
</table>


</td><td>Plot the points (5, 8), (10, 4) etc. from the table.</td><td rowspan="5">Plot the points from the table on a number plane and draw a smooth curve through them. Use the ‘doubling test’ to determine if the quantities are in proportion. Describe the features of the graph.
A graph showing inverse proportion is a smooth curve sloping down from left to right that does not cut either axis.</td></tr>
<tr><td>c i</td><td>No</td><td>If speed is 0, the cyclist cannot travel.</td></tr>
<tr><td>ii</td><td>No</td><td>There is no speed at which the cyclist can ride 40 km in 0 hours.</td></tr>
<tr><td>d</td><td>The quantities are in inverse proportion.</td><td>If <i>s</i> is doubled, <i>t</i> is halved.</td></tr>
<tr><td>e</td><td colspan="2">The graph is a smooth curve that slopes down from left to right (has a negative gradient). It does not cut either of the axes.</td></tr>
</table>

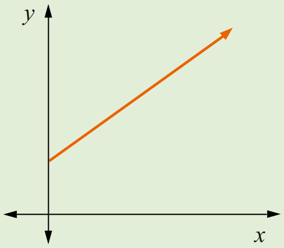
EXAMPLE 3

Determine whether the following graphs show direct proportion, inverse proportion or neither between the quantities x and y.

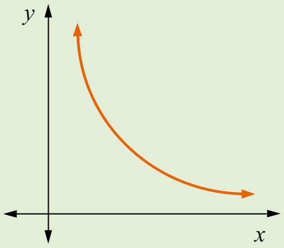
a



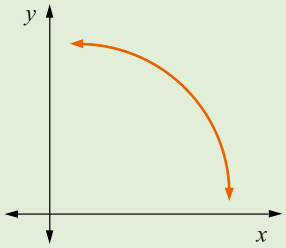
b



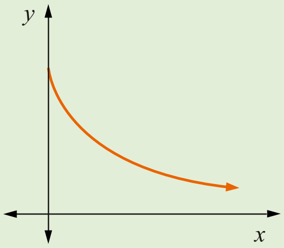
c



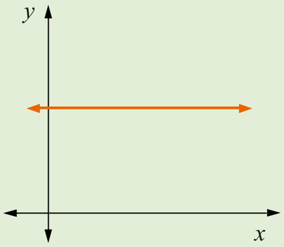
d

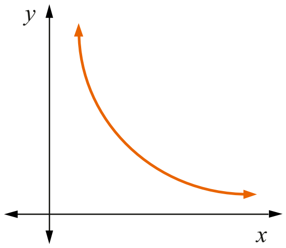
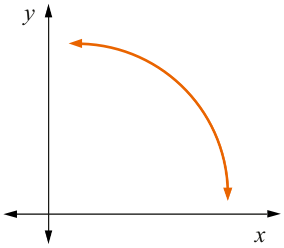


e



f



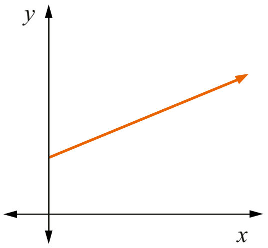
	Solve	Think	Apply
a	Direct proportion	The graph is a straight line passing through the origin and has a positive gradient.	If two quantities x and y are in direct proportion then the graph of y against x is a straight line that passes through the origin and has a positive gradient (slopes up from left to right). If two quantities x and y are in inverse proportion then the graph of y against x is a smooth curve that always slopes down from left to right and does not cut either of the axes.
b	Neither	The graph is a straight line with a positive gradient but it does not pass through the origin.	
c	Inverse proportion	The graph is a smooth curve that slopes down from left to right and does not cut either axis. 	
d	Neither	The graph is a smooth curve that slopes down from left to right but it cuts both axes. 	
e	Neither	The graph is a smooth curve that slopes down from left to right but it touches the y-axis.	
f	Neither	The graph is a straight horizontal line that does not pass through the origin. y does not change as x increases.	

Exercise 11C

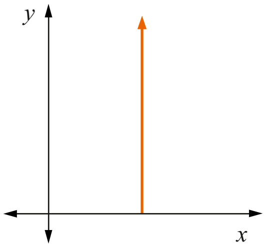
1 Consider the straight-line graphs shown.

- **i** Does the line pass through the origin?
- **ii** Does the line have a positive gradient?
- **iii** Are the quantities x and y in direct proportion?

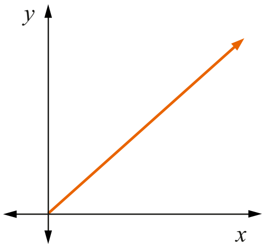
a



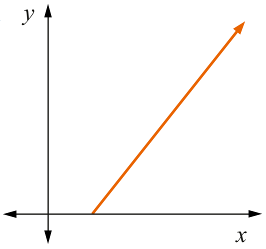
b



c



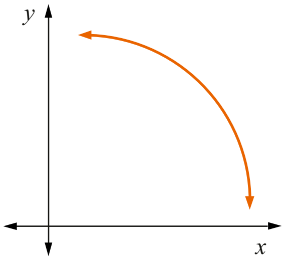
d



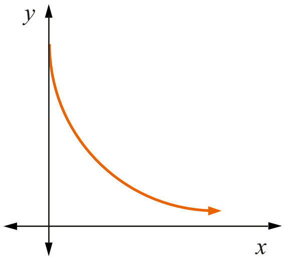
2 Consider the smooth curves shown.

- **i** Does the curve slope down from the left to the right?
- **ii** Does the curve cut the x-axis and the y-axis (extend if necessary)?
- **iii** Are x and y in inverse proportion?

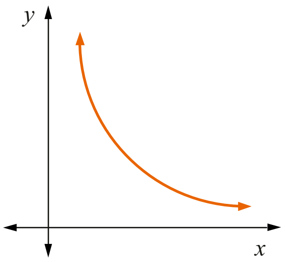
a



b



c



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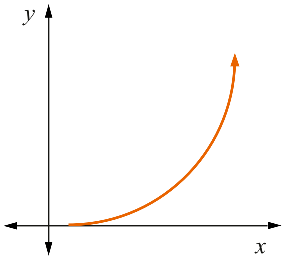
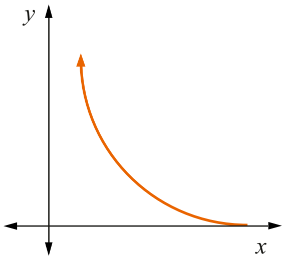
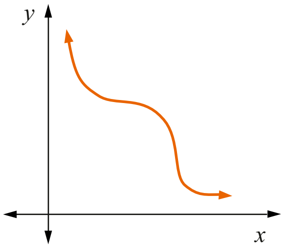


e

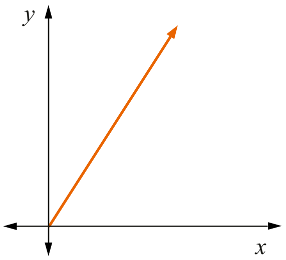


f

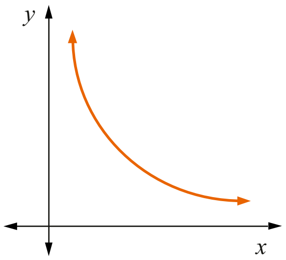


3 Determine whether the following graphs show direct proportion, inverse proportion or neither between the quantities x and y.

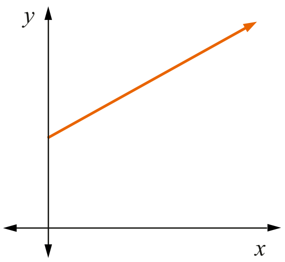
a



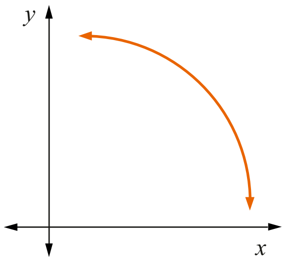
b



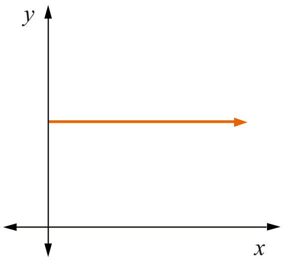
c



d



e



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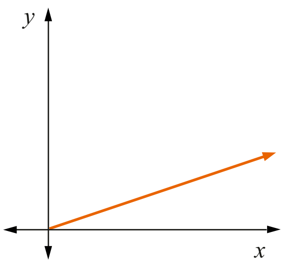

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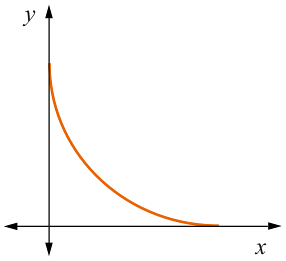

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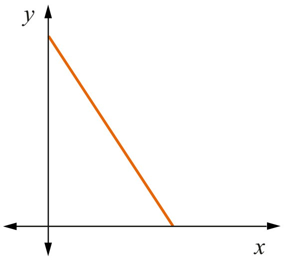

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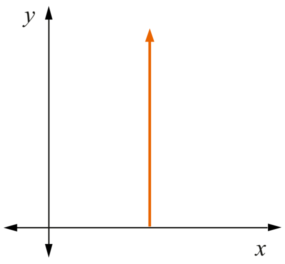

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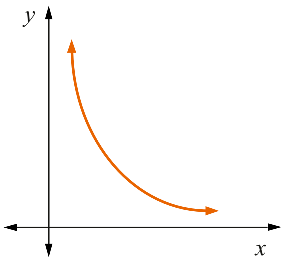

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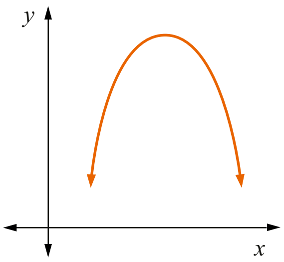

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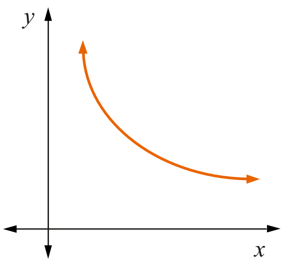

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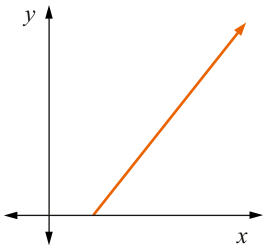

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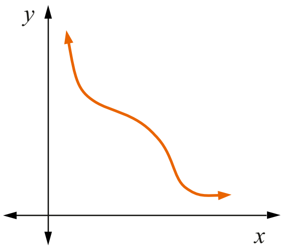

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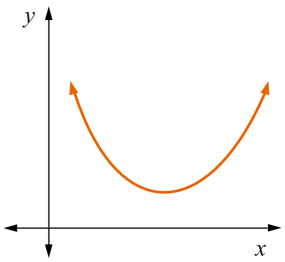


D Direct linear proportion

Formally, we define **direct linear proportion** as follows:

> Two variables are in direct linear proportion if the ratio of one to the other is constant.

This may be stated algebraically as:

> Two variables P and Q are in direct linear proportion if $\frac{P}{Q} = k$, where k is a constant.
> (k is called the **constant of proportionality**.)

This relationship may be written $P = kQ$.

Hence for direct linear proportion, as one quantity Q increases the other quantity P also increases in such a way that the ratio of P to Q is always the same.

EXAMPLE 1

The progress of a marathon runner over time is shown in the table.

Time taken (t min)	30	108	156	204	252
Distance travelled (d km)	5	18	26	34	42

a Determine whether or not the given variables are in direct linear proportion.

b If they are in direct linear proportion, write an equation that describes the relationship between the variables.

	Solve/Think	Apply
a	Add another row to the table and determine the values of $\frac{d}{t}$ for each case.	Determine if the ratio of one quantity to the other is constant by dividing the second quantity by the first quantity. Equate the ratio of quantities and the constant. Rearrange this equation if required.

Time taken (t min)	30	108	156	204	252
Distance travelled (d km)	5	18	26	34	42
$\frac{d}{t}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

As the ratio $\frac{d}{t}$ = a constant $\left(\frac{1}{6}\right)$ the variables d and t are in direct linear proportion.

b

$$\frac{d}{t} = \frac{1}{6}$$

$$d = \frac{1}{6}t \text{ or } d = \frac{t}{6}$$

Exercise 11D

1 **a** Complete the table on the right.

b Is $\frac{y}{x}$ a constant?

c Are x and y in direct linear proportion?

d Complete: $\frac{y}{x}$ = ____ $\therefore y$ = ____

x	1	2	3	4	5
y	3	6	9	12	15
$\frac{y}{x}$					

2 **i** Determine whether or not the variables in the tables given below are in direct linear proportion.

ii If they are in direct linear proportion, write an equation that describes the relationship between the variables.

a

N	15	25	35	70
M	9	15	21	42

b

q	5	8	12	14	19
p	7	11.2	16.8	19.6	26.6

c

x	1	2	3	4
y	19	16	11	4

d

m	5	12	14	18
C	18	43.2	50.4	64.8

3 **i** Graph the relationships shown in each of the tables in question **2**. (Put the first variable on the horizontal axis and the second variable on the vertical axis.)

ii Comment on the features of the graph if the variables are in direct linear proportion.

EXAMPLE 2

The circumference of a circle C can be approximated using the formula $C = 3.14D$, where D cm is the diameter. Determine whether or not C and D are in direct linear proportion.

Solve	Think	Apply
$C = 3.14D$ so $\frac{C}{D} = 3.14$ The ratio $\frac{C}{D}$ = a constant, hence the circumference and diameter are in direct linear proportion.	Divide both sides of the equation by D.	If two quantities P and Q can be written in the form $\frac{P}{Q}$ = a constant, then they are in direct linear proportion.

4 Complete the following.

$M = 1.2V$ Divide both sides by V.

$\frac{M}{V} = \frac{1.2V}{V} =$ ___ $\therefore$ M and V are/are not in direct linear proportion.

5 Sarah's wages, \$$W$, are calculated using the formula $W = 15t$, where t is the number of hours worked. Determine whether or not W and t are in direct linear proportion.

6 The perimeter (P) of a square is given by $P = 4s$ where s is the side length. Determine whether or not P and s are in direct proportion.

7 The cost \$$C$ of buying m kilograms of bananas is $C = 2.4m$. Determine whether or not C and m are in direct proportion.

8 The number of revolutions, n, that the wheel of a bicycle makes when travelling over a distance of d metres is given by the formula $n = \frac{d}{2}$. Complete the following.

$n \div d = \frac{d}{2} \div d$

$\frac{n}{d} = \frac{d}{2} \times \frac{1}{d} =$ ___

Hence n and d are/are not in direct linear proportion.

9 The extension, e, of a spring when a weight of m grams is placed on its end is given by the formula $e = \frac{2}{7}m$. Complete the following.

$e \div m = \frac{2}{7}m \div m$

$\frac{e}{m} = \frac{2}{7}m \times \frac{1}{m} =$ ___

Hence e and m are/are not in direct linear proportion.

EXAMPLE 3

The commission (\$$C$) earned by a salesperson is calculated using the formula $C = 100 + 0.08S$ where \$$S$ is the value of their sales.

a Determine algebraically whether or not C and S are in direct linear proportion.

b **i** Complete the table and graph the relationship $C = 100 + 0.08S$.

ii Comment on the features of the graph.

S	0	100	200	300	400
C					

Solve/Think

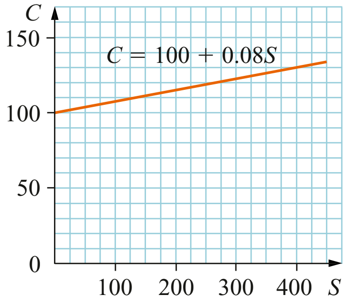
a The formula cannot be written in the form $\frac{C}{S}$ = a constant.

Dividing both sides by S gives $\frac{C}{S} = \frac{100 + 0.08S}{S}$ or $\frac{C}{S} = \frac{100}{S} + 0.08$.

Hence C and S are not in direct linear proportion.

b **i**

S	0	100	200	300	400
C	100	108	116	124	132



ii The graph is a straight line with a positive gradient, but it does not start at the origin.

Apply

If the relationship between the quantities P and Q cannot be written in the form $\frac{P}{Q}$ = a constant, then P and Q are not in direct linear proportion. This can be confirmed by graphing the relationship.

Note: From the table of the values in Example 3 it can be seen that as S increases C increases, but the ratio of C to S does not remain constant, again showing that the quantities are not in direct linear proportion.

S	100	200	300	400
C	108	116	124	132
$\frac{C}{S}$	1.08	0.58	0.41	0.33

10 The cost (\$$C$) of hiring a taxi in Sydney is $C = 3.4 + 2.45d$ where d is the distance travelled in kilometres.

a Is it possible to write the formula in the form $\frac{C}{d}$ = a constant? Determine whether C and d are in direct linear proportion.

b **i** Complete the following table and graph the relationship $C = 3.4 + 2.45d$.

d	1	2	5	10
C				

ii Comment on the features of the graph.

c Complete this table. Does the ratio $\frac{C}{d}$ remain constant?

d	1	2	5	10
C				
$\frac{C}{d}$				

11 Using algebra or a table of values, determine whether the given variables are in direct linear proportion.

a $M = 3N$ **b** $y = 0.5x$ **c** $p = 5.3s$ **d** $m = \frac{n}{5}$

e $y = \frac{1}{3}x$ **f** $s = \frac{3}{7}t$ **g** $y = x + 1$ **h** $v = z - 2$

i $p = 8 + 4q$ **j** $C = \frac{4b}{7}$ **k** $s = 3t - 2$ **l** $y = \frac{4}{x}$

E Conversion graphs

EXAMPLE 1

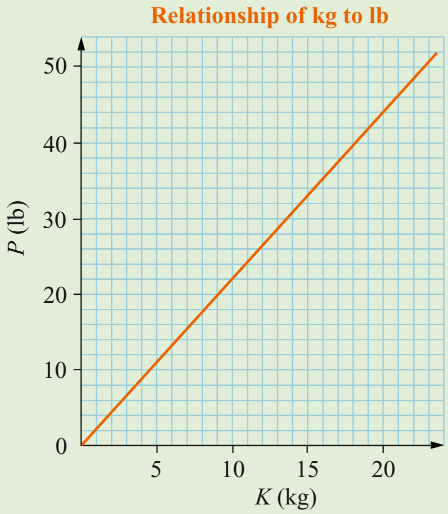
This graph can be used to convert kilograms (K) to pounds weight (P) and vice versa.

a Use the graph to complete the table.

K (kg)	5	10	15	20
P (lb)				

b **i** Are the variables in direct linear proportion? Why?

ii Find the relationship between P and K.



<table>
<tr><th></th><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr><td>a</td><td><table><tr><td>K (kg)</td><td>5</td><td>10</td><td>15</td><td>20</td></tr><tr><td>P (lb)</td><td>11</td><td>22</td><td>33</td><td>44</td></tr></table></td><td>The values are read from the graph.</td><td rowspan="3">Find the appropriate values of P and K from the graph.
The relationship between the quantities graphed can be determined by finding the ratio of one to the other. This ratio is the gradient of the line.</td></tr>
<tr><td>b i</td><td>Yes</td><td>From Section 11B, straight-line graphs that pass through the origin and have a positive gradient show that the quantities are in direct linear proportion.</td></tr>
<tr><td>ii</td><td>$\frac{P}{K} = 2.2$ or $P = 2.2K$</td><td>Add another row to the table.<table><tr><td>K (kg)</td><td>5</td><td>10</td><td>15</td><td>20</td></tr><tr><td>P (lb)</td><td>11</td><td>22</td><td>33</td><td>44</td></tr><tr><td>$\frac{P}{K}$</td><td>2.2</td><td>2.2</td><td>2.2</td><td>2.2</td></tr></table>From the table $\frac{P}{K} = 2.2$.</td></tr>
</table>

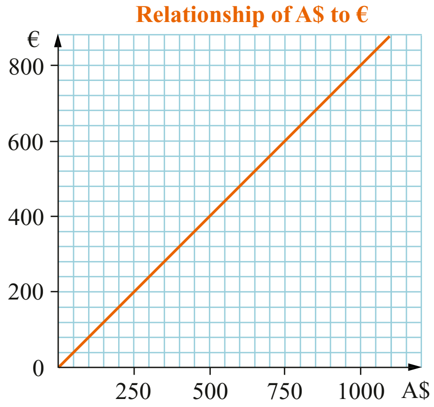
Exercise 11E

1 **a** This graph can be used to convert Australian dollars (A$) to euros (€). Use the graph to complete the table.

A	250	400		
€			560	800

b **i** Are the variables in direct linear proportion? Why?

ii Find the relationship between *A* and €.

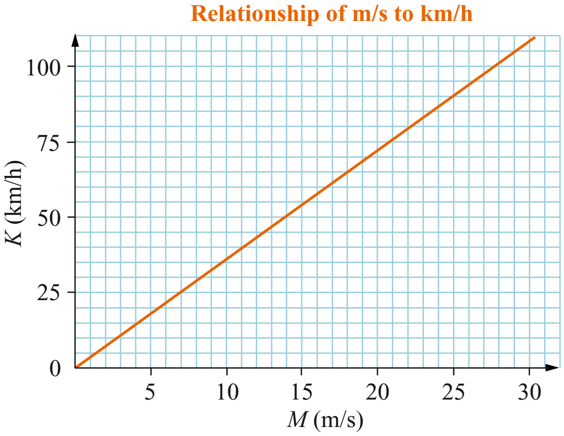

Relationship of A$ to €

2 **a** This graph can be used to convert metres per second (M) to kilometres per hour (K). Use the graph to complete the table.

M (m/s)	10	15		
K (km/h)			72	108

b **i** Are the variables in direct linear proportion? Why or why not?

ii Find the relationship between K and M.

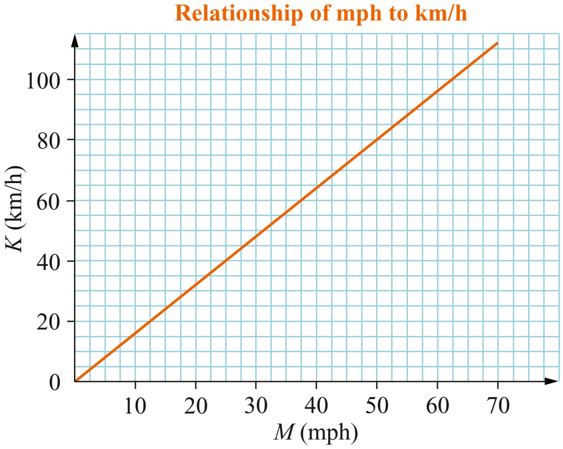

Relationship of m/s to km/h

3 **a** This graph can be used to convert from miles per hour (M) to kilometres per hour (K). Use the graph to complete the table.

M (mph)	20	50		
K (km/h)			96	112

b **i** Are the variables in direct linear proportion? Explain your answer.

ii Find the relationship between K and M.


Relationship of mph to km/h

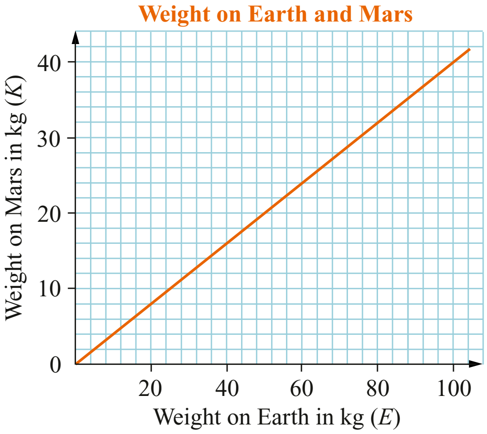
4 **a** This graph can be used to convert from weight (in kilograms) on Earth (E) to weight on Mars (M). Use the graph to complete the table.

E	30	40		
M			24	40

b **i** Are the variables in direct linear proportion? Why?

ii Find the relationship between M and E.

c How much would you weigh on Mars?


Weight on Earth and Mars

F Modelling direct linear proportion

EXAMPLE 1

Peter’s weekly wage ($\$W$) is in direct linear proportion to the number of hours he works (t).

a If the constant of proportionality is 14, write an equation that could be used to calculate his wage.

b Hence calculate his wage for a week in which he works 25 hours.

c How many hours does he work if he earns \$504?

	Solve	Think	Apply
a	$k = 14$ so $\frac{W}{t} = 14$ This may be written $W = 14t$.	As W and t are in direct linear proportion, $\frac{W}{t} = k$. Substitute $k = 14$ into the equation.	If P and Q are in direct linear proportion, $\frac{P}{Q} = k$, where k is the constant of proportionality. Substitute the value of the constant of proportion into the equation and use it to calculate the required values.
b	If $t = 25$, $W = 14 \times 25 = 350$. If Peter works 25 hours he earns \$350.	Substitute $t = 25$ into the equation and solve for W.	
c	If $W = 504$, $504 = 14t$. $\frac{504}{14} = W \therefore W = 36$ Peter works 36 hours if he earns \$504.	Substitute $W = 504$ into the equation and solve for t.	

Exercise 11F

1 The mass (m) in grams of a metallic object is directly proportional to its volume (V) in cubic centimetres.

a If the constant of proportionality is 7, complete the following to find an equation that could be used to find the mass of an object made from this metal.

$k = \_\_\_$ so $\frac{m}{V} = \_\_\_$ or $m = \_\_\_$

b If the volume of the metallic object is 45 cm^3, complete the following to find its mass.

$m = \_\_\_ \times V = \_\_\_ \times \_\_\_ = \_\_\_$ g

c If the mass of an object made from this metal is 840 g, complete the following to find its volume.

$840 = \_\_\_ \times V$

$V = \frac{840}{\square} = \_\_\_$ cm^3

2 The amount of fertiliser (F tonnes) used by a farmer on his crops is in direct linear proportion to the area (A hectares) he has to fertilise.

a If the constant of proportionality is 3, write an equation that could be used to calculate the amount of fertiliser needed.

b Hence calculate the amount of fertiliser he would need to cover an area of 2.3 ha.

c What area could he fertilise with 5.4 t?

3 The number of words (W) that a typist can type is in direct linear proportion to the time (t minutes) that she works.

a If the constant of proportionality is 30, write an equation which could be used to calculate the number of words typed in a given time.

b Hence calculate the number of words she types in 45 minutes.

c How long would it take her to type 600 words?

4 The interest ($\$I$) earned on an investment of $10 000 is in direct linear proportion to the number of months (n) the money is invested.

- **a** If the constant of proportionality is 50, write an equation that could be used to calculate the interest earned.
- **b** Hence calculate the interest earned for 15 months.
- **c** For how long would you have to invest this money to earn interest of $900?

EXAMPLE 2

The amount of fuel used by a car (F) is in direct linear proportion to the distance travelled (d).

- **a** Write an equation that could be used to calculate the amount of fuel used for a given journey.
- **b** Given that the car used 12 L on a journey of 150 km, calculate the constant of proportionality.
- **c** Hence calculate the amount of fuel that would be used for a journey of 400 km.
- **d** How far could the car travel on 60 L of fuel?

	Solve	Think	Apply
a	$\frac{F}{d} = k$ or $F = kd$	F and d are in direct linear proportion, so $\frac{F}{d} = k$.	Use the algebraic definition of direct linear proportion to write down an equation, involving k, connecting the two quantities. Use the given values of these quantities to determine the value of k. Use the equation to find the required values.
b	If $d = 150$ then $F = 12$. $\frac{12}{150} = k$ $k = 0.08$	Substitute $d = 150$ and $F = 12$ into the formula to find k.	
c	The equation can now be written as $\frac{F}{d} = 0.08$ or $F = 0.08d$. If $d = 400$, $F = 0.08 \times 400 = 32$ The car would use 32 L of fuel.	Substitute $d = 400$ into the equation and solve for F.	
d	If $F = 60$, $60 = 0.08d$ $d = \frac{60}{0.08} = 750$ The car could travel 750 km.	Substitute $F = 60$ into the equation and solve for d.	

5 The distance (d km) a cyclist can travel in a given time is in direct linear proportion to her speed (s km/h). Complete the following.

- **a** Write an equation that could be used to calculate the distance travelled at a given speed.

  $\frac{d}{s} = k$ or $d =$ ___
- **b** If the cyclist travels 60 km at a speed of 20 km/h, calculate the constant of proportionality.

  $d = 60$ when $s = 20$, hence $60 = k \times 20$

  $$k = \frac{60}{\square} = \_\_\_$$

  The equation can now be written as $d =$ ___ s.
- **c** Calculate the distance she can travel at 25 km/h.

  When $s = 25$, $d =$ ___ $\times 25 =$ ___ km
- **d** At what speed would she need to travel to cover 54 km?

  When $d = 54$, $54 =$ ___ $\times s$

  $$s = \frac{54}{\square} = \_\_\_ \text{ km/h}$$

6 The volume of paint (V L) needed to paint the interior walls of a house is directly proportional to the area (A m^2) to be painted.

a Write an equation that could be used to calculate the volume of paint needed.

b If 6 L of this paint will cover 75 m^2 of wall, calculate the constant of proportionality.

c Calculate the volume of paint needed to cover 120 m^2.

d What area could be covered with 12 L of paint?

7 The breaking strain (S units) of a steel wire is directly proportional to the circumference (C mm) of the wire.

a Write an equation that could be used to calculate the breaking strain of this steel wire.

b If a wire of circumference 8 mm has a breaking strain of 30 units, calculate the constant of proportionality.

c Calculate the breaking strain of a wire of circumference 12 mm.

d What would the circumference of the wire need to be to withstand a maximum strain of 60 units?

EXAMPLE 3

Given that the variables in the table are in direct linear proportion, calculate the missing values.

m	3	5	
T	12		32

Solve/Think	Apply
As the variables are in direct proportion, $\frac{T}{m} = k$. From the table, $T = 12$ when $m = 3$, hence $\frac{12}{3} = k \;\therefore k = 4$. Hence: $\frac{T}{m} = 4$ or $T = 4m$ If $m = 5$: $T = 4 \times 5 = 20$ If $T = 32$: $32 = 4 \times m$ $\therefore m = 8$	*Step 1:* Find the constant of proportionality. *Step 2:* Write down the equation from which T can be calculated, given the value of m. *Step 3:* Use the formula to calculate the missing values.

m	3	5	8
T	12	20	32

8 Given that the variables in the following tables are in direct linear proportion, calculate the missing values.

a

x	5	10	
y	9		27

b

x	4	8	
y	10		50

c

t	8	15	
M	4.8		15

d

x	7	14	
y	5.25		18

Language in mathematics

1 These sentences from this chapter have had the spaces removed and the letters collected in groups of four. Write the sentences correctly. Parts **a** and **b** are still in the correct order, but part **c** has been put into blocks of four and the blocks rearranged.

a Twoq uant itie sare dire ctly prop orti onal ifan incr ease inon eoft hemc ause sapr opor tion alin crea sein theo ther.

b Twoq uant itie sare inve rsel ypro port iona lifa ninc reas eino neof them caus esap ropo rtio nald ecre asei nthe othe r.

c igin dire opor heor Iftw iesa gain atpa thro of*y*a ntit then oqua ught sses st*x*i tion ctpr raph neth theg rein raig htli sast

Terms

constant of proportionality
conversion graphs
direct linear proportion
directly proportional
indirectly proportional
inverse proportion
linear proportion
quantities
rate

Check your skills

1 90 km/h is equivalent to:

A 2.5 m/s **B** 25 m/s **C** 150 m/s **D** 1500 m/s

2 700 mL/h is equivalent to:

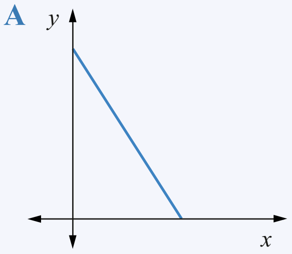
A 1.68 L/d **B** 16.8 L/d **C** 168 L/d **D** 16 800 L/d

3 The variables 'number of people sharing a lottery prize' and 'the size of the share' are an example of:

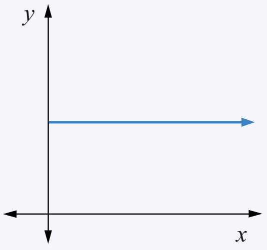
A direct proportion **B** inverse proportion
C statistical proportion **D** none of these

4 Which of the following graphs shows that y is in direct proportion to x?

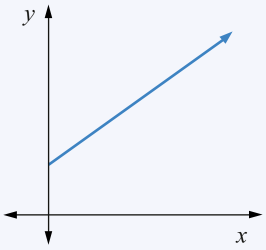
A



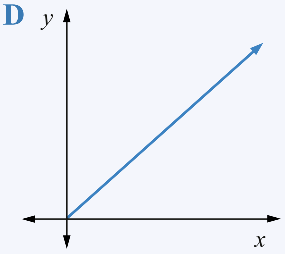
B



C



D



5 Which pair of variables is in direct linear proportion?

A

x	1	2	3
y	4	7	10

B

x	0	1	2
y	4	5	6

C

x	1	2	3
y	12	6	4

D

x	6	12	18
y	4	8	12

6 Which of the following equations shows that the variables are in direct linear proportion?

A $y = \frac{5}{x}$ **B** $P = Q + 1$ **C** $E = \frac{2m}{7}$ **D** $k = 4t - 1$

7 Using the graph for converting miles per hour (M) to kilometres per hour (K) in question **3** of Exercise 11E, 25 miles per hour is equivalent to:

A 8 km/h **B** 16 km/h **C** 20 km/h **D** 40 km/h

8 y is in direct linear proportion to x. When $x = 5$, $y = 3\frac{1}{3}$, so the value of the constant of proportionality is:

A $\frac{2}{3}$ **B** 1.5 **C** $1\frac{2}{3}$ **D** $16\frac{2}{3}$

9 The variables x and y are in direct linear proportion.
The missing values in the table are:

x	6	9	
y	16		40

A $y = 3\frac{3}{8}, x = 106\frac{2}{3}$ **B** $y = 3\frac{3}{8}, x = 15$

C $y = 24, x = 106\frac{2}{3}$ **D** $y = 24, x = 15$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3	4	5, 6	7	8, 9
Section	A	B	C	D	E	F

11A Review set

1 Convert:

a 1.8 t/ha to kg/m^2 **b** 90 km/h to m/s

2 Convert:

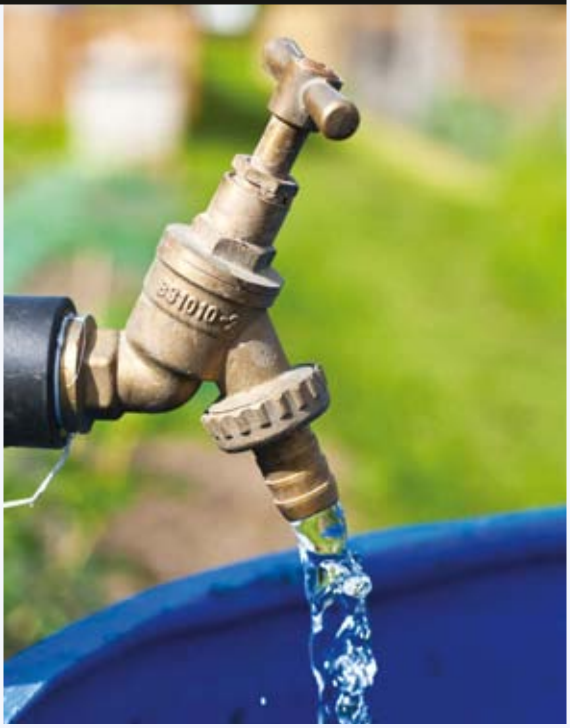
a 16 c/cm to \$/m **b** 20 m/s to km/h

3 Determine whether the following quantities are in direct proportion, inverse proportion or neither.

a rate of flow from a tap and time to fill a bucket

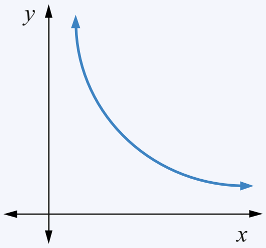
b number of employees and company profit

c side length of a square and its perimeter

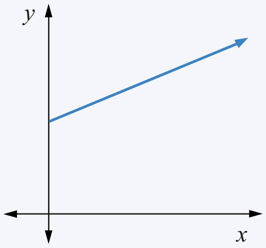


4 Do the following graphs show direct proportion, inverse proportion or neither?

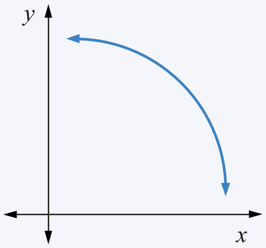
a



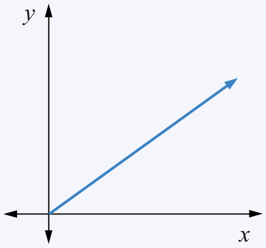
b



c



d



5 Determine whether or not the variables in the table are in direct linear proportion.

a

x	1	2	3
y	5	6	7

b

t	5	15	35
s	3	9	21

6 Determine whether or not the variables in each equation are in direct linear proportion.

a $y = 2x + 3$

b $P = \frac{4Q}{7}$

7 Use the graph for converting kilograms (K) to pounds weight (P) in Example 1 of Section 11E to convert:

a 8 kg to lb

b 40 lb to kg

8 p is in direct linear proportion to m and when $m = 2$, $p = 9$.

a Calculate the constant of proportionality.

b What is the value of p when $m = 5$?

c Determine the value of m when $p = 31.5$.

11B Review set

1 Convert:

a \$4.34/week to c/day

b 70 km/h to m/s

2 Convert:

a 15 kg/m^2 to t/ha

b 35 m/s to km/h

3 Determine whether the following quantities are in direct proportion, inverse proportion or neither.

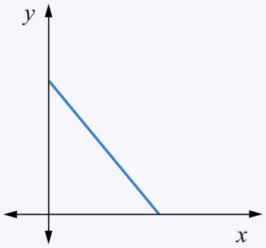
a the actual distance between two cities and the distance between the cities on a map

b the family income and the number of children in family

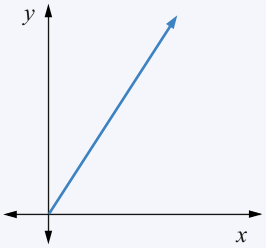
c the price of tomatoes and the weight of tomatoes you can buy for \$20

4 Do the following graphs show direct proportion, inverse proportion or neither?

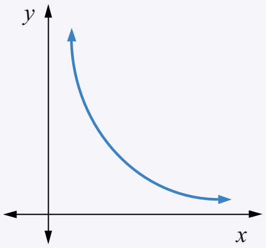
a



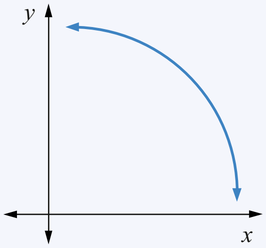
b



c



d



5 Determine whether or not the variables in the table are in direct linear proportion.

a

x	5	8	12
y	2.5	4	6

b

t	2	4	6
s	12	6	4

6 Determine whether or not the variables in each equation are in direct linear proportion.

a $z = \frac{3w}{5}$

b $p = 3w + 5$

7 Use the graph for converting miles per hour (M) to kilometres per hour (K) in question **3** of Exercise 11E to convert:

a 28 mph to km/h

b 90 km/h to mph

8 Given that the variables in the following table are in direct linear proportion, calculate the missing values.

x	8	11	
y	12		21

11C Review set

1 Convert:

a 72 L/h to mL/s

b 90 km/h to m/s

2 Convert:

a 6.4 g/mL to kg/L

b 18 m/s to km/h

3 Determine whether the following quantities are in direct proportion, inverse proportion or neither.

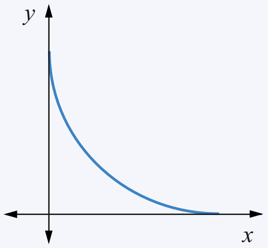
a the diameter of a circle and the circumference

b the number of equally sized hoses and the time to fill a swimming pool

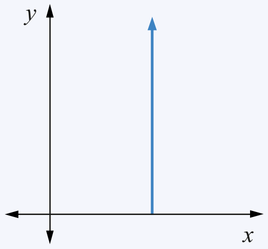
c the number of hotels in a town and the number of churches

4 Do the following graphs show direct proportion, inverse proportion or neither?

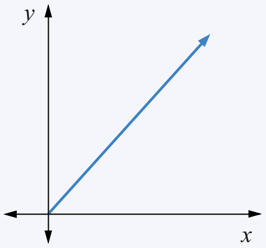
a



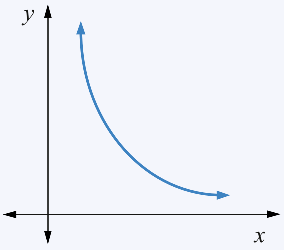
b



c



d



5 Determine whether or not the variables in the table are in direct linear proportion.

a

x	4	9	11
y	$1\frac{1}{3}$	3	$3\frac{2}{3}$

b

t	2	4	6
s	3.6	7.2	10.8

6 Determine whether or not the variables in each equation are in direct linear proportion.

a $d = \frac{3}{4}t$

b $s = 3t + 4$

7 Use the graph for converting from weight (in kilograms) on Earth (E) to weight (in kilograms) on Mars (M) in question **4** of Exercise 11E to convert:

a 65 kg on Earth to weight on Mars

b 30 kg on Mars to weight on Earth.

8 v is in direct linear proportion to r and when $r = 18$, $v = 144$.

a Calculate the constant of proportionality.

b Find the value of v when $r = 14$.

c Find the value of r when $v = 200$.

11D Review set

1 Convert:

a 5 mL/min to L/d

b 40 km/h to m/s

2 Convert:

a 5.2 c/s to $/h

b 8 m/s to km/h

3 Determine whether the following quantities are in direct proportion, inverse proportion or neither.

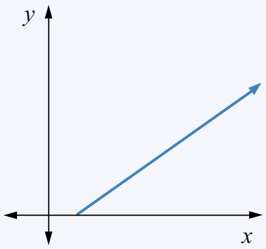
a the radius of a circle and the area of the circle

b the time taken to lay a brick wall and the number of bricklayers (assuming they all work at the same rate)

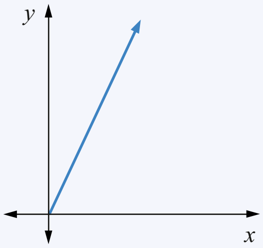
c the thickness of a book and the number of pages (ignore the cover)

4 Do the following graphs show direct proportion, inverse proportion or neither?

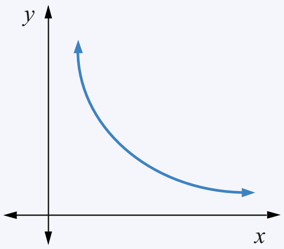
a



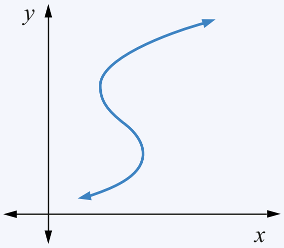
b



c



d



5 Determine whether or not the variables in the table are in direct linear proportion.

a

x	4	9	15
y	3.2	7.2	12

b

x	5	16	18
y	12.5	40	45

6 Determine whether or not the variables in each equation are in direct linear proportion.

a $y = 1.257k$

b $P = \frac{4M}{9}$

7 Use the graph for converting Australian dollars (A$) into euros (€) in question **1** of Exercise 11E to convert:

a A$700 to euros

b €260 to A$

8 Given that the variables in the following table are in direct linear proportion, calculate the missing values.

x	5	6	
y	4		7.2

12

Equations and inequalities

This chapter deals with the solution of linear equations and inequalities.
After completing this chapter you should be able to:

- solve linear equations
- solve word problems using linear equations
- substitute into formulas and solve
- explain why a particular value could be a solution to an equation
- solve linear inequalities.

NSW Syllabus references: 5.2 N&A Equations
Outcomes: MA5.2-1WM, MA5.2-2WM, MA5.2-3WM, MA5.2-8NA
NUMBER & ALGEBRA – ACMNA234, ACMNA235, ACMNA236, ACMNA240

A Linear equations review

Linear equations are equations of the form (or can be simplified to the form) $ax + b = 0$, where a and b are constants and x is the unknown (or variable).

EXAMPLE 1

Solve these linear equations.

a $7x - 9 = -5$ **b** $17 = 8 - 4x$

	Solve	Think	Apply
a	$7x - 9 = -5$ $7x - 9 + 9 = -5 + 9$ $7x = 4$ $\frac{7x}{7} = \frac{4}{7}$ $x = \frac{4}{7}$	Add 9 to both sides. Divide both sides by 7.	Add or subtract numbers to both sides until the pronumeral is on one side and a number is on the other side. Then multiply or divide to solve.
b	$17 = 8 - 4x$ $17 - 8 = 8 - 4x - 8$ $9 = -4x$ $\frac{9}{-4} = \frac{-4x}{-4}$ $\frac{-9}{4} = x$ $x = -\frac{9}{4} = -2\frac{1}{4}$	Subtract 8 from both sides. Divide both sides by −4. Write the solution with x on the left-hand side.	

Exercise 12A

1 Complete the following to solve these linear equations.

a

$$\begin{aligned} 3x - 5 &= 8 \\ 3x - 5 + \underline{\quad} &= 8 + \underline{\quad} \\ \frac{3x}{\square} &= \frac{\square}{3} \\ x &= \underline{\quad} \end{aligned}$$

b

$$\begin{aligned} 5x + 3 &= -11 \\ 5x + 3 - \underline{\quad} &= -11 - \underline{\quad} \\ \frac{5x}{5} &= \frac{\square}{\square} \\ x &= \underline{\quad} \end{aligned}$$

2 Solve for x in the following equations.

a $x + 3 = 10$
b $3x = -9$
c $3x + 6 = 0$
d $3x - 4 = -6$
e $5x + 8 = 2$
f $4x - 9 = 1$
g $8x - 6 = 10$
h $3x + 6 = 7$
i $6 + 7x = -2$
j $5 = 3x + 7$
k $6x - 7 = -1$
l $-1 = 2x + 6$
m $6 - x = -5$
n $-4x = 15$
o $3 - 2x = 7$
p $5 - 4x = -7$
q $3 - 7x = -2$
r $17 - 2x = -1$
s $11 = 3 - 2x$
t $15 - 2x = -1$
u $8 = 3 - 2x$
v $6 = -1 - 7x$
w $-15 = 3 - 6x$
x $11 = -4 - 3x$

EXAMPLE 2

Solve for m in the equation $\frac{m}{3} - 5 = -2$.

Solve	Think	Apply
$\frac{m}{3} - 5 = -2$ $\frac{m}{3} - 5 + 5 = -2 + 5$ $\frac{m}{3} = 3$ $\frac{m}{3} \times 3 = 3 \times 3$ $m = 9$	Add 5 to both sides. Multiply both sides by 3.	Add or subtract numbers first, then multiply to solve.

3 Complete to solve the following equation.

$$\begin{aligned} \frac{p}{5} - 3 &= 8 \\ \frac{p}{5} - 3 \underline{\qquad} &= 8 \underline{\qquad} \\ \frac{p}{5} &= \underline{\qquad} \\ \frac{p}{5} \times \underline{\qquad} &= 11 \times \underline{\qquad} \\ p &= \underline{\qquad} \end{aligned}$$

4 Solve these equations for x.

a $\frac{x}{2} + 3 = 8$

b $\frac{x}{3} - 1 = 4$

c $\frac{x}{5} + 2 = -3$

d $\frac{x}{6} + 3 = -4$

e $\frac{x}{7} - 2 = 4$

f $\frac{x}{10} - 6 = -1$

5 Check the given solution by substitution and say whether or not it is correct.

a $2x + 8 = 15 \quad (x = 7)$

b $7 + 5x = 9 \quad (x = 2)$

c $-15 = 6 - 7x \quad (x = 3)$

d $\frac{x}{5} - 3 = 6 \quad (x = \frac{9}{5})$

EXAMPLE 3

If $y = 5x - 3$ find x when $y = -18$.

Solve	Think	Apply
$y = 5x - 3$ $-18 = 5x - 3$ $-18 + 3 = 5x - 3 + 3$ $-15 = 5x$ $\frac{-15}{5} = \frac{5x}{5}$ $-3 = x$ $x = -3$	Substitute $y = -18$. Add 3 to both sides. Divide both sides by 5.	Often when substituting and solving an equation, the pronumeral is on the right-hand side. Solve as normal and then write the pronumeral on the left-hand side.

6 If $y = 4x + 7$, complete the following to find x when $y = 3$.

$$\underline{\quad\quad} = 4x + 7$$
$$3 - \underline{\quad\quad} = 4x + 7\ \underline{\quad\quad}$$
$$\underline{\quad\quad} = 4x$$
$$\frac{\square}{4} = \frac{4x}{\square}$$
$$x = \underline{\quad\quad}$$

7 **a** Given that $y = 3x - 5$, find x when $y = 5$.
b Given that $y = 4x + 2$, find x when $y = 11$.
c Given that $y = 7 - 5x$, find x when $y = 0$.
d Given that $y = 4 - 3x$, find x when $y = -3$.
e Given that $y = 5 - 7x$, find x when $y = -5$.
f Given that $y = 3x - 5$, find x when $y = 8$.

B Pronumerals on both sides

When you are solving equations with pronumerals on both sides, as well as adding and subtracting numbers from both sides, you may have to add and subtract pronumerals from both sides. The first step to adding or subtracting pronumerals is to move them to one side. It does not matter which side. Next add or subtract to move the numbers to the other side of the equation.

EXAMPLE 1

Solve the following equations.

a $5x + 2 = 3x - 5$
b $15 - 2x = 11 + x$

	Solve	Think	Apply
a	$5x + 2 = 3x - 5$ $5x + 2 - 3x = 3x - 5 - 3x$ $2x + 2 = -5$ $2x + 2 - 2 = -5 - 2$ $2x = -7$ $\frac{2x}{2} = \frac{-7}{2}$ $x = -\frac{7}{2}$ $= -3\frac{1}{2}$	Subtract $3x$ from both sides. Subtract 2 from both sides. Divide both sides by 2.	Eliminate the pronumeral from one side of the equation by adding or subtracting one of the pronumeral terms. Solve the resulting equation in the same way as in the previous exercise.
b	$15 - 2x = 11 + x$ $15 - 2x + 2x = 11 + x + 2x$ $15 = 11 + 3x$ $15 - 11 = 11 + 3x - 11$ $4 = 3x$ $\frac{4}{3} = \frac{3x}{3}$ $\frac{4}{3} = x$ $x = 1\frac{1}{3}$	Add $2x$ to both sides. Subtract 11 from both sides. Divide both sides by 3. Swap the pronumeral to the left-hand side.	

Exercise 12B

1 Complete the following to solve $7x - 3 = 2x + 1$.

$$\begin{aligned} 7x - \_\_\_ - 3 &= 2x - \_\_\_ + 1 \\ \_\_\_ - 3 &= 1 \\ 5x - 3 \_\_\_\_ &= 1 \_\_\_\_ \\ \frac{\square}{5} &= \frac{\square}{5} \\ x &= \_\_\_ \end{aligned}$$

2 Solve the following equations with integer solutions.

a $5x + 2 = 2x + 14$
b $3x + 7 = 11 - x$
c $5 + x = 8 - 2x$
d $3x - 4 = 5x - 2$
e $3 - x = x + 7$
f $4 - 2x = 3 - x$
g $2x - 3 = x + 6$
h $5x - 9 = 1 + 6x$
i $3x - 5 = 7 - x$

3 Solve the following equations.

a $8x + 7 = 4x - 2$
b $7x + 3 = 2x + 7$
c $5 + 2x = 11 - x$
d $x - 3 = 5x + 7$
e $3 + x = 17 + 4x$
f $15 - 3x = 2 - x$
g $2x + 5 = 9 - 2x$
h $3x - 5 = 5x + 9$
i $5 - 7x = 3x + 2$
j $5\mathrm{a} + 3 = a - 1$
k $4 - 3s = 2s + 17$
l $9x - 4 = 3 + 4x$
m $11a - 7 = 5a + 12$
n $3y - 5 = -14 - 2y$
o $7p = 15 - 3p$

EXAMPLE 2

By substituting, check the solutions to the following equations.

a $2x - 5 = 10 - 3x \quad (x = 3)$
b $5x + 2 = 2x - 7 \quad (x = 2)$

	Solve	Think	Apply
a	Does $2x - 5 = 10 - 3x$ when $x = 3$? LHS: $2 \times 3 - 5 = 1$ RHS: $10 - 3 \times 3 = 1$ LHS = RHS $\therefore x = 3$ is a solution.	Substitute 3 for x on both sides of the equation. Left-hand side = 1 Right-hand side = 1 $x = 3$ is a solution.	Substitute the value of x and evaluate both sides of the equation. Both sides must give the same value for that value of x to be a solution. The actual value of the sides is not relevant.
b	Does $5x + 2 = 2x - 7$ when $x = 2$? LHS: $5 \times 2 + 2 = 12$ RHS: $2 \times 2 - 7 = -3$ $12 \neq -3$ $\therefore x = 2$ is not a solution.	Substitute 2 for x on both sides of the equation. Left-hand side = 12 Right-hand side = −3 This is not a solution.	

4 Complete the following to check if $x = -3$ is a solution to $4x + 3 = 7 - 2x$.

Does $4x + 3 = 7 - 2x$ when $x = -3$?
LHS: $4 \times \_\_\_ + 3 = \_\_\_$
RHS: $7 - 2 \times \_\_\_ = \_\_\_$
LHS ___ RHS
$x = 3$ ___ a solution.

5 By substituting, check the solutions to the following equations.

a $3x + 9 = 4 + 2x \quad (x = 1)$

b $9a + 2 = 7a - 4 \quad (a = -3)$

c $7a - 5 = 3 - a \quad (a = 2)$

d $15 - 2x = 6 + x \quad (x = 3)$

e $2x - 3 = 7 - 4x \quad (x = \frac{5}{3})$

f $5x - 7 = 3 + x \quad (x = 3\frac{1}{2})$

EXAMPLE 3

Solve these equations.

a $5(x + 1) - 2(x - 2) = 7$

b $3(x + 1) = 5x + 3(2x - 1)$

	Solve	Think	Apply
a	$5(x + 1) - 2(x - 2) = 7$ $5x + 5 - 2x + 4 = 7$ $3x + 9 = 7$ $3x + 9 - 9 = 7 - 9$ $3x = -2$ $x = -\frac{2}{3}$	Expand the brackets. Collect the like terms. Subtract 9 from both sides. Divide both sides by 3.	The number and its sign in front of the brackets is multiplied by each term within the brackets. The most common error is to multiply the second term in the brackets incorrectly. Be vigilant with the signs.
b	$3(x + 1) = 5x + 3(2x - 1)$ $3x + 3 = 5x + 6x - 3$ $3x + 3 = 11x - 3$ $3x + 3 - 3x = 11x - 3 - 3x$ $3 = 8x - 3$ $3 + 3 = 8x - 3 + 3$ $\frac{6}{8} = \frac{8x}{8}$ $\frac{3}{4} = x$ $x = \frac{3}{4}$	Expand the brackets. Collect the like terms. Subtract $3x$ from both sides. Add 3 to both sides. Divide both sides by 8.	

6 Complete the following to solve $4(x - 3) - 3(2x + 1) = 2$.

$$\begin{aligned} 4x\,\_\_\_\_ - 6x\,\_\_\_\_ &= 2 \\ \_\_\_x - \_\_\_ &= 2 \\ -2x - 15 + \_\_\_ &= 2 + \_\_\_ \\ -2x &= \_\_\_ \\ \frac{-2x}{\square} &= \frac{\square}{\square} \\ x &= \_\_\_ \end{aligned}$$

7 Solve for x in these equations given that all answers are integers.

a $3(x + 1) - 2(x - 4) = 13$

b $2(x - 5) + 3(x + 2) = -9$

c $4(x - 5) + 5(x + 1) = 12$

d $2(x - 1) = 3(x + 5) - 22$

e $4(x - 2) = 3x + 4(x - 2)$

f $2(x - 1) = 4(2x + 1) - 9x$

g $4 - x = 2 - 3(x + 2)$

h $6 - 2(x + 5) = 2(2x - 1) - 5x$

8 Solve for x in each equation.

a $2(x + 1) - 1 = 8$

b $5(1 - 3x) = -4$

c $3(x + 2) - 7 = 11$

d $2(x + 1) + 3(x - 1) = 6$

e $4(2x - 1) + 7 = 0$

f $11 - 2(x - 1) = 7$

g $3 - 2(x + 1) = -4$

h $7 - (2 - x) = 2x$

i $5x - 4(4 - x) = x + 1$

j $3 - x = 5 - 2(x + 1)$

k $2(x - 1) = 1 - (3 - x)$

l $x + 7(4 - x) = 2x + 3(x - 1)$

EXAMPLE 4

If $y = 3 - 5(x + 4)$, find x when $y = -32$.

Solve	Think	Apply
$y = 3 - 5(x + 4)$ $-32 = 3 - 5(x + 4)$ $= 3 - 5x - 20$ $-32 = -17 - 5x$ $-32 + 17 = -17 - 5x + 17$ $-15 = -5x$ $\frac{-15}{-5} = \frac{-5x}{-5}$ $3 = x$ $x = 3$	Substitute $y = -32$. Expand. Collect like terms. Add 17 to both sides. Divide both sides by -5.	Substitute the value, simplify both sides if possible, then solve the equation. The pronumeral is often on the right-hand side of the equation.

9 If $y = -2 - 3(2x - 5)$, complete the following to find x when $y = 7$.

$\underline{\qquad} = -2 - 3(2x - 5)$

$\underline{\qquad} = -2 - 6x + \underline{\qquad}$

$\underline{\qquad} = \underline{\qquad} - 6x$

$7 \underline{\qquad} = 13 \underline{\qquad} - 6x$

$-6 = -6x$

$\frac{\square}{\square} = \frac{-6x}{-6}$

$\underline{\qquad} = x$

$x = \underline{\qquad}$

10 **a** Given that $y = 7 - 3(x + 2)$, find x when $y = -5$.

b Given that $y = 5 - 4(x - 3)$, find x when $y = 37$.

c Given that $y = 4 - 5(2x - 5)$, find x when $y = 12$.

d Given that $y = 14 - 3(2x - 8)$, find x when $y = 0$.

e Given that $y = 3x - 2(5x + 1)$, find x when $y = -16$.

f Given that $y = 4x - 3(5 - 2x)$, find x when $y = 8$.

g Given that $y = 3(2x - 1) - 4(x + 2)$, find x when $y = -3$.

h Given that $y = 4(1 - 3x) - 2(1 - x)$, find x when $y = 2$.

Investigation 1 Using a spreadsheet

You can use a spreadsheet program to solve the equation $3x - 7 = 9 - x$, given that the solution is an integer. The spreadsheet needs to have three columns labelled x, $3x - 7$ and $9 - x$.
In cell A1 enter x. In cell B1 enter $3x - 7$.
In cell C1 enter $9 - x$. In cell A2 enter 0.
In cell B2 enter = A2 * 3 − 7. In cell C2 enter = 9 − A2.
Enter the values in column A as shown in the table.
Use the fill down command to find the values for each side of the equation. The answer is the value of x that gives the same value in each column. It can be seen that $x = 4$ is the solution.
$x = 4$ gives the same value for both expressions.

	A	B	C
1	x	$3x - 7$	$9 - x$
2	0	−7	9
3	1	−4	8
4	2	−1	7
5	3	2	6
6	4	5	5
7	5	8	4
8	6	11	3

1 Change the spreadsheet to solve the following equations with integer solutions.

a $5x - 3 = 53 - 2x$
b $3x + 5 = 35 - 2x$
c $19 - 2x = 7x - 44$
d $6x + 11 = 41 - 4x$
e $3x - 17 = 33 - 7x$
f $9x + 15 = 79 - 7x$

2 Change the spreadsheet to solve the following equations with negative integer solutions.

a $4x - 3 = 13 + 6x$
b $7x - 3 = -25 + 5x$
c $8 - 7x = 2 - 8x$
d $3 - 5x = 39 - 3x$

3 Explain how to modify the spreadsheet to solve equations that do not have integer solutions.

4 Solve these equations.

a $3x - 7 = 6 - 9x$
b $5x + 23 = 2x - 8$
c $8 - 7x = 4x + 59$

C Equations with fractions

To solve equations involving fractions, use all of the equation-solving strategies; but if there are multiple terms involving fractions, multiply all terms by the lowest common denominator (LCD) to eliminate all fractions.

EXAMPLE 1

Solve these equations for x.

a $\dfrac{4x + 3}{5} = -2$
b $\frac{1}{3}(2x - 1) = -4$

	Solve	Think	Apply
a	$\dfrac{4x + 3}{5} = -2$ $\dfrac{4x + 3}{5} \times 5 = -2 \times 5$ $4x + 3 = -10$ $4x + 3 - 3 = -10 - 3$ $4x = -13$ $\dfrac{4x}{4} = -\dfrac{13}{4}$ $x = -\dfrac{13}{4} = -3\frac{1}{4}$	Multiply both sides by 5. Subtract 3 from both sides. Divide both sides by 4.	To solve this type of equation, multiply both sides of the equation by the denominator to eliminate the fraction. Solve the resulting equation as normal.

EXAMPLE 1 CONTINUED

	Solve	Think	Apply
b	$\frac{1}{3}(2x - 1) = -4$		To solve this type of equation, multiply both sides of the equation by the denominator to eliminate the fraction. Solve the resulting equation as normal.
	$\frac{1}{3}(2x - 1) \times 3 = -4 \times 3$	Multiply both sides by 3.	
	$2x - 1 = -12$	Expand the bracket.	
	$2x - 1 + 1 = -12 + 1$	Add 1 to both sides.	
	$2x = -11$		
	$\frac{2x}{2} = -\frac{11}{2}$	Divide both sides by 2.	
	$x = -\frac{11}{2} = -5\frac{1}{2}$		

Exercise 12C

1 Complete the following to solve $\frac{7x - 5}{3} = 4$ for x.

$$\begin{aligned} \frac{7x-5}{3} \times \underline{\qquad} &= 4 \times \underline{\qquad} \\ 7x - 5 &= \underline{\qquad} \\ 7x - 5 \underline{\qquad} &= 12 \underline{\qquad} \\ \frac{7x}{\square} &= \frac{\square}{\square} \\ x &= \underline{\qquad} \end{aligned}$$

2 Solve these equations for x.

a $\frac{x - 1}{2} = 6$

b $\frac{x - 5}{3} = -1$

c $\frac{x - 4}{2} = 6$

d $\frac{x + 5}{3} = -1$

e $\frac{4x + 2}{5} = 2$

f $\frac{3x - 2}{4} = -2$

g $\frac{2x + 1}{5} = 3$

h $\frac{3x - 1}{7} = 2$

i $\frac{2x + 7}{3} = 0$

j $\frac{1}{2}(3x + 1) = -1$

k $\frac{1 + 2x}{7} = 6$

l $\frac{1 - 2x}{2} = 3$

m $\frac{1}{5}(4 - 3x) = -1$

n $\frac{1}{4}(5 - 2x) = -2$

o $\frac{1}{3}(2x - 5) = -2$

EXAMPLE 2

Solve the following equations.

a $\frac{x}{2} + 3 = 6$

b $\frac{3x}{4} - 5 = 2$

c $\frac{4x - 5}{3} + 2 = 7$

	Solve	Think	Apply
a	$\frac{x}{2} + 3 = 6$		Use the usual equation-solving techniques to simplify the equation if possible before multiplying by the denominator.
	$\frac{x}{2} + 3 - 3 = 6 - 3$	Subtract 3 from both sides.	
	$\frac{x}{2} = 3$		
	$\frac{x}{2} \times 2 = 3 \times 2$	Multiply both sides by 2.	
	$x = 6$		

EXAMPLE 2 CONTINUED

	Solve	Think	Apply
b	$\frac{3x}{4} - 5 = 2$ $\frac{3x}{4} - 5 + 5 = 2 + 5$ $\frac{3x}{4} = 7$ $\frac{3x}{4} \times 4 = 7 \times 4$ $3x = 28$ $\frac{3x}{3} = \frac{28}{3}$ $x = 9\frac{1}{3}$	Add 5 to both sides. Multiply both sides by 4. Divide both sides by 3.	To solve this type of equation, multiply both sides of the equation by the denominator to eliminate the fraction. Solve the resulting equation as normal.
c	$\frac{4x - 5}{3} + 2 = 7$ $\frac{4x - 5}{3} + 2 - 2 = 7 - 2$ $\frac{4x - 5}{3} = 5$ $\frac{4x - 5}{3} \times 3 = 5 \times 3$ $4x - 5 = 15$ $4x - 5 + 5 = 15 + 5$ $4x = 20$ $\frac{4x}{4} = \frac{20}{4}$ $x = 5$	Subtract 2 from both sides. Multiply both sides by 3. Add 5 to both sides. Divide both sides by 4.	

3 Complete the following to solve $\frac{3x}{5} + 7 = -3$ for x.

$$\frac{3x}{5} + 7 \underline{\qquad} = -3 \underline{\qquad}$$
$$\frac{3x}{5} = \underline{\qquad}$$
$$\frac{3x}{5} \times 5 = \underline{\qquad} \times \underline{\qquad}$$
$$3x = \underline{\qquad}$$
$$\frac{3x}{3} = \frac{\square}{\square}$$
$$x = \underline{\qquad}$$

4 Solve these equations for x.

a $\frac{x}{3} - 1 = 4$

b $\frac{x}{5} + 3 = -3$

c $\frac{x}{7} - 4 = 1$

d $\frac{2x}{3} - 5 = 1$

e $\frac{3x}{5} + 2 = -1$

f $\frac{5x}{7} - 2 = 1$

g $\frac{2x - 1}{3} + 2 = 5$

h $\frac{3x + 2}{4} - 7 = 2$

i $\frac{4x + 3}{2} + 5 = -3$

j $6 + \frac{2x - 5}{3} = 4$

k $-7 + \frac{4 - 3x}{2} = -1$

l $3 + \frac{7 - 2x}{5} = -1$

EXAMPLE 3

Solve these equations for x.

a $\frac{2x-1}{3} = \frac{5}{2}$

b $\frac{3x-1}{5} = \frac{2x}{7}$

	Solve	Think	Apply
a	$\frac{2x-1}{3} = \frac{5}{2}$	LCD of 3 and 2 is 6.	The LCD is the most efficient number by which both sides can be multiplied to eliminate the fraction. Although it is less efficient, simply multiplying by the product of the denominators will enable the equation to be solved. Divide the LCD by the denominator of each fraction individually and multiply the result by the numerator of the fraction.
	$\frac{2x-1}{3} \times 6 = \frac{5}{2} \times 6$	Multiply both sides by 6.	
	$2(2x-1) = 15$	Expand the bracket.	
	$4x - 2 = 15$		
	$4x - 2 + 2 = 15 + 2$	Add 2 to both sides.	
	$4x = 17$		
	$\frac{4x}{4} = \frac{17}{4}$	Divide both sides by 4.	
	$x = \frac{17}{4}$		
	$x = 4\frac{1}{4}$		
b	$\frac{3x-1}{5} = \frac{2x}{7}$	LCD of 5 and 7 is 35.	
	$\frac{3x-1}{5} \times 35 = \frac{2x}{7} \times 35$	Multiply both sides by 35.	
	$7(3x-1) = 5(2x)$	Expand the bracket.	
	$21x - 7 = 10x$		
	$21x - 7 - 10x = 10x - 10x$	Subtract $10x$ from both sides.	
	$11x - 7 = 0$		
	$11x - 7 + 7 = 0 + 7$	Add 7 to both sides.	
	$11x = 7$		
	$\frac{11x}{11} = \frac{7}{11}$	Divide both sides by 11.	
	$x = \frac{7}{11}$		

5 Complete the following to solve $\frac{2x-1}{3} = \frac{3x}{5}$ for x.

LCD of 3 and 5 is ___.

Multiply both sides by ___.

$\frac{2x-1}{3} \times 15 = \frac{3x}{5} \times \underline{\qquad}$

$\underline{\qquad}(2x-1) = 3(3x)$

$10x \underline{\qquad} = 9x$

$10x - \underline{\qquad} - 5 = 9x - \underline{\qquad}$

$-5 = \underline{\qquad}x$

$x = \underline{\qquad}$

6 Solve these equations for x.

a $\frac{2x + 1}{3} = \frac{1}{2}$

b $\frac{3x - 1}{5} = \frac{1}{4}$

c $\frac{5x + 2}{3} = \frac{3}{4}$

d $\frac{4x + 5}{3} = \frac{4}{5}$

e $\frac{2x - 5}{4} = \frac{3}{5}$

f $\frac{x + 1}{2} = \frac{x}{3}$

g $\frac{2x - 1}{7} = \frac{3x}{5}$

h $\frac{4x - 1}{5} = \frac{2x}{3}$

i $\frac{4 - 3x}{2} = \frac{4x}{7}$

j $\frac{7x - 3}{4} = \frac{3x}{8}$

Extension

EXAMPLE 4

Solve these equations for x.

a $\frac{3x - 1}{2} = 4x - 1$

b $\frac{2x + 5}{4} = \frac{3x - 1}{5}$

	Solve	Think	Apply
a	$\frac{3x - 1}{2} = 4x - 1$		Use the LCD to remove the fractions and solve the resulting equation using previously learned techniques.
	$\frac{3x - 1}{2} \times 2 = (4x - 1) \times 2$	Multiply both sides by 2.	
	$3x - 1 = 2(4x - 1)$	Expand the brackets.	
	$3x - 1 = 8x - 2$		
	$3x - 1 - 8x = 8x - 2 - 8x$	Subtract $8x$ from both sides.	
	$-5x - 1 = -2$		
	$-5x - 1 + 1 = -2 + 1$	Add 1 to both sides.	
	$-5x = -1$		
	$\frac{-5x}{-5} = \frac{-1}{-5}$	Divide both sides by -5.	
	$x = \frac{1}{5}$		
b	$\frac{2x + 5}{4} = \frac{3x - 1}{5}$	LCD of 4 and 5 is 20.	
	$\frac{2x + 5}{4} \times 20 = \frac{3x - 1}{5} \times 20$	Multiply both sides by 20.	
	$5(2x + 5) = 4(3x - 1)$	Expand the brackets.	
	$10x + 25 = 12x - 4$		
	$10x + 25 - 12x = 12x - 4 - 12x$	Subtract $12x$ from both sides.	
	$-2x + 25 = -4$		
	$-2x + 25 - 25 = -4 - 25$	Subtract 25 from both sides.	
	$-2x = -29$		
	$\frac{-2x}{-2} = \frac{-29}{-2}$	Divide both sides by -2.	
	$x = 14\frac{1}{2}$		

7 Complete the following to solve $\frac{6x-5}{5} = 2x + 3$.

$$
\begin{aligned}
\frac{6x-5}{5} \times \underline{\quad} &= (2x+3) \times \underline{\quad} \\
6x - 5 &= 10x \underline{\qquad} \\
6x - 5 + 5 &= 10x + \underline{\quad} + \underline{\quad} \\
6x &= 10x + \underline{\quad} \\
6x - \underline{\quad} &= 10x - 10x + 20 \\
\underline{\quad} &= 20 \\
\frac{-4x}{\square} &= \frac{20}{-4} \\
x &= \underline{\quad}
\end{aligned}
$$

8 Solve these equations for x.

a $\frac{2x-5}{3} = 3x + 1$ **b** $\frac{5x-7}{3} = 2 - 4x$ **c** $\frac{8-3x}{5} = 2x + 3$

d $\frac{3x+2}{5} = 6x + 1$ **e** $\frac{3x+2}{5} = \frac{x-1}{4}$ **f** $\frac{1-x}{2} = \frac{x+2}{3}$

g $\frac{2x-1}{7} = \frac{3x+4}{5}$ **h** $\frac{x+1}{2} = \frac{2x-3}{3}$ **i** $\frac{7-3x}{4} = \frac{2x-1}{10}$

j $\frac{9+5x}{6} = \frac{4x-5}{12}$ **k** $\frac{5x+2}{6} = \frac{7x-4}{5}$ **l** $\frac{3x+7}{2} = \frac{4x+1}{11}$

Extension

EXAMPLE 5

Solve these equations for x.

a $\frac{2x}{3} - \frac{x}{2} = 5$ **b** $\frac{x}{5} - 3 = \frac{3x}{8}$

	Solve	Think	Apply
a	$\frac{2x}{3} - \frac{x}{2} = 5$ $6\left(\frac{2x}{3}\right) - 6\left(\frac{x}{2}\right) = 5 \times 6$ $2(2x) - 3(x) = 30$ $4x - 3x = 30$ $x = 30$	The LCD of 3 and 2 is 6. Multiply each term by 6. Divide and expand. Simplify.	Every term, whether it has a denominator or not, must be multiplied by the LCD. A whole number term has a denominator of 1, as does an algebraic term without a denominator.
b	$\frac{x}{5} - 3 = \frac{3x}{8}$ $40\left(\frac{x}{5}\right) - 3 \times 40 = 40\left(\frac{3x}{8}\right)$ $8(x) - 120 = 5(3x)$ $8x - 120 = 15x$ $8x - 120 - 8x = 15x - 8x$ $-120 = 7x$ $\frac{-120}{7} = \frac{7x}{7}$ $\frac{-120}{7} = x$ $x = -17\frac{1}{7}$	The LCD of 5 and 8 is 40. Multiply each term by 40. Divide and expand. Subtract $8x$ from both sides. Divide both sides by 7.	

9 Complete the following to solve $\frac{2x}{5} - \frac{3x}{7} = -2$ for x.

LCD of 5 and 7 is ____.

$$
\begin{aligned}
35\left(\frac{2x}{5}\right) - \underline{\qquad}\left(\frac{3x}{7}\right) &= -2 \times \underline{\qquad} \\
\underline{\qquad}(2x) - 5(\underline{\qquad}) &= -70 \\
14x - \underline{\qquad} &= -70 \\
\underline{\qquad} &= -70 \\
x &= \underline{\qquad}
\end{aligned}
$$

10 Solve these equations for x.

a $\frac{x}{2} + \frac{x}{5} = 2$ **b** $\frac{x}{2} - \frac{2x}{3} = \frac{5}{6}$ **c** $\frac{3x}{2} - \frac{x}{8} = 11$ **d** $\frac{x}{2} + \frac{x}{4} = 5$

e $\frac{x}{3} + \frac{5x}{6} = 14$ **f** $\frac{x}{3} + \frac{3x}{4} = 6\frac{1}{2}$ **g** $\frac{2x}{5} - \frac{x}{2} = -2$ **h** $\frac{x}{3} - 2 = \frac{7x}{12}$

Investigation 2 Equation solver

Use a graphics calculator to solve $3x + 8 = 6 - 2x$. Instructions for a Casio fx-9860G AU series are:

Select EQUA from the MAIN MENU. Select Type Solver by pressing F3 .

Enter the equation $3x + 8$ SHIFT = $6 - 2x$.Press EXE to store.

Press F6 to solve. This gives $x = -0.4$.

Press F1 to return, then F2 to delete the equation.

1 Solve these equations for x.

a $4x - 5 = 7 - 3x$ **b** $6 - 2x = 5x + 3$ **c** $6 - 5x = 3 + 2x$

d $4(x + 3) = 7(4 - 3x)$ **e** $\frac{4 - 5x}{3} = 8$ **f** $\frac{6 - 2x}{5} + \frac{4x + 1}{2} = 3$

2 Solve other equations.

D Practical equations (extension)

EXAMPLE 1

If twice a certain number is subtracted from 11, the result is 4 more than the number. Find the number.

Solve	Think	Apply
$11 - 2x = x + 4$ $11 - 2x + 2x = x + 4 + 2x$ $11 = 3x + 4$ $11 - 4 = 3x + 4 - 4$ $7 = 3x$ $\frac{7}{3} = \frac{3x}{3}$ $\therefore x = \frac{7}{3}$ The number is $\frac{7}{3}$ (or $2\frac{1}{3}$).	Let x be the number, so $2x$ is twice the number. $11 - 2x$ is twice the number subtracted from 11. Four more than the number is $x + 4$, so $11 - 2x = x + 4$.	Carefully define the pronumeral. Use the information to form an equation. Solve the equation. Check that the solution to the equation makes sense in the context of the question.

Exercise 12D

1 Solve the following problems to find the number.

- **a** When a number is multiplied by 3 and then increased by 7, the answer is 19.
- **b** When a number is subtracted from 11, the result is 5 more than the number.
- **c** When a number is decreased by 3 and the result doubled, the answer equals the original number.
- **d** When half a number is added to one-third of a number, the answer is 30.

EXAMPLE 2

The sum of three consecutive even numbers is 132. Find the smallest number.

Solve	Think	Apply
$x + (x + 2) + (x + 4) = 132$ $3x + 6 = 132$ $3x + 6 - 6 = 132 - 6$ $3x = 126$ $\frac{3x}{3} = \frac{126}{3}$ $x = 42$ 42 is the smallest even number.	Let x be the smallest even number. $(x + 2)$ and $(x + 4)$ are the next two even numbers.	Consecutive integers are whole numbers that follow one another.

2

- **a** If two consecutive integers have a sum of 127, find the numbers.
- **b** If three consecutive integers add to 27, find the smallest of them.
- **c** Four consecutive integers have a sum of −6. Find the largest of them.
- **d** Three consecutive odd numbers add to 33. Find the numbers.

EXAMPLE 3

If five more than a number is one more than twice the number, what is the number?

Solve	Think	Apply
$5 + n = 2n + 1$ $5 + n - n = 2n + 1 - n$ $5 = n + 1$ $5 - 1 = n + 1 - 1$ $4 = n$ The number is 4.	Let n be the number. $5 + n$ is 5 more than the number. $2n + 1$ is one more than twice the number.	Carefully define the pronumeral. Form an equation and solve it.

3 Solve the following problems to find the number.

- **a** Seven more than a number is three more than twice the number.
- **b** Four more than a number is eight more than three times the number.
- **c** Six more than twice a number is four more than four times the number.
- **d** Nine less than five times a number is three less than twice the number.

EXAMPLE 4

The sum of two numbers is 14. When one number is added to twice the other, the result is 25. Find the numbers.

<table>
<tr><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr>
<td>$n + 2(14 - n) = 25$
$n + 28 - 2n = 25$
$n + 28 - 2n - 28 = 25 - 28$
$-n = -3$
$n = 3$
As $14 - n = 14 - 3 = 11$,
the numbers are 3 and 11.</td>
<td>Let n be one number. As the numbers add to 14, the other number is $(14 - n)$.
LHS $= n + 2(14 - n)$.</td>
<td>Express both numbers in terms of the pronumeral first before using the other information to form an equation.</td>
</tr>
</table>

4 Solve the following problems.

- **a** The sum of two numbers is 10. When one number is added to twice the other, the result is 16. Find the numbers.
- **b** The sum of two numbers is 12. When one number is subtracted from three times the other, the result is 4. Find the numbers.
- **c** Two numbers differ by 2. Twice the smaller number is added to the larger number and the result is 14. Find the numbers.
- **d** Three consecutive even integers are such that the sum of the two smaller numbers is equal to six more than the largest number. Find the integers.
- **e** Three consecutive integers are such that three times the sum of the larger pair is equal to five times the sum of the smaller pair. Find the numbers.

EXAMPLE 5

At the moment Jack is 5 years older than Mim. In 7 years time Mim's age will be three-quarters of Jack's age. How old are they at present?

<table>
<tr><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr>
<td>$x + 7 = \frac{3}{4}(x + 12)$
$(x + 7) \times 4 = \frac{3}{4}(x + 12) \times 4$
$4(x + 7) = 3(x + 12)$
$4x + 28 = 3x + 36$
$4x + 28 - 3x = 3x + 36 - 3x$
$x + 28 = 36$
$x + 28 - 28 = 36 - 28$
$x = 8$
As $x + 5 = 8 + 5 = 13$,
Mim is 8 years old and
Jack is 13 years old.</td>
<td>Use a table to help develop the equation.
Let x be Mim's age and
$x + 5$ be Jack's age.
<table>
<tr><th></th><th>Age now (years)</th><th>Age in 7 years</th></tr>
<tr><td>Mim</td><td>x</td><td>$x + 7$</td></tr>
<tr><td>Jack</td><td>$x + 5$</td><td>$x + 12$</td></tr>
</table>
</td>
<td>Careful definition of the pronumeral is important. A table can assist in developing the equation. Solve the equation, making sure that the answer makes sense in the context.</td>
</tr>
</table>

5 Solve the following problems.

- **a** A man is currently three times as old as his son. In 11 years from now he will be twice as old as his son will be then. How old is his son now?
- **b** At present Guido is 8 years older than Bob. If Guido was 1 year younger, his age would be double Bob's age. How old is Bob?
- **c** In 5 years time Pam will be twice as old as Sam was 2 years ago. Pam is 8 years older than Sam. How old is Sam?
- **d** The sum of Peter's and Susan's ages is 20 years. If Peter's age was doubled, it would be 5 years more than three times Susan's age. How old is Susan?

E Substitution into formulas

A **formula** is an equation that connects two or more variables. It is normal for one of the variables to be expressed in terms of the other(s).

The **subject of the formula** is the variable that is written in terms of the other variables.

Formula substitution

If a formula contains two or more variables, and we know the value of all but one of them, we can use the formula to find the value of the unknown variable. Follow the method below.

1 Write the formula.
2 State the values of the known variables.
3 Substitute into the formula to obtain an equation with one variable.
4 Solve the equation for the unknown variable.

EXAMPLE 1

Find a given that $u = 50$, $v = 110$, $t = 6$ and $v = u + at$.

Solve	Think	Apply
$v = u + at$ $110 = 50 + a \times 6$ $110 - 50 = 50 + 6a - 50$ $60 = 6a$ $\frac{60}{6} = \frac{6a}{6}$ $a = 10$	Substitute the values into the formula. Subtract 50 from both sides. Divide both sided by 6.	Substitute the values, simplify and then solve the resulting equation.

Exercise 12E

1 Complete the following to find u, given that $s = ut + \frac{1}{2}at^2$ and $s = 200$ when $t = 5$ and $a = 10$.

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ 200 &= u(5) + \tfrac{1}{2} \times (\_\_\_) \times (\_\_\_)^2 \\ 200 &= \_\_\_ + 125 \\ 200 - \_\_\_ &= \_\_\_ + 125 - 125 \\ \frac{\square}{5} &= \frac{5u}{\square} \\ u &= \_\_\_ \end{aligned}$$

2 **a** Find a given that $v = u + at$, $u = 20$, $v = 150$ and $t = 5$.
b Find u given that $v = u + at$, $a = 20$, $v = 135$ and $t = 5$.
c Find the value of t given that $v = 102$, $u = 18$, $a = 7$ and $v = u + at$.
d Find the value of a given that $v = 54$, $u = 12$, $t = 14$ and $v = u + at$.

3 Use the formula $d = \frac{1}{2}ct$ to find the value of:
a c when $d = 100$ and $t = 8$
b t when $d = 320$ and $c = 16$

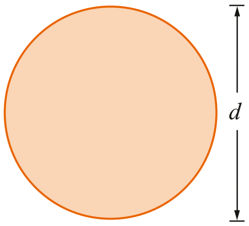
4 Use the formula $a = \dfrac{2Rn}{n + 1}$ to find the value of:
a R when $a = 12$ and $n = 24$
b R when $a = 10$ and $n = 36$

5 Use the formula $I = \dfrac{E}{R + r}$ to find the value of E when $I = 8$, $R = 15$ and $r = 3$.

6 **a** Find s given that $v = 20$, $a = 10$, $u = 10$ and $v^2 = u^2 + 2as$.
b Find a given that $u = 5$, $t = 5$, $s = 100$ and $s = ut + \frac{1}{2}at^2$.

7 The formula to convert temperature measurements from degrees Celsius, C, to degrees Fahrenheit, F, is $F = \frac{9}{5}C + 32$. Find C when:
a $F = 120°$
b $F = 60°$
c $F = 212°$

8 The formula for finding the circumference (perimeter) C of a circle of diameter d is $C = \pi d$.
a Find the circumference of a circle of diameter 11.4 cm.
b Find the diameter of a circle with circumference 250 cm.



9 The formula for calculating the circumference C of a circle of radius r is $C = 2\pi r$.
a Find the circumference of a circle of radius 8.6 cm.
b Find the radius of a circle of circumference 100 m.

10 The area of a rhombus is given by $A = \frac{1}{2}xy$.
a Given $A = 40$ and $x = 8$, find y.
b Given $A = 38$ and $y = 12$, find x.

11 The conversion from degrees Fahrenheit, F, to degrees Celsius, C, is $C = \frac{5}{9}(F - 32)$. Find:
a C when $F = 180$
b F when $C = 20$

12 When a car travels a distance of d km in time t h, the average speed, s km/h, for the journey is given by the formula $s = \frac{d}{t}$.

a Find the average speed of a car that travels 200 km in 2 h.

b What is the distance travelled by a car in $3\frac{1}{4}$ h if its average speed is 80 km/h?

c How long does it take for a car to travel 865 km at an average speed of 110 km/h?

13 The area of a rectangle is given by $A = lb$, where A is the area, l the length and b the breadth.

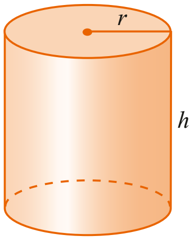
a Find the area of a rectangle with length 16 cm and breadth 5 cm.

b Find the length of a rectangle with area 30 cm^2 and breadth 5 cm.

14 A cylinder of radius r and height h has volume given by $V = \pi r^2 h$.

a Find the volume of a cylindrical tin can of radius 12 cm and height 17.5 cm.

b Find the height of a cylinder of radius 4 cm, given that its volume is 80 cm^3.



15 The formula for energy is $E = \frac{1}{2}mv^2$, where m is the mass of an object and v is its speed.

a Calculate E given $m = 100$ and $v = 5$.

b Calculate m given $E = 1400$ and $v = 18$.

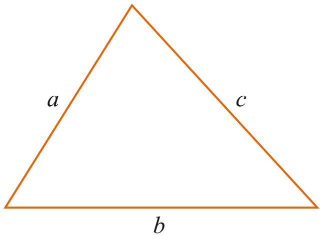
16 The surface area of a closed cylinder is given by $A = 2\pi r^2 + 2\pi rh$. Find h when $A = 1800$ and $r = 3$.

17 The formula for gradient, m, is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$. Find y_2 when $x_1 = 5$, $x_2 = 3$, $y_1 = 2$ and $m = -2$.

18 To find the area of a triangle with sides a, b and c units long, we find s, its semi-perimeter, using the formula $s = \frac{a + b + c}{2}$, and then use $A = \sqrt{s(s - a)(s - b)(s - c)}$.

A triangle has sides of length 5 cm, 6 cm and 7 cm.

Find its semi-perimeter and hence its area.



Investigation 3 Algebraic solutions

The formula for the perimeter of a rectangle is $P = 2l + 2b$.

a Find b when $P = 20$ and $l = 8$. Draw the rectangle.

b Find b when $P = 20$ and $l = 10$. Explain how this rectangle looks.

c Find b when $P = 20$ and $l = 15$. What would this rectangle look like?

This investigation shows that although an algebraic solution can be found, it might not make sense in the context of the question.

F Inequalities

The equation $2x - 1 = 5$ has one solution: $x = 3$. The inequality $2x - 1 > 5$ has an infinite number of solutions: $x = 4$, $x = 5$, $x = 6\frac{1}{2}$ and many more. In fact $x > 3$ describes all the solutions.

These symbols are convenient for representing inequalities.

Symbol	Meaning
$<$	less than
$>$	greater than
$\leqslant$	less than or equal to
$\geqslant$	greater than or equal to
$\neq$	not equal to

EXAMPLE 1

Write the following in algebraic form.

a Three times a number is always smaller than ten.

b Twice a number is larger than or equal to eight.

c Four more than three times a number is greater than fifteen.

d Eight more than half a number is less than or equal to three.

	Solve	Think	Apply
a	$3x < 10$	Let the unknown number be x.	Use the correct symbol for each inequality.
b	$2x \geqslant 8$		
c	$3x + 4 > 15$		
d	$\frac{x}{2} + 8 \leqslant 3$		

Exercise 12F

1 Write the following in algebraic form.

a Four times a number is always smaller than thirteen.

b Five times a number is always greater than fifty.

c Ten times a number is greater than or equal to eighty.

d Two more than three times a number is less than thirty-five.

e Three more than four times a number is more than seventy.

f Eight less than a number is less than or equal to thirty.

g Thirteen less than twice a number is greater than twenty.

h Nine less than twenty times a number is more than ten.

i Ten more than half a number is less than nineteen.

j Five more than one-third of a number is greater than six.

EXAMPLE 2

a Is $x = 3$ a solution to the inequality $3x - 2 > 5$?
b Is $x = -5$ a solution to the inequality $5 - 2x \leqslant 6$?
c Is $x = 7$ a solution to the inequality $3 - 5x > 2x + 1$?

	Solve	Think	Apply
a	Is $3(3) - 2 > 5$? $7 > 5$ is true. $\therefore x = 3$ is a solution.	Substitute $x = 3$ into the inequality. Evaluate the LHS. $7 > 5$ is true.	To check a solution, first substitute the value. Simplify both sides if necessary. Check that the resulting inequality is true.
b	Is $5 - 2(-5) \leqslant 6$? $15 \leqslant 6$ is false. $\therefore x = -5$ is not a solution.	Substitute $x = -5$. Evaluate the LHS. $15 \leqslant 6$ is not true.	
c	Is $3 - 5(7) > 2(7) + 1$? $-32 > 15$ is false. $\therefore x = 7$ is not a solution.	Substitute $x = 7$. Evaluate both sides. $-32 > 15$ is not true.	

2 Complete the following to find if $x = -2$ is a solution to the inequality $7 - 3x \leqslant 8$.

$7 - 3(\_\_\_) \leqslant 8$
$13 \leqslant 8$ is ___.
$\therefore x = -2$ is/is not a solution.

3 Check if the value in brackets is a solution to the inequality.

a $3x - 2 > 7$ $(x = 4)$
b $3 - 4x \geqslant 13$ $(x = -5)$
c $5x + 1 < 11$ $(x = 2)$
d $7x - 8 \geqslant 5$ $(x = 10)$
e $\frac{x}{2} + 3 > 0$ $(x = 10)$
f $\frac{3x - 1}{4} \leqslant 3$ $(x = -7)$
g $7 - 4x \leqslant 3$ $(x = -2)$
h $\frac{x - 3}{2} < 7$ $(x = 0)$
i $x + 6 > 0$ $(x = -3)$
j $3x + 7 < 11 - 2x$ $(x = 3)$
k $4x + 1 \geqslant 6$ $(x = 1)$
l $7x + 3 < 5 - x$ $(x = 3)$
m $2 > 5 - x$ $(x = -3)$
n $3(x + 1) - 2 > 5 - 5(1 - x)$ $(x = -1)$

EXAMPLE 3

Find a solution to these inequalities using the guess-and-check method.

a $5 - 7x \geqslant 10$
b $3x - 5 < 8 - 5x$

	Solve	Think	Apply
a	Try $x = 0$: $5 - 7(0) \geqslant 10$ $5 \geqslant 10$ is false. $\therefore x = 0$ is not a solution. Try $x = 5$: $5 - 7(5) \geqslant 10$ $-30 \geqslant 10$ is false. $\therefore x = 5$ is not a solution. Try $x = -10$: $5 - 7(-10) \geqslant 10$ $75 \geqslant 10$ is true. $\therefore x = -10$ is a solution.	Substitute the value $x = 0$. Substitute $x = 5$. Substitute $x = -10$.	Begin by guessing a value. Substitute and simplify to check if it is a solution. Test positive and negative values. Zero is often a first choice as it is very easy to evaluate.

EXAMPLE 3 CONTINUED

	Solve	Think	Apply
b	Try $x = 4$: $3(4) - 5 < 8 - 5(4)$ $7 < -12$ is false. $\therefore x = 4$ is not a solution. Try $x = 0$: $3(0) - 5 < 8 - 5(0)$ $-5 < 8$ is true. $\therefore x = 0$ is a solution.	Substitute $x = 4$. Substitute $x = 0$.	It may take a number of trials before a solution is found. Once one solution is found it is easy to find more.

4 Complete the following to find a solution to $3 + 2x > 8$ using the guess-and-check method.

Try $x = 0$: $3 + 2(\_\_\_) > 8$

$\_\_\_ > 8$ is ___ $\therefore x = 0$ is not a solution.

Try $x = -5$: $3 + 2(-5) > 8$

$\_\_\_ > 8$ is ___ $\therefore x = 5$ is ___ a solution.

Try $x = 5$: $3 + 2(\_\_\_) > 8$

$\_\_\_ > 8$ is ___ $\therefore x = 5$ is a solution.

5 Find three solutions to each of the following inequalities, using the guess-and-check method.

a $3x \leqslant -9$ **b** $4 - \frac{x}{5} \geqslant -3$ **c** $-2x \geqslant 5$

d $4 + 3x \geqslant 7 + 2x$ **e** $5 - 6x \leqslant -7$ **f** $3x - 4 > 5x - 2$

g $2 > 4 - x$ **h** $4 - 2x \geqslant 3 - x$ **i** $\frac{x}{3} > -2$

6 How many solutions are there to these inequalities?

a $5x - 12 > 3$ **b** $5 - 2x \leqslant 3 + 4x$

Investigation 4 Further inequalities

Change both sides of the inequality $5 < 7$ by the operation × (-3) and insert the $>$ or $<$ signs to make the new inequality true.

$5 < 7$

$5 \times -3 < 7 \times -3$ Multiply each number by -3.

$-15 \_\_ -21$

Thus $-15 > -21$ Insert the inequality sign.

1 Both sides of the inequality $6 > 4$ are changed by the operation in brackets. Insert $>$ or $<$ signs to make the new inequality true.

a $12 \_\_\_ 8$ $(\times 2)$ **b** $3 \_\_\_ 2$ $(\div 2)$

c $9 \_\_\_ 7$ $(+ 3)$ **d** $1 \_\_\_ -1$ $(- 5)$

e $-12 \_\_\_ -8$ $(\times (-2))$ **f** $-3 \_\_\_ -2$ $(\div (-2))$

g $4 \_\_\_ 2$ $(+ 2)$ **h** $8 \_\_\_ 6$ $(- (-2))$

2 Both sides of the inequality $-8 < 4$ are changed by the operation in brackets. Insert $<$ or $>$ signs to make the new inequality true.

a $-2 \_\_\_ 1$ $(\div 1)$ **b** $-24 \_\_\_ 12$ $(\times 3)$

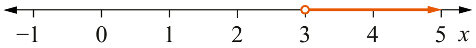
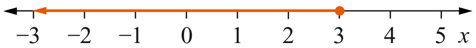
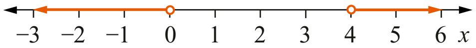
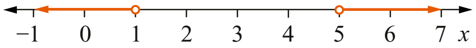
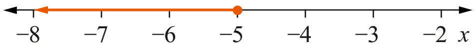
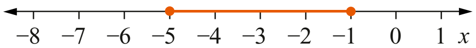
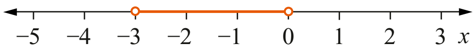
c $1 \_\_\_ 13$ $(+ 9)$ **d** $-15 \_\_\_ -3$ $(- 7)$

e $4 \_\_\_ -2$ $(\div (-2))$ **f** $24 \_\_\_ -12$ $(\times (-3))$

g $2 \_\_\_ -2$ $(\div (-4))$ **h** $-12 \_\_\_ 0$ $(+ (-4))$

3 Using your answers to questions **1** and **2**, write a set of rules for working with inequalities.

4 Inequalities may be written in equation form, as a number line graph, or in words. Complete this table.

Inequality	Graph	Description
$x > 3$		All numbers greater than 3.
$x \leqslant 3$		All numbers less than or equal to 3.
$-4 < x \leqslant 2$		All numbers between −4 and 2, including 2.
$x < 0$ or $x > 4$		All numbers greater than 4 or less than 0.
$x \geqslant -2$		
		
		All numbers between 3 and 8, including 3.
		
		
		All numbers greater than or equal to −8.
$x \leqslant -3$ or $x \geqslant 2$		
		
		All numbers less than 0.
		All numbers less than 2 or greater than 3.
$4 > x$		
$5 \leqslant x$		
		

5 a What does the closed (●) circle mean?

b What does the open (○) circle mean?

c Can $x < 0$ and $x > 4$ both be true at the same time?

d Why does $0 < x > 4$ not make sense?

G Solving inequalities

Inequalities are algebraic sentences containing at least one of the symbols $>$, $<$, $\geqslant$ or $\leqslant$. For example, $3x + 2 \geqslant -7$ is an inequality.

To solve inequalities, treat them as equations and carry out the same operation to each side of the inequality.

$3x + 2$	$\geqslant$	-7
Left-hand side		Right-hand side

In Investigation 4, these rules for inequalities were discovered.

- If we *add* or *subtract* from both sides of an inequality, we keep the same inequality sign.
- If we *multiply* or *divide* both sides of an inequality by a positive number, we keep the same inequality sign.
- If we *multiply* or *divide* both sides of an inequality by a negative number, we *reverse* the inequality sign.

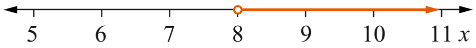
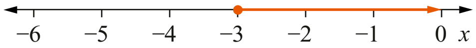
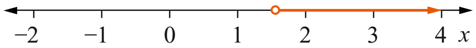
Note: The reverse of $>$ is $<$. The reverse of $\geqslant$ is $\leqslant$.

For example, if	$3x \geqslant 12$	then	$x \geqslant 4$	Divide both sides by 3.
But if	$-3x \leqslant 18$	then	$x \geqslant -6$	Divide both sides by -3 and reverse the inequality sign.

EXAMPLE 1

Solve these inequalities for x and graph the solutions on a number line.

a $3x - 16 > 8$ **b** $3 - 4x \leqslant 15$ **c** $2 > 5 - 2x$

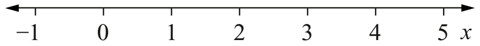
	Solve	Think	Apply
a	$3x - 16 > 8$ $3x - 16 + 16 > 8 + 16$ $3x > 24$ $\frac{3x}{3} > \frac{24}{3}$ $x > 8$ 	Add 16 to both sides. Divide both sides by 3.	Solve inequalities in exactly the same way as the equations earlier in this chapter. Be careful to remember to reverse the inequality sign if multiplying or dividing by a negative value. If the pronumeral is on the right-hand side, swap sides and reverse the inequality sign.
b	$3 - 4x \leqslant 15$ $3 - 4x - 3 \leqslant 15 - 3$ $-4x \leqslant 12$ $\frac{-4x}{-4} \geqslant \frac{12}{-4}$ $x \geqslant -3$ 	Subtract 3 from both sides. Divide both sides by -4 and reverse the inequality sign.	
c	$2 > 5 - 2x$ $2 - 5 > 5 - 2x - 5$ $-3 > -2x$ $\frac{-3}{-2} < \frac{-2x}{-2}$ $\frac{3}{2} < x$ $x > \frac{3}{2}$ 	Subtracting 5 from both sides. Divide both sides by -2 and reverse the inequality sign. Swap x to the LHS and reverse the inequality sign.	

Exercise 12G

1 Complete the following to solve these inequalities for x and graph the solutions on a number line.

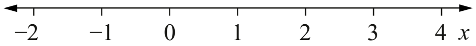
a

$$\begin{aligned} 5x - 12 &< 8 \\ 5x - 12 \,\_\_\_\_ &< 8 + 12 \\ 5x &< \_\_\_ \\ \frac{5x}{\square} &< \frac{\square}{\square} \\ x &< \_\_\_ \end{aligned}$$



b

$$\begin{aligned} 7 &> 4 - 3x \\ 7 - \_\_\_\_ &> 4 - 4 - 3x \\ \_\_\_ &> -3x \\ \frac{3}{\square} &< \frac{-3x}{-3} \\ \_\_\_ &< x \\ x &> \_\_\_ \end{aligned}$$



2 Solve these inequalities for x and graph the solutions on a number line.

a $x + 3 > 5$

b $x - 2 \leqslant 3$

c $3x \geqslant 18$

d $\frac{x}{3} < -2$

e $2x \leqslant -12$

f $x - 3 > -4$

g $\frac{x}{4} > -4$

h $x + 4 < -3$

i $x + 5 > 0$

j $3x \leqslant -9$

k $-2x \geqslant 6$

l $-5x < 10$

m $2x + 1 > 13$

n $3x - 2 \leqslant 7$

o $4x + 1 \geqslant 6$

p $3 - x < 0$

q $5 - 2x \geqslant 0$

r $2 > 5 - x$

s $3 - 4x \geqslant 13$

t $5 - 6x \leqslant -7$

u $1 - \frac{x}{7} \leqslant 2$

3 Solve these inequalities and graph the solutions on a number line.

a $2(y + 3) \geqslant 12$

b $4(2y - 1) \leqslant 3$

c $5(3a - 2) > 6$

d $7(2 - 3x) > 5$

e $4(8 - 3y) < 2$

f $9(y + 1) \leqslant 0$

4 Solve these inequalities for x and graph the solutions on a number line.

a $\frac{x}{2} > -3$

b $\frac{x}{-3} \leqslant 4$

c $\frac{x}{2} + 3 > 0$

d $\frac{2x + 1}{2} \geqslant 5$

e $\frac{3x - 1}{4} \leqslant 3$

f $\frac{1}{2}(3 - 2x) \geqslant -1$

g $\frac{x - 3}{-2} < 7$

h $\frac{4 - x}{5} \geqslant -2$

i $\frac{3 - 5x}{-4} > -1$

Extension

EXAMPLE 2

Solve these inequalities for x.

a $3 + 2x < 13 - 3x$

b $5 - 3x \geqslant x + 7$

	Solve	Think	Apply
a	$3 + 2x < 13 - 3x$ $3 + 2x + 3x < 13 - 3x + 3x$ $3 + 5x < 13$ $3 + 5x - 3 < 13 - 3$ $5x < 10$ $\frac{5x}{5} < \frac{10}{5}$ $x < 2$	Add $3x$ to both sides. Subtract 3 from both sides. Divide both sides by 5; the inequality sign remains the same.	Solve the inequality as you would an equation. If multiplying or dividing by a negative value, reverse the inequality sign.

EXAMPLE 2 CONTINUED

	Solve	Think	Apply
b	$5 - 3x \geqslant x + 7$ $5 - 3x - x \geqslant x + 7 - x$ $5 - 4x \geqslant 7$ $5 - 4x - 5 \geqslant 7 - 5$ $-4x \geqslant 2$ $\frac{-4x}{-4} \leqslant \frac{2}{-4}$ $-4x \leqslant -2$ $x \leqslant -\frac{1}{2}$	Subtract x from both sides. Subtract 5 from both sides. Divide both sides by -4 and reverse the inequality sign.	Solve the inequality as you would an equation. If multiplying or dividing by a negative value, reverse the inequality sign.

5 Solve these inequalities for x.

a $4 + 3x \geqslant 5 + x$
b $3 + 3x < 13 + x$
c $3x - 4 > 5x - 2$
d $x - 3 \leqslant 5x + 7$
e $3x + 7 < 11 - 2x$
f $4 - 2x \geqslant 3 - x$
g $3x + 2 > x - 5$
h $2x - 3 < 5 - 7x$
i $5 - 2x \geqslant x + 4$
j $7 - 3x \leqslant 5 - x$
k $3(x - 1) > x + 2$
l $5 - 2x \leqslant 2(x + 2)$
m $3x - 2 > 2(x - 1) + 5x$
n $1 - (x - 3) \geqslant 2(x + 5) - 1$
o $5 - (x + 2) \leqslant 2(2x - 1)$
p $3(x + 1) - 2 > 5 - 2(x - 1)$
q $3 - x \geqslant 5 - 2(x + 1)$
r $x + 7(4 - x) < 2 - 5(1 - x)$

6 Solve these inequalities for x.

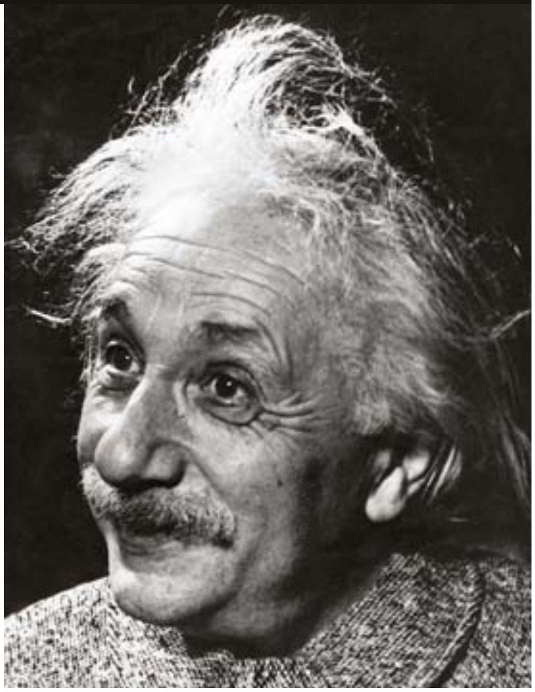
a $\frac{3 + x}{4} \geqslant \frac{2 - x}{3}$
b $\frac{6x - 5}{2} < \frac{3x + 1}{4}$
c $\frac{7x - 5}{5} \leqslant \frac{3x - 1}{4}$
d $\frac{4x + 3}{3} > \frac{1 - x}{5}$
e $\frac{5x - 3}{2} > \frac{4 - x}{3}$
f $\frac{5 - 3x}{7} \leqslant \frac{2 + x}{4}$
g $\frac{3 - 2x}{4} < \frac{7x + 1}{5}$
h $\frac{9 - 2x}{3} \geqslant \frac{2x + 1}{4}$
i $\frac{11 - 4x}{7} - 3 > 5$
j $\frac{2x - 7}{4} + 3 < 7$
k $\frac{4x - 3}{2} + \frac{7x - 5}{3} \geqslant -1$
l $\frac{8 - 3x}{2} - \frac{4x + 1}{3} \leqslant 1$

7 Solve these inequalities for x.

a $\frac{3 + x}{4} \geqslant \frac{2 - x}{5}$
b $\frac{6x - 5}{3} < \frac{3x + 1}{4}$
c $\frac{7x - 5}{5} \leqslant \frac{3x - 1}{3}$
d $\frac{4x + 3}{3} > \frac{1 - x}{2}$
e $\frac{11 - 4x}{7} - 3 > 9$
f $\frac{2x - 7}{4} - 3 < 2$

Language in mathematics

Albert Einstein (1879–1955)



Albert Einstein was born in Ulm, Germany. His family moved to Munich where Einstein started his education at a school that instilled a very rigid discipline.

Young Albert did not take well to strict regimentation and showed very little academic enthusiasm and ability. His family moved to Milan, Italy, leaving him behind in Munich to continue his schooling. Einstein left school with poor grades in a number of subjects. It was said that the reason he could not cope was because of the stifling education system, but in fact he could not bear to be away from the rest of his family.

A few years later he recommenced studying in Switzerland and finally spent four years at the Polytechnic Academy in Zurich concentrating on Physics. His first position after graduating was as a mathematics teacher, and he later worked as an examiner in the patents office.

In later years he proved to be one of the greatest thinkers of all time. He offered explanations for many observed physical phenomena that were previously unexplained, and predicted many useful results through mathematics that ultimately led to new discoveries. Einstein published three great papers that changed theoretical physics.

Einstein deduced the equation $E = mc^2$ where E represents energy, m mass and c the velocity of light, in 1905 in a paper on the special theory of relativity. This equation indicated the enormous energy content of small masses, and the validity of his deduction was demonstrated in 1945 by the enormous energy in atomic explosions.

In 1933 Einstein renounced his German citizenship and moved to Princeton in the USA. He spent the last 20 years of his life working there, serving as in inspiration to other scientists around him, and campaigning for the international control of atomic power.

1 **a** How old would Albert Einstein be today?
b Why did Einstein have difficulty at school?
c How can so much energy be released from something as small as an atom?
d Einstein believed that imagination is more important than knowledge. What do you think he might have meant by this?

2 Insert vowels to complete these glossary terms.

a s _ l _ t _ _ n
b _ n _ q _ _ l _ t _ _ s
c _ q _ _ t _ _ n
d v _ r _ _ bl _
e s _ bst _ t _ t _
f s _ lv _

3 Write a paragraph describing how a linear equation is solved.

Terms

consecutive
equation
formula
fractions
inequalities
like terms
linear
lowest common denominator (LCD)
pronumeral
solution
solve
subject
substitute
unknown
value
variable

Check your skills

1 The solution to $3x - 5 = -6$ is:

A $x = -\frac{1}{3}$ **B** $x = \frac{1}{3}$ **C** $x = -\frac{11}{3}$ **D** $x = \frac{11}{3}$

2 The solution to $\frac{x}{5} + 3 = 2$ is:

A $x = -\frac{1}{5}$ **B** $x = 1$ **C** $x = 25$ **D** $x = -5$

3 Given $y = 7 - 2x$ and $y = -8$, x is:

A $7\frac{1}{2}$ **B** $x = -\frac{1}{2}$ **C** $\frac{1}{2}$ **D** $-7\frac{1}{2}$

4 The solution to $8 - 5x = 2x + 3$ is:

A $x = \frac{7}{5}$ **B** $x = \frac{5}{7}$ **C** $x = \frac{5}{3}$ **D** $x = -\frac{5}{3}$

5 $x = -2$ is not a solution of:

A $3x + 5 = 3 + 2x$ **B** $7 - 2x = 3x + 17$

C $4x + 1 = 7 - 5x$ **D** $6x + 4 = 2x - 4$

6 The first line with an error in solving $\frac{x-3}{2} - \frac{4-2x}{3} = 2$ is:

A $3(x - 3) - 2(4 - 2x) = 12$

B $3x - 9 - 8 - 4x = 12$

C $-x - 17 = 12$

D $x = -29$

7 The solution to $\frac{6 - 5x}{3} = -2$ is:

A $x = 2\frac{2}{5}$ **B** $x = 0$ **C** $x = \frac{8}{5}$ **D** $x = \frac{5}{12}$

8 The first line with an error in the solution of $\frac{3x}{2} - \frac{x+1}{3} = -\frac{1}{5}$ is:

A $15(3x) - 10(x + 1) = -6$

B $45x - 10x - 10 = -6$

C $35x = -16$

D $x = -\frac{35}{16}$

9 When one-third of a number is added to twice a number, the answer is 10. The number is:

A $1\frac{3}{7}$ **B** $\frac{10}{7}$ **C** $3\frac{1}{3}$ **D** $4\frac{2}{7}$

10 A mother is currently twice as old as her son. In 5 years time the sum of their ages will be 70 years. An equation to find the son's age now is:

A $n + 2n = 5$ **B** $n + 2n = 70$

C $(n + 5) = 7 + (2n + 5)$ **D** $(n + 5) + (2n + 5) = 70$

11 The formula for the perimeter of a rectangle is $P = 2l + 2b$. Given that $P = 30$ and $l = 12$, the value of b is:

A 42 **B** 12 **C** 6 **D** 3

12 In symbols 'three less than five times a number is always greater than the number plus one' is:

A $5n - 3 > 1$ **B** $5(n - 3) > n + 1$

C $3 - 5n > 1$ **D** $5n - 3 > n + 1$

13 A solution to $6 - 7x > -8$ is:

A $x = 1$ **B** $x = 5$ **C** $x = 3$ **D** $x = 4$

14 The number that is not a solution of $\frac{4 - 3x}{5} > -1$ is:

A 3 **B** 2 **C** 1 **D** 0

15 A solution to $7 - 3x < 4 + 2x$ is:

A $x = -\frac{3}{5}$ **B** $x = 0$ **C** $x = \frac{3}{5}$ **D** $x = 1$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4, 5	6–8	9, 10	11	12	13–15
Section	A	B	C	D	E	F	G

12A Review set

1 Solve these equations for x.

a $2x + 5 = -3$ **b** $4x = 2x + 11$ **c** $\frac{x}{3} = \frac{1}{6}$

d $\frac{x}{7} + 9 = -3$ **e** $2 - x = 3x + 7$ **f** $\frac{x - 5}{4} = -7$

2 If $x = -3$, find y given that $y = \frac{4 - 2x}{5}$.

3 If $y = 3(2x - 1) - (x - 5)$, find x when $y = 7$.

4 Solve these equations for x.

a $3(2x + 1) = 4$ **b** $2(3 - x) = 3(x + 5)$ **c** $4 - 3(2 - x) = 7$

d $3(x - 2) + 5(x + 1) = 15$ **e** $\frac{x}{5} = \frac{3}{2}$ **f** $\frac{1}{2}(2x + 1) = -3$

g $\frac{2x - 1}{3} = 4$ **h** $\frac{x}{3} - \frac{x}{6} = 4$ **i** $x + \frac{x}{3} = \frac{3}{4}$

5 Solve the following problems.

a Five times a certain number x is equal to 12 more than the number. Find the number.

b When three consecutive integers are added the result is 48. Find the largest integer.

c One side of a rectangle is 2 cm shorter than the other side, and its perimeter is 96 cm. Find the length of the longer side.

6 **a** Find a given that $v = u + at$, $u = 15$, $v = 100$ and $t = 5$.

b The formula to convert degrees Celsius, C, to degrees Fahrenheit, F, is $F = \frac{9}{5}C + 32$. Find C when $F = 100$.

7 Solve these inequalities for x.

a $x + 5 < 3$ **b** $\frac{x}{2} \geqslant -3$ **c** $4x + 1 \geqslant 2$

d $\frac{2x - 3}{4} > 1$ **e** $\frac{4 - 2x}{5} \geqslant 3$ **f** $3 - 2x < 14 + 5x$

12B Review set

1 Solve these equations for x.

a $3x - 1 = 2$

b $6x = 11 - 3x$

c $\frac{2x}{3} = 5$

d $\frac{x}{5} - 3 = -6$

e $4 - 3x = 2x + 1$

f $\frac{x - 3}{4} = 2$

2 If $a = -2$, find t given that $t = \frac{5a - 4}{3}$.

3 If $y = 2(3x - 4) - 5(3 - x)$, find x when $y = -3$.

4 Solve these equations for x.

a $2(4x + 1) = 3$

b $3(5 - 2x) = 5(x - 3)$

c $5 - 2(5 - 2x) = 3$

d $4(x - 5) + 3(2x + 1) = 3$

e $\frac{x}{7} = \frac{5}{3}$

f $\frac{1}{3}(4 - 5x) = -2$

g $\frac{3x - 5}{2} = 8$

h $\frac{x}{5} - \frac{x}{2} = -4$

i $x - \frac{x}{4} = \frac{2}{3}$

5 Solve these problems.

a Four times a number is equal to 15 more than the number. Find the number.

b When four consecutive integers are added, the result is 54. Find the smallest integer.

6 **a** Find the value of t given that $v = 78$, $u = 12$, $a = 6$ and $v = u + at$.

b The area of a rhombus is given by $A = \frac{1}{2}xy$. Given that $A = 84$ and $x = 24$, find y.

7 Solve:

a $2x \geqslant -8$

b $x - 5 < -3$

c $-8x > 16$

d $4 - 3x < 5$

e $\frac{3x - 1}{4} \geqslant 2$

f $\frac{6 - x}{2} > 4$

12C Review set

1 Solve these equations for x.

a $4x + 5 = 12$

b $5 - 3x = -7$

c $\frac{x}{3} = -4$

d $\frac{x}{3} - 5 = 7$

e $3x - 2 = x + 6$

f $\frac{2x + 5}{4} = -1$

2 If $y = \frac{2x - 3}{5}$ find:

a y when $x = \frac{1}{2}$

b x when $y = 3$

3 Given that $y = 5 - 2(x - 3)$, find x when $y = 3$.

4 Solve these equations.

a $-3(2a + 5) = 15$

b $2(3a - 4) = a + 9$

c $3(2p - 3) = 4(p + 1)$

d $x + 5(3 - x) = 2x + 4(x - 1)$

e $\frac{x}{2} = \frac{4}{3}$

f $\frac{1}{3}(2x + 1) = -4$

g $\frac{2x + 3}{4} = \frac{3x - 2}{5}$

h $\frac{2x + 3}{3} - \frac{3x + 1}{4} = 2$

i $\frac{x + 1}{5} + x = \frac{1 - x}{2}$

5 **a** I think of a number, double it and add 7. The answer is −4. Find the number.

b An apple costs twice as much as a banana. The two of them cost 48 cents. Find the cost of the apple.

c When a number is increased by 5 then trebled, the result is six more than two-thirds of the number. Find the number.

d An athlete can run at 15 km/h and cycle at 48 km/h. In his training he runs and cycles the same distance. If his training time is 3 hours and 9 minutes, what distance does he cycle and run?

6 The formula to convert from degrees Fahrenheit, F, to degrees Celsius, C, is $C = \frac{5}{9}(F - 32)$. Find:

a C when $F = 88$ **b** F when $C = 50$

7 Solve:

a $-\frac{x}{3} > 2$ **b** $6 - x < 1$ **c** $4x + 1 \geqslant 7$

d $\frac{x + 1}{3} < 2$ **e** $\frac{4 - 3x}{5} \geqslant 1$ **f** $5 - 3x \leqslant 2(1 - x)$

12D Review set

1 Solve these equations for x.

a $3x - 2 = 5$ **b** $4 - 7x = 3$ **c** $\frac{x}{5} = -4$

d $\frac{x}{7} + 3 = -2$ **e** $4x - 7 = 3x + 5$ **f** $\frac{3x - 2}{5} = 2$

2 Given that $y = \frac{3x - 5}{3}$, find:

a y when $x = \frac{1}{3}$ **b** x when $y = \frac{7}{3}$

3 Given that $y = 7 - 5(4 - x)$, find x when $y = 0$.

4 Solve these equations.

a $-2(4x + 3) = 15$ **b** $4(3t + 1) = t - 2$ **c** $4(3p - 7) = 5(1 - p)$

d $x - 5(2x + 1) - 3 = 4$ **e** $\frac{4}{3} = \frac{x}{4}$ **f** $\frac{1}{4}(3x - 1) = -5$

g $\frac{3 - 4x}{2} = \frac{x + 5}{5}$ **h** $\frac{3x - 4}{5} - \frac{2 - 4x}{3} = 1$ **i** $\frac{x + 1}{2} + x = \frac{3 - x}{3} + 1$

5 Solve these problems.

a When 12 is added to twice a number, the answer is 8. Find the number.

b If a number is decreased by 3 then multiplied by 4, the result is 7 more than five times the number. Find the number.

6 The volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$.

a Find the volume when the radius is 8 cm and height is 15 cm.

b Find the height of a cylinder with radius 6 cm and volume 100 cm^3.

7 Solve:

a $x - 4 > 3$ **b** $-3x > 9$ **c** $4 - 3x \leqslant 14$

d $\frac{x}{3} + 2 \geqslant 0$ **e** $\frac{4 - 3x}{2} \geqslant 1$ **f** $3 - x \geqslant 2 - 4(x - 1)$

Chapters 11–12 Cumulative review

1 **a** Convert the following quantities.

- **i** 2.7 t/ha to kg/m^2
- **ii** 60 km/h to m/s
- **iii** 22 c/cm to \$/m
- **iv** 16 m/s to km/h

b Determine whether the following quantities are in direct proportion, inverse proportion or neither.

- **i** rate of flow from tap, time to fill bucket
- **ii** radius of circle, circumference
- **iii** the actual distance between two cities, the distance between the cities on a map
- **iv** family income, number of children in family
- **v** the price of apples, weight of apples you can buy for \$20

c Determine whether or not the variables in the table are in direct linear proportion.

i

x	8	9	10
y	5	6	7

ii

x	5	15	35
y	4	12	28

d Use the conversion graph in question **3** Exercise 11E to convert:

- **i** 40 mph to km/h
- **ii** 40 km/h to mph

e Determine whether or not the variables in each equation are in direct linear proportion.

- **i** $z = \frac{3w}{7}$
- **ii** $p = 4w + 5$

f The variables v and r are in direct linear proportion and when $r = 9$, $v = 144$.

- **i** Calculate the constant of proportionality.
- **ii** Find the value of v when $r = 15$.
- **iii** Find the value of r when $v = 320$.

2 **a** Solve these equations for x.

- **i** $4x - 3 = 2$
- **ii** $6 - 5x = -1$
- **iii** $\frac{x}{3} = -2$
- **iv** $\frac{x}{7} - 3 = 2$
- **v** $2x - 5 = 1 - 4x$
- **vi** $\frac{6x - 1}{3} = -1$

b If $y = \frac{2x - 3}{4}$, find:

- **i** y when $x = 2$
- **ii** x when $y = 5$

c Solve these equations.

- **i** $-3(2x - 1) = 12$
- **ii** $3(2t - 1) = 5 - t$
- **iii** $4(2r - 7) = 3(2 - r)$
- **iv** $\frac{3x - 5}{2} - \frac{x + 1}{4} = -2$

d When 20 is added to four times a number, the answer is 8. Find the number.

e If $s = \frac{n}{2}(2a + (n - 1)d)$, find s when $n = 50$, $a = 8$ and $d = 3$.

f Solve these inequalities.

- **i** $t - 5 > 4$
- **ii** $6 - 5x \geqslant 14$
- **iii** $\frac{x}{5} - 2 < 0$
- **iv** $\frac{5 - 3x}{7} \leqslant -5$
- **v** $4 - 2x \geqslant 3 - 5(x - 4)$

CHAPTER 1 REVIEW OF YEAR 8

Exercise 1A

1 A part of the whole population
2 Sample, as a census is too costly and time consuming
3 You would use a sample of people who like or go skateboarding.
4 **a** 16.5 **b** 19 **c** 17 **d** 10
5 **a** 41.48 **b** 44 **c** 41 **d** 36
6 **a** Class A: mean = 40.67, mode = 51, median = 41, range = 46
Class B: mode = 28 and 46, range = 39, mean = 43.13, median = 42
b Class B as mean and median are both higher
7 **a** $\frac{13}{50}$ **b** 208

Exercise 1B

1 1 : 3 **2** B
3 $x = \frac{20}{7} = 2\frac{6}{7}$
4 **a** 5 : 12 **b** 18 : 25
5 55
6 **a** 31 cm, 24.8 cm **b** 68.2 cm
7 0.175 km/min
8 A **9** 20.4 m
10 55 cm

Exercise 1C

1 **a** Reflection **b** Rotation
2 **a** SAS **b** AAS **c** SSS **d** RHS
3 **a** Three pairs of equal angles do not make triangles congruent.
b Equal angles are opposite different sides.
c Equal sides are not corresponding sides.
4 **a** 1 and 2, AAS **b** 1 and 3, SAS
5 **a** SSS, $x = 50°$, $y = 95°$, $z = 35°$
b SAS, $x = 15$, $y = 45°$, $z = 35°$
c AAS, $x = 9$, $y = 10$, $z = 50°$

Exercise 1D

1 **a** 7^5 **b** 9^7 **c** 2^8 **d** 10^3
2 **a** Base 3, index 8 **b** Base 5, index 2
c Base 8, index 4 **d** Base 6, index 0
3 **a** $6 \times 6 \times 6 \times 6$ **b** $7 \times 7 \times 7$
c 6×6 **d** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
4 **a** 512 **b** 729 **c** 625 **d** 343
5 **a** 3^{13} **b** 7^{12} **c** 4^5 **d** 6^{11} **e** 2^{30}
6 **a** False **b** False **c** False **d** False
7 **a** 36 **b** 6 **c** 1 **d** 1
8 **a** a^3 **b** $6r^4$ **c** x^2y^4
9 **a** $t \times t$ **b** $5 \times a \times a \times a \times a$
c $p \times p \times p \times p \times p \times p$ **d** $15 \times e \times e \times e \times e \times e$
10 **a** 9 **b** 36 **c** 144 **d** 4

Exercise 1E

2 **a** 270 mm **b** 4000 mm
3 **a** 34 cm **b** 7000 cm
4 76.4 cm
5 **a** 174.7 cm **b** 102.94 cm
6 126.52 cm
7 $w = 7.2$ cm, $x = 9.12$ cm, $y = 12.6$ cm, $z = 12.6$ cm; perimeter = 90.2 m
8 29 units^2
9 **a** 180 mm^2 **b** 49 cm^2
c 10 m^2 **d** 44 cm^2
10 **a** 104 cm^2 **b** 5400 m^2 **c** 243 m^2
11 **a** 22 m^2 **b** 2100 cm^2
12 **a** 50 mm **b** 8 m **c** 64 cm
d 1160 cm **e** 438 mm **f** 84 m
g 800 mm^2 **h** 72 000 cm^2 **i** 90 cm^2

Exercise 1F

1 48 h
2 **a** 4 min **b** 5 h
3 3 h 30 min
4 **a** 9 h 23 min **b** 1 h 39 min
5 7:17 am **6** 3 h 48 min
7 2 h 20 min **8** 3 h 17 min

Exercise 1G

1 Shade any 14 squares. **2** $\frac{11}{20}$
3 **a** $13\frac{3}{11}$ **b** $\frac{29}{8}$
4 **a** 100 **b** $\frac{35}{48}$
5 $\frac{4}{5}, \frac{2}{3}, \frac{8}{15}$
6 **a** $\frac{3}{8}$ **b** 222 kg
7 **a** \$120 **b** \$400 **c** $\frac{2}{15}$ **d** \$80
8 $\frac{2}{100}$ **9** 8.307
10 **a** $4\frac{1}{5}$ **b** 3.375
11 0.17
12 **a** 3.85 **b** 4
13 **a** 4.64 **b** 332.4 **c** 11.9
14 **a** 30.6 **b** 5.2
15 **a** \$48.30 **b** \$152 040
16 Shade any 3 squares. **17** 48%
18 **a** $\frac{37}{100}$ **b** 0.57
19 **a** 380% **b** 62.5%
20 0.65 (65%), 70%, $\frac{4}{5}$ (80%)
21 **a** 27% **b** $\frac{3}{20}$ **c** 4.25
22 **a** \$54 **b** 12 m
23 25%
24 **a** 130 **b** 240

Exercise 1H

1 **a** Centre, diameter **b** Chord, segment
c 2 radii, sector **d** Circumference
2 $\frac{3}{10}$ **3** $C = \pi d$
4 35.8 cm **5** $C = 2\pi r$
6 42.73 cm **7** $A = \pi r^2$
8 **a** 153.9 cm^2 **b** 11.9 cm^2
9 96.8 cm^2
10 **a** 1.312 m^3 **b** 42 654 cm^3 **c** 1078 cm^3

Exercise 1I

1 **a** 0 **b** 89
2 **a** A **b** B and C **c** C
3 **a** Not possible **b** Male **c** Not possible
4 **a** **i** 470 kg **ii** 420 kg **iii** 440 kg
b Median
5 11

Exercise 1J

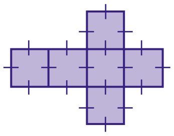
1 **a** **i** b **ii** $b^2 = a^2 + c^2$
b **i** PR **ii** $PR^2 = QR^2 + PQ^2$
2 **a** No **b** Yes
3 **a** 81 **b** 8.4
4 **a** 7.5 cm **b** 17.1 cm
5 **a** 6.0 cm **b** 38.7 cm
6 **a** 14.4 cm **b** 27.2 cm
7 50.91 cm

Exercise 1K

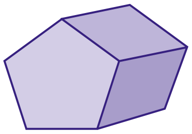
1 **a** $14x$ **b** $6y$ **c** $7a^2$ **d** $6ac$
2 **a** $60n$ **b** $-10a$ **c** $24m$ **d** $35p$
3 **a** $5a$ **b** $-4m$ **c** bc **d** $4m$
4 **a** $-wx - 3y$ **b** 0
5 **a** $\frac{5a}{7}$ **b** $\frac{2a}{15}$ **c** $\frac{q^2}{5}$ **d** 3
6 **a** $a^2 - an$ **b** $2mn^2 - 5mn$
c $12p^2 + 8p$ **d** $-8py + 4pw$
7 **a** $31a - 23$ **b** $23xy - 12x - 8y$
8 **a** $mn(n + 1)$ **b** $q(p - a)$
c $4(p - 3d)$ **d** $5(5f - 3)$
9 **a** $-3(k - 3)$ **b** $-4(p + 3d)$
10 **a** A pronumeral is a letter that stands for a number.
b A coefficient is a number that multiplies a pronumeral.
11 **a** 24 **b** $-\frac{28}{8} = -\frac{7}{2}$
c -59 **d** -51
12 **a** $6d + 23$ **b** $\frac{3(x - 7)}{8}$

Exercise 1L

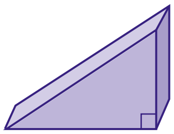
1 **a** $DCGH$ **b** $ABCD, EFGH$ **c** $AEHD, BFGC$
2 **a**



b 86.64 cm²
3 **a** 338.72 cm² **b** 492 cm²
4 **a**

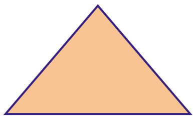


b



5 **a**

b



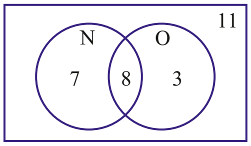
6 **a** 3163.5 cm³ **b** 3325 mm³
7 27 946 cm³
8 **a** 3546.9 cm³ **b** 793.3 cm³
9 **a** 1000 mm³ **b** 1 000 000 mL
c 1 000 000 cm³ **d** 1000 L
e 1 kL **f** 5 300 000 cm³

Exercise 1M

1 $-12, \div 4$
2 **a** $x = 6$ **b** $x = -15$ **c** $x = 9$ **d** $x = -7$
e $y = \frac{11}{3}$ **f** $p = 13$ **g** $d = -20$ **h** $c = \frac{14}{10} = \frac{7}{5}$
i $m = -20$ **j** $q = \frac{10}{17}$
3 **a** $p = \frac{15}{2}$ **b** $x = 24$
4 **a** No **b** No
5 $x + 23 = 114, x = 91$
6 **a** $x \geqslant -12$ **b** $m < 28$

Exercise 1N

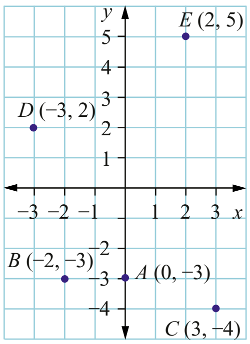
1 **a** {red, blue, green, yellow} **b** $\frac{1}{4}$
2 **a** {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} **b** $\frac{1}{10}$
3 **a** $\frac{7}{10}$, 70% **b** $\frac{1}{4}$, 0.25 **c** 0.625, 62.5%
4 **a** 20 **b** 9 **c** $\frac{9}{20}$
5 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{13}$
6 **a** Example: tossing a coin and getting a 1
b About 50%
c Example: high probability
7 **a** Example: getting a number from 1 to 6
b Example: getting a 7
c Example: getting an even number
8 **a** $\frac{1}{5}$ **b** $\frac{4}{5}$ **c** $\frac{2}{5}$ **d** $\frac{3}{5}$
9 **a** 29 girls



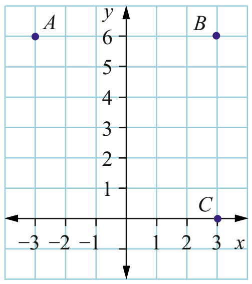
b **i** 7 **ii** 3 **iii** 18
iv 10 **v** 11

Exercise 1O

1



2 **a**



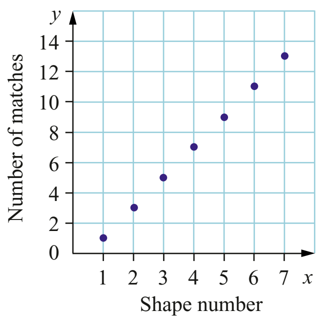
b $(-3, 0)$

3 **a**

Shape number	1	2	3	4	5
Number of matches	1	3	5	7	9

b Number of matches equals twice the shape number minus one.
c $y = 2x - 1$

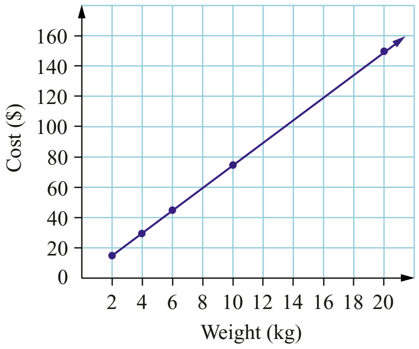
d, e



e (6, 11), (7, 13)

4 **a** (2, 15), (4, 30), (6, 45), (10, 75), (20, 150)

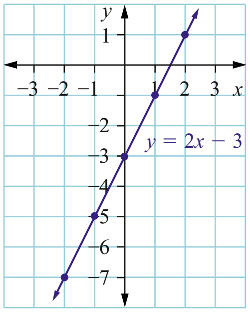
b Bulk minute steak



c \$120 **d** 12 kg

5

x	−2	−1	0	1	2
y	−7	−5	−3	−1	1



6 **a**

x	−2	−1	0	1
y	0	3	6	9

b $y = 3x + 6$

CHAPTER 2 INDICES

Exercise 2A

1 **a** $m^3 \times m^4 = (m \times m \times m) \times (m \times m \times m \times m)$
$= m \times m \times m \times m \times m \times m \times m$
$= m^7$

b $m^5 \div m^3 = \dfrac{m \times m \times m \times m \times m}{m \times m \times m} = \dfrac{m^2}{1}$
$= m^2$

c $(m^2)^3 = (m \times m) \times (m \times m) \times (m \times m)$
$= m \times m \times m \times m \times m \times m = m^6$

2 **a** $m^2 \times m^4 = m \times m \times m \times m \times m \times m$
$= m^6$

b $m^6 \div m^2 = \dfrac{m \times m \times m \times m \times m \times m}{m \times m} = \dfrac{m^4}{1}$
$= m^4$

c $(m^2)^4 = (m \times m) \times (m \times m) \times (m \times m) \times (m \times m)$
$= m^8$

3 **a** **i** $a^5 \times a^4 = 2^5 \times 2^4 = 32 \times 16 = 512$
ii $a^9 = 2^9 = 512$
b Yes

4 **a** **i** 15 625 **ii** 15 625 **b** Yes

5 **a** **i** 6561 **ii** 6561 **b** Yes

6 **a** $n^3 \times n^5 = n^{3+5} = n^8$ **b** $m^7 \div m^3 = m^{7-3} = m^4$
c $(k^2)^5 = k^{2 \times 5} = k^{10}$

7 **a** m^9 **b** q^{15} **c** t^{19} **d** b^{20} **e** v^{13}

8 **a** a^2 **b** x^{10} **c** w^6 **d** b **e** z

9 **a** b^8 **b** h^{15} **c** k^{16} **d** z^{60} **e** n^8

10 **a** m^6 **b** x^3 **c** b^{24} **d** m^{13} **e** v^{70}
f n **g** b^7 **h** y^{25} **i** t^{31} **j** a^6

11 **a, b** The bases are not the same.

12 **a** True **b** False, m^7 **c** False, p^9 **d** True
e False, a^4b^5 **f** True **g** False, p^9 **h** True
i False, w^{12} **j** False, $\dfrac{p^6}{q^2}$ **k** True **l** False, n^{30}

Exercise 2B

1 **a** $\dfrac{m^7 \times m^5}{m^8} = \dfrac{m^{12}}{m^8} = m^4$

b $\dfrac{(a^4)^6}{a^5 \times a^4} = \dfrac{a^{24}}{a^9} = a^{15}$

2 **a** x^6 **b** w^5 **c** m^2 **d** k^3
e a^3 **f** y^2 **g** z **h** x^7
i k^9 **j** m^{11} **k** a^{32} **l** t^{20}
m y^5 **n** a^5 **o** b^{40}

3 $5t^7 \times 6t^4 = 5 \times 6 \times t^7 \times t^4$
$= 30 \times t^{11} = 30t^{11}$

4 **a** $12m^{12}$ **b** $10p^{10}$ **c** $18t^{12}$
d $70a^{16}$ **e** $24w^{19}$ **f** $30b^9$
g $24z^{17}$ **h** $80q^{18}$ **i** $18d^{18}$

5 $\dfrac{10a^7}{6a^4} = \dfrac{10 \times a^7}{6 \times a^4} = \dfrac{10}{6} \times \dfrac{a^7}{a^4}$
$= \dfrac{5}{3} \times a^3 = \dfrac{5a^3}{3}$

6 **a** $2m^5$ **b** $2a^5$ **c** $3w^2$ **d** $\dfrac{4z^4}{3}$ **e** $\dfrac{4k^6}{3}$
f $\dfrac{3e^4}{2}$ **g** $\dfrac{m^5}{3}$ **h** $\dfrac{a^5}{2}$ **i** $\dfrac{3t^7}{4}$ **j** $\dfrac{3b^5}{4}$

7 **a** $\dfrac{a^2}{a^5} = \dfrac{a \times a}{a \times a \times a \times a \times a} = \dfrac{1}{a^3}$

b $\dfrac{12y^4}{8y^5} = \dfrac{12}{8} \times \dfrac{y \times y \times y \times y}{y \times y \times y \times y \times y}$
$= \dfrac{3}{2} \times \dfrac{1}{y} = \dfrac{3}{2y}$

8 **a** $\dfrac{1}{m^3}$ **b** $\dfrac{3}{k^4}$ **c** $\dfrac{2}{p^2}$ **d** $\dfrac{1}{3y^4}$ **e** $\dfrac{4}{3z^5}$ **f** $\dfrac{2}{3x^7}$

9 $(4b^2)^3 = 4b^2 \times 4b^2 \times 4b^2$
$= 4 \times 4 \times 4 \times b^2 \times b^2 \times b^2$
$= 4^3 \times (b^2)^3 = 64 \times b^6 = 64b^6$

10 **a** $27a^{12}$ **b** $64m^{18}$ **c** $49p^{10}$ **d** $10\,000k^8$
e $125t^{33}$ **f** $x^{15}y^{10}$ **g** $m^{12}n^{18}$ **h** $p^{28}q^{12}$
i a^8b^{20} **j** $8x^{15}y^6$ **k** $27x^{21}y^{12}$ **l** $125p^6q^{12}$

11 **a** $4p^2q^3 \times 5p^3q = 4 \times p^2 \times q^3 \times 5 \times p^3 \times q$
$= 4 \times 5 \times p^2 \times p^3 \times q^3 \times q$
$= 20 \times p^5 \times q^4 = 20p^5q^4$

b $\dfrac{15x^7y^5}{18x^4y^3} = \dfrac{15}{18} \times \dfrac{x^7}{x^4} \times \dfrac{y^5}{y^3}$
$= \dfrac{5}{6} \times x^3 \times y^2 = \dfrac{5x^3y^2}{6}$

12 **a** $8a^8b^5$ **b** $10m^{10}n^8$ **c** $12p^{11}q^{15}$ **d** $30x^{10}y^4$

e $12w^{14}z^{17}$ **f** $35a^3b^6c^6$ **g** $\frac{3a^2b^4}{2}$ **h** $3x^4y^7$

i $\frac{k^4m^6}{5}$ **j** $\frac{3a^3b^5}{4}$ **k** $\frac{4m}{5}$ **l** $\frac{5a}{7c^2}$

Exercise 2C

1 **a** $a^4 \div a^4 = a^{4-4} = a^0$ **b** $a^4 \div a^4 = 1, \therefore a^0 = 1$

2 **a** $k^7 \div k^7 = k^{7-7} = k^0$ **b** $k^7 \div k^7 = 1, \therefore k^0 = 1$

3 **a** 1 **b** 1 **c** 3 **d** 4 **e** 9

f 1 **g** 1 **h** 10 **i** 8 **j** 1

k 4 **l** 6 **m** 13 **n** 5 **o** 2

Exercise 2D

1

2^3	2^2	2^1	2^0
8	$8 \times \frac{1}{2} = 4$	$4 \times \frac{1}{2} = 2$	$2 \times \frac{1}{2} = 1$

2^{-1}	2^{-2}	2^{-3}
$1 \times \frac{1}{2} = \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Hence $2^{-1} = \frac{1}{2} = \frac{1}{2^1}$

$2^{-2} = \frac{1}{4} = \frac{1}{2^2}$

$2^{-3} = \frac{1}{8} = \frac{1}{2^3}$

2

10^3	10^2	10^1	10^0
1000	$1000 \times \frac{1}{10} = 100$	$100 \times \frac{1}{10} = 10$	$10 \times \frac{1}{10} = 1$

10^{-1}	10^{-2}	10^{-3}
$1 \times \frac{1}{10} = \frac{1}{10}$	$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$	$\frac{1}{100} \times \frac{1}{10} = \frac{1}{1000}$

Hence $10^{-1} = \frac{1}{10} = \frac{1}{10^1}$

$10^{-2} = \frac{1}{100} = \frac{1}{10^2}$

$10^{-3} = \frac{1}{1000} = \frac{1}{10^3}$

3 $2^2 \div 2^5 = 2^{2-5} = 2^{-3}$

$2^2 \div 2^5 = \frac{2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2^3}$

Hence, $2^{-3} = \frac{1}{2^3}$.

4 **a** $5^3 \div 5^7 = 5^{3-7} = 5^{-4}$

b $5^3 \div 5^7 = \frac{5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5^4}$

c From parts **a** and **b**, $5^{-4} = \frac{1}{5^4}$

5 **a** $\frac{1}{3}$ **b** $\frac{1}{4^3}$ **c** $\frac{1}{2^5}$ **d** $\frac{1}{8^2}$ **e** $\frac{1}{5^4}$

f $\frac{1}{12}$ **g** $\frac{1}{9^2}$ **h** $\frac{1}{6}$ **i** $\frac{1}{7^3}$ **j** $\frac{1}{3^6}$

k $\frac{1}{2^8}$ **l** $\frac{1}{5}$ **m** $\frac{1}{10^5}$ **n** $\frac{1}{5^{10}}$ **o** $\frac{1}{4^{15}}$

6 **a** $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ **b** $5^{-1} = \frac{1}{5^1} = \frac{1}{5}$

c $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

7 **a** $\frac{1}{9}$ **b** $\frac{1}{32}$ **c** $\frac{1}{64}$ **d** $\frac{1}{625}$ **e** $\frac{1}{1024}$

f $\frac{1}{216}$ **g** $\frac{1}{81}$ **h** $\frac{1}{81}$ **i** $\frac{1}{3125}$ **j** $\frac{1}{512}$

k $\frac{1}{343}$ **l** $\frac{1}{256}$ **m** $\frac{1}{729}$ **n** $\frac{1}{10\,000}$ **o** $\frac{1}{81}$

8 **a** 2^{-1} **b** 2^{-2} **c** 2^{-8} **d** 2^{-5} **e** 2^{-3}

f 5^{-1} **g** 7^{-2} **h** 4^{-3} **i** 3^{-4} **j** 5^{-6}

k 3^{-10} **l** 6^{-1} **m** 7^{-5} **n** 4^{-9} **o** 10^{-1}

9 $\frac{1}{7^{-2}} = \frac{1}{\frac{1}{7^2}} = 1 \times \frac{7^2}{1} = 7^2$ or $\frac{1}{7^{-2}} = \frac{7^0}{7^{-2}} = 7^{0-(-2)} = 7^2$

10 **a** 3^4 **b** 2^7 **c** 7^2 **d** $6^1 = 6$ **e** 4^5

11 $\left(\frac{5}{8}\right)^{-1} = \frac{1}{\frac{5}{8}} = 1 \times \frac{8}{5} = \frac{8}{5}$or $1\,\frac{3}{5}$

12 **a** $1\frac{1}{2}$ **b** $1\frac{1}{3}$ **c** $1\frac{1}{7}$ **d** 5

e 10 **f** $\frac{2}{3}$ **g** $\frac{4}{11}$

13 $\frac{b}{a}$

Exercise 2E

1 $\frac{a^3}{a^4} = a^{3-4} = a^{-1}$

$\frac{a^3}{a^4} = \frac{a \times a \times a}{a \times a \times a \times a} = \frac{1}{a}$

Hence, $a^{-1} = \frac{1}{a}$.

2 **a** $\frac{a^3}{a^5} = a^{3-5} = a^{-2}$

b $\frac{a^3}{a^5} = \frac{a \times a \times a}{a \times a \times a \times a \times a} = \frac{1}{a^2}$

c From parts **a** and **b**, $a^{-2} = \frac{1}{a^2}$.

3 **a** $\frac{a^2}{a^5} = a^{2-5} = a^{-3}$

b $\frac{a^2}{a^5} = \frac{a \times a}{a \times a \times a \times a \times a} = \frac{1}{a^3}$

c From parts **a** and **b**, $a^{-3} = \frac{1}{a^3}$.

4 **a** $\frac{a^2}{a^6} = a^{2-6} = a^{-4}$

b $\frac{a^2}{a^6} = \frac{a \times a}{a \times a \times a \times a \times a \times a} = \frac{1}{a^4}$

c From parts **a** and **b**, $a^{-4} = \frac{1}{a^4}$.

5 **a** $\frac{a}{a^6} = a^{1-6} = a^{-5}$

b $\frac{a}{a^6} = \frac{a}{a \times a \times a \times a \times a \times a} = \frac{1}{a^5}$

c From parts **a** and **b**, $a^{-5} = \frac{1}{a^5}$.

6 $a^{-n} = \frac{1}{a^n}$

7 **a** $\frac{1}{y^2}$ **b** $\frac{1}{k}$ **c** $\frac{1}{m^3}$ **d** $\frac{1}{x^6}$ **e** $\frac{1}{t^{10}}$

8 **a** a^{-8} **b** k^{-2} **c** x^{-11} **d** n^{-14} **e** z^{-20}

9 **a** $\frac{3}{k}$ **b** $\frac{1}{3k}$ **c** $\frac{2}{y^5}$ **d** $\frac{1}{32y^5}$ **e** $\frac{3}{t^4}$ **f** $\frac{1}{81t^4}$

10 **a** $\frac{1}{k^3} = k^{-3}$

b $\frac{2}{k^3} = 2 \times \frac{1}{k^3} = 2 \times k^{-3} = 2k^{-3}$

c $\frac{1}{2k^3} = \frac{1}{2} \times \frac{1}{k^3} = \frac{1}{2} \times k^{-3} = \frac{1}{2}k^{-3}$ or $\frac{k^{-3}}{2}$

11 **a** **i** p^{-5} **ii** $7p^{-5}$ **iii** $\frac{1}{7}p^{-5}$ or $\frac{p^{-5}}{7}$

b **i** m^{-10} **ii** $6m^{-10}$ **iii** $\frac{1}{6}m^{-10}$ or $\frac{m^{-10}}{6}$

c **i** y^{-7} **ii** $4y^{-7}$ **iii** $\frac{1}{4}y^{-7}$ or $\frac{y^{-7}}{4}$

Exercise 2F

1 **a** a^{-7} **b** y^4 **c** e^{-2} **d** n **e** b^{-8}

f w^5 **g** z^2 **h** k^{-4} **i** y^{-8} **j** t^{-20}

2 **a** $3a^5 \times 4a^{-2} = 3 \times 4 \times a^5 \times a^{-2}$
$= 12 \times a^{5+(-2)}$
$= 12 \times a^3 = 12a^3$
b $3x^2 \div 4x^{-3} = \frac{3x^2}{4x^{-3}} = \frac{3}{4} \times \frac{x^2}{x^{-3}}$
$= \frac{3}{4} \times x^{2-(-3)}$
$= \frac{3}{4} \times x^5 = \frac{3}{4}x^5$ or $\frac{3x^5}{4}$
c $(7m^{-3})^2 = 7m^{-3} \times 7m^{-3}$
$= 7 \times 7 \times m^{-3} \times m^{-3}$
$= 49 \times (m^{-3})^2$
$= 49 \times m^{-6} = 49m^{-6}$

3 **a** $90a^2$ **b** $18b^{-7}$ **c** $6v^{-4}$
d $4y^6$ **e** $3p^{-6}$ **f** $\frac{3}{8}k^{-2}$ or $\frac{3k^{-2}}{8}$
g $125z^{-12}$ **h** $32m^{-15}$ **i** $9w^{-12}$
j $2n^{-2}$ **k** $\frac{1}{9}x^{-2}$ or $\frac{x^{-2}}{9}$ **l** $\frac{1}{125}m^{-6}$ or $\frac{m^{-6}}{125}$

4 **a** False **b** False **c** True **d** False
e True **f** False **g** False **h** True
i True **j** False **k** False **l** True

5 **a** $a^2 = 9, 2a = 6$ **b** $a^3 = 27, 3a = 9$
c $a^{-2} = \frac{1}{9}, \frac{-2}{a} = -\frac{2}{3}$ **d** $a^{-3} = \frac{1}{27}, \frac{-3}{a} = -1$
e $a^2 \times a = 27, a^2 + a = 12$
f $a^2 + a^2 = 18, a^4 = 81$ **g** $a^2 - a^2 = 0, a^0 = 1$
h $5a^2 \times 3a = 405, 5a^2 + 3a = 54$

Exercise 2G

1 **a** $4(2x + 5) = 4 \times 2x + 4 \times 5 = 8x + 20$
b $3(4a - 2b) = 3 \times 4a + 3 \times (-2b)$
$= 12a + (-6b) = 12a - 6b$

2 **a** $6w + 15$ **b** $18z - 12$
c $20a + 15b$ **d** $8x - 6y$
e $10z^2 + 60$ **f** $7ab - 14a^2$
g $4m^2 + 4n^2$ **h** $2m^3 - 6mn$
i $20b + 10a + 15$ **j** $15x - 9y - 6z$
k $26 - 13x$ **l** $15a + 5b - 35c$

3 **a** $6ab + 12ac$ **b** $12x^2 - 8xy$ **c** $60k^2 - 40km$
d $m^3 + 2m$ **e** $12xy - 30x^3$ **f** $6k^4 + 15k^2$
g $5a^5 - 2a^3$ **h** $2p^7 + 6p^8$

4 **a** $-2(3x + 7) = -2 \times 3x + (-2) \times 7$
$= -6x + (-14) = -6x - 14$
b $-5(6a - 3b) = -5 \times 6a + (-5) \times (-3b)$
$= -30a + 15b$

5 **a** $-2y - 6$ **b** $-5a - 10$ **c** $-3w - 12$
d $-4m + 28$ **e** $-t - 3$ **f** $-b - 6$
g $-6k - 15$ **h** $-8m + 10$ **i** $-7w - 3$
j $-4x + 1$

6 **a** $-4x^2(3x^3 + 2xy) = -4x^2 \times 3x^3 + (-4x^2) \times 2xy$
$= -12x^5 + (-8x^3y)$
$= -12x^5 - 8x^3y$
b $-a^2(5a^2 - 4ab) = -a^2 \times 5a^2 + (-a^2) \times (-4ab)$
$= -5a^4 + 4a^3b$

7 **a** $-6a^3 - 4a^2b$ **b** $-8x^3 + 12x^2y$
c $-9p^4 - 12p^3q$ **d** $-4y^5 + 3xy^4$
e $-6m^6 - 15m^5n$ **f** $-y^5 + 4y^2$
g $-10x^5 + 15x^3y$ **h** $-mt^3 - t^2$

8 **a** $4(y + 5) - 3 = 4 \times y + 4 \times 5 - 3$
$= 4y + 20 - 3 = 4y + 17$
b $11 + 3(2x - 7) = 11 + 3 \times 2x + 3 \times (-7)$
$= 11 + 6x - 21 = 6x + 11 - 21$
$= 6x - 10$

9 **a** $4a + 18$ **b** $6b - 12$ **c** $12w - 1$
d $10y - 17$ **e** $18z - 2$ **f** $8x + 16$
g $6b + 2$ **h** $4y + 33$ **i** $6w - 8$

10 **a** $9 - 3(y - 2) = 9 - 3 \times y + (-3) \times (-2)$
$= 9 - 3y + 6 = -3y + 9 + 6$
$= -3y + 15$
b $-5(3x + 2) + 8 = -5 \times 3x + (-5) \times 2 + 8$
$= -15x - 10 + 8 = -15x - 2$

11 **a** $-2a + 2$ **b** $-3y + 14$
c $-4b - 3$ **d** $-2v + 19$
e $-6w + 5$ **f** $-15t + 22$
g $-15x - 2$ **h** $-6k + 12$
i $-12z - 4$

12 **a** $3(4k - 2) + 5(k + 3) = 12k - 6 + 5k + 15$
$= 12k + 5k - 6 + 15$
$= 17k + 9$
b $4(3y - 5) + 2(5y - 1) = 12y - 20 + 10y - 2$
$= 12y + 10y - 20 - 2$
$= 22y - 22$

13 **a** $13k + 9$ **b** $15m + 11$ **c** $14p + 6$
d $13a - 6$ **e** $25x - 11$ **f** $14y + 1$
g $12v - 16$ **h** $22x + 2y$ **i** $23a - 9b$

14 **a** $3(2x + 4y) - 2(4x - 5y) = 6x + 12y - 8x + 10y$
$= 6x - 8x + 12y + 10y$
$= -2x + 22y$
b $2(x - 5) - (x + 4) = 2x - 10 - x - 4$
$= 2x - x - 10 - 4 = x - 14$

15 **a** $4k + 9$ **b** $2w + 26$ **c** $9t - 10$
d $13z - 8$ **e** $-2a + 14$ **f** $-d - 18$
g $10x + 11y$ **h** $4a - 12b$ **i** $-2q + 10$

Language in mathematics

1 **a** Three to the power five or three to the fifth (power)
b x squared
c Seven to the power zero
d x to the power minus three

2 **a** 7^2 **b** y^3 **c** 6^5 **d** m^4

3 **a** Index **b** Exponent

4 **a** Base **b** Index, power or exponent

5 **a** When multiplying numbers with the same base, add the indices (index law). Hence, $m^3 \times m^2 = m^5$ (not m^6).
b When raising a power of a number by another power, multiply the indices (index law). Hence, $(x^3)^2 = x^6$ (not x^5).
c $y^n \div y^n = y^{n-n} = y^0$ by the index law. But $y^n \div y^n = 1$. Hence, $y^0 = 1$.
d By the index laws and part **c**, $a^{-2} = a^{0-2} = \frac{a^0}{a^2} = \frac{1}{a^2}$ (not $-2a$).
e Multiply both terms in the brackets by the number outside (distributive law). Hence, $3(k + 5) = 3k + 15$ (not $3k + 5$).
f By order of operations rules, clear brackets first. Hence, $5 + 4(n + 2) = 5 + 4n + 8 = 4n + 13$ (not $9(n + 2)$).

6 $3p^0 = 3 \times p^0 = 3 \times 1 = 3$
$(3p)^0 = 1$

Check your skills

1 D **2** B **3** C **4** C **5** B
6 C **7** B **8** B **9** D **10** C
11 B **12** D **13** B **14** A **15** D
16 A

Review set 2A

1 a y^{17} b k^6 c p^{14} d t^8
e $125m^{12}$ f $6a^6b^9$ g $\frac{m^2n^5}{2}$

2 a $\frac{1}{a^5}$ b $\frac{2}{a^5}$ c $\frac{1}{2a^5}$ d $\frac{2}{5a^5}$

3 a 1 b 5 c 1 d 3

4 a 8 b $1\frac{1}{3}$ c $\frac{4}{9}$

5 a $\frac{1}{z^3}$ b $\frac{2}{z^3}$ c $\frac{1}{8z^3}$

6 a k^{-7} b $2k^{-7}$ c $\frac{1}{2}k^{-7}$ or $\frac{k^{-7}}{2}$

7 a y^2 b e^8 c n^{-20} d $18b^5$ e $2k^{-2}$

8 a True b False c False d True e False

9 a $10v - 20w$ b $2a^5 + 4a^4$ c $-12x - 15$

10 a $10m + 7$ b $7a - 2b$ c $-x^2 + 4$

Review set 2B

1 a m^{20} b t^{20} c z^{24} d b^{19}
e $16m^{28}$ f $20p^7q^8$ g $\frac{3ab^8}{4}$

2 a $\frac{1}{m^3}$ b $\frac{3}{m^3}$ c $\frac{1}{3m^3}$ d $\frac{2}{3m^3}$

3 a 1 b 4 c 1 d 3

4 a 9 b $\frac{4}{5}$ c $1\frac{5}{6}$

5 a $\frac{1}{e^4}$ b $\frac{3}{e^4}$ c $\frac{1}{81e^4}$

6 a p^{-4} b $5p^{-4}$ c $\frac{1}{5}p^{-4}$ or $\frac{p^{-4}}{5}$

7 a k^{-4} b m^{-3} c n^{-15} d $20n^5$ e $\frac{a^3}{2}$

8 a False b True c False d False e False

9 a $-40p - 30$ b $3m^7 - m^5$ c $-2a^2 + 5a$

10 a $4q + 32$ b $4a^2 - 13a - 1$ c $6m - 6t$

Review set 2C

1 a p^{14} b y^8 c t^{30} d 1
e $27v^{21}$ f $18x^{10}y^{14}$ g $\frac{3m^2n}{2}$

2 a $\frac{1}{k^2}$ b $\frac{8}{k^2}$ c $\frac{1}{8k^2}$ d $\frac{3}{4k^2}$

3 a 1 b 7 c 1 d 12

4 a 27 b $1\frac{1}{7}$ c $\frac{5}{6}$

5 a $\frac{1}{b^5}$ b $\frac{3}{b^5}$ c $\frac{1}{243b^5}$

6 a t^{-3} b $2t^{-3}$ c $\frac{1}{2}t^{-3}$ or $\frac{t^{-3}}{2}$

7 a d^{-8} b n c k^6 d $15a^{-3}$ e $\frac{2m^7}{3}$

8 a True b False c False d True e False

9 a $20w + 8x$ b $-2k^7 + 4k^5$ c $-4s + 7$

10 a $t + 4$ b $-15y$ c $-21m + 22$

CHAPTER 3 COLLECTING AND ANALYSING DATA

Exercise 3A

1 a 2007 b 2005

2 a 121 mm b 10 mm c 148 mm

3 a October b February

4 a September b May

5 a 2006 b 2007 c 2007

6 a 2011 b 2008 c 2009

7 a 2007 b 2002

8 a 1215 mm b 1160 mm

9 a i 69 mm ii 53 mm
b September has the lowest average rainfall.

10 a June b September

11 a 0 mm b August

12 a 643 mm b June

13 September

14 2007, 2011

15 70%

16 a 10 944 b 3339 c 2988

17 a 20.2% b 11.4% c 8.3%
d 3.7% e 2.5%

18 a 22 784 b 7719

19 a 544 000 b 177 000 c 156 000
d 211 000 e 127 000

20 a UK b UK

21 a Hong Kong/Lebanon b South Korea

22 Italy

23 a 6.3% b 5.7% c 1.7% d 1.5% e 1.5%

24 a 1.7% b 1.5% c 0.4% d 0.4% e 0.4%

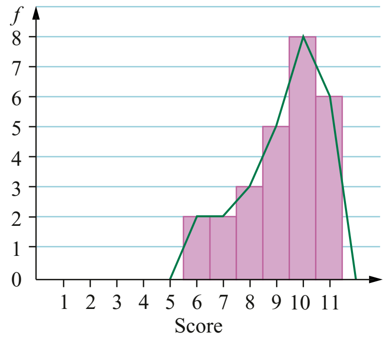
25 a 87.2% b 121.4% c 104.1% d 29.6% e 3.4%

Exercise 3B

1 a Negatively skewed b Bimodal
c Symmetric d Positively skewed
e Negatively skewed

2 a Symmetric b Positively skewed
c Negatively skewed d Bimodal
e Bimodal

3 a



b It is negatively skewed.

4 a Females: negatively skewed, males: positively skewed
b 9 Blue: positively skewed, 9 Red: bimodal

5 a Pre-test: positively skewed, post-test: negatively skewed
b Team A: positively skewed, team B: negatively skewed

Exercise 3C

1 a

Group 1	Stem	Group 2
6 4	0	7 7 8
6 6 3 3 1	1	2 2 5 7 8
9 9 7 4 3 3 2	2	1 3 5 6 7
9 8 4 1 0	3	2 4 6 6 7
0	4	1 3 6
1	5	

b Group 1 is positively skewed, group 2 is symmetric.

c Group 1: mean = 24.7, median = 24, range = 47
Group 2: mean = 24.9, median = 25, range = 39
The means and medians are approximately the same, indicating that both groups took about the same length of time. The range for group 1 is greater than that for group 2, indicating a greater spread of times. However, if we ignore the outlier of 51, the range of group 1 becomes 37, which is almost the same as for group 2.

2 a

Test 1	Stem	Test 2
	0	6
9	1	2 3 3 5 6 8 9
7 4 2	2	0 1 4 6 6
9 9 8 5 4 2 0	3	1 5
7 6 5 2 1	4	0

b Test 1 is negatively skewed, test 2 is positively skewed.

c Test 1: mean = 35, median = 36.5, range = 28
Test 2: mean = 20.9, median = 19.5, range = 34
The mean and median for test 1 were both considerably higher than those for test 2, indicating that, in general, the students scored much higher marks on test 1 than on test 2. The ranges indicate that the spread of marks for test 2 was slightly larger than for test 1.

d Test 2 was much more difficult than test 1.

3 a

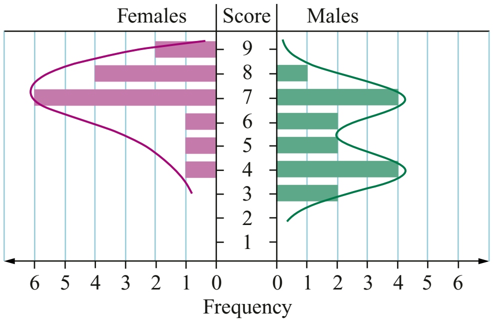
Sydney	Stem	Melbourne
	22	4
	23	1 8 9 9
	24	0 2 3 4
9 5 4 4 4 3 0	25	0 3 7
9 9 8 8 7 4 4 3 2 2	26	0 0 9
2 0	27	4
9	28	0 1 6
	29	5

22|4 = 22.4

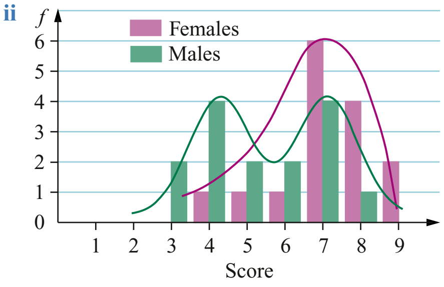
b Sydney is positively skewed; Melbourne is positively skewed and bimodal.

c Sydney: mean = 26.3, median = 26.35, range = 3.9
Melbourne: mean = 25.5, median = 25.15, range = 7.1
The mean and median for Sydney are both greater than for Melbourne indicating that, in general, Sydney's maximum temperatures are higher than Melbourne's. Melbourne has a greater spread of maximum temperatures than Sydney, where maximum temperatures are more consistent.

4 a i



ii



b Females: negatively skewed, males: bimodal

c Females: mean = 7.1, median = 7, range = 5
Males: mean = 5.3, median = 5, range = 5
The mean and median for females is greater than for males; that is, the females performed better than the males. The ranges are the same, but the female scores are skewed to the right whereas the male scores are more uniformly spread.

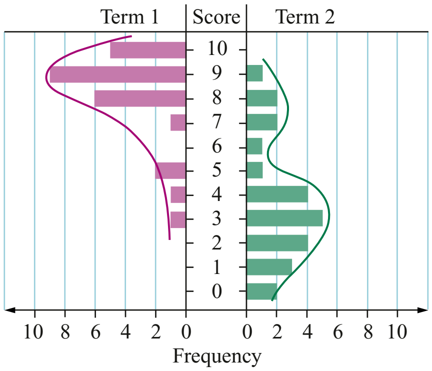
5 a

Term 1

Mark	Frequency
3	1
4	1
5	2
6	0
7	1
8	6
9	9
10	5

Term 2

Mark	Frequency
0	2
1	3
2	4
3	5
4	4
5	1
6	1
7	2
8	2
9	1



b The term 1 test scores are negatively skewed.
The term 2 test scores are bimodal and positively skewed.

c The term 2 test was harder than the term 1 test and perhaps there were some students who understood something the rest of the class did not understand.

d Term 1: mean = 8.1, median = 9, range = 7
Term 2: mean = 3.7, median = 3, range = 9
The mean and median of the term 2 test are much smaller than those for term 1, indicating that the class performance on this test was significantly less than on the term 1 test, possibly because the term 2 test was harder or because the students were not as well prepared. The spread of marks on the term 2 test was greater than on the term 1 test.

6 a Team A: distribution is positively skewed.
Team B: distribution is symmetric.

b Team A: mean = 1.8, median = 2, range = 5
Team B: mean = 2.25, median = 2, range = 5
The median score is the same for both teams, but the mean for team B is greater than that for team A, indicating that team B performed better. The ranges are the same, but as team A's scores are positively skewed, it has more low scores than team B.

7 a Team A: bimodal, team B: negatively skewed

b Team A: mean = 5.4, median = 5.5, range = 6
Team B: mean = 6.6, median = 7, range = 4
The range for team A was greater than for team B, but the mean and median for team B were greater than those for team A, indicating that team B's goalkeeper made many more saves than team A's goalkeeper over the season.

Language in mathematics

1 **a** A symmetric distribution is commonly a bell-shaped curve.
b Most of the data is on the left-hand side of the distribution.
c Most of the data is on the right-hand side of the distribution.
d It has two peaks, showing there are two modes.

2 **a** Hanna Neumann was born in Berlin in 1914.
b Her main areas of interest were mathematics, history and religion.
c She is remembered for her mathematical ability and the time she devoted to teaching her students.

Check your skills

1 A **2** B **3** B **4** D **5** C
6 A **7** B **8** C **9** C **10** C

Review set 3A

1 **a** **i** India **ii** Australia **iii** Bangladesh
b **i** Rwanda **ii** Rwanda **iii** Australia
c Rwanda, Netherlands, Belgium, Sri Lanka
d Because Australia has a large land area.

2 **a** Bimodal **b** Positively skewed
c Symmetric **d** Negatively skewed

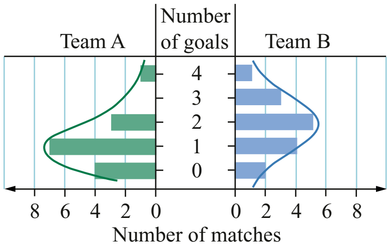
3 **a** Both distributions are approximately symmetric with the boys curve tending to a slight negative skew.
b Boys: mean = 169.4, median = 170, range = 29
Girls: mean = 162.9, median = 163, range = 24
Both the mean and the median for boys are greater than those for girls, indicating that, in general, the boys are taller than the girls in this class. The range of the boys' heights is greater than that of the girls' heights, indicating a slightly greater spread of heights.

Review set 3B

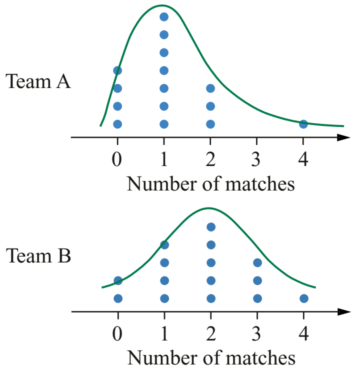
1 **a** **i** WA **ii** NSW **iii** ACT **iv** ACT
b **i** ACT **ii** ACT **iii** NT **iv** Tas
c It has a very small land area.

2 **a** Symmetric **b** Positively skewed
c Bimodal **d** Negatively skewed

3 **a** **i**



ii

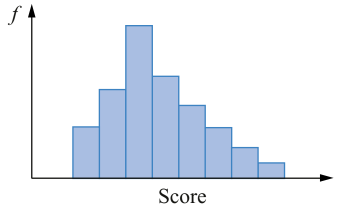


b Team A: positively skewed, team B: symmetric
c Team A: mean = 1.1, median = 1, range = 4
Team B: mean = 1.8, median = 2, range = 4
The mean and median for team B are greater than those for team A, indicating that, in general, team B scored more goals per match than team A. The ranges of the scores are the same but team A has more low scores.

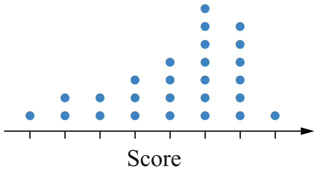
Review set 3C

1 **a** **i** 46% **ii** 58%
b **i** 18% **ii** 12%
c **i** 42% **ii** 40% **iii** 32%
d Out of the labour force
e **i** 38% **ii** 32% **iii** 22%
f Out of the labour force

2 **a**



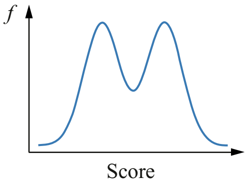
b



c

Stem	Leaf
2	0 1
3	2 5 7
4	3 4 6 6 8 9
5	4 6 7
6	1 3

d



3 **a**

Girls	Stem	Boys
9 9	0	
7 5 2	1	7 8
9 8 8 6 4 0	2	4 6
8 6 2	3	1 2 2 5
1	4	0 3 6 8 9 9
	5	1

b Girls: symmetric, boys: negatively skewed
c Girls: mean = 24.3, median = 26, range = 32
Boys: mean = 36.1, median = 35, range = 34
The mean and median for the boys are greater than those for the girls, indicating that the boys performed significantly better on this test. The ranges are almost the same but the results for the boys are skewed towards the higher numbers.

CHAPTER 4 NUMBERS OF ANY MAGNITUDE

Exercise 4A

1 a Yes b No c No d Yes e No
f Yes g No h No i No j Yes

2

10^0	10^1	10^2	10^3	10^4	10^5	10^6
1	10	100	1000	10 000	100 000	1 000 000

3 a 7000 = 7 × 1000 = 7×10^3
b 600 000 = 6 × 100 000 = 6×10^5

4 a 3×10^6 b 7×10^4 c 8×10^3
d 6×10^5 e 5×10^2

5 a 67 000 = 6.7 × 10 000 = 6.7×10^4
b 8 420 000 = 8.42 × 1 000 000 = 8.42×10^6

6 a 4.8×10^3 b 3.92×10^5 c 6.4×10^4
d 2.18×10^6 e 7.6×10^2

7 a 5×10^4 = 5 × 10 000 = 50 000
b 4.93×10^6 = 4.93 × 1 000 000 = 4 930 000

8 a 30 000 b 7000 c 9 000 000 d 400 000
e 800 f 460 000 g 6710 h 3 900 000
i 83 600 j 520 000

9

0.1	0.01	0.001	0.0001	0.000 01	0.000 001
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$	$\frac{1}{100\,000}$	$\frac{1}{1\,000\,000}$

10 a $0.0007 = 7 \times 0.0001 = 7 \times \frac{1}{10^4} = 7 \times 10^{-4}$
b $0.06 = 6 \times 0.01 = 6 \times \frac{1}{10^2} = 6 \times 10^{-2}$

11 a 3×10^{-3} b 7×10^{-6} c 5×10^{-4}
d 2×10^{-5} e 9×10^{-2}

12 a $4 \times 10^{-3} = 4 \times \frac{1}{10^3} = 4 \times \frac{1}{1000}$
$= 4 \times 0.001 = 0.004$
b $3 \times 10^{-2} = 3 \times \frac{1}{10^2} = 3 \times \frac{1}{100}$
$= 3 \times 0.01 = 0.03$

13 a 0.06 b 0.000 003 c 0.002
d 0.0005 e 0.000 009

14 a 3×10^4 = 30 000, $3^4 = 81$
b $5 \times 10^{-2} = 0.05$, $5^{-2} = \frac{1}{25}$
c $2 \times 10^3 = 2000$, $2^3 = 8$
d 2×10^{-5} = 0.000 02, $2^{-5} = \frac{1}{32} \approx 0.03$
e 4×10^6 = 4 000 000, $4^6 = 4096$
f 5×10^6 = 5 000 000, 5^6 = 15 625

15 *Step 1:* 3.97
Step 2: Number of places = 6 to the right
Step 3: 3 970 000 = 3.97×10^6

16 a 3.72×10^5 b 5.4×10^4 c 2.98×10^6
d 3.4×10^3 e 6.09×10^5 f 8.75×10^4
g 7.698×10^6 h 3.61×10^8 i 8×10^3
j 5.6×10^7

17 a 5.7×10^{-4} b 7.8×10^{-5} c 6.1×10^{-3}
d 2.96×10^{-6} e 8.01×10^{-4} f 5×10^{-7}
g 4.39×10^{-3} h 2.8×10^{-6} i 9×10^{-5}
j 4.9×10^{-9}

18 Move the decimal point 5 places to the right.
5.72×10^5 = 572 000

19 a 7 320 000 b 52 000 c 567 000
d 3800 e 92 700 000 f 69 140
g 3 275 000 h 700 000 i 200 000 000
j 308 000

20 a 0.000 003 98 b 0.000 53 c 0.000 070 9
d 0.0088 e 0.000 005 9 f 0.000 000 307
g 0.0006 h 0.000 003 i 0.000 027 1
j 0.000 000 000 36

21 a 1.29×10^5 b 1.52×10^8 km
c 2.54×10^{-9} cm d 2.6×10^{-7} m
e 1.07×10^5 km/h

22 a 78 300 000 km b 1 400 000 000
c 0.000 028 m d 0.000 000 000 28 h
e 10 000 000 000 000

Exercise 4B

1 a $(4 \times 10^7) \times (3 \times 10^8) = (4 \times 3) \times (10^7 \times 10^8)$
$= 12 \times 10^{15}$
$= 1.2 \times 10^1 \times 10^{15}$
$= 1.2 \times 10^{16}$
b $(6 \times 10^3) \div (4 \times 10^7) = (6 \div 4) \times (10^3 \div 10^7)$
$= 1.5 \times 10^{-4}$
c $(2 \times 10^{-4})^5 = 2^5 \times (10^{-4})^5$
$= 32 \times 10^{-20}$
$= 3.2 \times 10^1 \times 10^{-20}$
$= 3.2 \times 10^{-19}$

2 a 1.2×10^{15} b 2.4×10^{22} c 4.2×10^9
d 6×10^{-15} e 2×10^{12} f 3×10^{10}
g 3×10^{-10} h 2×10^{-12} i 8×10^{15}
j 4.9×10^{19} k 2.7×10^{-17} l 6.4×10^{-19}
m 3.5×10^{-12} n 7.5×10^{-5}

3 a 3.5×10^{13} b 1.8×10^{-8}
c 1.5×10^5 d 1.8×10^2

4 a 1.536×10^{20} b 2.43×10^{19} c 9×10^8
d 2.7×10^{-18} e 8×10^6 f 7.5×10^{-19}
g 2.5×10^{21} h 1.4×10^{-12} i 1.6×10^{33}
j 2.704×10^{-11} k 2.16×10^{38} l 3.125×10^{-37}
m 2.5×10^9 n 3.7×10^{-12} o 3×10^6

5 a 3.11×10^8, positive b Bigger

6 a -2.5928×10^8, negative b Smaller

7 a 4.37×10^{-4}, positive b Bigger

8 a $10^{-10} < 10^{-8}$
so $7.9 \times 10^{-10} < 6.5 \times 10^{-8}$
b $10^6 = 10^6$, $2.09 < 5.3$
so $2.09 \times 10^6 < 5.3 \times 10^6$

9 a $4.6 \times 10^{14} < 7.2 \times 10^{15}$
b $4.5 \times 10^{16} < 3.4 \times 10^{18}$
c $9.6 \times 10^{-12} < 6.8 \times 10^{-9}$
d $7.8 \times 10^{-8} < 3.8 \times 10^{-6}$
e $2.5 \times 10^{-4} < 7.1 \times 10^5$
f $2.9 \times 10^{16} < 3 \times 10^{16}$
g $6.4 \times 10^{-10} < 8.5 \times 10^{-10}$
h $8.1 \times 10^{14} < 2.8 \times 10^{15} < 5.9 \times 10^{16}$
i $8.9 \times 10^{-8} < 5 \times 10^{-6} < 3.9 \times 10^{-5}$
j $7.8 \times 10^{-5} < 8.3 \times 10^{-3} < 6.3 \times 10^6$

10 1.08×10^9 km

11 Alpha Centauri

12 2.54×10^{-3} cm or 0.002 54 cm

13 400 days

14 a $0.000\,000\,\dot{3}$ s or 3.3×10^{-7} s
b $0.\dot{3}\dot{0}$ s

15 285 000 m^2/person or 2.85×10^5 m^2/person

Exercise 4C

1 a 7.2 kL = 7.2×10^3 L (or 7200 L)
b 84 μL = 84×10^{-6} L (or 0.000 084 L)
$= (8.4 \times 10^1) \times 10^{-6}$ L $= 8.4 \times 10^{-5}$ L

2 **a** **i** 7×10^9 g **ii** 3.8×10^{-8} g
b **i** 3.1×10^6 J **ii** 1.24×10^1 or 12.4 J
3 **a** **i** 4×10^{12} W **ii** 4.5×10^{-5} W
b **i** 9×10^3 g **ii** 2×10^9 g
4 **a** $26\,000 \text{ g} = 26\,000 \div 10^3 \text{ kg}$
$= (2.6 \times 10^4) \div 10^3$
$= 2.6 \times 10$ or 26 kg
b $26\,000 \text{ g} = (2.6 \times 10^4) \div 10^6 \text{ Mg}$
$= 2.6 \times 10^{-2}$
$= 0.026$ Mg
5 **a** $0.000\,075 \text{ g} = 0.000\,075 \div 10^{-3} \text{ mg}$
$= 0.000\,075 \times 10^3 \text{ mg} = 0.075 \text{ mg}$
b $0.000\,075 \text{ g} = 0.000\,075 \div 10^{-9} \text{ ng}$
$= 0.000\,075 \times 10^9 \text{ ng} = 75\,000 \text{ ng}$
6 **a** **i** 6.7×10^4 or 67 000 kL
ii 6.7×10^{-2} or 0.067 GL
b **i** 8.2×10^{-1} or 0.82 µL
ii 8.2×10^5 or 820 000 pL
7 **a** **i** 6.4×10^2 or 640 Mm
ii 6.4×10^{-1} or 0.64 Gm
b **i** 9×10^{-5} or 0.000 09 cm
ii 9×10^2 or 900 nm
8 **a** 560 kJ **b** 0.56 MJ
9 $1 \text{ Tm} = 10^{12} \text{ m}$ $1 \text{ Mm} = 10^6 \text{ m}$
Hence $1.8 \text{ Tm} = 1.8 \times 10^{12} \div 10^6 \text{ Mm}$
$= 1.8 \times 10^6$ or 1 800 000 Mm
10 **a** 3.6×10^3 or 3600 km
b 5.2×10^6 or 5 200 000 nW
c 7.4×10^6 or 7 400 000 ML
d 8.9×10^3 or 8900 pL
e 4.8×10^9 Mg
f 1.8×10^6 or 1 800 000 kJ
11 **a** 7×10^3 or 7000 µW
b 2×10^6 or 2 000 000 mg
c 5×10^3 or 5000 µL
d 1.8×10^6 or 1 800 000 mJ
e 1.7×10^5 or 170 000 µm
f 3×10^{-4} TL or 0.0003 TL
12 **a** 0.1495 Tm **b** 1.495×10^8 km
13 **a** 5.97×10^{23} kg
b 5.97×10^5 or 597 000 Zg
14 **a** 5.6×10^{-12} mm **b** 5.6×10^{-6} nm
15 **a** **i** 1024 **ii** 1024 **iii** 1 048 576
b **i** 1.074×10^9 **ii** 1 048 576 **iii** 1024
c **i** 2^{20} **ii** 2^{30} **iii** 2^{40}

Exercise 4D

1 **a** 2900 ms **b** 0.73 ms **c** 7.4 ns
d 3 700 000 fs **e** 14.5 fs **f** 6.84×10^{-4} ms
2 **a** 3 weeks $= 3 \times 7 \times 24$ h $= 504$ h
b 3888 min $= 3888 \div 60$ h $= 64.8$ h
$= 64.8 \div 24$ days $= 2.7$ days
or 3888 min $= 3888 \div (60 \times 24) = 2.7$ days
3 **a** 3 024 000 s **b** 20 160 min **c** 3600 months
d 54 centuries **e** 75.6 decades
f 13 035.7 years (or 13 044.6 years counting leap years)
4 4.53×10^9 years **5** 2000 times

Exercise 4E

1 **a** 60 **b** 6 **c** 600 **d** 6000
e 6 000 000 **f** $\frac{6}{1000}$ **g** $\frac{6}{10}$ **h** $\frac{6}{100}$
i 600 **j** $\frac{6}{10\,000}$
2 **a** 6, 4, 536 000 **b** 7, 8, 58 000 **c** 7, 5, 8000
3 **a** 37 000 **b** 84 000 **c** 524 000
d 8000 **e** 19 000 **f** 623 000
g 181 000 **h** 6000 **i** 3000
j 401 000 **k** 1000 **l** 0
4 **a** 6, 9, 4700 **b** 2, 5, 5130 **c** 8, 3, 148
5 **a** 5400 **b** 16 800 **c** 20 400
d 800 **e** 400 **f** 240 200
g 3100 **h** 9600 **i** 500
j 100 **k** 100 **l** 0
6 **a** 670 **b** 2370 **c** 830
d 1060 **e** 70 **f** 30
g 310 **h** 20 060 **i** 410
j 1250 **k** 10 **l** 0
7 **a** 17 **b** 25 **c** 82 **d** 237
e 583 **f** 265 **g** 21 **h** 106
i 301 **j** 56 **k** 1 **l** 0
8 **a** **i** 47 000 **ii** 46 800 **iii** 46 780 **iv** 46 784
b **i** 28 000 **ii** 28 500 **iii** 28 460 **iv** 28 457
c **i** 39 000 **ii** 39 200 **iii** 39 170 **iv** 39 166
d **i** 8000 **ii** 8500 **iii** 8460 **iv** 8462
e **i** 183 000 **ii** 182 700 **iii** 182 680 **iv** 182 679
9 **a** 5, 2, 63.5 **b** 2, 0, 63.52 **c** 0, 5, 63.521
10 **a** 38.3 **b** 38.27 **c** 38.268
11 **a** **i** 8.4 **ii** 8.44 **iii** 8.438
b **i** 6.6 **ii** 6.58 **iii** 6.584
c **i** 0.9 **ii** 0.86 **iii** 0.863
d **i** 0.2 **ii** 0.19 **iii** 0.186
e **i** 18.6 **ii** 18.56 **iii** 18.556
f **i** 21.6 **ii** 21.60 **iii** 21.603
g **i** 4.1 **ii** 4.06 **iii** 4.061
h **i** 5.0 **ii** 5.04 **iii** 5.044
i **i** 7.0 **ii** 7.01 **iii** 7.007
j **i** 3.0 **ii** 3.00 **iii** 3.000
12 **a** 3.60 **b** 50.0 **c** 2.690 **d** 13.00
e 1.00 **f** 4.900 **g** 100.0 **h** 70.00
13 **a** 75 **b** <85 **c** $75 \leqslant \text{number} < 85$
14 **a** 350 **b** <450 **c** $350 \leqslant \text{number} < 450$
15 **a** **i** 27 500 **ii** $<28\,500$
iii $27\,500 \leqslant \text{number} < 28\,500$
b **i** 42.5 **ii** <43.5
iii $42.5 \leqslant \text{number} < 43.5$
c **i** 5.65 **ii** <5.75
iii $5.65 \leqslant \text{number} < 5.75$
d **i** 6.315 **ii** <6.325
iii $6.315 \leqslant \text{number} < 6.325$
16 $162.5 \text{ cm} \leqslant \text{height} < 163.5 \text{ cm}$
17 $415 \text{ g} \leqslant \text{weight} < 425 \text{ g}$
18 $12.35 \text{ s} \leqslant \text{time} < 12.45 \text{ s}$

Exercise 4F

1 **a** 2 **b** 5 **c** 1 **d** 7 **e** 8
2 **a** 2, tens, 10, 30
b 8, units, whole number, 28.
c 4, first, 1 decimal place, 28.5
d 7, second, 2 decimal places, 28.47
e 0, third, 3 decimal places, 28.471
3 **a** **i** 400 **ii** 430 **iii** 428
b **i** 6000 **ii** 6200 **iii** 6240
c **i** 8 **ii** 7.8 **iii** 7.82
d **i** 0.5 **ii** 0.53 **iii** 0.527
e **i** 50 000 **ii** 54 000 **iii** 53 700
f **i** 700 000 **ii** 730 000 **iii** 726 000

g i 0.04 ii 0.039 iii 0.0393
h i 0.005 ii 0.0051 iii 0.005 07
i i 6000 ii 6100 iii 6100
j i 2000 ii 2000 iii 2010
4 **a** 370 000 **b** 240 **c** 0.005 80
d 9.00 **e** 300 000 **f** 500
g 0.0400 **h** 0.300 **i** 0.002 00
j 1 000 000
5 **a** i 555 ii <565
iii 555 ≤ number < 565
b i 8.15 ii <8.25
iii 8.15 ≤ number < 8.25
c i 47.5 ii <48.5
iii 47.5 ≤ number < 48.5
d i 0.715 ii <0.725
iii 0.715 ≤ number < 0.725
e i 36 500 ii <37 500
iii 36 500 ≤ number < 37 500
f i 0.0835 ii <0.0845
iii 0.0835 ≤ number < 0.0845
6 **a** 482.5 ≤ number < 483.5
b 3.855 ≤ number < 3.865
c 14 450 ≤ number < 14 550
d 0.1275 ≤ number < 0.1285
e 56.85 ≤ number < 56.95
f 3205 ≤ number < 3215
7 **a** 295 ≤ number < 305
b 2950 ≤ number < 3050
c 5995 ≤ number < 6005
d 23 950 ≤ number < 24 050
e 499 500 ≤ number < 500 500
f 0.795 ≤ number < 0.805
8 **a** 2 **b** 2 **c** 4 **d** 2 **e** 2
f 4 **g** 2 **h** 2 **i** 1, 2 or 3
j 1, 2, 3 or 4 **k** 2, 3, 4 or 5 **l** 1, 2, 3, 4, 5, 6 or 7

Exercise 4G

1 **a** 29 200
b i 12 900 ii 700 iii 15 700
c 29 300 **d** Differ by 100.
2 **a** 5250
b i 8090 ii 2830
c 5260 **d** Differ by 10.
3 **a** 18.7 **b** 2.3 **c** 18.4
d Differ by 0.3.
4 **a** 143.4 **b** 142.6 **c** Differ by 0.8.
d 142.1 **e** Differ by 1.3.
5 **a** 75.172 **b** 75
c i 33 ii 17 iii 26
d 76 **e** Differ by 1.
6 **a** 60 **b** 4, 20 **c** 80 **d** Differ by 20.
7 **a** 2.18
b i 16.4 ii 7.50
c 2.19 **d** Differ by 0.01.
8 **a** 4220 **b** 4240 **c** Differ by 20.
9 **a** 0.18 **b** 0.19 **c** Differ by 0.01.
10 **a** 3.3 **b** 10.89, no
c i 3.32, 11.0224, no
ii 3.317, 11.002 489, no
iii 3.3166, 10.999 8…, no
iv 3.31662, 10.999 96…, no
11 **a** 3 s.f. **b** 7 s.f. **c** 4 s.f.
d 3 s.f. **e** 4 s.f.

Exercise 4H

1 **a** i 1 cm ii 0.5 cm
b i 1 g ii 0.5 g
c i 1 m ii 0.5 m
d i 1 min ii 0.5 min
e i 1 L ii 0.5 L
f i 0.1 kg ii 0.05 kg
g i 0.1 s ii 0.05 s
h i 0.1 m ii 0.05 m
i i 0.1 L ii 0.05 L
j i 0.01 m ii 0.005 m
2 **a** kg, 1 kg, limit of reading = 1 kg
b Greatest possible error = $\frac{1}{2}$ × 1 kg = 0.5 kg
c Limits of accuracy = 18 ± 0.5 kg = 17.5, 18.5 kg
d 17.5 kg ≤ true mass < 18.5 kg
3 **a** 0.1 m, 0.1 m, limit of reading = 0.1 m
b Greatest possible error = $\frac{1}{2}$ × 0.1 m = 0.05 m
c Limits of accuracy = 9.4 ± 0.05 m = 9.35 m, 9.45 m
d 9.35 m ≤ true length < 9.45 m
4 **a** i 1 mm ii 0.5 mm iii 11.5 mm, 12.5 mm
iv 11.5 mm ≤ true length < 12.5 mm
b i 1 g ii 0.5 g iii 347.5 g, 348.5 g
iv 347.5 g ≤ true mass < 348.5 g
c i 1 mL ii 0.5 mL iii 374.5 mL, 375.5 mL
iv 374.5 mL ≤ true capacity < 375.5 mL
d i 0.1 km ii 0.05 km
iii 8.15 km, 8.25 km
iv 8.15 km ≤ true length < 8.25 km
e i 0.1 s ii 0.05 s iii 18.35 s, 18.45 s
iv 18.35 s ≤ true time < 18.45 s
f i 0.1 kg ii 0.05 kg iii 4.85 kg, 4.95 kg
iv 4.85 kg ≤ true mass < 4.95 kg
g i 0.01 m ii 0.005 m
iii 2.365 m, 2.375 m
iv 2.365 m ≤ true length < 2.375 m
h i 0.01 L ii 0.005 L iii 5.805 L, 5.815 L
iv 5.805 L ≤ true capacity < 5.815 L
5 **a** i 20 g ii 10 g iii 410 g, 430 g
iv 410 g ≤ true mass < 430 g
b i 10 mL ii 5 mL iii 365 mL, 375 mL
iv 365 mL ≤ true capacity < 375 mL
c i 1000 ii 500 iii 37 500, 38 500
iv 37 500 ≤ true crowd < 38 500
d i $\frac{1}{2}$h ii $\frac{1}{4}$h iii $6\frac{1}{4}$h,$6\ \frac{3}{4}$h
iv $6\frac{1}{4}$h ≤ true time <$6\ \frac{3}{4}$h
6 **a** 50 m ÷ 7 = 7.142 85… m
i Correct to 1 s.f.: 7.142 85… m = 7 m
ii Correct to 3 s.f.: 7.142 85… m = 7.14 m
iii Correct to 4 s.f.: 7.142 85… m = 7.143 m
b i To the nearest metre, metre
ii To the nearest centimetre, centimetre
iii To the nearest millimetre, millimetre
7 **a** 133.333… g
i Correct to 1 s.f.: 133.333… g = 100 g
ii Correct to 2 s.f.: 133.333… g = 130 g
iii Correct to 3 s.f.: 133.333… g = 133 g
b i To the nearest 100 g, 100 g
ii To the nearest 10 g, 10 g
iii To the nearest g, 1 g
8 3, hundredths, hundredth, 10 g
4, thousandths, thousandth, 1 g, second

9 **a** 3.64 m is to the nearest cm, 3.640 m is to the nearest mm
b 5.8 kg is to the nearest 100 g, 5.80 kg is to the nearest 10 g
c 12 s is to the nearest second, 12.0 s is to the nearest tenth of a second
d 36 cm is to the nearest cm, 36.0 cm is to the nearest mm
e 23.8 s is to the nearest tenth of a second, 23.80 is to the nearest hundredth of a second
f 7.29 km is to the nearest 10 m, 7.290 km is to the nearest 1 m
g 1.5 t is to the nearest 100 kg, 1.50 t is to the nearest 10 kg
h 5.83 L is to the nearest 10 mL, 5.830 L is to the nearest mL

Exercise 4I

1 **a** Perimeter = 28 cm
b Greatest possible error = 0.5 cm
c 8.5 cm and 9.5 cm, 4.5 cm and 5.5 cm
d Lower limit of perimeter = 8.5 + 8.5 + 4.5 + 4.5 cm
= 26 cm
Upper limit of perimeter = 9.5 + 9.5 + 5.5 + 5.5 cm
= 30 cm
e Maximum error = 30 − 28 cm = 2 cm
2 **a** Total mass = 47 kg + 52 kg = 99 kg
b Greatest possible error = 0.5 kg
c 46.5 kg and 47.5 kg, 51.5 kg and 52.5 kg
d **i** Lower limit = 46.5 + 51.5 kg = 98 kg
ii Upper limit = 47.5 + 52.5 kg = 100 kg
e Maximum error = 100 − 99 kg = 1 kg
3 **a** Area of the rectangle is $5 \times 3\ \text{m}^2 = 15\ \text{m}^2$.
b The greatest possible error is 0.5 m.
c Limits of accuracy of the length are 4.5 m and 5.5 m.
Limits of accuracy of the breadth are 2.5 m and 3.5 m.
d Lower limit of area = $4.5 \times 2.5\ \text{m}^2 = 11.25\ \text{m}^2$
Upper limit of area = $5.5 \times 3.5\ \text{m}^2 = 19.25\ \text{m}^2$
e $19.25 - 15\ \text{m}^2 = 4.25\ \text{m}^2$
$15 - 11.25\ \text{m}^2 = 3.75\ \text{m}^2$
Maximum error is $4.25\ \text{m}^2$.
4 Length has 3 s.f. and breadth has 2 s.f.
The least number of significant figures in the data is 2.
Thus round to 2 s.f.
Area = $135 \times 42\ \text{m}^2 = 5670\ \text{m}^2 = 5700\ \text{m}^2$ to 2 s.f.
5 Mass has 4 s.f. and volume has 3 s.f.
The least number of significant figures in the data is 3.
Thus round to 3 s.f.

$$\text{Density} = \frac{2.894\ \text{kg}}{0.255\ \text{m}^3} = 11.349\ \ldots\ \text{kg/m}^3$$
$$= 11.3\ \text{kg/m}^3 \text{ to 3 s.f.}$$

6 **a** 48.9 has 3 s.f. and 3.156 has 4 s.f.
The least number of significant figures in the data is 3.
Thus round to 3 s.f.
$48.9 \times 3.156 = 154.32\ldots = 154$ to 3 s.f.
b 6784 has 4 s.f., 9.5 has 2 s.f.
The least number of significant figures in the data is 2.
Thus round to 2 s.f.
$6784 \div 9.5 = 714.10\ldots = 710$ to 2 s.f.
7 **a** 12.365 is accurate to 3 d.p.
14.92 is accurate to 2 d.p.
Least decimal place accuracy is ‘correct to 2 d.p.’.
Thus round the calculation to 2 d.p.
12.365 + 14.92 = 27.285
= 27.29 to 2 d.p.
b 86.89 is accurate to 2 d.p.
57 is accurate to the nearest whole number.
The least decimal place accuracy is to the nearest whole number. Thus round to the nearest whole number.
86.89 − 57 = 29.89
= 30 to the nearest whole number.
8 Perimeter = 348 to the nearest whole number.

Language in mathematics

1 Scientific notation is a convenient way of writing very large and very small numbers. The number is expressed as the product of a number between 1 and 10 and a power of 10.
2 There is always an error in measurements due to the limit of reading of the measuring instrument.
3 **a** The limit of reading is the smallest unit on the measuring instrument.
b The greatest possible error is half the limit of reading.
c Limits of accuracy = measurement ± greatest possible error.
4 9 s has been measured to the nearest second. 9.0 s has been measured to the nearest 0.1 of a second, so 9.0 s is a more accurate measurement.
5 **a** 7 ms **b** 4.6 GJ **c** 3.5 Tm
d 2.4 ng **e** 6 km **f** 5.2 ML
6 significant, digit, truncate
7 The second s.f. is 5. This is in the third column after the decimal point, so round to 3 d.p.
∴ 0.0358 = 0.036 to 2 s.f.

Check your skills

1 D **2** B **3** C **4** B **5** C
6 D **7** D **8** A **9** D **10** D
11 D **12** C **13** D **14** B **15** D
16 D **17** A **18** B **19** B **20** A
21 A **22** B **23** B

Review set 4A

1 **a** 1.7×10^{10} **b** 3.5×10^{-7}
2 **a** 286 000 **b** 0.000 306
3 $3 \times 10^4 = 30\ 000$, $3^4 = 81$
4 **a** 1.59×10^{-13} **b** 9×10^6
c 5.329×10^{31} **d** 5×10^{-3}
5 **a** 2.94×10^{15}, 2.94×10^{16}
b 6.5×10^{-14}, 1.4×10^{-10}
6 **a** 6.4×10^9 g **b** 6.4×10^{-9} g
7 **a** 37 km **b** 0.037 Mm
8 4.2×10^6 nW **9** 11.6 days to 3 d.p.
10 **a** 50 **b** 5 **c** $\frac{5}{100}$ **d** 500
11 **a** 2500 **b** 7930 **c** 34 **d** 50 000
12 **a** 8.5 **b** 8.46 **c** 8.463
13 **a** 3.50 **b** 20.0
14 **a** 35
b <45 as any number $\geqslant 45$ would be rounded to 50.
c $35 \leqslant \text{number} < 45$
15 **a** 3 **b** 4 **c** 2
16 **a** 50 **b** 48 **c** 48.4
d 48.35 **e** 48.351
17 **a** 3 **b** 1 **c** 3
d 2 **e** 2, 3, 4 or 5
18 **a** 1 s **b** 0.5 s **c** 16.5 s, 17.5 s
d $16.5\ \text{s} \leqslant \text{true time} < 17.5\ \text{s}$

19 3.65 m is accurate to 2 d.p. (to nearest cm). 3.650 m is accurate to 3 d.p. (to nearest mm), so 3.650 m is more accurate.

20 **a** 32 cm **b** 0.5 cm
c Length: 8.5 cm, 9.5 cm; breadth: 6.5 cm, 7.5 cm
d Lower = 30 cm, upper = 34 cm
e Maximum error = 2 cm

21 72 cm^2

Review set 4B

1 **a** 4.6×10^4 **b** 3×10^{-4}
2 **a** 40 000 000 **b** 0.000 001 8
3 $4 \times 10^6 = 4\ 000\ 000$, $4^6 = 4096$
4 **a** 1.56×10^9 **b** 5×10^{15}
c 2.43×10^{-28} **d** 1.4×10^{-5}
5 **a** 4.1×10^9, 5×10^9 **b** 3.1×10^{-15}, 4.5×10^{-11}
6 **a** 5.6×10^6 J **b** 8.6×10^{-2} J
7 **a** 0.7 nm **b** 7×10^5 nm
8 1.4×10^6 kg **9** 3.6 ks
10 **a** $\frac{6}{10}$ **b** 600 **c** $\frac{6}{1000}$ **d** 60
11 **a** 13 600 **b** 4060 **c** 148 **d** 100 000
12 **a** 1.6 **b** 1.56 **c** 1.561
13 **a** 2.70 **b** 40.0
14 163.5 cm ⩽ height < 164.5 cm
15 **a** 2 **b** 1 **c** 5
16 **a** 365 000 **b** 540 **c** 0.002 40 **d** 2.00
17 **a** 2 **b** 2 **c** 4 **d** 1
e 1, 2, 3 or 4
18 **a** 0.1 kg **b** 0.05 kg **c** 15.35 kg, 15.45 kg
d 15.35 kg ⩽ true mass < 15.45 kg

19 1.54 kg is accurate to 2 d.p. (nearest hundredth of a kg or 10 g). 1.540 kg is accurate to 3 d.p. (nearest thousandth of a kg or 1 g), so 1.540 kg is more accurate.

20 **a** 26 cm **b** 0.5 cm
c Length: 7.5 cm, 8.5 cm; breadth: 4.5 cm, 5.5 cm
d Lower = 24 cm, upper = 28 cm
e Maximum error = 2 cm

21 7.5 m/s

Review set 4C

1 **a** 2.3×10^7 **b** 5×10^{-5}
2 **a** 98 000 **b** 0.000 037
3 $7 \times 10^5 = 700\ 000$; $7^5 = 16\ 807$
4 **a** 1.19×10^{14} **b** 4×10^5
c 2.43×10^{47} **d** 1.5×10^6
5 **a** 4.6×10^{13}, 3.8×10^{15} **b** 7.7×10^{-16}, 3.1×10^{-12}
6 **a** 3.6×10^{13} m **b** 4.5×10^{-5} m
7 **a** 0.000 91 mg **b** 910 ng
8 5900 pL **9** 22 800 years
10 **a** 8 **b** $\frac{8}{10}$ **c** $\frac{8}{100}$ **d** 8000
11 **a** 13 800 **b** 770 **c** 24 **d** 90 000
12 **a** 13.1 **b** 13.07 **c** 13.065
13 **a** 4.20 **b** 21.0
14 **a** 3 **b** 4 **c** 2
15 **a** 20 **b** 18 **c** 17.6
d 17.63 **e** 17.631
16 **a** 425
b <435 as any number ⩾435 would be rounded to 440.
c 425 ⩽ number < 435
17 **a** 3 **b** 1 **c** 3
d 2 **e** 2, 3, 4 or 5

18 **a** 0.01 m **b** 0.005 m
c 4.375 m and 4.385 m
d 4.375 m ⩽ true length < 4.385 m

19 6 cm is accurate to the nearest whole number (nearest cm). 6.0 cm is accurate to 1 d.p. (nearest mm), so 6.0 cm is more accurate.

20 **a** 24 cm **b** 0.05 cm
c Length: 6.75 cm, 6.85 cm; breadth: 5.15 cm, 5.25 cm
d Lower = 23.8 cm, upper = 24.2 cm
e Maximum error = 0.2 cm

21 BMI = 24

CUMULATIVE REVIEW: 2–4

1 **a** **i** 6^6 **ii** 5^{11}
b **i** base = 7, index = 8
ii base = 4, index = −11
c **i** $4 \times 4 \times 4 \times 4 \times 4 \times 4$
ii $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
d **i** 625 **ii** 2048
e **i** 5^{20} **ii** 3^{84} **iii** 5^3
iv 4^{24} **v** 3^{48}
f **i** False **ii** False
g **i** $\frac{1}{7^5}$ **ii** $\frac{1}{24}$
h **i** $\frac{1}{81}$ **ii** $\frac{1}{81}$ **iii** $\frac{1}{4}$ **iv** 1

2 **a** **i** $\frac{15h}{25}$ **ii** $\frac{20xy}{15}$
b **i** $\frac{w}{4}$ **ii** $\frac{9m}{7}$
c **i** $\frac{1}{m}$ **ii** $\frac{1}{p^3}$ **iii** $\frac{1}{r^2}$ **iv** t^3
d **i** p^{12} **ii** y^{12} **iii** t^{35}
iv c^{10} **v** $27v^{15}$ **vi** $21x^{10}y^{15}$
e **i** 1 **ii** 7 **iii** 1 **iv** 12
f **i** $\frac{1}{c^5}$ **ii** $\frac{4}{c^5}$ **iii** $\frac{1}{(4c)^5}$
g **i** d^{-7} **ii** n^3 **iii** k^{10}
iv $15a^{-6}$ **v** $\frac{2m^5}{3}$ **vi** $\frac{3n}{2}$
h **i** True **ii** False **iii** False
iv True **v** False **vi** True

3 **a** **i** $35w + 14x$ **ii** $-2k^7 + 3k^5$ **iii** $-4s + 8$
b **i** $11t + 2$ **ii** $2x - 16y$

4 **a** **i**

Girls	Stem	Boys
	1	
7 4 4	2	
9 5 2 0	3	2 3 9
7 4 3 3 1	4	1 6 7 7 9
6 3 1	5	4 5 8
	6	1 3 4 6

ii Girls: symmetrical, boys: bimodal
iii Girls: mean 39.3, median 41, range 32
Boys: mean 51.1, median 49, range 34
Mean and median are greater for boys than for girls, indicating the boys performed better on this test.
b **i** Symmetrical **ii** Positively skewed
iii Bimodal **iv** Negatively skewed

5 **a** **i** 3.25×10^8 **ii** 7.4×10^{-5}
b **i** 62 100 000 **ii** 0.000 03
c $4 \times 10^6 = 4\ 000\ 000$ but
$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$

d **i** 1.95×10^{25} **ii** 4.86×10^{22}
iii 4.096×10^{75} **iv** 7×10^{5}
e 7.5×10^{-16}, 2.4×10^{-11}, 6.7×10^{-10}, 7.6×10^{-10}
f **i** 515 **ii** 525
iii $515 \leqslant \text{number} < 525$
g 7 cm is accurate to the nearest cm. 7.0 cm is accurate to the nearest 1 d.p. (nearest mm), so 7.0 cm is more accurate.

CHAPTER 5 FINANCIAL MATHEMATICS

Exercise 5A

1 **a** Fortnightly = \$842.66 × 2 = \$1685.32
b Yearly = \$842.66 × 52 = \$43 818.32
c Monthly = \$43 818.32 × 12 = \$3651.53
d Quarterly = \$842.66 × 13 = \$10 954.58
2 **a** \$1457.12 **b** \$37 885.12
c \$3157.09 **d** \$9471.28
3 **a** **i** \$1828 **ii** \$47 528
iii \$3960.67 **iv** \$11 882
b **i** \$1580 **ii** \$41 080
iii \$3423.33 **iv** \$10 270
c **i** \$2051.20 **ii** \$53 331.20
iii \$4444.27 **iv** \$13 332.80
d **i** \$1969.20 **ii** \$51 199.20
iii \$4266.60 **iv** \$12 799.80
e **i** \$2757.88 **ii** \$71 704.88
iii \$5975.41 **iv** \$17 926.22
4 **a** Weekly $= \frac{\$74\ 250}{52} = \$1427.88$
b Fortnightly $= \frac{\$74\ 250}{26} = \$2855.77$
c Monthly $= \frac{\$74\ 250}{12} = \$6187.50$
d Quarterly $= \frac{\$74\ 250}{4} = \$18\ 562.50$
5 **a** \$1246.15 **b** \$2492.31
c \$5400 **d** \$16 200
6 **a** **i** \$1007.69 **ii** \$2015.38
iii \$4366.67 **iv** \$13 100
b **i** \$703.85 **ii** \$1407.69
iii \$3050 **iv** \$9150
c **i** \$1834.04 **ii** \$3668.08
iii \$7947.50 **iv** \$23 842.50
d **i** \$1466.92 **ii** \$2933.85
iii \$6356.67 **iv** \$19 070
e **i** \$1594.23 **ii** \$3188.46
iii \$6908.33 **iv** \$20 725
7 **a** **i** \$1538.46 **ii** \$3076.92
iii \$6666.67
b **i** \$2115.38 **ii** \$4230.77
iii \$9166.67
c **i** \$769.23 **ii** \$1538.46
iii \$3333.33
8 Yearly salary = \$7320 × 12 = \$87 840
Weekly salary $= \frac{\$87\ 840}{52} = \$1689.23$
9 \$1592.31
10 **a** \$969.23 **b** \$666.92 **c** \$1300.38
d \$1615.38 **e** \$830.54
11 Lisa (\$70 200 p.a.)
12 Weekly wage = \$23.65 × 35 = \$827.75
13 \$759.50
14 **a** \$661.50 **b** \$780.50 **c** \$926.80
d \$1878.45 **e** \$2957.50
15 Hourly rate $= \frac{\$520.30}{22} = \$23.65$
16 \$28.60/h
17 **a** \$28.40/h **b** \$35.10/h **c** \$24.20/h
d \$19.40/h **e** \$17.90/h
18 **a** Weekly wages = \$19.65 × 31 = \$609.15
b Fortnightly wages = \$609.15 × 2 = \$1218.30
c Yearly wages = \$609.15 × 52 = \$31 675.80
d Monthly wages $= \frac{\$31\ 675.80}{12} = \$2639.65$ to the nearest cent
19 **a** \$619.20 **b** \$1238.40
c \$32 198.40 **d** \$2683.20
20 **a** **i** \$1634 **ii** \$3268
iii \$84 968 **iv** \$7080.67
b **i** \$1048.80 **ii** \$2097.60
iii \$54 537.60 **iv** \$4544.80
c **i** \$2010.20 **ii** 4020.40
iii \$104 530.40 **iv** \$8710.87
d **i** \$701.10 **ii** \$1402.20
iii \$36 457.20 **iv** \$3038.10
e **i** \$2861.40 **ii** \$5722.80
iii \$148 792.80 **iv** \$12 399.40
f **i** \$894.90 **ii** \$1789.80
iii \$46 534.80 **iv** \$3877.90

Exercise 5B

1 Normal pay = \$22.36 × 28 = \$626.08
Overtime = (\$22.36 × 1.5) × 3 = \$100.62
Total wages = \$626.08 + \$100.62 = \$726.70
2 \$959.40 **3** \$1186.25
4 \$696.60 **5** \$1090.05
6 \$790.50
7 Normal pay = \$29.60 × 30 = \$888
Overtime = (\$29.60 × 1.5) × 7 + (\$29.60 × 2) × 5
= \$606.80
Total wages = \$888 + \$606.80 = \$1494.80
8 \$1513.20 **9** \$1487.20
10 \$1735.20
11 **a** \$209.95 **b** \$247 **c** \$296.40
12 **a** \$236.70 **b** \$289.30 **c** \$341.90
13 **a** Weekend rate = \$31.68 × 1.5 = \$47.52
b Weekday wage = \$31.68 × 17 = \$538.56
Weekend wage = \$47.52 × 5 = \$237.60
Total wages = \$538.56 + \$237.60 = \$776.16
14 **a** \$40.92 **b** \$695.64
15 **a** \$29.10 **b** \$562.60
16 **a** \$62.16 **b** \$1087.80
17 **a** \$47.14 **b** \$1272.78
18 **a** \$61.80 **b** \$82.40 **c** \$1565.60
19 Bonus $= \frac{\$74\ 760}{12} = \$6230$
20 \$4883.33 **21** \$2138.50
22 \$1455 **23** \$3600
24 **a** 4 weeks pay = \$884 × 4 = \$3536
Holiday loading = 17.5% of \$3536
= 0.175 × \$3536 = \$618.80
b Total pay = normal pay + holiday loading
= \$3536 + \$618.80 = \$4154.80
25 **a** \$532 **b** \$3572
26 **a** \$438.55 **b** \$2944.55

27 **a** $861 **b** $5781
28 **a** $591.50 **b** $3971.50

Exercise 5C

1 Joseph's wages = $24.04 × 21 = $504.84
2 $591.84 **3** $360.60
4 **a** $701.40 **b** $140
5 $1019.34
6 Wages = $25.40 × 11 + $30.48 × 3 + $36.58 × 5
= $553.74
7 $498.78 **8** $379.44
9 $402.36 **10** $466 99
11 $452.88
12 **a** $242.32 **b** $296.24
13 $49.31

Exercise 5D

1 Income = 370 × $1.27 = $469.90
2 $216 **3** $199.20
4 $721.00 **5** $2340
6 $433.40

Exercise 5E

1 Commission = 4% of $6800= 0.04 × $6800 = $272
2 $800 **3** $1184
4 $1280
5 **a** $9900 **b** $4800 **c** $14 700
6 **a** $1620 **b** $972 **c** $414
7 **a** $375 **b** $675 **c** $6075
8 Excess = $5800 − $2000 = $3800
Commission = 0.185 × $3800 = $703
9 **a** Nil **b** $350 **c** $2450
10 Commission
= 4.5% of $8000 + 6.2% of ($12 000 − $8000)
= 0.045 × $8000 + 0.062 × $4000 = $608
11 **a** $260 **b** $399.80 **c** $400
d $575 **e** $1030
12 **a** $460 **b** $500 **c** $800
d $1100 **e** $1820
13 **a** $8700 **b** $10 000 **c** $10 600
d $11 825 **e** $13 250
14 Commission = 0.04 × $6240 = $249.60
Weekly earnings = retainer + commission
= $270 + $249.60 = $519.60
15 $848.80
16 **a** $433 **b** $340 **c** $520
d $374 **e** $445.20

17

Employee	Commission	Weekly earnings
R. Roberts	$504	$704
H. Low	$1290	$1440
J Thum	$1672	$1772
K Trau	$2322	$2322
G Flood	$2918.40	$2918.40

18 **a** $550 **b** $578 **c** $634
d $720.10 **e** $1597.20
19 **a** **i** A $480, B nil **ii** A $570, B $450
iii A $600, B $600 **iv** A $630, B $750
v A $780, B $1500
b If sales are more than $4000 per week, method B is better, otherwise method A is better.

Exercise 5F

1 Income = $45 320 + ($665 ÷ 2) + $800
= $46 452.50 = $46 452
Deductions = $340 + $60 + $45 = $445
Taxable income = $46 452 − $445 = $46 007
2 $39 731 **3** $29 156
4 $55 805

Exercise 5G

1 $47 953 is in the third row of the tax table.
Excess = $47 953 − $37 000 = $10 953
Tax = $3572 + 0.325 × $10 953 = $7131.73
2 **a** $19 975.68 **b** $7443.08
c $0 **d** $2432
3 **a** $34 596 **b** $3115.24
4 **a** $6984.50 **b** Refund of $2099.50
5 **a** $19 900 **b** $323
6 **a** Total income = $86 452 + $5203 = $91 655
b Taxable income = $91 655 − $2612 = $89 043
c Tax payable = $17 547 + $0.37 × ($89 043 − $80 000)
= $20 892.91
d Tax refund or balance payable
= $20 892.91 − $20 867.40 = $25.51 payable
7 **a** $32 260 **b** $31 170
c $2464.30 **d** Refund of $751.60
8 **a** $40 215 **b** $35 312
c $3251.28 **d** Refund of $1872.62
9 **a** $88 985 **b** $75 101
c $15 954.83 **d** Refund of $33.17

Exercise 5H

1 $I = PRT$ = $8450 × 0.047 × 3 = $1191.45
2 $700 **3** $1440

4

Principal	Simple interest
$5 800	$1624
$15 000	$1575
$24 000	$5400
$6500	$1950
$18 000	$1008
$9 300	$1264.80
$6 000	$540

5 Interest = $5500 × 0.045 × 4 = $990
Amount after 4 years = $5500 + $990 = $6490
6 $10 755 **7** $24 000
8 $17 496
9 $\text{Interest} = \$4500 \times 0.03 \times \frac{17}{12} = \$191.25$
10 **a** $675 **b** $700
c $937.50 **d** $1026
e $7020 **f** $412.80
11 Interest for 1 year = 0.04 × $2300 = $92
$\text{Number of years invested} = \frac{\$276}{\$92} = 3 \text{ years}$
12 6 years
13 5 years
14 Interest for 1 year = $504 ÷ 4 = $126
$\text{Annual interest rate} = \frac{\$126}{\$2300} \times 100\% = 5.48\%$
15 7% p.a. **16** 4.6% p.a.

Exercise 5I

1 **a** 75c **b** $5.30 **c** $2.80 **d** $7.30
e $3.95 **f** $8.50 **g** $16.25 **h** $22
i $54.85 **j** $39.15 **k** $17.35 **l** $70

2 **a** $4.15 **b** $5.00 **c** $4.80 **d** $7.15
e 15c **f** $1 **g** $11.45 **h** $1.80

3 Cash price = 89% of $799.90
= 0.89 × $799.90 = $711.91
The cash price is $711.90 to nearest 5 cents.

4 $421.50

5 **a** $138.20 **b** $435.20 **c** $210.55
d $169.75 **e** $64.85

Exercise 5J

1 **a** 10 Dec. 2012 to 11 Jan. 2013 **b** 20.240%
c 21.240% **d** 0.055 45% **e** $3000
f $1104.65 **g** 6 Feb. 2013 **h** $37

2 **a** $1895.35; yes
b Jan paid off the whole of the opening balance during the month, leaving no balance to be charged interest.
c The new purchases are still within the interest-free period.

3 **a** 1.95% **b** $1858.35

Exercise 5K

1 Total amount paid
= $120 + $130 + $75 + $110 + $190 + $75 = $700
Balance = $735 − $700 = $35

2 $34 **3** $40

4 **a** Balance to be paid = $184 − $40 = $144
b Amount of each monthly instalment = $\frac{\$144}{3}$ = $48

5 **a** $405 **b** $101.25

6 **a** $77.80 **b** $700.20 **c** $58.35

7 **a** $68.40 **b** $387.60 **c** $64.60

Exercise 5L

1 **a** Total cost of buying the snow skiing set on terms
= $75 + 24 × $34 = $891
b Amount saved by paying cash = $891 − $742
= $149

2 **a** $871 **b** $172

3 **a** $2669.80 **b** $371.80

4 **a** $1943 **b** $345

5 **a** $1171.33 **b** $292.33

6 **a** Total cost = $150 + 36 × $37.59 = $1503.24
b Total interest = $1503.24 − $1230 = $273.24
c Annual interest = $\frac{\text{total interest}}{\text{number of years}}$
= $\frac{\$273.24}{3}$ = $91.08
d Amount borrowed = balance owing after paying the deposit = $1230 − $150 = $1080
e Annual interest rate = $\frac{\text{annual interest}}{\text{amount borrowed}} \times 100\%$
= $\frac{\$91.08}{\$1080} \times 100\%$
= 8.43%

7 **a** $1992.56 **b** $493.56 **c** $246.78
d $1299 **e** 19%

8 **a** $2494 **b** $604 **c** $201.33
d $1790 **e** 11.25%

9 **a** 9503.84 **b** 3513.84 **c** 878.46
d 5490 **e** 16%

10 **a** Balance owing = $998 − $99 = $899
b Interest = $899 × 0.09 × 2
= $161.82
c Balance owing + interest = $899 + $161.82
= $1060.82
Monthly repayment = $\frac{\$1060.82}{24}$
= $44.20 to nearest cent

11 **a** $7600 **b** $1824 **c** $392.67

12 **a** $9900 **b** $2673 **c** $349.25

13 **a** $499 **b** $4491
c $1347.30 **d** $243.26

14 **a** $2985 **b** $16 915
c $8051.54 **d** $520.14

Exercise 5M

1 **a** $3174.72 **b** $132.28 **c** $1175.72

2 **a** $14 005.80 **b** $389.05 **c** $5406.80

3 **a** $2215.76 **b** $184.65 **c** $617.76

Exercise 5N

1 Monthly repayments for $7000
= $46.6078 × 7
= $326.25 to the nearest cent

2 $298.93

3 **a** $3233.77 **b** $1456.74 **c** $438.10
d $401.04 **e** $361.48 **f** $858.48

4 **a** Monthly repayment = $38.5127 × 4.7
= $181.01 to the nearest cent
b Total cost of loan = $181.01 × 30 + $220
= $5650.30

5 **a** $229.17 **b** $8430.12

6 **a** $419.68 **b** $20 394.77

7 **a** $6278.52 **b** $15 940.16 **c** $9720.80
d $12 613.80 **e** $21 597.40

8 **a** $1278.52 **b** $3940.16 **c** $1220.80
d $3213.80 **e** $3597.40

Language in mathematics

6 **a** fortnightly **b** retainer
c taxable **d** discount

7 piecework, service, investment

Check your skills

1 C **2** B **3** C **4** A **5** D
6 A **7** C **8** B **9** D **10** C
11 C **12** A **13** D **14** B **15** D
16 B **17** A **18** C **19** D

Review set 5A

1 **a** $927.80 **b** $24 122.80
c $2010.23 **d** $6030.70

2 **a** $1076.92 **b** $2153.85
c $4666.67 **d** $14 000

3 $878.90 **4** $2632

5 $338.50 **6** $760.32

7 $476 **8** $639.84

9 **a** $54 002 **b** $9097.65 **c** $946.79 refund

10 **a** $1296 **b** $540

11 $150.35 **13** $69

14 **a** $7999 **b** $3839.52 **c** $246.64

15 **a** $5908.55 **b** $246.19 **c** $1909.55

16 $230.24

Review set 5B

1 **a** $737.08 **b** $19 164.08
c $1597.01 **d** $4791.02
2 **a** $876.92 **b** $1753.85
c $3800 **d** $11 400
3 $1046.64 **4** $3196
5 $449.24 **6** $355.68
7 $596 **8** $525.02
9 $5520.38
10 **a** $3240 **b** $1350
11 $76.55 **12** $25.36
14 **a** $1455 **b** $436.50 **c** $78.81
16 $257.93

Review set 5C

1 **a** $692.31 **b** $1384.62
c $3000 **d** $9000
2 **a** $730 **b** $18 980
c $1581.67 **d** $4745
3 $854.40 **4** $1398.80
5 $596.40 **6** $544.50
7 **a** $10 350 **b** $14 850 **c** $17 200
8 $611.93 **9** $1658.18
10 **a** $3900 **b** $1365
12 No, because he always leaves most of the balance to attract interest. He would do better to pay as much as possible off the balance each month.
13 $98.50
14 **a** $10 000 **b** $2400 **c** $344.44
15 **a** $5769 **b** $158.17 **c** $2119
16 $643.86

CHAPTER 6 AREA AND SURFACE AREA

Exercise 6A

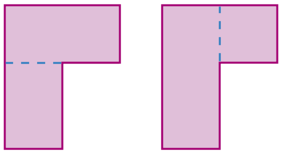
1 **a** 400 mm^2 **b** 310 000 cm^2
c 32 000 000 m^2 **d** 4 m^2
e 73 000 m^2 **f** 4.2 ha
g 1500 mm^2 **h** 3.2 m^2
i 32.8 cm^2 **j** 0.235 km^2
k 365 000 m^2 **l** 0.078 ha
2 **a** 7 m^2 **b** 5 300 000 mm^2
c 0.036 ha **d** 100 ha
e 230 ha **f** 420 000 cm^2
g 2000 mm^2 **h** 0.063 km^2
3 **a** 32 cm^2 **b** 180 m^2
c 80 cm^2 **d** 144 m^2
4 **a** 14 m^2 **b** 4 cm^2
c 880 m^2 **d** 36 cm^2
5 **a** 24 cm^2 **b** 36 cm^2
c 120 cm^2 **d** 7.8 m^2
6 **a** 201.06 cm^2 **b** 153.94 m^2
c 56.75 cm^2 **d** 30.19 cm^2
e 183.85 cm^2 **f** 1.25 km^2

Exercise 6B

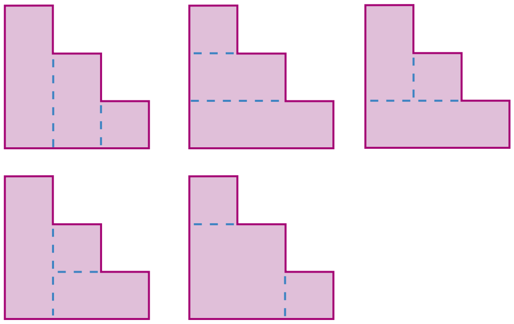
1 The lengths of the diagonals are 15.2 cm and 7.5 cm.
$A = \frac{1}{2} \times 15.2 \times 7.5 = 57\text{ cm}^2$
2 **a** 30 mm^2 **b** 29.925 m^2 **c** 15.48 cm^2
3 The lengths of the diagonals are 4.3 m and 1.8 m.
$A = \frac{1}{2} \times 4.3 \times 1.8 = 3.87\text{ m}^2$
4 **a** 67.5 cm^2 **b** 51.04 m^2 **c** 19.35 cm^2
5 The lengths of the parallel sides are 3 m and 8 m.
The height is 4 m.
$A = \frac{1}{2} \times 4 \times (3 + 8) = 22\text{ m}^2$
6 **a** $h = 4$ m, $A = 32\text{ m}^2$ **b** $h = 6$ cm, $A = 30\text{ cm}^2$
c $h = 7$ cm, $A = 98\text{ cm}^2$
d $h = 28$ mm, $A = 1190\text{ mm}^2$
7 *For triangle 1:* $b = 14$ cm, $h = 5$ cm
$A = \frac{1}{2} \times 14 \times 5 = 35\text{ cm}^2$
For triangle 2: $b = 14$ cm, $h = 4$ cm
$A = \frac{1}{2} \times 14 \times 4 = 28\text{ cm}^2$
Area of quadrilateral $= 35 + 28 = 63\text{ cm}^2$
8 **a** 20 cm^2 **b** 58.5 cm^2 **c** 266 m^2
9 **a** 51 m^2 **b** 48 cm^2 **c** 60 mm^2
d 82.5 km^2 **e** 108.375 m^2 **f** 23.165 m^2

Exercise 6C

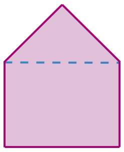
1 **a**



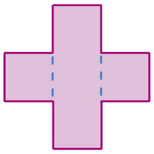
b



c



d **i**



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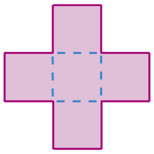
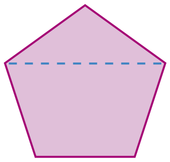
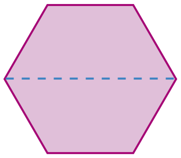


e



f **i**



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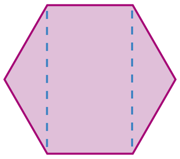

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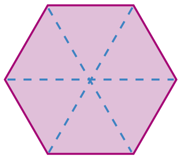
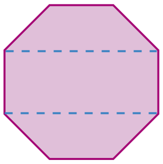
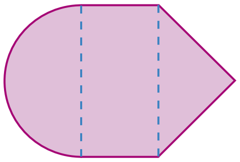


g



h



2 *For rectangle 1:*
$A_1 = 25 \times 8 = 200\text{ m}^2$
For rectangle 2:
$x = 26 - 8 = 18\text{ m}$
$A_2 = 18 \times 14 = 252\text{ m}^2$
Area of shape $= 200 + 252 = 452\text{ m}^2$
Or for rectangle 1:
$A_1 = 26 \times 14 = 364\text{ m}^2$
For rectangle 2:
$y = 25 - 14 = 11\text{ m}$
$A_2 = 11 \times 8 = 88\text{ m}^2$
Area of shape $= 364 + 88 = 452\text{ m}^2$

3 **a** 168 cm^2 **b** 165 cm^2 **c** 84 m^2
d 118 cm^2 **e** 65 mm^2 **f** 133 m^2
g 210 cm^2 **h** 126 cm^2 **i** 245.5 m^2
j 139.3 km^2 **k** 164.5 m^2 **l** 304.5 m^2
m 140 cm^2 **n** 73.1 mm^2 **o** 118 cm^2

4 *For the triangle:*
$b = 20 + 6 = 26\text{ m}$
$h = 17 + 3 = 20\text{ m}$
Area $= \frac{1}{2} \times 26 \times 20 = 260\text{ m}^2$
For the rectangle:
Area $= 6 \times 3 = 18\text{ m}^2$
Area of shape $= 260 - 18 = 242\text{ m}^2$

5 **a** 120 cm^2 **b** 56 m^2 **c** 86.5 m^2
d 70 cm^2 **e** 225 m^2 **f** 263.5 cm^2
g 364.5 cm^2 **h** 255.9 m^2 **i** 30.4 m^2

6 $\pi \times 10^2 - \pi \times 6^2 \approx 201.1\text{ cm}^2$

7 **a** 88.0 cm^2 **b** 100.5 m^2 **c** 100.5 m^2
d 50 m^2 **e** 16.9 cm^2 **f** 33 m^2

Exercise 6D

1 \$3256.97
2 **a** 400 m^2 **b** \$25 120
3 **a** 36.566 m^2 **b** \$4387.92
4 \$2784.80
5 **a** $263\,500\text{ m}^2$ **b** 26.35 ha **c** 18.445 ha
d 7.905 ha **e** \$1 897 200
6 38 m^2 **7** 6 m^2
8 1.914 t

Exercise 6E

1 $A = \frac{70}{360} \times \pi \times 5^2 \approx 15.3\text{ m}^2$

2 **a** 8.4 cm^2 **b** 26.2 cm^2 **c** 25.1 cm^2
d 215.1 m^2 **e** 69.8 m^2 **f** $23\,082.0\text{ m}^2$
g 38.5 cm^2 **h** 47.7 m^2

3 $A = \frac{65}{360} \times \pi \times 34^2 + \frac{1}{2} \times 30 \times 16 \approx 896\text{ cm}^2$

4 **a** 19.9 m^2 **b** 65.7 m^2 **c** 2165.9 cm^2

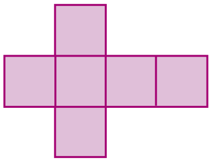
5 169.6 cm^2

Exercise 6F

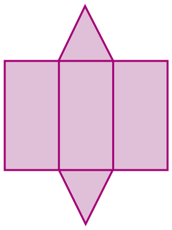
1 **a** **i** D **ii** Triangular prism
b **i** A **ii** Rectangular pyramid
c **i** E **ii** Cube
d **i** C **ii** Pentagonal prism
e **i** B **ii** Square prism

2 **a** Right **b** Oblique **c** Oblique
d Oblique **e** Right **f** Right

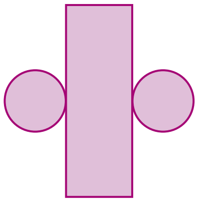
3 **a** Cube



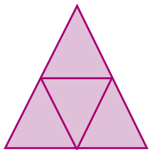
b Triangular prism



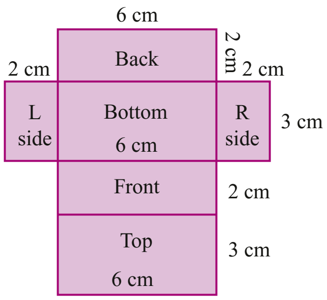
c Cylinder



d Triangular pyramid

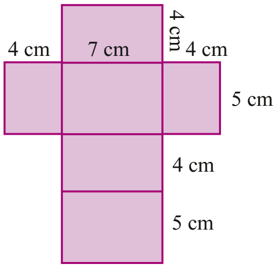


4 **a**



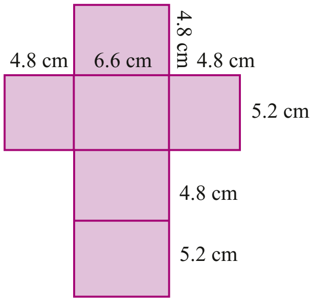
b $SA = (\text{bottom} + \text{top}) + (\text{front} + \text{back})$
$+ (\text{left side} + \text{right side})$
$SA = (6 \times 3) \times 2 + (6 \times 2) \times 2 + (3 \times 2) \times 2$
$= 72\text{ cm}^2$

5 **a** **i**



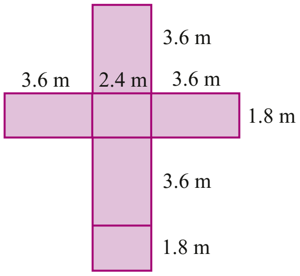
ii 166 cm^2

b **i**



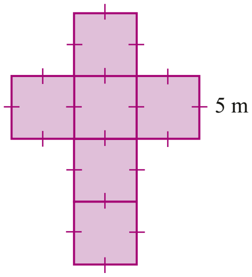
ii 181.92 cm^2

c **i**



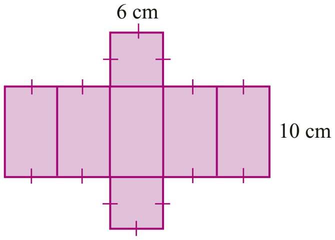
ii 38.88 m^2

d **i**



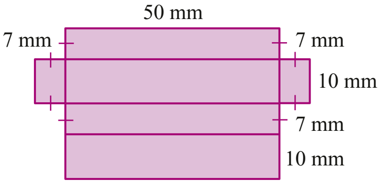
ii 150 m^2

e **i**



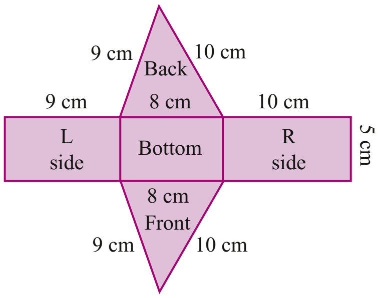
ii 312 cm^2

f **i**

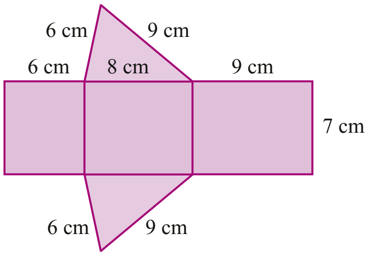


ii 1840 mm^2

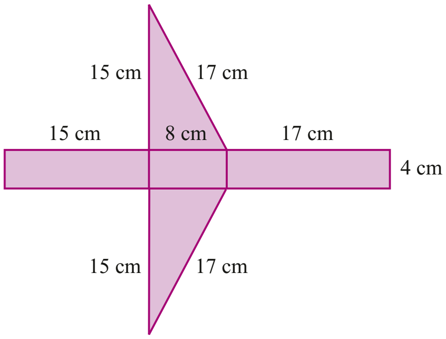
6



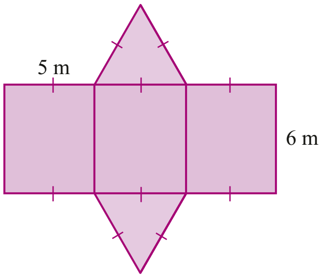
7 **a**



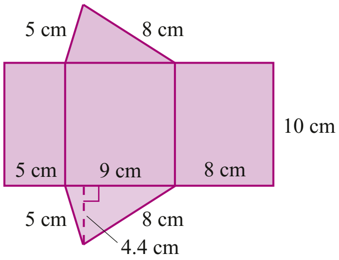
b



c



8 **a**



b SA = area of 2 triangles + area of 3 rectangles

$=(\frac{1}{2} \times 9 \times 4.4) \times 2 + 10 \times 5 + 10 \times 9 + 10 \times 8$

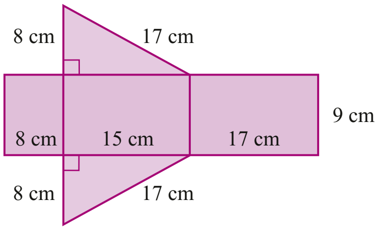
$= 259.6$ cm^2

9 **a** 294 cm^2 **b** 141 cm^2 **c** 319.8 cm^2

10 **a** $x^2 = 8^2 + 15^2 = 289$

$\therefore x = \sqrt{289} = 17$ cm

b



c $SA = (\frac{1}{2} \times 15 \times 8) \times 2 + 9 \times 8 + 9 \times 15 + 9 \times 17$

$= 480$ cm^2

11 **a** **i** $x = 5$ cm **ii** 96 cm^2

b **i** $x = 13$ m **ii** 300 m^2

c **i** $x \approx 18.0$ mm **ii** 279 mm^2

Exercise 6G

1 33 960 cm^2 (or 3.396 m^2)

2 7 000 000 mm^2 (or 7 m^2)

3 6000 cm^2 (or 0.6 m^3) **4** \$133.76

5 2320 cm^2 (or 0.232 m^2)

6 **a** 1422 cm^2 **b** 1602 cm^2

7 6 h 42 min

Language in mathematics

2 **a** circle **b** quadrilateral
c composite **d** rhombus
e kite **f** sector
g trapezium **h** triangle
i oblique

3 **a** A semicircle is a half of a circle.
b A quadrant is a quarter of a circle.
c Composite areas may be found in more than one way.

4 The area of a rhombus or a kite is half the product of the diagonals.

Check your skills

1 C **2** A **3** C **4** D **5** A
6 A **7** A **8** D **9** A **10** B
11 C **12** C **13** D **14** C **15** B

Review set 6A

1 **a** 8500 mm^2 **b** 1.5 ha **c** 3 500 000 m^2
2 **a** $A = \frac{1}{2}bh$ **b** $A = \pi r^2$ **c** $A = l^2$
3 **a** 40 m^2 **b** 147 cm^2 **c** 759 cm^2
4 **a** 23.4 m^2 **b** 52 cm^2 **c** 126.4 cm^2
5 9161 **6** Oblique triangular prism
7 **a** 214 cm^2 **b** 314.6 cm^2
8 75.48 m^2

Review set 6B

1 **a** 42 800 m^2 **b** 300 mm^2 **c** 0.43 m^2
2 **a** $A = \frac{1}{2}h(a + b)$ **b** $A = \frac{1}{2}xy$ **c** $A = bh$
3 **a** 13.65 cm^2 **b** 326.2 cm^2 **c** 19.6 m^2
4 **a** 46 m^2 **b** 102.7 cm^2 **c** 42.4 cm^2
5 \$886.52 **6** Oblique square prism
7 **a** 138 cm^2 **b** 180 cm^2
8 880 cm^2

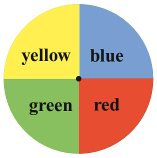
Review set 6C

1 **a** 210 ha **b** 0.065 km^2
c 800 000 mm^2
2 **a** $A = lb$ **b** $A = \frac{1}{2}xy$
c $A = \frac{\text{sector angle}}{360} \times \text{area of circle}$
3 **a** 129 cm^2 **b** 132 cm^2 **c** 280 cm^2
4 **a** 61.7 m^2 **b** 26.8 m^2 **c** 75.6 m^2
5 **a** 1664 cm^2 **b** 25 936 cm^2 (or 2.5936 m^2)
6 Right pentagonal prism
7 **a** 102 cm^2 **b** 154.3 cm^2
8 367.2 m^2

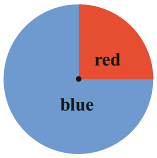
CHAPTER 7 PROBABILITY

Exercise 7A

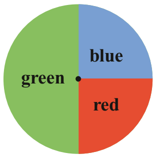
1 **a** **i** 50% **ii** 55% **iii** 43%
b Survey iii
2 **a** Group 1: 40%, Group 2: 36.7%, Group 3: 34%
b Group 3
3 **a** Point up 33.8%, point down 66.2%
b **i** ≈34% **ii** ≈66%
4 **a** 31% **b** 53% **c** 16% **d** 84%
5 **a** ≈46% **b** ≈54%
6 **a** ≈23% **b** ≈49% **c** ≈28% **d** ≈51%
7 ≈20%
8 **a** 19% **b** 15% **c** 32% **d** 47%
9 **a**



b



c



Exercise 7B

1 **a** Number of students in Year 9 = 27 + 6 + 19 + 88
= 140

b Number of students chosen for the athletics team
= number of students inside the athletics circle
= 27 +6 = 33
P(student was chosen for the athletics team) $= \frac{33}{140}$

c Number of students not chosen for athletics team
= number of students outside the athletics circle
= 19 + 88 = 107
P(student was not chosen for the athletics team) $= \frac{107}{140}$

d Number of students chosen for the athletics team but not the swimming team = number of students inside the athletics circle but not inside the swimming circle
= 27
P(student was chosen for the athletics team but not the swimming team) $= \frac{27}{140}$

e Number of students chosen for the swimming team
= number of students inside the swimming circle
= 6 + 19 = 25
P(student was chosen for the swimming team)
$= \frac{25}{140} = \frac{5}{28}$

f Number of students not chosen for the swimming team
= number of students outside the swimming circle
= 27 + 88 = 115
P(student was not chosen for the swimming team)
$= \frac{115}{140} = \frac{23}{28}$

g Number of students chosen for the swimming team but not the athletics team = number of students inside the swimming circle but not inside the athletics circle
= 19
P(student was chosen for the swimming team but not the athletics team) $= \frac{19}{140}$

h Number of students chosen for neither team
= number of students outside both circles = 88
P(student was chosen for neither team) $= \frac{88}{140} = \frac{22}{35}$

i Number of students chosen for both teams = number of students in the intersection of both circles = 6
P(student was chosen for the both teams) $= \frac{6}{140} = \frac{3}{70}$

j Number of students who were chosen for either the athletics team or the swimming team or both
= number of students inside both circles
= 27 + 6 + 19 = 52
P(student was chosen for either team or both)
$= \frac{52}{140} = \frac{13}{35}$

k Number of students who were chosen for either the athletics team or the swimming team but not both
= number of students inside both circles but excluding the intersection = 27 + 19 = 46
P(student was chosen for either team but not both)
$= \frac{46}{140} = \frac{23}{70}$

2 **a** $\frac{12}{29}$ **b** $\frac{17}{29}$ **c** $\frac{8}{29}$ **d** $\frac{11}{29}$ **e** $\frac{18}{29}$
f $\frac{7}{29}$ **g** $\frac{10}{29}$ **h** $\frac{4}{29}$ **i** $\frac{19}{29}$ **j** $\frac{15}{29}$

3 **a** $\frac{7}{30}$ **b** $\frac{23}{30}$ **c** $\frac{1}{5}$ **d** $\frac{4}{5}$ **e** $\frac{17}{30}$
f $\frac{13}{30}$ **g** 0

4 **a** $\frac{1}{10}$ **b** $\frac{17}{50}$ **c** $\frac{21}{50}$ **d** $\frac{7}{50}$ **e** $\frac{29}{50}$
f $\frac{11}{25}$

5 **a** $\frac{39}{197}$ **b** $\frac{26}{197}$ **c** $\frac{31}{197}$ **d** 0 **e** $\frac{143}{197}$
f $\frac{14}{197}$ **g** $\frac{183}{197}$ **h** $\frac{119}{197}$ **i** $\frac{54}{197}$

6 **a** $\frac{31}{60}$ **b** $\frac{1}{6}$ **c** $\frac{7}{60}$ **d** $\frac{3}{20}$ **e** $\frac{1}{12}$
f $\frac{47}{60}$ **g** $\frac{29}{60}$

7 **a** $\frac{17}{30}$ **b** $\frac{13}{30}$ **c** $\frac{11}{30}$ **d** $\frac{19}{30}$ **e** $\frac{1}{3}$
f $\frac{1}{10}$ **g** $\frac{3}{10}$ **h** $\frac{4}{15}$ **i** $\frac{9}{10}$ **j** $\frac{3}{5}$
k $\frac{7}{10}$ **l** $\frac{3}{5}$ **m** $\frac{3}{10}$ **n** $\frac{1}{10}$ **o** $\frac{4}{15}$
p $\frac{1}{3}$

8 **a** $\frac{58}{115}$ **b** $\frac{57}{115}$ **c** $\frac{9}{46}$ **d** $\frac{37}{46}$ **e** $\frac{29}{230}$
f $\frac{49}{115}$ **g** $\frac{66}{115}$ **h** $\frac{103}{230}$ **i** $\frac{49}{115}$ **j** $\frac{29}{230}$
k $\frac{49}{115}$ **l** $\frac{8}{115}$

9 **a** $\frac{1}{10}$ **b** $\frac{31}{100}$ **c** $\frac{29}{50}$ **d** $\frac{41}{100}$ **e** $\frac{17}{100}$
f $\frac{21}{50}$ **g** $\frac{69}{100}$ **h** $\frac{59}{100}$ **i** $\frac{31}{100}$

10 **a** $\frac{15}{28}$ **b** $\frac{5}{14}$ **c** $\frac{3}{4}$ **d** $\frac{1}{4}$ **e** $\frac{5}{28}$
f $\frac{3}{14}$

Language in mathematics

1 **a** 80% **b** 45% **c** 12% **d** 1%
2 The 0% in the table is the relative frequency of 55 year-old males who lived to 100, based on the records used to construct the table.
3 **a** 870 **b** 690 **c** 490 **d** 290

Check your skills

1 B **2** D **3** A

Review set 7A

1 **a** $\frac{60}{100} = \frac{3}{5}$ **b** $\frac{3}{100}$
2 **a** $\frac{1}{10}$ **b** $\frac{7}{15}$ **c** $\frac{17}{30}$ **d** $\frac{13}{30}$ **e** $\frac{4}{15}$
3 **a** $\frac{24}{29}$ **b** $\frac{17}{29}$ **c** $\frac{7}{29}$ **d** $\frac{5}{29}$ **e** $\frac{6}{29}$

Review set 7B

1 **a** Group 1: $\frac{6}{10} = \frac{3}{5}$, Group 2: $\frac{17}{30}$, Group 3: $\frac{55}{100} = \frac{11}{20}$
b Group 3
2 **a** $\frac{3}{25}$ **b** $\frac{14}{25}$ **c** $\frac{17}{25}$ **d** $\frac{8}{25}$ **e** $\frac{1}{5}$
3 **a** 0.27 **b** 0.17 **c** 0.42 **d** 0.69
e 0.59 **f** 0.31 **g** 0.1

Review set 7C

1 **a** $\frac{49}{150}$ **b** $\frac{101}{150}$
2 **a** $\frac{3}{29}$ **b** $\frac{21}{29}$ **c** $\frac{24}{29}$ **d** $\frac{5}{29}$ **e** $\frac{11}{29}$
3 **a** $\frac{13}{15}$ **b** $\frac{4}{5}$ **c** $\frac{29}{30}$ **d** $\frac{1}{6}$
e $\frac{1}{30}$ **f** $\frac{1}{15}$ **g** $\frac{1}{10}$

Review set 7D

1 **a** $\frac{17}{100}$ **b** $\frac{32}{100} = \frac{8}{25}$
c Perform more trials of the experiment
2 **a** $\frac{9}{20}$ **b** $\frac{3}{10}$ **c** $\frac{3}{20}$
d $\frac{23}{30}$ **e** $\frac{37}{60}$ **f** $\frac{7}{30}$
3 **a** $\frac{17}{40}$ **b** $\frac{3}{8}$ **c** $\frac{19}{40}$ **d** $\frac{2}{40} = \frac{1}{20}$
e $\frac{36}{40} = \frac{9}{10}$ **f** $\frac{17}{20}$ **g** $\frac{1}{10}$

CUMULATIVE REVIEW: 5–7

1 **a** **i** \$1288.46 **ii** \$2576.92 **iii** \$5583.33
b \$2192.67 **c** \$914.85 **d** \$3854
e \$406.70 **f** \$650 **g** \$285.40
h \$422.93
i **i** \$3528 **ii** \$1176
j \$214.14
k **i** \$13 999 **ii** \$6719.52
iii \$431.64
l **i** \$5328.55 **ii** \$222.02
2 **a** Trapezium
b **i** 44.31 cm^2 **ii** 7.82 cm^2
iii 44.84 m^2
c **i** 37.3 m^2 **ii** 15.9 m^2
d **i** 40.3 m^2 **ii** 26.9 m^2
3 **a** **i** $\frac{7}{14} = \frac{1}{2}$ **ii** $\frac{5}{14}$ **iii** $\frac{1}{2}$
iv $\frac{7}{14} = \frac{1}{2}$ **v** 1 **vi** 0
b **i** $\frac{2}{36} = \frac{1}{18}$ **ii** $\frac{1}{18}$ **iii** $\frac{1}{36}$ **iv** $\frac{1}{6}$
c **i** $\frac{27}{100}$ **ii** $\frac{3}{10}$
iii A tall person who does not wear glasses
d **i** $\frac{22}{40} = \frac{11}{20}$ **ii** $\frac{17}{40}$
iii $\frac{5}{40} = \frac{1}{8}$ **iv** $\frac{29}{40}$

CHAPTER 8 RIGHT-ANGLED TRIGONOMETRY

Exercise 8A

1 $x^2 = 8^2 + 13^2 = 233$
$x = \sqrt{233} \approx 15.3$ cm correct to 1 decimal place
2 **a** 17 cm **b** 23 cm **c** 14 cm
3 **a** 9.2 m **b** 34.9 m **c** 12.4 m
4 $x^2 = 14^2 - 9^2 = 115$
$x = \sqrt{115} \approx 10.7$ cm correct to 1 decimal place
5 **a** 11 cm **b** 14 m **c** 24 cm
d 8.0 cm **e** 30.4 m **f** 16.8 mm
6 $c^2 = 14^2 = 196$
$a^2 + b^2 = 8^2 + 12^2 = 208$
$c^2 \neq a^2 + b^2$
Triangle is not right-angled.
7 **a** No **b** Yes **c** No
d No **e** Yes **f** No

Exercise 8B

1 **a** **i** AB **ii** BC **iii** AC
b **i** RQ **ii** PR **iii** PQ
c **i** XZ **ii** XY **iii** ZY
d **i** r **ii** p **iii** q
e **i** n **ii** m **iii** l
f **i** EF **ii** ED **iii** FD
g **i** y **ii** x **iii** z
h **i** UV **ii** TV **iii** UT
i **i** TU **ii** SU **iii** ST
j **i** g **ii** h **iii** k
k **i** v **ii** u **iii** w
l **i** m **ii** k **iii** l

2 **a** b **b** a **c** a **d** b

3

$\frac{\text{opposite}}{\text{adjacent}}$	$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$
$\frac{g}{f}$	$\frac{g}{h}$	$\frac{f}{h}$

4

	$\frac{\text{opposite}}{\text{adjacent}}$	$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$
a	$\frac{BC}{AC}$	$\frac{BC}{AB}$	$\frac{AC}{AB}$
b	$\frac{PR}{PQ}$	$\frac{PR}{RQ}$	$\frac{PQ}{RQ}$
c	$\frac{XY}{ZY}$	$\frac{XY}{XZ}$	$\frac{ZY}{XZ}$
d	$\frac{p}{q}$	$\frac{p}{r}$	$\frac{q}{r}$
e	$\frac{m}{l}$	$\frac{m}{n}$	$\frac{l}{n}$
f	$\frac{ED}{FD}$	$\frac{ED}{EF}$	$\frac{FD}{EF}$
g	$\frac{x}{z}$	$\frac{x}{y}$	$\frac{z}{y}$
h	$\frac{TV}{UT}$	$\frac{TV}{UV}$	$\frac{UT}{UV}$
i	$\frac{SU}{ST}$	$\frac{SU}{TU}$	$\frac{ST}{TU}$
j	$\frac{h}{k}$	$\frac{h}{g}$	$\frac{k}{g}$
k	$\frac{u}{w}$	$\frac{u}{v}$	$\frac{w}{v}$
l	$\frac{k}{l}$	$\frac{k}{m}$	$\frac{l}{m}$

5 $\tan\theta = \frac{XY}{YZ}$, $\sin\theta = \frac{XY}{XZ}$, $\cos\theta = \frac{ZY}{ZX}$

6 **a** **i** $\frac{UT}{TS}$ **ii** $\frac{UT}{US}$ **iii** $\frac{TS}{US}$
b **i** $\frac{ED}{DF}$ **ii** $\frac{ED}{EF}$ **iii** $\frac{DF}{EF}$
c **i** $\frac{MN}{ML}$ **ii** $\frac{MN}{LN}$ **iii** $\frac{LM}{LN}$
d **i** $\frac{a}{b}$ **ii** $\frac{a}{c}$ **iii** $\frac{b}{c}$
e **i** $\frac{t}{s}$ **ii** $\frac{t}{u}$ **iii** $\frac{s}{u}$
f **i** $\frac{x}{z}$ **ii** $\frac{x}{y}$ **iii** $\frac{z}{y}$
g **i** $\frac{XY}{XZ}$ **ii** $\frac{XY}{YZ}$ **iii** $\frac{XZ}{YZ}$
h **i** $\frac{h}{j}$ **ii** $\frac{h}{i}$ **iii** $\frac{j}{i}$
i **i** $\frac{RT}{ST}$ **ii** $\frac{RT}{RS}$ **iii** $\frac{ST}{RS}$

7 **a** **i** $\frac{BC}{CA}$ **ii** $\frac{BA}{AC}$ **iii** $\frac{CB}{BA}$
b **i** $\frac{TV}{AV}$ **ii** $\frac{AT}{AV}$ **iii** $\frac{TV}{TA}$
c **i** $\frac{PR}{AP}$ **ii** $\frac{AR}{AP}$ **iii** $\frac{PR}{AR}$

Exercise 8C

1 **a** 0.4695 **b** 0.8988 **c** 0.1944
d 0.3907 **e** 0.6947 **f** 3.4874
g 4.3315 **h** 0.6691 **i** 0.4540
j 0.8910 **k** 0.7002 **l** 0.1564

2 **a** 6.1223 **b** 0.9748 **c** 18.0213
d 14.9227 **e** 29.8535 **f** 5.6732
g 4.4550 **h** 3.8684 **i** 5.6557
j 49.4798 **k** 1.2535 **l** 6.2655

3 **a** 31° **b** 65° **c** 47° **d** 17°
e 14° **f** 18° **g** 29° **h** 43°
i 84° **j** 33° **k** 27° **l** 1°

4 **a** 21° **b** 82° **c** 65° **d** 48°
e 76° **f** 6° **g** 2° **h** 33°
i 88° **j** 0° **k** 88° **l** 11°
m 49° **n** 39° **o** 13°

5 **a** 78° **b** 74° **c** 38° **d** 8°
e 41° **f** 35° **g** 3° **h** 66°
i 37° **j** 71° **k** 49° **l** 53°

Exercise 8D

1
$$\begin{aligned} \sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\ \sin 73° &= \frac{x}{45} \\ \therefore x &= 45 \sin 73° \\ &= 43.0337 \\ &= 43 \text{ m (1 decimal place)} \end{aligned}$$

2 **a** 10.8 cm **b** 2.8 mm **c** 11.3 cm
d 8.7 km **e** 8.7 cm **f** 1.7 m

3
$$\begin{aligned} \sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\ \sin 58° &= \frac{6}{x} \\ \therefore x &= \frac{6}{\sin 58°} \\ &= 7.0751 = 7.1 \text{ m (1 decimal place)} \end{aligned}$$

4 **a** 17.6 cm **b** 16.4 m **c** 25.6 mm
d 7.4 m **e** 10.6 m **f** 13.2 m

5 **a** 5.7 cm **b** 2.6 m **c** 47.6 cm

6
$$\begin{aligned} \cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ \cos 27° &= \frac{x}{34} \\ \therefore x &= 34 \cos 27° \\ &= 30.2942\ldots = 30.3 \text{ m (1 decimal place)} \end{aligned}$$

7 **a** 5.1 cm **b** 10.1 m **c** 81.3 km
d 4.0 cm **e** 7.4 cm **f** 38.7 mm

8 **a** 162.8 cm **b** 4.2 m **c** 184.0 mm
d 9.4 m **e** 60.1 m **f** 58.1 m

9 **a** 56.1 cm **b** 14.5 m **c** 17.3 cm

10
$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ \tan 46° &= \frac{x}{22} \\ \therefore x &= 22 \tan 46° \\ &= 22.7817\ldots = 22.8 \text{ m (1 decimal place)} \end{aligned}$$

11 **a** 2.7 m **b** 10.1 cm **c** 8.5 km
d 26.4 m **e** 20.0 m **f** 7.8 m
12 **a** 7.0 cm **b** 13.2 m **c** 229.5 mm
d 28.8 cm **e** 36.7 mm **f** 60.9 m
13 **a** $x = 2.9$ cm **b** $y = 13.8$ m
c $m = 25.5$ km **d** $t = 49.5$ m
e $y = 102.9$ mm **f** $x = 4.2$ m
g $a = 309.8$ cm **h** $c = 12.9$ cm
i $r = 19.6$ m

Exercise 8E

1 Use the sine ratio to find θ to the nearest degree.

$$\begin{aligned}\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}}\\ &= \frac{33}{51}\\ \therefore \theta &= 40.3202\ldots = 40^\circ\end{aligned}$$

2 **a** 39° **b** 40° **c** 24° **d** 43° **e** 32°
f 42° **g** 50° **h** 44° **i** 42°

3
$$\begin{aligned}\cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}}\\ &= \frac{17}{27}\\ \therefore \theta &= 50.9772\ldots = 51^\circ\end{aligned}$$

4 **a** 66° **b** 41° **c** 48° **d** 56° **e** 48°
f 74° **g** 20° **h** 32° **i** 67°

5
$$\begin{aligned}\tan\theta &= \frac{\text{opposite}}{\text{adjacent}}\\ &= \frac{7}{11}\\ \therefore \theta &= 32.4712\ldots \approx 32^\circ\end{aligned}$$

6 **a** 63° **b** 27° **c** 56°
d 56° **e** 21° **f** 42°
g 60° **h** 12° **i** 52°
7 **a** 52° **b** 25° **c** 38°
d 18° **e** 50° **f** 18°
g 53° **h** 76° **i** 38°
j 41° **k** 18° **l** 20°

Exercise 8F

1
$$\begin{aligned}\tan 74^\circ &= \frac{x}{21}\\ x &= 21\tan 74^\circ\\ &= 73.2357\ldots \approx 73\text{ m}\end{aligned}$$

The flagpole is about 73 m high.

2 $h = 39$ m

3
$$\begin{aligned}\tan 53^\circ &= \frac{x}{135}\\ x &= 135\tan 53^\circ\\ &= 179.1510\ldots \approx 179\text{ m}\end{aligned}$$

The boat is about 179 m out from the base of the cliff.

4 47 m

5
$$\begin{aligned}\sin 67^\circ &= \frac{35}{60}\\ \theta &= 35.6853\ldots \approx 36^\circ\end{aligned}$$

The angle of elevation is 36°.

6 108 m

7
$$\begin{aligned}\sin 67^\circ &= \frac{2.8}{x}\\ \therefore x\sin 67^\circ &= 2.8\\ x &= \frac{2.8}{\sin 67^\circ}\\ &= 3.0418\ldots \approx 3.0\text{ m}\end{aligned}$$

The ladder is about 3.0 m long.

8 11 cm **9** 2.5 m

Language in mathematics

1 **a** opposite **b** hypotenuse **c** cosine
d tangent **e** sine **f** adjacent

2 **a** Angles are measured in degrees.
b Sine, cosine and tangent are all ratios.
c The hypotenuse is opposite the right angle.
d Cosine is the adjacent side divided by the hypotenuse.

3 The origin of the word cosine is from the phrase complement of sine so the cosine of an angle equals the sine of ninety degrees minus the angle.

5 **a** 83 or 84 years **b** Probability theory
c Chance processes **d** Two systems of equations
e The mathematical education of schoolchildren

Check your skills

1 C **2** B **3** C **4** D **5** A
6 B **7** B **8** B **9** D **10** D
11 B **12** C **13** D **14** A **15** C
16 B **17** D

Review set 8A

1 **a** 7.6 cm **b** 4.1 m **c** 5.8 m

2 **a** $\sin\alpha = \frac{y}{x}$, $\cos\alpha = \frac{z}{x}$, $\tan\alpha = \frac{y}{z}$,
$\sin\beta = \frac{z}{x}$, $\cos\beta = \frac{y}{x}$, $\tan\beta = \frac{z}{y}$
b $\sin\alpha = \frac{l}{n}$, $\cos\alpha = \frac{m}{n}$, $\tan\alpha = \frac{l}{m}$,
$\sin\beta = \frac{m}{n}$, $\cos\beta = \frac{l}{n}$, $\tan\beta = \frac{m}{l}$
c $\sin\alpha = \frac{t}{r}$, $\cos\alpha = \frac{u}{r}$, $\tan\alpha = \frac{t}{u}$,
$\sin\beta = \frac{u}{r}$, $\cos\beta = \frac{t}{r}$, $\tan\beta = \frac{u}{t}$
d $\sin\alpha = \frac{b}{a}$, $\cos\alpha = \frac{c}{a}$, $\tan\alpha = \frac{b}{c}$,
$\sin\beta = \frac{c}{a}$, $\cos\beta = \frac{b}{a}$, $\tan\beta = \frac{c}{b}$

3 **a** 11.1 cm **b** 12.6 mm **c** 84.2 m
4 **a** 31° **b** 47° **c** 32°
5 **a** 26.0 m **b** 66 m **c** 90°, 37°, 53°

Review set 8B

1 **a** 14.8 cm **b** 25.6 m **c** 24.2 cm

2 **a** $\sin\theta = \frac{y}{z}$, $\cos\theta = \frac{x}{z}$, $\tan\theta = \frac{y}{x}$
b $\sin\theta = \frac{r}{q}$, $\cos\theta = \frac{p}{q}$, $\tan\theta = \frac{r}{p}$
c $\sin\theta = \frac{AB}{BC}$, $\cos\theta = \frac{AC}{BC}$, $\tan\theta = \frac{AB}{AC}$
d $\sin\theta = \frac{SV}{ST}$, $\cos\theta = \frac{VT}{ST}$, $\tan\theta = \frac{SV}{VT}$

3 **a** 70.7 cm **b** 31.7 cm **c** 25.6 m
4 **a** 51° **b** 49° **c** 27°
5 **a** 4.8 m **b** 384 m

Review set 8C

1 **a** 7.4 cm **b** 6.6 m **c** 10.2 cm

2 **a** $\sin\alpha = \frac{b}{a}$, $\cos\alpha = \frac{c}{a}$, $\tan\alpha = \frac{b}{c}$,
$\sin\beta = \frac{c}{a}$, $\cos\beta = \frac{b}{a}$, $\tan\beta = \frac{c}{b}$
b $\sin\alpha = \frac{q}{r}$, $\cos\alpha = \frac{p}{r}$, $\tan\alpha = \frac{q}{p}$,
$\sin\beta = \frac{p}{r}$, $\cos\beta = \frac{q}{r}$, $\tan\beta = \frac{p}{q}$

c $\sin \alpha = \frac{x}{z}, \cos \alpha = \frac{y}{z}, \tan \alpha = \frac{x}{y},$

$\sin \beta = \frac{y}{z}, \cos \beta = \frac{x}{z}, \tan \beta = \frac{y}{x}$

d $\sin \alpha = \frac{e}{f}, \cos \alpha = \frac{d}{f}, \tan \alpha = \frac{e}{d},$

$\sin \beta = \frac{d}{f}, \cos \beta = \frac{e}{f}, \tan \beta = \frac{d}{e}$

3 **a** 17.0 cm **b** 14.9 cm **c** 5.2 mm

4 **a** 23° **b** 41° **c** 64°

5 **a** 30.2 m, 19.6 m, 33° **b** 23°

Review set 8D

1 **a** Yes **b** No **c** Yes

2 **a** $\sin \theta = \frac{a}{b}, \cos \theta = \frac{c}{b}, \tan \theta = \frac{a}{c}$

b $\sin \theta = \frac{RT}{PT}, \cos \theta = \frac{PR}{PT,} \tan \theta = \frac{RT}{PR}$

c $\sin \theta = \frac{LM}{LT}, \cos \theta = \frac{TM}{LT}, \tan \theta = \frac{LM}{TM}$

d $\sin \theta = \frac{s}{t}, \cos \theta = \frac{r}{t}, \tan \theta = \frac{s}{r}$

3 **a** 1.5 m **b** 10.9 cm **c** 6.8 mm

4 **a** 55° **b** 23° **c** 16°

5 **a** 45 m **c** 187 m

CHAPTER 9 SIMILARITY

Exercise 9A

1 **b** The image of *PQRS* is *P′Q′R′S′*.

c The pairs of corresponding sides are *PQ* and *P′Q′*, *QR* and *Q′R′*, *RS* and *R′S′*, and *SP* and *S′P′*.

2 **a** **ii** *AB* and *A′B′*, *BC* and *B′C′*, *CD* and *C′D′*, *AD* and *A′D′*

b **ii** *PQ* and *P′Q′*, *QR* and *Q′R′*, *RS* and *R′S′*, *PS* and *P′S′*

c **ii** *XY* and *X′Y′*, *OX* and *OX′*, *OY* and *OY′*

3 **a** **ii** 2 **b** **ii** 1.5 **c** **ii** 2 **d** **ii** 2.5

Exercise 9B

1 **a** Similar, $k = 2$ **b** Similar, $k = 1.5$

c Not similar (matching angles are equal but not all matching sides are in proportion)

2 **a** Similar, $k = 3$ **b** Similar, $k = 1.5$

c Similar $k = 1.25$ **d** Not similar

3 **a, e** All rectangles are equiangular, but do not necessarily have their matching sides in proportion and so not all rectangles are similar.

4 **a** True **b** True **c** False **d** True **e** False

5 They are congruent when $k = 1$.

6 **a** $k = 3$ **b** $k = 1.25$

Exercise 9C

1 **a** *KM* and *XY*, *ML* and *XZ*, *KL* and *YZ* are matching sides.

b $k = \frac{XZ}{ML} = \frac{8}{5} = 1.6$

2 **a** **i** *AB* and *EF*, *AC* and *ED*, *BC* and *FD*

ii 2.2

b **i** *PQ* and *WU*, *QR* and *UV*, *PR* and *WV*

ii 0.8

c **i** *AB* and *EF*, *BC* and *FD*, *AC* and *ED*

ii 0.6

d **i** *QP* and *ST*, *QR* and *SU*, *PR* and *TU*

ii $\frac{1}{2}$

3 $k = \frac{8}{4} = 2, x = 2 \times 6 = 12$ cm

4 **a** $x = 9$ cm **b** $x = 12.5$ cm

c $x = 14$ cm **d** $y = 4$ cm

5 $k = \frac{12}{15} = \frac{4}{5}$ or 0.8, $x = \frac{4}{5} \times 10 = 8$ cm

6 **a** $x = 10$ cm **b** $x = 21$ cm

c $x = 7.5$ cm **d** $x = 12$ cm

7 *PS* and *AD* are matching sides. $k = \frac{5}{2.5} = 2$

RS and *CD* are matching sides. $x = 2 \times 3 =$ 6 cm

8 **a** $x = 12.5$ cm **b** $x = 4.5$ cm

c $x = 9\frac{1}{3}$c m **d** $x = 15$ cm

9 *ED* and *BA*, and *DF* and *AC* are matching sides.

$\frac{x}{15} = \frac{36}{12}, x = 15 \times \frac{36}{12} = 45$ cm

10 **a** $x = 11.2$ cm **b** $x = 16.25$ cm

c $x = 22.5$ cm **d** $x = 9\frac{1}{3}$c m

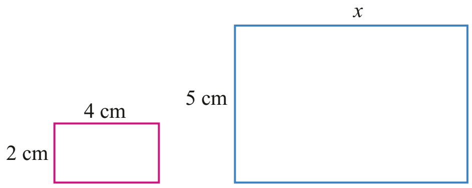
e $x = 8$ cm **f** $x = 5$ cm

g $x = 8.75$ cm

11 **a** $x = 12$ cm **b** $y = 21$ cm

c $x = 3.5$ cm, $y = 8$ cm **d** $x = 2.25$ cm, $y = 15$ cm

12



$\frac{x}{4} = \frac{5}{2}, x = 4 \times \frac{5}{2} = 10$ cm

13 **a** $x = 1.6$ cm **b** $x = 2.25$ cm

c $x = 11\frac{2}{3}$c m **d** $x = 7.2$ cm

e $x = 9$ cm

Exercise 9D

2 56 mm **3** 71 mm

4 $w = 400$ m, $x = 300$ m, $y = 560$ m, $z = 340$ m

5 **a** 1 : 400 **b** 12 m

6 **a** 1 : 4000 **b** 136 m

7 **a** 1 : 2000 **b** 67 m

8 8 m **9** 10.8 m

10 1 m **11** 100 m

Language in mathematics

1 **a** C **b** D **c** A **d** B

3 scale

Check your skills

1 B **2** B **3** C **4** B **5** B

6 B **7** C **8** A **9** D **10** D

Review set 9A

2 **a** Similar, $k = 3$ **b** Not similar

3 **a** True **b** False

4 **a** *AB* and *LK*, *BC* and *KM*, *CA* and *ML*

b $k = 2.4$

5 **a** $k = 3, x = 15, y = 6$ **b** $k = 0.8, x = 12, y = 8$

6 $k = 1.5, x = 9$ cm

7 **a** $x = 7.5, y = 3\frac{1}{3}$ **b** $x = 6, y = 12$

8 98 mm

9 $x = 2.8$ cm

10 $h = 5.6$ m

Review set 9B

2 **a** Not similar **b** Similar, $k = 2$

3 **a** True **b** False

c True

4 **a** PQ and YX, QR and XZ, PR and YZ

b $k = \frac{1}{5}$

5 **a** $x = 24, y = 6$ **b** $x = 6, y = 14$

6 $k = 1.5, x = 2.25$ cm

7 **a** $x = 15, y = 14$ **b** $x = 10, y = 12$

8 240 m × 160 m

9 $x = 12.8$ cm

10 $h = 6.5$ m

Review set 9C

2 **a** Similar, $k = \frac{2}{3}$ **b** Not similar

3 $k = 1.5$

4 **a** PQ and LK, QR and KM, PR and LM

b $k = 1.2$

5 **a** $k = 1.4, x = 25.2, y = 15$

b $x = 17.6, y = 20$

6 $k = 2.4, x = 9.6$ cm

7 **a** $x = 9.6, y = 10$ **b** $x = 20, y = 38.4$

8 73 mm

9 $k = \frac{5}{3}, x = 20$ cm

10 17.5 m

Review set 9D

2 **a** Similar, $k = \frac{2}{3}$ **b** Not similar

3 Base angles of both triangles equal or vertex angles equal

4 **a** AB and FD, BC and DE, CA and EF

b $k = 1.4$

5 **a** $k = 3, x = 27, y = 6.5$

b $k = 0.9, x = 18.9, y = 15$

6 $k = 2$ or $\frac{1}{2}, x = 4$ cm

7 **a** $x = 35, y = 30$ **b** $x = 16.8, y = 40$

8 18 m **9** $x = 3$ cm

10 **a** 5.3 m **b** 8.8 m

CHAPTER 10 LINEAR AND NON-LINEAR RELATIONSHIPS

Exercise 10A

Only midpoints are given.

1 **a** (5, 4) **b** (7, 3) **c** (5, 6)

2 **a** (2, 6) **b** (5, 5) **c** (3, 6)

3 **a** (1, 3) **b** (−2, 3) **c** (3, 1)

d (4, 1) **e** (1, 2) **f** (−4, 1)

g $(0,4\frac{1}{2})$ **h** $(1\frac{1}{2},0)$ **i** (2, 14)

4 The coordinates of the third vertex on the triangle are (7, 2).
The midpoint of the horizontal interval is (5, 2).
The midpoint of the vertical interval is $(7, 3\frac{1}{2})$.
The midpoint is $(5, 3\frac{1}{2})$.

5 **a** (3, 3) **b** (1, 4) **c** (2, −3)

d (0, 0) **e** $(\frac{1}{2},1)$ **f** (1, 3)

g $(-3\frac{1}{2},2)$ **h** $(-2\frac{1}{2}, -1\frac{1}{2})$ **i** (0, 0)

6 The x-coordinate of M will be halfway between the x-coordinates of A and B.

x-coordinate of $M = \dfrac{-2 + 6}{2} = 2$

The y-coordinate of M will be halfway between the y-coordinates of A and B.

y-coordinate of $M = \dfrac{5 + (-3)}{2} = 1$

Midpoint of the line interval is (2, 1).

7 **a** $(4, -5\frac{1}{2})$ **b** $(2, -2\frac{1}{2})$ **c** (3, 3)

d (16, 15) **e** $(7\frac{1}{2}, -\frac{1}{2})$ **f** $(-3\frac{1}{2}, -5\frac{1}{2})$

g $(3a, 5b)$ **h** $(p, 4q)$ **i** (0, 0)

9 The midpoint formula is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

$x_1 = -4, x_2 = 2$

$y_1 = 7, y_2 = -1$

$\left(\dfrac{-4 + 2}{2}, \dfrac{7 + -1}{2}\right) = \dfrac{-2}{2}, \dfrac{6}{2} = (-1, 3)$

The midpoint is (−1, 3).

10 **a** (3, 5) **b** $(2,1\frac{1}{2})$

c $(\frac{1}{4}, \frac{3}{4})$ **d** $(-1\frac{1}{2}, -1)$

11 It is the average of the two x-coordinates and two y-coordinates.

Exercise 10B

Only distances are given (in units).

1 The coordinates of the third vertex are (7, 4).

Vertical = 6 − 4 = 2 units

Horizontal = 7 − 2 = 5 units

$c^2 = 5^2 + 2^2 = 29$

$c = \sqrt{29} \approx 5.39$ cm (2 decimal places)

Distance between the points is 5.39 units (2 decimal places).

2 **a** 5 **b** $\sqrt{18} \approx 4.24$

c $\sqrt{41} \approx 6.40$ **d** $\sqrt{85} \approx 9.22$

e $\sqrt{34} \approx 5.83$ **f** $\sqrt{74} \approx 8.60$

3 Vertical length = 7 − (−4) = 11 units

Horizontal length = (−4) − 3 = −7 = 7 units

(distances must be positive)

$c^2 = 11^2 + 7^2 = 170$

$c = \sqrt{170} = 13.04$ cm (2 decimal places)

Distance between the points is 13.04 units (2 decimal places).

4 **a** $\sqrt{122} \approx 11.05$ **b** 13

c $\sqrt{97} \approx 9.85$ **d** $\sqrt{160} \approx 12.65$

e $\sqrt{73} \approx 8.54$ **f** 5

Exercise 10C

1 Vertical rise = 3

Horizontal run = 7

Slope of line $AB = \frac{3}{7}$

2 **a** $\frac{3}{20}$ **b** $\frac{1}{5}$ **c** $\frac{3}{4}$

3 **a** $\frac{3}{4}$ **b** $\frac{3}{6} = \frac{1}{2}$ **c** 2 **d** $\frac{5}{4}$

4 Vertical rise = 4

Horizontal run = 10

Gradient $= \frac{4}{10} = \frac{2}{5}$

5 **a** $\frac{7}{9}$ **b** $\frac{3}{8}$ **c** $\frac{1}{3}$ **d** 11

Exercise 10D

1 **a** The slope of AB is positive. Slope of $AB = \frac{3}{4}$
b The slope of CD is negative. Slope of $CD = -\frac{5}{3}$

2 **a** $+2$ **b** $-\frac{4}{3}$ **c** -4 **d** $\frac{1}{2}$
e $\frac{5}{3}$ **f** $-\frac{3}{4}$

3 **a** 0 **b** $\frac{1}{5}$ **c** $\frac{2}{5}$ **d** $\frac{7}{6}$
e $\frac{7}{3}$ **f** Undefined

4 **a** 0 **b** $-\frac{1}{3}$ **c** $-\frac{1}{2}$ **d** $-\frac{3}{4}$
e $-\frac{3}{2}$ **f** -7 **g** Undefined

5 **a** OP, PQ, RS, TU **b** QR, ST, UV
c TU **d** ST **e** VW **f** PQ

6 The slope is negative.
The rise is 6 and the run is 7.
Gradient $= -\frac{6}{7}$

7 Only gradients are given.
a $-\frac{4}{11}$ **b** $\frac{4}{9}$ **c** $\frac{4}{5}$ **d** $\frac{3}{5}$ **e** $\frac{1}{10}$ **f** $-\frac{4}{7}$

8 The rise is 5 and the run is 5.
Gradient $= \frac{5}{5} = 1$

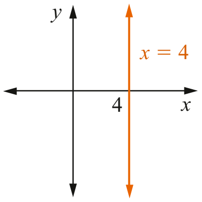
9 **a** $\frac{4}{3}$ **b** $-\frac{3}{4}$ **c** $\frac{4}{5}$ **d** $-\frac{4}{3}$

10 **a** 1 **b** $\frac{4}{3}$ **c** $-\frac{2}{3}$ **d** $\frac{1}{2}$
e -2 **f** $-\frac{1}{4}$

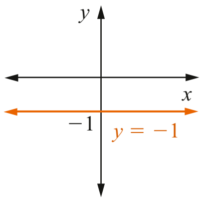
11 **a** 2 **b** $\frac{25}{2}$ **c** -6 **d** $-\frac{1}{125}$
e $\frac{5}{8}$ **f** $\frac{2}{7}$

Exercise 10E

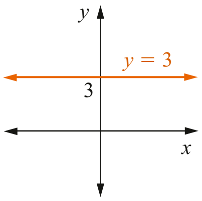
1 **a**



b



2 **a**



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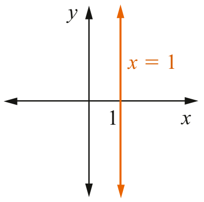

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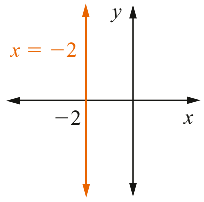

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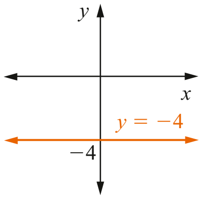

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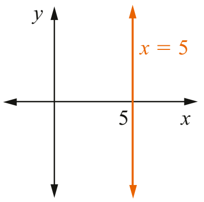

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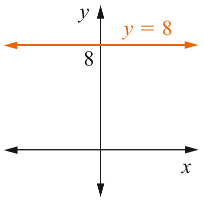
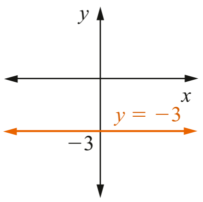
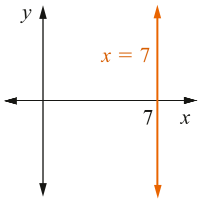


g



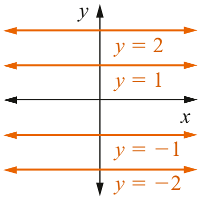
h



3 **a** $y = 3, y = -4, y = 8, y = -3$
b $(0, 3), (0, -4), (0, 8), (0, -3)$

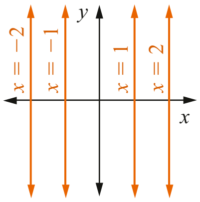
4 **a** $x = 1, x = -2, x = 5, x = 7$
b $(1, 0), (-2, 0), (5, 0), (7, 0)$

5 **a**



b $y = 0$ **c** All values of y are 0.

6 **a**



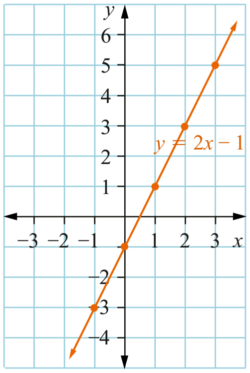
b $x = 0$ **c** All values of x are 0.

Exercise 10F

1 $y = 2x - 1$

x	−1	0	1	2	3
y	−3	−1	1	3	5

$y = 2(0) - 1 = -1$
$y = 2(2) - 1 = 3$
$y = 2(3) - 1 = 5$



2 **a** $y = x$

x	1	2	3	4	5
y	1	2	3	4	5

b $y = x - 2$

x	−1	0	1	2	3
y	−3	−2	−1	0	1

c $y = -x$

x	-2	-1	0	1	2
y	2	1	0	-1	-2

d $y = 5 - x$

x	-2	-1	0	1	2
y	7	6	5	4	3

e $y = 2x + 1$

x	-2	-1	0	1	2
y	-3	-1	1	3	5

f $y = -\frac{1}{2}x$

x	-2	-1	0	1	2
y	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1

g $y = 8 - 2x$

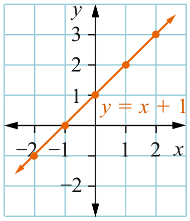
x	-2	-1	0	1	2
y	12	10	8	6	4

h $y = 1 - 3x$

x	-2	-1	0	1	2
y	7	4	1	-2	-5

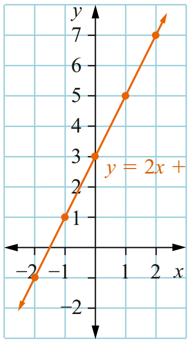
3 a $y = x + 1$

x	-2	-1	0	1	2
y	-1	0	1	2	3



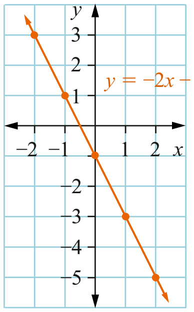
b $y = 2x + 3$

x	-2	-1	0	1	2
y	-1	1	3	5	7



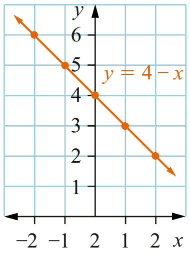
c $y = -2x - 1$

x	-2	-1	0	1	2
y	3	1	-1	-3	-5



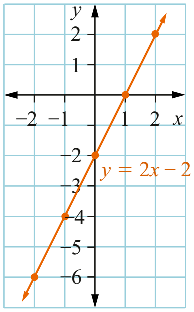
d $y = 4 - x$

x	-2	-1	0	1	2
y	6	5	4	3	2



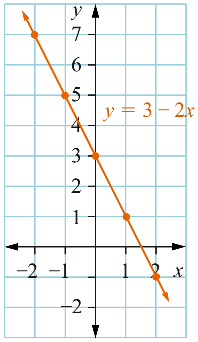
e $y = 2x - 2$

x	-2	-1	0	1	2
y	-6	-4	-2	0	2



f $y = 3 - 2x$

x	-2	-1	0	1	2
y	7	5	3	1	-1



4 **a** $y = x, y = x - 2, y = 2x + 1, y = x + 1,$
$y = 2x + 3, y = 2x - 2$

b $y = -x, y = 5 - x, y = -\frac{1}{2}x, y = 8 - 2x,$
$y = 1 - 3x, y = -2x - 1, y = 4 - x, y = 3 - 2x$

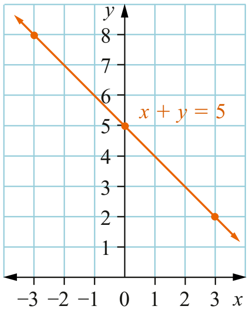
c The coefficient of x is positive for positive gradients and negative for negative gradients.

d **i** Positive **ii** Negative **iii** Negative
iv Positive **v** Positive **vi** Negative

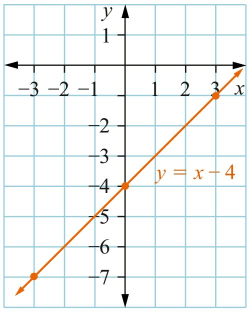
5 $x + y = 5$

x	−3	0	3
y	8	5	2

When $x = -3$: $-3 - y = 5$
$y = 5 + 3 = 8$
When $x = 0$: $0 + y = 5$
$y = 5$
When $x = 3$: $3 + y = 5$
$y = 5 - 3 = 2$



6 **a**



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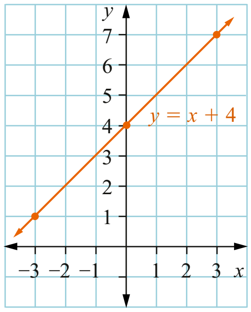

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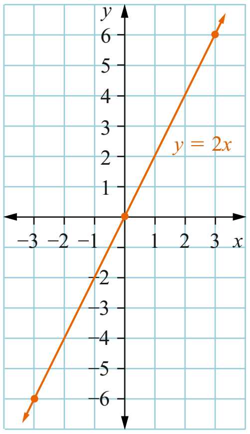

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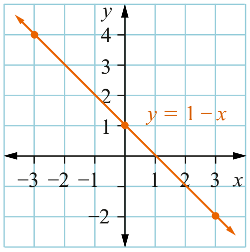

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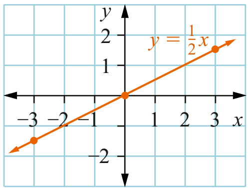

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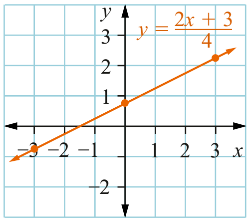

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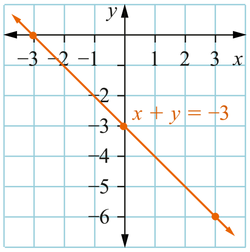

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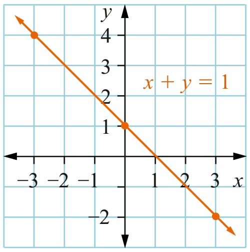
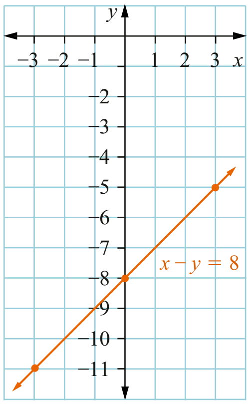
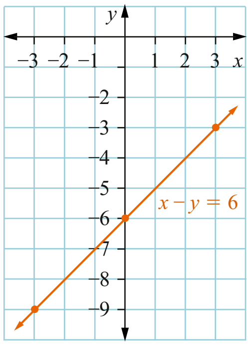


i



j



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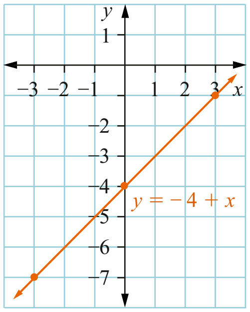

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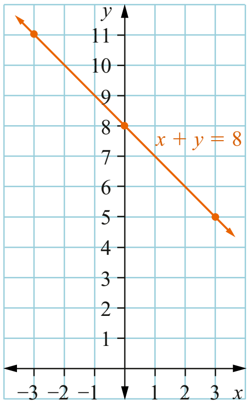

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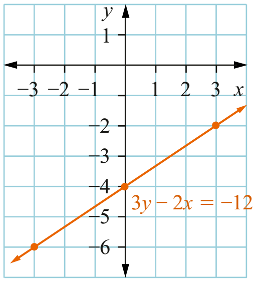
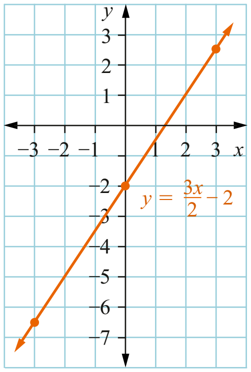
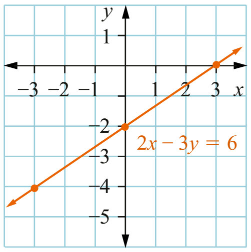


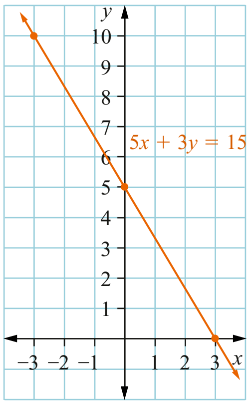
n



o



p



7 **a** Yes **b** No **c** No **d** Yes
e No **f** No **g** Yes **h** Yes
i Yes **j** Yes

8 **a** C **b** A **c** D **d** E
e B

9 When $x = 3$, $y = 3(3) - 4 = 9 - 4 = 5$.
As the y-values are equal, the point does lie on the line.

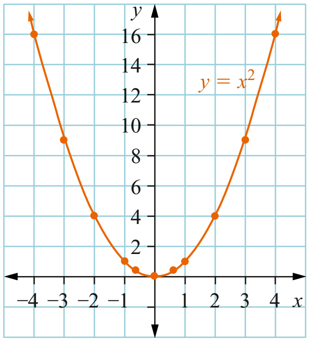
10 **a** Yes **b** Yes **c** No **d** No
e No **f** Yes **g** Yes **h** No
i Yes **j** No

Exercise 10G

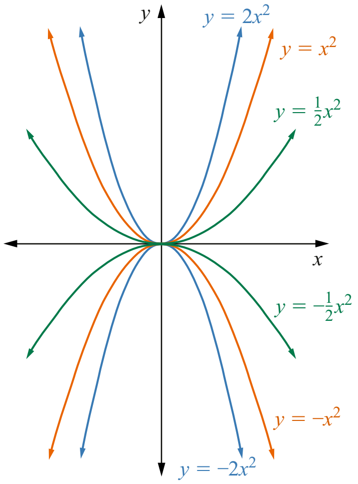
1 **a** $y = x^2$

x	-4	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	4
y	16	9	4	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1	4	9	16

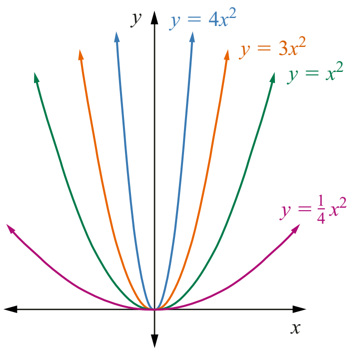
b



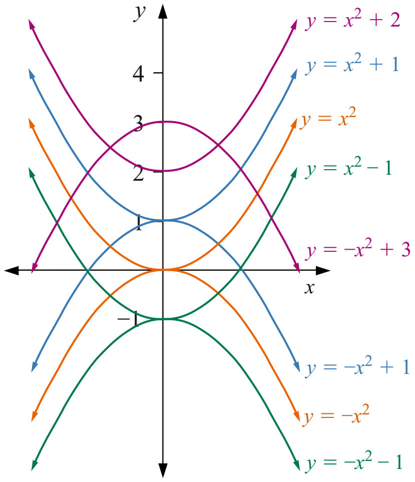
2 **a**



b



c

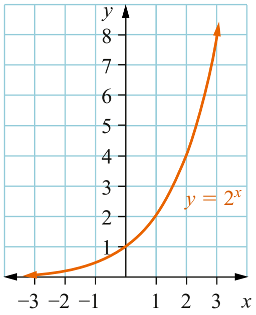


3 All are symmetrical about the y-axis. If the sign of the coefficient of x^2 is negative, the graph is inverted (the opposite way up) compared with $y = x^2$.

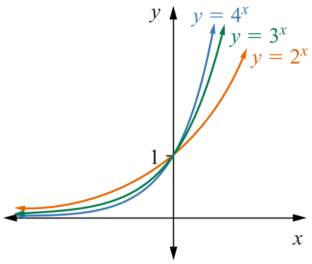
a, b The larger the coefficient of x, the narrower the graph. All graphs pass through (0, 0).

c The base graph $y = x^2$ passes through (0, 0); other graphs are raised or lowered by the value added to or subtracted from x^2.

4



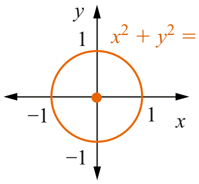
5



All graphs pass through (0, 1). All y-values are positive. All have positive slope and the gradient increases as x increases.

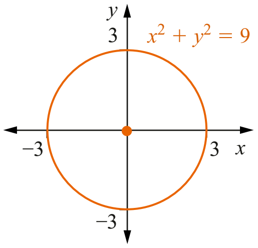
6 The centre of the circle is (0, 0).
The circle cuts the x-axis at ±5 and the y-axis at ±5.
The radius of the circle is $\sqrt{25} = 5$ units.

7 **a**



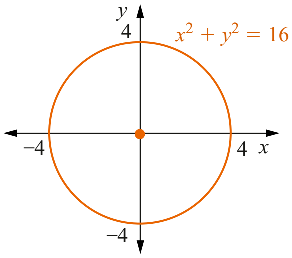
Centre (0, 0)
radius = 1 unit

b



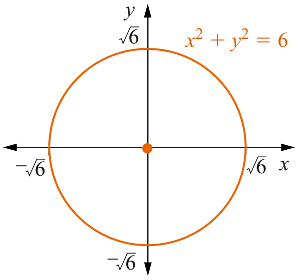
Centre (0, 0)
radius = 3 units

c



Centre (0, 0)
radius = 4 units

d



Centre (0, 0)
radius = $\sqrt{6}$ units

Language in mathematics

1 **a** 64 years
b Examples: Loci and set work, received a law degree and became King's Councillor
c One
d He did not publish much of his work while he was alive.
e *Concerning the Comparison of Curved and Straight Lines*

2 **a** vertical **b** horizontal **c** gradient
d interval **e** oblique **f** slope
g linear **h** midpoint

3 **a** The y-axis has the equation $x = 0$.
b The x-axis has the equation $y = 0$.
c An uphill gradient is a positive slope.
d The midpoint of a line is in the middle.
e If a line goes downhill the gradient is negative.
f The length of an interval is found using Pythagoras' rule.

4 In coordinate geometry an interval joins two points and has length, a midpoint and slope.

Check your skills

1 B **2** A **3** D **4** B **5** D
6 A **7** C **8** C **9** D **10** B
11 A **12** B **13** C **14** C **15** B
16 C

Review set 10A

1 **a** (7, 3) **b** (2, 7) **c** (3, 1)

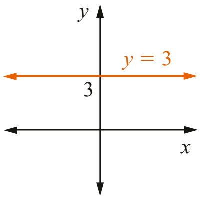
2 **a** $\sqrt{52}$ units **b** $\sqrt{20}$ units

3 $\frac{3}{5}$

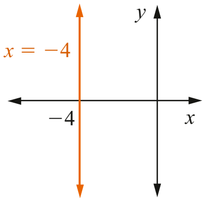
4 **a** $\frac{5}{11}$ **b** $-\frac{1}{2}$

5 2

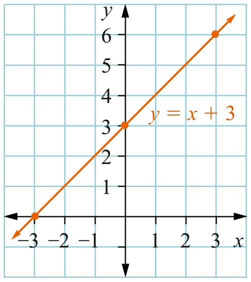
6 **a**



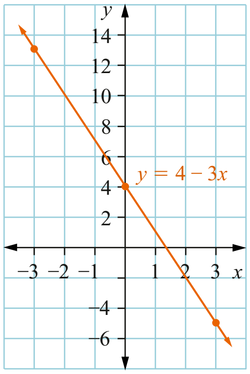
b



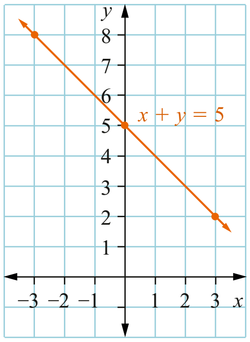
7 **a**



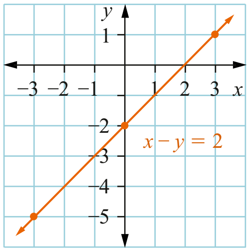
b



c



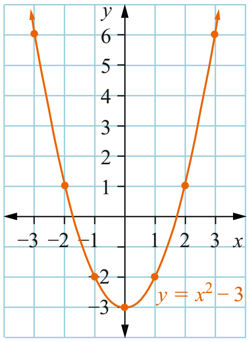
d



8 Yes, as $2(4) - 11 = -3$

9 $y = x^2 - 3$

x	−3	−2	−1	0	1	2	3
y	6	1	−2	−3	−2	1	6



Review set 10B

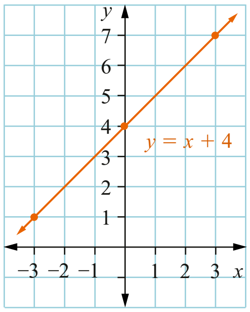
1 **a** $(3\frac{1}{2}, 3)$ **b** $(3, 5\frac{1}{2})$

2 **a** $\sqrt{65}$ units **b** $\sqrt{65}$ units

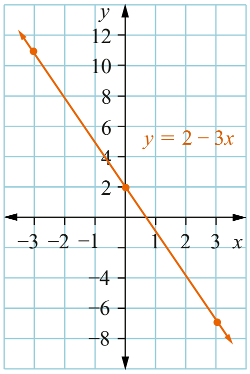
3 **a** $\frac{7}{4}$ **b** $\frac{5}{6}$

4 $-\frac{1}{3}$

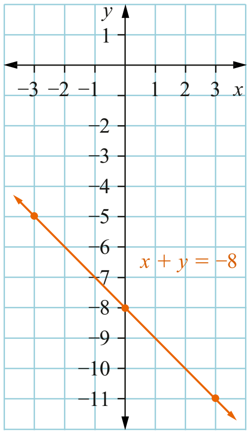
5 **a**



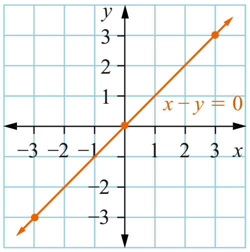
b



c



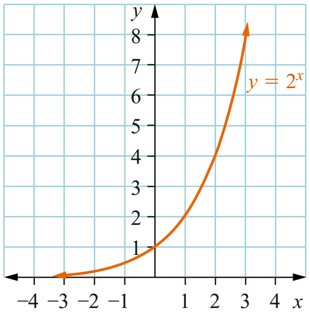
d



6 No, as $-1 - (-3) = 2$ not 4

7 $y = 2^x$

x	−2	−1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



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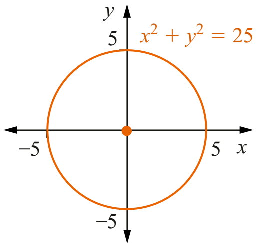


Review set 10C

1 **a** $(8, 3)$ **b** $(5\frac{1}{2}, 2\frac{1}{2})$ **c** $(2\frac{1}{2}, -\frac{1}{2})$

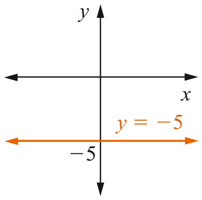
2 **a** $\sqrt{50}$ units **b** $\sqrt{41}$ units

3 $-\frac{4}{7}$

4 **a** $\frac{5}{13}$ **b** $-\frac{3}{11}$

5 $\frac{5}{2}$

6 **a**



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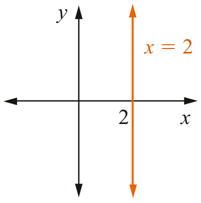
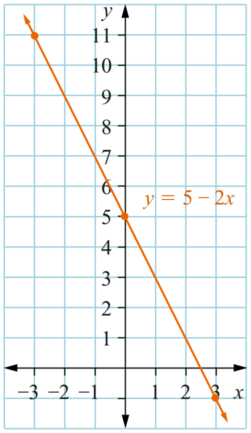
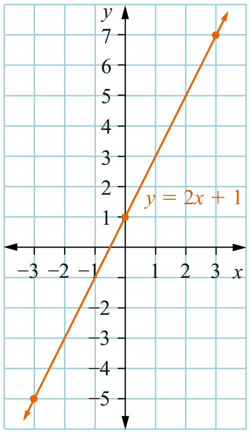


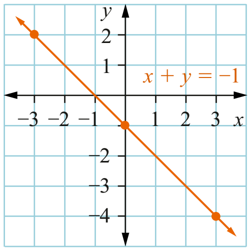
7 **a**



b



c



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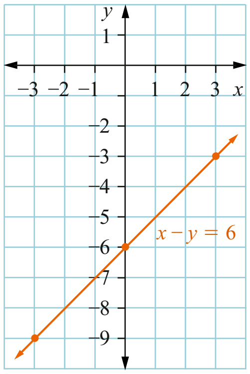
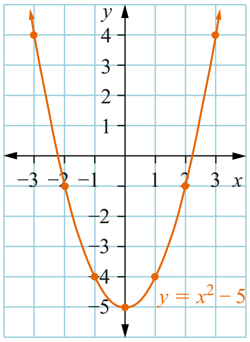


8 No, as $-3(2) - 1 = -7$ not 5

9 $y = x^2 - 5$

x	−3	−2	−1	0	1	2	3
y	4	−1	−4	−5	−4	−1	4



Review set 10D

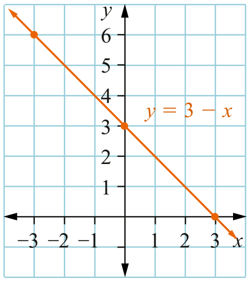
1 **a** (1, 1) **b** $(5\frac{1}{2}, 5)$

2 **a** $\sqrt{85}$ units **b** $\sqrt{20}$ units

3 **a** $-\frac{6}{7}$ **b** $\frac{1}{3}$

4 -1

5 **a**



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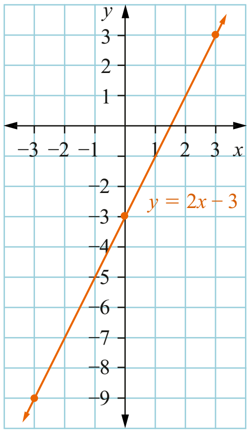
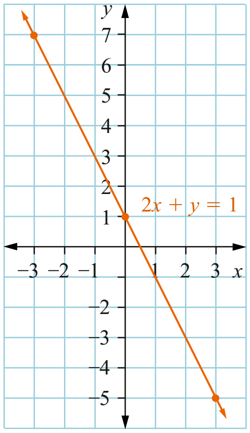


c



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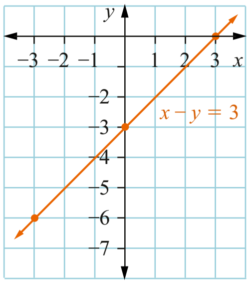
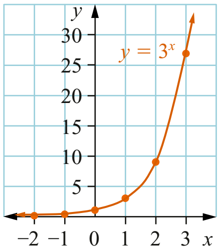


6 Yes, as $-1 + 5 = 4$

7 $y = 3^x$

x	-2	-1	0	1	2	3
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

ii



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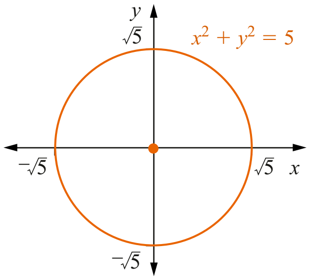


CUMULATIVE REVIEW: 8–10

1 **a** $AY, AR, RY; a, r, y$

b RT or TR; o

c **i** $\sin\theta = \frac{t}{m}$ **ii** $\cos\theta = \frac{a}{m}$ **iii** $\tan\theta = \frac{t}{a}$

d **i** $\tan\theta = \frac{AD}{DR} = \frac{r}{a}$ **ii** $\sin\theta = \frac{AD}{AR} = \frac{r}{d}$

iii $\cos\theta = \frac{DR}{AR} = \frac{a}{d}$

e **i** 0.6018 **ii** 0.9455 **iii** 0.5095

f **i** 74° **ii** 33° **iii** 70° **iv** 85°

g **i** $x = 13.3$ mm **ii** $x = 16.1$ cm

iii $x = 80.8$ mm **iv** $x = 44.5$ cm

h **i** 69° **ii** 52° **iii** 35° **iv** 69°

i 281 m **j** 169 m

2 **b** Yes, $k = 3$ **c** Yes

d $k = \frac{18}{7}, x = 20\frac{4}{7}, y = 5\frac{4}{9}$

3 **a** **i** $(4\frac{1}{2}, 5)$ **ii** $(3, 5)$

b **i** $(6, 3)$ **ii** $(2, 6\frac{1}{2})$ **iii** $(6\frac{1}{2}, 2\frac{1}{2})$ **iv** $(2\frac{1}{2}, -1\frac{1}{2})$

c **i** $d = \sqrt{52} \approx 7.2$ **ii** $d = \sqrt{58} \approx 7.6$

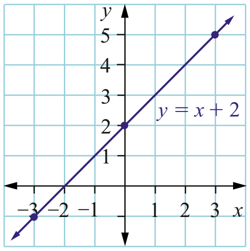
d **i** $d = \sqrt{50} \approx 7.1$ **ii** $d = \sqrt{41} \approx 6.4$

e **i** $\frac{6}{4} = 1\frac{1}{2}$ **ii** $\frac{6}{9} = \frac{2}{3}$

f **i** $-\frac{5}{4}$ **ii** 2

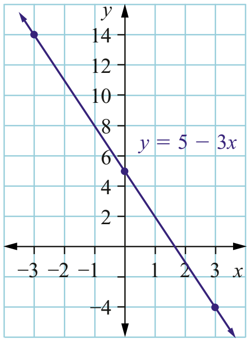
g **i** $y = x + 2$

x	-3	0	3
y	-1	2	5



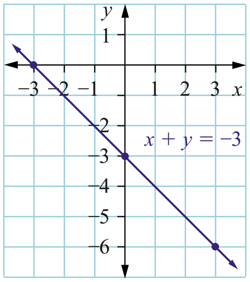
ii $y = 5 - 3x$

x	-3	0	3
y	14	5	-4



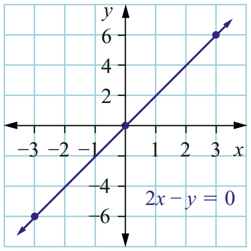
iii $x + y = -3$

x	-3	0	3
y	0	-3	-6



iv $2x - y = 0$

x	−3	0	3
y	−6	0	6

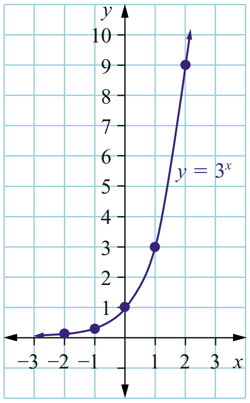


h Yes, since $-1 - (-3) = 2$

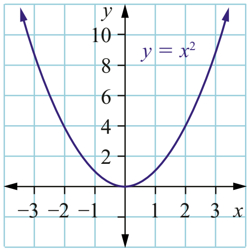
i **i**

x	−2	−1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

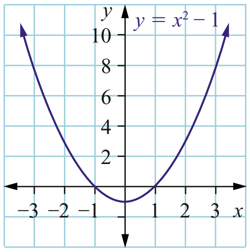
ii



j **i**



ii



CHAPTER 11 PROPORTION AND RATES

Exercise 11A

1 **a** 60 km/h **b** 60 words/min **c** 3.2 t/ha
d $3.16/kg **e** 45 c/call **f** 0.08 L/km
g $2.70/kL **h** 0.24 L/h

2 **a** $\frac{72\text{ L}}{1\text{ h}} = \frac{72 \times 1000\text{ mL}}{1 \times 60 \times 60\text{ s}} = 20\text{ mL/s}$

b $\frac{\$7}{1\text{ m}} = \frac{7 \times 100\text{c}}{1 \times 1000\text{ mm}} = 0.7\text{ c/mm}$

3 **a** 9 c/min **b** 8 c/cm **c** 0.8 kg/m^2
d 13.5 mL/s **e** 0.75%/month **f** 1.5 c/g
g $16\frac{2}{3}$ m/s **h** 625 g/h **i** 1.8 g/mL

4 **a** $\frac{7\text{ g}}{1\text{ mL}} = \frac{7 \div 1000\text{ kg}}{1 \div 1000\text{ L}}$

$= \frac{7}{1000}\text{ kg} \div \frac{1}{1000}\text{ L} = 7\text{ kg/L}$

b $\frac{8\text{ m}}{1\text{ s}} = \frac{8 \div 1000\text{ km}}{1 \div 3600\text{ h}}$

$= \frac{8}{1000}\text{ km} \div \frac{1}{3600}\text{ h} = 28.8\text{ km/h}$

5 **a** 6 kg/L **b** 54 km/h **c** $7.20/h
d 18 L/h **e** $9/m **f** 3 t/ha
g $45/kg **h** 100.8 km/h

Exercise 11B

1 **a** Direct **b** Direct **c** Inverse
d Unrelated **e** Inverse **f** Inverse
g Unrelated **h** Inverse **i** Direct
j Direct **k** Direct **l** Inverse

Exercise 11C

1 **a** **i** No **ii** Yes **iii** No
b **i** No **ii** No **iii** No
c **i** Yes **ii** Yes **iii** Yes
d **i** No **ii** Yes **iii** No

2 **a** **i** Yes **ii** Yes **iii** No
b **i** Yes **ii** Yes **iii** No
c **i** Yes **ii** No **iii** Yes
d **i** No **ii** Yes **iii** No
e **i** Yes **ii** Yes **iii** No
f **i** Yes **ii** No **iii** No

3 **a** Direct **b** Inverse **c** Neither **d** Neither
e Neither **f** Direct **g** Neither **h** Neither
i Neither **j** Inverse **k** Neither **l** Inverse
m Neither **n** Neither **o** Neither

Exercise 11D

1 **a**

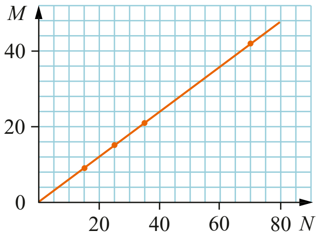
x	1	2	3	4	5
y	3	6	9	12	15
$\frac{y}{x}$	3	3	3	3	3

b Yes **c** Yes **d** $\frac{y}{x} = 3 \therefore y = 3x$

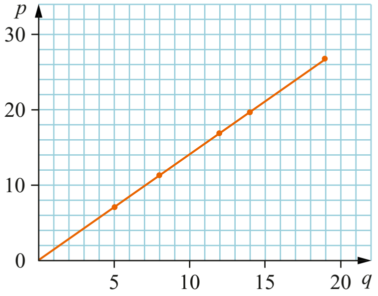
2 **a** **i** Yes **ii** $\frac{M}{N} = 0.6$ or $M = 0.6N$

b **i** Yes **ii** $\frac{p}{q} = 1.4$ or $p = 1.4q$
c **i** No
d **i** Yes **ii** $\frac{C}{m} = 3.6$ or $C = 3.6m$

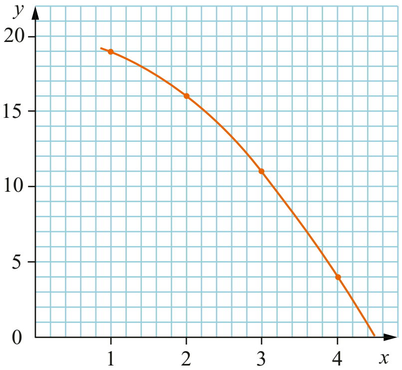
3 a i



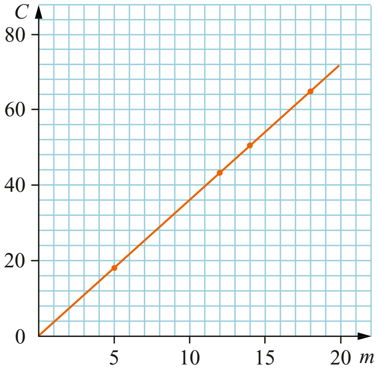
b i



c i



d i



a, b, d ii Graphs are straight lines with a positive gradient and they pass through the origin.

4 $\frac{M}{V} = \frac{1.2V}{V} = 1.2$

$\therefore$ M and V are in direct linear proportion.

5 Yes, $\frac{W}{t} = 15$

6 Yes, $\frac{P}{s} = 4$

7 Yes, $\frac{C}{m} = 2.4$

8 $\frac{n}{d} = \frac{d}{2} \times \frac{1}{d} = \frac{1}{2}$

Hence n and d are in direct linear proportion.

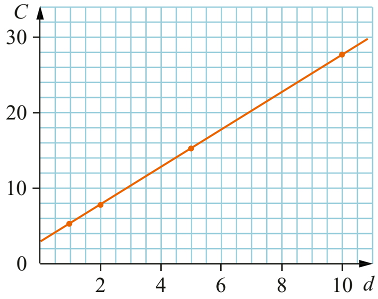
9 $\frac{e}{m} = \frac{2}{7}m \times \frac{1}{m} = \frac{2}{7}$

Hence e and m are in direct linear proportion.

10 a No

b i

d	1	2	5	10
C	5.85	7.90	15.65	27.90



ii Straight line with positive gradient but does not pass through the origin.

c

d	1	2	5	10
C	5.85	7.90	15.65	27.90
$\frac{C}{d}$	5.85	3.95	3.13	2.79

No

11 a Yes **b** Yes **c** Yes
d Yes **e** Yes **f** Yes
g No **h** No **i** No
j Yes **k** No **l** No

Exercise 11E

1 a

A	250	400	700	1000
€	200	320	560	800

b i Yes, $\frac{€}{A} = 0.8$ **ii** $€ = 0.8A$

2 a

M (m/s)	10	15	20	30
K (km/s)	36	54	72	108

b i Yes, $\frac{K}{M} = 3.6$ **ii** $K = 3.6M$

3 a

M	20	50	60	70
K	32	80	96	112

b i Yes, $\frac{K}{M} = 1.6$ **ii** $K = 1.6M$

4 a

E	30	40	60	100
M	12	16	24	40

b i Yes, $\frac{M}{E} = 0.4$ **ii** $M = 0.4E$

Exercise 11F

1 a $k = 7$ so $\frac{m}{V} = 7$ or $m = 7V$

b $m = 7 \times V = 7 \times 45 = 315$ g

c $840 = 7 \times V$

$V = \frac{840}{7} = 120$ cm^3

2 a $F = 3A$ **b** 6.9 t **c** 1.8 ha
3 a $W = 30t$ **b** 1350 words **c** 20 min
4 a $I = 50n$ **b** $750 **c** 18 months

5 **a** $\frac{d}{s} = k$ or $d = ks$

b $k = \frac{60}{20} = 3$

The equation can now be written as $d = 3s$.

c When $s = 25$, $d = 3 \times 25 = 75$ km

d When $d = 54$, $54 = 3 \times s$

$s = \frac{54}{3} = 18$ km/h

6 **a** $\frac{V}{A} = k$ or $V = kA$ **b** $k = 0.08$

c 9.6 L **d** 150 m^2

7 **a** $\frac{S}{C} = k$ or $S = kC$ **b** $k = 3.75$

c 45 units **d** 16 mm

8 **a**

x	5	10	15
y	9	18	27

b

x	4	8	20
y	10	20	50

c

t	8	15	25
M	4.8	9	15

d

x	7	14	24
y	5.25	10.5	18

Language in mathematics

1 **a** Two quantities are directly proportional if an increase in one of them causes a proportional increase in the other.

b Two quantities are inversely proportional if an increase in one of them causes a proportional decrease in the other.

c If two quantities are in direct proportion then the graph of y against x is a straight line that passes through the origin.

Check your skills

1 B **2** B **3** B **4** D **5** D
6 C **7** D **8** A **9** D

Review set 11A

1 **a** 0.18 kg/m^2 **b** 25 m/s
2 **a** \$16/m **b** 72 km/h
3 **a** Inverse **b** Neither **c** Direct
4 **a** Inverse **b** Neither
c Neither **d** Direct
5 **a** No **b** Yes
6 **a** No **b** Yes
7 **a** ≈18 pounds **b** ≈18 kg
8 **a** $k = 4.5$ **b** $p = 22.5$ **c** $m = 7$

Review set 11B

1 **a** 62c/d **b** $19\frac{4}{9}$m /s
2 **a** 150 t/ha **b** 126 km/h
3 **a** Direct **b** Neither **c** Inverse
4 **a** Neither **b** Direct
c Inverse **d** Neither
5 **a** Yes **b** No
6 **a** Yes **b** No
7 **a** ≈45 km/h **b** ≈56 mph

8

x	8	11	14
y	12	16.5	21

Review set 11C

1 **a** 20 mL/s **b** 25 m/s
2 **a** 6.4 kg/L **b** 64.8 km/h
3 **a** Direct **b** Inverse **c** Neither
4 **a** Neither **b** Neither
c Direct **d** Inverse
5 **a** Yes **b** Yes
6 **a** Yes **b** No
7 **a** 26 kg **b** 75 kg
8 **a** $k = 8$ **b** $r = 112$ **c** $v = 25$

Review set 11D

1 **a** 7.2 L/d **b** $11\frac{1}{9}$m /s
2 **a** \$187.20/h **b** 28.8 km/h
3 **a** Neither **b** Inverse **c** Direct
4 **a** Neither **b** Direct
c Inverse **d** Neither
5 **a** Yes **b** Yes
6 **a** Yes **b** Yes
7 **a** €560 **b** A\$325

8

x	5	6	9
y	4	4.8	7.2

CHAPTER 12 EQUATIONS AND INEQUALITIES

Exercise 12A

1 **a** $3x - 5 + 5 = 8 + 5$

$\frac{3x}{3} = \frac{13}{3}$

$x = \frac{13}{3} = 4\frac{1}{3}$

b $5x + 3 - 3 = -11 - 3$

$\frac{5x}{5} = \frac{-14}{5}$

$x = -\frac{14}{5} = -2\frac{4}{5}$

2 **a** $x = 7$ **b** $x = -3$ **c** $x = -2$
d $x = -\frac{2}{3}$ **e** $x = -\frac{6}{5}$ **f** $x = \frac{5}{2}$
g $x = 2$ **h** $x = \frac{1}{3}$ **i** $x = -\frac{8}{7}$
j $x = -\frac{2}{3}$ **k** $x = 1$ **l** $x = -\frac{7}{2}$
m $x = 11$ **n** $x = -\frac{15}{4}$ **o** $x = -2$
p $x = 3$ **q** $x = \frac{5}{7}$ **r** $x = 9$
s $x = -4$ **t** $x = 8$ **u** $x = -\frac{5}{2}$
v $x = -1$ **w** $x = 3$ **x** $x = -5$

3 $\frac{p}{5} - 3 + 3 = 8 + 3$

$\frac{p}{5} = 11$

$\frac{p}{5} \times 5 = 11 \times 5$

$p = 55$

4 **a** $x = 10$ **b** $x = 15$ **c** $x = -25$
d $x = -42$ **e** $x = 42$ **f** $x = 50$

5 **a** No **b** No **c** Yes **d** No

6 $3 = 4x + 7$
$3 - 7 = 4x + 7 - 7$
$-4 = 4x$
$\frac{-4}{4} = \frac{4x}{4}$
$x = -1$

7 **a** $x = \frac{10}{3}$ **b** $x = \frac{9}{4}$ **c** $x = \frac{7}{5}$
d $x = \frac{7}{3}$ **e** $x = \frac{10}{7}$ **f** $x = \frac{13}{3}$

Exercise 12B

1 $7x - 2x - 3 = 2x - 2x + 1$
$5x - 3 = 1$
$5x - 3 + 3 = 1 + 3$
$\frac{5x}{5} = \frac{4}{5}$
$x = \frac{4}{5}$

2 **a** $x = 4$ **b** $x = 1$ **c** $x = 1$
d $x = -1$ **e** $x = -2$ **f** $x = 1$
g $x = 9$ **h** $x = -10$ **i** $x = 3$

3 **a** $x = -\frac{9}{4}$ **b** $x = \frac{4}{5}$ **c** $x = 2$
d $x = -\frac{5}{2}$ **e** $x = -\frac{14}{3}$ **f** $x = \frac{13}{2}$
g $x = 1$ **h** $x = -7$ **i** $x = \frac{3}{10}$
j $a = -1$ **k** $s = -\frac{13}{5}$ **l** $x = \frac{7}{5}$
m $a = \frac{19}{6}$ **n** $y = -\frac{9}{5}$ **o** $p = \frac{3}{2}$

4 LHS: $4 \times (-3) + 3 = -9$
RHS: $7 - 2 \times (-3) = 13$
LHS ≠ RHS, $x = 3$ is not a solution.

5 **a** No **b** Yes **c** No
d Yes **e** Yes **f** No

6 $4x - 12 - 6x - 3 = 2$
$-2x - 15 = 2$
$-2x - 15 + 15 = 2 + 15$
$-2x = 17$
$\frac{-2x}{-2} = \frac{17}{-2}$
$x = -\frac{17}{2} = -8\frac{1}{2}$

7 **a** $x = 2$ **b** $x = -1$ **c** $x = 3$
d $x = 5$ **e** $x = 0$ **f** $x = 2$
g $x = -4$ **h** $x = -2$

8 **a** $x = \frac{7}{2}$ **b** $x = \frac{3}{5}$ **c** $x = 4$
d $x = \frac{7}{5}$ **e** $x = -\frac{3}{8}$ **f** $x = 3$
g $x = \frac{5}{2}$ **h** $x = 5$ **i** $x = \frac{17}{8}$
j $x = 0$ **k** $x = 0$ **l** $x = \frac{31}{11}$

9 $7 = -2 - 3(2x - 5)$
$7 = -2 - 6x + 15$
$7 = 13 - 6x$
$7 - 13 = 13 - 13 - 6x$
$-6 = -6x$
$\frac{-6}{-6} = \frac{-6x}{-6}$
$1 = x$
$x = 1$

10 **a** $x = 2$ **b** $x = -5$ **c** $x = \frac{17}{10}$
d $x = \frac{19}{3}$ **e** $x = 2$ **f** $x = \frac{23}{10}$
g $x = 4$ **h** $x = 0$

Exercise 12C

1 $\frac{7x - 5}{3} \times 3 = 4 \times 3$
$7x - 5 = 12$
$7x - 5 + 5 = 12 + 5$
$\frac{7x}{7} = \frac{17}{7}$
$x = \frac{17}{7}$ or $2\frac{3}{7}$

2 **a** $x = 13$ **b** $x = 2$ **c** $x = 16$
d $x = -8$ **e** $x = 2$ **f** $x = -2$
g $x = 7$ **h** $x = 5$ **i** $x = -\frac{7}{2}$
j $x = -1$ **k** $x = \frac{41}{2}$ **l** $x = -\frac{5}{2}$
m $x = 3$ **n** $x = \frac{13}{2}$ **o** $x = -\frac{1}{2}$

3 $\frac{3x}{5} + 7 - 7 = -3 - 7$
$\frac{3x}{5} = -10$
$\frac{3x}{5} \times 5 = -10 \times 5$
$3x = -50$
$\frac{3x}{3} = \frac{-50}{3}$
$x = -\frac{50}{3} = -16\frac{2}{3}$

4 **a** $x = 15$ **b** $x = -30$ **c** $x = 35$
d $x = 9$ **e** $x = -5$ **f** $x = \frac{21}{5}$
g $x = 5$ **h** $x = \frac{34}{3}$ **i** $x = -\frac{19}{4}$
j $x = -\frac{1}{2}$ **k** $x = -\frac{8}{3}$ **l** $x = \frac{27}{2}$

5 LCD of 3 and 5 is 15.
Multiply both sides by 15.
$\frac{2x - 1}{3} \times 15 = \frac{3x}{5} \times 15$
$5(2x - 1) = 3(3x)$
$10x - 5 = 9x$
$10x - 10x - 5 = 9x - 10x$
$-5 = -x$
$x = 5$

6 **a** $x = \frac{1}{4}$ **b** $x = \frac{3}{4}$ **c** $x = \frac{1}{20}$
d $x = -\frac{13}{20}$ **e** $x = \frac{37}{10}$ **f** $x = -3$
g $x = -\frac{5}{11}$ **h** $x = \frac{3}{2}$ **i** $x = \frac{28}{29}$
j $x = \frac{6}{11}$

7 $\frac{6x - 5}{5} \times 5 = (2x + 3) \times 5$
$6x - 5 = 10x + 15$
$6x - 5 + 5 = 10x + 15 + 5$
$6x = 10x + 20$
$6x - 10x = 10x - 10x + 20$
$-4x = 20$
$\frac{-4x}{-4} = \frac{20}{-4}$
$x = -5$

8 **a** $x = -\frac{8}{7}$ **b** $x = \frac{13}{17}$ **c** $x = -\frac{7}{13}$
d $x = -\frac{1}{9}$ **e** $x = -\frac{13}{7}$ **f** $x = -\frac{1}{5}$
g $x = -3$ **h** $x = 9$ **i** $x = \frac{37}{19}$
j $x = -\frac{23}{6}$ **k** $x = 2$ **l** $x = -3$

9 LCD of 5 and 7 is 35.

$$35\left(\frac{2x}{5}\right) - 35\left(\frac{3x}{7}\right) = -2 \times 35$$
$$7(2x) - 5(3x) = -70$$
$$14x - 15x = -70$$
$$-x = -70$$
$$x = 70$$

10 **a** $x = \frac{20}{7}$ **b** $x = -5$ **c** $x = 8$
d $x = \frac{20}{3}$ **e** $x = 12$ **f** $x = 6$
g $x = 20$ **h** $x = -8$

Exercise 12D

1 **a** 4 **b** 3 **c** 6 **d** 36
2 **a** 63, 64 **b** 8 **c** 0 **d** 9, 11, 13
3 **a** 4 **b** −2 **c** 1 **d** 2
4 **a** 6, 4 **b** 4, 8 **c** 4, 6 **d** 8, 10, 12
e 1, 2, 3
5 **a** 11 **b** 7 **c** 17 **d** 7

Exercise 12E

1 $s = ut + \frac{1}{2}at^2$

$$200 = u(5) + \frac{1}{2} \times (10) \times (5)^2$$
$$200 = 5u + 125$$
$$200 - 125 = 5u + 125 - 125$$
$$\frac{75}{5} = \frac{5u}{5}$$
$$u = 15$$

2 **a** $a = 26$ **b** $u = 35$
c $t = 12$ **d** $a = 3$
3 **a** $c = 25$ **b** $t = 40$
4 **a** $R = 6\frac{1}{4}$ **b** $R = \frac{185}{36} = 5\frac{5}{36}$
5 $E = 144$
6 **a** $s = 15$ **b** $a = 6$
7 **a** $C = 48\frac{8}{9}°$ **b** $C = 15\frac{5}{9}°$ **c** $C = 100°$
8 **a** $C = 35.81$ cm **b** $d = 79.58$ cm
9 **a** $C = 54.0$ cm **b** $r = 15.9$ m
10 **a** $y = 10$ **b** $x = 6\frac{1}{3}$
11 **a** $C = 82\frac{2}{9}°$ **b** $F = 68°$
12 **a** 100 km/h **b** 260 km **c** 7 h 52 min
13 **a** 80 cm² **b** 6 cm
14 **a** 7916.8 cm³ **b** 1.6 cm
15 **a** $E = 1250$ **b** $m = 8.64$
16 $h = 92.5$ **17** $y_2 = 6$
18 $s = 9, A = 14.7$ cm²

Exercise 12F

1 **a** $4x < 13$ **b** $5x > 50$
c $10x \geqslant 80$ **d** $3x + 2 < 35$
e $4x + 3 > 70$ **f** $x - 8 \leqslant 30$
g $2x - 13 > 20$ **h** $20x - 9 > 10$
i $\frac{x}{2} + 10 < 19$ **j** $\frac{x}{3} + 5 > 6$

2 $7 - 3(-2) \leqslant 8$
$13 \leqslant 8$ is false $\therefore x = -2$ is not a solution.

3 **a** Yes **b** Yes **c** No **d** Yes
e Yes **f** Yes **g** No **h** Yes
i Yes **j** No **k** No **l** No
m No **n** Yes

4 Try $x = 0$: $3 + 2(0) > 8$
$3 > 8$ is false $\therefore x = 0$ is not a solution.
Try $x = -5$: $3 + 2(-5) > 8$
$-7 > 8$ is false $\therefore x = 5$ is not a solution.
Try $x = 5$: $3 + 2(5) > 8$
$13 > 8$ is true $\therefore x = 5$ is a solution.

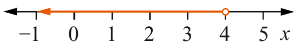
5 **a** $x \leqslant -3$ **b** $x \leqslant 35$ **c** $x \leqslant -2\frac{1}{2}$
d $x \geqslant 3$ **e** $x \geqslant 2$ **f** $x < -1$
g $x > 2$ **h** $x \leqslant 1$ **i** $x > -6$

6 **a** Infinitely many **b** Infinitely many

Exercise 12G

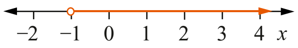
1 **a** $5x - 12 + 12 < 8 + 12$

$$5x < 20$$
$$\frac{5x}{5} < \frac{20}{5}$$
$$x < 4$$

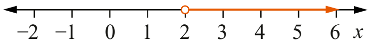


b $7 > 4 - 3x$

$$7 - 4 > 4 - 4 - 3x$$
$$3 > -3x$$
$$\frac{3}{-3} < \frac{-3x}{-3}$$
$$-1 < x$$
$$x > -1$$

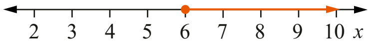


2 **a** $x > 2$

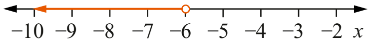


b $x \leqslant 5$

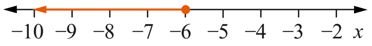
c $x \geqslant 6$



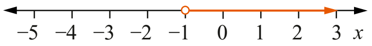
d $x < -6$



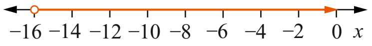
e $x \leqslant -6$



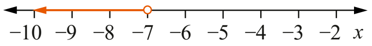
f $x > -1$



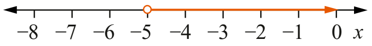
g $x > -16$



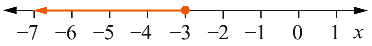
h $x < -7$



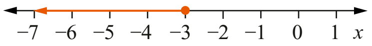
i $x > -5$



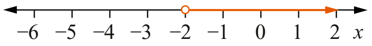
j $x \leqslant -3$



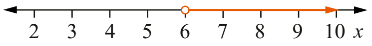
k $x \leqslant -3$



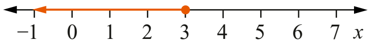
l $x > -2$



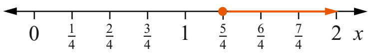
m $x > 6$



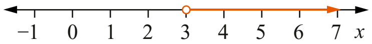
n $x \leqslant 3$



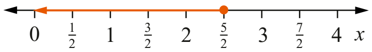
o $x \geqslant \frac{5}{4}$



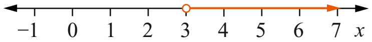
p $x > 3$



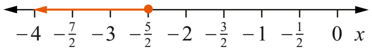
q $x \leqslant \frac{5}{2}$



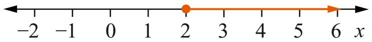
r $x > 3$



s $x \leqslant -\frac{5}{2}$

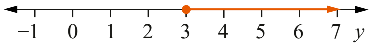


t $x \geqslant 2$

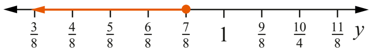


u $x \geqslant -7$

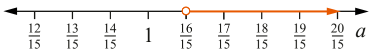
3 a $y \geqslant 3$



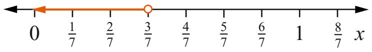
b $y \leqslant \frac{7}{8}$



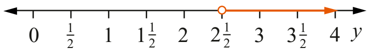
c $a > \frac{16}{15}$



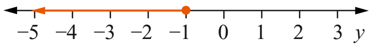
d $x < \frac{3}{7}$



e $y > 2\frac{1}{2}$

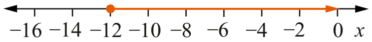


f $y \leqslant -1$



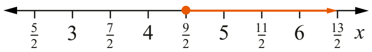
4 a $x > -6$

b $x \geqslant -12$

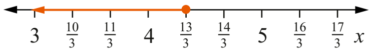


c $x > -6$

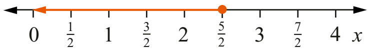
d $x \geqslant \frac{9}{2}$



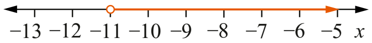
e $x \leqslant \frac{13}{3}$



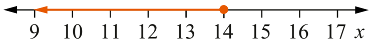
f $x \leqslant \frac{5}{2}$



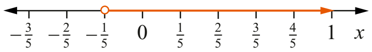
g $x > -11$



h $x \leqslant 14$



i $x > -\frac{1}{5}$



5 a $x \geqslant \frac{1}{2}$ **b** $x < 5$ **c** $x < -1$
d $x \geqslant -\frac{5}{2}$ **e** $x < \frac{4}{5}$ **f** $x \leqslant 1$
g $x > -\frac{7}{2}$ **h** $x < \frac{8}{9}$ **i** $x \leqslant \frac{1}{3}$
j $x \geqslant 1$ **k** $x > \frac{5}{2}$ **l** $x \geqslant \frac{1}{4}$
m $x < 0$ **n** $x \leqslant -\frac{5}{3}$ **o** $x \geqslant 1$
p $x > \frac{6}{5}$ **q** $x \geqslant 0$ **r** $x > \frac{31}{11}$

6 a $x \geqslant -\frac{1}{7}$ **b** $x < \frac{11}{9}$ **c** $x \leqslant \frac{15}{13}$
d $x > -\frac{12}{23}$ **e** $x > 1$ **f** $x \geqslant \frac{6}{19}$
g $x > \frac{11}{38}$ **h** $x \leqslant \frac{33}{14}$ **i** $x < -\frac{45}{4}$
j $x < \frac{23}{2}$ **k** $x \geqslant \frac{1}{2}$ **l** $x \geqslant \frac{16}{17}$

7 a $x \geqslant -\frac{7}{9}$ **b** $x < \frac{23}{15}$ **c** $x \leqslant \frac{5}{3}$
d $x > -\frac{3}{11}$ **e** $x < -\frac{73}{4}$ **f** $x < \frac{27}{2}$

Language in mathematics

2 a solution **b** inequalities **c** equation
d variable **e** substitute **f** solve

Check your skills

1 A **2** D **3** A **4** B **5** C
6 B **7** A **8** C **9** D **10** D
11 D **12** D **13** A **14** A **15** D

Review set 12A

1 a $x = -4$ **b** $x = \frac{11}{2}$ **c** $x = \frac{1}{2}$
d $x = -84$ **e** $x = -\frac{5}{4}$ **f** $x = -23$
2 $y = 2$
3 $x = 1$
4 a $x = \frac{1}{6}$ **b** $x = -\frac{9}{5}$ **c** $x = 3$
d $x = 2$ **e** $x = 7\frac{1}{2}$ **f** $x = -\frac{7}{2}$
g $x = 6\frac{1}{2}$ **h** $x = 24$ **i** $x = \frac{9}{16}$
5 a $x = 3$ **b** 17 **c** 25 cm
6 a $a = 17$ **b** $C = 37\frac{7}{9}°$
7 a $x < -2$ **b** $x \geqslant -6$ **c** $x \geqslant \frac{1}{4}$
d $x > \frac{7}{2}$ **e** $x \leqslant -\frac{11}{2}$ **f** $x > -\frac{11}{7}$

Review set 12B

1 a $x = 1$ **b** $x = \frac{11}{9}$ **c** $x = \frac{15}{2}$
d $x = -15$ **e** $x = \frac{3}{5}$ **f** $x = 11$
2 $t = -\frac{14}{3}$ **3** $x = \frac{20}{11}$
4 a $x = \frac{1}{8}$ **b** $x = \frac{30}{11}$ **c** $x = 2$
d $x = 2$ **e** $x = \frac{35}{3}$ **f** $x = 2$
g $x = 7$ **h** $x = \frac{40}{3}$ **i** $x = \frac{8}{9}$
5 a 5 **b** 12
6 a $t = 11$ **b** $y = 7$
7 a $x \geqslant -4$ **b** $x < 2$ **c** $x < -2$
d $x > -\frac{1}{3}$ **e** $x \geqslant 3$ **f** $x < -2$

Review set 12C

1 a $x = \frac{7}{4}$ **b** $x = 4$ **c** $x = -12$
d $x = 36$ **e** $x = 4$ **f** $x = -\frac{9}{2}$
2 a $y = -\frac{2}{5}$ **b** $x = 9$
3 $x = 4$
4 a $a = -5$ **b** $a = \frac{17}{5}$ **c** $p = \frac{13}{2}$
d $x = \frac{19}{10}$ **e** $x = \frac{8}{3}$ **f** $x = -\frac{13}{2}$
g $x = \frac{23}{2}$ **h** $x = -15$ **i** $x = \frac{3}{17}$
5 a $-\frac{11}{2}$ **b** 32 cents
c $-\frac{27}{7}$ **d** 36 km

6 **a** $C = 31\frac{1}{9}°$ **b** $F = 122°$

7 **a** $x < -6$ **b** $x > 5$ **c** $x \geqslant \frac{3}{2}$

d $x < 5$ **e** $x \leqslant -\frac{1}{3}$ **f** $x \geqslant 3$

Review set 12D

1 **a** $x = \frac{7}{3}$ **b** $x = \frac{1}{7}$ **c** $x = -20$

d $x = -35$ **e** $x = 12$ **f** $x = 4$

2 **a** $y = -\frac{4}{3}$ **b** $x = 4$

3 $x = \frac{13}{5}$

4 **a** $x = -\frac{21}{8}$ **b** $t = -\frac{6}{11}$ **c** $p = \frac{33}{17}$

d $x = -\frac{4}{3}$ **e** $x = \frac{16}{3}$ **f** $x = -\frac{19}{3}$

g $x = \frac{5}{22}$ **h** $x = \frac{37}{29}$ **i** $x = \frac{9}{11}$

5 **a** -2 **b** -19

6 **a** $V = 3016$ cm^3 **b** $h = 0.9$ cm

7 **a** $x > 7$ **b** $x < -3$ **c** $x \geqslant -\frac{10}{3}$

d $d \geqslant -6$ **e** $x \leqslant \frac{2}{3}$ **f** $x \geqslant 1$

CUMULATIVE REVIEW: 11–12

1 **a** **i** 0.27 kg/m^2 **ii** $16\frac{2}{3}$m /s

ii \$22/m **iv** 57.6 km/h

b **i** Inverse **ii** Direct **iii** Direct

iv Neither **v** Inverse

c **i** No **ii** Yes

d **i** 65 km/h **ii** 25 mph

e **i** Yes **ii** No

f **i** $k = 16$ **ii** $v = 240$ **iii** $r = 20$

2 **a** **i** $x = \frac{5}{4}$ **ii** $x = \frac{7}{5}$ **iii** $x = -6$

iv $x = 35$ **v** $x = 1$ **vi** $x = -\frac{1}{3}$

b **i** $y = \frac{1}{4}$ **ii** $x = \frac{23}{2} = 11\frac{1}{2}$

c **i** $x = -\frac{3}{2}$ **ii** $t = \frac{8}{7}$ **iii** $r = \frac{34}{11}$

iv $x = \frac{3}{5}$

d -3 **e** $s = 4075$

f **i** $t > 9$ **ii** $x \leqslant -\frac{8}{5}$ **iii** $x < 10$

iv $x \geqslant \frac{40}{3}$ **v** $x \geqslant \frac{19}{3}$

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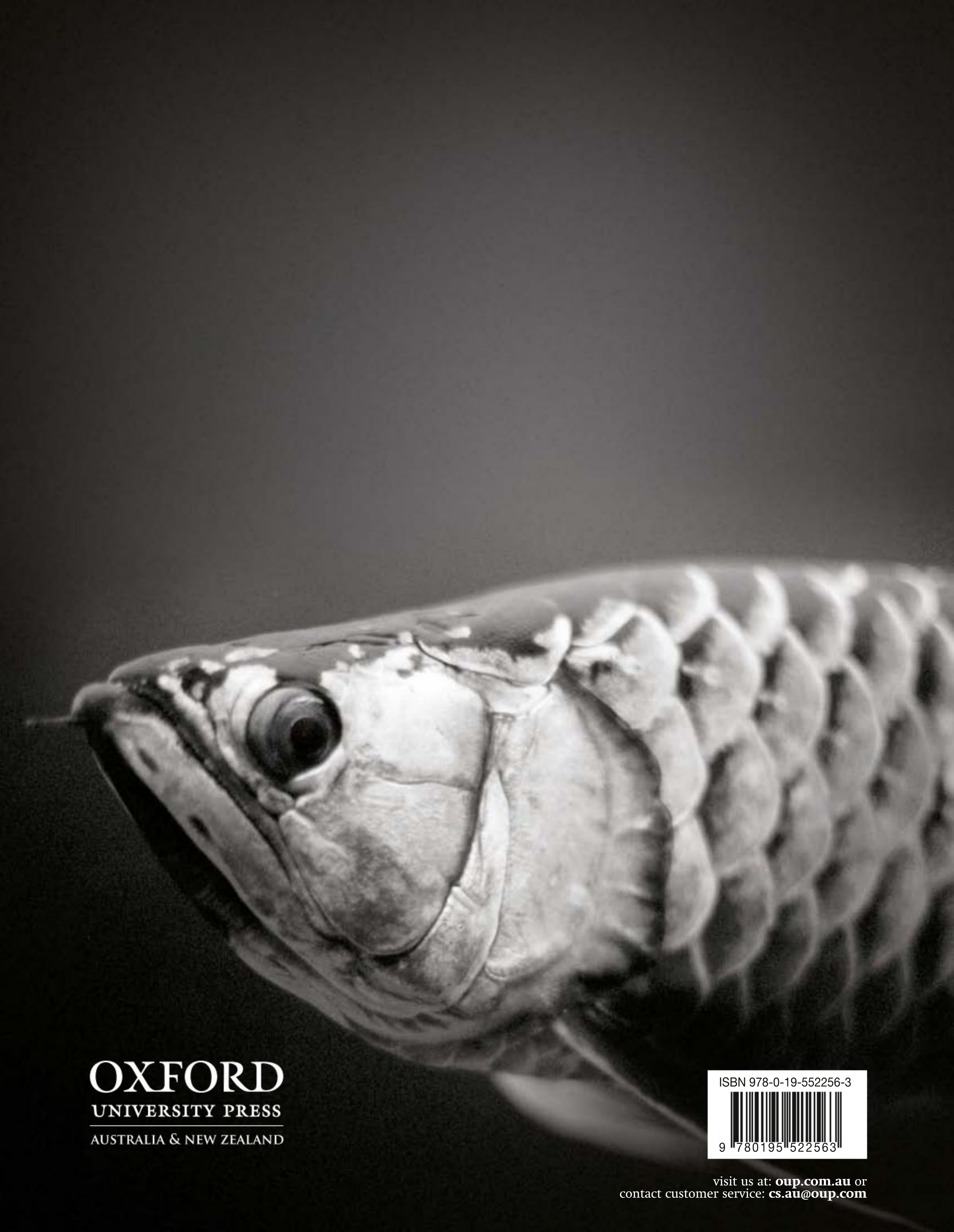

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