MATHEMATICS

OXFORD MATHS 8



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AUSTRALIAN CURRICULUM

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Warning to First Nations Australians
Aboriginal and Torres Strait Islander peoples are advised that this publication may include images or names of people now deceased.

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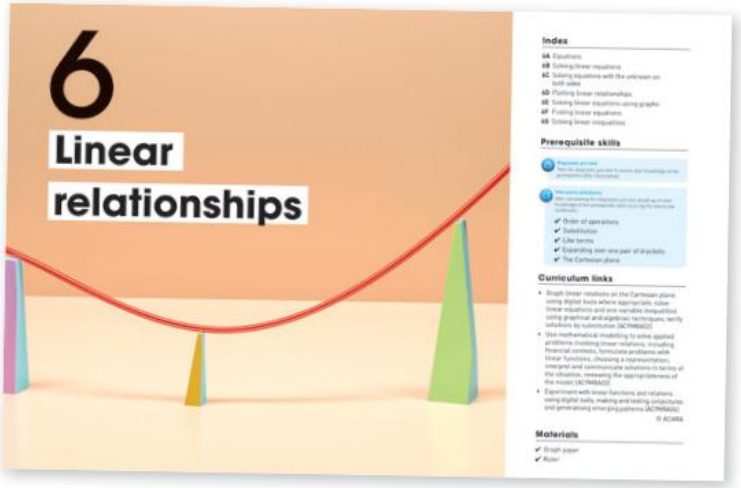
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Oxford Maths 7–10 Australian Curriculum utilises an innovative suite of print and digital resources to guide students on a focused mathematics journey. The series makes maths accessible to students with differing levels of understanding, increasing engagement by giving learners the opportunity to achieve success at their own skill level while also providing comprehensive syllabus coverage.

Key features of Student Books

- Complete access to all digital resources available on Student obook pro.
- Australian Maths Trust (AMT) spreads offer unique questions designed to challenge students and build engagement.
- STEAM projects encourage inter-disciplinary thinking.
- Semester reviews provide an opportunity to revise key concepts from each semester.
- An illustrated glossary of newly introduced mathematics terms.



Each chapter opens with:

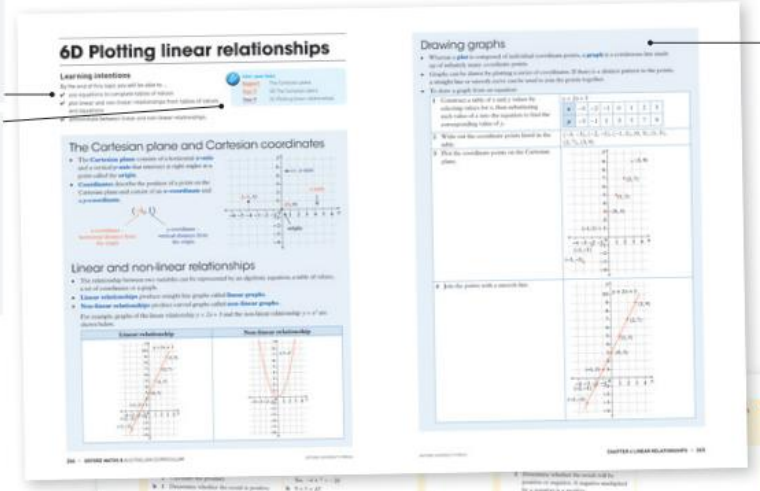
- **Prerequiste skills** with reference to an online diagnostic pre-test and interactive skillsheets.
- **Curriculum links** to all relevant content descriptions in the Australian Curriculum.
- **Materials** used to complete the exercises.

Learning intentions

- Signpost the foundational skills being developed in each section.

Inter-year links

- Provide easy access to support and extension material from each of the 7–10 Student Books as students build knowledge year on year.

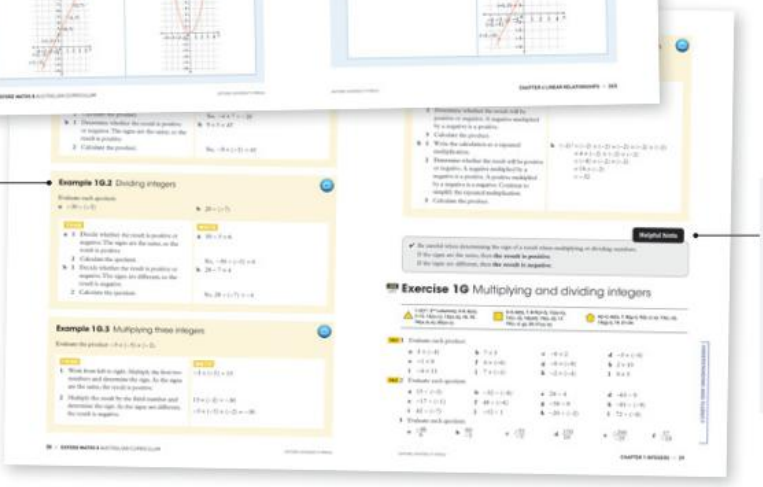


New theory

- Backed by the latest pedagogical research to promote engagement with the material.
- Filled with precise diagrams that bring key concepts to life and aid understanding.

Worked examples

- Outline a step-by-step thought process for solving essential questions with direct reference to the exercises.



Helpful hints

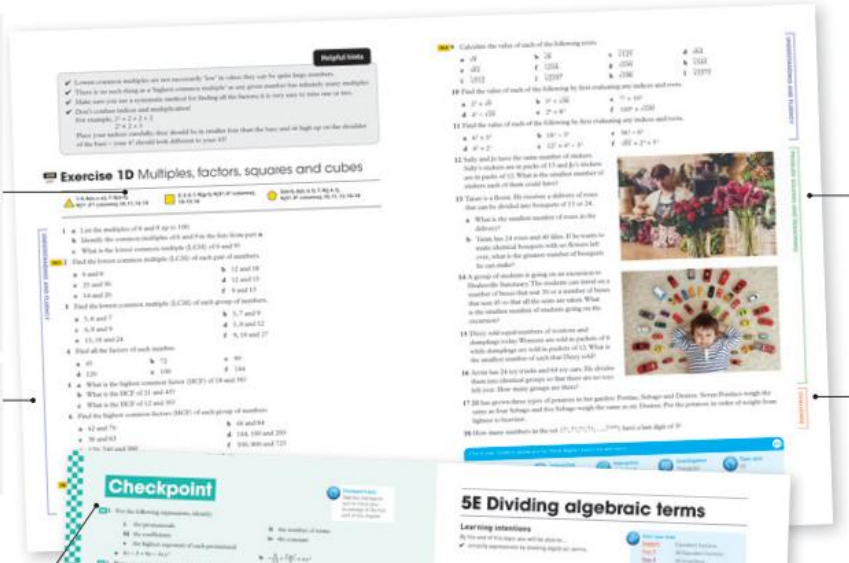
- Provide additional strategies for tackling problems.
- Highlight important elements of the theory.
- Point out common misconceptions.

Differentiated learning pathways

- Separate each exercise into three pathways, tailored for students of all skill levels.
- Each pathway can be assigned based on results of the diagnostic pre-tests that are recommended at the beginning of every chapter.

Understanding and fluency

- Basic exercises dedicated to practicing key concepts.



Problem solving and reasoning

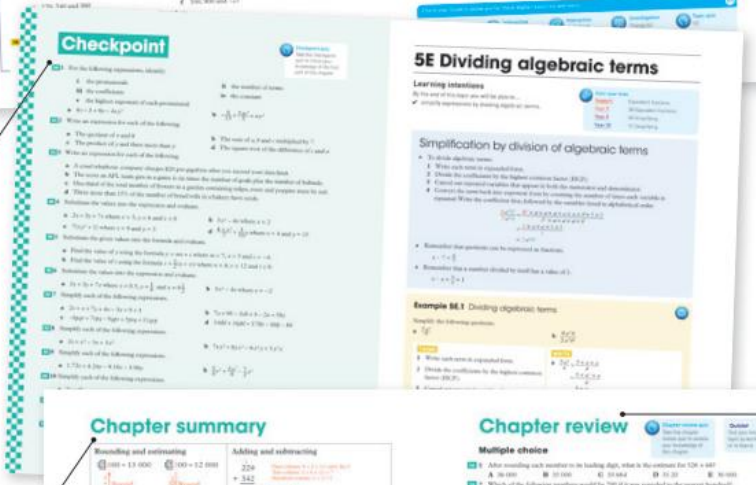
- Comprehensive exercises bring together new ideas and provide engaging contexts from real-world problems.

Challenge

- Advanced exercises designed to build engagment and anticipate future learning outcomes.

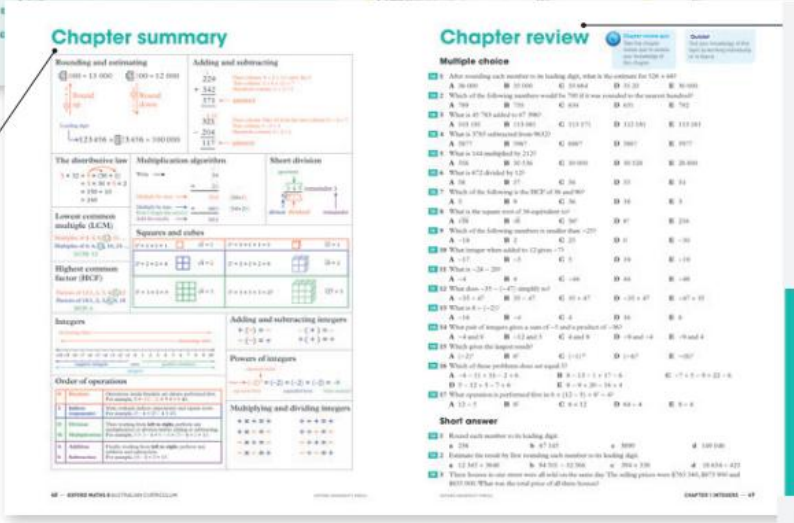
Checkpoint

- A section at the middle of each chapter dedicated to surmising key skills and encourage memory retention.
- Reference to an online checkpoint quiz to gauge student progress.



Chapter summary

- Condense all the theory from each section into one accessible revision page.



Chapter reviews

- Additional practice questions to further consolidate understanding at the end of each chapter.
- Reference to an online chapter review quiz to track results.
- Reference to Quizlet test to revise new terminology.

Integrated STEAM projects

- Take the hard work out of cross-curricular learning with engaging STEAM projects. Two fully integrated projects are included at the end of each book in the series, and are scaffolded and mapped to the Science, Maths and Humanities curricula. The same projects also feature in the corresponding Oxford Humanities and Oxford Science series to assist cross-curricular learning.



Problem solving through design thinking

- Each STEAM project investigates a real-world problem that students are encouraged to problem solve using design thinking.

Full digital support

- Each STEAM project is supported by a wealth of digital resources, including student booklets (to scaffold students through the design-thinking process of each project), videos to support key concepts and skills, and implementation and assessment advice for teachers.

Key features of Student obook pro

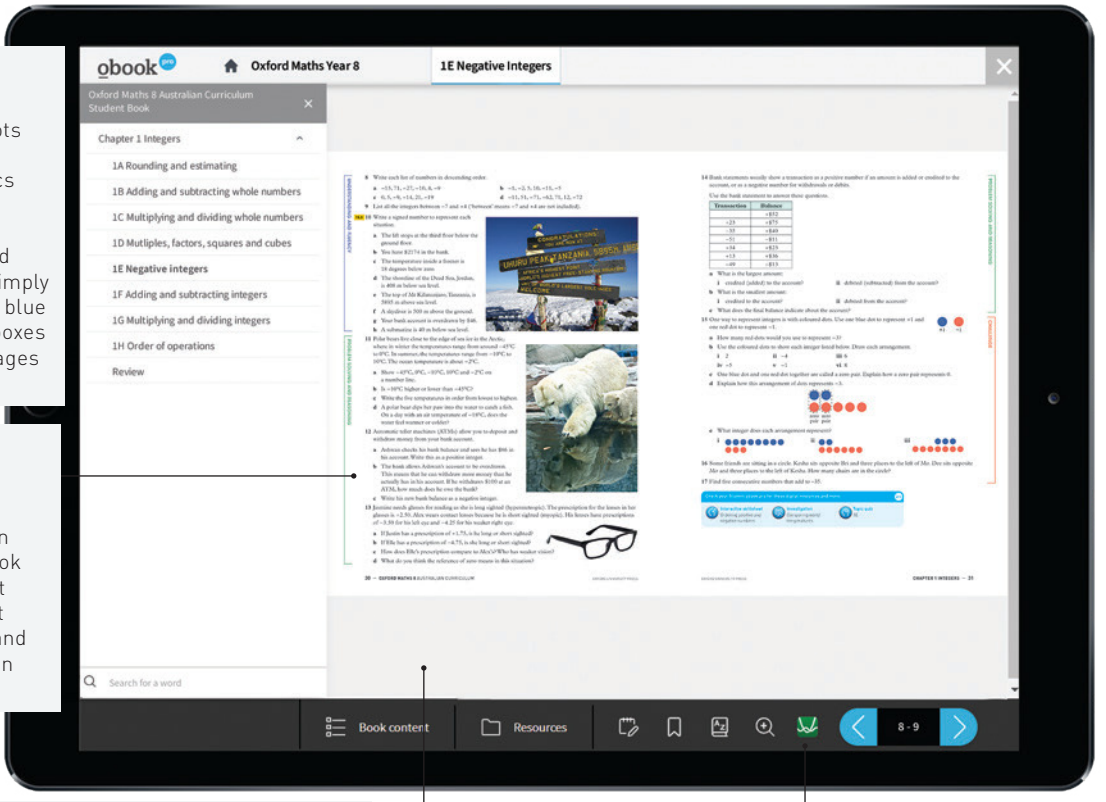
- Student obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- It offers a complete digital version of the Student Book with interactive note-taking, highlighting and bookmarking functionality, allowing students to revisit points of learning.
- A complete ePDF of the Student Book is also available for download for offline use and read-aloud functionality.

Integrated digital resources

- Integrated hotspots allow students to access diagnostics tests, quizzes, interactive skill sheets, videos and inter-year links simply by clicking on the blue digital resource boxes throughout the pages of the book.

Complete digital version of the Student Book

- The digital version of the Student Book is true to the print version, making it easy to navigate and transition between print and digital.



Toolbar features

- Notes can be added and saved to the text by simply selecting and highlighting.
- Bookmarks can be saved to any page.
- Australian Concise Oxford Dictionary can provide immediate definitions to any word within the text.

Desmos integration

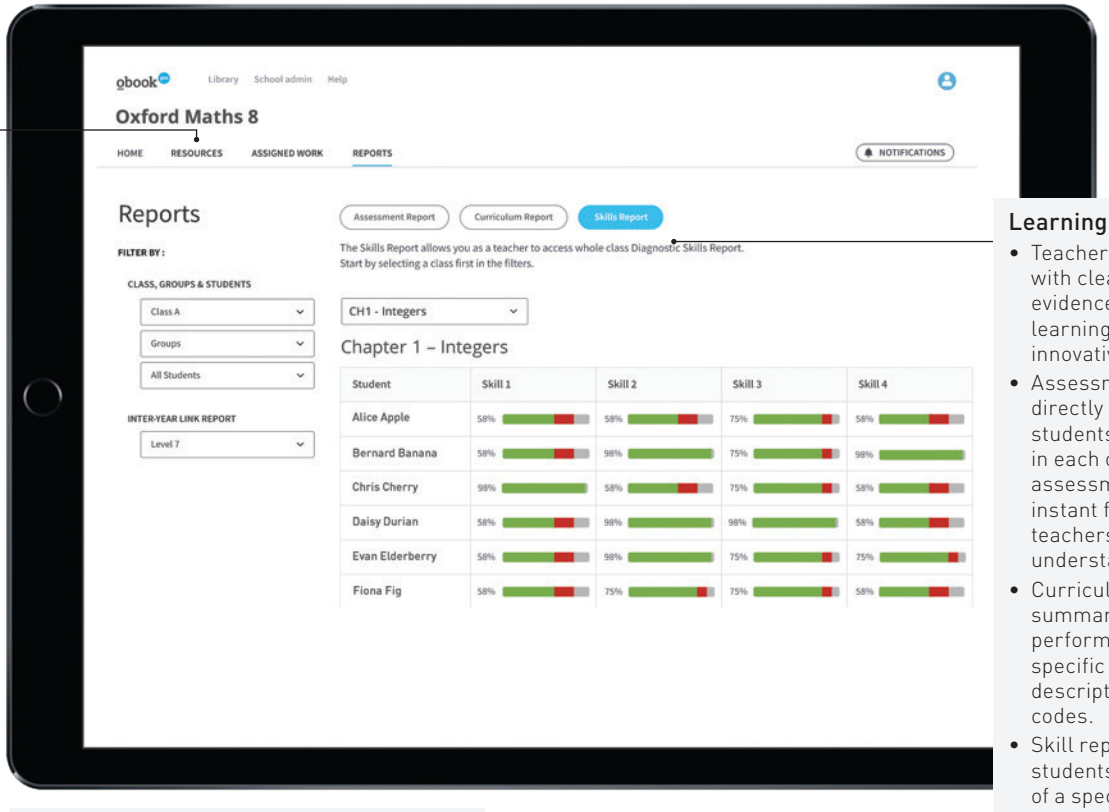
- Our partership with Desmos allows students to access a suite of calculator tools as they read through the text, providing convenient graphical support as well as the opportunity to investigate plane geometry and Cartesian coordinates.

Benefits for students

- Integrated Australian Concise Oxford Dictionary look up feature.
- Targeted instructional videos for every worked example question.
- Groundwork resources to support assumed knowledge.
- Interactive assessments to consolidate understanding.
- Auto-marked practice exam question sets.
- Integrated Quizlet sets, including real-time online quizzes with live leaderboards.
- Access to online assessment results to track progress.

Key features of Teacher obook pro

- Teacher obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- Each chapter and topic of the Student Book is accompanied by full teaching support, including assessment reporting, worked solutions, chapter tests, detailed teacher notes and lesson plans.
- Teachers can use their Teacher obook pro to share notes and easily assign resources or assessments to students, including due dates and email notifications.



Learning pathway reports

- Teachers are provided with clear and tangible evidence of student learning progress through innovative reports.
- Assessment reports directly show how students are performing in each online interactive assessment, providing instant feedback for teachers about areas of understanding.
- Curriculum reports summarise student performance against specific curriculum content descriptors and curriculum codes.
- Skill reports indicate the students understanding of a specific skill in mathematics.

Additional resources

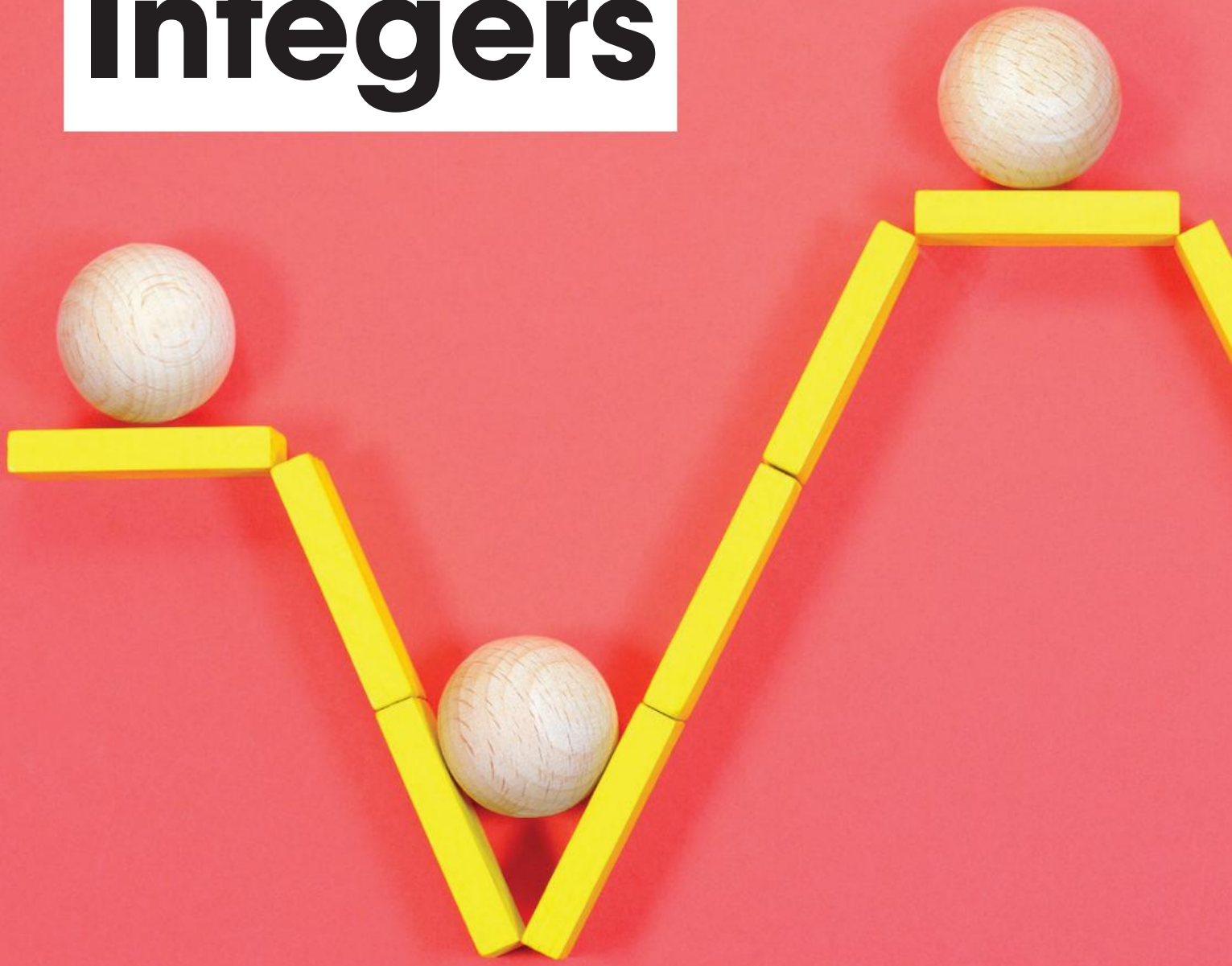
- Each chapter of the Student Book is accompanied by additional interactive skillsheets, worksheets, investigations and topic quizzes to help students progress.

Benefits for teachers

- Diagnostic pre-tests and chapter tests that track students' progress against Study Design key knowledge, providing detailed learning pathway reports that differentiate each student's ability in each skill.
- Assign reading and assessments to students either individually, or in groups – administration is taken care of!
- Set-up classes, monitor student progress and graph results.
- Worked solutions for every Student Book question.
- Detailed teacher notes, teaching programs and lesson plans.

1

Integers





Index

1A Rounding and estimating
1B Adding and subtracting whole numbers
1C Multiplying and dividing whole numbers
1D Multiples, factors, squares and cubes
1E Negative integers
1F Adding and subtracting integers
1G Multiplying and dividing integers
1H Order of operations

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Adding and subtracting two-digit numbers
- ✔ Times tables
- ✔ Multiplying and dividing by powers of 10
- ✔ Number lines

Curriculum links

- Use the 4 operations with integers and with rational numbers, choosing and using efficient strategies and digital tools where appropriate (AC9M8N04)
- Use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts; formulate problems, choosing efficient calculation strategies and using digital tools where appropriate; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model (AC9M8N05)

© ACARA

Materials

- ✔ Calculator

1A Rounding and estimating

Learning intentions

By the end of this topic you will be able to ...

- ✔ round whole numbers
- ✔ estimate the results of simple calculations
- ✔ assess the accuracy of estimations.

Inter-year links

Support — Place value

Year 7 — 1A Place value

Place value

- Place value charts can be used to determine the value of each digit in a number.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
3	2	8	7	2	6	3

The place value of 3 is 3 000 000
The place value of 2 is 200 000
The place value of 8 is 80 000
The place value of 7 is 7000
The place value of 2 is 200
The place value of 6 is 60
The place value of 3 is 3

→ 3 2 8 7 2 6 3

Rounding

- **Rounding** a number involves replacing a number with an approximation that is simpler and easier to use in calculations. Using rounded numbers makes calculations less accurate.
  - → 'Approximately equal' is shown using the symbol '≈'.
- To round to the nearest thousand, place a box around the digit in the thousands place.
  - → If the digit to the right of the box is equal to or greater than 5, **round up**.
  - → If the digit to the right of the box is less than 5, **round down**.

$12\,600 \approx 13\,000$

9 8 7 (6) 5 — Round up

$17\,300 \approx 17\,000$

4 (3) 2 1 0 — Round down

- Rounding to a number's **leading digit** involves boxing the first digit and checking if the second digit is less than or greater than 5.

Leading digit

$123\,456 = 123\,456 \approx 100\,000$

Estimation

- To **estimate** is to find an approximate value of a calculation, usually involving rounding.
  For example, a crowd of 10 213 on Day 1 and a crowd of 6156 on Day 2 at Bells Beach could be described as 10 213 + 6156 ≈ 10 000 + 6000, which is approximately 16 000 people. The actual number of people is 16 369.

Example 1A.1 Rounding a number



Write an approximation for each number by rounding to the nearest hundred.

a 17 845 **b** 4565 **c** 992

THINK	WRITE
a **1** Draw a box around the digit in the hundreds place.	**a** 17 $\boxed{8}$45
2 Consider the digit to the right of the boxed digit. This digit is less than 5, so do not change the boxed digit.	17 $\boxed{8}$45
3 All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero.	17 $\boxed{8}$00 17 845 ≈ 17 800
b **1** Draw a box around the digit in the hundreds place.	**b** 4$\boxed{5}$65
2 Consider the digit to the right of the boxed digit. This digit is greater than 5, so add one to the digit in the box.	4$\boxed{5}$65
3 All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero.	4$\boxed{6}$00 4565 ≈ 4600
c **1** Draw a box around the digit in the hundreds place.	**c** $\boxed{9}$92
2 Consider the digit to the right of the boxed digit. This digit is greater than 5, so add one to the digit in the box. The boxed digit changes from 9 to 10, so write 0 in the boxed digit's place and add one to the place to the left.	$\boxed{9}$92
3 Replace all digits to the right of the boxed digit with a zero.	1$\boxed{0}$00 992 ≈ 1000

Example 1A.2 Estimation by rounding



Estimate the result of each calculation by first rounding each number to its leading digit.

a 1307 + 4875 **b** 576 × 42 **c** 45 229 ÷ 5

THINK	WRITE
1 Round each number to its leading digit.	**a** $\boxed{1}$307 + $\boxed{4}$875 ≈ 1000 + 5000
2 Perform the calculation.	1000 + 5000 = 6000
3 Write the answer using the '≈' symbol.	1307 + 4875 ≈ 6000
	b $\boxed{5}$76 × $\boxed{4}$2 ≈ 600 × 40
	600 × 40 = 24 000
	576 × 42 ≈ 24 000
	c $\boxed{4}$5 229 ÷ $\boxed{5}$ ≈ 50 000 ÷ 5
	50 000 ÷ 5 = 10 000
	45 229 ÷ 5 ≈ 10 000

Helpful hints

- ✔ Remember that the place of a digit in a number indicates its value.
  For example, the 4 in 3452 represents 400, *not* 4.
- ✔ The digit 0 (zero) is important. It shows that there is nothing in that place value position and it keeps all other digits in the correct places.
- ✔ Be careful when rounding to a specific place value. If you draw a square around the wrong place, your answer will not be correct. Use the place value chart to double-check if you are unsure.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1 000 000	100 000	10 000	1000	100	10	1

ANS p490

Exercise 1A Rounding and estimating

▲ 1–6, 7(a, c, e), 8(1st, 2nd columns), 9–11, 13–15, 18 ■ 3–6, 7(b, d, f), 8(2nd, 3rd columns), 9–10(e–h), 12, 14, 16, 17, 19, 20 ⬟ 3, 4, 6, 8(i–l), 10, 12, 16, 17, 20–22

UNDERSTANDING AND FLUENCY

1 Decide whether each number is closer to 400 or 500.

a 438 **b** 477 **c** 462 **d** 455 **e** 433 **f** 449

2 Decide whether each number is closer to 6000 or 7000.

a 6789 **b** 6306 **c** 6010 **d** 6505 **e** 6880 **f** 6448

1A.1 **3** **a** Round each number to the nearest ten.

i 482 **ii** 6377 **iii** 56 026 **iv** 738 494 **v** 8075 **vi** 904 507

b Round each number in part **a** to the nearest hundred.

4 **a** Round each number to the nearest thousand.

i 36 428 **ii** 7510 **iii** 183 915 **iv** 50 703 **v** 6052 **vi** 825

b Round each number in part **a** to the nearest hundred.

5 Round each number to its leading digit.

a 77 **b** 42 **c** 81 **d** 347 **e** 160

f 555 **g** 909 **h** 2489 **i** 6902 **j** 22 117

6 Round each number to its leading digit.

a 94 **b** 99 **c** 952 **d** 92 949 **e** 9008 **f** 960

7 Give three examples of numbers that round to each of these approximations if you round to the leading digit.

a 40 **b** 700 **c** 3000 **d** 50 000 **e** 800 000 **f** 2 000 000

8 Calculate:

a 900 + 700 **b** 300 − 10 **c** 30 × 20

d 80 ÷ 4 **e** 500 × 40 **f** 800 + 3000

g 200 ÷ 5 **h** 6000 − 500 **i** 40 000 ÷ 20

j 7000 × 300 **k** 20 000 − 9000 **l** 5000 + 10 000

1A.2 **9** Estimate the result of each calculation by first rounding each number to its leading digit.

a 468 + 731 **b** 92 – 38 **c** 27 × 49 **d** 83 ÷ 2

e 582 × 17 **f** 245 + 6379 **g** 4512 ÷ 43 **h** 137 – 51

10 Estimate the result of each calculation by first rounding each number to its leading digit.

a 3694 ÷ 442 **b** 187 × 9364 **c** 7085 – 750 **d** 964 + 5803

e 8277 × 65 234 **f** 13 761 + 8036 **g** 94 113 ÷ 587 **h** 24 905 – 780

11 An office building has 35 782 panes of glass. Write an approximation for this number if you round to:

a the leading digit

b the nearest hundred

c the nearest thousand

d the nearest ten

e the nearest ten thousand.



12 Russia is the largest country in the world, with an area of 17 098 242 km^2. Round this value to:

a its leading digit

b the nearest hundred

c the nearest thousand

d the nearest ten thousand

e the nearest hundred thousand

f the nearest million.

13 Ida is making a budget for her family's groceries. She decides to base her budget on the previous week's grocery bill, in which she spent the following:
Supermarket $184
Fish market $37
Butcher $48
Fruit and vegetable market $29

a Estimate to the nearest $10 the cost of the groceries for Ida's family last week.

b If Ida uses this figure for her budget, will it be an overestimation or underestimation?

14 Tina thinks 9648 rounded to its leading digit is 10 000. Ruby isn't sure, as she thinks the rounded number should have the same number of digits as the original number. Who is correct? Explain.

15 Daniel is saving for a quad bike. He has saved $60 per month for the past 28 months.



a Write an approximation for the number of months by rounding the value to its leading digit.

b Using your answer to part **a**, estimate the amount of money Daniel has saved.

c Does he have enough money to buy the quad bike?

d If he doesn't have enough, estimate how many more months he needs to save.

16 Paving tiles cost $7 each. Antonia needs 385 tiles for the back patio.

a Write an approximation of the number of tiles she needs by rounding to the leading digit.

b Estimate the cost of the tiles, using your answer to part **a**.

c Compare your estimate to the exact cost of the tiles.

PROBLEM SOLVING AND REASONING

17 The school fundraising committee wants to raise money by buying boxes of sunscreen from a wholesaler to sell for a profit. Each box has a wholesale price of $47.

a Estimate the number of boxes the committee could buy with $2000.

b If the committee used this estimate, without performing the exact calculation decide whether there would be money left over or money owed to the wholesaler. Explain your answer.

18 Australia has an area of 7 692 024 km^2 and Indonesia has an area of 1 919 440 km^2.

a Round each value to its leading digit.

b Use approximations to estimate:

i the difference in area between the two countries

ii the area of Australia compared to the area of Indonesia.

c Check how close your estimations are to the exact answers for part **b**.

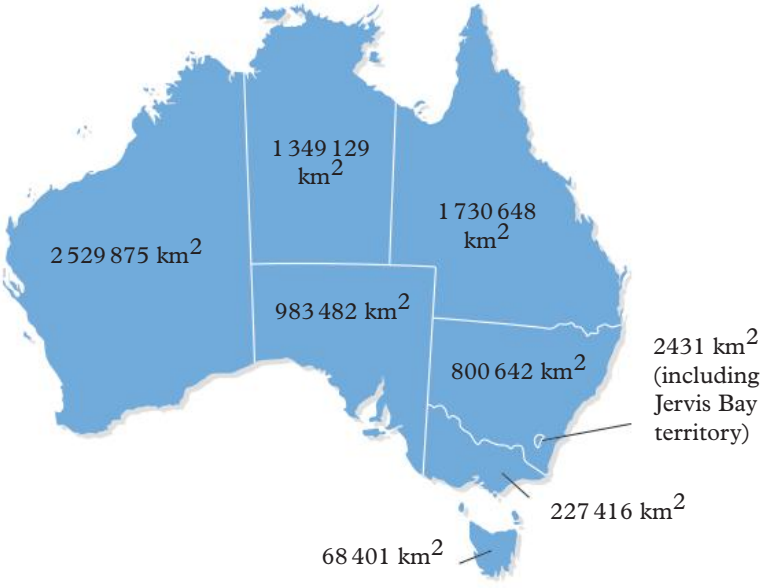
19 People attending a State of Origin rugby league match entered the stadium through one of five gates. The number of people passing through each gate is shown.

Gate	A	B	C	D	E
Number of people	9361	10 758	12 196	8844	11 037

a Estimate the number of people attending the match by rounding the numbers at each gate to the leading digit.

b Calculate the exact number of people attending the match.

20 Australia is the sixth largest country in the world. The areas of the states and territories (including islands) of Australia are shown on the map.



a Round each area to its leading digit.

b Estimate the area of Australia by adding the approximate values from part **a**.

c The area of NSW, rounded to the nearest thousand, is 801 000 km^2. Round each of the other areas to the nearest thousand.

d Estimate the area of Australia by adding the approximate values from part **c**.

e Compare the estimates you obtained in parts **b** and **d** with the accepted value for the area of Australia of 7 692 024 km^2.

CHALLENGE

21 The area of Australia is 7 692 024 km^2 and the population at the end of 2020 was 25 694 393.

a Estimate the area per person for the entire population by rounding both numbers to the leading digit.

b New Zealand has an area of 268 021 km^2 and the population at the end of 2020 was 5 106 400. Estimate the area per person for New Zealand by rounding both numbers to the leading digit and compare it with Australia.

22 Estimate the number of words you say in a day.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Place value
- Interactive skillsheet: Rounding integers
- Interactive skillsheet: Estimation by rounding
- Investigation: An Australian road trip
- Topic quiz: 1A

1B Adding and subtracting whole numbers

Learning intentions

By the end of this topic you will be able to ...

- ✔ use mental strategies for addition and subtraction
- ✔ use the addition and subtraction algorithms for large numbers.

Inter-year links

Support Adding whole numbers

Year 7 1B Adding whole numbers

Addition properties

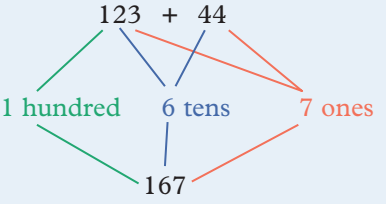
- The **associative law of addition** states that regardless of how the numbers are grouped in a calculation, the result does not change.
  For example, $(3 + 5) + 2 = 3 + (5 + 2)$.
- The **commutative law of addition** states that the order in which numbers are added does not change the result.
  For example, $2 + 8 = 8 + 2$.

Addition (+)	Subtraction (−)
• Sum	• Difference
• Plus	• Take away
• More than	• Less than
• Increase	• Fewer
• Total	• Reduce
• Together	• Minus

Mental strategies

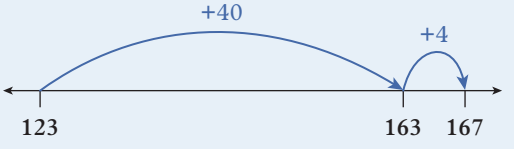
- The **by-parts method** involves adding the digits in each place value separately.

  For example, $123 + 44 = 100 + (20 + 40) + (3 + 4)$
  $= 100 + 60 + 7$
  $= 167$



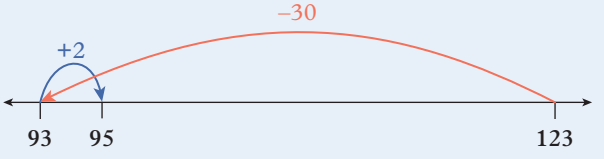
- The **jump method** involves breaking down one of the given numbers, then adding or subtracting each part of the number in stages.

  For example, $123 + 44 = 123 + 40 + 4$
  $= 163 + 4$
  $= 167$



- The **compensation method** involves rounding one number to make the calculations easier. The amount by which the number was originally rounded is then added or subtracted.

  For example, $123 - 28 = 123 - 30 + 2$
  $= 93 + 2$
  $= 95$



Addition algorithm

- In the **addition algorithm**, always start by adding the digits in the ones column first, then add the digits in the tens column, then the hundreds column and so on.

$$\begin{array}{r} {\scriptstyle 1} \\ 472 \\ +\ 846 \\ \hline 1318 \end{array}$$

Ones column: 2 + 6 = 8
Tens column: 7 + 4 = 11, carry the 10
Hundreds column: 4 + 8 + (1) = 13
← Answer

Subtraction algorithm

- In the **subtraction algorithm**, always start by subtracting the digits in the ones column first, then subtract the digits in the tens column, then the hundreds column and so on.
  - → If the subtraction in a particular column cannot be done, take a value from the column to the left.

$$\begin{array}{r} {\scriptstyle 7\ 11} \\ 381 \\ -\ 145 \\ \hline 236 \end{array}$$

Ones column: Take 10 from the tens column
11 − 5 = 6
Tens column: 7 − 4 = 3
Hundreds column: 3 − 1 = 2
← Answer

Example 1B.1 Using mental strategies to add two numbers

Work out each sum using the mental strategy given in brackets.

a 282 + 317 (by-parts method)
b 51 + 39 (jump method)
c 48 + 68 (compensation method)

THINK

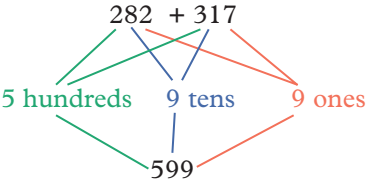
a **1** Add the hundreds together.
2 Add the tens together.
3 Add the ones together.
4 Combine these sums for the final answer.

b **1** Choose a starting number, then break down the second number into tens and ones.
2 Add the tens to the starting number, followed by the ones.

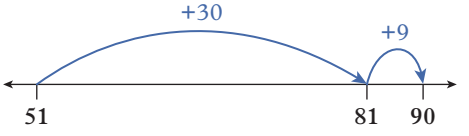
c **1** Choose a starting number, then round the second number to the nearest 10.
2 Add the rounded number to the starting number, then adjust for rounding.

WRITE

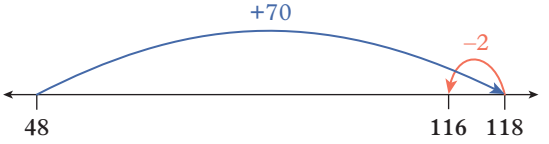
a 282 + 317 = 599



b 51 + 39 = 90



c 48 + 68 = 116



Example 1B.2 Using the addition algorithm

Calculate 781 + 965 using the addition algorithm:
$$\begin{array}{r} 781 \\ +\ 965 \\ \hline \end{array}$$

THINK

1. Add the digits in the ones column: 1 + 5 = 6.
2. Add the digits in the tens column: 8 + 6 = 14.
   Write 4 in the tens column of the answer and write 1 above the hundreds column.
3. Add the digits in the hundreds column: 1 + 7 + 9 = 17.

WRITE

$$\begin{array}{r} {\scriptstyle 1} \\ 781 \\ +\ 965 \\ \hline 1746 \end{array}$$

Example 1B.3 Using mental strategies to subtract two numbers

Work out each difference using the mental strategy given in brackets.

a 112 – 67 (jump method)

b 200 – 89 (compensation method)

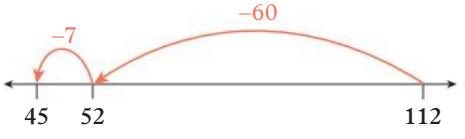
THINK

a
1. Start with the first number, then break down the second number into tens and ones.
2. Subtract the tens from that number, followed by the ones.

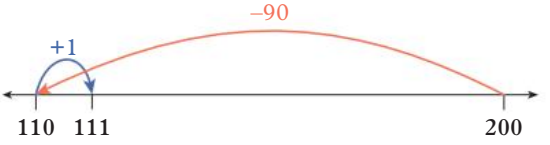
b
1. Start with the first number, then round the second number to the nearest 10.
2. Subtract the rounded number from the starting number, then adjust for rounding.

WRITE

a 112 – 67 is 45.



b 200 – 89 is 111.



Example 1B.4 Using the subtraction algorithm

Calculate 678 – 93 using the subtraction algorithm:
$$\begin{array}{r} 678 \\ -\ 93 \\ \hline \end{array}$$

THINK

1. Subtract the digits in the ones column: 8 – 3 = 5.
2. Subtract the digits in the tens column: 7 – 9.
   Take 1 from the hundreds column by reducing the hundreds to 5 and increasing the tens to 17.
   Subtract the digits in the tens column: 17 – 9 = 8.
3. Subtract the digits in the hundreds column: 5 – 0 = 5. Recall that if there is no digit in a place then that place has a value of zero.

WRITE

$$\begin{array}{r} {\scriptstyle 5\ 17} \\ 6\not{7}\not{8} \\ -\ 9\ 3 \\ \hline 5\ 8\ 5 \end{array}$$

Helpful hints

- ✔ Don't forget that addition is commutative – reordering sums can make them a lot easier!
- ✔ Subtraction is not commutative. Unlike addition problems, you cannot change the order of the numbers in subtraction problems. You must work through the problem from left to right.
- ✔ Make sure that your place value columns are well aligned so that you add or subtract the correct digits.
- ✔ Read each question carefully. You may find some subtraction questions easy to calculate in your head.

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Exercise 1B Adding and subtracting whole numbers

 1–3, 4(a–c), 5(a, c), 6, 7, 8–10(b, c), 11, 13, 15

 1a–d(ii, iii), 3(c, d), 4(d–f), 5(b, d), 6a–d(ii, iii), 8–10, 12, 14, 15, 17, 18(a)

 1a–d(iv), 4(e, f), 5(c, d), 6a–d(iv), 9(c, d), 10(c, d), 12, 14–19

UNDERSTANDING AND FLUENCY

1B.1 **1** **a** Use the by-parts method to work out each sum.

i 34 + 63 **ii** 89 + 72 **iii** 654 + 325 **iv** 1243 + 9654

b Use the jump method to work out each sum.

i 647 + 72 **ii** 768 + 51 **iii** 456 + 128 **iv** 3452 + 139

c Use the compensation method to work out each sum.

i 24 + 99 **ii** 472 + 56 **iii** 658 + 465 **iv** 7629 + 475

d Use any mental strategy to work out each sum.

i 87 + 90 **ii** 489 + 650 **iii** 8765 + 59 **iv** 5983 + 587

1B.2 **2** Calculate each sum using the addition algorithm.

a $\begin{array}{r} 243 \\ +\ 715 \\ \hline \end{array}$ **b** $\begin{array}{r} 538 \\ +\ 154 \\ \hline \end{array}$ **c** $\begin{array}{r} 758 \\ +\ 461 \\ \hline \end{array}$ **d** $\begin{array}{r} 697 \\ +\ 805 \\ \hline \end{array}$

3 Use the addition algorithm to calculate each of the following.

a 4765 + 476 **b** 7410 + 3065 **c** 16 239 + 2633 **d** 98 543 + 23 987

4 Use the addition algorithm to calculate each of the following.

a 456 + 280 + 487 **b** 876 + 387 + 594 **c** 986 + 386 + 2769

d 56 + 875 + 3987 **e** 6954 + 389 + 23 **f** 7632 + 9520 + 3198

5 Evaluate:

a 56 871 + 34 981 **b** 54 870 + 29 528 + 43 679

c 5498 + 2873 + 8921 + 7629 **d** 397 + 287 + 888 + 396 + 492

1B.3 **6** **a** Use the jump method to work out each difference.

i 93 – 30 **ii** 126 – 39 **iii** 145 – 51 **iv** 764 – 71

b Use the compensation method to work out each difference.

i 87 – 54 **ii** 763 – 68 **iii** 874 – 618 **iv** 433 – 67

c Use any mental strategy to work out each difference.

i 130 – 70 **ii** 5600 – 410 **iii** 390 – 180 **iv** 568 – 456

d Use any mental strategy to work out each problem.

i 79 + 39 – 56 **ii** 750 + 830 – 690 **iii** 78 + 565 – 120 **iv** 350 – 90 +876

1B.4 **7** Calculate each difference using the subtraction algorithm.

a $\begin{array}{r} 438 \\ -\ 399 \\ \hline \end{array}$ **b** $\begin{array}{r} 852 \\ -\ 576 \\ \hline \end{array}$ **c** $\begin{array}{r} 2356 \\ -\ 976 \\ \hline \end{array}$ **d** $\begin{array}{r} 3765 \\ -\ 387 \\ \hline \end{array}$

8 Use the subtraction algorithm to calculate each of the following.

a 487 – 458 **b** 3865 – 376 **c** 4652 – 234 **d** 6798 – 478

9 Use the subtraction algorithm to calculate each of the following.

a 7654 – 488 **b** 7421 – 6423 **c** 75 638 – 689 **d** 86 408 – 42 766

10 Calculate:

a 566 + 238 – 328 **b** 2776 + 5409 – 3111

c 76 409 + 45 629 – 83 888 **d** 6543 – 2389 + 4319

11 Yiyan is the manager of five city car parks. The capacities of the car parks are 356, 821, 489, 398 and 450. What is the total number of cars that can use Yiyan's car parks?

12 The population of Queensland is 5 250 000 and the population of Brisbane is 2 470 000.

a How many Queenslanders live outside Brisbane?

b How much does the population of Queensland have to grow for it to be 6 000 000?

c If the population of Queensland decreases by 32 000 and grows by 59 000 next year, what will the population be at the end of the year?



13 Two local libraries decide to merge their book collections and form one mega-library. Spring Hill Library has 27 356 books and Dove St Library has 54 237 books.

a How many books are there in total?

b The mega-library finds that there are two copies of 28 345 books so it is decided one copy of each will be given away. How many books will there be in the new library?

14 Raf and Mac go on a road trip around Queensland. They combine expenses for the trip and Raf keeps a tally of the distances they travel and the money they spend.

Stage	Distance travelled (km)	Accommodation (2 nights at each town)	Expenses (petrol, food, etc.)
Brisbane to Maryborough	253	$292	$128
Maryborough to Eidsvold	228	$320	$175
Eidsvold to Miles	275	$228	$184
Miles to Toowoomba	210	$248	$162
Toowoomba to Brisbane	125	$0	$85

a What was the total distance Raf and Mac travelled on their road trip?

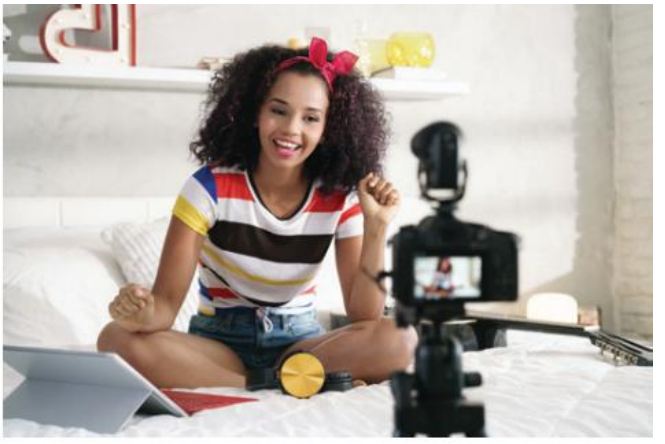
b How much in total did they spend on accommodation?

c What were their total expenses not including accommodation?

d Raf and Mac had each saved up $1000 for the trip. How much was left over when they got back to Brisbane?

PROBLEM SOLVING AND REASONING

15 Aya has 2437 subscribers to her YouTube channel and Akuac has 3762 subscribers.



a What is the total number of subscribers between the two girls?

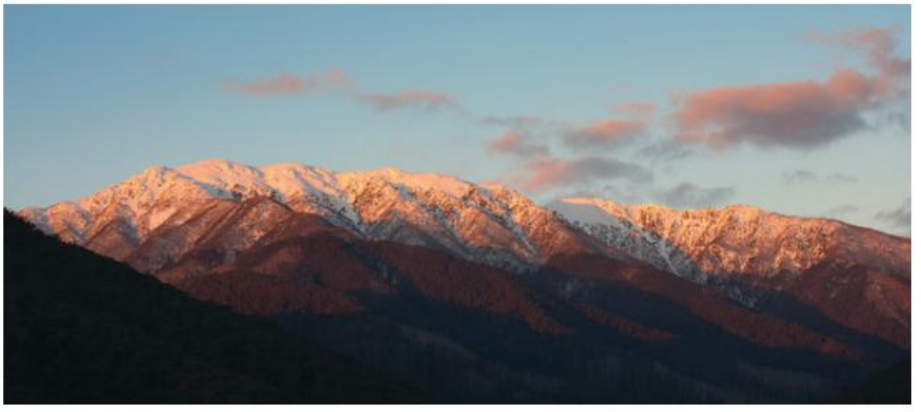
b Aya and Akuac decide to combine their channels into one new channel, thus combining their subscribers.
If there are 1562 people who initially subscribed to both channels, how many subscribers will the new combined channel have? (Assuming channels can be merged and subscribers can't be counted twice.)

16 The Australian Alps Walking Track goes from Mt Hotham to Mt Bogong in Victoria's high country. Jai and Rudra walk the track over 4 days in the summer. They walk the following distances: 13 km, 17 km, 14 km and 16 km.

a How long is the Australian Alps Walking Track?

Mt Bogong is 1986 m above sea level and Mt Hotham is 1861 m above sea level.

b What is the difference in height between the two mountains?



17 Shannon's shoe store has enough space for 568 pairs of shoes. If Shannon has 483 pairs of shoes left in her store, how many does she need to order to be at full capacity?

CHALLENGE

18 a What is the difference between the sum of the first five odd numbers (starting from 1) and the sum of the first five even numbers (starting from 2)?

b What is the difference between the sum of the first 2000 odd numbers (starting from 1) and the first 2000 even numbers (starting from 2)?

19 A cryptarithm is a type of mathematical puzzle in which the numbers have been replaced by letters. There are six solutions to the cryptarithm below. How many of them can you find?

$$\begin{array}{r} \text{SEED} \\ +\text{WATER} \\ \hline \text{FRUIT} \end{array}$$

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Adding and subtracting whole numbers
- Investigation: Palindromic numbers
- Topic quiz: 1B

1C Multiplying and dividing whole numbers

Learning intentions

By the end of this topic you will be able to ...

- ✔ use mental strategies for multiplication
- ✔ calculate products and quotients using the multiplication and division algorithms.

Inter-year links

Support	Multiplying whole numbers
Year 7	1D Multiplying whole numbers

Multiplication

- Repeated addition can be reduced to multiplication of **factors** to give a **product**.

$$\underbrace{3 + 3}_{\text{repeated addition}} = \overbrace{2 \times 3}^{\text{factors}} = \underbrace{6}_{\text{product}}$$

Multiplication strategies

- The **associative law of multiplication** states that regardless of how the numbers are grouped, the result does not change.

  For example, $(2 \times 4) \times 6 = 2 \times (4 \times 6)$.
- The **commutative law of multiplication** states that the order in which numbers are multiplied does not change the results.

  For example, $112 \times 4 = 4 \times 112$.
- The **distributive law** states that multiplication can be 'distributed' across a bracket. This means that large numbers can be broken down into a sum of smaller numbers, each of which can then be multiplied separately.

  For example,

$$\begin{aligned} 5 \times 32 &= 5 \times (30 + 2) \\ &= 5 \times 30 + 5 \times 2 \\ &= 150 + 10 \\ &= 160 \end{aligned}$$

Multiplication algorithm

- When two numbers are multiplied, the first number is multiplied by the individual digits of the second number, considering the place values of the digits, then the products are added together.

Write ⟶

$$\begin{array}{rrl} & {\scriptstyle 2}\,34 & \\ \times & 26 & \\ \hline & 204 & (34\times 6) \\ + & 680 & (34\times 20) \\ \hline & 884 & \end{array}$$

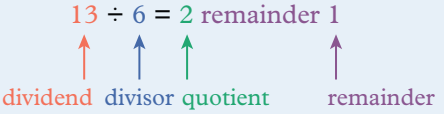
Multiply by ones ⟶

Multiply by tens ⟶
Don't forget the zero(s)

Add the results ⟶

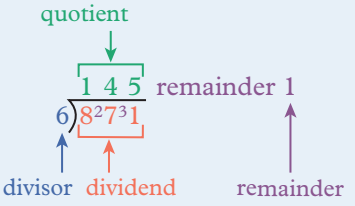
Division

- The **dividend** is the number that is divided by another number.
- The **divisor** is the number by which the dividend is divided.
- The **quotient** is the whole-number result of the division.
- The **remainder** is the part of the dividend that is left over when the number is divided by the divisor. The remainder can be zero.



Short division

- Short division is an algorithm used to simplify a division problem by breaking it down into easy steps.
- Start by dividing the first digit of the dividend by the divisor.



Multiplication (×)	Division (÷)
• Product	• Quotient
• Times	• Divide
• Multiply	• Shared
• Groups of	• Distributed

Example 1C.1 Using strategies to multiply numbers

Work out each product using the multiplication strategy given in brackets.

a 9 × 34 (distributive law)

b 12 × 4 × 2 (associative law)

THINK

a **1** Break down the second number into tens and ones.

2 Multiply each part of the second number by the first number.

3 Add the results together.

b **1** Use brackets to group pairs of numbers that are easy to multiply together and/or produce round numbers.

2 Multiply the numbers grouped in the brackets.

3 Multiply the result by the third number.

WRITE

a $9 \times 34 = 9 \times (30 + 4)$

$= 9 \times 30 + 9 \times 4$

$= 270 + 36$

$= 306$

b $12 \times 4 \times 2 = 12 \times (4 \times 2)$

$= 12 \times 8$

$= 96$

Example 1C.2 Using the multiplication algorithm

Calculate the product using the multiplication algorithm: $\begin{array}{r} 359 \\ \times\ \ 72 \\ \hline \end{array}$

THINK

1. Set out the multiplication problem by lining up the digits according to their place value.
2. Multiply the first number by the ones of the second number: 359 by 2 ($359 \times 2 = 718$)
3. Multiply the first number by the tens of the second number: 359 by 70 ($359 \times 70 = 25\,130$)
4. Add the products and write the final answer at the bottom of the calculation.

WRITE

$$\begin{array}{r} {\scriptstyle 4\ 6} \\ {\scriptstyle 1\ 1} \\ 359 \\ \times\ \ 72 \\ \hline 718 \quad (359 \times 2) \\ +\ 25130 \quad (359 \times 70) \\ \hline 25848 \end{array}$$

$359 \times 72 = 25\,848$

Example 1C.3 Using short division

Use short division to calculate the quotient and remainder of $1257 \div 8$.

THINK

1. Set up the problem using the division algorithm. The divisor is 8 and the dividend is 1257.
2. How many 8s go into 1? Write a zero above the 1.
3. How many 8s go into 12? Write 1 above the 2 on the quotient line. Put the remainder 4 in front of the next digit, 5.
4. How many 8s go into 45? Write a 5 above the 5. Put the remainder 5 in front of the next digit, 7.
5. How many 8s go into 57? Write 7 above the 7 on the quotient line. Work out the remainder. $7 \times 8 = 56$, so the remainder is $57 - 56 = 1$.

WRITE

$$\begin{array}{r} 0\ 1\ 5\ 7 \\ 8\overline{)1\ 2^{4}5^{5}7} \end{array}$$ remainder 1

$1257 \div 8 = 157$ remainder 1

Helpful hints

- ✔ When using the distributive law, don't forget to maintain the place value using zeros.
  For example, $32 \times 3 = (30 \times 3) + (2 \times 3)$
  $32 \times 3 \neq (3 \times 3) + (2 \times 3)$
- ✔ Carefully line up digits by place value when setting up long multiplication problems to avoid making errors when adding the products together.
- ✔ Remember that division goes from left to right.
- ✔ Remember that BIDMAS is a helpful acronym when considering the order of operations for an equation.

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Exercise 1C Multiplying and dividing whole numbers

 1–3(1st, 2nd columns), 4(a, c), 5, 6, 7, 8–9(a–c), 10-13, 17

 1–3(2nd, 3rd columns), 4(b, d), 6(c, f), 7, 8–9(d–f), 10(c, d), 11, 14, 15, 17, 19

 1(j–l), 2(g, i), 4(d), 6(d–f), 8–9(e, f), 10(c, d), 11, 14, 16, 18–21

UNDERSTANDING AND FLUENCY

1C.1 **1** Use the distributive law to calculate:

a 3×42 **b** 6×102 **c** 203×7 **d** 305×9

e 17×98 **f** 83×99 **g** 24×95 **h** 41×19

i 62×101 **j** 29×1001 **k** 11×97 **l** 15×999

2 Use mental strategies to calculate:

a $17 \times 8 \times 3$ **b** $21 \times 5 \times 6$ **c** $3 \times 74 \times 2$

d $66 \times 8 \times 10$ **e** $4 \times 6 \times 25$ **f** $921 \times 5 \times 20$

g $35 \times 4 \times 5$ **h** $29 \times 250 \times 4$ **i** $200 \times 186 \times 5$

3 Find each product.

a 9×70 **b** 12×800 **c** 11×700 **d** 126×50

e 356×900 **f** 2765×3000 **g** 2763×9000 **h** $1674 \times 50\,000$

4 Find each product.

a $60 \times 40 \times 300$ **b** $80 \times 700 \times 8000$ **c** $2400 \times 20 \times 200$ **d** $15\,000 \times 500 \times 50$

1C.2 **5** Multiply the following pairs of numbers using the multiplication algorithm.

a 589×76 **b** 498×49 **c** 4598×23

d 4623×32 **e** 9245×72 **f** 9430×520

6 Multiply the following pairs of numbers using the multiplication algorithm.

a 387×860 **b** 378×598 **c** 638×496

d 3769×458 **e** 4287×352 **f** 5923×5030

7 **a** Estimate these products by first rounding each number to its leading digit, then multiplying.

i 8462×95 **ii** 3299×87 **iii** $95\,687 \times 625$ **iv** $78\,438 \times 8345$

b Check how close your estimations are to the exact result by performing the calculations.

1C.3 **8** Use short division to calculate each quotient and remainder.

a $8\overline{)463}$ **b** $6\overline{)348}$ **c** $7\overline{)160}$

d $5\overline{)375}$ **e** $9\overline{)689}$ **f** $4\overline{)5682}$

9 Use short division to calculate each quotient and remainder.

a $5824 \div 4$ **b** $8399 \div 6$ **c** $4210 \div 3$

d $84\,276 \div 5$ **e** $56\,289 \div 9$ **f** $1489 \div 7$

10 Perform each division using short division.

a $\frac{143}{6}$ **b** $\frac{653}{8}$ **c** $\frac{246}{7}$ **d** $\frac{489}{3}$

11 The average of a set of scores is the total sum of the scores divided by the number of scores. For example, the average of 25, 30 and 23 is $(25 + 30 + 23) \div 3$ or $\frac{25 + 30 + 23}{3}$.

a Find the sum of 19, 22, 24 and 27.

b Divide your answer from part **a** by 4 to find the average of 19, 22, 24 and 27.

12 a The commutative law relates to the order of operations. Calculate:

i $3 + 6$

ii $6 + 3$

b Does the order matter when you add two numbers? Try two more examples.

Your answer to part **b** confirms that addition obeys the commutative law.

c Does subtraction obey the commutative law? Try these calculations.

i $6 - 3$

ii $3 - 6$

d Does multiplication obey the commutative law? Try these.

i 6×3

ii 3×6

e Does division obey the commutative law? Try these.

i $6 \div 3$

ii $3 \div 6$

13 a The associative law also relates to the order of operations. Calculate:

i $(5 + 6) + 7$

ii $5 + (6 + 7)$

iii $(5 + 7) + 6$

b Does it matter in which order you add three numbers? Try two more examples.

c Your answer to part **b** confirms that addition obeys the associative law. Do subtraction, multiplication and division obey the associative law? Try some examples to help you decide.

14 The commutative and associative laws can make mental calculations easier.

a To calculate $54 + 118 + 16$, you can add any two of the numbers together first.

i Which two numbers are easiest to add together first? Explain.

ii Perform the calculation.

b To calculate $87 \times 25 \times 4$, you can multiply any two of the numbers together first.

i Which two numbers are easiest to multiply together first? Explain.

ii Perform the calculation.

c Explain how these laws can make some calculations easier to perform.

15 a Calculate these, then compare your answers.

i $4 \times (5 + 6)$ **ii** $4 \times 5 + 4 \times 6$

b Calculate these, then compare your answers.

i $6 \times (10 - 2)$ **ii** $6 \times 10 - 6 \times 2$

c Parts **a** and **b** are examples of the application of the distributive law. Describe this law.

d Show how the distributive law works by providing two more examples.

16 The distributive law can be used to make some multiplication calculations easier to perform.

a The calculation 16×24 can be written as $16 \times (20 + 4)$ or $16 \times 20 + 16 \times 4$.

i Calculate 16×20.

ii Calculate 16×4.

iii Add your results and write the answer to 16×24.

b The calculation 45×998 can be written as $45 \times (1000 - 2)$ or $45 \times 1000 - 45 \times 2$.

i Calculate 45×1000.

ii Calculate 45×2.

iii Subtract your results and write the answer to 45×998.

c Explain how the distributive law can be used to make some multiplication calculations easier to perform.

PROBLEM SOLVING AND REASONING

17 Ms Lee spends $480 per school year on coffee from The Blue Cup Café.



a If the school year is 40 weeks, how much does Ms Lee spend per week?

b If The Blue Cup Café had a special teacher price of $4 per cup, how many times per week does Ms Lee buy coffee?

c Mr Lin prefers the cheaper coffee at the service station over the road. He pays $2 per cup and buys coffee five days per week.

How much does Mr Lin pay for coffee in one week?

d How much does Mr Lin pay for coffee in a school year?

18 The Year 8 class captains are tasked with providing icy poles for the whole school at the swimming sports carnival. There are 1289 students at the school.

a How many packs of icy poles do the class captains have to buy if there are 20 in each pack, so that no student misses out?

b How many icy poles will be left over if every student has one?

c If the packets of icy poles cost $3 each, how much money do the class captains need?

19 Astrid is planning to hire a car for a road trip from Melbourne to Cairns. The distance she plans to travel is 2814 km. The car hire company informs Astrid that the car can travel an average of 7 kilometres on one litre of fuel.

a Use this information to calculate how many litres of fuel Astrid will need to complete her journey.

b If Astrid's fuel tank is 50 litres, how many tanks of fuel does Astrid need for her journey?

c Astrid wants to make sure she overestimates the amount of money she will need to spend on fuel, so she rounds the price of fuel up to $3 per litre. Using this estimate, how much money will Astrid budget for her fuel expenses?

CHALLENGE

20 Darcy has a number of mini spring rolls. After eating one, she gives half the remainder to her sister. After eating another spring roll, she gives half of what is left to her brother. Darcy now has only five spring rolls left. How many did she start with?



21 A recipe for eight biscuits needs 20 g butter, 30 g sugar and 40 g rolled oats.

a How many grams of sugar do I need if I want to make 16 biscuits?

b How many biscuits can I make if I have 140 g butter, 150 g sugar and 160 g rolled oats?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Multiplying and dividing whole numbers

Investigation
Factorials

Topic quiz
1C

1D Multiples, factors, squares and cubes

Learning intentions

By the end of this topic you will be able to ...

- ✔ find the lowest common multiple and highest common factor of two numbers
- ✔ evaluate squares and cubes of numbers
- ✔ evaluate square roots and cube roots.



Inter-year links

Support	Multiples and factors
Year 7	2C Multiples and the lowest common multiple
Year 9	1F Surds

Multiples

- The **multiples** of a whole number are the numbers produced by multiplying that number by another whole number.
  For example, some multiples of 3 are:
  $3 \times 1 = 3$
  $3 \times 2 = 6$
  $3 \times 3 = 9$
- The **lowest common multiple (LCM)** of two or more numbers is the common multiple with the smallest value.

Multiples of 4: 4, 8, 12, 16 …
Multiples of 6: 6, 12, 18, 24 …
LCM: 12

Factors

- The **factors** of a whole number are the whole numbers which divide exactly into that number.
  For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.
- The divisibility rules are helpful when looking for factors of large numbers.

÷2 The last digit of the number is even.	÷3 The sum of all the digits in the number is divisible by 3.	÷4 The number made by the last two digits is divisible by 4.
÷5 The last digit is 5 or 0.	÷6 The number is divisible by 2 and 3.	÷7 There is no easy trick for 7. ☹
÷8 The number made by the last three digits is divisible by 8.	÷9 The sum of all the digits in the number is divisible by 9.	÷10 The last digit is 0.

- The **highest common factor (HCF)** of two or more whole numbers is the **common factor** with the greatest value.

Factors of 12: 1, 2, 3, 4, (6), 12
Factors of 18: 1, 2, 3, (6), 9, 18
HCF: 6

Exponents

- Repeated multiplications can be represented using exponents.

exponent/index
base → $2^3 = 2 \times 2 \times 2 = 8$
exponent form | expanded form | basic numeral

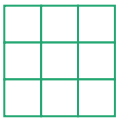
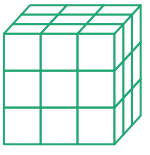
- The **base** is the number that is repeatedly multiplied.
- The **exponent** (or **index**) represents the number of times the base appears in **expanded form**.

Roots

- To find the **square root** of a number, find the number that when squared, or has 'an exponent of 2', results in the original number.

  For example, $\sqrt{9} = 3$ because $3^2 = 9$.
- To find the **cube root** of a number, find the number that when cubed, or has 'an exponent of 3', results in the original number.

  For example, $\sqrt[3]{8} = 2$ because $2^3 = 8$.

Squares	Cubes
$1^2 = 1 \times 1 = 1$  $\sqrt{1} = 1$	$1^3 = 1 \times 1 \times 1 = 1$  $\sqrt[3]{1} = 1$
$2^2 = 2 \times 2 = 4$  $\sqrt{4} = 2$	$2^3 = 2 \times 2 \times 2 = 8$  $\sqrt[3]{8} = 2$
$3^2 = 3 \times 3 = 9$  $\sqrt{9} = 3$	$3^3 = 3 \times 3 \times 3 = 27$  $\sqrt[3]{27} = 3$

Example 1D.1 Finding the LCM and HCF

Determine:

a the LCM of 14 and 35

b the HCF of 24 and 38.

THINK

a **1** List the multiples of 14 and the multiples of 35.

2 Draw a box around the LCM, the multiple with the lowest value that appears in both lists.

b **1** List all the factors of 24 and all the factors of 38.

2 Draw a box around the HCF, the factor with the highest value that appears in both lists.

WRITE

a Multiples of 14: 14, 28, 42, 56, 70 …
Multiples of 35: 35, 70, 105, 140 …
The LCM is 70.

b Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 38: 1, 2, 19, 38
The HCF is 2.

Example 1D.2 Calculating the square and cube

Calculate:

a the square of 5

b the cube of 5.

THINK

1 To 'square' 5, raise 5 to the power of 2. To 'cube' 5, raise 5 to the power of 3.

2 Identify the base and the exponent and multiply the base by itself as many times as specified by the exponent.

3 Perform the multiplication.

WRITE

a $5^2 = 5 \times 5$
$= 25$

b $5^3 = 5 \times 5 \times 5$
$= (5 \times 5) \times 5$
$= 25 \times 5$
$= 125$

Example 1D.3 Calculating the square root and cube root

Calculate the value of each of the following roots.

a $\sqrt{36}$

b $\sqrt[3]{64}$

THINK

a Use the times tables to determine which number, multiplied by itself, is equal to the number under the square root symbol.

b Use the times tables and estimation to determine which number, multiplied by itself three times, is equal to the number under the cube root symbol.

WRITE

a $6 \times 6 = 36$
So, $\sqrt{36} = 6$

b $4^3 = 4 \times 4 \times 4$
$= (4 \times 4) \times 4$
$= 16 \times 4$
$= 64$
So, $\sqrt[3]{64} = 4$

Helpful hints

- ✔ Lowest common multiples are not necessarily 'low' in value; they can be large numbers.
- ✔ There is no such thing as a 'highest common multiple' as any given number has infinitely many multiples.
- ✔ Make sure you use a systematic method for finding all the factors; it is very easy to miss one or two.
- ✔ Don't confuse exponents and multiplication!
  For example, $2^3 = 2 \times 2 \times 2$
  $2^3 \neq 2 \times 3$
  Place your exponents carefully; they should be in smaller font than the base and sit high up on the shoulder of the base – your 4^3 should look different to your 43!

ANS p491

Exercise 1D Multiples, factors, squares and cubes


1–5, 6(a, c, e), 7, 8(a–f), 9(1st, 2nd columns), 10, 11, 14, 15


2, 3, 6, 7, 8(g–l), 9(3rd, 4th columns), 10–13, 16


3(d–f), 6(b, d, f), 7, 8(j, k, l), 9(3rd, 4th columns), 10, 11, 13, 15–18

UNDERSTANDING AND FLUENCY

1 **a** List the multiples of 6 and 9 up to 100.
b Identify the common multiples of 6 and 9 in the lists from part **a**.
c What is the lowest common multiple (LCM) of 6 and 9?

1D.1 **2** Find the lowest common multiple (LCM) of each pair of numbers.

a 6 and 8 **b** 12 and 18
c 25 and 30 **d** 12 and 15
e 14 and 20 **f** 9 and 15

3 Find the lowest common multiple (LCM) of each group of numbers.

a 5, 6 and 7 **b** 5, 7 and 9
c 6, 8 and 9 **d** 5, 8 and 12
e 15, 18 and 24 **f** 9, 18 and 27

4 Find all the factors of each number.

a 45 **b** 72 **c** 90
d 120 **e** 100 **f** 144

5 **a** What is the highest common factor (HCF) of 18 and 36?
b What is the HCF of 21 and 45?
c What is the HCF of 12 and 30?

6 Find the highest common factors (HCF) of each group of numbers.

a 42 and 70 **b** 48 and 84
c 36 and 63 **d** 144, 180 and 200
e 120, 240 and 360 **f** 500, 900 and 725

7 Find the LCM and the HCF of each group of numbers.

a 24 and 45 **b** 16 and 36 **c** 6, 15 and 20 **d** 7, 12 and 22

1D.2 **8** Calculate the square and the cube of each of the following numbers.

a 3 **b** 2 **c** 10 **d** 7 **e** 6 **f** 12
g 8 **h** 4 **i** 1 **j** 9 **k** 11 **l** 14

1D.3 **9** Calculate the value of each of the following roots.

a $\sqrt{9}$ **b** $\sqrt[3]{8}$ **c** $\sqrt[3]{125}$ **d** $\sqrt{64}$

e $\sqrt{81}$ **f** $\sqrt[3]{216}$ **g** $\sqrt{100}$ **h** $\sqrt[3]{343}$

i $\sqrt[3]{512}$ **j** $\sqrt[3]{2197}$ **k** $\sqrt{196}$ **l** $\sqrt[3]{3375}$

10 Find the value of each of the following by first evaluating any exponents and roots.

a $2^3 \times \sqrt{9}$ **b** $3^3 + \sqrt{36}$ **c** $7^2 \times 10^3$

d $4^3 \div \sqrt{16}$ **e** $2^6 \times 8^2$ **f** $100^2 \times \sqrt{100}$

11 Find the value of each of the following by first evaluating any exponents and roots.

a $6^2 \times 3^3$ **b** $18^2 \div 3^2$ **c** $36^2 \div 6^3$

d $4^5 \times 2^1$ **e** $12^2 \times 4^2 \div 3^2$ **f** $\sqrt{81} \times 2^4 \times 5^2$

12 Sally and Jo have the same number of stickers. Sally's stickers are in packs of 15 and Jo's stickers are in packs of 12. What is the smallest number of stickers each of them could have?



13 Taran is a florist. He receives a delivery of roses that can be divided into bouquets of 15 or 24.

a What is the smallest number of roses in the delivery?

b Taran has 24 roses and 40 lilies. If he wants to make identical bouquets with no flowers left over, what is the greatest number of bouquets he can make?

14 A group of students is going on an excursion to Australia Zoo. The students can travel on a number of buses that seat 30 or a number of buses that seat 45 so that all the seats are taken. What is the smallest number of students going on the excursion?



15 Dizzy sold equal numbers of wontons and dumplings today. Wontons are sold in packets of 8 while dumplings are sold in packets of 12. What is the smallest number of each that Dizzy sold?

16 Arvin has 24 toy trucks and 64 toy cars. He divides them into identical groups so that there are no toys left over. What is the largest possible number of groups there might be?

17 Jill has grown three types of potatoes in her garden: Pontiac, Sebago and Desiree. Seven Pontiac potatoes weigh the same as four Sebago potatoes and five Sebago potatoes weigh the same as six Desiree potatoes. Put the potatoes in order of weight from lightest to heaviest.

18 How many numbers in the set $\{7^1, 7^2, 7^3, 7^4, \ldots, 7^{2000}\}$ have a last digit of 3?

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Lowest common multiple
- Interactive skillsheet: Highest common factor
- Interactive skillsheet: Exponents
- Interactive skillsheet: Squares and square roots
- Investigation: Triangular numbers
- Topic quiz: 1D

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

1A **1** Round each number to the nearest hundred.

a 576 **b** 934
c 7871 **d** 2839
e 45 621 **f** 76 618

1A **2** Estimate the result of each calculation by first rounding each number to its leading digit.

a 849 + 381 **b** 56 – 18
c 65 × 43 **d** 582 ÷ 16
e 493 × 28 **f** 825 + 9114
g 8402 ÷ 79 **h** 658 – 180

1B **3** Calculate each of the following.

a 64 + 98 **b** 745 + 832
c 6982 + 5176 **d** 28 + 743 + 6287
e 6287 + 341 + 89 **f** 3916 + 5328 + 6124

1B **4** Calculate each of the following.

a 63 – 45 **b** 582 – 96
c 846 – 757 **d** 5723 – 68
e 8743 – 374 **f** 7398 – 3572

1B **5** Calculate each of the following.

a 68 – 34 + 287 **b** 738 + 296 – 457
c 5921 + 4827 – 4277 **d** 83 422 + 82 113 – 57 329

1C **6** Calculate each of the following.

a 768 × 43 **b** 846 × 76
c 6214 × 57 **d** 9262 × 83
e 7248 × 82 **f** 6524 × 645

1C **7** Use short division to calculate each quotient and remainder.

a 395 ÷ 4 **b** 646 ÷ 7
c 8328 ÷ 5 **d** 76 398 ÷ 9
e 68 341 ÷ 11 **f** 653 789 ÷ 6

1D **8** Find the LCM of each pair of numbers.

a 7 and 6 **b** 4 and 9
c 5 and 12 **d** 30 and 45
e 18 and 24 **f** 36 and 54

1D **9** Find the highest common factor (HCF) of each pair of numbers.

a 24 and 30 **b** 27 and 72
c 45 and 75 **d** 120 and 185
e 450 and 600 **f** 288 and 432

1D **10** Find the value of each of the following by first evaluating any exponents and roots.

a $6^2 + 4^3$ **b** $9^2 \div 3^2$
c $\sqrt{81} \times 4^2$ **d** $2^6 \div 8$
e $12^2 + 6^2 \div 3^2$ **f** $\sqrt{49} \times 3^3 + 8^2$

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1E Negative integers

Learning intentions

By the end of this topic you will be able to ...

- ✔ compare the value of positive and negative numbers
- ✔ use negative numbers to represent real-life situations.

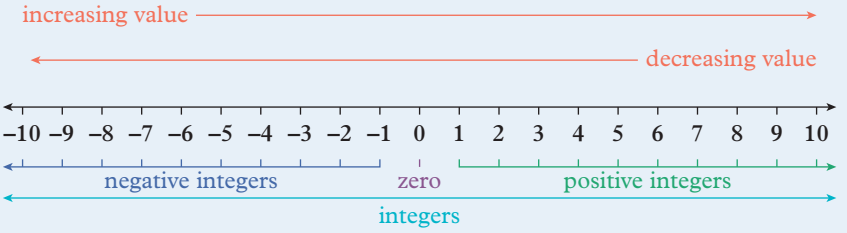
Inter-year links

Support Negative numbers

Year 7 5A Negative numbers

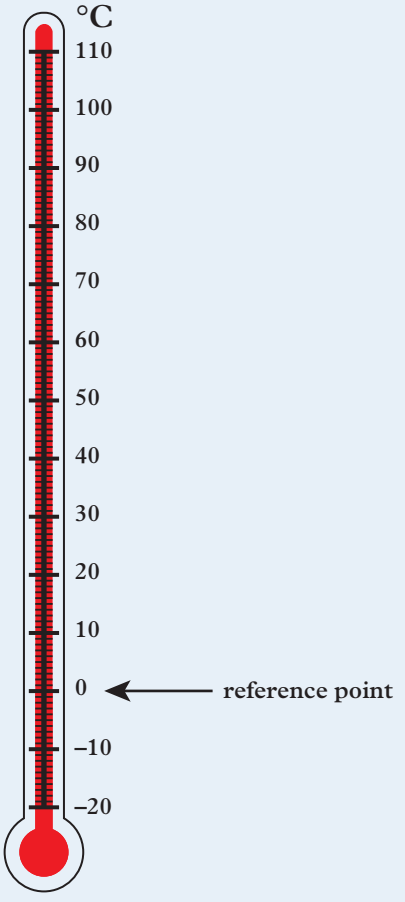
Integers

- The set of **integers** includes positive whole numbers, negative whole numbers and zero.
- **Negative integers** are less than zero in value and are always preceded by a negative sign, −, placed directly before the first digit.
- **Positive integers** are greater than zero in value and can be represented using a positive sign, +. The positive sign can also be left out.



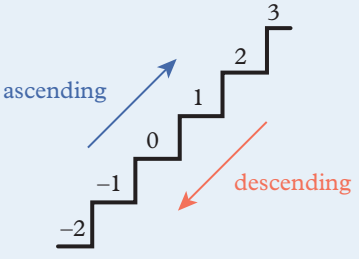
- In the real world, negative numbers are used to represent quantities that lie on a scale that goes below zero. A sign is used to indicate direction relative to the **reference point** of zero.

  For example, a negative temperature indicates that the temperature has dropped below the reference point of 0°.



Ordering numbers

- Numbers are in **ascending order** when they are arranged in order of increasing value.
- Numbers are in **descending order** when they are arranged in order of decreasing value.

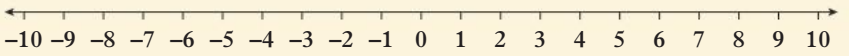


- 'Less than' is shown using the symbol '<'
- 'Greater than' is shown using the symbol '>'
- 'Equal to' is shown using the symbol '='

Example 1E.1 Identifying the greater of two integers

Identify which integer is greater in each pair of numbers. Use the number line to help you.

a −5 and 2 **b** −4 and −7



THINK

a **1** Locate the two numbers on the number line and compare their positions. 2 is to the right of −5 so 2 is greater than −5.

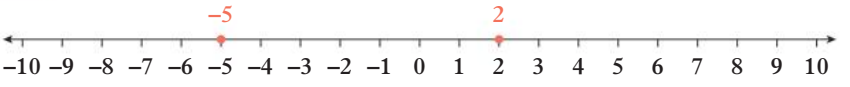
2 Use the 'greater than' symbol, >, in your answer.

b **1** Locate the two numbers on the number line and compare their positions. −4 is to the right of −7 so −4 is greater than −7.

2 Use the 'greater than' symbol, >, in your answer.

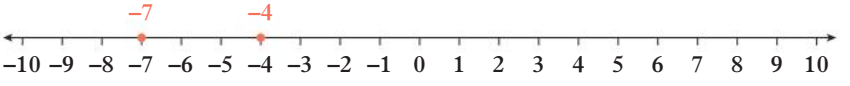
WRITE

a



$2 > -5$

b



$-4 > -7$

Example 1E.2 Representing a situation with a signed number

Write an integer to represent each situation.

a The temperature is 15 degrees above zero.

b A fish swims 4 m below the surface of the water.

c Your bank account is overdrawn by $30.

THINK

a **1** Identify the reference point.

2 Determine the sign of the integer based on the number's position relative to the reference point. '15 degrees above zero' is indicated as 15.

b **1** Identify the reference point.

2 Determine the sign of the integer based on the number's position relative to the reference point. '4 m below' is indicated as −4.

c **1** Identify the reference point.

2 Determine the sign of the integer based on the number's position relative to the reference point. 'Overdrawn by $30' is indicated as −30.

WRITE

a The reference point is 0° so the integer is 15.

b The reference point is the water surface, so the integer is −4.

c The reference point is $0, so the integer is −30.

Helpful hints

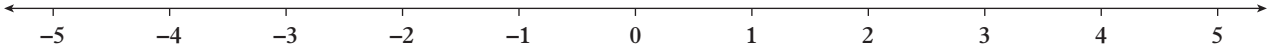
- ✔ The negative equivalent of a positive number is called the additive inverse.
  For example, −4 is the additive inverse of 4.
- ✔ Don't get your 'greater than' and 'less than' symbols mixed up! The 'mouth' of the symbol is always trying to 'eat' the greater number.
  For example, $4 > 3$ and $3 < 4$.

ANS p492

Exercise 1E Negative integers

1–5, 6(a–f), 7(a, c), 8(a, c), 9, 10(a–d), 11, 12 | 4, 5, 6(g–l), 7(b, d), 8(b, d), 9, 10(e–h), 12–15 | 6(g–l), 7(c, d), 8(c, d), 9, 10(e–h), 13–17

1 Look at the number line below.



As shown in the number line above, write all of:

a the negative integers **b** the positive integers **c** the integer that is neither positive nor negative.

1E.1 **2** Which integer is greater in each pair of numbers?

a 3 and 9 **b** 0 and 5 **c** −7 and 1

d 5 and −8 **e** 0 and −2 **f** −6 and −4

3 Which integer is smaller in each pair of numbers?

a 5 and 6 **b** 0 and 3 **c** −2 and 7

d 4 and −5 **e** 0 and −1 **f** −8 and −9

4 Which number is greater in each pair of numbers?

a 0 and −38 **b** −42 and 24 **c** 14 and −16

d 125 and −50 **e** −25 and −35 **f** −14 and 0

5 Which number is smaller in each pair of numbers?

a 7 and −70 **b** −21 and −12 **c** 55 and −68

d −40 and −50 **e** −354 and −345 **f** 450 and −450

6 Complete each number statement by writing < (is less than) or > (is greater than) in the space provided. You can draw a number line to help you think.

a −3 ____ 2 **b** −8 ____ −4 **c** 0 ____ −1

d 9 ____ −9 **e** −3 ____ −5 **f** −7 ____ −2

g 4 ____ −6 **h** 5 ____ −12 **i** 6 ____ −4

j −18 ____ −8 **k** −6 ____ −7 **l** −8 ____ 0

7 Write each list of integers in ascending order.

a 14, −20, 10, −7, 8, −11, −12

b −33, 42, −19, −41, 0, 6, 29

c −8, −88, 68, 8, −28, −18, −48

d 73, −82, 3, −140, 104, 145, −126

8 Write each list of numbers in descending order.

a −15, 71, −27, −10, 4, −9

b −1, −2, 5, 10, −11, −5

c 0, 5, −9, −14, 21, −19

d −11, 51, −71, −62, 71, 12, −72

9 List all the integers between −7 and +4 ('between' means −7 and +4 are not included).

1E.2 **10** Write a signed number to represent each situation.



a The lift stops at the third floor below the ground floor.

b You have $2174 in the bank.

c The temperature inside a freezer is 18 degrees below zero.

d The shoreline of the Dead Sea, Jordan, is 408 m below sea level.

e The top of Mt Kilimanjaro, Tanzania, is 5895 m above sea level.

f A skydiver is 500 m above the ground.

g Your bank account is overdrawn by $46.

h A submarine is 40 m below sea level.

11 Polar bears live close to the edge of sea ice in the Arctic, where in winter the temperatures range from around −45°C to 0°C. In summer, the temperatures range from −10°C to 10°C. The ocean temperature is about −2°C.



a Show −45°C, 0°C, −10°C, 10°C and −2°C on a number line.

b Is −10°C higher or lower than −45°C?

c Write the five temperatures in order from lowest to highest.

d A polar bear dips her paw into the water to catch a fish. On a day with an air temperature of −18°C, does the water feel warmer or colder?

12 Automatic teller machines (ATMs) allow you to deposit and withdraw money from your bank account.

a Ashwan checks his bank balance and sees he has $86 in his account. Write this as a positive integer.

b The bank allows Ashwan's account to be overdrawn. This means that he can withdraw more money than he actually has in his account. If he withdraws $100 at an ATM, how much does he owe the bank?

c Write his new bank balance as a negative integer.

13 Jasmine needs glasses for reading as she is long sighted (hypermetropic). The prescription for the lenses in her glasses is +2.50. Alex wears contact lenses because he is short sighted (myopic). His lenses have prescriptions of −3.50 for his left eye and −4.25 for his weaker right eye.



a If Justin has a prescription of +1.75, is he long or short sighted?

b If Elle has a prescription of −4.75, is she long or short sighted?

c How does Elle's prescription compare to Alex's? Who has weaker vision?

d What do you think the reference of zero means in this situation?

14 Bank statements usually show a transaction as a positive number if an amount is added or credited to the account, or as a negative number for withdrawals or debits.

Use the bank statement to answer these questions.

Transaction	Balance
	+$52
+23	+$75
−35	+$40
−51	−$11
+34	+$23
+13	+$36
−49	−$13

a What is the largest amount:

i credited (added) to the account? **ii** debited (subtracted) from the account?

b What is the smallest amount:

i credited to the account? **ii** debited from the account?

c What does the final balance indicate about the account?

15 One way to represent integers is with coloured dots. Use one blue dot to represent +1 and one red dot to represent −1.



a How many red dots would you use to represent −3?

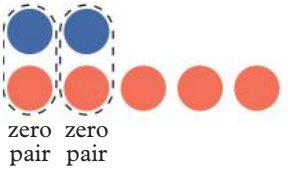
b Use the coloured dots to show each integer listed below. Draw each arrangement.

i 2 **ii** −4 **iii** 6

iv −5 **v** −1 **vi** 8

c One blue dot and one red dot together are called a zero pair. Explain how a zero pair represents 0.

d Explain how this arrangement of dots represents −3.



e What integer does each arrangement represent?

i 

ii 

iii 

16 Some friends are sitting in a circle. Kesha sits opposite Bri and three places to the left of Mo. Dee sits opposite Mo and three places to the left of Kesha. How many chairs are in the circle?

17 Find five consecutive numbers that add to −35.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Ordering positive and negative numbers

Investigation
Comparing world temperatures

Topic quiz
1E

1F Adding and subtracting integers

Learning intentions

By the end of this topic you will be able to ...

✔ solve problems involving addition and subtraction of positive and negative numbers.

Inter-year links

Year 7 — 5B Adding and subtracting positive numbers

Adding and subtracting positive integers

- When adding a positive integer, move to the right on the number line.

  For example, to calculate $-4 + 5$, start at -4 and move 5 steps to the right.

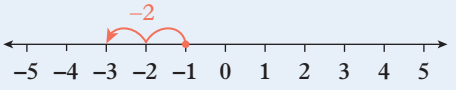
  $-4 + 5 = 1$



- When subtracting a positive integer, move to the left on the number line.

  For example, to calculate $-1 - 2$, start at -1 and move 2 steps to the left.

  $-1 - 2 = -3$

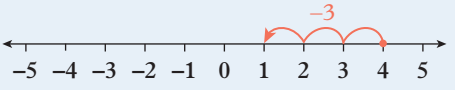


Adding and subtracting negative integers

- When adding a negative integer, move to the left on the number line.

  For example, to calculate $4 + (-3)$, start at 4 and move 3 steps to the left.

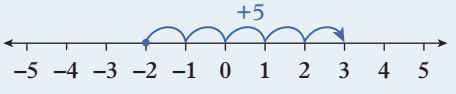
  $4 + (-3) = 4 - 3$
  $= 1$



- When subtracting a negative integer, move to the right on the number line.

  For example, to calculate $-2 - (-5)$, start at -2 and move 5 steps to the right.

  $-2 - (-5) = -2 + 5$
  $= 3$



- Adding a negative number can be simplified to subtracting its value. $+(-) = -$
- Subtracting a negative number can be simplified to adding its value. $-(-) = +$

Example 1F.1 Adding and subtracting positive numbers

Use the number line to calculate:

a $-6 + 8$ **b** $4 - 8$

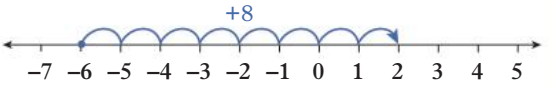
THINK

a 1 Draw a number line.
2 Start at the first number, then add the second number by moving right.

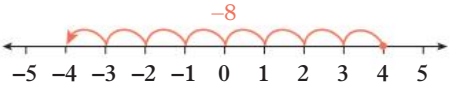
b 1 Draw a number line.
2 Start at the first number, then subtract the second number by moving left.

WRITE

a $-6 + 8 = 2$



b $4 - 8 = -4$



Example 1F.2 Adding and subtracting negative numbers

Use the number line to calculate:

a $4 + (-7)$ **b** $-2 - (-3)$

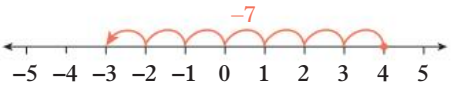
THINK

a 1 Simplify the calculation. Remember that $+(-) = -$
2 Draw a number line.
3 Start at the first number, then subtract the value by moving left.

b 1 Simplify the calculation. Write the problem in a simpler equivalent form by replacing $-(-)$ with $+$.
2 Draw a number line.
3 Start at the first number, then add the value by moving right.

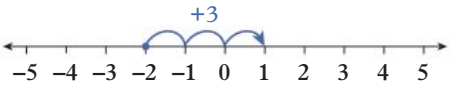
WRITE

a $4 + (-7) = 4 - 7$



$4 + (-7) = -3$

b $-2 - (-3) = -2 + 3$



$-2 - (-3) = 1$

Helpful hints

✔ The negative sign is identical to the minus sign. The function of the symbol depends on its context.

Example	Function of '−'	Read as
-8	Indicates the number is less than zero in value	'negative eight'
$2 - 8$	Indicates the operation of subtraction	'two minus eight'
$-8 + 2$	Indicates the number is less than zero in value	'negative eight plus two'

✔ Don't make assumptions about the sign of the answer – subtraction can produce positive numbers and addition can produce negative numbers!

✔ When simplifying calculations involving negative numbers, only combine the signs located in between the numbers.

$+(-) = -$ $\quad -(-) = +$ $\quad +(+) = +$ $\quad -(+) = -$

ANS p493

Exercise 1F Adding and subtracting integers

 1–3, 4–6(a–e), 7, 8–9(a–e), 10(a, b, d, e), 11(a), 12–14(1st column), 15, 17, 18

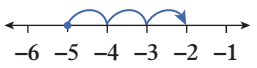
 4–6(e–i), 7(a, c, f), 8–10(c–f), 11(b), 12–14(2nd column), 16, 18, 19, 21, 22(a, b)

 8–9(f–i), 10(f–h), 11(b), 12(c, f, i), 13–14(2nd column), 18, 20–23

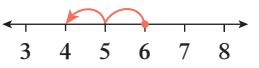
UNDERSTANDING AND FLUENCY

1 Write the addition or subtraction calculation shown in each diagram. Hint: Consider the starting point, the number of steps and the end point.

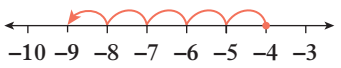
a



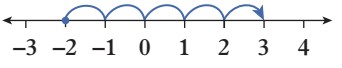
b



c

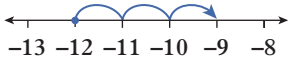


d

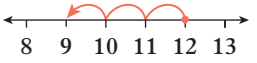


2 Match each diagram (**A**, **B**, **C**) to the appropriate problem (**a**, **b**, **c**).

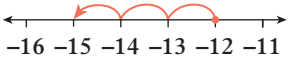
A



B



C



a $-12 - 3$ **b** $-12 + 3$ **c** $12 - 3$

3 Calculate each result for question **2**.

1F.1 **4** Use a number line to calculate:

a $-4 + 7$ **b** $1 + 5$ **c** $8 - 3$

d $-2 - 6$ **e** $-9 + 5$ **f** $3 - 6$

g $-5 - 1$ **h** $7 - 2$ **i** $2 - 6$

5 Draw a number line from −20 to 20 and use it to calculate:

a $15 - 12$ **b** $-17 + 11$ **c** $9 - 13$

d $-4 + 20$ **e** $-2 - 17$ **f** $8 - 18$

g $-14 - 3$ **h** $-16 + 14$ **i** $11 - 24$

j $-19 + 19$ **k** $-10 - 10$ **l** $-18 + 20$

6 Decide whether each result is positive, negative or zero.

a $-3 - 7$ **b** $16 + 14$ **c** $8 - 15$

d $-13 + 13$ **e** $-11 + 18$ **f** $19 - 12$

g $-17 + 4$ **h** $-14 - 10$ **i** $17 - 20$

7 Simplify the following calculations by using + or − to fill the gap.

a $-3 - (+7) = -3$ _____ 7 **b** $1 + (+6) = 1$ _____ 6

c $-4 + (-5) = -4$ _____ 5 **d** $2 - (-4) = 2$ _____ 4

e $5 - (+9) = 5$ _____ 9 **f** $-6 - (-8) = -6$ _____ 8

8 Simplify each problem.

a $-1 - (-3)$ **b** $8 - (+4)$ **c** $-5 + (-2)$

d $6 - (-1)$ **e** $4 + (+3)$ **f** $3 - (+9)$

g $-5 - (-8)$ **h** $-3 + (+1)$ **i** $-7 + (-3)$

1F.2 **9** Use a number line to calculate:

a $-4 - (-7)$ **b** $9 + (-3)$ **c** $2 - (+6)$

d $-5 - (-5)$ **e** $-8 + (+7)$ **f** $6 + (-1)$

g $-3 - (-4)$ **h** $4 - (+9)$ **i** $2 - (-7)$

10 Decide whether the result of each problem will be positive, negative or zero.

a $11 - (-8)$ **b** $-16 - (+4)$ **c** $13 - (+13)$ **d** $15 - (+18)$

e $-10 - (-7)$ **f** $-17 - (+11)$ **g** $8 - (-14)$ **h** $12 - (+19)$

11 Complete each addition table.

a

+	−5	−3	0	1	4
−3				−2	
−2					
−1			−1		
3					
6	1				

b

+	−10	−14	−7	13	19
11					
14					
−10					
−22					
−18					

12 Calculate:

a $-45 + 40$ **b** $-72 - 27$ **c** $56 - 88$

d $-34 + 43$ **e** $130 - 170$ **f** $-213 - 62$

g $-158 + 400$ **h** $286 - 168$ **i** $-505 + 505$

13 Calculate:

a $-2 + 5 + 7$ **b** $3 - 9 + 2$

c $6 - 1 - 8$ **d** $-10 + 4 - 7$

e $-12 + 19 - 7$ **f** $-20 - 11 - 2$

g $33 + 22 - 65$ **h** $70 - 81 - 15$

i $-24 - 18 + 12$ **j** $59 - 93 + 17$

14 Simplify each problem, and then calculate the result.

a $-5 - (+7) + (+6)$ **b** $9 + (-2) - (-8)$

c $-4 - (-3) + (+1)$ **d** $8 + (-6) - (-10)$

e $-22 - (+5) + (+13)$ **f** $-14 - (-8) + (-25)$

g $34 + (-19) - (+7)$ **h** $-9 - (+26) - (-37)$

15 You enter a lift at the 15th floor and travel down 19 floors. Which level do you finish at? Show how you calculated your answer.



16 Sarah enters a lift at the third floor and travels down nine floors and then up five floors. What floor does she finish at? Show the problem you used to get your answer.

17 What is the difference between a temperature of −18°C and a temperature of 33°C? Show the subtraction problem you used to calculate the result.

18 Jess owes her brother $60.

a Write this amount as an integer.

b She pays him $35. Write an addition problem to work out the amount she still owes him.

c How much does Jess still owe her brother?

PROBLEM SOLVING AND REASONING

19 A bungee jumper dives from a platform (position A) 50 m above the water. He dips 2 m under the water (position B) before rebounding to a height 18 m below the platform (position C).



a If the reference is the surface of the water, write positions A, B and C as integers.

b Write an expression to find the distance between:

i A and B

ii B and C

iii A and C.

c Calculate each distance.

d What is the largest distance for part **c**? Explain why that is the largest distance.

20 A deep sea submersible is lowered into the ocean from a ship's deck 15 m above sea level. The unmanned vessel is on a mission to take photos of deep sea animals.

For each part, write an addition or subtraction problem and then calculate the answer.



a A deep sea anglerfish is 900 m below sea level. What vertical distance has the submersible travelled from the deck of the ship to reach this fish?

b A deep sea amphipod is 2500 m below sea level. How much further has the submersible travelled vertically to reach it?

c What vertical distance does the submersible now need to travel to return to the deck of the ship?

d When the ship gets back to the dock, the crane on board the ship lowers the submersible onto the dock. The crane lifts the submersible 2 m above the ship's deck, moves it horizontally at that height and then lowers it 8 m onto the dock. What is the height of the dock above sea level?

CHALLENGE

21 What number should be subtracted from each of these to give the result of 20?

a 65 **b** −10 **c** 1281

d −347 **e** −1000 **f** −3240

22 Find the missing integer to make each number sentence true.

a $-54 + 33 + ____ = 12$ **b** $121 - ____ - 50 = -6$

c $-300 + 225 - ____ = 50$ **d** $____ - 58 + 73 = -25$

23 At the start of March, Tyra's bank account shows a balance of −$310. At the end of the month, it shows a balance of −$247.

a Calculate the difference between the two account balances. Show the subtraction problem you used to calculate the result.

b Interpret the result. Hint: What does this result mean?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Adding and subtracting with negative numbers

Investigation
Create a subtraction

Topic quiz
1F

1G Multiplying and dividing integers

Learning intentions

By the end of this topic you will be able to ...

- ✔ solve problems involving multiplication and division of negative numbers
- ✔ solve problems involving exponents of negative numbers.

Inter-year links

Support	Multiplying whole numbers
Year 7	1D Multiplying whole numbers

Multiplying integers

- Multiplying two integers can give a result that is positive or negative (or zero), according to these rules.
  - → The product of two positive integers is a positive integer.
  - → The product of a positive integer and a negative integer is a negative integer.
  - → The product of two negative integers is a positive integer.

$$+ \times + = +$$
$$+ \times - = -$$
$$- \times + = -$$
$$- \times - = +$$

Dividing integers

- Division of integers follows the same rules as multiplication of integers.
  - → The quotient of two positive integers is a positive integer.
  - → The quotient of a positive integer and a negative integer is a negative integer.
  - → The quotient of two negative integers is a positive integer.

$$+ \div + = +$$
$$+ \div - = -$$
$$- \div + = -$$
$$- \div - = +$$

Exponents of negative numbers

- When applying exponents to negative numbers, the sign needs to be taken into account.

exponent/index

base → $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

exponent form expanded form basic numeral

  - → If the base is negative and the exponent is an even number, the basic numeral is positive.
  - → If the base is negative and the exponent is an odd number, the basic numeral is negative.

Example 1G.1 Multiplying integers

Evaluate each product.

a -4×7 **b** $-9 \times (-5)$

THINK	WRITE
a 1 Determine whether the result is positive or negative. The signs are different, so the result is negative.	**a** $4 \times 7 = 28$
2 Calculate the product.	So, $-4 \times 7 = -28$
b 1 Determine whether the result is positive or negative. The signs are the same, so the result is positive.	**b** $9 \times 5 = 45$
2 Calculate the product.	So, $-9 \times (-5) = 45$

Example 1G.2 Dividing integers

Evaluate each quotient.

a $-30 \div (-5)$ **b** $28 \div (-7)$

THINK	WRITE
a 1 Decide whether the result is positive or negative. The signs are the same, so the result is positive.	**a** $30 \div 5 = 6$
2 Calculate the quotient.	So, $-30 \div (-5) = 6$
b 1 Decide whether the result is positive or negative. The signs are different, so the result is negative.	**b** $28 \div 7 = 4$
2 Calculate the quotient.	So, $28 \div (-7) = -4$

Example 1G.3 Multiplying three integers

Evaluate the product $-3 \times (-5) \times (-2)$.

THINK	WRITE
1 Work from left to right. Multiply the first two numbers and determine the sign. As the signs are the same, the result is positive.	$-3 \times (-5) = 15$
2 Multiply the result by the third number and determine the sign. As the signs are different, the result is negative.	$15 \times (-2) = -30$ $-3 \times (-5) \times (-2) = -30$

Example 1G.4 Calculating the value of a number in exponent form

Write each of these in expanded form and calculate their values.

a $(-3)^2$ **b** $(-2)^5$

THINK

a 1 Write the calculation as a repeated multiplication.

2 Determine whether the result will be positive or negative. A negative multiplied by a negative is a positive.

3 Calculate the product.

b 1 Write the calculation as a repeated multiplication.

2 Determine whether the result will be positive or negative. A negative multiplied by a negative is a positive. A positive multiplied by a negative is a negative. Continue to simplify the repeated multiplication.

3 Calculate the product.

WRITE

a $(-3)^2 = (-3) \times (-3)$
$= 9$

b $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2)$
$= 4 \times (-2) \times (-2) \times (-2)$
$= (-8) \times (-2) \times (-2)$
$= 16 \times (-2)$
$= -32$

Helpful hints

✔ Be careful when determining the sign of a result when multiplying or dividing numbers.
If the signs are the same, then **the result is positive**.
If the signs are different, then **the result is negative**.

ANS p493

Exercise 1G Multiplying and dividing integers

△ 1–2(1st, 2nd columns), 3–5, 6(a), 7–13, 14(a–c), 15(a, b), 16, 18, 19(a, b, e), 20(a–c)

□ 3–5, 6(b), 7, 8–9(d–f), 12(e–h), 13(c, d), 14(d–f), 15(c, d), 17, 19(c, d, g), 20, 21(a, b)

⬠ 4(i–l), 6(b), 7, 8(g–i), 9(b, d, e), 13(c, d), 14(g–i), 19, 21–24

1G.1 **1** Evaluate each product.

a $3 \times (-4)$ **b** 7×5 **c** -6×2 **d** $-3 \times (-9)$
e -1×8 **f** $4 \times (-6)$ **g** $-8 \times (-9)$ **h** 2×10
i -4×11 **j** $7 \times (-1)$ **k** $-2 \times (-4)$ **l** 9×5

1G.2 **2** Evaluate each quotient.

a $15 \div (-3)$ **b** $-32 \div (-8)$ **c** $24 \div 4$ **d** $-63 \div 9$
e $-17 \div (-1)$ **f** $48 \div (-6)$ **g** $-56 \div 8$ **h** $-81 \div (-9)$
i $42 \div (-7)$ **j** $-12 \div 1$ **k** $-20 \div (-2)$ **l** $72 \div (-8)$

3 Evaluate each quotient.

a $\frac{-48}{6}$ **b** $\frac{60}{-5}$ **c** $\frac{-51}{-3}$ **d** $\frac{150}{10}$ **e** $\frac{-200}{-25}$ **f** $\frac{57}{-19}$

UNDERSTANDING AND FLUENCY

4 Calculate each of the following.

a $5 \times (-7)$ **b** $-9 \div 3$ **c** $-28 \div (-4)$ **d** -10×6

e $36 \div (-9)$ **f** $-15 \times (-4)$ **g** $-100 \div 20$ **h** $18 \times (-2)$

i $-3 \times (-30)$ **j** $25 \div (-25)$ **k** -1×38 **l** $-45 \div (-1)$

5 Complete each statement.

a $-15 \times$ _____ $= 45$ **b** $56 \div$ _____ $= -8$ **c** _____ $\times (-4) = 36$

d _____ $\div (-25) = -4$ **e** $-140 \div$ _____ $= 2$ **f** _____ $\times 16 = -48$

6 Complete each multiplication table.

a

×	−2	−1	0	+1	+2
−4	8			−4	
−2					
0			0		
+2					
+4		−4			
+6					

b

×		−20	−10		20
7				70	
			−50		
0					
−2	50				
		80			
					−120

7 A short way of writing $-1 \times (-2)$ is $-(-2)$. Write each problem as the product of two numbers and then calculate the result.

a $-(-7)$ **b** $-(+12)$ **c** $-(-88)$

d $-(+25)$ **e** $-(8 - 5)$ **f** $-(-3 + 2)$

1G.3 **8** Evaluate each product.

a $-4 \times 3 \times (-5)$ **b** $3 \times (-2) \times 7$ **c** $6 \times (-1) \times 9$

d $8 \times 2 \times (-3)$ **e** $-5 \times (-2) \times (-13)$ **f** $-7 \times 3 \times (-4)$

g $4 \times 5 \times (-8)$ **h** $-11 \times 5 \times (-2)$ **i** $9 \times (-4) \times 5$

9 Evaluate each statement.

a $-2 \times 2 \times (-3) \times 3$ **b** $20 \div (-2) \div (-2) \div (-1)$ **c** $-4 \times (-1) \times 3 \times (-2)$

d $-18 \div (-2) \times 4 \div (-6)$ **e** $9 \times (-2) \div 3 \times (-5)$ **f** $-7 \times (-4) \div 2 \times (-3)$

10 Which of these five options is the expanded form of $(-4)^5$?

A $4 \times 4 \times 4 \times 4 \times 4$ **B** $5 \times 5 \times 5 \times 5$ **C** $-4 \times (-4) \times 4 \times 4 \times 4$

D $-4 \times (-4) \times (-4) \times (-4)$ **E** $-4 \times (-4) \times (-4) \times (-4) \times (-4)$

11 Without calculating each result, predict whether the basic numeral is positive or negative.

a $(-2)^9$ **b** $(-15)^6$ **c** $(+6)^7$ **d** $(-100)^4$

1G.4 **12** Write each of these in expanded form and calculate their values.

a $(-9)^2$ **b** 8^2 **c** $(-6)^3$ **d** 7^3

e $(-5)^4$ **f** 1^4 **g** 4^5 **h** $(-10)^5$

13 By collecting the like terms, write each of these multiplications in exponent form.

a $-5 \times (-5) \times (-5) \times (-9) \times (-9) \times (-9) \times (-9)$ **b** $-4 \times (-4) \times (-4) \times (-4) \times (-4) \times (-4) \times 3 \times 3 \times 3$

c $7 \times 7 \times (-6) \times (-6) \times (-6) \times (-6) \times (-6) \times (-6)$ **d** $-8 \times (-8) \times (-8) \times (-8) \times (-10) \times (-10)$

14 Write each of these in expanded form and calculate their values.

a $(-2)^3 \times (-4)^2$ **b** $(-5)^2 \times 3^4$ **c** $(-10)^3 \times (-2)^5$

d $(-3)^5 \times (-1)^2$ **e** $(-6)^2 \times (-2)^3$ **f** $(-1)^4 \times 3^2$

g $(-7)^2 \times (-1)^5$ **h** $(-3)^3 \times (-2)^2$ **i** $(-1)^3 \times (-2)^5$

15 Complete each set of problems. Describe the pattern you see.

a
$2 \times 5 =$ _____
$2 \times 4 =$ _____
$2 \times 3 =$ _____
$2 \times 2 =$ _____
$2 \times 1 =$ _____
$2 \times 0 =$ _____
$2 \times (-1) =$ _____
$2 \times (-2) =$ _____
$2 \times (-3) =$ _____
$2 \times (-4) =$ _____
$2 \times (-5) =$ _____

b
$-3 \times 5 =$ _____
$-3 \times 4 =$ _____
$-3 \times 3 =$ _____
$-3 \times 2 =$ _____
$-3 \times 1 =$ _____
$-3 \times 0 =$ _____
$-3 \times (-1) =$ _____
$-3 \times (-2) =$ _____
$-3 \times (-3) =$ _____
$-3 \times (-4) =$ _____
$-3 \times (-5) =$ _____

c
$-4 \times (-5) =$ _____
$-4 \times (-4) =$ _____
$-4 \times (-3) =$ _____
$-4 \times (-2) =$ _____
$-4 \times (-1) =$ _____
$-4 \times 0 =$ _____
$-4 \times 1 =$ _____
$-4 \times 2 =$ _____
$-4 \times 3 =$ _____
$-4 \times 4 =$ _____
$-4 \times 5 =$ _____

d
$5 \times (-5) =$ _____
$5 \times (-4) =$ _____
$5 \times (-3) =$ _____
$5 \times (-2) =$ _____
$5 \times (-1) =$ _____
$5 \times 0 =$ _____
$5 \times 1 =$ _____
$5 \times 2 =$ _____
$5 \times 3 =$ _____
$5 \times 4 =$ _____
$5 \times 5 =$ _____

16 a Dividing is the inverse operation to multiplying. Complete each statement.

i $2 \times 3 =$ _____ so _____ $\div 2 = 3$ or _____ $\div 3 = 2$

ii $2 \times (-3) =$ _____ so _____ $\div 2 = -3$ or _____ $\div (-3) = 2$

iii $-2 \times 3 =$ _____ so _____ $\div (-2) = 3$ or _____ $\div 3 = -2$

iv $-2 \times (-3) =$ _____ so _____ $\div (-2) = -3$ or _____ $\div (-3) = -2$

b Does division follow the same pattern as multiplication to give positive or negative results?

c Is the result positive or negative for each of these division types?

i positive ÷ positive

ii negative ÷ negative

iii positive ÷ negative

iv negative ÷ positive

17 a Calculate each of these by first writing them in expanded form.

i $(-1)^1$ **ii** $(-1)^2$ **iii** $(-1)^3$

iv $(-1)^4$ **v** $(-1)^5$ **vi** $(-1)^6$

b Which of these problems give a positive result?

c Which of these problems give a negative result?

d Explain your observations.

e Predict the result for:

i $(-1)^{13}$ **ii** $(-1)^{32}$ **iii** $(-1)^{100}$

iv $(-1)^{203}$ **v** $(-1)^{188}$ **vi** $(-1)^{555}$

18 a Using a calculator, determine:

i 8×0 **ii** -15×0 **iii** 0×65 **iv** $0 \times (-72)$

b What do you notice when you multiply any integer by zero?

c Using a calculator, determine:

i $0 \div (-9)$ **ii** $0 \div 26$ **iii** $0 \div (-11)$ **iv** $0 \div 39$

d What do you notice when you divide zero by any integer?

19 Find two integers that have:

a a sum of −9 and a product of 18

b a sum of 4 and a product of −45

c a sum of −5 and a product of −66

d a sum of 10 and a product of −56

e a difference of 2 and a product of 24

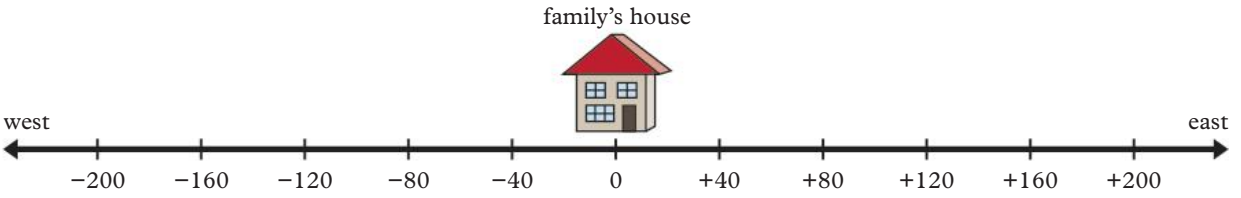
f a difference of 8 and a product of 9

g a difference of 18 and a product of −32.

PROBLEM SOLVING AND REASONING

20 Astro the dog has escaped from his yard. His owners decide to search along the road which runs east–west outside the house. This number line can be used to represent the road outside the family's house, with the house at zero.

a Tyler and Rhys walk 120 m west from the house and call Astro's name. What is their position on the number line?



b Sophie and Imogen walk 80 m east from the house and call Astro's name. What is their position on the number line?



c What is the distance between the pairs of searchers?

d Imogen now walks a further 30 m east. What is her new position on the number line?

e Rhys now walks 160 m east. What is his new position on the number line?

f What is the distance between:

i Sophie and Rhys **ii** Tyler and Imogen?

g Natalie starts at the house, then walks twice as far as Sophie but in the same direction as Tyler. What is her position on the number line?

h Hayden starts at the house, then walks one-third of the distance walked by Tyler and in the same direction. What is his position on the number line?

i A neighbour has found Astro in her yard, 60 m west of the family's house. Who is closest to Astro? How far and in which direction does he or she need to walk to collect the dog?

j If each member of the family now walks home, calculate the distance each person walked from the start of the search.

21 Squaring two different numbers can produce the same result. For example, $3^2 = 9$ and $(-3)^2 = 9$.

a Which two numbers, when squared (raised to the power of 2), give 25? Why are there two numbers?

b Is there a number that, when squared, gives −25? Explain.

c How many different numbers can be cubed (raised to the power of 3) to give 8?

d How many different numbers can be cubed (raised to the power of 3) to give −8?

e Predict how many numbers can be raised to the power of 4 to give the same positive result. Hint: Try to raise a number to the power of 4 to give 16. Repeat for a negative result.

f Similarly, predict what would happen for exponents of 5, 6 and so on. Provide examples to support your answer.

22 What number(s), when squared, give(s) each result?

a 49 **b** 81 **c** 1 **d** 4 **e** 64 **f** 100

23 What number(s), when cubed, give(s) each result?

a 27 **b** −125 **c** 64 **d** −1 **e** −64 **f** −1000

CHALLENGE

24 Consider $(+3)^2$, $(-3)^2$, 3^2 and -3^2.

a Which of these produce the same result?

b Explain how $(-3)^2$ is different from -3^2.

c Decide whether each pair produces the same result.

i $(-4)^3$ and -4^3 **ii** $(-2)^4$ and -2^4 **iii** $(-1)^6$ and -1^6 **iv** $(-10)^5$ and -10^5

d Describe the general pattern you have seen.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet — Multiplying and dividing negative numbers
- Interactive skillsheet — Exponents of negative numbers
- Investigation — Multiplication triangulation
- Topic quiz — 1G

1H Order of operations

Learning intentions

By the end of this topic you will be able to ...

- ✔ order operations in calculations involving two or more operations
- ✔ evaluate expressions including integers using the order of operations.

Inter-year links

Support	Order of operations
Year 7	1H Order of operations

Order of operations

- **'BIDMAS'** can be used to help remember the order of operations.

B	**Brackets**	Operations inside brackets are always performed first. For example, $2^2 \times (4 - 6) - 8 \div (-2) = 2^2 \times (-2) - 8 \div (-2)$.
I	**Indices (or exponents)**	Next, evaluate indices (exponents) and roots. For example, $2^2 \times (-2) - 8 \div (-2) = 4 \times (-2) - 8 \div (-2)$.
D **M**	**Division** **Multiplication**	Then working from **left to right**, perform any multiplication or division before adding or subtracting. For example, $4 \times (-2) - 8 \div (-2) = -8 - (-4)$.
A **S**	**Addition** **Subtraction**	Finally, working from **left to right**, perform any addition and subtraction. For example, $-8 - (-4) = -8 + 4 = -4$.

- Operations at the same ranking are performed in order from left to right.

  For example, $5 + 8 - 2 = 13 - 2$
  $= 11$

  $-20 \times \frac{3}{10} = -\frac{60}{10}$
  $= -6$

- Where there is more than one set of brackets in the problem, perform the operations inside the innermost brackets first.

  For example, $[2 \times (5 - 3)] - 7 = [2 \times (5 - 3)] - 7$
  $= (2 \times 2) - 7$
  $= 4 - 7$
  $= -3$

Addition (+)	Subtraction (−)	Multiplication (×)	Division (÷)
• Sum	• Difference	• Product	• Quotient
• Plus	• Take away	• Times	• Divide
• More than	• Less than	• Multiply	• Shared
• Increase	• Fewer	• Groups of	• Distributed
• Total	• Reduce		
• Together	• Minus		

Example 1H.1 Order of operations

Evaluate each of the following.

a $-3 + 4 \times (-2)$ **b** $7 - (-15) \div 5$ **c** $(-2)^3 \times 3 + 5$

THINK

a **1** Perform the multiplication and determine the sign of the result.

2 Simplify the signs and perform the subtraction.

b **1** Perform the division and determine the sign of the result.

2 Simplify the signs and perform the addition.

c **1** Apply the exponent and determine the sign of the result.

2 Perform the multiplication and determine the sign of the result.

3 Perform the addition.

WRITE

a $-3 + 4 \times (-2) = -3 + (-8)$

$= -3 - 8$

$= -11$

b $7 - (-15) \div 5 = 7 - (-3)$

$= 7 + 3$

$= 10$

c $(-2)^3 \times 3 + 5 = -8 \times 3 + 5$

$= -24 + 5$

$= -19$

Example 1H.2 Order of operations with brackets

Evaluate each of the following.

a $-18 \div (4 - 10) + 2 \times 7$ **b** $6 - [-12 \div (9 - 5)]^2 - 2$

THINK

a **1** Perform any operations inside brackets first.

2 Working from left to right, perform division and multiplication before addition.

3 Perform the addition.

b **1** There are two sets of brackets. Perform the operation in the innermost set of brackets first.

2 Perform the operation in the remaining set of brackets.

3 Apply the exponent before addition and subtraction.

4 Working from left to right, perform the subtraction.

WRITE

a $-18 \div (4 - 10) + 2 \times 7 = -18 \div -6 + 2 \times 7$

$= 3 + 2 \times 7$

$= 3 + 14$

$= 17$

b $6 - [-12 \div (9 - 5)]^2 - 2 = 6 - [-12 \div 4]^2 - 2$

$= 6 - [-3]^2 - 2$

$= 6 - 9 - 2$

$= -3 - 2$

$= -5$

Helpful hints

- ✔ Remember to use BIDMAS rather than working from left to right.
- ✔ It is helpful to determine the sign at each step of a calculation before progressing.
- ✔ Calculating the value of a number in exponent form uses the following rules:
  - If the base is negative and the exponent is an even number, **the result is positive**.
  - If the base is negative and the exponent is an odd number, **the result is negative**.
- ✔ For multiplying and dividing integers, recall the simplification rules for calculations involving two numbers.
  - If the signs are the same, then **the result is positive**.
  - If the signs are different, then **the result is negative**.

$+ (-) = -$ $\quad - (-) = +$ $\quad + (+) = +$ $\quad - (+) = -$

- ✔ For adding and subtracting integers, recall the simplification rules. You can also use the number line to help you.

ANS p495

Exercise 1H Order of operations

△ 1–3, 4(1st column), 5–8, 9, 11, 12(a, b) ▭ 3, 4(2nd column), 5–8, 10, 12–15 ⬠ 4(2nd column), 5–8, 12, 13, 15–17

1 For each calculation, list the operations in the correct order in which they should be completed.

a $-28 - 3 \times 7 + 11$ **b** $50 + (34 - 19) \div (-5)$ **c** $12 + 3 \times 4^2 - (-41)$

d $-20 - 2 \times 6 + 3^2$ **e** $8 + [9 - (-3 + 2)]$ **f** $24 \div 6 + (5 + (-1))^2$

1H.1 **2** Evaluate each of the following.

a $16 + 28 \div 4$ **b** $60 - 2 \times 10$ **c** $27 \div 3 + 6 \times 11$

d $22 + 30 \times 2 \div 15$ **e** $10 \times 8 - 7 \times 9$ **f** $6^2 - 8 \times 3 + 35$

3 Evaluate each of the following.

a $7 - 2 \times 8$ **b** $-1 + 16 \div 16$ **c** $28 - 3 \times 11$

d $-6 + 5 \times 3 - 2$ **e** $-5 + [-9 + (-6 + 4)]$ **f** $42 \div (-7) + [5 \times (-2)^2]$

1H.2 **4** Evaluate each of the following.

a $72 \div (7 + 2) - 2 \times 3$ **b** $(27 - 14) \times (19 + 11)$

c $11 - (4 \times 2 - 19)$ **d** $(-3 \times 7 + 1) \div (8 - 12)$

e $9^2 + 7 \times (34 - 28)$ **f** $10 \times 8 - 4 \times (3^2 + 2)$

g $60 \div [6 + (3 \times 5 - 1)] + 48$ **h** $-12 + [-48 \div (28 - 26)]^2 - 9$

5 Evaluate each of the following.

a $(-3)^2 \times (-14 + 10) \div (-6)$ **b** $(-19 + 16)^2 + (-23 + 17)^3$ **c** $(-2)^3 \times (-4) \div (-8)$

d $(-2)^2 \times (-2^2)$ **e** $(120 \div 6) - (-10) \div 2$ **f** $[-3 + (150 \div 50)] \times (-7)^2 \div 5^2$

6 Write an equivalent sum to the following using a single sign. Do not calculate the answers.

a $66 + (-7)$ **b** $415 - (-322)$ **c** $365 - (+109)$

d $1750 - (-3487)$ **e** $4376 + (-237)$ **f** $6548 - (+2399)$

7 Identify the mistakes that have been made in the following calculations.

a $(-10 + 18)^2 = -64$ **b** $-15 + 5 \times 5 = -50$ **c** $-18 \div 3 + 6 = -2$

d $(4 - 20) \div (-4) = -4$ **e** $(-4)^3 \div 8 = 8$ **f** $-5 \times 6 + 90 \div 10 = 39$

8 Find the correct answers to the problems in question **7**.

9 You have $195 to buy three shirts priced at $58 each.

a Write a calculation to show how to work out how much money you will have left over.

b Estimate how much money is left over by rounding each number to its leading digit then performing the calculation.

c Perform the calculation in part **a** and compare your estimate to the exact amount of money you have left over.

10 Using the images shown, calculate the cost of buying lunch for the class: 14 salad rolls, 11 samosa packs, 9 flavoured milks, 16 orange juices and 25 apples. Show the numbers and operations you used in the calculation.

$7 $6 for three $4 $5 $1







11 When standing with outstretched arms above her head, the height of an Olympic diver is 3 m. She dives from the 10 m platform and touches the bottom of the pool with her outstretched fingers. The depth of the pool is 6 m.



a If the pool surface is the reference point for zero, write the maximum height of the diver above the pool surface as a directed number.

b Write the depth of the pool as a directed number.

c Write a subtraction problem to calculate the vertical distance covered by the diver. Hint: Find the difference between the two positions.

d What vertical distance has the diver covered?

12 The daily maximum and minimum temperatures at Mt Buller were recorded over a week.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Maximum temperature (°C)	8	4	−3	−2	−1	0	1
Minimum temperature (°C)	2	−3	−5	−6	−6	−3	0

a Calculate the difference between the minimum and maximum temperatures for each day.

b Which day had the biggest range of temperatures?

c Calculate the average (or mean) of:

i the maximum temperatures

ii the minimum temperatures.

d What is the difference between the average minimum and average maximum temperature?

13 This incomplete statement shows some deposits and withdrawals made to Martin's bank account.

Date	Reference	Transaction	Balance
30 April	–	–	+$289
2 May	Tony's Newsagent	+$132	
15 May	Movie Place	−$17	
21 May	Bicycle City	−$500	
24 May	Tony's Newsagent	+$230	
26 May	DD Online Music Store	−$38	
27 May	Sports and Stuff Pty Ltd		+$69



a Is a transaction of +$132 a deposit or a withdrawal?

b Is a transaction of −$17 a deposit or a withdrawal?

c What does it mean if the balance in Martin's account is:

i positive

ii negative?

d Find the account balance after the transaction made on 21 May.

e Find the balance after the transaction made on 24 May.

f What transaction is made on 27 May so that the account balance is +$69?

g Was Martin's account overdrawn at any stage? What penalty do banks have for when an account is overdrawn?

14 If the sum of two numbers is zero, is the product of the numbers positive or negative? Explain.

15 Find three integers whose sum is −5 and product is 120.

16 Evaluate the following.

a $6 + \cfrac{52}{6 + \cfrac{63}{3 + \cfrac{48}{3 + \cfrac{55}{3 + \cfrac{96}{8 + \cfrac{40}{8 + \cfrac{4}{1 + \cfrac{5}{1 + \frac{8}{2}}}}}}}}}$

b $4 - \cfrac{-64}{10 - \cfrac{52}{15 - \cfrac{132}{-3 - \cfrac{-54}{-1 - \cfrac{50}{3 - \cfrac{7}{3 - \cfrac{36}{4 - \cfrac{10}{1 - \frac{24}{8}}}}}}}}}$

17 The 'Four Fours' problem uses four of the digit '4' and a combination of operations and brackets to write calculations that are equal to each integer.

For example, 4 + 4 + 4 + 4 = 16.

a Write a series of calculations, using four '4's and any combination of the operations +, −, ×, ÷ and brackets, that are equal to each of the integers 0 to 20. Are all of these integers possible using these operations?

b If we include the square root operation, $\sqrt{4}$, can you write a calculation for each of the integers 1 to 20? If you couldn't find all the numbers in part **a**, see if you can find them using the square root operation.

c The factorial, $n!$, is an operation that multiplies each of the integers from a number, n, down to 1.

For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.

Using the factorial with the above operations, which numbers are now possible?

d Explain how you could use your results above to write calculations that are equal to each of the integers from −20 to −1.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Order of operations

Investigation
Converting temperatures

Topic quiz
1H

Chapter summary

Rounding and estimating

$12\,600 \approx 13\,000$ — Round up (9, 8, 7, 6, 5)

$12\,300 \approx 12\,000$ — Round down (4, 3, 2, 1, 0)

Leading digit

$123\,456 = 123\,456 \approx 100\,000$

Adding and subtracting

$$\begin{array}{r} {\scriptstyle 1} \\ 229 \\ +\ 342 \\ \hline 571 \end{array}$$ ← answer

Ones column: 9 + 2 = 11, carry the 1
Tens column: 2 + 4 + (1) = 7
Hundreds column: 2 + 3 = 5

$$\begin{array}{r} {\scriptstyle 1\ 11} \\ 321 \\ -\ 204 \\ \hline 117 \end{array}$$ ← answer

Ones column: Take 10 from the tens column 11 − 4 = 7
Tens column: 1 − 0 = 1
Hundreds column: 3 − 2 = 1

The distributive law

$5 \times 32 = 5 \times (30 + 1)$
$= 5 \times 30 + 5 \times 2$
$= 150 + 10$
$= 160$

Multiplication algorithm

Write → 34

× 26

Multiply by ones → 204 (34×6)

Multiply by tens → + 680 (34×20)
Don't forget the zero(s)

Add the results → 884

Short division

quotient: 145 remainder 1

$6\overline{)8^{2}7^{3}1}$

divisor: 6 dividend: 8271 remainder: 1

Lowest common multiple (LCM)

Multiples of 4: 4, 9, (12), 16 …
Multiples of 6: 6, (12), 18, 24 …
LCM: 12

Highest common factor (HCF)

Factors of 12: 1, 2, 3, 4, (6), 12
Factors of 18: 1, 2, 3, (6), 9, 18
HCF: 6

Squares and cubes

$1^2 = 1 \times 1 = 1$	$\sqrt{1} = 1$	$1^3 = 1 \times 1 \times 1 = 1$	$\sqrt[3]{1} = 1$
$2^2 = 2 \times 2 = 4$	$\sqrt{4} = 2$	$2^3 = 2 \times 2 \times 2 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 3 \times 3 = 9$	$\sqrt{9} = 3$	$3^3 = 3 \times 3 \times 3 = 27$	$\sqrt[3]{27} = 3$

Integers

increasing value →
← decreasing value

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10

negative integers | zero | positive numbers

integers

Adding and subtracting integers

$+\,(-) = -$ $\quad -\,(+) = -$
$-\,(-) = +$ $\quad +\,(+) = +$

Powers of integers

exponent/index

base → $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

exponent form | expanded form | basic numeral

Order of operations

B	**Brackets**	Operations inside brackets are always performed first. For example, $5 \times (10 - 2) = 5 \times 8 = 40$.
I	**Indices (exponents)**	Next, evaluate indices (exponents) and square roots. For example, $3^3 - 4 = 27 - 4 = 23$.
D / M	**Division / Multiplication**	Then working from **left to right**, perform any multiplication or division before adding or subtracting. For example, $5 \times 3 - 4 + 8 \div 4 = 15 - 4 + 2 = 13$.
A / S	**Addition / Subtraction**	Finally, working from **left to right**, perform any addition and subtraction. For example, $15 - 4 + 2 = 13$.

Multiplying and dividing integers

$+ \times + = +$ $\quad + \div + = +$
$+ \times - = -$ $\quad + \div - = -$
$- \times + = -$ $\quad - \div + = -$
$- \times - = +$ $\quad - \div - = +$

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Multiple choice

1A **1** After rounding each number to its leading digit, what is the estimate for 526×68?
A 36 000 **B** 35 000 **C** 33 664 **D** 31 20 **E** 30 000

1A **2** Which of the following numbers would be 700 if it was rounded to the nearest hundred?
A 789 **B** 750 **C** 634 **D** 651 **E** 792

1B **3** What is 45 783 added to 67 398?
A 103 181 **B** 113 081 **C** 113 171 **D** 112 181 **E** 113 181

1B **4** What is 3765 subtracted from 9632?
A 5877 **B** 5967 **C** 6867 **D** 5867 **E** 5977

1C **5** What is 144 multiplied by 212?
A 356 **B** 30 536 **C** 30 000 **D** 30 528 **E** 28 800

1C **6** What is 672 divided by 12?
A 58 **B** 57 **C** 56 **D** 55 **E** 54

1D **7** Which of the following is the HCF of 36 and 90?
A 5 **B** 9 **C** 36 **D** 18 **E** 3

1D **8** What is the square root of 36 equivalent to?
A $\sqrt{36}$ **B** $\sqrt{6}$ **C** 36^2 **D** 6^2 **E** 216

1E **9** Which of the following numbers is smaller than -25?
A -18 **B** 2 **C** 25 **D** 0 **E** -30

1F **10** What integer when added to 12 gives -7?
A -17 **B** -5 **C** 5 **D** 19 **E** -19

1F **11** What is $-24 - 20$?
A -4 **B** 4 **C** -44 **D** 44 **E** -48

1F **12** What does $-35 - (-47)$ simplify to?
A $-35 - 47$ **B** $35 - 47$ **C** $35 + 47$ **D** $-35 + 47$ **E** $-47 + 35$

1G **13** What is $8 \div (-2)$?
A -16 **B** -4 **C** 4 **D** 16 **E** 6

1G **14** What pair of integers gives a sum of -5 and a product of -36?
A -4 and 9 **B** -12 and 3 **C** 4 and 9 **D** -9 and -4 **E** -9 and 4

1G **15** Which gives the largest result?
A $(-2)^6$ **B** 6^2 **C** $(-1)^{10}$ **D** $(-4)^3$ **E** $-(8)^2$

1H **16** Which of these problems does *not* equal 5?
A $-4 - 11 + 16 - 2 + 6$ **B** $8 - 13 - 1 + 17 - 6$ **C** $-7 + 5 - 9 + 22 - 6$
D $5 - 12 + 3 - 7 + 6$ **E** $8 - 9 + 20 - 18 + 4$

1H **17** What operation is performed first in $6 \times (12 - 5) + 8^2 \div 4$?
A $12 - 5$ **B** 8^2 **C** 6×12 **D** $64 \div 4$ **E** $8 \div 4$

Short answer

1A **1** Round each number to its leading digit.
a 236 **b** 67 145 **c** 3890 **d** 149 046

1A **2** Estimate the result by first rounding each number to its leading digit.
a $12\,345 + 3648$ **b** $94\,501 - 32\,566$ **c** 394×338 **d** $18\,654 \div 425$

1B **3** Three houses in one street were all sold on the same day. The selling prices were \$765 340, \$875 900 and \$655 000. What was the total price of all three houses?

1B **4** The City of Ipswich has 236 708 residents, while the Moreton Bay Region has 486 645 residents. What is the difference in the number of residents between the two local government areas?

1C **5** Quinn sold 14 bouquets of roses today. If each bouquet has 12 roses, how many roses in total did Quinn sell?

1C **6** Tyler has saved up $165. He intends to spend the money on $15 t-shirts. How many will he be able to buy?

1D **7** Aida sold equal numbers of jam donuts and cinnamon donuts today. Jam donuts are sold in boxes of 6 while cinnamon donuts are sold in packets of 9. What is the smallest number of each that Aida sold?

1D **8** Find the value of the following.

a 6^3 **b** 2^7 **c** 8 squared **d** 11^2

1D **9** Find the value of the following.

a square root of 4 **b** $\sqrt{81}$ **c** cube root of 27 **d** $\sqrt[3]{125}$

1E **10** Use the correct inequality sign to make each statement true.

a $-5 \square 2$ **b** $-5 \square -2$ **c** $7 \square -4$ **d** $4 \square -7$

1E **11** Write each list of integers in ascending order.

a 24, −40, 20, −14, 16, −22, −24 **b** −17, 21, −27, −35, 0, 9, 63

c −6, −66, 86, 6, −26, −16, −46 **d** 65, −87, 3, −300, 190, 45, −409

1F **12** Calculate:

a $-5 + (-4)$ **b** $2 + (7)$ **c** $22 + (-34)$ **d** $-50 + (69)$

1F **13** Tamara's bank account shows a balance of −$28. If she deposits $150, what is her new account balance?

1F **14** Overnight, the minimum temperature in Alice Springs was −3°C. By 2 pm, the temperature had climbed to a maximum of 18°C. Calculate the difference between the minimum and maximum temperatures.

1F **15** Calculate:

a $-8 + 7$ **b** $-4 - 6$ **c** $17 - 25$ **d** $-44 + 34$

e $-66 + 66$ **f** $-50 - 50$

1G **16** Calculate:

a $7 \times (-8)$ **b** $-5 \times (-9)$ **c** $-36 \div 4$ **d** $-100 \div (-20)$

e -12×0 **f** $4 \times (-15)$ **g** $\frac{18}{-6}$ **h** $\frac{-42}{-3}$

1G **17** Calculate:

a $-6 \times 3 \times (-10)$ **b** $[20 \div (-4)] \times 7$ **c** $[-18 \div (-3)] \times 2$ **d** $-5 \times (-2) \times (-4)$

1G **18** Calculate:

a $(-2)^5$ **b** $(-3)^2$ **c** $(-1)^9$ **d** $(-10)^4$

1H **19** Calculate:

a $33 + (-40) \times 3 \div 10$ **b** $75 - 4 \times 11 + 6^2 \div (-4)$

c $-10 \times 7 - 5 \times [(-4)^2 - 4]$ **d** $-120 \div [-9 + (4 \times 8 - 11)]$

1H **20** Calculate:

a $-15 \times (-4 + 16)$ **b** $(-4)^2 - (-3)^3$

c $[-6 \times 7 + (-10 + 7)^2] \div 3$ **d** $[-1 + (-2)^2 + (-3)^3] \div 3$

Analysis

1 The daily minimum and maximum temperatures at a ski resort were recorded over a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Minimum temperature (°C)	1	−2	−2	−3	−5	−2	−1
Maximum temperature (°C)	8	5	9	4	−1	5	5

a Which day had:

i the highest temperature **ii** the lowest temperature?

b Calculate the difference between the minimum and maximum temperatures for each day.

c Which day had the biggest range of temperatures?

d Calculate the average of the minimum and maximum temperatures.

e What is the difference between the average minimum and average maximum temperatures?

2 The population figures (rounded to the nearest 100) for each state and territory of Australia at the end of June 2020 are displayed in this table.

State or territory	Population
New South Wales	8 164 100
Victoria	6 694 900
Queensland	5 174 400
Western Australia	2 661 900
South Australia	1 769 300
Tasmania	540 600
Australian Capital Territory	431 100
Northern Territory	246 000

a Determine the actual difference in population between NSW and Victoria.

b Round each population figure to the leading digit.

Use your answers from part **b** to answer the following questions.

c Estimate the difference in population between NSW and Victoria. Compare this to part **a**.

d Estimate the population of Australia.

3 Jez is the elephant keeper at the zoo. He has two elephants to take care of: Mali and Man Jai. Mali weighs $3^3 \times 10^2$ kg while Man Jai weighs $2 \times 3^2 \times 10^2$ kg.

a What is the weight of each elephant as a basic numeral?

b What is the difference in weight between the two elephants?

c What is the sum of the weights of the two elephants?

d What is the HCF between the weight of the elephants?

Jez has to divide the food for the elephants according to their weight.

e The elephants need about one-tenth of their weight in food. How much food does Jez have to provide?

f How much food does each elephant need?

Man Jai gets sick and stops eating for a few days. As a result, he loses 5 kg each day. Jez writes the following expression to show Man Jai's weight after his illness: $2 \times 3^2 \times 10^2 - 5^n$ where n is the number of days he is sick. For example, if Man Jai is sick for 2 days, then Jez calculates his weight as $2 \times 3^2 \times 10^2 - 5^2$.

g Calculate Man Jai's weight after his illness using Jez's expression for 1, 2 and 3 days.

h Explain the problem with Jez's expression.

i Rewrite the expression so that it gives the correct weight of Man Jai after his illness.



2 Fractions and decimals





Index

2A Fractions
2B Adding and subtracting fractions
2C Multiplying and dividing fractions
2D Decimals
2E Adding and subtracting decimals
2F Multiplying and dividing decimals
2G Terminating, non-terminating and recurring decimals

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Place Value
- ✔ Number lines
- ✔ Adding and subtracting whole numbers
- ✔ Multiplying and dividing whole numbers
- ✔ Order of operations

Curriculum links

- Recognise irrational numbers in applied contexts, including square roots and π (AC9M8N01)
- Recognise terminating and recurring decimals, using digital tools as appropriate (AC9M8N03)
- Use the 4 operations with integers and with rational numbers, choosing and using efficient strategies and digital tools where appropriate (AC9M8N04)
- Use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts; formulate problems, choosing efficient calculation strategies and using digital tools where appropriate; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model (AC9M8N05)

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Materials

- ✔ Calculator

2A Fractions

Learning intentions

By the end of this topic you will be able to ...

- ✔ identify and find equivalent fractions
- ✔ find the lowest common denominator of two fractions
- ✔ simplify fractions.

Inter-year links
Support Fractions
Year 7 3A Fractions

Fractions

- **Proper fractions** have a **numerator** that is less than the **denominator**.
  Some examples are: $\frac{3}{5}$, $\frac{2}{6}$, $\frac{11}{15}$.
- **Improper fractions** have a numerator that is greater than or equal to the denominator.
  Some examples are: $\frac{13}{6}$, $\frac{9}{5}$, $\frac{31}{8}$.
- **Mixed numbers** consist of both a whole number and a proper fraction.
  For example, $2\frac{1}{6} = 2 + \frac{1}{6}$.

Proper fractions
Numerator < Denominator

$\frac{2}{6}$ →

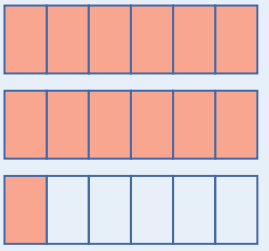


Improper fraction
Numerator ≥ Denominator

$\frac{13}{6}$ →

Mixed number
How many wholes?
How many parts left?

← $2\frac{1}{6}$



To convert a mixed number to an improper fraction:	To convert an improper fraction to a mixed number:
1. Multiply the denominator by the whole number and add the numerator. 2. Write the result over the denominator. $3\frac{1}{4} = \frac{(4\times3)+1}{4} = \frac{13}{4}$	1. Divide the numerator by the denominator. 2. Write down the whole number. Place the remainder over the denominator. 3. Simplify the fraction if possible. $\frac{13}{4} = 13 \div 4 = 3$ remainder 1 $= 3\frac{1}{4}$

- Whole numbers can be written as fractions where the denominator is a factor of the numerator.
  For example, $5 = \frac{5}{1}$, $2 = \frac{2}{1}$, $100 = \frac{100}{1}$ and $\frac{9}{3} = 3$, $\frac{28}{7} = 4$.

Negative fractions

- Just like integers, negative fractions can be added, subtracted, multiplied and divided.
- Fractions with a negative sign can be written as shown.

Negative fractions

$-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$

Positive fractions

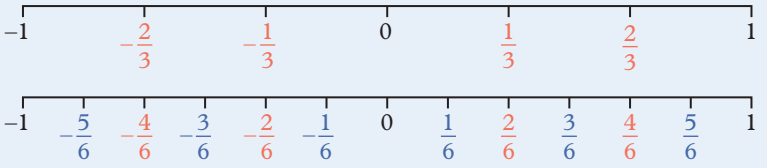
$\frac{-1}{-2} = \frac{1}{2}$

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Equivalent fractions

- **Equivalent fractions** are fractions that have the same numerical value. They mark the same point on the number line.

  For example, $-\frac{1}{3}$ and $-\frac{2}{6}$ mark the same point on the number line, so they are equivalent fractions.



- An equivalent fraction is produced by multiplying or dividing the numerator and the denominator of a fraction by the same value.
- The **sign** of a number is whether it is positive, negative or zero. Equivalent fractions always have the same sign.

$\frac{2}{3} \overset{\times 5}{=} \frac{10}{15}$ $\qquad$ $\frac{12}{10} \overset{\div 2}{=} \frac{6}{5}$

Comparing fractions using equivalent fractions

- To change a pair of fractions so they have the **lowest common denominator (LCD)**, find the lowest common multiple of the denominators and then convert each fraction into the equivalent fraction.

The LCM of 2 and 5 is 10.

$\frac{1}{2} \overset{\times 5}{=} \frac{5}{10}$ $\qquad$ $\frac{3}{5} \overset{\times 2}{=} \frac{6}{10}$

Simplifying fractions

- To find an equivalent fraction in its simplest form:
  - → Cancel out common factors until the only common factor between the numerator and denominator is 1.

    For example, $\frac{36}{72} = \frac{\cancel{36}^{18}}{\cancel{72}^{36}}$

    $= \frac{\cancel{18}^{3}}{\cancel{36}^{6}}$

    $= \frac{\cancel{3}^{1}}{\cancel{6}^{2}}$

    $= \frac{1}{2}$.
  - → Find the HCF of the numerator and the denominator and then divide the numerator and denominator by the HCF.

    For example, the HCF of 36 and 72 is 36, so $\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}}$

    $= \frac{1}{2}$.
- For mixed numbers, only simplify the fraction part; the whole number stays the same.

  For example, $2\frac{12}{24} = 2\frac{\cancel{12}^{1}}{\cancel{24}^{2}}$

  $= 2\frac{1}{2}$.

Example 2A.1 Writing equivalent fractions

Fill the gaps to produce equivalent fractions.

a $\frac{3}{7} = \frac{18}{\square}$ **b** $\frac{91}{35} = \frac{\square}{5}$ **c** $\frac{-2}{11} = \frac{\square}{33}$

THINK

1. Look for a relationship between the two numerators or the two denominators. Consider if the equivalent fraction has been found by using multiplication or division.
2. Use the same relationship to obtain the denominator or numerator of the second fraction.
3. Ensure that the sign of the equivalent fraction matches the first fraction.

WRITE

a $\frac{3}{7} \overset{\times 6}{=} \frac{18}{\square}$

$\frac{3}{7} \underset{\times 6}{\overset{\times 6}{=}} \frac{18}{42}$

b $\frac{91}{35} = \frac{\square}{5}$ ($\div 7$)

$\frac{91}{35} \underset{\div 7}{\overset{\div 7}{=}} \frac{13}{5}$

c $\frac{-2}{11} = \frac{\square}{33}$ ($\times 3$)

$\frac{-2}{11} \underset{\times 3}{\overset{\times 3}{=}} \frac{-6}{33}$

Example 2A.2 Ordering fractions by finding the lowest common denominator

Determine which fraction is greater. Write your answer as an inequality.

a $\frac{5}{6} \square \frac{7}{9}$ **b** $-\frac{7}{2} \square -\frac{13}{4}$

THINK

1. Find the LCM of the denominators.
2. For each fraction, find the equivalent fraction with a denominator equal to the LCM.
3. Compare the fractions with the same denominator and write the answer using the original fractions.

WRITE

a 18 is the LCD.

$\frac{5}{6} \underset{\times 3}{\overset{\times 3}{=}} \frac{15}{18}$ $\frac{7}{9} \underset{\times 2}{\overset{\times 2}{=}} \frac{14}{18}$

$\frac{15}{18} > \frac{14}{18}$; therefore, $\frac{5}{6} > \frac{7}{9}$.

b 4 is the LCD.

$-\frac{7}{2} \underset{\times 2}{\overset{\times 2}{=}} -\frac{14}{4}$

$-\frac{14}{4} < -\frac{13}{4}$; therefore, $-\frac{7}{2} < -\frac{13}{4}$.

Example 2A.3 Simplifying fractions

Write each fraction in its simplest form.

a $\frac{24}{40}$ **b** $2\frac{35}{105}$

THINK

a **1** Find a common factor of 24 and 40. 2 is a common factor.

2 Divide both the numerator and the denominator by 2.

3 Find another common factor and repeat the process. Repeat until the only common factor is 1.

b **1** Find the HCF of the numerator and the denominator of the fraction component.

2 Leave the whole number as it is and divide both the numerator and the denominator by 35.

WRITE

a $\frac{24}{40} = \frac{\cancel{24}^{12}}{\cancel{40}^{20}}$

$= \frac{\cancel{12}^{6}}{\cancel{20}^{10}}$

$= \frac{\cancel{6}^{3}}{\cancel{10}^{5}}$

$= \frac{3}{5}$

b HCF = 35

$2\frac{35}{105} = 2\frac{\cancel{35}^{1}}{\cancel{105}^{3}}$

$= 2\frac{1}{3}$

Helpful hints

- ✔ Remember that 'equivalent' is another way of saying 'equal to'.
- ✔ Each fraction has many equivalent fractions and the simplest form of that fraction is the one with no common divisors. For example,
  $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$ and so on.
- ✔ Use the divisibility rules from Chapter 1. The larger the common factor, the faster you'll find the equivalent fraction in its simplest form.

ANS p496

Exercise 2A Fractions

1–3, 4(a, c, e, f, k), 5, 6–9(1st, 2nd columns), 10–12, 13(1st column), 14, 15, 16(a, b), 17(a, b, f, h), 19	1(b, d, e), 4(b, d, g, h, l), 6–9(2nd, 3rd columns), 11–12(b, c), 13(2nd column), 16(c, d), 17(c, d, e, g), 18, 20, 21(a)	4(i–l), 6(3rd column), 7(g, h), 8–9(3rd column), 11–12(b, c), 13(g–l), 17(e, g, h), 19–22

1 Write all equivalent positive or negative fractions showing the possible positions of the negative sign.

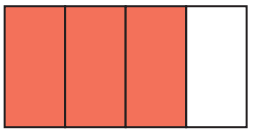
a $\frac{1}{-2}$ **b** $\frac{-10}{3}$ **c** $\frac{-5}{-6}$ **d** $\frac{-2}{-3}$ **e** $-\frac{2}{5}$ **f** $-\frac{-3}{-7}$

2 Draw a number line with a scale from −10 to 10 and show the position of each number.



a −3 **b** $\frac{5}{2}$ **c** $-\frac{1}{2}$ **d** 5 **e** $7\frac{3}{4}$ **f** $-4\frac{3}{4}$

3 Consider the rectangle below.



a What fraction of the rectangle has been shaded?

b Draw a rectangle that is divided into 12 equal sections. Shade the rectangle so that it represents an equivalent fraction to that in part **a**.

c Write the equivalent fraction for part **b**.

2A.1 **4** Fill the gaps to produce equivalent fractions.

a $\frac{3}{10} = \frac{6}{\square}$ **b** $\frac{2}{7} = \frac{\square}{35}$ **c** $\frac{18}{33} = \frac{\square}{11}$ **d** $\frac{16}{48} = \frac{2}{\square}$

e $-\frac{4}{5} = -\frac{\square}{30}$ **f** $\frac{-8}{36} = \frac{-2}{\square}$ **g** $-1\frac{3}{8} = \frac{\square}{40}$ **h** $3\frac{3}{4} = \frac{75}{\square}$

i $\frac{1}{4} = \frac{2}{\square} = \frac{\square}{20} = \frac{8}{\square} = \frac{20}{\square} = \frac{\square}{100}$

j $-\frac{3}{7} = \frac{-15}{\square} = \frac{\square}{49} = \frac{\square}{70} = \frac{-300}{\square} = -\frac{\square}{210}$

k $1\frac{1}{5} = \frac{\square}{10} = \frac{60}{\square} = \frac{\square}{250} = \frac{\square}{45} = \frac{72}{\square}$

l $-2\frac{1}{3} = -\frac{\square}{14} = \frac{\square}{15} = \frac{49}{\square} = -\frac{\square}{9} = -\frac{\square}{300}$

5 From the list below, select which fractions are equivalent to:

a $\frac{2}{5}$ **b** $\frac{4}{3}$.

$\frac{6}{10}$, $\frac{20}{15}$, $\frac{8}{20}$, $\frac{7}{10}$, $\frac{28}{21}$, $\frac{22}{55}$, $\frac{4}{10}$, $\frac{16}{9}$, $1\frac{4}{12}$, $\frac{10}{25}$, $\frac{18}{45}$, $2\frac{6}{15}$

2A.2 **6** Which fraction in each given pair is greater? Write your answer as an inequality. Hint: Write equivalent fractions with the same denominator.

a $\frac{3}{8} \square \frac{1}{2}$ **b** $\frac{4}{7} \square \frac{5}{14}$ **c** $\frac{-9}{10} \square \frac{-4}{5}$

d $-\frac{5}{12} \square -\frac{2}{3}$ **e** $1\frac{5}{6} \square 1\frac{19}{24}$ **f** $\frac{17}{15} \square \frac{7}{3}$

g $-\frac{2}{3} \square -\frac{3}{4}$ **h** $\frac{5}{6} \square \frac{4}{5}$ **i** $-\frac{11}{8} \square -\frac{5}{3}$

j $2\frac{3}{4} \square \frac{17}{6}$ **k** $-\frac{29}{3} \square -9\frac{1}{2}$ **l** $3\frac{5}{12} \square \frac{25}{8}$

7 Decide which fraction is greater in each pair.

a $\frac{5}{9}, \frac{7}{9}$ **b** $\frac{11}{4}, \frac{9}{4}$ **c** $-2\frac{3}{5}, -\frac{12}{5}$

d $-\frac{43}{6}, -7\frac{5}{6}$ **e** $3\frac{7}{11}, \frac{37}{11}$ **f** $-\frac{29}{2}, -8\frac{1}{2}$

g $4\frac{2}{3}, \frac{16}{3}$ **h** $-\frac{55}{9}, -6\frac{2}{9}$ **i** $3\frac{5}{7}, \frac{25}{7}$

2A.3 **8** Write each fraction in simplest form.

a $\frac{8}{10}$ **b** $\frac{14}{21}$ **c** $\frac{42}{54}$

d $\frac{45}{72}$ **e** $\frac{40}{12}$ **f** $\frac{55}{30}$

9 Write each fraction in simplest form.

a $\frac{6}{15}$ **b** $-\frac{33}{77}$ **c** $-\frac{18}{45}$

d $-\frac{30}{18}$ **e** $\frac{32}{36}$ **f** $\frac{18}{42}$

g $3\frac{24}{32}$ **h** $-5\frac{35}{63}$ **i** $4\frac{9}{33}$

j $-1\frac{24}{42}$ **k** $2\frac{15}{40}$ **l** $-7\frac{45}{54}$

10 Why does simplifying a fraction find an equivalent fraction?

11 Write each list of fractions in ascending order.

a $\frac{2}{5}, \frac{7}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{3}{5}, \frac{1}{5}$ **b** $1\frac{1}{4}, 2\frac{5}{8}, \frac{9}{8}, 2\frac{3}{4}, \frac{3}{8}, \frac{7}{4}$ **c** $-1\frac{1}{2}, -\frac{2}{5}, -1\frac{9}{10}, -\frac{9}{2}, -\frac{3}{10}, -\frac{9}{5}$

12 Write each list of fractions in descending order.

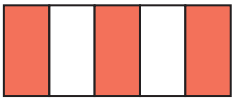
a $\frac{13}{11}, \frac{5}{11}, \frac{19}{11}, \frac{9}{11}, \frac{3}{11}, 1\frac{5}{11}$ **b** $-\frac{2}{3}, -1\frac{1}{2}, -\frac{3}{4}, -\frac{1}{2}, -\frac{4}{3}, -1\frac{1}{4}$ **c** $\frac{5}{6}, -2\frac{1}{6}, -\frac{11}{6}, \frac{1}{6}, -\frac{5}{12}, 1\frac{1}{3}$

13 Write each amount as a fraction of the total amount in simplest form. Hint: Make sure the units are the same before you simplify your fraction.

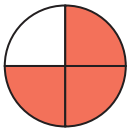
a 15 kg out of 25 kg
b 80 m out of 360 m
c \$35 out of \$120
d 18 seconds out of 60 seconds
e 50c out of \$3
f 14 days out of 5 weeks
g 20 mm out of 7 cm
h 300 g out of 2 kg
i 28 seconds out of 1 minute
j 75c out of \$4.25
k 20 cm out of 6 m
l 16 minutes out of 2 hours

14 Match one of the fractions $\frac{3}{4}, \frac{2}{3}, \frac{3}{5}$ or $\frac{5}{8}$ to the amount shaded in each diagram.

a



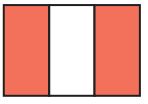
b



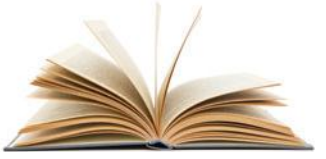
c



d



15 If you have read 56 out of 224 pages of a novel, what fraction have you read?



16 Consider these two rows of a multiplication grid.

	1	2	3	4	5	6	7	8	9
× 3	3	6	9	12	15	18	21	24	27
× 4	4	8	12	16	20	24	28	32	36

a Write each column of products as a fraction by making the first number the numerator and the second the denominator. For example, the first column will be $\frac{3}{4}$.

b What can be said about all of the fractions made by dividing a multiple of 3 by the corresponding multiple of 4?

Consider these two rows.

	1	2	3	4	5	6	7	8	9
× 7	7	14	21	28	35	42	49	56	63
× 5	5	10	15	20	25	30	35	40	45

c Write each column of products as a fraction by making the first number the numerator and the second the denominator. For example, the first column will be $\frac{7}{5}$.

d What can be said about all of the fractions made by dividing a multiple of 7 by the corresponding multiple of 5?

17 Find the fraction that is halfway between each pair of fractions.

a $\frac{3}{7}$ and $\frac{5}{7}$ **b** $-\frac{5}{11}$ and $-\frac{9}{11}$ **c** $\frac{3}{8}$ and $\frac{5}{8}$ **d** $-\frac{1}{3}$ and $-\frac{5}{9}$

e $\frac{4}{15}$ and $\frac{2}{5}$ **f** $\frac{1}{2}$ and $-\frac{3}{4}$ **g** $-1\frac{1}{4}$ and $1\frac{1}{3}$ **h** $2\frac{2}{3}$ and $3\frac{1}{5}$

PROBLEM SOLVING AND REASONING

18 Brodie scored 25 out of 30 on the first Maths test and 21 out of 28 on the second. On which test did Brodie perform better?

19 Tony and Lisbeth hit a target with a ball with the same level of accuracy. Tony hit the target four times in five shots. Lisbeth hit the target 12 times. Use equivalent fractions to work out how many shots Lisbeth took.



20 Fractions can be compared in several ways:

1 comparing with a common denominator or the lowest common denominator

2 comparing with a common numerator or the lowest common numerator

3 comparing with a benchmark such as 1 or $\frac{1}{2}$.

For each of the following pairs of fractions:

i decide which strategy would be the most efficient to compare the pair and explain why you chose that strategy

ii state which fraction is greater by writing an appropriate inequality.

a $\frac{4}{7}, \frac{7}{15}$

b $\frac{15}{17}, \frac{15}{4}$

c $\frac{99}{100}, \frac{100}{101}$

d $1\frac{1}{8}, 1\frac{1}{11}$

e $1\frac{3}{5}, 1\frac{2}{3}$

f $\frac{77}{10}, \frac{39}{5}$

CHALLENGE

21 The number of faults out of the total number of serves was recorded for each player in a tennis match.

	Naomi	Ash
Total number of serves	119	102
Number of serving faults	14	10



a Who had the better success rate in serving?

b If Naomi served at the same fault rate in her next match, how many faults did she make out of 85 serves?

c If Ash served at the same fault rate in her next match and made 15 faults, what was her total number of serves?

22 a Which one of the following fractions could not be placed between the fractions marked on the number line?



A $\frac{3}{7}$ **B** $\frac{5}{11}$ **C** $\frac{48}{96}$ **D** $\frac{31}{41}$ **E** $\frac{51}{121}$

b Explain how you checked each fraction.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet — Visual representations of fractions
- Interactive skillsheet — Equivalent fractions
- Investigation — Cross country fractions
- Topic quiz — 2A

2B Adding and subtracting fractions

Learning intentions

By the end of this topic you will be able to ...

- ✔ add and subtract fractions with different denominators
- ✔ add and subtract fractions and mixed numbers
- ✔ add and subtract negative fractions and mixed numbers.

Inter-year links

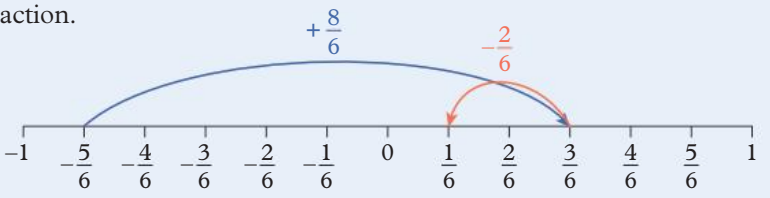
Support — Adding and subtracting fractions

Year 7 — 3E Adding and subtracting fractions

Adding and subtracting fractions

- To add and subtract fractions:
  1. Convert into equivalent fractions with a common denominator.
  2. Add or subtract the numerators and keep the same denominator.
  3. Simplify the fraction.

$$-\frac{5}{6}+\frac{4}{3}-\frac{1}{3}=-\frac{5}{6}+\frac{8}{6}-\frac{2}{6}$$
$$=\frac{1}{6}$$



- To add or subtract mixed numbers, first convert the mixed numbers to improper fractions. Convert the sum or difference back to a mixed number and simplify your answer.
- Simplify the calculation by considering the operation and the sign applied to each fraction.

$$+\,(-)=-$$
$$-\,(+)=-$$
$$-\,(-)=+$$
$$+\,(+)=+$$

Example 2B.1 Adding and subtracting fractions with common denominators

Determine the result of each sum or difference.

a $\frac{2}{7}+\frac{8}{7}$ **b** $\frac{13}{6}-\frac{4}{6}$

THINK

a Add the numerators and keep the same denominator.

b 1 Subtract the numerators and keep the same denominator.

2 Simplify the fraction by dividing the numerator and denominator by the HCF.

WRITE

a $\frac{2}{7}+\frac{8}{7}=\frac{10}{7}$

b $\frac{13}{6}-\frac{4}{6}=\frac{9^{3}}{6_{2}}$

$=\frac{3}{2}$

Example 2B.2 Adding and subtracting fractions with different denominators

Determine the result of each sum or difference.

a $\frac{5}{6}+\frac{1}{2}$ **b** $\frac{11}{5}-\frac{6}{4}$

THINK

a **1** Convert into equivalent fractions with a common denominator. A common multiple of 2 and 6 is 6.

2 Add the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

b **1** Convert into equivalent fractions with a common denominator. A common multiple of 5 and 4 is 20.

2 Subtract the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

WRITE

a $\frac{5}{6}+\frac{1}{2}=\frac{5}{6}+\frac{1\times 3}{2\times 3}$

$=\frac{5}{6}+\frac{3}{6}$

$=\frac{8^{4}}{6^{3}}$

$=\frac{4}{3}$

b $\frac{11}{5}-\frac{6}{4}=\frac{11\times 4}{5\times 4}-\frac{6\times 5}{4\times 5}$

$=\frac{44}{20}-\frac{30}{20}$

$=\frac{14^{7}}{20^{10}}$

$=\frac{7}{10}$

Example 2B.3 Adding and subtracting negative fractions

Determine the result of each sum or difference.

a $-\frac{12}{5}-\left(-\frac{7}{5}\right)$ **b** $-\frac{2}{3}+\left(-\frac{1}{2}\right)$

THINK

a **1** Simplify the calculation. Remember that subtracting a negative is the same as adding the positive.

2 Add the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

b **1** Simplify the calculation. Remember that adding a negative is the same as subtracting the positive.

2 Convert into equivalent fractions with a common denominator. A common multiple of 3 and 2 is 6.

3 Subtract the numerators and keep the same denominator.

WRITE

a $-\frac{12}{5}-\left(-\frac{7}{5}\right)=-\frac{12}{5}+\frac{7}{5}$

$=-\frac{5^{1}}{5^{1}}$

$=-1$

b $-\frac{2}{3}+\left(-\frac{1}{2}\right)=-\frac{2}{3}-\frac{1}{2}$

$=-\frac{2\times 2}{3\times 2}-\frac{1\times 3}{2\times 3}$

$=-\frac{4}{6}-\frac{3}{6}$

$=-\frac{7}{6}$

Example 2B.4 Adding and subtracting mixed numbers

Determine the result of each sum or difference.

a $2\frac{1}{3}+\frac{5}{6}$ **b** $1\frac{3}{4}-2\frac{5}{12}$

THINK

a **1** Convert the mixed numbers into improper fractions.

2 Convert into equivalent fractions with a common denominator. A common multiple of 3 and 6 is 6.

3 Add the numerators and keep the same denominator.

4 Convert the answer into a mixed number and simplify the fraction.

b **1** Convert the mixed numbers into improper fractions.

2 Convert into equivalent fractions with a common denominator. A common multiple of 4 and 12 is 12.

3 Subtract the numerators and keep the same denominator.

4 Simplify the fraction.

WRITE

a $2\frac{1}{3}+\frac{5}{6}=\frac{7}{3}+\frac{5}{6}$

$=\frac{7\times 2}{3\times 2}+\frac{5}{6}$

$=\frac{14}{6}+\frac{5}{6}$

$=\frac{19}{6}$

$=3\frac{1}{6}$

b $1\frac{3}{4}-2\frac{5}{12}=\frac{7}{4}-\frac{29}{12}$

$=\frac{7\times 3}{4\times 3}-\frac{29}{12}$

$=\frac{21}{12}-\frac{29}{12}$

$=-\frac{\cancel{8}^{2}}{\cancel{12}^{3}}$

$=-\frac{2}{3}$

Helpful hints

- ✔ Only add or subtract the numerators of fractions; the denominator stays the same.

$$\frac{3}{5}-\frac{2}{5}=\frac{3-2}{5}$$

- ✔ You can reduce the complexity of an addition or subtraction by finding the lowest common denominator. Always try to simplify first so that your lowest common multiple will be as small as possible.
- ✔ Adding does not always make a number larger. For example, adding a negative number will give a smaller result.
- ✔ Subtracting does not always make a number smaller. For example, subtracting a negative number will give a larger result.
- ✔ When simplifying calculations involving negative numbers, only combine the signs located in between the numbers.

$$+\,(-)=-$$
$$-\,(+)=-$$
$$-\,(-)=+$$
$$+\,(+)=+$$

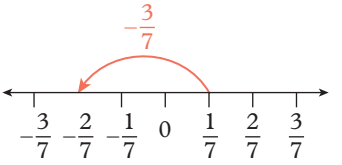
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Exercise 2B Adding and subtracting fractions

▲ 1–7, 8(a, b, c, f), 9, 10(a), 11(a–d), 14, 15 ■ 4, 5, 7, 8, 9, 10(b), 11(e–h), 12(a, b, c), 13, 16, 18, 19 ⬟ 4–5(e–h), 7–8(e, g, h), 9, 10(b, c), 11(g, h), 12(d–f), 16–20

UNDERSTANDING AND FLUENCY

1 Consider $\left(+\frac{1}{7}\right) + \left(-\frac{3}{7}\right)$.



a Explain why this can be simplified to $\frac{1}{7} - \frac{3}{7}$.

b Use this number line to calculate the result.

2B.1 **2** Evaluate the result of each sum or difference.

a $\frac{5}{17} + \frac{8}{17}$ **b** $\frac{7}{10} - \frac{3}{10}$ **c** $\frac{3}{8} + \frac{5}{8}$ **d** $\frac{6}{3} - \frac{5}{3}$

3 Find the lowest common denominator for each pair of fractions.

a $\frac{4}{5}$ and $\frac{2}{3}$ **b** $\frac{3}{7}$ and $\frac{1}{14}$ **c** $\frac{6}{10}$ and $\frac{4}{15}$ **d** $\frac{3}{8}$ and $\frac{5}{12}$

e $\frac{2}{15}$ and $\frac{2}{3}$ **f** $\frac{3}{4}$ and $\frac{2}{5}$ **g** $\frac{1}{6}$ and $\frac{3}{4}$ **h** $\frac{4}{9}$ and $\frac{1}{6}$

2B.2 **4** Evaluate the result of each sum or difference.

a $\frac{5}{22} + \frac{8}{11}$ **b** $\frac{3}{4} - \frac{2}{5}$ **c** $\frac{9}{10} + \frac{2}{3}$ **d** $\frac{3}{8} - \frac{1}{12}$

e $\frac{11}{15} - \frac{7}{10}$ **f** $\frac{5}{6} + \frac{3}{4}$ **g** $\frac{5}{6} - \frac{4}{9}$ **h** $\frac{3}{8} + \frac{2}{5}$

2B.3 **5** Evaluate the result of each sum or difference.

a $\frac{1}{7} - \left(-\frac{3}{7}\right)$ **b** $-\frac{2}{3} + \left(-\frac{5}{3}\right)$ **c** $\frac{5}{8} - \left(+\frac{7}{8}\right)$ **d** $-\frac{4}{5} + \left(+\frac{2}{5}\right)$

e $-\frac{1}{3} + \left(-\frac{4}{3}\right)$ **f** $-\frac{3}{4} - \left(+\frac{1}{2}\right)$ **g** $-\frac{1}{3} - \left(-\frac{5}{9}\right)$ **h** $\frac{2}{5} + \left(-\frac{3}{2}\right)$

6 Evaluate the result of each sum or difference.

a $-\frac{4}{5} + \frac{3}{5}$ **b** $\frac{2}{7} - \frac{5}{7}$ **c** $-\frac{3}{11} - \frac{6}{11}$ **d** $-\frac{4}{3} + \frac{1}{6}$ **e** $\frac{2}{7} - \frac{1}{2}$ **f** $-\frac{3}{4} + \frac{7}{8}$

2B.4 **7** Evaluate each problem and simplify where possible.

a $1\frac{1}{4} - \frac{7}{8}$ **b** $2\frac{3}{5} + \frac{13}{20}$ **c** $2\frac{2}{3} + 1\frac{4}{5}$ **d** $3\frac{7}{11} - 1\frac{1}{2}$

e $4\frac{3}{4} + 2\frac{2}{3}$ **f** $5\frac{5}{6} - 3\frac{2}{15}$ **g** $1\frac{2}{3} - 2\frac{1}{3}$ **h** $-4\frac{3}{5} - 1\frac{1}{10}$

8 Determine the result of each sum and difference.

a $\frac{2}{3} - \frac{4}{3} + \frac{1}{3}$ **b** $-\frac{3}{4} + \frac{1}{4} + \frac{1}{2}$ **c** $\frac{2}{3} - \frac{5}{6} - \frac{1}{2}$ **d** $-\frac{3}{5} - \frac{1}{4} + \frac{7}{10}$

e $\frac{5}{9} - \frac{2}{3} + \frac{1}{2}$ **f** $5\frac{2}{7} - 9\frac{6}{7} + 1\frac{5}{7}$ **g** $-4\frac{3}{4} + 3\frac{5}{8} + 2\frac{7}{12}$ **h** $10\frac{3}{10} - 4\frac{3}{4} - 5\frac{3}{5}$

PROBLEM SOLVING AND REASONING

9 Nine numbers are arranged in a square.

a Add the three numbers in:

i the first row **ii** the second row **iii** the third row.

b Add the three numbers in:

i the first column **ii** the second column **iii** the third column.

c Add the three numbers along each diagonal.

d What do you notice about your answers to parts **a**, **b** and **c**?

e Your answer to part **d** is the reason why the nine numbers form a magic square. The sum in each row, column and diagonal is called the magic sum. What is the magic sum for this set of numbers?

$\frac{4}{5}$	$\frac{1}{10}$	$\frac{3}{5}$
$\frac{3}{10}$	$\frac{1}{2}$	$\frac{7}{10}$
$\frac{2}{5}$	$\frac{9}{10}$	$\frac{1}{5}$

10 Copy and complete each magic square. Hint: First work out the magic sum.

a

		$\frac{2}{5}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{15}$

b

		$\frac{1}{12}$
$\frac{1}{8}$	$\frac{5}{24}$	
$\frac{1}{3}$		

c

		$-\frac{1}{3}$
$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$

11 Consider the following sums and differences. Without performing the calculation, state whether the sum or difference is positive, negative or zero.

a $\left(-\frac{2}{5}\right)+\frac{3}{4}$ **b** $\left(-\frac{7}{10}\right)-\left(-\frac{3}{10}\right)$ **c** $\frac{9}{11}-\left(-\frac{51}{17}\right)$ **d** $\frac{2}{3}+\left(-\frac{4}{6}\right)$

e $\left(-\frac{13}{25}\right)+\left(-\frac{81}{15}\right)$ **f** $\left(-\frac{5}{4}\right)-\left(+\frac{7}{3}\right)$ **g** $\left(-\frac{12}{5}\right)-\left(-\frac{36}{15}\right)$ **h** $-\frac{1}{77}-\frac{9}{15}$

12 We can take advantage of negative fractions to add and subtract mixed numbers without first converting to improper fractions by using the associative, commutative and distributive laws.

For example: $2\frac{1}{3}-1\frac{2}{3}=\left(2+\frac{1}{3}\right)-\left(1+\frac{2}{3}\right)$

$=2+\frac{1}{3}-1-\frac{2}{3}$

$=(2-1)+\left(\frac{1}{3}-\frac{2}{3}\right)$

$=1+\left(-\frac{1}{3}\right)$

$=\frac{2}{3}$

Calculate the following sums and differences without converting to improper fractions. Give your answer as a mixed number.

a $5\frac{7}{8}-4\frac{3}{8}$ **b** $5\frac{3}{8}-4\frac{7}{8}$ **c** $4\frac{7}{8}-5\frac{3}{8}$

d $4\frac{3}{8}-5\frac{7}{8}$ **e** $5\frac{7}{8}+4\frac{3}{8}$ **f** $-5\frac{7}{8}+\left(-4\frac{3}{8}\right)$

13 In subtraction of fractions problems, when is it better to convert to improper fractions first?

14 Craig wants to combine two open cartons of milk into one 2 L container. If one carton has $\frac{1}{3}$ L and the other has $1\frac{3}{4}$ L, will the 2 L container be large enough? Explain.

15 On Monday evening, Eleanor spent $1\frac{1}{4}$ hours on the computer researching an assignment for school, Adam spent $2\frac{1}{2}$ hours on the computer catching up with friends on Facebook, and their mother spent $\frac{5}{6}$ of an hour typing up a report for work. How much time was spent on the computer on Monday evening?

16 One-fifth of the student population at a local high school ride to school, one-tenth walk to school and two-thirds catch public transport. The remainder is driven to school.



What fraction of students:

a ride or walk to school

b catch public transport, ride or walk to school

c are driven to school?

17 John had a pizza delivered to his house and ate half of it before soccer training. Jen arrived home and ate half of what was left. Oscar then ate one-third of the remaining pizza. What fraction of the original pizza is left over?

PROBLEM SOLVING AND REASONING

18 Metal screws come in different sizes, with diameters often listed in inches.

a Find the difference in size between two screws with diameters $\frac{3}{50}$ inches and $\frac{41}{250}$ inches.

b Wes has drilled a hole in the wall of diameter $\frac{3}{20}$ inches to hang a painting. Which of these two screws would be best for him to use?

19 a Match each set of calculations with the corresponding number line. There is one set of calculations and one number line that do not have a match. For these, write a corresponding calculation and draw a corresponding number line diagram.

A $\frac{1}{2}+\frac{7}{6}=\left(\frac{1}{2}+\frac{3}{6}\right)+\frac{4}{6}$
$=1+\frac{2}{3}$
$=1\frac{2}{3}$

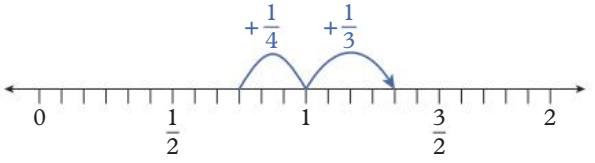
B $\frac{3}{4}+\frac{7}{12}=\left(\frac{3}{4}+\frac{3}{12}\right)+\frac{4}{12}$
$=1+\frac{1}{3}$
$=1\frac{1}{3}$

C $\frac{3}{2}-\frac{7}{10}=\left(\frac{3}{2}-\frac{5}{10}\right)-\frac{2}{10}$
$=1-\frac{1}{5}$
$=\frac{4}{5}$

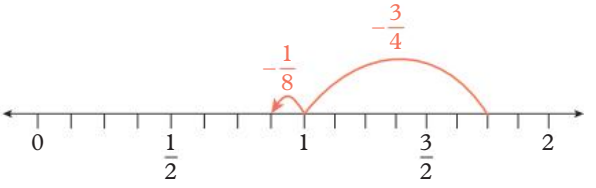
D $\frac{7}{4}-\frac{7}{8}=\left(\frac{7}{4}-\frac{6}{8}\right)-\frac{1}{8}$
$=1-\frac{1}{8}$
$=\frac{7}{8}$

E $\frac{3}{4}-\frac{1}{12}=\left(\frac{3}{4}+\frac{3}{12}\right)-\frac{4}{12}$
$=1-\frac{1}{3}$
$=\frac{2}{3}$

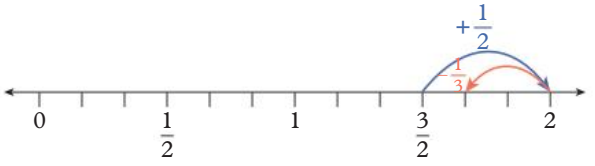
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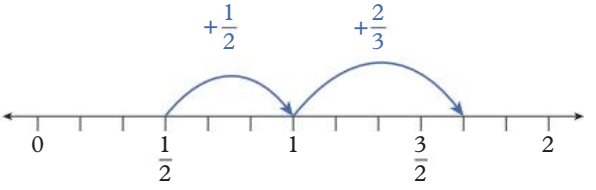
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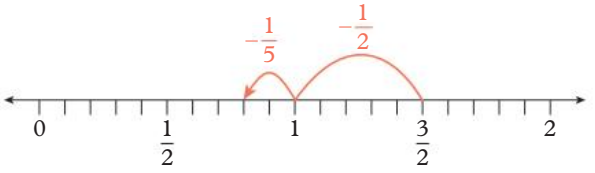
iii



iv



v



b Describe the strategy that was used to evaluate these calculations.

CHALLENGE

20 Consider the following calculations. Determine which one is not equivalent to the others.

i $\left(-\frac{5}{3}\right)+\left(+\frac{4}{7}\right)-\left(-\frac{7}{11}\right)$

ii $\left(-\frac{5}{3}\right)-\left(-\frac{4}{7}\right)+\left(+\frac{7}{11}\right)$

iii $\left(+\frac{4}{8}\right)-\left(+\frac{5}{3}\right)+\left(+\frac{4}{7}\right)-\left(-\frac{7}{11}\right)+\left(-\frac{33}{66}\right)$

iv $\left(-\frac{7}{11}\right)-\left(-\frac{4}{7}\right)+\left(-\frac{5}{3}\right)$

v $\left(\frac{-5}{3}\right)+\left(\frac{-4}{-7}\right)-\left(\frac{7}{-11}\right)$

vi $\left(+\frac{7}{11}\right)-\left(+\frac{5}{3}\right)+\left(+\frac{4}{7}\right)$

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Adding and subtracting fractions
- Investigation: Puzzling pumpkins
- Topic quiz: 2B

2C Multiplying and dividing fractions

Learning intentions

By the end of this topic you will be able to ...

- ✔ multiply and divide fractions and mixed numbers
- ✔ multiply and divide negative fractions and mixed numbers
- ✔ determine fractions of an amount.



Inter-year links

Year 7 3F Multiplying fractions

Multiplying fractions

- To multiply two fractions together:
  1. Look diagonally and vertically for any common factors between the numerators and denominators. Cancel any common factors in the numerators and denominators by dividing both numbers by the highest common factor (HCF).
  2. Multiply the numerators together and the denominators together.

$$\frac{2}{3}\times\frac{3}{4}=\frac{2\times\cancel{3}^{1}}{\cancel{3}^{1}\times 4}=\frac{\cancel{2}^{1}}{\cancel{4}^{2}}=\frac{1}{2}$$

Diagonally

Vertically

- When multiplying mixed numbers, convert them into improper fractions, multiply across and give the result as a mixed number.
- Whole numbers can be written as fractions with a denominator of 1. For example, $8=\frac{8}{1}$.
- When asked to find a fraction 'of' a whole number or fraction, replace the 'of' with '×'. For example, $\frac{2}{3}$ of 20 is the same as $\frac{2}{3}\times 20$.

Dividing fractions by fractions

- To find the **reciprocal** of a fraction, invert the fraction by swapping the numerator and the denominator.

Reciprocal: $5 \rightarrow \frac{1}{5}$ and $\frac{3}{4} \rightarrow \frac{4}{3}$

- Dividing by a fraction is the same as multiplying by the reciprocal of the fraction. Keep the first fraction the same, and multiply by the reciprocal of the second fraction.

$$\frac{3}{4}\div\frac{3}{8}=\frac{3}{4}\times\frac{8}{3}$$

- For both multiplying and dividing fractions:
  - → If the signs are the same, then the result is positive.
  - → If the signs are different, then the result is negative.

$+\times+=+$ $\quad +\div+=+$

$+\times-=-$ $\quad +\div-=-$

$-\times+=-$ $\quad -\div+=-$

$-\times-=+$ $\quad -\div-=+$

Example 2C.1 Multiplying fractions

Determine the result of each product.

a $\frac{8}{9} \times \frac{5}{6}$ **b** $\frac{23}{9} \times 3$ **c** $1\frac{2}{3} \times 2\frac{7}{10}$

THINK

a 1 Simplify the 8 and 6 by dividing both numbers by 2.

2 Multiply the numerators together, then multiply the denominators together.

b 1 Write 3 as a fraction with a denominator of 1.

2 Simplify the 9 and 3 by dividing both numbers by 3.

3 Multiply the numerators together, then multiply the denominators together.

c 1 Convert each mixed number into an improper fraction.

2 Simplify the 5 and 10 by dividing both numbers by 5 and then cancel the 3 and 27 by dividing both numbers by 3.

3 Multiply the numerators together, then multiply the denominators together.

4 Convert the improper fraction to a mixed number.

WRITE

a $\frac{8}{9} \times \frac{5}{6} = \frac{\cancel{8}^{4}}{9} \times \frac{5}{\cancel{6}_{3}}$

$= \frac{4 \times 5}{9 \times 3}$

$= \frac{20}{27}$

b $\frac{23}{9} \times 3 = \frac{23}{9} \times \frac{3}{1}$

$= \frac{23}{\cancel{9}^{3}} \times \frac{\cancel{3}^{1}}{1}$

$= \frac{23 \times 1}{3 \times 1}$

$= \frac{23}{3}$

c $1\frac{2}{3} \times 2\frac{7}{10} = \frac{5}{3} \times \frac{27}{10}$

$= \frac{\cancel{5}^{1}}{\cancel{3}^{1}} \times \frac{\cancel{27}^{9}}{\cancel{10}_{2}}$

$= \frac{1 \times 9}{1 \times 2}$

$= \frac{9}{2}$

$= 4\frac{1}{2}$

Example 2C.2 Dividing fractions

Determine the result of each quotient.

a $\frac{1}{7} \div \frac{3}{5}$ **b** $\frac{4}{5} \div 6$ **c** $3\frac{3}{8} \div 2\frac{1}{4}$

THINK

a 1 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

2 Multiply the numerators together and then multiply the denominators together.

WRITE

a $\frac{1}{7} \div \frac{3}{5} = \frac{1}{7} \times \frac{5}{3}$

$= \frac{1 \times 5}{7 \times 3}$

$= \frac{5}{21}$

b 1 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

2 Cancel out any common factors.

3 Multiply the numerators together and then multiply the denominators together.

c 1 Convert each mixed number to an improper fraction.

2 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

3 Cancel out any common factors.

4 Multiply the numerators together and then multiply the denominators together.

5 Change the improper fraction to a mixed number.

b $\frac{4}{5} \div 6 = \frac{4}{5} \times \frac{1}{6}$

$= \frac{\cancel{4}^{2}}{5} \times \frac{1}{\cancel{6}^{3}}$

$= \frac{2 \times 1}{5 \times 3}$

$= \frac{2}{15}$

c $3\frac{3}{8} \div 2\frac{1}{4} = \frac{27}{8} \div \frac{9}{4}$

$= \frac{27}{8} \times \frac{4}{9}$

$= \frac{\cancel{27}^{3}}{\cancel{8}^{2}} \times \frac{\cancel{4}^{1}}{\cancel{9}^{1}}$

$= \frac{3 \times 1}{2 \times 1}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

Example 2C.3 Multiplying and dividing negative fractions

Determine the result of each product or quotient.

a $-\frac{2}{9} \times \left(-\frac{13}{4}\right)$ **b** $\frac{12}{7} \div \left(-\frac{3}{14}\right)$ **c** $-2\frac{3}{4} \div \left(-4\frac{1}{2}\right)$

THINK

a 1 Determine whether the result is positive or negative. The signs are the same, so the result is positive.

2 Simplify the 2 and 4 by dividing both numbers by 2.

3 Multiply the numerators together, and then multiply the denominators together.

b 1 Determine whether the result is positive or negative. The signs are different, so the result is negative.

2 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

3 Cancel out any common factors.

4 Multiply the numerators together and then multiply the denominators together.

WRITE

a $\frac{\cancel{2}^{1}}{9} \times \frac{13}{\cancel{4}^{2}} = \frac{13}{18}$

So, $-\frac{2}{9} \times \left(-\frac{13}{4}\right) = \frac{13}{18}$

b $\frac{12}{7} \div \frac{3}{14} = \frac{\cancel{12}^{4}}{\cancel{7}^{1}} \times \frac{\cancel{14}^{2}}{\cancel{3}^{1}}$

$= \frac{8}{1}$

$= 8$

So, $\frac{12}{7} \div \left(-\frac{3}{14}\right) = -8$

c **1** Determine whether the result is positive or negative. The signs are the same, so the result is positive.

2 Convert each mixed number to an improper fraction.

3 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

4 Cancel out any common factors.

5 Multiply the numerators together and then multiply the denominators together.

c $2\frac{3}{4} \div 4\frac{1}{2} = \frac{11}{4} \div \frac{9}{2}$

$= \frac{11}{\cancel{4}^{2}} \times \frac{\cancel{2}^{1}}{9}$

$= \frac{11}{18}$

So, $-2\frac{3}{4} \div \left(-4\frac{1}{2}\right) = \frac{11}{18}$

Helpful hints

- ✔ You don't need a common denominator when multiplying and dividing fractions.
- ✔ Don't forget to change any whole number into a fraction with a denominator of 1.
- ✔ Make sure to simplify your fractions! You can cancel common factors vertically or diagonally.
- ✔ The phrase 'Keep Change Flip' can help you remember how to turn a 'dividing by a fraction' problem into a multiplication problem.
- ✔ When working with fractions, multiplying does not always make a number larger and dividing does not always make a number smaller.

Keep Change Flip

$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$

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Exercise 2C Multiplying and dividing fractions

▲ 1–5(1st, 2nd columns), 6–8, 9(a–c), 10–13, 14(a, b), 16(a)

■ 1–5(3rd, 4th columns), 6–8, 9(d–f), 11, 13, 14, 16(a, c, e), 17(a–c), 18(a, b)

⬟ 2(e–h), 5, 6(e, f), 8, 9, 11, 15–17, 18(c, d), 19, 20

UNDERSTANDING AND FLUENCY

1 Write the equivalent multiplication equation for each division equation by using the reciprocal.

a $\frac{9}{20} \div \frac{3}{5}$ **b** $\frac{9}{20} \div \frac{7}{15}$ **c** $\frac{20}{9} \div \frac{15}{7}$ **d** $\frac{20}{9} \div \frac{7}{15}$

e $\frac{2}{9} \div \frac{7}{4}$ **f** $\frac{2}{9} \div \frac{5}{21}$ **g** $\frac{9}{2} \div \frac{21}{5}$ **h** $\frac{9}{2} \div \frac{5}{21}$

2C.1 **2** Determine the result of each product. Hint: Simplify before multiplying.

a $\frac{3}{8} \times \frac{2}{5}$ **b** $\frac{9}{10} \times \frac{4}{3}$ **c** $\frac{18}{25} \times \frac{35}{36}$ **d** $\frac{32}{49} \times \frac{21}{40}$

e $\frac{6}{375} \times \frac{25}{20}$ **f** $\frac{64}{12} \times \frac{27}{16}$ **g** $\frac{10}{77} \times \frac{242}{33}$ **h** $\frac{75}{70} \times \frac{42}{45}$

3 Determine the result of each product.

a $1\frac{7}{8} \times \frac{3}{5}$ **b** $3\frac{1}{5} \times 1\frac{1}{4}$ **c** $9\frac{1}{6} \times 1\frac{1}{11}$ **d** $\frac{5}{18} \times 2\frac{7}{10}$

e $5\frac{1}{3} \times 2\frac{2}{5}$ **f** $\frac{3}{4} \times 8$ **g** $7 \times \frac{3}{8}$ **h** $2\frac{1}{6} \times 15$

i $42 \times 3\frac{9}{21}$ **j** $12 \times 5\frac{1}{60}$

2C.2 **4** Determine the result of each quotient.

a $\frac{5}{7} \div \frac{3}{2}$ **b** $\frac{3}{14} \div \frac{12}{35}$ **c** $\frac{9}{28} \div \frac{6}{7}$ **d** $\frac{15}{22} \div \frac{5}{33}$

e $\frac{21}{20} \div \frac{3}{20}$ **f** $\frac{17}{42} \div \frac{17}{6}$ **g** $\frac{27}{40} \div \frac{15}{40}$ **h** $\frac{12}{5} \div \frac{3}{50}$

5 Determine the result of each quotient.

a $2\frac{3}{4} \div \frac{5}{12}$ **b** $1\frac{5}{8} \div 2\frac{8}{9}$ **c** $2\frac{1}{3} \div 4\frac{2}{3}$ **d** $\frac{50}{63} \div 3\frac{4}{7}$

e $2\frac{1}{3} \div 1\frac{5}{9}$ **f** $\frac{12}{7} \div 6$ **g** $\frac{17}{4} \div 7$ **h** $12 \div \frac{6}{7}$

i $7 \div \frac{17}{4}$ **j** $20\frac{25}{27} \div 5$

6 Calculate each amount in the units shown in brackets.

a $\frac{1}{2}$ of 14 hours (hours) **b** $\frac{3}{4}$ of \$48 (dollars) **c** $\frac{2}{5}$ of 35 kg (kg)

d $\frac{4}{9}$ of 72 cm (cm) **e** $\frac{2}{7}$ of 3 weeks (days) **f** $\frac{3}{5}$ of 2 cm (mm)

2C.3 **7** Determine the result of each of the following.

a $-\frac{6}{11} \times \left(-\frac{2}{3}\right)$ **b** $\frac{4}{5} \times \left(-\frac{5}{12}\right)$ **c** $-\frac{15}{8} \times \frac{16}{25}$

d $-\frac{12}{35} \div \left(-\frac{7}{9}\right)$ **e** $-\frac{5}{6} \div \frac{10}{7}$ **f** $\frac{4}{9} \times \left(-\frac{5}{12}\right)$

g $-4\frac{1}{2} \times 3\frac{1}{3}$ **h** $-2\frac{1}{6} \div \left(-3\frac{1}{4}\right)$ **i** $7\frac{2}{3} \div \left(-5\frac{1}{9}\right)$

8 Determine the result of each of the following. Hint: Recall your knowledge of BIDMAS.

a $\left(\frac{6}{11} + \frac{2}{3}\right) \div \frac{5}{6}$ **b** $3\frac{1}{4} - 2\frac{1}{2} \times \frac{7}{10}$ **c** $\frac{8}{9} \times \left(\frac{4}{8} - \frac{3}{4}\right) \div 2\frac{2}{3}$ **d** $2\frac{3}{5} \times 3\frac{1}{3} + \frac{2}{11} \times 1\frac{3}{8}$

9 Calculate:

a $-\frac{5}{9} \times \frac{3}{5} - \frac{2}{3}$ **b** $-\frac{5}{6} \div \frac{10}{9} \times -\frac{4}{7}$ **c** $\frac{1}{2} + \frac{4}{7} \div -\frac{2}{5}$

d $\frac{4}{9} \times \frac{3}{8} - \frac{5}{3} \times \frac{7}{10}$ **e** $-\frac{5}{2} \times \left(\frac{7}{15} - \frac{3}{5}\right)$ **f** $-\frac{2}{3} \times \frac{6}{7} \div 2\frac{2}{3}$

10 The seating capacity of an interstate train carriage is 88 people. If the carriage is three-quarters full, how many people are seated in the train carriage?

11 To bake 12 chocolate cupcakes, you need these ingredients.

Chocolate cupcakes

(makes 12)

100 g butter

$\frac{1}{2}$ cup sugar

2 large eggs

$1\frac{1}{2}$ cups flour

1 teaspoon baking powder

$\frac{1}{2}$ cup milk

$\frac{3}{4}$ cup cocoa powder

1 cup boiling water

Chocolate icing

$1\frac{1}{2}$ cups icing sugar

150 g butter

120 g chocolate

$\frac{1}{2}$ cup thickened cream



a Work out the quantities of each ingredient (excluding the icing) if you want to bake:

i 24 cupcakes **ii** 6 cupcakes **iii** 8 cupcakes.

b Finn bakes 12 cupcakes but before he can ice them, one-quarter are eaten. Adjust the ingredient list for the chocolate icing so it makes only enough for the remaining cupcakes.

12 Grapes are to be sold in $2\frac{1}{2}$ kg bags.



a How many bags must Tricia pack if she has a 20 kg crate of grapes?

b From another crate, Tricia packs 11 bags and has $1\frac{1}{2}$ kg of grapes left over. How many kilograms of grapes were in the second crate?

13 Leanne has 20 red buttons amongst other coloured buttons to make an art piece. Leanne wants the red buttons to be $\frac{5}{6}$ of the entire design.



a How many buttons in total are needed for Leanne's art piece?

b Leanne stumbles across a box containing more red buttons and decides to make the art piece $\frac{1}{4}$ larger than its original size. How many red buttons are needed for the increased art piece?

c What fraction is the number of buttons in the increased art piece out of the number of buttons in the original-size art piece?

14 Mixed numbers can be multiplied without first writing them as improper fraction by using the distributive property.

For example,

$$\begin{aligned} 8 \times 2\tfrac{1}{2} &= 8 \times \left(2 + \tfrac{1}{2}\right) \\ &= 8 \times 2 + 8 \times \tfrac{1}{2} \\ &= 16 + 4 \\ &= 20 \end{aligned}$$

Multiply the following by using the distributive property. Write your answers using mixed numbers where necessary.

a $14 \times 2\frac{4}{7}$ **b** $30 \times 4\frac{9}{10}$ **c** $\frac{20}{3} \times 6\frac{2}{5}$ **d** $8 \times 4\frac{3}{5}$

15 A mixed number can be divided without first writing it as an improper fraction.

For example,

$$\begin{aligned} 3\tfrac{1}{2} \div \tfrac{4}{5} &= \left(3 + \tfrac{1}{2}\right) \div \tfrac{4}{5} \\ &= \left(3 \div \tfrac{4}{5}\right) + \left(\tfrac{1}{2} \div \tfrac{4}{5}\right) \\ &= \tfrac{15}{4} + \tfrac{5}{8} \\ &= 4\tfrac{3}{8} \end{aligned}$$

Divide the following by using the distributive property. Write your answers as mixed numbers where necessary.

a $25\frac{10}{19} \div 5$ **b** $6\frac{5}{9} \div \frac{1}{9}$ **c** $20\frac{8}{15} \div \frac{4}{3}$ **d** $5\frac{11}{13} \div \frac{10}{7}$

16 We can evaluate powers of fractions by using repeated multiplication of fractions.

For example,

$\left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1^2}{3^2} = \frac{1}{9}$.

Evaluate the following. Hint: Be careful with powers of negative fractions and write mixed numbers as improper fractions.

a $\left(\frac{2}{5}\right)^3$ **b** $\left(-\frac{2}{3}\right)^4$ **c** $\left(-\frac{2}{3}\right)^3$ **d** $\left(\frac{9}{7}\right)^2$ **e** $\left(3\frac{1}{3}\right)^3$

17 Since repeated multiplication of fractions repeatedly multiplies the numerator and repeatedly multiplies the denominator, we can reverse the process to find the square root of a fraction by finding the square root of the numerator and the square root of the denominator.

For example,

$\sqrt{\frac{4}{9}} = \sqrt{\frac{2^2}{3^2}} = \sqrt{\frac{2}{3} \times \frac{2}{3}} = \frac{2}{3}$.

Evaluate the following. Hint: Write mixed numbers as improper fractions and simplify first.

a $\sqrt{\frac{16}{25}}$ **b** $\sqrt{\frac{49}{9}}$ **c** $\sqrt{\frac{81}{100}}$ **d** $\sqrt{\frac{50}{18}}$ **e** $\sqrt{3\frac{13}{36}}$

18 Multiplying two mixed numbers can be performed by using the distributive law three times.

For example,

$$8\tfrac{1}{2} \times 2\tfrac{1}{2} = 8\tfrac{1}{2} \times \left[2 + \tfrac{1}{2}\right]$$
$$= \left[8\tfrac{1}{2} \times 2\right] + \left[8\tfrac{1}{2} \times \tfrac{1}{2}\right]$$
$$= \left[8 \times 2 + \tfrac{1}{2} \times 2\right] + \left[8 \times \tfrac{1}{2} + \tfrac{1}{2} \times \tfrac{1}{2}\right]$$

Multiply the following by using the distributive property. Write your answers using mixed numbers where necessary.

a $2\frac{3}{4} \times 6\frac{8}{9}$ **b** $14\frac{7}{15} \times 5\frac{3}{7}$ **c** $4\frac{1}{5} \times 9\frac{1}{3}$ **d** $3\frac{7}{12} \times 5\frac{3}{14}$

19 Explain why $3\frac{9}{10} \div 5\frac{2}{3} = \left(3 \div 5\frac{2}{3}\right) + \left(\frac{9}{10} \div 5\frac{2}{3}\right)$ but $3\frac{9}{10} \div 5\frac{2}{3} \neq \left(3\frac{9}{10} \div 5\right) + \left(3\frac{9}{10} \div \frac{2}{3}\right)$ by referring to the distributive property and the reciprocal.

20 a Which of the following operations with fractions is not made easier by using a common denominator?

A Addition **B** Subtraction **C** Multiplication
D Division **E** Raising to a power

b Explain why the answer(s) from part **a** are not made easier by using a common denominator.

c Explain why the other operations are made easier (or not made harder) by using a common denominator.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Multiplying and dividing fractions

Investigation
Fractions on the farm

Topic quiz
2C

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

2A **1** Fill the gaps to produce equivalent fractions.

a $\frac{6}{\square} = \frac{48}{80}$ **b** $\frac{\square}{20} = 2\frac{3}{4}$

c $\frac{63}{18} = \frac{\square}{8}$ **d** $\frac{6}{5} = \frac{\square}{-18}$

2A **2** Simplify the following fractions.

a $\frac{12}{15}$ **b** $\frac{60}{45}$

c $8\frac{16}{64}$ **d** $\frac{35}{-63}$

2A **3** Which fraction in each given pair is greater? Write your answer as an inequality.

a $\frac{2}{3} \square \frac{3}{4}$ **b** $\frac{7}{9} \square -\frac{8}{9}$

c $-\frac{17}{10} \square -\frac{9}{5}$ **d** $\frac{30}{36} \square \frac{36}{54}$

2B **4** Determine the LCD for each fraction pair.

a $\frac{3}{4}$ and $\frac{2}{3}$ **b** $\frac{5}{9}$ and $\frac{1}{6}$

c $\frac{3}{8}$ and $\frac{5}{12}$ **d** $\frac{7}{11}$ and $\frac{31}{33}$

2B **5** Evaluate the following sums.

a $\frac{2}{3} + \frac{2}{5}$ **b** $-\frac{7}{9} + 4\frac{2}{3}$

c $\frac{43}{10} + \left(-2\frac{3}{50}\right)$ **d** $-\frac{9}{7} + \left(-\frac{9}{4}\right)$

2B **6** Evaluate the following differences.

a $\frac{2}{3} - \frac{3}{4}$ **b** $-4\frac{1}{5} - 2\frac{8}{15}$

c $\frac{3}{16} - \left(-\frac{17}{24}\right)$ **d** $-3\frac{1}{3} - \left(-2\frac{1}{7}\right)$

2C **7** Evaluate the following products.

a $\frac{12}{5} \times \frac{25}{6}$ **b** $-\frac{4}{3} \times \frac{8}{9}$

c $4\frac{2}{5} \times \left(-\frac{1}{2}\right)$ **d** $-\frac{8}{6} \times \left(-3\frac{1}{3}\right)$

2C **8** Determine the reciprocal of the following.

a $\frac{3}{5}$ **b** $\frac{1}{7}$

c 9 **d** $5\frac{2}{3}$

2C **9** Evaluate the following quotients.

a $\frac{12}{35} \div \frac{4}{7}$ **b** $-2\frac{4}{5} \div \frac{2}{5}$

c $\frac{6}{7} \div \left(-1\frac{7}{8}\right)$ **d** $-\frac{3}{2} \div \left(-\frac{27}{16}\right)$

2B/2C **10** Evaluate the following using the order of operations.

a $\frac{3}{5} + \frac{6}{5} \times \frac{2}{5}$ **b** $-\frac{2}{3} - \frac{11}{15} + \frac{7}{10}$

c $\frac{4}{9} + \left(-\frac{6}{5}\right) \div \frac{8}{15}$ **d** $\frac{9}{10} \div \left(-\frac{5}{3}\right) \times \left(-\frac{2}{7}\right)$

2C **11** Find:

a $\frac{5}{8}$ of 104 GB **b** $\frac{3}{4}$ of 10 L

c $1\frac{1}{2}$ of 500 g **d** $\frac{17}{20}$ of \$5.00

2D Decimals

Learning intentions

By the end of this topic you will be able to ...

- ✔ convert between fractions and decimals
- ✔ compare and order decimals
- ✔ round decimals to the appropriate decimal place.

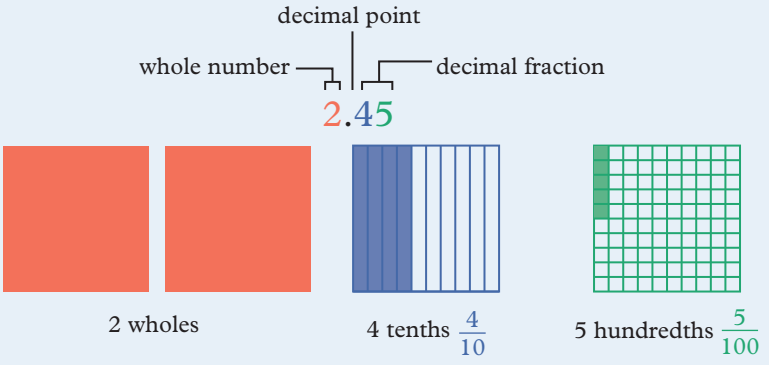
Inter-year links

Support — Fractions and decimals

Year 7 — 4A Decimals

Decimals

- A **decimal** is a number whose whole number part and fraction part are separated by a decimal point.



- The value of each digit depends on the place or position of the digit in the number.
  For example, the decimal 1234.5678 is shown in the place value table.

Thousands 1000	Hundreds 100	Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
1	2	3	4	.	5	6	7	8

→ Each place value to the right is 10 times smaller than the place before.

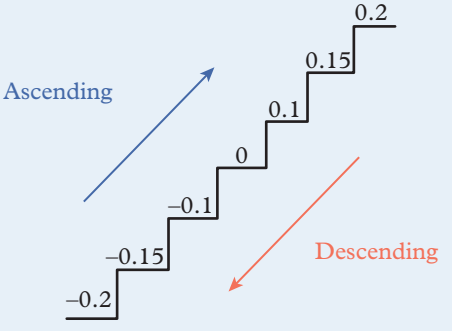
→ Each place value to the left is 10 times larger than the place before.

Ordering numbers

- To order decimals consider the digits from left to right.

  For example, 5.673 and 5.689 have the same whole number and tenth values. 8 hundredths is greater than 7 hundredths, so 5.673 < 5.689.

Ones	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
5	.	6	7	3
5	.	6	8	9



Rounding

- A decimal can be rounded to a given number of decimal places by considering the next digit to the right.
  For example, both numbers shown have been rounded to two decimal places.

$1.2\boxed{3}95 \approx 1.24$ (9, 8, 7, 6, 5 → Round up)

$1.2\boxed{3}25 \approx 1.23$ (4, 3, (2), 1, 0 → Round down)

Converting between fractions and decimals

- To convert decimals to fractions, place the decimal fraction part of the number as the numerator of the fraction. Write the fraction denominator as 10, 100, 1000 … with as many zeros as there are digits after the decimal point in the decimal fraction part of the number.

0.5	5 tenths	$\frac{5}{10}$
0.55	55 hundredths	$\frac{55}{100}$
0.555	555 thousandths	$\frac{555}{1000}$

- To convert from a fraction to a decimal, **trailing zeros** are added following digits after the decimal point in the dividend until there is no remainder. Trailing zeros do not change the value of the number.

Align decimal points

Dividend → $\frac{1}{8} = 8\overline{)1.0^{2}0^{4}0}$ ← Divisor

$0.1\ 2\ 5$

Trailing zeros

Example 2D.1 Ordering decimals

Arrange the following in ascending order.
5.74, 5.7, 5.47, 5.04, 5.407, 4.57

THINK

1 Compare the whole number parts. There are five numbers with a whole number of 5 and one with a whole number of 4.

2 Compare the tenths values of the numbers with the same whole number part. 7 tenths is greater than 4 tenths and 4 tenths is greater than 0 tenths.

3 Compare the hundredths values of the numbers with the same whole number and tenths parts. 7 hundredths is greater than 0 hundredths and 4 hundredths is greater than 0 hundredths. When a place value does not contain a digit, its value is zero.

4 Write the decimals in ascending order, from smallest to largest.

WRITE

4.57, 5.74
5.7,
5.47,
5.04,
5.407

4.57, 5.04, 5.47, 5.74,
5.407, 5.7

4.57, 5.04, 5.47, 5.74,
5.407, 5.7

4.57, 5.04, 5.407, 5.47, 5.7, 5.74

Example 2D.2 Rounding decimals

Round each number to two decimal places.

a 5.7323 **b** −12.099 76

THINK

a 1 Draw a box around the second decimal place.

2 Look at the digit to the right of the box. The digit to the right is a 2. Do not change the digit in the box.

3 Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same.

b 1 Draw a box around the second decimal place.

2 Look at the digit to the right of the box. The digit to the right is a 9. Add 1 to the digit in the box.

3 As the boxed digit changes from 9 to 10, write zero in the boxed digit's place and add one to the place to the left.

4 Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same.

Keep the zero in the second decimal place as the question is asking for two decimal places.

WRITE

a $5.7\boxed{3}23$

$5.7\boxed{3}23$

$5.7323 \approx 5.73$

b $-12.0\boxed{9}9\,76$

$-12.0\boxed{9}9\,76$

$-12.1\boxed{0}$

$-12.099\,76 \approx -12.10$

Example 2D.3 Converting a decimal to a fraction

Write the following decimals as fractions.

a 1.4 **b** −0.52 **c** 10.068

THINK

1 Identify any whole numbers to the left of the decimal point.

2 Place the decimal fraction part of the number as the numerator of the fraction. Make the fraction denominator 10, 100, 1000 … with as many zeros as there are digits after the decimal point in the decimal fraction part of the number.

3 Simplify the fraction by finding common factors or the HCF of the numerator and denominator.

WRITE

a $1.4 = 1\frac{4}{10}$

$= 1\frac{\cancel{4}^{2}}{\cancel{10}^{5}}$

$= 1\frac{2}{5}$

b $-0.52 = -\frac{52}{100}$

$= -\frac{\cancel{52}^{13}}{\cancel{100}^{25}}$

$= -\frac{13}{25}$

c $10.068 = 10\frac{68}{1000}$

$= 10\frac{\cancel{68}^{17}}{\cancel{1000}^{250}}$

$= 10\frac{17}{250}$

Example 2D.4 Converting fractions to decimals

Write $\frac{3}{8}$ as a decimal.

THINK

1. As the denominator of the fraction is not a power of 10, write the fraction as a division problem with a decimal point after the numerator.
2. Work out the division using short division. Include trailing zeros after the decimal point until you can divide exactly without a remainder. Align the decimal points.

WRITE

$\frac{3}{8} = 3 \div 8$

$$8\overline{)3.^{3}0^{6}0^{4}0} \quad 0.\ 3\ 7\ 5$$

$\frac{3}{8} = 0.375$

Helpful hints

- ✔ Decimals and fractions are actually two different ways of writing the same thing!
- ✔ Zeros between non-zero digits in a decimal (called placeholder zeros) must never be left out, or the value of the number will be changed.
- ✔ Zeros at the end of a decimal (called trailing zeros) do not change the value of the number.
- ✔ If you have trouble determining the place value of a digit, you can draw the place value table to help you.

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
1	2	.	3	4	5	6

- ✔ You may think that 0.000 19 is greater than 0.2 because there are more digits shown. Remember to compare from left to right. As 0 tenths is smaller than 2 tenths, 0.2 is greater than 0.000 19.
- ✔ Make sure to consider the real life context before rounding decimals.
  For example, if a chip packet costs $1 and I have $1.50, don't tell me that I can buy 1.5 chip packets – I can only buy 1!

ANS p499

Exercise 2D Decimals

△ 1–6, 7(a–c, g, i), 8(a–f), 9(a-d, g), 10(a–f, h), 11(a–e), 12(a–d, g, j), 13, 14, 17, 19

□ 2(d–f), 4–5(b, d), 7(d–i), 8(g–l), 9(e–h), 10(g–l), 11(f–h), 12(e–h, k, l), 15, 16, 18, 20, 21

⬠ 2(e, f), 4(d), 5(c, d), 7–8(3rd column), 9(g, h), 10(j–l), 11(g, h), 12(i–l), 15, 18, 20–22

UNDERSTANDING AND FLUENCY

1 Write the following numbers into the place value chart below.

a 0.0065 **b** 0.6500 **c** 6.005 **d** 60.5

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
		.				

2 Determine the place value of 5 in each number. Refer to the place value chart to help you.

a 4.705 **b** 59.300 **c** 605.9

d 1740.50 **e** 1.3745 **f** 88.352 00

3 How many decimal places does each number in question **2** have?

2D.1 **4** Write each list of numbers in ascending order.

a 0.214, 0.142, 0.2, 0.41, 0.104, 0.14

b 8.702, 9.87, 8.072, 9.782, 8.2, 8.97

c 150.4, –10.45, 15.04, 10.54, 105.4, –15.40

d –799.8, –800.07, 800.7, –800.8, 799.08, –800.08

5 Write each list of numbers in descending order.

a 13.7, 13.07, 13.173, 13.31, 13.137, 13.713

b 0.658, 0.6058, 0.6508, 0.685, 0.6085, 0.65

c –50.5050, 5.5555, 5.550 00, 555.55, 5.005 00, –50.0505

d 100, –99.90, –100, 999.099, –99.009, 999.90

6 Explain why 9.46 is larger than 9.452 even though 9.452 has more digits.

2D.2 **7** Round each number to two decimal places.

a 25.679 **b** −673.8234 **c** 0.784 231

d −101.0157 **e** 8148.255 **f** −54.2618

g 330.406 **h** −4.854 793 **i** 15.9999

8 Round each decimal to the number of decimal places shown in the brackets.

a 57.234 (2) **b** 8.9137 (3) **c** 92.087 12 (4)

d −0.756 825 (3) **e** −135.152 79 (1) **f** 65.071 226 33 (5)

g 7.510 429 (3) **h** −208.009 862 (4) **i** 1.727 727 727 (5)

j 845.845 845 (1) **k** −0.123 456 78 (4) **l** −0.123 456 78 (7)

2D.3 **9** Write the following decimals as fractions.

a 1.03 **b** 2.9 **c** 3.007 **d** −0.0001

e 10.7 **f** −0.000 03 **g** 12.000 009 **h** −0.000 000 7

10 Write each number as a fraction in simplest form.

a 0.12 **b** −0.54 **c** 0.803

d −0.04 **e** 0.025 **f** −0.0006

g −0.74 **h** 0.252 **i** −0.0015

j −0.048 **k** 0.000 17 **l** −0.85

11 Write each fraction as a decimal.

a $\frac{7}{10}$ **b** $-\frac{29}{100}$ **c** $\frac{451}{1000}$ **d** $-\frac{2047}{10\,000}$

e $\frac{4}{100}$ **f** $-\frac{9}{1000}$ **g** $\frac{5}{10\,000}$ **h** $-\frac{63}{1000}$

2D.4 **12** Write each fraction as a decimal.

a $\frac{4}{5}$ **b** $\frac{5}{8}$ **c** $-\frac{1}{2}$ **d** $\frac{5}{4}$

e $\frac{1}{8}$ **f** $-\frac{2}{5}$ **g** $\frac{7}{20}$ **h** $-\frac{9}{25}$

i $\frac{1}{40}$ **j** $-\frac{3}{5}$ **k** $\frac{11}{50}$ **l** $-\frac{5}{16}$

PROBLEM SOLVING AND REASONING

13 Wendy thinks 8.2735 rounded to three decimal places is 8.273. Rosa thinks it is 8.274. Who is correct? Explain.

14 Jayden thinks that 24.7963 rounded to two decimal places is 24.80. Oscar thinks it is 24.71. Who is correct? Explain.

15 Peter thinks that 15.998 33 rounded to two decimal places is 16. Kota thinks it is 16.00. Who is correct? Explain.

16 Sabine needs to round 2.448 to one decimal place. First, Sabine rounds 2.448 to two decimal places to get 2.45. Sabine then rounds to one decimal place to get 2.5. Explain what Sabine has done wrong.

17 The length of a mosquito is measured to be 0.68 cm. Write this value correct to one decimal place.

18 Six students' heights were measured as 1.651 m, 1.63 m, 1.568 m, 1.615 m, 1.58 m and 1.657 m.



a Which is the smallest measurement?

b Which is the largest measurement?

c Write the list of measurements in ascending order.

d Round each measurement to two decimal places.

e Write your list from part **d** in ascending order. What do you notice?

f Round each measurement to one decimal place.

g Write your list from part **f** in ascending order. What do you notice?

19 The width of a human hair is measured to be 0.018 35 mm. Write this value as an approximation to:

a two decimal places **b** three decimal places **c** four decimal places.

20 The diameter of an optical fibre is 0.2845 mm.



a Round this value to:

i one decimal place

ii two decimal places

iii three decimal places.

b Write the exact diameter as a fraction in simplest form.

21 Round each amount to the nearest five cents.

a $42.37 **b** $207.62

c $7.24 **d** $15.03

e $60.71 **f** $99.99

CHALLENGE

22 Use each digit from 0 to 9 only once to write a number with nine decimal places such that when rounded to an even number of decimal places, the number rounds up, and when rounded to an odd number of decimal places, the number rounds down.

□.□□□□□□□□□

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Ordering decimals
- Interactive skillsheet: Rounding decimals
- Interactive skillsheet: Fractions and decimals
- Investigation: Decimal history
- Topic quiz: 2D

2E Adding and subtracting decimals

Learning intentions

By the end of this topic you will be able to ...

✔ add and subtract positive and negative decimals.

Inter-year links

Support	Adding and subtracting decimals
Year 7	4C Adding and subtracting decimals

Adding and subtracting decimals

- Decimals can be added or subtracted in the same way as whole numbers using the vertical addition and subtraction algorithms. Add or subtract digits in each column from right to left.

$$\begin{array}{r} 3.02 \\ +\ 4.15 \\ \hline 7.17 \end{array}$$

- Make sure that the decimal points are aligned and the digits are in the correct place value columns.
- When working with negative decimals, simplify the calculation by considering the operation and whether each decimal is positive or negative.

$$+\,(+) = +$$
$$-\,(+) = -$$
$$+\,(-) = -$$
$$-\,(-) = +$$

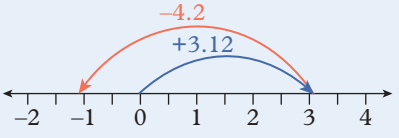
- If both numbers in an expression are negative, use the distributive law to rewrite the expression as a product of −1 and a sum of two positive numbers. Follow the order of operations to find the value of the original expression.
  - → Use the vertical addition algorithm if necessary to find the sum of the two positive numbers.

$$\begin{aligned} -3.12 - 4.2 &= -1 \times (3.12 + 4.2) \\ &= -1 \times 7.32 \\ &= -7.32 \end{aligned}$$

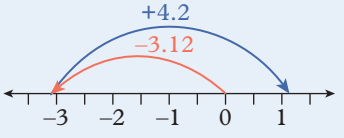
- The **magnitude** of a number is the value of a number ignoring the sign. For example, the magnitude of −3 is 3.
- If one number is positive and the other is negative, ignore the minus sign and subtract the number smaller in magnitude from the number greater in magnitude. The sign of the result is the same as the sign of the number greatest in magnitude.
  - → Use the vertical subtraction algorithm if necessary to find the difference between the two numbers.

$$\begin{aligned} 3.12 - 4.2 &= -(4.2 - 3.12) \\ &= -1.08 \end{aligned} \qquad \begin{aligned} -3.12 + 4.2 &= +(4.2 - 3.12) \\ &= +1.08 \end{aligned}$$

  - → It may be helpful to use a number line to determine whether the result of an operation is positive or negative.



3.12 − 4.2 = −1.08



−3.12 + 4.2 = (+)1.08

Example 2E.1 Adding and subtracting decimals

Use vertical addition and vertical subtraction to calculate each result.

a 14.507 + 8.63 **b** 45.93 − 30.498

THINK

1 Estimate the result by rounding to the nearest whole number.
2 Write the numbers underneath each other. Ensure the decimal points and digits are aligned.
3 Replace blank spaces with additional zeros so that each number contains the same number of decimal places.
4 Add or subtract the digits in each column as you would for whole numbers. Ensure the decimal point in the answer is lined up with the decimal points in the calculation.
5 Check that your estimate is a close approximation of the result and that the decimal point is in the correct position.

WRITE

a 14.507 + 8.63 ≈ 15 + 9
= 24

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ \ } \\ 14.507 \\ +\ 08.630 \\ \hline 23.137 \end{array}$$

14.507 + 8.63 = 23.137
24 ≈ 23.137

b 45.93 − 30.498 ≈ 46 − 30
=16

$$\begin{array}{r} {\scriptstyle \ \ 8\ 12\ 10} \\ 45.9\not{3}\not{0} \\ -\ 30.498 \\ \hline 15.432 \end{array}$$

45.93 − 30.498 = 15.432
16 ≈ 15.432

Example 2E.2 Adding decimals with negative signs

Calculate each result.

a −3.256 + 5.89 **b** −1.68 + (−3.45)

THINK

a 1 Estimate the result by rounding to the nearest whole number.
2 Determine whether to add or subtract the decimals. Since one is negative and one is positive, ignore the minus sign and subtract the smaller decimal from the larger decimal using the vertical subtraction algorithm.
3 Determine the sign of the result. Since the positive decimal is greater in magnitude than the negative decimal, the result is positive.
4 Check your answer against your estimate.

WRITE

a −3.256 + 5.89 ≈ −3 + 6
= 3

$$\begin{array}{r} {\scriptstyle \ \ 8\ 10} \\ 5.8\not{9}\not{0} \\ -3.256 \\ \hline 2.634 \end{array}$$

−3.256 + 5.89 = 2.634

3 ≈ 2.634

b **1** Simplify the signs. Estimate the result by rounding to the nearest whole number.	**b** $-1.68 + (-3.45) = -1.68 - 3.45$ $\approx -2 - 3$ $= -5$
2 Determine whether to add or subtract the decimals. Since both are negative, ignore the minus signs and add the two decimals together using the vertical addition algorithm.	$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 1.68 \\ +3.45 \\ \hline 5.13 \end{array}$
3 Determine the sign of the result. Since both decimals are negative, the result is negative.	$-1.68 + (-3.45) = -(1.68 + 3.45)$ $= -5.13$
4 Check your answer against your estimate.	$-5 \approx -5.13$

Example 2E.3 Subtracting decimals with positive and negative signs

Calculate each result.

a $7.58 - 8.6$ **b** $4.206 - (-13.797)$

THINK	WRITE
a **1** Estimate the result by rounding to the nearest whole number.	**a** $7.58 - 8.6 \approx 8 - 9$ $= -1$
2 Determine whether to add or subtract the decimals. Since one is negative and one is positive, ignore the minus sign and subtract the smaller decimal from the larger decimal using the vertical subtraction algorithm.	$\begin{array}{r} {\scriptstyle 5\ 10} \\ 8.\not{6}\not{0} \\ -\ 7.58 \\ \hline 1.02 \end{array}$
3 Determine the sign of the result. Since the negative decimal is greater in magnitude than the positive decimal, the result is negative.	$7.58 - 8.6 = -1.02$
4 Check your answer against your estimate.	$-1 \approx -1.02$
b **1** Simplify the signs. Estimate the result by rounding to the nearest whole number.	**b** $4.206 - (-13.797) = 4.206 + 13.797$ $\approx 4 + 14$ $= 18$
2 Add the two decimals together using the vertical addition algorithm.	$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 13.797 \\ +\ 4.206 \\ \hline 18.003 \end{array}$
3 Determine the sign of the result. Since both decimals in the simplified expression are positive, the result is positive.	$4.206 - (-13.797)$ $= 4.206 + 13.797 = 18.003$
4 Check your answer against your estimate.	$18 \approx 18.003$

Helpful hints

- ✔ Rounding to the nearest whole number is a good way to check your result makes sense.
- ✔ Make sure that the decimal points in each number are aligned when adding or subtracting decimals. For example,

$$\begin{array}{r} 100\,000.3 \\ -\quad 12.000\,003 \\ \hline \end{array}$$

- ✔ When adding trailing zeros after the decimal, you are not changing the number. You are just writing a zero that was already implicitly there.

ANS p500

Exercise 2E Adding and subtracting decimals

△ 1, 2, 3(a–c), 4, 5–8(1st column), 9–12, 13(a, b), 15, 16(a)

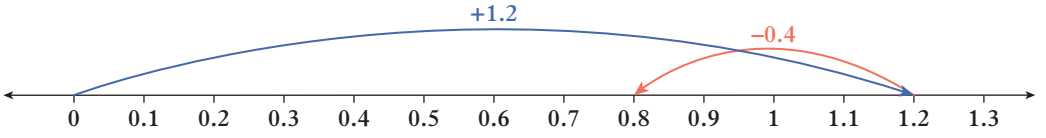
□ 2, 3(d–f), 4, 5–8(2nd column), 9(c, d), 12, 13(a, c), 14, 16, 17(a, b)

⬠ 3(d–f), 5–8(2nd column), 9(d), 13(c, d), 14, 17(c, d), 18, 19

UNDERSTANDING AND FLUENCY

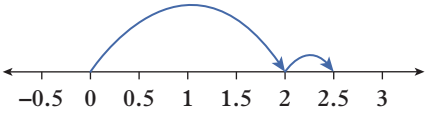
1 Adding and subtracting decimals can be shown on the number line. Instead of starting at the first number of the calculation, we can start at zero and treat the first number as a positive or negative jump along the number line.

For example, 1.2 – 0.4 is shown on the number line below, starting at zero.

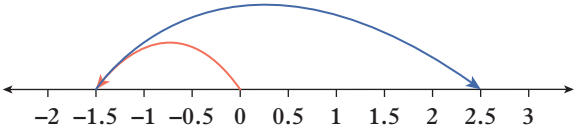


For each number line below, write the calculation shown, starting from zero.

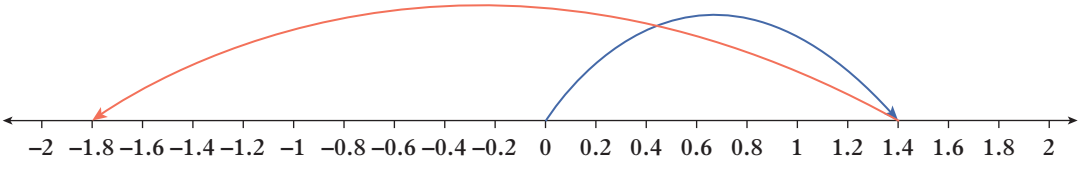
a



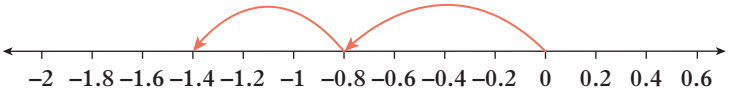
b



c



d



2E.1 **2** Use vertical addition to calculate each result.

a
$$\begin{array}{r} 5.27 \\ +4.83 \\ \hline \end{array}$$

b
$$\begin{array}{r} 18.9 \\ +\ 7.35 \\ \hline \end{array}$$

c
$$\begin{array}{r} 0.085 \\ 1.32 \\ +0.647 \\ \hline \end{array}$$

d
$$\begin{array}{r} 17.378 \\ 6.003 \\ +\ 5.999 \\ \hline \end{array}$$

3 Use vertical addition to calculate each result.

a 23.82 + 17.13 + 124.06 + 82.95

b 5.613 + 9.27 + 11.3856 + 7.5

c 324.7 + 76.29 + 4.93 + 285.064

d 74.0106 + 2.93 + 28.738 + 0.9167

e 0.0034 + 0.21 + 0.024 + 0.0402

f 99.999 + 14.24 + 4.748 + 0.0284

4 Use vertical subtraction to calculate each result.

a
$$\begin{array}{r} 158.29 \\ -\ 75.46 \\ \hline \end{array}$$

b
$$\begin{array}{r} 2051.384 \\ -\ 1265.73 \\ \hline \end{array}$$

c
$$\begin{array}{r} 0.35 \\ -\ 0.2156 \\ \hline \end{array}$$

d
$$\begin{array}{r} 70.308 \\ -\ 44.057 \\ \hline \end{array}$$

5 Use vertical subtraction to calculate each result.

a 9.104 − 6.3728

b 265.7 − 88.504

c 5061.03 −1266.735

d 73 412 − 8743.56

e 9000 − 0.555

f 6.5677 − 4.5678

2E.2 **6** Calculate each result.

a 3.2 + (−5.8)

b −3.2 + (−5.8)

c −3.2 + 5.8

d 51.294 + (−3.028)

e −4.1928 + 0.285

f −0.048 + (−495.19)

2E.3 **7** Calculate each result.

a 4.78 − 8.48

b −4.78 − 8.48

c −4.78 − (−8.48)

d −3.541 − 58.242

e −274.28 − (−0.4827)

f 9.402 − 10.611

8 Evaluate the following.

a 5.8 − 9.9 + 3.2

b 19.53 + 3.78 − 32.05

c 100 − 210.15 + 27.3

d 0.0032 + 0.1 − 0.352

9 Evaluate the following.

a (+3.7) − (+5.2) + (−6.1)

b (+4.8) + (−9.7) + (+10.11)

c (−17.3) − (+12.4) − (−30.8)

d (−85.23) + (−0.0573) − (+2.902 04)

10 The circuit of a fun run has five different stages. The lengths of the stages are: 1.435 km, 2.790 km, 1.902 km, 2.285 km and 3.613 km. First estimate the distance around the full circuit by rounding each length to the nearest whole number and then calculate the exact distance.



11 Lee Redmond once held the record for the longest fingernails on both hands (see table), after not cutting them for 30 years.



	Fingernail lengths (cm)	
	Right hand	**Left hand**
Thumb	76.4	80.0
Index finger	72.3	76.4
Middle finger	74.1	76.7
Ring finger	73.6	76.2
Little finger	71.6	73.6

a What is the total length of the fingernails on:

i her right hand

ii her left hand?

b Which hand has the longer fingernails? Suggest a reason for this.

c What is the total length of the fingernails on both hands?

12 Consider the calculation $7.2 - 9.4 + 2.3$.

a Which of the following is not equivalent to the original calculation?

A $(7.2 - 9.4) + 2.3$ **B** $(7.2 + 2.3) - 9.4$ **C** $7.2 - (9.4 - 2.3)$

D $7.2 - (9.4 + 2.3)$ **E** $2.3 - (9.4 - 7.2)$

b Explain why the answer to part **a** is not equivalent to $7.2 - 9.4 + 2.3$ and write a similar expression that would be equivalent.

c For any calculation involving only addition and subtraction, if all subtractions are performed first, will the answer always be correct? Explain why or why not.

13 Calculate the following sums and differences by using addition and subtraction of the equivalent fractions.

For example, $12.3 + 4.21 = \frac{123}{10} + \frac{421}{100} = \frac{1230}{100} + \frac{421}{100} = \frac{1651}{100} = 16.51$.

a $-17.32 + 4.0032$

b $9.999 + (-50.91)$

c $-0.003 - (-0.0502)$

d $-90.06 - 73.0012$

14 Jimmy currently has $32.50 saved up. He wants to buy a new game that costs $69.99. Jimmy's parents agree to pay the difference if Jimmy pays them back. Jimmy earns $5.50 a week in pocket money but can increase it by completing various chores. Each chore has a limit of how often Jimmy can complete it for pocket money. The chores and how much Jimmy earns from them are listed below.

- unload and hang the washing up to dry, $1.10 twice a week
- fold the laundry and put it away, $0.90 twice a week
- take out the rubbish bin, $0.45 daily
- dust the shelves and bookcases, $1.70 weekly
- help prepare dinner, $2.10 daily

a Can Jimmy pay his parents back in two weeks? State how much Jimmy still owes or how much extra he earns.

b If Jimmy wants to save up for a $79.99 game after he has paid his parents back, what is the minimum number of additional weeks it will take?

15 Look back over your calculations for adding decimals together. What do you notice about the number of decimal places in the calculation and the number of decimal places in the result? Determine the number of decimal places in the result of a sum where a number with x decimal places is added to a number with y decimal places, where x is greater than or equal to y.

16 a Use the fact that $-(3.12 + 4.2) = -3.12 + (-4.2)$ by the distributive law to help explain why we can add 3.12 and 4.2 to calculate $-3.12 - 4.2$.

b Use the fact that $-(4.2 - 3.12) = -4.2 - (-3.12)$ by the distributive law to explain why we can subtract 3.12 from 4.2 to calculate $3.12 - 4.2$.

17 Subtraction can be rewritten as adding a negative number. As addition is associative and commutative, this allows us to rearrange the order of calculations. We can then add all the positive numbers together and all the negative numbers together. The result can be determined by finding the difference between the two numbers. For example,

$$\begin{aligned} &-3.1 + (-2.4) + 4.9 - (-9.3) - 6.2 \\ &= (-3.1) + (-2.4) + (+4.9) + (+9.3) + (-6.2) \\ &= [(+4.9) + (+9.3)] + [(-3.1) + (-2.4) + (-6.2)] \\ &= 14.2 + (-11.7) \\ &= 2.5 \end{aligned}$$

Write the following as the difference of the total positive and negative sums by making use of the associative and commutative laws, then evaluate.

a $9.2 - 7.2 + (-4.1) - (-4.8) + 7 - 2.6$

b $-28.19 - (-49.18) + 99.24 + (-72.09) - (-13.25)$

c $0.028 - 0.18 - (-0.291) + (-2.59) - 12.01 + 9$

d $-70.18 - 81.8 + 29.09 - (-45.29) - 29.90 + 17.7$

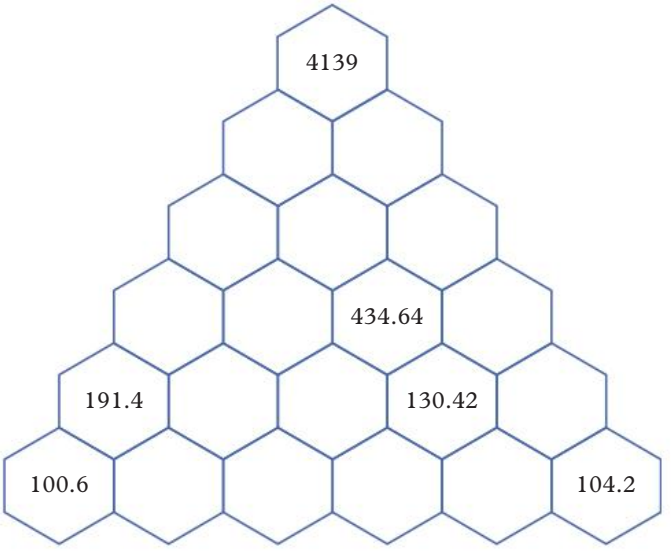


18 You begin by facing North. You turn 197.35° clockwise, then 87.194° anti-clockwise, then 3.813° clockwise, then 99.24° clockwise, then 100.01° anti-clockwise, and finally 32.021° anti-clockwise.

a In what direction (clockwise or anti-clockwise) and by how many degrees are you now facing relative to North?

b How many degrees did you turn in total, regardless of direction?

19 Fill in the blank boxes with the correct number. Every number is formed by adding the number left below and right below in the box. Your algebra skills may be useful for solving this problem.



Check your Student obook pro for these digital resources and more:

- **Interactive skillsheet** — Adding and subtracting decimals
- **Investigation** — How large is your hand?
- **Topic quiz** — 2E

PROBLEM SOLVING AND REASONING

CHALLENGE

2F Multiplying and dividing decimals

Learning intentions

By the end of this topic you will be able to ...

- ✔ multiply positive and negative decimals
- ✔ divide positive and negative decimals.

Inter-year links

Support	Multiplying decimals
Year 7	4D Multiplying decimals

Multiplying and dividing by powers of 10

- Multiplying a number by 10 increases each digit's place value by 1 column. Move the decimal point one place-value space to the right and insert a zero where necessary.

$$5.23 \times 10 = 52.3$$
$$5.23 \times 100 = 523.$$
$$5.23 \times 1000 = 5230.$$

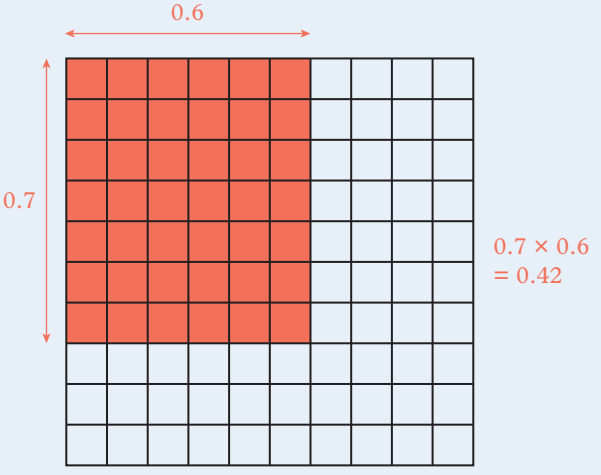
- Dividing a number by 10 decreases each digit's place value by 1 column. Move the decimal point one place value space to the left and insert a zero where necessary.

$$5.23 \div 10 = 0.523$$
$$5.23 \div 100 = 0.0523$$
$$5.23 \div 1000 = 0.00523$$

Multiplying decimals

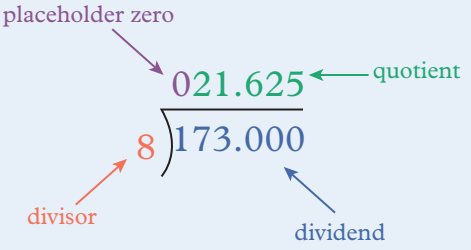
- Decimals can be multiplied in the same way as whole numbers.
- The result has the same number of decimal places as the total number of decimal places in the original problem.

1. Estimate the result. Round each decimal to the leading digit.
2. Count the decimal places in the problem.
3. Multiply the decimals as whole numbers.
4. Write the answer with this same number of decimal places. Add zeros where necessary.



Dividing a decimal by a whole number

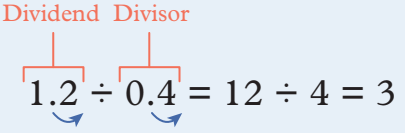
- Align the decimal point in the answer with the one in the dividend.
- Trailing zeros are added after the decimal point in the dividend until there is no remainder.



Dividing a decimal by a decimal

- When you divide by a decimal, you can create an equivalent problem by moving the decimal point to the right or multiplying by $\frac{10}{10}, \frac{100}{100}, \frac{1000}{1000}, \ldots$ (You are creating equivalent decimal fractions!)

1. Convert the divisor to a whole number by moving the decimal point to the right.
2. Move the decimal point the same number of spaces to the right on the dividend.
3. Perform short division and add trailing zeros when necessary. Make sure the decimal points are aligned.



Example 2F.1 Multiplying and dividing by powers of 10

Calculate:

a 2.27×100 **b** $3.14 \div 1000$

THINK

a When multiplying by 100, move the decimal point two place-value spaces to the right. Don't include the decimal point if there are no digits other than zero after it.

b When dividing by 1000, move the decimal point three place-value spaces to the left. Add zeros where necessary.

WRITE

a 2.27
$2.27 \times 100 = 227$

b 0003.14
$3.14 \div 1000 = 0.003\,14$

Example 2F.2 Multiplying decimals

Use long multiplication to calculate 20.45×-3.9.

THINK	WRITE
1 Estimate the result by rounding to the leading digit. Remember that a positive number multiplied by a negative number is a negative number.	$20.45 \times -3.9 \approx 20 \times -4$ $= -80$
2 Count the decimal places in the problem. There are three decimal places in total.	20.45×3.9
3 Multiply the decimals as whole numbers using long multiplication.	$\begin{array}{r} {\scriptstyle 1\ 1} \\ {\scriptstyle 4\ 4} \\ 2045 \\ \times\ \ 39 \\ \hline 18405 \\ +61350 \\ \hline 79755 \end{array}$ 79.755
4 Write the answer with this same number of decimal places. Add zeros where necessary and write the answer with a negative sign.	$20.45 \times -3.9 = -79.755$
5 Check that your estimate is a close approximation of the result and that the decimal point is in the correct position.	$-79.755 \approx -80$

Example 2F.3 Dividing a decimal by a whole number

Use short division to calculate $-37.45 \div 4$.

THINK	WRITE
1 Estimate the result by rounding to the leading digit. Remember that a negative divided by a positive is a negative.	$-37.45 \div 4 \approx -40 \div 4$ $= -10$
2 Evaluate the division in the same way as for whole numbers, using short division.	$\begin{array}{r} -09.3625 \\ 4\overline{)-37.{}^{1}4{}^{2}5{}^{1}0{}^{2}0} \end{array}$
3 Align the decimal point and the digits that have the same place value.	
4 Add trailing zeros after the last digit in the dividend to obtain an exact decimal answer without a remainder.	
5 Check that your estimate is a close approximation of the result and that the decimal point is in the correct position. Don't forget to include the appropriate sign.	$-37.45 \div 4 = -9.3625$ $-10 \approx -9.3625$

Example 2F.4 Dividing by decimals

Use short division to calculate $-9.46 \div -0.5$.

THINK

1. Convert the divisor to a whole number by moving the decimal point to the right.
2. Move the decimal point the same amount of spaces to the right on the dividend. Add a place-holding zero where necessary.
3. Perform short division and add trailing zeros when necessary. Make sure the decimal points are aligned. Remember that a negative divided by a negative is a positive.

WRITE

$-9.46 \div -0.5 = -94.6 \div -5$

$$\begin{array}{r} 18.92 \\ -5\overline{)-9^{4}4.^{4}6^{1}0} \end{array}$$

$-9.46 \div -0.5 = 18.92$

Helpful hints

- ✔ Moving the decimal point of the dividend and the divisor one place to the right is the same as multiplying by $\frac{10}{10}$.
- ✔ Remember that trailing zeros should be added to the dividend until you can divide exactly, without a remainder.

ANS p500

Exercise 2F Multiplying and dividing decimals

△ 1–5, 6(a, b, d, e, g), 7, 8(a, c, d), 9(a–c), 12(a, b), 13, 14

□ 1, 2, 3(e–h), 4(e–h), 6(c, d, f, h, i), 7, 8(b, e, f), 9, 10, 11(a, c), 12(c), 15(a, c), 16(a, d)

⬠ 3–4(c, f, h), 6(d, f, g, i), 7, 8(c, e, f), 9(e, f), 11(b, d), 12(c), 15–18

2F.1 **1** Evaluate the products and quotients.

a 0.0052×10 **b** 10.4×100 **c** $92.1 \div 10$

d $340 \div 10\,000$ **e** $0.057 \div 100$ **f** $45.78 \div 1000$

2F.2 **2** Use multiplication to calculate each result. Remember to estimate your result first, and to compare it with your answer.

a 24.35×8 **b** 305.6×4 **c** $17\,352.7 \times 3$ **d** $0.327\,51 \times 9$

3 Use long multiplication to calculate each result.

a 8.27×5.3 **b** -12.86×2.4 **c** $-235.2 \times (-9.7)$ **d** 62.513×1.8

e $9.2 \times (-0.46)$ **f** $3.8927 \times (-0.52)$ **g** $-56.361 \times (-0.14)$ **h** 157.93×7.12

2F.3 **4** Use short division to calculate each result.

a $72.596 \div 4$ **b** $6.724 \div (-5)$ **c** $-58.2743 \div 7$ **d** $65.0955 \div 3$

e $-4.73 \div (-8)$ **f** $0.627 \div (-4)$ **g** $0.1536 \div 6$ **h** $-138.0714 \div 5$

UNDERSTANDING AND FLUENCY

5 Dividing a decimal by another decimal is easier if you write an equivalent division problem where the divisor is a whole number.

a In $3.568 \div 0.4$, the divisor is 0.4. Should you multiply 0.4 by 10, 100 or 1000 to produce 4?

b Complete: $3.568 \div 0.4 =$ _____ $\div 4$

c Why is it important to multiply each decimal in the division problem by the same number?

d Use your answer to part **b** to work out the division.

2F.4 **6** Write each calculation as an equivalent division problem where the divisor is a whole number.

a $7.24 \div 0.2$ **b** $4.902 \div (-0.6)$ **c** $19.764 \div 0.03$

d $-1.728 \div (-0.08)$ **e** $1.4368 \div 0.004$ **f** $-7.491 \div 0.06$

g $4.872 \div (-1.5)$ **h** $2.808 \div 0.12$ **i** $-0.1638 \div (-0.009)$

7 Calculate the result of each division problem in question **6**.

8 Calculate:

a $(15.73 + 2.9) \times 1.5 - 26.07$ **b** $72.816 \div 8 + 4.9 \times 0.5$

c $6.9 + 0.528 \div 0.04 - 9.3$ **d** $12.08 - \dfrac{3.4 + 0.5}{4} + 8.217$

e $-3.2 \times (-9.51) + (-91.5) \div 1.5$ **f** $\dfrac{0.31 \times 0.031}{5} - \dfrac{5}{-0.25 \times (-0.4)}$

PROBLEM SOLVING AND REASONING

9 Calculate the cost of each item. Write your answers correct to the nearest cent.



a 4 kg of frozen berries at $7.35 per kg

b 3 kg of watermelon at $1.95 per kg

c 0.6 kg of snow peas at $5.62 per kg

d 2.5 kg of potatoes at $3.95 per kg

e 0.55 kg of bok choy at $8.99 per kg

f 1.74 kg of green beans at $6.89 per kg

10 a Estimate the total cost of the items listed in question **9** by first rounding the cost per kg of each item to the nearest dollar. Round your answer to the nearest dollar.

b Find the total cost of these items. Round your answer to the nearest five cents.

c How much change would you receive if you paid with a $100 note?

11 Calculate the following products and quotients by using multiplication and division of fractions.

For example, $12.3 \times 0.02 = \dfrac{123}{10} \times \dfrac{2}{100} = \dfrac{246}{1000} = 0.246$ and $0.02 \div 0.4 = \dfrac{2}{100} \div \dfrac{4}{10} = \dfrac{2}{100} \times \dfrac{10}{4} = \dfrac{1}{20} = \dfrac{5}{100} = 0.05$

a -0.17×0.0032 **b** $9.999 \times (-5.91)$ **c** $-0.003 \div (-7.5)$ **d** $-9.6 \div 0.0012$

12 a Calculate the cost of one item in each case.

i nine bread rolls cost $5.40 **ii** four cartons of milk cost $11.56 **iii** five DVDs cost $94.75

b Calculate the cost of 1 kg in each case.

i 3 kg of tomatoes costs $13.95 **ii** 7 kg of sausages costs $47.95 **iii** 6 kg of dog food costs $81.66

c Calculate the cost of 100 g in each case.

i 200 g of salami costs $25.90 **ii** 500 g of cheese costs $7.95 **iii** 300 g of cashews costs $5.85.

13 A box of 10 coloured pencils costs $7.50. The pencils can also be bought individually for 72 cents each.



a Find the cost per pencil in each case.

b Which option provides better value (or is the 'best buy')? Explain.

14 William bought a 400 g pack of mushrooms for $3.69. Kate bought 300 g of loose mushrooms for $2.98. In each of these calculations, round your answer to the nearest cent.



a Calculate the amount paid by each of them for 100 g of mushrooms.

b Who made the best buy? Explain.

c Why does a store offer both options?

15 We can evaluate powers of decimals by using repeated multiplication of decimals. However, evaluating the power of a decimal can be made easier by first writing the decimal as a fraction. For example,

$(-1.2)^2 = \left(-\frac{12}{10}\right)^2 = \left(-\frac{12}{10}\right) \times \left(-\frac{12}{10}\right) = \frac{144}{100} = 1.44$ and

$(-0.2)^3 = \left(-\frac{2}{10}\right)^3 = \left(-\frac{2}{10}\right) \times \left(-\frac{2}{10}\right) \times \left(-\frac{2}{10}\right) = -\frac{8}{1000} = -0.008$

Evaluate the following powers.

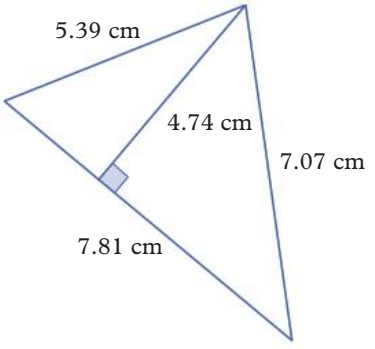
a $(-0.4)^3$ **b** $(-0.001)^3$ **c** $(-0.09)^2$ **d** $(2.5)^5$

16 Write the following division problems as multiplication by the reciprocal, then evaluate the quotient.

a $6.3 \div 0.5$ **b** $9.2 \div 0.2$ **c** $0.49 \div 0.25$

d $7.4 \div 1.25$ **e** $0.041 \div 2.5$ **f** $10.21 \div 6.25$

17 Calculate the area and perimeter of the triangle shown.



18 Use the digits 0 to 9 no more than once each to make the result of this calculation as close to 0 as possible.

□.□□ − □.□□ × □.□□

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Multiplying and dividing decimals

Investigation
Let's party!

Topic quiz
2F

PROBLEM SOLVING AND REASONING

CHALLENGE

2G Terminating, non-terminating and recurring decimals

Learning intentions

By the end of this topic you will be able to ...

- ✔ identify decimals as terminating, non-terminating and recurring
- ✔ convert fractions to recurring decimals
- ✔ write non-terminating and recurring decimals using appropriate notation.

Inter-year links

Support	Fractions, decimals and percentages
Year 7	4G Fractions, decimals and percentages

Terminating decimals

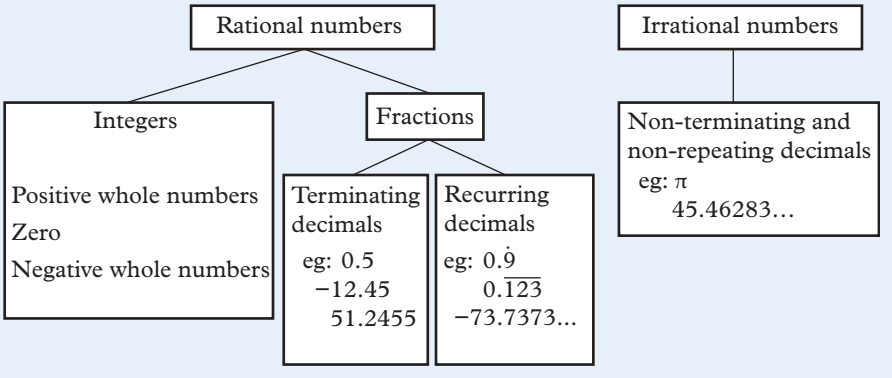
- A **terminating decimal** 'terminates' or stops after a number of decimal places.
  For example: 24.3, 0.49, 5.768234

Non-terminating decimals

- A **non-terminating decimal** has an endless (infinite) number of decimal places.
  For example: 17.32635…, 0.567743…, 0.788888…
  → The ellipses (three full stops) written after the last decimal place indicate that an endless number of digits follow.

Recurring decimals

- A **recurring decimal** is a special type of non-terminating decimal. The digits after the decimal point form a recurring or repeating pattern.
  For example: 8.999999…, 42.671671671…, 0.582323232…
- A recurring decimal can be written in an abbreviated form using dots or a vinculum (bar) over repeating digits.
  → If a single digit is repeated, place a dot over the digit that repeats forever.
  For example, $8.999999... = 8.\dot{9}$
  → If more than one digit is repeated, place a dot over the first and last digits in the repeating pattern, or place a bar over the digits in the repeating pattern.
  For example, $0.58232323… = 0.58\dot{2}\dot{3}$ and $42.671671… = 42.\overline{671}$
- **Rational numbers** are numbers that can be written exactly as fractions. This includes whole numbers, fractions, terminating decimals and recurring decimals.
- **Irrational numbers** are numbers that cannot be written exactly as fractions. These are non-terminating decimals that are not recurring decimals; for example, π (or '**pi**').



Example 2G.1 Writing recurring decimals in abbreviated form

Write each recurring decimal in abbreviated form using dots or a bar.

a 7.682222222…

b −24.503150315031…

THINK

a Look for a repeating pattern of digits. Since there is only one digit that repeats, show a dot (or a bar) above the 2.

b Look for a repeating pattern of digits. Show a bar above 5031 (or show a dot above the first and last digit in the pattern).

WRITE

a 7.682222222…
$= 7.68\dot{2}$

b −24.503150315031…
$= -24.\overline{5031}$ (or $-24.\dot{5}03\dot{1}$)

Example 2G.2 Writing recurring decimals to a number of decimal places

Write each recurring decimal to show 12 decimal places.

a $0.\dot{4}$

b $-159.8\overline{372}$

THINK

a 1 Identify the repeating digit. The digit 4 has a dot above it so it is a repeating digit.

2 Write the decimal to twelve decimal places and indicate that more digits follow.

b 1 Identify the repeating digits. The digits 372 have a bar above them, so all three digits are repeating.

2 Write the decimal to twelve decimal places and indicate that more digits follow.

WRITE

a $0.\dot{4}$
$= 0.444444444444…$

b $-159.8\overline{372}$
$= -159.837237237237…$

Example 2G.3 Expressing fractions as decimals

Convert $\frac{5}{3}$ to a decimal by dividing the numerator by the denominator.

THINK

a 1 Write the fraction as a division problem with a decimal point after the numerator.

2 Perform short division and add trailing zeros until a pattern occurs. Make sure the decimal points are aligned.

3 Write the decimal using the abbreviated form with a dot (or a bar) above 6.

WRITE

$\frac{5}{3} = 5 \div 3$

$$\begin{array}{r} 1.\ 6\ 6\ 6... \\ 3\overline{)5.^{2}0^{2}0^{2}0^{2}0} \end{array}$$

$\frac{5}{3} = 1.\dot{6}$

Example 2G.4 Expressing mixed numbers as decimals

Convert $-1\frac{2}{7}$ to a decimal by first converting to an improper fraction and then dividing the numerator by the denominator.

THINK

1. Convert the mixed number to an improper fraction.
2. Write the fraction as a division problem with a decimal point after the numerator.
3. Perform short division and add trailing zeros until a pattern occurs. Make sure the decimal points are aligned.
4. Write the decimal using the abbreviated form with a bar above the six recurring digits.

WRITE

$-1\frac{2}{7} = -\frac{9}{7}$

$= -9 \div 7$

$$\begin{array}{r} -1.\ 2\ 8\ 5\ 7\ 1\ 4\ 2\ldots \\ 7\overline{)-9.^{2}0^{6}0^{4}0^{5}0^{1}0^{3}0^{2}0^{6}0} \end{array}$$

$-1\frac{2}{7} = -1.\overline{285714}$

Helpful hints

- ✔ There are only two options for fractions converted to decimals. Either they terminate or they are recurring. There are no fractions with an integer numerator and denominator that do not terminate and have no recurring pattern. Such decimals can never be written as fractions.
- ✔ Some people think that decimals are more 'exact' than fractions. However, mathematicians prefer fractions as you cannot tell if a decimal has been rounded, truncated or only measured to a number of decimal places.
- ✔ Zero is a rational number as $0 = \frac{0}{1}$. In fact, all integers are rational numbers.
- ✔ There is no limit on the number of decimals that can make a repeating pattern in a recurring decimal. For example, $\frac{1}{7} = 0.\overline{142857}$ has a pattern of six repeating digits.

ANS p501

Exercise 2G Terminating, non-terminating and recurring decimals

1–3, 4–5(a–h), 6, 7(a, c, f), 8–10, 11(a–f), 12, 14, 15, 18	3, 4–5(i–l), 6–8, 9(d–f), 10, 11(g–k), 12, 13, 16–19	3, 4(j–l), 5(i–o), 6–7(f, g, h), 9(e, f), 10, 11(g, h, k, l), 12, 16–20

1 State whether each decimal is terminating (T) or non-terminating (N).

a 4.57235…	**b** 56.803	**c** 21.57777…	**d** −8.56139562
e −0.45173069…	**f** 17.5656565…	**g** −0.123412341234…	**h** −0.78787878
i −30.082174…	**j** 291.2912912	**k** 0.999…	**l** −5.043127

2 Which of the numbers in question **1** are recurring decimals?

3 Explain the differences between terminating, non-terminating and recurring decimals.

2G.1 **4** Write each recurring decimal in abbreviated form using dots or a bar.

a 0.999999… **b** −4.3636363… **c** 87.255555… **d** 3.476476476…
e −0.738222222… **f** 16.19666666… **g** 483.88888… **h** −6.729191919…
i −0.18367367367… **j** 1.356283562835… **k** 28.7070707… **l** −9.23071444444…

2G.2 **5** Write each recurring decimal to show twelve decimal places.

a $0.\dot{1}$ **b** $5.3\dot{7}$ **c** $-14.\overline{82}$
d $-21.5\dot{6}\dot{9}$ **e** $8.\overline{104}$ **f** $0.\overline{6513}$
g $-232.68\dot{4}$ **h** $44.761\ \overline{359}$ **i** $-94.57\dot{0}\dot{2}$
j $-1.\overline{73924}$ **k** $11.812\dot{1}$ **l** $-6.9\overline{04}$
m $14.2\dot{4}1\dot{3}$ **n** $0.5\overline{620}$ **o** $-139.\dot{2}74\dot{5}$

2G.3 **6** Convert each fraction to a decimal by dividing the numerator by the denominator.

a $\frac{7}{10}$ **b** $-\frac{1}{2}$ **c** $\frac{1}{3}$ **d** $\frac{5}{4}$
e $\frac{2}{11}$ **f** $-\frac{5}{8}$ **g** $\frac{8}{3}$ **h** $-\frac{3}{4}$

2G.4 **7** Convert each fraction to a decimal.

a $-1\frac{1}{8}$ **b** $1\frac{2}{5}$ **c** $2\frac{5}{6}$ **d** $-1\frac{1}{7}$
e $3\frac{3}{5}$ **f** $-4\frac{1}{9}$ **g** $1\frac{8}{11}$ **h** $-2\frac{5}{12}$

8 Consider the fractions and your answers to question **6**.

a Which fractions produce:
i terminating decimals
ii non-terminating decimals?
b Which fractions produce recurring decimals?
c Do any of the fractions produce a non-terminating decimal that isn't a recurring decimal?

9 Write each number as a fraction. Hint: Whole numbers can be written with a denominator of 1.

a 0.5 **b** 4 **c** 6.2
d 0.37 **e** 18 **f** 2.509

10 Numbers that can be written exactly as fractions are rational numbers.

a Use your answers to question **9** to explain why whole numbers and terminating decimals are rational numbers.
b Are recurring decimals rational numbers? Use your answers to questions **6** and **8** to help you decide. Explain your decision.

11 Classify each number as rational or irrational.

a 4.6 **b** 0.12121212… **c** 0.5871 **d** 2.35789621…
e 35 **f** 734.94651824… **g** $0.872\ \overline{503}$ **h** 96.73425
i $\frac{3}{17}$ **j** 500 000.000… **k** 0.117 843 26… **l** $10\frac{3}{4}$

12 An irrational number that is commonly used is π (read as pi). This number has intrigued mathematicians for thousands of years and is related to measurements on a circle.
Since it is a decimal with an endless number of decimal places and no obvious repeating pattern, an estimate or approximation of its value is used in calculations.

a Find the symbol π on your calculator. Press the appropriate keys to obtain a decimal approximation for π. Write the number shown on your calculator screen.
b Write an approximation for π to the following number of decimal places.
i two **ii** four **iii** five **iv** six

13 It is possible to determine if a fraction will have a recurring decimal or not without performing the division.

a Complete the table below.

Unit fraction	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{11}$	$\frac{1}{4}$	$\frac{1}{20}$	$\frac{1}{15}$	$\frac{1}{8}$	$\frac{1}{12}$
Decimal	$0.\dot{1}$	0.1	0.2								
Denominator's prime factorisation	3^2	2×5	5								
Is 2 and/or 5 a factor of the denominator?	No	Yes									
Any prime factors other than 2 and/or 5?	Yes	No									
Is the decimal terminating?	No										

b According to the table, when will a unit fraction have a terminating decimal?

c Use your rule from part **b** to decide if these unit fractions will have a terminating decimal or not. Convert each simplified fraction to a decimal to check your answers.

i $\frac{1}{25}$ **ii** $\frac{1}{24}$ **iii** $\frac{1}{18}$ **iv** $\frac{1}{40}$ **v** $\frac{1}{125}$

d Your rule from part **b** extends to all simplified fractions. Use the rule to decide if these fractions will have a terminating decimal or not. Remember to simplify your fractions first! Convert each simplified fraction to a decimal to check your answers.

i $\frac{24}{7}$ **ii** $\frac{7}{32}$ **iii** $\frac{21}{30}$ **iv** $\frac{4}{26}$ **v** $\frac{42}{15}$

14 Another irrational number is $\sqrt{2}$.

a Write the number shown on your calculator screen for $\sqrt{2}$.

b Write an approximation for $\sqrt{2}$ to the following number of decimal places.

i two **ii** four **iii** six **iv** seven

c Write the number shown on your calculator screen for $\sqrt{3}$.

d Is $\sqrt{3}$ an irrational number?

e Write an approximation for $\sqrt{3}$ to the following number of decimal places.

i two **ii** four **iii** five **iv** six

15 a Convert each fraction to a decimal.

i $\frac{1}{9}$ **ii** $\frac{2}{9}$ **iii** $\frac{3}{9}$ **iv** $\frac{4}{9}$

b What type of decimal do you get?

c Use the pattern from part **a** to write these as decimals.

i $\frac{5}{9}$ **ii** $\frac{6}{9}$ **iii** $\frac{7}{9}$ **iv** $\frac{8}{9}$

16 Other fractions, both with recurring and non-recurring decimal expansions, have patterns in their decimal expansion.

a Write down the first ten multiples of 9.

b Convert each fraction to a decimal.

i $\frac{1}{11}$ **ii** $\frac{2}{11}$ **iii** $\frac{3}{11}$ **iv** $\frac{4}{11}$

c Describe the pattern in the decimal expansion of the elevenths. Check your pattern holds by writing and then checking the decimal expansion of $\frac{9}{11}$.

17 a Write $\frac{1}{3}$ and $\frac{2}{3}$ as decimals, showing at least the first five decimal places.

b Add the two decimals you found in part **a**.

c Calculate $\frac{1}{3} + \frac{2}{3}$ and compare your answer to part **b**.

18 Before the use of calculators and computers, many mathematicians used a fraction as an estimate for the value of π. Some examples are shown in the table that follows. See some of the estimations in the table below.

Place, person and era of origin	Fraction estimate	Decimal approximation (to six decimal places)
Ancient Egypt (1700 BCE)	$\frac{256}{81}$	
Ancient Greece, Archimedes (287–212 BCE)	$\frac{223}{71}$	
	$\frac{22}{7}$	
China, Zu Chongzhi (429–500 CE)	$\frac{355}{113}$	
India, Bhāskara II (1114–1185 CE)	$\frac{3927}{1250}$	
Italy, Leonardo of Pisa (1170–1250 CE)	$\frac{12\,960}{4126}$	
	$\frac{7200}{2291}$	

a Use a calculator to complete the third column of the table.

b Which value is closest to the approximation of π you wrote for question **12b** part **iv**?

19 Archimedes proved that π was between $\frac{223}{71}$ and $\frac{22}{7}$. Can you use a computer to write the *exact* value of π as a fraction or a decimal? Explain.

20 Recurring decimals are able to be written as fractions. For example, $0.\dot{6}$ is equivalent to $\frac{2}{3}$. You can verify this using short division or a calculator. To write a recurring decimal as a fraction, there is a series of steps you can follow.

a In the calculation below, match each step (**1–6**) with its corresponding line of working (**A–F**) and then write the lines of working in the correct order.

Steps:

1 Let x represent the recurring decimal.

2 Multiply the recurring decimal by 10 to find an equation for $10x$.

3 Form a new subtraction equation from the two equations for x and $10x$.

4 Write the simplified equation that results from the subtraction.

5 Divide both sides of the equation by 9 to obtain x on the left-hand side of the equation and a fraction on the right-hand side.

6 Write the recurring decimal as a fraction, in simplest form.

Lines of working:

A $0.\dot{6} = \frac{2}{3}$

B $x = \frac{6}{9}$

C $10x = 6.666\,666\,\ldots$

D $x = 0.666\,666\,\ldots$

E $10x - x = 6.666\,666\,\ldots - 0.666\,666\,\ldots$

F $9x = 6$

b Why did you multiply x by 10 in the calculation above? How did this help you obtain a whole number on the right-hand side of the equation in step **4**?

c Use the method in part **a** to write each of these recurring decimals as a faction.

i $0.\dot{2}$ **ii** $0.\dot{7}$ **iii** $2.\dot{3}$ **iv** $5.\dot{6}$

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

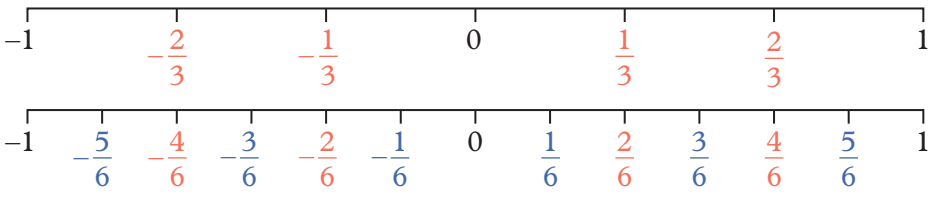
Interactive skillsheet
Terminating, non-terminating and recurring decimals

Investigation
Let the cycle begin!

Topic quiz
2G

Chapter summary

Equivalent fractions



$\frac{2}{3} = \frac{10}{15}$ (×5) $\qquad \frac{12}{10} = \frac{6}{5}$ (÷2)

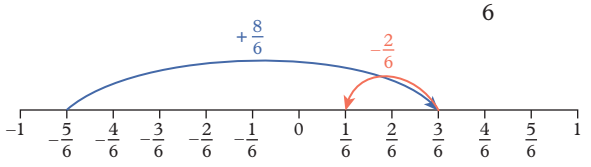
The HCF of 36 and 72 is 36.

$$\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}} = \frac{1}{2}$$

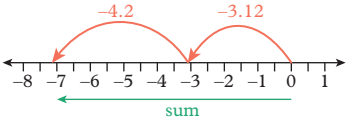
Adding and subtracting fractions

1. Convert into equivalent fractions with a common denominator.
2. Add or subtract the numerators and keep the same denominator.

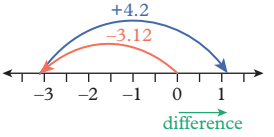
$$-\frac{5}{6} + \frac{4}{3} - \frac{1}{3} = -\frac{5}{6} + \frac{8}{6} - \frac{2}{6}$$
$$= \frac{1}{6}$$



Adding and subtracting decimals



$$-\left(\begin{array}{r} 3.12 \\ +\ 4.20 \\ \hline 7.32 \end{array}\right)$$



$$+\left(\begin{array}{r} {}^{1}\ {}^{10} \\ 4.\cancel{2}\,\cancel{0} \\ -\ 3.1\ 2 \\ \hline 1.0\ 8 \end{array}\right)$$

Fractions to decimals

Align decimal points

$$\text{Dividend} \rightarrow \frac{1}{8} = 8\overline{)1.0^{2}0^{4}0} \leftarrow \text{Divisor}$$

0.1 2 5

Trailing zeros

Multiplying fractions

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times \cancel{3}^{1}}{\cancel{3}^{1} \times 4}$$
$$= \frac{\cancel{2}^{1}}{\cancel{4}^{2}}$$
$$= \frac{1}{2}$$

Diagonally

Vertically

Multiplying decimals

2.75 ← two decimal places
×0.03 ← two decimal places
= four decimal places

$$\begin{array}{r} {}^{2}\ {}^{1} \\ 275 \\ \times\ \ 3 \\ \hline 825 \end{array}$$

2.75 × 0.03 = 0.0825

Dividing by a whole number

$$9\overline{)172.44} = 19.16$$

quotient (the answer)

divisor (the number we are dividing by)

dividend (the number being divided)

Reciprocals

$\frac{3}{4}$ and $\frac{4}{3}$

Dividing fractions

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$$

Multiplying and dividing by 10s

5.23 × 10 = 52.3
5.23 × 100 = 523.0
5.23 ÷ 10 = 0.523
5.23 ÷ 100 = 0.0523

Lowest common denominators

The LCM of 2 and 5 is 10.

$\frac{1}{2} = \frac{5}{10}$ (×5) $\qquad \frac{3}{5} = \frac{6}{10}$ (×2)

Dividing by a decimal

Dividend Divisor

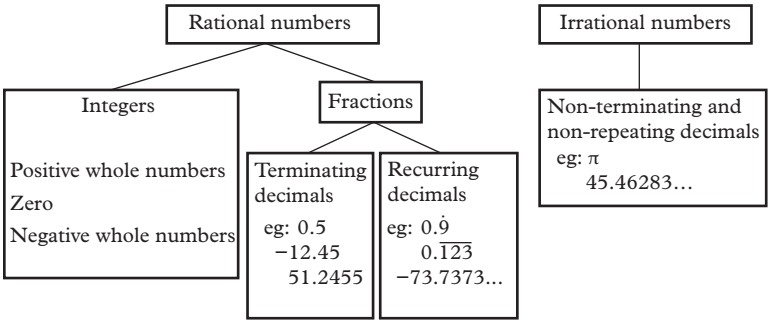
$$1.2 \div 0.4 = 12 \div 4 = 3$$

Improper fractions and mixed numbers

$$3\frac{1}{4} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$$

$$\frac{13}{4} = 13 \div 4 = 3 \text{ remainder } 1$$
$$= 3\frac{1}{4}$$

Rational and irrational numbers



Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Multiple choice

2A **1** What missing value in $\frac{7}{9} = \frac{49}{\square}$ will produce an equivalent fraction?

A 51 **B** 63 **C** 81 **D** 441 **E** 7

2A **2** Which of the following is incorrect?

A $\frac{5}{3} > \frac{4}{3}$ **B** $\frac{3}{5} < \frac{3}{4}$ **C** $\frac{2}{3} > \frac{32}{47}$ **D** $\frac{7}{9} < \frac{9}{11}$ **E** $\frac{9}{8} > \frac{13}{12}$

2B **3** What is the lowest common denominator for $\frac{3}{8} - \frac{1}{6}$?

A 48 **B** 16 **C** 24 **D** 2 **E** 18

2B **4** What is $\frac{1}{5} - \frac{4}{5}$?

A -3 **B** $-\frac{3}{5}$ **C** $\frac{3}{5}$ **D** $\frac{5}{5}$ **E** $-\frac{2}{5}$

2B/2C **5** Which problem has a positive answer?

A $-\frac{2}{3} + \left(-\frac{3}{4}\right)$ **B** $-\frac{2}{5} \times \frac{1}{2}$ **C** $-\frac{5}{2} - \left(-\frac{6}{7}\right)$ **D** $-\frac{4}{5} - \left(+\frac{2}{3}\right)$ **E** $-\frac{1}{7} \div \left(-\frac{8}{9}\right)$

2C **6** What is $\frac{9}{10} \div 1\frac{1}{5}$?

A $\frac{9}{2}$ **B** $\frac{3}{4}$ **C** $\frac{27}{25}$ **D** $\frac{21}{10}$ **E** $\frac{4}{3}$

2D **7** What is $-3.125\,84$ rounded to three decimal places?

A -3.125 **B** -3.13 **C** -3.128 **D** -3.126 **E** -3.584

2E **8** What is $-5 + 2.38$?

A -7.38 **B** -7.62 **C** -2.62 **D** -2.38 **E** -3.38

2F **9** How many decimal places are there in the answer to 26.018×15.4?

A 1 **B** 3 **C** 2 **D** 8 **E** 4

2G **10** Which number is an example of a recurring decimal?

A 32.85

B $-7.757\,575$

C $-6.666...$

D $3.654\,864\,321...$

E $-0.121\,232\,123\,432...$

2G **11** Which of the following is *not* equivalent to $\frac{5}{33}$?

A $0.\overline{15}$ **B** $0.1\dot{5}$ **C** $0.15\dot{}$ **D** $\frac{15}{99}$ **E** $0.15\overline{1}$

Short answer

2A **1** Write each fraction in its simplest form.

a $\frac{35}{60}$ **b** $-\frac{30}{16}$ **c** $-3\frac{18}{32}$ **d** $5\frac{48}{72}$

2B **2** Calculate:

a $\frac{3}{7} + \frac{2}{5}$ **b** $\frac{11}{15} - \frac{3}{10}$ **c** $5\frac{2}{3} + \frac{4}{11}$ **d** $6\frac{2}{5} - 3\frac{4}{7}$

2C **3** Calculate:

a $\frac{12}{25} \times \frac{5}{14}$ **b** $\frac{7}{18} \div \frac{7}{9}$ **c** $6\frac{1}{4} \times 3\frac{2}{5}$ **d** $3\frac{1}{2} \div 1\frac{5}{9}$

2C **4** Calculate:

a $-\frac{10}{9} \times \left(-\frac{27}{35}\right)$ **b** $\frac{15}{11} \div \left(-\frac{90}{44}\right)$ **c** $-3\frac{5}{6} \times 8\frac{2}{3}$ **d** $-2\frac{4}{5} \div \left(-5\frac{4}{9}\right)$

2B/2C **5** Calculate:

a $-\frac{3}{4}-\left(-\frac{1}{2}\right)$ **b** $\frac{2}{5}-\left(+\frac{1}{3}\right)$ **c** $\frac{3}{7}-1\frac{2}{7}$

d $-2\frac{1}{6}+6\frac{2}{3}$ **e** $\frac{1}{2}\div\frac{7}{4}-\frac{2}{5}$ **f** $-\frac{1}{8}+\frac{2}{3}\times\left(-\frac{9}{8}\right)$

2C **6** Find:

a $\frac{2}{5}$ of 60 km/h

b $\frac{1}{2}$ of $\frac{2}{3}$ of a cup

c $3\frac{1}{3}$ of 21 metres

d $\frac{18}{60}$ of 10 minutes

2D **7** Write in ascending order.

a 0.123, 0.120, 0.213, 0.0012, 0.231, 0.2

b 8.916, 9.6, 8.16, 8.9, 9.16, 9.916

c 0.05, 5, −0.5, −5, 0.50, −0.05

d −0.421, −2.41, −10.42, −14.2, −12.04, −2.14

2D **8** Write in descending order.

a 0.017, 0.7, 0.71, 0.0701, 0.717, 0.107

b 2.127, 2.1, 2.721, 2.27, 2.72, 2.7

c −0.89, 89, 90, −9.8, −0.899, 80.9

d −0.9, −1, −0.01, −0.99, −0.099, −0.1

2D **9** Round each decimal to the number of decimal places shown in brackets.

a 13.216 51 (2) **b** −0.001 215 (4)

c 54.545 545 (3) **d** −181.182 154 136 (1)

2E **10** Calculate:

a 23.56 + 45.98 + 74.13

b 53.15 − 29.67

c 123.5 + 46.12 + 9.0142 + 87.15

d 1284.068 − 294.4687

2E **11** Calculate:

a −5.42 + 1.59

b −8.30 − (−12.482)

c 49.207 + (−51.52)

d 99.29 − (−15.490)

2F **12** Calculate:

a 6.3128 × 4 **b** 65.589 ÷ (−6)

c −3.46 × 5.7 **d** −468.268 × (−0.89)

2G **13 a** Decide whether each decimal is terminating or non-terminating.

i 0.123123123

ii 6.562562...

iii −5.5

iv 3.645841235...

b Which of the numbers in part **a** are recurring decimals?

2G **14** Write each decimal in short form using dots or a horizontal bar.

a 0.333333...
b 5.385385...
c −45.685757575...
d −0.092451451...

2G **15** Write each number to show twelve decimal places.

a $-0.2\dot{3}$
b $3.\overline{27}$
c $46.35\dot{8}\dot{7}$
d $-17.36\overline{485}$

2G **16** Write each fraction as a recurring decimal using dots or a horizontal bar.

a $\frac{1}{30}$
b $-\frac{4}{7}$
c $-9\frac{5}{9}$
d $\frac{17}{11}$

Analysis

1 The daily minimum and maximum temperatures at a ski resort were recorded over a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Min. temp. (°C)	0.7	−2.2	−0.9	−3.4	−5.1	−1.8	0.2
Max. temp. (°C)	6.5	4.1	7.8	2.5	−0.6	3.9	5.3



a Which day had:
  i the highest temperature
  ii the lowest temperature?

b Calculate the difference between the minimum and maximum temperatures for Monday.

c Calculate the difference between the minimum and maximum temperatures for each other day.

d Which day had the biggest range of temperatures?

e Calculate the average of the following, correct to one decimal place.
  i the minimum temperatures
  ii the maximum temperatures

f What is the difference between the average minimum and average maximum temperatures?

2 A school has four house teams for sports events: Alpha, Beta, Gamma, and Delta, and every student is assigned into one of the four houses. The school tries to have as close to one-quarter of students in each house every year. In 2021, the school had 1596 students.



a In 2021, Alpha house had exactly $\frac{1}{4}$ of the students in the school. How many students were in Alpha house?

b In 2021, Beta house had $\frac{9}{38}$ of the students in the school.
  i How many students were in Beta house?
  ii If $\frac{11}{21}$ of Beta house are girls, how many girls are in Beta house?

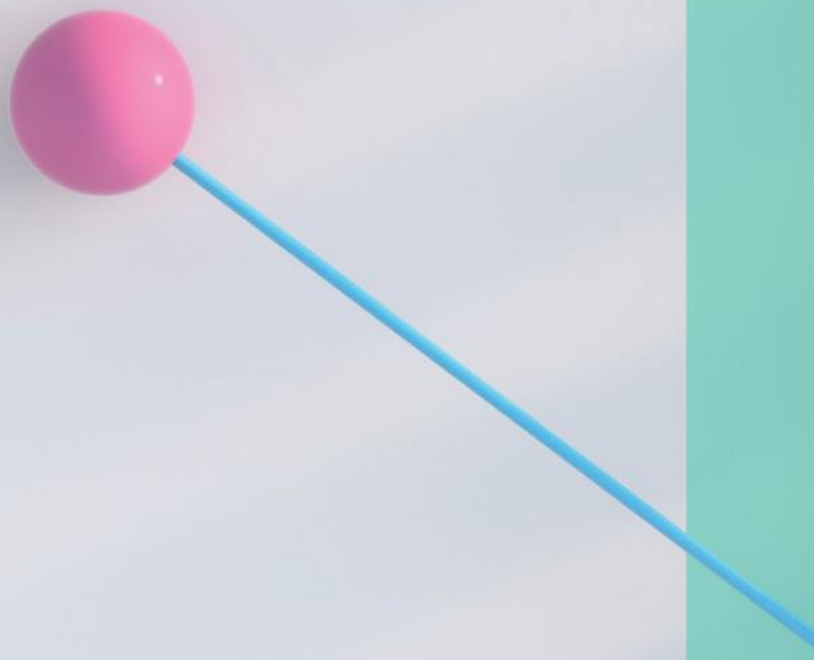
c In 2021, Gamma house had 402 students.
  i What fraction of students were in Gamma house?
  ii How many students did Gamma house have above or below one-quarter of the students at the school?

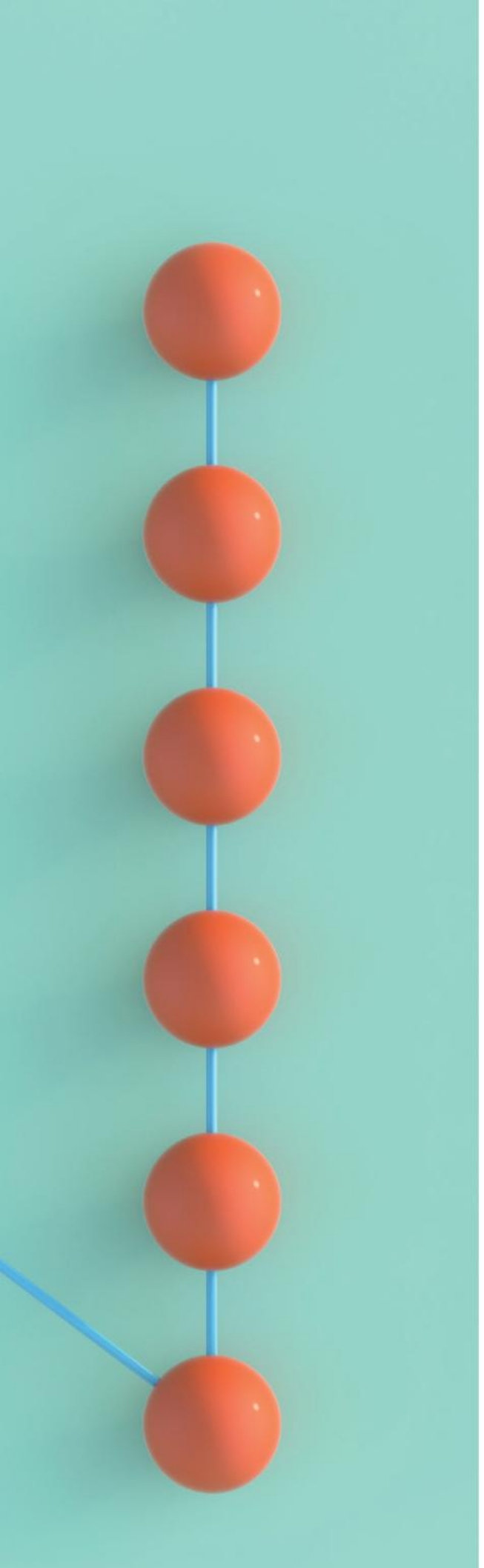
d In 2021,
  i how many students were in Delta house?
  ii what fraction of the school were in Delta house?

e In 2022, Delta house had 416 students. If Delta house had $\frac{13}{50}$ of the students at the school that year, how many students went to the school in 2022?

3

Percentages, ratios and rates





Index

3A Percentages, decimals and fractions
3B Calculating percentages
3C Mark-ups and discounts
3D Ratios
3E Equivalent ratios
3F Dividing a quantity in a given ratio
3G Rates

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Visual representation of fractions
- ✔ Equivalent fractions
- ✔ Multiplying and dividing fractions
- ✔ Multiplying and dividing decimals
- ✔ Rounding decimals

Curriculum links

- Use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts; formulate problems, choosing efficient calculation strategies and using digital tools where appropriate; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model (AC9M8N05)
- Recognise and use rates to solve problems involving the comparison of 2 related quantities of different units of measure (AC9M8M05)
- Use mathematical modelling to solve practical problems involving ratios and rates, including financial contexts; formulate problems; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model (AC9M8M07)

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Materials

- ✔ Calculator

3A Percentages, decimals and fractions

Learning intentions

By the end of this topic you will be able to ...

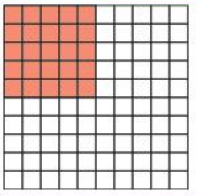
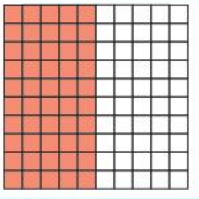
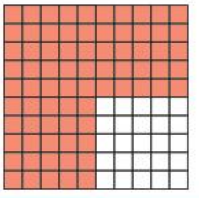
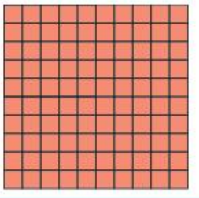
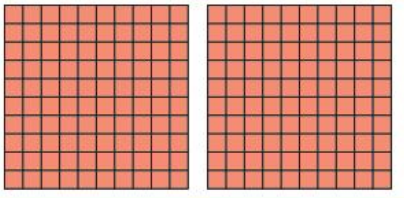
- ✔ explain that percentage means out of 100
- ✔ convert between percentages, fractions and decimals
- ✔ convert between a fractional percentage, fractions and decimals.

Inter-year links

Support	Fractions, decimals and percentages
Year 7	4G Fractions, decimals and percentages

Percentages, fractions and decimals

- A **percentage** is a proportion of a whole that has been divided into one hundred parts.
- The term 'per cent' means 'per hundred' or 'out of 100'. The symbol for percentage is %.
- Percentages, fractions and decimals are just different ways of representing the same value.

Percentage	Fraction	Decimal	
25%	$\frac{25}{100} = \frac{1}{4}$	0.25	
50%	$\frac{50}{100} = \frac{1}{2}$	0.5	
75%	$\frac{75}{100} = \frac{3}{4}$	0.75	
100%	$\frac{100}{100} = 1$	1	
200%	$\frac{200}{100} = 2$	2	

	Fraction	Decimal
Percentage to…	Write the percentage as a fraction with a denominator of 100. $15\% = \frac{15}{100}$ $= \frac{3}{20}$	Divide the percentage by 100. $23.4\% = \frac{23.4}{100}$ $= 23.4 \div 100$ $= 0.234$

	Percentage	Decimal
Fraction to…	Multiply the fraction by 100% and simplify. $\frac{1}{3} \times 100\% = \frac{100}{3}\%$ $= 33\frac{1}{3}\%$	Divide the numerator by the denominator. Add trailing zeros as required. Align decimal points Dividend → 1, Divisor → 8 $\frac{1}{8} = 8\overline{)1.0^{2}0^{4}0}$ → 0.1 2 5 Trailing zeros

	Percentage	Fraction
Decimal to…	Multiply the decimal by 100 and include the percentage symbol %. $0.65 = 65\%$	Place the decimal as the numerator of the fraction and the denominator 10, 100, 1000 … with as many zeros as there are digits after the decimal point. $0.65 = \frac{65}{100}$

Example 3A.1 Converting from a percentage

Write each percentage as a fraction in its simplest form and as a decimal.

a 40% **b** 220% **c** 25.3%

THINK

1. To convert a percentage to a fraction, divide by 100 and simplify the fraction.
2. To write a percentage as a decimal, we divide the percentage by 100.

WRITE

a Fraction: $40\% = \frac{\cancel{40}^{2}}{\cancel{100}^{5}}$

$= \frac{2}{5}$

Decimal: $40\% = 0.4$

b Fraction: $220\% = \frac{\cancel{220}^{11}}{\cancel{100}^{5}}$

$= \frac{11}{5}$

Decimal: $220\% = 2.2$

c Fraction: $25.3\% = \frac{25.3}{100}$

$\frac{25.3}{100} \overset{\times 10}{=} \frac{253}{1000}$ ($\times 10$ above and below)

Decimal: $25.3\% = 0.253$

Example 3A.2 Converting from a decimal

Write each decimal as a percentage and as a fraction in its simplest form.

a 0.86 **b** 3.2 **c** 0.0125

THINK

1 To convert a decimal to a percentage, multiply by 100%. 'Move' the decimal point two places to the right and insert a place-holding zero in the empty space if necessary.

2 To convert a decimal to a fraction, place the decimal fraction part of the number as the numerator of the fraction. Make the fraction denominator 10, 100, 1000 … with as many zeros as there are digits after the decimal point in the decimal fraction part of the number. Simplify the fraction.

WRITE

a Percentage: $0.86 = 86\%$

Fraction: $0.86 = \frac{\cancel{86}^{43}}{\cancel{100}^{50}}$

$= \frac{43}{50}$

b Percentage: $3.2 = 320\%$

Fraction: $3.2 = 3\frac{\cancel{2}^{1}}{\cancel{10}^{5}}$

$= 3\frac{1}{5}$

c Percentage: $0.0125 = 1.25\%$

Fraction: $0.0125 = \frac{\cancel{125}^{1}}{\cancel{10\,000}^{80}}$

$= \frac{1}{80}$

Example 3A.3 Converting from a fraction

Write each fraction as a percentage and as a decimal.

a $\frac{1}{25}$ **b** $\frac{27}{300}$ **c** $4\frac{2}{3}$

THINK

a 1 Multiply the fraction by 100% and divide the numerator by the denominator to simplify.

2 Convert the fraction to an equivalent fraction with a denominator of 100 and then convert to a decimal by 'moving' the decimal point of the numerator two places to the left.

b 1 Multiply the fraction by 100% and divide the numerator by the denominator to simplify.

2 Convert the fraction to an equivalent fraction with a denominator of 100 and then convert to a decimal by 'moving' the decimal point of the numerator two places to the left.

WRITE

a Percentage: $\frac{1}{25} = \frac{1}{25} \times 100\%$

$= \frac{100}{25}\%$

$= 4\%$

Decimal: $\frac{1}{25} \overset{\times 4}{=} \frac{4}{100}$

$= 0.04$

b Percentage: $\frac{27}{300} = \frac{27}{300} \times 100\%$

$= \frac{2700}{300}\%$

$= 9\%$

Decimal: $\frac{27}{300} \overset{\div 3}{=} \frac{9}{100}$

$= 0.09$

c **1** For mixed numbers, multiply the whole number and the fraction by 100%. Divide the numerator by the denominator to simplify, then add both values together again.

2 Express the mixed number as a whole number plus a fraction. If you cannot easily determine an equivalent fraction with a denominator of 100, then use short division to convert the fraction to a decimal. Add both values together again.

c Percentage: $4\frac{2}{3} = 4\frac{2}{3} \times 100\%$

$= 400\% + \frac{200}{3}\%$

$= 400\% + 66\frac{2}{3}\%$

$= 466\frac{2}{3}\%$

Decimal: $4\frac{2}{3} = 4 + \frac{2}{3}$

$= 4 + 3\overline{)2.^{2}0^{2}0^{2}0}$ (0 . 6 6 6 ...)

$= 4.\dot{6}$

Helpful hints

- ✔ Remember that one whole equals 100%. If you have a percentage that is greater than 100%, then you have more than one whole.
- ✔ It's easier to convert from a fraction to a decimal when the fraction has a denominator of 100, before you have simplified your answer.
- ✔ Don't forget to simplify your fractions.

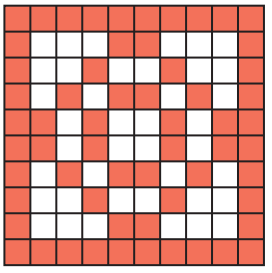
ANS p502

Exercise 3A Percentages, decimals and fractions

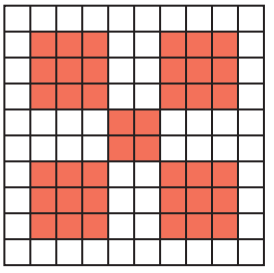
1–12, 13(a–d), 14–16, 17(a–c) | 3(d–f), 4–7, 8(b, d), 9–13(e–h), 14, 17–19, 20(a, b) | 5, 7, 9–13(e–h), 17–22

1 What percentage of each large square has been shaded?

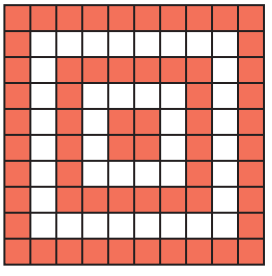
a



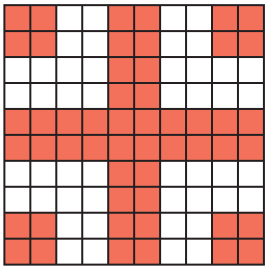
b



c



d



2 Write the percentage of each square in question **1** that is *not* shaded.

3 **a** Huynh has downloaded 76% of a song onto her iPhone. What percentage remains to be downloaded?

b If the selling price of an item is reduced by 23%, what percentage of the original price is to be paid?

c The Brisbane Lions win 17% of their games. What percentage do they lose?

d Forty-two per cent of homes were affected by floods. What percentage of homes were not affected?

e Morgan received 93% on her Japanese test. What percentage of marks did she lose?

f It is estimated that 54% of Australians do no formal exercise on a regular basis. What percentage of Australians exercise on a regular basis?

3A.1 **4** Write each percentage as a fraction in its simplest form and as a decimal.

a 17%	**b** 21%	**c** 35%	**d** 8%
e 215%	**f** 98%	**g** 174%	**h** 28%

5 Write each percentage as a fraction in its simplest form and as a decimal.

a 7%	**b** 3.1%	**c** 162%	**d** 72.1%
e 347%	**f** 81.09%	**g** 10.57%	**h** 6.78%

3A.2 **6** Write each decimal as a percentage and as a fraction in its simplest form.

a 0.71	**b** 0.28	**c** 0.05	**d** 0.08
e 0.9	**f** 0.31	**g** 0.54	**h** 0.95

7 Write each decimal as a percentage and as a fraction in its simplest form.

a 0.369	**b** 0.248	**c** 10.06	**d** 0.81
e 7.057	**f** 1.859	**g** 12.1	**h** 0.2446

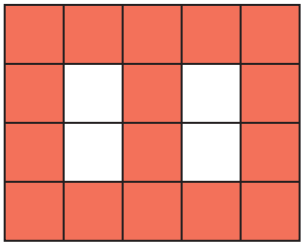
8 For each figure:

i write how many sections are shaded as a fraction of the total number of sections

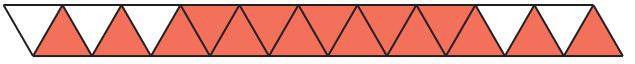
ii write this fraction as an equivalent fraction with a denominator of 100

iii write the percentage of sections that are shaded.

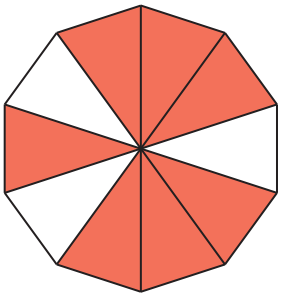
a



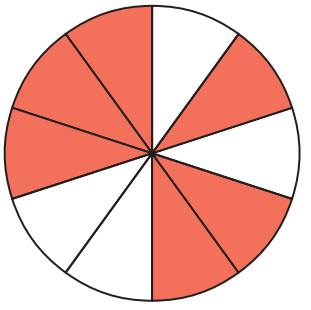
b



c



d



3A.3 **9** Write each fraction as a decimal and percentage by first finding an equivalent fraction that has a denominator of 100.

a $\frac{3}{5}$	**b** $\frac{7}{10}$	**c** $\frac{17}{200}$	**d** $\frac{23}{25}$
e $\frac{13}{10}$	**f** $\frac{3}{2}$	**g** $\frac{11}{400}$	**h** $\frac{7}{2}$

10 Write each mixed number as a percentage. Hint: Whole numbers are multiples of 100%.

a $2\frac{1}{4}$ **b** $6\frac{1}{200}$ **c** $7\frac{3}{10}$ **d** $3\frac{3}{4}$

e $4\frac{7}{25}$ **f** $5\frac{4}{5}$ **g** $1\frac{12}{500}$ **h** $13\frac{13}{50}$

11 Write each fraction as a percentage and a decimal.

a $\frac{5}{8}$ **b** $\frac{27}{40}$ **c** $5\frac{1}{3}$ **d** $\frac{3}{16}$

e $\frac{11}{125}$ **f** $3\frac{2}{9}$ **g** $\frac{11}{15}$ **h** $1\frac{3}{7}$

12 Write each percentage as a fraction in simplest form and as a decimal.

a $\frac{1}{4}\%$ **b** $\frac{11}{3}\%$ **c** $\frac{5}{8}\%$ **d** $2\frac{1}{2}\%$

e $8\frac{4}{5}\%$ **f** $15\frac{2}{3}\%$ **g** $3\frac{5}{9}\%$ **h** $13\frac{1}{6}\%$

13 Write a percentage to describe these situations by first finding an equivalent fraction that has a denominator of 100.

a Andrew ate three out of five cupcakes.

b Nine people out of twenty went home.

c Seventeen people out of twenty-five voted yes.

d Mark knew 46 people out of 200.

e Steffie obtained a score of 17 out of 20 for the quiz.

f The movie critic rated the movie one out of five stars.

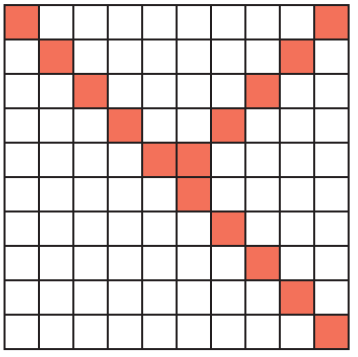
g A dam with a capacity of 600 megalitres currently holds 144 megalitres.

h Seventy-five of the 250 people surveyed were happy with their mobile phone provider.

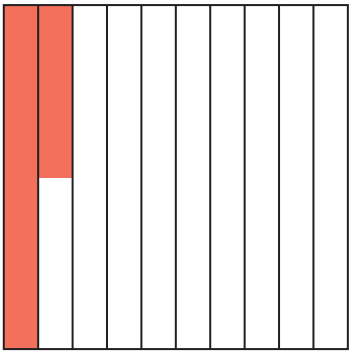
14 What percentage of its original value is a car worth if it depreciates (loses value) by these amounts?

a 20% **b** 19% **c** 8% **d** 12% **e** 28.5% **f** $33\frac{1}{3}\%$

15 Thomas and Matthew were asked to draw a diagram representing 15%. Their answers are shown below.



Matthew



Thomas

a Which diagram is correct? Explain your reasoning.

b Show another way of representing 15%.

16 These containers each contain 50% of their total capacity.





Explain how each container holds a different volume of liquid, but they are the same percentage full.

PROBLEM SOLVING AND REASONING

17 Percentages can be added and subtracted like other numbers. Calculate:

a $17\% + 49\%$ **b** $23\% + 32\frac{1}{2}\%$

c $53\frac{1}{2}\% + 16\frac{1}{2}\% + 9\frac{2}{3}\%$ **d** $7.9\% + 28.7\% + 46.5\%$

e $92\% - 68\%$ **f** $16\frac{4}{5}\% - 12\frac{2}{5}\%$

g $37.4\% + 52.9\% - 15.6\%$ **h** $75.95\% + 5.28\% - 32.13\% - 28.66\%$

18 A packet of gumballs is emptied onto a table. See the gumballs to the right.



a Write the number of blue gumballs as a fraction of the total.

b Convert this to a decimal and round to four decimal places.

c Convert this decimal to a percentage. What percentage of the gumballs are blue?

d What percentage of the gumballs (correct to two decimal places) is:

i orange **ii** green **iii** red or yellow **iv** not red?

19 Marigo received a $5\frac{1}{4}\%$ pay rise. What fraction of her salary does this represent?

CHALLENGE

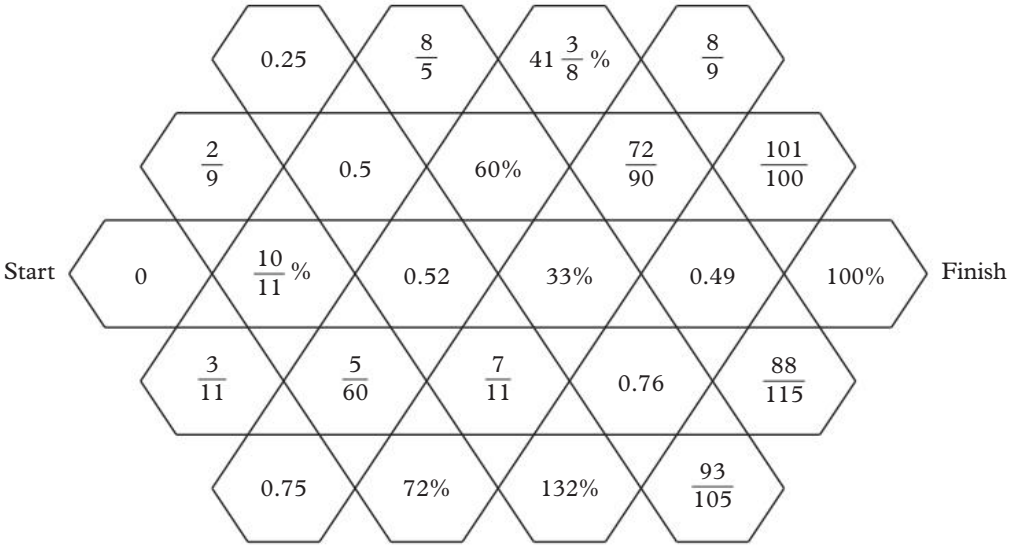
20 For each list, draw a number line and place each number in its correct position.

a $\frac{7}{5}, 0.89, \frac{3}{4}, 200\%, \frac{4}{5}, 1.3$

b $4.9, \frac{13}{2}, 560\%, 7.2, 4\frac{3}{10}, \frac{22}{4}$

c $\frac{16}{5}, \frac{3}{2}, 340\%, 0.28, 1\frac{4}{5}, 65\%$

21 Starting at zero in this diagram, find the shortest path to 100%, moving in ascending order.



22 Showing examples, explain how you could represent the percentages below using:

80% 92% 25% 17% 43% 66%

a a rectangle of length 10 cm and width 2 cm **b** a grid of 200 squares.

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Fractions, decimals and percentages

Investigation
Best one-percentages, decimals or fractions

Topic quiz
3A

3B Calculating percentages

Learning intentions

By the end of this topic you will be able to ...

- ✔ express a quantity as a percentage of another
- ✔ calculate the percentage of a quantity
- ✔ increase and decrease a number by a percentage.

Inter-year links

Support Calculating percentages

Year 7 4H Calculating percentages

Express a quantity as a percentage of another

- To express a quantity as a percentage of another quantity:
  1. Make sure that both quantities are expressed in the same units.
  2. Form a fraction with the numerator as the quantity you want to express as a percentage. Simplify the fraction if possible.
  3. Write the fraction as a percentage.

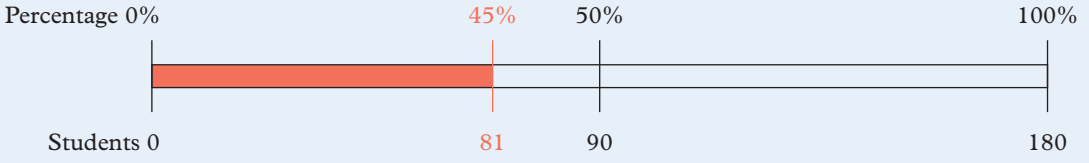
  For example, 81 students as a percentage of 180 students:

$$\frac{81}{180} = \frac{9}{20}$$
$$= \frac{45}{100} = 45\%$$

Percentage of a quantity

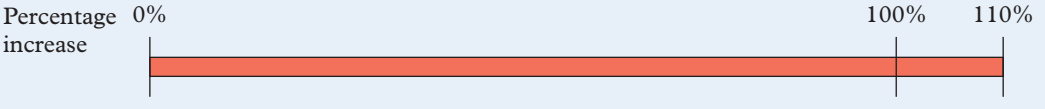
To calculate a percentage of a quantity, write the percentage as a decimal and multiply by the quantity. For example, 45% of 180 students like ice cream more than frozen yoghurt.

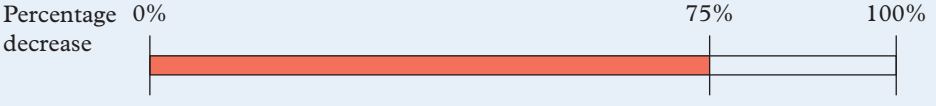
$$45\% \text{ of } 180 = 0.45 \times 180$$
$$= 81 \text{ students}$$



Percentage increase and decrease

- When a quantity is increased or decreased, we can express the change as a percentage of the original amount.
  - → A **percentage increase** occurs when a percentage amount is added to the original amount.
  - → A **percentage decrease** occurs when a percentage amount is subtracted from the original amount.





- To increase an amount by a percentage:
  1 Add the percentage to 100% and convert this new percentage to a decimal number.
  2 Multiply the amount to be increased by the decimal number.

  Increase 30 by 5% $= (100\% + 5\%) \times 30$
  $= 105\% \times 30$
  $= 1.05 \times 30$
  $= 31.5$
- To decrease an amount by a percentage:
  1 Subtract the percentage from 100% and convert this new percentage to a decimal number.
  2 Multiply the amount to be decreased by the decimal number.

  Decrease 20 by 30% $= (100\% - 30\%) \times 20$
  $= 70\% \times 20$
  $= 0.7 \times 20$
  $= 14$

Percentage error

- The error of a measurement is the positive difference between the true value and the measured value.

  For example, if a person is 120 cm tall and their height is measured to be 123 cm, then the error in the measurement is 3 cm.
- The **percentage error** is the error of a measurement written as a percentage of the true value of the quantity.

  For example, the percentage error from the example above is

$$\frac{3 \text{ cm}}{120 \text{ cm}} = \frac{1}{40}$$
$$= \frac{2.5}{100}$$
$$= 2.5\%$$

Example 3B.1 Expressing a quantity as a percentage of another

a Write 60 as a percentage of 150.

b Write 90 cents as a percentage of $12.

THINK

a 1 Express the amount as a fraction of the total and simplify.

2 To convert a fraction to a decimal or percentage, write the fraction with a denominator of 100.

3 Write your answer.

b 1 Convert $12 to 1200 cents and write 90 cents as a fraction out of 1200.

2 To convert a fraction to a decimal or percentage, write the fraction with a denominator of 100.

3 Write your answer using the units from the question.

WRITE

a $\frac{60}{150} \overset{\div 30}{=} \frac{2}{5}$

$= \frac{40}{100}$

$= 40\%$

60 is 40% of 150.

b $\frac{90}{1200} \overset{\div 12}{=} \frac{7.5}{100}$

$= 7.5\%$

90 cents is 7.5% of $12.

Example 3B.2 Calculating a percentage of a quantity

Calculate 7% of 220.

THINK	WRITE
1 Convert the percentage to a decimal.	7% of 220 = 0.07 × 220
2 Multiply by the quantity.	= 15.4
3 Write your answer.	7% of 220 is 15.4.

Example 3B.3 Changing a quantity by a percentage

a Increase 4500 by 40%.

b Decrease 240 by 15%.

THINK	WRITE
a 1 Add the percentage to 100% and convert this new percentage to a decimal number.	**a** Increase 4500 by 40% = (100% + 40%) × 4500 = 140% × 4500
2 Multiply the quantity to be increased by the decimal number.	= 1.4 × 4500 = 6300
b 1 Subtract the percentage from 100% and convert this new percentage to a decimal number.	**b** Decrease 240 by 15% = (100% − 15%) × 240 = 85% × 240
2 Multiply the quantity to be decreased by the decimal number.	= 0.85 × 240 = 204

Example 3B.4 Finding the original amount after a change

a After a 60% increase, a number is now 800. What was the original amount?

b After a 20% decrease, a number is now 160. What was the original amount?

THINK	WRITE
a 1 To increase a number by 60%, we multiply by 160%. To reverse the process, divide by 160%.	**a** Original amount = 800 ÷ (100% + 60%)
2 Convert this new percentage to a decimal number.	= 800 ÷ 160%
	= 800 ÷ 1.6
3 Divide the new amount by the decimal number to calculate the original amount.	= 500
b 1 To decrease a number by 20%, we multiply by 80%. To reverse the process, divide by 80%.	**b** Original amount = 160 ÷ (100% − 20%)
2 Convert this new percentage to a decimal number.	= 160 ÷ 80%
	= 160 ÷ 0.8
3 Divide the new amount by the decimal number to calculate the original amount.	= 200

Helpful hints

✔ You can use fractions or decimals to find the percentage of a quantity. Both methods will give you the same answer. For example, this is an alternative way to solve **Example 3B.1 a**.

$$\frac{60}{\cancel{150}^{5}} \times \frac{\cancel{100}^{2}\%}{1} = \frac{\cancel{60}^{20}}{\cancel{3}^{1}} \times \frac{2\%}{1}$$
$$= \frac{20}{1} \times \frac{2}{1}\%$$
$$= 40\%$$

✔ Can you see that writing a quantity as a percentage of a total and finding a percentage of a total are the opposite of each other?

20 as a percentage of 40	50% of 40
$\frac{20}{40} = 50\%$	$50\% \times 40 = 20$

ANS p503

Exercise 3B Calculating percentages

1–7, 8–9(a–d), 10–12, 14, 15 | 3–7, 8–9(e–h), 10, 11, 13, 14, 17, 18, 19(a) | 3, 5, 8(g–l), 9–11, 13, 14, 16–20

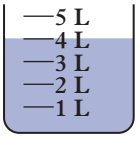
FLUENCY AND UNDERSTANDING

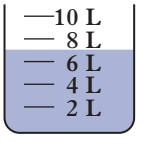
3B.1 **1** Write these amounts.

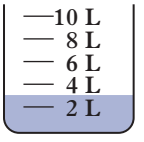
a 20 as a percentage of 80 **b** 45 as a percentage of 50 **c** 19 as a percentage of 25
d 90 as a percentage of 120 **e** 300 as a percentage of 250 **f** 135 as a percentage of 108

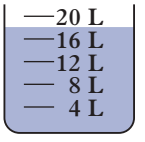
2 Calculate the percentage of the measuring jar:

i that has been filled **ii** that is empty.

a


b


c


d


3 Write each proportion:

i as a fractional percentage **ii** as a decimal percentage.

a 140 as a percentage of 800 **b** 72 as a percentage of 36 000 **c** 46 as a percentage of 125
d 57 as a percentage of 200 **e** 58 as a percentage of 80 **f** 282 as a percentage of 48

4 Write each amount as a percentage of the total.

a $1288 of a $5600 loan is repaid.
b 10.55 km of the 42.2 km marathon has been completed.
c 42 of the 120 lots of land have been sold.
d 63 likes have been received out of 90 views on YouTube.
e 3 of 40 training sessions have been missed.

5 Write each of these as a percentage.

a 50 g out of 4 kg **b** 48 minutes out of 8 hours **c** 9 months out of 3 years
d 18 hours out of one day **e** 35 cents out of $3.50 **f** 90 minutes out of 24 hours
g 60 mL out of 2 L **h** 800 mm out of 3.2 m

3B.2 **6** Calculate:

a 10% of 840 **b** 42% of 1800 **c** 6% of 150
d 18% of 6400 **e** 125% of 580 **f** 9% of 1250

7 Calculate:

a 63% of 720 m **b** 150% of 90 seconds **c** 35% of 660 cm

d 12% of 840 L **e** 40% of 190 buttons **f** 10.5% of 112 mL

8 Calculate:

a 30% of $1200 **b** 25% of $5000 **c** 40% of $250 **d** 15% of $10 500

e 8% of $200 **f** 21% of $2745 **g** 13% of $850 **h** 150% of $299

i $5\frac{1}{2}$% of $1260 **j** $10\frac{1}{4}$% of $1260 **k** 20.75% of $199 **l** 450% of $820

3B.3 **9** Increase or decrease each amount by the given percentage. Where appropriate, round answers to two decimal places.

a Increase $3700 by 20%. **b** Decrease 120 kg by 5%.

c Increase $980 by 5.2%. **d** Decrease $850 by 7.5%.

e Increase $87.50 by 1.75%. **f** Decrease 75 minutes by 10%.

g Increase 5.2 km by 12.4%. **h** Decrease 516 L by 26%.

3B.4 **10** Calculate the original amount if:

a after a 25% increase a number is now 500

b after a 30% decrease a number is now 595

c after a 16% decrease a number is now 2688

d after a $33\frac{1}{3}$% increase a number is now 1200.

11 Calculate the new amount if:

a a score of 400 was increased by 15%

b 2500 L of water in a tank was decreased by 30%

c a playlist of 60 songs was increased by 75%

d the waiting time of 15 minutes was decreased by 40%.

12 The tallest horse ever recorded is a Belgian draft horse named Big Jake, who measured 210 cm tall. The smallest horse ever recorded is a dwarf miniature horse named Thumbelina who measured 44.5 cm tall. As a comparison, an average horse is about 165 cm tall and an average Labrador dog is about 60 cm tall.



a Write Thumbelina's height as a fraction of Big Jake's in simplest form.

b Convert the answer from part **a** to a percentage rounded to two decimal places.

c Find the percentage of:

i Thumbelina's height compared to an average horse's height

ii Thumbelina's height compared to an average Labrador's height

iii Big Jake's height compared to an average horse's height.

The height of a horse (and dog) is measured at its withers (shoulders) because this height does not change when the animal moves, like the height of the head can.

d Estimate how tall these two horses would be measured to the top of their heads.

e How would this change the percentage you calculated in part **b**?

13 a What is 20% of 50?



b If a platter originally had 50 cupcakes and 20% were eaten, how many cupcakes would be left?

c By the end of the night, 20% of the remaining cupcakes were taken from the platter. How many cupcakes is this?

d Explain why the amounts obtained in parts **a** and **c** are different.

e Find how many cupcakes are still on the platter at the end of the night.

PROBLEM SOLVING AND REASONING

14 For each situation:

i find the amount of change that has occurred

ii state whether it is an increase or decrease

iii calculate the increase or decrease as a percentage of the original.

a A room that began with 25 people now has 31 people.

b A shelf that had 40 chocolate bars now has only 5 chocolate bars.

c Stacey moves house and instead of walking 1.5 km to school, she walks 2 km.

d Ahmed was 150 cm tall in Year 7 and is now 165 cm tall.

e A jogger improves her lap time from 20 minutes to 18 minutes.



15 Tom incorrectly measured the length of a table to be 43.6 cm when the actual length is 42.6 cm. Lisa measured the length of a bench to be 85.4 cm when the actual length is 86.4 cm.

a What is the size of the error made by each student?

b Calculate the percentage error for each student by finding the size of the error as a percentage of the actual length. Who has the greater percentage error and why?

16 Arthur is studying caterpillars for a science experiment. He measures one caterpillar as being 3.2 cm long. The actual length of the caterpillar is 3.15 cm.

a Calculate the error of Arthur's measurement.

b Calculate the error as a percentage of the actual length of the caterpillar.



17 In 2013 the wombat population in Epping Forest National Park was 196. The wombat population has been growing by 8% each year. How many wombats would be expected in the park in 2022 if this rate of growth continues?

CHALLENGE

18 a 50% of a number is equal to 32. Find the original number.

b 20% of a number is equal to 19.6. Find the original number.

c 70% of a number is equal to 322. Find the original number.

19 a The sides of a square increase by 20%. What is the percentage increase in area?

b The sides of a triangle increase by 20%. What is the percentage increase in area?

c Make a statement about the results you obtained in parts **a** and **b**.

d Investigate another shape and then make a general statement about the percentage increase in area when the dimensions of a shape are increased by 20%.

20 Jamie has 5 kg of flour. He uses 20% of the flour to bake bread, and then he uses 5% of the remaining flour to bake biscuits.

a How many kilograms of flour remained after he baked bread?

b How much flour did he use to bake biscuits?

c Calculate the total percentage of the flour that has been used after baking bread and biscuits.

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Calculating percentages

Investigation
Of and As – what's the diff?

Topic quiz
3B

3C Mark-ups and discounts

Learning intentions

By the end of this topic you will be able to ...

✔ calculate mark-ups and discounts including GST.

Inter-year links

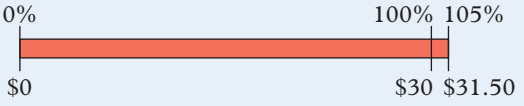
Support Calculating percentages

Year 7 4H Calculating percentages

Mark-ups, discounts and GST

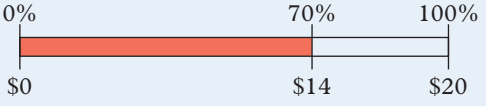
- The **cost price** (wholesale price) is the cost paid to create or obtain a good or service before it is sold to the consumer.
- The **selling price** is the price that a product or service is sold at by the seller.
- A **mark-up** is the amount the cost price is increased by to give a profit and is usually expressed as a percentage.
- Finding the price after a mark-up is the same as increasing a quantity by a given percentage.
  1. Add the percentage to 100% and convert this new percentage to a decimal number.
  2. Multiply the amount to be increased by the decimal number.

  Increase \$30 by 5% $= (100\% + 5\%) \times \$30$
  $= 105\% \times \$30$
  $= 1.05 \times \$30$
  $= \$31.50$

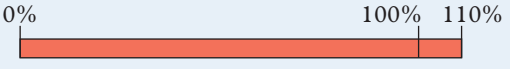


- A **discount** is the amount the selling price is decreased by to sell at a lower price and is usually expressed as a percentage.
- Finding the price after a discount is the same as decreasing a quantity by a given percentage.
  1. Subtract the percentage from 100% and convert this new percentage to a decimal number.
  2. Multiply the amount to be decreased by the decimal number.

  Decrease \$20 by 30% $= (100\% - 30\%) \times \$20$
  $= 70\% \times \$20$
  $= 0.7 \times \$20$
  $= \$14$



- **GST** (Goods and Services Tax) was introduced by the Australian Government in 1999. Currently, the rate is 10% of the selling price of the product or service.
- GST is added to the price of a good or service by performing a percentage increase of 10% to the selling price of a product or service.



→ The process to calculate the total amount paid by a customer is the same as calculating a 10% mark-up on the selling price.

Example 3C.1 Calculating mark-ups and discounts

Calculate the final price of:

a a $350 camera with 25% mark-up

b a $299 pair of shoes discounted 30%

c a $418 monitor with GST.

THINK		WRITE
a 1	Mark-ups are the same as percentage increases. Add the percentage to 100% and convert this new percentage to a decimal number.	**a** final price = (100% + 25%) × $350 = 125% × $350
2	Multiply the amount to be increased by the decimal number.	= 1.25 × $350
3	Write your answer using the correct units.	= $437.50
b 1	Discounts are the same as percentage decreases. Subtract the percentage from 100% and convert this new percentage to a decimal number.	**b** final price = (100% − 30%) × $299 = 70% × $299
2	Multiply the amount to be decreased by the decimal number.	= 0.7 × $299
3	Write your answer using the correct units.	= $209.30
c 1	GST is a tax applied to goods increasing the price of a good or service by 10%. Add the percentage to 100% and convert this new percentage to a decimal number.	**c** final price = (100% + 10%) × $418 = 110% × $418
2	Multiply the amount to be increased by the decimal number.	= 1.1 × $418
3	Write your answer using the correct units.	= $459.80

Example 3C.2 Calculating original price

Calculate the original price of:

a a mountain bike, if the selling price is $904 and the mark-up is 60%

b a notebook, if the selling price is $1125 and the discount is 25%.

THINK		WRITE
a 1	To increase a number by 60%, multiply by 160%. To reverse the process, divide by 160%.	**a** original price = $904 ÷ (100% + 60%) = $904 ÷ 160% = $904 ÷ 1.6 = $565
2	Convert this new percentage to a decimal number.	
3	Divide the new amount by the decimal number to calculate the original amount.	
b 1	To decrease a number by 25%, we multiply by 75%. To reverse the process, divide by 75%.	**b** original price = $1125 ÷ (100% − 25%) = $1125 ÷ 75% = $1125 ÷ 0.75 = $1500
2	Convert this new percentage to a decimal number.	
3	Divide the new amount by the decimal number to calculate the original amount.	

Helpful hints

✔ The prices you see in store already include GST. The selling price that is used to calculate a mark-up or discount is before GST.

ANS p504

Exercise 3C Mark-ups and discounts

1, 2–3(1st, 2nd columns), 4, 5, 6(a, c, e), 7, 8(a–e), 9, 10, 12, 13, 14(a, c, e, g), 16, 17, 21, 22	1(b, d, f, h), 2-3(2nd, 3rd columns), 4, 6(b, d, f), 7, 8(e–i), 9, 11(a–d), 12, 14(b, d, f, h), 15–18, 20, 21, 23	1(g, h), 2–3(3rd column), 4(2nd column), 6(e, f), 7, 8(e–i), 9, 11(e–h), 14(b, d, f, h), 15, 16, 19-21, 23, 24



1 Calculate each of these percentages.

a 10% of $360.50
b 25.25% of $4200
c 20% of $550.75
d 120% of $400
e 190% of $850
f 7.3% of $960
g 32% of $729
h 115.09% of $2900

3C.1 **2** Calculate the new price for each of these discounts.

a 20% discount on $150
b 15% discount on $300
c 25% discount on $840
d 40% discount on $680
e 50% discount on $1238
f 12% discount on $460
g 45% discount on $855
h 30% discount on $124.50
i 70% discount on $2075

3 Calculate the new price for each of these mark-ups.

a 20% mark-up on $420
b 50% mark-up on $668
c 65% mark-up on $120
d 18% mark-up on $924
e 87% mark-up on $1348
f 120% mark-up on $1600

4 Calculate the final selling price of these.

a a 30% mark-up on $1200
b a 25% discount on $449
c a 40% discount on $2250
d a 75% mark-up on $149
e a 47% mark-up on $645
f a 38% discount on $750
g a 54% discount on $1200
h a 25% discount on $1050
i an 85% mark-up on $175
j a 15% mark-up on $399

5 Lakmali works at a clothing store that applies a 45% mark-up on all items. What is the final selling price of:

a a pair of jeans with cost price $98
b a t-shirt with cost price $25
c a hoodie with cost price $65
d a shirt with cost price $102?

6 Calculate the prices paid for these items after GST is added, rounding to the nearest cent where appropriate.

a dining table and chairs $1285
b services provided by a plumber $240
c insurance purchased for a car $601.45
d five 3-m lengths of timber at $6.50 per metre
e electricity service and supply charge $314.65
f membership at a gymnasium at $72.95 per month

7 For each of these, determine:

i the selling price
ii the mark-up or discount amount.

a A camera is purchased for $120 and sold later at a mark-up of 62%.
b A laptop originally marked at $1198 is offered for sale at a discount of 35%.
c Work tools each marked at $49.90 are offered for sale with a 15% discount.

UNDERSTANDING AND FLUENCY

3C.2 **8** Calculate the original price in each of these sales. Where necessary, round your answer to the nearest five cents.



a A mobile phone sells for $450 after a mark-up of 50%.

b A pair of sports shorts sells for $25 after a discount of 20%.

c Eyeliner sells for $11.85 following a 15% discount.

d A hardware store sells an electric chainsaw for $169 after it is marked up by 95%.

e A furniture store offers a leather lounge suite for sale for $9995 after a discount of 12.5%.

f Fitness equipment retails for $1499 following a 140% mark-up.

g The digital copy of a video game is on sale for $4.99 after a 90% discount.

h A jumper is on sale for $31.25 after a 37.5% discount.

i A painting is being sold for $1 200 000 after a 250% mark-up.

9 The following items each include the GST charge in the price. Calculate the pre-GST price, rounding to the nearest cent where appropriate.

a telephone and internet services $155.65

b computer accessories purchased for $235.95

c garden maintenance provided for $182

d a necklace bought at a jewellery store for $120.50

10 The selling price of an item is also known as the retail price. Michael plans to buy a new external hard drive to store his games. The hard drive has a retail price of $157.95, but he receives a 12.5% discount because he has a customer loyalty card.

a If the discount is 12.5%, what percentage of the retail price will Michael pay?

b Calculate the price Michael pays after the discount. Round your answer to the nearest five cents.

11 The following represent the original prices and the percentage discount amounts offered on some goods. In each case, calculate:

i the selling price after the discount

ii the discount amount.

Where appropriate, round answers to the nearest cent.



a $500; 12% discount

b $179.50; 15% discount

c $249; 8% discount

d $895.95; 4% discount

e $624.60; 14% discount

f $29 995; 5.5% discount

g $12 680; 12.5% discount

h $1495.99; 17.5% discount

PROBLEM SOLVING AND REASONING

12 Melinda makes jewelled earrings and adds an 85% mark-up to her costs when determining the retail prices. Each individual earring contains a metal hook that costs $8.50 and three decorative stones that each cost $4.60.

a How much does it cost Melinda to make each pair of these earrings?

b What is the value of the mark-up?

c How much would Melinda advertise these pairs of jewelled earrings for?

13 A manufacturer advertises their football boots for a wholesale price of $89.90. A sports store plans to sell these boots to the public at a mark-up of 110%.



a If the mark-up is 110%, what percentage of the wholesale price will a member of the public pay for these boots?

b Calculate the retail price for these boots to the nearest five cents.

14 The following represent the wholesale prices and the percentage mark-up amounts offered on some goods. In each case, calculate:

i the retail price after the mark-up **ii** the mark-up amount.

Where appropriate, round answers to the nearest cent.

a $620; 24% mark-up
b $89.95; 45% mark-up
c $1269; 80% mark-up
d $450.50; 85.5% mark-up
e $6250; 140% mark-up
f $350.99; 125% mark-up
g $14 625; 112.5% mark-up
h $2295; 137.5% mark-up



15 A girl's bicycle is reduced to $198 after a discount of 20%. To determine the original price, Jane thinks that she needs to calculate 20% of $198 and add the result to $198. Tim thinks that Jane has it wrong and that the calculation is more complex. Which person do you think is correct? Show working to support your answer.

16 The unitary method can be used to solve percentage increase or decrease questions when the original amount is not known. Consider a television that has a retail price of $765 after a discount of 15%.

a A discount of 15% means you pay 85% of the original price. The unitary method requires you to find how much 1% represents (one unit). Calculate 1% of the original price.

b The original price of the television represents the full amount, or 100%. Use your answer to part **a** to calculate 100% of the original price. Hint: Multiply the amount for 1% by 100.

c What was the original price of the television?

17 The method outlined in question **16** can also be applied to calculate the original amount after a mark-up has been applied. Consider a different television that retails for $1800 after an 80% mark-up. Calculate the wholesale price of the television. Hint: A mark-up of 80% means you pay 180% of the original price.

18 Reconsider the scenario in question **15**. What was the price of the bike before the discount?

19 Julian accidentally discounted a $699 food mixer by 20% instead of the advertised 30%. Once he realised his mistake he took a further 10% off the discounted price. The customer claimed that this would not be the same as reducing the original price of the mixer by 30%. Who is correct? Show workings to support your answer.



20 Switched On Electrics is having a 'Scratch and Dent' sale. Any damaged stock will be discounted by 35%. A refrigerator is scratched and discounted to $1820.

a What was the original price of the refrigerator?

b How much money is saved because of the scratch on the door?

21 A grocery store applies a 30% mark-up on all its goods. A GST of 10% is then added to this marked-up price.

a Calculate the final price of each item once the mark-up and GST have been added.

i toothpaste, 160 g originally $3.88
ii tuna, 95 g originally $1.24
iii instant coffee originally $1.99
iv oil, 4 L originally $17.99
v rice crackers, 90 g originally $2.49
vi raisin toast, 520 g originally $4.59

b Compare the original price with the final price of each item and calculate the actual percentage increase correct to two decimal places.

c Explain why increasing a price by 30% then 10% is different from increasing it by 40%.

PROBLEM SOLVING AND REASONING

22 Consider the two statements: 'All stock discounted by 45%' and 'Stock discounted by up to 45%'. Explain whether these statements mean the same thing.

23 In Australia, workers pay income tax on their income to the government. The amount they pay depends on the amount they earn. The amount of tax paid is outlined in the following table.

Taxable income	Tax on this income
0 – $18 200	Nil ($0)
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

Source: https://www.ato.gov.au/rates/individual-income-tax-rates/

For example, a worker earning $46 000 a year would pay the following amount in tax:

$$\begin{aligned} 5092 + 32.5\% \times (46\,000 - 45\,000) &= 5092 + 32.5\% \times 1000 \\ &= 5092 + 0.325 \times 1000 \\ &= 5092 + 325 \\ &= \$5417 \end{aligned}$$

a Explain how the information in the table is used in the above example.

b Calculate the amount of tax paid by someone earning:

i $30 000

ii $65 000

iii $109 000

iv $135 000

v $205 000

vi $1 000 000.



c What do the amounts $5092, $29 467 and $51 667 represent? Show how they are calculated.

d For each amount in part **b**, calculate the percentage of the total income paid in tax correct to one decimal place.

e Explain why it is incorrect to say that people who earn over $180 000 pay 45% income tax.

CHALLENGE

24 David bought a new computer and paid $500 after several discounts. The computer was in a 45% off sale and his store membership rewarded him with an extra 10% off purchases more than $100. He also used a credit of $75 from the return of a faulty item.

a If the discount percentages were applied sequentially and then the store credit was applied, how much did the computer cost originally?

b If the store credit was applied first and then the discount percentages, how much did it originally cost?

c If the discounts were added and the total percentage was applied, and then the store credit was applied, how much did it cost originally?

d If the percentage discounts were applied sequentially, does it matter in which order they were applied? Explain why or why not.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Mark-ups, discounts and GST

Investigation
Inflating growth

Topic quiz
3C

Checkpoint



Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

3A **1** Write each percentage as a fraction in its simplest form and as a decimal.

a 25% **b** 76% **c** 4% **d** 0.5% **e** 120% **f** 2.56%

3A **2** Write each decimal as a percentage and as a fraction in its simplest form.

a 0.63 **b** 0.12 **c** 0.85 **d** 3.3 **e** 0.006 **f** 7.5

3A **3** Write each fraction as a decimal and a percentage.

a $\frac{3}{8}$ **b** $\frac{13}{40}$ **c** $5\frac{2}{3}$ **d** $\frac{5}{16}$ **e** $\frac{11}{25}$ **f** $8\frac{1}{6}$

3B **4** Write a percentage to describe these situations by first finding an equivalent fraction that has a denominator of 100.

a Hetty broke 3 out of 10 eggs.
b 3 students out of 25 were late to class.
c 41 people out of 50 like chocolate.
d 15 people out of 20 accepted the invitation.
e Jazz scored 36 out of 40 for her test.
f Reviewers rated the new toaster 4 stars out of 5.

3B **5** Write each of these as a percentage.

a 40 g out of 200 g **b** 45 minutes out of 180 minutes **c** 6 months out of 4 years
d 3 hours out of one day **e** 60 cents out of $3.00 **f** 960 mm out of 72 m

3B **6** Calculate:

a 15% of 960 **b** 45% of 2700 **c** 36% of 800
d 32% of 8400 **e** 250% of 960 **f** 12% of 2500

3B **7** Calculate:

a 23% of 580 L **b** 120% of $1500 **c** 2.8% of 700 cm
d 18% of 1700 m **e** 61% of 800 beans **f** $5\frac{1}{2}$% of $3500

3B **8** Increase or decrease each amount by the given percentage. Where appropriate, round answers to two decimal places.

a Increase $9600 by 20%. **b** Decrease 800 km by 12.5%. **c** Increase $4500 by 9.5%.
d Decrease $45.80 by 7.5%. **e** Increase 8.3 km by 1.75%. **f** Decrease 120 minutes by 10%.

3C **9** Calculate the selling price of these (rounded to the nearest 5 cents).

a a 60% mark-up on $1800 **b** a 15% discount on $950 **c** a 20% discount on $9870
d a 25% mark-up on $765 **e** a 21.5% mark-up on $954 **f** a 28% discount on $1250

3C **10** For each item, calculate:

i the GST to be added
ii the price it will be sold for (round to the nearest five cents).

a novel $32 **b** jeans $85 **c** laptop $1250
d pen $3.50 **e** purse $26 **f** chair $35

3C **11** Calculate the original price, if:

a 40% of an amount is $180
b 35% of an amount is $3430
c 16% of an amount is $1376
d 12% of an amount is $1014.60
e 85% of an amount is $595
f 24% of an amount is $6072.60.

3D Ratios

Learning intentions

By the end of this topic you will be able to ...

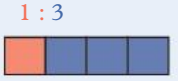
- ✔ identify the ratio shown in a diagram or written scenario
- ✔ write ratios comparing units of measure
- ✔ understand when a ratio is referring to a part-to-part comparison and when a ratio is referring to a part-to-whole comparison.

Inter-year links

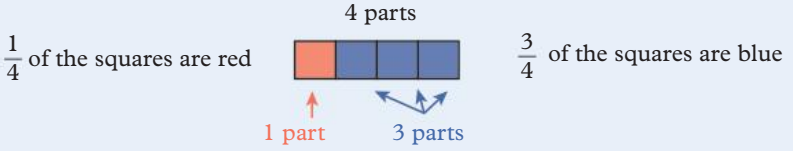
Year 7	3H Ratios
Year 9	6B Ratios, rates, direct proportion and scale
Year 10	6E Proportion and scaling

Ratios

- A **ratio** is a comparison of two or more quantities with the same units. A ratio must be written in the order of the given worded descriptions.
  For example, the ratio 1 : 3 reads '1 part compared with 3 parts' or '1 to 3'.



- Ratios can be viewed as fractions where the numerator is the number in the ratio and the denominator is the total number of parts.

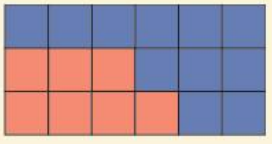


- Before writing a ratio, the numbers must be in the same unit of measurement.
- Often a part-to-part comparison is written as a ratio and a part-to-whole comparison as a fraction.

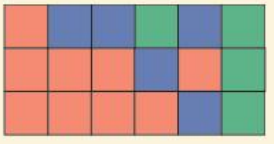
Example 3D.1 Determining ratios from a diagram

Write a ratio comparing the red to blue tiles in part **a** and red to blue to green tiles in part **b**.

a



b



THINK

1. Count the number of each coloured tile.
2. Write the numbers as they appear in the question: red then blue then green.
3. Write the ratio with a colon between each comparison.

WRITE

a There are 7 red tiles and 11 blue tiles.
red : blue

7 : 11

b There are 9 red tiles, 5 blue tiles and 4 green tiles.
red : blue : green

9 : 5 : 4

Example 3D.2 Writing ratios

Write each comparison as a ratio.

a 3 footballs to 8 soccer balls **b** 7 seconds to 3 minutes **c** 0.05 km to 23 cm to 6.73 m

THINK	WRITE
a Write the numbers as a ratio in the order they appear in the question.	**a** 3 : 8
b 1 Convert the quantities to the same unit. Converting the larger unit to the smaller unit is usually simpler. Convert minutes into seconds.	**b** 3 mins = 180 seconds
2 Write the numbers as a ratio in the order they appear in the question. Do not include units.	7 : 180
c 1 Convert the quantities to the same unit. Convert km to cm and m to cm.	**c** 0.05 km = 5000 cm 6.73 m = 673 cm
2 Write the numbers as a ratio in the order they appear in the question. Do not include units.	5000 : 23 : 673

Example 3D.3 Part-to-part versus part-to-whole comparisons

Purple paint is made with 4 parts blue paint and 5 parts red paint.

a Write a ratio comparing blue paint to red paint. **b** Write a ratio comparing blue paint to purple paint.

c What fraction of the purple paint is blue? **d** What fraction of the purple paint is red?

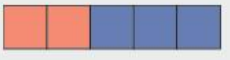
THINK	WRITE
a Write the numbers as a ratio in the order they appear in the question.	**a** blue : red 4 : 5
b Add up the total number of parts in the purple paint. Write the numbers as a ratio in the order they appear in the question.	**b** 4 + 5 = 9 blue : purple 4 : 9
c Write a fraction showing the numerator as the blue paint and the denominator as the purple paint total.	**c** Fraction of blue paint = $\frac{4}{9}$
d Write a fraction showing the numerator as the red paint and the denominator as the purple paint total.	**d** Fraction of red paint = $\frac{5}{9}$

Helpful hints

- ✔ Don't mix up the order of the numbers in the ratio. It tells you the order of the quantities you are comparing.
- ✔ Ratios are not written in size order, only in comparison order.
- ✔ When you are writing ratios, consider if you are comparing a part to part or a part to whole.
  For example, in the diagram shown, you can make both these comparisons:

  The ratio of red to blue tiles is 2 : 3.

  The ratio of blue tiles to the entire shape is 3 : 5.



ANS p505

Exercise 3D Ratios

1–4, 5–6(1st column), 7–13, 16, 18 | 1(e–h), 2, 5–6(2nd column), 9, 10, 13–15, 17, 19 | 2(b), 5–6(2nd column), 10, 14–17, 19, 20

FLUENCY AND UNDERSTANDING

1 Write each of these as a ratio.

a Dominic won 11 tournaments in a year compared to Stefanos, who won 7.

b Fedora kicked 89 goals last season while Sophie kicked 101 goals.

c Mark trains five times a week while Christian trains three times a week.

d Two-stroke petrol fuel is made by mixing 25 parts petrol to 1 part oil.

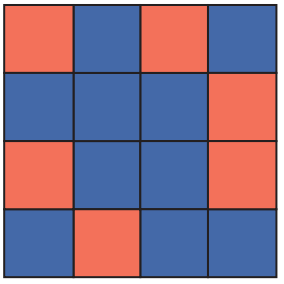
e A pancake mixture requires two cups of flour to three cups of milk.

f Concrete is made by mixing one part cement, two parts sand and three parts screenings.

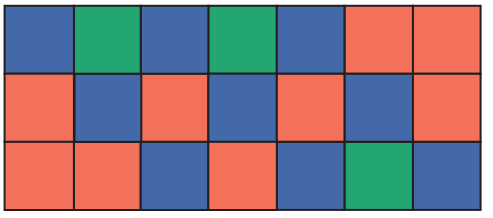
g A hair dye solution is made up in the ratio of three parts dye to seven parts water.

h Sarah works 8-hour shifts on the weekend and 3-hour shifts on a weekday.

3D.1 **2** **a** What is the ratio of red to blue tiles?



b What is the ratio of blue to red to green tiles?

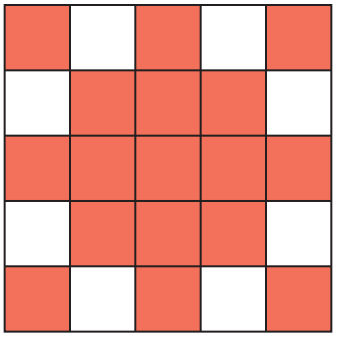


3 For each of these, write:

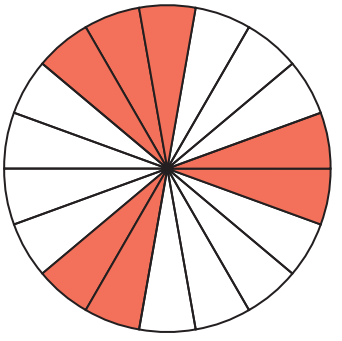
i the number of shaded sections to the number of non-shaded sections as a ratio

ii the number of non-shaded sections to total number of sections as a fraction.

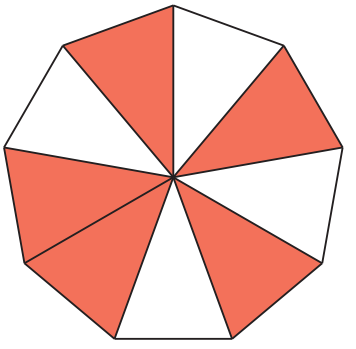
a



b



c



4 In question **3**, why is part **i** written as a ratio and part **ii** as a fraction?

3D.2 **5** Write each comparison as a ratio in the given order.

a 20 seconds to 33 seconds

b 95 kg to 57 kg

c 113 m to 167 m

d 256 mL to 175 mL

e $499 to $575

f 23 days to 2 days

g 4 months to 13 months

h 19 cans to 21 cans

i 42 km to 117 km

j 16 L to 39 L

k 19 wins to 11 losses

l 23 losses to 7 wins to 3 draws

m 15 minutes to 17 minutes to 19 minutes

n 5 red to 7 blue to 11 yellow

6 Write each comparison as a ratio.

a 13 days to 3 weeks
b 2 hours to 17 minutes
c 27 cents to $5
d 4000 mL to 3 L
e 1.7 km to 793 m
f 7 months to 3 years
g 509 g to 1.3 kg
h 472 kg to 2.371 tonnes
i 61 mm to 7.9 cm
j 38 minutes to 2 hours 11 minutes
k $4.20 to 97 cents
l 0.6 L to 53 mL
m 0.05 km to 23 cm to 6.73 m
n 5631 g to 1.753 kg to 1 tonne

7 Write the number of pins in each comparison as a ratio.



a **i** blue pins compared to yellow pins
ii white pins compared to green pins
iii yellow pins compared to total number of pins
iv red and green pins compared to blue pins

b Which of the ratios in part **a** could also be written as a fraction? Explain.

3D.3 **8** Green paint is made with 3 parts blue paint and 4 parts yellow paint.

a Write a ratio comparing blue paint to yellow paint.
b Write a ratio comparing blue paint to green paint.
c What fraction of the green paint is blue?
d What fraction of the green paint is yellow?

9 A showbag contains 11 Cherry Ripes, 9 Flakes, 7 Picnics, 5 Time Outs and 8 Turkish Delights.

a Write the ratio comparing the number of:
i Cherry Ripes to Turkish Delights to Picnics
ii Flakes to the total number of chocolate bars.

b Write the number of:
i Time Outs and Turkish Delights as a fraction of the total number of chocolate bars
ii Flakes and Picnics as a percentage of the total number of chocolate bars.

10 Consider this group of animals.



a What is the ratio of the numbers of:
i dogs to cats **ii** mammals to birds **iii** rabbits to fish?

b What fraction of the total number of animals are lizards?

c Write the number of mammals and birds as a percentage (to the nearest per cent) of the total number of animals.

d Write the number of dogs and fish as a percentage (to the nearest per cent) of the total number of animals.

e Dane said that the ratio of dogs to cats was 1 : 2. Explain where Dane went wrong and what he should have written.

11 Sofia's win–loss ratio against Bianca is 4 : 5. What does this mean?

12 Eighty-seven per cent of an iceberg is submerged in water.

a What percentage of the iceberg is visible?

b Write as a ratio the percentage of the iceberg submerged to the percentage visible.

13 A manager finds that in a box of 50 nectarines, 18% have blemishes. Write the:

a ratio of the numbers of unblemished nectarines to blemished nectarines

b the fraction of the total number of nectarines that have blemishes.

14 The mark-up on a pair of jeans purchased for $80 was 315%. What is the ratio of the original price to the selling price?

PROBLEM SOLVING AND REASONING

15 Anzac biscuits date back at least 100 years. One of the earliest versions contained the following ingredients:

2 cups rolled oats
1 cup plain flour
½ cup sugar
125 g butter
1 tablespoon golden syrup
2 tablespoons boiling water



The 'dry ingredients' of oats, flour, and sugar are mixed together first.

a What is the ratio of oats to flour to sugar?

b What fraction of the dry ingredients is flour?

This recipe will make 12 biscuits. Sam decides to make Anzac biscuits for his year level.

c When he combines the dry ingredients together he has 35 cups' worth. How many biscuits is he planning to make?

16 Consider the height of Melbourne's Rialto Tower (270 m) and Taiwan's Taipei 101 (1670 feet).

a Explain why the ratio for the heights of Rialto Tower and Taipei 101 cannot be written as 270 : 1670.

b What needs to be done to the measurements before a comparison can be made?

c Write the ratio of the Rialto Tower's height to Taipei 101's height to the nearest metre.
Hint: 1 foot = 0.3048 m

17 Whiter Than White bleach recommends three capfuls of bleach for $2\frac{1}{2}$ L of water.

a Can the two quantities be written as a ratio as they are?

b If a capful is 19 mL, what is the ratio of the quantities of bleach to water?

c Sharon thinks the solution of bleach and water is quite strong. Suggest an alternative ratio which would give a weaker solution. Explain your logic.

18 One hundred people attended an auction on the weekend. Seven people expressed interest in bidding but on the day only three actually bid.

a Write a fraction for the number of bidders to the total number of people.

b Write a fraction for the number of non-bidders to the total number of people.

c Write a ratio for the number of bidders to the number of non-bidders.

d Write a sentence explaining when a ratio can be written as a fraction.

19 Lydia, Sylvia and Julia are three sisters, aged 16, 14 and 12, respectively. Their grandparents give them $210 and tell them to divide it amongst the three of them. Lydia suggests the girls divide the money between them according to the ratio of their ages. Julia objects to this and suggests the money should be divided in thirds. Sylvia says she is happy with either way. Why do you think Sylvia feels this way? Use evidence to support your answer.

CHALLENGE

20 The Golden Ratio is an irrational number that is not quite as famous as π, but it also has a Greek letter as a symbol – Φ(phi). Many artists and architects believe that rectangles whose length and width are in the Golden Ratio make the most pleasing shape. We can define the Golden Ratio as the ratio of a shorter length, y, and a longer length, x, such that x divided by y is also equal to the sum of the two lengths divided by the longer length.

a Write out the two equations that define the Golden Ratio, Φ. When the shorter length, y, is equal to 1, what do you notice about the value of the Golden Ratio, Φ, and the longer length, x?

b Combine your equations from part **a** to create a new equation that does not contain x.

c The Golden Ratio, correct to 6 decimal places, is equal to 1.618034. Substitute this value into your equation. If both sides of the equation give the same value correct to 6 decimal places, then your equation is correct.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Visual representations of ratios
- Interactive skillsheet: Units of measurement
- Investigation: Aspects of framing
- Topic quiz: 3D

3E Equivalent ratios

Learning intentions

By the end of this topic you will be able to ...

- ✔ simplify ratios involving decimals and fractions
- ✔ use equivalent fractions to find unknown quantities.

Inter-year links

Support	Equivalent fractions
Year 7	3B Equivalent fractions
Year 9	6B Ratios, rates, direct proportion and scale

Equivalent ratios

- Numbers in a ratio can be multiplied or divided by the same value to create an **equivalent ratio**.

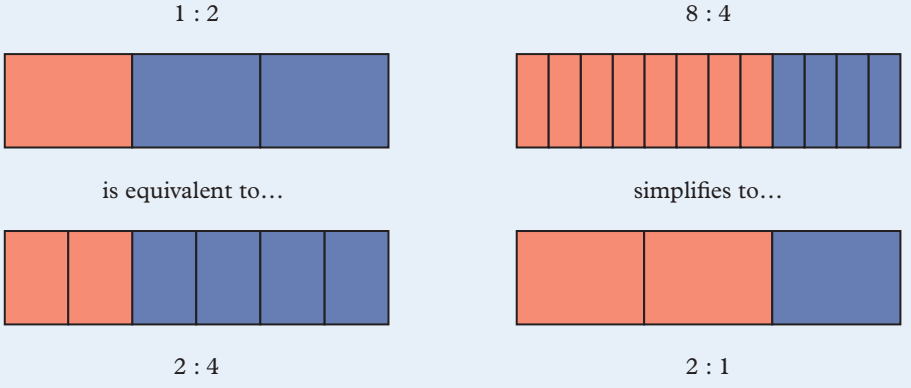
$$5 : 3 \overset{\times 4}{=} 20 : 12 \overset{\times 6}{=} 120 : 72$$

Simplifying ratios

- A ratio is written in simplest form when the only common factor in the ratio is 1.

$$120 : 72 \overset{\div 24}{=} 5 : 3 \quad \text{or} \quad 120 : 72 \overset{\div 6}{=} 20 : 12 \overset{\div 4}{=} 5 : 3$$

- Simplified ratios do not contain fractions or decimals.



Comparisons with decimals and fractions

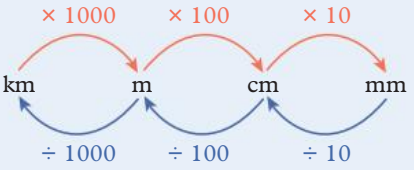
- Comparisons containing decimals can be written as ratios by first multiplying each part of the ratio by a power of 10 (that is, 10, 100, 1000, etc.) to form whole numbers.

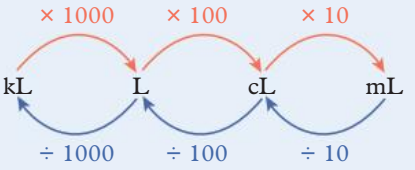
$$3.4 : 1.3 \overset{\times 10}{=} 34 : 13$$

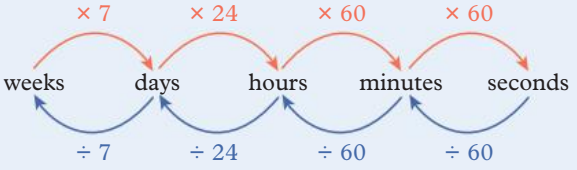
- Comparisons containing fractions can be written as ratios by first writing equivalent fractions with the lowest common denominator (LCD) and then multiplying each fraction by the value of the LCD to form whole numbers.
  - → Mixed numbers must first be converted to improper fractions.

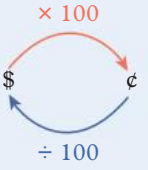
Converting units

- Converting between units of measurement is important when calculating ratios.









Example 3E.1 Simplifying ratios

Write the ratio in simplest form.

a 18 : 24 **b** 35 : 56

THINK

a **1** Find a common factor of 18 and 24. 2 is a common factor.
2 Divide the parts of the ratio by 2.
3 Find another common factor and repeat the process. Repeat until the only common factor is 1.

b **1** Find the highest common factor (HCF) of each number in the ratio.
2 Simplify by dividing each number in the ratio by the HCF.

WRITE

a 18 : 24 (÷2)
= 9 : 12 (÷3)
= 3 : 4

b The HCF of 35 and 56 is 7.
35 : 56 (÷7)
= 5 : 8

Example 3E.2 Simplifying ratios involving decimals and fractions

Write each comparison as a ratio in simplest form.

a 4.5 to 2.75 **b** $2\frac{1}{6}$ to $\frac{5}{8}$

THINK

a **1** Write the comparison as a ratio.
2 Multiply each number by 100 to obtain whole numbers.
3 Simplify the ratio by dividing each number in the ratio by the HCF of 25.

b **1** Convert the mixed number to an improper fraction. Write the comparison as a ratio.
2 Identify the lowest common denominator (LCD) of the two fractions. The LCD is 24. Write each fraction as an equivalent fraction with a denominator of 24.
3 Multiply each fraction by 24 to obtain whole numbers.
4 Check if it can be simplified.

WRITE

a 4.5 : 2.75 (×100)
= 450 : 275 (÷25)
= 18 : 11

b $2\frac{1}{6} = \frac{13}{6}$

$\frac{13}{6} : \frac{5}{8} = \frac{52}{24} : \frac{15}{24}$ (×24)
= 52 : 15

Example 3E.3 Simplifying ratios involving different units

Write the comparison 20 seconds to 2 minutes 15 seconds as a ratio in simplest form.

THINK

1 Convert the quantities to the same unit.

2 Write the comparison as a ratio in the order given.

3 Simplify by dividing each number in the ratio by the HCF. The HCF of 20 and 135 is 5.

WRITE

2 minutes = 2 × 60 seconds
= 120 seconds
2 minutes 15 seconds = 120 + 15 seconds
= 135 seconds

20 : 135 (÷5, ÷5)
= 4 : 27

Example 3E.4 Working with equivalent ratios

Find the value of a if $3 : 5 = 21 : a$.

THINK

1 Compare the first number in each ratio, 3 and 21. What should 3 be multiplied by to give 21?
3 multiplied by 7 is 21.

2 As the ratios are equivalent, 5 multiplied by 7 is a.

3 Identify the value of a.

WRITE

3 : 5 (×7, ×7)
= 21 : a

$5 \times 7 = a$

$a = 35$

Helpful hints

✔ It is okay to compare quantities that contain fractions or decimals in a ratio; however, ratios in their simplest form never contain fractions or decimals.

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Exercise 3E Equivalent ratios

△ 1–6(1st, 2nd columns), 7–9, 13–15 | □ 1–6(2nd, 3rd columns), 7(2nd column), 8(f–i), 10–12, 15, 16, 18, 20, 22 | ⬠ 1–3(3rd, 4th columns), 4(d, f, g, i), 7(2nd column), 8(f–i), 10, 15, 17, 19–21, 23, 24

3E.1 **1** Write each ratio in simplest form.

a 7 : 84	**b** 9 : 12	**c** 106 : 36	**d** 22 : 55
e 108 : 72	**f** 45 : 95	**g** 38 : 34	**h** 17 : 102
i 16 : 24 : 42	**j** 75 : 50 : 125	**k** 36 : 60 : 84	**l** 18 : 42 : 48

3E.2 **2** Write each comparison as a ratio in simplest form.

a 3.6 to 7.8	**b** 1.25 to 5.5	**c** 6.8 to 2.48	**d** 3.8 to 11.4
e 2.3 to 5.9	**f** 7.2 to 8.4	**g** 1.05 to 3.15	**h** 8.1 to 1.75
i 7.8 to 2.4 to 4.2	**j** 2.25 to 0.55 to 1.75	**k** 7.2 to 9.6 to 8.4	**l** 1.2 to 4.2 to 1.6

UNDERSTANDING AND FLUENCY

3 Write each comparison as a ratio in simplest form.

a $\frac{1}{6}$ to $\frac{1}{4}$ **b** $\frac{2}{3}$ to $\frac{3}{4}$ **c** $\frac{1}{3}$ to $\frac{2}{5}$ **d** $\frac{3}{7}$ to $\frac{2}{11}$

e $\frac{9}{22}$ to $\frac{5}{11}$ **f** $\frac{4}{7}$ to $\frac{6}{5}$ **g** $6\frac{1}{4}$ to $12\frac{1}{2}$ **h** $5\frac{2}{3}$ to $7\frac{5}{9}$

i $2\frac{1}{8}$ to $\frac{2}{5}$ **j** $3\frac{1}{5}$ to 4 **k** 0.25 to $\frac{1}{8}$ **l** $\frac{3}{4}$ to 0.125

3E.3 **4** Write each comparison as a ratio in simplest form.

a 65 mL to 3.2 L **b** 17 mm to 2.34 m **c** 92 cents to $17.28

d $\frac{2}{3}$ hours to 0.8 hours **e** 5 days to 10 weeks **f** 3800 L to 4 kL

g 0.9 km to 550 m **h** 1.28 cm to 5 mm **i** $1\frac{3}{4}$ minutes to 420 seconds

5 Which pairs of ratios are equivalent? Explain why you think each pair is equivalent.

a 1 : 2 and 3 : 6 **b** 8 : 5 and 2 : 3 **c** 4 : 10 and 2 : 5 **d** 2 : 9 and 10 : 45

e 16 : 25 and 4 : 5 **f** 63 : 42 and 9 : 7 **g** 2 : 3 and 22 : 34 **h** 15 : 9 and 5 : 3

i 1 : 5 and 15 : 30 **j** 17 : 51 and 68 : 204 **k** 49 : 64 and 7 : 8 **l** 27 : 18 and 3 : 2

3E.4 **6** Find the value of each pronumeral.

a $5 : 9 = 25 : a$ **b** $15 : 50 = 3 : b$ **c** $9 : 2 = c : 16$ **d** $8 : 36 = d : 9$

e $e : 55 = 19 : 11$ **f** $f : 4 = 42 : 24$ **g** $2 : g = 22 : 66$ **h** $20 : h = 5 : 2$

i $i : 23 = 12 : 92$ **j** $j : 6 = 30 : 60$ **k** $15 : k = 3 : 17$ **l** $18 : l = 2 : 13$

m $7 : 15 = 49 : m$ **n** $72 : 168 = 6 : n$ **o** $68 : 140 = x : 35$ **p** $96 : 80 = p : 5$

7 Fill in the gaps to complete the equivalent ratios.

a 2 : 5 = _____ : 20 = 20 : _____ **b** 6 : 7 = _____ : 35 = 54 : _____

c _____ : 360 = 70 : 60 = 7 : _____ **d** 225 : _____ = 9 : 12 = _____ : 4

e _____ : 104 = 68 : _____ = 17 : 13 **f** _____: 48 = 75 : _____ = 15 : 4

g 20 : _____ = 40 : 96 = _____ : 240 **h** _____: 636 = 72 : 212 = 18 : _____

8 Find the value of the letter in each equivalent ratio statement. Hint: Simplify the complete ratio in the statement first.

a $a : 24 = 20 : 30$ **b** $b : 52 = 20 : 13$ **c** $13 : c = 65 : 5$

d $25 : d = 10 : 14$ **e** $48 : 32 = 18 : e$ **f** $33 : 44 = f : 12$

g $33 : 54 = g : 72$ **h** $15 : 20 = 36 : h$ **i** $40 : 36 = i : 27$

PROBLEM SOLVING AND REASONING

9 Write each quantity as a ratio in simplest form.

a The number of students in a school is 975 while the number of teachers is 75.

b Mix 25 mL of cordial to 225 mL of water.

c 8500 Diamonds fans and 4500 Silver Ferns fans watched a netball match between the two teams.

d I have 8 gold bracelets and 14 silver bracelets.

10 Eric paid a deposit of $704.70 for his overseas trip, which cost $4698. Write the ratio, in simplest form, of the deposit paid to the amount owing.

11 Jasmine is helping her mother make pancakes for her class end-of-term breakfast. The ingredients are listed on the right and feed 10 people.

a Which ingredients can be written as a ratio? Clearly explain your choices.

b Write a ratio for the ingredients identified in part **a**.

c Repeat part **b** if 50 pancakes are required.

Pancakes
(serves 10)

500 mL full-cream milk
300 g self-raising flour
2 eggs
60 g butter
2 tablespoons sugar

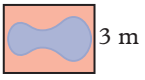
12 The ratio of rainy days to sunny days in a month was 2 : 3. If there were 18 sunny days, how many rainy days were there?

13 The ratio of girls to boys in a class is 5 : 4. If there are 15 girls in the class, find:

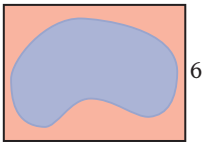
a how many boys are in the class

b how many students are there in total.

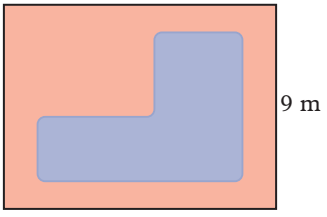
14 When you compare the ratio of a part to its total, the ratio can be written as a fraction. Consider the four different pool enclosures pictured.



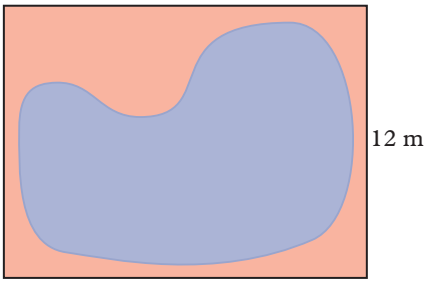
4 m
Enclosure A



8 m
Enclosure B



12 m
Enclosure C



16 m
Enclosure D

a Complete the table for each pool enclosure.

Pool enclosure	Length	Width	Perimeter
A			
B			
C			
D			

b For each enclosure, write the ratio of the length to the perimeter in simplest form.

c Describe a pattern in your answers to part **b**.

d Use your answers to part **b** to write each ratio as a fraction in simplest form.

e Compare the fractions you have obtained. What is special about them?

f Describe what this fraction tells you about the length and perimeter of each pool enclosure.

15 The ratio of blue pens to black pens to red pens in a classroom is 7 : 4 : 1. If there are 24 black pens in the classroom, how many pens are there altogether?



16 For best results, the instructions on a new cleaning product recommend the ratio of detergent to water be 2 : 7.

a How many litres of water are required if 300 mL of detergent are used?

b How many millilitres of detergent are required if 28 L of water are used?

17 A two-stroke fuel mixture is made by mixing oil and petrol in the ratio of 1 : 25.

a How many litres of petrol should be added to 850 mL of oil?

b How many millilitres of oil should be added to 15 L of petrol?

PROBLEM SOLVING AND REASONING

18 Gordon bought shares in a Telco company for $8.20 and sold them for $40.80. Find the ratio, in simplest form, of:

a the original price to the selling price

b the original price to the difference between the selling price and the original price.

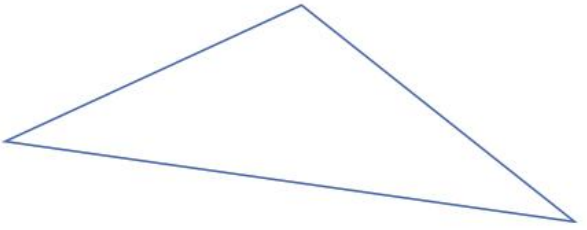
19 Imelda purchased a pair of shoes that had been discounted by 60%. The shoes were originally priced at $850. Find the ratio, in simplest form, of:

a the original price to the sale price

b the discount compared to the original price

c the discount compared to the sale price.

20 The side lengths of a scalene triangle are in the ratio 7 : 8 : 5.



a If the shortest side length is 17.4 cm, find the length of the longest side.

b Find the perimeter of the scalene triangle.

CHALLENGE

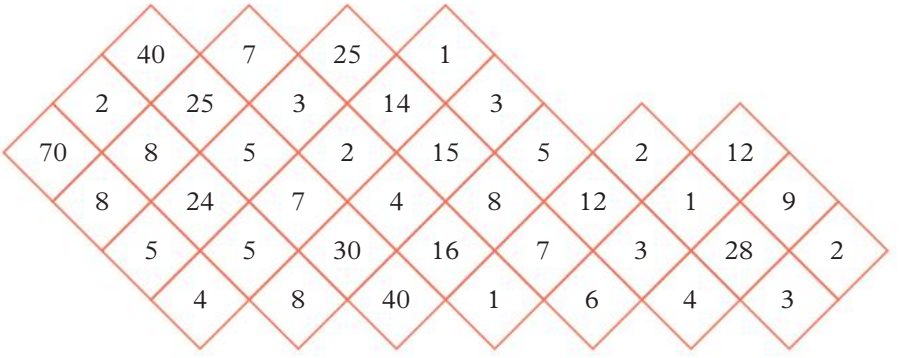
21 a Comment on how the part-to-whole ratio 22 : 27 compares with the part-to-whole ratios 8 : 9 and 12 : 15.

b Obtain three ratios that lie between the part-to-whole ratios 22 : 27 and 12 : 15.

22 Compare the part-to-whole ratios 4 : 5 and 10 : 13 and state which is larger. Explain your answer.

23 Without using equivalence, compare the part-to-whole ratios 17 : 33 and 12 : 25 and state which is larger. Explain your answer.

24 Using only the numbers below, find 10 pairs of equivalent ratios. You must use all the numbers in the diagram and each number can only be used once. You may like to present your answers in a table like the one below.



1st ratio	2nd ratio

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Ratios in simplest form

Investigation
Motorbike dilemmas

Topic quiz
3E

3F Dividing a quantity in a given ratio

Learning intentions

By the end of this topic you will be able to ...

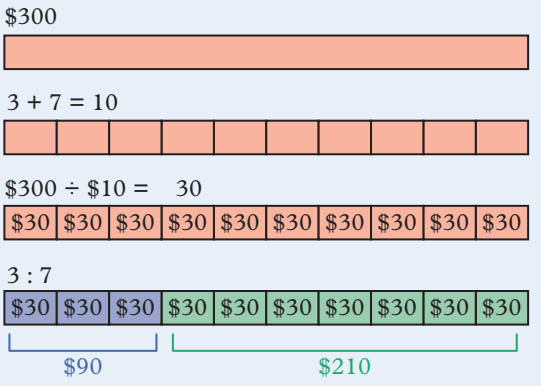
- ✔ divide a quantity into a given ratio
- ✔ find a quantity from a given ratio.

Inter-year links

Support	Dividing a decimal by a whole number
Year 7	4E Dividing decimals

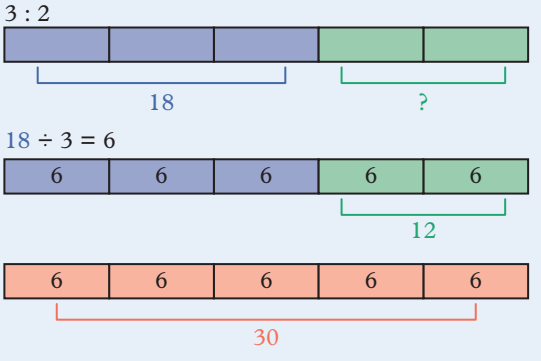
Dividing a quantity into a given ratio

- When dividing a quantity in a given ratio, follow these steps.
  1. Find the total number of parts in the ratio.
  2. Divide the quantity by the total number of parts.
  3. Multiply the result of the previous step by each of the ratio.
     For example, dividing $300 into 3 : 7 is $90 and $210.
- Remember to include the same units with your answer.
- One way to check that you have divided the quantity correctly is to add the individual amounts to see whether the result is the same as the original quantity.



Finding a quantity from a given ratio

- To find a quantity from a given ratio:
  1. Determine the value of 1 part of the ratio. Divide the quantity by the number of parts in the corresponding ratio.
  2. When the value of 1 part is found, any part of the ratio and the whole can be found.
     For example, if the ratio of rice to water in a recipe is 3 : 2 and a chef uses 18 cups of rice,
     - → 18 cups of rice is related to the 3 parts in the ratio.
     - → 18 divided by 3 gives 1 part equal to 6 cups.
     - → There are 2 parts water to 3 parts rice, so the water required to match the rice is 12 cups.
     - → 30 cups in total are required for this recipe.



Example 3F.1 Dividing a quantity into a ratio

Divide 4800 in the ratio 3 : 1.

THINK	WRITE
1 Find the total number of parts in the ratio.	Number of parts = 3 + 1 = 4
2 Divide the quantity by the total number of parts.	4800 ÷ 4 = 1200
3 Multiply 1200 by each part of the ratio.	1200 × 3 = 3600 1200 × 1 = 1200
	The ratio 3 : 1 divides 4800 into 3600 and 1200.

Example 3F.2 Dividing a decimal quantity into a ratio

Divide $20.25 into the ratio 4 : 5.

THINK	WRITE
1 Find the total number of parts in the ratio.	Total number of parts = 4 + 5 = 9
2 Divide by the total number of parts.	20.25 ÷ 9 = 2.25
3 Multiply 2.25 by each part of the ratio. Remember to give your answer in dollars.	2.25 × 4 = 9 2.25 × 5 = 11.25 The ratio 4 : 5 divides $20.25 into $9 and $11.25.

Example 3F.3 Dividing a fraction quantity into a ratio

Divide $\frac{11}{45}$ into the ratio 2 : 3.

THINK	WRITE
1 Find the total number of parts in the ratio.	Total number of parts = 2 + 3 = 5
2 Divide the quantity by the total number of parts.	$\frac{11}{45} \div 5 = \frac{11}{45} \times \frac{1}{5}$ $= \frac{11}{225}$
3 Multiply $\frac{11}{225}$ by each part of the ratio. Remember to simplify your fractions where possible.	$\frac{11}{225} \times 2 = \frac{22}{225}$ $\frac{11}{225} \times 3 = \frac{33}{225}$ $= \frac{11}{75}$ The ratio 2 : 3 divides $\frac{11}{45}$ into $\frac{22}{225}$ and $\frac{11}{75}$.

Example 3F.4 Finding a quantity from a given ratio

The ratio of cats to dogs in a neighbourhood is 11 : 6. If there are 102 dogs in the neighbourhood:

a how many cats are there **b** how many cats and dogs are there in total?

THINK	WRITE
a **1** Determine the total number of parts in the ratio.	**a** 11 + 6 = 17
2 Determine the number of pets in 1 part of the ratio. Consider the ratio of cats to dogs, 11 : 6. 6 parts equals 102 dogs. Divide by the number of parts.	102 ÷ 6 = 17
3 Multiply 17 by the parts for cats.	17 × 11 = 187 There are 187 cats in the neighbourhood.
b Add the number of cats to the number of dogs.	**b** 187 + 102 = 289 There are 289 cats and dogs in total.

Helpful hints

- ✔ Always add your ratio parts together to find out how many parts there are in total.
- ✔ If you need to refresh your skills on multiplying and dividing decimals and fractions, refer to chapter 2. There is some good stuff in there!
- ✔ You can consider the process of finding 1 part as simplifying. From **Example 3F.4**,

6 parts = 102 dogs
÷6 ↓ ÷6
1 part = 17 dogs

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Exercise 3F Dividing a quantity in a given ratio

△ 1–10, 12–15, 17, 22 □ 4–7, 10, 11, 13, 15, 16, 18, 19, 21, 23(a–d) ⬠ 5–7, 13, 15, 18, 20–24

1 Divide 500 in the following ratios. Use the diagram to help you.



a 1 : 9 **b** 2 : 8 **c** 3 : 7 **d** 6 : 4 **e** 3 : 2 **f** 4 : 1

3F.1 **2** Divide $9200 in each ratio.

a 3 : 2 **b** 5 : 3 **c** 3 : 1 **d** 1 : 1 **e** 21 : 25 **f** 2 : 8

3 Divide 36 000 km in each ratio.

a 5 : 7 **b** 7 : 1 **c** 10 : 8 **d** 4 : 5 **e** 12 : 3 **f** 1 : 1

4 Divide $240 in each ratio.

a 1 : 2 **b** 9 : 1 **c** 2 : 10 **d** 10 : 6 **e** 3 : 1 : 4 **f** 7 : 2 : 1

5 Divide 300 kg in each ratio.

a 4 : 6 **b** 1 : 9 **c** 13 : 2 **d** 4 : 11 **e** 4 : 5 : 1 **f** 8 : 5 : 3 : 4

UNDERSTANDING AND FLUENCY

UNDERSTANDING AND FLUENCY

3F.2 **6** Divide \$3.60 in each ratio.

a 3 : 1 **b** 1 : 8 **c** 11 : 1 **d** 5 : 4 **e** 1 : 3 : 5 **f** 2 : 3 : 7

3F.3 **7** Divide $\frac{8}{15}$ in each ratio.

a 2 : 1 **b** 3 : 2 **c** 3 : 5 **d** 6 : 2 **e** 2 : 3 : 3 **f** 6 : 5 : 4

8 Use the diagram in Question **1** to divide 35 cups of water in each ratio.

a 3 : 2 **b** 4 : 1 **c** 7 : 3 **d** 9 : 1 **e** 3 : 5 : 2 **f** 3 : 4 : 3

3F.4 **9** The ratio of fiction to non-fiction books in a library is 7 : 8. If there are 3500 fiction books in the library:

a how many non-fiction books are there **b** how many books are there in total?

10 The ratio of tomato seedlings to chilli seedlings in Paul's garden is 5 : 7. If there are 21 chilli seedlings:

a how many tomato seedlings are there **b** how many seedlings are there in total?

11 The ratio of germinated seeds to non-germinated seeds in Rita's garden is 9 : 4. If there are 4500 germinated seeds:

a how many non-germinated seeds are there

b how many seeds are there in total?

PROBLEM SOLVING AND REASONING

12 The ratio of female to male fans at a football training session is 5 : 2. If there are 224 fans at the training session, how many are male?

13 Morning Sunshine fruit juice drink contains fruit juice and water in the ratio 1 : 3.

a How much water (in mL) is in a 2 L container of Morning Sunshine? Hint: 1 L = 1000 mL.

b How much fruit juice is in 600 mL of Morning Sunshine?

14 The angles in a triangle are in the ratio 3 : 4 : 2. What is the size of each angle? Hint: The sum of angles in any triangle is 180°.

15 Divide \$300 using the ratio 4 : 6.

a Calculate the total number of parts in the ratio.

b Write each number in the ratio as a fraction of the total number of parts.

c Multiply each fraction from part **b** by the quantity (\$300).

d Do the amounts found in part **c** add to \$300?

e Decide if you prefer this method of dividing a quantity into a ratio.

16 Nadia divides her pocket money for April in the ratio 3 : 2 : 1 to cover buying a gift, entertainment expenses and savings. Calculate the amount she has allocated for entertainment expenses if her total pocket money for April is \$84.



17 Katie, Mike and Sam contributed different amounts to a \$5 raffle ticket. When they won, they distributed the winnings in the same ratio as they put in. If Katie won \$450, Mike won \$200 and Sam won \$350, find:

a the total amount won

b the ratio of their contribution

c how much they each put towards the ticket.

18 Sarah, Cameron and Hunter bought a \$5 raffle ticket. If Sarah contributed \$2.50, Cameron contributed \$1.50 and Hunter contributed \$1.00, find:

a the ratio of their contributions

b their individual winnings, if the ticket won:

i \$50 **ii** \$150 **iii** \$700 **iv** \$2000 **v** \$10 000 **vi** \$13 000.

19 Jia is the treasurer of her netball club which has $200 in its budget. She has decided that $\frac{3}{20}$ of the budget will be spent on a present for the coach, $\frac{1}{4}$ of the budget will be spent on trophies for all the players and the rest of the budget will be spent on snacks at the games.

- **a** What fraction of the budget will be spent on snacks?
- **b** Write a ratio representing the money Jia will spend on the present, trophies and snacks.
- **c** Find the percentage that is allocated to each part of the budget.
- **d** Find how much money Jia will spend on each part of the budget.

20 Bella wants to organise her time in the holidays. She decides she will spend $\frac{2}{5}$ of her time catching up with friends, $\frac{1}{4}$ of her time doing outdoor activities, $\frac{1}{10}$ of her time doing her homework and chores, and the remainder of her time relaxing at home.



- **a** Find the percentage of time that Bella will catch up with friends, do outdoor activities and do homework or chores. Hint: For each fraction find an equivalent fraction with a denominator of 100.
- **b** From part **a**, find the percentage of time she will relax at home.
- **c** Write these percentages as a ratio in simplest form.
- **d** Assuming that for the two-week holiday she has 10 'usable' hours a day, find the number of hours Bella spends on each activity.

21 The perimeter of a rectangular computer screen is 228 cm, with a width-to-height ratio of 12 : 7.

- **a** Calculate the width and height of the screen.
- **b** Calculate the area of the screen.

22 How do the questions in this topic highlight the difference between a ratio that shows the relationship of part to part, and a fraction that shows the relationship of part to whole? Why is it important?

23 To create two distinct shades of pink paint, the following ratios of white to red paint are required: 4 : 1 and 7 : 1. A 4 L can in each shade is purchased.



- **a** Calculate the amounts (in mL) of white and red paint in each can.
- **b** The two cans are accidentally mixed together. What is the new ratio of white to red paint?
- **c** Write the amount of red paint as a fraction, in simplest form, of the mixed paint.
- **d** Write the amount of white paint as a percentage of the mixed paint.
- **e** The mixed paint is divided evenly into two cans. How much red and while paint would have to go into each can for the shades to be exactly the same?

24 An aquarium has fish and amphibians. The amphibians are frogs and newts. There are 40 more fish than amphibians.

The ratio of frogs to newts to fish is 2 : 5 : 11. How many animals in total does the aquarium have?

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet — Dividing a quantity into a ratio
- Interactive skillsheet — Finding a quantity from a given ratio
- Investigation — On the ropes
- Topic quiz — 3F

3G Rates

Learning intentions

By the end of this topic you will be able to ...

- ✔ write a rate in its simplest form
- ✔ use a rate to solve a problem
- ✔ understand the relationship between speed, distance and time.

Inter-year links

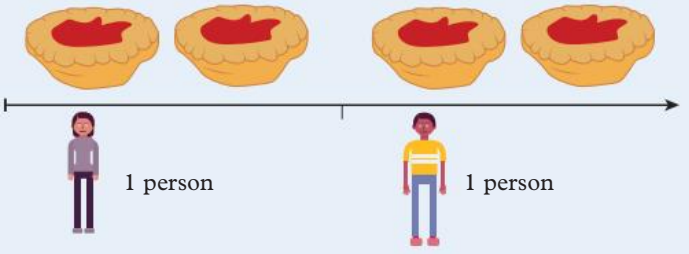
Year 7	4E Dividing decimals
Year 9	6B Ratios, rates, direct proportion and scale

Rates

- A **rate** is a comparison between two or more different quantities with different units. It is a measure of how a quantity changes for each unit of another quantity. The second quantity being compared is often a measure of time.
- The unit of a rate is two units separated by the word 'per' (meaning 'for each') or the symbol /.
  For example,
  → 100km/h means travelling 100 km in every hour.



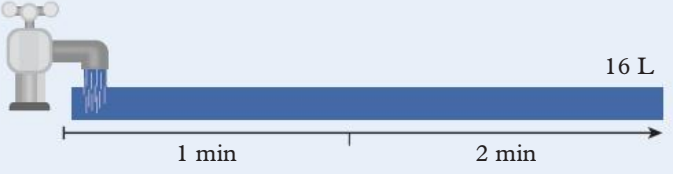
→ 2 meat pies per person means each person gets 2 meat pies.



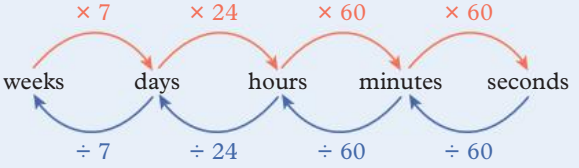
→ $500 per week means someone receives $500 each week.



→ 8 L/min means that for each minute, 8 litres flow.



- Order is important when writing a rate.
- Converting units of measurement is important when calculating rates, especially units of time.



Simplified rates

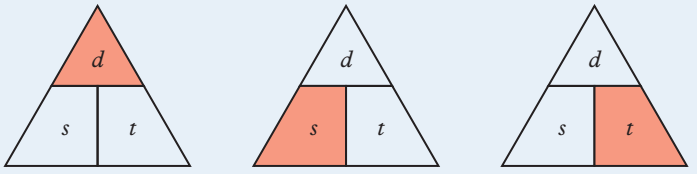
- For a rate to be in simplest form, the second of the two quantities being compared must have a value of 1.
  1 Write the rate as a fraction including the units.
  2 Simplify the fraction so that the denominator is 1.
  3 Write your answer as a rate using the '/' to indicate 'per'.

$$\frac{36\text{ km}}{12\text{ h}} \overset{\div 12}{=} \frac{3\text{ km}}{1\text{ h}} = 3\text{ km/h}$$

Average speed, distance and time

- Speed is an example of a rate. It compares distance and time.

  Average speed $= \frac{\text{distance travelled}}{\text{time taken}}$ or $s = \frac{d}{t}$



distance = speed × time speed = $\frac{\text{distance}}{\text{time}}$ time = $\frac{\text{distance}}{\text{speed}}$

Example 3G.1 Writing statements as rates in simplest form

Write each statement as a rate in simplest form.

a 30 m in 5 seconds

b $54 for 40 L of petrol

THINK

a 1 Write the two quantities as a fraction including the units.

2 Simplify the fraction so that the denominator is 1. For the rate to be in simplest form, the second quantity needs be 1. Divide both quantities by 5.

3 Write your answer as a rate using the '/' to indicate 'per'.

b 1 Write the two quantities as a fraction including the units.

2 Simplify the fraction so that the denominator is 1. Divide both quantities by 40.

3 Write your answer as a rate using the '/' to indicate 'per'. The dollar sign '$' must be written before the number.

WRITE

a 30 m in 5 seconds $= \frac{30\text{ m}}{5\text{ s}}$

$$\frac{30\text{ m}}{5\text{ s}} \overset{\div 5}{=} \frac{6\text{ m}}{1\text{ s}}$$

$\frac{6\text{ m}}{1\text{ s}} = 6\text{ m/s}$

b $54 for 40 L of petrol $= \frac{\$54}{40\text{ L}}$

$$\frac{\$54}{40\text{ L}} \overset{\div 40}{=} \frac{\$1.35}{1\text{ L}}$$

$\frac{\$1.35}{1\text{ L}} = \$1.35\text{/L}$

Example 3G.2 Solving problems using rates

Will loves to read. He can read 56 pages in half an hour.

a How many pages can he read in 2 hours?

b How many pages can he read in 3.5 hours?

c How long will it take him to read 1008 pages?

THINK

a **1** Write the two quantities as a fraction including the units. Simplify the fraction so that the denominator is 1.

2 Multiply the hourly rate by 2 to work out the new amount. The number of pages Will can read in 2 hours is found in the numerator of the equivalent rate.

b Multiply the hourly rate by 3.5 to work out the new amount. The number of pages Will can read in 3.5 hours is found in the numerator of the equivalent rate.

c Divide the number of pages by the number of pages Will can read in one hour.

WRITE

a 56 pages per half hour $= \frac{56 \text{ pages}}{\frac{1}{2} \text{ hour}}$

$= \frac{112 \text{ pages}}{1 \text{ hour}}$

$\frac{112 \text{ pages}}{1 \text{ hour}} \overset{\times 2}{=} \frac{224 \text{ pages}}{2 \text{ hours}}$

Will can read 224 pages in 2 hours.

b $\frac{112 \text{ pages}}{1 \text{ hour}} \overset{\times 3.5}{=} \frac{392 \text{ pages}}{3.5 \text{ hours}}$

Will can read 392 pages in 3.5 hours.

c $1008 \div 112 = 9$

It takes Will 9 hours to read 1008 pages.

Example 3G.3 Calculating average speed

Calculate the average speed of a truck that travels 750 km in 13.5 hours, rounding your answer to two decimal places.

THINK

1 Write the formula for average speed.

2 Substitute the values into the formula and calculate s.

3 Write your answer using the correct units of the rate.

WRITE

$s = \frac{d}{t}$

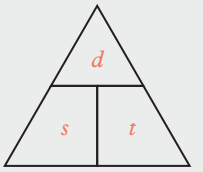
$s = \frac{750 \text{ km}}{13.5 \text{ h}}$

$= 55.555\ldots \text{ km/h}$

The average speed is 55.56 km/h.

Helpful hints

- ✔ Make sure that you identify the units of the rate. It is important to understand the units because they will determine how you deal with them.
- ✔ Choose multiplication or division to manipulate your rate to give your desired units.
- ✔ Don't forget to include units in your answer. It is helpful to use units in your working so that they also appear in your answer.
- ✔ Remember the distance–speed–time triangle can be very helpful for remembering the formula.



ANS p507

Exercise 3G Rates

△ 1–3(1st column), 4–6, 9–11, 13–15, 17 □ 3(2nd column), 4, 6–8, 10–12, 16–18 ⬠ 4, 7, 8, 10–12, 16–19

1 Write each statement as a rate with the appropriate unit.

a 60 km in each hour
b 591 L in each minute
c four quarters in each game
d $16 for each hour
e 45 cm each year
f five patients each hour
g $1.49 for each day
h $28.99 for each month
i $5.80 for each box
j 0.5 kg for each week
k 290 mm each year
l 14 km for each litre

2 What rate unit could be used in each situation?

a a cheetah chasing its prey
b Usain Bolt sprinting 100 m
c the cost of 10 apples
d the cost of tiling the bathroom
e filling a bath with water
f cricketer Pat Cummins bowling
g downloading songs on to an iPod
h driving on a freeway

3G.1 **3** Write each statement as a rate in simplest form.

a 200 m in 4 minutes
b 6 L in three bottles
c 80 cents for 20 seconds
d 135 students with nine teachers
e 468 seats in 18 rows
f 100 newspapers in four bundles
g 68 runs in 10 overs
h $20 for eight muffins
i 102 goals in 15 games
j 39 996 tickets sold in 9 minutes
k 77 goals over 22 games
l area of 33.6 m^2 requires 6.4 L of paint

4 Write a rate in its simplest form to represent each situation.

a A car uses 18 L of petrol to travel 252 km.
b The cost of 15 mangoes is $29.85.
c A bath fills up to 54 L in $4\frac{1}{2}$ minutes.
d A runner takes $2\frac{3}{4}$ hours to complete a 42.2 km marathon.
e Queensland's population is 5 281 905 and its area is 1 852 642 km^2. (Correct to two decimal places.)
f Light travels 1080 million km in 60 minutes.

UNDERSTANDING AND FLUENCY



3G.2 **5** Lina can type 1200 words in half an hour.

a How many words can she type in 1 hour?

b How many words can she type in 20 minutes?

c How long does it take Lina to type 1800 words?

6 Boris earns $182.40 for working 12 hours.

a How much does he earn for working 1 hour?

b How much does he earn for working 40 hours?

c How long does Boris have to work to earn $1459.20?

7 Sound travels 23 160 m in 60 seconds.

a How far does sound travel in 1 second?

b How far does sound travel in 1 hour?

c How long does it take for sound to travel 772 000 m?

8 Fabian worked 22 hours last weekend and received $396. At this rate, how much will he earn if he works $19\frac{3}{4}$ hours next weekend?

3G.3 **9** Calculate the average speed for each situation, rounding your answer to two decimal places where appropriate.

a a bus travels 876 km in 16 hours

b a jet travels 1965 km in 2.3 hours

c a cyclist rides 78 km in 2.4 hours

d an athlete runs 10 km in 26.44 minutes

e James Magnussen took 47.71 seconds to swim 100 m freestyle

f Libby Trickett took 52.88 seconds to swim 100 m freestyle

PROBLEM SOLVING AND REASONING



10 Alexis and Kym arrange to meet at the running track to train for an athletics carnival. They cycle to the track, leaving their homes at the same time.

It takes Alexis 8 minutes to travel 2800 m from her home to the track. Kym takes 18 minutes, after travelling a distance of 6.12 km.

Alexis thought she was faster, but Kym was not convinced.

a Calculate each girl's average speed.

b Who was fastest? How do you know?

c What unit did you use for speed? Explain.

11 Max's team scored 230 runs in 40 overs. William's team scored 280 runs in 50 overs.

a Calculate each team's run rate per over.

b Which team had a better run rate?

12 Leandro's resting heart rate is 14 beats in 15 seconds.

a Heart rate is often expressed in beats per minute (bpm). Write Leandro's heart rate in bpm.

b How many minutes does it take for Leandro's heart to beat 1512 times?

13 Usain Bolt recorded a world-record time of 9.58 seconds for the 100 m sprint. Compare his rate with that of:



a Michael Johnson's time of 19.32 seconds for the 200 m sprint

b The US 4×100m relay team of 1992, who recorded a time of 37.4 seconds

c A cheetah that was able to cover a distance of 600 m in 37.5 seconds.

14 Victor takes 48 minutes to run 15 km.

a What distance does he cover per minute?

b If Victor is able to maintain this rate, what distance would he cover in these times?

i 55 min **ii** 60 min **iii** 1 hour 25 min **iv** 1.5 h

c How long would it take Victor to cover these distances?

i 625 m **ii** 31.25 km **iii** 25 km **iv** 2 km

15 Compare each pair of grocery items and work out which option is the better value for money.



a 5 kg of potatoes for $4.95 or 7.5 kg for $7.15

b a packet of four batteries for $3.99 or a packet of ten for $9.89

c a 500 g tin of coffee for $9 or a 150 g jar for $2.99

d a 440 g tin of peaches for $1.85 or a 650 g tin for $2.56

16 A phone company offers its customers two plans.

Plan A: connection fee of 40 cents and then 40 cents per minute or part thereof

Plan B: connection fee of 50 cents and then 38 cents per minute or part thereof

a Calculate the cost of a 1-minute call for each plan. Which offers the best deal?

b Calculate the cost of a 3-minute call for each plan. Which offers the best deal?

c Calculate the cost of a 5-minute call for each plan. Which offers the best deal?

d Calculate the cost of a 6-minute 23-second call for each plan. Which offers the best deal?

e Use your answers from parts **a–d** to explain when it would be best to use each plan.

17 When repaying a loan from a bank, you pay interest on top of the amount of the loan. The amount of interest, I, depends on the original amount you borrowed, P, the interest rate charged per year, r, and the time, T, it takes to repay the loan in years. Simple interest is calculated using the formula:

$$I = P \times r \times T$$

For each of the following sets of values:

i convert the interest rate to a fraction

ii calculate the total simple interest charged

iii find the sum of the interest and the original loan amount, which is the total owed to the bank.

a $P = \$1000$, $r = 5\%$, $T = 2$ years

b $P = \$1200$, $r = 2.5\%$, $T = 3$ years

c $P = \$5000$, $r = 4\%$, $T = 5$ years

18 It takes 6 window washers 6 minutes to wash 6 windows. How long does it take 12 window washers to wash 12 windows?

19 Siv starts rowing across a 400 m wide river at a speed of $\frac{14}{9}$ m/s. At the same time Riley starts swimming from the other side of the river at a rate of 2 min per 100 metres. How far from Siv's starting point do they meet?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Rates

Investigation
Solution rates

Topic quiz
3G

Chapter summary

Percentage, fractions and decimals

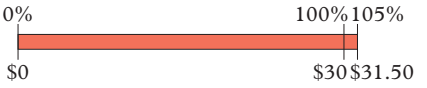
	Fraction	Decimal
Percentage to…	Write the percentage as a fraction with a denominator of 100. $15\% = \frac{15}{100}$ $= \frac{3}{20}$	Divide the percentage by 100. $23.4\% = \frac{23.4}{100}$ $= 23.4 \div 100$ $= 0.234$

	Percentage	Decimal
Fraction to…	Multiply the fraction by 100% and simplify. $\frac{1}{3} \times 100\% = \frac{100}{3}\%$ $= 33\frac{1}{3}\%$	Divide the numerator by the denominator. Add trailing zeros as required. Align decimal points Dividend → 1, Divisor → 8: $8\overline{)1.000}$ = 0.125 Trailing zeros

	Percentage	Fraction
Decimal to…	Multiply the decimal by 100 and include the percentage symbol %. $0.65 = 65\%$	Place the decimal as the numerator of the fraction and the denominator 10, 100, 1000 … with as many zeros as there are digits after the decimal point. $0.65 = \frac{65}{100}$

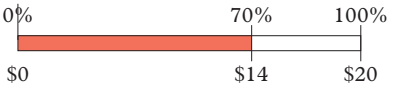
Mark-ups (percentage increase)

Increase \$30 by 5% = (100% + 5%) × \$30
= 105% × \$30
= 1.05 × \$30
= \$31.50



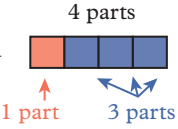
Discounts (percentage decrease)

Decreases \$20 by 30% = (100% – 30%) × \$20
= 70% × \$20
= 0.7 × \$20
= \$14



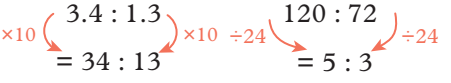
Ratios

$\frac{1}{4}$ of the squares are red



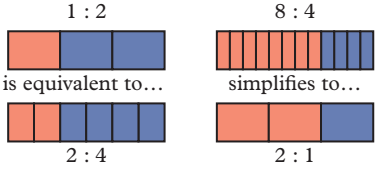
$\frac{3}{4}$ of the squares are blue

Simplifying ratios

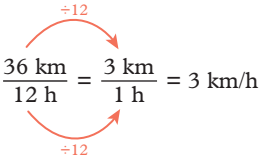


Equivalent ratios

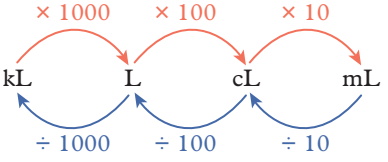


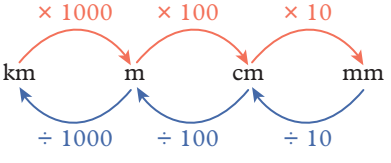


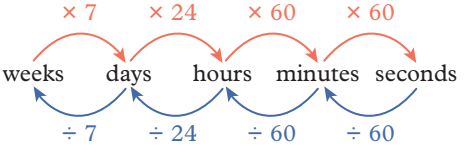
Rates



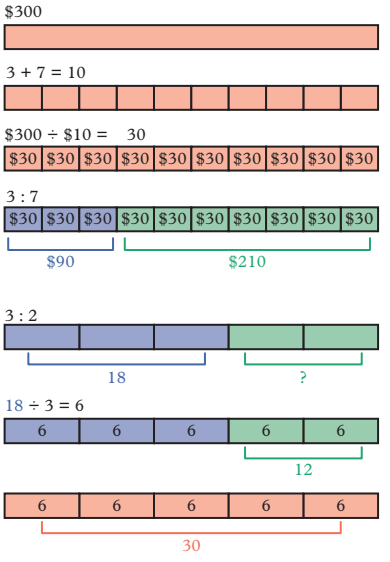
Unit conversions







Dividing a quantity into a ratio



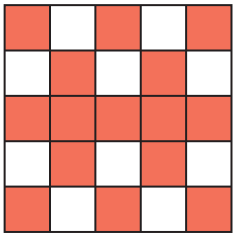
Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Compete in teams to test your knowledge of legal definitions

Multiple choice

3A **1** What percentage of this figure is shaded?



A 10% **B** 15% **C** 40% **D** 50% **E** 60%

3A **2** What is $\frac{1}{5}$ written as a percentage?

A 10% **B** 15% **C** 20% **D** 25% **E** 95%

3A **3** Which value is *not* the same as 75%?

A 0.75 **B** 75 **C** $\frac{75}{100}$ **D** $\frac{3}{4}$ **E** $\frac{15}{20}$

3B **4** What is 45% of 360?

A 162 **B** 198 **C** 162% **D** 315 **E** 315%

3C **5** After a 32% discount, what is the selling price of an item originally marked $480?

A $153.60 **B** $326.40 **C** $448 **D** $633.60 **E** $1500

3D **6** Written as a ratio, what is 3 hours to 37 minutes?

A 3 : 37 **B** 37 : 3 **C** 37 : 180 **D** 33 : 7 **E** 180 : 37

3E **7** Which ratio is the same as 16 : 28 : 12 written in simplest form?

A 4 : 7 : 3 **B** 3 : 7 : 4 **C** 8 : 14 : 6 **D** 6 : 18 : 2 **E** 32 : 56 : 24

3E **8** What is the value of a in 3 : 7 = a : 35?

A 7 **B** 14 **C** 15 **D** 21 **E** 33

3F **9** When $3600 is divided in the ratio 1 : 3, what are the parts?

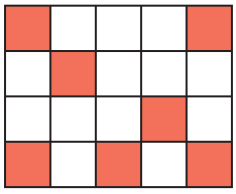
A $1200 and $2400 **B** $900 and $2700 **C** $1500 and $2100
D $1200 and $2700 **E** $3600 and $10 800

3G **10** What is 120 metres in 24 seconds written as a rate in its simplest form?

A $\frac{120}{24}$ m/s **B** $\frac{24}{120}$ m/s **C** $\frac{1}{5}$ m/s **D** 5 m/s **E** 6 m/s

Short answer

3A **1** Consider this figure.



a How many shaded sections are there?
b Write the number of shaded sections as a fraction of the total number of sections.
c Write the fraction in part **b** as an equivalent fraction with denominator 100.
d Write the shaded amount as a percentage.

3A **2** Write each percentage as a decimal.

a 23% **b** 62% **c** 16.52% **d** 291.6%

3A **3** Write each decimal as a percentage.

a 0.32 **b** 7.62 **c** 31.625 **d** 0.0003

3A **4** Write each fraction as a percentage.

a $\frac{3}{4}$ **b** $\frac{7}{10}$ **c** $\frac{17}{20}$ **d** $\frac{15}{32}$

3B **5** Write:

a 20 as a percentage of 50 **b** 24 as a percentage of 150 **c** 58 as a percentage of 125

3B **6** Calculate:

a 10% of 340 **b** 25% of 62 **c** 150% of 1500 **d** 37% of 2400

3C **7** Calculate the price to be paid after each mark-up or discount.

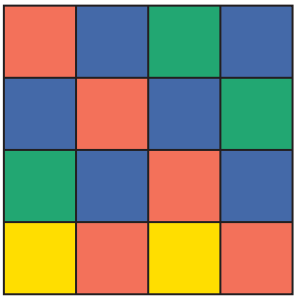
a a 10% mark-up on $950 **b** a 44% mark-up on $628 **c** a 28% discount on $1255

3C **8** Calculate the percentage mark-up or discount.

a a radio with cost price $40 and selling price $55 **b** a book with cost price $35 and selling price $21

c shares with cost price $2.98 and selling price $4.52 **d** a TV with cost price $1400 and selling price $958

3D **9** Refer to this coloured grid.



a Write each comparison as a ratio.

i red sections to blue sections

ii yellow sections to green sections

b Write each comparison as a fraction.

i green sections to total sections

ii yellow sections to total sections

3D **10** Write each comparison as a ratio.

a 35 kg to 47 kg **b** 7 weeks to 31 weeks **c** 8 months to 3 years **d** 45 cents to $1.25

3E **11** Simplify each ratio.

a 12 : 44 **b** 64 : 108 **c** 16 : 32 : 96 **d** 15 : 75 : 105

3E **12** Calculate the value of each pronumeral.

a $6 : 7 = a : 49$ **b** $13 : 4 = 52 : b$ **c** $c : 48 = 3 : 12$ **d** $15 : d = 3 : 7$

3F **13** Calculate the total number of parts for each ratio.

a 3 : 5 **b** 12 : 5 **c** 4 : 5 : 8 **d** 6 : 10 : 15

3F **14** Divide $1500 in each ratio.

a 2 : 3 **b** 3 : 7 **c** 3 : 4 : 8 **d** 4 : 5 : 11

3G **15** Write each statement as a rate with the appropriate unit.

a 100 km per hour **b** 120 L in 40 seconds

c $140 for 8 hours of work **d** 26 books read in 6 months

3G **16** Write each statement as a rate in simplest form.

a a football team scored 180 points in 120 minutes of play

b the cost of 45 L of petrol was $62.55

c a 375 mL can of drink costs $1.80

d a worker earned $218.40 for 13 hours of work

Analysis



1 A market stall holder bought a variety of coloured 'hoodies' to sell. Each hoodie cost the stall holder $20 and she intends to sell them to the public at a mark-up of 65%.

a Write the percentage mark-up value as a:

i fraction in simplest form

ii decimal.

b Calculate the value of the mark-up on each hoodie.

c What price will the hoodies be?

d If the stall holder sells 35 hoodies on the first day, what is the difference between the total cost price of the hoodies and total amount they sell for?

To improve the sales of the hoodies for her second day at the market, the stall holder decides to offer a 10% discount on the selling price.

e What is the discount amount for each hoodie?

f What is the new selling price?

g What profit does the stall holder make on the sale of each hoodie?

h The stall holder sells 55 hoodies at the discounted amount. Write the profit from the sale of these as a percentage of the cost she paid for them.



2 Kris is doing some landscaping in her backyard.

Kris buys 3 cubic metres of Terracotta mulch at $75 per cubic metre and 5 cubic metres of Black mulch at $60 per cubic metre.

a What is the ratio of Terracotta mulch to Black mulch in simplest form?

b What price per cubic metre (to the nearest 5 cents) did Kris pay overall for mulch?

For her rock garden Kris decides she needs to buy River Rocks at $170 per cubic metre and Honeycomb Rocks at $100 per cubic metre. The ratio of River Rocks to Honeycomb Rocks will be 2 : 3 and Kris buys 1.6 cubic metres of River Rocks.

c How many cubic metres of Honeycomb Rocks does she buy?

d What is the total cost of the rocks?

Kris bought a total of 36 plants to put in her garden. The three types of plants were Grevillea, Callistemon and Banksia in the ratio 2 : 3 : 4.

e How many Callistemon plants did she buy?

f How many Banksia plants did she buy?

g Kris decides to increase the number of plants by 25%, while keeping the same ratio. How many extra of each plant does she buy?

3 This is my go-to Trail Mix recipe:

$\frac{3}{4}$ cup pecans
$\frac{3}{4}$ cup cashews
$\frac{1}{2}$ cup sunflower seeds
$\frac{1}{2}$ cup pumpkin seeds
$\frac{1}{2}$ cup cherries
$\frac{1}{3}$ cup raisins
$\frac{2}{3}$ cup chocolate

a Find the ratio of nuts to seeds in simplest form.

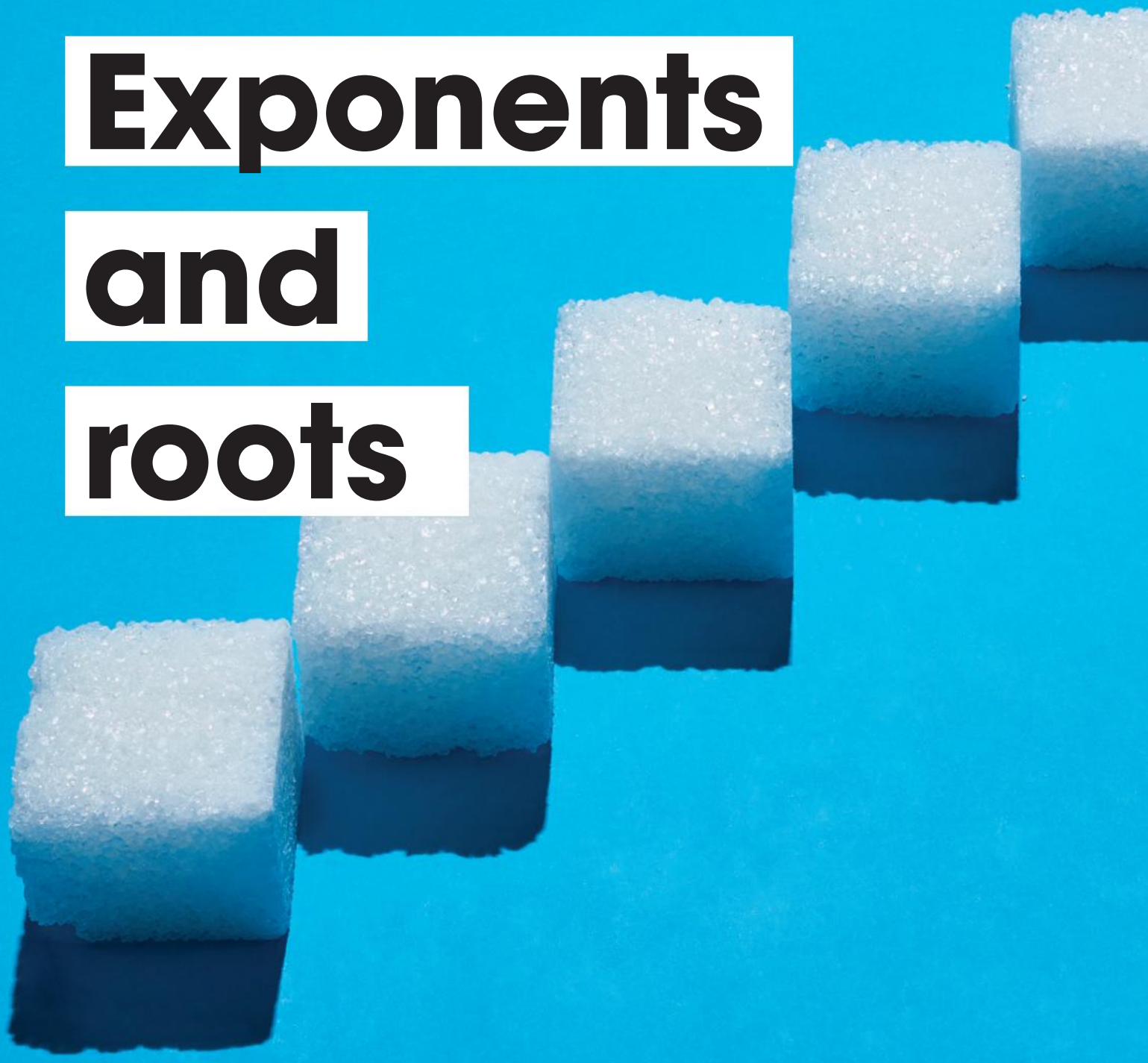
b Find the ratio of nuts to fruits in simplest form.

c How much chocolate would I need for 12 cups of trail mix?

d I decide to leave out the fruit and chocolate but want to have the same amount of trail mix as the original recipe. How much extra of each of the nuts and seeds do I need if I keep to the same ratio?

4

Exponents and roots





Index

4A Exponents
4B Products and quotients of powers
4C Raising exponents and the zero exponent
4D Combining the exponent laws
4E Roots

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Times tables
- ✔ Order of operations
- ✔ Prime and composite numbers
- ✔ Multiplying and dividing fractions
- ✔ Multiplying decimals

Curriculum links

- Recognise irrational numbers in applied contexts, including square roots and π (AC9M8N01)
- Establish and apply the exponent laws with positive integer exponents and the zero-exponent, using exponent notation with numbers (AC9M8N02)

© ACARA

Materials

- ✔ Calculator

4A Exponents

Learning intentions

By the end of this topic you will be able to ...

- ✔ convert numbers between exponent and expanded form
- ✔ calculate the value of numbers in exponent form
- ✔ express integers as a product of prime factors.

Inter-year links

Year 7	1G Exponents and square roots
Year 9	1A Exponents
Year 10	1A Exponents

Exponent notation

- **Exponent notation** or **exponent form** is used to represent repeated multiplication.
  - → 3^4 is read as '3 to the power of 4'.
- The base is the number that is repeatedly multiplied.
- The exponent or index indicates how many times the base appears in expanded form.
- If no exponent is shown, the implied exponent is 1.
  For example, $2 = 2^1$.
- The **square** of a number is the result of multiplying a number by itself.
  For example, 3^2 can be read as '3 to the power of 2', '3 squared' or 'the square of 3'.
- The **cube** of a number is the result of raising that number to the power of 3.
  For example, 4^3 can be read as '4 to the power of 3', '4 cubed' or 'the cube of 4'.

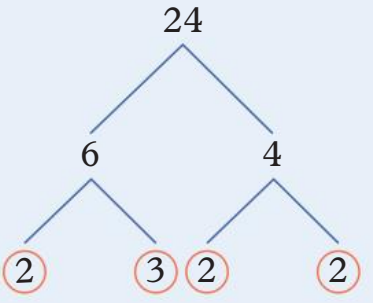
index/exponent

base → $3^4 = 3 \times 3 \times 3 \times 3 = 81$

exponent form | expanded form | basic numeral

Prime factorisation

- The **prime factorisation** of an integer is the product of the primes that make up that integer.
  - → Prime factorisation is often expressed in exponent form with the bases listed in ascending order.
    For example, the prime factorisation of $24 = 2^3 \times 3$.
- Prime factorisation can be performed using **factor trees**. In factor trees, composite numbers are broken down into pairs until prime numbers are found.



Example 4A.1 Calculating the value of a number in exponent form

Write 2^5 in expanded form and calculate its value.

THINK	WRITE
1 Identify the base and the exponent. Write the base in expanded form by multiplying five twos together.	$2^5 = 2 \times 2 \times 2 \times 2 \times 2$
2 Perform the multiplication.	$= 32$

Example 4A.2 Calculating squares and cubes

Calculate:

a 4^2 **b** the square of 6 **c** 3 cubed

THINK

1. Identify the base and the exponent. 'Square' refers to an exponent of 2 and 'cube' refers to an exponent of 3.
2. Perform the multiplication.

WRITE

a $4^2 = 4 \times 4$
$= 16$

b $6^2 = 6 \times 6$
$= 36$

c $3^3 = 3 \times 3 \times 3$
$= 27$

Example 4A.3 Prime factorisation using factor trees

Use factor trees to express each number as the product of its prime factors. Write your answer in exponent form.

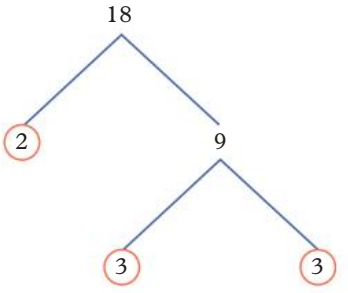
a 18 **b** 225

THINK

1. Divide the composite number by its lowest prime factor to identify a factor pair. The lowest prime factor of even numbers is always 2. Remember that if the sum of all the digits in a number is divisible by 3, that number is also divisible by 3.
2. Repeat step **1** until each branch ends in a prime number.
3. Write the initial composite number as a product of its prime factors. Write the answer in exponent form and list the bases in ascending order.

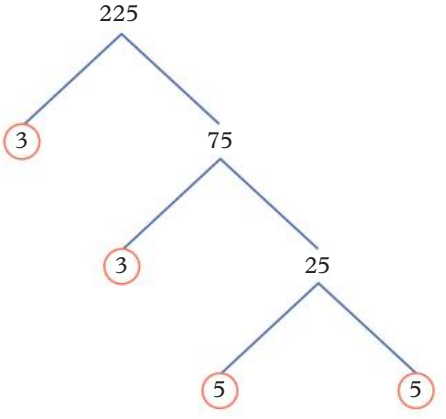
WRITE

a



$18 = 2 \times 3 \times 3$
$= 2 \times 3^2$

b



$225 = 3 \times 3 \times 5 \times 5$
$= 3^2 \times 5^2$

Helpful hints

- ✔ Take care when writing exponents – they should be written smaller than the base and sit high up on the shoulder of the base to avoid confusion between 3^4 and 34.
- ✔ Remember that exponentiation and multiplication are two different operations.
  For example, $3^2 \neq 3 \times 2$, $3^2 = 3 \times 3$.
- ✔ Read questions involving exponents carefully. You may be asked to:
  - → Write a number in expanded form; for example, $3^2 = 3 \times 3$
  - → Calculate or evaluate; for example, $3^2 = 9$

ANS p508

Exercise 4A Exponents

▲ 1–11, 12(a, c), 13, 14 ■ 3–11, 12(c, d), 13, 15–17 ⬟ 5–7, 10, 12(c, d), 15–18

UNDERSTANDING AND FLUENCY

1 The number 5^3 is written in exponent form.

a State the value of the base.

b State the value of the exponent.

c Write the number in expanded form, as a repeated multiplication.

d Perform the repeated multiplication to convert 5^3 into a basic numeral.

4A.1 **2** Write each number in expanded form and calculate its value.

a 2^5 **b** 4^3 **c** 1^9 **d** 3^3 **e** 10^7

f 2^8 **g** 5^4 **h** 7^2 **i** 8^3 **j** 6^4

4A.2 **3** Calculate:

a 8^2 **b** 7 squared **c** the square of 5

d 11^2 **e** 6 squared **f** the square of 1

g 60^2 **h** 100 squared **i** the square of 20

4 Calculate:

a 3^3 **b** 2 cubed **c** the cube of 4

d 100^3 **e** 10 cubed **f** the cube of 5

5 Complete the table below.

	Exponent form	Base	Exponent	Expanded form	Basic numeral
a	3^4	3	4	$3 \times 3 \times 3 \times 3$	
b	2^5	2			32
c			2	8×8	
d				6×6	

6 Write each number in expanded form and calculate its value.

a $(0.8)^2$ **b** $(3.4)^2$ **c** $(0.5)^3$ **d** $(0.3)^4$ **e** $(0.1)^5$ **f** $(0.2)^7$

7 Write each number in expanded form and calculate its value.

a $\left(\frac{1}{4}\right)^3$ **b** $\left(\frac{7}{9}\right)^2$ **c** $\left(\frac{5}{6}\right)^3$ **d** $\left(\frac{3}{10}\right)^4$ **e** $\left(\frac{1}{2}\right)^5$ **f** $\left(3\frac{1}{3}\right)^6$

8 Use a calculator to evaluate the following numbers.

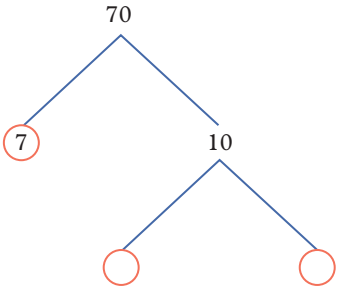
a 57^2 **b** 101^3 **c** 13^7

d $(6.85)^3$ **e** $(1.9)^4$ **f** $(12.4)^5$

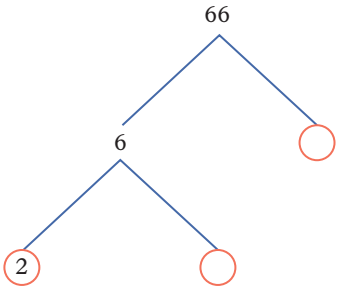
g $\left(\frac{18}{23}\right)^3$ **h** $\left(\frac{7}{11}\right)^6$ **i** $\left(\frac{3}{5}\right)^9$

9 Complete each factor tree, then write the prime factorisation of each composite number in exponent form.

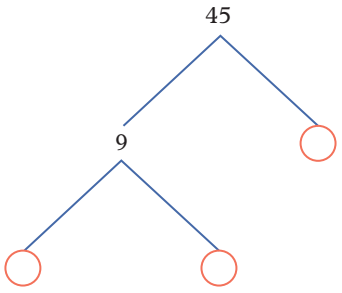
a



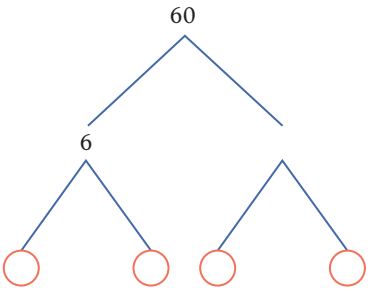
b



c



d



4A.3 **10** Use a factor tree to express each number as the product of its prime factors. Write your answer in exponent form.

a 75 **b** 48 **c** 90

d 126 **e** 288 **f** 1092

11 A student incorrectly evaluates 4 squared as $4^2 = 8$. Identify the student's error and explain why $4^2 \neq 8$.

12 Use the correct order of operations to complete the following calculations.

a $2^3 + 8^2 \div 4$ **b** $5^2 \times 0.2 + 3^4$ **c** $5.8 - (1.2)^2 + (0.1)^4$ **d** $\frac{7}{10} + \left(\frac{2}{5}\right)^3 \times 6\frac{1}{4}$

13 a Find:

i 1^2 **ii** 11^2 **iii** 111^2 **iv** 1111^2

b Describe the pattern you see.

c Without using a calculator, use the pattern to write the results for:

i $11\,111^2$ **ii** $111\,111^2$ **iii** $1\,111\,111^2$ **iv** $11\,111\,111^2$

14 For the six weeks of the school holidays, Sarah's parents offer to pay her \$30 per week for doing some extra jobs around the house.

a How much would Sarah earn over the holidays if she agrees to this arrangement?

b Instead, Sarah suggests that her parents pay her \$3 in the first week, then three times as much as the previous week in each of the following weeks.

How much would Sarah earn in each week of the holidays under her suggested arrangement?

c Compare the total amount that Sarah could earn over the school holidays from each arrangement. Which plan do you think Sarah's parents will agree to?

d If Sarah's parents agree to her suggested arrangement over a longer period, express the amount that Sarah would earn in her eleventh week using exponent notation.

e Calculate how much Sarah would earn in the eleventh week of her arrangement.

PROBLEM SOLVING AND REASONING

15 The size of a colony of bacteria containing 1000 cells doubles every hour.

a How large is the colony after:

i one hour

ii two hours

iii three hours?

b Use your answers to part **a** to complete the second row of the table below.

Number of hours	1	2	3	4	5
Size of colony					
Number of times larger than original amount		4		16	

c How many times larger than the original colony is the new colony after:

i one hour

ii two hours

iii three hours?

d Use your answers to part **c** to complete the third row of the table.

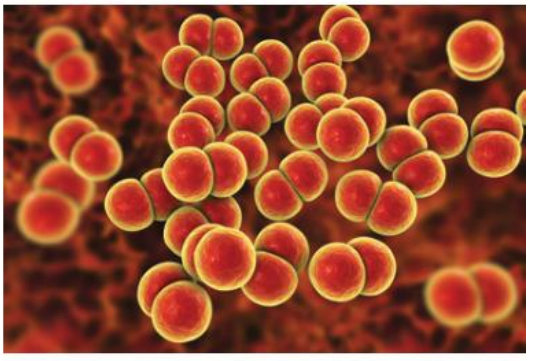
e Express each of the numbers in the third row of the table in exponent form with a base of 2.

f In exponent form, how many times larger than the original colony is the new colony after:

i 6 hours

ii 10 hours

iii 24 hours?



g Use a calculator to work out how many times larger the colony will be after 24 hours.

h Use your answer to part **g** to determine the number of cells in the colony after 24 hours.

16 Prime factorisation can be used to find highest common factors (HCF).

a Write 144 as the product of its prime factors.

b Write 270 as the product of its prime factors.

c Identify the prime factors that are shared by both numbers, then multiply those factors together for the HCF.

d Use prime factorisation to find the HCF of the following pairs of numbers.

i 117 and 182 **ii** 420 and 132

iii 495 and 220 **iv** 432 and 405

CHALLENGE

17 A single parasite invades a host. The number of parasites doubles every 3 days. How many days until there are over 1 million parasites in the host?

18 The half-life of a chemical substance refers to the time it takes for the amount of that chemical to decrease by a half. The half-life of caffeine in your body is 6 hours. If a single cup of coffee contains c milligrams of caffeine and you drink one cup of coffee at 8 am, then two more cups at 2 pm, how much caffeine is still in your body at 8 pm in terms of c?



Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Exponents

Interactive skillsheet
Prime factorisation

Investigation
What are the odds?

Topic quiz
4A

4B Products and quotients of powers

Learning intentions

By the end of this topic you will be able to ...

- ✔ simplify products of numbers with the same base using an exponent law
- ✔ simplify quotients of numbers with the same base using an exponent law.

Inter-year links

Support	Adding and subtracting whole numbers
Year 7	1B Adding whole numbers
Year 9	1B Products and quotients of powers
Year 10	1A Exponents

The exponent laws

- **Simplification** involves reducing the complexity of a calculation.
- The **exponent laws** are a set of rules that can be used to simplify calculations involving numbers in exponent form.
- Exponent laws only apply to numbers with the *same base*.

Product of powers law

- When multiplying numbers in exponent form with the same base, add the exponents.

$$\begin{aligned} 2^3 \times 2^5 &= (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^{3+5} \\ &= 2^8 \end{aligned}$$

Quotient of powers law

- When dividing numbers in exponent form with the same base, subtract the second exponent from the first exponent.

$$\begin{aligned} 2^5 \div 2^3 &= (2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2 \times 2) \\ &= 2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2} \div (\cancel{2} \times \cancel{2} \times \cancel{2}) \\ &= 2^{5-3} \\ &= 2^2 \end{aligned}$$

- Remember that quotients can be written as fractions. When simplifying fractional quotients, subtract the exponent of the number in the denominator from the exponent of the number in the numerator.

$$\begin{aligned} \frac{2^5}{2^3} &= \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} \\ &= \frac{2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2}} \\ &= 2^{5-3} \\ &= 2^2 \end{aligned}$$

Example 4B.1 Simplifying products using an exponent law

Use an exponent law to simplify the following products.

a $4^6 \times 4^4$ **b** 9×9^7

THINK

1 Check that the bases are the same.

2 Write the base and add the exponents. Remember that $9 = 9^1$.

WRITE

a $4^6 \times 4^4 = 4^{(6+4)}$
$= 4^{10}$

b $9 \times 9^7 = 9^{(1+7)}$
$= 9^8$

Example 4B.2 Simplifying quotients using an exponent law

Use an exponent law to simplify the following quotients.

a $3^{11} \div 3^{10}$ **b** $\frac{8^5}{8^2}$

THINK

a **1** Check that the bases are the same.

2 Write the base and subtract the second exponent from the first exponent. Simplify the answer by leaving out the exponent of 1.

b **1** Check that the bases are the same.

2 Write the base and subtract the exponent of the number in the denominator from the exponent of the number in the numerator.

WRITE

a $3^{11} \div 3^{10} = 3^{(11-10)}$
$= 3^1$
$= 3$

b $\frac{8^5}{8^2} = 8^{(5-2)}$
$= 8^3$

Helpful hints

✔ 'Simplify' and 'evaluate' are two different command terms:
- → To simplify a calculation in this chapter, use exponent laws to combine the terms, and hence reduce the complexity of the calculation.
- → To evaluate, or 'find the value' of a calculation, convert that calculation from exponent form into a basic numeral.

ANS p510

Exercise 4B Products and quotients of powers

△ 1–8, 9–10(a–c), 12, 13(a, b) □ 5–8, 9–10(d–f), 11, 13, 14(a, b) ⬠ 5–7, 9–10(d–f), 11, 13, 14

UNDERSTANDING AND FLUENCY

4B.1 **1** Use an exponent law to simplify the following products:

a $3^3 \times 3^2$ **b** $4^5 \times 4^3$ **c** $6^2 \times 6^4$

d $5^3 \times 5^6$ **e** $8^2 \times 8$ **f** 2×2^4

2 Evaluate each of your answers to question **1** by converting each number into a basic numeral.

4B.2 **3** Use an exponent law to simplify the following quotients:

a $9^{14} \div 9^{11}$ **b** $4^6 \div 4^5$ **c** $6^4 \div 6$

d $\frac{4^7}{4^2}$ **e** $\frac{10^8}{10^5}$ **f** $\frac{6^{11}}{6^7}$

4 Evaluate each of your answers to question **3**.

5 Use an exponent law to first simplify, then evaluate the value of each of the following calculations.

a $2^5 \times 2^6$ **b** $5^2 \times 5^4$ **c** $3^4 \times 3^3$ **d** $4^6 \times 4^2$

e $5^7 \div 5^4$ **f** $\frac{3^9}{3^3}$ **g** $9^6 \div 9^3$ **h** $2^{12} \div 2^5$

i $6^3 \times 6^2$ **j** $\frac{8^5}{8^4}$ **k** $10^4 \times 10^5$ **l** $\frac{7^8}{7^6}$

6 The exponent laws also apply to non-integer bases. Use an exponent law to simplify each of the following calculations.

a $(1.5)^3 \times (1.5)^6$ **b** $(0.7)^4 \times (0.7)^2$ **c** $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^4$ **d** $\left(\frac{7}{5}\right)^6 \times \left(\frac{7}{5}\right)^8$

e $(9.2)^8 \div (9.2)^5$ **f** $(0.63)^{11} \div (0.63)^4$ **g** $\left(\frac{3}{4}\right)^6 \div \left(\frac{3}{4}\right)^2$ **h** $\left(\frac{5}{2}\right)^9 \div \left(\frac{5}{2}\right)^7$

7 Determine the value of the missing exponent in the calculations below.

a $7^5 \times 7^{\square} = 7^{15}$ **b** $6^{\square} \times 6 = 6^6$ **c** $2^{13} \div 2^{\square} = 2^5$

d $\frac{9^7}{9^{\square}} = 9^3$ **e** $\frac{9^{\square}}{9^7} = 9^{14}$ **f** $\frac{12^{\square}}{12^{15}} = 144$

8 Complete the following calculations by filling in the missing exponents.

a $5^2 \times 5^5 \times 5^4 = 5^7 \times 5^4$
$= 5^{\square}$

b $20^8 \times 20 \times 20^{10} = 20^{\square} \times 20^{10}$
$= 20^{\square}$

c $8^{32} \div 8^7 \div 8^{10} = 8^{25} \div 8^{\square}$
$= 8^{\square}$

d $7^{24} \div 7^6 \div 7 = 7^{\square} \div 7^{\square}$
$= 7^{\square}$

9 Use an exponent law to simplify these. Leave your answer in exponent form.

a $2^5 \times 2^4 \times 2^3$ **b** $3^3 \times 3 \times 3^2$ **c** $10^3 \times 10^2 \times 10^5$

d $4^2 \times 4^4 \times 4^3$ **e** $5^7 \times 5^3 \times 5^6$ **f** $9 \times 9^5 \times 9^8$

10 Use an exponent law to simplify these. Leave your answer in exponent form.

a $6^{10} \div 6^5 \div 6^3$ **b** $7^{12} \div 7^2 \div 7^6$ **c** $12^{16} \div 12^9 \div 12^6$

d $8^{10} \div 8^3 \div 8^3$ **e** $5^{15} \div 5^6 \div 5^8$ **f** $11^{11} \div 11 \div 11^5$



11 Light-years are a distance unit defined by the distance that light travels in one year. One light-year is approximately 10^{13} km. Icarus is the farthest star from Earth to have been detected so far. If Icarus is 10^{23} km from Earth, how many light-years is Icarus from Earth? Express your answer in exponent form.

12 Identify and explain the mistake made in each of the following calculations, then give the correct answer in each instance.

a $5^8 \div 5^2 = 5^4$ **b** $2^3 \times 2^3 = 2^9$ **c** $\frac{4^7}{4^7} = 4^1$ **d** $9^8 \div 9^{10} = 9^2$

13 a Given $(-3)^2 = -3 \times (-3) = 9$, evaluate:

i $(-2)^2$ **ii** $(-2)^3$ **iii** $(-2)^4$ **iv** $(-2)^5$

b Describe the pattern in your answers to part **a**.

c Explain how you can determine the sign of your answer when evaluating a negative number raised to a given power.

d Evaluate:

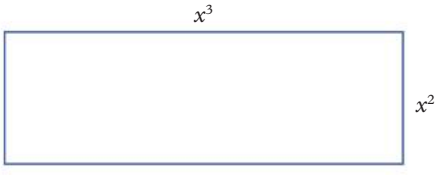
i $(-3)^3$ **ii** $(-2)^6$ **iii** $(-10)^4$ **iv** $(-10)^9$

14 The exponent laws can also be applied to terms containing pronumerals.

a Express the area of this rectangle as a product.

b Simplify your answer to part **a**.

c If the area of the rectangle is 7776 mm^2, what are the dimensions of the rectangle?



Check your Student obook pro for these digital resources and more:

- **Interactive skillsheet** — Products and quotients of powers
- **Worksheet** — Multiplying and dividing using exponent form
- **Investigation** — Growing cubes
- **Topic quiz** — 4B

4C Raising exponents and the zero exponent

Learning intentions

By the end of this topic you will be able to ...

- ✔ raise a number in exponent form by another exponent using an exponent law
- ✔ evaluate calculations involving the zero exponent.

Inter-year links

Support	Multiplying and dividing whole numbers
Year 7	1D Multiplying whole numbers
Year 9	1C Raising exponents and the zero exponent
Year 10	1A Exponents

Power of a power law: Raising exponents by another exponent

- When raising a number in exponent form by another exponent, multiply the exponents.
- Every base number inside brackets should have its exponent multiplied by the exponent outside the brackets.

$$(2^3)^5 = 2^{(3 \times 5)} = 2^{15}$$

$$(2 \times 3)^5 = 2^5 \times 3^5$$

$$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$$

The zero exponent

- Excluding 0, any number to the power of 0 is equal to 1.

$2^0 = 1$

Example 4C.1 Raising an exponent by another exponent

Use an exponent law to simplify $(3^4)^5$.

THINK

Write the base and multiply the exponents.

WRITE

$(3^4)^5 = 3^{(4 \times 5)}$

$= 3^{20}$

Example 4C.2 Distributing exponents across brackets

Simplify:

a $(4 \times 7)^2$ **b** $\left(\frac{5}{2}\right)^4$

THINK

Multiply the exponent of every base inside the brackets by the exponent outside the brackets. Remember that base numbers that do not appear to have an exponent have an exponent of 1, so $4 = 4^1$.

WRITE

a $(4 \times 7)^2 = 4^{(1 \times 2)} \times 7^{(1 \times 2)}$

$= 4^2 \times 7^2$

b $\left(\frac{5}{2}\right)^4 = \frac{5^{(1 \times 4)}}{2^{(1 \times 4)}}$

$= \frac{5^4}{2^4}$

Example 4C.3 Evaluating calculations involving the zero exponent

Evaluate:

a 23^0 **b** $8^2 \div 8^2$

THINK

a Any number, excluding 0, to the exponent of 0 is equal to 1.

b When dividing numbers in exponent form with the same base, subtract the second exponent from the first exponent.

WRITE

a $23^0 = 1$

b $8^2 \div 8^2 = 8^{(2-2)}$
$= 8^0$
$= 1$

Helpful hints

✔ Take care not to mix up the exponent laws.
- → across a multiplication sign, add exponents
- → across a division sign, subtract exponents
- → across brackets, multiply exponents

✔ Remember that $2^0 = 1$, *not* 0.

Exponent law	Example
Product of powers law	$2^5 \times 2^3 = 2^{5+3}$
Quotient of powers law	$2^5 \div 2^3 = 2^{5-3}$
Power of a power law	$(2^5)^3 = 2^{5\times 3}$

ANS p510

Exercise 4C Raising exponents and the zero exponent

△ 1–7, 9(a–c), 12 □ 1, 2, 4–6, 8, 9, 11–13 ⬠ 1, 2, 5, 8–10, 12–15

4C.1 **1** Use an exponent law to simplify:

a $(6^4)^5$ **b** $(2^6)^2$ **c** $(7^4)^3$

d $(11^2)^4$ **e** $(8^5)^3$ **f** $(3^7)^2$

4C.2 **2** Simplify:

a $(5 \times 9)^3$ **b** $\left(\frac{3}{4}\right)^6$ **c** $\left(\frac{9}{5}\right)^4$

d $(11 \times 4)^7$ **e** $\left(\frac{7}{10}\right)^3$ **f** $(3 \times 8)^9$

4C.3 **3** Evaluate:

a 4^0 **b** 15^0 **c** 144^0

4 Use an exponent law to simplify, then evaluate:

a 3.2^0 **b** $(2 \times 3)^0$ **c** $\left(\frac{7}{8}\right)^0$

d $9^3 \div 9^3$ **e** $\frac{8^5}{8^5}$ **f** $\frac{4}{4}$

5 Use an exponent law to simplify each of the following calculations.

a $(0.5^4)^3$ **b** $(1.2^8)^0$ **c** $(3.8^5)^5$

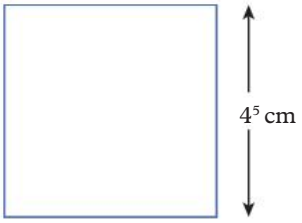
d $(2^5 \times 2.5^2)^4$ **e** $(0.7^7 \times 5.4^3)^0$ **f** $(4.8^0 \times 6.6^6)^3$

UNDERSTANDING AND FLUENCY

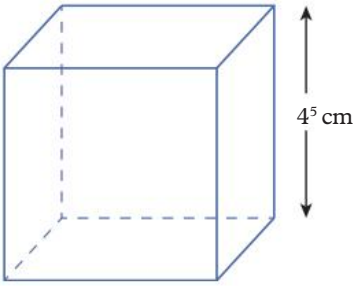
PROBLEM SOLVING AND REASONING

6 Writing your answer in exponent form, determine the value of:

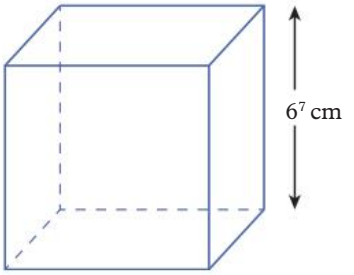
a the area of this square



b the volume of this cube



c the surface area of this cube.



7 Determine the value of the missing exponent in the calculations below.

a $(7^3)^{\square} = 7^{18}$ **b** $(9^{\square})^5 = 9^{20}$ **c** $(8^4)^{\square} = 1$

d $(6 \times 8)^{\square} = 6^5 \times 8^5$ **e** $\left(\frac{4}{15}\right)^{\square} = \frac{4^{10}}{15^{10}}$ **f** $\left(\frac{2}{5}\right)^{\square} = 1$

8 Complete the following calculations by filling in the missing numbers.

a $(2^3)^4 = \square^4 = \square^3 = 2^{\square}$

b $10^{5 \times 4} = 100\,000^{\square} = 10\,000^{\square} = 10^{\square}$

c $8^8 = \square^{24} = 4^{\square} = 16^{\square}$

9 Complete the following calculations by filling in the missing exponent.

a $(2^2)^{\square} = 32$ **b** $(2 \times 3)^{\square} = 36$ **c** $\left(\frac{1}{5}\right)^{\square} = \frac{1}{125}$

d $(4^{\square})^3 = 64$ **e** $(3 \times 3)^{\square} = 81$ **f** $\left(\frac{2}{3}\right)^{\square} = \frac{16}{81}$

10 Without evaluating, list the following numbers in ascending order.

3^{14}, $(3^2)^5$, $(3^4)^3$, $(3^{15})^0$, $((3^2)^3)^3$

11 Consider the expression $\frac{2^5}{2^5}$.

a Calculate the value of the expression.

b Use an exponent law to simplify the expression. Leave your answer in exponent form.

c Use this method to write a calculation showing that $15^0 = 1$.



12 Decide whether the numerical statements are true or false.

a $(5^3)^4 = (5^2)^6$

b $(2^4)^6 = (2^8)^3$

c $(4 \times 5)^6 = (20^3)^3$

d $(2 \times 3)^8 = (6^2)^4$

CHALLENGE

13 Find the value of x:

a $(7^x)^5 = 7^{20}$

b $(7^x \times 7^x)^5 = 7^{20}$

14 Show that $(2^3)^6 = 4^9$.

15 If $x^y = y^x$ and $x \neq y$, what are the smallest, positive integer values of x and y?

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Raising exponents and the zero exponent

Investigation
Bert and Ernie's exponent laws

Topic quiz
4C

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

4A 1 Write each number in expanded form and calculate its value.

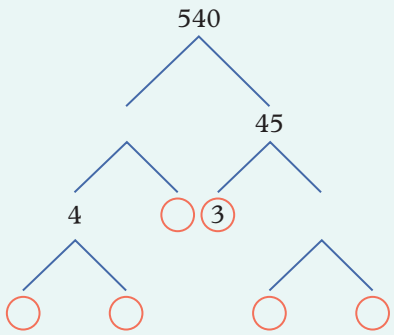
a 2^4 **b** 3^4 **c** 5^3 **d** 10^6

4A 2 Calculate:

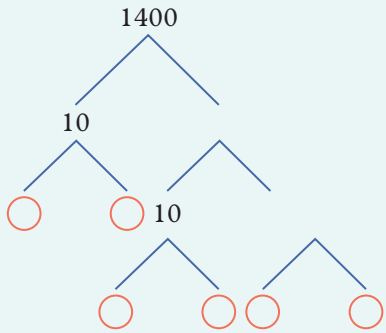
a 5 squared **b** the square of 9 **c** 4 cubed **d** the cube of 6

4A 3 Complete each factor tree, then write the prime factorisation of each number in exponent form.

a



b



4A 4 Use a factor tree to express each number as the product of its prime factors. Write your answer in exponent form.

a 24 **b** 72 **c** 195 **d** 350

4A 5 A scientist studies a colony of bacteria in a petri dish and notes the following information.

Number of minutes	0	1	2	3	4	5
Size of colony	10	30	90			
Number of times as large as original amount	1	3	9			

a Complete the table.

b Express each of the numbers in the third row of the table in exponent form with a base of 3.

c In exponent form, how many times larger than the original colony will the new colony be after:

i 10 minutes **ii** 1 hour **iii** 24 hours?

d Use a calculator to determine how many bacteria will be in the colony after 10 minutes.

4B 6 Use an exponent law to simplify the following products.

a $2^5 \times 2^3$ **b** $7^{13} \times 7^7$ **c** $11^5 \times 11$ **d** $9^9 \times 9^9$

4B 7 Use an exponent law to simplify the following quotients.

a $3^{10} \div 3^6$ **b** $10^{11} \div 10$ **c** $\frac{14^6}{14^2}$ **d** $\frac{8^{17}}{8^9}$

4B 8 Simplify then evaluate each of the following calculations.

a $2^2 \times 2^4$ **b** $5^3 \times 5$ **c** $6^7 \div 6^5$ **d** $\frac{10^{13}}{10^6}$

4C 9 Use an exponent law to simplify:

a $(4^3)^6$ **b** $(9^2)^3$ **c** $(13^{10})^4$ **d** $(10^{12})^8$

4C 10 Simplify:

a $(3 \times 5)^6$ **b** $\left(\frac{2}{7}\right)^4$ **c** $(9 \times 15)^5$ **d** $\left(\frac{9}{8}\right)^{10}$

4C 11 Evaluate:

a 16^0 **b** $(4 \times 5)^0$ **c** $8^4 \div 8^4$ **d** $\left(\frac{10}{3}\right)^0$

4D Combining the exponent laws

Learning intentions

By the end of this topic you will be able to ...

✔ use multiple exponent laws to simplify complex calculations.

Inter-year links

Support	Order of operations
Year 7	1H Order of operations
Year 9	1C Raising exponents and the zero exponent
Year 10	1A Exponents

Using multiple exponent laws to simplify complex calculations

- Multiple exponent laws can be used to simplify complex calculations.
- The order of operations also applies to the simplification of expressions containing exponents. Calculations contained in grouping symbols should be simplified first.

	Exponent law
Product of powers law	$a^m \times a^n = a^{m+n}$
Quotient of powers law	$\frac{a^m}{a^n} = a^m \div a^n$ $= a^{m-n}$
Power of a power law	$(a^m)^n = a^{m \times n}$ $(a \times b)^m = a^m \times b^m$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Example 4D.1 Using exponent laws to simplify complex calculations

Use exponent laws to simplify:

a $(2^3)^3 \times (2^2)^{11}$ **b** $\frac{3^7 \times 3^2}{3^5}$

THINK

a 1 To raise the exponent, write each base and multiply the exponents.

2 Check that the bases are the same. To simplify the product, write the base and add the exponents.

b 1 The fraction line groups the terms in the numerator and the denominator, so the expressions in the numerator and denominator should be simplified before dividing the numerator by the denominator. The bases in the numerator are the same, so to simplify the numerator, write the base and add the exponents.

2 The bases in the numerator and the denominator are the same, so to simplify the fraction, write the base and subtract the exponent of the number in the denominator from the exponent of the number in the numerator.

WRITE

a $(2^3)^3 \times (2^2)^{11} = 2^{(3 \times 3)} \times 2^{(2 \times 11)}$
$= 2^9 \times 2^{22}$
$= 2^{(9+22)}$
$= 2^{31}$

b $\frac{3^7 \times 3^2}{3^5} = \frac{3^{(7+2)}}{3^5}$
$= \frac{3^9}{3^5}$
$= 3^{(9-5)}$
$= 3^4$

Example 4D.2 Simplifying calculations by writing numbers in exponent form

Consider the calculation $\frac{1024}{32 \times 8}$

a Express each of the numbers in the calculation in exponent form.

b Simplify the calculation using the exponent laws.

c Evaluate the calculation.

THINK

a Rewrite each number in exponent form.

b Rewrite the fraction. All numbers are expressed in exponent form with a base of 2, so the calculation can be simplified with the exponent laws. Simplify the denominator by adding the exponents, then simplify the fraction by subtracting the exponent of the number in the denominator from the exponent of the number in the numerator.

c Calculate the value of the number by performing the multiplication.

WRITE

a $8 = 2^3$, $32 = 2^5$, $1024 = 2^{10}$

b $\frac{1024}{32 \times 8} = \frac{2^{10}}{2^5 \times 2^3}$
$= \frac{2^{10}}{2^{(5+3)}}$
$= \frac{2^{10}}{2^8}$
$= 2^{(10-8)}$
$= 2^2$

c $2^2 = 2 \times 2$
$= 4$

Helpful hints

Always check that the bases are the same before simplifying products and quotients of numbers in exponent form!

ANS p511

Exercise 4D Combining the exponent laws

△ 1–5, 6(a, c), 7(a, d, e), 8 ▢ 1–6, 7(c, e, f), 8(b, d, f, h), 9–11 ⬠ 3–6, 7(d, f), 8(b, d, f, h), 9–12

4D.1

1 Use exponent laws to simplify:

a $(3^2)^3 \times (3^4)^2$ **b** $(9^5)^4 \times (9^3)^7$ **c** $(12^3)^{12} \times (12^4)^0$ **d** $(8^0)^5 \times (8^0)^{21}$

2 Use exponent laws to simplify:

a $\frac{4^5 \times 4^3}{4^6}$ **b** $\frac{7^6 \times 7^6}{7^{10}}$ **c** $\frac{6^4 \times 6^7}{6^0}$ **d** $\frac{11^8 \times 11^3}{11^5}$

e $\frac{5^6 \times 5^{10}}{5^9 \times 5^3}$ **f** $\frac{8^7 \times 8^2}{8 \times 8^5}$ **g** $\frac{2^8 \times 2^6}{2^0 \times 2^4}$ **h** $\frac{4^{12} \times 4^6}{4^{12} \times 4^3}$

3 Simplify:

a $\frac{(3^2)^4 \times 3^5}{3^4 \times 3^3}$ **b** $\frac{3^5 \times 3^6 \times 3^7}{3^{11} \times 3^2}$ **c** $\frac{4^{14} \times 4^8}{4 \times (4^5)^0}$ **d** $\frac{2^{13} \times (2^4)^3 \times 2^5}{2^{12} \times 2^{10}}$

UNDERSTANDING AND FLUENCY

4 Complete this table.

Base 2	**Exponent form**	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
	Basic numeral	2	4	8							
Base 3	**Exponent form**	3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9	3^{10}
	Basic numeral	3	9	27							

4D.2 **5** **a** Use the table of values from question **4** to express each of the numbers in the following calculations in exponent form.

i 32×16

ii $1024 \div 256$

iii $59\,049 \div 729$

iv $\dfrac{128 \times 32}{16}$

v $\dfrac{27 \times 243}{2187}$

vi $64 \times 256 \div 512$

vii $\dfrac{512 \times 64}{128 \times 16}$

viii $\dfrac{128 \times 1024}{256 \times 64}$

b Use the exponent laws to simplify each calculation.

c Refer to the table to write the exponent forms from part **b** as basic numerals.

6 Evaluate each of the following calculations without a calculator.

a $512 \times 32 \div 64$ **b** $9 \times 243 \div 27$ **c** $\dfrac{32 \times 256}{8 \times 16}$ **d** $\dfrac{6561 \times 19\,683}{729 \times 81}$

PROBLEM SOLVING AND REASONING

7 Use exponent laws to simplify, then evaluate the following calculations.

a $\dfrac{3^8 \times 2^6}{3^6 \times 2^3}$ **b** $\dfrac{5^9 \times 3^6}{5^7 \times 3^5}$ **c** $\dfrac{6^4 \times 4^7}{6^3 \times 4^5}$

d $\dfrac{8^0 \times 9^8 \times 10^5}{9^7 \times 10^2}$ **e** $\dfrac{7^6 \times 5^{10}}{7^6 \times 5^7}$ **f** $\dfrac{10^7 \times 8^4}{10 \times 8^2 \times 7^0}$

8 Fill in the missing exponents in the following calculations.

a $(2^3)^4 \times (2^4)^{\square} = 2^{20}$ **b** $\dfrac{6^7 \times 6^7}{6^{\square}} = 6^2$

c $\dfrac{3^{\square} \times 3^2}{3^6} = 1$ **d** $(15^{\square})^8 \times (15^{10})^2 = 15^{20}$

e $\dfrac{(5^2)^4}{5^{\square}} = 5^3$ **f** $\dfrac{4 \times 4^{16}}{4^7 \times 4^{\square}} = 4$

g $(9^{\square} \times 9^4)^2 = 9^{14}$ **h** $\dfrac{(11^{\square})^6 \times 11^6}{11^8 \times 11^7} = 11^9$

9 Explain why $(-5)^2 \neq -(5)^2$, but $(-5)^3 = -(5)^3$.

10 Without evaluating, decide whether the following numerical statements are true or false.

a $(-9)^4 \neq -(9)^4$ **b** $(-11)^9 \neq -(11)^9$ **c** $(-8)^5 = -(2^3)^5$

d $(-100)^4 \neq -(10)^8$ **e** $(-2 \times 3)^9 \neq -(6^3)^3$ **f** $(-3 \times 5)^{16} = -(15^3)^8$

CHALLENGE

11 Use exponent laws to find the value of x.

$$\frac{(9^x)^3 \times (9^4)^x}{(9^x)^2} = 9^{45}$$

12 Simplify then evaluate the following expression by first writing each base as the product of its prime factors.

$$\frac{96^4 \times 1296^3}{864^5}$$

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Exponent laws

Investigation
Bases of 4 and 5

Topic quiz
4D

4E Roots

Learning intentions

By the end of this topic you will be able to ...

- ✔ find the square root of perfect squares and the cube root of perfect cubes
- ✔ identify surds.

Inter-year links

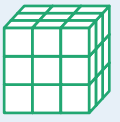
Year 7	1G Exponents and square roots
Year 9	1F Surds
Year 10	2A Rational and irrational numbers

Roots

- Squaring and taking the square root are inverse (opposite) operations. Similarly, cubing and taking the cube root are inverse operations.
- The square root, $\sqrt{}$, of a number is the number that, when squared, results in the original number.

  For example, $\sqrt{16} = 4$ because $4^2 = 16$.
- The cube root, $\sqrt[3]{}$, of a number is the number that, when cubed, results in the original number.

  For example $\sqrt[3]{8} = 2$ because $2^3 = 8$.

Squares			Cubes		
$1^2 = 1 \times 1 = 1$		$\sqrt{1} = 1$	$1^3 = 1 \times 1 \times 1 = 1$		$\sqrt[3]{1} = 1$
$2^2 = 2 \times 2 = 4$		$\sqrt{4} = 2$	$2^3 = 2 \times 2 \times 2 = 8$		$\sqrt[3]{8} = 2$
$3^2 = 3 \times 3 = 9$		$\sqrt{9} = 3$	$3^3 = 3 \times 3 \times 3 = 27$		$\sqrt[3]{27} = 3$

Surds

- A **perfect square** is the square of an integer and a **perfect cube** is the cube of an integer.

Bases, a	Perfect squares, a^2	Perfect cubes, a^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000

- A square or cube root of a number that produces an irrational number (a non-terminating, non-repeating decimal) is called a **surd**.

  For example, $\sqrt{2}$ is a surd because $\sqrt{2} = 1.414\,21...$

Example 4E.1 Calculating the value of square roots and cube roots

Calculate the value of:

a the square root of 49

b $\sqrt{121}$

c $\sqrt[3]{125}$

THINK

a Use the times tables to determine which number, multiplied by itself, is equal to 49.

b Use the times tables to determine which number, multiplied by itself, is equal to 121.

c The '3' above the root sign denotes 'the cube root' of 125. Determine which number, when cubed, is equal to 125.

WRITE

a $7 \times 7 = 49$
$\sqrt{49} = 7$

b $11 \times 11 = 121$
$\sqrt{121} = 11$

c $5 \times 5 \times 5 = 125$
$\sqrt[3]{125} = 5$

Example 4E.2 Identifying surds

Use a calculator to classify the following roots as rational or irrational.

a $\sqrt{0.36}$

b $\sqrt{1.25}$

THINK

a $\sqrt{0.36} = 0.6$, which is a terminating decimal, so $\sqrt{0.36}$ is a rational number.

b $\sqrt{1.25} = 1.118\,03...$ which is a non-terminating, non-recurring decimal, so $\sqrt{1.25}$ is an irrational number.

WRITE

a $\sqrt{0.36}$ is a rational number.

b $\sqrt{1.25}$ is an irrational number.

Helpful hints

- ✔ As with exponents, take care when writing cube roots. In a cube root, the '3' should be small and sit high up on the shoulder of the root sign to avoid confusion between $\sqrt[3]{8}$ and $3\sqrt{8}$.
- ✔ Remember that if no number is specified to the left of the root sign, it stands for 'square root'.
  For example, $\sqrt{4} = 2$.
- ✔ Recall that the root sign is a grouping symbol, so operations contained within the root sign should be completed before applying the root.
  For example: $\sqrt{16 + 9} = \sqrt{25}$
  $= 5$

ANS p511

Exercise 4E Roots

1–10, 11(a, b), 13(a, b) | 2, 3, 5–10, 13, 14–15(a) | 2, 3, 5–8, 11–15

1 Complete the table below by evaluating the numbers in exponent form.

Exponent form	Perfect square	Exponent form	Perfect cube
2^2		2^3	
3^2		3^3	
4^2		4^3	
5^2		5^3	
6^2		6^3	
7^2		7^3	
8^2		8^3	
9^2		9^3	
10^2		10^3	

4E.1 **2** Calculate the value of:

a the square root of 16 **b** the square root of 9 **c** the square root of 25

d $\sqrt{1}$ **e** $\sqrt{4}$ **f** $\sqrt{100}$

g $\sqrt{81}$ **h** $\sqrt{225}$ **i** $\sqrt{144}$

j $\sqrt{400}$ **k** $\sqrt{169}$ **l** $\sqrt{900}$

3 Calculate the value of:

a the cube root of 8 **b** the cube root of 64 **c** the cube root of 125 **d** the cube root of 1

e $\sqrt[3]{27}$ **f** $\sqrt[3]{1000}$ **g** $\sqrt[3]{216}$ **h** $\sqrt[3]{8000}$

4 Use a calculator to evaluate the following square roots.

a $\sqrt{196}$ **b** $\sqrt{256}$ **c** $\sqrt{1089}$

d $\sqrt{26\,244}$ **e** $\sqrt{2.25}$ **f** $\sqrt{53.29}$

g $\sqrt{67.24}$ **h** $\sqrt{475.24}$ **i** $\sqrt{25.3009}$

4E.2 **5** Use a calculator to classify the following roots as rational or irrational.

a $\sqrt{144}$ **b** $\sqrt{20}$ **c** $\sqrt{54.76}$

d $\sqrt{331\,776}$ **e** $\sqrt{8.3}$ **f** $\sqrt{0.49}$

6 To evaluate the square root of a fraction, take the square root of the numerator and the denominator individually.

For example, $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$.

Evaluate the following fractional square roots.

a $\sqrt{\frac{1}{16}}$ **b** $\sqrt{\frac{1}{64}}$ **c** $\sqrt{\frac{4}{25}}$

d $\sqrt{\frac{9}{49}}$ **e** $\sqrt{\frac{81}{100}}$ **f** $\sqrt{\frac{36}{121}}$

7 Use the correct order of operations to complete the following calculations. Hint: the root sign is a grouping symbol, so operations contained within the root sign should be completed before applying the root.

a $4\left(\sqrt{25} + 15\right)$ **b** $\left(\sqrt{36} - \sqrt{25}\right)^4$ **c** $2\sqrt{5^2 - 4^2}$ **d** $3\left(\sqrt{13^2 - 12^2}\right)^2$

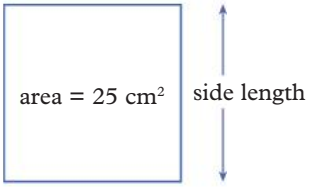
PROBLEM SOLVING AND REASONING

8 Identify and explain the errors in the following calculations. Give the correct answer in each instance.

a $\sqrt{16} = 8$ **b** $\sqrt[3]{64} = 8$

9 Explain how finding the square and square root of a number are related.

10 a If a square has area 25 cm^2, explain how you can calculate its side length.



b Calculate the side length of a square with area:

i 169 cm^2 **ii** 200 m^2 **iii** 268.96 km^2.

c Which of your answers is irrational?

11 The fourth root of a number can be determined using a similar method to the one used to find the square root.

a If $5^4 = 625$, what is the fourth root of 625, $\sqrt[4]{625}$?

b If $7^5 = 16\,807$, what is the fifth root of 16 807, $\sqrt[5]{16\,807}$?

c Evaluate the following roots without using a calculator.

i $\sqrt[4]{16}$ **ii** $\sqrt[4]{81}$ **iii** $\sqrt[4]{10\,000}$

iv $\sqrt[5]{32}$ **v** $\sqrt[5]{1}$ **vi** $\sqrt[5]{243}$

12 a Use a calculator to evaluate the following roots.

i $\sqrt[3]{3375}$ **ii** $\sqrt[3]{68.921}$ **iii** $\sqrt[3]{99}$

iv $\sqrt[4]{2401}$ **v** $\sqrt[4]{73}$ **vi** $\sqrt[4]{0.1}$

vii $\sqrt[5]{32\,768}$ **viii** $\sqrt[5]{0.000\,32}$ **ix** $\sqrt[5]{200}$

x $\sqrt[6]{729}$ **xi** $\sqrt[6]{496}$ **xii** $\sqrt[6]{15\,625}$

b Classify each root in part **a** as rational or irrational.

13 a If $(-3)^3 = -27$, what is $\sqrt[3]{-27}$?

b If $(-2)^5 = -32$, what is $\sqrt[5]{-32}$?

c Explain why it is not possible to calculate $\sqrt{-4}$.

CHALLENGE

14 Prime factorisation can be used to determine if a number is a perfect square.

For example, 196 is a perfect square because the prime factors of 196 can be divided into two identical groups.

$196 = 2 \times 2 \times 7 \times 7$
$= 2 \times 7 \times 2 \times 7$
$= 14 \times 14$
$= 14^2$

a Use prime factorisation to determine whether the following numbers are perfect squares. Find the square root of each perfect square.

i 168 **ii** 289 **iii** 576 **iv** 786

b Use prime factorisation to determine whether the following numbers are perfect cubes. Find the cube root of each perfect cube.

i 216 **ii** 512 **iii** 730 **iv** 1331

15 The square root of a composite number can be expressed as the product of the square root of the number's positive factors.

$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

Use this property to simplify and evaluate the following square roots without using a calculator.

a $\sqrt{324}$ **b** $\sqrt{676}$ **c** $\sqrt{1936}$ **d** $\sqrt{5184}$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Roots

Investigation
To square or square root?

Topic quiz
4E

Chapter summary

Terminology

index/exponent

base → $3^4 = 3 \times 3 \times 3 \times 3 = 81$

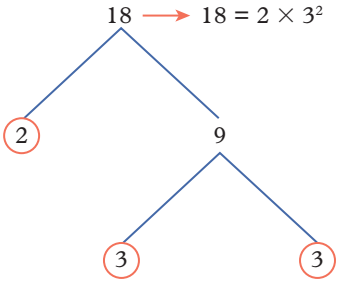
exponent form | expanded form | basic numeral

3^2 can be read as '3 to the power of 2', '3 squared' or 'the square of 3'.

4^3 can be read as '4 to the power of 3', '4 cubed' or 'the cube of 4'.

Prime factorisation

- The prime factorisation of an integer is the product of the primes that make up that integer.



Exponent laws

	Exponent law	Example
Product of powers law	$a^m \times a^n = a^{m+n}$	$2^5 \times 2^3 = 2^{5+3} = 2^8$
Quotient of powers law	$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$	$\frac{2^8}{2^5} = 2^8 \div 2^5 = 2^{8-5} = 2^3$
Power of a power law	$(a^m)^n = a^{m \times n}$ $(a \times b)^m = a^m \times b^m$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$(2^3)^5 = 2^{3 \times 5} = 2^{15}$

Combining exponent laws

BIDMAS

- Multiple exponent laws can be used to simplify complex calculations.

$$\frac{3^7 \times 3^2}{3^5} = \frac{3^{(7+2)}}{3^5} = \frac{3^9}{3^5} = 3^{(9-5)} = 3^4$$

$$(2^3)^3 \times (2^2)^{11} = 2^{(3 \times 3)} \times 2^{(2 \times 11)} = 2^9 \times 2^{22} = 2^{(9+22)} = 2^{31}$$

The zero exponent

- Excluding 0, any number to the power of 0 is equal to 1.

$a^0 = 1$

$8^2 \div 8^2 = 8^{(2-2)} = 8^0 = 1$

Surds

- Irrational numbers are non-recurring and non-terminating decimals.
- A square or cube root that produces an irrational number is a surd.

$\sqrt{2} = 1.414\,21...$

Perfect squares and perfect cubes

Squares		Cubes	
$1^2 = 1 \times 1 = 1$	$\sqrt{1} = 1$	$1^3 = 1 \times 1 \times 1 = 1$	$\sqrt[3]{1} = 1$
$2^2 = 2 \times 2 = 4$	$\sqrt{4} = 2$	$2^3 = 2 \times 2 \times 2 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 3 \times 3 = 9$	$\sqrt{9} = 3$	$3^3 = 3 \times 3 \times 3 = 27$	$\sqrt[3]{27} = 3$

Bases, a	Perfect squares, a^2	Perfect cubes, a^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Multiple choice

4A **1** 3^4 is equal to

A 1 **B** 12 **C** 34 **D** 64 **E** 81

4A **2** The cube of 8 is

A 800 **B** 512 **C** 64 **D** 24 **E** 2

4B **3** Which of these is the simplest form of $3^5 \times 3^4$?

A $\frac{1}{3}$ **B** 3^0 **C** 3 **D** 3^9 **E** 3^{20}

4B **4** Simplify $\frac{12^6}{12^2}$.

A 12 **B** 12^3 **C** 12^4 **D** 12^8 **E** 12^{12}

4C **5** Evaluate $\left(\frac{2}{5}\right)^3$.

A $\frac{8}{5}$ **B** $\frac{6}{5}$ **C** $\frac{6}{15}$ **D** $\frac{8}{125}$ **E** $\frac{2}{125}$

4C **6** Which of the following expressions does **not** equal 1?

A 10^0 **B** $10^0 \times 10$ **C** $\frac{10^3}{10^3}$ **D** $(10^3)^0$ **E** $(10^3 \times 10)^0$

4D **7** Use exponent laws to simplify $(5^2)^7 \times (5^6)^4$.

A 54 **B** 5^7 **C** 5^{19} **D** 5^{38} **E** 5^{90}

4D **8** The missing exponent in the calculation $\frac{12^3 \times 12^{\square}}{12^4} = 12^5$ is:

A 2 **B** 3 **C** 4 **D** 5 **E** 6

4E **9** The cube root of 64 is:

A 3 **B** 4 **C** 8 **D** 16 **E** 24

4E **10** Which of the following roots is an irrational number?

A $\sqrt{8}$ **B** $\sqrt{16}$ **C** $\sqrt{64}$ **D** $\sqrt[3]{27}$ **E** $\sqrt[3]{216}$

Short answer

4A **1** Write each number in expanded form, then calculate its value.

a 3^3 **b** 2^4 **c** 8^2 **d** 10^5

4A **2** Evaluate:

a 6^2 **b** 8 squared **c** the cube of 2 **d** 4 cubed

4A **3** Use a factor tree to find the prime factorisation of the following numbers. Write your answer in exponent form.

a 45 **b** 124

4B **4** Use an exponent law to simplify:

a $3^5 \times 3^7$ **b** $2^7 \times 2^3$ **c** $12^3 \times 12$ **d** $4^6 \times 4^2 \times 4^3$

4B **5** Simplify the following calculations using an exponent law.

a $6^8 \div 6^2$ **b** $5^7 \div 5^7$ **c** $\frac{10^{15}}{10^8}$ **d** $\frac{7^2}{7}$

4C **6** Use an exponent law to simplify:

a $(9^5)^3$ **b** $(6 \times 3)^{10}$ **c** $\left(\frac{3}{10}\right)^3$ **d** $\left(\frac{2 \times 7}{9}\right)^8$

4C **7** Evaluate the following calculations.

a 4×4^0 **b** $(7 \times 5)^0$ **c** $\frac{20^{10}}{20^{10}}$ **d** $\left(\frac{5}{5^0}\right)^2$

4D **8** Simplify, then evaluate the following calculations.

a $\frac{7^6 \times 7^5}{7^9}$ **b** $\frac{2^8 \times (2^4)^2 \times 2^3}{2^{14}}$

4D **9** For each of the following calculations:

i express each number in exponent form with the same base

ii simplify using exponent laws

iii evaluate.

a $\frac{5^4}{25}$ **b** $\frac{3^2 \times 81}{27}$ **c** $\frac{64 \times 256}{4 \times 16}$ **d** $16 \times (4^2)^2$

4E **10** Calculate the value of:

a the square root of 4

b $\sqrt{81}$

c the cube root of 27

d $\sqrt[3]{125}$

4E **11 a** Use a calculator to determine the value of the following to two decimal places where necessary.

i $\sqrt{529}$ **ii** $\sqrt{700}$ **iii** $\sqrt{156.25}$ **iv** $\sqrt[3]{42}$

b Classify each number in part **a** as rational or irrational.

Analysis

1 A radioactive metal has a half-life of 1 day. This means that each day, half the mass of the metal disintegrates into the atmosphere as a result of radioactive decay.

A physicist observes a 240 gram piece of the metal over a number of days and records the following data.

Number of days	Mass of metal (grams)	$\frac{\text{mass of metal}}{\text{original mass of metal}}$
0	240	
1		$\frac{1}{2}$
2		
3		



a Complete the table.

b Express each of the numbers in the third row in exponent form with a base of $\frac{1}{2}$.

c Write a calculation in exponent form for the amount of metal remaining after one week.

d Use an exponent law to simplify, then evaluate your answer to part **c**. Express your answer as a fraction.

e How much metal will be left after 1 week? Express your answer as a decimal.

f How long until there is less than 1 gram of metal remaining?

2 Consider the fraction $\frac{490 \times 36^2}{756 \times 507^0}$

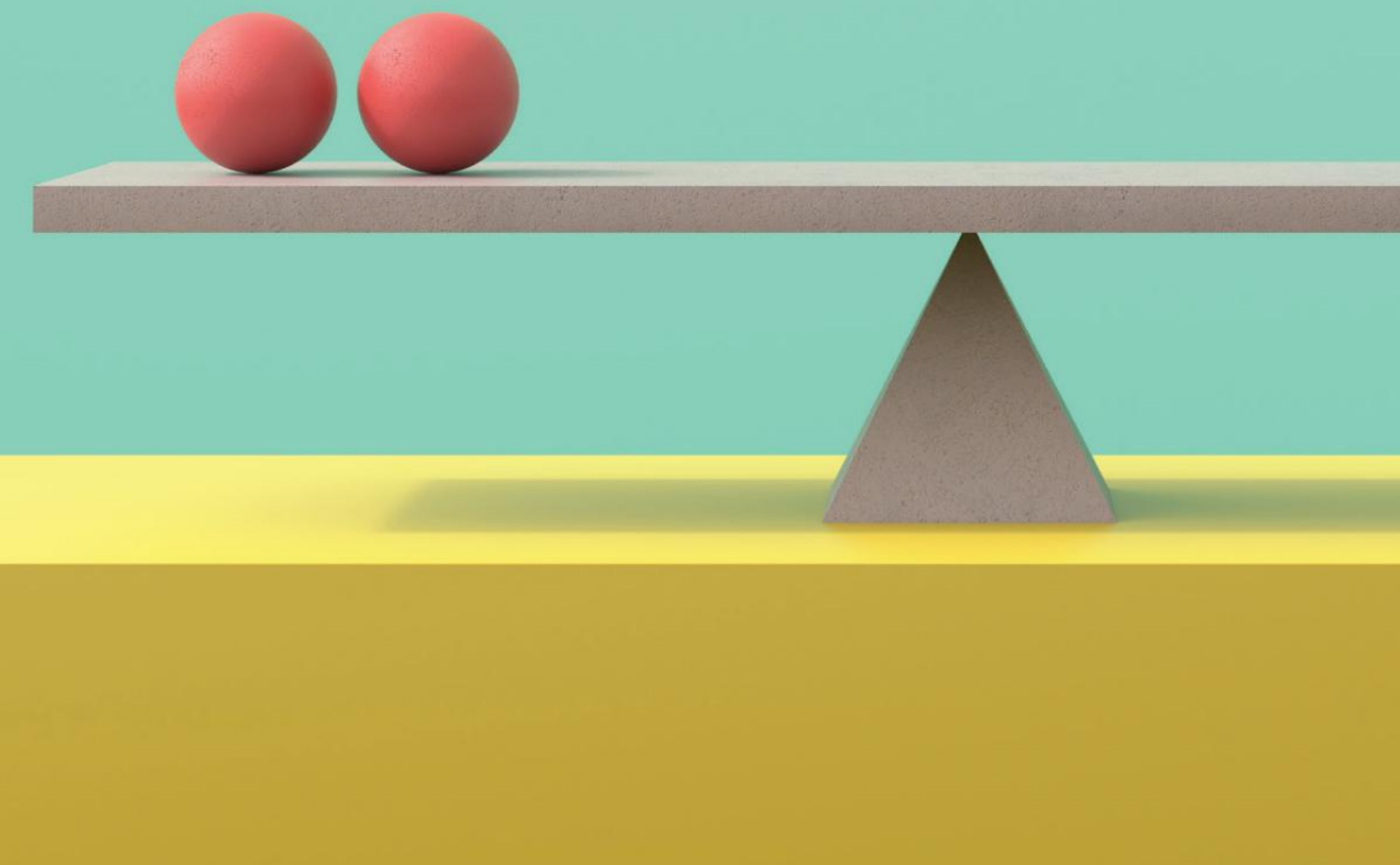
a Use prime factorisation to rewrite the fraction, expressing each number as the product of its prime factors.

b Simplify the fraction using exponent laws.

c Evaluate your answer to part **b**.

5

Algebra





Index

5A Pronumerals
5B Substitution
5C Adding and subtracting algebraic terms
5D Multiplying algebraic terms
5E Dividing algebraic terms
5F Expanding
5G Factorising

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Variables
- ✔ Writing formulas
- ✔ Grouping symbols
- ✔ Order of operations
- ✔ Highest common factor

Curriculum links

- Create, expand, factorise, rearrange and simplify linear expressions, applying the associative, commutative, identity, distributive and inverse properties (AC9M8A01)

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Materials

- ✔ Calculator

5A Pronumerals

Learning intentions

By the end of this topic you will be able to ...

- ✔ identify terms and expressions
- ✔ write expressions to represent worded problems.

Inter-year links

Support	Writing formulas
Year 7	6B Writing formulas

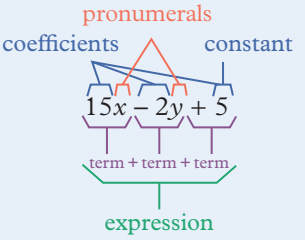
Pronumerals

- A **pronumeral** is a letter or symbol that is used in place of a number. Pronumerals can be used to represent an unknown number or a variable.
  - → An **unknown number** has one value.
  - → A **variable** is a quantity that can have different values.

Terms and expressions

- An **expression** is a quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations.
- A **term** is part of an expression that is separated from the other parts by a plus or minus sign (as long as the plus or minus sign is not inside any brackets).

  Note: If the term is separated on the left by a minus sign, then the term is negative.



- A **coefficient** is the number acting as a multiplier in an algebraic term, usually written before the pronumeral. A pronumeral without a number preceding it has a coefficient of 1.
- A **constant** is a term without any pronumerals, also called the constant coefficient or the constant term.

Algebraic notation

- Products are simplified by leaving out the multiplication sign and placing the number first.

  $7 \times x = 7x$ and $7 \times (x + 2) = 7(x + 2)$
- When a pronumeral is multiplied by 1, the 1 is left out.

  $1 \times x = x$
- Quotients are represented using fractions.

  $x \div 7 = \frac{x}{7}$ and $(x + 2) \div 7 = \frac{(x + 2)}{7}$
- Terms with fractional coefficients can be written in two ways.

  $\frac{x}{7} = \frac{1}{7}x$
- **Grouping symbols** include brackets, the fraction line in a quotient and the root sign. Grouping symbols are used to preserve order of operations in algebraic expressions.

  $7(x + 2)$

  $\frac{x + 2}{7}$

  $7\sqrt{x + 2}$

Example 5A.1 Identifying terms, coefficients and constants

Consider the expression $4a + \frac{b}{2} - c - 11$.

a List the terms in the expression.
b List any variables in the expression.
c Identify the coefficient of each term that contains a variable.
d List any constant terms.

THINK	WRITE
a Identify the individual terms, which are separated by a + or – sign. A minus sign can be read as a negative sign and therefore 'belongs' to the term to the right of the sign.	**a** The terms are $4a$, $\frac{b}{2}$, $-c$ and -11.
b Variables are represented by pronumerals. List the pronumerals.	**b** The variables are a, b and c.
c Identify the number that comes before each pronumeral. Terms with fractional coefficients can be written in two ways. $\frac{b}{2} = \frac{1}{2}b$, so the coefficient of the b term is $\frac{1}{2}$. As the negative sign is a property of the c term, the coefficient of c is -1.	**c** The coefficient of a is 4. The coefficient of b is $\frac{1}{2}$. The coefficient of c is -1.
d List the terms that do not contain pronumerals.	**d** The constant term is -11.

Example 5A.2 Writing expressions

Write an expression for each statement.

a 5 less than a
b A quarter of b

THINK	WRITE
a Think about which operation is taking place. 'Less than' means subtract, so 5 less than a is a minus 5.	**a** $a - 5$
b 'A quarter' of b means b divided by 4. Simplify the quotient by rewriting it as a fraction. Terms with fractional coefficients can be written in two ways – either form is acceptable.	**b** $b \div 4$ $= \frac{b}{4}$ or $\frac{1}{4}b$

Example 5A.3 Writing expressions using grouping symbols

Write an expression for each statement using grouping symbols where appropriate.

a Add 7 to b, then multiply the result by 4.

b The product of a and d, all divided by 2.

THINK	WRITE
a **1** Think about which operations are taking place. 'Add 7' means +7.	**a** $b + 7$
2 Look for words that refer to the order of operations. '*Then* multiply *the result* by 4' means the multiplication should be performed *after* the addition. Group $b + 7$ using brackets, then multiply the brackets by 4.	$(b + 7) \times 4$
3 Simplify the product by placing the number first and removing the multiplication sign.	$= 4(b + 7)$
b **1** Finding 'the product' means to multiply.	**b** $a \times d$
2 '*All* divided by 2' means the whole product should be divided by 2.	$(a \times d) \div 2$
3 Simplify the product and the quotient. The fraction line is a grouping symbol, so the brackets are no longer necessary.	$= \frac{ad}{2}$

Example 5A.4 Writing an expression from a worded problem

Lee works casually as a nurse. He earns $35 an hour, plus a bonus of $50 for each night shift.
Write an expression to represent the amount of money Lee will earn for a night shift of t hours.

THINK	WRITE
1 Identify the variable in the problem.	t = number of hours in the shift
2 Think about which operations are taking place. If Lee earns $35 per hour, he will earn 35 times the number of hours he works. 'Plus' means add 50 to Lee's total earnings for a given night shift.	$35 \times t + 50$
3 Write an expression to represent the amount of money Lee will earn for a night shift of t hours. Simplify the product.	$= 35t + 50$

Helpful hints

- ✔ In the expression $5 + x$, the coefficient of x is 1, not 0.
  Watch out for negative coefficients in expressions containing two or more terms. In the expression $5 - 3x$, for example, the coefficient of x is -3.
- ✔ The order of operations changes with the placement of grouping symbols. Take care when placing brackets and writing fractions and roots! For example:
  $\frac{x+1}{2}$ is not the same as $\frac{x}{2} + 1$.

ANS p512

Exercise 5A Pronumerals

1–6, 7(b, c, f, h), 8–11, 13, 15 | 1, 3, 5, 6, 8, 11, 12, 14, 16, 17, 19 | 1, 3, 5, 8, 12, 16–20

5A.1 **1** For each of the following expressions:

i list the terms in the expression

ii list any variables in the expression

iii identify the coefficient of each term that contains a variable

iv list any constant terms.

a $6a + b - 1$ **b** $2xy - 3x$ **c** $11f - g + 15h - 10$ **d** $-\frac{9}{4}p$

e $\frac{3}{4}c + d + \frac{1}{3}$ **f** $\frac{x}{2} + \frac{y}{11} - \frac{7}{10} - \frac{z}{8}$ **g** $-\frac{m}{5} - \frac{n}{7} - 8p$ **h** $\frac{5x}{8} - 9y + \frac{z}{4} - \frac{2}{5}$

2 State whether each of these is an expression or an equation.

a $2a + 3$ **b** $x - 6 = 1$ **c** $7 = 5 + \frac{5}{6}h$

d $x + y + z = 9$ **e** $3a + b - 7$ **f** $k = \frac{4m + 1}{8}$

3 Answer true or false for each statement. For the statements that are false, provide a reason for your answer.

a $2x + \frac{1}{3} = 4$ is an expression.

b $5 - 12a + b$ has 3 terms.

c $7 - y$ is a term.

d $\frac{5}{7}$ is a term.

e The coefficient of $-d$ is 1.

f The coefficient of $\frac{b}{12}$ is 12.

g $-\frac{5f}{4}$ is an expression.

h The coefficient of $\frac{2x}{3}$ is $\frac{2}{3}$.

5A.2 **4** Write an expression for each statement.

a 5 more than a

b z divided by 3

c 23 less than y

d 50 times m

e The product of x and $\frac{2}{5}$

f Twice the value of p

g Half of s

h The sum of three-quarters of w and 5

5 Write an expression for each statement. Use x to represent the unknown number.

a A number is multiplied by 7 and then 2 is added.

b A number is divided by 3 and then 6 is subtracted.

c 10 less than three-quarters of a number

d The sum of 13 and a third of a number

e 28 less than the product of 15 and a number

f 5 less than half of a number

UNDERSTANDING AND FLUENCY

6 Write a number sentence to describe the order of operations in each of the following expressions.

a $c + 8$ **b** $11 - x$ **c** $\frac{x}{5}$

d $\frac{9}{10}q$ **e** $3x - 5$ **f** $\frac{10a}{3} + 16$

5A.3 **7** Write an expression for each statement using grouping symbols where appropriate

a Add 6 to x, then multiply the result by 3

b Subtract 10 from n, then divide the result by 7

c The product of f and g, all multiplied by $\frac{3}{5}$

d 1 less than the square root of t

e The sum of b and 12, all divided by 15

f The square root of the sum of z and 9

g 7 less two-thirds of v

h Triple the sum of 33 and y

8 Write an expression to represent each of the following number sentences. Use x for the unknown number.

a Subtract 3 from a number, then multiply the result by 8

b The sum of a number and 20, all multiplied by 6

c 2 less than the product of $\frac{5}{6}$ and a number

d 12 more than four-fifths of a number

e Double the sum of a number and 10

f The sum of a number and 4 is divided by 12, then 16 is added to the result

PROBLEM SOLVING AND REASONING

5A.4 **9** Amir has n songs on his phone. He downloads a further 18 songs. Write an expression to represent the number of songs he now has on his phone.

10 Four friends bought a lottery ticket together and agreed to share the prize evenly. If they win $\$c$ in total, how much does each person get?

11 Pritesh finds x old records in his shed and decides to sell them for \$8 each. Write an expression for the total price of the records.



12 Ash is running low on free space on his laptop. He deletes 10 GB (gigabytes) of data that he doesn't need anymore. Write an equation for the amount of data remaining on his laptop, y, compared to the original amount of data, x.

13 Hayden is y years old. Answer each of the following questions using an algebraic expression.

a How old was Hayden:

i 6 years ago **ii** 10 years ago?

b How old will Hayden be in:

i 5 years' time **ii** 23 years' time?

c How old is Hayden's sister if she is x years younger than him?

d How old is Hayden's uncle if he is twice as old as Hayden?



14 A kitchen hand is washing up c cups and p plates.

a How many items does he have to wash?

b If he breaks a plate, how many plates are left?

c After he breaks a plate, how many cups and plates does he now have to dry?

15 An art teacher orders n brushes from an online store that charges \$12 for delivery. The total cost of his order can be represented by the algebraic expression $8n + 12$.

a List the terms in the expression.

b What does the constant term represent?

c What does the coefficient of the term with n represent?

d What will the total cost of the order be if the art teacher orders 10 brushes?

16 Gillian hires a photobooth for a birthday party. The photobooth company charges \$130 per hour and an extra flat fee of \$25 for props and backdrops.

a Write an expression for the total cost of hiring the photobooth for t hours.

b How much will it cost Gillian to hire the photobooth, props and backdrops for 3 hours?



17 At the end of autumn, Ron has x chickens on his farm. 7 baby chickens hatch over winter. By the end of spring, more chickens have hatched: Ron now has double the number of chickens than he did at the end of winter.

a Write an expression for the total number of chickens on Ron's farm at the end of spring.

b If Ron had 13 chickens at the end of autumn, how many chickens did he have at the end of spring?

18 Naomi and Rupi are working on the following question:

'A container of x puzzle blocks is divided between three students. If each student has the same number of blocks and one block is left over, write an expression for the number of blocks each student has to work with.'

Naomi believes the answer is $\frac{x-1}{3}$ while Rupi thinks the answer should be $\frac{x+1}{3}$.

a Decide who is correct. Explain why their expression is correct.

b Rewrite the question to represent the other expression.

19 Consider the expression $\frac{4x-5}{9}$.

a Rewrite the expression as two separate fractions.

b What is the coefficient of the term with variable x?

c What is the value of the constant term?

20 During a sale, a shop advertises 20% off the full price of t-shirts and 30% off the full price of pants. A customer buys x t-shirts that cost \$$y$ full price, and a pants that cost \$$b$ full price.

a Write a two-term expression for the total cost of the customer's purchase.

b If the customer bought twice as many t-shirts as pants, the t-shirts cost \$20 each at full price and the pants cost \$50 each at full price, rewrite the expression in terms of one pronumeral only.

c How many t-shirts and pairs of pants did the customer buy if they spent a total of \$134?

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Expressions and equations
- Worksheet: Writing algebraic expressions
- Investigation: Pronumerals in mark-ups and discounts
- Topic quiz: 5A

5B Substitution

Learning intentions

By the end of this topic you will be able to ...

- ✔ substitute values into and evaluate expressions
- ✔ substitute values into formulas to find unknown values.



Inter-year links

Year 7 6E Substitution

Substitution

- **Substitution** involves replacing each pronumeral in an algebraic expression with a number.

Let $x = -2$

$5x^2 - x = 5(-2)^2 - (-2)$

- For clarity, the number being substituted into an expression is enclosed in brackets when:
  - → the number is negative
  - → the expression involves multiplication or exponents.
- To **evaluate** an algebraic expression, substitute the known values of each pronumeral into the expression and complete the operations in the correct order.
- A **formula** is an algebraic statement that describes the relationship between two or more variables.

  For example, the formula for the area of a triangle is:

  $A = \frac{1}{2}bh$

  Where A = area

  b = base length

  h = height.

B	**Brackets**
I	**Indices or exponents**
D	**Division**
M	**Multiplication**
A	**Addition**
S	**Subtraction**

- Substitution can be used to find the value of an unknown variable when the values of all the other variables in a formula are known.

Example 5B.1 Evaluating expressions



Evaluate each expression by substituting 3 for x.

a $\frac{x}{3} + 1$ **b** $4x^2 + 1$

THINK

a Replace x with 3 and complete the operations in the correct order. Division is performed before addition.

b Replace x with 3 and complete the operations. As the expression involves multiplication and exponents, enclose the 3 in brackets for clarity. Exponents are applied first, then multiplication, then addition.

WRITE

a $\frac{x}{3} = \frac{3}{3} + 1$

$= 1 + 1$

$= 2$

b $4x^2 + 1 = 4(3)^2 + 1$

$= 4(9) + 1$

$= 36 + 1$

$= 37$

Example 5B.2 Substituting negative values into expressions

Evaluate each expression by letting $m = 5$ and $n = -2$.

a $3m - n$ **b** $2mn^2$

THINK

a Replace m with 5 and n with -2 and complete the operations in the correct order. Multiplication is performed before subtraction. Remember that subtracting a negative integer is equivalent to addition.

b Replace m with 5 and n with -2 and complete the operations. Exponents are applied before multiplication. Remember that the product of two negative numbers gives a positive number, so $(-2)^2 = -2 \times -2 = 4$.

WRITE

a $3m - n = 3(5) - (-2)$
$= 15 + 2$
$= 17$

b $2mn^2 = 2(5)(-2)^2$
$= 2(5)(4)$
$= 40$

Example 5B.3 Evaluating expressions containing grouping symbols

Substitute $a = 3$ and $b = 4$ into each expression and evaluate.

a $7(a - b)$ **b** $\frac{2a + b}{5}$

THINK

a Replace a with 3 and b with 4 and complete the operations in the correct order. Bracketed operations are performed first, so complete the subtraction in the brackets before the multiplication. Remember that the product of a positive number and a negative number is a negative number.

b Replace a with 3 and b with 4 and complete the operations. The fraction line is a grouping symbol, so the expression in the numerator of the fraction should be evaluated before dividing through by the denominator.

WRITE

a $7(a - b) = 7(3 - 4)$
$= 7(-1)$
$= -7$

b $\frac{2a + b}{5} = \frac{2(3) + 4}{5}$
$= \frac{6 + 4}{5}$
$= \frac{10}{5}$
$= 2$

Example 5B.4 Using formulas to find unknown values

Use the formula $y = 2\sqrt{x + 9}$ to find the value of y if $x = 7$.

THINK

To find the value of y, substitute $x = 7$ into the expression on the right-hand side of the equation and evaluate. Remember that the root sign is a grouping symbol, so the expression contained within the root sign should be evaluated before multiplying by 2.

WRITE

$y = 2\sqrt{x + 9}$
$= 2\sqrt{7 + 9}$
$= 2\sqrt{16}$
$= 2 \times 4$
$= 8$

Helpful hints

✔ Take care when substituting negative numbers into expressions.
→ Recall that adding a negative number is equivalent to subtraction and subtracting a negative number is equivalent to addition.
→ Writing calculations out in full will help you to simplify calculations involving negative numbers correctly.

$+(+)=+$ $\quad +(-)=-$ $\quad -(+)=-$ $\quad -(-)=+$

$+\times+=+$ $\quad +\times-=-$ $\quad -\times+=-$ $\quad -\times-=+$

ANS p513

Exercise 5B Substitution

▲ 1–8, 9(a, b, e, f), 10, 11, 13 ■ 2, 3, 5, 6, 8, 9, 11, 12, 14, 16(a, b) ⬟ 5, 6, 8, 9, 12, 14, 15, 17

UNDERSTANDING AND FLUENCY

5B.1 **1** Evaluate each expression by substituting 4 for x.

a $x+8$ **b** $\frac{x}{2}$ **c** $\sqrt{x}$ **d** $5x-10$ **e** $\frac{2x}{5}$

f $5x^2$ **g** $4x^3-2$ **h** $2\sqrt{x}-5$ **i** $5x^2-x$ **j** $\frac{1}{2}x^2-8$

5B.2 **2** Evaluate each expression by letting $m=4$ and $n=-3$.

a $m+n+4$ **b** $2m+5n$ **c** $7m-n$ **d** $\frac{1}{4}mn$

e $3m^2+6n^2$ **f** $\frac{1}{8}m^2-2n$ **g** $\frac{m^2n}{12}$ **h** $6mn+n^2$

3 Evaluate $3a+5$ by substituting each value for a into the expression.

a $a=2$ **b** $a=7$ **c** $a=1$ **d** $a=0$

e $a=-1$ **f** $a=-3$ **g** $a=\frac{1}{3}$ **h** $a=-\frac{2}{3}$

4 Evaluate $6y^2-y$ by substituting each value for y.

a $y=1$ **b** $y=3$ **c** $y=-1$ **d** $y=-4$

e $y=0$ **f** $y=\frac{1}{2}$ **g** $y=\frac{1}{3}$ **h** $y=2.5$

5B.3 **5** Substitute $a=5$ and $b=2$ into each expression and evaluate.

a $10(a-b)$ **b** $\frac{2a+2b}{7}$ **c** $\frac{1}{7}(3a-4b)$ **d** $2\sqrt{2a+3b}$

e $\frac{3b+1}{a}$ **f** $\sqrt{a+10b}-12$ **g** $\frac{ab-5}{9}$ **h** $\frac{3(a-b^2)}{7}$

5B.4 **6** Use each of the following formulas to find the value of y if $x=3$.

a $y=2x+15$ **b** $y=\frac{x}{3}+5$ **c** $y=5\sqrt{x+6}$ **d** $y=\frac{4x-15}{5}$

e $y=\frac{2}{9}(10-x^2)$ **f** $y=\frac{1}{x}+\frac{5}{3}$ **g** $y=\frac{2}{5}x-2$ **h** $y=\sqrt{12x}-11$

7 Find the value of y for each of these values of x using the formula $y=2x+7$.

a $x=3$ **b** $x=8$ **c** $x=-6$ **d** $x=-1$

e $x=4.5$ **f** $x=0$ **g** $x=\frac{1}{2}$ **h** $x=-\frac{3}{2}$

8 Find the value of k for each of these values of m using the formula $k=4(m^2-3)$.

a $m=1$ **b** $m=6$ **c** $m=-2$ **d** $m=50$

e $m=1.5$ **f** $m=\frac{1}{4}$ **g** $m=0$ **h** $m=-\frac{3}{4}$

9 Recall the rules for carrying out the four operations with negative numbers and evaluate each expression by letting $p = -6$, $q = 2$ and $r = -3$.

a $2(q - r)$ **b** $\sqrt{14q - 6p}$ **c** $q(p + r)$ **d** $\frac{r - p}{q}$

e $\frac{pq}{r}$ **f** $\frac{p^2 - q^2}{r}$ **g** $4(p^2 - q) - 5r$ **h** $-\frac{3p}{q - r}$

10 The relationship between average speed, s, distance, d, and time, t, can be represented by the formula $s = \frac{d}{t}$.

a Find the average speed, in metres per second, of a bicycle that travels 120 m in 40 seconds.

b Find the average speed, in kilometres per hour, of a motorcyclist who travels 282 km in 3 hours.

11 The distance a skydiver falls through the air, d, in metres after a given amount of time, t, in seconds can be roughly calculated using the formula $d = 4.9t^2$.



a How far does the skydiver fall in the first second of his dive?

b How far does the skydiver fall:

i in the first 3 seconds of his dive

ii in the first 5 seconds of his dive?

c Does the skydiver's speed increase or decrease over the first 5 seconds of his dive? Explain.

12 Use the formulas in the table to answer the following questions.

a What is the area of a square that has a side length of 2 cm?

b What is the area of a rectangle that has a length of 10 cm and a width of 7 cm?

c What is the area of a triangle that has a base length of 5 cm and a height of $\frac{5}{2}$ cm?

d **i** Which has the greater area – a square with a side length of 13 cm, or a rectangle with a length of 5 cm and a width of 3 cm?

ii Which has the greater area – a rectangle with a length of 10 cm and a width of $\frac{3}{5}$ cm, or a triangle with a base length of $\frac{5}{4}$ cm and a height of 15 cm?

Shape	Area formula, A	Dimensions
Square	$A = l^2$	l = side length
Rectangle	$A = lw$	l = length w = width
Triangle	$A = \frac{1}{2}bh$	b = base length h = height

13 For each of the following:

i identify and explain the error made in each calculation

ii determine the correct result.

a $2x + 6$

Let $x = 5$

$2x + 6 = 25 + 6$
$= 31$

b $3x^2 + 6$

Let $x = 1$

$3x^2 + 6 = 3(1)^2 + 6$
$= 9 + 6$
$= 15$

c $2^x + 4x$

Let $x = 0$

$2^x + 4x = 2^{(0)} + 2(0)$
$= 0 + 0$
$= 0$

d $x^2 - 4x$

Let $x = -3$

$x^2 - 4x = -(3)^2 - 4(-3)$
$= -9 + 12$
$= 3$

PROBLEM SOLVING AND REASONING

14 If $\$P$ is kept in a bank account that earns interest at a rate of $r\%$ per year, the amount of money in the bank account, $\$A$, after n years is given by:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

a If \$1000 is kept in a bank account for 1 year at an interest rate of 5%, calculate how much money there will be in the account at the end of the year.

b \$2000 is invested in a bank account with an interest rate of 4% per year. Use a calculator to find the amount of money in the account after 5 years, correct to 2 decimal places.

15 Consider the expression $2x^2 - 16x$. There are two values of x that, when substituted into the expression, give 0. Fill in the following table to find the two values of x that reduce the expression to 0.

x	x^2	$2x^2$	$16x$	$2x^2 - 16x$
0				
2				
4				
6				
8				

CHALLENGE

16 In most countries of the world, temperature is measured in degrees Celsius. Some countries, such as the USA, use degrees Fahrenheit. A formula can be used to convert a temperature from one unit to the other. Let C represent the temperature in degrees Celsius and F represent the temperature in degrees Fahrenheit.

a Use the formula $C = \frac{5}{9}(F - 32)$ to convert these temperatures into degrees Celsius. Write each answer to the nearest degree.

i 77°F **ii** −4°F **iii** 41°F **iv** 100°F **v** −1°F **vi** 62°F

b Use the formula $F = \frac{9}{5}C + 32$ to convert these temperatures into degrees Fahrenheit. Write each answer to the nearest degree.

i 30°C **ii** −10°C **iii** 0°C **iv** 27°C **v** −8°C **vi** 42°C

c The temperature in Chicago is 23°F. What is it in degrees Celsius?

d Which temperature is lower: 86°F or 32°C? Justify your answer.

17 The salespeople at a certain car dealership are permitted to lower the selling price of different cars by different set amounts when bargaining with customers.

For a car marketed at $\$x$, the salespeople can lower the price by $\$y$. After this, they can take 5% off the reduced value. Finally, if the customer still insists on a lower price, they can take off a further $\$z$.



a Write a formula for the lowest possible selling price, $\$P$, of a car marketed at $\$x$.

b A customer haggles on the price of a hatchback marketed at \$19 990. The salesperson first reduces the price by \$1000, then reduces it by a further 5% and finally takes off \$500. What is the final price of the hatchback?

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Substitution
- Worksheet: Substitution
- Investigation: Arithmetic and geometric sequences
- Topic quiz: 5B

5C Adding and subtracting algebraic terms

Learning intentions

By the end of this topic you will be able to ...

- ✔ identify like terms
- ✔ simplify expressions by collecting like terms.

Inter-year links

Year 9	2A Simplifying
Year 10	1C Simplifying

The laws of arithmetic

- All the laws of arithmetic apply to algebraic terms and expressions.

	Arithmetic	Algebra
The associative law	$(3 + 5) + 2 = 3 + (5 + 2)$ $(3 \times 5) \times 2 = 3 \times (5 \times 2)$	$(a + b) + c = a + (b + c)$ $(a \times b) \times c = a \times (b \times c)$
The commutative law	$3 + 2 = 2 + 3$ $3 \times 2 = 2 \times 3$	$a + b = b + a$ $a \times b = b \times a$

Like terms

- **Like terms** are terms that contain exactly the same pronumerals.
  - → Following the associative law, the order of pronumerals can be different in two like terms.
  - → It can be helpful to write terms containing powers out in **expanded form** to determine whether they are like terms.
    For example, $a^2b = a \times a \times b$ and $ab^2 = a \times b \times b$ so a^2b and ab^2 are not like terms.

$3a^2b = 3 \times a \times a \times b$

exponent form — expanded form

Pronumeral(s)	Examples of like terms
x	x, $3x$, $\frac{2}{3}x$, $0.7x$ ...
ab	ba, $-2ab$, $\frac{ab}{5}$, $6.6ab$ ...
m^2n	$-m^2n$, $18m^2n$, $\frac{9m^2n}{4}$, $0.05m^2n$ ...

Simplification by addition and subtraction of like terms

- Simplification involves reducing the complexity of an expression. This can be done by adding or subtracting like terms, multiplying terms or dividing terms.
- To simplify an algebraic expression by adding and subtracting like terms, or 'collecting like terms':
  1. Circle the like terms in the expression.
  2. Group the like terms together.
  3. Add and subtract the coefficients of each set of like terms.

$$6ab + \frac{3}{5}a + ab - \frac{2}{5}a = 6ab + ab + \frac{3}{5}a - \frac{2}{5}a$$

$$= 7ab + \frac{1}{5}a$$

Example 5C.1 Identifying like terms

Identify and list the like terms in the following groups of terms.

a $36x, -x, 5y, \frac{3x}{4}, 0.5y, \frac{1}{2}y$

b $7xy, \frac{5}{2}yx, -2x, -4x^2, \frac{x^2}{6}, 10x$

THINK

a Circle the like terms, or terms that contain exactly the same pronumerals. The values of the coefficients do not affect whether or not two or more terms are like terms.

b Circle the like terms. According to the commutative law, $xy = yx$, so $7xy$ and $\frac{5}{2}yx$ are like terms. Remember that $x^2 = x \times x$, so x and x^2 are *not* like terms.

WRITE

a $36x, -x, 5y, \frac{3x}{4}, 0.5y, \frac{1}{2}y$

The two sets of like terms in the list are:

$36x$, $-x$ and $\frac{3x}{4}$

$5y$, $0.5y$ and $\frac{1}{2}y$

b $7xy, \frac{5}{2}yx, -2x, -4x^2, \frac{x^2}{6}, 10x$

The three sets of like terms in the list are:

$7xy$ and $\frac{5}{2}yx$

$-2x$ and $10x$

$-4x^2$ and $\frac{x^2}{6}$

Example 5C.2 Adding and subtracting like terms

Simplify each expression where possible.

a $10x - 3x + 3 + x$

b $-6xy + 2x - 3yx - \frac{x}{2}$

c $2 + xy^2 - 5x^2y - 12xy^2 - 7 - 2y^2x$

THINK

a **1** Circle the like terms in the expression.

2 Group the like terms together.

3 Add and subtract the coefficients of like terms. Note that '3' is a constant and there are no other constants in the expression, so 3 cannot be combined with any other terms.

b **1** Circle the like terms.

2 Group the like terms.

3 Add and subtract the coefficients of each set of like terms. Remember that fractional coefficients can be written in two ways, so $-\frac{x}{2} = -\frac{1}{2}x$.

c **1** Circle the like terms.

2 Group the like terms.

3 Add and subtract the coefficients of each set of like terms. Note that $xy^2 = x \times y \times y$, but $x^2y = x \times x \times y$, so xy^2 and x^2y are *not* like terms and $-5x^2y$ cannot be combined with any other terms.

WRITE

a $10x - 3x + 3 + x = 10x - 3x + x + 3$
$= 8x + 3$

b $-6xy + 2x - 3yx - \frac{x}{2} = -6xy - 3yx + 2x - \frac{x}{2}$
$= -9xy + \frac{3}{2}x$

c $2 + xy^2 - 5x^2y - 12xy^2 - 7 - 2y^2x = 2 - 7 + xy^2 - 12xy^2 - 2y^2x - 5x^2y$
$= -5 - 13xy^2 - 5x^2y$

Helpful hints

- ✔ For two terms to be like terms, the pronumeral components of the terms must be *exactly* the same, but following the commutative law, the order of the pronumerals can vary.
  For example, xy and yx are like terms.
- ✔ The plus and minus signs in an algebraic expression also indicate when a coefficient is positive or negative. When grouping like terms, remember that the signs 'belong' to the terms to the right of them, so minus signs should move with their terms.
  For example, $x + 7y - 3x + 5y = x - 3x + 7y + 5y$.

ANS p514

Exercise 5C Adding and subtracting algebraic terms

△ 1–4, 5(1st column), 6, 7(b, d, f, h), 8, 9(b, c, e), 10, 11(a–c)
□ 1(b, d, f, h), 3, 5, 7, 9(d–f), 11–13, 14–16(a–c)
⬠ 1(g, h), 3(g, h), 5, 7, 9, 14(b, d, f), 15–16(c, d), 17–19

5C.1

1 Identify and list the like terms in the following groups of terms.

a $20x, 8y, -15y, y, -12x, 3x$

b $5x, \frac{3}{7}x, \frac{5}{7}y, -25x, 6y, \frac{x}{7}$

c $4xy, -yx, 0.1x, \frac{1}{8}xy, -\frac{yx}{10}, \frac{y}{15}$

d $-a, 5a^2, \frac{4}{9}a^2, 12.5a, 2.3a^2, -\frac{9a^2}{10}$

e $\frac{11}{18}p^3, \frac{p^2}{5}, 5.5p^3, p^3, -11p^2, -0.05p^2$

f $\frac{9ab}{10}, 7abc, -cab, \frac{5}{6}ab, 13cab, 7.2ba$

g $-9fg, 120f^2g, \frac{20fg^2}{3}, -\frac{14}{3}f^2g, 0.33fg^2, \frac{4fg}{5}$

h $\frac{2}{9}xy^2z, 0.095zyx, -12y^2xz, 18.6xyz, \frac{x^2y^2z}{24}, 99zy^2x$

2 Answer true or false to each statement. Explain why each false statement is incorrect.

a $2x$ and $-2x$ are like terms.

b $15a$ and $15a^2$ are like terms.

c $8p$ and $\frac{p}{5}$ are like terms.

d $\frac{5}{8}def$ and $5fed$ are like terms.

e $4ab$ and $6a^2b$ are like terms.

f $-3a^2b$ and $5ba^2$ are like terms.

UNDERSTANDING AND FLUENCY

UNDERSTANDING AND FLUENCY

3 Simplify each pair of like terms by adding or subtracting the coefficients of each term.

a $5x + 4x$

b $10kp - 2kp$

c $3a^2 - 7a^2$

d $5g - g$

e $yx - 3xy$

f $-5g^2h + 7g^2h$

g $\frac{3}{4}abc - \frac{1}{2}cab$

h $\frac{2xyz^2}{5} - \frac{2}{3}xz^2y$

4 Answer true or false to each statement. For the statements that are false, write the correct simplified expression.

a $8x + 6x$ simplifies to $14x$.

b $11k - k$ simplifies to 11.

c $3p + 6q$ simplifies to $9pq$.

d $5m^2 + 2n + m^2$ simplifies to $6m + 2n$.

e $2ab + 3a + 7ba$ simplifies to $12ab$.

f $\frac{1}{4}a + \frac{1}{3}c + \frac{1}{2}a + c$ simplifies to $\frac{3}{4}a + \frac{4}{3}c$.

g $x^2y + 2x + y$ cannot be simplified.

h $\frac{2}{9}p^2q - \frac{5}{6}p^2 + \frac{1}{6}qp^2$ cannot be simplified.

5C.2 **5** Simplify each expression where possible.

a $3b + 5 + 5b - 2b$

b $3n + 4mn - 9mn - 4n$

c $12 - 5c^2 - 8 - 3c^2$

d $4gh + 3hg - 3 - 6gh$

e $5pq - 6pq - 10q^2p + 8p^2q$

f $2x^2y + 10yx^2 - x^2y - 12xy^2$

g $\frac{5}{2}str + \frac{1}{3}st - \frac{1}{6}ts - \frac{2}{7}rst$

h $\frac{abc}{3} - \frac{2}{3}ab - \frac{5}{6}cab - 2 + \frac{ab}{6}$

i $\frac{g^3h}{5} - \frac{5}{3} + \frac{2}{10}g^3h + \frac{7}{2} + \frac{4g^2h}{5} - \frac{1}{15}gh$

j $\frac{5}{4}m^2n + \frac{1}{4}m^2n^4 + m^2n - \frac{3}{2} - m^2n^4 + \frac{5}{3}$

6 Consider the expression $7x^2 + 2xy + 9x^2 - yx$.

a Evaluate the expression by substituting $x = 3$ and $y = 7$ into the four terms.

b Simplify the expression by adding like terms.

c Check your answer to part **b** by substituting $x = 3$ and $y = 7$ into the simplified expression. Do you obtain the same answer as part **a**?

d Which expression was easier to evaluate? Explain.

7 Simplify then evaluate each expression by letting $a = 5$ and $b = 3$.

a $4a + 2a + 6b + 8b$

b $9a - 6a + 7b + 2b$

c $5ab + 4b - 2a - b$

d $6ab - ba - 7ba + a^2$

e $-4b^2 - b^2 + 8a^2 - a^2$

f $\frac{3}{4}a^2 - \frac{1}{2}a^2 + \frac{2}{3}b^2 - \frac{4}{3}b^2$

g $-ab + 3ab^2 - 10ba - 5b^2a$

h $7ab^2 + 2a^2b - 9ab^2 - 5ba^2$

i $\frac{3}{4}a^2b - 2ab^2 + \frac{a^2b}{5} - \frac{7}{6}ab^2$

PROBLEM SOLVING AND REASONING

8 When grouping the like terms in an expression, Tania writes:

$$3x - 2y + 7x - 4y = 3x - 7x + 2y - 4y$$

a Identify and explain Tania's mistake.

b Show the correct grouping of like terms in the expression $3x - 2y + 7x - 4y$.

9 Simplify each expression by collecting the like terms.

a $5x + 3y + 6y - 2x - 7y + 8x$

b $\frac{1}{4}y + \frac{1}{6} - \frac{1}{2}y - \frac{4}{3} + \frac{5}{4}y - y$

c $-2xy + 5yx + 4y - 8y + 2y - 3xy$

d $xy - x + 5yx - 3xy + 9x - 5yx$

e $8x^2y + 3yx^2 - 6xy^2 + 4 + 5yx^2 - 9$

f $\frac{3x^2y}{4} + 2xy^2 - 2x^2y - \frac{3}{5}yx^2 - \frac{xy^2}{2} + \frac{7}{4}y^2x - xy^2$

10 Paree is planning to build a rectangular garden bed.

a Draw a diagram of Paree's garden bed. If l = length and w = width, label each of the four sides of the diagram with the appropriate pronumeral.

b Write an expression for the perimeter of the garden bed by adding the lengths of each side.

c Simplify the expression by collecting the like terms.

d If P = perimeter, write a formula for the perimeter of the garden bed.

e Use the formula to calculate the total perimeter of three different garden beds with the following dimensions.

i $l = 4$ m, $w = 3$ m

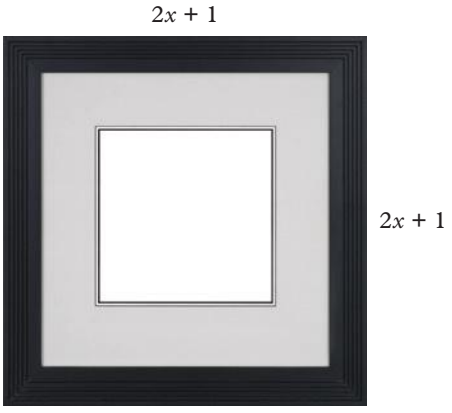
ii $l = 6.4$ m, $w = 2.8$ m

iii $l = 15\frac{1}{2}$ m, $w = 7\frac{1}{2}$ m

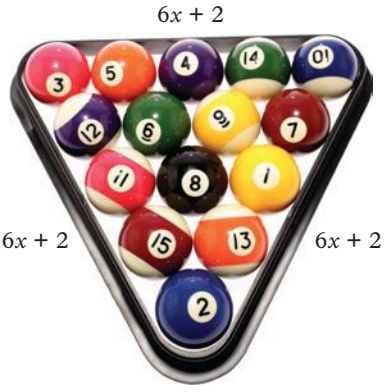
f Paree decides to plant a box hedge around the edge of her garden bed. She has enough plants to extend around a perimeter of 20 m. Suggest one possible set of length and width measurements for her garden bed.

11 Write an expression for the perimeter of each of the objects shown below.

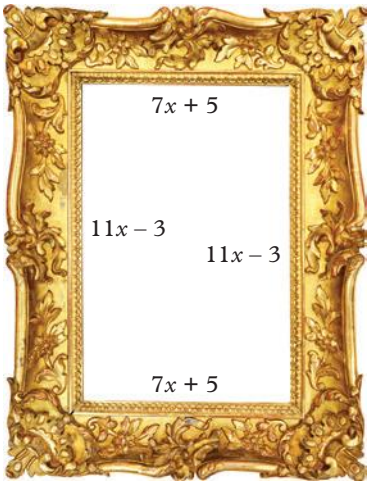
a length of outer edge of frame



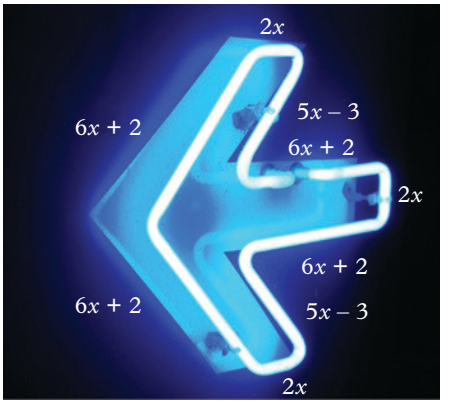
b perimeter of triangle



c perimeter of inner frame



d length of neon used



12 Calculate the perimeter for each object shown in question **11** if $x = 4$ cm.

13 Consider the following working out and explain the error made in each calculation. Give the correct simplified version of the left-hand side in each case.

a $a + a + a + a = a^4$

b $2x + 3y = 5xy$

c $9x + 4x = 13x^2$

d $8x - x = 8$

PROBLEM SOLVING AND REASONING

14 We can consider terms as like terms if they contain a factor of the exact same grouped term.

For example, $2(x+3)$, $5(x+3)$, $\frac{x+3}{2}$ are like terms because they all contain $(x+3)$ as a factor. However, $(x+3)$, $\frac{1}{x+3}$, $\sqrt{x+3}$ are not like terms. Simplify the following expressions.

a $3(2x+3)+9(2x+3)+4(2x+5)+7(2x+5)$

b $7(x-3y+4)-4(4-3y+x)+9(x+4-3y)-2(-3y+4+x)$

c $7(x+2)+2(x-2)-5(x+2)+6(x-2)$

d $\frac{6}{3x+1}+\frac{7}{3x+1}+\frac{8}{3x-1}+\frac{4}{3x-1}$

e $5\sqrt{7-2x}+4\sqrt{7+2x}-8\sqrt{7+2x}+2\sqrt{7-2x}$

f $-7\sqrt{a+b}+8(a+b)+\frac{2}{a+b}+6\sqrt{a+b}-\frac{5}{a+b}+(a+b)$

15 For each of the following statements, substitute the given values into each expression and evaluate them.

a $\frac{4}{3x}$ and $\frac{4}{3}x$

i $x=1$ **ii** $x=-1$ **iii** $x=2$

b $(-x)^2$ and $-x^2$

i $x=0$ **ii** $x=1$ **iii** $x=-1$

c $(-x)^3$ and $-x^3$

i $x=0$ **ii** $x=1$ **iii** $x=-1$

d $\frac{-3x}{5}$, $\frac{3x}{-5}$, $-\frac{3x}{5}$, and $\frac{-3x}{-5}$

i $x=1$ **ii** $x=-1$ **iii** $x=2$

CHALLENGE

16 For each set of expressions, state whether the expressions are equivalent or not equivalent and provide reasons to support your claim.

a $\frac{4}{3x}$ and $\frac{4}{3}x$ **b** $(-x)^2$ and $-x^2$

c $(-x)^3$ and $-x^3$ **d** $\frac{-3x}{5}$, $\frac{3x}{-5}$, $-\frac{3x}{5}$ and $\frac{-3x}{-5}$

17 Simplify the following expression.

$2a^2b^7c^5d^5e^9f^{11}g^{14}+4f^{11}g^{14}b^7d^5c^5a^2e^9+6a^2e^9g^{14}b^5f^{11}c^5d^7+5b^7f^{11}a^2c^5e^9g^{14}d^5+3f^{11}g^{14}d^7c^5b^5a^2e^9$

18 Simplify the expression $1.\dot{6}x+0.1\dot{6}x+0.41\dot{6}x$.

19 The price of an adult ticket to a theme park is $\$x$. The price of a concession ticket is three-fifths of an adult ticket and the price of a child ticket is five-eighths of a concession ticket. Write an expression for the total cost for a group visiting the theme park with three adults, five concession card holders and two children.



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Interactive skillsheet
Like terms

Investigation
Help the animals

Topic quiz
5C

5D Multiplying algebraic terms

Learning intentions

By the end of this topic you will be able to ...

- ✔ convert algebraic terms from expanded form to exponent form
- ✔ simplify algebraic products by multiplying algebraic terms.

Inter-year links

Year 7	1G Exponents and square roots
Year 9	2A Simplifying
Year 10	1C Simplifying

The exponent laws

- Repeated multiplication of variables can be written in exponent form.

$$\underbrace{3 \times a \times a \times a \times b}_{\text{expanded form}} = \underbrace{3a^3b}_{\text{exponent form}}$$

Simplification by multiplication of algebraic terms

- To multiply algebraic terms:
  1. Write each term in expanded form.
  2. Group the coefficients and like variables.
  3. Convert the term back into exponent form by counting the number of times each variable is repeated. Write the coefficient first, followed by the variables listed in alphabetical order.

$$\begin{aligned} 3a^3b \times (-2a^2b^2) &= 3 \times a \times a \times a \times b \times (-2) \times a \times a \times b \times b \\ &= 3 \times (-2) \times a \times a \times a \times a \times a \times b \times b \times b \\ &= -6a^5b^3 \end{aligned}$$

Example 5D.1 Writing algebraic terms in exponent form

Simplify the following terms by expressing them in exponent form.

a $2 \times a \times a \times a$ **b** $-9 \times b \times a \times a$

THINK

a 1 Write any repeated multiplications in exponent form. Identify the exponent of each base by counting the number of times that base is repeated in expanded form. a is repeated 3 times in expanded form, so $a \times a \times a = a^3$.

2 Leave out the multiplication signs.

b 1 Write any repeated multiplications in exponent form.

2 Leave out the multiplication signs and list the variables in alphabetical order.

WRITE

a $2 \times a \times a \times a = 2 \times a^3$

$= 2a^3$

b $-9 \times b \times a \times a = -9 \times b \times a^2$

$= -9a^2b$

Example 5D.2 Multiplying algebraic terms

Simplify the following products.

a $4ab \times 3c$ **b** $5a^2 \times (-a^4)$ **c** $-2p^2q^3 \times 7qp^5$

THINK

1 Write each term in expanded form. Remember the coefficient of $-a^4$ is -1.

2 Group the coefficients and variables.

3 Convert the term back into exponent form by counting the number of times each variable is repeated. Write the coefficient first, followed by the variables, listed in alphabetical order.

WRITE

a $4ab \times 3c = 4 \times a \times b \times 3 \times c$
$= 4 \times 3 \times a \times b \times c$
$= 12abc$

b $5a^2 \times (-a^4) = 5 \times a \times a \times (-1) \times a \times a \times a \times a$
$= 5 \times (-1) \times a \times a \times a \times a \times a \times a$
$= -5a^6$

c $-2p^2q^3 \times 7qp^5 = -2 \times p \times p \times q \times q \times q \times 7 \times q \times p \times p \times p \times p \times p$
$= -2 \times 7 \times p \times p \times p \times p \times p \times p \times p \times q \times q \times q \times q$
$= -14p^7q^4$

Helpful hints

- ✔ Exponents only apply to the number or variable immediately to the left of the exponent.
  For example, in the term $4gh^3$, the exponent of 3 only applies the variable h, hence: $4gh^3 = 4 \times g \times h \times h \times h$
- ✔ Use brackets to indicate that the exponent applies to more than one number or variable.
  For example, $(4gh)^3 = 4^3g^3h^3$
  $= 64g^3h^3$

ANS p515

Exercise 5D Multiplying algebraic terms

△ 1–3, 4(1st column), 5, 6, 7(1st column), 8(b), 9–10(2nd, 3rd columns), 11, 12(a), 13, 15(a–d)

□ 2, 4(2nd column), 6, 7, 9–10(2nd, 3rd columns), 12, 14–16

⬠ 4(g–j), 6(e, f), 7(2nd column), 9–10(3rd column), 12, 14–17

1 Simplify the following terms by determining the sign of the product, placing the coefficient first, and leaving out the multiplication sign.

a $4 \times x$ **b** $6 \times k$ **c** $d \times 7$ **d** $m \times 5$
e $1 \times p$ **f** $-3 \times h$ **g** $a \times (-9)$ **h** $-y \times 10$
i $8 \times (-c)$ **j** $-1 \times e$ **k** $-2 \times (-f)$ **l** $-m \times (-3)$

2 Simplify:

a $3 \times a \times b$ **b** $4 \times m \times n$ **c** $p \times 2 \times t$ **d** $d \times (-8) \times c$ **e** $x \times y \times 9$
f $k \times f \times 5$ **g** $a \times b \times c$ **h** $h \times y \times e$ **i** $2 \times q \times 4$ **j** $m \times 3 \times (-6)$

3 Match each of the terms written in expanded form with its exponent form.

Expanded form	Exponent form
a $2 \times a \times a \times a \times b \times c$	**A** $2a^2bc$
b $2 \times a \times a \times b \times c$	**B** $2a^3bc$
c $2 \times a \times b \times b \times c \times c$	**C** $2ab^2c^2$

5D.1 **4** Simplify the following terms by expressing them in exponent form.

a $5 \times m \times m$
b $-2 \times a$
c $-1 \times x \times y$
d $n \times 2 \times n$
e $-6 \times d \times d \times c \times c$
f $7 \times a \times a \times c \times b$
g $s \times s \times t \times t \times t \times (-2)$
h $-10 \times g \times g \times g \times f \times f$
i $1 \times q \times p \times q \times r \times q \times r \times r$
j $z \times x \times y \times (-11) \times x \times x \times y \times z$

5 Complete each of the multiplication problems below.

a $2^3 \times 2^4$
$= 2 \times 2 \times 2 \times ___ \times ___ \times ___ \times ___$
$= 2^7$

b $a^3 \times a^4$
$= a \times ___ \times ___ \times a \times ___ \times ___ \times ____$
$= ___$

c $m^2 \times m^6$
$= m \times ___ \times ___ \times ___ \times ___ \times ___ \times ___ \times ___$
$= ___$

d $x^5 \times x^2 \times x^3$
$= ___ \times ___ \times ___ \times ___ \times ___ \times ___ \times ___ \times ___ \times ___ \times ___$
$= ___$

6 Complete each of the multiplication problems below.

a $a^2 \times a^3$
b $k^4 \times k$
c $c^4 \times c \times c^2$
d $e \times e^5 \times f^2 \times f^3$
e $a^3 \times d^5 \times a^2 \times d$
f $m \times n \times n^3 \times m^4 \times m^5 \times n$

5D.2 **7** Simplify the following products.

a $6a \times 2bc$
b $-a^2 \times h^3$
c $3wz \times 7xy$
d $6x^4 \times 2y^5$
e $7a^5 \times 3a^2$
f $-5p \times p^{11}$
g $8n \times 3n$
h $5x^6y^5 \times 3x^2y^5$
i $a^5b^4 \times (-a^3b^2)$
j $9xy \times (-2xz)$
k $\frac{1}{6}w^4x^3y^5 \times \frac{6}{5}x^2yz$
l $\frac{3}{7}a^3b^3c \times (-6ca^2b^4)$

8 Complete the multiplication tables below by multiplying each term listed in the left-hand column by each of the terms in the top row.

a

×	$4a$	3	$6m$	$2abc$
$3d$				
ab^2				
$5mn$				
$2ac$				

b

×	$3x$	$4xyz$	$-5y$	$2x^2$
$-2x$				
xy^2				
$6xy$				
$-7yz$				

UNDERSTANDING AND FLUENCY

9 Simplify each of the following products.

a $-2gh \times 5b \times 7k$ **b** $3g^4 \times 5h^3 \times 2g^6$ **c** $4b^3 \times 3b \times (-b^5)$

d $mn \times 6m \times 8q$ **e** $7gh \times (-3gh) \times fgh$ **f** $\frac{1}{2}m^7 \times (-\frac{3}{4}m^3n) \times \frac{3}{5}mn^2$

g $4a^2b^5 \times 3ac \times 2a^3$ **h** $-2y^5wx^2 \times 5x^5w^2y \times (-xy^2)$ **i** $\frac{5}{4}ab^9d \times (-3a^5b^4c) \times \frac{2}{3}d^3c^6a^2$

10 Complete the following products by finding the missing factor in each of the factor pairs.

a $2 \times __ = 6$ **b** $8 \times __ = -16$ **c** $__ \times k = 6k$

d $3 \times __ = 3x$ **e** $a \times __ = ab$ **f** $x \times __ = -7xy$

g $__ \times c = 5cd$ **h** $4a \times __ = 8ab^2$ **i** $__ \times 3mn = 15mn^4$

j $6e \times __ = -12e^3f$ **k** $__ \times 4l^2 = 20jk^2l^3$ **l** $5xy \times __ = -5x^2yz^5$

PROBLEM SOLVING AND REASONING

11 There are a few conventions mathematicians follow when using fractional coefficients. Use your understanding of fractions and expanding expressions to answer the following.

a Which of the following are equal to $\frac{3}{2} \times 5$?

A $\frac{3 \times 5}{2 \times 5}$ **B** $\frac{3}{2 \times 5}$ **C** $\frac{3 \times 5}{2}$ **D** $\frac{3}{2} \times \frac{1}{5}$ **E** $\frac{2}{3} \times 5$

b Which of the following are equal to $\frac{3}{2}x$?

A $\frac{2}{3}x$ **B** $\frac{3}{2} \times \frac{1}{x}$ **C** $\frac{3}{2x}$ **D** $\frac{3x}{2}$ **E** $\frac{3x}{2x}$

c State the coefficient of x and y for $\frac{2 + 7x - y}{5}$.

d Which of the following are equal to $3\frac{1}{2}$?

A $3 - \frac{1}{2}$ **B** $3 \times \frac{1}{2}$ **C** $3 \div \frac{1}{2}$ **D** $3 + \frac{1}{2}$ **E** $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

e Which of the following are equal to 32?

A $30 - 2$ **B** 3×2 **C** $3 \div 2$ **D** $30 + 2$ **E** $2 \times 2 \times 2$

f Which of the following are equal to $3x$?

A $3 - x$ **B** $3 \times x$ **C** $3 \div x$ **D** $3 + x$ **E** $x \times x \times x$

g Using your answers from part **c** and **d**, explain why mathematicians would choose to not use a mixed number, where possible, as a coefficient, such as $3\frac{1}{2}x$.

12 a Complete the following calculations.

i $(3x^2)^4 = (3x^2) \times (___) \times (___) \times (___)$
$= 3 \times __ \times __ \times __ \times x^2 \times __ \times __ \times __$
$= ____$

ii $\left(\frac{x^2}{3}\right)^4 = \left(\frac{x^2}{3}\right) \times \left(\frac{__}{__}\right) \times \left(\frac{__}{__}\right) \times \left(\frac{__}{__}\right)$
$= \frac{x^2 \times __ \times __ \times __}{3 \times __ \times __ \times __}$
$= \frac{__}{__}$

b Use the method in part **a** to simplify the following expressions by writing them without brackets.

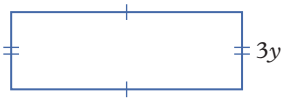
i $(2x^5)^6$ **ii** $\left(\frac{y^9}{2}\right)^5$ **iii** $(4a^2b^7)^3$ **iv** $\left(\frac{5u^6}{2v^5}\right)^4$

13 Write a simplified expression for the areas of the following shapes.

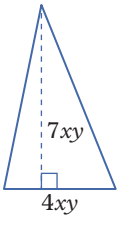
a



b



c



14 For each of the following,

i draw a labelled diagram

ii write a simplified expression for the area of the shape.

a A rectangle has a length three times its width.

b A triangle has a perpendicular height that is 4.3 times the width of the base.

c An isosceles triangle has a height, perpendicular to one of the two equal side lengths, that is half the length of the two equal side lengths.

15 The top of this box is rectangular, with a length three times its width. All measurements are in centimetres.



a If the x is equal to the width of the top of the box, write an algebraic term to represent the length of the top of the box.

b The area of a rectangle is equal to its length times its width. Write an algebraic multiplication problem to represent the area of the top of the box, then simplify.

c Calculate the area of the top of the box if $x = 8$ cm.

d How can you check that your answer to part **c** is correct?

e The height of the box is five times the width. Write an algebraic term to represent the height of the box.

f Draw a diagram of the box and label the length, width and height of the box with their respective algebraic terms.

g Complete this table to obtain the simplified algebraic term for the area of each of the six faces of the box.

Face	Area
Top	$3x \times x = 3x^2$
Bottom	___ × ___ = ___
Front	___ × ___ = ___
Back	___ × ___ = ___
Left side	___ × ___ = ___
Right side	___ × ___ = ___

h Use your answers to part **g** to write an expression for the surface area of the box. Simplify the expression by collecting the like terms.

i Calculate the surface area of the box if $x = 4$ cm.

j Calculate the surface area of the box for each value of x.

i 5 **ii** 12 **iii** 0.5 **iv** 2.1

k Volume = length × width × height. Write an algebraic term to represent the volume of the box.

l Use a calculator to work out the volume of the box for each x value given in part **j**.

16 The expression $a^3 \times a^4 = a^{3+4}$ as we are multiplying three a's and four a's to make a total of seven a's multiplied together. Simplify $a^{58} \times a^{97} \times a^{45}$ and explain why your answer is correct without writing the repeated multiplication out but referring to it.

17 Simplify $2^{5a+1} \times 3^{8+10a} \times 2^{7a} \times 2^4 \times 3^{3a+8}$.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Multiplying terms

Topic quiz
5D

PROBLEM SOLVING AND REASONING

CHALLENGE

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

5A 1 For the following expressions, identify:

i the pronumerals
ii the number of terms
iii the coefficients
iv the constant
v the highest exponent of each pronumeral.

a $4x - 3 + 9y - 4xy^2$
b $-\frac{9}{13} + \frac{5m^2}{3} + nv^3$

5A 2 Write an expression for each of the following.

a The quotient of x and 8
b The sum of a, b and c multiplied by 7
c The product of y and three more than y
d The square root of the difference of c and a

5A 3 Write an expression for each of the following.

a A cruel telephone company charges $20 per gigabyte after you exceed your data limit.
b The score an AFL team gets in a game is six times the number of goals plus the number of behinds.
c One-third of the total number of flowers in a garden containing tulips, roses and poppies must be red.
d Three more than 15% of the number of bread rolls in a bakery have seeds.

5B 4 Substitute the values into the expression and evaluate.

a $2x + 3y + 7z$ where $x = 5$, $y = 6$ and $z = 8$
b $3x^2 - 4x$ where $x = 2$
c $7(xy^2 + 1)$ where $x = 9$ and $y = 3$
d $\frac{4 + x^2}{5} + \frac{3}{10}y$ where $x = 4$ and $y = 15$

5B 5 Substitute the given values into the formula and evaluate.

a Find the value of y using the formula $y = mx + c$ where $m = 7$, $x = 5$ and $c = -4$.
b Find the value of s using the formula $s = \frac{1}{2}(u + v)t$ where $u = 4$, $v = 12$ and $t = 8$.

5B 6 Substitute the values into the expression and evaluate.

a $2x + 3y + 7z$ where $x = 0.5$, $y = \frac{1}{6}$ and $z = 8\frac{1}{2}$
b $3x^2 - 4x$ where $x = -2$

5C 7 Simplify each of the following expressions.

a $2x + x + 7y + 4x - 3y + 9 + 1$
b $7a + 9b - 3ab + b - 2a + 5ba$
c $-6pqr + 7rpq - 9qpr + 5prq + 11qrp$
d $14ikl + 16jkl + 17lki - 8klj - kli$

5C 8 Simplify each of the following expressions.

a $2x + x^2 - 5x + 3x^2$
b $7xy^2 + 8yx^2 - 6x^2y + 5y^2x$

5C 9 Simplify each of the following expressions.

a $1.72x + 4.24y - 9.18x - 3.98y$
b $\frac{5}{4}x^2 + \frac{3x^2}{8} - \frac{7}{2}x^2$

5D 10 Simplify each of the following expressions.

a $7x \times 9y$
b $-3ac \times (-9bd)$
c $2(5gh)(-11pq)$
d $4xy \times abc \times 9uv$

5D 11 Simplify each of the following expressions.

a $12x^2 \times 3x^3$
b $7xy^2 \times 8x^3y^2$

5D 12 Simplify each of the following expressions.

a $0.12x \times 3.1x$
b $\frac{25x^8}{16} \times \frac{24}{15}x^4$

5E Dividing algebraic terms

Learning intentions

By the end of this topic you will be able to ...

✔ simplify expressions by dividing algebraic terms.

Inter-year links

Support	Equivalent fractions
Year 7	3B Equivalent fractions
Year 9	2A Simplifying
Year 10	1C Simplifying

Simplification by division of algebraic terms

- To divide algebraic terms:
  1. Write each term in expanded form.
  2. Divide the coefficients by the highest common factor (HCF).
  3. Cancel out repeated variables that appear in both the numerator and denominator.
  4. Convert the term back into exponent form by counting the number of times each variable is repeated. Write the coefficient first, followed by the variables listed in alphabetical order.

$$\frac{6a^5b^3}{3a^3b} = \frac{\cancel{6}^{2} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a \times \cancel{b}^{1} \times b \times b}{\cancel{3}^{1} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{b}^{1}}$$
$$= \frac{2 \times a \times a \times b \times b}{1}$$
$$= 2a^2b^2$$

- Remember that quotients can be expressed as fractions.

$$x \div 7 = \frac{x}{7}$$

- Remember that a number divided by itself has a value of 1.

$$x \div x = \frac{x}{x} = 1$$

Example 5E.1 Dividing algebraic terms

Simplify the following quotients.

a $\frac{5a^2}{a}$ **b** $\frac{8a^3b}{2a^2b^2}$

THINK

1. Write each term in expanded form.
2. Divide the coefficients by the highest common factor (HCF).
3. Cancel out repeated variables that appear in both the numerator and denominator. Remember that any term divided by itself is equal to 1, so $\frac{a}{a} = 1$ and $\frac{b}{b} = 1$.
4. Convert the term(s) back into exponent form by counting the number of times each variable is repeated. Write the coefficient first, followed by the variables listed in alphabetical order.

WRITE

a $\frac{5a^2}{a} = \frac{5 \times a \times a}{a}$
$= \frac{5 \times \cancel{a}^{1} \times a}{\cancel{a}^{1}}$
$= \frac{5 \times a}{1}$
$= 5a$

b $\frac{8a^3b}{2a^2b^2} = \frac{8 \times a \times a \times a \times b}{2 \times a \times a \times b \times b}$
$= \frac{\cancel{8}^{4} \times \cancel{a}^{1} \times \cancel{a}^{1} \times a \times \cancel{b}^{1}}{\cancel{2}^{1} \times \cancel{a}^{1} \times \cancel{a}^{1} \times \cancel{b}^{1} \times b}$
$= \frac{4 \times a}{b}$
$= \frac{4a}{b}$

Helpful hints

- ✔ Take care when writing variables with a large exponent in expanded form.
- ✔ The divisibility rules can help you to find the HCF. Recall that a number is:
  - → divisible by 2 if the number is even
  - → divisible by 3 if the number's digits add to a multiple of three
  - → divisible by 5 if the number ends in zero or five.

ANS p516

Exercise 5E Dividing algebraic terms

△ 1(2nd, 3rd columns), 2, 3(1st, 3rd columns), 4–10 ▢ 1(3rd, 4th columns), 3(1st, 4th columns), 4–8, 10, 11, 12 ⬠ 3(j–o), 6–8, 10, 11, 13–15

UNDERSTANDING AND FLUENCY

5E.1

1 Simplify the following quotients.

a $\frac{5a}{5}$ **b** $\frac{15x}{3y}$ **c** $\frac{9d}{d}$ **d** $\frac{12w}{4w}$ **e** $\frac{3m}{9m}$

f $\frac{4ab}{a}$ **g** $\frac{18c^3}{6c}$ **h** $\frac{10b^5}{15b^2}$ **i** $\frac{4mnp}{24mn}$ **j** $\frac{35wx}{21wxy}$

k $\frac{6a^4b^2}{ab^2}$ **l** $\frac{5m^3}{m^2n^4}$ **m** $\frac{6m^3bc}{9mc}$ **n** $\frac{14s^3t^4}{7s^3t^2}$ **o** $\frac{8h^5k^2j}{10h^3k}$

2 Complete the following division problems.

a $xy \div y$ **b** $8k \div 4$ **c** $18ab \div 2b$

d $9x^2y \div xy$ **e** $3mn^2 \div 12mn$ **f** $ab^2c^3d \div abc^2$

3 Simplify each algebraic fraction where possible.

a $\frac{15k}{12km}$ **b** $\frac{2jk^5}{k^4}$ **c** $\frac{3xy^4}{y^4}$ **d** $\frac{m^7}{p^3}$ **e** $\frac{b^5c^4}{b^3c^2}$

f $\frac{4k^8}{2a^5}$ **g** $\frac{18x^4}{22y^4}$ **h** $\frac{6a^3b}{2c^3}$ **i** $\frac{x^2y}{4x}$ **j** $\frac{8mn^2}{2m}$

k $\frac{2m^6n^7}{2m^5n^2}$ **l** $\frac{6x^6y^5}{6x^2y^5}$ **m** $\frac{18w^4x}{3x^5y^4}$ **n** $\frac{16pq}{20p^2q}$ **o** $\frac{ab^2cd}{bc^2}$

4 Simplify:

a $\frac{-9xy}{3x}$ **b** $30a^3b^2 \div (-6ab^2)$ **c** $\frac{-12uv^3}{-8uv}$ **d** $-18rsv \div 21svp$

5 Simplify:

a $\frac{x^4 \times x^3}{x^2}$ **b** $\frac{m^7 \times m^6}{m^9}$ **c** $\frac{6a^2 \times a^8}{a^4}$ **d** $\frac{n^5 \times n^7}{n^3 \times n^4}$ **e** $\frac{5d^6 \times d^3}{d^9}$

6 Fill in the blanks to complete the equation.

a $16m \div ___ = 2m$ **b** $\frac{27jk}{___} = -3k$ **c** $___ \div 2xy = 8x^2y$ **d** $\frac{___}{10a^2b^3} = \frac{a}{5b}$

7 Simplify:

a $\frac{8x^3y}{4x} + \frac{3x^2y^2}{y} - \frac{18x^5y^3}{3x^3y^2}$ **b** $\frac{16b^5a^3 - 3a^3b^5 + 7a^3b^5}{2a^2b}$

c $\frac{9x^2y}{3x^2} + \frac{16x^3y^5}{8x^2y^5} + \frac{36x^{11}y^9}{4x^{11}y^8} + \frac{15x^7y}{x^7} + \frac{33xy^{15}}{11y^{15}}$ **d** $\frac{4mn^2 + 15nm^2 - 6m^2n^2 + 17n^2m - 11m^2n}{mn}$

8 Simplify the following quotients, then evaluate by substituting the given value(s).

a $\frac{42c^2d^6}{35c^5d^4}$ where $d = 5$ and $c = 6$ **b** $a^5z^3 \div az^2$ where $a = 2$ and $z = 4$

c $\frac{25pq}{5p}$ where $p = 3.5$ and $q = 8.1$ **d** $\frac{28x^4y^9}{4y^9x^4}$ where $x = 31$ and $y = 78$

9 Fabian and his father are laying cement pavers in the backyard to create a smooth surface for skateboarding. The pavers come in different sizes but are rectangular in shape and have a length twice their width.

a Let x represent the width of a paver in centimetres. Write the length in terms of x and hence write an algebraic term for the area of one paver.

b The ground to be paved is square with a length represented by $10x$ cm. Write an algebraic term for the area of the square to be paved.



c How many pavers are required to pave the ground?

d To check whether your answer to part **c** is correct, substitute a value for x. For example, let $x = 40$, and evaluate each of the following using this value of x.

i width of paver **ii** length of paver

iii area of paver **iv** length of square to be paved

v area of square to be paved

e Use your answers to part **d iii** and **v** to calculate the number of pavers required. How does this compare to your answer to part **c**?

f Will the size of the pavers affect the number of pavers needed? Explain.

10 To divide two terms, we can multiply the first term by the reciprocal of the second term. Complete the following division problems.

a $\frac{3x}{8} \div \frac{5x}{4}$ **b** $\frac{9x^3}{4y} \div \frac{4x}{3y^2}$ **c** $\frac{x^2y^3}{z^5} \div \frac{1}{xyz}$ **d** $\frac{1}{ab^2c^3} \div a^2bc^3$

11 The lowest common multiple (LCM) of two numbers is the common multiple with the lowest value. When written as the product of prime factors, the LCM is the product of the highest power of each factor.

For example, the LCM of $2^3 \times 3^2 \times 5^6$ and $2^4 \times 3^5 \times 5$ is $2^4 \times 3^5 \times 5^6$. Similarly, the LCM of $x^3y^2z^6$ and x^4y^5z is $x^4y^5z^6$. Determine the LCM of the following:

a $2^9 \times 13^5 \times d^{12}$ and $2^{14} \times 13^2 \times d^6$

b $3^2 \times 7^4 \times a^9 \times b^2$ and $3^4 \times 7^2 \times a^5 \times b^6$

c $p^6q^7r^{11}s^6$ and $p^9q^2r^{15}s^2$

d $r^{42}s^{51}v^{99}$ and $r^{45}t^{78}u^{84}$

12 Simplify each expression.

a $5g \times 3h + \frac{8g^2h}{4g}$

b $\frac{3t \times 7u - 5 \times ut}{4t}$

c $8r^2 - 9r \times (2r - 7r)$

d $a^2 \times 2a - 10a^4 \div 2a^2 + \frac{6a^5}{3a^3} + 11a \times a$

13 Consider the expression $\frac{x}{y^4z^2} \div \frac{z}{xy^2} \times \frac{y^2z^3}{x^2}$.

a Evaluate the expression when:

i $x = 2$, $y = 3$ and $z = 5$ **ii** $x = 3$, $y = 5$ and $z = 2$ **iii** $x = 2$, $y = 2$ and $z = 2$

b Simplify the expression and use the simplified form to explain the answers to part **a**.

14 Simplify the following expression:

$$\frac{2x - 2x^2 \div 2x + 2x \times x - x^5 \div x^3}{3x + x^3 \div \left(5x \times \frac{x}{5}\right)}$$

15 a Use six of the digits 1–9 only once such that the quotient below has the largest sum of the exponents of x and y possible. State this sum.

$x^{\square}y^{\square} \div x^{\square}y^{\square} = x^{\square}y^{\square}$

b How do you know this is the largest sum?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Dividing terms

Topic quiz
5E

PROBLEM SOLVING AND REASONING

CHALLENGE

5F Expanding

Learning intentions

By the end of this topic you will be able to ...

✔ expand algebraic expressions.

Inter-year links

Year 7	6D Order of operations in algebra
Year 9	2B Expanding
Year 10	1D Expanding

Expanding algebraic expressions

- To **expand** an algebraic product containing brackets using the distributive law:
  1. Multiply each term inside the brackets by the term outside the brackets.
  2. Simplify the results by performing any multiplication, addition and subtraction.

$$a(b + c) = a \times b + a \times c$$
$$= ab + ac$$

The distributive law

- The distributive law applies to both numerical and algebraic expressions.

	Arithmetic	Algebra
The distributive law	$7(30 + 2) = 7 \times 30 + 7 \times 2$ $= 210 + 14$ $= 224$	$a(b + c) = ab + ac$

The area model

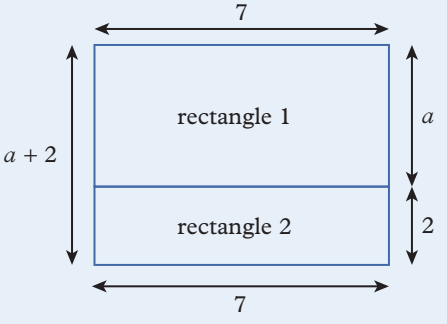
- The application of the distributive law to algebraic expressions can also be demonstrated using the area of a rectangle.

  large rectangle = rectangle 1 + rectangle 2

  length × width = length × width_1 + length × width_2

  $7 \times (a + 2) = 7 \times a + 7 \times 2$

  $7(a + 2) = 7a + 14$



Example 5F.1 Expanding by a whole number

Expand and simplify each expression using the distributive law.

a $6(a + 4)$ **b** $2(3x - y)$ **c** $3(2x + 4y + 5)$

THINK	WRITE
a 1 Multiply each term inside the brackets by the term outside the brackets.	**a** $6(a + 4) = 6 \times a + 6 \times 4$
2 Simplify the results by performing the multiplications.	$= 6a + 24$
b 1 Multiply each term inside the brackets by the term outside the brackets. Note that the second term inside the brackets is negative.	**b** $2(3x - y) = 2 \times 3x + 2 \times (-y)$
2 Simplify the results by performing the multiplications.	$= 6x - 2y$
c 1 Multiply each of the three terms inside the brackets by the term outside the brackets.	**c** $3(2x + 4y + 5) = 3 \times 2x + 3 \times 4y + 3 \times 5$
2 Simplify the results by performing the multiplications.	$= 6x + 12y + 15$

Example 5F.2 Expanding by a variable

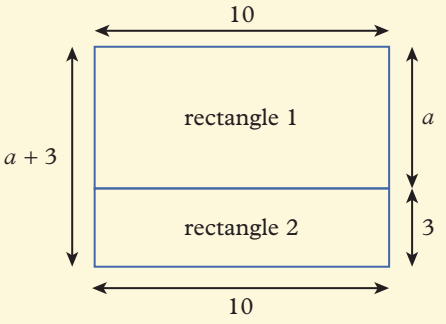
Expand and simplify each expression.

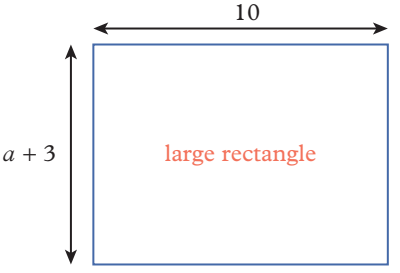
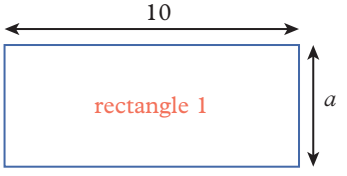
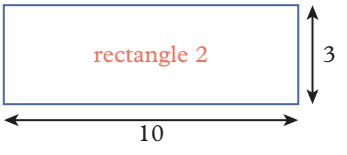
a $-7p(3q - 4)$ **b** $2m(5m - 4n)$ **c** $5a(a - 3b - 5)$

THINK	WRITE
a 1 Multiply each term inside the brackets by the term outside the brackets. Enclose the second term in brackets to separate the plus sign and the negative coefficient of the p term.	**a** $-7p(3q - 4) = -7p \times 3q + (-7p) \times (-4)$
2 Simplify the results by performing the multiplications. Remember the product of two negative numbers gives a positive number.	$= -21pq + 28p$
b 1 Multiply each term inside the brackets by the term outside the brackets.	**b** $2m(5m - 4n) = 2m \times 5m + 2m \times (-4n)$
2 Simplify the results by performing the multiplications. Remember that $m \times m = m^2$.	$= 10m^2 - 8mn$
c 1 Multiply each of the three terms inside the brackets by the term outside the brackets.	**c** $5a(a - 3b - 5) = 5a \times a + 5a \times (-3b) + 5a \times (-5)$
2 Simplify the results by performing the multiplications. Remember the product of a negative and positive number is a negative.	$= 5a^2 - 15ab - 25a$

Example 5F.3 Demonstrating the expansion of algebraic expressions using the area model

Demonstrate that $10(a + 3) = 10a + 30$ is true by finding the area of each rectangle in the diagram.



THINK	WRITE
1 Write an expression for the area of the large, composite rectangle.	area of large rectangle = length × width $= 10 \times (a + 3)$ $= 10(a + 3)$
	
2 Write an expression for the area of rectangle 1.	area of rectangle 1 $= 10 \times a$ $= 10a$
	
3 Calculate the area of rectangle 2.	area of rectangle 2 $= 10 \times 3$ $= 30$
	
4 Show that the area of the large rectangle is equal of the sum of the area of rectangle 1 and rectangle 2.	area of large rectangle = area of rectangle 1 + area of rectangle 2 Therefore, $10(a + 3) = 10a + 30$

Helpful hints

- ✔ When multiplying through by a negative term, use brackets for clarity.
  For example, $-7p(3q - 4) = -7p \times 3q + (-7p) \times (-4)$
- ✔ Show all your working out to avoid making arithmetical errors and errors related to sign.
- ✔ After expanding expressions containing brackets, always simplify your results.
  Simplify, simplify, simplify!

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Exercise 5F Expanding

△ 1(a, b, g, h, i), 2–6, 7(a, b, e, h), 8, 10–12 | □ 2, 3, 4(c, d), 5(b, d, f, h), 6, 7(c, f, g, h), 9, 12–14, 16 | ⬠ 2(2nd, 3rd columns), 3(e, f), 5(b, d, f, h), 7(c, f, g, h), 9(c, d), 13–18

5F.1 **1** Expand and simplify each expression using the distributive law.

a $3(a + 2)$ **b** $5(k - 7)$ **c** $2(5 + a)$

d $8(a + b)$ **e** $4(2x + 6)$ **f** $2(-1 + 8m)$

g $7(5a + 3b)$ **h** $6(8c - 5d)$ **i** $3(-5m - 2n)$

5F.2 **2** Expand and simplify each expression.

a $a(b + 4)$ **b** $x(3 + w)$ **c** $d(c - e)$

d $p(2p + 3)$ **e** $k(1 - 4k)$ **f** $2x(4y + 3w)$

g $3x(1 - 4x)$ **h** $\frac{1}{3}m(2m + 7n)$ **i** $-3a(2a - 4b)$

j $-2e^2(-4e - 5)$ **k** $5a^2\left(\frac{1}{10}a^2 - 3ab\right)$ **l** $-11y(x^2 + y^2)$

3 The distributive law can also be applied to the expansion of brackets containing more than two terms. Use the distributive law to expand and simplify the following expressions.

a $2(3x + y + 7)$ **b** $5(2ab + 4d - 3)$ **c** $7(2c - d + 5ef)$

d $-6(3g - 2h + gh)$ **e** $-4(5k + 2km - 9)$ **f** $-3(2xy + 4 - 7w)$

4 Apply the distributive law, then simplify by collecting any like terms to complete the following calculations.

a $5(a + 4) + 2(a + 7)$
$= 5 \times __ + 5 \times __ + 2 \times __ + 2 \times __$
$= 5a + __ + 2a + __$
$= _a + __$

b $3b(c + 2) + 2c(a + 5b)$
$= __ \times c + __ \times 2 + 2c \times __ + 2c \times __$
$= 3bc + __ + __ + __$
$= __ + __ + __$

c $d(d + 8) + 6(d - 3)$
$= d \times __ + d \times __ + __ \times d + __ \times (-3)$
$= __ + 8d + __ - __$
$= __ + __ - __$

d $4k(2k + 3) - 2(k - 5)$
$= 4k \times __ + 4k \times __ + (-2) \times __ + (-2) \times (-5)$
$= 8k^2 + __ - __ + __$
$= __ + __ + __$

5 Expand and simplify each expression.

a $2(a + 6) + 5(a + 3)$ **b** $4(b - 3) + 6(b + 2)$

c $5m(n + 4) + 3n(k + 2m)$ **d** $7x(3 + y) + \frac{2}{3}y(8 - 3x)$

e $\frac{k}{4}(k - 1) + 5(k + 4)$ **f** $c(c + 4) - 7(c - 5)$

g $rst(2 + 5t) + 9r(4t + st)$ **h** $h(h^2 - 2) - 4h(h + 3)$

UNDERSTANDING AND FLUENCY

6 Complete the following expansion.

$-(a+2) = -1 \times (a+2)$
$= -1 \times ___ - 1 \times ___$
$= -___ - ___$

7 Expand and simplify each expression.

a $6(p+3) + (p+2)$
b $3(d-2) - (d+8)$
c $x(x+2) + (x+5)$
d $k(k+4) - (3k+1)$
e $2e\left(e+\frac{1}{2}\right) - \left(e-\frac{3}{2}\right)$
f $4a(b+1) - (6+ab)$
g $\frac{1}{3}w(y-3) - (wy+2)$
h $mp(n-m) + (n-3m^2p)$

8 Consider the expression $5a(3a+10)$.

a Calculate the value of the expression if $a = 4$.
b Expand the expression.
c Substitute $a = 4$ into your answer to part **b**.
d Was it easier to evaluate the expression containing brackets, or its expanded form? Explain.

PROBLEM SOLVING AND REASONING

9 Expand and then evaluate each expression by letting $a = 2$.

a $2a(a+5)$ **b** $3a(10-2a)$ **c** $5a(3a-13)$ **d** $\frac{a}{3}(6a-9)$

10 In three years, Emily will be twice as old as Chantelle.

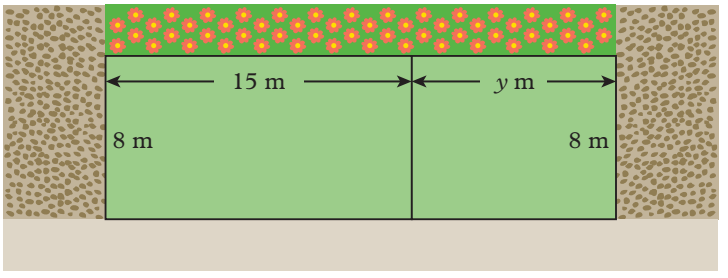
a If Chantelle is currently c years old, write an expression for Chantelle's age in three years' time.
b Use your answer to part **a** to write an expression for Emily's age in three years' time.
c Use the distributive law to expand the expression for Emily's age in three years' time.
d Determine Emily's age in three years' time if Chantelle is currently 18 years old.

11 One of Andre's jobs is to mow the front lawn. The front lawn is rectangular and measures 15 m by 8 m.

a Calculate the area of lawn to be mown.

A neighbour notices Andre is about to start mowing and asks Andre to mow his front lawn as well.

b Let y be equal to the width of the neighbour's front lawn. Write an expression for the area of lawn in the neighbour's front yard.

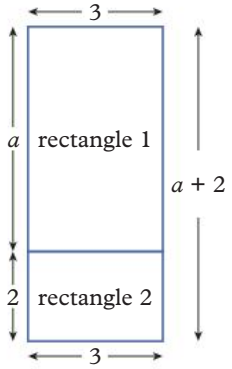




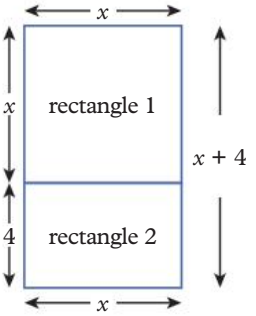
c Add your answers to parts **a** and **b** to write an expression for the total area of lawn to be mown.
d Now consider both lawns as one large rectangle. Draw a diagram of this rectangle and label the length and width measurements.
e Use brackets to write an expression for the total area of the large rectangle.
f Expand the expression and compare the expanded expression to your answer to part **c**. Are the expressions the same? Why or why not?

5F.3 **12** Demonstrate that each statement is true by finding the area of each rectangle in the diagrams below.

a $3(a + 2) = 3a + 6$



b $x(x + 4) = x^2 + 4x$



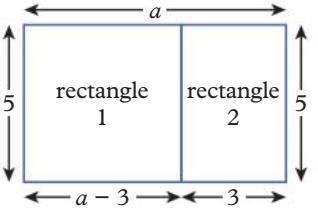
13 Draw diagrams that could be used to demonstrate that each of the following statements is true.

a $6(k + 8) = 6k + 48$

b $4(2d + 9) = 8d + 36$

c $2p(p + 3) = 2p^2 + 6p$

14 The diagram below can be used to demonstrate that $5(a - 3) = 5a - 15$.



a What is the length and width of rectangle 1?

b Use your answer to part **a** to write an expression for the area of rectangle 1 using brackets.

c Another way to calculate the area of rectangle 1 is to subtract the area of rectangle 2 from the total area of the large rectangle.

i State the length and width of the large rectangle.

ii Use these length and width measurements to write an expression for the area of the large rectangle.

iii State the length and width of rectangle 2.

iv Use these length and width measurements to write an expression for the area of rectangle 2.

v Use your answers to parts **ii** and **iv** to write an expression for the area of rectangle 1.

15 Draw a diagram and demonstrate $c(d - 5) = cd - 5c$.

16 If a and b are both positive integers, is $5a \times b + 3 < 5a(b + 3)$? Use expansion to justify your answer.

17 Expand and simplify each expression.

a $3x^2(4x^3 - 7) - 5x^4(x + 2)$

b $a^3(ab + c - 1) + a^2(3ac - a^2b + a)$

c $2m(3m^2np^2 + 5mn) - 6n(m^2 + m^2p^2 - 4)$

d $5w(4w^2x - 3y) + 2y(9w - 6xy) - 4x(5w^3 - 3y^2)$

18 Evaluate each expression in question **17** using these values:

$a = -3, b = 7, c = 4, m = 2, n = -4, p = -1, w = -5, x = 2, y = -6$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Expanding over one pair of brackets

Investigation
Andrew's vegie patch

Topic quiz
5F

5G Factorising

Learning intentions

By the end of this topic you will be able to ...

- ✔ find the HCF of two or more algebraic terms
- ✔ factorise algebraic expressions by taking out the HCF.

Inter-year links

Support	Multiples and factors
Year 7	2D Factors and the highest common factor
Year 9	2C Factorising using the HCF
Year 10	1F Factorising

The highest common factor (HCF) of algebraic terms

- A common factor of two or more algebraic terms is an algebraic term that divides exactly into each of those terms.
- The highest common factor (HCF) of two or more algebraic terms is the 'highest' or 'largest' combination of coefficients, pronumerals and algebraic expressions that are factors of every term.
  The HCF of a set of algebraic terms can have both a numerical and a pronumeral component.

Set of algebraic terms	HCF
x, $7x$	x
$6m$, $12mn$, $8mp$	$2m$
$15a^3bc$, $9a^2c$	$3a^2c$

- To identify the HCF of a set of algebraic terms:
  1. Write each term as the product of its prime and algebraic factors.
  2. Circle the common factors.
  3. Multiply the common factors for the HCF.

$$15a^3bc = (3) \times 5 \times (a) \times (a) \times a \times b \times (c)$$
$$9a^2c = (3) \times 3 \times (a) \times (a) \times (c)$$
$$\text{HCF} = 3 \times a \times a \times c$$
$$= 3a^2c$$

Factorising algebraic expressions

- **Factorising** an algebraic expression is the opposite to expanding an algebraic expression.

expanding →
$$7(a + 2) = 7a + 14$$
← factorising

- To factorise an algebraic expression:
  1. Identify the HCF of all the terms in the expression.
  2. Write the HCF in front of the brackets.
  3. Inside the brackets, divide each term in the original expression by the HCF.
  4. Simplify the expression inside the brackets.

$$7a + 14 = 7\left(\frac{\cancel{7}^{1}a}{\cancel{7}^{1}} + \frac{\cancel{14}^{2}}{\cancel{7}^{1}}\right)$$
$$= 7(a + 2)$$

Original expression: $7a + 14$
HCF: 7
Factorised expression: $7(a + 2)$

Example 5G.1 Identifying the highest common factor

Find the highest common factor (HCF) of each pair of terms.

a $2a$ and $5a$

b $12xy$ and $-18x^3$

THINK

a 1 Write each term as the product of its prime and algebraic factors.

2 Circle the common factors.

3 The only common factor is 'a', so the HCF is a.

b 1 Write each term as the product of its prime and algebraic factors. The negative sign can be represented using the factor '-1'.

2 Circle the common factors.

3 Multiply the common factors for the HCF. Simplify the product.

WRITE

a $2a = 2 \times (a)$

$5a = 5 \times (a)$

HCF $= a$

b $12xy = (2) \times 2 \times (3) \times (x) \times y$

$-18x^3 = -1 \times (2) \times (3) \times 3 \times (x) \times x \times x$

HCF $= 2 \times 3 \times x$

$= 6x$

Example 5G.2 Factorising algebraic expressions

Factorise the following expressions.

a $8x + 28$

b $9xyz - 12wxy$

THINK

a 1 Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term in the original expression by the HCF.

4 Simplify the bracketed expression.

b 1 Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term in the original expression by the HCF. Note that the minus sign remains from the original expression.

4 Simplify the bracketed expression.

WRITE

a HCF $= 4$

$8x + 28 = 4\left(\frac{8^2 x}{4^1} + \frac{28^7}{4^1}\right)$

$= 4(2x + 7)$

b HCF $= 3xy$

$9xyz - 12wxy = 3xy\left(\frac{9^3 x^1 y^1 z}{3^1 x^1 y^1} - \frac{12^4 w x^1 y^1}{3^1 x^1 y^1}\right)$

$= 3xy(3z - 4w)$

Example 5G.3 Factorising algebraic expressions containing exponents

Factorise each expression.

a $a^2 - 6a$ **b** $10m^3n + 15m^2n^2$

THINK

a **1** Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term by the HCF. It is helpful to write the pronumerals with exponents in expanded form. Note that the minus sign remains from the original expression.

4 Simplify the bracketed expression.

b **1** Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term by the HCF. It is sometimes helpful to write the pronumerals with exponents in expanded form. Alternatively, an exponent law can be used to simplify the exponents.

4 Simplify the bracketed expression.

WRITE

a HCF = a

$$a^2 - 6a = a\left(\frac{a^1 \times a}{a^1} - \frac{6 \times a^1}{a^1}\right)$$
$$= a(a - 6)$$

b HCF = $5m^2n$

$$10m^3n + 15m^2n^2 = 5m^2n\left(\frac{\cancel{10}^{2} \times m^1 \times m^1 \times m \times n^1}{\cancel{5}^{1} \times m^1 \times m^1 \times n^1} + \frac{\cancel{15}^{3} \times m^1 \times m^1 \times n^1 \times n}{\cancel{5}^{1} \times m^1 \times m^1 \times n^1}\right)$$
$$= 5m^2n(2m + 3n)$$

Helpful hints

- ✔ When looking for the highest common factor, remember to consider any coefficients and all pronumerals.
- ✔ Simplify, simplify, simplify!

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Exercise 5G Factorising

△ 1, 2(1st column), 3, 4(1st, 2nd columns), 5, 6(1st, 2nd columns), 7, 8–9(1st column), 10–13

□ 2(2nd column), 4(2nd, 3rd columns), 6(3rd, 4th columns), 7, 8–11(2nd column), 14, 15–16(a, c), 18

⬠ 2(h, j, l, n), 4(3rd column), 6(4th column), 7, 8–11(2nd column), 14, 15, 16(b, d), 17, 18

1 Match each pair of terms with their HCF.

Terms	HCF
a $15a$ and 10	**A** $15b$
b $5ab$ and $5a$	**B** $10ab$
c $20a^2b$ and $10ab$	**C** 5
d $15a^5b^3$ and $30bc^2$	**D** $5a$

5G.1 **2** Find the highest common factor (HCF) of each pair of terms.

a $4x$ and 12
b $10d$ and $-5d$
c $6j$ and $8jk$
d $3ab$ and $-3ab$
e $-9cd$ and $9ef$
f $7xy$ and $14yz$
g abc and bcd
h $10abcd$ and $-14bcde$
i $6xy$ and $2y^2$
j $3c^2$ and $9b^2c$
k $30g^2h$ and $-24g^4$
l $4ab^3$ and $8bc^3$
m $2m^2n^2p^2$ and $3m^3p^2$
n $15r^2s^4t$ and $-20r^3s^4t^5$

3 Complete the following factorised expressions.

a $2x + 6 = 2\left(\frac{2x}{__} + \frac{6}{__}\right) = 2(__+__)$

b $8a - 36 = __\left(\frac{8a}{4} - \frac{36}{4}\right) = __(2a - 9)$

c $25mn + 30n = 5n\left(\frac{__}{5n} + \frac{__}{5n}\right) = 5n(__+__)$

d $kmn - mn = __\left(\frac{kmn}{__} - \frac{__}{__}\right) = __(k - __)$

e $12ef - 18df = 6f\left(\frac{__}{__} - \frac{__}{__}\right) = 6f(__-__)$

f $-26ab + 24cd = __\left(\frac{-26ab}{__} + \frac{__}{__}\right) = __(13ab + __)$

g $-abc + acd = -ac\left(\frac{__}{__} + \frac{__}{__}\right) = -ac(__-__)$

h $-21xyz - 28wxy = __\left(\frac{__}{__} - \frac{__}{__}\right) = __(__+4w)$

5G.2 **4** Factorise each expression.

a $4a + 8$
b $7e + 21$
c $45 + 5k$
d $3 + 12y$
e $27 - 9p$
f $24ab + 18b$
g $12st - 14t$
h $5mn + 10n$
i $3g - 9gh$
j $22xy - 4xz$
k $3ab + 3bc$
l $16mn - 8mp$
m $27xyz - 18wxy$
n $12kmn + 15mn$
o $-21efg + 24fgh$

5 Complete the following factorised expressions.

a $h^2 + 7h = h\left(\frac{h^2}{__} + \frac{7h}{__}\right) = h(__+__)$

b $10a + 5a^2 = __\left(\frac{10a}{__} + \frac{5a^2}{__}\right) = __(__+a)$

c $6p^3 + 9p^5 = 3p^3\left(\frac{__}{__} + \frac{__}{__}\right) = 3p^3(__+__)$

d $6xy + 4x^2y = __\left(\frac{6xy}{__} + \frac{4x^2y}{__}\right) = 2xy(__+__)$

e $mn^2 + 5mn = __\left(\frac{__}{__} + \frac{5mn}{__}\right) = __(__+5)$

f $24efg^2 + 32fg^7h^3 = __\left(\frac{24efg^2}{__} + \frac{__}{__}\right) = __(3e + __)$

5G.3 **6** Factorise each expression.

a $c^2 + 4c$
b $6k - k^2$
c $a^2 - 2a$
d $3m^2 + 3m^4$
e $10p^3 + 11p^5$
f $4h - 7h^4$
g $6x^2y + 3x^3y^6$
h $5b^2c^5 - 10bc^2$
i $8mn^2 + 36m^2n^3$
j $27xyz^2 - 15x^2yz$
k $14a^3bc^{10} + 16a^2b^7c^4$
l $12e^2fg^2 + 18ef^5g^2$

UNDERSTANDING AND FLUENCY

7 Check your answers to question **6** by expanding each expression.

8 In some cases, it can be best to factorise using a HCF with a negative coefficient. Find the HCF that has a negative coefficient for each pair of terms.

a $-5a$ and -10
b $-2pq$ and $-6p$
c $-9hk$ and $-15jk$
d $-14rst$ and $-7st$
e $-4x^2$ and $4x$
f $-12m^3$ and $20m^2n^7p^5$
g $-x^2y$ and $-xy$
h $-8b^2c^2$ and $28abc^4$

9 Factorise each expression by finding the HCF that has a negative coefficient.

a $-4a - 12$
b $-6 - 8k$
c $-7m + 14mn$
d $-15b + 35ab$
e $-3st - 9tr$
f $-18abc - 24bcd$
g $-12x^{12} + 21x^{21}$
h $-h^2 - 2h$
i $-16x^2y^7 + 8x^5y^{10}$
j $-45ab^6 - 10a^{15}b^{10}c^8$
k $-42xyz^2 - 36x^4y^{13}z^2$
l $-11a^2bc^3 + 9ab^2c^9$

10 Find the HCF of the following sets of terms.

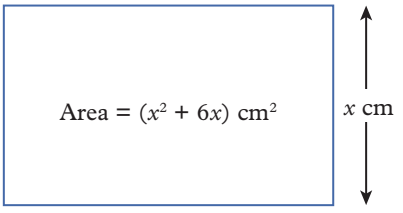
a $12a$, $12b$, $3a$
b $5abcd$, $6bdfh$, $-7bdhp$
c $15rm$, $30ms$, $40nmq$
d $36ad$, $-42ab$, $-24ac$, $18ae$
e $-8xyz$, $4uxz$, $16vzx$, $32zwx$
f $21i^4j^2k$, $49j^5i^2k^3$, $28k^4i^3j^2$

11 Factorise:

a $15p + 15q - 5$
b $6jk - 3ik - 10kl$
c $-8mnpq + 4jknp - 2pmkn$
d $27fg - 54gf + 45fh - 36hf$
e $17uvwx + 34xvu - 51vzxu - 68uv$
f $56a^8b^{12}c^4 + 32a^9b^7c^9 - 80a^7b^{11}c^7$

PROBLEM SOLVING AND REASONING

12 Consider this rectangle, which has a width of x cm and an area of $(x^2 + 6x)$ cm^2.



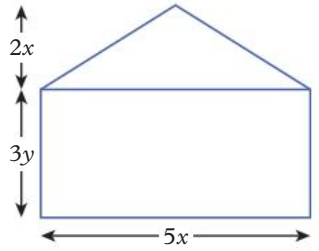
a Factorise the expression for the area of the rectangle.
b List the two factors that multiply to give the expression for the area.
c What is the expression for the length of this rectangle?
d Demonstrate you have the correct expression for the length of this rectangle by multiplying the length by the width to obtain the expression for its area.
e Another way of showing that you have the correct expression for the length of this rectangle is to use substitution. Try this method by letting $x = 3$.

13 Since factorising is the opposite of expanding, we can check our answers by doing the opposite manipulations to our answers. That is, expand after we factorise or factorise after we expand.

i Perform the following manipulations (expand or factorise).
ii Use the opposite manipulation (factorise or expand) to check if your answer is correct.

a Expand $8(4 - 3x)$.
b Factorise $7y + 49$.
c Expand $3b^3(2b^2 - 5b^5)$.
d Factorise $25a^2 - 125ab$.

14 Consider the composite shape shown below.



a Write an expression for the area of the composite shape.

b Factorise your answer to part **a**.

15 It is possible to take any term or expression out as a factor, even if it's not the highest common factor or even a common factor.

For example, to take 7 out as a factor of $x + 5y + 11z$, write 7 in front of $(x + 5y + 11z)$ and divide each term in the brackets by 7: $7\left(\frac{x}{7} + \frac{5y}{7} + \frac{11z}{7}\right)$. If possible, simplify the fractions.

For each of the expressions, take out the following terms as a factor.

i 6 **ii** -1 **iii** $\frac{1}{2}$ **iv** x **v** $-3x$

a $x + 5y + 11z$

b $2x - 3y + 6z$

c $18x^2 + 36x^3y - 54x^4z$

d $4xy - 6x - 10xz$

16 Each of the following problems appeared on a class test. Students were asked to fully factorise the expression. Below are some calculations where a student has made a mistake.



i Describe the mistake in each calculation.

ii Factorise the expression correctly.

a $-18p + 12q = -6\left(\frac{-18p}{-6} + \frac{12q}{-6}\right) = -6(3p + 2q)$

b $8r - 16r^3 = 4r\left(\frac{8r}{4r} - \frac{16r^3}{4r}\right) = 4r(2 - 4r^2)$

c $20y^4 + 5y^3 = 5y^3\left(\frac{20y^4}{5y^3} + \frac{5y^3}{5y^3}\right) = 5y^3(4y)$

d $6x^3y + 7xy^3 = 6x^3y\left(\frac{6x^3y}{6x^3y} + \frac{7xy^3}{6x^3y}\right) = 6x^3y\left(1 + \frac{7y^2}{6x^2}\right)$

17 Consider $y(3x + 2) + y(4x - 7)$ and $(x + 2)(3x + 2) + (x + 2)(4x - 7)$.

a Determine the HCF of each expression.

b Factorise and simplify each expression.

18 Factorise the expression $\frac{5}{8}x^{12}y^{19} - \frac{25}{8}y^{20}x^{21} + \frac{15}{8}x^{17}y^{17} - \frac{10}{8}y^{40}x^{32}$.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Factorising using the HCF

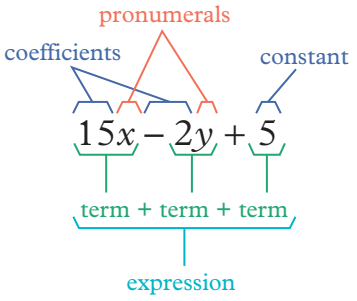
Topic quiz
5G

PROBLEM SOLVING AND REASONING

CHALLENGE

Chapter summary

Terms and expressions



Algebraic notation

$7 \times x = 7x$

$7 \times (x + 2) = 7(x + 2)$

$1 \times x = x$

$x \div 7 = \frac{x}{7}$

$(x + 2) \div 7 = \frac{(x + 2)}{7}$

$\frac{x}{7} = \frac{1}{7}x$

Grouping symbols

$7(x + 2)$ $\quad \frac{x + 2}{7}$ $\quad 7\sqrt{x + 2}$

Substitution

Let $x = -2$

$5x^2 - x = 5(-2)^2 - (-2)$

Simplifying signs

$+(+) = +$ $\quad +\times + = +$

$+(-) = -$ $\quad +\times - = -$

$-(+) = -$ $\quad -\times + = -$

$-(-) = +$ $\quad -\times - = +$

Like terms

Pronumeral(s)	Examples of like terms
x	x, $3x$, $\frac{2}{3}x$, $0.7x$...
ab	ba, $-2ab$, $\frac{ab}{5}$, $6.6ab$...
m^2n	$-m^2n$, $18m^2n$, $\frac{9m^2n}{4}$, $0.05m^2n$...

Adding and subtracting like terms

$6ab + \frac{3}{5}a + ab - \frac{2}{5}a = 6ab + ab + \frac{3}{5}a - \frac{2}{5}a$

$= 7ab + \frac{1}{5}a$

Exponent and expanded form

$3a^2b = 3 \times a \times a \times b$

exponent form — expanded form

Expanding and factorising

expanding →

$7(a + 2) = 7a + 14$

← factorising

Expanding

$3(2a - 6) = 3 \times 2a + 3 \times (-6)$

$= 6a + (-18)$

$= 6a - 18$

Multiplying algebraic terms

$3a^3b \times (-2a^2b^2) = 3 \times a \times a \times a \times b \times (-2) \times a \times a \times b \times b$

$= 3 \times (-2) \times a \times a \times a \times a \times a \times b \times b \times b$

$= -6a^5b^3$

Identifying the HCF

$15a^3bc = 3 \times 5 \times a \times a \times a \times b \times c$

$9a^2c = 3 \times 3 \times a \times a \times c$

HCF $= 3 \times a \times a \times c$

$= 3a^2c$

Dividing algebraic terms

$\frac{6a^5b^3}{3a^3b} = \frac{2 \times 3 \times a \times a \times a \times a \times a \times b \times b \times b}{3 \times a \times a \times a \times b}$

$= \frac{2 \times a \times a \times b \times b}{1}$

$= 2a^2b^2$

Factorising

$7a + 14 = 7\left(\frac{7a}{7} + \frac{14}{7}\right)$

$= 7(a + 2)$

Original expression: $7a + 14$

HCF: 7

Factorised expression: $7(a + 2)$

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Multiple choice

5A **1** What is the coefficient of the second term in the expression $10a^2 - 6a^2b + 13b - 15$?

A 6 **B** -6 **C** 13 **D** 15 **E** -15

5A **2** On a rainy day, the volume of water in a rainwater tank increases by 10 litres every hour. If there was originally 5 litres of water in the tank and it rains for t hours, this total volume of water in the tank is best represented by the expression:



A $15t$ **B** $10t$ **C** $10(t + 5)$ **D** $t + 5$ **E** $10t + 5$

5B **3** If $x = 4$ and $y = -3$, $2xy - 9y$ is equal to:

A 3 **B** 51 **C** -3 **D** -51 **E** -60

5C **4** Which list contains only like terms?

A $3a, ab, -5a$ **B** $4mn, mn, 2nm$ **C** $-8p, 6p^2, 5p^3$ **D** $2cd, de, -7ce$ **E** $5xy, 5x, 5y$

5C **5** Simplify $7m + 2n - 5m - 2n$.

A $2m$ **B** $12m + 4n$ **C** $2mn$ **D** $2m + 4n$ **E** $12mn$

5C **6** What does $2x^2 - 3x^2$ simplify to?

A $5x^2$ **B** 1 **C** $-x^2$ **D** x^2 **E** -1

5D **7** What is the expanded form of $-5abc^2$?

A $-5 \times a \times a \times b \times b \times c \times c$ **B** $-5 \times a \times b \times c$ **C** $-5 \times a \times a \times b \times c$
D $-5 \times a \times b \times b \times c$ **E** $-5 \times a \times b \times c \times c$

5D **8** Which of these is *not* equivalent to $24a^6b^3c^5$?

A $2a^2b^2 \times 3a^2c^3 \times 4a^2bc^2$ **B** $3a^4b \times 8b^2c \times a^2c^4$ **C** $3a^4c^5 \times 4a^2b \times 2ab^2$
D $bc^4 \times 6a^3 \times 4a^3b^2c$ **E** $4abc \times 3a^4bc^3 \times 2abc$

5E **9** What does $\frac{4ab^2}{8ab}$ simplify to?

A $2b$ **B** $\frac{1}{2b}$ **C** $\frac{2}{b}$ **D** $\frac{b}{2}$ **E** $\frac{1}{2}ab$

5F **10** When expanded, $5m(3m + 2n - 6)$ equals:

A $15m^2 + 2n - 6$ **B** $8m^2 + 7mn - 11m$ **C** $15m^2 + 10mn + 30m$
D $8m^2 + 10mn - 6$ **E** $15m^2 + 10mn - 30m$

5F **11** Which expression simplifies to $12mn + 5m$?

A $m(3n - 4) + 2m(5n + 4)$ **B** $4n(2m - 3) + 2m(2n + 4)$ **C** $m(2n - 3) + 2m(5n + 4)$
D $3m(3n - 3) + m(3n + 4)$ **E** $6m(n - 1) + 6m(n + 4)$

5G **12** What is the HCF of $24ab^2c$ and $18bc$?

A $2bc$ **B** $3c$ **C** $6bc$ **D** $8ab$ **E** $3ab$

Short answer

5A **1** Write an expression for each statement. Use m to represent the unknown number.

a A number is multiplied by 4 and then 3 is added.

b Half of the product of a number and 5.

c Add 3 to a number, then divide the result by 7.

d 6 less than a number all multiplied by $\frac{3}{5}$.

5A **2** Zoo entry is \$25 per adult, \$13 per child and \$20 per pensioner. Write an expression for each of the following, in dollars.

a the cost of entry for x adults

b the cost of entry for y children

c the cost of entry for p pensioners

d the total cost of entry for x adults, y children and p pensioners

5B **3** Evaluate each expression by substituting $a = 2$, $b = 5$ and $c = -3$.

a $7a + 2b$ **b** $5a^2 - 4bc$ **c** $-\frac{1}{3}(6ab - 12)$ **d** $\frac{b^2 - a^2}{c}$

5B **4** A cyclist is travelling with a constant acceleration, a m/s^2. Their final speed, v, is equal to the square root of the sum of the square of their initial speed, u, and twice the product of their acceleration and the distance they cover, s.

a Write a formula for the final speed of the cyclist.

b If the cyclist is travelling initially at 2 m/s and is accelerating at 3 m/s^2 for 42 m, what is the final speed?

5C **5** Identify and list the like terms in the following groups of terms.

a $6a, \frac{1}{7}a, b, -b, \frac{a}{10}, \frac{3}{8}b$

b $0.5x, 95y, \frac{1}{2}y, 1.7x, 10x, \frac{5}{6}x$

c $5de, -ed, \frac{2ef}{15}, 0.01de, 12fe, ef$

d $p^2, 2p^3, 5.1p^2, -15p^2, \frac{3p^3}{2}, \frac{3}{2}p^2$

e $\frac{9}{10}ab, 6.5abc, -6.5ba, 0.9cab, \frac{11}{20}bca, -ab$

f $\frac{4}{9}f^2g, \frac{gf^2}{3}, -23g^2f, -2.3g^2, \frac{6}{7}f^2, -3.14fg^2$

5C **6** Simplify:

a $4m + 5n + 8m + 3n$

b $3cd + 2c + 5dc + 4$

c $2k^2 + 6 - 9k^2 - 1$

d $5ab^2 - 2a^2b + 4a^2b - 3ab^2$

e $2xyz + 5xy - 7zyx - yx + \frac{3}{2}xy - \frac{2}{5}y$

f $\frac{3}{4}pq^2 - 7q^2 - \frac{1}{2}q^2p - q^2 - pq^2$

5D **7** Simplify the following products.

a $x^6 \times x^8$

b $m \times m$

c $h^7 \times h \times h^3$

d $q^5 \times p^{10} \times q \times p^2$

e $x^5 \times y^{12} \times x \times x^{19} \times y \times y^2$

f $a \times c^9 \times b^{21} \times a^5 \times b^9 \times c^3$

5D **8** Simplify each product.

a $-5 \times f \times g$ **b** $p^2 \times 2s$ **c** $7g \times 8g^9$

d $3ab \times 8bc$ **e** $2m^5n^8 \times (-3m^7n^4)$ **f** $-2ef^2 \times \frac{2}{5}efg^2 \times (-3f^7g^{16})$

5E **9** Simplify the following quotients.

a $k^{11} \div k^7$ **b** $8c^9 \div c$ **c** $x^9y^2 \div x^6y$

d $m^6n^7 \div m^3n^7$ **e** $p^{20}q^8r^{20} \div p^9q^6r^2$ **f** $a^3b^{24}c^{11} \div a^2b^{12}c$

5E **10** Simplify:

a $\frac{15a}{20}$ **b** $\frac{6cd}{2d}$ **c** $\frac{6abcd}{10bc}$

d $\frac{2e^2hk^2}{5ek}$ **e** $\frac{16x^{13}y^6}{28x^6y}$ **f** $\frac{2t^4 \times 4t^5}{8t^9}$

5F **11** Expand:

a $5(a + 3)$ **b** $4(7b - c)$ **c** $\frac{c}{2}(6a + 10d)$ **d** $-6m(4m^2 + n)$

5F **12** Expand and simplify:

a $3(k + 7) + -5(3k - 2)$

b $2y(y - 6) - (y + 4)$

5G **13** Find the highest common factor (HCF) of each pair of terms.

a 2 and $6a$ **b** $9p$ and $12pq$ **c** $10b^2$ and $-5bc$

d $16rst$ and $24st$ **e** $-a^4b^3c^{10}$ and $11a^2b$ **f** $-42mn^9p^8$ and $18m^6n^{12}p$

5G **14** Factorise:

a $14h - 18$ **b** $20ab + 10ac$ **c** $8c^2 + 12c$

d $-5efg - 5fgh$ **e** $15a^2b - 40ab$ **f** $18x^2y^5 - 24wx^{10}y^3$

Analysis



1 A roadside stall sells fresh, free-range eggs daily. The price varies every week depending on the time of year and the number of eggs that were laid. Johnny shops at the stall daily to buy fresh eggs to always have a fresh supply. Johnny buys two more eggs than the day before each day for the entire week to have enough for the different meals he cooks on different days.

Let x be the number of eggs Johnny buys on Monday and let y be the price the stall sells eggs at for the week.

a Write an expression for the total cost for Johnny after a week as the sum of the costs of each day. Do not expand or simplify the expression.

b If the price of an egg was \$0.43 and Johnny bought 4 eggs on Monday, what was the total cost for the week?

c To make Johnny's life easier with budgeting, simplify the total cost expression from part **a** by expanding and collecting like terms.

d The next week, the price of an egg is \$0.48 and Johnny buys 3 eggs on Monday. Use the expression from part **c** to determine the total cost for the second week.

e Johnny thinks the expression will be even simpler if it is factorised. Factorise the expression from part **c**.

f For the third week, the price of an egg is \$0.39 and Johnny buys 5 eggs on Monday. Use the expression from part **e** to determine the total cost for the third week.



2 A gardening supplier is designing a rectangular prism–shaped planter box that has a length and width that are square numbers, but not necessarily the same square number, and a height that is a cube number.

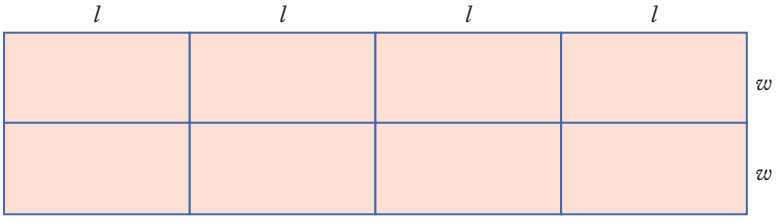
Let a be the number squared for the length, b be the number squared for the width, and c be the number cubed for the height.

a Write an expression for the perimeter of the base of one planter box.

b Write an expression for the area of the base of one planter box.

c Write an expression for the volume of one planter box.

A gardener buys eight of these planter boxes and arranges them into two rows and four columns as shown.



d Write an expression for the perimeter of the base of the array of eight planter boxes.

e Write an expression for the area of the base of the array of eight planter boxes.

f Write an expression for the volume of the array of eight planter boxes.

Semester 1 review

Short answer

1 Round the following numbers according to the description in the brackets.

a 569 238.472 940 (correct to the nearest thousand)

b 569 238.472 940 (correct to the leading digit)

c 208 492.175 207 (correct to the nearest thousandth)

d 208 492.175 207 (correct to two decimal places)

e 0.000 047 19 (correct to the leading digit)

f 28 174.201 (correct to the nearest multiple of 20)

2 Fill the gaps to produce equivalent fractions.

a $\frac{7}{8} = \frac{21}{\square}$ **b** $\frac{-8}{10} = \frac{\square}{15}$ **c** $2\frac{4}{5} = \frac{70}{\square}$ **d** $\frac{-1.2}{-0.07} = \frac{\square}{70}$

3 Simplify the following.

a $\frac{-90}{-120}$ **b** $\frac{300}{18}$ **c** $7\frac{45}{60}$ **d** 56 : 48

e 36 : 24 : 72 **f** 0.5 : 0.025 **g** $\frac{1}{9} : \frac{2}{3}$ **h** 150 km per 2.5 hours

4 Evaluate the following sums.

a 579 + 643 + 87 **b** 14 + (−52) **c** $\frac{5}{9} + \frac{5}{6}$

d 12.41 + 9.015 **e** $2^4 + 2^3$ **f** $3\frac{4}{5} + 6\frac{2}{3}$

5 Evaluate the following differences.

a 1041 − 828 **b** (−16) − (−23) **c** $\frac{1}{7} - \frac{5}{8}$

d 48.029 − 9.994 **e** $3^4 - 3^2$ **f** $6\frac{3}{8} - 2\frac{7}{12}$

6 Evaluate the following products.

a 245 × 18 **b** −5 × 12 **c** $\frac{12}{10} \times \frac{25}{18}$

d −4.2 × (−31.8) **e** $2^2 \times 2^3$ **f** $3\frac{1}{5} \times 4\frac{1}{6}$

7 Evaluate the following quotients.

a 5144 ÷ 8 **b** −255 ÷ (−3) **c** $\frac{12}{25} \div \left(-\frac{9}{10}\right)$

d 0.042 ÷ 0.6 **e** $3^8 \div 3^5$ **f** $4\frac{1}{12} \div 2\frac{5}{6}$

8 Evaluate the following.

a 6^3 **b** 3^5 **c** $\left(\frac{5}{4}\right)^3$

d $(-0.2)^4$ **e** $\left(3\frac{1}{2}\right)^4$ **f** 7^0

9 Evaluate the following roots.

a $\sqrt{36}$ **b** $\sqrt[3]{8}$ **c** $\sqrt{\frac{81}{25}}$ **d** $\sqrt{0.49}$

10 Evaluate the following calculations.

a $15 - 3^2 \times 8 \div 6 + 2$ **b** $25 \div (-5) - (-9) \times 7$

c $\frac{5}{6} - \frac{2}{3} \div \frac{4}{7} - \frac{3}{4}$ **d** $3.4 \times (2.5 + 3.7) - 8 \times 1.1^3$

11 Simplify the following.

a $5y + (-8xy) + 4x + 3yx + x + (-2y)$ **b** $12b - 8a - (-3b) - 7b$

c $2^4 \times 2^3$ **d** $7x^2y \times 5xy^5$

e $3^{12} \div 3^4$ **f** $\frac{24a^5b^6}{3b^2}$

g $(3^6)^5$ **h** $2^8 \times 3^{10} \div 2^2 \div 3^5$

i $\frac{6x + 12}{3}$ **j** $\frac{12p^6q^3}{4p^2q^3} - \frac{18p^{12}q^8}{2p^8q^8}$

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12 For the following calculations:

i round each value to the leading digit (first non-zero digit) and estimate the calculation

ii evaluate the actual result of the calculation

iii determine the percentage error between the estimation and the actual result, correct to two decimal places.

a $710\,284 \times 0.021\,83$

b $9107.4029 \div 2.145$

13 Write these lists of numbers in ascending order.

a 842.01, 8.4201, 824.01, 24.01, 24.1, 842.1

b $-5.31, 3.15, 5.13, -5.1, -5.031, 5.103$

c $\frac{7}{8}, \frac{3}{4}, \frac{4}{5}, \frac{11}{12}, \frac{9}{10}, \frac{2}{3}$

d $-\frac{3}{2}, \frac{6}{-2}, \frac{-3}{4}, \frac{-3}{-4}, \frac{4}{3}, -\frac{4}{3}$

14 State if the following are:

i terminating or non-terminating

ii recurring or non-recurring, if they are non-terminating.

a 2.33333…

b 5.194 817 53…

c 2.849

d 0.001 212 121 2…

15 Complete the table below.

Fraction	Decimal / recurring decimal	Percentage
	1.45	
$\frac{7}{8}$		
	2.5	
$\frac{21}{5}$		
		$33.\dot{3}\% = 33\frac{1}{3}\%$
		108%
$\frac{4}{33}$		

16 Write the following.



a 12 games played as a fraction out of 15 games owned

b 16 quests completed as a percentage out of 40 possible quests

c 8 episodes watched as a percentage out of 25 episodes

d the ratio of connected to unconnected puzzle pieces when 840 pieces are connected in a 1000-piece puzzle

e 57 eggs cooked in three weeks as a rate per week

f the average speed of a car that travels 276.25 km in 3.25 hours

17 Calculate:

a $\frac{5}{8}$ of 72

b 0.8 of 55

c 110% of 99

d 58 increased by 12%

e 120 decreased by 20%

f $2700 shared in the ratio 4 : 5

g the number of blue counters in a bag, if the bag has 35 red counters and the ratio of blue counters to red counters is 3 : 5

h the mass of a block if its density is 20 kg/m^3 and the volume of the block is 0.04 m^3

i the distance a bike will travel when ridden at 8 km/h for 30 minutes

j the time taken for someone walking at 2 m/s to walk 1 km.



18 Write an expression for the following:

a the difference of the squares of two numbers, p and q

b the sum of the product of 3 and a number, x, and the quotient of another number, y, and 12

c the quotient of 6 more than a number, x, and 6 less than the number x

d the square root of the number 7 times a number k

19 Expand and simplify:

a $8(4 - 3x)$ **b** $5 - 3(8x - 11) + 2x$ **c** $2b(6a + 5b + 3)$ **d** $5x^2y(6xy - 9y^4)$

20 Factorise:

a $36x + 27$ **b** $12a + 36b - 18c$ **c** $\frac{10}{3}p^3 + \frac{25}{3}p^7$ **d** $-\frac{3}{8}u^2v + \frac{5}{4}uv^2$

21 Calculate the following:



a the cost price of a cupcake, if it was marked up by 280% and to be sold for \$2.10

b the cost price of a book, if it was discounted by 40% and to be sold for \$15.00

c the selling price of a bag of lollies that is bought at \$0.90 per bag and marked up by 120%

d the selling price of an indoor plant that was \$80 and is currently discounted by 20%

e the cost, including GST, of a meal that was \$45 not including GST

22 Calculate the LCM and HCF of the following.

a 18 and 30

b 210 and 700

c $2^4 \times 3^6 \times 5^4$ and $2^3 \times 3^9 \times 5^4$

d $2^7 \times 5^8 \times 7^4$ and $2^3 \times 3^8 \times 7 \times 11^2$

23 Substitute the value in brackets into each expression, then evaluate.

a $8d^2 + 5d + 4$ $(d = 10)$ **b** $\frac{3}{4}x + x^2$ $(x = -2)$

c $6y - \frac{2}{y}$ $\left(y = \frac{5}{3}\right)$ **d** $\frac{1.03 - r^2}{r + 0.7}$ $(r = 0.5)$

24 State if the following are true or false.

a $(8 - 3) - 4 = 8 - (3 - 4)$ **b** $(120 \div 6) \div 2 = 120 \div (6 \times 2)$

c $(3^4)^5 = 3^{(4^5)}$ **d** $3\frac{1}{2} \times 8 = 3 \times 8 + \frac{1}{2} \times 8$

e $4 - (-3) = 4 + 3$ **f** $3^8 - 3^4 = 3^2$

25 Express the following numbers as the product of their prime factors. Write your answer in exponent form.

a 120 **b** 875

Analysis

1 Barry owns a \$2 shop and uses a simple cash register where the prices of items must be entered manually. Part of the stock is fake flowers. He stocks three types of fake flowers.



- lower quality flowers that cost \$0.50 per flower
- medium quality flowers that cost \$1.25 per flower
- higher quality flowers that cost \$1.95 per flower

Barry marks up his stock 80% from the cost price to the selling price.

a Determine the selling price of the three types of flowers.

b A customer buys 5 lower quality flowers, 3 medium quality flowers, and 4 higher quality flowers. How much does the customer pay?

c Barry uses a different colour price tag each year to keep track of how long the stock has been in the store. Any item that has been in the store for two or more years is sold at a discount of 30%.

A customer buys 10 lower quality flowers, 8 medium quality flowers, and 5 higher quality flowers, all of which have been in the store for two or more years.

i How much does the customer pay?

ii How much more money would Barry have made if the flowers were sold at full price?

d Barry gets a delivery of 100 lower quality flowers, 75 medium quality flowers, and 50 higher quality flowers. The delivery cost was $20. Calculate:

i the total cost of purchasing the flowers including delivery

ii the difference between the total selling price and the total cost price if Barry sells all the flowers within two years.

Another supplier sells their flowers for the following prices.

- lower quality flowers that cost $0.35 per flower
- medium quality flowers that cost $1.15 per flower
- higher quality flowers that cost $2.10 per flower
- delivery cost of $25 per shipment

e How much cheaper or more expensive would Barry's purchase in part **d** be from this supplier?

2 A teacher notices a number of mistakes on a recent revision quiz.

i $372 + 654 = 926$ **ii** $1025 - 856 = 231$ **iii** $632 \times 72 = 5688$

iv $2760 \div 8 = 37$ **v** $4^4 = 16$ **vi** $\sqrt{16} = 8$

vii $\sqrt[3]{27} = 9$ **viii** $2^3 \times 2^2 = 2^6$ **ix** $3^3 \div 3^3 = 0$

a Determine the correct answer for each calculation.

b Calculate the percentage error between the correct and incorrect answers, correct to two decimal places.

c Modify the first number in each calculation so that the answers are correct.

3 Harrison was investigating the exponent laws and how they apply with algebra. He sets up the following pairs of products of prime factors:

i $2^5 \times 3^2 \times 5$ and $2^3 \times 3^5 \times 5^3$

ii $3^5 \times 2^2 \times 7$ and $3^3 \times 2^5 \times 7^3$

iii $3^5 \times 5^2 \times 7$ and $3^3 \times 5^5 \times 7^3$

iv $a^5 b^2 c$ and $a^3 b^5 c^3$

For each of the following, leave your answer as the product of prime factors.

a Determine the product of each pair.

b Determine the quotient of each pair.

c Determine the highest common factor of each pair.

d Determine the lowest common multiple of each pair.

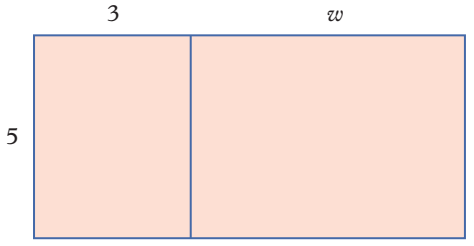
4 Gavin is extending his patio. Currently, his patio is 5 metres by 3 metres.

a If Gavin wants to extend his patio by w metres, write an expression, using brackets, for the area of his new patio in m^2.

b Expand your answer to part **a**.

c If Gavin can only extend his patio to a total width of 8 m, what is the maximum area of his extension?

d Gavin wants to check the total area of the patio if he extends the patio by 1 m, 2 m, 3 m and 4 m. Calculate the area of the patio for these extension lengths.



Gavin's partner wants to plant flowers along the two shorter sides and one of the longer sides of the patio.

e Write an expression for the length of the flowerbed that will surround three sides of the patio.

f Gavin digs a trench with a width x metres for the flowerbed. When Gavin's partner sees the flowerbed, he asks Gavin to double the width of the flowerbed. Use brackets to write an expression for the final area of the flowerbed in m^2.

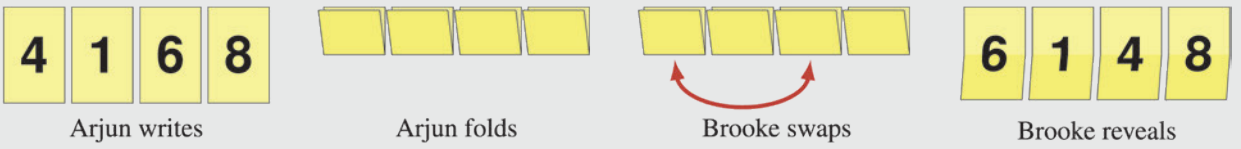
g Expand your answer to part **f**.

h If Gavin decides to extend the patio by 3 metres, and the flowerbed has a width of 0.5 metres, what will the total area of the flowerbed be?

EXPLORATIONS 1

1 Note swap

Arjun and Brooke make up a game called 'note swap'. Arjun takes a number of sticky notes and places them in a line on the desk. While Brooke is not looking, he writes a single digit on each note to form a number, read left to right, that is divisible by the number of notes used. Arjun then folds each note in half so Brooke cannot see the digits. Brooke's job is to swap two sticky notes of her choosing and then reveal all the digits to form her own number, again reading left to right. If the new number is also divisible by the number of notes used, Brooke wins. If not, Arjun wins.



a Arjun uses 3 sticky notes. Explain why Brooke will always win, no matter how she swaps the notes.

b Arjun uses 4 sticky notes. What strategy should Brooke use to guarantee she wins?

c Arjun uses 7 sticky notes. Explain why swapping the first and last digit is a winning strategy for Brooke.

d From 2 to 12 inclusive, there is only one number of notes which allows Arjun to come up with a winning strategy. What is the number and what is his winning strategy?

e Show that Brooke has a winning strategy for all remaining cases from 2 to 12 inclusive.

f Do either Arjun or Brooke have a winning strategy for 14 notes?

2 Recurring themes

a By long or short division, show that $\frac{1}{7}$ as a decimal is $0.\overline{142857}$.

b Because every possible remainder from 1 to 6 occurs in the above calculation, it follows that the same pattern of recurring digits must also occur in the decimal forms of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$. Use this fact to write each of these fractions as a decimal, without doing any division.

c Convert $\frac{1}{14}$ to an equivalent (but unsimplified) fraction with a multiple of 10 in the denominator. Use this and part **b** to help you write $\frac{1}{14}$ as a decimal.

d What is $\frac{3}{14} - \frac{1}{5}$ as a simplified fraction? Use this to help you write 3 as a decimal.

In part **d**, it made sense to subtract $\frac{1}{3}$ for two reasons: it is close to but slightly less than $\frac{3}{14}$, and its own decimal form terminates because 5 is a factor of 10.

e Adapt the ideas above to write $\frac{5}{14}, \frac{9}{14}, \frac{11}{14}$ and $\frac{13}{14}$ as decimals.

f Find a clever way to work out $\frac{2}{63}$ as a decimal without doing the division.

3 Parcel packers

In the AMT warehouse, it takes 8 professional packers 15 hours to pack the parcels of competition papers to send to schools.



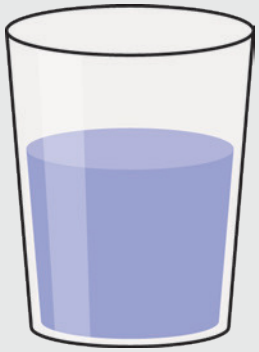
a Assuming that all professional packers work at the same constant rate and do not get in each others' way, how long would it take 12 professionals to pack all the parcels?

b The 12 professionals are joined by 8 trainee packers who can each work at only half the rate of a professional. How long would it take the professionals and trainees to pack all the parcels?

c Some professionals and 6 trainees started packing the parcels and would have taken 12 hours to complete the job. However, after $2\frac{1}{2}$ hours two professionals and all trainees had to leave. How many professionals were there? How much longer after the 8 packers left would the remaining packers take to finish the job?

d Suppose x professionals and y trainees require t hours to complete a job, but after T hours only X professionals and Y trainees continue. Find a formula for the number of hours needed by the remaining packers to finish the job.

4 Sharesies



Seven girls sit around a circular table. Each has a glass and some of the glasses contain water. Each glass can hold up to 720 mL. In turn, going clockwise around the table, each girl divides the water in her glass into six equal shares and adds these to the glasses of each of the other girls. No water is ever spilt.

a On one occasion, the first girl begins with a full glass. When all seven girls have completed their turns, the first girl ends up with a full glass again. How much water did the third girl have at the beginning?

b On another occasion, the total amount of water is 2.1 L. When all seven girls have completed their turns, each of them ends up with the same amount of water as they had originally. How much water did each girl have at the beginning?

c What is the answer to part **b** if each girl divides the water in her glass into seven equal parts and keeps one of the parts for herself?

5 Powerful stuff

a Without using a calculator, write the number 4096 as a:

i power of 2 **ii** power of 4 **iii** perfect square **iv** perfect cube.

b Explain why the number 1 has all of the properties in part **a**: it is a power of 2, a power of 4, a perfect square and a perfect cube. What is the next smallest number with all of these properties?

c Write the number $12^3 \times 18^5$ as the product of a perfect square and a perfect cube. Now find a second way to do this.

d Find all powers of 20 that *cannot* be written as the product of a perfect square and a perfect cube. What about powers of 24? Can you generalise your results?

e The sum $256^2 = 32^3 + 8^5$ shows a perfect square that can be written as the sum of a perfect cube and a fifth power. Another example is $3^2 = 2^3 + 1^5$, but we are not interested in the trivial case $0^2 = 0^3 + 0^5$.

i Find a perfect cube that can be written as the sum of a perfect square and a fifth power.

ii Find a fifth power that can be written as the sum of a perfect square and a perfect cube.

f Do some research to find out about *Pythagorean triples* and *Fermat's last theorem*.

i Can a fourth power be written as the sum of two perfect squares?

ii Can a sixth power be written as the sum of two perfect cubes?

6 House points

Year 8 Maths teacher Mr Stan Deliver has his class practising mental arithmetic. Each student chooses a handful of consecutive numbers, none of which ends in zero. They add up the numbers and then divide the total by the last digit of each number. For each division which results in a whole number answer, the student earns a house point.

For example, Zeba chooses 23, 24 and 25 with total 72. She then calculates $72 \div 3$, $72 \div 4$ and $72 \div 5$. Two of these result in a whole number, so Zeba earns two house points.

a Andy chooses 15, 16 and 17. How many house points does he win? Is his score improved by adding one more number to his list?

b Basma chooses eight consecutive numbers. Explain why she always wins at least 2 house points. What is the smallest possible total of Basma's eight numbers if she only wins 2 house points?

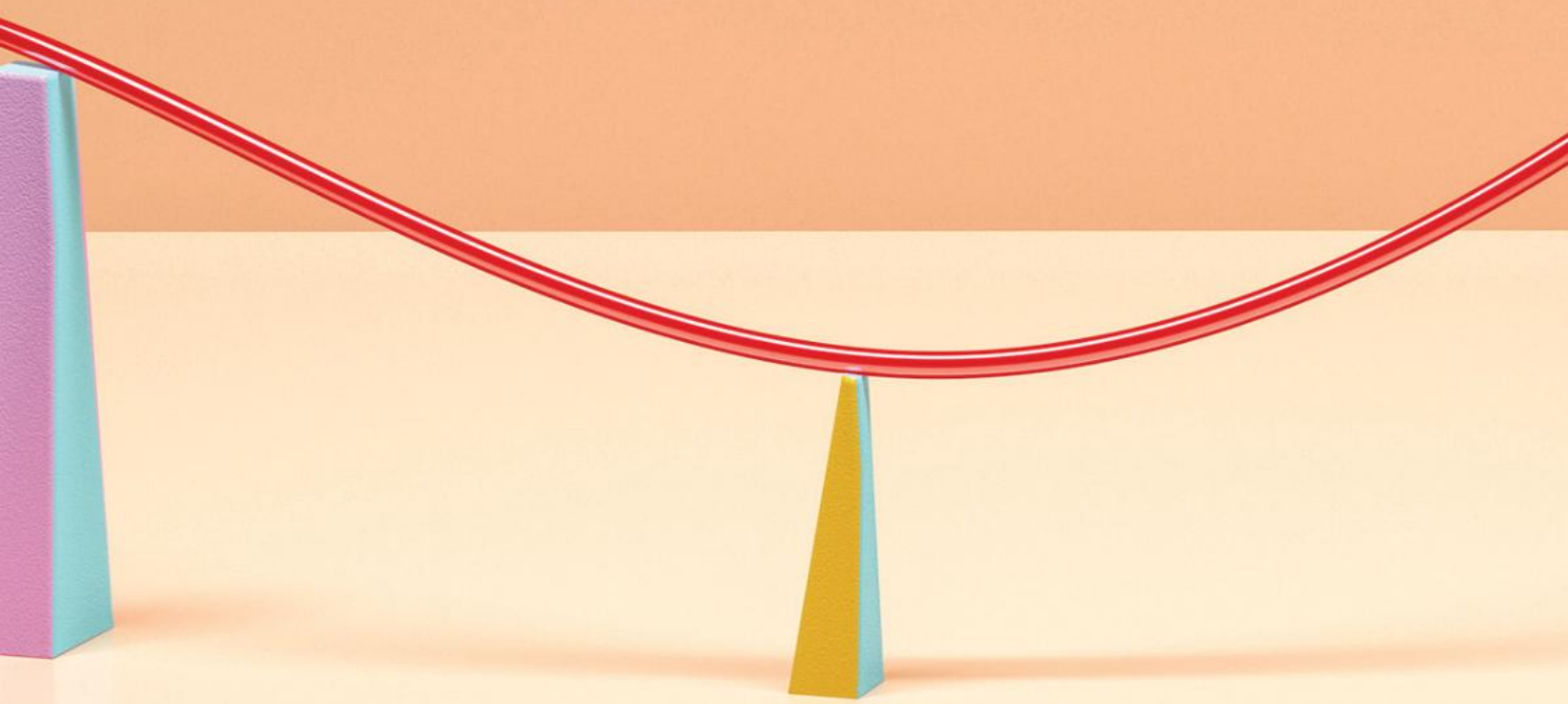
c Cheren chooses six consecutive numbers. Explain why she cannot win more than 2 house points. What is the smallest number of house points Cheren can win?

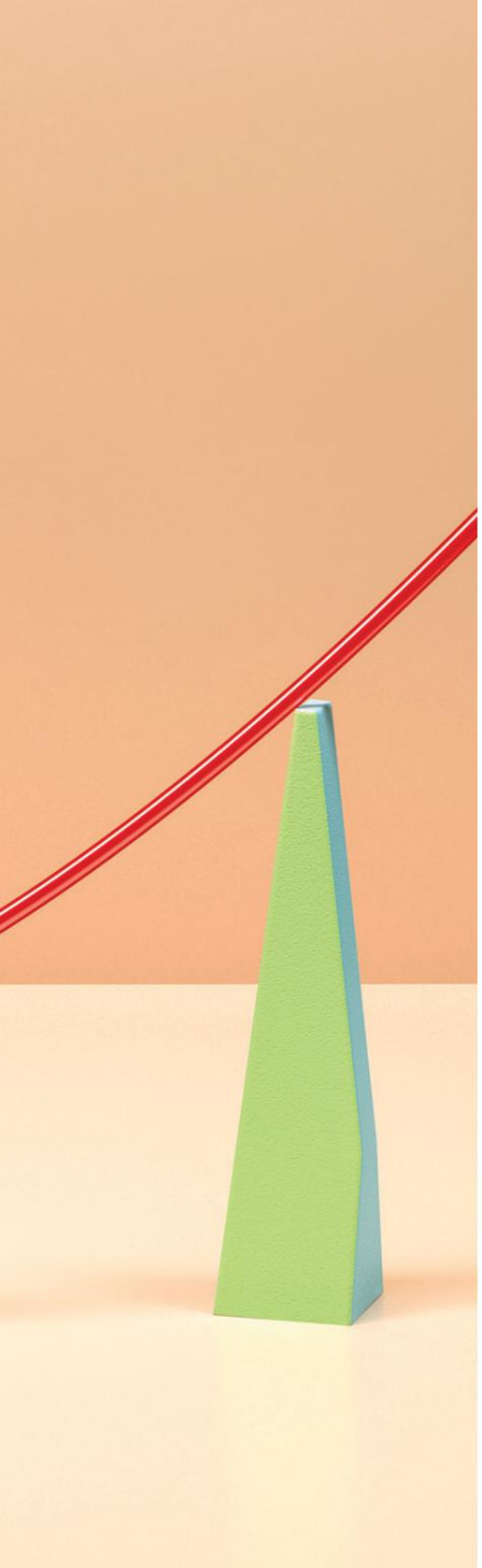
d Dmitri chooses nine consecutive numbers. Explain why he cannot win more than 5 house points. What is the smallest possible total of Dmitri's numbers if he does win the maximum 5 house points?

e Eli chooses well and wins a house point for every number in his list. What is the largest possible number of consecutive numbers he could have chosen? What is the smallest possible total of Eli's numbers?

6

Linear relationships





Index

6A Equations
6B Solving linear equations
6C Solving equations with the unknown on both sides
6D Plotting linear relationships
6E Solving linear equations using graphs
6F Finding linear equations
6G Solving linear inequalities

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Order of operations
- ✔ Substitution
- ✔ Like terms
- ✔ Expanding over one pair of brackets
- ✔ The Cartesian plane

Curriculum links

- Graph linear relations on the Cartesian plane using digital tools where appropriate; solve linear equations and one-variable inequalities using graphical and algebraic techniques; verify solutions by substitution (AC9M8A02)
- Use mathematical modelling to solve applied problems involving linear relations, including financial contexts; formulate problems with linear functions, choosing a representation; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model (AC9M8A03)
- Experiment with linear functions and relations using digital tools, making and testing conjectures and generalising emerging patterns (AC9M8A04)

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Materials

- ✔ Graph paper
- ✔ Ruler

6A Equations

Learning intentions

By the end of this topic you will be able to...

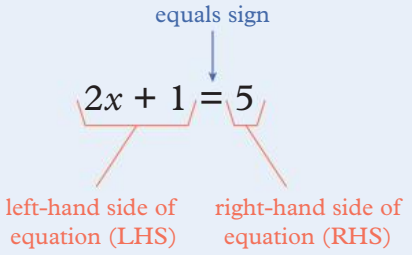
- ✔ check solutions using substitution
- ✔ solve simple equations by inspection
- ✔ write equations to represent worded problems.

Inter-year links

Year 7 — 6C Terms, expressions and equations

Equations

- An **equation** is a mathematical statement that shows equivalence between the expression on the left-hand side (LHS) of the equation and the right - hand side (RHS) of the equation.
  → If two expressions are *not* equivalent, this is denoted using the 'not equal to' symbol, ≠.



Solving equations

- To find a **solution** to an equation, find a value which, when substituted for the unknown variable or pronumeral, makes the equation a true statement.
- Solutions are usually given with the pronumeral on the left- and the value on the right-hand side of an equals sign, for example $x = 3$.
- To **solve by inspection**, examine the equation and mentally identify the number that when substituted for the unknown pronumeral makes the equation a true statement.
- To check if a value is a solution to an equation, substitute that value into the equation to see whether it makes a true statement.

Example 6A.1 Checking a solution

Determine whether $x = -3$ is the solution to the equation $2x + 5 = 1$.

THINK

1. Substitute $x = -3$ into the expression on the left-hand side of the equation and evaluate. Remember that the product of a positive and a negative number gives a negative number.
2. Determine whether the statement is true or false by comparing the value of the left- and right-hand sides of the equation.

WRITE

$2x + 5 = 1$

LHS $= 2(-3) + 5$
$= -6 + 5$
$= -1$

RHS $= 1$

LHS $\neq$ RHS

$x = -3$ is not the solution to $2x + 5 = 1$.

Example 6A.2 Solving equations by inspection



Solve each equation by inspection.

a $a + 5 = 12$

b $7c = -35$

THINK	WRITE
a **1** Consider what number, when added to 5, gives 12.	**a** $a + 5 = 12$
2 Check the solution.	Let $a = 7$ LHS $= 7 + 5$ $= 12$ RHS $= 12$ LHS = RHS
3 State the solution to the equation.	$a = 7$
b **1** Consider what number, when multiplied by 7, gives -35. As the coefficient of $7c$ is positive and the number on the right-hand side of the equation is negative, c must be a negative value.	**b** $7c = -35$
2 Check the solution.	Let $c = -5$ LHS $= 7(-5)$ $= -35$ RHS $= -35$ LHS = RHS
3 State the solution to the equation.	$c = -5$

Example 6A.3 Writing equations from number sentences



Write an equation to represent each of these number sentences. Use n for the unknown number in each instance.

a 3 more than a number is equal to 8.

b 5 less than a number is equal to -2.

c Twice a number is equal to 14.

THINK	WRITE
a Read through the number sentence from left to right. The left-hand side of the equation is '3 more than a number' or $n + 3$. The right-hand side of the equation is 8.	**a** $n + 3 = 8$
b Read through the number sentence from left to right. The left-hand side of the equation is '5 less than a number' or $n - 5$. The right-hand side of the equation is -2.	**b** $n - 5 = -2$
c Read through the number sentence from left to right. The left-hand side of the equation is 'twice a number' or $2n$. The right-hand side of the equation is 14.	**c** $2n = 14$

Example 6A.4 Writing equations to represent worded problems

A rose bush grows three times as many roses as last year. Erin cuts 5 roses to fill a vase.

a If r roses grew on the bush last year, write an expression for the number of roses on the bush after Erin fills the vase.

b After Erin fills the vase, there are 13 roses left on the bush. Write an equation that can be used to determine how many roses grew on the bush last year.

c Solve the equation by inspection.

d How many roses grew on the bush last year?

THINK	WRITE
a Think about which operations are taking place. 'Three times' means multiply by 3. If Erin removes 5 roses to fill the vase, there will be 5 fewer roses on the bush.	**a** $3r - 5$
b If there are '13 roses left on the bush', the number of roses is equal to 13.	**b** $3r - 5 = 13$
c Consider what number makes the equation a true statement. Check the solution.	**c** Let $r = 6$ LHS $= 3r - 5$ $= 3(6) - 5$ $= 18 - 5$ $= 13$ RHS $= 13$ LHS = RHS $r = 6$
d Interpret the solution. As r = the number of roses that grew on the bush last year, 6 roses grew on the bush last year.	**d** 6 roses grew on the bush last year.

Helpful hints

- ✔ Remember to define your pronumerals before writing equations to represent a worded problem.
- ✔ If you have difficulty in solving equations by inspection, try using either the 'guess, check and improve' or 'balance model' methods from Year 7.

ANS p521

Exercise 6A Equations

△ 1–4, 5(1st, 2nd columns), 6, 8, 10–14, 16, 17, 19 ▢ 4, 5(3rd, 4th columns), 7, 9–13, 15, 18, 19, 21, 23 ⬠ 4, 5(3rd, 4th columns), 11–13, 17, 20–24

1 a State whether each of these is an equation or an expression.

i $5x + 4$ **ii** $8k = 24$ **iii** $14 - 3p = 5$

iv $\frac{3k - 1}{2}$ **v** $2 = 9 - x$ **vi** $4(7d + 3) = 8$

b Explain how you can tell the difference between an equation and an expression.

2 Substitute $x = 6$ into each equation. Does this value of x make the equation a true statement?

a $x + 3 = 9$ **b** $x - 5 = -1$ **c** $2x = 8$

d $\frac{x}{3} = 2$ **e** $4x + 5 = 19$ **f** $3x - 7 = 11$

3 Substitute $y = -9$ into each equation. Does this value of y make the equation a true statement?

a $y - 1 = -8$ **b** $3y = 27$ **c** $\frac{18}{y} = -2$

d $-5y = 45$ **e** $4y + 1 = -35$ **f** $2y + 6 = 24$

6A.1 **4** Determine if the value given in brackets for each pronumeral is the solution to the equation.

a $a - 5 = 6$ $(a = 11)$ **b** $4k = -16$ $(k = -4)$ **c** $m + 3 = 11$ $(m = 7)$ **d** $\frac{h}{5} = 2$ (h $= -10$)

e $6p - 4 = 2$ $(p = 1)$ **f** $3n + 7 = -1$ $(n = -2)$ **g** $\frac{b + 2}{3} = 2$ $(b = 7)$ **h** $15 - f = 16$ $(f = -1)$

6A.2 **5** Solve each equation by inspection.

a $a + 3 = 10$ **b** $b - 2 = 4$ **c** $3c = 24$ **d** $\frac{d}{4} = 3$

e $-4e = -8$ **f** $\frac{f}{3} = -1$ **g** $g - 5 = -4$ **h** $h + 6 = 6$

i $9 - i = 12$ **j** $8j = -40$ **k** $4 + k = 11$ **l** $l - 15 = 20$

m $\frac{24}{m} = 6$ **n** $-10n = 100$ **o** $\frac{x}{2} = -7$ **p** $p - 4 = -4$

q $8 = q + 2$ **r** $48 = -4r$ **s** $9 = 13 + s$ **t** $5 = \frac{t}{5}$

6 Which solution makes $2x + 5 = 13$ a true statement?

A $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$ **E** $x = 5$

7 Which solution makes $3y - 1 = -7$ a true statement?

A $y = -7$ **B** $y = 4$ **C** $y = -4$ **D** $y = 2$ **E** $y = -2$

8 Which solution makes $\frac{m + 3}{8} = 1$ a true statement?

A $m = 5$ **B** $m = 6$ **C** $m = 7$ **D** $m = 8$ **E** $m = 9$

9 Which solution makes $\frac{k}{3} - 1 = 0$ a true statement?

A $k = 1$ **B** $k = 2$ **C** $k = 3$ **D** $k = 4$ **E** $k = 5$

10 What is the value of the unknown that makes each equation a true statement?

a $3a + 9 = 15$ **b** $5b - 3 = 2$ **c** $7c - 2 = 19$ **d** $2d + 10 = 10$

e $\frac{e + 4}{5} = 2$ **f** $\frac{f - 1}{3} = 3$ **g** $\frac{g}{2} + 6 = 8$ **h** $\frac{h}{7} - 1 = -2$

11 Solve each equation by inspection.

a $4x + 1 = 21$ **b** $3x - 2 = 10$ **c** $7x + 1 = -6$ **d** $2x - 5 = 13$

e $\frac{x}{5} - 6 = -8$ **f** $\frac{x}{3} + 11 = 11$ **g** $\frac{x + 9}{2} = 3$ **h** $\frac{x - 4}{4} = -2$

6A.3 **12 a** Write an equation to represent each of these number statements. Use n for the unknown number.

i 4 added to a number gives 11.

ii 3 subtracted from a number gives −2.

iii 15 more than a number is equal to 21.

iv 8 less than a number is equal to −4.

v Twice a number is equal to 14.

vi One-quarter of a number is 9.

vii The sum of a number and −7 is equal to −20.

viii The product of a number and 11 is equal to 22.

ix When a number is multiplied by 5, the result is 55.

x When a number is divided by 6, the result is −3.

b Solve each equation by inspection.

13 a Write an equation to represent each of these number sentences. Use x for the unknown number.

i A number is multiplied by 3, then 8 is added to give 11.

ii A number is divided by 6, then 2 is subtracted to give 1.

iii The sum of a number and 7 all divided by 2 is equal to 5.

iv 9 less than a number all times 3 gives 3.

b Solve each equation.

PROBLEM SOLVING AND REASONING

6A.4 **14** Evie and her older brother Harry earn pocket money weekly. Harry receives $38 each week.

a Write an expression for the total amount of money they receive each week if Evie receives x dollars per week.

b Write an equation that can be used to find how much Evie receives each week if they receive a total of $62 per week.

c Solve the equation by inspection.

d How much pocket money does Evie receive each week?

15 Stella has 26 rolls of washi tape. A delivery containing more rolls of washi tape has just arrived.



a Write an expression for the total number of rolls of washi tape Stella will have if there are x rolls in the delivery.

b Write an equation that can be used to find how many rolls of washi tape are in the delivery if Stella has a total of 81 rolls of tape after the delivery.

c Solve the equation by inspection.

d How many rolls of washi tape were in the delivery?

16 Theo has invited some friends over for dinner. He has bought enough dumplings so that each person will have 8.

a If x represents the number of people at Theo's dinner, write an equation that can be used to find the number of people if Theo bought a total of 96 dumplings.

b Solve the equation by inspection.

c How many people were at the dinner at Theo's house?

17 A bag of macadamia nuts is divided amongst four students.



a Define a pronumeral to represent the variable in this problem.

b Write an expression for the number of macadamia nuts each student receives.

c Write an equation for finding the number of macadamia nuts in the bag if each person receives nine macadamia nuts.

d Solve the equation by inspection to find the value of the pronumeral.

e How many macadamia nuts are in the bag?

18 The selection of truffles shown to the right costs a total of $24.



a Assuming each truffle costs the same amount of money, write an equation that can be used to determine the cost of each truffle.

b Solve the equation by inspection to calculate the cost of one truffle.

19 Nakkiah takes half of the fruit in a fruit bowl to basketball training, then her dad puts 3 new pieces of fruit into the bowl.

a If there are now 7 pieces of fruit in the bowl, write an equation to represent this problem.

b Solve the equation to calculate how many pieces of fruit were originally in the fruit bowl.

20 The number of cars in a car park changes during the day. At 11 am, there are twice as many cars as there were at 9 am. At 4 pm, there are 7 more cars in the car park than at 11 am.

a If there are 11 cars in the car park at 4 pm, write an equation that can be used to calculate how many cars were in the car park at 9 am.

b Use the equation to determine how many cars were in the car park at 9 am.

21 Tickets to Australia Zoo cost $25 per child and $40 per adult.

a Write an expression for the cost of c children's tickets.

b Write an expression for the cost of a adult tickets.

c Write an equation for the number of children's and adult tickets you could buy with $155.

d Poh's family is made up of 2 adults and 3 children. He thinks it will cost $155 to purchase tickets to Australia Zoo for his whole family. Use your equation to check if Poh's calculations are correct.

22 Sia starts a fitness routine and records the number of minutes she spends exercising each week. In the third week, Sia exercises 30 minutes more than she did in the first week. In the fifth week she exercises double the number of minutes she did in the third week.

a If Sia exercises for 270 minutes in the fifth week, write an equation that can be used to calculate how long she exercised in the first week.

b Use the equation to find how long Sia exercised in the first week.

23 Arvan records the amount of time he spends practising the guitar. Unfortunately, he spills some water on his record and some of the information is lost.

Day	Time spent practising	Comments
Monday		Good start!
Tuesday		Same as Monday
Wednesday		Double Monday!
Thursday		20 minutes less than yesterday
Friday		Same as Monday
Total	190 minutes	

Arvan thinks he can write an equation using the information in his table to find the missing amounts. Arvan's equation is $x + x + x + x + x = 190$.

a Explain why Arvan's equation is incorrect.

b Write a new equation that will find the number of minutes Arvan practises on Monday.

c Fill in the table for each day.

d Arvan's equation does calculate a certain value that could be useful for him. What is it?

24 Lina visited the Milton Market one Sunday. At one stall she bought a handbag. At another stall she bought 3 t-shirts. At a food van she bought 3 donuts for $10. Lina went to the market with $100, came home with no money, and the handbag cost double the amount of all 3 t-shirts.

a Write an equation to solve the cost of the handbag.

b Solve the equation by inspection.

c Check your answer by substitution.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Writing formulas

Interactive skillsheet
Solving equations by inspection

Investigation
Circus antics

Topic quiz
6A

6B Solving linear equations

Learning intentions

By the end of this topic you will be able to ...

✔ solve equations using inverse operations.

Inter-year links

Year 7	6G Solving equations using inverse operations
Year 9	3A Solving linear equations
Year 10	3A Solving linear equations

Equivalent equations

- **Equivalent equations** are equations that have the same solution.
- When the same operation is performed on both sides of an equation, the equality between the two sides of the equation is preserved.

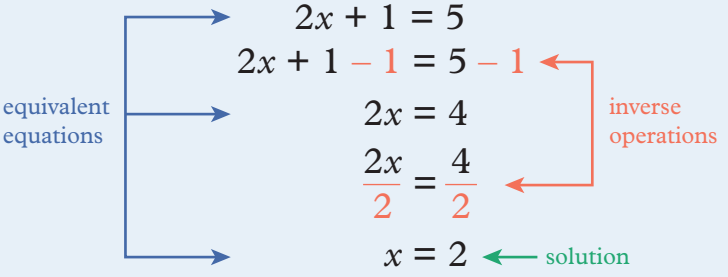
Inverse operations

- **Inverse operations**, or opposite operations, reverse the effect of another operation.

Operation	Inverse operation
+ 3	− 3
− 3	+ 3
× 3	÷ 3
÷ 3	× 3

- Inverse operations can be used to manipulate the expressions on the left- and right-hand side of an equation until we obtain an equation with the pronumeral by itself on one side of the equation, and therefore determine the solution to the equation.

  Each manipulation of the expressions produces an equivalent equation, with the simplest equivalent equation revealing the solution.
- To solve equations using inverse operations:
  1. Identify the operation(s) acting on the pronumeral.
  2. Identify the inverse operation(s) required to reverse the operation(s) and get x by itself on one side of the equation.
  3. Apply the inverse operations to both sides of the equation or 'do the same to both sides' to maintain equivalence.



- For equations involving more than one operation, inverse operations must be performed in the *reverse order* to BIDMAS.
  - → For example, in the equation $2(x + 5) = 17$ the first inverse operation to perform would be that required to reverse the multiplication (÷ 2), and the second would be that required to reverse the addition within the brackets (− 5).



Example 6B.1 Solving one-step equations using inverse operations

Solve the following equations using inverse operations.

a $x + 3 = 11$

b $\frac{x}{5} = -8$

c $-4x = 16$

d $x - 12 = 7$

THINK	WRITE
a 1 Identify the operation acting on the pronumeral. In this case, 3 is being added to the x term.	**a** The operation acting on x is + 3.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, subtract 3 from both sides of the equation.	The inverse operation to + 3 is − 3. $x + 3 = 11$ $x + 3 - 3 = 11 - 3$
3 Simplify both sides of the equation by completing the inverse operation.	$x = 8$
4 Write the solution to the equation.	
b 1 Identify the operation acting on the pronumeral. In this case, the x term is being divided by 5.	**b** The operation acting on x is ÷ 5.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, multiply both sides of the equation by 5.	The inverse operation to ÷ 5 is × 5. $\frac{x}{5} = -8$ $\frac{x}{5} \times 5 = -8 \times 5$
3 Simplify both sides of the equation by completing the inverse operation.	$\frac{5x}{5} = -40$
4 Write the solution to the equation.	$x = -40$
c 1 Identify the operation acting on the pronumerals. In this case, the x term is being multiplied by −4.	**c** The operation acting on x is × −4.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, divide both sides of the equation by −4.	The inverse operation to × −4 is ÷ −4. $-4x = 16$
3 Simplify both sides of the equation by completing the inverse operation.	$\frac{-4^{1}x}{-4^{1}} = \frac{16^{-4}}{-4^{1}}$
4 Write the solution to the equation.	$x = -4$
d 1 Identify the operation acting on the pronumerals. In this case, 12 is subtracted from the x term.	**d** The operation acting on x is −12.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, add 12 to both sides of the equation.	The inverse operation to −12 is +12. $x - 12 = 7$ $x - 12 + 12 = 7 + 12$
3 Simplify both sides of the operation by completing the inverse operation.	
4 Write the solution to the equation.	$x = 19$

Example 6B.2 Solving two-step equations using inverse operations

Use inverse operations to solve the following equations.

a $10 - 4x = 26$ **b** $\frac{x+3}{7} = 6$

THINK

a **1** Apply the inverse operations to both sides of the equation in the reverse order to BIDMAS. First, identify any addition/subtraction acting on the x term. In this case, reordering the terms so that the term containing the variable appears first clarifies that 10 is being added to the x term. Apply the inverse operation to both sides of the equation.

2 Second, identify any multiplication/division acting on x and apply the inverse operation to both sides of the equation to obtain the solution. Remember that dividing a positive number by a negative number gives a negative number.

b **1** Apply the inverse operations to both sides of the equation in the reverse order to BIDMAS. Note that the '+ 3' is contained within a fraction and the fraction line acts like a bracket. The '+ 3' is therefore considered a bracketed operation, not an addition/subtraction. As there is no addition/subtraction, begin by identifying any multiplication/division and apply the inverse operation.

2 The '+ 3' is no longer contained within a fraction, so apply the inverse operation to obtain the solution.

WRITE

a
$$10 - 4x = 26$$
$$-4x + 10 = 26$$
$$-4x + 10 - 10 = 26 - 10$$
$$-4x = 16$$
$$\frac{-4^{1}x}{-4^{1}} = \frac{16^{-4}}{-4^{1}}$$
$$x = -4$$

b
$$\frac{x+3}{7} = 6$$
$$\frac{x+3}{7} \times 7 = 6 \times 7$$
$$\frac{7^{1}(x+3)}{7^{1}} = 42$$
$$x + 3 = 42$$
$$x + 3 - 3 = 42 - 3$$
$$x = 39$$

Helpful hints

✔ Take care when solving equations containing grouping symbols. Inverse operations must be performed in the reverse order to BIDMAS, meaning operations contained within grouping symbols are often addressed towards the end of the calculations.

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Exercise 6B Solving linear equations

▲ 1–11, 12(a–c), 13, 17(a–c) ■ 4–9, 11, 12(c, e, f), 13(c, d), 14, 15, 17(c, e, g, h), 19, 20 ⬟ 5–9, 12, 13(c, d), 16–20

1 Identify the inverse operation for each of the operations listed below.

a $+ 6$ **b** $\times 4$ **c** $- 7$ **d** $\div 3$

e $\times (-2)$ **f** $\div (-8)$ **g** $- 11$ **h** $\times (-20)$

2 Apply the inverse operation given in the brackets to both sides of the equation to solve each equation.

a $x + 6 = 19$ $(- 6)$ **b** $x - 13 = 29$ $(+ 13)$ **c** $11x = 132$ $(\div 11)$ **d** $\frac{x}{3} = 9$ $(\times 3)$

e $-5x = -5$ $(\div -5)$ **f** $x - 1 = -2$ $(+ 1)$ **g** $\frac{x}{5} = -5$ $(\times 5)$ **h** $-12x = 36$ $(\div -12)$

3 Identify the inverse operation required to solve each of these equations.

a $x - 6 = 22$ **b** $x + 9 = 11$ **c** $3x = 21$ **d** $\frac{x}{4} = 6$

e $x - 2 = 9$ **f** $\frac{x}{5} = -9$ **g** $7x = -49$ **h** $x + 1 = 12$

4 Identify the two inverse operations required (in order) to solve each of these equations.

a $3x + 3 = 24$ **b** $2x - 8 = 16$ **c** $\frac{x}{2} + 5 = 11$ **d** $5(x + 5) = 35$

e $5x - 11 = -36$ **f** $\frac{2x}{5} = 8$ **g** $9(x + 4) = -63$ **h** $\frac{x + 4}{3} = 10$

6B.1 **5** Solve the following equations using inverse operations.

a $x + 4 = 16$ **b** $x - 11 = 3$ **c** $6x = 42$ **d** $\frac{x}{2} = 5$

e $x + 5 = 5$ **f** $\frac{x}{11} = -2$ **g** $9x = -45$ **h** $x + 2 = -1$

i $-2x = 4$ **j** $-5x = 0$ **k** $x - 7 = -10$ **l** $-x = 17$

6 Solve the following questions.

a $x + 4.5 = 7$ **b** $x - 0.6 = -5.6$ **c** $1.2x = -6$ **d** $-1.4x = 2.8$

6B.2 **7** Use inverse operations to solve the following equations.

a $2x + 7 = 23$ **b** $9x - 4 = 5$ **c** $\frac{x}{6} + 3 = 7$ **d** $8(x - 2) = 32$

e $\frac{x + 2}{5} = 4$ **f** $\frac{x}{3} - 9 = 2$ **g** $\frac{x - 8}{2} = 6$ **h** $3(x + 7) = 27$

i $7x - 4 = 45$ **j** $\frac{x + 9}{5} = 3$ **k** $10x + 5 = 15$ **l** $2(x - 6) = 14$

8 Solve the following equations for x.

a $-5x + 8 = 3$ **b** $-4x - 3 = 5$ **c** $11 + 2x = 19$ **d** $7 - 3x = 1$

e $-2(x - 6) = -18$ **f** $-8x + 1 = -15$ **g** $-x + 4 = 13$ **h** $6 - x = -4$

i $-x - 5 = 17$ **j** $-3(x + 8) = 21$ **k** $-(x + 3) = -2$ **l** $-(x - 7) = 1$

9 The solution to a given equation will not always be an integer. Leave your answers as fractions.

a $5x = 1$ **b** $6x = 27$ **c** $3x - 5 = 12$

d $-5x + 7 = 3$ **e** $\frac{x - 2}{3} = \frac{4}{5}$ **f** $-\frac{2}{3}(x + 4) = 5$

10 Caleb decided that a quick way to find the solution to $x - 37 = 85$ is to subtract 37 from both sides of the equation. Is he correct? Explain.

11 Yuhan and Connor take different approaches to solving $\frac{2x}{3} = 4$ for x. Their approaches are shown below.

Yuhan

$$\frac{2x}{3} = 4$$
$$\frac{2x}{3} \times 3 = 4 \times 3$$
$$2x = 12$$
$$\frac{2x}{2} = \frac{12}{2}$$
$$x = 6$$

Connor

$$\frac{2x}{3} = 4$$
$$\frac{2}{3}x = 4$$
$$x = 4 \div \frac{2}{3}$$
$$= 4 \times \frac{3}{2}$$
$$= \frac{12}{2}$$
$$= 6$$

a Check each student's working. Has each student solved the equation correctly?

b Describe one advantage and one disadvantage of each approach.

12 Solve the following questions:

a $\frac{4x}{7} = 8$ **b** $\frac{8x}{3} = -16$ **c** $-\frac{3}{5}x = 6$

d $\frac{2}{3}x = \frac{1}{2}$ **e** $\frac{2x}{9} = 0.4$ **f** $\frac{8x}{9} = \frac{5}{3}$

PROBLEM SOLVING AND REASONING

13 **i** Write an equation to represent each problem.

ii Solve the equation using inverse operations to answer each question.

a The temperature rises by 4°C in the first hour after sunrise and then doubles by midday. If it is 26°C at midday, what was the temperature at sunrise?

b Hiring a party venue costs $280. There is a $15 per person charge for finger food. If Marita has a budget of $1600, how many people can attend a party at this venue?

c Ryan is saving to buy a sound mixing board that costs $649. He saves $56 per month. If he currently has $257, in how many months will he be able to buy the mixing board?

d Reese cooks some donuts. While they are still hot, his friends eat six of them. Half of the donuts that are left are iced and the other half are filled with jam. If there are 11 donuts filled with jam, how many donuts did Reese make?

14 By the end of Tuesday Jin is up to page 256 in her book. On Monday Jin read $\frac{1}{3}$ the number of pages she read on Tuesday. If Jin started Monday on page 216, how many pages did she read on Tuesday? Write an equation and solve it using inverse operations.

15 Henry plants x seedlings each week, but $\frac{1}{4}$ of the seedlings he plants die. At the end of 5 weeks Henry has 75 seedlings. Write an equation and solve it to find x.



16 Jenny's mum awards her 10 points for doing a job and deducts 5 points if she forgets to do a job. At the end of one week, Jenny has forgotten to do $1\frac{1}{2}$ as many jobs as she has remembered to do.

a Write an expression to represent the total number of points Jenny will have at the end of the week.

b Explain why you do not have enough information to determine the number of jobs that Jenny completed that week.

c Explain why Jenny will not end up with negative points.

d If Jenny ends up with 20 points, find how many jobs she was expected to do that week.

17 Solve each equation.

a $4(x + 2) - 5 = 15$ **b** $\frac{2x - 6}{5} = 4$ **c** $\frac{2x}{3} + 8 = 14$ **d** $\frac{5x + 1}{6} = -4$

e $\frac{6 - x}{5} = 2$ **f** $-\frac{x}{4} + 9 = 7$ **g** $9(x - 4) + 3 = 57$ **h** $\frac{4x}{7} - 3 = 5$

CHALLENGE

18 **i** Identify the inverse operations (in order) required to solve these equations.

ii Find the positive solutions to the equations.

a $3x^2 + 5 = 32$ **b** $4(x^2 - 2) = 56$ **c** $\sqrt{2x + 5} = 5$

19 Sophie is $\frac{1}{4}$ the age she will be in 36 years' time. How old is she now?

20 Giovanni solved the following equation and got $b = 12$ as his solution: $\frac{b + 12}{4} = 15$

a Prove by substitution that the solution is wrong. **b** Identify the mistake. **c** What is the correct solution?

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Interactive skillsheet
Solving equations using inverse operations

Investigation
Household bills

Topic quiz
6B

6C Solving equations with the unknown on both sides

Learning intentions

By the end of this topic you will be able to ...

✔ solve equations with the unknown on both sides.

Inter-year links

Year 7	6G Solving equations using inverse operations
Year 9	3A Solving linear equations
Year 10	3A Solving linear equations

Solving equations with the unknown on both sides

- As a pronumeral is just a letter standing in for a number, inverse operations can also be applied to terms containing a pronumeral.
- To solve an equation in which the unknown appears on both sides of the equation:
  1. Use inverse operations to remove the pronumeral term from one side of the equation.
  2. Solve the equation using inverse operations.
     For example, to remove $3x$ from the right-hand side of the equation to the right, subtract $3x$ from both sides of the equation.

$$\begin{aligned} 4x - 2 &= 3x + 1 \\ 4x - 3x - 2 &= 3x - 3x + 1 \\ x - 2 &= 1 \\ x - 2 + 2 &= 1 + 2 \\ x &= 3 \end{aligned}$$

- When using inverse operations, it is important to show your working. This can be done by explicitly applying the inverse operations to both sides or, alternatively, by writing each inverse operation in brackets to the right of each equivalent equation.

$$\begin{aligned} 4x - 2 &= 3x + 1 \\ x - 2 &= 1 && (-3x) \\ x &= 3 && (+2) \end{aligned}$$

Example 6C.1 Solving equations with the unknown on both sides – subtracting the pronumeral term

Solve each equation using inverse operations.

a $6x - 3 = 5x + 7$ **b** $8x - 5 = 10x + 3$

THINK

1. Remove the pronumeral term from one side of the equation by subtracting the pronumeral term with the smaller coefficient from both sides of the equation.
2. Solve the equation using inverse operations.

WRITE

a
$$\begin{aligned} 6x - 3 &= 5x + 7 \\ 6x - 5x - 3 &= 5x - 5x + 7 \\ x - 3 &= 7 \\ x - 3 + 3 &= 7 + 3 \\ x &= 10 \end{aligned}$$

b
$$\begin{aligned} 8x - 5 &= 10x + 3 \\ 8x - 8x - 5 &= 10x - 8x + 3 \\ -5 &= 2x + 3 \\ -5 - 3 &= 2x + 3 - 3 \\ -8 &= 2x \\ \frac{-8^{-4}}{2^{1}} &= \frac{2^{1}x}{2^{1}} \\ -4 &= x \text{ or } x = -4 \end{aligned}$$

Example 6C.2 Solving equations with the unknown on both sides – adding the pronumeral term

Find the value of x in the following equations.

a $4x + 7 = -2x - 23$ **b** $10 - 2x = 4x - 12$

THINK

1 Remove the negative pronumeral term from one side of the equation by adding the pronumeral term to both sides of the equation.

2 Solve the equation using inverse operations.

WRITE

a
$$\begin{aligned} 4x + 7 &= -2x - 23 \\ 4x + 2x + 7 &= -2x + 2x - 23 \\ 6x + 7 &= -23 \\ 6x + 7 - 7 &= -23 - 7 \\ 6x &= -30 \\ \frac{\cancel{6}^{1}x}{\cancel{6}^{1}} &= \frac{\cancel{-30}^{-5}}{\cancel{6}^{1}} \\ x &= -5 \end{aligned}$$

b
$$\begin{aligned} 10 - 2x &= 4x - 12 \\ 10 - 2x + 2x &= 4x + 2x - 12 \\ 10 &= 6x - 12 \\ 10 + 12 &= 6x - 12 + 12 \\ 22 &= 6x \\ \frac{\cancel{22}^{11}}{\cancel{6}^{3}} &= \frac{\cancel{6}^{1}x}{\cancel{6}^{1}} \\ \frac{11}{3} &= x \text{ or } x = \frac{11}{3} \end{aligned}$$

Helpful hints

- ✔ When solving an equation with the unknown on both sides, first remove the pronumeral with the smaller coefficient to avoid having to work with negative coefficients.
- ✔ To find the solution, the pronumeral does not have to appear on the left-hand side of the equation – as long as the pronumeral is by itself on one side of the equation, you have found the solution!

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Exercise 6C Solving equations with the unknown on both sides

▲ 1–3(1st, 2nd columns), 4–8, 10–12, 15 ■ 1–3(3rd, 4th columns), 4, 5, 7, 9, 12, 13, 15, 16(a, d, f, g) ⬟ 2–3(3rd, 4th columns), 4, 7, 10, 12, 14–17

6C.1

1 Solve each equation using the inverse operations.

a $9x - 2 = 8x + 11$	**b** $6x + 5 = 5x + 9$	**c** $2x + 7 = x + 12$	**d** $7x - 3 = 6x - 1$
e $4x - 9 = 5x - 3$	**f** $4x - 10 = 4 + 3x$	**g** $2x + 18 = 8 + 3x$	**h** $2 + 7x = 8x - 13$
i $3x + 9 = 5x - 7$	**j** $x - 12 = 1 + 2x$	**k** $11x - 5 = 10x + 3$	**l** $19x + 3 = 20x + 3$

2 Solve:

a $5a + 4 = 3a + 8$	**b** $4b - 1 = 2b + 7$	**c** $2c - 3 = 6c + 1$	**d** $3d + 10 = 7d + 2$
e $11e - 2 = 6 + 13e$	**f** $5f + 9 = 7f - 2$	**g** $2g + 6 = 5g - 4$	**h** $10 + 5h = h - 2$
i $7i + 2 = 3i + 12$	**j** $12 + j = 6j - 13$	**k** $8k + 3 = k - 3$	**l** $8l - 9 = 12l + 9$

6C.2 **3** Find the value of x in the following equations.

a $3x - 11 = -2x + 9$ **b** $2x + 1 = -5x - 13$ **c** $8x + 3 = -2x - 7$ **d** $3x - 8 = -6x + 19$

e $5x + 6 = -3x + 21$ **f** $3x - 5 = 7 - 9x$ **g** $-4x + 9 = -x - 9$ **h** $6 - x = x - 4$

i $29 - 2x = 5 - 6x$ **j** $2x + 2 = 30 - 4x$ **k** $43 - x = x + 1$ **l** $-12 - 5x = 24 - 13x$

4 **i** Solve the following equations for x.

ii Check your solutions using substitution.

a $8x - 5 = 5x + 13$ **b** $2x - 5 = 15 - 3x$ **c** $4x - 15 = -4x + 17$ **d** $-2x + 12 = -x - 7$

e $4 + 7x = 5 + 12x$ **f** $-19 - 3x = 13 - x$ **g** $x - 6 = 9x + 10$ **h** $19 - 3x = 15 + 3x$

5 **a** Simplify $2x + 3 + 4x + 1 = 3x + 7 + x - 5$ by adding like terms on each side.

b Solve the simplified equation.

6 Three students produce different steps of working to solve $6x + 11 = 2x - 13$.

Andy

$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 - 2x &= 2x - 13 - 2x \\ 8x + 11 &= -13 \\ 8x + 11 - 11 &= -13 - 11 \\ 8x &= -24 \\ \frac{8x}{8} &= \frac{-24}{8} \\ x &= -3 \end{aligned}$$

Kwame

$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 &= 2x - 13 - 2x \\ 6x + 11 &= -13 \\ 6x + 11 - 11 &= -13 - 11 \\ 6x &= -24 \\ \frac{6x}{6} &= \frac{-24}{6} \\ x &= -4 \end{aligned}$$

Hayley

$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 - 2x &= 2x - 13 - 2x \\ 4x + 11 &= 13 \\ 4x + 11 - 11 &= 13 - 11 \\ 4x &= 2 \\ \frac{4x}{4} &= \frac{2}{4} \\ x &= \frac{1}{2} \end{aligned}$$

a Check each student's work. Identify and describe any mistakes.

b Solve the equation correctly. Show your working.

7 Solve each equation.

a $3p + 2p - 5 = p + 7$ **b** $5q + 2 + 3q + 4 = 13 + 2q + 5$

c $6r + 7 - 3r + 3 = r + 4$ **d** $4s + 5 - 2s + 1 = 26 - 3s$

e $6t + 2 + 4t + 9 = 2t + 3 + 3t - 2$ **f** $9 - 5u + 3u - 7 = 5 - 3u + 10$

8 Selena and Hayden have the same amount of money. Selena buys 5 kg of cherries and has $2 left over. Hayden buys 2 kg of cherries and has $20 left over.



a If x represents the cost of 1 kg of cherries, which equation best suits this situation?

A $5x + 20 = 2x + 2$ **B** $5x + 2 = 2x + 20$

C $5x - 2 = 2x - 20$ **D** $5x - 20 = 2x - 2$

E $5x - 20 = 2x + 2$

b Solve the equation to find the cost of 1 kg of cherries.

9 The number of biscuits in a box is unknown. However, Kayla can fill her biscuit tin with five more than the contents of three boxes or seven less than the contents of four boxes.

a If n represents the number of biscuits in a box, which equation best suits this situation?

A $3n - 5 = 4n - 7$ **B** $3n - 5 = 4n + 7$

C $3n + 5 = 4n + 7$ **D** $3n + 5 = 4n - 7$

E $5 - 3n = 7 - 4n$

b Solve the equation to find the number of biscuits in a box.

10 At the school canteen, the cost of three apples and five oranges is the same as the cost of seven apples and two oranges. Each orange costs 80 cents. Let a represent the cost of an apple in cents.

a Write an equation to represent this problem.

b Solve the equation to find the cost of an apple.

PROBLEM SOLVING AND REASONING

11 Sean poses this problem to his neighbour: 'In four years, I will be three times as old as my cousin is now. Two years ago, I was twice my cousin's current age. Can you work out how old I am?' If a represents the current age of Sean's cousin:

a One expression for Sean's current age is $3a - 4$. Write another expression for Sean's current age.

b Use the expressions in part **a** to write an equation.

c Solve the equation to determine the current age of Sean's cousin.

d How old is Sean?

12 Jada and Tom have an equal number of marbles. Jada has four small marbles and Tom has ten small marbles. Jada has three times as many large marbles as Tom. Let n represent the number of large marbles Tom has.



a Write an expression for how many marbles Tom has.

b Write an expression for how many marbles Jada has.

c Form an equation using these expressions.

d Solve the equation to find the value of n.

e How many large marbles does Tom have?

f How many large marbles does Jada have?

g If they agree to swap all their marbles, how many marbles will each person swap?

13 In an AFL match, each goal is worth 6 points and each behind is worth 1 point. An AFL match ends in a draw, with the home team scoring twice as many goals as the away team. The home team scored 3 behinds while the away team scored 27 behinds.

a Write an expression for the number of points scored by the away team using g to represent the number of goals scored by the away team.

b Write an expression for the number of points scored by the home team.

c Form an equation using these expressions.

d Solve the equation to find the value of g and hence list the final score of each team.

14 In rugby league, each try is worth 4 points and each penalty goal is worth 2 points. In a rugby league match, the away team scored twice as many points as the home team. The away team scored two penalty goals and three times as many tries as the home team. The home team scored four penalty goals.

a Write an expression for the number of points scored by the home team using t to represent the number of tries scored by the home team.

b Write an expression for the number of points scored by the away team.

c Form an equation using these expressions. Hint: Remember, the away team scored twice as many points.

d Solve the equation to find the value of t.

e List the number of tries scored by each team.

CHALLENGE

15 a Expand the expressions on the left- and right-hand sides of the equation $5(x - 2) = 3(x + 8)$.

b Solve the equation for x.

16 Solve each equation for x.

a $2(x - 5) = x + 19$ **b** $7(x + 2) = 3x + 2$ **c** $4(x - 1) = 2 - 2x$

d $3(x + 4) = 2(x + 9)$ **e** $5(x - 7) = 3(x + 5)$ **f** $6(x + 3) - x + 5 = 2(x + 1)$

g $4(2x - 3) = 3(7 - x)$ **h** $7(3x + 5) = 5(2x + 7)$ **i** $11(3x + 2) = 4(2x - 7)$

17 a Explain how you would solve each of the following equations.

b Find the positive solutions to the equations.

i $5(x^2 + 2) = 2(2x^2 + 7)$ **ii** $3(x^2 - 4) = 2(x^2 - 6)$

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Solving equations with the unknown on both sides

Investigation
Profit and loss

Topic quiz
6C

Checkpoint



Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

6A **1** Solve the following equations by inspection.

a $3a = 12$ **b** $\frac{d}{5} = 4$ **c** $f - 7 = -3$

d $-4m = 20$ **e** $\frac{p}{3} = -7$ **f** $z + 8 = -9$

6A **2** Solve the following equations by inspection.

a $7a + 5 = 12$ **b** $\frac{b}{2} + 5 = 10$ **c** $3k - 8 = 13$

d $-6x + 9 = 33$ **e** $\frac{2j}{5} = -6$ **f** $10y + 8 = 78$

6A **3** Write an equation to represent each of these number sentences. Use n for the unknown number.

a 6 added to a number gives 18.
b 8 subtracted from a number gives −10.
c 7 more than a number is equal to 36.
d 10 less than a number is equal to −4.

6A **4** Write an equation to represent each of these number sentences. Use x for the unknown number.

a A number is multiplied by 4, then 9 is added to give 33.
b A number is divided by 5, then 4 is subtracted to give 0.
c The sum of a number and 12 all divided by 3 is equal to 6.
d The sum of a number and 3 all multiplied by 8 is equal to 64.

6B **5** Solve the following equations using inverse operations.

a $x + 11 = 25$ **b** $x - 8 = 17$ **c** $12x = 96$

d $\frac{x}{7} = 5$ **e** $x + 23 = -18$ **f** $-6x = 54$

6B **6** Use inverse operations to solve the following equations.

a $3x + 8 = 20$ **b** $9x - 2 = 16$ **c** $\frac{x}{4} + 5 = 0$

d $6(x - 4) = 48$ **e** $\frac{x}{6} + 9 = 18$ **f** $\frac{x + 7}{2} = 10$

6B **7** Solve the following equations for x.

a $-6x + 4 = -20$ **b** $-8x - 3 = -51$ **c** $-3(x + 12) = 60$

d $\frac{x - 8}{6} = -4$ **e** $-9(x + 3) = 27$ **f** $\frac{x + 8}{3} = -11$

6B **8** Solve the following equations for x.

a $7x = 1$ **b** $-2x + 5 = 14$ **c** $\frac{5x}{6} = 7$

d $5(x + 3) = 21$ **e** $\frac{x - 4}{5} = \frac{2}{3}$ **f** $\frac{1}{3}(x + 6) = -5$

6C **9** Solve the following equations for x.

a $8x + 3 = 5x + 15$ **b** $9x + 4 = 6x + 13$ **c** $2x + 7 = x + 15$

6C **10** Solve the following equations for x.

a $10x - 8 = 18x + 14$ **b** $6x + 9 = 8x - 11$ **c** $3x - 6 = 12 + 9x$

6C **11** Solve the following equations for x.

a $9x - 8 = 13x + 13$ **b** $2x - 21 = 9x + 13$ **c** $-7x + 4 = 5x - 8$

6C **12** Solve the following equations for x.

a $4(x + 6) = 3x - 15$ **b** $7(x - 5) = 3x + 6$ **c** $5(x + 3) - 4x + 6 = 3(x + 7)$

6D Plotting linear relationships

Learning intentions

By the end of this topic you will be able to ...

- ✔ use equations to complete tables of values
- ✔ plot linear and non-linear relationships from tables of values and equations
- ✔ differentiate between linear and non-linear relationships.

Inter-year links

Support	The Cartesian plane
Year 7	5D The Cartesian plane
Year 9	3C Plotting linear relationships

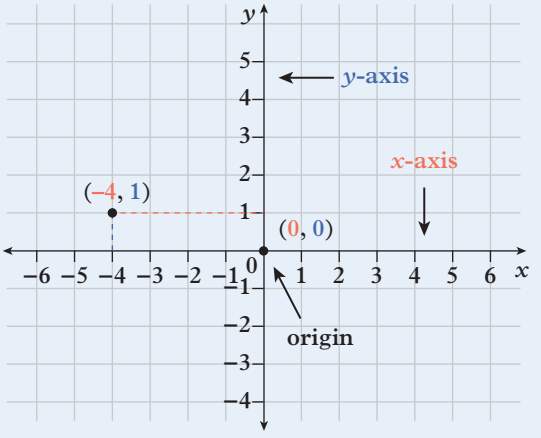
The Cartesian plane and Cartesian coordinates

- The **Cartesian plane** consists of a horizontal ***x*-axis** and a vertical ***y*-axis** that intersect at right angles at a point called the **origin**.
- **Coordinates** describe the position of a point on the Cartesian plane and consist of an ***x*-coordinate** and a ***y*-coordinate**.

$(-4, 1)$

x-coordinate – horizontal distance from the origin

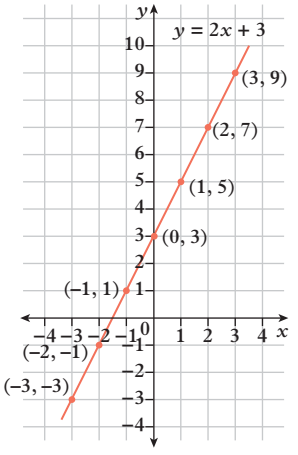
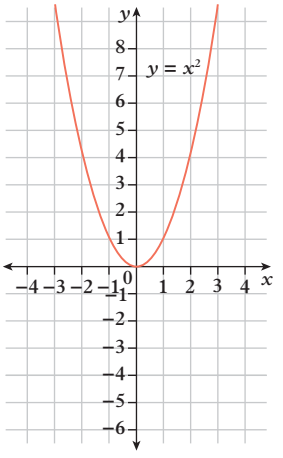
y-coordinate – vertical distance from the origin



Linear and non-linear relationships

- The relationship between two variables can be represented by an algebraic equation, a table of values, a set of coordinates or a graph.
- **Linear relationships** produce straight line graphs called **linear graphs**.
- **Non-linear relationships** produce curved graphs called **non-linear graphs**.

For example, graphs of the linear relationship $y = 2x + 3$ and the non-linear relationship $y = x^2$ are shown below.

Linear relationship	Non-linear relationship
	

Drawing graphs

- Whereas a **plot** is composed of individual coordinate points, a **graph** is a continuous line made up of infinitely many coordinate points.
- Graphs can be drawn by plotting a series of coordinates. If there is a distinct pattern to the points, a straight line or smooth curve can be used to join the points together.
- To draw a graph from an equation:

1 Construct a table of x and y values by selecting values for x, then substituting each value of x into the equation to find the corresponding value of y.

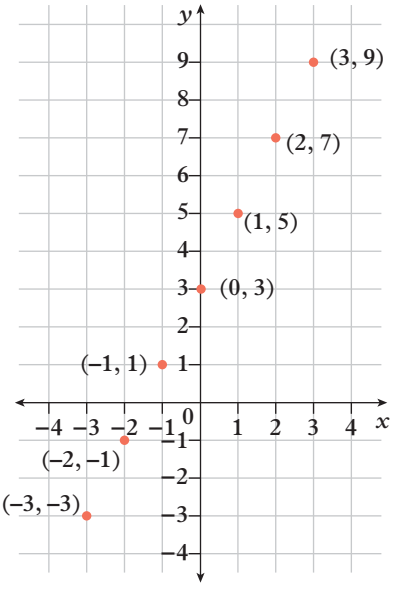
$y = 2x + 3$

x	−3	−2	−1	0	1	2	3
y	−3	−1	1	3	5	7	9

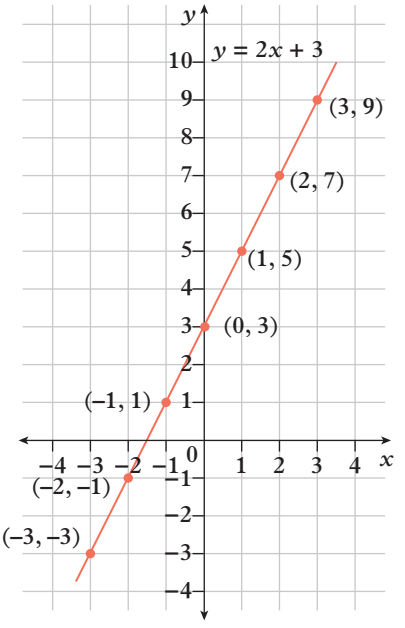
2 Write out the coordinate points listed in the table.

(−3, −3), (−2, −1), (−1, 1), (0, 3), (1, 5), (2, 7), (3, 9)

3 Plot the coordinate points on the Cartesian plane.



4 Join the points with a smooth line.



Example 6D.1 Plotting points on the Cartesian plane

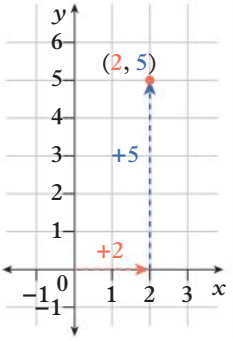
Plot the following points on the Cartesian plane and label each point with its coordinates.

a (2, 5) **b** (−5, −1)

THINK

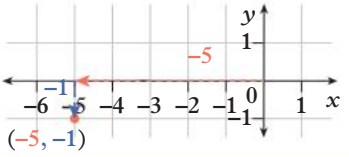
1 Draw a Cartesian plane. The x- and y-axes should extend far enough to accommodate the largest coordinates.

2 Mark each point with a dot and label the dot with its coordinates.

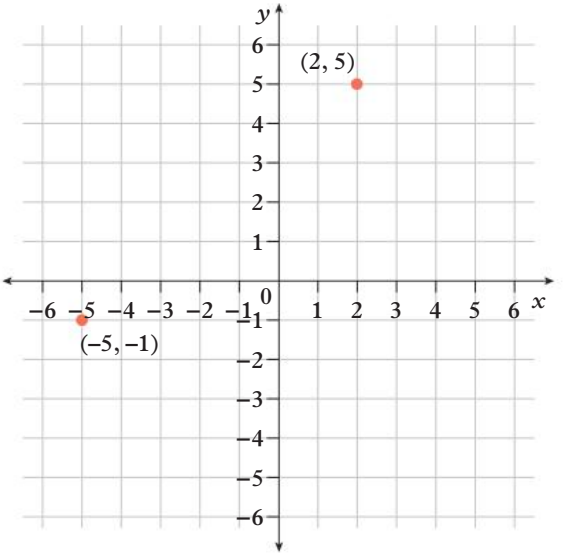


a The point (2, 5) has an x-coordinate of 2 and a y-coordinate of 5. Start at the origin, move 2 units to the right and 5 units up.

b The point (−5, −1) has an x-coordinate of −5 and a y-coordinate of −1. Start at the origin, move 5 units to the left and 1 unit down.



WRITE



Example 6D.2 Using an equation to complete a table of values

Complete the table of values for $y = 2x - 1$.

x	−2	−1	0	1	2
y					

THINK

1 Substitute each value of x into the equation to find the corresponding value of y.

2 Complete the table of values.

WRITE

Let $x = -2$
$y = 2(-2) - 1$
$= -4 - 1$
$= -5$

Let $x = -1$
$y = 2(-1) - 1$
$= -2 - 1$
$= -3$

Let $x = 0$
$y = 2(0) - 1$
$= 0 - 1$
$= -1$

Let $x = 1$
$y = 2(1) - 1$
$= 2 - 1$
$= 1$

Let $x = 2$
$y = 2(2) - 1$
$= 4 - 1$
$= 3$

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

Example 6D.3 Graphing linear relationships

Plot a graph of $y = 5 - x$ by first completing a table of values for integer values of x from -3 to 3.

THINK

1 Construct a table of values for x from -3 to 3. Substitute each value of x into the equation to find the corresponding values of y.

2 Write out the coordinates of each point listed in the table.

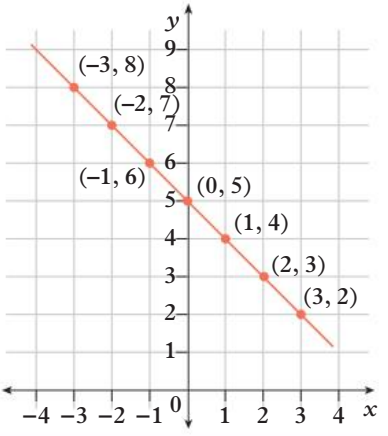
3 Plot the points on the Cartesian plane.

4 Join the points with a smooth line.

WRITE

x	-3	-2	-1	0	1	2	3
y	8	7	6	5	4	3	2

$(-3, 8), (-2, 7), (-1, 6), (0, 5), (1, 4), (2, 3), (3, 2)$



Example 6D.4 Identifying linear and non-linear relationships

Use the tables of values to plot graphs and determine whether the relationships are linear or non-linear.

a

x	-2	-1	0	1	2
y	7	4	1	-2	-5

b

x	-2	-1	0	1	2
y	9	4	1	0	1

THINK

a 1 Write the coordinates of each point listed in the table.

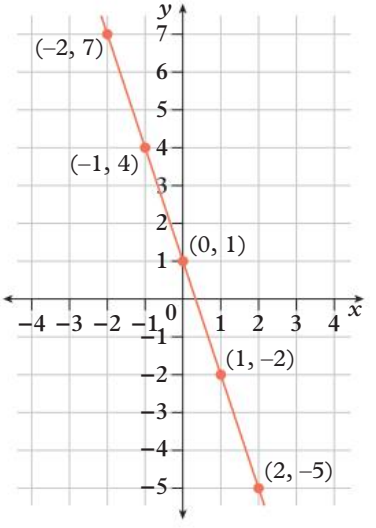
2 Plot the points on the Cartesian plane.

3 Join the coordinate points with a straight line or smooth curve.

4 The coordinates form a straight line, so the table of values represents a linear relationship.

WRITE

a $(-2, 7), (-1, 4), (0, 1), (1, -2), (2, -5)$



The table of values represents a linear relationship.

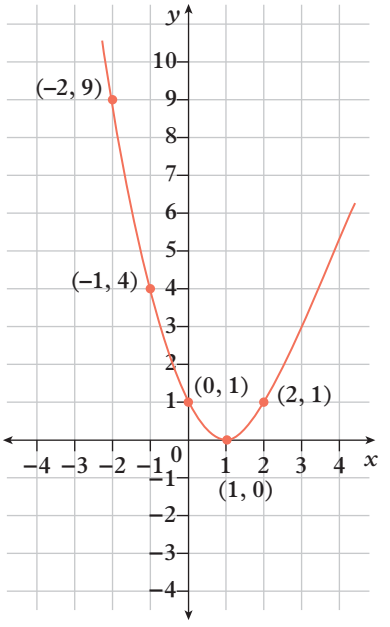
b 1 Write the coordinates of each point listed in the table.

2 Plot the points on the Cartesian plane.

3 Join the coordinate points with a straight line or smooth curve.

4 The coordinates do not form a straight line, so the table of values represents a non-linear relationship.

b (−2, 9), (−1, 1), (0, 0), (1, 1), (2, 4)



The table of values represents a non-linear relationship.

Helpful hints

- ✔ When plotting points, locate the value of the x-coordinate first, then move up or down to the value on the y-coordinate.
- ✔ When plotting a linear graph, use a ruler when joining the points with a straight line.
- ✔ Vertical and horizontal straight lines are also linear relationships.

ANS p523

Exercise 6D Plotting linear relationships

△ 1–3, 4, 5(1st, 2nd columns), 6–11, 15 □ 2–4, 5(2nd, 3rd columns), 6–12, 14, 15, 17(a) ⬠ 5(2nd, 3rd columns), 6–11, 13, 15–17

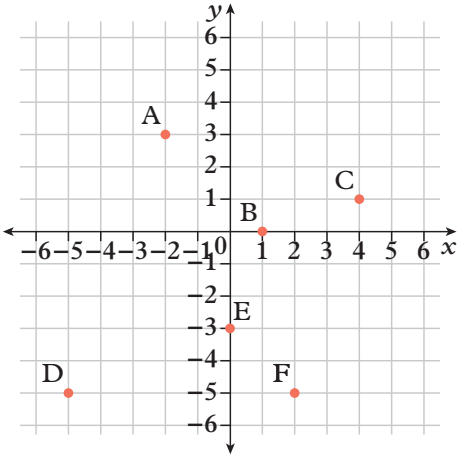
UNDERSTANDING AND FLUENCY

1 Match the coordinates to the points shown on the Cartesian plane.

a (−2, 3) **b** (4, 1) **c** (2, −5)
d (1, 0) **e** (0, −3) **f** (−5, −5)

6D.1 **2** Plot the following points on the Cartesian plane and label each point with its coordinates.

a (3, 5) **b** (−4, −5) **c** (2, −2)
d (6, 0) **e** (−1, 4) **f** (0, 3)



6D.2 **3** Complete the table of values for each of the following linear equations.

a $y = x + 1$

x	−2	−1	0	1	2
y					

b $y = 2x + 2$

x	−3	−2	−1	0	1
y					

c $y = 3 - x$

x	−1	0	1	2	3
y					

d $y = 3x - 4$

x	−1	0	1	2	3
y					

4 Plot a graph of each of the linear relationships in question **3**.

6D.3 **5** Plot a graph of each linear relationship by first completing a table of values for integer values of x from −3 to 3.

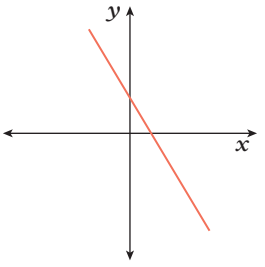
a $y = x + 4$ **b** $y = x - 3$ **c** $y = -x$ **d** $y = 2 - x$

e $y = 5 - x$ **f** $y = -x - 1$ **g** $y = 3x$ **h** $y = 2x + 3$

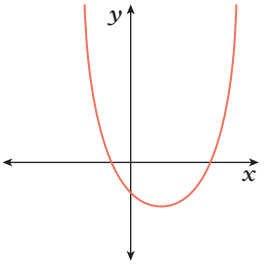
i $y = 4x - 2$ **j** $y = 3x + 1$ **k** $y = 6 - 3x$ **l** $y = 4 - 2x$

6 Determine whether the following relationships are linear or non-linear.

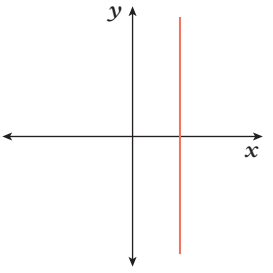
a



b



c



6D.4 **7** Use the tables of values to plot graphs and determine whether the following relationships are linear or non-linear.

a

x	−2	−1	0	1	2
y	0	1	2	3	4

b

x	−2	−1	0	1	2
y	8	5	2	−1	−4

c

x	0	1	2	3	4
y	0	2	5	10	17

8 **a** Plot the graph of each relationship using the table of values.

i $y = 2$

x	−2	−1	0	1	2
y	2	2	2	2	2

ii $y = -1$

x	−2	−1	0	1	2
y	−1	−1	−1	−1	−1

iii $y = 8$

x	−2	−1	0	1	2
y	8	8	8	8	8

b Are these relationships linear? Explain.

c What do you notice about the equation and its matching graph?

9 **a** Plot the graph of each relationship using the table of values.

i $x = 3$

x	3	3	3	3	3
y	−2	−1	0	1	2

ii $x = 0$

x	0	0	0	0	0
y	−2	−1	0	1	2

iii $x = -6$

x	−6	−6	−6	−6	−6
y	−2	−1	0	1	2

b Are these relationships linear? Explain.

c What do you notice about the equation and its matching graph?

10 Without using a table of values, draw the graph of each relationship.

a $x = 2$ **b** $y = 4$ **c** $y = -3$ **d** $x = -1$ **e** $x = 5$ **f** $y = 0$

11 Two hot air balloons lift off at the same time. Their heights over the first 10 seconds of flight are recorded in these tables.

Red balloon

Time (s)	0	2	4	6	8	10
Height (m)	0	4	8	12	16	20

Blue balloon

Time (s)	0	2	4	6	8	10
Height (m)	0	8	14	18	19	20

a What are the two variables shown in each table?

When two variables are related, the variable that 'depends' on the other is called the dependent variable and the other variable is called the independent variable. Generally, the independent variable is shown in the first row in a table of values, and on the horizontal axis on the Cartesian plane.

b In the hot air balloon scenario, which is the:

i independent variable **ii** dependent variable?

c Write the coordinates of the points listed in each table.

d Showing time on the horizontal axis and height on the vertical axis, plot the points on the Cartesian plane and join each set of points with a smooth line to produce two graphs. Label each graph.

e Use the graphs to estimate:

i the height of each balloon after 5 seconds

ii the time it takes each balloon to reach a height of 7 metres.

f Which graph shows a linear relationship? Explain.

g Graphs can be used to make predictions. Which graph allows for more accurate predictions? Justify your choice.

12 This table shows the distance Jamie covers in each hour of a road trip in her car.

Time (h)	0	1	2	3	4	5
Distance (km)	0	80	160	240	320	400



a What is the independent variable in this relationship? What is the dependent variable?

b Plot a graph of this relationship and label the axes appropriately.

c Is the relationship linear? Explain.

d Use the graph to determine how far Jamie travelled in 2.5 hours.

e Use the graph to determine how long it took her to travel 260 km.

f If Jamie travelled for 7 hours in total, what total distance would she cover? What assumption do you need to make for this prediction?

g Write an equation to represent the relationship between the distance, d, Jamie travels and the time, t, it takes her to cover that distance. Explain your thinking.

h Use the equation to check your answer to part **f**.

13 The water consumption for a household can be described by the linear equation $w = 900d$, where w represents the amount of water (in L) used for the month and d represents the number of days from the start of the month.

a Identify the independent and dependent variables in this relationship.

b Plot a graph of this relationship for the first ten days of the month and label the axes appropriately.

c Is this relationship linear? Explain.

d Use the graph to determine the amount of water used in the first week of the month.

e A resident of the household wants to determine how many days it takes to consume 20 000 L of water.

i Should the resident use the equation or a graph to determine the time taken to consume 20 000 L of water? Justify your choice.

ii Determine how many days it will take to consume 20 000 L of water.

14 Gavin's water tank has sprung a leak and water is leaking out at a constant rate of 800 L/h. He used the following table to record how much water is left in the tank.

Time after the leakage started (h)	0	0.25	0.5	0.75	1
Water left in the tank (L)	2000	1800	1600	1400	1200

a How much water was in the tank before the leakage started?

b Write a formula to represent the relationship between the water left in the tank (w) and time (t).

c In this relationship, which is the:

i dependent variable **ii** independent variable?

d Plot a graph of this relationship and label the axes appropriately.

e Gavin called a plumber immediately after the leakage started. If it takes 3 hours for the plumber to travel to Gavin's place, how much water would be left in the tank by the time the plumber arrives?

15 The following table shows the perimeter and area for squares of different side lengths.

Side length (l)	1 cm	2 cm	3 cm	4 cm	5 cm
Perimeter (P)	4 cm	8 cm	12 cm	16 cm	20 cm
Area (A)	1 cm^2	4 cm^2	9 cm^2	16 cm^2	25 cm^2

If you were to graph P vs l and A vs l, which one would be a linear graph? How can you tell without graphing first?

16 The following table shows the number of cows in a paddock, the number of fence posts required for that many cows, and the area of the paddock.

Number of cows (C)	10	20	30	40
Number of fence posts (P)	50	100	150	200
Area of paddock (A)	300 m^2	1200 m^2	2700 m^2	4800 m^2

a Which graph would be linear: P vs C or A vs C? Justify your answer.

b What does your answer to part **a** tell you about the relationship between the number of cows and the area of a paddock? Explain your answer.

17 Both of Lily's parents invested $1000 in a bank account when she was born. Lily's mum invested with 'compound interest' and her dad with 'simple interest'.

The following table shows the value of the investment over the years.

Lily's age, A (years)	**Mum's investment (M)**	**Dad's investment (D)**
0	$1000	$1000
1	$1100	$1120
2	$1210	$1240
3	$1331	$1360
4	$1464.10	$1480
5	$1610.51	$1600

a If you graphed M vs A and D vs A, which graph would be linear?

b Plot the graphs for both investments and use them to predict the value of the investments when Lily is 10.

c Based on this question do you think compound interest or simple interest is a better choice? Are there other factors to consider?

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Plotting graphs

Investigation
Graphing length and width of a rectangle of fixed area

Topic quiz
6D

6E Solving linear equations using graphs

Learning intentions

By the end of this topic you will be able to ...

✔ solve linear equations using a graph.

Inter-year links

Support — Interpreting graphs

Year 7 — 5E Interpreting graphs

Solving linear equations using graphs

- Each point on a graph is made up of an x-coordinate and a corresponding y-coordinate. The equation of a graph describes the relationship between the x- and y-coordinates of all the points that make up the graph.

Graph

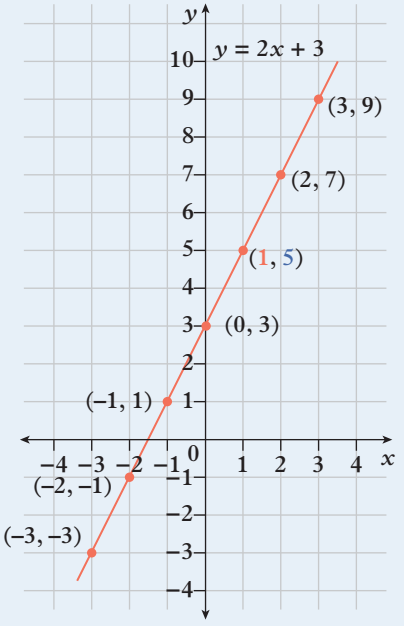


Table of values

x	–3	–2	–1	0	1	2	3
y	–3	–1	1	3	5	7	9

Equation

$y = 2x + 3$

Coordinate

(1, 5)

Solution

Let $y = 5$

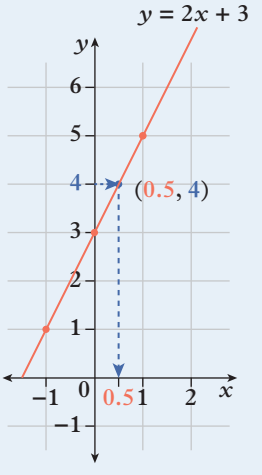
$5 = 2x + 3$

$2 = 2x$

$x = 1$

- The x-coordinate of each point on a graph is the solution to a specific equation related to the equation of the graph.

  For example, to solve the equation $2x + 3 = 4$ using the graph of $y = 2x + 3$, locate 4 on the y-axis, then trace across to the graph and down to the x-axis to find the corresponding value of x.



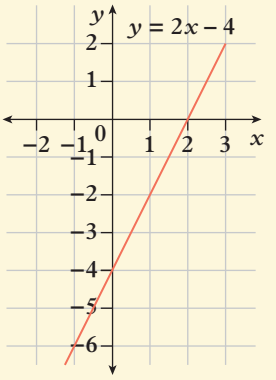
Example 6E.1 Solving linear equations using a graph

Solve the following equations using the graph of $y = 2x - 4$.

a $2x - 4 = 2$

b $2x - 4 = 0$

c $2x - 4 = 1$

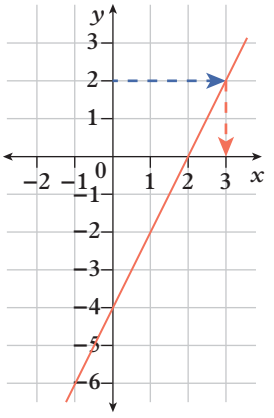


THINK

a Use the graph to find the value of x when $y = 2$. Locate 2 on the y-axis, trace across to the equation line, then down to the x-axis to find the corresponding value of x.

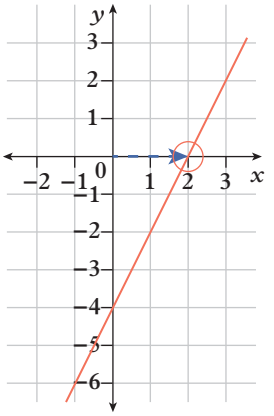
WRITE

a When $y = 2$, $x = 3$.
The solution to $2x - 4 = 2$ is $x = 3$.



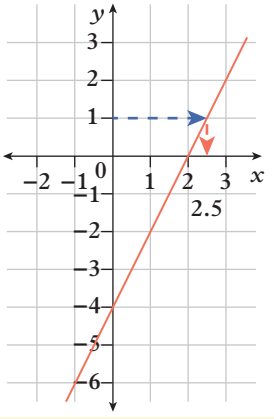
b Use the graph to find the value of x when $y = 0$. $y = 0$ is on the x-axis, so trace along the x-axis to the equation line to find the value of x when $y = 0$.

b When $y = 0$, $x = 2$.
The solution to $2x - 4 = 0$ is $x = 2$.



c Use the graph to find the value of x when $y = 1$. Locate 1 on the y-axis, trace across to the equation line, then down to the x-axis to find the corresponding value of x.

c When $y = 1$, $x = 2.5$ or $2\frac{1}{2}$.
The solution to $2x - 4 = 1$ is $x = 2.5$ or $2\frac{1}{2}$.



Example 6E.2 Solving linear equations by plotting graphs

Consider the following set of linear equations.

i $5 - x = 2$ **ii** $5 - x = -3$ **iii** $5 - x = 0$

a Write the equation of the graph required to solve this set of equations graphically.

b List the y values that need to be included on the graph to solve each equation graphically.

c Use a table of values for x from -2 to 2 to plot the graph, and hence find the solution to each equation.

THINK

a Identify the expression that is shared by all the equations in the set. The expression on the left-hand side of every equation in the set is '$5 - x$', so a graph of $y = 5 - x$ is required to solve this set of equations graphically.

b Each equation in the set is written in the form $5 - x = y$. List the value of y in each equation.

c **1** Construct a table of values for x is equal to $-2, -1, 0, 1, 2$.

2 Plot the coordinates on the Cartesian plane and join the points with a smooth line to draw the graph.

3 Use the graph to find the value of x for each y value listed in step **b**.

4 Write the solution to each equation.

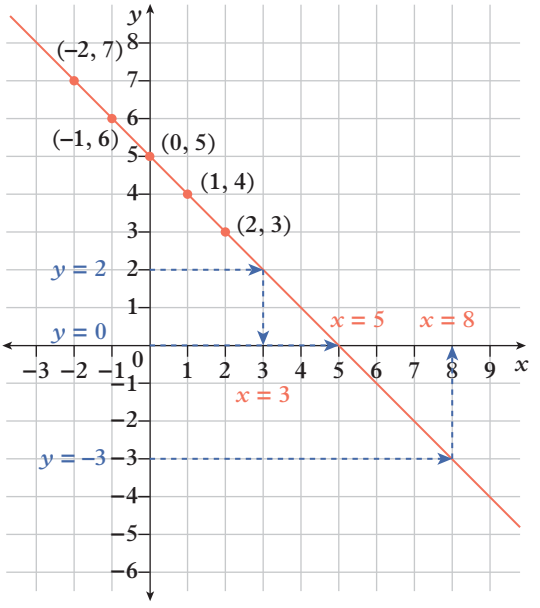
WRITE

a $y = 5 - x$

b $y = 2, -3, 0$

c

x	-2	-1	0	1	2
y	7	6	5	4	3



i $x = 3$ **ii** $x = 8$ **iii** $x = 5$

Helpful hints

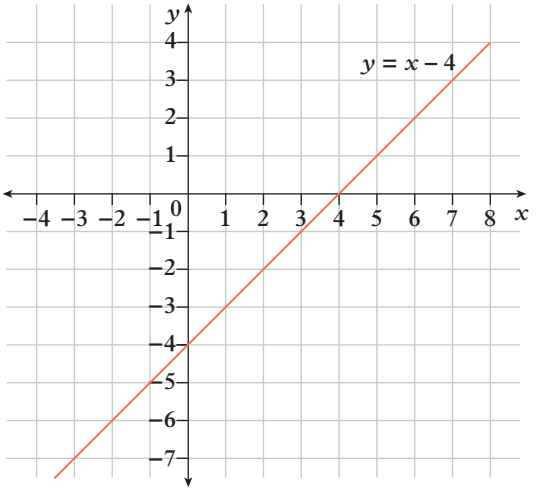
- ✔ The coordinates of graphs do not have to be integer values. For example, in the graph $y = 2x + 3$, $x = 1.5$, $x = 1.2$ and $x = -11.37$ all have a corresponding y-value.
- ✔ Although x and y are commonly used to represent the independent and dependent variables when working on the Cartesian plane, remember that variables can be represented using any pronumeral.
- ✔ When the value of a solution read from a graph is not an exact value, the graph can still be used to provide an estimate for a solution which can be calculated exactly by solving an equation.

ANS p527

Exercise 6E Solving linear equations using graphs

▲ 1–6, 8, 10, 11 ■ 2–4, 6–9, 11–13 ⬟ 3, 5, 8, 9, 12–16

1 Consider the graph of $y = x - 4$ shown to the right.



a Use the graph to find the value of x that corresponds to each of the following values of y.

i $y = 2$ **ii** $y = -3$

iii $y = -6$ **iv** $y = -2$

b Use your answers to part **a** to solve each equation.

i $x - 4 = 2$ **ii** $x - 4 = -3$

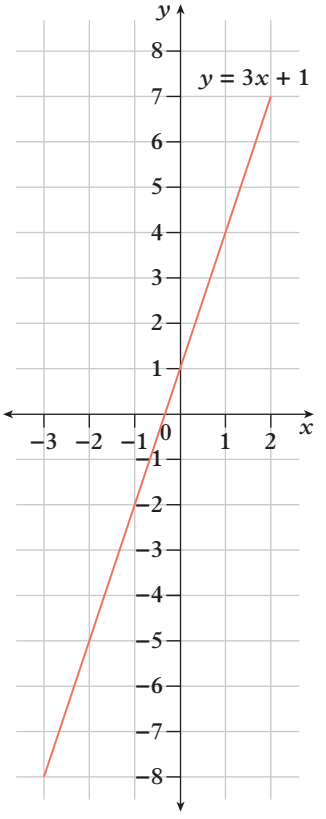
iii $x - 4 = -6$ **iv** $x - 4 = -2$

c Use the graph to solve the following equations.

i $x - 4 = 4$ **ii** $x - 4 = 0$

iii $x - 4 = -4$ **iv** $x - 4 = -6$

6E.1 **2** Solve the following equations using the graph of $y = 3x + 1$ shown to the right.



a $3x + 1 = 7$ **b** $3x + 1 = -2$

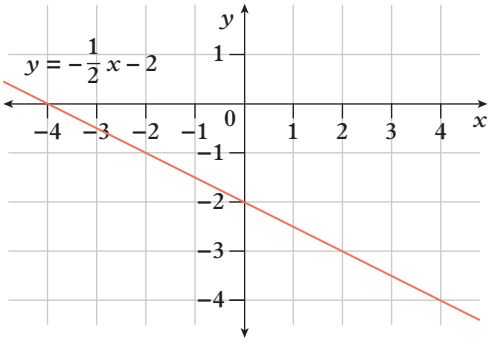
c $3x + 1 = 4$ **d** $3x + 1 = 1$

e $3x + 1 = -5$ **f** $3x + 1 = -8$

3 Solve the following equations using the graph of $y = -\frac{1}{2}x - 2$ shown below.

a $-\frac{1}{2}x - 2 = -3$ **b** $-\frac{1}{2}x - 2 = -1$ **c** $-\frac{1}{2}x - 2 = -2$

d $-\frac{1}{2}x - 2 = 0$ **e** $-\frac{1}{2}x - 2 = -\frac{1}{2}$ **f** $-\frac{1}{2}x - 2 = -\frac{5}{2}$



4 **a** Plot the graph of $y = 2 - x$ for x is equal to -3 to 3.

b Use the graph to solve the following set of equations.

i $2 - x = 1$ **ii** $2 - x = -1$ **iii** $2 - x = 0$

iv $2 - x = 2$ **v** $2 - x = 1.5$ **vi** $2 - x = 0.5$

5 **a** Plot the graph of $y = 2x + 3$ for x is equal to -3 to 3.

b Use the graph to solve the following set of equations.

i $2x + 3 = 1$ **ii** $2x + 3 = -3$ **iii** $2x + 3 = 6$

iv $2x + 3 = 3$ **v** $2x + 3 = -2$ **vi** $2x + 3 = 8$

UNDERSTANDING AND FLUENCY

6E.2 **6** Consider the following set of linear equations.

i $3x - 2 = 7$ **ii** $3x - 2 = -5$ **iii** $3x - 2 = -20$

a Write the equation of the graph required to solve this set of equations graphically.

b List the y value that needs to be used to solve each equation.

c Plot the graph and hence find the solution to each equation.

7 Consider the following set of linear equations.

i $4x - 5 = 3$ **ii** $4x - 5 = -13$ **iii** $4x - 5 = -7$

a Write the equation of the graph required to solve this set of equations graphically.

b List the y values that need to be included on the graph to solve each equation graphically.

c Plot the graph, and hence find the solution to each equation.

8 Solve each equation graphically.

a $1 - 2x = 3$ **b** $1 - 2x = -5$ **c** $1 - 2x = 7$

d $1 - 2x = -9$ **e** $1 - 2x = 9$ **f** $1 - 2x = -4$

9 Solve each equation graphically.

a $8x - 3 = 5$ **b** $8x - 3 = 13$ **c** $8x - 3 = 21$

d $8x - 3 = -3$ **e** $8x - 3 = -11$ **f** $8x - 3 = -7$

PROBLEM SOLVING AND REASONING

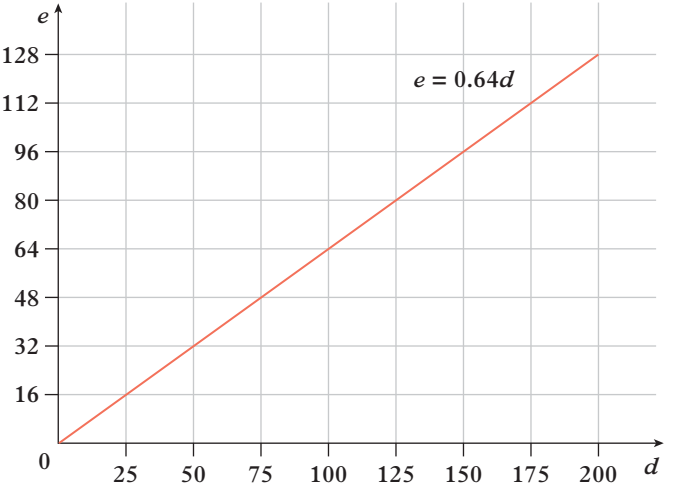
10 Consider the equation $-4x + 6 = -4$.

a Solve the equation using inverse operations.

b Solve the equation graphically.

c Describe one advantage and one disadvantage of the graphical solution method compared to the algebraic solution method.

11 Matteo has been comparing the prices of goods in Italy with those in Australia. To save time, he decides to plot a graph to convert Euros, e, to Australian dollars, d. He looks up the exchange rate and finds that \$1 = 0.64 Euros, or $e = 0.64d$.



a Use the graph to solve each of the following equations.

i $0.64d = 32$

ii $0.64d = 96$

iii $0.64d = 128$

b A designer belt costs 50 Euros.

i Write an equation that can be used to work out the cost of the belt in Australian dollars.

ii Use the graph to solve the equation. What is the equivalent cost of the belt in Australian dollars, to the nearest dollar?



c Matteo compiles a list of items for comparison.

shirt: 64 Euros

shoes: 100 Euros

jeans: 42 Euros

leather jacket: 140 Euros

i Write an equation that can be used to work out the equivalent cost of each item in Australian dollars.

ii Use the graph to solve each equation, stating the value of d to the nearest whole number.

iii Write the cost of each item in Australian dollars.

12 In Australia, temperature is measured in degrees Celsius. Some countries, such as the USA, measure temperature in degrees Fahrenheit. The formula $F = \frac{9}{5}C + 32$ can be used to convert temperatures in degrees Celsius, C, into degrees Fahrenheit, F.

a Use the formula to complete the table of values below.

C	−25	0	25	50	75
F					

b Use the table of values to plot a graph of the relationship between degrees Celsius, C, and degrees Fahrenheit, F.

c Use the graph to solve each equation to find the value of C, rounded to the nearest integer.

i $\frac{9}{5}C + 32 = 40$ **ii** $\frac{9}{5}C + 32 = 104$ **iii** $\frac{9}{5}C + 32 = 14$

Olivia is watching an American cooking show when the host gives the following advice on how to store food bought from the supermarket.

'Frozen meat should be stored in the freezer at −5°F, dairy products and vegetables should be stored in the fridge at about 37°F and tomatoes should be kept at room temperature, ideally around 68°F.'

d **i** Write three equations that can be used to calculate the equivalent temperatures in degrees Celsius.

ii Use the graph to solve each equation, rounded to the nearest integer.

iii Rewrite the advice for an Australian audience.

e The host recommends keeping hot, cooked food above 140°F. What temperature is this in degrees Celsius?

13 Equations with the unknown on both sides can also be solved graphically. To solve $4x - 3 = 2x + 1$ graphically:

a Plot the graph of $y_1 = 4x - 3$ for x in the range −1 to 3.

b Plot the graph of $y_2 = 2x + 1$ on the same set of axes.

c Find the point where these lines intersect and write the coordinates of the point where $y_1 = y_2$.

d Write the solution to $4x - 3 = 2x + 1$.

e Check your solution by substituting the value for x into both sides of the equation.

14 To solve each of the linear equations below graphically:

i write the equations of the graphs to be plotted

ii plot each pair of graphs on the same set of axes

iii read the solution to the equation off the graph

iv use substitution to check your solution.

a $x - 2 = 7 - 2x$ **b** $2x + 4 = -3x - 6$

15 The number of lamingtons in a packet is unknown. However, Adam can fill a plastic container with two more than the contents of one packet or four less than the contents of two packets.

a Write an equation to represent this problem.

b Write the equations of the graphs that need to be plotted to solve the equation graphically.

c Solve the equation graphically.

d How many lamingtons are in a packet?



16 Solve each equation graphically:

a $3x - 4 = x - 1$ **b** $2x - 5 = -x - 5$

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Solving linear equations graphically

Investigation
Integer solutions to linear equations

Topic quiz
6E

6F Finding linear equations

Learning intentions

By the end of this topic you will be able to ...

- ✔ find the linear equation for a table of values
- ✔ find the linear equation for a graph when the coordinates are known.

Inter-year links

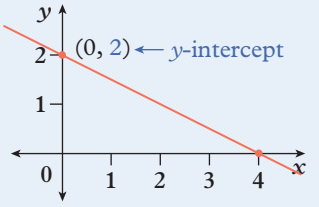
Support	The Cartesian plane
Year 7	5D The Cartesian plane
Year 9	3F Determining linear equations
Year 10	3C Determining linear equations

Finding the equation for a linear relationship

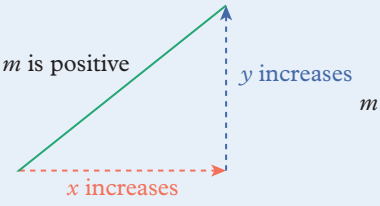
- Linear equations can be written in the form:

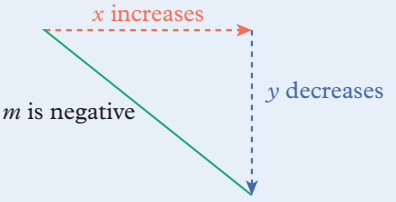
  $y = mx + c$

- The constant, c, in $y = mx + c$ is equal to the value of y when $x = 0$. This is also known as the ***y*-intercept**.



- The coefficient, m, in $y = mx + c$ is the slope of the relationship, which is equal to the change in the value of y as x increases by 1.
  - → When y increases as x increases, m is positive.
  - → When y decreases as x increases, m is negative.
  - → The magnitude of m (the value ignoring the sign) indicates the steepness of the slope. A larger magnitude means a steeper slope.





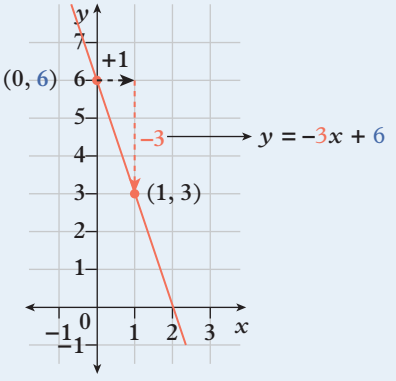
For example, the equation for the table of values below is $y = 2x + 3$ because:

- → each time the value of x increases by 1, the value of y increases by 2
- → when $x = 0$, $y = 3$.

		+1	+1	+1	+1	
x	−2	−1	0	1	2	
y	−1	1	3	5	7	→ $y = 2x + 3$
		+2	+2	+2	+2	

The equation for the graph below is $y = -3x + 6$ because:

- → each time the value of x increases by 1, the value of y decreases by 3
- → when $x = 0$, $y = 6$.



Example 6F.1 Finding the linear equation for a table of values

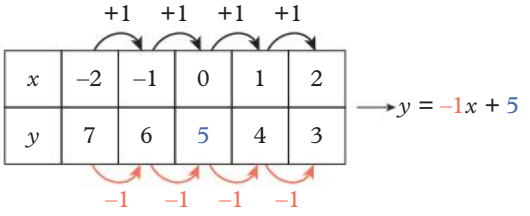
Use the table below to find the linear equation which represents the relationship between x and y.

x	-2	-1	0	1	2
y	7	6	5	4	3

THINK

1. The coefficient of x is equal to the change in y when x increases by 1. Each time the value of x increases by 1, the value of y decreases by 1 so the coefficient of x, $m = -1$.
2. The constant is equal to the value of y when $x = 0$. When $x = 0$, $y = 5$ so the constant $c = 5$.
3. Write the equation for the table.

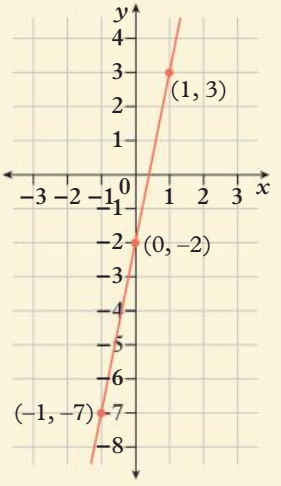
WRITE



The linear equation for the table is $y = -x + 5$.

Example 6F.2 Finding the linear equation of a graph

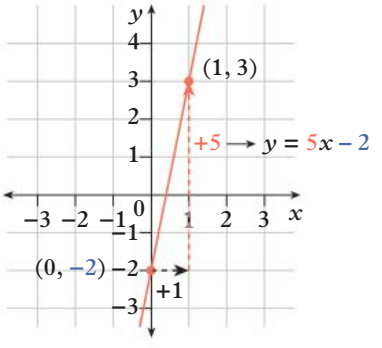
Find the equation of the graph below.



THINK

1. The coefficient of x is equal to the change in y when x increases by 1. Select two consecutive coordinate points and note the change in y when x increases by 1. Each time the value of x increases by 1, the value of y increases by 5, so the coefficient of x, $m = 5$.
2. The constant is equal to the value of y when $x = 0$. When $x = 0$, $y = -2$ so the constant $c = -2$.
3. Write the equation for the graph.

WRITE



The linear equation for the graph is $y = 5x - 2$.

Helpful hints

✔ If you're unsure about a question involving a table, plot the points and sketch the graph; this may help you find the equation!

ANS p530

Exercise 6F Finding linear equations

▲ 1–9, 10(a, c) ■ 1–6, 8–12 ⬟ 1(d–f), 2–4, 8–13

UNDERSTANDING AND FLUENCY

6F.1 **1** Use the following tables to find the linear equations which represent the relationships between x and y.

a

x	0	1	2	3	4
y	2	4	6	8	10

b

x	0	1	2	3	4
y	4	1	−2	−5	−8

c

x	−2	−1	0	1	2
y	−3	−1	1	3	5

d

x	−2	−1	0	1	2
y	1	−1	−3	−5	−7

e

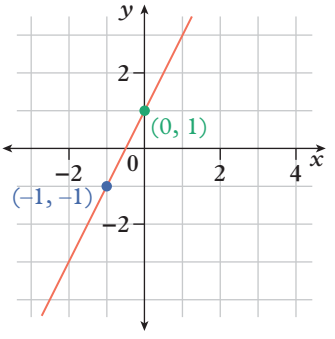
x	−4	−3	−2	−1	0
y	−4	−2	0	2	4

f

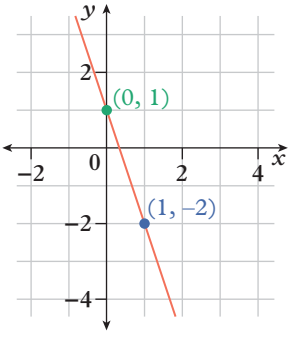
x	−4	−3	−2	−1	0
y	7	4	1	−2	−5

6F.2 **2** Find the equations for each of the graphs below.

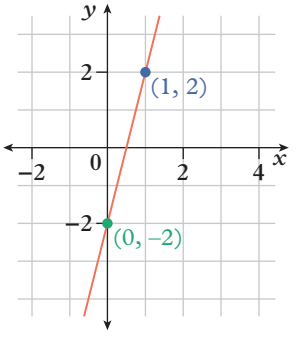
a



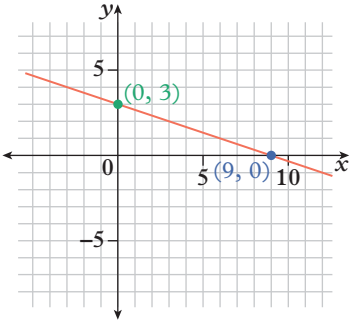
b



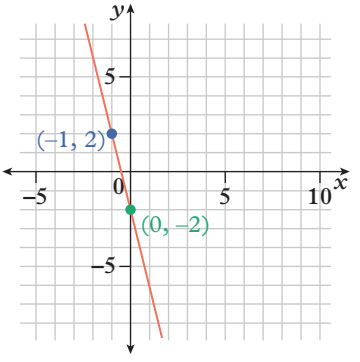
c



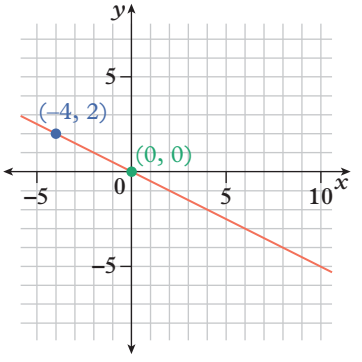
d



e



f



3 Draw the graph of $y = 2x - 3$ by plotting the y-intercept and one other point.

4 Draw the graph of $y = -3x + 2$ by plotting the y-intercept and one other point.

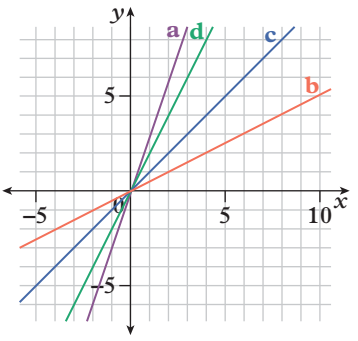
5 Match the graph with the equation.

A $y = 0.5x$

B $y = x$

C $y = 2x$

D $y = 3x$



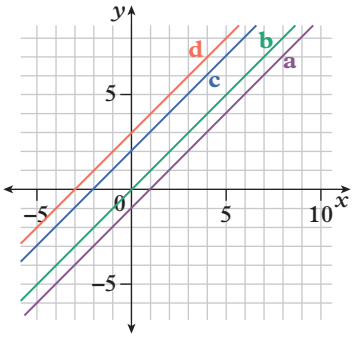
6 Match the graph with the equation.

A $y = x + 2$

B $y = x$

C $y = x - 1$

D $y = x + 3$



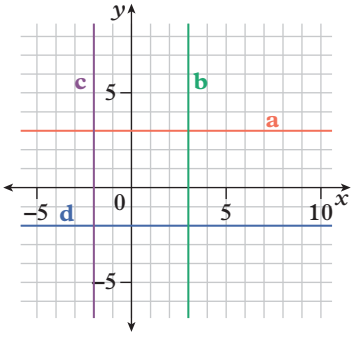
7 Match the graph with the equation.

A $y = 3$

B $x = 3$

C $y = -2$

D $x = -2$



8 Find the linear equations which are represented in the following tables.

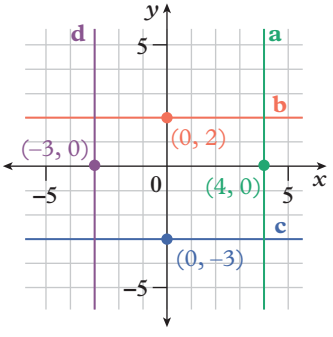
a

x	−4	−2	0	2	4
y	21	13	5	−3	−11

b

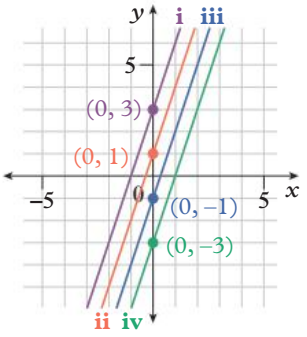
x	0	3	6	9	12
y	−5	−14	−23	−32	−41

9 Write the equations of these horizontal and vertical lines.

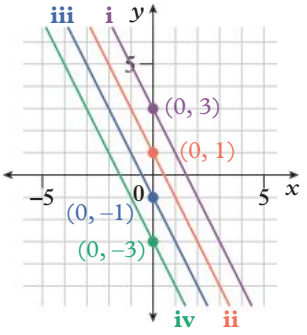


PROBLEM SOLVING AND REASONING

10 a Write the equations of these graphs.



b Write the equations of these graphs.



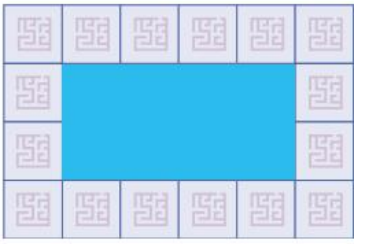
c What do the relationships in part **a** have in common?

d What do the relationships in part **b** have in common?

11 Consider your answers to question **10**. What can you say about the geometry of linear graphs that have the same value of m in the equation $y = mx + c$?

CHALLENGE

12 Tori used 16 tiles to make one row of tiles around her pool as shown. Tori wants to add more rows to the path and needs to know how many tiles she will need for each additional row.

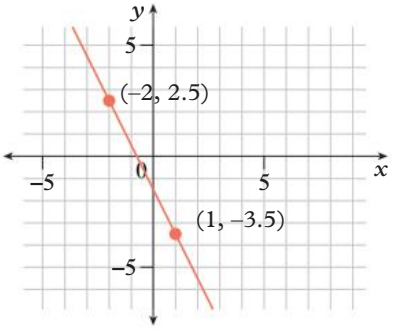


a Complete the following table.

Row	1	2	3	4
Tiles in the row	16			

b Use the table to write an equation that will give Tori the number of tiles she needs for each row.

13 Find the equation of this straight line.



Check your Student obook pro for these digital resources and more:

- **Interactive skillsheet** — Determining linear equations by inspection
- **Investigation** — Up, up and away!
- **Topic quiz** — 6F

6G Solving linear inequalities

Learning intentions

By the end of this topic you will be able to ...

- ✔ solve linear inequalities using inverse operations
- ✔ graph the solutions of one variable linear inequalities.

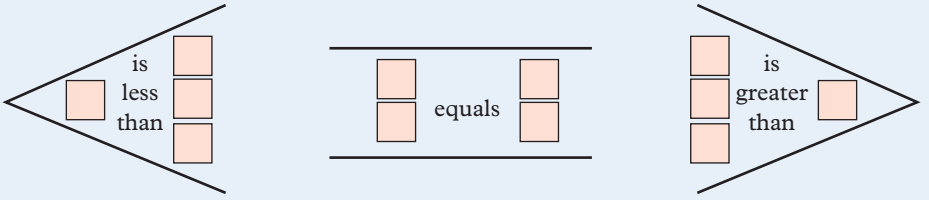
Inter-year links

Year 7	6G Solving equations using inverse operations
Year 9	3B Solving linear inequalities
Year 10	3D Solving linear inequalities

Linear inequalities

- An **inequality** is a mathematical statement that compares the values of two unequal expressions. Inequalities are written using inequality signs, like the ones shown in the table below.

Inequality sign	Meaning	Example	Meaning of example
$<$	'is less than'	$x < 2$	x is less than 2
$>$	'is greater than'	$x > 2$	x is greater than 2
$\leq$	'is less than or equal to'	$x \leq 2$	x is less than or equal to 2
$\geq$	'is greater than or equal to'	$x \geq 2$	x is greater than or equal to 2



- A linear inequality is an inequality containing pronumerals only raised to the power of 1.

Solving linear inequalities

- If the same operation is performed on both sides of an inequality sign, this results in an equivalent inequality of a different form. The possible values of the unknown remain the same.
- Inverse operations can be used to solve linear inequalities.

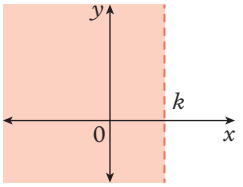
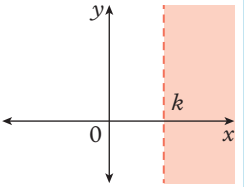
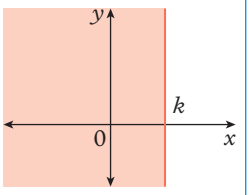
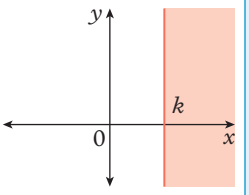
For example, $5 + 2x \leq 1 \quad (-5)$

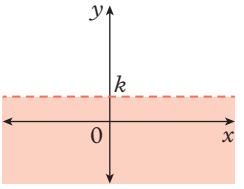
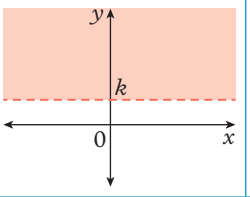
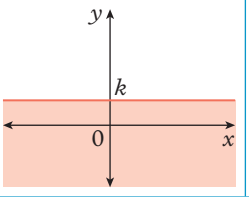
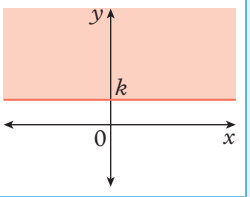
$2x \leq -4 \quad (\div 2)$

$x \leq -2$

Representing linear inequalities on the Cartesian plane

- Linear inequalities can be represented on the Cartesian plane using a boundary line and shading. On the Cartesian plane:
  - → a solid boundary line indicates that the points on the line are included, meaning the value is either 'less than or equal to' ($\leq$) or 'greater than or equal to' ($\geq$) the value represented by the boundary line on the Cartesian plane.
  - → a dotted boundary line indicates that the values on the line are not included, meaning the value is either 'less than' ($<$) or 'greater than' ($>$) the value represented by the boundary line on the Cartesian plane.

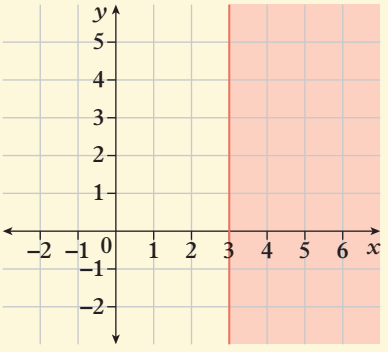
Inequality	$x < k$	$x > k$	$x \leq k$	$x \geq k$
Cartesian plane representation				
Meaning	x is less than k	x is greater than k	x is less than or equal to k	x is greater than or equal to k

Inequality	$y < k$	$y > k$	$y \leq k$	$y \geq k$
Cartesian plane representation				
Meaning	y is less than k	y is greater than k	y is less than or equal to k	y is greater than or equal to k

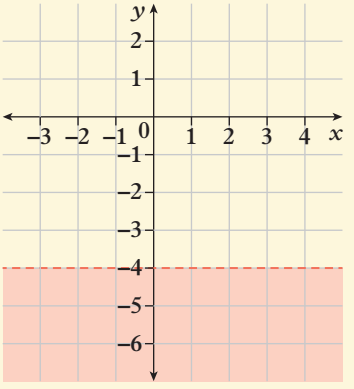
Example 6G.1 Finding linear inequalities from a graph

Write the inequality represented by each of these graphs.

a



b



THINK

a The solid boundary line passes through $x = 3$, indicating that x can be equal to 3. The shaded area is on the right of the line, indicating that x can be any value greater than 3. So x is greater than or equal to 3.

b The dotted boundary line passes through $y = -4$, indicating that y cannot be equal to –4. The shaded area is below the line, indicating that y can be any value less than –4. So y is less than –4.

WRITE

a $x \geq 3$

b $y < -4$

Example 6G.2 Solving one-variable linear inequalities

Solve each of these inequalities.

a $\frac{x}{4} - 3 \leq 0$ **b** $-2 > 5x + 3$ **c** $3(x + 6) \geq 12$

THINK

a 1 Apply the inverse operations to both sides in the correct order.

2 Check your solution. Substitute $x = 12$ into the LHS of the inequality. The result should equal the RHS. Then substitute any value for x that is less than 12 into the original inequality to check the inequality sign. The result should be less than or equal to the RHS.

b 1 Apply the inverse operations to both sides in the correct order.

2 Write the solution with x on the LHS. Reverse the inequality sign if needed.

3 Check your solution. Substitute $x = -1$ into the RHS of the inequality. The result should equal the LHS. Then substitute any value for x that is less than -1 into the original inequality to check the inequality sign. The result should be less than the LHS.

c 1 Apply the inverse operations to both sides in the correct order.

2 Write the solution with x on the LHS.

3 Check your solution. Substitute into the RHS of the inequality. The result should equal the LHS. Then substitute any value for x that is greater than -2 into the original inequality to check the inequality sign. The result should be less than the LHS.

WRITE

a $\frac{x}{4} - 3 \leq 0 \quad (+ 3)$

$\frac{x}{4} \leq 3 \quad (\times 4)$

$x \leq 12$

Check by substituting $x = 12$ into $\frac{x}{4} - 3 \leq 0$.

LHS $= \frac{12}{4} - 3$

$= 0$

$=$ RHS (For $x = 12$, LHS = RHS as required.)

Check by substituting a value less than 12 (for example, $x = 8$).

LHS $= \frac{8}{4} - 3$

$= -1$

$-1 < 0$ (For $x < 12$, LHS < RHS as required.)

b $-2 > 5x + 3 \quad (- 3)$

$-5 > 5x \quad (\div 5)$

$-1 > x$

$x < -1$

Check by substituting $x = -1$ into $-2 > 5x + 3$.

RHS $= 5(-1) + 3$

$= -2$

$=$ LHS (For $x = -1$, LHS = RHS as required.)

Check by substituting a value less than -1 (for example, $x = -2$).

RHS $= 5(-2) + 3$

$= -7$

$-7 < -2$ (For $x < -1$, LHS > RHS as required.)

c $3(x + 6) \geq 12 \quad (\div 3)$

$x + 6 \geq 4 \quad (- 6)$

$x \geq -2$

Check by substituting $x = -2$ into $3(x + 6) \geq 12$.

LHS $= 3(-2 + 6)$

$= 12$

$=$ RHS (For $x = -2$, LHS = RHS as required.)

Check by substituting a value greater than -2 (for example, $x = 0$).

LHS $= 3(0 + 6)$

$= 18$

$18 \geq 12$ (For $x = 0$, LHS ≥ RHS as required.)

Example 6G.3 Solving and graphing one-variable linear inequalities

Solve and graph the following linear inequalities.

a $\frac{x}{3} + 4 \leq -2$ **b** $1 < 2x - 4$

THINK

a **1** Apply inverse operations to both sides in the correct order.

2 Draw a Cartesian plane, with the x-axis long enough to include $x = -18$. As x can be equal to -18, draw a solid vertical line at $x = -18$.

3 Shade the area to the left of $x = -18$ to represent $x < -18$.

b **1** Apply inverse operations to both sides in the correct order.

2 Write the solution with x on the LHS. Reverse the inequality sign if needed.

3 Draw a Cartesian plane, with the x-axis long enough to include $x = \frac{5}{2}$. As x cannot be equal to $\frac{5}{2}$, draw a dotted vertical line at $x = \frac{5}{2}$.

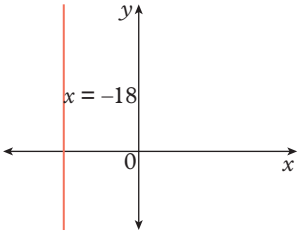
4 Shade the area to the right of $x = \frac{5}{2}$ to represent $x > \frac{5}{2}$.

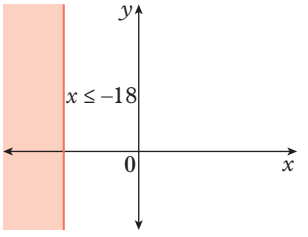
WRITE

a $\frac{x}{3} + 4 \leq -2$

$\frac{x}{3} \leq -6$ (-4)

$x \leq -18$ $(\times 3)$



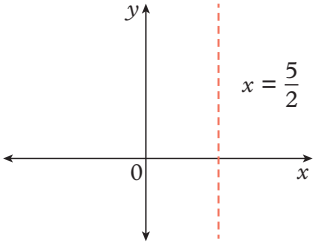


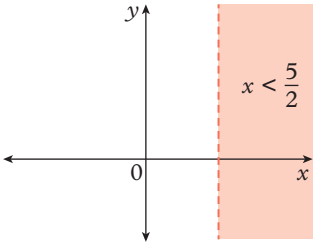
b $1 < 2x - 4$

$5 < 2x$ $(+4)$

$\frac{5}{2} < x$ $(\div 2)$

$x > \frac{5}{2}$





Helpful hints

✔ If you need to swap the LHS and RHS of an inequality, remember to reverse the inequality sign so that it is still pointing towards the lesser expression. For example, if $5 \leq x$, then $x \geq 5$.

ANS p530

Exercise 6G Solving linear inequalities

1, 2, 3(1st column), 4, 5(1st, 2nd columns), 6, 8, 9 | 2–3(2nd column), 5–7, 12 | 2–3(2nd column), 6, 11, 13, 14

1 Write each of the following statements as an inequality.

a Two is less than three.

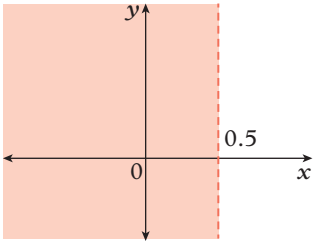
b Three is greater than two.

c Three is greater than or equal to two.

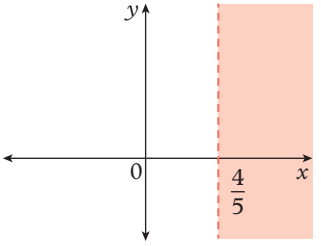
d Negative three is less than or equal to negative two.

6G.1 **2** Write the inequality represented on each of these graphs.

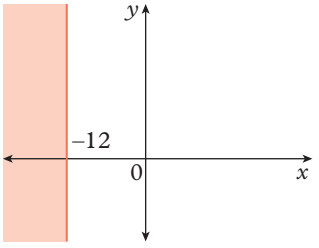
a



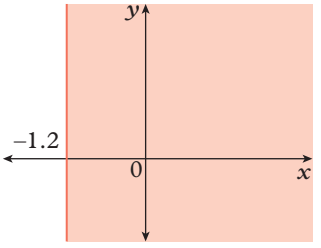
b



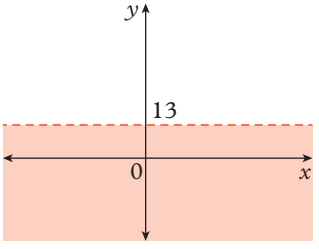
c



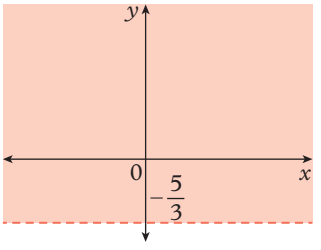
d



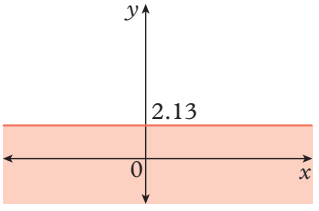
e



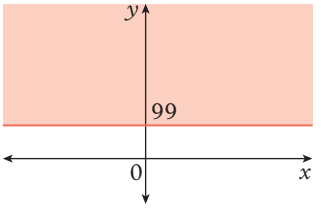
f



g



h



3 For each of these inequalities, show the values of x on the Cartesian plane.

a $x > 4$ **b** $x \leq 3$ **c** $x \geq -2$ **d** $x < 0$

e $y > 3$ **f** $y \leq -1$ **g** $y \geq -2$ **h** $y < 2$

4 Apply the operations below to each side of the following inequalities.

a $-4 < 9$ **b** $x \geq 6$

i add five **ii** subtract nine **iii** multiply by three **iv** divide by three

6G.2 **5** Solve each of these inequalities.

a $x + 5 > 9$ **b** $\frac{x}{10} \leq 3$ **c** $-20 > 4x$ **d** $\frac{x}{5} + 8 \leq 11$

e $2x - 9 \geq -5$ **f** $5(x + 3) \leq 25$ **g** $\frac{x - 8}{12} > -3$ **h** $5 < 10x + 14$

UNDERSTANDING AND FLUENCY

6G.3 **6** Solve each of these inequalities and show the solution on the Cartesian plane.

a $3x + 2 \geq -13$ **b** $\frac{2+x}{5} > 3$ **c** $\frac{3x}{2} - 5 \leq 4$ **d** $\frac{x-4}{2} < 5$

7 **a** How many solutions does a linear equation have?

b How many solutions can a linear inequality have? Explain.

PROBLEM SOLVING AND REASONING

8 Write an inequality statement to represent each of the following situations. Use a pronumeral for the unknown quantity each time.



a A house with a reserve price of $650 000 is sold at auction. What price could the house have sold for?

b To go on a theme park ride, a person must be over 97 cm tall. What could be the height of a person on this ride?

c The speed limit along a road is 60 km/h. At what speeds could a person legally travel along this road?

9 Todd is deciding how many packs of chewing gum he should buy. Each pack costs $3 and he has $20 in his wallet.

a Write an inequality to represent this situation and then solve it.

b List the number of packs of gum Todd could possibly buy.

10 Emily and Ethan are selling watermelons at a market. It costs $60 to set up the stall, and they agree to share the money they make evenly. Each watermelon is priced at $23.

a Write an expression for the amount of money they each will take home if they sell x watermelons.

b Emily and Ethan aim to sell enough watermelons to take home at least $100 each. Use your expression from part **a** to write an inequality for this situation.

c How many watermelons do they need to sell to meet their goal? Note that they only sell whole watermelons.

11 To make calls from Australia to friends and family travelling overseas, Jada has a pre-paid phone card with a credit of $50. For each call, she is charged a connection fee of 49c and a per minute rate depending on the country she is calling. The per minute charges are shown in the table on the right.

Country	Per minute rate
Egypt	$1.47
India	$1.35
Japan	$0.80
Morocco	$1.95

a For how many minutes could Jada talk to her friend travelling in Morocco and not go over her $50 limit?

b Another of Jada's friends is in Egypt. For how many minutes could Jada talk to her if she wants to save at least $25 of her $50 credit for another call?

CHALLENGE

12 Solve each of these inequalities. Use substitution to check your solutions.

a $4x - 3 > 2x + 5$ **b** $7x + 1 \leq 3x - 7$ **c** $5x - 9 \geq 9 - x$ **d** $2x + 4 < 5x - 8$

e $3 - x > 4x - 2$ **f** $x + 11 \geq 7 - 3x$ **g** $8 - 3x \leq 18 - 5x$ **h** $1 - 6x > -3x - 5$

13 Solve each of these inequalities by first expanding to remove brackets.

a $2(x - 3) > x + 5$ **b** $4x - 7 \leq 3(x + 2)$ **c** $5(x + 1) \geq -3x - 11$

d $3(2x - 5) < 7x + 4$ **e** $9(x + 5) < 10(x + 6)$ **f** $3(3x - 1) \leq -2(x - 4)$

g $4(x + 3) \geq 2(5x - 3)$ **h** $2(2 - x) > 3(1 - 2x)$ **i** $5(3 - 2x) \leq -4(2x - 7)$

14 Solve each of these inequalities.

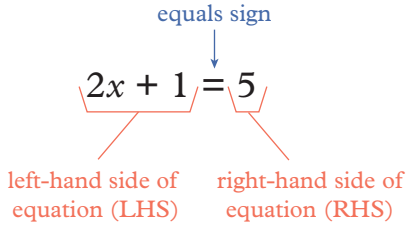
a $-3 \leq 2x - 5 \leq 7$ **b** $15 < \frac{5(1-2x)}{3} < 25$

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Representing simple linear inequalities on the Cartesian plane
- Interactive skillsheet: Solving simple linear inequalities
- Investigation: Sign and direction of inequalities
- Topic quiz: 6G

Chapter summary

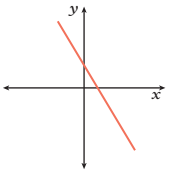
Equations



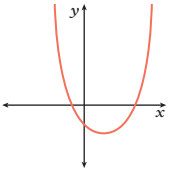
- A solution to an equation makes the equation a true statement.
- To solve by inspection, identify the correct value of the pronumeral.

Linear and non-linear relationships

Linear



Non-linear

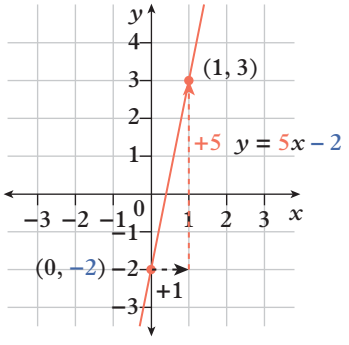


Finding linear equations

All linear equations can be written in the form

$$y = mx + c$$

- m is equal to the change in value of y as x increases by 1.
- c is equal to the value of y when $x = 0$.



Solving an equation using inverse operations

1. Identify the operation(s) acting on the pronumeral.
2. Identify the inverse operations.
3. Apply the inverse operations to both sides of the equation.

$x + 1 = 5$
$x + 1 - 1 = 5 - 1$ ← inverse operations
$x = 4$ ← solution

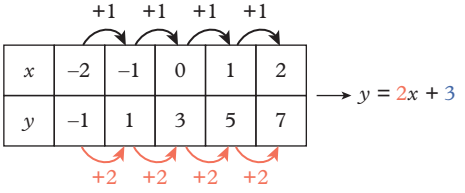
Solving equations with the unknown on both sides

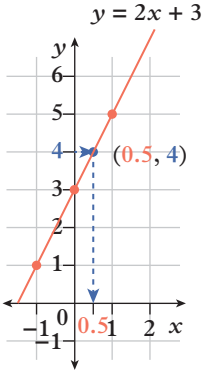
1. Use inverse operations to remove the pronumeral term from one side.
2. Solve the equation using inverse operations.

$4x - 2 = 3x + 1$
$4x - 3x - 2 = 3x - 3x + 1$
$x - 2 = 1$
$x - 2 + 2 = 1 + 2$
$x = 3$

Solving linear equations using graphs

1. Make the side of the equation containing the unknown equal y.
2. Plot the graph of the equation from step 1.
3. Locate the value from the other side of the equation on the y-axis, and read off the corresponding value of x.





$2x + 3 = 4$
$x = 0.5$

Solving linear inequalities

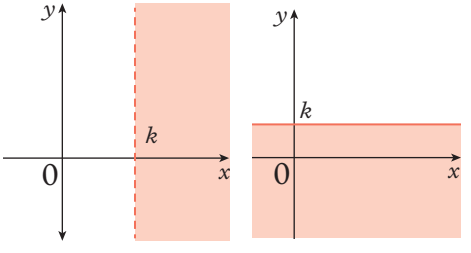
1. Apply the inverse operations to both sides in the correct order.
2. Write the solutions with x on the LHS. Reverse the inequality sign if needed.
3. Check the solution by substitution.

$-2 > 5x + 3$ (-3)
$-5 > 5x$ $(\div 5)$
$-1 > x$
$x < -1$

Check by substituting a value less than –1

Representing linear inequalities on the Cartesian plane

- A solid boundary line indicates that the points on the line are included (≤ or ≥)
- A dotted boundary line indicates that the points on the line are not included (< or >)



$x > k$ $y \leq k$

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Multiple choice

6A **1** The solution to $x + 5 = 12$ is:

A $x = 17$ **B** $x = 12$ **C** $x = 7$ **D** $x = 5$ **E** $x = 1$

6A **2** $y = -4$ is the solution to which equation?

A $y + 4 = 0$ **B** $2y - 3 = 4$ **C** $3y - 12 = 0$ **D** $4 - y = 0$ **E** $8 - 2y = 0$

6A **3** A dance class has double the number of students that it had last week. Four of the students have to leave early, leaving 28 students in the class.

Which of the following equations can be used to determine how many students were in the class last week?

A $2s + 4 = 28$ **B** $2s - 4 = 28$ **C** $\frac{s}{2} + 4 = 28$ **D** $\frac{s}{2} - 4 = 28$ **E** $\frac{s}{4} = 28$

6B **4** Which equivalent equation is formed when 5 is subtracted from both sides of $2x - 6 = 14$?

A $2x - 1 = 14$ **B** $2x - 6 = 9$ **C** $2x - 11 = 9$ **D** $2x - 1 = 9$ **E** $x = 10$

6B **5** Which of the following operation(s) should be applied to both sides of the equation to solve $\frac{3x}{4} = 5$?

A Subtract 5 **B** Divide by $\frac{4}{3}$

C Divide by 4, then multiply by 3 **D** Multiply by $\frac{3}{4}$

E Multiply by 4, then divide by 3

6C **6** What is the solution to $3x + 7 = 2x + 9$?

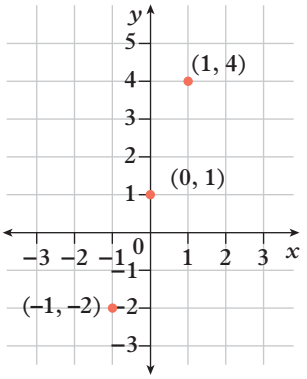
A $x = 16$ **B** $x = -16$ **C** $x = \frac{16}{5}$ **D** $x = 2$ **E** $x = -2$

6C **7** The value of x in the equation $8x + 15 = 3 - x$ is:

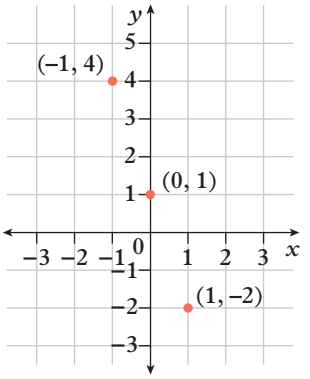
A $x = -3$ **B** $x = 2$ **C** $x = -2$ **D** $x = \frac{4}{3}$ **E** $x = -\frac{4}{3}$

6D **8** Which of the following plot points satisfy the equation of $y = 1 - 3x$?

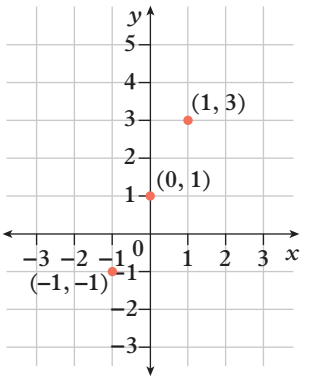
A



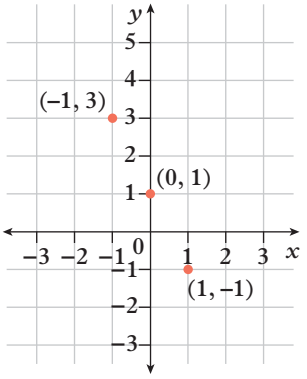
B



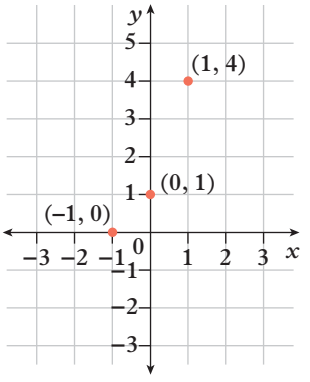
C



D

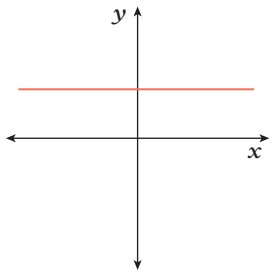


E

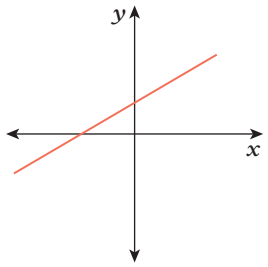


6D **9** Which of the following graphs represents a non-linear relationship?

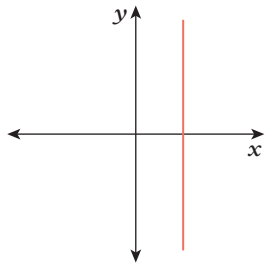
A



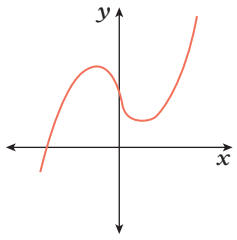
B



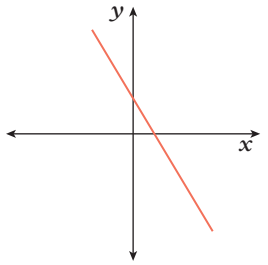
C



D



E



6E **10** Consider the set of related equations below.

$\frac{x}{2} + 5 = 7$ $\frac{x}{2} + 5 = 0$ $\frac{x}{2} + 5 = -1$

The equation of the graph that can be used to solve this set of equations is:

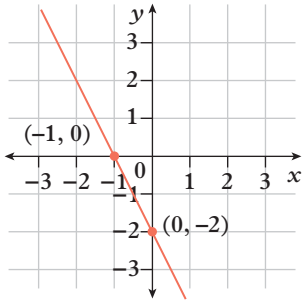
A $\frac{x}{2} + 5 = 7$ **B** $\frac{x}{2} + 5 = 0$ **C** $\frac{x}{2} + 5 = -1$ **D** $y = \frac{x}{2}$ **E** $y = \frac{x}{2} + 5$

6F **11** The equation for the table of values below is:

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

A $y = x + 1$ **B** $y = 2x - 1$ **C** $y = 2x + 1$ **D** $y = -2x - 1$ **E** $y = -2x + 1$

6F **12** The equation of this graph is:



A $y = -2x - 2$
B $y = 2x - 2$
C $y = 2x + 2$
D $y = -2x + 2$
E $y = -x - 2$

6G **13** What is the solution to $2x - 1 < 5$?

A $x > 3$ **B** $x < 3$ **C** $x > 2$ **D** $x < 2$ **E** $x < 6$

6G **14** Which of the following values of x satisfies the inequality $60 < 2x + 12$?

A 36 **B** 24 **C** 12 **D** 6 **E** 0

Short answer

6A **1** Use substitution to determine whether $x = -3$ is the solution to the following equations.

a $x + 11 = 8$ **b** $5x = -15$ **c** $4x + 12 = 0$ **d** $2x - 3 = 9$

6A **2** Solve the following equations by inspection.

a $a + 7 = 16$ **b** $b - 6 = 8$ **c** $4c = 12$ **d** $\frac{d - 7}{4} = 2$

6B **3** Use inverse operations to solve the equations.

a $5x - 2 = 13$ **b** $-\frac{1}{3}x + 8 = 11$ **c** $\frac{4c}{7} = 8$

d $4(3d + 5) = 44$ **e** $\frac{4x}{5} + 12 = 13$ **f** $\frac{6 - 8x}{7} = 10$

6B **4** A bookstore owner is shipping small books and large books in a box. Small books weigh half as much as large books. There are 12 of each type of book in the box.

a If a small book weighs s kg, write an equation to represent this situation given that a box weighs 36 kg.

b Solve the equation and find the weight of a large and small book.

6C **5** Solve:

a $5x + 6 = 3x - 4$ **b** $8x + 3 = 6x + 7$

c $-4x - 15 = 6x + 15$ **d** $x + 3 = -2 - x$

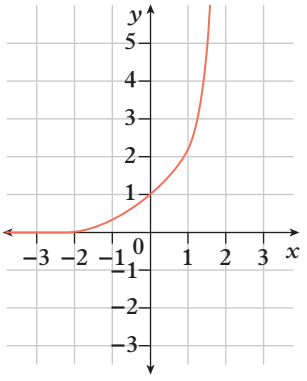
6C **6** Suzi's age in 11 years will be equal to five times her age five years ago. Write an equation and solve it to find how old Suzi is now.

6D **7** Plot a graph of each of the following relationships by first completing a table of values for integer values of x from -3 to 3.

a $y = x - 4$ **b** $y = 4x + 1$ **c** $y = 3 - x$ **d** $y = 6 - 3x$

6D **8** Determine whether the following relationships are linear or non-linear.

a



b

x	−2	−1	0	1	2
y	−15	−11	−7	−3	1

6E **9** Write an equation for each of the tables below.

a

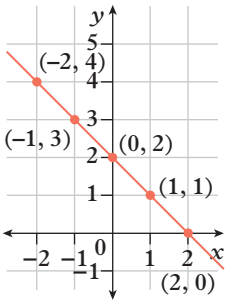
x	−2	−1	0	1	2
y	−13	−9	−5	−1	3

b

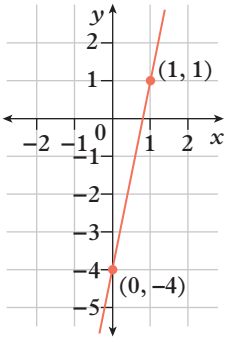
x	−2	−1	0	1	2
y	2	1	0	−1	−2

6E **10** Determine the equations of the graphs below.

a



b

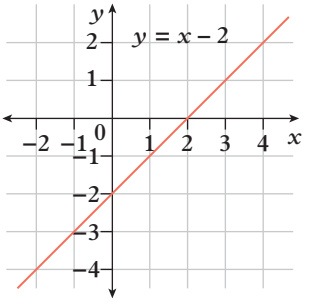


6F **11** Refer to the graph of $y = x - 2$ on the right to find the value of x that corresponds to each of the following values of y.

a $y = 2$ **b** $y = -3$ **c** $y = 1$ **d** $y = 0$

6F **12** Refer to the graph of $y = x - 2$ on the right to solve each equation.

a $x - 2 = -4$ **b** $x - 2 = -1$ **c** $x - 2 = -2$



6G **13** Solve the following inequalities and represent the result on the Cartesian plane.

a $4x - 3 > 5$ **b** $7x + 1 \leq 7$ **c** $9 \geq 5x + 10$

Analysis

1 A suburban party venue costs $150 to hire, plus $20 per person for games and food.



a If p people attend the party, write an equation for the total cost, C, of the venue hire, games and food.

b Kacie's mother gives her $450 for venue hire, games and food. Substitute this value into the equation to determine how many people Kacie can have at her party.

Kacie finds another party venue in the city that offers a similar service. Their costs consist of a hire fee of $250 plus $15 per person for games and food.

c Write a second equation for the total cost, C, of venue hire, games and food for a party attended by p people at the second venue.

d Complete the tables below for each venue.

Suburban venue

P	0	10	20	30	40	50
C						

City venue

P	0	10	20	30	40	50
C						

e Use the tables of values to plot a graph of each relationship on the same set of axes. Label each graph.

f Do these graphs show a linear relationship? Why or why not?

g Use the graphs to find:

i the cost of inviting 25 people to a party at each venue

ii how many people Kacie can invite to a party at each venue with a budget of $450.

Although the city venue has a higher hire fee, if Kacie invites more than a certain number of people, the city venue will be cheaper than the suburban venue.

h Use your graph to determine which venue will be cheaper if Kacie invites:

i 10 people **ii** 50 people **iii** 20 people

2 Joss is the manager of a bookstore. He uses Crate Couriers to deliver his orders. Crate Couriers charge Joss $5 per book plus $3 flat fee per delivery.

a Write an equation to find the total cost, C, of delivering x books.

b How much does it cost Joss to have 10 books delivered to a customer?

c How many books can Joss have delivered for $38?

d Complete the table below.

Number of books	0	1	2	3	4	5
Cost of delivery						

e Plot a graph of cost of delivery vs number of books for Crate Couriers.

A new courier approaches Joss. Crown Couriers don't have a flat fee but they charge $6 per book delivery. At first Joss thinks Crown will be too expensive but he decides he should investigate further.

f Fill in the table for Crown Couriers below.

Number of books	0	1	2	3	4	5
Cost of delivery						

g On the same graph as part **e**, plot the graph of cost of delivery vs number of books for Crown Couriers.

h After looking at the two graphs Joss can see that the situation is not so simple. Explain when Joss should use Crate Couriers and when he should use Crown Couriers.

Algorithmic Thinking

Algorithmic thinking: Number and algebra

An algorithm is a set of precise instructions that when followed will achieve a goal or solve a problem. Algorithms can be described using diagrams called flowcharts, which contain actions written inside shapes, such as boxes and diamonds, and use arrows to indicate which actions need to be completed next. Once the actions in an algorithm have been identified, it may be possible to encode the algorithm in a coding language so it can be automated by a computer or machine.

The term iteration refers to repetitive actions in algorithms. Flowcharts use arrows to show iteration by looping back to previous actions. Some algorithms contain actions that are only done if certain conditions are true; these are called conditional actions. Conditional actions are usually shown in diamonds, with arrows directed to the appropriate actions depending on the result of the condition.

Example 1

The decimal number system, also known as base-10, uses ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Decimal numbers are defined using positional notation that corresponds to powers of 10.
This is 5127 in base-10:

$5127 = 5 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 7 \times 10^0$

Computers are built using electrical circuits that can be in two states: either OFF represented by a 0, or ON represented by a 1.

The binary number system, also known as base-2, uses two digits: 0 and 1. Binary numbers are defined using positional notation that corresponds to powers of 2. Binary numbers are identified with a subscript of 2 after the number, as shown below:

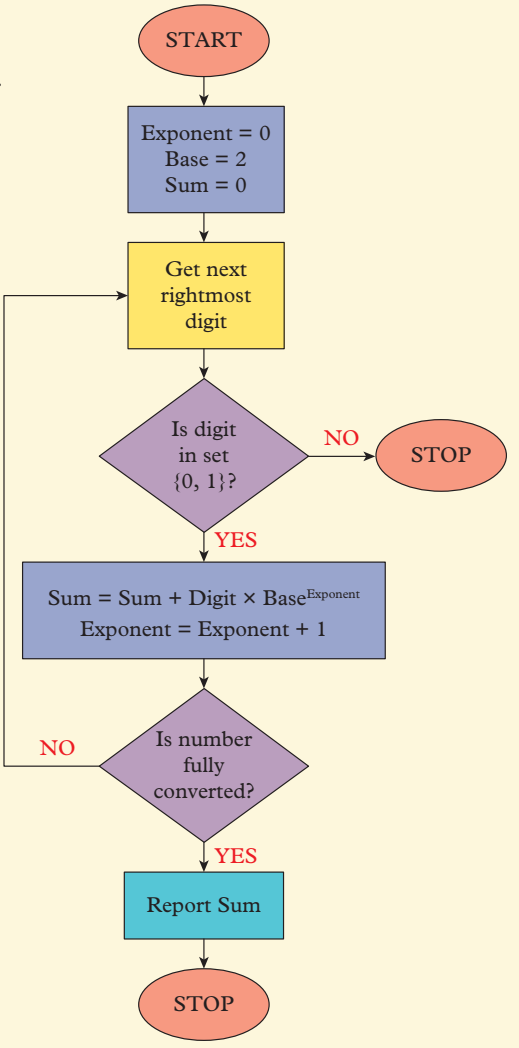
$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$1101_2 = 13$

The flowchart on the right shows how to convert a binary number into a decimal number. The flowchart uses the variables Exponent, Base and Sum to keep track of the solution in the Sum variable, which changes value as the decimal number is calculated.

The following steps show the actions done using the flowchart to convert the binary number 11 into a decimal number.

1 Start
2 Exponent = 0, Base = 2, Sum = 0
3 Get the next rightmost digit, which is 1
4 Check if digit is in {0, 1}, then follow YES path in flowchart
5 Update the variable Sum to Sum + 1×2^0, giving Sum = 1
6 Increase the Exponent variable by 1 to Exponent = 1
7 Check if the number is fully converted, then follow NO path in flowchart
8 Get the next rightmost digit, which is 1
9 Check if digit is in {0, 1}, then follow YES path in flowchart
10 Update the variable Sum to Sum + 1×2^1, giving Sum = 3



11 Increase the Exponent variable by 1 to Exponent = 2
12 Check if number is fully converted, then follow YES path in flowchart
13 Report the contents of the Sum variable, which is 3, a decimal number
14 Stop

Task 1

1 Use the flowchart in Example 1 to convert the binary number 1101 into a decimal number.
2 What will happen if the flowchart tries to convert the number 170?
3 What are the conditional actions in the flowchart?
4 Where is the iteration in the flowchart?
5 Which variables remain constant in the flowchart and which variables change?
6 What makes the flowchart stop and report the solution?
7 **CHALLENGE:** Code the flowchart into a coding language and try it out.

Example 2



The quinary number system, also known as base-5, uses five digits: 0, 1, 2, 3 and 4. The quinary number system was used by the indigenous Gumatj people of Australia, which are part of the Yolgnu language group in the Northern Territory Australia. Quinary numbers are identified with a subscript 5 after the number, as shown below:

$431_5 = 4 \times 5^2 + 3 \times 5^1 + 1 \times 5^0$

$431_5 = 116$

The following table shows a step-by-step conversion of the decimal number 127 into the quinary number 1002 by using repeated division by 5, and then using the quotient, remainder and exponent to build up the quinary number.

Decimal 127	Division by 5	Quotient	Remainder	Exponent	Quinary 1002
127 (Starting number)	$127 \div 5$	25	2	0	2×5^0
25 (Copy prior quotient)	$25 \div 5$	5	0	1	0×5^1
5 (Copy prior quotient)	$5 \div 5$	1	0	2	0×5^2
1 (Copy prior quotient)	$1 \div 5$	0	1	3	1×5^3
0 (Stop)					

Task 2

1 Convert the decimal number 82 into a quinary number system using a table as shown in Example 2.
2 What kind of information would need to be stored in variables for an algorithm to convert decimal numbers to quinary numbers?
3 Using ideas from Examples 1 and 2, create a flowchart to convert a decimal number into a quinary number.
4 Test your flowchart with the decimal numbers 127, 116 and 82.
5 **CHALLENGE:** Code your flowchart for converting decimal numbers to quinary numbers into a coding language and try it out.

7

Triangles and quadrilaterals





Index

7A Angles
7B Triangles
7C Pythagoras' theorem
7D Using Pythagoras' theorem
7E Transformations
7F Congruence
7G Congruent triangles
7H Dilations and similar figures
7I Similar triangles
7J Quadrilaterals

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Solving equations by inspection
- ✔ Like terms
- ✔ Solving equations using inverse operations
- ✔ The Cartesian plane

Curriculum links

- Use Pythagoras' theorem to solve problems involving the side lengths of right-angled triangles (AC9M8M06)
- Identify the conditions for congruence and similarity of triangles and explain the conditions for other sets of common shapes to be congruent or similar, including those formed by transformations (AC9M8SP01)
- Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related problems explaining reasoning (AC9M8SP02)

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Materials

- ✔ Compass
- ✔ Ruler
- ✔ Protractor

7A Angles

Learning intentions

By the end of this topic you will be able to ...

- ✔ calculate and use complementary and supplementary angles
- ✔ calculate and use angles at a point
- ✔ calculate and use related angles.

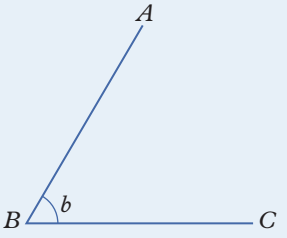
Inter-year links

Support — Understanding angles

Year 7 — 7B Angles at a point

Types of angles

- Angles can be named using either a pronumeral, for example b, or an angle symbol followed by capital letters which represent the vertices of the angle, for example $\angle ABC$.
- Angles are classified according to their size:
  - → **acute:** between 0° and 90°
  - → **right:** 90°
  - → **obtuse:** between 90° and 180°
  - → **straight:** 180°
  - → **reflex:** between 180° and 360°
  - → **revolution:** 360°

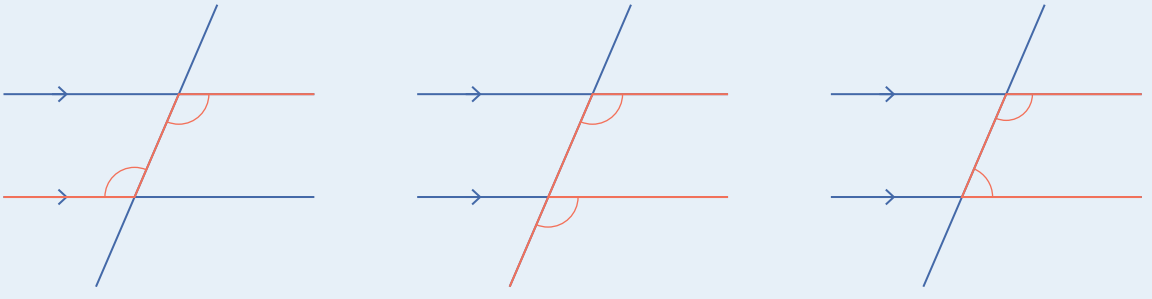


Angle properties

- Angles at a point that form a straight line add to 180°.
- Angles at a point that form a revolution add to 360°.
- Two angles that add to 90° are called **complementary angles**.
- Two angles that add to 180° are called **supplementary angles**.
- Angles at a point form **vertically opposite angles** that are equal in size.

Angles and parallel lines

- When two or more lines have arrowheads on them, this means that they are parallel. Multiple arrowheads may be used if there is more than one pair of parallel lines in a diagram.
- A **transversal** is a straight line that cuts across two or more other lines.
- When two or more **parallel lines** are intersected by a transversal:
  - → **Alternate angles** are equal.
  - → **Corresponding angles** are equal.
  - → **Co-interior angles** are supplementary.

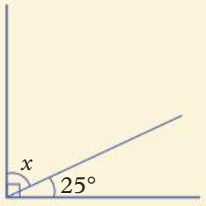


- Related angles can be used to determine if two lines are parallel. If any of the relationships listed above are true, then the lines crossed by a transversal are parallel.

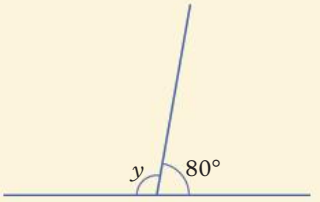
Example 7A.1 Using complementary and supplementary angles

Find the size of each labelled unknown angle.

a



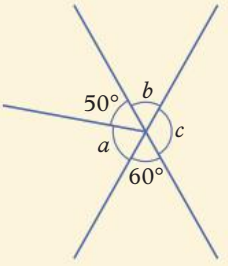
b



THINK	WRITE
a 1 Identify the relationship between the unknown and known angles. x and 25° are complementary angles.	**a** $x = 90° - 25°$
2 Calculate the size of the unknown angle. Complementary angles add to 90°, so $x = 90° - 25°$.	$= 65°$
b 1 Identify the relationship between the unknown and known angles. y and 80° are supplementary angles.	**b** $y = 180° - 80°$
2 Calculate the size of the unknown angle. Supplementary angles add to 180°, so $y = 180° - 80°$.	$= 100°$

Example 7A.2 Using angles at a point

Find the size of each labelled unknown angle.



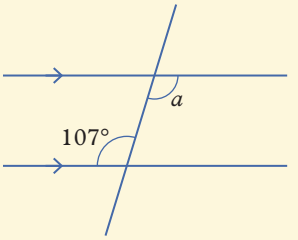
THINK	WRITE
1 Identify any relationships between unknown and known angles. b is vertically opposite to 60° and vertically opposite angles are equal in size.	$b = 60°$
2 c and 60° are supplementary angles, so $c = 180° - 60°$.	$c = 180° - 60°$ $= 120°$
3 50°, a and 60° form a straight angle, so $a = 180° - 50° - 60°$.	$a = 180° - (50° + 60°)$ $= 70°$

Example 7A.3 Finding unknown angles on parallel lines

For the following diagram:

a Identify the relationship between the unknown and known angles.

b Find the size of the angle a.



THINK

a a and 107° are the internal angles of a reverse 'Z' shape, so they are alternate angles.

b Alternate angles formed by parallel lines are equal in size.

WRITE

a a and 107° are alternate angles.

b $a = 107°$

Helpful hints

- ✔ Unless stated, you will not need to measure any of the angles in a diagram. All the information required to answer questions is already provided.
- ✔ Don't assume that two given lines are parallel if they look like they are parallel in a diagram! Pay attention to whether lines have parallel marks or if you are given information in a question.

ANS p533

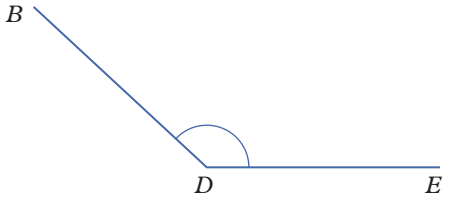
Exercise 7A Angles

1–9, 11–13	1, 2(a, d, e), 3–5, 6(e–i), 7–9, 12–15, 17(a)	1(d–f), 3–5, 6(f–i), 7(d–f), 8, 10–12, 16–18

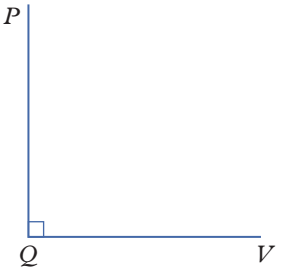
UNDERSTANDING AND FLUENCY

1 Name each angle and state what type of angle it is.

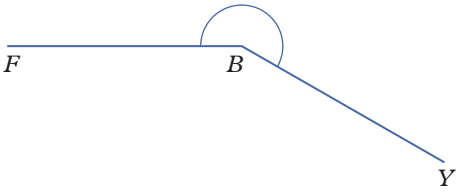
a



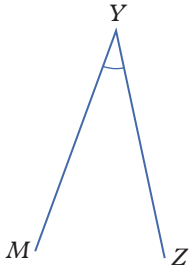
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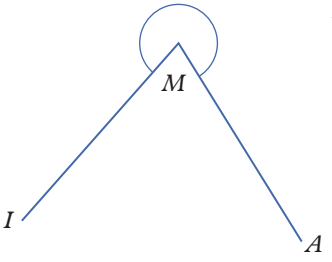
c



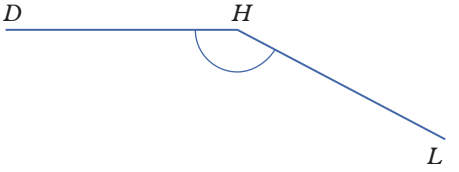
d



e

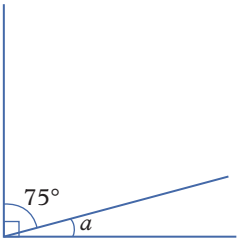


f



7A.1 **2** Find the size of each labelled unknown angle.

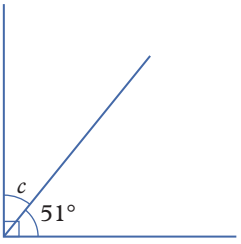
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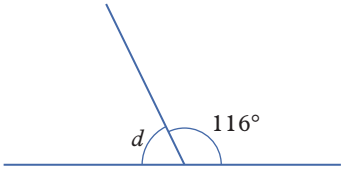
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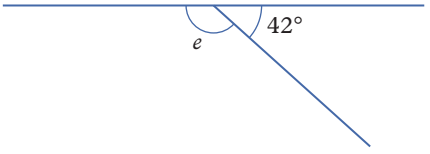
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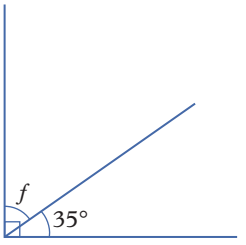
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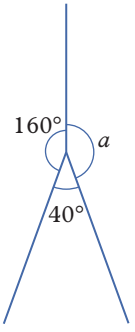


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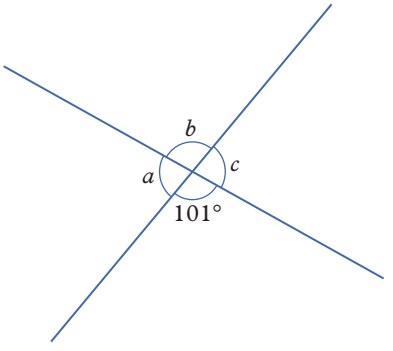


7A.2 **3** Find the size of each labelled unknown angle.

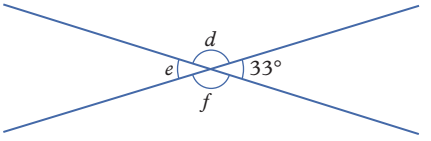
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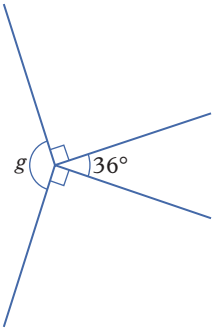
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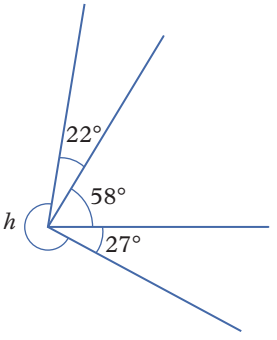
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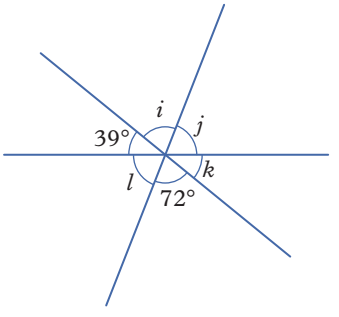
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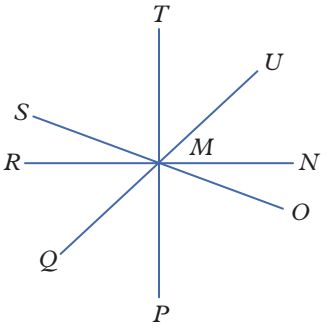
4 Identify the relationship (complementary, supplementary or vertically opposite) between each pair of angles in this diagram.

a $\angle UMT$ and $\angle NMU$

b $\angle OMT$ and $\angle PMO$

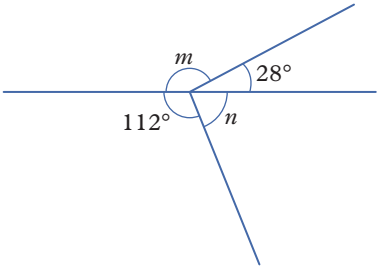
c $\angle SMR$ and $\angle NMO$

d $\angle TMQ$ and $\angle PMQ$

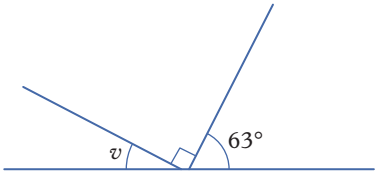


5 Find the value of each pronumeral without using a protractor.

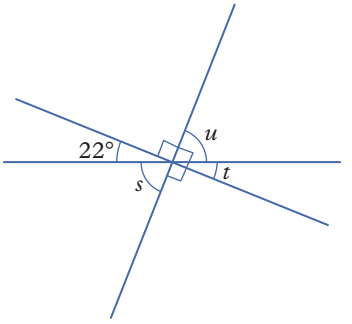
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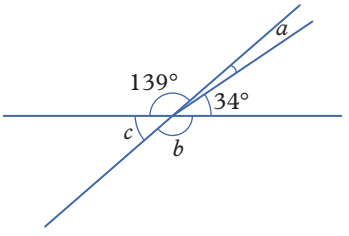
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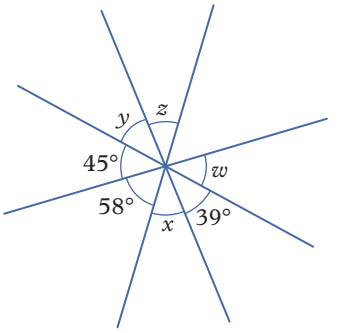
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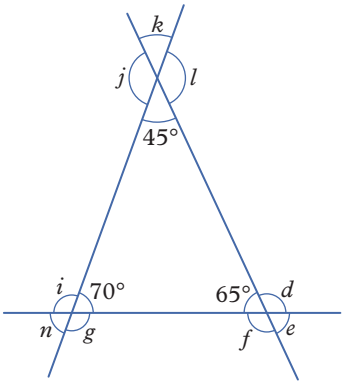
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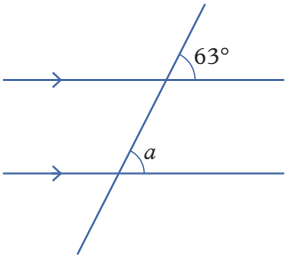


7A.3 **6** For each of the following diagrams:

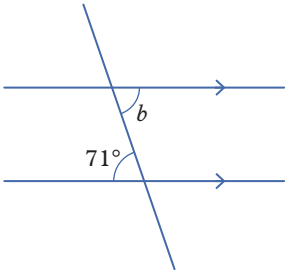
i Identify the relationship between the unknown and known angles.

ii Find the size of the labelled unknown angle.

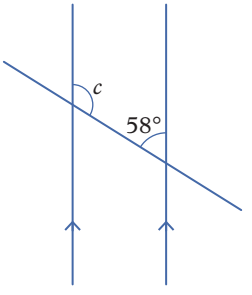
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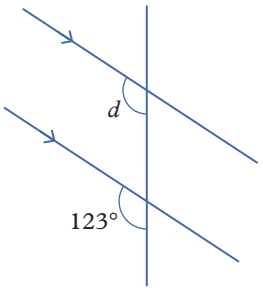
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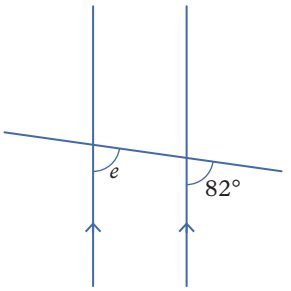
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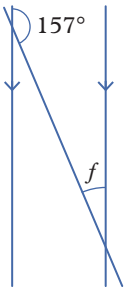
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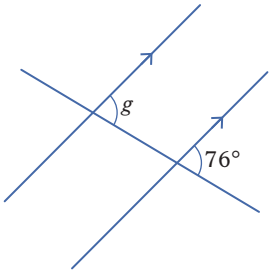
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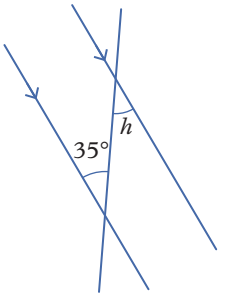
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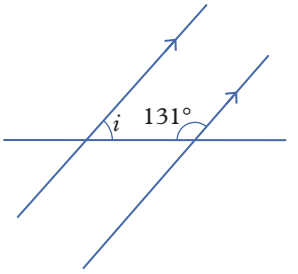
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h

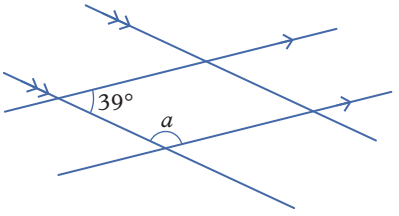


i

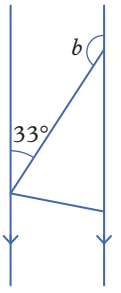


7 Find the size of each labelled unknown angle. In each case, state whether the angle is alternate, corresponding or co-interior to the given angles.

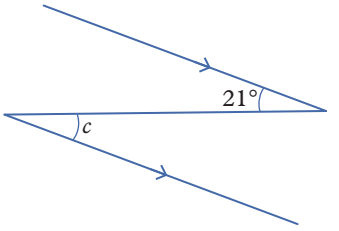
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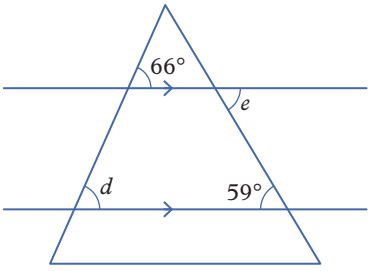
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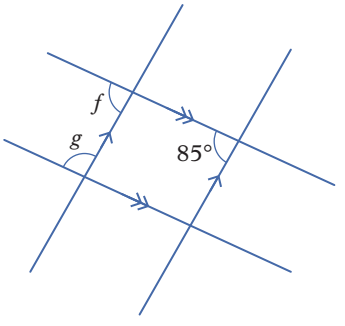
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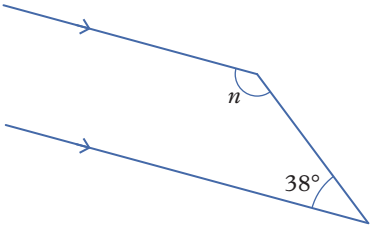
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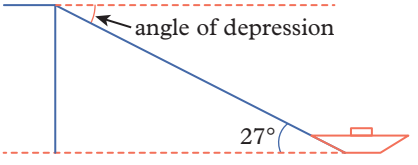
e



f



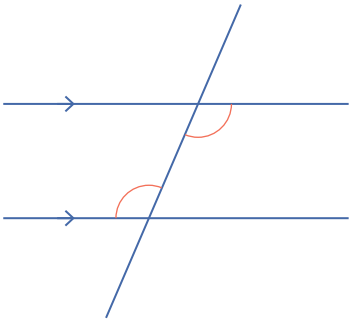
8 The angle of elevation (from a boat to a cliff top) is 27°. What is the angle of depression from the cliff top to the boat? Justify your answer.



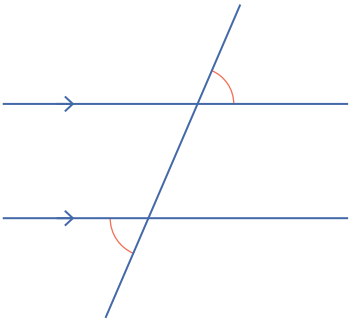
9 The Australian standard for staircases prefers that the angle that the stairs make with the horizontal be between 30° and 38°, and states that it must be between 20° and 45°. The handrail is parallel to the stairs and all vertical support posts are parallel to each other. If one of the angles between the handrail and the support posts is 67°, determine if this staircase meets the Australian standard and if it is in the preferred range.



10 Alternate angles usually refer to the angles on opposite sides of the transversal between two lines; however, the angles on the outside of the lines on opposite sides of the transversal are also alternate angles. To distinguish between the two, we call the ones between the lines alternate interior angles and the ones outside of the lines alternate exterior angles. Explain why alternate exterior angles will be equal for parallel lines.
Note: You can assume the term 'alternate angles' always refers to alternate interior angles, unless specified otherwise.



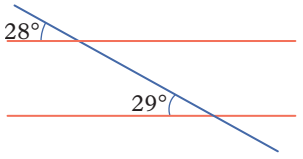
alternate interior angles



alternate exterior angles

11 Use your knowledge of co-interior, alternate and corresponding angles to check whether the pairs of red lines are parallel or not. Give a reason why each pair is or is not parallel.

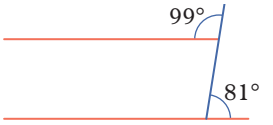
a



b



c

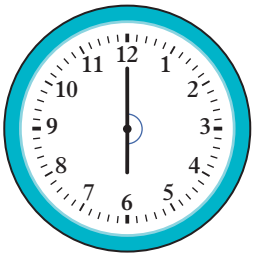


12 An understanding of angles around a point can help you to draw analogue clocks accurately.

a How many degrees are in a circle?

b Find the size of the angle indicated on each clock.

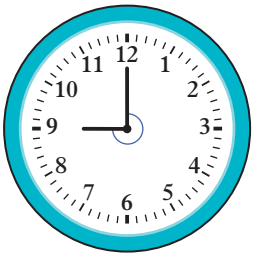
i



ii



iii

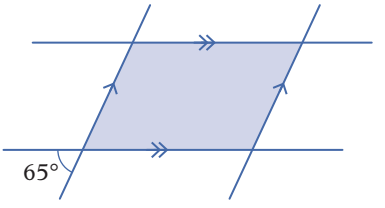


c What size angle would be between the hands of a clock displaying these times? Start from 12 o'clock and move clockwise to the other hand.

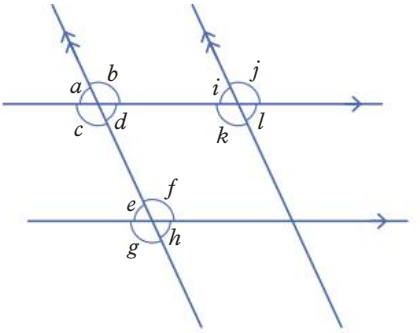
i 4:00 pm **ii** 11:00 am **iii** 8:00 pm **iv** 1:00 am

13 a What shape is formed in the diagram on the right?

b Use your knowledge of angles and parallel lines to state the size of all the internal angles of the shape.



14 In the diagram to the right, identify all pairs of:

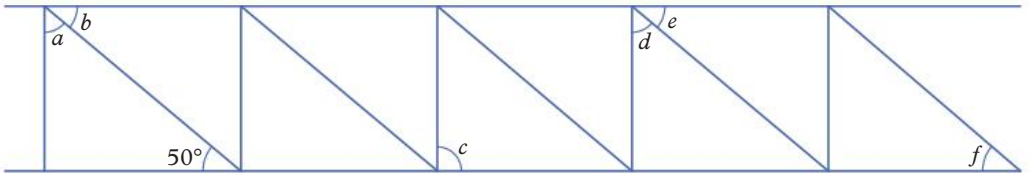


a alternate angles

b corresponding angles

c co-interior angles.

15 The diagram below shows a building's structural support beams. If you know that the main two horizontal beams are at right angles to the vertical beams, and the slanted beams are all parallel to each other, use the information provided to find the size of each angle labelled with a pronumeral, giving reasons for your answers.



16 a If a pair of parallel lines is cut by a single transversal, explain how knowing the size of one angle in the diagram means that you can find the size of every other angle.

b If a pair of parallel lines is cut by a number of transversals that are parallel to each other, explain how knowing the size of one angle in the diagram still means that you can find the size of every other angle.

17 a What is the size of the angle between two consecutive hour marks on a clock face?

b What is the size of:

i half this angle

ii a quarter of this angle

iii three quarters of this angle?

c Use your answers to parts **a** and **b** to find the size of the angle indicated on these clocks.

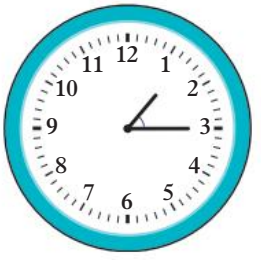
i



ii



iii



18 Find the size of the smaller angle between the two hands of a clock displaying these times.

a 10:20 **b** 6:55 **c** 4:35 **d** 2:27 **e** 11:09

Use these hints to help you.

- How many minute marks are displayed on a clock?
- What is the size of the angle between two of the minute marks?
- What is $\frac{1}{60}$ of the size of the angle between two of the minute marks?
- How far does the small hand move every time the big hand moves 1 minute?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Classifying angles

Interactive skillsheet Angles at a point

Interactive skillsheet Angles and parallel lines

Worksheet Identifying and drawing angles

Investigation Angles and parallel lines using technology

Topic quiz 7A

7B Triangles

Learning intentions

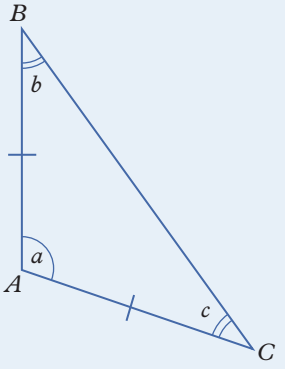
By the end of this topic you will be able to ...

- ✔ classify triangles
- ✔ calculate unknown interior and exterior angles of triangles.

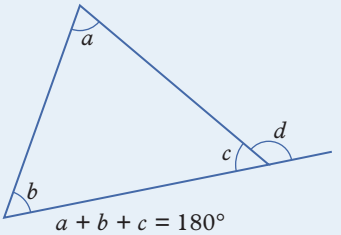
Inter-year links
Year 7 7D Classifying triangles

Triangle properties

- A triangle is a two-dimensional (2D) shape with three straight sides.
- The **interior angles** of a triangle add to 180°.
- Triangles can be classified by their side lengths and interior angles.
  - → An **equilateral triangle** has three sides of equal length and three interior angles of 60°.
  - → An **isosceles triangle** has two sides of equal length and two interior angles of equal size. The equal angles are opposite the sides of equal length.
  - → A **scalene triangle** has three sides of different length and three different interior angles.
  - → An **acute triangle** has three interior acute angles (less than 90°).
  - → A **right triangle** has one interior right angle (90°) and two interior acute angles (less than 90°).
  - → An **obtuse triangle** has one interior obtuse angle (more than 90°) and two interior acute angles (less than 90°).
- When drawing geometric shapes, sides of equal length are marked using hatch marks or dashes, and angles of equal size are marked using the same number of arcs.
  For example, in this triangle, sides AB and AC are of equal length and angles b and c are equal in size.



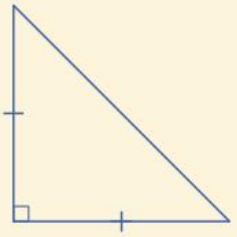
- An **exterior angle** is an angle between any side of a shape and a line extended from the adjacent side.
- The exterior angle of a triangle is supplementary to the adjacent interior angle; therefore, it is also equal to the sum of the two opposite interior angles.



$a + b + c = 180°$
$c + d = 180°$
$d = a + b$

Example 7B.1 Classifying triangles

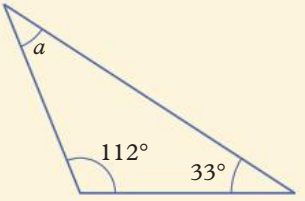
Classify the following triangle according to its sides and angles.



THINK	WRITE
1 Determine how many sides of the triangle are equal in length. The triangle has two sides equal in length, so it is an isosceles triangle. 2 Look at the angles in the triangle. The triangle has a right angle, so it is a right-angled triangle. 3 Write your answer.	This is a right-angled isosceles triangle.

Example 7B.2 Finding an unknown angle in a triangle

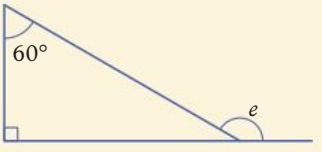
Find the size of angle a.



THINK	WRITE
The interior angles in a triangle add to 180°. Subtract each of the known angles from 180° to calculate the third angle, a.	$a = 180° - 112° - 33°$ $= 35°$

Example 7B.3 Finding the exterior angle of a triangle

Find the size of angle e.



THINK	WRITE
The exterior angle of a triangle is equal to the sum of the two opposite interior angles.	$e = 60° + 90°$ $= 150°$

Helpful hints

- ✔ Don't guess that a triangle is equilateral or isosceles based on how it looks! This should only be determined once you know the side lengths and/or angles of a triangle.
- ✔ Every triangle with exactly two equal angles will have two sides of equal length opposite those angles and be an isosceles triangle.

ANS p534

Exercise 7B Triangles

△ 1–6, 7(a, b, d), 8, 9 ■ 2–6, 7(b, c, e), 10–13 ⬠ 2(d, f), 3(e, f), 4, 5, 7(e, f), 10, 11, 13, 14

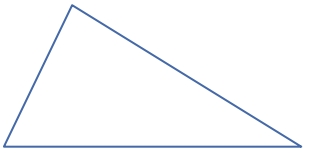
UNDERSTANDING AND FLUENCY

7B.1 **1** Classify each triangle according to its sides and angles.

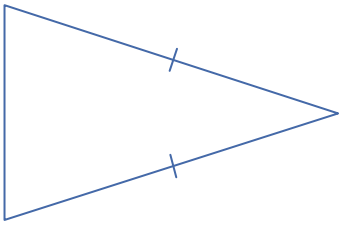
a



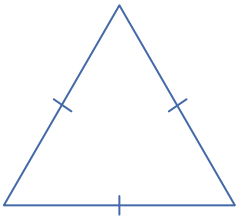
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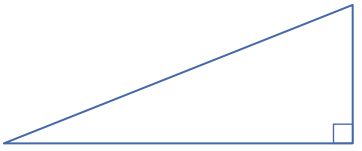
c



d



e

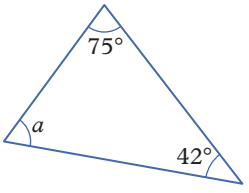


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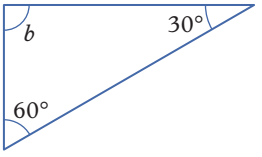


7B.2 **2** Find the size of each unknown angle.

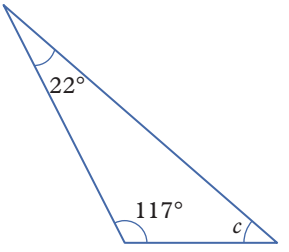
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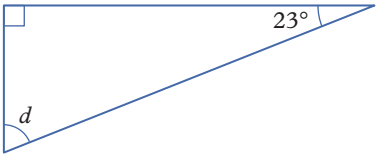
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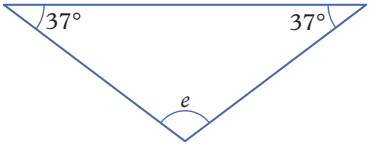
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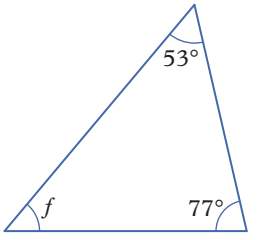
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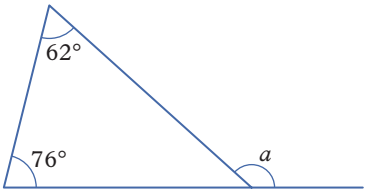


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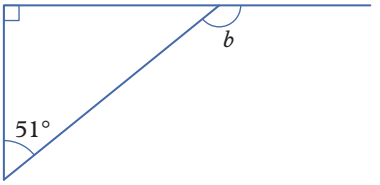


7B.3 **3** Find the size of each labelled unknown angle.

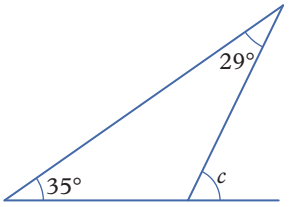
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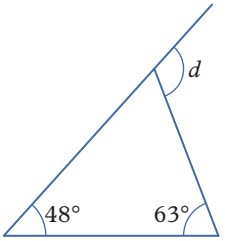
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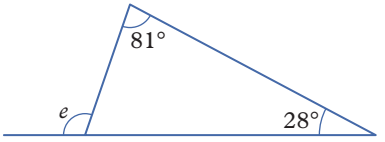
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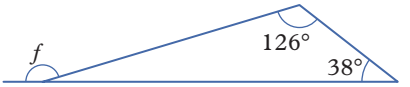
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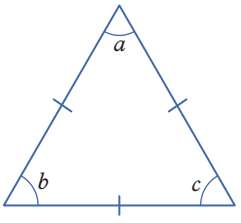


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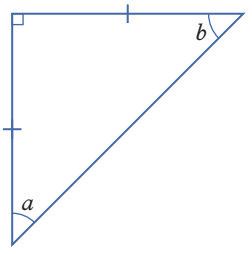


4 Use your knowledge of triangle properties to find the size of each labelled unknown angle.

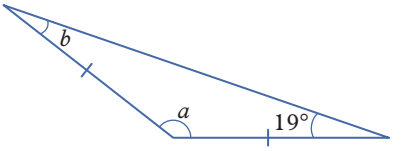
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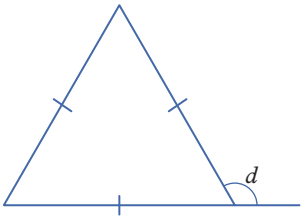
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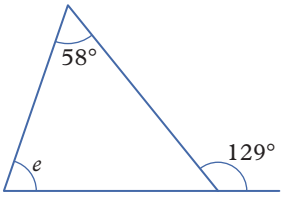
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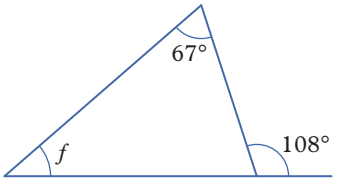
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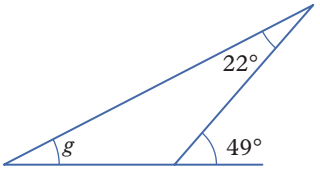
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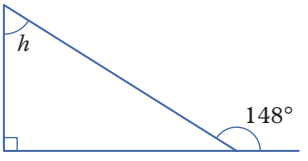
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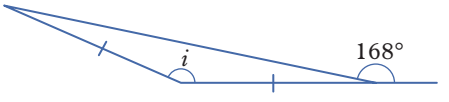
g



h



i



5 Classify each of the triangles from question **4** according to its sides and angles.

6 Answer true or false to the following statements, giving a reason for your answers.

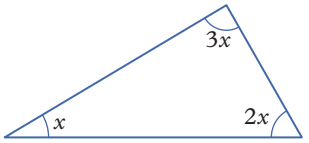
a An equilateral triangle can have a right angle.

b An isosceles triangle can have an obtuse angle.

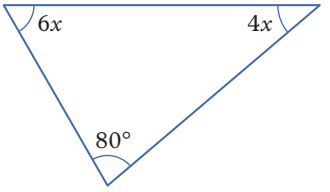
c A scalene triangle can have a right angle.

7 Use your knowledge of triangle properties and algebra to find the value of x in each triangle.

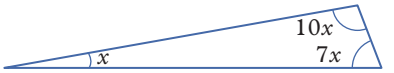
a



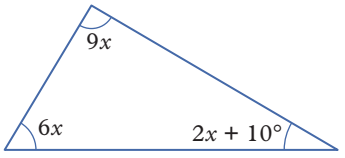
b



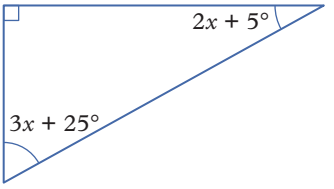
c



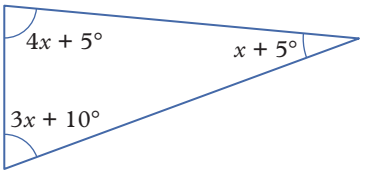
d



e



f

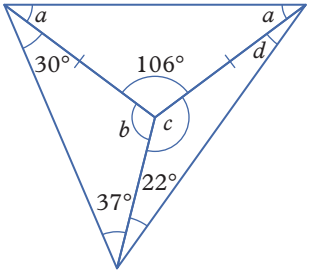


8 Rebecca measures the angles in a triangle and finds they are in a ratio of 1 : 2 : 3. That is, angle b is twice as big as angle a and angle c is three times as big as angle a.

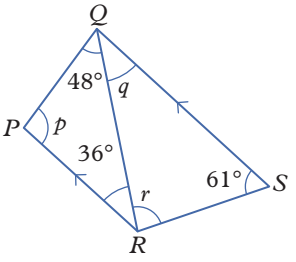
a Find the size of each angle in this triangle.

b Classify this triangle, justifying your answer.

9 Determine the values of the pronumerals a, b, c and d.



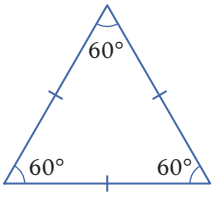
10 a Determine the values of the pronumerals p, q and r.



b What angle must $\angle RSQ$ be changed to such that PQ and RS would be parallel?

c Which other angle will change if PQ and RS become parallel by changing $\angle RSQ$, and to what size?

11 a Consider this equilateral triangle.

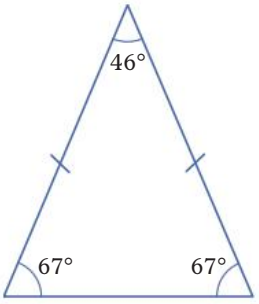


i Extend each side to create an exterior angle at each vertex.

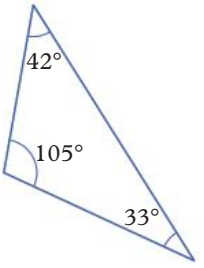
ii Find the size of each of the exterior angles and add them together. What do they add to?

Repeat **i** and **ii** for these two triangles.

b



c



d What can you say about the sum of the exterior angles of a triangle?

12 A classmate was given a number of triangles to construct, but is having trouble drawing them. For each set of instructions, draw a diagram of the triangle or provide a reason why it cannot be drawn.

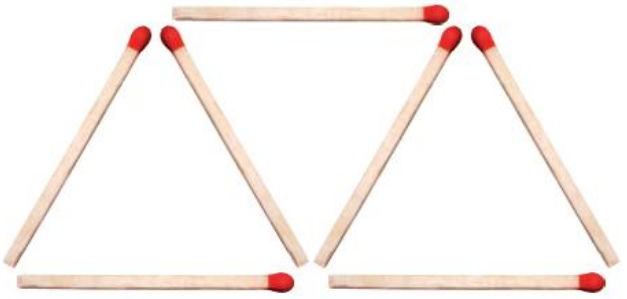
a a triangle with internal angles of 90°, 50° and 60°

b a triangle with two internal angles of 40°

c a triangle with side lengths of 10 cm, 3 cm and 5 cm

d a triangle with two obtuse angles

13 You can construct equilateral triangles using matches, as shown here.



a How many matches were used for these three triangles?

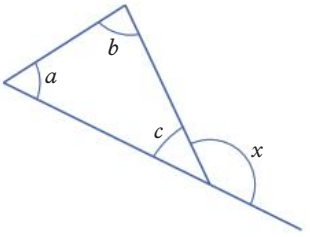
b Use:

i nine matches to construct four equilateral triangles

ii nine matches to construct five equilateral triangles

iii six matches to construct four equilateral triangles.

14 Use an algebraic process to show that $x = a + b$.



PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Triangle properties

Investigation Stained glass triangles

Topic quiz 7B

7C Pythagoras' theorem

Learning intentions

By the end of this topic you will be able to ...

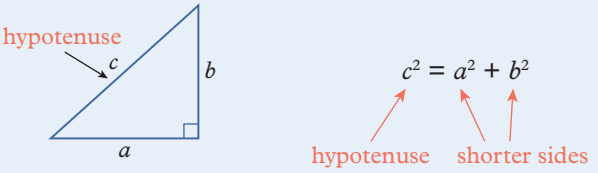
- ✔ use Pythagoras' theorem to determine whether a triangle is right-angled
- ✔ use Pythagoras' theorem to find the hypotenuse of a right-angled triangle.

Inter-year links

Year 7	1G Exponents and square roots
Year 9	6A Pythagoras' theorem

Pythagoras' theorem in right-angled triangles

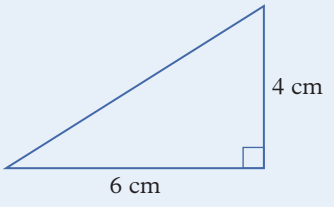
- Pythagoras of Samos was an ancient Greek philosopher and mathematician who lived from approximately 570 to 495 BCE. He has been credited with many mathematical and scientific discoveries, although the origins of his most famous theorem are disputed to this day.
- The **hypotenuse** is the longest side of a right-angled triangle. It is the side directly opposite the right angle.
- **Pythagoras' theorem** states the relationship between the three side lengths of a right-angled triangle. That is, the sum of the squares of the shorter sides is equal to the square of the hypotenuse. The relationship for the triangle shown is:



- A triangle is right-angled if it has side lengths that satisfy Pythagoras' theorem.
- Any set of three positive whole numbers that satisfy Pythagoras' theorem is called a **Pythagorean triple** (or Pythagorean triad).
  For example, 3, 4, 5 is a Pythagorean triple as $5^2 = 3^2 + 4^2$.

Finding the length of the hypotenuse

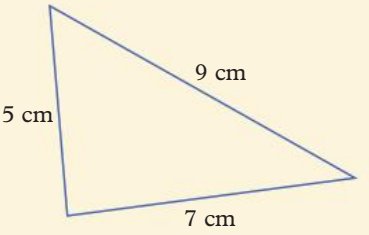
- To find the length of the hypotenuse using Pythagoras' theorem:
  1. State Pythagoras' theorem and define *a*, *b* and *c*.
  2. Substitute the values for *a*, *b* and *c* and simplify the equation.
  3. Solve for *c* by finding the square root of both sides of the equation using inverse operations.
- The result for the length of the hypotenuse can be written as an exact value (whole number, terminating decimal or in surd form) or approximated to a specific number of decimal places.
  For example, the side length of the hypotenuse in this triangle can be written as $\sqrt{52}$ cm or 7.21 cm correct to two decimal places.



$$\begin{aligned} c^2 &= a^3 + b^2 \\ &= 6^2 + 4^2 \\ &= 36 + 16 \\ &= 52 \\ c &= \sqrt{52} \text{ cm} \\ &\approx 7.21 \text{ cm} \end{aligned}$$

Example 7C.1 Using Pythagoras' theorem to determine whether a triangle is right-angled

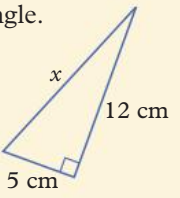
Use Pythagoras' theorem to decide whether this is a right-angled triangle.



THINK	WRITE
1 Identify the length of the longest side and define as c.	Let $c = 9$ cm.
2 Calculate the value of c^2.	$c^2 = 9^2$ $= 81$
3 Identify the lengths of the two shorter sides and define as a and b.	Let $a = 5$ cm and $b = 7$ cm.
4 Calculate the value of $a^2 + b^2$.	$a^2 + b^2 = 5^2 + 7^2$ $= 25 + 49$ $= 74$
5 Compare c^2 to $a^2 + b^2$. If the two values are equal, then the triangle is right-angled. If the two values are not equal, then it is not a right-angled triangle.	$c^2 \neq a^2 + b^2$ The triangle is not a right-angled triangle.

Example 7C.2 Calculating the length of the hypotenuse (whole number)

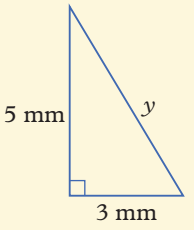
Use Pythagoras' theorem to calculate the length of the hypotenuse of this triangle.



THINK	WRITE
1 The triangle is right-angled, so state Pythagoras' theorem and define a, b and c.	$c^2 = a^2 + b^2$, where $a = 5$ cm, $b = 12$ cm and $c = x$.
2 Substitute the values for a, b and c, and simplify the equation.	$x^2 = 5^2 + 12^2$ $= 25 + 144$ $= 169$
3 Solve for x by finding the square root of both sides of the equation using inverse operations.	$x = \sqrt{169}$ $= 13$ cm

Example 7C.3 Calculating the length of the hypotenuse (decimal value)

Calculate the length of the hypotenuse of this triangle correct to two decimal places.



THINK	WRITE
1 The triangle is right-angled, so state Pythagoras' theorem and define a, b and c.	$c^2 = a^2 + b^2$, where $a = 3$ mm, $b = 5$ mm and $c = y$.
2 Substitute the values for a, b and c, and simplify the equation.	$y^2 = 3^2 + 5^2$ $= 9 + 25$ $= 34$
3 Solve for y by finding the square root of both sides of the equation.	$y = \sqrt{34}$
4 Use a calculator to find the approximate length. Round to two decimal places. (Note that $\sqrt{34}$ mm is the exact length.)	≈ 5.83 mm

Helpful hints

- ✔ Make sure you correctly identify the hypotenuse. It is the longest side and is opposite the right angle. It may not be obvious, so always look at the numbers carefully.
- ✔ Pythagoras' theorem is an excellent formula to memorise. You will use it many times in your mathematics studies. Writing the formula before substituting the values in will help you to memorise it quicker.
- ✔ It doesn't matter which shorter sides you assign to be a or b, but the hypotenuse must always be c!

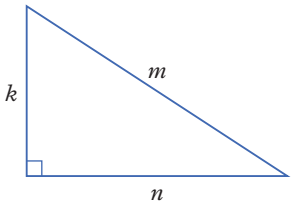
ANS p536

Exercise 7C Pythagoras' theorem

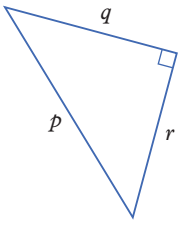
1–8, 11, 13, 14, 15(a, b, f), 16	2, 3, 5, 7, 9, 12, 15(c, e, f), 16, 17, 20(a, b), 23	2, 6, 7, 9, 10, 12, 16, 18, 19, 20(c, d), 21, 22

1 Identify the pronumeral shown on the hypotenuse in each triangle.

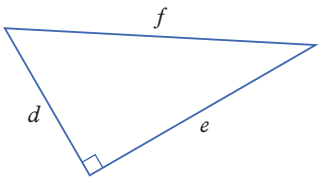
a



b

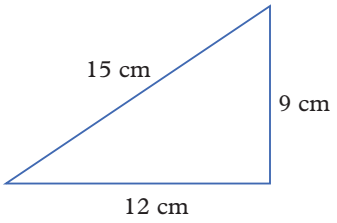


c

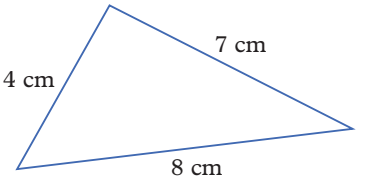


7C.1 **2** Use Pythagoras' theorem to decide whether these are right-angled triangles.

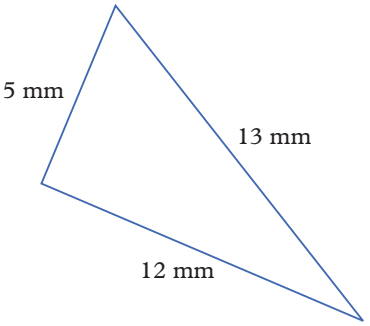
a



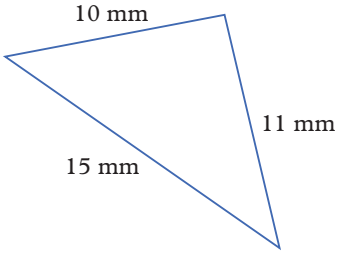
b



c



d



3 Decide whether each triangle with the given side lengths is right-angled. Hint: The longest side in a right-angled triangle is the hypotenuse.

a 6 mm, 8 mm, 10 mm

b 7 cm, 24 cm, 25 cm

c 8 cm, 10 cm, 14 cm

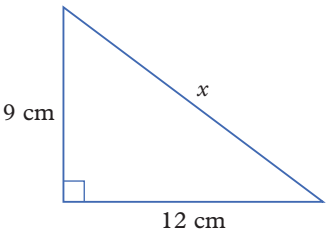
d 11 mm, 15 mm, 19 mm

e 20 cm, 21 cm, 29 cm

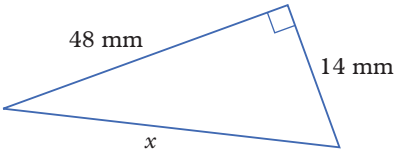
f 10 mm, 18 mm, 20 mm

7C.2 **4** Use Pythagoras' theorem to calculate the length of the hypotenuse in each triangle.

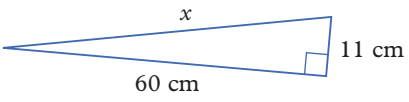
a



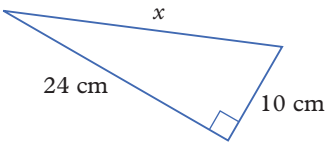
b



c

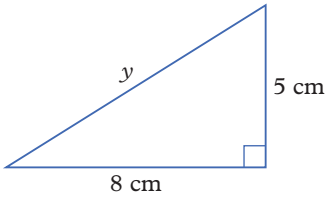


d

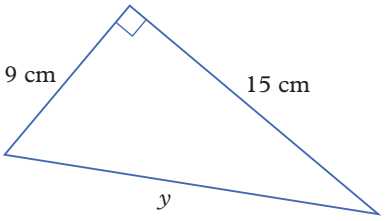


7C.3 **5** Calculate the length of the hypotenuse in each triangle correct to two decimal places.

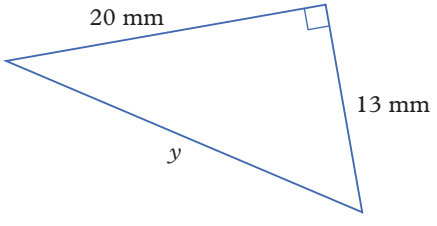
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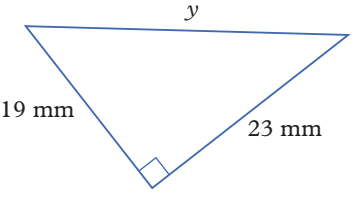
b



c



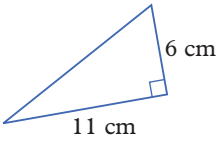
d



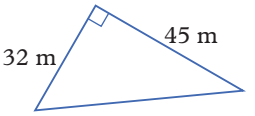
UNDERSTANDING AND FLUENCY

6 Find the unknown length in each right-angled triangle, correct to two decimal places.

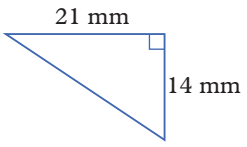
a



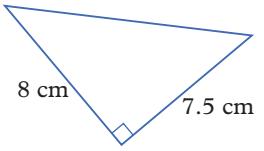
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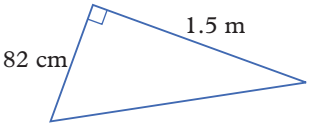


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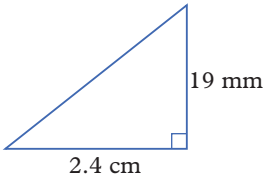


7 Find the length of the hypotenuse in each right-angled triangle, correct to the nearest centimetre.
Hint: Convert all length measurements to centimetres first.

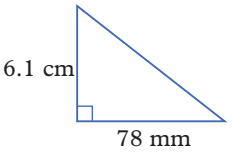
a



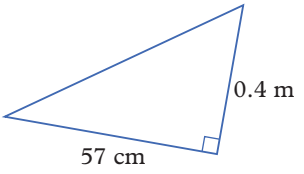
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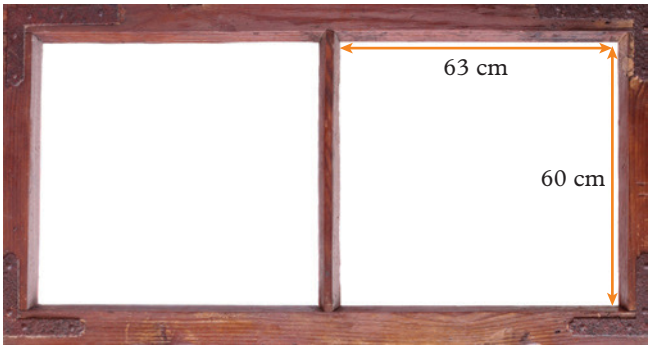


d



PROBLEM SOLVING AND REASONING

8 The timber frames in a window should meet at right angles. The diagonal length of the glass in one section of the window below is measured to be 87 cm. Use Pythagoras' theorem to check whether this window frame has been made correctly.



9 A rectangular picture frame measures 70 cm by 24 cm. The picture framer checks that the frame is 'square' (right-angled) by measuring the distance from the top left corner to the bottom right corner of the frame. The measured distance is 73 cm.

a Draw a diagram of the frame and label it with all the known information.

b Use Pythagoras' theorem to check whether the frame is 'square'. If it is not, should the diagonal distance be longer or shorter?

c Suggest a length for the diagonal distance.

10 Both multiplication and addition are commutative. That is, when numbers or variables are multiplied or added, the order doesn't matter, as the result will be the same.
For example: $2 \times 3 = 3 \times 2$ and $2 + 3 = 3 + 2$

In Pythagoras' theorem, a and b refer to the lengths of the two shorter sides of a right-angled triangle. Does it matter which of the shorter sides is labelled a and which is labelled b? Explain by referring to commutative properties.

11 Pythagorean triples can be generated using formulas. One method for finding the Pythagorean triples uses three formulas and two known values, m and n where $m > n$, to generate the shorter side lengths a and b, and the hypotenuse c. The formulas are:

- $a = m^2 - n^2$
- $b = 2mn$
- $c = m^2 + n^2$

Use the formulas to complete the table below to generate Pythagorean triples. The first row has been completed for you.

Values		Pythagorean triple		
m	***n***	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$
2	1	$2^2 - 1^2 = 3$	$2 \times 2 \times 1 = 4$	$2^2 + 1^2 = 5$
3	1			
3	2			
4	1			
4	2			
4	3			
5	1			
5	2			
5	3			
5	4			
6	1			

12 Pythagorean triples form the lengths of right-angled triangles. Some triples are multiples of another triple and they form similar triangles.

For example, (6, 8, 10) and (9, 12, 15) are multiples of (3, 4, 5).

A Pythagorean triple that does not have a common factor between the three sides is called a primitive Pythagorean triple.

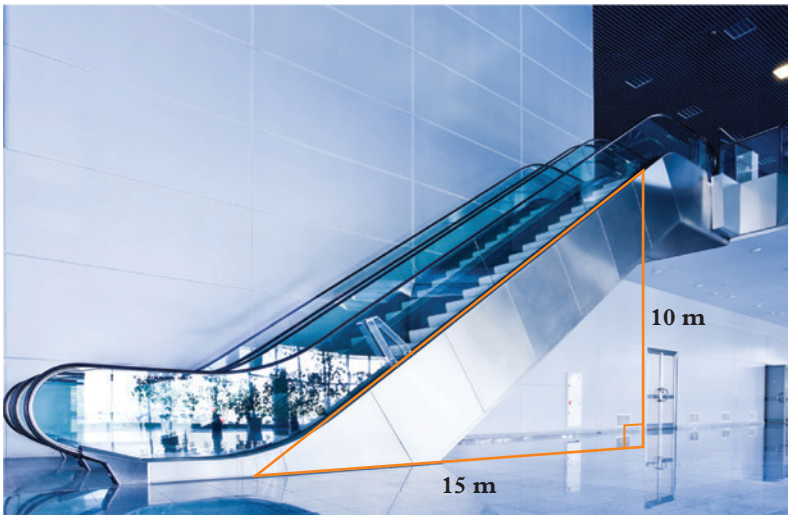
a List the primitive Pythagorean triples from question **11**.

b The Pythagoreans triples below are all multiples of a primitive Pythagorean triple. State the primitive Pythagorean triple in each case.

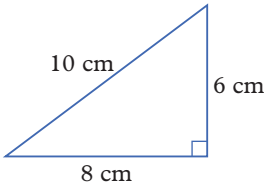
i (50, 120, 130) **ii** (24, 45, 51) **iii** (27, 36, 45)

iv (36, 160, 164) **v** (390, 432, 582) **vi** (231, 2420, 2431)

13 An escalator rises a vertical distance of 10 m for a horizontal distance of 15 m. What distance would you travel from the bottom to the top of the escalator? Give your answer correct to two decimal places.



14 Consider Triangle A shown.



Triangle A

a Use Pythagoras' theorem to confirm that Triangle A is right-angled.

b A new triangle, Triangle B, is formed by doubling the side lengths of Triangle A. Does Pythagoras' theorem still work for Triangle B?

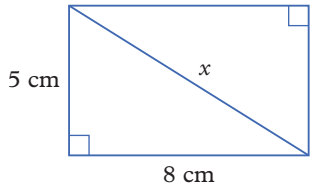
c Triangle C is formed by tripling the side lengths of Triangle A. Does Pythagoras' theorem still work for Triangle C?

d Triangle D is formed by halving the side lengths of Triangle A. Does Pythagoras' theorem still work for Triangle D?

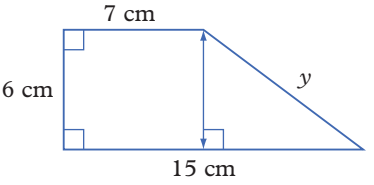
e Will Pythagoras' theorem still work if the side lengths of Triangle A are multiplied by 10?

15 Use Pythagoras' theorem to calculate the unknown length indicated by the pronumeral in each diagram, correct to two decimal places.

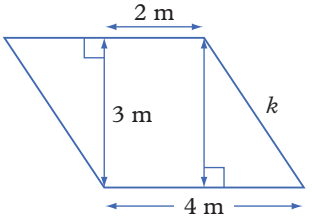
a



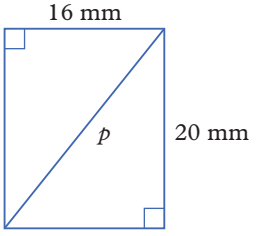
b



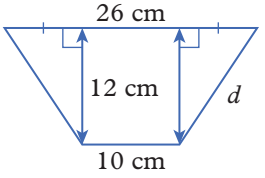
c



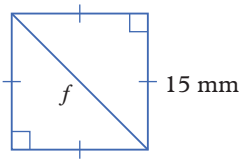
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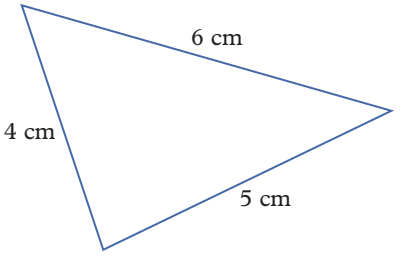
e



f



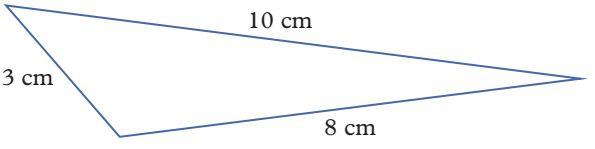
16 a Calculate the square of the side lengths of this acute triangle.



b Complete the following sentence.

In an acute triangle, the square of the longest side is _____ the sum of the squares of the two other sides.

c Calculate the square of the side lengths of this obtuse triangle.

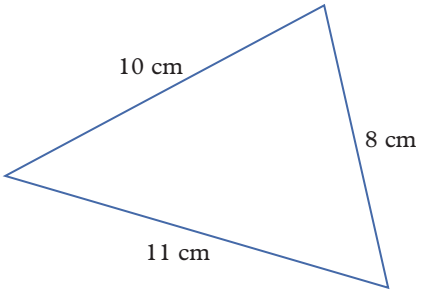


d Complete the following sentence.

In an obtuse triangle, the square of the longest side is _____ the sum of the squares of the two other sides.

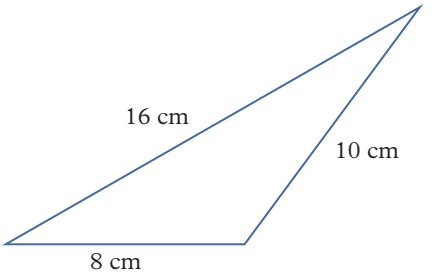
e Confirm that your sentences work for the following triangles.

i



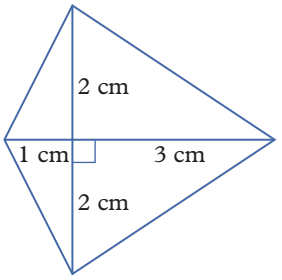
Acute triangle

ii



Obtuse triangle

17 Calculate the perimeter of the kite shown, correct to two decimal places.



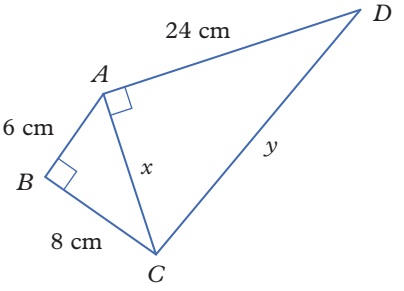
18 Consider the shape *ABCD* shown on the right.

a How many right-angled triangles can you see? Name them.

b Calculate the length of *AC*. Which side lengths did you use in your calculation?

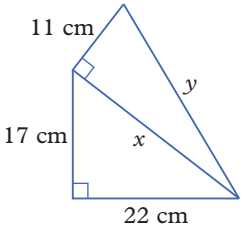
c Calculate the length of *CD*. Explain how you were able to do this.

d Find the perimeter of the shape *ABCD*.

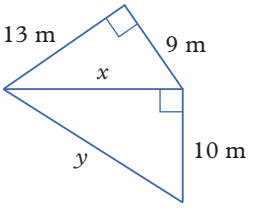


19 Find the lengths indicated by x and y in each diagram, correct to two decimal places.

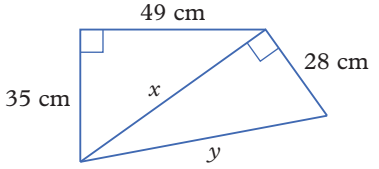
a



b

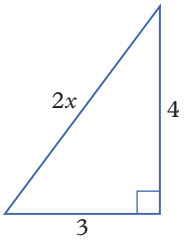


c

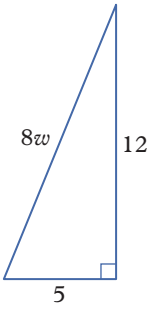


20 Determine the value of the pronumeral in each of the following. Write your answer as a fraction where appropriate.

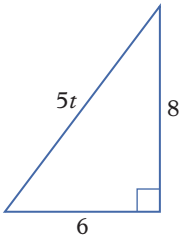
a



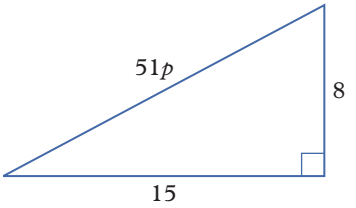
b



c



d



CHALLENGE

21 Builders in ancient Egypt used a piece of rope with 12 equally spaced knots as a tool. How did this show that a corner was 'square'? Explain using a diagram.

22 Pythagoras' theorem is one of the most thoroughly proved theorems with hundreds of unique proofs (mathematical explanations). One proof, which we will look at, is known as 'Proof by rearrangement'.

Consider Diagram 1 and Diagram 2. Both diagrams have the same four right-angled triangles with shorter sides of a and b, and a hypotenuse with length c.

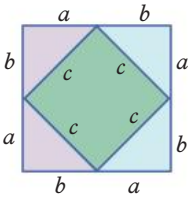


Diagram 1

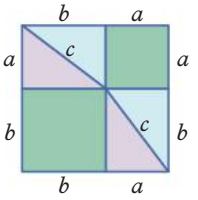


Diagram 2

a Explain why the two squares have the same area.

b Using your ideas from part **a**, explain how this pair of diagrams shows that $c^2 = a^2 + b^2$.

We can prove this algebraically by writing an equation equating the areas of each square.

c Write an equation for the total area in each diagram by adding the area of their smaller sections.

d The area in both diagrams is the same, so we can equate the equations from part **c.** Show that this equation is equivalent to $c^2 = a^2 + b^2$.

Another proof for Pythagoras' theorem can be shown using Diagram 3.

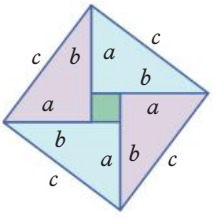


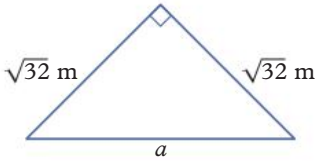
Diagram 3

e Using this diagram, explain why $c^2 = (b - a)^2 + 4 \times \frac{1}{2}ab$ by considering the area of each part.

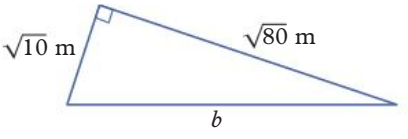
f Show that the equation from part **e** is equivalent to $c^2 = a^2 + b^2$.

23 Squaring and taking the square root of a number are inverse operations, so $(\sqrt{a})^2 = a$. Use this information to determine the values of the pronumerals in these diagrams, giving your answers correct to two decimal places where required.

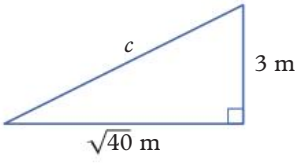
a



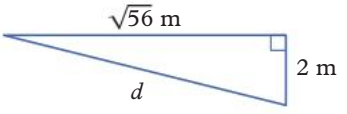
b



c



d



Check your Student obook pro for these digital resources and more:

- Interactive skillsheet — Pythagoras' theorem
- Interactive skillsheet — Finding the length of the hypotenuse
- Investigation — Proving Pythagoras' theorem
- Topic quiz — 7C

7D Using Pythagoras' theorem

Learning intentions

By the end of this topic you will be able to ...

✔ use Pythagoras' theorem to determine the shorter side of a right-angled triangle.

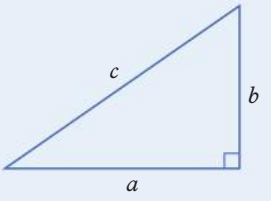
Inter-year links

Year 7 6G Solving equations using inverse operations

Year 9 6A Pythagoras' theorem

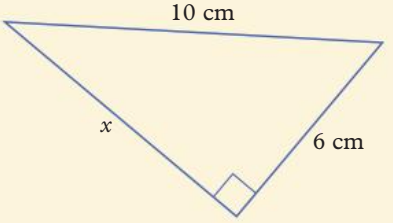
Finding the length of a shorter side

- Pythagoras' theorem states that in a right-angled triangle with side lengths a, b and c, where c is the length of the hypotenuse, $c^2 = a^2 + b^2$.
- To find the length of one of the shorter sides using Pythagoras' theorem:
  1. State Pythagoras' theorem and define a, b and c.
  2. Substitute the values for a, b and c, and simplify the equation.
  3. Solve for a or b by finding the square root of both sides of the equation using inverse operations. Write the positive solution only and include the correct unit for the side length.



Example 7D.1 Calculating the length of a shorter side (whole number)

Use Pythagoras' theorem to calculate the unknown side length in this triangle.



THINK

1. The triangle is right-angled, so state Pythagoras' theorem and define a, b and c.
2. Substitute the values for a, b and c, and simplify the equation.
3. Solve for x by first subtracting 36 from both sides of the equation and then finding the square root.

WRITE

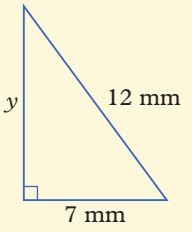
$c^2 = a^2 + b^2$, where $a = x$, $b = 6$ cm and $c = 10$ cm.

$10^2 = x^2 + 6^2$

$100 = x^2 + 36$
$64 = x^2$
$\sqrt{64} = x$
$8 = x$
$x = 8$ cm

Example 7D.2 Calculating the length of a shorter side (decimal value)

Calculate the unknown side length in this triangle, correct to two decimal places.



THINK

1. The triangle is right-angled, so state Pythagoras' theorem and define a, b and c.
2. Substitute the values for a, b and c, and simplify the equation.
3. Solve for y by first subtracting 49 from both sides of the equation and then finding the square root.
4. Use a calculator to find the approximate length correct to two decimal places. (Note that $\sqrt{95}$ mm is the exact length.)

WRITE

$c^2 = a^2 + b^2$, where $a = 7$ mm, $b = y$ and $c = 12$ mm.

$12^2 = 7^2 + y^2$

$144 = 49 + y^2$

$95 = y^2$

$\sqrt{95} = y$

$y \approx 9.75$ mm

Helpful hints

- ✔ A common mistake when using Pythagoras' theorem often occurs in the second-last step. When looking at $y^2 = 95$, instead of taking the square root of both sides, some students will halve the result. The 2 is an index, not a factor!
- ✔ Remember that c represents the length of the hypotenuse in $c^2 = a^2 + b^2$, not the unknown side. So when finding the length of the shorter side, substitute the pronumeral into a or b.
- ✔ The solution of an equation $x^2 = 64$ has two solutions, $x = \pm 8$, but you only write the positive solution if x represents a length, as lengths can't be negative.

ANS p537

Exercise 7D Using Pythagoras' theorem

△ 1, 2–4(1st column), 5–9, 10(a, b) □ 2–4(2nd column), 5, 6–7(2nd column), 9, 11, 12, 15, 18(a, b) ⬠ 4, 5, 6–7(2nd column), 11–14, 15(b), 16, 17, 18(c, d)

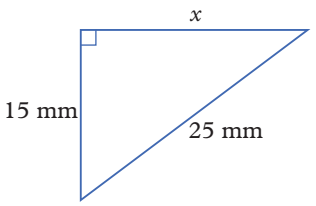
1 Solve the following equations for x. Write your answers correct to two decimal places.

a $x^2 + 25 = 36$ **b** $x^2 + 7^2 = 10^2$ **c** $16 + x^2 = 64$

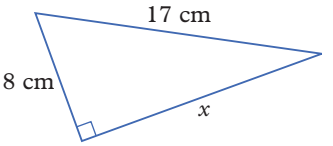
d $81 = x^2 + 9$ **e** $12^2 = 5^2 + x^2$ **f** $36 = x^2 + x^2$

7D.1 **2** Use Pythagoras' theorem to calculate the unknown side length in each triangle.

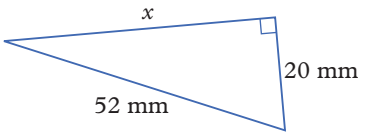
a



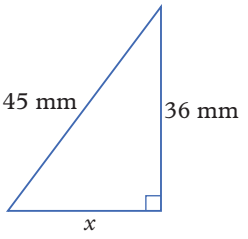
b



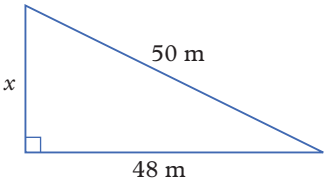
c



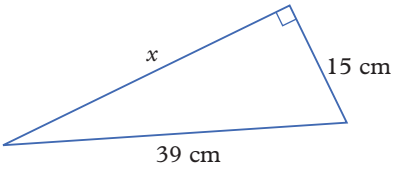
d



e

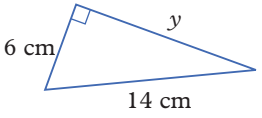


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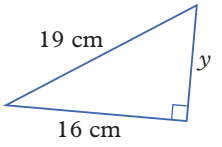


7D.2 **3** Calculate the unknown side length in each triangle, correct to two decimal places.

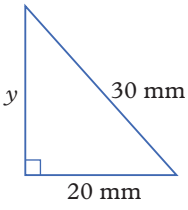
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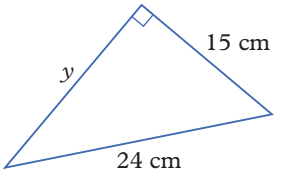
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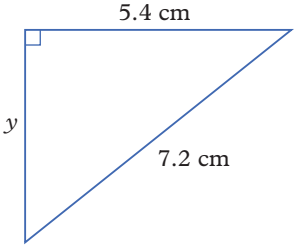
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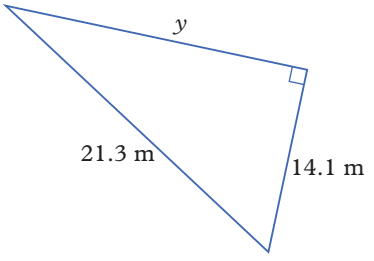
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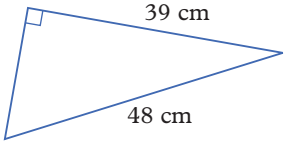


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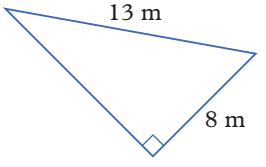


4 Find the unknown length in each right-angled triangle, correct to two decimal places.

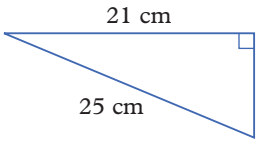
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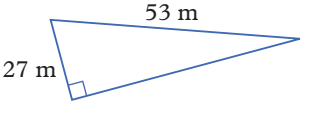
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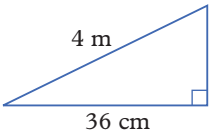


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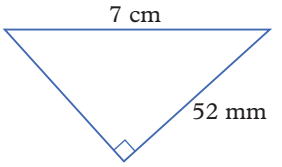


5 Find the unknown length in each right-angled triangle, correct to the nearest centimetre. Hint: Convert all length measurements to centimetres first.

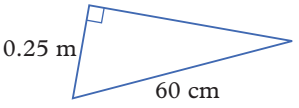
a



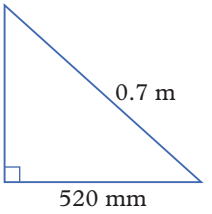
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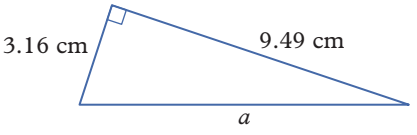


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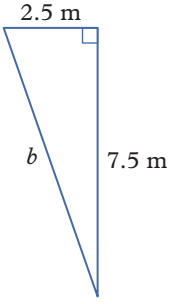


6 Calculate the length of the unknown side in each right-angled triangle, correct to two decimal places.

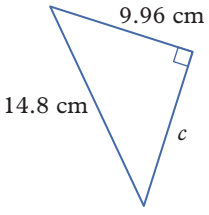
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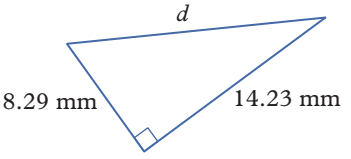
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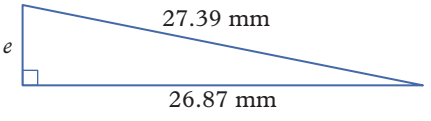
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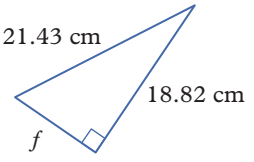
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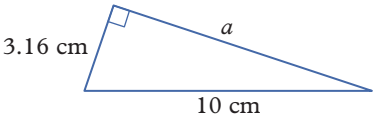


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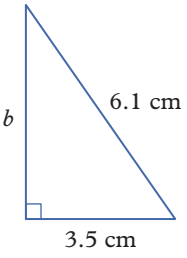


7 Calculate the area of the following right-angled triangles, correct to two decimal places.

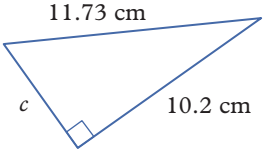
a



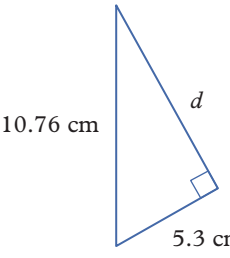
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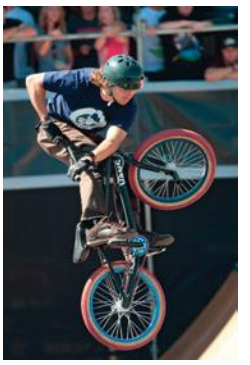
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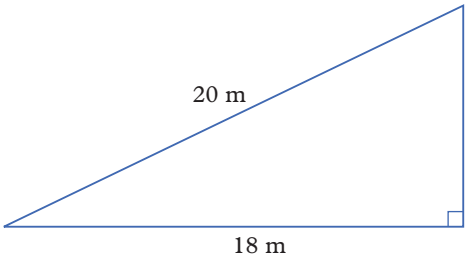


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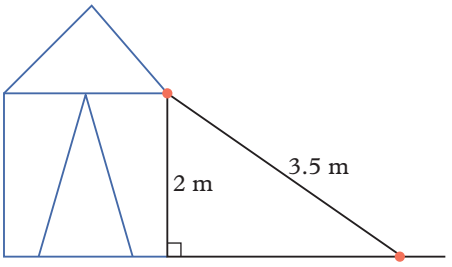


8 A BMX rider uses the ramp shown in this diagram to perform stunts. How high is the ramp? Give your answer correct to two decimal places.



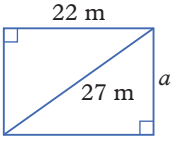


9 A guy rope on a tent is 3.5 m long and is attached to the tent at a height of 2 m. How far from the tent does the guy rope reach the ground? Give your answer correct to two decimal places.

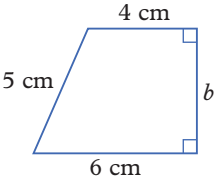


10 Use Pythagoras' theorem to calculate the unknown length indicated by the unknown in each diagram, correct to two decimal places.

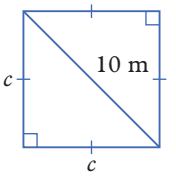
a



b

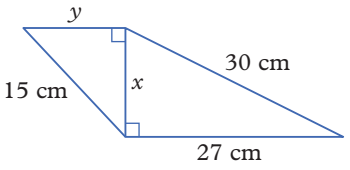


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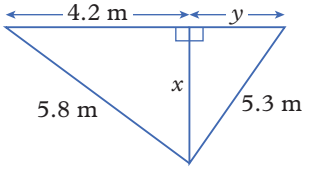


11 Find the lengths labelled x and y in each diagram, correct to two decimal places.

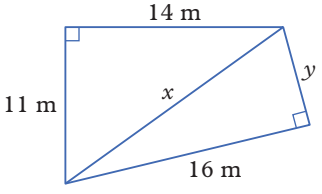
a



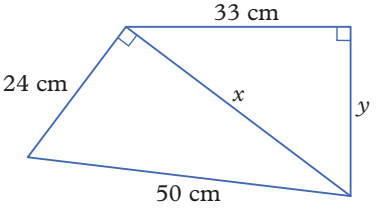
b



c



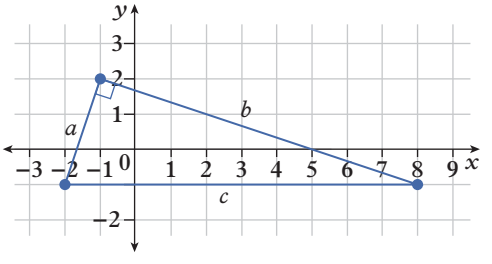
d



12 For each shape in question **11**, find correct to two decimal places:

i the perimeter **ii** the area.

13 Consider the triangle shown on the Cartesian plane.



a State the length of c.

b Determine the square of the lengths of a and b.

c Show that the triangle is right-angled.

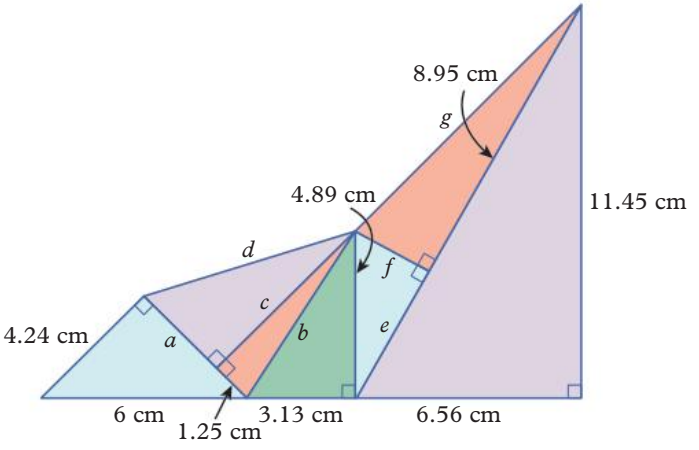
d Determine the midpoint of each side length. Write your answer using decimals.

e Determine the squares of the lengths between each pair of midpoints.

f Determine whether the triangle formed by the midpoints is right-angled.

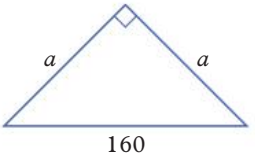
PROBLEM SOLVING AND REASONING

14 Determine the value of all pronumerals, a–g, correct to two decimal places.

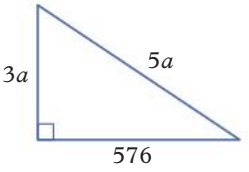


15 Determine the value of the unknown, correct to two decimal places where appropriate.

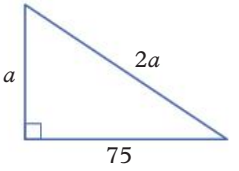
a



b



c



16 Joey is determining the unknown side length of a right-angled triangle with the two side lengths 7 cm and 15 cm. Joey writes down all possible calculations for the third length.

i $\sqrt{7^2 + 15^2}$ **ii** $\sqrt{15^2 + 7^2}$ **iii** $\sqrt{7^2 - 15^2}$ **iv** $\sqrt{15^2 - 7^2}$

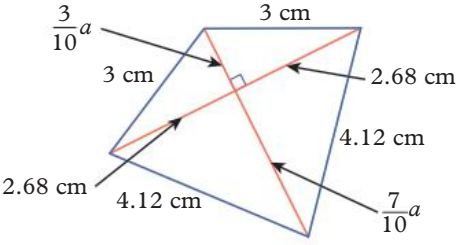
a If the missing side length is the hypotenuse, which calculation(s) could be used?

b If the missing side length is not the hypotenuse, which calculation(s) could be used?

c Is there a calculation listed that does not result in the side length of a right-angled triangle? Explain why or why not.

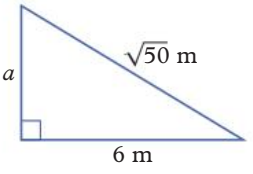
CHALLENGE

17 Calculate the area of the kite on the right. Give your answer correct to the nearest integer.

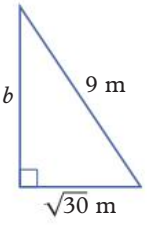


18 Determine the values of the pronumerals in these diagrams, giving your answers correct to two decimal places where required.

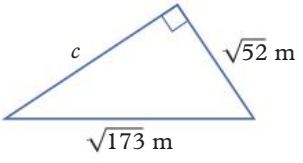
a



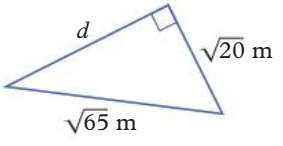
b



c



d



Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Finding the length of a shorter side

Topic quiz
7D

7E Transformations

Learning intentions

By the end of this topic you will be able to ...

✔ describe transformations

✔ perform transformations.

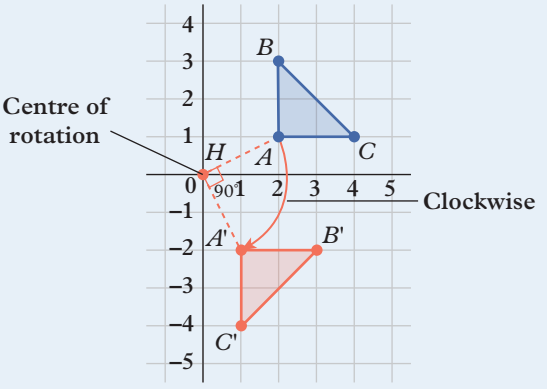
Inter-year links

Support Transformations

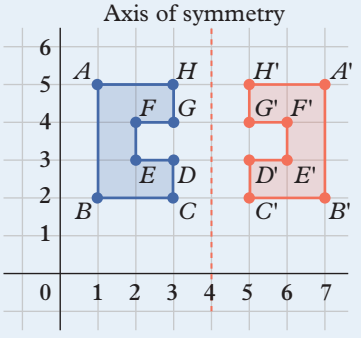
Year 7 8D Translations

Transformations

- **Rotations**, **reflections** and **translations** are **isometric transformations**. Isometric transformations do not change the shape or size of a shape or object.
  → The angle and side measurements of a shape do not change after an isometric transformation.
- A transformed shape or object (also called an **image**) should always be given a name that indicates that it is a transformation; for example, the image of A is A′.
- A rotation turns a shape about a fixed point. A rotation requires:
  1 the centre of rotation
  2 the angle of rotation
  3 the direction of the rotation.
  → When performing a rotation, first identify a single point to concentrate on. This is called the **point of focus**.

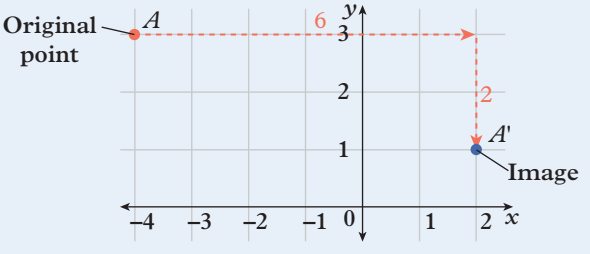


- A reflection is performed in an axis of symmetry. Each point in the image must be the same distance from the axis of symmetry as the corresponding original point.



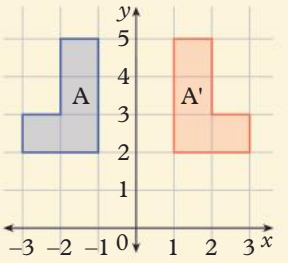
- A translation is when a shape or object is moved without turning or changing size.
  → Translations are recorded by counting how many units the shape slides up or down and left or right.

Translation 6 units right and 2 units down



Example 7E.1 Describing transformations

Provide a full description of the transformation shown. Include the type of transformation as well as a description of the specific movement.



THINK

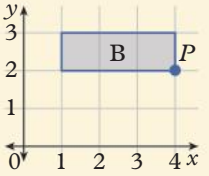
1. Determine if the shape has been translated, rotated or reflected.
2. Determine which line the shape has been reflected in.
3. Write your answer.

WRITE

The shape has been reflected in the y-axis.

Example 7E.2 Performing a transformation

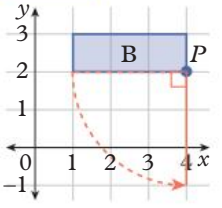
Rotate this shape 90° anti-clockwise around point P.

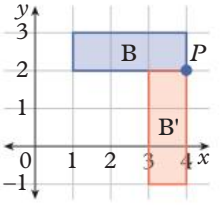


THINK

1. Select a point of focus on the shape. Since the given point of rotation is on the shape, it is easier to choose a point of focus that has an edge in common with it. Rotate this edge 90° anti-clockwise around the point of rotation.
2. Draw in the remainder of the image. The rotated shape must be the same size as the original shape.
3. Name the final image.

WRITE





Helpful hints

- ✔ When transforming a shape, every point on the original shape must be transformed.
- ✔ Start by transforming a single point on a shape, rather than trying to transform the whole shape in one go.

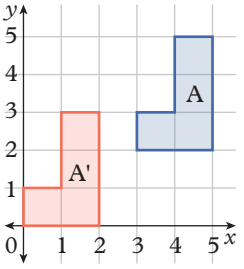
ANS p537

Exercise 7E Transformations

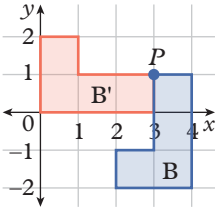
1–4, 6, 7(a, b), 12 | 1, 3–5, 7, 8, 10–13 | 1, 3, 5, 7–9, 13, 14

7E.1 **1** Provide a full description of each of the transformations shown. Include the type of transformation as well as a description of the specific movement.

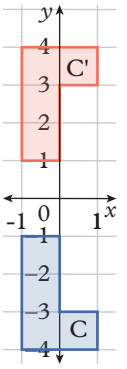
a



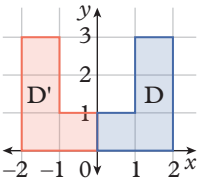
b



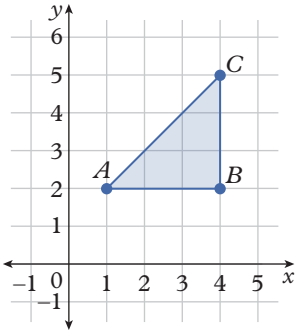
c



d



7E.2 **2** Transform the triangle shown using the descriptions provided. Start from the original triangle each time.



a Translate 3 units left and 2 units up.

b Reflect in the x-axis.

c Rotate 90° clockwise about A.

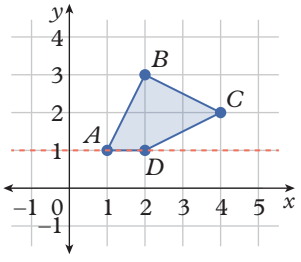
d Reflect in the y-axis.

e Rotate 180° clockwise about the origin.

f Translate 1 unit right and 4 units up.

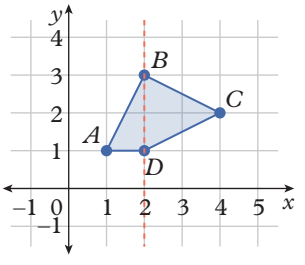
3 Transform the quadrilateral shown using the descriptions provided. Draw each image on its own Cartesian plane.

a



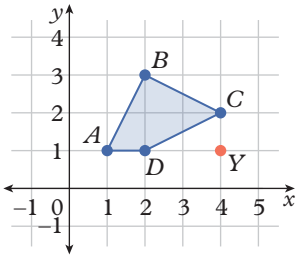
Reflect in the line AD.

b



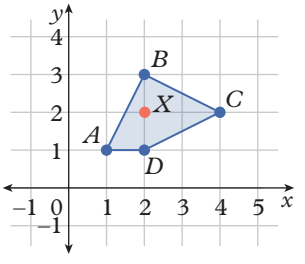
Reflect in the line BD.

c



Rotate 90° anti-clockwise about point Y.

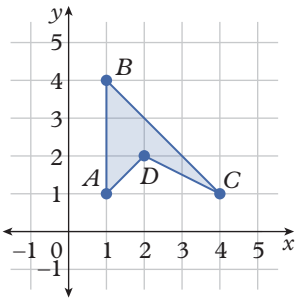
d



Rotate 90° clockwise about point X.

UNDERSTANDING AND FLUENCY

4 Transform the quadrilateral shown using the descriptions provided. Start from the original quadrilateral each time. Draw each image on its own Cartesian plane.



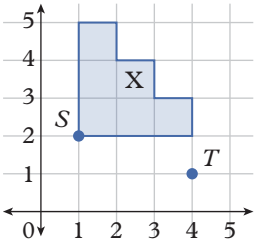
a Rotate 90° clockwise about the origin, then translate 2 units left and 3 units down.

b Translate 2 units left and 3 units down, then rotate 90° clockwise about the origin.

c Reflect in the y-axis, then rotate 90° anti-clockwise about the origin.

d Rotate 90° anti-clockwise about the origin, then reflect in the y-axis.

5 Perform each transformation on this shape. Start from the original shape each time.



a Reflect in the x-axis.

b Translate 4 units left and 3 units up.

c Reflect in the y-axis.

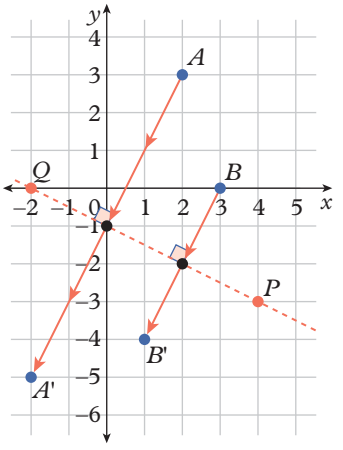
d Rotate 90° clockwise around point S.

e Translate 1 unit right and 2 units down.

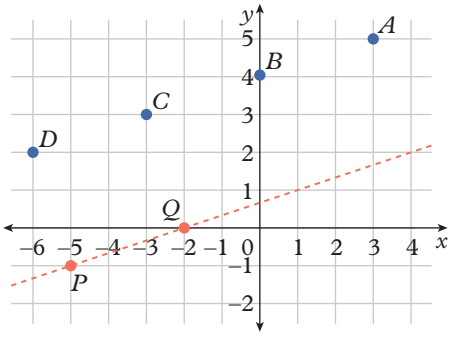
f Rotate 90° anti-clockwise around point T.

PROBLEM SOLVING AND REASONING

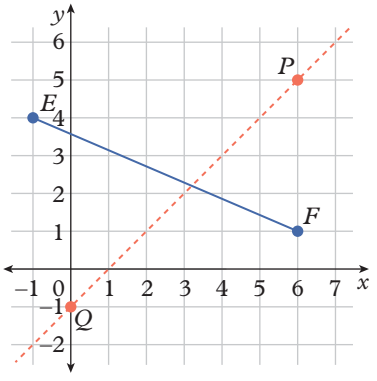
6 When we reflect in a line, we use the point on the line that is at a right angle to the point being reflected. To ensure you reflect at a right angle, draw an arrow between two points on the mirror line. Rotate this arrow 90° and follow that direction from the point you are reflecting, A or B. Repeat the number of times you draw the arrow from the point to the mirror line to find the location of the image.



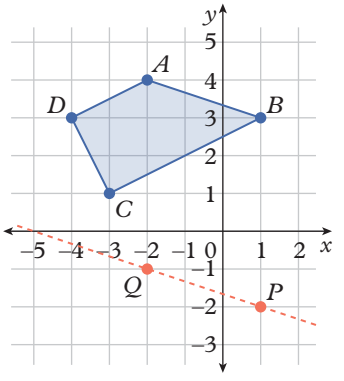
a Reflect points A, B, C and D in the line PQ.



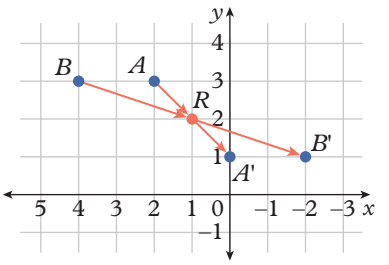
b Reflect the line segment EF in the line PQ.



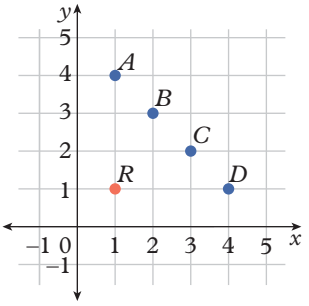
c Reflect the quadrilateral $ABCD$ in the line PQ.



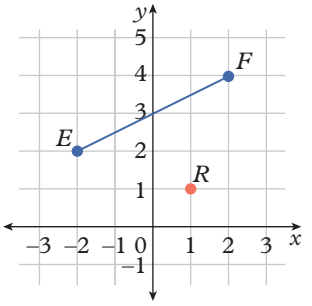
7 Instead of reflecting in a line, we can reflect in a point. To do this, draw an arrow from the point you are reflecting, A or B, to the point you are reflecting in, R, then repeat the arrow going from the point you are reflecting in, R, to where the image, A' and B', will be.



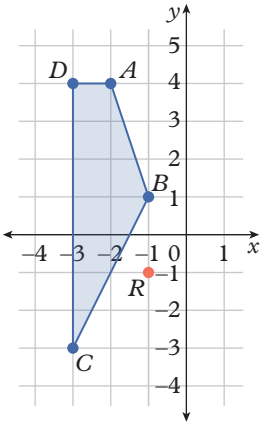
a Reflect the points A, B, C and D in the point R.



b Reflect the line segment EF in the point R.

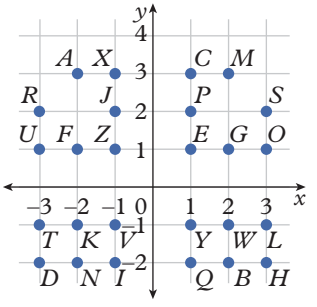


c Reflect the quadrilateral $ABCD$ in the point R.



8 Begin at point (2, 3). Apply the following transformations in order from start to finish and write down the letter at each new point to reveal a hidden phrase.

→ Reflect in the y-axis
→ Rotate 90° anti-clockwise about the origin
→ Translate 4 units right and 3 units up
→ Rotate 90° degrees clockwise about the origin
→ Translate 2 units right and 2 units up
→ Reflect in the y-axis
→ Rotate 180° anti-clockwise about the origin
→ Reflect in the x-axis
→ Rotate 360° clockwise about the origin
→ Translate 5 units left and 2 units down



9 Consider $\triangle ABC$ and the two lines PQ and RS.

a Reflect $\triangle ABC$ in the line PQ.

b Reflect the image of $\triangle ABC$ from part **a** in the line RS.

c Measure the following angles correct to the nearest degree.

i $\angle AXA''$

ii $\angle BXB''$

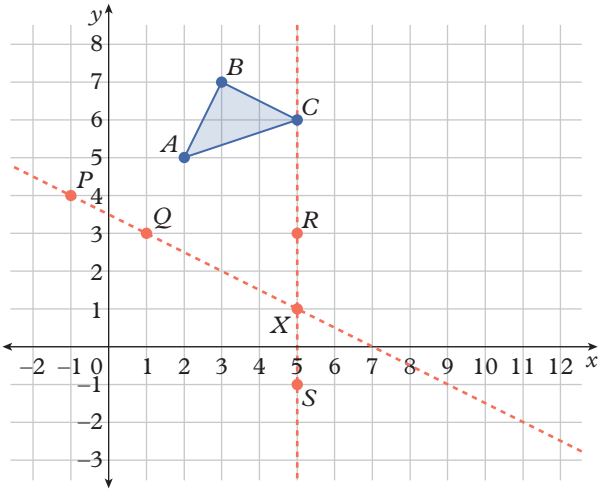
iii $\angle CXC''$

d Measure the following lengths correct to one decimal place.

i AX and XA''

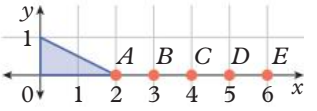
ii BX and BX''

iii CX and XC''



e Describe the single transformation from $\triangle ABC$ to $\triangle A''B''C''$.

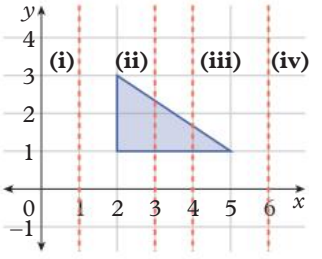
10 Consider the diagram.



a Rotate the original shape 90° clockwise around each point to form five images. For each rotation, start from the original shape.

b Describe how moving the centre of rotation further away from a shape changes the position of the image.

11 Consider the diagram on the right.



a Copy the triangle and reflect it in each of the four mirror lines shown. For each reflection, start from the original image.

b What do you notice about the difference between images created by a mirror line that is on one side of the object as opposed to images created by a mirror line that cuts through the object?

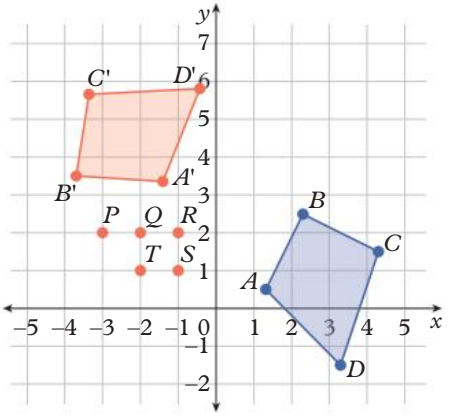
12 Answer true or false for the following statements.

a When a shape undergoes an isometric transformation, the angle and side measurements of the shape stay the same.

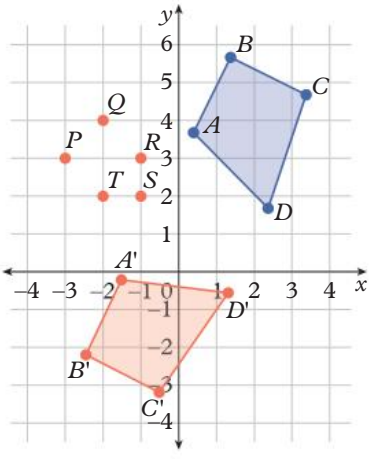
b When a shape undergoes a translation, the orientation of the shape does not change.

CHALLENGE

13 Identify which point the quadrilateral *ABCD* was rotated anti-clockwise about and by what angle to the nearest degree.



14 Identify the two points that lie on the axis of symmetry that the quadrilateral *ABCD* was reflected in.



Check your Student obook pro for these digital resources and more:

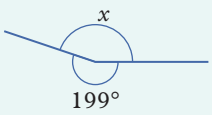
- **Interactive skillsheet** Translations
- **Interactive skillsheet** Reflections
- **Interactive skillsheet** Rotations
- **Investigation** Treasure hunt
- **Topic quiz** 7E

Checkpoint

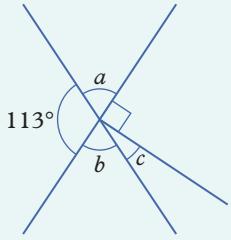
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

7A **1** Determine the value of each of the pronumerals.

a

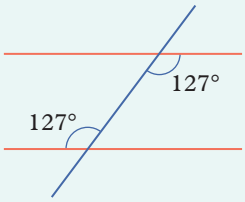


b

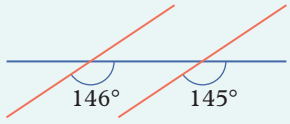


7A **2** Determine if the red lines are parallel or not parallel. State why or why not.

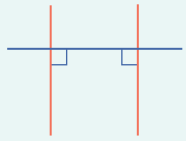
a



b

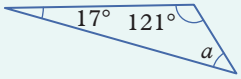


c



7B **3** Determine the size of the missing angles and classify the type of triangle according to its sides and angles.

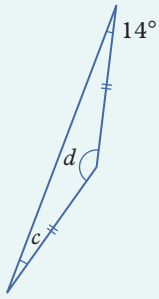
a



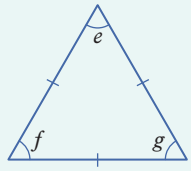
b



c

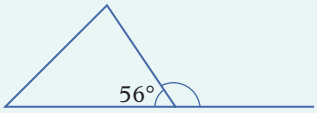


d

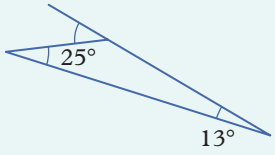


7B **4** Determine the size of the exterior angle marked.

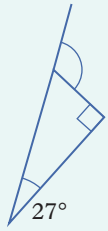
a



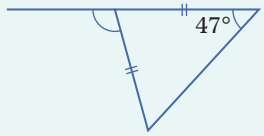
b



c



d



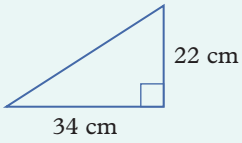
7C **5** Determine whether the following triples could be side lengths of a right-angled triangle.

a (21, 28, 35)

b (24, 121, 123)

7C **6** Determine the length of the missing side, correct to two decimal places.

a



b



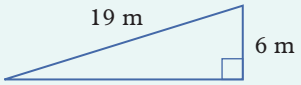
7D **7** Solve the following equations for x.

a $25 = x^2 + 16$

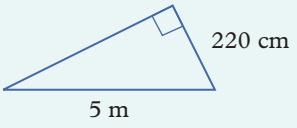
b $13^2 = x^2 + 12^2$

7D **8** Determine the length of the missing side, correct to two decimal places.

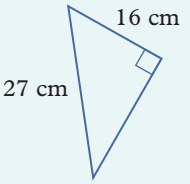
a



b

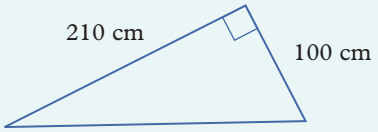


c

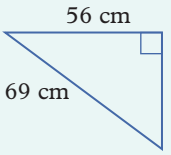


7D **9** Determine the length of the missing side, correct to two decimal places.

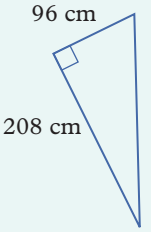
a



b

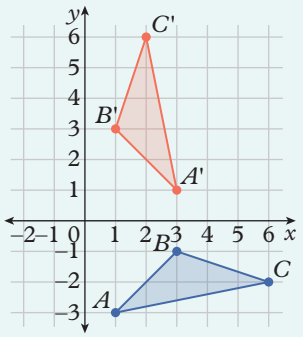


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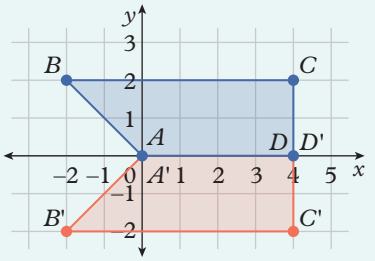


7E **10** Provide a full description of each of the transformations shown.
Include the type of transformation as well as a description of the specific movement.

a

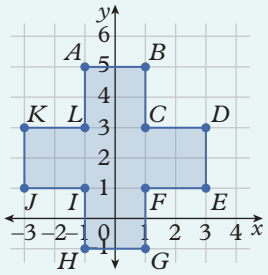


b

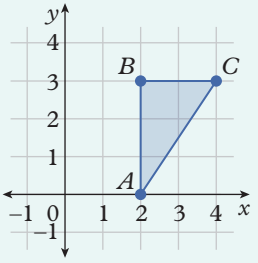


7E **11** Draw the image of the following shapes after the transformation is applied.

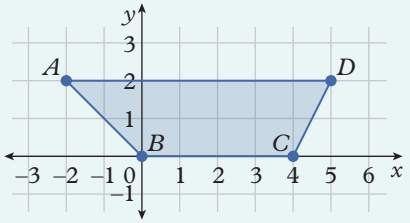
a Translate 5 units down and 2 units right.



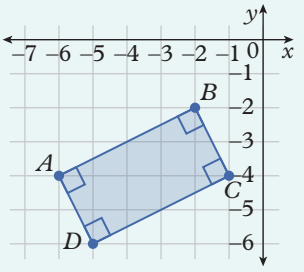
b Rotate 90° clockwise about the origin.



c Reflect in the *y*-axis.



d Rotate 90° clockwise about the origin then translate 6 units right.



OXFORD UNIVERSITY PRESS

7F Congruence

Learning intentions

By the end of this topic you will be able to ...

- ✔ determine if shapes are congruent
- ✔ identify congruent shapes
- ✔ match vertices and sides of congruent pairs.

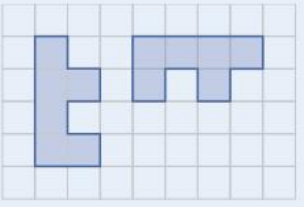
Inter-year links

Support 2D shapes

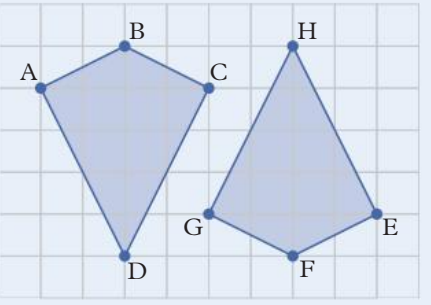
Year 7 8A Polygons

Congruence

- Two figures are **congruent** if they have the same shape and size, or if one shape is the mirror image of the other.
  - → Congruent figures can be in any position or orientation.
  - → Corresponding side lengths and angles are equal in congruent figures.
  - → For example, these shapes are congruent even though they are not in the same orientation.

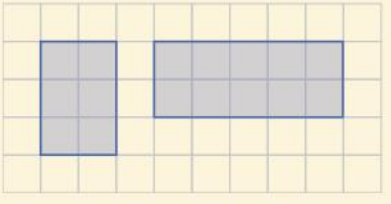


- Isometric transformations (rotations, reflections and translations) produce congruent figures.
- The symbol ≅ is used to indicate congruence. When making a congruence statement, matching vertices must be put in the same order.
  - → For example, ABCD ≅ EFGH.



Example 7F.1 Determining if two shapes are congruent

Determine if these two shapes are congruent.



THINK

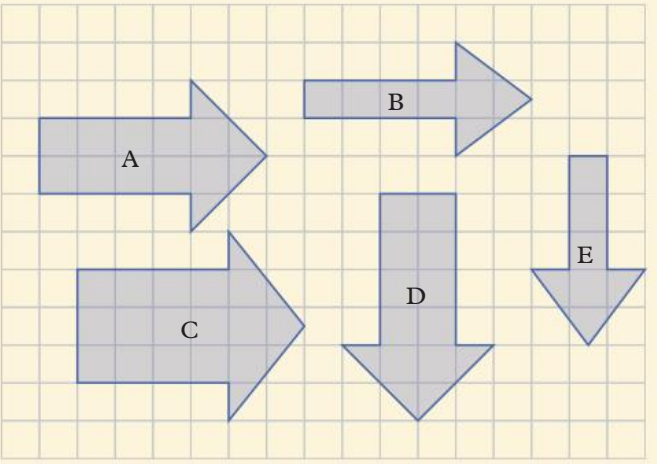
1. Determine if the basic shapes are the same.
2. Check if the corresponding sides are equal.
3. Write your answer.

WRITE

The shapes are not congruent because they do not have equal corresponding sides.

Example 7F.2 Identifying congruent pairs

Identify any pairs of congruent shapes.



THINK

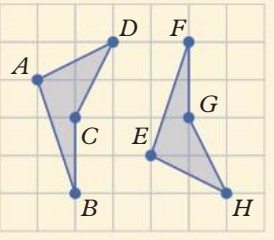
1. Compare the length and width of all the shapes.
2. E is shorter than the other shapes, so it does not have a congruent pair.
3. C is wider than the other shapes, so it does not have a congruent pair.
4. B is thinner than both A and D, so it does not have a congruent pair.
5. Shapes A and D are the same shape and the corresponding sides are equal in length, so they are a congruent pair.

WRITE

Shapes A and D are congruent.

Example 7F.3 Matching sides of congruent pairs

Match the corresponding sides of this pair of congruent quadrilaterals.



THINK

1. Pick a side from the first shape that has a different length from all other sides. For example, side *AB* is the only side that goes down by 3 units and across by 1 unit. Find the corresponding side on the second shape (*EF*).
2. Pick an adjacent side to that chosen in step 1; for example, *BC*. Find the corresponding side on the second shape (*FG*).
3. Continue picking adjacent sides on the first shape and finding the corresponding sides on the second shape until all sides have been matched.

WRITE

AB and *EF*, *BC* and *FG*, *CD* and *GH*, *DA* and *HE*

Helpful hints

✔ Remember that orientation does not matter when determining congruence, but the size of the shapes does matter!

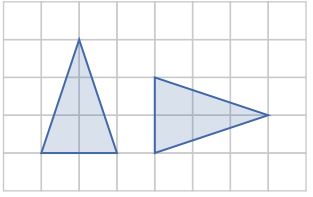
ANS p541

Exercise 7F Congruence

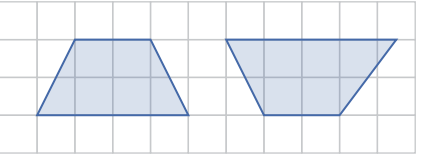
△ 1–5, 6(a, b), 7, 8 ▢ 1–4, 6, 7, 9, 11 ⬠ 2, 3(c, d), 4(b, c), 6(c, d), 8–12

7F.1 **1** Decide whether each pair of shapes is congruent.

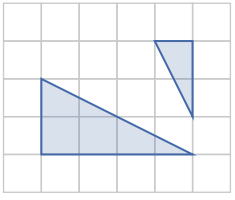
a



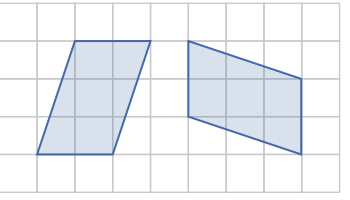
b



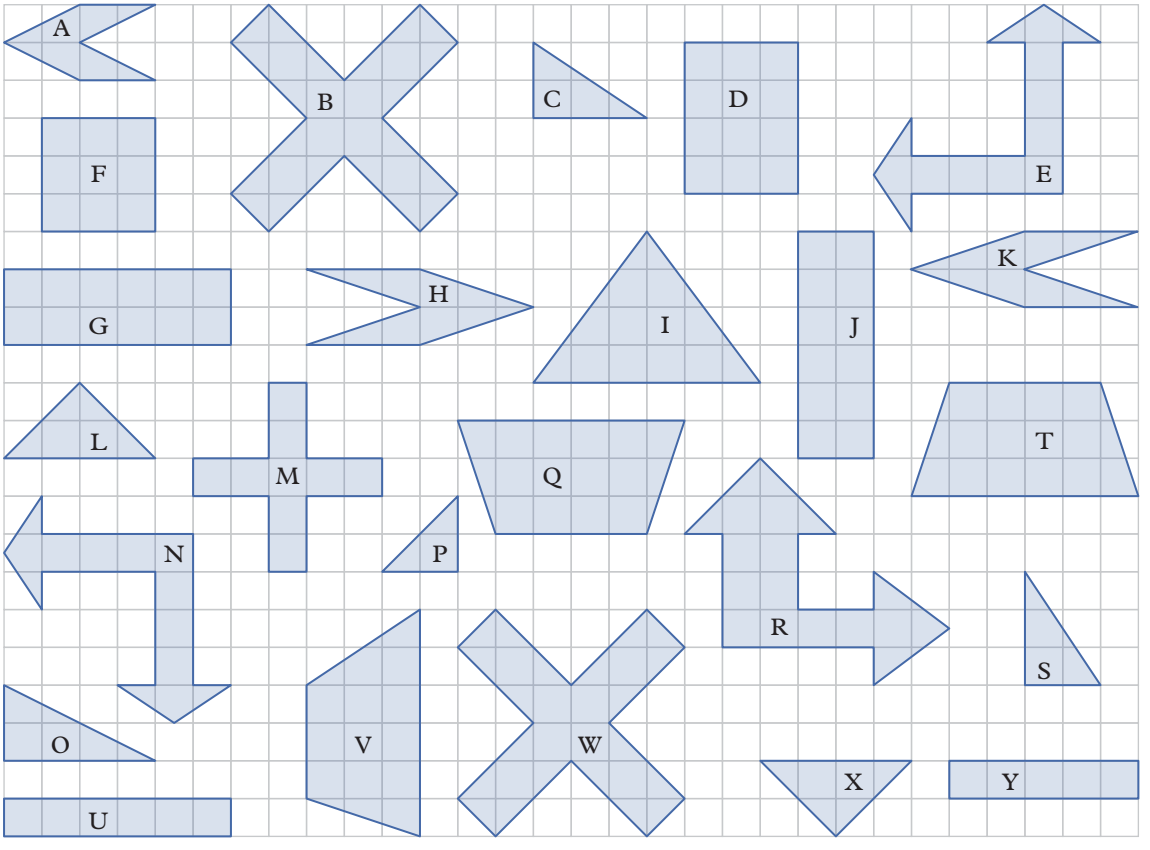
c



d



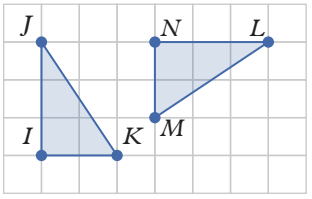
7F.2 **2** Identify any pairs of shapes that are congruent.



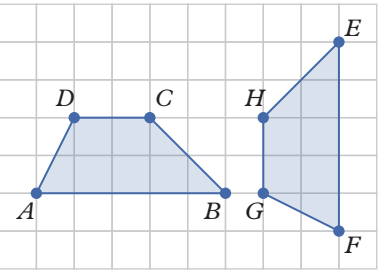
UNDERSTANDING AND FLUENCY

7F.3 **3** Match the corresponding sides of each pair of congruent shapes.

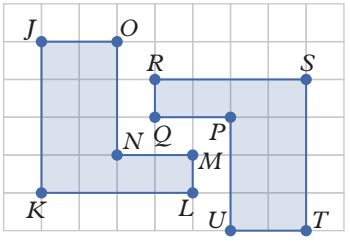
a



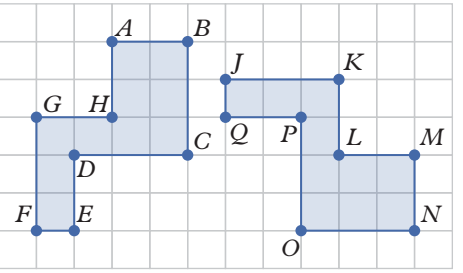
b



c

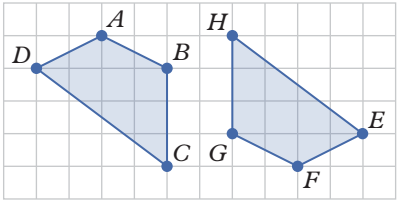


d

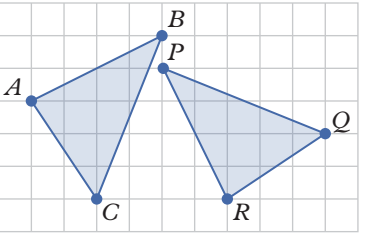


4 Match the corresponding vertices of each pair of congruent shapes.

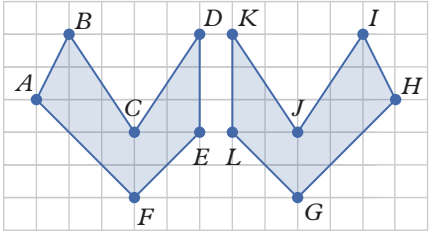
a



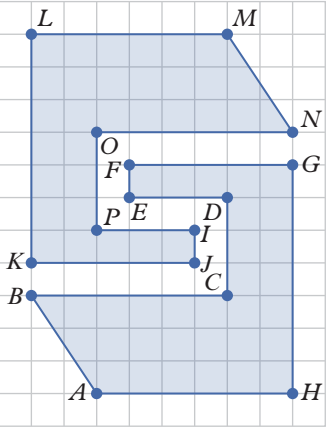
b



c

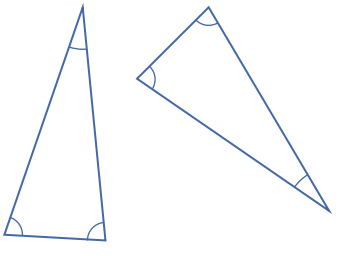


d

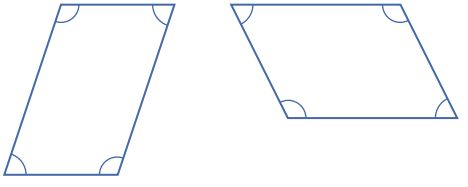


5 For each pair of figures, measure all sides and angles and state if the figures are congruent.

a

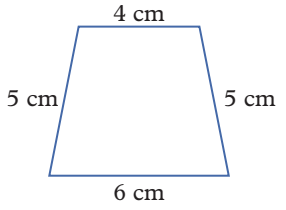


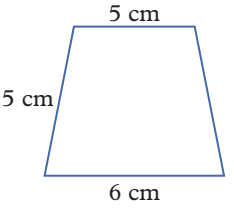
b



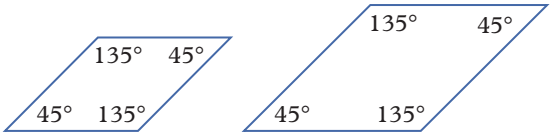
6 Determine if each pair of figures is congruent, justifying your decision.

a

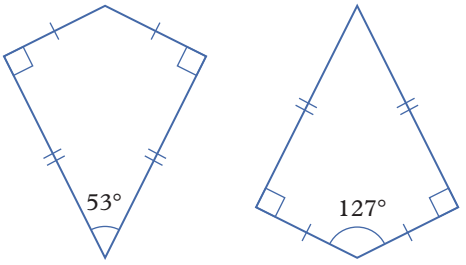




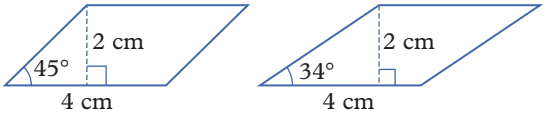
b



c

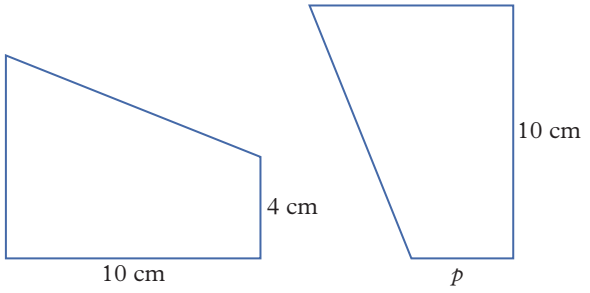


d

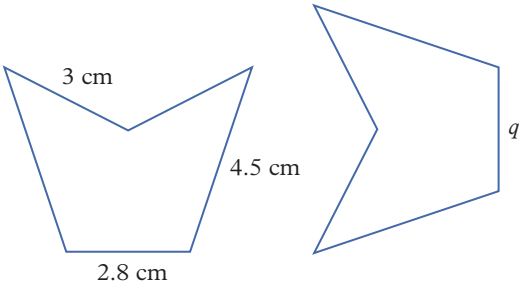


7 Find the value of the pronumerals, given that these pairs of shapes are congruent.

a

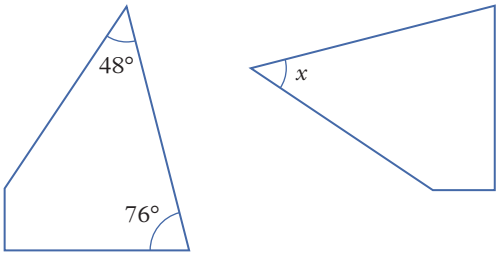


b

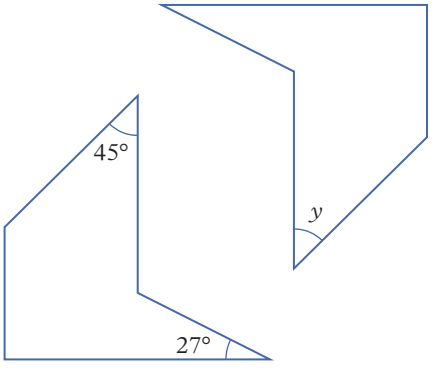


8 Find the value of the pronumerals, given that these pairs of shapes are congruent.

a



b



9 Answer true or false to the following statements, giving reasons for your answers.

- **a** All squares with the same side length are congruent with each other.
- **b** All rectangles with the same side lengths are congruent with each other.
- **c** All triangles with the same base length and height are congruent with each other.
- **d** All parallelograms with the same base length and height are congruent with each other.
- **e** All circles with the same radii are congruent with each other.

10 Explain why it is not possible to match corresponding sides and vertices on congruent squares, rectangles or parallelograms.

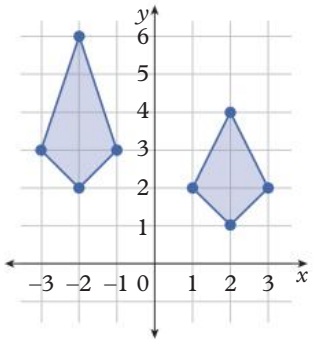
PROBLEM SOLVING AND REASONING

11 For each of the following pairs of shapes:

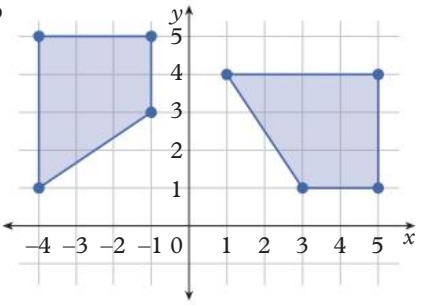
i identify the single transformation that can be applied to the left shape to show if the two shapes are congruent or not congruent

ii state whether the shapes are congruent or not congruent.

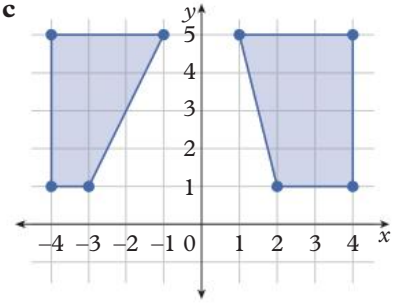
a



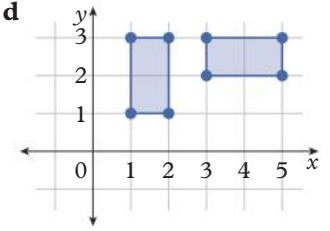
b



c



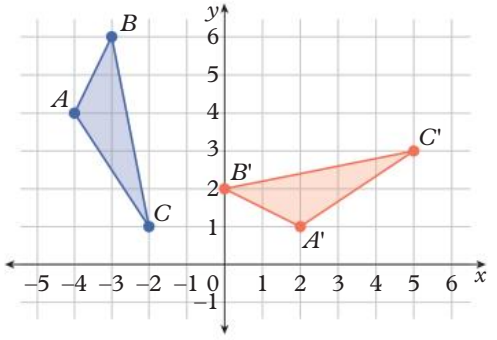
d



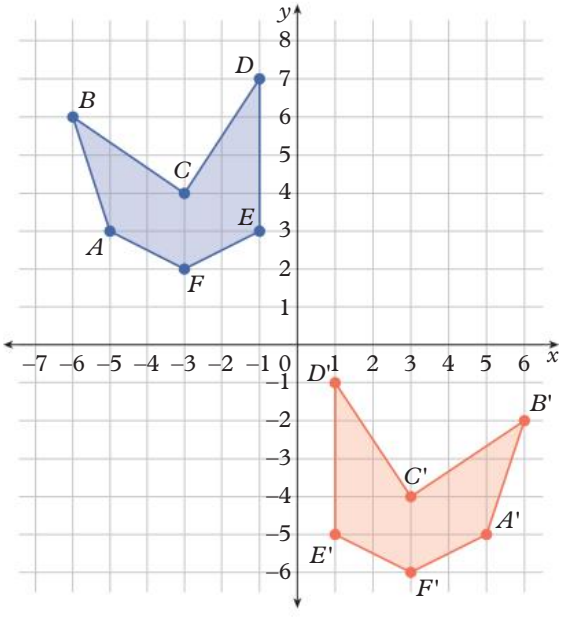
CHALLENGE

12 Describe a series of transformations that can be applied to the left shape to show the two shapes are congruent.

a



b



Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Congruence

Investigation
Tiling patterns in art and design

Topic quiz
7F

7G Congruent triangles

Learning intentions

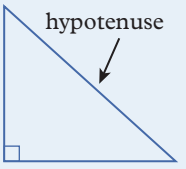
By the end of this topic you will be able to ...

✔ use congruence conditions to determine if two triangles are congruent.

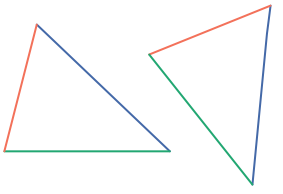
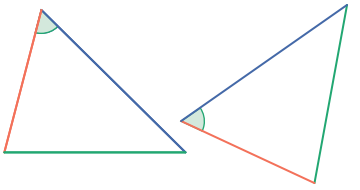
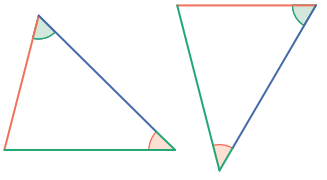
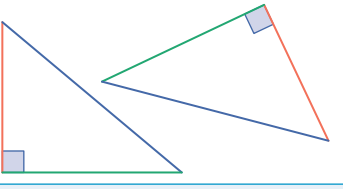
Inter-year links
Year 7 7D Classifying triangles

Congruent triangles

- Two triangles are congruent if all their sides and angles are equal.
- The symbol Δ can be used instead of writing 'triangle'; for example, ΔABC.
- An **included angle** is an angle between two sides.
- The hypotenuse of a right-angled triangle is the side that appears opposite the right angle.



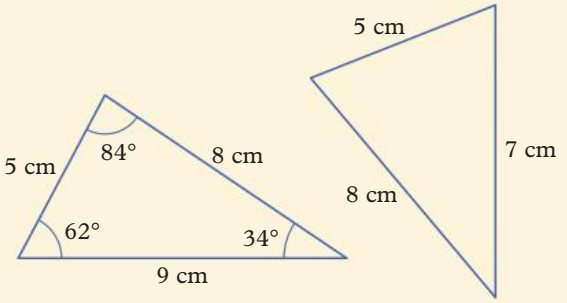
- If a pair of triangles meet any of the following conditions, they are congruent:

Condition	Name	Meaning	Example
SSS	side–side–side	All three side lengths are equal.	
SAS	side–angle–side	Two side lengths and the included angle (between these sides) are equal.	
AAS (or ASA or SAA)	angle–angle–side	Two angles and a corresponding side length are equal.	
RHS	right angle–hypotenuse–side	The hypotenuse and another side length of two right-angled triangles are equal.	

- In the condition AAS, any corresponding side length is sufficient to prove congruence. This rule may also be known as ASA or SAA.
- Other specifications, such as AAA and SSA, are not sufficient to prove congruence.

Example 7G.1 Using the SSS condition

Use the congruence condition SSS to decide if these triangles are congruent.



THINK

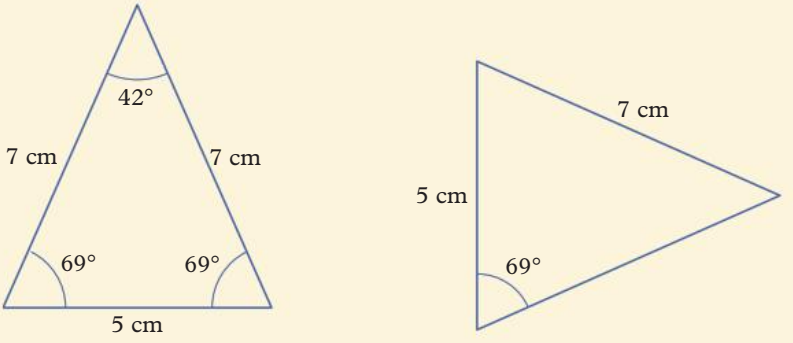
1. SSS means the triangles are congruent if all three side lengths are equal.
2. Go through each of the side lengths on the first shape and check whether there is a matching side length on the second triangle. In the second triangle there is not a matching side length for the 9 cm side in the first triangle.
3. If the condition is met, the triangles are congruent; if the condition fails, the triangles are not congruent.

WRITE

The two triangles are not congruent.

Example 7G.2 Using the SAS condition

Use the congruence condition SAS to decide if these triangles are congruent.



THINK

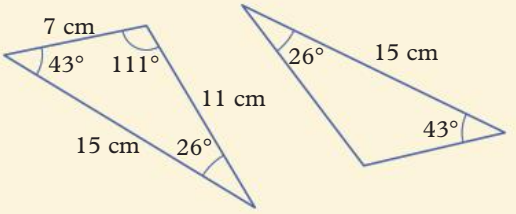
1. SAS means the triangles are congruent if two side lengths and the included angle are equal.
2. Check that there are two matching side lengths on each triangle.
3. Look at the angles between the two matching sides. In the second triangle the angle is not between the sides, so we cannot determine if the SAS condition is met.

WRITE

The two triangles may or may not be congruent.

Example 7G.3 Using the AAS condition

Use the congruence condition AAS to decide if these triangles are congruent.



THINK

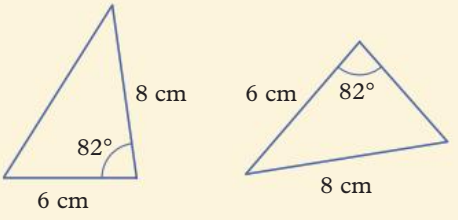
1. AAS means the triangles are congruent if two angles and a corresponding side length are equal.
2. Check that there are two pairs of equal angles.
3. Identify the corresponding sides of each triangle by using the matching angles from step 2.
4. If any two corresponding side lengths are equal, the condition is met, and the triangles are congruent.

WRITE

The two triangles are congruent.

Example 7G.4 Determining if two triangles are congruent

Decide if this pair of triangles is congruent, giving a reason for your answer.



THINK

1. Identify what information is provided in each triangle. Both triangles contain two equal sides and an equal angle.
2. Check whether any of the conditions for congruence are met. The first triangle meets the SAS condition; however, the angle is not between corresponding sides in the second triangle.
3. Provide the answer.

WRITE

The given information is insufficient to determine whether the triangles are congruent or not.

Helpful hints

- ✔ When using the SAS condition, make sure the provided angle is between the corresponding sides!
- ✔ When using the AAS condition, make sure that the matching side lengths are corresponding sides!
- ✔ When there is a pair of right-angled triangles, if the information for the RHS condition is not provided remember to check if there is sufficient information for either the SAS or AAS condition.

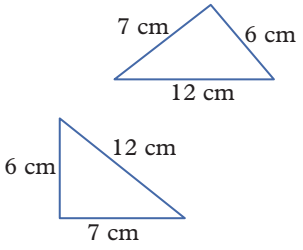
ANS p542 Exercise 7G Congruent triangles

1–3, 5, 6, 7(a, b), 9, 10 | 1–4, 5(c, d), 6, 8, 10, 11, 12(a, b) | 1–3, 5(c, d), 6, 10–13

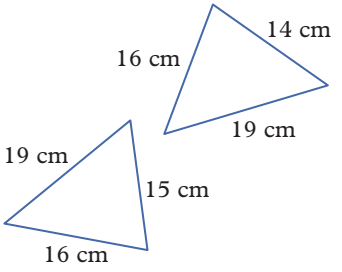
UNDERSTANDING AND FLUENCY

7G.1 **1** Use the congruence condition SSS to decide if these triangles are congruent.

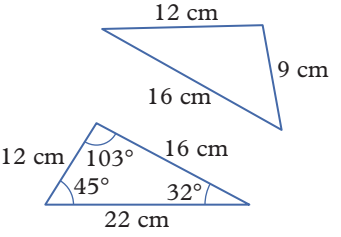
a



b

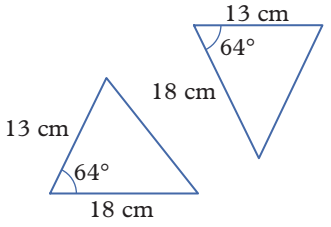


c

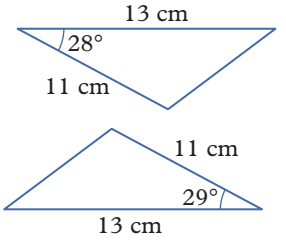


7G.2 **2** Use the congruence condition SAS to decide if these triangles are congruent.

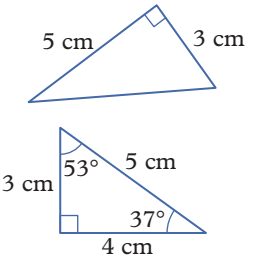
a



b

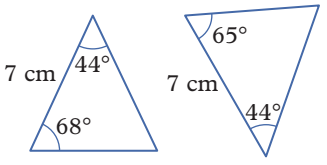


c

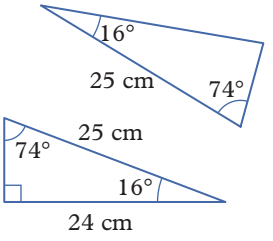


7G.3 **3** Use the congruence condition AAS to decide if these triangles are congruent.

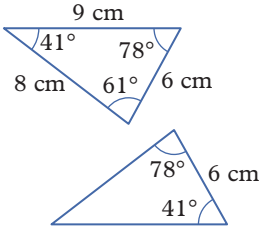
a



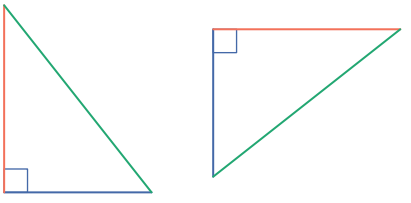
b



c



4 The congruence condition RHS only applies to right-angled triangles. In every right-angled triangle the longest side (known as the hypotenuse) appears opposite the right angle.



a Why must the hypotenuse be one of the sides for this condition? Hint: If the right angle is between the two sides, what condition does it become?

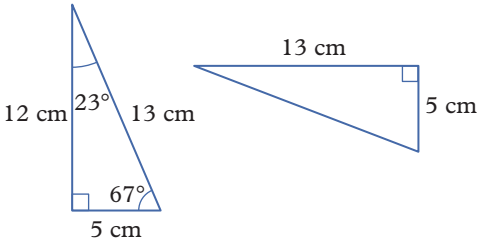
When the triangle is right-angled and you know the length of two sides, the length of the third side can be calculated using Pythagoras' theorem.

b If you calculated the length of the third side, which condition would this now be? Hint: You would know the lengths of all the sides.

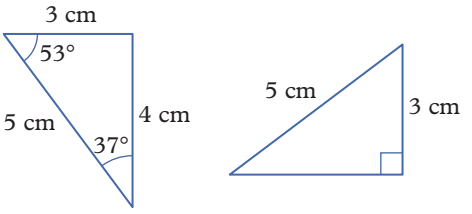
c How does this show that RHS is a condition for congruence?

5 Use the condition RHS to decide if these pairs of triangles are congruent.

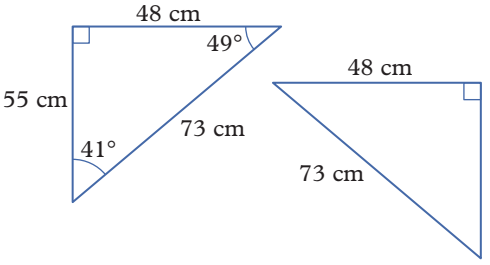
a



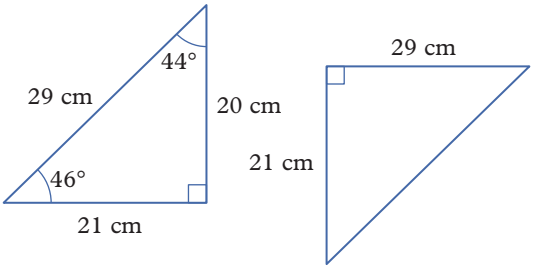
b



c

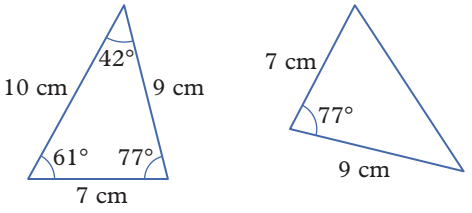


d

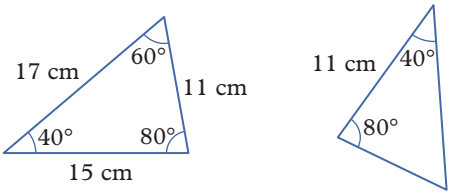


7G.4 **6** Decide if these pairs of triangles are congruent, giving a reason for your answer.

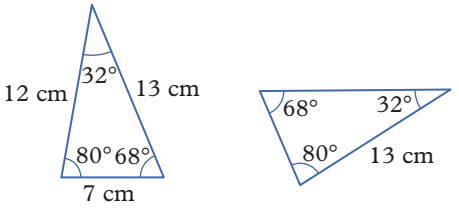
a



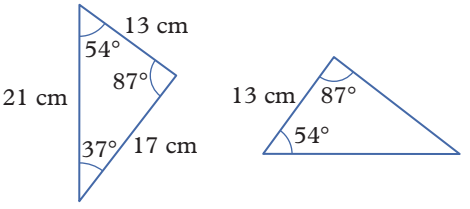
b



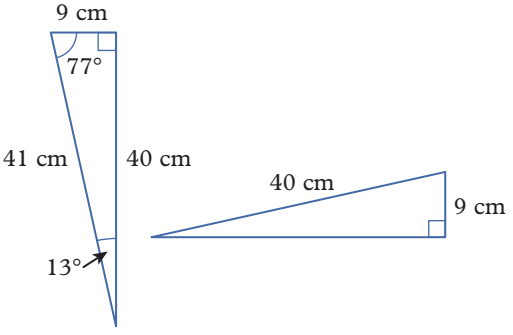
c



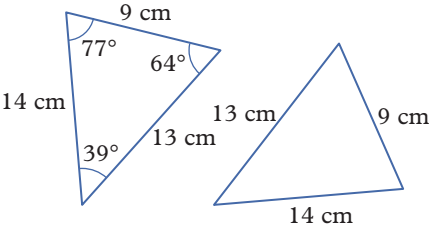
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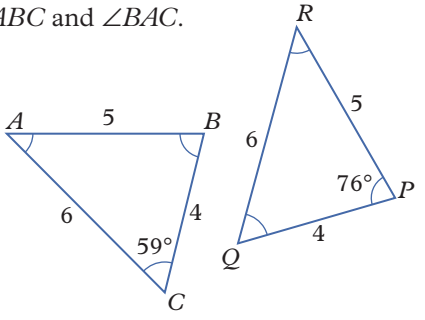


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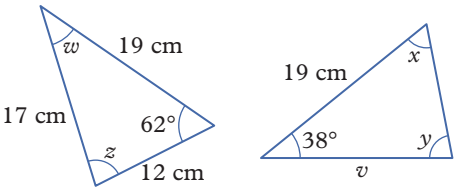
7 **a** State the condition that explains why this pair of triangles is congruent.

b Determine the size of the angles $\angle ABC$ and $\angle BAC$.

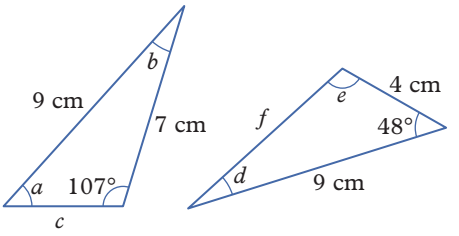


8 Each of these pairs of triangles are congruent.

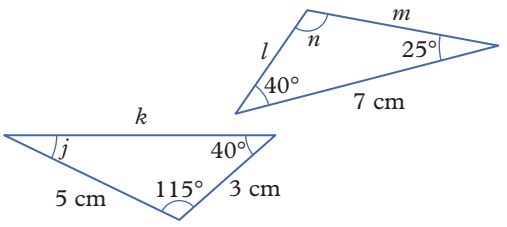
a



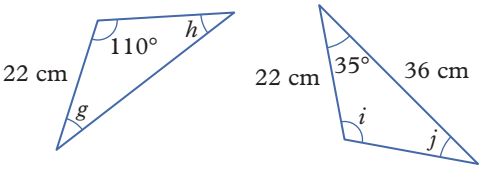
b



c



d



i Find the unknown side lengths and angles in each triangle.

ii State the condition(s) that you used to solve each problem.

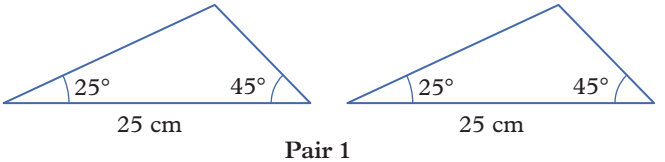
9 Another condition for congruence is ASA.

a If you know two of the angles in a triangle, how do you find the third angle?

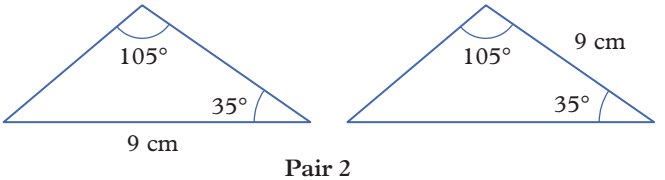
b Using your answer to part **a**, explain how you know that two triangles that have two pairs of equal angles must meet the specification AAA.

ASA refers to two matching pairs of angles and the pair of sides in between.

c Explain why pair 1 meets the condition ASA but pair 2 does not.



Pair 1



Pair 2

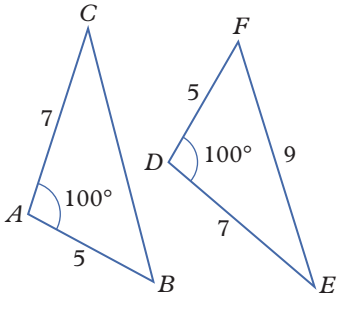
d Can you explain how, if a pair of triangles meet the condition ASA, they also meet the condition AAS? Hint: You need to find the third angle of the triangles.

10 For each of the following:

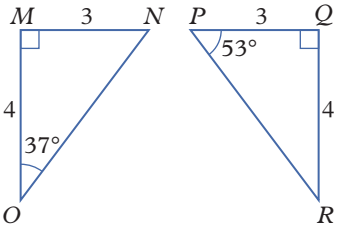
i explain why the given condition is or is not able to show the triangles are congruent

ii if the condition cannot show the triangles are congruent, state a condition that can.

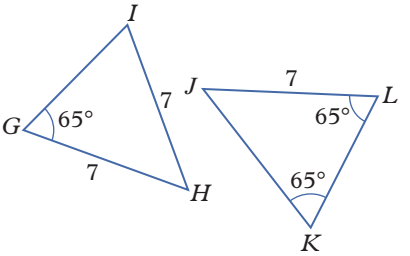
a SSS



b RHS



c AAS

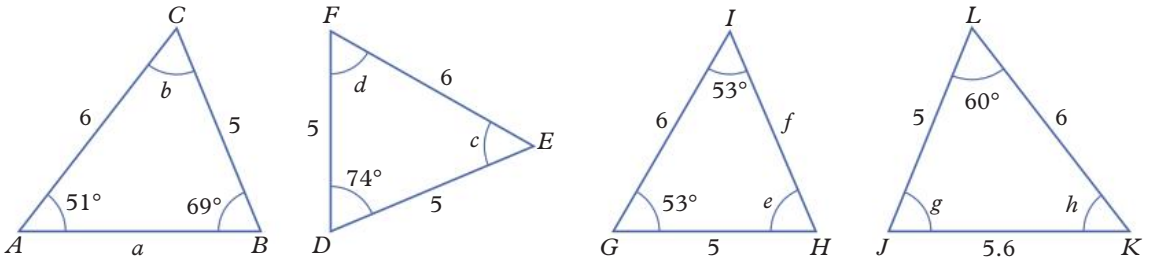


11 For each group of triangles,

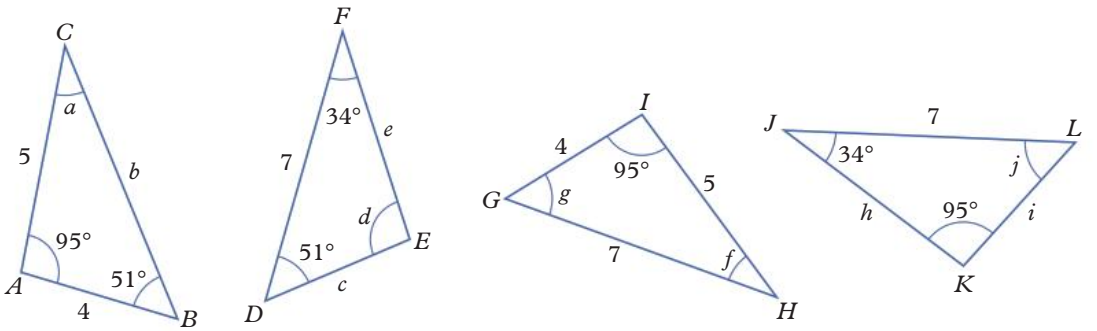
i determine which triangles are congruent

ii determine the values of the pronumerals.

a



b

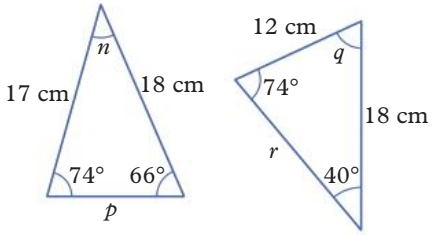


12 For each pair of triangles below:

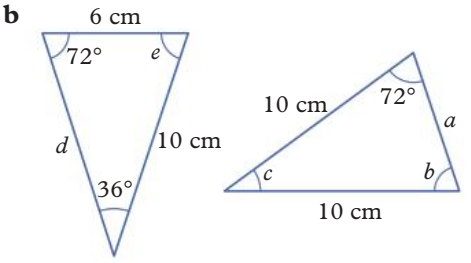
i decide if they are congruent, stating which condition you used

ii find the value of the pronumerals if the pair is congruent.

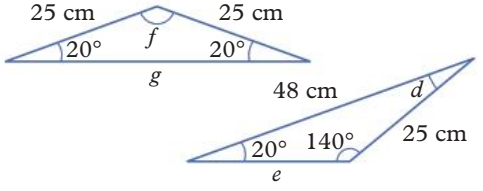
a



b



c



13 There are up to 12 values that can be given for a pair of congruent triangles (six angles and six side lengths, three each per triangle). SSS, SAS, AAS and RHS all require at least six values to be given. What is the largest number of values that can be given about two congruent triangles and still not be certain the two are congruent, without determining other lengths or angles? Provide an example pair of triangles using hatch marks and arcs to label equal lengths and angles.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Congruent triangles

Investigation Congruence in construction

Topic quiz 7G

PROBLEM SOLVING AND REASONING

CHALLENGE

7H Dilations and similar figures

Learning intentions

By the end of this topic you will be able to ...

- ✔ determine scale factors
- ✔ find unknown lengths in similar figures
- ✔ perform dilations.

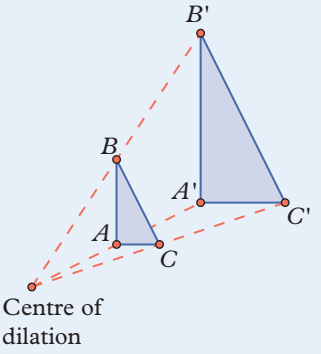
Inter-year links

Year 7 — 3B Equivalent fractions

Year 9 — 6C Dilations and similar figures

Dilations

- A **dilation** is a transformation that changes the size of an object. For a dilation, the shape and orientation of the image remains the same as the original shape.
  - → The angle measurements of a shape do not change after a dilation.
- A dilation can be either an **enlargement** or a **reduction**.
  - → An enlargement creates an image that is larger than the original shape.
  - → A reduction creates an image that is smaller than the original shape.
- A **centre of dilation** and **scale factor** need to be specified to perform a dilation.



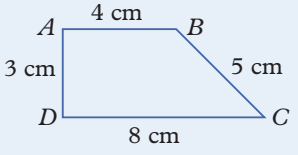
- The scale factor can be found using the formula:

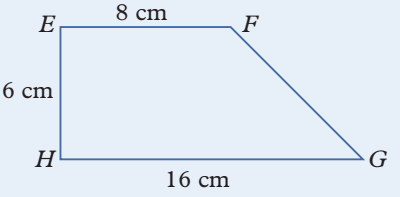
  $$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

  - → If the scale factor is between 0 and 1, the image will be a reduction.
  - → If the scale factor is greater than 1, the image will be an enlargement.
- The side length of a dilation can be found by multiplying the original side length by the scale factor.
  - → For example, when dilating a shape with a side AB:
    Length of $A'B'$ = length of AB × scale factor.

Similar figures

- Two figures are said to be **similar** if they are the same shape. Similar figures do not need to be the same size.
  - → An image produced by dilation will be a similar figure with the original shape. For example, $ABCD$ and $EFGH$ are similar figures.

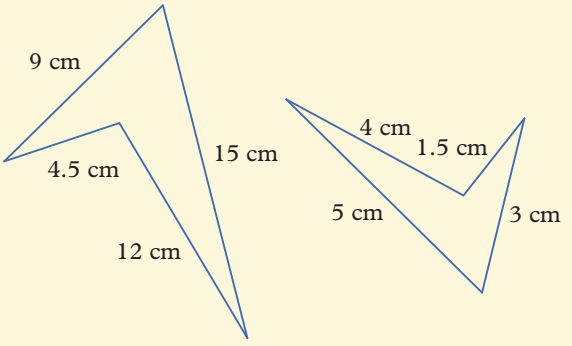




- If two figures are similar, all corresponding angles are equal in size and all corresponding sides will be in the same ratio.
- If the scale factor between two similar figures is 1, then the two figures are congruent.
- The symbol ~ can be used to indicate similarity. When making a similarity statement, corresponding vertices must be put in the same order.
  → For example, *ABCD* ~ *EFGH*.
- If two figures are known to be similar, an unknown side length can be found by using the scale factor.

Example 7H.1 Finding the scale factor

Find the scale factor for this pair of similar figures if the figure on the left is the original shape.



THINK

1 Identify two corresponding sides. The longest side of the original figure is 15 cm and the longest side of the image is 5 cm.

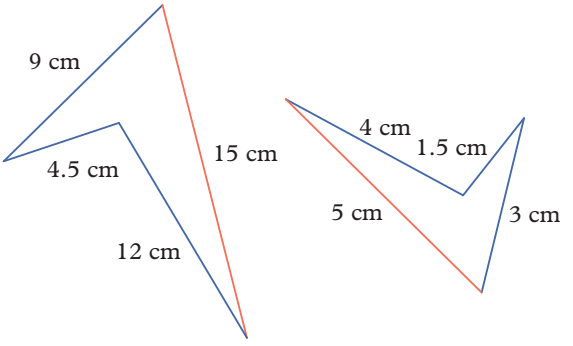
2 Substitute these lengths into the formula:

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

3 Match the remaining corresponding sides and check that each pair of sides is in the same ratio.

4 Check that the scale factor seems reasonable (between 0 and 1 means a reduction).

WRITE



$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{5 \text{ cm}}{15 \text{ cm}}$$
$$= \frac{1}{3}$$

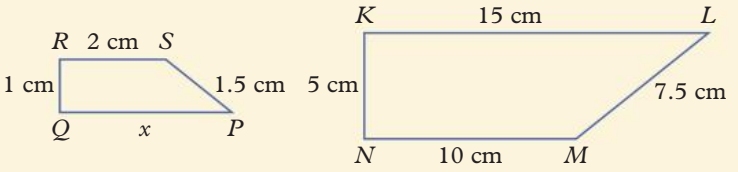
Check each corresponding pair of sides:

$$\text{Scale factor} = \frac{3}{9} = \frac{1.5}{4.5} = \frac{4}{12} = \frac{1}{3}$$

The scale factor of $\frac{1}{3}$ is a reduction, as required.

Example 7H.2 Finding an unknown side length in similar figures

Find the value of x, given that these two figures are similar.



THINK

1. Identify two corresponding sides and calculate the scale factor. Use corresponding vertices to write the side names in the same order. As R corresponds to N and S corresponds to M, then side RS corresponds to side NM (not MN).
2. Find a pair of corresponding sides that involve x.
3. Substitute in the known values and solve for x.

WRITE

Sides RS and NM are corresponding.

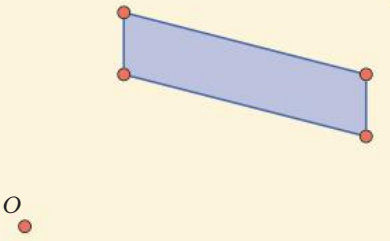
$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{10 \text{ cm}}{2 \text{ cm}}$$
$$= 5$$

Side QP has length x. Sides QP and KL are corresponding.

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$5 = \frac{15}{x}$$
$$5 \times \frac{x}{5} = \frac{15}{x} \times \frac{x}{5}$$
$$x = \frac{15}{5}$$
$$x = 3 \text{ cm}$$

Example 7H.3 Performing a dilation

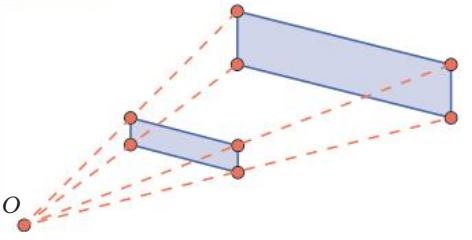
Dilate this figure by a scale factor of $\frac{1}{2}$ using the centre of dilation O.



THINK

1. Use dashed lines to connect the centre of dilation with each of the vertices on the original shape.
2. The scale factor is $\frac{1}{2}$ so measure half the distance from the centre of dilation to each of the vertices on the original shape and mark these points. These are the corresponding vertices of the image.
3. Label the vertices of the image and join them together to complete the dilation.

WRITE



Helpful hints

✔ The image from a dilation will always be in the same orientation as the original shape; however, similar shapes do not need to be in the same orientation.

ANS p542

Exercise 7H Dilations and similar figures

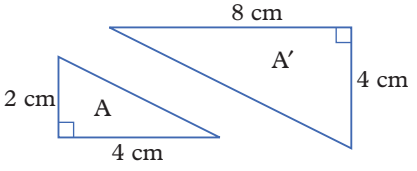
1–5, 6(a–c), 7, 10–12 | 2–5, 6(d–f), 11–13, 18 | 3–5, 6(e, f), 8–10, 12, 14–17

1 Decide whether each scale factor produces an enlargement or a reduction.

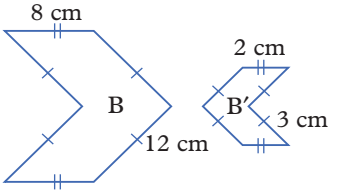
a 4 **b** $\frac{1}{3}$ **c** 2 **d** 6 **e** $\frac{1}{10}$ **f** $\frac{1}{5}$

7H.1 **2** Find the scale factor for each pair of similar figures.

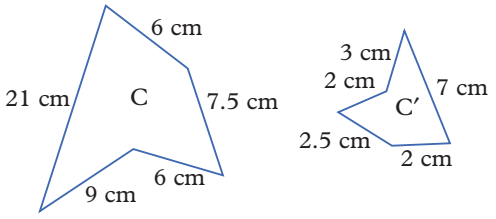
a



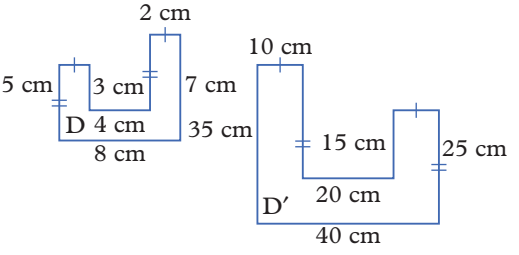
b



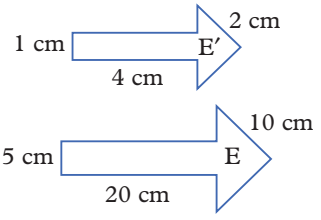
c



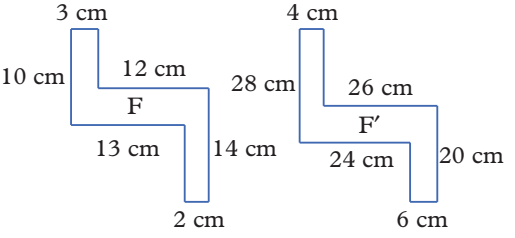
d



e

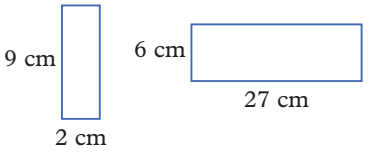


f

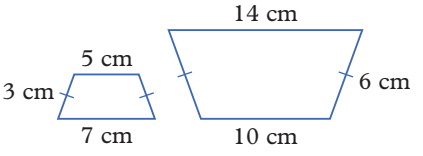


3 Find the scale factor for the similar figures if the figure on the left of each pair is the original shape.

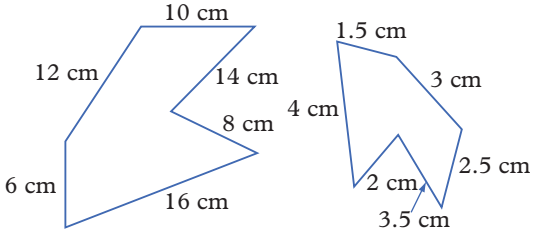
a



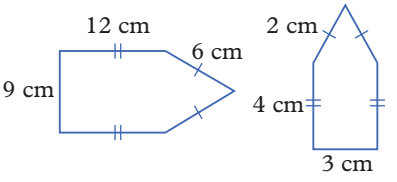
b



c



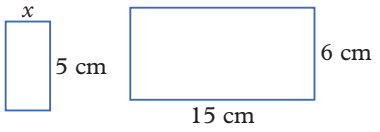
d



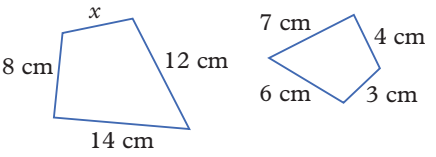
UNDERSTANDING AND FLUENCY

7H.2 **4** Find the value of x, given that these pairs of shapes are similar.

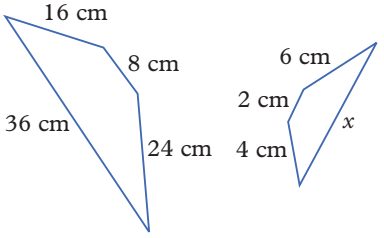
a



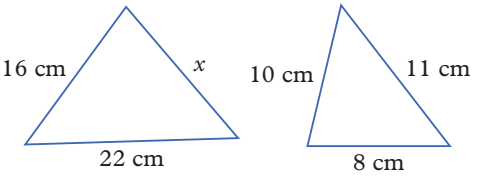
b



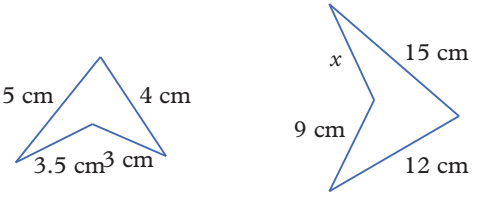
c



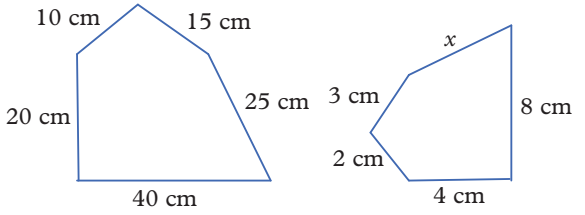
d



e



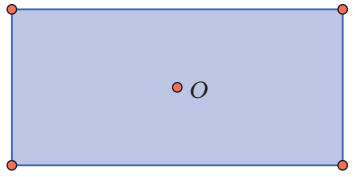
f



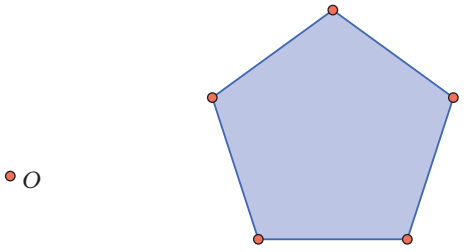
7H.3 **5** Use the worksheet available on your obook pro to dilate each figure using the centre of dilation O and a scale factor of:

i 3 **ii** $\frac{1}{3}$

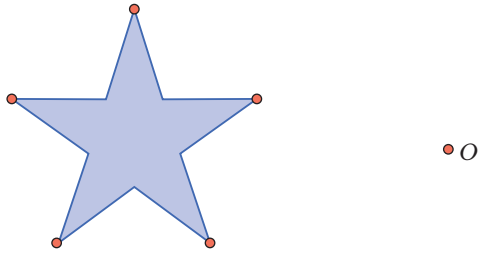
a



b

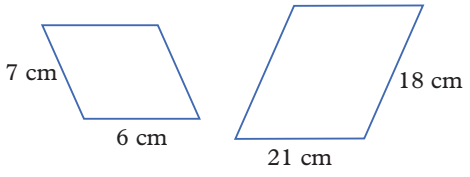


c

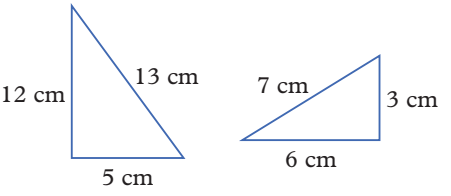


6 Decide whether each pair of shapes is similar by calculating and comparing the scale factor for each pair of corresponding sides.

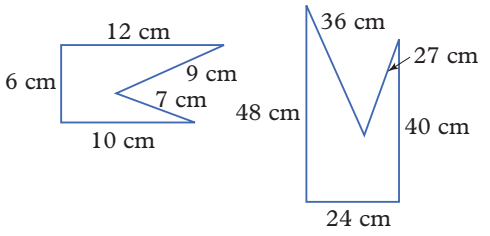
a



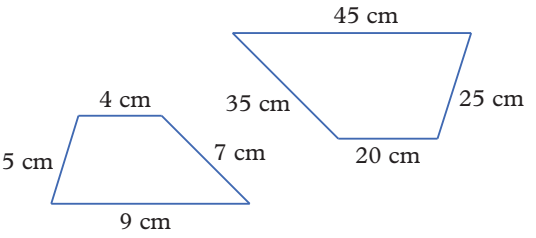
b



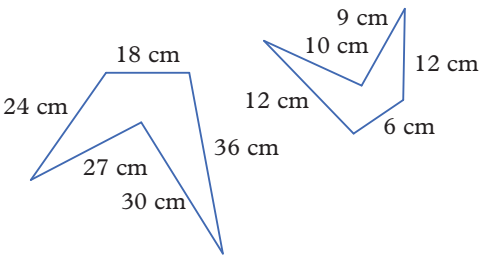
c



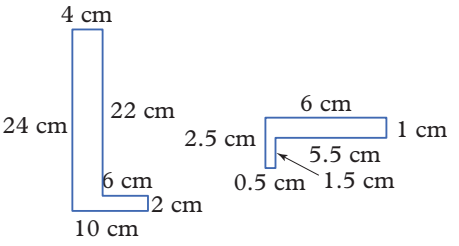
d



e



f



7 A microscope with a scale factor of 20 000 was used to view a bacterial cell, which appeared 5.2 cm long. What was the actual length of the bacterial cell?

8 Explain why a circle is similar to all other circles, why a square is similar to all other squares, but a rectangle is not similar to all other rectangles.

9 Finn was explaining a dilation to a classmate and said that a dilation of $\frac{2}{3}$ was the same as dilating by a factor of 2 and then dilating by a factor of $\frac{1}{3}$. Can you explain why he is correct?

10 Explain the difference between congruent figures and similar figures.

11 One standard print size for photographs is 10 cm × 15 cm. Alice wanted to enlarge a photograph to place it onto a canvas measuring 40 cm × 50 cm. Are the photograph and its corresponding canvas art similar figures?

12 A ratio can be used instead of a fraction to describe a dilation. For example, a figure that is 10 times as small as the original has a scale factor of $\frac{1}{10}$, which is the ratio 1 : 10.

a How would a scale factor of $\frac{1}{2}$ be written as a ratio?

b Explain the relationship between the fraction and the ratio. Hint: What parts of the ratio relate to the numerator and denominator?

c Consider an enlargement with a scale factor of 2. What is 2 written as a fraction?

d Use your answer to part **c** to write a scale factor of 2 as a ratio.

e Write these ratios as scale factors.

i 1 : 4 **ii** 1 : 100 **iii** 1 : 2500

iv 3 : 4 **v** 2 : 1 **vi** 7 : 2

f Explain how you can tell if a similar figure is an enlargement or reduction by the order of the ratio.

13 Timothy got a model plane for his birthday, which had a scale factor of 1 : 100.



a What does this scale factor mean?

b If the real-life plane has a length of 34 m, what is the length of the model plane?

c If the model plane has a wingspan of 27 cm, what is the wingspan of the real-life plane?

14 A model car is constructed using a ratio of 1 : 40.



a If the model car has a length of 10 cm, what is the length of the real car?

b If the real car has a height of 1.5 m, what is the height of the model car?

c Do you think that a model car can be a true similar figure to a real car? Explain.

15 Marc has a hiking map with a ratio scale of 1 : 75 000.

a Marc planned to walk the length of a 15 km track. What distance would this be on the map?

b At lunchtime, he had walked a distance shown as 7 cm on the map. How far did he have left to walk on the track?

16 Elliot drew a regular hexagon with side lengths of 3 cm.

a What is the difference between a regular and an irregular hexagon?

b If Elliot drew another regular hexagon with different side lengths, explain how you know without measuring anything that the two hexagons must be similar.

CHALLENGE

17 Emily bought Timothy a model plane to go into a special glass case that allows for a maximum height of 15 cm.

a If the plane was 30 m tall in real life, what maximum scale factor could be applied to the model plane?

When the plane arrives, Emily finds that the plane actually measures 16 cm in height.

b What scale factor has actually been applied to the model plane?

Assuming that all dimensions must stay in the same ratio, it is important that the plane fits into the glass case.

c What scale factor would the model need to be dilated by to fit into the space? Hint: Which measurement is the original and which is the image?

d What scale factor would the space need to be dilated by to fit the current model?

e How are the answers to parts **c** and **d** different and how are they similar?



18 This table shows the dimensions of standard paper sizes, rounded to the nearest millimetre. Use the table to decide whether any or all of the following sizes are similar figures. Compare:

a sizes within the A group (e.g. A2, A4 and A7)

b sizes within the B group (e.g. B0, B3 and B6)

c sizes within the C group (e.g. C1, C5 and C8)

d sizes between different groups (e.g. A0, B0 and C0).

	A sizes (mm)	B sizes (mm)	C sizes (mm)
0	841 × 1189	1000 × 1414	917 × 1297
1	594 × 841	707 × 1000	648 × 917
2	420 × 594	500 × 707	458 × 648
3	297 × 420	353 × 500	324 × 458
4	210 × 297	250 × 353	229 × 324
5	148 × 210	176 × 250	162 × 229
6	105 × 148	125 × 176	114 × 162
7	74 × 105	88 × 125	81 × 114
8	52 × 74	62 × 88	57 × 81
9	37 × 52	44 × 62	40 × 57
10	26 × 37	31 × 44	28 × 40

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Similar figures

Worksheet
Dilations

Topic quiz
7H

7I Similar triangles

Learning intentions

By the end of this topic you will be able to ...

- ✔ find unknown angles and lengths in similar triangles
- ✔ determine whether two triangles are similar.

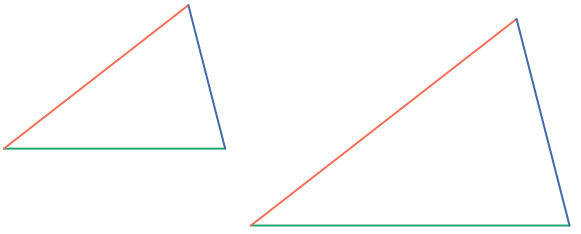
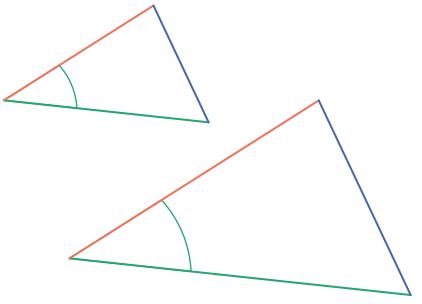
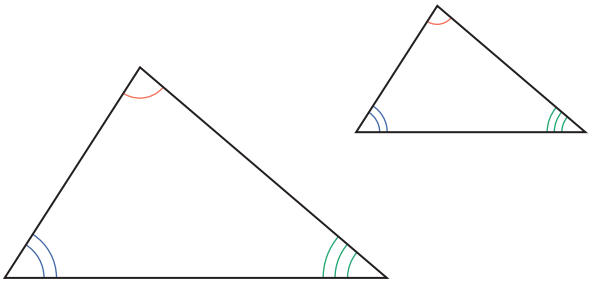
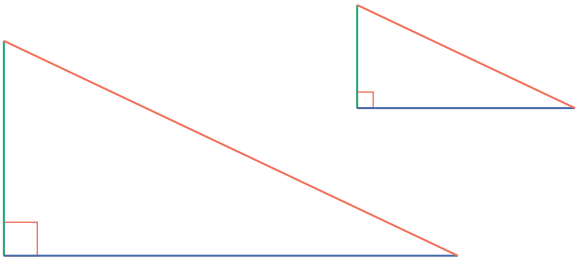


Inter-year links

Year 7 7D Classifying triangles

Similar triangles

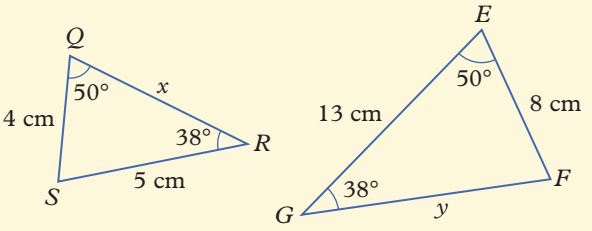
- Two triangles are similar if:
  - → corresponding pairs of angles are equal
  - → corresponding pairs of side lengths are in the same ratio.
- If a pair of triangles meets any of the following conditions, they are similar:

Condition	Meaning	Example
SSS	All three pairs of corresponding side lengths are in the same ratio.	
SAS	Two corresponding pairs of side lengths are in the same ratio and the included angles are equal.	
AAA	All three pairs of corresponding angles are equal.	
RHS	The lengths of the pair of hypotenuses are in the same ratio as another pair of sides in a right-angled triangle.	

- In the condition SAS the matching angles must be between the corresponding pairs of side lengths to demonstrate similarity.
- Other specifications, such as SSA and AAS, are not sufficient to demonstrate similarity.
- The similarity conditions are the minimum set of conditions required to demonstrate similarity. In some questions more information may be provided, however if any of the conditions are met then a pair of triangles will be similar.
- If a pair of triangles are congruent then they will also be similar (with a scale factor of 1).

Example 7I.1 Finding unknown side lengths in similar triangles

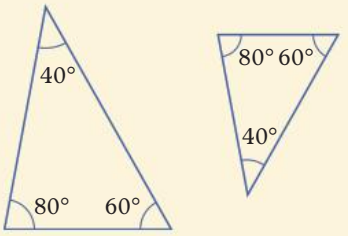
Find the value of the unknown side lengths in this pair of similar triangles.



THINK	WRITE
1 Identify the equal corresponding angles and then match the corresponding sides opposite these angles. Sides QS and EF are both opposite the 38° angle. Sides RS and GF are both opposite the 50° angle.	
2 Using the pair of corresponding sides that have known measurements (QS and EF), find the scale factor between $\triangle QSR$ and $\triangle EFG$. Consider $\triangle QSR$ as the original triangle and $\triangle EFG$ as the image.	Corresponding pairs: $QS = 4$ cm and $EF = 8$ cm Scale factor $= \frac{\text{image length}}{\text{original length}}$ $= \frac{8}{4}$ $= 2$
3 Use the scale factor formula to find y in $\triangle EFG$.	Corresponding pairs: $RS = 5$ cm and $GF = y$ Scale factor $= \frac{\text{image length}}{\text{original length}}$ $2 = \frac{y}{5}$ $y = 2 \times 5$ $y = 10$ cm
4 Use the scale factor formula to find x in $\triangle QSR$.	Corresponding pairs: $QR = x$ and $EG = 13$ cm Scale factor $= \frac{\text{image length}}{\text{original length}}$ $2 = \frac{13}{x}$ $x = \frac{13}{2}$ $x = 6.5$ cm

Example 7I.2 Identifying similarity conditions

Which condition would you use to check that these triangles are similar?



THINK

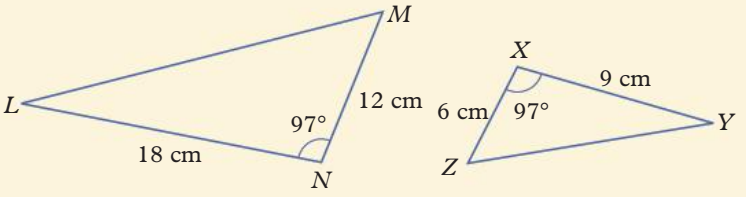
Look at the information provided in each triangle. All angles are given in both triangles.

WRITE

The similarity condition AAA can be used to check whether these triangles are similar.

Example 7I.3 Determining if two triangles are similar

Determine whether these two triangles are similar.



THINK

1 Both triangles show two sides and an included angle, so we can use the similarity condition SAS to determine whether the two triangles are similar.

2 Calculate the scale factor between the two shorter sides (MN and XZ) and the two longer sides (LN and XY).

3 Compare the scale factors to determine whether the similarity condition is met.

4 Write your final answer.

WRITE

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{XZ}{MN}$$
$$= \frac{6\text{ cm}}{12\text{ cm}}$$
$$= \frac{1}{2}$$

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$
$$= \frac{XY}{LN}$$
$$= \frac{9\text{ cm}}{18\text{ cm}}$$
$$= \frac{1}{2}$$

The two triangles are similar as they meet the similarity condition SAS.

Helpful hints

✔ Make sure that you identify the corresponding sides and angles correctly. This is a key skill. Remember that the given triangles may not be in the same orientation.

Exercise 7I Similar triangles

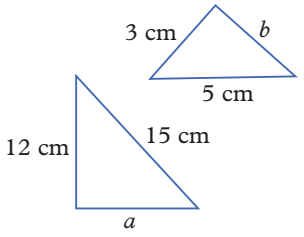
ANS p543

1, 2, 4–6, 9, 11, 12, 14 | 1, 3–5, 7, 8, 10, 13, 15, 16 | 1, 4, 5, 7, 11, 13, 14, 16–19

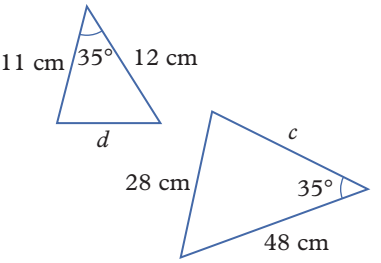
UNDERSTANDING AND FLUENCY

7I.1 **1** Find the values of the unknown side lengths in each pair of similar triangles.

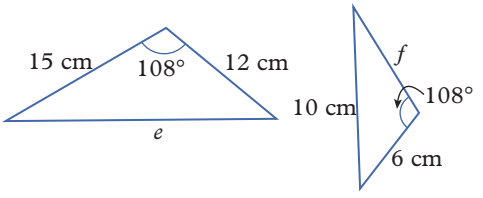
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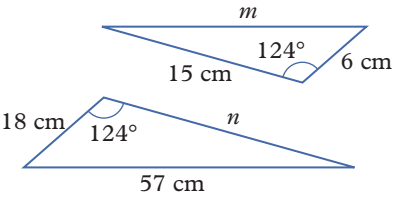
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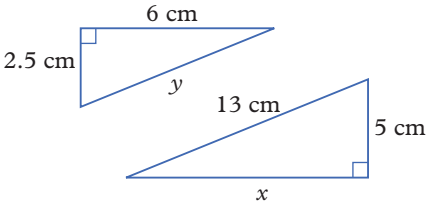
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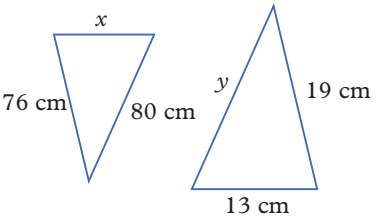
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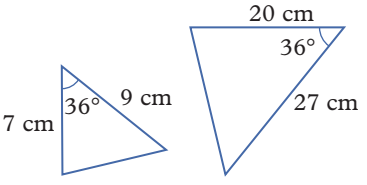


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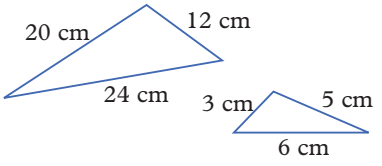


7I.2 **2** State the condition you would use to decide whether each pair of triangles is similar.

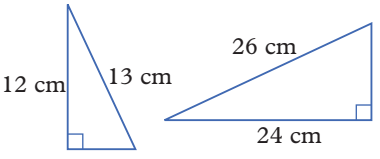
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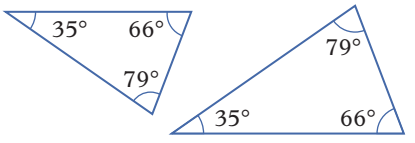
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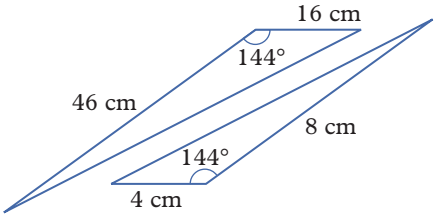
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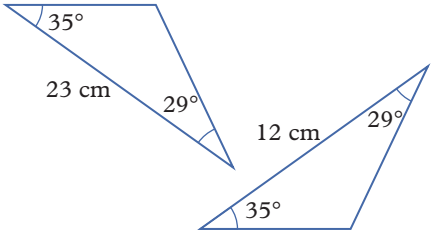
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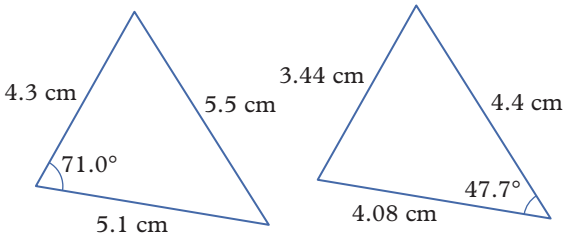


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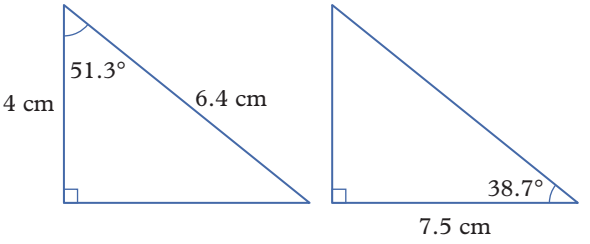


3 Which condition(s) could you use to check that these triangles are similar?

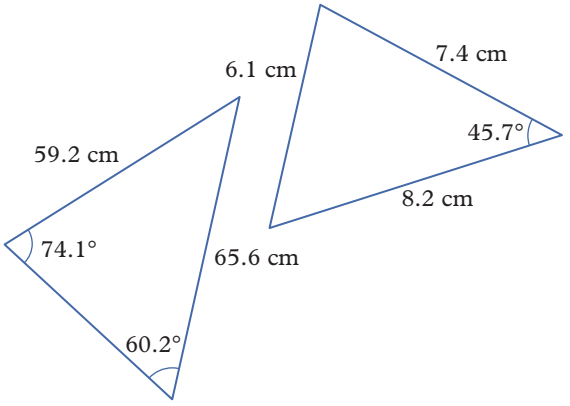
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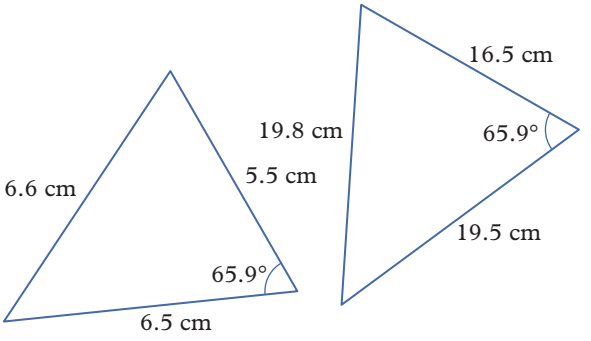
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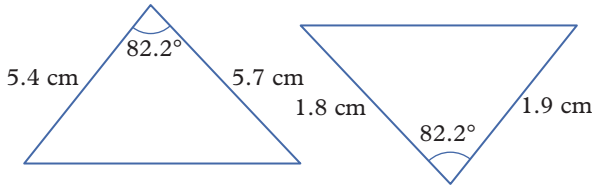


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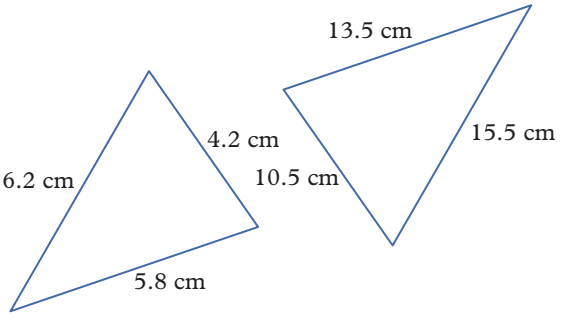


71.3 **4** Determine if these pairs of triangles are similar. State the similarity condition used or explain why the triangles are not similar.

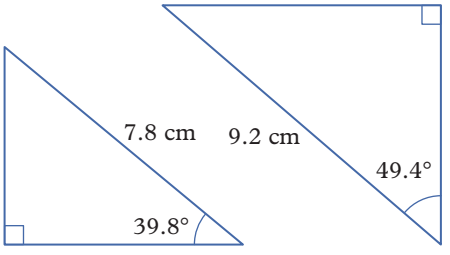
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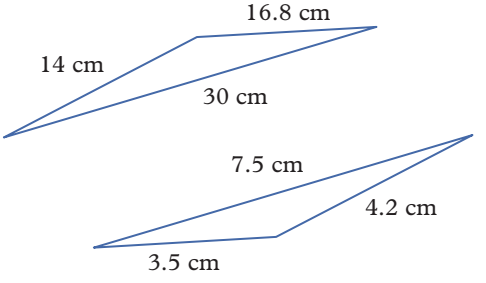
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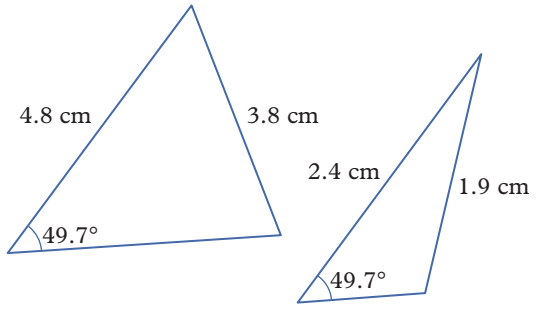
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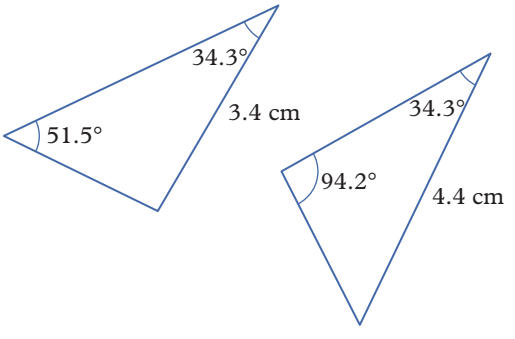
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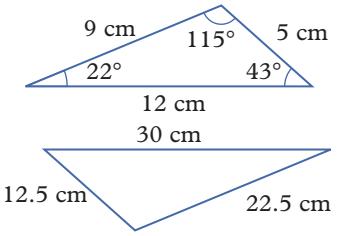
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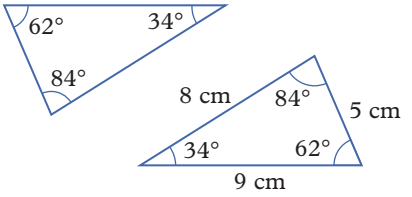
UNDERSTANDING AND FLUENCY

5 Determine whether these pairs of triangles are similar.

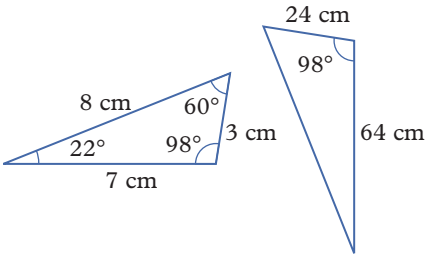
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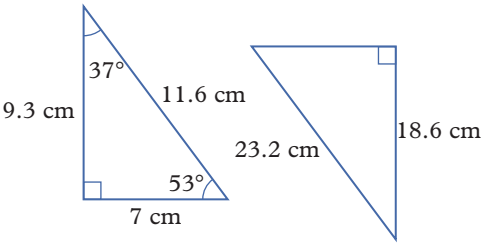
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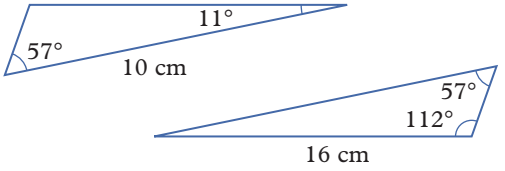
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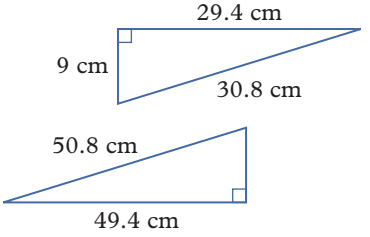
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PROBLEM SOLVING AND REASONING

6 Decide if the pairs of triangles from question **2** are similar.

7 Consider the four similar-triangle conditions. What conditions need to be added to each of the conditions for the similar triangles to be congruent?

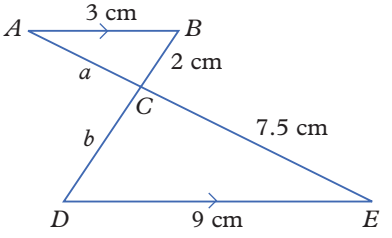
8 Why are all equilateral triangles similar to each other?

9 Explain why only two angles need to be checked when using AAA as a test for similarity.

10 a Using an understanding of angles and parallel lines, explain why the pair of triangles on the right are similar.

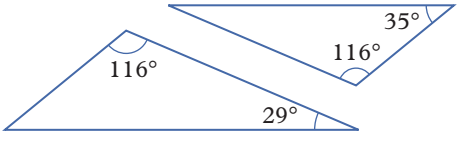
b If triangle *ABC* is the original figure, find the scale factor between the two triangles.

c Find the length of the unknown sides.

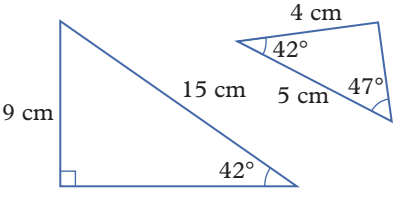


11 Use your understanding of triangle properties to determine whether these pairs of triangles are similar.

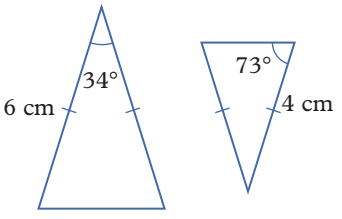
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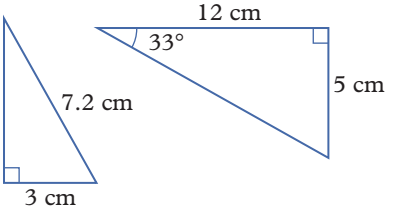
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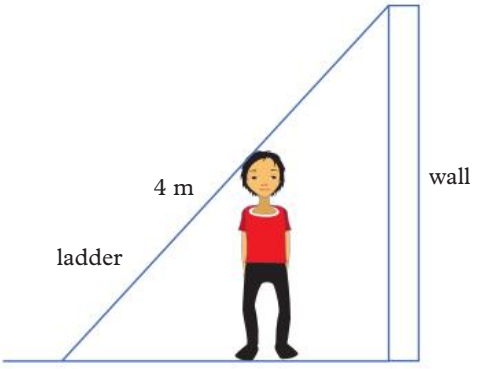
12 Similar triangles can be used to find the heights or lengths of objects that are difficult to measure. Consider this diagram showing the shadows cast by a person and a tree. The person is 1.76 m tall and casts a shadow of 2.1 m. If the tree casts a shadow of 7.8 m, how tall is the tree?



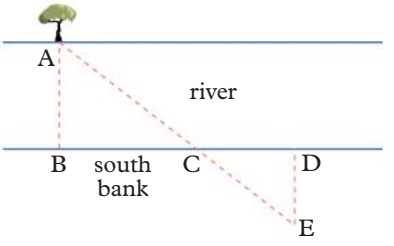
13 Luke wants to measure the heights of the walls in his house and then order some wall decorations. He places his 4 m ladder on an angle so that it hits the top of the wall, as shown in the diagram. The ladder has rungs every 50 cm: at 50 cm, 1 m, etc. If Luke, at 170 cm tall, stood directly under the ladder and his head came into contact with the fifth rung, find:

a the length of the ladder where it touched Luke's head

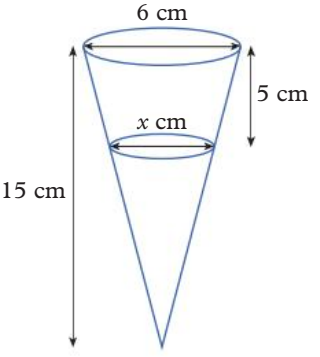
b the height of Luke's walls.



14 In order to find the width of a river, Ahmed places three markers along its southern bank. Marker B is directly opposite a tree at position A. Marker C is 50 m east of marker B and 40 m west of marker D. Ahmed then stands at point E, 30 m south of marker D and he can see marker C in a direct line with the tree at point A. Mark in these measurements on the diagram and then use similar triangles to find the width of the river.



15 An ice-cream cone is 15 cm high with an opening of diameter 6 cm. If chocolate coats the inside of the cone from the bottom to 5 cm below its top, find the largest diameter of the chocolate coating inside the cone.



16 Draw diagrams to help you solve these similar triangle problems.

a Scott has a red triangular flag that is 10 cm high and 15 cm long. He wants to create a larger version from 2 m of red material. If he uses its full length, what height would this larger flag be?

b Christy bought a new netball ring but she wasn't sure that it was the official height (3.05 m). She measured her shadow as 117 cm and the netball ring's shadow as 195 cm. If Christy is 1.8 m tall, is the netball ring the official height?

c Oscar wants to know the height of a flagpole. A guide wire positioned 3.5 m from its base reached to exactly half its height. Oscar stood exactly halfway between the flagpole and where the guide wire attached to the ground and the guide wire touched the top of his head. If Oscar is 1.75 m tall, how tall is the flagpole?

17 Sean and Tania want to know whose house is taller. One morning, Sean measures his shadow as 1.9 m and the shadow of his house as 3.42 m. Later that day, Tania measures her shadow as 2.3 m and the shadow of her house as 3.8 m. If Sean is 172 cm tall and Tania is 165 cm tall, whose house is taller?

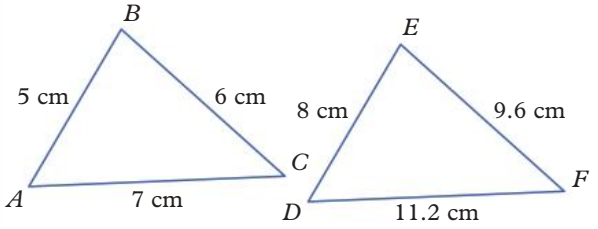
18 Explain why you cannot just compare the shadow lengths of the houses in question **17** to decide which house is taller.

CHALLENGE

19 The process of showing that two triangles are similar can be formalised and generalised further. We want to prove beyond a shadow of a doubt that a given fact is true by writing up bulletproof reasoning behind every step we take. We call these geometric proofs.

Follow the worked examples and then complete the proofs.

Example: Prove that $\triangle ABC \sim \triangle DEF$.



$\frac{DE}{AB} = \frac{8 \text{ cm}}{5 \text{ cm}} = 1.6$

$\frac{EF}{BC} = \frac{9.6 \text{ cm}}{6 \text{ cm}} = 1.6$

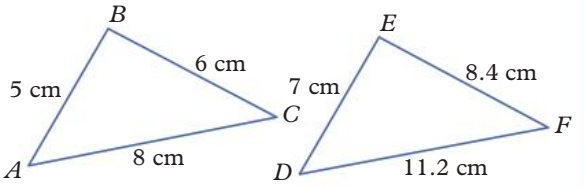
$\frac{FD}{CA} = \frac{11.2 \text{ cm}}{7 \text{ cm}} = 1.6$

$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$

Corresponding sides are in the same ratio.

$\therefore \triangle ABC \sim \triangle DEF$ by SSS

a Prove that $\triangle ABC \sim \triangle DEF$.



$\frac{DE}{AB} = \frac{\square}{\square} =$

$\frac{EF}{BC} = \frac{\square}{\square} =$

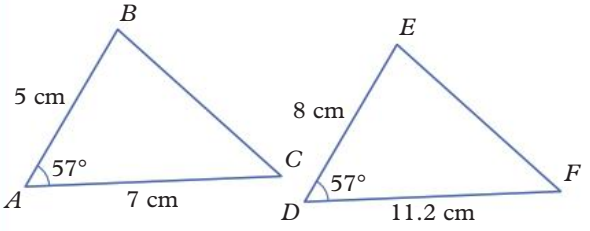
$\frac{FD}{CA} = \frac{\square}{\square} =$

$\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

Corresponding sides are in the same ratio.

$\therefore \triangle$_____ $\sim \triangle$____ by ____

Example: Prove that $\triangle ABC \sim \triangle DEF$.



$\frac{DE}{AB} = \frac{8 \text{ cm}}{5 \text{ cm}} = 1.6$

$\frac{FD}{CA} = \frac{11.2 \text{ cm}}{7 \text{ cm}} = 1.6$

$\angle EDF = 57°$

$\angle BAC = 57°$

$\frac{DE}{AB} = \frac{FD}{CA}$

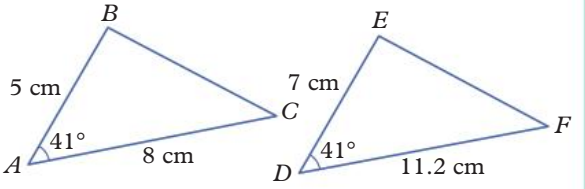
Corresponding sides are in the same ratio.

$\angle EDF = \angle BAC$

Corresponding included angles are equal.

$\therefore \triangle ABC \sim \triangle DEF$ by SAS

b Prove that $\triangle ABC \sim \triangle DEF$.



$\frac{DE}{AB} = \frac{\square}{\square} =$

$\frac{FD}{CA} = \frac{\square}{\square} =$

$\angle EDF =$ ____°

$\angle BAC =$ ____°

$\frac{\square}{\square} = \frac{\square}{\square}$

Corresponding sides are in the same ratio.

$\angle$____ = $\angle$_____

Corresponding included angles are equal.

$\therefore \triangle$_____ $\sim \triangle$____ by ____

Check your Student obook pro for these digital resources and more: pro

- Interactive skillsheet — Similar triangles
- Investigation — Using mirrors to calculate length
- Topic quiz — 7I

7J Quadrilaterals

Learning intentions

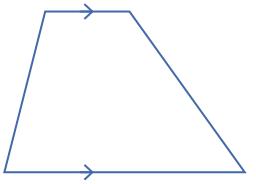
By the end of this topic you will be able to ...

- ✔ classify quadrilaterals
- ✔ find missing angles in quadrilaterals
- ✔ use congruent triangles to establish properties of quadrilaterals.

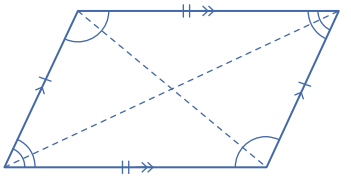
Inter-year links
Year 7 7E Classifying quadrilaterals

Properties of quadrilaterals

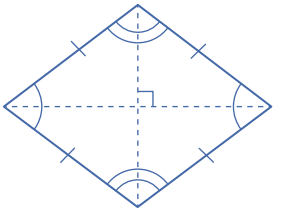
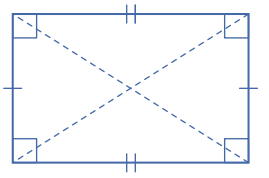
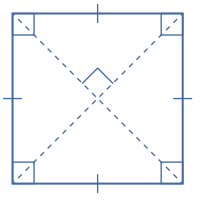
- A **quadrilateral** is a two-dimensional (2D) shape with four straight sides.
- The angles in a quadrilateral add to 360°.
- Quadrilaterals can be classified by their side lengths and interior angles.

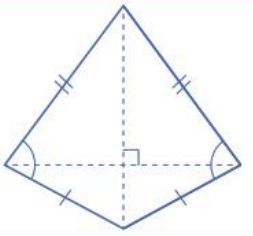
Type of quadrilateral	Properties	Example
Trapezium	• at least one pair of parallel sides	

A parallelogram is a special type of trapezium.

Parallelogram	• two pairs of parallel sides • two pairs of opposite angles that are equal in size • opposite sides are equal in length • diagonals bisect each other (divide each other into two equal parts)	

Three special types of parallelogram are the rhombus, rectangle and square.

Rhombus	• four sides of equal length • diagonals are perpendicular • diagonals bisect the vertex angles	
Rectangle	• four right angles, 90° • diagonals are equal in length	
Square	• four sides of equal length • four right angles, 90° • diagonals are equal in length • diagonals are perpendicular • diagonals bisect the vertex angles	

Type of quadrilateral	Properties	Example
Kite	• two pairs of sides that are equal in length • one pair of opposite angles that are equal in size • diagonals are perpendicular • one diagonal bisects the other diagonal • one diagonal bisects a pair of vertex angles	
Irregular quadrilateral	• four different internal angles • no parallel sides	

- Congruent triangles can be used to establish the properties of special quadrilaterals.

Example 7J.1 Classifying quadrilaterals

Classify this quadrilateral.



THINK

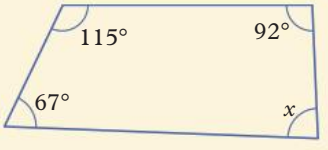
Examine the properties of this quadrilateral. The quadrilateral has opposite sides that are equal in length and it has four right angles, so it is a rectangle.

WRITE

The quadrilateral is a rectangle.

Example 7J.2 Finding an internal angle in a quadrilateral

Find the size of angle x.



THINK

The internal angles in a quadrilateral add to 360°. Subtract each of the known angles from 360° to determine the size of the fourth angle, x.

WRITE

$x = 360° - 115° - 92° - 67°$
$= 86°$

Helpful hints

- ✔ The diagonals of a non-square rectangle are not perpendicular (at right angles to each other)!
- ✔ As rhombuses, rectangles and squares are all parallelograms, they also have the properties of parallelograms.
- ✔ Every square is also a rhombus and a rectangle, as well as being a parallelogram.

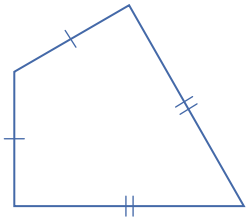
ANS p544

Exercise 7J Quadrilaterals

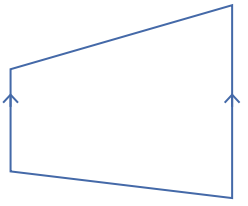
1–4, 5(a–c), 7, 8, 11 | 1–4, 5(d–f), 6, 8, 10, 12, 13 | 3–5, 9, 12–15

7J.1 **1** Classify each quadrilateral. Give a reason for your answer.

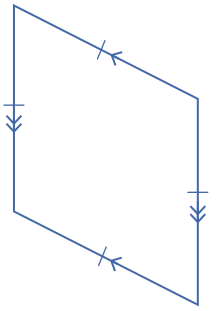
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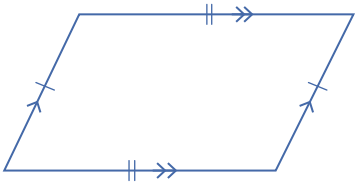
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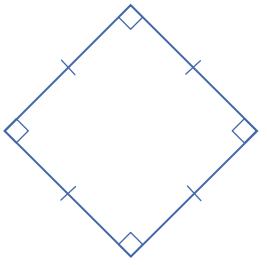
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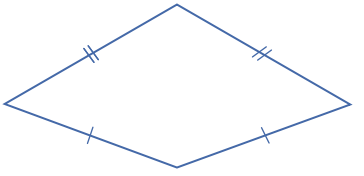
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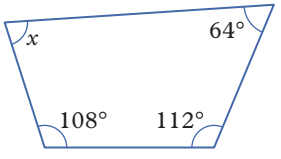


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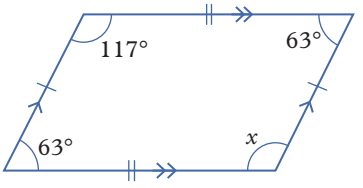


7J.2 **2** Find the size of angle x in each quadrilateral.

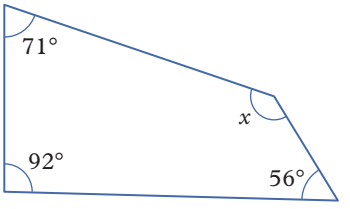
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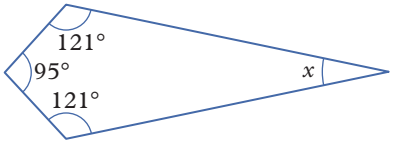
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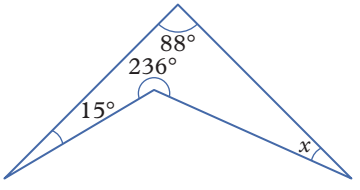
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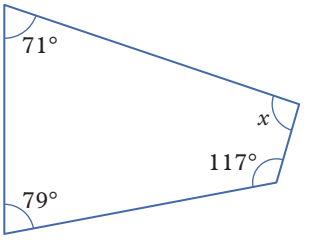
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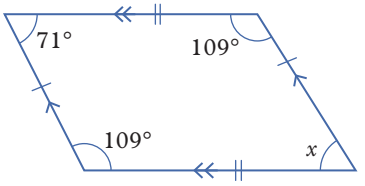
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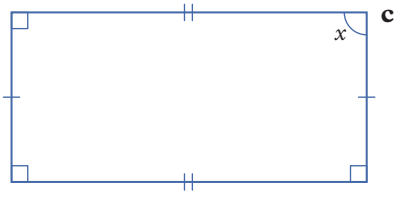
UNDERSTANDING AND FLUENCY

3 Classify each quadrilateral and find the size of angle x.

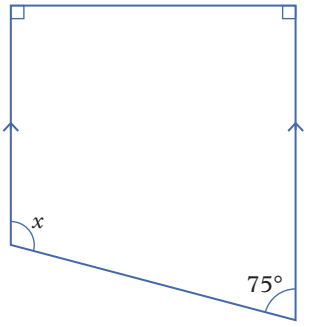
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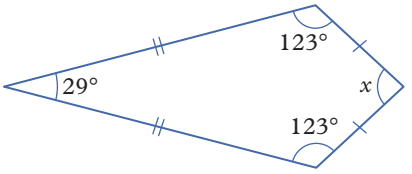
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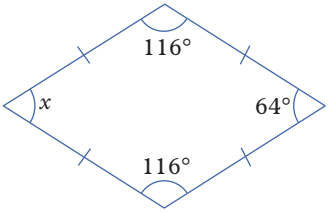
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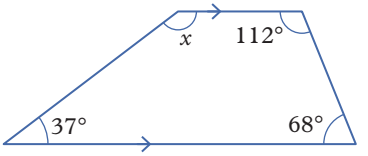
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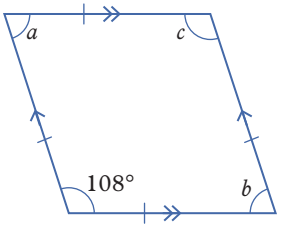


4 Use your knowledge of quadrilaterals to:

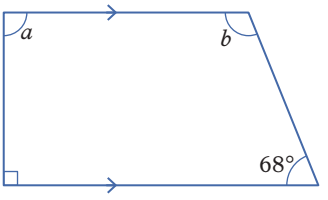
i name each figure

ii find the size of the unknown angles.

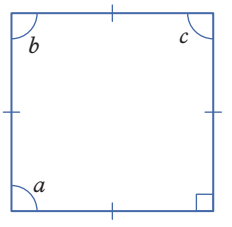
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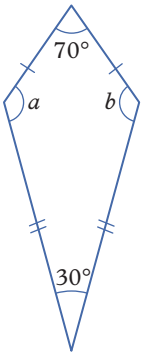
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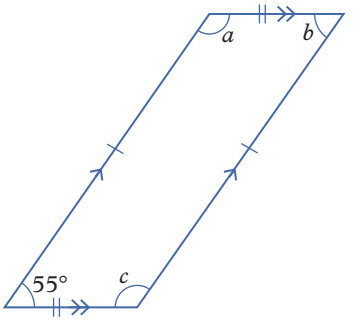
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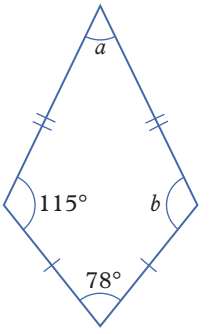
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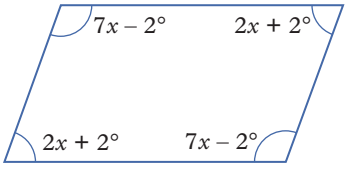


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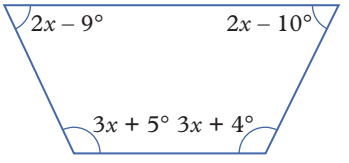


5 Use your understanding of quadrilateral properties and basic algebra to find the value of x in each quadrilateral.

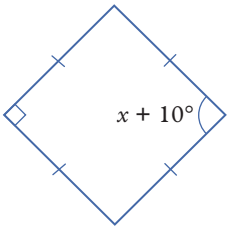
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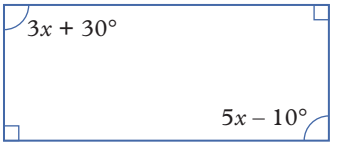
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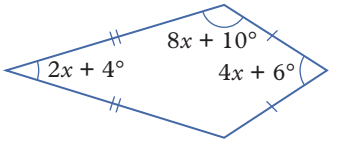
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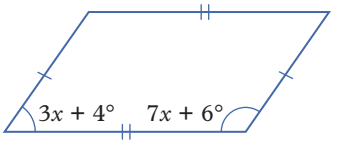
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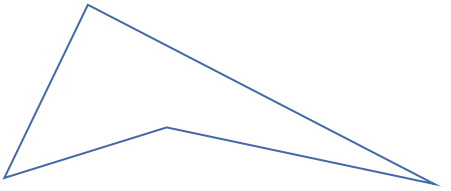
e



f



6 An irregular shape can be convex or concave. A convex shape has internal angles all smaller than 180°. A concave shape has at least one reflex angle (greater than 180°).



a Explain why a concave quadrilateral can have only one internal reflex angle.

b Find the size of the largest angle if the other three angles measure 20°, 20° and 50°.

7 What name is given to a parallelogram with interior angles of 90°?

8 Decide if each statement is true or false. If they are false, give a reason why.

a A square is a type of parallelogram.

b A kite has a pair of parallel sides.

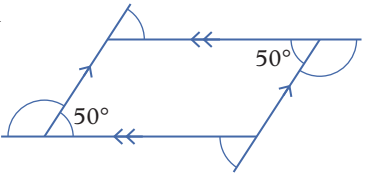
c A parallelogram is a type of rectangle.

d A trapezium has exactly one pair of parallel sides.

e A rhombus always has four equal sides and four equal angles.

f A rectangle always has four equal sides and four equal angles.

9 Exterior angles can be found for quadrilaterals as well as for triangles, as shown in this parallelogram.

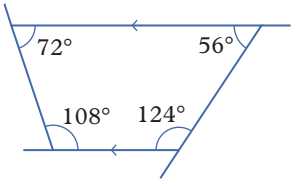


a **i** What is the size of each exterior angle in this parallelogram?

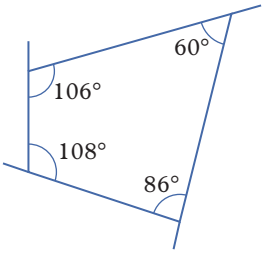
ii What is the sum of the exterior angles?

Repeat **i** and **ii** for these two quadrilaterals.

b



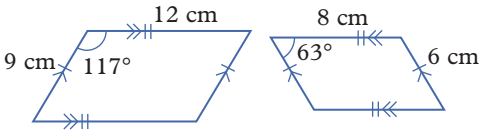
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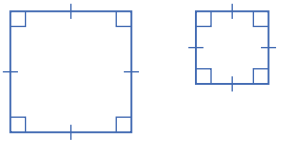
d What can you say about the sum of the exterior angles of a quadrilateral?

10 Use your understanding of quadrilateral properties to decide whether each pair of shapes is similar.

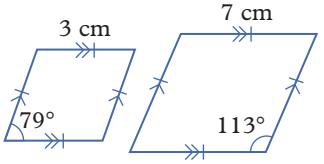
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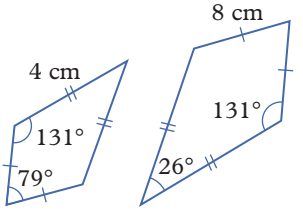
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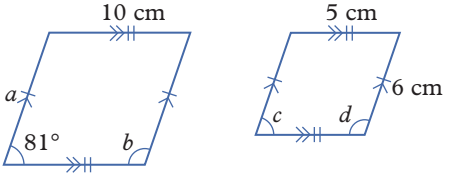


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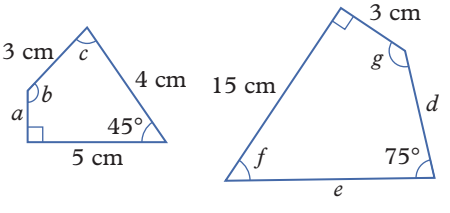


11 Use your understanding of quadrilateral properties to find the value of the pronumerals in each pair of similar figures.

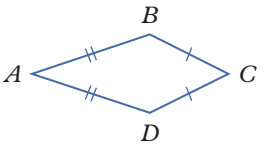
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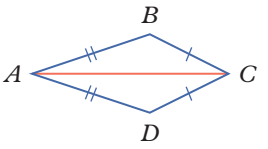
b



12 Consider the kite *ABCD* shown with equal side lengths marked. We can prove other properties of kites by using congruent triangles. Copy and complete the explanations for each part.



a Add the diagonal *AC* to the kite to cut it into two triangles.
Explain why $\triangle ABC \cong \triangle ADC$ by stating the three pairs of congruent sides.



$\triangle$___ $\cong$ $\triangle$____ by _____ as __ $\cong$ __, __ $\cong$ __, and __ is common to both triangles.

b Explain why $\angle ABC \cong \angle ADC$, $\angle BCA \cong \angle DCA$, and $\angle BAC \cong \angle DAC$.

$\angle$___ $\cong$ $\angle$____, $\angle$___ $\cong$ $\angle$____, and $\angle$___ $\cong$ $\angle$___ because they are _____ angles in _____ triangles.

c Use the congruent angles in part **b** to explain why diagonal *AC* bisects (cuts in half) $\angle BAD$ and $\angle BCD$.

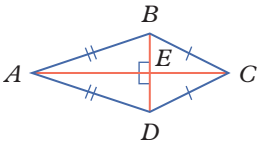
$\angle BAD = \angle$___ + $\angle$____.

Since $\angle BAC \cong \angle DAC$, $\angle BAD = 2\angle$___ and so $\angle$___ has been bisected into two equal angles by diagonal __.

Likewise, $\angle BCD = \angle$___ + $\angle$___.

Since $\angle BCA \cong \angle DCA$, $\angle BCD = 2\angle$___ and so $\angle$___ has been bisected into two equal angles by diagonal __.

d Add the perpendicular height to $\triangle ABC$ and $\triangle ADC$ if the base is *AC* for both.
Explain why the two perpendicular heights, *BE* and *DE*, form the diagonal *BD*.



Since $\triangle$___ $\cong$ $\triangle$___, the perpendicular heights __ and __ will be in the same relative position in the triangles, at point _ to matching vertices _ and _.

These two line segments align to form the diagonal __.

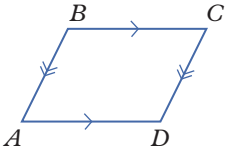
e Explain why $BE \cong DE$ and why diagonal *AC* bisects diagonal *BD* at a right angle.

Since $\triangle$___ $\cong$ $\triangle$___, __ and __ are _____ lengths in _____ triangles, which are therefore _____.

$\angle$___ and $\angle$____ are right angles since __ and __ are perpendicular heights to __ on which point E lies.

Since __ intersects __ at point E, diagonal __ has _____ diagonal __ at a _____ angle.

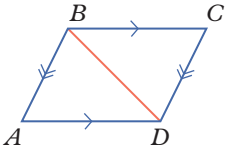
13 Consider the parallelogram *ABCD* shown with parallel sides as marked. We can prove other properties of a parallelogram by using congruent triangles. Copy and complete the explanations for each part.



a Determine which pairs of angles in *ABCD* are equal.

$\angle$___ = $\angle$___, and $\angle$____ = $\angle$___

b Add diagonal *BD* to the parallelogram. Explain why $\angle BDA \cong \angle DBC$ and $\angle DBA \cong \angle BDC$.



__ and __ are parallel and __ is a transversal.

$\angle$___ $\cong$ $\angle$___ as they are _____ angles on _____ lines.

Likewise, __ and __ are parallel and __ is a _____ so $\angle$___ $\cong$ $\angle$___ as they are _____ angles on _____ lines.

c Explain why it does not matter which diagonal (*AC* or *BD*) was added to find two congruent triangles.

Either diagonal will act as a _____ to both pairs of _____ sides, so the _____ angles formed by the _____ will be congruent.

d Explain why $\triangle ABD \cong \triangle CDB$.

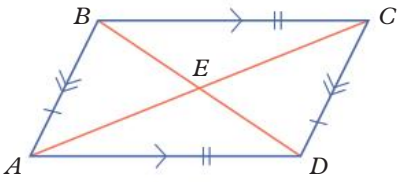
$\triangle$___ $\cong$ $\triangle$___ by ___ as $\angle$____ $\cong$ $\angle$____, $\angle$____ $\cong$ $\angle$____, and __ is common to both triangles.

e Use the fact that $\triangle ABD \cong \triangle CDB$ to explain why $AB \cong CD$ and $AD \cong CB$.

$\triangle$___ $\cong$ $\triangle$___, where side _____ corresponds to side _____ and side _____ corresponds to side _____, so ___ $\cong$ ___ and ___ $\cong$ ___.

14 In question **13**, we showed that for the parallelogram $ABCD$, given the parallel sides shown:

- $\angle BDA \cong \angle DBC$
- $\angle DBA \cong \angle BDC$
- $AB \cong CD$
- $AD \cong CB$.



Copy and complete the explanations for each part.

a Add diagonal AC to the parallelogram. Explain why $\angle AED \cong \angle BEC$ and $\angle BEA \cong \angle CED$.

$\angle$___ $\cong$ $\angle$___ as they are _____ angles.

Likewise, $\angle$___ $\cong$ $\angle$___ as they are _____ angles.

b State which two pairs of triangles are congruent and why they are congruent.

Δ___ $\cong$ Δ___ by ___ as $\angle$____ $\cong$ $\angle$____, $\angle$____ $\cong$ $\angle$____, and __ $\cong$ ___.

Δ___ $\cong$ Δ___ by ___ as $\angle$____ $\cong$ $\angle$____, $\angle$____ $\cong$ $\angle$____, and __ $\cong$ ___.

c Explain why $BE \cong DE$ and $AE \cong CE$.

__ $\cong$ __ and __ $\cong$ __ as they are _____ lengths in _____ triangles, which are _____.

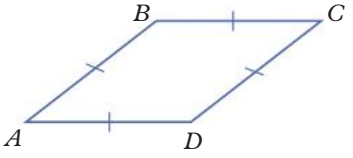
d Explain why the diagonals AC and BD bisect each other.

Diagonal __ = __ + __, and since __ $\cong$ __, diagonal __ = 2__ and so has been _____ into two equal line segments by diagonal __.

Likewise, diagonal __ = __ + __, and since __ = __, diagonal __ = 2__ and so has been _____ into _____ by diagonal __.

Therefore, _____.

15 Consider the rhombus $ABCD$ with equal sides marked.



a Add diagonal AC and explain why $\Delta ABC \cong \Delta ADC$.

b State what type of triangles ΔABC and ΔADC are. Explain how you know.

c Explain why $\angle BAC \cong \angle BCA$ and $\angle DAC \cong \angle DCA$.

d Explain why $\angle ABC \cong \angle ADC$, $\angle BCA \cong \angle DCA$, and $\angle BAC \cong \angle DAC$.

e Explain why the diagonal AC bisects $\angle BAD$ and $\angle BCD$.

f Explain why a similar process to parts **a–e** can be used to show that the diagonal BD bisects $\angle ABC$ and $\angle ADC$.

16 Each part below gives a clue about a different quadrilateral. Use the clues to:

i find the size of each angle **ii** name the quadrilateral.

a All angles are equal and all sides are equal.

b Two pairs of angles are equal and one pair of angles is twice the size of the other pair. Two pairs of side lengths are equal and one pair is twice the length of the other pair.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Quadrilateral properties

Investigation
Stained glass quadrilaterals

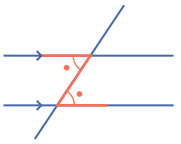
Topic quiz
7J

PROBLEM SOLVING AND REASONING

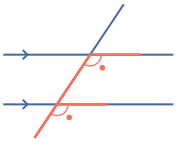
CHALLENGE

Chapter summary

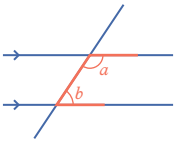
Angles and parallel lines



alternate 'Z'



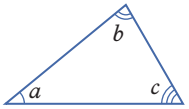
corresponding 'F'



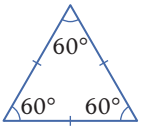
$a + b = 180°$
supplementary

Triangles

$a + b + c = 180°$



scalene



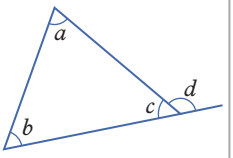
equilateral



right-angled



isosceles

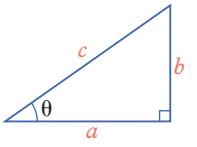


$d = a + b$

Pythagoras' theorem

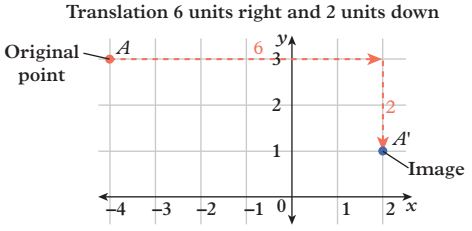
$c^2 = a^2 + b^2$

hypotenuse shorter sides

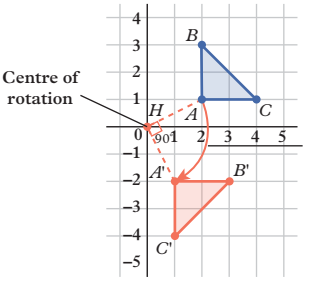


- If a triangle satisfies Pythagoras' theorem it is right-angled.
- Any set of three whole numbers that satisfies Pythagoras' theorem is a Pythagorean triple. For example, 3, 4, 5 is a Pythagorean triple.

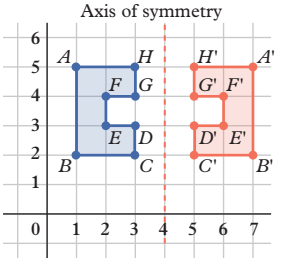
Translations



Rotations



Reflections



Congruent figures

- Same shape and size. Isometric translations produce congruent figures.
- A tessellation is a repeating pattern of shapes with no overlaps or gaps

Similar figures

- Same shape but do not need to be the same size.
- All corresponding angles are equal in size.
- All corresponding sides are in the same ratio.

Congruent triangles

- SSS (side–side–side): all three side lengths are equal.
- SAS (side–angle–side): two side lengths and the included angle are equal.
- AAS (angle–angle–side): two angles and a corresponding side are equal.
- RHS (right angle–hypotenuse–side): the hypotenuse and another side of a right-angled triangle are equal.

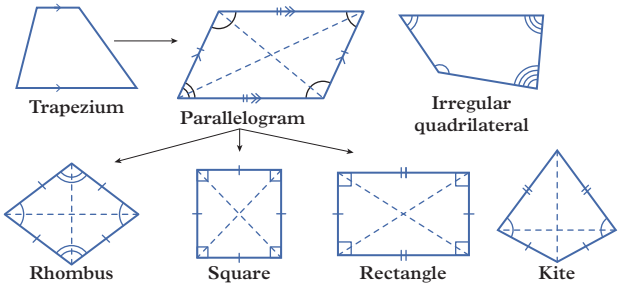
Similar triangles

- SSS (side–side–side): all three pairs of corresponding side lengths are in the same ratio.
- SAS (side–angle–side): two corresponding pairs of side lengths are in the same ratio and the included angles are equal.
- AAA (angle–angle–angle): all three pairs of corresponding angles are equal.
- RHS (right angle–hypotenuse–side): the hypotenuses are in the same ratio as another set of sides in a right-angled triangle.

Dilations

- An enlargement creates an image that is larger than the original shape.
- A reduction creates an image that is smaller than the original shape.
- A centre of dilation and scale factor are needed to perform a dilation.
- scale factor $= \dfrac{\text{image length}}{\text{original length}}$
- Length of $A'B'$ = length of AB × scale factor

Quadrilaterals



Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

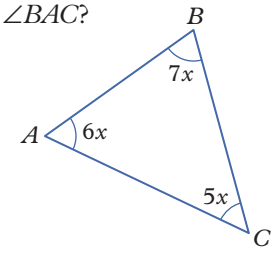
Quizlet
Test your knowledge of this topic by working individually or in teams

Multiple choice

7A **1** Which statement is *not* correct?

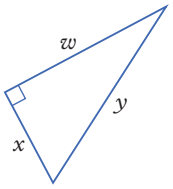
A Vertically opposite angles are equal in size.
B The sum of two obtuse angles will always be less than a revolution.
C Complementary angles add to 180°.
D Two supplementary angles make a straight angle.
E A reflex angle is always larger than an obtuse angle.

7B **2** What size is angle $\angle BAC$?



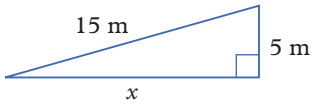
A 6°
B 10°
C 18°
D 60°
E 120°

7C **3** Using Pythagoras' theorem, which statement shows the relationship between the side lengths of the triangle?



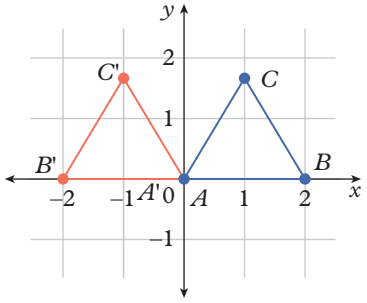
A $w^2 = x^2 + y^2$
B $x^2 = w^2 + y^2$
C $y^2 = x^2 + w^2$
D $y^2 = x^2 - w^2$
E $y^2 = w^2 - x^2$

7D **4** The unknown side length x in this triangle, correct to the nearest m, is:



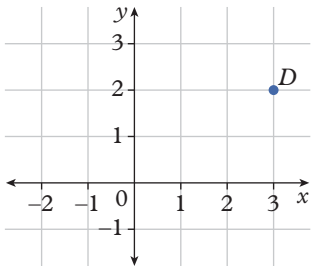
A 3 m
B 10 m
C 14 m
D 16 m
E 20 m

7E **5** Which of the following transformations would *not* have the image of the equilateral triangle shown?



A Translate 2 units left.
B Translate 2 units right.
C Rotate 120° anti-clockwise.
D Rotate 240° clockwise.
E Reflect in the y-axis.

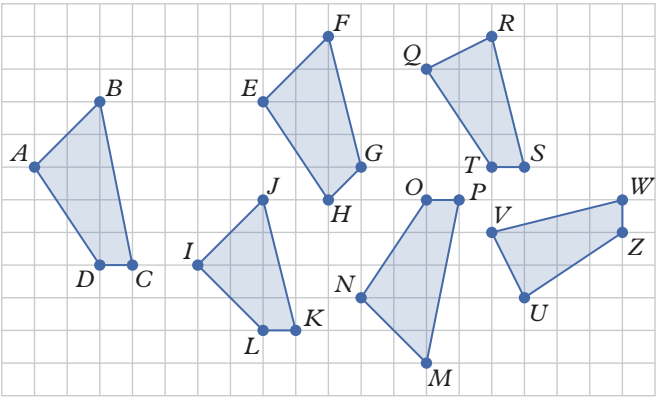
7E **6** What are the coordinates of the point D if it is translated by 2 units left and 1 unit down, and then reflected in the x-axis?



A (2, 0)
B (−3, 1)
C (−1, 1)
D (1, −3)
E (1, −1)

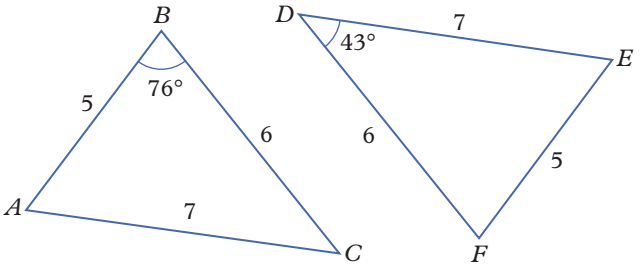
7F **7** Which of the following is congruent to *ABCD*?

A *EFGH*
B *IJKL*
C *NMPO*
D *QRST*
E *UVWZ*



7G **8** Using the triangles shown, determine the size of $\angle BAC$.

A 76°
B 43°
C 14°
D 61°
E 47°

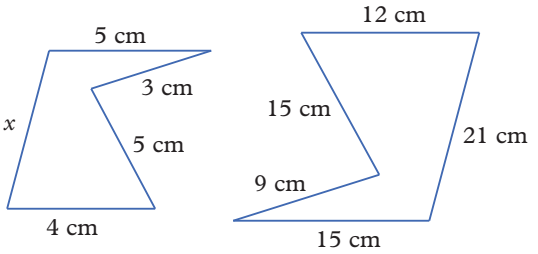


7H **9** Which scale factor will not result in an enlargement?

A $\frac{1}{3}$ **B** $\frac{5}{4}$ **C** 2 **D** $\frac{10}{3}$ **E** 4

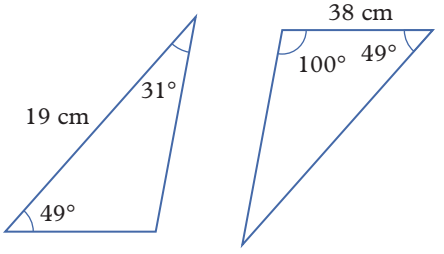
7H **10** Find the value of x for these similar figures.

A 21 cm
B 63 cm
C 7 cm
D 8.75 cm
E 12 cm



7I **11** Which test would you use to decide whether these two triangles are similar?

A AAA
B SSS
C SAS
D RHS
E AAS



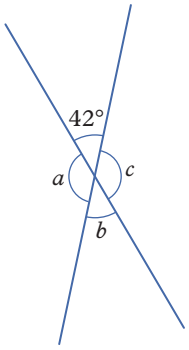
7J **12** Which statement is true?

A A rhombus is a rectangle.
B A parallelogram is a square.
C A square is a rhombus.
D A trapezium cannot have a right angle.
E A kite has parallel sides.

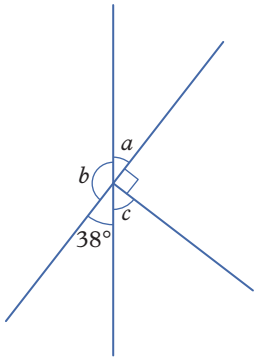
Short answer

7A **1** Without using a protractor find the size of each unknown angle.

a

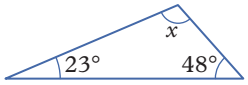


b

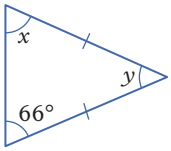


7B **2** Find the size of angles x and y.

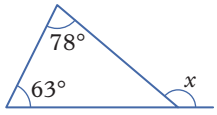
a



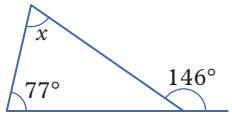
b



c



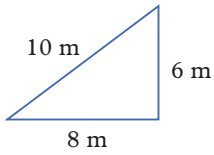
d



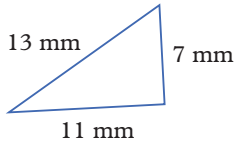
7B **3** For each triangle shown in question **2**, identify whether it is isosceles or scalene, and whether it is acute-angled or obtuse-angled.

7C **4** Use Pythagoras' theorem to decide whether these are right-angled triangles.

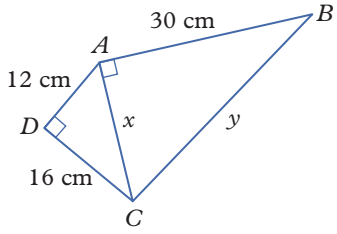
a



b

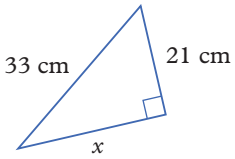


7C **5** Find the length of the sides represented by x and y in this diagram, correct to two decimal places.

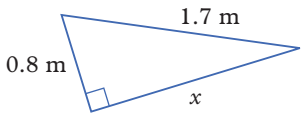


7D **6** Calculate the side length x in each triangle, correct to two decimal places.

a



b



7E **7** Transform the triangle shown below right using the descriptions provided. Start from the original triangle each time.

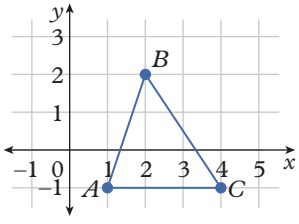
a Translate 2 units right and 3 units down.

b Reflect in the x-axis.

c Rotate 90° clockwise about A.

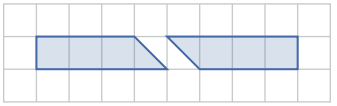
d Rotate 180° clockwise about the origin, then translate 3 units up.

e Translate 2 units right and 1 unit down, then reflect in the y-axis.

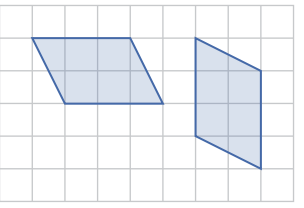


7F **8** Decide whether each pair of shapes is congruent.

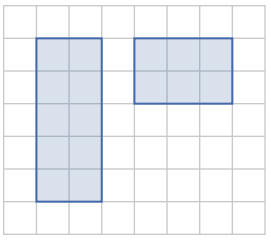
a



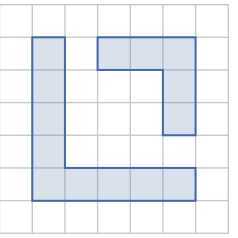
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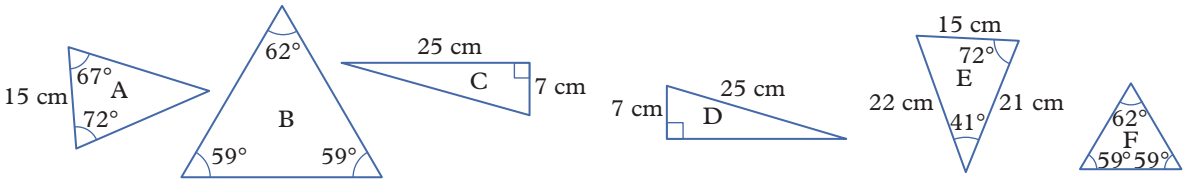
c



d



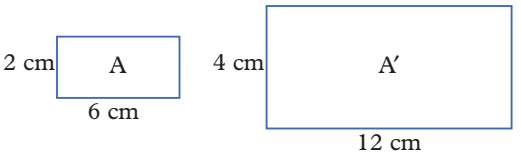
7G **9** Identify any pairs of congruent triangles.



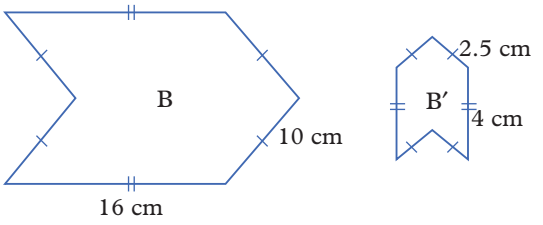
7G **10** Explain your answers to question **9** using one of the conditions for congruence (SSS, SAS, AAS, RHS).

7H **11** Describe each transformation as a reduction or an enlargement and state the scale factor.

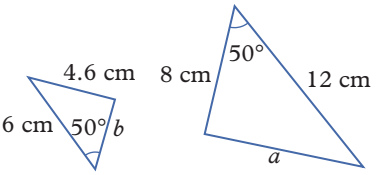
a



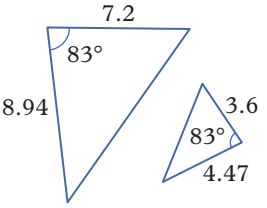
b



7I **12** Given that these two triangles are similar, find the value of the unknown side lengths.

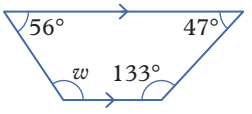


7I **13** Determine whether these two triangles are similar. If similar, state the similarity condition you used, or explain why they are not similar.

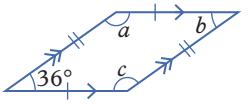


7J **14** Find the size of the unknown angles in these quadrilaterals.

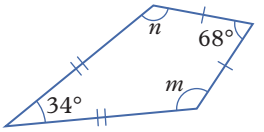
a



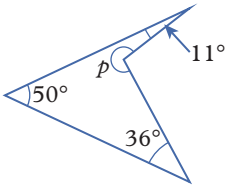
b



c



d

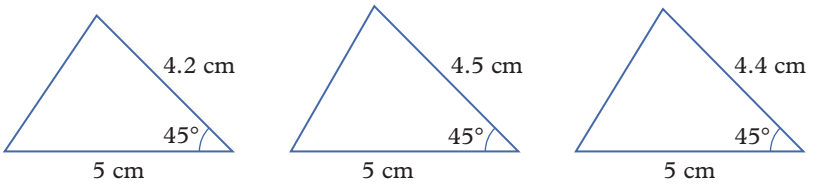


7J **15** Name the quadrilaterals shown in question **14**.

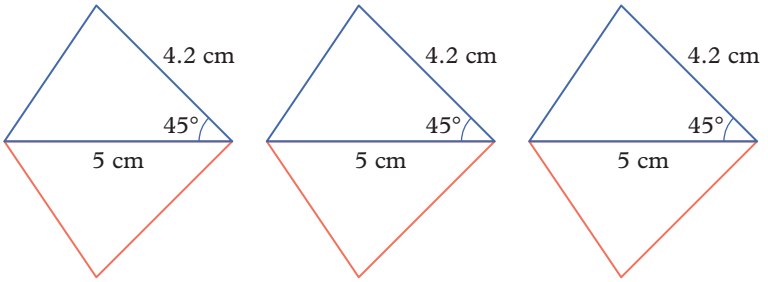
Analysis

1 Jess is constructing congruent kites to use for an art project.

a She starts by constructing three triangles using the lengths 5 cm and 4.2 cm and the included angle 45°. Jess thinks her compass may have slipped when constructing the triangles and so decides to measure her lengths and angles. Explain why her triangles are not congruent.

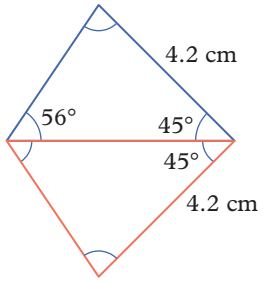


b Jess reconstructs her incorrectly drawn triangles so that they are all congruent. Her plan is to draw the reflection of these triangles below the 5 cm length to create her kites. Explain why these shapes will be kites referring to congruent lengths and triangles.



c Construct another kite for Jess using two congruent triangles with the same 5 cm length, 4.2 cm lengths, and included angles of 45°.

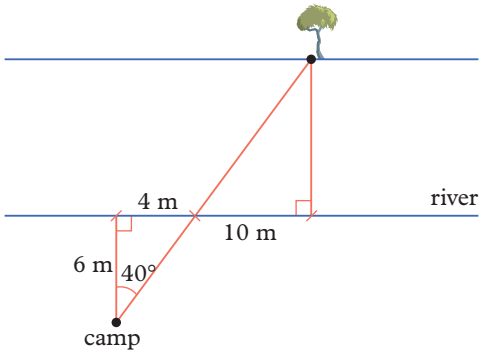
Jess measures the other base angle of her original triangle to find it is 56°. She says it is now possible to determine the size of all the interior angles in her kite.



d Explain why the size of all the other angles can be determined.

e Determine the size of the four interior angles of the kite.

2 While preparing to study a species of gum tree on a field trip, Stephanie set up a camp 6 m from a river's edge. From the camp, she measures the angle to a gum tree she wishes to get to and records various measurements, as shown in the diagram.



a Considering both the triangles shown in Stephanie's diagram, what are the angle sizes in each triangle?

b Are the two triangles congruent or only similar? Provide a reason to support your answer.

c What is the scale factor linking the triangle attached to the camp site to the triangle attached to the gum tree?

d Briefly explain how the triangles can be used to determine the width of the river.

e Calculate the width of the river.

f Explain how Pythagoras' theorem can be used to calculate the straight line distance between the camp and the gum tree.

g Calculate the straight line distance between the camp and the gum tree correct to two decimal places.

8

Units of measurement





Index

8A Length and perimeter
8B Area of triangles and rectangles
8C Area of quadrilaterals
8D Area of a circle
8E Volume and capacity
8F Three-dimensional space
8G Time

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Substitution
- ✔ Multiplying and dividing by powers of 10
- ✔ Adding and subtracting decimals
- ✔ The Cartesian plane
- ✔ Understanding time

Curriculum links

- ✔ Solve problems involving the area and perimeter of irregular and composite shapes using appropriate units (AC9M8M01)
- ✔ Solve problems involving the volume and capacity of right prisms using appropriate units (AC9M8M02)
- ✔ Solve problems involving the circumference and area of a circle using formulas and appropriate units (AC9M8M03)
- ✔ Solve problems involving duration, including using 12- and 24-hour time across multiple time zones (AC9M8M04)
- ✔ Describe the position and location of objects in 3 dimensions in different ways, including using a three-dimensional coordinate system with the use of dynamic geometric software and other digital tools (AC9M8SP03)

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Materials

- ✔ Calculator

8A Length and perimeter

Learning intentions

By the end of this topic you will be able to ...

- ✔ convert between units of length
- ✔ find the perimeter of parallelograms, trapeziums, rhombuses, kites and circles
- ✔ find the perimeter of two-dimensional shapes.

Inter-year links

Support Length and perimeter

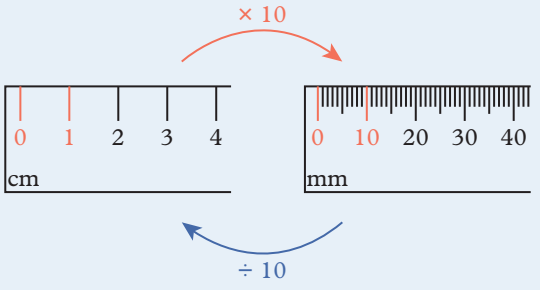
Year 7 9A Length and perimeter

Units of length

- To specify a **length**, you need a numerical value as well as a unit of length (for example, 5 km).

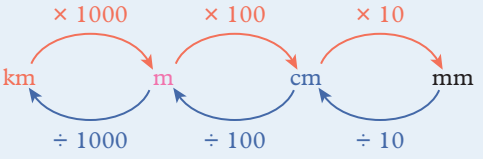
Metric units of length	
Name	**Abbreviation**
Millimetre	mm
Centimetre	cm
Metre	m
Kilometre	km

- To convert between units of length, multiply or divide by the appropriate **conversion factor**.
  For example, a length of 1 cm is the same as 10 mm; therefore, the conversion factor when converting from cm to mm is 10.



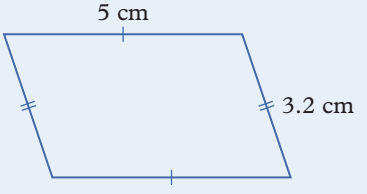
Conversion factors

1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m

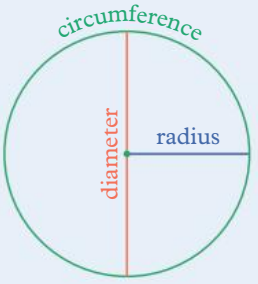


Perimeter

- The **perimeter** of a two-dimensional (2D) shape is the length of the shape's boundary.
- To calculate the perimeter of a shape, add the lengths of each side together.
  For example, the perimeter of the parallelogram is 5 cm + 3.2 cm + 5 cm + 3.2 cm = 16.4 cm.
  - → All measurements must be in the same unit before being added together.
  - → Sides of equal length are marked using hatch marks.



- The perimeter of a circle is called its **circumference** (C). To find the circumference of a circle, multiply the diameter by π. The irrational number π has an infinite number of non-recurring decimal places and is approximately equal to 3.14.
  - → The **diameter** of a circle (d) is the length from one point on the circumference to another through the centre of the circle.
  - → The **radius** of a circle (r) is the length from the centre of the circle to any point on the circumference. The radius is half the length of the diameter.
- $C = \pi d$ or $C = 2\pi r$



Example 8A.1 Converting between units of length

Convert 1.23 m into millimetres.

THINK

1 Convert from metres to centimetres.
To convert from a larger unit to a smaller unit, multiply by the conversion factor.

2 Convert from centimetres to millimetres.

WRITE

1 m = 100 cm

$1.23 \text{ m} = 1.23 \times 100 \text{ cm}$
$= 123 \text{ cm}$

1 cm = 10 mm

$123 \text{ cm} = 123 \times 10 \text{ mm}$
$= 1230 \text{ mm}$

Example 8A.2 Calculate the circumference of a circle

Calculate the circumference of a circle with diameter 8 m, correct to two decimal places.

THINK

1 Identify which formula to use.

2 Substitute the measurement for the diameter.

3 Calculate using π on your calculator.

4 Round to two decimal places and include the appropriate unit.

WRITE

$C = \pi d$

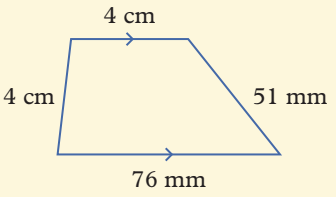
$= \pi \times 8$

$= 25.132\,740\,8 \ldots$

$\approx 25.13 \text{ cm}$

Example 8A.3 Calculating perimeter

Calculate the perimeter of the trapezium in centimetres.



THINK

1 Convert all measurements into the unit specified by the question.

2 Write out the sum of the side lengths. There are four sides so there should be four lengths in the sum.

3 Add all the lengths. Include the appropriate unit in the answer.

WRITE

51 mm = 51 ÷ 10 cm
= 5.1 cm

76 mm = 76 ÷ 10 cm
= 7.6 cm

perimeter = 4 cm + 51 mm + 76 mm + 4 cm
= 4 cm + 5.1 cm + 7.6 cm + 4 cm

= 20.7 cm

Helpful hints

- ✔ Once the units are the same, you can leave them out of calculations, but you must remember to include them in your final answer!
- ✔ Always estimate your answer first. If your calculated answer is significantly different than your estimate, check your working to ensure mistakes haven't been made.
- ✔ When converting from a larger unit to a smaller unit (e.g. cm to m) you multiply, and when converting from a smaller unit to a larger unit (e.g. m to km) you divide.
- ✔ Unless instructed otherwise, use π on your calculator instead of an approximation such as 3.14. You should only round your answers at the final step of the calculations.

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Exercise 8A Length and perimeter

1, 2, 3–5(a–e), 6, 7(a–e), 9–10(1st column), 13, 15, 18, 19 | 1(b, c, e, g), 2(b, d, f, h), 3–5(b, d, f), 7(b, d, f), 9–10(c–e), 12, 16, 17, 20, 21, 24 | 1–2(d, f, h), 3–5(d–f), 7(d–f), 8, 9–10(e–f), 14, 16, 22, 24, 26

8A.1

1 Convert these lengths to the units shown in brackets.

a 58 m (cm) **b** 2.7 km (m) **c** 521 cm (m) **d** 398 m (km)

e 1.2 cm (mm) **f** 3987 m (km) **g** 423 mm (cm) **h** 7 mm (cm)

2 Convert these lengths to the units shown in brackets.

a 5.34 m (mm) **b** 3.95 km (cm) **c** 235 000 mm (m) **d** 145 000 000 mm (km)

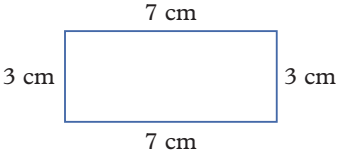
e 2368 cm (km) **f** 287 mm (m) **g** 1.934 km (cm) **h** 0.91 km (mm)

8A.2 **3** Find the circumference of a circle with:

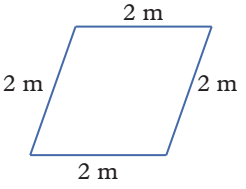
a diameter 1.2 m
b radius 2.5 cm
c diameter 46 mm
d radius 0.8 cm
e radius 91 cm
f radius 1.3 mm
g diameter 1.75 m
h diameter 2.6 cm
i diameter 82 km.

4 Calculate the perimeter of these quadrilaterals.

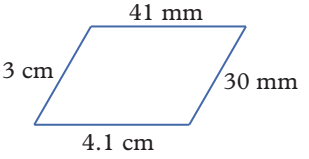
a



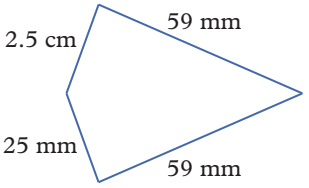
b



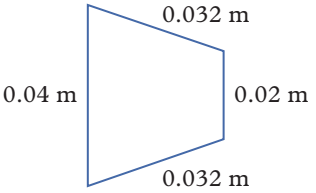
c



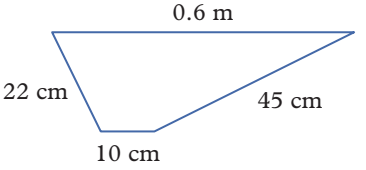
d



e

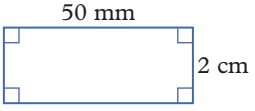


f

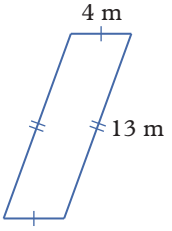


5 Calculate the perimeter of these quadrilaterals.

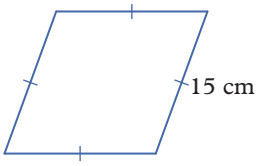
a



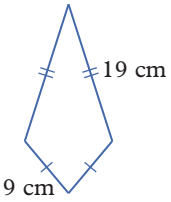
b



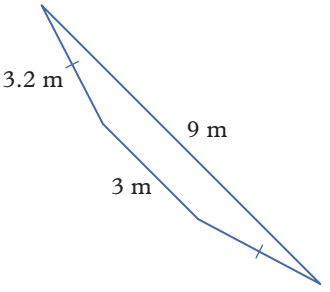
c



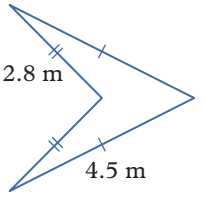
d



e



f

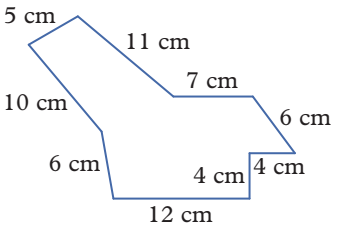


6 Calculate the perimeter of:

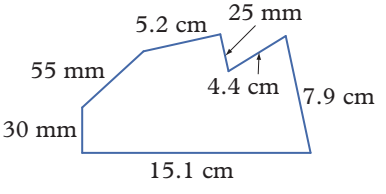
a a square with side length 4 cm
b a rhombus with base 15 mm
c a rectangle with length 10 mm and width 15 mm
d a parallelogram with side lengths 18 cm and 12 cm.

8A.3 **7** Calculate the perimeter of these shapes.

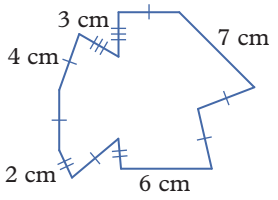
a



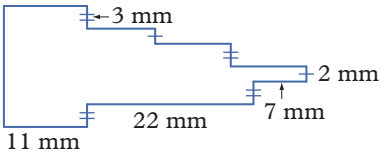
b



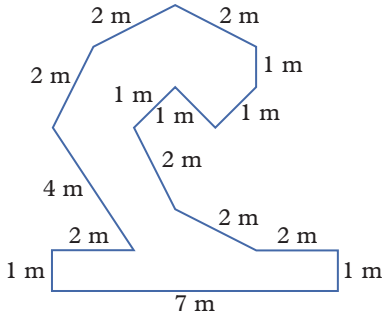
c



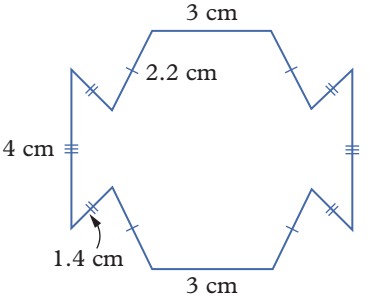
d



e



f



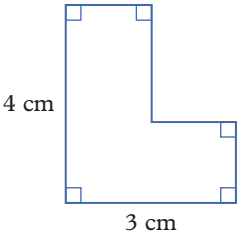
8 Calculate the perimeter of:

a an equilateral triangle with side length 2.5 cm

b a regular hexagon with side length 2.5 m

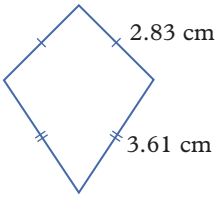
c an L-shaped hexagon with longer sides 3 cm and 4 cm

d a regular octagon with side length 7.7 cm.

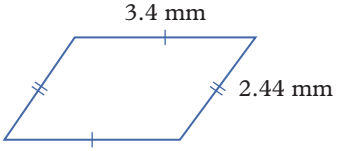


9 Calculate the perimeter of the following shapes, correct to two decimal places.

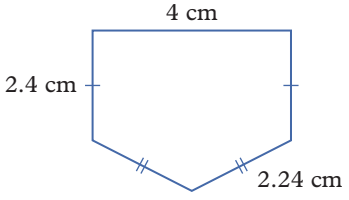
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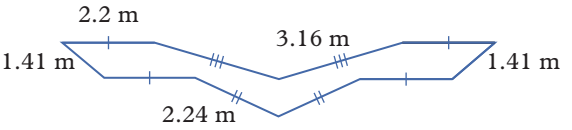
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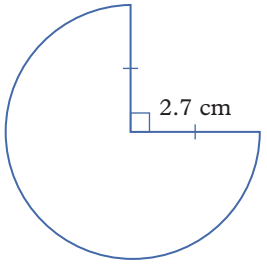
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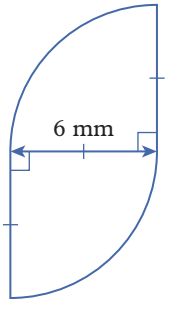
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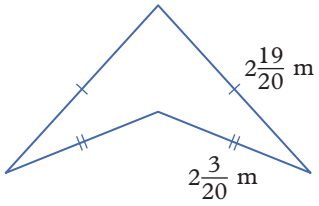


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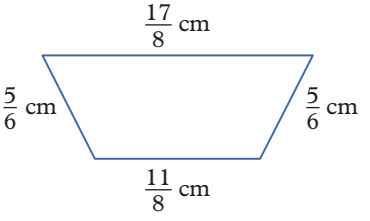


10 Calculate the perimeter of the following shapes. Leave your answers as exact values.

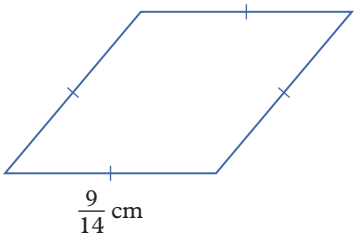
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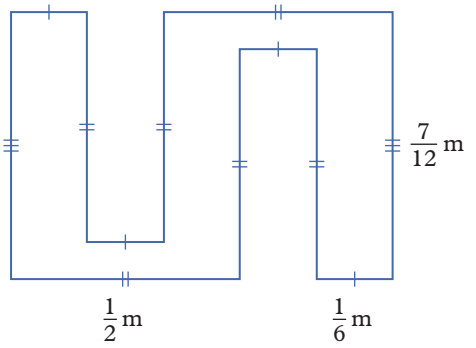
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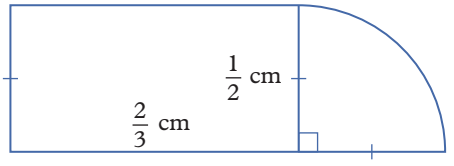
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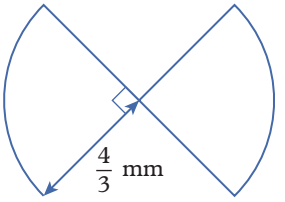
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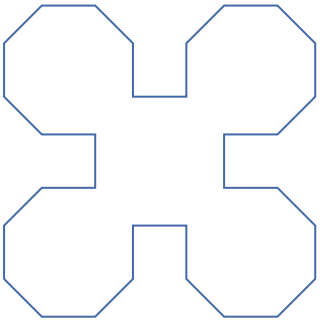
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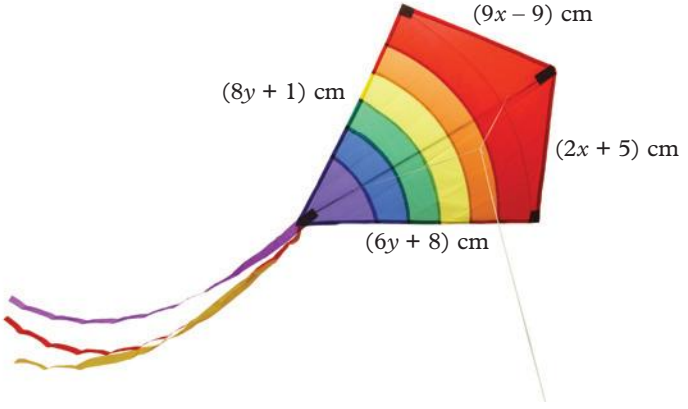
11 Trish makes earrings. One design is a rectangular jewel with wire wrapped around its perimeter twice. If the jewel is 4.5 cm long and 1.5 cm wide, how much wire (in metres) is used for 20 pairs of earrings?

12 A road has a white line painted on each edge, and a dashed line down the middle. If every 10 m section of the road has eight 650 mm long dashes down its centre, calculate the total length of painted line required (in kilometres) for a 10 km road.

13 The perimeter of each of the congruent regular octagons that make up this shape is 48 cm. Determine the perimeter of this shape, justifying your answer.



14 Determine the values of x and y and hence calculate the perimeter, P, of this kite.



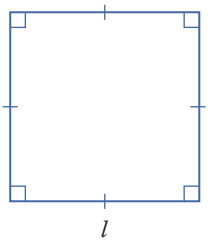
15 A square has a perimeter of 36 cm. What is its side length? Explain your answer.

16 A new television screen has a perimeter of 6.1 m. If it has a length of 1.9 m, find its width.

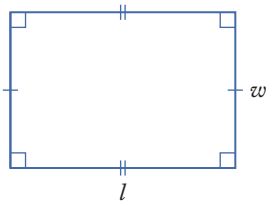
17 A circular flower bed has a circumference of 4.71 m. What is the diameter correct to two decimal places?

18 Write an expression for the perimeter of the following quadrilaterals.

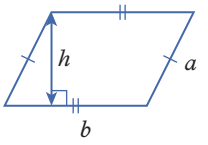
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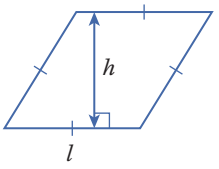
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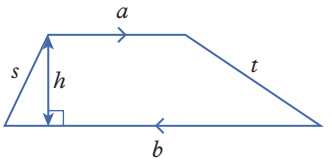
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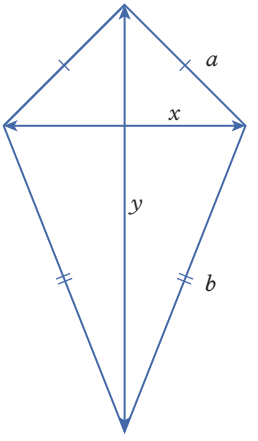
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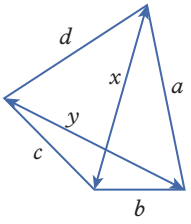
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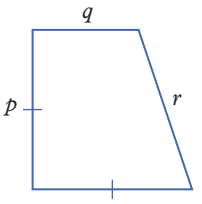
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h

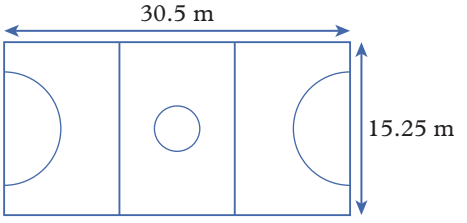


19 To warm up before a netball match, Jordan has the option of either running six laps around the netball court or doing four shuttle runs.

a How far is one lap around the netball court?

b How far is six laps around the netball court?

A shuttle run involves starting at the left of the court and running hard to the first third-line then back to the start, running to the second third-line and back to the start, and finally running up to the end of the court and back.



c How far is the first part of the shuttle run (to the first third-line and back)? Give your answer correct to two decimal places.

d What distance does a single shuttle run cover?

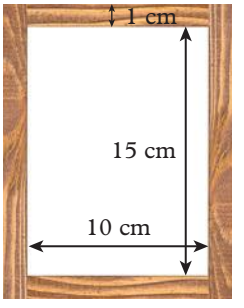
e How far is four shuttle runs? Explain how you calculated the answer.

f Which warm-up option is shorter?

20 Alice wants to make some photo frames for a friend. She buys some wood to fit around the outside of a standard-sized photo.

a What is the perimeter of the photo that fits in this frame?

Alice wants the frame to be 1 cm wide, as shown in the image.



b Why is the length of wood needed greater than the perimeter of the photo?

Hint: Are the corner parts of the frame included in the photo perimeter?

c What length of wood is needed?

d What is the perimeter around the photo frame? Why is this different from part **c**?

e Investigate and explain how the results to parts **c** and **d** would change if:

i the frame was 2 cm wide

ii the frame was 3 cm wide

iii the frame was for a 15 cm × 20 cm photo

iv the frame was for a 20 cm × 25 cm photo.

21 Alice packs the photo frames from question **20** into a gift box with the dimensions shown in the photo and wants to tie a ribbon around the present. Assuming that she will need a minimum of 60 cm of ribbon for the bow, at least how much ribbon will she need to buy in order to wrap the present as shown?



22 A rectangle has a perimeter of 30 m. List all the possible whole number dimensions for the rectangle.

23 A rectangular pool is 12.5 m long and 4.7 m wide.

a What length of paving is required around the pool's edge if the pavers used are 50 cm wide?

b Jordan incorrectly measured the length of the pool as 13.4 m and the width as 5.2 m. What is the size of the error in each measurement?

c Find the percentage error in each measurement by calculating the size of the error as a percentage of the actual measurement. Give your answers correct to one decimal place.

d Which measurement has a greater:

i error **ii** percentage error?

e Calculate the length of paving required if Jordan's measurements are used.

f Find the percentage error in the length of paving if using Jordan's measurements. Give your answer correct to one decimal place.

24 Jackson takes on a scrapbooking project. He decides to cover the internal perimeter of each page in ribbon that is 2 cm wide. If there are 56 square pages that measure 25 cm in the book, find the length of ribbon (in metres) required to place ribbon inside the perimeter of each page. Justify your answer.

25 A rectangular vegetable garden has a perimeter of 28 m.

a Write down a possible set of whole number dimensions for this garden.

A fence is to be placed 1 m from the edge around the vegetable garden to allow space for more vegetables.

b What length of fencing is required?

c What length of fencing would be required if the distance between the fencing and the edge of the garden was:

i 2 m **ii** 3 m?

d Can you see a pattern or rule that you could follow in order to find the length of fencing required, no matter how close or far from the edge it is placed?

e Repeat parts **a–d** using a different set of dimensions. Do your findings match the rule you found in part **d**? Explain your answer.

26 An inch is 2.54 cm.

a A foot is equal to 12 inches. What is the conversion factor from feet to centimetres?

b A yard is equal to 3 feet. What is the conversion factor from yards to metres?

c One mile is equal to 1760 yards. What is the conversion factor from miles to kilometres?

d The official distance for a marathon is 26 miles and 385 yards. What is this distance in kilometres?

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Converting between units of length

Interactive skillsheet Perimeter

Interactive skillsheet Circumference

Investigation In the extreme

Topic quiz 8A

8B Area of triangles and rectangles

Learning intentions

By the end of this topic you will be able to ...

- ✔ calculate the area of rectangles and triangles
- ✔ convert between units of area.

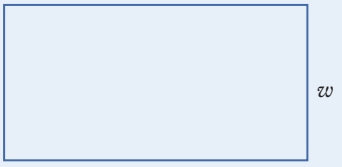
Inter-year links

Support	Understanding area
Year 7	9C Area of a rectangle
Year 9	5A Area of composite shapes
Year 10	6A Area review

Area

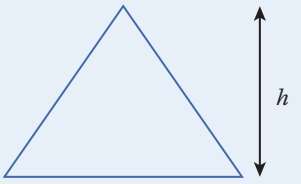
- The **area** of a 2D shape is the space occupied by that shape.
- To find the area of a rectangle, multiply its length (l) by its width (w).
- To find the area of a triangle, multiply its base (b) by its height (h) then divide by 2.
  → The **base** length and height must be perpendicular (at a right angles) to each other.

Rectangle



$A = lw$

Triangle



$A = \frac{1}{2}bh$

- Measurements must be in the same unit before they are substituted into area formulas.

Metric units of length	
Name	**Abbreviation**
Square millimetre	mm^2
Square centimetre	cm^2
Square metre	m^2
Square kilometre	km^2

Surface area

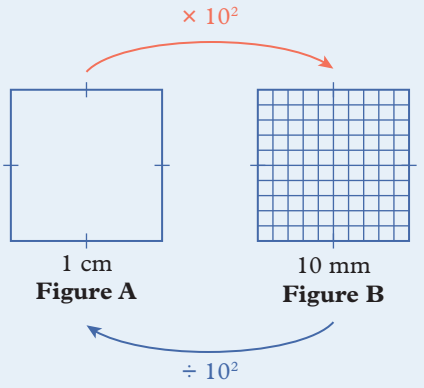
- The **surface area** of a 3D object is the total area of the outer surface of the object.
  → To find the surface area of a 3D object, first identify the shape and dimensions of each face, then find the area of each individual face and add the values together.

Converting between units of area

- To convert between square units of area, multiply or divide by the square of the conversion factor between the lengths.

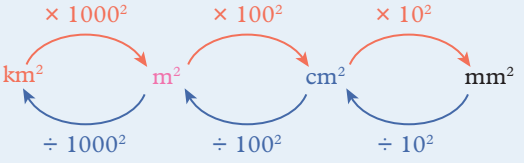
  For example, a square centimetre (1 cm × 1 cm = 1 cm^2) is equivalent to 100 square millimetres (10 mm × 10 mm = 10^2 mm^2 = 100 mm^2). That means we can multiply the number of cm^2 by 10^2 to determine the number of mm^2.

  Likewise we can divide the number of mm^2 by 10^2 to determine the number of cm^2, since each mm^2 is $\frac{1}{100}$ th of a cm^2.



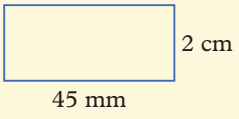
Conversion factors

1 cm^2 = 10^2 mm^2
= 100 mm^2
1 m^2 = 100^2 cm^2
= 10 000 cm^2
1 km^2 = 1000^2 m^2
= 1 000 000 m^2



Example 8B.1 Calculating the area of a rectangle

Calculate the area of this rectangle in cm^2.



THINK

1. Identify the measurements for l and w and convert the length into the unit specified by the question.
2. Substitute the values for l and w into the formula for the area of a rectangle.
3. Calculate the area of the rectangle. Remember to include the appropriate unit in the answer.

WRITE

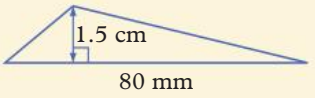
l = 45 mm
= 4.5 cm
w = 2 cm

$A = l \times w$
$= 4.5 \times 2$

$= 9$ cm^2

Example 8B.2 Calculating the area of a triangle

Calculate the area of this triangle in mm^2.



THINK	WRITE
1 Identify the measurements for b and h and convert the height into the units specified by the question.	$b = 80$ mm $h = 1.5$ cm $= 15$ mm
2 Substitute the values for b and h into the formula for the area of a triangle.	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 80 \times 15$
3 Calculate the area of the triangle. Remember to include the appropriate unit in the answer.	$= 600$ mm^2

Example 8B.3 Converting to a smaller area unit

Convert 3.8 cm^2 into mm^2.

THINK	WRITE
1 Identify the appropriate conversion factor. To convert from a larger unit to a smaller unit, multiply by the conversion factor.	1 cm^2 = 10^2 mm^2 = 100 mm^2
2 Complete the calculation.	3.8 cm^2 = 3.8 × 100 = 380 mm^2

Example 8B.4 Converting to a larger area unit

Convert 120 000 mm^2 into m^2.

THINK	WRITE
1 First convert from mm^2 to cm^2. To convert from a smaller unit to a larger unit, divide by the conversion factor.	1 cm^2 = 10^2 mm^2 = 100 mm^2 1 m^2 = 100^2 cm^2 = 10 000 cm^2 120 000 mm^2 = 120 000 ÷ 100 = 1200 cm^2
2 Complete the calculation by converting from cm^2 to m^2.	1200 cm^2 = 1200 ÷ 10 000 = 0.12 m^2

Helpful hints

✔ When multiplying by a power of ten, move the decimal point to the *right*.

5.23 × 10 = 52.3
5.23 × 100 = 523.
5.23 × 1000 = 5230.

✔ When dividing by a power of ten, move the decimal point to the *left*.

5.23 ÷ 10 = 0.523
5.23 ÷ 100 = 0.0523
5.23 ÷ 1000 = 0.005 23

✔ Remember that larger units take up more area for each square unit, so there will be less of them than for smaller units. Likewise, smaller units take up less area for each square unit, so there will be more of them than for larger units.

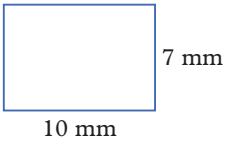
ANS p547

Exercise 8B Area of triangles and rectangles

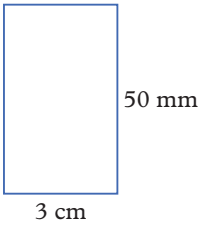
△ 1, 2, 3(a–e), 4–7, 8–9(a, c, f), 10–12, 13(a, b), 15, 17, 22, 24 ▭ 1, 2, 3(f–i), 4(b, c, f), 5–7, 8–9(d–f), 12, 13, 16, 18, 23, 25, 26(a, b) ⬠ 3(f–i), 4–7, 8–9(d–f), 13, 14, 19–21, 25, 26

8B.1 **1** Calculate the area of each rectangle. Where more than one unit is used, give your answer in terms of the smaller unit.

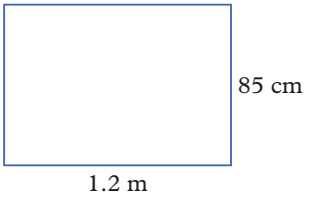
a



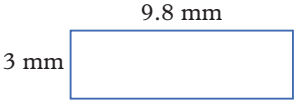
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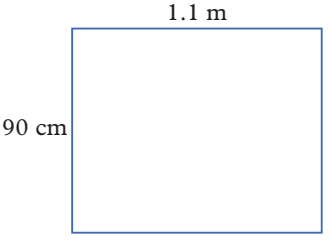
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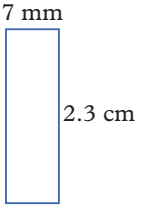
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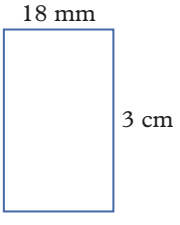
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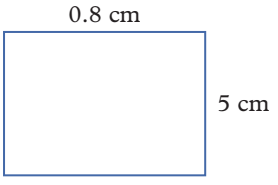
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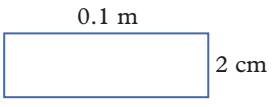
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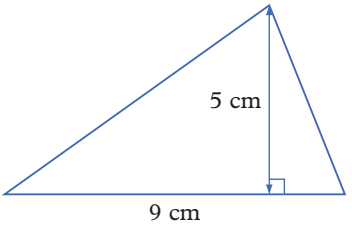
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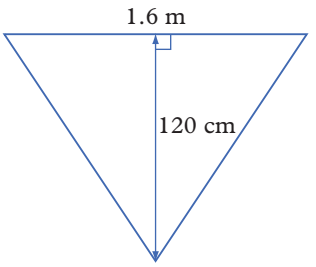
UNDERSTANDING AND FLUENCY

8B.2 **2** Calculate the area of each triangle. Where more than one unit is used, give your answer in terms of the larger unit.

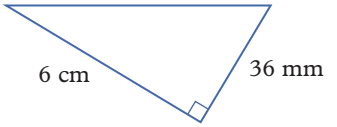
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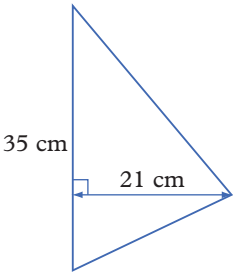
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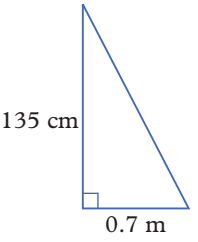
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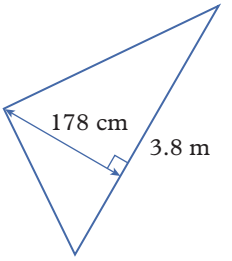
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e



f



3 Calculate the area of each shape in the unit specified in brackets.

a a rectangle with length 35 mm, width 4 cm (mm^2)

b a rectangle with length 12 mm, width 17 cm (cm^2)

c a triangle with base 15 mm, height 2 cm (mm^2)

d a square with length 7.2 mm (mm^2)

e a triangle with base 25 cm, height 10 mm (cm^2)

f a triangle with base 8 cm, height 42 mm (cm^2)

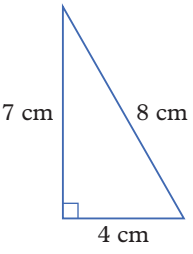
g a square with base 19 cm (cm^2)

h a rectangle with base 4.2 cm, height 11 mm (mm^2)

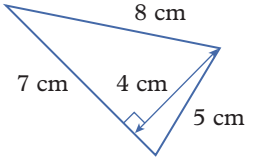
i a triangle with base 0.7 m, height 49 cm (cm^2)

4 Calculate the area of each shape.

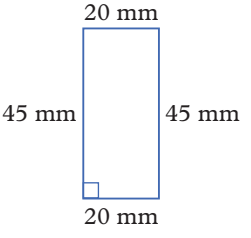
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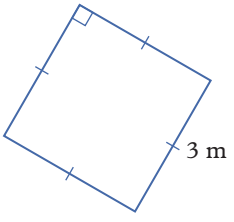
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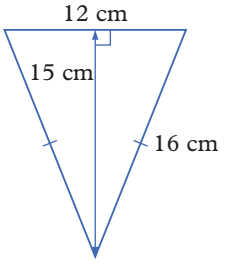
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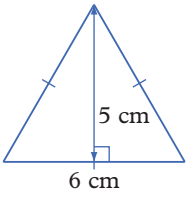
d



e



f



8B.3 **5** Convert each area measurement.

a 3 m^2 into cm^2 **b** 10 cm^2 into mm^2 **c** 6.5 m^2 into cm^2

d 19.7 cm^2 into mm^2 **e** 1.6 km^2 into m^2 **f** 0.75 km^2 into m^2

8B.4 **6** Convert each area measurement.

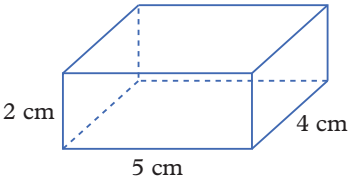
a 25 000 cm² into m²
b 750 000 mm² into cm²
c 400 mm² into cm²
d 125 000 m² into km²
e 675 cm² into m²
f 500 000 000 m² into km²

7 Convert each area measurement.

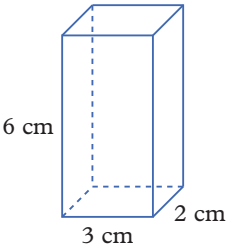
a 150 000 mm² into m²
b 0.002 km² into cm²
c 1.5 m² into mm²
d 379 000 cm² into km²

8 Calculate the surface area of each of these rectangular prisms.

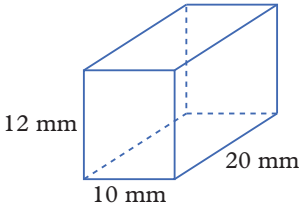
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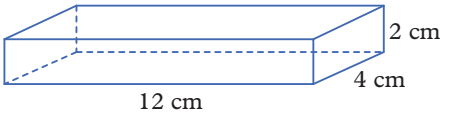
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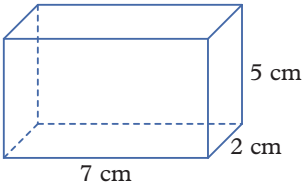
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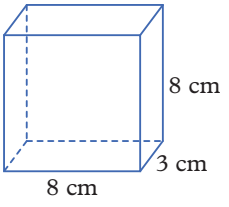
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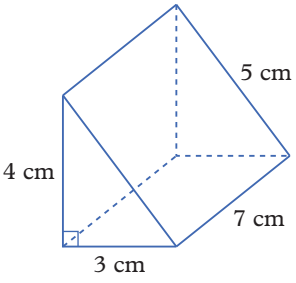


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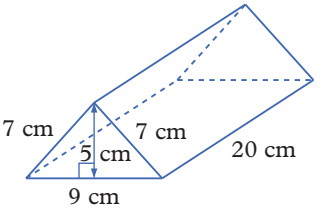


9 Calculate the surface area of each of these triangular prisms.

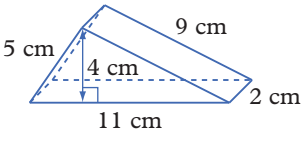
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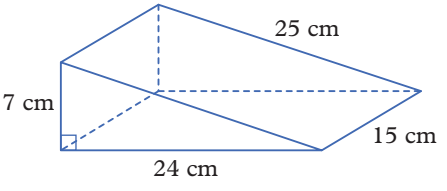
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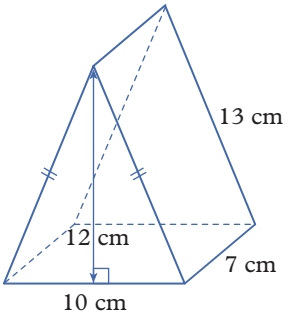
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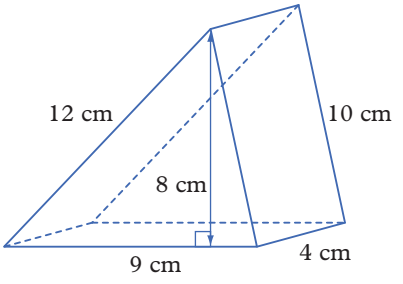
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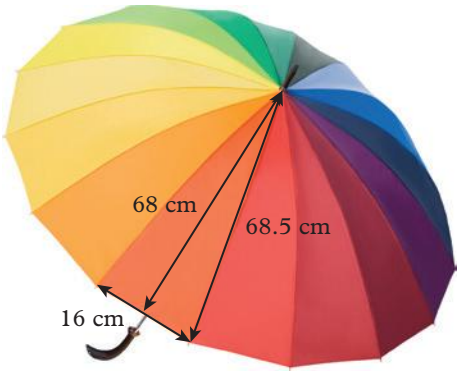
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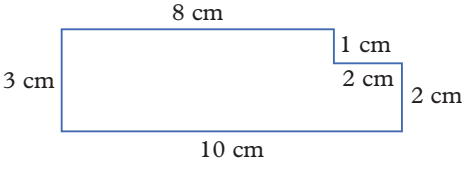


10 How much material is needed to make the multi-coloured umbrella shown?



11 A classmate attempted question **10** and after using 68.5 cm as the height of each triangle, obtained an answer of 8768 cm^2. Explain where they went wrong.

12 A composite shape is a shape that can be split up into two or more basic shapes. For example, the composite shape below can be split up into rectangles as shown in Figures A and B.



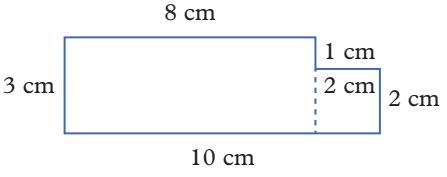


Figure A

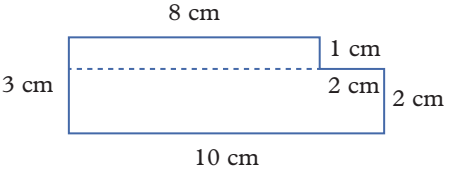


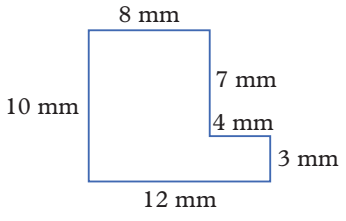
Figure B

a Find the area of each rectangle in Figure A and hence the area of the entire shape.

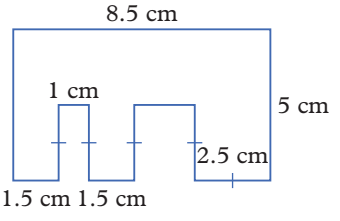
b Find the area of each rectangle in Figure B and hence the area of the entire shape.

13 Find the area of each of these composite shapes. Hint: For some parts it may be easier to subtract shapes.

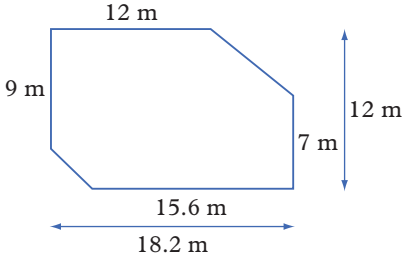
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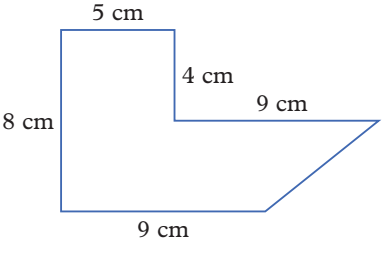
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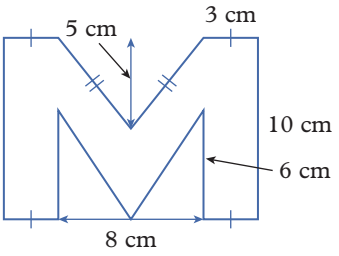
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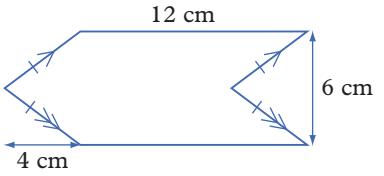
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f



14 Which unit of area (mm^2, cm^2, m^2 or km^2) would be best for measuring the area of these? Justify your answers.

a basketball court **b** wristwatch face **c** national park

d five-dollar note **e** surface of a skateboard ramp **f** family-size pizza

15 Ahmed needed to convert 11 cm^2 into mm^2. He said that because there are 10 mm in 1 cm, then there are 110 mm^2 in 11 cm^2. Explain where he went wrong and provide the correct answer.

16 Alex was completing a worksheet on conversions and provided the following answer to a question:

$40\,000\ cm^2 = (40\,000 \div 100)\ m^2$
$= 400\ m^2$

Explain where Alex made a mistake and provide the correct answer.

17 Jess wants to make a fitted valance for the new bed base she bought. Assuming that the bed base is a rectangular prism, determine the minimum amount of material that she needs to buy to make the fitted valance, which will need to cover the sides and top of the bed base.



18 A company produces painted wooden toy boxes. All six faces of the box are to be painted.



a Calculate the surface area of the outside of the box (the sum of the area of the six faces).

b If each box needs two coats of paint and one tin of paint covers 15 m^2, how many toy boxes can be painted with one tin?

19 If a pool measures 18 m by 11 m by 2 m deep, and it costs $5 per square metre to paint, how much will it cost to paint the interior of the swimming pool?

PROBLEM SOLVING AND REASONING

20 A hectare is 10 000 m^2.

a How many hectares are in 1 km^2?

There are many other units of area that are still used today that are not metric, such as the acre. A hectare is roughly equal to 2.5 acres.

b Approximately how many square metres are there in an acre?

21 Aaron harvests wheat on his farm and gets a yield of about 3.2 tonnes (3200 kg) per hectare.



a Write this in:

i kilograms per hectare

ii kilograms per square metre

iii grams per square metre

iv grams per square centimetre.

b Which unit do you think makes the most sense to use? Explain your answer.

c If Aaron is paid 35 cents per kilogram of wheat, find how much money he will earn if he harvests 3000 hectares of wheat.

22 A rectangle has an area of 20 cm^2 and a length of 4 cm. Find its width and explain why there is only one rectangle that you can draw that fits this description.

23 A rectangle has a perimeter of 28 cm. If the dimensions are whole numbers, find three possible areas this rectangle could have.

24 A square has an area of 64 mm^2. Sketch this square, stating its dimensions, its perimeter and why there is only one square that matches this description.

25 A wooden frame is to go around a rectangular mirror with an area of 3600 cm^2.

a If the mirror has a length of 90 cm, what is its width?

b Calculate the area of the wooden surface if the frame is to be 5 cm wide? Hint: Draw a picture to help you with your calculations.

CHALLENGE

26 Mohammed is making a rectangular vegetable garden and has 24 m of fencing to put around the garden.

a List three possible sets of dimensions and corresponding areas for the garden.

b What is the maximum area that you can find using 24 m of fencing? What shape is this?

Mohammed realises he can enclose a bigger area if he puts the garden against the shed wall and only has to fence three sides of the garden.

c List another three possible sets of dimensions and corresponding areas for the veggie garden if one side is against the shed wall.

d What is the maximum area that you can find now using 24 m of fencing for three sides? What shape is this? How does this relate to your answer to part **b**?

Check your Student obook pro for these digital resources and more:

- **Interactive skillsheet** Area of a rectangle
- **Interactive skillsheet** Area of a triangle
- **Interactive skillsheet** Converting between units of area
- **Investigation** Units of area and accuracy
- **Topic quiz** 8B

8C Area of quadrilaterals

Learning intentions

By the end of this topic you will be able to ...

✔ calculate the area of parallelograms, kites, rhombuses and trapeziums.

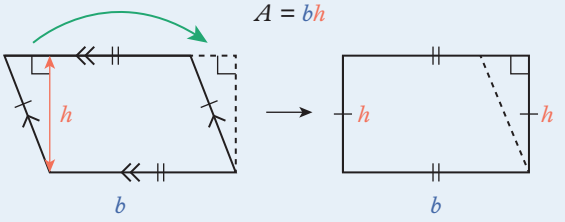
Inter-year links

Support	Understanding area
Year 7	9D Area of a parallelogram
Year 9	5A Area of composite shapes
Year 10	6A Area review

Area of quadrilaterals

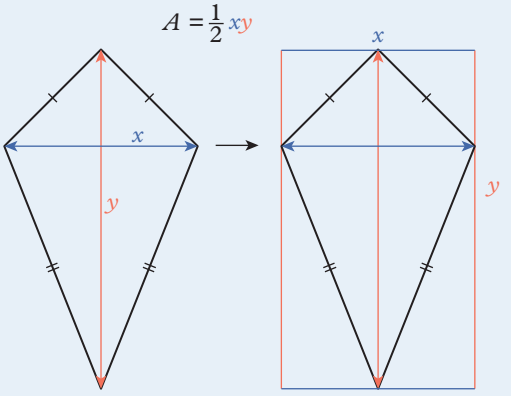
- Every parallelogram can be rearranged into a rectangle with the same base length and height; therefore, the area of a parallelogram can be calculated using the formula $A = bh$, where b is the base length and h is the perpendicular height.

Parallelogram



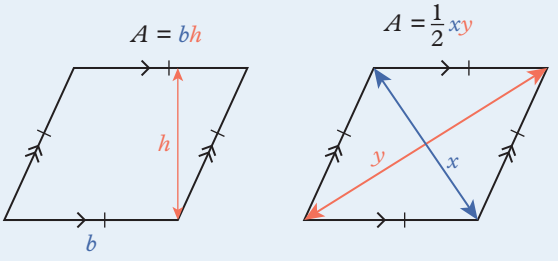
- The area of a kite is exactly half the area of the rectangle that encloses it. Therefore, the area of a kite can be calculated using the formula $A = \frac{1}{2}xy$, where x and y are the lengths of the diagonals of the kite.

Kite



- As every rhombus is both a kite and a parallelogram, the area of a rhombus can be calculated using either the formula for the area of a parallelogram, $A = bh$, or the formula for the area of a kite, $A = \frac{1}{2}xy$, depending on the information provided.

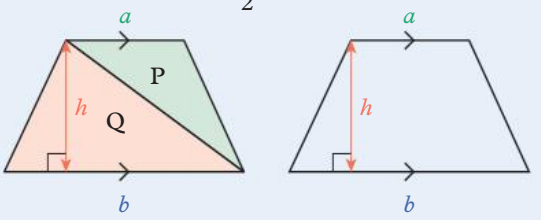
Rhombus



- Every trapezium can be split into two triangles, both with the same perpendicular height. The area of triangle P is $\frac{1}{2}ah$ and the area of triangle Q is $\frac{1}{2}bh$; therefore, the area of the trapezium is $A = \frac{1}{2}ah + \frac{1}{2}bh$, or more concisely, $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is the perpendicular height.

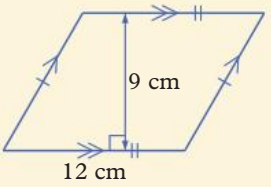
Trapezium

$A = \frac{1}{2}(a + b)h$



Example 8C.1 Calculating the area of a parallelogram

Calculate the area of this parallelogram.



THINK

1. Identify the measurements for b and h.
2. Substitute the values for b and h into the formula for the area of a parallelogram.
3. Calculate the result. Remember to include the appropriate unit in the answer.

WRITE

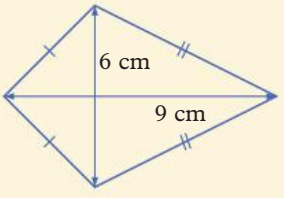
$A = bh$

$= 12 \times 9$

$= 108 \text{ cm}^2$

Example 8C.2 Calculating the area of a kite

Calculate the area of this kite.



THINK

1. Identify the measurements for x and y.
2. Substitute the values for x and y into the formula for the area of a kite.
3. Calculate the result. Remember to include the appropriate unit in the answer.

WRITE

$x = 6$ cm, $y = 9$ cm

$A = \frac{1}{2}xy$

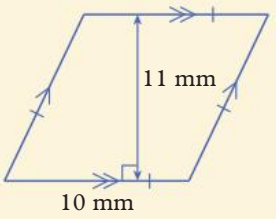
$= \frac{1}{2} \times 6 \times 9$

$= 27 \text{ cm}^2$

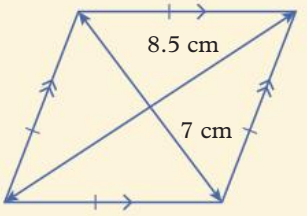
Example 8C.3 Calculating the area of a rhombus

Calculate the area of these rhombuses.

a



b



THINK

a
1. Determine which formula to use. b and h are given, so use the formula for the area of a parallelogram.
2. Identify the given measurements.
3. Substitute the values for b and h into the formula for the area of a parallelogram.
4. Calculate the result and include the appropriate unit.

b
1. Determine which formula to use. x and y are given, so use the formula for the area of a kite.
2. Identify the given measurements.
3. Substitute the values for x and y into the formula for the area of a kite.
4. Calculate the result and include the appropriate unit.

WRITE

a $b = 10$ mm, $h = 11$ mm

$A = bh$

$b = 10$ mm, $h = 11$ mm

$A = 10 \times 11$

$= 110$ mm^2

b $x = 7$ cm, $y = 8.5$ cm

$A = \frac{1}{2}xy$

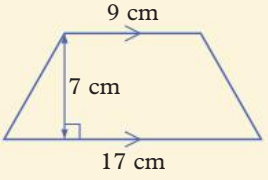
$x = 7$ cm, $y = 8.5$ cm

$A = \frac{1}{2} \times 7 \times 8.5$

$= 29.75$ cm^2

Example 8C.4 Calculating the area of a trapezium

Calculate the area of this trapezium.



THINK

1. Identify the given measurements.
2. Substitute the values for a, b and h into the formula for the area of a trapezium.
3. Calculate the result and include the appropriate unit.

WRITE

$a = 9$ cm, $b = 17$ cm, $h = 7$ cm

$A = \frac{1}{2}(a + b)h$

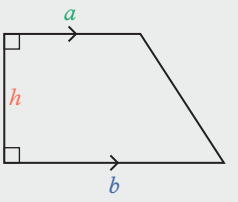
$= \frac{1}{2} \times (9 + 17) \times 7$

$= \frac{1}{2} \times 26 \times 7$

$= 91$ cm^2

Helpful hints

- ✔ All area formulas must include the multiplication of two lengths to create an area.
- ✔ In the formulas for the area of a parallelogram and trapezium, the height must be the perpendicular distance between parallel sides.
- ✔ In a right-angled trapezium, the height will be the length of the side joining the right angles.
- ✔ In the formula for the area of a kite, it doesn't matter which diagonal length you label x or y, since $xy = yx$! Likewise, in the formula for the area of a trapezium it doesn't matter which of the parallel sides you label a or b, since $a + b = b + a$!



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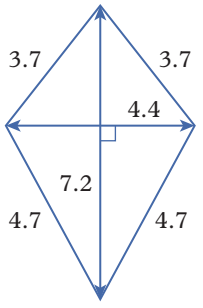
Exercise 8C Area of quadrilaterals

1–8, 10, 11, 13 | 2–5(b, c), 6–8, 11, 12, 14, 17(a), 20 | 6–8, 12, 15, 17(b), 18–20

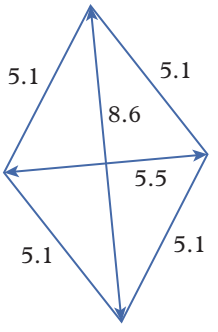
UNDERSTANDING AND FLUENCY

1 Which of the lengths in the following shapes are needed to determine the area?

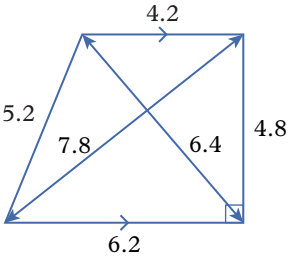
a



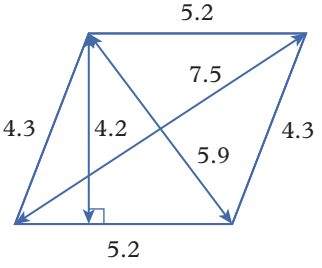
b



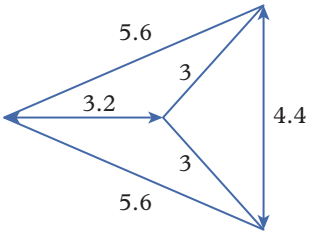
c



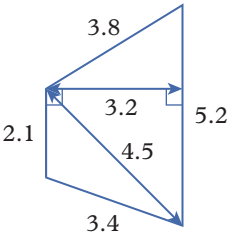
d



e

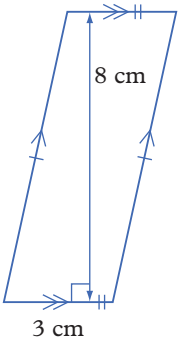


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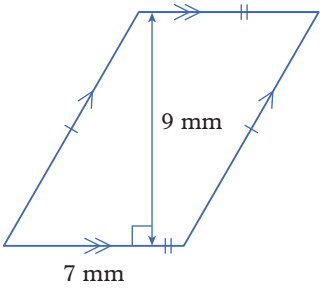


8C.1 **2** Calculate the area of each parallelogram.

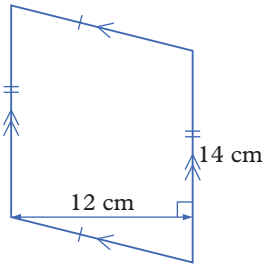
a



b



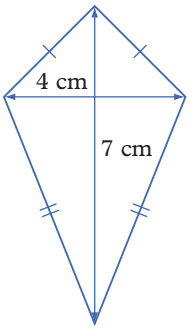
c



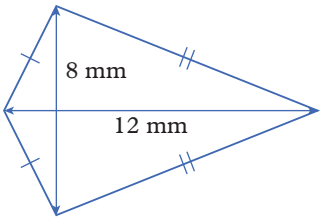
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8C.2 **3** Calculate the area of each kite.

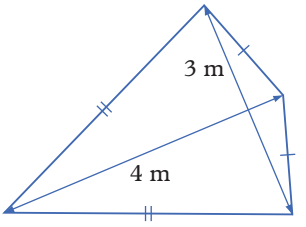
a



b

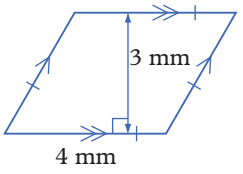


c

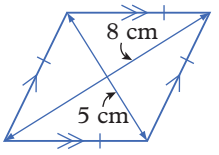


8C.3 **4** Calculate the area of each rhombus.

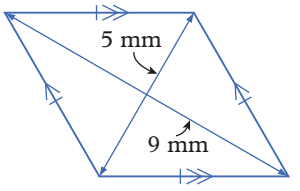
a



b

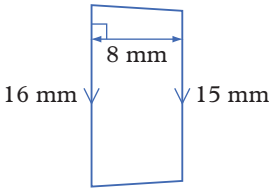


c

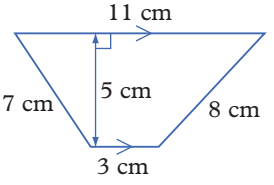


8C.4 **5** Calculate the area of each trapezium.

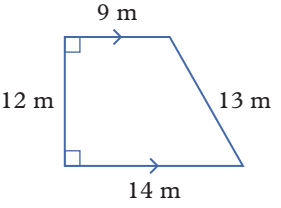
a



b



c



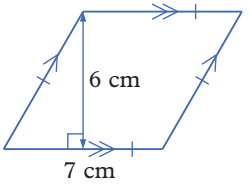
6 For each shape:

i identify if it is a parallelogram, kite, rhombus or trapezium

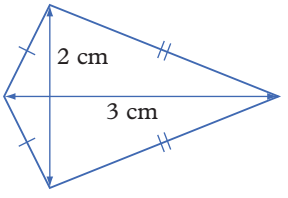
ii write the formula to find its area

iii calculate its area.

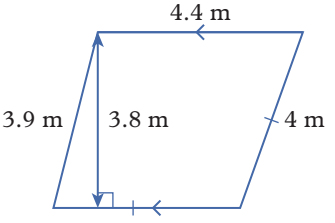
a



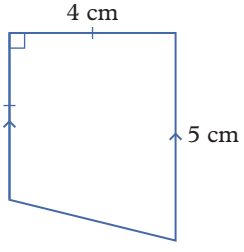
b



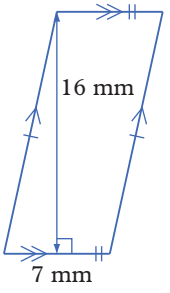
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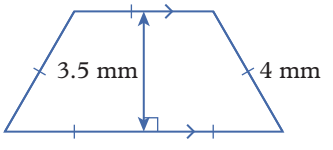
d



e

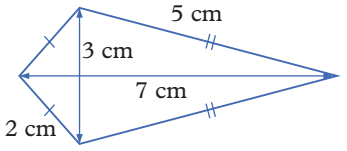


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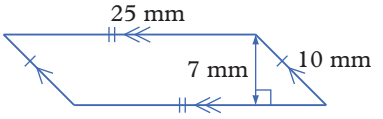


7 Calculate the area of each quadrilateral.

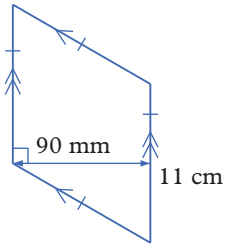
a



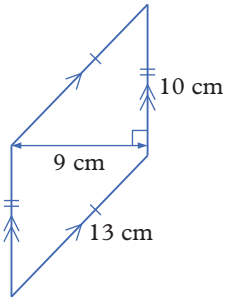
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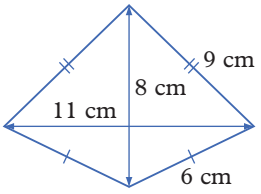
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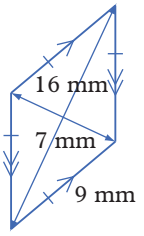
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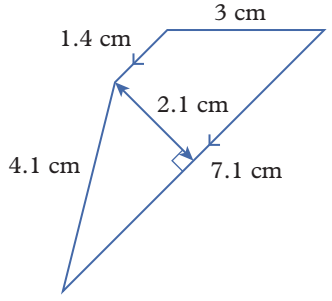


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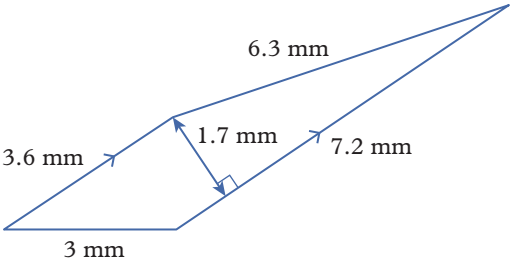


8 Evaluate the areas of the following shapes.

a



b

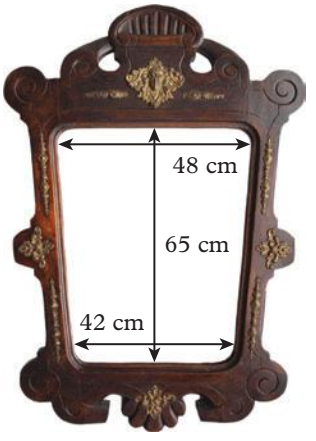


9 Amanda wants to make some kites. She decides to make two types, a large blue kite and a smaller red kite.

a The blue kite is 95 cm long and 65 cm wide. What is its area?

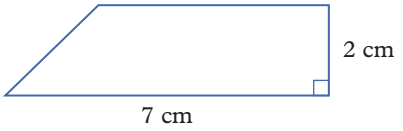
b The red kite has an area of 400 cm^2. If the kite is 40 cm long, how wide must it be?

10 Find the cost of fitting the frame on the right with a mirror if your local hardware store charges \$0.25 per cm^2 for mirror glass.

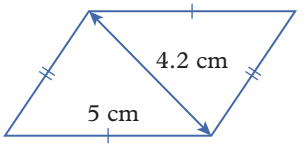


11 If no other side lengths are determined, is there enough information to calculate the area of the following shapes? Give reasons with your answers.

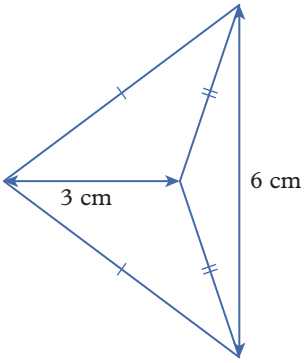
a



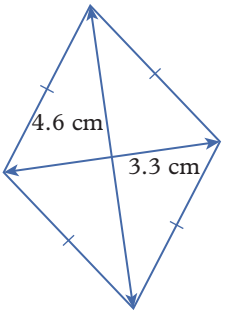
b



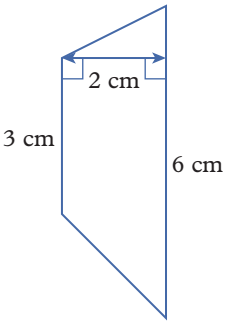
c



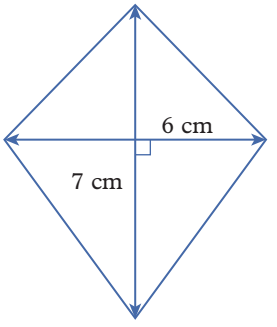
d



e



f

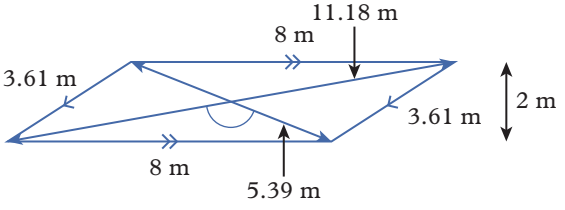


12 Some quadrilaterals can be named in multiple ways; for example, every square is also a rectangle. For each of the following quadrilaterals:

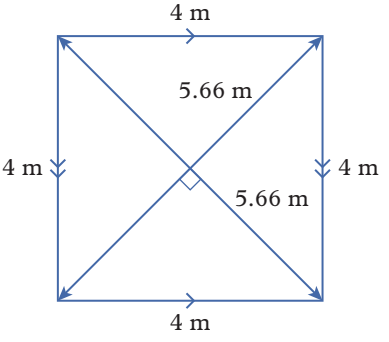
i state if the shape is a parallelogram, kite, rhombus and/or trapezium

ii determine the area of the quadrilateral, correct to one decimal place where necessary.

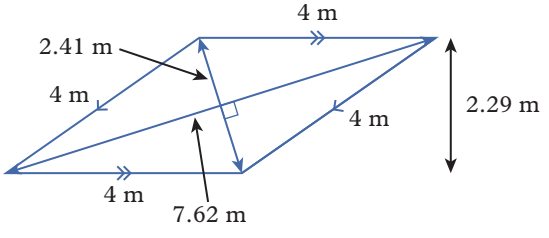
a



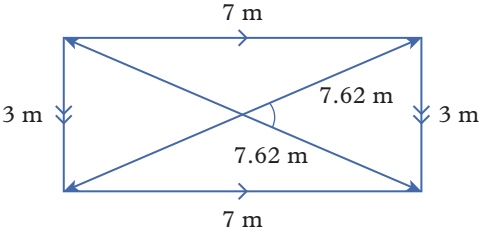
b



c



d



13 Consider this building in Hamburg.



a What shape is it?

b If the glass section is 21 m tall and the base is 86 m long, what is the area of this entire section?

c Is every window equal in area? Explain your answer.

d Which area formulas would you need to calculate the area of the windows?

14 Use the information provided to determine the missing lengths.

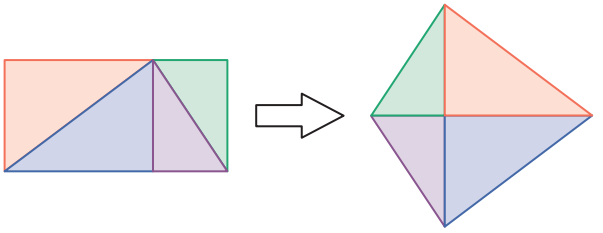
a The area of a kite is 72 cm^2 and the length of one of the diagonals is 16 cm. Determine the length of the other diagonal.

b The area of a rhombus is 17 cm^2 and the base length is 4 cm. Determine the height of the rhombus.

c The area of a trapezium is 22.5 cm^2 and the sum of the lengths of the parallel sides is 9 cm. Determine the height of the trapezium.

d The area of a trapezium is 48 mm^2, the height is 3 mm, and the length of one of the parallel sides is 1.3 cm. Determine the length of the other parallel side.

15 A rectangle has an area of 48 cm^2. The rectangle is cut into two pairs of congruent triangles which are rearranged to form a kite. If the lengths of the diagonals are integers, what possible dimensions (lengths of the diagonals) can the kite have?



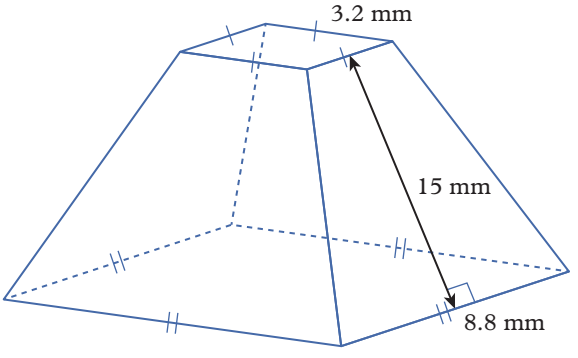
16 a The perimeter of a square is 36 cm. Determine the area of the square.

b The area of a square is 36 cm^2. Determine the perimeter of the square.

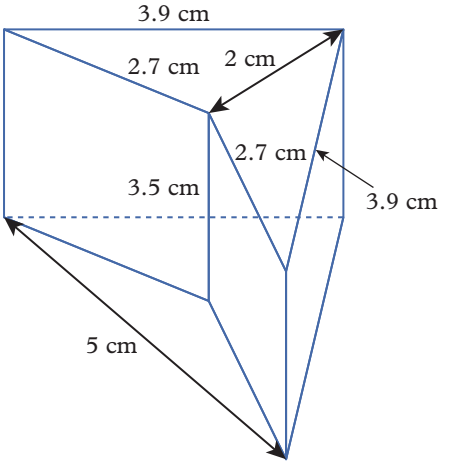
c Determine the side length of a square (in cm) such that the value of the area and perimeter are the same.

17 Calculate the surface area of the following objects.

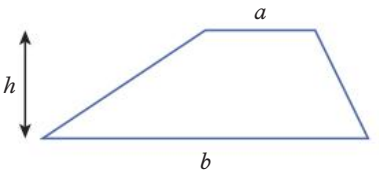
a truncated square-based pyramid



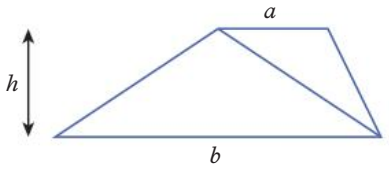
b kite-based prism



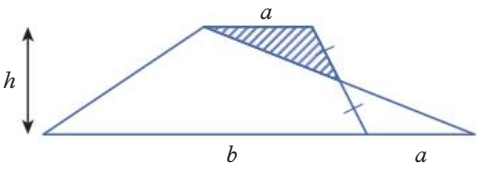
18 The formula for the area of a trapezium can be derived in multiple ways. Describe how each diagram has been constructed from the trapezium shown, and how it could be used to derive the formula for the area of the trapezium.



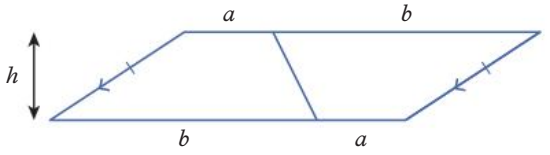
a



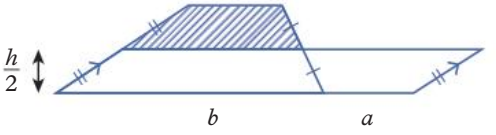
b



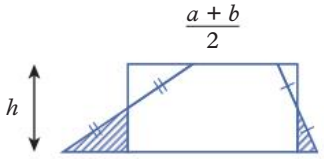
c



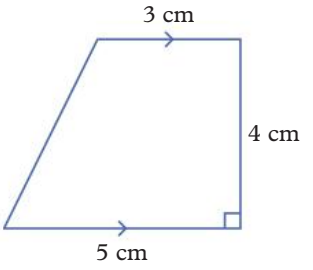
d



e



19 Consider the trapezium shown. The shorter parallel side is increased by 150%, the longer parallel side is increased by 20%, and the height is decreased by 75%.



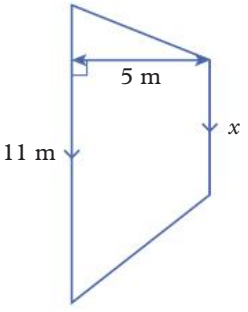
a Determine the area of:

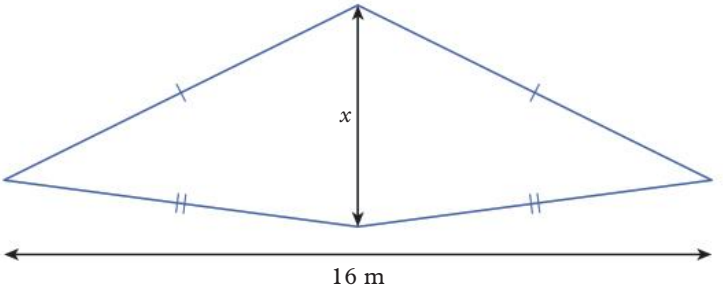
i the original trapezium

ii the altered trapezium.

b Determine the percentage change of the area, stating whether it is an increase or decrease.

20 Determine the value of x such that these two quadrilaterals have the same area. State this area.





Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Area of quadrilaterals

Investigation
Land to value

Topic quiz
8C

8D Area of a circle

Learning intentions

By the end of this topic you will be able to ...

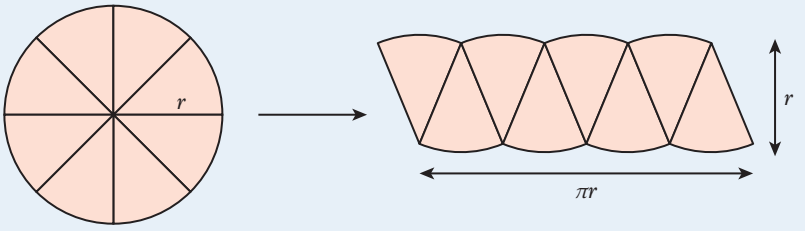
- ✔ estimate the area of a circle by using a square grid
- ✔ calculate the area of a circle
- ✔ calculate the radius, diameter or circumference, given the area of a circle.

Inter-year links

Support	Understanding area
Year 7	9E Area of a triangle
Year 9	5A Area of composite shapes
Year 10	6A Area review

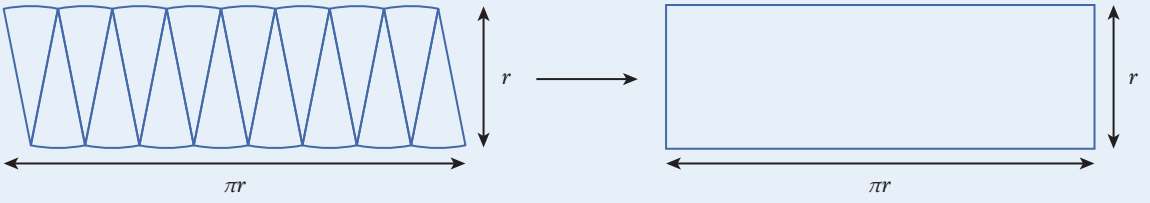
Area of a circle

- By dividing a circle into equal parts and rearranging these parts as shown, we can find the area of a circle.

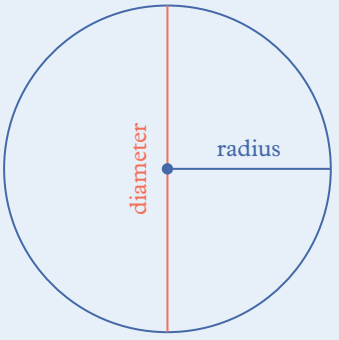


→ The base of the rearranged shape is approximately equal to half of the circumference (πr).

- As the number of parts increases, the resulting figure becomes rectangular in shape.



- Therefore, the formula for the area of a circle with radius r is



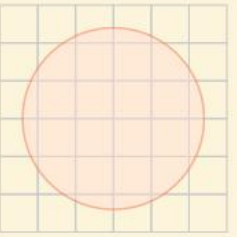
$$A = \pi r^2$$

→ If you are given the diameter, find the radius before using the formula.

→ This formula can be rearranged to find the radius of a circle, given the area.

Example 8D.1 Estimating the area of a circle

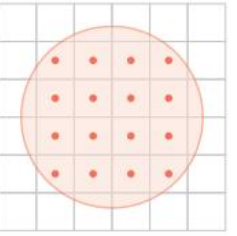
Estimate the area of this circle shown on 1 cm grid paper.



THINK

1. Count the number of squares that are completely enclosed by the shape.
2. Count the number of squares that are at least half covered by the shape. We'll include these squares in our final tally and exclude any squares that are less than half covered.
3. Give your answer for the area. To show your answer is an estimate, use the symbol ≈, which means 'is approximately equal to'.

WRITE

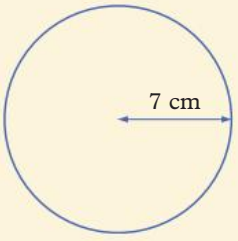


Area ≈ 16 cm^2

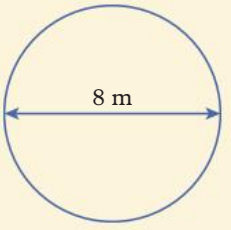
Example 8D.2 Calculating the area of a circle

Calculate the area of these circles correct to two decimal places.

a



b



THINK

a
1. Identify the value of the radius, r.
2. Substitute the value for r into the formula for the area of a circle.
3. Calculate the result using π on your calculator.
4. Round to two decimal places and include the appropriate unit.

b
1. Identify the value of the radius, r. Divide the diameter by 2 to find the radius.
2. Substitute the value for r into the formula for the area of a circle.
3. Calculate the result using π on your calculator.
4. Round to two decimal places and include the appropriate unit.

WRITE

a $r = 7$ cm

$A = \pi r^2$

$= \pi \times 7^2$

$= 153.938...$

≈ 153.94 cm^2

b $r = d \div 2$

$= 8 \div 2$

$= 4$ m

$A = \pi r^2$

$= \pi \times 4^2$

$= 50.265...$

≈ 50.27 m^2

Example 8D.3 Calculating the radius of a circle, given the area

Find the radius of a circle which has an area of 184 cm^2, correct to two decimal places.

THINK

1 Identify both the given value and the required value.

2 Substitute the value for A into the formula for the area of a circle.

3 Rearrange the formula to calculate the value of the radius. Remember to follow the order of operations.

4 Round to two decimal places and include the appropriate unit.

WRITE

$A = 184$ cm^2, $r = ?$

$$A = \pi r^2$$
$$184 = \pi r^2$$
$$\frac{184}{\pi} = r^2$$
$$\sqrt{\frac{184}{\pi}} = r$$
$$r = 7.653...$$
$$r \approx 7.65 \text{ cm}$$

Helpful hints

✔ The formulas for the circumference of a circle ($2\pi r$) and the area of a circle (πr^2) are similar. To tell them apart look at where the 2 is: when the 2 is an exponent, it's an area formula; when the 2 is a coefficient, it's a perimeter formula.

ANS p549

Exercise 8D Area of a circle

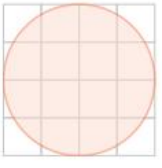
△ 1–8, 10, 15, 16, 18(a, b, e), 19(a, b) □ 1, 3–7, 9, 11, 12, 14, 18–19(a, c, e), 22 ⬠ 1(c), 3(d–f), 4–7, 11–14, 18(e, f), 19(d–f), 20–23

UNDERSTANDING AND FLUENCY

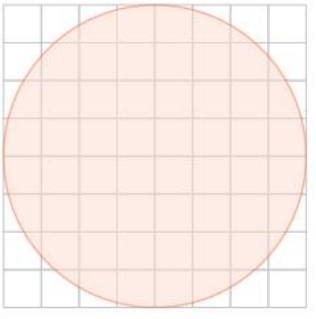
For all relevant questions in this topic, use π on your calculator and unless stated otherwise round answers to two decimal places.

8D.1 **1** Estimate the area of these circles shown on 1 cm grid paper.

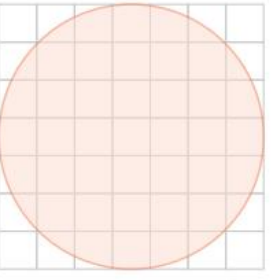
a



b



c



2 For each of the circles in question **1**:

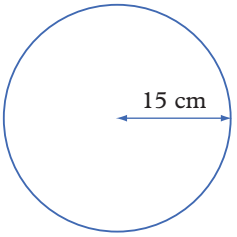
i give the diameter in cm

ii give the radius in cm

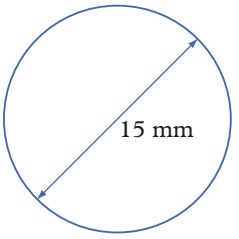
iii divide the estimated area by the square of the radius.

8D.2 **3** Calculate the area of each circle.

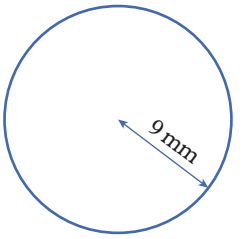
a



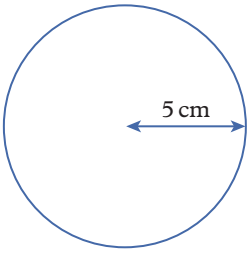
b



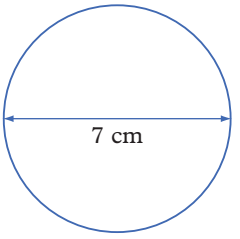
c



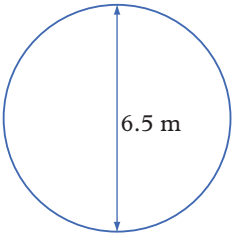
d



e

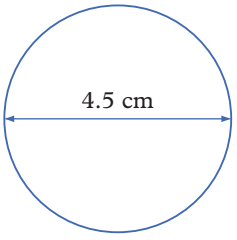


f

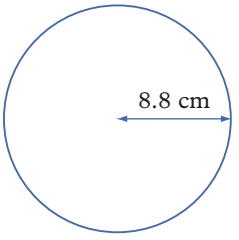


4 Calculate the exact area of each circle by leaving π in the answer.

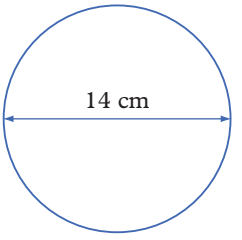
a



b



c



5 Calculate the area and perimeter of a circle with:

a radius 4.4 cm **b** diameter 7.5 m **c** diameter 15 mm **d** radius 29 mm.

8D.3 **6** Find the radius of a circle which has an area of:

a 12 cm^2 **b** 25 mm^2 **c** 100 cm^2 **d** 68 cm^2.

7 Find the diameter of a circle which has an area of:

a 80 cm^2 **b** 49 cm^2 **c** 20 mm^2 **d** 135 cm^2.

8 Anika wants to make a circular placemat which has a diameter of 55 cm. What is the area of the placemat?

9 What is the total area covered by a sprinkler which has a range of 7.5 m?

10 A pizzeria sells two sizes of pizza: medium with a diameter of 12 inches and large with a diameter of 18 inches. Explain why you get more pizza by ordering one large rather than two medium pizzas.

11 a The area of a circle can be written as $A = \frac{1}{4}\pi d^2$, where d is the diameter of the circle. Show that $\frac{1}{4}\pi d^2 = \pi r^2$.

b The area of a circle can be written as $A = \frac{1}{2}Cr$, where C is the circumference of the circle and r is the radius. Show that $\frac{1}{2}Cr = \pi r^2$.

12 Use $A = \pi r^2$ or $A = \frac{1}{4}\pi d^2$ to solve the following problems.

a Find A when $d = 8$ m.

b Find A when $r = 3$ cm.

c Find A when $C = 40$ mm.

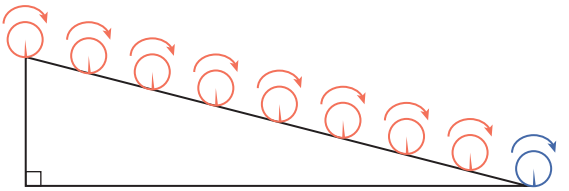
d Find A when $C = 50$ cm.

e Find d when $A = 125$ cm^2.

f Find C when $A = 90$ mm^2.

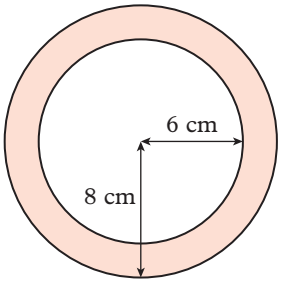
g Find C when $A = 20$ cm^2.

13 A circular object with an area of 58 cm^2 rolls down an inclined ramp after exactly 8 revolutions. How long is the ramp correct to the nearest integer?

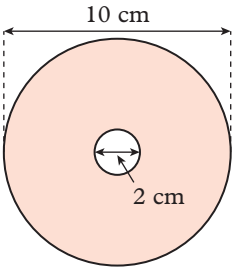


14 You can subtract the area of one circle from another to find the area of a ring. Another name for a ring is an annulus. Find the area for each annulus.

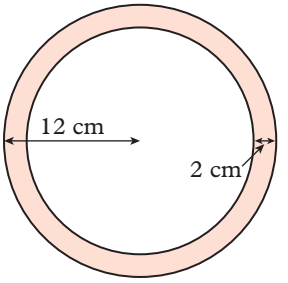
a



b

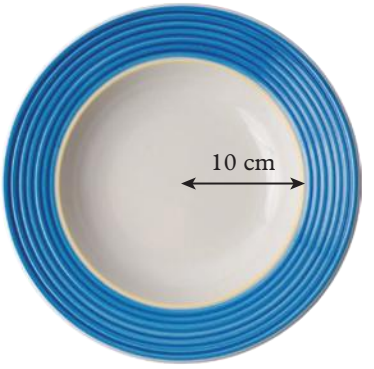


c



15 What is the largest area of pancake that could be cooked in a frying pan with a base diameter of 24 cm?

16 Adrian wants to cook a pancake using the frying pan from question **15** and place it on this plate.



a What is the maximum area a pancake could have if it had to fit on the white section of this plate, and the white section has a radius of 10 cm?

b To make a pancake the size you found in part **a**, what distance should be between the edges of the pancake and the side of the frying pan?

c What is the entire area of the plate if it has a diameter of 30 cm?

d What is the area of the painted section of the plate?

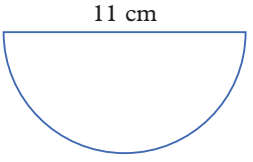
17 What is the total area that this button covers if it has a diameter of 20 mm and each hole has a radius of 1.5 mm?



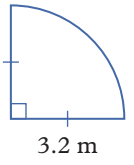
18 A semicircle is exactly half of a circle, and a quadrant is exactly a quarter of a circle. To find the area of semicircles and quadrants, find the area of the full circle and then divide by 2 or 4, respectively.

Find the area of these sections of circles.

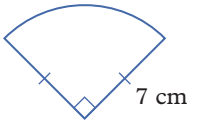
a



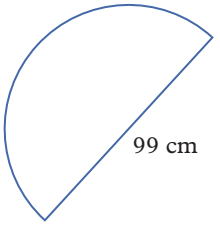
b



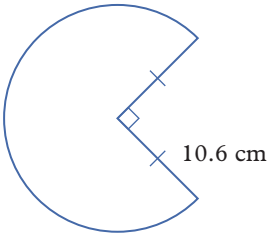
c



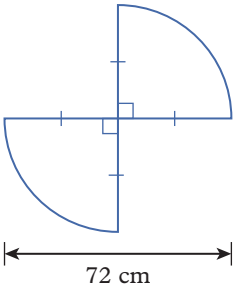
d



e

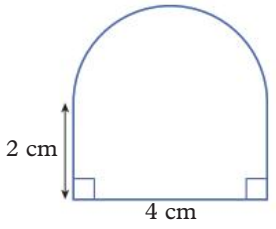


f

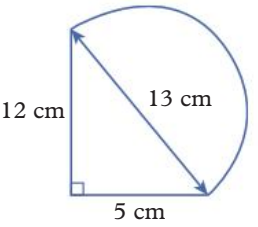


19 Use your understanding of the area of a circle to find the area of each composite shape.

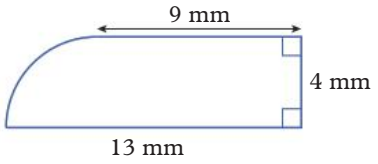
a



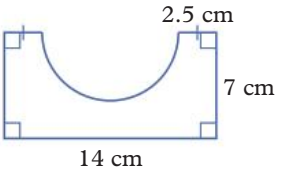
b



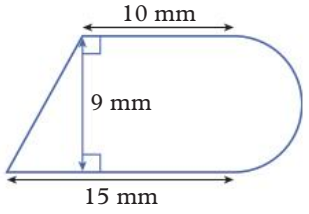
c



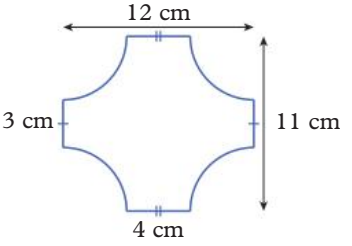
d



e



f



20 Explain why you can only find the exact length of the radius of a circle if the area is given with π still included in the value. Support your answer by finding the radius of a circle with an area of:

a 16 mm^2 **b** 16π mm^2.

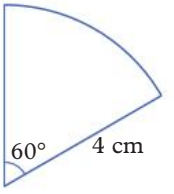
21 a A circle has an area of 49π cm^2. Find its:

i diameter **ii** circumference (keeping π in the answer).

b A circle has a circumference of 8π m. Find its:

i radius **ii** area (keeping π in the answer).

22 A sector is a portion of a circle formed by two radii and part of the circumference (an arc). You can use knowledge of angles to calculate the area of a sector. Look at the sector shown here.



a What is the angle at the vertex of the sector (where the two radii meet)?

b How many degrees are in a circle?

c Use your answers to parts **a** and **b** to write the angle for this sector as a fraction of the degrees in a circle.

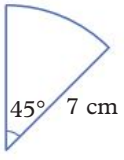
d What is the radius of this sector?

e What would be the area of the circle if it was complete?

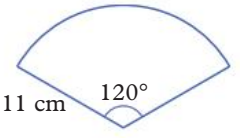
f Use your answers to parts **c** and **e** to find the area of the sector.

23 Calculate the area of each sector.

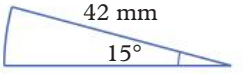
a



b



c



Check your Student obook pro for these digital resources and more:

Interactive skillsheet Area of a circle

Investigation Using arrays and rectangles to approximate areas

Topic quiz 8D

PROBLEM SOLVING AND REASONING

CHALLENGE

Checkpoint

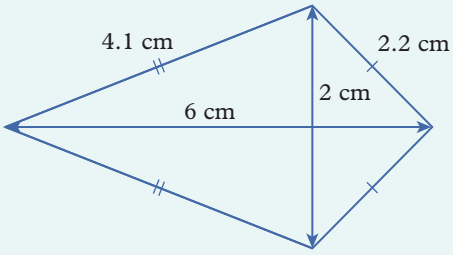
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

8A **1** Convert the following.

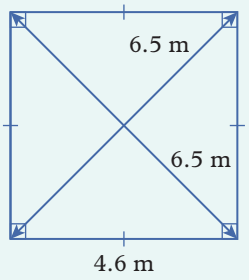
a 45 cm to m **b** 5.9 km to m **c** 351 mm² to cm² **d** 0.2 m² to cm²

8A **2** Calculate the perimeter of the following shapes.

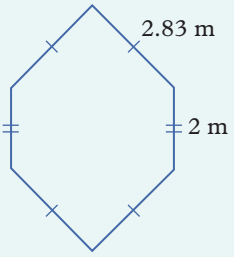
a



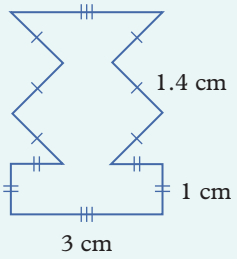
b



c

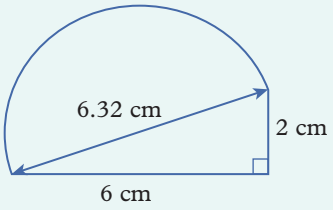


d

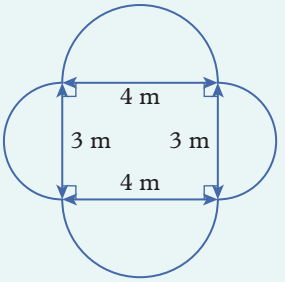


8A **3** Calculate the perimeter of the following composite shapes correct to two decimal places.

a



b

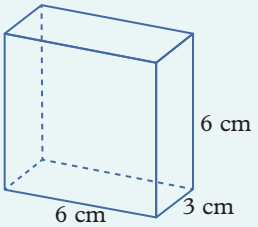


8A **4** Calculate the circumference of the following circles. Give your answers correct to two decimal places.

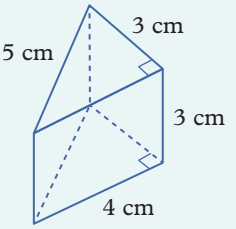
a circle with diameter 31 m **b** circle with radius 7 cm

8B **5** Calculate the surface area of the following objects.

a



b



8C **6** Use the given information to find the missing length.

a The perimeter of a square is 22 m. Determine the side length of the square.

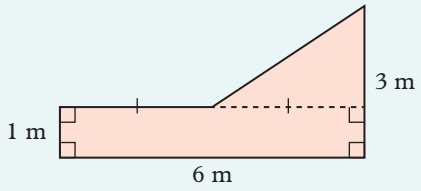
b The area of a rectangle with a width of 4 m is 28.8 m². Determine the length of the rectangle.

c The area of a kite is 31.5 cm². If one of the diagonals is 7 cm long, determine the length of the other diagonal.

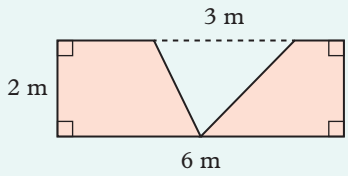
d The area of a trapezium is 37.8 cm². If one of the parallel sides is 9.5 cm long and the perpendicular height is 6 cm, determine the length of the other parallel side.

8B **7** Calculate the shaded area of the following composite shapes.

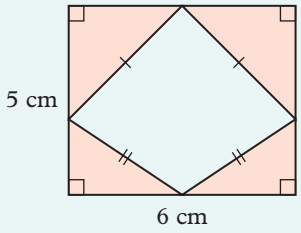
a



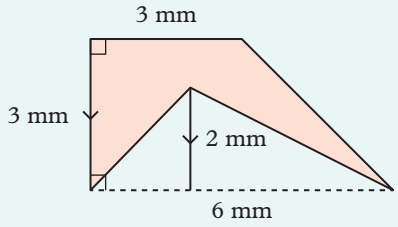
b



c



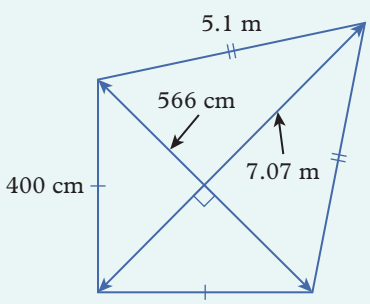
d



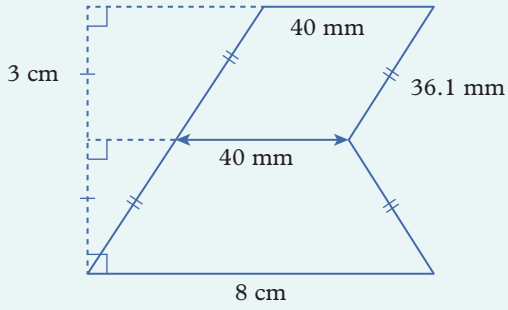
8C **8** For each of the following shapes, calculate the:

i perimeter **ii** area.

a

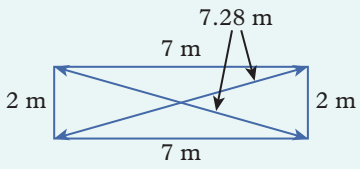


b

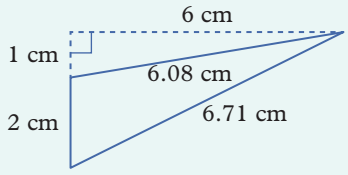


8C **9** Calculate the area of the following shapes, giving your answers correct to two decimal places where necessary.

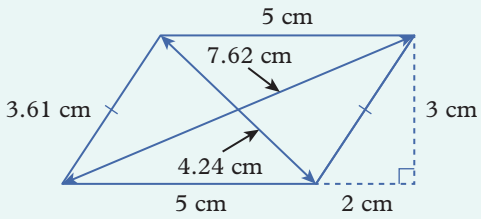
a



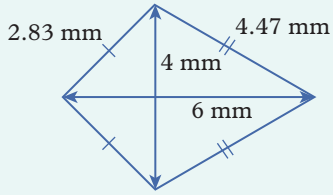
b



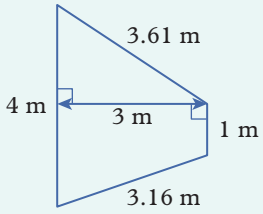
c



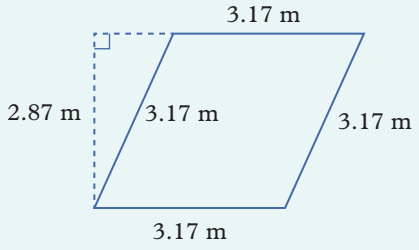
d



e



f



8D **10** Calculate the area of each shape in question **3**. Give your answers correct to two decimal places.

8E Volume and capacity

Learning intentions

By the end of this topic you will be able to ...

- ✔ calculate the volume and capacity of right prisms
- ✔ convert units of volume and capacity.

Inter-year links

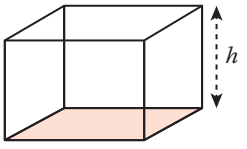
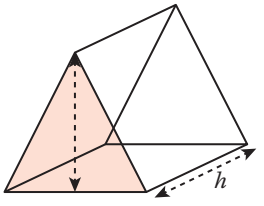
Support	Volume and capacity
Year 7	9G Volume of prisms
Year 9	5C Volume
Year 10	6C Volume

Volume of prisms

- The **volume** of a 3D object is the amount of three-dimensional space that it occupies.

Metric units of volume	
Name	**Abbreviation**
Cubic millimetre	mm^3
Cubic centimetre	cm^3
Cubic metre	m^3

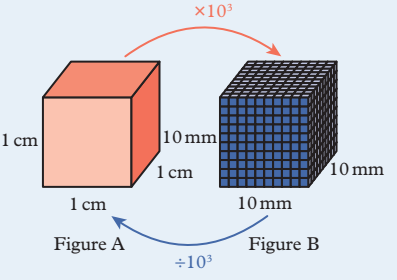
- A **prism** is a 3D object with a consistent cross section. The cross section has the same shape as the **base** of the prism.
  → The shape of the cross section must be a **polygon**.
- The volume (V) of any prism can be found by multiplying the area of the base (A) by the height (h), $V = Ah$.

Shape	Diagram	Formula
Rectangular prism		V = (area of rectangle) × h
Triangular prism		V = (area of triangle) × h

- The base and height of a prism must be perpendicular (at right angles) to each other.

Converting between units of volume

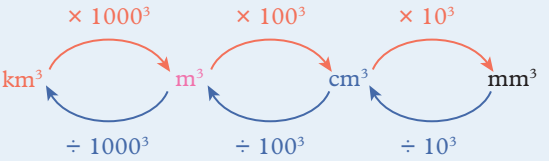
- To convert between units of volume, multiply or divide by the cube of the conversion factor.
  For example, a cubic centimetre (1 cm × 1 cm × 1 cm = 1 cm^3) is equivalent to 1000 cubic millimetres (10 mm × 10 mm × 10 mm = 10^3 mm^3 = 1000 mm^3). That means we can multiply the number of cm^3 by 10^3 to determine the number of mm^3. Likewise we can divide the number of mm^3 by 10^3 to determine the number of cm^3, since each mm^3 is $\frac{1}{1000}$ of a cm^3.



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Conversion factors

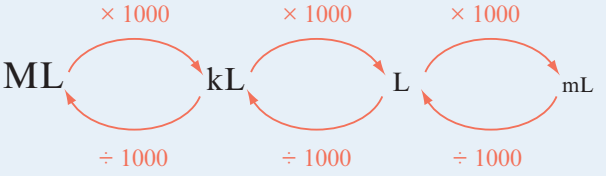
$1\ cm^3 = 10^3\ mm^3$
$= 1000\ mm^3$
$1\ m^3 = 100^3\ cm^3$
$= 1\ 000\ 000\ cm^3$
$1\ km^3 = 1000^3\ m^3$
$= 1\ 000\ 000\ 000\ m^3$



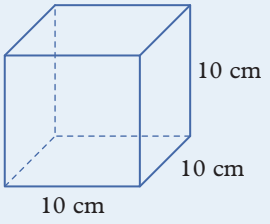
Capacity

- The **capacity** of a 3D object is a measure of how much that object can hold.

Metric units of capacity	
Name	**Abbreviation**
Millilitres	mL
Litres	L
Kilolitres	kL
Megalitres	ML



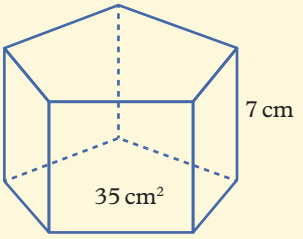
→ A container with an inside volume of 1 cm^3 holds 1 mL of liquid.
→ A container with an inside volume of 1000 cm^3 holds 1 L of liquid.
→ A container with an inside volume of 1 m^3 holds 1 kL of liquid.



Volume = 1000 cm^3
Capacity = 1 L

Example 8E.1 Calculating the volume of a prism

Calculate the volume of this prism.



THINK

1 Identify the values for A and h.

2 Substitute the values for A and h into the formula for the area of a prism.

3 Calculate the volume of the prism. Remember to include the appropriate unit in the answer.

WRITE

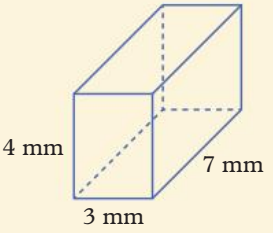
$A = 35\ cm^2$, $h = 7\ cm$

$V = Ah$
$= 35 \times 7$

$= 245\ cm^3$

Example 8E.2 Calculating the volume of a rectangular prism

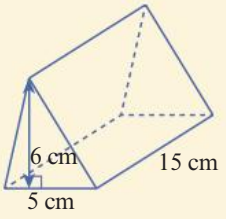
Calculate the volume of this rectangular prism.



THINK	WRITE
1 Identify the base of the prism. In this example we will designate the 3 mm by 4 mm side as the base.	$l = 3$ mm, $w = 4$ mm
2 Use the formula for the area of a rectangle, $A = lw$, to calculate the area of the base, A.	$A = lw$ $= 3 \times 4$ $= 12$ mm^2
3 Identify the value of the height of the prism and substitute the values for the area of the base and the height into the formula for the volume of a prism.	$h = 7$ mm $V = Ah$ $= 12 \times 7$
4 Calculate the volume of the rectangular prism. Remember to include the appropriate unit in the answer.	$= 84$ mm^3

Example 8E.3 Calculating the volume of a triangular prism

Calculate the volume of this triangular prism.



THINK	WRITE
1 Identify the base of the prism. In this example the base is the triangular face. As we will be dealing with two heights, we will refer to them as h_1 and h_2.	$b = 5$ cm, $h_1 = 6$ cm
2 Calculate the area of the base, A. In this example use the formula for the area of a triangle, $A = \frac{1}{2}bh$.	$A = \frac{1}{2}bh_1$ $= \frac{1}{2} \times 5 \times 6$ $= 15$ cm^2
3 Identify the value of the height of the prism and substitute the values for the area of the base and the height into the formula for the area of a prism.	$h_2 = 15$ cm $V = Ah_2$ $= 15 \times 15$
4 Calculate the volume of the triangular prism. Remember to include the appropriate unit in the answer.	$= 225$ cm^3

Example 8E.4 Converting units of volume

a Convert 2 m^3 into cm^3.

b Convert 216 mm^3 into cm^3.

THINK

a **1** Identify the appropriate conversion factor. To convert from a larger unit to a smaller unit, multiply by the conversion factor.

2 Complete the calculation.

b **1** Identify the appropriate conversion factor. To convert from a smaller unit to a larger unit, divide by the conversion factor.

2 Complete the calculation.

WRITE

a $1\ m^3 = 100^3\ cm^3$
$= 1\ 000\ 000\ cm^3$

$2\ m^3 = 2 \times 1\ 000\ 000$
$= 2\ 000\ 000\ cm^3$

b $1\ cm^3 = 10^3\ mm^3$
$= 1000\ mm^3$

$216\ mm^3 = 216 \div 1000$
$= 0.216\ cm^3$

Example 8E.5 Converting units of capacity

a Convert 3 L into mL.

b Convert 500 L into kL.

THINK

a **1** Identify the appropriate conversion factor. To convert from a larger to a smaller unit, multiply by the conversion factor.

2 Complete the calculation.

b **1** Identify the appropriate conversion factor. To convert from a smaller to a larger unit, divide by the conversion factor.

2 Complete the calculation.

WRITE

a 1 L = 1000 mL
3 L = 3 × 1000
= 3000 mL

b 1 kL = 1000 L
500 L = 500 ÷ 1000
= 0.5 kL

Helpful hints

- ✔ For rectangular prisms, any of the sides can be designated as the base, but remember that the height must be perpendicular (at right angles) to the base.
- ✔ Recall the following formulas for area:
  - → Rectangle: $A = lw$
  - → Triangle: $A = \frac{1}{2}bh$
  - → Parallelogram: $A = bh$
  - → Kite: $A = \frac{1}{2}xy$
  - → Rhombus: $A = bh$ or $A = \frac{1}{2}xy$
  - → Trapezium: $A = \frac{1}{2}(a + b)h$
- ✔ Volume and capacity are not the same! Volume is a measure of how much space an object occupies (expressed in mm^3, cm^3 or m^3), whereas capacity is a measure of how much an object can hold (expressed in mL, L, kL or ML).

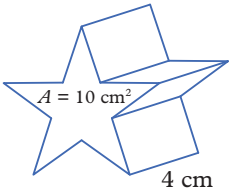
ANS p550 Exercise 8E Volume and capacity

▲ 1–4, 5(1st column), 6, 7, 9, 10, 11, 14, 17 ■ 1(c, d), 2–4(c), 5–7(a–c), 8, 11, 13–18 ⬟ 1–6(2nd column), 7–10, 16, 18, 19

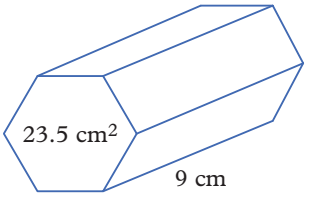
UNDERSTANDING AND FLUENCY

8E.1 **1** Calculate the volume of each prism. The base area is shown.

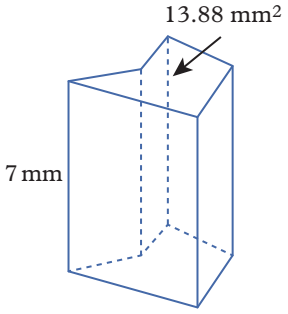
a



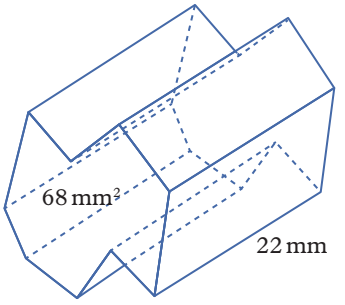
b



c

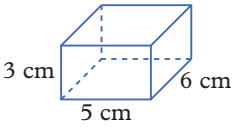


d

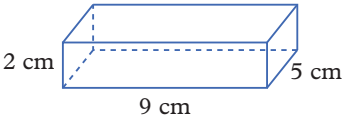


8E.2 **2** Calculate the volume of each rectangular prism.

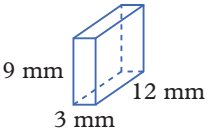
a



b

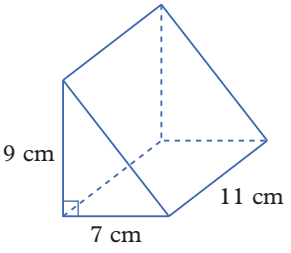


c

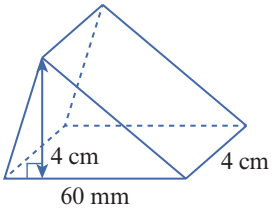


8E.3 **3** Calculate the volume of each triangular prism.

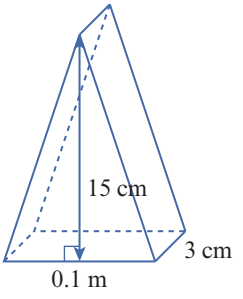
a



b

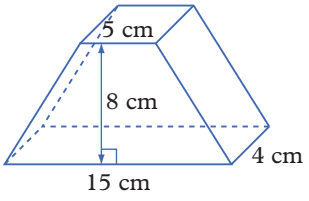


c

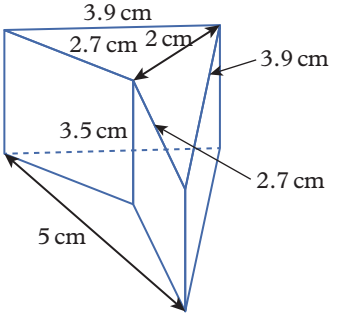


4 Use the formula $V = Ah$ to calculate the volume of each object.

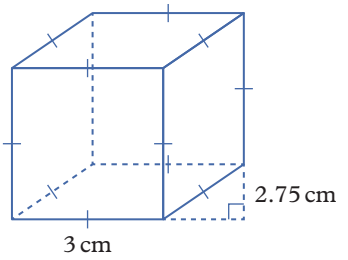
a



b



c



8E.4 **5** Convert each measurement.

a 9 cm^3 into mm^3 **b** 18 m^3 into cm^3 **c** 16 000 mm^3 into cm^3

d 1 750 000 m^3 into km^3 **e** 4.2 km^3 into m^3 **f** 4 000 000 cm^3 into m^3

g 4.56 m^3 into cm^3 **h** 845 000 cm^3 into m^3 **i** 0.15 km^3 into m^3

j 6.1 cm^3 into mm^3 **k** 28 mm^3 into cm^3 **l** 1 700 000 000 000 m^3 into km^3

8E.5 **6** Use the conversion factors to complete these statements.

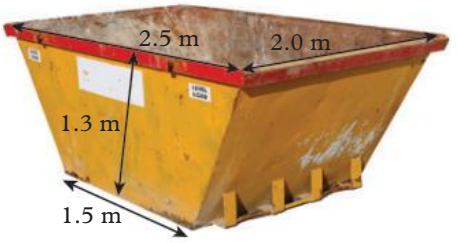
a 8 L = _____ mL **b** 15 kL = _____ L **c** 70 mL = _____ L

d 6.4 ML = _____ kL **e** 2500 L = _____ kL **f** 3 kL = _____ mL

7 Complete the table by working out the missing value for each container.

	Inside volume of container	Capacity of container
a	14 cm^3	_____ mL
b	_____ cm^3	3 L
c	5.8 m^3	_____ kL
d	750 cm^3	_____ L
e	_____ m^3	1000 kL
f	_____ m^3	6.4 ML

8 A skip has the dimensions shown. Calculate its volume in cubic metres.



9 A tissue box is 21 cm long, 11.4 cm wide and 12.3 cm tall.

a What is its volume?

b If each tissue laid flat in the box is 1 mm high, how many tissues would it fit?

10 A rectangular prism has a square base with sides that measure 5 cm. If the prism has a volume of 100 cm^3, what is its height?

11 The volume of a rectangular prism can also be given by using the formula $V = lwh$, where l, w and h represent the length, width and height, respectively. Explain why it doesn't matter which of the measurements you designate as the length, width or height.

12 Priya was completing some volume conversions and wrote the following answers.

a

$7\ m^3 = (7 \times 100^3)\ cm^3$
$= 2100\ cm^3$

b

$3000\ mm^3 = (3000 \div 100^3)\ cm^3$
$= 30\ cm^3$

i Explain the mistake Priya made in each instance.

ii Provide the correct working for the conversion.

13 a Calculate the volume of this perfume bottle in cubic centimetres.

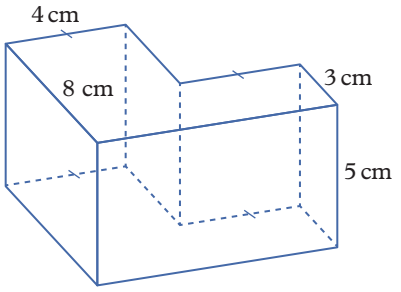
b Convert the volume from part **a** into cubic millimetres.

c Another way to do this is to first convert all the dimensions into millimetres and then find the volume. Show how doing the conversion first gives you the same answer as in part **b**.

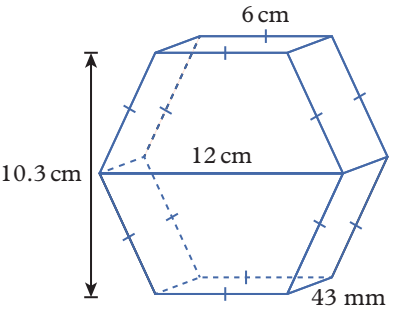


14 Calculate the volume of these hexagonal prisms.

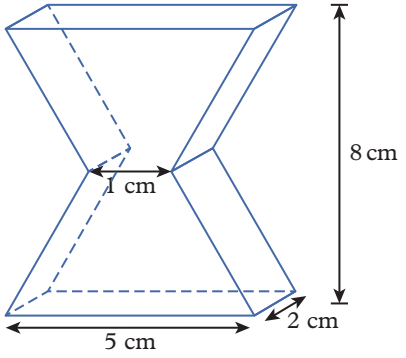
a



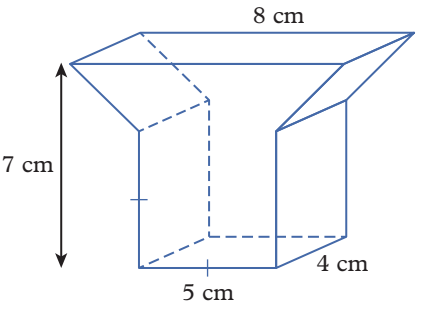
b



c



d



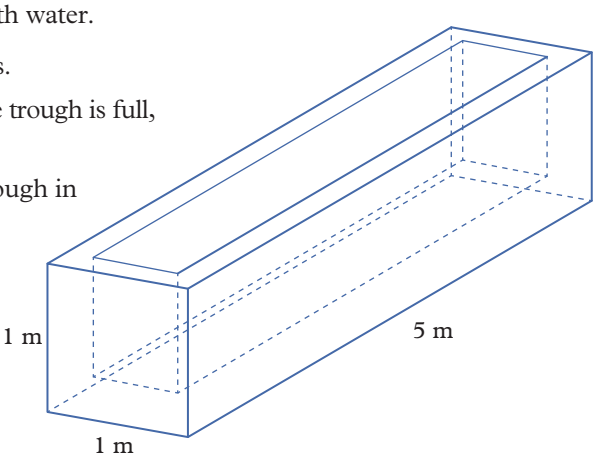
15 A concrete water trough is designed as follows. A 1 m × 5 m × 1 m concrete block is formed around a 80 cm × 480 cm × 90 cm mould to create a cavity to fill with water.

a Determine the maximum capacity of the trough in litres.

b Determine the total volume of trough and water when the trough is full, in cubic centimetres.

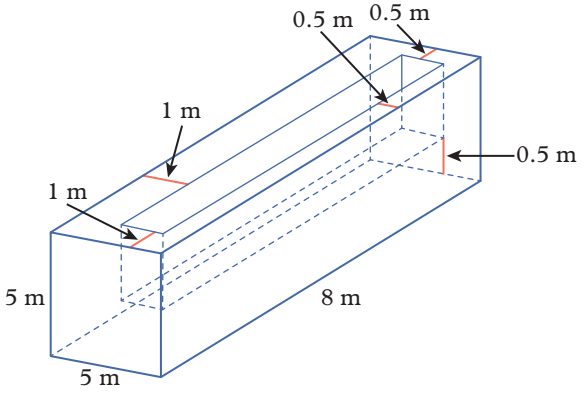
c Determine the volume of concrete used to create the trough in cubic centimetres.



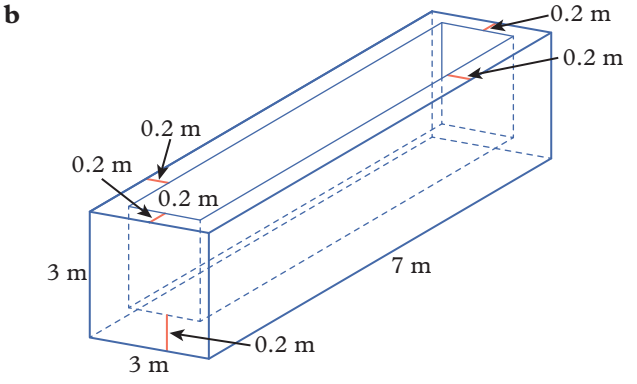


16 Calculate the capacity of each of the following troughs in litres.

a



b

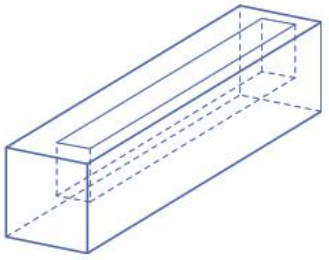


17 Consider the following three troughs made from the same sized block.

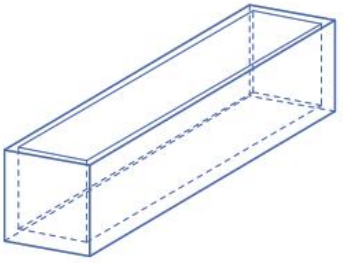
a Put these troughs in ascending order according to their capacity.

b Put these troughs in ascending order according to their volume.

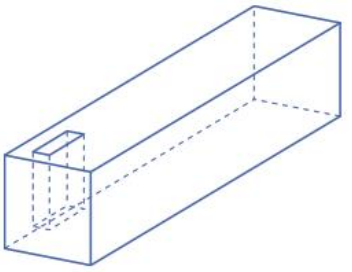
A



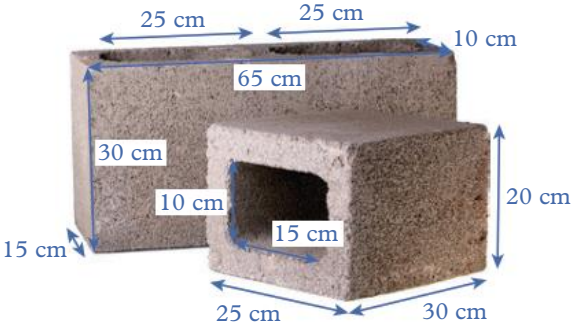
B



C



18 Find the volume of concrete needed to make these two blocks.

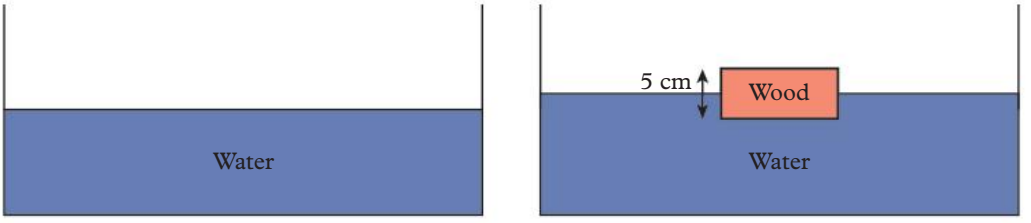


19 Archimedes knows that when an object is submerged under water, the water level will rise as the water becomes *displaced* by the object. He also knows that the volume of the submerged object is equal to the volume of the displaced water.

Archimedes has a rectangular glass container with base dimensions 30 cm × 40 cm. He fills it with 12 L of water and finds that the water level is exactly halfway up the total height of the glass container.

a Determine the height of Archimedes' container.

Archimedes then floats a rectangular wooden block on the surface of the water. The wooden block has dimensions 10 cm × 8 cm × 5 cm. He measures $\frac{3}{5}$ of the wooden block is submerged under water.



b Determine the volume of the displaced water by first finding the volume of the submerged object.

c Find the new water level by considering the relationship: volume of water in the container remains the same = base of the tank × water level − volume of the block submerged under water.

d Determine the difference in the water level in cm.

e Archimedes keeps adding water to the container until the surface of the wooden block is at the same level as the edge of the container, what's the water level now?

f How much water has been added?

g Archimedes added the water at a constant rate of 5 L/minute. How long did it take for the surface of the block to be at the same level as the edge of the container?

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet — Volume of prisms
- Interactive skillsheet — Converting between units of volume
- Investigation — A bit of trash
- Topic quiz — 8E

8F Three-dimensional space

Learning intentions

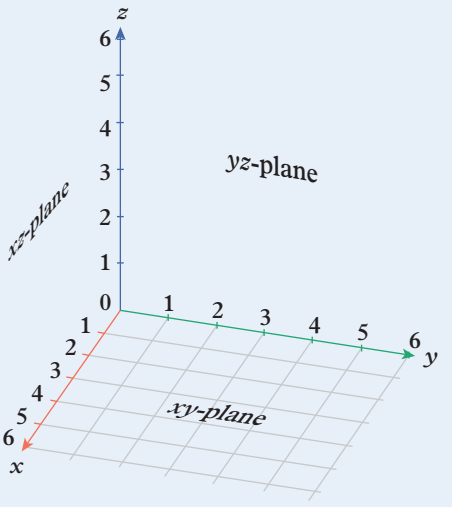
By the end of this topic you will be able to...

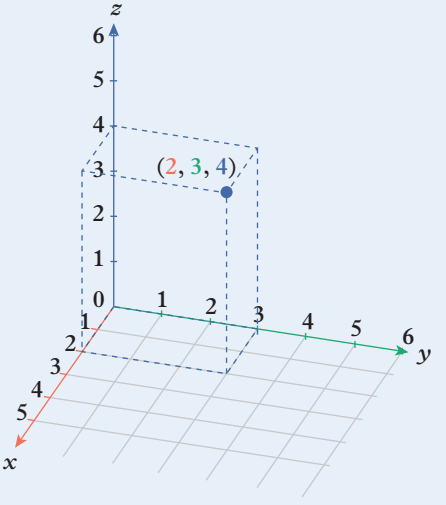
- ✔ identify the coordinates of a point in three dimensions
- ✔ plot a point in three dimensions using its coordinates
- ✔ work with shapes and objects whose vertices are given as coordinates in three dimensions.

Inter-year links
Year 7 5D The Cartesian plane

Three-dimensional coordinates

- A third axis, the **z-axis**, can be added perpendicular to the Cartesian plane (the xy-plane) to allow for three dimensions of coordinates to be displayed.
- When displaying three-dimensional space on a two-dimensional page, we use other cues to ensure points appear where they are intended, such as perspective (closer looks larger, further away looks smaller) and dotted line segments.
- To ensure all three axes are clearly visible, we show the space at an angle that skews the xy-plane, making right angles on the plane not appear as right angles. Similarly, right angles between the x- and z-axes (the xz-plane) as well as y- and z-axes (the yz-plane) do not appear as right angles.
- **Three-dimensional coordinates** consist of three values to specify a location.
- The coordinates of a point in three dimensions are written (x, y, z), where:
  - → the **x-coordinate** represents how far forward (positive) or backward (negative) the point is from the origin
  - → the **y-coordinate** represents how far right (positive) or left (negative) the point is from the origin
  - → the **z-coordinate** represents how far up (positive) or down (negative) the point is from the origin.
- We can give the coordinates of the vertices of two-dimensional shapes and three-dimensional objects in three-dimensional space that can be joined up to create an image of the shape or object.

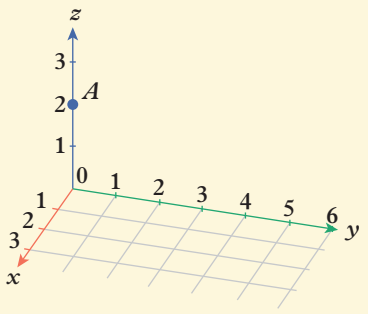




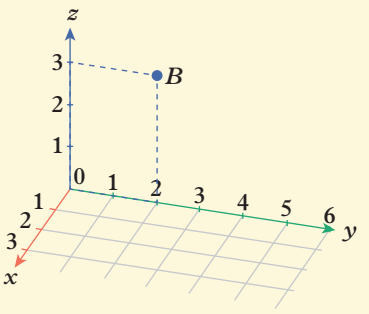
Example 8F.1 Writing coordinates of points in three-dimensional space

Write the coordinates of the points below.

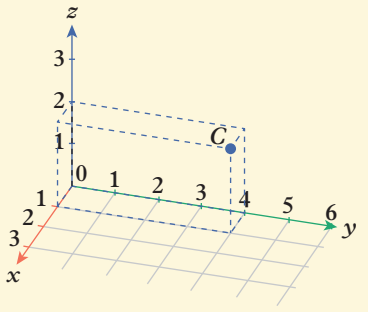
a



b



c



THINK

a **1** The point is on the z-axis, so the x-coordinate and y-coordinate are both 0.

2 Identify the z-coordinate by reading how far up or down the point is from the origin.

3 Write the coordinate as (x, y, z).

b **1** The point is on the yz-plane, so the x-coordinate is 0.

2 Identify the y-coordinate by reading how far right or left the point is from the origin.

3 Identify the z-coordinate by reading how far up or down the point is from the origin.

4 Write the coordinate as (x, y, z).

c **1** Identify the x-coordinate by reading how far forward or back the point is from the origin.

2 Identify the y-coordinate by reading how far right or left the point is from the origin.

3 Identify the z-coordinate by reading how far up or down the point is from the origin.

4 Write the coordinate as (x, y, z).

WRITE

a x-coordinate is 0
y-coordinate is 0
z-coordinate is 2

$A(0, 0, 2)$

b x-coordinate is 0

2 units in the positive y-direction

3 units in the positive z-direction

$B(0, 2, 3)$

c 1 unit in the positive x-direction

4 units in the positive y-direction

2 units in the positive z-direction

$C(1, 4, 2)$

Example 8F.2 Plotting points in three-dimensional space

Plot each of the following points on its own diagram. Clearly show the x-, y- and z-coordinates using dotted lines.

a $A(4, 5, 2)$ **b** $B(0, 4, 3)$

THINK

a **1** Mark the x-, y- and z-coordinates on their respective axes.

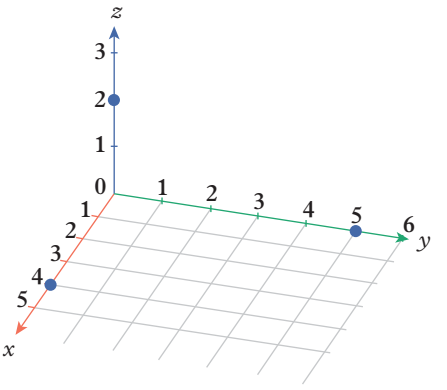
2 Mark the point $(4, 5, 0)$ on the xy-plane with the corresponding x- and y-coordinates. Add dashed line segments from the x-axis and the y-axis to indicate the position of the point on the xy-plane.

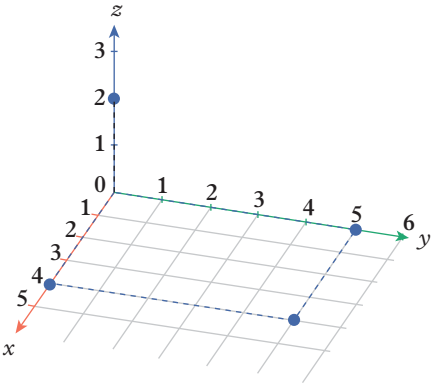
3 Move the points on the xy-plane marked in steps 1 and 2 two units up in the z-direction, as the z-coordinate of A is 2. Add dashed line segments to connect corresponding points.

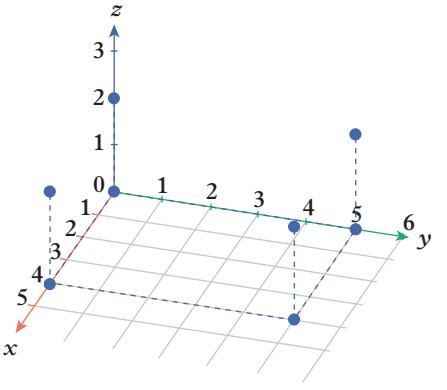
4 Add dashed line segments between the points to indicate the position of the point $A(4, 5, 2)$ in three-dimensional space.

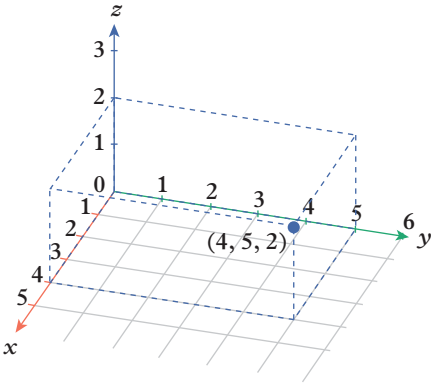
WRITE

a









THINK

b **1** Mark the x-, y- and z-coordinates on their respective axes.

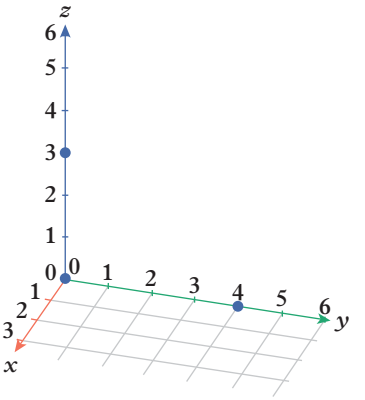
2 Mark the point $(0, 4, 0)$ on the xy-plane with the corresponding x- and y-coordinates. The x-coordinate is 0, so the position of $(0, 4, 0)$ is on the y-axis.

3 Move the points on the xy-plane marked in steps 1 and 2 three units up in the z-direction, as the z-coordinate of A is 3. Add dashed line segments to connect corresponding points.

4 Add dashed line segments between the points to indicate the position of the point $B(0, 4, 3)$ in three-dimensional space.

WRITE

b



Example 8F.3 Drawing shapes and objects in three-dimensional space

Draw the following shapes and objects by first plotting the vertices in three-dimensional space.

a triangle with vertices at (3, 1, 2), (1, 4, 3) and (5, 4, 5).

b rectangular prism with vertices at (0, 0, 0), (3, 0, 0), (0, 4, 0), (3, 4, 0), (0, 0, 2), (3, 0, 2), (3, 4, 2), (0, 4, 2).

THINK

a **1** Plot the points that are the vertices of the shape.

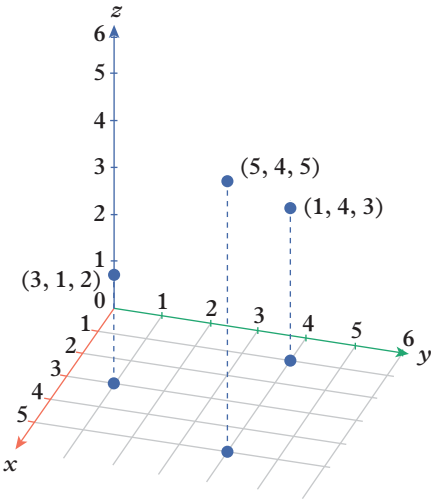
2 Join the vertices up to form the correct shape.

b **1** Plot the points that are the vertices of the object.

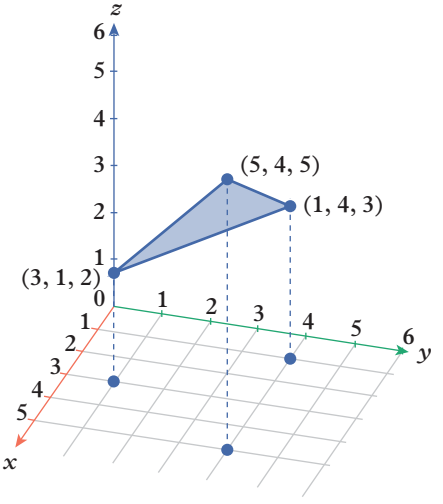
2 Join the vertices up to form the correct object. Use dashed line segments for hidden edges.

WRITE

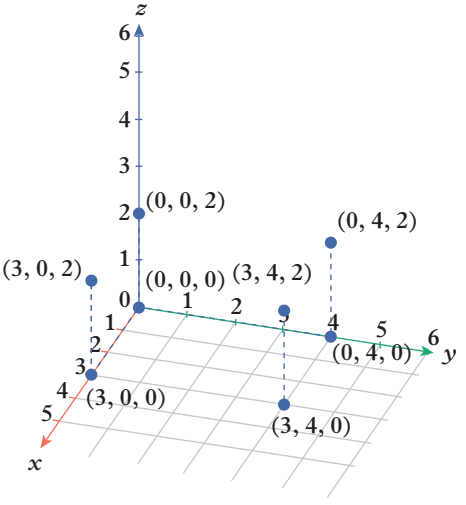
a **1**



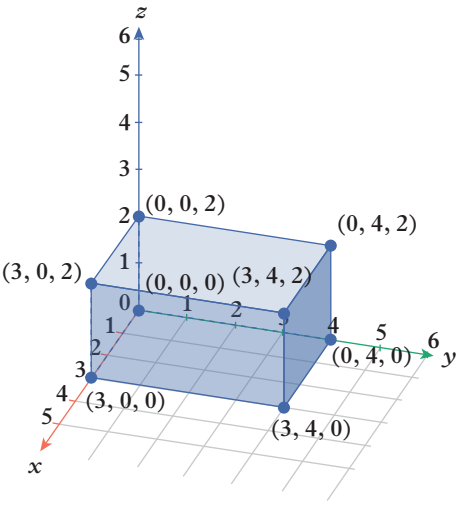
2



b **1**



2



Helpful hints

- ✔ Several points in three-dimensional space can look the same when drawn on a two-dimensional page. Clarify the position of your points by using dashed line segments.
- ✔ Use a ruler to make sure your points are the correct distance from each axis and parallel to the appropriate axis by sliding the ruler from the axis to the correct position without rotating it.

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Exercise 8F Three-dimensional space

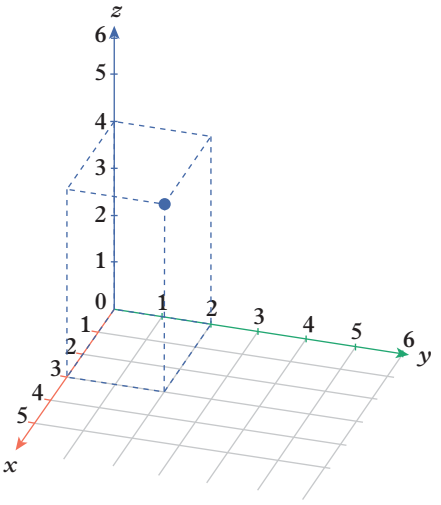
1, 2, 3–4(a), 5, 7–10(a, b)

1–3, 4–5(b–d), 6, 7(2nd column), 8–12

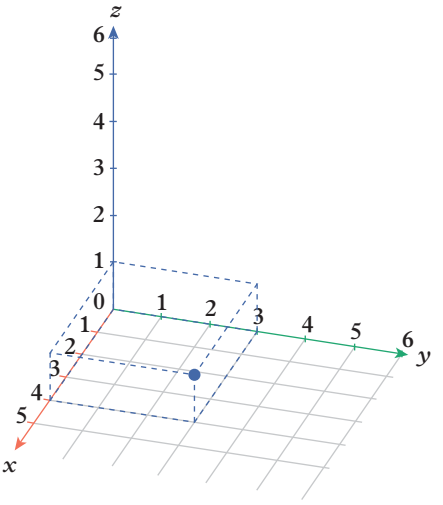
1, 2(d–f), 3–4(c), 5, 6, 7(d–f), 8–10(c, d), 11–13

8F.1 **1** Write the coordinates of the points shown below.

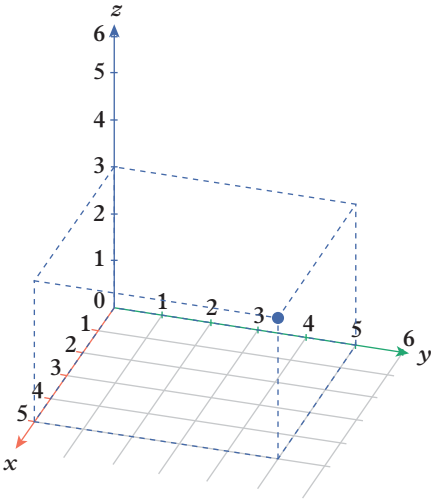
a



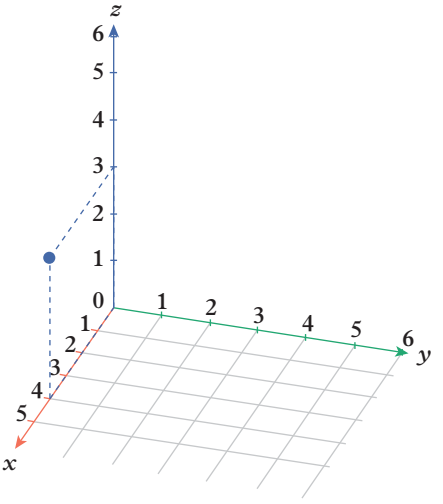
b



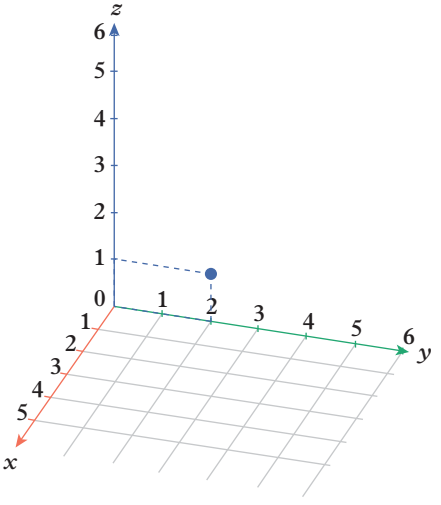
c



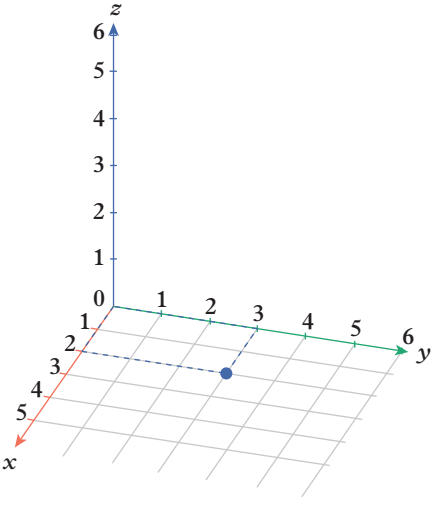
d



e



f



UNDERSTANDING AND FLUENCY

8F.2 **2** Plot each of the following points on its own diagram. Clearly show the x-, y- and z-coordinates using dotted lines.

a (3, 4, 2) **b** (2, 1, 5) **c** (5, 1, 4) **d** (4, 3, 0) **e** (3, 0, 0) **f** (0, 4, 5)

8F.3 **3** Draw the following shapes and objects by first plotting the vertices in three-dimensional space.

a triangle with vertices at (4, 5, 5), (4, 3, 4) and (1, 1, 4).

b rectangular prism with vertices at (0, 0, 0), (5, 0, 0), (0, 2, 0), (5, 2, 0), (0, 0, 4), (5, 0, 4), (0, 2, 4), (5, 2, 4).

c rectangular prism with vertices at (2, 1, 0), (3, 1, 0), (2, 5, 0), (3, 5, 0), (2, 1, 3), (3, 1, 3), (2, 5, 3), (3, 5, 3).

4 Determine the coordinates of the images of the points after the listed translations.

a (1, 2, 5) is translated 5 units in the positive x-direction, 3 units in the positive y-direction, and 2 units in the negative z-direction.

b (6, 0, 5) is translated 4 units in the negative x-direction, 7 units in the positive y-direction, and 4 units in the positive z-direction.

c (12, 8, 3) is translated 8 units in the negative x-direction, 4 units in the negative y-direction, and 5 units in the positive z-direction.

d (0, 10, 14) is translated 4 units in the positive x-direction, 6 units in the negative y-direction, and 6 units in the negative z-direction.

5 Describe the location in three-dimensional space of points with the following characteristics.

a an x- and y-coordinate of 0

b a y- and z-coordinate of 0

c an x- and z-coordinate of 0

d an x-, y-, and z-coordinate of 0

e a z-coordinate of 0

f an x-coordinate of 0

g a y-coordinate of 0

6 Multistorey car parks use a three-dimensional coordinate system to identify individual car parks. An example of this is 'floor, row, position'. For example, a car on the third floor, second row, six parking spaces along is written as 326.

a Write the coordinates of the following cars in the car park.

i fifth floor, seventh row, eight parking spaces along

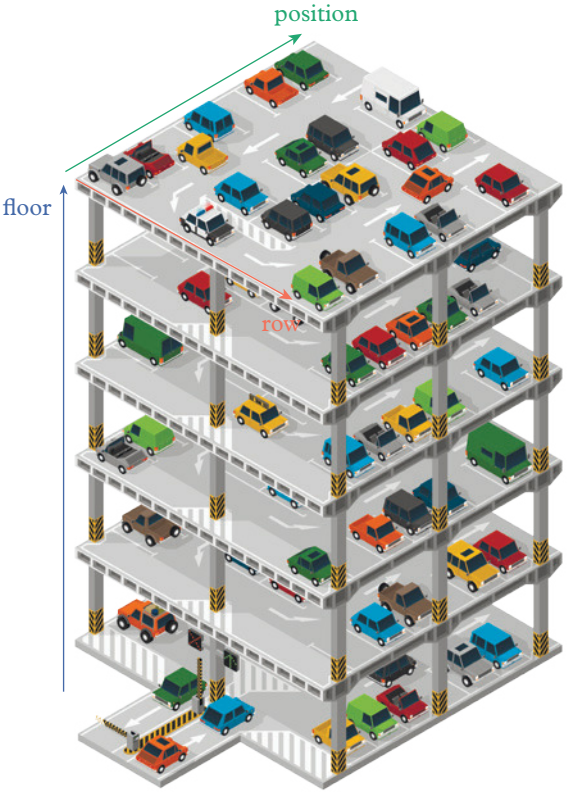
ii seventh floor, second row, one parking space along

iii second floor, eighth row, five parking spaces along

iv first floor, seventh row, nine parking spaces along

v ninth floor, seventh row, five parking spaces along

vi ground floor, fourth row, three parking spaces along



b Describe the position of the following cars in the car park.

i 512 **ii** 491 **iii** 149

iv 725 **v** 028 **vi** 294

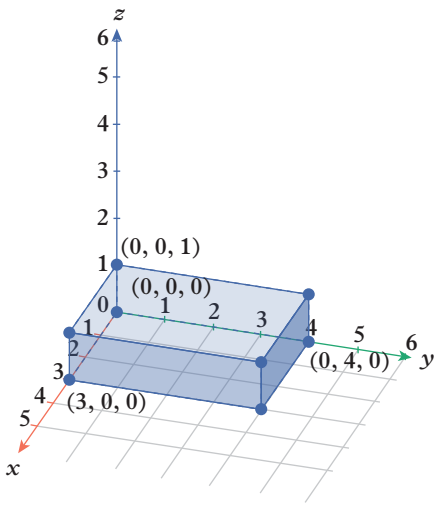
c Describe how the following people can get from where they are to where they want to go. Describe movement between floors using 'up' and/or 'down', between rows using 'left' and/or 'right', and between positions using 'forwards' and/or 'backwards'.

i Jacinta enters the car park from the connected shopping centre adjacent to 408 and they want to return to their car that is parked at 591.

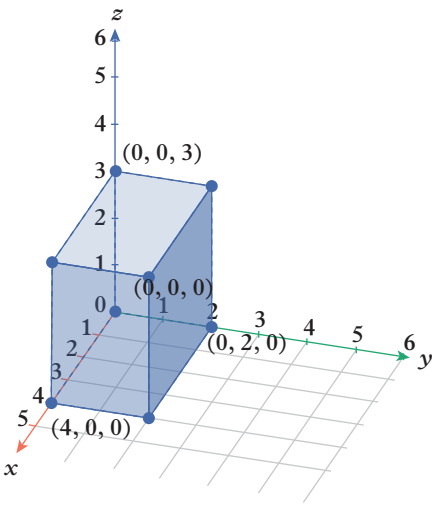
ii Lucinda parked their car at 789 and they want to go to the entrance of the shopping centre adjacent to 203.

7 Write down the coordinates of the remaining unlabelled vertices of the following prisms.

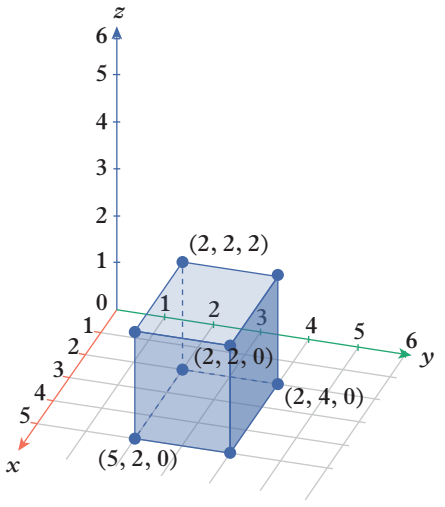
a



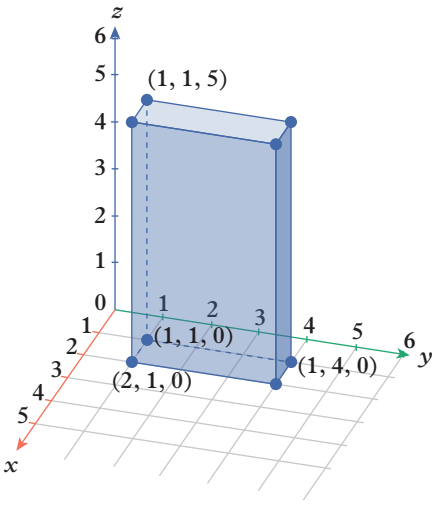
b



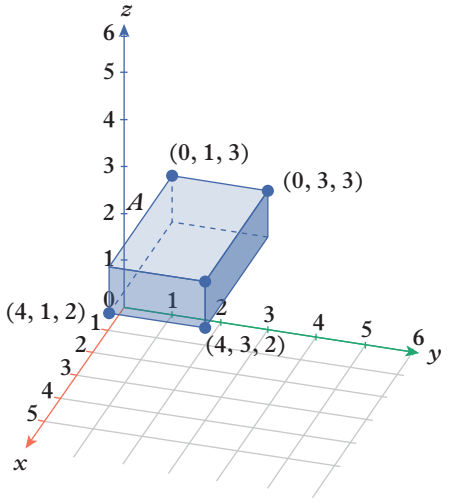
c



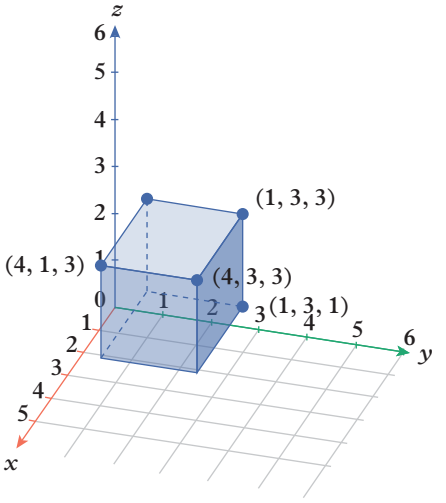
d



e



f



8 Write down the coordinates of the remaining four vertices of the following rectangular prisms.

a (0, 0, 0), (0, 2, 0), (4, 0, 0), (0, 0, 4)

b (0, 0, 0), (0, 4, 2), (5, 4, 0), (5, 4, 2)

c (2, 2, 0), (5, 2, 0), (2, 3, 0), (2, 2, 1)

d (3, 1, 0), (5, 1, 0), (3, 3, 0), (3, 1, 3)

9 Calculate the volume of the prisms in question **8**, if each unit is equivalent to 1 cm.

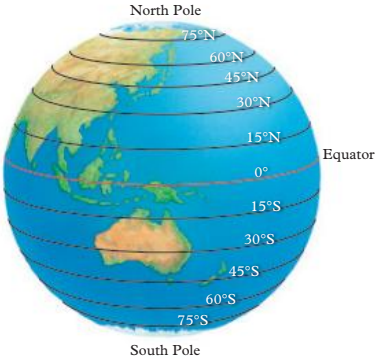
10 Calculate the surface area of the prisms in question **8**, if each unit is equivalent to 1 cm.

CHALLENGE

11 a Determine the distance of the point (3, 4, 0) from the origin.
Hint: Make a right-angled triangle where the hypotenuse joins the origin and the point (3, 4, 0)

b Determine the distance of the point (3, 4, 12) from the origin.

12 Another system for giving coordinates in three dimensions is spherical coordinates. A common example of spherical coordinates is coordinates on Earth written as (latitude, longitude, altitude). Longitude measures the angle, in degrees, north or south from the equator; longitude, in degrees, measures the angle east or west from the Prime Meridian; Altitude measures how far, in metres, the position is above sea level, where a negative altitude references a point below sea level. For example, a point that is 30° north of the equator, 45° east of the Prime Meridian and 50 m above sea level is written as (30° N, 45° E, 50 m).



a Write the coordinates of the following landmarks using their latitude, longitude and altitude.

i The Eiffel Tower is 48.8584° north of the equator, 2.2945° east of the Prime Meridian and 35 m above sea level.

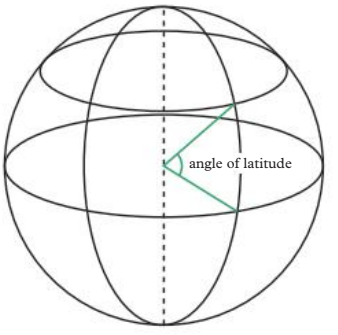
ii The Statue of Liberty is 40.6892° north of the equator, 74.0445° west of the Prime Meridian and 7.2 m above sea level.

iii Uluru is 25.3444° south of the equator, 131.0369° east of the Prime Meridian and 863 m above sea level.

iv The Macchu Picchu is 13.1631° south of the equator, 72.5450° west of the Prime Meridian and 2430 m above sea level.

v Mt. Fuji is 35.3606° north of the equator, 138.7274° east of the Prime Meridian and 3776 m above sea level.

vi The Eureka Skydeck is 37.8213° south of the equator, 144.9647° east of the Prime Meridian and 300 m above sea level.



b Describe the position of the following points in terms of their latitude, longitude and altitude.

i (60° N, 171° E, 709 m)

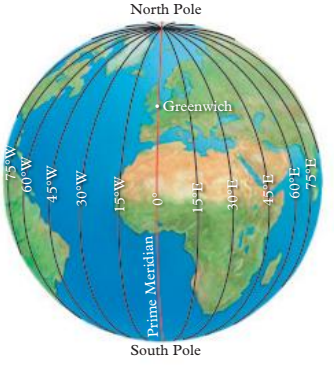
ii (25° N, 128° W, 545 m)

iii (5° S, 47° W, 135 m)

iv (79° S, 120° E, 642 m)

v (16° N, 63° W, 97 m)

vi (34° S, 152° E, 13 m)



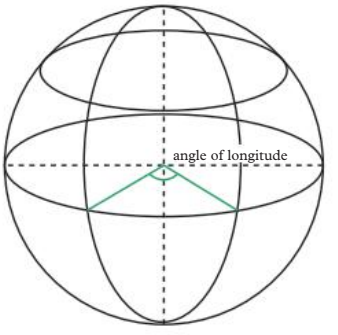
13 Determine the coordinates of the images of the points after the listed transformations.

a (15° N, 124° E, 25 m) is rotated 30° north and 60° west, and translated 130 m up.

b (28° S, 48° W, 40 m) is rotated 30° north and 60° west, and translated 12 m down.

c (78° N, 4° W, 100 m) is rotated 11° north and 140° east, and translated 55 m up.

d (41° N, 19° W, 320 m) is rotated 120° south and 120° east, and translated 231 m down.



Check your Student obook pro for these digital resources and more:

- Interactive skillsheet — Three-dimensional coordinates
- BLM — 3D coordinate system
- Investigation — Virtual maps versus street views
- Topic quiz — 8F

8G Time

Learning intentions

By the end of this topic you will be able to ...

- ✔ convert between units of time
- ✔ calculate the duration between two given times
- ✔ use time zones to calculate the time in different locations.

Inter-year links
Support Understanding time

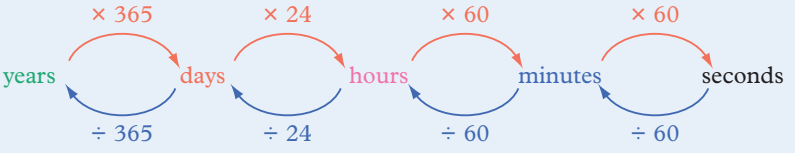
Units of time

- To convert between units of time, multiply or divide by the appropriate conversion factor.

  For example, 5 hours = 5 × 60
  = 300 minutes

Conversion factors

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 year = 365 day

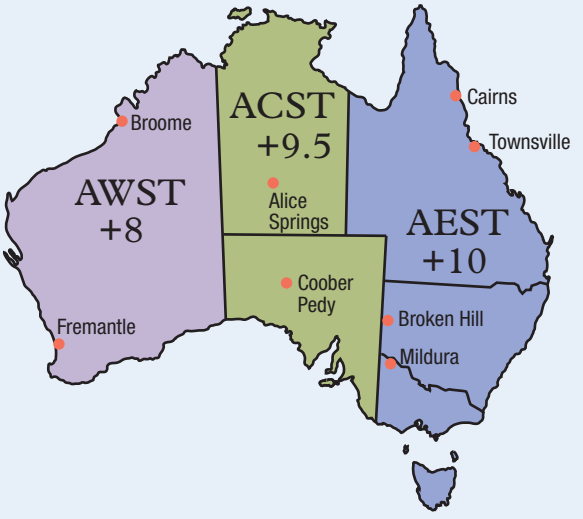


12- and 24-hour time

- In **12-hour time**, am and pm are used to describe whether a time is before or after midday. For example, 7 am represents 7 o'clock in the morning.
- In **24-hour time**, the time is given as a four-digit number. Each day starts at 0000 (midnight) and finishes at 2359. Times after midday have 12 hours added to them. For example, 2 pm is 1400 hours.
- A **duration** is the length between two given times.

Time zones

- The Earth is divided into 24 major **time zones**. A time zone map shows how many hours ahead or behind a time zone is from **Coordinated Universal Time (UTC)**, which is the time in Greenwich (in the UK). For example, UTC+10 means that the time is 10 hours ahead of UTC time.
- Australia covers three time zones:
  - → Australian Western Standard Time (AWST) is UTC+8.
  - → Australian Central Standard Time (ACST) is UTC+9.5.
  - → Australian Eastern Standard Time (AEST) is UTC+10.



Example 8G.1 Converting between units of time

Convert 2 hours into seconds.

THINK	WRITE
1 Convert from hours to minutes. To convert from a larger unit to a smaller unit, multiply by the conversion factor.	1 hour = 60 minutes 2 hours = 2 × 60 = 120 minutes
2 Convert from minutes to seconds.	1 minute = 60 seconds 120 minutes = 120 × 60 = 7200 seconds

Example 8G.2 Converting between 12- and 24-hour time

Convert:

a 9:20 pm into 24-hour time

b 1345 into 12-hour time.

THINK	WRITE
a As the time is after midday (pm), add 1200 to this value to express the time in 24-hour time.	**a** 9:20 pm = 0920 + 1200 = 2120
b As the given time is greater than 1200, subtract 1200 from 1345 and include pm to express the time in 12-hour time.	**b** 1345 = 1345 – 1200 = 1:45 pm

Example 8G.3 Calculating durations of elapsed time

A car leaves home at 11:07 am and arrives at its destination at 4:12 pm. How long did it take to get there?

THINK	WRITE
1 Count the number of minutes from the departure time to the next whole hour.	Time from 11:07 am to 12:00 pm = 53 minutes
2 Count the number of hours from the time at the end of the previous step to the whole hour of the arrival time.	Time from 12:00 pm to 4:00 pm = 4 hours
3 Count the number of minutes from the time at the end of the previous step to the arrival time.	Time from 4:00 pm to 4:12 pm = 12 minutes
4 Calculate the total duration by adding the individual time intervals. Remember that 60 minutes = 1 hour.	53 mins + 4 hrs + 12 mins = 4 hrs + 65 mins = 5 hrs and 5 mins

Example 8G.4 Calculating times in different locations

The time is 8:35 am in Perth (AWST). Calculate the time in Brisbane (AEST).

THINK	WRITE
1 Calculate the difference between the two time zones. Eastern Standard Time (AEST) is UTC+10 and Western Standard Time (AWST) is UTC+8, so AEST is 2 hours ahead of WST.	AEST is 2 hours ahead of AWST.
2 As AEST is 2 hours ahead of AWST, add 2 hours to the time in Perth to determine the time in Brisbane.	8:35 am + 2 hours = 10:35 am The time in Brisbane is 10:35 am.

Helpful hints

- ✔ All times expressed in 24-hour time must be given as four-digit numerals. For example, 7:35 am is 0735, not 735!
- ✔ When calculating the duration of elapsed time remember that there are only 60 minutes in 1 hour; for example, 73 minutes = 1 hour and 13 minutes.

ANS p552

Exercise 8G Time

△ 1–7, 8(1st, 2nd columns), 9(a, c, e), 10–12, 14, 17 □ 1–7, 8(2nd, 3rd columns), 9(b, d, f), 10, 13, 15, 19(a–d) ⬠ 4–7, 8(3rd column), 9(b, d, f), 16, 18–20

8G.1

1 Convert each of these into minutes.

a 3 hours **b** 5 hours **c** 10 hours
d 120 seconds **e** 600 seconds **f** 6000 seconds

2 Convert each of these.

a 10 minutes into seconds **b** 5 minutes into seconds **c** 60 minutes into seconds
d 120 minutes into hours **e** 1800 minutes into hours **f** 240 minutes into hours

3 Convert each of these.

a 30 minutes into hours **b** 45 minutes into hours **c** 20 minutes into hours
d 15 seconds into minutes **e** 6 seconds into minutes **f** 48 seconds into minutes

4 Convert each of these.

a 3 hours into seconds **b** 10 hours into seconds **c** 6 hours into seconds
d 1.5 hours into seconds **e** 14 400 seconds into hours **f** 9000 seconds into hours

5 How many:

a minutes in a day **b** hours in a week **c** seconds in a day
d hours in a leap year **e** seconds in a week **f** hours in June?

UNDERSTANDING AND FLUENCY

8G.2 **6** Convert these times into 24-hour time.

a 2:30 am **b** 4:15 pm **c** 1:48 pm **d** 11:11 pm

7 Convert these times into 12-hour time.

a 1925 **b** 1200 **c** 2218 **d** 0958

8G.3 **8** Calculate the duration between each pair of times.

a 4:00 pm and 9:00 pm **b** 8:00 am and 11:00 pm **c** 11:30 pm and 2:30 am

d 0900 and 1400 **e** 1330 and 2230 **f** 1600 and 2130

g 5:27 pm and 7:45 pm **h** 12:52 pm and 6:05 pm **i** 6:43 pm and 2:37 am

j 0004 and 1931 **k** 1919 and 2222 **l** 0636 and 1421

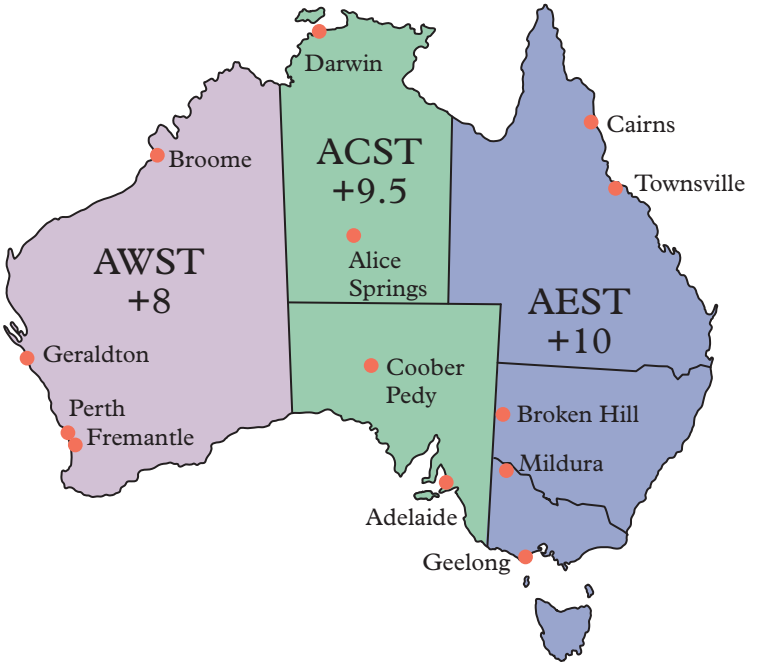
8G.4 **9** Find the time in the given time zone.

a 3:45 pm (UTC+8) in UTC+10 **b** 3:45 pm (UTC+10) in UTC+8

c 11:05 am (UTC+9.5) in UTC+10 **d** 11:05 am (UTC+9.5) in UTC+8

e 2345 (UTC+10) in UTC+9.5 **f** 2345 (UTC+8) in UTC+9.5

10 The time is given in one of Australia's time zones. Determine what time it is in the other places.



a The time is 1:49 pm AEST. What is the time in:

i Broome, WA **ii** Alice Springs, NT **iii** Cairns, QLD?

b The time is 2:28 am ACST. What is the time in:

i Coober Pedy, SA **ii** Broken Hill, NSW **iii** Fremantle, WA?

c The time is 1417 AWST. What is the time in:

i Geelong, VIC **ii** Perth, WA **iii** Adelaide, SA?

d The time is 0040 AEST. What is the time in

i Darwin, NT **ii** Townsville, QLD **iii** Geraldton, WA?

11 a Convert 15 minutes into a fraction of an hour.

b Write this fraction as a decimal.

c Explain why 1.25 hours is not equivalent to 1 hour and 25 minutes.

12 A family takes a road trip from Melbourne to Cairns. Their trip is plotted in the table below, showing when they leave, arrive and when they stop for lunch.

	Day 1	Day 2	Day 3	Day 4
Departure time	8:30 am	8:00 am	8:15 am	8:20 am
Start lunch	12:30 pm	12:30 pm	12:55 pm	1:05 pm
Finish lunch	1:00 pm	1:00 pm	1:40 pm	1:30 pm
Arrival time	7:30 pm	6:00 pm	7:45 pm	7:15 pm

a How long did they take from departure to arrival on their first day?

b How long was their lunch break on the first day?

c Use your answers to parts **a** and **b** to find how long they drove for on the first day.

d Find how long they drove for on the second, third and fourth days.

e How many hours driving did they do for the whole trip?

13 **i** Determine the error each person made when calculating the following durations.

ii Determine the correct duration.

a 2:45 pm to 6:25 pm

2:45 + 00:15 = 2:00
2:00 + 00:25 = 2:25
15 + 25 = 40 min
2:25 + 4:00 = 6:25
4 h 40 min

b 2:45 am to 6:25 pm

2:45 + 00:15 = 3:00
3:00 + 00:25 = 3:25
15 + 25 = 40 min
3:25 + 3:00 = 6:25
3 h 40 min

c 2:45 pm to 6:25 am

2:45 + 00:15 = 3:00
3:00 + 00:25 = 3:25
15 + 25 = 40 min
3:25 pm + 16:00 = 6:25 am
16 h 40 min

14 Andrew was studying for an upcoming test. He spent 85 minutes making flash cards, had a 10 minute break, spent 30 minutes quizzing himself using the flash cards, had a 30 minute lunch break, spent another 30 minutes quizzing himself using the flash cards, then sat a practice test for 50 minutes. If it was 11:10 am when he started, at what time did he finish the practice test?

15 Jaime has a bike with an odometer on it that tells him how many metres he has travelled. When he leaves for a bike ride along the Toogoom Trail, the odometer is at 1391 and the time is 1104. Upon arriving at Toogoom, the odometer reads 5403 and the time is 1334.

a How far did Jaime ride?

b How long did it take him to get to Toogoom?

c What is Jaime's average speed in m/s correct to two decimal places?

16 Harry and family are driving from Cairns, QLD to Eucla, WA overnight. They leave Cairns at 8 pm and Harry falls asleep during the drive. He is awoken by his parents when they arrive at the hotel in Eucla. He sees the clock in the lobby of the hotel that says it is 10 am. Harry assumes it took them 14 hours to get there.

a Why is Harry wrong?

b How long did it actually take Harry and family to get to the hotel?

c If the car travelled at an average speed of 80 km/h over the whole trip, approximately how many kilometres did they drive?

d Harry checks the route on his phone and sees it was 1490 km. Calculate the error between the approximation in part **c** and the actual distance as a percent of the actual distance, correct to the whole number.



17 Use the provided time zone map to decide which place is not in the same time zone as the others.

a **A** Coral Sea Islands **B** Papua New Guinea **C** Guam
D Solomon Islands **E** Tasmania

b **A** Jakarta, Indonesia **B** Makassar, Indonesia **C** Manila, Philippines
D Shanghai, China **E** Perth, Australia

18 Calculate the duration between the following times. Write all answers in hours and minutes.

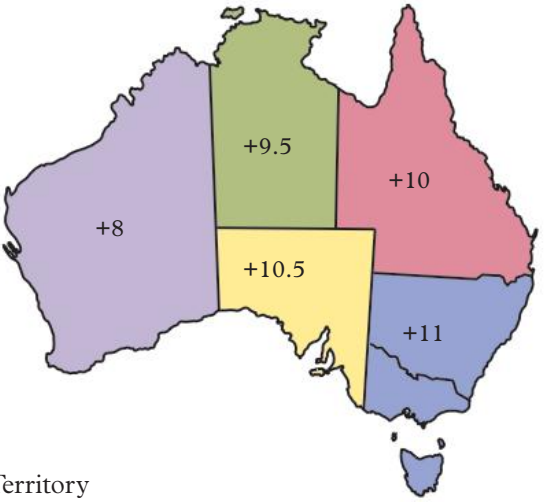
a 2:45 pm today and 11:10 am tomorrow

b 10:15 am on 22/08 and 11:00 am on 24/08

c 3:30 pm on 19/10 and 9:00 am on 22/10

d 1114 on 14/11/2022 and 1530 on 14/11/2023. Hint: 2023 is not a leap year.

19 Daylight saving time is when people put their clocks forward 1 hour on the first Sunday in October to have an 'extra' hour of daylight in the evenings. The clocks then move backwards 1 hour on the first Sunday in April. In Australia, the Australian Capital Territory, New South Wales, South Australia, Tasmania and Victoria observe daylight saving time. The Northern Territory, Queensland and Western Australia do not. This diagram shows the different time zones in summer when daylight saving time is observed.



a In summer, how many hours is:

i Victoria ahead of Queensland

ii South Australia ahead of Western Australia

iii the Australian Capital Territory ahead of the Northern Territory

iv Tasmania ahead of Western Australia?

b In winter, if it is 6 pm in South Australia, what time is it in:

i Western Australia

ii the Northern Territory

iii Victoria

iv Queensland?

c Repeat part **b**, but for summer.

d What is special about the time difference between South Australia and Queensland in summer and winter?

e Find the time in Darwin when it is:

i 4 pm on 27 January in Melbourne

ii 3 am on 16 July in Cairns

iii midday on 4 December in Perth

iv 9:30 am on 25 December in Sydney

v 5:45 pm on 30 June in Hobart

vi 11 am on 30 November in Adelaide.

20 Zoë and her friend Wes want to take a holiday in Thailand (UTC+7) during July.



a Zoë's flight leaves Melbourne (UTC+10) at 1:00 pm and arrives in Thailand at 7:00 pm. How long will the flight be?

b Wes has to take a different flight. His flight leaves at 1445. If his flight takes the same time, what time will he land in Thailand?

c On the way home they plan to stop in Bali (UTC+8). If they leave Thailand at 1000 and arrive in Bali at 1530, how long is this flight?

d What time will Zoë and Wes land home if they leave Bali at 1330 and their flight is 8 hours long?

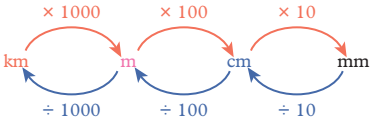
CHALLENGE

Check your Student obook pro for these digital resources and more:

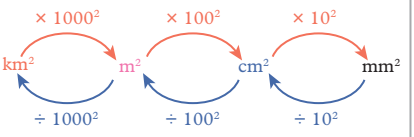
- Interactive skillsheet: Converting between units of time
- Interactive skillsheet: Durations of time
- Interactive skillsheet: Time zones
- Investigation: Daylight saving time
- Topic quiz: 8G

Chapter summary

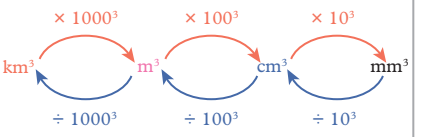
Units of length



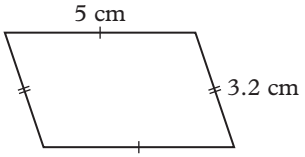
Units of area



Units of volume

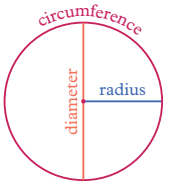


Perimeter



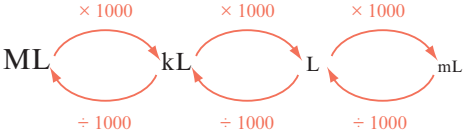
5 + 3.2 + 5 + 3.2 = 16.4 cm

Circumference and area of a circle



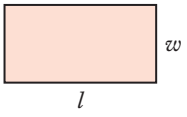
$C = \pi d$ or
$C = 2\pi r$
$A = \pi r^2$

Units of capacity



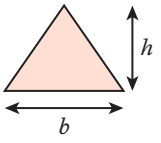
Area

Rectangle



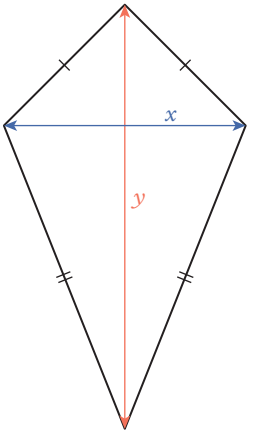
$A = lw$

Triangle



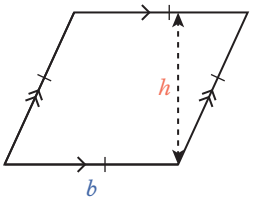
$A = \frac{1}{2}bh$

Kite



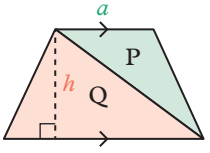
$A = \frac{1}{2}xy$

Rhombus



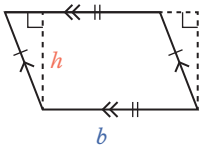
$A = bh$

Trapezium

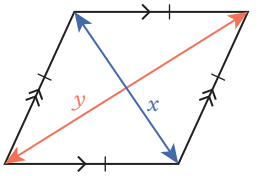


$A = \frac{1}{2}(a+b)h$

Parallelogram

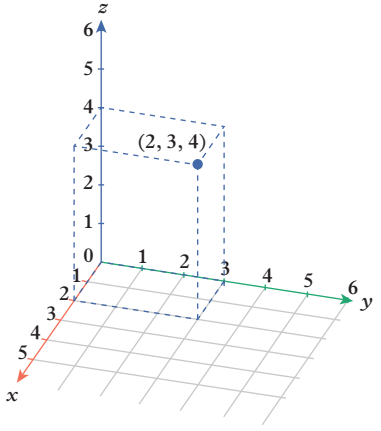


$A = bh$



$A = \frac{1}{2}xy$

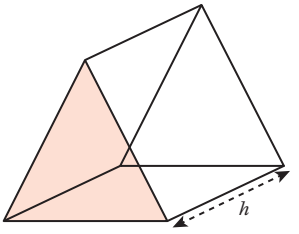
3D coordinates



Units of time

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 year = 365 days

Volume of prisms



$V = Ah$

Where A is the area of the base and h is the height

12- and 24-hour time

7 am = 0700

2 pm = 1400

Time zones

Australian Western Standard Time (AWST) is UTC+8.

Australian Central Standard Time (ACST) is UTC+9.5.

Australian Eastern Standard Time (AEST) is UTC+10.

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

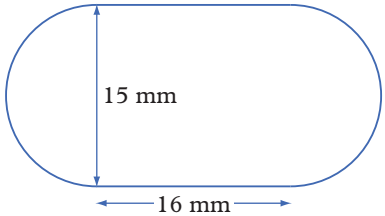
Quizlet
Test your knowledge of this topic by working individually or in teams

Multiple choice

8A **1** What is 7.84 m equal to?
A 0.784 km **B** 7840 mm **C** 78.4 cm **D** 0.0784 km **E** 7840 cm

8B **2** A circle has a diameter of 6 cm. What is its radius?
A 12 cm **B** 6π cm **C** 36π cm **D** 3 cm **E** 3π cm

8A **3** Which of these is closest to the total perimeter of this shape?



A 31 mm **B** 47 mm **C** 62 mm **D** 63.1 mm **E** 79.1 mm

8B **4** A triangle has a base length of 10 cm and an area of 20 cm^2. What is its height?
A 1 cm **B** 2 cm **C** 4 cm **D** 12 cm **E** 200 cm

8B **5** 2000 m^2 is not equal to which of the following?
A 40 m × 50 m **B** 2 000 000 000 mm^2 **C** 20 000 000 cm^2
D 2 km^2 **E** 0.2 hectares

8B **6** What is the surface area of a rectangular prism with length 15 cm, width 4 cm and height 9 cm?
A 231 cm^2 **B** 192 cm^2 **C** 390 cm^2 **D** 462 cm^2 **E** 540 cm^2

8C **7** A rhombus has diagonals of lengths 5 cm and 6 cm. What is its area?
A 30 cm^2 **B** 11 cm^2 **C** 56 cm^2 **D** 15 cm^2 **E** 60 cm^2

8D **8** Which of these is closest to the area of the shape in question **3**?
A 946.9 mm^2 **B** 754 mm^2 **C** 480 mm^2 **D** 416.7 mm^2 **E** 240 mm^2

8E **9** What is the volume of an irregular prism with base area 2.5 cm^2 and length 10 mm?
A 250 mm^3 **B** 25 mm^3 **C** 2.5 cm^3 **D** 25 cm^3 **E** 125 mm^3

8E **10** What is 3 cm^3 equal to?
A 30 mm^3 **B** 300 mm^3 **C** 300 m^3 **D** 3000 mm^3 **E** 30 000 mm^3

8E **11** Which pair of volume and capacity measurements are equivalent?
A 40 mL and 4 cm^3 **B** 1 m^3 and 10 L **C** 250 mL and 2.5 L
D 500 cm^3 and 0.5 L **E** 150 mL and 1.5 mm^2

8F **12** Which of the following points is on the y-axis?
A (2, 0, 6) **B** (0, 2, 4) **C** (2, 4, 0)
D (0, 0, 2) **E** (0, 4, 0)

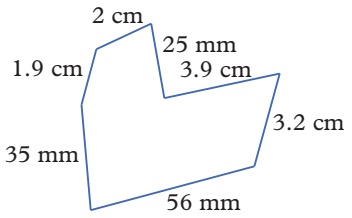
8G **13** What is the time duration between 1156 and 2014?
A 7 hours, 18 minutes **B** 8 hours, 42 minutes **C** 8 hours, 18 minutes
D 9 hours, 18 minutes **E** 7 hours, 42 minutes

8G **14** If the time is 4:45 pm in Melbourne (UTC+10), what is the time in Perth (UTC+8)?
A 2:45 pm **B** 4:15 pm **C** 4:45 pm **D** 5:15 pm **E** 6:45 pm

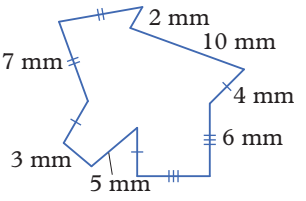
Short answer

8A **1** Calculate the perimeter of each shape.

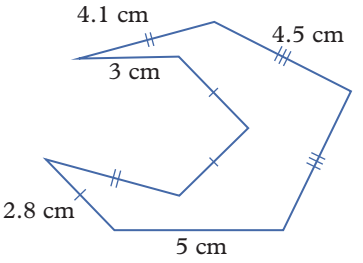
a



b

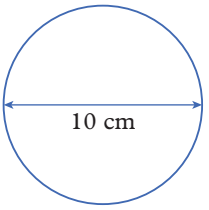


c

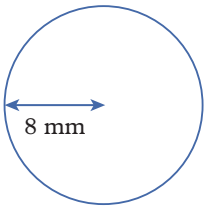


8A **2** Calculate the circumference of each circle correct to two decimal places.

a

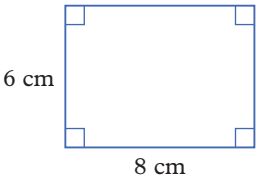


b

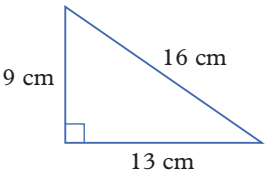


8B **3** Calculate the area of each shape.

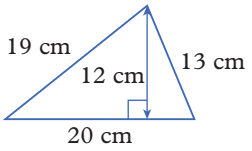
a



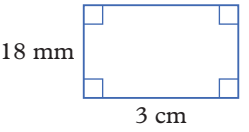
b



c



d



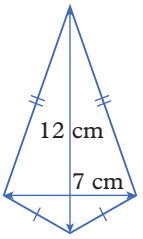
8B **4** Convert each area measurement.

a 5 m^2 into cm^2

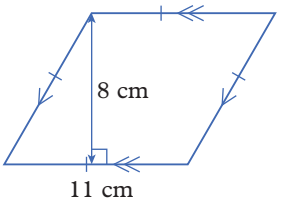
b 50 mm^2 into cm^2

8C **5** Calculate the area of each shape.

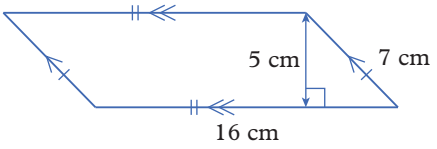
a



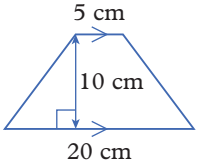
b



c



d



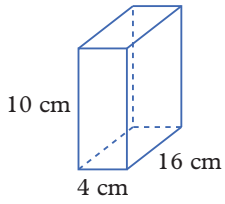
8D **6** Calculate the area of each circle in question **2**. Give your answers correct to two decimal places.

8E **7** Convert each measurement.

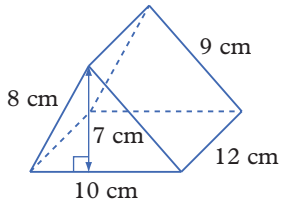
a 3.5 cm^3 into mm^3
b 400 cm^3 into m^3
c 35 000 mL to L
c 0.124 L to mL

8E **8** Calculate the volume of each object.

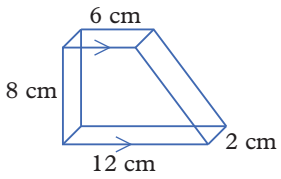
a



b



c



8F **9** Plot each of the following points on its own diagram. Clearly show the x-, y- and z-coordinates using dotted lines.

a (3, 4, 2)
b (0, 1, 5)
c (3, 0, 4)
d (4, 3, 0)

8G **10** Calculate the duration between the following times.

a 3:45 pm to 7:30 pm
b 10:05 pm to 6:37 am
c 0712 to 1546
d 2055 to 0322

8G **11** Convert the following times.

a 7:23 am UTC+10 to UTC+8
b 2:49 pm UTC+9.5 to UTC+10
c 9:10 am AWST to AEST
d 1:20 pm AEST to ACST

Analysis

A friend wants to have a ball pit at their birthday party. They have set aside a rectangular space in their backyard that is 3 m long and 2 m wide.

a What area would this cover?
b How much fencing would be needed to go around the ball pit?

Your friend can only buy the wood that they need for the fencing in planks that are 50 cm wide. The walls of the pit need to stand 1 m tall.

c What length of wood is needed?
d Your friend wants to paint the wooden fencing inside and out with two coats of paint. What is the total surface area to be painted? Ignore the thickness of the wood.

They also want to paint a diamond on the front fence of the ball pit. From point to point (that is, its diagonals) it measures 55 cm vertically and 30 cm horizontally.

e What is the area of this diamond?

Around the diamond will be six circles representing the coloured balls in the ball pit. Each circle has a diameter of 6.6 cm.

f What is the circumference of each circle correct to two decimal places?
g Find the total area to be painted for the six circles correct to two decimal places.
h What is the volume of the pit?
i If each ball to go into the pit has a volume of 150 cm^3, how many could theoretically fit into the pit? Be careful with unit conversions!
j Your friend finds that only about 25 000 balls fit into the pit. What explanation can you offer for this difference?

Algorithmic Thinking

Algorithmic thinking: Measurement and space

An algorithm is a set of precise instructions that when followed will achieve a goal or solve a problem. Algorithms can be described using diagrams called flowcharts, which contain actions written inside shapes such as boxes and diamonds, and use arrows to indicate which actions need to be completed next.

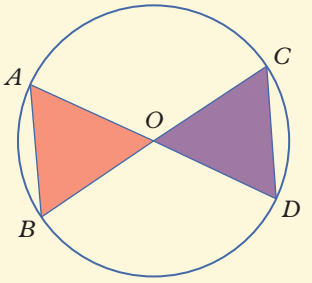
Example 1

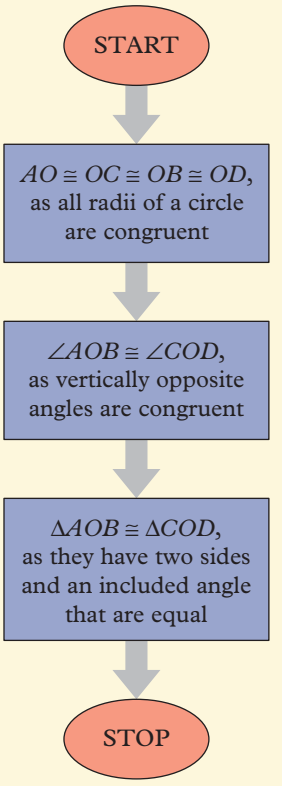
A flowchart can be used to show the steps for a geometric proof, by providing sequential statements supported with geometric reasoning.

Geometric proofs build on axioms, which are mathematical statements that are taken to be true. Axioms provide a starting point for further reasoning in a proof.

The circle below contains triangles ΔAOB and ΔCOD. The point O is at the centre of the circle and the points A, B, C and D lie on the circumference.

A flowchart can be used to prove that these triangles are congruent. The flowchart on the right shows the sequence of statements and arguments needed to demonstrate this proof.





Task 1

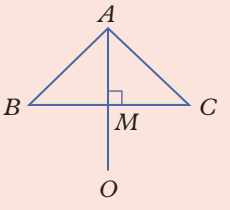
1 Discuss the statement and reasons presented in each of the sequential steps of the proof shown in Example 1 with someone else.

2 Make a list of all the properties that define congruent triangles.

3 Consider the following geometric construction.

The line segment OA bisects (divides into two equal parts) the angle BAC and is a perpendicular bisector of the line segment BC.

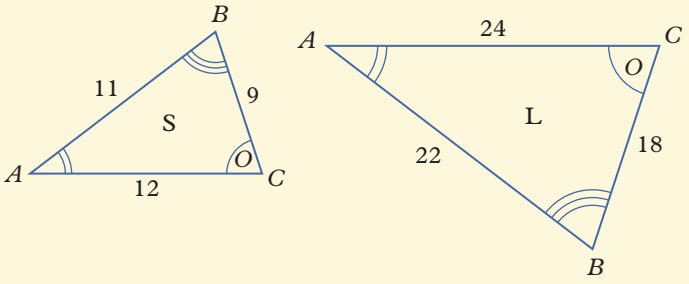
Construct a flowchart proof with a statement and reason for each step to demonstrate ΔABM and ΔAMC are congruent.



Some algorithms contain actions that are only done if certain conditions are true; these are called conditional actions. Flowcharts usually show conditions in diamonds and use arrows to direct to the appropriate actions depending on the result. Variables are used to determine if conditions are true or false.

Example 2

The following diagram shows two similar triangles. The triangle on the left is labelled S (for small) and the triangle on the right is labelled L (for large).



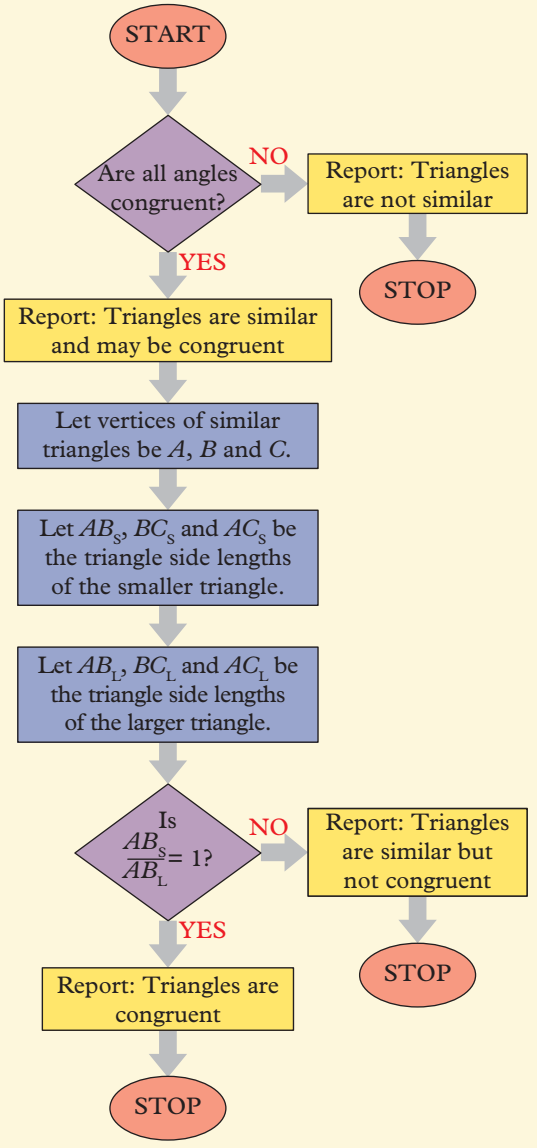
The flowchart on the right shows a sequence of statements with reasoned arguments in each box to determine if these triangles are congruent.

The condition in the first diamond checks if the angles are congruent. If the angles are not congruent, then the triangles are not similar.

For the smaller triangle, the side lengths have the values $AB_S = 11$, $BC_S = 9$ and $AC_S = 12$.

For the larger triangle, the side lengths have the values $AB_L = 22$, $BC_L = 18$ and $AC_L = 24$.

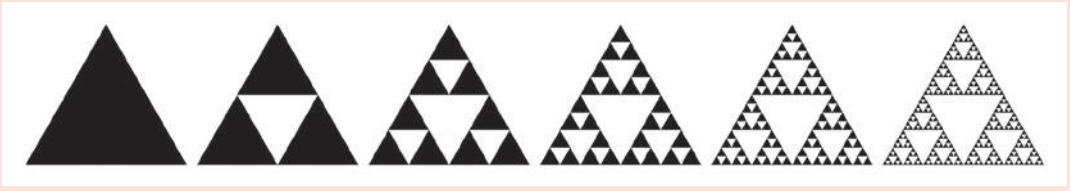
When the condition 'Is $\frac{AB_S}{AB_L} = 1$?' is tested in the diamond, the value of $\frac{AB_S}{AB_L} = \frac{1}{2}$. Therefore, the NO path is followed, and the triangles are reported as being similar but not congruent.



Task 2

1 Check that the flowchart in Example 2 works with other pairs of triangles.

The diagram below shows the first six stages in the evolution of a geometric pattern known as Sierpinski's triangle.

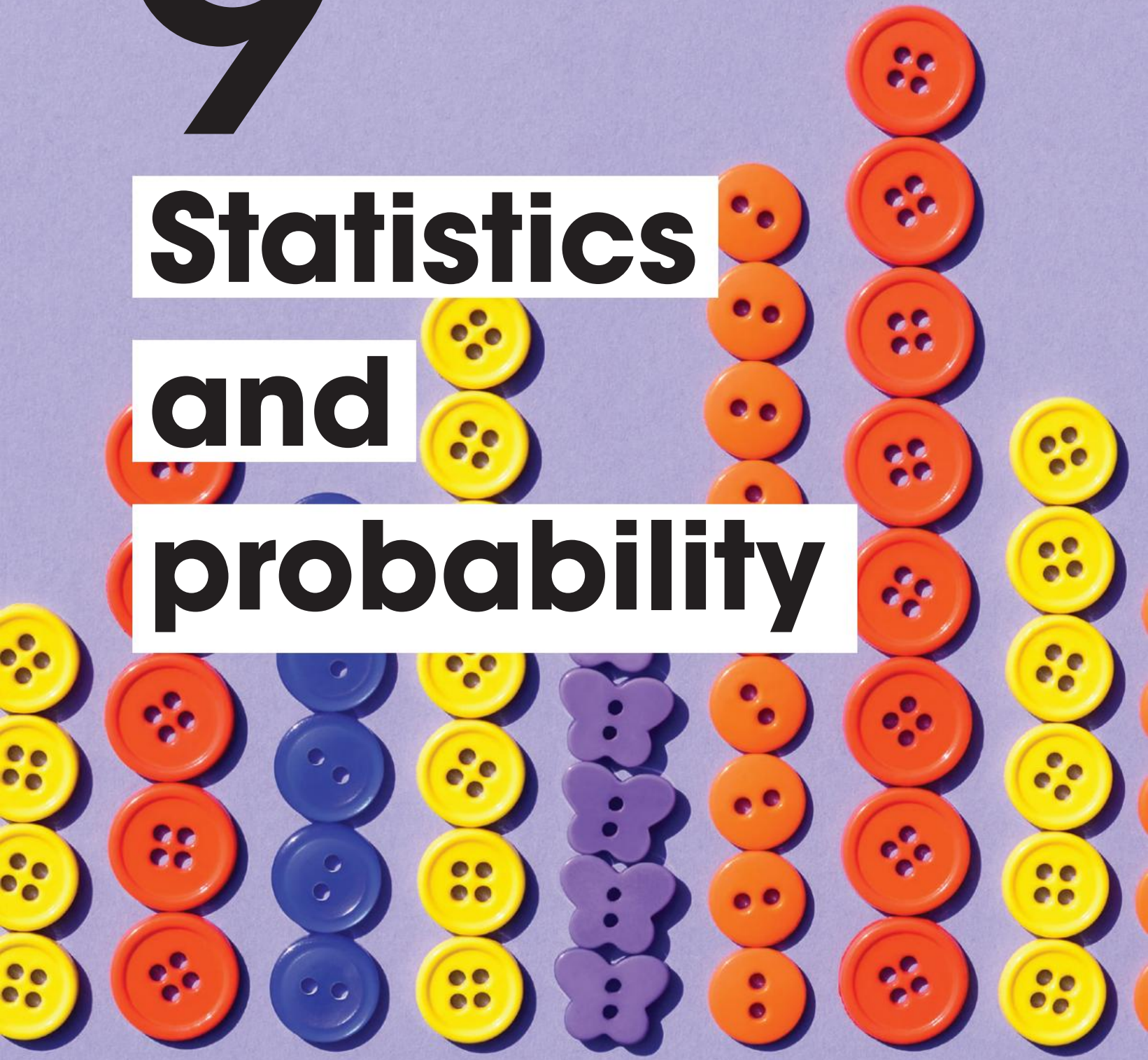


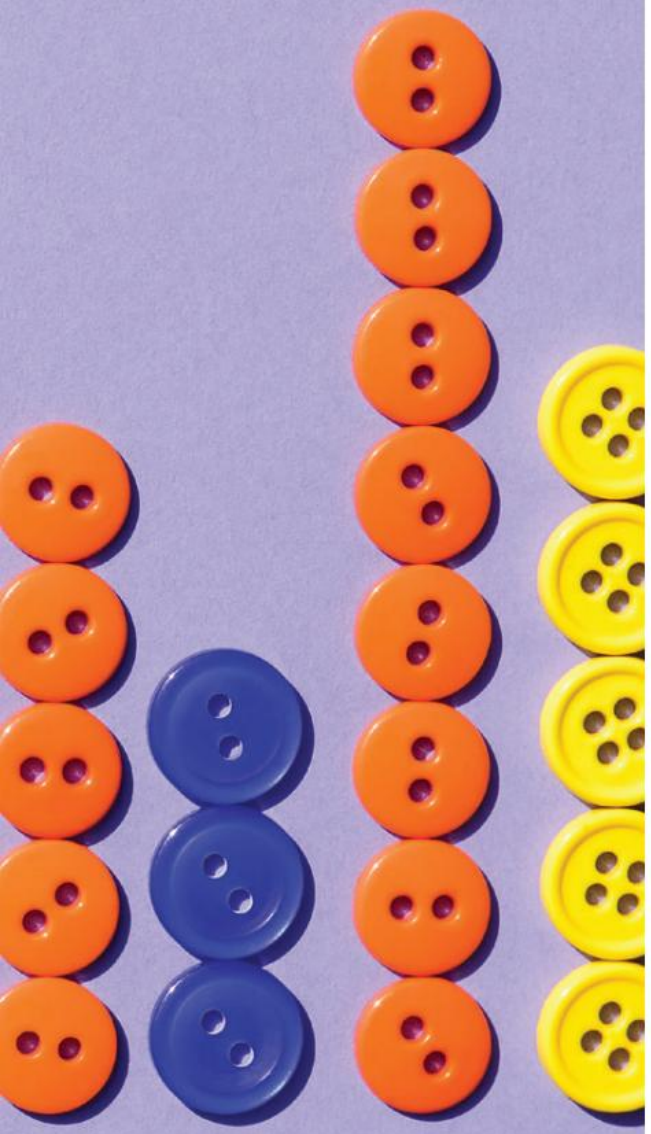
The first stage is a black equilateral triangle. The next stage is created by removing from the centre of the black triangle the equilateral triangle formed by joining the midpoints of the edges of the black triangle. This process is then repeated for each black equilateral triangle at every stage.

2 **CHALLENGE:** Using your list of congruent triangle properties from Task 1 and ideas from Examples 1 and 2, construct a flowchart proof with a statement and reason for each step to show that each black triangle in the Sierpinski triangle pattern is similar at all stages of the pattern.

9

Statistics and probability





Index

9A Collecting data and sampling methods
9B Summary statistics
9C Analysing data
9D Theoretical probability
9E Two-way tables
9F Venn diagrams

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Interpreting graphs
- ✔ Fractions, decimals and percentages
- ✔ Adding and subtracting fractions
- ✔ Multiplying and dividing fractions

Curriculum links

- Investigate techniques for data collection including census, sampling, experiment and observation ... (AC9M8ST01)
- Analyse and report on the distribution of data from primary and secondary sources using random ... (AC9M8ST02)
- Compare variations in distributions and proportions obtained from random samples of the same size ... (AC9M8ST03)
- Plan and conduct statistical investigations involving samples of a population; ... (AC9M8ST04)
- Recognise that complementary events have a combined probability of one; ... (AC9M8P01)
- Determine all possible combinations for 2 events, using two way tables, tree diagrams and Venn diagrams, ... (AC9M8P02)
- Conduct repeated chance experiments and simulations, using digital tools ... (AC9M8P03)

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9A Collecting data and sampling methods

Learning intentions

By the end of this topic you will be able to ...

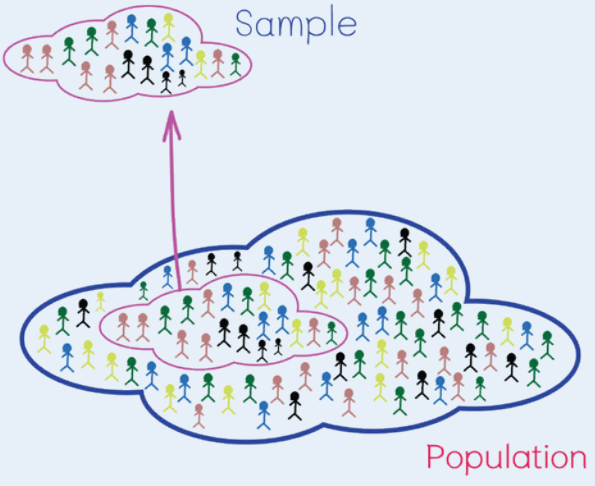
- ✔ distinguish between a population and a sample
- ✔ distinguish between sampling methods
- ✔ identify sources of bias in data collection methods.

Inter-year links

Support	Collecting data
Year 9	7A Surveys and sampling methods
Year 10	9F Evaluating statistical reports

Collecting data

- When collecting data, the **population** refers to all potential pieces of data under consideration.
- A **census** is a survey of an entire population.
  → Taking a census can be impractical, expensive and time consuming.
- A **sample** is a selection of data that is part of the population.
  → Samples can be used to estimate characteristics of the population.
- When taking a sample, every piece of data should have an equal probability of being selected. Otherwise, the sample will be **biased**.
  → A biased sample may be systematically different from the population. For example, a biased sample may favour one outcome over others.
- A fair (unbiased) sample is likely to represent the characteristics of the population.
  → Fair samples must be of a reasonable size to accurately reflect the population.



- Collecting data by observation is when a researcher observes events as they naturally occur. For example, counting the number of kicks and handballs each player in a football game makes.
- Data collected by the researcher is known as primary data. If the data being studied was previously collected by someone else, it is called secondary data.

Surveys

- For a survey to be successful, the questions asked must be meaningful and allow for all possible answers. Poor or badly designed questions can lead to biased answers.

Sampling methods

- **Simple random sampling** techniques ensure that every potential piece of data has an equal chance of being selected.
  - → **Systematic sampling** selects data points at fixed intervals (for example, every 10th person). The starting point should be random.
  - → **Stratified sampling** divides the population into separate categories (for example, age) and takes a random sample from each category. The size of each sample is proportional to how large the category is.
- **Convenience sampling** is when a sample that is easily accessible is selected. For example, selecting students from your class to complete a survey.
- **Judgemental sampling** is when a sample is selected by the researcher. For example, a researcher may ask students to take part in a study.
- **Quota sampling** divides the population into certain categories (like stratified sampling) and takes a specified number from each category. The size of each sample is chosen by the researcher, meaning that potential pieces of data may have different chances of being selected.
- **Cluster sampling** divides the population into groups and then a random sample of the groups (or clusters) is selected. The population within a cluster should be diverse but the composition of all clusters should be similar (for example, all clusters are pairs consisting of one male and one female).

Example 9A.1 Census or sample?



Zoe wants to find out what the favourite local park of the residents of her town is. She asks everybody in her school what their favourite park in town is. Is this a census or a sample?

THINK

1 Identify the population. The population is Zoe's town.

2 Identify who is surveyed. Zoe's school has been surveyed.

3 Determine whether the survey taken is of the whole population (census) or if it is a sample.

WRITE

This is a sample.

Example 9A.2 Determining whether a sample is biased



Mohamed surveyed his friends and family to find out the most popular radio station in their hometown. Decide whether Mohamed's survey is fair or biased, giving reasons.

THINK

Determine whether the survey is a random sample of the population, or if it is biased and potentially unrepresentative of the population.

WRITE

Mohamed's friends and family may share common interests and opinions that do not truly reflect the population, so this is a biased sample.

Example 9A.3 Identifying the sampling method

Maria selects a sample of 10 people to ask which brand of shampoo they use. She selects people for her sample according to their hair colour, in proportion to how common that hair colour is in the population. Determine the sampling method Maria has used.

THINK

1 Determine whether this is a simple random sample, a convenience sample, a judgemental sample, a cluster sample or a quota sample. Every person has an equal chance of being selected, so this is a simple random sample.

2 Determine whether this is a systematic sample or a stratified sample. The population has been divided into categories with an equal proportion taken from each category, so this is a stratified sample.

WRITE

This is a stratified sample.

Example 9A.4 Writing meaningful survey questions

Alec is investigating the favourite sport of students at his school. He asks 50 students 'Do you prefer the AFL or NRL?' Will his question lead to biased answers? If so, rewrite the question so that it provides fair answers.

THINK

1 Does the question provide an equal opportunity for all possible answers? This question only asks about the AFL and NRL, so it will produce biased answers.

2 Rewrite the question so that it allows for all possible answers.

WRITE

This question will lead to biased answers.

A better question would be 'What is your favourite sport?'

Helpful hints

- ✔ Handpicking a sample will not provide a random sample of a population, even if the person picking believes that their sample is representative of the population.
- ✔ The only way to take an unbiased sample is to take a random sample from the whole population, for example by using systematic sampling or stratified sampling.
- ✔ A census will always be fair as it covers the whole population.

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Exercise 9A Collecting data and sampling methods

1–7, 9, 10 | 1–2(a, c, d), 3–6, 8, 11–13, 16 | 1–2(c, d), 4(d), 5, 8, 10, 12, 14–17

1 Identify the population in each situation.

a Surveying students at a local school to find what the town's Year 8 students think of a local issue

b Surveying people at a supermarket to find national attitudes to war

c Surveying every thousandth person from a Western Australian phone book to find the favourite movie in Western Australia

d Surveying people randomly to find a town's attitudes to a new mall being built



9A.1 **2** Decide whether a census or a sample has been taken in each of these situations.

a Nikita questions every person in her class to find out the favourite chocolate of the entire class.

b Jacqui questions every person in all her year level to find out the favourite film for the school.

c Travis asks everyone he knows how many pets they have in order to predict the average number of pets in any household in the state.

d Aaron asks everybody in his family what their favourite dessert is to decide what to cook for a family dinner.

3 Decide whether a census or a sample would be most appropriate for gathering information in each situation.

a Deciding who will be the next school captain

b Getting feedback on the latest version of Windows

c Finding which store in your town (or suburb) sells PlayStations at the cheapest price

d Finding the average number of blue M&Ms in a packet

9A.2 **4** For the following situations decide whether the survey is fair or biased, giving reasons.



a Dane asks every 100th person from the local phone book if they prefer skiing or snowboarding to find out the town's favourite winter sport.

b Jacey asks everybody in the school what they think of the uniform to see how popular it is with the students.

c Ryan asks everybody in a casino what they think about laws to restrict certain forms of gambling to find out the common opinion in Australia.

d Tracey asks every 10th person in Woolworths which supermarket they prefer in order to find the town's favourite.

9A.3 **5** Determine the sampling method used in each of these situations.

a Jamie asks every fifth person in a store their opinion on a new magazine.

b Helen selects a panel that consists of three girls and two boys out of a group of 15 girls and 10 boys.

c Xavier calls every phone number in the phone book ending in a randomly chosen digit to complete a survey.

d Mia decides to survey 20 students from public schools and 30 students from private schools.

e Devon chooses 20 people on the oval at lunchtime to ask if they enjoy sport.

f Rita posts a survey she needs filled in for class to her Instagram story for her followers to complete.

g Sara measures the diameter of all the roses in her family's well-maintained garden to determine the average diameter of any rose.

UNDERSTANDING AND FLUENCY

9A.4 **6** For each of these survey questions identify if they will lead to biased answers, and if so, rewrite the question so that it provides fair answers.

	Topic	Question
a	Finding the most popular animal	Do you like cats or dogs?
b	Deciding which film to see in the holidays	Do you like horror movies?
c	Investigating the most popular chocolate	What is your favourite chocolate?
d	Finding the most common hobby amongst Year 8 students	What do you think the most common hobby might be?
e	Deciding where to go for a holiday	Where are three places you might like to go for the holidays?
f	Investigating voting intentions	Who do you think will win the election?

PROBLEM SOLVING AND REASONING

7 Lizzie wants to conduct a survey looking at the attitudes of people in town towards graffiti.



a What is the population?

b Somebody suggests Lizzie ask random people that she sees at the local mall. What potential problems are there with this method?

c Somebody else suggests that Lizzie ask every fifth person who walks past the local post office. What kind of sampling is this?

d What potential problems are there with this second method?

e Give an example of how Lizzie might collect data using stratified sampling.

8 Write two different questions that provide fair answers if you were to investigate:

a favourite soft drinks

b number of pets in a household

c internet usage per week

d student opinions about a new school rule.

9 A survey is to be done on a new school uniform. Provide a method of sampling the students at the school that uses:

a systematic sampling

b stratified sampling

c quota sampling

d cluster sampling.

10 For each survey:

i identify the population

ii provide an example of how you might collect a fair sample of the population.

a Investigating the top five favourite celebrities of Year 8s at your school

b Investigating a town's opinion on a new national law

c Investigating the average number of people in an Australian family

11 The Queensland government wants to determine the view of state residents on their new policies. Instead of surveying the population, they choose to use cluster sampling.

a What is an advantage of using cluster sampling instead of surveying the entire population?

b The government decides to survey the residents of three randomly chosen local government areas in Queensland. Why might the results of the survey not accurately represent the views of the whole population?

12 A new video game for a popular franchise was announced with a video of the gameplay on social media. To determine the fanbase's reaction to the announcement, the developers checked the top 20 posts with the hashtag they included in the announcement and found 18 negative posts and 2 positive posts.

a What type of sample did the developers use?

b Is this sample fair or biased? Explain.

In the lead up to the launch of the game, the developers checked 20 reviews of the game. They had paid 5 of these reviewers and provided review copies to 10 others. They found the majority of the reviews were positive.

c What type of sample did the developers use?

d Is this sample fair or biased? Explain.

13 Judgemental selection for a sample involves deliberately choosing people or objects for your sample.

a Why might deliberately selecting people or objects give a better sample than a randomly selected sample?

b Why might deliberately selecting people or objects give a worse sample than a randomly selected sample?

14 Sally wants to find out what people in town think of a local issue. She makes a post on her social media, linking to an online survey she has created. How might this method of surveying be biased?

15 All the gamers at a school are asked to identify the main equipment they use to play video games. Marci samples this population to determine the most popular type of device/console that gamers at her school use. The number of students Marci interviewed is also shown in the table. Determine if Marci's sample is perfectly stratified, approximately stratified, or not stratified. Explain why.

Equipment used to play video games	Population (percentage)	Marci's sample (frequency)
PlayStation	23%	13
Xbox	17%	8
Nintendo	27%	17
Mobile	15%	7
PC	18%	9

16 A class of 25 students state whether they prefer half-hour- or full-hour-long episodes for TV shows. Their preferences are: full, half, half, full, full, half, half, half, half, half, half, half, half, half, full, full, half, half, full, full, full, half, half, full, half.

a Select every second response, starting from the second response, and determine the percentage who prefer half-hour episodes, correct to the nearest percent.

b What type of sample is this?

c Calculate the percentage of the population who prefer half-hour episodes.

d Calculate the percentage error between the sample percentage and the population percentage, correct to the nearest tenth of a percent.

17 A new vaccine has been developed and is ready for human trials in a sample of the population. The developers of the vaccine are considering how large the sample should be.

a Give a possible consequence of the sample being too small.

b Give a possible consequence of the sample being too large.

Check your Student obook pro for these digital resources and more:


Interactive skillsheet
Sampling methods


Investigation
First day disasters


Topic quiz
9A

PROBLEM SOLVING AND REASONING

CHALLENGE

9B Summary statistics

Learning intentions

By the end of this topic you will be able to ...

- ✔ calculate the mean, median, mode and range of a numerical data set
- ✔ evaluate how individual values affect the mean, median and range.

Inter-year links

Support	Understanding data
Year 7	10A Summary statistics
Year 9	7D Summary statistics from tables and displays
Year 10	9A Five-number summary and interquartile range

Summary statistics

- The **mean** is a numerical average of the values in a data set. To find the mean, add all the numbers together, then divide by the number of values in the set.

$$\text{mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$$

- The **median** is the middle of a data set when the numbers are listed in order. The median divides a data set in half.
  - → If there are n values in a data set, the median is the $\frac{(n+1)}{2}$ th value.
    For example, if there are 11 values, the median is the 6th value.
    If there are 16 values, the median is the 8.5th number, meaning it is halfway between the 8th and 9th values.

2, 3, 4, 5, 6
median = 4

1, 2, 3, 4, 5, 6
median = $\frac{3+4}{2}$ = 3.5

- The **mode** is the most common value in a data set. There can be more than one mode in a data set.
- The **range** is the difference between the largest (maximum) and smallest (minimum) values in a data set.
  range = maximum value – minimum value

Outliers

- An **outlier** is an extreme data point that is much higher or lower than the other values in a data set.

Example 9B.1 Calculating summary statistics

Find the **a** mean, **b** median, **c** mode and **d** range for this data set: 8, 4, 1, 2, 9, 2, 4, 13, 2

THINK

a Add the numbers together and divide by the total number of values.

b List the numbers in the data set in ascending order and find the middle value.

c List the numbers in the data set in ascending order and identify the number which occurs most often.

d Identify the minimum and maximum values of the data set. Subtract the minimum value from the maximum value.

WRITE

a $\text{mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$

$= \frac{8+4+1+2+9+2+4+13+2}{9} = \frac{45}{9} = 5$

b 1, 2, 2, 2, (4), 4, 8, 9, 13
The median is 4.

c 1, 2, 2, 2, 4, 4, 8, 9, 13
The mode is 2.

d range = max – min
= 13 – 1
= 12

Example 9B.2 Summary statistics from a frequency table

For the data shown in the following frequency table, find:

a the mean

b the median

c the mode

d the range.

Score (x)	Frequency (f)
0	4
1	6
2	8
3	7
Total	**25**

THINK

a Add a 'score × frequency' column to the table and complete it.
The total of this column is the sum of all data points, whereas the total of the frequency column is the total number of data points.

b Add a cumulative frequency column to the table and complete it.
Add 1 to the number of values and divide by 2 to find the position of the median. Then identify the row which contains the median score by using the cumulative frequency column.

c Identify the score with the highest frequency.

d Identify the minimum and maximum values in the score column of the data set. Subtract the minimum value from the maximum value.

WRITE

Score (x)	Frequency (f)	Score × frequency	Cumulative frequency
0	4	$0 \times 4 = 0$	4
1	6	$1 \times 6 = 6$	$4 + 6 = 10$
2	8	$2 \times 8 = 16$	$10 + 8 = 18$
3	7	$3 \times 7 = 21$	$18 + 7 = 25$
Total	**25**	**43**	

a $\text{mean} = \dfrac{\text{sum of all data points}}{\text{total number of data points}}$

$= \dfrac{43}{25}$

$= 1.72$

b $\dfrac{(25+1)}{2} = 13$, so the median is the 13th score.

Therefore the median is 2.

c The mode is 2.

d range = max − min

$= 3 - 0$

$= 3$

Helpful hints

- ✔ If every value in a data set is different then there is no mode, but if at least one value is repeated, then there is a mode! Remember that there can be more than one mode.
- ✔ Remember that the median is the middle number of the data set, not the number in the middle of the range.
- ✔ If a data set has an odd number of values then the median is one of the values; otherwise, the median does not have to appear in the data set.
- ✔ Like the median, the mean does not have to appear in the data set.

Exercise 9B Summary statistics

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1–4, 5(a–c), 6–9, 11, 14 | 3, 4, 5(d–f), 6, 8, 10, 12, 16, 17(a) | 4, 5(d–f), 8, 11, 13, 15–18

UNDERSTANDING AND FLUENCY

9B.1 **1** Consider this data set: 5, 8, 7, 11, 4, 9, 5. Write the numbers in order. Then for this data, find:

a the mean **b** the median **c** the mode **d** the range.

2 Find the mean (to one decimal place), median, mode and range for each set of data.

a 11, 19, 18, 12, 12 **b** 2, 8, 4, 9, 7, 6, 2, 4, 1 **c** 8, 7, 3, 5, 7, 7, 4, 8, 6, 0, 9, 7

3 Monica collected this data on the heights (in centimetres) of her classmates.

161 152 148 167 162 149 159 155 161 145 155 156

For this data, find:

a the mean (to two decimal places) **b** the median **c** the mode **d** the range.

4 Mario collected data on the heights (in centimetres) of his classmates.

154 157 158 161 162 198 168 147 152 152 153 150

For this data, find:

a the mean (to two decimal places) **b** the median **c** the mode **d** the range.

9B.2 **5** For the data shown in each frequency table, find:

i the mean (to two decimal places) **ii** the median **iii** the mode **iv** the range.

a

Score (x)	Frequency (f)
1	9
2	11
3	6
4	4
Total	**30**

b

Score (x)	Frequency (f)
2	9
3	6
4	5
5	4
Total	**24**

c

Score (x)	Frequency (f)
10	4
20	8
30	6
40	2
Total	**20**

d

Score (x)	Frequency (f)
1	4
2	8
3	6
4	2
5	5
Total	**25**

e

Score (x)	Frequency (f)
13	2
14	4
15	1
16	2
17	1
Total	**10**

f

Score (x)	Frequency (f)
0	8
10	12
20	20
30	6
40	4
Total	**50**

6 A Year 8 class were asked how many online quizzes they did last week. The results were:

0 1 6 3 5 1 2 0 0 2 1 3
2 1 0 0 1 1 2 18 2 3 5 3

a Create a frequency table to represent the data.

b Find the **i** mean (to one decimal place), **ii** median and **iii** mode.

c Find the range.

d Identify the outlier.

7 Calculate summary statistics for the data shown in this table. Give your answers to one decimal place where necessary.

Score (x)	Frequency (f)
11	4
12	8
13	16
14	17
15	13
16	4
17	2
Total	**64**

8 Calculate summary statistics for the data shown in this table.

Score (x)	Frequency (f)
–0.3	4
–0.2	9
–0.1	9
0	2
0.1	6
Total	**30**

9 The weights (to the nearest kilogram) of school bags of a Year 8 class were recorded for an investigation.

2 6 11 4 2 4 8 5
9 3 4 2 3 5 6 5
2 4 3 5 5 7 10 10

a Arrange the data into a frequency table.

b Calculate the four summary statistics for this data. Round your answers to one decimal place where necessary.

c Use your answers to part **b** to write two sentences about the weights of these school bags.

10 Monique collected this data on the number of text messages people in her class sent per day.

6 39 15 26 18 37 48 14 88 9 22 74
42 22 56 33 16 28 31 48 52 17 21 55

By calculating the summary statistics for this data, write two sentences about Monique's data.



11 The weights of 10 oranges are recorded to the nearest gram.

110 125 118 136 104 142 123 202 130 128

a Identify the outlier in this data set.

b Calculate the value of the mean **i** with and **ii** without the outlier included in the data set.

c Calculate the value of the range **i** with and **ii** without the outlier included in the data set.

d Discuss how the outlier affects the value of the mean and the range.

12 The mean is often given as a value to represent a data set; however, the presence of an outlier can significantly affect the mean.

a Explain why an outlier can affect the mean, using the comparison of the heights in Monica's and Mario's classes from questions **3** and **4** to help you.

b Would you use mean or median to describe 'average' height in Mario's class? Explain your answer.

c Identify the outlier in Mario's class.

d Recalculate the mean for Mario's class without the outlier. How does this compare to the median?

e What is the range for Mario's class without the outlier? Compare to your answer for question **4b**.

13 A green bag of 10 grocery items weighs 3 kg.

a What is the average (mean) weight of the grocery items?

b A bag of chicken fillets is removed from the green bag and the average weight changes to 200 g. What was the weight of the chicken fillets?

14 For each of the following, a mistake has been made in determining the summary statistic.

i Determine the error made for each statistic.

ii Determine the difference between the actual statistic and the incorrect statistic. Calculate each difference as 'actual statistic – incorrect statistic'.

a Calculate the median of this list of data: 8, 3, 5, 1, 2, 7, 5.

8, 3, 5, 1, 2, 7, 5

The number in the middle of the list is 1, so the median is 1.

b Calculate the range of this list of data: 8, 3, 5, 1, 2, 7, 5.

8, 3, 5, 1, 2, 7, 5

The first and last numbers of the list are 8 and 5, so the range is $5 - 8 = -3$.

c Calculate the mean from this table.

Score (x)	Frequency (f)
1	3
2	4
4	7
7	2

$\text{mean} = \dfrac{1 + 2 + 4 + 7}{4} = 3.5$

The mean is the sum of the scores (1, 2, 4, 7) divided by the number of scores, 4, which is 3.5.

d Calculate the range from this table.

Score (x)	Frequency (f)
1	24
2	15
3	52
4	17

The largest value in the table is 52 and the smallest is 1, so the range is $52 - 1 = 51$.

e Calculate the median from this table.

Score (x)	Frequency (f)
4	1
6	0
8	4
10	12
12	13

The median is the middle number, and the number in the middle of the score column is 8.

15 Consider this set of 15 values: $10, 12, 12, 13, 13, 13, 14, 15, 15, 15, 16, 16, 17, 19, x$, where x is the final number in the set but has not been determined yet. The value of x is not necessarily the largest value in the set.

a Complete the table below, where in each row x is the value given in the left-hand cell.

x	Mean (correct to two decimal places)	Median	Mode(s)	Range
13	14.20	14		9
8	13.87		13, 15	11
1		14		
−3	13.13			
15	14.33			9
25			13, 15	

b Which statistics changed the most as the value of x changed? Which changed the least?

c For what value(s) of x did the median change from 14? Explain why this happened.

d For what value(s) of x did the range change from 9? Explain why this happened.

e For what value(s) of x did the mode change from both 13 and 15? Explain why this happened.

f For what other values of x will the mode change to being a different value from 13 and 15?

g Complete these sentences.

i As the value of x decreases, the value of the mean _____.

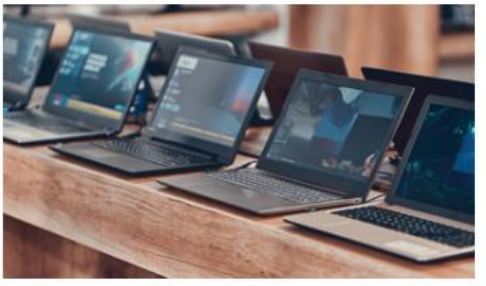
ii As the value of x increases, the value of the mean _____.

h If $x = 100$, how do you expect the statistics will change? Calculate the mean, median, mode and range to check your expectation.

16 Jacinta is looking to buy a laptop and has narrowed her choices down to two. To help her make her decision, she looks at the customer ratings for both laptops on the website she will purchase from.

Laptop A	
Rating	**Frequency**
★★★★★	10
★★★★	1
★★★	0
★★	1
★	10

Laptop B	
Rating	**Frequency**
★★★★★	1
★★★★	3
★★★	10
★★	3
★	1



a Calculate the mean, median and range for both laptops' ratings.

b Give reasons, related to the ratings, why Jacinta may want to and may not want to buy Laptop A.

c Give reasons, related to the ratings, why Jacinta may want to and may not want to buy Laptop B.

17 a A set of 4 numbers has a mean, median and mode of 7, and a range of 4. Write down the four numbers.

b A set of 5 numbers has a median of 10 and modes of 3 and 12. Calculate the mean and range of the data.

18 Consider the set of data 20, 24, 29, 34, 40, 55, x, y, where x and y are not necessarily the largest values in the set. Determine the values of x and y such that the mean = 37.125, the median = 31.5, and the range = 50.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet The mean

Interactive skillsheet The median

Interactive skillsheet The mode

Interactive skillsheet The range

Investigation Pool usage

Topic quiz 9B

9C Analysing data

Learning intentions

By the end of this topic you will be able to ...

- ✔ use sample properties to predict population parameters
- ✔ use summary statistics to describe key features of data.

Inter-year links

Year 7	10B Describing data with dot plots
Year 9	7E Describing data
Year 10	9C Distributions of data

Using samples to make predictions

- Random samples drawn from the same population will all vary from each other.
- A sample can be used to predict **population parameters** (characteristics of a population). To be confident of these predictions, a sample must be:
  - → gathered using a fair and unbiased technique
  - → large enough to represent the population.
- The size of the sample required to make accurate predictions will depend on the size of the population. For example, a sample of 50 students may be large enough to make accurate predictions about Year 8 students in your school but won't be large enough to make accurate predictions about Year 8 students in Victoria.

Suggested sample sizes	
Population size	**Minimum sample size**
0–20	Census
21–100	20
101–500	30
500–1000	50
1001–5000	100
5001–50 000	500
50 000+	1000+

Describing data

- Summary statistics can be used to describe data.
  - → The mean, median and mode are **measures of centre** as they each identify a central position in a data set.
  - → The range is a **measure of spread** as it provides an indication of how spread out the data is.
- When deciding which measure of centre should be used to describe data, the presence of outliers should be taken into consideration.
  - → Outliers can skew the value of the mean but will not impact the median or mode.
  - → A small sample size with the mean being significantly higher or lower than the median may indicate an outlier skewing the value of the mean.
- When describing a population or sample, people use words such as:
  - → 'average', which usually refers to the mean, but sometimes median
  - → 'most common', 'most popular' and 'most often', which refer to the mode
  - → 'middle' and 'centre', which refer to the median
  - → 'spread' or 'span', which usually refer to the range.

Example 9C.1 Identifying summary statistics

Identify which summary statistics are being used in these sentences.

a The most popular team was the Brisbane Broncos.

b The levels of a chemical in the water varied by up to 20 g.

THINK

a The term 'most popular' refers to the mode.

b The word 'varied' refers to the spread of the data, which is the range.

WRITE

a mode

b range

Example 9C.2 Identifying whether a sample can predict population parameters

The table below shows the levels a group of Year 8 students reached when playing a computer game on their first attempt.

Given this data, could you reasonably predict population parameters? Explain your answer.

Population	Sampling technique	Sample	Mean	Median	Mode	Range
Year 8 students (90)	Draw names out of a hat	35 students from Year 8	4.4	4	4	8

THINK

1. Look at the population and sample size. Is the sample size large enough to accurately represent the population?
2. Look at the sampling technique. Was the sample gathered using a fair and unbiased technique?
3. Determine if you would be confident enough to use these results to predict the population parameters.

WRITE

You can be confident in using this sample to predict the population parameters as the sample size is sufficient and the sampling technique is random.

Helpful hints

✔ Outliers have more of an impact on the mean for small samples than they do for large samples! For example, in a sample of the heights of 10 students, a very tall student would significantly affect the mean, whereas in a sample of 100 students, a very tall student would not have much impact on the mean.

ANS p556

Exercise 9C Analysing data

1–7, 9 | 1, 3–6, 8–10 | 1, 3, 6, 8–11

UNDERSTANDING AND FLUENCY

9C.1 **1** Identify which summary statistics are being used in these sentences.

a The average height for a 14 year old is 162 cm.
b The temperature covered a span of 10°C.
c The most popular music in the class was hip-hop.
d The times varied by up to 40 minutes.
e The train is most often 2 minutes late.
f Andrew is in the middle of his height range.

2 Tanya wants to investigate the most popular sports team in South Australia. Which samples do you think would better reflect the population? Explain your answers.

a A sample of 20 people or a sample of 100 people
b A sample of 619 people or a sample of 500 people
c A sample of 20 randomly selected people or a sample of 40 students
d A sample of 100 people in Australia or a sample of 50 people in South Australia

9C.2 **3** Given the data in these tables, state whether you could reasonably predict population parameters. Explain your answer.

	Population	Sampling technique	Sample	Mean	Median	Mode	Range
a	Your school	Asking your friends	50 of your friends	8	7	8	3
b	Australia	Stratified sampling	1000 people in Canberra	16	16	14	20
c	Victoria	Systematic sampling	30 people in Victoria	8	3	3	26
d	Audience at a circus (200 people)	Stratified sampling	100 people at the circus	4	1	1	10
e	Your suburb	Systematic sampling	500 people in your suburb	9	8	9	3

4 Laani reads that there are 2.5 people in an average Australian household. She decides to conduct a survey to see if her results match. She asks everybody that she knows at school and displays her results in this table.

Number of people in household	2	3	4	5	6	7	8
Frequency	6	19	38	20	14	2	1

a Find the mean, median and mode for this data.
b What is the population of the survey? What is the sample?
c How many people has Laani surveyed?
d Why might this sample not truly reflect the population?

5 Laani repeats the survey from question **4**, but this time she uses a random sampling technique of the population. She surveys 20 people and finds an average of 2.9 people per household.

a How is this different from her original sample?
b Why might you still not be confident that this sample reflects the population? Hint: How might an outlier affect this sample?

In the end Laani surveys 200 people, finding an average of 2.6 people per household.

c Why might Laani be more confident of her results with a larger sample?
d As sample size increases, you can see that the sample mean more closely resembles the population mean. Do you think this would be the same for other summary statistics? Explain your answer.

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6 Three different samples are taken from a population. The results are summarised below.

Sample: 20 people	
Category	**Frequency**
A	5
B	2
C	7
D	4
E	2

Sample: 50 people	
Category	**Frequency**
A	14
B	4
C	19
D	7
E	6

Sample: 200 people	
Category	**Frequency**
A	52
B	23
C	84
D	24
E	17

a The percentage frequency is the frequency divided by the size of the sample and is then written as a percentage. Convert each table to percentage frequency.

b Which category(s) is most likely to be the mode for the population? Explain by referring to the sample percentage frequencies.

c Which category(s) is most likely to be the least common for the population? Explain by referring to the sample percentage frequencies.

7 Jade conducted an online survey about Australians' favourite season of the year. She received the following results.

Favourite season	Frequency
Spring	51
Summer	78
Autumn	43
Winter	28

a What proportion of people preferred each of the seasons in Jade's survey?

Karl conducted a similar online survey and received the following results.

Favourite season	Frequency
Spring	99
Summer	134
Autumn	94
Winter	73

b What proportion of people preferred each of the seasons in Karl's survey?

c Explain why there is a variation in the proportions from the two surveys.

d Assuming each survey was collected randomly, how might you report on the favourite seasons of Australians?

8 Chris wants to find the average number of mobile phones in Australian households. He decides to take a sample of 50 people. The results are shown in the frequency table below.



a Find the mean, median and mode for this data.

b What might you need to consider before using this data to predict the population average?

c Use either the sample mean or median to estimate the average number of mobile phones in Australian households. Explain your decision.

Number of mobile phones	0	1	2	3	4	5	6
Frequency	1	1	11	8	16	9	4

PROBLEM SOLVING AND REASONING

9 Lauren wants to know the opinion of people at her school on reality TV. She takes five samples of people and tabulates the percentages.

Opinion	20 random Year 8s	20 random students	4 random students from each year level	20 random teachers	15 random students and 5 random teachers
Love it	10%	15%	25%	40%	25%
Like it	15%	20%	12.5%	25%	15%
No opinion	30%	15%	12.5%	15%	20%
Don't like	40%	30%	37.5%	15%	15%
Despise it	5%	20%	12.5%	5%	25%

a How many Year 8s in the first sample like reality TV?

b Using the percentages in the table, what is the mean percentage of people who love reality TV at Lauren's school?

c Explain why the percentages varied so much from sample to sample.

d Based on your answer from part **c**, do these samples give a clear picture of the opinions on reality TV at Lauren's school?

10 The Lyric Theatre in Brisbane has 2000 seats. Over four consecutive shows, all the seats were filled. The country of origin of every attendee was noted. The data for the four shows is given in the following table.

	Thursday night	Friday night	Saturday night	Sunday night
Australia	1119	1340	1461	900
China	131	98	79	116
UK	273	245	182	320
USA	370	215	170	553
Other	107	102	108	111
Total	**2000**	**2000**	**2000**	**2000**

a Find the average number of attendees from each country (including other) over the four nights.

Historical data can be used to make predictions about future events. The theatre manager uses the data above to predict the country of origin of theatre attendees the following week on any night.

b Use your answers from part **a** to find the average proportion of attendees from each country (including other) written as a percentage.

c If only 1200 seats are available in the theatre due to required maintenance, how many people from each country would the theatre manager expect to be in attendance? Round your answers to the nearest integer.

The theatre manager now wants to only consider the Sunday night shows.

d Should the theatre manager continue using the answers to part **b** to make predictions? If not, what could they do instead?

e How could the theatre manager improve their predictions for parts **b** to **d**?

CHALLENGE

11 Suzie takes a sample of 100 Year 8s at her school, calculates the mean height to be 1.52 m and determines there are 85% who are right-handed. Suzie claims that the average height of all Year 8s in Australia is 1.52 m and 85% of Year 8s in Australia are right-handed.

a Explain why Suzie does not have enough information to make that claim.

b What claim can Suzie make given her sample?

Check your Student obook pro for these digital resources and more:


Worksheet
Connecting samples and populations


Investigation
Ice cream flavours


Topic quiz
9C

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

9A **1** Identify whether the following situations use a sample or a census.

a Checking what every third person leaving a shop bought to determine the most popular item that shop sells
b Walking the entire length of a path to determine if there are any problems to fix along the path
c Interviewing all Year 8 students at a school about the best place to have lunch at their school

9A **2** Identify whether the following questions contain bias.

a Asking 'What is your opinion of the Prime Minister? Please rate on a scale from 1 to 10, where 1 is dreadful and 10 is amazing' to determine random people's opinions of the Prime Minister.
b Asking 'Are you smart?' to determine if someone is intelligent.
c Asking 'What is your favourite chocolate, and why is it Crunchie?' to determine your classmates' favourite type of chocolate.

9A **3** Identify the sampling method in each of the following situations.

a Ringing every 5th number on the school's phone directory to survey families at the school
b Using the most recent Australian census to determine the number of random people from different nationalities to interview
c Posting a survey to a group page on Facebook for people in the group to answer if they want to

9B **4** Calculate the **i** mean, **ii** median, **iii** mode and **iv** range of the following data sets.

a 17, 20, 6, 11, 20, 14, 20, 14, 18, 7
b 3.3, 5, 1.5, 0.3, 3.2, 3.3, 0.3, 2.9, 4.1, 2.8

9B **5** The mean for a set of data is 30 and the median for the same set is 18. Which statistic should be used for the average? Explain why.

9C **6** Can these sample statistics be used to predict population parameters? Explain why or why not.

	Population	Sampling technique	Sample	Mean	Median	Mode	Range
a	Students at your school (1200 students)	Randomly selected from a list of all students	120 students	4	5	4	10
b	Students at your school (1200 students)	Randomly selected from a list of all students	12 students	4	5	4	10
c	Students at your school (1200 students)	Online survey attached to school newsletter	120 students	4	5	4	10
d	Students at your school (1200 students)	All students who had been given a detention this month	120 students	4	5	4	10

9C **7** Thirty people entering a shopping centre are asked if they are going to Woolworths. The responses are:

Yes, Yes, Yes, Yes, No, No, No, Yes, No, No, Yes, Yes, Yes, No, No, No, No, Yes, Yes, Yes, Yes, Yes, Yes, No, No, No, Yes, Yes, No, No.

a Select every second response, starting from the second response, and determine the percentage who are planning to go to Woolworths, correct to the nearest percent.
b Select every third response, starting from the third response, and determine the percentage who are planning to go to Woolworths.
c Explain why the two samples have different percentages despite being taken from the same population.

9D Theoretical probability

Learning intentions

By the end of this topic you will be able to ...

- ✔ describe events using the language of probability
- ✔ identify complementary events
- ✔ calculate the probability of an event and its complement.

Inter-year links

Support	Understanding probability
Year 7	10E Theoretical probability
Year 9	8A Two-step chance experiments
Year 10	10A Theoretical probability

Theoretical probability

- The **probability** of an **event** is how likely that event is to happen. A probability must have a value in the range of 0 (impossible) to 1 (certain).
- The **sample space** of an **experiment** is a list of the possible outcomes in the experiment.
  - → An event is a collection of one or more outcomes from the sample space of an experiment.
- The probabilities of all outcomes in a sample space add up to 1.
- A **favourable outcome** is an outcome that you want.
- If all outcomes in the sample space are equally likely to occur, the theoretical probability of an event is calculated by:

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes in event}}{\text{total number of possible outcomes}}$$

  - → If outcomes within the sample space are not equally likely to occur, you can write them as a list of outcomes that reflect their chance of occurring.

    For example, for selecting a letter at random from the word ENERGY, the list can be written as E, N, E, R, G, Y, where each outcome is equally likely to occur.

Describing events

- 'at least' means equal to or above the given value. For example, 'rolling at least 3' means 3, 4, 5, …
- 'at most' means equal to or below the given value. For example, 'rolling at most 5' means 5, 4, 3, …
- 'more than' means above the given value. For example, 'more than 3' means 4, 5, 6, …
- 'fewer than' means below the given value. For example, 'fewer than 5' means 4, 3, 2, …
- 'and' means both outcomes must be met. For example, 'selecting a red card and a king' means selecting a red king.
- 'or' means either of the given outcomes.
  - → 'or' can be inclusive, meaning if both outcomes are met the outcome is favourable. For example, 'rolling a prime number or more than 4' when rolling a standard six-sided die means 2, 3, 5 or 6.
  - → 'or' can also be exclusive, meaning if both outcomes are met the outcome is not favourable. For example, 'rolling a prime number or more than 4 but not both' when rolling a standard six-sided die means 2, 3 or 6 (5 is not included).

Complementary events

- The complement of event A is the event that A does not occur. For example, the complement of the event 'rolling a 1 on a standard six-sided die' is 'not rolling a 1 on a standard six-sided die.'
  - → If the probability of event A is Pr(A), then the probability of the complement of event A is Pr(A').
- The probabilities of **complementary events** always add to 1. This means that Pr(A) + Pr(A') = 1.

Example 9D.1 Interpreting the language of probability

Identify the favourable outcomes in each of the following situations.

a Rolling at least 5 on a standard six-sided die

b Landing on a 1 or a prime number when this spinner is spun

c Landing on a multiple of 2 or a multiple of 3, but not both, when this spinner is spun

d Getting a multiple of 2 and a multiple of 3 when randomly picking a number from 1–12



THINK	WRITE
a 'at least' means any outcome equal to or above the value is favourable.	**a** {5, 6}
b 'or' means any section which satisfies either condition is favourable.	**b** {1, 2, 3, 5, 7}
c 'not both' means that any section which satisfies both conditions is not favourable.	**c** {2, 3, 4, 8}
d 'and' means that both conditions must be met for an outcome to be favourable.	**d** {6, 12}

Example 9D.2 Calculating theoretical probability

The letters from the word FRANCE are placed in a bag and a letter is selected at random from the bag.

a State the number of possible outcomes.

b What is the probability of selecting a C?

c What is the probability of selecting a vowel?

THINK	WRITE
a Count the total number of outcomes, with each letter counting as a separate outcome.	**a** total number of outcomes = 6
b Count the number of favourable outcomes and substitute into the formula with the total number of outcomes.	**b** $\Pr(C) = \frac{1}{6}$
c Count the number of favourable outcomes (A and E) and substitute into the formula with the total number of outcomes.	**c** $\Pr(\text{vowel}) = \frac{2}{6}$ $= \frac{1}{3}$

Example 9D.3 Finding the complement of probabilities

Find Pr(A') if Pr(A) is equal to:

a 0.3

b $\frac{5}{7}$

THINK	WRITE
Complementary events add to 1. Subtract the given probability from 1.	**a** $1 - 0.3 = 0.7$
	b $1 - \frac{5}{7} = \frac{2}{7}$

Example 9D.4 Using complementary events

A standard six-sided die is rolled and the number on the upper face is noted.

a List the sample space.

b What is the probability of 'rolling at most 2'?

c What is the complementary event to 'rolling at most 2'?

d What is the probability of this complementary event in part **c**?

THINK

a List all the possible outcomes within curly brackets.

b The favourable outcomes are rolling a 1 and rolling a 2.

c The complement to 'at most' a given number is 'more than' that number.

d Complementary events add to 1. Subtract the original probability from 1.

WRITE

a $\{1, 2, 3, 4, 5, 6\}$

b $\Pr(\text{at most } 2) = \frac{2}{6}$

$= \frac{1}{3}$

c More than 2

d $\Pr(\text{more than } 2) = 1 - \frac{1}{3}$

$= \frac{2}{3}$

Helpful hints

✔ When using 'at least', 'at most', 'more than' or 'fewer than', make sure you identify whether or not the given value is included in the favourable outcome.

ANS p557

Exercise 9D Theoretical probability

1, 2, 4–9, 11, 14–16 | 1, 3–7, 9–13, 16, 19, 20, 22(a, b) | 1(c–f), 5, 6, 7(c–f), 11, 13, 14, 17, 18, 20–22

UNDERSTANDING AND FLUENCY

9D.1 **1** A 10-sided die numbered from 1 to 10 is rolled. Identify the favourable outcomes in each of the following situations.

a Getting at least a 5

b Getting at most a 5

c Getting an odd number and a prime number

d Getting an odd number and not a prime number

e Getting an odd number or a prime number or both

f Getting an odd number or a prime number, but not both

9D.2 **2** A letter is selected at random from the word LEMON. State:

a the number of possible outcomes

b the probability of selecting an L

c the probability of selecting a vowel.

3 Calculate the theoretical probability of:

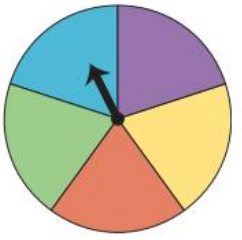
a rolling a standard six-sided die and obtaining a 6

b selecting the letter Z from the word CRAZY

c selecting a blue marble from a bag containing three red, two blue, one yellow and four green marbles

d randomly guessing the correct answer to a multiple-choice question with options A–E.

4 Consider this spinner if spun once.



a State the total number of possible outcomes.

b What is the probability of landing on blue?

c What is the probability of landing on yellow or green?

d What is the probability of landing on anything except red?

5 Calculate the theoretical probability of:

a selecting a consonant from the word CRAZY

b selecting a green or yellow marble from a bag containing four blue, three yellow, two red and six green marbles

c rolling a standard six-sided die and obtaining a number greater than 4

d selecting a C, E or S from the word SUPERCALIFRAGILISTICEXPIALIDOCIOUS

e rolling a standard six-sided die and obtaining any number except 6.

9D.3 **6** Find Pr(A') if Pr(A) is equal to:

a 0.4 **b** $\frac{3}{4}$ **c** 0.1 **d** $\frac{1}{2}$ **e** $\frac{1}{8}$ **f** 0.7 **g** $\frac{2}{3}$ **h** 0.25

7 A 10-sided die numbered from 1 to 10 is rolled. State, in words (without writing 'not event') and as a set of values, the complementary event of each of the following.

a Getting at least a 5

b Getting at most a 5

c Getting an odd number and a prime number

d Getting an odd number and not a prime number

e Getting an odd number or a prime number or both

f Getting an odd number or a prime number, but not both

9D.4 **8** A standard six-sided die is rolled. State:

a the sample space

b the probability of rolling at least a 5

c the complementary event to rolling at least a 5

d the probability of the complementary event.

9 A letter is selected at random from the word EXCITEMENT.

a What is the probability of selecting a consonant?

b What is the complementary event to selecting a consonant?

c What is the probability of the complementary event?

10 In the image shown there are 5 blue, 5 red, 4 yellow and 1 pink button. A button is selected at random from the collection. Assume that every button has an equally likely chance of being selected.



a How many buttons are there?

b What is the probability of selecting a blue button?

c State the complementary event to selecting a blue button and find its probability.

d What is the probability of selecting a white button?

e What is the probability of selecting a button with four holes?

f State the complementary event to part **e** and find its probability.

11 A number of households were surveyed about the number of eggs they used in total on Sunday morning. The data collected is summarised in the table below.

Number of eggs	0	1	2	3	4	5	6	7	8
Frequency	5	2	4	3	5	2	6	3	4

Calculate the probability that a household selected at random will have used:

a at least one egg **b** at most four eggs **c** at least four eggs **d** an even number of eggs.

e Calculate the probability of the complementary event of part **a**. State how many eggs were used in these households.

f Are parts **b** and **c** complementary events? Explain why or why not. If they are not complementary, describe the event that is complementary to part **b** and to part **c**.

12 Explain why, when calculating theoretical probability, the outcomes listed in the sample space must be equally likely to occur for the number of elements to be used as the total number of possible outcomes.

13 A standard deck of cards has four suits with 13 cards each: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king.



a How many cards are there in total?

b What is the probability of selecting an ace?

c What is the probability of the complementary event to 'selecting an ace'?

d What is the probability of selecting a picture card (jack, queen or king)?

e Are you more likely to select a queen of hearts or an ace? Explain.

f What is the probability of selecting a picture card that is not a heart?

g What is the probability of the complementary event to part **f**?

h What is the probability of selecting an even number from the deck?

14 Consider the phrase YEAR EIGHT IS BETTER THAN YEAR SEVEN.

a How many letters are there in total? **b** What is the probability of selecting an E?

c What is the probability of the complementary event to part **b**?

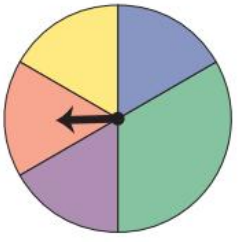
d What is the probability of selecting an A or a B? **e** What is the complementary event to part **d**?

Now consider the letters as vowels and consonants.

f How many vowels are in the phrase? **g** State the probability of selecting a vowel.

h State the complement of this event and its probability.

15 Consider this spinner, where the green section is twice the size of every other section. If it is spun once:



a What is the probability that you will land on red or blue?

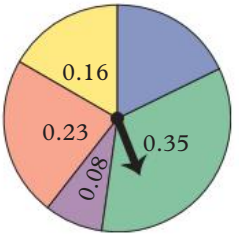
b Explain why the answer to part **a** is not $\frac{2}{5}$, even though there are five different colours.

c What is the probability that you will land on yellow?

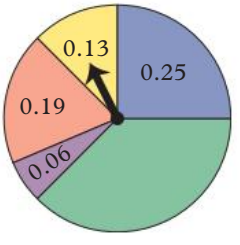
d State the complementary event to 'landing on yellow' and state its probability.

16 Use your knowledge of the sum of probabilities to find the probability of landing on the unlabelled sector in each spinner.

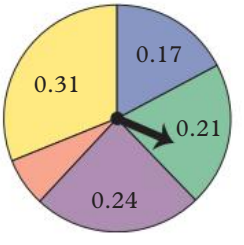
a



b



c



17 A spinner has Pr(blue) $= \frac{1}{2}$, Pr(red) $= \frac{1}{4}$, Pr(green) $= \frac{1}{8}$ and the remaining portion is yellow. State the probability of spinning yellow.

18 At a sporting match there are 200 spectators. Ninety-five people are there to support Team A, eighty-six people are there to support Team B and the remainder don't have a preference.

a Find the probability that a randomly chosen spectator doesn't have a preference who wins.

b If you are there to support Team B, what is the probability that a person who sits next to you will also support Team B?

19 Maynard flipped two coins and got two tails. He said that the complementary event of 'flipping two tails' is 'flipping two heads'. Can you explain where he may have gone wrong?

20 A packet of M&Ms contains 55 individual pieces. You count out 13 blue, 7 brown, 9 green, 11 orange, 7 red and the rest yellow.



a What is the probability of selecting a yellow M&M?

b What is the probability of selecting a blue or green M&M?

c Imagine you select a yellow M&M.

i How many M&Ms are left in the packet now?

ii How many of these are yellow?

iii What is the probability of selecting a yellow M&M from the packet now?

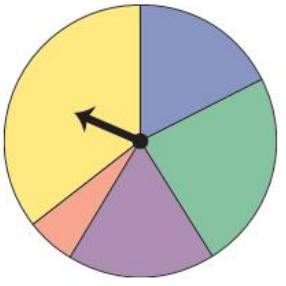
d Imagine that the second M&M you select is blue.

i How many M&Ms are left in the packet now?

ii How many of these are blue?

iii What is the probability of selecting a blue or green M&M now?

21 Use a protractor and your knowledge of angles in a circle to find the probability of spinning the spinner shown on the right and landing in each of the sectors.



22 A bag contains a number of marbles coloured blue, yellow, green and red. You have a $\frac{1}{4}$ chance of selecting a blue marble, $\frac{1}{5}$ chance of selecting green and $\frac{3}{5}$ chance of not selecting yellow.

a What is the probability of selecting a yellow marble?

b What is the probability of selecting a red marble?

c Hence find the smallest number of marbles that could be in the bag.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet The language of probability

Interactive skillsheet Calculating theoretical probability

Interactive skillsheet Complementary events

Investigation Scrabble probabilities

Topic quiz 9D

9E Two-way tables

Learning intentions

By the end of this topic you will be able to ...

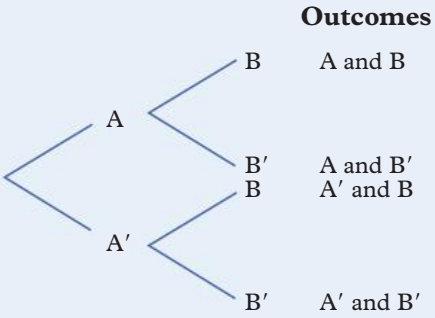
- ✔ determine all possible combinations of two events
- ✔ interpret and create two-way tables
- ✔ calculate relative frequencies using two-way tables.

Inter-year links

Year 9 8E Two-way tables

Year 10 10C Two-way tables and Venn diagrams

Combinations of two events



- When two events A and B are being considered, there are four possible combinations of outcomes.
  - → Both A and B occur
  - → A occurs but not B
  - → B occurs but not A
  - → Neither A nor B occur
- The notation for complementary events A′ and B′ is used in the outcomes where A and B do not occur.
- These four possible outcomes are usually displayed in either a two-way table or a Venn diagram.

Two-way tables

- **Two-way tables** display frequencies for two different categories. For example, this two-way table shows whether students at two different schools play musical instruments.

	School A	School B	Total
Musical instrument	10	4	14
No musical instrument	3	13	16
Total	13	17	30

  - → The values in the total column are calculated by summing the values in each row and column, respectively. The total in the bottom right-hand corner is also equal to the size of the data set.
- Two-way tables can also be used to calculate probabilities and **relative frequencies**.

Example 9E.1 Understanding two-way tables

Consider this two-way table, which shows whether students in two classes prefer cereal or toast for breakfast.

a How many students were surveyed?

b How many students in class A prefer toast for breakfast?

c How many students prefer cereal for breakfast?

	Class A	Class B	Total
Cereal	4	9	13
Toast	7	5	12
Total	11	14	25

THINK

a The total number of people surveyed is displayed in the bottom right-hand corner.

b Identify the cell that represents both class A and toast.

c Identify the cell that shows the total of the cereal row.

WRITE

a 25 people

b 7 students

c 13 students

Example 9E.2 Creating and completing a two-way table

50 students from Year 7 and Year 8 were surveyed on whether they took public or private transport to travel to school. 19 Year 7 students took private transport and 13 of 27 Year 8 students took public transport.

a Create a two-way table and input the data that is given in this question.

b Complete the two-way table.

THINK

a **1** Draw the table with one of the categories (year level) placed in the rows and the other category (type of transport) placed in the columns. Then add the third row and column to write the totals.

2 Insert the information from the question into the relevant cells in the table.

b **1** The numbers in each row and column add to give the value in the total cells, so wherever there are two out of three cells in a row or column, the third can be found using addition or subtraction.

2 The total column contains two cells with information. Identify the value of the missing cell (A).

3 The Year 8 row has two cells with information. Identify the value of the missing cell (B).

4 The Year 7 row now has two cells with information. Identify the value of the missing cell (C).

5 The private transport column now has two cells with information. Identify the value of the missing cell (D).

6 Identify the value of the missing cell in the public transport column (E).

7 Complete the table by writing the values of A, B, C, D and E in the table.

WRITE

a

	Public transport	Private transport	Total
Year 7			
Year 8			
Total			

	Public transport	Private transport	Total
Year 7		19	
Year 8	13		27
Total			50

b

	Public transport	Private transport	Total
Year 7	C	19	A
Year 8	13	B	27
Total	E	D	50

A: $50 - 27 = 23$

B: $27 - 13 = 14$

C: $23 - 19 = 4$

D: $19 + 14 = 33$

E: $4 + 13 = 17$

	Public transport	Private transport	Total
Year 7	4	19	23
Year 8	13	14	27
Total	17	33	50

Example 9E.3 Using two-way tables to calculate probability

Consider this two-way table. What is the probability a person chosen randomly from the group:

a is from Year 7 and prefers toast

b prefers cereal

c is from Year 8 or prefers cereal, but not both?

	Year 7	Year 8	Total
Cereal	4	9	13
Toast	7	5	12
Total	11	14	25

THINK

a Identify the cell which represents both Year 7 and toast. The value in this cell is the number of favourable outcomes. The total number of possible outcomes is the number in the bottom right-hand corner.

b Identify the row which represents cereal. The total at the end of this row is the number of favourable outcomes.

c Identify the cells which represent Year 8 or cereal. Add the values in these cells, not including the cell which represents Year 8 and cereal.

WRITE

a Number of favourable outcomes = 7

$\Pr(\text{Year 7 and prefers toast}) = \frac{7}{25}$

b Number of favourable outcomes = 13

$\Pr(\text{prefers cereal}) = \frac{13}{25}$

c Number of favourable outcomes = 4 + 5
= 9

$\Pr(\text{Year 8 or prefers cereal, but not both}) = \frac{9}{25}$

Helpful hints

- ✔ Calculate all missing values in a two-way table before identifying which values are being asked for!
- ✔ When filling in a two-way table, check your values by making sure the sum of each row and column equals the values in the total row and column.
- ✔ When creating a two-way table each category must cover all the data. Using complementary events can help!

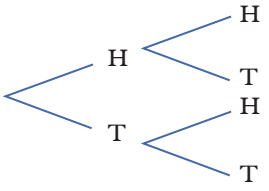
ANS p558

Exercise 9E Two-way tables

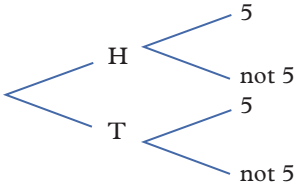
▲ 1–4, 6–9, 11–13 ■ 2, 3, 5–10, 13, 15, 16 ⬟ 2(c, d), 3, 5, 6, 9, 13–18

1 For each of the following diagrams, describe the two events that may have taken place and write a list of the possible outcomes.

a



b



2 Draw diagrams to find the possible outcomes of the following combinations of two events.

a Two events C and D

b Flipping a coin and rolling a 4 (using a 6-sided die)

c Drawing a red card (from a standard deck of 52 cards) and flipping a coin

d Rolling a prime number (using a 6-sided die) and drawing a black card (from a standard deck of 52 cards).

9E.1 **3** Consider this two-way table.

	Year 7	Year 8	Total
Play sport	11	17	28
Don't play sport	12	10	22
Total	23	27	50



a What are the four different categories?

b How many students were surveyed?

c How many Year 8 students play sport?

d How many Year 7 students are there?

e What does the number 12 in the table represent?

f How many students don't play sport?

9E.2 **4** 50 students from Year 7 and Year 8 were surveyed on whether they prefer action or comedy movies. 23 Year 7 students preferred comedy movies and 10 of 18 Year 8 students preferred action movies.

a Create a two-way table to represent this data.

b Use the given information to complete the two-way table.

5 100 people from two Brisbane suburbs were surveyed on whether they prefer cats or dogs. 48 of 59 people from Burbank prefer dogs and 19 people from Pinjarra Hills prefer cats.



a Create a two-way table to represent this data.

b Use the given information to complete the two-way table.

6 Complete these two-way tables.

a

	Week day	Weekend	Total
Volleyball	14		31
Tennis			36
Total		48	

b

	Year 8	Year 9	Total
CD	11		26
Digital		19	
Total	29		

7 Use the objects in this photo to complete the two-way table below. Note that the paper clip is red.



	Lollies	Not lollies	Total
Red			
Not red			
Total			

9E.3 **8** Consider this two-way table which shows whether Year 5 and Year 10 students prefer Facebook or Instagram.

	Year 5	Year 10	Total
Facebook	27	43	70
Instagram	20	10	30
Total	47	53	100

What is the probability a person chosen randomly from the group:

a is a Year 10 student and prefers Facebook

b prefers Instagram

c is a Year 5 student or prefers Facebook, or both?

9 Consider this two-way table which shows whether action or comedy films were first released at the cinema or on a streaming service.

	Cinema	Streaming	Total
Action	5	6	11
Comedy	19	10	29
Total	24	16	40

a How many films were recorded in total?

b How many action films were released on streaming services?

c How many cinema releases were comedies?

d What is the probability of a film chosen randomly from the group being:

i a streaming action release **ii** a comedy cinema release

iii released at the cinema **iv** a comedy or released on streaming, but not both?

10 Consider the two-way table shown in question **3**. What is the probability of a person chosen randomly being:

a a Year 8 student who plays sport **b** a Year 7 student who doesn't play sport

c a Year 7 student **d** a student who plays sport

e a student who is in Year 7 or who plays sport, but not both

f a student who is in Year 8 or who doesn't play sport, or both?

PROBLEM SOLVING AND REASONING

11 A group of 100 people were surveyed about whether they prefer to put Vegemite or jam or their toast. In total, 64 people said they prefer Vegemite. Of the 64 people who prefer Vegemite, 40 said they prefer to have it on white bread. This is exactly the same as the total number of people who prefer non-white bread.

a Construct a two-way table to represent the survey data.

b Find the probability that a person chosen randomly from the group prefers jam on non-white bread.

12 Two classes of students were surveyed on their chocolate preference. In a Year 8 class of 24 students, 16 preferred M&Ms over Smarties. Of the 18 people who preferred Smarties, 10 came from the Year 7 class.

	Year 7	Year 8	Total
M&Ms		16	
Smarties	10		18
Total		24	50

a Copy and complete the two-way table.

b What is the probability that a person chosen randomly from this group:

i prefers Smarties over M&Ms **ii** is in Year 7 **iii** is in Year 8 and prefers M&Ms?

c A person is chosen randomly from the Year 8 class. What is the probability that they prefer M&Ms?

d Explain how part **c** is different from part **b iii**.

13 Two-way tables don't always have exactly two columns and two rows. This table, which shows the preferred sports of people in northern and southern Australia, has three columns.

	Soccer	AFL	Rugby	Total
Northern	10	12	28	50
Southern	17	30	3	50
Total	27	42	31	100

a What is the probability that a person chosen randomly from the group:

i likes AFL best **ii** likes rugby but is not from northern Australia

iii likes soccer or rugby **iv** is from northern Australia and likes soccer?

b What can you say about the difference in sport preference between northern and southern Australia?

14 A group of 200 students were surveyed about the number of siblings they have and if they have pets. Of the 120 students who have pets, 18 have no siblings, 53 have one sibling, 37 have two siblings, and the rest have three or more siblings. In total, 24 students have no siblings, 65 students have one sibling, 48 students have two siblings, and the rest have three or more siblings.

a Create a two-way table to represent this information.

b Find the probability that a student chosen at random from this group:

i has pets and at least one sibling

ii has at most two siblings and has no pets

iii has three or more siblings or has pets but not both.

15 Consider this table which shows the hair and eye colours of a random sample of 100 Australians.

	Blond hair	Brown hair	Total
Blue eyes	16		
Brown eyes		31	
Total		61	100

a Copy and complete the table.

b Find the probability that a person chosen randomly from the group:

i has blond hair **ii** has blue eyes **iii** has blond hair and blue eyes.

c A person is randomly chosen from the group. You know that they have blond hair.

i How many people have blond hair?

ii How many people with blond hair have blue eyes?

iii What is the probability that somebody with blond hair has blue eyes?

iv Explain why this is different from part **b iii**.

This kind of problem is usually described with the word 'given'. This means instead of considering the whole group, you consider only the limited group in the question. Part **c iii** can be asked as 'What is the probability somebody has blue eyes, given they have blond hair?' This tells you to look at the blond group as the total.

d To answer the question 'What is the probability that somebody has blue eyes, given that they have brown hair?', you can follow these steps.

i The overall group that you are looking at is people with brown hair. How many people are in this group?

ii The specific group that you want is people with brown hair and blue eyes. How many people are in this group?

iii Use your answers to parts **i** and **ii** to find the probability that somebody has blue eyes, given that they have brown hair.

16 Consider the table in question **13**. A person is randomly chosen from the group.

a Find the probability that the person likes AFL, given that they are from:

i southern Australia **ii** northern Australia.

b Find the probability that the person is from northern Australia, given that they like:

i rugby **ii** AFL.

17 Use the following information to construct a two-way frequency table for the events A, B and their complements. 'A' and 'not A' should be the columns, whereas 'B' and 'not B' should be the rows.

- The probability of A is $\frac{3}{10}$.
- The probability of A and B is $\frac{1}{6}$.
- The probability of A or B or both is $\frac{2}{3}$.

The numbers in your two-way table must be the smallest possible integers for the given information.

18 As you have seen, not every value needs to be known to complete a two-way table. How many cells with values need to be provided to complete a three row by three column two-way table? What restrictions are there on where these numbers need to go?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Two-way tables

Investigation
Pizza menus

Investigation
Tree diagrams

Topic quiz
9E

9F Venn diagrams

Learning intentions

By the end of this topic you will be able to ...

- ✔ interpret and create Venn diagrams
- ✔ calculate relative frequencies using Venn diagrams.

Inter-year links

Year 9 8F Venn diagrams

Year 10 10C Two-way tables and Venn diagrams

Venn diagrams

- A **Venn diagram** is used to display the relationship between different categories.
- Venn diagrams are named after John Venn, who conceived of them in 1880, so the 'V' in 'Venn' should always be capitalised.
- In a Venn diagram each circle represents a different category, and the rectangle containing the circles represents the entire data set.
- Each of the regions in a Venn diagram represents a different section of the data. If there are two categories, A and B, in a data set then:

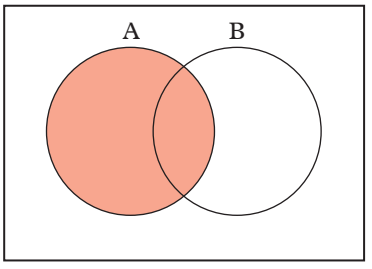
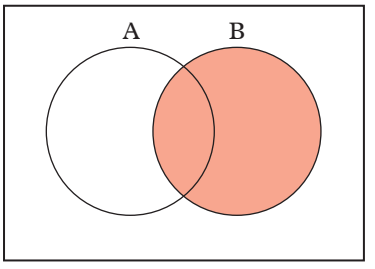
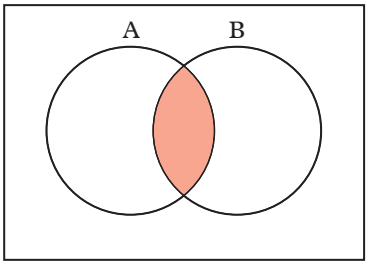
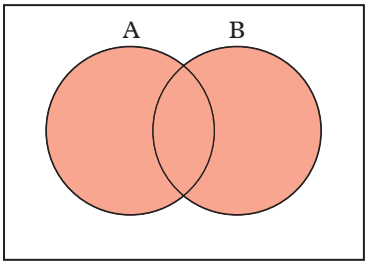
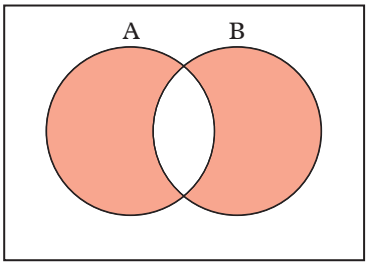
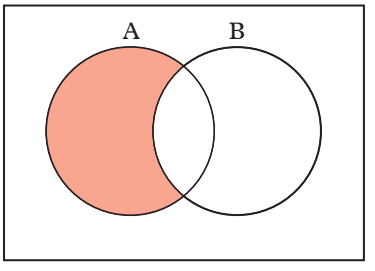
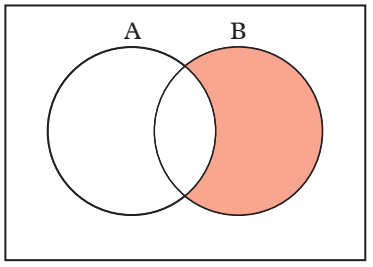
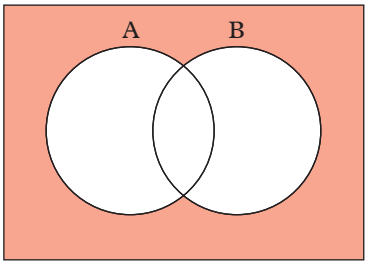
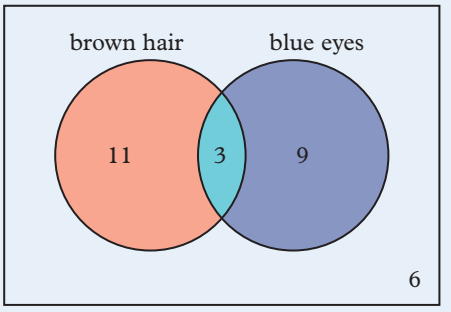
Diagram	What the shaded region represents	Diagram	What the shaded region represents
	A		B
	A and B		A or B
	A or B but not both		A and not B

Diagram	What the shaded region represents	Diagram	What the shaded region represents
	B and not A		Neither A nor B

- Venn diagrams can be labelled by either putting the frequency of items in each section, or by putting the individual items in each section.
  For example, in the diagram on the right representing whether people have brown hair and blue eyes:
  - → 11 people have brown hair without blue eyes
  - → 3 people have brown hair and blue eyes
  - → 9 people have blue eyes without brown hair
  - → 6 people have neither brown hair nor blue eyes.



Example 9F.1 Understanding Venn diagrams

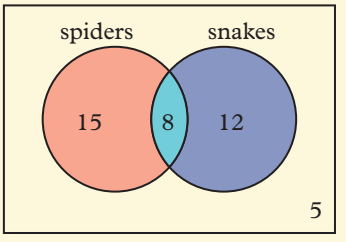
This Venn diagram shows whether people are afraid of spiders and snakes.

a How many people are afraid of both snakes and spiders?

b How many people are afraid of spiders?

c How many people are not afraid of either?

d How many people were surveyed?



THINK

a Identify the required region(s). Those afraid of both snakes and spiders are the number in the middle section, where the two circles overlap.

b Identify the required region(s). Those afraid of spiders are the numbers in the left-hand circle. Add together the numbers within this circle.

c Identify the required region(s). Those who are not afraid of either are the number outside the two circles.

d Add all the numbers within the Venn diagram.

WRITE

a 8 people are afraid of both snakes and spiders.

b 15 + 8 = 23
23 people are afraid of spiders.

c There are five people who are not afraid of either snakes or spiders.

d 15 + 8 + 12 + 5 = 40
40 people were surveyed.

Example 9F.2 Using Venn diagrams to calculate probability

Consider the Venn diagram shown in Example 9F.1. What is the probability that a person chosen at random from the group is afraid of:

a both snakes and spiders
b snakes
c spiders only
d snakes or spiders, but not both?

THINK

a Identify the region which represents both snakes and spiders. The value in this region is the number of favourable outcomes. Sum all the numbers in the Venn diagram to get the total number of outcomes.

b Identify the region which represents snakes. Add the values in this region to get the number of favourable outcomes.

c Identify the region which represents spiders only. The value in this region is the number of favourable outcomes.

d Identify the regions which represent snakes or spiders. Add the values in these regions, not including the regions which represent snakes and spiders.

WRITE

a Number of favourable outcomes = 8

Total number of outcomes = 15 + 8 + 12 + 5
= 40

$\Pr(\text{snakes and spiders}) = \frac{8}{40}$
$= \frac{1}{5}$

b Number of favourable outcomes = 8 + 12
= 20

$\Pr(\text{snakes}) = \frac{20}{40}$
$= \frac{1}{2}$

c Number of favourable outcomes = 15

$\Pr(\text{spiders only}) = \frac{15}{40}$
$= \frac{3}{8}$

d Number of favourable outcomes = 15 + 12
= 27

$\Pr(\text{snakes or spiders, but not both}) = \frac{27}{40}$

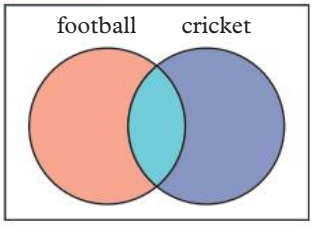
Example 9F.3 Creating a Venn diagram

50 people were surveyed about whether they play football or cricket. 24 people in total play football, 21 people in total play cricket, and 11 people play neither sport.
Draw a Venn diagram to represent this situation.

THINK

1. Draw two circles, one representing football and the other representing cricket. Draw a rectangle around the two diagrams.
2. Enter the value representing the people who play neither sport (11) outside the two circles.
3. Determine the number of people who play either sport by subtracting the number of people who play neither (11) from the total number of people (50).

WRITE



either sport = 50 − 11
= 39

4 Add the number of people who play football (24) to the number of people who play cricket (21). Subtract the number of people who play either sport (39) from this value to find the number of people who play both sports. Enter this value in the region where the circles overlap.

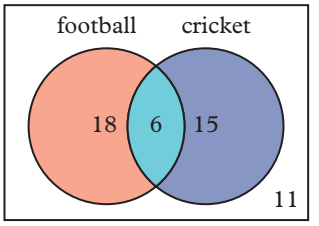
both sports = 24 + 21 − 39
= 6

5 Determine the number of people who play football but not cricket by subtracting the number who play both sports (6) from the number who play football (24). Enter this value in the appropriate region of the diagram.

football but not cricket = 24 − 6
= 18

6 Determine the number of people who play cricket but not football by subtracting the number who play both sports (6) from the number who play cricket (21). Enter this value in the appropriate region of the diagram.

cricket but not football = 21 − 6
= 15



Helpful hints

- ✔ Don't forget to put the frequency which belongs to none of the categories outside the circles!
- ✔ When creating a Venn diagram, a common mistake is to forget that the total frequency for each category is the sum of the sections that circle has been split up into.

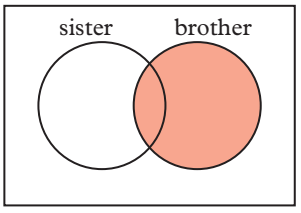
ANS p559

Exercise 9F Venn diagrams

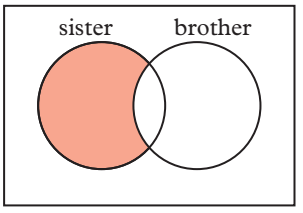
1–3, 5, 6, 8–12, 14(a, b), 15 | 1, 3, 4, 6, 7, 9–11, 13–15, 17 | 1, 5, 6, 10, 11, 14–18

1 These Venn diagrams represent whether people have brothers or sisters. Write what the shaded section in each diagram represents.

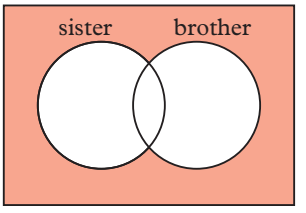
a



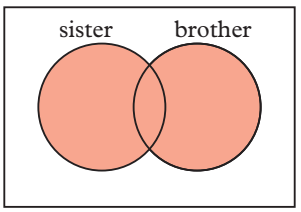
b



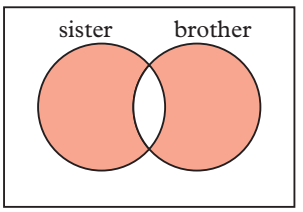
c



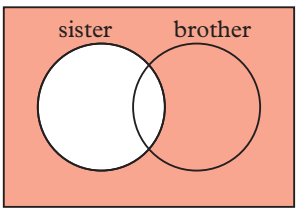
d



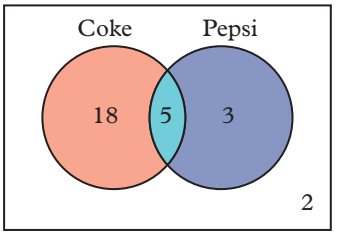
e



f



9F.1 **2** Consider this Venn diagram which shows the cola soft drink preferences of a group of people.





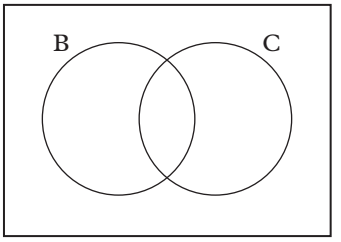
a How many people like only Coke?

b How many people like Coke?

c How many people like both Coke and Pepsi?

d How many people like neither?

3 This Venn diagram represents whether people support Brisbane (AFL) or the North Queensland Cowboys (NRL). Section B represents those who support Brisbane and section C represents those who support the Cowboys.



Shade the correct section of the diagram for each of these groups.

a Brisbane supporters who also support the Cowboys

b Brisbane supporters who don't support the Cowboys

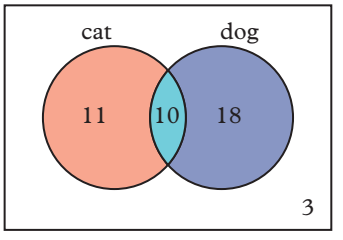
c All people who support the Cowboys

d People who don't support either Brisbane or the Cowboys

e People who support Brisbane or the Cowboys, but not both

f The Cowboys supporters who don't support Brisbane

4 Consider this Venn diagram.



a How many people own a cat and a dog?

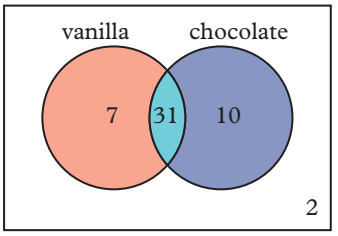
b How many people own a cat?

c How many people do not own either a cat or a dog?

d How many people were surveyed?



5 Consider this Venn diagram which shows whether people like a new chocolate and vanilla frosting.





a How many people:

i like both chocolate and vanilla

ii like chocolate only

iii like vanilla

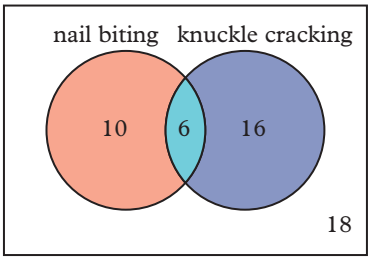
iv do not like either

v like chocolate or vanilla

vi do not like chocolate?

b How many people were surveyed?

9F.2 **6** Consider this Venn diagram, showing the incidence of two bad habits.



What is the probability that a person chosen at random from the group:

a bites their nails and cracks their knuckles

b cracks their knuckles but doesn't bite their nails

c bites their nails

d bites their nails or cracks their knuckles

e doesn't do either?

7 Consider the Venn diagram shown in question **4**. What is the probability that a person chosen at random from the group owns:

a both a cat and a dog

b a dog but not a cat

c a cat

d neither a dog nor a cat

e a cat or a dog, but not both?

8 Consider the Venn diagram shown in question **5**. What is the probability that a person chosen at random from the group likes:

a vanilla but not chocolate

b chocolate

c vanilla and chocolate

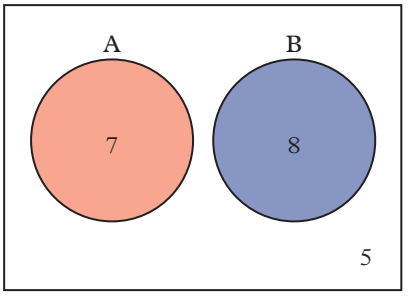
d vanilla or chocolate?

9F.3 **9** In a group of 40 people, 26 people like KFC and 30 people like McDonald's. There are four people who don't like either. Draw a Venn diagram to represent this situation.

10 Construct a Venn diagram using the objects in this photo, using the categories 'yellow' and 'round fruit'. Add your numbers together when you finish to check that you have completed it correctly.



11 Use the Venn diagram shown to calculate the probability of randomly selecting:



a A
b A but not B
c B
d B but not A
e A and B
f A or B or both
g A or B but not both
h neither A nor B
i A and B or neither A nor B.

12 A survey found that $\frac{9}{20}$ people go for daily walks, $\frac{7}{10}$ of people binge watch TV, and $\frac{1}{5}$ of people do both.

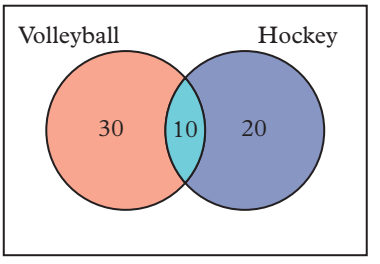
a If 100 people were surveyed, how many would you expect to:
i go for daily walks but not binge watch TV
ii binge watch TV but not go for daily walks
iii either go for daily walks or binge watch TV or both
iv neither go for daily walks nor binge watch TV?

b If 60 people were surveyed, how many:
i go for daily walks or binge watch TV or both
ii neither go for daily walks nor binge watch TV?

c What fraction of people surveyed:
i both go for daily walks and binge watch TV, or neither go for daily walks nor binge watch TV
ii either go for daily walks or binge watch TV but not both?

13 Clyde has created a Venn diagram to represent the following information:

30 people surveyed want to play volleyball, 20 people surveyed want to play hockey, and 10 people surveyed want to play both.



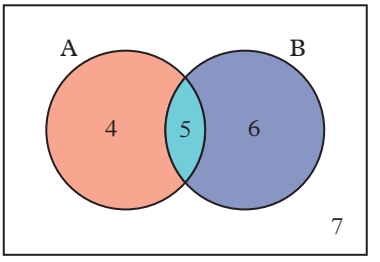
Clyde is unsure if his Venn diagram is correct as the wording of the number of people did not clearly state if the numbers represented the people who only voted for volleyball, hockey or both, or if the people who voted both were included in the people who only voted for volleyball or hockey.

a If 50 people were surveyed, Clyde's Venn diagram cannot be correct. Explain why and create the correct Venn diagram.

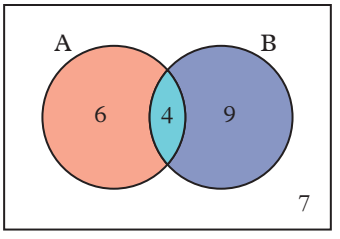
b If 60 people were surveyed, can Clyde's Venn diagram be correct? Explain why or why not.

14 Two-way tables and Venn diagrams display much of the same information. Consider how this two-way table and this Venn diagram are based on the same information.

	A	**Not A**	**Total**
B	5	6	11
Not B	4	7	11
Total	9	13	22



a Draw a two-way table that represents the information provided in this Venn diagram.

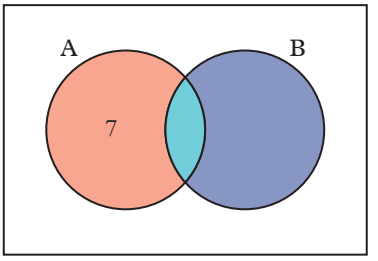


b Draw a Venn diagram that represents the information provided in this two-way table.

	A	Not A	Total
B	10	12	22
Not B	15	8	23
Total	25	20	45

c Complete the two-way table and Venn diagram shown so that they represent the same set of information.

	A	Not A	Total
B	5		
Not B		11	
Total			50



d For the information in the two-way tables and Venn diagrams in parts **a**, **b** and **c**, calculate the relative frequency of choosing the following out of the universal set.

i A and B

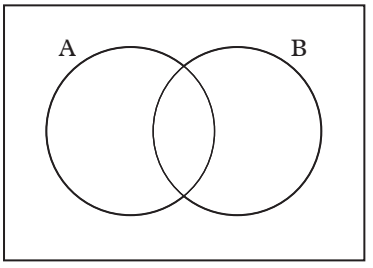
ii A and not B

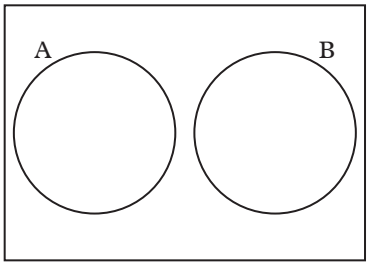
iii A or B or both

iv A or B but not both

v not B

15 Consider the following Venn diagrams.





a Shade the regions of both Venn diagrams that represent each of the following.

i A and B

ii A and not B

iii not A and B

iv not A and not B

b What do you notice about the regions you shaded in part **a**?

c Use your observation in part **b** to write a rule about the sum of the probabilities of the four events listed in part **a** for the first Venn diagram. Hint: What must the probabilities add up to?

d We say that the events in the second Venn diagram are 'mutually exclusive'. How does your rule from part **c** change when A and B are mutually exclusive events?

16 Consider this Venn diagram. If A is the set of people who like white chocolate and B is the set of people who like dark chocolate, find the probability that a person chosen randomly from the group:

a likes white chocolate

b likes white chocolate but not dark chocolate

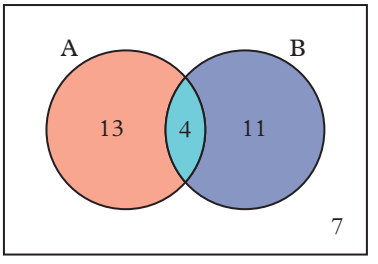
c likes both white and dark chocolate

d likes white chocolate, given that they like dark chocolate

e likes dark chocolate, given that they like white chocolate

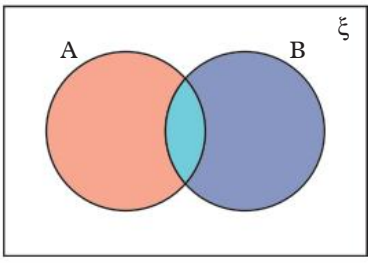
f likes white chocolate or dark chocolate but not both

g does not like dark chocolate.



CHALLENGE

17 Set notation can be used to describe categories in Venn diagrams. This Venn diagram shows sets A and B. Copy the diagram five times.



a On the first diagram, shade the overlapping section of A and B. In set notation, this is the intersection of sets A and B. Label this $A \cap B$.

b On the second diagram, shade inside the circles. In set notation, this is the union of sets A and B. Label this $A \cup B$.

c On the third diagram, shade everything that does not belong to set A; that is, everything outside this circle. This is the complement of set A. Label this A'.

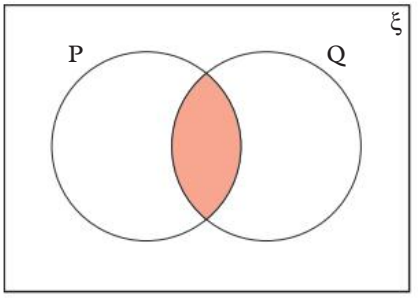
d Everything inside the rectangle is said to belong to the universal set. Label this with ξ. Shade everything inside the rectangle to represent ξ on the fourth diagram.

e Use what you have learnt to shade B' on the fifth diagram.

18 The following diagrams show sets P and Q.

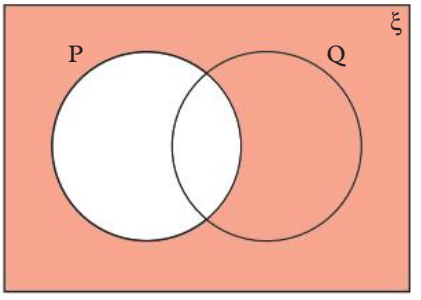
Identify what each of the shaded regions represent.

a



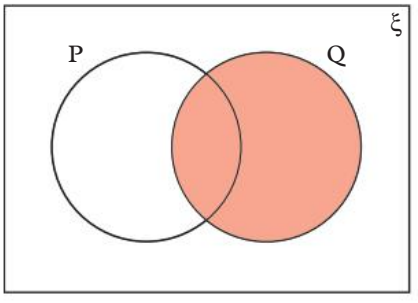
A P
B Q
C $P \cap Q$
D $P \cup Q$
E P'

b



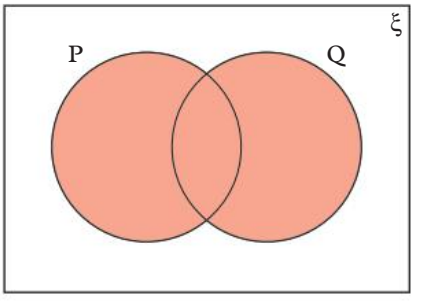
A P
B Q
C $P \cap Q$
D $P \cup Q$
E P'

c



A P
B Q
C $P \cap Q$
D $P \cup Q$
E P'

d



A P
B Q
C $P \cap Q$
D $P \cup Q$
E P'

Check your Student obook pro for these digital resources and more:

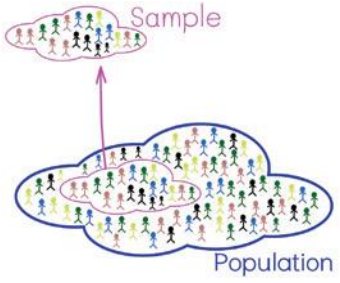
Interactive skillsheet
Venn diagrams

Investigation
Venn diagrams and social networks

Topic quiz
9F

Chapter summary

Collecting data



- Fair samples are likely to accurately represent the characteristics of the population.

Using samples to make predictions

- Random samples should be:
  - → fair and unbiased
  - → large enough to represent the population.
- The size of a sample should depend on the size of the population.

Sampling methods

- Simple random sampling techniques: every potential piece of data has an equal chance of being selected
  - → Systematic sampling: selecting data at fixed intervals
  - → Stratified sampling: dividing the population into categories and taking a random sample from each category
- Non-random sampling techniques: different pieces of data have an unequal chance of being selected
  - → Convenience sampling: taking a sample that is easily accessible
  - → Judgemental sampling: a sample is selected by a researcher
  - → Quota sampling: dividing the population into categories and taking a specified number from each category
  - → Cluster sampling: dividing the population into similar but diverse groups and taking a random sample of the groups

Theoretical probability

- Assuming all outcomes are equally likely:

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes in event}}{\text{total number of possible outcomes}}$$

- The complement of an event is the probability that the event does not occur.
- The probabilities of complementary events add to 1.

Summary statistics

- mean = $\frac{\text{sum of all data points}}{\text{total number of data points}}$
- median = middle value of an ordered data set
- mode = most common value in a data set
- range = maximum value – minimum value
- outlier = extreme data point much higher or lower than the other values

Describing data

- A small sample size with the mean being significantly different to the median may indicate the presence of an outlier.
- 'average' usually refers to the mean.
- 'most common', 'most popular' and 'most often' refer to the mode.
- 'middle' and 'centre' usually refer to the median.
- 'spread' or 'span' usually refer to the range.

Two-way tables

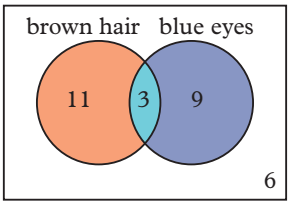
	School A	School B	Total
Musical instrument	10	4	14
No musical instrument	3	13	16
Total	13	17	30

- 10 students at school A play a musical instrument.
- 17 students from school B are in the data set.

Describing events

- 'at least' – equal to or above the given value
- 'at most' – equal to or below the given value
- 'more than' – above the given value
- 'fewer than' – below the given value
- 'and' – both outcomes must be met
- 'or' – either of the given outcomes

Venn diagrams



- 3 students have brown hair and blue eyes.
- 9 students have blue eyes but not brown hair.
- 11 students have brown hair but not blue eyes
- 29 students were surveyed

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Multiple choice

9A **1** Which of these is stratified sampling?

A Selecting every third person who walks past
B Drawing names out of a hat
C Surveying 15 people in a class of 25
D Surveying 3 boys and 2 girls out of a group of 12 boys and 8 girls
E Selecting the first 15 people who walk past

9B **2** For the data set 1, 5, 4, 3, 2, 5, 5, 3, 4, 8, the mean is:

A 3 **B** 4 **C** 5 **D** 7 **E** 8

9B **3** For the data set in question **2**, the median is:

A 3 **B** 4 **C** 5 **D** 7 **E** 8

9C **4** Belinda wants to investigate her local town's opinion on a current issue. Which of these is the most appropriate method of taking a sample to use?

A Asking everybody in the entire town (population 15 000)
B Surveying every fourth person at the local mall until 200 are surveyed
C Asking everybody she knows
D Surveying every 10th person in the local phone book until 100 are surveyed
E Asking every second house in the street where she lives

9D **5** What is the complementary probability to rolling a 1 or 2 on a standard die?

A $\frac{1}{6}$ **B** $\frac{2}{6}$ **C** $\frac{3}{6}$ **D** $\frac{4}{6}$ **E** $\frac{5}{6}$

9E **6** Consider this two-way table.

	Brown eyes	Not brown eyes	Total
Blond	3	12	15
Not blond	19	16	35
Total	22	28	50

What is the probability of a person chosen randomly from the group having brown eyes?

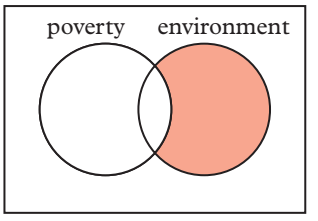
A $\frac{3}{22}$ **B** $\frac{3}{50}$ **C** $\frac{19}{50}$ **D** $\frac{14}{25}$ **E** $\frac{11}{25}$

9E **7** This two-way table shows the ice cream preferences of Year 7 and Year 8 students out of chocolate or vanilla. How many Year 8 students prefer chocolate? Complete the two-way table to find the answer.

	Chocolate	Vanilla	Total
Year 7		65	170
Year 8			
Total		110	350

A 105 **B** 135 **C** 45 **D** 145 **E** 115

9F **8** A group of students were asked what they believed the two biggest issues facing the world in the 20th century are. What does the shaded section represent?

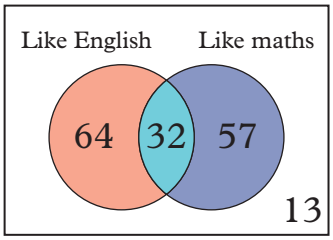


A the environment
B poverty and the environment
C poverty
D the environment only
E not the environment

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9F **9** The Year 8 students at a school were asked whether they like English and maths. The results are displayed in this Venn diagram. How many students don't like English?

A 64
B 96
C 13
D 57
E 70



Short answer

9A **1** For the following surveys, identify:

i if it is a census or sample

ii if the sampling technique is fair or biased, providing reasons. If it is biased, provide a fair alternative.

a Liana asks her class members who their favourite celebrity is to find the most popular celebrity in the state.

b Joe asks every fifth person on the school enrolment list how many people are in their family to find the average number of people in a family at his school.

c Gerard asks everybody in his class if they like chocolate to find out the most popular snack in his class.

9A **2** Write a fair question that could be used on a survey on each of the following topics.

a The most popular AFL team in Queensland

b The most common way that students travel to your school

c The preferred flavour in a Neapolitan tub of ice cream (strawberry, vanilla, chocolate) of Year 8 students at your school

9B **3** Calculate summary statistics (mean, median, mode and range) for each of these data sets.

a 1, 4, 3, 2, 5, 2, 4, 1, 1, 6, 4

b 11, 18, 21, 17, 13, 11, 16, 2, 9, 11

c 2, 2, 2, 2, 29, 3

9B **4** Consider this frequency table.

Calculate summary statistics for this data.

Score	Frequency
12	3
13	6
14	8
15	2
16	0
17	0
18	1

9B **5** Recalculate the summary statistics in the frequency table from question **4** without including the 18. Which summary statistics change, and which stay the same?

9C **6** Explain why the following sampling methods should *not* be used to find the most popular teacher in school.

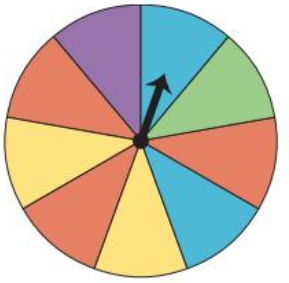
a Asking everybody in Year 8

b Asking 200 people randomly from the phone book

c Asking one person from each year level

9C **7** Explain whether you could reasonably predict the population parameters from this table.

Population	Tasmania
Sampling technique	Stratified sampling
Sample	50 people from Tasmania
Mean	7
Median	10
Mode	11
Range	26



9D **8** Consider this spinner.

a What is the theoretical probability of spinning blue?

b What is the theoretical probability of spinning green or red?

c Find the theoretical probability of the complementary event to part **b**.

9D **9** Consider the following heights of 10 students in a class:

159 cm, 181 cm, 159 cm, 161 cm, 168 cm, 147 cm, 166 cm, 167 cm, 158 cm, 155 cm.

Find the probability that a randomly selected student from the class:



a is at least 166 cm tall

b is at most 166 cm tall

c is less than the mean height.

9E **10** Consider this two-way table.

	Like Vegemite	**Don't like Vegemite**	**Total**
Like Milo	97	56	153
Don't like Milo	18	29	47
Total	115	85	200

What is the probability that a person chosen randomly from the group:

a likes Vegemite

b likes Vegemite but not Milo

c likes Milo but not Vegemite

d doesn't like Milo

e likes Milo or Vegemite, but not both?

9F **11** Consider this Venn diagram.

a How many people were surveyed?

b How many people like Italian food?

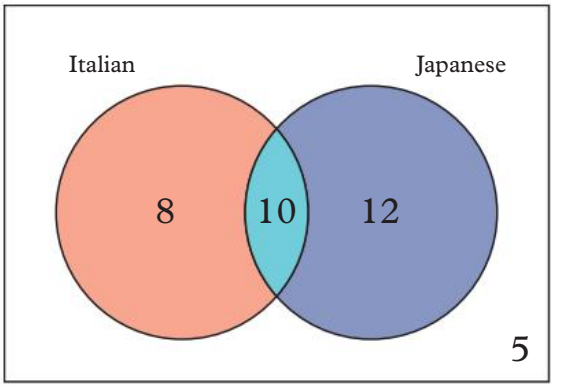
c What is the probability that a person chosen randomly from the group:

i likes both food types

ii likes Italian food only

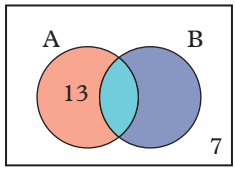
iii does not like Japanese food

iv likes Japanese or Italian food?



9F **12** The following Venn diagram and two-way table represent the same information. Complete both diagrams.

	B	B'	Total
A	11		
A'			
Total			41



Analysis

1 Drew wanted to find the average age of people who went to the local beach. One Saturday he went to the beach and asked the age of every fifth person who walked past. The results are shown.

14	17	18	23	21	19	23	45	23	46
61	15	16	17	39	32	9	19	27	26
31	5	21	19	18	16	15	26	22	59
72	13	49	43	16	15	19	23	22	21
63	56	8	24	35	16	19	25		

a Describe the sampling method. Is Drew taking a census or a sample?

b Calculate summary statistics (mean, median, mode and range) for the data. Round to one decimal place where necessary.

c What limitations does this sample have if you want to use it to predict the population parameters?

d Make a prediction about what you would expect to see in the population.

2 A carnival game involves selecting a counter from a bag that contains numbered counters from 1 to 10. The number of each counter is shown in the table below.

Number	1	2	3	4	5	6	7	8	9	10
Frequency	4	6	8	10	12	12	10	8	6	4

You must pay 5 tickets to play and you get the number of tickets on the counter as a prize.

a Calculate the probability of playing and ending up with at least one more ticket than you started with.

If you choose, you can up the ante and possibly win more tickets by paying an extra 10 tickets. If you get an even number you get 8 tickets, if you get a prime number you get 9 tickets, you get 30 tickets if you get an even prime number, but you get nothing if the number is not even and not prime.

b Construct a two-way table to show the frequency of each outcome.

c What is the probability of not winning any tickets?

d What is the probability of not losing tickets?

If you choose, you can pay another 10 tickets to remove all of the counters that are not a prime or all of the counters that are not even.

e Removing which type of counters increases the chance of winning tickets the most?

f Is it worth paying two extra lots of 10 tickets to remove both the not-prime and not-even-numbered counters? Why or why not?



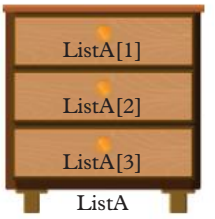
Algorithmic Thinking

Algorithmic thinking: Statistics and probability

An algorithm is a set of precise instructions that when followed will achieve a goal or solve a problem. Once the actions in an algorithm have been identified, it may be possible to encode the algorithm in a coding language so it can be automated by a computer or machine.

Information for algorithms is held in variables, which can be thought of as containers that hold values for computations, as in algebra. An algorithm can contain many different variables for computations. For example, a variable called Sum can be thought of a box that holds a number, such as 27.

Lists are variables that can be thought of as multi-drawer containers with numbered drawers. For example, a variable called ListA can be thought of a chest of drawers with different values in each drawer (labelled 1, 2 and 3), where the contents of each drawer are ListA[1], ListA[2] and ListA[3] respectively.

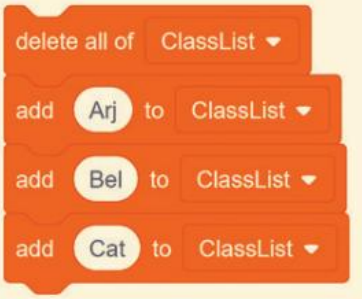


Pseudocode is a language used to describe the steps in an algorithm, as shown in Examples 1 and 2. Algorithms written in pseudocode which use iteration and variables can be coded in a coding language such as Scratch, which is a block-based visual coding language.

Example 1

This example shows how you can create a list variable in pseudocode, with the corresponding code in Scratch.

Create list of ClassList = {Arj,Bel,Cat}

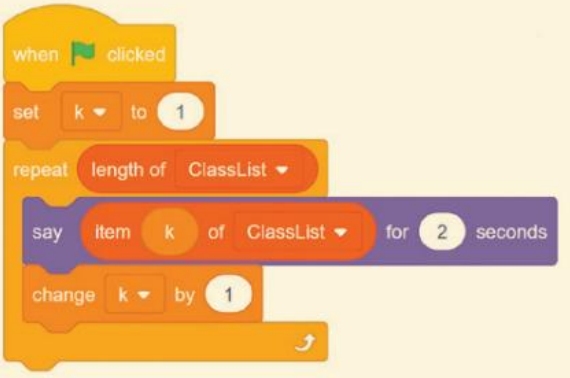


The term iteration refers to repetitive actions, as shown in Example 2. Actions that need to be repeated are grouped together and indented with directives such as Repeat, which defines the iteration. Iteration is very useful for working on lists of numbers collected as statistics.

Example 2

This example shows how you can use the Repeat directive in pseudocode, with the corresponding code in Scratch.

Create list of ClassList = {Arj,Bel,Cat}
Create variable k
Set k to 1
Repeat length of ClassList times:
    Report the k^{th} item of ClassList
    Add 1 to k



Task 1

1 Identify the variables used in each of the algorithms defined in Examples 1 and 2.
2 What is the output reported in the algorithm defined in Example 2?

Example 3



The collection and processing of data for statistical analysis can be defined in an algorithm with the following basic steps.

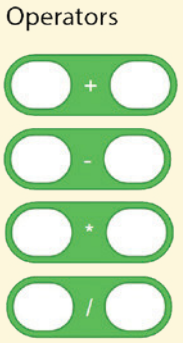
1 **Collect Data** Survey or observation
2 **Identify Data** Numeric or categorical data
3 **Sort Numeric Data** Stem-and-leaf plot or frequency table
4 **Analyse Data** Calculate measures of central tendency (mean, median, mode)

The butterfly Vanessa Atalanta, known more commonly as the red admiral butterfly, is a medium-sized butterfly with black wings, orange stripes and white spots. It has a wingspan of around 50 mm.

A butterfly researcher observes and records a sample of 15 random observations of Vanessa Atalanta butterfly wingspans from a small regional area. The researcher records her observations of the wingspans in mm in a list variable called Sample1.

Sample1 = {50.1, 49.7, 48.7, 51.1, 50.1, 51.3, 49.1, 49.8, 50.2, 50.7, 50.4, 47.9, 51.3, 48.2, 51.1}

When numeric data such as the butterfly wingspan measurements is stored in variables, operations can be applied to these variables. The Scratch coding language uses the operators shown on the right for numeric variables.

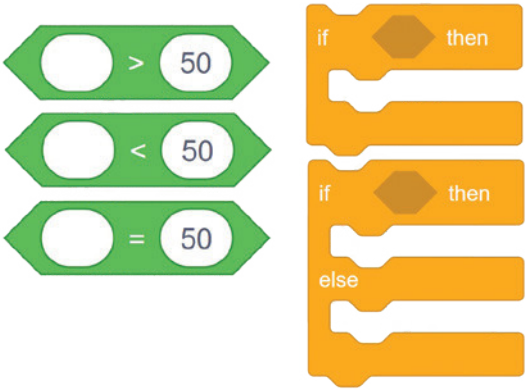


Task 2

1 Create a list variable in Scratch or another coding language to hold the Sample1 data.
2 Using ideas from Examples 1 and 2, create an algorithm to find the mean of the wingspans recorded by the researcher.
3 **CHALLENGE:** Code your algorithm for find the mean of the wingspan observations into Scratch or another coding language.

The term conditional refers to conditional actions that are only done if certain conditions are true. Conditional actions are grouped together and indented in pseudocode with directives such as If Then or If Then Else.

The Scratch coding language uses the operators on the right for comparing variables with an if-then or if-then-else control condition.



Task 3

1 **CHALLENGE:** Using ideas from Example 2 and If Then or If Then Else statements, write an algorithm in pseudocode to find the minimum and maximum wingspans of the sample observations.
2 **CHALLENGE:** Code your algorithm for finding the minimum and maximum for the butterfly wingspan observations into Scratch or another coding language.

Semester 2 review

Short answer

1 Check if the bracketed values are the solution to the following equations.

a $3x - 5 = 19$ $(x = 8)$

b $3 - 8x = -1$ $\left(y = \frac{1}{2}\right)$

c $-7x + 9 = 37$ $(x = -3)$

d $3w + 8 = 5w - 2$ $(w = 5)$

2 Solve the following equations for x.

a $5x - 3 = 47$

b $8(x + 3) = 48$

c $8x + 8 = 7x - 3$

d $4x + 7 = 9x - 2$

3 Plot the following points on the Cartesian plane and state whether the plot is linear or non-linear.

a $(-2,5), (-1,2), (0,-1), (1,-4)$

b $(0,1), (1,2), (2,4), (3,8)$

c $(-1,-1), (0,0), (1,1), (2,8)$

d $\left(\frac{1}{6},-\frac{2}{3}\right), \left(\frac{1}{4},-\frac{1}{2}\right), \left(\frac{1}{3},-\frac{1}{3}\right), \left(\frac{1}{2},0\right)$

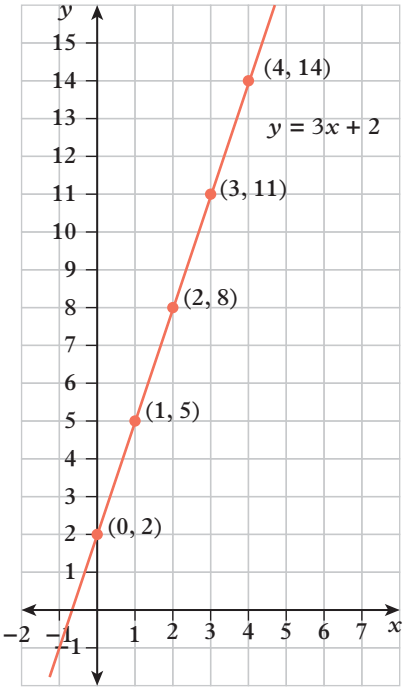
4 Solve the equations using the graphs below.

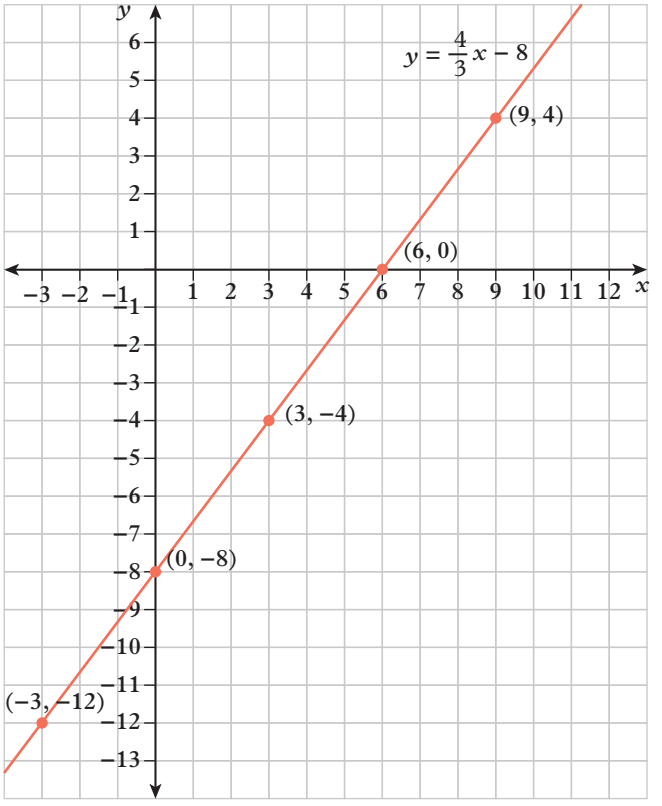
a i $3x + 2 = 11$

ii $3x + 2 = 2$

b i $\frac{4}{3}x - 8 = -12$

ii $\frac{4}{3}x - 8 = 0$





5 a i Create a table of values for $y = 2x + 3$ for $x = 0$ to 3.

ii Plot the points on the Cartesian plane and state whether $y = 2x + 3$ is linear or non-linear.

b i Create a table of values for $y = 2^x + 3$ for $x = 0$ to 3.

ii Plot the points on the Cartesian plane and state whether $y = 2^x + 3$ is linear or non-linear.

6 Determine the equation of each of the following linear relationships.

a

x	0	1	2	3	4
y	7	5	3	1	-1

b

x	–2	–1	0	1	2
y	–5	–1	3	7	11

c

x	0	3	6	9	12
y	–2	–5	–8	–11	–14

d

x	–2	0	2	4	6
y	5	6	7	8	9

7 Solve each of these inequalities and show the solutions on the Cartesian plane

a $3x + 2 < -4$ **b** $2 + 4x \geq 5$

8 State if the following angles are complementary, supplementary or equal.

a vertically opposite angles

b the non-right angles in a right-angled triangle

c corresponding angles on parallel lines cut by a transversal

d co-interior angles on parallel lines cut by a transversal

e the exterior angle and the interior angle at the same vertex

f the included angles between unequal sides in a kite

g adjacent angles in parallelograms

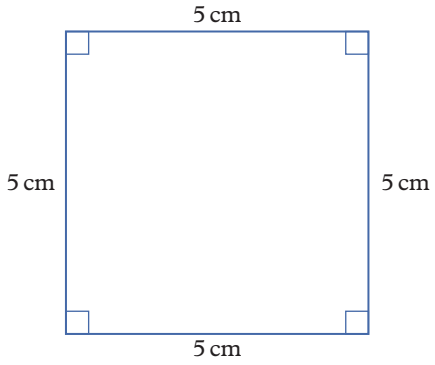
h corresponding angles in congruent shapes

9 For each of the following quadrilaterals:

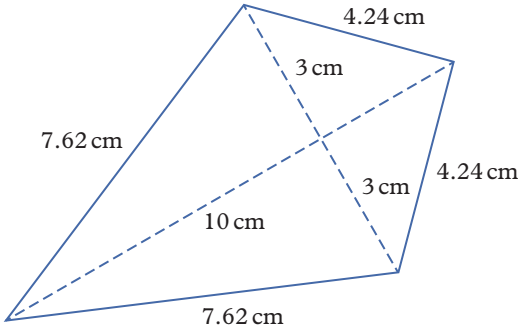
i determine the type of quadrilateral, giving the most specific name

ii determine the area.

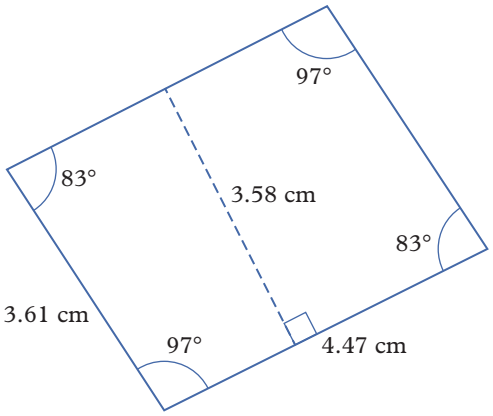
a



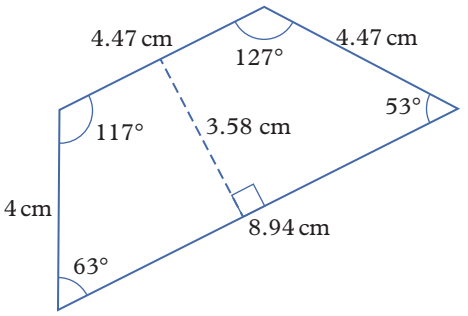
b



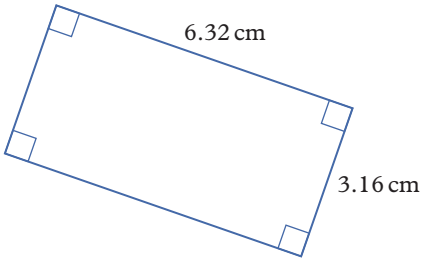
c



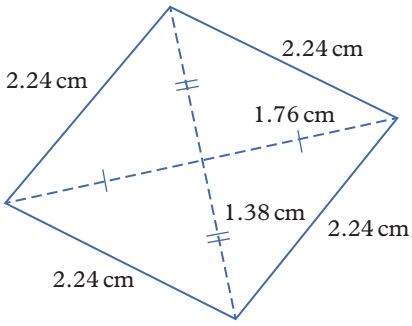
d



e

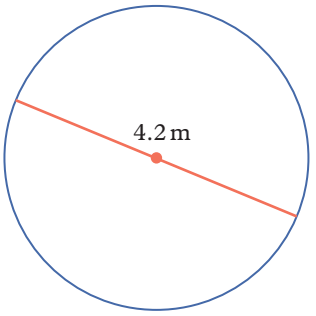


f

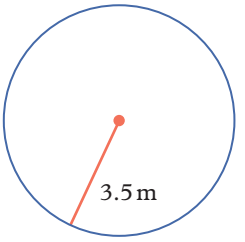


10 Determine the area and perimeter of the following shapes, correct to two decimal places.

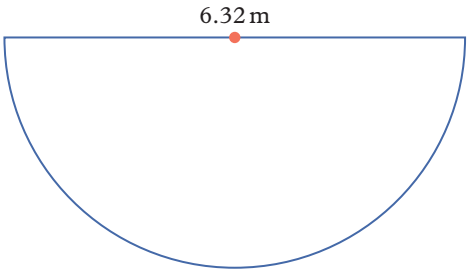
a



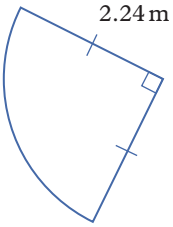
b



c



d



11 For each of the following, write and then solve an equation using the given information.

a 5 more than 3 times a number is equal to 29.

b The perimeter of a rectangle with a length of 3.4 m is 10.1 m. Find the width of the rectangle.

c The circumference of a circle is 30 cm. Find the length of the diameter of the circle, correct to two decimal places.

d The area of a triangle with a side length of 1.5 cm is 2.25 cm^2. Find the perpendicular height to the 1.5 cm side length.

e The area of a parallelogram with a base length of 7.2 cm is 24.48 cm^2. Find the perpendicular height to the base.

f The area of a circle is 30 cm^2. Find the length of the radius of the circle, correct to two decimal places.

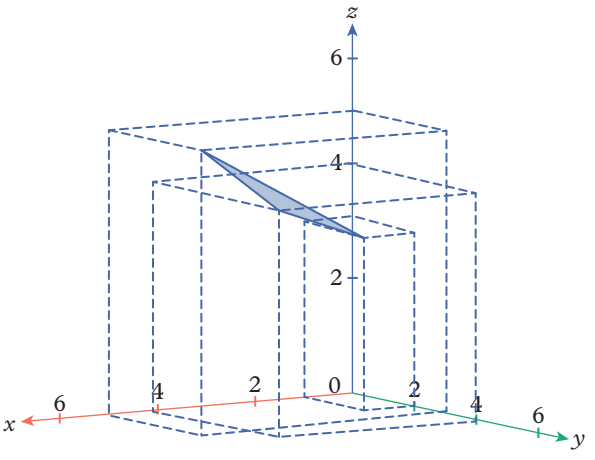
g Two of the interior angles of a triangle are 34° and 57°. Find the size of the third interior angle.

h The mean of 5, 8, 13, 2 and one other number is 6.2. Find the last number.

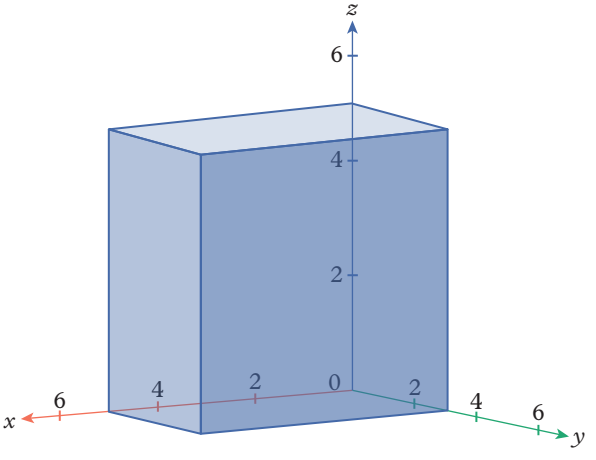
i The probability of three out of four outcomes from a sample space are 0.3, 0.12, 0.44. Find the probability of the fourth outcome.

12 List the coordinates of the vertices of the following shapes and objects.

a

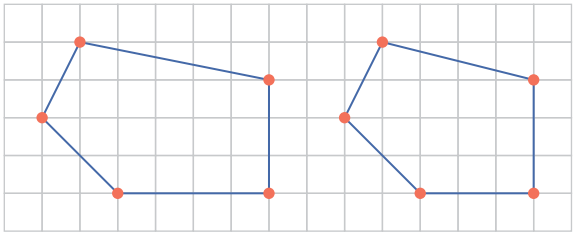


b

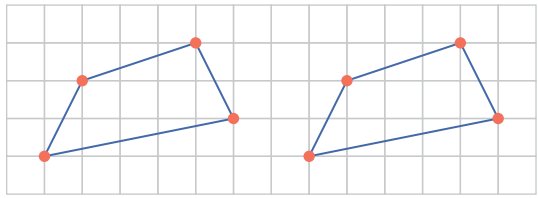


13 State if the two shapes are congruent or not.

a

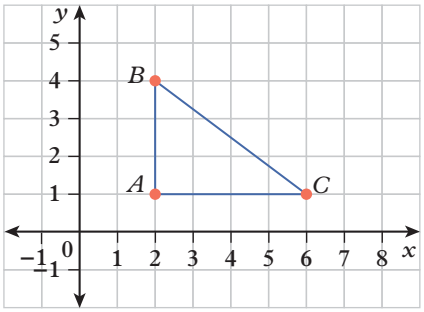


b

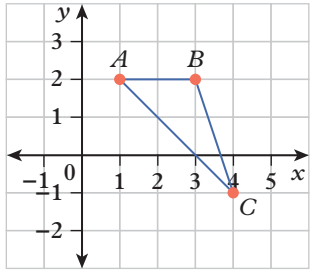


14 Apply the transformation(s) to the shape shown.

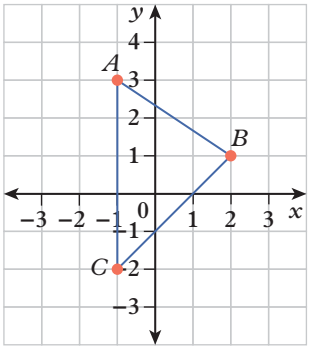
a Rotate 90° clockwise about the origin.



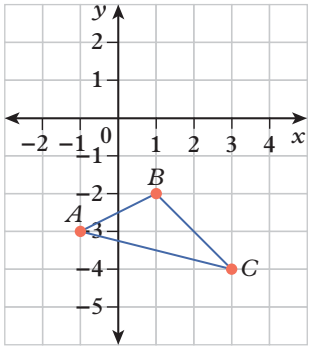
b Reflect across the x-axis.



c Translate 3 units left and 4 units up.

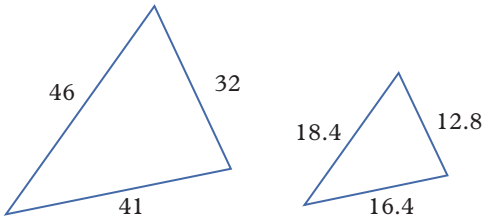


d Rotate 90° anti-clockwise about the origin, then reflect in the y-axis.

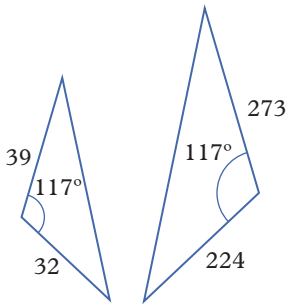


15 Determine if the following pairs of triangles are similar. If they are similar, state the condition that explains why.

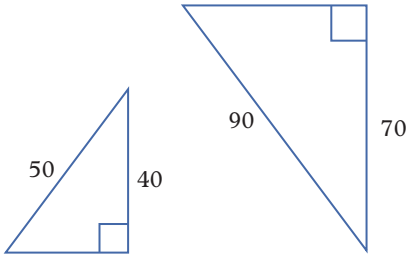
a



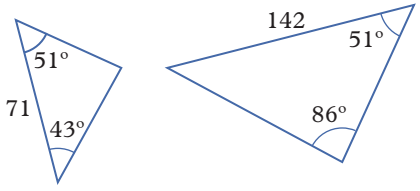
b



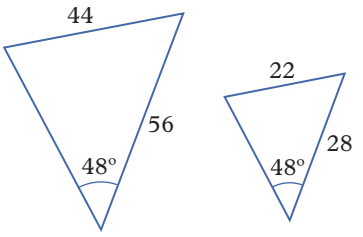
c



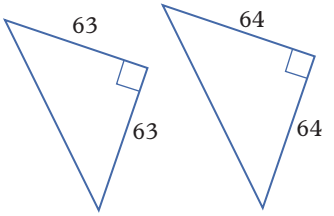
d



e

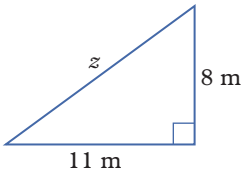


f

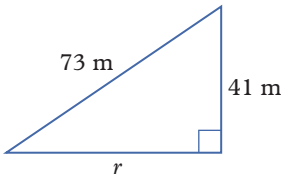


16 Determine the length of the unknown side lengths, correct to two decimal places.

a



b



17 Determine whether these side lengths can form right-angled triangles.

a 7.7 cm, 41.7 cm, 42.7 cm

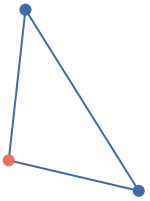
b 29 mm, 128 mm, 131 mm

c 20 cm, 48 cm, 52 cm

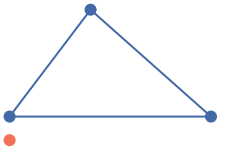
d 4 m, 4.2 m, 5.8 m

18 Dilate each figure using the orange centre of dilation and the scale factor provided.

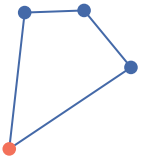
a Scale factor: $\frac{1}{2}$



b Scale factor: 2



c Scale factor: 3



19 Find the probability of the following.

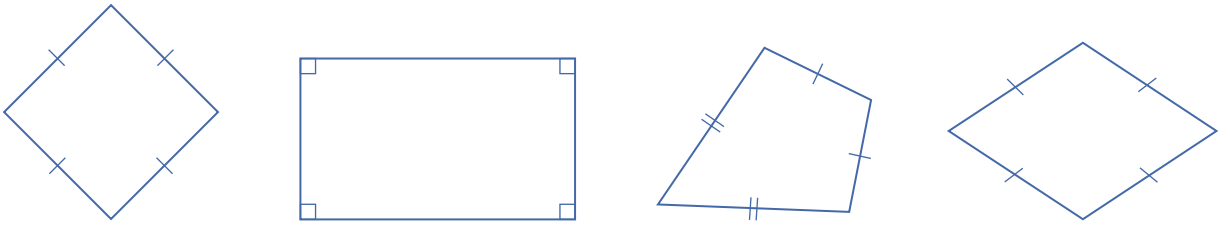
a selecting a number with a recurring decimal from the following list

$\frac{2}{3}, \frac{5}{7}, \frac{3}{10}, \frac{3}{6}, \frac{7}{25}, \frac{5}{12}, \frac{11}{20}, \frac{18}{120}, \pi$

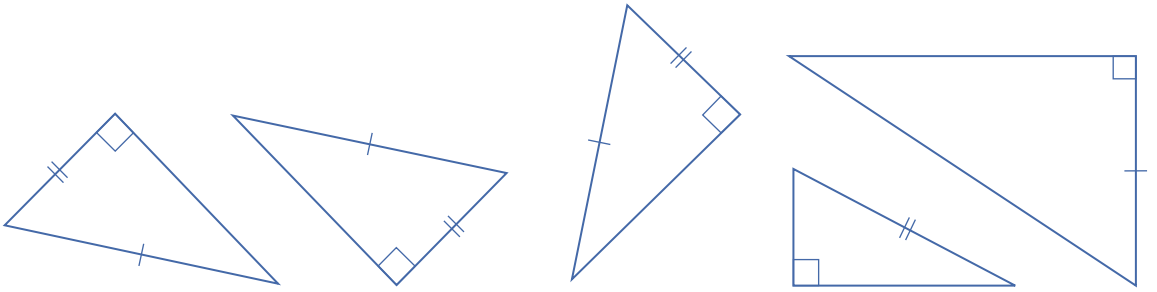
b selecting an expression from the following list

$2x + 4y, -\frac{4}{3}, 8x + 1 = 15, y = 4 - 3x, 3 \times 4 + 5 = 17, 0.2z$

c selecting a quadrilateral from the shapes below that has perpendicular diagonals

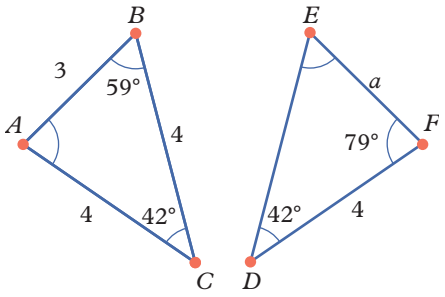


d selecting a triangle that is congruent to at least one other triangle from the following shapes

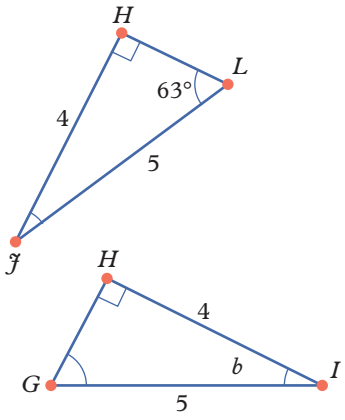


20 Give the condition why the pair of triangles are congruent, then find the values of the pronumerals.

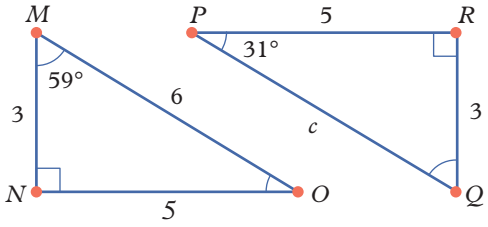
a



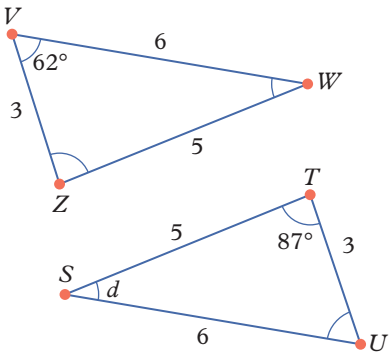
b



c

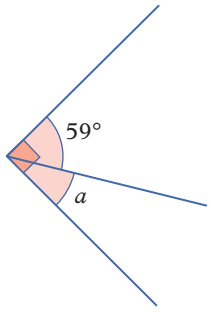


d

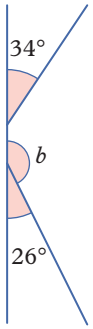


21 Find the value of the pronumerals.

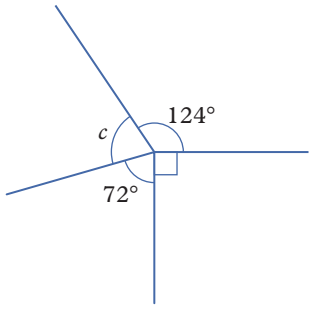
a



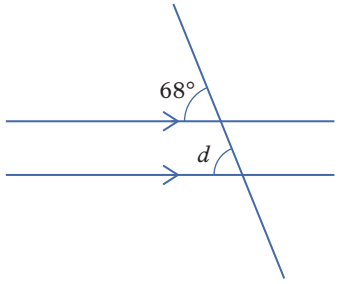
b



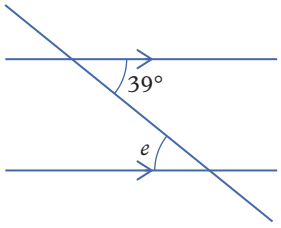
c



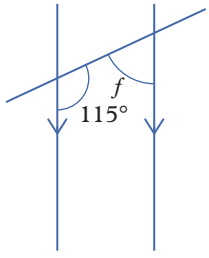
d



e



f

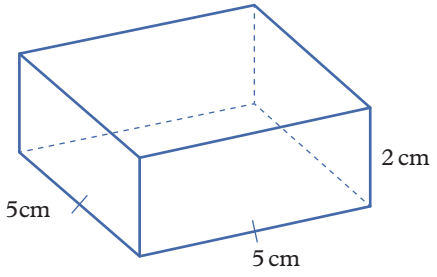


22 Convert the following into the unit in brackets.

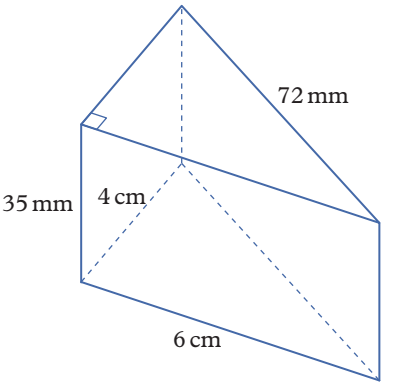
a 24 mm (m)
b 5 m^3 (cm^3)
c 8001 km^2 (m^2)
d 4240 mm^3 (cm^3)
e 0.2 cm^2 (m^2)
f 30 km (cm)
g 4320 seconds (hours)
h 2.5 hours (minutes)
i 1823 (12-hour time)
j 12:30 am (24-hour time)
k 1240 mL (L)
l 35L (mL)

23 Calculate the surface area and volume of the following.

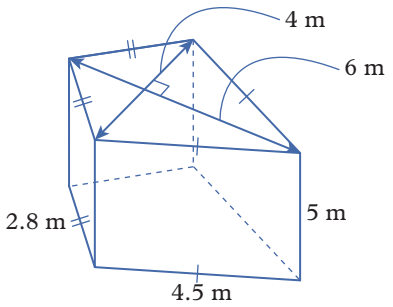
a



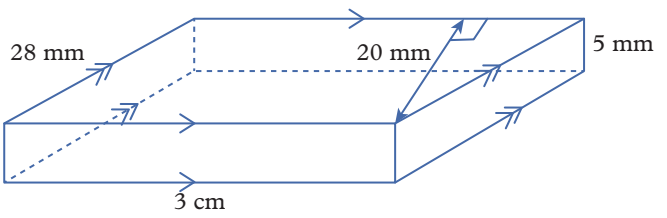
b



c



d

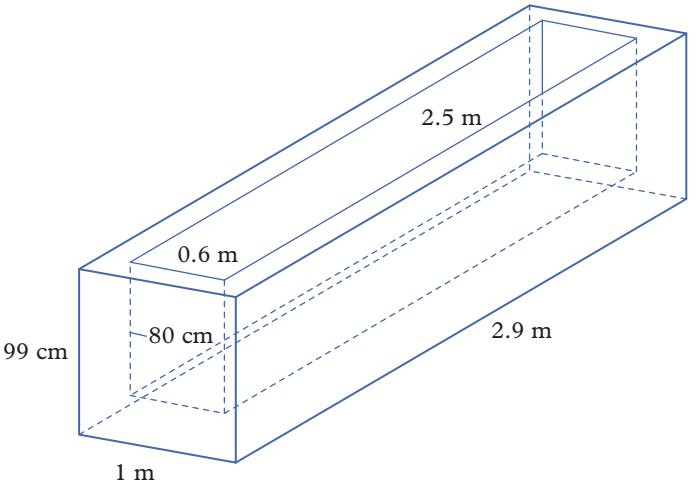


24 A rectangular prism has side lengths 1 m, 2.9 m and 99 cm.

a Calculate the volume of the rectangular prism.

b A rectangular prism with side lengths 0.6 m, 2.5 m and 80 cm is carved out of the original rectangular prism. Calculate the capacity of the carved out prism.

c Calculate the new volume of the object.



25 Convert the following times into the time zone in brackets.

a 2:03 pm AEST (ACST)

b 5:41 am AWST (AEST)

c 9:00 pm UTC+10 (UTC+1)

d 8:31 am UTC−2 (UTC+5)

26 Determine the length of time elapsed when these times are in the same time zone.

a 0847 to 1423

b 9:45 pm to 1:10 am

27 Determine the length of time elapsed for these pairs of times.

a 6:38 pm AEST to 6:12 pm AWST

b 0555 UTC+1 to 2320 UTC+10

28 Calculate the mean of the following data set.

Score	Frequency
3	2
5	4
7	6
9	8

29 a Complete the following two-way table of a survey of students and their preferred hobbies.

	Reading books	Playing board games	Playing video games	Total
Week day	25			65
Weekend			71	135
Total		30	105	

b If a student from the survey is selected at random, find the probability that they:

i prefer playing board games on the weekend

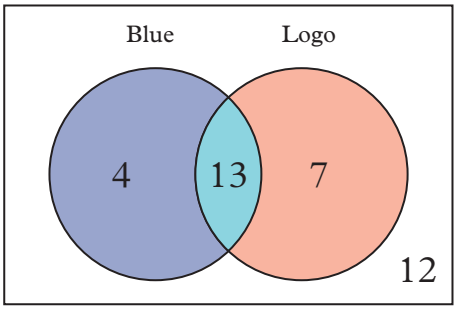
ii prefer playing board games or video games on a week day

iii do not prefer reading books

iv prefer doing their hobby on the weekend, or prefer playing video games, but not both.



30 The Venn diagram shows the number of counters in a bag that are blue or are marked with a logo. Calculate the relative frequency of the following.



a blue counters that are marked with a logo

b blue counters that are not marked with a logo

c blue counters or counters marked with a logo, or both

d blue counters or counters marked with a logo, but not both

e neither blue counters nor counters marked with a logo

31 For each of the following, determine:

i if it is a sample or a census

ii if it is fair or biased (if it is a sample)

iii what sampling method is being used (if it is a sample)

iv whether the sample could be used to predict population characteristics (if it is a sample).

a 1000 randomly selected students in Victoria were surveyed about their opinions of mathematics to determine the opinions Victorian students have about mathematics.

b A group of 10 friends throw discuses to determine the average distance they can throw a discus.

c 50 people at a shopping centre in Victoria on a weekday are selected to participate in a survey to determine the state average number of sick days people take.

32 A number of households were asked how many bathrooms they have. The data collected is shown below.

3 2 1 1 2 1 8 2 2 1.5 2 3 2.5 1 1.5 2

a Calculate the mean (to two decimal places), median, mode and range.

b Recalculate the mean, median, mode and range excluding the outlier. Which summary statistics change and which stay the same?

33 The Queensland government wants to know the approximate number of households with pet cats in the state. They need to choose a sampling method.



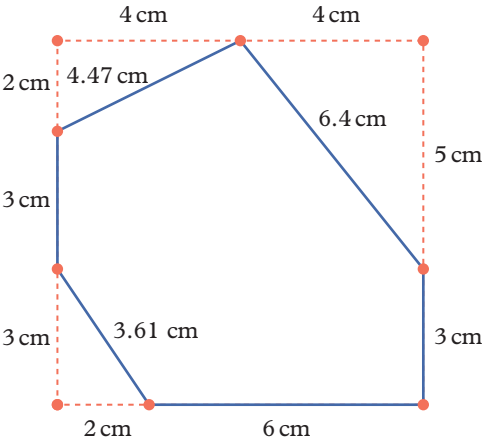
- **a** Why might they choose not to conduct a census?
- **b** Suggest one way they could use quota sampling.
- **c** They decide to use cluster sampling by taking a random sample of both rural and urban local government areas. Suggest one advantage and one disadvantage of this method.

34 Write a sentence to describe the following statistics.

- **a** survey of shoppers about how long they spent at the shops, mean = 1 h 20 m
- **b** survey about the number of board games people owned, median = 4
- **c** survey about people's favourite colour, mode = blue
- **d** sample of the length of a number of chess matches, range = 34 turns

Analysis

1 Kala is constructing a prism using cardboard with an irregular hexagonal base. She plans to construct the base using right-angled triangles. Her plan is shown at right.



- **a** Use a ruler and protractor to construct the hexagonal base.
- **b** Calculate the area of the hexagonal base.

Kala initially wants the prism to be 8 cm tall.

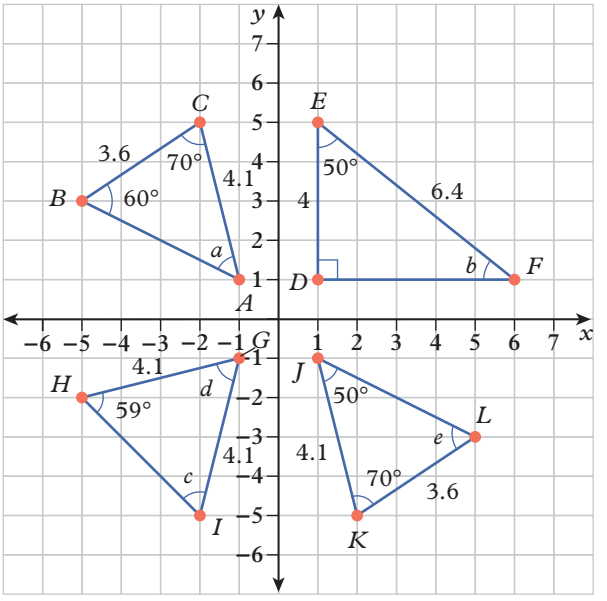
- **c** What is the total amount of cardboard Kala will need?
- **d** How much space will Kala's prism take up?

Kala instead decides to choose the height of the prism such that the value of its surface area and volume are equal.

Let h be the height of Kala's prism.

- **e** Write expressions, in terms of h, for the surface area and volume of the prism.
- **f** Determine the height of the prism, correct to two decimal places, if the value of the surface area and volume are equal.

2 Consider the triangles to the right.



- **a** Determine the value of all pronumerals.
- **b** Determine the mean, median, mode and range of the interior angles of the triangles.
- **c** What type of data are the angles in the triangles?
- **d** If one interior angle is selected at random from any of the triangles, what is the probability that it is acute?
- **e** Two of the triangles are congruent. Write the congruency statement and the condition that explains why they are congruent.
- **f** What single transformation could be applied to one of the congruent triangles to create the other congruent triangle?
- **g** Which triangle has the largest area?

3 A survey is being conducted by Molly in Brisbane (UTC+10) and Lester in London (UTC+1). They want to see if people in Brisbane and London have similar or different preferences between going to the beach, camping or to the snow. Every day for four days both Molly and Lester survey people from their respective cities. The results are shown below.

Destination	Brisbane	London
Beach	21	14
Camping	17	8
Snow	12	18

B	L
38	19
25	12
37	23

B	L
34	60
21	22
25	68

B	L
14	44
12	30
9	46

a Convert the tables to percentage frequency, correct to the nearest percent.

b Write a short paragraph about whether the preferences of people in Brisbane and London are similar or different. Include references to the percentages from part **a**.

Molly and Lester decide to organise a video call to discuss their results. Lester suggests 9 am London time.

c Determine the local time in Brisbane in 24-hour time for Molly.

During their video call, they decide to check if the season influenced people's opinions by repeating the experiment in six months' time. The percentage frequencies from each day in the later surveys are shown below.

Destination	Brisbane	London
Beach	37%	32%
Camping	33%	24%
Snow	30%	44%

B	L
40%	29%
27%	25%
33%	53%

B	L
31%	30%
39%	20%
30%	50%

B	L
33%	33%
40%	22%
27%	45%

d By referring to the tables, explain whether the change of season did or did not impact people's preferences.

4 Kate works at a toy store which specialises in knitted children's toys. She earns $20 per hour. Each of her shifts lasts eight hours and she works five shifts a week.



a What is the base rate that Kate earns per week?

Kate earns a bonus of $15 if she opens the store and a bonus of $20 if she is responsible for closing the store. She opens the store twice a week and closes the store three times a week.

b How much does Kate earn per week including bonuses?

The regional manager offers Kate a promotion where she can choose between earning an extra 15% per hour or increasing the store opening and closing bonuses by $5 each. The number of times she opens and closes the store would stay the same. Kate wants to use her knowledge of linear relationships to compare the amount she would earn in one week with each option.

c Write the equation of the linear relationship between Kate's pay, P_1, and hours worked, h, if she chooses the first option.

d Write the equation of the linear relationship between Kate's pay, P_2, and hours worked, h, if she chooses the second option.

e Sketch both linear relationships on the same set of axes for values of h between 0 and 40.

f Based on your graphs, which option should Kate choose?

Kate is offered a job by a rival toy company where she would earn $30 per hour but there would be no bonuses for opening and closing the store.

g Write the equation representing the linear relationship between Kate's potential pay, P_3, and hours worked, h.

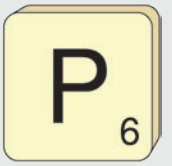
h Graph the linear relationship you found in part **g** on the same set of axes as the other two linear relationships for values of h between 0 and 40.

i Assume Kate will continue to work the same number of hours each week. Will she earn more per week if she accepts the new job?

EXPLORATIONS 2

1 Foodle

Foodle is a word game in which players place letter tiles in a row to make the name of a food. A player's score is the sum of the numbers on the tiles that make up the word. Each letter of the alphabet scores a different number of points from 1 to 26. The letter P scores 6 points.



a The word PEA scores 10 points, PEAR scores 18 points, PEPPER scores 28 points and PAELLA scores 23 points. How many points is the letter L worth?

b The word PEACH scores 19 points and CELERY scores 27 points. How many points does CHERRY score?

c The word CARROT scores 44 points. How many points does POTATO score?

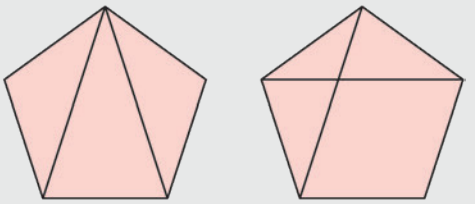
d The word GRAPE scores 22 points, GARLIC scores 31 points, ONION scores 57 points and CREAM scores 29 points. How many points does MELON score?

e Find the lowest possible score for COCONUT.

f The word OCTOPUS scores 89 points and BURGER scores 69 points. Find the lowest and highest possible scores for BRUSSELS SPROUT.

g Explain why the score for BEANS must be even.

2 Pentagon wedges



a What is the angle sum of any pentagon? Hint: What is the smallest number of triangles into which a pentagon can be divided?

b What is the angle at each corner of a regular pentagon?

c The diagram shows the two cases when a pair of diagonals is drawn on a regular pentagon. What special types of triangles and quadrilaterals are formed by the diagonals? Find the angles in each shape.

Given an isosceles triangle, we call it *narrow* if it has the same angles as the acute-angled triangles in the diagrams above. We call it *wide* if it has the same angles as the obtuse-angled triangles.

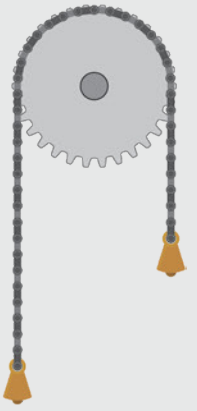
d Show how to divide a narrow triangle into two smaller triangles, one of which is narrow and the other wide.

e Show that a regular pentagon can be divided into 33 narrow triangles and 67 wide triangles. Can you generalise this result to other pairs of numbers?

f There are four types of isosceles triangle that can be divided into two smaller isosceles triangles. Two of these are the narrow and wide triangles described above. Find the other two types.

3 Bicycle chimes

Max pulls apart an old bicycle and builds a machine using the bicycle chain and one of the sprocket wheels. He breaks the chain, hangs it over the wheel and attaches a bell to each end, as shown. He then attaches a motor that turns the wheel clockwise for exactly one hour and then anticlockwise for exactly one hour, both at the same constant rate.



Max positions the centre of the wheel on the wall so that at 12 noon the bell on the left hits the floor and rings. The wheel turns clockwise for one hour so that at exactly 1 pm the right bell hits the floor and rings. At 2 pm the left bell rings again, and this process continues alternately every hour.

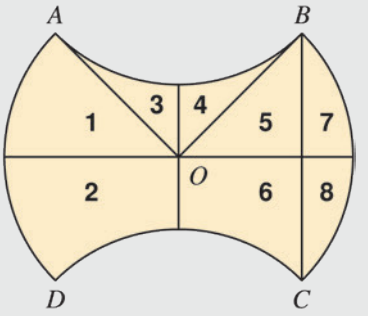
a At 2:10 pm the difference in height between the two bells is 80 cm. How far off the floor is the left bell at 3 pm?

b The diameter of the sprocket wheel is 10 cm. To two decimal places, how many revolutions does the wheel make per hour?

c As the years go by, the sprockets on the wheel wear out and the chain starts to slip. Max replaces the old wheel with a 14 cm–diameter sprocket wheel from the old bicycle. The motor still turns the wheel clockwise and anticlockwise at the same constant rate as before. To the nearest millimetre, by how much does Max need to raise or lower the centre of the new wheel so that the bells still ring every hour?

4 Dogbones

In the dogbone shown, points A, B, C and D are the vertices of a square with centre O. The four circular arcs which join the vertices all have radius 4 units.



a Find the exact area of each numbered piece, in square units. Hint: Find the areas of shapes 1 and 5 first, then shapes 7 and 4.

b Show how to rearrange the 8 pieces to make the following shapes:

i two identical rectangles

ii a right-angled isosceles triangle

iii a symmetrical hexagon

5 Freaky frequencies

Sebastian is practising converting lists of data into frequency distribution tables. For example, starting with the list {1, 2, 2, 3, 3, 2, 4, 1}, he puts all possible scores in ascending order in the first column, namely 1, 2, 3 and 4. He then records the frequency of each score (that is, the number of times it occurs in the list) in the second column, as shown.

Score	Freq.
1	2
2	3
3	2
4	1

Sebastian notices something unusual about this example: reading the entries in the table from left to right then top to bottom produces the list {1, 2, 2, 3, 3, 2, 4, 1}, which exactly matches the one he started with! He calls such a collection of data a *freaky-frequency list*.

a Find, with reasons, the only freaky-frequency list of length 2.

b Show that there are exactly two freaky-frequency lists that involve only the numbers 1, 2, 3 and 4.

Given any list of data which is *not* a freaky-frequency list, we can still read off the entries in its table to produce another list. For example, starting with {1}, the table's only row is $\boxed{1}\boxed{1}$ which produces the list {1, 1}. This in turn has only the row $\boxed{1}\boxed{2}$ in its table, producing the list {1, 2}. Continuing in this way, we get the following sequence of lists:

$$\{1\} \rightarrow \{1,1\} \rightarrow \{1,2\} \rightarrow \{1,1,2,1\} \rightarrow \{1,3,2,1\} \rightarrow \ldots$$

c Find the 100th list in the above sequence.

d Find the 100th list in the sequence which starts with the list {1, 2, 3, 4, 5}.

e The fourth list in a sequence is {1, 1, 2, 3, 3, 1, 5, 1}. Find the length of the first list.

6 Robot designs

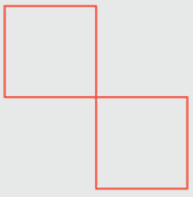
My drawing robot accepts the following commands:

- Fx : move forward x units.
- Lx : turn x degrees to the left.
- Rn [...] : repeat the instructions in the brackets n times.

For example, the instructions R3 [F30 L120] tell the robot to draw an equilateral triangle with a side length 30 units.

I tried to program my robot to draw the three designs shown below, but each set of instructions I came up with had a single mistake. Find and correct each mistake.

a R2 [R4 [F50 L90] L90]



b R3 [R3 [F50 L120] L60]



c R6 [R6 [F50 L60] L120]



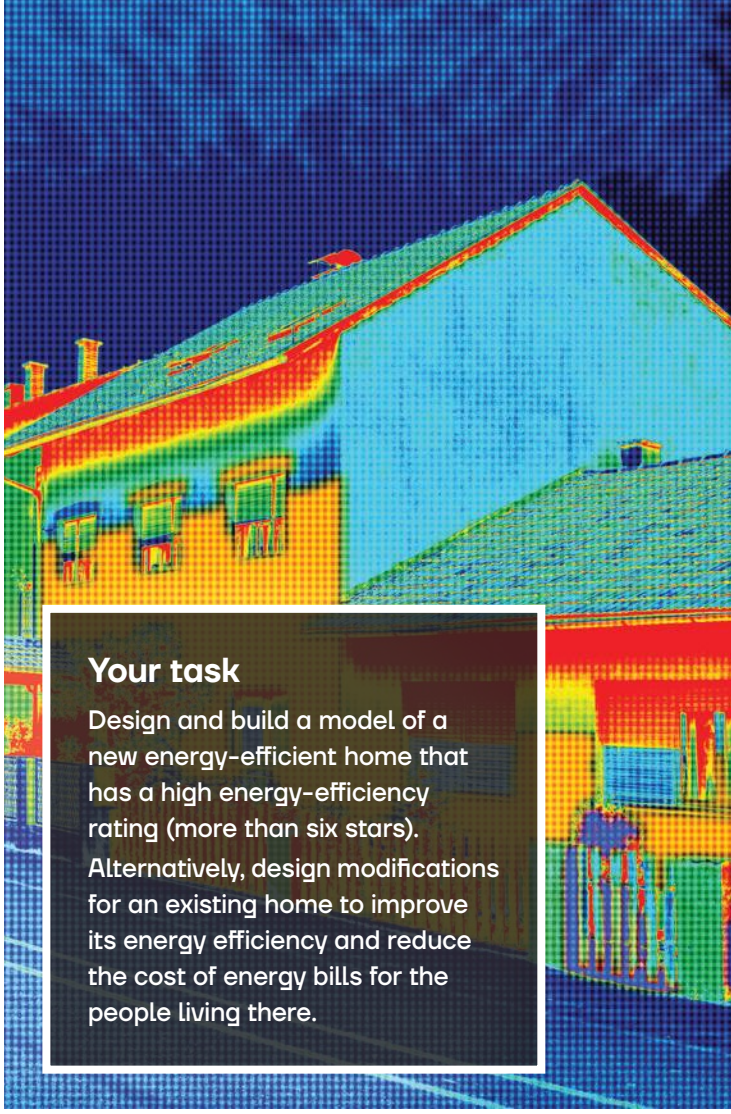
[STEAM project 1]

How can we build more energy-efficient homes so that we live more sustainably?

Have you ever considered how much energy it takes to heat or cool your home? A study by the Australian Government found that homes contribute 11 per cent of Australia's total carbon emissions. Reducing the amount of energy needed to heat or cool a home will not only save people money on their gas and electricity bills, it will make housing more sustainable.

NatHERS

The Nationwide House Energy Rating Scheme (NatHERS) is a rating system that is used across Australia to identify the energy efficiency of homes (where ten stars is the most energy efficient). Computer modelling software uses the local climate, the orientation of the home and materials used in construction to estimate the amount of energy needed to heat or cool the home. In most states and territories of Australia, new homes must reach a minimum of six stars to be approved for construction. There are many ways to improve the energy efficiency of your home. For example, if you insulate the walls, floors and roof, and use awnings (covers that extend over windows or doors), you can reduce the amount of thermal energy needed to cool the home in summer. Or in cooler climates, if you design the home so that living areas are on the northern side, you can maximise the sunlight that is available to heat the home in winter.



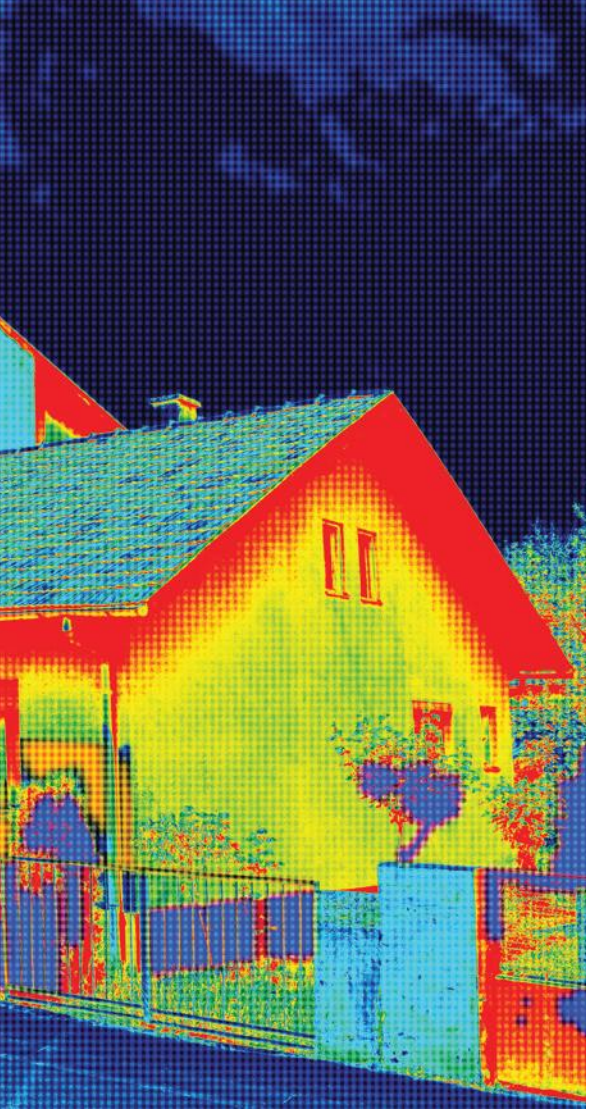
Your task

Design and build a model of a new energy-efficient home that has a high energy-efficiency rating (more than six stars). Alternatively, design modifications for an existing home to improve its energy efficiency and reduce the cost of energy bills for the people living there.

Figure 1 Infrared imaging shows warmer temperatures as red and cooler temperatures as blue. A building with insulation will lose less thermal energy to the outside environment in winter and will gain less thermal energy from the outside environment in summer.

Retrofitting homes

Not everyone wants to or can afford to build a new home. People who rent sometimes have little choice when it comes to the energy efficiency of their home, but this doesn't mean that they need to have large gas and electricity bills. There are things that they can do to reduce the movement of thermal energy in summer or winter. Covering the floor with rugs, preventing heat from moving in or out of windows, and controlling windows to manage airflow are all ways to make a home more energy-efficient and liveable.



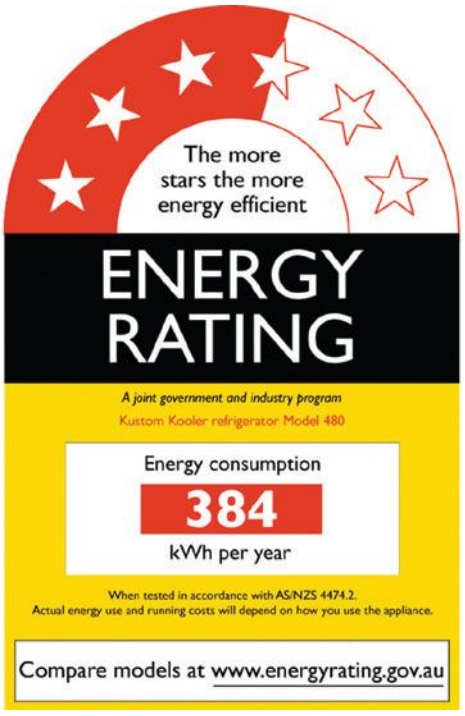


Figure 2 Energy rating schemes help consumers to understand the energy efficiency of products or appliances. The more stars, the more energy efficient the product or appliance is.

HUMANITIES

In Geography this year, you will learn about the importance of sustainable housing in Australia's urban centres and major cities. You will explore how the forecasted growth of Australia's cities could impact their liveability, and the environmental issues resulting from urbanisation.

In Economics and Business, you will study the housing market, the interaction between businesses and consumers in meeting consumer demands and when the government intervenes in markets.

To complete this task successfully, you will need to study seasonal weather patterns at the location of your energy-efficient home, to improve energy use.

You will find more information on this in Chapter 5 'Urban life' and Chapter 19 'Australian markets' of *Oxford Humanities and Social Sciences 8 Australian Curriculum*.

MATHS

In Maths this year, you will learn how to determine the area and volume of different shapes, using and converting between appropriate units. You will also learn skills for dealing with percentage changes in financial contexts. These skills will help you to quantify the costs and benefits of design features and predict the popularity of market incentives. You will perform calculations with and without digital technology.

To complete this task successfully, you will need to perform calculations for your model home, and then scale up to estimate the potential benefits of your design at a national level.

You will find help for applying these maths skills in sections 3B 'Calculating percentages', 3C 'Mark-ups and discounts', 8C 'Area of quadrilaterals' and 8D 'Area of a circle' of *Oxford Maths 8 Australian Curriculum*.

SCIENCE

In Science this year, you will learn about how energy can be transferred between objects and transformed (such as from the Sun's light energy to electrical energy). You will also examine earthquake resistant materials used in construction.

To complete this task successfully, you will need to identify how each element of a home's design can affect its heating and cooling needs, and how construction materials can affect a home's stability.

You will also need to identify the elements that can be changed in a new home design, and compare these to the elements that can be changed in an established home.

You will find more information on this in Chapter 3 'Plate tectonics' and Chapter 4 'Energy' of *Oxford Science 8 Australian Curriculum*.

[STEAM project 1]

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.



Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for? How big is their family and home?
- What factors might affect the liveability of the home?
- What do they need? What do they not need?
- What does it feel like to face these limitations on liveability? What words would you use to describe how it would feel to face these limitations?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your home modifications, you need to define the parameters you are working towards.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide.

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1. Describe the orientation of the land where the home will be built or modified (include the location of sunrise and sunset, the angle of sun in the middle of summer and winter and any existing shadows).
2. Describe any existing energy-efficient features of the current home or design.
3. Describe how you will measure any improvement in energy-efficient features of the home.

Ideate

Once you know who you're designing for, and what the criteria are, it's time to get creative!

- Outline the criteria or requirements your design must fulfil (for example, improving heating or cooling of the home).
- Brainstorm at least one idea per person that fulfils the criteria.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Build

Draw each individual design.

Include in the individual designs:

a labels for each part of the design

b the materials you will use for the home's construction

c a description of how the modifications will improve the energy star rating of the home

d an estimation of the cost of applying the modifications.

Present your design to your group.

Build the prototype

Choose and build two or three model homes for your group design (scale: 2 cm = 1 m).

Use the following questions as a guideline for your prototype.

- What materials will you need to build your model home?
- What skills will you need to build your model home?
- How will you test the effectiveness of your model home? (What will you compare it to?)
- How will you identify each energy-efficient feature of your model home?
- How will you collect data that supports your claims about energy efficiency?

Test

Use the scientific method to design and test the energy efficiency of your model home. You will test more than one design so that you can compare them, so you will need to control your variables between tests.

What criteria will you use to determine the success of your model home?

Conduct your tests and record your results in an appropriate table.

Communicate

Present your home design as though you are trying to get your peers to invest in it.

In your presentation, you will need to:

- construct a labelled diagram of your model home in its orientation, including the location of sunrise and sunset, the angle of sun in the middle of summer and winter, and any existing shadows
- describe the key energy-efficient features of your model home
- explain how each energy-efficient feature affects the liveability of your model home
- explain the principles that support your design (the importance of energy efficiency and how the existing landscape affected the design of the home)
- estimate the cost of implementing your design
- explain and quantify the benefits on a national scale if all new homes were to include your design features.

Check your Student obook pro for the following digital resources to help you with this STEAM project:

Student booklet
This helpful booklet will guide you step-by-step through the project.

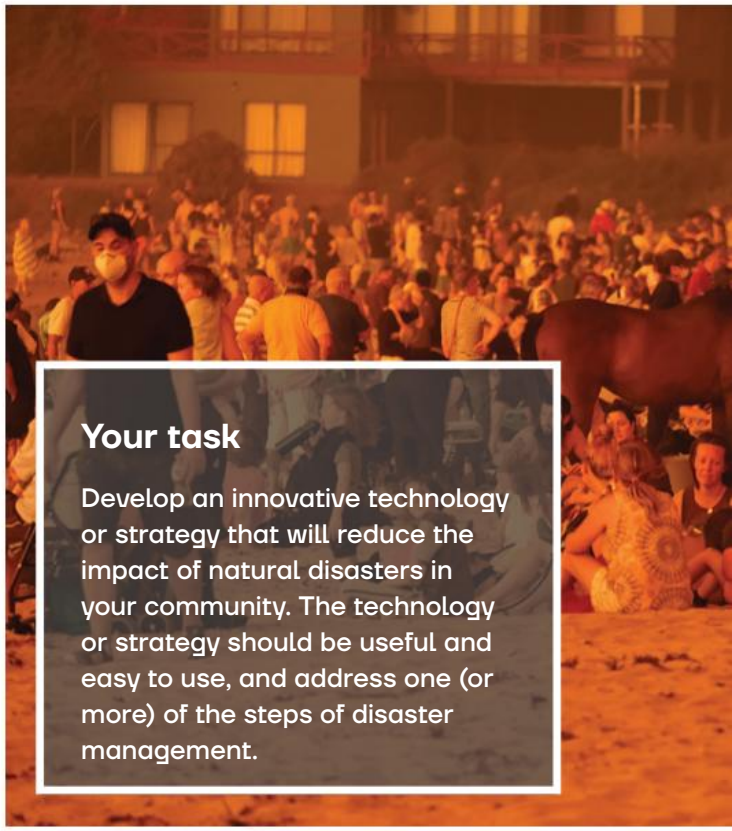
What is the design cycle?
This video will help you to better understand each phase of the design cycle.

How to manage a project
This 'how-to' video will help you to manage your time throughout the design cycle.

How to pitch your idea
This 'how-to' video will help you with the 'Communicate' phase of your project.

[STEAM project 2]

How can we use technology so that the impact of natural disasters is reduced?



Your task

Develop an innovative technology or strategy that will reduce the impact of natural disasters in your community. The technology or strategy should be useful and easy to use, and address one (or more) of the steps of disaster management.

Figure 1 Hundreds of people (and animals) huddled on Malua Bay beach in NSW for almost 24 hours, as bushfires tore through communities in the nearby Batemans Bay area.

Across the world, climate patterns are changing and weather events are becoming more extreme. In 2020, 20 per cent of Australia's forests were burnt, a record-breaking 30 named hurricanes developed over the Atlantic Ocean, and an extreme monsoon season in Asia caused flooding across a quarter of Bangladesh. Australia is particularly prone to natural disasters. Bushfires, flooding, drought periods and cyclones dramatically affect people's lives, and alter natural landscapes.

Natural disasters can be devastating for communities affected by them. Recovering from natural disasters also comes with huge economic costs. This can sometimes lead to a harmful cycle in which a nation gets stuck between experiencing disaster and responding to disaster.

Disaster management

Climate action is one of the 17 Sustainable Development Goals agreed to in 2015 by world leaders. One of the identified targets in combating climate change is to strengthen the resilience and adaptive capacity of all countries to natural disasters and hazards related to climate.

It is essential that communities prepare for disasters so as to reduce their impact. The United Nations Office for Disaster Risk Reduction (UNDRR) recommends using technology as part of disaster management, for example:

- warning systems in Japan that trigger emergency breaks in bullet trains if earthquakes are detected
- systems in sub-Saharan Africa that monitor rainfall data and analyse trends to forecast and build resilience to drought
- atlases in China that record the risk of natural disasters, using location data from spatial technologies such as GPS.

After the 2019–20 Australian bushfires, it was argued that satellite technology, drones and mobile phone apps would have been extremely helpful for fighting the fires and communicating with the people affected.

Disaster management should include four steps:

- prevention – reducing hazards before a disaster takes place, so its impact is not as severe (e.g. building schools that are earthquake resistant)
- preparation – training people so they know how to act when a disaster happens (e.g. running evacuation drills)
- response – taking action during a disaster (e.g. emergency crews and volunteers taking on emergency operations)
- recovery – taking action to help people rebuild their lives (e.g. restoring services in a community).

Disaster management is an ongoing process that can occur like a cycle between the phases of prevention, preparation, response and recovery.

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Figure 2 Floodwaters near Sydney

HUMANITIES

In Geography this year, you will learn about landscapes and landforms and how they can be degraded by both human and natural causes. You will study a geomorphic hazard, its impacts on a place and various ways of responding to it.

In History, you might also study how past societies have dealt with disasters and managed their responses; for example, how Rapa Nui (Easter Island) inhabitants adapted to life without trees, how Shogunate Japan developed policies to sustain its forests, or how an unstable climate (including drought and heavy monsoons) affected the Khmer Empire.

To complete this task successfully, you will need to investigate a potential natural hazard in your local area. You will research any current disaster-response plans related to managing such an event, and gain an understanding of how various people and businesses in the community would be impacted.

You will find more information on this in Chapters 2, 17, 15 and 16 of *Oxford Humanities and Social Sciences 8 Australian Curriculum*.

MATHS

In Maths this year, you will consolidate and extend your skills in describing and interpreting. This will include creating and analysing plots of non-linear data, investigating the use of sampling methods, and broadening your understanding of measures of spread. You will also perform calculations with percentage changes in financial contexts. You will analyse and represent data, both with and without digital technology.

To complete this task successfully, you will need to weigh up the costs of your disaster-management technology or strategy against its potential benefits, including by estimating the likelihood of natural disasters and the severity of their effects.

You will find help for applying these maths skills and statistics in sections 3B 'Calculating percentages', 3C 'Mark-ups and discounts', 6D 'Plotting linear relationships' and 9A 'Collecting data and sampling methods' of *Oxford Maths 8 Australian Curriculum*.

SCIENCE

In Science this year, you will learn about how the energy of the Earth over long periods of time generates forces that can melt rocks, produce volcanoes and make diamonds. You will also learn how the kinetic energy in the air and waves can cause damage to the surrounding environments during cyclones and tsunamis. The Australian bush also contains large amounts of chemical energy that is transformed into thermal energy during the summer fire season.

To complete this task successfully, you will need to consider how energy is transferred and transformed during natural disasters. This understanding will allow you to predict and potentially reduce the impact of the disaster in your local community.

You will find more information on this in Chapter 2 'The rock cycle' and Chapter 4 'Energy' of *Oxford Science 8 Australian Curriculum*.

[STEAM project 2]

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.



Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for?
- What natural disaster could they face?
- How often could this disaster occur, and on what scale?
- What do they need? What do they not need?
- What does it feel like to face these problems? What words would you use to describe these feelings?
- What could the cost of such a disaster be in terms of lives lost, income lost, damage to private property and public infrastructure, and environmental impact?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your innovative strategy or technology, you need to define the parameters you are working towards.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide.

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1. Define the term 'innovation'. Describe how much a strategy needs to differ from current practice to be considered innovative.
2. Describe how the natural disaster will affect the people in the community.
3. Describe how you could measure whether your solution will reduce the impact of the natural disaster.
4. Explain how you could determine what cost (price) should be acceptable for implementing your solution.

Ideate

Once you know who you're designing for, and you know what the criteria are, it's time to get creative!

- Outline the criteria or requirements your designed strategy or technology must fulfil (for example, how many people will be helped, how much they will be helped, how long they will need the help).
- Brainstorm at least one idea per person that fulfils the criteria.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

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Build

Draw each design strategy or technological idea. Label each stage of the strategy or part of the technology and explain how it will be used by the community.

Include in the individual strategy or technology:

a the timeline for the activation of the strategy or technology (i.e. how long after the disaster will the idea be ready to be used?)

b the number of people and materials that will be needed for the strategy or technology to reduce the impact of the natural disaster

c a description of how the community will benefit from the strategy or technology.

Present your strategy or technology to your group.

Build the prototype

Choose and build two or three versions of the prototype strategy or technology for your group design.

Use the following questions as a guideline for your prototype.

- What equipment do you have access to?
- What skills do you have, or will you need, to make your prototype strategy or technology?
- How could you model your strategy or technology if equipment is not available?
- How could you test the effectiveness of the prototype strategy or technology with the community?

Test

Use the scientific method to design an experiment that will test the effectiveness of your prototype strategy or technology. Alternatively, conduct a survey of the community to determine their opinion of the usefulness of the prototype.

Conduct your tests or survey your community, and record your results in an appropriate table.

Consider how you could use the results of the experiment or survey to modify your design.

Communicate

Present your design to the class as though you are trying to get your peers to invest in your strategy or technology.

In your presentation, you will need to:

- describe how the natural disaster will affect your community
- describe your prototype strategy or technology
- describe how the prototype will reduce the impact of the natural disaster on the community
- describe the materials, people and money that will be required to have the required effect on the natural disaster.

Check your Student obook pro for the following digital resources to help you with this STEAM project:

Student booklet
This helpful booklet will guide you step-by-step through the project.

What is the design cycle?
This video will help you to better understand each phase in the design cycle.

How to manage your project
This 'how-to' video will help you to manage your time throughout the design cycle.

How to define a problem
This 'how-to' video will help you to narrow your ideas down and define a specific problem.

Answers

CHAPTER 1 Integers

EX p6 1A Rounding and estimating

1 **a** 400 **b** 500 **c** 500 **d** 500
e 400 **f** 400

2 **a** 7000 **b** 6000 **c** 6000 **d** 7000
e 7000 **f** 6000

3 **a** **i** 480 **ii** 6380 **iii** 56 030
iv 738 490 **v** 8080 **vi** 904 510
b **i** 500 **ii** 6400 **iii** 56 000
iv 738 500 **v** 8100 **vi** 904 500

4 **a** **i** 36 000 **ii** 8000 **iii** 184 000
iv 51 000 **v** 6000 **vi** 1000
b **i** 36 400 **ii** 7500 **iii** 183 900
iv 50 700 **v** 6100 **vi** 800

5 **a** 80 **b** 40 **c** 80 **d** 300
e 200 **f** 600 **g** 900 **h** 2000
i 7000 **j** 20 000

6 **a** 90 **b** 100 **c** 1000
d 90 000 **e** 9000 **f** 1000

7 Sample answers:
a 37, 42, 35 **b** 692, 705, 744
c 2995, 3125, 3451 **d** 52 168, 46 854, 50 369
e 795 002, 755 036, 827 039
f 1 758 364, 2 498 567, 2 215 000

8 **a** 1600 **b** 290 **c** 600
d 20 **e** 20 000 **f** 3800
g 40 **h** 5500 **i** 2000
j 2 100 000 **k** 11 000 **l** 15 000

9 **a** 1200 **b** 50 **c** 1500
d 40 **e** 12 000 **f** 6200
g 125 **h** 50

10 **a** 10 **b** 1 800 000 **c** 6200 **d** 7000
e 560 000 000 **f** 18 000 **g** 150 **h** 19 200

11 **a** 40 000 **b** 35 800 **c** 36 000
d 35 780 **e** 40 000

12 **a** 20 000 000 km^2 **b** 17 098 200 km^2
c 17 098 000 km^2 **d** 17 100 000 km^2
e 17 100 000 km^2 **f** 17 000 000 km^2

13 **a** $300
b overestimation

14 Tina is correct. Since the leading digit is 9 and the second digit is 6, the first digit increases by one and the digits that follow are replaced with zero. Increasing 9 by one gives 10 so there is a total of five digits in the answer.

15 **a** 30 **b** $1800 **c** no
d 20 months

16 **a** 400 **b** $2800
c $2695; estimate is $105 more

17 **a** 40
b The estimate of 40 would be lower than the actual number of boxes as the approximate price ($50) is higher than the actual price ($47). There would be money left over.

18 **a** Australia: 8 000 000 km^2
Indonesia: 2 000 000 km^2
b **i** 6 000 000 km^2
ii approximately 4 times the area of Indonesia
c **i** 5 772 584 km^2
ii 4.0074 times the area of Indonesia

19 **a** 48 000 **b** 52 196

20 **a** WA: 3 000 000 km^2, Qld: 2 000 000 km^2, NT: 1 000 000 km^2, SA: 1 000 000 km^2, NSW: 800 000 km^2, Vic: 200 000 km^2, Tas: 70 000 km^2, ACT: 2000 km^2
b 8 072 000 km^2
c WA: 2 530 000 km^2, Qld: 1 731 000 km^2, NT: 1 349 000 km^2, SA: 983 000 km^2, NSW: 801 000 km^2, Vic: 227 000 km^2, Tas: 68 000 km^2, ACT: 2000 km^2
d 7 691 000 km^2
e Values rounded to the nearest thousand produce an estimate much closer to the actual values.

21 **a** $\frac{8}{30} = \frac{4}{15} \approx 0.27$ km^2 per person
b New Zealand has approximately $\frac{3}{50} \approx 0.06$ km^2 per person. So, Australia has approximately 4.5 times more area per person than New Zealand.

22 Record yourself speaking for a minute and count the number of words you say. Consider how many minutes of the last hour you have spoken for. Multiplying by 12, is this a good estimate of your day?

EX p12 1B Adding and subtracting whole numbers

1 **a** **i** 97 **ii** 161 **iii** 979 **iv** 10897
b **i** 719 **ii** 819 **iii** 584 **iv** 3591
c **i** 123 **ii** 528 **iii** 1123 **iv** 8104
d **i** 177 **ii** 1139 **iii** 8824 **iv** 6570

2 **a** 958 **b** 692 **c** 1219 **d** 1502

3 **a** 5241 **b** 10 475 **c** 18 872 **d** 122 530

4 **a** 1223 **b** 1857 **c** 4141 **d** 4918
e 7366 **f** 20 350

5 **a** 91 852 **b** 128 077 **c** 24 921 **d** 2460

6 **a** **i** 63 **ii** 87 **iii** 94 **iv** 693
b **i** 33 **ii** 695 **iii** 256 **iv** 366
c **i** 60 **ii** 5190 **iii** 210 **iv** 112
d **i** 62 **ii** 890 **iii** 523 **iv** 1136

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7 **a** 39 **b** 276 **c** 1380 **d** 3378

8 **a** 29 **b** 3489 **c** 4418 **d** 6320

9 **a** 7166 **b** 998 **c** 74 949 **d** 43 642

10 **a** 476 **b** 5074 **c** 38 150 **d** 8473

11 2514

12 **a** 2 780 000 **b** 750 000 **c** 5 277 000

13 **a** 81 593 **b** 53 248

14 **a** 1091 km **b** $1088 **c** $734 **d** $178

15 **a** 6199 **b** 4637

16 **a** 60 km **b** 125 m

17 85 pairs of shoes

18 **a** 5 **b** 2000

19 S=4, E=5, D=1, W=6, A=8, T=3, R=2, F=7, U=9, I=0
S=5, E=8, D=9, W=3, A=6, T=1, R=2, F=4, U=0, I=7
S=6, E=8, D=9, W=3, A=5, T=1, R=2, F=4, U=0, I=7
S=7, E=1, D=3, W=4, A=8, T=9, R=6, F=5, U=0, I=2
S=8, E=1, D=3, W=4, A=8, T=9, R=6, F=5, U=0, I=2
S=8, E=5, D=1, W=6, A=4, T=3, R=2, F=7, U=9, I=0

EX p18 1C Multiplying and dividing whole numbers

1 **a** 126 **b** 612 **c** 1421 **d** 2745
e 1666 **f** 8217 **g** 2280 **h** 779
i 6262 **j** 29 029 **k** 1067 **l** 14 985

2 **a** 408 **b** 630 **c** 444 **d** 5280
e 600 **f** 92 100 **g** 700 **h** 29 000
i 186 000

3 **a** 630 **b** 9600 **c** 7700 **d** 6300
e 320 400 **f** 8 295 000
g 24 867 000 **h** 83 700 000

4 **a** 720 000 **b** 448 000 000
c 9 600 000 **d** 375 000 000

5 **a** 44 764 **b** 24 402 **c** 105 754 **d** 147 936
e 665 640 **f** 4 903 600

6 **a** 332 820 **b** 226 044
c 316 448 **d** 1 726 202
e 1 509 024 **f** 29 792 690

7 **a** **i** 800 000 **ii** 270 000 **iii** 60 000 000
iv 640 000 000
b **i** 803 890 **ii** 287 013 **iii** 59 804 375
iv 654 565 110

8 **a** 57 remainder 7 **b** 58
c 22 remainder 6 **d** 75
e 76 remainder 5 **f** 1420 remainder 2

9 **a** 1456 **b** 1399 remainder 5
c 1403 remainder 1 **d** 16 855 remainder 1
e 6254 remainder 3 **f** 212 remainder 5

10 **a** 23 remainder 5 **b** 81 remainder 5
c 35 remainder 1 **d** 163

11 **a** 92
b 23

12 **a** **i** 9 **ii** 9 **b** no
c **i** 3
ii −3; subtraction does not obey the commutative law.
d **i** 18
ii 18; multiplication obeys the commutative law.
e **i** 2
ii 0.5; division does not obey the commutative law.

13 **a** **i** 18 **ii** 18 **iii** 18
b no
c Multiplication obeys the associative law, but subtraction and division do not.
For example, multiplication:
$5 \times 4 \times 2 = 4 \times 5 \times 2 = 2 \times 5 \times 4 = 40$;
subtraction: $(7 - 2) - 3 = 2$ and $7 - (2 - 3) = 8$ (does not obey the associative law);
division: $(8 \div 4) \div 2 = 1$ and $8 \div (4 \div 2) = 4$ (does not obey the associative law).

14 **a** **i** 54 + 16, since the result (70) is a multiple of 10 and easier to add
ii 188
b **i** 25 × 4, since the result (100) is easier to multiply by
ii 8700
c As order is not important, any two of the three numbers can be added or multiplied first. If this results in a number that is a multiple of 10, it can make the final operation easier.

15 **a** **i** 44 **ii** 20 + 24 = 44; the answers are the same
b **i** 48 **ii** 60 − 12 = 48; the answers are the same
c Each number inside the brackets is multiplied by the number at the front of the brackets.
d Sample answer:
$10 \times (5 + 4) = 10 \times 5 + 10 \times 4 = 50 + 40 = 90$
$10 \times (5 + 4) = 10 \times 9 = 90$

16 **a** **i** 320 **ii** 64 **iii** 384
b **i** 45 000 **ii** 90 **iii** 44 910
c Write one of the numbers to be multiplied as either a sum or difference that involves a multiple of 10 or power of 10 and a single-digit number.

17 **a** $12 **b** 3 **c** $10 **d** $400

18 **a** 65 **b** 11 **c** $195

19 **a** 402 L **b** 9 **c** $1206

20 23

21 **a** 60 g **b** 32

EX p24 1D Multiples, factors, squares and cubes

1 **a** 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96
9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
b 18, 36, 54, 72, 90
c 18

2 **a** 24 **b** 36 **c** 150 **d** 60
e 140 **f** 45

3 **a** 210 **b** 315 **c** 72 **d** 120
e 360 **f** 54

4 **a** 1, 3, 5, 9, 15, 45
b 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
c 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
d 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
e 1, 2, 4, 5, 10, 20, 25, 50, 100
f 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

5 **a** 18 **b** 3 **c** 6

6 **a** 14 **b** 12 **c** 9 **d** 4
e 120 **f** 25

7 **a** LCM = 360, HCF = 3
b LCM = 144, HCF = 4
c LCM = 60, HCF = 1
d LCM = 924, HCF = 1

8 **a** 9, 27 **b** 4, 8 **c** 100, 1000
d 49, 343 **e** 36, 216 **f** 144, 1728
g 64, 512 **h** 16, 64 **i** 1, 1
j 81, 729 **k** 121, 1331 **l** 196, 2744

9 **a** 3 **b** 2 **c** 5 **d** 8
e 9 **f** 6 **g** 10 **h** 7
i 8 **j** 13 **k** 14 **l** 15

10 **a** 24 **b** 33 **c** 49 000 **d** 16
e 4096 **f** 100 000

11 **a** 972 **b** 36 **c** 6 **d** 2048
e 256 **f** 3600

12 60

13 **a** 120 **b** 8

14 90

15 24

16 8

17 Pontiac, Desiree, Sebago

18 500

Checkpoint

1 **a** 600 **b** 900 **c** 7900 **d** 2800
e 45 600 **f** 76 600

2 **a** 1200 **b** 40 **c** 2800 **d** 30
e 15 000 **f** 9800 **g** 100 **h** 500

3 **a** 162 **b** 1577 **c** 12 158 **d** 7058
e 6717 **f** 15 368

4 **a** 18 **b** 486 **c** 89 **d** 5655
e 8369 **f** 3826

5 **a** 321 **b** 577 **c** 6471 **d** 108 206

6 **a** 33 024 **b** 64 296 **c** 354 198 **d** 768 746
e 594 336 **f** 4 207 980

7 **a** 98 remainder 3 **b** 92 remainder 2
c 1665 remainder 3 **d** 8488 remainder 6
e 6212 remainder 9 **f** 108 964 remainder 5

8 **a** 42 **b** 36 **c** 60 **d** 90
e 72 **f** 108

9 **a** 6 **b** 9 **c** 15 **d** 5
e 150 **f** 144

10 **a** 100 **b** 9 **c** 144 **d** 8
e 148 **f** 253

EX p29

1E Negative integers

1 **a** negative integers: −1, −2, −3, −4, −5
b positive integers: +1, +2, +3, +4, +5
c 0

2 **a** 9 **b** 5 **c** 1 **d** 5
e 0 **f** −4

3 **a** 5 **b** 0 **c** −2 **d** −5
e −1 **f** −9

4 **a** 0 **b** 24 **c** 14 **d** 125
e −25 **f** 0

5 **a** −70 **b** −21 **c** −68 **d** −50
e −354 **f** −450

6 **a** −3 < 2 **b** −8 < −4 **c** 0 > −1
d 9 > −9 **e** −3 > −5 **f** −7 < −2
g 4 > −6 **h** 5 > −12 **i** 6 > −4
j −18 < −8 **k** −6 > −7 **l** −8 < 0

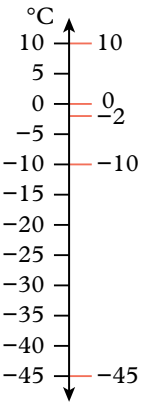
7 **a** −20, −12, −11, −7, 8, 10, 14
b −41, −33, −19, 0, 6, 29, 42
c −88, −48, −28, −18, −8, 8, 68
d −140, −126, −82, 3, 73, 104, 145

8 **a** 71, 4, −9, −10, −15, −27
b 10, 5, −1, −2, −5, −11
c 21, 5, 0, −9, −14, −19
d 71, 51, 12, −11, −62, −71, −72

9 −6, −5, −4, −3, −2, −1, 0, 1, 2, 3

10 **a** −3 **b** +2174 **c** −18 **d** −408
e +5895 **f** +500 **g** −46 **h** −40

11 **a**



b higher
c −45°C, −10°C, −2°C, 0°C, 10°C
d warmer

12 **a** +86 **b** $14 **c** −14

13 **a** long sighted
b short sighted
c Elle has weaker vision.
d normal vision; that is, neither long nor short sighted

14 a i $34 **ii** $51

b i $13 **ii** $35

c Account is overdrawn; the person owes money to the bank.

15 a three

b i

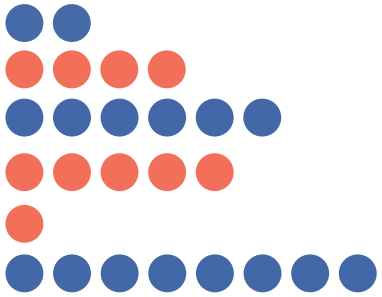
ii

iii

iv

v

vi



c The reference point for opposite integers is always zero as both integers are the same number of units away from 0. Moving forward to the positive integer and then moving back as indicated by the opposite integer will always result in zero. When combined, opposite integers 'cancel' each other to result in zero. This can be modelled by equal numbers of blue and red dots.

d Two blue dots and two red dots produce two zero pairs; the remaining three red dots represent the integer −3.

e i 5 **ii** −4 **iii** −4

16 12

17 −9, −8, −7, −6, −5

EX p34 1F Adding and subtracting integers

1 a −5 + 3 = −2 **b** 6 − 2 = 4

c −4 − 5 = −9 **d** −2 + 5 = 3

2 a C **b** A **c** B

3 a −15 **b** −9 **c** 9

4 a 3 **b** 6 **c** 5 **d** −8

e −4 **f** −3 **g** −6 **h** 5

i −4

5 a 3 **b** −6 **c** −4 **d** 16

e −19 **f** −10 **g** −17 **h** −2

i −13 **j** 0 **k** −20 **l** 2

6 a negative **b** positive **c** negative

d zero **e** positive **f** positive

g negative **h** negative **i** negative

7 a −3 − 7 **b** 1 + 6 **c** −4 − 5

d 2 + 4 **e** 5 − 9 **f** −6 + 8

8 a −1 + 3 **b** 8 − 4 **c** −5 − 2

d 6 + 1 **e** 4 + 3 **f** 3 − 9

g −5 + 8 **h** −3 + 1 **i** −7 − 3

9 a 3 **b** 6 **c** −4 **d** 0

e −1 **f** 5 **g** 1 **h** −5

i 9

10 a positive **b** negative **c** zero

d negative **e** negative **f** negative

g positive **h** negative

11 a

+	−5	−3	0	1	4
−3	−8	−6	−3	−2	1
−2	−7	−5	−2	−1	2
−1	−6	−4	−1	0	3
3	−2	0	3	4	7
6	1	3	6	7	10

b

+	−10	−14	−7	13	19
11	1	−3	4	24	30
14	4	0	7	27	33
−10	−20	−24	−17	3	9
−22	−32	−36	−29	−9	−3
−18	−28	−32	−25	−5	1

12 a −5 **b** −99 **c** −32 **d** 9

e −40 **f** −275 **g** 242 **h** 118 **i** 0

13 a 10 **b** −4 **c** −3 **d** −13 **e** 0

f −33 **g** −10 **h** −26 **i** −30 **j** −17

14 a −6 **b** 15 **c** 0 **d** 12 **e** −14

f −31 **g** 8 **h** 2

15 four levels below ground; 15 − 19 = −4

16 first level below ground; 3 − 9 + 5 = −1

17 51°C; 33 − (−18) = 51

18 a −60 **b** −60 + (+35) **c** $25

19 a A: 50, B: −2, C: 32

b i 50 − (−2) = 50 + 2

ii 32 − (−2) = 32 + 2

iii 50 − (+32) = 50 − 32

c i 52 m **ii** 34 m **iii** 18 m

d 52 m; it shows the greatest distance the bungee rope extends to before rebounding a lesser distance each time after.

20 a −900 − (+15) = −915; 915 m down

b −2500 − (−900) = −1600; 1600 m down

c 15 − (−2500) = +2515; 2515 m up

d 9 m above the sea level

21 a 45 **b** −30 **c** 1261 **d** −367

e −1020 **f** −3260

22 a 33 **b** 77 **c** −125 **d** −40

23 a $63; (−247) − (−310) = 63

b Initially, account is overdrawn by $310. During the month, $63 is added to the account and the overdrawn amount is reduced from $310 to $247.

EX p39 1G Multiplying and dividing integers

1 a −12 **b** 35 **c** −12 **d** 27 **e** −8 **f** −24

g 72 **h** 20 **i** −44 **j** −7 **k** 8 **l** 45

2 a −5 **b** 4 **c** 6 **d** −7 **e** 17 **f** −8

g −7 **h** 9 **i** −6 **j** −12 **k** 10 **l** −9

3 a −8 **b** −12 **c** 17 **d** 15 **e** 8 **f** −3

4 a −35 **b** −3 **c** 7 **d** −60 **e** −4 **f** 60

g −5 **h** −36 **i** 90 **j** −1 **k** −38 **l** 45

5 **a** −3 **b** −7 **c** −9 **d** 100 **e** −70 **f** −3

6 **a**

×	−2	−1	0	+1	+2
−4	8	4	0	−4	−8
−2	4	2	0	−2	−4
0	0	0	0	0	0
+2	−4	−2	0	2	4
+4	−8	−4	0	4	8
+6	−12	−6	0	6	12

b

×	−25	−20	−10	10	20
7	−175	−140	−70	70	140
5	−125	−100	−50	50	100
0	0	0	0	0	0
−2	50	40	20	−20	−40
−4	100	80	40	−40	−80
−6	150	120	60	−60	−120

7 **a** $-1 \times (-7) = 7$ **b** $-1 \times +12 = -12$
c $-1 \times (-88) = 88$ **d** $-1 \times 25 = -25$
e $-1 \times (8 - 5) = -1 \times 3 = -3$
f $-1 \times (-3 + 2) = -1 \times (-1) = 1$

8 **a** 60 **b** −42 **c** −54 **d** −48 **e** −130
f 84 **g** −160 **h** 110 **i** −180

9 **a** 36 **b** −5 **c** −24 **d** −6 **e** 30
f −42

10 E

11 **a** negative **b** positive **c** positive
d positive

12 **a** $-9 \times (-9) = 81$
b $8 \times 8 = 64$
c $-6 \times (-6) \times (-6) = -216$
d $7 \times 7 \times 7 = 343$
e $-5 \times (-5) \times (-5) \times (-5) = 625$
f $1 \times 1 \times 1 \times 1 = 1$
g $4 \times 4 \times 4 \times 4 \times 4 = 1024$
h $-10 \times (-10) \times (-10) \times (-10) \times (-10) = -100\,000$

13 **a** $(-5)^3 \times (-9)^4$ **b** $(-4)^6 \times 3^3$
c $7^2 \times (-6)^6$ **d** $(-8)^4 \times (-10)^2$

14 **a** $-2 \times (-2) \times (-2) \times (-4) \times (-4) = -128$
b $-5 \times (-5) \times 3 \times 3 \times 3 \times 3 = 2025$
c $-10 \times (-10) \times (-10) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = 32\,000$
d $-3 \times (-3) \times (-3) \times (-3) \times (-3) \times (-1) \times (-1) = -243$
e $-6 \times (-6) \times (-2) \times (-2) \times (-2) = -288$
f $-1 \times (-1) \times (-1) \times (-1) \times 3 \times 3 = 9$
g $-7 \times (-7) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -49$
h $-3 \times (-3) \times (-3) \times (-2) \times (-2) = -108$
i $-1 \times (-1) \times (-1) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = 32$

15 **a** 10, 8, 6, 4, 2, 0, −2, −4, −6, −8, −10; the terms decrease by 2, or you get the next number by subtracting 2 (or adding −2)
b −15, −12, −9, −6, −3, 0, 3, 6, 9, 12, 15; the terms increase by 3, or you get the next number by adding 3 (or subtracting −3)
c 20, 16, 12, 8, 4, 0, −4, −8, −12, −16, −20; the terms decrease by 4, or you get the next number by subtracting 4 (or adding −4)
d −25, −20, −15, −10, −5, 0, 5, 10, 15, 20, 25; the terms increase by 5, or you get the next number by adding 5 (or subtracting −5)

16 **a** **i** $2 \times 3 = 6$ so $6 \div 2 = 3$ or $6 \div 3 = 2$
ii $2 \times (-3) = -6$ so $-6 \div 2 = -3$ or $-6 \div (-3) = 2$
iii $-2 \times 3 = -6$ so $-6 \div (-2) = 3$ or $-6 \div 3 = -2$
iv $-2 \times (-3) = 6$ so $6 \div (-2) = -3$ or $6 \div (-3) = -2$
b yes, same pattern
c **i** positive **ii** positive
iii negative **iv** negative

17 **a** **i** −1 **ii** 1 **iii** −1
iv 1 **v** −1 **vi** 1
b those with even exponent or power (**ii**, **iv**, **vi**)
c those with odd exponent or power (**i**, **iii**, **v**)
d Negative numbers raised to an even power will give a positive result and negative numbers raised to an odd power will give a negative result.
e **i** −1 **ii** 1 **iii** 1
iv −1 **v** 1 **vi** −1

18 **a** **i** 0 **ii** 0 **iii** 0 **iv** 0
b The answer is always zero.
c **i** 0 **ii** 0 **iii** 0 **iv** 0
d The answer is always zero.

19 **a** −6, −3 **b** 9, −5 **c** −11, 6 **d** −4, 14
e 4, 6 or −6, −4 **f** −9, −1 or 1, 9
g −16, 2 or −2, 16

20 **a** −120 **b** +80 **c** 200 m **d** +110 **e** +40
f **i** 40 m **ii** 230 m
g −160 **h** −40
i Hayden. He needs to walk 20 m west.
j Natalie: 320 m; Tyler: 240 m; Hayden: 120 m; Rhys: 320 m; Sophie: 160 m; Imogen: 220 m

21 **a** 5 and −5; both numbers give the result of 25 when squared.
b No; to square a number is to multiply the number by itself. Either a negative number is squared (− × − → +) or a positive number is squared (+ × + → +). In both cases the result is positive.
c only one number (2)
d only one number (−2)

e Two; raising both 2 and −2 to the power of 4 gives the result of 16. No number can be raised to the power of 4 to give a negative result.

f There will be two numbers that when raised to an even power (such as 2, 4, 6, 8, etc.) give the same positive result. For example, $2^6 = 64$ and $(-2)^6 = 64$. No number raised to an even power gives a negative result. There will only be one number that when raised to an odd power (such as 3, 5, 7, etc.) results in a given answer. For example, $2^5 = 32$ and $(-2)^5 = -32$.

22 a 7 and −7 **b** 9 and −9 **c** 1 and −1
d 2 and −2 **e** 8 and −8 **f** 10 and −10

23 a 3 **b** −5 **c** 4 **d** −1 **e** −4 **f** −10

24 a $(+3)^2$, $(-3)^2$ and 3^2

b $(-3)^2 = -3 \times (-3) = +9$
while $-3^2 = -(3^2) = -(3 \times 3) = -9$

c **i** yes **ii** no **iii** no **iv** yes

d A number of the form $-x^a$ will always produce a negative result regardless of whether the power is odd or even. A negative base raised to an odd power will also produce a negative result.

EX p45 1H Order of operations

1 a multiplication, subtraction, addition

b subtraction (within brackets), division, addition

c squaring, multiplication, addition, subtraction

d squaring, multiplication, subtraction, addition

e addition (within inner brackets), subtraction (within outer brackets), addition

f addition (within brackets), squaring, division, addition

2 a 23 **b** 40 **c** 75 **d** 26 **e** 17 **f** 47

3 a −9 **b** 0 **c** −5 **d** 7 **e** −16 **f** 14

4 a 2 **b** 390 **c** 22 **d** 5 **e** 123
f 36 **g** 51 **h** 555

5 a 6 **b** −207 **c** −4 **d** −16 **e** 25 **f** 0

6 a 66 − 7 **b** 415 + 322 **c** 365 − 109
d 1750 + 3487 **e** 4376 − 237 **f** 6548 − 2399

7 a A negative has been added to the answer.

b Addition has been calculated before the multiplication.

c Addition has been calculated before the division.

d A negative divided by a negative is a positive.

e Negative 4 to the power of 3 is −64 so the answer is also negative.

f Ignored the negative in front of the 5.

8 a 64 **b** 10 **c** −3 **d** 4 **e** −8 **f** −21

9 a 195 − 3 × 58

b 200 − 3 × 60 = 20; $20

c $21; the estimate is $1 less.

10 14 × 7 + 11 × 6 + 9 × 4 + 16 × 5 + 25 × 1 = $305

11 a 13 **b** −6
c 13 − (−6) **d** 19 m

12 a

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Difference	6	7	2	4	5	3	1

b Monday

c **i** 1°C **ii** −3°C

d 4°C

13 a deposit **b** withdrawal

c **i** He has money in the bank.

ii He owes the bank money.

d −$96 **e** $134

f −$27, a withdrawal of $27

g Yes, from 21 May to 24 May. A fee and/or interest is charged.

14 Negative. For the sum of two numbers to be zero, they must have different signs. As one number is positive and the other negative, the product will be negative (+ × − → −), unless they were both zero.

15 −6, −4, 5

16 a 10 **b** 12

17 a Sample answer:

Number	Equation
0	4 ÷ 4 × 4 − 4
1	4 ÷ 4 + 4 − 4
2	4 − (4 + 4) ÷ 4
3	(4 × 4 − 4) ÷ 4
4	4 + 4 × (4 − 4)
5	(4 × 4 + 4) ÷ 4
6	(4 + 4) ÷ 4 + 4
7	4 + 4 − 4 ÷ 4
8	4 ÷ 4 × 4 + 4
9	4 ÷ 4 + 4 + 4
10	Not possible

Number	Equation
11	Not possible
12	4 × (4 − 4 ÷ 4)
13	Not possible
14	Not possible
15	4 × 4 − 4 ÷ 4
16	4 × 4 + 4 − 4
17	4 × 4 + 4 ÷ 4
18	Not possible
19	Not possible
20	4 × (4 ÷ 4 + 4)

b Sample answer (considering the numbers not possible in part **a**):

Number	Equation
10	$(4 + 4 + 4) - \sqrt{4}$
11	Not possible
13	Not possible

Number	Equation
14	$4 \times 4 - 4 \div \sqrt{4}$
18	$4 \times 4 + 4 - \sqrt{4}$
19	Not possible

c Sample answer (considering the numbers not possible in part **a** or **b**):

Number	Equation
11	$(4! \times \sqrt{4} - 4) \div 4$
13	$(4! \times \sqrt{4} + 4) \div 4$
19	$4! - (4 + 4 \div 4)$

d Place brackets around each calculation and add a negative sign in front. This will keep the value of the calculation the same but give a negative result.

CHAPTER 1 review

Multiple choice

1	B	**2**	D	**3**	E	**4**	D	**5**	D	**6**	C
7	D	**8**	A	**9**	E	**10**	E	**11**	C	**12**	D
13	B	**14**	E	**15**	A	**16**	D	**17**	A		

Short answer

1 **a** 200 **b** 70 000 **c** 4000
d 100 000

2 **a** 14 000 **b** 60 000 **c** 120 000 **d** 50

3 $2 296 240

4 249 937

5 168

6 11

7 18

8 **a** 216 **b** 128 **c** 64 **d** 121

9 **a** 2 **b** 9 **c** 3 **d** 5

10 **a** $-5 < 2$ **b** $-5 < -2$
c $7 > -4$ **d** $4 > -7$

11 **a** −40, −24, −22, −14, 16, 20, 24
b −35, −27, −17, 0, 9, 21, 63
c −66, −46, −26, −16, −6, 6, 86
d −409, −300, −87, 3, 45, 65, 190

12 **a** −9 **b** +9 **c** −12 **d** +19

13 $122

14 21°C

15 **a** −1 **b** −10 **c** −8 **d** −10
e 0 **f** −100

16 **a** −56 **b** +45 **c** −9 **d** +5
e 0 **f** −60 **g** −3 **h** +14

17 **a** 180 **b** −35 **c** 12 **d** −40

18 **a** −32 **b** 9 **c** −1 **d** 10 000

19 **a** 21 **b** 22 **c** −130 **d** −10

20 **a** −180 **b** 43 **c** −11 **d** −8

Analysis

1 **a** **i** Wednesday **ii** Friday
b Monday: 7°C, Tuesday: 7°C, Wednesday: 11°C, Thursday: 7°C, Friday: 4°C, Saturday: 7°C, Sunday: 6°C
c Wednesday
d **i** −2°C **ii** 5°C
e 7°C

2 **a** 1 469 200
b 8 000 000, 7 000 000, 5 000 000, 3 000 000, 2 000 000, 500 000, 400 000, 200 000
c 1 000 000; the difference is 469 200 people
d 26 100 000

3 **a** 2700 kg, 1800 kg **b** 900 kg **c** 4500 kg **d** 900 kg
e 450 kg **f** 270 kg and 180 kg
g 1795 kg, 1775 kg, 1675 kg
h The elephant should be losing 5 kg each day, but Jez's expression suggests the elephant lost 20 kg on the second day and 100 kg on the third day.
i $2 \times 3^2 \times 10^2 - n \times 5$

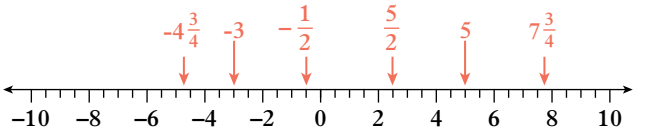
CHAPTER 2 Fractions and decimals

EX p57

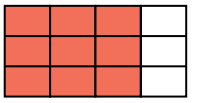
2A Fractions

1 **a** $\frac{1}{-2} = -\frac{1}{2} = \frac{-1}{2}$ **b** $\frac{-10}{3} = -\frac{10}{3} = \frac{10}{-3}$
c $\frac{-5}{-6} = \frac{5}{6}$ **d** $\frac{-2}{-3} = \frac{2}{3}$
e $-\frac{2}{5} = \frac{-2}{5} = \frac{2}{-5}$ **f** $-\frac{-3}{-7} = -\frac{3}{7} = \frac{-3}{7} = \frac{3}{-7}$

2



3 **a** $\frac{3}{4}$
b Sample answer:



c $\frac{9}{12}$

4 **a** $\frac{3}{10} = \frac{6}{20}$ **b** $\frac{2}{7} = \frac{10}{35}$ **c** $\frac{18}{33} = \frac{6}{11}$ **d** $\frac{16}{48} = \frac{2}{6}$
e $-\frac{4}{5} = -\frac{24}{30}$ **f** $\frac{-8}{36} = \frac{-2}{9}$ **g** $-1\frac{3}{8} = -\frac{55}{40}$
h $3\frac{3}{4} = \frac{75}{20}$ **i** $\frac{1}{4} = \frac{2}{8} = \frac{5}{20} = \frac{8}{32} = \frac{20}{80} = \frac{25}{100}$
j $-\frac{3}{7} = \frac{-15}{35} = \frac{-21}{49} = \frac{-30}{70} = \frac{-300}{700} = -\frac{90}{210}$
k $1\frac{1}{5} = \frac{12}{10} = \frac{60}{50} = \frac{300}{250} = \frac{54}{45} = \frac{72}{60}$
l $-2\frac{1}{3} = -\frac{14}{6} = \frac{-35}{15} = \frac{-49}{21} = -\frac{21}{9} = -\frac{700}{300}$

5 **a** $\frac{8}{20}, \frac{22}{55}, \frac{4}{10}, \frac{10}{25}, \frac{18}{45}$ **b** $\frac{20}{15}, \frac{28}{21}, 1\frac{4}{12}$

6 **a** $\frac{3}{8} < \frac{1}{2}$ **b** $\frac{4}{7} > \frac{5}{14}$ **c** $\frac{-9}{10} < \frac{-4}{5}$
d $-\frac{5}{12} > -\frac{2}{3}$ **e** $1\frac{5}{6} > 1\frac{19}{24}$ **f** $\frac{17}{15} < \frac{7}{3}$
g $-\frac{2}{3} > -\frac{3}{4}$ **h** $\frac{5}{6} > \frac{4}{5}$ **i** $-\frac{11}{8} > -\frac{5}{3}$
j $2\frac{3}{4} < \frac{17}{6}$ **k** $-\frac{29}{3} < -9\frac{1}{2}$ **l** $3\frac{5}{12} > \frac{25}{8}$

7 **a** $\frac{7}{9}$ **b** $\frac{11}{4}$ **c** $-\frac{12}{5}$
d $-\frac{43}{6}$ **e** $3\frac{7}{11}$ **f** $-8\frac{1}{2}$
g $\frac{16}{3}$ **h** $-\frac{55}{9}$ **i** $3\frac{5}{7}$

8 **a** $\frac{4}{5}$ **b** $\frac{2}{3}$ **c** $\frac{7}{9}$
d $\frac{5}{8}$ **e** $\frac{10}{3}$ $\left(\text{or } 3\frac{1}{3}\right)$ **f** $\frac{11}{6}$ $\left(\text{or } 1\frac{5}{6}\right)$

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9 **a** $\frac{2}{5}$ **b** $-\frac{3}{7}$ **c** $-\frac{2}{5}$

d $-\frac{5}{3}\left(\text{or } -1\frac{2}{3}\right)$ **e** $\frac{8}{9}$ **f** $\frac{3}{7}$

g $3\frac{3}{4}$ **h** $-5\frac{5}{9}$ **i** $4\frac{3}{11}$

j $-1\frac{4}{7}$ **k** $2\frac{3}{8}$ **l** $-7\frac{5}{6}$

10 Sample answer: An equivalent fraction is created by dividing (or multiplying) the numerator and denominator by the same number, creating a fraction with the same value.

11 **a** $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{7}{5}$

b $\frac{3}{8}, \frac{9}{8}, 1\frac{1}{4}, \frac{7}{4}, 2\frac{5}{8}, 2\frac{3}{4}$

c $-\frac{9}{2}, -1\frac{9}{10}, -\frac{9}{5}, -1\frac{1}{2}, -\frac{2}{5}, -\frac{3}{10}$

12 **a** $\frac{19}{11}, 1\frac{5}{11}, \frac{13}{11}, \frac{9}{11}, \frac{5}{11}, \frac{3}{11}$

b $-\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4}, -1\frac{1}{4}, -\frac{4}{3}, -1\frac{1}{2}$

c $1\frac{1}{3}, \frac{5}{6}, \frac{1}{6}, -\frac{5}{12}, -\frac{11}{6}, -2\frac{1}{6}$

13 **a** $\frac{3}{5}$ **b** $\frac{2}{9}$ **c** $\frac{7}{24}$ **d** $\frac{3}{10}$

e $\frac{1}{6}$ **f** $\frac{2}{5}$ **g** $\frac{2}{7}$ **h** $\frac{3}{20}$

i $\frac{7}{15}$ **j** $\frac{3}{17}$ **k** $\frac{1}{30}$ **l** $\frac{2}{15}$

14 **a** $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{5}{8}$ **d** $\frac{2}{3}$

15 $\frac{1}{4}$

16 **a** $\frac{3}{4}, \frac{6}{8} = \frac{3}{4}, \frac{9}{12} = \frac{3}{4}, \frac{12}{16} = \frac{3}{4}, \frac{15}{20} = \frac{3}{4}, \frac{18}{24} = \frac{3}{4}, \frac{21}{28} = \frac{3}{4}, \frac{24}{32} = \frac{3}{4}, \frac{27}{36} = \frac{3}{4}$

b All of the fractions made by dividing a multiple of 3 by the corresponding multiple of 4 will simplify to $\frac{3}{4}$ as they are equivalent fractions.

c $\frac{7}{5}, \frac{14}{10} = \frac{7}{5}, \frac{21}{15} = \frac{7}{5}, \frac{28}{20} = \frac{7}{5}, \frac{35}{25} = \frac{7}{5}, \frac{42}{30} = \frac{7}{5}, \frac{49}{35} = \frac{7}{5}, \frac{56}{40} = \frac{7}{5}, \frac{63}{45} = \frac{7}{5}$

d All of the fractions made by dividing a multiple of 7 by the corresponding multiple of 5 will simplify to $\frac{7}{5}$ as they are equivalent fractions.

17 **a** $\frac{4}{7}$ **b** $-\frac{7}{11}$ **c** $\frac{1}{2}$ **d** $-\frac{4}{9}$

e $\frac{1}{3}$ **f** $-\frac{1}{8}$ **g** $\frac{1}{24}$ **h** $2\frac{14}{15}$

18 The first test

19 15

20 **a** **i** Since $\frac{4}{7} > \frac{1}{2}$ and $\frac{7}{15} < \frac{1}{2}$, we can compare the fractions using the benchmark $\frac{1}{2}$.

ii $\frac{4}{7} > \frac{7}{15}$

b **i** Since the numerators are the same, we can compare the denominators.

ii $\frac{15}{17} < \frac{15}{4}$

c **i** Both fractions are one piece away from 1; we can compare the difference between the fractions and the benchmark 1.

ii $\frac{99}{100} < \frac{100}{101}$

d **i** Both mixed numbers have a whole number part of 1, so we only need to compare the fraction parts. Since the numerators are the same, we can compare the denominators.

ii $1\frac{1}{8} > 1\frac{1}{11}$

e **i** Since the numerators and denominators are quite small, finding any equivalent fractions will be simple.

ii $1\frac{3}{5} < 1\frac{2}{3}$

f **i** Since 10 is a multiple of 5, we can compare using an equivalent fraction of $\frac{39}{5}$ with a denominator of 10.

ii $\frac{77}{10} < \frac{39}{5}$

21 **a** Ash (lower value is better rate in serving)

b 10 **c** 153

22 **a** D

b Answers will vary but may include some of the strategies from question **20**.

EX p64 2B Adding and subtracting fractions

1 **a** The first number is written more simply as $\frac{1}{7}$ and the operations + (−) can be replaced with −.

b $-\frac{2}{7}$

2 **a** $\frac{13}{17}$ **b** $\frac{2}{5}$ **c** 1 **d** $\frac{1}{3}$

3 **a** 15 **b** 14 **c** 30 **d** 24

e 15 **f** 20 **g** 12 **h** 18

4 **a** $\frac{21}{22}$ **b** $\frac{7}{20}$ **c** $1\frac{17}{30}$ **d** $\frac{7}{24}$

e $\frac{1}{30}$ **f** $1\frac{7}{12}$ **g** $\frac{7}{18}$ **h** $\frac{31}{40}$

5 **a** $\frac{4}{7}$ **b** $-2\frac{1}{3}$ **c** $-\frac{2}{8} = -\frac{1}{4}$ **d** $-\frac{2}{5}$

e $-1\frac{2}{3}$ **f** $-1\frac{1}{4}$ **g** $\frac{2}{9}$ **h** $-1\frac{1}{10}$

6 **a** $-\frac{1}{5}$ **b** $-\frac{3}{7}$ **c** $-\frac{9}{11}$

d $-1\frac{1}{6}$ **e** $-\frac{3}{14}$ **f** $\frac{1}{8}$

7 **a** $\frac{3}{8}$ **b** $3\frac{1}{4}$ **c** $4\frac{7}{15}$ **d** $2\frac{3}{22}$

e $7\frac{5}{12}$ **f** $2\frac{7}{10}$ **g** $-\frac{2}{3}$ **h** $-5\frac{7}{10}$

8 **a** $-\frac{1}{3}$ **b** 0 **c** $-\frac{2}{3}$ **d** $-\frac{3}{20}$

e $\frac{7}{18}$ **f** $-2\frac{6}{7}$ **g** $1\frac{11}{24}$ **h** $-\frac{1}{20}$

9 **a** **i** $1\frac{1}{2}$ **ii** $1\frac{1}{2}$ **iii** $1\frac{1}{2}$

b **i** $1\frac{1}{2}$ **ii** $1\frac{1}{2}$ **iii** $1\frac{1}{2}$

c Each diagonal adds to $1\frac{1}{2}$.

d They are all the same. **e** $1\frac{1}{2}$

10 a

$\frac{2}{15}$	$\frac{7}{15}$	$\frac{2}{5}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{15}$
$\frac{4}{15}$	$\frac{1}{5}$	$\frac{8}{15}$

b

$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{12}$
$\frac{1}{8}$	$\frac{5}{24}$	$\frac{7}{24}$
$\frac{1}{3}$	$\frac{1}{24}$	$\frac{1}{4}$

c

$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{3}$
$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$
0	$-\frac{5}{6}$	$\frac{1}{3}$

11 a Positive **b** Negative **c** Positive **d** Zero
e Negative **f** Negative **g** Zero **h** Negative

12 a $1\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $-\frac{1}{2}$ **d** $-1\frac{1}{2}$
e $10\frac{1}{4}$ **f** $-10\frac{1}{4}$

13 It is better to convert to improper fractions first when the fraction part of the mixed number you are subtracting is greater than the fraction part of the mixed number you are subtracting from.

14 $\frac{1}{3} + 1\frac{3}{4} = 2\frac{1}{12}$ L; the container is not large enough.

15 $4\frac{7}{12}$ hours

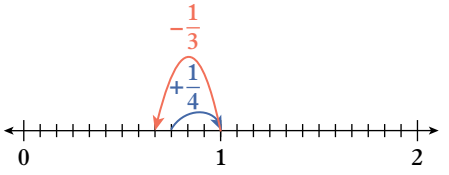
16 a $\frac{3}{10}$ **b** $\frac{29}{30}$ **c** $\frac{1}{30}$

17 $\frac{1}{6}$

18 a $\frac{13}{125}$ inches **b** The $\frac{41}{250}$ inch screw

19 a i B **ii** D **iv** A **v** C

E



iii $\frac{3}{2} + \frac{1}{6} = \left(\frac{3}{2} + \frac{3}{6}\right) - \frac{2}{6} = 2 - \frac{1}{3} = 1\frac{2}{3}$

b Each calculation partitioned the second fraction into the part that would make a whole number and the remainder to avoid changing denominators multiple times. This is the compensation method for fractions.

20 iv

EX p70 2C Multiplying and dividing fractions

1 a $\frac{9}{20} \times \frac{5}{3}$ **b** $\frac{9}{20} \times \frac{15}{7}$ **c** $\frac{20}{9} \times \frac{7}{15}$ **d** $\frac{20}{9} \times \frac{15}{7}$
e $\frac{2}{9} \times \frac{4}{7}$ **f** $\frac{2}{9} \times \frac{21}{5}$ **g** $\frac{9}{2} \times \frac{5}{21}$ **h** $\frac{9}{2} \times \frac{21}{5}$

2 a $\frac{3}{20}$ **b** $\frac{6}{5}$ $\left(\text{or } 1\frac{1}{5}\right)$ **c** $\frac{7}{10}$ **d** $\frac{12}{35}$
e $\frac{1}{50}$ **f** 9 **g** $\frac{20}{21}$ **h** 1

3 a $\frac{9}{8}$ or $1\frac{1}{8}$ **b** 4 **c** 10 **d** $\frac{3}{4}$
e $12\frac{4}{5}$ or $\frac{64}{5}$ **f** 6 **g** $2\frac{5}{8}$ or $\frac{21}{8}$
h $32\frac{1}{2}$ or $\frac{65}{2}$ **i** 144 **j** $60\frac{1}{5}$ or $\frac{301}{5}$

4 a $\frac{10}{21}$ **b** $\frac{5}{8}$ **c** $\frac{3}{8}$ **d** $4\frac{1}{2}$ or $\frac{9}{2}$
e 7 **f** $\frac{1}{7}$ **g** $1\frac{4}{5}$ or $\frac{9}{5}$ **h** 40

5 a $6\frac{3}{5}$ or $\frac{33}{5}$ **b** $\frac{9}{16}$ **c** $\frac{1}{2}$ **d** $\frac{2}{9}$
e $1\frac{1}{2}$ or $\frac{3}{2}$ **f** $\frac{2}{7}$ **g** $\frac{17}{28}$ **h** 14
i $\frac{28}{17}$ or $1\frac{11}{17}$ **j** $4\frac{5}{27}$ or $\frac{113}{27}$

6 a 7 hours **b** \$36 **c** 14 kg
d 32 cm **e** 6 days **f** 12 mm

7 a $\frac{4}{11}$ **b** $-\frac{1}{3}$ **c** $-1\frac{1}{5}$
d $\frac{108}{245}$ **e** $-\frac{7}{12}$ **f** $-\frac{5}{27}$
g -15 **h** $\frac{2}{3}$ **i** $-1\frac{1}{2}$

8 a $1\frac{5}{11}$ **b** $\frac{3}{2}$ $\left(\text{or } 1\frac{1}{2}\right)$ **c** $-\frac{1}{12}$ **d** $8\frac{11}{12}$

9 a -1 **b** $\frac{3}{7}$ **c** $-\frac{13}{14}$
d -1 **e** $\frac{1}{3}$ **f** $-\frac{3}{14}$

10 66

11 a i 200 g butter, 1 cup sugar, 4 large eggs, 3 cups flour, 2 tsp baking powder, 1 cup milk, $1\frac{1}{2}$ cups cocoa powder, 2 cups boiling water

ii 50 g butter, $\frac{1}{4}$ cup sugar, 1 large egg, $\frac{3}{4}$ cup flour, $\frac{1}{2}$ tsp baking powder, $\frac{1}{4}$ cup milk, $\frac{3}{8}$ cup cocoa powder, $\frac{1}{2}$ cup boiling water

iii $66\frac{2}{3}$ g butter, $\frac{1}{3}$ cup sugar, $1\frac{1}{3}$ large eggs, 1 cup flour, $\frac{2}{3}$ tsp baking powder, $\frac{1}{3}$ cup milk, $\frac{1}{2}$ cup cocoa powder, $\frac{2}{3}$ cup boiling water

b $1\frac{1}{8}$ cups icing sugar, $112\frac{1}{2}$ g butter, 90 g chocolate, $\frac{3}{8}$ cup thickened cream

12 a 8 **b** 29 kg

13 a 24 buttons **b** 25 red buttons **c** $\frac{5}{4}$

14 a 36 **b** 147 **c** $42\frac{2}{3}$ **d** $36\frac{4}{5}$

15 a $5\frac{2}{19}$ **b** 59 **c** $15\frac{2}{5}$ **d** $4\frac{6}{65}$

16 a $\frac{8}{125}$ **b** $\frac{16}{81}$ **c** $-\frac{8}{27}$ **d** $\frac{81}{49}$ **e** $\frac{1000}{27}$

17 a $\frac{4}{5}$ **b** $\frac{7}{3}$ **c** $\frac{9}{10}$ **d** $\frac{5}{3}$ **e** $\frac{11}{6}$

18 a $18\frac{17}{18}$ **b** $78\frac{8}{15}$ **c** $39\frac{1}{5}$ **d** $18\frac{115}{168}$

19 By writing a division calculation as a multiplication by the reciprocal, we can apply the distributive law to the dividend (left fraction) by multiplying the whole part and the fraction part of the sum by the reciprocal of the divisor (right fraction). We can then confirm this is equivalent to dividing each part by the original divisor by writing the multiplications as divisions by the reciprocal. However, we cannot apply the distributive law to the divisor as it is no longer the sum of 5 and $\frac{2}{3}$ after we get its reciprocal.

$$3\frac{9}{10} \div 5\frac{2}{3} = 3\frac{9}{10} \div \frac{17}{3} = 3\frac{9}{10} \times \frac{3}{17}$$
$$= \left(3 \times \frac{3}{17}\right) + \left(\frac{9}{10} \times \frac{3}{17}\right)$$
$$= \left(3 \div 5\frac{2}{3}\right) + \left(\frac{9}{10} \div 5\frac{2}{3}\right)$$

20 a C and E

b Multiplying fractions that do not have a common denominator already will generally have larger numerators and denominators to then multiply after finding the equivalent fractions with common denominators. The quotient will also then need to be simplified. The same applies to raising a fraction to a power.

c Addition and subtraction need the same-sized pieces (denominator) to calculate the sum or difference. When dividing fractions, if the divisor has a common denominator with the dividend, then they will divide to 1: $\frac{5}{10} \div \frac{3}{10} = \frac{5}{\cancel{10}^{1}} \times \frac{\cancel{10}^{1}}{3} = \frac{5}{3}$.

Checkpoint

1 a $\frac{6}{10} = \frac{48}{80}$ **b** $\frac{55}{20} = 2\frac{3}{4}$ **c** $\frac{63}{18} = \frac{28}{8}$ **d** $\frac{6}{5} = \frac{-18}{-15}$

2 a $\frac{4}{5}$ **b** $\frac{4}{3}$ **c** $8\frac{1}{4}$ **d** $-\frac{5}{9}$

3 a $\frac{2}{3} < \frac{3}{4}$ **b** $\frac{7}{9} > -\frac{8}{9}$ **c** $-\frac{17}{10} > -\frac{9}{5}$ **d** $\frac{30}{36} > \frac{36}{54}$

4 a 12 **b** 18 **c** 24 **d** 33

5 a $\frac{16}{15} = 1\frac{1}{15}$ **b** $\frac{35}{9} = 3\frac{8}{9}$

c $\frac{56}{25} = 2\frac{6}{25}$ **d** $-\frac{99}{28} = -3\frac{15}{28}$

6 a $-\frac{1}{12}$ **b** $-\frac{101}{15} = -6\frac{11}{15}$

c $\frac{43}{48}$ **d** $-\frac{25}{21} = -1\frac{4}{21}$

7 a 10 **b** $-\frac{32}{27} = -1\frac{5}{27}$

c $-2\frac{1}{5}$ **d** $\frac{40}{9} = 4\frac{4}{9}$

8 a $\frac{5}{3}$ **b** 7 **c** $\frac{1}{9}$ **d** $\frac{3}{17}$

9 a $\frac{3}{5}$ **b** -7 **c** $-\frac{16}{35}$ **d** $\frac{8}{9}$

10 a $\frac{27}{25} = 1\frac{2}{25}$ **b** $-\frac{7}{10}$ **c** $-\frac{65}{36} = -1\frac{29}{36}$ **d** $\frac{27}{175}$

11 a 65 GB **b** $7\frac{1}{2}$ L **c** 750 g **d** \$4.25

EX p78

2D Decimals

1

	Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
a		0	.	0	0	6	5
b		0	.	6	5	0	0
c		6	.	0	0	5	
d	6	0	.	5			

2 a 5 thousandths or $\frac{5}{1000}$ **b** 5 tens or 50

c 5 ones or 5 **d** 5 tenths or $\frac{5}{10}$

e 5 ten thousandths or $\frac{5}{10\,000}$

f 5 hundredths or $\frac{5}{100}$

3 a 3 **b** 3 **c** 1 **d** 2 **e** 4 **f** 5

4 a 0.104, 0.14, 0.142, 0.2, 0.214, 0.41

b 8.072, 8.2, 8.702, 8.97, 9.782, 9.87

c –15.40, –10.45, 10.54, 15.04, 105.4, 150.4

d –800.8, –800.08, –800.07, –799.8, 799.08, 800.7

5 a 13.713, 13.7, 13.31, 13.173, 13.137, 13.07

b 0.685, 0.658, 0.6508, 0.65, 0.6085, 0.6058

c 555.55, 5.5555, 5.55000, 5.005 00, –50.0505, –50.5050

d 999.90, 999.099, 100, –99.009, –99.90, –100

6 Comparing each place value from left to right, 9.46 contains 6 hundredths whereas 9.452 only contains 5 hundredths. (The first two place values are identical.)

7 a 25.68 **b** −673.82 **c** 0.78

d −101.02 **e** 8148.26 **f** −54.26

g 330.41 **h** −4.85 **i** 16.00

8 a 57.23 **b** 8.914 **c** 92.0871

d −0.757 **e** −135.2 **f** 65.071 23

g 7.510 **h** −208.0099 **i** 1.727 73

j 845.8 **k** −0.1235 **l** −0.123 456 8

9 a $1\frac{3}{100}$ **b** $2\frac{9}{10}$ **c** $3\frac{7}{1000}$ **d** $-\frac{1}{10\,000}$

e $10\frac{7}{10}$ **f** $-\frac{3}{100\,000}$ **g** $12\frac{9}{1\,000\,000}$

h $-\frac{7}{10\,000\,000}$

10 a $\frac{3}{25}$ **b** $-\frac{27}{50}$ **c** $\frac{803}{1000}$ **d** $-\frac{1}{25}$ **e** $\frac{1}{40}$

f $-\frac{3}{5000}$ **g** $-\frac{37}{50}$ **h** $\frac{63}{250}$ **i** $-\frac{3}{2000}$

j $-\frac{6}{125}$ **k** $\frac{17}{100\,000}$ **l** $-\frac{17}{20}$

11 a 0.7 **b** −0.29 **c** 0.451 **d** −0.2047

e 0.04 **f** −0.009 **g** 0.0005 **h** −0.063

12 a 0.8 **b** 0.625 **c** −0.5 **d** 1.25

e 0.125 **f** −0.4 **g** 0.35 **h** −0.36

i 0.025 **j** −0.6 **k** 0.22 **l** −0.3125

13 Rosa; the digit in the fourth decimal place is 5 so you must round up.

14 Jayden; the digit in the third decimal place is 6 so you must round up. Adding one to the digit in the second decimal place gives 10 which means you write 0 in the second decimal place and add one to the first decimal place (7 + 1 = 8).

15 Kota; Peter correctly rounded to 16 but did not write the two decimal places that were asked for, so it appears he rounded to the nearest whole number.

16 Sabine has not rounded directly to one decimal place. For this number, rounding to two decimal places meant that the digit in the hundredths place changed from 4 to 5 which results in rounding up rather than rounding down.

17 0.7 cm

18 a 1.568 m **b** 1.657 m

c 1.568 m, 1.58 m, 1.615 m, 1.63 m, 1.651 m, 1.657 m

d 1.65 m, 1.63 m, 1.57 m, 1.62 m, 1.58 m, 1.66 m

e 1.57 m, 1.58 m, 1.62 m, 1.63 m, 1.65 m, 1.66 m; the measurements are still all different and there is no change to the order.

f 1.7 m, 1.6 m, 1.6 m, 1.6 m, 1.6 m, 1.7 m

g 1.6 m, 1.6 m, 1.6 m, 1.6 m, 1.7 m, 1.7 m; there are only two different measurements.

19 **a** 0.02 mm **b** 0.018 mm **c** 0.0184 mm

20 **a** **i** 0.3 mm **ii** 0.28 mm **iii** 0.285 mm

b $\frac{569}{2000}$ mm

21 **a** \$42.35 **b** \$207.60 **c** \$7.25

d \$15.05 **e** \$60.70 **f** \$100.00

22 Sample answer: 0.918273645

EX p84 2E Adding and subtracting decimals

1 **a** $2 + 0.5$ **b** $-1.5 + 4$

c $1.4 - 3.2$ **d** $-0.8 - 0.6$

2 **a** 10.1 **b** 26.25

c 2.052 **d** 29.38

3 **a** 247.96 **b** 33.7686 **c** 690.984

d 106.5953 **e** 0.2776 **f** 119.0154

4 **a** 82.83 **b** 785.654 **c** 0.1344 **d** 26.251

5 **a** 2.7312 **b** 177.196 **c** 3794.295

d 64668.44 **e** 8999.445 **f** 1.9999

6 **a** -2.6 **b** -9 **c** 2.6

d 48.266 **e** -3.9078 **f** -495.238

7 **a** -3.7 **b** -13.26 **c** 3.7

d -61.783 **e** -273.7973 **f** -1.209

8 **a** -0.9 **b** -8.74

c -82.85 **d** -0.2488

9 **a** -7.6 **b** 5.21

c 1.1 **d** $-88.189\,34$

10 Estimate: 12 km, exact: 12.025 km

11 **a** **i** 368 cm **ii** 382.9 cm

b Sample answer: Left hand; Lee may be right-handed and use her left hand less, so there is less wear on the nails.

c 750.9 cm

12 **a** D

b When you expand the brackets using the distributive law, the calculation becomes $7.2 - 9.4 - 2.3$; however, the original calculation calls for $+ 2.3$. A similar expression that is equivalent to the original calculation is $7.2 + (-9.4 + 2.3)$.

c Not when one subtraction follows another. For example, the correct answer to $10 - 5 - 3$ is 2, but if the second subtraction $(5 - 3)$ is performed first the result is $10 - 2 = 8$, which is incorrect.

13 **a** -13.3168 **b** -40.911

c 0.0472 **d** -163.0612

14 **a** Yes, \$20.61 extra **b** 3 weeks

15 The sum will have at most x decimal places.

16 **a**

$$-(3.12 + 4.2) = -3.12 + (-4.2)$$
$$= -3.12 - 4.2$$

If you find the sum of 3.12 and 4.2, then multiply by negative 1, you get the same answer as $-3.12 - 4.2$.

b

$$-(4.2 - 3.12) = -4.2 - (-3.12)$$
$$= -4.2 + 3.12$$
$$= 3.12 + (-4.2)$$
$$= 3.12 - 4.2$$

If you subtract 3.12 from 4.2, then multiply by negative 1, you get the same answer as $3.12 - 4.2$.

17 **a** $21 - 13.9 = 7.1$

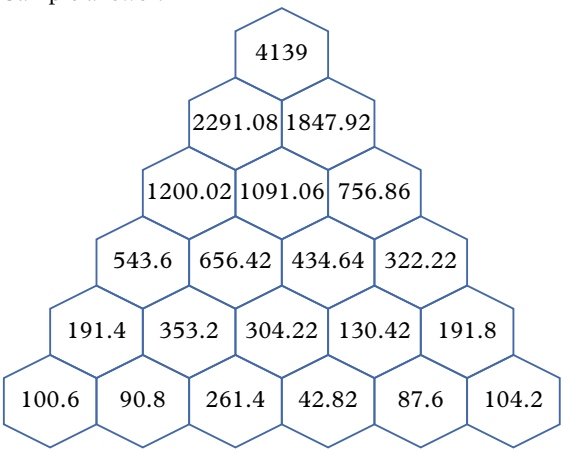
b $161.67 - 100.28 = 61.39$

c $9.319 - 14.78 = -5.461$

d $92.08 - 181.88 = -89.8$

18 **a** 81.178° clockwise **b** 519.628°

19 Sample answer:



EX p91 2F Multiplying and dividing decimals

1 **a** 0.052 **b** 1040 **c** 9.21

d 0.034 **e** 0.000 57 **f** 0.045 78

2 **a** 194.8 **b** 1222.4 **c** 52 058.1

d 2.947 59

3 **a** 43.831 **b** -30.864 **c** 2281.44

d 112.5234 **e** -4.232 **f** $-2.024\,204$

g 7.890 54 **h** 1124.4616

4 **a** 18.149 **b** -1.3448 **c** -8.3249

d 21.6985 **e** 0.591 25 **f** $-0.156\,75$

g 0.0256 **h** $-27.614\,28$

5 **a** 10 **b** 35.68

c To keep an equivalent division problem

d 8.92

6 **a** $72.4 \div 2$ **b** $49.02 \div (-6)$ **c** $1976.4 \div 3$

d $-172.8 \div (-8)$ **e** $1436.8 \div 4$ **f** $-749.1 \div 6$

g $48.72 \div (-15)$ **h** $280.8 \div 12$ **i** $-163.8 \div (-9)$

7 **a** 36.2 **b** -8.17 **c** 658.8

d 21.6 **e** 359.2 **f** -124.85

g -3.248 **h** 23.4 **i** 18.2

8 **a** 1.875 **b** 11.552 **c** 10.8

d 19.322 **e** -30.568 **f** $-49.998\,078$

9 **a** \$29.40 **b** \$5.85 **c** \$3.37

d \$9.88 **e** \$4.94 **f** \$11.99

10 **a** $65 **b** $65.45 **c** $34.55

11 **a** −0.000 544 **b** −59.094 09

c 0.0004 **d** −8000

12 **a** **i** $0.60 **ii** $2.89 **iii** $18.95

b **i** $4.65 **ii** $6.85 **iii** $13.61

c **i** $12.95 **ii** $1.59 **iii** $1.95

13 **a** box: $0.75, individual: $0.72

b Buying individual pencils, as the cost per pencil is less

14 **a** 400 g pack: $0.92, loose: $0.99

b William, as the cost is cheaper per 100 g

c Someone may need more or less than the 400 g in a package.

15 **a** −0.064 **b** −0.000 000 001

c 0.0081 **d** 97.656 25

16 **a** $6.3 \times 2 = 12.6$ **b** $9.2 \times 5 = 46$

c $0.49 \times 4 = 1.96$ **d** $7.4 \times \frac{4}{5} = 5.92$

e $0.041 \times \frac{2}{5} = 0.0164$ **f** $10.21 \times \frac{4}{25} = 1.6336$

17 perimeter = 20.27 cm, area = 18.5097 cm^2

18 Sample answer: $5.07 - 3.96 \times 1.28 = 0.0012$

EX p96

2G Terminating, non-terminating and recurring decimals

1 **a** N **b** T **c** N **d** T **e** N **f** N

g N **h** T **i** N **j** T **k** N **l** T

2 **c**, **f**, **g**, **k**

3 Sample answer: A terminating decimal 'terminates' or stops after a number of decimal places. A non-terminating decimal has an endless (infinite) number of decimal places. A recurring decimal is a special type of non-terminating decimal in which some or all of the digits after the decimal point form a recurring (repeating) pattern.

4 **a** $0.\dot{9}$ (or $0.\overline{9}$) **b** $-4.\overline{36}$ (or $-4.\dot{3}\dot{6}$)

c $87.2\dot{5}$ **d** $3.\overline{476}$ (or $3.\dot{4}7\dot{6}$)

e $-0.738\dot{2}$ **f** $16.19\dot{6}$

g $483.\dot{8}$ **h** $-6.72\overline{91}$ (or $6.72\dot{9}\dot{1}$)

i $-0.18\overline{367}$ (or $-0.18\dot{3}67\dot{}$)

j $1.\overline{35628}$ (or $1.\dot{3}562\dot{8}$)

k $28.\overline{70}$ (or $28.\dot{7}\dot{0}$)

l $-9.230714\dot{}$

5 **a** 0.111 111 111 111…

b 5.377 777 777 777…

c −14.828 282 828 282…

d −21.569 696 969 696…

e 8.104 104 104 104…

f 0.651 365 136 513…

g −232.684 444 444 444…

h 44.761 359 359 359…

i −94.570 202 020 202…

j −1.739 247 392 473…

k 11.812 111 111 111…

l −6.904 040 404 040…

m 14.241 341 341 341…

n 0.562 062 062 062…

o −139.274 527 452 745…

6 **a** 0.7 **b** −0.5 **c** $0.\dot{3}$ **d** 1.25

e $0.\overline{18}$ **f** −0.625 **g** $2.\dot{6}$ **h** −0.75

7 **a** −1.125 **b** 1.4

c $2.8\dot{3}$ **d** $-1.\overline{142857}$

e 3.6 **f** $-4.\dot{1}$ **g** $1.\overline{72}$ **h** $-2.41\dot{6}$

8 **a** **i** a, b, d, f, h **ii** c, e, g

b c, e, g **c** No

9 **a** $\frac{1}{2}$ **b** $\frac{4}{1}$ **c** $6\frac{1}{5}$ $\left(\text{or } \frac{31}{5}\right)$

d $\frac{37}{100}$ **e** $\frac{18}{1}$ **f** $2\frac{509}{1000}$ $\left(\text{or } \frac{2509}{100}\right)$

10 **a** They can be expressed as fractions.

b Yes, because they can be expressed as fractions.

11 **a** rational **b** rational **c** rational

d irrational **e** rational **f** irrational

g rational **h** rational **i** rational

j rational **k** irrational **l** rational

12 **a** Sample answer: 3.141 592 654

b **i** 3.14 **ii** 3.1416

iii 3.141 59 **iv** 3.141 593

13 **a**

Unit fraction	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{11}$
Decimal	$0.\dot{1}$	0.1	0.2	0.5	$0.1\dot{6}$	$0.\overline{09}$
Denominator's prime factorisation	3^2	2×5	5	2	2×3	11
Is 2 or 5 a factor of the denominator?	No	Yes	Yes	Yes	Yes	No
Any prime factors other than 2 and/or 5?	Yes	No	No	No	Yes	Yes
Is the decimal terminating?	No	Yes	Yes	Yes	No	No

Unit fraction	$\frac{1}{4}$	$\frac{1}{20}$	$\frac{1}{15}$	$\frac{1}{8}$	$\frac{1}{15}$
Decimal	0.25	0.05	$0.0\dot{6}$	0.125	$0.08\dot{3}$
Denominator's prime factorisation	2^2	$2^2 \times 5$	3×5	2^3	$2^2 \times 3$
Is 2 or 5 a factor of the denominator?	Yes	Yes	Yes	Yes	Yes
Any prime factors other than 2 and/or 5?	No	No	Yes	No	Yes
Is the decimal terminating?	Yes	Yes	No	Yes	No

b The fraction has a terminating decimal when the only factors of the denominator are powers of 2 or 5.

c **i** terminating, 0.04

ii non-terminating, $0.041\dot{6}$
iii non-terminating, $0.0\dot{5}$
iv terminating, 0.025
v terminating, 0.008

d **i** non-terminating, $3.\overline{428\,571}$
ii terminating, 0.21875
iii terminating, 0.7
iv non-terminating, $0.1\overline{53\,846}$
v terminating, 2.8

14 **a** Sample answer: 1.414 213 562
b **i** 1.41 **ii** 1.4142
iii 1.414 214 **iv** 1.414 213 6
c One possible answer is 1.732 050 807.
d yes
e **i** 1.73 **ii** 1.7321
iii 1.732 05 **iv** 1.732 051

15 **a** **i** $0.\dot{1}$ **ii** $0.\dot{2}$ **iii** $0.\dot{3}$ **iv** $0.\dot{4}$
b recurring decimal
c **i** $0.\dot{5}$ **ii** $0.\dot{6}$ **iii** $0.\dot{7}$ **iv** $0.\dot{8}$

16 **a** 9, 18, 27, 36, 45, 54, 63, 72, 81, 90
b i $0.\overline{09}$ **ii** $0.\overline{18}$ **iii** $0.\overline{27}$ **iv** $0.\overline{36}$
c The recurring digits are the multiples of 9. $\frac{9}{11} = 0.\overline{81}$

17 **a** 0.333 33…, 0.666 66…
b 0.999 99… **c** 1. 0.999 99… is equivalent to 1.

18 **a** 3.160 494, 3.140 845, 3.142 857, 3.141 593, 3.141 600, 3.141 057, 3.142 732
b $\frac{355}{113}$

19 Sample answer: It is an irrational number (non-terminating but not recurring decimal), so it cannot be written as an exact fraction or decimal.

20 **a** 1D, 2C, 3E, 4F, 5B, 6A
b To find another number with the same recurring part, so that we could subtract to eliminate the decimal part.
c **i** $\frac{2}{9}$ **ii** $\frac{7}{9}$ **iii** $\frac{7}{3}$ **iv** $\frac{17}{3}$

CHAPTER 2 review

Multiple choice

1 B **2** C **3** C **4** B **5** E **6** B
7 D **8** C **9** E **10** C **11** C

Short answer

1 **a** $\frac{7}{12}$ **b** $-1\frac{7}{8}\left(\text{or } -\frac{15}{8}\right)$
c $-3\frac{9}{16}$ **d** $5\frac{2}{3}$

2 **a** $\frac{29}{35}$ **b** $\frac{13}{30}$ **c** $6\frac{1}{33}$ **d** $2\frac{29}{35}$ EX p109

3 **a** $\frac{6}{35}$ **b** $\frac{1}{2}$ **c** $21\frac{1}{4}$ **d** $\frac{9}{4}$

4 **a** $\frac{6}{7}$ **b** $-\frac{2}{3}$ **c** $-\frac{299}{9} = -33\frac{2}{9}$ **d** $\frac{18}{35}$

5 **a** $-\frac{1}{4}$ **b** $\frac{1}{15}$ **c** $-\frac{6}{7}$
d $4\frac{1}{2}$ **e** $-\frac{4}{35}$ **f** $-\frac{7}{8}$

6 **a** 24 km/h **b** $\frac{1}{3}$ of a cup
c 70 metres **d** 3 minutes

7 **a** 0.0012, 0.120, 0.123, 0.2, 0.213, 0.231
b 8.16, 8.9, 8.916, 9.16, 9.6, 9.916
c −5, −0.5, −0.05, 0.05, 0.50, 5
d −14.2, −12.04, −10.42, −2.41, −2.14, −0.421

8 **a** 0.717, 0.71, 0.7, 0.107, 0.0701, 0.017
b 2.721, 2.72, 2.7, 2.27, 2.127, 2.1
c 90, 89, 80.9, −0.89, −0.899, −9.8
d −0.01, −0.099, −0.1, −0.9, −0.99, −1

9 **a** 13.22 **b** −0.0012 **c** 54.546 **d** −181.2

10 **a** 143.67 **b** 23.48
c 265.7842 **d** 989.5993

11 **a** −3.83 **b** 4.182 **c** −2.313 **d** 114.78

12 **a** 25.2512 **b** −10.9315
c −19.722 **d** 416.758 52

13 **a** **i** terminating **ii** non-terminating
iii terminating **iv** non-terminating
b only part **ii**

14 **a** $0.\dot{3}$ or $0.\overline{3}$ **b** $5.\overline{385}$ or $5.\dot{3}8\dot{5}$
c $-45.68\overline{57}$ or $45.68\dot{5}\dot{7}$
d $-0.092\overline{451}$ or $-0.092\dot{4}5\dot{1}$

15 **a** −0.233333333333… **b** 3.272727272727…
c 46.358787878787…
d −17.364854854854…

16 **a** $0.0\dot{3}$ **b** $-0.\overline{571428}$ **c** $-9.\dot{5}$ **d** $1.\overline{54}$

Analysis

1 **a** **i** Wednesday **ii** Friday
b 5.8°C
c Tuesday: 6.3°C, Wednesday: 8.7°C, Thursday: 5.9°C, Friday: 4.5°C, Saturday: 5.7°C, Sunday: 5.1°C
d Wednesday
e **i** −1.8°C **ii** 4.2°C
f 6.0°C

2 **a** 399 students
b **i** 378 students **ii** 198 students
c **i** $\frac{67}{266}$ **ii** 3 students above one-quarter
d **i** 417 students **ii** $\frac{139}{532}$
e 1600 students

CHAPTER 3 Percentages, ratios and rates

3A Percentages, decimals and fractions

1 **a** 60% **b** 40% **c** 60% **d** 52%

2 **a** 40% **b** 60% **c** 40% **d** 48%

3 **a** 24% **b** 77% **c** 83% **d** 58%
e 7% **f** 46%

4 **a** $\frac{17}{100}$, 0.17 **b** $\frac{21}{100}$, 0.21

c $\frac{7}{20}$, 0.35 **d** $\frac{2}{25}$, 0.08

e $2\frac{3}{20}$, 2.15 **f** $\frac{49}{50}$, 0.98

g $1\frac{37}{50}$, 1.74 **h** $\frac{7}{25}$, 0.28

5 a $\frac{7}{100}$, 0.07 **b** $\frac{31}{1000}$, 0.031

c $1\frac{31}{50}$, 1.62 **d** $\frac{721}{1000}$, 0.721

e $3\frac{47}{100}$, 3.47 **f** $\frac{8109}{10\,000}$, 0.8109

g $\frac{1057}{10\,000}$, 0.1057 **h** $\frac{339}{5000}$, 0.0678

6 a 71%, $\frac{71}{100}$ **b** 28%, $\frac{7}{25}$

c 5%, $\frac{1}{20}$ **d** 8%, $\frac{2}{25}$

e 90%, $\frac{9}{10}$ **f** 31%, $\frac{31}{100}$

g 54%, $\frac{27}{50}$ **h** 95%, $\frac{19}{20}$

7 a 36.9%, $\frac{369}{1000}$ **b** 24.8%, $\frac{31}{125}$

c 1006%, $10\frac{3}{50}$ **d** 81%, $\frac{81}{100}$

e 705.7%, $7\frac{57}{1000}$ **f** 185.9%, $1\frac{859}{1000}$

g 1210%, $12\frac{1}{10}$ **h** 24.46%, $\frac{1223}{5000}$

8 a i $\frac{16}{20}$ **ii** $\frac{80}{100}$ **iii** 80%

b i $\frac{15}{20}$ **ii** $\frac{75}{100}$ **iii** 75%

c i $\frac{7}{10}$ **ii** $\frac{70}{100}$ **iii** 70%

d i $\frac{6}{10}$ **ii** $\frac{60}{100}$ **iii** 60%

9 a 0.6, 60% **b** 0.7, 70%

c 0.085, 8.5% **d** 0.92, 92%

e 1.3, 130% **f** 1.5, 150%

g 0.0275, 2.75% **h** 3.5, 350%

10 a 225% **b** 600.5% **c** 730% **d** 375%

e 428% **f** 580% **g** 102.4% **h** 1326%

11 a 62.5% or 0.625 **b** 67.5% or 0.675

c $533\frac{1}{3}\%$ or $5.3\dot{3}$ **d** 18.75% or 0.1875

e 8.8% or 0.088 **f** $322\frac{2}{9}\%$ or $3.2\dot{2}$

g $73\frac{1}{3}\%$ or $0.7\dot{3}$ **h** $142\frac{6}{7}\%$ or $1.\overline{428571}$

12 a $\frac{1}{400}$, 0.0025 **b** $\frac{11}{300}$, $0.03\dot{6}$

c $\frac{1}{160}$, 0.00625 **d** $\frac{1}{40}$, 0.025

e $\frac{11}{125}$, 0.088 **f** $\frac{47}{300}$, $0.15\dot{6}$

g $\frac{8}{225}$, $0.03\dot{5}$ **h** $\frac{79}{600}$, $0.131\dot{6}$

13 a $\frac{3}{5} = \frac{60}{100} = 60\%$ **b** $\frac{9}{20} = \frac{45}{100} = 45\%$

c $\frac{17}{25} = \frac{68}{100} = 68\%$ **d** $\frac{46}{200} = \frac{23}{100} = 23\%$

e $\frac{17}{20} = \frac{85}{100} = 85\%$ **f** $\frac{1}{5} = \frac{20}{100} = 20\%$

g $\frac{144}{600} = \frac{24}{100} = 24\%$ **h** $\frac{75}{250} = \frac{3}{10} = \frac{30}{100} = 30\%$

14 a 80% **b** 81% **c** 92%

d 88% **e** 71.5% **f** $66\frac{2}{3}\%$

15 a Both are correct.

b Sample answers include: shade 30 out of 200, shade 60 out of 400.

16 As the capacity of each container is different, 50% (or 50 mL out of every 100 mL) would be a different volume of liquid for each.

17 a 66% **b** $55\frac{1}{2}\%$ **c** $79\frac{2}{3}\%$ **d** 83.1%

e 24% **f** $4\frac{2}{5}\%$ **g** 74.7% **h** 20.44%

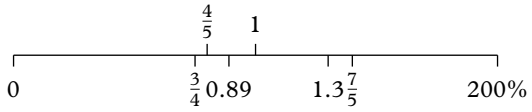
18 a $\frac{3}{13}$ **b** 0.2308 **c** 23.08%

d i 30.77% **ii** 7.69%

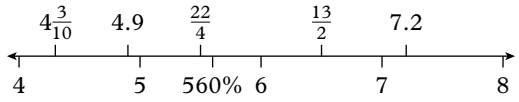
iii 38.46% **iv** 84.62%

19 $\frac{21}{400}$

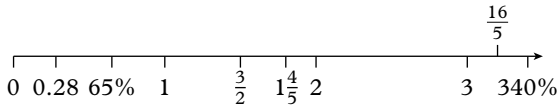
20 a



b



c



21 0, $\frac{10}{11}\%$, 0.52, $\frac{7}{11}$, 0.76, $\frac{88}{115}$, 100%

22 a 10 cm = 100 mm, so a length of 1 mm would represent 1%. For 80%, shade 80 mm (or 8 cm); for 92%, shade 92 mm (or 9.2 cm); for 25%, shade 25 mm (or 2.5 cm); for 17%, shade 17 mm (or 1.7 cm).

b Double the percentage amount. For example, 80% means 80 out of 100 or 160 out of 200. Shade 160 of the 200 squares in the grid to represent 80%.

3B Calculating percentages

EX p116

1 a 25% **b** 90% **c** 76% **d** 75%

e 120% **f** 125%

2 a i 80% **ii** 20%

b i 70% **ii** 30%

c i 30% **ii** 70%

d i 90% **ii** 10%

3 a i $17\frac{1}{2}\%$ **ii** 17.5%

b i $\frac{1}{5}\%$ **ii** 0.2%

c i $36\frac{4}{5}\%$ **ii** 36.8%

d i $28\frac{1}{2}\%$ **ii** 28.5%

e i $72\frac{1}{2}\%$ **ii** 72.5%

f i $587\frac{1}{2}\%$ **ii** 587.5%

4 a 23% **b** 25% **c** 35%

d 70% **e** 7.5%

5 **a** $1\frac{1}{4}$% or 1.25% **b** 10%
c 25% **d** 75%
e 10% **f** $6\frac{1}{4}$% or 6.25%
g 3% **h** 25%

6 **a** 84 **b** 756 **c** 9 **d** 1152
e 725 **f** 112.5

7 **a** 453.6 m **b** 135 seconds
c 231 cm **d** 100.8 L
e 76 buttons **f** 11.76 mL

8 **a** $360 **b** $1250 **c** $100 **d** $1575
e $16 **f** $576.45 **g** $110.50 **h** $448.50
i $69.30 **j** $129.15 **k** $41.29 **l** $3690

9 **a** $4440 **b** 114 kg **c** $1030.96
d $786.25 **e** $89.03 **f** 67.5 min
g 5.84 km **h** 381.84 L

10 **a** $400 **b** $850 **c** $3200
d $900

11 **a** 460 **b** 1750 L
c 105 songs **d** 9 minutes

12 **a** $\frac{89}{420}$ **b** 21.19%
c **i** 26.97% **ii** 74.17% **iii** 127.27%
d Sample answer: Big Jake may be 252 cm and Thumbelina may be 53 cm.
e Assuming the proportions of the horses remain the same, using the heights in part **d**, the percentage from part **b** changes to 21.03% (not much difference!).

13 **a** 10 **b** 40 **c** 8
d Part **c** calculates 20% of the remaining cupcakes not the original amount.
e 32

14 **a** **i** six people **ii** increase **iii** 24%
b **i** 35 chocolate bars **ii** decrease
iii 87.5% or $87\frac{1}{2}$%
c **i** 0.5 km **ii** increase
iii $33.\dot{3}$% or $33\frac{1}{3}$%
d **i** 15 cm **ii** increase **iii** 10%
e **i** 2 minutes **ii** decrease **iii** 10%

15 **a** Tom: 1 cm, Lisa: 1 cm
b Tom: 2.3%, Lisa: 1.2%; Tom, because his error is compared to a smaller value.

16 **a** 0.05 cm **b** 1.59%

17 392 wombats

18 **a** 64 **b** 98 **c** 460

19 **a** 44% **b** 44%
c The percentage increase is the same for both shapes.
d When the side lengths of any shape are increased by 20%, the area will increase by 44%.

20 **a** 4 kg **b** 1.2 kg **c** 24%

EX p121 3C Mark-ups and discounts

1 **a** $36.05 **b** $1060.50 **c** $110.15 **d** $480
e $1615 **f** $70.08 **g** $233.28 **h** 3337.61

2 **a** $120 **b** $255 **c** $630
d $408 **e** $619 **f** $404.80
g $470.25 **h** $87.15 **i** $622.50

3 **a** $504 **b** $1002 **c** $198
d $1090.32 **e** $2520.76 **f** $3520

4 **a** $1560 **b** $336.75 **c** $1350 **d** $260.75
e $948.15 **f** $465 **g** $552 **h** $787.50
i $323.75 **j** $458.85

5 **a** $142.10 **b** $36.25 **c** $94.25 **d** $147.90

6 **a** $1413.50 **b** $264 **c** $661.60
d $107.25 **e** $346.12 **f** $80.25

7 **a** **i** $194.40 **ii** $74.40
b **i** $778.70 **ii** $419.30
c **i** $42.42 **ii** $7.48

8 **a** $300 **b** $31.25 **c** $13.95
d $86.65 **e** $11 422.85 **f** $624.60
g $49.90 **h** $50 **i** $342 857.15

9 **a** $141.50 **b** $214.50 **c** $165.45
d $109.55

10 **a** 87.5% **b** $138.20

11 **a** **i** $440 **ii** $60
b **i** $152.58 **ii** $26.92
c **i** $229.08 **ii** $19.92
d **i** $860.11 **ii** $35.84
e **i** $537.16 **ii** $87.44
f **i** $28 345.28 **ii** $1649.72
g **i** $11 095 **ii** $1585
h **i** $1234.19 **ii** $261.80

12 **a** $44.60 **b** $37.91 **c** $82.51

13 **a** 210% **b** $188.80

14 **a** **i** $768.80 **ii** $148.80
b **i** $130.43 **ii** $40.48
c **i** $2284.20 **ii** $1015.20
d **i** $835.68 **ii** $385.18
e **i** $15 000 **ii** $8750
f **i** $789.73 **ii** $438.74
g **i** $31 078.13 **ii** $16 453.13
h **i** $5450.63 **ii** $3155.63

15 Jane is finding 20% of the discount price, not the original price. Her calculations will give: $0.2 \times \$198 = \$39.60$
Original price = $198 + $39.60 = $237.60
Find 20% of $237.60 and see if it gives a reduced price of $198.
A 20% discount means you pay 80% or 0.8 of the original price of the bike:
$198 ÷ 0.8 = $247.50
20% of $247.50 gives a reduced price of $198.
Hence, Tim is correct.

16 **a** $9 **b** $900 **c** $900

17 $1000

18 $247.50

19 70% of $699 = $489.30; 80% of $699 is $559.20, then 90% of $559.20 is $503.28; the customer is correct.

20 **a** $2800 **b** $980

21 **a** **i** $5.55 **ii** $1.77 **iii** $2.85
iv $25.73 **v** $3.56 **vi** $6.56

b **i** 43.04% **ii** 42.74% **iii** 43.22%
iv 43.02% **v** 42.97% **vi** 42.92%

c When you increase the price by 40%, you find 140%, or 1.4 times the original. When you increase the price by 30% then 10%, you first find 130% of, or 1.3 times, the original price, then you find 110% of, or 1.1 times, this second price. Increasing by 30% then 10% means multiplying by 1.3, then 1.1, which is equivalent to multiplying by $1.3 \times 1.1 = 1.43 = 143\% = 100\% + 43\%$. Increasing by 30%, then 10% is the same as increasing by 43%. The percentage is not exactly 43% because of errors introduced by rounding.

22 'All stock discounted by 45%' means all stock will receive a discount of 45%. 'Stock discounted by up to 45%' means the discount can be any amount from 0% to 45%; for example, 10%, 20%, 35%, 45%.

23 **a** $46 000 is between $45 001 and $120 000, so tax paid is $5092 plus 32.5 cents per $1 over $45 000, which works out to be 32.5% of $46 000 – $45 000 = $1000.

b **i** $2242 **ii** $11 592 **iii** $25 892
iv $35 017 **v** $67 917 **vi** $420 667

c They are the amount of tax paid on earnings below the corresponding row in the table.
$5092 = 19\% \times (45\,000 - 18200)$
$29\,467 = 5092 + 32.5\% \times (120\,000 - 45\,000)$
$51\,667 = 29\,467 + 37\% \times (180\,000 - 120\,000)$

d **i** 7.4% **ii** 17.8% **iii** 23.8%
iv 25.9% **v** 33.1% **vi** 42.1%

e They only pay 45% tax on income over $180 000, not on their entire income. The more they earn, the closer the percentage paid in tax is to 45%.

24 **a** $1161.62 **b** 1085.10 **c** 1277.78

d It does not matter as multiplication is commutative. That is, a 45% discount then a 10% discount is the same as a 10% discount then a 45% discount:
$0.55 \times 0.90 = 0.90 \times 0.55$

Checkpoint

1 **a** $\frac{1}{4}$, 0.25 **b** $\frac{19}{25}$, 0.76 **c** $\frac{1}{25}$, 0.04
d $\frac{1}{200}$, 0.005 **e** $1\frac{1}{5}$, 1.2 **f** $\frac{16}{625}$, 0.0256

2 **a** 63%, $\frac{63}{100}$ **b** 12%, $\frac{3}{25}$ **c** 85%, $\frac{17}{20}$
d 330%, $3\frac{3}{10}$ **e** 0.6%, $\frac{3}{500}$ **f** 750%, $7\frac{1}{2}$

3 **a** 0.375, 37.5% **b** 0.325, 32.5%
c $5.\dot{6}$, $566\frac{2}{3}\%$ **d** 0.3125, 31.25%
e 0.44, 44% **f** $8.1\dot{6}$, $816\frac{2}{3}\%$

4 **a** 30% **b** 12% **c** 82% **d** 75%
e 90% **f** 80%

5 **a** 20% **b** 25% **c** 12.5% **d** 12.5%
e 20% **f** $1\frac{1}{3}\%$

6 **a** 144 **b** 1215 **c** 288 **d** 2688
e 2400 **f** 300

7 **a** 133.4 L **b** $1800 **c** 19.6 cm **d** 306 m
e 488 beans **f** $192.50

8 **a** $11 520 **b** 700 km **c** $4927.50 **d** $42.37
e 8.45 km **f** 108 minutes

9 **a** $2880 **b** $807.50 **c** $7896
d $956.25 **e** $1159.10 **f** $900

10 **a** **i** $3.20 **ii** $35.20
b **i** $8.50 **ii** $93.50
c **i** $125 **ii** $1375
d **i** $0.35 **ii** $3.85
e **i** $2.60 **ii** $28.60
f **i** $3.50 **ii** $38.50

11 **a** $450 **b** $9800 **c** $8600 **d** $8455
e $700 **f** $25 302.50

EX p128

3D Ratios

1 **a** 11 : 7 **b** 89 : 101 **c** 5 : 3 **d** 25 : 1
e 2 : 3 **f** 1 : 2 : 3 **g** 3 : 7 **h** 8 : 3

2 **a** 6 : 10 **b** 9 : 9 : 3

3 **a** **i** 17 : 8 **ii** $\frac{8}{25}$
b **i** 7 : 11 **ii** $\frac{11}{18}$
c **i** 5 : 4 **ii** $\frac{4}{9}$

4 Part-to-part comparisons are best shown as ratios and part-to-whole comparisons are best shown as fractions.

5 **a** 20 : 33 **b** 95 : 57 **c** 113 : 167
d 256 : 175 **e** 499 : 575 **f** 23 : 2
g 4 : 13 **h** 19 : 21 **i** 42 : 117
j 16 : 39 **k** 19 : 11 **l** 23 : 7 : 3
m 15 :17 : 19 **n** 5 : 7 : 11

6 **a** 13 : 21 **b** 120 : 17 **c** 27 : 500
d 4 : 3 **e** 1700 : 793 **f** 7 : 36
g 509 : 1300 **h** 472 : 2371 **i** 61 : 79
j 38 : 131 **k** 420 : 97 **l** 600 : 53
m 5000 : 23 : 673
n 5631 : 1753 : 1 000 000

7 **a** **i** 11 : 9 **ii** 2 : 7 **iii** 9 : 40 **iv** 18 : 11

b The ratio 9 : 40 could be written as the fraction $\frac{9}{40}$ as it represents how a part compares to the whole.

8 **a** 3 : 4 **b** 3 : 7 **c** $\frac{3}{7}$ **d** $\frac{4}{7}$

9 **a** **i** 11 : 8 : 7 **ii** 9 : 40
b **i** $\frac{13}{40}$ **ii** 40%

10 **a** **i** 2 : 1 **ii** 5 : 1 **iii** 1 : 2
b $\frac{1}{9}$ **c** 67% **d** 44%

e They are in the incorrect order; 2 : 1.

11 For every four games she wins, she loses five; or out of every nine games she plays, she wins four and loses five.

12 **a** 13% **b** 87 : 13

13 **a** 41 : 9 **b** $\frac{9}{50}$

14 80 : 332 (or 20 : 83)

15 **a** 4 : 2 : 1 **b** $\frac{2}{7}$ **c** 120 biscuits

16 **a** Measurements must be put into same units.

b Convert one to same unit as the other measurement.

c 1670 feet = (1670 × 0.3048) m ≈ 509 m; 270 : 509

17 **a** No, units are not the same.

b 57 : 2500

c More water will result in a weaker solution so increase the second value in the ratio. Less bleach will also result in a weaker solution so decrease the first value in the ratio. Both options could also apply.

18 **a** $\frac{3}{100}$ **b** $\frac{97}{100}$ **c** 3 : 97

d A ratio can be written as a fraction when comparing a part to the whole.

19 The ratio of the ages simplifies to 8 : 7 : 6. Adding them up and dividing \$210 by the total gives one part equal to \$10. The money divided by the girls' ages would be \$80 to Lydia, \$70 to Sylvia and \$60 to Julia. Dividing the money into thirds gives each girl \$70 each. In both cases, Sylvia gets the same amount of money.

20 **a** $\Phi = \frac{x}{y}, \Phi = \frac{x + y}{x}$

When $y = 1$, $\Phi = x$, $\Phi = \frac{x + 1}{x}$

When $y = 1$, the value of the Golden Ratio $\Phi = x$, and the longer length $x = \frac{(x + 1)}{x}$.

b $\Phi^2 = \Phi + 1$

c $1.618\,034^2 \simeq 2.618\,034$

$1.618\,034 + 1 = 2.618\,034$

Both sides of the equation give the same value.

EX p133 3E Equivalent ratios

1 **a** 1 : 12 **b** 3 : 4 **c** 53 : 18 **d** 2 : 5

e 3 : 2 **f** 9 : 19 **g** 19 : 17 **h** 1 : 6

i 8 : 12 : 21 **j** 3 : 2 : 5 **k** 3 : 5 : 7 **l** 3 : 7 : 8

2 **a** 6 : 13 **b** 5 : 22 **c** 85 : 31 **d** 1 : 3

e 23 : 59 **f** 6 : 7 **g** 1 : 3 **h** 162 : 35

i 13 : 4 : 7 **j** 45 : 11 : 35 **k** 6 : 8 : 7 **l** 6 : 21 : 8

3 **a** 2 : 3 **b** 8 : 9 **c** 5 : 6

d 33 : 14 **e** 9 : 10 **f** 10 : 21 (20 : 42)

g 1 : 2 **h** 3 : 4 (51 : 68) **i** 85 : 16

j 4 : 5 (16 : 20) **k** 2 : 1 **l** 6 : 1

4 **a** 13 : 640 **b** 17 : 2340 **c** 23 : 432

d 5 : 6 **e** 1 : 14 **f** 19 : 20

g 18 : 11 **h** 64 : 25 **i** 1 : 4

5 **a** Yes. The second ratio is obtained by multiplying the first ratio by 3.

b No

c Yes. The second ratio is obtained by dividing the first ratio by 2.

d Yes. The second ratio is obtained by multiplying the first ratio by 5.

e No **f** No **g** No

h Yes. The second ratio is obtained by dividing the first ratio by 3.

i No

j Yes. The second ratio is obtained by multiplying the first ratio by 4.

k No

l Yes. The second ratio is obtained by dividing the first ratio by 9.

6 **a** 45 **b** 10 **c** 72 **d** 2

e 95 **f** 7 **g** 6 **h** 8

i 3 **j** 3 **k** 85 **l** 117

m 105 **n** 14 **o** 17 **p** 6

7 **a** 2 : 5 = 8 : 20 = 20 : 50

b 6 : 7 = 30 : 35 = 54 : 63

c 420 : 360 = 70 : 60 = 7 : 6

d 225 : 300 = 9 : 12 = 3 : 4

e 136 : 104 = 68 : 52 = 17 : 13

f 180 : 48 = 75 : 20 = 15 : 4

g 20 : 48 = 40 : 96 = 100 : 240

h 216 : 636 = 72 : 212 = 18 : 53

8 **a** 16 **b** 80 **c** 1

d 35 **e** 12 **f** 9

g 44 **h** 48 **i** 30

9 **a** 13 : 1 **b** 1 : 9 **c** 17 : 9 **d** 4 : 7

10 3 : 17

11 **a** Flour and butter because the quantities are in the same units

b Flour to butter is 300 : 60 = 5 : 1. (Butter to flour is 60 : 300 = 1 : 5.)

c For 50 pancakes (10 × 5), multiply each number in the ratio by 5 to keep in proportion. 300 : 60 = 1500 : 300

12 12

13 **a** 12 **b** 27

14 **a**

Pool enclosure	Length	Width	Perimeter
A	4 m	3 m	14 m
B	8 m	6 m	28 m
C	12 m	9 m	42 m
D	16 m	12 m	56 m

b A 4 : 14, B 8 : 28, C 12 : 42, D 16 : 56

c The ratios are multiples of each other.

d $\frac{4}{14} = \frac{2}{7}, \frac{8}{28} = \frac{2}{7}, \frac{12}{42} = \frac{2}{7}, \frac{16}{56} = \frac{2}{7}$

e Each ratio simplifies to $\frac{2}{7}$.

f The length is always $\frac{2}{7}$ of the perimeter.

15 72 pens

16 **a** 1.05 L **b** 8000 mL

17 **a** 21.25 L **b** 600 mL

18 **a** 41 : 204 **b** 41 : 163

19 **a** 5 : 2 **b** 3 : 5 **c** 3 : 2

20 **a** 27.84 cm **b** 69.6 cm

21 **a** The ratio 22 : 27 is larger than 12 : 15 but smaller than 8 : 9.

b Sample answers include: 81 : 100, 161 : 200, 101 : 125.

22 4 : 5 is larger.

23 17 : 33 is the larger ratio, as 17 is more than half of 33 and 12 is less than half of 25.

24 Sample answer

1st ratio	2nd ratio
3 : 5	9 : 15
7 : 20	14 : 40
4 : 7	40 : 70
5 : 25	1 : 5
25 : 30	5 : 6
3 : 7	12 : 28
8 : 16	4 : 8
8 : 12	2 : 3
8 : 24	1 : 3
2 : 4	1 : 2

EX p139 3F Dividing a quantity in a given ratio

1 **a** 50 and 450 **b** 100 and 400

c 150 and 350 **d** 300 and 200

e 300 and 200 **f** 400 and 100

2 **a** \$5520 and \$3680 **b** \$5750 and \$3450

c \$6900 and \$2300 **d** \$4600 and \$4600

e \$4200 and \$5000 **f** \$1840 and \$7360

3 **a** 15 000 km and 21 000 km

b 31 500 km and 4500 km

c 20 000 km and 16 000 km

d 16 000 km and 20 000 km

e 28 800 km and 7200 km

f 18 000 km and 18 000 km

4 **a** \$80 and \$160 **b** \$216 and \$24

c \$40 and \$200 **d** \$150 and \$90

e \$90, \$30 and \$120 **f** \$168, \$48 and \$24

5 **a** 120 kg and 180 kg **b** 30 kg and 270 kg

c 260 kg and 40 kg **d** 80 kg and 220 kg

e 120 kg, 150 kg and 30 kg

f 120 kg, 75 kg, 45 kg and 60 kg

6 **a** \$2.70 and \$0.90 **b** \$0.40 and \$3.20

c \$3.30 and \$0.30 **d** \$2.00 and \$1.60

e \$0.40 and \$1.20 and \$2.00

f \$0.60 and \$0.90 and \$2.10

7 **a** $\frac{16}{45}$ and $\frac{8}{45}$ **b** $\frac{8}{25}$ and $\frac{16}{75}$

c $\frac{1}{5}$ and $\frac{1}{3}$ **d** $\frac{2}{5}$ and $\frac{2}{15}$

e $\frac{2}{15}$ and $\frac{1}{5}$ and $\frac{1}{5}$ **f** $\frac{16}{75}$ and $\frac{8}{45}$ and $\frac{32}{225}$

8 **a** 21 cups and 14 cups **b** 28 cups and 7 cups

c $24\frac{1}{2}$ cups and $10\frac{1}{2}$ cups

d $31\frac{1}{2}$ cups and $3\frac{1}{2}$ cups

e $10\frac{1}{2}$ cups and $17\frac{1}{2}$ cups and 7 cups

f $10\frac{1}{2}$ cups and 14 cups and $10\frac{1}{2}$ cups

9 **a** 4000 non-fiction books **b** 7500 books

10 **a** 15 tomato seedlings **b** 36 seedlings

11 **a** 2000 non-germinated **b** 6500 seeds

12 64

13 **a** 1500 mL **b** 150 mL

14 60°, 80°, 40°

15 **a** 10

b $\frac{4}{10}$ and $\frac{6}{10}$ or $\frac{2}{5}$ and $\frac{3}{5}$

c \$120 and \$180 **d** Yes

16 \$28

17 **a** \$1000 **b** 9 : 4 : 7

c Katie \$2.25; Mike \$1.00; Sam \$1.75

18 **a** 5 : 3 : 2

b **i** \$25, \$15, \$10 **ii** \$75, \$45, \$30

iii \$350, \$210, \$140 **iv** \$1000, \$600, \$400

v \$5000, \$3000, \$2000 **vi** \$6500, \$3900, \$2600

19 **a** $\frac{3}{5}$ **b** 3 : 5 : 12

c 15% to a present, 25% to trophies, 60% to snacks

d \$30 on a present, \$50 on trophies, \$120 on snacks

20 **a** Catching up with friends 40%; outdoor activities 25%; doing homework/chores 10%

b 25% **c** 8 : 5 : 2 : 5

d 56 hours, 35 hours, 14 hours, 35 hours

21 **a** width 72 cm, height 42 cm **b** 3024 cm^2

22 The ratio that shows the relationship of part to part compares one part to another. For example, in 2 : 3 there is a total of 5 parts. The first number represents $\frac{2}{5}$ of the total and the second number represents $\frac{3}{5}$ of the total.
The ratio that shows the relationship of a part to a whole is better expressed as a fraction as it compares one number to the total.

23 **a** Can A has 3200 mL of white paint and 800 mL of red paint.
Can B has 3500 mL of white paint and 500 mL of red paint.

b 67 : 13 **c** $\frac{13}{80}$ **d** $83\frac{3}{4}$% or 83.75%

e To make 4 : 1 from 67 : 13, one option could be to add 1 part red and 3 parts white. To make 7 : 1 from 67 : 13, one option could be to add 24 parts white.

24 Sample answer: 20 frogs, 50 newts and 110 fish.

EX p145 3G Rates

1 **a** 60 km/h **b** 591 L/min

c four quarters/game **d** \$16/h

e 45 cm/year **f** five patients/h

g \$1.49/day **h** \$28.99/month

i $5.80/box **j** 0.5 kg/week
k 290 mm/year **l** 14 km/L

2 **a** m/s **b** m/s **c** cents/apple or $/kg
d $/m^2 **e** L/min **f** wickets/over or km/h
g kB/s or GB/min **h** km/h

3 **a** 50 m/min **b** 2 L/bottle
c 4 cents per second **d** 15 students/teacher
e 26 seats/row **f** 25 newspapers/bundle
g 6.8 runs/over **h** $2.50/muffin
i 6.8 goals/game **j** 4444 tickets/min
k 3.5 goals/game **l** 5.25 m^2/L

4 **a** 14 km/L **b** $1.99/mango
c 12 L/min **d** 15.3 km/h
e 2.85 people/km^2 **f** 18 million km/min

5 **a** 2400 words **b** 800 words **c** 45 minutes

6 **a** $15.20 **b** $608 **c** 96 hours

7 **a** 386 m **b** 1 389 600 m **c** $33\frac{1}{3}$ minutes

8 $355.50

9 **a** 54.75 km/h **b** 854.35 km/h **c** 32.5 km/h
d 0.38 km/min **e** 2.10 m/s **f** 1.89 m/s

10 **a** Alexis 350 m per min, Kym 340 m per min
b Alexis, as she travelled a greater distance per minute
c Speed is distance (m) per one unit of time (min) so speed unit is m per min or m/min.

11 **a** Max 5.75 runs/over, William 5.6 runs/over
b Max's team

12 **a** 56 bpm **b** 27 min

13 Usain Bolt's rate is 10.4384 m/s.
a 10.35 m/s **b** 10.70 m/s **c** 16 m/s

14 **a** 0.3125 km (312.5 m)
b **i** 17.1875 km **ii** 18.75 km
iii 26.5625 km **iv** 28.125 km
c **i** 2 min **ii** 1 h 40 min
iii 1 h 20 min **iv** 6 min 24 s

15 **a** 7.5 kg of potatoes ($0.95/kg compared to $0.99/kg)
b packet of 10 batteries ($0.989/battery compared to $0.9975/battery)
c 500 g tin of coffee ($0.018/g compared to $0.01993/g)
d 650 g tin of peaches ($0.0039/g compared to $0.0042/g)

16 **a** Plan A: 80c, Plan B: 88c; Plan A
b Plan A: $1.60, Plan B: $1.64; Plan A
c Plan A: $2.40, Plan B: $2.40; both plans are the same.
d Plan A: $3.20, Plan B: $3.16; Plan B
e Use Plan A if most calls are shorter than 5 min, use Plan B if most calls are longer than 5 min, use either plan for calls of exactly 5 min.

17 **a** **i** $\frac{1}{20}$ (or equivalent) **ii** $100 **iii** $1100
b **i** $\frac{1}{40}$ (or equivalent) **ii** $90 **iii** $1290
c **i** $\frac{2}{50}$ (or equivalent) **ii** $1000 **iii** $6000

18 6 minutes. When doubling the amount of window washers and the number of windows, the minutes taken will remain the same.

19 260.5 m

CHAPTER 3 review

Multiple choice

1 E **2** C **3** B **4** A **5** B
6 E **7** A **8** C **9** B **10** D

Short answer

1 **a** 7 **b** $\frac{7}{20}$ **c** $\frac{35}{100}$ **d** 35%

2 **a** 0.23 **b** 0.62 **c** 0.1652 **d** 2.916

3 **a** 32% **b** 762% **c** 3162.5% **d** 0.03%

4 **a** 75% **b** 70% **c** 85% **d** 46.875%

5 **a** 40% **b** 16% **c** 46.4%

6 **a** 34 **b** 15.5 **c** 2250 **d** 888

7 **a** $1045 **b** $904.32 **c** $903.60

8 **a** 37.5% profit **b** 40% loss
c 51.68% profit **d** 31.57% loss

9 **a** **i** 5 : 6 **ii** 2 : 3
b **i** $\frac{3}{16}$ **ii** $\frac{1}{8}$

10 **a** 35 : 47 **b** 7 : 31 **c** 2 : 9 **d** 9 : 25

11 **a** 3 : 11 **b** 16 : 27 **c** 1 : 2 : 6 **d** 1 : 5 : 7

12 **a** 42 **b** 16 **c** 12 **d** 35

13 **a** 8 **b** 17 **c** 17 **d** 31

14 **a** $600, $900 **b** $450, $1050
c $300, $400, $800 **d** $300, $375, $825

15 **a** 100 km/h **b** 3 L/s
c $17.50/h **d** 4.333 books/month

16 **a** 1.5 points/min **b** $1.39/L
c $0.0048/mL or 0.48 c/mL **d** $16.80/h

Analysis

1 **a** **i** $\frac{13}{20}$ **ii** 0.65
b $13 **c** $33 **d** $455 **e** $3.30
f $29.70 **g** $9.70 **h** 48.5%

2 **a** 3 : 5 **b** $65.60 **c** 2.4 cubic metres
d $512 **e** 12 **f** 16
g 2 Grevilleas, 3 Callistemons and 4 Banksias

3 **a** 3 : 2 **b** 9 : 5 **c** 2 cups
d $1\frac{1}{5}$ cups of pecans, $1\frac{1}{5}$ cups of cashews, $\frac{4}{5}$ cup of sunflower seeds and $\frac{4}{5}$ cup of pumpkin seeds.

CHAPTER 4 Exponents and roots

EX p156

4A Exponents

1 **a** 5 **b** 3 **c** 5 × 5 × 5 **d** 125

2 **a** 2 × 2 × 2 × 2 × 2 = 32
b 4 × 4 × 4 = 64
c 1 × 1 × 1 × 1 × 1 × 1 × 1 × 1 × 1 × 1 = 1
d 3 × 3 × 3 = 27
e 10 × 10 × 10 × 10 × 10 × 10 × 10 = 10 000 000

f $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

g $5 \times 5 \times 5 \times 5 = 625$

h $7 \times 7 = 49$

i $8 \times 8 \times 8 = 512$

j $6 \times 6 \times 6 \times 6 = 1296$

3 **a** 64 **b** 49 **c** 25 **d** 121 **e** 36

f 1 **g** 3600 **h** 10 000 **i** 400

4 **a** 27 **b** 8 **c** 64 **d** 1 000 000

e 1000 **f** 125

5

	Exponent form	Base	Exponent	Expanded form	Basic numeral
a	3^4	3	4	$3 \times 3 \times 3 \times 3$	81
b	2^5	2	5	$2 \times 2 \times 2 \times 2 \times 2$	32
c	8^2	8	2	8×8	64
d	6^2	6	2	6×6	36

6 **a** $0.8 \times 0.8 = 0.64$

b $3.4 \times 3.4 = 11.56$

c $0.5 \times 0.5 \times 0.5 = 0.125$

d $0.3 \times 0.3 \times 0.3 \times 0.3 = 0.0081$

e $0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 = 0.000\,01$

f $0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2 = 0.000\,012\,8$

7 **a** $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$

b $\frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$

c $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

d $\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{81}{10\,000}$

e $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$

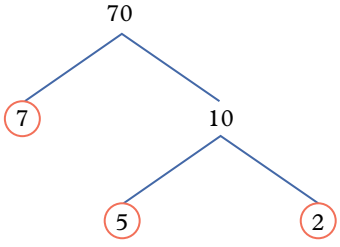
f $3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} = \frac{1\,000\,000}{729}$

8 **a** 3249 **b** 1 030 301 **c** 62 748 517

d 321. 419 125 **e** 13.0321 **f** 293 162.50624

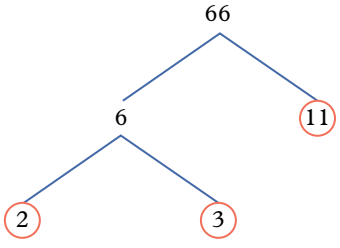
g $\frac{5832}{12\,167}$ **h** $\frac{117\,649}{1\,771\,561}$ **i** $\frac{19\,683}{1\,953\,125}$

9 **a**



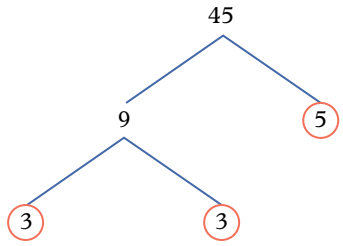
$70 = 2 \times 5 \times 7$

b



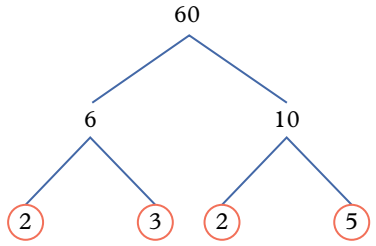
$66 = 2 \times 3 \times 11$

c



$45 = 3^2 \times 5$

d



$60 = 2^2 \times 3 \times 5$

10 **a** $75 = 3 \times 5^2$

b $48 = 2^4 \times 3$

c $90 = 2 \times 3^2 \times 5$

d $126 = 2 \times 3^2 \times 7$

e $288 = 2^5 \times 3^2$

f $1092 = 2^2 \times 3 \times 7 \times 13$

11 The student calculated 4^2 as $4 \times 2 = 8$, but $4^2 = 4 \times 4$.

12 **a** 24 **b** 86

c 4.3601 **d** $\frac{11}{10}$ $\left(\text{or } 1\frac{1}{10}\right)$

13 **a** **i** 1 **ii** 121

iii 12 321 **iv** 1 234 321

b Sample answer: The resulting number displays digits in a pattern with 1 as the first digit, 2 as the second digit, 3 as the third digit, etc, increasing by one each time until it reaches a value that is the same as the number of digits in the base. The digits that follow then decrease by one to finish the pattern at 1. For example, the base of 111^2 contains three digits so the first three digits in the number are 123 followed by 21 to give 12 321.

c **i** 123 454 321

ii 12 345 654 321

iii 1 234 567 654 321

iv 123 456 787 654 321

14 **a** \$180 **b** \$3, \$9, \$27, \$81, \$243, \$729

c \$30 per week **d** $\$3^{11}$

e \$177 147

15 **a** **i** 2000 cells **ii** 4000 cells **iii** 8000 cells

b 2000, 4000, 8000, 16 000, 32 000

c **i** 2 **ii** 4 **iii** 8

d 2, 4, 8, 16, 32 **e** $2^1, 2^2, 2^3, 2^4, 2^5$

f **i** 2^6 **ii** 2^{10} **iii** 2^{24}

g 16 777 216 **h** 16 777 216 000

16 **a** $144 = 2^4 \times 3^2$

b $270 = 2 \times 3^3 \times 5$

c HCF = $2 \times 3^2 = 18$

d **i** 13 **ii** 12 **iii** 55 **iv** 27

17 60 days

18 $\frac{5}{4}c$

4B Products and quotients of powers

EX p160

1 **a** 3^5 **b** 4^8 **c** 6^6 **d** 5^9 **e** 8^3 **f** 2^5

2 **a** 243 **b** 65 536 **c** 46 656

d 1 953 125 **e** 512 **f** 32

3 **a** 9^3 **b** 4 **c** 6^3

d 4^5 **e** 10^3 **f** 6^4

4 **a** 729 **b** 4 **c** 216

d 1024 **e** 1000 **f** 1296

5 **a** $2^{11} = 2048$ **b** $5^6 = 15\,625$

c $3^7 = 2187$ **d** $4^8 = 65\,536$

e $5^3 = 125$ **f** $3^6 = 729$

g $9^3 = 729$ **h** $2^7 = 128$

i $6^5 = 7776$ **j** $8^1 = 8$

k $10^9 = 1\,000\,000\,000$ **l** $7^2 = 49$

6 **a** $(1.5)^9$ **b** $(0.7)^6$ **c** $\left(\frac{1}{2}\right)^7$ **d** $\left(\frac{7}{5}\right)^{14}$

e $(9.2)^3$ **f** $(0.63)^7$ **g** $\left(\frac{3}{4}\right)^4$ **h** $\left(\frac{5}{2}\right)^2$

7 **a** 10 **b** 5 **c** 8 **d** 4 **e** 21 **f** 17

8 **a** $5^2 \times 5^5 \times 5^4 = 5^7 \times 5^4$
$= 5^{11}$

b $20^8 \times 20 \times 20^{10} = 20^9 \times 20^{10}$
$= 20^{19}$

c $8^{32} \div 8^7 \div 8^{10} = 8^{25} \div 8^{10}$
$= 8^{15}$

d $7^{24} \div 7^6 \div 7 = 7^{18} \div 7^1$
$= 7^{17}$

9 **a** 2^{12} **b** 3^6 **c** 10^{10} **d** 4^9 **e** 5^{16} **f** 9^{14}

10 **a** 6^2 **b** 7^4 **c** 12 **d** 8^4 **e** 5 **f** 11^5

11 10^{10} light-years

12 **a** Exponents have been divided instead of subtracted; $5^8 \div 5^2 = 5^6$.

b Exponents have been multiplied instead of added; $2^3 \times 2^3 = 2^6$.

c Exponents have been divided instead of subtracted; $\frac{4^7}{4^7} = 4^0$.

d The first exponent has been subtracted from the second, instead of the other way round; $9^8 \div 9^{10} = 9^{-2}$.

13 **a** **i** 4 **ii** −8 **iii** 16 **iv** −32

b Sample answer: The sign of the answers alternates between positive and negative and the number doubles in value.

c Sample answer: When a negative number is raised to a power, if the exponent is even the answer will be positive, and if the exponent is odd the answer will be negative.

d **i** −27 **ii** 64 **iii** 10 000

iv −1 000 000 000

14 **a** $x^2 \times x^3$ **b** x^5 **c** 36×216 mm

4C Raising exponents and the zero exponent

EX p163

1 **a** 6^{20} **b** 2^{12} **c** 7^{12} **d** 11^8 **e** 8^{15} **f** 3^{14}

2 **a** $5^3 \times 9^3$ **b** $\frac{3^6}{4^6}$ **c** $\frac{9^4}{5^4}$

d $11^7 \times 4^7$ **e** $\frac{7^3}{10^3}$ **f** $3^9 \times 8^9$

3 **a** 1 **b** 1 **c** 1

4 **a** 1 **b** 1 **c** 1 **d** 1 **e** 1 **f** 1

5 **a** 0.5^{12} **b** 1 **c** 3.8^{25}

d $2^{20} \times 2.5^8$ **e** 1 **f** 6.6^{18}

6 **a** 4^{10} cm^2 **b** 4^{15} cm^3 **c** 6^{15} cm^2

7 **a** 6 **b** 4 **c** 0 **d** 5 **e** 10 **f** 0

8 **a** $(2^3)^4 = 8^4 = 16^3 = 2^{12}$

b $10^{5 \times 4} = 100\,000^4 = 10\,000^5 = 10^{20}$

c $8^8 = 2^{24} = 4^{12} = 16^6$

9 **a** 2 **b** 2 **c** 3 **d** 1 **e** 2 **f** 4

10 $(3^{15})^0$, $(3^2)^5$, $(3^4)^3$, 3^{14}, $((3^2)^3)^3$

11 **a** 1 **b** 2^0

c Sample answer: $\frac{15^5}{15^5} = 15^{(5-5)}$
$= 15^0$
$= 1$

12 **a** true **b** true **c** false **d** true

13 **a** $x = 4$ **b** $x = 2$

14 Sample answer: $(2^3)^6 = 2^{18}$
$= (2^2)^9$
$= 4^9$

15 $x = 2, y = 4$, or $x = 4, y = 2$

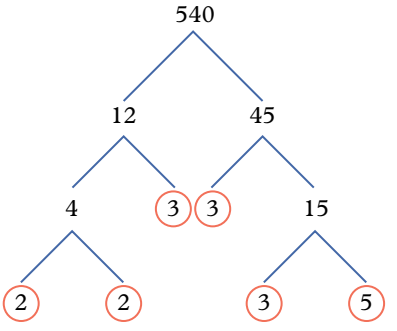
Checkpoint

1 **a** 16 **b** 81

c 125 **d** 1 000 000

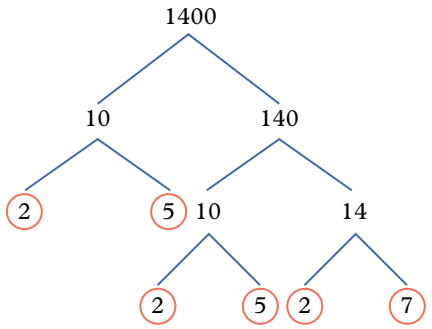
2 **a** 25 **b** 81 **c** 64 **d** 216

3 **a**



$540 = 2^2 \times 3^3 \times 5$

b



$1400 = 2^3 \times 5^2 \times 7$

4 **a** $2^3 \times 3$ **b** $2^3 \times 3^2$

c $3 \times 5 \times 13$ **d** $2 \times 5^2 \times 7$

5 **a**

Number of minutes	0	1	2	3	4	5
Size of colony	10	30	90	270	810	2430
Number of times as large as original amount	1	3	9	27	81	243

b $3^0, 3^1, 3^2, 3^3, 3^4, 3^5$

c **i** 3^{10} **ii** 3^{60} **iii** 3^{1440}

d 590 490

6 **a** 2^8 **b** 7^{20} **c** 11^6 **d** 9^{18}

7 **a** 3^4 **b** 10^{10} **c** 14^4 **d** 8^8

8 **a** 64 **b** 625

c 36 **d** 10 000 000

9 **a** 4^{18} **b** 9^6 **c** 13^{40} **d** 10^{96}

10 **a** $3^6 \times 5^6$ **b** $\frac{2^4}{7^4}$ **c** $9^5 \times 15^5$ **d** $\frac{9^{10}}{8^{10}}$

11 **a** 1 **b** 1 **c** 1 **d** 1

EX p167 4D Combining the exponent laws

1 **a** 3^{14} **b** 9^{41} **c** 12^{36} **d** 8^0

2 **a** 4^2 **b** 7^2 **c** 6^{11} **d** 11^6

e 5^4 **f** 8^3 **g** 2^{10} **h** 4^3

3 **a** 3^6 **b** 3^5 **c** 4^{21} **d** 2^8

4

Base 2	Exponent form	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
	Basic numeral	16	32	64	128	256	512	1024
Base 3	Exponent form	3^4	3^5	3^6	3^7	3^8	3^9	3^{10}
	Basic numeral	81	243	729	2187	6561	19 683	59 049

5 **a** **i** $2^5 \times 2^4$ **ii** $2^{10} \div 2^8$ **iii** $3^{10} \div 3^6$

iv $\frac{2^7 \times 2^5}{2^4}$ **v** $\frac{3^3 \times 3^5}{3^7}$ **vi** $2^6 \times 2^8 \div 2^9$

vii $\frac{2^9 \times 2^6}{2^7 \times 2^4}$ **viii** $\frac{2^7 \times 2^{10}}{2^8 \times 2^6}$

b **i** 2^9 **ii** 2^2 **iii** 3^4 **iv** 2^8

v 3^1 **vi** 2^5 **vii** 2^4 **viii** 2^3

c **i** 512 **ii** 4 **iii** 81 **iv** 256

v 3 **vi** 32 **vii** 16 **viii** 8

6 **a** 256 **b** 81 **c** 64 **d** 2187

7 **a** 72 **b** 75 **c** 96 **d** 9000

e 125 **f** 64 000 000

8 **a** 2 **b** 12 **c** 4 **d** 0

e 5 **f** 9 **g** 3 **h** 3

9 Sample answer: The number on the right-hand side of each equation is negative because the negative sign is located outside the brackets. When a negative number is raised to a power, if the exponent is even the answer will be positive, and if the exponent is odd the answer will be negative, so $(-5)^2$ is positive, but $(-5)^3$ is negative.

10 **a** true **b** true **c** true **d** true

e false **f** false

11 $x = 9$

12 384

EX p171 4E Roots

1

Exponent form	Perfect square	Exponent form	Perfect cube
2^2	4	2^3	8
3^2	9	3^3	27
4^2	16	4^3	64
5^2	25	5^3	125
6^2	36	6^3	216
7^2	49	7^3	343
8^2	64	8^3	512
9^2	81	9^3	729
10^2	100	10^3	1000

2 **a** 4 **b** 3 **c** 5 **d** 1

e 2 **f** 10 **g** 9 **h** 15

i 12 **j** 20 **k** 13 **l** 30

3 **a** 2 **b** 4 **c** 5 **d** 1

e 3 **f** 10 **g** 6 **h** 20

4 **a** 14 **b** 16 **c** 33 **d** 162 **e** 1.5

f 7.3 **g** 8.2 **h** 21.8 **i** 5.03

5 **a** rational **b** irrational **c** rational

d rational **e** irrational **f** rational

6 **a** $\frac{1}{4}$ **b** $\frac{1}{8}$ **c** $\frac{2}{5}$ **d** $\frac{3}{7}$ **e** $\frac{9}{10}$ **f** $\frac{6}{11}$

7 **a** 80 **b** 1 **c** 6 **d** 75

8 **a** The value has been divided by 2 instead of finding the square root; $\sqrt{16} = 4$.

b The square root has been found instead of the cube root; $\sqrt[3]{64} = 4$.

9 Sample answer: Finding the square root is the inverse or opposite of squaring a number.

10 **a** Sample answer: The area of a square is found by squaring the side length, so the side length can be found by taking the square root of the area.

b **i** 13 cm

ii 14.142 135 623… m

iii 16.4 km

c **ii** is irrational

11 a 5 **b** 7

c **i** 2 **ii** 3 **iii** 10

iv 2 **v** 1 **vi** 3

12 a **i** 15 **ii** 4.1

iii 4.626 065 009… **iv** 7

v 2.923 012 786... **vi** 0.562 341 325…

vii 8 **viii** 0.2

ix 2.885 399 8… **x** 3

xi 2.813 500 172... **xii** 5

b **i** rational **ii** rational **iii** irrational

iv rational **v** irrational **vi** irrational

vii rational **viii** rational **ix** irrational

x rational **xi** irrational **xii** rational

13 a -3

b -2

c It is not possible to take the square root of a negative number as the square of any negative number is positive.

14 a **i** not a perfect square

ii perfect square; $\sqrt{289} = 17$

iii perfect square; $\sqrt{576} = 24$

iv not a perfect square

b **i** perfect cube; $\sqrt[3]{216} = 6$

ii perfect cube; $\sqrt[3]{512} = 8$

iii not a perfect cube

iv perfect cube; $\sqrt[3]{1331} = 11$

15 a 18 **b** 26 **c** 44 **d** 72

CHAPTER 4 review

Multiple choice

1 E **2** B **3** D **4** C **5** D

6 B **7** D **8** E **9** B **10** A

Short answer

1 a $3 \times 3 \times 3 = 27$

b $2 \times 2 \times 2 \times 2 = 16$

c $8 \times 8 = 64$

d $10 \times 10 \times 10 \times 10 \times 10 = 100\,000$

2 a 36 **b** 64 **c** 8 **d** 64

3 a $45 = 3^2 \times 5$

b $124 = 2^2 \times 31$

4 a 3^{12} **b** 2^{10} **c** 12^4 **d** 4^{11}

5 a 6^6 **b** 5^0 **c** 10^7 **d** 7^1

6 a 9^{15} **b** $6^{10} \times 3^{10}$ **c** $\frac{3^3}{10^3}$ **d** $\frac{2^8 \times 7^8}{9^8}$

7 a 4 **b** 1 **c** 1 **d** 25

8 a $7^2 = 49$ **b** $2^5 = 32$

9 a **i** $\frac{5^4}{5^2}$ **ii** 5^2 **iii** 25

b **i** $\frac{3^2 \times 3^4}{3^3}$ **ii** 3^3 **iii** 27

c **i** Sample answer: $\frac{2^6 \times 2^8}{2^2 \times 2^4}$

ii 2^8 **iii** 256

d **i** Sample answer: $2^4 \times ((2^2)^2)^2$

ii 2^{12} **iii** 4096

10 a 2 **b** 9 **c** 3 **d** 5

11 a **i** 23 **ii** 26.46

iii 12.5 **iv** 3.48

b **i** rational **ii** irrational

iii rational **iv** irrational

Analysis

1 a

Number of days	Mass of metal (grams)	$\frac{\text{mass of metal}}{\text{original mass of metal}}$
0	240	1
1	120	$\frac{1}{2}$
2	60	$\frac{1}{4}$
3	30	$\frac{1}{8}$

b $\left(\frac{1}{2}\right)^0, \left(\frac{1}{2}\right)^1, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3$ **c** $240 \times \left(\frac{1}{2}\right)^7$

d $\frac{15}{8}$ **e** 1.875 grams **f** 8 days

2 a $\frac{(2 \times 5 \times 7^2) \times (2^2 \times 3^2)^2}{(2^2 \times 3^3 \times 7) \times (3 \times 13^2)^0}$ **b** $2^3 \times 3 \times 5 \times 7$

c 840

CHAPTER 5 Algebra

EX p181

5A Pronumerals

1 a **i** $6a$, b and -1

ii a and b

iii The coefficient of $6a$ is 6. The coefficient of b is 1.

iv -1

b **i** $2xy$ and $-3x$

ii x and y

iii The coefficient of $2xy$ is 2. The coefficient of $-3x$ is -3.

iv No constant terms

c **i** $11f$, $-g$, $15h$ and -10

ii f, g and h

iii The coefficient of $11f$ is 11. The coefficient of $-g$ is -1. The coefficient of $15h$ is 15.

iv -10

d **i** $-\frac{9}{4}p$

ii p

iii The coefficient of $-\frac{9}{4}p$ is $-\frac{9}{4}$.

iv No constant terms

e **i** $\frac{3}{4}c$, d and $\frac{1}{3}$

ii c and d

iii The coefficient of $\frac{3}{4}c$ is $\frac{3}{4}$. The coefficient of d is 1.

iv $\frac{1}{3}$

f **i** $\frac{x}{2}$, $\frac{y}{11}$, $-\frac{7}{10}$ and $-\frac{z}{8}$

ii x, y and z

iii The coefficient of $\frac{x}{2}$ is $\frac{1}{2}$. The coefficient of $\frac{y}{11}$ is $\frac{1}{11}$. The coefficient of $-\frac{z}{8}$ is $-\frac{1}{8}$.

iv $-\frac{7}{10}$

g **i** $-\frac{m}{5}$, $-\frac{n}{7}$ and $-8p$

ii m, n and p

iii The coefficient of $-\frac{m}{5}$ is $-\frac{1}{5}$. The coefficient of $-\frac{n}{7}$ is $-\frac{1}{7}$. The coefficient of $-8p$ is -8.

iv No constant terms

h **i** $\frac{5x}{8}$, $-9y$, $\frac{z}{4}$ and $-\frac{2}{5}$

ii x, y and z

iii The coefficient of $\frac{5x}{8}$ is $\frac{5}{8}$. The coefficient of $-9y$ is -9. The coefficient of $\frac{z}{4}$ is $\frac{1}{4}$.

iv $-\frac{2}{5}$

2 **a** expression **b** equation **c** equation **d** equation **e** expression **f** equation

3 **a** false, it is an equation

b true

c false, there are 2 terms

d true

e false, the coefficient of $-d$ is -1

f false, the coefficient of $\frac{b}{12}$ is $\frac{1}{12}$

g true

h true

4 **a** $a + 5$ **b** $\frac{z}{3}$ **c** $y - 23$ **d** $50m$

e $\frac{2}{5}x$ **f** $2p$ **g** $\frac{s}{2}$ **h** $\frac{3}{4}w + 5$

5 **a** $7x + 2$ **b** $\frac{x}{3} - 6$ **c** $\frac{3}{4}x - 10$

d $13 + \frac{1}{3}x$ **e** $15x - 28$ **f** $\frac{1}{2}x - 5$

6 **a** The sum of c and 8

b x less than 11

c x divided by 5

d the product of nine-tenths and q

e 5 less than the product of 3 and x

f 16 more than the product of ten-thirds and a

7 **a** $3(x + 6)$ **b** $\frac{n - 10}{7}$ **c** $\frac{3}{5}fg$ **d** $\sqrt{t} - 1$

e $\frac{b + 12}{15}$ **f** $\sqrt{z + 9}$

g $\frac{2}{3}v - 7$ **h** $3(33 + y)$

8 **a** $8(x - 3)$ **b** $6(x + 20)$ **c** $\frac{5}{6}x - 2$

d $\frac{4}{5}x + 12$ **e** $2(x + 10)$ **f** $\frac{x + 4}{12} + 16$

9 $n + 18$

10 $\frac{c}{4}$

11 $8x$

12 $y = x - 10$

13 **a** **i** $y - 6$ **ii** $y - 10$

b **i** $y + 5$ **ii** $y + 23$

c $y - x$ **d** $2y$

14 **a** $c + p$ **b** $p - 1$ **c** $c + p - 1$

15 **a** $8n$ and 12

b the delivery fee

c the price of each brush

d \$92

16 **a** $130t + 25$ **b** \$415

17 **a** $2(x + 7)$ **b** 40 chickens

18 **a** Naomi is correct. If one block is left over, then $x - 1$ blocks are divided between the three students.

b Sample answer: An extra block is added to the container so that the blocks can be divided equally between three students.

19 **a** $\frac{4x}{9} - \frac{5}{9}$ **b** $\frac{4}{9}$ **c** $-\frac{5}{9}$

20 **a** $0.8xy + 0.7ab$ **b** Sample answer: $67a$

c 4 t-shirts, 2 pairs of pants

EX p186

5B Substitution

1 **a** 12 **b** 2 **c** 2 **d** 10 **e** $1\frac{3}{5}$ $\left(\text{or } \frac{8}{5}\right)$

f 80 **g** 254 **h** -1 **i** 76 **j** 0

2 **a** 5 **b** -7 **c** 31 **d** -3 **e** 102

f 8 **g** -4 **h** -63

3 **a** 11 **b** 26 **c** 8 **d** 5 **e** 2

f -4 **g** 6 **h** 3

4 **a** 5 **b** 51 **c** 7 **d** 100 **e** 0

f 1 **g** $\frac{1}{3}$ **h** 35

5 **a** 30 **b** 2 **c** 1 **d** 8 **e** $1\frac{2}{5}$ $\left(\text{or } \frac{7}{5}\right)$

f -7 **g** $\frac{5}{9}$ **h** $\frac{3}{7}$

6 **a** 21 **b** 6 **c** 15 **d** $-\frac{3}{5}$ **e** $\frac{2}{9}$

f 2 **g** $-\frac{4}{5}$ **h** -5

7 **a** 13 **b** 23 **c** -5 **d** 5 **e** 16

f 7 **g** 8 **h** 4

8 **a** -8 **b** 132

c 4 **d** 9988

e -3 **f** $-11\frac{3}{4}$ $\left(\text{or } -\frac{47}{4}\right)$

g -12 **h** $-9\frac{3}{4}$ $\left(\text{or } -\frac{39}{4}\right)$

9 **a** 10 **b** 8

c -18 **d** $1\frac{1}{2}$ $\left(\text{or } \frac{3}{2}\right)$

e 4 **f** $-10\frac{2}{3}$ $\left(\text{or } -\frac{32}{3}\right)$

g 151 **h** $3\frac{3}{5}$ $\left(\text{or } \frac{18}{5}\right)$

10 **a** 3 m/s **b** 94 km/h

11 a 4.9 m

b **i** 44.1 m **ii** 122.5 m

c increase; average speed in first second is 4.9 m/s, in first 3 seconds is $\frac{44.1}{3}$ or 14.7 m/s, in first 5 seconds is $\frac{122.5}{5}$ or 24.5 m/s.

12 a 4 cm² **b** 70 cm² **c** $6\frac{1}{4}$ cm² $\left(\text{or } \frac{25}{4} \text{ cm}^2\right)$

d **i** the square **ii** the triangle

13 a **i** Instead of substituting $x = 5$ into $2x$ and multiplying 2 by 5, x has been replaced with 5 in the expression $2x$ without multiplying.

ii 16

b **i** Incorrect order of operations. Multiplication has been performed before applying the exponent.

ii 9

c **i** 2^0 does not equal 0, $2^0 = 1$.

ii 1

d **i** The negative sign has been left outside the brackets.

ii 21

14 a \$1050 **b** \$2433.31

15

x	x^2	$2x^2$	$16x$	$2x^2 - 16x$
0	0	0	0	0
2	4	8	32	−24
4	16	32	64	−32
6	36	72	96	−24
8	64	128	128	0

$x = 0$ and $x = 8$

16 a **i** 25 °C **ii** −20 °C **iii** 5 °C **iv** 38 °C **v** −18 °C **vi** 17 °C

b **i** 86 °F **ii** 14 °F **iii** 32 °F **iv** 81 °F **v** 18 °F **vi** 108 °F

c −5 °C

d 86 °F, as 32 °C = 89.6 °F

17 a $P = 0.95(x - y) - z$ **b** \$17 540.50

EX p191 5C Adding and subtracting algebraic terms

1 a $20x$, $-12x$ and $3x$
$8y$, $-15y$ and y

b $5x$, $\frac{3}{7}x$, $-25x$ and $\frac{x}{7}$
$\frac{5}{7}y$ and $6y$

c $4xy$, $-yx$, $\frac{1}{8}xy$ and $-\frac{yx}{10}$

d $-a$ and $12.5a$
$5a^2$, $\frac{4}{9}a^2$, $2.3a^2$ and $-\frac{9a^2}{10}$

e $\frac{11}{18}p^3$, $5.5p^3$ and p^3
$\frac{p^2}{5}$, $-11p^2$ and $-0.05p^2$

f $\frac{9ab}{10}$, $\frac{5}{6}ab$ and $7.2ba$
$7abc$, $-cab$ and $13cab$

g $-9fg$ and $\frac{4fg}{5}$
$120f^2g$ and $-\frac{14}{3}f^2g$
$\frac{20fg^2}{3}$ and $0.33fg^2$

h $\frac{2}{9}xy^2z$, $-12y^2xz$ and $99zy^2x$
$0.095zyx$ and $18.6xyz$

2 a true

b false; the powers of the pronumerals are not the same

c true **d** true

e false; the powers of the pronumerals are not the same

f true

3 a $9x$ **b** $8kp$ **c** $-4a^2$ **d** $4g$

e $-2xy$ **f** $2g^2h$ **g** $\frac{1}{4}abc$ **h** $-\frac{4}{15}xyz^2$

4 a true

b false; $10k$

c false; $3p + 6q$ (does not simplify further)

d false, $6m^2 + 2n$

e false, $9ab + 3a$

f true

g true

h false; $\frac{7}{18}p^2q - \frac{5}{6}p^2$

5 a $6b + 5$ **b** $-n - 5mn$ **c** $4 - 8c^2$ **d** $gh - 3$

e $-pq - 10q^2p + 8p^2q$

f $11x^2y - 12xy^2$

g $\frac{31}{14}rst + \frac{1}{6}st$

h $-\frac{1}{2}abc - \frac{1}{2}ab - 2$

i $\frac{2}{5}g^3h + \frac{11}{6} + \frac{4g^2h}{5} - \frac{1}{15}gh$

j $\frac{9}{4}m^2n - \frac{3}{4}m^2n^4 + \frac{1}{6}$

6 a 165 **b** $16x^2 + xy$ **c** yes

d simplified expression; only had to substitute values into two terms

7 a $6a + 14b$; 72 **b** $3a + 9b$; 42

c $5ab + 3b - 2a$; 74 **d** $-2ab + a^2$; −5

e $-5b^2 + 7a^2$; 130 **f** $\frac{1}{4}a^2 - \frac{2}{3}b^2$; $\frac{1}{4}$

g $-11ab - 2ab^2$; −255 **h** $-2ab^2 - 3a^2b$; −315

i $\frac{19}{20}a^2b - \frac{19}{6}ab^2$; $-\frac{285}{4}$

8 a Tania didn't keep the minus sign with $2y$ and the addition sign with $7x$.

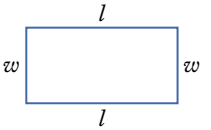
b $3x + 7x - 2y - 4y$

9 a $11x + 2y$ **b** $-\frac{7}{6}$

c $-2y$ **d** $-2xy + 8x$

e $16x^2y - 6xy^2 - 5$ **f** $-\frac{37}{20}x^2y + \frac{9}{4}xy^2$

10 a



b $l + w + l + w$

c $2l + 2w$

d $P = 2l + 2w$

e **i** 14 m **ii** 18.4 m **iii** 46 m

f Sample answer: length 8 m, width 2 m

11 a $8x + 4$ **b** $18x + 6$ **c** $36x + 4$ **d** $40x + 2$

12 **a** 36 cm **b** 78 cm **c** 148 cm **d** 162 cm

13 **a** The a has been multiplied four times instead of added four times. The expression simplifies to $4a$.

b The terms have been treated as like terms and added together instead of as unlike terms. The expression $2x + 3y$ is already in its simplest form.

c The x in each term has been multiplied together instead of treated as like terms. The expression simplifies to $13x$.

d The $-x$ term has been interpreted as dividing by x to cancel out the x in the $8x$ term. Instead, $-x$ is the same as $-1x$ so the expression simplifies to $7x$.

14 **a** $12(2x + 3) + 11(2x + 5)$ **b** $10(x - 3y + 4)$

c $2(x + 2) + 8(x - 2)$ **d** $\frac{13}{3x+1} + \frac{12}{3x-1}$

e $7\sqrt{7 - 2x} - 4\sqrt{7 + 2x}$ **f** $9(a + b) - \frac{3}{a+b} - \sqrt{a + b}$

15 **a** **i** $\frac{4}{3}$ and $\frac{4}{3}$ **ii** $-\frac{4}{3}$ and $-\frac{4}{3}$ **iii** $\frac{2}{3}$ and $\frac{8}{3}$

b **i** 0 and 0 **ii** 1 and -1 **iii** 1 and -1

c **i** 0 and 0 **ii** -1 and -1 **iii** 1 and 1

d **i** $-\frac{3}{5}$, $-\frac{3}{5}$, $-\frac{3}{5}$, and $\frac{3}{5}$

ii $\frac{3}{5}$, $\frac{3}{5}$, $\frac{3}{5}$, and $-\frac{3}{5}$

iii $-\frac{6}{5}$, $-\frac{6}{5}$, $-\frac{6}{5}$, and $\frac{6}{5}$

16 **a** The expressions are not equivalent. The expressions are equal for $x = 1$ and $x = -1$, $\frac{4}{3x}$ is dividing x and $\frac{4}{3}x$ is multiplying x, hence why the expressions are different for $x = 2$.

b The expressions are not equivalent. Despite the fact that the expressions are equal for $x = 0$, squaring a positive or negative number always results in a positive number whereas taking the negative of a squared number always results in a negative: $(-x)^2 = (-x)(-x) = xx = x^2$ but $-x^2 = -xx$.

c The expressions are equivalent. Cubing the negative of a number always results in the negative of the cube of that number: $(-x)^3 = (-x)(-x)(-x) = -xxx = -x^3$.

d The first three expressions are equivalent but $\frac{-3x}{-5}$ is not equivalent. Looking at the coefficients of the first three expressions, they all simplify to the same negative fraction. The coefficient of the final expression simplifies to a positive.

17 $11a^2b^7c^5d^5e^9f^{11}g^{14} + 9a^2b^5c^5d^7e^9f^{11}g^{14}$

18 $2.25x$ or $\frac{9}{4}x$

19 $\frac{27}{4}x$

EX p196 5D Multiplying algebraic terms

1 **a** $4x$ **b** $6k$ **c** $7d$ **d** $5m$

e p **f** $-3h$ **g** $-9a$ **h** $-10y$

i $-8c$ **j** $-e$ **k** $2f$ **l** $3m$

2 **a** $3ab$ **b** $4mn$ **c** $2pt$ **d** $-8cd$

e $9xy$ **f** $5fk$ **g** abc **h** ehy

i $8q$ **j** $-18m$

3 **a** B **b** A **c** C

4 **a** $5m^2$ **b** $-2a$ **c** $-xy$ **d** $2n^2$

e $-6c^2d^2$ **f** $7a^2bc$ **g** $-2s^2t^3$

h $-10f^2g^3$ **i** pq^3r^3 **j** $-11x^3y^2z^2$

5 **a** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

b $a \times a \times a \times a \times a \times a \times a = a^7$

c $m \times m \times m \times m \times m \times m \times m \times m = m^8$

d $x \times x \times x \times x \times x \times x \times x \times x \times x \times x \times x \times x \times x = x^{10}$

6 **a** a^5 **b** k^5 **c** c^7 **d** e^6f^5 **e** a^5d^6 **f** $m^{10}n^5$

7 **a** $12abc$ **b** $-a^2h^3$ **c** $21wxyz$

d $12x^4y^5$ **e** $21a^7$ **f** $-5p^{12}$

g $24n^2$ **h** $15x^8y^{10}$ **i** $-a^8b^6$

j $-18x^2yz$ **k** $\frac{1}{5}w^4x^5y^6z$ **l** $-\frac{18}{7}a^5b^7c^2$

8 **a**

×	$4a$	3	$6m$	$2abc$
$3d$	$12ad$	$9d$	$18dm$	$6abcd$
ab^2	$4a^2b^2$	$3ab^2$	$6ab^2m$	$2a^2b^3c$
$5mn$	$20amn$	$15mn$	$30m^2n$	$10abcmn$
$2ac$	$8a^2c$	$6ac$	$12acm$	$4a^2bc^2$

b

×	$3x$	$4xyz$	$-5y$	$2x^2$
$-2x$	$-6x^2$	$-8x^2yz$	$10xy$	$-4x^3$
xy^2	$3x^2y^2$	$4x^2y^3z$	$-5xy^3$	$2x^3y^2$
$6xy$	$18x^2y$	$24x^2y^2z$	$-30xy^2$	$12x^3y$
$-7yz$	$-21xyz$	$-28xy^2z^2$	$35y^2z$	$-14x^2yz$

9 **a** $-70bghk$ **b** $30g^{10}h^3$ **c** $-12b^9$ **d** $48m^2nq$

e $-21fg^3h^3$ **f** $-\frac{9}{40}m^{11}n^3$ **g** $24a^6b^5c$

h $10w^3x^8y^8$ **i** $-\frac{5}{2}a^8b^{13}c^7d^4$

10 **a** $2 \times 3 = 6$ **b** $8 \times -2 = -16$

c $6 \times k = 6k$ **d** $3 \times x = 3x$

e $a \times b = ab$ **f** $x \times -7y = -7xy$

g $5d \times c = 5cd$ **h** $4a \times 2b^2 = 8ab^2$

i $5n^3 \times 3mn = 15mn^4$ **j** $6e \times -2e^2f = -12e^3f$

k $5jk^2l \times 4l^2 = 20jk^2l^3$ **l** $5xy \times -xz^5 = -5x^2yz^5$

11 **a** C **b** D

c The coefficient of x is $\frac{7}{5}$; the coefficient of y is $-\frac{1}{5}$.

d D **e** D **f** B

g When using algebraic terms, placing pronumerals or a numeral and a pronumeral next to each other generally implies multiplication. However, for mixed numbers, placing a numeral in front of a fraction implies addition. Putting the algebraic and mixed numbers together leads to conflicting conventions presented simultaneously. For clarity, we do not generally use mixed numbers as coefficients.

12 **a** **i** $(3x^2)^4 = (3x^2) \times (3x^2) \times (3x^2) \times (3x^2)$

$= 3 \times 3 \times 3 \times 3 \times x^2 \times x^2 \times x^2 \times x^2$

$= 81x^8$

ii $\left(\frac{x^2}{3}\right)^4 = \left(\frac{x^2}{3}\right) \times \left(\frac{x^2}{3}\right) \times \left(\frac{x^2}{3}\right) \times \left(\frac{x^2}{3}\right)$

$= \frac{x^2 \times x^2 \times x^2 \times x^2}{3 \times 3 \times 3 \times 3}$

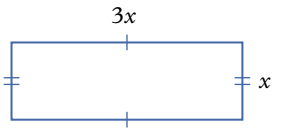
$= \frac{x^8}{81}$

b **i** $64x^{30}$ **ii** $\frac{y^{45}}{32}$

iii $64a^6b^{21}$ **iv** $\frac{625u^{24}}{16v^{20}}$

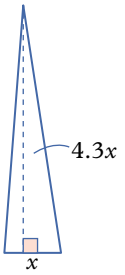
13 **a** $48xy$ **b** $12x^2y^2$ **c** $14x^2y^2$

14 **a** **i**



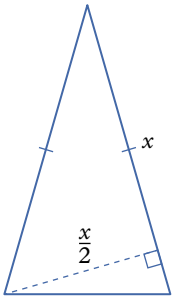
ii $3x^2$

b **i**



ii $2.15x^2$

c **i**



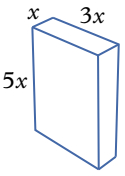
ii $\frac{x^2}{4}$

15 **a** $3x$ **b** $3x^2$ **c** 192 cm²

d When $x = 8$, width = 8 cm and length = 24 cm.
Area = length × width = 24 × 8 = 192 cm².
This gives the same result as substituting $x = 8$ directly into $3x^2$.

e $5x$

f



g

Face	Area
top	$3x \times x = 3x^2$
bottom	$3x \times x = 3x^2$
front	$3x \times 5x = 15x^2$
back	$3x \times 5x = 15x^2$
left side	$x \times 5x = 5x^2$
right side	$x \times 5x = 5x^2$

h $3x^2 + 3x^2 + 15x^2 + 15x^2 + 5x^2 + 5x^2 = 46x^2$

i 736 cm²

j **i** 1150 cm² **ii** 6624 cm² **iii** 11.5 cm²

iv 202.86 cm²

k $15x^3$

l **i** 1875 cm³ **ii** 25 920 cm³

iii 1.875 cm³ **iv** 138.915 cm³

16 a^{200} since we are multiplying 58 a's, 97 a's, and 45 a's to make a total of 200 a's multiplied together.

17 $2^{12a+5} \times 3^{13a+16}$

Checkpoint

1 **a** **i** x and y **ii** 4 **iii** 4, 9, −4

iv −3 **v** x: 1, y: 2

b **i** m, n, and v **ii** 3 **iii** $\frac{5}{3}$, 1

iv $-\frac{9}{13}$ **v** m: 2, n: 1, v: 3

2 **a** $\frac{x}{8}$ **b** $7(a+b+c)$ **c** $y(y+3)$

d $\sqrt{c+a}$ or $\sqrt{a+c}$

3 **a** $20g$ **b** $6g+b$ **c** $\frac{1}{3}(t+r+p)$

d $0.15b+3$ or $\frac{3b}{20}+3$

4 **a** 84 **b** 4 **c** 574 **d** $\frac{17}{2}$

5 **a** $y = 31$ **b** $s = 64$

6 **a** 61 **b** 20

7 **a** $7x+4y+10$ **b** $2ab+5a+10b$

c $8pqr$ **d** $30ikl+8jkl$

8 **a** $-3x+4x^2$ **b** $12xy^2+2x^2y$

9 **a** $-7.46x+0.26y$ **b** $-\frac{15}{8}x^2$

10 **a** $63xy$ **b** $27abcd$

c $-110ghpq$ **d** $36abcuvxy$

11 **a** $36x^5$ **b** $56x^4y^4$

12 **a** $0.372x^2$ **b** $\frac{5}{2}x^{12}$ or $\frac{5x^{12}}{2}$

EX 5E Dividing algebraic terms

p202

1 **a** a **b** $\frac{5x}{y}$ **c** 9 **d** 3 **e** $\frac{1}{3}$

f $4b$ **g** $3c^2$ **h** $\frac{2b^3}{3}$ **i** $\frac{p}{6}$ **j** $\frac{5}{3y}$

k $6a^3$ **l** $\frac{5m}{n^4}$ **m** $\frac{2m^2b}{3}$ **n** $2t^2$ **o** $\frac{4h^2kj}{5}$

2 **a** x **b** $2k$ **c** $9a$ **d** $9x$ **e** $\frac{n}{4}$ **f** bcd

3 **a** $\frac{5}{4m}$ **b** $2jk$ **c** $3x$ **d** $\frac{m^7}{p^3}$ (can't be simplified)

e b^2c^2 **f** $\frac{2k^8}{a^5}$ **g** $\frac{9x^4}{11y^4}$ **h** $\frac{3a^3b}{c^3}$ **i** $\frac{xy}{4}$ **j** $4n^2$

k mn^5 **l** x^4 **m** $\frac{6w^4}{x^4y^4}$ **n** $\frac{4}{5p}$ **o** $\frac{abd}{c}$

4 **a** $-3y$ **b** $-5a^2$ **c** $\frac{3v^2}{2}$ **d** $-\frac{6r}{7p}$

5 **a** x^5 **b** m^4 **c** $6a^6$ **d** n^5 **e** 5

6 **a** $16m \div 8 = 2m$ **b** $\frac{27jk}{-9j} = -3k$

c $16x^3y^2 \div 2xy = 8x^2y$ **d** $\frac{2a^3b^2}{10a^2b^3} = \frac{a}{5b}$

7 **a** $-x^2y$ **b** $10ab^4$ **c** $5x+27y$

d $21n+4m-6mn$

8 **a** $\frac{5}{36}$ **b** 64 **c** 40.5 **d** 7

9 **a** length = $2x$ cm; area = $2x^2$ cm²

b $100x^2$ cm² **c** 50 pavers

d **i** 40 cm **ii** 80 cm **iii** 3200 cm²

iv 400 cm **v** 160 000 cm²

e 50; same

f No; the expression used to calculate the number of pavers simplifies to 50 in each case as the pronumeral x is cancelled in the fraction.

10 a $\frac{3}{10}$ **b** $\frac{27x^2y}{16}$ **c** $\frac{x^3y^4}{z^4}$ **d** $\frac{1}{a^3b^3c^6}$

11 a $2^{14} \times 13^5 \times d^{12}$ **b** $3^4 \times 7^4 \times a^9 \times b^6$

c $p^9q^7r^{15}s^6$ **d** $r^{45}s^{51}t^{78}u^{84}v^{99}$

12 a $17gh$ **b** $4u$

c $53r^2$ **d** $2a^3 + 8a^2$

13 a **i** 1 **ii** 1 **iii** 1

b The expression simplifies to 1. It doesn't matter what the values of x, y and z are, the expression will always simplify to 1.

14 $\frac{x+1}{4}$

15 a Sample answer: $x^9y^7 \div x^1y^2 = x^8y^5$, sum is 8 + 5 = 13

b 9 cannot be an exponent of the divisor (second term) otherwise a subtraction would result in a negative exponent. The largest exponent for the quotient that can be created using a 9 is 8 by subtracting 1. Using the remaining digits, the three largest possible sums are 15 = 8 + 7, 14 = 8 + 6 and 13 = 8 + 5. Since 9 and 8 are already in use, it is not possible to have a difference of 7 with the remaining digits: 7 = $\mathcal{9} - 2 = \mathcal{8} - \mathcal{1}$. Similarly, since 9, 8 and 1 are in use, it is not possible to have a difference of 6 with the remaining digits: 6 = $\mathcal{9} - 3 = \mathcal{8} - 2 = 7 - \mathcal{1}$. However, it is possible to have a difference of 5 as 5 = 7 − 2. If instead we started with 9 − 2 = 7, the largest sum is still 13 since 15 = 7 + 8 but 8 = $\mathcal{9} - 1$ and 14 = 7 + $\mathcal{7}$.

EX p207 5F Expanding

1 a $3a + 6$ **b** $5k - 35$

c $10 + 2a$ **d** $8a + 8b$

e $8x + 24$ **f** $-2 + 16m$

g $35a + 21b$ **h** $48c - 30d$

i $-15m - 6n$

2 a $ab + 4a$ **b** $3x + wx$

c $cd - de$ **d** $2p^2 + 3p$

e $k - 4k^2$ **f** $8xy + 6wx$

g $3x - 12x^2$ **h** $\frac{2}{3}m^2 + \frac{7}{3}mn$

i $-6a^2 + 12ab$ **j** $8e^3 + 10e^2$

k $\frac{1}{2}a^4 - 15a^3b$ **l** $-11x^2y - 11y^3$

3 a $6x + 2y + 14$ **b** $10ab + 20d - 15$

c $14c - 7d + 35ef$ **d** $-18g + 12h - 6gh$

e $-20k - 8km + 36$ **f** $-6xy - 12 + 21w$

4 a $5 \times a + 5 \times 4 + 2 \times a + 2 \times 7$
$= 5a + 20 + 2a + 14 = 7a + 34$

b $3b \times c + 3b \times 2 + 2c \times a + 2c \times 5b$
$= 3bc + 6b + 2ac + 10bc = 13bc + 6b + 2ac$

c $d \times d + d \times 8 + 6 \times d + 6 \times (-3)$
$= d^2 + 8d + 6d - 18 = d^2 + 14d - 18$

d $4k \times 2k + 4k \times 3 + (-2) \times k + (-2) \times (-5)$
$= 8k^2 + 12k - 2k + 10$
$= 8k^2 + 10k + 10$

5 a $7a + 27$ **b** $10b$

c $11mn + 20m + 3kn$ **d** $21x + 5xy + \frac{16}{3}y$

e $\frac{k^2}{4} + \frac{19k}{4} + 20$ **f** $c^2 - 3c + 35$

g $11rst + 5rst^2 + 36rt$ **h** $h^3 - 4h^2 - 14h$

6 $-(a + 2) = -1 \times (a + 2)$
$= -1 \times a - 1 \times 2$
$= -a - 2$

7 a $7p + 20$ **b** $2d - 14$

c $x^2 + 3x + 5$ **d** $k^2 + k - 1$

e $2e^2 + \frac{3}{2}$ **f** $3ab + 4a - 6$

g $-\frac{2}{3}wy - w - 2$ **h** $mnp - 4m^2p + n$

8 a 440 **b** $15a^2 + 50a$ **c** 440

d Sample answer: It was easier to evaluate the expression in expanded form as there are just two terms to evaluate.

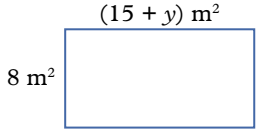
9 a $2a^2 + 10a$; 28 **b** $30a - 6a^2$; 36

c $15a^2 - 65a$; −70 **d** $2a^2 - 3a$; 2

10 a $c + 3$ **b** $2(c + 3)$ **c** $2c + 6$ **d** 42

11 a 120 m² **b** $8y$ m² **c** $(120 + 8y)$ m²

d



e $8(15 + y)$ m²

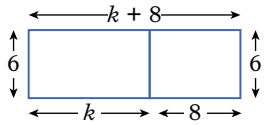
f $(120 + 8y)$ m², the expressions are the same as they represent the same area of lawn.

12 area of large rectangle = sum of areas of two smaller rectangles

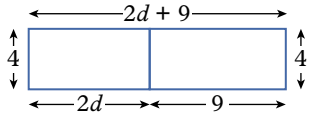
a area of large rectangle = $(a + 2) \times 3$ or $3(a + 2)$
sum of areas of smaller rectangles =
$3 \times a + 3 \times 2 = 3a + 6$

b area of large rectangle = $(x + 4) \times x$ or $x(x + 4)$
sum of areas of smaller rectangles =
$x \times x + x \times 4 = x^2 + 4x$

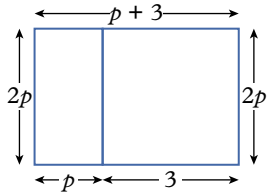
13 a



b



c

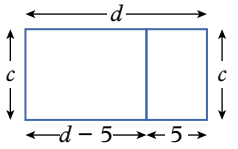


14 **a** $a - 3, 5$ **b** $5(a - 3)$

c **i** $a, 5$ **ii** $5a$ **iii** $3, 5$ **iv** 15

v $5a - 15$

15



16 $5a(b + 3) = 5ab + 15a$, so if a and b are both positive integers, $5a \times b + 3$ will always be less than $5a(b + 3)$.

17 **a** $7x^5 - 10x^4 - 21x^2$ **b** $4a^3c$

c $6m^3np^2 + 4m^2n - 6m^2np^2 + 24n$ **d** $3wy$

18 **a** -20 **b** -432 **c** -256 **d** 90

EX 5G Factorising

p212

1 **a** C **b** D **c** B **d** A

2 **a** 4 **b** $5d$ **c** $2j$ **d** $3ab$ **e** 9

f $7y$ **g** bc **h** $2bcd$ **i** $2y$ **j** $3c$

k $6g^2$ **l** $4b$ **m** m^2p^2 **n** $5r^2s^4t$

3 **a** $2x + 6 = 2\left(\frac{2x}{2} + \frac{6}{2}\right) = 2(x + 3)$

b $8a - 36 = 4\left(\frac{8a}{4} - \frac{36}{4}\right) = 4(2a - 9)$

c $25mn + 30n = 5n\left(\frac{25mn}{5n} + \frac{30n}{5n}\right) = 5n(5m + 6)$

d $kmn - mn = mn\left(\frac{kmn}{mn} - \frac{mn}{mn}\right) = mn(k - 1)$

e $12ef - 18df = 6f\left(\frac{12ef}{6f} - \frac{18df}{6f}\right) = 6f(2e - 3d)$

f $-26ab + 24cd = -2\left(\frac{-26ab}{-2} + \frac{24cd}{-2}\right)$
$= -2(13ab - 12cd)$

g $-abc + acd = -ac\left(\frac{-abc}{-ac} + \frac{acd}{-ac}\right) = -ac(b - d)$

h $-21xyz - 28wxy = -7xy\left(\frac{-21xyz}{-7xy} - \frac{28wxy}{-7xy}\right)$
$= -7xy(3z + 4w)$

4 **a** $4(a + 2)$ **b** $7(e + 3)$ **c** $5(9 + k)$

d $3(1 + 4y)$ **e** $9(3 - p)$ **f** $6b(4a + 3)$

g $2t(6s - 7)$ **h** $5n(m + 2)$ **i** $3g(1 - 3h)$

j $2x(11y - 2z)$ **k** $3b(a + c)$ **l** $8m(2n - p)$

m $9xy(3z - 2w)$ **n** $3mn(4k + 5)$ **o** $-3fg(7e - 8h)$

5 **a** $h^2 + 7h = h\left(\frac{h^2}{h} + \frac{7h}{h}\right) = h(h + 7)$

b $10a + 5a^2 = 5a\left(\frac{10a}{5a} + \frac{5a^2}{5a}\right) = 5a(2 + a)$

c $6p^3 + 9p^5 = 3p^3\left(\frac{6p^3}{3p^3} + \frac{9p^5}{3p^3}\right) = 3p^3(2 + 3p^2)$

d $6xy + 4x^2y = 2xy\left(\frac{6xy}{2xy} + \frac{4x^2y}{2xy}\right) = 2xy(3 + 2x)$

e $mn^2 + 5mn = mn\left(\frac{mn^2}{mn} + \frac{5mn}{mn}\right) = mn(n + 5)$

f $24efg^2 + 32fg^7h^3 = 8fg^2\left(\frac{24efg^2}{8fg^2} + \frac{32fg^7h^3}{8fg^2}\right)$
$= 8fg^2(3e + 4g^5h^3)$

6 **a** $c(c + 4)$ **b** $k(6 - k)$ **c** $a(a - 2)$

d $3m^2(1 + m^2)$ **e** $p^3(10 + 11p^2)$ **f** $h(4 - 7h^3)$

g $3x^2y(2 + xy^5)$ **h** $5bc^2(bc^3 - 2)$ **i** $4mn^2(2 + 9mn)$

j $3xyz(9z - 5x)$ **k** $2a^2bc^4(7ac^6 + 8b^6)$

l $6efg^2(2e + 3f^4)$

8 **a** -5 **b** $-2p$ **c** $-3k$ **d** $-7st$

e $-4x$ **f** $-4m^2$ **g** $-xy$ **h** $-4bc^2$

9 **a** $-4(a + 3)$ **b** $-2(3 + 4k)$ **c** $-7m(1 - 2n)$

d $-5b(3 - 7a)$ **e** $-3t(s + 3r)$ **f** $-6bc(3a + 4d)$

g $-3x^{12}(4 - 7x^9)$ **h** $-h(h + 2)$

i $-8x^2y^7(2 - x^3y^3)$ **j** $-5ab^6(9 + 2a^{14}b^4c^8)$

k $-6xyz^2(7 + 6x^3y^{12})$ **l** $-abc^3(11a - 9bc^6)$

10 **a** 3 **b** bd **c** $5m$ **d** $6a$

e $4xz$ **f** $7i^2j^2k$

11 **a** $5(3p + 3q - 1)$

b $k(6j - 3i - 10l)$

c $-2np(4mq - 2jk + km)$

d $-9f(3g - h)$

e $17uv(wx + 2x - 3xz - 4)$

f $8a^7b^7c^4(7ab^5 + 4a^2c^5 - 10b^4c^3)$

12 **a** $x(x + 6)$ **b** x and $x + 6$ **c** $x + 6$

d $(x + 6) \times x = x \times (x + 6) = x \times x + x \times 6 = x^2 + 6x$

e length $= 3 + 6 = 9$ cm; width $= 3$ cm so
area = length × width $= 9 \times 3 = 27$ cm^2
using $x^2 + 6x$, area $= 3^2 + 6 \times 3 = 9 + 18 = 27$ cm^2

13 **a** **i** $32 - 24x$ **ii** $8(4 - 3x)$

b **i** $7(y + 7)$ **ii** $7y + 49$

c **i** $6b^5 - 15b^8$ **ii** $3b^3(2b^2 - 5b^5)$

d **i** $25a(a - 5b)$ **ii** $25a^2 - 125ab$

14 **a** $5x \times 3y + \frac{1}{2} \times 5x \times 2x = 15xy + 5x^2$

b $5x(3y + x)$

15 **a** **i** $6\left(\frac{x}{6} + \frac{5y}{6} + \frac{11z}{6}\right)$ **ii** $-(-x - 5y - 11z)$

iii $\frac{1}{2}(2x + 10y + 22z)$ **iv** $x\left(1 + \frac{5y}{x} + \frac{11z}{x}\right)$

v $-3x\left(-\frac{1}{3} - \frac{5y}{3x} - \frac{11z}{3x}\right)$

b **i** $6\left(\frac{x}{3} - \frac{y}{2} + z\right)$ **ii** $-(-2x + 3y - 6z)$

iii $\frac{1}{2}(4x - 6y + 12z)$ **iv** $x\left(2 - \frac{3y}{x} + \frac{6z}{x}\right)$

v $-3x\left(-\frac{2}{3} + \frac{y}{x} - \frac{2z}{x}\right)$

c **i** $6(3x^2 + 6x^3y - 9x^4z)$

ii $-(-18x^2 - 36x^3y + 54x^4z)$

iii $\frac{1}{2}(36x^2 + 72x^3y - 108x^4z)$

iv $x(18x + 36x^2y - 54x^3z)$

v $-3x(-6x - 12x^2y + 18x^3z)$

d **i** $6\left(\frac{2xy}{3} - x - \frac{5xz}{3}\right)$ **ii** $-(-4xy + 6x + 10xz)$

iii $\frac{1}{2}(8xy - 12x - 20xz)$ **iv** $x(4y - 6 - 10z)$

v $-3x\left(-\frac{4y}{3} + 2 + \frac{10z}{3}\right)$

16 a i When evaluating $\frac{12q}{-6}$, the student got $2q$, not $-2q$, and used a positive sign instead of a negative sign.

ii $-6(3p - 2q)$

b i The student did not take out the HCF, $8r$, as a factor so the expression is not fully factorised.

ii $8r - 16r^3 = 8r\left(\frac{8r}{8r} - \frac{16r^3}{8r}\right) = 8r(1 - 2r^2)$

c i The student simplified $\frac{5y^3}{5y^3}$ to 0, not 1, as the numerator and denominator are the same.

ii $20y^4 + 5y^3 = 5y^3\left(\frac{20y^4}{5y^3} + \frac{5y^3}{5y^3}\right) = 5y^3(4y + 1)$

d i The student took out the first term as the factor, not the HCF.

ii $6x^3y + 7xy^3 = xy\left(\frac{6x^3y}{xy} + \frac{7xy^3}{xy}\right) = xy(6x^2 + 7y^2)$

17 a y and $(x + 2)$

b $y(7x - 5)$ and $(x + 2)(7x - 5)$

18 $\frac{5}{8}x^{12}y^{17}(y^2 - 5x^9y^3 + 3x^5 - 2x^{20}y^{23})$

CHAPTER 5 review

Multiple choice

1	B	**2**	E	**3**	A	**4**	B	**5**	A	**6**	C
7	E	**8**	C	**9**	D	**10**	E	**11**	C	**12**	C

Short answer

1 a $4m + 3$ **b** $\frac{5m}{2}$ **c** $\frac{m + 3}{7}$

d $\frac{3}{5}(m - 6)$

2 a $25x$ **b** $13y$ **c** $20p$

d $25x + 13y + 20p$

3 a 24 **b** 80 **c** -16 **d** -7

4 a $v = \sqrt{u^2 + 2as}$ **b** 16 m/s

5 a $6a, \frac{1}{7}a, \frac{a}{10}$ and $b, -b, \frac{3}{8}b$

b $0.5x, 1.7x, 10x, \frac{5}{6}x$ and $95y, \frac{1}{2}y$

c $5de, -ed, 0.01de$ and $\frac{2ef}{15}, 12fe, ef$

d $p^2, 5.1p^2, -15p^2, \frac{3}{2}p^2$ and $2p^3, \frac{3p^3}{2}$

e $\frac{9}{10}ab, -6.5ba, -ab$ and $6.5abc, 0.9cab, \frac{11}{20}bca$

f $\frac{4}{9}f^2g, \frac{gf^2}{3}$ and $-23g^2f, -3.14fg^2$

6 a $12m + 8n$ **b** $8cd + 2c + 4$

c $-7k^2 + 5$ **d** $2ab^2 + 2a^2b$

e $-5xyz + \frac{11}{2}xy - \frac{2}{5}y$ **f** $-\frac{3}{4}pq^2 - 8q^2$

7 a x^{14} **b** m^2 **c** h^{11}

d $p^{12}q^6$ **e** $x^{25}y^{15}$ **f** $a^6b^{30}c^{12}$

8 a $-5fg$ **b** $2p^2s$ **c** $56g^{10}$

d $24ab^2c$ **e** $-6m^{12}n^{12}$ **f** $\frac{12}{5}e^2f^{10}g^{18}$

9 a k^4 **b** $8c^8$ **c** x^3y

d m^3 **e** $p^{11}q^2r^{18}$ **f** $ab^{12}c^{10}$

10 a $\frac{3a}{4}$ **b** $3c$ **c** $\frac{3ad}{5}$

d $\frac{2ehk}{5}$ **e** $\frac{4x^7y^5}{7}$ **f** 1

11 a $5a + 15$ **b** $28b - 4c$

c $3ac + 5cd$ **d** $-24m^3 - 6mn$

12 a $-12k + 31$ **b** $2y^2 - 13y - 4$

13 a 2 **b** $3p$ **c** $5b$

d $8st$ **e** a^2b **f** $6mn^9p$

14 a $2(7h - 9)$ **b** $10a(2b + c)$ **c** $4c(2c + 3)$

d $-5fg(e + h)$ **e** $5ab(3a - 8)$ **f** $6x^2y^3(3y^2 - 4wx^8)$

Analysis

1 a $yx + y(x + 2) + y(x + 4) + y(x + 6) + y(x + 8) + y(x + 10) + y(x + 12)$

b \$30.10 **c** $7xy + 42y$

d \$30.24 **e** $7y(x + 6)$

f \$30.03

2 a $2a^2 + 2b^2$ **b** a^2b^2

c $a^2b^2c^3$ **d** $8a^2 + 4b^2$

e $8a^2b^2$ **f** $8a^2b^2c^3$

SEMESTER 1 Review

Short Answer

1 a 569 000 **b** 600 000 **c** 208 492.175

d 208 492.18 **e** 0.000 05 **f** 28 180

2 a $\frac{7}{8} = \frac{21}{24}$ **b** $\frac{-8}{10} = \frac{-12}{15}$

c $2\frac{4}{5} = \frac{70}{25}$ **d** $\frac{-1.2}{-0.07} = \frac{1200}{70}$

3 a $\frac{3}{4}$ **b** $\frac{50}{3} = 16\frac{2}{3}$ **c** $7\frac{3}{4}$ **d** 7 : 6

e 3 : 2 : 6 **f** 20 : 1 **g** 1 : 6 **h** 60 km/h

4 a 1309 **b** -38 **c** $\frac{25}{18} = 1\frac{7}{18}$

d 21.425 **e** 24 **f** $10\frac{7}{15} = \frac{157}{15}$

5 a 213 **b** 7 **c** $-\frac{27}{56}$

d 38.035 **e** 72 **f** $\frac{91}{24} = 3\frac{19}{24}$

6 a 4410 **b** -60 **c** $\frac{5}{3} = 1\frac{2}{3}$

d 133.56 **e** 32 **f** $\frac{40}{3} = 13\frac{1}{3}$

7 a 643 **b** 85 **c** $-\frac{8}{15}$

d 0.07 **e** 27 **f** $\frac{49}{34} = 1\frac{15}{34}$

8 a 216 **b** -243

c $\frac{125}{64} = 1\frac{61}{64}$ **d** 0.0016

e $\frac{2401}{16} = 150\frac{1}{16}$ **f** 1

9 a 6 **b** -2

c $\frac{9}{5} = 1\frac{4}{5}$ **d** 0.7

10 a 5 **b** 58

c $-\frac{13}{12} = -1\frac{1}{12}$ **d** 10.432

11 **a** $3y - 5xy + 5x$ **b** $8b - 8a$
c 2^7 **d** $35x^3y^6$
e 3^8 **f** $8a^5b^4$
g 3^{30} **h** $2^6 \times 3^5$
i $2x + 4$ **j** $-6p^4$

12 **a** **i** 14 000 **ii** 15 505.499 72
iii 9.71%
b **i** 4500 **ii** 4245.875 477...
iii 5.99%

13 **a** 8.4201, 24.01, 24.1, 824.01, 842.01, 842.1
b −5.31, −5.1, −5.031, 3.15, 5.103, 5.13
c $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{7}{8}, \frac{9}{10}, \frac{11}{12}$
d $\frac{6}{-2}, -\frac{3}{2}, -\frac{4}{3}, \frac{-3}{4}, \frac{-3}{-4}, \frac{4}{3}$

14 **a** **i** non-terminating **ii** recurring
b **i** non-terminating **ii** non-recurring
c **i** terminating
d **i** non-terminating **ii** recurring

15

Fraction	Decimal / recurring decimal	Percentage
$\frac{29}{20} = 1\frac{9}{20}$	1.45	145%
$\frac{7}{8}$	0.875	87.5%
$\frac{5}{2} = 2\frac{1}{2}$	2.5	250%
$\frac{21}{5}$	4.2	420%
$\frac{1}{3}$	$0.\dot{3}$	$33.\dot{3}\% = 33\frac{1}{3}\%$
$\frac{27}{25} = 1\frac{2}{25}$	1.08	108%
$\frac{4}{33}$	$0.\overline{12}$	$12.\overline{12}\% = 12\frac{4}{33}\%$

16 **a** $\frac{4}{5}$ **b** 40% **c** 32%
d 21 : 4 **e** 19 eggs/week **f** 85 km/h

17 **a** 45 **b** 44 **c** 108.9
d 64.96 **e** 96 **f** \$1200 and \$1500
g 21 blue **h** 0.8 kg **i** 4 km
j 8 min 20 seconds

18 **a** $p^2 - q^2$ **b** $3x + \frac{y}{12}$ **c** $\frac{x+6}{x-6}$ **d** $\sqrt{7k}$

19 **a** $32 - 24x$ **b** $-22x + 38$
c $12ab + 10b^2 + 6b$ **d** $30x^3y^2 - 45x^2y^5$

20 **a** $9(4x + 3)$ **b** $6(2a + 6b - 3c)$
c $\frac{5}{3}p^3(2 + 5p^4)$
d $\frac{uv}{4}\left(-\frac{3u}{2} + 5v\right) = -\frac{uv}{4}\left(\frac{3u}{2} - 5v\right)$

21 **a** 55c or \$0.55 **b** \$25
c \$1.98 **d** \$64
e \$49.50

22 **a** LCM = 90, HCF = 6
b LCM = 2100, HCF = 70
c LCM = $2^4 \times 3^9 \times 5^4$, HCF = $2^3 \times 3^6 \times 5^4$
d LCM = $2^7 \times 3^8 \times 5^8 \times 7^4 \times 11^2$, HCF = $2^3 \times 7$

23 **a** 854 **b** $2\frac{1}{2} = \frac{5}{2}$
c $8\frac{4}{5} = \frac{44}{5}$ **d** 0.65

24 **a** false **b** true **c** false
d true **e** true **f** false

25 **a** $2^3 \times 3 \times 5$ **b** $5^3 \times 7$

Analysis

1 **a** lower quality flowers \$0.90
medium quality flowers \$2.25
higher quality flowers \$3.51
b \$25.29
c **i** \$31.19 **ii** \$13.36
d **i** \$261.25 **ii** \$173
e \$10 cheaper

2 **a** **i** 372 + 654 = 1026 **ii** 1025 − 856 = 169
iii 632 × 72 = 45 504 **iv** 2760 ÷ 8 = 345
v $4^4 = 256$ **vi** $\sqrt{16} = 4$
vii $\sqrt[3]{27} = 3$ **viii** $2^3 \times 2^2 = 2^5$
ix $3^3 \div 3^3 = 1$
b **i** 9.75% **ii** 36.69% **iii** 87.5% **iv** 89.28%
v 93.75% **vi** 100% **vii** 200% **viii** 100%
ix 100%
c **i** 272 + 654 = 926 **ii** 1087 − 856 = 231
iii 79 × 72 = 5688 **iv** 296 ÷ 8 = 37
v $2^4 = 16$ **vi** $\sqrt{64} = 8$
vii $\sqrt[3]{729} = 9$ **viii** $2^4 \times 2^2 = 2^6$
ix $0^3 \div 3^3 = 0$

3 **a** **i** $2^8 \times 3^7 \times 5^4$ **ii** $3^8 \times 2^7 \times 7^4$
iii $3^8 \times 5^7 \times 7^4$ **iv** $a^8b^7c^4$
b **i** $\frac{2^2}{3^3 \times 5^2}$ **ii** $\frac{3^2}{2^3 \times 7^2}$
iii $\frac{3^2}{5^3 \times 7^2}$ **iv** $\frac{a^2}{b^3c^2}$
c **i** $2^3 \times 3^2 \times 5$ **ii** $3^3 \times 2^2 \times 7$
iii $3^3 \times 5^2 \times 7$ **iv** a^3b^2c
d **i** $2^5 \times 3^5 \times 5^3$ **ii** $3^5 \times 2^5 \times 7^3$
iii $3^5 \times 5^5 \times 7^3$ **iv** $a^5b^5c^3$

4 **a** $5(w + 3)$ **b** $5w + 15$
c 25 m²
d $w = 1$: $A = 20$ m²
$w = 2$: $A = 25$ m²
$w = 3$: $A = 30$ m²
$w = 4$: $A = 35$ m²
e $w + 13$ **f** $2x(w + 13)$
g $2xw + 26x$ **h** 16 m²

EXPLORATIONS 1

1 a The sum of digits is always a multiple of 3.

b Swap the first two digits.

c The value changes by a multiple of 999 999, which is itself a multiple of 7.

d With 2 notes, put an odd digit first and an even digit second.

e For 5, 6, 8, 9, 10 and 12 notes, swap the first two digits, and for 11 notes, swap the 1st and 3rd digits (among other possibilities).

f Brooke can always win by swapping the 1st and 7th digits (among other possibilities).

2 a $7\overline{)1.{}^{1}0{}^{3}0{}^{2}0{}^{6}0{}^{4}0{}^{5}0{}^{1}0}$ $0.1428 57\ldots$

b $\frac{2}{7} = 0.\overline{285714}$, $\frac{3}{7} = 0.\overline{428571}$, $\frac{4}{7} = 0.\overline{571428}$, $\frac{5}{7} = 0.\overline{714285}$, $\frac{6}{7} = 0.\overline{857142}$

c $\frac{1}{14} = \frac{5}{70} = 0.0\overline{714285}$

d $\frac{1}{70}$, so $\frac{3}{14} = 0.2\overline{142857}$

e $\frac{5}{14} = 0.3\overline{571428}$, $\frac{9}{14} = 0.6\overline{428571}$, $\frac{11}{14} = 0.7\overline{857142}$ and $\frac{13}{14} = 0.9\overline{285714}$

f $\frac{2}{63} = \frac{1}{7} - \frac{1}{9} = 0.\overline{031746}$

3 a 10 hours **b** 71 hours

c 7 professionals and 19 hours

d $\frac{(x + y/2)(t - T)}{X + Y/2} = \frac{(2x + y)(t - T)}{2X + T}$

4 a 480 mL

b 600 mL, 500 mL, 400 mL, 300 mL, 200 mL, 100 mL and 0 mL

c 525 mL, 450 mL, 375 mL, 300 mL, 225 mL, 150 mL and 75 mL

5 a **i** 2^{12} **ii** 4^6 **iii** 64^2 **iv** 16^3

b $1 = 2^0 = 4^0 = 1^2 = 1^3$ and $64 = 2^6 = 4^3 = 8^2 = 4^3$

c $18^2 \times 216^3$, $3888^2 \times 6^3$, $144^2 \times 54^3$ or 486×24^3

d Only 20^1 and 24^1 cannot be written this way (1 is both a square and a cube). For any whole number n, the only power which may not be able to be written this way is n^1, since all higher powers always can.

e There are many solutions. Two are given for each part: one which can be constructed from powers of 2 and a second which is essentially a coincidence.

i $128^3 = 1024^2 + 16^5$ (and $7^3 = 10^2 + 3^5$)

ii $32^5 = 4096^2 + 256^3$ (and $8^5 = 104^2 + 28^3$)

f **i** Yes. For example, 7-24-25 is a Pythagorean triple, so $5^4 = 25^2 = 7^2 + 24^2$.

ii No (aside from $0^6 = 0^3 + 0^3$). The sixth power is also a perfect cube, but Fermat's last theorem says that no cube is the sum of two smaller cubes.

6 a 1 house point. No, adding 14 or 18 will not improve his score.

b If the first number is n, the total is $4(2n + 7)$ which is divisible by at least 2 and 4. The smallest total with only 2 house points is 44, when $n = 2$.

c If the first number ends in 1, the total is of the form $60n+21$, which is only divisible by last digits 1 and 3; similarly for the other cases. The smallest number of house points is 0 (for example, from 4, 5, 6, 7, 8, 9).

d The total is always odd, so cannot be divided by 2, 4, 6 or 8. The smallest total is 315.

e 5 consecutive numbers, with smallest total 120.

CHAPTER 6 Linear relationships

6A Equations

EX p230

1 a **i** expression **ii** equation **iii** equation **iv** expression **v** equation **vi** equation

b Equations contain an equals sign (=).

2 a yes **b** no **c** no **d** yes **e** no **f** yes

3 a no **b** no **c** yes **d** yes **e** yes **f** no

4 a yes **b** yes **c** no **d** no **e** yes **f** no **g** no **h** yes

5 a $a = 7$ **b** $b = 6$ **c** $c = 8$ **d** $d = 12$ **e** $e = 2$ **f** $f = -3$ **g** $g = 1$ **h** $h = 0$ **i** $i = -3$ **j** $j = -5$ **k** $k = 7$ **l** $l = 35$ **m** $m = 4$ **n** $n = -10$ **o** $x = -14$ **p** $p = 0$ **q** $q = 6$ **r** $r = -12$ **s** $s = -4$ **t** $t = 25$

6 D

7 E

8 A

9 C

10 a $a = 2$ **b** $b = 1$ **c** $c = 3$ **d** $d = 0$ **e** $e = 6$ **f** $f = 10$ **g** $g = 4$ **h** $h = -7$

11 a $x = 5$ **b** $x = 4$ **c** $x = -1$ **d** $x = 9$ **e** $x = -10$ **f** $x = 0$ **g** $x = -3$ **h** $x = -4$

12 a **i** $n + 4 = 11$ **ii** $n - 3 = -2$ **iii** $n + 15 = 21$ **iv** $n - 8 = -4$ **v** $2n = 14$ **vi** $\frac{n}{4} = 9$ **vii** $n + (-7) = -20$ or $n - 7 = -20$ **viii** $11n = 22$ **ix** $5n = 55$ **x** $\frac{n}{6} = -3$

b **i** $n = 7$ **ii** $n = 1$ **iii** $n = 6$ **iv** $n = 4$ **v** $n = 7$ **vi** $n = 36$ **vii** $n = -13$ **viii** $n = 2$ **ix** $n = 11$ **x** $n = -18$

13 a **i** $3x + 8 = 11$ **ii** $\frac{x}{6} - 2 = 1$ **iii** $\frac{x + 7}{2} = 5$ **iv** $3(x - 9) = 3$

b **i** $x = 1$ **ii** $x = 18$ **iii** $x = 3$ **iv** $x = 10$

14 **a** $x + 38$ **b** $x + 38 = 62$
c $x = 24$ **d** \$24

15 **a** $26 + x$ **b** $26 + x = 81$
c $x = 55$ **d** 55 rolls

16 **a** $8x = 96$ **b** $x = 12$ **c** 12 people

17 **a** m = number of macadamia nuts
b $\frac{m}{4}$ **c** $\frac{m}{4} = 9$
d $m = 36$ **e** 36 macadamia nuts

18 **a** $8t = 24$, where t = cost of each chocolate
b \$3

19 **a** $\frac{f}{2} + 3 = 7$, where f = number of pieces of fruit initially in the bowl
b 8 pieces of fruit

20 **a** $2c + 7 = 11$, where c = number of cars in the car park at 9 am
b 2 cars

21 **a** $25c$ **b** $40a$ **c** $25c + 40a = 155$
d Substituting $a = 2$ and $c = 3$ into the equation shows Poh's calculations are correct.

22 **a** $2(x + 30) = 270$ **b** 105 minutes

23 **a** The time Arvan spends practising every day is not the same every day.
b $x + x + 2x + 2x - 20 + x = 190$ (or $7x - 20 = 190$)
c

Day	Time spent practising	Comments
Monday	30	Good start!
Tuesday	30	Same as Monday
Wednesday	60	Double Monday!
Thursday	40	20 minutes less than yesterday
Friday	30	Same as Monday
Total	190	

d The average (mean) time Arvan spends practising

24 **a** $h + \frac{h}{2} + 10 = 100$
b $h = 60$ **c** $60 + \frac{60}{2} + 10 = 100$

EX p236 6B Solving linear equations

1 **a** -6 **b** $\div 4$ **c** $+7$ **d** $\times 3$
e $\div -2$ **f** $\times -8$ **g** $+11$ **h** $\div -20$

2 **a** $x = 13$ **b** $x = 42$ **c** $x = 12$ **d** $x = 27$
e $x = 1$ **f** $x = -1$ **g** $x = -25$ **h** $x = -3$

3 **a** $+6$ **b** -9 **c** $\div 3$ **d** $\times 4$
e $+2$ **f** $\times 5$ **g** $\div 7$ **h** -1

4 **a** $-3, \div 3$ **b** $+8, \div 2$ **c** $-5, \times 2$ **d** $\div 5, -5$
e $+11, \div 5$ **f** $\times 5, \div 2$ or $\div 2, \times 5$
g $\div 9, -4$ **h** $\times 3, -4$

5 **a** $x = 12$ **b** $x = 14$ **c** $x = 7$ **d** $x = 10$
e $x = 0$ **f** $x = -22$ **g** $x = -5$ **h** $x = -3$
i $x = -2$ **j** $x = 0$ **k** $x = -3$ **l** $x = -17$

6 **a** $x = 2.5$ **b** $x = -5$ **c** $x = -5$ **d** $x = -2$

7 **a** $x = 8$ **b** $x = 1$ **c** $x = 24$ **d** $x = 6$
e $x = 18$ **f** $x = 33$ **g** $x = 20$ **h** $x = 2$
i $x = 7$ **j** $x = 6$ **k** $x = 1$ **l** $x = 13$

8 **a** $x = 1$ **b** $x = -2$ **c** $x = 4$ **d** $x = 2$
e $x = 15$ **f** $x = 2$ **g** $x = -9$ **h** $x = 10$
i $x = -22$ **j** $x = -15$ **k** $x = -1$ **l** $x = 6$

9 **a** $x = \frac{1}{5}$ **b** $x = 4\frac{1}{2}$ $\left(\text{or } x = \frac{9}{2}\right)$
c $x = 5\frac{2}{3}$ $\left(\text{or } x = \frac{17}{3}\right)$ **d** $x = \frac{4}{5}$
e $x = 4\frac{2}{5}$ $\left(\text{or } x = \frac{22}{5}\right)$
f $x = -11\frac{1}{2}$ $\left(\text{or } x = -\frac{23}{2}\right)$

10 No, as Caleb has not performed the inverse operation on both sides of the equation. He needs to add 37 to obtain the solution.

11 **a** Both students have solved the equation correctly.
b Yuhan's method: Eliminating the fraction first means you don't need to work with fractions after that step; however, you need to consistently work on both sides of the equation.
Connor's method: By completing one calculation at each step, the solution is easy to follow; however, you need to be confident working with fractions.

12 **a** $x = 14$ **b** $x = -6$ **c** $x = -10$
d $x = \frac{3}{4}$ **e** $x = 1.8$ **f** $x = 1\frac{7}{8}$ $\left(\text{or } x = \frac{15}{8}\right)$

13 **a** **i** $2(x + 4) = 26$ **ii** $x = 9$
b **i** $280 + 15x = 1600$ **ii** $x = 88$
c **i** $257 + 56x = 649$ **ii** $x = 7$
d **i** $\frac{x - 6}{2} = 11$ **ii** $x = 28$

14 $216 + \frac{t}{3} + t = 256$, $t = 30$

15 $5\left(x \times \frac{3}{4}\right) = 75$, $x = 20$

16 **a** $10x - 5\left(\frac{3}{2}x\right)$
b You do not know how many points Jenny had at the end of the week.
c The expression simplifies to $\frac{5}{2}x$, which will always be a positive value as the number of jobs completed will be positive (or zero).
d 8

17 **a** $x = 3$ **b** $x = 13$ **c** $x = 9$ **d** $x = -5$
e $x = -4$ **f** $x = 8$ **g** $x = 10$ **h** $x = 14$

18 **a** **i** $-5, \div 3, \surd$ **ii** 3
b **i** $\div 4, +2, \surd$ **ii** 4
c **i** square, $-5, \div 2$ **ii** 10

19 12 years old

20 **a** $\frac{12 + 12}{4} = \frac{24}{4}$
$= 6$
b Giovanni performed the inverse operations in the incorrect order. He subtracted 12 from each side first, before multiplying by 4, whereas he should've multiplied each side by 4 first.
c 48

EX p240 6C Solving equations with the unknown on both sides

1 **a** $x = 13$ **b** $x = 4$ **c** $x = 5$
d $x = 2$ **e** $x = -6$ **f** $x = 14$
g $x = 10$ **h** $x = 15$ **i** $x = 8$
j $x = -13$ **k** $x = 8$ **l** $x = 0$

2 **a** $a = 2$ **b** $b = 4$ **c** $c = -1$ **d** $d = 2$
e $e = -4$ **f** $f = 5\frac{1}{2}$ $\left(\text{or } f = \frac{11}{2}\right)$
g $g = 3\frac{1}{3}$ $\left(\text{or } g = \frac{10}{3}\right)$ **h** $h = -3$
i $i = 2\frac{1}{2}$ $\left(\text{or } i = \frac{5}{2}\right)$ **j** $j = 5$
k $k = -\frac{6}{7}$ **l** $l = -4\frac{1}{2}$ $\left(\text{or } l = -\frac{9}{2}\right)$

3 **a** $x = 4$ **b** $x = -2$
c $x = -1$ **d** $x = 3$
e $x = 1\frac{7}{8}$ $\left(\text{or } x = \frac{15}{8}\right)$ **f** $x = 1$
g $x = 6$ **h** $x = 5$
i $x = -6$ **j** $x = 4\frac{2}{3}$ $\left(\text{or } x = \frac{14}{3}\right)$
k $x = 21$ **l** $x = 4\frac{1}{2}$ $\left(\text{or } x = \frac{9}{2}\right)$

4 **a** $x = 6$ **b** $x = 4$ **c** $x = 4$
d $x = 19$ **e** $x = -\frac{1}{5}$ **f** $x = -16$
g $x = -2$ **h** $x = \frac{2}{3}$

5 **a** $6x + 4 = 4x + 2$ **b** $x = -1$

6 **a** Andy: added $2x$ to left side instead of subtracting; Kwame: did not subtract $2x$ from both sides of equation; Hayley: simplified right side incorrectly (-13, not $+13$)

b
$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 - 2x &= 2x - 13 - 2x \\ 4x + 11 &= -13 \\ 4x + 11 - 11 &= -13 - 11 \\ 4x &= -24 \\ \frac{4x}{4} &= \frac{-24}{4} \\ x &= -6 \end{aligned}$$

7 **a** $p = 3$ **b** $q = 2$ **c** $r = -3$
d $s = 4$ **e** $t = -2$ **f** $u = 13$

8 **a** B **b** \$6

9 **a** D **b** 12

10 **a** $3a + 400 = 7a + 160$ **b** 60 cents

11 **a** $2a + 2$ **b** $3a - 4 = 2a + 2$
c $a = 6$; Sean's cousin is 6 years old.
d Sean is 14 years old.

12 **a** $n + 10$ **b** $3n + 4$ **c** $3n + 4 = n + 10$
d $n = 3$ **e** 3 **f** 9 **g** 13

13 **a** $6g + 27$ **b** $12g + 3$ **c** $12g + 3 = 6g + 27$
d $g = 4$; home team: 8 goals 3 behinds; away team: 4 goals 27 behinds; each team scored 51 points

14 **a** $4t + 8$ **b** $12t + 4$
c $12t + 4 = 2(4t + 8)$ or $12t + 4 = 8t + 16$
d $t = 3$
e home team: 3 tries; away team: 9 tries

15 **a** $5x - 10 = 3x + 24$ **b** $x = 17$

16 **a** $x = 29$ **b** $x = -3$ **c** $x = 1$
d $x = 6$ **e** $x = 25$ **f** $x = -7$
g $x = 3$ **h** $x = 0$ **i** $x = -2$

17 **a** Expand the brackets, apply inverse operations to get x^2 alone on one side of the equation, then take the square root of both sides.
b **i** $x = 2$ **ii** $x = 0$

Checkpoint

1 **a** $a = 4$ **b** $d = 20$ **c** $f = 4$
d $m = -5$ **e** $p = -21$ **f** $z = -17$

2 **a** $a = 1$ **b** $b = 10$ **c** $k = 7$
d $x = -4$ **e** $j = -15$ **f** $y = 7$

3 **a** $n + 6 = 18$ **b** $n - 8 = -10$ **c** $n + 7 = 36$
d $n - 10 = -4$

4 **a** $4x + 9 = 33$ **b** $\frac{x}{5} - 4 = 0$ **c** $\frac{x + 12}{3} = 6$
d $8(x + 3) = 64$

5 **a** $x = 14$ **b** $x = 25$ **c** $x = 8$
d $x = 35$ **e** $x = -41$ **f** $x = -9$

6 **a** $x = 4$ **b** $x = 2$ **c** $x = -20$
d $x = 12$ **e** $x = 54$ **f** $x = 13$

7 **a** $x = 4$ **b** $x = 6$ **c** $x = -32$
d $x = -16$ **e** $x = -6$ **f** $x = -41$

8 **a** $x = \frac{1}{7}$ **b** $x = -\frac{9}{2}$ **c** $x = \frac{42}{5}$
d $x = \frac{6}{5}$ **e** $x = \frac{22}{3}$ **f** $x = -21$

9 **a** $x = 4$ **b** $x = 3$ **c** $x = 8$

10 **a** $x = -\frac{11}{4}$ **b** $x = 10$ **c** $x = -3$

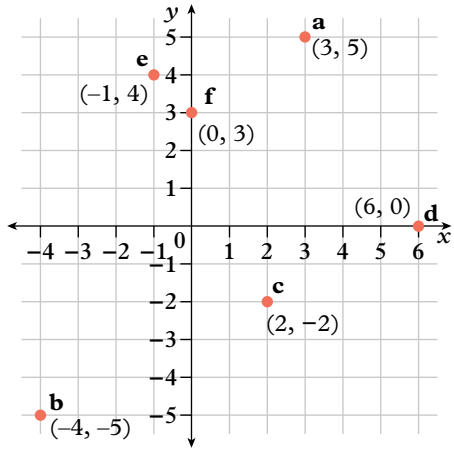
11 **a** $x = -\frac{21}{4}$ **b** $x = -\frac{34}{7}$ **c** $x = 1$

12 **a** $x = -39$ **b** $x = \frac{41}{4}$ **c** $x = 0$

EX p248 6D Plotting linear relationships

1 **a** A **b** C **c** F **d** B **e** E **f** D

2



3 **a**

x	−2	−1	0	1	2
y	−1	0	1	2	3

b

x	−3	−2	−1	0	1
y	−4	−2	0	2	4

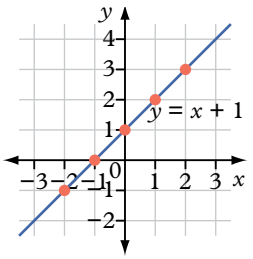
c

x	−1	0	1	2	3
y	4	3	2	1	0

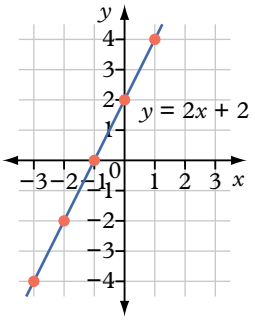
d

x	−1	0	1	2	3
y	−7	−4	−1	2	5

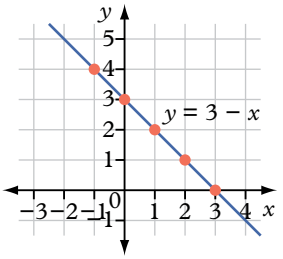
4 a



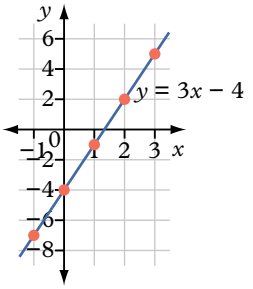
b



c

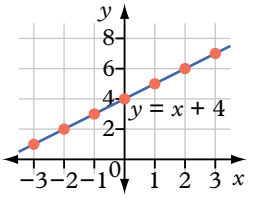


d



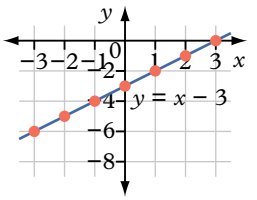
5 a

x	−3	−2	−1	0	1	2	3
y	1	2	3	4	5	6	7



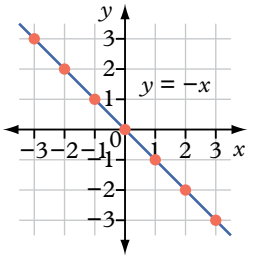
b

x	−3	−2	−1	0	1	2	3
y	−6	−5	−4	−3	−2	−1	0



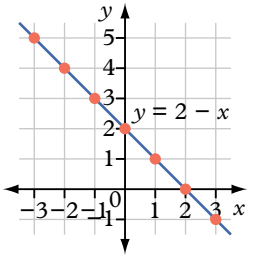
c

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3



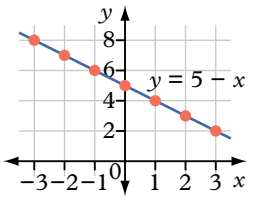
d

x	−3	−2	−1	0	1	2	3
y	5	4	3	2	1	0	−1



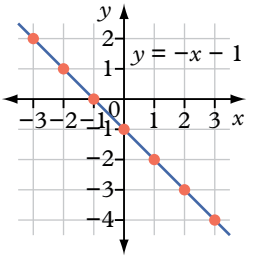
e

x	−3	−2	−1	0	1	2	3
y	8	7	6	5	4	3	2



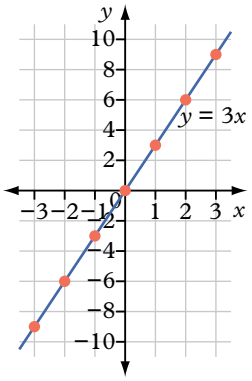
f

x	−3	−2	−1	0	1	2	3
y	2	1	0	−1	−2	−3	−4



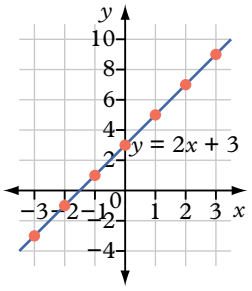
g

x	−3	−2	−1	0	1	2	3
y	15	12	9	6	3	0	−3



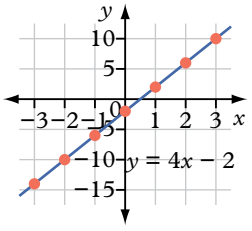
h

x	−3	−2	−1	0	1	2	3
y	−3	−1	1	3	5	7	9



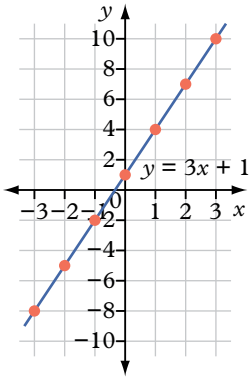
i

x	−3	−2	−1	0	1	2	3
y	−14	−10	−6	−2	2	6	10



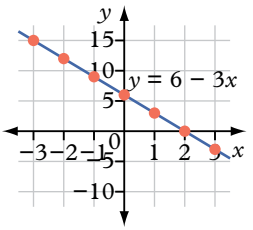
j

x	−3	−2	−1	0	1	2	3
y	−8	−5	−2	1	4	7	10



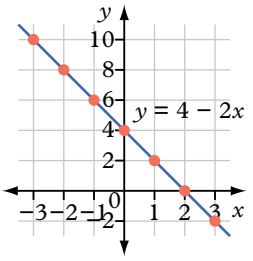
k

x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9



l

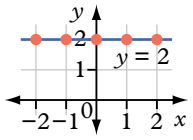
x	−3	−2	−1	0	1	2	3
y	10	8	6	4	2	0	−2



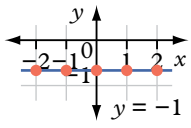
6 **a** linear **b** non-linear **c** linear

7 **a** linear **b** linear **c** non-linear

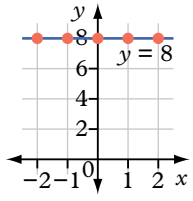
8 **a** **i**



ii



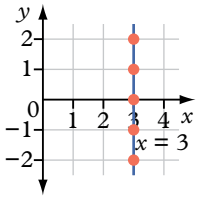
iii



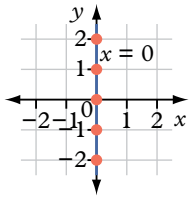
b yes, they are straight lines

c The horizontal line passes through the y-value described by the equation.

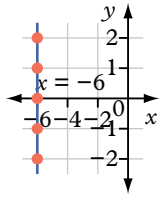
9 **a** **i**



ii



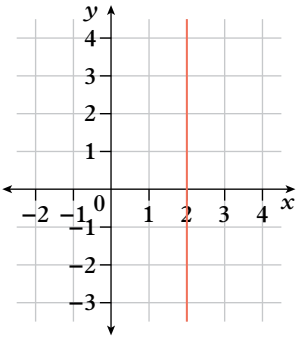
iii



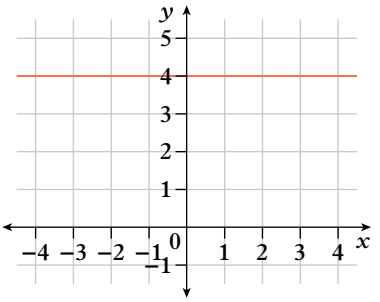
b yes, they are straight lines

c The vertical line passes through the x-value described by the equation.

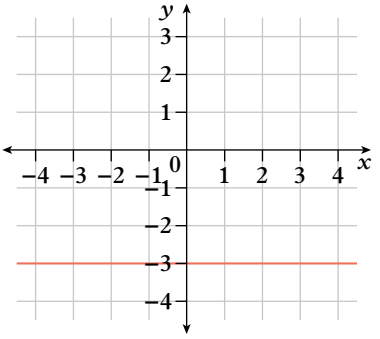
10 a



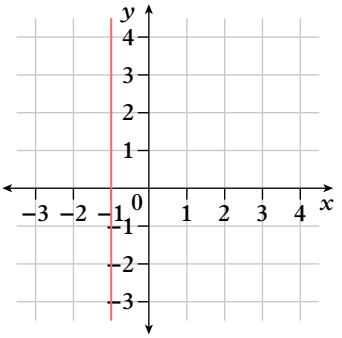
b



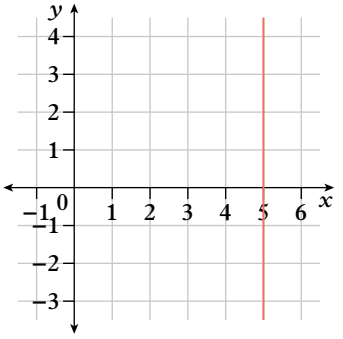
c



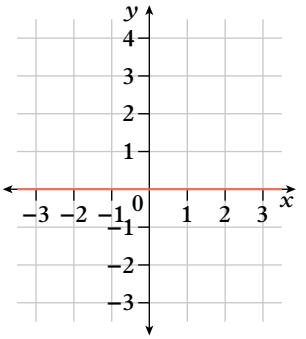
d



e



f



11 a time and height

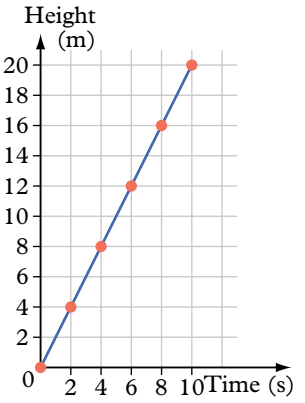
b **i** time; shown first in table of values

ii height

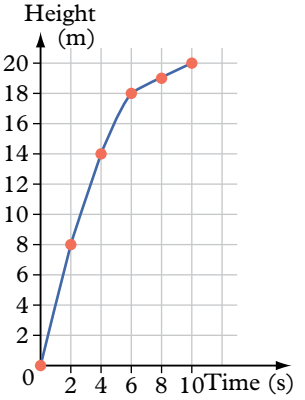
c Red balloon: (0, 0), (2, 4), (4, 8), (6, 12), (8, 16), (10, 20)

Blue balloon: (0, 0), (2, 8), (4, 14), (6, 18), (8, 19), (10, 20)

d Red balloon:



Blue balloon:



e **i** red balloon: 10 m; blue balloon: 16 m

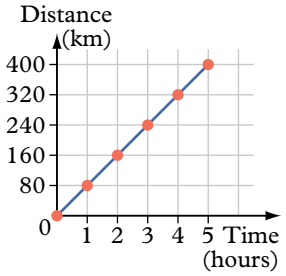
ii red balloon: 3.5 s; blue balloon: approximately 1.75 s

f the red balloon, as the points form a straight line

g the red balloon, as the behaviour is more predictable than the blue balloon

12 a independent = time, dependent = distance

b



c yes, as the points form a straight line

d 200 km **e** $3\frac{1}{4}$ hours

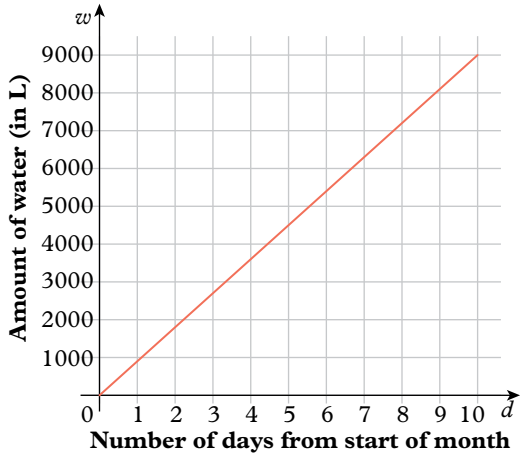
f 560 km; assume Jamie maintains the same speed

g $d = 80t$; multiply the number of hours by 80 to determine the distance Jamie travels.

h $d = 80 \times 7 = 560$

13 a independent = number of days, dependent = amount of water

b



c yes, as the points form a straight line

d 6300 L

e **i** the equation, as the result can be calculated exactly and because the graph doesn't cover the range needed

ii approx. 22 days $\left(22\frac{2}{9} \text{ days}\right)$

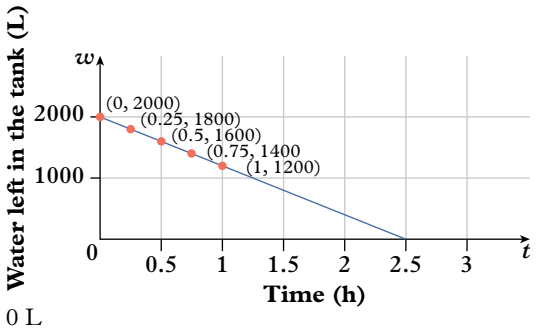
14 a 2000 L

b $w = 2000 - 800t$

c **i** water left (w)

ii time passed (t)

d



e 0 L

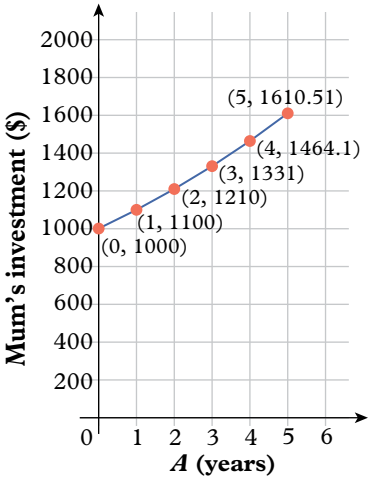
15 P vs l; the size of the perimeter increases by a constant amount each time.

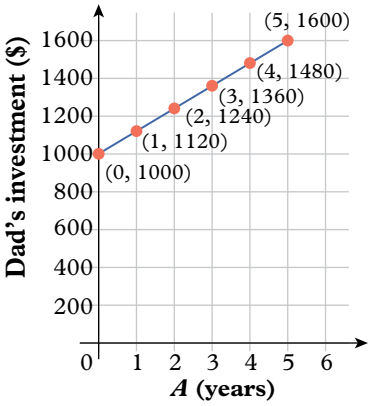
16 a P vs C; the number of fence posts increases by a constant amount each time.

b This relationship is non-linear. While the number of cows increases by a constant amount, the area of the paddock increases by different amounts.

17 a D vs A

b





Mum's investment: approx. $2600, Dad's investment: $2200

c Sample answer: Compound interest seems like the better choice. Other factors to consider are the interest rate and the terms and conditions of the investments.

EX 6E Solving linear equations using graphs

p255

1 a **i** $x = 6$ **ii** $x = 1$ **iii** $x = -2$ **iv** $x = 2$

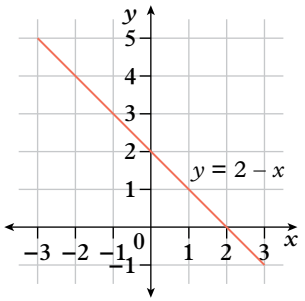
b **i** $x = 6$ **ii** $x = 1$ **iii** $x = -2$ **iv** $x = 2$

c **i** $x = 8$ **ii** $x = 4$ **iii** $x = 0$ **iv** $x = -2$

2 a $x = 2$ **b** $x = -1$ **c** $x = 1$ **d** $x = 0$ **e** $x = -2$ **f** $x = -3$

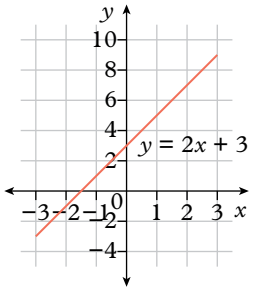
3 a $x = 2$ **b** $x = -2$ **c** $x = 0$ **d** $x = -4$ **e** $x = -3$ **f** $x = 1$

4 **a**



b **i** $x = 1$ **ii** $x = 3$ **iii** $x = 2$
iv $x = 0$ **v** $x = 0.5$ **vi** $x = 1.5$

5 **a**

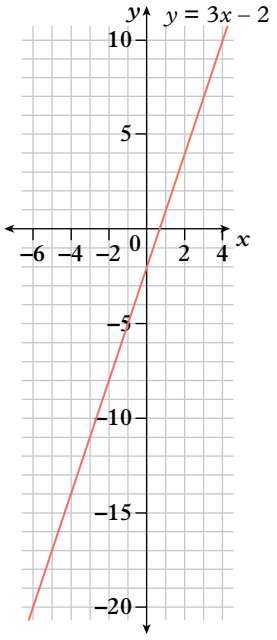


b **i** $x = -1$ **ii** $x = -3$ **iii** $x = 1.5$
iv $x = 0$ **v** $x = -2.5$ **vi** $x = 2.5$

6 **a** $y = 3x - 2$

b **i** $y = 7$ **ii** $y = -5$ **iii** $y = -20$

c

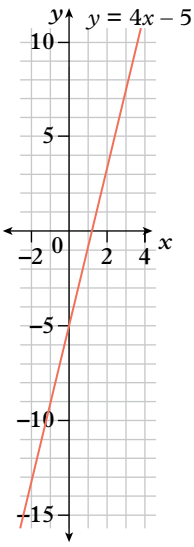


i $x = 3$ **ii** $x = -1$ **iii** $x = -6$

7 **a** $y = 4x - 5$

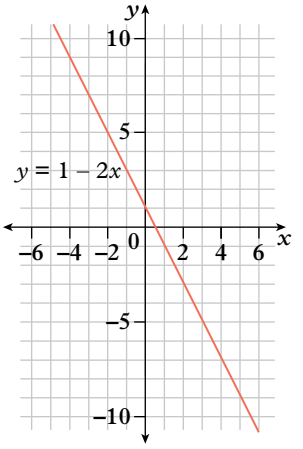
b **i** $y = 3$ **ii** $y = -13$ **iii** $y = -7$

c



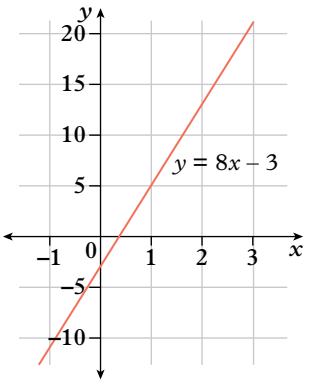
i $x = 2$ **ii** $x = -2$ **iii** $x = -0.5$

8



a $x = -1$ **b** $x = 3$ **c** $x = -3$
d $x = 5$ **e** $x = -4$ **f** $x = 2.5$

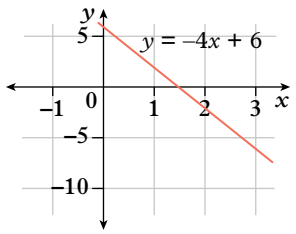
9



a $x = 1$ **b** $x = 2$ **c** $x = 3$
d $x = 0$ **e** $x = -1$ **f** $x = -0.5$

10 a $x = 2.5$

b



$x = 2.5$

c Sample answer: The graphical solution method can be used to solve many similar equations quickly; however, it takes longer to draw a graph and it is difficult to read non-integer values accurately from a graph.

11 a **i** $d = 50$ **ii** $d = 150$ **iii** $d = 200$

b **i** $0.64d = 50$ **ii** $d = 78.125$, \$78

c shirt:

i $0.64d = 64$ **ii** $d = 100$ **iii** \$100

shoes:

i $0.64d = 100$ **ii** $d = 156$ **iii** \$156

jeans:

i $0.64d = 42$ **ii** $d = 66$ **iii** \$66

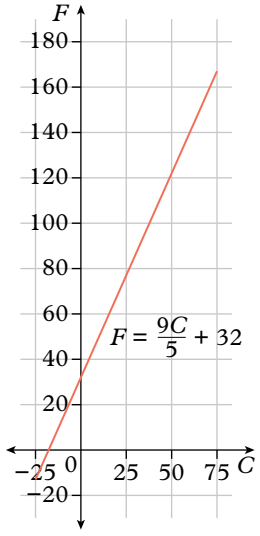
leather jacket:

i $0.64d = 140$ **ii** $d = 219$ **iii** \$219

12 a

C	−25	0	25	50	75
F	−13	32	77	122	167

b



c **i** $C \approx 4$ **ii** $C = 40$ **iii** $C = -10$

d meat:

i $\frac{9}{5}C + 32 = -5$ **ii** $C \approx -21$

dairy products and vegetables:

i $\frac{9}{5}C + 32 = 37$ **ii** $C \approx 3$

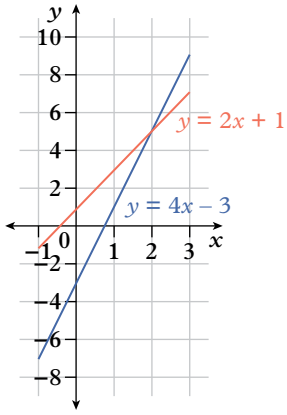
tomatoes:

i $\frac{9}{5}C + 32 = 68$ **ii** $C = 20$

iii 'Meat to be frozen should be stored in the freezer at −21°C, dairy products and vegetables stored in the fridge at about 3°C and tomatoes kept at room temperature, ideally around 20°C.'

e 60°C

13 a, b



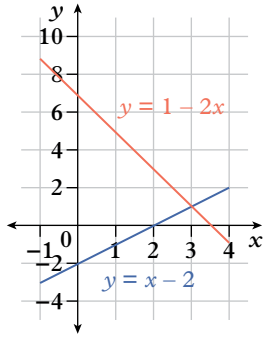
c (2, 5)

d $x = 2$

e $4x - 3 = 2x + 1$
$4(2) - 3 = 2(2) + 1$
$8 - 3 = 4 + 1$
$5 = 5$

14 a **i** $y_1 = x - 2$ and $y_2 = 7 - 2x$

ii

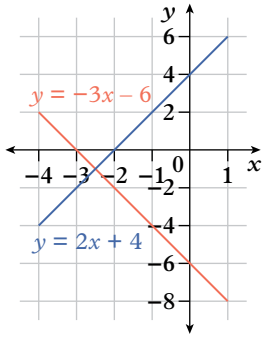


iii $x = 3$

iv $x - 2 = 7 - 2x$
$3 - 2 = 7 - 2(3)$
$1 = 7 - 6$
$1 = 1$

b **i** $y_1 = 2x + 4$ and $y_2 = -3x - 6$

ii



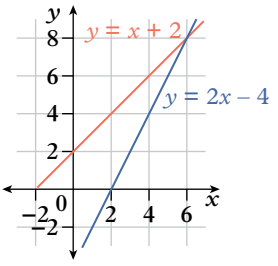
iii $x = -2$

iv $2x + 4 = -3x - 6$
$2(-2) + 4 = -3(-2) - 6$
$-4 + 4 = 6 - 6$
$0 = 0$

15 a $x + 2 = 2x - 4$

b $y_1 = x + 2$ and $y_2 = 2x - 4$

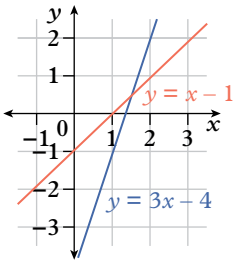
c



$x = 6$

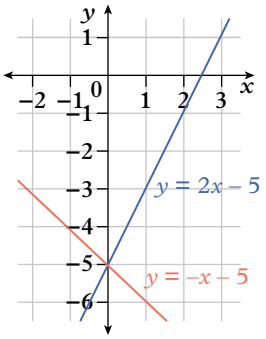
d six lamingtons

16 a



$x = 1.5$

b



$x = 0$

EX p260 6F Finding linear equations

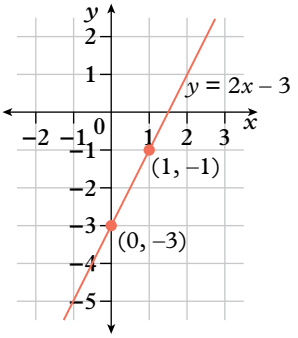
1 a $y = 2x + 2$ **b** $y = -3x + 4$ **c** $y = 2x + 1$

d $y = -2x - 3$ **e** $y = 2x + 4$ **f** $y = -3x - 5$

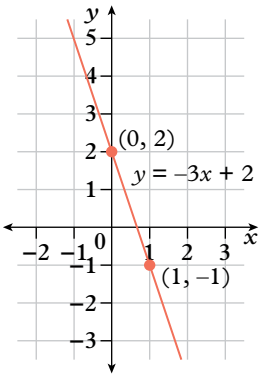
2 a $y = 2x + 1$ **b** $y = -3x + 1$ **c** $y = 4x - 2$

d $y = -\frac{1}{3}x + 3$ **e** $y = -4x - 2$ **f** $y = -\frac{x}{2}$

3



4



5 a D **b** A **c** B **d** C

6 a C **b** B **c** A **d** D

7 a A **b** B **c** D **d** C

8 a $y = -4x + 5$ **b** $y = -3x - 5$

9 a $x = 4$ **b** $y = 2$ **c** $y = -3$ **d** $x = -3$

10 a i $y = 3x + 3$ **ii** $y = 3x + 1$ **iii** $y = 3x - 1$

iv $y = 3x - 3$

b i $y = -2x + 3$ **ii** $y = -2x + 1$ **iii** $y = -2x - 1$

iv $y = -2x - 3$

c The equations all have the same x coefficient

d The equations all have the same x coefficient

11 Linear graphs that have the same value for m in the equation $y = mx + c$ are parallel.

12 a

Rows	Tiles in the row
1	16
2	24
3	32
4	40

b $t = 8r + 8$

13 $y = -2x - 1.5$

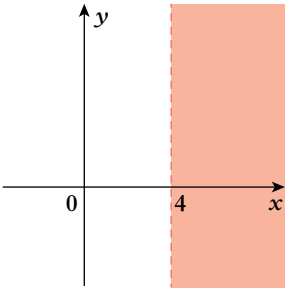
EX p267 6G Solving linear inequalities

1 a $2 < 3$ **b** $3 > 2$ **c** $3 \geq 2$ **d** $-3 \leq -2$

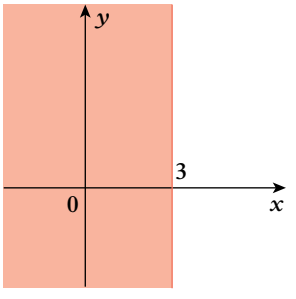
2 a $x < 0.5$ **b** $x > \frac{4}{5}$ **c** $x \leq -12$ **d** $x \geq -1.2$

e $y < 13$ **f** $y > -\frac{5}{3}$ **g** $y \leq 2.13$ **h** $y \geq 99$

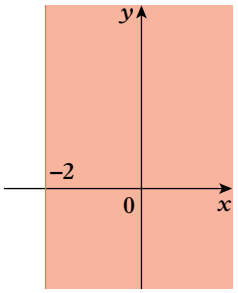
3 a



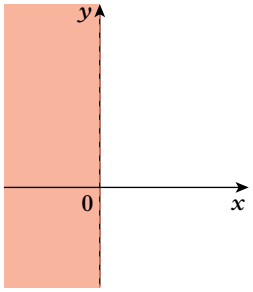
b



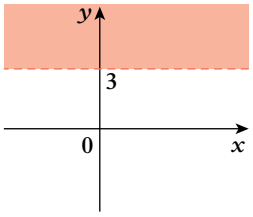
c



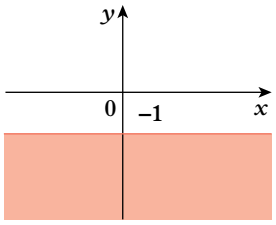
d



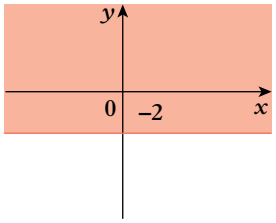
e



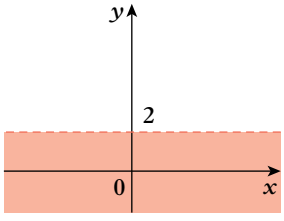
f



g



h

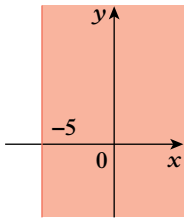


4 a **i** $1 < 14$ **ii** $-13 < 0$
iii $-12 < 27$ **iv** $-\frac{4}{3} < 3$

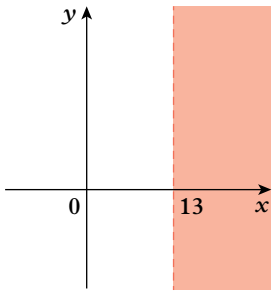
b **i** $x + 5 \geq 11$ **ii** $x - 9 \geq -3$
iii $3x \geq 18$ **iv** $\frac{x}{3} \geq 2$

5 a $x > 4$ **b** $x \leq 30$ **c** $x < -5$ **d** $x \leq 15$
e $x \geq 2$ **f** $x \leq 2$ **g** $x > -28$ **h** $x > -\frac{9}{10}$

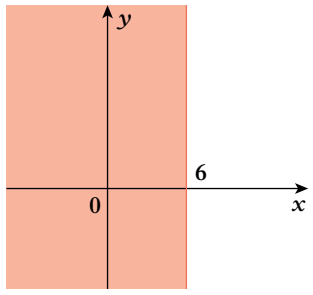
6 a $x \geq -5$



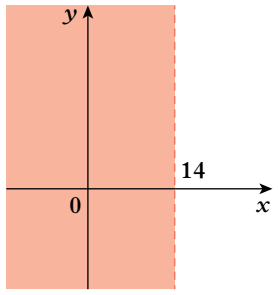
b $x > 13$



c $x \leq 6$



d $x < 14$



7 a one

b Linear inequalities have an infinite number of solutions within a given range.

8 a $p \geq 650\,000$, where p is the selling price in dollars

b $h > 97$, where h is the height of a person in centimetres

c $s \leq 60$, where s is the speed in km/h

9 a $3p \leq 20$, where p is the cost of a pack of gum in dollars; $p \leq 6\frac{2}{3}$

b Todd could buy 1, 2, 3, 4, 5 or 6 packs of gum.

10 a $\frac{23x - 60}{2}$ **b** $\frac{23x - 60}{2} \geq 100$

c 12

11 a 25 min **b** 16 min

12 a $x > 4$ **b** $x \leq -2$ **c** $x \geq 3$ **d** $x > 4$
e $x < 1$ **f** $x \leq -1$ **g** $x \leq 5$ **h** $x < 2$

13 **a** $x > 11$ **b** $x \leq 13$ **c** $x \geq -2$
d $x > -19$ **e** $x > -15$ **f** $x \leq 1$
g $x \leq 3$ **h** $x > -\frac{1}{4}$ **i** $x \geq -\frac{13}{2}$

14 **a** $1 \leq x \leq 6$ **b** $-4 > x > -7$ or $-7 < x < -4$

CHAPTER 6 review

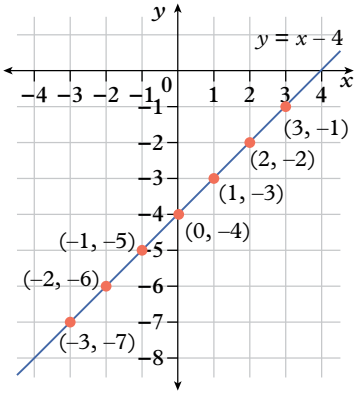
Multiple choice

1 C **2** A **3** B **4** C **5** E **6** D
7 E **8** B **9** D **10** E **11** B **12** A
13 B **14** A

Short answer

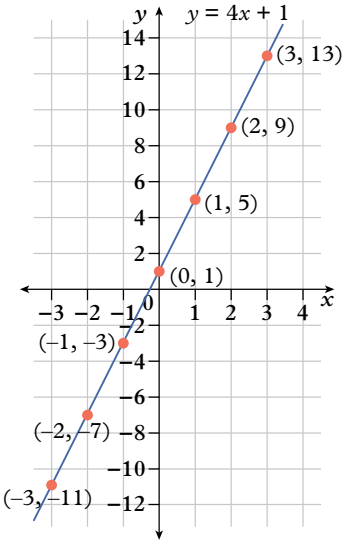
1 **a** yes **b** yes **c** yes **d** no
2 **a** $a = 9$ **b** $b = 14$ **c** $c = 3$ **d** $d = 15$
3 **a** $x = 3$ **b** $x = -9$ **c** $c = 14$ **d** $d = 2$
e $x = 1\frac{1}{4}$ $\left(\text{or } x = \frac{5}{4}\right)$ **f** $x = -8$
4 **a** $12s + 24s = 36$
b $s = 1$; small book = 1 kg, large book = 2 kg
5 **a** $x = -5$ **b** $x = 2$ **c** $x = -3$
d $x = -2\frac{1}{2}$ $\left(\text{or } x = -\frac{5}{2}\right)$
6 $s + 11 = 5(s - 5)$, Suzi is 9 years old.
7 **a**

x	−3	−2	−1	0	1	2	3
y	−7	−6	−5	−4	−3	−2	−1



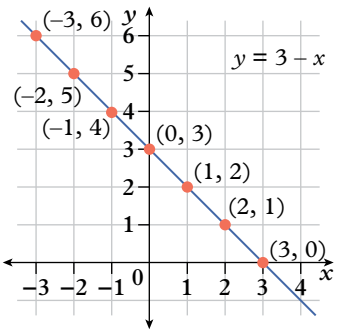
b

x	−3	−2	−1	0	1	2	3
y	−11	−7	−3	1	5	9	13



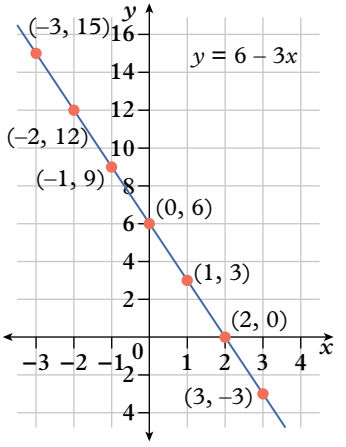
c

x	−3	−2	−1	0	1	2	3
y	6	5	4	3	2	1	0

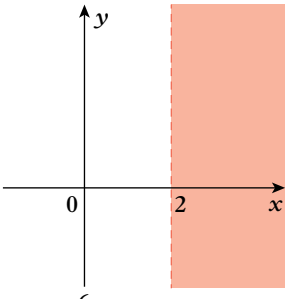


d

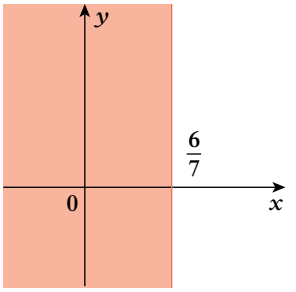
x	−3	−2	−1	0	1	2	3
y	15	12	9	6	3	0	−3



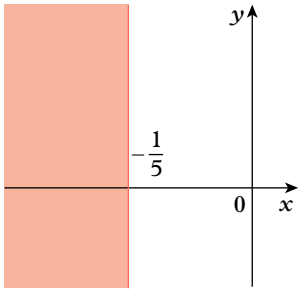
8 **a** non-linear **b** linear
9 **a** $y = 4x - 5$ **b** $y = -x$
10 **a** $y = -x + 2$ **b** $y = 5x - 4$
11 **a** $x = 4$ **b** $x = -1$ **c** $x = 3$ **d** $x = 2$
12 **a** $x = -2$ **b** $x = 1$ **c** $x = 0$
13 **a** $x > 2$



b $x \leq \frac{6}{7}$



c $x \leq -\frac{1}{5}$



Analysis

1 a $C = 150 + 20p$

b 15 people can attend the party.

c $C = 250 + 15p$

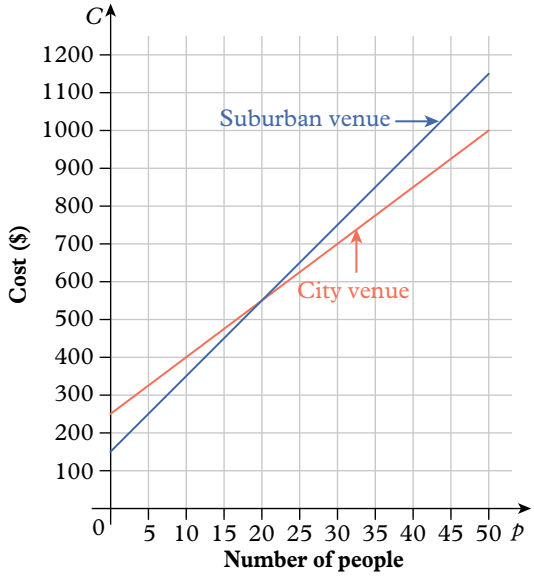
d Suburban venue

p	0	10	20	30	40	50
C	150	350	550	750	950	1150

City venue

p	0	10	20	30	40	50
C	250	400	550	700	850	1000

e



f yes, as the points form a straight line

g i suburban = \$650, city = \$625

ii suburban = 15, city = 13

h i suburban **ii** city **iii** same cost

2 a $C = 5x + 3$

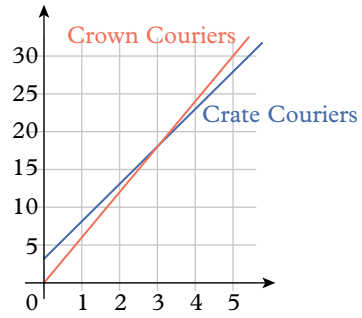
b \$53

c 7

d

Number of books	0	1	2	3	4	5
Cost of delivery	\$3	\$8	\$13	\$18	\$23	\$28

e, g



f

Number of books	0	1	2	3	4	5
Cost of delivery	\$0	\$6	\$12	\$18	\$24	\$30

h Joss should use Crate Couriers for delivering more than 3 books and he should use Crown Couriers for less than 3 books. For 3 books it makes no difference.

EX p275

Algorithmic Thinking 1 Number and algebra

Task 1

1 13

2 The flowchart will stop when it reaches the second digit (7).

3 'Is digit in set {0, 1}?' and 'Is number fully converted?'

4 At the 'Is number fully converted?' action

5 Exponent remains constant, while Base and Sum change.

6 The flowchart stops and reports the number when it is fully converted.

7 Teacher to check.

Task 2

1 312_5

Decimal 82	Division by 5	Quotient	Remainder	Exponent	Quinary 312
82	82 ÷ 5	16	2	0	2×5^0
16	16 ÷ 5	3	1	1	1×5^1
3	3 ÷ 5	0	3	2	3×5^2

2 The exponent and sum

3 Teacher to check.

4 Teacher to check.

5 Teacher to check.

CHAPTER 7 Triangles and quadrilaterals

7A Angles

EX p280

1 a $\angle BDE$ or $\angle EDB$, obtuse angle

b $\angle PQV$ or $\angle VQP$, right angle

c $\angle FBY$ or $\angle YBF$, reflex angle

d $\angle MYZ$ or $\angle ZYM$, acute angle

e $\angle IMA$ or $\angle AMI$, reflex angle

f $\angle DHL$ or $\angle LHD$, obtuse angle

2 **a** $a = 15°$ **b** $b = 15°$ **c** $c = 39°$
d $d = 64°$ **e** $e = 138°$ **f** $f = 55°$

3 **a** $a = 160°$
b $a = 79°$, $b = 101°$, $c = 79°$
c $d = 147°$, $e = 33°$, $f = 147°$
d $g = 144°$
e $h = 253°$
f $i = 72°$, $j = 69°$, $k = 39°$, $l = 69°$

4 **a** complementary **b** supplementary
c vertically opposite **d** supplementary

5 **a** $m = 152°$, $n = 68°$
b $v = 27°$
c $s = 68°$, $t = 22°$, $u = 68°$
d $a = 7°$, $b = 139°$, $c = 41°$
e $w = 45°$, $x = 38°$, $y = 39°$, $z = 38°$
f $d = 115°$, $e = 65°$, $f = 115°$, $g = 110°$, $i = 110°$, $j = 135°$, $k = 45°$, $l = 135°$, $n = 70°$

6 **a** **i** corresponding **ii** $a = 63°$
b **i** alternate **ii** $b = 71°$
c **i** co-interior **ii** $c = 122°$
d **i** corresponding **ii** $d = 123°$
e **i** corresponding **ii** $e = 82°$
f **i** co-interior **ii** $f = 23°$
g **i** corresponding **ii** $g = 76°$
h **i** alternate **ii** $h = 35°$
i **i** co-interior **ii** $i = 49°$

7 **a** $a = 141°$, co-interior
b $b = 147°$, co-interior
c $c = 21°$, alternate
d $d = 66°$, corresponding; $e = 59°$, alternate
e $f = 85°$, corresponding; $g = 95°$, co-interior
f $n = 142°$, co-interior

8 27°, angles are alternate and equal

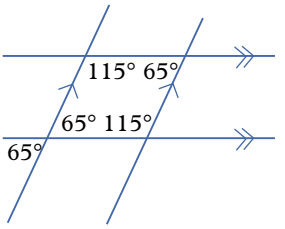
9 The stairs have an angle of 23° with respect to the horizontal and meet the standards but are not in the preferred range.

10 Alternate exterior angles are supplementary to the alternate interior angles (which are equal). The supplements of equal angles will be equal to each other, so alternate exterior angles are equal for parallel lines.

11 **a** Angles are corresponding but not equal, so the lines are not parallel.
b Angles are co-interior and supplementary, so the lines are parallel.
c 99° is supplementary to the angle alternate to 81°, so the lines are parallel.

12 **a** 360°
b **i** 180° **ii** 90° **iii** 270°
c **i** 120° **ii** 330° **iii** 240°
iv 30°

13 **a** parallelogram
b



14 **a** c and f; d and e; i and d; b and k
b a and e; b and f; c and g; d and h; j and b; i and a; l and d; k and c
c c and e; d and f; i and b; d and k

15 $a = 40°$ (angles in a triangle), $b = 50°$ (alternate angles), $c = 90°$ (right angle), $d = 40°$ (corresponding angle), $e = 50°$ (corresponding angle), $f = 50°$ (corresponding angle)

16 **a** The transversal creates two intersections. Around each intersection there will be two pairs of vertically opposite equal angles and they are related to each other by being supplementary. The pairs of angles from one intersection and the other are related and either equal or supplementary by being co-interior, corresponding or alternate. So all angles are either equal or supplementary to a given angle.
b From part **a**, the angles formed by one transversal are all related and either equal or supplementary to a given angle. For each new parallel transversal, all of its angles correspond to an angle from the previous transversal. So no matter how many parallel transversals are added, all angles are equal or supplementary to a given angle.

17 **a** 30°
b **i** 15° **ii** 7.5° **iii** 22.5°
c **i** 165° **ii** 97.5° **iii** 52.5°

18 **a** 170° **b** 122.5° **c** 72.5° **d** 88.5° **e** 79.5°

EX p288

7B Triangles

1 **a** obtuse-angled isosceles
b acute-angled scalene
c acute-angled isosceles
d acute-angled equilateral
e right-angled scalene
f obtuse-angled scalene

2 **a** 63° **b** 90° **c** 41°
d 67° **e** 106° **f** 50°

3 **a** 138° **b** 141° **c** 64°
d 111° **e** 109° **f** 164°

4 **a** $a = 60°$, $b = 60°$, $c = 60°$
b $a = 45°$, $b = 45°$ **c** $a = 142°$, $b = 19°$
d $d = 120°$ **e** $e = 71°$ **f** $f = 41°$
g $g = 27°$ **h** $h = 58°$ **i** $i = 156°$

5 **a** acute-angled equilateral
b right-angled isosceles
c obtuse-angled isosceles

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d acute-angled equilateral

e acute-angled scalene

f acute-angled scalene

g obtuse-angled scalene

h right-angled scalene

i obtuse-angled isosceles

6 **a** False; the angles in an equilateral must be 60° (acute).

b True; the other angles in an obtuse-angled isosceles triangle will both be less than 45°.

c True; the other angles must sum to 90° and not be equal.

7 **a** $x = 30°$ **b** $x = 10°$ **c** $x = 10°$

d $x = 10°$ **e** $x = 12°$ **f** $x = 20°$

8 **a** $a = 30°$, $b = 60°$, $c = 90°$

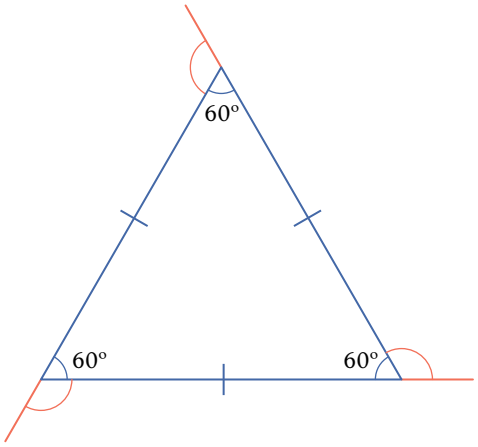
b Right-angled scalene; the triangle has three different interior angles including a right angle.

9 $a = 37°$, $b = 113°$, $c = 141°$, $d = 17°$

10 **a** $p = 96°$, $q = 36°$, $r = 83°$

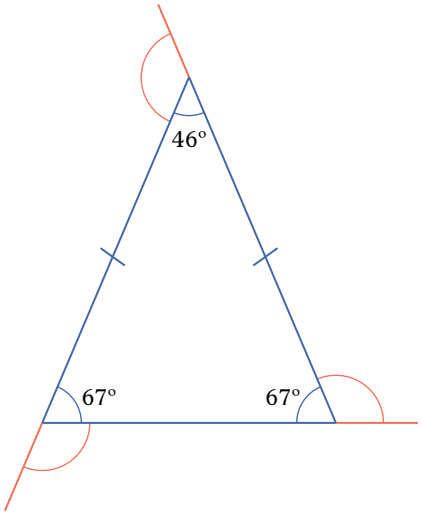
b 96° **c** $\angle QRS = 48°$

11 **a** **i**



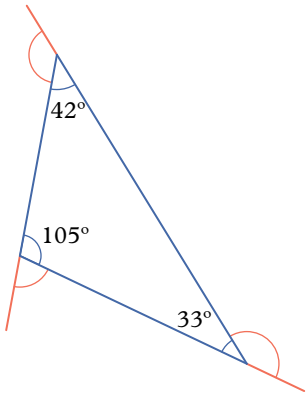
ii Each exterior angle is 120°; they add to 360°.

b **i**



ii Exterior angles are 134°, 113° and 113°; they add to 360°.

c **i**

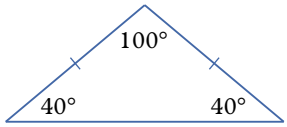


ii Exterior angles are 138°, 147° and 75°; they add to 360°.

d The exterior angles of any triangle add to 360°.

12 **a** Not possible. The angle sum of a triangle is 180°. These angles add to 200°.

b

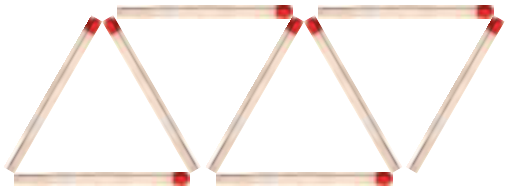


c Not possible. If the base length is 10 cm, the other two sides are not long enough to meet.

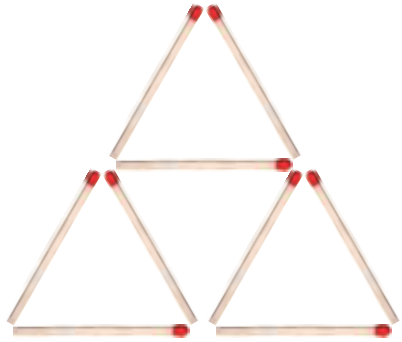
d Not possible: The angle sum of a triangle is 180°. Two obtuse angles add to more than 180°.

13 **a** seven matches

b **i**

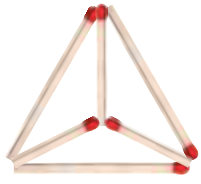


ii



(The large triangle is the fifth triangle.)

iii



(triangular pyramid)

14 $x + c = 180°$

$a + b + c = 180°$

$x + c = a + b + c$

$x = a + b$

EX 7C Pythagoras' theorem

p294

1 **a** m **b** p **c** f

2 **a** right-angled **b** not right-angled
c right-angled **d** not right-angled

3 **a** right-angled **b** right-angled
c not right-angled **d** not right-angled
e right-angled **f** not right-angled

4 **a** 15 cm **b** 50 mm **c** 61 cm **d** 26 cm

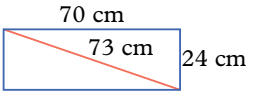
5 **a** 9.43 cm **b** 17.49 cm **c** 23.85 mm **d** 29.83 mm

6 **a** 12.53 cm **b** 55.22 m **c** 25.24 mm **d** 10.97 cm

7 **a** 171 cm **b** 3 cm **c** 10 cm **d** 70 cm

8 Yes, because $60^2 + 63^2 = 87^2$.

9 **a**



b The frame is not square. The diagonal distance should be longer.

c 74 cm

10 No, because $a^2 + b^2$ is the same as $b^2 + a^2$.

11

Values		Pythagorean triple		
m	n	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$
2	1	$2^2 - 1^2 = 3$	$2 \times 2 \times 1 = 4$	$2^2 + 1^2 = 5$
3	1	8	6	10
3	2	5	12	13
4	1	15	8	17
4	2	12	16	20
4	3	7	24	25
5	1	24	10	26
5	2	21	20	29
5	3	16	30	34
5	4	9	40	41
6	1	35	12	37

12 **a** (3, 4, 5), (5, 12, 13), (15, 8, 17), (7, 24, 25), (21, 20, 29), (9, 40, 41), (35, 12, 37)

b **i** (5, 12, 13) **ii** (8, 15, 17)
iii (3, 4, 5) **iv** (9, 40, 41)
v (65, 72, 97) **vi** (21, 220, 221)

13 18.03 m

14 **a** $10^2 = 6^2 + 8^2$, $100 = 36 + 64$

b yes; $20^2 = 12^2 + 16^2$

c yes; $30^2 = 18^2 + 24^2$

d yes; $5^2 = 3^2 + 4^2$

e yes; $100^2 = 60^2 + 80^2$

15 **a** 9.43 cm **b** 10 cm **c** 3.61 m
d 25.61 mm **e** 14.42 cm **f** 21.21 mm

16 **a** 16, 25, 36

b In an acute triangle, the square of the longest side is less than the sum of the squares of the two other sides.

c 9, 64, 100

d In an obtuse triangle, the square of the longest side is greater than than the sum of the squares of the two other sides.

e **i** $121 < 164$ $(64 + 100)$

ii $169 > 100$ $(36 + 64)$

17 11.68 cm

18 **a** Triangle ABC and Triangle ACD

b $AC = 10$ cm; $AB = 6$ cm and $BC = 8$ cm

c $CD = 26$ cm; the hypotenuse in triangle ABC becomes one of the smaller sides in triangle ACD.

d 64 cm

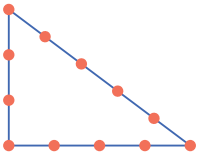
19 **a** $x = 27.80$ cm, $y = 29.90$ cm

b $x = 15.81$ m, $y = 18.71$ m

c $x = 60.22$ cm, $y = 66.41$ cm

20 **a** $x = \frac{5}{2}$ **b** $w = \frac{13}{8}$ **c** $t = 2$ **d** $p = \frac{1}{3}$

21 The rope with 12 equally spaced knots forms a right-angled triangle with sides of 3 intervals, 4 intervals and 5 intervals. Pythagoras' theorem applies $(3^2 + 4^2 = 5^2)$.



22 **a** Both squares have side lengths of $a + b$, so they both have an area of $(a + b)^2$.

b Both squares have the same area and include four identical triangles. So, the remaining space must be equal. The remaining area (shaded green) in Diagram 1 is a square with side lengths of c so it has an area of c^2. The remaining area of Diagram 2 is two squares, one with side length a and the other with side length b. So their areas are a^2 and b^2 respectively. Therefore, equating the remaining areas, $c^2 = a^2 + b^2$.

c Diagram 1: $c^2 + 4 \times \frac{1}{2}ab = c^2 + 2ab$
Diagram 2: $a^2 + b^2 + 4 \times \frac{1}{2}ab = a^2 + b^2 + 2ab$

d $c^2 + 2ab = a^2 + b^2 + 2ab$
$c^2 = a^2 + b^2$

e The total area of Diagram 3 with side lengths of c is c^2. The square is made up of four right-angled triangles with an area of $\frac{1}{2}ab$ and a square with side lengths of $b - a$ and an area of $(b - a)^2$.

f $c^2 = (b - a)^2 + 4 \times \frac{1}{2}ab$
$c^2 = b^2 - 2ab + a^2 + 2ab$
$c^2 = a^2 + b^2$

23 **a** $a = 8$ m **b** $b = 9.49$ m
c $c = 7$ m **d** $d = 7.75$ m

EX p302 7D Using Pythagoras' theorem

1 **a** $x = \pm 3.32$ **b** $x = \pm 7.14$ **c** $x = \pm 6.93$
d $x = \pm 8.49$ **e** $x = \pm 10.91$ **f** $x = \pm 4.24$

2 **a** 20 mm **b** 15 cm **c** 48 mm
d 27 mm **e** 14 m **f** 36 cm

3 **a** 12.65 cm **b** 10.25 cm **c** 22.36 mm
d 18.73 cm **e** 4.76 cm **f** 15.96 m

4 **a** 27.98 cm **b** 10.25 m **c** 13.56 cm
d 45.61 m

5 **a** 398 cm **b** 5 cm **c** 55 cm **d** 47 cm

6 **a** 10.00 cm **b** 7.91 m **c** 10.95 cm
d 16.47 mm **e** 5.31 mm **f** 10.25 cm

7 **a** 14.99 cm^2 **b** 8.74 cm^2 **c** 29.54 cm^2
d 24.82 cm^2

8 8.72 m

9 2.87 m

10 **a** 15.65 m **b** 4.58 cm **c** 7.07 m

11 **a** $x = 13.08$ cm, $y = 7.35$ cm
b $x = 4$ m, $y = 3.48$ m
c $x = 17.80$ m, $y = 7.81$ m
d $x = 43.86$ cm, $y = 28.90$ cm

12 **a** **i** 79.35 cm **ii** 224.65 cm^2
b **i** 18.78 m **ii** 15.36 m^2
c **i** 48.81 m **ii** 139.48 m^2
d **i** 135.90 cm **ii** 1003.17 cm^2

13 **a** $c = 10$ **b** $a^2 = 10$ and $b^2 = 90$
c A triangle is right-angled if its side lengths satisfy Pythagoras' theorem. $c^2 = 10^2 = 100$ and $a^2 + b^2 = 10 + 90 = 100$.
Therefore, the triangle is right-angled.
d $(-1.5, 0.5)$, $(3.5, 0.5)$, $(3, -1)$
e 22.5, 2.5, 25
f $c^2 = 25$ and $a^2 + b^2 = 22.5 + 2.5 = 25$. So, the triangle is right-angled.

14 $a = 3.00$ cm, $b = 5.81$ cm, $c = 5.67$ cm, $d = 6.41$ cm, $e = 4.25$ cm, $f = 2.43$ cm, $g = 9.27$ cm

15 **a** $a = 8.94$ **b** $a = 144$ **c** $a = 43.30$

16 **a** **i** and **ii** **b** **iv**
c **iii** The calculation subtracts the square of the shorter side from the square of the hypotenuse. This is the wrong way around. The calculation is the square root of a negative number, which is not a real number as needed for the length of a right-angled triangle.

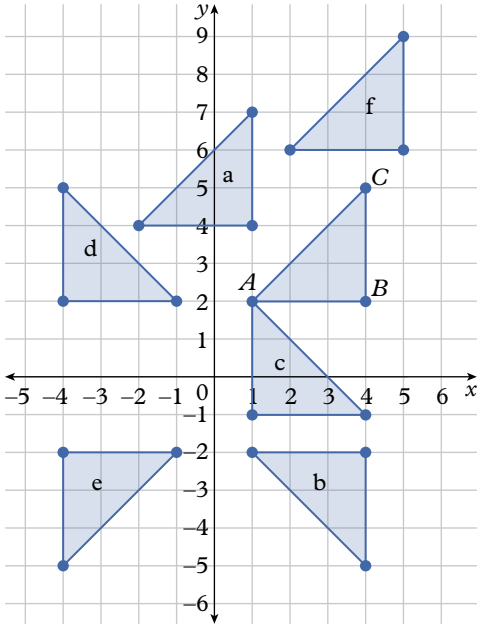
17 12 cm^2

18 **a** $a = 3.74$ m
b $b = 7.14$ m
c $c = 11$ m
d $d = 6.71$ m

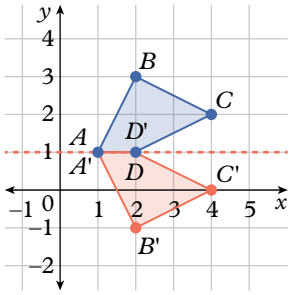
EX p309 7E Transformations

1 **a** translated 3 units left and 2 units down
b rotated 90° clockwise (or 270° anti-clockwise) about P
c reflected in the x-axis
d reflected in the y-axis

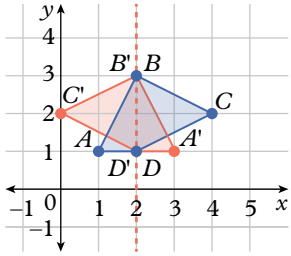
2



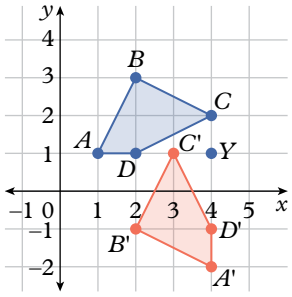
3 **a**



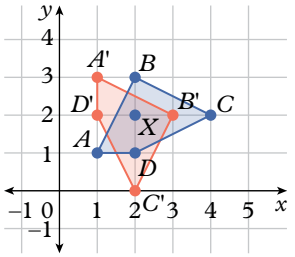
b



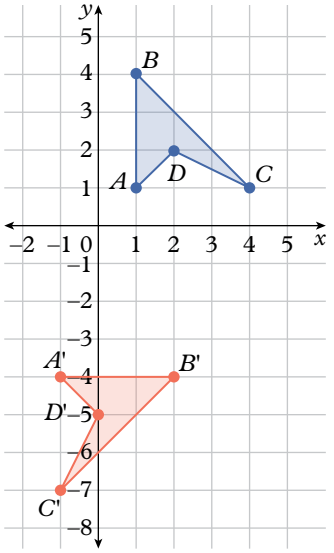
c



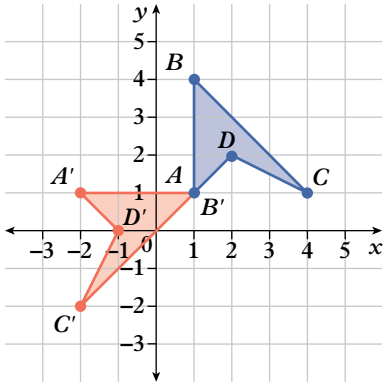
d



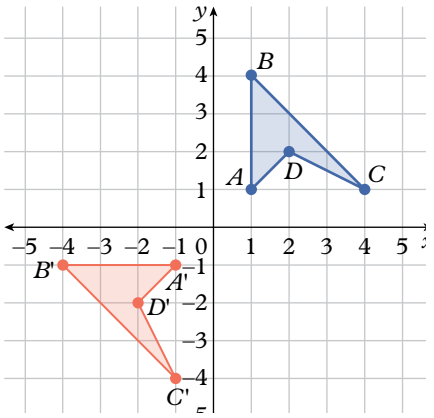
4 a



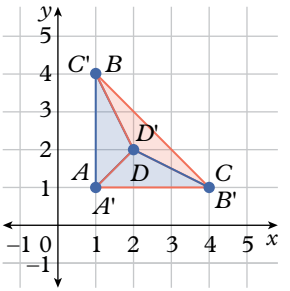
b



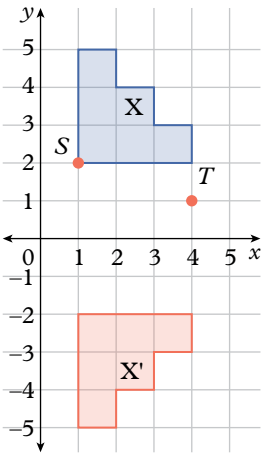
c



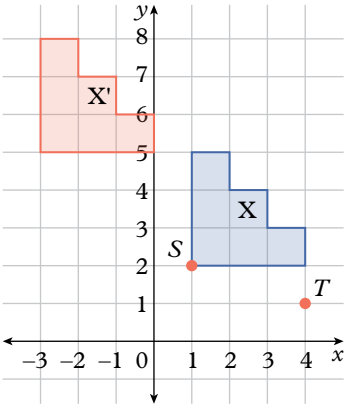
d



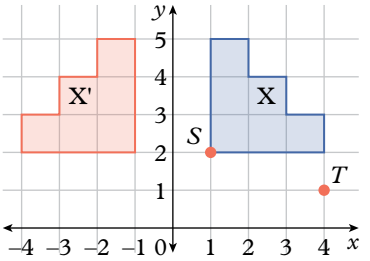
5 a



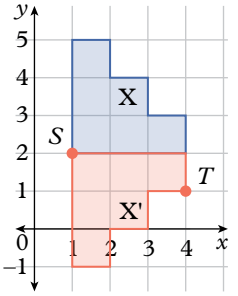
b



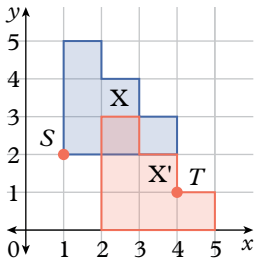
c



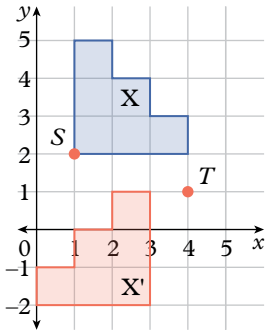
d



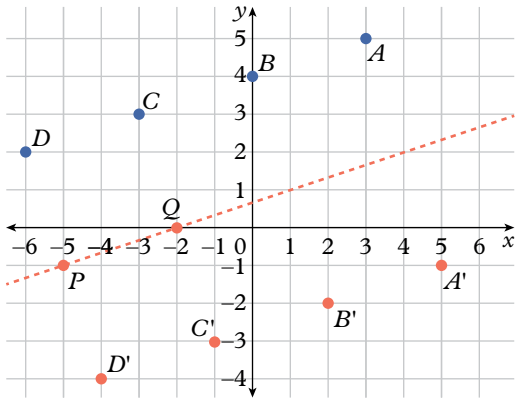
e


y
x
X
X'
S
T

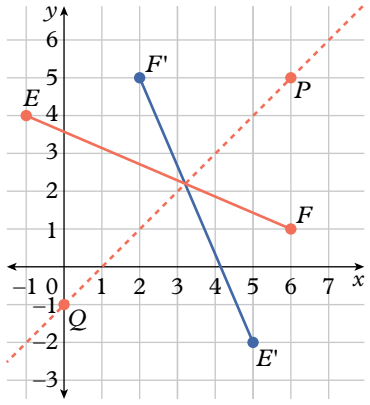
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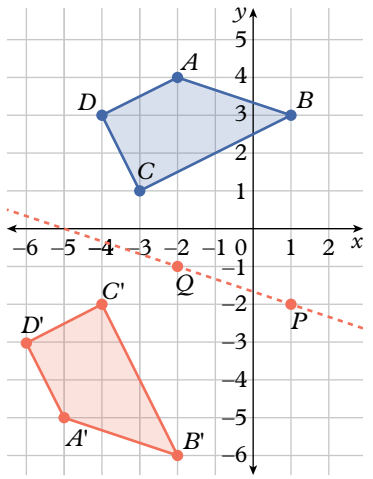
6 a


y
x
A
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C
D
Q
P
A'
B'
C'
D'

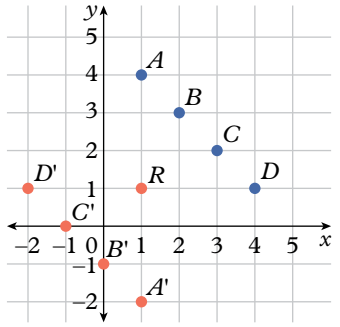
b


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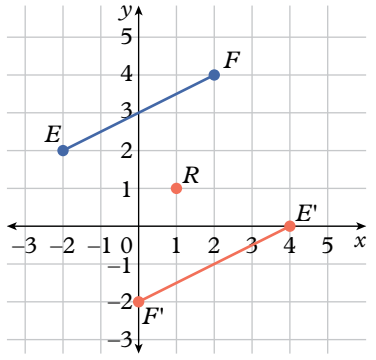
c


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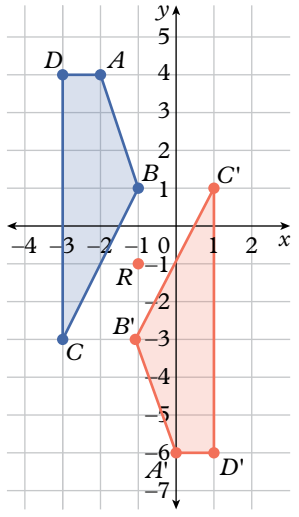
7 a


y
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D'
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D
C'
B'
A'

b

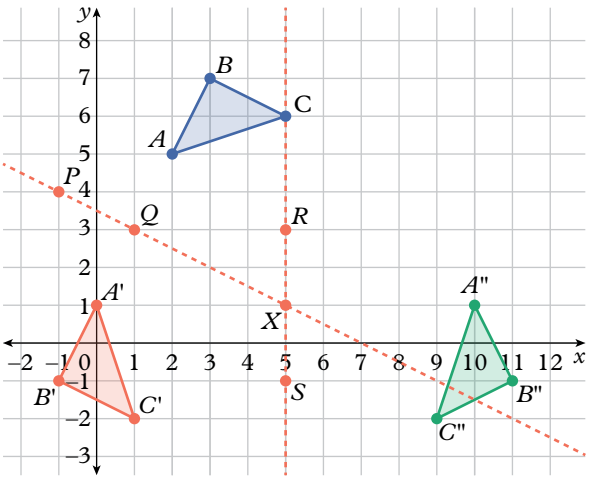

y
x
F
E
R
E'
F'

c


y
x
D
A
B
C'
R
B'
C
A'
D'

8 MADE YOU LOOK

9 a, b



c **i** 127° (or 233°) **ii** 127° (or 233°)

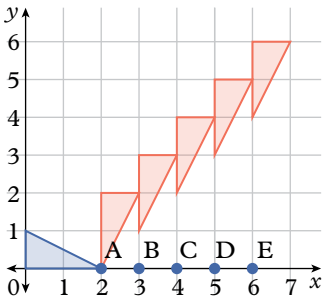
iii 127° (or 233°)

d **i** 5.0 and 5.0 **ii** 6.3 and 6.3

iii 5.0 and 5.0

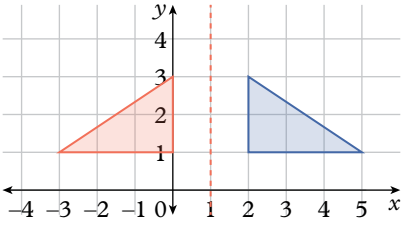
e Rotation of 127° clockwise (or 233° anticlockwise) about point X

10 a

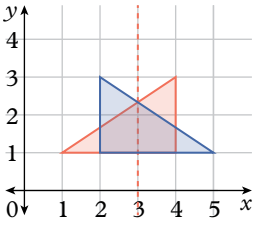


b Moving the centre of rotation further away from a shape moves the image further away from the original shape in both dimensions.

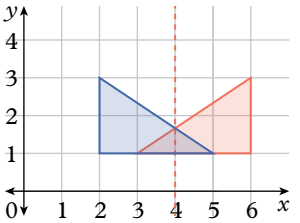
11 a **i**



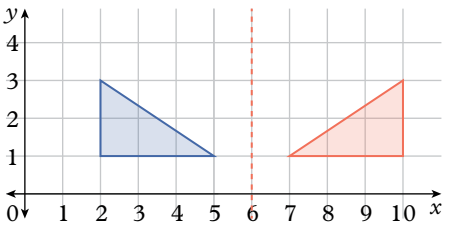
ii



iii



iv



b When a mirror line is on one side of an object the original object and image do not overlap, unlike when the mirror line cuts through an object.

12 a True

b True

13 Point S by 111°

14 P and S

Checkpoint

1 a $x = 161°$

b $a = 67°$, $b = 67°$, $c = 23°$

2 a parallel; the alternate angles are equal

b not parallel; the corresponding angles are not equal

c parallel; the co-interior angles are supplementary

3 a $a = 42°$, obtuse-angled scalene

b $b = 90°$, right-angled scalene

c $c = 14°$, $d = 152°$, obtuse-angled isosceles

d $e = 60°$, $f = 60°$, $g = 60°$, acute-angled equilateral

4 a 124°

b 38°

c 117°

d 94°

5 a yes **b** no

6 a 40.50 cm **b** 112.36 mm or 11.24 cm

7 a $x = \pm 3$ **b** $x = \pm 5$

8 a 18.03 m **b** 449.00 cm or 4.49 m

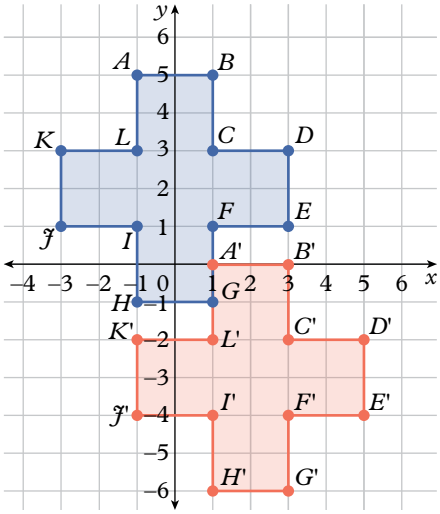
c 21.75 cm

9 a 232.59 cm **b** 40.31 cm **c** 229.09 cm

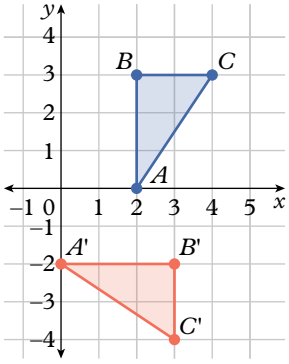
10 a rotated 90° clockwise about the origin

b reflected in the x-axis

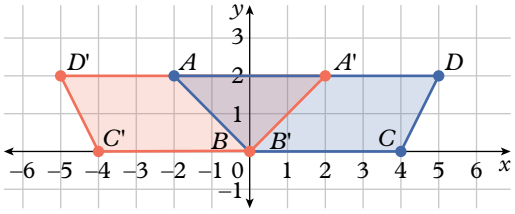
11 a



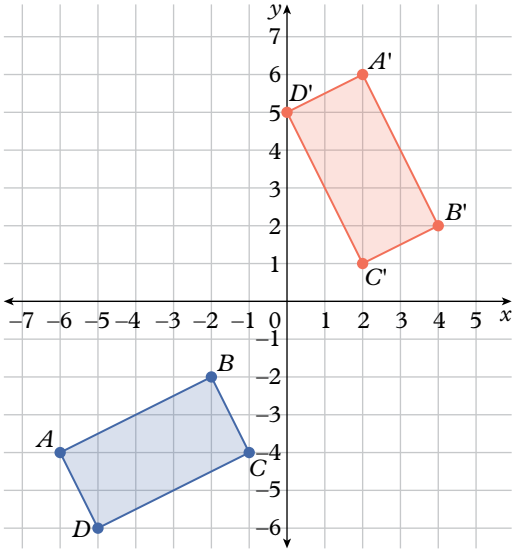
b



c



d



EX 7F Congruence

p317

1 **a** yes **b** no **c** no **d** yes

2 *B* and *W*, *C* and *S*, *E* and *N*, *G* and *J*, *H* and *K*, *L* and *X*, *Q* and *T*

3 **a** *IJ* and *NL*, *JK* and *LM*, *KI* and *MN*

b *AD* and *FG*, *DC* and *GH*, *CB* and *HE*, *BA* and *EF*

c *JK* and *TS*, *KL* and *SR*, *LM* and *RQ*, *MN* and *QP*, *NO* and *PU*, *OJ* and *UT*

d *AB* and *MN*, *BC* and *NO*, *CD* and *OP*, *DE* and *PQ*, *EF* and *QJ*, *FG* and *JK*, *GH* and *KL*, *HA* and *LM*

4 **a** *A* and *F*; *B* and *G*; *C* and *H*; *D* and *E*

b *A* and *R*; *B* and *P*; *C* and *Q*

c *A* and *H*; *B* and *I*; *C* and *J*; *D* and *K*; *E* and *L*; *F* and *G*

d *A* and *M*; *B* and *N*; *C* and *O*; *D* and *P*; *E* and *I*; *F* and *J*; *G* and *K*; *H* and *L*

5 **a** congruent **b** not congruent

6 **a** not congruent; corresponding sides not equal length

b not congruent; corresponding angles equal but corresponding sides not equal

c congruent; corresponding angles and corresponding side lengths are equal.

d not congruent; corresponding angles are not equal.

7 **a** $p = 4$ cm **b** $q = 2.8$ cm

8 **a** $x = 48°$ **b** $y = 45°$

9 **a** True; all angles in squares are 90° so the corresponding sides and angles are equal.

b True; all angles in rectangles are 90° so the corresponding sides and angles are equal.

c False; the angles in triangles with the same base length and height are not always equal.

d False; the angles in parallelograms with the same base length and height are not always equal.

e True; all circles with the same radii are the same shape and size.

10 Rotating a square, rectangle and parallelogram by 180° will give an identical shape in the same orientation; therefore, there is more than one possible set of matching sides and vertices for these shapes.

11 **a** **i** Translate 4 units right and 1 unit down.

ii not congruent

b **i** Rotate 90° clockwise about the origin.

ii congruent

c **i** Reflect in the *y*-axis.

ii not congruent

d **i** Rotate 90° clockwise about (3, 1).

ii congruent

12 **a** Rotate 90° anticlockwise about the origin, then translate 6 units right and 5 units up.

b Reflect in the *y*-axis, then translate 8 units down.

EX p324 7G Congruent triangles

1 **a** congruent **b** not congruent
c not congruent

2 **a** congruent **b** not congruent
c not congruent

3 **a** not congruent **b** congruent
c not congruent

4 **a** If the right angle is between the two shorter sides, it becomes the condition SAS.
b SSS
c SSS is a condition for congruence.

5 **a** not congruent **b** congruent
c congruent **d** not congruent

6 **a** congruent, SAS (7 cm, 77°, 9 cm)
b not congruent, ASA **c** not congruent, ASA
d congruent, ASA (54°, 13 cm, 87°)
e not congruent, RHS
f congruent, SSS (9 cm, 13 cm, 14 cm)

7 **a** SSS
b $\angle ABC = 76°$ and $\angle BAC = 45°$

8 **a** **i** $v = 17$ cm, $w = 38°$, $x = 62°$, $y = 80°$, $z = 80°$
ii AAS and angle sum of a triangle
b **i** $a = 48°$, $b = 25°$, $c = 4$ cm, $d = 25°$, $e = 107°$, $f = 7$ cm
ii AAS and angle sum of a triangle
c **i** $j = 25°$, $k = 7$ cm, $l = 3$ cm, $m = 5$ cm, $n = 115°$
ii angle sum of a triangle and SAS
d **i** $g = 35°$, $h = 35°$, $i = 110°$, $j = 35°$
ii isosceles triangle facts and SAS

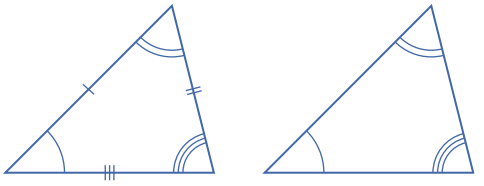
9 **a** angle sum of a triangle (three angles sum to 180°)
b Both angle pairs will require the same value to make the angle sum equal 180°.
c Pair 1 supports ASA as the corresponding sides between the angles are equal whereas the equal sides in Pair 2 are not in corresponding positions.
d Once you find the third angle, you have the requirements for AAS (two angles and one corresponding side).

10 **a** **i** No, because not all three sides are known on both triangles.
ii SAS
b **i** No, because the length of the hypotenuse is not known.
ii SAS or AAS
c **i** Yes, because both are isosceles triangles, so all angles can be determined and there is a pair of congruent side lengths.

11 **a** **i** $\triangle ABC \cong \triangle JKL$ and $\triangle DEF \cong \triangle GHI$
ii $a = 5.6$, $b = 60°$, $c = 53°$, $d = 53°$, $e = 74°$, $f = 5$, $g = 69°$, $h = 51°$
b **i** $\triangle ABC \cong \triangle DEF \cong \triangle GHI \cong \triangle JKL$
ii $a = 34°$, $b = 7$, $c = 4$, $d = 95°$, $e = 5$, $f = 34°$, $g = 51°$, $h = 5$, $i = 4$, $j = 51°$

12 **a** **i** congruent; angle sum of a triangle is 180°, ASA
ii $n = 40°$, $p = 12$ cm, $q = 66°$, $r = 17$ cm
b **i** congruent; isosceles triangles, ASA
ii $a = 6$ cm, $b = 72°$, $c = 36°$, $d = 10$ cm, $e = 72°$
c **i** congruent; isosceles triangles, ASA
ii $d = 20°$, $e = 25$ cm, $f = 140°$, $g = 48$ cm

13 9 values: all 6 angles and 3 different side lengths placed on either triangle



EX p331 7H Dilations and similar figures

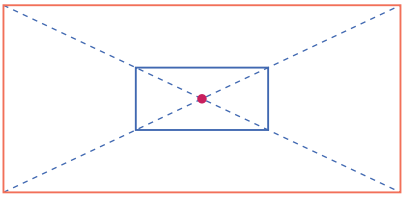
1 **a** enlargement **b** reduction **c** enlargement
d enlargement **e** reduction **f** reduction

2 **a** 2 **b** $\frac{1}{4}$ **c** $\frac{1}{3}$
d 5 **e** $\frac{1}{5}$ **f** 2

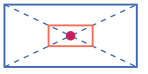
3 **a** 3 **b** 2 **c** $\frac{1}{4}$ **d** $\frac{1}{3}$

4 **a** $x = 2$ cm **b** $x = 6$ cm **c** $x = 9$ cm
d $x = 20$ cm **e** $x = 10.5$ cm **f** $x = 5$ cm

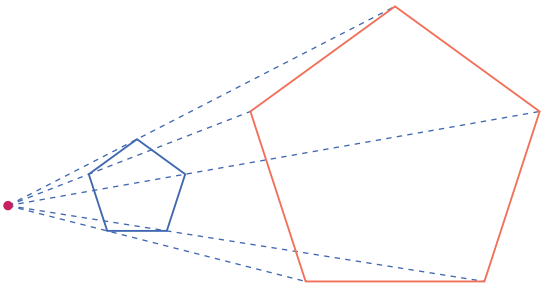
5 **a** **i**



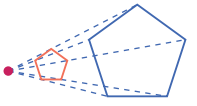
ii



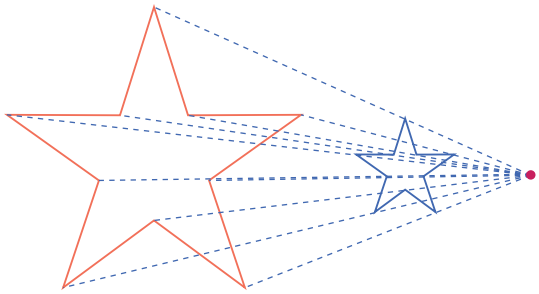
b **i**



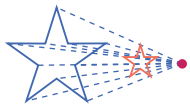
ii



c **i**



ii



6 **a** similar **b** not similar **c** not similar
d similar **e** not similar **f** similar

7 0.0026 mm

8 All circles and squares are exactly the same shape, whereas rectangles can vary in shape.

9 Finn is correct because $\frac{2}{3}$ is the same as $2 \times \frac{1}{3}$.

10 Congruent figures are the same size and shape. Similar figures are the same shape, but not necessarily the same size.

11 No

12 **a** 1 : 2

b The first part of the ratio is the numerator and the second part is the denominator.

c $\frac{2}{1}$ **d** 2 : 1

e **i** $\frac{1}{4}$ **ii** $\frac{1}{100}$ **iii** $\frac{1}{2500}$
iv $\frac{3}{4}$ **v** 2 **vi** $\frac{7}{2}$

f If the first part of the ratio is smaller than the second part, it is a reduction. If the first part of the ratio is larger than the second part, it is an enlargement.

13 **a** Size of plane is reduced by a scale factor of 100.

b 34 cm **c** 27 m

14 **a** 4 m **b** 3.75 cm

c If all proportions are in same ratio, yes; however, it would be very difficult to manufacture a model car with every aspect in the same ratio.

15 **a** 20 cm **b** 9750 m or 9.75 km

16 **a** A regular hexagon has all side lengths equal and all angles equal (120°); an irregular hexagon does not.

b Because all angles will be the same (they are for every regular hexagon) and all side lengths must be equal, any regular hexagon is similar to any other regular hexagon.

17 **a** $\frac{1}{200}$ **b** $\frac{2}{375}$ **c** $\frac{15}{16}$ **d** $\frac{16}{15}$

e The numerators and denominators are swapped because in part **c** the plane is the original, but in part **d** the case is the original.

18 **a** Looking at all paper sizes (from A0 to A8), the sizes are almost all similar (scale factor around 0.71), though rounding limits exact dilations to even numbers and odd numbers.

b similar relationship in B-sized paper

c similar relationship in C-sized paper

d B series covers sizes that are not found in the A series, so they are not similar. The C series sizes are meant to be similar to A sizes, but there are some slight differences due to rounding errors.

EX p338 71 Similar triangles

1 **a** $a = 9$ cm, $b = 4$ cm

b $c = 44$ cm, $d = 7$ cm

c $e = 20$ cm, $f = 7.5$ cm

d $m = 19$ cm, $n = 45$ cm

e $x = 12$ cm, $y = 6.5$ cm

f $x = 52$ cm, $y = 20$ cm

2 **a** SAS **b** SSS **c** RHS
d AAA **e** SAS **f** AAA

3 **a** SSS **b** AAA **c** SAS
d SSS or SAS

4 **a** Yes; SAS

b No; all corresponding side pairs are not in the same ratio

c No; corresponding angles are not equal.

d Yes; SSS

e No; SSA is not a condition for similarity

f Yes; AAA

5 **a** similar **b** similar **c** not similar
d similar **e** similar **f** not similar

6 **a** not similar **b** similar **c** similar
d similar **e** not similar **f** similar

7 With all the similarity conditions, if one pair of corresponding sides is equal then the triangles will be congruent.

8 All equilateral triangles have three equal angles (60°), meeting similarity condition AAA.

9 The third angle is equal to 180° minus the sum of the other two angles. Therefore, since the two triangles have the same two angles, their third angles will also be equal.

10 **a** Using knowledge of alternate angles within parallel lines ($\angle ABC = \angle EDC$ and $\angle BAC = \angle DEC$) and vertically opposite angles ($\angle ACB = \angle ECD$), these triangles have three corresponding equal pairs of angles and hence are similar.

b scale factor = 3

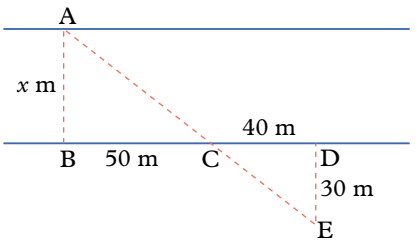
c $a = 2.5$ cm, $b = 6$ cm

11 **a** similar **b** not similar **c** similar
d not similar

12 $x = 6.54$ m

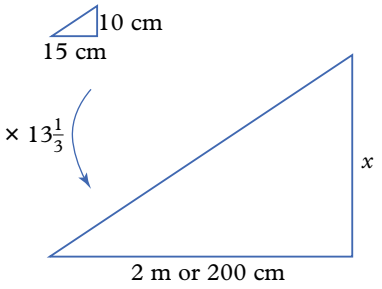
13 **a** 2.5 m **b** 2.72 m tall

14 $x = 37.5$ m

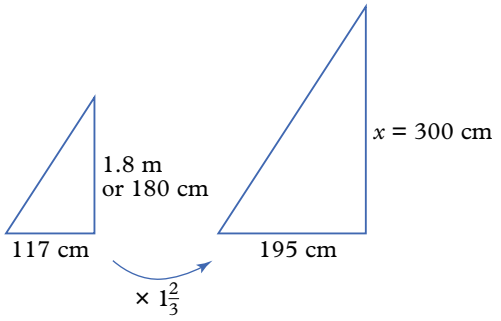


15 4 cm

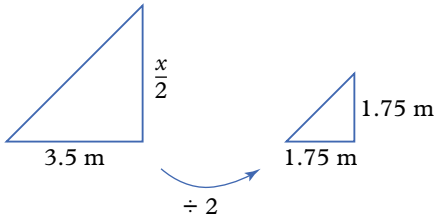
16 **a** 133.33 cm



b Ring is 5 cm too short, at 300 cm tall.



c 7 m



17 Sean's house is 3.1 m tall; Tania's house is 2.7 m tall. Sean's house is taller.

18 You need to have the second similar triangle to compare house heights and shadows because the measurements were taken at different times of day so the lengths of the shadows would change.

19 **a** $\frac{DE}{AB} = \frac{7}{5} = 1.4$
$\frac{EF}{BC} = \frac{8.4}{6} = 1.4$
$\frac{FD}{CA} = \frac{11.2}{8} = 1.4$
$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$
Corresponding sides are in the same ratio
$\therefore \triangle ABC \sim \triangle DEF$ by SSS

b $\frac{DE}{AB} = \frac{7}{5} = 1.4$
$\frac{FD}{CA} = \frac{11.2}{8} = 1.4$
$\angle EDF = 41°$
$\angle BAC = 41°$
$\frac{DE}{AB} = \frac{FD}{CA}$
Corresponding sides are in the same ratio.
$\angle EDF = \angle BAC$
Corresponding angles are equal
$\therefore \triangle ABC \sim \triangle DEF$ by SAS

EX p345 7J Quadrilaterals

1 **a** kite; two pairs of adjacent sides equal in length
b trapezium; one pair of opposite sides that are parallel
c rhombus; all four sides equal in length and opposite sides parallel, opposite angles equal, but not 90°
d parallelogram; opposite sides are parallel and equal in length, opposite angles are equal
e square; all four sides equal in length and all angles are 90°
f kite; two pairs of adjacent sides equal in length

2 **a** $x = 76°$ **b** $x = 117°$ **c** $x = 141°$
d $x = 23°$ **e** $x = 21°$ **f** $x = 93°$

3 **a** parallelogram; $x = 71°$
b rectangle; $x = 90°$ **c** trapezium; $x = 105°$
d kite; $x = 85°$ **e** rhombus; $x = 64°$
f trapezium; $x = 143°$

4 **a** **i** rhombus
ii $a = 72°$, $b = 72°$ and $c = 108°$
b **i** trapezium **ii** $a = 90°$, $b = 112°$
c **i** square **ii** $a = 90°$, $b = 90°$, $c = 90°$
d **i** kite **ii** $a = 130°$, $b = 130°$
e **i** parallelogram **ii** $a = 125°$, $b = 55°$, $c = 125°$
f **i** kite **ii** $a = 52°$, $b = 115°$

5 **a** $x = 20°$ **b** $x = 37°$ **c** $x = 80°$
d $x = 20°$ **e** $x = 15°$ **f** $x = 17°$

6 **a** Having two internal reflex angles would make the angle sum exceed 360°.
b 270°

7 rectangle

8 **a** True
b False. A kite has no parallel sides. It contains two pairs of adjacent sides equal in length, but not parallel.

c False. A rectangle is a type of parallelogram. The reverse is not true as a parallelogram does not necessarily have four angles equal to 90°.

d False. A trapezium has at least one pair of parallel sides. It can have two pairs of parallel sides.

e False. A rhombus has four equal sides and two pairs of angles equal in size. When all four angles are equal in size it is called a square.

f False. A rectangle has two pairs of opposite equal sides and four equal angles.

9 a i 130°, 50°, 130°, 50° **ii** 360°

b i 108°, 124°, 56°, 72° **ii** 360°

c i 74°, 120°, 94°, 72° **ii** 360°

d The exterior angles of any quadrilateral add to 360°.

10 a similar **b** similar **c** not similar

d not similar

11 a $a = 12$ cm, $b = 99°$, $c = 81°$, $d = 99°$

b $a = 1$ cm, $b = 150°$, $c = 75°$, $d = 9$ cm, $e = 12$ cm, $f = 45°$, $g = 150°$

12 a $\triangle ABC \cong \triangle ADC$ by SSS as $AB \cong AD$, $BC \cong DC$, and AC is common to both triangles.

b $\angle ABC \cong \angle ADC$, $\angle BCA \cong \angle DCA$, and $\angle BAC \cong \angle DAC$ because they are corresponding angles in congruent triangles.

c $\angle BAD = \angle BAC + \angle DAC$. Since $\angle BAC \cong \angle DAC$, $\angle BAD = 2\angle BAC$ and so $\angle BAD$ has been bisected into two equal angles by diagonal AC. Likewise, $\angle BCD = \angle BCA + \angle DCA$. Since $\angle BCA \cong \angle DCA$, $\angle BCD = 2\angle BCA$ and so $\angle BCD$ has been bisected into two equal angles by diagonal AC.

d Since $\triangle ABC \cong \triangle ADC$, the perpendicular heights BE and DE will be in the same relative position in the triangles, at point E to matching vertices B and D. These two line segments align to form the diagonal BD.

e Since $\triangle ABC \cong \triangle ADC$, BE and DE are corresponding lengths in congruent triangles, which are therefore congruent. $\angle BEA$ and $\angle DEA$ are right angles since BE and DE are perpendicular heights to AC on which point E lies. Since AC intersects BD at point E, diagonal AC has bisected diagonal BD at a right angle.

13 a $\angle BCD = \angle DAB$, and $\angle ABC = \angle CDA$

b BC and AD are parallel and BD is a transversal. $\angle BDA \cong \angle DBC$ as they are alternate angles on parallel lines. Likewise, AB and CD are parallel and BD is a transversal so $\angle DBA \cong \angle BDC$ as they are alternate angles on parallel lines.

c Either diagonal will act as a transversal to both pairs of parallel sides, so the alternate angles formed by the transversal will be congruent.

d $\triangle ABD \cong \triangle CDB$ by AAS as $\angle BDA \cong \angle DBC$, $\angle DBA \cong \angle BDC$, and BD is common to both triangles.

e $\triangle ABD \cong \triangle CDB$, where side AB corresponds to side CD and side AD corresponds to side CB, so $AB \cong CD$ and $AD \cong CB$.

14 a $\angle AED \cong \angle BEC$ as they are vertically opposite angles. Likewise, $\angle BEA \cong \angle CED$ as they are vertically opposite angles.

b $\triangle AED \cong \triangle CEB$ by AAS as $\angle AED \cong \angle BEC$, $\angle BDA \cong \angle DBC$, and $AD \cong CB$.
$\triangle AEB \cong \triangle CED$ by AAS as $\angle ABD \cong \angle CDB$, $\angle BEA \cong \angle CED$, and $AB \cong DC$.

c $BE \cong DE$ and $AE \cong CE$ as they are corresponding lengths in congruent triangles, which are congruent.

d Diagonal $BD = BE + DE$, and since $BE \cong DE$, diagonal $BD = 2BE$ and so has been bisected into two equal line segments by diagonal AC. Likewise, diagonal $AC = AE + CE$, and since $AE \cong CE$, diagonal $AC = 2AE$ and so has been bisected into two equal line segments by diagonal BD. Therefore, the diagonals AC and BD bisect each other.

15 a $\triangle ABC \cong \triangle ADC$ by SSS as $AB = AD$, $BC = DC$, and AC is common to both triangles.

b $\triangle ABC$ and $\triangle ADC$ are isosceles triangles because they each have two equal lengths, $AB = BC$ and $AD = CD$.

c $\angle BAC \cong \angle BCA$ and $\angle DAC \cong \angle DCA$ as they are angles opposite equal lengths in an isosceles triangle.

d $\angle ABC \cong \angle ADC$, $\angle BCA \cong \angle DCA$, and $\angle BAC \cong \angle DAC$ because they are corresponding angles in congruent triangles.

e $\angle BCD = \angle BCA + \angle DCA$, and since $\angle BCA \cong \angle DCA$, $\angle BCD = 2\angle BCA$ and so $\angle BCD$ has been bisected into two equal angles by diagonal AC. Likewise $\angle BAD = \angle BAC + \angle DAC$, and since $\angle BAC \cong \angle DAC$, $\angle BAD = 2\angle BAC$ and so $\angle BAD$ has been bisected into two equal angles by diagonal AC. Therefore, diagonal AC bisects $\angle BAD$ and $\angle BCD$.

f Diagonal DB forms two congruent isosceles triangles $\triangle ABD$ and $\triangle CBD$ in the same way diagonal AC did. So all subsequent steps can be repeated for $\triangle ABD$ and $\triangle CBD$ and the associated angles.

16 a i 90° **ii** square

b i 60°, 60°, 120°, 120°

ii parallelogram

CHAPTER 7 review

Multiple choice

1	C	**2**	D	**3**	C	**4**	C	**5**	B	**6**	E
7	C	**8**	D	**9**	A	**10**	C	**11**	A	**12**	C

Short answer

1 a $a = 138°$, $b = 42°$, $c = 138°$

b $a = 38°$, $b = 142°$, $c = 52°$

2 a $x = 109°$ **b** $x = 66°$, $y = 48°$

c $x = 141°$ **d** $x = 69°$

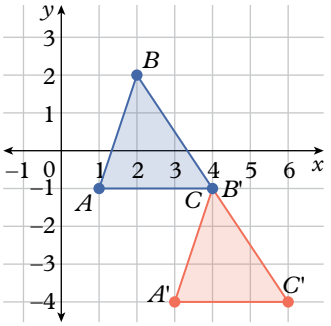
3 **a** obtuse-angled scalene
b acute-angled isosceles
c acute-angled scalene
d acute-angled scalene

4 **a** right-angled
b not right-angled

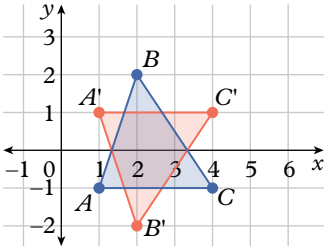
5 $x = 20$ cm, $y = 36.06$ cm

6 **a** 25.46 cm
b 1.50 m

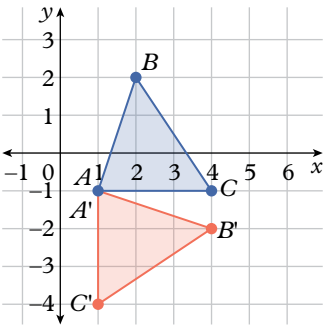
7 **a**



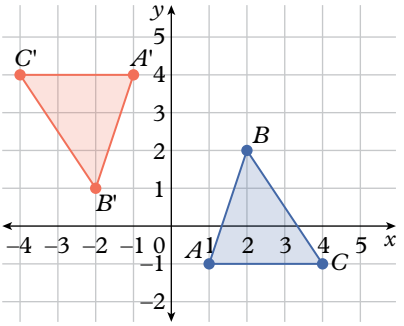
b



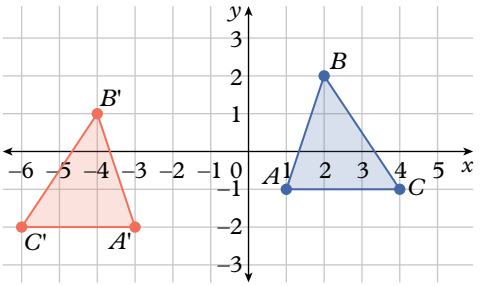
c



d



e



8 **a** congruent
b congruent
c not congruent
d not congruent

9 A and E

10 A and E are congruent due to the condition ASA.

11 **a** enlargement; scale factor = 2
b reduction; scale factor = $\frac{1}{4}$

12 $a = 9.2$ cm and $b = 4$ cm

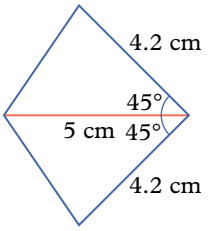
13 Yes, by SAS

14 **a** $w = 124°$
b $a = 144°$, $b = 36°$, $c = 144°$
c $m = 129°$, $n = 129°$
d $p = 263°$

15 **a** trapezium **b** parallelogram
c kite **d** irregular quadrilateral

Analysis

1 **a** The triangles have an equal length and included angle but the remaining lengths are not equal, so the triangles are not congruent by SAS.
b A reflection creates a congruent image with congruent lengths and angles. The quadrilateral formed will therefore have pairs of congruent adjacent sides (the two 4.2 cm and the two lengths opposite the 45° angles). So the quadrilateral is a kite.
c



d Given two interior angles in one triangle, the third must make the total 180°. The two triangles are congruent, so the corresponding angles must be equal.
e 90°, 112°, 79°, 79°

2 **a** camp triangle: 40°, 90°, 50° gum-tree triangle: 40° (gum tree), 90°, 50°
b Corresponding angles are the same but side lengths are not; triangles are similar (AAA) but not congruent.
c 2.5

d Side length of 6 m in 'camp triangle' corresponds to the side in 'gum tree triangle' that represents width of the river. To determine width of the river, multiply 6 m by scale factor (6 × 2.5).

e 15 m

f We can use Pythagoras' theorem to calculate the lengths of the hypotenuses of both similar triangles and add them together to find the straight-line distance between the camp and gum tree.

g 25.24 m

CHAPTER 8 Units of measurement

EX 8A Length and perimeter

p360

1 **a** 5800 cm **b** 2700 m **c** 5.21 m
d 0.398 km **e** 12 mm **f** 3.987 km
g 42.3 cm **h** 0.7 cm

2 **a** 5340 mm **b** 395 000 cm **c** 235 m
d 145 km **e** 0.023 68 km **f** 0.287 m
g 193 400 cm **h** 910 000 mm

3 **a** 3.77 m **b** 15.71 cm **c** 144.51 mm
d 5.03 cm **e** 571.77 cm **f** 8.17 mm
g 5.50 m **h** 8.17 cm **i** 257.61 km

4 **a** 20 cm **b** 8 m
c 142 mm (14.2 cm) **d** 168 mm (16.8 cm)
e 0.124 m **f** 137 cm (1.37 m)

5 **a** 14 cm (or 140 mm) **b** 34 m
c 60 cm **d** 56 cm
e 18.4 m **f** 14.6 m

6 **a** 16 cm **b** 60 mm **c** 50 mm **d** 60 cm

7 **a** 65 cm **b** 43.6 cm (436 mm)
c 47 cm **d** 112 mm
e 31 m **f** 28.4 cm

8 **a** 7.5 cm **b** 15 m **c** 14 cm **d** 61.6 cm

9 **a** 12.88 cm **b** 11.68 mm **c** 13.28 cm **d** 22.42 m
e 18.12 cm **f** 30.85 mm

10 **a** $10\frac{1}{5}$ m **b** $5\frac{1}{6}$ cm **c** $2\frac{4}{7}$ cm
d $4\frac{5}{6}$ m or $\frac{29}{6}$ m **e** $\frac{\pi}{4} + \frac{7}{3}$ cm **f** $\frac{16 + 4\pi}{3}$ mm

11 480 cm = 4.8 m

12 25.2 km

13 192 cm, the shape is made up of 32 edges of the octagons, each with a side length of 6 cm.

14 $x = 2$, $y = 3.5$, $P = 76$ cm

15 9 cm, the side length of a square is a quarter of the perimeter.

16 1.15 m

17 1.50 m

18 **a** $4l$ **b** $2w + 2l = 2(w + l)$
c $2a + 2b = 2(a + b)$ **d** $4l$
e $a + b + s + t$ **f** $2a + 2b = 2(a + b)$
g $a + b + c + d$ **h** $2p + q + r$

19 **a** 91.5 m **b** 549 m **c** 20.33 m **d** 122 m
e 488 m, four times the distance of one shuttle run (calculated in part **d**)
f four shuttle runs

20 **a** 50 cm
b The corners of the frame are where the frame is longer than the width and length of the photo. Because of this, the sides of the frame must be longer than the sides of the photo.
c 54 cm
d 58 cm; the perimeter is longer because the corner lengths are counted twice, horizontally and vertically.
e **i** length of wood = 58 cm, perimeter = 66 cm
ii length of wood = 62 cm, perimeter = 74 cm
iii length of wood = 74 cm, perimeter = 78 cm
iv length of wood = 94 cm, perimeter = 98 cm

21 20 × 4 + 15 × 4 + 60 = 200 cm

22 1 cm × 14 cm, 2 cm × 13 cm, 3 cm × 12 cm, 4 cm × 11 cm, 5 cm × 10 cm, 6 cm × 9 cm, 7 cm × 8 cm

23 **a** 36.4 m
b length: 0.9 m, width 0.5 m
c length: 7.2%, width: 10.6%
d **i** length **ii** width
e 39.2 m
f 7.7%

24 51.52 m; there are 56 pages, each of which will require 92 cm of ribbon.

25 **a** Some possible answers are: 6 m × 8 m, 10 m × 4 m, 10.2 m × 3.8 m.
b 8 m longer than garden perimeter: 36 m
c **i** 44 m **ii** 52 m
d Multiply the distance by 8 and add the answer to the initial perimeter (28 m).
e The rule works for all cases. Start with the initial rectangle or the inside rectangle. Once you've found this perimeter, the outer perimeter can be found by adding 8 × the distance to the fence to the inside perimeter.

26 **a** 1 foot = 30.54 cm
b 1 yard = 0.9144 m
c 1 mile = 1.609 344 km
d 42.194 988 km

EX 8B Area of triangles and rectangles

p369

1 **a** 70 mm^2 **b** 1500 mm^2 **c** 10 200 cm^2
d 29.4 mm^2 **e** 9900 cm^2 **f** 161 mm^2
g 540 mm^2 **h** 4 cm^2 **i** 20 cm^2

2 **a** 22.5 cm^2 **b** 0.96 m^2 **c** 10.8 cm^2
d 367.5 cm^2 **e** 0.4725 m^2 **f** 3.382 m^2

3 **a** 1400 mm^2 **b** 20.4 cm^2 **c** 150 mm^2
d 51.84 mm^2 **e** 12.5 cm^2 **f** 16.8 cm^2
g 361 cm^2 **h** 462 mm^2 **i** 1715 cm^2

4 **a** 14 cm² **b** 14 cm² **c** 900 mm²
d 9 m² **e** 90 cm² **f** 15 cm²

5 **a** 30 000 cm² **b** 1000 mm² **c** 65 000 cm²
d 1970 mm² **e** 1 600 000 m² **f** 750 000 m²

6 **a** 2.5 m² **b** 7500 cm² **c** 4 cm²
d 0.125 km² **e** 0.0675 m² **f** 500 km²

7 **a** 0.15 m² **b** 20 000 000 cm²
c 1 500 000 mm² **d** 0.000 037 9 km²

8 **a** 76 cm² **b** 72 cm² **c** 1120 mm²
d 160 cm² **e** 118 cm² **f** 224 cm²

9 **a** 96 cm² **b** 505 cm² **c** 94 cm²
d 1008 cm² **e** 372 cm² **f** 196 cm²

10 8704 cm²

11 They used the slant height (68.5 cm) as the height of each trianglular section rather than the perpendicular height (68 cm).

12 **a** 24 cm² and 4 cm²; 28 cm²
b 8 cm² and 20 cm²; 28 cm²

13 **a** 92 mm² **b** 35 cm² **c** 199 m²
d 66 cm² **e** 96 cm² **f** 72 cm²

14 The area units should give numerical values in the units, tens and hundreds which people can easily understand.
a m² **b** mm² **c** km² **d** m² **e** m² **f** cm²

15 11 cm² = (11 × 10²) mm² = 1100 mm²; Ahmed forgot to square the conversion factor.

16 Alex divided by the conversion factor instead of the square of the conversion factor.

$$40\ 000\ \text{cm}^2 = (40\ 000\ \text{cm} \div 100^2)\ \text{m}^2$$
$$= 4\ \text{m}^2$$

17 8.295 m²

18 **a** 27 450 cm²
b 2 boxes

19 $1570

20 **a** 100 **b** 4000 m²

21 **a** **i** 3200 kg/hectare **ii** 0.32 kg/m²
iii 320 g/m² **iv** 0.032 g/cm²
b Grams per square metre will make the most sense for most people, as they are units people are more familiar with. For farmers, tonnes per hectare makes the most sense, as a farmer can have thousands of hectares of land.
c $3 360 000

22 Width is 5 cm. There is only one answer because only one number gives 20 when multiplied by 4.

23 Possible answers include: 40 cm² (10 cm × 4 cm), 48 cm² (6 cm × 8 cm), 24 cm² (12 cm × 2 cm).

24 A square has equal side lengths and its area is the square of its side length. There is only one positive number that when multiplied by itself gives 64: × 8 = 64, so the dimensions must be 8 mm × 8 mm. The perimeter is 32 mm.

8 mm



8 mm

25 **a** 40 cm **b** 1400 cm²

26 **a** Possible answers include: 8 m × 4 m (32 m²), 7 m × 5 m (35 m²), 6 m × 6 m (36 m²), 2.4 m × 9.6 m (23.04 m²)
b 36 m²; the maximum area corresponds to a square with side lengths 6 m.
c Possible answers include: 2 m × 20 m (20 m side on shed); 40 m², 4 m × 16 m (16 m side on shed); 64 m², 5 m × 14 m (14 m side on shed); 70 m².
d 72 m² is the maximum area for a rectangle with dimensions 12 m × 6 m. Its shape is a half square. Its area is twice the area of the square in part **b**.

8C Area of quadrilaterals

EX p378

1 **a** 4.4 and 7.2 **b** 8.6 and 5.5 **c** 4.2, 4.8 and 6.2
d 4.2 and 5.2 **e** 3.2 and 4.4 **f** 2.1, 3.2, and 5.2

2 **a** 24 cm² **b** 63 mm² **c** 168 cm²

3 **a** 14 cm² **b** 48 mm² **c** 6 m²

4 **a** 12 mm² **b** 20 cm² **c** 22.5 mm²

5 **a** 124 mm² **b** 35 cm² **c** 138 m²

6 **a** **i** rhombus **ii** $A = bh$ **iii** 42 cm²
b **i** kite **ii** $A = \frac{1}{2}xy$ **iii** 3 cm²
c **i** trapezium **ii** $A = \frac{1}{2}(a + b)h$
iii 15.96 m²
d **i** trapezium **ii** $A = \frac{1}{2}(a + b)h$
iii 18 cm²
e **i** parallelogram **ii** $A = bh$ **iii** 112 mm²
f **i** trapezium **ii** $A = \frac{1}{2}(a + b)h$
iii 21 mm²

7 **a** 10.5 cm² **b** 175 mm²
c 99 cm² (or 9900 mm²) **d** 90 cm²
e 44 cm² **f** 56 mm²

8 **a** 8.925 cm² **b** 9.18 mm²

9 **a** 3087.5 cm² **b** 20 cm

10 $731.25

11 **a** No, there is not enough information to confirm it is a trapezium, as we do not know the two sides that appear horizontal are parallel.
b No, there is enough information to confirm it is a parallelogram, but not enough information to determine its area as the perpendicular height is not known.
c Yes, there is enough information to confirm it is a kite, and the length of the perpendicular diagonals are both known.

d Yes, there is enough information to confirm it is a rhombus and both diagonal lengths are known (which are perpendicular since it is a rhombus).

e Yes, there is enough information to confirm it is a trapezium as the two vertical sides are both at right angles to the 2 cm length, which acts as a transversal indicating the vertical sides are parallel.

f No, there is not enough information to confirm it is a kite even though the diagonals are perpendicular.

12 a i parallelogram, trapezium

ii $8 \times 2 = 16$ m^2, $\frac{1}{2}(8 + 8) \times 2 = 16$ m^2

b i parallelogram, kite, rhombus, trapezium

ii $4 \times 4 = 16$ m^2, $\frac{1}{2} \times 5.66 \times 5.66 \approx 16.0$ m^2 (kite and rhombus), $\frac{1}{2}(4 + 4) \times 4 = 16$ m^2

c i parallelogram, kite, rhombus, trapezium

ii $4 \times 2.29 = 9.2$ m^2, $\frac{1}{2} \times 2.41 \times 7.63 = 9.2$ m^2 (kite and rhombus), $\frac{1}{2}(4 + 4) \times 2.29 = 9.2$ m^2

d i parallelogram, trapezium

ii $3 \times 7 = 21$ m^2, $\frac{1}{2}(7 + 7) \times 3 = 21$ m^2

13 a parallelogram **b** 1806 m^2

c No, as the windows on the far left or far right sides are not similar to others away from the side edges.

d For the rectangular windows, $A = lw$. For the trapezium windows, $A = \frac{1}{2}(a + b)h$. For the triangular windows, $A = \frac{1}{2}bh$.

14 a 9 cm **b** 4.25 cm **c** 5 cm

d 19 mm (1.9 cm)

15 1 cm × 96 cm, 2 cm × 48 cm, 3 cm × 32 cm, 4 cm × 24 cm, 6 cm × 16 cm, 8 cm × 12 cm

16 a 81 cm^2 **b** 24 cm **c** 4 cm

17 a 447.68 mm^2 **b** 56.2 cm^2

18 a The trapezium has been cut into two triangles, one with base length a, the other with base length b, and both with height h. The area of the trapezium is the sum of the area of the two triangles $\frac{1}{2}ah + \frac{1}{2}bh$, which is equal to $\frac{1}{2}(a + b)h$.

b Part of the trapezium has been cut off and rotated to form a triangle with a base length of $(a + b)$ and height h. The area of this triangle is equal to the area of the trapezium, which is $\frac{1}{2}(a + b)h$.

c The trapezium has been duplicated and rotated to form a parallelogram with a base length of $(a + b)$ and height h. The area of the trapezium is half of the area of the parallelogram, which is $\frac{1}{2}(a + b)h$.

d The top half of the trapezium has been cut off and rotated to form a parallelogram with a base length of $(a + b)$ and height $\frac{h}{2}$. The area of the trapezium is equal to the area of the parallelogram, which is $\frac{1}{2}(a + b)h$.

e Two triangles have been cut off and rotated to form a rectangle with a length of the average of the parallel sides, $\frac{a + b}{2}$, and width h. The area of the trapezium is equal to the area of the rectangle, which is $\frac{1}{2}(a + b)h$.

19 a i 16 cm^2 **ii** 6.75 cm^2

b 57.8125% decrease

20 $x = 5$ m, $A = 40$ m^2

8D Area of a circle

EX p386

1 a 12 cm^2 **b** 52 cm^2 **c** 37 cm^2

2 a i 4 cm **ii** 2 cm **iii** 3

b i 8 cm **ii** 4 cm **iii** 3.25

c i 7 cm **ii** 3.5 cm **iii** 3.02

3 a 706.86 cm^2 **b** 176.71 mm^2 **c** 254.47 mm^2

d 78.54 cm^2 **e** 38.48 cm^2 **f** 33.18 m^2

4 a 5.0625π cm^2 **b** 77.44π cm^2 **c** 49π cm^2

5 a area = 60.82 cm^2, perimeter = 27.65 cm

b area = 44.18 m^2, perimeter = 23.56 m

c area = 176.71 mm^2, perimeter = 47.12 mm

d area = 2642.08 mm^2, perimeter = 182.21 mm

6 a 1.95 cm **b** 2.82 mm

c 5.64 cm **d** 4.65 cm

7 a 10.09 cm **b** 7.90 cm

c 5.05 mm **d** 13.11 cm

8 2375.83 cm^2

9 176.71 m^2

10 A medium pizza has an area of 113.10 inch^2, so two medium pizzas have an area of 226.19 inch^2, whereas a large pizza has an area of 254.47 inch^2 which is greater than the area of two medium pizzas.

11 a $\frac{1}{4}\pi d^2 = \frac{1}{4}\pi(2r)^2 = \frac{1}{4}\pi \times 4r^2 = \pi r^2$

b $\frac{1}{2}Cr = \frac{1}{2}(2\pi r)r = \pi r^2$

12 a 50.27 m^2 **b** 28.27 cm^2 **c** 127.32 mm^2

d 198.94 cm^2 **e** 12.62 cm **f** 33.63 mm

g 15.85 cm

13 216 cm

14 a 87.96 cm^2 **b** 75.40 cm^2 **c** 138.23 cm^2

15 452.39 cm^2

16 a 314.16 cm^2 **b** 2 cm **c** 706.86 cm^2

d 392.70 cm^2

17 285.88 mm^2

18 a 47.52 cm^2 **b** 8.04 m^2 **c** 38.48 cm^2

d 3848.84 cm^2 **e** 264.74 mm^2 **f** 2035.75 cm^2

19 a 14.28 cm^2 **b** 96.37 cm^2 **c** 48.57 mm^2

d 66.19 cm^2 **e** 144.31 mm^2 **f** 81.73 cm^2

20 To find the length of the radius, you first need to divide A by π. Unless the value of A contains π, the result of this calculation will be irrational; therefore, the exact length of the radius cannot be found.

a $\sqrt{\frac{16}{\pi}} \approx 2.26$ mm **b** $\sqrt{16} = 4$ mm

21 a i 14 cm **ii** 14π cm

b i 4 m **ii** 16π m^2

22 a 60° **b** 360° **c** $\frac{60}{360}$ $\left(\text{or } \frac{1}{6}\right)$

d 4 cm **e** 50.27 cm^2 **f** 8.38 cm^2

23 a 19.24 cm^2 **b** 126.71 cm^2 **c** 230.91 mm^2

Checkpoint

1 **a** 0.45 m **b** 5900 m
c 3.51 cm^2 **d** 2000 cm^2

2 **a** 12.6 cm **b** 18.4 m **c** 15.32 m **d** 18.4 cm

3 **a** 17.93 cm **b** 21.99 m

4 **a** 97.39 m **b** 43.98 cm

5 **a** 144 cm^2 **b** 48 cm^2

6 **a** 5.5 m **b** 7.2 m **c** 9 cm **d** 3.1 cm

7 **a** 9 m^2 **b** 9 m^2 **c** 15 cm^2 **d** 7.5 mm^2

8 **a** **i** 1820 cm = 18.2 m
ii 200 081 cm^2 = 20.0081 m^2
b **i** 264.4 mm = 26.44 cm
ii 3000 mm^2 = 30 cm^2

9 **a** 14 m^2 **b** 6 cm^2 **c** 15 cm^2 **d** 12 mm^2
e 7.5 m^2 **f** 9.10 m^2

10 **a** 21.69 cm^2 **b** 31.63 m^2

EX 8E Volume and capacity

p396

1 **a** 40 cm^3 **b** 211.5 cm^3 **c** 97.16 mm^3
d 1496 mm^3

2 **a** 90 cm^3 **b** 90 cm^3 **c** 324 mm^3

3 **a** 346.5 cm^3 **b** 48 cm^3 **c** 225 cm^3

4 **a** 320 cm^3 **b** 17.5 cm^3 **c** 24.75 cm^3

5 **a** 9000 mm^3 **b** 18 000 000 cm^3
c 16 cm^3 **d** 0.00175 km^3
e 4 200 000 000 m^3 **f** 4 m^3
g 4 560 000 cm^3 **h** 0.845 m^3
i 150 000 000 m^3 **j** 6100 mm^3
k 0.028 cm^3 **l** 1700 km^3

6 **a** 8000 mL **b** 15 000 L **c** 0.07 L
d 6400 kL **e** 2.5 kL **f** 3 000 000 mL

7 **a** 14 mL **b** 3000 cm^3 **c** 5.8 kL
d 0.75 L **e** 1000 m^3 **f** 6400 m^3

8 5.2 m^3

9 **a** 2944.62 cm^3 **b** 123

10 4 cm

11 Multiplication is commutative, so it doesn't matter in which order you multiply the length, width and height; therefore, it doesn't matter which of the measurements you designate as the length, width or height.

12 **a** **i** Priya multiplied by the power (3) instead of mutliplying by 100 three times.
ii 7 m^3 = (7 × 100^3) cm^3 = 7 000 000 cm^3
b **i** Priya used the incorrect conversion factor.
ii 3000 mm^3 = (3000 ÷ 10^3) cm^3 = 3 cm^3

13 **a** 82.5 cm^3 **b** 82 500 mm^3
c 25 × 30 × 110 = 82 500 mm^3

14 **a** 220 cm^3 **b** 398.61 cm^3
c 48 cm^3 **d** 152 cm^3

15 **a** 3 456 000 mL **b** 5 000 000 cm^3
c 1 544 000 cm^3

16 **a** 102 375 L **b** 48 048 L

17 **a** B, A, C **b** C, A, B

18 small block: 10 500 cm^3, large block: 14 250 cm^3

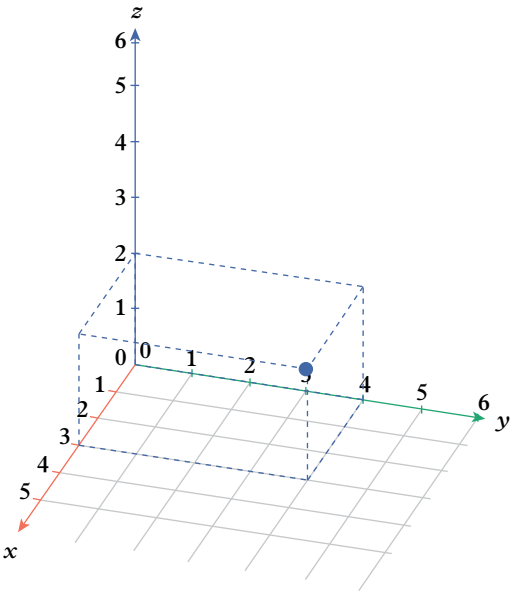
19 **a** 20 cm **b** 240 cm^3 **c** 10.2 cm
d 0.2 cm **e** 19.2 cm **f** 10.8 L
g 2.16 minutes

EX 8F Three-dimensional space

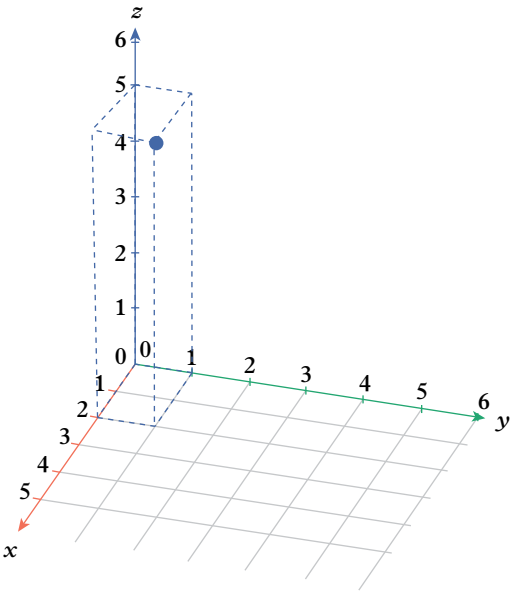
p405

1 **a** (3, 2, 4) **b** (4, 3, 1) **c** (5, 5, 3)
d (4, 0, 3) **e** (0, 2, 1) **f** (2, 3, 0)

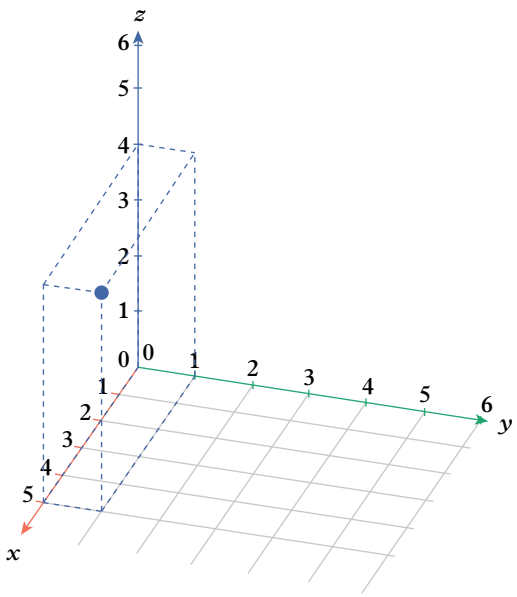
2 **a**



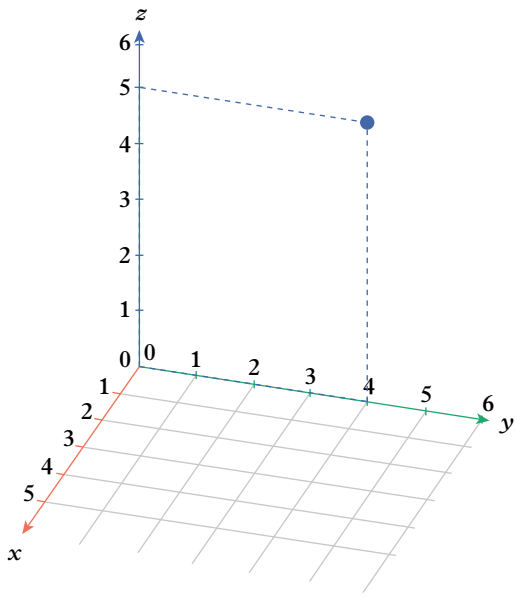
b



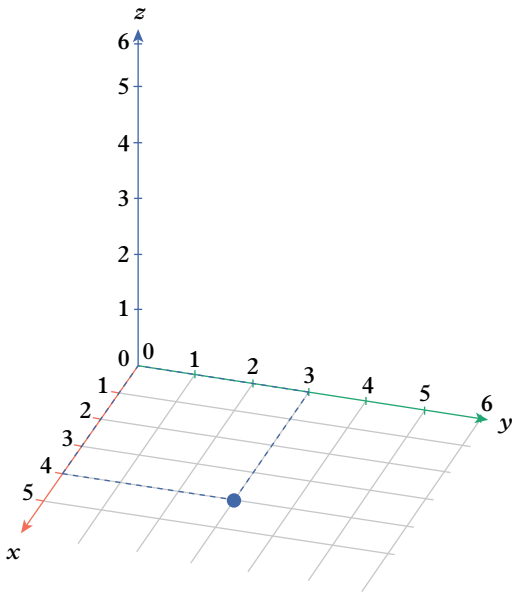
c


z
y
x

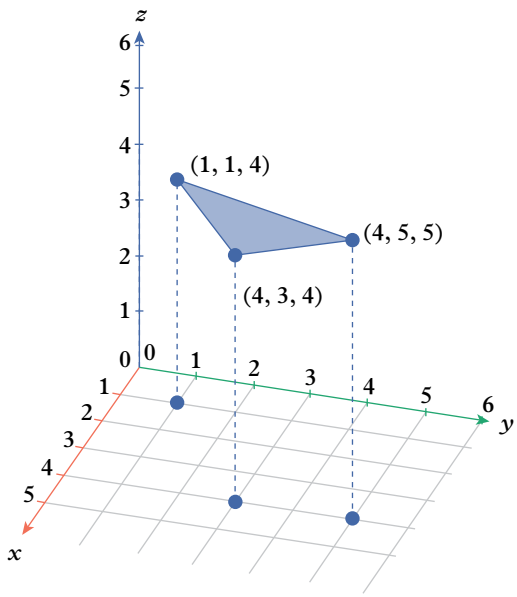
f


z
y
x

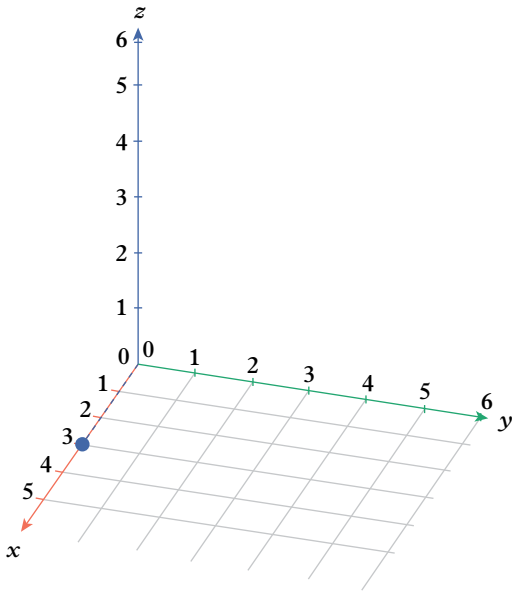
d


z
y
x

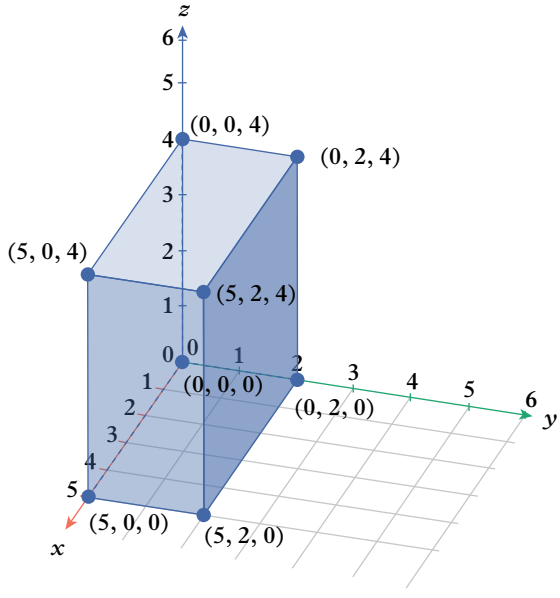
3 a


(1, 1, 4)
(4, 5, 5)
(4, 3, 4)
z
y
x

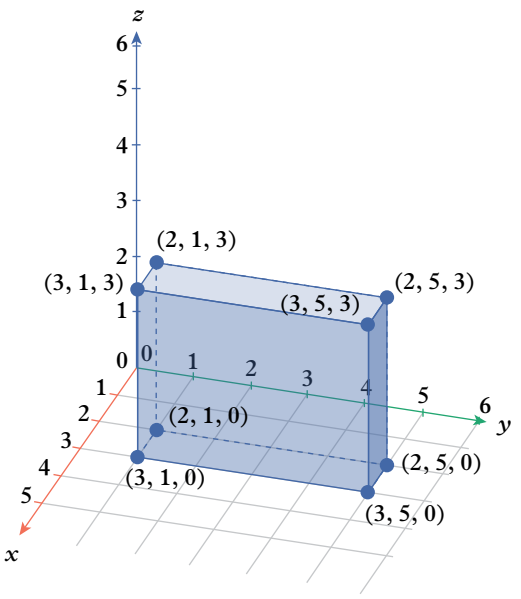
e


z
y
x

b


(0, 0, 4)
(0, 2, 4)
(5, 0, 4)
(5, 2, 4)
(0, 0, 0)
(0, 2, 0)
(5, 0, 0)
(5, 2, 0)
z
y
x

c



4 **a** (6, 5, 3) **b** (2, 7, 9) **c** (4, 4, 8) **d** (4, 4, 8)

5 **a** *z*-axis **b** *x*-axis **c** *y*-axis **d** origin
e *xy*-plane **f** *yz*-plane **g** *xz*-plane

6 **a** **i** 578 **ii** 721 **iii** 285
iv 179 **v** 975 **vi** 043

b **i** fifth floor, first row, two parking spaces along
ii fourth floor, ninth row, one parking space along
iii first floor, fourth row, nine parking spaces along
iv seventh floor, second row, five parking spaces along
v ground floor, second row, eight parking spaces along
vi second floor, ninth row, four parking spaces along

c **i** one floor up, nine rows right, seven spaces backwards
ii five floors down, eight rows left, six spaces forwards

7 **a** (3, 4, 0), (3, 0, 1), (0, 4, 1), (3, 4, 1)
b (4, 2, 0), (4, 0, 3), (0, 2, 3), (4, 2, 3)
c (5, 4, 0), (5, 2, 2), (2, 4, 2), (5, 4, 2)
d (2, 4, 0), (2, 1, 5), (1, 4, 5), (2, 4, 5)
e (0, 1, 2), (0, 3, 2), (4, 1, 3), (4, 3, 3)
f (1, 1, 3), (1, 1, 1), (4, 1, 1), (4, 3, 1)

8 **a** (4, 2, 0), (0, 2, 4), (4, 0, 4), (4, 2, 4)
b (0, 4, 0), (5, 0, 0), (5, 0, 2), (0, 0, 2)
c (5, 3, 0), (5, 2, 1), (2, 3, 1), (5, 3, 1)
d (5, 3, 0), (3, 3, 3), (5, 1, 3), (5, 3, 3)

9 **a** 32 cm **b** 40 cm **c** 3 cm **d** 12 cm

10 **a** 64 cm **b** 76 cm **c** 14 cm **d** 32 cm

11 **a** 5 units **b** 13 units

12 **a** **i** (48.8584° N, 2.2945° E, 35 m)
ii (40.6892° N, 74.0445° W, 7.2 m)
iii (25.3444° S, 131.0369° E, 863 m)
iv (13.1631° S, 72.5450° W, 2430 m)
v (35.3606° N, 138.7274° E, 3776 m)
vi (37.8213° S, 144.9647° E, 300 m)

b **i** 60° north of the equator, 171° east of the Prime Meridian, 709 m above sea level
ii 25° north of the equator, 128° west of the Prime Meridian, 545 m above sea level
iii 5° south of the equator, 47° west of the Prime Meridian, 135 m above sea level
iv 79° south of the equator, 120° east of the Prime Meridian, 642 m above sea level
v 16° north of the equator, 63° west of the Prime Meridian, 97 m above sea level
vi 34° south of the equator, 152° east of the Prime Meridian, 13 m above sea level

13 **a** (45° N, 64° E, 155 m)
b (2° N, 108° W, 28 m)
c (89° N, 136° E, 155 m)
d (79° S, 101° E, 89 m)

EX p411

8G Time

1 **a** 180 minutes **b** 300 minutes **c** 600 minutes
d 2 minutes **e** 10 minutes **f** 100 minutes

2 **a** 600 seconds **b** 300 seconds **c** 3600 seconds
d 2 hours **e** 30 hours **f** 4 hours

3 **a** 0.5 or $\frac{1}{2}$ hour **b** 0.75 or $\frac{3}{4}$ hour
c $\frac{1}{3}$ hour **d** 0.25 or $\frac{1}{4}$ minute
e 0.1 or $\frac{1}{10}$ minute **f** 0.8 or $\frac{4}{5}$ minute

4 **a** 10 800 seconds **b** 36 000 seconds
c 21 600 seconds **d** 5400 seconds
e 4 hours **f** 2.5 hours

5 **a** 1440 minutes **b** 168 hours
c 86 400 seconds **d** 8784 hours
e 604 800 seconds **f** 720 hours

6 **a** 0230 **b** 1615 **c** 1348 **d** 2311

7 **a** 7:25 pm **b** 12:00 pm **c** 10:18 pm **d** 9:58 am

8 **a** 5 hours **b** 15 hours
c 3 hours **d** 5 hours
e 9 hours **f** 5 hours 30 minutes
g 2 hours 18 minutes **h** 5 hours 13 minutes
i 7 hours 54 minutes **j** 19 hours 27 minutes
k 3 hours 3 minutes **l** 7 hours 45 minutes

9 **a** 5:45 pm **b** 1:45 pm
c 11:35 am **d** 9:35 am
e 2315 **f** 0115 (next day)

10 **a** **i** 11:49 am **ii** 1:19 pm **iii** 1:49 pm
b **i** 2:28 am **ii** 2:58 am **iii** 12:58 am
c **i** 1617 **ii** 1417 **iii** 1547
d **i** 0010 **ii** 0040 **iii** 2240

11 **a** $\frac{1}{4}$ hour **b** 0.25 hour
c 1.25 hours = 1 hour 15 minutes. Decimals can be written as fractions out of 100, but minutes are written as fractions of 60 (of an hour).

12 **a** 11 hours **b** $\frac{1}{2}$ hour **c** 10.5 hours
d 9.5 hours, 10.75 hours, 10.5 hours
e 41 hours 15 minutes

13 **a** **i** Didn't change the hour after adding 15 mins to 2:45.
ii 3 hours 40 minutes
b **i** Didn't consider am/pm difference.
ii Duration = 15 h 40 min
c **i** Miscounted from 3:25 pm to 6:25 am.
ii Duration = 15 h 40 min

14 3:05 pm

15 **a** 4012 m **b** 2 hours 30 minutes **c** 0.45 m/s

16 **a** 14 hours after 8 pm AEST is 10 am AEST, not 10 am AWST, which is the time zone the clock is likely set to.
b 16 hours **c** 1280 km **d** 14%

17 **a** D **b** A

18 **a** 20 hours 25 minutes **b** 48 hours 45 minutes
c 65 hours 30 minutes **d** 8762 hours 57 minutes

19 **a** **i** 1 hour **ii** 2.5 hours **iii** 1.5 hours **iv** 3 hours
b **i** 4:30 pm **ii** 6 pm **iii** 6:30 pm **iv** 6:30 pm
c **i** 3:30 pm **ii** 5 pm **iii** 6:30 pm **iv** 5:30 pm
d In winter, Queensland is ahead, but in summer, South Australia is ahead.
e **i** 2:30 pm **ii** 2:30 am **iii** 1:30 pm **iv** 8 am **v** 5:15 pm **vi** 10 am

20 **a** 9 hours **b** 2045
c 4 hours 30 minutes **d** 2330

CHAPTER 8 review

Multiple choice

1	B	**2**	D	**3**	E	**4**	C	**5**	D	**6**	D
7	D	**8**	D	**9**	C	**10**	D	**11**	D	**12**	E
13	C	**14**	A								

Short answer

1 **a** 22.6 cm (or 226 mm) **b** 58 mm **c** 33.6 cm

2 **a** 31.42 cm **b** 50.27 mm

3 **a** 48 cm^2 **b** 58.5 cm^2 **c** 120 cm^2
d 5.4 cm^2 (540 mm^2)

4 **a** 50 000 cm^2 **b** 0.5 cm^2

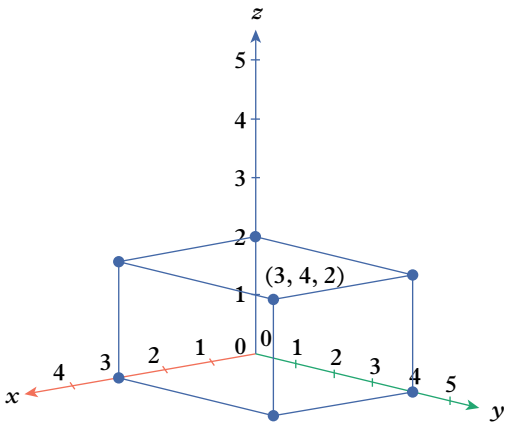
5 **a** 42 cm^2 **b** 88 cm^2 **c** 80 cm^2 **d** 125 cm^2

6 **a** 78.54 cm^2 **b** 201.06 mm^2

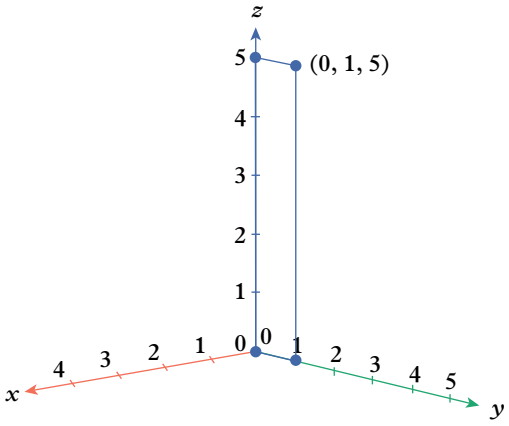
7 **a** 3500 mm^3 **b** 0.0004 m^3 **c** 35 L **d** 124 mL

8 **a** 640 cm^3 **b** 420 cm^3 **c** 144 cm^3

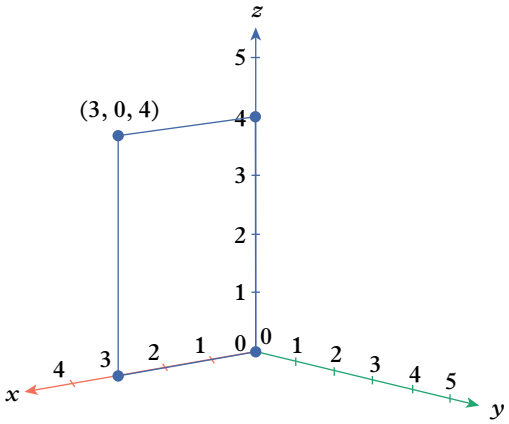
9 **a**



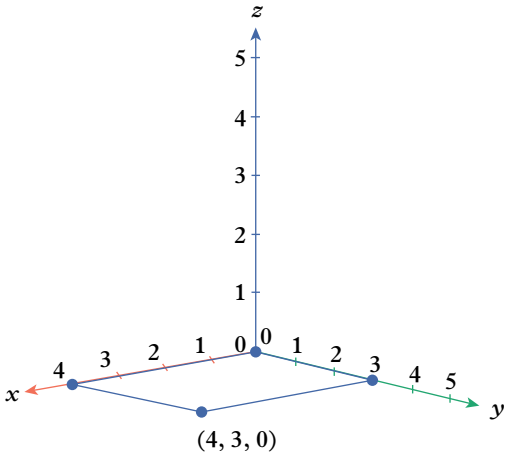
b



c



d



10 **a** 3 hours 45 minutes **b** 8 hours 32 minutes
c 8 hours 34 minutes **d** 6 hours 27 minutes

11 **a** 5:23 am **b** 3:19 pm
c 11:10 am **d** 12:50 pm

Analysis

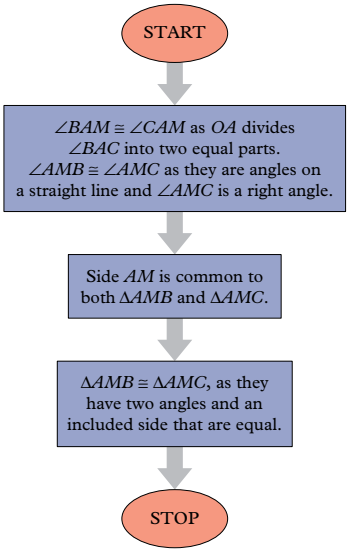
a 6 m^2 **b** 10 m **c** 20 m
d 40 m^2 for two coats **e** 825 cm^2
f 20.73 cm **g** 205.27 cm^2
h 6 m^3 **i** 40 000
j The balls will not take up the entire space as there will be space between them. The theoretical calculation does not take into account this space.

EX p420

Algorithmic Thinking 2 Measurement and space

Task 1

1 Teacher to check.
2 Congruent triangles have the same internal angles and side lengths.
3 Sample answer:



Task 2

1 Teacher to check.
2 Teacher to check.

CHAPTER 9 Statistics and probability

EX p427

9A Collecting data and sampling methods

1 **a** all students in Year 8 in that town
b all people in the country
c everybody in Western Australia
d all people living in the town
2 **a** census **b** sample **c** sample **d** census
3 **a** census **b** sample **c** census **d** sample
4 **a** Biased, as the question does not allow all possible answers
b Fair, as every student has an opportunity to give their opinion
c Biased, as only getting opinions from people who visit casinos, not across the country
d Biased, as only Woolworths customers are targeted
5 **a** systematic **b** stratified **c** cluster
d quota **e** judgemental **f** convenience
g convenience **h** cluster
6 **a** Biased. What is your favourite animal?
b Biased. From this selection of films, which do you recommend?
c Fair
d Biased. What is your favourite hobby?
e Fair
f Biased. Who do you plan to vote for?
7 **a** The people in the town
b People at the local mall may have different views on graffiti from the wider population, or Lizzie may choose to ask a certain type of person.
c Systematic sampling
d It is possible that people walking past the post office may also have different views on graffiti.
e Samples from different income brackets according to the size of each income bracket, choosing random samples from different age groups according to the size of each age group
8 Sample answers:
a What is your favourite soft drink? Which soft drink flavour do you recommend?
b How many pets do you have at home? Give the type and total number of pets at home.
c How many hours do you spend using the internet each week? How much is your weekly download?
d Are you in favour of the new school rule? What are your thoughts about the new school rule?
9 Sample answers:
a Each student is numbered from 1 to the end. Depending on the numbers required for the sample, every 20th (for example) student is selected from the list.
b A sample is selected with a fair representation of girls and boys across each year level, and in proportion to the numbers of students at each year level.
c A sample of 20 girls and 20 boys is taken from each year level (for example).
d One year level is chosen at random and all students in that year are surveyed (for example).
10 **a** **i** Year 8 students at your school
ii Sample answer: select every fifth student from the Year 8 list of students.
b **i** the people in the town
ii Sample answer: select every 10th (or agreed number) household from town records.
c **i** the Australian population

ii Sample answer: select a sample of families across Australia in proportion to the population in each state (targeting families in proportion to those living in the country as well as in cities).

11 a Cluster sampling is likely to be much cheaper than surveying the entire population (for example).

b The composition and diversity of local government areas may vary significantly depending on whether they are rural, urban, suburban or inner city (for example).

12 a Convenience sampling

b Biased. The most popular posts are likely to have more extreme views (strongly like and strongly dislike) than posts that are more neutral (like, dislike, don't care). The most popular posts are also more likely to be from accounts with a large number of followers rather than a random viewer of the announcement.

c Judgemental

d Biased. They paid for one-quarter of the reviews they checked, which are more likely to be positive. The reviewers that were sent review copies may be less biased, but also less critical so they receive review copies of other games. These 15 reviews make up three-quarters of the reviews the developers checked and therefore they are likely to be skewed towards being more positive.

13 a Deliberately selecting people or objects allows you to choose people or objects that fit the criteria you need for your sample, which may not be possible when only randomly selecting people or objects.

b Deliberately selecting people or objects may bias your selection if you choose people or objects that are a better fit for your desired result than a randomly selected sample of the population would be.

14 Those with a certain opinion of the local issue may be more likely to complete the survey.

15 Marci's sample has percentages that are close to the population percentages (24%, 15%, 31%, 13%, 17%) but are not exactly the same, so her sample is approximately stratified.

16 a 67% **b** systematic

c 64% **d** 4.7% error

17 a There may not be enough people to find all side effects of the vaccine (for example).

b There may be negative side effects that affect many of the participants in the trial, which could have been avoided in a smaller trial (for example).

EX p432 9B Summary statistics

1 a 7 **b** 7 **c** 5 **d** 7

2 a mean = 14.4, median = 12, mode = 12, range = 8

b mean ≈ 4.8, median = 4, mode = 2 and 4, range = 8

c mean ≈ 5.9, median = 7, mode = 7, range = 9

3 a 155.83 cm **b** 155.5 cm

c 155 cm and 161 cm **d** 22 cm

4 a 159.33 cm **b** 155.5 cm

c 152 cm **d** 51 cm

5 a **i** 2.17 **ii** 2 **iii** 2 **iv** 3

b **i** 3.17 **ii** 3 **iii** 2 **iv** 3

c **i** 23 **ii** 20 **iii** 20 **iv** 30

d **i** 2.84 **ii** 3 **iii** 2 **iv** 4

e **i** 14.6 **ii** 14 **iii** 14 **iv** 4

f **i** 17.2 **ii** 20 **iii** 20 **iv** 40

6 a

Number of quizzes	Frequency (f)
0	5
1	6
2	5
3	4
4	0
5	2
6	1
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	0
17	0
18	1
Total	24

b **i** 2.6 **ii** 2 **iii** 1

c 18 **d** 18

7 mean ≈ 13.7, median = 14, mode = 14, range = 6

8 mean = –0.11, median = –0.1, mode = –0.2 and –0.1, range = 0.4

9 a

Weight (kg)	Frequency
2	4
3	3
4	4
5	5
6	2
7	1
8	1
9	1
10	2
11	1

b mean = 5.2, median = 5, mode = 5, range = 9

c Sample answer: The middle school bag in weight was 5 kg. The school bags varied in weight by 9 kg.

10 Sample answer: There was a difference of 82 between the student who sent the fewest text messages and the

student who sent the most text messages. The average number of text messages sent was 34.

11 a 202

b i 131.8 **ii** 124

c i 98 **ii** 38

d The outlier increased both the mean and the range, but it had a much larger effect on the range than the mean.

12 a A large outlier value will increase the mean and a low outlier value will decrease the mean. Both Monica and Mario's classes have the same median, but the outlier in Mario's class has increased the mean compared to Monica's class.

b The median, as it is the middle value and is not affected by the presence of outliers

c 198 cm

d Mean ≈ 155.82. There is a 3.51 cm difference in the mean when the outlier is disregarded. The result is much closer to the median value.

e 21 cm. There is a 30 cm difference in the range when the outlier is disregarded.

13 a 0.3 kg or 300 g

b 1.2 kg or 1200 g

14 a i They did not order the data first. The median is only in the middle of the list if the data is in ascending or descending order.

ii difference = 4

b i They did not order the data first. The maximum and minimum are only at the ends of the list if the data is in ascending or descending order.

ii difference = 10

c i They did not take the frequencies into account in the numerator or denominator in the calculation of the mean.

ii difference = –0.1875

d i They looked at all numbers in the table rather than only in the score column. The range is the difference between the maximum and minimum score, not any value in the table.

ii difference = –48

e i They did not take the frequencies into account to determine the middle number, they only looked at the scores.

ii difference = 2

15 a

x	Mean (correct to two decimal places)	Median	Mode(s)	Range
13	14.20	14	13	9
8	13.87	14	13, 15	11
1	13.40	14	13, 15	18
–3	13.13	14	13, 15	22
15	14.33	15	15	9
25	15.00	15	13, 15	15

b The mean and the range changed the most. The median and the mode changed the least.

c 15 and 25. For x = 13, 8, 1, and −3 (any number 14 or less), 14 was in the 8th position and so was the median, whereas if x is 15 or more, there is one less number below 14, which now takes the 7th position, and the 8th position is now taken by 15, so 15 becomes the median. This is the case for x = 15 and x = 25. Changing x to be a number greater than 14 increases the median.

d 8, 1, −3 and 25. The values of 13 and 15 are within the range of the data from 10 to 19. All other values are outside of the range and so change the minimum or maximum value, which affects the value of the range.

e 13 and 15. When x = 13, there are four 13s, which is now the most frequent number by itself. The same thing occurs when x = 15. If x = 12 or 16, then 12 or 16 would have equal frequency with 13 and 15 and become a third mode. For any other value of x, the highest frequency numbers do not change.

f x = 12 and x = 16

g i As the value of x decreases, the value of the mean decreases.

ii As the value of x increases, the value of the mean increases.

h The mean and range will increase, the median will be 15 and the mode will be 13 and 15. Mean = 20. Range = 90.

16 a Mean = 3, median = 3, range = 4 for laptops A and B

b Laptop A has nearly half of its ratings as 5 stars. However, it also has nearly half of its ratings as 1 star. It is a love-it or hate-it laptop, so Jacinta will likely either really like Laptop A or really dislike it.

c Laptop B has most of its ratings as 3 stars and only one 1 star and 5 stars rating. So, most people find the laptop rather average, not amazing, but also not terrible. Jacinta will have a better idea how she will find the laptop.

17 a 5, 7, 7, 9 **b** mean = 8, range = 9

18 x = 25 and y = 70 or x = 70 and y = 25

EX p438 9C Analysing data

1 a mean **b** range **c** mode

d range **e** mode **f** median

2 a sample of 100 people, as larger is more reliable

b sample of 619 people, as larger is more reliable

c sample of 20 randomly selected people, as this sample is more likely to be representative of the population

d 50 people in South Australia, as they are all in the population

3 a The sample is probably taken from the population; however, it is not a random sample so it should not be used to predict the population parameters.

b The sample is limited to Canberra instead of the whole of Australia, so it should not be used to predict the population parameters.

c The sample of 30 is too small to allow you to accurately predict the population parameters of Victoria.

d The high range and higher mean in comparison with median and mode indicate the presence of an outlier. Ignoring the outlier, the results may predict the population parameters.

e The sampling technique seems fair and the sample size is not too small, so these results could be used to predict population parameters.

4 a mean = 4.27, median = 4, mode = 4

b Population: every Australian household; sample: students Laani knows at her school.

c 100

d The people Laani knows are all part of a family. This means that she is not including the households in Australia with no school age children.

5 a Her sample is not restricted to her school or people she knows, and people are selected in a random process.

b The sample size (20) is very small. One outlier would affect the mean.

c The larger the sample, the more reliable the result.

d All of the summary statistics should be more reliable as the sample size increases.

6 a

Sample: 20 people	
Category	**%**
A	25
B	10
C	35
D	20
E	10

Sample: 50 people	
Category	**%**
A	28
B	8
C	38
D	14
E	12

Sample: 200 people	
Category	**%**
A	26
B	11.5
C	42
D	12
E	8.5

b Category C. It is the most common category in every sample.

c Category B or E. They are the least common categories in every sample.

7 a Spring: 25.5%, Summer: 39%, Autumn: 21.5%, Winter: 14%

b Spring: 24.75%, Summer: 33.5%, Autumn: 23.5%, Winter: 18.25%

c There is a natural variation when samples are taken from the same population.

d Sample answer: Summer appears to be Australia's favourite season, with winter the least popular.

8 a mean: 3.6 mobiles, median: 4 mobiles, mode: 4 mobiles

b Consider how the survey was conducted, who was in the sample and how large the sample is compared to the population.

c Either measure would be satisfactory for a fair survey. However, care needs to be taken in the event of outliers, which may make the median a more reliable measure.

9 a 3 Year 8s

b 23%

c The samples are from different subgroups of people within the school (students from different year levels and teachers) who may have different opinions based on their age.

d No, as the samples have widely varied percentages between categories and each sample considers different subgroups

10

	Australia	China	UK	USA	Other
a	1205	106	255	327	107
b	60.25%	5.3%	12.75%	16.35%	5.35%
c	723	64	153	196	64

d No, as the proportions of attendees on the Sunday night differ greatly from the average over the four shows. They should use only the Sunday night data to make predictions.

e They could collect more data from future shows on the same four consecutive nights as the proportions may vary. The larger the data set, the more accurate the predictions should be.

11 a Suzie only sampled 100 Year 8s and they were all from her school. Her school may not be representative of all Year 8s in Australia.

b The average height of all Year 8s in Suzie's school is 1.52 m and 85% of Year 8s in Suzie's school are right-handed.

Checkpoint

1 a sample **b** census

c sample

2 a no bias **b** bias

c bias

3 a systematic **b** stratified

c convenience

4 a **i** 14.7 **ii** 15.5

iii 20 **iv** 14

b **i** 2.67 **ii** 3.05

iii 0.3 and 3.3 **iv** 4.7

5 The median. The mean is much higher than the median, so it was likely increased by the presence of an outlier, which has little to no effect on the median.

6 a Yes. The sample is randomly chosen and is of a large size (10%).

b No. The sample size is too small (1%).

c No. The sample is opt-in and will only favour a particular subset of students (those who read the newsletter and want to fill in the survey).

d No. The sample favours a particular subset of students (those who had detention).

7 **a** 53%

b 50%

c The percentage of people who are planning to go to Woolworths will vary from sample to sample by random chance, even when taken from the same population.

EX 9D Theoretical probability

p444

1 **a** {5, 6, 7, 8, 9, 10} **b** {1, 2, 3, 4, 5}

c {3, 5, 7} **d** {1, 9}

e {1, 2, 3, 5, 7, 9} **f** {1, 2, 9}

2 **a** 5 **b** $\frac{1}{5}$ **c** $\frac{2}{5}$

3 **a** $\frac{1}{6}$ **b** $\frac{1}{5}$ **c** $\frac{2}{10} = \frac{1}{5}$ **d** $\frac{1}{5}$

4 **a** 5 **b** $\frac{1}{5}$ **c** $\frac{2}{5}$ **d** $\frac{4}{5}$

5 **a** $\frac{4}{5}$ **b** $\frac{9}{15} = \frac{3}{5}$ **c** $\frac{2}{6} = \frac{1}{3}$

d $\frac{8}{34} = \frac{4}{17}$ **e** $\frac{5}{6}$

6 **a** 0.6 **b** $\frac{1}{4}$ **c** 0.9 **d** $\frac{1}{2}$

e $\frac{7}{8}$ **f** 0.3 **g** $\frac{1}{3}$ **h** 0.75

7 **a** Getting at most 4 or getting less than 5 {1, 2, 3, 4}

b Getting at least 6 or getting less than 5 {6, 7, 8, 9, 10}

c Getting an odd but not a prime, a prime but not an odd, or neither an odd nor a prime {1, 2, 4, 6, 8, 9, 10}

d Getting an odd and a prime, a prime and not an odd, or neither an odd nor a prime; alternatively, getting either an even number or a prime number {2, 3, 4, 5, 6, 7, 8, 10}

e Getting neither an odd nor a prime {4, 6, 8, 10}

f Getting an odd and a prime or neither an odd nor a prime {3, 4, 5, 6, 7, 8, 10}

8 **a** {1, 2, 3, 4, 5, 6} **b** $\frac{1}{3}$

c probability of rolling less than 5

d $\frac{2}{3}$

9 **a** $\frac{6}{10}$ or $\frac{3}{5}$ **b** selecting a vowel **c** $\frac{4}{10}$ or $\frac{2}{5}$

10 **a** 15 **b** $\frac{5}{15}$ or $\frac{1}{3}$

c selecting a non-blue button; $\frac{10}{15}$ or $\frac{2}{3}$

d 0 **e** $\frac{7}{15}$

f not selecting a button with four holes; $\frac{8}{15}$

11 **a** $\frac{29}{34}$ **b** $\frac{19}{34}$ **c** $\frac{10}{17}$ **d** $\frac{12}{17}$

e $\frac{5}{34}$, 0 eggs

f At most four and no less than four are not complementary as the sum of their probabilities is not equal to one (they both include four). More than four eggs (or at least five eggs) is complementary to at most four eggs. Less than four eggs is complementary to at least four eggs.

12 The sample space usually lists every different outcome rather than every possible outcome; these must be equally likely or an incorrect answer will be obtained.

13 **a** 52 **b** $\frac{1}{13}$ **c** $\frac{12}{13}$ **d** $\frac{3}{13}$

e ace; there are four aces to choose from whereas there is only one queen of hearts.

f $\frac{9}{52}$ **g** $\frac{43}{52}$ **h** $\frac{5}{13}$

14 **a** 30 **b** $\frac{7}{30}$ **c** $\frac{23}{30}$ **d** $\frac{4}{30}$ or $\frac{2}{15}$

e selecting a Y, E, R, I, G, H, T, S, H, N or V

f 12 **g** $\frac{12}{30}$ or $\frac{2}{5}$

h selecting a consonant; $\frac{18}{30}$ or $\frac{3}{5}$

15 **a** $\frac{2}{6}$ or $\frac{1}{3}$

b The green sector is twice the size of other sectors, so the five colours are not all equally likely. It is like having six outcomes but two of these are green.

c $\frac{1}{6}$ **d** not spinning yellow; $\frac{5}{6}$

16 **a** 0.18 **b** 0.37 **c** 0.07

17 $\frac{1}{8}$

18 **a** $\frac{19}{200}$ **b** $\frac{85}{199}$

19 He forgot the two outcomes of one head, one tail.

20 **a** $\frac{8}{55}$ **b** $\frac{22}{55}$ or $\frac{2}{5}$

c **i** 54 **ii** 7 **iii** $\frac{7}{54}$

d **i** 53 **ii** 12 **iii** $\frac{21}{53}$

21 blue: $\frac{1}{6}$, green: $\frac{17}{72}$, purple: $\frac{13}{72}$, orange: $\frac{1}{18}$, yellow: $\frac{13}{36}$

22 **a** $\frac{2}{5}$ **b** $\frac{3}{20}$ **c** 20

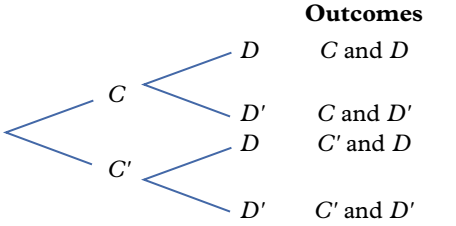
EX 9E Two-way tables

p450

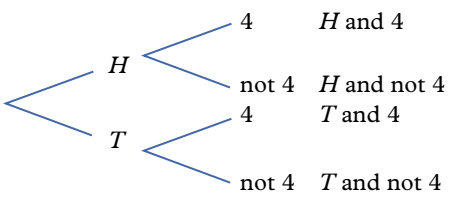
1 **a** Flipping two coins, {HH, HT, TH, TT}

b For example, flipping a coin and rolling a 5 (using a 6-sided die), {H and 5, H and not 5, T and 5, T and not 5}

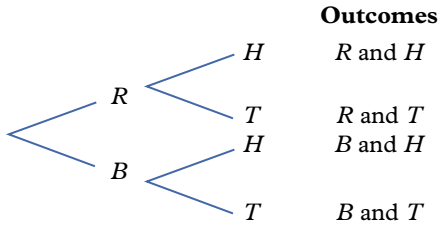
2 **a**



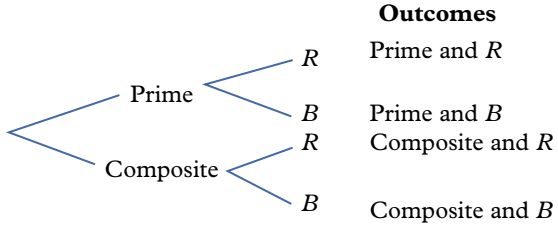
b



c



d



3 **a** Year 7 students who play sport, Year 8 students who play sport, Year 7 students who don't play sport, Year 8 students who don't play sport

b 50 **c** 17 **d** 23

e Year 7 students who don't play sport

f 22

4 **a, b**

	Year 7	Year 8	Total
Action	9	10	19
Comedy	23	8	31
Total	32	18	50

5 **a, b**

	Cats	Dogs	Total
Burbank	11	48	59
Pinjarra Hills	19	22	41
Total	30	70	100

6 **a**

	Weekday	Weekend	Total
Volleyball	14	17	31
Tennis	5	31	36
Total	19	48	67

b

	Year 8	Year 9	Total
CD	11	15	26
Digital	18	19	37
Total	29	34	63

7

	Lollies	Not lollies	Total
Red	5	3	8
Not red	4	2	6
Total	9	5	14

8 **a** $\frac{43}{100}$ **b** $\frac{30}{100}$ or $\frac{3}{10}$ **c** $\frac{90}{100}$ or $\frac{9}{10}$

9 **a** 40 **b** 6 **c** 19

d **i** $\frac{6}{40}$ or $\frac{3}{20}$ **ii** $\frac{19}{40}$

iii $\frac{24}{40}$ or $\frac{3}{5}$ **iv** $\frac{25}{40}$ or $\frac{5}{8}$

10 **a** $\frac{17}{50}$ **b** $\frac{12}{50}$ or $\frac{6}{25}$ **c** $\frac{23}{50}$

d $\frac{28}{50}$ or $\frac{14}{25}$ **e** $\frac{29}{50}$ **f** $\frac{39}{50}$

11 **a**

	White bread	Non-white bread	Total
Vegemite	40	24	64
Jam	20	16	36
Total	60	40	100

b probability(jam on non-white toast) $\frac{16}{100}$ or $\frac{4}{25}$

12 **a**

	Year 7	Year 8	Total
M&Ms	16	16	32
Smarties	10	8	18
Total	26	24	50

b **i** $\frac{18}{50}$ or $\frac{9}{25}$ **ii** $\frac{26}{50}$ or $\frac{13}{25}$ **iii** $\frac{16}{50}$ or $\frac{8}{25}$

c $\frac{16}{24}$ or $\frac{2}{3}$

d Part **c** asked for students who preferred M&Ms from the Year 8 class only, whereas part **b** asked who is in Year 8 and prefers M&Ms as a proportion of the whole group.

13 **a** **i** $\frac{42}{100}$ or $\frac{21}{50}$ **ii** $\frac{3}{100}$

iii $\frac{58}{100}$ or $\frac{29}{50}$ **iv** $\frac{10}{100}$ or $\frac{1}{10}$

b The northern part has a preference for rugby and the southern part has a preference for AFL.

14 **a**

	0	1	2	3+	Total
Pets	18	53	37	12	120
No pets	6	12	11	51	80
Total	24	65	48	63	200

b **i** $\frac{51}{100}$ **ii** $\frac{29}{200}$ **iii** $\frac{159}{200}$

15 **a**

	Blond hair	Brown hair	Total
Blue	16	30	46
Brown	23	31	54
Total	39	61	100

b **i** $\frac{39}{100}$ **ii** $\frac{46}{100}$ or $\frac{23}{50}$ **iii** $\frac{16}{100}$ or $\frac{4}{25}$

c **i** 39 **ii** 16 **iii** $\frac{16}{39}$

iv Part **b iii** asks for the proportion of the entire group who have blond hair and blue eyes, whereas part **c iii** asks for the proportion of blonds who have blue eyes.

d **i** 61 **ii** 30 **iii** $\frac{30}{61}$

16 **a** **i** $\frac{30}{50}$ or $\frac{3}{5}$ **ii** $\frac{12}{50}$ or $\frac{6}{25}$

b **i** $\frac{28}{31}$ **ii** $\frac{12}{42}$ or $\frac{2}{7}$

17

	A	Not A	Total
B	5	11	16
Not B	4	10	14
Total	9	21	30

18 8. The numbers, by themselves, should not fill a row or column.

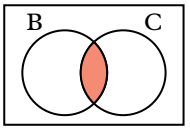
EX 9F Venn diagrams

p457

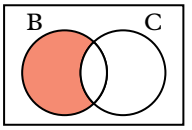
1 **a** people with brothers
b people with sisters but not brothers
c people without sisters or brothers
d people with sisters or brothers or both
e people with sisters or brothers but not both
f people without sisters

2 **a** 18 **b** 23 **c** 5 **d** 2

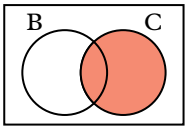
3 **a**



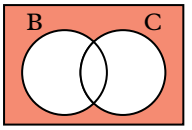
b



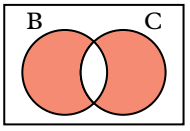
c



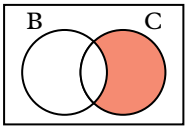
d



e



f



4 **a** 10 **b** 21 **c** 3 **d** 42

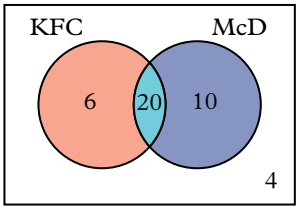
5 **a** **i** 31 **ii** 10 **iii** 38
iv 2 **v** 48 **vi** 9
b 50

6 **a** $\frac{6}{50}$ or $\frac{3}{25}$ **b** $\frac{16}{50}$ or $\frac{8}{25}$ **c** $\frac{16}{50}$ or $\frac{8}{25}$ **d** $\frac{32}{50}$ or $\frac{16}{25}$
e $\frac{18}{50}$ or $\frac{9}{25}$

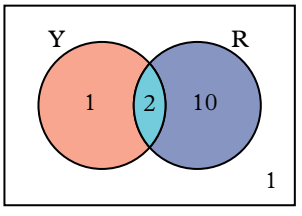
7 **a** $\frac{10}{42}$ or $\frac{5}{21}$ **b** $\frac{18}{42}$ or $\frac{3}{7}$ **c** $\frac{21}{42}$ or $\frac{1}{2}$ **d** $\frac{3}{42}$ or $\frac{1}{14}$
e $\frac{29}{42}$

8 **a** $\frac{7}{50}$ **b** $\frac{41}{50}$ **c** $\frac{31}{50}$ **d** $\frac{48}{50}$ or $\frac{24}{25}$

9



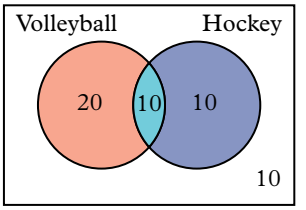
10



11 **a** $\frac{7}{20}$ **b** $\frac{7}{20}$ **c** $\frac{2}{5}$ **d** $\frac{2}{5}$ **e** 0
f $\frac{3}{4}$ **g** $\frac{3}{4}$ **h** $\frac{1}{4}$ **i** $\frac{1}{4}$

12 **a** **i** 25 **ii** 50 **iii** 95 **iv** 5
b **i** 57 **ii** 3
c **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$

13 **a** The total of the numbers in the Venn diagram is more than 50 so it cannot represent the votes of 50 people.

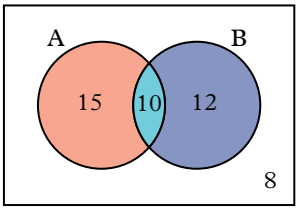


b If the 30 people who wanted to play volleyball does not include the 10 people who want to play both then it can be correct as the total of the numbers in the Venn diagram is 60. If the 30 people includes the people who want to play both then it is not correct and there would be 20 people who want to play neither volleyball nor hockey.

14 **a**

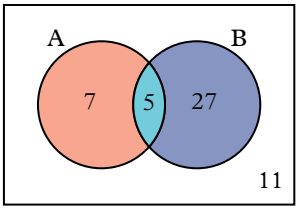
	A	Not A	Total
B	4	9	13
Not B	6	7	13
Total	10	16	26

b



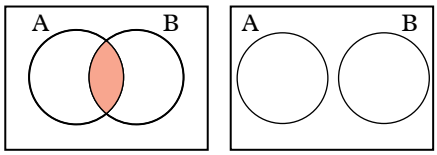
c

	A	Not A	Total
B	5	27	32
Not B	7	11	18
Total	12	38	50

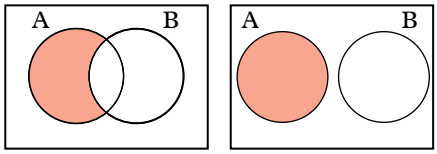


d **a** **i** $\frac{2}{13}$ **ii** $\frac{3}{13}$ **iii** $\frac{19}{26}$
iv $\frac{15}{26}$ **v** $\frac{1}{2}$
b **i** $\frac{2}{9}$ **ii** $\frac{1}{3}$ **iii** $\frac{37}{45}$
iv $\frac{3}{5}$ **v** $\frac{23}{45}$
c **i** $\frac{1}{10}$ **ii** $\frac{7}{50}$ **iii** $\frac{39}{50}$
iv $\frac{17}{25}$ **v** $\frac{9}{25}$

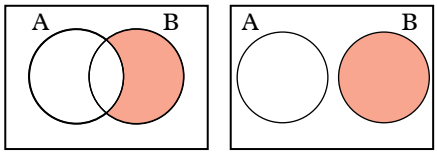
15 a i



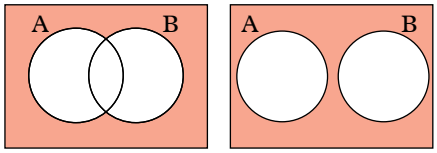
ii



iii



iv



b When all of the shaded regions of each Venn diagram are put together, they cover the entire Venn diagram.

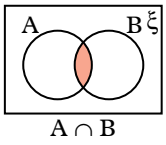
c Pr(A and B) + Pr(A and not B) + Pr(Not A and B) + Pr(Not A and Not B) = 1

d Since Pr(A and B) = 0, the rule simplifies to Pr(A and not B) + Pr(Not A and B) + Pr(Not A and Not B) = 1

16 a $\frac{17}{35}$ **b** $\frac{13}{35}$ **c** $\frac{4}{35}$ **d** $\frac{4}{15}$

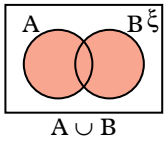
e $\frac{4}{17}$ **f** $\frac{24}{35}$ **g** $\frac{20}{35}$ or $\frac{4}{7}$

17 a



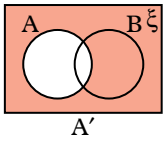
$A \cap B$

b



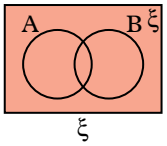
$A \cup B$

c



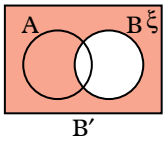
A'

d



ξ

e



B'

18 a C **b** E **c** B **d** D

CHAPTER 9 review

Multiple choice

1 D **2** B **3** B **4** D **5** D

6 E **7** B **8** D **9** E

Short answer

1 a i sample

ii Biased; increase the sample size and sample randomly from the state's population.

b i sample **ii** fair

c i census

ii Biased; Gerard should ask people which snack they like best.

2 a What is your favourite AFL team?

b How do you travel to school?

c What is your preferred ice cream flavour in a Neapolitan tub containing strawberry, vanilla and chocolate?

3 a mean = 3, median = 3, mode = 1 and 4, range = 5

b mean = 12.9, median = 12, mode = 11, range = 19

c mean ≈ 6.67, median = 2, mode = 2, range = 27

4 mean = 13.7, median = 14, mode = 14, range = 6

5 Mean = 13.47, median = 14, mode = 14, range = 3. The mean and range change, and the median and mode stay the same.

6 a The sample does not include teachers who do not teach Year 8.

b The sample will almost certainly include people who do not know the teachers in school.

c The sample is too small and they may not all have experience with all the teachers in school.

7 The sampling technique appears fair, although 50 is too small a sample for the population and there appears to be at least one outlier from looking at the value of the range compared to the mean and the median. The results could not be reliably used to predict the population parameters.

8 a $\frac{2}{9}$ **b** $\frac{4}{9}$ **c** $\frac{5}{9}$

9 a $\frac{2}{5}$ **b** $\frac{7}{10}$ **c** $\frac{3}{5}$

10 a $\frac{115}{200}$ or $\frac{23}{40}$ **b** $\frac{18}{200}$ or $\frac{9}{100}$

c $\frac{56}{200}$ or $\frac{7}{25}$ **d** $\frac{47}{200}$ **e** $\frac{74}{200}$ or $\frac{37}{100}$

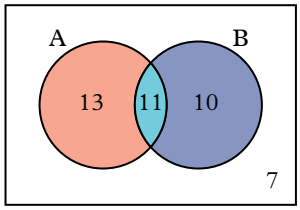
11 a 35 **b** 18

c i $\frac{10}{35}$ or $\frac{2}{7}$ **ii** $\frac{8}{35}$

iii $\frac{13}{35}$ **iv** $\frac{30}{35}$ or $\frac{6}{7}$

12

	B	B'	Total
A	11	13	24
A'	10	7	17
Total	21	20	41



Analysis

1 **a** systematic, sample

b mean ≈ 26.7, median = 21.5, mode = 19, range = 67

c Details such as type of beach, weather on the day, time of year or if this beach is popular for holiday makers are not known.

d The results indicate a person chosen randomly at the beach is most likely to be between about 13 and 27.

2 **a** $\frac{1}{2}$

b

	Even	Not even	Total
Prime	6	30	36
Not prime	34	10	44
Total	40	40	80

c $\frac{1}{8}$

d $\frac{3}{40}$

e The not prime counters

f No. You pay 30 tickets total to win 30 tickets. That is no different to not playing the game.

EX p468 Algorithmic Thinking 3 Statistics and probability

Task 1

1 Example 1: ClassList

Example 2: Classlist and k

2 Arj,Bel,Cat

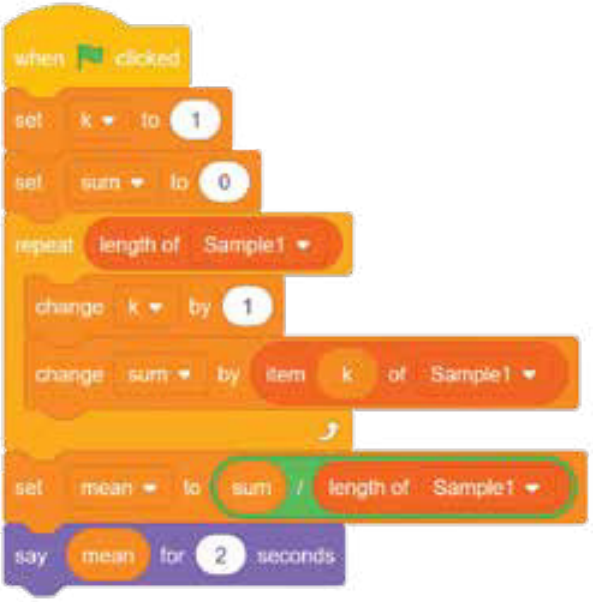
Task 2

1 Sample answer:



2 Teacher to check.

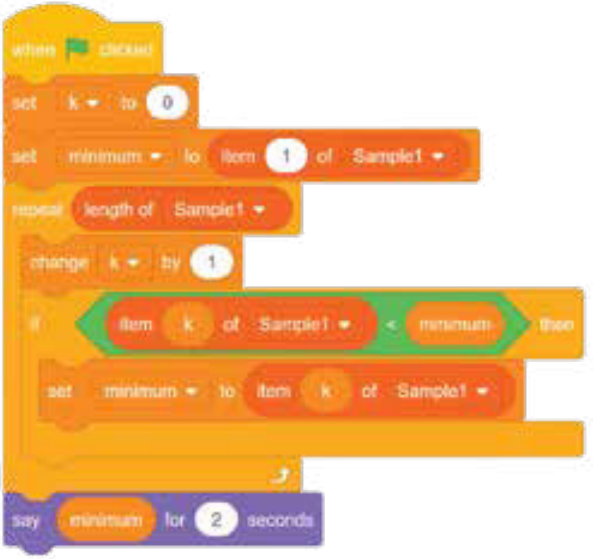
3 Sample answer in Scratch:



Task 3

1 Teacher to check.

2 Sample answer in Scratch (to find the minimum):



SEMESTER 2 Review

Short Answer

1 **a** solution **b** solution

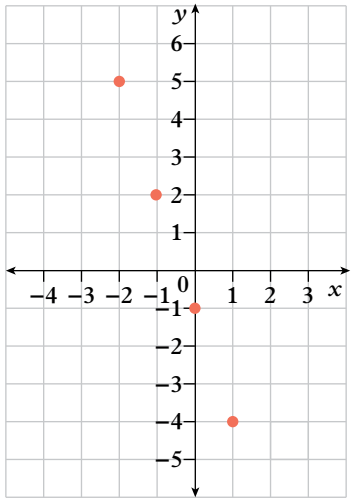
c not the solution **d** solution

2 **a** $x = 10$ **b** $x = 3$

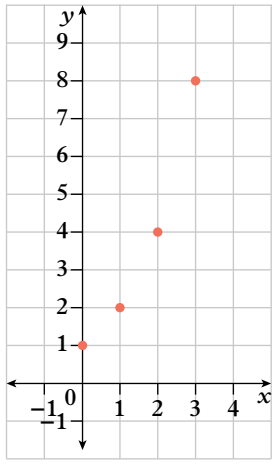
c $x = -11$

d $x = \frac{9}{5} = 1\frac{4}{5}$

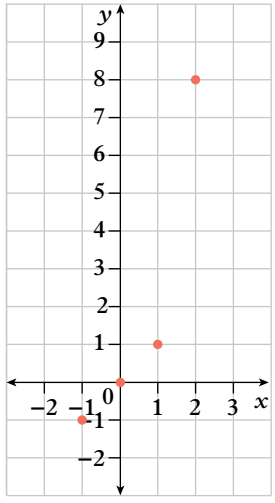
3 a linear



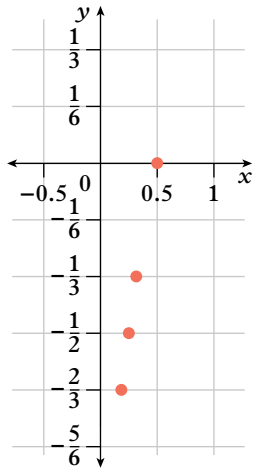
b non-linear



c non-linear



d linear



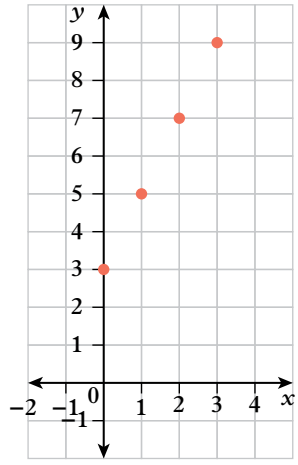
4 a i $x = 3$ **ii** $x = 0$

b i $x = -3$ **ii** $x = 6$

5 a i $x = 6$

x	0	1	2	3
y	3	5	7	9

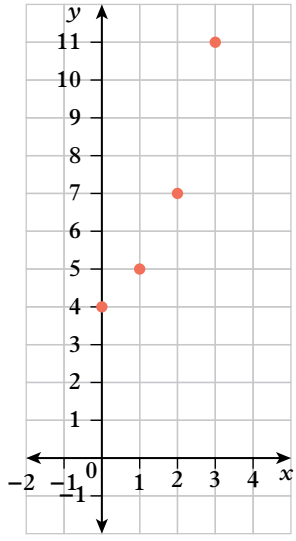
ii linear



b

x	0	1	2	3
y	4	5	7	11

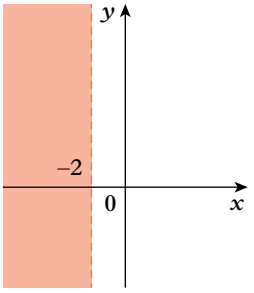
ii non-linear



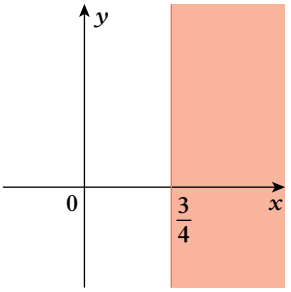
6 **a** $y = -2x + 7$ **b** $y = 4x + 3$

c $y = -x - 2$ **d** $y = \frac{1}{2}x + 6$

7 **a** $x < -2$



b $x \geq \frac{3}{4}$



8 **a** equal **b** complementary

c equal **d** supplementary

e supplementary **f** equal

g supplementary **h** equal

9 **a** **i** square **ii** $A = 25$ cm^2

b **i** kite **ii** $A = 30$ cm^2

c **i** parallelogram **ii** $A = 16.0026$ cm^2

d **i** trapezium **ii** $A = 24.0039$ cm^2

e **i** rectangle **ii** $A = 19.9712$ cm^2

f **i** rhombus **ii** $A = 4.8576$ cm^2

10 **a** $A = 13.85$ m^2, $P = 13.19$ m

b $A = 38.48$ m^2, $P = 21.99$ m

c $A = 15.69$ m^2, $P = 16.25$ m

d $A = 3.94$ m^2, $P = 8.00$ m

11 **a** $3x + 5 = 29$
$x = 8$

b $2 \times 3.4 + 2w = 10.1$
$w = 1.65$ m

c $\pi D = 30$
$D = 9.55$ cm

d $\frac{1}{2} \times 1.5 \times h = 2.25$
$h = 3$ cm

e $7.2h = 24.48$
$h = 3.4$ cm

f $\pi r^2 = 30$
$r = 3.09$ cm

g $34 + 57 + x = 180°$
$x = 89°$

h $\frac{5 + 8 + 13 + 2 + x}{5} = 6.2$
$x = 3$

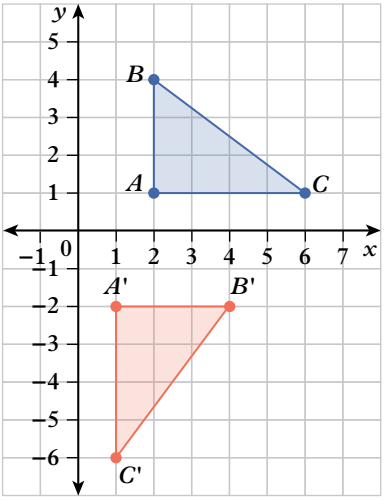
i $0.3 + 0.12 + 0.44 + x = 1$
$x = 0.14$

12 **a** (1, 2, 3), (4, 4, 4), (5, 3, 5)

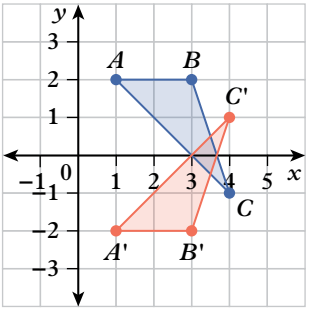
b (0, 0, 0), (5, 0, 0), (5, 3, 0), (0, 3, 0), (0, 3, 5), (5, 3, 5), (5, 0, 5), (0, 0, 5)

13 **a** not congruent **b** congruent

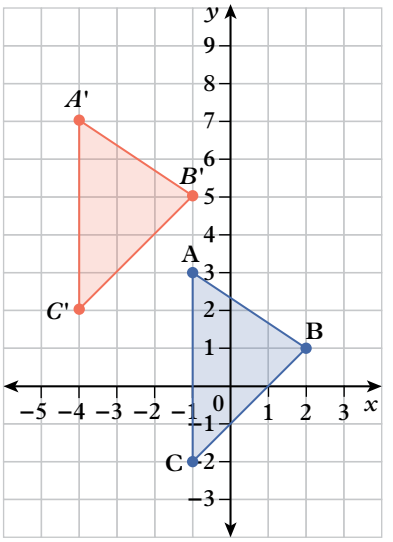
14 **a**



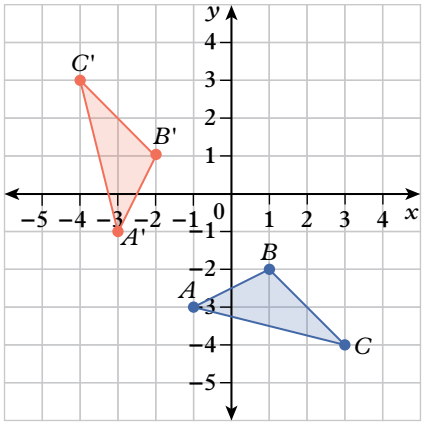
b



c



d

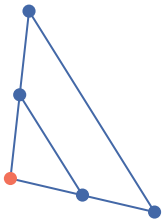


15 **a** yes, SSS **b** yes, SAS **c** no
d yes, AAA **e** no **f** yes, SAS

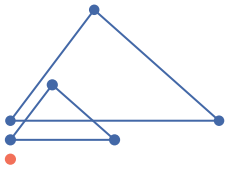
16 **a** $z = 13.60$ m **b** $r = 60.40$ m

17 **a** no **b** no **c** yes **d** yes

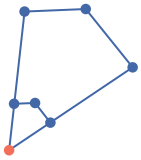
18 **a**



b



c



19 **a** $\frac{3}{9} = \frac{1}{3}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$ **d** $\frac{3}{5}$

20 **a** AAS or ASA, $a = 3$
b RHS, $b = 27°$
c SAS, $c = 6$
d SSS, $d = 31°$

21 **a** $a = 31°$ **b** $b = 154°$
c $c = 74°$ **d** $d = 68°$
e $e = 39°$ **f** $f = 65°$

22 **a** 0.024 m
b 5 000 000 cm^3
c 8 001 000 000 m^2
d 4.24 cm^3
e 0.000 02 m^2
f 3 000 000 cm
g 1.2 hours
h 150 minutes
i 6:23 pm
j 0030
k 1.24 L
l 35 000 mL

23 **a** $SA = 90$ cm^2, $V = 50$ cm^3
b $SA = 84.2$ cm^2, $V = 42$ cm^3
c $SA = 97$ m^2, $V = 60$ m^3
d $SA = 17.8$ cm^2, $V = 3$ cm^3

24 **a** 2.871 m^3 or 2 871 000 cm^3
b 1 200 000 mL or 1200 L
c 1.671 m^3 or 1 671 000 cm^3

25 **a** 1:33 pm ACST
b 7:41 am AEST
c 12:00 pm UTC+1
d 3:31 pm UTC+5

26 **a** 5 h 36 m **b** 3 h 25 m

27 **a** 1 hour 34 minutes **b** 8 hours 25 minutes

28 mean = 7

29 **a**

	Reading books	Playing board games	Playing video games	Total
Week day	25	6	34	65
Weekend	40	24	71	135
Total	65	30	105	200

b **i** $\frac{3}{25} = 0.12$ **ii** $\frac{1}{5} = 0.2$
iii $\frac{27}{40} = 0.675$ **iv** $\frac{49}{100} = 0.49$

30 **a** $\frac{13}{36}$ **b** $\frac{1}{9}$ **c** $\frac{2}{3}$ **d** $\frac{11}{36}$ **e** $\frac{1}{3}$

31 **a** **i** sample
ii fair
iii simple random sampling
iv yes
b **i** census
c **i** sample **ii** biased
iii judgemental **iv** no

32 **a** mean = 2.22, median = 2, mode = 2, range = 7
b mean = 1.83, median = 2, mode = 2, range = 2. The median and range change. The mode and median stay the same.

33 **a** It may be too expensive (for example).
b Choose a number of both rural and urban households to survey (for example).
c This method is much cheaper than a census, but it doesn't take into account how location and climate may impact the species of pets in a household (for example).

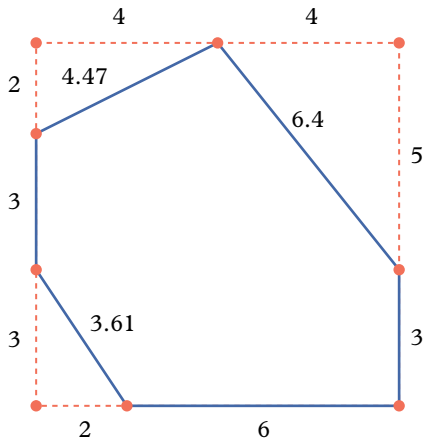
34 **a** The average length of time shoppers spent at the shops was 1 h 20 m.
b The average number of board games people owned was 4.

c The most common favourite colour was blue.

d The length of a number of chess matches spanned 34 turns.

Analysis

1 a



b 47 cm^2 **c** 305.84 cm^2 **d** 376 cm^3

e surface area: $26.48h + 94$, volume: $47h$

f $h = 4.58$ cm

2 a $a = 50°$, $b = 40°$, $c = 59°$, $d = 62°$, $e = 60°$

b mean = 60°, median = 59.5°, mode = 50°, range = 50°

c continuous numerical data

d $\frac{11}{12}$

e $\triangle ABC \cong \triangle JLK$, SAS

f rotation of 180° about the origin

g $\triangle DEF$

3 a

Destination	Brisbane	London
Beach	42%	35%
Camping	34%	20%
Snow	24%	45%

B	L
38%	35%
25%	22%
37%	43%

B	L
43%	40%
26%	15%
31%	45%

B	L
40%	37%
34%	25%
26%	38%

b People in Brisbane and London have similar preferences about going to the beach, with sample percentages in the high 30s to low 40s in both cities. However, the sample percentages for camping and going to the snow vary much more significantly. Camping in Brisbane varies between 25% and 34% whereas in London it varies between 15% and 25%. Going to the snow in Brisbane varies between 24% and 37% whereas in London it varies between 38% and 45%. Going to the beach was most popular in Brisbane while going to the snow was most popular in London. The least popular activity in London is camping, while in Brisbane camping and snow were similar in preference.

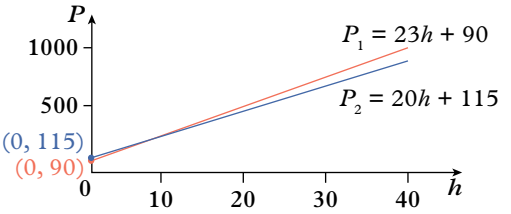
c 1800

d While some variation would still be expected if the seasons did not impact people's preferences, the percentages changed consistently across the samples. Most notably, the sample percentages for going to the snow increased in London to vary between 45% and 53% while in Brisbane they stayed between 27% and 33% (within the variation of the previous samples). The sample percentages for going to the beach in Brisbane and London dropped to vary between 29% and 40%. Camping increased in Brisbane from 27% to 40% but stayed within the same variation for London. For this time period, going to the snow became Brisbane's least popular activity, but the most popular and least popular activities in London remained the same: snow and camping, respectively.

4 a \$800 **b** \$890 **c** $P_1 = 23h + 90$

d $P_2 = 20h + 115$

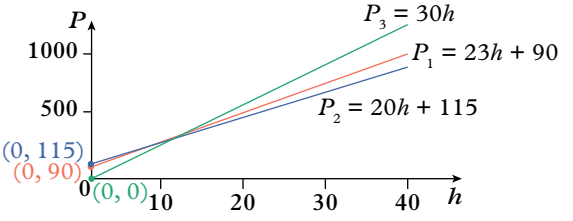
e



f Option 1

g $P_3 = 30h$

h



i Yes

EXPLORATIONS 2

1 **a** 5

b 36

c 55

d 45

e 65

f 209 and 221

g The word BEANS scores
$b + 1 + 3 + 13 + s = b + s + 17$ points.
$s = b + 1$; therefore, $b + s$ is always odd, and $b + s + 17$ is always even.

2 **a** 540°

b 108°

c Isosceles triangles with apex angle 36° and base angles 72°, isosceles triangles with apex angle 108° and base angles 36°, and a rhombus with angles 72° and 108°.

d



e Show that a wide triangle can also be divided into one narrow and one wide triangle. It generalises to any pair of numbers as long as there is at least one narrow triangle and two wide triangles.

f Apex angle 90° and base angles 45° (it divides into two more of the same type) and apex angle $25\frac{5}{7}°$ and base angles $77\frac{1}{7}°$ (it divides into two different types).

3 **a** 120 cm

b 3.82 revolutions per hour

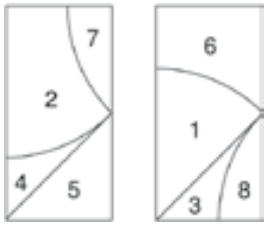
c Raise 209 mm

4 **a**

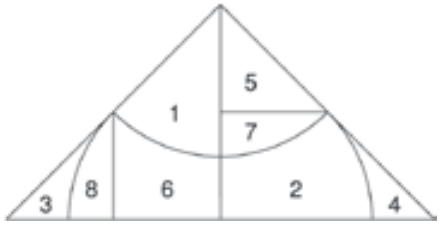
Number	1	2	3 and 4	5	6	7 and 8
Area	2π	8	$8 - 2\pi$	4	$12 - 2\pi$	$2\pi - 4$

b One possible arrangement is shown for each shape. There are many others.

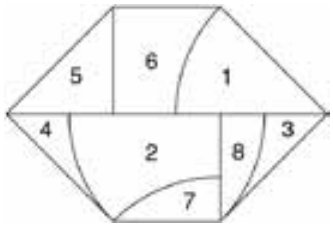
i



ii



iii



5 **a** {2, 2}

b The other freaky frequency list involving only the numbers 1, 2, 3 and 4 is {1, 3, 2, 1, 3, 3, 4, 1}

c {1, 2, 2, 3, 3, 2, 4, 1}

d {1, 4, 2, 2, 3, 2, 4, 2, 5, 1, 6, 1}

e 9

6 Corrections are underlined:

a R2 [R4 [F50 L90] <u>L180</u>]

b <u>R6</u> [R3 [F50 L120] L60]

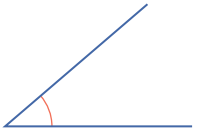
c R6 [R6 [F50 L60] <u>L60</u>]

Glossary

AAA
abbreviation for angle–angle–angle. Triangles are similar if the three internal angles are the same

AAS
abbreviation for angle–angle–side. Triangles are congruent if two angles and a corresponding side length are equal

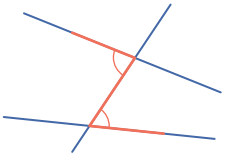
acute angle
angle greater than 0° but less than 90°



acute triangle
triangle with three interior acute angles (less than 90°)

addition algorithm
an algorithm in which two or more numbers are added by first adding the ones values, then tens values, then hundreds values and so on

alternate angles
pair of angles not adjacent to (next to) each other on opposite sides of a transversal between two lines, sometimes called 'Z angles'



area
amount of space enclosed by a two-dimensional (2D) shape. Common units are mm^2, cm^2, m^2, km^2, ha

ascending order
where a list of numbers is written from smallest to largest

associative law
regardless of how three or more numbers are grouped when added or multiplied, the result is the same; for example,

$(4 + 5) + 6 = 9 + 6 = 15$

$4 + (5 + 6) = 4 + 11 = 15$

$(4 \times 5) \times 6 = 20 \times 6 = 120$

$4 \times (5 \times 6) = 4 \times 30 = 120$

base
1. for a value expressed in exponent form, the base is the number which is repeatedly multiplied; for example, 2^4 has a base of 2 and can be written as $2 \times 2 \times 2 \times 2$
2. in a 2D shape, it is a nominated side of the shape. The base and height are perpendicular to each other
3. in a 3D object, it is a nominated face of the object. The base and height are perpendicular to each other

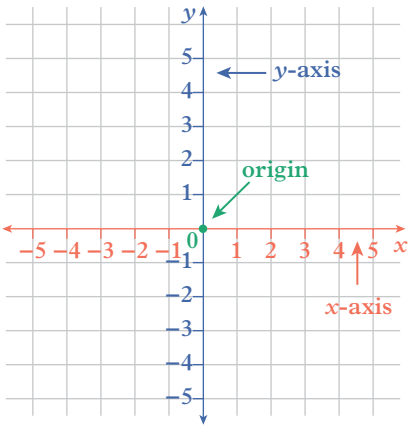
biased
a biased sample is one where the method of collecting data produces a sample that does not accurately reflect the population

BIDMAS
word used to remember the correct order of operations: Brackets, Indices (exponents) and roots, Division and Multiplication, and Addition and Subtraction

by-parts method
involves adding the digits in each place-value separately

capacity
amount of liquid that a three-dimensional object (container) can hold

Cartesian plane
number plane or region formed by a pair of horizontal and vertical axes that allows any point on the plane to be described. It consists of a horizontal x-axis and a vertical y-axis that intersect at right angles at a point called the origin



census
survey of an entire population

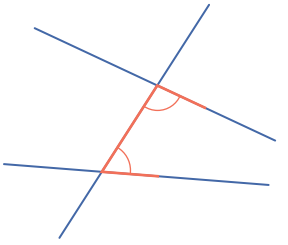
centre of dilation
the point from which a dilation is made

circumference
perimeter of a circle

cluster sampling
dividing the population into similarly diverse groups and then taking a random sample of the groups

coefficient
the number by which a pronumeral is multiplied; for example, the coefficient of $5m$ is 5 and the coefficient of $-3a$ is -3

co-interior angles
pair of angles on the same side of a transversal between two lines, sometimes called 'C angles'



common factors
factors that divide exactly into two or more whole numbers; for example, 12 and 9 have a common factor of 3

commutative law
order in which two numbers are added or multiplied is not important; for example, $3 + 4 = 7$ and $4 + 3 = 7$; $3 \times 4 = 12$ and $4 \times 3 = 12$

compensation method
involves rounding one number to make the calculations easier. The amount by which the number was originally rounded is then added or subtracted

complementary angles
angles that add to make a right angle (90°)

complementary events
a pair of events A and A' (not A) where only one of the two events can take place and their probabilities sum to 1

congruence
when figures are identical in shape and size but can be in any position or orientation

constant
a term without any pronumeral component; for example, the constant in the expression $3b - 4c + 10$ is 10

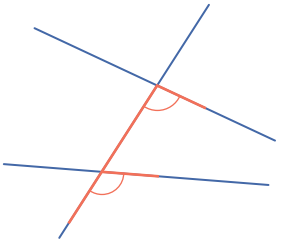
convenience sampling
when a sample that is easily accessible is selected; for example, selecting students from your class to complete a survey

conversion factor
a multiplier for converting a quantity expressed in one set of units into another quantity expressed in a different set of units; for example, multiply by 10 to convert from cm to mm

Coordinated Universal Time (UTC)
time zone referenced from Greenwich in London at 0° longitude (also known as Greenwich Mean Time). Parts of the world to the east of time zone '0' are ahead of UTC, parts of the world to the west are behind UTC. For example, Melbourne is in the UTC+10 time zone. This means it is 10 hours ahead of UTC

coordinates
pair (or triple) of numbers that describe the position of a point on a plane, graph or in 3D space

corresponding angles
pair of angles on the same side of a transversal and in corresponding positions on the two lines (both above a line or both below a line), sometimes called 'F angles'



cost price (or wholesale price)
the cost paid to create or obtain a good or service before it is sold to consumers

cube
to raise to the power of three; for example, $2^3 = 8$

cube root
number that when cubed results in the original number; for example, the cube root of 8 (or $\sqrt[3]{8}$) is 2

decimals
numbers with a whole number component and a decimal component. The digits to the left of the decimal point indicate the whole number and the digits to the right of the decimal point indicate a value less than one; for example, 3.456

denominator
bottom number of a fraction. For example, the denominator of $\frac{2}{3}$ is 3

descending order
where a list of numbers is written from largest to smallest

diameter
line segment from one side of a circle to the other through the centre of the circle. Also defined as the length of this line segment

dilation
transformation that produces an enlargement or reduction of the original figure

discount
an amount off the original price; the difference between the regular price and a lower price

distributive law
states that the same result is obtained whether you add the numbers inside a pair of brackets first before multiplying by another number or multiply each number inside the brackets by the number outside the brackets before adding, $a(b + c) = ab + ac$

dividend
number that is being divided by another number (divisor); for example, 6.84 is the dividend, 2 is the divisor and 3.42 is the quotient in $6.84 \div 2 = 3.42$

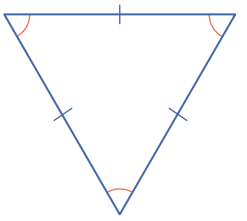
divisor
number that divides into another number (dividend); for example, in $15 \div 3 = 5$, 3 is the divisor

duration
the length between two given times

enlargement
an image produced by a dilation that is larger than the original figure

equation
a collection of two or more algebraic terms separated by operation symbols and an equals sign

equilateral triangle
triangle with all three sides equal in length. All three angles are also equal in size



equivalent equations
equations that have the same solution

equivalent fractions
different fractions that have the same value. An equivalent fraction is formed by multiplying or dividing the numerator and the denominator of a fraction by the same number. For example, $\frac{24}{36} = \frac{4}{6} = \frac{2}{3}$

equivalent ratio
can be produced by multiplying or dividing all numbers in a ratio by the same value

estimation (estimate)
a rough calculation of a value or number

evaluate
substitute the known values of each pronumeral into an expression and complete operations in the correct order

event
a specified result that may be obtained in a probability experiment

expand
remove the brackets in an expression by using the distributive law

expanded form

1 power written as a repeated multiplication; for example, 4^3 written in expanded form is $4 \times 4 \times 4$

2 algebraic expression written without brackets, as opposed to factorised form

experiment

repeatable procedure that has a clearly defined set of possible results

exponent (index)

for a value expressed in exponent form, the exponent indicates the number of times the base is written as a repeated multiplication; for example, 2^4 has an exponent of 4 and can be written as $2 \times 2 \times 2 \times 2$. We read 2^4 as "2 to the power of 4"

exponent form (exponent notation)

shorter form of writing a repeated multiplication. A number in exponent form is written with a base and an exponent; for example, 2^4 is written in exponent form

exponent laws

a set of rules that can be used to simplify calculations involving numbers in exponent form

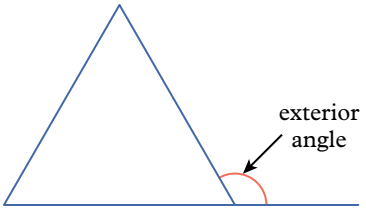
	Exponent law
Product of powers	$a^m \times a^n = a^{m+n}$
Quotient of powers	$\frac{a^m}{a^n} = a^m + a^n = a^{m-n}$
Power of a power	$(a^m)^n = a^{m \times n}$ $(a \times b)^m = a^m \times b^m$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

expression

either a single term or the sum or difference of two or more terms; for example, $3b - 4c + 10$ is an expression

exterior angle

an angle between any side of a shape and a line extended from the adjacent side



factor trees

tree diagrams where composite numbers are broken down into factor pairs until prime numbers are found

factorise

to write an algebraic expression as the product of its factors; for example, $6x + 3$ factorises to $3(2x + 1)$ where the two factors are 3 and $2x + 1$

factors

whole numbers that divide exactly into a given whole number; for example, factors of 8 are 1, 2, 4 and 8

favourable outcome

outcome within the sample space that we are interested in

formula

relationship or rule between two or more variables, usually represented using pronumerals; for example, $s = \frac{d}{t}$ is the formula for speed equals distance divided by time

GST (Goods and Services Tax)

introduced by the Australian Government in 1999. The rate is 10% on the selling price of the products or service

graph

a continuous line made up of infinitely many coordinate points

grouping symbols

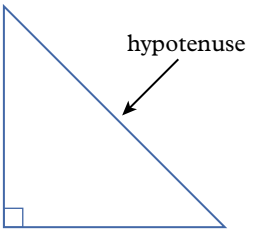
include brackets, radical and fraction lines to show where a group starts and ends, and help to establish the order used to apply mathematical operations

highest common factor (HCF)

largest factor that is common to two or more given numbers; for example, factors of 8 are 1, 2, 4, 8 and factors of 12 are 1, 2, 3, 4, 6, 12 so HCF of 8 and 12 is 4

hypotenuse

the side opposite the right angle of a right-angled triangle



image

result of completing a transformation on a shape or object. If the original shape is labelled A, then the image (new shape) is labelled A'

improper fraction

fraction where the numerator is greater than (or equal to) the denominator; for example, $\frac{9}{7}$ or $\frac{6}{6}$

included angle

the angle between two given sides

index (plural: indices)

see exponent

inequality

a mathematical statement that compares the values of two expressions

integer

a whole number that is positive, negative or zero

interior angle

an angle inside a shape or object

inverse operation

operation that reverses the effect of a previous operation; for example, multiplication and division are inverse operations and addition and subtraction are inverse operations

irrational numbers

numbers that cannot be written exactly as a fraction. These are non-terminating decimals that are not recurring decimals; for example, π, $\sqrt{2}$, $\sqrt{3}$

irregular quadrilateral

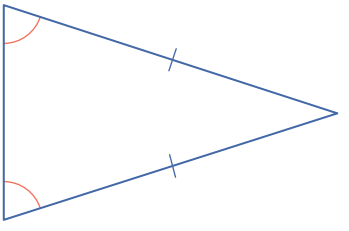
quadrilateral with four different internal angles and no parallel sides

isometric transformation

type of transformation (such as translation, rotation or reflection) applied to a shape or object that does not change its shape or size

isosceles triangle

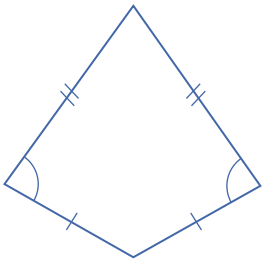
triangle with two sides equal in length. The angles opposite the equal sides are also equal



judgemental sampling
when a sample is selected by the researcher; for example, a researcher may ask students to take part in a study

jump method
involves breaking down one of the given numbers, then adding or subtracting each part of the number in stages

kite
quadrilateral with two pairs of adjacent sides equal in length, one pair of opposite angles equal in size and no parallel sides

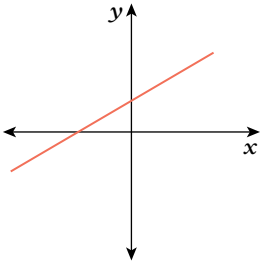


leading digit
first non-zero digit of a number; for example, the leading digit of 328 is 3

length
measurement from one end to the other of a shape or object. In a rectangle, it is the side measurement that is not defined as the width. Common units are mm, cm, m, km

like terms
algebraic terms that contain exactly the same pronumerals in any order

linear graph
a graph that forms a straight line on the Cartesian plane



linear relationship
relationship between two variables that produces a linear graph

lowest common denominator (LCD)
lowest common multiple of the denominators in two or more fractions. For example, $\frac{1}{3}$ and $\frac{1}{2}$ have an LCD of 6

lowest common multiple (LCM)
lowest (first) multiple that is common in the multiples list of two or more numbers; for example, multiples of 3 are 3, 6, 9, 12, 15, 18, … and multiples of 4 are 4, 8, 12, 16, 20, 24, … so LCM of 3 and 4 is 12

magnitude
the value of a number ignoring the sign; for example the magnitude of −3 is 3

mark-up
amount added to a cost price; for example, an original price has a 25% mark-up added to obtain the selling price

mean
a measure of the centre of a data set, also referred to as the average, calculated by adding all of the data values together and dividing by the number of values

measures of centre
describes the data by identifying the central position. The mean, median and mode are measures of centre

measures of spread
describes the spread of the data. The range is a measure of spread

median
a measure of the centre of a data set. The middle of a data set when the numbers are listed in order. The median divides a data set in half

mixed number
made up of a whole number component and a proper fraction component; for example, $3\frac{4}{5}$

mode
a measure of the centre of a data set determined by finding the value, or values, that occur most frequently in the data set

multiples
result of multiplying a given number by whole numbers; for example, multiples of 4 are 4, 8, 12, 16, … (4×1, 4×2, 4×3, 4×4, …)

negative integers
whole numbers that are less than zero in value and are always preceded by a negative sign, −, placed directly before the first digit

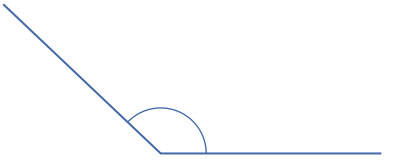
non-linear graph
graph of a non-linear relationship. The graph is not a straight line

non-linear relationship
relationship between two variables where the coordinate points describing this relationship do not lie in a straight line when plotted on the Cartesian plane

non-terminating decimal
decimals with an endless (infinite) number of decimal places; for example, 5.761 238… or 4.266 666…

numerator
top number of a fraction; for example, the numerator of $\frac{2}{3}$ is 2

obtuse angle
angle greater than 90° but less than 180°

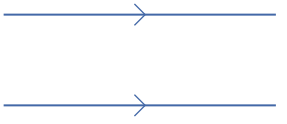


obtuse triangle
triangle with one interior obtuse angle (more than 90°)

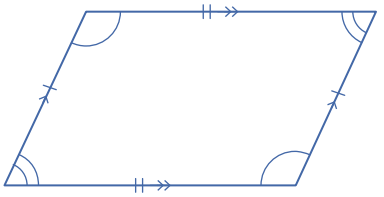
origin
point where the x- and y-axes cross on a Cartesian plane. Coordinates are (0, 0)

outlier
extreme piece of data that is much higher or lower than the rest of the data in the data set

parallel lines
lines (or rays or segments) that are drawn side by side and are the same distance apart so they never touch are parallel. Matching arrowheads are used to indicate that lines are parallel



parallelogram
quadrilateral with two pairs of opposite sides that are parallel and equal in length, and two pairs of opposite angles that are equal in size. Rectangles, squares and rhombuses are also classified as parallelograms



percentage
a portion of a whole divided into one hundred parts. The symbol for percentage is %

percentage decrease
occurs when a percentage amount is subtracted from the original amount

percentage error
the error of a measurement written as a percentage of the true value of the quantity

percentage increase
occurs when a percentage amount is added to the original amount

perfect cube
can be expressed as the product of three equal numbers; for example, $2 \times 2 \times 2 = 8$, so 8 is a perfect cube

perfect square
can be expressed as the product of two equal numbers; for example, $3 \times 3 = 9$, so 9 is a perfect square

perimeter
length of the boundary of a two-dimensional (2D) shape. Common units are mm, cm, m, km

pi
irrational number (3.141 592 653 …) used in calculations involving circumference and area of a circle. It is the ratio of the circumference of a circle to its diameter. The symbol for pi is π

plot
a Cartesian plane composed of individual coordinate points

point of focus
a point to concentrate on when rotating a shape

polygon
general name for a two-dimensional shape with straight sides

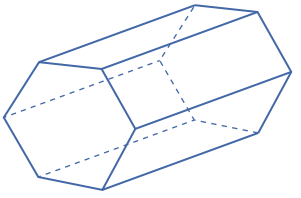
population
complete set of individuals or things that we are seeking information about

population parameters
the characteristics of a population, including the mean, median, mode and range

positive integers
whole numbers that are greater than zero in value and can be represented using a positive sign, +. The positive sign can also be left out

prime factorisation
the product of the primes that make up that integer

prism
object with two ends that are identical polygons and joined by straight edges



probability
chance of an event occurring. The event can be very likely and have a high probability or not likely and have a low probability. Words or numbers can be used to describe the chance of an event occurring

product
result of a multiplication; for example, $10 \times 3 = 30$ where 30 is the product

pronumeral
letter or symbol that takes the place of a number

proper fraction
fraction where the numerator is smaller than the denominator; for example, $\frac{4}{5}$

Pythagoras' theorem
theorem that states that the of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the other side length

Pythagorean triple
any set of three positive whole numbers that satisfy Pythagoras' theorem

quadrilateral
two-dimensional shape with four straight sides

quota sampling
dividing the population into separate categories (such as gender or age) and then taking a specified sample from each category that is chosen by the researcher

quotient
result of a division. For example, $28 \div 4 = 7$ where 7 is the quotient

radius (plural: radii)
line segment from the centre of a circle to the edge of the circle. Also defined as the length of this line segment and half the diameter of the circle

range
a measure of spread determined by finding the difference between the highest and lowest values in a data set

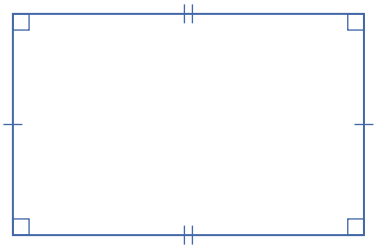
rate
a comparison between two or more different quantities. It is a measure of how a quantity changes for each unit of another quantity

ratio
a comparison of two or more quantities of the same kind; for example, when comparing the number of red jellybeans to blue jellybeans, the ratio is 2 : 3

rational numbers
numbers that can be written exactly as fractions. This includes whole numbers, fractions, terminating decimals and recurring decimals

reciprocal
the multiplicative inverse of a number which can be found by write the number as a fraction and swapping the numerator and denominator. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. When a number is multiplied by its reciprocal, the result is 1

rectangle
quadrilateral with two pairs of opposite sides that are parallel and equal in length, and four angles of 90°. A rectangle is also classified as a parallelogram



rectangular prism
prism with three pairs of identical rectangular faces

recurring decimal
a special type of non-terminating decimal. The digits after the decimal point form a recurring or repeating pattern. For example,

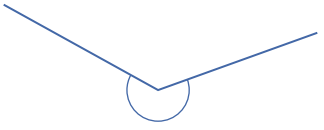
0.333 33… = 0.3. and
5.476 767 6… = $5.\overline{476}$

reduction
an image produced by a dilation that is smaller than the original figure

reference point
a point or number used to define the location of another point relative to the reference point. This point is often zero on a number line or Cartesian Plane

reflection
transformation where a shape or object is reflected (flipped) in a mirror line to produce its exact mirror image

reflex angle
angle that is greater than 180° but less than 360°



relative frequency
the number of times a favourable outcome occurs divided by the total number of outcomes

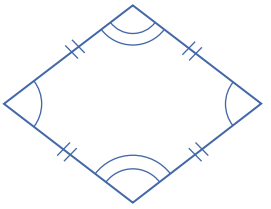
$$\text{Relative frequency} = \frac{\text{number of occurrences}}{\text{total number of outcomes}}$$

remainder
the part of the dividend that is left over when the number is not exactly divisible by the divisor. For example, 22 divided by 7 is 3 remainder 1

revolution
angle that is exactly 360°. Also known as a perigon

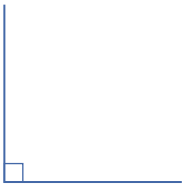


rhombus
quadrilateral with two pairs of opposite sides that are parallel, all four sides equal in length and two pairs of opposite angles that are equal in size. A rhombus can also be classified as a parallelogram

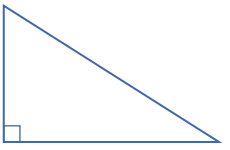


RHS
abbreviation for right angle–hypotenuse–side. Right-angled triangles are congruent if their hypotenuses are the same length and another pair of side lengths are equal, and are similar if the lengths of their hypotenuses are in the same ratio as another pair of sides

right angle
angle that is exactly 90°. Often represented by a small box drawn where the lines meet



right triangle
triangle where the size of one interior angle is 90°



rotation
transformation where a shape or object is rotated (turned) in either a clockwise or anticlockwise direction around a given point

round down
to find an approximation that is less than the original number

rounding
involves replacing the number with an approximation that is simpler and easier to use in calculations, but which makes any calculations less accurate. For example,
123 343 ≈ 120 000

round up
to find an approximation that is greater than the original number

sample
selection from a population

sample space
list of all the different outcomes possible for a probability experiment; for example, sample space when rolling a die is {1, 2, 3, 4, 5, 6}

SAS
abbreviation for side–angle–side. Triangles are congruent if two corresponding side lengths are equal and the angle between them is equal, and are similar if two corresponding side lengths are in the same ratio and the angle between them is equal

scale factor
how many times larger or smaller an image is after a figure has been dilated:

$$\text{scale factor} = \frac{\text{image length}}{\text{original length}}$$

scalene triangle
triangle with three sides of different lengths. All three angles are different in size



selling price
the price that a product or service is sold at by the seller

sign (of a number)
whether a number is positive, negative or zero

similar
figures with the same shape but they are not necessarily the same size. Dilations produce similar figures

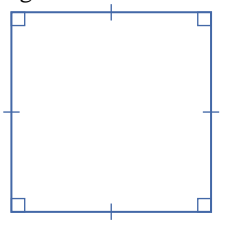
simple random sampling
techniques that ensure every potential piece of data has an equal chance of being selected. Types of simple random sampling include systematic sampling and stratified sampling

simplification
process of reducing the complexity of a calculation by adding, subtracting, multiplying and dividing terms

solution
value of the pronumeral that makes an equation a true statement

solve by inspection
a mental strategy to solve simple equations. To inspect or use observation to work out the solution to an equation in your head

square
1 quadrilateral with all four sides equal in length, two pairs of opposite sides that are parallel and four angles of 90°



2 to raise to the power of 2; for example, $3^2 = 9$

square root
the square root of a number is the number that when multiplied by itself produces the original number; for example, the square root of 9 (or $\sqrt{9}$) is 3

SSS
abbreviation for side–side–side. Triangles are congruent if all three side lengths are equal, and are similar if all pairs of corresponding side lengths are in the same ratio

straight angle
angle that is exactly 180°



stratified sampling
dividing the population into separate categories (such as gender or age) and then taking a random sample from each category that is proportional to its size

substitution
replacement of a pronumeral with a given number; for example, substituting $x = 2$ in $3x + 4$ gives $3 \times 2 + 4$

subtraction algorithm
an algorithm in which two or more numbers are subtracted by first subtracting the ones values, then tens values, then hundreds values and so on

supplementary angles
angles that add to make a straight angle (180°)

surd
the square root or cube root of a number that is not a perfect square or perfect cube. Surds are irrational numbers

surface area
total area of the outer surface of an object. Surface area of a prism is the sum of the areas of each face of the prism

systematic sampling
selecting a sample at fixed intervals (such as every third person). The starting point should be random

term
a single pronumeral, single number, or the product of a number and one or more pronumerals. For example, the expression $3b - 4c + 10$ has three terms

terminating decimal
decimal that 'terminates' or stops after a set number of decimal places; for example, 0.625

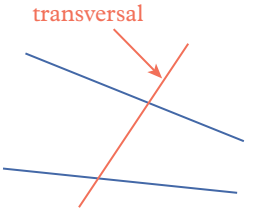
three-dimensional coordinates
three numbers that describe the position of a point in three-dimensional space

time zones
bands on the surface of the Earth where countries experience the same time. Each time zone usually spans a difference in longitude of 15°

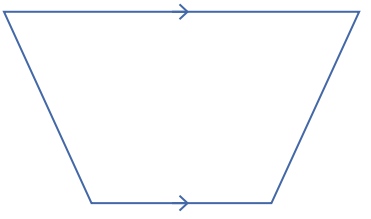
trailing zeros
zeros to the right of the last non-zero digit after the decimal point in a decimals; for example, 3.900 has two trailing zeros

translation
transformation where a shape or object is translated (moved) in a straight line without turning or changing size

transversal
line that crosses (intersects) a pair or set of lines



trapezium
quadrilateral with at least one pair of opposite sides that are parallel



triangular prism
3D object with five faces including two identical triangular faces whose respective vertices are joined by parallel lines

12-hour time
time measured over a 12-hour period from midnight and then from midday. am and pm are used to describe whether a time is before or after midday

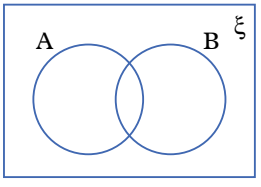
24-hour time
time measured over a 24-hour period from midnight. Usually shown with four digits to indicate the number of hours and minutes from midnight; for example, 1530 means 15 hours and 30 minutes after midnight – or 3:30 pm

two-way tables
a table that displays frequencies for two different categories

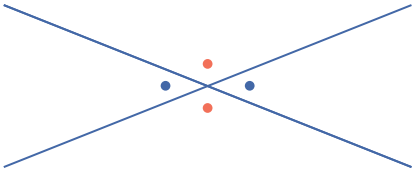
unknown number
a number that is represented using a pronumeral; for example, the x in $x + 3 = 5$

variable
quantity that can have different values and can be represented with a word, symbol or pronumeral

Venn diagram
a display of the relationship between different sets of data. The universal set, ξ, is represented by a rectangle and each set of data is represented by a circle



vertically opposite angles
pair of equal angles not adjacent to (next to) each other, formed where two lines intersect



volume
amount of space that a three-dimensional (3D) object occupies

x-axis
horizontal number line that forms part of the Cartesian plane

x-coordinate
1 first of a pair of numbers (coordinates) that describes the point's horizontal distance from the origin on the Cartesian plane

$(-4, 1)$

x-coordinate – horizontal distance from the origin

2 first of a triple of numbers (coordinates) that describes how far in front or behind a point is from the origin in a three-dimensional coordinate system

y-axis
vertical number line that forms part of the Cartesian plane

y-coordinate
1 second of a pair of numbers (coordinates) that describes the point's vertical distance from the origin on the Cartesian plane

$(-4, 1)$

y-coordinate – vertical distance from the origin

2 second of a triple of numbers (coordinates) that describes how left or right a point is from the origin in a three-dimensional coordinate system

y-intercept
the value of the y-coordinate where a line meets the y-axis (when $x = 0$). In a linear equation of the form $y = mx + c$, the y-intercept is equal to the constant, c

z-axis
axis perpendicular to the Cartesian plane in a three-dimensional coordinate system

z-coordinate
third of a triple of numbers (coordinates) that describes how far up or down the point is from the origin in a three-dimensional coordinate system

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Given a fixed perimeter, the circle is the shape with the largest possible area. Similarly, given a fixed surface area, the object with the largest possible volume is a sphere – like a bubble!

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