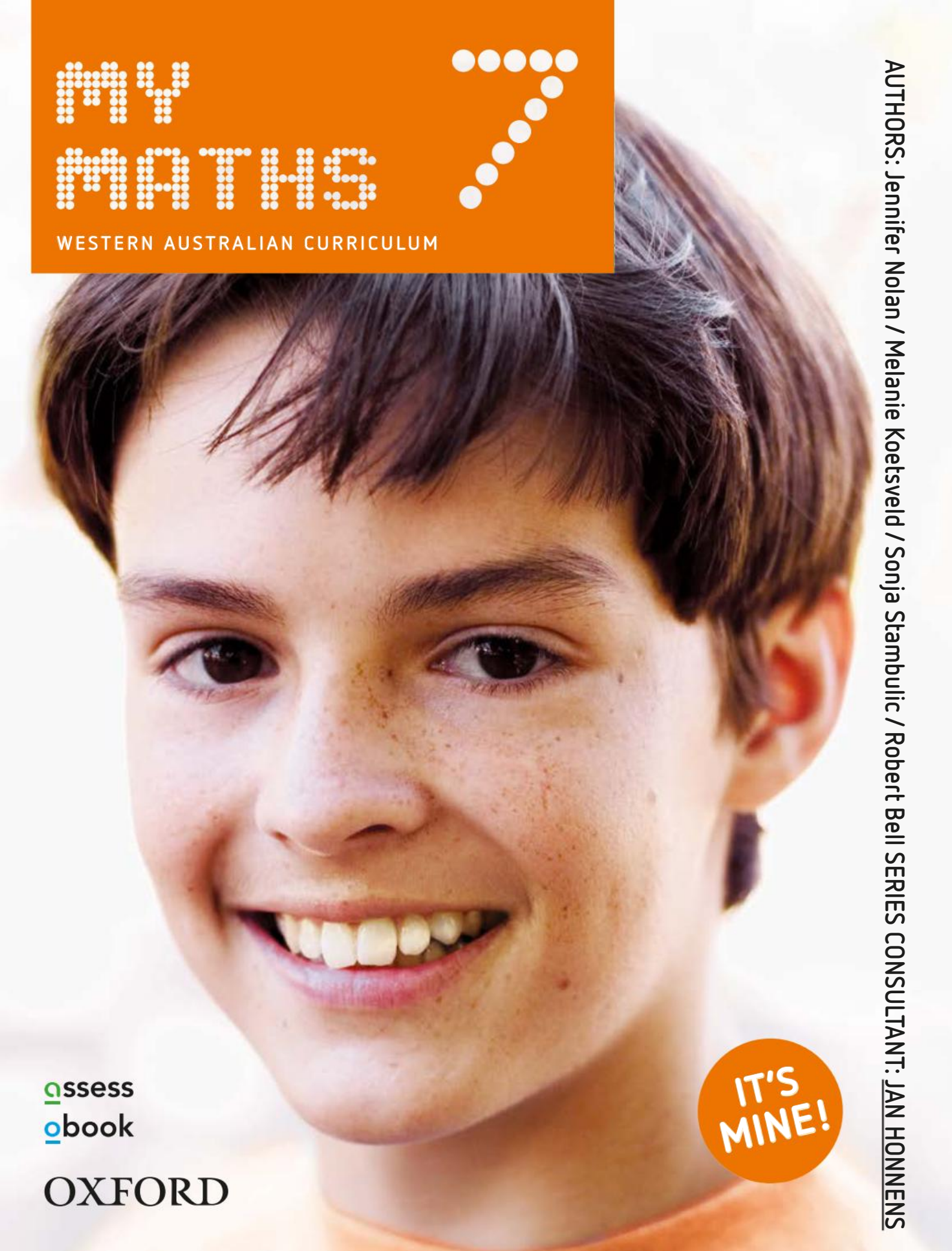
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WESTERN AUSTRALIAN CURRICULUM



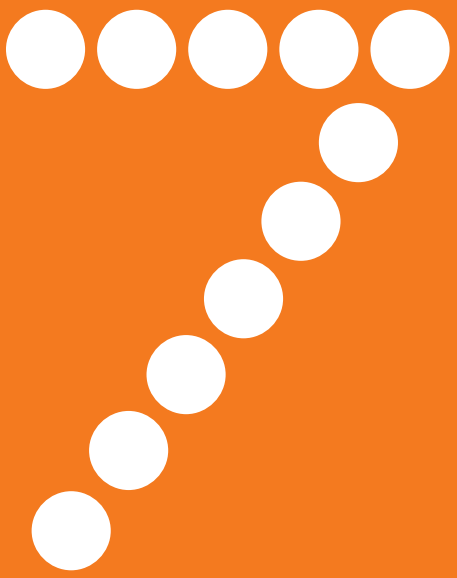
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# MY MATHS



AUTHORS: Jennifer Nolan / Melanie Koetsveld / Sonja Stambulic / Robert Bell SERIES CONSULTANT: JAN HONNENS

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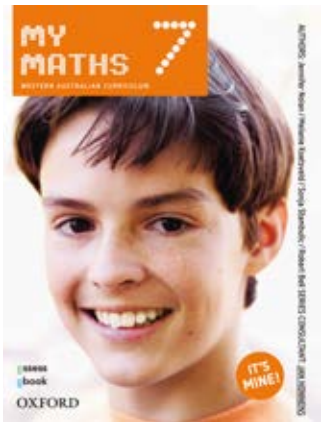
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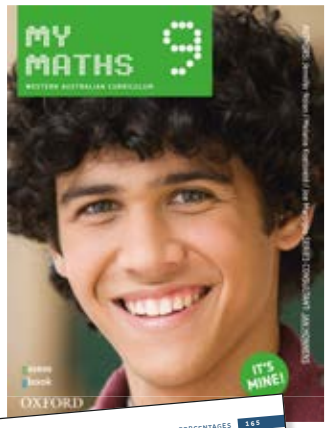
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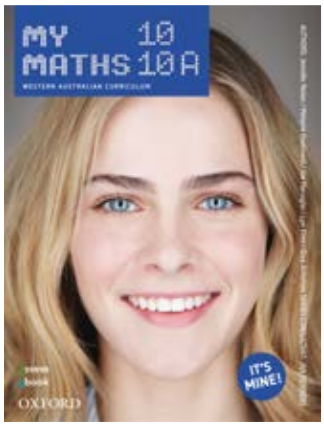
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***Oxford MyMaths Western Australian Curriculum*** has been specifically developed to support students wherever and whenever learning happens: in class, at home, with teacher direction or in independent study.

## STUDENT BOOK + OBOOK/ASSESS

- Finely levelled exercises to ensure smooth progress
- Integrated worked examples – right where your students need them
- Learning organised around the 'big ideas' of mathematics
- Discovery, practice, thinking and problem-solving activities promote deep understanding
- A wealth of revision material to consolidate and prove learning
- Rich tasks to apply understanding
- Highly accessible and easy to navigate
- Comprehensive digital tutorials and guided examples to support independent progress




The student book accurately and carefully delivers the course.

Every question matched to the Western Australian Curriculum proficiencies.

Worked examples are clearly laid out and located where students need them most.

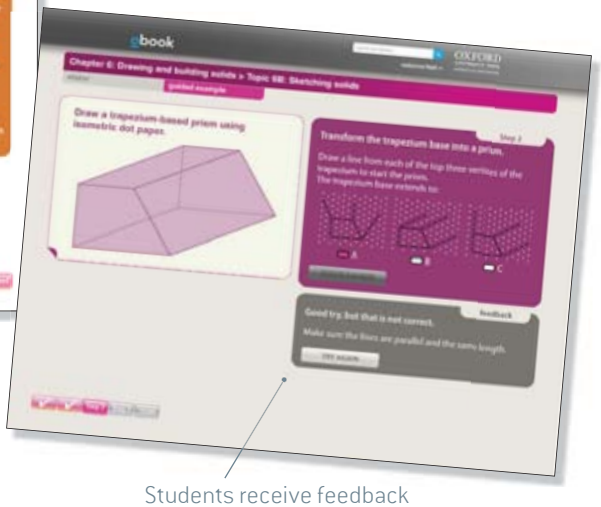
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E-tutors scaffold understanding of key concepts and build confidence.

Self-discovery opportunities for students through guided exploration.



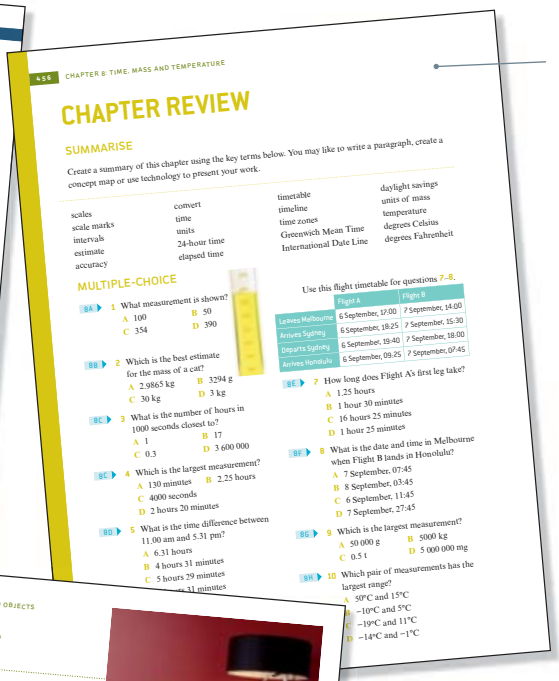

Guided examples support practice and fluency



Students receive feedback for incorrect responses

Finely levelled content enables students to progress with ease

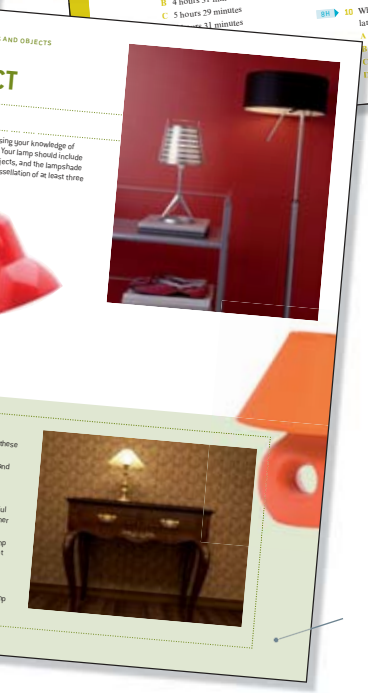
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Optimise understanding and performance.

Personalised learning: tailor the very best learning experiences for all.





Rich tasks where students can demonstrate understanding

Intervention and extension worksheets supplied for every topic.



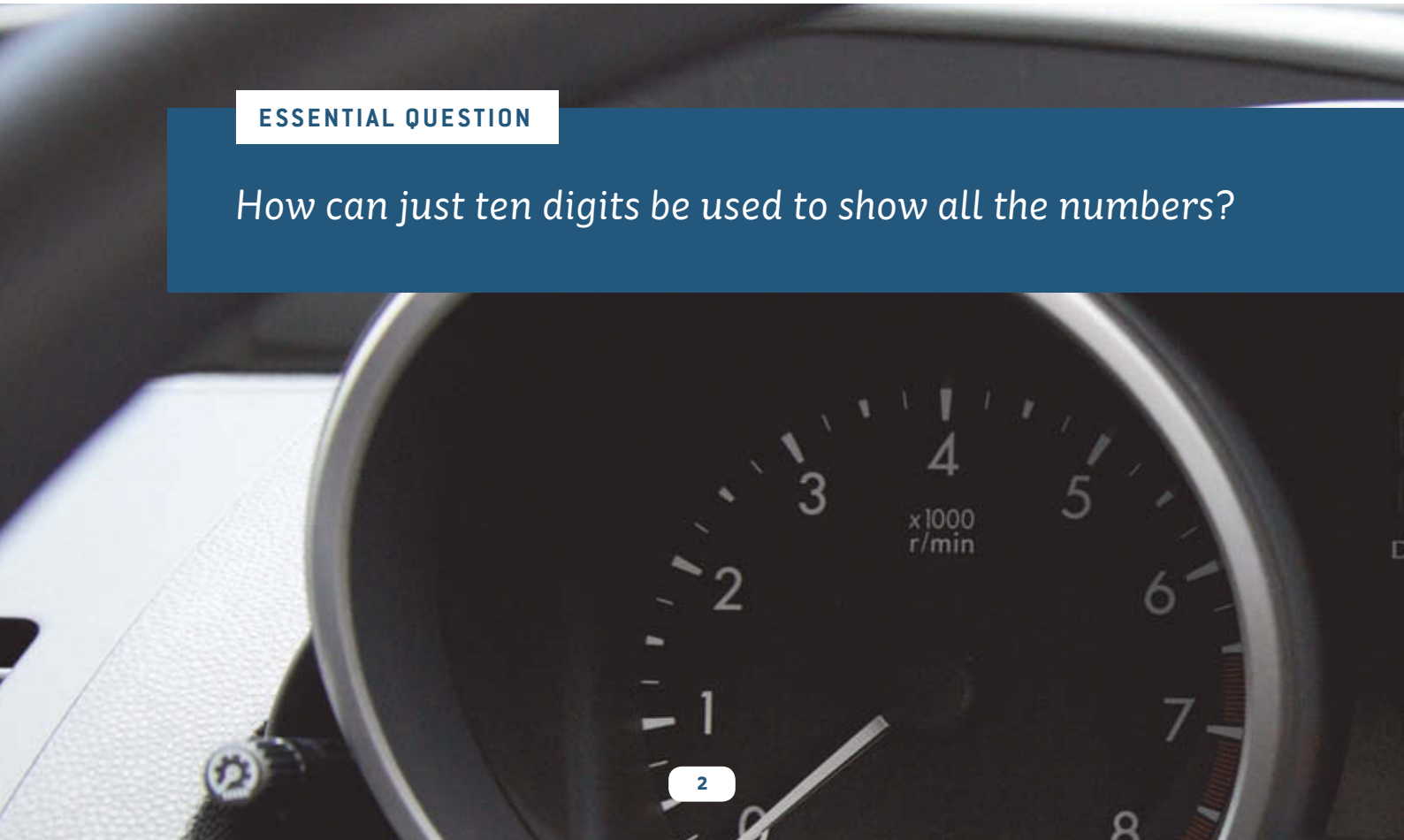

## TEACHER OBOOK/ASSESS

Practical classroom resources and tools:

- Manage student differentiation
- Correct common misconceptions
- Assign work
- Set tests
- Monitor results
- Any device, anytime, anywhere.

# 1 WHOLE NUMBERS


1A Understanding place value
1B Adding whole numbers
1C Subtracting whole numbers
1D Multiplying whole numbers
1E Dividing whole numbers
1F Powers and square roots
1G Order of operations
1H Multiples and factors
1I Prime and composite numbers


**ESSENTIAL QUESTION**

*How can just ten digits be used to show all the numbers?*

# Are you ready?

1A **1** Written in words, what is 82 150?

**A** eighty thousand, two hundred and fifteen

**B** eighty-two thousand, one hundred and fifteen

**C** twenty-eight thousand, one hundred and five

**D** eighty-two thousand, one hundred and fifty

1A **2** Written in digits, what is one hundred and twenty-four thousand, five hundred and eight?

**A** 124 580

**B** 124 508

**C** 10 024 508

**D** 10 020 458

1A **3** What is 236 457 written in place-value notation?

**A** 200 000 + 30 000 + 6000 + 400 + 50 + 7

**B** 2 + 3 + 6 + 4 + 5 + 7

**C** 200 + 30 + 6 + 400 + 50 + 7

**D** 200 000 + 30 000 + 6000 + 400 + 5 + 7

1A **4** Which number is the largest in each list of numbers?

**a** 23, 41, 18, 39

**b** 135, 153, 133, 150

1A **5** Which of these numbers are odd?
24, 31, 15, 50, 63, 48.

**A** 24, 50, 48 **B** 31, 15, 50

**C** 31, 15, 63 **D** 24, 15, 63

1B **6** Calculate:

**a** 32 + 45 **b** 18 + 57

1C **7** Calculate:

**a** 89 − 36 **b** 64 − 28

1D **8** Calculate:

**a** 32 × 4 **b** 84 × 6

1D **9** Calculate:

**a** 42 × 10 **b** 75 × 100

1D **10** What is 3040 × 1000?

**A** 34 000 **B** 304 000

**C** 3 040 000 **D** 30 400 000

1E **11** Calculate:

**a** 48 ÷ 4 **b** 5300 ÷ 10

1E **12** What is 263 ÷ 7?

**A** 39

**B** 37

**C** 38 remainder 3

**D** 37 remainder 4

1F **13** What is 2 × 2 × 2 × 2 × 2?

**A** 10 **B** 32

**C** 64 **D** 128

1F **14** What is 3 × 3 × 3 × 3?

1H **15** What are the next three numbers in the pattern 4, 8, 12, 16, …?

**A** 18, 20, 22

**B** 20, 28, 36

**C** 20, 24, 28

**D** 32, 64, 128

1H **16** Which list gives all the factors of 30?

**A** 1, 3, 10, 30

**B** 1, 2, 3, 4, 5, 6, 10, 30

**C** 1, 2, 5, 6, 15, 30

**D** 1, 2, 3, 5, 6, 10, 15, 30

1H **17** One factor pair for 12 is 3 and 4. What is another?

**A** 3 and 9 **B** 2 and 6

**C** 2 and 10 **D** 5 and 7

# 1A Understanding place value

## Start thinking!



Huge crowds attend the AFL grand final each year at the Melbourne Cricket Ground (MCG). In three consecutive years, the crowd numbers were 97 531, 97 302 and 100 012.

1 Which of the three numbers is the smallest? Give a reason for your answer.

2 Which of the three numbers is the largest? Give a reason for your answer.

3 One student claims the crowd number for the third year is the smallest as it starts with the digit 1 compared to the other two numbers, which start with the digit 9. Explain why this is incorrect.

## KEY IDEAS

- Whole numbers are made up of combinations of digits. The value of each digit depends on the place or position of the digit in the number.
- The digit 0 (zero) is important even though it shows that there is nothing in that place-value position. It keeps the other digits in the right places in the number.
- < means 'is less than' (or 'is smaller than')
- > means 'is greater than' (or 'is larger than')
- Always read mathematical statements from left to right.
- One way to write an approximate value for a number is to round the number to its leading (or first) digit. To do this, look at the second digit in the number.
- If the second digit is 0, 1, 2, 3 or 4, the first digit stays the same and each digit that follows is replaced with zero.
  For example, 328 ≈ 300.

leading or first digit
3 2 8
second digit

- If the second digit is 5, 6, 7, 8 or 9, the first digit is increased by one and each digit that follows is replaced with zero.
  For example, 372 ≈ 400.

leading or first digit
3 7 2
second digit

# EXERCISE 1A Understanding place value

## EXAMPLE 1A-1 Writing numbers in place-value notation

Write each number in place-value notation.

a 369 b 28 104 c 5070

**THINK**

1 You may like to show the numbers in a place-value chart. Remember to include zero in the appropriate columns.

2 Write each number in place-value notation by showing the value of each digit.

**WRITE**

| Ten thousands 10 000 | Thousands 1000 | Hundreds 100 | Tens 10 | Ones 1 |
|---|---|---|---|---|
| | | 3 | 6 | 9 |
| 2 | 8 | 1 | 0 | 4 |
| | 5 | 0 | 7 | 0 |

a 369 = 300 + 60 + 9
b 28 104 = 20 000 + 8000 + 100 + 4
c 5070 = 5000 + 70

**1** Write these numbers in place-value notation.

a 56 b 238 c 4751
d 12 649 e 8507 f 63 044

**2** State whether each number in question **1** is an odd or an even number.

## EXAMPLE 1A-2 Writing the value of a digit in a number

Write the value of the digit 4 in each of these numbers.

a 347 b 48 052

**THINK**

a Consider the place value of 4.

b Consider the place value of 4.

**WRITE**

a Place value is 4 tens.
Value of 4 in 347 is 40.

b Place value is 4 ten thousands.
Value of 4 in 48 052 is 40 000.

UNDERSTANDING AND FLUENCY

**3** Write the value of the digit 9 in each of these numbers.

| | | |
|---|---|---|
| **a** 298 | **b** 957 | **c** 39 |
| **d** 59 406 | **e** 7891 | **f** 970 412 |




**4** For each number shown on these signs, write the value of the digit that is listed in brackets.

**a** distance where animals may be near the road (9)

**b** height above sea level of Mt Kosciuszko (8)

**c** **i** distance to Alice Springs (1)

**ii** distance to Tennant Creek (6)

**iii** distance to Darwin (5)

**5** Write these numbers in digit form.

**a** sixty-two thousand

**b** nine hundred and seventy-eight

**c** three hundred and four

**d** two hundred and fifty thousand, one hundred and twelve

**e** twelve thousand, five hundred and forty-three

**f** nine thousand and twenty-six

**6** Write these numbers in words.

| | | |
|---|---|---|
| **a** 362 | **b** 7215 | **c** 45 733 |
| **d** 234 601 | **e** 6 420 058 | **f** 55 555 |

## EXAMPLE 1A-3 Ordering numbers

Write this list of numbers in order from smallest to largest: 23 706, 2376, 23 678.

| THINK | WRITE |
|---|---|
| **1** Count the number of digits in each number. The four-digit number is smallest. | 23 706 has five digits, 2376 has four digits and 23 678 has five digits. So 2376 is the smallest number. |
| **2** Compare the size of each digit in corresponding place values for the other two numbers, starting from the highest place value. | 2 3 7 0 6<br>2 3 6 7 8<br>same different |
| **3** In the hundreds place value, 7 is larger than 6 (or 700 is larger than 600), so 23 706 is larger than 23 678. | 23 706 > 23 678 |
| **4** Write the numbers from smallest to largest. | 2376, 23 678, 23 706 |

**7** Write each list of numbers in order from smallest to largest.

**a** 4562, 439, 45 629
**b** 35 218, 53 176, 25 786
**c** 6754, 67 554, 67 454
**d** 9103, 9130, 9013
**e** 24 701, 24 007, 24 071
**f** 613 548, 613 583, 613 538

**8** Copy and complete these number sentences by writing < or > in the space provided.

**a** 8530 ____ 7503
**b** 46 249 ____ 64 249
**c** 317 294 ____ 37 294
**d** 709 ____ 4503
**e** 5678 ____ 5876
**f** 10 462 ____ 10 248
**g** 360 ____ 306
**h** 8245 ____ 8254
**i** 214 007 ____ 210 007
**j** 989 000 ____ 998 000

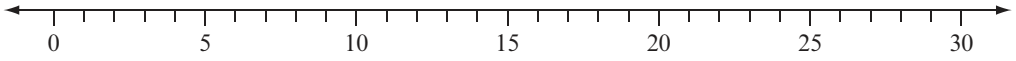
**9** Write each list of numbers in ascending order (from smallest to largest).

**a** 58, 72, 9, 40, 88, 15, 28
**b** 856, 805, 890, 806, 846
**c** 625, 9472, 6105, 10 417, 9902
**d** 2374, 23 074, 23 704, 234, 2347

**10** Write each list of numbers in descending order (from largest to smallest).

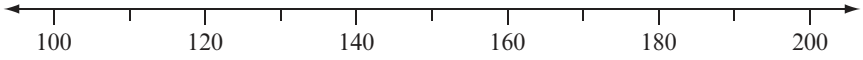
**a** 870, 8000, 87, 1800, 807
**b** 3999, 3909, 399, 309, 3099
**c** 72 156, 75 126, 75 561, 75 516
**d** 2 567 291, 256 291, 1 967 219

**11** Copy this number line and mark the position of these numbers.




**a** 20
**b** 5
**c** 11
**d** 28
**e** 2
**f** 16

**12** Copy this number line and mark the position of these numbers.




**a** 150
**b** 185
**c** 132
**d** 200
**e** 108
**f** 173

**13** Draw a number line with a scale from 2000 to 3000 and mark the position of these numbers.

**a** 2500
**b** 2850
**c** 2100
**d** 2920
**e** 2250
**f** 2070

UNDERSTANDING AND FLUENCY

**14** Sometimes when you estimate or describe amounts, you don't need to know the exact number. An approximate value can be just as valuable. For example, the crowd of 100 012 at the MCG could be described as approximately 100 000 people.




**a** Decide whether each number is closer to 200 or 300.
**i** 228 **ii** 252 **iii** 280 **iv** 219 **v** 266

**b** Decide whether each number is closer to 5000 or 6000.
**i** 5743 **ii** 5086 **iii** 5617 **iv** 5508 **v** 5499

**c** Explain your thinking for parts **a** and **b**.

## EXAMPLE 1A-4 Rounding a number to its leading (first) digit

Write an approximation for each number by rounding to its leading digit.

**a** 719 **b** 4802

### THINK

**a** Look at the second digit (1). Since it is less than 5, keep the first digit (7) and replace all other digits with zero.

**b** Look at the second digit (8). Since it is 5 or more, increase the first digit by one (4 + 1 = 5) and replace all other digits with zero.

### WRITE

**a** 719 ≈ 700

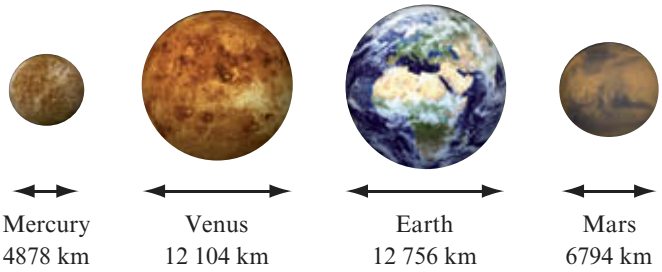
**b** 4802 ≈ 5000

**15** Write an approximation to each number by rounding to its leading digit.

**a** 784 **b** 45 **c** 103 **d** 6522
**e** 38 405 **f** 580 **g** 8521 **h** 22 199
**i** 174 011 **j** 962 **k** 73 730 **l** 95 874

PROBLEM SOLVING AND REASONING

**16** The four planets closest to the Sun are shown with their diameter measurements.




**a** Write their diameters as approximate values by rounding to the leading digit.

**b** Draw a number line and mark these approximate values on it. Label each value with the name of the planet.

**c** Use your approximate values to write a sentence comparing the sizes of the planets.

**17** **a** Write these measurements as approximate values by rounding to the leading digit.

**i** diameter of the Moon

**ii** distance from the Earth to the Moon, if the distance between them is 384 402 km.

**b** Is the distance from the Earth to the Moon approximately 10, 100 or 1000 times greater than the diameter of the Moon? Explain your thinking.




**18** A novel contains 112 pages, numbered 1 to 112. How many times does the digit 1 appear in the page numbers shown on each page of the book? Show your reasoning.

**19** **a** How many different two-digit numbers can you make from 3 and 5 if you cannot repeat digits? List them in ascending order.

**b** How many can you make if you can repeat digits? List them in descending order.

**20** **a** How many different two-digit numbers can you make from 2, 4 and 7 if you cannot repeat digits? List them in descending order.

**b** How many can you make if you can repeat digits? List them in ascending order.

CHALLENGE

**21** Consider the digits 1, 3, 6 and 9. Use these digits to write:

**a** the largest four-digit number without repeating any digits

**b** the smallest four-digit number if digits can be repeated

**c** the largest even number without repeating any digits

**d** all the four-digit numbers between 3620 and 6350 if no digits can be repeated

**e** the third largest number if digits can be repeated

**f** the second largest odd number if digits can be repeated.

**Reflect**

Explain why the place value of the digits is important when comparing the size of numbers.

# 1B Adding whole numbers

## Start thinking!

Two young elephants arrive at the zoo. To work out how much food is needed, zookeepers must calculate the total mass of the elephants. Can you calculate this for them?

You need to find the **sum** of 523 and 246. This is the same as calculating 523 + 246. There are a number of ways to do this.




1 Try at least two different ways to work out 523 + 246.

2 Compare your working for each method. Which method did you find easiest to use? Explain.

3 Write your answer to the original problem. What is the total mass of the elephants?

## KEY IDEAS

- Whole numbers can be added 'in your head' using a mental strategy. For example, you do not need to use pen and paper or a calculator to work out 500 + 200 = 700 or 42 + 16 = 58.
- In the **vertical addition method**, the numbers are written one under the other with the digits lined up according to their place value.

$$\begin{array}{r} 523 \\ +\ 246 \\ \hline \quad\quad \end{array} \leftarrow \text{answer}$$

- Always start by adding the digits in the ones column first, then the tens column, followed by the hundreds column and so on.
- Each digit in the answer also lines up in the appropriate column according to its place value.

# EXERCISE 1B Adding whole numbers

UNDERSTANDING AND FLUENCY

**1** Use a mental strategy to work out each sum.

| | | |
|---|---|---|
| **a** 50 + 30 | **b** 100 + 200 | **c** 3000 + 4000 |
| **d** 120 + 60 | **e** 360 + 20 | **f** 50 + 250 |
| **g** 25 + 13 | **h** 16 + 61 | **i** 56 + 34 |
| **j** 48 + 22 | **k** 65 + 35 | **l** 123 + 345 |

**2** Use a mental strategy to work out each sum.

| | | |
|---|---|---|
| **a** 20 + 40 + 10 | **b** 500 + 100 + 300 | **c** 4000 + 1000 + 1000 |
| **d** 18 + 12 + 50 | **e** 24 + 30 + 16 | **f** 73 + 7 + 20 |
| **g** 52 + 11 + 27 | **h** 35 + 29 + 35 | **i** 13 + 22 + 31 |
| **j** 130 + 44 + 6 | **k** 115 + 205 + 80 | **l** 203 + 203 + 203 |

## EXAMPLE 1B-1 Using the vertical addition method to add two numbers

Copy and complete this addition problem to calculate 634 + 891 using the vertical addition method.

$$\begin{array}{r} 634 \\ +\ 891 \\ \hline \\ \hline \end{array}$$

**THINK**

**1** Ones column: 4 + 1 = 5.

**2** Tens column: 3 + 9 = 12. Write 2 in the tens column of the answer and 1 in the hundreds column (shown in green).

**3** Hundreds column: 1 + 6 + 8 = 15.

**WRITE**

$$\begin{array}{r} {\scriptstyle 1}\phantom{0} \\ 634 \\ +\ 891 \\ \hline 1525 \end{array}$$

**3** Copy and complete each addition problem using vertical addition.

| | | | |
|---|---|---|---|
| **a** 243 + 715 | **b** 538 + 154 | **c** 758 + 461 | **d** 697 + 805 |
| **e** 1413 + 3706 | **f** 8520 + 5179 | **g** 2044 + 671 | **h** 56 052 + 79 418 |

**4** Use vertical addition to calculate each of these.

| | | | |
|---|---|---|---|
| **a** 641 + 478 | **b** 157 + 296 | **c** 2438 + 5160 | **d** 3762 + 1489 |
| **e** 2175 + 485 | **f** 96 + 5743 | **g** 16 407 + 782 | **h** 8009 + 35 714 |

## EXAMPLE 1B-2 Using the vertical addition method to add three numbers

Use the vertical addition method to calculate 3108 + 547 + 1619.

**THINK**

1 Set out the addition problem in columns according to place value.

2 Ones column: 8 + 7 + 9 = 24. Write 4 in the ones column of the answer and 2 at the top of the tens column (shown in green).

3 Tens column: 2 + 0 + 4 + 1 = 7.

4 Hundreds column: 1 + 5 + 6 = 12. Write 2 in the hundreds column of the answer and 1 at the top of the thousands column (shown in blue).

5 Thousands column: 1 + 3 + 0 + 1 = 5.

**WRITE**

$$\begin{array}{r} {\scriptstyle 1}\ {\scriptstyle 2} \\ 3108 \\ 547 \\ +\ 1619 \\ \hline 5274 \end{array}$$

UNDERSTANDING AND FLUENCY

**5** Use vertical addition to calculate each of these.

**a** 385 + 461 + 723 **b** 749 + 218 + 837 **c** 516 + 705 + 192

**d** 927 + 56 + 614 **e** 869 + 21 + 70 **f** 24 + 503 + 988

**g** 2764 + 8293 + 451 **h** 7605 + 246 + 38 **i** 51 + 4032 + 876

**6** Find each result without using a calculator.

**a** 23 748 + 61 392 + 53 709 **b** 82 407 + 9385 + 411

**c** 34 715 + 306 937 + 8256 **d** 947 + 600 411 + 85 103

**e** 78 294 + 19 + 6844 **f** 3007 + 458 + 27 090

**7** Find each result without using a calculator.

**a** 28 + 7640 + 459 + 7 + 834 + 2406 **b** 92 762 + 547 + 85 + 2942 + 6073

**c** 397 + 16 + 69 005 + 8 + 6255 + 20 **d** 891 546 + 6509 + 65 + 101 + 7043

**8** Check your answers to questions **6** and **7** with a calculator.

**9** You decide to train for a local cycling race. On the first weekend you cycle 32 km, on the second you cycle 45 km and on the third you cycle 59 km. To find the total distance, you can write the calculation in a number of ways.

**a** Copy and complete these sentences to describe the same calculation in different ways.

**i** Add _____ and _____ and _____ together.

**ii** What is _____ plus _____ plus _____?

**iii** Find the result of _____ + _____ + _____.

**iv** Find the sum of _____, _____ and _____.

**b** Are there any other ways of describing the calculation? List them.

**c** Perform the calculation to find the total distance.

**10** The Tour de France is an annual international cycling race that finishes in Paris, France. In 2011, Cadel Evans won the race, following the itinerary shown.

**a** Answer these questions without using a calculator.

**i** How far do the competitors ride in the first five days of the race?

**ii** How far do the competitors ride before their first rest day?

**iii** Which stage has the longest distance for a cyclist to ride in a day? Would this be the hardest day of cycling? Explain.

**iv** How far do the cyclists ride between leaving Gap and finishing in Paris?

**v** What is the total distance covered in the Tour de France?

**b** Check your answers using a calculator.

**c** Find out about this year's Tour de France. What total distance is covered?

| Date in 2011 | Start and finish | Distance |
|---|---|---|
| 2 July | Passage du Gois → Mont des Alouettes | 192 km |
| 3 July | Les Essarts → Les Essarts (team time trial) | 23 km |
| 4 July | Olonne-sur-Mer → Redon | 198 km |
| 5 July | Lorient → Mûr-de-Bretagne | 172 km |
| 6 July | Carhaix → Cap Fréhel | 165 km |
| 7 July | Dinan → Lisieux | 226 km |
| 8 July | Le Mans → Châteauroux | 218 km |
| 9 July | Aigurande → Super-Besse Sancy | 189 km |
| 10 July | Issoire → Saint-Flour | 208 km |
| 11 July | Le Lioran Cantal | rest |
| 12 July | Aurillac → Carmaux | 158 km |
| 13 July | Blaye-les-Mines → Lavaur | 168 km |
| 14 July | Cugnaux → Luz-Ardiden | 211 km |
| 15 July | Pau → Lourdes | 152 km |
| 16 July | Saint-Gaudens → Plateau de Beille | 169 km |
| 17 July | Limoux → Montpellier | 192 km |
| 18 July | Département de la Drôme | rest |
| 19 July | Saint-Paul-Trois-Châteaux → Gap | 163 km |
| 20 July | Gap → Pinerolo | 179 km |
| 21 July | Pinerolo → Galibier Serre-Chevalier | 200 km |
| 22 July | Modane Valfréjus → Alpe-d'Huez | 110 km |
| 23 July | Grenoble → Grenoble (time trial) | 42 km |
| 24 July | Créteil → Paris Champs-Élysées | 95 km |



**11** To enter the stadium for a soccer match, patrons used one of four gates. The number of people passing through the turnstiles of each gate is shown.



| Gate | Number of people |
|---|---|
| A | 8759 |
| B | 9042 |
| C | 10 365 |
| D | 11 008 |

**a** Estimate the number of people attending the match by first rounding the numbers at each gate to the leading (or first) digit.

**b** Calculate the exact number of people that attended the match.

**c** Which answer would a sports commentator be more likely to use when reporting on the match?

**12** Find two whole numbers that add to 53.

**13** Find two whole numbers that add to 386 and meet these conditions.

**a** both numbers are odd **b** both numbers are even

**14** Find three whole numbers that add to 5207. Suggest another set of three numbers that add to the same total.

**15** Depending on the numbers being added, a strategy can be used to make things easier. For example, to find the result of 24 + 37 + 16, you can add the first and third numbers together (24 + 16 = 40) and then add the second number to this total (40 + 37 = 77). Use this strategy to find each sum without using pen and paper or a calculator.

**a** 17 + 29 + 3 **b** 246 + 38 + 12

**c** 85 + 13 + 15 + 7 **d** 151 + 77 + 29

**e** 1 + 2 + 98 + 99 **f** 5 + 8 + 95 + 92

PROBLEM SOLVING AND REASONING

**16** A ski lift is carrying two boys with their ski equipment.

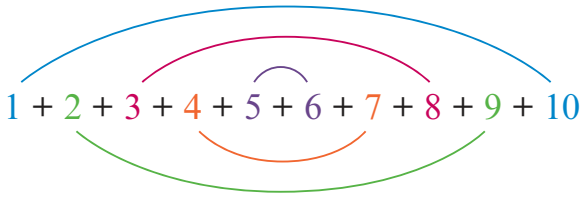
- **a** What is the total mass of the boys?
- **b** What is the total mass of their boots, snowboard and skis? The mass is shown for one of each item.
- **c** What is the total mass on the ski lift?
- **d** During summer, the ski lift is used for sightseeing. Suggest what the mass of each person and their belongings, such as a picnic basket or knapsack, could be to have a total mass of 150 kg on the lift.




CHALLENGE

**17** Consider adding the numbers from 1 to 10.

- **a** Add the numbers in order: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = ?
- **b** Now try another way by first writing the numbers in suitable pairs. That is, write the smallest number and the largest number together, then the second smallest and second largest together and so on.




Copy and complete this calculation for adding the numbers from 1 to 10

(1 + 10) + (2 + 9) + (3 + __) + (4 + __) + (5 + __)= __ + __ + __ + __ + __ = __

**18** Use the strategy of grouping numbers in suitable pairs to add the numbers from 1 to 20.

**19** **a** Use the strategy of grouping numbers in suitable pairs to add these numbers.

- **i** from 1 to 9
- **ii** from 1 to 19

- **b** Explain what is different about using this strategy with an odd number of numbers.
- **c** Use a suitable strategy to add these numbers.
  - **i** from 2 to 8
  - **ii** from 3 to 17
  - **iii** from 5 to 25

**Reflect**

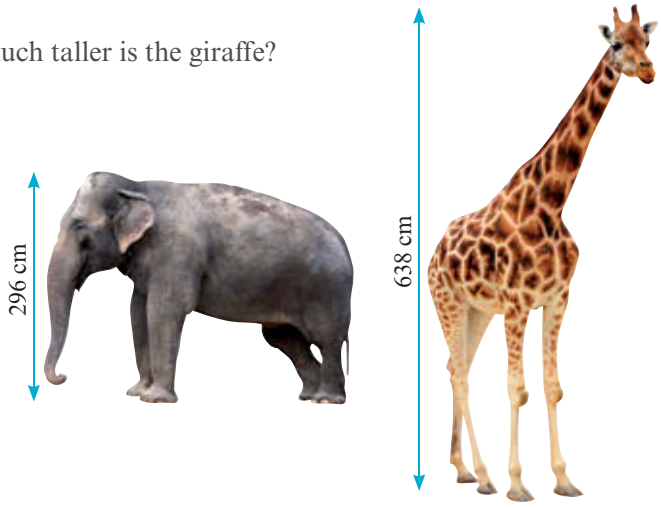
When adding numbers together, what do you need to remember?

# 1C Subtracting whole numbers

## Start thinking!

The height of these two animals is shown. How much taller is the giraffe?

You need to find the **difference** between 638 and 296. This is the same as calculating 638 − 296. There are a number of ways to do this.




1 Try at least two different ways to work out 638 − 296.

2 Compare your working for each method. Which method did you find easiest to use? Explain.

3 Write your answer to the original problem. How much taller is the giraffe?

## KEY IDEAS

- Whole numbers can be subtracted 'in your head' using a mental strategy. For example, you do not need to use pen and paper or a calculator to work out 500 − 200 = 300 or 28 − 15 = 13.
- In the **vertical subtraction method**, the numbers are written one under the other with the digits lined up according to their place value. The larger number is written above the smaller number.
- Always start by subtracting the digits in the ones column first, then the tens column, followed by the hundreds column and so on.

$$\begin{array}{r} 638 \\ -\ 296 \\ \hline \\ \hline \end{array} \leftarrow \text{answer}$$

- If the subtraction in a particular column cannot be done, rename the top digit in the next column to the left.
- Each digit in the answer also lines up in the appropriate column according to its place value.

# EXERCISE 1C Subtracting whole numbers

UNDERSTANDING AND FLUENCY

**1** Use a mental strategy to work out each difference.

| | | |
|---|---|---|
| **a** 60 − 40 | **b** 300 − 200 | **c** 8000 − 5000 |
| **d** 170 − 30 | **e** 250 − 40 | **f** 710 − 20 |
| **g** 41 − 20 | **h** 35 − 12 | **i** 96 − 44 |
| **j** 178 − 58 | **k** 270 − 35 | **l** 393 − 281 |

**2** Use a mental strategy to work out each problem.

| | | |
|---|---|---|
| **a** 50 + 20 − 10 | **b** 700 + 100 − 300 | **c** 2000 + 6000 − 4000 |
| **d** 14 + 16 − 20 | **e** 98 + 12 − 1 | **f** 570 + 25 − 80 |
| **g** 28 − 8 − 5 | **h** 45 − 10 − 11 | **i** 120 − 40 − 30 |
| **j** 36 − 12 + 8 | **k** 63 − 41 + 20 | **l** 400 − 50 + 85 |

## EXAMPLE 1C-1

**Using the vertical subtraction method to subtract two numbers with same number of digits**

Copy and complete this subtraction problem to calculate 426 − 281 using the vertical subtraction method.

$$\begin{array}{r} 426 \\ -\ 281 \\ \hline \\ \hline \end{array}$$

**THINK**

**1** Ones column: 6 − 1 = 5.

**2** Tens column: 2 − 8. You cannot subtract 8 tens from 2 tens so rename the hundreds of the first number as 3 (shown in green) and the tens as 12 (shown in blue). So 12 − 8 = 4.

**3** Hundreds column: 3 − 2 = 1.

**WRITE**

$$\begin{array}{r} {\scriptstyle 3\,12} \\ 4\not{2}6 \\ -\ 281 \\ \hline 145 \\ \hline \end{array}$$

**3** Copy and complete each subtraction problem using vertical subtraction.

| | | | |
|---|---|---|---|
| **a** 358 − 125 = ___ | **b** 867 − 496 = ___ | **c** 624 − 359 = ___ | **d** 406 − 134 = ___ |
| **e** 8527 − 2365 = ___ | **f** 5031 − 4216 = ___ | **g** 7349 − 1072 = ___ | **h** 3415 − 2838 = ___ |

**4** Use vertical subtraction to calculate each of these.

| | | | |
|---|---|---|---|
| **a** 568 − 143 | **b** 814 − 362 | **c** 456 − 234 | **d** 938 − 651 |
| **e** 624 − 185 | **f** 3846 − 1724 | **g** 7508 − 5631 | **h** 8135 − 6479 |

## EXAMPLE 1C-2

**Using the vertical subtraction method to subtract numbers that have different numbers of digits**

Use the vertical subtraction method to calculate 4083 − 627.

### THINK

1 Set out the problem in columns according to place value.

2 Ones column: 3 − 7. You cannot subtract 7 ones from 3 ones so rename the tens of the first number as 7 (shown in blue) and the ones as 13 (shown in green). So 13 − 7 = 6.

3 Tens column: 7 − 2 = 5.

4 Hundreds column: 0 − 6. Rename the thousands as 3 (shown in orange) and hundreds as 10 (shown in pink). So 10 − 6 = 4.

5 Thousands column: 3 − 0 = 3.

### WRITE

$$\begin{array}{r} {\scriptstyle 3\ 10\ 7\ 13} \\ 4\not{0}\not{8}\not{3} \\ -\ \ 627 \\ \hline 3456 \end{array}$$

UNDERSTANDING AND FLUENCY

**5** Use vertical subtraction to calculate each of these.

**a** 6327 − 215 **b** 5962 − 647
**c** 2475 − 728 **d** 7836 − 908
**e** 4025 − 462 **f** 5002 − 431
**g** 1471 − 86 **h** 9806 − 79

**6** Find the result without using a calculator.

**a** 36 274 − 28 093 **b** 508 246 − 137 651
**c** 211 537 − 36 409 **d** 81 752 − 8362
**e** 4526 − 75 **f** 614 803 − 7024
**g** 920 517 − 925 **h** 100 265 − 1078

**7** Check your answers to question **6** with a calculator.

**8** For each calculation:

**i** find an estimate of the answer by first rounding each number to its leading digit
**ii** use pen and paper to work out the exact answer
**iii** use a calculator to check the result you obtained for part **ii**.

**a** 61 + 48 − 77 **b** 254 − 123 + 448
**c** 4708 − 369 + 532 **d** 29 071 + 8275 − 17 466
**e** 77 − 25 + 89 − 60 + 41 **f** 809 + 1252 − 754 − 36

UNDERSTANDING AND FLUENCY

**9** The longest river in the world is the Nile in Africa, with a length of 6650 km. The longest river in Australia is the Darling River, with a length of 2740 km. To find the difference in length between these two rivers, you can write the calculation in a number of ways.

**a** Copy and complete these sentences to describe the same calculation in different ways.

**i** Subtract _____ from _____.

**ii** What is _____ minus _____?

**iii** Find the result of _____ − _____.

**iv** Find the difference between _____ and _____.

**b** Are there any other ways of describing the calculation? List them.

**c** Perform the calculation to find the difference between the two lengths.

**10** Josh is comparing the distance to travel by plane from Melbourne to Rome using two different routes. One journey stops at Hong Kong for refuelling, while another stops at Singapore.

**a** Find the total flight distance from Melbourne to Rome if the plane stops at Hong Kong on the way.

**b** Find the total flight distance from Melbourne to Rome if the plane stops at Singapore on the way.

| Flight sector | Distance by air |
|---|---|
| Melbourne–Hong Kong | 7435 km |
| Melbourne–Singapore | 6064 km |
| Hong Kong–Rome | 9307 km |
| Singapore–Rome | 10 048 km |

**c** Which flight distance is the shortest and by how much?

PROBLEM SOLVING AND REASONING

**11** The 162-storey building named Burj Khalifa, located in Dubai, United Arab Emirates, was completed in 2010.




**a** How does the height of this building compare with other structures? Find the difference in height between Burj Khalifa and each of the structures shown in this table.

| Structures | Date completed | Height |
|---|---|---|
| Washington Monument (Washington DC, USA) | 1884 | 169 m |
| Eiffel Tower (Paris, France) | 1889 | 300 m |
| Empire State Building (New York, USA) | 1931 | 381 m |
| Sydney Tower (Sydney, Australia) | 1981 | 309 m |
| Petronas Towers (Kuala Lumpur, Malaysia) | 1998 | 452 m |
| Taipei 101 (Taipei, Taiwan) | 2003 | 509 m |
| Q1 (Gold Coast, Australia) | 2005 | 323 m |
| Eureka Tower (Melbourne, Australia) | 2006 | 297 m |

**b** Which two structures could have their heights added to give a result closest to the height of Burj Khalifa?

**c** Which three structures could have their heights added to give a result closest to the height of Burj Khalifa?

**12** The highest mountain in the world is Mount Everest, which has a height above sea level of 8848 m.



**a** How does this compare to Mount Kosciuszko, the highest mountain in Australia, at 2228 m above sea level?

**b** Compare the height of Mount Everest with the three mountains in our solar system listed below.

| Mountain | Location | Height above surface |
|---|---|---|
| Mons Huygens | the Moon | 4700 m |
| Maxwell Montes | Venus | 11 000 m |
| Olympus Mons | Mars | 21 171 m |

**c** Find the difference in height between the mountains listed for Mars and the Moon.

**d** Find the difference in height between Mount Kosciuszko and Maxwell Montes.

**13** There are 191 people travelling on a train. At the next station, 52 people leave the train and another 48 board the train. At the following station, 69 people get off the train and 75 get on. How many people are now on the train?

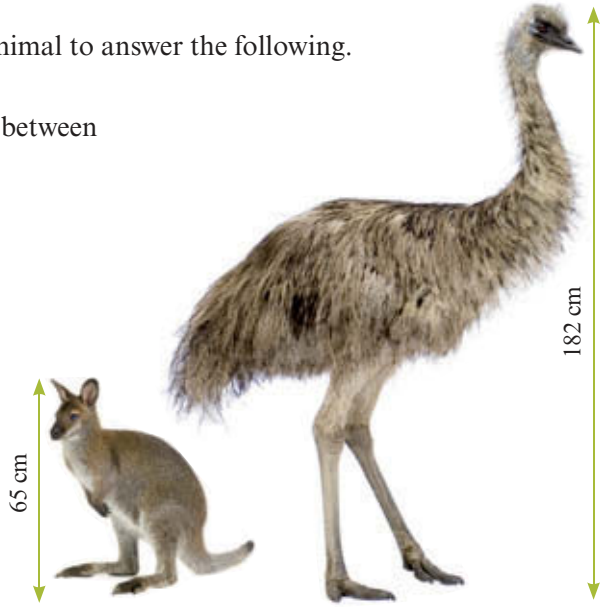
**14** Work with a classmate to measure your height and your partner's height to the nearest centimetre. What is the difference in your heights?

**15** Use the heights shown with each animal to answer the following. Clearly show your working.

**a** What is the difference in height between you and the emu?

**b** What is the difference in height between you and the wallaby?

**c** Explain why the order in which you subtract the heights in part **a** might be different from that in part **b**.



CHALLENGE

**16** In May 2000, five planets in our solar system reached a point in their orbit around the Sun where they were roughly positioned in a straight line on the same side of the Sun (as viewed from the Sun). Earth was positioned on the opposite side of the Sun. The average distance from the Sun for each planet is shown.

**a** Using this information, find the distance between these planets.

**i** Jupiter and Saturn

**ii** Mercury and Saturn

**iii** Venus and Mercury

**b** If the Sun has a diameter of about 1 392 000 km, what was the distance between Jupiter and Earth?

**c** In June 2010, Uranus, Jupiter and Mercury were roughly lined up on one side of the Sun while Venus, Mars and Saturn were lined up on the other side. At this time, what was the approximate distance between:

**i** Jupiter and Mercury?

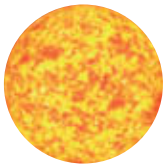
**ii** Saturn and Venus?

**iii** Jupiter and Saturn?

**d** The film *War of the Worlds* is based on the idea that Martians travelled to Earth when the distance between Mars and Earth was the shortest. About what distance would the Martian spacecraft have travelled?



Earth
149 600 000 km



Sun



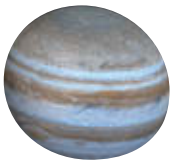
Mercury
57 909 000 km



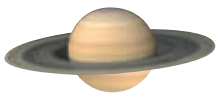
Venus
108 200 000 km



Mars
227 940 000 km



Jupiter
778 400 000 km



Saturn
1 423 600 000 km

**17** Find two numbers that, when subtracted from each other, produce a result of 43.

**18** Find two numbers that have a difference of 168 and meet these conditions.

**a** both numbers are odd

**b** both numbers are even

**19** Find two numbers that have a difference of 286 and meet these conditions.

**a** one number must be more than twice the other

**b** one number must be triple the other

**Reflect**

When subtracting numbers, what do you need to remember?

# 1D Multiplying whole numbers

## Start thinking!

Pandas spend around 16 hours a day eating up to 18 kg of bamboo leaves, stems and shoots. For a group of nine pandas, how much food would be eaten in one day?



You need to find the **product** of 18 and 9. This is the same as calculating 18 × 9.

1 Work out 18 × 9. Describe the method you used to obtain your answer.

2 Write your answer to the original problem. How much food will nine pandas eat in a day?

3 Use this result to calculate how much food these pandas will eat in 28 days. Describe the method you used.

## KEY IDEAS

- Use short multiplication when multiplying by a single-digit number.
- When multiplying a whole number by 10, the result will have a zero in the ones position. For example, 3 × 10 = 30.
- When multiplying a whole number by 100, the result will have a zero in both the ones and tens positions. For example, 3 × 100 = 300.
- When multiplying a whole number by 1000, the result will have a zero in the ones, tens and hundreds positions. For example, 3 × 1000 = 3000.
- Use long multiplication when multiplying by a number with more than one digit.
- In long multiplication, the two numbers are arranged vertically in columns according to place value.

```
  34
× 26   ← this is the same as 20 + 6
 204   ← this is the result of multiplying 34 by 6
 680   ← this is the result of multiplying 34 by 20
 884     (Notice that you place a zero in the
         ones column and then multiply 34 by 2.)
```

The final answer is obtained by adding the two multiplication results together.

# EXERCISE 1D Multiplying whole numbers

**1** Use short multiplication to work out each product.

| | | | |
|---|---|---|---|
| **a** 74 × 8 | **b** 93 × 2 | **c** 45 × 3 | **d** 26 × 7 |
| **e** 192 × 5 | **f** 804 × 9 | **g** 532 × 6 | **h** 281 × 4 |
| **i** 6158 × 3 | **j** 3075 × 7 | **k** 24 159 × 5 | **l** 413 083 × 9 |

## EXAMPLE 1D-1 Multiplying by 10, 100 or 1000

Write the result of each problem.

**a** 62 × 10 **b** 153 × 100 **c** 27 × 1000

| THINK | WRITE |
|---|---|
| **a** Use the short cut of placing 0 in the ones position and moving 2 to the tens position and 6 to the hundreds position. Since there is one zero in 10, move the digits one place to the left. | **a** 62 × 10 = 620 |
| **b** Use the short cut. Since there are two zeros in 100, move the digits two places to the left. | **b** 153 × 100 = 15 300 |
| **c** Use the short cut. Since there are three zeros in 1000, move the digits three places to the left. | **c** 27 × 1000 = 27 000 |

UNDERSTANDING AND FLUENCY

**2** Write the result of each problem.

| | | | |
|---|---|---|---|
| **a** 5 × 10 | **b** 32 × 10 | **c** 639 × 10 | **d** 51 623 × 10 |
| **e** 7 × 100 | **f** 2784 × 100 | **g** 48 × 100 | **h** 103 × 100 |
| **i** 85 × 1000 | **j** 936 × 1000 | **k** 15 × 1000 | **l** 7020 × 1000 |

**3** Write the result of each problem.

| | | | |
|---|---|---|---|
| **a** 4 × 3 × 10 | **b** 6 × 7 × 10 | **c** 2 × 10 × 9 | **d** 15 × 4 × 100 |
| **e** 28 × 6 × 100 | **f** 13 × 100 × 7 | **g** 8 × 5 × 1000 | **h** 41 × 3 × 1000 |
| **i** 356 × 1000 × 2 | **j** 64 × 100 × 9 | **k** 125 × 1000 × 4 | **l** 7103 × 5 × 10 |

**4** Work out each product. Use the strategy of multiplying by 10 or 100 or 1000. For example, 60 is the same as 10 × 6 or 6 × 10.

| | | | |
|---|---|---|---|
| **a** 8 × 60 | **b** 4 × 30 | **c** 12 × 70 | **d** 93 × 40 |
| **e** 217 × 50 | **f** 6345 × 20 | **g** 52 × 800 | **h** 428 × 300 |
| **i** 7493 × 600 | **j** 86 × 4000 | **k** 319 × 7000 | **l** 1542 × 2000 |

**5** Rewrite each calculation so that the second number is written in place-value notation. For example, 94 × 57 = 94 × (50 + 7).

**a** 65 × 48 **b** 283 × 54 **c** 415 × 23 **d** 1059 × 18

## EXAMPLE 1D-2 Using long multiplication to multiply by a two-digit number

Calculate 68 × 37 using long multiplication.

### THINK

**1** Write the numbers vertically. 37 = 30 + 7, so multiplying by 37 is the same as multiplying by 7 then 30 and adding the results.

**2** Multiply 68 by 7 (68 × 7 = 476).

**3** Next multiply 68 by 30. Multiply by 10 (place a zero in the ones column under the previous result) and then multiply by 3 (68 × 3 = 204).

**4** Add the results of the two multiplications and write the answer underneath.

### WRITE

$$\begin{array}{r} {\scriptstyle 2\ \ \ } \\ {\scriptstyle 5\ \ \ } \\ 68 \\ \times\ \ 37 \\ \hline 476 \\ 2040 \\ \hline 2516 \end{array}$$

So 68 × 37 = 2516.

UNDERSTANDING AND FLUENCY

**6** Calculate each problem using long multiplication.

**a** 38 × 15 **b** 62 × 24 **c** 46 × 32 **d** 85 × 73

**e** 123 × 37 **f** 231 × 56 **g** 782 × 49 **h** 506 × 81

**i** 2654 × 42 **j** 9417 × 25 **k** 31 464 × 78 **l** 10 851 × 94

**7** Perform the calculations listed in question **5** using long multiplication.

**8** Check your answers to questions **6** and **7** using a calculator.

**9** From a part time job, James has saved $37 each week. To find the amount he has saved after 28 weeks, you can write the calculation in a number of ways.

**a** Copy and complete these sentences to describe the same calculation in different ways.

**i** Multiply ____ and ____ together. **ii** What is ____ times ____?

**iii** Find the result of ____ × ____. **iv** Find the product of ____ and ____.

**v** Find ____ lots of ____.

**b** Are there any other ways of describing the calculation? List them.

**c** Perform the calculation to find the total amount that James has saved.

**10** Rewrite each calculation so that the second number is written in place-value notation. For example, 716 × 245 = 716 × (200 + 40 + 5).

**a** 538 × 124 **b** 361 × 253 **c** 4825 × 627 **d** 9141 × 382

## EXAMPLE 1D-3 Using long multiplication to multiply by a three-digit number

Calculate 543 × 286 using long multiplication.

**THINK**

1. Write the numbers vertically.
2. Multiply 543 by 6 (543 × 6 = 3258).
3. Next multiply 543 by 80. Multiply by 10 (place a zero in the ones column under the previous result) and then multiply by 8 (543 × 8 = 4344).
4. Next multiply 543 by 200. Multiply by 100 (place two zeros under the previous result) and then multiply by 2 (543 × 2 = 1086).
5. Add the results of the three multiplications and write the answer underneath.

**WRITE**

$$\begin{array}{r} {\scriptstyle 3\,2} \\ {\scriptstyle 2\,1} \\ 543 \\ \times\ 286 \\ \hline 3258 \\ 43\,440 \\ 108\,600 \\ \hline 155\,298 \end{array}$$

So 543 × 286 = 155 298.

UNDERSTANDING AND FLUENCY

**11** Calculate each problem using long multiplication.

| | | | |
|---|---|---|---|
| **a** 346 × 125 | **b** 865 × 347 | **c** 624 × 253 | **d** 937 × 625 |
| **e** 497 × 516 | **f** 702 × 281 | **g** 1302 × 374 | **h** 5896 × 892 |
| **i** 2475 × 403 | **j** 70 219 × 210 | **k** 6555 × 9385 | **l** 10 461 × 7254 |

**12** Perform the calculations listed in question **10** using long multiplication.

**13** Check your answers to questions **11** and **12** using a calculator.

**14** Copy this table into your workbook.

| First number × second number | Product | Number of zeros in the first number | Number of zeros in the second number | Number of zeros in the product |
|---|---|---|---|---|
| 10 × 10 | | | | |
| 100 × 10 | | | | |
| 1000 × 10 | | | | |
| 10 000 × 10 | | | | |

**a** Write your answer to each multiplication in the product column.

**b** Complete each row by writing the number of zeros in the first number, the second number and the product.

**c** Can you see a pattern? Explain how this pattern provides a quick method of doing multiplications like this.

**d** Use this method to calculate each product.

**i** 10 × 100 **ii** 100 × 100 **iii** 100 × 1000 **iv** 1000 × 1000

UNDERSTANDING AND FLUENCY

**15** **a** To find 4000 × 100, the calculation can be written as 4 × 1000 × 100 or 1000 × 100 × 4.

**i** Calculate 1000 × 100.

**ii** Multiply this result by 4 to obtain your final answer.

**b** Use this strategy to calculate each product.

**i** 300 × 10 **ii** 700 × 100 **iii** 6000 × 100 **iv** 2000 × 1000

**v** 100 × 50 **vi** 1000 × 400 **vii** 10 000 × 8000 **viii** 100 × 9000

**16** **a** One way to calculate 300 × 20 is to calculate 3 × 100 × 2 × 10 or 3 × 2 × 100 × 10.

**i** Calculate 3 × 2. **ii** Calculate 100 × 10.

**iii** Multiply the results you found in parts **i** and **ii** and write your answer to 300 × 20.

**b** Use this strategy to calculate each product.

**i** 400 × 20 **ii** 3000 × 30 **iii** 200 × 600 **iv** 9000 × 500

**v** 70 × 800 **vi** 600 × 4000 **vii** 30 000 × 7000 **viii** 800 × 20 000

**17** Find each product.

**a** 40 × 10 × 200 **b** 60 × 900 × 3000 **c** 700 × 20 × 400 **d** 5000 × 300 × 80

**18** Another strategy for multiplying is to look for pairs of numbers that are easy to multiply together first. For example, 25 × 18 × 4 is the same as 25 × 4 × 18. The product of 25 × 4 is 100, which makes the calculation 100 × 18 easier to work out without using long multiplication.

Calculate each of these without using a calculator or long multiplication.

**a** 25 × 18 × 4 **b** 5 × 679 × 2 **c** 20 × 5 × 4016 **d** 793 × 50 × 2

**e** 358 × 25 × 4 **f** 250 × 4 × 17 **g** 2 × 891 × 500 **h** 6055 × 5 × 200

**19** **a** Estimate the result for each problem by first rounding each number to its leading digit, then multiplying. (Hint: use your strategy from question **16**.)

**i** 591 × 82 **ii** 2175 × 93 **iii** 7856 × 304 **iv** 63 019 × 5647

**b** Check how close your estimations are to the exact result.

PROBLEM SOLVING AND REASONING

**20** A human heart beats around 72 times in a minute.

**a** How many times does it beat in one hour?

**b** How many times does it beat in one day?

**21** To measure your pulse rate, place two fingers (not your thumb) on the inside of your wrist or at the side of your neck.



**a** Count the number of beats in 20 seconds, using a stopwatch or clock.

**b** A pulse rate is the number of beats in 1 minute. What is your pulse rate?

**c** Use your pulse rate to calculate the approximate number of times your heart beats in 1 hour.

**d** About how many times does your heart beat in 1 day?

PROBLEM SOLVING AND REASONING

**22** A school has 25 students in each of its 32 classes.

**a** How many students are enrolled at this school?

**b** On a particular day, 15 classes each have three students away from school and 11 classes have two students away. How many students are at school on this day?

**23** Over the school holidays, a team of eight teenagers deliver take-away menus to homes near a pizza restaurant.
How many menus are delivered in a week if each teenager visits 46 homes each day?

**24** Galápagos tortoises move extremely slowly, covering a distance of about 260 m in 1 hour. These tortoises can store food and water so well that they can go without eating or drinking for up to a year.



**a** What distance could a tortoise travel in 4 hours?

**b** Compare this result with the distance that a human could walk in 4 hours, assuming humans walk about 4500 m per hour.

**c** If a tortoise didn't eat or drink for 1 year, how long would this be in hours, assuming there are 365 days in a year?

**d** The longest lifespan on record belongs to a male tortoise kept in a British military fort for 154 years until he died from an accidental fall.
Work out how long he lived in:

**i** months **ii** days **iii** hours **iv** minutes **v** seconds

CHALLENGE

**25** **a** Calculate how old you will be at your next birthday in:

**i** months **ii** days **iii** hours **iv** minutes **v** seconds

**b** Use your pulse rate from question **21** to work out the approximate number of heart beats you will have experienced in your life up to your next birthday.

**NOTE** Assume there are 365 days in a year. As a super challenge, consider leap years in your working.

**c** About how many hours are there between now and your next birthday?

**26** Earth travels a distance of about 2 575 200 km each day.
Assuming there are about 365 days in a year, estimate the distance Earth travels in one complete orbit around the Sun; that is, estimate how far it travels in one year.
(Hint: first round each number to its leading digit.)

**Reflect**

Why is the skill of multiplying by 10, 100, 1000, etc. so useful?

# 1E Dividing whole numbers

## Start thinking!



As a treat, chimpanzees at the zoo are given some bananas to share. If there are 20 bananas to be shared among five chimpanzees, how many will each chimp get?

You need to find the result of dividing 20 by 5 or 20 ÷ 5.
This result is called the **quotient**.

1 Write the quotient for 20 ÷ 5.

2 Explain how you can check your answer using multiplication.

3 If there were six chimpanzees instead, how many bananas would each receive? Write the quotient and the remainder for 20 ÷ 6.

$$20 \div 6 = ?$$

**dividend** ÷ **divisor** = quotient and remainder

## KEY IDEAS

- When dividing, always start with the leading or first digit. This is different from adding, subtracting and multiplying, where you start with the ones digits.
- Use short division when dividing by a number between 1 and 10.
- Use long division when dividing by a number larger than 10.
- In both methods, you set up the calculation for 871 ÷ 6 like this.

divisor → $6\overline{)871}$ ← dividend; ← quotient and remainder

- Calculating the remainder after each division stage is an important step in both methods. In the calculations shown, the remainders at each stage are in colour.
- One number is said to divide exactly into another when there is no remainder (the remainder is zero).

**Short division**

$$6\overline{)8^{2}7^{3}1}\quad 145 \text{ remainder } 1$$

**Long division**

$$\begin{array}{r} 145 \text{ remainder } 1 \\ 6\overline{)871} \\ -6\downarrow\phantom{1} \\ \hline 27 \\ -24\downarrow \\ \hline 31 \\ -30 \\ \hline 1 \end{array}$$

# EXERCISE 1E Dividing whole numbers

UNDERSTANDING AND FLUENCY

**1** For each division, identify:

**i** the dividend **ii** the divisor **iii** the quotient **iv** the remainder.

**a** 9 ÷ 2 = 4 remainder 1 **b** 17 ÷ 7 = 2 remainder 3 **c** 30 ÷ 5 = 6

**2** Copy and complete this table. The first row has been done for you.

| | think | quotient | quotient × divisor | remainder |
|---|---|---|---|---|
| 7 ÷ 3 | How many 3s in 7? | 2 | 2 × 3 = 6 | 7 − 6 = 1 |
| 19 ÷ 5 | How many 5s in 19? | 3 | 3 × 5 = ____ | 19 − ____ = 4 |
| 33 ÷ 4 | How many 4s in 33? | | ____ × 4 = _ | 33 − ____ = ____ |
| 62 ÷ 15 | How many 15s in 62? | 4 | 4 × 15 = ____ | 62 − ____ = ____ |

**3** Find the quotient and the remainder for each division problem.

**a** 13 ÷ 2 **b** 23 ÷ 5 **c** 17 ÷ 3 **d** 25 ÷ 10

**e** 38 ÷ 8 **f** 26 ÷ 4 **g** 66 ÷ 9 **h** 48 ÷ 6

## EXAMPLE 1E-1 Using short division

Use short division to calculate each division problem. **a** 98 ÷ 4 **b** 459 ÷ 7

### THINK

**a** **1** How many 4s in 9? (2) Write 2 above 9 in the quotient line. Work out the remainder. 4 × 2 = 8, so remainder = 9 − 8 = 1.

**2** How many 4s in 18? (4) Write 4 above 8 in the quotient line. Work out the remainder. 4 × 4 = 16, so remainder = 18 − 16 = 2.

**3** Write the answer.

**b** **1** How many 7s in 4? (none) How many 7s in 45? (6) Write 6 above 5 in the quotient line. Work out the remainder. 7 × 6 = 42, so remainder = 45 − 42 = 3.

**2** How many 7s in 39? (5) Write 5 above 9 in the quotient line. Work out the remainder. 7 × 5 = 35, so remainder = 39 − 35 = 4.

**3** Write the answer.

### WRITE

**a** 2 4 remainder 2
4 ) 9 1 8

98 ÷ 4 = 24 remainder 2

**b** 6 5 remainder 4
7 ) 45 3 9

459 ÷ 7 = 65 remainder 4

**4** Use short division to calculate each division problem.

**a** 538 ÷ 4 **b** 756 ÷ 6 **c** 172 ÷ 3 **d** 1229 ÷ 5

**e** 3048 ÷ 8 **f** 9812 ÷ 7 **g** 67 059 ÷ 2 **h** 286 347 ÷ 9

**5** Another way is to write a division calculation is as a fraction. For example, 57 ÷ 3 is the same as $\frac{57}{3}$. The vinculum (horizontal line between the two numbers) replaces the division sign. Perform each division.

**a** $\frac{63}{9}$ **b** $\frac{1470}{6}$ **c** $\frac{658}{7}$ **d** $\frac{1251}{3}$

**6** Copy this table.

| First number ÷ second number | $\frac{\text{first number}}{\text{second number}}$ | Quotient | Number of zeros in the first number | Number of zeros in the second number | Number of zeros in the quotient |
|---|---|---|---|---|---|
| 10 ÷ 10 | $\frac{10}{10}$ | | | | |
| 100 ÷ 10 | $\frac{100}{10}$ | | | | |
| 1000 ÷ 10 | $\frac{1000}{10}$ | | | | |
| 10 000 ÷ 10 | $\frac{10\,000}{10}$ | | | | |
| 100 ÷ 100 | $\frac{100}{100}$ | | | | |
| 1000 ÷ 100 | $\frac{1000}{100}$ | | | | |
| 10 000 ÷ 100 | $\frac{10\,000}{100}$ | | | | |

**a** Write your answer to each division in the quotient column.

**b** Complete each row of the table by writing the number of zeros in the first number, the second number and the quotient.

**c** Can you see a pattern? Explain how this pattern provides a shortcut for calculations like this.

**d** Use this method to work out the quotient for each calculation.

**i** 10 000 ÷ 10 **ii** 100 000 ÷ 100

**iii** 1000 ÷ 1000 **iv** 10 000 ÷ 1000

**7 a** To work out 5000 ÷ 100, the calculation can be written as $\frac{5000}{100}$ or $\frac{5 \times 1000}{100}$ or $5 \times \frac{1000}{100}$.

**i** Calculate 1000 ÷ 100.

**ii** Multiply this result by 5 to obtain your final answer.

**b** Use this strategy to work out the quotient for each calculation.

**i** 200 ÷ 10 **ii** 600 ÷ 100

**iii** 9000 ÷ 100 **iv** 4000 ÷ 1000

**v** 800 ÷ 10 **vi** 3000 ÷ 100

**vii** 50 000 ÷ 1000 **viii** 6000 ÷ 10

**8** **a** One way to work out 600 ÷ 20 is to write the calculation as $\frac{600}{20}$ or $\frac{6 \times 100}{2 \times 10}$ or $\frac{6}{2} \times \frac{100}{10}$.

**i** Calculate 6 ÷ 2. **ii** Calculate 100 ÷ 10.

**iii** Multiply the results obtained in parts **i** and **ii** and write your answer to 600 ÷ 20.

**b** Use this strategy to work out each division.

**i** $\frac{900}{30}$ **ii** $\frac{8000}{20}$

**iii** $\frac{1200}{60}$ **iv** $\frac{25\,000}{500}$

**v** 6000 ÷ 300 **vi** 80 000 ÷ 4000

**vii** 70 000 ÷ 700 **viii** 1600 ÷ 80

**9** **a** Estimate the quotient to each division problem by first rounding each number to its leading digit before dividing. (Hint: use your strategy from question **8**.)

**i** 627 ÷ 33 **ii** 5940 ÷ 18

**iii** 3852 ÷ 214 **iv** 83 490 ÷ 3795

**b** Use a calculator to check how close your estimations are to the exact result.

## EXAMPLE 1E-2 Using long division

Use long division to calculate 6492 ÷ 19.

### THINK

**1** How many 19s in 6? (none) How many 19s in 64? (3) Write 3 (shown in orange) above 4 in the quotient line. Work out the remainder. 19 × 3 = 57, so remainder = 64 − 57 = 7.

**2** Bring down 9 and write it beside the remainder of 7. This makes the next number to divide into become 79.

**3** How many 19s in 79? (4) Write 4 (shown in blue) above 9 in the quotient line. Work out the remainder. 19 × 4 = 76, so remainder = 79 − 76 = 3.

**4** Bring down 2 and write it beside the remainder of 3. This makes the next number to divide into become 32.

**5** How many 19s in 32? (1) Write 1 (shown in green) above 2 in the quotient line. Work out the remainder. 19 × 1 = 19, so remainder = 32 − 19 = 13.

**6** Write the answer.

### WRITE

```
      3
19 ) 6 4 9 2
   − 5 7
     7
```

```
      3 4
19 ) 6 4 9 2
   − 5 7 ↓
     7 9
   − 7 6
     3
```

```
      3 4 1 remainder 13
19 ) 6 4 9 2
   − 5 7 ↓ ↓
     7 9
   − 7 6 ↓
     3 2
   − 1 9
     1 3
```

6492 ÷ 19 = 341 remainder 13

UNDERSTANDING AND FLUENCY

**10** Use long division to calculate each division problem.

**a** 542 ÷ 21 **b** 739 ÷ 18 **c** 884 ÷ 26 **d** 7462 ÷ 35

**e** 1658 ÷ 43 **f** 37 610 ÷ 50 **g** 43 803 ÷ 31 **h** 90 300 ÷ 28

**11** Work out each division.

**a** $\frac{272}{17}$ **b** $\frac{3350}{25}$ **c** $\frac{47\,136}{32}$ **d** $\frac{36\,088}{52}$

**12** Georgia is helping to arrange a birthday party for her younger brother. She has 165 sweets to share among 15 party bags. To work out the number of sweets in each party bag, you can write the calculation in a number of ways.



**a** Copy and complete these sentences to describe the same calculation in different ways.

**i** Divide _____ by _____.

**ii** How many times does _____ go into _____?

**iii** Find the result of _____ ÷ _____.

**iv** Find the quotient of _____ divided by _____.

**b** Are there any other ways of describing the calculation? List them.

**c** Find the number of sweets in each party bag.

PROBLEM SOLVING AND REASONING

**13** Tanya has 144 minutes of 'talk time' left on her mobile phone. She wants to phone six of her friends. How long should a call be if she talks to each friend for the same amount of time?

**14** Chris can type messages via SMS on his mobile phone at a rate of 68 characters each minute.

**a** How long would it take him to type a message of 272 characters?

**b** Explain how you could check the answer to this division problem using multiplication.



PROBLEM SOLVING AND REASONING

**15** Charlotte swam 1500 m in a 50-m swimming pool. How many laps of the pool did she complete?

**16** The ancestors of the common wombat and the southern hairy-nosed wombat were the giant wombats (diprotodons) that lived in Australia from two million years ago, disappearing around 40 000 years ago. A giant wombat was about 3 m long and 180 cm high, with a mass of about 2000 kg. The mass of a common wombat is about 37 kg. The photo shows a hairy-nosed wombat and its mass.




**a** How many common wombats would be approximately equivalent in mass to one giant wombat?

**b** How many hairy-nosed wombats would be approximately equivalent in mass to one giant wombat?

**17** Isobel and Conor are helping to make up food parcels for a charity organisation. There are 1350 bags of rice to be shared among 84 food parcels.

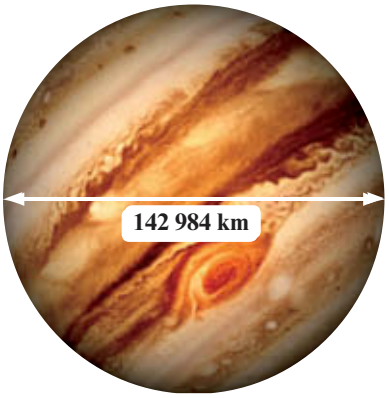
**a** How many bags of rice will be in each food parcel?

**b** How many bags of rice will be left over?

**c** Suggest the number of food parcels that could be made up with an equal share of the bags of rice so that no bags are left over.

CHALLENGE

**18** Earth orbits the Sun in about 365 days, while the planet Jupiter completes an orbit around the Sun in about 4333 Earth days. Earth has a diameter of approximately 12 756 km.




**a** Estimate the number of times that Earth would complete a full orbit around the Sun for each one of Jupiter's orbits. (Hint: first round each number to its leading digit.)

**b** If Earth was drawn to the same scale as Jupiter is shown in the photo, about how many Earths would fit across the equator of Jupiter?

**Reflect**

Why is finding the remainder so useful in each step of a long division calculation?

# 1F Powers and square roots

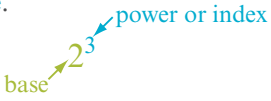
## Start thinking!

**1** A short way of writing the repeated multiplication $2 \times 2 \times 2$ is $2^3$, which is read as '2 to the power of 3'.

**a** What number is being repeatedly multiplied? This number is called the **base**.

**b** How many times has the same number (or the base) been written in the multiplication problem? This value is called the **power** or the **index**.

**c** Work out the product of $2 \times 2 \times 2$. This result is called the **basic numeral**.




**2** Consider $3^5$, which is a repeated multiplication written in **index form** or **index notation**.

**a** How is this number read?

**b** What is the base?

**c** What is the index or power?

**d** How many times is the base written, in the expanded form of the number?

**e** Carry out the repeated multiplication to find the basic numeral.

**f** Copy and complete this statement.

$3^5 = 3 \times ____ \times ____ \times ____ \times ____ = ____$

index form | expanded form | basic numeral

## KEY IDEAS

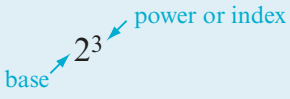
- A repeated calculation can be written in index form or index notation.

  $2^3 = 2 \times 2 \times 2 = 8$

  index form | expanded form | basic numeral

  **NOTE** The plural of the word index is *indices*. For example, the powers or indices in the calculation $2^3 + 2^5$ are 3 and 5.

- The base is the number to be repeatedly multiplied and the power or index indicates the number of times the base is written. (Other names for power or index are *exponent* or *logarithm*.)

- To square a number is to multiply it by itself or raise it to the power of 2. For example, $3^2 = 9$.

- To find the **square root** of a number is to find the number that when squared (raised to the power of 2) results in the original number. For example, $\sqrt{9} = 3$.

# EXERCISE 1F Powers and square roots

**1** The number $6^5$ is in index form.
- **a** What does the 6 indicate?
- **b** What does the 5 indicate?

## EXAMPLE 1F-1 Converting to index form

Write each repeated multiplication problem in index form.

**a** $8 \times 8 \times 8 \times 8 \times 8 \times 8$ **b** $3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4$

**THINK**

**a** Write the number that is being repeatedly multiplied as the base (8). Count the number of times the base is written and write it as the index (6).

**b** Notice that there are two different numbers to be repeatedly multiplied. Write the first base (3) with its power (2) multiplied to the second base (4) with its power (5).

**WRITE**

**a** $8 \times 8 \times 8 \times 8 \times 8 \times 8$
$= 8^6$

**b** $3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4$
$= 3^2 \times 4^5$

UNDERSTANDING AND FLUENCY

**2** Write each repeated multiplication problem in index form.
- **a** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
- **b** $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
- **c** $11 \times 11 \times 11$
- **d** $20 \times 20 \times 20 \times 20 \times 20$
- **e** $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
- **f** $300 \times 300 \times 300 \times 300$

**3** How would you read each answer you obtained in question **2**? Write each answer using words.

**4** Write each multiplication problem in index form.
- **a** $7 \times 7 \times 7 \times 9 \times 9 \times 9 \times 9$
- **b** $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2$
- **c** $3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
- **d** $8 \times 8 \times 8 \times 8 \times 13 \times 13$
- **e** $2 \times 2 \times 2 \times 2 \times 6 \times 6 \times 7 \times 7$
- **f** $19 \times 19 \times 23 \times 23 \times 23 \times 23 \times 31$

## EXAMPLE 1F-2 Calculating the value of a number in index form

Write $9^3$ in expanded form and calculate its value.

| THINK | WRITE |
|---|---|
| **1** Write the repeated multiplication in expanded form. Identify the base (9) and the index (3). This means that 9 is written 3 times. | $9^3 = 9 \times 9 \times 9$ |
| | $= 81 \times 9$ |
| **2** Calculate the basic numeral. | $= 729$ |

UNDERSTANDING AND FLUENCY

**5** Write each of these in expanded form and calculate its value.

**a** $3^2$ **b** $2^5$ **c** $7^3$ **d** $5^2$

**e** $9^4$ **f** $4^3$ **g** $1^5$ **h** $6^3$

**6** Copy and complete this table.

| Index form | Base | Index or power | Expanded form | Basic numeral |
|---|---|---|---|---|
| $2^4$ | 2 | 4 | $2 \times 2 \times 2 \times 2$ | |
| $6^3$ | 6 | | | |
| | | 2 | $7 \times 7$ | |
| | | | $5 \times 5 \times 5$ | |

**7** Which number is bigger: $3^5$ or $5^3$?

**8** Arrange the numbers in each list from smallest to largest. You may like to use a calculator to help you.

**a** $3^2, 4^5, 2^3, 5^4$ **b** $7^6, 6^7, 1^{50}, 50^3$

**c** $3^{10}, 9^4, 6^5, 10^3$ **d** $8^3, 3^8, 4^6, 12^2$

## EXAMPLE 1F-3 Calculating the product of two numbers in index form

Write $2^4 \times 3^2$ in expanded form and calculate its value.

| THINK | WRITE |
|---|---|
| **1** Write the multiplication calculation in expanded form. | $2^4 \times 3^2 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$ |
| **2** Perform the calculation. | $= 16 \times 9$ |
| | $= 144$ |

**9** Write each of these in expanded form and calculate its value.

**a** $2^3 \times 4^2$ **b** $3^4 \times 5^2$ **c** $8^2 \times 10^3$ **d** $3^5 \times 1^4$

**e** $2^6 \times 7^2$ **f** $10^2 \times 3^3$ **g** $9^4 \times 11^2$ **h** $6^2 \times 3^2$

**10** Calculate each of these by first working out the value of any numbers written in index form.

**a** $3^4 + 2^3$ **b** $10^6 + 5^2$ **c** $8^2 - 2^6$ **d** $4^3 - 1^9$

**e** $10^2 \div 5^2$ **f** $6^4 \div 3^3$ **g** $4^2 + 2^4 + 3^3$ **h** $9^2 + 2^2 - 1^5$

**11 a** Copy and complete this table.

| Index form | Expanded form | Basic numeral | Number of zeros in the basic numeral |
|---|---|---|---|
| $10^1$ | 10 | 10 | 1 |
| $10^2$ | $10 \times 10$ | 100 | |
| $10^3$ | | | |
| $10^4$ | | | |
| $10^5$ | | | |

**b** Can you see a pattern? This pattern provides a quick method of finding the value of a number in index form with a base of 10. Describe this method.

**c** Use this method to write the basic numeral for each of these.

**i** $10^8$ **ii** $10^6$ **iii** $10^{15}$ **iv** $10^{10}$

**d** What is the basic numeral for $10^0$? Use the pattern seen in the table.

**e** Write each number in index form with a base of 10 and the appropriate power.

**i** 100 **ii** 10 000 **iii** 1 **iv** 10 000 000

**12** For this task, you will need at least 25 counters. You may like to work with one or more classmates.

**a** Arrange four counters to make a square pattern. How many counters are there in each row and column of the square? Draw a diagram of this.

**b** Try a different number of counters to make another square pattern. How many counters are there in each row and column? Draw a diagram of this.

**c** Repeat part **b** with other groups of counters to make all the possible squares you can with the counters you have. Draw a diagram of each square pattern.

**d** Each number you have modelled with counters in parts **a**, **b** and **c** is called a **square number**. Why do you think this is? Write the square number next to its matching diagram.

**e** How many square numbers are there from 1 to 25? (Hint: is 1 a square number? Does it have the same number of counters in each row and column?)

**f** List the next five square numbers.

**13** **a** Copy and complete this table.

| Index form | Expanded form | Basic numeral |
|---|---|---|
| $1^2$ | $1 \times 1$ | 1 |
| $2^2$ | $2 \times 2$ | |
| $3^2$ | | 9 |
| $4^2$ | | |
| $5^2$ | | |

**b** Compare the square numbers you found in question **12** with the basic numerals you have listed in the table. What do you notice?

**c** The number $3^2$ is read as '3 to the power of 2'. It can also be read as '3 squared' or 'the square of 3'. Can you see why this is? Explain.

**d** Write each description in both index form and as a basic numeral.

**i** 9 squared **ii** 4 squared

**iii** the square of 10 **iv** the square of 7

**14** Find the value of each of these.

**a** $11^2$ **b** $25^2$ **c** $100^2$ **d** $32^2$

**e** $3^2 + 1^2$ **f** $8^2 - 6^2$ **g** $7^2 + 9^2$ **h** $10^2 - 5^2$

**15** What number, when it is squared, gives each of these results?

**a** 25 **b** 81 **c** 64 **d** 9

**e** 1 **f** 100 **g** 4 **h** 36

**16** Another way of asking 'what number is squared to give a result of 25?' is 'find the square root of 25'.

**a** What is the square root of 25?

**b** A short way of writing the square root of 25 is $\sqrt{25}$. Notice that the symbol $\sqrt{}$ means 'the square root of' the given number. Write this problem and your answer using the square root symbol.

**c** Write each of these using the square root symbol and find its value.

**i** the square root of 16 **ii** the square root of 81

**iii** the square root of 4 **iv** the square root of 100

**17** Find the value of each of these.

**a** $\sqrt{9}$ **b** $\sqrt{36}$ **c** $\sqrt{64}$ **d** $\sqrt{1}$

**e** $\sqrt{49}$ **f** $\sqrt{121}$ **g** $\sqrt{144}$ **h** $\sqrt{400}$

**18** Without using a calculator, copy and complete each statement.

**a** $12^2 = 144$ so $\sqrt{144} =$ ___ **b** $35^2 = 1225$ so $\sqrt{1225} =$ ___

**c** $61^2 = 3721$ so $\sqrt{\phantom{xxx}} = 61$ **d** $298^2 = 88\,804$ so $\sqrt{\phantom{xxx}} = 298$

**19** Explain how finding the square of a number and finding the square root of a number are related.

PROBLEM SOLVING AND REASONING

**20** A colony of bacteria grows very quickly and triples its size each day.

**a** Copy and complete this table.

| Number of days | Amount of growth | Amount of growth in index form | Number of times larger than original size |
|---|---|---|---|
| 1 | 3 | $3^1$ | 3 |
| 2 | $3 \times 3$ | $3^2$ | 9 |
| 3 | $3 \times 3 \times 3$ | | |
| 4 | | | |
| 5 | | | |

**b** How many times larger is the bacteria colony after:

**i** 6 days? **ii** 1 week? **iii** 2 weeks?

**c** How long does it take for the bacteria colony to be:

**i** 27 times larger? **ii** 243 times larger? **iii** 6561 times larger?

**21** After 1 week, the weight of a baby mouse is double its weight at birth. For the next few weeks, its weight continues to double each week.



**a** What number do you multiply by to double a quantity?

**b** After 1 week, the mouse would have a weight that is twice as big as the birth weight. After 2 weeks, the weight would be $2 \times 2$ or 4 times as big as the birth weight. Write each number in index form.

**c** In index form, write the number of times the weight is bigger than the birth weight after three weeks.

**d** How much bigger is the baby mouse after five weeks?

**22** Diana sends a text message to four of her friends. Each friend forwards it to another four people, who each then send it to another four people.

**a** How many text messages have been sent?

**b** Explain how powers can be used to solve this problem.

CHALLENGE

**23** Over the summer school holidays, Taylor was offered a part-time job earning her $100 per week. The job was available for 4 weeks. Taylor decided to discuss a different payment plan with her prospective boss. 'How about paying me only $5 in the first week and then in each of the other weeks paying me five times as much as the week before.' The boss thought this new plan might save her money. Which pay offer do you think the boss should go with? Show your calculations to justify your answer.




**Reflect**

How are powers and multiplication related?

# 1G Order of operations

## Start thinking!

**1** Consider this problem: $5 + 3 \times 4 - 6 \div 2$.

**a** Find the answer without using a calculator.

**b** Write a list of the operations (+, −, ×, etc.) you used and the order in which you used them.

**c** Compare your answer to part **a** with those of two classmates. Did you all get the same?

**d** Discuss the steps you each used to obtain your answer. Which answer do you think is correct?

To avoid confusion in calculations like this, mathematicians agree to use a particular order when performing a number of operations in the one problem. This is the **order of operations** or set of rules to follow:

| | |
|---|---|
| First | **B**rackets (operations inside grouping symbols) |
| Second | **I**ndices (powers and square roots) |
| Third | **D**ivision and **M**ultiplication |
| Fourth | **A**ddition and **S**ubtraction |

**2** One way to remember this order is to think of **BIDMAS**. Can you see why?

**NOTE** Some people use **BODMAS**, which has 'O' for 'power of' in place of 'I' for 'indices'.

**3** Follow the correct order of operations to calculate the answer to the problem in question **1**.

## KEY IDEAS

- The order of operations is:

| | |
|---|---|
| first: | **B**rackets (operations inside grouping symbols are always calculated first) |
| second: | **I**ndices (powers and square roots) |
| third: | **D**ivision and **M**ultiplication (work from left to right) |
| fourth: | **A**ddition and **S**ubtraction (work from left to right) |

- Operations at the same ranking are performed in order from left to right.
- Where there is more than one set of grouping symbols in the problem, calculate the operations inside the innermost brackets first.
- One way of remembering the order is to think of **BIDMAS** (or **BODMAS**).

# EXERCISE 1G Order of operations

## EXAMPLE 1G-1 Order of operations with no grouping symbols

Calculate each of these.

**a** $3 + 8 \div 2$ **b** $7 \times 2^3 - 9 \times 5$

| THINK | WRITE |
|---|---|
| **a** **1** Write the problem. | **a** $3 + 8 \div 2$ |
| **2** Division is performed before addition. ($8 \div 2 = 4$) | $= 3 + 4$ |
| **3** Perform the addition. | $= 7$ |
| **b** **1** Write the problem. | **b** $7 \times 2^3 - 9 \times 5$ |
| **2** Powers and square roots (indices) are worked out before $\times$, $\div$, + and −. ($2^3 = 8$) | $= 7 \times 8 - 9 \times 5$ |
| **3** Multiplication is performed before subtraction so, working from left to right, calculate $7 \times 8$ first. ($7 \times 8 = 56$) | $= 56 - 9 \times 5$ |
| **4** Perform the other multiplication next. ($9 \times 5 = 45$) | $= 56 - 45$ |
| **5** Perform the subtraction. | $= 11$ |

UNDERSTANDING AND FLUENCY

**1** Calculate each of these.

**a** $6 + 15 \div 3$
**b** $9 - 4 \times 2$
**c** $7 + 3 \times 6 - 4$
**d** $12 \div 4 + 7 \times 8$
**e** $16 + (9 - 3) \div 2$
**f** $2 + 9 \times 4 \div 6$
**g** $10 \times 6 - 5 \times 9$
**h** $8^2 - 5 \times 3 + 11$
**i** $21 - 3^2 \times 2 + 1$
**j** $4 \times 3^3 - 6^2 \times 2 + 1$
**k** $\sqrt{9} + 9 \times 8 - 2^3$
**l** $22 - 2 \times 11 + 4^2 \div \sqrt{16}$

## EXAMPLE 1G-2 Order of operations with grouping symbols

Calculate each of these.

**a** $16 \div (10 - 2) + 3 \times 7$ **b** $8 + [12 \div (9 - 5)]^2 - 1$

| THINK | WRITE |
|---|---|
| **a 1** Write the problem. | **a** $16 \div (10 - 2) + 3 \times 7$ |
| **2** Perform any operations inside brackets first. $(10 - 2 = 8)$ | $= 16 \div 8 + 3 \times 7$ |
| **3** Perform ÷ and × before +. Working from left to right, calculate $16 \div 8$ first. $(16 \div 8 = 2)$ | $= 2 + 3 \times 7$ |
| **4** Perform × before +. $(3 \times 7 = 21)$ | $= 2 + 21$ |
| **5** Perform the addition. | $= 23$ |
| **b 1** Write the problem. | **b** $8 + [12 \div (9 - 5)]^2 - 1$ |
| **2** There are two sets of grouping symbols (brackets). Perform the operation in the innermost set of brackets first. $(9 - 5 = 4)$ | $= 8 + [12 \div 4]^2 - 1$ |
| **3** Perform the operation in the remaining set of brackets. $(12 \div 4 = 3)$ | $= 8 + 3^2 - 1$ |
| **4** Perform the operation of squaring as 'indices' comes before + and −. | $= 8 + 9 - 1$ |
| **5** Working from left to right, perform the addition then the subtraction. | $= 17 - 1$<br>$= 16$ |

UNDERSTANDING AND FLUENCY

**2** Calculate each of these.

**a** $28 \div (12 - 5) + 2 \times 6$

**b** $(9 + 3) \div (15 - 13)$

**c** $4^3 - 10 \times (3 + 1)$

**d** $6 + [18 \div (10 - 1^5)]^2 - 7$

**e** $\sqrt{100} \times 5 - 2 \times (2^4 + 3)$

**f** $3 \times [8 + (2 \times 9 - 4)] + 20$

**3** Perform the calculations in question **2** using a calculator. Enter each number, operation and grouping symbol from left to right. Does your calculator perform the operations in the correct order?

**4** The **average** of a set of scores is found by working out this calculation:

(total sum of the scores) ÷ number of scores.

For example, the average of 9, 10 and 14 is found by working out $(9 + 10 + 14) \div 3$.

**a** Use the correct order of operations to find the average of 9, 10 and 14.

**b** Find the average of each set of numbers.

**i** 15, 18 and 24
**ii** 3, 5, 6, 8, 9 and 11
**iii** 278 and 356
**iv** 7, 7, 11 and 15
**v** 50, 51, 52, 52 and 60
**vi** 50, 20, 30, 40, 60, 50 and 30

**5** The set of rules called the order of operations is also related to some other mathematical laws. You will have used these laws in your calculations without realising.

**a** Calculate 8 + 2.
**b** Calculate 2 + 8.
**c** Compare your answers to parts **a** and **b**. Are they the same?
**d** Does it matter in which order you add numbers? This is called the **commutative law**. Write another example to show that the commutative law (where order does not matter) works for addition.
**e** Does it matter in which order you subtract numbers? For example, does 8 − 2 give the same answer as 2 − 8?
**f** Does it matter in which order you multiply numbers? Provide some examples to help explain your answer.
**g** Does it matter in which order you divide numbers? Provide some examples to help explain your answer.
**h** Look at your answers to parts **d**–**g**. Which operations obey the commutative law?

**6** Consider what happens if you add three numbers in a different order. Let's try it with 6 + 3 + 5.

**a** Calculate 6 + 3 first and then add 5.
**b** Calculate 6 + 5 first and then add 3.
**c** Calculate 3 + 5 first and then add 6.
**d** Compare your answers to parts **a**, **b** and **c**. What does this indicate about the order in which you can add these three numbers?
**e** The **associative law** states that you can add numbers in any order. Apply this law to find the sum of 25 + 37 + 13. Which two numbers are easier to add together first?

**7** Consider what happens if you multiply three numbers in a different order. Let's try it with 6 × 3 × 5.

**a** Calculate 6 × 3 first and then multiply by 5.
**b** Calculate 6 × 5 first and then multiply by 3.
**c** Calculate 3 × 5 first and then multiply by 6.
**d** Compare your answers to parts **a**, **b** and **c**. What does this indicate about the order in which you can multiply these three numbers?
**e** The associative law also states that you can multiply numbers in any order. Explain how the associative law can make the calculation 25 × 58 × 4 easier to perform.

**8** Do you think that the associative law would apply to the operations of subtraction and division? Try some examples to help you decide.

UNDERSTANDING AND FLUENCY

**9** **a** Calculate $3 \times (4 + 5)$ using the correct order of operations.

**b** Calculate $3 \times 4 + 3 \times 5$.

**c** Compare your answers to parts **a** and **b**. This demonstrates another law called the **distributive law**.

**d** Compare your answers to $7 \times (10 + 6)$ and $7 \times 10 + 7 \times 6$. Does this demonstrate the distributive law? Explain why or why not.

**e** Explain how the distributive law can make a calculation like $23 \times 16$ easier to perform without using long multiplication. (Hint: $23 \times 16$ is the same as $23 \times (10 + 6)$.)

**f** Use the distributive law to calculate each of these without using long multiplication.

**i** $35 \times 14$ **ii** $68 \times 19$ **iii** $41 \times 102$ **iv** $87 \times 106$

PROBLEM SOLVING AND REASONING

**10** In Australian Rules football, each team scores goals and behinds. Each goal has a value of six points and each behind has a value of one point. During a match, the scoreboard usually shows the number of goals, behinds and points both teams have scored. This photo taken by a football fan does not show the number of points.




**a** Copy and complete these statements to show the numbers and operations needed to calculate the total points scored by each team.

Points scored by Sharks = _____ × 6 + _____ × 1

Points scored by Visitors = _____ × _____ + _____ × _____

**b** Calculate the number of points each team has scored.

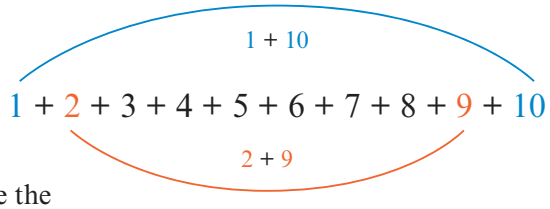
**c** At this stage of the match, which team is winning and by how much?

PROBLEM SOLVING AND REASONING

**11** **a** Add the numbers in order from 1 to 10.

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 =$ _____.

**b** Is there a quicker way of working out the sum? Let's try a strategy where pairs of numbers are matched together. (You tried a version of this earlier in the chapter.)




Copy and complete this diagram, where the smallest number and the largest number are paired together, then the second smallest number and the second largest number are paired together and so on. Remember that you can add numbers in any order and still obtain the same total.

**c** What is the result when each pair of numbers is added?

**d** How many pairs of numbers are there?

**e** Use the operation of multiplication and your answers to parts **c** and **d** to calculate the sum of all the numbers from 1 to 10.

**12** Use the strategy from question **11** to find the sum of all the whole numbers in these groups.

**a** 1 to 20 **b** 1 to 40 **c** 10 to 50 **d** 15 to 35

**13** In the late 1780s, students in a class were asked to add the whole numbers from 1 to 100. A student named Johann Carl Friedrich Gauss (who later became a famous mathematician) correctly answered the question in less than one minute by using the strategy of adding pairs of numbers. Try this calculation yourself. Can you find the answer in less than one minute?

**14** For your birthday, you and a group of friends go to the cinema. At the cinema kiosk, you order eight medium soft drinks, four jumbo containers of popcorn and seven ice creams.

**a** Write a mathematical calculation to show how to work out the total cost of the food and drink.

**b** Perform the operations in the calculation to find the total cost.




CHALLENGE

**15** Pick a number between 1 and 10. Use this number as many times as you like with any of the operations +, −, × and ÷ to make a mathematical calculation that gives a result of 7. You can also use brackets and square root symbols or use the number as a power. For example, $(3^3 + 3) \div 3 - 3 = 7$.

**Reflect**

Why is it important to perform operations in the correct order?

# 1H Multiples and factors

## Start thinking!

**1** **a** Copy and complete this table for the first eight numbers of the pattern 3, 6, 9, 12, 15, …

| Number | 3 × ? |
|---|---|
| 3 | 3 × 1 |
| 6 | 3 × 2 |
| 9 | 3 × ____ |
| 12 | 3 × ____ |
| 15 | 3 × ____ |
| | |
| | |
| | |

**b** What would be the 40th number in this pattern? Describe a quick way to work this out.

**c** These numbers are called **multiples** of 3. They are the result of multiplying 3 by another whole number. Give five more examples of numbers that are multiples of 3.

**2** Multiplying the two whole numbers 4 and 9 gives a product of 36.

**a** List another two whole numbers that give a product of 36.

**b** Are there are other pairs of whole numbers that multiply to give 36? List them.

**c** List all the whole numbers that have been used to give the product 36. These whole numbers are called **factors**.

**3** Explain how multiples and factors are different.

## KEY IDEAS

- The multiples of a number result from multiplying that number by another whole number. For example, the multiples of 3 are 3, 6, 9, 12, 15, 18, …
- The factors of a number are all the whole numbers that divide exactly into that number. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.
- It is often easier to find factors in pairs. For example, the factor pairs that give a product of 12 are 1 and 12, 2 and 6, 3 and 4.
- A number divides exactly into another number if there is no remainder (the remainder is zero).
- The **lowest common multiple (LCM)** of two or more numbers is the smallest number that both or all the numbers divide into.
- The **highest common factor (HCF)** of two or more numbers is the highest number that will divide into both or all the numbers.

# EXERCISE 1H Multiples and factors

**1** Copy and complete each number pattern.

**a** 2, 4, 6, ___, ___, ___
**b** 9, 18, 27, ___, ___, ___
**c** 5, 10, 15, ___, ___, ___
**d** 4, 8, 12, ___, ___, ___
**e** 8, 16, 24, ___, ___, ___
**f** 7, 14, 21, ___, ___, ___

## EXAMPLE 1H-1 Finding multiples

List the first six multiples of 7.

**THINK**

**1** Each multiple of 7 results from multiplying 7 by a whole number. For the first six multiples, 7 is multiplied first by 1, then 2, then 3, then 4, then 5 and then 6.

**2** List the first six multiples of 7.

**WRITE**

$7 \times 1 = 7$
$7 \times 2 = 14$
$7 \times 3 = 21$
$7 \times 4 = 28$
$7 \times 5 = 35$
$7 \times 6 = 42$

The first six multiples of 7 are 7, 14, 21, 28, 35 and 42.

UNDERSTANDING AND FLUENCY

**2** List the first eight multiples of each number.

**a** 10 **b** 6 **c** 9 **d** 5 **e** 11 **f** 30

**3** Write the multiples of 8 between 23 and 65.

**4** Write the multiples of 7 between 50 and 100.

**5** The first ten multiples of 3 and 4 are shown.

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, …
Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, …

**a** Copy the two lists and circle the numbers that occur in both.
**b** The numbers that are the same or common to each list are called **common multiples**. Write the common multiples you have found.
**c** Are these the only common multiples of 3 and 4? Explain.
**d** Write the first six common multiples of 3 and 4.

**6** **a** List the first ten multiples of 4.
**b** List the first ten multiples of 5.
**c** Circle the common multiples of 4 and 5 that you can see in the lists.
**d** Find the first four common multiples of 4 and 5.

**7** **a** List the multiples of 5 and 6 that are less than 100.
**b** Write the common multiples of 5 and 6 that you can see in the lists.
**c** What is the smallest number that is a common multiple of 5 and 6? This is called the lowest common multiple (or LCM) of 5 and 6.

## EXAMPLE 1H-2 Finding the lowest common multiple (LCM)

Find the lowest common multiple (LCM) of 2 and 3.

| THINK | WRITE |
|---|---|
| **1** List the multiples of 2. | multiples of 2:<br>2, 4, (6), 8, 10, (12), 14, 16, (18), 20, … |
| **2** List the multiples of 3. | multiples of 3:<br>3, (6), 9, (12), 15, (18), 21, … |
| **3** Write the multiples that are the same in the two lists. You can circle them first. | common multiples: 6, 12, 18, … |
| **4** Identify the lowest number in the list of common multiples. | LCM is 6. |

UNDERSTANDING AND FLUENCY

**8** Find the lowest common multiple (LCM) of each pair of numbers.
**a** 2 and 5 **b** 4 and 7 **c** 6 and 9 **d** 8 and 12
**e** 5 and 8 **f** 7 and 10 **g** 15 and 20 **h** 8 and 6

**9** Find the LCM of each group of numbers.
**a** 2, 3 and 4 **b** 5, 6 and 20 **c** 3, 4 and 9 **d** 4, 6 and 8
**e** 5, 3 and 2 **f** 30, 9 and 2 **g** 6, 7 and 8 **h** 2, 3, 4 and 5

**10** Copy and complete each of these.
**a** 15 = 1 × 15 or 3 × ___ **b** 18 = 1 × ___ or 2 × ___ or 3 × ___
**c** 32 = 1 × ___ or ___ × 16 or ___ × ___ **d** 28 = 1 × ___ or ___ × 14 or 4 × ___
**e** 49 = 1 × ___ or ___ × 7 **f** 50 = ___ × 50 or 2 × ___ or 5 × ___

**11** Use your answers to question **10** to list the factors of these numbers.
**a** 15 **b** 18 **c** 32 **d** 28 **e** 49 **f** 50

## EXAMPLE 1H-3 Finding factors

Find the factors of 48.

| THINK | WRITE |
|---|---|
| **1** List pairs of whole numbers that multiply to give a product of 48. | factor pairs for 48: $1 \times 48$, $2 \times 24$, $3 \times 16$, $4 \times 12$, $6 \times 8$ |
| **2** Write the factors of 48 as a list in ascending order. These are all the whole numbers that divide exactly into 48. | Factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48. |

UNDERSTANDING AND FLUENCY

**12** Find the factors of each number.

**a** 6 **b** 20 **c** 56 **d** 81 **e** 27 **f** 100

**13** Which factors are common (the same) in each group of numbers? Use your answers to questions **11** and **12** to help you.

**a** 6 and 20 **b** 81 and 27 **c** 50 and 100
**d** 56 and 100 **e** 6 and 81 **f** 32 and 56
**g** 6, 15 and 18 **h** 20, 28 and 32 **i** 28, 49 and 56

**14** The factors of 24 and 36 are shown.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

**a** Copy the two lists and circle the numbers that are the same in each list.
**b** The numbers that are the same or common to each list are called **common factors**. Write the common factors you have found.
**c** Are these the only factors that are common for 24 and 36? Explain.

**15** **a** List the factors of 18.
**b** List the factors of 45.
**c** Circle the common factors of 18 and 45 that you can see in the lists.
**d** Write the common factors of 18 and 45.

**16** **a** List the factors of 12.
**b** List the factors of 30.
**c** Write the common factors of 12 and 30.
**d** What is the largest number that is a common factor of 12 and 30? This is called the highest common factor (or HCF) of 12 and 30.

## EXAMPLE 1H-4 Finding the highest common factor (HCF)

Find the highest common factor (HCF) of 24 and 36.

| THINK | WRITE |
|---|---|
| **1** List the factors of 24. | factors of 24:<br>1, (2), (3), (4), (6), 8, (12), 24 |
| **2** List the factors of 36. | factors of 36:<br>1, (2), (3), (4), (6), 9, (12), 18, 36 |
| **3** Write the factors that are the same in the two lists. You can circle them first. | common factors: 2, 3, 4, 6, 12 |
| **4** Identify the highest number in the list of common factors. | HCF is 12. |

**NOTE** The HCF is also called the greatest common divisor. Can you see why?

**17** Find the highest common factors (HCF) of each pair of numbers.

**a** 8 and 24 **b** 15 and 27 **c** 24 and 42 **d** 5 and 20
**e** 36 and 32 **f** 45 and 30 **g** 63 and 35 **h** 50 and 100

**18** Find the HCF of each group of numbers.

**a** 4, 8 and 12 **b** 10, 25 and 30 **c** 16, 24 and 32 **d** 15, 6 and 27
**e** 42, 36 and 18 **f** 20, 30 and 50 **g** 38, 26 and 14 **h** 9, 6, 12 and 18

**19** Find the LCM and the HCF of each group of numbers.

**a** 12 and 30 **b** 10 and 35 **c** 4, 12 and 16 **d** 2, 9 and 18

PROBLEM SOLVING AND REASONING

**20** Every fourth paling of this fence is painted blue.



**a** There are 84 palings in the fence. How many of them are painted blue?
**b** The fence is to be extended and will now have 128 palings. How many more palings need to be painted blue?
**c** Have you used multiples or factors to help you find the answers?

**21** According to the timetable, a train is expected to depart from the station every 12 minutes. The first train of the day departs at 5.12 am.

**a** List the times in the first hour that a train should depart from the station.
**b** When will these trains depart from the station?
**i** the third train of the day **ii** the sixth train of the day
**c** Which train will depart from the station at 8.24 am?

PROBLEM SOLVING AND REASONING

**22** Max is arranging 18 pieces of sushi on a plate for his guests. He would like to arrange them so that there is the same number of pieces in each row.



- **a** Suggest one way that Max could arrange the sushi on a plate. You may like to draw a diagram to show your answer.
- **b** How many possible ways are there? Explain the other arrangements that are possible.
- **c** Have you used multiples or factors of 18 to help find the answers?

**23** For a school fete, Hanna and Josh are deciding how to divide up a batch of 120 cookies so that there is an equal number of cookies in each bag. Suggest at least three ways this could be done.

**24** **a** What is the lowest common multiple of 3 and 4?

- **b** Use the LCM to write a list of the first six common multiples of 3 and 4.
- **c** Is there a number that is the highest common multiple of 3 and 4? Give a reason for your answer.

**25** **a** What is the highest common factor of 18 and 45?

- **b** Is there a number that is the lowest common factor of 18 and 45? If so, write the number.
- **c** Explain why the lowest common factor of two (or more) numbers is not particularly useful to us. Think about the lowest common factors for different pairs of numbers.

CHALLENGE

**26** Imogen and Olivia begin jogging around an oval at exactly the same time from the same starting point. Imogen runs each lap of the oval in 6 minutes and Olivia in 8 minutes.

- **a** When will they next pass the starting point at exactly the same time?
- **b** How many laps will each of them have jogged at this time?
- **c** Assuming they keep up the same pace, how many full laps will each girl have jogged when they stop after 1 hour 30 minutes?
- **d** How many times will they have both passed the starting point at exactly the same time during their run?

**27** Find the smallest whole number that has only:

- **a** one factor
- **b** two factors
- **c** three factors
- **d** four factors
- **e** five factors
- **f** six factors.

**Reflect**

How can you remember the difference between finding multiples and finding factors?

# 1I Prime and composite numbers

## Start thinking!

Any counting number (1, 2, 3, 4, …), except for the number 1, can be described as either a **prime number** or a **composite number**.

**1** The prime numbers from 1 to 20 are 2, 3, 5, 7, 11, 13, 17, 19.
- **a** For each prime number listed, write its factors.
- **b** How many factors does each prime number have?

**2** The composite numbers from 1 to 20 are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.
- **a** For each composite number listed, write its factors.
- **b** How many factors does each composite number have?
- **c** Does a composite number have more, less or the same number of factors as a prime number?

**3** Explain how prime numbers and composite numbers are different.

**4** How many factors does the number 1 have? Is this the same number of factors that a prime number has or a composite number has? Explain why the number 1 is a special number that is neither prime nor composite.

## KEY IDEAS

- A prime number is a counting number (whole number) that has exactly two factors: itself and 1.
- A composite number is a counting number that has more than two factors.
- The number 1 is neither a prime number nor a composite number.
- You can use the fact that all even numbers, except 2, are composite numbers to help you decide whether a number is prime or composite.
- A composite number can be written as the product of factors that are prime numbers. These factors are called prime factors.
- For larger composite numbers, it can be useful to produce a factor tree to find the prime factors.

For example:

$6 = 2 \times 3$

composite number; prime factors

# EXERCISE 1I Prime and composite numbers

UNDERSTANDING AND FLUENCY

**1** A Greek mathematician named Eratosthenes (pronounced 'E-rah-toss-thee-nees'), who lived from 276 BCE to 195 BCE, is famous for his easy method of using multiples to separate out all the composite numbers and leave only the prime numbers. This method is called 'The sieve of Eratosthenes'.

**a** Copy the numbers from 1 to 100 in a grid.

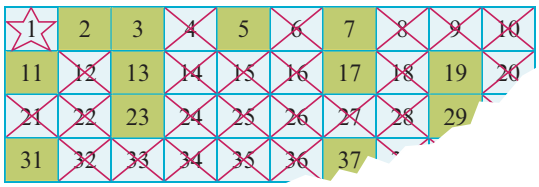
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

**b** Draw a star around the number 1, as it is neither a prime number nor a composite number.

**c** Move to the next number, which is 2, and highlight the number by drawing a circle around it or colouring the grid square containing 2. Then cross out all the multiples of 2. (That is, cross out 4, 6, 8, 10, …, 100.)

**d** Move to the next number, which is 3, and highlight the number. Then cross out all the multiples of 3 that have not already been crossed out.

**e** Move to the next number that is not crossed out and highlight it. Then cross out all the multiples of that number that have not already been crossed out.




**f** Repeat the step in part **e** until all the numbers from 2 to 100 have been highlighted or crossed out.

**2** Look at your completed grid from question **1**. The highlighted numbers are prime numbers and those crossed out are composite numbers.

**a** List all the prime numbers from 20 to 100.

**b** List all the composite numbers from 11 to 19.

**c** How many one-digit prime numbers are there? List them.

**d** How many even prime numbers are there? List them.

**e** What is the largest two-digit prime number in which each digit is a prime number?

**3** List the factors for each number.

**a** 24 **b** 49 **c** 73 **d** 125 **e** 37 **f** 82

**4** Use your answers to question **3** to decide whether each number is prime or composite.

## EXAMPLE 1I-1 Identifying prime and composite numbers

State whether each number is prime or composite. Give a reason to support your answer.

**a** 66 **b** 43 **c** 57

| THINK | WRITE |
|---|---|
| **a** Is the number even? (yes) All even numbers except 2 are composite. | **a** 66 is a composite number since it is an even number (that is not 2) and so has more than two factors. |
| **b** Is the number even? (no) List its factors. | **b** Factors of 43 are 1, 43.<br>43 is a prime number since it has exactly two factors. |
| **c** Is the number even? (no) List its factors. | **c** Factors of 57 are 1, 3, 19, 57.<br>57 is a composite number since it has more than two factors. |

UNDERSTANDING AND FLUENCY

**5** State whether each number is prime or composite. Give a reason for your answer.

**a** 28 **b** 47 **c** 79 **d** 145
**e** 131 **f** 200 **g** 203 **h** 303

**6** Find two prime numbers that add to give each sum.

**a** 24 **b** 36 **c** 82 **d** 144

**7** Find two prime numbers that multiply to give each product.

**a** 21 **b** 55 **c** 26 **d** 115

**8** Repeat the method used in question **1** to find all the prime numbers from 101 to 200.

**9** Answer true or false to each statement. Give a reason for your answer.

**a** All even numbers are prime numbers.
**b** All odd numbers are prime numbers.
**c** The first two prime numbers are 2 and 3.
**d** All composite numbers are even numbers.
**e** The sum of two prime numbers is always even.
**f** The product of two prime numbers is always a composite number.
**g** The number 1 is neither a prime number nor a composite number.
**h** Between 1 and 30, there are more composite numbers than prime numbers.

## EXAMPLE 1I-2 Writing a composite number as the product of prime factors

For each composite number:

i find its prime factors

ii write it as a product of prime factors.

a 8 b 12 c 30

### THINK

a 1 Find the factors of 8.

2 Identify the prime factors of 8.

3 Use the prime factor to write 8 as a product.

b 1 Find the factors of 12.

2 Identify the prime factors of 12.

3 Use the prime factors to write 12 as a product.

c 1 Find the factors of 30.

2 Identify the prime factors of 30.

3 Use the prime factors to write 30 as a product.

### WRITE

a factors of 8: 1, 2, 4, 8

i prime factor of 8: 2

ii $8 = 2 \times 2 \times 2$

b factors of 12: 1, 2, 3, 4, 6, 12

i prime factors of 12: 2 and 3

ii $12 = 2 \times 2 \times 3$

c factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

i prime factors of 30: 2, 3 and 5

ii $30 = 2 \times 3 \times 5$

UNDERSTANDING AND FLUENCY

**10** For each composite number:

i find its prime factors

ii write it as a product of prime factors.

a 4 b 10 c 18 d 35

e 16 f 20 g 27 h 40

**11** From earlier work in this chapter, you will have seen that repeated factors can be expressed in a form using powers. Factors that are not repeated remain the same.

For example, the composite numbers in Example 1I-2 can be written as:

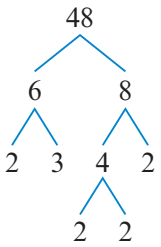
$8 = 2 \times 2 \times 2 = 2^3$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$30 = 2 \times 3 \times 5$

For each composite number in question **10**, write the product of prime factors using powers.

**12** For larger composite numbers, it is useful to produce a factor tree to find the prime factors. The factor tree for 48 starts with two branches showing a product of two factors (say, $6 \times 8$) and continues with pairs of branches until all factors are prime numbers.




Use the factor tree to answer the following.

**a** What is the pair of factors branching from 6? Are they prime numbers?

**b** What is the pair of factors branching from 8? Are they prime numbers?

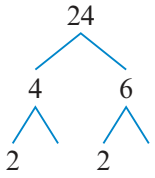
**c** Explain why you need to have a further pair of factors branching from 4 but not from the other numbers.

**d** List the prime factors of 48.

**e** Write 48 as a product of its prime factors.

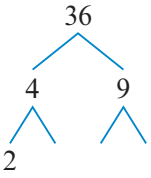
**13** Copy and complete each factor tree and show each number as a product of prime factors.

**a**



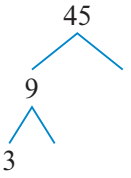

Prime factors of 24 are:
2 and ___
$24 = 2 \times 2 \times ___ \times ___$
$= 2^3 \times ___$

**b**



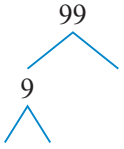

Prime factors of 36 are:
___ and ___
$36 = 2 \times ___ \times ___ \times ___$
$= 2^2 \times ___$

**c**



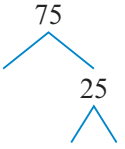

Prime factors of 45 are:
___ and 5
$45 = ___ \times 3 \times ___$
$= ___ \times ___$

**d**



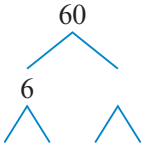

Prime factors of 99 are:
3 and ___
$99 = 3 \times ___ \times ___$
$= ___ \times ___$

**e**




Prime factors of 75 are:
___ and ___
$75 = ___ \times ___ \times ___$
$= 3 \times ___$

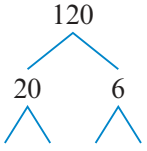
**f**



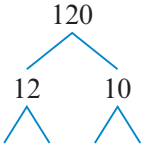

Prime factors of 60 are:
___, ___ and ___
$60 = ___ \times ___ \times$
$___ \times ___$
$= ___ \times ___ \times ___$

**14** Complete each factor tree to show that the same product of prime factors will be obtained for 120 regardless of the factors you begin with.

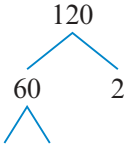
**a**




**b**




**c**



**15** For each composite number:

**i** find its prime factors by first drawing a factor tree

**ii** write it as a product of prime factors.

| | | | |
|---|---|---|---|
| **a** 28 | **b** 56 | **c** 44 | **d** 80 |
| **e** 132 | **f** 52 | **g** 250 | **h** 90 |
| **i** 72 | **j** 210 | **k** 100 | **l** 400 |

PROBLEM SOLVING AND REASONING

**16** Look at the pizza shown. It is to be divided into 12 equal pieces. Writing 12 as the product of prime factors shows you an easy way of cutting the pizza. As $12 = 2 \times 2 \times 3$, you can use the numbers 2, 2 and 3 to guide you in dividing the pizza. Here are the steps to follow.



**a** Use this method to work out how to divide the pizza into six equal pieces. Draw a diagram to show each step.

**b** Describe how this method could be used to divide the pizza into 18 equal pieces.

**c** Does it matter in which order you use the prime factors to divide the pizza? Explain your answer.

**Step 1:**
cut the pizza into 2 equal pieces.



**Step 2:**
cut each piece into 2 equal pieces.



**Step 3:**
cut each piece into 3 equal pieces.



**17** A chocolate cake mixture has been baked in a large rectangular container. The chef wishes to cut the cake into 36 individual serves.

**a** Describe how prime factors can be used to divide the cake into 36 equal pieces.

**b** Draw a diagram to show a way of cutting the cake.

**c** Are there different ways to cut the cake into 36 equal pieces? If so, draw diagrams to show another two possible ways.

CHALLENGE

**18** A pair of prime numbers that differ by 2 are called twin primes. For example, 3 and 5 are twin primes, as $5 - 3 = 2$.

**a** List the three other pairs of twin primes that occur for numbers from 1 to 20.

**b** How many pairs of twin primes occur for numbers from 21 to 100? List them.

**c** How many pairs of twin primes occur for numbers from 101 to 200? List them.

**Reflect**

How can you identify which numbers are prime and which are composite?

# CHAPTER REVIEW

## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

| | | | |
|---|---|---|---|
| place value | sum | long division | order of operations |
| odd | vertical addition | power | multiples |
| even | difference | index | factors |
| ascending order | vertical subtraction | base | lowest common multiple |
| descending order | product | index form | highest common factor |
| number line | long multiplication | expanded form | prime number |
| approximate | quotient | basic numeral | composite number |
| leading digit | remainder | square numbers | prime factors |
| rounding | short division | square root | factor tree |

## MULTIPLE-CHOICE

1A **1** Which list of numbers is written in order from smallest to largest?

**A** 1053, 531, 510, 501
**B** 345, 354, 3045, 3405
**C** 2461, 2614, 2641, 2611
**D** 85, 83, 81, 82, 87, 89

1B **2** What is 14 985 + 486 + 1987 + 9135?

**A** 14 373 **B** 25 483
**C** 26 593 **D** 174 805

1C **3** Which calculation gives an answer of 42?

**A** 96 − 42 **B** 1569 − 584
**C** 7859 − 7813 **D** 9026 − 8984

1D **4** Which multiplication problem produces the largest answer?

**A** 48 × 49 **B** 13 × 78
**C** 26 × 54 **D** 89 × 21

1E **5** Which division problem does *not* have a remainder of 4?

**A** 79 ÷ 5 **B** 36 ÷ 8
**C** 43 ÷ 10 **D** 22 ÷ 6

1F **6** What is 5 × 5 × 5 × 8 × 8 in index form?

**A** $3^5 \times 2^8$ **B** $5^3 + 8^2$
**C** $40^5$ **D** $5^3 \times 8^2$

1G **7** What is 5 + 4 × 9 − 3?

**A** 15 **B** 38 **C** 54 **D** 78

1H **8** What is the highest common factor of 10 and 25?

**A** 5 **B** 10 **C** 25 **D** 50

1I **9** Which number is *not* a prime number?

**A** 11 **B** 25 **C** 37 **D** 53

1I **10** How is 60 written as the product of prime factors?

**A** 3 × 4 × 5 **B** 2 × 2 × 15
**C** 2 × 2 × 3 × 5 **D** 2 × 3 × 10

## SHORT ANSWER

1A **1** Decide whether each statement is true or false.
- **a** $467 > 647$ **b** $0 < 78$
- **c** $930 < 9030$ **d** $15\,452 > 15\,542$

1A **2** Write each list of numbers in ascending order.
- **a** 5347, 547, 53 047, 57
- **b** 87 605, 87 506, 87 056

1A **3** Write an approximation to each number by rounding to its leading digit.
- **a** 392 **b** 9488
- **c** 554 **d** 12 345

1B **4** Calculate each of these.
- **a** 467 + 56 + 7801 + 943
- **b** 383 604 + 2557 + 16 092

1B **5** Find two whole numbers that add to give 478 if:
- **a** both numbers are odd
- **b** both numbers are even.

1C **6** Calculate each of these.
- **a** 9564 − 5381 **b** 371 625 − 38 047
- **c** 17 659 − 9816 **d** 101 011 − 59 678

1C **7** Calculate each of these.
- **a** 4895 − 1625 − 325
- **b** 978 − 486 − 239

1D **8** Calculate each of these.
- **a** $54 \times 37$ **b** $6135 \times 429$
- **c** $9856 \times 11$ **d** $12\,652 \times 43$

1D **9** Use a strategy to calculate each of these.
- **a** $389 \times 100$ **b** $412 \times 6000$
- **c** $3400 \times 200$ **d** $500 \times 300 \times 40$
- **e** $25 \times 48 \times 4$ **f** $931 \times 50 \times 2$

1E **10** Calculate each of these.
- **a** $567\,204 \div 7$ **b** $850 \div 16$
- **c** $\frac{7521}{23}$ **d** $\frac{68\,445}{45}$

1E **11** Use a strategy to calculate each of these.
- **a** $7000 \div 10$ **b** $80\,000 \div 2000$

1F **12** Write each in expanded form and calculate its value.
- **a** $2^6$ **b** $5^3 \times 3^2$

1F **13** Write 10 000 in index form with a base of 10.

1F **14** Write each of these in both index form and as a basic numeral.
- **a** seven squared **b** the square of six

1F **15** Find the value of each of these.
- **a** $\sqrt{25}$ **b** $\sqrt{81} + 4^2$

1G **16** Calculate each of these.
- **a** $7^2 - 6 \times 4 + 3$
- **b** $8 \times (19 - 13) + 2 \times \sqrt{36}$
- **c** $4 + 3 \times (5 - 2)^2$
- **d** $16 - 2 \times (3 + 4) + 9$

1H **17** Write the multiples of 4 between 22 and 45.

1H **18** Write all the factors of 60.

1H **19** Find the lowest common multiple of each group of numbers.
- **a** 12 and 18 **b** 6, 9 and 15

1H **20** Find the highest common factor of each group of numbers.
- **a** 12 and 18 **b** 6, 9 and 15

1H **21** Find three numbers that have:
- **a** a factor of 7
- **b** four different factors
- **c** an odd number of factors.

1I **22**
- **a** Is 40 a prime or composite number? Give a reason to support your answer.
- **b** Draw a factor tree and list the prime factors of 40.
- **c** Write 40 as a product of its prime factors in index notation.

## NAPLAN-STYLE PRACTICE

**1** Which number is six thousand and forty-eight?

- ◯ 6480
- ◯ 6048
- ◯ 6408
- ◯ 6084

**2** Which group of numbers is listed in ascending order?

- ◯ 236, 245, 254, 263, 236
- ◯ 7503, 7053, 7350, 7530
- ◯ 4965, 4695, 4659, 4569
- ◯ 603, 623, 630, 632, 662

**3** Jane's total electricity bill this year is $2386. Last year it was $1857. What is the difference in the two yearly amounts?

☐

Questions **4** and **5** refer to this road sign.




**4** What is the distance between Miles and Surat?

☐ km

**5** If a person drives from this intersection to Miles, then to Injune and then to Mitchell, how far have they travelled in total?

☐ km

**6** Hayden kept a record of how many laps of the pool he swam each day.

What is the total number of laps he swam in a week?

☐

| Day | Number of laps |
|---|---|
| Monday | 18 |
| Tuesday | 25 |
| Wednesday | 19 |
| Thursday | 32 |
| Friday | 28 |
| Saturday | 15 |
| Sunday | 24 |

**7** A set of DVDs contains 14 episodes of a television show. Each episode runs for 86 minutes. What is the total running time?

☐

**8** Alexis bought five books, three photo frames, seven magazines and two board games. If the books cost $11 each, photo frames $14 each, magazines $6 each and board games $21 each, how much did she spend?

$ ☐

**9** To estimate the value of 751 × 329, each number is first rounded to its leading digit.

Which calculation must be performed?

- ◯ 700 × 300
- ◯ 700 × 400
- ◯ 800 × 300
- ◯ 800 × 400

**10** This carton contains eggs of about the same size. The total mass inside the carton is 660 g.



What is the mass of one egg?

| 55 g | 66 g | 648 g | 672 g |
|---|---|---|---|
| ◯ | ◯ | ◯ | ◯ |

**11** Two families paid $10 each to buy one lottery ticket. Their ticket was one of five winners in a $1 000 000 prize. If each family has four members, how much did each person receive?

- ◯ $250 000
- ◯ $125 000
- ◯ $50 000
- ◯ $25 000

**12** Zahra goes for a bike ride and after 17 minutes she has ridden 4250 metres. How many kilometres will she cover in an hour?

| 15 | 17 | 72 | 255 |
|---|---|---|---|
| ◯ | ◯ | ◯ | ◯ |

**13** What is another way of writing $4^3$?

- ◯ 3 × 3 × 3 × 3
- ◯ 4 × 3
- ◯ 4 × 4 × 4
- ◯ 4 + 4 + 4

**14** What is the value of $\sqrt{100}$?

**15** If $17^2 = 289$, what is the value of $\sqrt{289}$?

2 ○ 17 ○ 289 ○ 578 ○

**16** Which calculation produces the largest result?

○ $2^4 \times 5^2$ ○ $5^2 \times 3^2$
○ $4^2 \times 6^2$ ○ $3^2 \times 2^5$

**17** Which calculation has the same value as $12 \times 4$?

○ $3 + 9 \times 5$ ○ $6 \times 9 - 5$
○ $6 \times 3 + 5$ ○ $(3 + 6) \times 5$

**18** What is the value of $8 \times (3 + 6^2 \div 9) - 2$?

**19** Which number comes next in this sequence?
48, 72, 96, 120, 144, ___

148 ○ 160 ○ 168 ○ 192 ○

**20** Christos walks around an athletic track in 6 minutes while Lisa jogs around the same track in 4 minutes. They both begin their laps of the oval at the same time and from the same starting position. When will they next pass the starting position at the same time?

**21** Which of these is *not* a prime number?

2 ○ 43 ○ 51 ○ 67 ○

**22** How is 72 written as a product of its prime factors?

○ $8 \times 9$ ○ $2 \times 2 \times 2 \times 3 \times 3$
○ $3 \times 4 \times 6$ ○ $2 \times 3 \times 3 \times 4$

**23** How many prime factors does 30 have?

1 ○ 2 ○ 3 ○ 8 ○

## ANALYSIS

Jordan wrote a list of the type and quantity of food he needed for his birthday party.
He carefully noted the price per item during his shopping trip.

| Item | Quantity | Price per item |
|---|---|---|
| Party Pie 24 pack | 4 | $4 |
| Allen's mixed lollies | 3 | $3 |
| Chips 100 g | 4 | $2 |
| M&Ms 250 g | 2 | $4 |
| Freddo frogs 20 pack | 2 | $5 |
| Cocktail sausages 2 kg pack | 1 | $6 |
| Frozen pizza | 1 | $10 |
| Mudcake | 3 | $4 |

**a** Work out how much Jordan spent on each item.

**b** How much did he spend in total?

**c** If he paid with two $50 notes, how much change did he get?

**d** What is the cheapest item on the list?

**e** Which item did he spend the least money on? Is this different from the answer to part **d**? Explain why or why not.

**f** Each bag of Allen's mixed lollies has 40 lollies. How many lollies in total does Jordan have?

**g** Jordan wants lolly bags for himself and his friends with at least 10 lollies in each bag. What is the maximum number of friends he can invite?

**h** Jordan adds Freddo frogs to the lolly bags. How many Freddos does each friend get? How many Freddos are left over?

**i** Jordan and his friends need to be divided into equal groups for games. Using your answer from part **g**, what size could groups be if there must be at least three people in each?

**j** Explain how Jordan can easily cut the pizza into 12 equal slices.

# CONNECT

## Working at a zoo

Many zoos offer the opportunity to work at a zoo for the day. A zookeeper's role includes many responsibilities. Here is some information about a few of the animals that you might look after at a zoo.

MEERKATS

**Meerkats:**

- eat a wide variety of food, including termites, crickets, spiders, eggs and mice
- have a mass at birth of about 35 g
- have a mass as an adult of about 950 g
- have an adult body length of about 30 cm
- have a lifespan in the wild of about 10 years, up to 13 years in a zoo
- are given special treats such as toys, live insects scattered in the sand and food hidden in logs or wrapped in special parcels to keep them occupied and active in zoos.



HIPPOPOTAMUSES

**Hippos:**

- eat about 40 kg of grass and fruit each night
- in zoos are often fed herbivore pellets (pressed hay), alfalfa, lettuce and, as a special treat, melons
- have a mass at birth of 25–45 kg
- have an adult mass of 1600–4500 kg for males, and about 1400 kg for females
- have a lifespan of about 45 years
- can run up to 30 km/h on land and hold their breath underwater for up to 30 minutes
- like to mark their territory by spreading their faeces and urine around with their tails, so it can be quite a messy job for zoo keepers to clean up!



### Your task

You are to investigate some of the ways animals are looked after at zoos. This includes consideration of the food they eat, the environment they live in and other factors that make their lives comfortable. Use the information provided for the animals described here, and then research information on a further three animals.

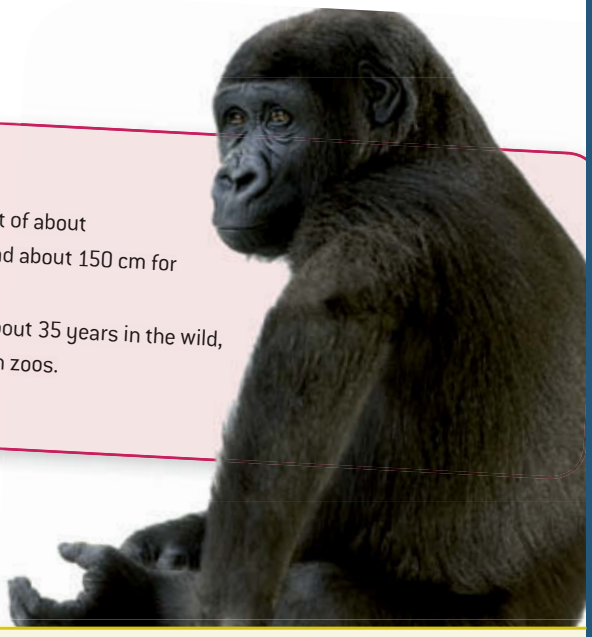
As a starting point, consider:

- the type of food each animal needs
- the amount of food needed to feed all the animals in the one enclosure
- the height requirements for an animal enclosure
- the amount of living space needed for different animals
- any special environmental requirements
- any special treats that zookeepers give animals to keep them happy.



You will need to show your calculations as evidence to back up your findings.

GROUPS



## GORILLAS

**Gorillas:**

- eat up to 18 kg of fruits and vegetables each day
- have a mass at birth of about 2 kg
- have an adult mass of 136–227 kg for males and 60–91 kg for females
- have an adult height of about 175 cm for males and about 150 cm for females
- have a lifespan of about 35 years in the wild, and up to 50 years in zoos.



## CAMELS

**Camels:**

- eat vegetation, including hay, and alfalfa pellets and carrots
- have a mass at birth of 37 kg
- have an adult mass of 300–690 kg
- can easily carry 90 kg and walk 32 km in a day
- can drink up to 145 L of water at one drinking session
- can survive a week or more in the wild without water, and several months without food.



## GIRAFFES

**Giraffes:**

- eat up to 80 kg of acacia leaves each day
- have a height at birth of 200 cm
- have an adult height of about 500 cm for males
- can run at speeds up to 60 km/h.



### Groups of animals

Collective nouns are used to describe groups of animals. In some cases, there can be more than one noun used to describe a particular animal. Here are just a few.

- a mob of emus
- a band of gorillas
- a herd of elephants
- an ambush of tigers
- a dazzle of zebras
- a crash of rhinoceroses
- a mob of meerkats
- a colony of penguins
- a coalition of cheetahs
- a bask of crocodiles

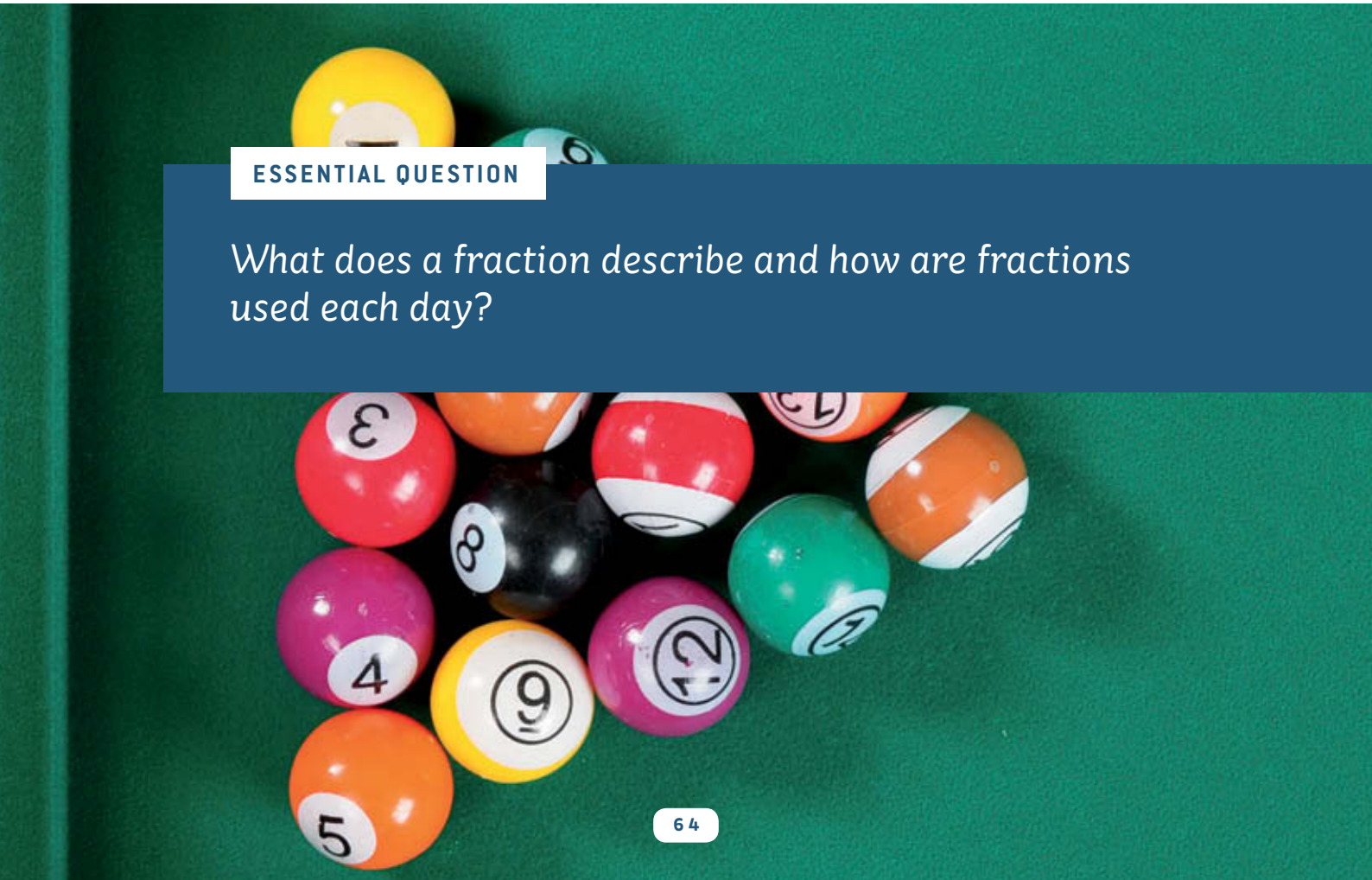
Complete the 1 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a PowerPoint presentation
- a newspaper article
- a video diary
- other (check with your teacher).

# 2 FRACTIONS AND RATIOS


2A Understanding fractions
2B Equivalent fractions
2C Adding and subtracting fractions
2D Multiplying fractions
2E Dividing fractions
2F Powers and square roots of fractions
2G Understanding ratios
2H Working with ratios

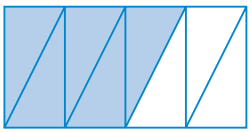



**ESSENTIAL QUESTION**

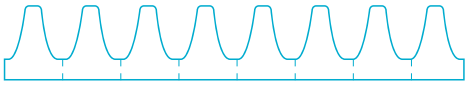
*What does a fraction describe and how are fractions used each day?*

2A **1** What fraction of each shape has been shaded?

**a** 

**b** 

2A **2** Copy this diagram. Shade $\frac{1}{2}$ of the chocolate bar.



2A **3** Describe each number as a proper fraction, an improper fraction or a mixed number.

**a** $\frac{5}{7}$ **b** $3\frac{2}{3}$ **c** $\frac{5}{18}$

**d** $\frac{9}{28}$ **e** $\frac{28}{9}$ **f** $2\frac{9}{11}$

2B **4** What is the highest common factor of 48 and 64?

**A** 4 **B** 8 **C** 16 **D** 24

2B **5** Which fraction is equivalent to $\frac{1}{3}$?

**A** $\frac{5}{16}$ **B** $\frac{9}{18}$ **C** $\frac{2}{15}$ **D** $\frac{4}{12}$

2B **6** Which fraction is equivalent to $\frac{5}{2}$?

**A** $\frac{35}{14}$ **B** $\frac{10}{6}$ **C** $\frac{14}{35}$ **D** $\frac{10}{10}$

2B **7** **a** Write this list of numbers in order from smallest to largest.

$\frac{5}{11}, \frac{13}{11}, 1, \frac{7}{11}, \frac{2}{11}$

**b** Write this list of numbers in order from largest to smallest.

$1, \frac{8}{17}, \frac{3}{17}, \frac{19}{17}, \frac{2}{17}, \frac{21}{17}$

2C **8** Calculate:

**a** $\frac{2}{15} + \frac{11}{15}$ **b** $\frac{13}{23} + \frac{4}{23}$

**c** $\frac{7}{9} - \frac{5}{9}$ **d** $\frac{19}{21} - \frac{8}{21}$

2C **9** What is the lowest common multiple of 3, 4 and 5?

**A** 120 **B** 60 **C** 20 **D** 12

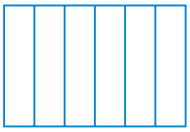
2D **10** Use this diagram to answer the question.



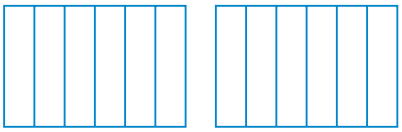
**a** What is $\frac{1}{2}$ of 16?

**b** What is $\frac{1}{4}$ of 16?

2E **11** **a** How many lots of $\frac{1}{6}$ are in the rectangle below?



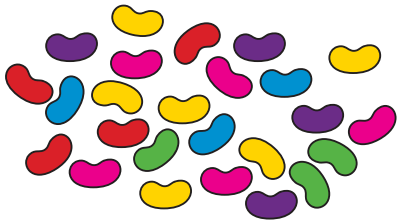
**b** How many lots of $\frac{1}{6}$ are in the two rectangles below?



2F **12** Calculate:

**A** $7^2$ **B** $5^3$ **C** $2^4$ **D** $\sqrt{36}$

2G **13** Use this diagram to answer the questions.



**a** Describe the number of red jellybeans compared to the number of blue jellybeans.

**A** 5 to 3 **B** 4 to 3

**C** 3 to 5 **D** 3 to 4

**b** Describe the number of green jellybeans compared to the number of yellow and pink jellybeans.

**A** 3 to 5 **B** 3 to 6

**C** 11 to 3 **D** 3 to 11

# 2A Understanding fractions

## Start thinking!

There are different types of fractions. An example of a **proper fraction** is $\frac{3}{4}$ and an example of an **improper fraction** is $\frac{7}{4}$.

**1** What is the difference between a proper and an improper fraction?

An improper fraction can also be written as a **mixed number**.
The improper fraction $\frac{7}{4}$ can be written as the mixed number $1\frac{3}{4}$.

**2** What is the difference between an improper fraction and a mixed number?

**3** What do an improper fraction and a mixed number have in common?

Mary, David and Xavier are to share three rectangular pizzas. Each pizza is cut into eight pieces.



**4** Each slice is $\frac{1}{8}$ of a pizza. If David eats seven slices, what fraction of a pizza has he eaten?

**5** Is your answer to question **4** a proper fraction, an improper fraction or a mixed number?

**6** Xavier eats 11 slices of pizza. What fraction of pizza has Xavier eaten?
Write your answer as an improper fraction and as a mixed number if you can.

## KEY IDEAS

- A fraction is a part or portion of a whole. It can be written as one whole number (**numerator**) over another (**denominator**), separated by a horizontal line (**vinculum**).
- Proper fractions have a numerical value less than 1. The numerator in a proper fraction is smaller than the denominator.
  Some examples are: $\frac{3}{5}, \frac{2}{9}, \frac{11}{15}$.

vinculum → $\frac{4}{5}$ ← the **numerator** shows the number of equal parts out of the whole

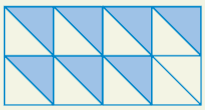
← the **denominator** shows the total number of parts in the whole

- Improper fractions have a numerical value greater than 1. The numerator in an improper fraction is larger than the denominator. Some examples are: $\frac{7}{2}, \frac{9}{5}, \frac{31}{8}$.
- Mixed numbers also have a numerical value greater than 1. They contain a whole number and proper fraction component. Some examples are: $4\frac{2}{3}, 7\frac{1}{8}, 6\frac{12}{17}$.
- Whole numbers can be written as fractions. For example, $\frac{2}{2} = 1, \frac{9}{3} = 3, \frac{28}{7} = 4, \frac{5}{1} = 5$.

# EXERCISE 2A Understanding fractions

## EXAMPLE 2A-1 Writing proper fractions

What fraction of the shape is shaded?



### THINK

1 Count the number of equal segments in the shape. (16). This is the denominator of the fraction.

2 Count the number of shaded segments in the shape (7). This is the numerator of the fraction.

3 Write the proper fraction.

### WRITE

The shape is divided into 16 segments.

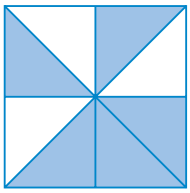
Seven of the 16 segments are shaded.

The fraction of the shape that is shaded is $\frac{7}{16}$.

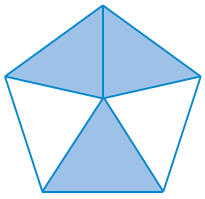
UNDERSTANDING AND FLUENCY

**1** What fraction of each shape is shaded?

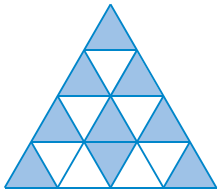
**a**



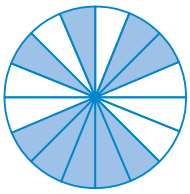
**b**



**c**



**d**



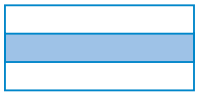
**2** What fraction of each shape in question **1** is *not* shaded?

**3** Which diagram correctly displays that $\frac{1}{3}$ of the shape has been shaded? Provide a reason for your selection.

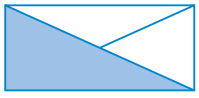
**A**



**B**



**C**



## EXAMPLE 2A-2 Writing improper fractions and mixed numbers

Write the number that is represented by the shading in the diagram as:
**a** an improper fraction **b** a mixed number.

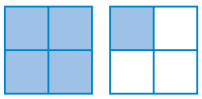


| THINK | WRITE |
|---|---|
| **a 1** Write the total number of quarters that are shaded. | **a** There are seven quarters that are shaded. |
| **2** Write the answer as an improper fraction. | The number represented is $\frac{7}{4}$. |
| **b 1** Look at any shapes in the diagram that are fully shaded. A fully shaded shape represents the whole number of 1. | **b** The whole of the first rectangle is shaded and three quarters of the second rectangle is shaded. |
| **2** Write the answer as a mixed number. | The number represented is $1\frac{3}{4}$. |

UNDERSTANDING AND FLUENCY

**4** Write the number that is represented by the shading in each diagram as:
**i** an improper fraction **ii** a mixed number.

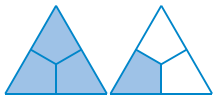
**a**



**b**



**c**



**d**



**5** Rule up a table with three columns. Title the columns 'Proper fractions', 'Improper fractions' and 'Mixed numbers'. Place each of these fractions into the appropriate column: $\frac{5}{2}$, $2\frac{3}{5}$, $\frac{7}{8}$, $1\frac{5}{9}$, $\frac{17}{2}$, $\frac{16}{31}$, $\frac{29}{15}$, $12\frac{1}{7}$, $\frac{13}{22}$.

## EXAMPLE 2A-3 Converting a mixed number to an improper fraction

Convert the mixed number $2\frac{2}{7}$ to an improper fraction.

| THINK | WRITE |
|---|---|
| **1** Write the mixed number as the sum of a whole number and a fraction. | $2\frac{2}{7}$ |
| | $= 2 + \frac{2}{7}$ |
| **2** Since the denominator is 7, write each whole number as seven sevenths. | $= \frac{7}{7} + \frac{7}{7} + \frac{2}{7}$ |
|  | |
| **3** Add all of the sevenths (7 + 7 + 2 = 16) and write the answer. | $= \frac{16}{7}$ |

**6** Convert each mixed number to an improper fraction.

**a** $1\frac{2}{9}$ **b** $3\frac{1}{6}$ **c** $5\frac{8}{9}$ **d** $2\frac{6}{11}$ **e** $10\frac{4}{5}$ **f** $8\frac{3}{4}$

**g** $2\frac{3}{7}$ **h** $12\frac{1}{3}$ **i** $4\frac{5}{8}$ **j** $5\frac{3}{10}$ **k** $7\frac{1}{12}$ **l** $2\frac{4}{15}$

## EXAMPLE 2A-4 Converting an improper fraction to a mixed number

Convert the improper fraction $\frac{25}{8}$ to a mixed number.

### THINK

**1** As 8 eighths make a whole, group 25 eighths into three lots of 8 eighths and 1 eighth left over.



**2** Rewrite each lot of 8 eighths as a whole number.

**3** Write the total as a whole number and fraction.

### WRITE

$\frac{25}{8}$

$= \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$

$= 1 + 1 + 1 + \frac{1}{8}$

$= 3\frac{1}{8}$

UNDERSTANDING AND FLUENCY

**7** Convert each improper fraction to a mixed number.

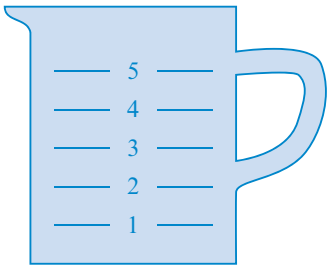
**a** $\frac{7}{5}$ **b** $\frac{26}{3}$ **c** $\frac{25}{9}$ **d** $\frac{43}{10}$ **e** $\frac{75}{8}$ **f** $\frac{88}{13}$

**g** $\frac{22}{7}$ **h** $\frac{17}{4}$ **i** $\frac{49}{6}$ **j** $\frac{50}{11}$ **k** $\frac{67}{5}$ **l** $\frac{111}{2}$

**8** Copy this diagram of a jug. The scale indicates measurements in litres.

Show the water level for a measurement of $\frac{4}{3}$ L on your jug.

(Hint: first convert the improper fraction to a mixed number.)




**9** Repeat the method used in question **8** to show the water level on a jug for each of these measurements.

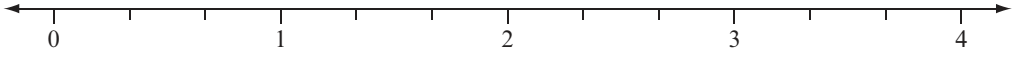
**a** $\frac{7}{2}$ L **b** $\frac{8}{5}$ L **c** $\frac{17}{4}$ L

**10** Convert both fractions to improper fractions and use the symbol < (is less than) or > (is greater than) to make a true statement.

**a** $2\frac{4}{7}$ $\frac{17}{7}$ **b** $8\frac{2}{3}$ $\frac{28}{3}$

**c** $\frac{38}{11}$ $2\frac{6}{11}$ **d** $9\frac{4}{5}$ $\frac{51}{5}$

**11** The number line below has been divided into four equal intervals from 0 to 4. Each interval has been further subdivided into three equal parts.



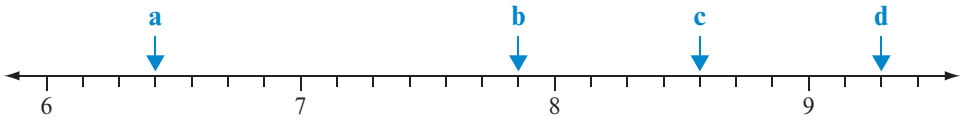

**a** What fraction of an interval is each part?

**b** Label each mark on the scale with the value it represents.

**12** Draw a number line from 0 to 4 and mark the position of each of these fractions.

**a** $\frac{2}{5}$ **b** $\frac{5}{5}$ **c** $\frac{8}{5}$ **d** $\frac{13}{5}$ **e** $\frac{17}{5}$

**13** Write down the values shown for **a–d** on the number line.




**14** The Australian state and territory flags are shown at right.

What fraction of the flags:

**a** include the Southern Cross in their design?

**b** include a flower in their design?

**c** include blue in their colours?

**d** do not include animals in their design?


Australian Capital Territory


New South Wales


Northern Territory


Queensland


South Australia


Tasmania


Victoria


Western Australia

**15** Write the fraction that is described by each statement.

**a** 17 correct answers on a quiz containing 25 questions

**b** the number of months in a year consisting of 31 days

**c** $23 change from a $50 note

**16** There are 60 minutes in 1 hour. What fraction of an hour is:

**a** 17 minutes? **b** 31 minutes?

**c** 119 minutes (as an improper fraction)? **d** 311 minutes (as a mixed number)?

**17** Georgina, Roisin and Vanessa decide to order large pizzas for a class lunch. Each pizza will be cut into eight equal slices. The girls estimate that each person will eat two slices and there will be a maximum of 22 people.

**a** Write the number of pizzas required as:

**i** an improper fraction **ii** a mixed number.

**b** Will each person be able to receive the estimated two slices of pizza if five large pizzas have been ordered?

**c** The girls' budget does not allow them to order extra pizzas. What is the minimum number of equal slices each pizza can be cut into, if everyone is to receive two slices?

**d** What fraction of the pizza will be left over if the pizzas are cut in the way suggested in part **c**?

**e** On the day of the lunch, three people were away and $4\frac{5}{8}$ of the pizzas were eaten. If each pizza was cut into eight slices, how many slices were eaten?

PROBLEM SOLVING AND REASONING

**18** Look at the image of the Colosseum shown here.



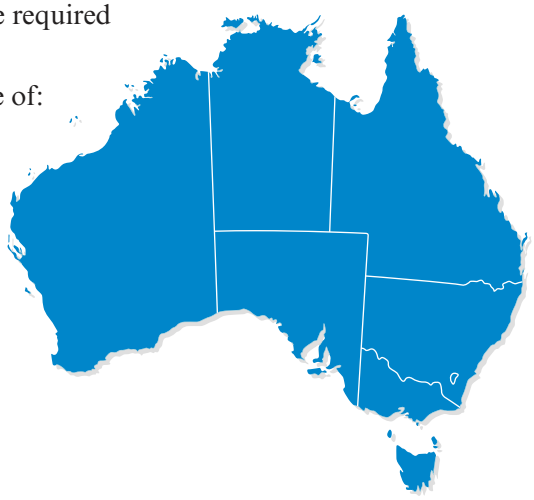
- **a** By estimation, how many levels high is the tallest point of the Colosseum?
- **b** What fraction of the height of the Colosseum is one level?
- **c** What fraction of the levels contain arches?

**19** Estimate what fraction of this ant's whole body length is made up of its head.



CHALLENGE

**20** Use this map of Australia to estimate the required fraction in each question.



- **a** What fraction of Australia is the state of:
  - **i** Victoria?
  - **ii** Tasmania?
  - **iii** Queensland?
  - **iv** New South Wales?
  - **v** Western Australia?
- **b** What fraction of Western Australia is the state of New South Wales?
- **c** What fraction of South Australia is the state of Tasmania?

**21** Gemma and Cindy disagree on which fraction is larger, $\frac{1}{4}$ or $\frac{1}{5}$. Describe how number lines can be used to compare the fractions and decide which is larger.

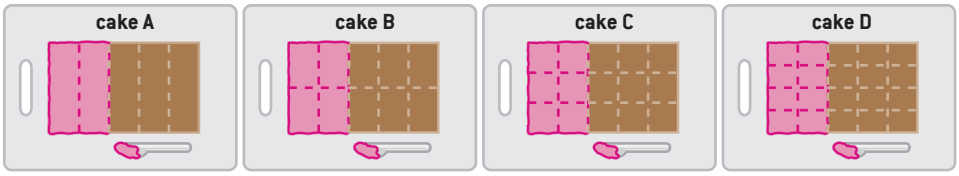
**Reflect**

Explain how you can compare the size of fractions.

# 2B Equivalent fractions

## Start thinking!

Look at each cake at right. Icing has been spread over some of the top of each cake and the cake has been cut into equal pieces.




1 What do you notice about the amount of cake that has been iced in each case?

2 Copy and complete this table.

| Cake | Total number of equal pieces | Number of pieces that have been iced | Fraction of the cake that has been iced |
|---|---|---|---|
| A | | | |
| B | | | |
| C | | | |
| D | | | |

3 Without drawing a diagram, explain how the next cake in the series would look in terms of the total number of pieces, the number of pieces that have been iced and the fraction of the cake that has been iced.

4 The fractions representing the amount of each cake that has been iced are called **equivalent fractions**. Explain why.

## KEY IDEAS

- Equivalent fractions have the same numerical value.
- An equivalent fraction can be formed by multiplying both the numerator and the denominator of a fraction by the same value (see Example A).
- Dividing both the numerator and the denominator of a fraction by the same value also produces an equivalent fraction (see Example B). This process is called simplifying or **cancelling**.
- A fraction is in its simplest form once the numerator and the denominator have been divided by the **highest common factor (HCF)**.
- Mixed numbers can be simplified by keeping the whole number the same and simplifying the fraction component.

**Example A**

$\frac{4}{5} = \frac{4}{5} \times \frac{3}{3} = \frac{12}{15}$

Multiplying by $\frac{3}{3}$ is the same as multiplying by 1, so the value of the equivalent fraction is the same.

**Example B**

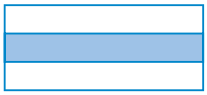
$$\frac{24}{54} \overset{\div 6}{=} \frac{4}{9}$$

# EXERCISE 2B Equivalent fractions

UNDERSTANDING AND FLUENCY

**1** What fraction of each shape is shaded?

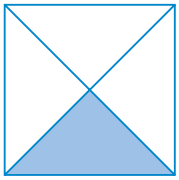
**a**



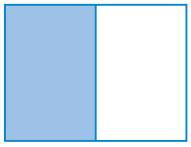
**b**



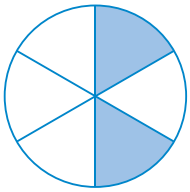
**c**



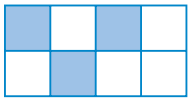
**d**



**e**



**f**



**2** What fraction of each shape in question **1** is *not* shaded?

**3** Create two equivalent fractions to represent each fraction found in:

**i** question **1**

**ii** question **2**.

## EXAMPLE 2B-1 Making equivalent fractions

Fill in the gap to make an equivalent fraction: $\frac{2}{3} = \frac{10}{__}$.

**THINK**

**1** Since $2 \times 5 = 10$, the numerator of the second fraction is obtained by multiplying the numerator of the first fraction by 5.

$$\frac{2}{3} \overset{\times 5}{=} \frac{10}{__}$$

**2** To find the equivalent fraction, use the same relationship (multiply by 5) to obtain the denominator of the second fraction.

$$\frac{2}{3} \overset{\times 5}{=} \frac{10}{__} \quad (\times 5 \text{ below})$$

**WRITE**

$\frac{2}{3} = \frac{10}{__}$

$\frac{2}{3} = \frac{10}{15}$

**4** Fill in the gaps to make equivalent fractions. (Hint: change mixed numbers to improper fractions first.)

**a** $\frac{4}{9} = \frac{}{36}$ **b** $\frac{24}{8} = \frac{12}{}$ **c** $2\frac{5}{11} = \frac{}{33}$

**d** $\frac{42}{70} = \frac{}{10}$ **e** $\frac{144}{48} = \frac{}{12}$ **f** $3\frac{2}{8} = \frac{}{72}$

**g** $\frac{2}{3} = \frac{}{9} = \frac{}{90}$ **h** $\frac{5}{6} = \frac{}{18} = \frac{20}{}$ **i** $\frac{2}{5} = \frac{10}{} = \frac{}{15}$

**j** $5\frac{1}{13} = \frac{}{39} = \frac{330}{}$ **k** $\frac{1}{10} = \frac{}{20} = \frac{}{30} = \frac{8}{}$ **l** $\frac{8}{7} = \frac{24}{} = \frac{}{35} = \frac{}{77}$

**m** $3\frac{3}{4} = \frac{}{24} = \frac{45}{} = \frac{}{72}$ **n** $\frac{1}{12} = \frac{6}{} = \frac{8}{} = \frac{}{144} = \frac{15}{} = \frac{}{240}$ **o** $\frac{2}{3} = \frac{}{6} = \frac{10}{} = \frac{}{30} = \frac{}{75} = \frac{400}{}$

## EXAMPLE 2B-2 Simplifying fractions

Write each fraction in its simplest form.

**a** $\frac{21}{49}$ **b** $2\frac{45}{65}$

### THINK

**a** **1** Write the fraction and find the highest common factor (HCF) of the numerator and the denominator. HCF = 7.

**2** Cancel by dividing both the numerator and the denominator by 7, the HCF.

**3** Write the answer.

**b** **1** Write the mixed number. Find the HCF of the numerator and the denominator of the fraction component. HCF = 5.

**2** Leave the whole number as it is and divide both the numerator and the denominator by 5, the HCF.

**3** Write the answer.

### WRITE

**a** $\frac{21}{49}$

$= \frac{\cancel{21}^{3}}{\cancel{49}^{7}}$

$= \frac{3}{7}$

**b** $2\frac{45}{65}$

$= 2\frac{\cancel{45}^{9}}{\cancel{65}^{13}}$

$= 2\frac{9}{13}$

UNDERSTANDING AND FLUENCY

**5** Write each fraction in its simplest form.

**a** $\frac{10}{16}$ **b** $\frac{11}{33}$ **c** $\frac{28}{36}$

**d** $\frac{30}{48}$ **e** $\frac{63}{56}$ **f** $\frac{116}{94}$

**g** $2\frac{12}{30}$ **h** $7\frac{20}{45}$ **i** $4\frac{56}{64}$

**j** $5\frac{33}{36}$ **k** $6\frac{15}{75}$ **l** $3\frac{16}{54}$

**6** Larni scored 22 of her team's 34 goals. What fraction of the team's goals did Larni score? Write your answer in simplest form.





**7** Cristiano spent 75 minutes of a 90-minute football game on the field. What fraction of time, in simplest form, did he spend off the field?

**8** Write four equivalent fractions for $\frac{4}{7}$.

**9** Create three proper fractions that are equivalent to $\frac{5}{9}$.

**10** Create three improper fractions that are equivalent to $\frac{7}{3}$.

**11** Create three mixed numbers that are equivalent to $1\frac{3}{5}$.

**12** **a** Write a fraction that is smaller than 1.

**b** Write a fraction that is greater than 1 with the same denominator as that in part **a**.

**c** Place these fractions on a number line and comment on their position with respect to 1.

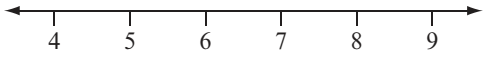
**13** **a** Draw a number line and mark the position of each of these fractions.

$\frac{12}{7}, \frac{10}{7}, \frac{15}{7}, \frac{5}{7}, \frac{21}{7}, \frac{32}{7}, \frac{1}{7}, 4\frac{3}{7}$

**b** Using the number line, list the fractions in ascending order (smallest to largest). What do you notice?

**14** Explain how to order fractions that have the same denominator.

**15** **a** Copy the number line on the right and divide each interval into three equal parts. Label each of these parts.




**b** Convert each fraction to a mixed number, simplifying where necessary, and then mark its position on the number line.

**i** $\frac{23}{3}$ **ii** $\frac{20}{3}$ **iii** $5\frac{2}{6}$ **iv** $7\frac{4}{6}$ **v** $\frac{24}{4}$ **vi** $\frac{12}{3}$

**c** Compare the position of the largest fraction to the other fractions.

**d** Compare the position of the smallest fraction to the other fractions.

**16** Convert both fractions to mixed numbers and use the < (is less than) or > (is greater than) symbol to make a true statement.

**a** $\frac{28}{3}$ $\frac{17}{5}$ **b** $\frac{49}{6}$ $\frac{38}{4}$ **c** $\frac{13}{2}$ $\frac{26}{9}$ **d** $\frac{28}{5}$ $\frac{61}{10}$

**e** $\frac{42}{7}$ $\frac{63}{9}$ **f** $\frac{27}{6}$ $\frac{15}{4}$ **g** $\frac{32}{5}$ $\frac{63}{8}$ **h** $\frac{21}{2}$ $\frac{41}{4}$

**17** Write each fraction in simplest form and list those that are equivalent fractions.

$\frac{1}{3}, \frac{5}{10}, \frac{12}{15}, \frac{6}{18}, \frac{17}{25}, \frac{30}{45}, \frac{16}{48}, \frac{18}{54}, \frac{20}{60}, \frac{52}{156}$

## EXAMPLE 2B-3 Comparing fractions using the lowest common denominator

Which fraction is larger: $\frac{3}{5}$ or $\frac{4}{7}$?

**THINK**

1 Find the lowest common denominator (LCD) for the two fractions. It may help to list all the multiples of 5 and 7 and highlight the lowest multiple that is common to both.

2 Write the LCD. Notice that both 5 and 7 are factors of 35.

3 For each fraction, find the equivalent fraction with a denominator of 35.

4 Compare the size of the numerators to decide which fraction is larger. $\frac{21}{35}$ is larger than $\frac{20}{35}$.

**WRITE**

5, 10, 15, 20, 25, 30, 35…
7, 14, 21, 28, 35, 42…

LCD = 35

$\frac{3 \times 7}{5 \times 7} = \frac{21}{35}$

$\frac{4 \times 5}{7 \times 5} = \frac{20}{35}$

$\frac{3}{5}$ is larger than $\frac{4}{7}$.

UNDERSTANDING AND FLUENCY

**18** Which fraction in each given pair is larger?

**a** $\frac{3}{6}$ or $\frac{4}{9}$ **b** $\frac{7}{10}$ or $\frac{12}{16}$ **c** $\frac{32}{5}$ or $\frac{45}{7}$

**d** $\frac{5}{6}$ or $\frac{7}{8}$ **e** $1\frac{6}{11}$ or $1\frac{11}{20}$ **f** $\frac{138}{12}$ or $\frac{180}{14}$

**g** $3\frac{7}{9}$ or $\frac{44}{12}$ **h** $\frac{17}{4}$ or $4\frac{1}{5}$ **i** $\frac{37}{6}$ or $6\frac{1}{12}$

**19** **a** Convert these fractions to equivalent fractions using the LCD and then arrange them in ascending order. $\frac{1}{4}, \frac{1}{6}, \frac{1}{3}, \frac{3}{4}, \frac{5}{6}, \frac{2}{3}, \frac{2}{4}$

**b** Draw a number line and mark the position of the equivalent fractions. What length is represented by the interval between each mark on the scale?

**c** Write the original fraction below its equivalent fraction on the number line.

**20** Write each list of fractions in ascending order (smallest to largest).

**a** $\frac{7}{5}, \frac{2}{5}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \frac{16}{5}$ **b** $\frac{5}{8}, \frac{11}{4}, \frac{2}{6}, \frac{7}{6}, \frac{5}{4}, \frac{9}{8}$ **c** $\frac{2}{9}, \frac{4}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{3}, \frac{9}{6}$ **d** $\frac{8}{15}, \frac{5}{4}, \frac{6}{3}, \frac{11}{10}, \frac{9}{20}, \frac{10}{6}$

**21** Write each list of fractions in descending order (largest to smallest).

**a** $\frac{3}{2}, \frac{8}{5}, \frac{7}{6}, \frac{10}{6}, \frac{5}{2}, \frac{12}{5}$ **b** $\frac{1}{8}, \frac{11}{12}, \frac{5}{6}, \frac{3}{8}, \frac{7}{12}, \frac{2}{6}$ **c** $\frac{10}{7}, \frac{9}{8}, \frac{3}{4}, \frac{3}{2}, \frac{7}{8}, \frac{6}{7}$ **d** $\frac{2}{9}, \frac{1}{3}, \frac{5}{7}, \frac{4}{3}, \frac{11}{7}, \frac{13}{9}$

**22** Write three fractions that are:

**a** smaller than $\frac{1}{4}$ **b** larger than $2\frac{1}{3}$ **c** between $\frac{2}{7}$ and $\frac{2}{5}$

**d** between $\frac{5}{9}$ and $\frac{4}{3}$ **e** between $2\frac{2}{11}$ and $\frac{7}{2}$ **f** between $\frac{1}{2}$ and $\frac{1}{3}$

**23** Philip was comparing two of his test results. He scored $\frac{22}{25}$ on the first test and $\frac{45}{50}$ on the second. On which test did Philip perform better?

PROBLEM SOLVING AND REASONING



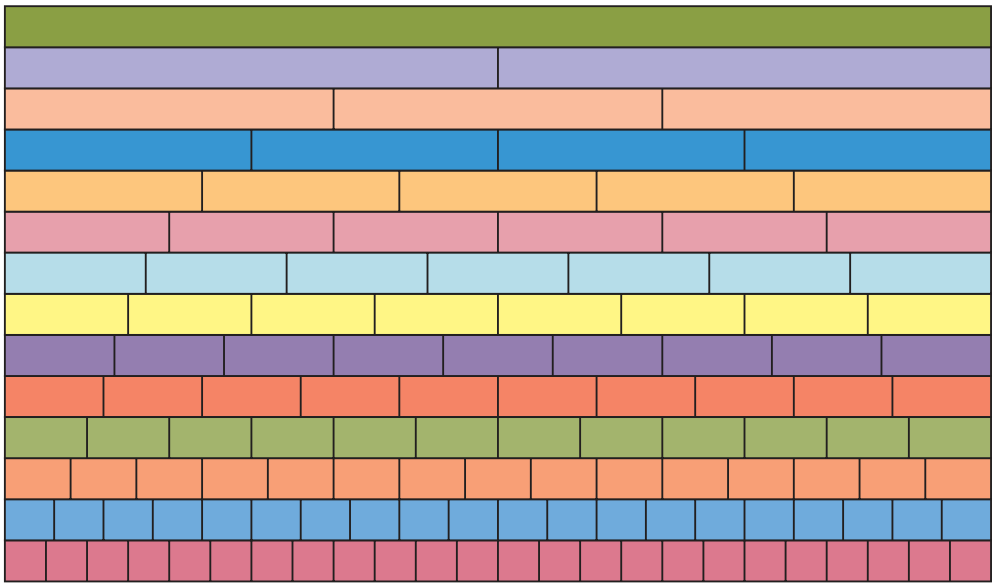
**24** Zach and Melanie are comparing their results after an afternoon of archery. From 35 attempts, Zach scored 20 bullseyes. Melanie scored 24 bullseyes from 40 attempts. Who hit the bullseye on the archery target with more accuracy? Justify your answer.

**25** The common denominator of three different fractions is 24. What could the denominators of the fractions be?

**26** The common denominator of three different fractions is 52. What could the denominators of the fractions be?

CHALLENGE

**27** This diagram shows a section of a fraction wall. Fraction walls can be used to obtain equivalent fractions or to compare fractions.



**a** What do you notice about each horizontal section of the wall?

**b** What fraction is shown by each segment in the second section down?

**c** What fraction is shown by each segment in the third section down?

**d** What do you notice about each segment of a particular horizontal section?

**e** Use the fraction wall provided to list the set of equivalent fractions for $\frac{1}{4}$.

**f** Use the fraction wall provided to list four fractions that are larger than $\frac{2}{5}$.

**g** Use graph or grid paper and follow the teacher's instructions to create your own fraction wall.

**h** Work in pairs and use the fraction wall to design three questions you could ask your classmates.

**Reflect**

What calculations could be performed using the fraction wall?

# 2C Adding and subtracting fractions

## Start thinking!

This diagram shows two fractions being added together.

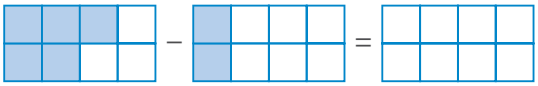


$\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$

**1** What do you notice about the denominator of each fraction?

**2** What do you notice about the numerators of the fractions being added and the numerator of the answer?

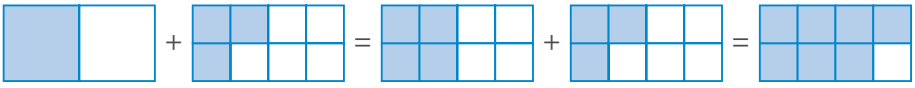
**3** Copy and complete this diagram to represent $\frac{5}{8} - \frac{2}{8}$.



$\frac{5}{8} - \frac{2}{8} =$ _____

**4** Describe how to add or subtract fractions that have the same denominator.

This diagram shows how to add fractions with different denominators.



$\frac{1}{2} + \frac{3}{8} = \frac{4}{8} + \frac{3}{8} = \frac{7}{8}$

**5** Look carefully at the diagram. Explain what must be done to add fractions with different denominators.

**6** Draw a diagram similar to the one in question **3** to represent $\frac{1}{2} - \frac{3}{8}$.

**7** Describe how to add or subtract fractions that have different denominators.

## KEY IDEAS

- To add (or subtract) fractions that have the same denominator, keep the denominator the same and add (or subtract) the numerators.
- To add (or subtract) fractions that have different denominators, first find the equivalent fractions with the lowest common denominator (LCD) and then add (or subtract) the numerators.
- Before adding or subtracting mixed numbers, convert them to improper fractions and check that they are written with a common denominator.
- If the answer is an improper fraction it should be converted to a mixed number.

# EXERCISE 2C Adding and subtracting fractions

## EXAMPLE 2C-1 Adding and subtracting fractions with the same denominator

Calculate:

**a** $\frac{3}{8} + \frac{7}{8}$ **b** $\frac{9}{14} - \frac{3}{14}$

**THINK**

**a** **1** Check if the denominators are the same (yes). Keep the denominator the same and add the numerators.

**2** Simplify the fraction if necessary.

**3** Convert the improper fraction to a mixed number.

**b** **1** Check if the denominators are the same (yes). Keep the denominator the same and subtract the numerators.

**2** Simplify the fraction if necessary.

**WRITE**

**a** $\frac{3}{8} + \frac{7}{8}$

$= \frac{3+7}{8}$

$= \frac{10}{8}$

$= \frac{5}{4}$

$= 1\frac{1}{4}$

**b** $\frac{9}{14} - \frac{3}{14}$

$= \frac{9-3}{14}$

$= \frac{6}{14}$

$= \frac{3}{7}$

UNDERSTANDING AND FLUENCY

**1** Calculate:

**a** $\frac{1}{5} + \frac{2}{5}$ **b** $\frac{4}{7} + \frac{2}{7}$

**c** $\frac{5}{12} + \frac{3}{12}$ **d** $\frac{5}{18} + \frac{6}{18}$

**e** $\frac{3}{37} + \frac{16}{37}$ **f** $\frac{9}{23} + \frac{16}{23}$

**g** $\frac{15}{11} + \frac{2}{11} + \frac{8}{11}$ **h** $\frac{2}{13} + \frac{4}{13} + \frac{5}{13}$

**i** $\frac{23}{100} + \frac{7}{100} + \frac{14}{100}$ **j** $\frac{11}{7} - \frac{5}{7}$

**k** $\frac{25}{21} - \frac{8}{21}$ **l** $\frac{19}{13} - \frac{5}{13}$

**m** $\frac{13}{15} - \frac{3}{15}$ **n** $\frac{26}{27} - \frac{25}{27}$

**o** $\frac{21}{33} - \frac{5}{33}$ **p** $\frac{48}{17} - \frac{13}{17} - \frac{10}{17}$

**q** $\frac{55}{3} - \frac{47}{3} - \frac{7}{3}$ **r** $\frac{21}{8} - \frac{3}{8} - \frac{15}{8}$

## EXAMPLE 2C-2 Adding and subtracting mixed numbers with the same denominator

Calculate: a $1\frac{4}{9} + 2\frac{1}{9}$ b $3\frac{1}{4} - 1\frac{3}{4}$

**THINK**

**a** **1** Convert each mixed number to an improper fraction. Check if the denominators are the same (yes).

**2** Add the numerators. The denominator stays the same.

**3** Convert to a mixed number.

**NOTE** Alternatively, $1\frac{4}{9} + 2\frac{1}{9} = 1 + 2 + \frac{4}{9} + \frac{1}{9} = 3 + \frac{5}{9} = 3\frac{5}{9}$

**b** **1** Convert each mixed number to an improper fraction. Check if the denominators are the same (yes).

**2** Subtract the numerators. The denominator stays the same.

**3** Convert to a mixed number and simplify.

**WRITE**

**a** $1\frac{4}{9} + 2\frac{1}{9}$
$= \frac{13}{9} + \frac{19}{9}$
$= \frac{32}{9}$
$= 3\frac{5}{9}$

**b** $3\frac{1}{4} - 1\frac{3}{4}$
$= \frac{13}{4} - \frac{7}{4}$
$= \frac{6}{4}$
$= 1\frac{2}{4}$
$= 1\frac{1}{2}$

**2** Calculate:

**a** $3\frac{1}{7} + 2\frac{2}{7}$ **b** $2\frac{1}{3} - 1\frac{2}{3}$ **c** $3\frac{2}{5} + 4\frac{1}{5}$ **d** $8\frac{1}{9} - 4\frac{4}{9}$

**e** $2\frac{8}{11} + 1\frac{5}{11}$ **f** $6\frac{4}{5} - 3\frac{2}{5}$ **g** $2\frac{2}{9} + 1\frac{7}{9} + 3\frac{4}{9}$ **h** $4\frac{5}{6} - 2\frac{1}{6} - 1\frac{5}{6}$

**i** $12\frac{2}{3} - 2\frac{1}{3} - 8\frac{2}{3}$ **j** $3\frac{2}{5} + 4\frac{1}{5} + 1\frac{4}{5}$ **k** $6\frac{5}{12} + 2\frac{1}{12} - 7\frac{11}{12}$ **l** $3\frac{1}{8} + 2\frac{5}{8} - 1\frac{7}{8}$

## EXAMPLE 2C-3 Adding and subtracting fractions with different denominators

Calculate: a $\frac{5}{6} + \frac{3}{4}$ b $\frac{2}{3} - \frac{1}{5}$

**THINK**

**a** **1** Check if the denominators are the same (no). Identify the LCD by finding the lowest common multiple of the denominators (12). Write each fraction as an equivalent fraction with a denominator of 12.

**2** Add the numerators together. The denominator stays the same.

**3** Convert the improper fraction to a mixed number.

**b** **1** Check if the denominators are the same (no). Write each fraction as an equivalent fraction with a denominator of 15.

**2** Subtract the numerators. The denominator stays the same.

**WRITE**

**a** $\frac{5}{6} + \frac{3}{4}$
$= \frac{5}{6} \times \frac{2}{2} + \frac{3}{4} \times \frac{3}{3}$
$= \frac{10}{12} + \frac{9}{12}$
$= \frac{19}{12}$
$= 1\frac{7}{12}$

**b** $\frac{2}{3} - \frac{1}{5}$
$= \frac{10}{15} - \frac{3}{15}$
$= \frac{7}{15}$

UNDERSTANDING AND FLUENCY

**3** Calculate:

**a** $\frac{1}{2}+\frac{2}{3}$ **b** $\frac{3}{5}-\frac{1}{4}$ **c** $\frac{3}{8}+\frac{5}{6}$ **d** $\frac{5}{9}-\frac{1}{2}$ **e** $\frac{3}{4}+\frac{2}{3}$ **f** $\frac{1}{3}-\frac{1}{5}$

**g** $\frac{4}{7}+\frac{1}{2}$ **h** $\frac{5}{6}-\frac{3}{5}$ **i** $\frac{6}{7}-\frac{1}{3}$ **j** $\frac{2}{3}+\frac{7}{10}$ **k** $\frac{2}{11}+\frac{5}{6}$ **l** $\frac{2}{5}-\frac{3}{8}$

**m** $\frac{3}{4}-\frac{3}{14}$ **n** $\frac{2}{7}+\frac{4}{5}$ **o** $\frac{4}{15}+\frac{7}{10}$ **p** $\frac{5}{8}-\frac{5}{12}$ **q** $\frac{3}{5}+\frac{1}{8}$ **r** $\frac{17}{23}-\frac{1}{4}$

**4** For the fractions in each part, write the LCD you would use to work out the result.

**a** $\frac{8}{9}+\frac{2}{3}$ **b** $\frac{21}{8}+\frac{17}{32}$ **c** $\frac{14}{19}-\frac{5}{38}$

**d** $\frac{49}{15}-\frac{14}{5}$ **e** $\frac{17}{6}+\frac{7}{12}$ **f** $\frac{17}{10}-\frac{2}{15}$




**5** Work out the result to each calculation shown in question **4**.

**6** Dean completed a 10 km run in $\frac{5}{7}$ of an hour, while Mark completed the run in $\frac{3}{4}$ of an hour.

**a** Which runner had the faster time?

**b** What was the difference between the two times?

**7** Calculate:

**a** $\frac{2}{3}+\frac{11}{12}+\frac{3}{4}$ **b** $\frac{3}{5}+\frac{13}{20}+\frac{12}{15}$ **c** $\frac{7}{8}+\frac{5}{3}-\frac{5}{12}$

**d** $\frac{5}{6}-\frac{1}{2}+\frac{3}{8}$ **e** $\frac{21}{5}+\frac{8}{3}-\frac{13}{10}$ **f** $\frac{8}{5}-\frac{11}{8}+\frac{1}{4}$

## EXAMPLE 2C-4 Adding and subtracting mixed numbers with different denominators

Calculate: **a** $4\frac{1}{2}+5\frac{2}{3}$ **b** $3\frac{7}{10}-2\frac{3}{4}$

### THINK

**a** **1** Convert each mixed number to an improper fraction.

**2** Check if the denominators are the same (no). Identify the LCD (6) and write each fraction as an equivalent fraction with a denominator of 6.

**3** Add the numerators together. The denominator stays the same.

**4** Convert to a mixed number.

**b** **1** Convert each mixed number to an improper fraction.

**2** Check if the denominators are the same (no). Identify the LCD (20) and write equivalent fractions with a denominator of 20.

**3** Subtract the numerators. The denominator stays the same.

### WRITE

**a** $4\frac{1}{2}+5\frac{2}{3}$
$=\frac{9}{2}+\frac{17}{3}$
$=\frac{27}{6}+\frac{34}{6}$
$=\frac{61}{6}$
$=10\frac{1}{6}$

**b** $3\frac{7}{10}-2\frac{3}{4}$
$=\frac{37}{10}-\frac{11}{4}$
$=\frac{74}{20}-\frac{55}{20}$
$=\frac{19}{20}$

UNDERSTANDING AND FLUENCY

**8** Calculate:

**a** $2\frac{2}{3} + 1\frac{1}{2}$ **b** $4\frac{4}{7} - 1\frac{2}{3}$ **c** $3\frac{2}{5} + 2\frac{1}{4}$

**d** $5\frac{2}{3} - 3\frac{4}{5}$ **e** $1\frac{3}{5} + 7\frac{1}{2}$ **f** $2\frac{1}{3} + 3\frac{3}{4}$

**g** $6\frac{5}{6} - 4\frac{3}{10}$ **h** $7\frac{1}{4} - 3\frac{5}{6}$ **i** $3\frac{1}{7} + 2\frac{2}{21}$

**j** $1\frac{5}{6} + 5\frac{1}{2}$ **k** $6\frac{1}{8} - 4\frac{3}{4}$ **l** $4\frac{1}{6} - 2\frac{7}{24}$

**9** Calculate:

**a** $1\frac{2}{3} + \frac{8}{7} + \frac{5}{2}$ **b** $\frac{8}{3} + 5 + 1\frac{1}{8}$ **c** $\frac{9}{2} + 4\frac{3}{7} - 2\frac{1}{6}$

**d** $\frac{28}{5} - 4\frac{1}{7} + \frac{1}{2}$ **e** $\frac{14}{3} - \frac{16}{9} + 2\frac{4}{5}$ **f** $3\frac{5}{12} - \frac{10}{5} + \frac{17}{6}$

**g** $3\frac{1}{11} + 2\frac{3}{4} + 1\frac{1}{2}$ **h** $1\frac{1}{7} + 2\frac{1}{2} - 3\frac{5}{14}$ **i** $4\frac{1}{5} + 1\frac{7}{25} - \frac{5}{6}$

PROBLEM SOLVING AND REASONING

**10** A factory worker is employed to make t-shirts. On average, each t-shirt takes $3\frac{1}{2}$ minutes for cutting, $2\frac{4}{5}$ minutes for sewing and $4\frac{1}{6}$ minutes for finishing. What is the total time required to make one t-shirt?

**11** Ramona has a part-time job in a café. This table displays the hours she worked in one week of the school holidays.

| Day | Number of hours and minutes worked | Number of minutes worked | Number of hours worked |
|---|---|---|---|
| Monday | 3 h 45 min | | |
| Tuesday | 1 h 50 min | | |
| Wednesday | | 330 | |
| Thursday | | 420 | |
| Friday | 10 h 30 min | | |
| Saturday | | | $8\frac{1}{6}$ |

**a** Copy the table.

**b** Complete the second and third columns of the table by converting the hours and minutes worked into minutes and vice versa.

**c** Write the time worked each day in hours in simplest fractional form and complete the last column of the table. (Hint: write the minutes as a fraction of an hour.)

**d** How many hours (in simplest fractional form) has Ramona spent working at the café in this week?

**12** A group of friends organise to catch up for lunch and decide to order pizza. Each person estimates what fraction of a pizza he or she can eat. Luisa can eat $\frac{1}{3}$, Declan $\frac{1}{2}$, Nisa $\frac{3}{8}$, Tomas $\frac{2}{3}$ and Christopher $\frac{3}{2}$ of a pizza.

**a** How many full pizzas will need to be ordered?

**b** What fraction of a pizza will be left over?

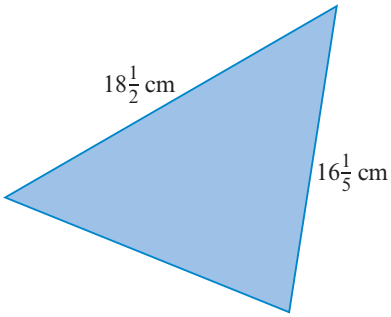
**13** Laila is planning to travel overseas at the end of the year and needs to save money so she devises a budget. She decides to allocate $\frac{2}{5}$ of her weekly pay to living expenses, $\frac{1}{3}$ to rent, $\frac{1}{8}$ to going out and the remainder to savings.

**a** Calculate what fraction of Laila's pay will go towards living expenses, rent and going out.

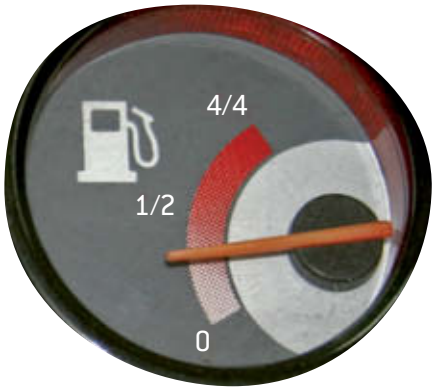
**b** What fraction of her pay will go towards savings? Explain your answer.

PROBLEM SOLVING AND REASONING

**14** The total length around the outside of the triangle at right is $49\frac{2}{5}$ cm.
The length measurements of two of the sides are known. What is the length of the unknown side?




**15** Jason checked the petrol gauge on his motorbike at the beginning of the day and it showed the tank was $\frac{9}{10}$ full. At the end of the day, the petrol gauge showed the tank was $\frac{1}{2}$ full. What fraction of petrol was used?




**16** While on camp, students were required to complete a 21 km bushwalk over three days. They covered $8\frac{2}{5}$ km on the first day and $7\frac{1}{3}$ km on the second day.

**a** What distance will they need to walk on the third day?

**b** How much further did they walk on the first day than on the second?

**17** **a** Calculate each of these and describe what you see.

**i** $\frac{1}{2} - \frac{1}{4}$ **ii** $\frac{1}{2} - \frac{1}{4} - \frac{1}{8}$ **iii** $\frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16}$ **iv** $\frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \frac{1}{32}$

**b** Predict the answer for the next two calculations in the pattern. Test your prediction.

**18** Complete each of these by filling in the gaps. Explain the pattern you see.

**a** $\frac{2}{7}, \frac{5}{7}, \frac{8}{7}, ____, ____, ____, ____$ **b** $10, 11\frac{2}{5}, 12\frac{4}{5}, ____, ____, ____, ____$

**c** $\frac{19}{3}, \frac{16}{3}, \frac{13}{3}, ____, ____, ____, ____$ **d** $12\frac{14}{15}, 10\frac{10}{15}, 8\frac{6}{15}, ____, ____, ____$

CHALLENGE

**19** Find the fraction that is halfway between each pair of fractions.

**a** $\frac{3}{10}$ and $\frac{7}{10}$ **b** $\frac{3}{5}$ and $\frac{4}{5}$ **c** $\frac{4}{11}$ and $\frac{5}{11}$ **d** $\frac{3}{7}$ and $\frac{6}{7}$

**e** $\frac{1}{6}$ and $\frac{1}{4}$ **f** $\frac{2}{5}$ and $\frac{2}{3}$ **g** $\frac{1}{4}$ and $\frac{1}{3}$ **h** $\frac{1}{10}$ and $\frac{1}{8}$

**20** Find a pair of fractions with different denominators that:

**a** add to give 1 **b** add to give $\frac{5}{8}$ **c** have a difference of $\frac{1}{5}$.

**21** The sum of three fractions is $\frac{25}{6}$. Given that one of the fractions is $\frac{10}{3}$, find the other two fractions. (Note: there is more than one answer.)

**22** Use a single pair of brackets to make each equation true.

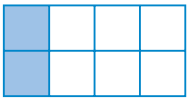
**a** $\frac{1}{2} - \frac{5}{8} - \frac{2}{5} = \frac{11}{40}$ **b** $\frac{19}{28} - \frac{3}{7} + \frac{3}{14} = \frac{1}{28}$

**Reflect**

Explain why you cannot add or subtract the numerators of fractions that have different denominators.

# 2D Multiplying fractions

## Start thinking!



**1** One quarter of this rectangle has been shaded blue.

**a** Explain how this diagram shows $\frac{1}{4}$?

**b** Can you think of a way to show half of one quarter on this diagram?



**2** The dark blue shading on this diagram shows half of one quarter.

**a** What fraction of the rectangle has been shaded dark blue?

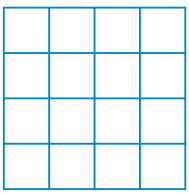
**b** What is $\frac{1}{2}$ of $\frac{1}{4}$? (Hint: Use your answer to part **a**.)

**c** Another way to write this is: What is $\frac{1}{2} \times \frac{1}{4}$? Copy and complete: $\frac{1}{2} \times \frac{1}{4} =$ ___.

**3** **a** Copy the diagram from question **1** and shade three quarters of the rectangle.

**b** Highlight half of the shading in a different colour to show half of three quarters.

**c** What is $\frac{1}{2} \times \frac{3}{4}$?



**4** **a** Copy this diagram and shade three quarters of it.

**b** Highlight a quarter of the shaded area in a different colour to show a quarter of three quarters.

**c** What is $\frac{1}{4} \times \frac{3}{4}$?

**5** Can you see a pattern to follow when multiplying two fractions? Write a description and compare it with a friend's.

## KEY IDEAS

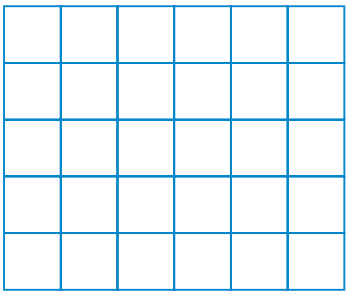
- To multiply fractions, follow these steps.
  1. First look for any factors common to the numerator and denominator.
  2. Cancel a number in the numerator and a number in the denominator by dividing both numbers by the HCF. This will simplify the calculation.

     Cancelling can occur vertically; for example, $\frac{\overset{1}{\cancel{3}}}{\underset{6}{\cancel{30}}} \times \frac{7}{8}$ or diagonally; for example, $\frac{3}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{13}$.
  3. Then multiply the numerators together and multiply the denominators together.
- The denominators do not need to be the same when multiplying fractions.
- Mixed numbers must be converted to improper fractions before multiplying.
- Whole numbers can be written as fractions with a denominator of 1. For example, $8 = \frac{8}{1}$.

# EXERCISE 2D Multiplying fractions

UNDERSTANDING AND FLUENCY

**1** **a** Copy this diagram.



**b** Shade $\frac{1}{6}$ of the rectangle.

**c** Using a different colour, shade $\frac{1}{5}$ of $\frac{1}{6}$.

**d** What fraction of the diagram does $\frac{1}{5}$ of $\frac{1}{6}$ represent?

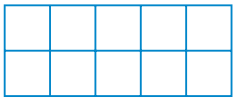
**e** What is $\frac{1}{5} \times \frac{1}{6}$?

**f** Repeat parts **a**–**e** for $\frac{2}{5}$ of $\frac{5}{6}$.

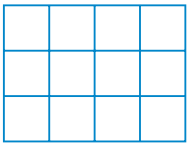
**g** Copy and complete: In mathematics, the word *of* can be replaced by _____.

**2** Use the diagrams below and the method in question **1** to work out each multiplication problem.

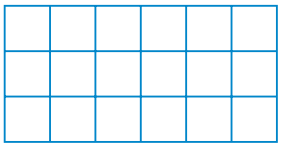
**a** $\frac{1}{2} \times \frac{3}{5}$



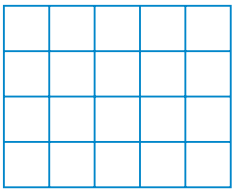
**b** $\frac{1}{3} \times \frac{1}{4}$



**c** $\frac{1}{3} \times \frac{5}{6}$



**d** $\frac{3}{4} \times \frac{2}{5}$



## EXAMPLE 2D-1 Multiplying fractions

Calculate $\frac{3}{5} \times \frac{2}{7}$.

### THINK

**1** Look for any common factors between numerators and denominators (none).

**2** Multiply the numerators together ($3 \times 2 = 6$) and then multiply the denominators together ($5 \times 7 = 35$).

### WRITE

$\frac{3}{5} \times \frac{2}{7}$

$= \frac{6}{35}$

**3** Calculate:

**a** $\frac{1}{5} \times \frac{1}{2}$ **b** $\frac{2}{3} \times \frac{1}{7}$ **c** $\frac{3}{11} \times \frac{2}{5}$

**d** $\frac{1}{3} \times \frac{4}{9}$ **e** $\frac{3}{8} \times \frac{1}{2}$ **f** $\frac{11}{13} \times \frac{6}{7}$

**g** $\frac{2}{5} \times \frac{4}{5}$ **h** $\frac{1}{3} \times \frac{7}{15}$ **i** $\frac{2}{9} \times \frac{5}{9}$

**j** $\frac{4}{11} \times \frac{5}{9}$ **k** $\frac{3}{10} \times \frac{7}{8}$ **l** $\frac{5}{13} \times \frac{8}{9}$

**4** Check your answers to question **2** using the method shown in Example 2D-1.
Calculate:

**a** $\frac{1}{2} \times \frac{3}{5}$ **b** $\frac{1}{3} \times \frac{1}{4}$

**c** $\frac{1}{3} \times \frac{5}{6}$ **d** $\frac{3}{4} \times \frac{2}{5}$

## EXAMPLE 2D-2 Multiplying fractions with cancelling

Calculate $\frac{8}{5} \times \frac{25}{12}$.

### THINK

**1** Look for any common factors between numerators and denominators. The numbers 8 and 12 have an HCF of 4. The numbers 25 and 5 have an HCF of 5.

**2** Cancel 8 and 12 by dividing both numbers by 4 (shown in green). Then cancel 25 and 5 by dividing both numbers by 5 (shown in blue).

**3** Write the result obtained after cancelling.

**4** Multiply the numerators together ($2 \times 5 = 10$) and then multiply the denominators together ($1 \times 3 = 3$).

**5** Convert the improper fraction to a mixed number.

### WRITE

$\frac{8}{5} \times \frac{25}{12}$

$= \frac{{}^{2}\cancel{8}}{{}_{1}\cancel{5}} \times \frac{\cancel{25}^{5}}{\cancel{12}_{3}}$

$= \frac{2}{1} \times \frac{5}{3}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

UNDERSTANDING AND FLUENCY

**5** Calculate:

**a** $\frac{8}{9} \times \frac{1}{4}$ **b** $\frac{2}{3} \times \frac{6}{11}$ **c** $\frac{3}{5} \times \frac{10}{13}$

**d** $\frac{1}{8} \times \frac{4}{17}$ **e** $\frac{7}{9} \times \frac{12}{35}$ **f** $\frac{20}{27} \times \frac{9}{15}$

**g** $\frac{3}{4} \times \frac{18}{24}$ **h** $\frac{9}{35} \times \frac{20}{21}$ **i** $\frac{14}{28} \times \frac{8}{13}$

**j** $\frac{8}{12} \times \frac{2}{5}$ **k** $\frac{10}{11} \times \frac{22}{15}$ **l** $\frac{6}{5} \times \frac{4}{12}$

**m** $\frac{35}{17} \times \frac{6}{42}$ **n** $\frac{21}{25} \times \frac{15}{12}$ **o** $\frac{20}{50} \times \frac{3}{18}$

## EXAMPLE 2D-3 Multiplying mixed numbers

Calculate $1\frac{2}{3} \times 2\frac{7}{10}$.

| THINK | WRITE |
|---|---|
| **1** Convert each mixed number to an improper fraction. | $1\frac{2}{3} \times 2\frac{7}{10}$<br>$= \frac{5}{3} \times \frac{27}{10}$ |
| **2** Cancel 5 and 10 by dividing both numbers by 5 (shown in green). Then cancel 3 and 27 by dividing both numbers by 3 (shown in blue). | $= \frac{{}^{1}\cancel{5}}{\cancel{3}_{1}} \times \frac{\cancel{27}^{9}}{\cancel{10}_{2}}$ |
| **3** Write the result obtained after cancelling. | $= \frac{1}{1} \times \frac{9}{2}$ |
| **4** Multiply the numerators together ($1 \times 9 = 9$) and then multiply the denominators together ($1 \times 2 = 2$). | $= \frac{9}{2}$ |
| **5** Convert the improper fraction to a mixed number. | $= 4\frac{1}{2}$ |

UNDERSTANDING AND FLUENCY

**6** Calculate:

**a** $\frac{3}{5} \times 1\frac{4}{11}$ **b** $\frac{22}{4} \times \frac{6}{7}$ **c** $1\frac{13}{20} \times \frac{7}{11}$ **d** $4\frac{2}{3} \times 3$

**e** $\frac{7}{9} \times 3\frac{5}{7}$ **f** $5\frac{2}{3} \times 2\frac{2}{5}$ **g** $5\frac{3}{5} \times 3\frac{1}{4}$ **h** $3\frac{1}{3} \times 4\frac{1}{8}$

**7** Calculate each of these. (Hint: a whole number can be written as a fraction with a denominator of 1.)

**a** $\frac{3}{8} \times 24$ **b** $\frac{5}{11} \times 66$ **c** $72 \times \frac{4}{9}$ **d** $\frac{5}{2} \times 16$

**e** $\frac{7}{6} \times 18$ **f** $3\frac{1}{6} \times 24$ **g** $\frac{12}{15} \times 4$ **h** $5 \times \frac{16}{25}$

**i** $9 \times \frac{1}{24}$ **j** $6 \times \frac{5}{36}$ **k** $\frac{8}{3} \times 15$ **l** $9 \times \frac{4}{81}$

**8** Refer to your answers to question **7**. Copy and complete the following using the words *larger* or *smaller*.

When multiplying a whole number by a proper fraction the answer is always _____ than the whole number.

When multiplying a whole number by an improper fraction or mixed number the answer is always _____ than the whole number.

**9** Calculate:

**a** $\frac{5}{9} \times \frac{9}{10} \times \frac{12}{17}$ **b** $\frac{3}{8} \times \frac{15}{26} \times \frac{8}{9}$ **c** $\frac{28}{5} \times \frac{11}{6} \times \frac{24}{7}$ **d** $1\frac{1}{12} \times \frac{8}{3} \times \frac{15}{4}$

**e** $\frac{22}{13} \times 10\frac{2}{5} \times 2\frac{1}{11}$ **f** $4\frac{1}{2} \times 5\frac{2}{5} \times \frac{21}{8}$ **g** $\frac{12}{5} \times 7\frac{1}{2} \times 3\frac{1}{4}$ **h** $6\frac{2}{3} \times 5\frac{1}{4} \times 1\frac{2}{7}$

UNDERSTANDING AND FLUENCY



**10** Look at these Vespa motorscooters.

**a** How could you use this diagram to explain that '$\frac{1}{4}$ of the Vespas are red'?

**b** How many Vespas are there?

**c** How many Vespas are red?

**d** What is the relationship between $\frac{1}{4}$ and the answers obtained in parts **b** and **c**?

**e** Using your answers to parts **b** and **c**, copy and complete this statement: $\frac{1}{4}$ of 12 equals _____.

**f** Rewrite the statement in part **e** as a mathematical equation. What operation has replaced the word *of*?

**11** Calculate:

**a** $\frac{2}{5}$ of $\frac{1}{8}$ **b** $\frac{3}{4}$ of $\frac{3}{7}$ **c** $\frac{2}{11}$ of $\frac{7}{9}$ **d** $\frac{9}{4}$ of $\frac{5}{11}$ **e** $2\frac{7}{13}$ of 26 **f** $5\frac{1}{2}$ of $3\frac{1}{3}$

PROBLEM SOLVING AND REASONING

**12** Hilaire works at the local swimming pool as a lifeguard. Her shift begins at 3.45 pm and ends at 6.15 pm.



**a** Write the hours Hilaire works in one shift as:

**i** a mixed number **ii** an improper fraction.

**b** If Hilaire has the same shift five nights a week, calculate the total number of hours she works as a fraction.

**13** Whilst correcting his students' homework, a teacher found three different answers to the question 'Find $2\frac{4}{9} \times 3\frac{3}{8}$ in simplest form'. The answers given were: $6\frac{12}{72}$, $8\frac{1}{4}$ and $6\frac{12}{17}$.

**a** Which answer is correct?

**b** Comment on the error that is likely to have been made in each case.

## EXAMPLE 2D-4 Multiplying fractions with unit conversions

Calculate $\frac{1}{5}$ of 2 hours. Give your answer in minutes.

| THINK | WRITE |
|---|---|
| **1** Convert the number of hours into minutes. | $\frac{1}{5}$ of 2 hours<br>$= \frac{1}{5}$ of 120 minutes |
| **2** Replace 'of' with '×' and cancel any common factors. | $= \frac{1}{5} \times 120$<br>$= \frac{1}{\cancel{5}_{1}} \times \frac{\cancel{120}^{24}}{1}$ |
| **3** Multiply numerators and denominators. | $= \frac{24}{1}$ |
| **4** Write your answer. | = 24 minutes |

PROBLEM SOLVING AND REASONING

**14** Calculate each fraction amount in the units shown in the brackets.

**a** $\frac{1}{4}$ of 3 hours (minutes)

**b** $\frac{2}{3}$ of 1 year (months)

**c** $\frac{4}{5}$ of \$80 (dollars)

**d** $\frac{6}{13}$ of the year (weeks)

**e** $\frac{2}{5}$ of 2 minutes (seconds)

**f** $\frac{3}{4}$ of 100 L (litres)

**g** $\frac{3}{7}$ of 2 weeks (days)

**h** $\frac{9}{4}$ of \$200 (dollars)

**i** $\frac{2}{3}$ of $6\frac{1}{2}$ hours (hours)

**j** $\frac{7}{5}$ of $11\frac{1}{4}$ km (km)

**k** $\frac{5}{6}$ of 10 years (months)

**l** $\frac{4}{7}$ of \$2.80 (cents)

**15** Amy baked a batch of 36 cookies and arranged them on a tray in a six by six pattern.

**a** Draw a diagram of the arrangement and shade $\frac{5}{6}$ of the diagram. How many cookies does this represent?

**b** Amy decides to take $\frac{2}{3}$ of the cookies to school and share them with her classmates. How many cookies will she take to school?

**c** When she reached school, Amy realised that $\frac{1}{8}$ of the cookies were broken in her bag. How many cookies were broken?



**16** A store takes $\frac{2}{5}$ off the price of its televisions during the stock-take sales. What will an LCD television normally sold for \$6320 now cost?

**17** Your friend manages to complete $\frac{3}{8}$ of the $5\frac{1}{2}$ km run before stopping. How far did she run prior to stopping?

CHALLENGE

**18** Boris spent $\frac{4}{5}$ of an hour training for his upcoming tennis match. Roger spent $3\frac{1}{2}$ times longer than Boris in preparing for his match.

**a** How long did each boy spend training?

**b** How much longer than Boris did Roger spend training?

**19** When two fractions are multiplied, their product (or answer) is $\frac{5}{7}$. What possible values could the two fractions be? List three pairs of possible fractions.

**20** Calculate:

**a** $3\frac{1}{5} \times \frac{3}{8} - \frac{2}{7}$

**b** $\frac{2}{7} + 3\frac{1}{2} \times \frac{3}{7}$

**c** $\left(\frac{3}{2} + 5\frac{1}{3}\right) \times 4\frac{1}{7}$

**Reflect**

Explain why it is useful to first simplify fractions before multiplying them.

# 2E Dividing fractions

## Start thinking!

**1** Look at the rectangles below.

**a** How many lots of $\frac{1}{4}$ are in one whole?



**b** How many lots of $\frac{1}{4}$ are in two wholes?



**c** How many lots of $\frac{1}{4}$ are in three wholes?



**2** What do you notice about your answers to question **1**?

**3** Repeat questions **1** and **2**, but this time draw rectangles that are divided into fifths. Consider how many lots of $\frac{1}{5}$ are in one whole, two wholes and three wholes.

**4** Asking how many lots of $\frac{1}{4}$ are in one whole is the same as asking 'What is 1 is divided by $\frac{1}{4}$?' This can be written as $1 \div \frac{1}{4}$.

Use the diagrams and your results for questions **1** and **3** to answer the following.

**a** $1 \div \frac{1}{4} =$ **b** $2 \div \frac{1}{4} =$ **c** $3 \div \frac{1}{4} =$ **d** $1 \div \frac{1}{5} =$ **e** $2 \div \frac{1}{5} =$ **f** $3 \div \frac{1}{5} =$

**5** Copy and complete these sentences.

**a** Dividing a number by $\frac{1}{4}$ is the same as _____ that number by 4.

**b** Dividing a number by $\frac{1}{5}$ is the same as _____ that number by _____.

## KEY IDEAS

- To divide fractions, follow these steps.
  1. Change from a division to a multiplication problem by replacing the division sign with a multiplication sign and turning the fraction that follows upside down.
  2. Proceed as for a multiplication problem by first cancelling, if possible, then multiplying the numerators together and the denominators together (see Example A).

  The process of turning a fraction upside down is called taking the **reciprocal** (see Example B).
- To divide mixed numbers, first convert them to improper fractions.
- Where appropriate, convert the answer to a mixed number.

**Example A**

$\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3}$

$= \frac{14}{15}$

**Example B**

The reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$.

The reciprocal of 4 is $\frac{1}{4}$ (since 4 can be written as $\frac{4}{1}$).

The reciprocal of $2\frac{1}{5}$ is $\frac{5}{11}$ (since $2\frac{1}{5}$ when written as an improper fraction is $\frac{11}{5}$).

# EXERCISE 2E Dividing fractions

**1** Rewrite each division problem in the form 'How many lots of ____ are there in ____?'

**a** $2 \div \frac{1}{8}$ **b** $4 \div \frac{1}{3}$ **c** $\frac{1}{2} \div \frac{1}{12}$ **d** $\frac{1}{3} \div \frac{1}{9}$

**2** Draw a diagram to help you find the answer to each division problem in question **1**.

**3** Write each division problem in question **1** as a multiplication problem.

**4** Check your answers to question **2** by working out the results to question **3**. Remember that whole numbers can be written with a denominator of 1.

## EXAMPLE 2E-1 Writing reciprocals

Write the reciprocal of each of these.

**a** $\frac{3}{7}$ **b** $\frac{1}{8}$ **c** 5

**THINK**

**a** Turn the fraction 'upside down'. That is, swap the numerator and the denominator.

**b** Turn the fraction 'upside down'.

**c** Write 5 as a fraction and then turn the fraction 'upside down'.

**WRITE**

**a** The reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$.

**b** The reciprocal of $\frac{1}{8}$ is $\frac{8}{1}$ or 8.

**c** $5 = \frac{5}{1}$. The reciprocal of 5 is $\frac{1}{5}$.

**5** Write the reciprocal of each of these.

**a** $\frac{4}{9}$ **b** $\frac{11}{3}$ **c** $\frac{2}{5}$ **d** $\frac{27}{4}$ **e** $\frac{1}{100}$ **f** 8 **g** $\frac{34}{1}$ **h** $\frac{65}{31}$

## EXAMPLE 2E-2 Dividing fractions

Calculate $\frac{1}{7} \div \frac{3}{5}$.

**THINK**

**1** Change the division problem to a multiplication problem. Replace ÷ with × and turn the fraction that follows the division sign upside down.

**2** Multiply the numerators together and then multiply the denominators together.

**WRITE**

$\frac{1}{7} \div \frac{3}{5}$

$= \frac{1}{7} \times \frac{5}{3}$

$= \frac{5}{21}$

UNDERSTANDING AND FLUENCY

**6** Calculate:

**a** $\frac{1}{8} \div \frac{2}{5}$ **b** $\frac{3}{4} \div \frac{3}{10}$ **c** $\frac{1}{12} \div \frac{1}{11}$

**d** $\frac{1}{9} \div \frac{4}{17}$ **e** $\frac{1}{10} \div \frac{4}{25}$ **f** $\frac{6}{7} \div \frac{8}{21}$

**g** $\frac{2}{3} \div \frac{8}{9}$ **h** $\frac{9}{14} \div \frac{4}{7}$ **i** $\frac{3}{4} \div \frac{5}{8}$

**j** $\frac{5}{9} \div \frac{10}{3}$ **k** $\frac{7}{10} \div \frac{5}{4}$ **l** $\frac{4}{11} \div \frac{3}{22}$

**m** $\frac{6}{7} \div \frac{2}{9}$ **n** $\frac{16}{15} \div \frac{12}{5}$ **o** $\frac{3}{8} \div \frac{9}{20}$

**p** $\frac{1}{24} \div \frac{13}{18}$ **q** $\frac{14}{3} \div \frac{7}{11}$ **r** $\frac{4}{13} \div \frac{20}{3}$

**7** Calculate:

**a** $6 \div \frac{1}{7}$ **b** $14 \div \frac{7}{6}$ **c** $12 \div \frac{8}{9}$

**d** $5 \div \frac{10}{13}$ **e** $\frac{2}{9} \div 6$ **f** $\frac{3}{4} \div 9$

**g** $\frac{5}{2} \div 10$ **h** $\frac{8}{9} \div 2$ **i** $4 \div \frac{7}{5}$

## EXAMPLE 2E-3 Dividing mixed numbers

Calculate $4\frac{3}{8} \div 2\frac{6}{7}$.

### THINK

**1** Convert each mixed number to an improper fraction.

**2** Change to a multiplication problem by replacing ÷ with × and turning the fraction that follows upside down.

**3** Cancel 35 and 20 by dividing each by the HCF of 5.

**4** Write the result after cancelling.

**5** Multiply the numerators together and then multiply the denominators together.

**6** Change the improper fraction to a mixed number.

### WRITE

$4\frac{3}{8} \div 2\frac{6}{7}$

$= \frac{35}{8} \div \frac{20}{7}$

$= \frac{35}{8} \times \frac{7}{20}$

$= \frac{{}^{7}\cancel{35}}{8} \times \frac{7}{\cancel{20}_{4}}$

$= \frac{7}{8} \times \frac{7}{4}$

$= \frac{49}{32}$

$= 1\frac{17}{32}$

**8** Calculate:

**a** $3\frac{1}{2} \div \frac{4}{5}$ **b** $1\frac{4}{7} \div \frac{8}{11}$ **c** $6\frac{1}{3} \div 1\frac{1}{4}$

**d** $7\frac{1}{5} \div 2\frac{3}{4}$ **e** $1\frac{2}{7} \div \frac{15}{21}$ **f** $2\frac{7}{9} \div \frac{4}{18}$

**g** $\frac{13}{4} \div 2\frac{6}{10}$ **h** $3\frac{3}{5} \div 1\frac{4}{5}$ **i** $4 \div 1\frac{1}{5}$

**j** $2\frac{1}{3} \div 5$ **k** $9\frac{1}{11} \div 10$ **l** $7\frac{3}{5} \div 19$

PROBLEM SOLVING AND REASONING

**9** Calculate:

**a** $\frac{15}{27} \div \frac{11}{21}$ **b** $\frac{13}{24} \div \frac{9}{40}$ **c** $2\frac{1}{12} \div 1\frac{5}{6}$

**d** $1\frac{1}{8} \div 10\frac{1}{2}$ **e** $3\frac{2}{10} \div 1\frac{1}{3}$ **f** $\frac{5}{27} \div 2\frac{7}{9}$

**g** $12\frac{2}{5} \div 8\frac{4}{15}$ **h** $3\frac{3}{20} \div 4\frac{3}{10}$ **i** $5\frac{7}{9} \div 6\frac{2}{3}$

**10** When dividing a mixed number by a proper fraction, do you obtain a smaller or larger number? Explain.

**11** The ingredients to make a sticky date pudding for four people are shown below.



$\frac{3}{4}$ cup of chopped pitted dates
1 teaspoon of bicarbonate of soda
60 g butter
$\frac{2}{3}$ cup of sugar
2 eggs
1 cup of self-raising flour

**a** A triple mixture of sticky date pudding is needed to feed 12 people. How much of each ingredient would be required?

**b** To feed six people, a one-and-a-half mixture of sticky date pudding is needed. How much of each ingredient is required?

**c** How much of each ingredient is required for the recipe to feed:

**i** two people?

**ii** three people?

**12** A batch of a chocolate pudding recipe requires $\frac{1}{6}$ kg of sugar. How many batches of chocolate pudding can be made from this 1 kg packet of sugar?




**13** Over the school holidays Brianna worked a total of 44 hours stacking shelves at the supermarket. If she worked $5\frac{1}{2}$ hour shifts, how many shifts did she work?

**14** Students in a technology and design class are making costumes for the school production. A total of 35 m of fabric has been purchased. If each garment requires $1\frac{3}{4}$ m of fabric, how many garments can be made?

**15** Rhys shares $3\frac{3}{8}$ pizzas equally between himself and eight friends. What fraction of a pizza will each person receive?

**16** A new housing estate is to be developed on $43\frac{1}{5}$ hectares of land. If each block measures $\frac{3}{5}$ hectares, how many blocks will there be?

**17** Madusha and Cynthia have been training for their school's upcoming walkathon. It takes them $\frac{1}{10}$ of an hour to complete one lap.



**a** If the time allowed for the walkathon is $7\frac{1}{2}$ hours, what is the maximum number of laps the girls could complete?

**b** Do you think the girls will complete the maximum number of laps? Explain.

**18** Gianluca must pack 15 kg of apples into plastic bags that can hold a maximum of $1\frac{1}{3}$ kg.



**a** How many $1\frac{1}{3}$ bags can he fill?

**b** How many kilograms of apples will be left over?

**19** After dividing two proper fractions, a mixed number is obtained. Write three possible division problems that show this.

**20** After dividing two proper fractions, a whole number is obtained. Write three possible division problems that show this.

**21** After dividing two mixed numbers, a proper fraction is obtained. Write three possible division problems that show this.

**22** After one fraction is divided by another, the answer is $\frac{5}{8}$. Write three possible division problems that show this.

CHALLENGE

**23** While marking test papers, a teacher obtained three different answers to the question 'Divide $4\frac{1}{14}$ into $2\frac{5}{7}$'.



The answers obtained were $\frac{2}{3}$, $\frac{1}{6}$ and $\frac{1083}{98}$.

**a** Which is the correct answer?

**b** Explain how the two incorrect answers might have been obtained.

**24** Use what you have learnt from question **23** to help you answer the following.

**a** Divide $\frac{2}{3}$ into $1\frac{5}{12}$.

**b** Divide $\frac{8}{9}$ into $6\frac{2}{3}$.

**c** Divide $1\frac{2}{5}$ into $\frac{8}{5}$.

**d** Divide $12\frac{3}{8}$ into $\frac{11}{8}$.

**e** Divide 3 into $2\frac{2}{15}$.

**f** Divide 8 into $5\frac{2}{6}$.

**g** Divide $\frac{8}{21}$ into $1\frac{1}{7}$.

**h** Divide $4\frac{1}{4}$ into $5\frac{2}{3}$.

**25** Calculate each of these.
Remember to think carefully about which operations are performed first.

**a** $\frac{5}{3} \div 1\frac{3}{7} + 1\frac{3}{4}$

**b** $2\frac{1}{3} \div \left(2\frac{2}{4} \div 1\frac{4}{5}\right)$

**c** $\frac{7}{3} + 2\frac{1}{2} \times \frac{7}{10} \div 1\frac{1}{2}$

**d** $\left(\frac{4}{5} - \frac{2}{3}\right) \div \left(1\frac{1}{3} + 3\frac{2}{9}\right) \times \left(5 \div \frac{2}{3}\right)$

**e** $\left(1\frac{5}{7} \times 1\frac{3}{4}\right) + \left(\frac{13}{3} - 3\frac{2}{3}\right) \div \left(2\frac{1}{3} \times \frac{1}{7}\right)$

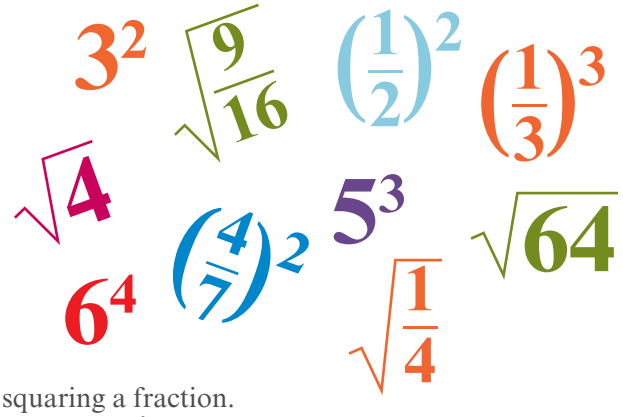
**f** $\left(1\frac{1}{2} - \frac{11}{12}\right) + \frac{5}{7} \times \left(\frac{1}{4} \div \frac{3}{7}\right)$

**Reflect**

What would you tell your friend it's important to remember when dividing fractions?

# 2F Powers and square roots of fractions

## Start thinking!




**1** Calculate these, then explain how you got your answers.

**a** $2^2$ **b** $5^2$ **c** $9^2$ **d** $10^2$

**2** Calculate:

**a** $\frac{1}{2} \times \frac{1}{2}$ **b** $\frac{1}{5} \times \frac{1}{5}$ **c** $\frac{1}{9} \times \frac{1}{9}$ **d** $\frac{1}{10} \times \frac{1}{10}$

**3** Write what you expect the answer to be for each of these.

**a** $\left(\frac{1}{2}\right)^2$ **b** $\left(\frac{1}{5}\right)^2$ **c** $\left(\frac{1}{9}\right)^2$ **d** $\left(\frac{1}{10}\right)^2$

**4** Copy and complete each of these to show two methods of squaring a fraction.

**a** $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2}$ or $\left(\frac{1}{2}\right)^2 = \frac{\square^2}{\square^2}$

$= ____$ $= \frac{1}{4}$

**b** $\left(\frac{1}{5}\right)^2 = \frac{1}{5} \times \frac{1}{5}$ or $\left(\frac{1}{5}\right)^2 = \frac{\square^2}{\square^2}$

$= ____$ $= \frac{1}{25}$

**5** Calculate these, then explain how you got your answers.

**a** $\sqrt{4}$ **b** $\sqrt{25}$ **c** $\sqrt{81}$ **d** $\sqrt{100}$

**6** Use your answers to question **5** to help you calculate each of these.

**a** $\sqrt{\frac{1}{4}}$ **b** $\sqrt{\frac{1}{25}}$ **c** $\sqrt{\frac{1}{81}}$ **d** $\sqrt{\frac{1}{100}}$

**7** Copy and complete the following to show how to calculate the square root of each fraction.

**a** $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}}$

$= \frac{1}{\square}$

**b** $\sqrt{\frac{1}{25}} = \frac{\sqrt{1}}{\sqrt{\square}}$

$= \frac{1}{\square}$

## KEY IDEAS

- A fraction can be squared in two ways.
  1. Multiply the fraction by itself. For example, $\left(\frac{7}{9}\right)^2 = \frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$.
  2. Square the numerator and then square the denominator. For example, $\left(\frac{7}{9}\right)^2 = \frac{7^2}{9^2} = \frac{49}{81}$.
- To obtain the square root of a fraction, find the square root of the numerator and then find the square root of the denominator. For example, $\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$.
- A mixed number must first be converted to an improper fraction before squaring or finding the square root. Where possible, the final answer can then be written as a mixed number.
- Fractions can also be raised to higher powers. For example, $\left(\frac{4}{7}\right)^3 = \frac{4^3}{7^3} = \frac{64}{343}$ and $\left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4} = \frac{16}{625}$.

# EXERCISE 2F Powers and square roots of fractions

## EXAMPLE 2F-1 Squaring fractions

Calculate $(\frac{3}{5})^2$.

| THINK | WRITE |
|---|---|
| | $(\frac{3}{5})^2$ |
| **1** Square the numerator and square the denominator. (Alternatively, write as the multiplication of two fractions.) | $= \frac{3^2}{5^2}$ or $\frac{3}{5} \times \frac{3}{5}$ |
| **2** Calculate the numerator and then the denominator. | $= \frac{9}{25}$ |

**1** Calculate:

**a** $(\frac{2}{5})^2$ **b** $(\frac{3}{8})^2$ **c** $(\frac{9}{7})^2$ **d** $(\frac{10}{3})^2$

**e** $(\frac{4}{5})^2$ **f** $(\frac{11}{6})^2$ **g** $(\frac{2}{11})^2$ **h** $(\frac{5}{7})^2$

## EXAMPLE 2F-2 Squaring mixed numbers

Calculate $(2\frac{1}{3})^2$.

| THINK | WRITE | |
|---|---|---|
| | $(2\frac{1}{3})^2$ | |
| **1** Convert the mixed number to an improper fraction. | $= (\frac{7}{3})^2$ | or $= (\frac{7}{3})^2$ |
| **2** Square the numerator and square the denominator. (Alternatively, write as the multiplication of two fractions.) | $= \frac{7^2}{3^2}$ | $= \frac{7}{3} \times \frac{7}{3}$ |
| **3** Calculate the numerator and then the denominator. | $= \frac{49}{9}$ | $= \frac{49}{9}$ |
| **4** Convert the improper fraction to a mixed number. | $= 5\frac{4}{9}$ | $= 5\frac{4}{9}$ |

**2** Calculate:

**a** $(2\frac{3}{5})^2$ **b** $(6\frac{1}{4})^2$ **c** $(1\frac{5}{7})^2$ **d** $(3\frac{1}{2})^2$

**e** $(2\frac{2}{3})^2$ **f** $(1\frac{3}{8})^2$ **g** $(1\frac{3}{7})^2$ **h** $(2\frac{6}{7})^2$

## EXAMPLE 2F-3 Finding the square root of fractions

Calculate $\sqrt{\frac{100}{144}}$.

**THINK**

1 Write as the square root of the numerator divided by the square root of the denominator.

2 Calculate the square root of the numerator and the square root of the denominator.

3 Simplify by cancelling. Divide the numerator and the denominator by the HCF of 2.

4 Write the answer.

**WRITE**

$\sqrt{\frac{100}{144}}$

$= \frac{\sqrt{100}}{\sqrt{144}}$

$= \frac{10}{12}$

$= \frac{\cancel{10}^{5}}{\cancel{12}_{6}}$

$= \frac{5}{6}$

**3** Calculate:

**a** $\sqrt{\frac{4}{81}}$ **b** $\sqrt{\frac{25}{64}}$ **c** $\sqrt{\frac{9}{121}}$ **d** $\sqrt{\frac{49}{64}}$

**e** $\sqrt{\frac{64}{144}}$ **f** $\sqrt{\frac{9}{81}}$ **g** $\sqrt{\frac{16}{25}}$ **h** $\sqrt{\frac{36}{100}}$

## EXAMPLE 2F-4 Find the square root of mixed numbers

Calculate $\sqrt{11\frac{1}{9}}$.

**THINK**

1 Convert the mixed number to an improper fraction.

2 Write as the square root of the numerator divided by the square root of the denominator.

3 Calculate the square root of the numerator and the square root of the denominator.

4 Write the answer as a mixed number.

**WRITE**

$\sqrt{11\frac{1}{9}}$

$= \sqrt{\frac{100}{9}}$

$= \frac{\sqrt{100}}{\sqrt{9}}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

**4** Calculate:

**a** $\sqrt{\frac{144}{25}}$ **b** $\sqrt{\frac{196}{49}}$ **c** $\sqrt{5\frac{4}{9}}$ **d** $\sqrt{7\frac{1}{9}}$

**e** $\sqrt{3\frac{6}{25}}$ **f** $\sqrt{3\frac{13}{36}}$ **g** $\sqrt{7\frac{9}{16}}$ **h** $\sqrt{1\frac{19}{81}}$

UNDERSTANDING AND FLUENCY

**5** First simplify each fraction and then calculate the result.

**a** $\sqrt{\frac{8}{32}}$ **b** $\sqrt{\frac{50}{72}}$ **c** $\sqrt{\frac{48}{75}}$ **d** $\sqrt{\frac{45}{80}}$

**e** $\sqrt{\frac{24}{54}}$ **f** $\sqrt{\frac{288}{50}}$ **g** $\sqrt{\frac{72}{162}}$ **h** $\sqrt{\frac{400}{25}}$

**6** Explain why it was important to first simplify each fraction in question **5** before finding its square root.

**7** Calculate:

**a** $\frac{3}{5} + \frac{2}{10} + \left(\frac{3}{5}\right)^2$ **b** $\frac{5}{9} - \frac{4}{8} + \left(\frac{4}{3}\right)^2$ **c** $\frac{4}{3} - \frac{2}{5} - \left(\frac{2}{3}\right)^2$

**d** $\frac{5}{2} - \frac{8}{9} + \left(\frac{5}{6}\right)^2$ **e** $\left(\frac{7}{8}\right)^2 - \frac{3}{4} + \frac{3}{16}$ **f** $\frac{9}{4} - \frac{4}{8} + \sqrt{\frac{16}{25}}$

**8** Calculate each of these. Remember to think carefully about which operations are performed first.

**a** $\frac{4}{9} \times \frac{14}{8} \times \left(\frac{3}{7}\right)^2$ **b** $\frac{2}{3} \div \frac{4}{5} \times \left(\frac{3}{2}\right)^2$ **c** $3\frac{1}{4} \times \frac{4}{5} \div \sqrt{\frac{16}{49}}$

**d** $\sqrt{\frac{16}{25}} \div \frac{8}{3} + \frac{5}{8} \div 2\frac{1}{2}$ **e** $3\frac{3}{5} \div 2\frac{1}{4} - \sqrt{\frac{27}{12}}$ **f** $\left(\frac{5}{8}\right)^2 - \frac{3}{4} \div 5\frac{1}{3}$

## EXAMPLE 2F-5 Raising fractions to the power of three

Calculate $\left(\frac{2}{3}\right)^3$.

### THINK

**1** Write the numerator to the power of 3 and the denominator to the power of three. (Alternatively, write as three lots of the fraction multiplied together.)

**2** Calculate the numerator and then the denominator.

### WRITE

$\left(\frac{2}{3}\right)^3$

$= \frac{2^3}{3^3}$ or $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

$= \frac{8}{27}$

**9** Calculate:

**a** $2^3$ **b** $4^3$ **c** $7^3$ **d** $\left(\frac{1}{2}\right)^3$

**e** $\left(\frac{1}{4}\right)^3$ **f** $\left(\frac{2}{7}\right)^3$ **g** $\left(\frac{3}{11}\right)^3$ **h** $\left(\frac{4}{5}\right)^3$

**10** Describe how to calculate the result when a fraction is raised to any power.

**11** Calculate:

**a** $\left(\frac{3}{4}\right)^3$ **b** $\left(\frac{2}{5}\right)^3$ **c** $\left(\frac{4}{3}\right)^3$ **d** $\left(\frac{9}{7}\right)^3$

**e** $\left(2\frac{1}{4}\right)^3$ **f** $\left(3\frac{1}{2}\right)^3$ **g** $\left(5\frac{2}{3}\right)^3$ **h** $\left(2\frac{5}{8}\right)^3$

UNDERSTANDING AND FLUENCY

**12** Calculate:

**a** $\left(\frac{1}{2}\right)^4$ **b** $\left(\frac{2}{3}\right)^4$ **c** $\left(\frac{1}{4}\right)^5$ **d** $\left(\frac{2}{5}\right)^5$

**e** $\left(\frac{1}{3}\right)^6$ **f** $\left(\frac{3}{2}\right)^6$ **g** $\left(\frac{3}{4}\right)^4$ **h** $\left(\frac{1}{10}\right)^8$

**13** Calculate:

**a** $\left(2\frac{1}{5}\right)^4$ **b** $\left(\frac{10}{3}\right)^4$ **c** $\left(\frac{5}{2}\right)^5$ **d** $\left(\frac{5}{3}\right)^5$

**e** $\left(2\frac{1}{3}\right)^6$ **f** $\left(3\frac{1}{4}\right)^6$ **g** $\left(1\frac{2}{5}\right)^4$ **h** $\left(1\frac{1}{8}\right)^4$

**14** Carefully study each of these fractions: $\frac{3}{7}$, $\left(\frac{3}{7}\right)^2$, $\left(\frac{3}{7}\right)^3$, $\left(\frac{3}{7}\right)^4$.

**a** Arrange them in ascending order.

**b** What did you notice? Is this what you expected?

**15** Carefully study each of these fractions: $\frac{6}{5}$, $\left(\frac{6}{5}\right)^2$, $\left(\frac{6}{5}\right)^3$, $\left(\frac{6}{5}\right)^4$.

**a** Arrange them in ascending order.

**b** What did you notice?

**16** Using your results from questions **14** and **15**, explain what happens when proper fractions and improper fractions are raised to powers of 2 or more.

PROBLEM SOLVING AND REASONING

**17** Your grandfather has a number of vinyl LP records in his music collection. The cover of one is a square. Work out the area in square metres by calculating the square of the length or (length × length).



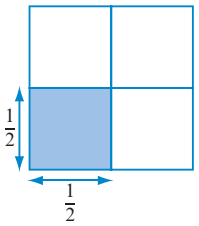

**18** In each diagram, a square has been shaded. Use the length shown to answer the following.

**a** Calculate the value of (length × length) or $(\text{length})^2$ to find the area of the shaded region.

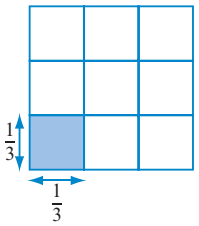
**b** Write the fraction of the large square that is shaded.

**c** Compare your answers to parts **a** and **b**. What do you notice?

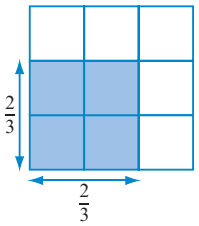
**i**



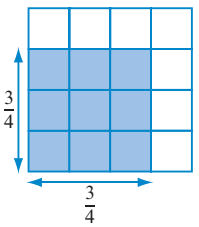

**ii**




**iii**




**iv**



PROBLEM SOLVING AND REASONING

**19** Look at each pair of fractions and work out whether they will give the same answer. Explain your reasoning.

**a** $(\frac{5}{8})^2$ and $\frac{5^2}{8}$ **b** $(\frac{2}{9})^2$ and $\frac{2}{9^2}$

**c** $(\frac{7}{3})^2$ and $\frac{7^2}{3^2}$ **d** $(\frac{1}{2}+\frac{1}{4})^2$ and $(\frac{1}{2})^2+(\frac{1}{4})^2$

**20** Two students produced two different answers when asked to calculate $(2\frac{1}{5})^2$. Look at each student's working and explain which is correct.

Olivia's calculation

$$(2\tfrac{1}{5})^2 = 2^2 + (\tfrac{1}{5})^2$$
$$= 4 + \tfrac{1}{25}$$
$$= 4\tfrac{1}{25}$$

Kate's calculation

$$(2\tfrac{1}{5})^2 = (\tfrac{11}{5})^2$$
$$= \tfrac{121}{25}$$
$$= 4\tfrac{21}{25}$$

**21** Jack produced these workings and answer when asked to calculate $\sqrt{9\frac{1}{16}}$. Comment on whether his answer and the process he used are correct.

Jack's workings

$$\sqrt{9\tfrac{1}{16}} = \sqrt{9} + \sqrt{\tfrac{1}{16}}$$
$$= 3 + \tfrac{1}{4}$$
$$= 3\tfrac{1}{4}$$

**22** A real estate agent states the size of a square block of land is $2\frac{7}{9}$ square kilometres.

**a** Explain how the length of the block of land can be worked out if you know that the area of a square is length × length.

**b** Calculate the length of the block of land for sale.

**c** How long is the fence that runs along the boundary of the land?

**d** If the land is for sale at $100 000, calculate the cost per square kilometre.




CHALLENGE

**23** Calculate:

**a** $(2\frac{2}{3})^2 - \sqrt{\frac{25}{81}}$ **b** $\frac{3}{14} \times \sqrt{\frac{169}{36}} \times (1\frac{1}{7})^2$

**c** $\frac{3}{4} \div \frac{9}{10} + \sqrt{1\frac{19}{81}} - \frac{1}{3}$ **d** $((2\frac{1}{4})^2 - \sqrt{\frac{196}{64}}) \div (1-\frac{1}{4})^2$

**e** $\sqrt{1\frac{21}{25} - 1\frac{1}{5}} \times 2\frac{2}{5} - \frac{3}{10}$ **f** $\sqrt{\sqrt{\frac{49}{144}} \times 2\frac{1}{3}}$

**Reflect**

How do you know in which order to perform operations on fractions?

# 2G Understanding ratios

## Start thinking!



1 Use the photo to copy and complete:
   - **a** There are _____ males compared to _____ females.
   - **b** There are _____ adults compared to _____ children.

2 What do you notice about the total in each comparison made in question **1**?

3 Write four more comparisons using the photo.

A comparison of two or more quantities is called a **ratio**. In the photo, there are 2 children compared to 15 adults, so the ratio can be written as 2:15. Notice that a ratio has numbers separated by a colon.

4 Rewrite the comparisons from question **1** as ratios using numbers and a colon.

5 Rewrite the comparisons from question **3** as ratios using numbers and a colon.



To make a particular shade of pink, one part of red paint and four parts of white paint are mixed together.

6 Write this information as a ratio.

7 Describe the colour that would be produced with a ratio of 2:3.

8 Would a mixture with a ratio of 3:2 produce the same shade as a mixture with a ratio of 2:3? Explain.

9 Explain why the order of each amount (part) listed in the ratio is important.

## KEY IDEAS

- A ratio is a comparison of two or more quantities of the same kind.
- Before writing a ratio, the numbers must be in the same unit of measurement.
- Ratios do not require units. That is, they are written as whole numbers with no units shown.
- A ratio must be written in the order of the worded description given. For example, one part cordial to five parts water is written as 1:5.
- Ratios do not contain fractions or decimals. They only contain whole numbers.

# EXERCISE 2G Understanding ratios



**1** Write each comparison as a ratio.

**a** the number of red jelly beans compared to the number of yellow jelly beans

**b** the number of orange jelly beans compared to the total number of jelly beans

**c** the number of pink jelly beans compared to the number of yellow jelly beans compared to the number of blue jelly beans

**d** the number of red and blue jelly beans compared to the number of pink and black jelly beans

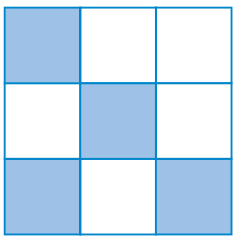


**2** Write each comparison as a ratio.

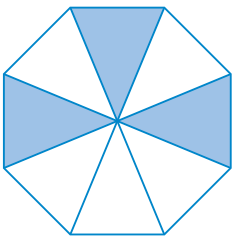
**i** the number of shaded parts to the number of non-shaded parts

**ii** the number of shaded parts to the total number of parts

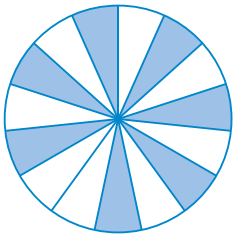
**a**



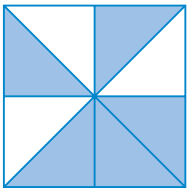
**b**



**c**



**d**



**3** Write each comparison as a ratio in the given order.

**a** 132 cm to 207 cm
**b** 16 kg to 35 kg
**c** 18 min to 11 min
**d** $85 to $121
**e** 12 weeks to 39 weeks
**f** 25 L to 17 L

**4** Write each comparison as a ratio in the given order.

**a** Collingwood scored 83 points compared to St Kilda's score of 67 points.
**b** Kayla spent 5 hours on Facebook and 1 hour doing homework.
**c** The fastest recorded tennis serve for a male is 247 km/h compared to 209 km/h for a female.
**d** Australia won three gold, four silver and nine bronze medals in the World Swimming Championships.
**e** Twenty-eight students tried out for the volleyball team while 23 students tried out for the netball team.
**f** To make a drink of cordial, use one part cordial to four parts water.

**5** Write the number of football players shown to the number of umpires as a ratio.




**6** Write each comparison of quantities as a ratio.

**a** A recipe requires five cups of flour to one cup of fruit and chopped nuts.
**b** A mixture contains 4 kg of sand, 2 kg of cement and 5 kg of gravel.
**c** In a group of 100 people, 13 are left-handed, 5 are both left- and right-handed and the rest are right-handed.
**d** Brittany and Lauren contribute equally to a friend's present.
**e** There are approximately three male passengers to every five female passengers on the train in the morning.

**7** List an example for each ratio.
You may like to use a diagram to show your example.

**a** 5:11 **b** 16:9 **c** 1:1 **d** 7:3

**8** A cupcake recipe lists these ingredients:



1 cup of milk
2 cups of sugar
3 cups of flour
4 eggs
5 tablespoons of melted butter
1 teaspoon of vanilla essence

**a** Write each comparison of quantities as a ratio in the order given.

**i** flour to sugar

**ii** milk to flour

**iii** dry ingredients (flour and sugar) to liquid ingredients (milk)

**b** Why does it not make sense to write a ratio of eggs to butter for this recipe?

**c** What would you need to know before you could write a ratio of melted butter to vanilla essence?

**9** From the given seven-day forecast, write ratios for:

**a** the number of windy days to the number of sunny days

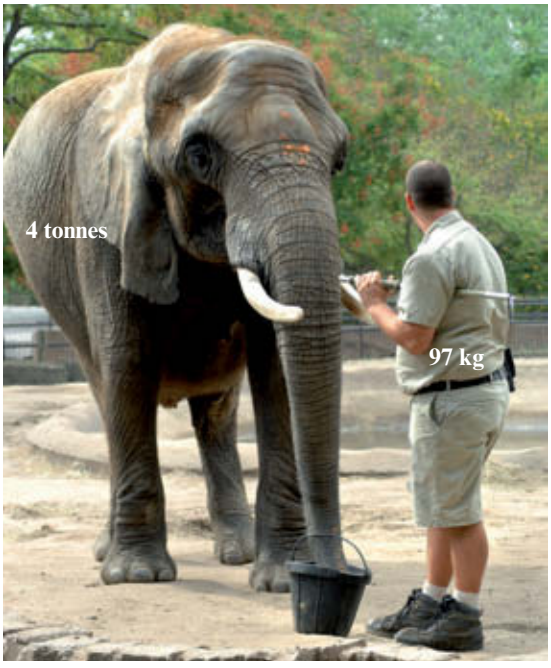
**b** the number of showery days to the total number of days

**c** the number of days with a maximum temperature of 13°C to the number of days with a maximum temperature of 15°C to the number of days with a maximum temperature of 18°C.

7 DAY FORECAST

| | | |
|---|---|---|
| today | max 13 | showers |
| tomorrow | max 15 | showers |
| Monday | max 18 | windy |
| Tuesday | max 13 | showers |
| Wednesday | max 18 | sunny |
| Thursday | max 18 | sunny |
| Friday | max 18 | sunny |

**10** Look at this elephant and its handler. The mass of each is shown.




**a** Explain why the ratio of the elephant handler's mass to the elephant's mass cannot be written as 97:4.

**b** Suggest what could be done to one of the measurements so that a ratio comparing their masses can be written.

**c** Discuss which measurement was selected and why.

**d** Write the ratio of the elephant handler's mass to the elephant's mass.
(Hint: 1000 kg = 1 tonne.)

**11** Can each of these comparisons be written as a ratio? Explain.

- **a** 45 m compared to 67 m
- **b** 375 mL of juice compared to 5 kg of chicken
- **c** \$5 compared to 50 cents
- **d** 19 km compared to 3 hours

**12** Convert the quantities to the same unit and then write the ratio for each comparison.

- **a** 10 days to 7 weeks
- **b** 23 minutes to 4 hours
- **c** 19 cm to 3.6 m
- **d** 25 cents to \$1.99
- **e** 157 kg to 5.2 tonne
- **f** 239 mL to 1.7 L
- **g** 51 months to 5 years
- **h** 11 bananas to 3 dozen bananas
- **i** 49 mL to 1 L
- **j** 5 hours to 47 minutes
- **k** 13.5 km to 8729 m
- **l** 23 seconds to 1.5 hours

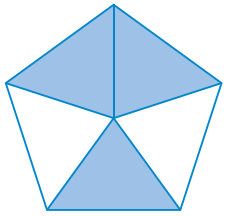
**13** In a survey, 173 out of 200 people preferred Dazzle toothpaste to their regular brand. The remainder preferred their regular brand.
Write a ratio for each of these.

- **a** the number of people that prefer Dazzle toothpaste to the total number of people surveyed
- **b** the number of people that prefer their regular brand of toothpaste to the total number of people surveyed
- **c** the number of people that prefer Dazzle toothpaste to the number of people who prefer their regular brand

**14** Max scored $\frac{87}{100}$ (or 87 out of 100) for his Geography test.
Write a ratio for:

- **a** the marks received compared to the total number of marks
- **b** the marks lost compared to the total number of marks
- **c** the marks received compared to the marks lost.

**15** Look at the pentagon shown.



- **a** What is the ratio of shaded sections to non-shaded sections?
- **b** What is the ratio of non-shaded sections to the total number of sections?
- **c** What is the ratio of shaded sections to the total number of sections?
- **d** Can any of the ratios in parts **a**–**c** be written as fractions? Explain your reasoning.
- **e** Can any of the ratios in parts **a**–**c** *not* be written as fractions? Explain your reasoning.
- **f** Copy and complete: Ratios can be written as fractions when comparing the ratio of a part to its _____.

PROBLEM SOLVING AND REASONING



**16** Geoff mixes 4 L of blue paint with 5 L of yellow paint to create a shade of green for his studio.

- **a** Write the ratio of the amount of blue paint to the amount of green paint.
- **b** What fraction of the new shade is blue paint?
- **c** What fraction of the new shade is yellow paint?



**17** Elio and Gaetano work at the local supermarket and are called in after school to cover some shifts. They both start at 4 pm. Elio is able to work till 9 pm while Gaetano must leave at 7 pm.

- **a** What is the ratio of the number of hours worked by Elio compared to those worked by Gaetano?
- **b** What is the total amount of time worked by both boys?
- **c** What fraction of the total amount of time did Elio work?

CHALLENGE

**18** Barry could not understand why his response to an Aussie rules question was marked incorrect. The question asked whether '10 goals and 3 behinds compared to 158 points' could be written as a ratio, to which Barry replied 'No, since the quantities involved different units'. Explain to Barry why his response was incorrect.

**19** Make up four comparisons and ask your friends whether a ratio can be written for each, stating reasons for their answers.

**Reflect**

What type of comparisons can be written as a ratio?



# 2H Working with ratios

## Start thinking!

For a family lunch, four trays of lasagne were baked.

The lasagne in the first tray was cut into 5 equal rectangular slices, the second into 10 equal rectangular slices, the third into 15 equal rectangular slices and the fourth into 20 equal rectangular slices. After the lunch, it was noticed that the same amount of lasagne had been eaten from each tray.

**1** For each tray of lasagne, write as a ratio the number of slices eaten compared to the original number of slices. For example, the ratio for Tray A is 2:5.

**2** Look for a pattern in your answers to question **1**. Write the next three ratios you would expect.

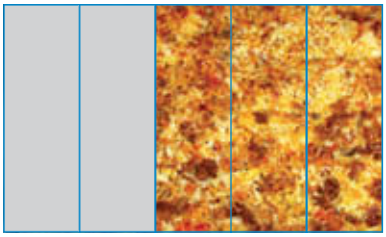
**3** Use your answers to question **1** to write each ratio as a fraction. For example, the fraction for Tray A is $\frac{2}{5}$.

**4** Explain what each fraction represents.

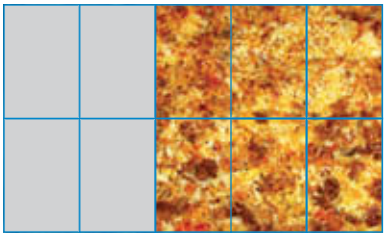
**5** Compare the fractions you have obtained. What is special about them?

**6** Copy and complete:
Numbers in a ratio can be _____ or _____ by the same value to create an equivalent ratio.

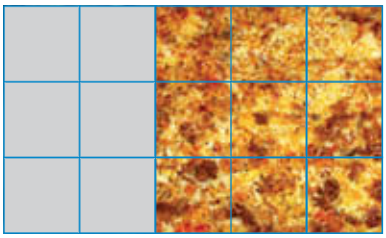
**A**



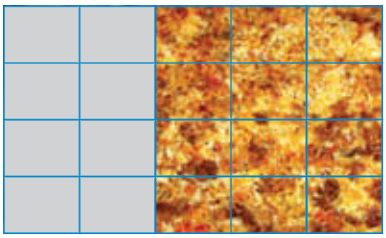
**B**



**C**



**D**



## KEY IDEAS

- Numbers in a ratio can be multiplied or divided by the same value to create an equivalent ratio.
- Ratios are written with whole numbers only.
- A ratio may be written in simplest form by dividing the numbers in the ratio by the HCF.

# EXERCISE 2H Working with ratios

## EXAMPLE 2H-1 Simplifying ratios

Write the ratio 35:56 in simplest form.

| THINK | WRITE |
|---|---|
| **1** Look for the HCF of each number in the ratio. HCF = 7. | 35:56 |
| **2** Divide each number in the ratio by the HCF of 7. (35 ÷ 7 = 5 and 56 ÷ 7 = 8.) | = 5:8 |

UNDERSTANDING AND FLUENCY

**1** Write each ratio in simplest form.

| | | |
|---|---|---|
| **a** 21:6 | **b** 12:18 | **c** 52:64 |
| **d** 36:96 | **e** 88:48 | **f** 40:45 |
| **g** 65:13 | **h** 22:132 | **i** 20:50:20 |
| **j** 125:75:25 | **k** 27:45:63 | **l** 24:56:40 |

**2** Fill in the gaps to complete the equivalent ratios.

| | |
|---|---|
| **a** 3:2 = ___:4 = 15:___ | **b** 4:5 = ___:25 = 16:___ |
| **c** ___:210 = 10:70 = 2:___ | **d** 144:___ = 24:9 = 8:___ |
| **e** ___:121 = 150:___ = 15:11 | **f** 147:___ = ___:63 = 7:9 |

## EXAMPLE 2H-2 Converting units to write comparisons as ratios

Write the comparison 6 cm to 42 mm as a ratio in simplest form.

| THINK | WRITE |
|---|---|
| **1** Write the quantities in the order given. | 6 cm to 42 mm |
| **2** Convert the quantities to the same unit. Write them in the smaller unit (mm). 1 cm = 10 mm. | = 60 mm to 42 mm |
| **3** Write the comparison as a ratio now that the quantities are in the same unit. | = 60:42 |
| **4** Divide each number in the ratio by the HCF. HCF = 6. (60 ÷ 6 = 10 and 42 ÷ 6 = 7.) | = 10:7 |

**3** Write each comparison as a ratio in simplest form.

**a** 42 mm to 7 cm
**b** 2 km to 400 m
**c** 50c to $3
**d** 5 weeks to 15 days
**e** 500 g to 3.6 kg
**f** 335 m to 0.6 km
**g** 55 seconds to $3\frac{1}{2}$ minutes
**h** 2 h 35 min to 45 min
**i** 4 weeks to 2 years
**j** 1.05 million to 950 000
**k** 72 mL to 2.5 L
**l** 65 c to $11.85

**4** A fruit bowl contains apples, oranges and bananas.

Write each comparison as a ratio in simplest form.

**a** the number of apples to the number of oranges
**b** the number of apples to the number of bananas
**c** the number of oranges to the number of bananas to the number of apples.



**5** In an upset win in an AFL round of football, Carlton beat Geelong with a score of 96 points to 84 points. Write the ratio of the scores in simplest form.

**6** In a class of 28 students, 16 are girls.

**a** How many students are boys?
**b** Write the ratio of girls to boys in simplest form.

**7** A breakfast cereal mix is made using 8 kg of oats, 4 kg of dried fruit and 12 kg of wheat flakes.

**a** Write the ratio of oats to dried fruit to wheat flakes in simplest form.
**b** Write each ingredient as a fraction of the whole mix.



**8** Last week, Shannen completed extra chores around home and earned $32. This week, she earned $14 less. Write as a ratio, in simplest form, the amount of last week's pocket money to this week's pocket money.

**9** The table shows the attendance at each performance of the local school production.

| Performance | Number of people attending |
|---|---|
| Thursday evening | 486 |
| Friday evening | 549 |
| Saturday matinee | 390 |
| Saturday evening | 520 |

**a** Find the total attendance to the school production over the three days.

**b** Write as a ratio in simplest form:

**i** the number attending Thursday's performance to the number attending Friday's performance

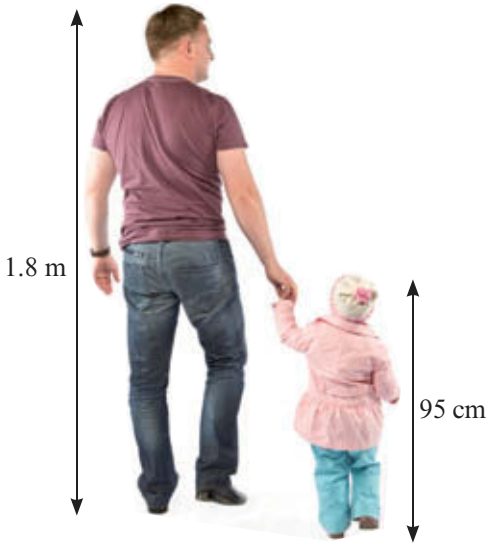
**ii** the number attending Saturday's matinee performance to the number attending Saturday's evening performance.

**c** If the venue can seat a maximum of 550 people, write as a ratio in simplest form the total attendance to the total possible attendance over the three days.

**10** Use the photos to write each ratio in simplest form.

**a** The ratio of the height of the taller person to the height of the shorter person.

**b** The ratio of the amount of liquid in the orange juice carton to the amount of liquid in the milk carton.







**11** Write three ratios that simplify to 6:7.

**12** Write three ratios that are equivalent to 48:60.

**13** Brett is unsure if the ratios 20:32 and 45:72 are equivalent. Explain how he can determine whether they are equivalent.

## EXAMPLE 2H-3 Identifying equivalent ratios

Is each pair of ratios equivalent?

**a** 3:16 and 9:48 **b** 18:12 and 36:28

### THINK

**a** **1** Write each ratio in simplest form. Divide the second ratio by 3.

**2** Since the simplest form of each ratio is the same, they are equivalent.

**b** **1** Write each ratio in simplest form. Divide the first ratio by 6 and the second by 4.

**2** Since the simplest form of each ratio is not the same, they are not equivalent.

### WRITE

**a** 3:16 is in simplest form
9:48 = 3:16

The ratios 3:16 and 9:48 are equivalent.

**b** 18:12 = 3:2
36:28 = 9:7

The ratios 18:12 and 36:28 are not equivalent.

**14** Is each pair of ratios equivalent? Give a reason for your answer.

**a** 7:8 and 21:24 **b** 20:32 and 5:8

**c** 63:18 and 9:2 **d** 30:45 and 90:60

**15** Create equivalent ratios by finding the missing numbers.

**a** 5:2 = 25:_____ **b** 280:350 = 4:_____

**c** 1800:2400 = _____:8 **d** _____:7 = 45 000:105 000

**e** 378:126 = 6:_____ **f** 44:_____ = 220:40

**16** The ratio of cats to dogs at the animal shelter is 7:4. If there are 21 cats, how many dogs are at the animal shelter?



PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING



**17** The ratio of boys to girls in a school orchestra is 2:3, and there are 15 girls in it.

**a** How many boys are in the orchestra?

**b** How many students are in the school orchestra?

**18** Ratios are useful for caterers because a recipe can be scaled up for a large group or scaled down for just a few people.



To make enough dough for two pizzas, a chef uses these ingredients.

1 cup of water
2 cups of plain flour
1 teaspoon of yeast
1 teaspoon of salt

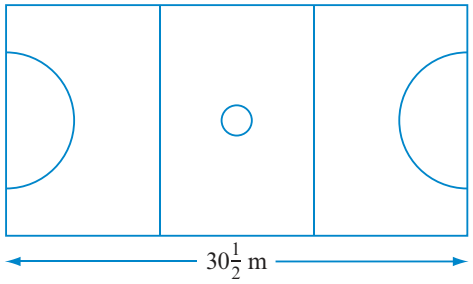
How much of each ingredient would the chef need to make:

**a** four pizzas? **b** twelve pizzas? **c** one pizza?

**19** The ratio of apple juice to soda water for a fruit punch recipe is 5:3. If the recipe requires 750 mL of apple juice, how much soda water is required?

CHALLENGE

**20** The ratio of the length of a netball court to its width is 2:1. What is the width of the court?




**21** The ratio of the number of supporters for the visiting team to the number of supporters for the home team at Saturday's game was 10:11. If there were 32 500 supporters for the visiting team, how many supporters were there for the home team?

**22** Provide three different real-world examples of ratios that are equivalent to 3:4. You may like to use diagrams to illustrate your ratios.

**Reflect**

How can equivalent ratios be used to solve problems?

# CHAPTER REVIEW

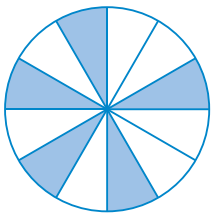
## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

fraction
numerator
denominator
vinculum
mixed number
improper fraction
proper fraction
equivalent fractions
simplifying
cancelling
highest common factor (HCF)
lowest common denominator (LCD)
reciprocal
power
square
square root
ratio
equivalent ratios

## MULTIPLE-CHOICE

2A **1** What fraction of the shape is coloured blue?



A 5 B $\frac{5}{10}$
C $\frac{7}{12}$ D $\frac{5}{12}$

2B **2** What is an equivalent fraction to $\frac{2}{5}$?

A $\frac{3}{5}$ B $\frac{6}{15}$
C $\frac{2}{10}$ D $\frac{1}{10}$

2C **3** What is $\frac{2}{5} + \frac{1}{3}$ equal to?

A $\frac{3}{8}$ B $\frac{2}{15}$
C $\frac{1}{2}$ D $\frac{11}{15}$

2C **4** What is $1\frac{1}{2} + 2\frac{5}{6} - 1\frac{2}{3}$?

A $2\frac{2}{3}$ B 4
C $2\frac{4}{5}$ D 6

2D **5** What is $\frac{2}{7}$ of $\frac{8}{9}$?

A $\frac{16}{63}$ B $\frac{10}{16}$
C $\frac{6}{2}$ D $\frac{18}{56}$

2D **6** 6 What is $\frac{14}{5} \times \frac{3}{21} \times \frac{25}{30}$ in simplest form?

A 3 B $\frac{5}{3}$
C $\frac{3}{5}$ D $\frac{1}{3}$

2E **7** What is $\frac{7}{20} \div \frac{14}{5}$ equal to?

A $\frac{1}{2}$ B $\frac{1}{8}$
C $\frac{7}{15}$ D $\frac{21}{25}$

2F **8** What is $\left(\frac{2}{3}\right)^3$?

A $\frac{6}{9}$ B $\frac{8}{27}$
C $\frac{5}{6}$ D $\frac{8}{9}$

2F **9** What is $\sqrt{1\frac{7}{9}}$?

A $\frac{16}{9}$ B $\frac{3}{4}$
C $\frac{4}{3}$ D $1\frac{2}{3}$

2G **10** Written as a ratio, what is 20 cm to 1 m?

A 20:1 B 20:10
C 20:100 D 20:1000

2H **11** What is a mixture of 100 mL cordial to 1.4 L of water written as a ratio in its simplest form?

A 100:1.4 B 100:1400
C 1:14 D 101.4

## SHORT ANSWER

2A **1** Copy and complete this table.

| | Mixed number | Improper fraction |
|---|---|---|
| a | $2\frac{3}{8}$ | |
| b | | $\frac{11}{2}$ |
| c | | $\frac{17}{9}$ |
| d | $5\frac{1}{3}$ | |

2B **2** Simplify each fraction.

a $\frac{16}{20}$ B $\frac{5}{45}$

C $\frac{50}{100}$ D $\frac{21}{49}$

2B **3** Place these fractions in ascending order.
$\frac{1}{4}, \frac{2}{7}, \frac{3}{5}, 1\frac{2}{3}, \frac{9}{2}, \frac{11}{20}$

2C **4** Calculate:

a $\frac{1}{9} + \frac{4}{9}$ b $\frac{7}{11} - \frac{5}{11}$

c $\frac{2}{3} - \frac{1}{2}$ d $\frac{3}{10} + \frac{2}{5}$

2C **5** Calculate:

a $2\frac{5}{7} + 3\frac{1}{7}$ b $2\frac{1}{5} - 1\frac{2}{5}$

c $1\frac{3}{8} - \frac{1}{4}$ d $1\frac{4}{5} + 2\frac{2}{3}$

2D **6** Calculate:

a $\frac{2}{5} \times \frac{4}{7}$ b $\frac{3}{10} \times \frac{5}{9}$

c $\frac{2}{3} \times \frac{10}{11}$ d $\frac{12}{5} \times \frac{4}{9}$

2D **7** Calculate:

a $\frac{2}{3} \times 1\frac{4}{5}$ b $2\frac{3}{7} \times \frac{1}{5}$

c $2\frac{1}{4} \times 3\frac{2}{3}$ d $1\frac{3}{8} \times 4\frac{1}{3}$

2E **8** Calculate:

a $\frac{3}{4} \div \frac{2}{3}$ b $\frac{25}{12} \div \frac{5}{3}$

c $\frac{49}{24} \div \frac{7}{16}$ d $\frac{1}{4} \div \frac{1}{3}$

2E **9** Calculate:

a $3\frac{4}{7} \div 2\frac{1}{7}$ b $\frac{8}{9} \div 1\frac{1}{3}$

c $3\frac{3}{5} \div \frac{6}{15}$ d $2\frac{3}{5} \div 1\frac{9}{15}$

2F **10** Calculate:

a $(\frac{3}{4})^2$ b $(\frac{5}{6})^2$

c $(\frac{3}{10})^3$ d $(\frac{1}{2})^4$

2F **11** Calculate:

a $\sqrt{\frac{25}{49}}$ b $\sqrt{\frac{144}{81}}$

c $\sqrt{\frac{1}{9}}$ d $\sqrt{1\frac{28}{36}}$

2G **12** Use the photo to write each comparison as a ratio.



a the number of pink Smarties to the number of yellow Smarties

b the number of yellow Smarties to the number of green Smarties

c the number of orange Smarties to the total number of Smarties

d the number of orange and pink Smarties to the number of green Smarties

2H **13** Write as ratios in their simplest form.

a 24:36

b 15:20

c 75c to $1.20

d 8 weeks to 4 days

2H **14** The ratio of snakes to lizards in a reptile zoo is 7:5. If there are 40 lizards, how many snakes are there in the zoo?

## NAPLAN-STYLE PRACTICE

**1** What is $3\frac{4}{7}$ as an improper fraction?

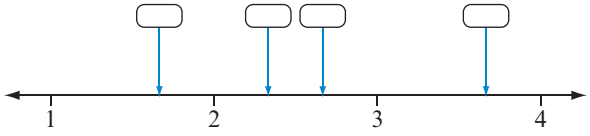
**2** Angela needs $\frac{33}{5}$ m of material. How many whole metres should she buy?

33 ○ 7 ○ 6 ○ 5 ○

**3** What is $\frac{24}{5}$ as a mixed number?

$2\frac{4}{5}$ ○ $4\frac{4}{5}$ ○ $\frac{5}{24}$ ○ $4\frac{2}{5}$ ○

**4** Which arrow is pointing closest to $2\frac{2}{3}$?




**5** Which fraction is equivalent to $\frac{5}{12}$?

$\frac{15}{25}$ ○ $\frac{7}{14}$ ○ $\frac{10}{24}$ ○ $\frac{10}{25}$ ○

**6** Which list is ordered smallest to largest?

○ $\frac{3}{4}, \frac{7}{2}, 2\frac{1}{4}, \frac{5}{8}, \frac{1}{2}$ ○ $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$

○ $1\frac{1}{3}, \frac{12}{5}, 2\frac{4}{5}, \frac{13}{3}, \frac{26}{15}$ ○ $\frac{2}{5}, 1\frac{3}{4}, \frac{9}{4}, 2\frac{7}{10}, \frac{16}{5}$

**7** Write $\frac{24}{16}$ in simplest form.

**8** Hayley has 18 red and 6 orange icy-poles. What fraction are orange?

$\frac{1}{2}$ ○ $\frac{1}{3}$ ○ $\frac{1}{4}$ ○ $\frac{1}{6}$ ○

**9** What is $3\frac{1}{5} - 2\frac{4}{5}$?

**10** What is $\frac{64}{5} - \frac{16}{3}$?

$7\frac{3}{5}$ ○ $7\frac{7}{15}$ ○ $3\frac{7}{15}$ ○ $7\frac{1}{3}$ ○

**11** What is $\frac{4}{5} \times \frac{2}{3}$?

**12** Angelina has a recipe that uses 3 cups of flour. She decides to make a $\frac{3}{4}$ batch. How much flour will Angelina need?

○ 3 cups ○ $\frac{3}{4}$ of a cup

○ 2 cups ○ $2\frac{1}{4}$ cups

**13** What is $\frac{8}{5} \div \frac{4}{9}$?

**14** What is $\frac{7}{8} \div 3\frac{1}{2}$?

$\frac{7}{48}$ ○ $\frac{49}{16}$ ○ $4\frac{3}{8}$ ○ $\frac{1}{4}$ ○

**15** If $4\frac{1}{2}$ L of juice is divided evenly between five friends, how much does each person get?

4 L ○ $\frac{9}{10}$ L ○ $1\frac{1}{8}$ L ○ $\frac{4}{5}$ L ○

**16** What fraction is halfway between $\frac{7}{11}$ and $\frac{8}{11}$?

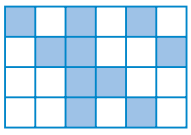
**17** What is $\left(\frac{3}{7}\right)^2$?

**18** What is $\left(2\frac{1}{4}\right)^2$?

$\frac{1}{4}$ ○ $\frac{81}{16}$ ○ $4\frac{1}{16}$ ○ $\frac{16}{81}$ ○

**19** What is $\sqrt{\frac{16}{25}}$?

Questions **20** and **21** refer to this diagram.



**20** In simplest form, what fraction is *not* shaded?

$\frac{14}{24}$ ○ $\frac{7}{12}$ ○ $\frac{10}{24}$ ○ $\frac{5}{12}$ ○

**21** What is the ratio of shaded segments to unshaded segments?

**22** Monique is 1.21 m tall. Lachlan is 98 cm tall. What is the ratio of Lachlan's height to Monique's?

- ◯ 98:1.21
- ◯ 98:121
- ◯ 1.21:98
- ◯ 121:98

**23** Alex has 16 jellybeans: 7 pink, 5 white and the rest blue. What is the ratio of blue to pink jellybeans?

**24** Which ratio is *not* equivalent to the ratio 3:7?

9:21 ◯ 12:28 ◯ 6:10 ◯ 30:70 ◯

**25** A netball game consists of four quarters and three rest breaks. The ratio of game time to rest time is 15:4. If total rest time is 16 minutes, what is the game's duration?

- ◯ 16 minutes
- ◯ 44 minutes
- ◯ 60 minutes
- ◯ 76 minutes

**26** Write the ratio 36:54 in its simplest form.

**27** The ratio of girls to boys at camp is 7:6. If there are 54 boys, how many girls are there?

## ANALYSIS

**1** A survey of 300 families found that 150 have one pet and 100 have two pets.

- **a** What fraction of families have one pet? Write this in simplest form.
- **b** What fraction of families have two pets? Write this in simplest form.
- **c** Write a ratio for the number of families with one pet compared to those with two pets.
- **d** If $\frac{1}{12}$ of families have three or more pets, how many families is this?
- **e** What fraction of families do *not* have pets? How many families is this?
- **f** A news reporter said that 5 in every 10 people have one pet, and 3 in every 10 people have two pets. Are they correct? Use your knowledge of equivalent fractions to support your answer.

**2** Brendan wants to buy a new mp3 player worth $50. He receives $10 pocket money every week.

- **a** If he saves $\frac{1}{5}$ of his pocket money each week, how much is this?
- **b** How many weeks would it take to save $50?
- **c** How much would he save in 12 weeks?
- **d** Write your answer to part **c** as a fraction of his weekly pocket money in the form of:
  - **i** an improper fraction
  - **ii** a mixed number.
- **e** If, after 12 weeks, Brendan spends $\frac{2}{3}$ of his savings, how much does he have left?
- **f** Brendan has saved $\frac{42}{5}$ of his weekly pocket money. How much is this and how long would it take at the rate mentioned in part **a**?
- **g** If he instead saved $\frac{3}{5}$ of his pocket money each week, how long would it take to achieve the amount of savings calculated in part **f**? (Hint: divide $\frac{42}{5}$ by $\frac{3}{5}$.)
- **h** Write the initial amount saved each week (part **a**) compared to the second amount saved each week (part **g**) as a ratio in simplest form.

# CONNECT

## Catering for a birthday party

Imagine you are planning a birthday party. You need to know how many people are attending and to estimate how much food and drink each person may want. How are fractions and ratios related to preparing for a party?

Assume that 24 people (including you) are expected. Of these 24 people, 14 are boys and 10 are girls. The menu will include: pizza, chicken, salad, fruit, drinks and cake. Ingredients required for some of the recipes are given.





**Smoky barbecue chicken kebabs**

*(serves 8)*

2 kg skinless and boneless chicken thighs
2 tablespoons olive oil
pita bread
mixed salad leaves

Sauce:

2 brown onions
$\frac{1}{2}$ cup golden syrup
1 cup cider vinegar
$\frac{2}{3}$ cup water
$1\frac{1}{4}$ cups tomato puree
$\frac{1}{3}$ cup Worcestershire sauce
2 tablespoons hot English mustard
$\frac{1}{4}$ cup olive oil
2 teaspoons smoked ground paprika
$\frac{1}{2}$ teaspoon hot chilli powder
$\frac{1}{3}$ cup brown sugar
sea salt and cracked black pepper

### Your task

Carry out these steps to prepare for the party.

- Calculate the number of pizzas required for the party.
- Work out the new quantities of ingredients needed to cater for 24 people, given that each recipe serves 8 people.
- Work out the new quantities of ingredients needed to cater for any given number of guests.
- Obtain a recipe for fruit punch and adjust it to cater for the number of guests.
- Calculate the ratio of boys to girls at the party.
- Work out which quantities can and cannot be compared as a ratio.





## Chocolate brownie and raspberry trifles

*(serves 8)*

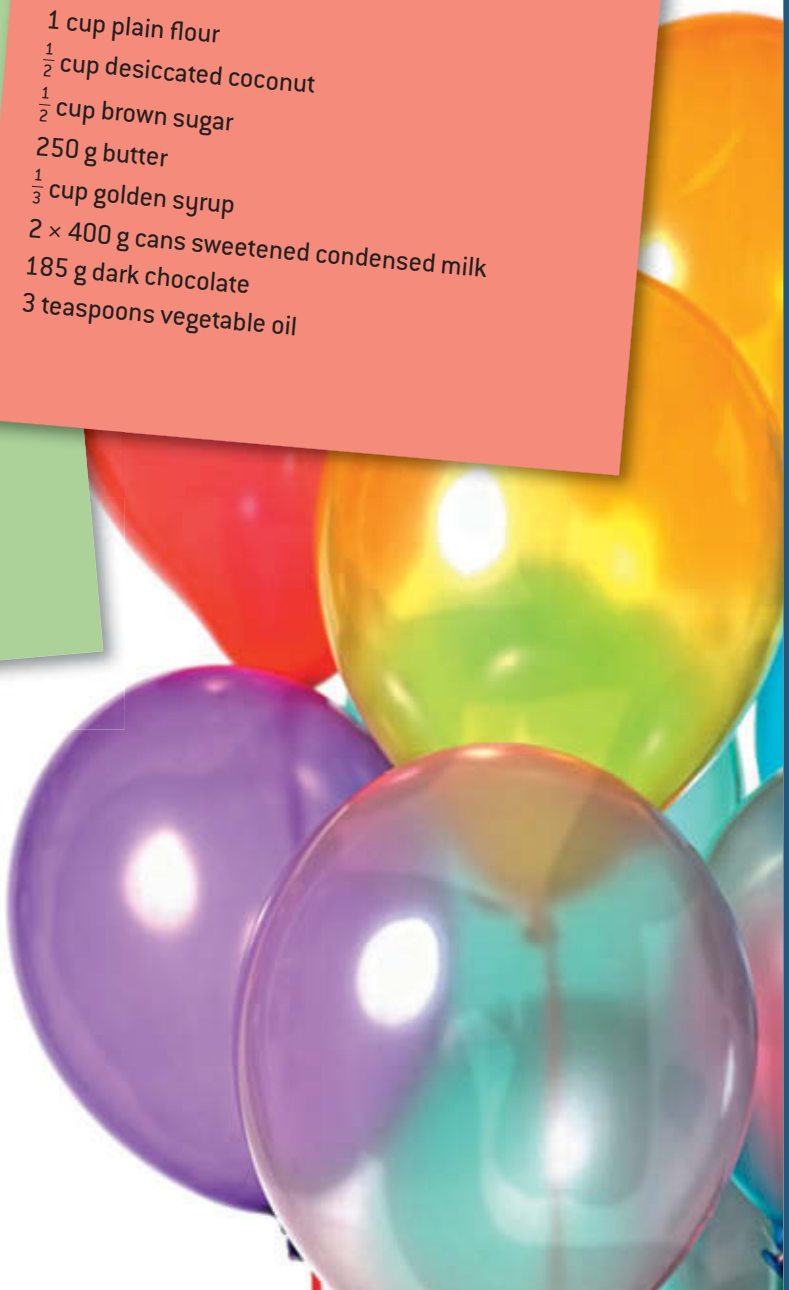
200 g dark chocolate
250 g unsalted butter
$1\frac{1}{3}$ cups brown sugar
4 eggs
$\frac{1}{3}$ cup cocoa powder
1 cup plain flour
$\frac{1}{4}$ teaspoon baking powder
4 × 120 g punnets raspberries
1 cup crème de cassis
2 cups cream
$\frac{1}{4}$ cup icing sugar



## Chocolate caramel slice

*(serves 8)*

1 cup plain flour
$\frac{1}{2}$ cup desiccated coconut
$\frac{1}{2}$ cup brown sugar
250 g butter
$\frac{1}{3}$ cup golden syrup
2 × 400 g cans sweetened condensed milk
185 g dark chocolate
3 teaspoons vegetable oil



Complete the 2 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a booklet
- a PowerPoint presentation
- a video
- other (check with your teacher).

# 3 DECIMALS AND PERCENTAGES


**3A** Understanding decimals

**3B** Adding and subtracting decimals

**3C** Multiplying decimals

**3D** Dividing a decimal by a whole number

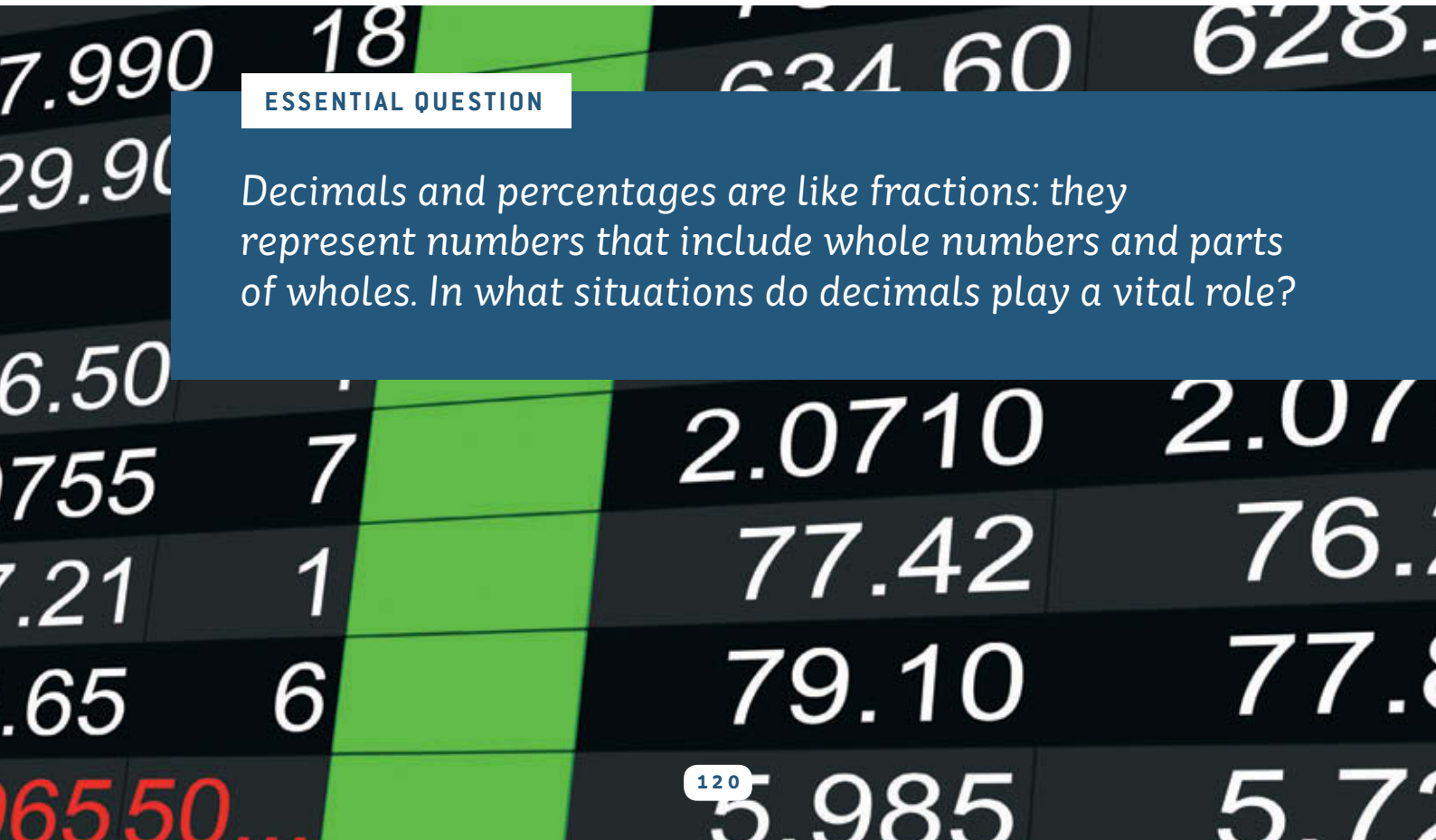
**3E** Dividing a decimal by a decimal

**3F** Converting between fractions and decimals

**3G** Understanding percentages

**3H** Converting between fractions, decimals and percentages

**3I** Calculating percentages

**ESSENTIAL QUESTION**

*Decimals and percentages are like fractions: they represent numbers that include whole numbers and parts of wholes. In what situations do decimals play a vital role?*

# Are you ready?

3A **1** **a** What is the place value of the digit 8 in 12 865?

**A** 8 **B** hundreds
**C** thousands **D** ten thousands

**b** What is the place value of the digit 5 in 12.053?

**A** hundreds **B** tenths
**C** hundredths **D** thousandths

**c** Which digit indicates the number of tenths in 234.567?

**A** 3 **B** 4 **C** 5 **D** 6

3A **2** What is 34.702 written in place-value notation?

**A** $3 \times 10 + 4 \times 1 + 7 \times \frac{1}{10} + 2 \times \frac{1}{100}$

**B** $3 \times 100 + 4 \times 10 + 7 \times \frac{1}{10} + 2 \times \frac{1}{100}$

**C** $3 \times 10 + 4 \times 1 + 7 \times \frac{1}{100} + 2 \times \frac{1}{1000}$

**D** $3 \times 10 + 4 \times 1 + 7 \times \frac{1}{10} + 2 \times \frac{1}{1000}$

3A **3** **a** Write seven tenths as a decimal.

**b** Write one hundredth as a decimal.

3B **4** **a** Round 562 to its leading digit.

**b** Round 24.75 to its leading digit.

3B **5** **a** Alex has used vertical addition to calculate 54 327 + 6849 + 37 014. Which answer is correct?

**A** 9819 **B** 98 190
**C** 64 890 **D** 159 831

**b** What is 75 341 – 8205?

3C **6** **a** What is 95 × 37?

**b** What is 124 × 50?

**A** 1245 **B** 6200
**C** 7200 **D** 12 450

3D **7** **a** When 32 is divided by 5, what is the remainder?

**A** 1 **B** 2 **C** 3 **D** 4

**b** What is 2512 ÷ 8?

3F **8** **a** What is 0.01 as a fraction?

**A** $\frac{1}{1}$ **B** $\frac{1}{10}$ **C** $\frac{1}{100}$ **D** $\frac{1}{1000}$

**b** What is 0.9 as a fraction?

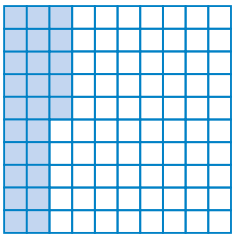
**A** $\frac{9}{10}$ **B** $\frac{1}{9}$ **C** $\frac{0}{9}$ **D** $\frac{9}{100}$

**c** What is $\frac{3}{10}$ as a decimal?

**d** What is $\frac{1}{2}$ as a decimal?

**A** 0.12 **B** 0.2 **C** 0.5 **D** 1.2

3G **9** Look at this grid.



**a** What fraction of the grid is shaded?

**A** $\frac{1}{25}$ **B** $\frac{3}{10}$
**C** $\frac{1}{5}$ **D** $\frac{25}{100}$

**b** What fraction of the grid is unshaded?

**c** If six more small squares are shaded, what fraction of the grid is now shaded?

3G **10** **a** What equivalent fraction of $\frac{7}{25}$ has a denominator of 100?

**A** $\frac{35}{100}$ **B** $\frac{82}{100}$ **C** $\frac{11}{100}$ **D** $\frac{28}{100}$

**b** What equivalent fraction of $\frac{7}{10}$ has a denominator of 100?

3I **11** **a** Which of these is the same as $\frac{4}{5}$ of 20?

**A** $\frac{4}{5} \times \frac{20}{1}$ **B** $\frac{4}{5} \div \frac{20}{1}$
**C** $\frac{4}{5} \times \frac{1}{20}$ **D** $\frac{4}{5} \div \frac{1}{20}$

**b** What is $\frac{23}{100} \times 600$?

# 3A Understanding decimals

## Start thinking!

| Thousands 1000 | Hundreds 100 | Tens 10 | Ones 1 | Decimal point | Tenths $\frac{1}{10}$ 0.1 | Hundredths $\frac{1}{100}$ 0.01 | Thousandths $\frac{1}{1000}$ 0.001 |
|---|---|---|---|---|---|---|---|
| | | | | . | | | |
| | | | | . | | | |

**Decimal numbers** (or **decimals** for short) have digits after the **decimal point**. You can look at the **place value** of each digit in a number.

**1** Copy the place-value chart and write the digits of 5218.347 in the correct positions.

**2** Explain what happens to place value as you move to the right across the chart.

**3** Decimal numbers are also called **decimal fractions**. They can be written as a fraction with a denominator of 10, 100, 1000 or other powers of 10, as shown in the chart.

**a** To write 5218.347 in place-value notation, copy and complete

$5218.347 = 5 \times 1000 + 2 \times ____ + 1 \times ____ + 8 \times ____ + 3 \times ____ + 4 \times ____ + 7 \times \frac{1}{1000}$

$= 5000 + ____ + ____ + ____ + ____ + ____ + \frac{7}{1000}$

$= 5000 + 200 + 10 + 8 + 0.3 + 0.04 + 0.007$

**b** What pattern is there between the decimal place and the number of zeros in the denominator of each fraction?

**4** The digit 3 is the first digit after the decimal point. This position is called the first **decimal place**.

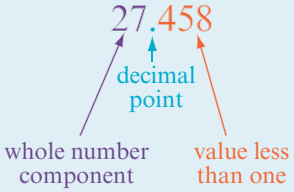
**a** What digit is in the:

**i** second decimal place? **ii** third decimal place?

**b** How many decimal places does 5218.347 have?

## KEY IDEAS

- Decimal numbers are also called decimal fractions or decimals.
- The position of a digit in a number determines its value and is called the place value.
- A decimal number can be written in place-value notation; for example, $27.458 = 20 + 7 + \frac{4}{10} + \frac{5}{100} + \frac{8}{1000}$.
- It is important that zeros between non-zero digits in a decimal (called **placeholder zeros**) are never left out; for example, the zeros in 103.120 56.
- Zeros at the end of a decimal (called **trailing zeros**) do not change the value of the decimal; for example, the zeros in 345.160 00.
- The number of decimal places in a number is the number of digits after the decimal point; for example, 27.458 has three decimal places.



# EXERCISE 3A Understanding decimals

## EXAMPLE 3A-1 Finding the place value of digits

What is the place value of the digit 9 in each number?

**a** 5.7839 **b** 0.981

**THINK**

**a** The digit 9 is four places to the right of the decimal point so it is in the ten-thousandths column.

**b** The digit 9 is one place to the right of the decimal point so it is in the tenths column.

**WRITE**

**a** nine ten-thousandths or $\frac{9}{10\,000}$

**b** nine tenths or $\frac{9}{10}$

**1** What is the place value of the digit 8 in each number?

**a** 28.61 **b** 304.128 **c** 927.835 **d** 16.0058 **e** 84.761 **f** 62.1852

**g** 821.793 **h** 71.085 **i** 0.0018 **j** 3.8024 **k** 8236.12 **l** 183.049

## EXAMPLE 3A-2 Writing a number in place-value notation

Write 23.814 in place-value notation.

**THINK**

**1** Draw a place-value chart and place the digits into the correct place.

**2** Use the place-value chart to write the number in place-value notation.

**WRITE**

| Tens 10 | Ones 1 | Decimal point . | Tenths $\frac{1}{10}$ 0.1 | Hundredths $\frac{1}{100}$ 0.01 | Thousandths $\frac{1}{1000}$ 0.001 |
|---|---|---|---|---|---|
| 2 | 3 | . | 8 | 1 | 4 |

$$23.814 = 2 \times 10 + 3 \times 1 + 8 \times \frac{1}{10} + 1 \times \frac{1}{100} + 4 \times \frac{1}{1000}$$
$$= 20 + 3 + \frac{8}{10} + \frac{1}{100} + \frac{4}{1000}$$

**2** Write each number in question **1** in place-value notation.

**3** Write each of these as a decimal number. Refer to a place-value chart to help you.

**a** $5000 + 700 + 80 + 5 + \frac{1}{10} + \frac{9}{100} + \frac{5}{1000}$ **b** $400 + 90 + 6 + 0.2 + 0.08 + 0.003$

**c** 6 thousandths **d** 5 ones, 7 tenths and 3 thousandths

**e** 3 tens, 6 ones and 7 hundredths **f** 4 tens, 1 tenth and 7 ten-thousandths

## EXAMPLE 3A-3 Identifying the number of decimal places

How many decimal places does 13.5023 have?

**THINK**

Count the number of digits (including any zeros) to the right of the decimal point.

**WRITE**

13.5023 has four decimal places.

UNDERSTANDING AND FLUENCY

**4** How many decimal places does each decimal have?

| | | | |
|---|---|---|---|
| **a** 6.214 | **b** 0.388 82 | **c** 16.03 | **d** 198.413 020 |
| **e** 28 | **f** 11.2 | **g** 2.000 102 84 | **h** 53.0 |
| **i** 70.5236 | **j** 108.4503 | **k** 22.250 620 | **l** 1.05 301 |

**5** Insert < or > between each pair of decimals to make the statement true.

| | | |
|---|---|---|
| **a** 4.73 ___ 3.473 | **b** 3.21 ___ 3.24 | **c** 9.6841 ___ 9.684 09 |
| **d** 0.017 ___ 0.107 | **e** 55.03 ___ 55.003 | **f** 12.218 ___ 12.2108 |
| **g** 6.9891 ___ 6.8999 | **h** 1378.5 ___ 1378.499 | **i** 800.78 ___ 800.778 |

**6** Write each list of decimals in ascending order.

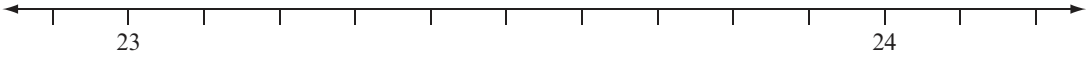
**a** 0.641, 0.461, 0.146, 0.328, 0.823, 0.238

**b** 2.341, 2.143, 2.431, 4.132, 1.234, 3.412

**c** 38.162, 36.218, 38.261, 36.812, 36.182, 32.861

**d** 18.4251, 18.2145, 18.4125, 18.1524, 18.4152,18.2154

**7** Copy this number line. Each division on the number line should be 1 cm apart.




**a** Mark the decimal numbers 23.15, 23.3, 23.45, 23.55, 23.7 and 23.95 on the number line.

**b** Which whole number is closest to each of the decimals less than 23.5?

**c** Which whole number is closest to each of the decimals greater than 23.5?

**d** Explain to a friend how you would round decimals to the nearest whole number.

**8** Rounding a number provides an approximation to that number. For example, 4.5382 rounded to two decimal places means that 4.5382 has an approximate value of 4.54. It is said to be accurate to the nearest one hundredth.

**a** Explain why 4.5382 is closer to 4.54 than to 4.53 when it is rounded to two decimal places.

**b** Copy and complete these instructions for rounding a decimal number to two decimal places using words from this list: *does*, *doesn't*, *second*, *third*, *one*.

**1** Write the number and draw or imagine a vertical line after the ___ decimal place.

UNDERSTANDING AND FLUENCY

2 Look at the digit to the right of the line; that is, the digit in the ___ decimal place.
If it is 0, 1, 2, 3 or 4 then the digit in the second decimal place ___ change.
If it is 5, 6, 7, 8 or 9 then add ___ to the digit in the second decimal place.
3 Discard any digits after the line and write your answer.

4.53|42 ≈ 4.53
4.53|82 ≈ 4.54

**c** Write 4.5382 rounded to three decimal places.
**d** Explain the strategy you used to obtain your answer to part **c**.
**e** Write a set of instructions for a classmate to explain how to round a decimal number to any given number of decimal places. Compare your instructions with a classmate's instructions.

## EXAMPLE 3A-4 Rounding numbers

Round 0.426 53 to three decimal places.

| THINK | WRITE |
|---|---|
| **1** Write the number and draw a vertical line after the third decimal place. | 0.426\|53 |
| **2** Look at the digit to the right of that line (5 – shown in green). | = 0.426\|53 |
| **3** The digit in the fourth decimal place is 5, so add 1 to the digit in the third decimal place. | |
| **4** Discard the digits after the line and write your answer. | ≈ 0.427 |

**9** Round 3.546 52 to:
**a** three decimal places **b** four decimal places **c** two decimal places.

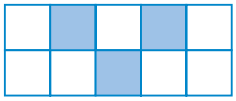
**10** Round each of these to the number of decimal places indicated in the brackets.
**a** 82.631 (2) **b** 59.111 78 (3) **c** 0.263 418 (4)
**d** 0.266 66 (2) **e** 15.0003 (3) **f** 2.175 498 7 (6)
**g** 0.218 638 2 (3) **h** 165.545 47 (4) **i** 43.806 251 (5)

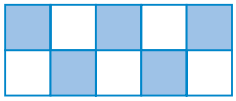
**11** During an athletics carnival, Mark threw a javelin the following distances: 22.81 m, 21.98 m, 21.89 m and 22.76 m.
**a** What was Mark's shortest throw?
**b** What was Mark's longest throw?
**c** Explain how you compared the distances to decide which is largest and which is smallest.



**12** For each rectangle, write the proportion that is shaded as a:

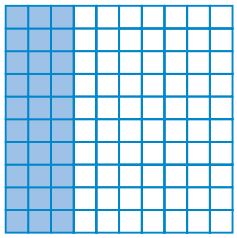
**i** fraction **ii** decimal fraction.

**a** 

**b** 

**c** 

**13** Look at the grid shown. Three of the ten columns are shaded. The fraction that has been shaded is $\frac{3}{10}$.



**a** Write this as a decimal number.

**b** How many small squares are there in total? How many of these are shaded?

**c** Write the number of squares that are shaded as a fraction.

**d** As a decimal number, this is 0.30. Explain why this is the same answer as your answer to part **a**.

**14** There are a number of ways of writing a decimal using place-value notation. For example, 0.30 = 3 tenths + 0 hundredths (or $\frac{3}{10} + \frac{0}{100}$) or 0.30 = 30 hundredths (or $\frac{30}{100}$).

**a** Write 0.30 in a place-value chart.

**b** Write 0.47 in a place-value chart and then copy and complete this statement.
0.47 = 4 tenths + _____ hundredths (or $\frac{4}{10} + \frac{7}{100}$) or 0.47 = 47 _____ (or $\frac{47}{100}$)

**c** Write 0.163 in a place-value chart and then copy and complete this statement.
0.163 = 1 tenth + _____ hundredths + _____ thousandths (or $\frac{1}{10} + \frac{6}{100} + \frac{3}{1000}$)o r
0.163 = _____ thousandths (or $\frac{163}{1000}$)

**d** Explain how you might use place value to write $\frac{681}{1000}$ as a decimal.

**15** Write each fraction as a decimal. You may like to refer to a place-value chart to help you.

**a** $\frac{7}{10}$ **b** $\frac{9}{10}$ **c** $\frac{33}{100}$ **d** $\frac{531}{1000}$ **e** $\frac{27}{100}$

**f** $\frac{11}{100}$ **g** $\frac{2978}{10\,000}$ **h** $\frac{804}{1000}$ **i** $\frac{80}{100}$ **j** $\frac{63}{1000}$

**16** Copy this table and allow enough rows for seven decimal numbers.

| Decimal number | Number of decimal places | Value of digit in first decimal place | Value of digit in second decimal place | Value of digit in third decimal place | Fractional form |
|---|---|---|---|---|---|
| 0.275 | 3 | $\frac{2}{10}$ | $\frac{7}{100}$ | $\frac{5}{1000}$ | $\frac{275}{1000}$ |
| 0.58 | 2 | $\frac{5}{10}$ | $\frac{8}{100}$ | | $\frac{58}{100}$ |

**a** Complete a row in the table for 0.634.

**b** How many decimal places does 0.634 have?

**c** How many zeros are in the denominator of 0.634 when it is in fractional form?

**d** Repeat parts **a**–**c** for the following.

**i** 0.983 **ii** 0.71 **iii** 0.407 **iv** 0.026

**e** Write a statement that describes the relationship between the number of decimal places in a number and the number of zeros in the denominator of the corresponding fraction.

PROBLEM SOLVING AND REASONING

**17** Use your results from question **16** to write these numbers as fractions.

**a** 0.127 **b** 0.401 **c** 0.03 **d** 0.0289 **e** 0.2001 **f** 0.0107

**18** Consider the decimal numbers 12.021, 12.201 and 12.210.

**a** Write these numbers in place-value notation. You may like to write them in a place-value chart first to help you.

**b** What does the zero represent in each number?

**c** What would happen if you wrote each number without its zero? For example, if you wrote 12.021 as 12.21, is the value the same?

**d** Zeros between non-zero digits, such as the zero in 12.021, are placeholder zeros. Why is it important that these zeros are never left out?

**e** Zeros at the end of a decimal (such as the zero in 12.210) are trailing zeros. Does it matter if these are left out? Explain your answer.

**f** Write the three numbers in ascending order (from smallest to largest).

**19** A friend wrote that $\frac{42}{1000} = 0.42$. Can you help explain where he might have gone wrong?

**20** Mia had to round 0.726 53 to three decimal places. Explain why her answer of 0.726 is incorrect.

**21** Round 168.246 588 to the following.

**a** the nearest hundred **b** the nearest ten **c** the nearest unit (one)

**d** the nearest tenth **e** the nearest hundredth **f** the nearest thousandth

**22** Write four examples of a decimal number with three decimal places that, when rounded to two decimal places, equals 5.82.

CHALLENGE

**23** Look at the section between 7 cm and 8 cm on your ruler. It is divided into tenths. So each small mark represents $\frac{1}{10}$ of 1 cm or 0.1 cm.

**a** If you magnified the section between 7.3 cm and 7.4 cm to see it divided into tenths, each small mark would represent $\frac{1}{10}$ of $\frac{1}{10}$ cm. What is this as a decimal number?

**b** If you magnified the section between 7.30 cm and 7.31 cm to see it divided into tenths, each small mark would represent $\frac{1}{10}$ of $\frac{1}{100}$ cm. What is this as a decimal number?

**c** If you repeated this process another two times, what would each of the small marks represent?

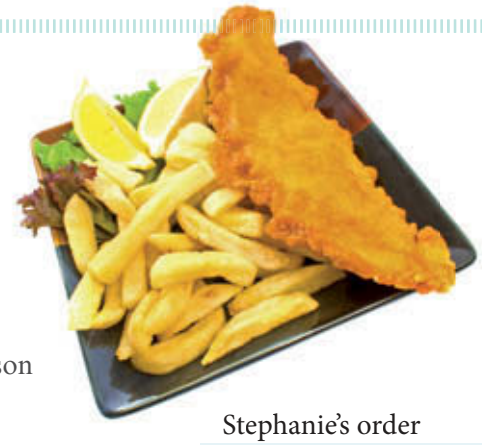


**Reflect**

Why is it important that a place value between the first and last digit of a number is never left 'empty'?

# 3B Adding and subtracting decimals

## Start thinking!



Friday night is fish and chips night at the Vlahos household.

**1** Round each value to the nearest dollar and get an estimate of the total cost of:
- **a** Stephanie's order
- **b** Phil's order.

**2** Use your estimates from question **1** to decide whether each person will receive any change from $10.

**3** Stephanie calculates the total cost of her order as $11.65.
- **a** Do you agree with Stephanie's answer?
- **b** Which item could Stephanie remove from her order so she doesn't spend more than $10? Explain.

**4** Phil calculates the total cost of his order as $1230.
- **a** Do you agree with Phil's answer?
- **b** Explain how he reached his answer.
- **c** Phil recalculated the cost of his order and this time, got $170.70. Explain how he may have got the second answer.
- **d** Show Phil how to perform the calculation so that he gets the correct answer.
- **e** What is the actual cost of Phil's order?

Stephanie's order

| |
|---|
| *1 King George whiting $4.80* |
| *1 dim sim $0.80* |
| *1 potato cake $0.75* |
| *minimum chips $2.50* |
| *small garden salad $2.80* |

Phil's order

| |
|---|
| *1 king burger $6.50* |
| *1 calamari ring $0.80* |
| *1 onion ring $0.80* |
| *minimum chips $2.50* |
| *1 pineapple fritter $1.70* |

## KEY IDEAS

- Decimals can be added or subtracted in the same way as whole numbers.
- When adding or subtracting decimals, the digits must be positioned in their correct place-value columns and the decimal points must be lined up. Blank spaces in the columns may be replaced with trailing zeros to minimise mistakes. The digits in each column working from right to left may then be added or subtracted.
- Estimation can be used to check decimal calculations. This is done by first rounding each decimal to a whole number and then adding or subtracting.

# EXERCISE 3B Adding and subtracting decimals

## EXAMPLE 3B-1 Using vertical addition to add decimals

Use vertical addition to calculate 8 + 13.21 + 140.6 + 3.089.

**THINK**

**1** Write the numbers underneath each other with the digits lined up in columns according to their place value. Ensure each decimal point is lined up.

**2** Replace blank spaces with trailing zeros so that each decimal contains the same number of decimal places. This will minimise mistakes.

**3** Add the digits in each column as you would for whole numbers. Ensure the decimal point in the answer is lined up with the decimal points in the calculation.

**WRITE**

```
   8
  13.21
 140.6
+  3.089
-------
```

```
   8.000
  13.210
 140.600
+  3.089
-------
 164.899
```

UNDERSTANDING AND FLUENCY

**1** Estimate the result of each calculation by rounding each number to the nearest 10.

**a**
```
  38.62
+ 49.59
-------
```

**b**
```
  132.61
+ 183.86
--------
```

**c**
```
   16.345
+ 175.928
---------
```

**d**
```
   54.756
+ 237.309
---------
```

**e**
```
    9.58
  117.371
+  22.65
-------
```

**f**
```
   46.2863
 1152.93
+    9
---------
```

**g**
```
  120.146
   11.06
+ 813.705
---------
```

**h**
```
    8.05
  261.841
+ 7832.5
--------
```

**2** Work out each calculation in question **1** and check how the answers compare with your estimates.

**3** Use vertical addition to calculate each of these.

**a** 12.582 + 9.81 + 13.62

**b** 1482 + 37.31 + 0.487

**c** 48.799 + 3.218 + 14

**d** 3826.1 + 0.4326 + 28.571

**e** 0.9453 + 128.62 + 40.1 + 8.3279

**f** 12.063 + 0.004 + 3.904

**NOTE** Remember to line up the digits according to place value.

## EXAMPLE 3B-2 Using vertical subtraction to subtract two decimals

Use vertical subtraction to calculate 29.43 − 16.785.

**THINK**

1 Write the two numbers underneath each other with the decimal points lined up and the digits lined up according to their place value.

2 Replace any blank spaces with trailing zeros so that each decimal contains the same number of decimal places.

3 Subtract the digits in each column as you would for whole numbers. Ensure the decimal point in the answer is lined up with the decimal points in the calculation.

**WRITE**

$$\begin{array}{r} 29.43 \\ -16.785 \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle 13\,12\quad\quad} \\ {\scriptstyle 8\ \ 3\ \ 2\,10} \\ 2\not{9}.\not{4}\not{3}\not{0} \\ -16.785 \\ \hline 12.645 \end{array}$$

UNDERSTANDING AND FLUENCY

**4** Estimate the result of each calculation by rounding each number to the nearest 10.

**a** $\begin{array}{r} 79.84 \\ -\ 23.61 \\ \hline \end{array}$ **b** $\begin{array}{r} 163.88 \\ -\ 129.16 \\ \hline \end{array}$ **c** $\begin{array}{r} 487.6 \\ -\ 92.7 \\ \hline \end{array}$ **d** $\begin{array}{r} 33.7 \\ -\ 10.48 \\ \hline \end{array}$

**e** $\begin{array}{r} 1492.6 \\ -\ 15.92 \\ \hline \end{array}$ **f** $\begin{array}{r} 172 \\ -\ 58.73 \\ \hline \end{array}$ **g** $\begin{array}{r} 23.078 \\ -\ 7.49 \\ \hline \end{array}$ **h** $\begin{array}{r} 2031.703 \\ -\ 798.0912 \\ \hline \end{array}$

**5** Work out each calculation in question **4** and check how the answers compare with your estimates.

**6** Use vertical subtraction to calculate each of these.

**a** 172.91 − 48.984 **b** 16.872 − 9.385
**c** 2014.23 − 1998.76 **d** 414.9 − 389.76
**e** 17 − 10.325 **f** 14.008 − 3.67

**7** Check your answers to questions **3** and **6** with a calculator.

**8** Shanti bought five items, priced at $3.69, $14.99, $7.25, $10.59 and $5.99.

**a** Estimate the total cost of the five items.
**b** Use your estimate from part **a** to work out how much change she should receive from $50.
**c** Find the sum of the five items and hence the exact amount of change she should receive.
**d** How accurate was your estimate?

UNDERSTANDING AND FLUENCY



**9** Natasha scored 8.3, 9.1, 8.5, 7.9 and 8.8 in her gymnastics competition.

- **a** Calculate Natasha's total score.
- **b** Monique competed in the same gymnastics competition and received scores of 8.6, 7.5, 9.2, 8.3 and 8.5. Calculate Monique's total score.
- **c** Who received the highest score and what was the difference in their totals?

**10** The minimum temperature on a particular day was recorded as 10.7°C while the maximum was 28.3°C. What is the difference between the minimum and maximum temperatures?



**11** Oliver, Sam and Than competed in the high jump championships, clearing heights of 1.43 m, 1.39 m and 1.38 m, respectively. If the school record for the high jump was 1.52 m, calculate the difference between each competitor's jump and the record.

**12** Daphne was asked to do this sum: 8.02 + 5.90 + 102.620 + 8.703. Her setting out is shown at right. Rewrite the sum to show Daphne how to set it out correctly.

```
 8.2
 5.9
12.62
+8.73
------
```

**13** Angelo needs these lengths of fencing for repairs on his farm: 15.36 m, 28.42 m, 5.8 m and 17.1 m. How much fencing is required?

PROBLEM SOLVING AND REASONING



**14** Matthew is planning to paint four bedrooms over a long weekend. He needs these amounts of paint: 4.2 L, 3.89 L, 2.5 L and 1.9 L.

- **a** How many litres of paint will Matthew need?
- **b** If the paint comes in 5-L tins, how many tins will he need to buy?

**15** Melbourne's average total monthly rainfall for the month of September is 65.7 mm. In the past year, it rained on five days during the month of September with daily totals of 28.7 mm, 12.8 mm, 1.3 mm, 0.9 mm and 20.98 mm. Comment on how this past year's monthly total compares with the average monthly total for September.

**16** The combined weight of Anna and her cat Corey is 69.8 kg. If Anna weighs 62.3 kg, calculate Corey's weight.

**17** Adrian is a sales rep and spends much of his time on the road travelling from one country town to the next. He is required to record the car's odometer readings at the end of each day, as shown in this log sheet.

| Day | Odometer reading | Distance travelled (km) |
|---|---|---|
| Start | 28 071.5 | 0 |
| Day 1 | 28 258.7 | |
| Day 2 | 28 407.3 | |
| Day 3 | 28 657.1 | |
| Day 4 | 28 801.9 | |
| Day 5 | 29 053.4 | |

**a** Copy and complete the log sheet.

**b** What was the odometer reading at the beginning of Adrian's travels?

**c** Which day did Adrian travel the least distance? How far did he travel on this day?

**d** Which day did Adrian travel the greatest distance? How far did he travel on this day?

**e** Calculate the total distance Adrian travelled in the five days.

**18** When travelling internationally you usually need to make sure that your checked luggage is less than 20 kg. Elio is checking in three bags, weighing 4.5 kg, 13.79 kg and 3.8 kg.



**a** What is the total weight of his checked luggage?

**b** How much weight will he need to remove to stay under 20 kg?

**19** A lift has a maximum capacity of 265 kg. A group of people with masses of 75.3 kg, 28.7 kg, 54.6 kg and 80.92 kg enter the lift.

**a** Has the maximum capacity of the elevator been exceeded? Explain.

**b** Can another person join the four people in the lift? If so, what could their maximum mass be?

**20** Sarah hopes to compete in her first 10 km fun run and plans to train every second day.

**a** In the first week, she runs 900 m, 1.3 km, 1.46 km and 1.81 km. How far did she run in total?

**b** In the second week Sarah plans to increase each distance covered in the first week by 500 m. What individual distances does Sarah plan to run in the second week?

**c** How far does Sarah plan to run in total over the second week?

PROBLEM SOLVING AND REASONING

**21** Elisa's monthly bank statement is shown below. An opening balance of $328.56 states how much money is in Elisa's account at the beginning of the month.

| Date | Transaction details | Debit (−) | Credit (+) | Balance |
|---|---|---|---|---|
| 28 Feb | Opening balance | | | 328.56 CR |
| 05 Mar | Personal cheque 531 287 | | 125.42 | |
| 08 Mar | FJ Jewellers | 58.99 | | |
| 09 Mar | Athletique Gym Membership | 65.00 | | |
| 16 Mar | ABWDL Smith St | 250.00 | | |
| 23 Mar | Officeworks | 28.53 | | |
| 30 Mar | Interest | | 1.78 | |
| 30 Mar | Monthly fee | 5.00 | | |
| 31 Mar | Closing balance | | | |

**a** Calculate the following.
- **i** the balance after each transaction
- **ii** the total debits
- **iii** the total credits
- **iv** thefi nal balance at the end of the month



**b** Elisa wishes to purchase an iPod nano, retailing for $199. Comment on whether she will have enough money in her account.

CHALLENGE

**22** Calculate each of these.
- **a** 12 + 629.1 + 34.75 − 392.7
- **b** 68.419 − 23.8 + 17.75
- **c** 37.941 − 37.899 + 15.371
- **d** 648 − (58.63 − 28.21)
- **e** 192.62 − (154 + 38.39)
- **f** (84.7 + 93.2) − (128.8 − 109.6)

**23** Design a problem for your classmates to calculate that meets these descriptions.
- **a** three decimal numbers, each containing four decimal places, that add up to 9.857
- **b** four decimal fractions, each containing three decimal places, with two addition signs and one subtraction sign, that give a total of 15.83

**Reflect**

Why is it important that decimal points line up when adding and subtracting decimals?

# 3C Multiplying decimals



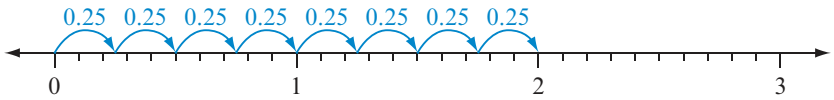
## Start thinking!

Albina has to make 14 costumes, each requiring 0.6 m of fabric. How can you work out how much fabric Albina needs?

**1** This number line shows three lots of 0.3. What do three lots of 0.3 equal? Explain how you obtained your answer.




**2** This number line shows eight lots of 0.25. What do eight lots of 0.25 equal?




**3** The words 'lots of' can be replaced by the multiplication sign (×). So 'three lots of 0.3 equals' can be written as '3 × 0.3 ='. Rewrite the statements 'four lots of 0.6 equals' and 'eight lots of 0.25 equals' using the multiplication and equals signs.

**4** Use the number lines to calculate:

**a** 3 × 0.3 = _____ **b** 8 × 0.25 = _____ **c** 6 × 0.3 = _____ **d** 11 × 0.25 = _____

**5** Work out each product.

**a** 3 × 3 = _____ **b** 8 × 25 = _____ **c** 6 × 3 = _____ **d** 11 × 25 = _____

**6** What do you notice about your answers to questions **4** and **5**?

**7** Copy and complete: When multiplying decimals by a whole number, the number of decimal places in the calculation is the _____ as the number of decimal places in the answer.

**8** How much fabric does Albina need?

## KEY IDEAS

- Decimals can be multiplied in the same way as whole numbers.
  1. Set out the problem as you would for whole numbers. Ignore the decimal points at this stage.
  2. Work out the multiplication.
  3. Count the total number of decimal places in the original calculation.
  4. Write the answer with this same number of decimal places.
- Estimating an answer before performing the calculation helps you to check your result. First round each number to its leading digit (the first non-zero digit) and then multiply.

# EXERCISE 3C Multiplying decimals

## EXAMPLE 3C-1 Finding the number of decimal places in an answer

How many decimal places will there be in the answer to 34.506 × 17.26?

| THINK | WRITE |
|---|---|
| **1** Count the number of decimal places in the first number. | 34.506 has three decimal places. |
| **2** Count the number of decimal places in the second number. | 17.26 has two decimal places. |
| **3** Add the two counts together. | 3 + 2 = 5<br>There will be five decimal places in the answer. |

**1** How many decimal places will there be in the answer to each calculation?

| | | |
|---|---|---|
| **a** 1.56 × 2.489 | **b** 0.4896 × 1.356 | **c** 12.364 29 × 3.8 |
| **d** 15.6308 × 46.75 | **e** 0.0056 × 46.21 | **f** 0.167 × 0.007 |
| **g** 3.2006 × 0.0801 | **h** 23.510 × 2.9084 | **i** 0.0301 × 126.5030 |
| **j** 9.008 023 × 1.5806 | **k** 13.206 408 06 × 1.2 | **l** 38.0530 × 1.3080 |

## EXAMPLE 3C-2 Multiplying decimals

Calculate 0.7 × 0.03.

| THINK | WRITE |
|---|---|
| **1** Ignore the decimal points and multiply the non-zero digits. | 7 × 3 = 21 |
| **2** Count the number of decimal places in each number to find the number of decimal places in the answer. | 0.7 has one decimal place.<br>0.03 has two decimal places.<br>1 + 2 = 3<br>There will be three decimal places in the answer. |
| **3** Use the digits 21 to write the answer with three decimal places. Add as many placeholder zeros as required. | 0.7 × 0.03 = 0.021 |

UNDERSTANDING AND FLUENCY

**2** Use your knowledge of multiplication facts (times tables) and decimal places to calculate each product.

| | | |
|---|---|---|
| **a** $0.2 \times 0.6$ | **b** $0.3 \times 0.8$ | **c** $0.6 \times 0.7$ |
| **d** $0.08 \times 0.9$ | **e** $0.004 \times 0.9$ | **f** $0.7 \times 0.7$ |
| **g** $1.2 \times 0.8$ | **h** $0.006 \times 0.04$ | **i** $0.0001 \times 0.007$ |
| **j** $0.03 \times 0.005$ | **k** $0.42 \times 0.02$ | **l** $1.1 \times 0.008$ |

**3** Calculate each of these. Remember to find an estimate of the answer first.

| | | |
|---|---|---|
| **a** $3.8 \times 7$ | **b** $9.8 \times 4$ | **c** $0.63 \times 5$ |
| **d** $1.14 \times 8$ | **e** $4.0008 \times 9$ | **f** $3.28 \times 6$ |
| **g** $2.759 \times 3$ | **h** $51.4 \times 7$ | **i** $19.86 \times 4$ |

## EXAMPLE 3C-3 Using long multiplication with decimals

Use long multiplication to calculate $18.63 \times 4.9$.

### THINK

**1** Estimate the answer by rounding each number to its leading digit. 18.63 is close to 20 and 4.9 is close to 5.

**2** Write the numbers under each other without the decimal points and perform the long multiplication.

**3** Count the number of decimal places in the original calculation (2 + 1 = 3).
There will be the same number of decimal places in the answer (3).

**4** Write the answer with the decimal point in the correct position. Check that it matches with your estimate (91.287 is close to 100).

### WRITE

Estimate is $20 \times 5 = 100$.

$$\begin{array}{r} {\scriptstyle 3\ 2\ 1} \\ {\scriptstyle 7\ 5\ 2} \\ 1\,8\,6\,3 \\ \times \quad 4\,9 \\ \hline 1\,6\,7\,6\,7 \\ 7\,4\,5\,2\,0 \\ \hline 9\,1\,2\,8\,7 \end{array}$$

$18.63 \times 4.9 = 91.287$

UNDERSTANDING AND FLUENCY

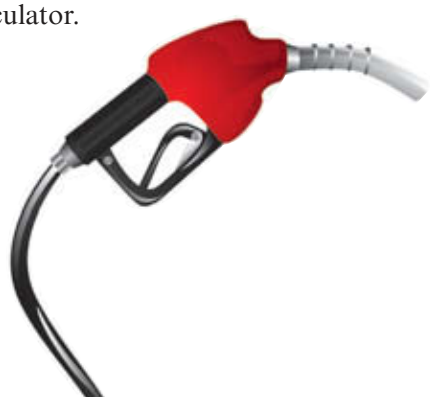
**4** Use long multiplication to calculate:

| | | |
|---|---|---|
| **a** $24.7 \times 2.8$ | **b** $63.4 \times 7.2$ | **c** $128.4 \times 9.8$ |
| **d** $13.64 \times 2.1$ | **e** $15.78 \times 8.3$ | **f** $148.42 \times 6.5$ |
| **g** $298.61 \times 0.16$ | **h** $13.213 \times 1.84$ | **i** $52.475 \times 42.9$ |

**5** Calculate each of these.

| | | |
|---|---|---|
| **a** $1.37 \times 24$ | **b** $2.41 \times 36$ | **c** $13.21 \times 15$ |
| **d** $8.64 \times 72$ | **e** $92.84 \times 0.002$ | **f** $186.71 \times 0.04$ |
| **g** $0.05 \times 248$ | **h** $0.006 \times 617$ | **i** $20.03 \times 52$ |

**6** Check your answers to questions **4** and **5** with a calculator.



**7** Barry purchases 58.7 L of unleaded petrol at a cost of $1.32 per litre.

- **a** Estimate the total cost of the petrol.
- **b** Calculate the total cost of the petrol rounded to two decimal places.
- **c** Comment on the difference between the estimated cost and the actual cost of the petrol purchased.

**8** Calculate the cost of each of these items, rounding your answers to two decimal places.



- **a** 0.75 kg of cherries at $12.99 per kg
- **b** 1.56 kg of Granny Smith apples at $3.90 per kg
- **c** 2.78 kg of bananas at $3.50 per kg
- **d** 1.2 kg of grapes at $6.90 per kg
- **e** three punnets of strawberries at $2.99 per punnet
- **f** 2.89 kg oranges at $3.49 per kg

**9** Look at the calculations in the first column of this table.

| Original calculation | Estimated answer | Answer |
|---|---|---|
| 38 × 21 | 40 × 20 = 800 | 798 |
| 3.8 × 21 | | |
| 38 × 2.1 | | |
| 3.8 × 2.1 | | |
| 380 × 210 | | |

- **a** What do the calculations have in common?
- **b** How are these calculations different?
- **c** Copy the table and complete the second column by estimating the answer. Round each number to its leading digit. The first one has been done for you.
- **d** Complete the third column by using the digits 798 and a decimal point to create a number close to the estimated answer obtained in part **d**.
- **e** What is the relationship between the total number of decimal places in the original calculation and the number of decimal places in the answer?

**10** You can use shortcuts when multiplying by a power of ten.

- **a** Calculate each product.
  - **i** 2 × 10 **ii** 2 × 100 **iii** 2 × 1000 **iv** 2 × 10 000
- **b** What happens to the place value of 2 after each calculation is completed?

**c** Using your results from parts **a** and **b**, copy and complete each sentence.

**i** Multiplying by 10 (or $10^1$) results in the 2 moving _____ place to the _____.

**ii** Multiplying by 100 (or $10^2$) results in the 2 moving _____ places to the _____.

**iii** Multiplying by 1000 (or $10^3$) results in the 2 moving _____ places to the _____.

**iv** Multiplying by 10 000 (or $10^4$) results in the 2 moving _____ places to the _____.

**d** Calculate each product.

**i** $5.2634 \times 10$ **ii** $5.2634 \times 100$

**iii** $5.2634 \times 1000$ **iv** $5.2634 \times 10\,000$

**e** What happens to the place value of the digits and the position of the decimal point after each calculation is completed?

**11** Your results for question **10** show a short cut to use when multiplying by 10, 100, 1000 and so on. Use this strategy to write the result for each of these.

**a** $6 \times 1000$ **b** $42 \times 100$ **c** $0.1284 \times 10$

**d** $15.721 \times 100$ **e** $0.0004 \times 1000$ **f** $0.0702 \times 100$

**g** $6.1 \times 10$ **h** $15.0 \times 10$ **i** $0.31 \times 1000$

**j** $64.813 \times 100$ **k** $4.8 \times 100\,000$ **l** $128.468 \times 1\,000\,000$

**m** $0.000\,456 \times 10\,000$ **n** $1.0009 \times 100$ **o** $1.003\,01 \times 1000$

PROBLEM SOLVING AND REASONING

**12** The local gym offers its clients the following payment plans for a yearly membership.

Option 1: one payment of $645

Option 2: monthly instalments of $59.45



**a** Find how much option 2 costs for a year.

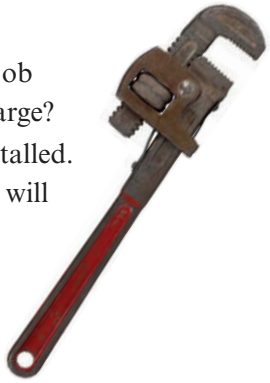
**b** Which option is the cheapest?

**c** How much money is saved if the cheaper option is chosen?

**13** Colin is a plumber. He charges an initial fee of $120 and an additional $36.80 per hour.

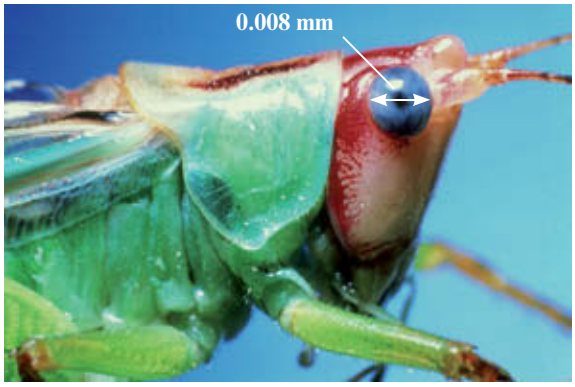
**a** The Grant family need a pipe to be unblocked. If this job takes Colin a total of 3.75 hours, how much will he charge?

**b** The Torcasios require a new hot water system to be installed. If this job takes Colin a total of 0.75 hours, how much will he charge?



PROBLEM SOLVING AND REASONING

**14** The magnification on a microscope is 1000×. This means that objects appear 1000 times larger than they actually are. How wide would an insect's eye that is 0.008 mm in diameter appear when viewed through this microscope?




**15** Multiplying by 60 is the same as multiplying by 6 and then multiplying by 10, as shown in the example. Use this strategy to calculate each product.

**Example**

$4.86 \times 60 = 4.86 \times 6 \times 10$
$= 29.16 \times 10$
$= 291.6$

**a** $12.98 \times 700$ **b** $37.2 \times 8000$
**c** $43.62 \times 2400$ **d** $106.2 \times 320$
**e** $6.5 \times 15\,000$ **f** $0.27 \times 45\,000$
**g** $176.31 \times 2900$ **h** $0.8463 \times 180\,000$
**i** $7.561 \times 6300$ **j** $51.68 \times 11\,000$

**16** Colleen bought 7000 shares at $15.80 each.

**a** How much did she pay in total?
**b** She decides to sell all her shares when they peak at $28.42 each. How much does Colleen receive?
**c** The difference between how much Colleen receives when she sells her shares and what she paid for them is called a **profit**. How much profit did Colleen receive from the sale of her shares?

CHALLENGE

**17** Calculate each of these.

**a** $(2.387 + 4.21 - 2.7) \times 3.1$
**b** $6.85 \times 7.2 - (4.5 + 8.2)$
**c** $14.85 - (6.57 + 12.35) \times 0.1$
**d** $62.87 \times 0.002 + 17.8 \times 0.02$

**18** **a** Calculate each of these.

**i** $0.2^2$ **ii** $0.4^2$
**iii** $0.25^2$ **iv** $0.008^2$

**b** Comment on the number of decimal places in the number to be squared and the number of decimal places in the answer. What interesting point do you observe?

**Reflect**

How are decimal places important when multiplying decimal numbers together?

# 3D Dividing a decimal by a whole number

## Start thinking!



Tyler pays $57 for four tickets to the cinema. He needs to let his three friends know how much he is owed for each ticket.

**1** He starts by considering how many 4s are in 57.

**a** Does 4 divide into 57 exactly?

**b** Write the division result (**quotient**) as a whole number and a remainder.

**2** Next he sets up a short division.

**a** Copy and complete this division to find the price of each ticket.

$$4\overline{)5^{1}7.^{1}0^{2}0}\quad 1\,4.$$

**b** The number to be divided into (the **dividend**) has been written as 57.00. Why do you think this is?

**c** Could Tyler have obtained the result by dividing into 57.0? Explain.

**d** Could he have obtained the result by dividing into 57.000 or 57.0000? Explain your answer.

**3** Tyler has written two trailing zeros after the decimal point so that he can obtain an exact decimal answer without a remainder.

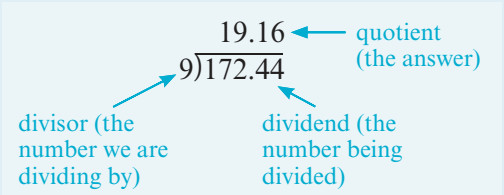
**a** Calculate:

**i** $7 \div 2$ **ii** $35 \div 8$ **iii** $43.5 \div 6$ **iv** $683.2 \div 4$.

**b** What was the least number of trailing zeros you needed to use?

## KEY IDEAS

- When dividing a decimal by a whole number, use the same method as that used for dividing whole numbers. Remember to line up the digits with the same place value in both the dividend and quotient in the division calculation.




- When performing the division calculation, trailing zeros should be added to follow digits after the decimal point in the dividend until you can divide exactly without a remainder.
- To find the cost of one item if you know the cost of a number of items, use the skill of dividing a decimal (the cost in dollars) by a whole number (number of items).

# EXERCISE 3D Dividing a decimal by a whole number

## EXAMPLE 3D-1 Using short division to divide a decimal by a whole number

Calculate 93.94 ÷ 7.

**THINK**

1 Work out the division in the same way as for whole numbers, but line up the decimal point and the digits that have the same place value.

2 Write your answer.

**WRITE**

$$7\overline{)9^{2}3.^{2}9^{1}4} \quad 1\ 3.\ 4\ 2$$

93.94 ÷ 7 = 13.42

**1** Calculate:

**a** 24.92 ÷ 4 **b** 68.605 ÷ 5 **c** 72.816 ÷ 8
**d** 161.98 ÷ 7 **e** 100.035 ÷ 9 **f** 4.9194 ÷ 3
**g** 26.2953 ÷ 9 **h** 20.75 ÷ 5 **i** 24.57 ÷ 7
**j** 0.1536 ÷ 6 **k** 0.4832 ÷ 4 **l** 1.456 08 ÷ 2

## EXAMPLE 3D-2 Dividing a decimal using trailing zeros

Calculate: 16.5 ÷ 6.

**THINK**

1 Use trailing zeros after the last digit in the dividend to obtain an exact decimal answer without a remainder. In this case, you need one trailing zero.

2 Write your answer.

**WRITE**

$$6\overline{)16.^{4}5^{3}0} \quad 2.\ 7\ 5$$

16.5 ÷ 6 = 2.75

**2** Calculate each of these. Remember to use trailing zeros if needed.

**a** 15 ÷ 6 **b** 27 ÷ 4 **c** 63 ÷ 5
**d** 17.4 ÷ 4 **e** 12.3 ÷ 6 **f** 2.031 ÷ 5
**g** 4.83 ÷ 8 **h** 234.3 ÷ 6 **i** 56.23 ÷ 4
**j** 9.55 ÷ 8 **k** 6.87 ÷ 6 **l** 25.9 ÷ 8

## EXAMPLE 3D-3 Calculating the cost of a single item

Calculate the cost of one nectarine if nine nectarines cost $12.60.

**THINK**

1 Write the calculation to show which operation (+, −, × or ÷) is needed.

2 Use short division to find the result.

3 Write your answer.

**WRITE**

cost of one nectarine
= $12.60 ÷ 9

$$9\overline{)12.^{3}6\ 0}\quad 1.\ 4\ 0$$

One nectarine costs $1.40.

UNDERSTANDING AND FLUENCY

**3** Calculate the cost of one item in each case.

**a** seven exercise books cost $8.96
**b** four tyres cost $870
**c** 12 pens cost $15.36
**d** 10 icy-poles cost $7.50
**e** 15 gym sessions cost $177
**f** eight movie passes for $58
**g** five chocolates cost $6.50
**h** five soft drinks cost $9
**i** nine candles cost $22.77
**j** three festival tickets cost $369.99

**4** Calculate the cost of 1 kg in each case.

**a** 6 kg of oranges cost $23.28
**b** 5 kg of nectarines cost $24.90
**c** 3 kg of mangoes cost $11.94
**d** 6 kg of scotch fillet costs $71.82
**e** 2 kg of capsicums cost $13.96
**f** 4 kg of apples cost $31.92
**g** 8 kg potatoes cost $12.96
**h** 7 kg chicken wings for $29.89
**i** 12 kg firewood cost $2.04
**j** 9 kg rice for $19.98

**5** A group of Year 7 students decides to divide the $558.75 they have raised throughout the year equally between three charities. How much will each charity receive?

**6** Eight tickets to a rock concert cost a total of $1087.20. What was the cost of each ticket?

**7** Four friends agree to contribute equally to buying a birthday present for their friend Winston. If the gift is $98.60, how much should each person pay?



**8** Milk can be bought in different sizes. At a supermarket, a 1-L container of milk costs $2.97 and a 2-L container costs $4.98.

- **a** Calculate the cost for 1 L if you buy the 2-L container of milk.
- **b** Compare this cost to the cost of the 1-L container of milk. Which option is the **best buy**? Explain why it is better value.

**9** Pizza can be bought by the slice or as a whole pizza that is equivalent to six slices. Compare the cost per slice. Which option is better value and by how much?



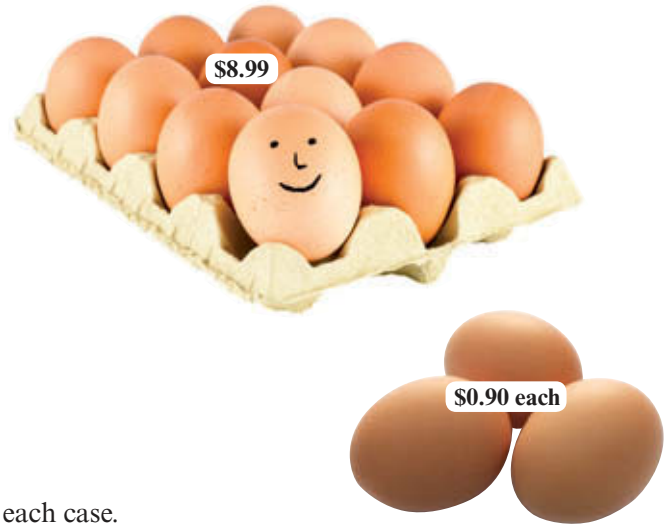

**10** Lorena has the option of buying a weekly travel ticket for $13.75 or a daily ticket costing $2.95.

- **a** How much does her weekly ticket cost per day if she uses it five days a week?
- **b** How does the price of a daily ticket compare with that of a weekly ticket? Which is the better option?
- **c** Are there occasions when buying daily tickets would be better than buying a weekly ticket? Explain.

**11** The cost of a 5 kg bag of potatoes is $6.30, while loose potatoes cost $1.45 a kilogram. Which is the better buy? Explain.

**12** Customers can buy single eggs or a carton of a dozen eggs at a health food store.

- **a** Compare the cost per egg in each case. Which option is better value?
- **b** How much would customers save per egg if they bought this option?
- **c** Why does the store offer both options? Explain.




**13** Calculate the cost of 100 g in each case. Where appropriate, round to the nearest cent.

- **a** 300 g of leg ham costs $11.19
- **b** 500 g of cheese costs $7.80
- **c** 900 g of green beans costs $4.76
- **d** 200 g of aniseed rings cost $2.99




**14** Simon bought a 500-g pack of tomatoes for $3.98. Julia bought three tomatoes that weighed a total of 300 g.

- **a** Calculate the amount paid by each of them for 100 g of tomatoes.
- **b** Who made the best buy?

**15** A recipe requires 800 g of pasta sauce. Katie considers two options:

| | |
|---|---|
| Barilla pasta sauce | 400-g jar for $3.73 |
| Leggos pasta sauce | 500-g jar for $4.09 |

- **a** Compare the value of 100 g of each product.
- **b** How many jars should Katie buy for her recipe? Which brand of sauce should Katie buy if she wants value for money? Explain.

**16** The first division prize of $6 428 212 in Lotto is to be shared amongst 18 winners. Exactly how much will each winner receive? Explain how you were able to calculate this.

**17** One hundred litres of cordial has been made to share amongst 180 students. State whether each student would receive more or less than 1 L. Explain your reasoning.

**18** Write a number with three decimal places that, when divided by 8, gives an answer between 2 and 3.

**19** Finn has the option of three different summer jobs: picking grapes, delivering pamphlets or packing boxes. He tries each of them out. He earns $51.20 picking grapes for 8 hours, $46.40 delivering pamphlets for 6 hours, and $32.50 packing boxes for 4 hours. Which job pays better by the hour?

**20** The best way to find the length of your pace is to take a number of steps, measure the distance and then use division to find the length of one step.

- **a** Find the length of one pace by using a measuring tape and taking:
  - **i** 10 paces
  - **ii** 12 paces
  - **iii** 20 paces.
- **b** Do you get the same answer for each trial?
- **c** Explain why there might be differences, and which trial you think is most accurate.

PROBLEM SOLVING AND REASONING

**21** Sometimes you may need to use both division and multiplication to find the answer. Roland would like to buy 10 mangoes. His friend Brianna bought three mangoes for $6.30.

- **a** How much did Brianna pay for one mango?
- **b** Use your answer to part **a** to find how much Roland should pay for 10 mangoes.
- **c** Explain how this problem uses both division and multiplication.

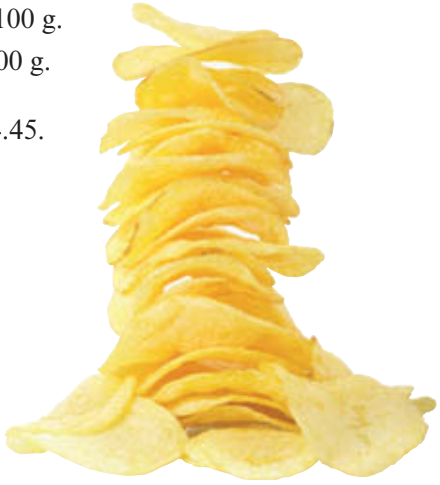
**22** Work out the cost of 10 mangoes in each situation.

- **a** 8 mangoes cost $5.60
- **b** 16 mangoes cost $9.60
- **c** 32 mangoes cost $20.80
- **d** 16 mangoes cost $13.60



**23** Work out each of these using a calculator. Where appropriate, round to the nearest cent.

- **a** 24 scones cost $29.76. Calculate the cost of 10 scones.
- **b** 15 kiwi fruits cost $31.80. Calculate the cost of 12 kiwi fruits.
- **c** 150 g of salami costs $5.60. Calculate the cost of 100 g.
- **d** 375 g of sugar costs $2.56. Calculate the cost of 100 g.



**24** A 175-g bag of crinkle cut chips can be bought for $4.45.
Tubes of Pringles chips come in two sizes:
150 g for $4.27 and 190 grams for $5.34.
Red Rock Deli chips come in 185-g bags for $4.16.

- **a** Use a calculator to work out the value of 100 grams of each product.
- **b** Which is the best buy?
- **c** What other factors could influence what you buy?

CHALLENGE

**25** Ava walks 8 km in 1 hour and 19 minutes, Caleb walks 6 km in 1 hour and 7 minutes, and Isaac walks 10 km in 1 hour 44 minutes. Who is the fastest walker and what is their pace in kilometres per hour?

**26** Sienna works at a veterinary clinic and needs to feed the dogs. Each dog eats 200 g of food every day. If the bin contains 4.38 kg of food, how many dogs can she feed?

**Reflect**

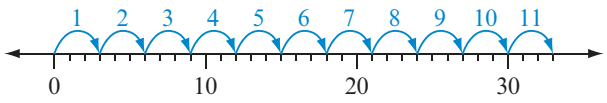
How can you compare the cost of items to decide which one is the best buy?

# 3E Dividing a decimal by a decimal

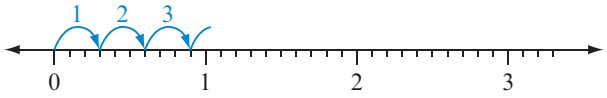
## Start thinking!

Let's look at what happens if you divide by a decimal number instead of a whole number.

**1** **a** Copy and complete this number line up to 60 to show a way of calculating 60 ÷ 3. How many times does 3 go into 60?




**b** Copy and complete this number line up to 6 to show a way of calculating 6 ÷ 0.3. How many times does 0.3 go into 6?




**c** Could this method be used to divide any number by a decimal? Explain.

**2** Using a number line can take too long. Let's investigate another method that relies on seeing a pattern.

**a** Calculate: **i** 600 000 ÷ 30 000 **ii** 60 000 ÷ 3000 **iii** 6000 ÷ 300 **iv** 600 ÷ 30

**b** What do you notice about the answer each time? Explain why this happens.

**c** What is the result of dividing 60 by 3? Does this match the pattern you can see in part **a**?

**d** What do you expect 6 ÷ 0.3 to be? Explain.

**e** Does your answer to part **d** match the result you obtained in question **1**?

**3** All of the division problems in question **2** are said to be equivalent, as they produce the same answer. Copy and complete these equivalent division problems.

**a** 60 ÷ ___ = 20 **b** 6 ÷ ___ = 20 **c** 0.6 ÷ ___ = 20 **d** ___ ÷ 0.003 = 20

## KEY IDEAS

- When you divide by a decimal, convert the division problem to an equivalent problem that has the number you are dividing by (**divisor**) written as a whole number.
- One way to convert the division problem is to multiply both numbers by a power of 10 (10 or 100 or 1000, etc.) so that the divisor becomes a whole number. In the example at right, you can write an equivalent problem by multiplying *both* numbers by 100. The divisor is now a whole number (4).

**Example**
12.328 ÷ 0.04
= 1232.8 ÷ 4

- Another way to convert a division problem is to use a short cut for multiplying by 10 or 100 or 1000, etc. Move the digits in each number the same number of places to the left. The decimal point appears to move to the right. In the example, the digits in each number are moved two places to the left. (The decimal point appears to move two places to the right.)
- Remember that trailing zeros should be added to the dividend until you can divide exactly, without a remainder.

# EXERCISE 3E Dividing a decimal by a decimal

## EXAMPLE 3E-1 Making the divisor a whole number

For 1.734 ÷ 0.06, describe how you can make the divisor (the number you are dividing by) become a whole number.

**THINK**

Decide what power of 10 to multiply by to get a whole number. The divisor has two decimal places so multiply by 100. A short cut is to move the decimal point two places to the right.

**WRITE**

0.06 has two decimal places.
Multiplying 0.06 by 100 makes it a whole number.
0.06 × 100 = 6

**1** For each division problem, describe how you can make the divisor (the number you are dividing by) become a whole number.

| | | |
|---|---|---|
| **a** 15.851 ÷ 0.5 | **b** 16.482 ÷ 0.04 | **c** 0.195 84 ÷ 0.3 |
| **d** 0.365 05 ÷ 0.7 | **e** 50.5956 ÷ 0.02 | **f** 30.456 ÷ 0.8 |
| **g** 10.473 24 ÷ 0.006 | **h** 5125 ÷ 0.4 | **i** 3.222 ÷ 0.09 |

## EXAMPLE 3E-2 Writing an equivalent division problem

Write an equivalent division problem for 1.734 ÷ 0.06 so that you are dividing by a whole number.

**THINK**

**1** Count the number of decimal places in the divisor. The divisor needs to be multiplied by 100 to make it a whole number.

**2** To write an equivalent division problem, the dividend (the number you are dividing into) must also be multiplied by 100.

**3** Write the equivalent division problem.

**NOTE** You can also use the short cut of moving the decimal point two places to the right in both numbers.

**WRITE**

0.06 has two decimal places.
0.06 × 100 = 6

1.734 × 100 = 173.4

1.734 ÷ 0.06
= 173.4 ÷ 6

**2** Write an equivalent division problem for each of these so that you are dividing by a whole number.

| | | |
|---|---|---|
| **a** 62.356 ÷ 0.8 | **b** 9.84 ÷ 0.008 | **c** 19.5674 ÷ 0.06 |
| **d** 0.462 ÷ 0.3 | **e** 0.042 ÷ 0.0004 | **f** 0.005 ÷ 0.02 |
| **g** 46.2085 ÷ 0.000 09 | **h** 0.056 ÷ 0.2 | **i** 2.108 ÷ 0.005 |

**3** Write an equivalent division problem for each problem in question **1**.

## EXAMPLE 3E-3 Dividing by a decimal number

Calculate 21.78 ÷ 0.5.

### THINK

**1** The decimal 0.5 must be multiplied by 10 to become a whole number. If you multiply the divisor by 10, you must also multiply the dividend by 10.

**2** Estimate the answer by rounding each number to its leading digit.

**3** Work out the division problem. Include trailing zeros until you can divide exactly without a remainder.

**4** Write the answer. Check that it matches with your estimate (43.56 is close to 40).

### WRITE

21.78 ÷ 0.5
= (21.78 × 10) ÷ (0.5 × 10)
= 217.8 ÷ 5

Estimate is 200 ÷ 5 = 40.

$$\begin{array}{r} 43.56 \\ 5\overline{)21^{1}7.^{2}8^{3}0} \end{array}$$

21.78 ÷ 0.5 = 43.56

UNDERSTANDING AND FLUENCY

**4** Calculate each of these. (Hint: first write each as an equivalent division problem.)

| | | |
|---|---|---|
| **a** 3.86 ÷ 0.2 | **b** 0.843 ÷ 0.003 | **c** 5.929 ÷ 0.07 |
| **d** 71.82 ÷ 0.6 | **e** 6.512 ÷ 0.04 | **f** 0.9265 ÷ 0.005 |
| **g** 27.32 ÷ 0.0008 | **h** 42.39 ÷ 0.9 | **i** 7.296 ÷ 0.02 |

**5** Calculate each of these. Remember to use trailing zeros if needed.

| | | |
|---|---|---|
| **a** 9.524 ÷ 0.8 | **b** 1.735 ÷ 0.04 | **c** 32.475 ÷ 0.006 |
| **d** 0.041 ÷ 0.05 | **e** 6.375 79 ÷ 0.002 | **f** 51.374 ÷ 0.08 |
| **g** 0.007 53 ÷ 0.000 04 | **h** 1.369 47 ÷ 0.0006 | **i** 0.0573 ÷ 0.004 |

**6** Calculate each division problem in question **1**.

**7** Use your knowledge of dividing by a decimal and your multiplication facts (times tables) to calculate each of these.

| | | |
|---|---|---|
| **a** 1.8 ÷ 0.2 | **b** 5.4 ÷ 0.9 | **c** 4.2 ÷ 0.6 |
| **d** 18 ÷ 0.09 | **e** 0.021 ÷ 0.007 | **f** 320 ÷ 0.4 |
| **g** 7.2 ÷ 0.08 | **h** 0.000 6 ÷ 0.000 02 | **i** 28.4 ÷ 0.004 |

**8** Lacey has $8.40 to buy muffins.

**a** Write a division problem to calculate the number of muffins she could buy with this money.

**b** Write an equivalent division problem so you are dividing by a whole number.

**c** Calculate the number of muffins Lacey could buy.




**9** How many 0.375 L bottles can be filled from a 32.25-L water dispenser?

**10** Using the cost shown on each item:

**i** write a division problem to calculate the number of each item you could buy with $12.00

**ii** write an equivalent division problem so you are dividing by a whole number

**iii** calculate the number of each item you could buy.

**a**




**b**




**c**




**d**




**11** Copy and complete the following so that each calculation gives the same answer.

**a** 240 000 ÷ 3000 = _____

**b** 24 000 ÷ _____ = 80

**c** 2400 ÷ _____ = _____

**d** 240 ÷ _____ = _____

**e** _____ ÷ _____ = 80

**f** _____ ÷ 0.03 = _____

**g** _____ ÷ 0.003 = _____

**h** 0.024 ÷ _____ = _____

**12** **a** Calculate each of these.

**i** 72 000 ÷ 800 **ii** 7200 ÷ 80 **iii** 720 ÷ 8

**b** Copy and complete each equivalent statement to obtain the same answer produced in part **a**.

**i** 72 ÷ ____ = ____ **ii** 7.2 ÷ ____ = ____ **iii** 0.72 ÷ ____ = ____

**c** Write three equivalent statements that will produce the same answer as 0.99 ÷ 0.11 = 9.

**13** **a** Calculate:

**i** $200 \div 40$ **ii** $200 \div 4$

**iii** $200 \div 0.4$ **iv** $200 \div 0.04$.

**b** What overall pattern do you notice as the divisor becomes smaller?

**c** What in particular do you notice about the answers when you are dividing by a decimal number less than 1?

**d** Copy and complete: When dividing by decimal numbers less than 1, the smaller the divisor, the _____ the answer.

**14** Some of the division problems (A–H) listed below produce the same answer.

**A** $840\,000 \div 12\,000$

**B** $84\,000 \div 1200$

**C** $84\,000 \div 120$

**D** $84 \div 1200$

**E** $840 \div 12$

**F** $84 \div 12$

**G** $8.40 \div 0.12$

**H** $8.4 \div 12$

**a** Which ones produce the same answer?

**b** Explain why they give the same answer.



**15** Sophia believes $6500 \div 130$ will produce the same answer as $0.65 \div 0.13$.

**a** Is she correct?

**b** How can you check (without doing the actual calculation) whether she is correct?

**16** Hamish considers two options when buying dog food: 1.2 kg bag for $2.88 and 0.8 kg bag for $2.20.

**a** Divide $2.88 by 1.2 to calculate the cost per kg for the 1.2 kg bag.

**b** Divide $2.20 by 0.8 to calculate the cost per kg for the 0.8 kg bag.

**c** Which is the best buy?

**17** As part of an awareness-raising campaign for childhood cancer, gold ribbon is to be cut into lengths and sold to supporters to wear.

**a** How many pieces 0.15 m long can be cut from a 21.7 m spool of ribbon?

**b** Will any ribbon be left over? If so, how much?

**18** One litre of petrol costs $1.54. How many litres of petrol were purchased if the cost was $70.07?

PROBLEM SOLVING AND REASONING

**19** Describe how you can decide which bottle of drink is the better value for money. Is the 2-L bottle of orange mineral water or the 1.5-L bottle of cola the better buy?




**20** **a** Find the answers to:

**i** 38 765.2 ÷ 10

**ii** 38 765.2 ÷ 100

**iii** 38 765.2 ÷ 1000

**iv** 38 765.2 ÷ 10 000.

**b** What do you notice about the position of the decimal point in each result?

**c** Copy and complete these sentences.

**i** Dividing by 10 (or $10^1$) results in the decimal point moving _____ place to the _____.

**ii** Dividing by 100 (or $10^2$) results in the decimal point moving _____ places to the _____.

**iii** Dividing by 1000 (or $10^3$) results in the decimal point moving _____ places to the _____.

**iv** Dividing by 10 000 (or $10^4$) results in the decimal point moving _____ places to the _____.

**d** Use the shortcut of 'moving' the decimal point to write the result for each problem.

| | | |
|---|---|---|
| **i** 678.543 ÷ 1000 | **ii** 953 ÷ 100 | **iii** 288.94 ÷ 10 |
| **iv** 15.721 ÷ 100 | **v** 293.4 ÷ 1000 | **vi** 70.2 ÷ 100 |
| **vii** 68 942.1 ÷ 1000 | **viii** 1509.5 ÷ 10 000 | **ix** 4.36 ÷ 100 |

**21** Dividing by 900 is the same as dividing by 9 and then dividing by 100 (see example). Use this strategy to calculate:

| | |
|---|---|
| **a** 393.946 ÷ 700 | **b** 231 120 ÷ 2400 |
| **c** 16 790.4 ÷ 320 | **d** 1248.9 ÷ 1500 |
| **e** 307 152 ÷ 45 000 | **f** 8192.5 ÷ 2900 |
| **g** 937 800 ÷ 180 000 | **h** 5620.23 ÷ 6300 |

**Example**

$5342.4 \div 900 = 5342.4 \div 9 \div 100$
$= 593.6 \div 100$
$= 5.936$

CHALLENGE

**22** Calculate each of these. Remember to use the correct order of operations.

| | |
|---|---|
| **a** $(12.6 \div 0.003) \times 3.8$ | **b** $(8.34 - 2.18) \div 0.28$ |
| **c** $1.4^2 + 8.38 \div 0.004$ | **d** $\sqrt{(0.0048 \div 0.03) \times 100}$ |
| **e** $\sqrt{(3.2 \times 5.1) - (0.16 \times 2)} \div 0.008$ | **f** $2.3(1.08 \times 7.23 - 4.201 \div 1.2)^2$ |

**23** Place a decimal point in 324 and 405 so that when the larger decimal is divided by the smaller decimal the result is 8.

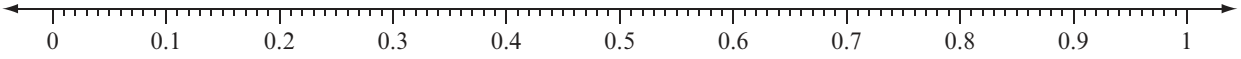
**Reflect**

How is dividing by a whole number different from dividing by a decimal number? How is it the same?

# 3F Converting between fractions and decimals

## Start thinking!

**1** Copy this number line. List the decimals 0.31, 0.27, 0.84, 0.75, 0.06 and 0.45 in ascending order then place them onto the number line.




**2** The decimal 0.57 represents 57 hundredths and can be written as $\frac{57}{100}$. Write each decimal in question **1** as a fraction with a denominator of 100 and write it directly below its equivalent decimal on the number line.

**3** Why do these fractions have a denominator of 100?

**4** Can you simplify any of these fractions? Write each simplified fraction under its equivalent fraction and decimal on the number line.

**5** Why do you think that a decimal can have more than one fraction that is of equal value?

**6** Write three equivalent fractions for 0.5.

**7** How might you convert 0.1587 to a fraction? (Hint: it represents 1587 ten-thousandths.)

## KEY IDEAS

- To write a decimal as a fraction:
  - use your knowledge of place value; for example, 0.7 is seven tenths or $\frac{7}{10}$, and 0.351 is 351 thousandths or $\frac{351}{1000}$
  - simplify fractions where possible; for example, $0.15 = \frac{15}{100} = \frac{3}{20}$.
- To write a fraction as a decimal:
  - if a fraction has a denominator that is a power of 10, use place value to write the equivalent decimal; for example, $\frac{43}{100}$ is 43 hundredths. The denominator of 100 tells you that the last digit in the numerator can be written in the hundredths place or second decimal place; so $\frac{43}{100} = 0.43$
  - if the denominator is not a power of 10, divide the numerator by the denominator; for example, $\frac{5}{8} = 5 \div 8 = 0.625$.

Other examples of fractions with denominators that are powers of 10:

$\frac{3}{10}$ is three tenths or 0.3

$\frac{51}{1000}$ is 51 thousandths or 0.051

# EXERCISE 3F Converting between fractions and decimals

**1** Write each decimal as a fraction with a denominator of 100.

| | | | |
|---|---|---|---|
| **a** 0.25 | **b** 0.50 | **c** 0.35 | **d** 0.88 |
| **e** 0.23 | **f** 0.08 | **g** 0.12 | **h** 0.95 |

**2** Write each fraction found in question **1** in simplest form.

## EXAMPLE 3F-1 Writing a decimal as a fraction

Write 0.42 as a fraction.

**THINK**

**1** Write the digits after the decimal point as the numerator (42). The place value of the last digit in 0.42 is hundredths, so the denominator is 100.

**2** Look for common factors to simplify the fraction. Divide both the numerator and denominator by the highest common factor of 2.

**3** Write the answer.

**WRITE**

$$0.42 = \frac{42}{100}$$

$$= \frac{\cancel{42}^{21}}{\cancel{100}_{50}}$$

$$= \frac{21}{50}$$

**3** Write each decimal as a fraction in simplest form.

| | | | | | |
|---|---|---|---|---|---|
| **a** 0.2 | **b** 0.7 | **c** 0.13 | **d** 0.26 | **e** 0.009 | **f** 0.426 |
| **g** 0.512 | **h** 0.720 | **i** 0. 0004 | **j** 0.56 | **k** 0.375 | **l** 0.28 |

## EXAMPLE 3F-2 Writing a fraction as a decimal when the denominator is a power of 10

Write $\frac{37}{1000}$ as a decimal.

**THINK**

**1** Using place value, $\frac{37}{1000}$ is 37 thousandths. This means there are three decimal places in the matching decimal.

**2** The last digit in the numerator (7) will be in the third decimal place and the other digit (3) will be in the second decimal place. Use a placeholder zero to fill the 'empty' place.

**WRITE**

$$\frac{37}{1000} = 0.037$$

**4** Write each fraction as a decimal.

**a** $\frac{3}{100}$ **b** $\frac{17}{100}$ **c** $\frac{9}{10}$ **d** $\frac{2010}{10\,000}$ **e** $\frac{375}{10\,000}$ **f** $\frac{156}{100\,000}$

**g** $\frac{7}{100}$ **h** $\frac{921}{1000}$ **i** $\frac{42}{1000}$ **j** $\frac{3}{10\,000}$ **k** $\frac{1981}{100\,000}$ **l** $\frac{86}{10\,000}$

## EXAMPLE 3F-3 Writing a fraction as a decimal

Write $\frac{5}{8}$ as a decimal.

**THINK**

1 As the denominator of the fraction is not a power of 10, write the fraction as a division problem.

2 Work out the division. Include trailing zeros until you can divide exactly without a remainder.

3 Write the answer.

**WRITE**

$\frac{5}{8} = 5 \div 8$

$$8\overline{)5.0^{2}0^{4}0}\ 0.6\ 2\ 5$$

$\frac{5}{8} = 0.625$

UNDERSTANDING AND FLUENCY

**5** Write each fraction as a decimal.

**a** $\frac{7}{8}$ **b** $\frac{3}{4}$ **c** $\frac{4}{5}$ **d** $\frac{9}{8}$ **e** $\frac{57}{4}$ **f** $\frac{15}{8}$

**g** $6\frac{3}{5}$ **h** $7\frac{1}{4}$ **i** $\frac{3}{20}$ **j** $\frac{1}{25}$ **k** $\frac{7}{50}$ **l** $\frac{9}{40}$

**6 a** In the previous division problems, you may have needed to add trailing zeros until you reached a stage where there was no remainder. The resulting decimal is called a **terminating decimal**. For example, $\frac{3}{8}$ produces a terminating decimal as 0.375 'terminates' or stops after three decimal places. List another three examples of a terminating decimal and explain your reasoning for choosing them.

**b** Sometimes when you are dividing, you cannot reach a stage where there is no remainder, no matter how many trailing zeros you add. The result has an endless number of decimal places. Use division to write $\frac{3}{11}$ as a decimal. Do you get to a stage where there is no remainder?

**c** What do you notice about the answer? (Hint: go to at least 10 decimal places.)

**d** When division results in a decimal number with an infinite number of decimal places *and* the digits after the decimal point form a repeating pattern, it is called a **recurring decimal**. What are the repeating digits in the decimal number for $\frac{3}{11}$?

**e** To save space, dots or horizontal lines are used to indicate the digits that recur (repeat). For example, $\frac{5}{6} = 0.833\,333\,33\ldots$ is written as $0.8\dot{3}$ and $\frac{1}{13} = 0.076\,923\,076\,923\,0\ldots$ is written as either $0.\dot{0}76\,923\dot{0}$ or $0.\overline{076\,923}$. Write $\frac{3}{11}$ as a decimal number using either dots or a horizontal line.

**NOTE** The three full stops you can see at the end of 0.833 333 33... indicate that the digit 3 continues forever. For 0.076 923 076 923 0..., they indicate that the pattern of digits continues forever.

**7** State whether each decimal is terminating (T) or recurring (R).

**a** 0.312 **b** 0.444 444 … **c** 0.128 712 87…
**d** 0.829 15 **e** 0.111 111 … **f** 0.583 333 …
**g** 1.933 333 … **h** 0.770 866 **i** 0.464
**j** 0.363 636 … **k** 0.2791 **l** 8.314 555 …

**8** Rewrite each recurring decimal in abbreviated form using dots or a horizontal line.

**a** 0.888 888 … **b** 0.212 121 …
**c** 0.467 467 … **d** 0.127 444 …
**e** 8.316 666 … **f** 1.333 333 …
**g** 9.102 464 646 … **h** 0.461 538 461 538 …

## EXAMPLE 3F-4 Writing a fraction as a recurring decimal

Write $\frac{4}{7}$ as a decimal.

### THINK

**1** Write the fraction as a division problem.

**2** Work out the division. Include trailing zeros until you can divide exactly without a remainder or you can see a pattern in the digits after the decimal point.

**3** Identify the pattern by showing one copy of the repeating digits.

**4** Write the decimal with a line shown over the repeating digits.

### WRITE

$$\begin{array}{r} 0.5\ 7\ 1\ 4\ 2\ 8\ 5\ 7\ 1 \\ \hline 7)4.0^{5}0^{1}0^{3}0^{2}0^{6}0^{4}0^{5}0^{1}0 \end{array}$$

Pattern in digits is 571 428.

$\frac{4}{7} = 0.\overline{571428}$

**9** Write each fraction as a decimal. Use a dot or horizontal line to indicate which digits recur.

**a** $\frac{2}{9}$ **b** $\frac{5}{6}$ **c** $\frac{6}{7}$ **d** $\frac{8}{11}$
**e** $\frac{7}{6}$ **f** $\frac{2}{3}$ **g** $\frac{11}{15}$ **h** $4\frac{5}{12}$

**10** You are offered 0.3 of this block of chocolate or a piece that is $\frac{1}{4}$ of the block of chocolate. Which portion is larger? Show how you worked this out.





**11** Jacey bought some lollipops for a car trip: two raspberry, two apple, one cola, two watermelon and three strawberries and cream.

- **a** How many lollipops are there in total?
- **b** Write the number of raspberry lollipops as a fraction of the total number of lollipops.
- **c** Write your answer to part **b** as a decimal.
- **d** Write the number of each of the other lollipops as both a fraction and a decimal of the total number of lollipops.

**12** When the denominator of a fraction is a power of 10, the fraction can be written as a decimal without performing division; for example, $\frac{7}{10} = 0.7$. Convert the given fractions into tenths, hundredths or thousandths. Then write each equivalent fraction as a decimal (see example).

| | | |
|---|---|---|
| **a** $\frac{1}{5}$ | **b** $\frac{3}{4}$ | **c** $\frac{7}{20}$ |
| **d** $\frac{4}{5}$ | **e** $\frac{11}{50}$ | **f** $\frac{2}{4}$ |
| **g** $\frac{13}{20}$ | **h** $\frac{1}{2}$ | **i** $\frac{19}{50}$ |
| **j** $\frac{1}{8}$ | **k** $\frac{4}{125}$ | **l** $\frac{53}{2500}$ |

**Example**

$\frac{4}{5} = \frac{4}{5} \times \frac{2}{2}$

$= \frac{8}{10}$

$= 0.8$

**13** Why is it possible to write the fractions in question **12** as fractions with denominators of powers of 10?

**14** Which of these fractions can be converted to decimals without first dividing? For those that can't, explain why.

| | | | |
|---|---|---|---|
| **a** $\frac{3}{5}$ | **b** $\frac{11}{15}$ | **c** $\frac{5}{6}$ | **d** $\frac{19}{22}$ |
| **e** $\frac{5}{25}$ | **f** $\frac{8}{30}$ | **g** $\frac{497}{500}$ | **h** $\frac{12}{13}$ |
| **i** $\frac{7}{25}$ | **j** $\frac{7}{9}$ | **k** $\frac{6}{11}$ | **l** $\frac{103}{125}$ |

**15** Not all decimal numbers are less than one. Consider 8.605.

- **a** Write the number in place-value notation.
- **b** To convert this decimal number to a fraction, what part of the number do you need to look at?
- **c** Write the decimal part as a decimal fraction and simplify as much as possible.
- **d** Use your answer to part **c** to write 8.605 in fractional form.

**16** Write each decimal in fractional form. Remember to simplify as much as possible.

| | | |
|---|---|---|
| **a** 5.83 | **b** 3.902 | **c** 6.45 |
| **d** 8.35 | **e** 2.0008 | **f** 3.9 |
| **g** 12.88 | **h** 1.706 | **i** 40.375 |
| **j** 1.73 | **k** 3.15 | **l** 26.842 |

PROBLEM SOLVING AND REASONING



**17** Salwa is making a teddy bear for a project at school. She has 4.25 m and $1\frac{4}{5}$ m of material.

- **a** Convert one of the measurements so that they are both in the same form.
- **b** How much material does Salwa have?
- **c** If she needs a total of 6 m of material, does she have enough?

**18** **a** Convert each of these fractions to a decimal by dividing the numerator by the denominator.

**i** $\frac{1}{7}$ **ii** $\frac{2}{7}$ **iii** $\frac{3}{7}$ **iv** $\frac{4}{7}$

- **b** Comment on the pattern observed in part **a**.
- **c** Assuming this pattern continues, write $\frac{5}{7}$ and $\frac{6}{7}$ as decimals without dividing.
- **d** Check your answers to part **c** using division.

**19** Investigate the pattern of recurring digits with fractions that have a denominator of 9. What do you find?

**20** **a** Repeat the investigation described in questions **18** and **19** with another fraction.

- **b** Can you find a further two examples of fractions with a particular denominator that produce recurring decimals?

**21** Lou scored 28 of her team's 36 goals last week and 38 of her team's 46 goals this week. In which week did Lou perform better?



**22** Taree received the following scores on her Science tests: Science at work $\frac{32}{36}$; Solids, liquids and gases $\frac{38}{45}$; Separating mixtures $\frac{35}{40}$;an d Forces and motion $\frac{72}{90}$.
Convert each of her test scores to a decimal number and then place them in ascending order.

**23** Write three fractions between 0.75 and 0.82.

**24** Write three decimals between $\frac{1}{5}$ and $\frac{2}{5}$.

CHALLENGE

**25** Consider this list of fractions: $3\frac{7}{9}, 3\frac{11}{12}, 3\frac{8}{10}, 3\frac{162}{200}, 3\frac{7}{8}$.
Which fraction is closest to 3.82?

**26** Write each list of numbers in ascending order.

- **a** $\frac{1}{10}$, $\frac{15}{25}$, $\frac{14}{16}$, 0.62, 0.08, $\frac{22}{20}$, 1.0 5, $\frac{16}{34}$
- **b** 2.21, 0.28, 5.6, $\frac{37}{17}$, $\frac{61}{90}$, 0.82, $5\frac{12}{20}$, $\frac{58}{75}$

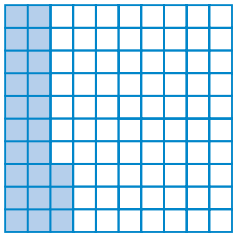
**Reflect**

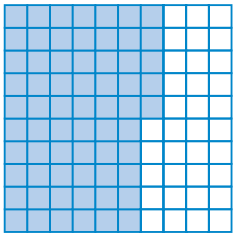
Why are powers of ten important when converting between decimals and fractions?

# 3G Understanding percentages

## Start thinking!

You will have seen and heard references to **percentages** before. For example, a reservoir is at 63% capacity or a bank offers an interest rate of 3.2% on your savings. What is a percentage? Let's look more closely at what this word means.


Figure A


Figure B

**1** Each of these two squares has been divided into 100 smaller squares. Count the number of smaller squares that have been coloured, and then copy and complete these sentences.

**a** In figure A, _____ out of the 100 squares have been shaded.

**b** In figure B, _____ out of the 100 squares have been shaded.

**2** Each amount found in question **1** is called a percentage. The term '**per cent**' means 'for every one hundred' or 'out of 100'. The symbol for percentage is %. So 17% means '17 out of 100'. What percentage of the square has been shaded:

**a** in figure A? **b** in figure B?

**3** Count how many squares are *not* shaded in each figure. What percentage of the square has *not* been shaded:

**a** in figure A? **b** in figure B?

**4** Look again at figure A.

**a** What percentage of figure A is: **i** shaded? **ii** not shaded?

**b** Add these two percentages together. What do you find?

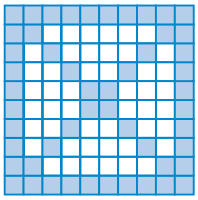
## KEY IDEAS

- A percentage is used to show how part of an amount is related to the total amount.
- The term 'per cent' means 'for every one hundred' or 'out of 100'. The symbol for percentage is %.
- A percentage may be written as a fraction with a denominator of 100.
  For example, $\frac{17}{100} = 17\%$.
- If a fraction does not have a denominator of 100, find an equivalent fraction that does.
  For example, $\frac{9}{25} = \frac{9}{25} \times \frac{4}{4} = \frac{36}{100} = 36\%$.

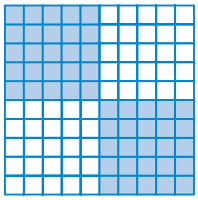
# EXERCISE 3G Understanding percentages

**1** What percentage of each square has been shaded?

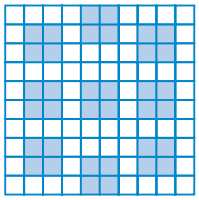
**a**



**b**



**c**



**2** Write the percentage of each square in question **1** that is unshaded.

## EXAMPLE 3G-1 Writing a percentage as a fraction with a denominator of 100

Write 27% as a fraction with a denominator of 100.

**THINK**

Write the digits in the percentage as the numerator over a denominator of 100.

**WRITE**

$$27\% = \frac{27}{100}$$

**3** Write each percentage as a fraction with a denominator of 100.

**a** 7% **b** 19% **c** 43% **d** 57% **e** 3% **f** 63%

**g** 11% **h** 91% **i** 79% **j** 21% **k** 99% **l** 33%

## EXAMPLE 3G-2 Writing a percentage as a fraction in simplest form

Write 48% as a fraction in simplest form.

**THINK**

1. Write the digits in the percentage as the numerator over a denominator of 100.
2. To simplify the fraction, look for common factors. Divide both the numerator and denominator by the highest common factor of 4.
3. Write the answer.

**WRITE**

$$48\% = \frac{48}{100}$$

$$= \frac{\cancel{48}^{12}}{\cancel{100}_{25}}$$

$$= \frac{12}{25}$$

**4** Write each percentage as a fraction in simplest form.

| | | | |
|---|---|---|---|
| **a** 64% | **b** 18% | **c** 27% | **d** 80% |
| **e** 5% | **f** 95% | **g** 10% | **h** 45% |
| **i** 60% | **j** 75% | **k** 35% | **l** 88% |

## EXAMPLE 3G-3 Writing a fraction with a denominator of 100 as a percentage

Write $\frac{19}{100}$ as a percentage.

**THINK**

The denominator of the fraction is 100, so write the digits of the numerator as the percentage.

**WRITE**

$\frac{19}{100} = 19\%$

**5** Write each fraction as a percentage.

| | | | |
|---|---|---|---|
| **a** $\frac{23}{100}$ | **b** $\frac{78}{100}$ | **c** $\frac{35}{100}$ | **d** $\frac{99}{100}$ |
| **e** $\frac{13}{100}$ | **f** $\frac{7}{100}$ | **g** $\frac{82}{100}$ | **h** $\frac{4}{100}$ |

## EXAMPLE 3G-4 Writing a fraction as a percentage

Write $\frac{3}{5}$ as a percentage.

**THINK**

1 Multiply both the numerator and denominator by 20 to create an equivalent fraction with a denominator of 100.

2 Write the digits of the numerator as the percentage.

**WRITE**

$\frac{3}{5} \times \frac{20}{20}$
$= \frac{60}{100}$
$= 60\%$

**6** Write each fraction as a percentage.

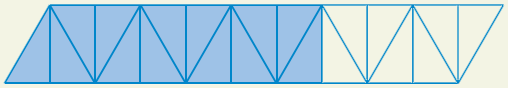
| | | |
|---|---|---|
| **a** $\frac{9}{20}$ | **b** $\frac{11}{50}$ | **c** $\frac{3}{4}$ |
| **d** $\frac{17}{25}$ | **e** $\frac{3}{5}$ | **f** $\frac{5}{25}$ |
| **g** $\frac{12}{20}$ | **h** $\frac{46}{50}$ | **i** $\frac{18}{50}$ |
| **j** $\frac{1}{4}$ | **k** $\frac{160}{200}$ | **l** $\frac{40}{200}$ |

**NOTE** First find an equivalent fraction that has a denominator of 100.

UNDERSTANDING AND FLUENCY

## EXAMPLE 3G-5 Finding the percentage of a figure that is shaded

What percentage of this figure has been shaded?



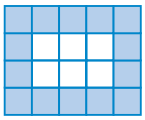
| THINK | WRITE |
|---|---|
| **1** Count the number of sections that are shaded, and the total number of sections. | 13 out of 20 sections are shaded. |
| **2** Write the proportion of the figure that has been shaded as a fraction. | $\frac{13}{20}$ |
| **3** Find an equivalent fraction that has a denominator of 100. | $= \frac{13}{20} \times \frac{5}{5}$<br>$= \frac{65}{100}$ |
| **4** Write the fraction as a percentage. | 65% of the figure has been shaded. |

UNDERSTANDING AND FLUENCY

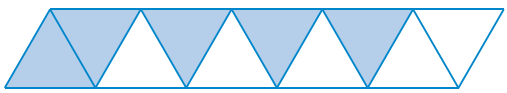
**7** Complete these tasks for the shapes below.

- **i** Count the number of sections that are shaded and the total number of sections.
- **ii** Write the proportion that is shaded as a fraction.
- **iii** Find an equivalent fraction that has a denominator of 100.
- **iv** Write this as a percentage.

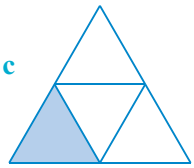
**a**



**b**

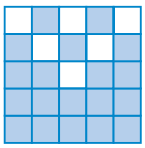


**c**

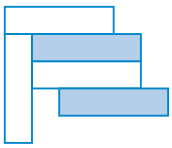


**8** What percentage of each figure has been shaded?

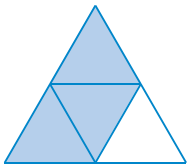
**a**



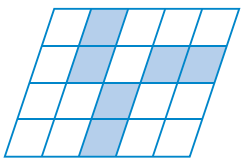
**b**



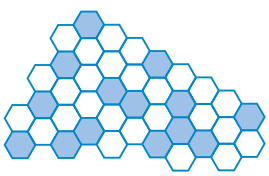
**c**



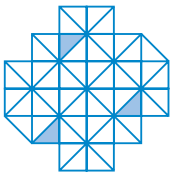
**d**



**e**



**f**

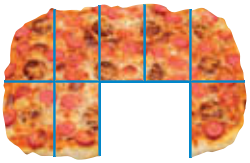


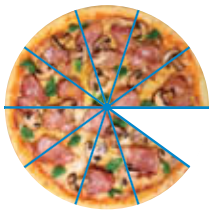
UNDERSTANDING AND FLUENCY

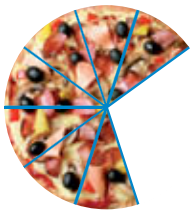
**9** **a** Write the percentage eaten from each of the three pizzas (assume all pieces are equal sized).

**b** Write as a percentage the portion of pizza not eaten from each tray.

**c** Write a brief explanation for a friend of how you calculated as a percentage the portion of pizza that was eaten.

**i** 

**ii** 

**iii** 

**10** Find the percentage that remains if:

**a** Jordan eats 45% of a muesli bar

**b** Robert watches 18% of a movie

**c** Jayne reads 75% of a book

**d** Gerard drinks 67% of a bottle of lemonade.

**11** Draw a grid of 100 squares and shade:

**a** 45% **b** 3% **c** 78% **d** 22%.

PROBLEM SOLVING AND REASONING

**12** While downloading songs to his iPod, Geoff noticed a horizontal bar on the screen. The bar's shading changed according to what percentage of the song had downloaded.

**a** Draw a rectangle 10 cm long and 1.5 cm wide to represent the horizontal bar.

**b** **i** Why do you think you were instructed to draw the rectangle 10 cm long?

**ii** What will each 1 cm represent along the rectangle?

**iii** What will each 1 mm represent along the rectangle?

**c** Using the horizontal bar from part **a**, show how the bar would look when 15% of a song has been downloaded.

**d** Draw four rectangles of the same dimensions as part **a** to show these percentages.

**i** 85% **ii** 38% **iii** 23% **iv** 96%

**13** Match each jug with the percentage it has been filled.

**a** 

**b** 

**c** 

**d** 

**A** 20% **B** 5% **C** 85% **D** 45%

**14** Write a percentage to describe each situation by first finding an equivalent fraction that has a denominator of 100.

**a** four students out of ten are blonde **b** 18 people out of 20 come to a party

**c** 2 people out of 50 miss the train **d** one person out of five walks to school

PROBLEM SOLVING AND REASONING

**15** Find the percentage that remains in each situation.

- **a** Renee finishes six out of ten homework problems.
- **b** Elise eats 35 out of 50 M&M'S.
- **c** Xavier completes 7 out of 20 levels on a game.
- **d** Peter uses one out of four spare batteries.

**16** Use percentages to estimate how full each object is.

**a**



**b**



**c**



**17** Reuben is having difficulty understanding how it can be that these two containers are both 50% full, yet the volume of liquid in each is different. Explain the relationship between the volume and the percentage of liquid in the containers.





**18** Percentages can be more than 100%. This can only happen when it is possible to have more than the whole amount.

- **a** If 200% means double the whole amount, what does 300% mean?
- **b** To write 100% as a number, first write it as a fraction: $100\% = \frac{100}{100} = 1$.

Use this method to write each percentage as a number.

**i** 200% **ii** 300% **iii** 400% **iv** 150%

**19** A Greasy Joe's hamburger has about 450% of the daily recommended fat intake.

- **a** Write this percentage as a number.
- **b** If you decide to get chips with the burger, the meal would have 5.25 times the amount of the daily recommended fat. Write this amount as a percentage.

CHALLENGE

**20** To write a percentage as a fraction, the digits become the numerator and the denominator is 100. Consider these examples showing percentages that involve fractions.

$$\frac{1}{8}\% = \frac{\frac{1}{8}}{100} = \frac{1}{8} \div \frac{100}{1} = \frac{1}{8} \times \frac{1}{100} = \frac{1}{800}$$

$$22\frac{1}{3}\% = \frac{67}{3}\% = \frac{\frac{67}{3}}{100} = \frac{67}{3} \div \frac{100}{1} = \frac{67}{3} \times \frac{1}{100} = \frac{67}{300}$$

Look at the original percentage and the final answer. Write a short cut method to convert a percentage containing a fraction to a fraction.

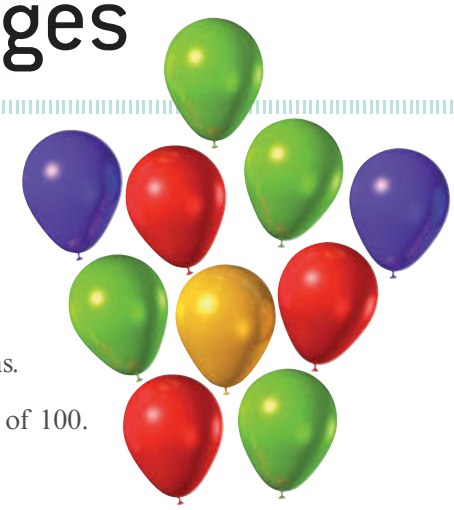
**21** Convert these percentages into fractions in simplest form.

| | | | |
|---|---|---|---|
| **a** $\frac{1}{7}\%$ | **b** $\frac{2}{9}\%$ | **c** $\frac{4}{5}\%$ | **d** $\frac{1}{2}\%$ |
| **e** $\frac{7}{10}\%$ | **f** $\frac{3}{4}\%$ | **g** $\frac{8}{13}\%$ | **h** $\frac{5}{6}\%$ |
| **i** $4\frac{1}{8}\%$ | **j** $7\frac{2}{3}\%$ | **k** $18\frac{1}{2}\%$ | **l** $10\frac{1}{4}\%$ |
| **m** $9\frac{3}{5}\%$ | **n** $8\frac{5}{7}\%$ | **o** $11\frac{1}{6}\%$ | **p** $4\frac{2}{11}\%$ |

**Reflect**

How is the number 100 important when working with percentages?

# 3H Converting between fractions, decimals and percentages



## Start thinking!

1 How many balloons are there in total?

2 How many red balloons are there?

3 Write the number of red balloons as a fraction of the total number of balloons.

4 Write your answer to question **3** as an equivalent fraction with a denominator of 100.

5 Write this fraction as: **a** a percentage **b** a decimal.

6 What do you notice about your answers to question **5**?

7 Complete this table.

8 Write a sentence describing the relationship between percentages and decimals.

9 Stephanie said that 2 out of 10 balloons were purple, Maria said that 20% were purple and Ben said that 0.2 were purple. Explain how they are all correct.

| Balloon colour | Fraction of total | Fraction with a denominator of 100 | Percentage | Decimal |
|---|---|---|---|---|
| Red | | | | |
| Purple | | | | |
| Yellow | | | | |
| Green | | | | |

## KEY IDEAS

- Percentages, decimals and fractions are all closely related.
- To write a percentage as a decimal, write the percentage as a fraction with a denominator of 100 and then write this as a decimal. For example, $23\% = \frac{23}{100} = 0.23$.
- To write a decimal as a percentage, write the decimal as a proportion of a whole amount or 100%. For example, $0.65 = 0.65 \times 1 = 0.65 \times 100\% = 65\%$.
- Alternatively, write the decimal as a fraction with a denominator of 100 and then write this as a percentage. For example, $0.65 = \frac{65}{100} = 65\%$.
- To write a fraction as a decimal or percentage, it is easier to write the fraction with a denominator of 100 first if possible. For example, $\frac{3}{4} = \frac{75}{100} = 0.75 = 75\%$.

A short cut is to divide by 100. For example, $23\% = 23 \div 100 = 0.23$.

A short cut is to multiply by 100. For example, $0.65 = 0.65 \times 100\% = 65\%$.

# EXERCISE 3H Converting between fractions, decimals and percentages

## EXAMPLE 3H-1 Writing a percentage as a decimal

Write 37% as a decimal.

**THINK**

1 Write 37% as a fraction.

2 Divide the numerator (37) by the denominator (100).

3 Write your answer. Show a digit before the decimal point. There are zero ones, so write 0.

**WRITE**

$37\% = \frac{37}{100}$

$= 37 \div 100$

$= 0.37$

**1** Write each percentage as a decimal.

| | | |
|---|---|---|
| **a** 46% | **b** 13% | **c** 99% |
| **d** 25% | **e** 20% | **f** 50% |
| **g** 5% | **h** 8% | **i** 1% |

## EXAMPLE 3H-2 Writing a decimal percentage as a decimal

Write 6.25% as a decimal.

**THINK**

1 Write 6.25% as a fraction.

2 Divide the numerator (6.25) by the denominator (100). A short cut to dividing by 100 is to 'move' the decimal point two places to the left.

3 Insert a placeholder zero in the 'empty' space (tenths place).

4 Write your answer. Show a digit before the decimal point.

**WRITE**

$6.25\% = \frac{6.25}{100}$

$= 6.25 \div 100$

$= .625$

$= .0625$

$= 0.0625$

**2** Write each percentage as a decimal.

| | | |
|---|---|---|
| **a** 23.84% | **b** 19.65% | **c** 46.7% |
| **d** 3.09% | **e** 567.4% | **f** 0.467% |
| **g** 12.895% | **h** 73.28% | **i** 200.5% |
| **j** 10.92% | **k** 404.04% | **l** 0.0101% |

## EXAMPLE 3H-3 Writing a decimal as a percentage

Write each decimal as a percentage. **a** 0.36 **b** 0.872

**THINK**

**a** **1** Write 0.36 as a proportion of a whole amount (0.36 × 1). A whole amount can also be shown as 100%.

**2** Multiply 0.36 by 100. A short cut is to 'move' the digits two places left, or the decimal point two places right.

**b** **1** Write 0.872 as a proportion of a whole amount (0.872 × 1). A whole amount can also be shown as 100%.

**2** Multiply 0.872 by 100. A short cut is to 'move' the decimal point two places right.

**WRITE**

**a** 0.36 = 0.36 × 1
= 0.36 × 100%
= 36%

**b** 0.872 = 0.872 × 1
= 0.872 × 100%
= 0.872
= 87.2%

**3** Write each decimal as a percentage.

| | | | |
|---|---|---|---|
| **a** 0.28 | **b** 0.88 | **c** 0.15 | **d** 0.62 |
| **e** 0.46 | **f** 0.72 | **g** 0.09 | **h** 0.04 |

**4** Write each decimal as a percentage.

| | | | |
|---|---|---|---|
| **a** 0.518 | **b** 9.02 | **c** 0.105 | **d** 0.7 |
| **e** 21.5 | **f** 8.41 | **g** 7.1 | **h** 0.009 |
| **i** 3.001 | **j** 5 | **k** 0.176 54 | **l** 0.000 045 |

## EXAMPLE 3H-4 Writing a fraction as a percentage

Write each fraction as a percentage by first converting to a decimal.
**a** $\frac{5}{8}$ **b** $\frac{1}{6}$ (correct to two decimal places)

**THINK**

**a** **1** Write the fraction as a decimal by dividing the numerator by the denominator.

**2** Write the decimal as a percentage.

**3** Multiply 0.625 by 100.

**b** **1** Write the fraction as a decimal. The decimal is recurring, so have the pattern repeat to at least five decimal places.

**2** Write the decimal as a percentage.

**3** Multiply 0.166 66... by 100.

**4** Round to two decimal places.

**WRITE**

**a** $\frac{5}{8} = 5 \div 8$
= 0.625
= 0.625 × 100%
= 62.5%

**b** $\frac{1}{6} = 1 \div 6$
= 0.166 66 …
= 0.166 66 … × 100%
= 16.666 …%
= 16.67%

UNDERSTANDING AND FLUENCY

**5** Write each fraction as a percentage by first converting to a decimal.

**a** $\frac{3}{8}$ **b** $\frac{1}{4}$ **c** $\frac{7}{16}$ **d** $\frac{21}{40}$

**e** $\frac{7}{8}$ **f** $\frac{5}{16}$ **g** $\frac{6}{125}$ **h** $\frac{9}{80}$

**6** Write each fraction as a percentage correct to two decimal places.

**a** $\frac{1}{3}$ **b** $\frac{3}{7}$ **c** $\frac{5}{6}$ **d** $\frac{9}{11}$

**e** $\frac{5}{9}$ **f** $\frac{5}{7}$ **g** $\frac{8}{13}$ **h** $\frac{7}{12}$

**7** Check your answers to questions **5** and **6** with a calculator.

**8** Eclectus parrots are found in north-eastern Australia. The male is green and the female is red and blue.



**a** Write the number of male parrots pictured as a fraction of the total number of parrots.

**b** What percentage of the group is:

**i** male? **ii** female?

**c** Write each answer to part **b** as a decimal.

**9** Copy and complete the table at right to show the equivalent forms of each amount.

| Fraction | Decimal | Percentage |
|---|---|---|
| $\frac{1}{2}$ | | |
| | 0.25 | |
| | | 75% |
| $\frac{1}{3}$ | | |
| | 0.125 | |
| | | 62.5% |
| $\frac{1}{5}$ | | |
| | 0.4 | |
| | | 60% |

PROBLEM SOLVING AND REASONING

**10** Lachlan scored 18 out of 25 on his first test and 23 out of 30 for his next test.

**a** Calculate what percentage he scored for his first test.

**b** Calculate what percentage he scored for his second test.

**c** Which test did Lachlan perform better on? Explain your answer.

**11** Create your own incomplete table like the one in question **9** with fraction, decimal and percentage equivalents of given amounts. Swap with your classmates and work out the missing values.

**12** What proportion of this season's sunglass collection has been sold?



Write the answer as a:

**a** fraction (in simplest form)

**b** decimal

**c** percentage.



**13** Melbourne's total water storage in December 2000 was 62.2% of full capacity. However, in December 2009 it had dropped to 37.8%. Calculate the difference between these two values as a:

**a** percentage

**b** fraction (in simplest form)

**c** decimal.

**14** During the stocktake sale, the price of certain items are reduced by 47%.

**a** Write this percentage as a fraction.

**b** What fraction is an item being sold for?

**15** The elements that make up the Sun are listed in the table.

| Element | Percentage of total mass |
|---|---|
| H (hydrogen) | 71.0% |
| He (helium) | 27.1% |
| O (oxygen) | 0.97% |
| C (carbon) | 0.40% |
| Si (silicon) | 0.099% |
| N (nitrogen) | 0.096% |
| Mg (magnesium) | 0.076% |
| Ne (neon) | 0.058% |
| S (sulphur) | 0.040% |
| Fe (iron) | 0.014% |

**a** Write the percentage of H as a fraction and a decimal.

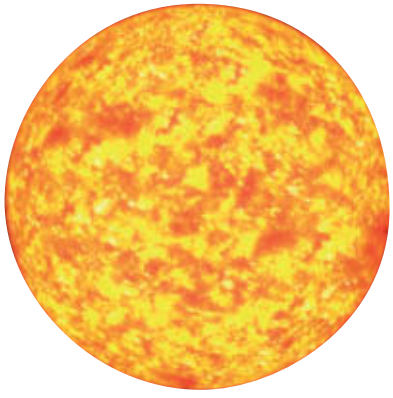
**b** If Fe and Mg are metals, find how much of the Sun is made of metal and write it as a:

**i** percentage

**ii** decimal

**iii** fraction.

**c** What do the percentages for the listed elements add to? Why do you think this might be?



**16** Eliza had cycled 496 km of the gruelling 620 km Great Victorian Bike Ride. What percentage of the ride did she still have to complete?



**17** **a** Write three fractions between 0.1 and 11%.

**b** Write three percentages between 0.716 and $\frac{22}{30}$.

**c** Write three decimals between $\frac{85}{96}$ and 89%.

CHALLENGE

**18** Write each percentage as a decimal.
(Hint: change the fractional part into a decimal first and then convert the percentage to a decimal.)

For example,
$37\frac{1}{2}\% = 37.5\% = 0.375$.

**a** $12\frac{1}{5}\%$ **b** $28\frac{1}{4}\%$ **c** $5\frac{7}{10}\%$ **d** $20\frac{6}{25}\%$

**e** $17\frac{13}{50}\%$ **f** $8\frac{7}{20}\%$ **g** $75\frac{1}{2}\%$ **h** $21\frac{4}{5}\%$

**19** Laura and Rosie were comparing their working of a particular problem. They were required to convert $17\frac{1}{8}\%$ to a decimal. Their setting out is slightly different; however, both methods produce the same result. Comment on whether the answer and setting out of each is correct.

***Laura***

$$17\frac{1}{8}\% = 17.125\% = \frac{17.125}{100} = \frac{17.125}{100} \times \frac{1000}{1000} = \frac{17125}{100\,000} = 0.171\,25$$

***Rosie***

$$17\frac{1}{8}\% = 17.125\% = \frac{17.125}{100} = 17.125 \div 100 = 0.171\,25$$

**20** Nick is able to hit the centre of a dartboard with an accuracy rate of 0.165, while Phil's accuracy rate is $15\frac{3}{8}\%$. Who has a greater chance of hitting the centre on their next attempt?

**21** Write each list of amounts in descending order.

**a** 12.8%, $\frac{9}{7}\%$, $\frac{6}{13}$, 0.249, $\frac{3}{7}$, 0.4231

**b** $385\frac{2}{3}\%$, 3.826, $3\frac{7}{8}$, 388%, 3.87, $3\frac{9}{11}$

**Reflect**

How are decimals, percentages and fractions related?

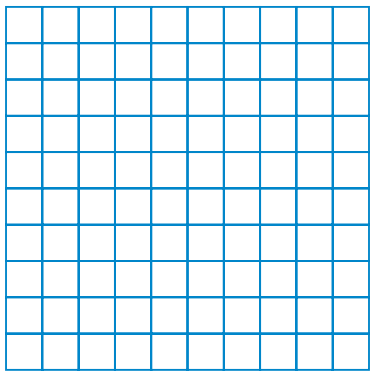
# 3I Calculating percentages

## Start thinking!

The price of this surfboard is to be discounted by 15%.
Think about how you can work out the discount as an amount of money.




**1** Copy this 10 × 10 grid.



- **a** Shade 15% of the grid. Remember that 15% means '15 out of every 100'.
- **b** How many of the smaller squares does 15% represent?
- **c** Copy and complete this statement: 15% of 100 equals _____.

**2** **a** Draw two more 10 × 10 grids and show what 15% of 200 is equal to.

- **b** What is 15% of 200?

**3** **a** Explain how you could show 15% of 600 using 10 × 10 grids.

- **b** What is 15% of 600?

The statement '15% of 600 equals' can be written as the multiplication problem $\frac{15}{100} \times 600 =$.

**4** Calculate $\frac{15}{100} \times 600$. Compare the result to your answer for question **3b**.

**5** What dollar amount is the surfboard discounted by?

**6** What is the sale price of the surfboard?

## KEY IDEAS

- To find the percentage of an amount, follow these steps.
  1. Convert the percentage to a fraction.
  2. Replace 'of' with a multiplication sign.
  3. Write the amount as a fraction with a denominator of 1.
  4. Work out the calculation and write the final answer in the correct form.

# EXERCISE 3I Calculating percentages

## EXAMPLE 3I-1 Calculating a percentage of an amount

Calculate 12% of $160.

**THINK**

1 Write the percentage as a fraction and replace 'of' with '×'. Write the dollar amount as a fraction with a denominator of 1.

2 Cancel 100 and 160 by dividing both numbers by 20 (shown in green).

3 Multiply the numerators together and then multiply the denominators together.

4 Divide the numerator by the denominator to convert the fraction to a decimal (correct to two decimal places). Write your final answer in dollars.

**WRITE**

12% of $160

$= \frac{12}{100} \times \frac{160}{1}$

$= \frac{12}{\cancel{100}_{5}} \times \frac{\cancel{160}^{8}}{1}$

$= \frac{12}{5} \times \frac{8}{1}$

$= \frac{96}{5}$

$= 96 \div 5$

$= \$19.20$

UNDERSTANDING AND FLUENCY

1 Calculate each of these.

a 5% of 3900
b 10% of 120
c 20% of 520
d 50% of 80
e 15% of 50
f 90% of 20
g 60% of 65
h 75% of 16
i 25% of 400
j 80% of 600
k 40% of 500
l 100% of 132

2 Calculate each of these.

a 12% of $40
b 17% of $450
c 11% of $500
d 15% of $1326
e 8% of $8750
f 9% of $1870
g 22% of $6750
h 20% of $19.90
i 25% of $35.80



## EXAMPLE 3I-2 Calculating a decimal percentage of an amount

Calculate 7.5% of 330.

**THINK**

1 Write the percentage as a fraction and replace 'of' with '×'. Write the amount as a fraction with a denominator of 1.

2 Cancel 100 and 330 by dividing both numbers by 10 (shown in green).

3 Multiply the numerators together and then multiply the denominators together.

4 Divide the numerator by the denominator to convert the fraction to a decimal.

**WRITE**

$$7.5\% \text{ of } 330$$
$$= \frac{7.5}{100} \times \frac{330}{1}$$
$$= \frac{7.5}{\cancel{100}^{10}} \times \frac{\cancel{330}^{33}}{1}$$
$$= \frac{7.5}{10} \times \frac{33}{1}$$
$$= \frac{247.5}{10}$$
$$= 24.75$$

**3** Calculate each amount. Write your answer as a decimal.

| | | |
|---|---|---|
| **a** 2.5% of 180 | **b** 5.5% of 70 | **c** 75.8% of 120 |
| **d** 20.4% of 150 | **e** 42.8% of 65 | **f** 17.5% of 1650 |
| **g** 5.3% of 960 | **h** 16.8% of 150 | **i** 85.2% of 95 |
| **j** 17.6% of 120 | **k** 12.5% of 99 | **l** 10.2% of 78 |

## EXAMPLE 3I-3 Calculating a percentage and writing the result as a fraction

Calculate 15% of 63.

**THINK**

1 Write the percentage as a fraction and replace 'of' with '×'. Write the amount as a fraction with a denominator of 1.

2 Cancel 15 and 100 by dividing both numbers by 5 (shown in green).

3 Multiply the numerators together and then multiply the denominators together.

4 Convert the improper fraction to a mixed number.

**WRITE**

$$15\% \text{ of } 63$$
$$= \frac{15}{100} \times \frac{63}{1}$$
$$= \frac{\cancel{15}^{3}}{\cancel{100}^{20}} \times \frac{63}{1}$$
$$= \frac{3}{20} \times \frac{63}{1}$$
$$= \frac{189}{20}$$
$$= 9\frac{9}{20}$$

UNDERSTANDING AND FLUENCY

**4** Calculate each amount. Write your answer as a fraction.

| | | |
|---|---|---|
| **a** 7% of 980 | **b** 56% of 120 | **c** 16% of 360 |
| **d** 12% of 120 | **e** 21% of 320 | **f** 85% of 950 |
| **g** 29% of 40 | **h** 35% of 125 | **i** 63% of 225 |
| **j** 49% of 170 | **k** 92% of 320 | **l** 45% of 225 |

**5** A survey showed 92% of commuters were not satisfied with public transport.

**a** If 6500 commuters took part in the survey, how many people were not satisfied with the public transport system?

**b** How many of the 6500 commuters were satisfied with the system?

**6** Calculate each of these.

| | | |
|---|---|---|
| **a** 110% of 40 | **b** 105% of 70 | **c** 300% of 15.2 |
| **d** 250% of 160 | **e** 120% of 8.9 | **f** 460% of 98 |
| **g** 165% of 90 | **h** 220% of 140 | **i** 180% of 630 |

PROBLEM SOLVING AND REASONING

**7** This laptop has a recommended price of $2199. A store has a sale offering 30% off.




**a** Calculate the value of the discount.

**b** Subtract this from the original price to find the sale price of the item.

**c** If you could only spend $1500 on a new laptop, would you be able to buy this?

**8** A camping store has discounted its tents by 20%.

**a** Calculate the discount on a tent that normally retails at $570.

**b** What is the sale price of the tent?

**9** Look at the percentage problems and your answers to question **6**.

**a** What you notice about each percentage?

**b** Compare each answer to the corresponding original amount.

**10** Sevi checks two stores for a new enclosure for her pet snake. 'Ripper Reptiles' has enclosures that were originally priced at $600 discounted by 18%. 'Enclosed' stocks them for $650 discounted by 25%.



**a** What is the value of the discount at each store?

**b** What is the new price of the enclosure at each store?

**c** Which store should Sevi buy the enclosure from?

**11** Angus read that the human body contains 65% water, 18% protein, 10% fat, 6% minerals and vitamins and 1% carbohydrate. Find the mass of each component in people of the following weights.

**a** 45 kg **b** 63 kg **c** 98 kg

**12** A food store decides the prices of items by marking up (or increasing) the wholesale price by 25% and then rounding up to the next 5 cents. Calculate the final price of items with these wholesale prices.

**a** \$2.99 **b** \$1.02 **c** \$5.85
**d** \$7.34 **e** \$1.37 **f** 95c

**13** Many items that you buy include a goods and service tax (GST). This means that 10% is added to the cost of most goods and services.
The items listed have not been taxed yet. For each item:

**i** find the amount of GST to be added
**ii** find the price it will be sold for.

**a** \$1199 Notebook
**b** \$209 calculator
**c** \$499 Nintendo Wii console
**d** \$2.99 packet of biscuits
**e** \$29.99 DVD
**f** \$54 backpack

**14** From question **13**, what do you notice about the amount of GST in relation to the original price? Write a sentence describing a shortcut to finding 10% of a price.

**15** Each of these quantities is to be decreased by a given percentage. Calculate:

**i** the amount each value has been decreased by
**ii** the new value.

**a** \$580 by 12%
**b** \$4500 by 25%
**c** 42.2 km by 5%
**d** 8.6 L by 10%
**e** 148 kg by 15%
**f** 176 kg by 20%



**16** This snorkelling set has been reduced by 30%. If the original price was \$79.98, what is the sale price, correct to the nearest cent?

**17** Explain why it is possible to have a 200% increase in price, but not a 200% reduction in price.

**18** In the past year, the price of a 1-L carton of milk has increased by 6%.

**a** If the price was \$2.50 last year, what amount has it increased by?
**b** What is the new price of a 1-L carton of milk?
**c** Write the new price as a fraction of the previous year's price, in simplest form.
**d** Write the fraction found in part **c** as a percentage.

PROBLEM SOLVING AND REASONING

**19** Andy was required to mark prices up by 25%; however, some of the final prices are incorrect. Check Andy's prices, comment on how the mistakes may have occurred and what the correct values should be.

| | Wholesale price | Marked price |
|---|---|---|
| **a** | $4.99 | $6.20 |
| **b** | $2.06 | $2.60 |
| **c** | $2.89 | $3.60 |
| **d** | $3.70 | $3.70 |
| **e** | $0.80 | $20.80 |

**20** Meredith is impressed at how quickly her mother can mentally calculate percentages. One of her mother's strategies is shown below.

> To obtain 10%, divide the amount by 10.
> This is because 10% can be written as $\frac{1}{10}$.
> So 10% of 72 is 7.2 (since 72 ÷ 10 = 7.2).

You can extend this idea to find other values. For example, to find 20% of an amount, find 10% and then double the value. Use this logic to copy and complete these sentences.

**a** To find 5% of an amount, first obtain 10% and then _____ the value.
**b** To find 30% of an amount, first obtain 10% and then _____ the value.
**c** To find 40% of an amount, first obtain 10% and then _____ the value.

**21** Explain how you would obtain 1% of an amount and 50% of an amount.

CHALLENGE

**22** Question **15** requires two calculations to find the final reduced value. When only the new (reduced) value is required, an alternative method can be used. Consider the tent in question **8**. It had been reduced by 20% so the percentage to be paid is 80%. (100% − 20% = 80%.)

$$\begin{aligned}\text{Sale price} &= 80\% \text{ of } \$570 \\ &= \frac{80}{100} \times \frac{570}{1} \\ &= 8 \times 57 \\ &= \$456\end{aligned}$$

Use the method shown in the box to obtain the reduced values in question **15**. How do these values compare with your original answers to question **15**?

**23** Use a similar strategy to that shown in question **22** to explain to a friend how to calculate the new price of an item after GST has been added.

**24** Work with a partner and use the results obtained in questions **20** and **21** to devise a strategy to calculate each of these percentages of an amount.

**a** 15% **b** 13%
**c** 55% **d** 80%

**Reflect**

Why are percentages useful?

# CHAPTER REVIEW

## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

| | | | |
|---|---|---|---|
| decimal number | decimal place | trailing zeros | recurring decimal |
| decimal fraction | rounding | best buys | percentage |
| place value | placeholder zeros | terminating decimal | discount |

## MULTIPLE-CHOICE

3A **1** What is 13.054 872 6 rounded to three decimal places?

A 13.0 B 13.054
C 13.055 D 13.0549

3A **2** Which statement is false?

A $12.345 > 12.309$
B $183.506 < 183.099$
C $48.2189 < 48.950$
D $22.4 > 22.056$

3B **3** Which has the smallest answer?

A $1.568 + 6.238 - 4.22$
B $1.456 + 0.853 + 1.0899$
C $10.9856 - 7.53 + 2.2$
D $11.236 - 4.2689 - 3.0501$

3C **4** What is $6.04 \times 2.1$?

A 1.2684 B 12.684
C 126.84 D 1268.4

3D **5** What is $8.21 \div 4$?

A 2.0525 B 2.21
C 4.21 D 32.84

3D **6** If 10 apples cost $9.85, how much would six apples cost?

A $0.985 B $1.64
C $5.91 D $16.42

3E **7** Which has the largest answer?

A $23 \div 0.4$ B $48 \div 1.2$
C $60.5 \div 1.1$ D $15 \div 0.3$

3F **8** What is $\frac{1}{5}$ written as a decimal?

A 0.2 B 0.5 C 1.5 D 0.15

3F **9** What is 0.26 as a fraction in simplest form?

A 0.26 B $\frac{13}{50}$ C $\frac{26}{100}$ D 26%

3G **10** What percentage of this figure is shaded?



A $\frac{5}{8}$% B 5% C 50% D 62.5%

3G **11** What is 48% as a fraction in simplest form?

A $\frac{48}{100}$ B $\frac{24}{50}$ C 0.48 D $\frac{12}{25}$

3G **12** What is $\frac{9}{20}$ written as a percentage?

A 9% B $\frac{9}{20}$% C 45% D 180%

3H **13** The fraction $\frac{7}{8}$ can also be written as:

A 78% B 7.8 C 0.75 D 87.5%

3H **14** What is 64.5% written as a decimal?

A 0.645 B 64.5
C 0.0645 D 64500

3H **15** What is 0.1265 written as a percentage?

A 0.1265% B 12.65%
C 1265% D 126500%

3I **16** What is 25% of 220?

A 5500 B 245 C 195 D 55

## SHORT ANSWER

3A **1** What is the place value of the digit 7 in each number?

a 10.578 b 2.075
c 0.000 517 d 16.794

3A **2** For each number:

i write the number of decimal places
ii round to two decimal places.

a 1589.231 57 b 12.709 66
c 0.9286 d 152.875 964 3

3A **3** Place this list in descending order.
12.5089, 12.8905, 18.0958, 12.0985, 10.9852

3B **4** Calculate:

a 23.25 + 84.15 + 13.06
b 1.025 + 3.508 + 4.999
c 14.562 − 12.869
d 5.239 − 2.506 + 1.5555

3C **5** Calculate:

a 12.63 × 100 b 15.3298 × 10
c 4.2 × 50 d 3.047 × 200

3C **6** Calculate:

a 0.7 × 0.3 b 0.9 × 0.04
c 1.1 × 0.08 d 0.08 × 0.04

3C **7** Calculate:

a 25.8 × 0.1 b 46.8 × 0.01
c 6.5 × 7.2 d 4.23 × 1.9

3D **8** Calculate:

a 78.94 ÷ 10 b 1256.359 ÷ 100
c 124.56 ÷ 10 d 189 765.6 ÷ 1000

3D **9** Calculate:

a 18.654 ÷ 2 b 94.56 ÷ 6
c 148.63 ÷ 5 d 327.45 ÷ 8

3D **10** Evie considers two packs of cheese: 500 g for $7.80 and 700 g for $12.60.

a Calculate the value of 100 g of each product.
b Which product is the best buy? Explain.

3E **11** Calculate:

a 6.2 ÷ 0.5 b 0.75 ÷ 0.2
c 8.62 ÷ 0.4 d 0.0081 ÷ 0.05

3F **12** Write each decimal as a fraction in simplest form.

a 0.75 b 0.18 c 0.489 d 0.003

3F **13** Write each fraction as a decimal.

a $\frac{87}{100}$ b $\frac{117}{1000}$ c $\frac{3}{10}$ d $\frac{7}{100}$

3F **14** Write each fraction as a decimal.

a $\frac{16}{25}$ b $\frac{19}{50}$ c $\frac{15}{20}$ d $\frac{3}{8}$

3G **15** Write each percentage as a fraction:

i with denominator of 100
ii in simplest form.

a 49% b 22% c 75% d 5%

3G **16** What percentage of a pie remains if Caleb eats 23% and Hogan eats 31%?

3H **17** Write each decimal as a percentage.

a 0.56 b 0.08 c 1.79 d 0.2748

3H **18** Write each percentage as a decimal.

a 78% b 46.5%
c 9% d 10.679%

3H **19** Write each number as a percentage.

a $\frac{3}{5}$ b $\frac{7}{8}$ c 1 d $2\frac{3}{4}$

3I **20** Calculate each amount.

a 10% of 450 b 30% of 500
c 45% of 300 d 85% of 650
e 6% of 1499 f 41% of 347

3I **21** Calculate each amount.

a 272% of 550 km
b 34.75% of 950 g
c 5.8% of $120
d 16.2% of 450 L

## NAPLAN-STYLE PRACTICE

Questions 1 and 2 refer to 164.506 483 2.

**1** What is the place value of the digit 0?

- ◯ hundreds
- ◯ tens
- ◯ tenths
- ◯ hundredths

**2** Round the number to two decimal places.

**3** Which list is written in ascending order?

- ◯ 1.26, 1.623, 1.86, 1.88
- ◯ 2.1, 2.89, 2.506, 2.4896
- ◯ 8.0175, 8.454, 9.9, 8.64
- ◯ 3.84, 3.521, 3.1133, 3.005

**4** What is 8.605 – 2.077?

**5** Which is the correct setting out for the sum of 1.3, 4.06 and 0.008?

- ◯
  $$\begin{array}{r} 1.3 \\ 4.6 \\ +\ 0.8 \\ \hline \end{array}$$
- ◯
  $$\begin{array}{r} 1.3 \\ 4.06 \\ +\ 0.008 \\ \hline \end{array}$$
- ◯
  $$\begin{array}{r} 1.03 \\ 4.06 \\ +\ 0.008 \\ \hline \end{array}$$
- ◯
  $$\begin{array}{r} 1.300 \\ 4.060 \\ +\ 0.008 \\ \hline \end{array}$$

**6** Andrew, James and Mark want to buy a new kettle. If Andrew has $16.55, James has $13.20 and Mark has $12.60, how much more money do they need to buy this kettle for $49?

$

**7** What is 1.26 × 0.55?

- ◯ 0.693
- ◯ 0.0693
- ◯ 1.2655
- ◯ 1.81

**8** What is the cost of 3 m of material if the price is $5.99 per metre?

**9** What is 0.548 ÷ 0.2?

- ◯ 0.1096
- ◯ 0.274
- ◯ 1.096
- ◯ 2.74

**10** If eight DVDs cost $47.92, how much do ten DVDs cost?

**11** Which option to buy tea bags is better value?

- ◯ 10 for $1.77
- ◯ 25 for $2.98
- ◯ 50 for $4.08
- ◯ 100 for $7.48

**12** Which option is the best buy for milk?

- ◯ 0.6 L for $1.98
- ◯ 1 L for $2.98
- ◯ 2 L for $5.92
- ◯ 3 L for $8.90

**13** What is $\frac{11}{20}$ as a decimal number?

**14** What is 0.875 as a fraction in simplest form?

- ◯ $\frac{875}{100}$
- ◯ $\frac{825}{1000}$
- ◯ $\frac{175}{200}$
- ◯ $\frac{7}{8}$

Questions 15–17 refer to this information.

Chelsea wants to buy some tights. One store is selling three pairs for $9.99 or $3.99 each. Chelsea has $35.20 and buys as many pairs as she can.

**15** How many pairs of tights can she buy?

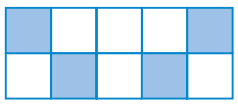
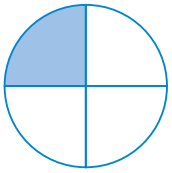
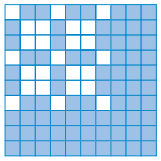
**16** How much money does she have left afterwards?

- ◯ $1.24
- ◯ $5.23
- ◯ $29.97
- ◯ $33.96

**17** On average, how much did each pair cost?

- ◯ $3.00
- ◯ $3.33
- ◯ $3.40
- ◯ $8.49

**18** Which diagram is 40% unshaded?

- ◯ 
- ◯ 
- ◯ 
- ◯ 

**19** An 18-carat gold necklace is made of 75% gold, 12.5% silver and the rest is copper. What percentage of the necklace is copper?

**20** Melinda notices that three out of every ten passing cars are red. What percentage of cars that pass are *not* red?

%

**21** Enrique scored a mark of $\frac{52}{80}$ on his maths test. What is this as a percentage?

**22** What is 59.23% written as a decimal number?

**23** Which list correctly shows the numbers 45%, $\frac{1}{8}$, 0.76, $\frac{4}{5}$, 0.12 and 9% in descending order?

- ☐ $\frac{4}{5}$, 0.76, 45%, $\frac{1}{8}$, 0.12, 9%
- ☐ 0.76, $\frac{4}{5}$, 45%, 0.12, $\frac{1}{8}$, 9%
- ☐ 9%, 0.12, $\frac{1}{8}$, 45%, 0.76, $\frac{4}{5}$
- ☐ 45%, 0.76, $\frac{4}{5}$, 0.12, $\frac{1}{8}$, 9%

**24** What is 35% of $620?

**25** What is the sale price of an item that costs $49.99 before it is discounted by 25%?

- ☐ $12.50
- ☐ $24.99
- ☐ $37.49
- ☐ $1249.75

Questions **26** and **27** refer to this information.

Michelle plans to buy 25 cupcakes. Her local bakery sells cupcakes for $2.10 each. The closest supermarket sells 10 cupcakes for $14.99.

**26** If the bakery has a 30% off everything sale, how much would it cost to buy 25 cupcakes?

- ☐ $52.50
- ☐ $36.75
- ☐ $31.48
- ☐ $15.75

**27** If the supermarket has 20% off bakery items, how much would it cost per cupcake if Michelle bought three packets?

- ☐ $35.98
- ☐ $1.50
- ☐ $1.20
- ☐ $1.05

## ANALYSIS

Rob, Carina, Belinda and Mario want to buy Alicia a bunch of white roses for her birthday.

**a** Rob has $10.35, Carina has $8.25, Belinda has $9.90 and Mario has $10.80. How much money do they have altogether?

**b** Their local florist, Fantastic Flowers, has white roses for $2.99 each. How many can they buy? How much change will they have left over?

Fantastic Flowers has a special offer, that if you buy a dozen white roses you get a 25% discount.

**c** How much would a dozen white roses cost without the discount?

**d** How much would they cost with the discount?

Mario bargains with the storekeeper and manages to buy 18 white roses for a $37\frac{1}{2}$% discount.

**e** Convert this percentage to a decimal.

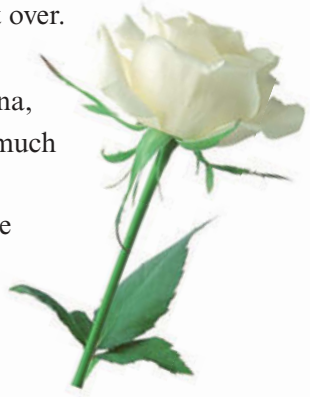
**f** How much would 18 white roses cost without the discount?

**g** What is 37.5% of this amount?

**h** How many decimal places does this number have? Round this number to two decimal places.

**i** Find how much Mario paid for the flowers and the amount of change left over.

**i** If the change was divided evenly between Rob, Carina, Belinda and Mario, how much would they each get? (Round your answer to the nearest cent.)



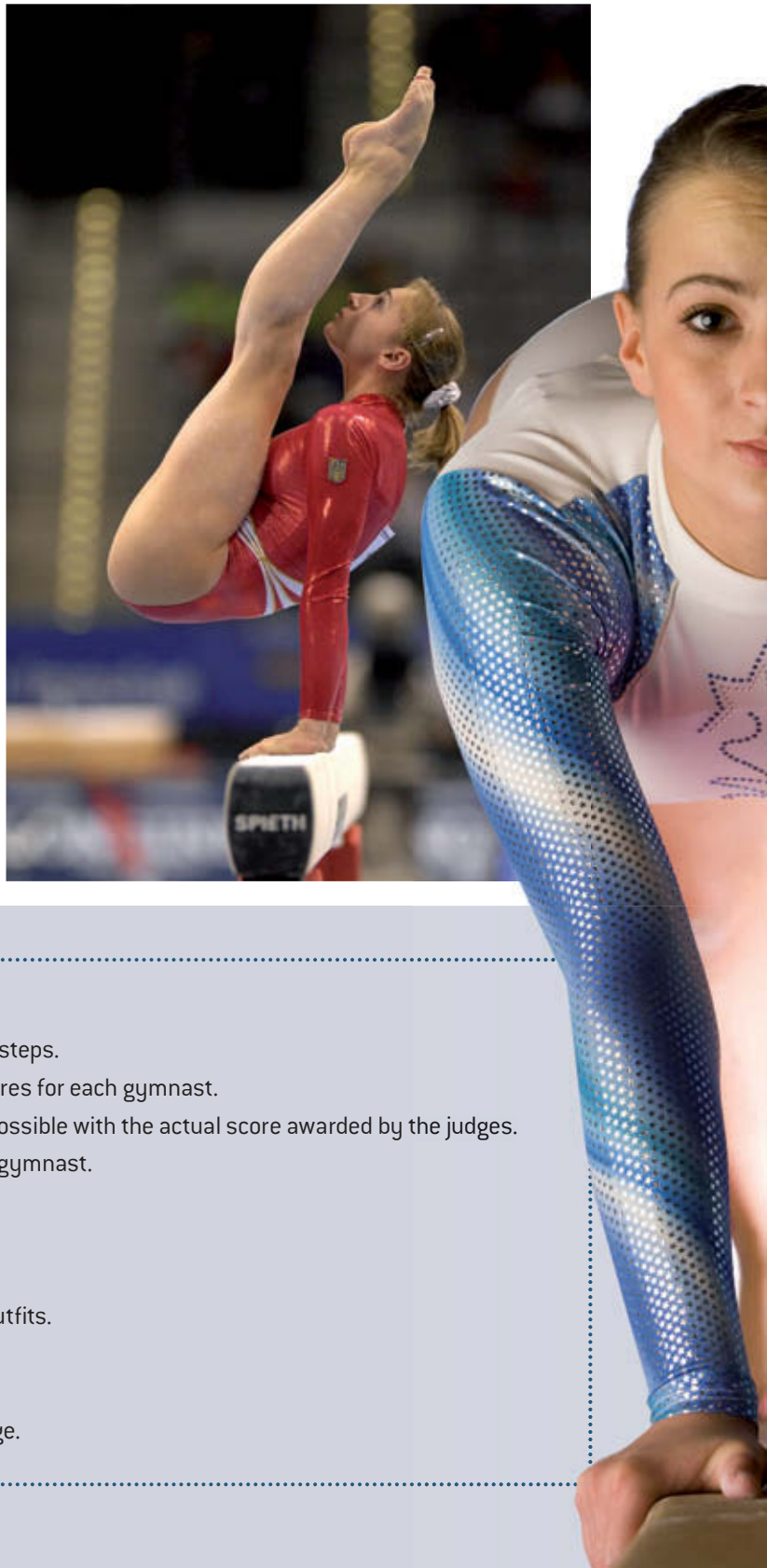
# CONNECT

## And the winner is…

Gymnastics is an exciting sport to watch. However, the scoring system means competitors and spectators must wait for the results to be calculated.

How are decimals used in finding a gymnast's final score?

Gymnasts are rated on how difficult their routine is and their execution (how well they perform it). Two judges decide the degree of difficulty and six independent judges rate the execution of the routine.



### Your task

For your investigation of gymnastics, follow these steps.

- Identify the smallest and largest execution scores for each gymnast.
- Compare the maximum total execution score possible with the actual score awarded by the judges.
- Work out the average execution score for each gymnast.
- Calculate each gymnast's final score.
- Rank each player within the team.
- Calculate the overall team score.
- Calculate the cost of materials for the team's outfits.
- Calculate the scores for the opposing team.
- Decide which team has won the championship.
- Convert each player's final score to a percentage.

Two teams are competing in the championships. These are the results for Team Carello.

| Team Carello | Degree of difficulty | Execution scores | | | | | |
|---|---|---|---|---|---|---|---|
| | | Judge 1 | Judge 2 | Judge 3 | Judge 4 | Judge 5 | Judge 6 |
| Antonia | 2.9 | 9 | 8.5 | 9.7 | 8.8 | 9.6 | 8.6 |
| Francesca | 2.4 | 7 | 7 | 8 | 7.6 | 6.9 | 8.2 |
| Olivia | 2.1 | 6.5 | 7.2 | 8.3 | 7.6 | 6.4 | 6.7 |
| Lorena | 1.9 | 8 | 8.8 | 8 | 8.2 | 7.9 | 8.2 |



These steps describe the scoring system.

1 Determine the degree of difficulty.
2 Obtain an execution score out of 10 from each of the six judges.
3 Discard the highest and lowest of the six execution scores.
4 Find the average of the remaining four scores.
5 Multiply the average of the scores by the degree of difficulty.
6 Add each team member's final score to find the team's total score.

Diving is another sport involving a specific scoring system. A further task is to investigate the scoring system of diving and compare it to the scoring system for gymnastics.



Complete the 3 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a poster
- an Excel spreadsheet
- a PowerPoint presentation
- other (check with your teacher).

# 4 INTEGERS AND THE CARTESIAN PLANE


**4A** Understanding negative numbers

**4B** Adding integers

**4C** Subtracting integers

**4D** Simplifying addition and subtraction of integers

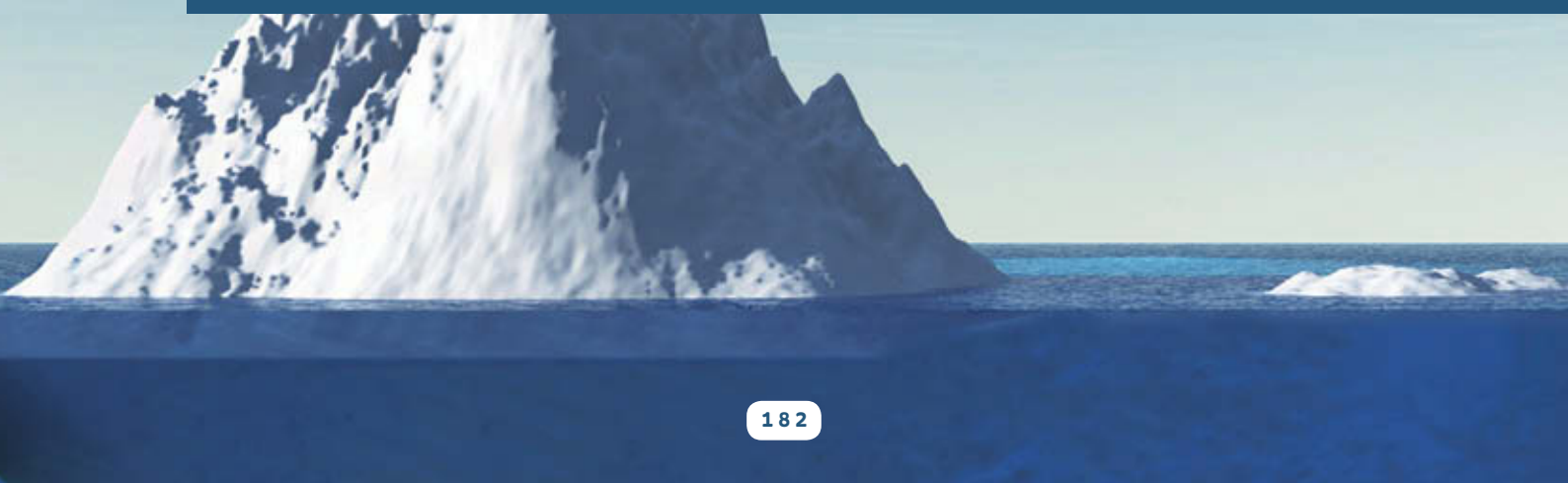
**4E** Introducing the Cartesian plane

**4F** Negative numbers and the Cartesian plane

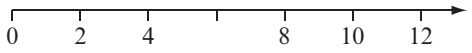
**4G** Interpreting graphs


**ESSENTIAL QUESTION**

*Sea level or the ground floor is often referred to as zero. How could you describe numbers lower or less than zero?*

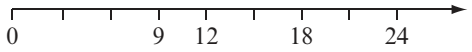


4A **1** Consider this number line.




What number is missing on the scale?

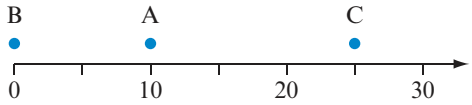
4A **2** Consider this number line.




Which set of numbers could be used to complete the scale?

**A** 2, 3, 13, 19 **B** 3, 6, 15, 21
**C** 3, 6, 16, 22 **D** 6, 8, 15, 22

4A **3** List the positions of the points shown on this number line.




4A **4** Write < (is less than) or > (is greater than) to make these statements true.

**a** 5 ___ 3 **b** 0 ___ 9

4A **5** Which statement is *not* true?

**A** 4 > 14 **B** 8 < 10
**C** 7 > 4 **D** 11 < 21

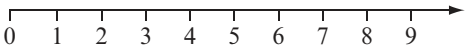
4A **6** Which set of numbers is in order from smallest to largest?

**A** 3, 8, 15, 7, 21, 11, 25
**B** 3, 7, 8, 11, 15, 21, 25
**C** 3, 7, 8, 15, 12, 21, 25
**D** 3, 6, 7, 11, 20, 17, 25

4A **7** Which set of numbers is in order from largest to smallest?

**A** 0, 5, 10, 15, 20, 25, 30
**B** 30, 25, 20, 10, 15, 0, 5
**C** 0, 25, 20, 15, 10, 5, 30
**D** 30, 25, 20, 15, 10, 5, 0

Questions **8**–**10** refer to the number line below.



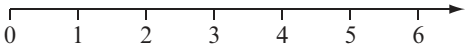

4B **8** What number is 2 units to the right of 6?

**A** 2 **B** 4 **C** 6 **D** 8

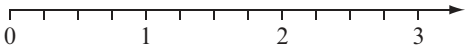
4B **9** What number is 3 units to the left of 5?

4B **10** What number is 4 units to the left of 4?

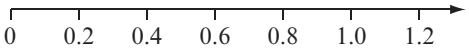
4E **11** Plot the point A at 3 and the point B at 0 on this number line.




4F **12** Plot the point C at $1\frac{1}{2}$ and the point D at $\frac{3}{4}$ on this number line.




4F **13** Plot the point E at 0.8 and the point F at 0.3 on this number line.




4G **14** Dan walks at a steady pace. If he travels 200 m in 4 minutes, what is his speed?

**A** 800 m/min **B** 200 m/min
**C** 100 m/min **D** 50 m/min

4G **15** Calculate the speed of a train that travels 150 km in 3 hours.

# 4A Understanding negative numbers



## Start thinking!

In Antarctica, the average temperature at the South Pole in winter is −60°C and in summer it is around −27°C. The numbers −60 and −27 are called **negative numbers**.

**1** How can you recognise that a number is negative?

**2** Which number do you think is larger: −60 or −27? Give a reason for your answer.

Positive numbers are written with a + sign or no sign in front of the first digit.

**3** Write three different examples of:

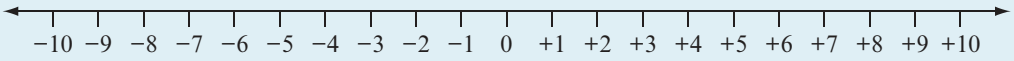
**a** a positive whole number **b** a negative whole number.

**4** Whole numbers that are positive, negative or zero are called **integers**.

Write three different examples of an integer.

## KEY IDEAS

- Negative numbers are always shown with a negative or minus sign (−) directly in front of the first digit; for example, −37. This is read as negative thirty-seven or minus thirty-seven.
- Positive numbers are shown with a positive or plus sign (+) or no sign in front of the first digit; for example, +42 or 42. This is read as positive forty-two or just forty-two.
- Whole numbers that are negative, positive or zero are called integers.
- Integers can be shown on a number line.

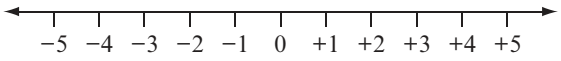



- On a horizontal number line, positive numbers are shown to the right of 0 on the scale and negative numbers are shown to the left of 0.
- Number lines can help you compare the size of integers. A number is larger if it is located further to the right than another number. A number is smaller if it is located further to the left than another number.

# EXERCISE 4A Understanding negative numbers

UNDERSTANDING AND FLUENCY

**1** This number line shows whole numbers from −5 to +5.




**a** From it, write three examples of:
**i** negative whole numbers **ii** positive whole numbers.

**b** Which whole number shown on the number line is neither positive nor negative?

**c** Copy the number line and mark a dot above each of these numbers on the scale.
**i** 0 **ii** 2 **iii** 4 **iv** −5 **v** −2 **vi** −1

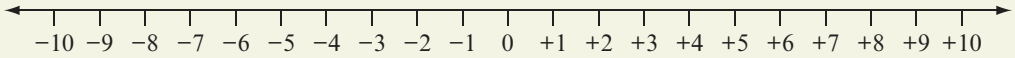
**d** Which number is larger in each pair of numbers? Use the number line to help you.
**i** 2 and 4 **ii** 0 and 2 **iii** 0 and −5 **iv** −2 and 4
**v** −5 and 2 **vi** −2 and −5 **vii** 4 and −1 **viii** −1 and −2

**e** Which number is smaller in each pair of numbers?
**i** 1 and 3 **ii** 0 and 1 **iii** 0 and −4 **iv** −1 and 5
**v** 2 and −2 **vi** −2 and −3 **vii** 3 and −1 **viii** −4 and −3

## EXAMPLE 4A-1 Comparing numbers

Copy and complete each number statement by writing < (is less than) or > (is greater than) in the space provided. Use the number line to help you.

**a** 3 _____ −7 **b** −8 _____ −5




### THINK

**a** **1** Find the two numbers on the number line and compare their positions.

**2** Complete the statement with the correct symbol as read from left to right. (3 is greater than −7.)

**b** **1** Locate the two numbers on the number line and compare their positions.

**2** Complete the statement with the correct symbol. (−8 is less than −5)

### WRITE

**a** 3 is positioned to the right of −7, so 3 is larger than −7.

$3 > -7$

**b** −8 is positioned to the left of −5, so −8 is smaller than −5.

$-8 < -5$

**2** Copy and complete each number statement by writing < (is less than) or > (is greater than) in the space provided.

**a** $-4 ____ 6$ **b** $-5 ____ -9$ **c** $7 ____ 0$

**d** $0 ____ -3$ **e** $-2 ____ -1$ **f** $-8 ____ -9$

**g** $6 ____ -6$ **h** $-7 ____ -1$ **i** $-4 ____ -3$

**3** Which number is smaller in each pair of numbers?

**a** 20 and −25 **b** −17 and −27 **c** 0 and −75

**d** 0 and 38 **e** −69 and −81 **f** −300 and −200

**g** −835 and −853 **h** −2000 and −1000 **i** −444 and −4444

**4** Copy and complete each number statement by writing < or > in the space provided.

**a** $-40 ____ 30$ **b** $0 ____ -22$ **c** $-65 ____ -55$

**d** $78 ____ -87$ **e** $127 ____ -134$ **f** $-500 ____ 0$

**g** $-120 ____ -170$ **h** $248 ____ -300$ **i** $-362 ____ -326$

**5** Write each list of integers in ascending order (from smallest to largest).

**a** 4, −6, 6, −2, 0 **b** −13, 10, −3, 4, −15 **c** −1, −2, 1, −11, −12

**d** 16, −18, 10, −5, 5, −9 **e** −20, 35, 0, −45, −1, 3 **f** −7, −77, 57, 7, −37, −17

**6** Most thermometers have scales that show positive and negative temperatures.

**a** What is the temperature shown on this thermometer?

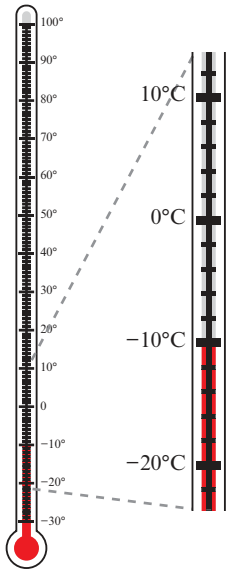
**b** Is each temperature listed below higher or lower than the temperature shown on the thermometer?

**i** 8°C **ii** 0°C **iii** −15°C **iv** −8°C

**c** Explain how you can tell whether a number is larger or smaller on the thermometer scale.

**d** Write three different values for temperatures that, compared to the one shown, are:

**i** larger **ii** smaller.




**7** List all the integers between −5 and +7.

**8** List three integers that are between −45 and −25.

**9** List five integers that are between −30 and 30. Write them in descending order.

**10** Write an integer to represent each situation.

**a** The temperature at a ski resort is 3 degrees below zero.

**b** You have $360 in the bank.

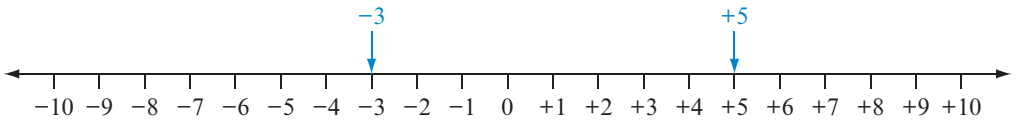
**c** The lift stops at the second floor below the ground floor.

**d** Lake Eyre is 15 m below sea level.

**e** The top of Mt Kosciuszko is 2230 m above sea level.

**f** Your bank account is overdrawn by $28.

**11** The reference point on this number line is 0. Each integer can be described in relation to this reference point. The integer −3 is 3 units to the left of 0 and the integer +5 (or just 5) is 5 units to the right of 0.




**a** Describe the position of each integer from 0 on the number line.

**i** −6 **ii** +8 **iii** +2

**iv** −2 **v** −4 **vi** +4

**b** Which pairs of integers in part **a** are the same number of units away from 0 but on opposite sides of the number line? These integers are called **opposite integers**.

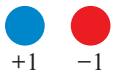
**c** What is the opposite of each of these numbers?

**i** −9 **ii** +7 **iii** −25

**iv** 10 **v** +99 **vi** −53

**d** List three different pairs of opposite integers.

**12** One way to model integers is to use coloured counters.
Use one blue counter to represent +1 and one red counter to represent −1 (any two colours will do).




Collect 10 blue counters and 10 red counters.

**a** To model positive numbers, use the blue counters. How many blue counters would you use to model +2?

**b** What integer is modelled in each diagram?

**i** 

**ii** 

**iii** 

**iv** 

**c** Use the counters to model each integer listed below. Describe your model in each case.

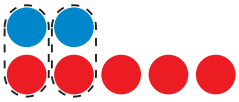
**i** +4 **ii** −3 **iii** +7 **iv** −8

**v** −2 **vi** +5 **vii** −9 **viii** +10

**d** Think about how you could use the counters to model the number 0. Explain why having one blue counter and one red counter together models 0 and is called a zero pair. (Hint: think about opposite integers.)

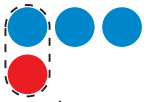

zero pair

**e** Explain how this diagram models −3.

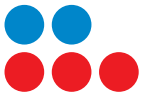

zero pair zero pair

**f** What integer is modelled in each diagram? (Remember to make zero pairs.)

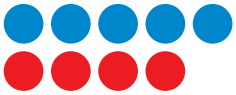
**i**



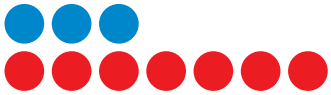

**ii**



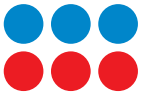
**iii**



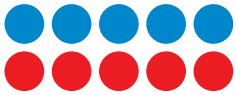
**iv**



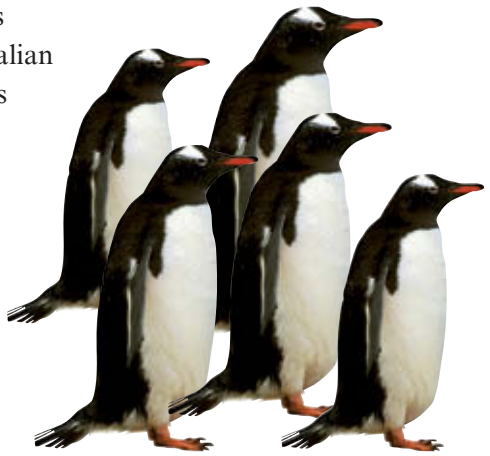
**v**



**vi**



**13** A group of King penguins and Gentoo penguins is living in a specially designed enclosure at an Australian aquarium. The temperature of their environment is controlled between −10°C and −5°C.



**a** Show these two temperatures on a number line.

**b** Which temperature is lower?

**c** The pool contains water kept at a temperature of −7°C. Write the three temperatures in ascending order (from smallest to largest).

**d** If the air temperature in the enclosure is −9°C, would it be warmer or cooler to a penguin leaving the pool?

**14** Pressing a button on the panel inside a lift allows people to travel to different floors in a building.




**a** How many levels does this building have?

**b** What is the highest level you can travel to in the lift?

**c** What is the lowest level you can travel to in the lift?

**d** If you enter the lift at ground level (Level 0) and press the button labelled 3, what do you expect to happen?

**e** If you enter the lift at ground level (Level 0) and press the button labelled −2, what do you expect to happen?

**f** Is Level 1 higher or lower than Level −1? Give a reason for your answer.

**g** Which buttons move you the same number of levels from ground level but in opposite directions?

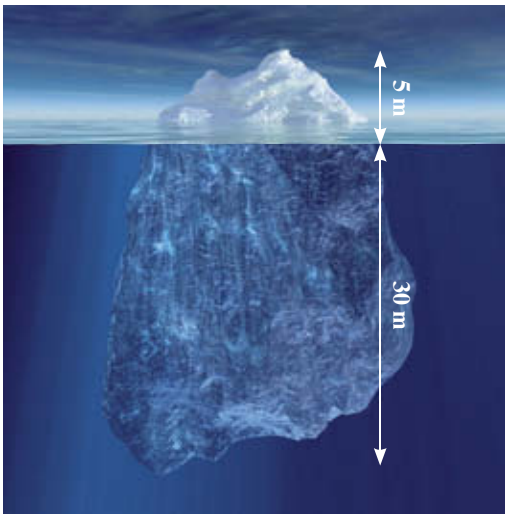
**h** Why do you think the buttons have been labelled as they are? Explain why this lift panel has used negative numbers for some of the buttons. (Hint: ground level, Level 0, is the reference level.)

PROBLEM SOLVING AND REASONING

**15** Automatic teller machines (ATMs) allow you to deposit and withdraw money from your bank account.

- **a** Katie checks her bank balance and sees that she has $100 in her account. Write this amount as a positive integer.
- **b** Katie's bank allows her account to be overdrawn. This means that she can withdraw more money than she actually has in her account. If she withdraws $120 at an ATM, how much does she owe the bank?
- **c** Write her new bank balance as a negative integer.

**16** In 1912, the luxury liner *Titanic* sank on its maiden voyage after its hull was damaged by an iceberg, which lay mostly under water. For this question, the water surface or sea level can be chosen as the reference value of 0.




- **a** The top of an iceberg reaches a height of 5 m above sea level. Write this value as a positive integer.
- **b** The iceberg extends 30 m below sea level. Write this value as a negative integer.
- **c** How tall is this iceberg? (Hint: find the vertical distance from top to bottom.)
- **d** The wreck of the *Titanic* now lies 3800 m below sea level on the sea floor. Write this value as an integer.

**17** In financial terms, you are said to be 'in the black' if you have money and 'in the red' if you owe money. Some students either have money in their accounts or owe money to a bank in the following amounts.

| Student | Financial status | Amount |
|---|---|---|
| Paulo | red | $85 |
| Kamilla | black | $98 |
| Amad | black | $135 |
| Jessica | red | $172 |

- **a** Write each student's bank balance as an integer.
- **b** Who has the most money in the bank?
- **c** Who owes the most money to the bank?

CHALLENGE

**18** Integers can be modelled in many ways. For example, −4 can be modelled with four red counters, or five red counters and one blue counter, or nine red counters and five blue counters and so on. Describe three different ways each of these integers can be modelled.

- **a** +3
- **b** −2
- **c** −6
- **d** +4
- **e** 0
- **f** −5

**Reflect**

Why do you need negative numbers?

# 4B Adding integers

## Start thinking!

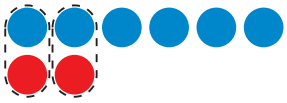
Collect 10 blue counters and 10 red counters (or use any two colours). A blue counter represents +1 and a red counter represents −1.

1 Use six blue counters and two blue counters to model (+6) + (+2).



   a How many counters do you have in total?
   b What integer does this represent?
   c Copy and complete: (+6) + (+2) = _____.

2 Use six blue counters and two red counters to model (+6) + (−2).




   a What integer does this represent?
   b Copy and complete: (+6) + (−2) = _____.

3 Follow the methods used in questions **1** and **2** to find the result for:
   **a** (−4) + (+3) **b** (−4) + (−3) **c** (−3) + (+5) **d** (−5) + (+5)

4 Use the words *positive* and *negative* to complete these statements.
   a Adding two positive integers produces a result that is _____.
   b Adding two negative integers produces a result that is _____.
   c Adding a positive integer and a negative integer produces a result that is either _____ or _____ or zero, depending on the numbers involved.

## KEY IDEAS

- Coloured counters can be grouped to model the adding of two integers (see Example A).
- Grouping +1 and −1 (a zero pair) gives 0. The remaining counters show the result (see Example B).

**Example A**



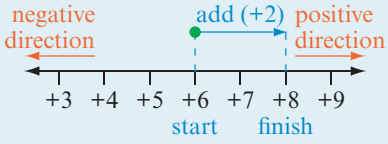
(−3) + (−2) = −5

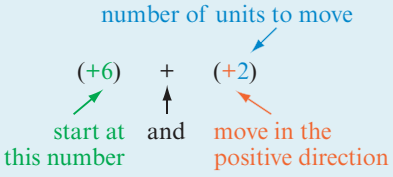
**Example B**



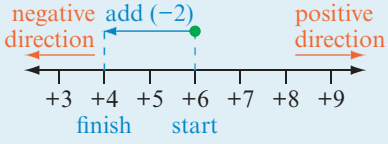
(−2) + (+5) = +3

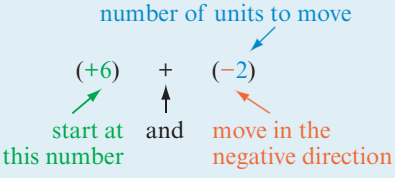
- Adding a positive integer using a number line: start at the first number and move right. For example, (+6) + (+2) = +8.




- Adding a negative integer using a number line: start at the first number and move left. For example, (+6) + (−2) = +4.



# EXERCISE 4B Adding integers

## EXAMPLE 4B-1 Using counters to add integers

Use counters to model each calculation and find the result.

a (−5) + (−3) b (+3) + (−7)

### THINK

a 1 Usefi ve red counters to model −5 and three red counters to model −3. Combine the counters as one group to model the addition.

2 Since the counters have the same colour, the total number of counters represents the result of the addition. (8 red = −8.) Write your answer.

b 1 Use three blue counters to model +3 and seven red counters to model −7. Combine the counters as one group to model the addition.

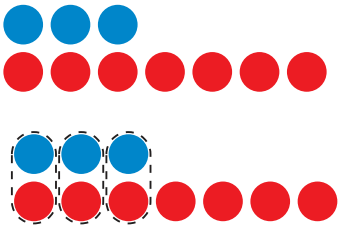
2 Match a blue counter with a red counter to form a zero pair. There are three zero pairs with four red counters remaining.

3 The remaining counters represent the result of the calculation. (4 red = −4.) Write your answer.

### WRITE

a 

(−5) + (−3) = −8

b 

(+3) + (−7) = −4

UNDERSTANDING AND FLUENCY

1 Copy and complete each number sentence using the diagrams provided.

a 

(−6) + (−1) = ____

b 

(−8) + (+3) = ____

c 

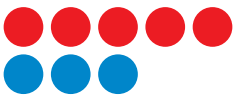
(+7) + (−4) = ____

d 

(−3) + (−____)= ____

e 

(____) + (+4) = ____

f 

(−5) + (____)= ____

UNDERSTANDING AND FLUENCY

**2** Use counters to model each calculation and find the result.

| | | |
|---|---|---|
| **a** (+2) + (+3) | **b** (−4) + (−5) | **c** (+3) + (−1) |
| **d** (+1) + (−6) | **e** (−5) + (+8) | **f** (+7) + (−9) |
| **g** (−8) + (+5) | **h** (−2) + (−4) | **i** (+6) + (−6) |
| **j** (−3) + (+4) | **k** (−7) + (+7) | **l** (+9) + (−9) |

**3** Use counters to model each calculation and find the result.

| | | |
|---|---|---|
| **a** (+3) + (+5) | **b** (−2) + (+7) | **c** (−9) + (+1) |
| **d** (+6) + (+4) | **e** (+3) + (−5) | **f** (−2) + (−6) |
| **g** (−1) + (−3) | **h** (+9) + (−4) | **i** (−4) + (+4) |
| **j** (+7) + (−7) | **k** (+5) + (−5) | **l** (−2) + (+2) |

**4** Explain how you tell by looking at the numbers to be added whether the result will be positive or negative or zero.

## EXAMPLE 4B-2 Identifying positive, negative and zero results

Decide whether the result of each problem will be positive, negative or zero.

**a** (+12) + (+17) **b** (−19) + (−13) **c** (−23) + (+23)
**d** (−15) + (+11) **e** (−27) + (+35)

### THINK

**a** Combining two lots of blue counters makes a result of blue counters.

**b** Combining two lots of red counters makes a result of red counters.

**c** Combining the same amount of red and blue counters makes zero.

**d** Combining more red counters than blue counters means there are red counters left over when zero pairs are made.

**e** Combining more blue counters than red counters means there are blue counters left over when zero pairs are made.

### WRITE

**a** The result for (+12) + (+17) is positive.

**b** The result for (−19) + (−13) is negative.

**c** The result for (−23) + (+23) is zero.

**d** The result for (−15) + (+11) is negative.

**e** The result for (−27) + (+35) is positive.

**5** Decide whether the result of each calculation will be positive, negative or zero.

| | | |
|---|---|---|
| **a** (−12) + (+9) | **b** (+19) + (−19) | **c** (+23) + (−21) |
| **d** (−30) + (+20) | **e** (+25) + (+18) | **f** (−31) + (−12) |
| **g** (−45) + (+45) | **h** (−29) + (+15) | **i** (−19) + (+24) |
| **j** (+78) + (−83) | **k** (+50) + (−30) | **l** (+49) + (−49) |

## EXAMPLE 4B-3 Using a number line to add integers

Use a number line to calculate:

a (−4) + (+3) b (−4) + (−3)

### THINK

**a** **1** Draw a number line. Start at the first number (−4) and move 3 units to the right to add +3.

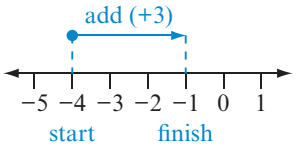
**2** Identify the number you finish at and write your answer.

**b** **1** Draw a number line. Start at the first number (−4) and move 3 units to the left to add −3.

**2** Identify the number you finish at and write your answer.

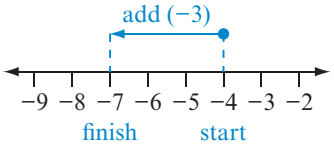
### WRITE

**a**




(−4) + (+3) = −1

**b**

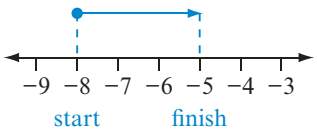



(−4) + (−3) = −7

UNDERSTANDING AND FLUENCY

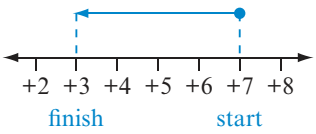
**6** Copy and complete each number sentence using the diagrams provided.

**a**



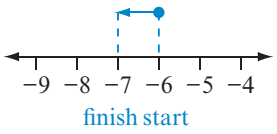

(−8) + (+3) = ____

**b**



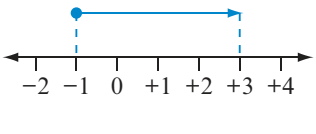

(+7) + (−4) = ____

**c**



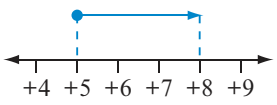

(−6) + (−1) = ____

**d**



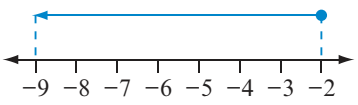

(−1) + (+____) = ____

**e**




(____) + (+3) = ____

**f**




(____) + (____) = ____

**7** Use the words *left*, *right*, *positive* and *negative* to copy and complete these statements about using a horizontal number line.

**a** To add a positive number, move to the _____ (or in a _____ direction).

**b** To add a negative number, move to the _____ (or in a _____ direction).

UNDERSTANDING AND FLUENCY

**8** Use a number line to calculate each of these.

| | | |
|---|---|---|
| **a** (−3) + (+4) | **b** (+2) + (+5) | **c** (−9) + (+7) |
| **d** (+1) + (−8) | **e** (−5) + (−4) | **f** (+4) + (−1) |
| **g** (−8) + (+5) | **h** (−2) + (+4) | **i** (−4) + (+4) |
| **j** (−2) + (+3) | **k** (−7) + (+7) | **l** (+6) + (−5) |
| **m** (−5) + (+5) | **n** (+8) + (−4) | **o** (−6) + (+1) |

**9** Use a number line to calculate each of these.

| | | |
|---|---|---|
| **a** (+3) + (+5) | **b** (−2) + (+7) | **c** (−9) + (+1) |
| **d** (+6) + (+4) | **e** (+3) + (−5) | **f** (−2) + (−6) |
| **g** (−1) + (−3) | **h** (+9) + (−4) | **i** (−4) + (+4) |
| **j** (+7) + (−7) | **k** (+5) + (−5) | **l** (−2) + (+2) |

**10** Compare your results for question **9** with those obtained using counters in question **3**.

**11** Calculate each of these. (Hint: imagine using lots of counters or a longer number line.)

| | | |
|---|---|---|
| **a** (−11) + (+6) | **b** (+20) + (−7) | **c** (−18) + (+5) |
| **d** (−13) + (−10) | **e** (−14) + (−15) | **f** (+23) + (−11) |
| **g** (−68) + (+9) | **h** (−82) + (−14) | **i** (−55) + (+28) |
| **j** (+21) + (−43) | **k** (−77) + (+79) | **l** (+126) + (−18) |

## EXAMPLE 4B-4 Writing an addition problem

An ice cream starts to melt. From a starting temperature of −4°C, the temperature of the outer surface of the ice cream increases by 9°C.

**a** Write an addition problem that helps you calculate the new temperature of the ice cream.
**b** What temperature will the ice cream be after it warms up?

### THINK

**a** **1** Identify the starting temperature. This means that −4 is the first integer in the problem.

**2** An increase of 9°C means to add 9°C so +9 is the second integer in the problem.

**3** Write the addition problem.

**b** **1** Calculate the problem.

**2** Write the answer.

### WRITE

**a** Starting temperature is −4°C.
first integer = −4

Need to add 9°C.
second integer = +9

(−4) + (+9)

**b** (−4) + (+9) = +5

After it warms up, the temperature of the ice cream is 5°C.

PROBLEM SOLVING AND REASONING

**12** Party pies are stored in a freezer at −18°C. To serve them, the party pies are heated up by 98°C.



**a** Write an addition problem that helps you calculate the temperature of the pies after heating.

**b** What will be the new temperature of the party pies?

**13** The temperature at 6 am is −8°C. One hour later the temperature has increased by 5°C.



**a** Write an addition problem that helps you calculate the temperature at 7 am.

**b** What will be the temperature at 7 am?

**14** Lachlan has overdrawn his bank account by $26.

**a** Represent this amount as an integer.

**b** Lachlan deposits $40 into his account. Write an addition problem that helps you calculate the new account balance.

**c** What is his new account balance?

**15** Jess owes her brother $50.

**a** Represent this amount as an integer.

**b** Jess pays back $35. Write an addition problem that helps you calculate how much Jess still owes.

**c** How much does Jess now owe her brother?

CHALLENGE

**16** Give three different examples where the sum of two integers is negative.

**17** Find three different integers that, when added to 6, give a negative result.

**18** What number should be added to each of these to give a result of 0?

**a** −4 **b** 7

**c** 1 **d** −10

**Reflect**

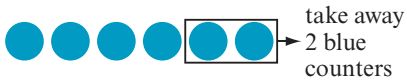
How do counters or a number line help when adding integers?

# 4C Subtracting integers

## Start thinking!

Collect 10 blue counters and 10 red counters (or any two colours). A blue counter represents +1 and a red counter represents −1. When adding integers you combine groups of counters, but to subtract you do the opposite: you remove or take away counters from a group.

**1** Use counters to model (+6) − (+2). Start with six blue counters to model +6 and then take away two blue counters to subtract +2.




**a** How many counters are left?

**b** What integer does this represent?

**c** Copy and complete: (+6) − (+2) = _____.

**2** Use counters to model (−4) − (−3). Start with four red counters.

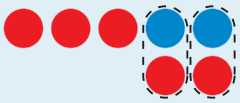
**a** How many counters are left?

**b** What integer does this represent?

**c** Copy and complete: (−4) − (−3) = _____.

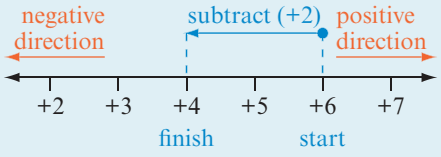
## KEY IDEAS

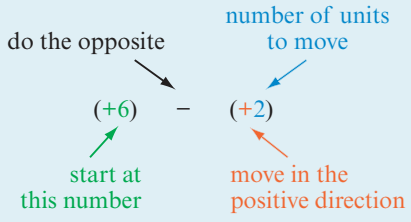
- Zero pairs are useful when subtracting integers as you can add equal numbers of blue and red counters to the model and not change the overall value. For example, the overall value of the integer in this model is still −3.



- Subtracting a positive integer using a number line: start at the first number and, instead of moving right, do the opposite and move left.

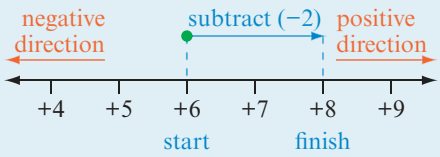
For example,
(+6) − (+2) = +4.

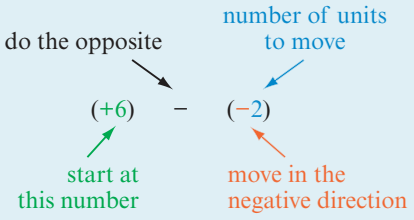



- Subtracting a negative integer using a number line: start at the first number and, instead of moving left, do the opposite and move right.

For example,
(+6) − (−2) = +8.



# EXERCISE 4C Subtracting integers

## EXAMPLE 4C-1 Using counters to subtract integers

Use counters to model (−5) − (−2) and find the result.

**THINK**

1 Start with five red counters to model −5. Take away two red counters to subtract −2.

2 The remaining counters represent the result. (3 red = −3.) Write your answer.

**WRITE**



(−5) − (−2) = −3

UNDERSTANDING AND FLUENCY

**1** Use these diagrams to find each result.

**a**



(+5) − (+3) = _____

**b**



(−7) − (−4) = _____

**c**



(−8) − (−2) = _____

**d**

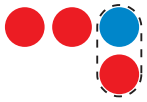


(+4) − (+1) = _____

**2** Use counters to model each calculation and write the result.

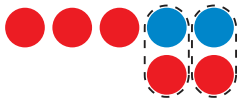
| | | | | | |
|---|---|---|---|---|---|
| **a** | (+9) − (+5) | **b** | (−5) − (−2) | **c** | (−8) − (−1) |
| **d** | (+7) − (+6) | **e** | (−4) − (−1) | **f** | (+8) − (+3) |
| **g** | (−3) − (−3) | **h** | (+9) − (+9) | **i** | (−1) − (−1) |

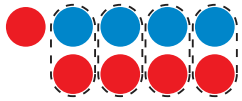
**3** What happens if you add a zero pair to an integer? Does its value change? Try it for the integer −2 and then copy and complete: (−2) + (0) = _____.

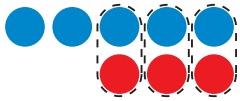


**4** What integer is represented in each diagram?

**a** 

**b** 

**c** 

**d** 

## EXAMPLE 4C-2 Using counters to subtract integers (using zero pairs)

Use counters to model (−4) − (+3) and find the result.

### THINK

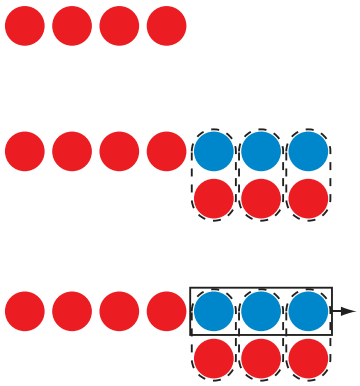
**1** Start with four red counters to model −4. To subtract +3, you need to take away three blue counters. There are no blue counters to remove.

**2** Add three zero pairs by placing three blue counters and three red counters with your four red counters. You now have three blue counters in the group without changing the overall value of −4.

**3** Remove three blue counters to model subtracting +3.

**4** The remaining counters represent the result. (7 red = −7.) Write your answer.

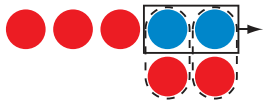
### WRITE



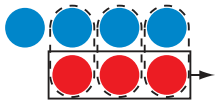
(−4) − (+3) = −7

UNDERSTANDING AND FLUENCY

**5** Use these diagrams to find each result.

**a** 

(−3) − (+2) = _____

**b** 

(+1) − (−3) = _____

**6** How do you decide the number of zero pairs to be added?

**7** Use counters to model each calculation and find the result.

**a** (+4) − (−2)
**b** (−1) − (+6)
**c** (−2) − (+4)
**d** (+8) − (−1)
**e** (−3) − (+2)
**f** (+1) − (−3)
**g** (+3) − (−5)
**h** (−4) − (+4)
**i** (+5) − (−5)

## EXAMPLE 4C-3 Using counters to subtract integers (using zero pairs)

Use counters to model (−3) − (−5) and find the result.

### THINK

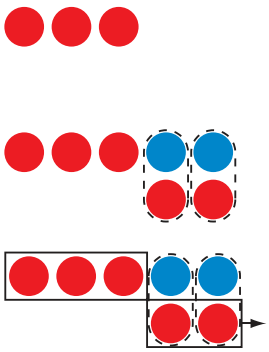
1 Start with three red counters to model −3. To subtract −5, you need to take away five red counters. You need two more red counters.

2 Add two zero pairs by placing two red counters and two blue counters with your three red counters. You now have five red counters in the group without changing the overall value of −3.

3 Remove five red counters to model subtracting −5.

4 The remaining counters represent the result. (2 blue = +2.) Write your answer.

### WRITE

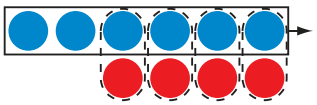


(−3) − (−5) = +2

UNDERSTANDING AND FLUENCY

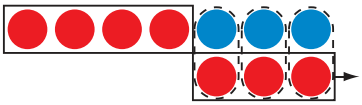
**8** Use these diagrams to find each result.

**a**



(+2) − (+6) = _____

**b**



(−4) − (−7) = _____

**9** How do you decide the number of zero pairs to be added?

**10** Use counters to model each calculation and find the result.

| | | |
|---|---|---|
| **a** (+3) − (+6) | **b** (−2) − (−7) | **c** (+4) − (+5) |
| **d** (−5) − (−9) | **e** (+1) − (+3) | **f** (−1) − (−6) |
| **g** (−3) − (−4) | **h** (−4) − (−8) | **i** (+2) − (+4) |

**11** In some subtraction problems you can take away the required counters straight away. In others, you first need to add some zero pairs. Explain when you need to add zero pairs to your model.

**12** Use counters to model each calculation and find the result.

| | | |
|---|---|---|
| **a** (−8) − (−5) | **b** (+7) − (+2) | **c** (−3) − (−2) |
| **d** (+4) − (+4) | **e** (+3) − (−1) | **f** (−4) − (+5) |
| **g** (+1) − (−6) | **h** (−5) − (+4) | **i** (+2) − (+3) |
| **j** (−7) − (−9) | **k** (+3) − (+8) | **l** (−2) − (−4) |
| **m** (−5) − (−5) | **n** (−3) − (+3) | **o** (+2) − (−2) |

## EXAMPLE 4C-4 Using a number line to subtract integers

Use a number line to calculate:

**a** $(-4) - (+3)$ **b** $(-4) - (-3)$

### THINK

**a** **1** Draw a number line. Start at the first number $(-4)$ and move 3 units to the left to subtract $+3$. (This is the opposite to adding $+3$.)

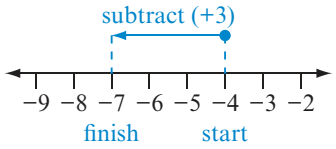
**2** Identify the number you finish at and write your answer.

**b** **1** Draw a number line. Start at the first number $(-4)$ and move 3 units to the right to subtract $-3$. (This is the opposite to adding $-3$.)

**2** Identify the number you finish at and write your answer.

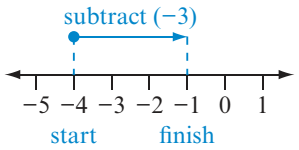
### WRITE

**a**




$(-4) - (+3) = -7$

**b**

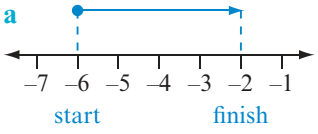



$(-4) - (-3) = -1$

UNDERSTANDING AND FLUENCY

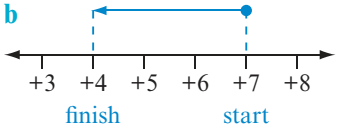
**13** Copy and complete each number sentence using the diagrams provided.

**a**



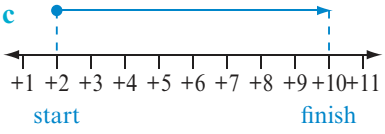

$(-6) - (-4) =$ ____

**b**



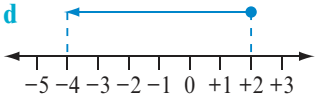

$(+7) - (+3) =$ ____

**c**



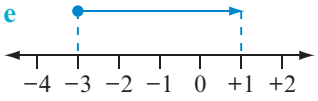

$(+2) - (-8) =$ ____

**d**



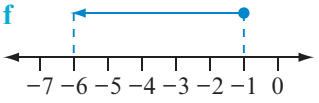

$(+2) - (____) =$ ____

**e**




$(____) - (-4) =$ ____

**f**




$(____) - (____) =$ ____

**14** Copy these statements about using a horizontal number line and use the words *left* or *right* and *positive* or *negative* to complete them.

**a** To subtract a positive number, move to the ____ (or in a ____ direction).

**b** To subtract a negative number, move to the ____ (or in a ____ direction).

**15** Use a number line to calculate each of these.

| | | |
|---|---|---|
| **a** $(+7) - (+2)$ | **b** $(-2) - (-3)$ | **c** $(-4) - (+6)$ |
| **d** $(+1) - (-5)$ | **e** $(-4) - (+4)$ | **f** $(-3) - (-1)$ |
| **g** $(+3) - (+6)$ | **h** $(-7) - (-7)$ | **i** $(+5) - (+9)$ |
| **j** $(+4) - (-1)$ | **k** $(+5) - (-5)$ | **l** $(-1) - (+5)$ |
| **m** $(+7) - (+8)$ | **n** $(-6) - (-9)$ | **o** $(+2) - (+8)$ |

UNDERSTANDING AND FLUENCY

**16** Use a number line to calculate each of these.

**a** $(-8) - (-5)$ **b** $(+7) - (+2)$ **c** $(-3) - (-2)$
**d** $(+4) - (+4)$ **e** $(+3) - (-1)$ **f** $(-4) - (+5)$
**g** $(+1) - (-6)$ **h** $(-5) - (+4)$ **i** $(+2) - (+3)$
**j** $(-7) - (-9)$ **k** $(+3) - (+8)$ **l** $(-2) - (-4)$
**m** $(-5) - (-5)$ **n** $(-3) - (+3)$ **o** $(+2) - (-2)$

**17** Compare your results for question **16** with those obtained using counters in question **12**.

**18** Calculate each of these. (Hint: imagine using lots of counters or a longer number line.)

**a** $(+10) - (+15)$ **b** $(-24) - (-13)$ **c** $(-19) - (+30)$
**d** $(+20) - (+11)$ **e** $(-50) - (+30)$ **f** $(+38) - (-27)$
**g** $(+62) - (+72)$ **h** $(-115) - (-44)$ **i** $(+250) - (-50)$

PROBLEM SOLVING AND REASONING

**19** Food in the freezer is frozen to a temperature of −15°C while food in the main section of the fridge is cooled to a temperature of 4°C.

**a** Write a subtraction problem to help you calculate the difference between the temperature in the main section and the temperature in the freezer.

**b** What is the difference in temperature between the two sections of the fridge?



**20** The temperature is 14°C. After 2 hours, the temperature decreases by 20°C.

**a** Write a subtraction problem that helps you calculate the new temperature.

**b** What is the new temperature?

**21** Hayley has $74 in her bank account.

**a** Represent this account balance as a positive integer.

**b** Hayley withdraws some money from her account. Her account balance is now −$31. Write a subtraction problem that helps you calculate the amount of money Hayley withdrew from her account. (Hint: find the difference between the two account balances.)

**c** How much money did Hayley withdraw from her account?

CHALLENGE

**22** Give three examples where the difference between two integers is negative.

**23** Find three different integers that, when subtracted from 15, give a negative result.

**24** What number should be subtracted from each of these to give a result of 0?

**a** −5 **b** +2 **c** −8 **d** 7

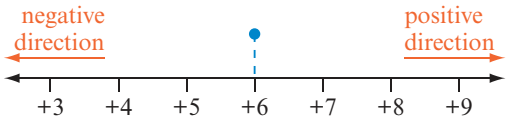
**Reflect**

What explanation would you give a friend to explain how to subtract integers?

# 4D Simplifying addition and subtraction of integers

## Start thinking!

So far you have added positive and negative integers and subtracted positive and negative integers using counters and number lines. Is there an easier way?




**1** **a** Use the number line to calculate: **i** (+6) + (+2) **ii** (+6) − (−2).
**b** Which direction did you move from +6 in each case?
**c** What does this show you about adding a positive number and subtracting a negative number?
**d** If (+6) + (+2) is the same as 6 + 2, explain why you can also write (+6) − (−2) more simply as 6 + 2.

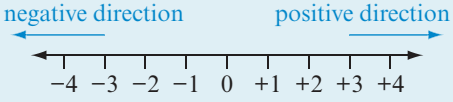
**2** **a** Use a number line to calculate: **i** (+6) − (+2) **ii** (+6) + (−2).
**b** Which direction did you move from +6 in each case?
**c** What does this show you about subtracting a positive number and adding a negative number?
**d** If (+6) − (+2) is the same as 6 − 2, explain why you can also write (+6) + (−2) more simply as 6 − 2.

**3** Copy and complete these sentences using *positive*, *negative*, +, −.
**a** To simplify adding a _____ number, replace + (+) with +.
**b** To simplify adding a negative number, replace + (−) with _____.
**c** To simplify subtracting a positive number, replace − (+) with _____.
**d** To simplify subtracting a _____ number, replace − (−) with _____.

## KEY IDEAS

+ (+) is equivalent to +
− (−) is equivalent to +
+ (−) is equivalent to −
− (+) is equivalent to −

- Addition and subtraction problems can be simplified by replacing two operation signs that are next to each other with one equivalent sign. For example, (−4) − (−3) simplifies to −4 + 3, because − (−) is equivalent to +.
- When using a number line, start at the position of the first number and then move left or right the number of units indicated by the second number. The final position is the result.
- When the sign between the two numbers is +, move to the right.
- When the sign between the two numbers is −, move to the left.



# EXERCISE 4D Simplifying addition and subtraction of integers

## EXAMPLE 4D-1 Simplifying addition and subtraction problems

Simplify each problem.

**a** $(-6) - (-9)$ **b** $(+7) - (+2)$

| THINK | WRITE |
|---|---|
| **a** Write the problem in a simpler equivalent form by replacing $-(-)$ with $+$. | **a** $(-6) - (-9)$<br>$= -6 + 9$ |
| **b** Write the problem in a simpler equivalent form by replacing $-(+)$ with $-$. | **b** $(+7) - (+2)$<br>$= 7 - 2$ |

UNDERSTANDING AND FLUENCY

**1** Copy and complete these statements about two operation signs written next to each other using the words *the same* and *different*.

**a** You can replace two operation signs that are ____ with $+$.

**b** You can replace two operation signs that are ____ with $-$.

**2** Copy and complete each of these to write an equivalent problem that is simpler. Use $+$ or $-$ to fill the gap.

**a** $(-5) - (+4)$
$= -5 __ 4$

**b** $(+2) + (+7)$
$= 2 __ 7$

**c** $(-3) + (-6)$
$= -3 __ 6$

**d** $(+1) - (-3)$
$= 1 __ 3$

**e** $(+8) - (+9)$
$= 8 __ 9$

**f** $(-5) - (-7)$
$= -5 __ 7$

**3** Write each problem more simply.

**a** $(-3) - (-5)$

**b** $(+6) - (+5)$

**c** $(-2) + (-4)$

**d** $(+5) + (+7)$

**e** $(+7) - (-1)$

**f** $(+4) - (+9)$

## EXAMPLE 4D-2

### Adding and subtracting integers after first simplifying the problem

Calculate each of these.

**a** (+3) + (−5) **b** (−4) − (−7)

**THINK**

**a** **1** Write the problem in a simpler equivalent form by replacing + (−) with −.

**2** Locate the first number (3) on the number line and move 5 units to the left (negative direction).

**3** Identify the number you finish at and write your answer.

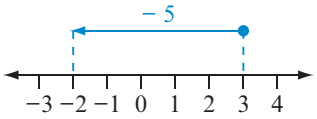
**b** **1** Write the problem in a simpler equivalent form by replacing − (−) with +.

**2** Locate the first number (−4) on the number line and move 7 units to the right (positive direction).

**3** Identify the number you finish at and write your answer.

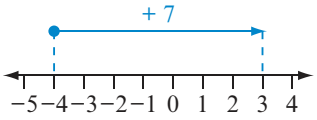
**WRITE**

**a** (+3) + (−5)
= 3 − 5




(+3) + (−5) = −2

**b** (−4) − (−7)
= −4 + 7




(−4) − (−7) = 3 (or +3)

UNDERSTANDING AND FLUENCY

**4** Calculate each of these.

| | | |
|---|---|---|
| **a** (+4) + (−2) | **b** (−3) − (−7) | **c** (+2) − (+6) |
| **d** (−6) − (−1) | **e** (−5) + (+4) | **f** (+8) + (−3) |
| **g** (−1) − (−9) | **h** (+4) − (+5) | **i** (+3) − (−8) |
| **j** (−5) − (+3) | **k** (−4) + (+2) | **l** (−6) + (−1) |

**5** Calculate each problem in question **2**.

**6** Calculate each problem in question **3**.

**7** Calculate each of these.

| | | |
|---|---|---|
| **a** 8 − 3 | **b** 4 − 7 | **c** −3 + 9 |
| **d** −5 − 2 | **e** 6 − 10 | **f** −1 − 3 |
| **g** 0 − 5 | **h** −10 + 4 | **i** −7 + 7 |
| **j** 9 − 12 | **k** −8 − 6 | **l** −15 + 20 |

**8** Copy and complete these addition tables.

**a**

| + | −2 | −1 | 0 | +1 | +2 |
|---|---|---|---|---|---|
| −2 | | | | −1 | |
| −1 | | | | | |
| 0 | | | 0 | | |
| +1 | | | | | |
| +2 | | +1 | | | |

**b**

| + | −5 | −3 | −2 | 2 | 4 |
|---|---|---|---|---|---|
| 7 | | | | | |
| 5 | | | | | |
| 0 | | | | | |
| −2 | | | | | |
| −4 | | | | | |

**9** Calculate each of these.

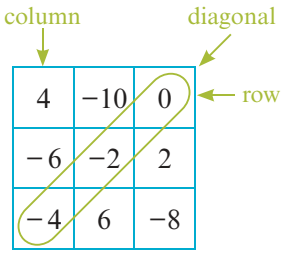
**a** −28 + 30 **b** −48 − 26 **c** 77 − 81

**d** −62 + 41 **e** 150 − 160 **f** −133 − 58

**g** −67 + 415 **h** 256 − 328 **i** 1000 − 4000

**10** Explain how you perform addition and subtraction on large integers.

**11** Nine numbers have been arranged as a square.




**a** Add the three numbers in:

**i** the first row

**ii** the second row

**iii** the third row.

**b** Add the three numbers in:

**i** the first column

**ii** the second column

**iii** the third column.

**c** Add the three numbers in each diagonal.

**d** What do you notice about your answers to parts **a**, **b** and **c**?

**e** Your answer to part **d** is why the nine numbers are said to form a magic square. The sum you have found for each row, column and diagonal is called the magic sum. What is the magic sum for this set of numbers?

**12** Copy and complete these magic squares. (Hint: first work out the magic sum.)

**a**

| | | |
|---|---|---|
| 4 | −3 | 2 |
| −1 | | 3 |
| 0 | 5 | |

**b**

| | | |
|---|---|---|
| | | −4 |
| | −1 | |
| 2 | −2 | |

**c**

| | | |
|---|---|---|
| 6 | | |
| −18 | | |
| 3 | | −12 |

**13** Write the number that is:

**a** 5 more than −3 **b** 7 less than 2 **c** 6 more than −6

**d** 8 less than −1 **e** 4 more than −5 **f** 3 less than 1

**14** At Thredbo, the daily maximum and minimum temperatures were recorded over a week.

| | Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|---|---|---|---|---|---|---|---|
| Maximum temperature (°C) | 7 | 3 | 8 | 2 | −1 | 4 | 5 |
| Minimum temperature (°C) | 1 | −2 | −1 | −3 | −5 | −2 | 0 |



**a** Calculate the difference between the minimum and maximum temperatures for Sunday. (Hint: subtract the minimum temperature from the maximum temperature.)

**b** Calculate the difference between the minimum and maximum temperatures for Monday.

**c** Calculate the difference between the minimum and maximum temperatures for each of the remaining days.

**d** Which day had the biggest range of temperatures?

**15** Raj has $182 in his bank account.

**a** Represent this amount as a positive integer.

**b** Raj withdraws $200 from his account. Write a subtraction problem that helps you calculate the new account balance.

**c** What is his new account balance as an integer?

**d** Interpret your answer to part **c**. Does Raj have money in his account or does he owe money to the bank?

## EXAMPLE 4D-3 Writing a calculation for a problem

You enter a lift at the second floor and travel down six floors. Which floor level do you finish at? Show the calculation you used to obtain your answer.

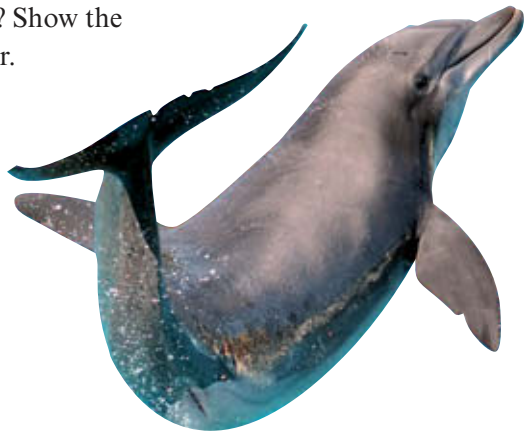
| THINK | WRITE |
|---|---|
| **1** Identify the floor you start at. This means that +2 (or 2) is the first integer in the calculation. | Starting floor is second floor (above ground level). First integer is +2 or 2. |
| **2** Moving down six floors means to subtract 6. Write the calculation and work out the result. | $2 - 6 = -4$ |
| **3** Write the answer. | You finish at the fourth floor below ground level. |

**16** You enter a lift at the seventh floor and travel down 10 floors. Which floor level do you finish at? Show the calculation you used to obtain your answer.

PROBLEM SOLVING AND REASONING

**17** A stone is dropped from the top of a cliff 50 m above the water and falls to the bottom of the sea. The depth of the water is 12 m. How far has the stone dropped? Show the calculation you used to obtain your answer.

**18** A dolphin is swimming 2 m below the surface of the water and then leaps up a vertical distance of 3 m. How high above the water's surface does the dolphin reach? Show the calculation you used to obtain your answer.



**19** This incomplete statement shows some of the deposits and withdrawals made to Lily's bank account.

| Date | Transaction | Balance |
|---|---|---|
| 30 June | – | +$75 |
| 5 July | −$24 | |
| 13 July | −$68 | |
| 24 July | +$45 | |
| 31 July | | +$19 |

**a** Does a transaction of +$45 represent a deposit or a withdrawal?
**b** Does a transaction of −$24 represent a deposit or a withdrawal?
**c** Explain what it means if the balance in Lily's account is positive.
**d** Explain what it means if the balance in Lily's account is negative.
**e** Find the balance of Lily's account after the transaction made on 24 July.
**f** What transaction is made on 31 July so that her account balance is +$19?

CHALLENGE

**20** List three examples of pairs of numbers where the sum of the two numbers is zero.

**21** Calculate each of these. Remember to work from left to right.

**a** $-1 + 3 + 5$ **b** $2 - 8 + 9$ **c** $4 - 1 - 7$
**d** $-5 + 3 - 8$ **e** $-7 + 2 + 4$ **f** $-6 - 1 - 3$
**g** $3 - 2 - 4$ **h** $1 - 11 - 5$ **i** $-8 + 3 - 9$

**22** Calculate each of these. Remember to first write each problem as a simpler equivalent calculation.

**a** $(-2) - (+9) + (+3)$
**b** $(+7) + (-4) - (-6)$
**c** $(-5) - (-1) + (+8)$
**d** $(+6) + (-3) - (-11)$
**e** $(-10) - (+7) + (+2)$
**f** $(-4) - (-4) + (-5)$
**g** $(+1) + (-9) - (+3)$
**h** $(-8) - (+6) - (-17)$

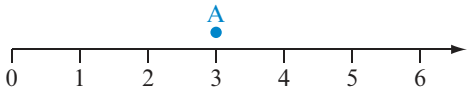
**Reflect**

What tips would you give to simplify addition and subtraction problems?

# 4E Introducing the Cartesian plane

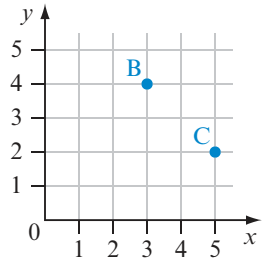
## Start thinking!

A number line can be used to describe the position of a point. For example, the position of point A on this number line is 3.




1 The reference point on a number line is 0. How many units is point A from 0?

2 Copy this diagram, which has two number lines drawn at right angles to each other. These number lines are also called a **set of axes**. The space between the set of axes is called a number plane or the **Cartesian plane** (named after the mathematician René Descartes, who invented it in 1619).




3 Label these elements on your diagram.

**a** ***x*-axis** (the horizontal number line) **b** ***y*-axis** (the vertical number line)

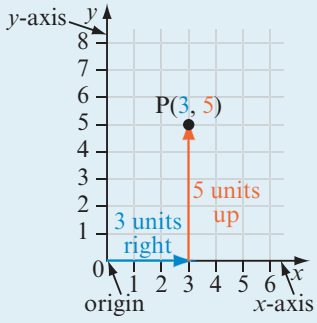
**c** the **origin** (the point where the two number lines meet)

4 A point, labelled B, has been drawn on the number plane. Suggest a way that the position of this point could be described.

5 The position of a point, C, is described by the **Cartesian coordinates** (5, 2). Explain how these coordinates can be used to locate the point.

## KEY IDEAS

- A Cartesian plane is formed when a horizontal number line (*x*-axis) and a vertical number line (*y*-axis) intersect at right angles at the origin.
- Cartesian coordinates are used to describe the location of a point on the Cartesian plane. The horizontal coordinate (***x*-coordinate**) is always listed first, followed by a comma then the vertical coordinate (***y*-coordinate**). For example, the point P has the coordinates (3, 5).
- The *x*-coordinate tells you how many units to move along the *x*-axis from the origin and the *y*-coordinate tells you how many units to move up parallel to the *y*-axis. For example, (3, 5) means to move 3 units right from the origin and then 5 units up.



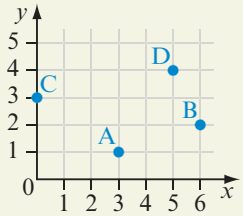

**NOTE** The point P has coordinates (3, 5).

x-coordinate y-coordinate

# EXERCISE 4E Introducing the Cartesian plane

## EXAMPLE 4E-1 Writing coordinates for points on a Cartesian plane

Write the coordinates of the points A, B, C and D, shown on this Cartesian plane.




**THINK**

1 For point A, identify the $x$-coordinate by finding the number of units you need to move along the $x$-axis from the origin to be under the point (3). Identify the $y$-coordinate by finding the number of units you then need to move up to the point (1). Write the $x$-coordinate first.

2 Repeat step **1** for points B, C and D.

**WRITE**

Point A is at (3, 1).
Point B is at (6, 2).
Point C is at (0, 3).
Point D is at (5, 4).

UNDERSTANDING AND FLUENCY

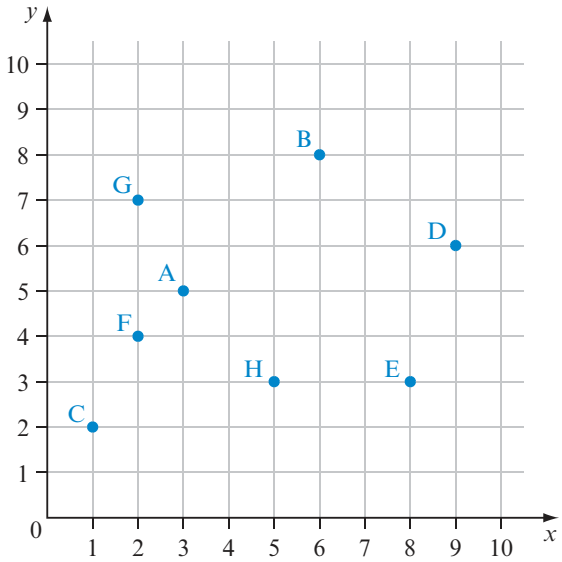
**1** When you write Cartesian coordinates, does the $x$-coordinate or $y$-coordinate come first? Can you think of a way to help you remember which coordinate comes first?

**2** Write the coordinates of the points A to H shown on this Cartesian plane.

**3** Write the coordinates of the origin.

**4** List the points in question **2** that have:

- **a** the same $x$-coordinate
- **b** the same $y$-coordinate
- **c** an $x$-coordinate of 6
- **d** a $y$-coordinate of 6.



## EXAMPLE 4E-2 Plotting points on a Cartesian plane

Plot these points on the Cartesian plane.

**a** E(2, 3) **b** F(1, 4)
**c** G(5, 2) **d** H(3, 0)

### THINK

**a** E has an $x$-coordinate of 2 and a $y$-coordinate of 3. Move 2 units along the $x$-axis from the origin and then 3 units up parallel to the $y$-axis. Plot the point by drawing a dot at this final position. Label the point as E.

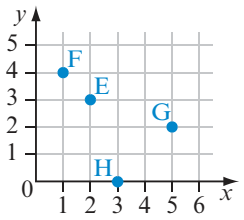
**b** F has an $x$-coordinate of 1 and a $y$-coordinate of 4. (1 unit right and 4 units up)

**c** G has an $x$-coordinate of 5 and a $y$-coordinate of 2. (5 units right and 2 units up)

**d** H has an $x$-coordinate of 3 and a $y$-coordinate of 0. (3 units right and 0 units up)

### WRITE

**a–d**




UNDERSTANDING AND FLUENCY

**5** Consider the coordinates of the points I to P, listed below.

**i** Describe the position of each point from the origin. For example, point I is 3 units right from the origin and then 9 units up.

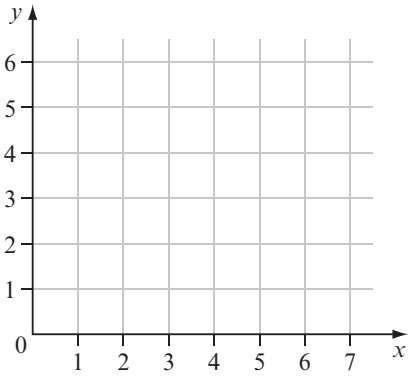
**ii** Copy the set of $x$- and $y$-axes shown in question **2**. Plot these points on the Cartesian plane. Label each point with its letter.

**a** I(3, 9) **b** J(7, 7) **c** K(4, 1) **d** L(6, 4)
**e** M(0, 7) **f** N(4, 0) **g** O(0, 0) **h** P(2, 5)

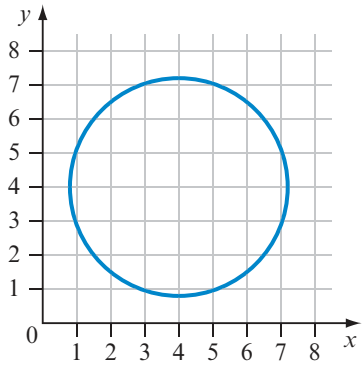
**6** List the points in question **5** that have:

**a** the same $x$-coordinate
**b** the same $y$-coordinate
**c** an $x$-coordinate of 0
**d** a $y$-coordinate of 0
**e** the coordinates of the origin
**f** a $y$-coordinate of 4.

**7** Copy this Cartesian plane and plot each point. Join the points together in the order shown. Describe the shape you have created.
(4, 6), (5, 4), (7, 4), (5, 3), (6, 1), (4, 2), (2, 1), (3, 3), (1, 4), (3, 4), (4, 6)



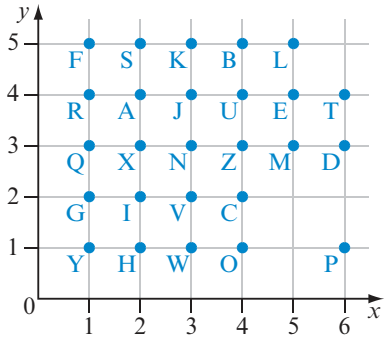
**8** On a Cartesian plane like the one shown, give instructions to draw a 'smiley' face (or another type of face). That is, give the coordinates of points to be joined to form the eyes, nose and mouth.




**9** You and your spy friend have created an unbreakable code. The two of you are the only ones with copies of the code-key grid. This allows you to send each other messages in coordinate code.

**a** Decode the following message by writing down the letter that matches each coordinate point.

(6, 4) (2, 1) (5, 4)
(5, 4) (5, 3) (2, 4) (2, 2) (5, 5)
(6, 1) (2, 4) (2, 5) (2, 5) (3, 1) (4, 1) (1, 4) (6, 3) (2, 2) (2, 5)
(6, 4) (2, 2) (4, 2) (3, 5) (5, 5) (5, 4) (2, 5)




**b** Now encode the following message: 'THE GATE IS OPEN'.

**10** Your spy friend sends you a postcard asking you to meet under the bridge at midnight. You notice the following words and numbers along the bottom of the card.



Join (0, 4) to (2, 4).
Join (1, 4) to (1, 1).
Join (0, 1) to (2, 1).
Join (3, 3) to (3, 1).
Join (2, 3) to (4, 3).
Join (5, 4) to (5, 3).
Join (6, 3) to (5, 2) to (6, 2) to (5, 1).
Join (10, 2) to (9, 3) to (8, 2) to (9, 1) to (10, 2) to (10, 1).
Join (11, 3) to (13, 3). Join (12, 3) to (12, 1).
Join (14, 1) to (14, 3) to (15, 3) to (15, 2) to (14, 2) to (15, 1).
Join (18, 2) to (17, 3) to (16, 2) to (17, 1) to (18, 2) to (18, 1).
Join (19, 1) to (19, 3) to (20, 3) to (20, 2) to (19, 2).

Draw a Cartesian plane with the scale along the $x$-axis extending from 0 to 21 and the scale along the $y$-axis extending from 0 to 5. Plot the points to decode the message.

**11** Plot the following points on a Cartesian plane and join them as described.
(Hint: Decide what scale you will need on each axis. For the $x$-axis scale, look for the largest $x$-coordinate and for the $y$-axis scale, look for the largest $y$-coordinate.)
Join (6, 7), (10, 11), (11, 8), (10, 6), (6, 7), (10, 5), (9, 0), (6, 7).
Join (7, 11), (6, 8), (6, 3), (5, 3), (5, 8), (4, 11).
Join (5, 8), (6, 8).
Join (5, 7), (1, 11), (0, 8), (1, 6), (5, 7), (1, 5), (4, 0), (5, 7).
Describe the shape you have created.

## EXAMPLE 4E-3 Plotting points from a table of values

For this table of values:

| $x$-coordinate | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$-coordinate | 2 | 3 | 4 | 5 | 6 | 7 |

a list the coordinates for each point
b plot each point on a Cartesian plane and join them with a smooth line
c describe whether the points lie on a curved or straight line.

### THINK

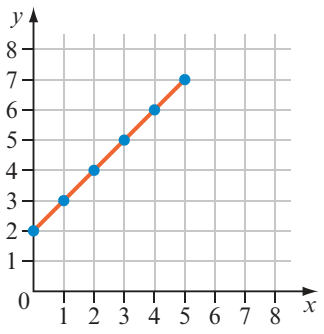
a Write the coordinates for each point. Write the $x$-coordinate first.

b Plot the points and join them in order.

c Describe the line you have drawn.

### WRITE

a (0, 2), (1, 3), (2, 4), (3, 5), (4, 6), (5, 7)

b 


c The points lie on a straight line.

**12** For each of these tables of values:
i list the coordinates for each point
ii plot each point on a Cartesian plane and join them with a smooth line (remember to first decide on the scale to be used on each axis)
iii describe whether the points lie on a curved or straight line.

a

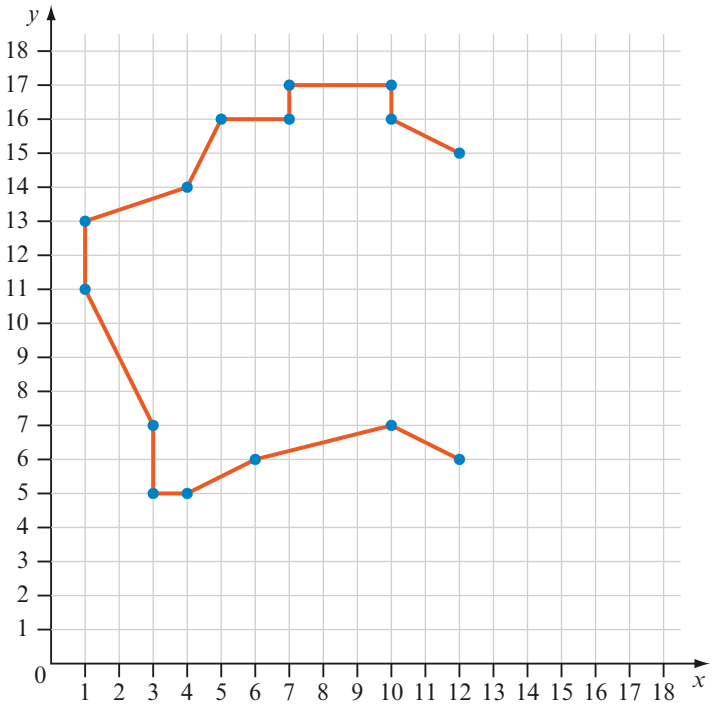
| $x$-coordinate | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $y$-coordinate | 0 | 3 | 8 | 15 |

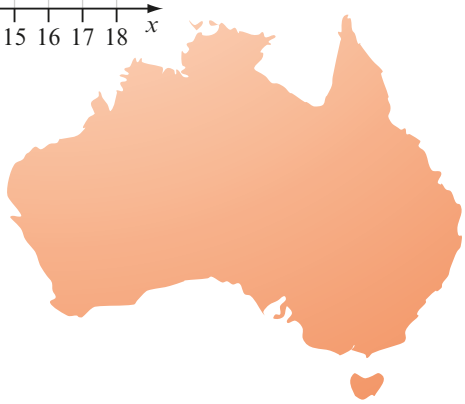
b

| $x$-coordinate | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$-coordinate | 6 | 4 | 2 | 0 |

PROBLEM SOLVING AND REASONING

**13** An incomplete map of Australia is drawn on this Cartesian plane.



**a** List the coordinates of the points to be joined in the correct order (like in question **7**) to produce the map of Australia drawn so far.

**b** Continue the coordinate instructions to complete the map.

**c** List the coordinates of all of the capital cities.

CHALLENGE

**14** Imagine that the floor of your classroom is a Cartesian plane with one axis located where the floor meets the back wall of the room and the other where the floor meets a side wall. Decide on the scale you will use. Give coordinates for everyone in your class based on where they are sitting.

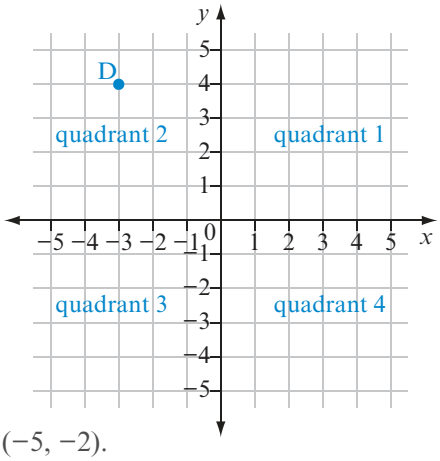
**15** Draw a Cartesian plane that extends from 0 to 12 on both axes. Connect the points (12, 0) and (0, 1) with a straight line. Then connect (11, 0) and (0, 2) and then (10, 0) and (0, 3). Continue this pattern.

**Reflect**

Why is the horizontal coordinate always written first in the coordinates of a point?

# 4F Negative numbers and the Cartesian plane

## Start thinking!




1 Copy this number plane, which is formed by drawing two number lines at right angles to each other. There are four different spaces, called **quadrants**, formed by the axes.

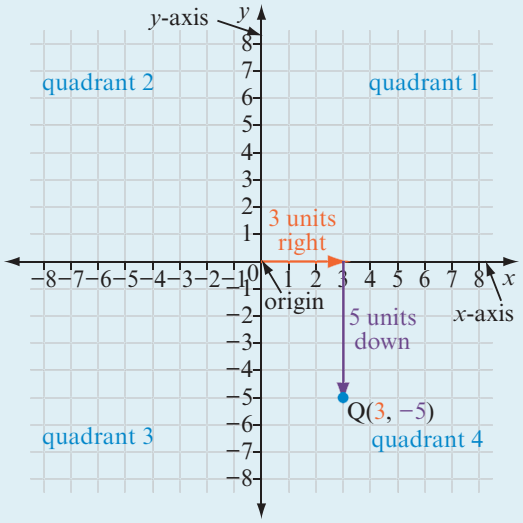
2 A point, D, has been drawn on the number plane.
- **a** Describe the position of point D from the origin.
- **b** List the coordinates of point D.
- **c** In which quadrant is point D located?

3 The position of a point, E, is described by the Cartesian coordinates (−5, −2).
- **a** Plot point E on your Cartesian plane.
- **b** Describe the method you used.
- **c** In which quadrant is point E located?

4 Plot a point of your choice in each quadrant of your Cartesian plane and then list the quadrant number and the coordinates of each point.

## KEY IDEAS

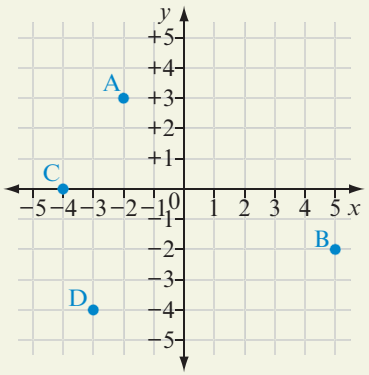
- A Cartesian plane is formed when a horizontal number line ($x$-axis) and a vertical number line ($y$-axis) intersect at right angles at the origin. When positive and negative numbers are used on the number lines, the axes divide the plane into four quadrants.
- Cartesian coordinates describe the location of a point on the Cartesian plane. The horizontal coordinate ($x$-coordinate) is always listed first, followed by a comma, then the vertical coordinate ($y$-coordinate).
- The $x$-coordinate tells you how many units left or right to move along the $x$-axis from the origin and the $y$-coordinate tells you how many units to move up or down parallel to the $y$-axis. For example, (3, −5) means to move 3 units right from the origin and then 5 units down.



# EXERCISE 4F Negative numbers and the Cartesian plane

## EXAMPLE 4F-1 Writing coordinates for points on a Cartesian plane

Write the coordinates of the points A, B, C and D, shown on this Cartesian plane.




### THINK

1 For point A, identify the $x$-coordinate by finding the number of units and direction you need to move along the $x$-axis from the origin to be under or above the point (−2). Identify the $y$-coordinate by finding the number of units and the direction you then need to move to reach the point (3). Write the $x$-coordinate first.

2 Repeat step 1 for points B, C and D.

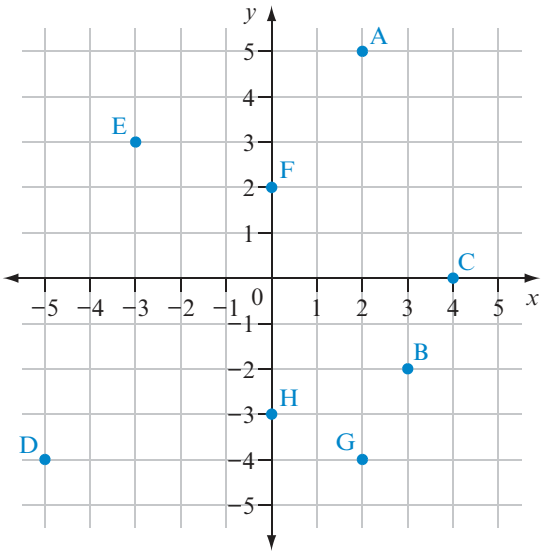
### WRITE

A is at (−2, 3).
B is at (5, −2).
C is at (−4, 0).
D is at (−3, −4).

UNDERSTANDING AND FLUENCY

1 Write the coordinates of the points A to H, shown on this Cartesian plane.

2 List the quadrant or the axis of the Cartesian plane that each point in question 1 is located.



## EXAMPLE 4F-2 Plotting points on a Cartesian plane

Plot these points on the Cartesian plane.

**a** E(4, −3) **b** F(−2, 5) **c** G(−4, −3) **d** H(0, −1)

### THINK

**a** E has an $x$-coordinate of 4 and a $y$-coordinate of −3. Move 4 units right along the $x$-axis from the origin and then 3 units down parallel to the $y$-axis. Plot the point by drawing a dot at this final position. Label the point as E.

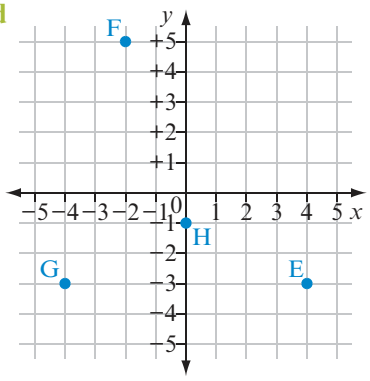
**b** F has an $x$-coordinate of −2 and a $y$-coordinate of 5. (2 units left and 5 units up)

**c** G has an $x$-coordinate of −4 and a $y$-coordinate of −3. (4 units left and 3 units down)

**d** H has an $x$-coordinate of 0 and a $y$-coordinate of −1. (0 units left or right and 1 unit down)

### WRITE

**a–d**




UNDERSTANDING AND FLUENCY

**3** Consider the coordinates of the points I to P, listed below.

**i** Describe the position of each point from the origin. For example, point I is 2 units right from the origin and then 3 units down.

**ii** Copy the set of $x$- and $y$-axes shown in question **1**. Plot these points on the Cartesian plane. Label each point with its letter.

**a** I(2, −3) **b** J(−4, −4) **c** K(−4, 3) **d** L(−1, 0)
**e** M(0, −2) **f** N(−3, 2) **g** O(5, −4) **h** P(0, 5)

**4** List the quadrant or the axis of the Cartesian plane where each point in question **3** is located.

**5** Draw a Cartesian plane and plot each set of points, then join them in the order given. Name each shape created.

**a** (−3, 5), (2, 5), (2, 3), (−3, 3), (−3, 5)
**b** (−2, −1), (1, 2), (−2, 2), (−5, −1), (−2, −1)
**c** (2, −2), (−2, −2), (−2, −6), (2, −6), (2, −2)
**d** (5, −2), (2, 0), (4, 5), (5, −2)

**6** A Cartesian plane that has positive and negative numbers on the scale for the axes has four quadrants.

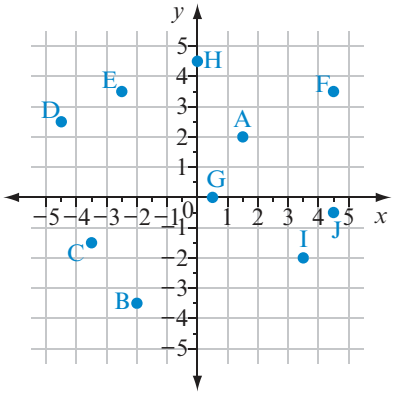
**a** Which of the quadrants has points with:

**i** all negative coordinates? **ii** all positive coordinates?

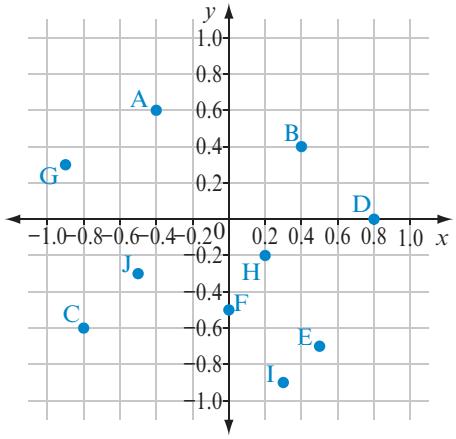
**b** Which of the quadrants has points with both positive and negative coordinates?

**7** Write the coordinates of the points A to J shown on each Cartesian plane.

**a**




**b**




**8** Draw a Cartesian plane with a scale for the axes that is suitable to plot each set of points. Label each point with its coordinates.

**a** $(1\frac{1}{3}, -2\frac{1}{3})$, $(0, -1\frac{2}{3})$, $(-\frac{2}{3}, 3)$, $(-2\frac{2}{3}, -1\frac{1}{3})$, $(2\frac{1}{3}, \frac{1}{3})$, $(\frac{1}{3}, 0)$

**b** $(-0.2, 0.4)$, $(1.8, -0.6)$, $(0.6, 0)$, $(-1.2, -1.4)$, $(0, 1.6)$, $(2, 0.8)$

**9** Draw a Cartesian plane with a horizontal axis from $-8$ to 10 and a vertical axis from $-12$ to 15. Use 1-cm grid paper to make plotting easier. Plot each point and join them together in the order shown.

Join $(5, 15)$, $(5, 9)$, $(5\frac{1}{2}, 8)$, $(5\frac{1}{2}, 7)$, $(6, 4\frac{1}{2})$, $(6, 4)$, $(4, 3\frac{1}{2})$, $(3, 3\frac{1}{2})$, $(3, 3)$, $(2, 3)$, $(1\frac{1}{2}, 2)$, $(2\frac{1}{2}, 2\frac{1}{2})$, $(3, 2)$, $(4\frac{1}{2}, -1)$, $(2, -1)$, $(2, -2)$, $(3, -3)$, $(1, -6)$, $(-1\frac{1}{2}, -5)$, $(-2, -6)$, $(-4, -6)$, $(-7, -5)$, $(-6, -4)$, $(-5\frac{1}{2}, -2\frac{1}{2})$, $(-6, -2)$, $(-5, 1)$, $(-6, 2)$, $(-4, 4)$, $(-3\frac{1}{2}, 5)$, $(-2\frac{1}{2}, 4)$, $(-2, 4)$, $(-1, 5)$, $(-2, 5\frac{1}{2})$, $(-2\ , 6\frac{1}{2})$, $(-2, 7)$, $(-1\frac{1}{2}, 7\frac{1}{2})$, $(-2, 10)$, $(-2\frac{1}{2}, 12)$, $(-3, 13\frac{1}{2})$, $(-2, 13)$, $(-2, 14)$, $(-1, 13\frac{1}{2})$, $(-\frac{1}{2}, 14\frac{1}{2})$, $(0, 14)$, $(\frac{1}{2}, 15)$, $(1, 14)$, $(1\frac{1}{2}, 15)$, $(2, 14)$, $(2\frac{1}{2}, 15)$, $(3, 14)$, $(3\frac{1}{2}, 15)$, $(4, 14)$, $(5, 15)$. Stop!

Join $(3, 3\frac{1}{2})$, $(2, 3\frac{1}{2})$, $(0, 4)$, $(-\frac{1}{4}, 3\frac{1}{4})$, $(\frac{1}{4}, 4\frac{1}{4})$. Stop!

Join $(-2, 4)$, $(-2, 3\frac{1}{2})$, $(-1\frac{1}{2}, 2\frac{1}{2})$, $(0, 2)$, $(1\frac{1}{2}, 2)$. Stop!

Join $(-5\frac{1}{2}, -2\frac{1}{2})$, $(-4, -3)$, $(-2, -3\frac{1}{2})$, $(-1, -3\frac{1}{2})$, $(1, -3)$, $(2, -2)$. Stop!

Join $(-5\ , 2\frac{1}{2})$, $(-7, 7\frac{1}{2})$, $(-8, 8\frac{1}{2})$, $(-7\frac{1}{2}, 9)$, $(-7, 8\frac{1}{2})$, $(-7, 9\frac{1}{2})$, $(-6\frac{1}{2}, 9\frac{1}{2})$, $(-6, 9)$, $(-6, 10)$, $(-5\frac{1}{2}, 10)$, $(-5, 9)$, $(-4, 9)$, $(-4, 8\frac{1}{2})$, $(-5, 7\frac{1}{2})$, $(-4\frac{1}{2}, 6)$, $(-3\frac{1}{2}, 5)$. Stop!

Join $(3, 2)$, $(6, 3)$, $(6\frac{1}{2}, 4\frac{1}{2})$, $(7, 4\frac{1}{2})$, $(7\frac{1}{2}, 3\frac{1}{2})$, $(8, 4)$, $(8\frac{1}{2}, 3\frac{1}{2})$, $(8, 3)$, $(9, 2\frac{1}{2})$, $(9, 2)$, $(8, 2)$, $(8, 1)$, $(7\frac{1}{2}, 1)$, $(7, 1\frac{1}{2})$, $(4, 0)$. Stop!

Join $(3, -3)$, $(4, -3\frac{1}{2})$, $(4\frac{1}{2}, -4)$, $(5, -5)$, $(6, -6)$, $(8, -6\frac{1}{2})$, $(8\frac{1}{2}, -8)$, $(7, -9)$, $(5, -9\frac{1}{2})$, $(3, -9)$, $(3, -6\frac{1}{2})$, $(3\frac{1}{2}, -6)$, $(3, -5)$, $(2, -4\frac{1}{2})$. Stop!

Join $(6, -6)$, $(5, -6\frac{1}{2})$, $(3\frac{1}{2}, -6)$. Stop!

Join $(-2\frac{1}{2}, -6)$, $(-1\frac{1}{2}, -7)$, $(-1\frac{1}{2}, -10)$, $(-2, -11)$, $(-3, -11\frac{1}{2})$, $(-4, -10)$, $(-4, -9)$, $(-5\frac{1}{2}, -8)$, $(-6, -7)$, $(-6, -6)$, $(-5\frac{1}{2}, -5\frac{1}{2})$. Stop!

Join $(-4, -9)$, $(-3, -8)$, $(-3, -7)$, $(-4, -6\frac{1}{2})$. Stop!

Join $(-4, -7)$ and $(-4, -6)$. Stop!

## EXAMPLE 4F-3 Plotting points from a table of values

For this table of values:

| *x*-coordinate | −4 | −2 | 0 | 2 | 4 | 6 |
|---|---|---|---|---|---|---|
| *y*-coordinate | −3 | −2 | −1 | 0 | 1 | 2 |

**a** list the coordinates for each point
**b** plot each point on a Cartesian plane and join them with a smooth line
**c** describe whether the points lie on a curved or straight line.

### THINK

**a** Write the coordinates for each point. Write the *x*-coordinate first.

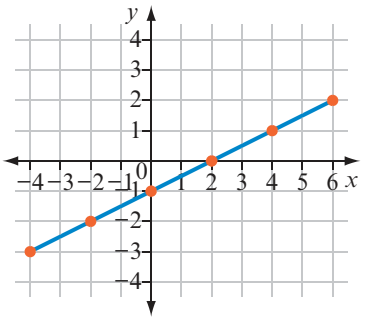
**b** Plot the points and join them in order.

**c** Describe the line you have drawn.

### WRITE

**a** (−4, −3), (−2, −2), (0, −1), (2, 0), (4, 1), (6, 2)

**b**




**c** The points lie on a straight line.

PROBLEM SOLVING AND REASONING

**10** For each of the following tables of values:
- **i** list the coordinates for each point
- **ii** plot each point on a Cartesian plane and join them with a smooth line (remember to first decide on the scale to be used on each axis)
- **iii** describe whether the points lie on a curved or straight line.

**a**

| *x*-coordinate | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| *y*-coordinate | 4 | −1 | −4 | −5 | −4 | −1 | 4 |

**b**

| *x*-coordinate | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|
| *y*-coordinate | $-1\frac{1}{2}$ | 1 | $2\frac{1}{2}$ | 3 | $2\frac{1}{2}$ | 1 | $-1\frac{1}{2}$ | −5 |

PROBLEM SOLVING AND REASONING

**11** The temperature at a ski resort was recorded every 2 hours starting at midnight. The results are shown in the table.

| Time (number of hours after midnight) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temperature (degrees Celsius) | −4 | −5 | −6 | −3 | −1 | 3 | 4 | 7 | 5 | 4 | 0 | −2 |



**a** Draw a Cartesian plane and label the horizontal axis as 'time' and the vertical axis as 'temperature'.

**b** Decide on the scale to use for your horizontal axis. Will you need negative numbers on the scale? Explain why or why not. Mark your scale on the time axis.

**c** Decide on the scale to use for the vertical axis. Mark your scale on the temperature axis.

**d** List the coordinates for each point. Remember that the horizontal coordinate is always written first.

**e** Plot the points provided in the table of values.

**f** What was the coldest recorded temperature? At what time of the day was this temperature recorded?

**g** What was the warmest recorded temperature? At what time of the day was this temperature recorded?

CHALLENGE

**12** Draw a Cartesian plane with both axes extending from −5 to 5.

**a** Plot the points A(−3, 4) and B(−3, −5).

**b** What is the distance between points A and B?

**c** Plot the point C(2, −5).

**d** What is the distance between points B and C?

**e** Find the coordinates of point D so that the points A, B, C and D form the corners of a rectangle.

**13** Plot the points with coordinates (−2, 3) and (4, 3) on a Cartesian plane.

**a** What is the distance between the points?

**b** If these two points form the corners of a square, list the coordinates of the other two vertices.

**c** Is there only one answer to part **b**? Investigate whether there are other possible answers.

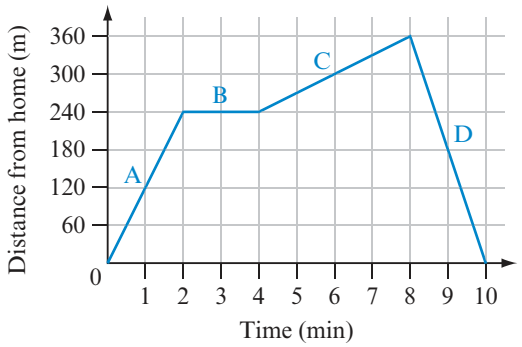
**Reflect**

Describe how to work out a suitable scale for the axes of a Cartesian plane when plotting a set of points.

# 4G Interpreting graphs

## Start thinking!

Kurt leaves home and walks first to the local shop to buy a magazine and then to his friend's house. His friend is not there so he jogs home. This **graph** represents his journey. This type of graph is often called a **travel graph**.




**1** This travel graph shows the relationship between two quantities.

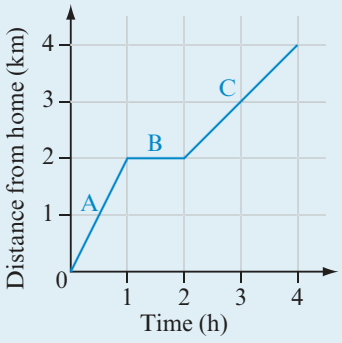
**a** What quantity is shown on the horizontal axis?

**b** What quantity is shown on the vertical axis?

**2** Use the travel graph to fully describe Kurt's journey. Think about how far he has travelled and how fast he is travelling at different times.

## KEY IDEAS

- A travel graph (or distance–time graph) shows the relationship between distance and time.
- The slope of a graph shows how much one amount is changing compared to the other. The steeper the slope is, the bigger the change.
- The slope of a travel graph indicates how quickly the distance is changing.
- The steeper the slope is, the bigger the change in distance for a given time interval (or the higher the speed). For example, section A has a steeper slope than section C in the graph shown, so the person is travelling faster in section A.
- A horizontal slope shows that there is no change in the distance. For example, section B shows that the person is stationary, with a speed of zero.
- Speed can be calculated by dividing the distance travelled by the time taken.



# EXERCISE 4G Interpreting graphs



Questions 1–7 relate to the travel graph of Kurt's journey, described opposite. The graph is made up of line segments which are labelled A, B, C and D.



**1** Look at section A of the graph.

**a** How far from home is Kurt after:

**i** 0 minutes? **ii** 1 minute?

**iii** 2 minutes?

**b** What distance did he travel between:

**i** 0 and 1 minute? **ii** 1 and 2 minutes?

**c** What does this tell you about his journey in the first 2 minutes?

**2** Look at section B of the graph.

**a** How far from home is Kurt after:

**i** 2 minutes? **ii** 3 minutes? **iii** 4 minutes?

**b** What distance did he travel between:

**i** 2 and 3 minutes? **ii** 3 and 4 minutes?

**c** What does this tell you about his journey during the time interval from 2 minutes to 4 minutes?

**3** Look at section C of the graph.

**a** How far from home is Kurt after:

**i** 4 minutes? **ii** 5 minutes? **iii** 6 minutes?

**iv** 7 minutes? **v** 8 minutes?

**b** What distance did he travel in each minute of the time between 4 minutes and 8 minutes?

**c** What does this tell you about his journey during this time interval?

**4** Look at section D of the graph.

**a** What distance does Kurt travel from his friend's house to his home?

**b** How long does this part of the journey take?

**5** In which section of the graph (A, B, C or D) is Kurt travelling the fastest? How can you tell?

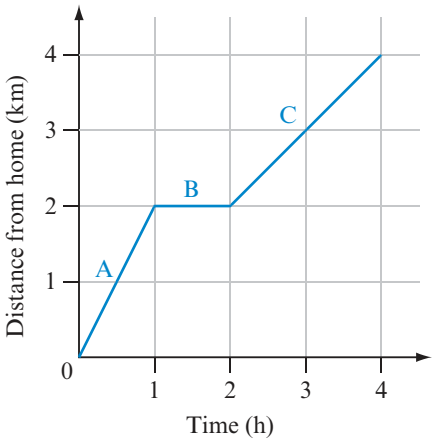
**6** What is different about section B compared to the other sections? What do sections like this on a travel graph tell you?

UNDERSTANDING AND FLUENCY

**7** Compare the slope of each section. In section A, Kurt travels a distance of 120 m in each minute compared to travelling 30 m in each minute in section C.

- **a** Work out the distance travelled in each minute for sections B and D.
- **b** Which section of the graph has the steepest slope?
- **c** Compare your answer to part **b** with the result you found in question **5**. Is there a connection between the slope of the graph and Kurt's speed? Explain.

**8** This travel graph represents Sarah's journey from her home to the music store after stopping to visit her friend along the way.




- **a** How far from home was Sarah after each of these times?
  - **i** 0 hours **ii** 1 hour **iii** 2 hours
  - **iv** 3 hours **v** 4 hours
- **b** When did Sarah reach her friend's house?
- **c** How long did she stay at her friend's house?
- **d** How far did Sarah travel from her friend's house to the music store?
- **e** In which section of the graph (A, B or C) did Sarah walk the fastest?
- **f** What does the horizontal section on the graph indicate?
- **g** Describe Sarah's journey.

## EXAMPLE 4G-1 Calculating speed from a travel graph

Use the travel graph on page 220 to calculate Kurt's speed in section C.

### THINK

1. Work out the time interval for section C.
2. Work out the distance travelled in this time interval.
3. Calculate the speed.
4. Check that your answer is reasonable. Look at the distance travelled in each minute in section C of the graph.

### WRITE

time interval = 8 − 4 = 4 min

distance travelled = 360 − 240
= 120 m

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$
$$= \frac{120 \text{ m}}{4 \text{ min}}$$
= 30 m/min

Kurt's speed in section C is 30 m/min.

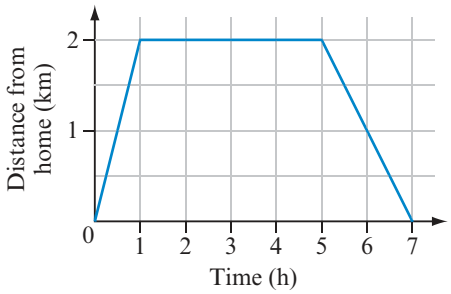
**9** Use the graph in question **8** to calculate Sarah's speed in:

- **a** section A **b** section B **c** section C.

**10** Use the graph on page 220 to calculate Kurt's speed in:

**a** section A **b** section B **c** section D.

**11** This graph represents Luke's journey to and from school.



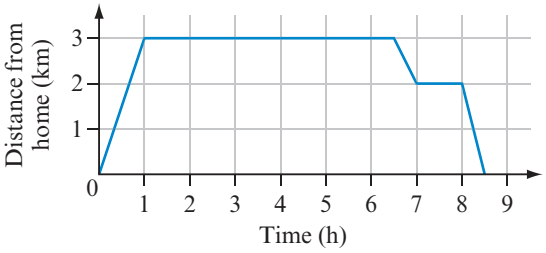

**a** What is the distance between Luke's home and school?

**b** How long was Luke at school?

**c** Did he walk faster from home to school or from school to home? Explain.

**d** According to the graph, what was the total distance he walked to and from school?

**12** Nina walked from her home to school. On the way home, she stopped for her dental appointment. This graph represents her journey.




**a** How far did Nina walk from home to school?

**b** When did she leave school to walk to the dentist?

**c** How far did she travel to get to the dental clinic?

**d** How long did she spend at her dental appointment?

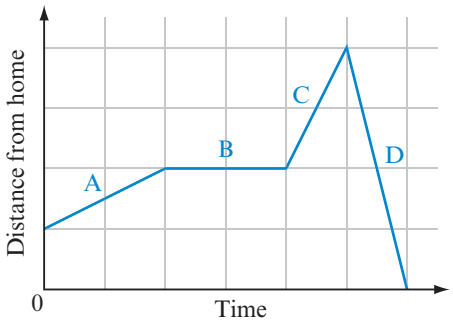
**e** What time did Nina arrive home if she left home at 7.30 am?

**f** In which section did Nina walk the fastest? What was her speed?

**g** Interpret the graph by describing Nina's journey.



**13** Use this graph of Zac's journey to answer true or false to each statement. Discuss your answers with a classmate.




**a** Section A shows that Zac is walking home.

**b** Section B shows that Zac is stationary.

**c** Section C shows that Zac is walking slower than in section A.

**d** Section D shows that Zac is walking faster than in section C.

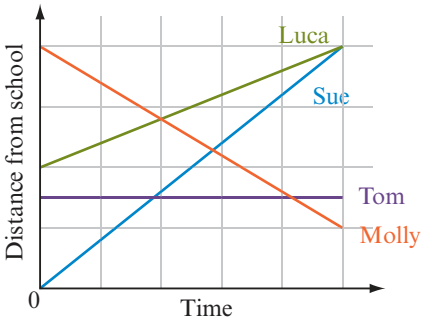
**14** Use the travel graph in question **13** to write a story about Zac's journey.

**15** Draw a graph to represent your own travel between home and school.

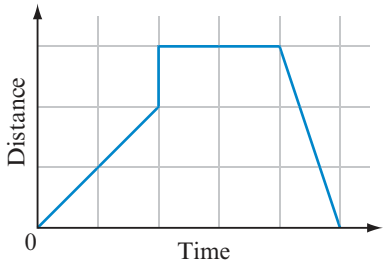
**16** This graph shows four students' travel.

**a** Which student is stationary (not moving)?

**b** Which student is travelling the fastest?




**17** Does this graph represent a journey that could really take place? Explain.




**18** Frozen soup is heated in a microwave oven. At different stages, the heating is paused so that the soup can be stirred. This graph shows the temperature change of the soup over the first 6 minutes.

**a** What is the temperature of the soup when it is first placed in the microwave oven?

**b** What is the temperature of the soup after 1 minute?

**c** When is the soup first stirred?

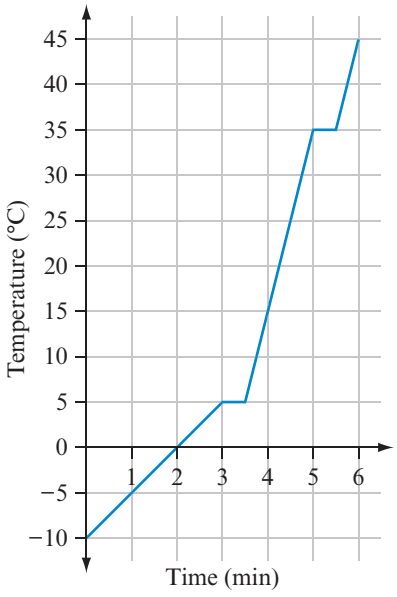
**d** The soup is stirred again at 5 minutes. What is its temperature?

**e** In which time interval/s on the graph is the temperature of the soup constant? (Hint: look for when the temperature remains the same).

**f** In which time interval/s is the temperature of the soup increasing the fastest? Explain.

**g** The soup needs to be heated to 65°C before it is ready to eat. Estimate how much longer the soup needs to be in the microwave oven.

**h** Interpret the graph by describing what happened to the soup over the time it was heated.

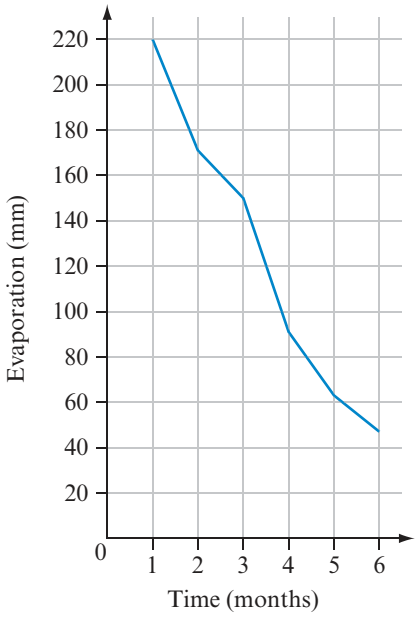





**19** Draw a graph of your own. Have a classmate interpret what the graph represents.

CHALLENGE

**20** Farmers and water authorities use information about the evaporation rate of water to manage water storage effectively. This graph shows data for evaporation recorded in Albany, Western Australia, over the first six months of a year. The values (in millimetres) are obtained using a standard-sized container (or 'Class A pan') and equipment that measures the change in the water level due to evaporation.




**a** What is the amount of evaporation in January (month 1)?

**b** What is the amount of evaporation in March (month 3)?

This table shows the amount of evaporation for the second half of the year.

| Month | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|
| Evaporation (mm) | 59 | 67 | 84 | 106 | 150 | 199 |



**c** Copy the graph above and extend the horizontal axis to include all the remaining months of the year. (Hint: show July as 7, August as 8 and so on.)

**d** Plot the values for July to December on your graph and join each point with a line.

**e** Which month of this year had the highest evaporation? What is the amount?

**f** Which month of this year had the lowest evaporation? What is the amount?

**g** Between which two months does the amount of evaporation change the most? How can you see this on the graph?

**h** Comment on your answers to parts **e**, **f** and **g**. What factors affect evaporation?

**Reflect**

What information does the slope of a graph give you?

# CHAPTER REVIEW

## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

| | | | |
|---|---|---|---|
| positive | set of axes | $x$-coordinate | distance |
| negative | $x$-axis | $y$-coordinate | time |
| integers | $y$-axis | coordinates | slope |
| opposite integers | origin | table of values | speed |
| Cartesian plane | quadrants | travel graph | stationary |

## MULTIPLE-CHOICE

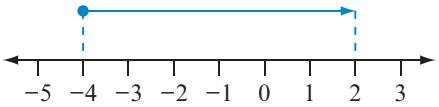
4A **1** Which integer lies between −3 and 3?

**A** −5 **B** −1.5
**C** −1 **D** 4

4B **2** Which integer, when added to +5, gives −3?

**A** −8 **B** −2
**C** +2 **D** +8

4C **3** Which calculation best represents this diagram?




**A** (−4) − (+6) = (+2)
**B** (−4) + (+2) = (−2)
**C** (−4) − (+2) = (−6)
**D** (−4) − (−6) = (+2)

4D **4** Which two integers give a sum of −9 and a difference of 1?

**A** −4 and 5 **B** 4 and 5
**C** 4 and −5 **D** −4 and −5

4D **5** Which of these calculations gives a positive answer?

**A** −12 − 24 **B** −12 + 24
**C** 12 − 24 **D** −12 + 12

4E **6** What are the coordinates of the origin?

**A** (1, 1) **B** (1, 0)
**C** (0, 0) **D** (0, 1)

4F **7** Which quadrant of the Cartesian plane does the point with coordinates (−3, 1) lie in?

**A** 1 **B** 2 **C** 3 **D** 4

4G **8** What does a travel graph with a horizontal line showing Justin's distance from home mean?

**A** his distance from home did not change over this time period
**B** his distance from home did change over this time period
**C** his speed increased over this time period
**D** his speed decreased over this time period

## SHORT ANSWER

4A **1** Decide whether each statement is true or false.

**a** $-3 > +3$ **b** $-12 < -5$

**c** $0 > -7$ **d** $1 < -1$

4A **2** Write each list in ascending order.

**a** 25, −5, 0, 12, −10, 2

**b** −3, 9, 6, −6, 0, −12

4A **3** Write an integer to represent each of these situations.

**a** Death Valley in California, USA, is 86 m below sea level.

**b** You have $615 in the bank.

**c** The top of Mt Everest is 8840 m above sea level.

**d** Your bank account is overdrawn by $35.

4B **4** Calculate:

**a** $(-6) + (-3)$ **b** $(+7) + (+8)$

**c** $(+2) + (-4)$ **d** $(-10) + (+9)$

4C **5** Calculate:

**a** $(+1) - (+9)$ **b** $(-4) - (-5)$

**c** $(-8) - (+6)$ **d** $(+11) - (+3)$

4D **6** Calculate:

**a** $-5 + 7$ **b** $-1 - 6$ **c** $9 - 12$

**d** $-8 + 3$ **e** $-15 + 15$ **f** $-20 - 20$

4D **7** A camel can survive temperatures that range from −29°C to 49°C. Find the difference between these two temperatures.

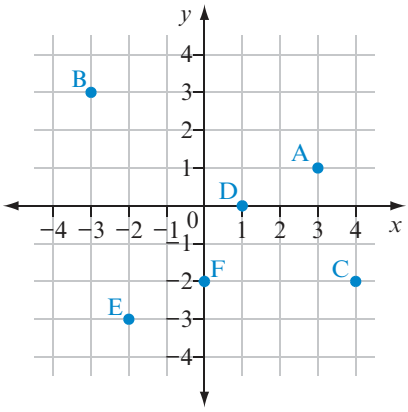
4D **8** A ship's anchor is dropped from 23 m above the water to rest on the sea bed. The depth of the water is 15 m. How far has the anchor dropped?

4E **9** **a** Copy and complete this table of values.

| *x*-coordinate | 0 | 1 | 2 | | 4 | |
|---|---|---|---|---|---|---|
| *y*-coordinate | 3 | 4 | | 6 | 7 | |
| coordinates | (0, 3) | (1, 4) | (2, 5) | (3, 6) | | (5, 8) |

**b** Draw a Cartesian plane with a suitable scale on each axis and plot each point shown in the table. Label each point with its coordinates.

4F **10** Write the coordinates of the points A to F shown on this Cartesian plane.

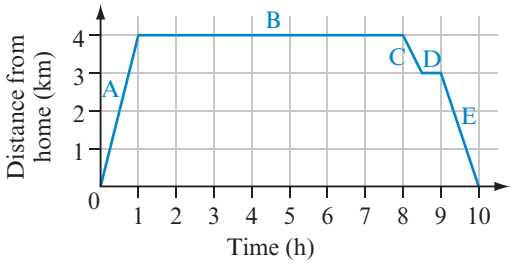



4F **11** Draw a Cartesian plane with a scale extending from −6 to 6 on both axes. Plot these points and label each point with its letter.

**a** A(−4, −2) **b** B(−1, 4)

**c** C(0, 3.5) **d** D($3\frac{1}{2}$, $-5\frac{1}{2}$)

4G **12** Ben walks from his home to school. On his way home from school, he stops at a shop with friends. This graph represents his journey.




**a** How far does Ben walk from home to school?

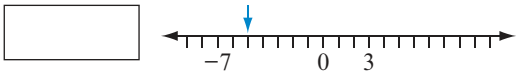
**b** In which section did Ben walk the fastest? What was his speed?

**c** Interpret the travel graph by describing what each section (A to E) represents.

## NAPLAN-STYLE PRACTICE

**1** The arrow is pointing to a position on the number line below.

What number belongs at this position?




**2** Which number is the largest?

1 ◯ −5 ◯ 2 ◯ −9 ◯

**3** Which number is the smallest?

0 ◯ −20 ◯ −10 ◯ 5 ◯

**4** When Andrew wakes up it is −3°C. By midday, it has warmed by 7°C. What is the temperature at midday?

Questions **5** and **6** refer to the following information.

Miners are working at Level 5 below the surface of the ground. Ground level is Level 0.

**5** At the end of a shift, workers use a lift that travels six levels up the vertical mine shaft. At what level do the miners leave the lift?

- ◯ Level 1 above the surface
- ◯ Level 1 below the surface
- ◯ Level 11 above the surface
- ◯ Level 11 below the surface

**6** On the way, the lift stops after travelling two levels up the mine shaft. At which level do the lift doors open?

**7** Complete the following number statement.

−5 + ☐ = −11

**8** What is the result of (−4) − (−3)?

−7 ◯ −1 ◯ 1 ◯ 7 ◯

**9** A cool room is kept at 4°C. Somebody turns the temperature down by 10°C. What is the new temperature of the cool room?

**10** A scuba diver jumps from the side of a boat and travels a vertical distance of 16 m to touch the bottom of the sea bed. If the boat's deck is 2 m above sea level, how deep is the water?

☐ m

**11** Complete the following number statement.

−4 − ☐ = 12

**12** Transactions for Peter's bank account are shown below.

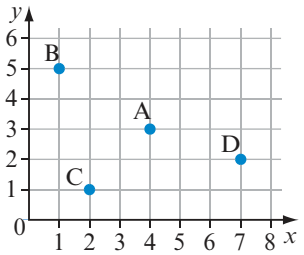
| Date | Reference | Transaction | Balance |
|---|---|---|---|
| 1 June | – | – | +$86 |
| 5 June | Deposit | +$25 | |
| 7 June | Cinemas | −$15 | |
| 11 June | Safeway | −$33 | |
| 21 June | JB Hifi | −$243 | |
| 30 June | Deposit | +$200 | |

How much is in Peter's bank account at the end of the month?

**13** Tess catches an elevator from floor 6 and gets off at floor −2. How many floors did Tess travel?

Questions **14**–**16** refer to the following figure.

The points A, B, C and D are plotted on this Cartesian plane.




**14** What are the coordinates of the point A?

(1, 5) ◯ (3, 4) ◯ (1, 2) ◯ (4, 3) ◯

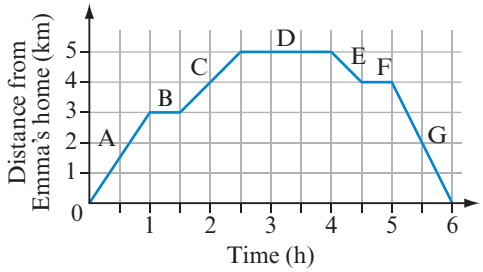
**15** Which point has the coordinates (1, 5)?

A ◯ B ◯ C ◯ D ◯

**16** A point on the Cartesian plane has the coordinates (2, 1).

The $x$-coordinate is ☐

Questions 17–19 refer to this travel graph.




Emma walks to her friend's house and together they walk to the cinema. After the film, they go to a café before returning home.

**17** Which section of the graph represents their time at the cinema?

B ☐ C ☐ D ☐ E ☐

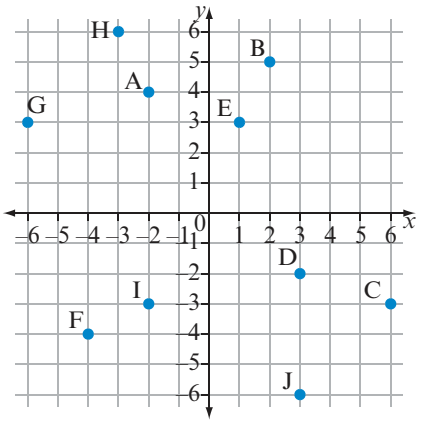
**18** Answer true or false. Emma's home is closer to the café than it is to her friend's house.

☐

**19** In which section did Emma walk the fastest?

☐

Questions 20–23 refer to the following figure. The points A to J are plotted on this Cartesian plane.




**20** Write the coordinates of point C.

☐

**21** Which point has the coordinates (−3, 6)?

A ☐ C ☐ F ☐ H ☐

**22** Which points are in the first quadrant?

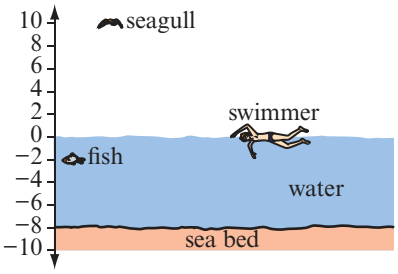
☐

**23** Which point has the same $x$-coordinate as point I?

☐

## ANALYSIS

Refer to the scale shown with the diagram to answer each of these questions.




**a** Write the position of each of these as an integer.

**i** seagull **ii** fish

**iii** swimmer **iv** sea bed

**b** Calculate the distance between the

**i** seagull and fish **ii** sea bed and fish.

**c** A parasailor is moving through the air at half the height of the seagull. Write the height of the parasailor as an integer.

**d** A sting ray is swimming three times as deep as the fish. Write its position as an integer.

**e** The seagull spots the fish under the water and dives towards it. This table of values shows the height of the seagull after spotting the fish.

| time (seconds) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| height (metres) | 10 | 8.5 | 5 | 2.5 | 0 | −1.5 | −2.5 | −1.5 | 0.5 |

The first set of values for time and height can be written as (0, 10). Write the information in the table as nine coordinate points.

**f** Plot the set of points on a Cartesian plane. Carefully label the axes.

**g** Describe the actions of the seagull during this 8 seconds.

# CONNECT

## Temperatures around the world

Temperature is a quantity that involves positive and negative numbers. The unit commonly used for temperature is degrees Celsius (°C). However, some countries, like the USA, use the temperature unit of degrees Fahrenheit (°F). How are these units different?

How can you compare different temperatures around the world?

You and your family are planning a holiday over the December and January holidays. If cost is not a consideration, where would you like to travel? Do you want a winter holiday, perhaps with snow, or a warm, summer environment with plenty of sun?

One way of considering where to travel is to look at the average temperatures in January at different places around the world. Your task is to compare temperatures at possible holiday destinations around the world to help you decide where to go.

The table shows just a few destinations and their temperatures. Notice that some of the average temperatures are given in degrees Fahrenheit. For example, the average maximum temperature in Los Angeles is 64°F. What is this temperature in degrees Celsius? Do you know a way to convert from one unit to the other?

| City, Country | Average maximum temperature in January | Average minimum temperature in January |
|---|---|---|
| Acapulco, Mexico | 31°C | 22°C |
| Athens, Greece | 12°C | 6°C |
| Bangkok, Thailand | 32°C | 21°C |
| Beijing, China | 2°C | −9°C |
| Cairo, Egypt | 18°C | 15°C |
| Las Vegas, USA | 54°F | 32°F |
| London, UK | 7°C | 2°C |
| Los Angeles, USA | 64°F | 45°F |
| Miami, Florida, USA | 75°F | 57°F |
| Moscow, Russia | −6°C | −13°C |
| New York, USA | 39°F | 27°F |
| Oslo, Norway | −1°C | −7°C |
| Paris, France | 6°C | 0°C |
| Rome, Italy | 12°C | 4°C |
| San Francisco, USA | 55°F | 41°F |
| Singapore | 30°C | 23°C |
| Sydney, Australia | 26°C | 18°C |
| Tokyo, Japan | 9°C | −1°C |
| Vancouver, Canada | 6°C | 0°C |
| Zurich, Switzerland | 2°C | −3°C |

### Your task

To investigate where to go for your holiday, follow these steps.

- Compare the temperatures of selected cities around the world.
- Convert temperatures to the same unit for easier comparison.
- Decide which of these holiday destinations suits you best.
- Investigate other holiday destinations.
- Decide which holiday destinations may suit you better at other times of the year.

Include all necessary working to justify your answers.



Of course, the average maximum temperature of a city can vary across the year. If you could select any time of the year for your holiday, would this influence your choice of holiday destination?

For example, here are the average maximum temperatures (in °C) for each month in Stockholm, Sweden.

| Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −3 | −3 | −1 | 4 | 10 | 15 | 17 | 16 | 12 | 7 | 3 | −1 |

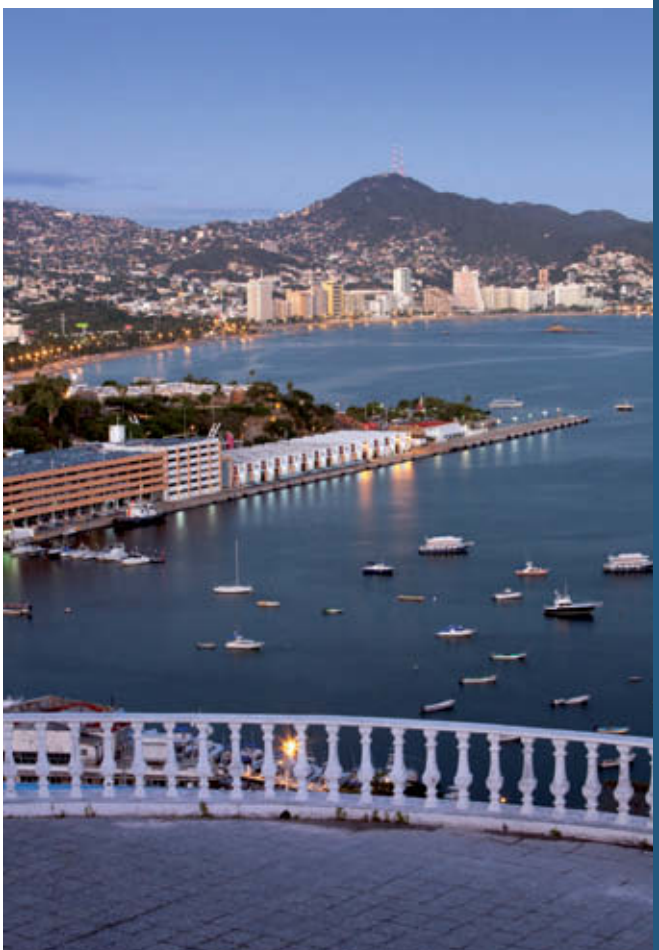










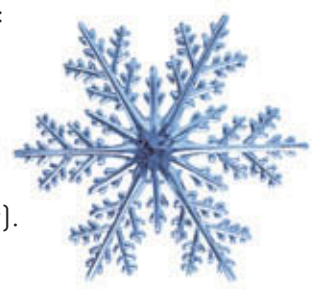



Complete the 4 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report.

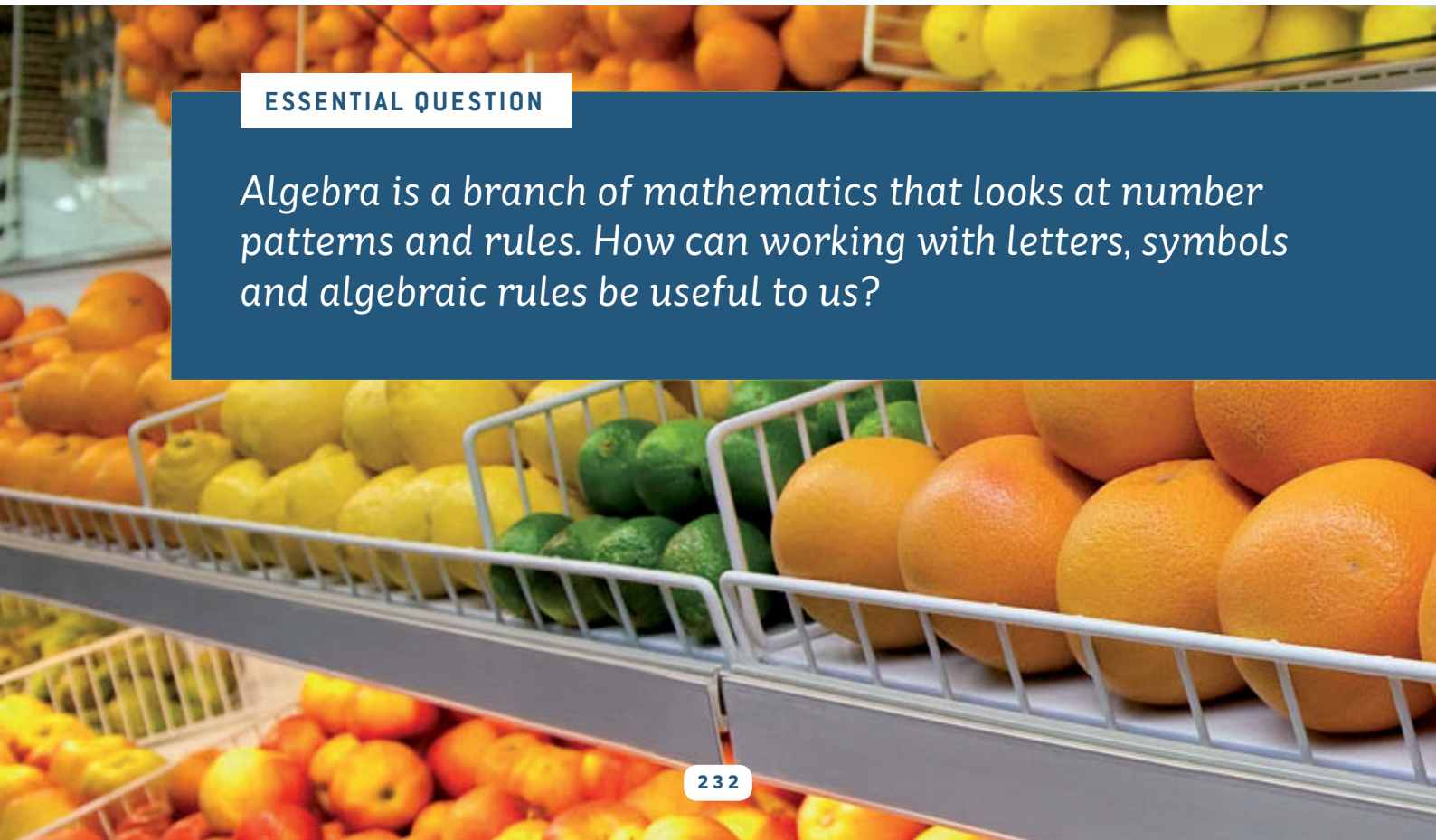
Your report could be in the form of:

- a booklet
- a poster
- a PowerPoint presentation
- a travel brochure
- other (check with your teacher).



# 5 ALGEBRA AND EQUATIONS


5A Understanding rules
5B Using pronumerals
5C Terms, expressions and equations
5D Evaluating expressions
5E Strategies for solving equations
5F Using flowcharts
5G Building expressions using flowcharts
5H Solving equations using backtracking
5I Solving equations using a balance model

**ESSENTIAL QUESTION**

*Algebra is a branch of mathematics that looks at number patterns and rules. How can working with letters, symbols and algebraic rules be useful to us?*

# Are you ready?

5A **1** Add 4 to each number.

**a** 3 **b** 5 **c** 9 **d** 1

5A **2** Subtract 3 from each number.

**a** 7 **b** 4 **c** 12 **d** 3

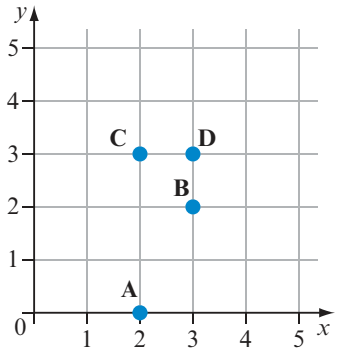
5A **3** Multiply each number by 6.

**a** 2 **b** 5 **c** 8

5A **4** Divide each number by 5.

**a** 20 **b** 5 **c** 35

5B **5** This Cartesian plane shows the position of four points. Which letter labels the point with coordinates (2, 3)?




5C **6** Rewrite each statement using numbers and one of the four operations +, –, × or ÷.

**a** the sum of 4 and 7

**b** the product of 6 and 5

**c** the difference between 8 and 3

**d** the quotient of 12 and 2

5D **7** What is 4 + 2 × 5?

**A** 11 **B** 14 **C** 30 **D** 40

5D **8** Calculate (6 + 12) ÷ 3.

5E **9** What is the missing number that makes each statement true?

**a** 4 + __ = 9 **b** 7 × __ = 42

**c** __ ÷ 3 = 8 **d** __ – 6 = 5

5E **10** Are these mathematical statements true or false?

**a** 12 + 9 = 21 **b** 7 – 2 = 4

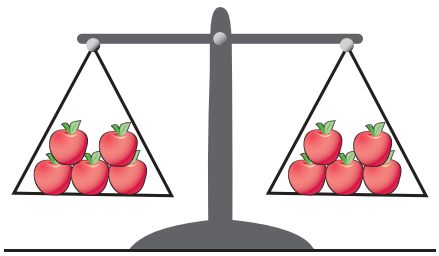
5F **11** Start with 10, subtract 4 then add 6. Divide the result by 3. What is your final answer?

**A** 2 **B** 4 **C** 6 **D** 8

5F **12** Start with 2, add 7 and multiply the result by 5. What is your final answer?

5H **13** A number is multiplied by 4 and then has 3 added to it to give 11. What number did you start with?

5I **14** This set of scales has five apples on each tray. Each apple has the same mass. The scales are balanced as each tray has the same mass. Two apples are taken off the left tray.



**a** What happens to the scales?

**A** They are balanced.

**B** They are unbalanced, with the right tray higher than the left tray.

**C** They are unbalanced, with the left tray higher than the right tray.

**D** The left tray stays at the same height as the right tray.

**b** How many apples should be taken off the right tray to balance the scales?

# 5A Understanding rules

## Start thinking!



A pasta machine flattens and stretches a piece of dough to produce a new piece that is longer.

**1** One piece of dough is 10 cm long when it goes in. It comes out of the machine 30 cm long. Another 20-cm piece goes into the pasta machine and comes out 60 cm long.
- **a** Can you see a pattern? Discuss with a partner.
- **b** Can you predict what length will come out of the machine if you know what length goes in?

**2** A third piece of dough goes in 30 cm long and comes out 90 cm long. The information for all three pieces of pasta dough is shown in this table.

| IN | 10 | 20 | 30 |
|---|---|---|---|
| OUT | 30 | 60 | 90 |

- **a** The IN and OUT values are known as **variables**. Why are they called this?
- **b** The relationship between the variables can be written as a rule, either as a sentence or as a mathematical statement. Which rule matches this pasta machine?

**A** OUT = IN + 20 **B** OUT = IN × 2 **C** OUT = IN × 3

**3** Copy and complete the table at right for this pasta machine.

| IN | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|
| OUT | 30 | 60 | 90 | | | |

**4** Use the rule to work out the length of dough:
- **a** coming out of the machine if the length going in is 100 cm
- **b** going into the machine if the length coming out is 240 cm.

**5** Another pasta machine makes the dough twice as long.
- **a** Create a table that shows the relationship between the two variables. Make up your own IN values for the table.
- **b** Write the rule for the relationship between the IN values and the OUT values for this pasta machine.

**6** Explain why a rule is useful when describing a relationship between two or more variables.

## KEY IDEAS

- Algebra is a branch of mathematics that looks at number patterns and rules.
- A rule describes a relationship between two or more variables.
- A variable is a quantity that can have varying or different values.
- A rule with two variables can be used to find the value of one variable if you know the value of the other variable.

# EXERCISE 5A Understanding rules

**1** The rule for the relationship between a set of IN and OUT numbers is: add 9 to the IN number to get the OUT number.
Find the OUT number for each of these IN numbers.

**a** 4 **b** 1 **c** 18 **d** 24
**e** 7 **f** 11 **g** 9 **h** 0

## EXAMPLE 5A-1 Using a simple rule to find missing OUT values

Use this rule to find the missing OUT numbers in the table: add 3 to each IN number.

| IN | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| OUT | | | | | |

### THINK

**1** Apply the rule to each IN number. (add 3)

**2** Write the missing OUT numbers in the table.

### WRITE

1 + 3 = 4
2 + 3 = 5
3 + 3 = 6
4 + 3 = 7
5 + 3 = 8

| IN | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| OUT | 4 | 5 | 6 | 7 | 8 |

UNDERSTANDING AND FLUENCY

**2** Use the given rule to find the missing OUT numbers in each table.

**a** Add 5 to each IN number.

| IN | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| OUT | | | | | |

**b** Multiply each IN number by 2.

| IN | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| OUT | | | | | |

**c** Subtract 6 from each IN number.

| IN | 8 | 11 | 14 | 20 | 28 |
|---|---|---|---|---|---|
| OUT | | | | | |

**d** Add 10 to each IN number.

| IN | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| OUT | | | | | |

**e** Divide each IN number by 3.

| IN | 3 | 12 | 33 | 42 | 60 |
|---|---|---|---|---|---|
| OUT | | | | | |

**f** Multiply each IN number by 8.

| IN | 0 | 1 | 5 | 8 | 11 |
|---|---|---|---|---|---|
| OUT | | | | | |

## EXAMPLE 5A-2 Using a rule with two operations to find missing OUT values

Use this rule to find the missing OUT numbers in this table: multiply each IN number by 5 and then subtract 4.

| IN | 1 | 3 | 4 | 7 | 10 |
|---|---|---|---|---|---|
| OUT | | | | | |

### THINK

1 Apply the rule to each IN number. (multiply by 5 and then subtract 4)

2 Write the missing OUT numbers in the table.

### WRITE

$1 \times 5 = 5$ then $5 - 4 = 1$
$3 \times 5 = 15$ then $15 - 4 = 11$
$4 \times 5 = 20$ then $20 - 4 = 16$
$7 \times 5 = 35$ then $35 - 4 = 31$
$10 \times 5 = 50$ then $50 - 4 = 46$

| IN | 1 | 3 | 4 | 7 | 10 |
|---|---|---|---|---|---|
| OUT | 1 | 11 | 16 | 31 | 46 |

**3** Use each given rule to find the missing OUT numbers in this table.

| IN | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| OUT | | | | | |

- **a** Multiply each IN number by 3 and then add 2.
- **b** Multiply each IN number by 7 and then subtract 5.
- **c** Subtract 3 from each IN number and then multiply by 10.
- **d** Add 2 to each IN number and then multiply by 4.
- **e** Multiply each IN number by itself and then add 6.
- **f** Multiply each IN number by itself and then subtract 9.

## EXAMPLE 5A-3 Using a rule to find missing IN and OUT values

Use this rule to find the missing values in this table: add 4 to the IN number to get the OUT number.

| IN | 0 | 3 | 15 | | | |
|---|---|---|---|---|---|---|
| OUT | | | | 5 | 26 | 10 |

### THINK

1 Apply the rule to each of the three given IN numbers (0, 3 and 15) to find the OUT number.

2 Find the IN numbers that, when 4 is added to them, gives the OUT numbers (5, 26 and 10).

3 Write the missing numbers in the table.

### WRITE

$0 + 4 = 4$
$3 + 4 = 7$
$15 + 4 = 19$

$? + 4 = 5$ so IN number is 1.
$? + 4 = 26$ so IN number is 22.
$? + 4 = 10$ so IN number is 6.

| IN | 0 | 3 | 15 | 1 | 22 | 6 |
|---|---|---|---|---|---|---|
| OUT | 4 | 7 | 19 | 5 | 26 | 10 |

**4** Use each given rule (which describes how to get the OUT number) to find the missing numbers in this table.

| IN | 6 | 8 | 12 | | | |
|---|---|---|---|---|---|---|
| OUT | | | | 8 | 28 | 16 |

**a** Add 7 to each IN number.

**b** Subtract 1 from each IN number.

**c** Multiply each IN number by 4.

**d** Divide each IN number by 2.

**e** Add 2 to each IN number.

**f** Multiply each IN number by 2.

**5** Use the given rule to find the missing numbers in each table.

**a** To get the OUT number, add 3 to each IN number and then multiply by 2.

| IN | 0 | 4 | 7 | | | |
|---|---|---|---|---|---|---|
| OUT | | | | 16 | 22 | 30 |

**b** To get the OUT number, multiply each IN number by 4 and then add 1.

| IN | 2 | 3 | 4 | | | |
|---|---|---|---|---|---|---|
| OUT | | | | 25 | 29 | 33 |

**c** To get the OUT number, subtract 2 from each IN number and then multiply by 3.

| IN | 3 | 5 | 7 | | | |
|---|---|---|---|---|---|---|
| OUT | | | | 21 | 24 | 36 |

**d** To get the OUT number, divide each IN number by 5 and then subtract 2.

| IN | 10 | 15 | 20 | | | |
|---|---|---|---|---|---|---|
| OUT | | | | 3 | 6 | 8 |

**6** Use each table to complete the rule for the relationship between the set of IN and OUT numbers.

**a** Rule: add ____ to each IN number to get the OUT number.

| IN | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| OUT | 13 | 14 | 15 | 16 | 17 |

**b** Rule: divide each IN number by ____ to get the OUT number.

| IN | 4 | 8 | 12 | 16 | 20 |
|---|---|---|---|---|---|
| OUT | 1 | 2 | 3 | 4 | 5 |

**7** Use each table to write the rule for the relationship between the set of IN and OUT numbers.

**a**

| IN | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| OUT | 10 | 11 | 12 | 13 | 14 |

**b**

| IN | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| OUT | 3 | 6 | 9 | 12 | 15 |

**c**

| IN | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|
| OUT | 11 | 12 | 13 | 14 | 15 |

**d**

| IN | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| OUT | 14 | 28 | 42 | 56 | 70 |

**e**

| IN | 8 | 16 | 24 | 32 | 40 |
|---|---|---|---|---|---|
| OUT | 1 | 2 | 3 | 4 | 5 |

**f**

| IN | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| OUT | 15 | 17 | 19 | 21 | 23 |

**g**

| IN | 20 | 22 | 24 | 26 | 28 |
|---|---|---|---|---|---|
| OUT | 6 | 8 | 10 | 12 | 14 |

**h**

| IN | 0 | 33 | 55 | 77 | 99 |
|---|---|---|---|---|---|
| OUT | 0 | 3 | 5 | 7 | 9 |

**8** Match each rule written as a sentence with its corresponding mathematical statement.

| Sentence | Mathematical statement |
|---|---|
| **a** Multiply the IN number by 7 to get the OUT number. | **A** OUT = IN + 7 |
| **b** Add 7 to the IN number to get the OUT number. | **B** OUT = IN − 7 |
| **c** Multiply the IN number by 7 and then add 7 to get the OUT number. | **C** OUT = IN × 7 |
| **d** Subtract 7 from the IN number. | **D** OUT = IN × 7 + 7 |

**9** Write as a mathematical statement each rule described in:

- **i** question **4**
- **ii** question **5**



**10** Apples are put in a machine to produce juice. For each apple that goes in, 100 mL of juice comes out.

The rule for this relationship is: multiply the number of apples by 100 to get the amount of juice in millilitres.

**a** Use the rule to complete this table.

| IN (number of apples) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| OUT (amount of juice in mL) | | | | | | | |

**b** What are the two variables in this relationship?

**c** What amount of juice comes out of the machine:

- **i** if six apples are put in?
- **ii** if 13 apples are put in?
- **iii** if 20 apples are put in?

**d** How many apples would need to be put in the machine:

- **i** to have 400 mL of juice come out?
- **ii** to have 900 mL of juice come out?
- **iii** to have 3000 mL of juice come out?



**11** The total cost of buying muesli bars is related to the number of muesli bars bought, as shown in this table.

| Number of muesli bars | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Total cost ($) | 2 | 4 | 6 | 8 | 10 | 12 |

**a** This relationship can be represented by a rule. Complete this sentence:
Rule: _____ the number of muesli bars by _____ to find the total cost in dollars.

**b** What are the two variables in this relationship?

**c** What is the total cost of buying:
**i** two muesli bars? **ii** five muesli bars?

**d** What is the total cost of buying:
**i** 11 muesli bars? **ii** 38 muesli bars?

**e** To get your answers to part **c**, was it easier to use the table or the rule? Explain.

**f** To get your answers to part **d**, was it easier to use the table or the rule? Explain.

**g** How many muesli bars could you buy for:
**i** $6? **ii** $34?




CHALLENGE

**12** A pattern can be seen when shapes containing different numbers of triangles are made using toothpicks.



1 triangle 2 triangles 3 triangles

**a** Copy and complete this table to show the relationship between the number of triangles and the number of toothpicks needed to make the shape.

| Number of triangles | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of toothpicks | | | | | | |

**b** What are the two variables?

**c** How many toothpicks are needed to make a shape containing:
**i** five triangles?
**ii** 14 triangles?

**d** Write the rule for the relationship between the number of triangles and the number of toothpicks in a sentence.

**e** Write this rule as a mathematical statement.

**f** Use the rule to find:
**i** the number of toothpicks needed to make a shape containing 120 triangles
**ii** the number of triangles that could be made with 237 toothpicks.

**Reflect**

Why is a rule more useful than just creating a table?

# 5B Using pronumerals



## Start thinking!

Three pencils from your pencil case have been placed on the desk.
The total number of pencils you have depends on how many pencils are left in the case.

**1** How many pencils do you have in total, if there are:
**a** two pencils in the case? **b** five pencils in the case?

**2** Copy and complete this table to show the relationship between the number of pencils in the case and the total number of pencils.

| IN (number of pencils in the case) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| OUT (total number of pencils) | | | | | | |

**3** What are the two variables in this relationship?

**4** Write the rule for the relationship using the words IN and OUT. (Hint: what happens to the IN number to get the OUT number?)

Mathematicians have developed a shorthand way of writing variables, using pronumerals instead of IN and OUT. A **pronumeral** is a letter or symbol that takes the place of a number. For example, replace IN with $x$ and OUT with $y$ to write the rule. When a rule is written like this, it is called a **formula**.

**5** Which formula matches your rule?
**A** $x = y + 3$ **B** $y = x + 3$ **C** $y = x \times 3$

**6** Copy and complete this new version of the table of values using the pronumerals $x$ and $y$ as the variables.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**7** Use the relationship to work out how many pencils you have in total if there are 11 pencils in the case.

## KEY IDEAS

- A pronumeral is a letter or symbol that is used in place of a number.
- A variable can be represented by a pronumeral.
- When a rule is written using pronumerals, it is called a formula.
- Products or quotients with pronumerals can be simplified.
  For example, $x \times 7 = 7x$ and $x \div 7 = \frac{x}{7}$.
- The relationship between two variables can be shown as:
  - a rule (or formula)
  - a table of values or
  - a graph.

**NOTE** Pronumerals are printed in italics so they are not confused with units of measurement. For example, m is for metre but *m* is a pronumeral representing a variable.

# EXERCISE 5B Using pronumerals

## EXAMPLE 5B-1 Writing a rule as a formula

Write each rule as a formula using $a$ for the IN number and $b$ for the OUT number.

**a** OUT = IN × 4 **b** OUT = IN ÷ 5

| THINK | WRITE |
|---|---|
| **a** **1** Replace OUT with $b$ and IN with $a$. | **a** OUT = IN × 4<br>$b = a \times 4$ |
| **2** Simplify the formula by writing $a \times 4$ as $4a$. Notice that the number is written before the letter in the product and the multiplication sign is left out. | $b = 4a$ |
| **b** **1** Replace OUT with $b$ and IN with $a$. | **b** OUT = IN ÷ 5<br>$b = a \div 5$ |
| **2** Simplify the formula by writing $a \div 5$ as $\frac{a}{5}$. Notice that the vinculum in the fraction is equivalent to a division sign. | $b = \frac{a}{5}$ |

UNDERSTANDING AND FLUENCY

**1** Write each rule as a formula using $a$ for the IN number and $b$ for the OUT number.

**a** OUT = IN + 8 **b** OUT = IN − 5 **c** OUT = IN × 6
**d** OUT = IN ÷ 3 **e** OUT = IN × 2 **f** OUT = IN + 1
**g** OUT = IN ÷ 4 **h** OUT = IN − 9 **i** OUT = 11 × IN
**j** OUT = 6 + IN **k** OUT = 3 − IN **l** OUT = 4 ÷ IN
**m** OUT = 2 × IN + 5 **n** OUT = 6 × IN − 2 **o** OUT = IN × 3 + 7

**2** Copy and complete the formula for each relationship from the table of values.

**a** $b = a +$ ____

| $a$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $b$ | 8 | 9 | 10 | 11 | 12 |

**b** $n = m -$ ____

| $m$ | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| $n$ | 1 | 2 | 3 | 4 | 5 |

**c** $y =$ ____ $x$

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 10 | 15 | 20 | 25 | 30 |

**d** $f =$ ____

| $e$ | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|
| $f$ | 1 | 2 | 3 | 4 | 5 |

## EXAMPLE 5B-2 Writing a formula from a table of values

Use the pronumerals in the table to write a formula relating the variables.

| $x$ | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| $y$ | 3 | 4 | 5 | 6 | 7 |

**THINK**

1 Look for a pattern in the two variables.

2 Check the pattern or rule works with all values.

3 Write the rule as a formula.

**WRITE**

$5 - 2 = 3 \quad 6 - 2 = 4$
It looks like $y$ is obtained by subtracting 2 from $x$.

$7 - 2 = 5 \quad 8 - 2 = 6 \quad 9 - 2 = 7$

$y = x - 2$

**3** Use the pronumerals in each table to write a formula relating the variables.

**a**

| $x$ | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| $y$ | 5 | 6 | 7 | 8 | 9 |

**b**

| $p$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $t$ | 0 | 3 | 6 | 9 | 12 |

**c**

| $c$ | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|
| $d$ | 6 | 7 | 8 | 9 | 10 |

**d**

| $k$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $n$ | 14 | 28 | 42 | 56 | 70 |

**e**

| $x$ | 6 | 10 | 18 | 26 | 30 |
|---|---|---|---|---|---|
| $y$ | 3 | 5 | 9 | 13 | 15 |

**f**

| $m$ | 4 | 9 | 11 | 16 | 19 |
|---|---|---|---|---|---|
| $h$ | 8 | 13 | 15 | 20 | 23 |

## EXAMPLE 5B-3 Plotting the graph of a relationship from a table of values

Plot the graph of the relationship, shown as a table of values.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 6 | 7 | 8 | 9 | 10 |

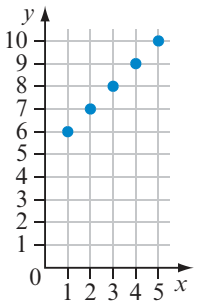
**THINK**

1 List the values as a set of coordinate points.

2 Plot the points on a Cartesian plane.

**WRITE**

(1, 6), (2, 7), (3, 8), (4, 9), (5, 10)



UNDERSTANDING AND FLUENCY

**4** Plot the graph for each relationship, shown as a table of values.

**a**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 4 | 5 | 6 | 7 | 8 |

**b**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 3 | 4 | 5 | 6 | 7 |

**c**

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 1 | 2 | 3 | 4 | 5 |

**d**

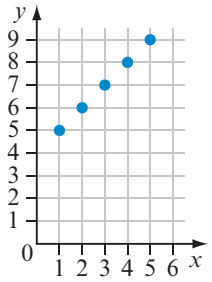
| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | 3 | 6 | 9 | 12 |

**5** Write the formula for each relationship in question **4**.

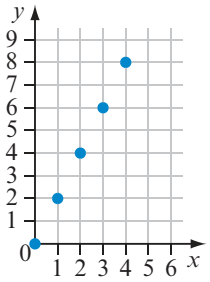
**6** For each graph below, copy and complete a table of values like this.

| $x$ | | | | | |
|---|---|---|---|---|---|
| $y$ | | | | | |

**a**




**b**




**7** Write the formula for each relationship in question **6**.

PROBLEM SOLVING AND REASONING

**8** Tobias has a full box of biscuits and four leftover biscuits from the last box. His friend wants to know how many biscuits there are altogether. Since the number of biscuits in the unopened box is unknown and can vary from one box to another, the variables can be described as:

$x$ represents the number of biscuits in the box

$n$ represents the total number of biscuits




**a** Copy and complete this table of values to show the relationship between $x$ and $n$.

| $x$ | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|
| $n$ | | | | | |

**b** Use the pronumerals to write a formula for the relationship between the two variables.

**c** Why is it important to know what the pronumerals represent?

**9** A recipe for scrambled eggs requires two eggs per person. How many eggs will be needed to make scrambled eggs for any given number of people?

- **a** What are the two variables in this situation?
- **b** Write a pronumeral for each variable and define (describe) what each one represents.
- **c** Complete a table of values that shows the relationship.
- **d** Write the formula for this relationship.

**10** Svetlana has some money in her wallet. She receives $20 as a gift. How much money does she now have? Write a formula for the relationship between the amount of money she originally had and the amount she now has. Remember to define the pronumerals used to represent the two variables.

**11** Laurence earns $50 each week from his part-time job. The formula $t = 50 - m$ represents a relationship between the amount of money Laurence spends during the week and the amount he saves at the end of the week.

- **a** What does $m$ represent?
- **b** What does $t$ represent?
- **c** Write five sets of possible values for $m$ and $t$ that would fit this formula.
- **d** Explain why $m$ and $t$ are considered to be variables.

**12** The formula $c = 5n$ relates to the cost of buying hamburgers.

- **a** What could $n$ represent?
- **b** What could $c$ represent?
- **c** Write five sets of possible values for $n$ and $c$ that would fit this formula.




**13** Write the formula for three different relationships of your own choice and explain what the pronumerals represent in each case.

**14** Copy and complete the formula for each relationship from the table of values. Notice that the rule involves more than one operation on a variable and so finding the formula can be more difficult.

**a** $y = 2x +$ ____

| | | | | | |
|---|---|---|---|---|---|
| | | +1 | +1 | +1 | +1 |
| $x$ | 1 | 2 | 3 | 4 | 5 |
| $y$ | 5 | 7 | 9 | 11 | 13 |
| | | +2 | +2 | +2 | +2 |

**b** $y = 5x -$ ____

| | | | | | |
|---|---|---|---|---|---|
| | | +1 | +1 | +1 | +1 |
| $x$ | 1 | 2 | 3 | 4 | 5 |
| $y$ | 3 | 8 | 13 | 18 | 23 |
| | | +5 | +5 | +5 | +5 |

**c** $y = 3x +$ ____

| | | | | | |
|---|---|---|---|---|---|
| | | +1 | +1 | +1 | +1 |
| $x$ | 4 | 5 | 6 | 7 | 8 |
| $y$ | 16 | 19 | 22 | 25 | 28 |
| | | +3 | +3 | +3 | +3 |

PROBLEM SOLVING AND REASONING

**15** Look at the numbers written above and below each table of values in question **14**. They show a pattern in the $x$ values and a pattern in the $y$ values that can help you find the formula.

**a** Describe the pattern in the $x$ values.

**b** Describe the pattern in the $y$ values.

The numbers shown above and below each table are called differences.
The $x$ differences show the difference between one $x$ value and the next;
the $y$ differences show the difference between one $y$ value and the next.

**c** Compare the pattern in the $y$ values with the formula in each case.

**d** Explain how differences can help you work out the formula in more difficult cases.

**16** Use the strategy of finding differences to work out the formula for each relationship.

**a**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 5 | 8 | 11 | 14 | 17 |

**b**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 1 | 5 | 9 | 13 | 17 |

**c**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 8 | 10 | 12 | 14 | 16 |

**d**

| $x$ | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| $y$ | 22 | 29 | 36 | 43 | 50 |

**e**

| $x$ | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| $y$ | 4 | 7 | 10 | 13 | 16 |

**f**

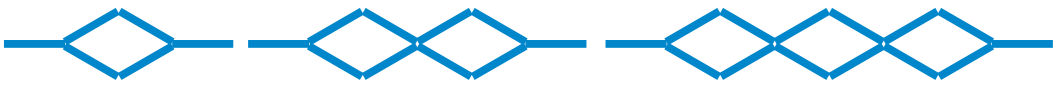
| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 6 | 11 | 16 | 21 | 26 |

**17** Plot the graph of each relationship in question **16**. Label each graph with its formula.

CHALLENGE

**18** The formula $y = 4x + 2$ represents a relationship that relates to designs of a rollerblading track. Each track is made up of a different number of identical concrete slabs.

Aerial plan of some possible tracks



one diamond | two diamonds | three diamonds

**a** What does $x$ represent?

**b** What does $y$ represent?

**c** If there are 22 concrete slabs available, describe the design of the largest rollerblading track that could be made.

**19** Write a similar formula to that shown in question **18** for another relationship of your own choice and explain what the pronumerals represent.



**Reflect**

What are pronumerals and how are they useful?

# 5C Terms, expressions and equations



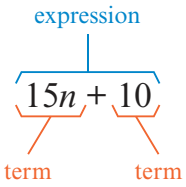
## Start thinking!

The cost of hiring a canoe is $15 per hour plus an extra booking fee of $10.

**1** What would be the total cost of hire if you booked a canoe for: **a** 1 hour? **b** 3 hours?

**2** If $n$ represents the number of hours the canoe is to be hired for, what would $15n$ represent?

**3** You could write $15n + 10$ to represent this situation. This is an example of an algebraic **expression**. It contains two **terms**. What does $15n + 10$ represent?




**4** The first term is made up of a number and a pronumeral multiplied together. The number is called the **coefficient** of the term.

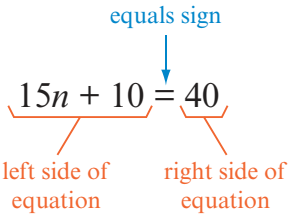
**a** Write the first term of the expression.

**b** In the first term, what is: **i** the variable? **ii** the coefficient?

**5** The second term is a number. As it does not contain a pronumeral, it is known as a **constant**.

**a** Write the constant term of the expression. **b** How is it different from the first term?

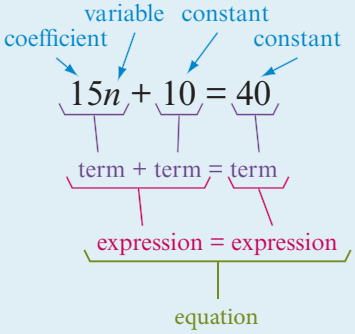
**6** You have $40 to spend on hiring a canoe. This means that $15n + 10$ needs to be 40, or $15n + 10 = 40$. Now the statement is called an **equation**. What is the value of $n$ that would make the equation a true statement?




**7** For how long can you hire a canoe if you have $40?

## KEY IDEAS

- Expressions and equations are made up of terms.
- A term can be a number, or the product of a number and one or more pronumerals. Some examples of terms are: 2, $6a$, $5mn$, $p$, $-3x$.
- Terms can be added to form an expression; for example, $4a + 5bc + 6$.
- Equations always have an equals sign. Expressions do not.
- A formula is a special type of equation where there is more than one variable; for example, $y = 2x + 3$.
- The number multiplied to a variable is known as the coefficient. For example, the coefficient of $3x$ is 3.
- A term without any pronumerals is known as a constant.



# EXERCISE 5C Terms, expressions and equations

UNDERSTANDING AND FLUENCY

**1** Answer true or false for each statement. For the statements that are false, provide a reason for your answer.

**a** $y = 3x + 2$ is an expression
**b** $4d$ is a term
**c** $7k - 3$ is an expression
**d** the constant in $5a + 3$ is 5
**e** $m = n + 9$ is a formula
**f** the coefficient of $x$ is 1
**g** $8n = 24$ is an expression
**h** the coefficient of $6y$ is 6
**i** the constant in $4c + 5$ is 5
**j** $8 + 3a$ is an equation
**k** $6x + 2$ has two terms
**l** $5 - 2y$ is an expression

**2** State whether each of these is an expression or an equation.

**a** $3x + 2 = 5$
**b** $x - 6$
**c** $8 = 4 + y$
**d** $a + b + c = 11$
**e** $m + 2n - 5$
**f** $5 - 2abc$
**g** $d = 3f + 7$
**h** $x + 2y + 4 - k$
**i** $k = 8m$
**j** $p + q + 2r + 5$

**3** Which of the equations in question **2** is also a formula?

## EXAMPLE 5C-1 Understanding expressions

For the expression $3a + b + 7$, identify:

**a** the number of terms in the expression
**b** any variables used in the expression
**c** the coefficient of each term containing a variable
**d** any constant terms.

| THINK | WRITE |
|---|---|
| **a** Count the number of terms separated by + (or −) signs. | **a** There are three terms. |
| **b** The variables are represented by pronumerals. Identify the pronumerals. | **b** The variables are $a$ and $b$. |
| **c** Look for the terms containing variables ($3a$ and $b$). As $3a = 3 \times a$, the coefficient of $3a$ is 3. As $b = 1 \times b$, the coefficient of $b$ is 1. | **c** The coefficient of $3a$ is 3. The coefficient of $b$ is 1. |
| **d** Identify any term that contains no pronumerals. | **d** The constant term is 7. |

**4** For each expression, identify:

**i** the number of terms in the expression
**ii** any variables used in the expression
**iii** the coefficient of each term containing a variable
**iv** any constant terms.

**a** $5x + 6$ **b** $2a + 4b + 3$ **c** $7mn + 4$
**d** $2 + 11k$ **e** $4c + 7d$ **f** $13y$
**g** $f + 5$ **h** $6x + w + 1 + 3y$ **i** $8p + 10 + m$

## EXAMPLE 5C-2 Writing expressions

Write an expression for each statement.

**a** the sum of $a$ and 3 **b** 9 less than $b$
**c** twice the value of $c$ **d** $d$ divided by 5

### THINK

**a** Finding the sum means to add. Write $a$ plus 3.

**b** Finding a number less than another is to subtract. Subtract 9 from $b$.

**c** Finding twice the value of a number is to multiply by 2. Multiply $c$ by 2. Write the product in a simpler way with the × sign left out.

**d** Write $d$ divided by 5 as a fraction. Remember that the ÷ sign is equivalent to the vinculum in a fraction.

### WRITE

**a** Expression is $a + 3$.

**b** Expression is $b - 9$.

**c** $c \times 2 = 2c$
Expression is $2c$.

**d** $d \div 5 = \frac{d}{5}$
Expression is $\frac{d}{5}$.

UNDERSTANDING AND FLUENCY

**5** Write an expression for each statement.

**a** the sum of $a$ and 7 **b** $b$ divided by 2 **c** 5 less than $c$
**d** 3 times the value of $d$ **e** $e$ subtracted from 8 **f** twice the value of $f$
**g** 4 divided by $g$ **h** 9 more than $h$ **i** the sum of $k$, $m$ and $n$

**6** How many terms are there in each expression for question **5**?

**7** Write an expression that has four terms.

**8** Write an expression for each statement. Use $a$ for the unknown number.

**a** 8 more than a number
**b** 15 less than a number
**c** a number is multiplied by 3
**d** a number is divided by 7

**9** Write an expression for each statement. Use $n$ for the unknown number.

- **a** a number is multiplied by 4 and then 5 is added to the result
- **b** a number is divided by 3 and then 7 is added to the result
- **c** a number is doubled and then 9 is subtracted from the result
- **d** a number is multiplied by 6 and then 1 is subtracted from the result

**10** Imagine that you repeat question **9** using $x$ instead of $n$. What difference would this make to your answers?

**11** Daniel starts with an unknown number which he represents with $x$.

- **a** He adds 3 to the number. What expression does he make?
- **b** To multiply the result by 4, he uses a pair of brackets to group the result before multiplying. Copy and complete this statement: $(x + __) \times 4$ or $__ \times (x + __)$.
- **c** Copy and complete this statement to show how Daniel can write this expression more simply: $__(x + __)$.

> **NOTE** The × sign between 4 and the expression in brackets can be left out. This is the same as when you write $4 \times a$ more simply as $4a$.

**12** Caitlin starts with an unknown number which she represents with $k$.

- **a** She adds 7 to the number. What expression does she make?
- **b** To divide the result by 3, she can use a pair of brackets to group the result before dividing. Copy and complete this expression: $(k + __) \div __$.
- **c** Another way of writing this expression is as a fraction where the vinculum replaces the brackets used for grouping terms together. Copy and complete this expression: $\frac{k + __}{__}$.

> **NOTE** Using a pair of brackets or a vinculum in a fraction are two ways of grouping a number of terms together. They are often called grouping symbols.

**13** Write each expression more simply.

- **a** $7 \times (x + 6)$
- **b** $2 \times (x - 3)$
- **c** $(x + 1) \times 5$
- **d** $(x - 2) \times 8$
- **e** $(x + 4) \div 7$
- **f** $(x - 5) \div 11$
- **g** $(9 + x) \div 2$
- **h** $(1 - x) \div 6$

**14** Use a pair of brackets or a vinculum to write expressions for these statements. Choose a pronumeral for the unknown number.

- **a** add 4 to a number and then multiply the result by 5
- **b** subtract 11 from a number and then multiply the result by 2
- **c** add 6 to a number and then divide the result by 7
- **d** subtract 3 from a number and then divide the result by 4
- **e** subtract a number from 10 and then multiply the result by 3
- **f** subtract a number from 6 and then divide the result by 9

**15** Write five different expressions of your own choice.

UNDERSTANDING AND FLUENCY

**16** Write an equation for each statement. Use $x$ for the unknown number.

- **a** 5 more than a number is equal to 11.
- **b** 3 less than a number is equal to 4.
- **c** a number multiplied by 6 is equal to 18.
- **d** a number divided by 2 is equal to 5.
- **e** twice a number is equal to 24.
- **f** half a number is equal to 9.

**17** Write five different equations of your own choice.

**18** What is different and what is the same for equations and formulas?

**19** Write five different formulas of your own choice.

PROBLEM SOLVING AND REASONING

**20** Teresa has $n$ Easter eggs.
Write an expression for each situation.



- **a** Her friend has 10 more eggs.
- **b** Her cousin has three times as many eggs.
- **c** Her younger brother has half as many eggs.
- **d** Her sister has four eggs less.

**21** Conor receives $p$ dollars per week as pocket money.

- **a** How much pocket money does he receive:
  - **i** in a fortnight?
  - **ii** in a year?
- **b** If he also earns $w$ dollars per week for washing his neighbour's car, how much does he receive each week?

**22** Your family has a weekly order of ten apples, seven bananas and six oranges from the local fruit market. The cost varies each week.



- **a** Choose a pronumeral to represent the cost of each type of fruit.
- **b** Write an expression for the cost of:
  - **i** the apples
  - **ii** the bananas
  - **iii** the oranges.
- **c** Write an expression for the total weekly cost of the fruit.
- **d** Use $w$ (for the total weekly cost of the fruit) and your answer to part **c** to write a formula for this relationship.

PROBLEM SOLVING AND REASONING

**23** Let the number of girls in a class be represented by $g$ and the number of boys in the class be represented by $b$. Write an expression for each of these, using $g$ and $b$.



**a** the number of students in the class

**b** the number of pens in the class if each student has three pens

**c** the number of people in the class, including the teacher

**d** the number of keys in the class if each girl has three keys, each boy has two keys and the teacher has four keys



**e** the number of boys in the class when three boys are away

**f** the number of students in the class when half the class are attending a sporting event

CHALLENGE

**24** Suggest a real-life situation that could be represented by each of these expressions.

**a** $a + 4$ **b** $20 - b$ **c** $6c$

**d** $\frac{d}{5}$ **e** $6e - 7$ **f** $5f + 2$

**25** If each expression in question **24** is equal to 12, write an equation for each case.

**26** Which expression in each pair of expressions represents the larger number?

**a** $x + 3$ and $x + 5$ **b** $x + 1$ and $x - 1$

**c** $x - 7$ and $x - 4$ **d** $x - 2$ and $x$

**e** $2x + 7$ and $2x + 4$ **f** $3x - 6$ and $3x + 1$

**27** If $y$ represents an unknown number, write an expression for the number multiplied by the same number. Can you write it in index form?

**Reflect**

What is the difference between an expression and an equation?

# 5D Evaluating expressions

## Start thinking!



In Australian Rules football, a team scores six points for a goal and one point for a behind.

**1** How many points are scored if a team kicks:
- **a** two goals?
- **b** three behinds?
- **c** two goals and three behinds?
- **d** five goals and eight behinds?

**2** How many points are scored if a team kicks:
- **a** $g$ goals?
- **b** $b$ behinds?

**3** Write an expression for the number of points scored by a team that kicks $g$ goals and $b$ behinds.

**4** What do the pronumerals $g$ and $b$ represent? Explain why they are useful.

**5** Replace $g$ with 2 and $b$ with 3 in the expression. This is called **substitution**. You are substituting (or replacing) each pronumeral with a particular number.

**6** Find the value of the expression by multiplying 2 by 6 and adding 3. This is called **evaluating** the expression.

**7** Compare your answers to question **6** and question **1c**.

**8** Evaluate the expression for these $g$ and $b$ values.
- **a** $g = 5, b = 8$
- **b** $g = 9, b = 6$
- **c** $g = 11, b = 7$
- **d** $g = 20, b = 12$

## KEY IDEAS

- To evaluate an expression, substitute (or replace) each pronumeral with a number and then work out the value.
- It is important to follow the correct order of operations when evaluating an expression.
- When substituting into a term that is a product of a number and one or more pronumerals, it may be easier to show the × signs as a reminder to multiply. For example, $5ab$ means $5 \times a \times b$.
- You can substitute into a formula to find the value of a variable (or pronumeral) when you know the value of the other variables.

The order of operations is:

| | |
|---|---|
| first: | **B**rackets (operations inside grouping symbols are always calculated first) |
| second: | **I**ndices (powers and square roots) |
| third: | **D**ivision and **M**ultiplication (work from left to right) |
| fourth: | **A**ddition and **S**ubtraction (work from left to right) |

# EXERCISE 5D Evaluating expressions

## EXAMPLE 5D-1 Evaluating expressions containing one pronumeral

Evaluate each expression by substituting 4 for $x$.

**a** $x + 2$ **b** $7x$ **c** $3x - 1$

**THINK**

**a** **1** Replace $x$ with 4.

**2** Write your answer.

**b** **1** Show the operation between 7 and $x$.

**2** Write your answer.

**c** **1** Replace $x$ with 4.

**2** Perform the multiplication first then the subtraction.

**WRITE**

**a** $x + 2$
$= 4 + 2$
$= 6$

**b** $7x$
$= 7 \times x$
$= 7 \times 4$
$= 28$

**c** $3x - 1$
$= 3 \times 4 - 1$
$= 12 - 1$
$= 11$

UNDERSTANDING AND FLUENCY

**1** Evaluate each expression by substituting 6 for $x$.

**a** $x + 5$ **b** $x - 2$ **c** $3x$
**d** $\frac{x}{2}$ **e** $5x + 8$ **f** $21 - x$

**2** Evaluate each expression by substituting 5 for $y$.

**a** $y + 12$ **b** $y - 1$ **c** $8y$
**d** $\frac{y}{5}$ **e** $3y - 2$ **f** $5 - y$

**3** Evaluate $3k + 5$ by substituting each of these values for $k$.

**a** $k = 1$ **b** $k = 3$ **c** $k = 4$
**d** $k = 7$ **e** $k = 0$ **f** $k = 10$

**4** Evaluate $4m - 1$ by substituting each of these values for $m$.

**a** $m = 2$ **b** $m = 5$ **c** $m = 1$
**d** $m = 8$ **e** $m = 10$ **f** $m = 9$

## EXAMPLE 5D-2 Evaluating expressions containing two pronumerals

Evaluate each expression by substituting $a = 4$, $b = 3$ and $c = 5$.

**a** $2a + b$ **b** $7(c - b)$ **c** $\frac{b + c}{2}$

**THINK**

**a** **1** Replace $a$ with 4 and $b$ with 3. Remember that $2a$ means $2 \times a$.

**2** Perform the multiplication first then the addition.

**b** **1** Replace $c$ with 5 and $b$ with 3.

**2** Perform the subtraction inside the brackets first then the multiplication.

**c** **1** Replace $b$ with 3 and $c$ with 5.

**2** Perform the addition first then the division. The vinculum of the fraction acts like a pair of brackets and groups terms together.

**WRITE**

**a** $2a + b$
$= 2 \times 4 + 3$
$= 8 + 3$
$= 11$

**b** $7(c - b)$
$= 7(5 - 3)$
$= 7 \times 2$
$= 14$

**c** $\frac{b + c}{2}$
$= \frac{3 + 5}{2}$
$= \frac{8}{2}$
$= 4$

UNDERSTANDING AND FLUENCY

**5** Evaluate each expression by substituting $a = 2$, $b = 3$ and $c = 6$.

**a** $a + 4$ **b** $b + c$ **c** $5c$

**d** $3a - c$ **e** $7a + b$ **f** $\frac{c}{b}$

**g** $6(a + c)$ **h** $\frac{a + b}{5}$ **i** $3(c - a)$

**j** $4b + 3c$ **k** $\frac{c + 4}{a}$ **l** $8a - 2b$

**6** Copy and complete this table by evaluating each expression using these values for $x$. The first two have been done for you.

| $x$ | $x + 6$ | $5x$ | $13 - x$ | $6x + 7$ | $3(x - 1)$ | $\frac{x + 4}{2}$ | $4x - 5$ | $\frac{x}{2} + 8$ | $\frac{120}{x}$ |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 8 | 10 | | | | | | | |
| 4 | | | | | | | | | |
| 6 | | | | | | | | | |
| 8 | | | | | | | | | |
| 10 | | | | | | | | | |
| 12 | | | | | | | | | |

UNDERSTANDING AND FLUENCY

**7** The term $ab$ means $a \times b$. What does $a^2$ mean?

**8** Evaluate these expressions by substituting $m = 3$ and $n = 5$.

**a** $m^2$ **b** $n^2$ **c** $m^2 + 5$

**d** $n^2 - 8$ **e** $4m^2$ **f** $2n^2 + 3$

**9** Find the value of $p$ for each of these $t$ values using the formula $p = 6t - 2$.

**a** $t = 1$ **b** $t = 9$

**c** $t = 20$ **d** $t = 6.4$

**e** $t = 5.3$ **f** $t = 0.8$

**g** $t = \frac{1}{2}$ **h** $t = \frac{2}{3}$

**NOTE** To find the value of the pronumeral on the left side of the formula, evaluate the expression on the right side of the formula.

**10** Find the value of $y$ if $x = 4$ is substituted into each formula.

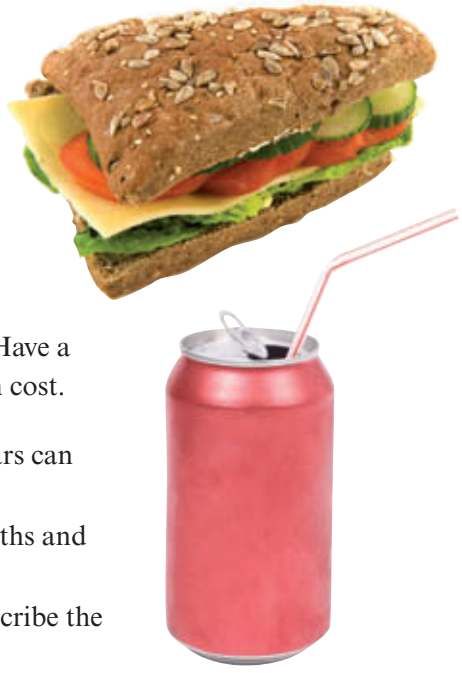
**a** $y = 19 - 3x$ **b** $y = \frac{1}{4}x$ **c** $y = \frac{3}{2}x + 7$

**d** $y = \frac{1}{5}(x + 6)$ **e** $y = \frac{2}{3}(x - 1)$ **f** $y = \frac{4}{x}$

**g** $y = \frac{28}{x} + 3$ **h** $y = 3x^2 - 15$ **i** $y = 19 - x^2$

PROBLEM SOLVING AND REASONING

**11** At the school canteen, a salad roll costs $5 and a can of drink costs $2.



**a** Write an expression for the cost of $r$ salad rolls and $d$ cans of drink.

**b** Evaluate the expression to find the cost of three salad rolls and five cans of drink.

**c** Find the cost of eight salad rolls and seven cans of drink.

**d** Choose another three sets of $r$ and $d$ values. Have a classmate evaluate the expression to find each cost.

**12** The number of months in a given number of years can be written as a formula.

**a** Choose a pronumeral for the number of months and another for the number of years.

**b** Write a formula using the pronumerals to describe the relationship between the two variables.

**c** Use your formula to find the number of months in:

**i** 10 years

**ii** 17 years

**iii** 8.5 years

**iv** 0.75 years.



**13** The number of legs on $n$ spiders can be represented by $l$.

**a** Write a formula using $l$ and $n$ to describe the relationship between them.

**b** Find the value of $l$ for each of these $n$ values.

**i** 2 **ii** 5

**iii** 8 **iv** 11

**c** Use the formula to calculate how many legs there would be on 231 spiders.

**14** The relationship between average speed ($s$), distance ($d$) and time ($t$) can be represented by the formula $s = \frac{d}{t}$.

**a** Calculate the value of $s$ for these $d$ and $t$ values.

**i** $d = 10$, $t = 2$

**ii** $d = 36$, $t = 9$

**iii** $d = 6.3$, $t = 3$

**iv** $d = 22$, $t = 4$

**b** Find the average speed (in metres per second) of a car that travels 100 m in 5 seconds.



**15** In most countries of the world, temperature is measured in degrees Celsius. Some countries, like the USA, use degrees Fahrenheit.

A formula can be used to convert from one temperature unit to the other. Let $C$ represent the temperature in degrees Celsius and $F$ represent the temperature in degrees Fahrenheit.

**a** Use the formula $C = \frac{5}{9}(F - 32)$ to convert these temperatures to degrees Celsius.

**i** 59°F

**ii** 86°F

**iii** 32°F

**iv** 95°F

**b** Use the formula $F = \frac{9}{5}C + 32$ to convert these temperatures to degrees Fahrenheit.

**i** 20°C

**ii** 100°C

**iii** 25°C

**iv** 60°C

**c** If the temperature in New York is 41°F, what is it in degrees Celsius?

**d** Which temperature is higher: 50°F or 15°C? Justify your answer.

PROBLEM SOLVING AND REASONING

**16** Copy and complete these tables of values. Substitute each $x$ value into the formula to find the corresponding $y$ value.

**a** $y = x + 3$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**b** $y = x - 2$

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**c** $y = 4x$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**d** $y = \frac{12}{x}$

| $x$ | 1 | 2 | 3 | 4 | 6 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**e** $y = 2x - 3$

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**f** $y = x^2$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**17** For each relationship shown in question **16**, draw a Cartesian plane and plot the coordinate points. Label each graph with its formula.

**18** A graph shows a **linear relationship** if the points lie in a straight line. If the points do not lie in a straight line, it is a **non-linear relationship**.

**a** To make it easier to see the relationship, draw a smooth line or curve through the points you have plotted for each graph in question **17**.

**b** Decide whether each graph is showing a linear or non-linear relationship.

CHALLENGE

**19** Follow these steps for each relationship described by its formula.

| $x$ | 1 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**i** Copy and complete a table of values like the one shown.

**ii** Plot the graph of the relationship.

**iii** Decide whether the relationship is linear or non-linear.

**a** $y = 3x + 1$

**b** $y = \frac{24}{x} - 2$

**c** $y = 2x^2 + 6$

**d** $y = \frac{x}{2} + 5$

**20** What do you notice about the variables in a formula that shows a linear relationship?

**21** Write three examples of a formula that shows a linear relationship.

**Reflect**

What is important to remember when evaluating an expression?

# 5E Strategies for solving an equation

## Start thinking!



Isabella bought an ice cream and a cola. Tania wants to buy an ice cream but isn't sure she has enough money. Isabella's cola cost $3 and she received $2 change from $10. Use $x$ to represent the unknown cost of an ice cream, in dollars.

**1** Write an expression for the cost of the ice cream plus the cola.

**2** How much did Isabella pay for both items?

**3** The relationship can be written as an equation. Use your answers to questions **1** and **2** to copy and complete: The equation is $x +$ ___ = ___.

**4** How can you work out the value of $x$?

Finding the value of $x$ is called solving the equation. To **solve** an equation is to find the value of the pronumeral that makes the equation a true statement. This value is called the **solution** to the equation.

**5** Would the solution to the equation be 4? Explain.

You may have used a strategy like 'think of a number that, when you add 3, gives 8' to work out the value of $x$. This strategy is called **solving by inspection**. It means to inspect or use observation to find the solution.

**6** Write the solution to your equation.

**7** How can you check whether your solution is correct?

## KEY IDEAS

- An equation contains an equals sign where one expression is equal to another. That is, the value of the left side of an equation equals the value of the right side of the equation.
- To solve an equation is to find the value of the variable (pronumeral) that makes the equation a true statement. This value is called the solution to the equation.
- One strategy for solving an equation is solving by inspection. This means to inspect or use observation to find the solution.
- Another strategy is 'guess, check and improve'. It is where you guess a value for the solution and check if this value makes the equation a true statement. If it doesn't, you improve the guess and try again.
- You can check if a solution is correct by substituting the value found for the pronumeral into the equation and seeing whether it makes the equation a true statement.

# EXERCISE 5E Strategies for solving an equation

**1** Substitute 4 for $x$ in the expression on the left side of each equation. Does it make the equation true?

**a** $x + 9 = 13$ **b** $x - 3 = 2$

**c** $9 - x = 1$ **d** $5x = 20$

**e** $\frac{x}{2} = 2$ **f** $3x = 7$

## EXAMPLE 5E-1 Checking a solution

Decide if $x = 11$ is the solution to the equation $x - 4 = 7$.

| THINK | WRITE |
|---|---|
| **1** Write the equation. | $x - 4 = 7$ |
| **2** Substitute 11 for $x$ in the left side (LS) of the equation and evaluate. | LS $= x - 4$<br>$= 11 - 4$ when $x = 11$<br>$= 7$ |
| **3** Write the value of the right side (RS) of the equation. | RS $= 7$ |
| **4** Compare the left and right sides of the equation. | LS = RS |
| **5** Write your decision. As $11 - 4 = 7$ is a true statement, the solution is $x = 11$. | So $x = 11$ is the solution. |

UNDERSTANDING AND FLUENCY

**2** Decide if the value given in brackets for each pronumeral is the solution to the equation.

**a** $m - 8 = 5$ $(m = 13)$

**b** $7p = 28$ $(p = 3)$

**c** $k + 5 = 14$ $(k = 8)$

**d** $11 - n = 7$ $(n = 4)$

**e** $\frac{t}{4} = 6$ $(t = 24)$

**f** $9y = 28$ $(y = 3)$

**g** $6 + c = 7$ $(c = 1)$

**h** $f - 4 = 4$ $(f = 6)$

**i** $\frac{x}{5} = 1$ $(x = 5)$

**NOTE** You know that the correct solution to an equation has been found if the value of the left side (LS) equals the value of the right side (RS) when the solution is substituted for the pronumeral.

## EXAMPLE 5E-2 Solving equations by inspection

Solve each equation by inspection.

**a** $a + 8 = 14$ **b** $b - 3 = 4$ **c** $4c = 20$ **d** $\frac{d}{5} = 9$

| THINK | WRITE |
|---|---|
| **a 1** Think of a number that, when 8 is added to it, gives 14 (6 + 8 = 14). | **a** $a + 8 = 14$ |
| **2** Write the solution. | $a = 6$ |
| **b 1** Think of a number that, when 3 is subtracted from it, gives 4 (7 − 3 = 4). | **b** $b - 3 = 4$ |
| **2** Write the solution. | $b = 7$ |
| **c 1** Think of a number that, when multiplied by 4, gives 20 (4 × 5 = 20). | **c** $4c = 20$ |
| **2** Write the solution. | $c = 5$ |
| **d 1** Think of a number that, when divided by 5, gives 9 (45 ÷ 5 = 9). | **d** $\frac{d}{5} = 9$ |
| **2** Write the solution. | $d = 45$ |

UNDERSTANDING AND FLUENCY

**3** Solve each equation by inspection.

| | | |
|---|---|---|
| **a** $a + 7 = 12$ | **b** $b - 1 = 5$ | **c** $2c = 16$ |
| **d** $\frac{d}{3} = 6$ | **e** $5e = 50$ | **f** $\frac{f}{8} = 2$ |
| **g** $g - 6 = 3$ | **h** $h + 4 = 11$ | **i** $10 - i = 6$ |
| **j** $7j = 56$ | **k** $3 + k = 13$ | **l** $l - 5 = 6$ |
| **m** $\frac{20}{m} = 5$ | **n** $6n = 42$ | **o** $\frac{x}{6} = 9$ |
| **p** $p + 11 = 11$ | **q** $9 = q - 2$ | **r** $33 = 3r$ |
| **s** $17 = 8 + s$ | **t** $2 = \frac{t}{4}$ | **u** $\frac{6}{3u} = 1$ |

**4** Check that the solutions for question **3** are correct.

**5** Explain what it means to solve an equation.

**6** Write an equation for each statement. Use $x$ for the unknown number.

**a** When 6 is added to a number, the result is 13.

**b** When 4 is subtracted from a number, the result is 8.

**c** When a number is multiplied by 9, the result is 54.

**d** When a number is divided by 2, the result is 5.

UNDERSTANDING AND FLUENCY

**e** 3 more than a number equals 14.
**f** 7 less than a number is equal to 5.
**g** Twice a number equals 18.
**h** One third of a number is 4.
**i** The sum of a number and 8 equals 21.
**j** The product of a number and 5 equals 30.

**7** Solve each equation in question **6**. Check your solutions.

**8** Which solution will make $4x + 3 = 11$ a true statement?
**A** $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$

**9** What value of $x$ will make $2x + 1 = 7$ a true statement?

**10** Solve each equation by inspection.
**a** $3x + 1 = 16$ **b** $2x - 3 = 1$ **c** $5x + 2 = 22$
**d** $4x - 1 = 11$ **e** $9 - 2x = 7$ **f** $\frac{x}{2} + 3 = 4$
**g** $\frac{x}{3} - 1 = 3$ **h** $\frac{x+1}{4} = 2$ **i** $\frac{x-2}{6} = 1$

**11** Write three different equations that have a solution of $x = 6$.

## EXAMPLE 5E-3 Solving an equation using the 'guess, check and improve' strategy

Use the 'guess, check and improve' strategy to solve $x + 6 = 15$.

### THINK

**1** Set up a table to show each guess for $x$ and a way to check if it is the solution. Start with a guess of 3 in this case.

**2** Does your next guess need to be higher or lower? Try a guess of 7 and check the solution.

**3** Repeat step 2 until the value for LS equals the value for RS.

**4** Write the solution.

### WRITE

| Guess $x$ | LS $x + 6$ | RS 15 | Check Is LS = RS? |
|---|---|---|---|
| 3 | 9 | 15 | no |
| | | | |
| | | | |
| | | | |

| Guess $x$ | LS $x + 6$ | RS 15 | Check Is LS = RS? |
|---|---|---|---|
| 3 | 9 | 15 | no |
| 7 | 13 | 15 | no |
| 10 | 16 | 15 | no |
| 9 | 15 | 15 | yes |

$x = 9$ is the solution to $x + 6 = 15$.

**12** Copy and complete this table to solve $4x + 11 = 35$ using the 'guess, check and improve' strategy.

| Guess $x$ | LS $4x + 11$ | RS 35 | Check Is LS = RS? |
|---|---|---|---|
| 3 | 23 | 35 | no |
| 9 | 47 | 35 | no |
| 5 | | 35 | |
| 7 | | 35 | |
| | | | |

**NOTE** The working for calculating LS when $x = 3$ is shown below.

$LS = 4x + 11$
$= 4 \times 3 + 11$
$= 12 + 11$
$= 23$

**13** Use the 'guess, check and improve' strategy to solve each equation.

**a** $2x + 7 = 29$ **b** $3x - 5 = 19$
**c** $6x - 4 = 74$ **d** $7x + 9 = 121$
**e** $4x - 3 = 89$ **f** $5x + 11 = 106$

**14** Copy and complete this table to solve $5x + 3 = 9$ using the 'guess, check and improve' strategy. (Hint: the solution is not a whole number.)

| Guess $x$ | LS $5x + 3$ | RS 9 | Check Is LS = RS? |
|---|---|---|---|
| 1 | 8 | 9 | no |
| 2 | 13 | 9 | no |
| 1.5 | | | |
| 1.3 | | | |
| | | | |

**15** Use the 'guess, check and improve' strategy to solve each equation. (Hint: the solutions will not be whole numbers.)

**a** $2x - 5 = 14$ **b** $4x + 0.4 = 10$
**c** $5x + 3 = 27$ **d** $3x - 14 = 7.3$



**16** T-shirts are on sale for $15 each.

**a** Write an expression for the cost of $n$ t-shirts.
**b** Write an equation for finding how many t-shirts you could buy with $60.
**c** Solve the equation to find the value of $n$.
**d** How many t-shirts can you buy with $60?



**17** A bus has 17 unoccupied seats.

**a** Write an expression for the seating capacity on the bus if $x$ passengers are on board.
**b** Write an equation for the number of passengers on board if the seating capacity is 48.
**c** Solve the equation.
**d** How many people are on board the bus?

PROBLEM SOLVING AND REASONING

**18** A box of Smarties is divided among six people.



**a** Write an expression for the number of Smarties each person receives, if the number of Smarties in the box is unknown. (Remember to define the pronumeral you are using.)

**b** Write an equation for finding the number of Smarties in the box if each person receives eight Smarties.

**c** Solve the equation.

**d** How many Smarties are in the box?

CHALLENGE

**19** Niccola has $5 and buys a sausage in bread at a fundraising event.



**a** Write an expression for the amount of change she receives.

**b** Write an equation for finding the cost of the sausage in bread if she receives $3 change.

**c** Solve the equation.

**d** How much is the sausage in bread?

**20** So far, you have looked at two different strategies for solving equations. (More strategies are described later in this chapter.) Write a description to help another student understand how each of these strategies can be used to solve equations.

**21** Use the 'guess, check and improve' strategy to solve each equation.

**a** $x^2 + 9 = 6x$ **b** $x^2 + 10 = 7x$

(Hint: there are two possible solutions to part **b**. Can you find both?)

**Reflect**

When you solve an equation, how can you be sure that your solution is correct?

# 5F Using flowcharts

## Start thinking!

In what order do you put on socks and runners?

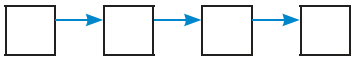

A


B


C


D

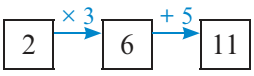
**1** **a** Copy this flowchart and arrange the photos in the correct order by writing each letter (A, B, C and D) in the boxes.



**b** Write the operations, listed at right, above the matching arrow to show the correct order of steps to follow.

Operations to follow:
tie laces
put on socks
put on runners

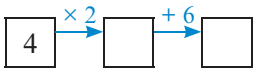
**2** Look at this flowchart, which shows what happens to a starting number (an IN number) to produce the final result (the OUT number).




**a** What is the starting number?
**b** What are the two operations shown?
**c** Which operation is performed first? Explain.
**d** What is the final result?

**3** Copy and complete these flowcharts by performing each operation in the order shown by the arrows.

**a**




**b**




**4** Following a flowchart from the final result back to the starting number is called **backtracking**.

**a** What do you notice about the flowcharts in question **3**?

**b** Compare the operations used to backtrack with those for each forward step. Which could be described as **inverse operations**? Explain.

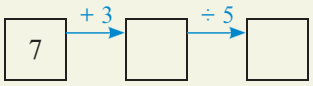
## KEY IDEAS

- A flowchart shows the correct order of operations to be performed on a starting number.
- Following a flowchart from the final result back to the starting number is called backtracking.
- To backtrack, undo the original operation at each step by performing the inverse operation.
- Adding and subtracting are inverse operations. For example, + 2 is the inverse of − 2.
- Multiplying and dividing are inverse operations. For example, ÷ 5 is the inverse of × 5.

# EXERCISE 5F Using flowcharts

## EXAMPLE 5F-1 Completing a flowchart

Copy and complete this flowchart.

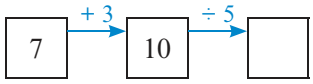


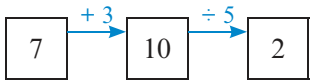

### THINK

1 Perform the first operation (add 3). Write the result in the second box (7 + 3 = 10).

2 Perform the next operation (divide by 5). Write the result in the last box (10 ÷ 5 = 2).

### WRITE

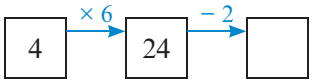


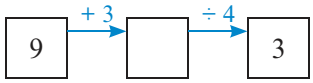

UNDERSTANDING AND FLUENCY

**1** Copy and complete each flowchart.

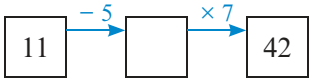
**a**



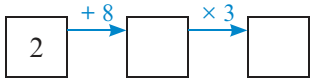

**b**



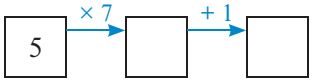

**c**



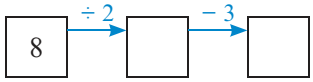

**d**




**e**




**f**




**2** Describe the steps shown in each flowchart of question **1** in words. For example, begin the description for part **a** by writing: Start with 4, multiply by 6, …

**3** Draw a flowchart to represent each description.

- **a** Start with 3, multiply by 2 and then add 8.
- **b** Start with 2, add 10 and then divide by 3.
- **c** Start with 11, subtract 5 and then multiply by 4.
- **d** Start with 8, divide by 4 and then add 1.
- **e** Start with 20, add 5 and then subtract 9.
- **f** Start with 1, multiply by 6 and then subtract 6.
- **g** Start with 35, divide by 7 and then subtract 5.
- **h** Start with 18, multiply by 2 and then divide 9.

**4** Copy and complete each flowchart.

**a** 3 → (× 7) → ☐ → (− 9) → ☐ → (÷ 2) → ☐

**b** 10 → (− 4) → ☐ → (× 5) → ☐ → (+ 1) → ☐

**c** 4 → (÷ 2) → ☐ → (− 1) → ☐ → (× 8) → ☐

**d** 9 → (+ 6) → ☐ → (÷ 3) → ☐ → (× 4) → ☐

**5** Copy and complete each flowchart.

**a** 5 → (+ 8) → ☐ → (× 2) → ☐ → (− 4) → ☐ → (÷ 11) → ☐

**b** 2 → (× 7) → ☐ → (+ 6) → ☐ → (÷ 5) → ☐ → (− 4) → ☐

**6** Copy and complete each flowchart.

**a** 6 → (× 8) → ☐ → (− 3) → 45

**b** ☐ → (× 4) → 36 → (− 7) → ☐

**c** ☐ → (÷ 3) → 5 → (+ 4) → ☐

**d** ☐ → (+ 6) → ☐ → (− 5) → 3

**e** ☐ → (× 2) → ☐ → (+ 4) → 20

**f** ☐ → (÷ 3) → ☐ → (× 4) → 24

**7** Copy and complete each flowchart.

**a** ☐ ← (− 5) ← 9 ← (+ 2) ← 7

**b** 6 ← (+ 1) ← ☐ ← (÷ 2) ← 10

**c** 15 ← (× 3) ← ☐ ← (÷ 5) ← 25

**d** ☐ ← (− 4) ← ☐ ← (× 9) ← 2

**e** ☐ ← (÷ 7) ← ☐ ← (− 6) ← 13

**f** ☐ ← (+ 8) ← ☐ ← (× 4) ← 3

**8** Answer the following questions for each flowchart.

**i** What operation is performed on 12 to get the final result?

**ii** To go from the final result back to 12, what operation is performed to undo the original operation?

**iii** Use your answers to parts **i** and **ii** to complete this sentence: The inverse operation to _____ is _____.

**a** 12 ⇄ (+ 3 / − 3) ⇄ 15

**b** 12 ⇄ (− 3 / + 3) ⇄ 9

**c** 12 ⇄ (× 3 / ÷ 3) ⇄ 36

**d** 12 ⇄ (÷ 3 / × 3) ⇄ 4

**9** Write the inverse operation to each of these.

**a** + 2 **b** × 6 **c** ÷ 5 **d** − 9 **e** × 3 **f** + 1

**10** Copy and complete these sentences using the words *add*, *subtract*, *multiply* and *divide*.

**a** To undo a step where you have added, you _____.

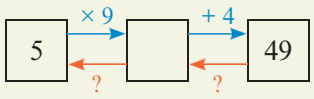
**b** To undo a step where you have subtracted, you _____.

**c** To undo a step where you have multiplied, you _____.

**d** To undo a step where you have divided, you _____.

## EXAMPLE 5F-2 Completing a flowchart and showing inverse operations

Copy and complete this flowchart. Show the inverse operations needed to backtrack to the starting number.



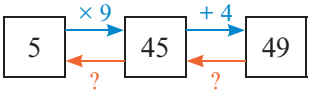

### THINK

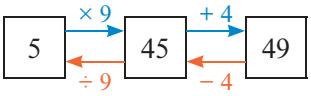
**1** Perform the first operation to find the missing value in the second box ($5 \times 9 = 45$).

**2** Write the inverse operations needed to backtrack to the starting number. The inverse of + 4 is − 4. The inverse of × 9 is ÷ 9.

**3** Check that the same values are obtained when tracking forwards or backwards.

### WRITE

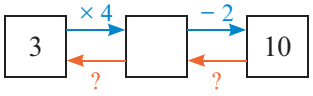


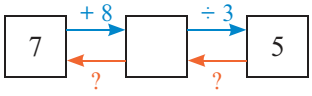

UNDERSTANDING AND FLUENCY

**11** Copy and complete each flowchart. Show the inverse operations to backtrack to the starting number.

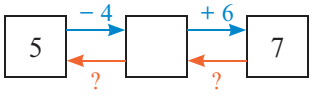
**a**



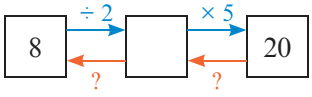

**b**




**c**

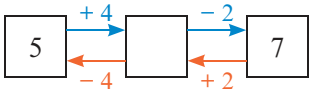



**d**

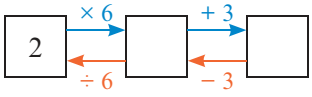



**12** Copy and complete each flowchart.

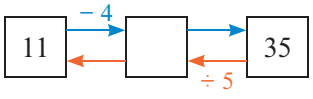
**a**



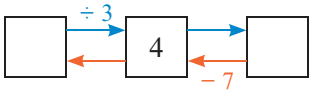

**b**



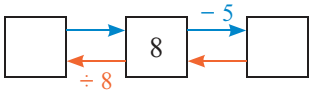

**c**



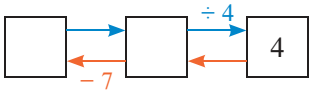

**d**




**e**

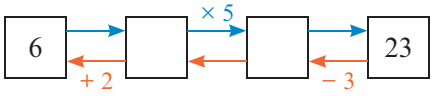



**f**

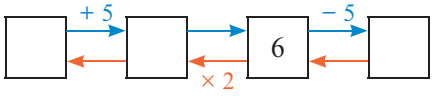


**13** Copy and complete each flowchart.

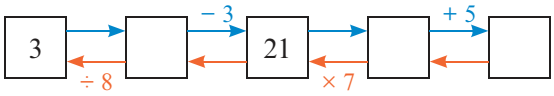
**a**



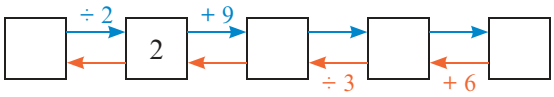

**b**




**c**




**d**




## EXAMPLE 5F-3 Using backtracking to find a mystery number

Draw a flowchart and use backtracking to find the mystery number in this description.
A mystery number is multiplied by 3 and then has 5 subtracted from it to get a final result of 19.

### THINK

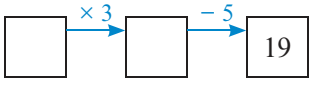
**1** Draw a flowchart with the first box empty for the mystery number. Show the operations of × 3 and − 5. Show the final result (19) in the last box.

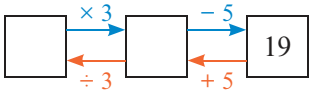
**2** Add arrows and labels to show the inverse operations.

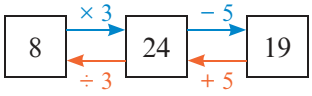
**3** Backtrack using the inverse operations to work out the missing numbers in the flowchart.

**4** Write the mystery number.

### WRITE




The mystery number is 8.

PROBLEM SOLVING AND REASONING

**14** Draw a flowchart and use backtracking to find the mystery number in each description.

- **a** A mystery number is multiplied by 5 and then has 7 subtracted from it to get a final result of 23.
- **b** A mystery number is divided by 2 and then has 11 added to it to get a final result of 18.
- **c** A mystery number has 8 added to it and then is multiplied by 3 to get a final result of 36.
- **d** A mystery number has 10 subtracted from it and then is divided by 6 to get a final result of 7.
- **e** A mystery number is multiplied by 2, then divided by 3 and then has 12 added to it to get a final result of 22.
- **f** A mystery number has 13 added to it, then is multiplied by 4 and then has 5 subtracted from it to get a final result of 55.

PROBLEM SOLVING AND REASONING

**15** Write your own mystery number problem and ask a classmate to solve it using a flowchart.

**16** Kayla received $70 for her birthday. She spent half of this on a paint set and then bought a canvas for $19.



**a** Draw a flowchart for this situation.

**b** Use the flowchart to work out how much money Kayla has left.



**17** An animal shelter provides care to injured wildlife before they are released back to their habitat. Two animals are brought in on Tuesday evening. Three are released on Wednesday. Half the remaining animals are released on Friday. Five animals are brought in at the weekend. At the start of the next week there are 12 animals in the animal shelter.

**a** Draw a flowchart for this situation.

**b** How many animals were at the shelter on Tuesday morning?

CHALLENGE

**18** A bus collects passengers from the terminal and travels towards the next town. At the first stop, four extra passengers get on. At the second stop, six passengers get on. The number of passengers on the bus is doubled after picking people up at the third stop. At the next town, 14 passengers get off, leaving 16 passengers still on board.

**a** Draw a flowchart for this situation.

**b** How many people were on the bus when it left the terminal?

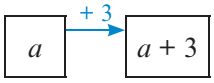
**Reflect**

What are inverse operations?

# 5G Building expressions using flowcharts

## Start thinking!

Flowcharts can be used to 'build' algebraic expressions.




**1** This flowchart shows how to build $a + 3$ from $a$.

**a** What is the starting value? **b** What operation is performed on $a$?

**c** What is the final result?

**2** Draw a flowchart to build:

**a** $b + 7$ from $b$ **b** $c - 2$ from $c$.

**3** **a** What is the variable in the expression $5d$?

**b** What operation is performed on $d$ to get $5d$? (Hint: $5d = 5 \times d$.)

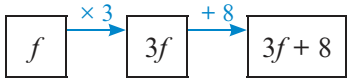
**c** Draw a flowchart to build $5d$ from $d$.

**4** **a** What is the variable in the expression $\frac{e}{4}$?

**b** What operation is performed on $e$ to obtain $\frac{e}{4}$? (Hint: $\frac{e}{4} = e \div 4$.)

**c** Draw a flowchart to build $\frac{e}{4}$ from $e$.

**5** This flowchart shows how to build $3f + 8$.




**a** What is the starting value?

**b** Which operation is performed first?

**c** What is the result of performing the first operation?

**d** Which operation is performed next?

**6** Draw a flowchart to build $2g - 5$ from $g$.

**7** Explain how to build an algebraic expression using a flowchart. Why is the order in which the operations are performed important?

## KEY IDEAS

- You can use a flowchart to build an expression starting from the variable (represented by a pronumeral).
- Each operation is performed on the entire expression in the previous box of the flowchart.
- The order in which the operations are performed is important.

# EXERCISE 5G Building expressions using flowcharts

**1** Draw a flowchart to build each expression from $x$.

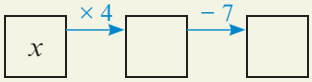
**a** $x + 5$ **b** $x - 4$ **c** $9x$

**d** $\frac{x}{3}$ **e** $15x$ **f** $x - 61$

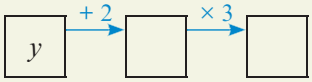
## EXAMPLE 5G-1 Completing a flowchart to build an expression

Copy and complete each flowchart to build an expression.

**a**




**b**




### THINK

**a** **1** Perform the first operation (multiply $x$ by 4) and write the result in the second box.

**2** Perform the next operation on the contents of the second box (subtract 7 from $4x$) and write the final result in the last box.

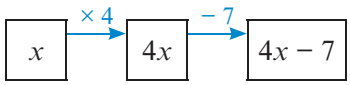
**b** **1** Perform the first operation (add 2 to $y$) and write the result in the second box.

**2** Perform the next operation on the contents of the second box (multiply $y + 2$ by 3) and write the final result in the last box.

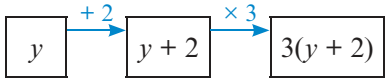
**NOTE** All of $y + 2$ is multiplied by 3, so brackets are used.

### WRITE

**a**



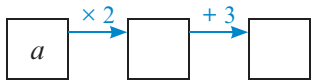

**b**



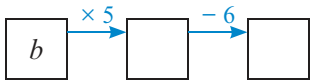

UNDERSTANDING AND FLUENCY

**2** Copy and complete each flowchart to build an expression.

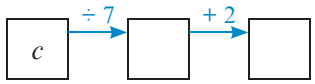
**a**



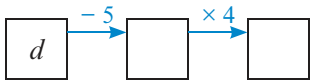

**b**



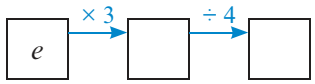

**c**



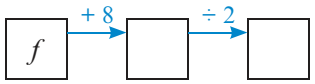

**d**




**e**




**f**



**3** Write the missing operations to complete each flowchart.

**a** $a \rightarrow \boxed{6a} \rightarrow 6a - 2$

**b** $b \rightarrow \boxed{\frac{b}{7}} \rightarrow \frac{b}{7} + 3$

**c** $c \rightarrow \boxed{c + 4} \rightarrow 9(c + 4)$

**d** $d \rightarrow \boxed{11d} \rightarrow 11d + 8$

**e** $e \rightarrow \boxed{e - 5} \rightarrow \frac{e - 5}{2}$

**f** $f \rightarrow \boxed{\frac{f}{3}} \rightarrow \frac{f}{3} - 1$

**4** Write the operations to be performed in the correct order to build each expression.

**a** $7x + 1$ **b** $3x - 5$ **c** $4x + 8$

**d** $\frac{x}{5} - 7$ **e** $\frac{x}{2} + 4$ **f** $\frac{x}{9} - 1$

**g** $4(x - 3)$ **h** $2(x + 10)$ **i** $7(x - 2)$

**j** $\frac{x + 4}{7}$ **k** $\frac{x - 3}{2}$ **l** $\frac{x + 9}{5}$

## EXAMPLE 5G-2 Drawing a flowchart to build an expression

Draw a flowchart to build each expression.

**a** $6(j - 1)$ **b** $\frac{k + 5}{3}$

### THINK

**a** **1** Write the variable in the first box.

**2** Work out the first operation to be performed. Since $j - 1$ is grouped in brackets, the first operation is to subtract 1 from $j$.

**3** The next operation is performed on the entire contents of the second box. You need to multiply $j - 1$ by 6 to get $6(j - 1)$.

**b** **1** Write the variable in the first box.

**2** Work out the first operation to be performed. The vinculum groups $k + 5$ together, so the first operation is to add 5 to $k$.

**3** The next operation is performed on the entire contents of the second box. You need to divide $k + 5$ by 3 to get $\frac{k + 5}{3}$.

### WRITE

**a** $j \xrightarrow{-1} \boxed{j - 1} \rightarrow \boxed{\phantom{x}}$

$j \xrightarrow{-1} \boxed{j - 1} \xrightarrow{\times 6} \boxed{6(j - 1)}$

**b** $k \xrightarrow{+5} \boxed{k + 5} \rightarrow \boxed{\phantom{x}}$

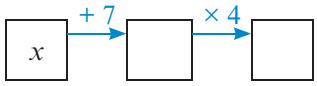
$k \xrightarrow{+5} \boxed{k + 5} \xrightarrow{\div 3} \boxed{\frac{k + 5}{3}}$

**5** Draw a flowchart to build each expression in question **4**.

**6** Draw a flowchart to build each expression.

| | | |
|---|---|---|
| **a** $5(x + 4)$ | **b** $\frac{x}{3} - 2$ | **c** $4x - 3$ |
| **d** $\frac{x + 6}{8}$ | **e** $9x + 1$ | **f** $\frac{x}{4} + 6$ |
| **g** $\frac{x - 5}{9}$ | **h** $3(x - 1)$ | **i** $\frac{5x}{7}$ |
| **j** $\frac{x}{6} - 4$ | **k** $9(x + 5)$ | **l** $\frac{x - 1}{3}$ |

**7** This flowchart shows the operations to build an expression from $x$.




Sophia writes the final expression as $4x + 7$.
Is she correct?
Give a reason for your answer.

**8** Draw a flowchart to build each expression.

**a** Start with $a$, multiply by 4 and then add 7.
**b** Start with $b$, divide by 3 and then add 5.
**c** Start with $c$, multiply by 8 and then divide by 5.
**d** Start with $d$, add 4 and then divide by 3.
**e** Start with $e$, subtract 1 and then multiply by 5.
**f** Start with $f$, add 8 and then multiply by 7.
**g** Start with $g$, multiply by 2 and then subtract 9.
**h** Start with $h$, multiply by 3 and then divide by 11.

**9** Write the expressions you have built in question **8**.

**10** Write the missing operations needed to complete each flowchart.

**a** $\boxed{j} \rightarrow \boxed{j + 2} \rightarrow \boxed{7(j + 2)} \rightarrow \boxed{7(j + 2) - 3}$

**b** $\boxed{k} \rightarrow \boxed{9k} \rightarrow \boxed{9k - 3} \rightarrow \boxed{\frac{9k - 3}{5}}$

**c** $\boxed{m} \rightarrow \boxed{6m} \rightarrow \boxed{6m + 1} \rightarrow \boxed{2(6m + 1)}$

**d** $\boxed{n} \rightarrow \boxed{n - 4} \rightarrow \boxed{\frac{n - 4}{3}} \rightarrow \boxed{\frac{n - 4}{3} + 9}$

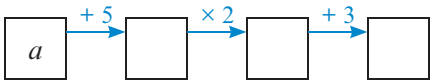
**e** $\boxed{p} \rightarrow \boxed{5p} \rightarrow \boxed{\frac{5p}{6}} \rightarrow \boxed{\frac{5p}{6} - 11}$

**f** $\boxed{q} \rightarrow \boxed{\frac{q}{4}} \rightarrow \boxed{\frac{q}{4} + 3} \rightarrow \boxed{8\left(\frac{q}{4} + 3\right)}$

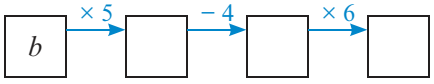
UNDERSTANDING AND FLUENCY

**11** Copy and complete each flowchart to build an expression.

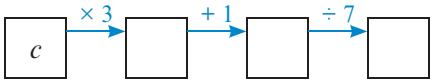
**a**



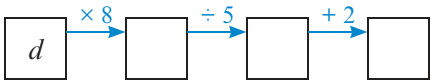

**b**



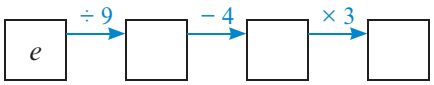

**c**



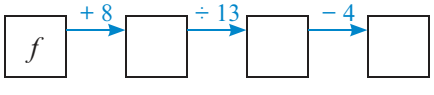

**d**



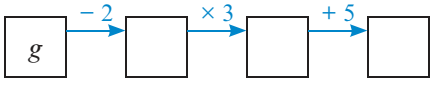

**e**




**f**

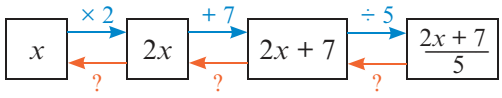



**g**




**NOTE** Each operation is performed on the entire contents of the previous box in the flowchart. Use grouping symbols such as a pair of brackets or a vinculum to show the correct order of operations.

**12** Copy and complete this flowchart to show the inverse operations needed to backtrack from the final expression to the starting variable.




**13** On your flowcharts for question **11**, draw and label arrows with inverse operations to backtrack from the final expression to the starting variable.

PROBLEM SOLVING AND REASONING

**14** Write an expression and have a classmate draw a flowchart to show how this expression is built.

**15** You have $y$ dollars in your pocket. Your mother gives you \$30. You spend \$25 on a CD and then spend half of what is left on a new shirt.

**a** Draw a flowchart for this situation.

**b** What is the expression for the final amount of money in your pocket?

**c** If $y$ is 53, what is the final amount of money in your pocket?

**d** Check that you get the same result whether you substitute into the final expression or perform the operations shown in the flowchart on the starting value of 53.



PROBLEM SOLVING AND REASONING

**16** There are $x$ fish in an aquarium. A customer buys five fish. The pet shop owner then adds more fish and doubles the number of fish in the tank.

- **a** Draw a flowchart for this situation.
- **b** What is the expression for the final number of fish in the aquarium?
- **c** If $x$ is 11, what is the final number of fish in the aquarium?
- **d** Check that you get the same result whether you substitute into the final expression or perform the operations shown in the flowchart on the starting value of 11.



CHALLENGE

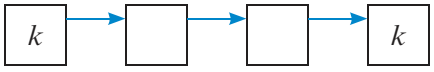
**17** What is the first operation performed on $x$ in each expression?

- **a** $7(3x + 4)$
- **b** $\frac{x - 5}{2} + 9$
- **c** $5(x + 6) - 1$
- **d** $\frac{3x - 2}{7}$
- **e** $9(\frac{x}{8} + 3)$
- **f** $4(x - 3) + 5$
- **g** $\frac{x + 7}{5} - 2$
- **h** $3(8x - 1)$
- **i** $\frac{6x + 3}{11}$

**18** Draw a flowchart to build each expression in question **17**.

**19** On your flowcharts for question **18**, draw and label arrows with inverse operations to backtrack from the final expression to the starting variable.

**20** Copy and complete this flowchart so that the starting variable and the final expression are both $k$.




**21** Write an expression that has three or more operations performed on the variable and have a classmate draw a flowchart to show how this expression is built.

**Reflect**

What is important to remember when using flowcharts to build expressions?

# 5H Solving equations using backtracking

## Start thinking!



There are $x$ mice in a cage. The pet-shop owner moves more mice into the cage so that there is now double the number of mice for sale. Monique buys three mice.

**1** What does the variable $x$ represent?

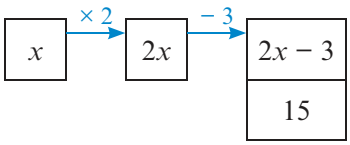
**2** Write an expression for the final number of mice in the cage.

**3** Draw a flowchart to build this expression.

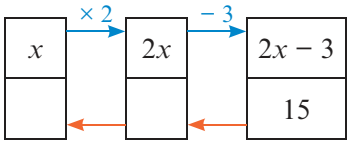
**4** What is the final number of mice in the cage when:

**a** $x$ is 5? **b** $x$ is 13?

How can you work out the original number of mice when you know the final number? If the final number of mice in the cage is 15, write 15 under $2x - 3$ on the flowchart.




**5** On your flowchart, copy the extra boxes and arrows shown in this diagram. Write the inverse operations to backtrack from the final expression to the starting variable.




**6** Use backtracking to fill in the missing values in the boxes on your flowchart.

**7** What is the value of $x$ when backtracking from 15? (Hint: look at the number written underneath.)

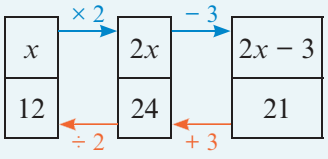
**8** You have solved the equation $2x - 3 = 15$ by backtracking. What is the solution to this equation?

**9** Explain what the equation represents and what the solution tells us.

## KEY IDEAS

- An equation can be solved by drawing a flowchart and backtracking.
- Backtracking involves using inverse operations to work backwards from the final expression to the variable.
- The solution to an equation is the value of the variable (pronumeral) that makes the equation a true statement.
- You can check if a solution is correct by substituting it into the equation and seeing whether it makes the equation a true statement.

This flowchart can be used to solve $2x - 3 = 21$.
The solution is $x = 12$.



# EXERCISE 5H Solving equations using backtracking

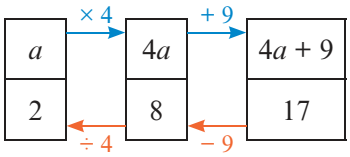
UNDERSTANDING AND FLUENCY

**1** For each flowchart, write:

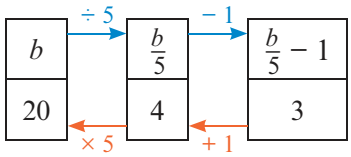
**i** the equation to be solved

**ii** the solution to the equation.

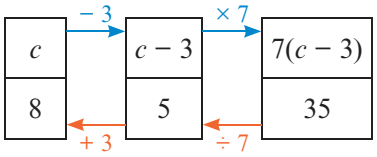
**a**



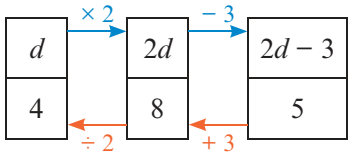

**b**



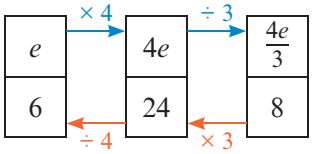

**c**



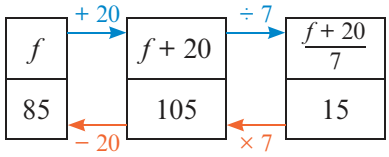

**d**




**e**



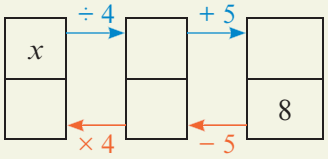

**f**




## EXAMPLE 5H-1

### Using a flowchart to write the equation to be solved and its solution

Copy and complete this flowchart. Write the equation to be solved and its solution.



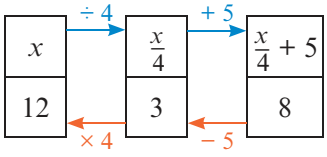

**THINK**

**1** Perform the operations shown by the arrows to find the missing values.

**2** Write the equation. Use the final expression and the number that is equivalent to it.

**3** Write the solution to the equation. This is the variable (the pronumeral $x$) and the equivalent value shown underneath.

**WRITE**



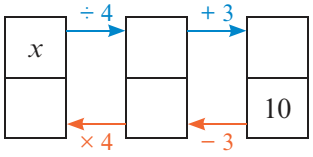

$\frac{x}{4} + 5 = 8$

$x = 12$

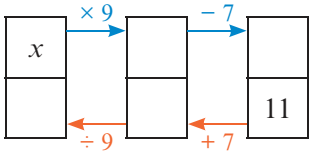
UNDERSTANDING AND FLUENCY

**2** Copy and complete each flowchart. Write the equation to be solved and its solution.

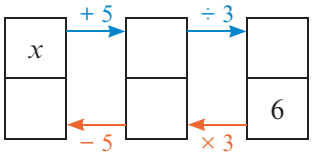
**a**



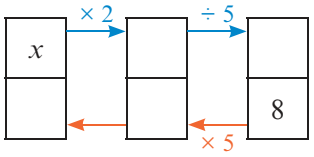

**b**



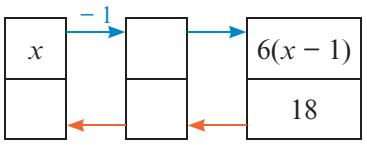

**c**



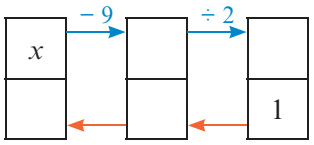

**d**



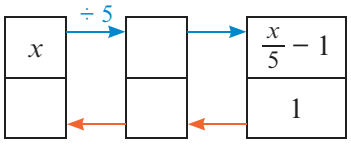

**e**



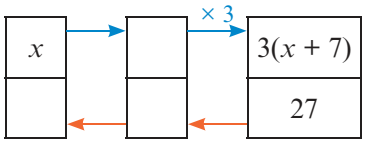

**f**




**g**




**h**




**3** Describe how to solve an equation using backtracking.

## EXAMPLE 5H-2 Solving an equation using backtracking

Solve $3(x - 1) = 24$ using backtracking.

### THINK

**1** Draw a flowchart to build $3(x - 1)$. Write the equivalent value of the expression underneath.

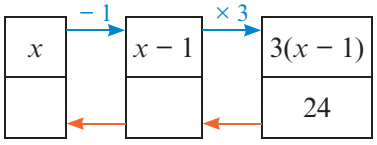
**2** Show the inverse operations needed to backtrack from the expression to the variable.

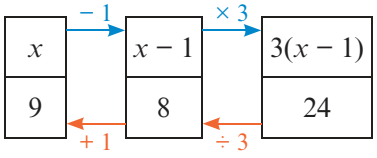
**3** Work out the missing values by backtracking.

**4** Write the solution.

**5** Check that the solution is correct. Is the left side (LS) equal to the right side (RS)?
$3(9 - 1) = 24$ is a true statement.

### WRITE




$x = 9$

Check: LS $= 3(x - 1)$
$= 3(9 - 1)$ when $x = 9$
$= 3 \times 8$
$= 24$
$=$ RS

**4** Solve each equation using backtracking.

**a** $4a + 5 = 29$ **b** $2b - 3 = 11$ **c** $5c + 1 = 16$

**d** $3(d + 2) = 21$ **e** $5(e - 7) = 10$ **f** $2(f + 6) = 28$

**g** $\frac{g}{2} - 1 = 5$ **h** $\frac{h}{5} + 6 = 9$ **i** $\frac{i}{4} - 2 = 7$

**j** $\frac{j - 4}{5} = 1$ **k** $\frac{k + 3}{2} = 4$ **l** $\frac{l - 1}{6} = 2$

**m** $\frac{3m}{2} = 9$ **n** $\frac{2n}{5} = 4$ **o** $\frac{5x}{8} = 10$

**5** Find the value of $x$ using backtracking.

**a** $5x - 3 = 17$ **b** $2(x - 6) = 6$ **c** $\frac{x + 2}{3} = 4$

**d** $\frac{x}{7} + 9 = 11$ **e** $4(x + 5) = 24$ **f** $\frac{3x}{5} = 9$

**g** $\frac{x}{3} - 4 = 5$ **h** $6x + 5 = 23$ **i** $\frac{x + 9}{2} = 8$

**j** $2x - 7 = 11$ **k** $\frac{x}{2} + 7 = 10$ **l** $3(x - 8) = 9$

**m** $\frac{x - 1}{7} = 3$ **n** $\frac{2x}{11} = 4$ **o** $6x + 7 = 19$

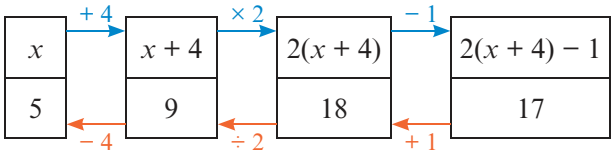
**6** Check that the solutions for question **5** are correct. Substitute the value of $x$ into the equation to see if you get a true statement.

**7** For each flowchart, write:

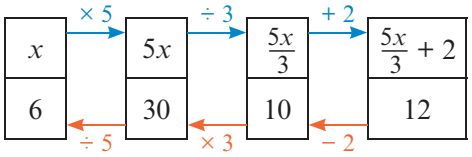
**i** the equation to be solved

**ii** the solution to the equation.

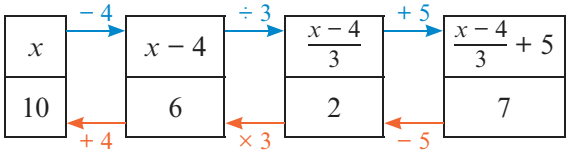
**a**



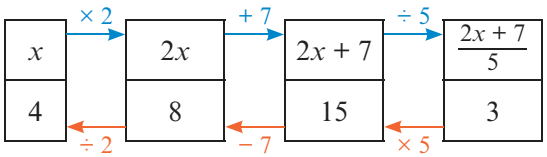

**b**




**c**



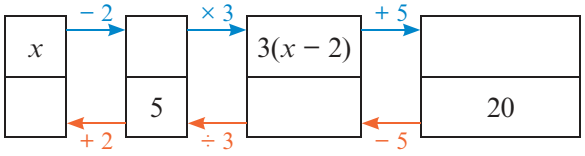

**d**



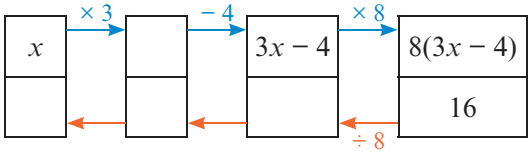
UNDERSTANDING AND FLUENCY

**8** Copy and complete each flowchart. Write the equation to be solved and its solution.

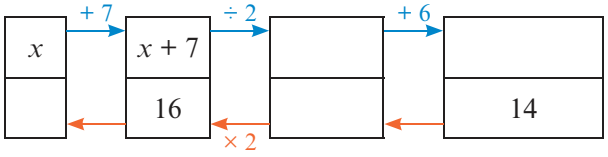
**a**



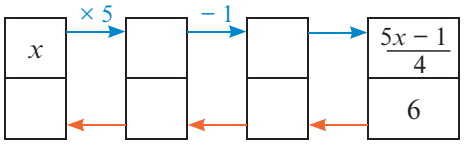

**b**



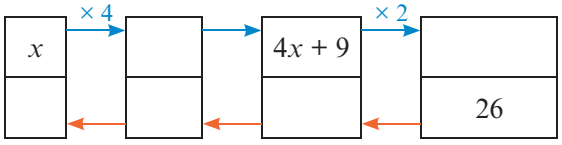

**c**



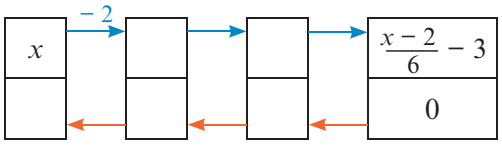

**d**




**e**

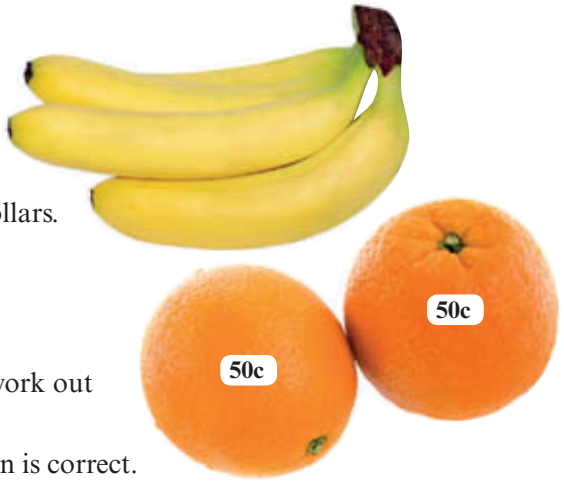



**f**




PROBLEM SOLVING AND REASONING

**9** The cost of three bananas and two oranges is \$7. Let the unknown cost of a banana be $x$ dollars.




**a** Which equation matches this description?

**A** $3(x + 1) = 7$ **B** $3x + 1 = 7$

**C** $3x + 2 = 7$ **D** $2x + 3 = 7$

**b** Solve the equation using backtracking to work out the cost of one banana.

**c** Use substitution to check that your solution is correct.

**10** A number has 5 added to it and then the result is divided by 3 to give an answer of 8. Let $x$ represent the unknown number.

**a** Which equation matches this description?

**A** $\frac{x}{3} + 5 = 8$ **B** $3(x + 5) = 8$

**C** $\frac{x + 3}{5} = 8$ **D** $\frac{x + 5}{3} = 8$

**b** Solve the equation using backtracking to find the unknown number.

**c** Use substitution to check that your solution is correct.

PROBLEM SOLVING AND REASONING

**11** Write an equation for each description.

**a** A number is multiplied by 2, then 5 is added to give an answer of 23.

**b** A number is divided by 3, then 2 is subtracted to give an answer of 5.

**c** A number has 4 added to it, then the result is multiplied by 6 to give an answer of 30.

**d** A number has 7 subtracted from it, then the result is divided by 2 to give an answer of 3.

**e** A number is doubled, then 1 is subtracted to give an answer of 9.

**f** A number is multiplied by 3, then the result is divided by 4 to give an answer of 9.

**NOTE** Remember to use a pronumeral to represent the unknown number.

**12** Solve each equation in question **11** using backtracking to find the unknown number.

**13** Fabian buys two cinema tickets and this box of popcorn for $40.




**a** Write an equation for this situation. Use a pronumeral for the cost of one cinema ticket.

**b** Solve the equation to find the cost of a cinema ticket.

**14** Kiara counts the money she has saved and decides to place half of it in her bank account. She then earns a further $20 for mowing the lawn and now has $59 in cash to spend. Work out the amount of money Kiara had originally saved by writing an equation and then solving it by backtracking.

**15** Write an equation and have a classmate solve it by backtracking.

CHALLENGE

**16** Solve each equation using backtracking.

**a** $2(5x + 8) = 36$ **b** $\frac{x-4}{3} + 7 = 9$ **c** $4(x + 5) - 2 = 42$

**d** $\frac{2x-3}{5} = 5$ **e** $7(\frac{x}{4} + 6) = 70$ **f** $\frac{8x}{3} + 1 = 9$

**g** $6(x - 2) + 7 = 43$ **h** $\frac{x+1}{4} - 3 = 2$ **i** $3(4x - 9) = 33$

**j** $\frac{7x+6}{2} = 3$ **k** $\frac{4x}{5} - 2 = 6$ **l** $\frac{5x-2}{3} = 1$

**17** Check that the solutions for question **16** are correct.

**18** A number is multiplied by 3, then 4 is added to the result, then the result is multiplied by 5 and finally 7 is subtracted from the result to get an answer of 43. What is the number you started with? Write an equation to represent this problem and use backtracking to find the unknown number.

**19** Use backtracking to solve:

**a** $\frac{3(5x-2)}{7} + 6 = 18$ **b** $4(\frac{x}{3} + 1) - 5 = 19$

**Reflect**

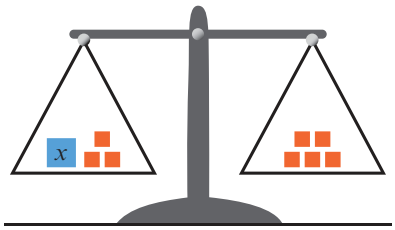
Why are inverse operations important when backtracking to find the solution to an equation?

# 5I Solving equations using a balance model



## Start thinking!

A balanced set of scales has equal masses on both sides. Equations are said to be balanced because, like balanced scales, their left sides and right sides are equal.




**1** This diagram models the equation $x + 3 = 5$.
The left side shows masses of $x$ g and 3 g while the right side shows 5 g.

- **a** What would happen if 2 g were removed from the left side?
- **b** What would you do to the right side to balance the scales?
- **c** What equation does the balanced set of scales now model?

**2** Repeat question **1** but this time, remove 3 g from the left side in part **a**.

**3** Find the solution to each equation by inspection.

**a** $x + 3 = 5$ **b** $x + 5 = 7$ **c** $x + 1 = 3$ **d** $x + 4 = 6$

**4** Why do you think the equations in question **3** are called **equivalent equations**?

**5** What operation could be performed on both sides of $x + 3 = 5$ to get the equivalent equation $x = 2$?

**6** Copy and complete the steps shown at right to solve $x + 3 = 5$ using the **balance method**. Each line of working shows an equivalent equation.

$$x + 3 = 5$$
$$x + 3 - 3 = 5 - ____$$
$$x = ____$$

**7** Why do you think using the balance method to solve equations is also known as 'doing the same thing to both sides'?

## KEY IDEAS

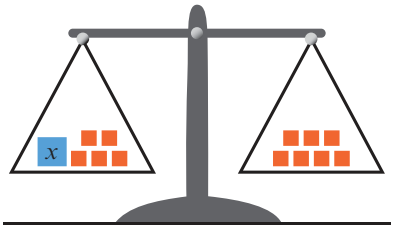
- Equivalent equations have the same solution.
- To solve an equation using the balance method, undo the operations performed on the pronumeral (say, $x$) to get $x$ by itself on one side of the equation. For the equation to stay balanced, the operation performed on one side of the equation must also be performed on the other side.
- An operation can be undone by performing the inverse operation.
- To produce equivalent equations, perform operations until you get the equation '$x = \ldots$.' This is the solution to the equation.

| Operation | Inverse operation |
|---|---|
| + | − |
| − | + |
| × | ÷ |
| ÷ | × |

# EXERCISE 5I Solving equations using a balance model

UNDERSTANDING AND FLUENCY

**1** An equation is modelled by this set of scales.




**a** Is this set of scales balanced? How can you tell?
**b** What equation does it model?
**c** If 3 g are removed from the left tray, what should you do to the right tray to make the scales balance again?
**d** What equivalent equation do the scales now model?
**e** How can you keep the scales balanced if 5 g are removed from the left side of the scales?
**f** What is the value of $x$?

**2** List the inverse operation to each of these.

**a** $+ 5$ **b** $- 3$
**c** $\times 8$ **d** $\div 2$

## EXAMPLE 5I-1 Solving an equation using the balance method (one step)

Solve $x + 7 = 11$ using the balance method.

**THINK**

**1** Decide what operation needs to be undone so that $x$ is by itself on one side of the equation. You need to undo '+ 7'.

**2** Undo '+ 7' by performing the inverse operation of '− 7' on both sides of the equation.

**3** Simplify the equation to get the solution.

**4** Check your solution.

**WRITE**

$x + 7 = 11$

$x + 7 - 7 = 11 - 7$

$x = 4$

LS $= x + 7$
$= 4 + 7$ when $x = 4$
$= 11$
$=$ RS

UNDERSTANDING AND FLUENCY

**3** Solve each equation using the balance method. Copy and complete the steps shown.

**a** $a + 5 = 12$

$a + 5 - __ = 12 - __$

$a = __$

**b** $b - 3 = 8$

$b - 3 + __ = 8 + __$

$b = __$

**c** $8c = 32$

$\frac{8c}{__} = \frac{32}{__}$

$c = __$

**d** $\frac{d}{3} = 5$

$\frac{d}{3} \times __ = 5 \times __$

$d = __$

**4** Solve each equation using the balance method.

**a** $x + 8 = 13$

**b** $x - 4 = 2$

**c** $3x = 24$

**d** $\frac{x}{5} = 6$

**e** $7x = 56$

**f** $x - 6 = 11$

**g** $\frac{x}{3} = 4$

**h** $x + 1 = 8$

**i** $x - 15 = 4$

**j** $11x = 33$

**k** $x + 23 = 50$

**l** $\frac{x}{10} = 18$

NOTE Show all steps of working.

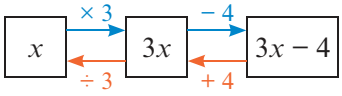
**5** Check that the solutions for question **4** are correct.

## EXAMPLE 5I-2 Solving an equation using the balance method (two steps)

Solve $3x - 4 = 11$ using the balance method.

### THINK

**1** Decide which operations need to be performed on $3x - 4$ to get $x$. You can draw a quick diagram like this to help you work out the order (+ 4 then ÷ 3).




**2** Undo '− 4' by performing the inverse operation of '+ 4' on both sides of the equation.

**3** Simplify the equation.

**4** Undo '× 3' by performing the inverse operation of '÷ 3' on both sides of the equation.

**5** Simplify the equation to get the solution.

### WRITE

$3x - 4 = 11$

$3x - 4 + 4 = 11 + 4$

$3x = 15$

$\frac{3x}{3} = \frac{15}{3}$

$x = 5$

**6** Solve each equation using the balance method.

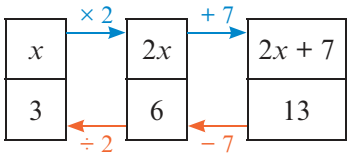
**a** $4x + 3 = 27$ **b** $2x - 5 = 1$

**c** $3x + 7 = 19$ **d** $7x - 2 = 12$

**e** $5x - 4 = 31$ **f** $6x + 1 = 31$

**g** $2x + 8 = 10$ **h** $3x - 4 = 20$

**7** The equation $2x + 7 = 13$ can be solved by backtracking along a flowchart.




**a** Write the order of the operations needed to build $2x + 7$ from $x$.

**b** Write the order of the operations needed to backtrack from $2x + 7$ to $x$.

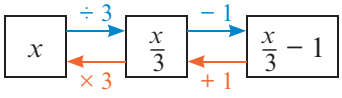
**c** Your answer to part **b** shows the inverse operations and the order in which they should be performed to get $x$. These are the same operations to use in the balance method. Copy and complete the steps shown to solve $2x + 7 = 13$ using the balance method.

$$\mathbf{2x + 7 = 13}$$
$$2x + 7 - __ = 13 - __$$
$$\mathbf{2x = 6}$$
$$\frac{2x}{__} = \frac{6}{__}$$
$$\mathbf{x = 3}$$

**d** Why are the equations highlighted in bold the same as those seen in the flowchart?

**e** What is the solution to $2x + 7 = 13$?

**8** **a** Which operations must be performed on $\frac{x}{3} - 1$ to get $x$? Use this diagram to help you.




**b** To solve $\frac{x}{3} - 1 = 4$ using the balance method, which operation should be performed first on both sides of the equation?

**c** Which operation should be performed next?

**d** Copy and complete the steps shown to find the solution to $\frac{x}{3} - 1 = 4$ using the balance method.

$$\frac{x}{3} - 1 = 4$$
$$\frac{x}{3} - 1 + __ = 4 + __$$
$$\frac{x}{3} = __$$
$$\frac{x}{3} \times 3 = __ \times __$$
$$x = __$$

**e** Solve $\frac{x}{3} - 1 = 4$ using backtracking. Compare the equivalent equations produced using each method.

**9** Solve each equation using the balance method.

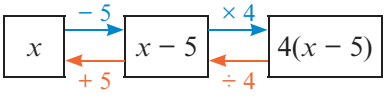
**a** $\frac{x}{5} + 1 = 3$ **b** $\frac{x}{3} - 4 = 2$

**c** $\frac{x}{2} + 6 = 9$ **d** $\frac{x}{4} - 1 = 6$

**e** $\frac{x}{9} + 2 = 5$ **f** $\frac{x}{6} + 9 = 13$

**g** $\frac{x}{3} - 5 = 6$ **h** $\frac{x}{7} - 2 = 4$

**10** **a** Which operations need to be performed on $4(x - 5)$ to get $x$? Use this diagram to help you.



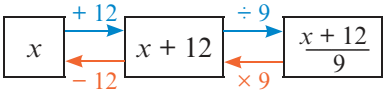

**b** To solve $4(x - 5) = 8$ using the balance method, which operation should be performed first on both sides of the equation?

**c** Which operation should be performed next?

**d** Copy and complete the steps shown to find the solution to $4(x - 5) = 8$ using the balance method.

$$4(x - 5) = 8$$
$$\frac{4(x - 5)}{4} = \frac{8}{__}$$
$$x - 5 = __$$
$$x - 5 + __ = __ + __$$
$$x = __$$

**11** **a** Which operations must be performed on $\frac{x + 12}{9}$ to get $x$? Use this diagram to help you.




**b** To solve $\frac{x + 12}{9} = 2$ using the balance method, which operation should be performed first on both sides of the equation?

**c** Which operation should be performed next?

**d** Copy and complete the steps shown to find the solution to $\frac{x + 12}{9} = 2$ using the balance method.

$$\frac{x + 12}{9} = 2$$
$$\frac{x + 12}{9} \times __ = 2 \times __$$
$$x + 12 = __$$
$$x + 12 - __ = __ - __$$
$$x = __$$

**12** Solve each equation using the balance method.

**a** $2(x + 3) = 16$

**b** $\frac{x - 4}{5} = 3$

**c** $3(x - 2) = 27$

**d** $\frac{x + 3}{7} = 2$

**e** $\frac{x - 1}{6} = 1$

**f** $5(x + 1) = 20$

**g** $4(x - 7) = 12$

**h** $\frac{x + 2}{3} = 6$

**13** Find the solution to each equation using the balance method.

**a** $9x + 4 = 13$

**b** $\frac{x}{2} + 5 = 12$

**c** $7(x + 1) = 49$

**d** $\frac{x + 4}{3} = 5$

**e** $\frac{x}{6} - 3 = 4$

**f** $6(x - 4) = 30$

**g** $3x - 7 = 17$

**h** $\frac{x - 8}{7} = 3$

**i** $11x + 3 = 47$

**j** $\frac{x}{5} + 8 = 14$

**k** $\frac{x + 6}{4} = 4$

**l** $\frac{2x}{7} = 6$

**m** $3(x - 8) = 6$

**n** $\frac{5x}{4} = 15$

**o** $4x - 8 = 0$

**p** $\frac{x - 5}{2} = 11$

PROBLEM SOLVING AND REASONING

**14** For each problem, write an equation and solve it using the balance method. Show all steps of your working.

**a** A restaurant has an outdoor terrace it sets up with tables during summer. Each table seats four people. How many tables should the restaurant set up if 64 people book for Sunday lunch?

**b** The cost of hiring a tennis court in the evening is $6 plus $15 per hour. For how many hours can you hire the tennis court if you have $81?

**c** You have $20 and buy a drink for $3. The cost of a standard pizza is $9. Extra toppings are $2 each. How many extra toppings could you get on a pizza?

**d** Alisa is saving to buy a digital camera that costs $369. She is able to save $24 per week. If she currently has $105, in how many weeks can she buy the camera?



CHALLENGE

**15** You have some money in your wallet and your older sister gives you $5 for cleaning her room. Since you are saving money to buy new runners, your mother agrees to match 'dollar for dollar' the amount you have. That is, she will give you extra money so that you now have twice the amount of money you had. You just have enough money to buy runners for $136. How much money did you originally have in your wallet?

**16** You have now used four different strategies for solving an equation.

**a** Describe each strategy. Explain the advantages or disadvantages of each.

**b** Compare each strategy. What is similar and what is different?

**c** Which strategy do you prefer? Give reasons for your answer.

**17** Solve each equation.

**a** $5(x + 3) - 2 = 33$

**b** $\frac{2(x - 1)}{5} + 17 = 21$

**c** $\frac{4(3x + 7)}{11} - 6 = 14$

**Reflect**

Why are equivalent equations useful in the balance method?

# CHAPTER REVIEW

## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

| | | | |
|---|---|---|---|
| rule | expression | evaluate | flowchart |
| IN number | equation | solve | inverse operations |
| OUT number | formula | solution | backtracking |
| variable | coefficient | inspection | equivalent equations |
| pronumeral | constant | checking solutions | balance method |
| term | substitute | guess, check and improve | table of values |

## MULTIPLE-CHOICE

5A **1** Which rule describes the relationship between these IN and OUT numbers?

| IN | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| OUT | 6 | 9 | 12 | 15 | 18 |

**A** OUT = IN + 4
**B** OUT = IN × 3
**C** IN = OUT + 4
**D** IN = OUT × 3

5B **2** Which formula describes the relationship between the two variables in the table?

| $a$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $b$ | 8 | 9 | 10 | 11 | 12 |

**A** $b = a + 6$ **B** $b = 4a$
**C** $b = a + 8$ **D** $b = 6 - a$

5C **3** The statement 'the sum of $a$, $b$ and $c$' is best represented by which expression?

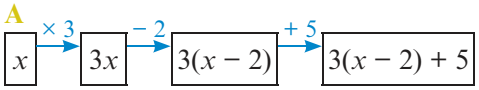
**A** $abc$ **B** $a + bc$
**C** $ab + c$ **D** $a + b + c$

5D **4** If $a = 4$, $b = 3$ and $c = 1$, what does the expression $2a + b - c$ equal?

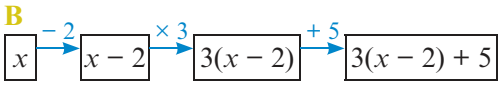
**A** 26 **B** 10
**C** 8 **D** 12

5G **5** Which flowchart correctly represents the expression $3(x - 2) + 5$?

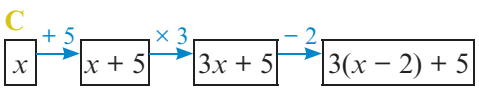
**A**



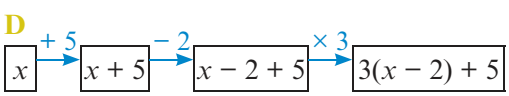

**B**



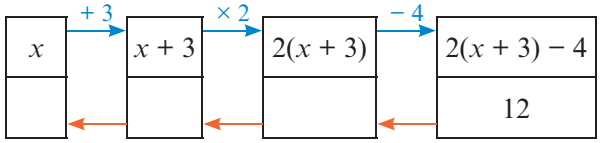

**C**




**D**




5H **6** Using this flowchart, what is the solution to the equation?




**A** $x = 5$ **B** $x = 11$
**C** $x = 12$ **D** $x = 16$

5I **7** What is the solution to $\frac{x - 6}{5} = 2$?

**A** $x = 4$ **B** $x = 8$
**C** $x = 10$ **D** $x = 16$

## SHORT ANSWER

5A **1** Use each rule to find the missing OUT numbers in this table.

| IN | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| OUT | | | | | |

a Multiply each IN number by 4 and then add 2.

b Subtract 1 from each IN number then multiply by 3.

5A **2** Write a rule for the relationship between the IN and OUT numbers shown.

| IN | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|
| OUT | 6 | 7 | 8 | 9 | 10 |

5B **3** Consider this table of values.

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $y$ | 3 | 5 | 7 | 9 | 11 |

a Write a formula for the relationship between the two variables.

b Plot the graph of this relationship.

c Is the relationship linear or non-linear? Explain.

5C **4** For each expression, identify:

i the number of terms

ii any variables used

iii the coefficient of each term containing a variable

iv any constant terms.

a $7x + 3y$ b $a + 3$

c $5d + 2c + 9$ d $4 + e + 2t + 6s$

5C **5** Write an equation for each statement.

a a number multiplied by 7 equals 21

b a number divided by 3 equals 6

c 7 more than a number equals 10

d 2 fewer than a number equals 17

5D **6** Evaluate each expression by substituting $x = 1$, $y = 4$ and $z = 2$.

a $x + y + z$ b $2x + y$

c $3y - z$ d $\frac{z}{x}$

5D **7** Find the value of $b$ if $a = 3$ is substituted into each formula.

a $b = 3a + 5$ b $b = 4a - 7$

c $b = 2(a + 6)$ d $b = \frac{a}{3} + 1$

5E **8** Solve each equation by inspection.

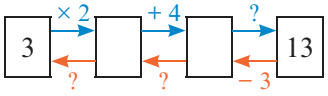
a $4a = 28$ b $b - 10 = 21$

c $c + 8 = 11$ d $\frac{d}{2} = 5$

5E **9** Solve each equation using the guess, check and improve strategy.

a $5x - 8 = 47$ b $2x + 13 = 22$

5F **10** Copy and complete this flow chart.



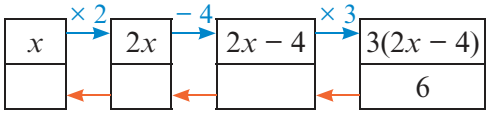

5F **11** Draw a flowchart and use backtracking to find a number that is multiplied by 4 then has 9 subtracted from it to equal 11.

5G **12** Draw a flowchart to build each expression.

a $2x + 7$ b $3(x - 1)$

c $\frac{x}{5} - 2$ d $\frac{x + 4}{3}$

5H **13** Copy and complete this flowchart. Write the equation to be solved and its solution.




5H **14** Solve each equation using backtracking.

a $3x - 2 = 10$ b $4(x + 2) = 20$

c $\frac{x}{3} + 8 = 10$ d $\frac{x + 5}{4} = 3$

5I **15** Solve each equation using the balance method.

a $x + 9 = 21$ b $\frac{x}{4} = 12$

c $3x + 5 = 17$ d $2(x - 1) = 24$

e $\frac{x + 5}{7} = 6$ f $\frac{x}{5} - 9 = 3$

## NAPLAN-STYLE PRACTICE

**1** The cost of buying slurpees is recorded in this table.

| Number of slurpees | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| Cost ($) | 15 | 20 | 25 | 30 | 35 |

What is the cost of nine slurpees?

Questions **2**–**4** refer to this information.

An artist builds sculptures out of old tin cans. The relationship between the number of cans used ($n$) and the number of sculptures produced ($s$) is shown in the table.

| $n$ | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| $s$ | 1 | 2 | | 4 | 5 |

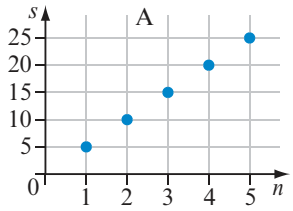
**2** What is the missing value in the table?

**3** Which formula best describes the relationship?

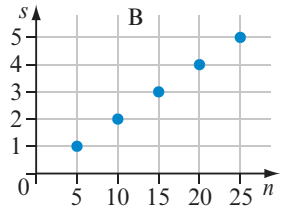
- ◯ $s = n + 5$
- ◯ $s = 5n$
- ◯ $s = \frac{n}{5}$
- ◯ $s = n - 5$

**4** Which graph best shows the relationship?

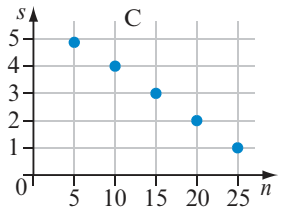
◯



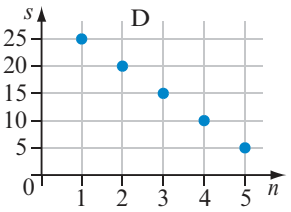

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**5** Mario has five more chocolates than Bill. If Mario has $m$ chocolates and Bill has $b$ chocolates, which of these statements best shows the relationship between $m$ and $b$?

- ◯ $m = b + 5$
- ◯ $m = 5b$
- ◯ $b = 5 + m$
- ◯ $5bm$

**6** The cost of lunch (in dollars) at a bakery is described by the expression $5s + 3d + 2p$, where $s$ is number of sandwiches, $d$ is number of drinks and $p$ is number of pastries. Some people buy six sandwiches, eight drinks and four pastries. What is the total cost?

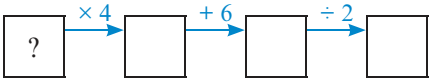
- ◯ $28
- ◯ $62
- ◯ $118
- ◯ $180

Questions **7** and **8** refer to this information.

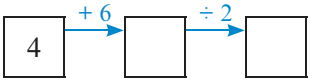
A mystery number is multiplied by 4, then has 6 added to it and the result is divided by 2.

**7** Which flowchart shows the operations performed on the number?

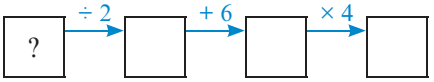
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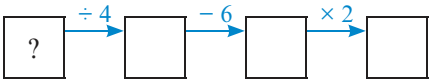

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**8** The last box shows 17. What is the mystery number?

Questions **9** and **10** refer to this information.

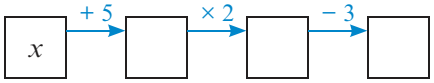
Entry to an animal farm is $4 per adult and $2 per child.

**9** Which expression describes the total entry cost (in dollars) for $a$ adults and $c$ children?

- ◯ $4 + 2$
- ◯ $6n$
- ◯ $6ac$
- ◯ $4a + 2c$

**10** A family of five (two adults and three children) visit the farm. What is their total entry cost?

**11** Sally builds a flowchart to show the operations performed on a mystery number.

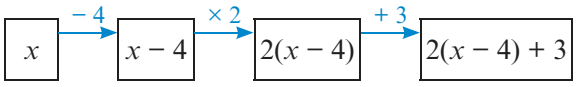



What should go in the last box?

- ◯ $2x - 2$
- ◯ $2x + 5 - 3$
- ◯ $2(x + 5) - 3$
- ◯ $5(x + 2) + 3$

Questions **12** and **13** refer to this information.
James has a number of lollies. He eats four of them, then buys more to double what was left. His mother then gives him three more lollies.

**12** Answer true or false. A flowchart that could represent this situation is:




**13** If James ends up with seven lollies, how many did he originally have?

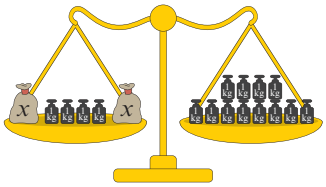
Questions **14** and **15** refer to this information.
Erica buys five apples and then gives $3 to a busker. In total, she has spent $13.

**14** Which equation matches this situation?

- ◯ $5a + 3 = 10$
- ◯ $8a = 13$
- ◯ $5a = 16$
- ◯ $5a + 3 = 13$

**15** What is the cost of a single apple?

Questions **16** and **17** refer to this diagram of a set of scales that shows two identical mystery items and a number of 1-kg weights. The scales are balanced.




**16** Which equation matches this situation?

- ◯ $x + 4 = 12$
- ◯ $2x = 12$
- ◯ $2x + 4 = 12$
- ◯ $2x - 4 = 12$

**17** What is the mass of a single mystery item?

kg

## ANALYSIS

Tess takes a group of her friends to get ice creams. The total cost of buying the ice creams is related to the number of ice creams purchased, as shown in this table.

| Number of ice creams | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Total cost ($) | 3 | 6 | 9 | 12 | 15 | 18 |

**a** What are the two variables in this relationship?

**b** Write a sentence describing the relationship between the two variables.

**c** Choose a pronumeral for each variable.

**d** Write a formula to describe the relationship between the two variables.

**e** Plot a graph of the relationship.

**f** Find the total cost of buying eight ice creams.

The store owner offers a deal to Tess. If she pays $5 up front, she can order any number of ice creams for $2 each.

**g** Explain how the expression $2n + 5$ could describe this situation.

**h** Find the total cost of buying eight ice creams in this new scenario.

Tess buys a number of ice creams and spends $35.

**i** Write an equation for this situation.

**j** Draw a flowchart to represent this situation.

**k** Use backtracking or another method to find how many ice creams Tess bought.

# CONNECT

## Tenpin bowling

At a tenpin bowling venue, the cost depends on the number of games you play and whether you take up any special deals on offer. How can you decide when and where to play to give you the best value? Does this decision change depending on how many games you play? In this task, you will investigate the cost of each special deal and then compare them to decide which suits you best.

Game offers from two different venues are provided so that you can decide on the best deal.



### Your task

For your investigation of the cost of bowling, follow these steps.

- Develop a relationship for the cost of playing a given number of games for each special offer and for normal prices.
- Work out the cost of playing a given number of games using the different offers.
- Work out how many games you can play for $38.
- Compare the cost of different special offers.
- Compare the cost of the two different venues.
- Research the costs and special offers at another tenpin bowling venue.
- Compare all the special offers to decide on the deal that suits you best.

Include all necessary working to justify your answers.

## Striker Tenpin Bowling

**Regular price**

$9 per game **plus** $2 shoe hire

**Discount Tuesdays**

All day Tuesdays

$6 per game **plus** $2 shoe hire

**School Holiday Special**

Family of four

$30 per game **includes** shoe hire

## Alley Bowling Lanes

| | Adults | Juniors*/ Students/Seniors |
|---|---|---|
| 1 game | $15 | $14 |
| 2 games | $22 | $19 |

**includes** shoe hire

all additional games $5 per person

*up to 14 years






Complete the 5 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- an information booklet
- a PowerPoint presentation
- an advertising brochure
- other (check with your teacher).

# 6 LINES, ANGLES AND LOCATION


6A Lines, rays and segments

6B Types of angles

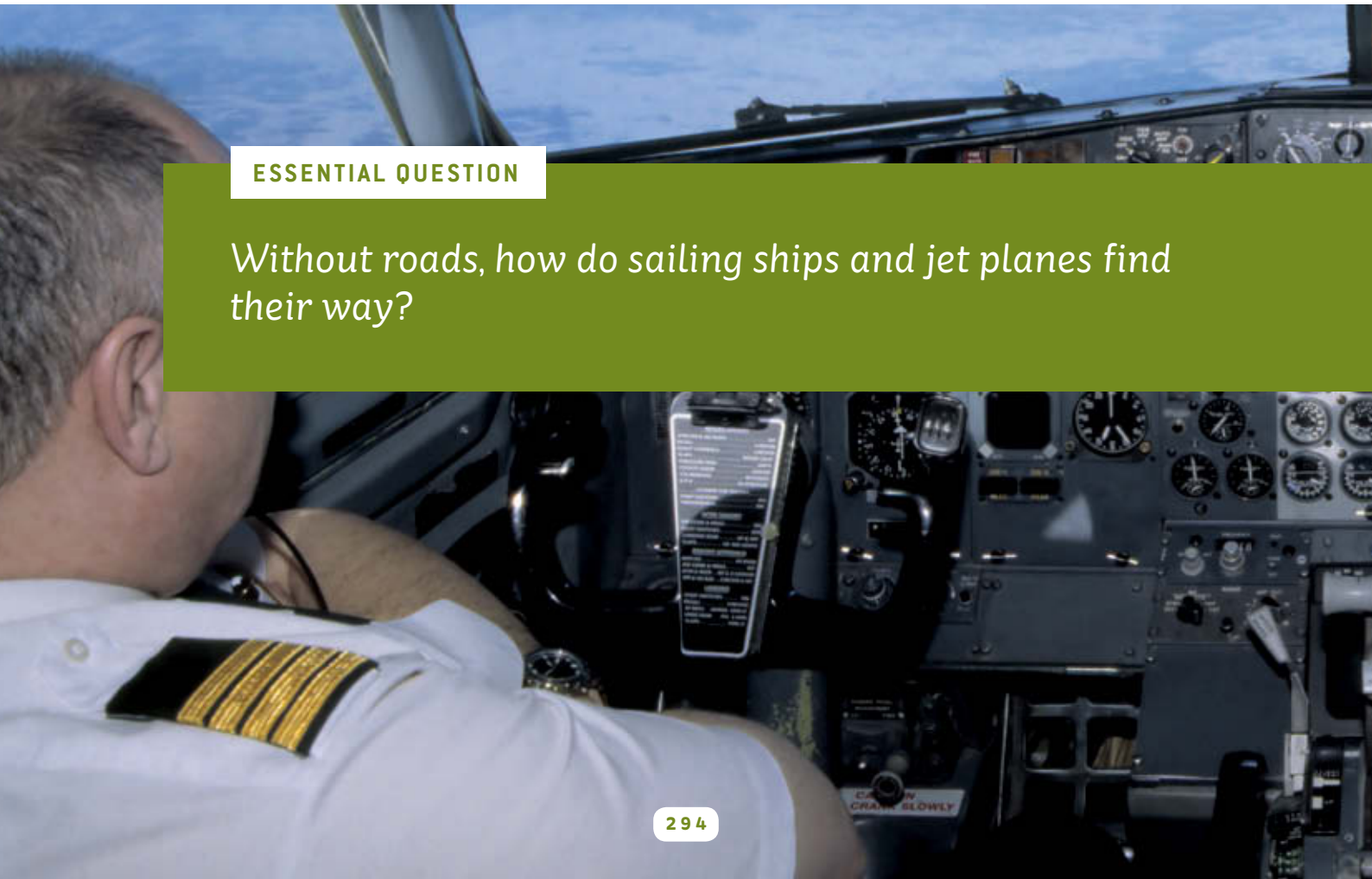
6C Measuring and drawing angles

6D Angles around a point

6E Angles and parallel lines

6F Understanding bearings

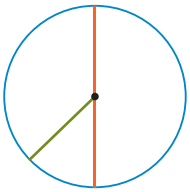
6G Understanding scale

**ESSENTIAL QUESTION**

*Without roads, how do sailing ships and jet planes find their way?*

6A **1** The black dot is in the centre of the circle. If the orange line is 10 cm long, how long is the green line?



**A** 10 cm **B** 5 cm
**C** 3.14 cm **D** 15 cm

6A **2** Which of these diagrams shows an angle?

**A**



**B**



**C**



**D**



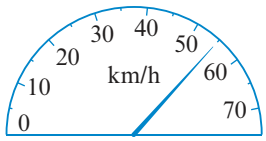
6B **3** **a** How many degrees in a right angle?
**A** 45° **B** 180° **C** 360° **D** 90°

**b** How many degrees in a revolution?
**A** 0° **B** 90° **C** 180° **D** 360°

**c** How many degrees in a straight line?
**A** 0° **B** 90° **C** 180° **D** 360°

6C **4** What value is shown on this scale?

**A** 50 km/h
**B** 55 km/h
**C** 60 km/h
**D** 65 km/h

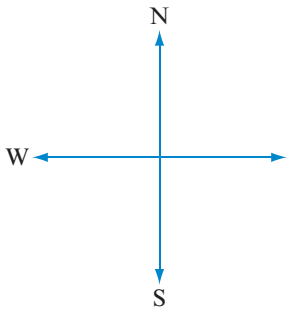



6D **5** **a** Which number makes this equation true? 150 + ___ = 180
**A** 10 **B** 20 **C** 30 **D** 40

**b** Which number makes this equation true? 37 + ___ = 90
**A** 53 **B** 43 **C** 33 **D** 23

**c** Which number makes this equation true? 245 + ___ = 360
**A** 15 **B** 95 **C** 105 **D** 115

6F **6** Refer to this diagram of a compass.




**a** What letter is missing from the compass?
**A** R **B** E **C** O **D** N

**b** What does the letter N stand for?
**A** Nature **B** Night
**C** North **D** Next

**c** What is the direction opposite to west?
**A** S **B** E **C** N **D** NW

6G **7** Which of these is not a unit of length measurement?
**A** millimetres **B** centimetres
**C** millilitres **D** kilometres

6G **8** Consider the following line.

**a** How long is the line in centimetres?
**A** 12 cm **B** 28 cm
**C** 8 cm **D** 6 cm

**b** If this line were 100 times longer, how long would it be?
**A** 800 cm **B** 80 cm
**C** 8 cm **D** 8000 cm

**c** How long is this in metres?
**A** 80 m **B** 0.8 m
**C** 800 m **D** 8 m

# 6A Lines, rays and segments

## Start thinking!

1 Copy this logo.

2 Look closely at the two red lines. If they continued in both directions, would the lines ever touch?

3 Compare the red lines with the grey lines. How are they different?

4 Work with a partner to locate these features. Label or highlight each feature on your diagram.

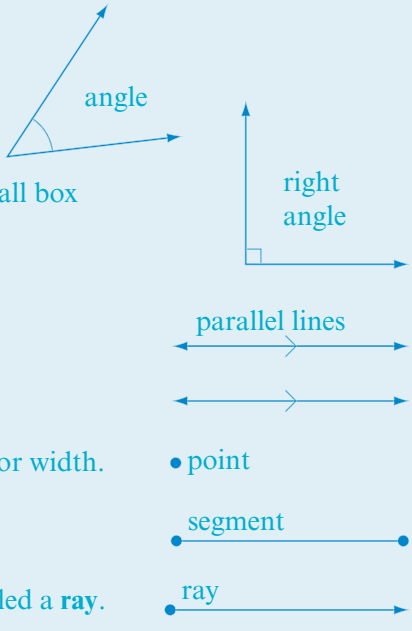
point
circle
parallel lines
perpendicular lines
segment
ray
angle
right angle




## KEY IDEAS

- When two lines meet they form an **angle**.
- Lines that form a **right angle** are **perpendicular**. The small box where the lines meet is the symbol for a right angle.
- Lines that never meet are **parallel**.

  To indicate that two lines are parallel, mark each line with matching arrowheads.
- A **point** is often shown as a dot. A point has no length or width.
- A line between two points is called a **segment**.
- A line that starts at a point and continues forever is called a **ray**.



# EXERCISE 6A Lines, rays and segments

UNDERSTANDING AND FLUENCY

**1** Draw an example and write your own definitions to complete the table below. Allow plenty of space to draw your examples.

| Term | Example | Definition |
|---|---|---|
| point | | an exact position in space |
| ray | | |
| segment | | |
| | | two lines the same distance apart over their entire length |
| perpendicular lines | | |
| | | a shape formed by two lines or rays that meet at a point |
| | | a line with every point the same distance from the centre |
| right angle | | |

**2** Draw a diagram to show each of these.

**a** two parallel rays

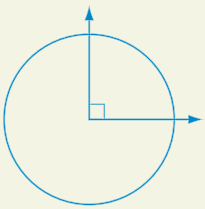
**b** a ray perpendicular to a segment

**c** five rays that share the same point

**d** two parallel lines and a line perpendicular to them

## EXAMPLE 6A-1 Describing lines, rays and segments

Use mathematical language to describe this diagram.



### THINK

**1** Decide whether the diagram shows lines, rays or segments.

**2** Describe how the rays meet.

**3** Write a statement to describe the diagram, including all the relevant information.

### WRITE

There are two rays.

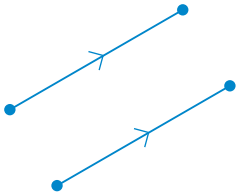
The rays are perpendicular.

There are two perpendicular rays from the centre of a circle.

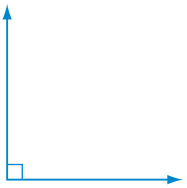
UNDERSTANDING AND FLUENCY

**3** Use mathematical terms to describe these diagrams.

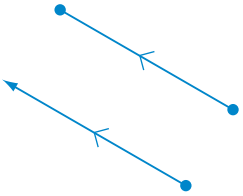
**a**



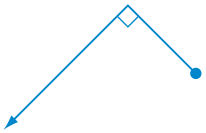
**b**



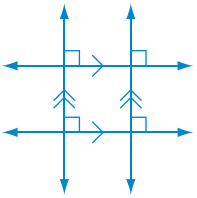
**c**



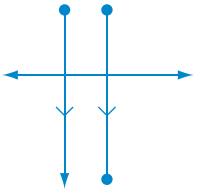
**d**



**e**



**f**



## EXAMPLE 6A-2 Drawing lines, rays and segments

Add a parallel segment above this line.

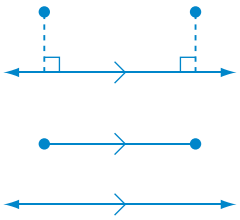


### THINK

**1** Parallel lines are always the same distance apart. Use a ruler to draw two points the same distance above the line.

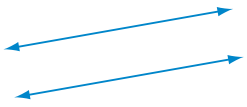
**2** A segment is a line between two points. Use a ruler to join the points with a segment.

### WRITE

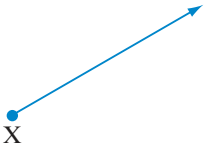


**4** Copy each diagram and include the features listed.

**a** Add a third parallel line above these two.



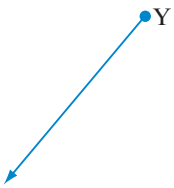
**b** Add a perpendicular ray from point X.



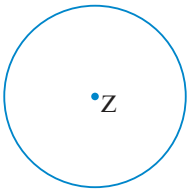

**c** Add a parallel segment.



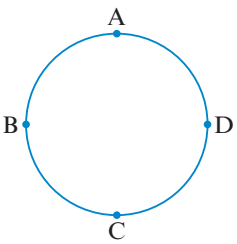
**d** Add three perpendicular rays from point Y.




**e** Add a smaller circle with point Z at the centre.



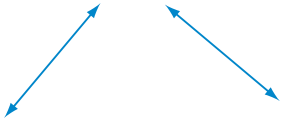

**f** Add segments from A to B, from A to C and from A to D.



UNDERSTANDING AND FLUENCY

**5** **a** If these lines were extended, would they be perpendicular?

**i**



**ii**

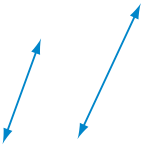


**b** Are these lines parallel?

**i**



**ii**



**6** Which term – line, ray or segment – would best describe the paths of each of these?

**a** A billiard ball rolled from one end of the table to the other.

**b** A beam of light from the sun.

**c** An arrow fired from a bow at a target.

**7** **a** Name two streets near your school or home that are parallel to each other.

**b** Name two more streets that are perpendicular to those in part **a**. Are these two streets parallel to each other?

**c** Draw a simple map to show parallel and perpendicular streets in your area.

PROBLEM SOLVING AND REASONING

**8** You are going to learn how to draw parallel and perpendicular lines with nothing more than a **compass** and a straight edge, just like the navigator of a ship would have hundreds of years ago.



You will need:

- a sharp pencil,
- a pair of compasses (commonly called a compass)
- a straight edge (a ruler is fine but any object with a straight edge will do).

**a** Perpendicular line construction. (Use half a page for this construction.)

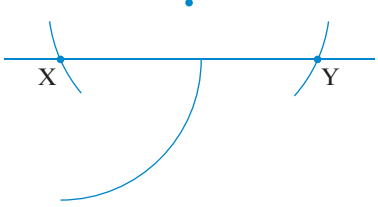
**Step 1:** Draw a straight line across your page. Draw a point, W, a few centimetres above the line.



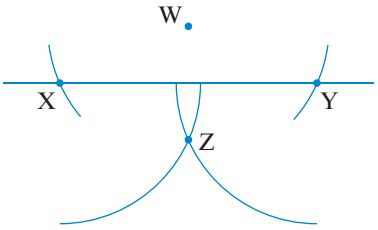

**Step 2:** Set your compass so that it is larger than the distance between W and the line. Draw two arcs crossing the line. Label these points X and Y.



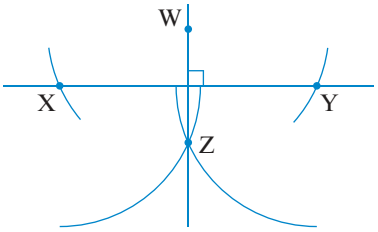

**Step 3:** Without changing the width, place the point of your compass on point X. Draw an arc below the line.



**Step 4:** Again without changing the width, place your compass on point Y. Draw an arc that crosses the one you drew in Step 3. Label the point where the two arcs cross as Z.

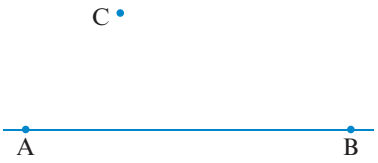



**Step 5:** Draw a straight line through W and Z. This line is perpendicular to the first line. It is the **perpendicular bisector** of the segment from X to Y. To bisect a segment is to divide it in half.

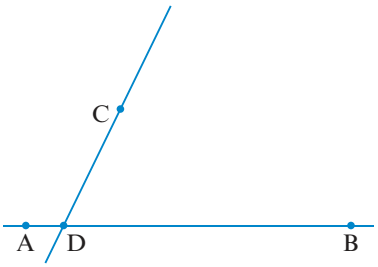



**b** Parallel lines construction. (Use half a page for this construction.)

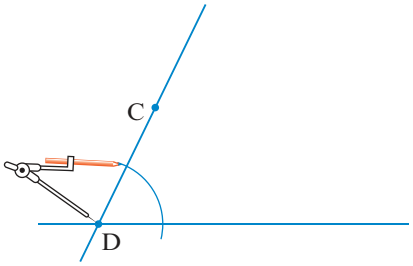
**Step 1:** Draw a straight line across your page through points A and B. Draw a point C a few centimetres above the line.



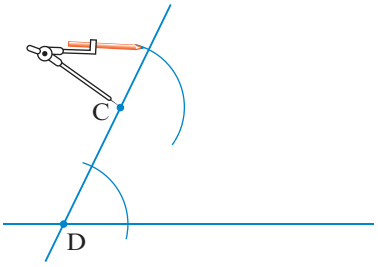

**Step 2:** Draw a straight diagonal line through C across your first line. Label the point where it crosses the first line as D.



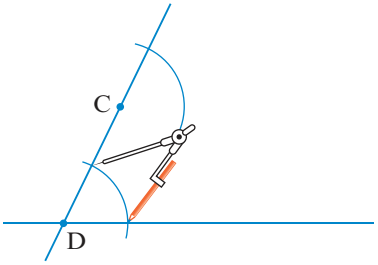

**Step 3:** Set your compass about half the width from D to C. Centre the compass as point D and draw an arc across the two lines.




**Step 4:** Move the compass to C and make a similar arc.

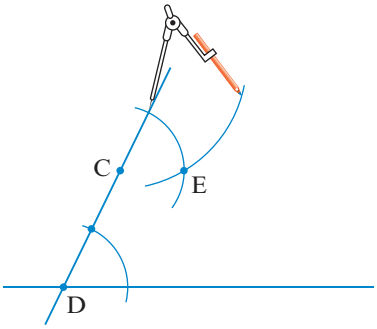



**Step 5:** Set the compass width to the distance between the points where your first arc crossed the two lines.

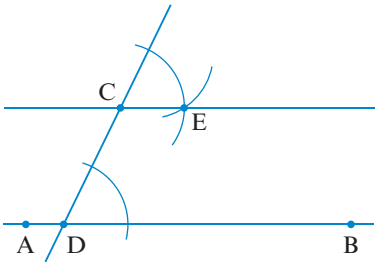


PROBLEM SOLVING AND REASONING

**Step 6:** Move the compass to where the second arc crosses the diagonal line and draw a new arc that crosses the second arc.




**Step 7:** Label the point where the two arcs meet as E. Draw a straight line through C and E. It should be parallel to your original line.



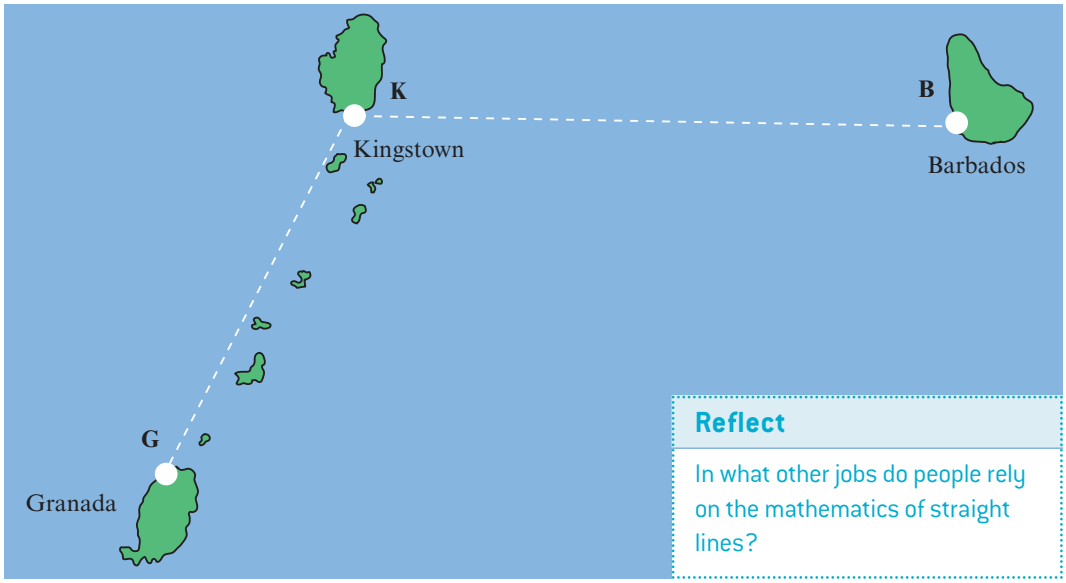

**9** Draw a line and a point in your book. Using the methods described in question **8**, construct lines perpendicular and parallel to your line through the point.

CHALLENGE

**10** It is 1705 and you are the navigator on Blackbeard's pirate ship in the Caribbean. Your spies tell you there is a cargo ship filled with treasure about to leave Barbados (point B). It will sail straight to Kingstown (point K) and then to Granada (point G). Blackbeard's plan is to disguise his ship and sail parallel to the cargo ship until just before Granada, when you will sail perpendicular to their course and board it.



Copy the map below and, from a starting point of your choice near Barbados, plot the course. Use your skill with compass constructions to draw the parallel and perpendicular lines required.




**Reflect**

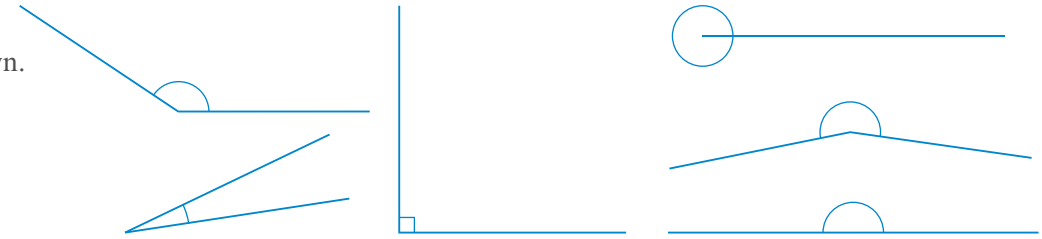
In what other jobs do people rely on the mathematics of straight lines?

# 6B Types of angles

## Start thinking!

An angle is formed anywhere that two lines meet. Different sized angles have different names. From smallest to largest they are: **acute**, right, **obtuse**, **straight**, **reflex** and **revolution**.

An example of each type is shown.



1 Draw each angle in order from smallest to largest.

2 Label each angle with its name.

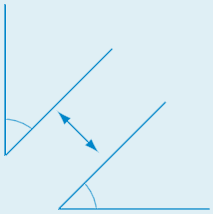
**NOTE** Arrowheads are not shown on lines that form angles.

3 From the list below, match each description of the size of the angle with your six angles.

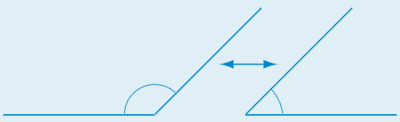
90°, 180°, 360°, between 0° and 90°, between 90° and 180°, between 180° and 360°

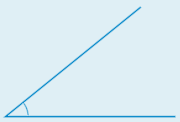
## KEY IDEAS

- Angles are classified according to their size.
- Right and straight angles are important because they make square corners and straight lines.
- Angles that add to make a right angle (90°) are called **complementary angles**.



- Angles that add to make a straight angle (180°) are called **supplementary angles**.



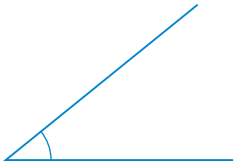
| Name | Size of angle | Example |
|---|---|---|
| acute | between 0° and 90° |  |
| right | 90° |  |
| obtuse | between 90° and 180° |  |
| straight | 180° |  |
| reflex | between 180° and 360° |  |
| revolution | 360° |  |

# EXERCISE 6B Types of angles

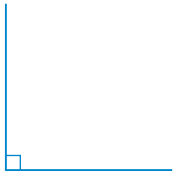
UNDERSTANDING AND FLUENCY

**1** Describe each angle as acute, right, obtuse, straight, reflex or a revolution.

**a**



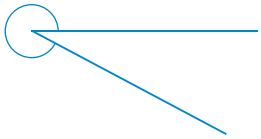
**b**



**c**



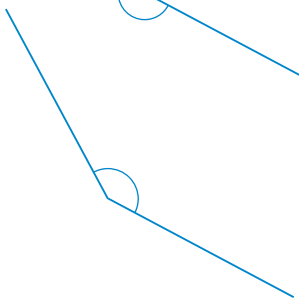
**d**



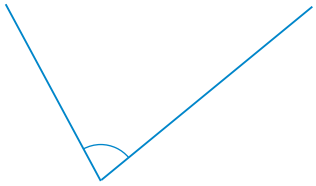
**e**



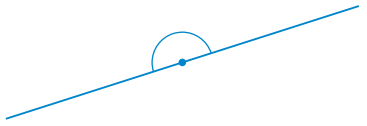
**f**



**g**



**h**



**i**



## EXAMPLE 6B-1 Finding the complement and the supplement of an angle

For the angle 42°, find the angle that is its:

**a** complement **b** supplement.

### THINK

**a** Complementary angles add to make a right angle (90°). Subtract 42° from 90° to find the complement.

**b** Supplementary angles add to make a straight angle (180°). Subtract 42° from 180° to find the supplement.

### WRITE

**a** 90° − 42° = 48°
48° is the complement of 42°.

**b** 180° − 42° = 138°
138° is the supplement of 42°.

**2** Find the complementary angle for each of these.

**a** 23° **b** 77° **c** 88° **d** 9°
**e** 45° **f** 57° **g** 34° **h** 61°

**3** Find the supplementary angle for each of these.

**a** 158° **b** 21° **c** 100° **d** 97°
**e** 149° **f** 135° **g** 81° **h** 90°

**4** For each angle listed, find the angle that is its:

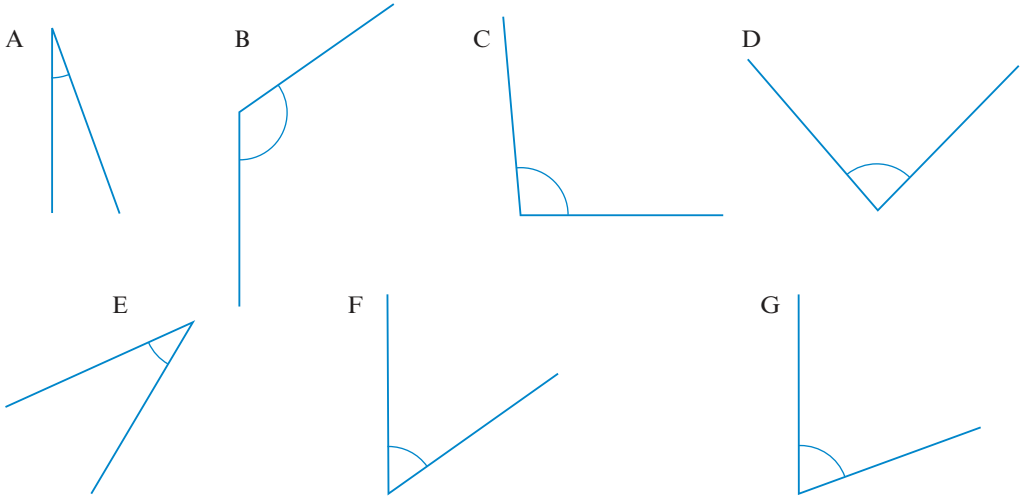
**i** complement **ii** supplement.

**a** 46° **b** 12° **c** 85° **d** 27°
**e** 33° **f** 64° **g** 72° **h** 5°

**5** Identify pairs of angles from the following list that are:

**a** complementary **b** supplementary.

(The same angle may be used in answering both parts **a** and **b**.)

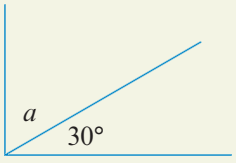



## EXAMPLE 6B-2

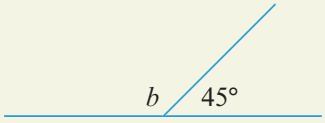
### Finding angle size using complementary and supplementary angles

Find the size of the labelled unknown angle.

**a**




**b**




**THINK**

**a** $a$ and 30° are complementary angles so they add to 90°.

**b** $b$ and 45° are supplementary angles so they add to 180°.

**WRITE**

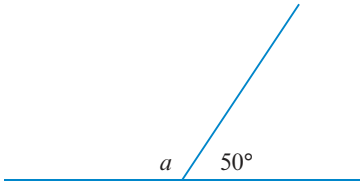
**a** $a + 30° = 90°$
$a = 90° - 30°$
$= 60°$

**b** $b + 45° = 180°$
$b = 180° - 45°$
$= 135°$

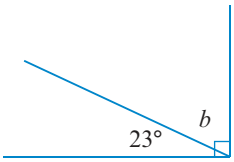
UNDERSTANDING AND FLUENCY

**6** Find the size of each labelled unknown angle.

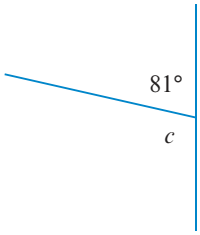
**a**



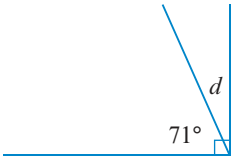

**b**



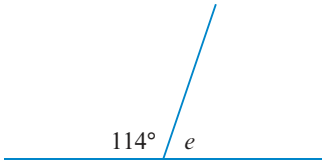

**c**



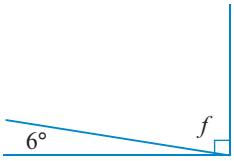

**d**



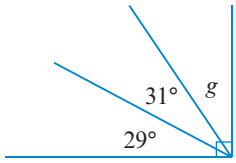

**e**



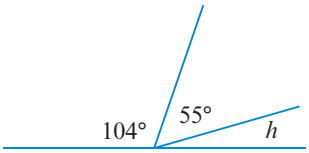

**f**



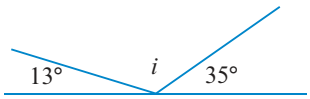

**g**



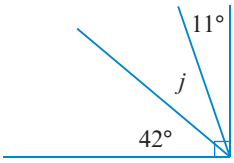

**h**




**i**



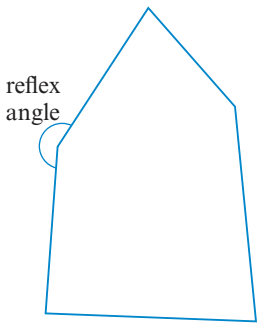

**j**




PROBLEM SOLVING AND REASONING

**7** This photo shows the door to a fashion boutique in Tokyo's Harajuku district.




**a** Copy the shape of the doorway and label all of the angles on the inside of the doorway as acute or obtuse.

**b** Label all of the angles on the outside of the doorway. One angle has already been labelled.

**c** What types of angles does the door behind the doorway have? How many are there?

**8** Angles are also used to describe the amount of turn when something changes direction.



**a** Point one arm straight out in front of you and point the other arm towards the classroom door.
How would you describe the angle between your arms (acute, right, obtuse, etc.)?

**b** Point one arm at the rubbish bin. Move the same arm so that it now points towards the front left corner of the room.
How would you describe the angle that your arm moved?

**c** Stand and face the direction of the school's front office. Now turn to face your teacher.
How would you describe the angle that you turned through?

**d** How else could you have turned and ended up facing the same way?
How would you describe that angle?

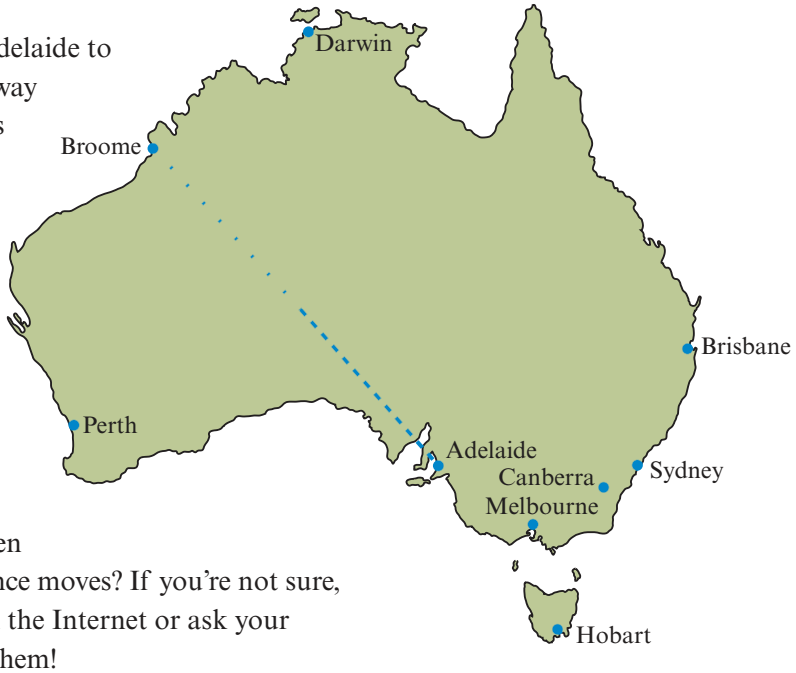
**9** The Google Earth plane takes off from Brisbane airport and flies to Sydney. It takes a photo over Sydney and then flies to Melbourne for another photo. From Melbourne it flies to Adelaide and then it turns around to fly back to Sydney, where it lands.

**a** Sketch a map of the plane's route.

**b** What type of angle did it turn through above each city?

PROBLEM SOLVING AND REASONING

**10** A plane is flying from Adelaide to Broome. When it is halfway there, bad weather forces it to change direction and land in Perth. Sketch the two possible angles the plane could turn through and label them as acute, obtuse or reflex.




**11** What type of angles do you turn through when you do the following dance moves? If you're not sure, research these dances on the Internet or ask your teacher to demonstrate them!

**a** the Nutbush **b** the Hokey-Pokey **c** the Macarena

**12** Derek is a male model who cannot turn left. When he wants to walk around a left corner he has to turn right until he is facing the correct direction.
How would you describe the angle that Derek rotates through when he does this?



**13** For the situations described below:

**i** state whether there are complementary or supplementary angles

**ii** find the size of the remaining angle each person needs to turn through.

**a** Peter turns through 56° in an attempt to make a right-hand turn.

**b** Gloria leaves her house but forgets something and turns through 87° in order to turn back.

**c** Michelle completes 146° of a half-pirouette.

**d** Leigh completes $\frac{2}{3}$ of a left-hand turn.

**14** Draw two angles that are complementary.

**15** Draw two angles that are supplementary.

**16** Can two acute angles be supplementary? Explain.

CHALLENGE

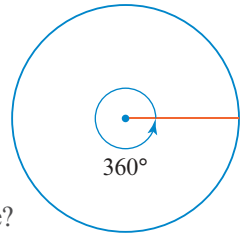
**17** Draw two angles – one obtuse, the other reflex – that add to make a revolution. Can you think of your own name for this pair of angles?

**Reflect**

Why are right-angled turns so common in dancing?

# 6C Measuring and drawing angles

## Start thinking!



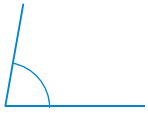

Over 3000 years ago, Babylonian astronomers estimated that there were 360 days in a year and so they divided circles into 360 degrees (360°). We still use this system today.

**1** Look at the angle in the centre of this circle. Use this to answer the following.

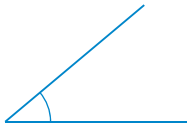
**a** How many degrees in a right angle? **b** How many degrees in a straight angle?

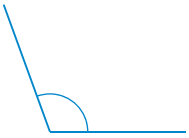
**c** How many degrees in a revolution?

**2** Estimate how many degrees are in each angle.

**a** 

**b** 

**c** 

**d** 

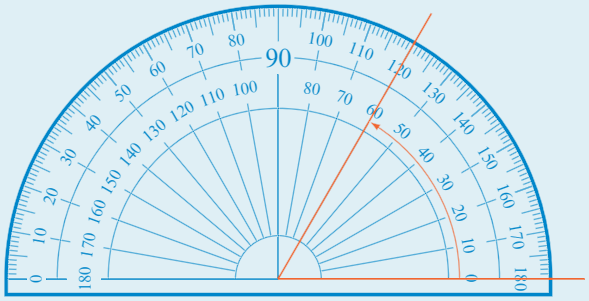
**3** To measure exactly how many degrees are in an angle, use a protractor. Some protractors are a full circle but most are semicircle or 180° protractors.

A 180° protractor has two scales. One runs from 0° to 180° clockwise, the other runs anti-clockwise. (See the diagram in Key ideas below.) How do you know which scale (inner or outer) to use?

**4** Trace each angle in question **2** to make a copy and extend the arms to be 6 cm long. Measure each angle to see how accurate you were.

## KEY IDEAS

- Angles are commonly measured in degrees. There are 360° in a full circle.
- To measure an angle, follow these steps to use a protractor.
  1. Estimate the size of the angle.
  2. Place the centre cross of the protractor over the point of the angle.
  3. Place the horizontal line that reads 0° on one arm of the angle.
  4. Read the scale from zero to the other arm of the angle.




**NOTE** Remember to use the correct scale (outer or inner) when measuring an angle with a protractor. Always measure from 0°.

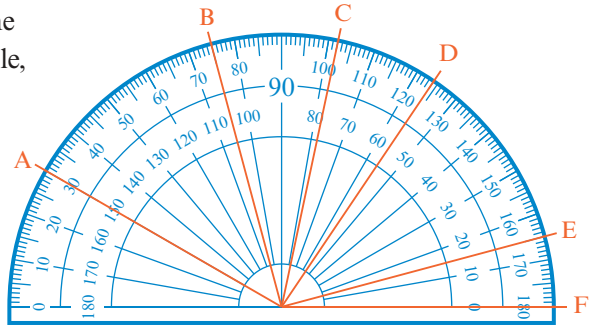
# EXERCISE 6C Measuring and drawing angles

UNDERSTANDING AND FLUENCY

**1** **a** How many right angles make up a straight angle?
**b** How many right angles make up a revolution?
**c** How many straight angles make up a revolution?

**2** Copy and complete these sentences.
**a** An acute angle must be smaller than a _____ angle and be between _____ and _____ degrees.
**b** An obtuse angle is larger than a _____ angle but smaller than a _____ angle. It must be between _____ and _____ degrees.
**c** A reflex angle is larger than a _____ angle but smaller than a _____. It must be between _____ and _____ degrees.

**3** **a** What two angles could be indicated by each position from A to F on the 180° protractor shown? For example, A could be 30° on the outer scale or 150° on the inner scale.
**b** Take one arm of an angle as the horizontal line running through F. What is the size of the angle if the other arm runs through:
**i** A? **ii** B? **iii** C? **iv** D? **v** E?




## EXAMPLE 6C-1 Estimating and measuring angles

Estimate and then measure this angle.



**THINK**

**1** Decide what type of angle it is.

**2** Estimate the size of the angle.

**3** Place the centre cross on the point of the angle and line up the zero line. Read the scale from 0°.

**4** Check your answer. The scale could also read 45°. Which is closer to your estimation?

**WRITE**

obtuse angle

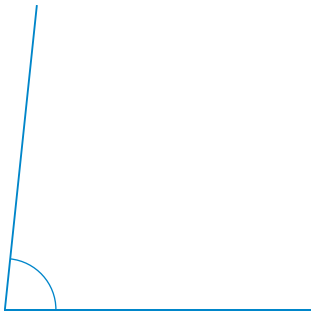
My estimate is about 140°.

The scale shows 135°.

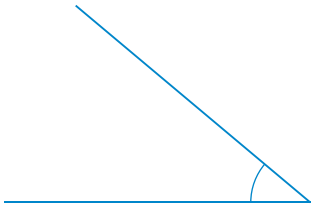
The actual measurement is 135°.

**4** First estimate and then measure these acute angles.

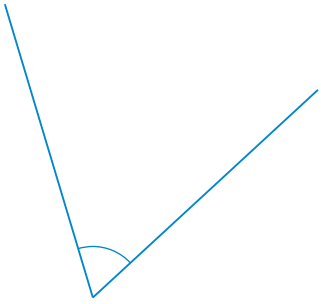
**a**



**b**



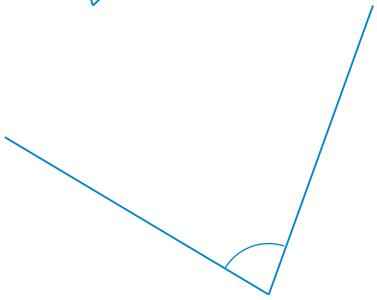
**c**



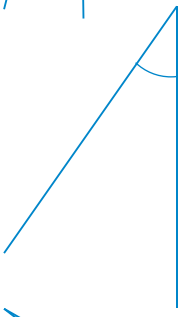
**d**



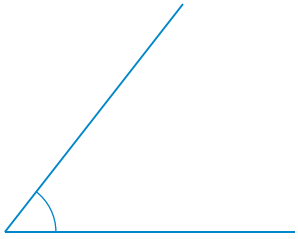
**e**



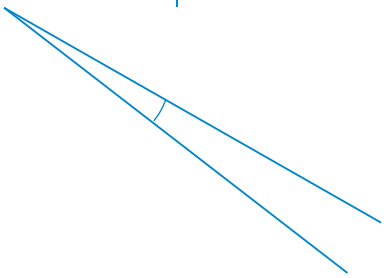
**f**



**g**



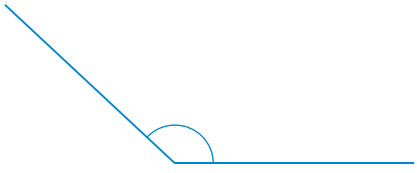
**h**



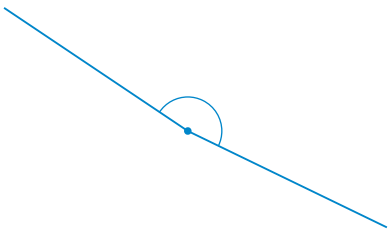
**5** How many angles did you estimate to within 10° of the actual measurement? Give yourself a mark out of 10 for estimating acute angles.

**6** Estimate and then measure these obtuse angles.

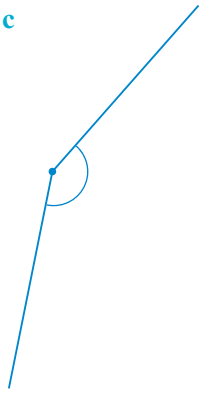
**a**



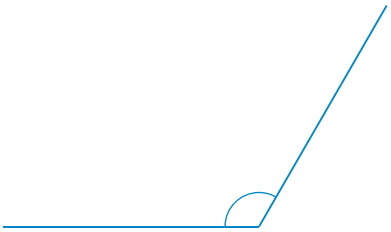
**b**



**c**

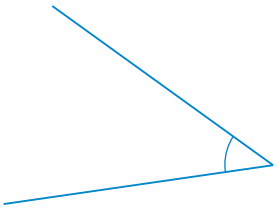


**d**



**7** State the angle type, estimate, and then measure these angles.

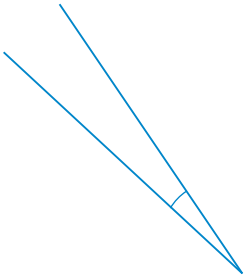
**a**



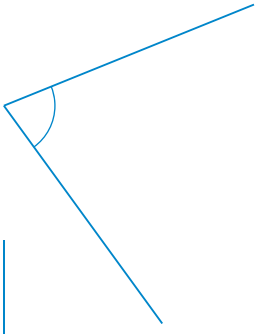
**b**



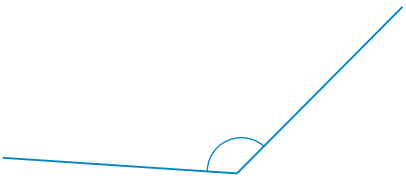
**c**



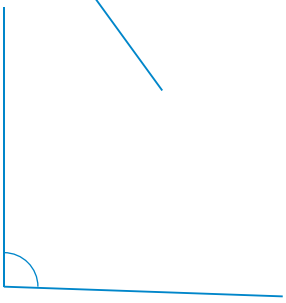
**d**



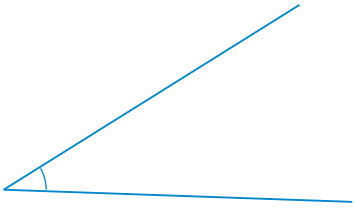
**e**



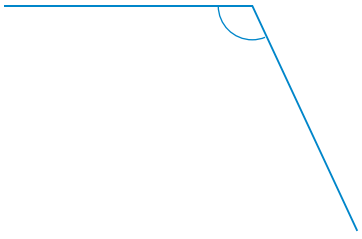
**f**



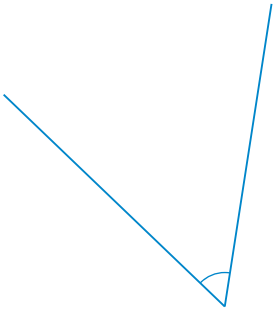
**g**



**h**



**i**



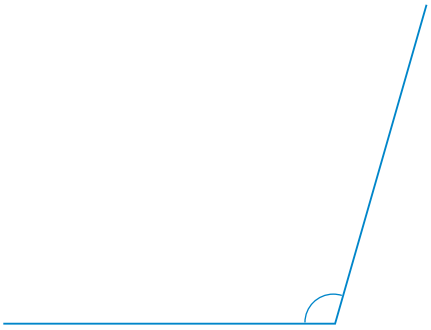
**j**



UNDERSTANDING AND FLUENCY

**8** How many of the angles in question **7** did you estimate correctly, to within 10° of the actual measurement? Give yourself a mark out of ten for estimating mixed angles.

**9** A distracted student measured this angle as 74°. What mistake did he make and what should the answer have been?



**10** Use a ruler (but not a protractor) to draw what you estimate each angle would look like.

| | | | | |
|---|---|---|---|---|
| **a** 80° | **b** 42° | **c** 173° | **d** 10° | **e** 131° |
| **f** 29° | **g** 100° | **h** 55° | **i** 164° | **j** 66° |

**11** Now use a protractor to measure each angle you drew for question **10**. Write the actual angle next to each drawing. How many angles did you draw to within 10° of the actual measurement? Give yourself a mark out of ten for drawing angles.

## EXAMPLE 6C-2 Drawing angles

Draw an angle of 60°.

### THINK

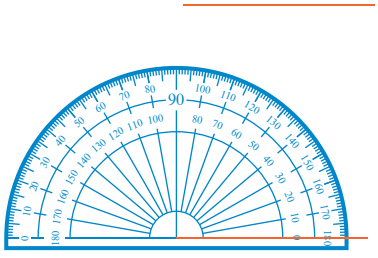
**1** Start with a straight line.

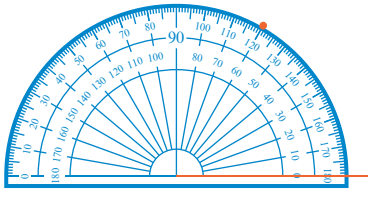
**2** Place your protractor over the line, with the line directly under the zero line and one end of it on the centre point.

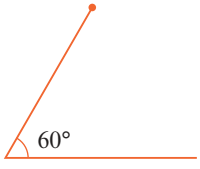
**3** Use the scale that starts at 0. Place a dot above the 60° mark on your protractor.

**4** Remove the protractor and use a ruler to join the end of your line with the dot.

### WRITE







**12** Draw each angle.

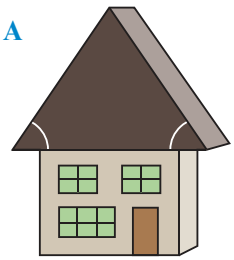
| | | | | |
|---|---|---|---|---|
| **a** 45° | **b** 80° | **c** 23° | **d** 120° | **e** 165° |
| **f** 9° | **g** 56° | **h** 147° | **i** 94° | **j** 112° |

PROBLEM SOLVING AND REASONING

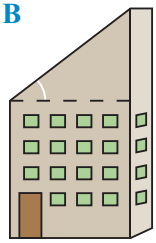
**13** Houses in alpine areas need to have roofs that form an angle of at least 50° with the horizontal so that snow will slide off. Which of these building designs will be suitable? Explain your reasoning.



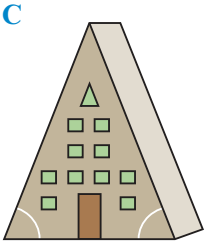
**A**



**B**



**C**



**D**

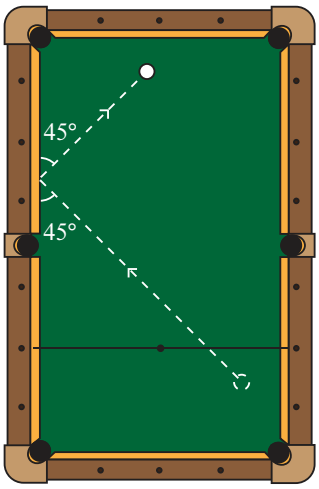


CHALLENGE

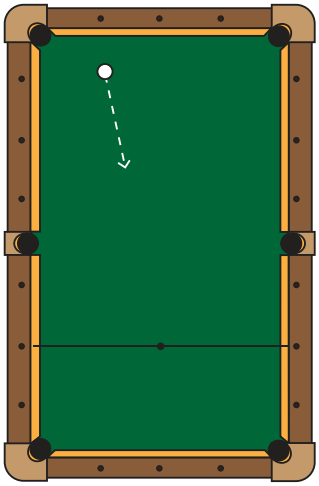
**14** Pool players make use of angles when planning their shots. They know that when a ball bounces off the side of a table it rebounds at the same angle.

The ball on the pool table shown at right hits the side of the table at an angle of 45° and bounces off at an angle of 45°.

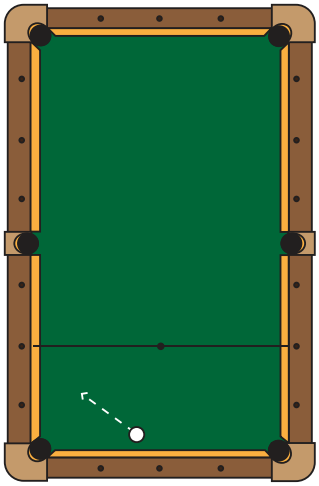
Use your knowledge of drawing angles to show which pocket the white ball will end up in.




**a**



**b**



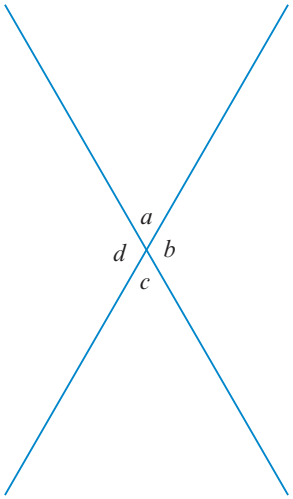
**Reflect**

What do you think is the most common mistake students make when using a 180° protractor?

# 6D Angles around a point

## Start thinking!

A small village has grown around the place where two straight roads cross each other, as shown in the diagram.




**1** Use a protractor to measure the four angles ($a$, $b$, $c$ and $d$) formed by the roads.

**2** Perform the following additions.

**a** angle $a$ + angle $b$ = _____°

**b** angle $c$ + angle $d$ = _____°

**c** angle $b$ + angle $c$ = _____°

**d** angle $a$ + angle $d$ = _____°

**e** angle $a$ + angle $b$ + angle $c$ + angle $d$ = _____°

**3** Name two pairs of angles in this diagram that are supplementary.

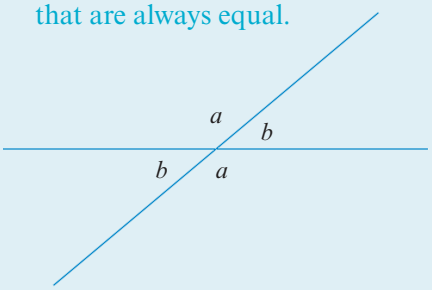
**4** Describe any patterns you see in the angle sizes.

**5** Draw another set of crossroads and measure the four angles. Do you notice the same pattern?

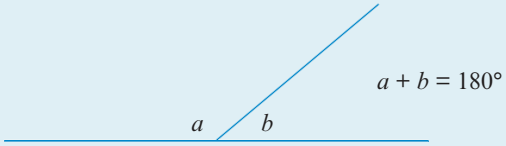
**6** On your drawing, use coloured pens or pencils to show which angles are identical. These are called **vertically opposite angles**.

## KEY IDEAS

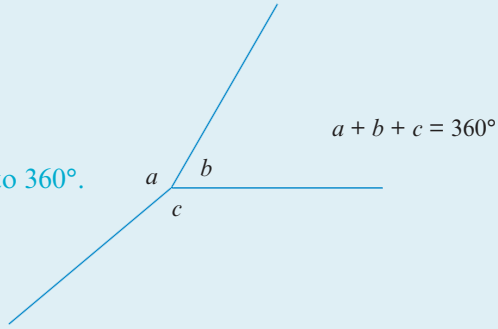
- Where two or more straight lines meet at a point, they form vertically opposite angles that are always equal.




- Angles around a straight line are supplementary and add to 180°.




- Angles around a point add to 360°.



# EXERCISE 6D Angles around a point

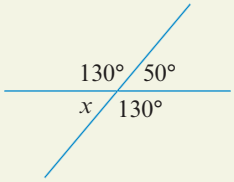
**1** Complete the following definitions.

**a** Angles around a point add to _____°. **b** Vertically opposite angles are _____.

**c** Angles around a line are supplementary and add to _____°.

## EXAMPLE 6D-1 Finding angle size using vertically opposite angles

Find the size of angle $x$.




**THINK**

Vertically opposite angles are equal. $x$ is vertically opposite 50°.

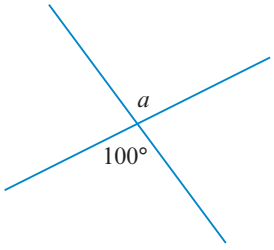
**WRITE**

$x = 50°$

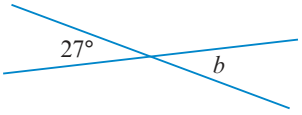
UNDERSTANDING AND FLUENCY

**2** Find the size of each labelled unknown angle.

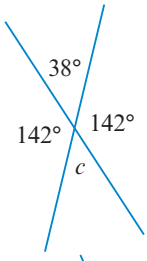
**a**



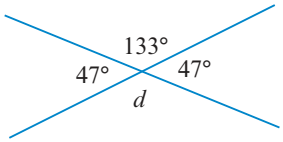

**b**



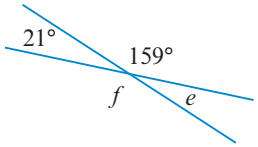

**c**



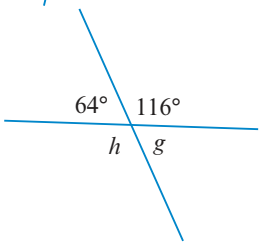

**d**




**e**

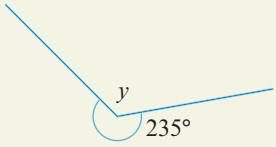



**f**




## EXAMPLE 6D-2 Finding angle size using angles around a point

Find the size of angle $y$.




**THINK**

Angles around a point add to 360°. Subtract 235° from 360°.

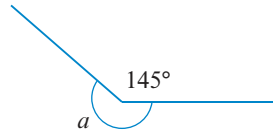
**WRITE**

$y = 360° - 235°$
$= 125°$

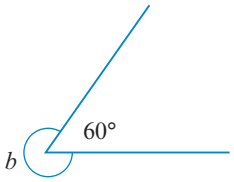
UNDERSTANDING AND FLUENCY

**3** Find the size of each unknown angle.

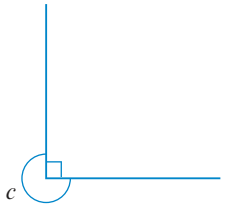
**a**



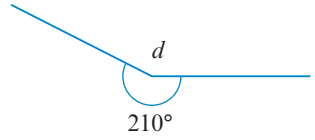

**b**



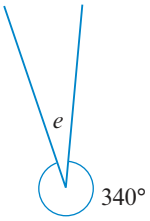

**c**



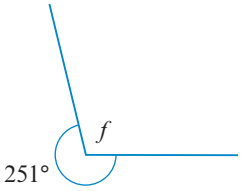

**d**




**e**



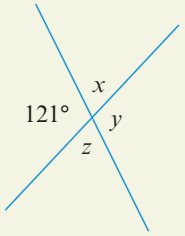

**f**




## EXAMPLE 6D-3

### Finding angle size using vertically opposite angles and straight angles

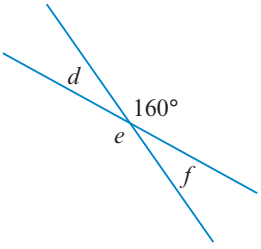
Find the size of each unknown angle.



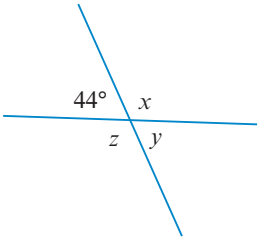

| THINK | WRITE |
|---|---|
| **1** Vertically opposite angles are equal. | $y = 121°$ |
| **2** Angles around a line are supplementary (add to 180°). | $x = 180° - 121°$<br>$= 59°$ |
| **3** Vertically opposite angles are equal. | $z = 59°$ |

**4** Find the size of each labelled unknown angle.

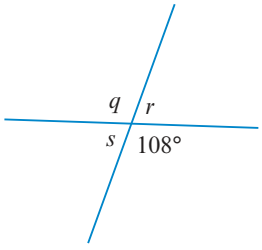
**a**



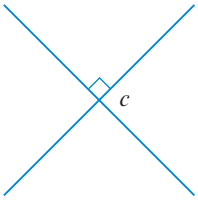

**b**



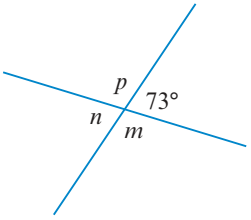

**c**



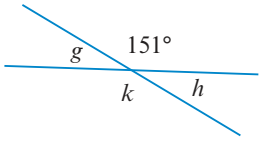

**d**




**e**

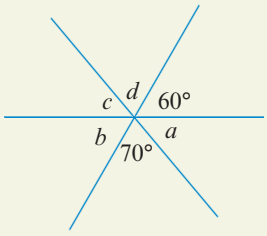



**f**



## EXAMPLE 6D-4 Finding size of angles around a point

Find the size of each unknown angle.




**THINK**

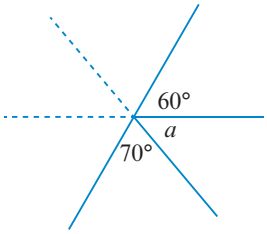
**1** Vertically opposite angles are equal.

**2** Angles around a line add to 180°.

**3** Vertically opposite angles are equal.

**WRITE**

$b = 60°$, $d = 70°$



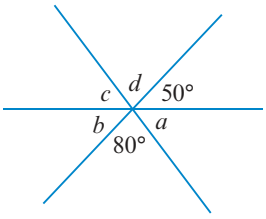

$a = 180° - 70° - 60°$
$= 50°$

$c = 50°$

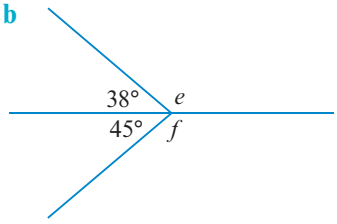
UNDERSTANDING AND FLUENCY

**5** Find the size of each unknown angle.

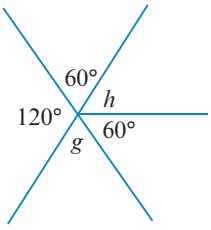
**a**



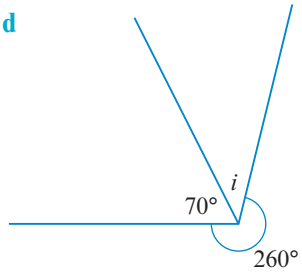

**b**



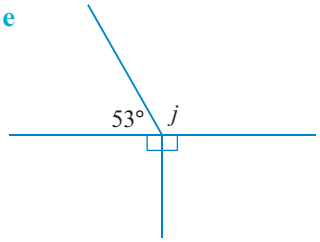

**c**



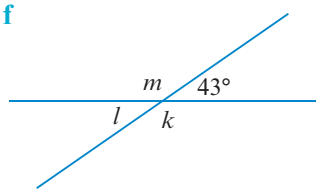

**d**



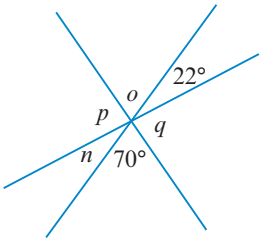

**e**



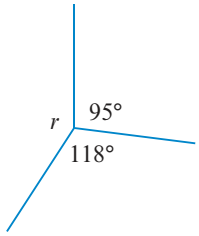

**f**



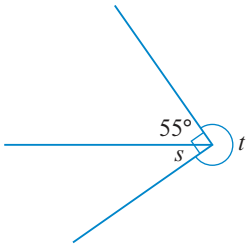

**g**




**h**



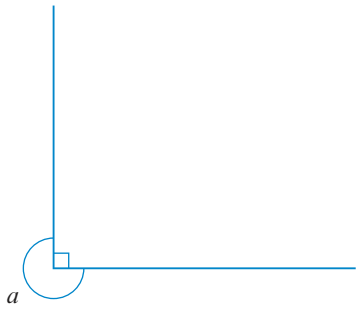

**i**



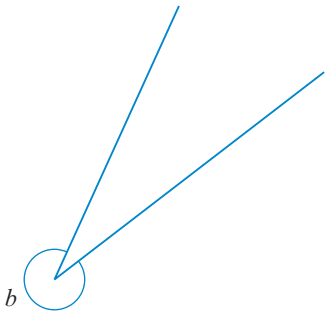
UNDERSTANDING AND FLUENCY

**6** **i** Estimate the size of each reflex angle.
**ii** Find the size of the reflex angle by measuring the acute or obtuse angle first.

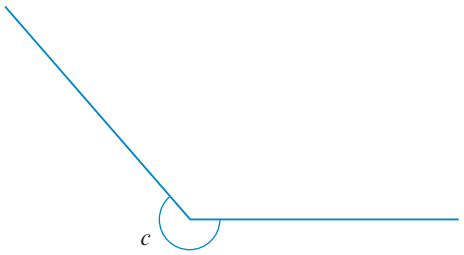
**a**



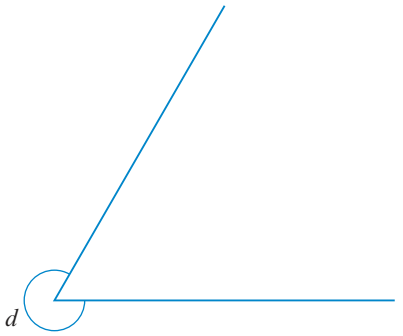

**b**



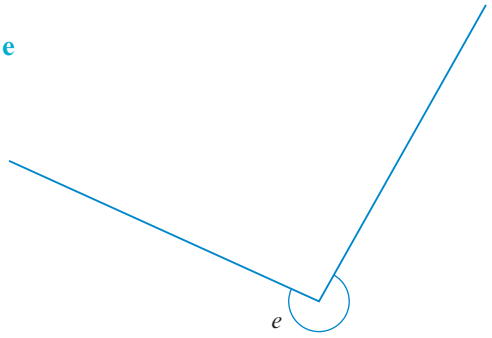

**c**



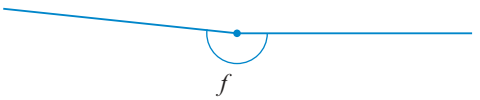

**d**




**e**




**f**




## EXAMPLE 6D-5 Drawing reflex angles

Use a protractor to draw an angle of 240°.

### THINK

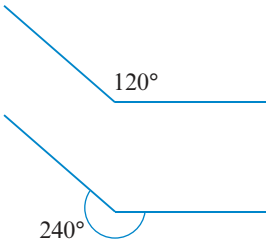
1 With a 180° protractor, draw an acute or obtuse angle first. Work out this angle.

2 Use a protractor to draw an angle of 120°.

3 Mark the 240° angle.

### WRITE

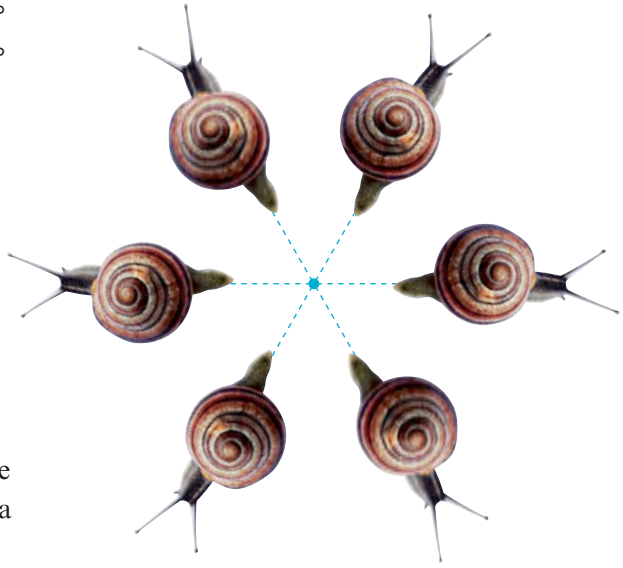
360° − 240° = 120°



PROBLEM SOLVING AND REASONING

**7** Draw these reflex angles. (Hint: draw an acute or obtuse angle first.)

**a** 270° **b** 200° **c** 335°
**d** 185° **e** 349° **f** 300°
**g** 227° **h** 312°



**8** In a snail race, six snails start at the centre of a circle and race to the outside. If the snails spread out as widely as possible and travel in straight lines, what angle is made between the trails they leave?

**9** Chelsea, Kane and Sara are sharing a large round pizza. Chelsea eats one piece, Kane eats four pieces and Sara eats three pieces.

**a** How many pieces do they need to cut the pizza into?

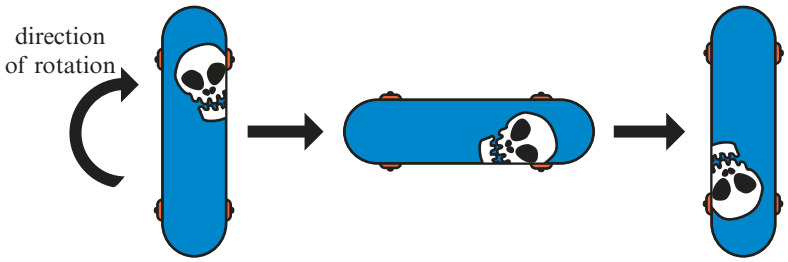
**b** What is the angle size of each piece of pizza?

The next weekend Chelsea's friend Ben joins them for dinner, so they decide to order two pizzas. This time Chelsea eats one piece, Kane eats four pieces, Sara eats two pieces and Ben eats three pieces. All the pizza is eaten.

**c** How many pieces is this in total?

**d** What is the angle size of each piece of pizza? Remember that there are two pizzas to share.

**10** Skateboarding, snowboarding and wakeboarding tricks often involve rotation through the air. A '180 aerial' means the rider spins through 180° and then lands with the board facing the opposite direction.






Show the start and end position of a rider's board after the following aerial rotations.

**a** 360° **b** 540° **c** 720°

CHALLENGE

**11** An aerial freestyle skier launches herself into the air but is knocked off balance by a gust of wind. She estimates that she has already turned through 415°. To land safely she needs to have turned through a multiple of 180°. How much further does she need to rotate if she is to land safely?

**Reflect**

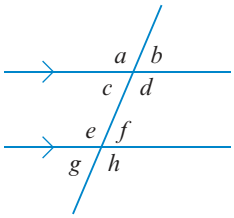
Why are 270° rotations uncommon in board sports?

# 6E Angles and parallel lines

## Start thinking!

1 Draw a pair of parallel lines and then rule another line across it, as shown in the diagram. This line is called a **transversal**. Label the angles in your diagram exactly the same as shown.

2 Use a protractor to measure all of the angles $a$–$h$. What do you notice?




3 Angles $d$ and $e$ are called **alternate angles**. Can you see why?

- **a** Name the other pair of alternate angles.
- **b** Make a copy of your diagram and highlight the pairs of alternate angles.

4 Angles $a$ and $e$ are called **corresponding angles**. Can you see why?

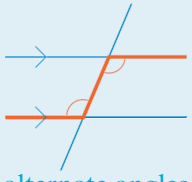
- **a** There are three other pairs of corresponding angles. Can you name them?
- **b** Make a copy of your diagram and highlight the pairs of corresponding angles.

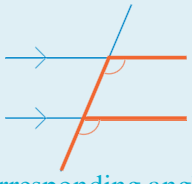
5 Angles $c$ and $e$ are called **co-interior angles** (or **allied angles**). Can you see why?

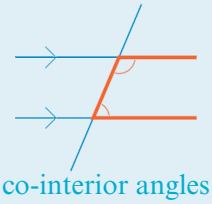
- **a** Name the other pair of co-interior angles.
- **b** Make a copy of your diagram and highlight the pairs of co-interior angles.

## KEY IDEAS

- A straight line that crosses a pair of parallel lines is called a transversal.
- When a transversal crosses a pair of parallel lines it creates a number of angles, many of which are equal.
- Alternate angles are on opposite (alternate) sides of the transversal between the parallel lines and are equal in size.
- Corresponding angles are on the same side of the transversal and in the same position on the parallel lines, either both above or both below a line. They are equal in size.
- Co-interior (or allied) angles are on the same side of the transversal between the parallel lines and they are supplementary (they add to 180°).


alternate angles


corresponding angles

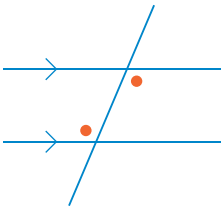

co-interior angles

# EXERCISE 6E Angles and parallel lines

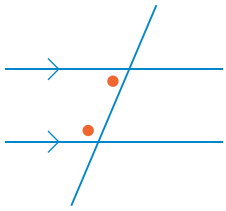
UNDERSTANDING AND FLUENCY

**1** Identify each pair of angles as either alternate, corresponding or co-interior.

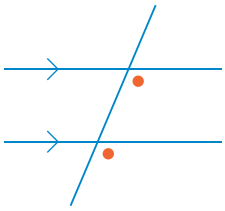
**a**



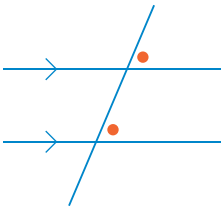
**b**



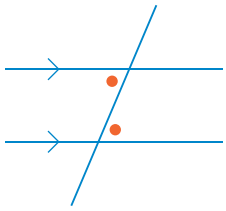
**c**



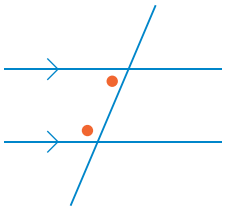
**d**



**e**



**f**



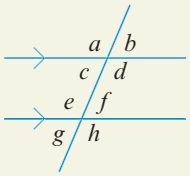
**2** Use your knowledge of angles and parallel lines to state whether the pairs of angles in question **1** are equal or supplementary.

## EXAMPLE 6E-1 Naming alternate, corresponding and co-interior angles

Name the angle that is:

**a** alternate to $c$

**b** corresponding to $e$.




### THINK

**a** **1** Alternate angles are on opposite sides of the transversal. Draw a 'Z' enclosing $c$.

**2** The other angle inside the 'Z' is $f$.

**3** Write your answer.

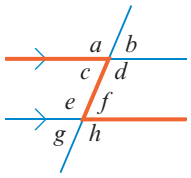
**b** **1** Corresponding angles are on the same side of the transversal. Draw an 'F' enclosing $e$.

**2** The other angle inside the 'F' is $a$.

**3** Write your answer.

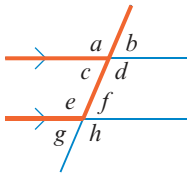
### WRITE

**a**




$f$ is alternate to $c$.

**b**



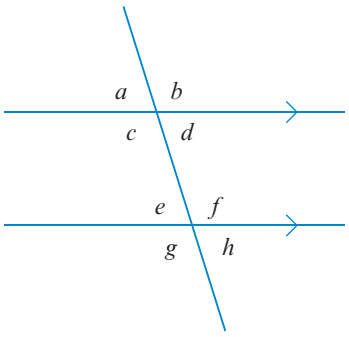

$a$ is corresponding to $e$.

**3** Use this diagram to name the angle that is:

**a** corresponding to $b$ **b** alternate to $e$

**c** corresponding to $a$ **d** co-interior to $c$

**e** co-interior to $f$ **f** alternate to $d$.




**4** Use the diagram given in question **3** to name the angle that is vertically opposite to each of these.

**a** $c$ **b** $h$ **c** $f$

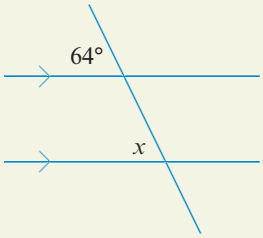
**d** $a$ **e** $b$ **f** $e$

## EXAMPLE 6E-2

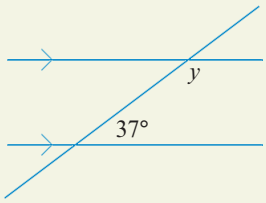
### Finding angle size using alternate, corresponding and co-interior angles

Find the size of each labelled unknown angle.

**a**




**b**




**THINK**

**a** Look for a relationship between $x$ and 64°. Corresponding angles are equal.

**b** Look for a relationship between $y$ and 37°. Co-interior angles add to 180°.

**WRITE**

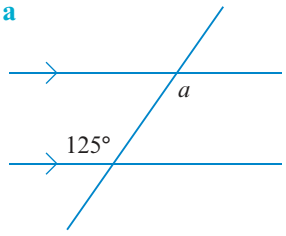
**a** $x$ and 64° are corresponding angles.
$x = 64°$

**b** $y$ and 37° are co-interior angles.
$y = 180° - 37°$
$= 143°$

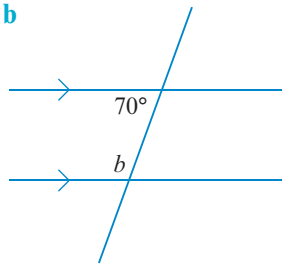
UNDERSTANDING AND FLUENCY

**5** Find the size of each labelled unknown angle.

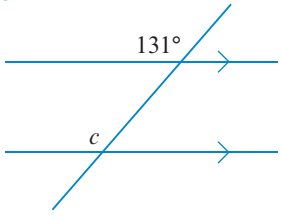
**a**



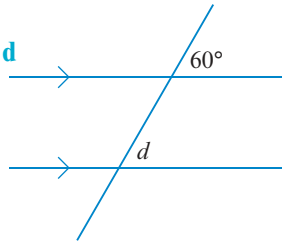

**b**



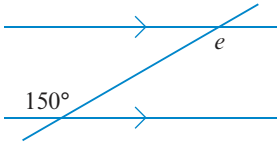

**c**



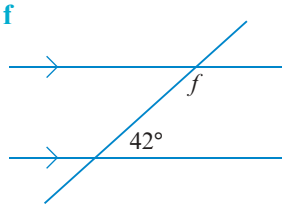

**d**




**e**

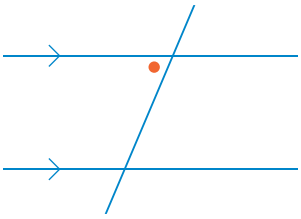



**f**



UNDERSTANDING AND FLUENCY

**6** For each part below, copy the diagram at right and mark in the angle required.

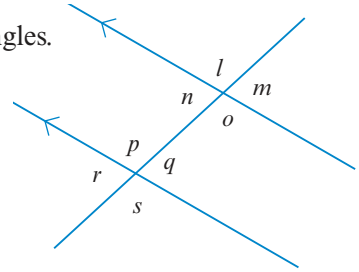


**a** alternate

**b** corresponding

**c** co-interior

**7** Find all the pairs of these angles.




**a** alternate angles

**b** co-interior angles

**c** corresponding angles

PROBLEM SOLVING AND REASONING

**8** Identify the pairs of angles marked in these photos.

**a**



**b**

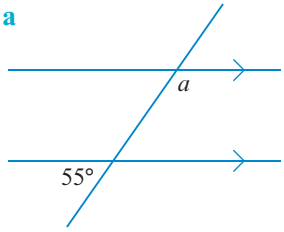


**c**

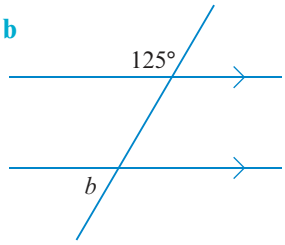


**9** Find the size of each labelled unknown angle. Explain which angle relationships you used in each case.

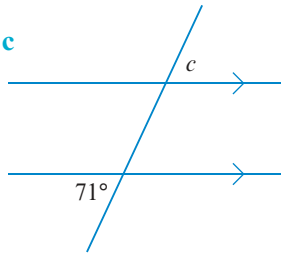
**a**



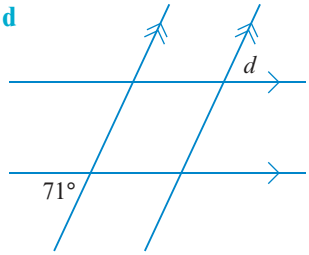

**b**



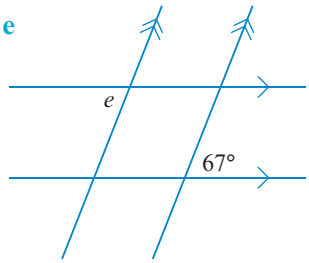

**c**



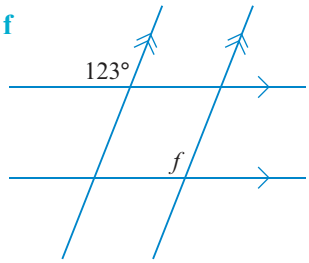

**d**



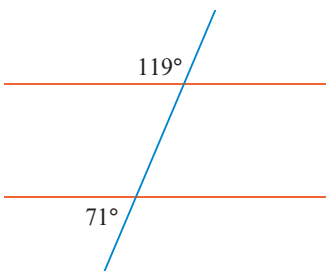

**e**




**f**

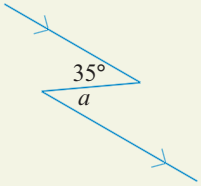



**10** Use your knowledge of angle relationships between parallel lines and a transversal to decide whether the two orange lines are parallel. Explain your reasoning.



## EXAMPLE 6E-3 Highlighting parallel lines to identify angle relationships

Find the size of angle $a$. Explain how you obtained your answer.



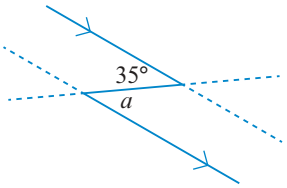

**THINK**

**1** Copy the diagram and make the parallel lines more obvious.

**2** Look for a relationship between $a$ and 35°.

**3** Write the size of $a$ and give a reason.

**WRITE**



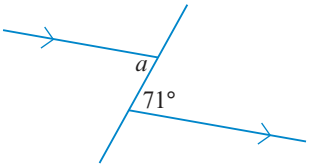

$a$ and 35° are alternate angles.

$a = 35°$ because alternate angles are equal.

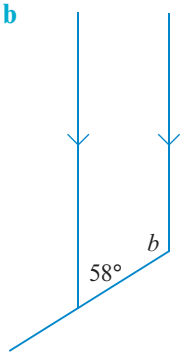
PROBLEM SOLVING AND REASONING

**11** Find the size of each labelled unknown angle. Explain how you obtained your answer.

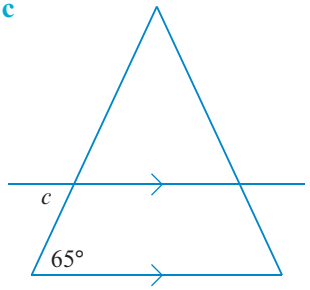
**a**



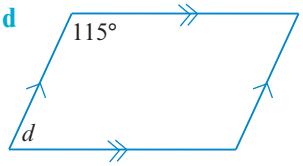

**b**



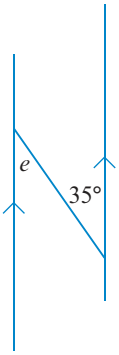

**c**



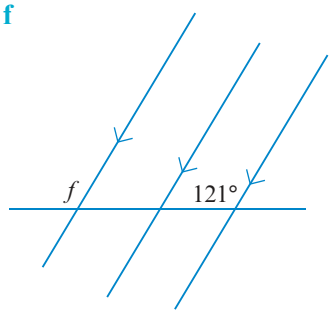

**d**



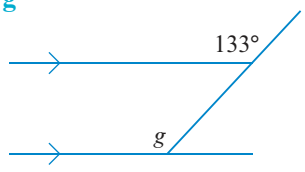

**e**



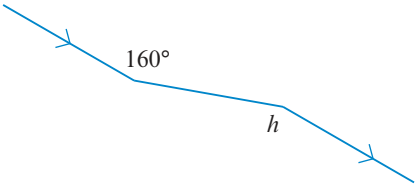

**f**




**g**

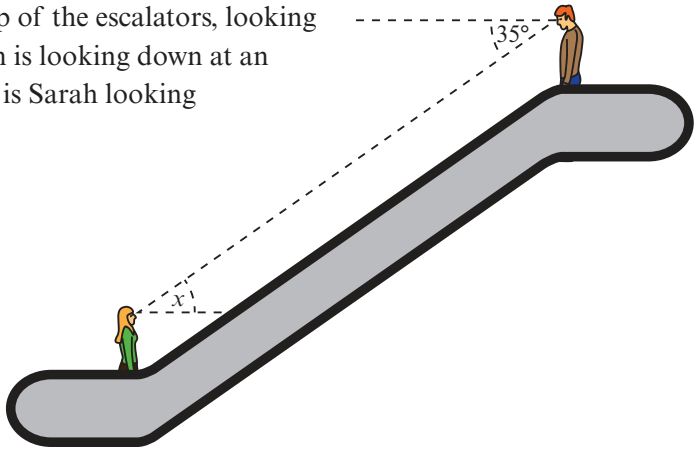



**h**

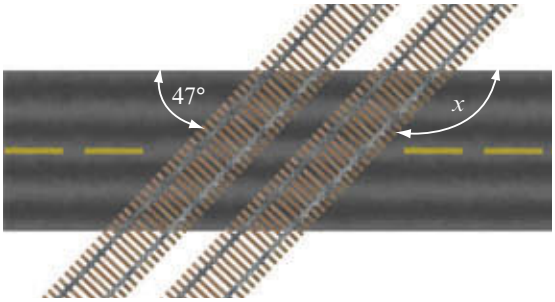


PROBLEM SOLVING AND REASONING

**12** Cameron is standing on top of the escalators, looking down at Sarah. If Cameron is looking down at an angle of 35°, at what angle is Sarah looking up at him?




**13** A road crosses a pair of railway lines as shown in the diagram. If a driver approaches the railway lines at an angle of 47° as shown, at what angle does the car leave the tracks?




CHALLENGE

**14** Amal is putting up some paintings over a straight crack in the wall. She knows that the top painting is parallel with the edge of the ceiling.




**a** Find the size of the angles marked $a$, $b$ and $c$ if the paintings are parallel with one another.

**b** Amal measures angle $c$ and finds that it is 66°. Are the paintings parallel with one another?

**c** How will she have to move the bottom painting in order to make it parallel to the top one?

**Reflect**

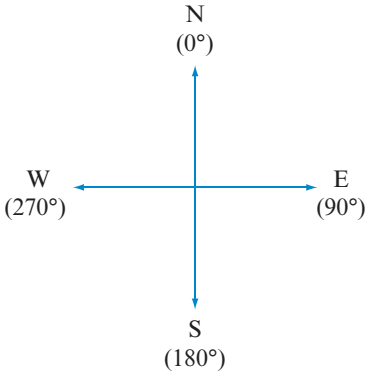
When is it useful to know about angles and parallel lines?

# 6F Understanding bearings



## Start thinking!

Directions and **bearings** are important for activities such as hiking, driving and flying. A **compass** has four major (or cardinal) points: north (N), east (E), south (S) and west (W). Each of these directions makes an angle (moving clockwise) with north. For example, east is 90° from north.




**1** What is the size of the angle, moving clockwise:

**a** from north to south? **b** from north to west?

**c** from east to west? **d** from north around to north again?

**2** Between each of the four cardinal points are four more points.
For example, between north and east is north-east (NE).
What name and abbreviation is given to the three remaining points?

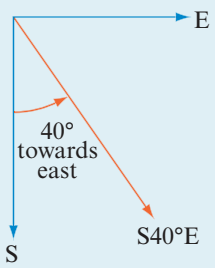
**3** What is the size of the angle, moving clockwise:

**a** from north to north-east? **b** from south to north-west?

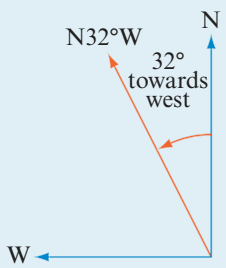
**c** from south-east to south-west? **d** from north-east to north-west?

## KEY IDEAS

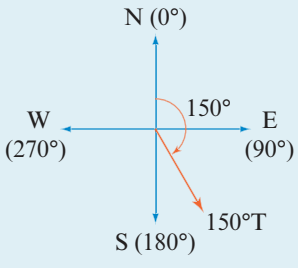
- A bearing is the direction from one object to another, using the angles around a compass.
- The standard bearings on a compass rose are north (N), east (E), south (S) and west (W).
- Two main types of bearings are commonly used: compass bearings and true bearings.
- A **compass bearing** starts from either north or south, whichever is closer, then moves towards either east or west.
  For example, S40°E means: face south, then turn 40° towards east.




  And N32°W means: face north, then turn 32° towards west.



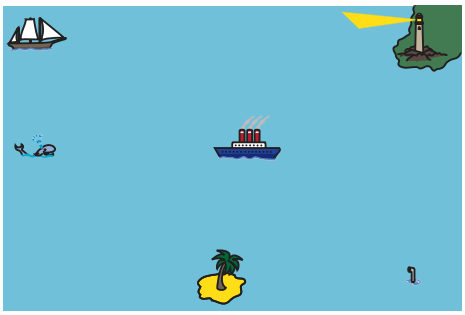

- A **true bearing** is always written with three digits and a T, and starts at north, moving clockwise around the compass in a circle.
  For example, the direction shown has a true bearing of 150°T.



# EXERCISE 6F Understanding bearings

UNDERSTANDING AND FLUENCY

**1** The diagram at right shows the positions of different objects.



**a** From the steamship at the centre, what compass direction (N, E, SW, etc) is:

**i** the whale?

**ii** the palm tree?

**iii** the lighthouse?

**b** What is located in the following directions from the steamship?

**i** north-west **ii** south **iii** south-east

**c** What is located:

**i** west of the lighthouse? **ii** north of the whale?

**iii** north-west of the periscope?

**2** Identify each of these as either a compass bearing or a true bearing.

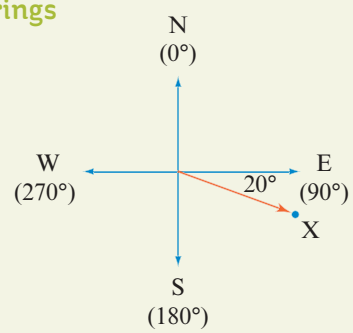
**a** 035°T **b** N66°W **c** 149°T **d** S24°E

## EXAMPLE 6F-1 Writing bearings

Give the direction to point X as:

**a** a compass bearing

**b** a true bearing.




### THINK

**a** **1** Is the direction to X closer to north or south? (south)
Work out the angle from south towards X.

**2** Write the bearing. To get to X, face south and then turn 70° towards east.

**b** **1** Work out the angle turning clockwise from north to X.

**2** Write the bearing. X is 110° clockwise from north.

### WRITE

**a** angle from south to X
= 90° − 20°
= 70°

Compass bearing is S70°E.

**b** 90° + 20° = 110°

True bearing is 110°T.

UNDERSTANDING AND FLUENCY

**3** Write the direction to each point in this diagram as:

**a** a compass bearing

**b** a true bearing.

**4** Use the diagram in question **1** to write a true bearing for:

**a** the steam ship from the palm tree

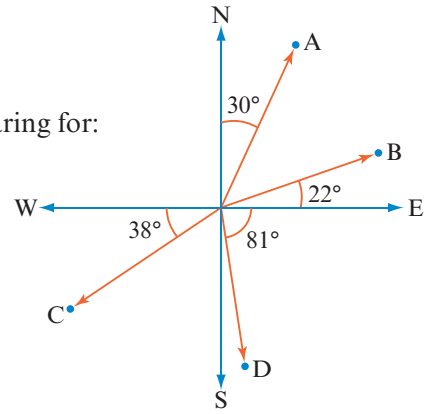
**b** the yacht from the lighthouse

**c** the periscope from the lighthouse

**d** the lighthouse from the steam ship

**e** the periscope from the steam ship

**f** the whale from the palm tree.




## EXAMPLE 6F-2 Showing a bearing on a compass rose

Draw a compass rose and show each bearing.

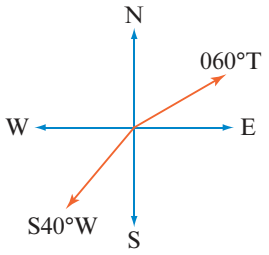
**a** S40°W **b** 060°T

### THINK

**a** S40°W means to face south and turn 40° towards west. Measure an angle of 40° from the south line in a clockwise direction.

**b** 060°T means to turn 60° in a clockwise direction from north (or 0°). Measure an angle of 60° from the north line in a clockwise direction.

### WRITE




**5** Draw a compass rose and show these compass bearings.

**a** N20°E **b** S50°W **c** S15°E **d** N48°W

**6** Draw a compass rose and show these true bearings.

**a** 150°T **b** 310°T **c** 256°T **d** 024°T

**7** **a** Draw a compass rose and by estimation show these bearings.

**i** S45°E **ii** N60°W **iii** S30°W **iv** N10°E

**b** Use a protractor to check the estimations you made.

**8** **a** Draw a compass rose and by estimation show these bearings.

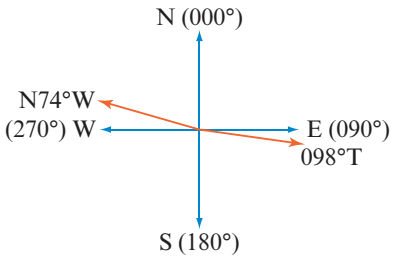
**i** 200°T **ii** 080°T **iii** 275°T **iv** 350°T

**b** Use a protractor to check the estimations you made.

## EXAMPLE 6F-3 Converting between true bearings and compass bearings

Convert:

**a** 098°T to a compass bearing **b** N74°W to a true bearing.

| THINK | WRITE |
|---|---|
| Draw a diagram to show the bearings. |   |
| **a 1** 098°T is closer to south (180°) than it is to north (000°).Work out the angle between 098°T and south (180°). | **a** 180° − 98° = 82° |
| **2** Write the compass bearing. To get to 098°T, start at south and turn 82° towards east. | 098°T is the same as S82°E. |
| **b 1** Work out the angle turning clockwise from north to N74°W. | **b** 90° − 74° = 16°<br>270° + 16° = 286°<br>(or 360° − 74° = 286°) |
| **2** Write the true bearing. | N74°W is the same as 286°T. |

UNDERSTANDING AND FLUENCY

**9** Write each true bearing as a compass bearing.

**a** 010°T **b** 260°T **c** 110°T **d** 330°T
**e** 146°T **f** 301°T **g** 215°T **h** 084°T

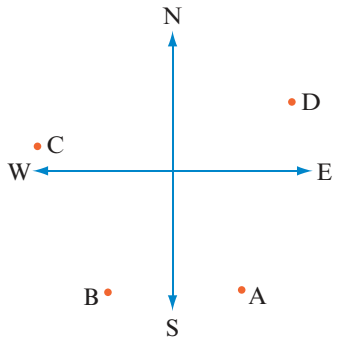
**10** Write each compass bearing as a true bearing.

**a** N50°E **b** S78°E **c** N18°W **d** S10°W
**e** S40°E **f** N89°E **g** S59°W **h** N27°W

**11** Cody attempted to find the bearings for the points A–D on the diagram at right.
For each point:

**i** explain what mistake he may have made
**ii** provide the correct answer.

**a** for A: E30°S
**b** for B: 155°T
**c** for C: N20°W
**d** for D: 60°






**12** Imagine you are a pilot flying between European capitals. You need to know what bearings to take. One flight is from London to Rome.

- **a** Copy the positions of the cities shown on the map into your workbook and draw a compass rose with its centre on London.
- **b** Draw a line from London to Rome.
- **c** Which two cardinal points is this line in between?
- **d** Estimate the bearing from London to Rome as:
  - **i** a compass bearing **ii** a true bearing.
- **e** Use your protractor to measure the bearing from London to Rome. Write your answer as both a true bearing and a compass bearing.
- **f** How does this compare to your original estimate?

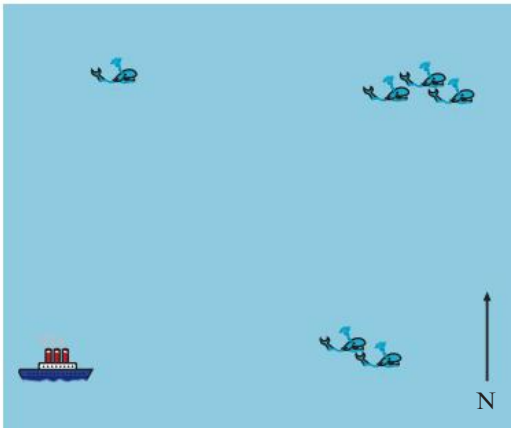
**13** Using question **12** as a guide, estimate then measure the bearings for the following trips. State your answer both as a true bearing and as a compass bearing.

- **a** London to Moscow **b** Moscow to Madrid **c** Madrid to Amsterdam
- **d** Amsterdam to Rome **e** Rome to Amsterdam

**14** Parts **d** and **e** of question **13** make up a return trip, flying in opposite directions.

- **a** What is the opposite direction to north?
- **b** If you are facing north and turn to face south, through what angle do you turn?
- **c** Find the difference between the true bearings in question **13** parts **d** and **e**.
- **d** How does your answer to part **c** compare to your answer to part **b**?
- **e** Can you write a rule to help you find the true bearing of any return trip?
- **f** Use this rule to quickly calculate the return trips for parts **a**–**c** in question **13**.
- **g** Write your answers to part **f** as compass bearings.
- **h** What rule can you see for the compass bearing of any return trip?

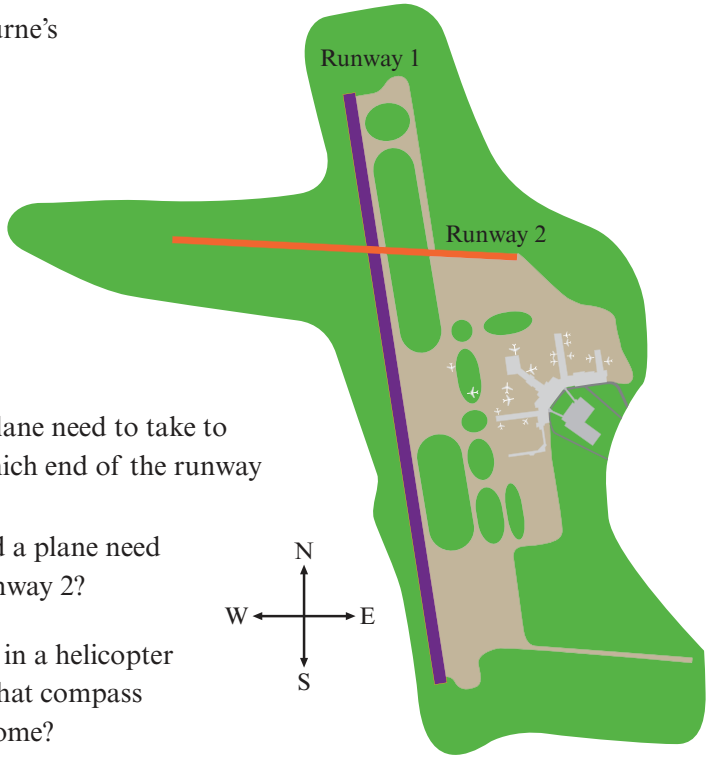
**15** You and your brother start a whale-watching business. You fly a hot air balloon over the harbour to spot the whales, and radio the bearings to your brother on the boat. Here is a map of the harbour with the location of the whales.




- **a** If your brother wants to visit the single whale first, what true bearing should he take?
- **b** What true bearing would he then need to take to reach the pair of whales?
- **c** From the pair of whales, what true bearing would it be to the group of three whales?

PROBLEM SOLVING AND REASONING

**16** On the right is a map of Melbourne's Tullamarine airport runways.




**a** What true bearing would a plane need to take to land on Runway 1 from the north?

**b** What compass bearing would a plane need to take to land on Runway 1 from the south?

**c** What true bearing would a plane need to take to land on Runway 2? (Hint: which end of the runway is closest to the terminal?)

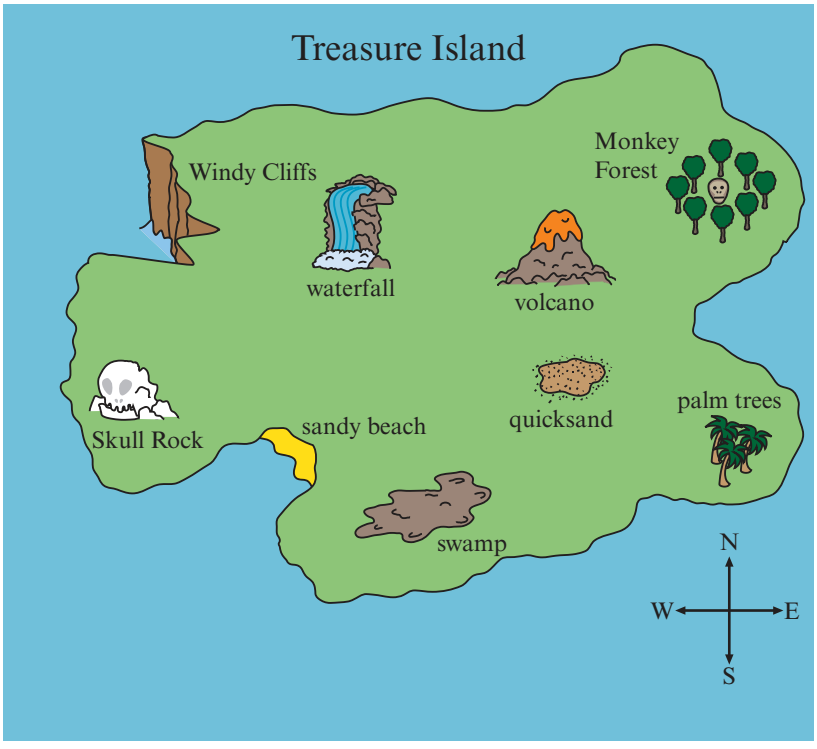
**d** What compass bearing would a plane need to be on to take off from Runway 2?

**17** If you got collected from school in a helicopter this afternoon, approximately what compass bearing would you take to get home?

CHALLENGE

**18** **a** A treasure map has the following directions. Can you decode them and work out which landmark the treasure is near? (One pace = 1 mm on the map.)

From the centre of sandy beach, walk 12 paces 310° then 32 paces 000° then 40 paces 090° then 16 paces 200° then 24 paces 180° then 28 paces 080° then 28 paces 355°.




**b** You can't carry all the treasure with you so you hide it somewhere else on the island. Write some directions to your own hiding spot.

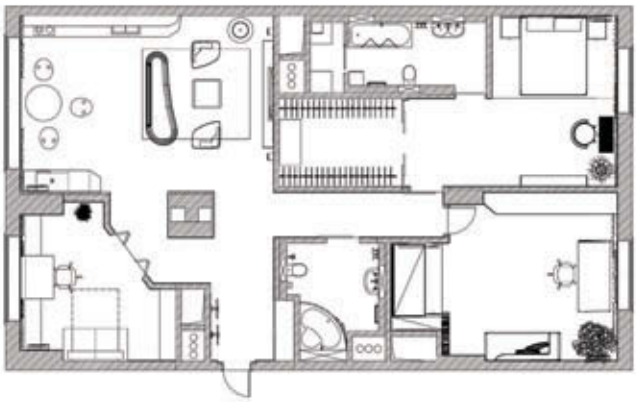
**Reflect**

What information would you need to make bearings useful in space?

# 6G Understanding scale

## Start thinking!

Ratios compare things in the same units. This map shows a **ratio scale** of 1:100. This means 1 cm on the plan represents 100 cm in the real world.



1 What distance would 2 cm on the map represent?

2 Write the distance found in question **1** in more meaningful units such as metres.

3 This map has a **scale factor** of $\frac{1}{100}$. How many times smaller is the distance on the plan compared to the real thing?

4 Sometimes a **map scale** is shown instead of a ratio scale. Which of these map scales matches the map?

**A** 100 m

**B** 5 m

**C** 50 m

**D** 10 m

## KEY IDEAS

- Maps or plans can be used to represent the real world.
- Map distances and real world distances can be calculated using a scale factor, which tells you how many times bigger or smaller the real world is than the map.

  For example, if 1 cm on a map represents 100 cm in the real world, the scale factor is $\frac{1}{100}$. This means the distance on the map is 100 times smaller than the real distance.
- Scale factors can be written as a ratio scale. The order in the ratio is written as distance on map: distance in real world.
- For the scale factor of $\frac{1}{100}$, the ratio scale is 1: 100.
- Scale factors can also be written as a map scale.
- For the scale factor of $\frac{1}{100}$, a map scale can be drawn as: 100 cm or 1 m

**NOTE** To work out the ratio scale, the distances on the map and in the real world need to be in the same unit.

# EXERCISE 6G Understanding scale

## EXAMPLE 6G-1 Converting between scale factors and ratio scales

Write: **a** $\frac{1}{1000}$ as a ratio scale **b** 1:200 as a scale factor.

### THINK

**a** $\frac{1}{1000}$ means the map distance is 1000 times smaller.

**b** 1:200 means the ratio of map distance to real world distance is 1 unit to 200 units.

### WRITE

**a** Ratio of map distance to real world distance is 1 unit to 1000 units.
Ratio scale is 1:1000.

**b** The map distance is 200 times smaller.
Scale factor is $\frac{1}{200}$.

UNDERSTANDING AND FLUENCY

**1** Write each of these scale factors as a ratio scale.

**a** $\frac{1}{300}$ **b** $\frac{1}{50}$ **c** $\frac{1}{20\,000}$ **d** $\frac{1}{4000}$

**2** Write each of these ratio scales as a scale factor.

**a** 1:800 **b** 1:70 **c** 1:200 000 **d** 1:5000

**3** Copy and complete this table. The first one is done for you.

| | Scale factor | Ratio scale | 1 cm on the map represents _____ in the real world | Measurement in meaningful units |
|---|---|---|---|---|
| | $\frac{1}{1000}$ | 1:1000 | 1000 cm | 10 m |
| **a** | $\frac{1}{500}$ | 1:500 | | |
| **b** | | 1:250 | 250 cm | |
| **c** | $\frac{1}{10}$ | | | 10 cm |
| **d** | | 1:100 000 | | 1 km |

**4** The map scale at right shows that 5 cm on a map represents 50 km in the real world.

50 km

**a** Copy and complete these sentences.

**i** This map scale shows that 1 cm on the map represents ___ km in the real world.

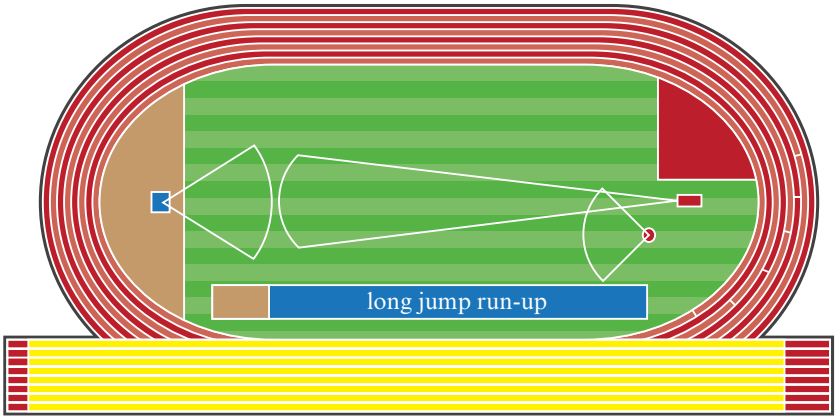
**ii** This map scale shows that 1 cm on the map represents ___ cm in the real world.

**b** Write the map scale as a ratio scale.

**c** What is the scale factor for this map?

UNDERSTANDING AND FLUENCY

**5** The map shows an athletics field. The 100 m running track is shown in yellow.



100 m track

- **a** Measure the length of the 100 m track.
- **b** Copy and complete: _____ cm on the map represents 100 m.
- **c** What does 1 cm on the map represent in metres?
- **d** What does 1 cm on the map represent in centimetres?
- **e** What is the ratio scale for this map?
- **f** What is the scale factor for this map?
- **g** Which of these map scales matches the map?

**A** 100 m

**B** 5 m

**C** 50 m

**D** 10 m

- **h** If the map scale is drawn 1 cm long instead, what will be the distance label?

?

**6** Draw a map scale for each of the scale factors in question **3**.

## EXAMPLE 6G-2 Finding distances from a map or plan

How wide is the yellow running track on the athletics field shown in question **5**?

| THINK | WRITE |
|---|---|
| **1** Use a ruler to measure the width of the track on the map. | On the map, the width of track is 1 cm. |
| **2** The ratio scale is 1:1000 which means 1 cm on the map represents 1000 cm. | The actual width of the track is 1000 cm. |
| **3** Divide by 100 to convert from centimetres to metres. | The track is 10 m wide. |

**7** Use the map in question **5** to answer these questions.

- **a** Use a ruler to measure the length of the long jump run-up on the map.
- **b** How many centimetres does this represent on the field?
- **c** How long is the long jump run-up in metres?
- **d** The coach of the athletics club makes his athletes sprint across the field. How many metres do the athletes sprint?

**8** This photo shows a scale model of a car. The ratio scale of model to car is 1: 80.

**a** How much bigger is the car compared to its model?

**b** Use a ruler to measure the length of the model.

**c** What is the length of the car in metres?

**d** How high is the car?



## EXAMPLE 6G-3 Using ratio scales

A map has a ratio scale of 1:200 000.

**a** What distance in real life does 1 cm on the map represent?

**b** What distance in real life does 7.5 cm on the map represent?

**c** What measurement on the map would represent a distance of 60 km?

### THINK

**a** 1:200 000 means that 1 cm represents 200 000 cm. Convert 200 000 cm to metres and then to kilometres (if necessary).

**b** For 7.5 cm, multiply 1 cm by 7.5, so multiply 2 km by 7.5.

**c** For 60 km, multiply 2 km by 30, so multiply 1 cm by 30.

### WRITE

**a** 1 cm represents 200 000 cm.
1 cm represents 2000 m.
1 cm represents 2 km.
1 cm on the map represents 2 km in real life.

**b** 1 cm × 7.5 = 7.5 cm
2 km × 7.5 = 15 km
7.5 cm on the map represents 15 km in real life.

**c** 2 km × 30 = 60 km
1 cm × 30 = 30 cm
30 cm on the map represents 60 km in real life.

UNDERSTANDING AND FLUENCY

**9** A map has a scale of 1:3 000 000.

**a** What meaningful distance does 1 cm on the map represent?

**b** What distance in real life does 4 cm on the map represent?

**c** What distance in real life does 9.5 cm on the map represent?

**d** What measurement on the map would represent 60 km?

**10** Repeat question **9** for ratio scales of:

**a** 1:200 000 **b** 1:100 000 **c** 1:500 000 **d** 1:60 000 000.

**11** Use the map and ratio scale from question **5** to answer these questions.

**a** The club record for the discus throw is 73.5 m. How many centimetres on the map is this?

**b** The club shot put record is 18.64 m. How many centimetres on the map is this?

**c** Find the length and width of the green section of the field.





**12** An athletics club wants to share the field with the local soccer club. Follow the instructions below to draw a soccer field using the same scale as the athletics field map in question **5**.

- **a** If the field is 100 m long, how many centimetres is this on the map?
- **b** If the field is 60 m wide, how many centimetres is this on the map?
- **c** Draw the soccer oval in your workbook, adding a centre line across its width.
- **d** In soccer there is a centre circle on the centre line. On the field the centre circle has a radius of 9.1 m. How many centimetres is this on the map?
- **e** Add the centre circle to complete your scale drawing of the soccer field.

Questions **13**–**16** refer to the scale map of Australia.




**13** Look at the scale for the map. Measure 1 cm on the map scale.

- **a** How many km does 1 cm represent on this map?
- **b** Multiply this number by the number of centimetres in 1 km. Copy and complete this sentence: 1 cm on the map represents ____ cm.
- **c** Write the scale factor for this map as a ratio.

**14** Use the map scale and a ruler to calculate the distance between the following cities.

- **a** Adelaide and Melbourne
- **b** Sydney and Hobart
- **c** Perth and Darwin
- **d** Brisbane and Canberra

**15** **a** **i** Which two Australian capital cities are furthest apart?

**ii** How far apart are they?

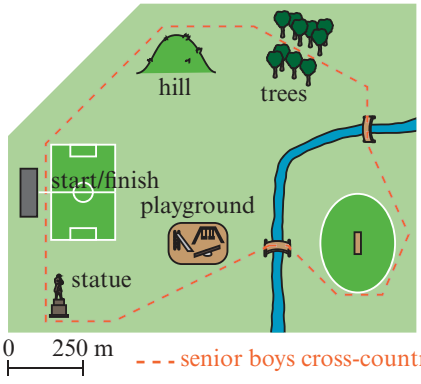
**b** **i** Which two Australian capital cities are closest together?

**ii** How far apart are they?

**16** In 1802 and 1803, Matthew Flinders was the first person to sail around Australia. Use a ruler to estimate how far he must have sailed.

**17** This map shows a park where your school plans to have its cross-country running race. The senior boys course is marked with a dotted line. Some boys are complaining that it is too long and should be less than 4 km.

- **a** Use a ruler and the scale to calculate how far the boys must run. Is the course fair?
- **b** The junior boys need a course between 2 and 3 km. Describe a possible course for them to take.
- **c** The school's Mountain Bike Club wants to hold a 10 km race in the park. Describe a course for them, starting at the top of the hill.



PROBLEM SOLVING AND REASONING

**18** Not all maps and plans are scaled-down versions of real life. Some are scaled up.

- **a** **i** If a scale factor of $\frac{1}{1000}$ means that a map is 1000 times smaller than real life, what would a scale factor of 1000 mean?
  - **ii** Which of these scale factors is a reduction and which is an enlargement?
- **b** **i** If a ratio scale of 1:100 means that a map is 100 times smaller than real life, what would a ratio scale of 100:1 mean?
  - **ii** Which of these ratio scales is a reduction and which is an enlargement?
- **c** Think of two examples where a map or plan might represent an enlargement of real life.
- **d** Identify the following scale factors as either reductions or enlargements.
  **i** 2 **ii** $\frac{1}{10}$ **iii** 100 **iv** $\frac{1}{5\,000\,000}$ **v** 5
- **e** For each of the scale factors in part **d**, state what distance in real life 1 cm on the map would represent.

**19** A drawing has a ratio scale of 10:1.

- **a** What is its scale factor?
- **b** What distance in real life does 1 cm on the drawing represent?
- **c** What distance in real life does 4.8 cm on the drawing represent?
- **d** What measurement on the drawing would represent 25 mm?
- **e** Challenge! Repeat parts **a–d** for a ratio scale of:
  **i** 1000:1 **ii** 100 000:1.
- **f** Do you think that the answers to part **e** are realistic? Explain.

**20** Logan has a scale model of a Mercedes SLK. The model has a scale factor of $\frac{1}{10}$.

- **a** What is this as a ratio scale?
- **b** If the width of the scale model is 17.9 cm, how wide is an actual Mercedes SLK?
- **c** If the height of an actual Mercedes SLK is about 1.3 m, how high is the scale model?

**21** Teagan has a toy car which is modelled on a Mercedes SLK.

- **a** If the toy car has a width of 3.6 cm, find the scale factor (round the denominator to the nearest whole number).
- **b** Express the scale factor as a ratio scale.
- **c** Use this scale factor to find how high the toy car is.
- **d** If the toy car has a length of 8.2 cm, how long is a Mercedes SLK in real life?

CHALLENGE

**22** Draw a map of your classroom to scale. Take some approximate measurements and decide what scale to use. Include both a map scale and a ratio scale.

**23** Repeat question **22** for a place (for example, netball court, town) or an object (for example, building, butterfly).

**Reflect**

How is a scale on a map useful?

# CHAPTER REVIEW

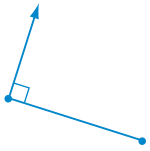
## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

line
point
ray
segment
parallel
perpendicular
right angle
acute angle
obtuse angle
straight angle
reflex angle
revolution
measuring an angle
complementary angles
supplementary angles
vertically opposite angles
transversal
alternate angles
corresponding angles
co-interior angles
bearings
compass rose
cardinal points
true bearings
compass bearings
scale factor
ratio scale
map scale

## MULTIPLE-CHOICE

6A **1** What does this diagram show?

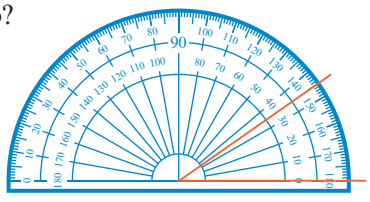


- **A** a ray parallel to a segment
- **B** a line parallel to a ray
- **C** a segment perpendicular to a ray
- **D** two parallel lines

6B **2** What are angles that add to 90° called?

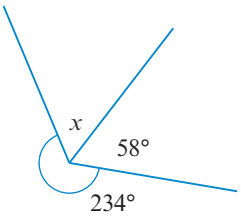
- **A** acute angles **B** right angles
- **C** complementary angles
- **D** supplementary angles

6C **3** What is this angle closest to?



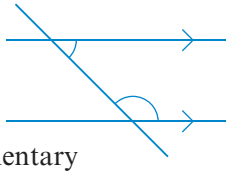
- **A** 30°
- **B** 35°
- **C** 45°
- **D** 145°

6D **4** What is the size of angle $x$ on the diagram?



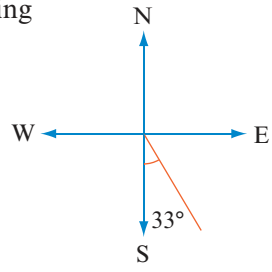

- **A** 42° **B** 360°
- **C** 122° **D** 68°

6E **5** What are the angles marked in this diagram?



- **A** alternate and supplementary
- **B** corresponding and complementary
- **C** co-interior and supplementary
- **D** co-interior and complementary

6F **6** What is the bearing shown on this compass rose?




- **A** 033°T
- **B** E33°S
- **C** 213°T
- **D** S33°E

Questions **7** and **8** refer to this map scale.




6G **7** What does 1 cm represent?

- **A** 2 km **B** 10 km
- **C** 5 km **D** 10 000

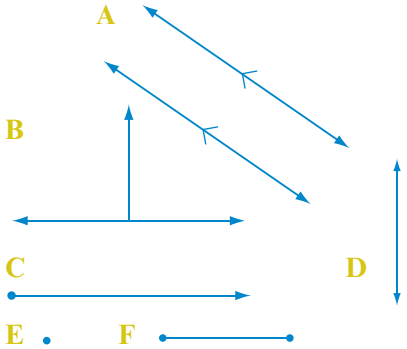
6G **8** Written as a ratio, what is this scale?

- **A** 1:10 000 **B** 1:200 000
- **C** 1:200 **D** 1:1 000 000

## SHORT ANSWER

6A **1** Match these definitions to Figures A–F.

**a** line **b** ray **c** segment
**d** point **e** parallel lines
**f** perpendicular lines




6B **2** Classify the angles below as acute, right, obtuse, straight, reflex or a revolution.

**a**



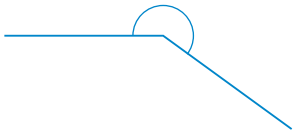
**b**



**c**



**d**



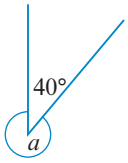
6B **3** **a** Find the complementary angle to:
**i** 23° **ii** 79° **iii** 55°
**b** Find the supplementary angle to:
**i** 127° **ii** 42° **iii** 9°

6C **4** Using a protractor, measure and record each angle in question **2**.

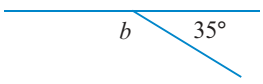
6C **5** Draw each angle.
**a** 40° **b** 115° **c** 200°

6D **6** Find the size of the unknown angles.

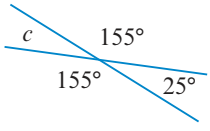
**a**



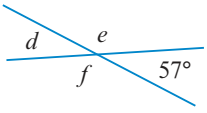

**b**



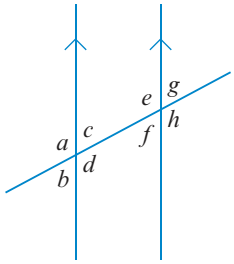

**c**




**d**




6E **7** Use this diagram to find:




**a** the angle corresponding to $c$
**b** the alternate angle to $e$
**c** the co-interior angle to $f$
**d** $b$, if $f$ is 77°
**e** $d$
**f** $a$.

6F **8** **a** Convert each bearing into the form shown in brackets.
**i** 055°T (compass bearing)
**ii** 200°T (compass bearing)
**iii** N60°W (true bearing)
**iv** S10°E (true bearing)
**b** Draw each bearing on a compass rose.

6G **9** A map has a ratio scale of 1:200 000.
**a** What is the scale factor of the map?
**b** What meaningful distance in real life is represented by 1 cm on the map?
**c** What distance in real life does 13.1 cm on the map represent?
**d** What measurement on the map would represent 4.4 km?

6G **10** A map of a town is missing its scale. The map shows the main street covering a distance of 11.2 cm. You know that the main street is actually 1.68 km long.
**a** Write 1.68 km in centimetres.
**b** Find the scale factor of the map.
**c** Write this as a ratio scale.
**d** Find the distance between two points in town if on the map the distance is 29.4 cm.

## NAPLAN-STYLE PRACTICE

**1** Which of the following is a ray?

- ☐ 
- ☐ 
- ☐ 
- ☐ 

**2** Which of these statements is true?

- ☐ Parallel lines meet at a 90° angle.
- ☐ A ray has a definite start and finish.
- ☐ A segment starts from a point and continues infinitely.
- ☐ A pair of lines that meet at right angles are called perpendicular.

**3** Line A is perpendicular to line B. Ray C is perpendicular to ray D. If line B is parallel to ray D, which of these statements is false?

- ☐ Line B extends in both directions forever.
- ☐ Ray C is parallel to line A.
- ☐ Ray D starts from a point.
- ☐ Line A is parallel to ray D.

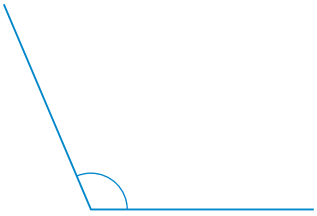
**4** Supplementary angles add to:

☐

**5** Complementary angles add to:

☐

Questions **6** and **7** refer to this angle.



**6** What is the best description of the angle above?

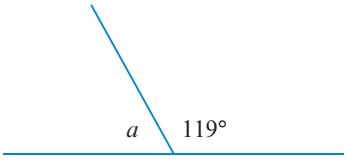
- ☐ obtuse
- ☐ acute
- ☐ reflex
- ☐ revolution

**7** What is the size of the angle?

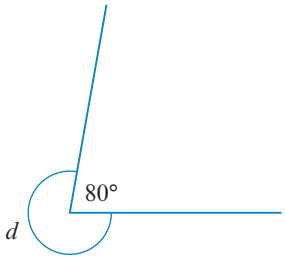
| 113° | 67° | 247° | 90° |
|---|---|---|---|
| ☐ | ☐ | ☐ | ☐ |

**8** What is the size of angle $a$ in this diagram?

☐

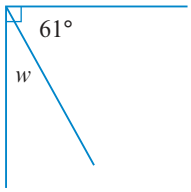



**9** What is the size of angle $d$ in this diagram?

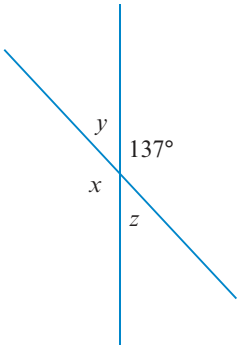



**10** What is the size of angle $w$ in this diagram?

☐




Questions **11**–**13** refer to this diagram.




**11** What is the size of angle $x$?

- ☐ 137°
- ☐ 360°
- ☐ 43°
- ☐ 223°

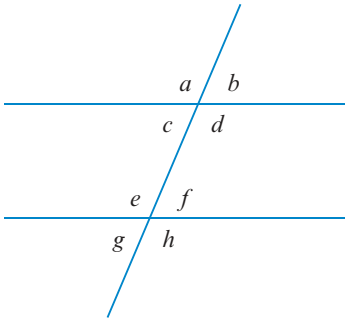
**12** What is the size of angle $y$?

- ☐ 137°
- ☐ 360°
- ☐ 43°
- ☐ 223°

**13** What is the size of angle $z$?

| 137° | 360° | 43° | 223° |
|---|---|---|---|
| ☐ | ☐ | ☐ | ☐ |

Questions **14**–**16** refer to this diagram.




**14** Which pair of angles is corresponding?

- ☐ $e$ and $h$
- ☐ $c$ and $f$
- ☐ $c$ and $e$
- ☐ $a$ and $e$

**15** Which pair of angles is alternate?

- ☐ $e$ and $h$
- ☐ $c$ and $f$
- ☐ $c$ and $e$
- ☐ $a$ and $e$

**16** Which pair of angles is co-interior?

- ☐ $e$ and $h$
- ☐ $c$ and $f$
- ☐ $c$ and $e$
- ☐ $a$ and $e$

Questions **17**–**20** refer to the diagram opposite that shows angles $a$ to $h$. Angle $a$ is 125°.

**17** What is the size of angle $d$?

**18** What is the size of angle $f$?

**19** What is the size of angle $c$?

**20** What is the size of angle $g$?

**21** What is N75°E written as a true bearing?

**22** What is S18°W written as a true bearing?

**23** What is 295°T written as a compass bearing?

- ☐ N295°E
- ☐ N25°W
- ☐ W25°N
- ☐ N65°W

**24** What is 100°T written as a compass bearing?

Questions **25** and **26** refer to this map scale.




**25** How many kilometres does 1 cm represent?

**26** How many centimetres would represent 10 km?

- ☐ 2 cm
- ☐ 2.5 cm
- ☐ 5 cm
- ☐ 10 cm

Questions **27** and **28** refer to this information.

Sally goes on a hike. She takes a large walking map that shows a ratio scale of 1:200 000.

**27** If she is going for an 8 km hike north, how far is this on her map?

- ☐ 8 cm
- ☐ 4 cm
- ☐ 12 cm
- ☐ 16 cm

**28** Sally misreads the scale as 1:100 000. Which of these statements is true?

- ☐ Sally thinks she is 4 cm further north on the map than she actually is.
- ☐ Sally thinks she is 4 cm further south on the map than she actually is.
- ☐ Sally has actually walked 16 km.
- ☐ Sally has actually walked 4 km.

## ANALYSIS

Use the treasure map on page 331 to answer the following questions.

Imagine that you are in the centre of the island. You measure that the volcano is on a bearing of 064°T.

**a** What is this as a compass bearing?

**b** Copy the treasure map and use your protractor to mark where you would be standing.

**c** Use your answer to part **b** to find the bearing of Skull Rock from where you are standing. Write this as both a true bearing and a compass bearing.

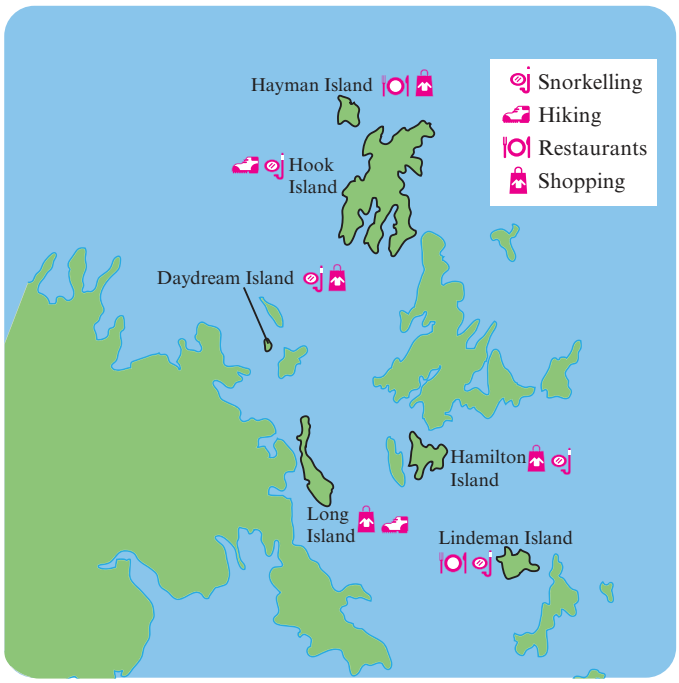
**d** What type of angle do you turn through when moving from the volcano to Skull Rock if you turn clockwise?

**e** The scale on the map gives 1 mm : 1 pace. If 1 pace = 1 m, write this map scale as a ratio scale.

# CONNECT

## An island cruise

For your birthday your parents are taking you on a tropical holiday to Queensland. You are allowed to take four friends and you've decided to hire a private yacht and cruise through the Whitsunday Islands. The boat is yours for five nights and comes with a captain to sail it for you. You just need to decide where to go!




Your friends all have different ideas about what to do and where to go. As group leader your job is to make them all happy by planning a route that includes everything that they are looking forward to.

Nam loves to eat. He is looking forward to eating at a special restaurant.

Sally loves shopping. She wants to spend at least one day browsing markets or shopping centres.

Yusuf is active and loves nature. He is keen to go hiking.

Sue has never been to Queensland before and really wants to do some snorkelling.

You are looking forward to all of these things but you've always dreamed of watching the sun set on a deserted island.



### Your task

Your task is first to plan a course that will include all of these activities.

Also, for reasons of safety, you need to leave a trip plan with the local police before you leave. They need to know exactly where you will anchor each night.

The boat's captain can take you anywhere but because he's a crusty old seadog he insists you use proper navigational language. He wants the directions in writing, too.

- Plan a course that allows you and your guests to do your chosen activities.
- Show the planned course on a map.
- Write the directions for your trip plan using distances and bearings.

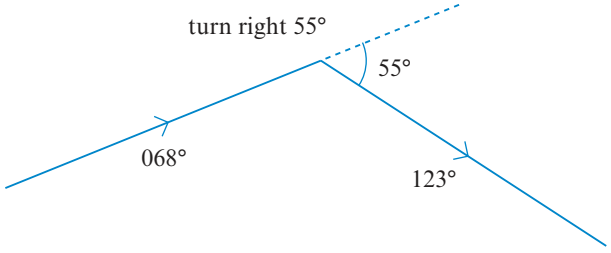








In case something happens to the captain, each person on the boat needs to be able to steer and follow the course. Your friends know all about angles but don't know much about bearings.




Complete the 6 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report.
Your report could be in the form of:

- a poster-sized annotated map of the trip
- a 'captain's log'
- a story told by one (or all) of the crew
- other (check with your teacher).

# 7 SHAPES AND OBJECTS


7A Classifying triangles

7B Classifying quadrilaterals

7C Identifying 2D shapes

7D Identifying 3D objects

7E Drawing 2D shapes and 3D objects

7F Planning and constructing 3D objects

7G Symmetry of 2D shapes and 3D objects

7H Describing transformations

7I Performing transformations




**ESSENTIAL QUESTION**

*How can you classify shapes and objects?*

# Are you ready?

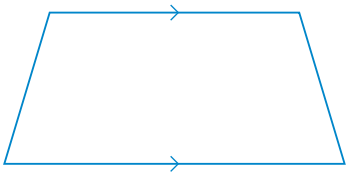
7A **1** How many degrees in a right angle?

7A **2** Which type of angle is the largest?

**A** reflex **B** acute
**C** obtuse **D** right

7B **3** How many sides does a quadrilateral have?

Questions **4** and **5** refer to this figure.



7C **4** What is this shape?

**A** triangle **B** trapezium
**C** rectangle **D** kite

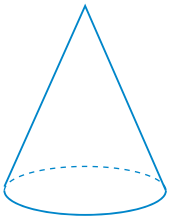
7C **5** What do the arrows on the shape mean?

**A** the lines are equal in length
**B** the lines are perpendicular
**C** the lines are straight
**D** the lines are parallel

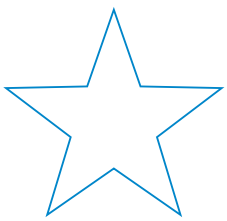
7D **6** How many faces does a cube have?

7D **7** Which of the following is not a drawing of a 3D object?

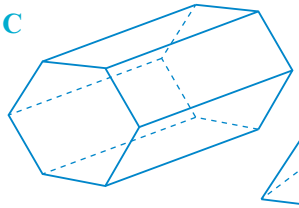
**A**



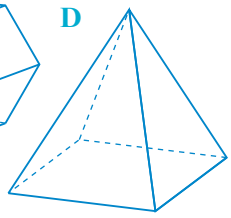
**B**



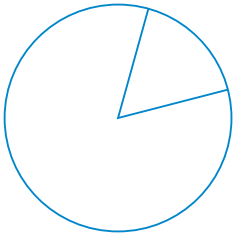
**C**



**D**

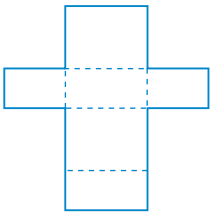


7E **8** What is the size of the angle in the small sector at the centre of the circle?



**A** 60° **B** 65°
**C** 70° **D** 120°

7F **9** This net folds to make which object?



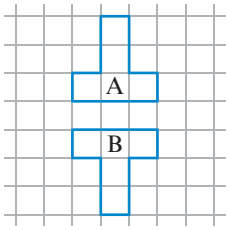
**A** rectangular prism
**B** cube
**C** pyramid
**D** triangular prism

7G **10** What type of symmetry does this shape have?



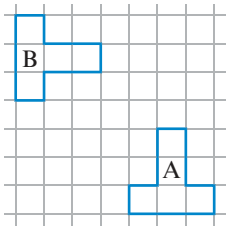
**A** no symmetry
**B** horizontal symmetry
**C** vertical symmetry
**D** both horizontal symmetry and vertical symmetry

7H **11** What has been done to shape A to produce shape B? It has been:




**A** rotated
**B** translated
**C** reflected in a horizontal line
**D** stretched

7H **12** What has happened to shape A to produce shape B?

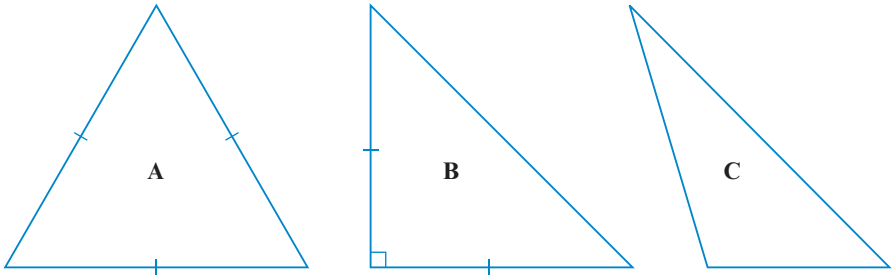



**A** rotated
**B** translated
**C** reflected in a horizontal line
**D** stretched

# 7A Classifying triangles

## Start thinking!

A **triangle** is a two-dimensional (2D) shape with three sides. Triangles can be described in different ways. One way to classify triangles is by looking at the length of their sides.




1 Look at the sides of triangle A. This type of triangle is called an **equilateral triangle**. Why do you think this is?

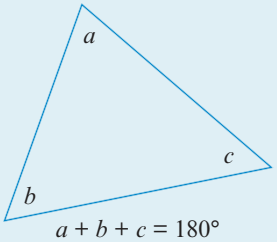
2 Write a sentence about the sides of an equilateral triangle. Can you write a similar sentence describing the angles in an equilateral triangle?

3 An **isosceles triangle** has two sides that are equal. Which of the triangles is an isosceles triangle? How do you know?

4 A third type of triangle has sides of all different lengths. This is called a **scalene triangle**. Which triangle is it?

## KEY IDEAS

- A triangle is a 2D shape with three sides.
- Triangles can be classified by their side lengths as either equilateral, isosceles or scalene.
- Triangles can be classified by their **internal angles (interior angles)** as either acute-angled, right-angled or obtuse-angled.
- The angles in a triangle add to 180°.
- The size of an angle in a triangle can be calculated if you are given the other two angles.
- The properties of a triangle can also be used to help you calculate missing angles.

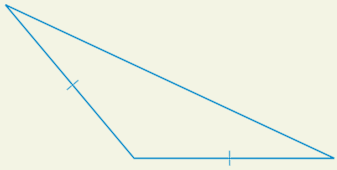



$a + b + c = 180°$

# EXERCISE 7A Classifying triangles

## EXAMPLE 7A-1 Classifying triangles

Classify this triangle according to its sides and angles.



**THINK**

1 Look at the sides of the triangle. Two of them are equal in length, so it must be an isosceles triangle.

2 Look at the angles of the triangle. One of them is obtuse, so it is an obtuse-angled triangle.

3 Write your answer.

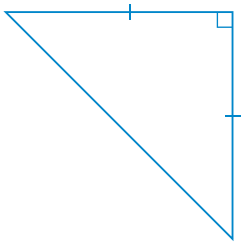
**WRITE**

This is an obtuse-angled isosceles triangle.

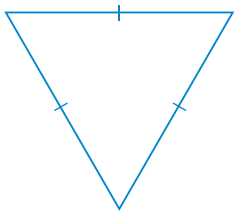
UNDERSTANDING AND FLUENCY

**1** Classify each triangle according to its sides (equilateral, isosceles or scalene).

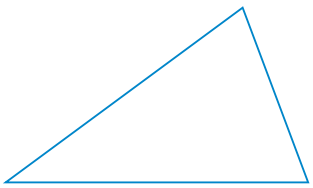
**a**



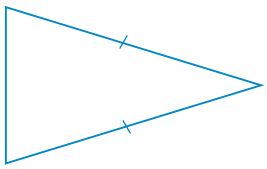
**b**



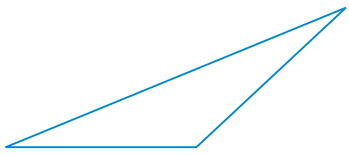
**c**



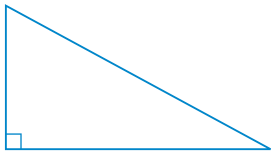
**d**



**e**



**f**

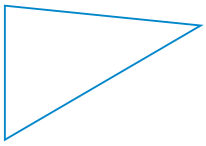


**2** Classify each triangle in question **1** according to its angles (acute, obtuse or right-angled).

UNDERSTANDING AND FLUENCY

**3** Classify each triangle according to its sides and angles.

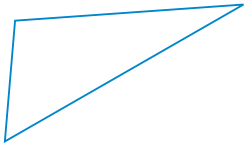
**a**



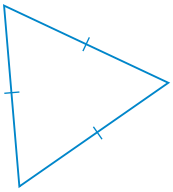
**b**



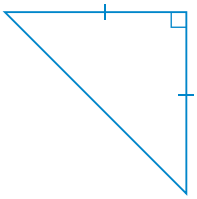
**c**



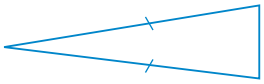
**d**



**e**

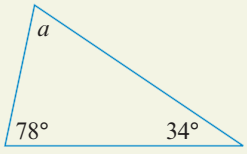


**f**



## EXAMPLE 7A-2 Finding the size of an angle in a triangle

Find the size of angle $a$.




**THINK**

1 Find the sum of the two known angles.

2 Angles in a triangle add to 180°. Subtract 112° from 180° to calculate $a$.

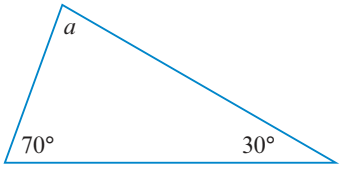
**WRITE**

$78° + 34° = 112°$

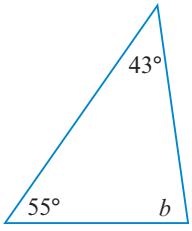
$a = 180° - 112°$
$= 68°$

**4** Find the size of each labelled unknown angle.

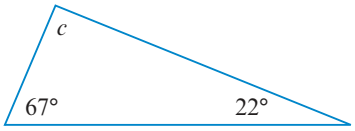
**a**



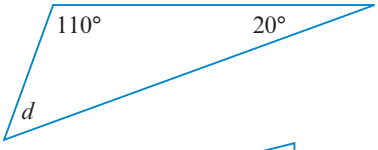

**b**



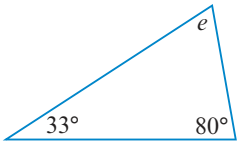

**c**



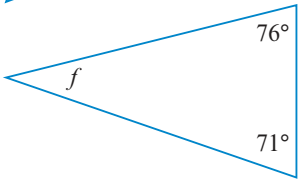

**d**




**e**

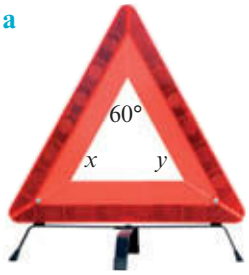



**f**

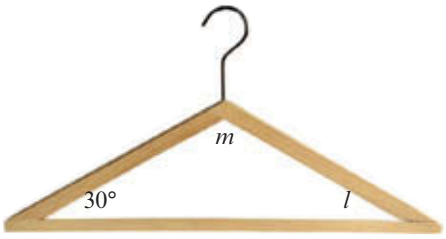


**5** Find the size of each labelled unknown angle.

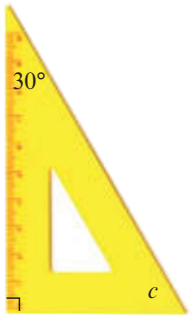
**a**




**b**

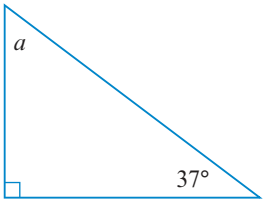



**c**

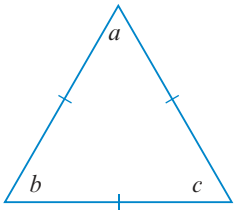



**6** Use your knowledge of the properties of a triangle to find the size of each labelled unknown angle.

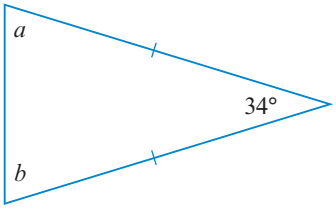
**a**



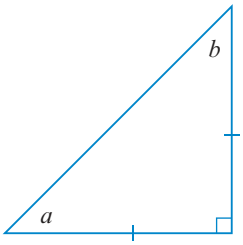

**b**



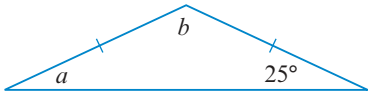

**c**



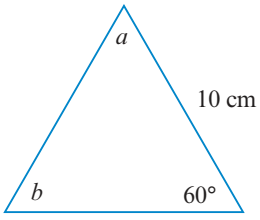

**d**




**e**




**f**

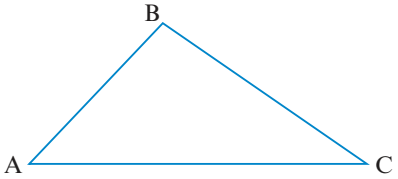



**7** Draw at least three different triangles containing an angle of 50°.

**8** Consider each of the six different types of triangle covered in this section.

- **a** Look around the classroom and find an example of each type. Write down what the object is.
- **b** Draw a diagram.
- **c** Label the triangle according to its sides and angles.

**9** Triangles are often named according to letters on their corners. The triangle shown can be named ΔABC. What other names could it be given with these letters?



**10** This photo was taken at Federation Square in Melbourne. How many triangles can you find?



**11** What angle does the roof on this house make with the horizontal?




PROBLEM SOLVING AND REASONING

**12** **a** Use a ruler to draw a triangle and cut it out.

**b** Show that the sum of the internal angles is equal to 180° by tearing off the corners of the triangle and placing them together.

**c** Repeat for another two different triangles. Do you think this proof will work for any triangle?

**13** The triangle below right has had one of its sides extended and an angle marked in. This is called an **exterior angle**.

**a** Why do you think it is called this?

**b** Use your knowledge of the angle sum of a straight line to find the size of the exterior angle.

**c** Add the two opposite interior angles together (shown in orange). What do you find?

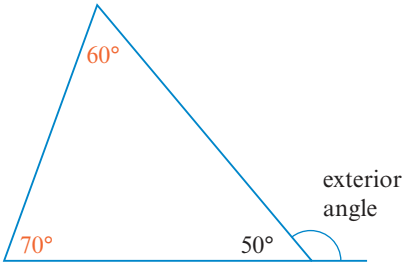
**d** Copy the triangle and extend the sides to mark in the other two exterior angles.

**e** Find these other two exterior angles.

**f** For each of these exterior angles, find the sum of the 'opposite' two interior angles (shown in orange). How does this relate to the size of the exterior angle?

**g** Copy and complete: The size of the _____ angle is equal to the sum of the two _____ interior angles.

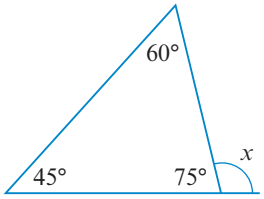
**h** Explain why this works, using your knowledge of supplementary angles.



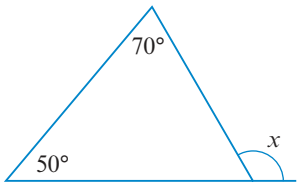
PROBLEM SOLVING AND REASONING

**14** Use your findings from question **13** to calculate the size of angle $x$ for these triangles.

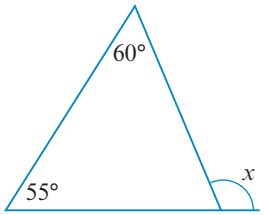
**a**



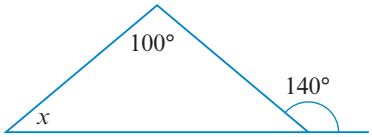

**b**



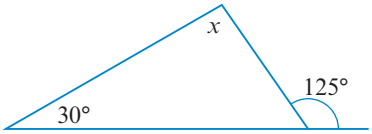

**c**



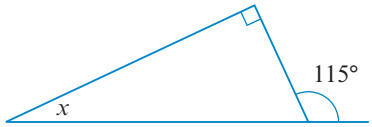

**d**




**e**




**f**




**15** Write a step-by-step tutorial for a friend explaining how to calculate angles inside a triangle. You may also like to include the calculation of exterior angles.

CHALLENGE

**16** Look at the triangle in question **13**.

**a** What is the sum of the exterior angles?

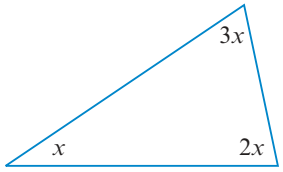
**b** Draw another two triangles and find:

**i** the exterior angles **ii** the sum of the exterior angles.

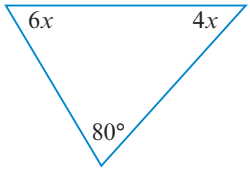
**c** Write a sentence describing what you find about the sum of the exterior angles of a triangle.

**17** Use your knowledge of triangle properties and basic algebra to find the value of the pronumerals in these triangles.

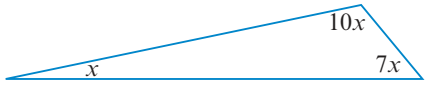
**a**



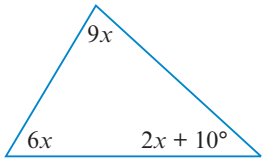

**b**



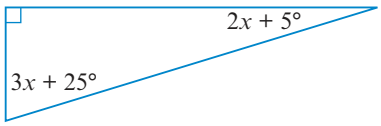

**c**



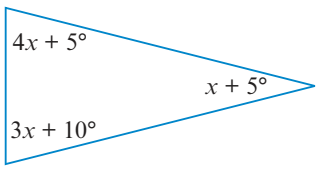

**d**




**e**




**f**

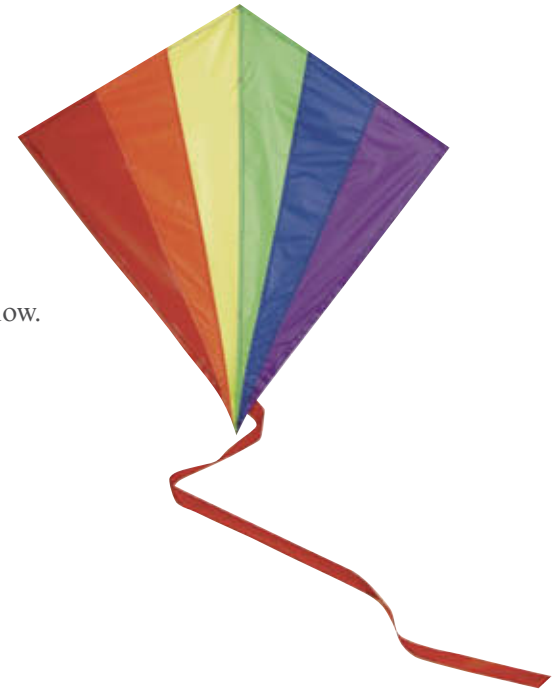



**Reflect**

How does the angle sum of a triangle relate to supplementary angles?

# 7B Classifying quadrilaterals

## Start thinking!



A **quadrilateral** is a 2D shape with four sides. This kite is an example of a quadrilateral.

1 What part of its name tells you that a quadrilateral has four sides?

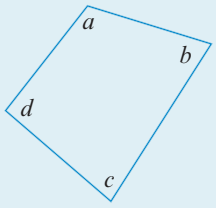
2 Draw and name each of the six special quadrilaterals.

3 Investigate these quadrilaterals according to the properties listed below. You may like to create a table to help you.

- **a** Are opposite sides parallel?
- **b** Are opposite sides equal?
- **c** Are all sides equal?
- **d** Are opposite angles equal?
- **e** Are angles fixed in size?
- **f** Is there anything else special about this quadrilateral?

## KEY IDEAS

- A quadrilateral is a 2D shape with four sides.
- Some quadrilaterals have special properties involving sides and angles. These are the **parallelogram**, **rhombus** (also known as a diamond), **rectangle**, **square**, **kite** and **trapezium**.
- Quadrilaterals without special properties are known as **irregular quadrilaterals**.
- The angles in a quadrilateral add to 360°.
- The size of an angle in a quadrilateral can be calculated if you are given the other three angles.
- The properties of special quadrilaterals can also be used to help you calculate missing angles.

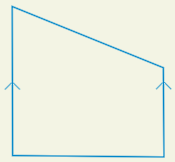



$a + b + c + d = 360°$

# EXERCISE 7B Classifying quadrilaterals

## EXAMPLE 7B-1 Classifying quadrilaterals

Classify this quadrilateral.



**THINK**

Identify the quadrilateral by considering its properties.
The quadrilateral has a pair of opposite sides that are parallel.

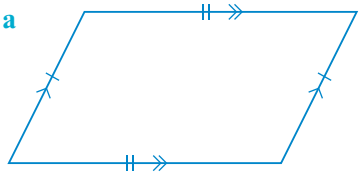
**WRITE**

The quadrilateral is a trapezium.

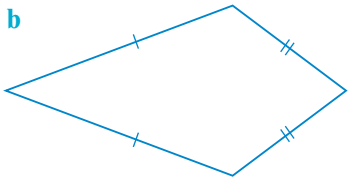
UNDERSTANDING AND FLUENCY

**1** Classify each quadrilateral. Give a reason for your answer.

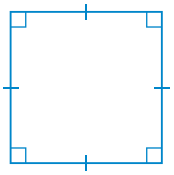
**a**



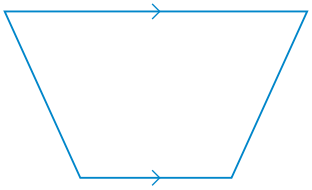
**b**



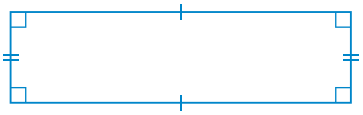
**c**



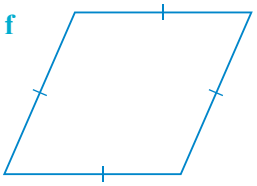
**d**



**e**

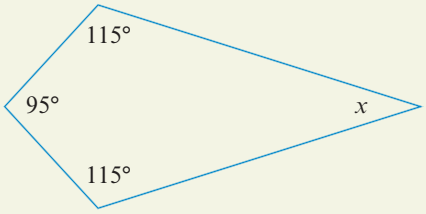


**f**



## EXAMPLE 7B-2 Finding the size of an angle in a quadrilateral

Find the size of angle $x$.




**THINK**

**1** Find the sum of the three known angles.

**2** Angles in a quadrilateral add to 360°.
Subtract 325° from 360°.

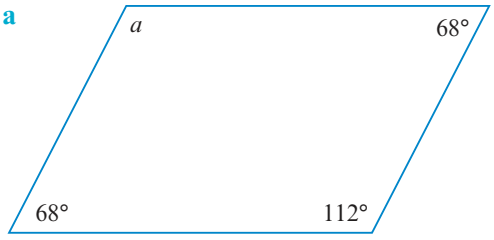
**WRITE**

$115° + 115° + 95° = 325°$

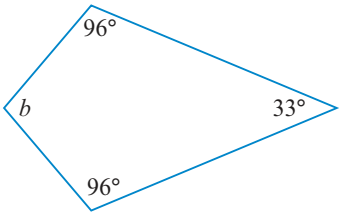
$x = 360° - 325°$
$= 35°$

**2** Find the size of each labelled unknown angle.

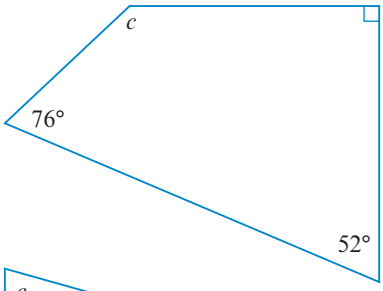
**a**



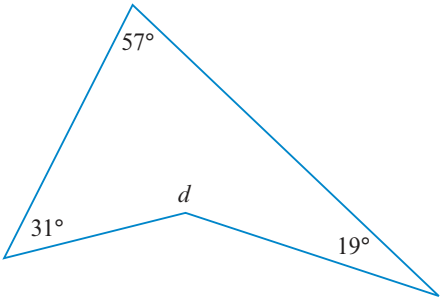

**b**



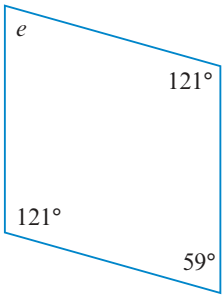

**c**



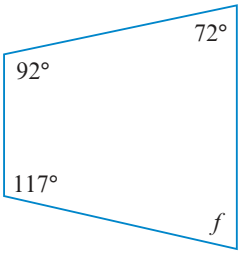

**d**




**e**

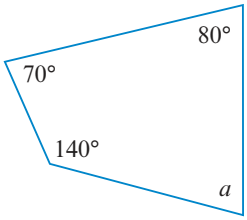



**f**

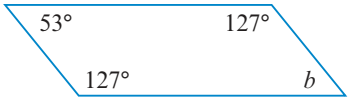



**3** Classify these quadrilaterals and find the size of each labelled unknown angle.

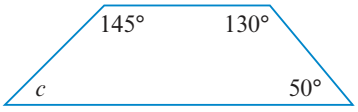
**a**



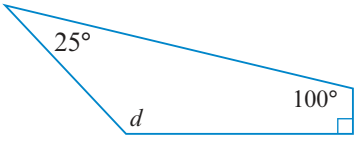

**b**



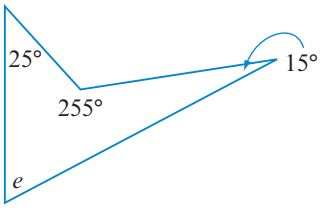

**c**



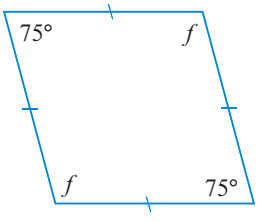

**d**




**e**

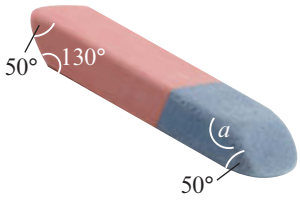



**f**



**4** Find the size of each labelled unknown angle.

**a**




**b**

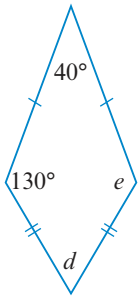



**c**

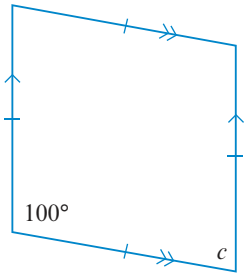



**5** Use your knowledge of quadrilateral properties to find the size of each labelled unknown angle.

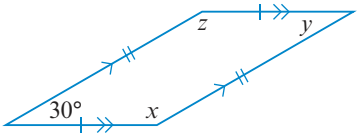
**a**



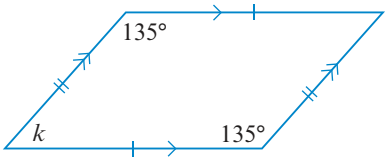

**b**




**c**



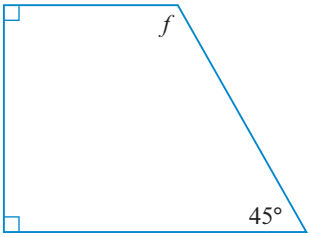

**d**



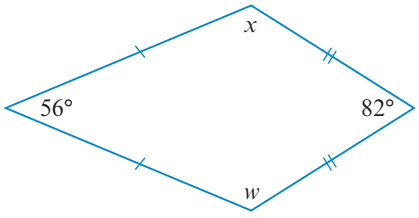

**e**



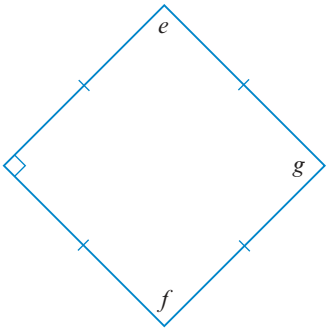

**f**



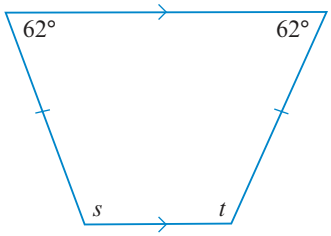

**g**




**h**




**i**




**6** Find an example of each of the six special quadrilaterals in the classroom or at home. Write down what object you found them in and draw a diagram.

**7** Match the following descriptions to one of the six special quadrilaterals.

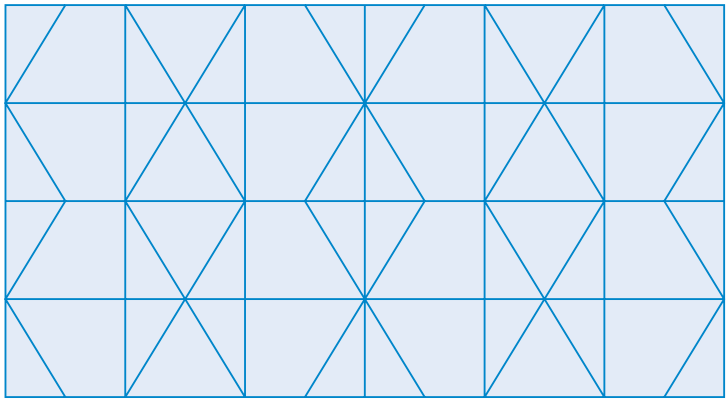
- **a** all sides are equal in length; opposite angles are equal
- **b** one pair of opposite sides is parallel
- **c** opposite sides are parallel and equal in length; all angles are 90°
- **d** opposite sides parallel and equal in length; opposite angles are equal
- **e** all sides are equal in length; all angles are 90°
- **f** two pairs of adjacent sides equal in length; one pair of opposite angles is equal

**8** Decide whether each statement is true or false.

- **a** A kite has two pairs of opposite sides equal in length.
- **b** A rhombus has four equal angles.
- **c** A trapezium has one pair of opposite sides equal in length.
- **d** The necklace at right is made up of parallelograms.



**9** How many quadrilaterals can you find in this grid? List the different quadrilaterals you can find.

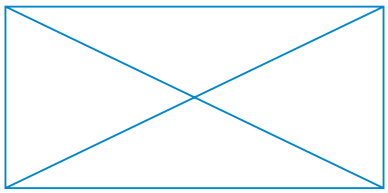


**10** Some of the special quadrilaterals are related.

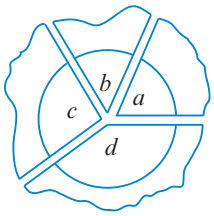
- **a** Is a square a rectangle?
- **b** Is a rectangle a square?
- **c** Is a rectangle a parallelogram?
- **d** Create a concept map linking squares, rectangles, parallelograms and rhombuses.
- **e** Explain why a square is a special type of rhombus.

**11** **a** Draw a rectangle into your workbook and draw a diagonal line. How many triangles have you formed?

- **b** What is the sum of the angles in a triangle?
- **c** Copy and complete: Sum of angles in a rectangle = 2 × _____° = _____°
- **d** Repeat for the other quadrilaterals. What do you find?
- **e** Write a sentence about the sum of the angles in a quadrilateral.
- **f** A friend added another diagonal line to the rectangle as shown on the right. This forms four triangles. Explain why it is not correct to say that the angle sum of a rectangle is $4 \times 180° = 720°$.



**12** Another way to demonstrate the angle sum of a quadrilateral is to follow a method used previously.



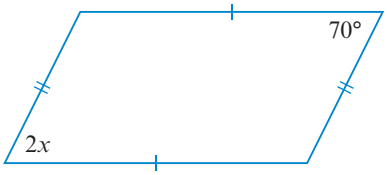

- **a** Draw a parallelogram and cut it out.
- **b** Tear off each of the angles and place them together as shown. What type of angle does this form?
- **c** How many degrees are in this type of angle? Does this match the angle sum you found in question **11**?
- **d** Repeat the activity with the five other special quadrilaterals. What do you find?

**13** Create a visual chart that allows you to classify quadrilaterals. Examples include a flowchart or a concept map.

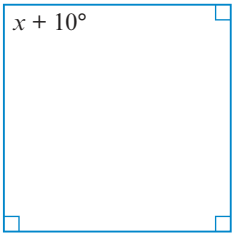
CHALLENGE

**14** Use your knowledge of quadrilateral properties and basic algebra to find the value of the pronumerals in these quadrilaterals.

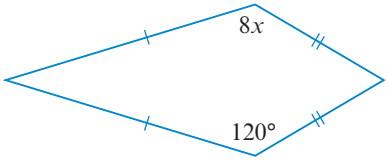
**a**



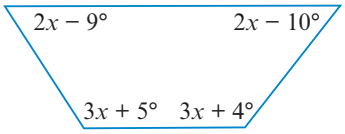

**b**



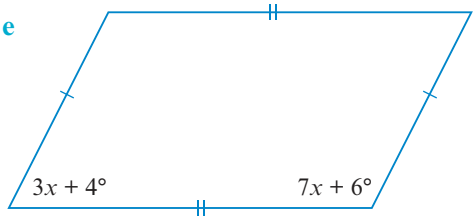

**c**



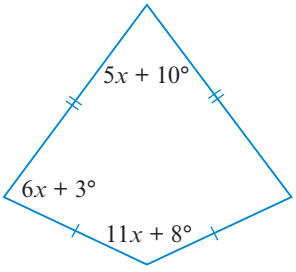

**d**




**e**




**f**




**15** Exterior angles can be found for quadrilaterals as well as triangles.

- **a** Choose one of the shapes on this page and copy it into your book, extending the sides to mark in the exterior angles.
- **b** Calculate the four exterior angles for the shape.
- **c** Add these angles together. What do you find? How does this compare to the sum of exterior angles for a triangle?
- **d** Repeat the activity for at least two different quadrilaterals. Write a sentence about what you find.

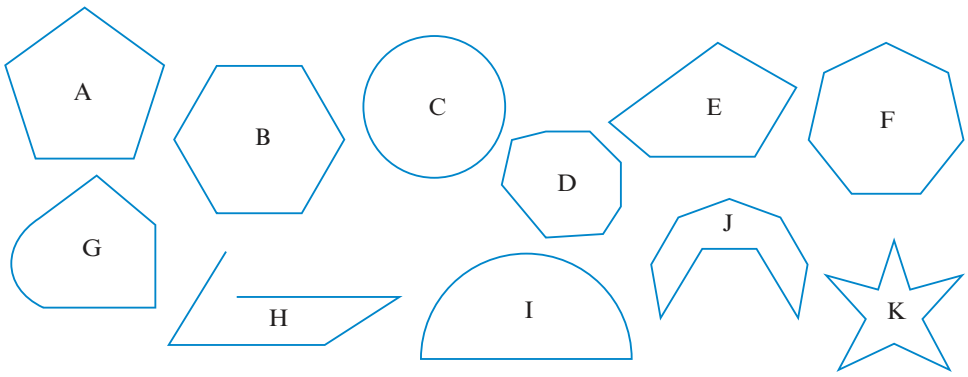
**Reflect**

What is a strategy you can use to help you identify types of quadrilaterals?

# 7C Identifying 2D shapes

## Start thinking!

There are many different ways to classify shapes. Look at these shapes.




**1** How might you sort them into two groups? (Hint: each group does not have to have the same number of shapes in it.)

A **polygon** is a closed shape with all straight sides.

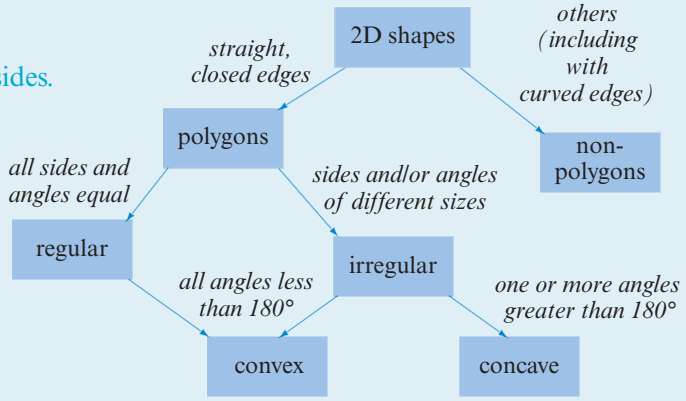
**2** Sort the shapes into polygons and non-polygons. How does this compare to what you did in question **1**?

**3** Look at shapes A and D. How are they different from each other?

**4** One of these two shapes (A or D) is a **regular polygon**. Which do you think is regular and why?

## KEY IDEAS

- A polygon is a closed shape with straight sides.
- Regular polygons have all sides equal in length and all angles equal in size, whereas **irregular polygons** do not.
- **Concave polygons** have at least one interior reflex angle. **Convex polygons** have all interior angles less than 180°




- Polygons are named according to how many angles (or how many sides) they have.

# EXERCISE 7C Identifying 2D shapes

UNDERSTANDING AND FLUENCY

**1 a** Copy and complete this table. The first and last names have been done for you.

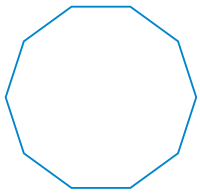
| Greek word | English meaning | Name of shape |
|---|---|---|
| penta | 5 | pentagon |
| hexa | 6 | |
| hepta | 7 | |
| octa | 8 | |
| nona | 9 | |
| deca | 10 | |
| dodeca | 12 | |
| poly | many | polygon |

**b** What shape is this clock?

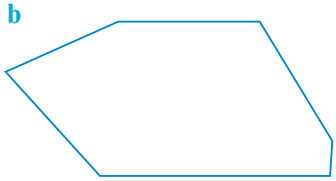


**2** Which of these shapes are polygons?

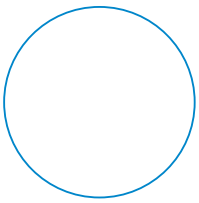
**a**



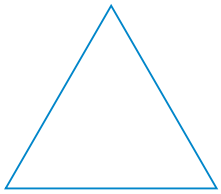
**b**



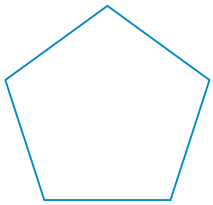
**c**



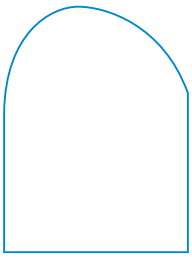
**d**



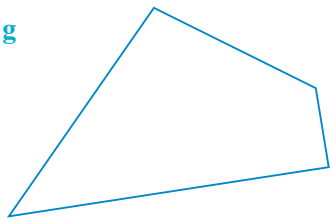
**e**



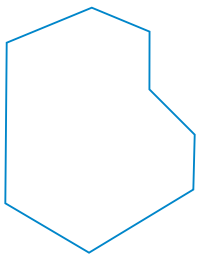
**f**



**g**



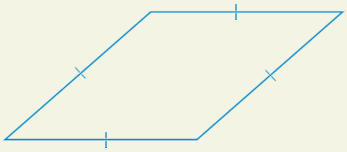
**h**



**i**



## EXAMPLE 7C-1 Describing polygons



Decide if this shape is:

**a** regular or irregular **b** convex or concave.

**THINK**

**a** All sides are equal in length but all angles are not equal in size, so the shape is irregular.

**b** There are no internal angles larger than 180°, so the shape is convex.

**WRITE**

**a** The shape is irregular.

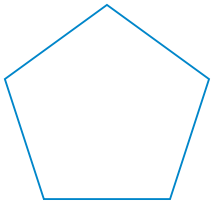
**b** The shape is convex.

UNDERSTANDING AND FLUENCY

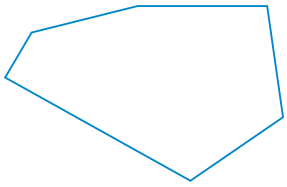
**3** Decide if each shape is:

**i** regular or irregular **ii** convex or concave.

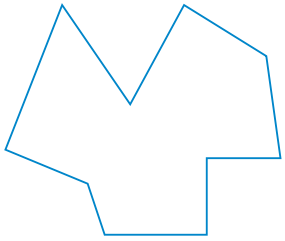
**a**



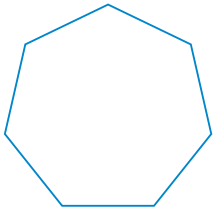
**b**



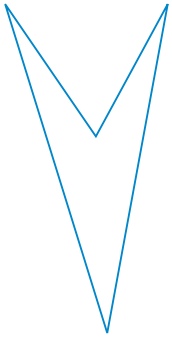
**c**



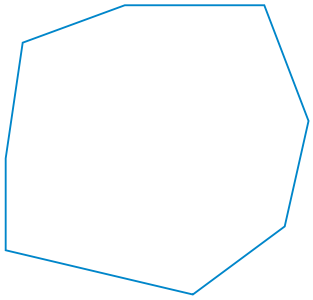
**d**



**e**

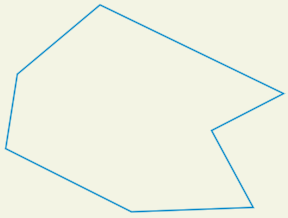


**f**



## EXAMPLE 7C-2 Naming polygons

Describe and name this shape.



**THINK**

All sides are straight, so it is a polygon.
All sides are not equal in length, so it is irregular.
There is a reflex angle, so it is concave.
It has seven sides, so it is a heptagon.

**WRITE**

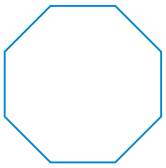
The shape is an irregular, concave heptagon.

**4** Consider each polygon in question **2**.

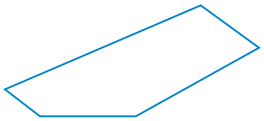
- **a** State if it is regular or irregular.
- **b** Write the number of sides it has.
- **c** Give it a name using your answer to parts **a** and **b**.

**5** Describe and name each polygon.

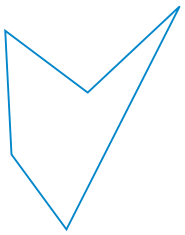
**a**



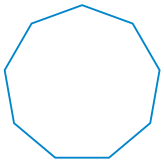
**b**



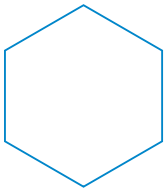
**c**



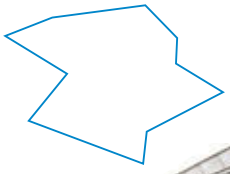
**d**



**e**



**f**



**6** What polygons can you identify in the wings of this dragonfly?



**7** Draw two examples of each of the following.

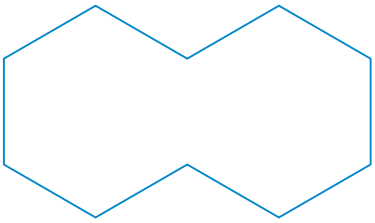
- **a** a hexagon
- **b** a concave polygon
- **c** an irregular polygon

**8** Use your knowledge of describing polygons to name each shape.

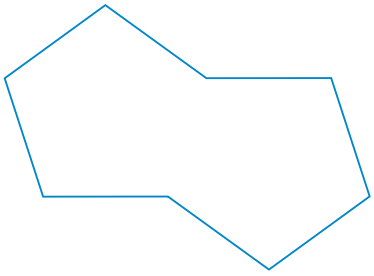
**a**



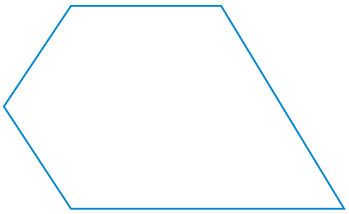
**b**



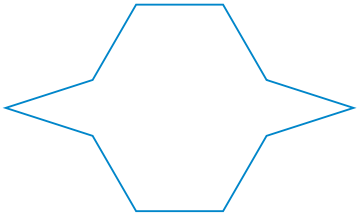
**c**



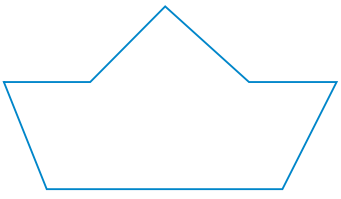
**d**



**e**



**f**



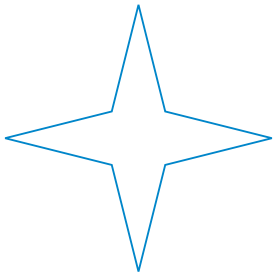
**9** Find examples of at least six different polygons in real life. You might like to look around the classroom, at home or even outside. Try to include some natural structures as well as built structures.

**10** A familiar type of concave polygon is the star. The star on page 358 (labelled K) can be called either a five-pointed star or a concave decagon.

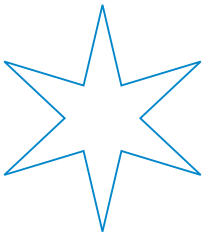
**a** What is the difference between these two names? What are they based on?

**b** Name the following star shapes according to each method.

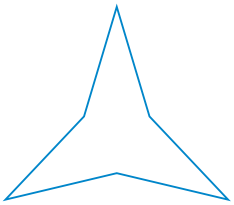
**i**



**ii**



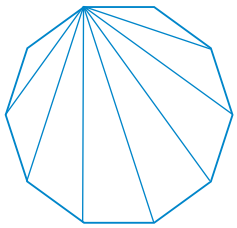
**iii**



**c** What relationship can you see between the two names?



**11** To calculate the angle sum of any polygon, divide the shape into triangles, as shown in this decagon.



**a** What property of triangles makes it useful to do this?

**b** How many sides does a decagon have?

**c** How many triangles are in the decagon?

**d** Use your answer from part **c** to calculate the number of degrees in a decagon.

**12** Draw each shape listed below and use this triangle method to find:

**i** the number of triangles **ii** the number of degrees.

**a** a pentagon **b** a hexagon **c** a heptagon **d** a nonagon

**13** **a** Are triangles and quadrilaterals considered to be polygons? Explain.

**b** If *tri-* means three and *tetra-* means four, rename the triangle and quadrilateral to match the other polygon names.

**14** Look at these photographs of a honeycomb and of the National Aquatic Center (known as the Water Cube) in Beijing.





**a** What shapes are in these two photos?

**b** What is the same and what is different about these two patterns?

PROBLEM SOLVING AND REASONING

**15** The two images shown in question **14** are examples of **tessellations**. A tessellation is a pattern of shapes that has no overlaps or gaps. There are three main types of tessellations:

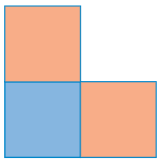
- regular tessellations are made from identical, regular polygons
- semi-regular tessellations are made from a number of regular polygons
- irregular tessellations are made from any shape.

**a** Describe the types of tessellations shown in question **14**.

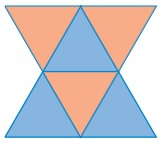
Aside from the hexagonal pattern shown in question **14**, there are only two other regular tessellations: using a square and using an equilateral triangle.

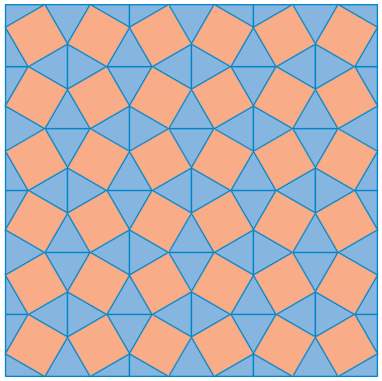
**b** Copy and complete these two other regular tessellations.

**i**



**ii**





**16** Here is an example of a semi-regular tessellation.

**a** What polygons are included in this tessellation?

**b** Create another tessellation with these same shapes. Draw and colour this in.

**c** Create a tessellation of your own using two types of quadrilaterals.

**17** Mosaics are a common example of an irregular tessellation. Describe and draw another two examples of tessellations that you see in everyday life.

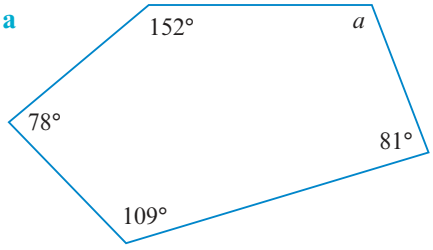
CHALLENGE

**18** Use your findings from question **11** to find the size of an individual internal angle in each of these shapes if they are regular polygons.

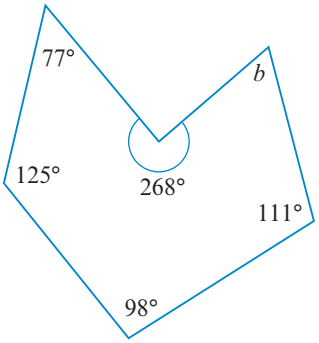
**a** pentagon **b** hexagon **c** heptagon **d** nonagon

**19** Find the size of each unknown angle in these irregular polygons.

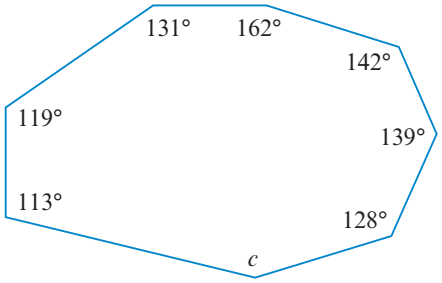
**a**




**b**




**c**



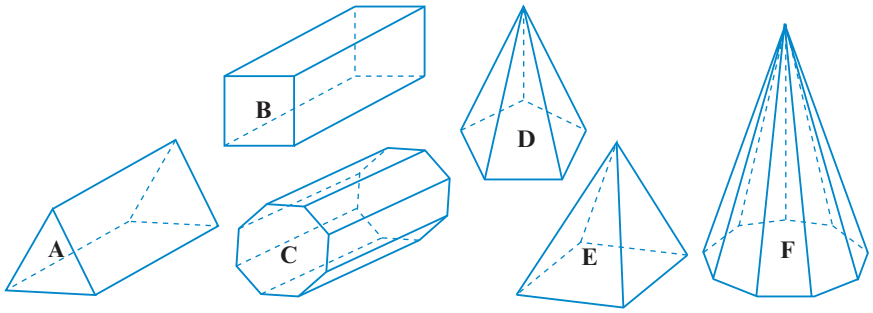

**Reflect**

What properties do you look for when classifying polygons?

# 7D Identifying 3D objects

## Start thinking!

Look at this group of objects.




1 Copy each object onto paper and cut them out.

2 Sort the objects into two different groups, explaining how you have chosen to do so.

3 All of these objects have a polygon as the **base**. Identify the 2D shape that each object has as its base.

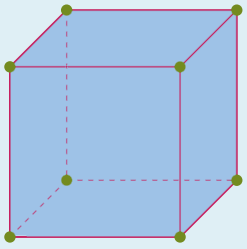
A **prism** is a **three-dimensional (3D)** object with ends that are matching polygons. It looks like the polygon has been pushed through space. A **pyramid** is similar to a prism but one end finishes in a point.

4 Copy and complete this table for each object A to F (placing the cut outs into the first column).

| Object | Base shape | Does it end in a point? | Prism or pyramid? |
|---|---|---|---|
| | | | |

## KEY IDEAS

- 3D objects can be classified in a number of ways. Many can be classed as either prisms or pyramids.
- A prism is an object with ends that are matching polygons. A pyramid is similar but one end finishes in a point.
- A 3D object that contains only polygons is called a **polyhedron** (plural = **polyhedra**). A polyhedron does not have to be either a prism or a pyramid. All polyhedra have:
  flat faces
  straight edges
  vertices (singular vertex).

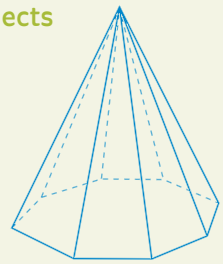


- A **face** is one of the flat 2D shapes that make up a side of an object (shaded blue). An **edge** is a line segment that joins two faces (coloured pink). A **vertex** is a corner point of three or more edges (coloured green).
- **Cylinders**, **cones** and **spheres** are examples of 3D objects that are not polyhedra.

# EXERCISE 7D Identifying 3D objects

## EXAMPLE 7D-1 Classifying objects

Classify this object as either a pyramid or prism.



**THINK**

It does not have ends that are matching polygons, but it ends in a point, so it must be a pyramid.

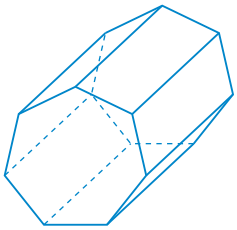
**WRITE**

It is a pyramid.

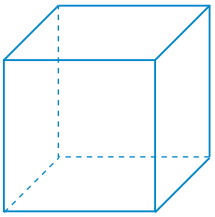
UNDERSTANDING AND FLUENCY

**1** Classify each object as either a prism or a pyramid.

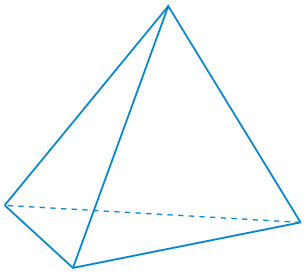
**a**



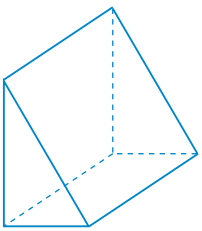
**b**



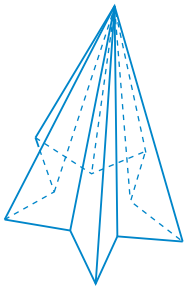
**c**



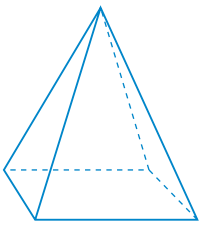
**d**



**e**



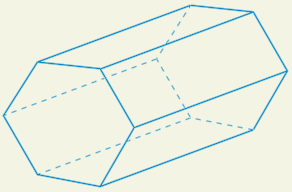
**f**



**2** What 2D shape forms the base of each object in question **1**?

## EXAMPLE 7D-2 Naming objects

Name this object.



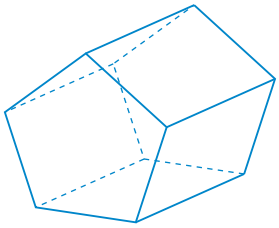
| THINK | WRITE |
|---|---|
| **1** Decide whether the 3D object is a prism or a pyramid. (prism) | |
| **2** Identify the shape of the base. (hexagon) | |
| **3** Name it using 'hexagon' and 'prism'. | It is a hexagonal prism. |

UNDERSTANDING AND FLUENCY

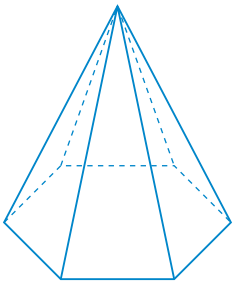
**3** Use your answers to questions **1** and **2** to name the objects in question **1**.

**4** Name each object.

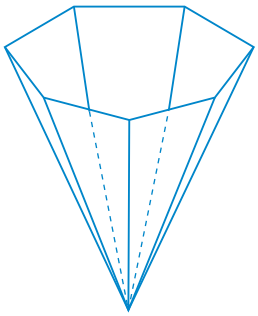
**a**



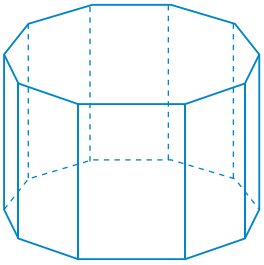
**b**



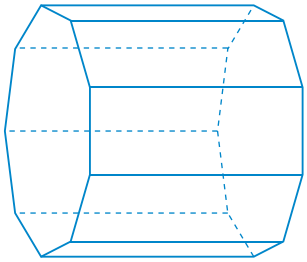
**c**



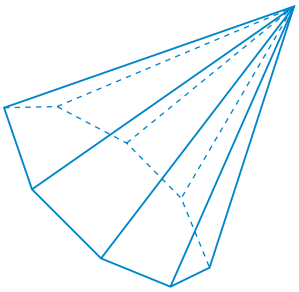
**d**



**e**



**f**



**5** Look at each object and provide its mathematical name.

**a**



**b**



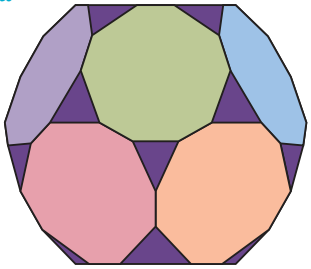

**c**



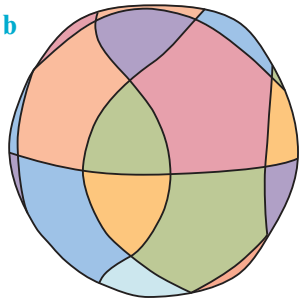
**6** Think of another three real-life objects that are prisms or pyramids and provide their mathematical names.

**7** Decide whether each 3D object is a polyhedron. Explain.

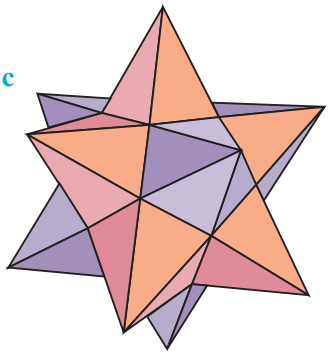
**a**



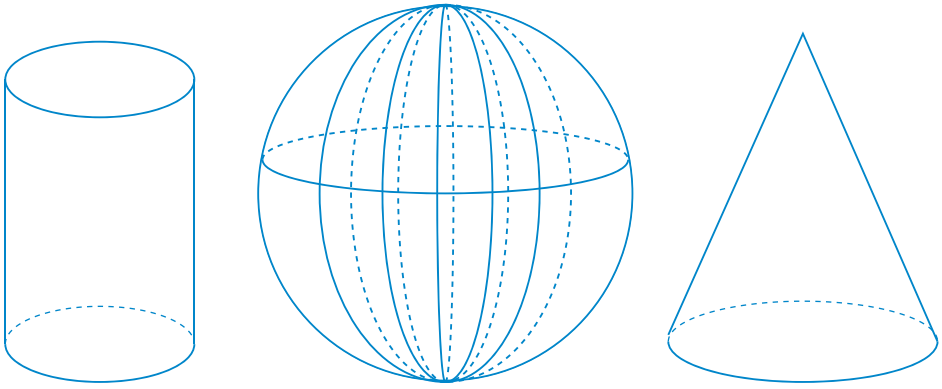
**b**



**c**



**8** Consider these three 3D objects.



**a** Name each object.

**b** What is different about these 3D objects compared to the ones you've seen so far?

**c** Explain why they are not polyhedra.

**9** Here are three composite objects.

- **i** Copy each object into your book.
- **ii** Split the object up into its parts.
- **iii** Name each part.

**a**



**b**



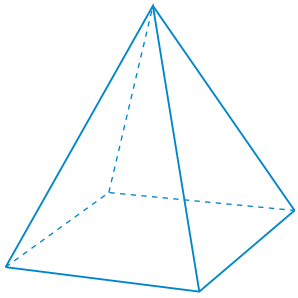
**c**



**10** Draw an image of yourself using only 3D objects. Make sure you label each object that you use. How many different types of 3D objects did you use?



**11** Look at this square-based pyramid.



- **a** How many faces does it have?
- **b** How many edges does it have?
- **c** How many vertices does it have?
- **d** Draw the square-based pyramid.
  - **i** Shade the faces green.
  - **ii** Highlight the edges in red.
  - **iii** Place a blue dot on the vertices.

**12** Look at all the 3D objects listed in questions **1** and **4**. Create a table listing the number of faces ($F$), the number of edges ($E$) and the number of vertices ($V$) for each object.



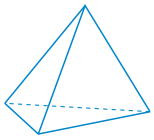
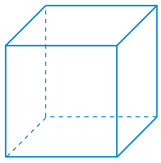
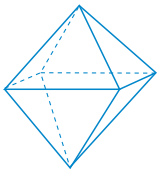
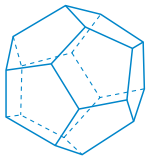
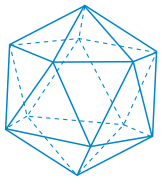
**13** How can you tell if a 3D object is a polyhedron?

**14** Is a soccer ball a polyhedron? Explain.

CHALLENGE

**15** There is a group of special polyhedra named the Platonic solids.
These polyhedra are all regular. This means that all their faces are the same shape, their edges are the same length and the vertices meet at the same angles.

**a** Copy and complete the table below.

| | Tetrahedron | Cube (Hexahedron) | Octahedron | Dodecahedron | Icosahedron |
|---|---|---|---|---|---|
| Image |  |  |  |  |  |
| Number of faces ($F$) | | | | 12 | 20 |
| Name of polygon on face | | | | | |

**b** Add two rows to the table and find the number of vertices ($V$) and the number of edges ($E$) for each Platonic solid.

**c** Add another row to the table and calculate $F + V - E$.

**d** What do you find?




**16** Euler's rule states that for any polyhedron, $F + V - E = 2$.

**a** Is this true for all the Platonic solids above?

**b** Investigate Euler's rule further by finding $F + V - E$ for all the objects in questions **1** and **4**. What do you find?

**17** Find the number of faces on each polyhedron, given the object has:

**a** 10 edges, 4 vertices

**b** 6 edges, 3 vertices

**c** 18 edges, 6 vertices

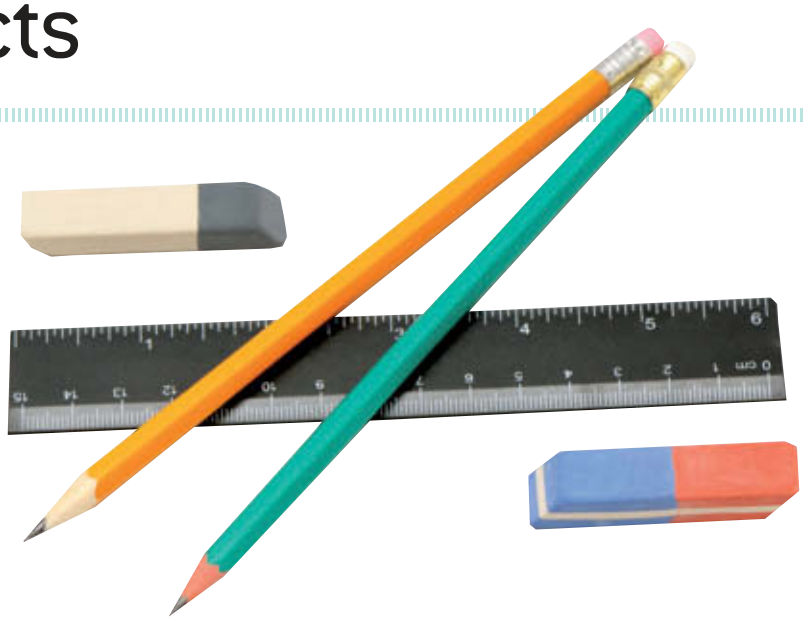
**d** 20 edges, 10 vertices.

**18** Draw one of the polyhedra from question **17**.

**Reflect**

How can you classify 3D objects?

# 7E Drawing 2D shapes and 3D objects



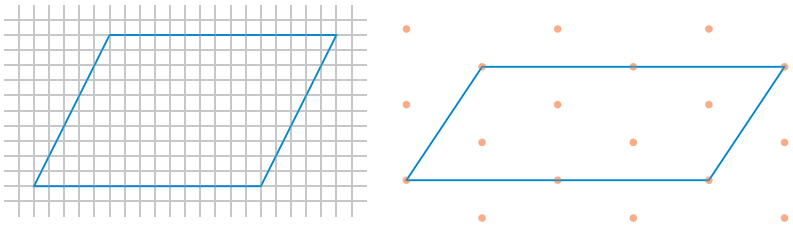
## Start thinking!

You can use many tools to draw 2D shapes and 3D objects. One very simple type of tool you can use is special paper. The two most common types are grid or graph paper and isometric dot paper.

1 Collect a sheet of graph paper and a sheet of isometric dot paper from your teacher.

2 Make a list of the basic triangles and quadrilaterals you have covered.

3 Draw one of each type of triangle and quadrilateral on both the graph paper and the isometric dot paper. A parallelogram has been done as an example.



4 Are there any triangles or quadrilaterals that are difficult to draw on either type of paper? Explain.

## KEY IDEAS

- Using graph paper or isometric dot paper can help you draw 2D shapes and 3D objects.
- Always use a ruler and pencil even when using graph paper.
- Sometimes a pair of compasses and protractor are useful when drawing shapes and objects.
- There are also various types of software that can be used to draw shapes and objects easily.



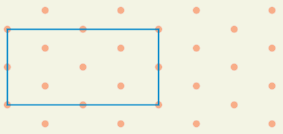
# EXERCISE 7E Drawing 2D shapes and 3D objects

**1** Draw these 2D shapes on isometric paper or graph paper.

**a** parallelogram **b** trapezium **c** rhombus
**d** isosceles triangle **e** hexagon **f** pentagon

## EXAMPLE 7E-1 Using a 2D shape to draw a 3D object

Extend this 2D shape to form a 3D prism.

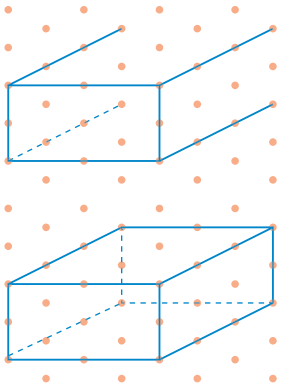


### THINK

**1** Draw a line from each vertex using the dots on the paper as a guide. Make sure each line is the same length and that all three lines are parallel. Show the hidden edge as a dashed line.

**2** Complete the prism by drawing two horizontal lines and two vertical lines that join the ends of the lines you have just drawn. Show any hidden edges as dashed lines.

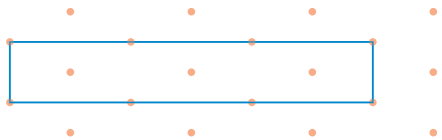
### WRITE



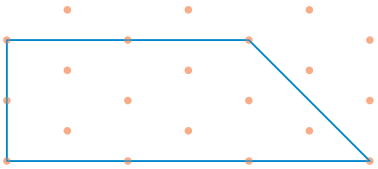
UNDERSTANDING AND FLUENCY

**2** Extend each 2D shape to represent a 3D object using isometric dot paper.

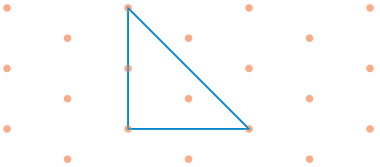
**a**



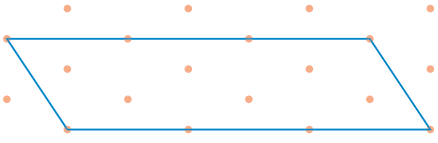
**b**



**c**



**d**



**3** Draw each 2D shape onto either isometric dot paper or graph paper and then transform it to represent a 3D prism.

**a** equilateral triangle **b** hexagon **c** rhombus
**d** trapezium **e** octagon **f** pentagon

## EXAMPLE 7E-2 Drawing a 3D object

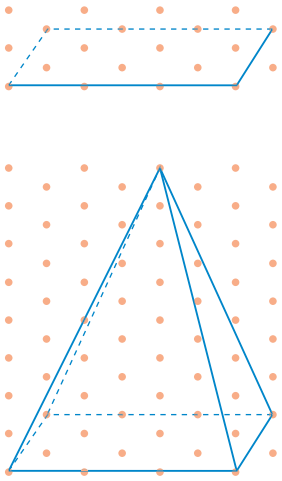
Draw a rectangular pyramid using isometric dot paper.

### THINK

**1** Although the base of the pyramid is a rectangle, looking at the base from the side and at an angle, it looks like a parallelogram. Draw a thin parallelogram by joining four dots on isometric paper with straight lines.

**2** Transform this into a pyramid by drawing a line from each of four vertices to a single dot. Use a dashed line for the hidden edge, and make the hidden edges of the base into a dashed line, too.

### WRITE



UNDERSTANDING AND FLUENCY

**4** Draw each 3D object using graph or isometric dot paper.

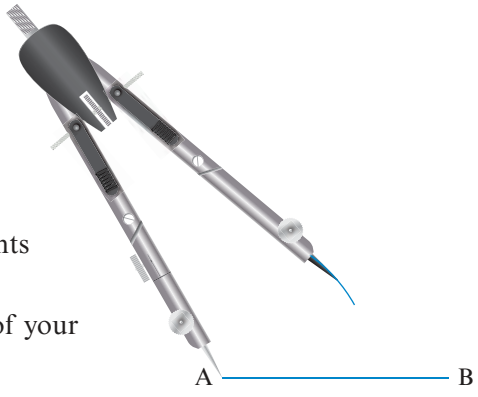
**a** cube **b** square-based pyramid
**c** kite-based prism **d** octagonal prism
**e** pentagonal pyramid **f** right-angled triangular prism

**5** Sometimes you may need to draw a triangle with certain side lengths. You can easily do this using a pair of compasses, commonly called a **compass** (notice that this is different from a directional compass used for bearings). Consider a triangle with side lengths of 3 cm, 5 cm and 6 cm.

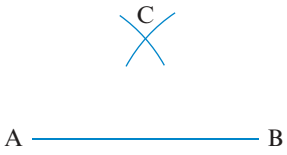
**a** What is the longest side? Rule a straight horizontal line of this length.

**b** Select one of the shorter side measurements and set your compass to this length.

**c** Place one tip of the compass on the end of your line and make a mark like that shown.



**d** Change the width of the compass to the length of the third side and repeat the step described in part **c** from the other end of the line so that your marks cross as shown.




**e** Complete the construction of the triangle and write the side lengths in the appropriate places.

**f** Do you have to start with the longest side? Explain.

**6** Use a compass to construct triangles with these sets of side lengths.

**a** 3 cm, 6 cm and 7 cm

**b** 4 cm, 4 cm and 6 cm

**c** 8 cm, 8 cm and 8 cm

**d** 2 cm, 9 cm and 10 cm



**7** You can construct an equilateral triangle within a circle using a compass, protractor, and your knowledge of angles in a revolution.

**a** Draw a circle with a radius of 3 cm using your compass.

**b** You need to divide the circle into three equal sectors. Why do you think this is necessary?

**c** How many degrees are there in a revolution?

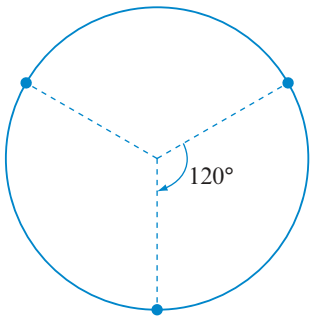
**d** How many degrees should be in each sector? (Hint: divide your answer to part **c** by the number of sides in a triangle).

**e** Draw a faint line from the centre of the circle to its circumference and mark the circumference with a dot. Place your protractor on the centre of the circle and measure 120° from this line, drawing another faint line and marking another dot on the circumference.

**f** Repeat part **e** from the second line you drew and mark in a third dot on the circumference of the circle, as shown in the diagram. You should now have divided the circle into three equal sectors.

**g** What is the angle at the centre of the circle for each sector?

**h** Join these three dots to form an equilateral triangle. Erase the circle and lines so that you are left with just the triangle.




**i** Compare the method used here with that used for question **6c**. Which method did you find easier for constructing an equilateral triangle? Why?

**j** Why would this method not work for an isosceles or scalene triangle?

**8** Explain why it is not possible to construct a triangle with side lengths of 4 cm, 7 cm and 12 cm.

**9** We can extend the method found in question **7** to help us draw regular polygons. Consider the regular hexagon.

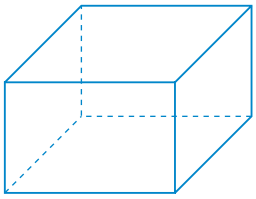
- **a** How many sides does a hexagon have?
- **b** How many sectors would a circle need to be split into?
- **c** What would be the size of the angle at the centre of the circle for each sector?
- **d** Copy and complete the following. To draw a regular polygon, first draw a _____. Split this into a number of equal sectors determined by the number of _____ in the polygon. Use a _____ to measure these sectors and mark dots on the _____ of the circle. Join the dots and _____ the circle to complete the diagram.
- **e** Draw a regular hexagon using this method.

**10** Use a compass and a protractor to construct each polygon.

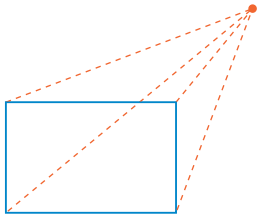
- **a** regular octagon
- **b** regular pentagon
- **c** regular nonagon
- **d** regular decagon

**11** How might you construct a cylinder using a compass and a ruler?

**12** Three-dimensional objects can also be drawn from a single-point perspective, representing how they look in real life. **Perspective drawings** make use of a **vanishing point**. Consider this rectangular prism.

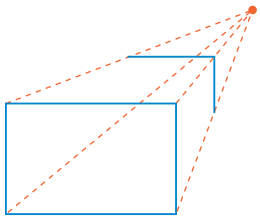


- **a** To draw a perspective drawing, first draw the front face of the prism: in this case, a rectangle.
- **b** Select a point above the rectangle to be the vanishing point.
- **c** Rule faint lines from each vertex of the rectangle towards the vanishing point, as shown in A.



A

- **d** Choose how long you want the prism to be by drawing in a top and side within the faint lines. Make sure these are parallel to the edges on the rectangle, as shown in B.
- **e** Complete the rectangular prism by making the remaining edges solid and erasing the left over faint lines and vanishing point.
- **f** How is this different from the rectangular prism shown at the start of the question?
- **g** Why are perspective drawings difficult to do for objects that are not prisms, such as pyramids?

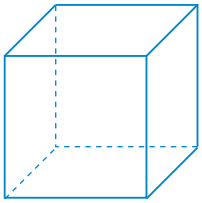


B

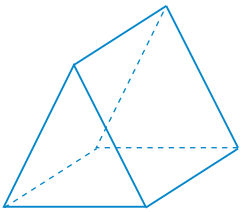
PROBLEM SOLVING AND REASONING

**13** Using the method described in question **12**, create two perspective drawings for each shape below. Select varying vanishing points and lengths.

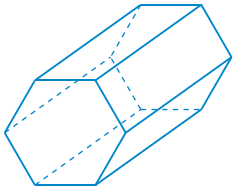
**a**



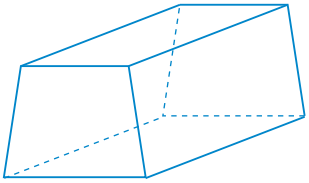
**b**



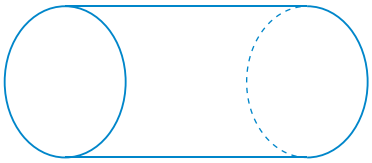
**c**



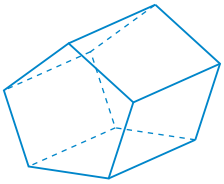
**d**



**e**



**f**



**14** There are a large number of software programs that help you to draw shapes and objects. Ask your teacher about what software is available and investigate drawing different shapes and objects with at least two different programs. Even Microsoft Word has a drawing toolbar you can use.

CHALLENGE

**15** Accurately draw a polyhedron of your choice. Explain the method you used.

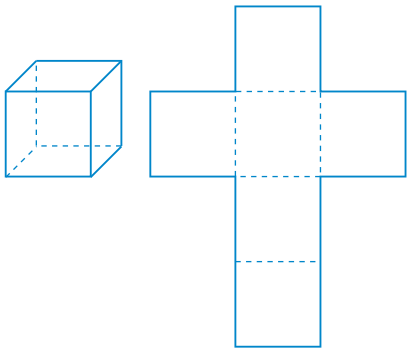


**Reflect**

When might it be important to be able to draw shapes and objects accurately?

# 7F Planning and constructing 3D objects

## Start thinking!



To construct a simple 3D object, you can make a **net**. A net is a 2D plan that folds to make the 3D object. Consider a cube.

1 How many faces does a cube have?

2 How many *different* faces does a cube have?

3 What shape are the faces of a cube?

4 Draw each face of the cube.

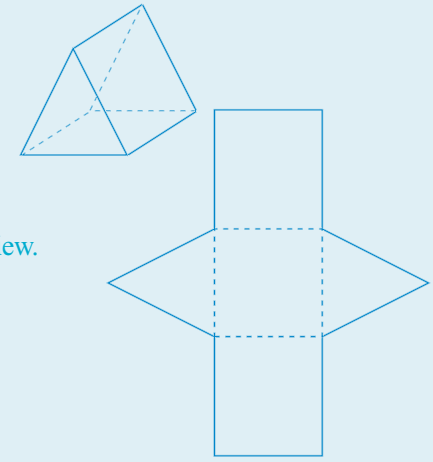
It is important when drawing a net that the faces are put together in such a way that they will fold to make the 3D object. For prisms, the most common net is the 'hopscotch' style net, as pictured above.

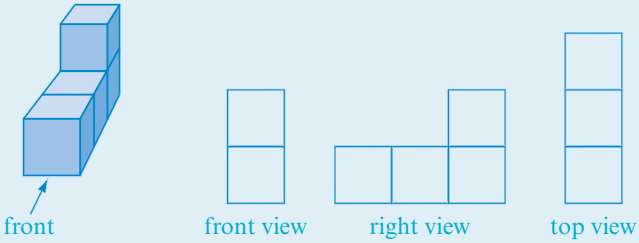
5 If you lined up all the faces in a straight line, would this work as a net? Explain why or why not.

6 There are 11 different nets for a cube. How many can you find? Be careful of mirror images and rotations!

## KEY IDEAS

- A net is a 2D plan that can be folded to form a 3D object.
- Nets are best used for simple 3D objects.
- A set of plans consists of a front view, a side view and a top view.
- A set of plans are best used for more complex 3D objects.



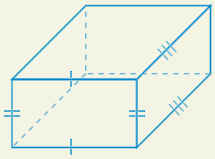



**NOTE** If you are uncertain that your net will work, cut it out and fold it up. Remember to include tabs if you are going to construct a 3D object.

# EXERCISE 7F Planning and constructing 3D objects

## EXAMPLE 7F-1 Drawing nets

Draw a net for this 3D object.



### THINK

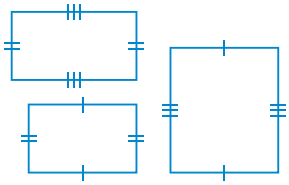
1 A rectangular prism has six faces, made up of three different kinds. Draw each different one.

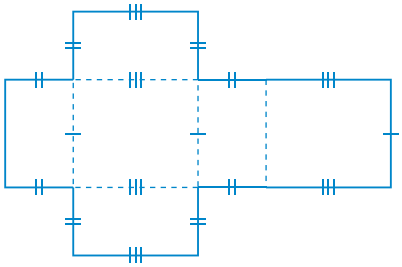
2 Try to visualise the rectangular prism unfolding – which face is on the bottom? This will form the centre of the net.

3 Arrange the remaining faces in a hopscotch style net. Look carefully at which faces are touching the centre face.

4 Add tabs if you wish to construct the object.

### WRITE

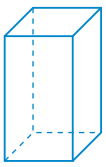




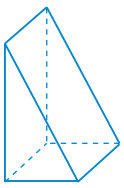
UNDERSTANDING AND FLUENCY

**1** Draw a net for each 3D object.

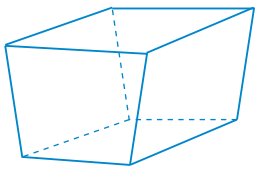
**a**



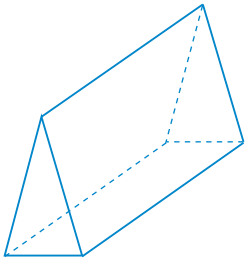
**b**



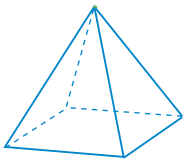
**c**



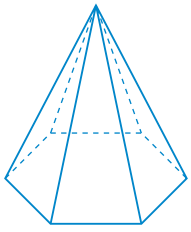
**d**



**e**



**f**



**2** Draw a net for each object.

**a**




**b**

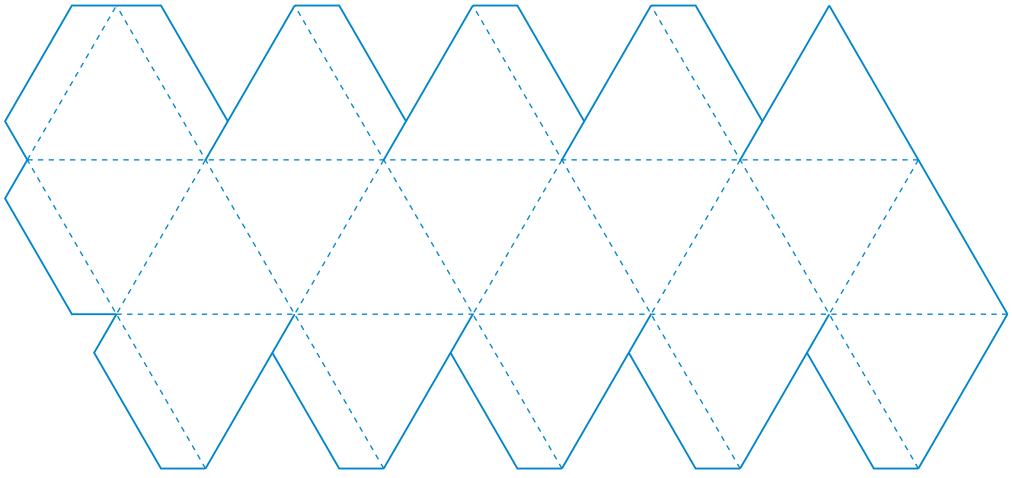



**c**



**3** Find a 3D object in your classroom. Draw a net and construct it.

**4** It is possible to draw nets for many other complex 3D objects. One example is the icosahedron.



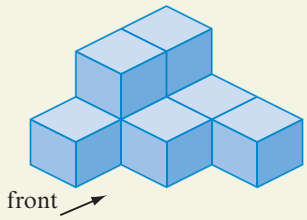
**a** Copy the net of the icosahedron and construct it.

**b** How many faces does the icosahedron have? What shape are these faces?

**c** Use the Internet or another resource, such as the library, to find nets for at least five other objects not mentioned here.

## EXAMPLE 7F-2 Drawing plans of 3D objects

Draw a set of plans for this object.



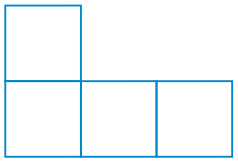

### THINK

**1** Look at the front of the object. From this angle, three cubes are visible on the bottom and only one on the top (to the left). Draw this.

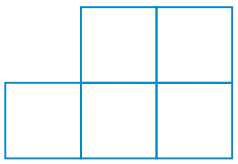
**2** Look at the right side of the object. Three cubes are visible on the bottom and two on the top (the middle and right). Draw this.

**3** Look at the top of the object. From this angle, three rows of cubes are visible. Draw this.

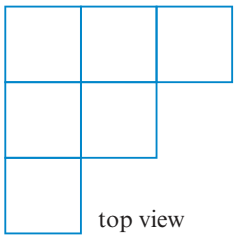
### WRITE



front view



right-side view

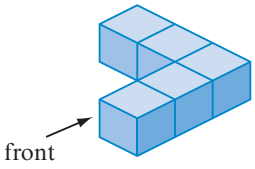


top view

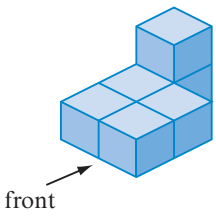
UNDERSTANDING AND FLUENCY

**5** Draw a set of plans for each object. You may like to use cubes to construct the object first.

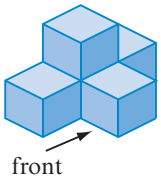
**a**



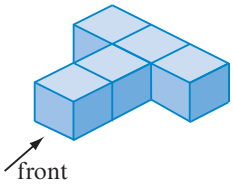

**b**



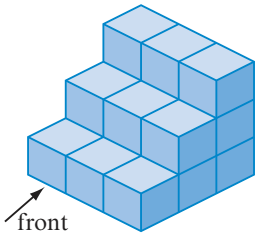

**c**



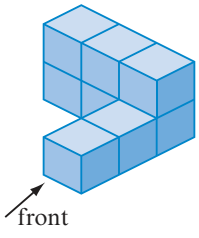

**d**




**e**

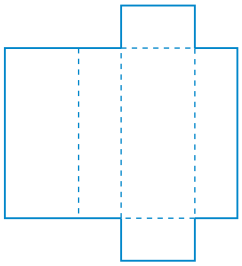



**f**

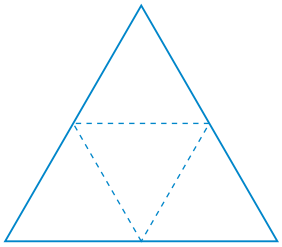


**6** What object comes from each net?

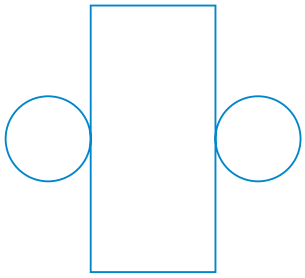
**a**



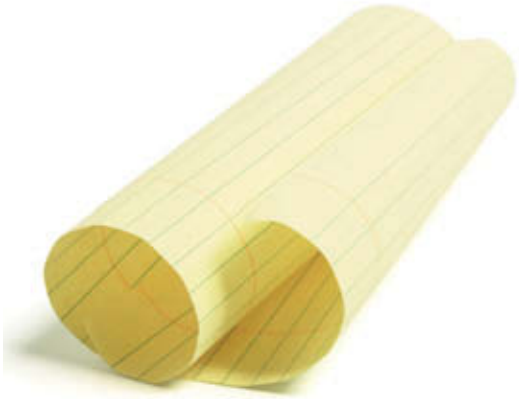
**b**



**c**



**7** Use a rolled-up piece of paper to show a classmate that the net in question **6c** does in fact belong to a cylinder.



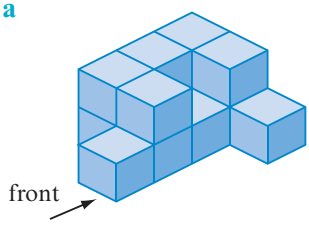
**8** How many different nets for a tetrahedron can you find?

**9** Nets can also be made for open objects. For example, this box has no lid. Redraw the nets for the objects in question **1a** and **c** as if they were open objects.

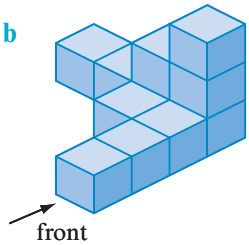


**10** Draw a set of plans for each of these more complex objects.

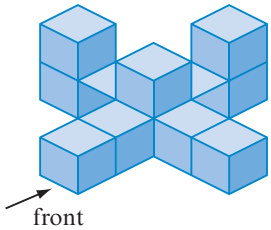
**a**




**b**




**c**

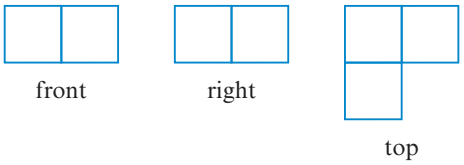


PROBLEM SOLVING AND REASONING

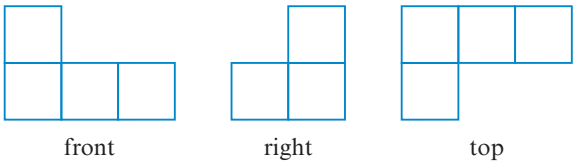
**11** Use cubes to build your own objects and see if your classmates can draw a set of plans for them.

**12** Use each set of plans to construct the matching 3D object.

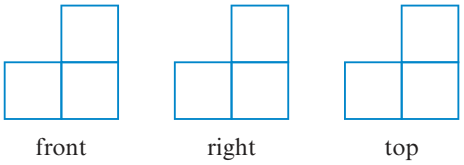
**a**



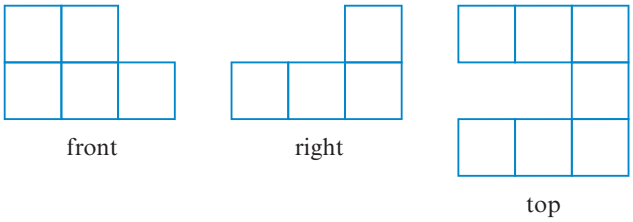

**b**




**c**

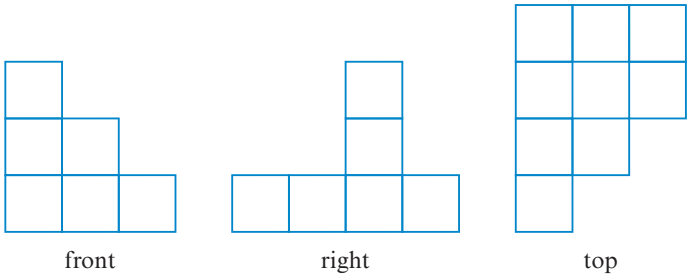



**d**




CHALLENGE

**13** Can you use this set of plans to construct the matching 3D object?




**14** Here is a challenge for your classmates. Without them seeing it, first construct an object using cubes and draw a set of plans for it. Knock down your object. Now ask your classmates to reconstruct the object using only the given plans.

**Reflect**

In what situations might it be important to make plans before construction?

# 7G Symmetry of 2D shapes and 3D objects

## Start thinking!



An object is **symmetrical** if you can cut it in half and the two halves are reflections of each other. This is also known as line symmetry. An easy way to find if an object is symmetrical is to use a mirror.

1 Where would you place a mirror on the photo of the ladybird to test if it is symmetrical?

2 Why do you think the line where you would place the mirror is called the line of symmetry?

3 Locate at least five objects around the classroom and, using a mirror, decide if each is symmetrical.

4 Look at this die.



**a** Is it symmetrical vertically?

**b** Is it symmetrical horizontally?

**c** Is it symmetrical diagonally?

**d** How many lines of symmetry can you find?

It is obviously possible to have more than one line of symmetry. Another name for a line of symmetry is an **axis of symmetry**. The face of the die has four axes of symmetry (remember that axes is the plural of axis).

5 Look again at the objects you found around the classroom. How many axes of symmetry does each object have?

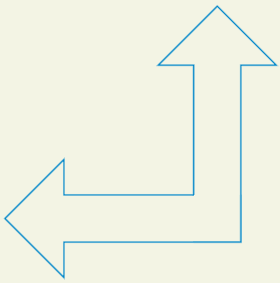
## KEY IDEAS

- An object is symmetrical if it can be divided into two halves that are mirror reflections of each other.
- A line where an object is symmetrical is called a line or axis of symmetry.
- Objects can have none, one or many axes of symmetry.
- An object has **rotational symmetry** if it looks the same after being rotated less than 360°.

# EXERCISE 7G Symmetry of 2D shapes and 3D objects

## EXAMPLE 7G-1 Looking for symmetry

Is this shape symmetrical?
If so, draw in the line of symmetry.

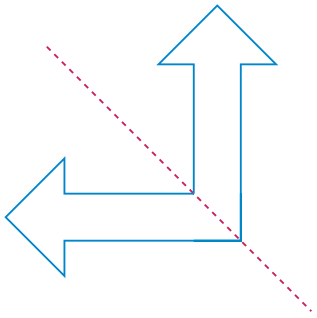


### THINK

1 Look to see if you can draw a **mirror line** somewhere on the shape.

2 The shape is symmetrical along a diagonal mirror line. Draw this in.

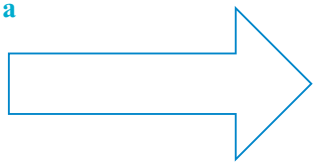
### WRITE



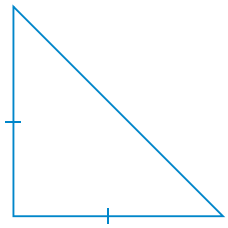
UNDERSTANDING AND FLUENCY

**1** Is each object symmetrical? If so, draw in the line of symmetry.

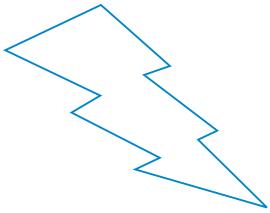
**a**



**b**



**c**



**d**



**e**



**f**



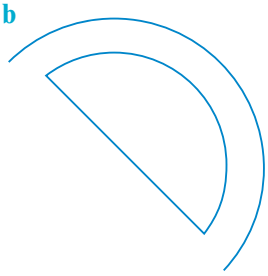
UNDERSTANDING AND FLUENCY

**2** Copy and complete these symmetrical figures.

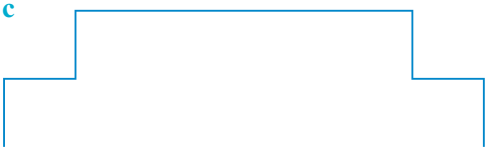
**a**



**b**



**c**



**d**



**e**

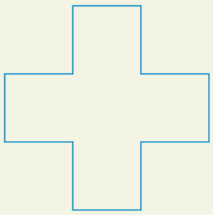


**f**



## EXAMPLE 7G-2 Finding axes of symmetry

How many axes of symmetry does this shape have?



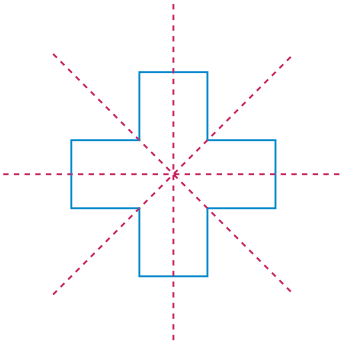
### THINK

**1** Look at the shape. How many places can you draw a line that divides the shape into two halves that are mirror reflections of each other?

**2** Is it symmetrical horizontally? Vertically? Diagonally? Anywhere else?

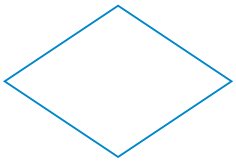
### WRITE

There are four axes of symmetry: horizontal, vertical, and diagonal both ways.

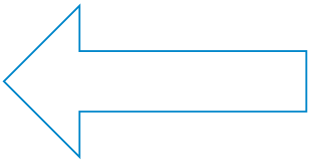


**3** How many axes of symmetry does each shape have?

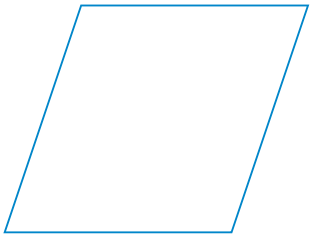
**a**



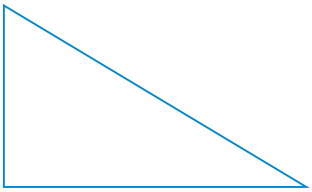
**b**



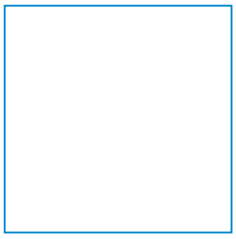
**c**



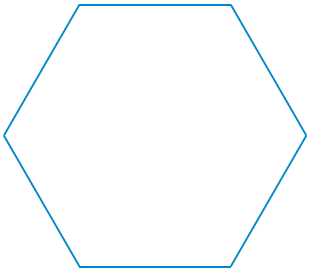
**d**



**e**



**f**



**4** Consider the alphabet.

**a** Write out the alphabet (in upper case) in your book.

**b** Find the axis or axes of symmetry for each letter (if they exist).

**c** Which letters have no axes of symmetry?

**d** Which letters have more than one axis of symmetry?

**e** Repeat the activity, but this time use the lower case alphabet.

**f** Write a paragraph describing any differences and similarities you find.

**g** Many different styles of writing and different fonts exist. Do your results differ from your classmates? Provide an explanation for your answer.

**h** Use a computer with a word processor such as Microsoft Word and investigate what happens to the axes of symmetry of different letters when fonts change.

**5** How many axes of symmetry does each object have?

**a**



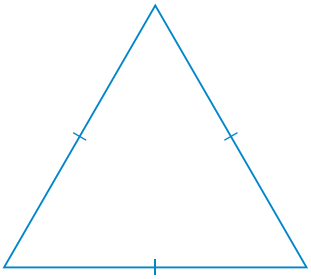
**b**



**c**

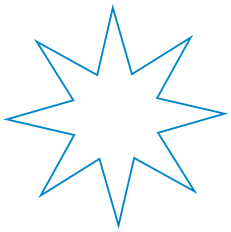


**6** Another kind of symmetry is called rotational symmetry. An object has rotational symmetry if it looks the same after being rotated a certain amount.



- **a** Copy and cut out this equilateral triangle.
- **b** Rotate the triangle until it looks exactly the same as when you started.
- **c** How many times can you rotate it before it is back in its beginning position?

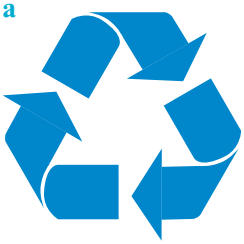
The order of rotational symmetry is the number of times a figure can be rotated and still look the same, before it is back to its original position.



- **d** What is the order of rotational symmetry for an equilateral triangle?
- **e** Repeat the activity with this star. What is its order of rotational symmetry?
- **f** What letters of the alphabet have an order of rotational symmetry that is two or more? List these with their orders.

**7** Find the order of rotational symmetry for the figures below.

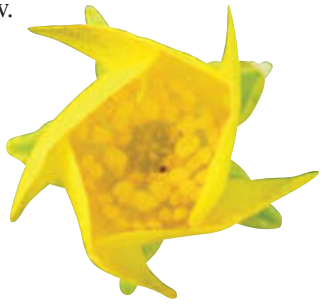
**a**



**b**



**c**



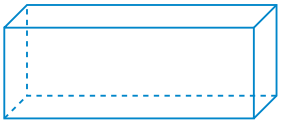


**8** Many animals have a basic symmetry. Consider the zebra at right. Some people would consider this to be a symmetrical photo, whilst others would disagree.



- **a** What reasons can you think of for both arguments?
- **b** What is your opinion?

**9** Symmetry can also be applied to 3D objects. Consider this rectangular prism. You may find it helpful to locate or construct a rectangular prism in the classroom to assist you with this question.



- **a** How many axes of symmetry does a rectangle have?
- **b** Rotate the rectangular prism. Is there another place where the prism can be divided into two mirror halves?
- **c** Draw a diagram of a rectangular prism showing all the axes of symmetry.

**10** How many axes of symmetry does a cube have? Find an example of a cube and rotate it carefully. Draw a diagram showing all the axes of symmetry of a cube.

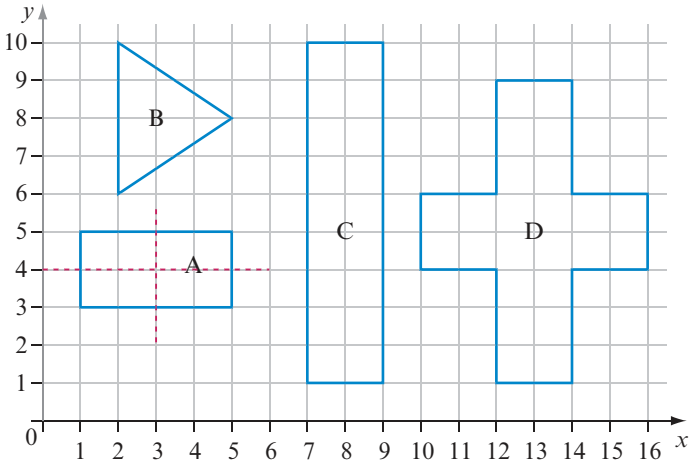
PROBLEM SOLVING AND REASONING

**11** Investigate the symmetry of other 3D objects. Can you find a pattern?

**12** A classmate completed question **6** and decided that the order of rotational symmetry was the same as the number of axes of symmetry. Explain why they are incorrect, using your results from question **7**.

**13** Is it possible to have an order of rotational symmetry of 1? Explain.

**14** You can use the Cartesian plane to help to identify lines of symmetry of various shapes. Look at the shapes on the plane. Shape A has a vertical line of symmetry that runs through 3 on the $x$-axis and a horizontal line of symmetry that runs through 4 on the $y$-axis. Find the lines of symmetry for the other three shapes.




**15** An example of a word which has an axis of symmetry is 'CHECKBOOK'.

**a** What kind of symmetry does 'CHECKBOOK' have?

**b** What other words can you find that have a horizontal axis of symmetry? (Hint: think about which *letters* have a horizontal axis of symmetry.)

**16** Another example of a word which has an axis of symmetry is 'MUM'.

**a** What kind of symmetry does 'MUM' have?

**b** Does this symmetry exist in lowercase letters?

**c** What other words can you find that have this kind of symmetry? Remember to think about both upper- and lowercase letters.

**17** What kind of symmetry does the word 'SWIMS' have? Can you find any other words like this?

CHALLENGE

**18** There are certain fonts that allow you to write many words in a style that allows rotational symmetry. These are called ambigrams. Dan Brown used such a font in his bestselling book, *The Da Vinci Code*. Use the Internet to research other fonts or programs and see if you can write your name so that it reads the same upside down as it does the right way up.



**Reflect**

Where might you find symmetry in real life?

# 7H Describing transformations

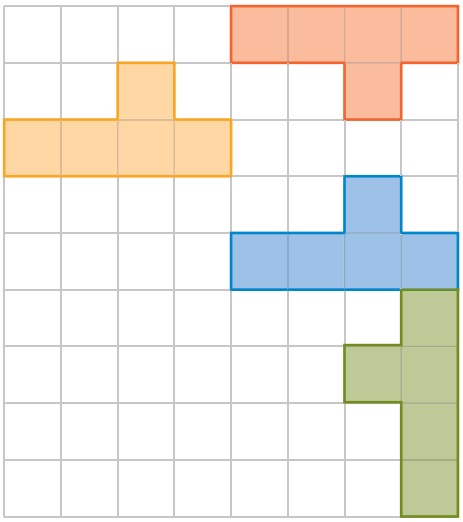
## Start thinking!

1 What do you think of when you hear the word 'transformation'?

2 How can you transform shapes and objects?

3 Look at the four shapes drawn on this grid. How are the shapes different and how are they the same?

4 Copy the grid and shapes on to a piece of grid or graph paper.

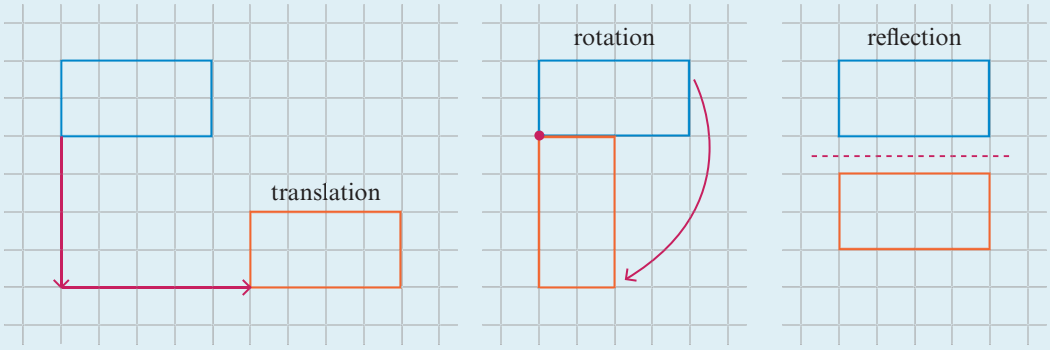


**Translation** is when a shape or object is moved without turning or changing size. You can record the size of the translation by counting how many units the shape has moved up or down and left or right.

5 Which shape is a translation of the blue shape?

## KEY IDEAS

- An **isometric transformation** does not change the shape or size of a shape or object.
- **Translation**, **rotation** and **reflection** are all isometric transformations.



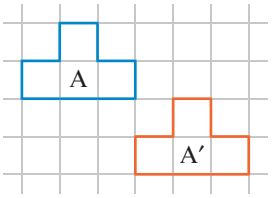

- The transformed shape or object (also called the **image**) should always be given a name that indicates that it is a transformation – usually by adding a dash to the original name. For example, the image of shape A is called A′.

# EXERCISE 7H Describing transformations

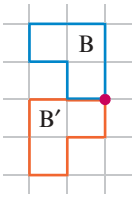
UNDERSTANDING AND FLUENCY

**1** Copy and complete the following sentences to describe the transformation shown in each figure.

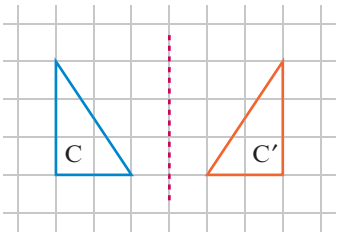
**a** The shape has been translated _____ units *up*/*down* and _____ units *left*/*right*.




**b** The shape has been rotated _____° in a _____ direction.

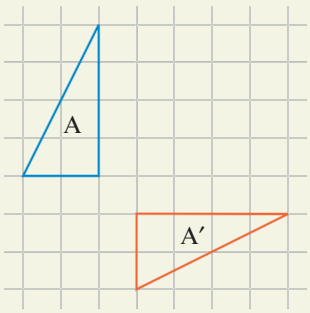



**c** The shape has been reflected in a _____ mirror line.




## EXAMPLE 7H-1 Identifying transformations

Identify the transformation shown as a translation, rotation or reflection of shape A.




**THINK**

**1** A′ is not in the same orientation as A, so it cannot be a translation.

**2** A′ appears to be flipped rather than turned (relative to A), so it cannot be a rotation.

**3** A′ appears to be a mirror image of A, so it is a reflection.

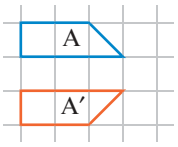
**WRITE**

A′ is a reflection of A.

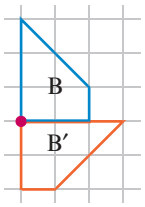
UNDERSTANDING AND FLUENCY

**2** Identify each transformation shown as a translation, rotation or reflection.

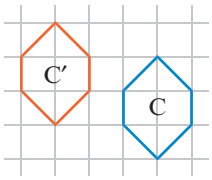
**a**



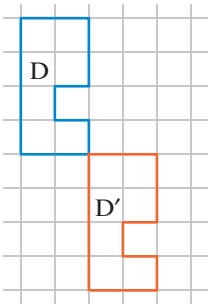

**b**



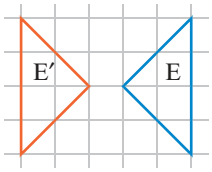

**c**



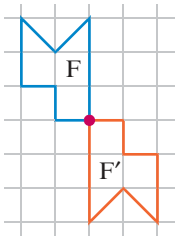

**d**




**e**

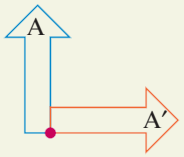



**f**




## EXAMPLE 7H-2 Describing transformations

Describe the transformation of the shape shown.




### THINK

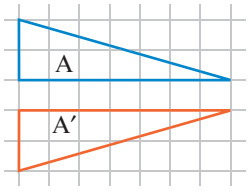
1. Has the shape been translated, rotated or reflected?
2. Has it been rotated in a clockwise or anticlockwise direction?
3. How much has it been rotated? It has been rotated a quarter turn – this is 90°.
4. Write your answer.

### WRITE

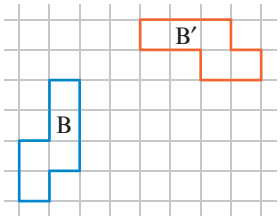
The shape has been rotated 90° in a clockwise direction.

**3** Describe the transformation of each shape.

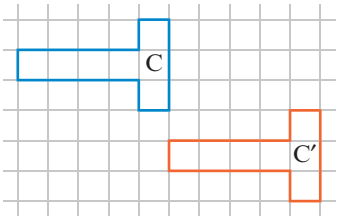
**a**



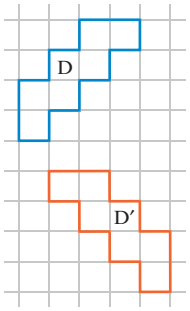

**b**



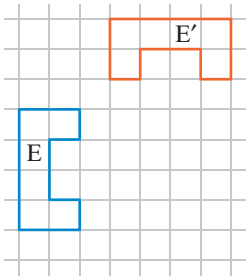

**c**



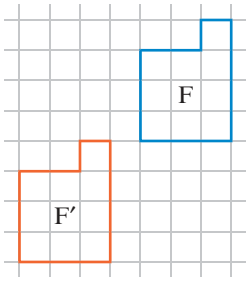

**d**




**e**



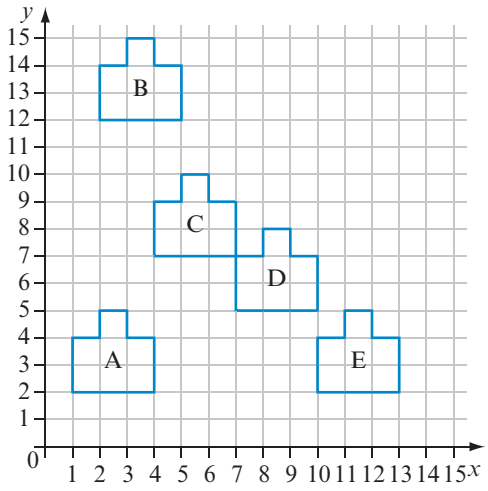

**f**



UNDERSTANDING AND FLUENCY

**4** Fully describe each transformation shown in question **2**.

**5** To identify translations more easily, you can use coordinates. The bottom left corner of shape A has the coordinates (1, 2).




**a** Write the coordinates of the matching point on shape B.

**b** Copy and complete: Shape A has moved _____ unit to the right and _____ units up to become shape B.

**c** Which shape has been translated 3 units to the right and 5 units up from shape A?

**d** Write the coordinates of the bottom left corner of this shape.

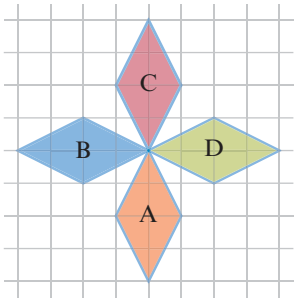
**e** A friend said that shape E has been translated 6 units to the right and 0 units up from shape A. Can you explain where they may have gone wrong?

**f** Describe these translations.

**i** shape B to shape C **ii** shape B to shape E **iii** shape D to shape A

**iv** shape E to shape B **v** shape D to shape C **vi** shape A to shape C

**6** This figure is made up of a number of rotations of one shape.




**a** Describe the transformations for the shapes B, C and D given that they are rotations of shape A.

**b** Make up your own pattern using a rotated shape.

**7** Copy these figures and draw in the mirror line that produces these shapes after reflection of one half.

**a**



**b**



PROBLEM SOLVING AND REASONING

**8** The figure in question **5** consists of a shape and four images. It was assumed that shape A was the original figure. Use the clues below to find the real original figure and the final image.

Clue 1: The first image has been translated 3 units to the left and 2 units up from the original figure.

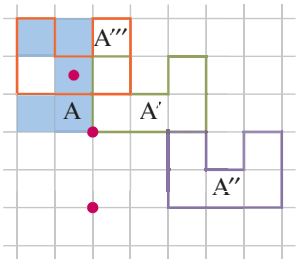
Clue 2: The second image has been translated 6 units to the right and 5 units down from the first image.

Clue 3: The third image has been translated 8 units to the left and 10 units up from the second image.

Clue 4: The final image has been translated 1 unit to the left and 10 units down from the third image.

**9** Explain why a rotation of 90° clockwise can also be described as rotation of 270° anti-clockwise.

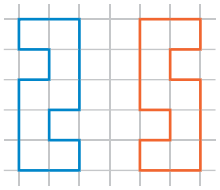
**10** Use your answer to question **9** to describe the rotations in parts **b** and **f** of question **2** in a different way.



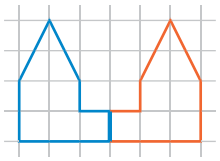

**11** The blue shape has been rotated 90° clockwise but around different points (shown on diagram at right). What is the difference between the rotations?

**12** Where would you place a mirror to form the given reflections? Copy the figures and draw in the mirror line.

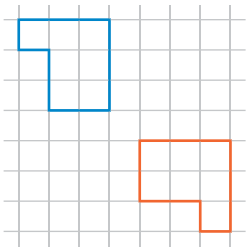
**a**



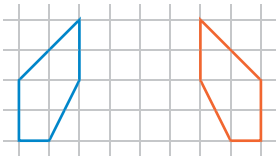
**b**



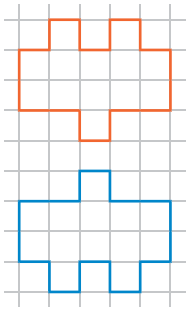
**c**



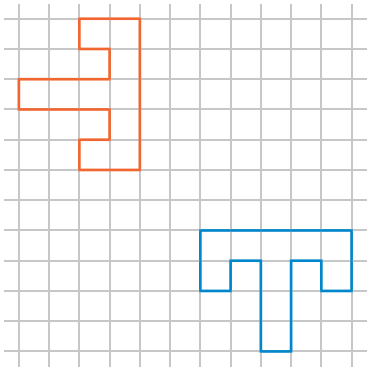
**d**



**e**

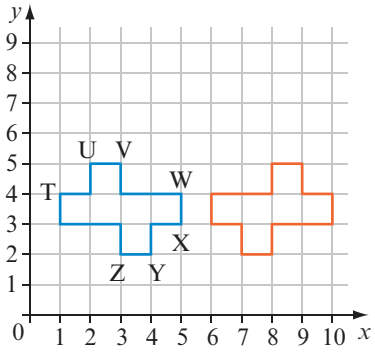


**f**

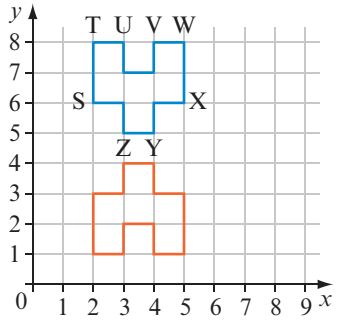


**13** Write the coordinates of the point V′ for each of these reflections.

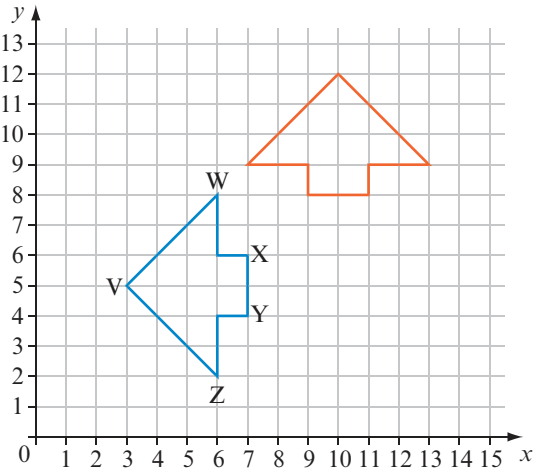
**a**




**b**




**c**



PROBLEM SOLVING AND REASONING

**14** Is reflecting a shape in a diagonal mirror the same as reflecting a shape in a horizontal mirror and then a vertical mirror? Explain.



**15** Describe how translation, rotation and reflection are shown in this Moroccan mosaic.



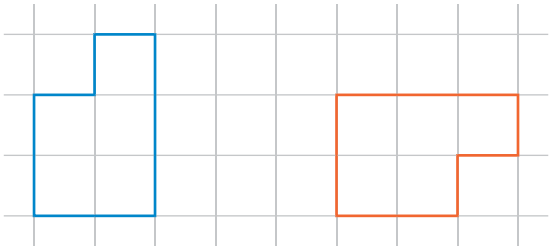
**16** The same transformation can be described in more than one way. For example, a translation could produce the same image as a reflection followed by another reflection.

- **a** Draw an example of this.
- **b** Find another way to describe a translation.
- **c** Find another way to describe:
  - **i** a reflection
  - **ii** a rotation.

**17** Using coordinates to map transformations has many practical applications, including software engineering for graphic design, video games and architecture. Use the Internet or other resources to find out how this is useful. Find other areas where it is also used.

CHALLENGE

**18** The shape on the right has been translated, rotated and then reflected. Can you work out the transformations? Describe them. (Hint: start with a reflection and work backwards.)

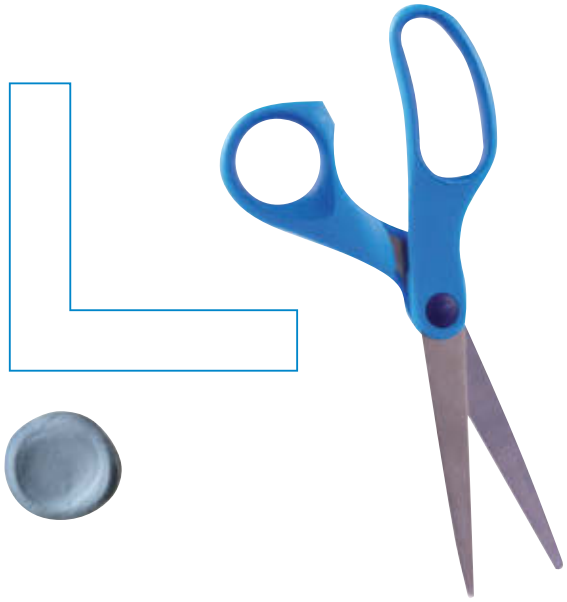


**Reflect**

How are transformations related to symmetry?

# 71 Performing transformations

## Start thinking!



1 What are the three isometric transformations?

2 How can you show the difference between a shape and its image?

3 Copy the shape 'L' shown at right and cut it out. Place it onto a piece of graph paper or 1-cm grid paper with some Blu-Tack. Use a pencil to draw the outline of the shape on the graph paper.

4 How can you change the shape but keep it the same?

5 Explain what translation means.

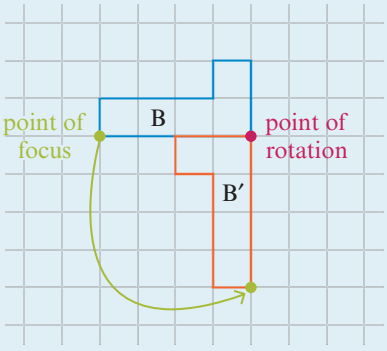
6 Translate the shape to somewhere else on the graph paper and draw its outline on the paper.

7 How many units has your shape moved up or down? Left or right? Be sure to start and finish in the same position; for example, the right corner. Copy and complete this statement: I translated the shape _____ units *up*/*down* and _____ units *left*/*right*.

**NOTE** Always remember to give your image a name that identifies it as a transformation, such as B′.

## KEY IDEAS

- Translation, rotation and reflection are isometric transformations.
- Transformations are easiest to do when the shape or object is on a grid.
- To perform a transformation, always identify a single point to concentrate on. This is called the point of focus. This is particularly important when it comes to rotations.
- For an object B, the image is named B′.

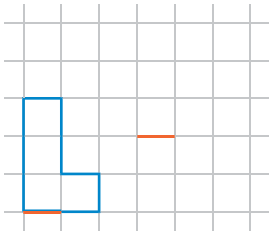


# EXERCISE 7I Performing transformations



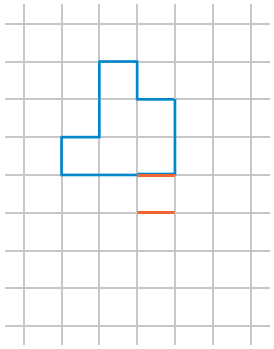
**1** Transform each shape according to the directions given. The image of part of each shape is shown in orange to help you get started.

**a**



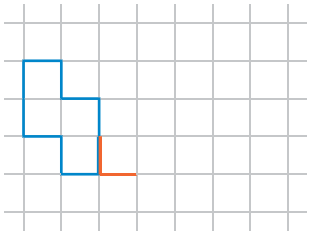
translate 2 units up and 3 units to the right

**b**



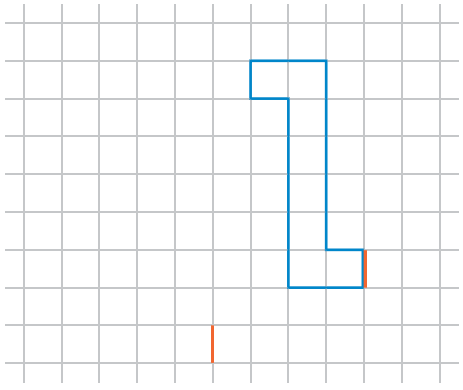
reflect in a horizontal mirror line

**c**



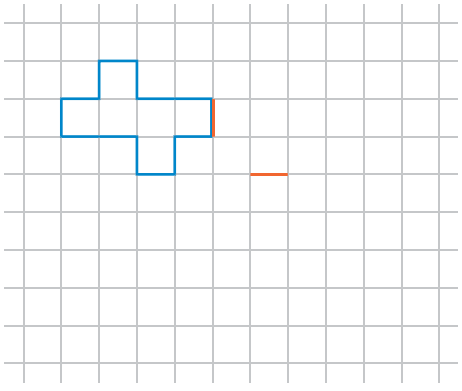
rotate 90° clockwise from the bottom right corner

**d**



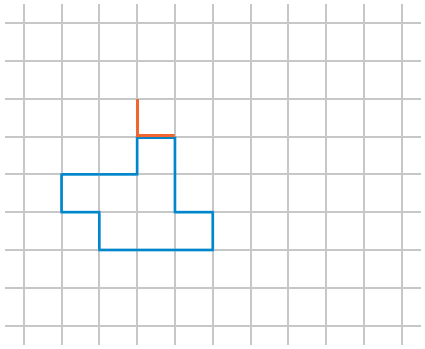
translate 2 units down and 4 units to the left

**e**



reflect in a diagonal mirror line

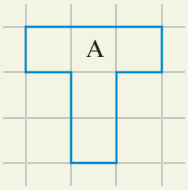
**f**



rotate 90° anticlockwise from the top right corner

## EXAMPLE 7I-1 Performing transformations

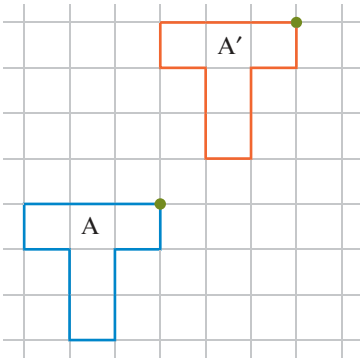
Translate this shape 4 units up and 3 units to the right.




### THINK

1. Choose a point of focus on shape A – you are translating up and to the right so pick the upper right corner.
2. Move this point 4 units up and then 3 units to the right.
3. Redraw the shape starting from the point which should be the upper right corner.
4. Call the new shape A′ to show that it is a transformation of the original shape.

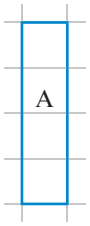
### WRITE




UNDERSTANDING AND FLUENCY

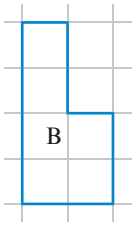
**2** Transform each shape according to the directions given.

**a**



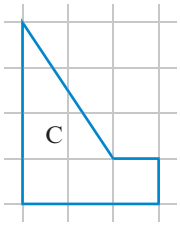

translate 3 units down and 2 units to the right

**b**



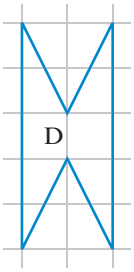

reflect in a vertical mirror line

**c**



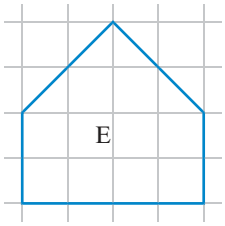

rotate 90° clockwise from the bottom right corner

**d**



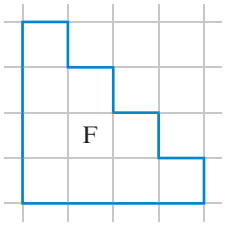

translate 1 unit up and 6 units to the left

**e**



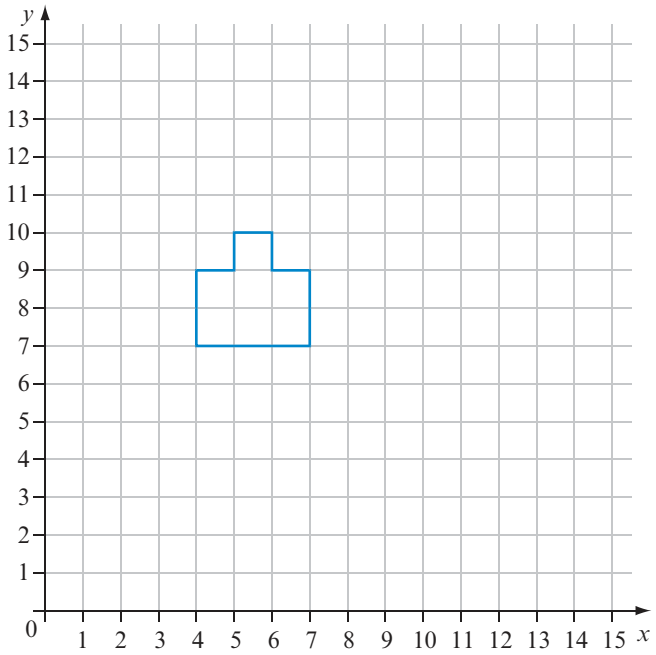

rotate 90° anti-clockwise from the top right corner

**f**




reflect in a horizontal mirror line below the shape

**3** Consider the shape on this grid.




**a** Copy the shape from the grid and place coordinates on each of its vertices.

**b** Complete each of the following translations and place coordinates on the vertices of the images.

**i** 2 units to the right, 3 units up

**ii** 5 units to the right, 6 units down

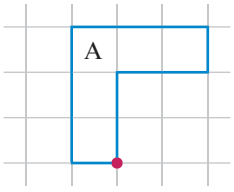
**iii** 2 units to the left, 3 units up

**iv** 3 units to the left, 4 units down

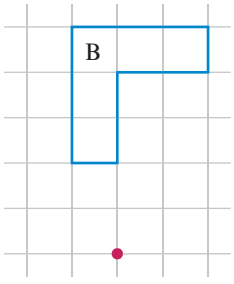
**NOTE** Start from the original shape each time.

**4** Rotate each shape 90° clockwise around the point given.

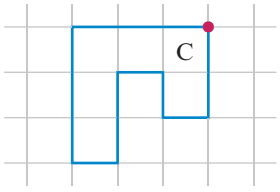
**a**



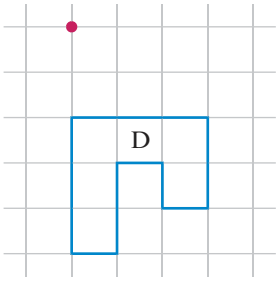

**b**



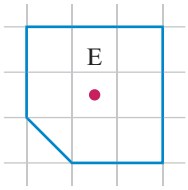

**c**



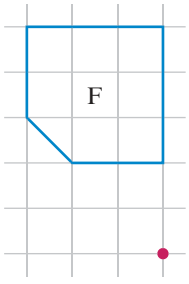

**d**




**e**




**f**

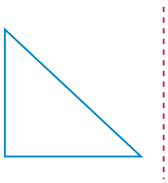



**5** Rotate the shapes in question **4** 90° anticlockwise around the point given.

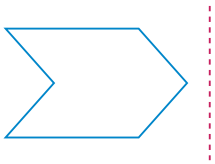
UNDERSTANDING AND FLUENCY

**6** Reflect these shapes in the mirror lines shown.

**a**



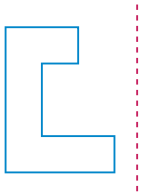
**b**



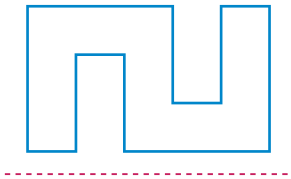
**c**



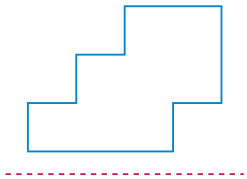
**d**



**e**



**f**

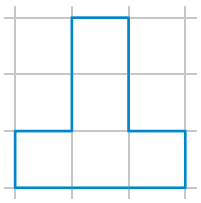


PROBLEM SOLVING AND REASONING

**7** Combinations of transformations can produce the same image. For example, 3 units right and 2 units up then 1 unit left and 1 unit up produces the same image as the transformation in question **3b** part **i**.
Write two examples of transformation combinations that produce the same image for each of the remaining parts of question **3b**.

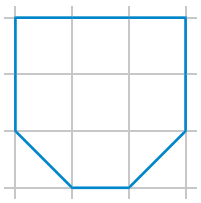
**8** Translate each shape according to the steps listed. State the overall translation.

**a**



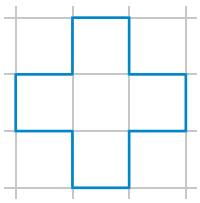
2 units right, 5 units up,
then 3 units right,
2 units down

**b**



5 units right, 1 unit up,
then 4 units left,
6 units down

**c**

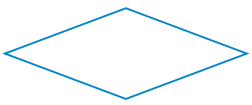


1 units left, 2 units up,
then 2 units left,
8 units up

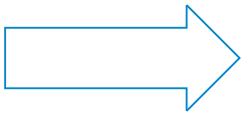
**9** Do you find it harder to rotate a shape around a point that is not on the shape? Explain.

**10** For each shape, use a series of rotations to form a flower-like figure similar to the one in Exercise 7H question **6** (page 391).

**a**



**b**



**c**



PROBLEM SOLVING AND REASONING

**11** You can use folded paper to simulate the effect of reflecting in more than one mirror.

**a** Take a sheet of A4 paper and fold it in half twice, as shown in the diagram.

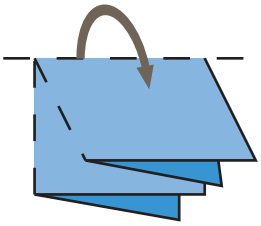
**b** Draw a shape in one of the corners, pressing down hard. It is better if the shape is not symmetrical.

**c** Unfold the paper and use your pencil to go over the imprints on the rest of the paper.

**d** Repeat the activity, but this time:

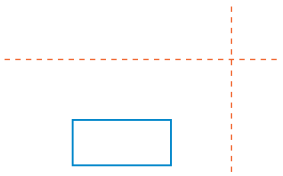
**i** use two shapes

**ii** before you draw the shapes, on a separate piece of paper draw a prediction of how you think they will be reflected on the paper after unfolding.

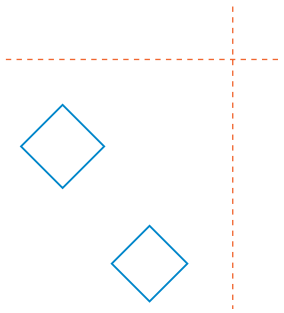


**12** Using folded paper or any other means, find the reflected images for the following.

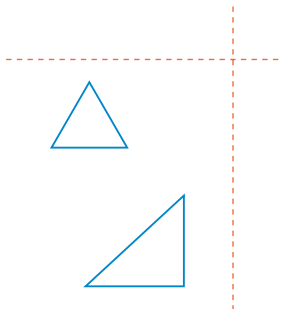
**a**



**b**



**c**

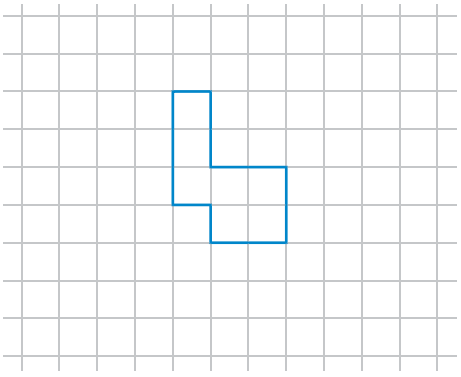


**13** Paper dolls are made using a series of joined reflections. Design and cut out your own paper images. How many different ways can you keep them joined?



CHALLENGE

**14** Transform this shape according to the steps listed.



**a** Translate 3 units to the left and 4 units down, then rotate 90° clockwise from the bottom right corner, then reflect in a vertical mirror line along its left side. Label this image A′.

**b** Translate 2 units to the right and 2 units up, then rotate 90° anti-clockwise from the bottom right corner, then reflect in a horizontal mirror line along its bottom. Label this image A″.

**c** Create your own set of steps involving at least one translation, one rotation and one reflection, in any order.

Draw each image on the same grid as the original shape.

**Reflect**

What common mistakes do you think people make when performing transformations?

# CHAPTER REVIEW

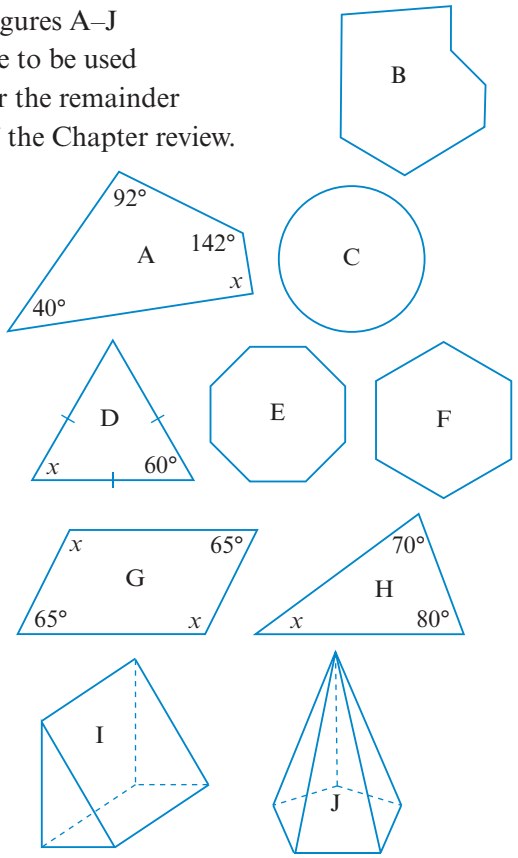
## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

| | | | |
|---|---|---|---|
| equilateral triangle | irregular | edge | symmetrical |
| isosceles triangle | convex | vertex | axis of symmetry |
| scalene triangle | concave | graph paper | rotational symmetry |
| exterior angle | tessellations | isometric dot paper | isometric transformation |
| quadrilateral | prism | perspective drawings | translation |
| adjacent | pyramid | vanishing point | rotation |
| polygon | polyhedron | net | reflection |
| regular | face | plan | mirror line |

## MULTIPLE-CHOICE

Figures A–J are to be used for the remainder of the Chapter review.




7A **1** What is the best description of figure H?

**A** acute-angled scalene triangle
**B** right-angled scalene triangle
**C** acute-angled isosceles triangle
**D** right-angled isosceles triangle

7B **2** What is the best description of figure G?

**A** quadrilateral **B** parallelogram
**C** rhombus **D** diamond

7C **3** Which 2D shape is not a polygon?

**A** A **B** B **C** C **D** D

7D **4** Which figure is a prism?

**A** B **B** E **C** I **D** J

7G **5** How many axes of symmetry does figure F have?

**A** 2 **B** 4
**C** 6 **D** infinite number

## SHORT ANSWER

7A **1** Find the size of angle $x$ in each shape.

**a** D **b** H

7B **2** Find the size of angle $x$ in each shape.

**a** A **b** G

7C **3** Give an appropriate name for each figure.

**a** A **b** B **c** E **d** F

7D **4** Describe the two 3D objects shown.

7E **5** Draw each of the following.

**a** a regular pentagon

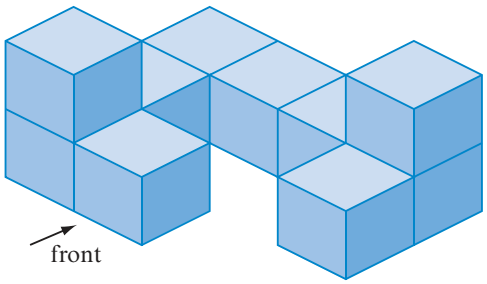
**b** a prism using the shape in figure E

**c** a pyramid using the shape in figure G

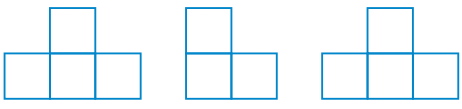
**d** a perspective drawing of the object in figure I

**e** a net for the object in figure J

7F **6** Draw a set of plans for this object.




7F **7** Use this set of plans to construct a matching 3D object.



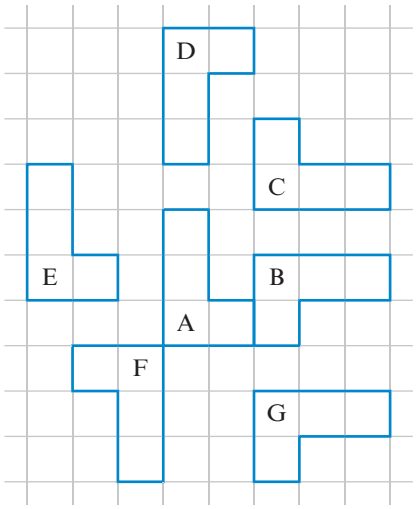

7G **8** Complete these steps for each of figures B, E and F.

**a** Copy the shape.

**b** Mark the axes of symmetry.

**c** State the number of axes of symmetry.

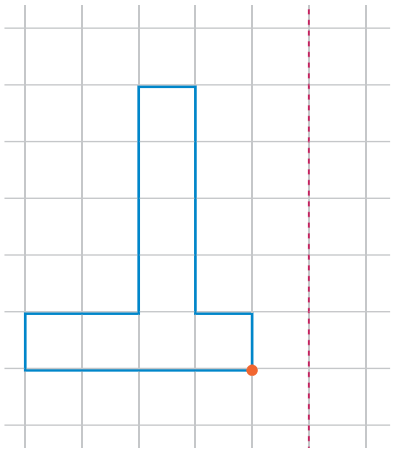
7H **9** Consider the shapes shown on this grid.




Describe the transformation(s) that produce each of shapes C to G if the original shape is:

**a** A **b** B.

7I **10** Consider the shape shown below.



**a** Copy the shape onto grid paper.

For parts **b**–**d**, start from the original shape each time.

**b** Translate the shape 3 units down and 2 units to the left.

**c** Reflect the shape in the mirror line shown.

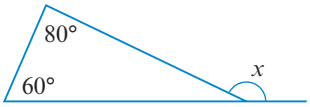
**d** Rotate the shape 90° anticlockwise around the point shown.

## NAPLAN-STYLE PRACTICE

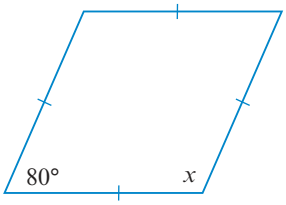
**1** What type of triangle has exactly two equal sides?

- ◯ right-angled triangle
- ◯ equilateral triangle
- ◯ scalene triangle
- ◯ isosceles triangle

**2** What is the size of angle $x$ in this diagram?




Questions **3** and **4** refer to this figure.

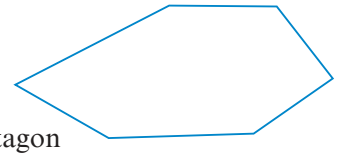



**3** What is the best description of this shape?

- ◯ quadrilateral
- ◯ rhombus
- ◯ parallelogram
- ◯ kite

**4** What is the size of angle $x$?

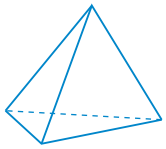
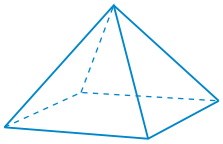
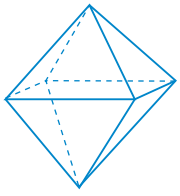
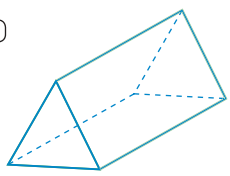
**5** What is the best description of this shape?



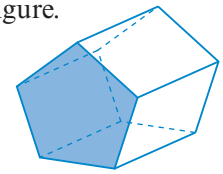
- ◯ irregular heptagon
- ◯ irregular hexagon
- ◯ irregular kite
- ◯ irregular concave hexagon

**6** How many sides does a decagon have?

**7** Which of these is a triangular prism?

- ◯ 
- ◯ 
- ◯ 
- ◯ 

Questions **8** and **9** refer to this figure.

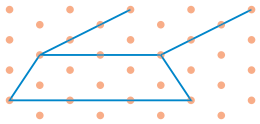


**8** What 2D shape forms the base of this object?

**9** What is the best name for this object?

- ◯ pentagonal prism
- ◯ pentagonal pyramid
- ◯ triangular prism
- ◯ rectangular pyramid

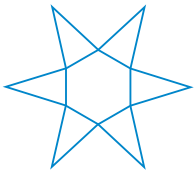
**10** When completed, what will this object be?



- ◯ parallelogram-based prism
- ◯ rectangular pyramid
- ◯ trapezoidal prism
- ◯ rectangular prism

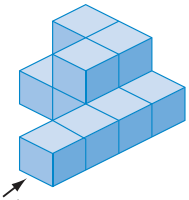
**11** To draw an octagon using the circle method, into how many parts should you divide the circle?

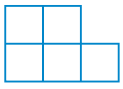
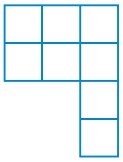
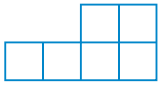
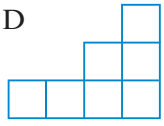
**12** Which 3D object does this net represent?



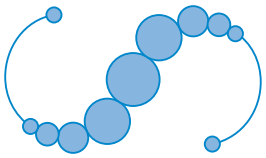
- ◯ triangular prism
- ◯ hexagonal prism
- ◯ rectangular pyramid
- ◯ hexagonal pyramid

**13** Which figure best represents the top view of this object?




- ◯ A 
- ◯ B 
- ◯ C 
- ◯ D 

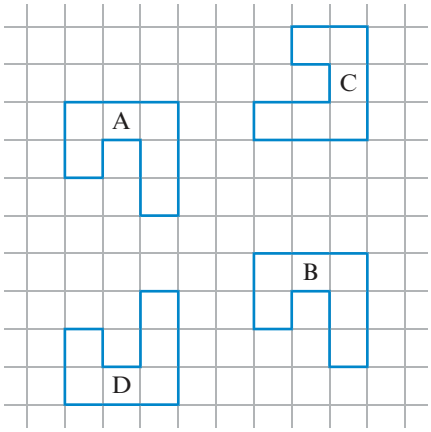
Questions **14** and **15** refer to this figure.



**14** How many axes of symmetry does the object have?

**15** What is the order of rotational symmetry for the object?

Questions **16**–**19** refer to this figure. Shapes B, C and D are all transformations of shape A.




**16** How has shape A been translated to produce shape B?

- ☐ 2 units right and 1 unit down
- ☐ 2 units right and 4 units down
- ☐ 5 units right and 4 units down
- ☐ 5 units left and 4 units up

**17** To produce shape C, shape A has been rotated in a clockwise direction by

**18** What has been done to shape A to produce shape D?

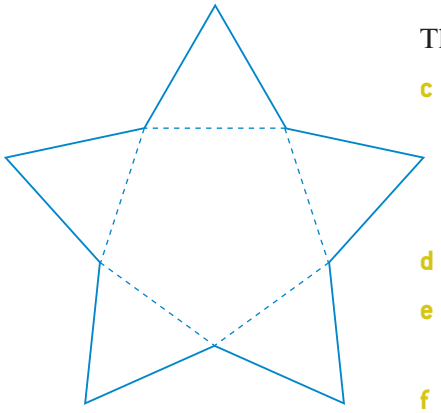
- ☐ reflected in a vertical mirror line
- ☐ translated 2 units down
- ☐ rotated 180° anti-clockwise
- ☐ reflected in a horizontal mirror line

**19** A fifth shape (shape E) was drawn onto the grid. It was produced by translating shape A 4 units right and 3 units down, then 2 units right and 1 unit up, and finally 3 units left and 2 units down. The total translation can be described as follows.

- ☐ 3 units left and 2 units down
- ☐ 9 units left and 6 units down
- ☐ 9 units right and 6 units up
- ☐ 3 units right and 4 units down

## ANALYSIS

Look at the shape shown.



**a** Give it a name according to the number of sides and size of its angles.

**b** How many axes of symmetry does it have?

This is actually a net for a 3D object.

**c** What individual shapes make up the net? Describe them accurately. You may like to measure the lengths of the sides with a ruler and the size of the angles with a protractor.

**d** What 3D object does this net make?

**e** Using graph or isometric paper, draw what the object will look like when it is constructed.

**f** Construct the object. You can make it as big as you like.

# CONNECT

## Lamp design

You are to design a lamp using your knowledge of 2D shapes and 3D objects. Your lamp should include at least two different 3D objects, and the lampshade must be designed with a tessellation of at least three different shapes.







### Your task

To design your lamp, follow these steps.

- Decide what 2D shapes and 3D objects will make up your lamp.
- Choose an appropriate tessellation that is colourful and attractive to cover either the base or lampshade.
- Draw a diagram of your lamp using graph or isometric dot paper.
- Draw a set of plans for the lamp.
- Construct a model of the lamp using a series of nets.







You will need:

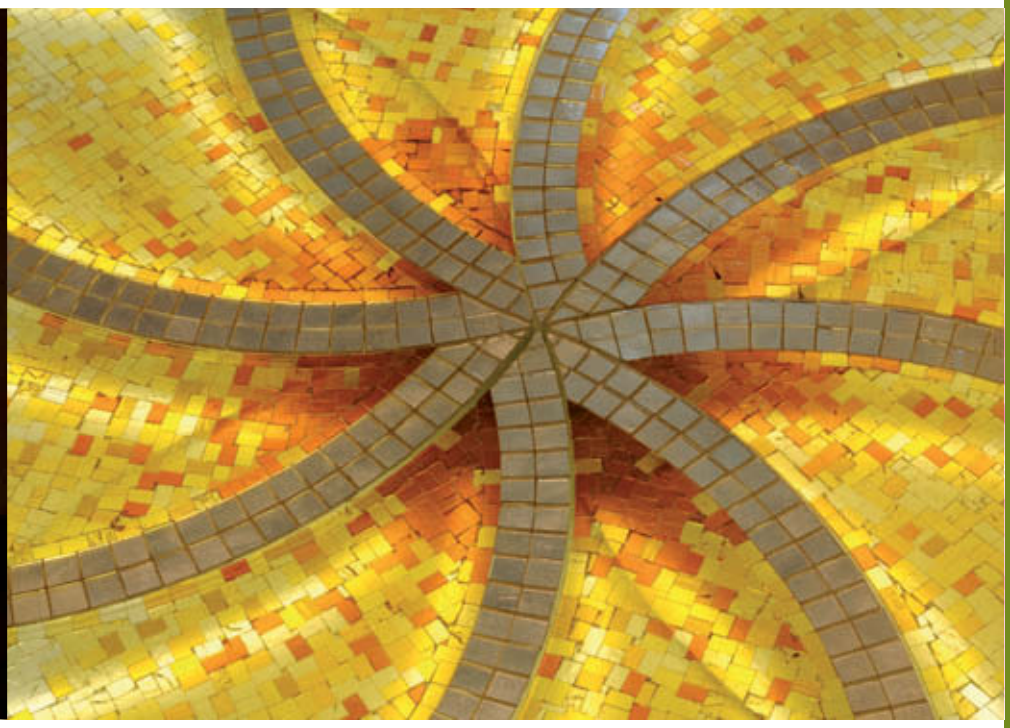
- paper
- scissors
- coloured pencils or textas
- glue or sticky tape

Complete the 7 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a set of blueprints and a model
- a PowerPoint presentation
- a video
- other (check with your teacher).





# 8 TIME, MASS AND TEMPERATURE

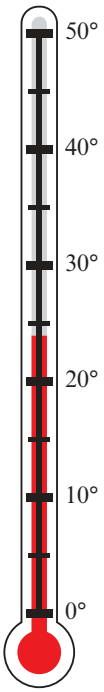

8A Reading scales
8B Estimation and accuracy
8C Understanding time
8D Working with time
8E Reading time
8F Time zones
8G Understanding mass
8H Understanding temperature




**ESSENTIAL QUESTION**

*How might knowledge of time, mass and temperature be important?*

8A **1** The thermometer at right measures the temperature in a classroom. What temperature does it show?



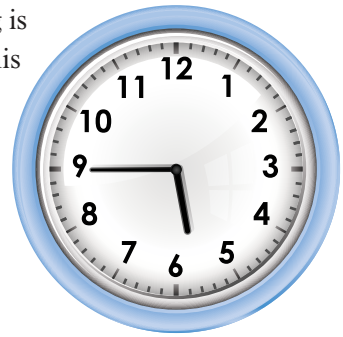

8C **2** What units would you use to measure the time it takes to run 100 m?

**A** milliseconds
**B** seconds
**C** minutes
**D** hours

8C **3** How many seconds in a minute?

8C **4** How many hours in a day?

8C **5** What time in the morning is shown on this clock face?




8C **6** The clock below displays the time in 24-hour time.




What time is this in 12-hour time?

8D **7** How much time has passed between 10.00 am and 5.30 pm?

**A** 5 hours 30 minutes
**B** 7 hours 30 minutes
**C** 4 hours 30 minutes
**D** 5 hours

8D **8** How many days between 17 February and 13 April in a leap year?

**A** 54 **B** 55 **C** 56 **D** 57

Questions **9** and **10** refer to this section of a train timetable.

| | | | |
|---|---|---|---|
| **Perth** | 0658 | 0728 | 0758 |
| **City West** | 0700 | 0730 | 0800 |
| **West Leederville** | 0702 | 0732 | 0802 |
| **Subiaco** | 0704 | 0734 | 0804 |

8E **9** What time does the first train leave Perth?

8E **10** How long does it take the second train to get from City West to Subiaco?

8G **11** What should you do to convert from grams to kilograms?

**A** multiply by 1000
**B** divide by 1000
**C** multiply by 100
**D** divide by 100

8G **12** Which of these masses is the heaviest?

**A** 400 g **B** 1 kg
**C** 90 g **D** 0.3 kg

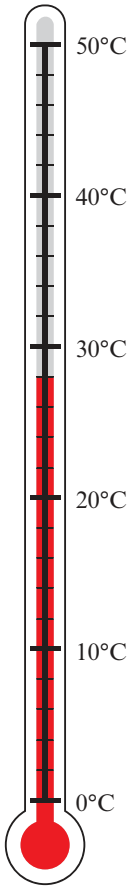
8H **13** The maximum daytime temperature is 26°C. Overnight the temperature drops to 9°C.
How much did the temperature drop by?

8H **14** Which of these temperatures is the coldest?

**A** 1°C **B** –4°C
**C** 23°C **D** –10°C

# 8A Reading scales

## Start thinking!




Look at this thermometer.

1 What is the major **interval** on the scale?

2 Which two numbers is the temperature between?

3 What do you estimate the temperature is? Why?

4 How many minor marks are there in between the major marks?

5 How many minor intervals are there between the major marks?

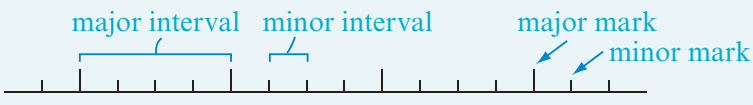
6 Copy and complete: The major interval of _____ °C is divided into _____ parts by the minor marks. This means that the minor interval is worth _____ ÷ _____ = _____ °C.

7 What is the temperature recorded on the thermometer?

8 Explain why somebody might think that the temperature is recorded as 20.4°C.

## KEY IDEAS

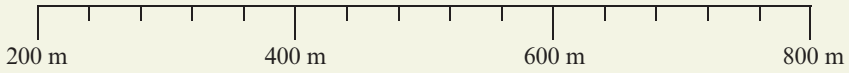
- When working with scales:
  1. first identify the major interval (the interval between two major marks)
  2. count the number of intervals in between the major marks (not the number of minor marks: remember, these are different)
  3. divide the major interval by the number of minor intervals to find the minor interval
  4. use the scale to record the measurement required.



# EXERCISE 8A Reading scales

## EXAMPLE 8A-1 Understanding intervals

Consider this scale.




a What is the major interval?
b How many minor intervals are there between each major mark on the scale?
c What is the minor interval?

### THINK

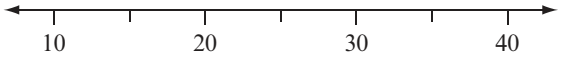
a Identify the major interval on the scale.
b Count the number of intervals between two major marks.
c Divide the size of the major interval by the number of minor intervals.

### WRITE

a major interval = 200 m
b There are five minor intervals between each major interval.
c minor interval = 200 ÷ 5
= 40 m

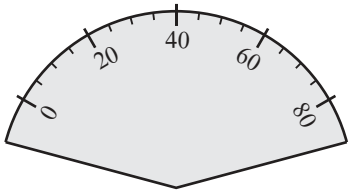
UNDERSTANDING AND FLUENCY

**1** Consider this scale.



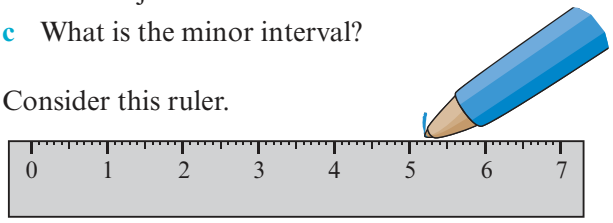

a What is the major interval?
b How many minor intervals are there between each major mark on the scale?
c What is the minor interval?

**2** Consider this scale.




a What is the major interval?
b How many minor intervals are there between each major mark on the scale?
c What is the minor interval?

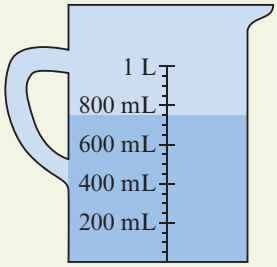
**3** Consider this ruler.




a What is the major interval?
b What is the semi-major interval? (Hint: here it is half the major interval.)
c What is the minor interval?
d What is the measurement recorded on the scale?

## EXAMPLE 8A-2 Reading scales

How much water is in the jug?




### THINK

1. Identify the major interval on the scale.
2. Work out the minor interval. There are four minor intervals in between each major mark.
3. Locate where the water level is on the scale. Which major mark is it above? How many minor marks higher is it than this major mark? (Alternatively, subtract one minor interval of 50 mL from the higher major mark of 800 mL.)

### WRITE

major interval = 200 mL

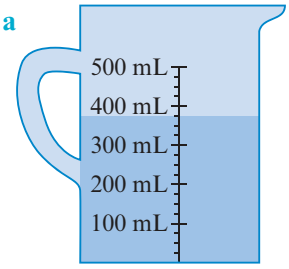
minor interval = 200 mL ÷ 4
= 50 mL

water level = 600 mL + 3 × 50 mL
= 750 mL

UNDERSTANDING AND FLUENCY

**4** Find the measurement shown on each scale.

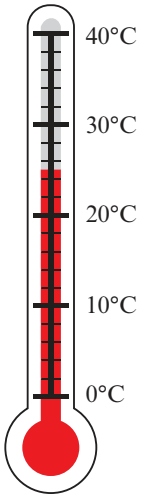
**a**




**b**




**c**

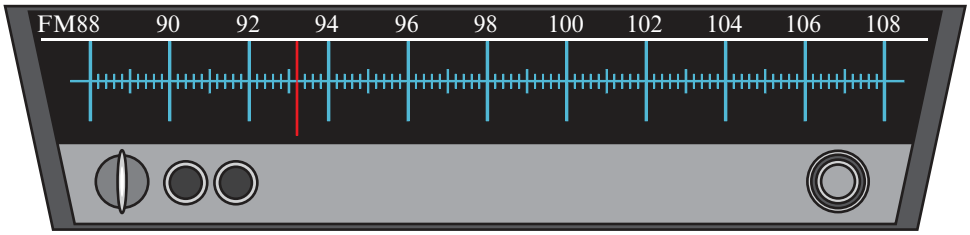



**5** **a** Copy the scales from questions **1** and **2** and record a measurement of 25 on each of them.

**b** Do you have to estimate for either measurement? Explain.

UNDERSTANDING AND FLUENCY

**6** Another type of scale is the one you use to tune the radio (though these are displayed digitally now). Below is a picture of an FM tuning scale. Place these stations on the tuning scale.

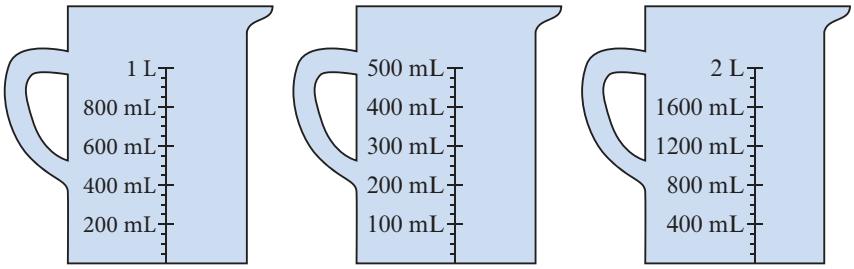



**a** Triple J 107.5FM
**b** Fox FM 101.9FM
**c** Nova 100 100.0FM
**d** Triple M 105.1FM
**e** Syn Radio 90.7FM
**f** your favourite radio station

PROBLEM SOLVING AND REASONING

**7** A recipe calls for the use of 750 mL of milk.

**a** Which of these jugs would you use to measure the milk?




**b** Copy your chosen jug and mark where 750 mL would be.

**c** The recipe asks you first to use 250 mL of milk. How much milk is left in the jug?

**d** Draw this new level on your jug.

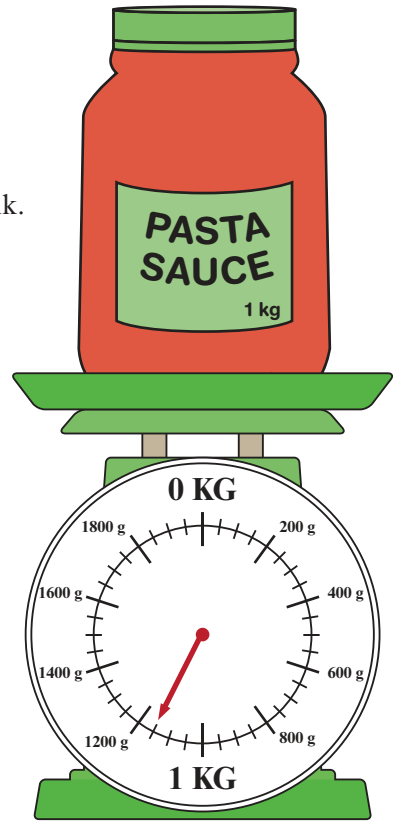
**e** You are then asked to use a further 75 mL of milk.

**i** How much is now left in the jug?

**ii** Draw this new level on your jug.

**iii** Explain why it is hard to be accurate.

**8** Look at this set of scales.




**a** What is the measurement shown on the scales?

**b** Is this in grams or kilograms?

**c** If you know that the jar holding the sauce has a mass of 200 g, what is the mass of the sauce inside the jar?

**d** The jar says that it holds 1 kg of pasta sauce. Does it? Explain how you know.

**9** This photo shows a speedometer from a car. It measures how fast the car is going.



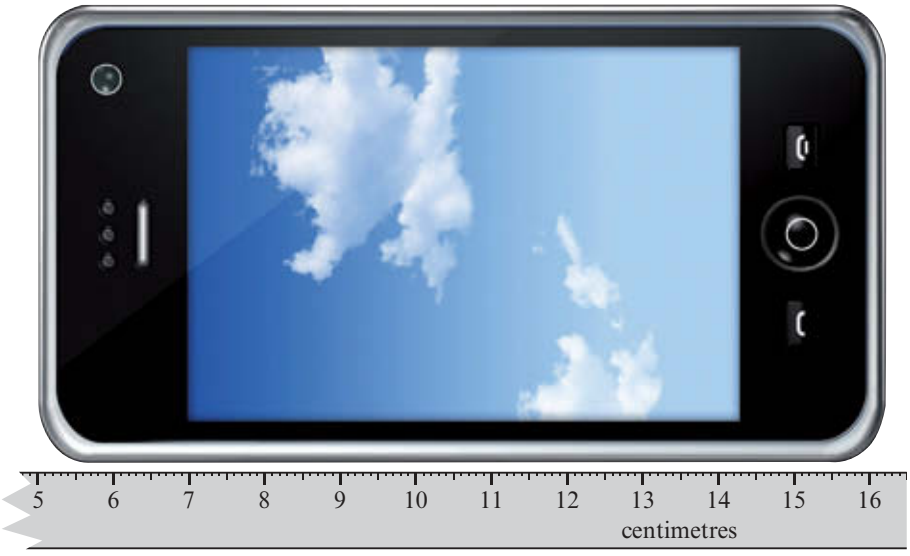

- **a** What are the major intervals (the ones with numbers)?
- **b** What are the semi-major intervals?
- **c** What are the minor intervals?
- **d** What speed is shown on this speedometer?
- **e** If the speed drops back three minor intervals, what is the new speed?

The car decelerates as it moves into a different speed zone and ends up with half the speed shown on the speedometer.

- **f** What is the new speed of the car?
- **g** Copy the speedometer and show this new speed.

**10** A classmate wanted to know whether his mobile phone would fit into the pocket of his bag, but he only had a broken ruler to measure with (shown below the phone).

- **a** What is the lower measurement for the phone on the ruler?
- **b** What is the higher measurement?
- **c** How can you work out how long the mobile phone is?
- **d** Find how long the mobile phone is. Write your answer in both millimetres and centimetres.
- **e** If the length is approximately twice that of the width, write the width of the phone.
- **f** If the pocket for the phone measures 7 cm by 10 cm, does the phone fit in it? Explain your answer.




**11** Draw your own scales to represent measurements that range from:

**a** 0 L to 2.5 L **b** 0 mm to 30 mm **c** 0°C to 60°C **d** 0 kg to 40 kg.

Think carefully about what to have as both your major and minor intervals.

PROBLEM SOLVING AND REASONING

**12** A barometer measures air pressure, which can tell you about the weather. Air pressure is measured in millibars (shown on the inner scale of this barometer).




- **a** What measurement is shown on the barometer?
- **b** What does this tell you about the weather?
- **c** If the air pressure dropped by 50 millibars, what would be the new measurement?
- **d** What might the weather be like at this new pressure?
- **e** What level of air pressure would you expect to see in the middle of a storm?

**13** Look at the measuring flask shown.




- **a** Why are the marks unevenly spaced?
- **b** Give an estimate of how much liquid is in the flask.

CHALLENGE

**14** The Richter scale is used to measure the strength of earthquakes. It is different from many other types of scale. Rather than each interval being equal (e.g. goes up in 10s), it uses multiplication so that each interval is different. A measurement of 2.0 on the Richter scale is 10 times larger than a measurement of 1.0, and a measurement of 3.0 on the Richter scale is 100 times larger than a measurement of 1.0. Investigate the Richter scale and write an explanation for your classmates about how the scale works. You might like to include examples of real earthquakes that have occurred in history.



**Reflect**

How are minor and major marks important in determining a scale measurement?

# 8B Estimation and accuracy

## Start thinking!




**1** Estimate the length of this iPod Shuffle.

**2** Use a ruler to measure its length.

**3** Describe the **accuracy** of your estimate.

**4** As a class, record all the measurements in a large table.

**a** What is:

**i** the smallest measurement? **ii** the largest measurement?

**b** Why might you get different measurements for the same object?

**5** Use a 1-m ruler to measure the length of the iPod (to the nearest minor mark on the ruler).

**6** Add a column to your class table and record your new results.

**a** What is:

**i** the smallest measurement? **ii** the largest measurement?

**b** Do these results 'agree' more closely? Why do you think this is?

**c** Are these results more accurate? Why or why not?

**7** Would you use a 1-m ruler to measure the length of:

**a** this iPod? **b** a car? **c** a walking track?

Explain the differences in your answers.

**8** Why is it important to have an appropriate measuring device to take accurate measurements?

## KEY IDEAS

- Always estimate before taking a measurement.
- If a recorded measurement is accurate it is very close to the exact measurement of an object.
- Always use a measurement tool that allows you to be as accurate and reasonable as possible.

# EXERCISE 8B Estimation and accuracy

## EXAMPLE 8B-1 Choosing appropriate devices and units

To measure your head circumference:

a what is the appropriate unit to measure in?
b what is an appropriate measuring device to use?

### THINK

a Head circumference is a reasonably small thing to measure, though not tiny. Which unit would be appropriate?

b Consider the unit to be used (cm) and the shape of the object (curved). What device would make this easiest?

### WRITE

a Head circumference is best measured in centimetres.

b A measuring tape would be the best device to use.

UNDERSTANDING AND FLUENCY

**1** For each object, select the appropriate unit and measuring device from the lists provided.

| | Object | Units | Measuring device |
|---|---|---|---|
| a | length of computer | mm, cm, m, km | 30-cm ruler, 1-m ruler, measuring tape, 5-cm ruler |
| b | your mass | mg, g, kg, tonne | 2-kg scales, bathroom scales, truck scales |
| c | winning time of 100 m race | ms, s, m, h, d | sundial, clock, stopwatch, watch |
| d | volume of cola in a bottle | mL, L, kL, ML | measuring cup, 2-L water jug, 10-kL water tank |
| e | length of tennis court | mm, cm, m, km | 30-cm ruler, measuring tape, trundle wheel, 1-m ruler |
| f | time taken to get to school | s, min, h, d, w | sundial, hourglass, stopwatch, watch |
| g | mass of chocolate bar | mg, g, kg, tonne | 2-kg scales, kitchen scales, truck scales |

**2** For each object, choose:

i an appropriate unit to measure in
ii an appropriate measuring device.

a length of your fingernails
b volume of milk in a cooking recipe
c time taken to watch a film
d mass of a smart phone
e length of your arm span
f mass of a grain truck

## EXAMPLE 8B-2 Estimating and measuring length

Estimate and then measure the length of this pencil. Comment on the accuracy of your estimate.



| THINK | WRITE |
|---|---|
| **1** The pencil looks to be more than 5 cm but less than 10 cm. Estimate about 8 cm. | estimate = 8 cm |
| **2** Use a ruler with millimetre markings as you are dealing with a smaller measurement. Line up the ruler so that one end of the pencil is at '0' and read the measurement at the other end of the pencil. | The pencil measures 8.8 cm or 88 mm. |
| **3** Write a sentence to comment on the accuracy of your estimate. | The estimate of 8 cm was fairly close to the actual measured length of 8.8 cm. |

UNDERSTANDING AND FLUENCY

**3** Estimate then measure each object with a ruler, giving your answers in both centimetres and millimetres. Comment on the accuracy of your estimates.

**a**



**b**



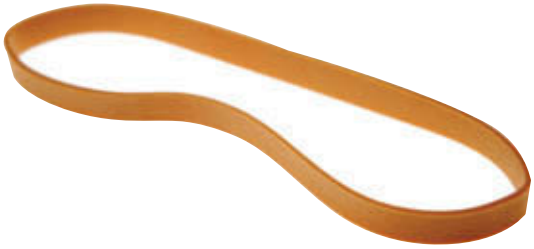
**c**



**d**



**e**



**f**



**g**



## EXAMPLE 8B-3 Checking measurements are reasonable

Consider this list of measurements: 169, 168, 172, 167.25, 171, 165, 164.
Which measurement do you think does not belong in the list? Explain.

| THINK | WRITE |
|---|---|
| **1** Identify which measurement is distinctly different from the rest. | 167.25 does not belong in the list. |
| **2** Give a reason for your answer. | The measurement is too precise to match the rest of the data. |

UNDERSTANDING AND FLUENCY

**4** For each of these lists of measurements:
- **i** identify which one may not belong
- **ii** give a reason why you think it doesn't belong.

**a** 13.1, 13.2, 13.4, 13.0, 12.9, 13.0005
**b** 7, 8, 6, 13, 7, 9, 7
**c** 43.74, 43.76, 437.5, 43.76, 43.75, 43.75
**d** 900, 1000, 800, 912, 1100, 700, 900, 800, 1000
**e** 18.25005, 18.2, 18.25001, 18.24999, 18.25010, 18.24996, 18.25003

**5** Find five objects of various lengths around the classroom.
**a** Estimate the length of each object.
**b** Measure the length of each object as accurately as you can.
**c** How accurate are your estimates?
**d** Use a different measuring device and comment on your results.
**e** Compare your results with a classmate. Are his or her results more accurate or reasonable than yours? Why or why not?

**6** Draw up a table like this one and sort these measuring devices into it. There may be more than one device in each cell.

| | Time | Mass | Temperature |
|---|---|---|---|
| Small measurements | | | |
| Medium measurements | | | |
| Large measurements | | | |

**a** stopwatch
**b** basic thermometer
**c** oven temperature scale
**d** wall clock
**e** bathroom scales
**f** atomic clock
**g** digital thermometer
**h** calendar
**i** kitchen scales
**j** 2-g balance scales
**k** weigh station



**7** For the following measurements, give:

- **i** an estimate
- **ii** an example of an appropriate measuring device.

**a** length of a basketball court

**b** time taken to run a marathon

**c** mass of a mobile phone

**d** temperature on a warm summer's day

**8** For the measurements listed in question **7**, explain whether the measurement given below is reasonable or not.

**a** 27.9965 m **b** 4 hours 35 minutes

**c** 0.1 kg **d** 15.2°C

**9** For the measurements in question **8** that are not reasonable, suggest a better measurement.

**10** Is it always better to be highly accurate? Explain.

**11** How is measuring temperature different from measuring time and mass?

**12** Look at each object and complete the following questions.

- **i** Record the measurement.
- **ii** Give a minimum and maximum value that you would accept as a measurement.
- **iii** Explain the reasoning behind your minimum and maximum values.

**a**



**b**




**c**




PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

**d**



**e**




**13** A measurement has been given below for each of the objects in question **12**.

**i** Does this measurement fall inside your minimum and maximum values?

**ii** Do you think that this measurement is reasonable? Explain.

**a** 75 kg **b** 17.355°C **c** 150 g **d** 4.12 pm **e** 35 mL

**14** What effect do minor intervals have on your ability to estimate?

**15** A common piece of advice is to measure twice when you are taking important measurements. Why might this be an advantage?

CHALLENGE

**16** When you are deciding which measuring device to use, you should consider its limit of accuracy. This tells you the range of values that the actual measurement lies within. To find the limit of accuracy of a device, find half of the minor interval on the scale. Consider the millilitre scale on the measuring cup shown.

**a** What is the major interval?

**b** What is the minor interval?

**c** What is half of the minor interval? This is the limit of accuracy.

**d** What is the volume of water in the cup?

**e** Subtract the limit of accuracy from your answer to part **d** to find the lower value of the possible range.

**f** Add the limit of accuracy to your answer to part **d** to find the upper value of the possible range.

**g** State the range of values that the actual volume of water lies within.




**17** Find the limit of accuracy and hence the range of values for the measurements shown on the instruments in question **12**.

How do these compare with your answers to question **12**?

**Reflect**

Why is it important to estimate before taking a measurement?



# 8C Understanding time

## Start thinking!

**1** **a** How many seconds in:
**i** 1 minute? **ii** 2 minutes? **iii** 3 minutes?
**b** Describe how to convert from minutes to seconds.

**2** **a** How many minutes in:
**i** 1 hour? **ii** 2 hours? **iii** 3 hours?
**b** Describe how to convert from hours to minutes.

**3** Explain what you need to do to convert to a smaller unit.

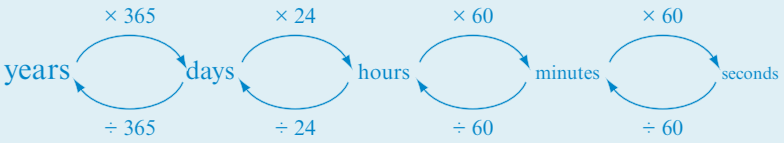
**4** **a** How many minutes in:
**i** 60 seconds? **ii** 120 seconds? **iii** 180 seconds?
**b** Describe how to convert from seconds to minutes.

**5** **a** How many hours in:
**i** 60 minutes? **ii** 120 minutes? **iii** 180 minutes?
**b** Describe how to convert from minutes to hours.

**6** Explain what you need to do to convert to a larger unit.

## KEY IDEAS

- There are 60 seconds in 1 minute.
  There are 60 minutes in 1 hour.
  There are 24 hours in 1 day.
  There are 365 days in 1 year.
  There are 366 days in a leap year.
- To convert to a larger unit, divide by the conversion factor. (There will be fewer of the larger unit.)
- To convert to a smaller unit, multiply by the conversion factor. (There will be more of the smaller unit.)




- In 24-hour time, the time is given as a four-digit number in 24 continuous hours. The day starts at 0000 (midnight) and finishes at 2359. Times after midday have 12 hours added to them. For example, 2 pm is 1400.

# EXERCISE 8C Understanding time

## EXAMPLE 8C-1 Converting time units in one step

Convert:

**a** 420 seconds into minutes **b** 2.5 hours into minutes.

### THINK

**a** To convert to a larger unit, divide by the conversion factor of 60. (60 seconds = 1 minute)

**b** To convert to a smaller unit, multiply by the conversion factor of 60. (60 minutes = 1 hour)

### WRITE

**a** 420 seconds
= (420 ÷ 60) minutes
= 7 minutes

**b** 2.5 hours
= (2.5 × 60) minutes
= 150 minutes

UNDERSTANDING AND FLUENCY

**1** Convert into minutes.

**a** 3 hours **b** 5 hours
**c** 10 hours **d** 120 seconds
**e** 600 seconds **f** 6000 seconds

**2** Convert each of these.

**a** 10 minutes into seconds
**b** 5 minutes into seconds
**c** 60 minutes into seconds
**d** 120 minutes into hours
**e** 1800 minutes into hours
**f** 240 minutes into hours

**3** Convert each of these.

**a** 30 minutes into hours
**b** 45 minutes into hours
**c** 20 minutes into hours
**d** 15 seconds into minutes
**e** 6 seconds into minutes
**f** 48 seconds into minutes



## EXAMPLE 8C-2 Converting time units in two steps

Convert 2 hours into seconds.

### THINK

1 First convert 2 hours into minutes. To convert to a smaller unit, multiply by the conversion factor of 60. (60 minutes = 1 hour)

2 Next convert the minutes into seconds. To convert to a smaller unit, multiply by the conversion factor of 60. (60 seconds = 1 minute)

3 Write your answer.

### WRITE

2 hours
= (2 × 60) minutes
= 120 minutes

= (120 × 60) seconds
= 7200 seconds

There are 7200 seconds in 2 hours.

**4** Convert each of these.

**a** 3 hours into seconds
**b** 10 hours into seconds
**c** 6 hours into seconds
**d** 1.5 hours into seconds
**e** 14400 seconds into hours
**f** 9000 seconds into hours

**5** How many:

**a** minutes in a day?
**b** hours in a week?
**c** seconds in a day?
**d** hours in a leap year?
**e** seconds in a week?
**f** hours in June?

## EXAMPLE 8C-3 Converting between 12- and 24-hour time

Convert:

**a** 9.20 pm into 24-hour time **b** Convert 1345 into 12-hour time.

### THINK

**a** 9.20 pm is after midday. Add 12 hours to 9.20 and write as a four-digit number.

**b** 1345 is higher than 1200, so the time must be after midday. Subtract 12 hours from 1345 and write pm after the time.

### WRITE

**a** 9.20 am + 12.00 hours
= 2120

**b** 1345 – 1200
= 1.45 pm

**6** Convert these times into 24-hour time.

**a** 2.30 am
**b** 4.15 pm
**c** 1.48 pm
**d** 11.11 pm

**7** Convert these times into 12-hour time.

**a** 1925 **b** 1200 **c** 2218 **d** 0958

**8** Write down the time displayed on these clocks in 12-hour time in words.

**a**



**b**



**c**




**d**




**e**



**f**




**9** What would be the best unit of time to use for each of these?

**a** the time it takes you to get to school
**b** the time it takes to read a book
**c** the time it takes an athlete to run a 100 m sprint
**d** the period of time a light bulb lasts
**e** the time it takes to finish your schooling
**f** the time it takes to wait for an online order



## EXAMPLE 8C-4 Converting time from decimal hours to hours and minutes

Convert 3.78 hours into hours and minutes.

| THINK | WRITE |
|---|---|
| **1** Write 3.78 as a sum of whole and decimal parts. | 3.78 = 3 hours + 0.78 hours |
| **2** Convert the decimal part to minutes by multiplying by 60. | = 3 hours + (0.78 × 60) minutes |
| **3** Write your answer correct to the nearest whole minute. | = 3 hours 47 minutes |

**10** Convert these into hours and minutes.

**a** 2.35 hours **b** 10.05 hours **c** 5.67 hours
**d** 8.48 hours **e** 18.984 hours **f** 33.296 hours

**11** Convert these into hours and minutes.

**a** 157 minutes **b** 451 minutes **c** 123.45 minutes
**d** 93.25 minutes **e** 763 000 seconds **f** 11 001 100 seconds

**12** **a** Convert 15 minutes into a fraction of an hour.
**b** Write this fraction as a decimal.
**c** Explain why 1.25 hours is not equivalent to 1 hour and 25 minutes.

**13** In question **12**, you may have automatically recognised that 15 minutes is $\frac{1}{4}$ of an hour. What other recognisable fractions of an hour do you know?

PROBLEM SOLVING AND REASONING

**14** To obtain your driver's licence in Victoria, you must do 120 hours of practice first.

**a** If you drive for 1 hour per week, how many weeks would it take?
**b** How long is this in years?
**c** If you drive for 2 hours per week, how many weeks would it take?
**d** How long is this in years?
**e** If you only have 6 months to get your licence, how many weeks is this?
**f** How many hours per week would you need to drive in order to make 120 hours practice in 6 months? Write your answer correct to two decimal places.
**g** Write your answer to part **f** in hours and minutes.
**h** Repeat parts **a**–**g** for the number of practice hours required in one of these states: NSW 100 hours, Qld 100 hours, SA 75 hours, Tas. 50 hours, WA 25 hours.



**15** How many:

**a** days have you been alive (including today)?
**b** hours of school do you attend a year?
**c** minutes of homework do you do a week?
**d** seconds of TV do you watch a day?

**16** You have some time to spare and decide to watch a movie. You have the following DVDs available.




**a** Convert all film times into hours and minutes.
**b** If you want to sit down for a minimum of 1.5 hours, which films could you watch?
**c** If you can only sit down for a maximum of 1.5 hours, which films could you watch?

PROBLEM SOLVING AND REASONING

**17** The devices below (a, b, c and d) can all be used to measure time. Match each device to its name (A, B, C or D) and the primary unit of time (i, ii, iii or iv) that it measures.

**a**



**b**



**c**




**d**




| | | | |
|---|---|---|---|
| **A** | sundial | **i** | seconds |
| **B** | stopwatch | **ii** | hours |
| **C** | egg timer | **iii** | days |
| **D** | calendar | **iv** | minutes |

**18** Represent 9.25 am on an analogue clock by following these steps.

- **a** Draw a clock face.
- **b** Draw the minute hand at 25 minutes. This is at the number 5.
- **c** Think carefully about where the hour hand will go. 9.25 am is almost halfway between 9 am and 10 am, so place the hour hand close to halfway between 9 and 10 on the clock face.

**19** Represent the following times on an analogue clock using the steps in question **18** to guide you.

**a** 3.15 pm **b** 11.50 am **c** 2.27 pm **d** 6.30 pm

CHALLENGE

**20** To accurately place the hour hand, use your understanding of minutes in an hour.

- **a** What fraction of an hour is 1 minute?
- **b** How many minor intervals are there between each hour?
- **c** Explain why, in relation to the hour hand, each minor mark relates to 12 minutes.
- **d** Represent the following times on an analogue clock.
  - **i** 7.25 pm **ii** 12.21 am **iii** 5.48 pm **iv** 10.09 am

**21** The clock at right has no numbers and only an hour hand. How can you use your skills in reading analogue clocks to estimate the time?



**Reflect**

How is the number 60 important when dealing with time?

# 8D Working with time

## Start thinking!

For this task, work in groups of four: two runners and two timers. Each group needs two stopwatches.

1 Choose somewhere in the school where you can safely run a distance of at least 50 m, such as an oval or basketball court.

2 Arrange yourselves so you have a runner and a timer at each end of your chosen distance. Each runner should estimate how long they think it will take them to run the required distance.

3 The first runner should start and *both* timers should start their stopwatches. When the first runner gets to the end, one timer should stop his or her stopwatch (without looking at it) and the second runner should run to the other end.

4 When the second runner gets to the end, the second timer should stop his or her stopwatch.

5 All team members should estimate the time it took each runner to run the distance and record their answer *before* looking at the timed stopwatches.




6 What was the time shown on the first stopwatch? How long did the first runner take?

7 What was the time shown on the second stopwatch?

8 How do you find out how long it took the second runner? Write this down.

9 Who was faster? Were your estimates accurate?

10 Explain how to calculate **elapsed time** when you have two different measurements.

## KEY IDEAS

- To calculate elapsed time, find the difference between the start time and the end time. You may find it easier to break the interval between the two times into smaller time intervals
- Always estimate your answer first.
- February has 28 days, or 29 days in a leap year.
  April, June, September and November have 30 days.
  All other months have 31 days.
  A leap year has 366 days instead of 365 days. Every fourth year is a leap year (unless it is divisible by 100 but not by 400).
- To calculate average speed, divide total distance by total time. For example, if a car travels 160 km in 2 hours, its average speed is $\frac{160}{2}$ km per hour or 80 km/h.

# EXERCISE 8D Working with time

## EXAMPLE 8D-1 Calculating elapsed time

A car leaves home at 11.37 am and arrives at its destination at 4.12 pm.
How long did it take to get there?

| THINK | WRITE |
|---|---|
| **Method 1** | |
| **1** Find the number of whole hours from 11.37 am to 4.37 pm. Count from 11 am until you reach 4 pm (5 hours). | time from 11.37 am to 4.37 pm<br>= 5 hours |
| **2** The time from 11.37 am to 4.12 pm will be less than 5 hours. Find the time difference between 4.12 pm and 4.37 pm. | time from 4.12 pm to 4.37 pm<br>= 37 minutes − 12 minutes<br>= 25 minutes |
| **3** Subtract the number of minutes from 5 hours to find the total time spent driving. Remember you are dealing with minutes, not decimal places. | total travel time from 11.47 am to 4.12 pm<br>= 5 hours − 25 minutes<br>= 4 hours 35 minutes |
| **Method 2** | |
| **1** Consider smaller time intervals between 11.37 am and 4.12 pm. Find the number of minutes from 11.37 am to the next whole hour. | time from 11.37 am to 12.00 pm<br>= 23 minutes |
| **2** Work out the number of hours from 12.00 pm to 4.00 pm. | time from 12.00 pm to 4.00 pm<br>= 4 hours |
| **3** Work out the number of minutes from 4.00 pm to 4.12 pm. | time from 4.00 pm to 4.12 pm<br>= 12 minutes |
| **4** Find the total time between 11.37 am and 4.12 pm by adding the individual times. | total travel time from 11.37 am to 4.12 pm<br>= 23 minutes + 4 hours + 12 minutes<br>= 4 hours 35 minutes. |

**1** Calculate the time difference between each pair of times.

**a** 4.00 pm and 9.00 pm
**b** 8.00 am and 11.00 pm
**c** 11.30 pm and 2.30 am
**d** 0900 and 1400
**e** 1330 and 2230
**f** 1600 and 2130

**2** Calculate the time difference between each pair of times.

**a** 5.27 pm and 7.45 pm
**b** 12.52 pm and 6.05 pm
**c** 6.43pm and 2.37am
**d** 0004 and 1931
**e** 1919 and 2222
**f** 0636 and 1421

**3** Find the time for each of these.

**a** 3 hours after 4.30 pm
**b** 6 hours 15 minutes after 11.00 am
**c** 1 hour 45 minutes after 2100
**d** 2 hours before 11.15 pm
**e** 4 hours before 3.20 pm
**f** 5 hours 15 minutes before 2230

## EXAMPLE 8D-2 Finding the difference in two dates

Find the number of days from 23 April until 10 October.

| THINK | WRITE |
|---|---|
| **1** Count the number of days to the end of April. There are 30 days in April. | 30 – 23 = 7 days |
| **2** List the number of days in each month between April and October. | May has 31 days<br>June has 30 days<br>July has 31 days<br>August has 31 days<br>September has 30 days |
| **3** State the number of days in October to be included. | 10 days from 1 to 10 October |
| **4** Add all the days together to get the total number of days from 23 April until 10 October. | 7 + 31 + 30 + 31 + 31 + 30 + 10<br>= 170 days |

UNDERSTANDING AND FLUENCY

**4** How many days are there from the first date until the second date in these pairs of dates?

**a** 13 April and 27 May
**b** 15 August and 19 December
**c** 17 May and 18 September
**d** 7 March and 2 October
**e** 3 July and 21 November
**f** 30 June and 4 January

Thirty days has September,
April, June and November.
All the rest have 31,
Excepting February alone,
And that has 28 days clear,
And 29 in a leap year.

**5** Find the date for each of these.

**a** 17 days after 6 January
**b** 42 days after 21 May
**c** 91 days after 19 November
**d** 18 days before 29 March
**e** 39 days before 8 August
**f** 24 days before 12 April

UNDERSTANDING AND FLUENCY

**6** A family takes a road trip from Melbourne to Perth. Their trip is plotted in the table below, showing when they leave, arrive and when they stop for lunch.

| | Day 1 | Day 2 | Day 3 | Day 4 |
|---|---|---|---|---|
| Departure time | 8.30 am | 8.00 am | 8.15 am | 8.20 am |
| Start lunch | 12.30 pm | 12.30 pm | 12.55 pm | 1.05 pm |
| Finish lunch | 1.00 pm | 1.00 pm | 1.40 pm | 1.30 pm |
| Arrival time | 7.30 pm | 6.00 pm | 7.45 pm | 7.15 pm |

**a** How long was their first day?

**b** How long was their lunch break on the first day?

**c** Use your answers to parts **a** and **b** to find how long they drove for on the first day.

**d** Find how long they drove for on the second, third and fourth days.

**e** How many hours driving did they do for the whole trip?

**f** If it is about 3430 km from Melbourne to Perth, find the average speed that the family drove.

**g** If the family also took two 15-minute breaks each day, how long does this mean they were actually driving for?

**h** Calculate their new average speed for the entire trip.



PROBLEM SOLVING AND REASONING

**7** I was born on 7 September 1998. One year and 213 days later, my sister was born. My brother was born 2 years and 3 days after that, and we got our pet dog on 25 July 2006. (Hint: remember to consider leap years.)

**a** When was my sister born?

**b** When was my brother born?

c How old was I when we got our dog?

**d** How much older am I than my brother?

**e** How old was my brother when we got our dog?

**f** How old will my sister be when the dog is 10 years old?

**8** Write your own problem like that in question **7** and swap it with a classmate for them to solve.

**9** The average walking speed is about 4 km/h.

- **a** How far would you walk in $\frac{1}{2}$ hour?
- **b** How far would you walk in 3 hours?
- **c** How long would it take to walk 8 km?
- **d** If you wanted to cover 20 km in 4 hours, at what average speed would you have to walk?

**10** Sixteen people tried out for the school relay team. The time shown on a stop watch at the end of each leg of the relay was recorded. The results (in seconds) are shown in the table.

| | Trial 1 | Trial 2 | Trial 3 | Trial 4 |
|---|---|---|---|---|
| First person | 15.1 | 14.9 | 16.5 | 15.1 |
| Second person | 32.8 | 32.0 | 28.9 | 30.9 |
| Third person | 46.3 | 48.9 | 45.1 | 43.7 |
| Fourth person | 60.0 | 63.6 | 62.6 | 58.9 |



- **a** Explain why the times increase with each person.
- **b** How can you calculate the time for each person?
- **c** Calculate the time for each person and put it into a separate table.
- **d** Who was the fastest person for each group?
- **e** If you were going to pick four people to be in the relay team, would you pick the fastest out of each group? Explain.
- **f** List the four people you would choose to have in your relay team.

**11** A new mobile phone states that its battery lasts for 220 minutes of talking time.



- **a** How long is this in hours and minutes?
- **b** If you talked on your phone for 10 minutes per day, how long would it last?
- **c** If you talked on your phone for 20 minutes per day, how long would it last?
- **d** If you wanted your phone to last 6 days between charges, how long could you talk on the phone each day?
- **e** Why do these answers not really reflect real battery life? What other factors are there to consider?

PROBLEM SOLVING AND REASONING

**12** You are waiting impatiently for 7 pm so that you can meet a friend you haven't seen in a long time. You look at the clock four times while waiting.

**a** Calculate how much time passed between each clock reading.

**b** For each clock reading, calculate how much time there is until 7 pm.



CHALLENGE

**13** Countdown timers can be used to save time calculating how long until a particular event. They are usually represented in digital form showing days, hours, minutes and seconds as dd:hh:mm:ss. Calculate the date and time of events for the following countdown timers, viewed at the given time and date.

**a** 4.21 pm, 25 July, 2009

| DAY | HOUR | MIN | SEC |
|---|---|---|---|
| 42 | 23 | 01 | 58 |

**b** 9.28 am, 13 December, 2010

| DAY | HOUR | MIN | SEC |
|---|---|---|---|
| 93 | 11 | 27 | 42 |

**c** 2122, 25-04-1998

| DAY | HOUR | MIN | SEC |
|---|---|---|---|
| 17 | 19 | 38 | 24 |

**14** Calculate what a countdown timer should show for these dates and times.

| | Current time and date | Time and date of event |
|---|---|---|
| **a** | 7.37 pm, 19 August 2012 | 9.30 am, 25 December 2012 |
| **b** | 3.21 pm, 26 October 2011 | 5.00 pm, 17 February 2012 |
| **c** | 0900, 12-08-2013 | 1600, 18-10-2014 |



**15** Write a countdown timer for three important events coming up in your life.

**Reflect**

What is important to remember when calculating elapsed time?

# 8E Reading time

## Start thinking!

Look at the train timetable opposite and focus on the Saturday section.




**1** **a** Where does the service start?
**b** Where does it finish?
**c** What do the blank sections mean?

**2** **a** List all the trains (by service number) that leave Traralgon on Saturday.
**b** How many leave in the morning? Underline them.
**c** How many of these arrive in the morning? Circle them.

**3** Look at the first IC service.
**a** What time does this leave Traralgon?
**b** What time does it arrive in Melbourne (Southern Cross)?
**c** Find the difference between the two times and hence state how long the train takes.

**4** What advice would you give somebody wanting to arrive in Melbourne before midday on a Saturday?

## KEY IDEAS

- Tables are a useful way of representing a lot of information easily and quickly.
- Timetables are tables containing information about scheduled events.
- When reading timetables check these points.
  1. Is it the right timetable or section? (Check the heading.)
  2. Should I read it in rows or columns?
  3. Is there a key to symbols, abbreviations or shading?

# EXERCISE 8E Reading time



Use this train timetable for Example 8E-1 and questions 1–3.

| | MONDAY – FRIDAY | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Service No.** | 8400 | 8402 | 8404 | 8406 | 8408 | 8410 | 8412 | 8414 | 8416 | 8418 | 8420 | 8422 | 8424 | 8426 | 8428 | C404 | 8430 | 8434 | 8432 |
| **Train/Coach** | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | COACH | TRAIN | TRAIN | TRAIN |
| **Seating/Catering** | | | ★ | | | ★ 🍷 | | | | | | | ★ 🍷 | | | | | | ★ 🍷 |
| | | | | | | IC | | | | | | | IC | | | | | | IC |
| | | | | | | BD | | | | | | | BD | | | | | | BD |
| TRARALGON dep | 04:50 | 05:38 | 06:00 | 06:43 | 07:15 | 07:47R | 08:15 | 09:10 | 10:20 | 11:15 | 12:15 | 13:15 | 14:07R | 15:12 | 16:00 | | 17:05 | 17:50 | 19:12R |
| Morwell Stn | 05:00 | 05:46 | 06:11 | 06:52 | 07:24 | 07:58 | 08:24 | 09:20 | 10:29 | 11:24 | 12:24 | 13:24 | 14:18 | 15:21 | 16:10 | | 17:13 | 17:59 | 19:23 |
| Moe | 05:11 | 05:56 | 06:22 | 07:02 | 07:34 | 08:09 | 08:34 | 09:31 | 10:43 | 11:34 | 12:39 | 13:35 | 14:30 | 15:36 | 16:27 | | 17:28 | 18:14 | 19:34 |
| Trafalgar | 05:18 | | 06:29 | | 07:40 | | 08:41 | 09:38 | 10:50 | 11:40 | 12:45 | 13:41 | GL | 15:43 | 16:34 | | 17:35 | | 19:41 |
| Yarragon | 05:24 | | 06:35 | | 07:45 | | 08:46 | 09:44 | 10:55 | 11:45 | 12:50 | 13:46 | | 15:49 | 16:40 | | 17:41 | | 19:47 |
| WARRAGUL | 05:33 | 06:11 | 06:45 | 07:17 | 07:53 | 08:28 | 08:54 | 09:53 | 11:03 | 11:53 | 12:58 | 13:54 | 14:49 | 15:57 | 16:48 | | 17:49 | 18:31 | 19:57 |
| Drouin | 05:39 | 06:17 | 06:51 | 07:23 | 07:58 | 08:34 | 08:59 | 09:59 | 11:09 | 11:58 | 13:03 | 13:59 | 14:55 | 16:03 | 16:54 | | 17:55 | 18:37 | 20:03 |
| Longwarry | 05:45 | | 06:58 | | 08:04 | | 09:05 | 10:05 | | 12:04 | | 14:05 | | 16:12 | 17:01 | | 18:08 | | 20:10 |
| Bunyip | 05:49 | | 07:02 | | 08:08 | | 09:08 | 10:09 | | 12:07 | | 14:08 | | 16:16 | 17:05 | | 18:12 | | 20:14 |
| Garfield | 05:53 | 06:27 | 07:06 | 07:33 | 08:12 | 08:47 | 09:11 | 10:13 | 11:19 | 12:10 | 13:12 | 14:12 | 15:08 | 16:20 | 17:09 | 18:03A | 18:16 | 18:49 | 20:18 |
| Tynong | 05:57 | | 07:10 | | 08:16 | | 09:15 | 10:17 | | 12:14 | | 14:16 | | 16:24 | 17:13 | 18:06 | 18:20 | | 22:22 |
| Nar Nar Goon | 06:01 | | 07:14 | | 08:20 | | 09:19 | 10:21 | | 12:18 | | 14:20 | | 16:28 | 17:17 | 18:12 | 18:24 | | 20:26 |
| PAKENHAM | 06:08 | | 07:24 | 07:44 | 08:26 | 09:00 | 09:26 | 10:29 | 11:30 | 12:26 | 13:25 | 14:28 | 15:25 | 16:37 | 17:26 | | 18:31 | 19:05 | 20:35 |
| Berwick | | | | | 08:35d | | | | | | | | | | | | | | |
| DANDENONG | 06:26d | 06:54d | 07:41d | 08:01d | 08:44d | 09:18d | 09:43d | 10:46d | 11:47d | 12:43d | 13:42d | 14:44d | 15:42d | 16:54d | 17:43d | | 18:48d | 19:22d | 20:52d |
| Clayton | 06:40d | 07:08d | 07:52d | 08:15d | 08:54d | | 09:53d | 10:57d | 11:58d | 12:53d | 13:56d | 14:55d | | 17:05d | 17:56d | | 19:00d | 19:32d | |
| Caulfield | 06:52d | 07:16d | 08:05d | 08:28d | 09:06d | 09:35d | 10:07d | 11:07d | 12:07d | 13:07d | 14:07d | 15:06d | 16:04d | 17:18d | 18:08d | | 19:12d | 19:42d | 21:12d |
| Richmond | 07:01d | 07:24d | 08:14d | 08:38d | 09:15d | 09:44d | 10:15d | 11:15d | 12:15d | | | | | | | | | | |
| MELBOURNE | | | | | | | | | | | | | | | | | | | |
| (Flinders Street) arr | 07:06d | 07:29d | 08:18d | 08:42d | 09:22d | 09:48d | 10:19d | 11:19d | 12:19d | 13:19d | 14:19d | 15:19d | 16:18d | 17:31d | 18:23d | | 19:24d | 19:55d | 21:25d |
| (Southern Cross) arr | 07:12 | 07:34 | 08:24 | 08:51 | 09:28 | 09:54 | 10:26 | 11:26 | 12:26 | 13:24 | 14:24 | 15:26 | 16:26 | 17:38 | 18:28 | | 19:29 | 20:00 | 21:31 |

| | SATURDAY | | | | | | | | | | | SUNDAY | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Service No.** | 8404 | 8408 | 8410 | 8414 | 8418 | 8422 | 8424 | 8426 | 8430 | 8432 | 8434 | 8404 | 8408 | 8410 | 8418 | 8422 | 8424 | 8430 | 8432 | 8434 |
| **Train/Coach** | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN | TRAIN |
| **Seating/Catering** | | | ★ 🍷 | | | | ★ 🍷 | | | ★ 🍷 | | | | ★ 🍷 | | | | | ★ 🍷 | |
| | | | IC | | | | IC | | | IC | | | | IC | | | IC | | IC | |
| | | | BD | | | | BD | | | BD | | | | BD | | | BD | | BD | |
| TRARALGON dep | 06:05 | 07:20 | 08:07R | 09:20 | 10:57 | 12:55 | 13:52R | 14:48 | 16:48 | 17:57R | 18:45 | 06:05 | 08:03 | 09:17R | 10:57 | 12:55 | 14:48R | 16:48 | 17:57R | 18:45 |
| Morwell | 06:14 | 07:29 | 08:18 | 09:29 | 11:06 | 13:04 | 14:03 | 14:57 | 16:57 | 18:08 | 18:54 | 06:14 | 08:12 | 09:28 | 11:06 | 13:04 | 14:57 | 16:57 | 18:08 | 18:54 |
| Moe | 06:23 | 07:38 | 08:29 | 09:42 | 11:15 | 13:13 | 14:21 | 15:06 | 17:06 | 18:20 | 19:03 | 06:23 | 08:21 | 09:39 | 11:15 | 13:13 | 15:06 | 17:06 | 18:20 | 19:03 |
| Trafalgar | 06:29 | 07:44 | | 09:48 | 11:21 | 13:19 | GL | 15:13 | 17:13 | | 19:10 | 06:29 | 08:27 | | 11:21 | 13:19 | 15:13 | 17:13 | | 19:10 |
| Yarragon | 06:34 | 07:49 | | 09:53 | 11:26 | 13:24 | | 15:19 | 17:19 | | 19:16 | 06:34 | 08:32 | | 11:26 | 13:24 | 15:19 | 17:19 | GL | 19:16 |
| WARRAGUL | 06:42 | 07:57 | 08:48 | 10:01 | 11:34 | 13:32 | 14:40 | 15:27 | 17:27 | 18:39 | 19:24 | 06:42 | 08:40 | 09:58 | 11:34 | 13:32 | 15:27 | 17:27 | 18:39 | 19:24 |
| Drouin | 06:47 | 08:02 | 08:54 | 10:06 | 11:39 | 13:37 | 14:46 | 15:33 | 17:33 | 18:45 | 19:30 | 06:47 | 08:45 | 10:04 | 11:39 | 13:37 | 15:33 | 17:33 | 18:45 | 19:30 |
| Longwarry | 06:53 | 08:08 | | 10:12 | 11:45 | 13:43 | | 15:40 | 17:40 | | 19:37 | 06:53 | 08:51 | | 11:46 | 13:43 | 15:40 | 17:40 | | 19:37 |
| Bunyip | 06:56 | 08:11 | | 10:15 | 11:48 | 13:46 | | 15:44 | 17:44 | | 19:41 | 06:56 | 08:54 | | 11:49 | 13:46 | 15:44 | 17:44 | | 19:41 |
| Garfield | 06:59 | 08:14 | 09:07 | 10:18 | 11:51 | 13:49 | 14:59 | 15:48 | 17:48 | 18:58 | 19:45 | 06:59 | 08:57 | 10:18 | 11:52 | 13:49 | 15:48 | 17:48 | 18:58 | 19:45 |
| Tynong | 07:03 | 08:18 | | 10:22 | 11:55 | 13:53 | | 15:52 | 17:52 | | 19:49 | 07:03 | 09:01 | | 11:56 | 13:53 | 15:52 | 17:52 | | 19:49 |
| Nar Nar Goon | 07:07 | 08:22 | | 10:26 | 11:59 | 13:57 | | 15:56 | 17:56 | | 19:53 | 07:07 | 09:05 | | 12:00 | 13:57 | 15:56 | 17:56 | | 19:53 |
| PAKENHAM | 07:13 | 08:29 | 09:23 | 10:33 | 12:05 | 14:04 | 15:12 | 16:04 | 18:04 | 19:12 | 20:02 | 07:13 | 09:12 | 10:32 | 12:05 | 14:04 | 16:04 | 18:04 | 19:12 | 20:02 |
| DANDENONG | 07:30d | 08:46d | 09:40d | 10:50d | 12:22d | 14:21d | 15:29d | 16:21d | 18:21d | 19:29d | 20:19d | 07:30d | 09:29d | 10:49d | 12:22d | 14:21d | 16:21d | 18:21d | 19:29d | 20:19d |
| Clayton | 07:40d | 08:56d | | 11:00d | 12:33d | 14:32d | | 16:32d | 18:32d | | 20:30d | 07:40d | 09:39d | | 12:33d | 14:32d | 16:32d | 18:32d | | 20:30d |
| Caulfield | 07:49d | 09:05d | 10:03d | 11:09d | 12:43d | 14:43d | 15:46d | 16:43d | 18:43d | 19:46d | 20:41d | 07:50d | 09:49d | 11:06d | 12:43d | 14:43d | 16:43d | 18:43d | 19:46d | 20:41d |
| Richmond | | 09:14d | 10:11d | 11:17d | 12:51d | | | | | | | | 09:57d | 11:14d | 12:51d | | | | | |
| MELBOURNE | | | | | | | | | | | | | | | | | | | | |
| (Flinders Street) arr | 08:01d | 09:19d | 10:15d | 11:21d | 12:55d | 14:55d | 16:00d | 16:55d | 18:55d | 20:00d | 20:53d | 08:03d | 10:01d | 11:20d | 12:55d | 14:55d | 16:55d | 18:55d | 20:00d | 20:53d |
| (Southern Cross) arr | 08:07 | 09:25 | 10:21 | 11:27 | 13:01 | 15:01 | 16:05 | 17:01 | 19:01 | 20:07 | 20:59 | 08:09 | 10:07 | 11:25 | 13:01 | 15:01 | 17:01 | 19:01 | 20:07 | 20:59 |

## EXAMPLE 8E-1 Reading timetables

What is the latest time you could leave Trafalgar to be at Flinders Street Station, Melbourne, by 10 am on Wednesday?

| THINK | WRITE |
|---|---|
| **1** Identify the latest train to come in to Flinders Street station *before* 10 am. | The last train to arrive at Flinders Street Station before 10 am arrives at 9.48 am. |
| **2** Look up when this train departs Trafalgar. There is a gap, so it doesn't stop at Trafalgar. | This train does not stop at Trafalgar. |
| **3** Look to the previous train – it leaves Trafalgar at 7.40 am to arrive at Flinders Street Station at 9.22 am. | The previous train does stop at Trafalgar. |
| **4** Write your answer. | The latest Trafalgar train to arrive at Flinders Street before 10 am on Wednesday leaves at 7.40 am. |

UNDERSTANDING AND FLUENCY

**1** Focus on the Sunday section of the timetable on page 433.

- **a** How many trains run between Traralgon and Melbourne on Sundays?
- **b** How many trains stop at Nar Nar Goon on a Sunday?
- **c** What is the earliest time you can leave Nar Nar Goon to arrive in Melbourne on a Sunday?
- **d** What is the latest train you can take from Nar Nar Goon on a Sunday to arrive in Melbourne by 5 pm?
- **e** What is the latest train you can take from Traralgon to be in Nar Nar Goon at 4.30 pm on Sunday?

**2** Focus on the Monday–Friday section of the timetable on page 433.

- **a** How many trains run from Traralgon to Melbourne during the week?
- **b** How many trains stop at Bunyip on the way to Melbourne?
- **c** The first four services are shaded darker. What might this mean?
- **d** How many services arrive in Melbourne before 10 am?
- **e** What time does the last service leave Traralgon? What time does it get into Southern Cross Station?
- **f** How long does the train in part **e** take?
- **g** Look at the train that leaves Traralgon at 07:15. How long does this take to reach Southern Cross Station?
- **h** How long does the train that leaves at 05:38 take to reach Southern Cross Station?
- **i** Find how much faster the 05:38 train is than the last train of the day.

**3** Look at the timetable on page 433 as a whole.

**a** What is the latest train you can take from Nar Nar Goon on a Wednesday to arrive in Melbourne by 5 pm?

**b** What is the latest train you can take from Traralgon to be in Nar Nar Goon at 4.30 pm on Saturday?

**c** What is the latest time you can leave Bunyip to be in Melbourne at 8.30 pm on Friday night?

**d** Assuming that you take a train that stops at all stations, what is the shortest time it takes to travel from:

**i** Traralgon to Warragul?

**ii** Pakenham to Dandenong?

**iii** Garfield to Clayton?

**iv** Moe to Richmond?

**4** Consider the following TV guide.

| Freeview | 6.00 pm | 6.30 pm | 7.00 pm | 7.30 pm | 8.00 pm | 8.30 pm | 9.00 pm | 9.30 pm |
|---|---|---|---|---|---|---|---|---|
| ABC1 | Country House Rescue | | ABC News | 7.30 | Kitchen Cabinet | Upper Middle Bogan | It's a Date | Would I Lie to You? |
| ABC2 | Giggle and Hoot | In the Night Garden | Spicks and Specks | Dr Who | | Kroll Show | Mock the Week | Alan Carr: Chatty Man |
| SBSONE | Metropolis | World News Australia | | Curry Nation | Luke Nguyen's Greater Mekong | Heston's Feasts | | Vikings |
| Seven | Seven News | Today Tonight | Home and Away | | Dynamo: Magician Impossible | | Please Marry My Boy | |
| 7MATE | Seinfeld | How I Met Your Mother | | Ultimate Factories | | Megastructures | | America's Hardest Prisons |
| Nine | Nine News | A Current Affair | Big Brother | Nine's Live Thursday Night Football | | | | The Footy Show |
| GO! | Big Brother | | The Big Bang Theory | The Big Bang Theory | The Big Bang Theory | Mike & Molly | Mike & Molly | Jay and Silent Bob Strike Back |
| Ten | The Simpsons | The Project | | Save with Jamie | | Law & Order S.V.U. | | Law & Order S.V.U. |
| ELEVEN | Sabrina, the Teenage Witch | Neighbours | Everybody Loves Raymond | Futurama | Futurama | Star Trek: Deep Space 9 | | Star Trek: Deep Space 9 |

**a** How many different channels are listed in this guide?

**b** What is the start time of this guide?

**c** What channel and what time is *The Simpsons* on?

**d** What is on ABC2 at 8.30 pm?

**e** What is on Seven at 2100?

**f** How long does *The Project* run for?

**g** What is the longest program in this guide? Which channel is it on and how long does it run for?

**h** If you wanted to watch *Big Brother*, what options do you have?

**i** How many sports programs are shown in this guide?

**j** Why is *The Big Bang Theory* split into three sections?

**k** How long does *Neighbours* go for?

**l** If you were allowed to watch 2 hours of television according to this guide, what would you pick?

**5** Over the school holidays (2 weeks) you have a lot of things to do. Make a timetable so that you can fit everything from the table in. Make sure you include times for meals and other standard activities.

| Activity | Time needed |
|---|---|
| Homework | 10 hours |
| Swimming training | 6 hours |
| Walking the dog | 1 hour a day |
| Using MSN | $\frac{1}{2}$ hour a day |
| Housework | 4 hours |
| Shopping | 4 hours |
| Watching TV | $\frac{1}{2}$ hour a day |

**6** Consider the timetable on page 433.

**a** Compare the IC trains to the normal trains. Are they faster? Why?

**b** What is the fastest train that goes from Traralgon to Melbourne? State the day and time for this train.

**7** Create a timetable to represent a standard week in your family. Include any individual or family events or commitments so that you can see where everybody will be at a certain time.

**8** Consider this bus timetable from an area in Brisbane.

## Strathpine/Bracken Ridge to Toombul

### Monday to Friday

| Route Number | 327 | 327 | 327 | 327 | 326 | 326 | 327 | 327 | 326 | 327 | 326 | 327 | 326 | 327 | 326 | 327 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | am | am | am | am | am | am | am | am | am | am | am | am | am | am | pm | pm |
| Westfield Strathpine | | | | | | | | 8:50 | | 9:50 | | 10:50 | | 11:52 | | 12:55 |
| North Wind Estate | 6:05 | 6:35 | 7:00 | 7:30 | | | 8:35 | 9:00 | | 10:00 | | 11:00 | | 12:02 | | 1:05 |
| Mustang St | \| | \| | \| | \| | 7:40 | 8:20 | \| | \| | 9:17 | \| | 10:20 | \| | 11:20 | \| | 12:23 | \| |
| Bracken Ridge Plaza | 6:10 | 6:40 | 7:05 | 7:35 | 7:42 | 8:22 | 8:40 | 9:05 | 9:19 | 10:05 | 10:22 | 11:05 | 11:22 | 12:07 | 12:25 | 1:10 |
| Sandgate Station | \| | \| | \| | \| | 8:02 | 8:40 | \| | \| | 9:37 | \| | 10:40 | \| | 11:40 | \| | 12:41 | \| |
| *Train to CITY* | \| | \| | \| | \| | *8:06* | *8:45* | \| | \| | *9:45* | \| | *10:45* | \| | *11:45* | \| | *12:45* | \| |
| Taigum Shopping Centre | 6:21 | 6:51 | 7:16 | 7:49 | 8:19 | 8:56 | 8:53 | 9:18 | 9:53 | 10:20 | 10:53 | 11:18 | 11:53 | 12:19 | 12:54 | 1:20 |
| Geebung Station | 6:29 | 6:59 | 7:26 | 8:03 | 8:27 | 9:04 | 9:05 | 9:30 | 10:01 | 10:32 | 11:01 | 11:30 | 12:01 | 12:30 | 1:02 | 1:31 |
| *Train to CITY* | *6:49* | *7:06* | *7:35* | *8:09* | *8:44* | *9:07* | *9:07* | *9:37* | *10:07* | *10:37* | *11:07* | *11:37* | *12:07* | *12:37* | *1:07* | *1:37* |
| Toombul Interchange | 6:45 | 7:15 | 7:45 | 8:21 | 8:45 | 9:18 | 9:20 | 9:45 | 10:16 | 10:47 | 11:16 | 11:45 | 12:16 | 12:45 | 1:17 | 1:46 |

| Route Number | 326 | 327 | 326 | 327 | 326 | 327 | 326 | 327 | 327 |
|---|---|---|---|---|---|---|---|---|---|
| | pm | pm | pm | pm | pm | pm | pm | pm | pm |
| Westfield Strathpine | | 1:55 | | 2:55 | | 3:55 | | 4:55 | 5:35 |
| North Wind Estate | | 2:05 | | 3:05 | | 4:05 | | 5:05 | 5:45 |
| Mustang St | 1:23 | \| | 2:23 | \| | 3:23 | \| | 4:23 | \| | \| |
| Bracken Ridge Plaza | 1:25 | 2:10 | 2:25 | 3:10 | 3:25 | 4:10 | 4:25 | 5:10 | 5:50 |
| Sandgate Station | 1:41 | \| | 2:41 | \| | 3:41 | \| | 4:41 | \| | \| |
| *Train to CITY* | *1:45* | \| | *2:45* | \| | *3:45* | \| | *4:45* | \| | \| |
| Taigum Shopping Centre | 1:54 | 2:20 | 2:59 | 3:20 | 3:56 | 4:23 | 4:56 | 5:23 | 6:00 |
| Geebung Station | 2:02 | 2:31 | 3:07 | 3:31 | 4:04 | 4:35 | 5:04 | 5:34 | 6:11 |
| *Train to CITY* | *2:07* | *2:37* | - | *3:37* | *4:08* | *4:37* | *5:07* | *5:37* | *6:37* |
| Toombul Interchange | 2:17 | 2:46 | 3:22 | 3:46 | 4:19 | 4:50 | 5:19 | 5:49 | 6:26 |

### Saturday

| Route Number | 327 | 327 | 326 | 327 | 326 | 327 | 326 | 327 | 326 | 327 | 326 | 327 | 326 | 327 | | 326 | 327 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | am | am | am | am | am | am | am | am | am | am | am | am | am | pm | | pm | pm |
| Westfield Strathpine | | | | 8:50 | | 9:50 | | 10:50 | | 11:52 | | 12:55 | | 1:55 | | | 4:55 |
| North Wind Estate | 7:05 | 8:05 | | 9:00 | | 10:00 | | 11:00 | | 12:02 | | 1:05 | | 2:05 | | | 5:05 |
| Mustang St | \| | \| | 8:20 | \| | 9:20 | \| | 10:20 | \| | 11:20 | \| | 12:23 | \| | 1:23 | \| | Each | 4:23 | \| |
| Bracken Ridge Plaza | 7:10 | 8:10 | 8:22 | 9:05 | 9:22 | 10:05 | 10:22 | 11:05 | 11:22 | 12:07 | 12:25 | 1:10 | 1:25 | 2:10 | | route | 4:25 | 5:10 |
| Sandgate Station | \| | \| | 8:40 | \| | 9:40 | \| | 10:40 | \| | 11:38 | \| | 12:40 | \| | 1:40 | \| | operates | 4:40 | \| |
| *Train to CITY* | \| | \| | *8:45* | \| | *9:45* | \| | *10:45* | \| | *11:45* | \| | *12:45* | \| | *1:45* | \| | hourly | *4:45* | \| |
| Taigum Shopping Centre | 7:22 | 8:22 | 8:53 | 9:19 | 9:53 | 10:19 | 10:53 | 11:19 | 11:51 | 12:19 | 12:53 | 1:22 | 1:53 | 2:20 | | until: | 4:53 | 5:20 |
| Geebung Station | 7:32 | 8:32 | 9:01 | 9:30 | 10:01 | 10:30 | 11:01 | 11:30 | 11:59 | 12:29 | 1:01 | 1:32 | 2:01 | 2:30 | | | 5:01 | 5:30 |
| *Train to CITY* | *7:37* | *8:37* | *9:07* | *9:37* | *10:07* | *10:37* | *11:07* | *11:37* | *12:07* | *12:37* | *1:07* | *1:37* | *2:07* | *2:37* | | | *5:07* | *5:37* |
| Toombul Interchange | 7:47 | 8:47 | 9:16 | 9:45 | 10:16 | 10:45 | 11:16 | 11:45 | 12:14 | 12:49 | 1:16 | 1:47 | 2:16 | 2:45 | | | 5:16 | 5:45 |

**a** How many buses leave from Westfield Strathpine on a weekday?

**b** What is special about the highlighted rows?

**c** What do the sections with '|' mean?

**d** If you wanted to catch a train from Geebung Station at 10.37 am on Monday, what time would you have to catch the bus at Westfield Strathpine?

**e** If you wanted to catch the same train but were at the Mustang St stop, which bus would you have to catch?

**f** Would you still take the 10:37 train? Explain.

**g** What is the latest bus you could catch from Mustang St to get into Toombul by 10 am on a Tuesday?

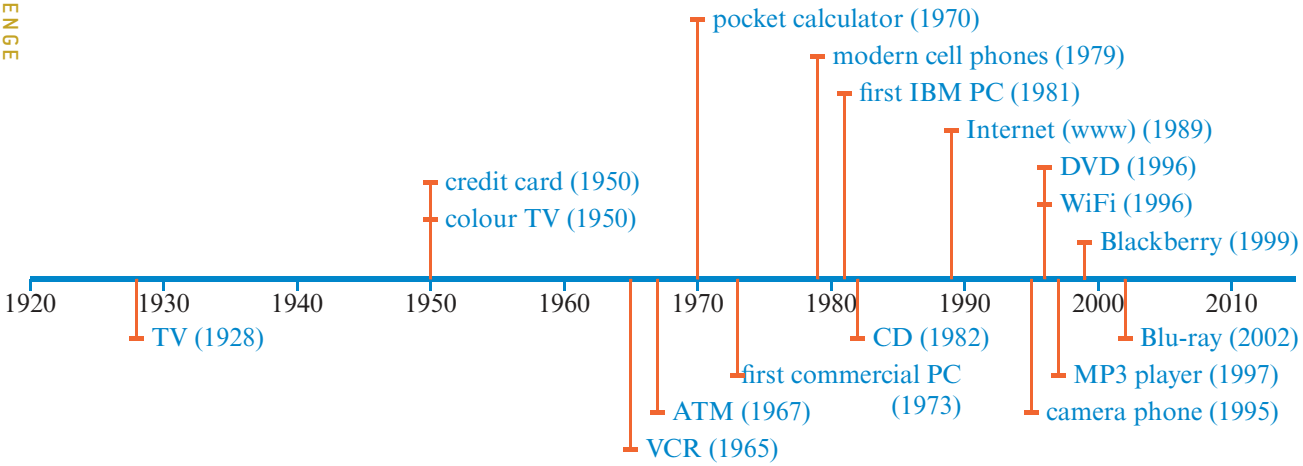
**h** Calculate how long the bus trip takes from North Wind Estate to Toombul at:

**i** 6.05 am (weekdays) **ii** 7.30 am (weekdays) **iii** 7.05 am (Saturday).

**i** What explanation can you offer for the differences in times?

CHALLENGE

**9** Another way to show information is by using a timeline instead of a timetable. Consider this timeline.




**a** When was the Internet invented?

**b** Which technology came first, DVDs or MP3 players?

**c** How many years were there between the invention of television and the invention of colour television?

**d** How long does this timeline span?

**e** Write three questions of your own about this timeline.

**f** What are some advantages and disadvantages of using a timeline to display information?

**g** Do you prefer timelines or timetables? Explain.

**h** Investigate when these technologies became popular. What do you notice?

**10** Create your own timeline detailing important events in your life.

**Reflect**

What is important to remember when reading timetables?

# 8F Time zones

## Start thinking!

Find a globe and a torch and shine the torch on the globe so that Australia is 'in the sun'.

**1** When it is daytime in Australia, is it day or night in Africa?

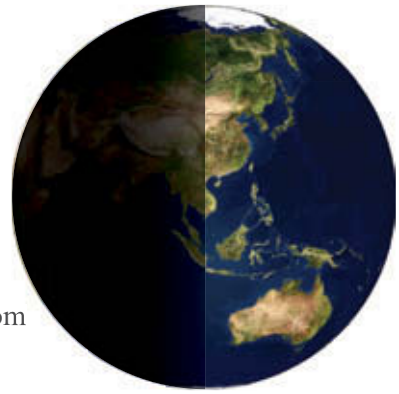
**2** List three places that are in daytime and three places where it is night when it is day in Australia.

Rotate the globe so that the sun rises in the east and sets in the west.

**3** Explain why it can't be the same time everywhere in the world.

Look at the world map opposite. It has colour bands for each **time zone** and the numbers along the top indicate how many hours ahead or behind a time zone is from the time zone '0'. Time zone '0' is called UTC (in English, Coordinated Universal Time) or GMT (Greenwich Mean Time).

**4** How do the colour bands on this world map help you see where one time zone finishes and another starts?



**5** Why do you think that the colours on the countries might not line up exactly with each colour band? For example, Greenland is mostly yellow (UTC −3) but stretches from UTC −5 to UTC −1.

**6** How many hours are there in a day?

**7** How many time zones are there?

**8** Why do you think you get the same answer for questions **6** and **7**?

Look at the purple band that shows UTC (time '0'). The United Kingdom (UK) is in this time zone (Greenwich is in the UK).

**9** List two other countries that are in this time zone.

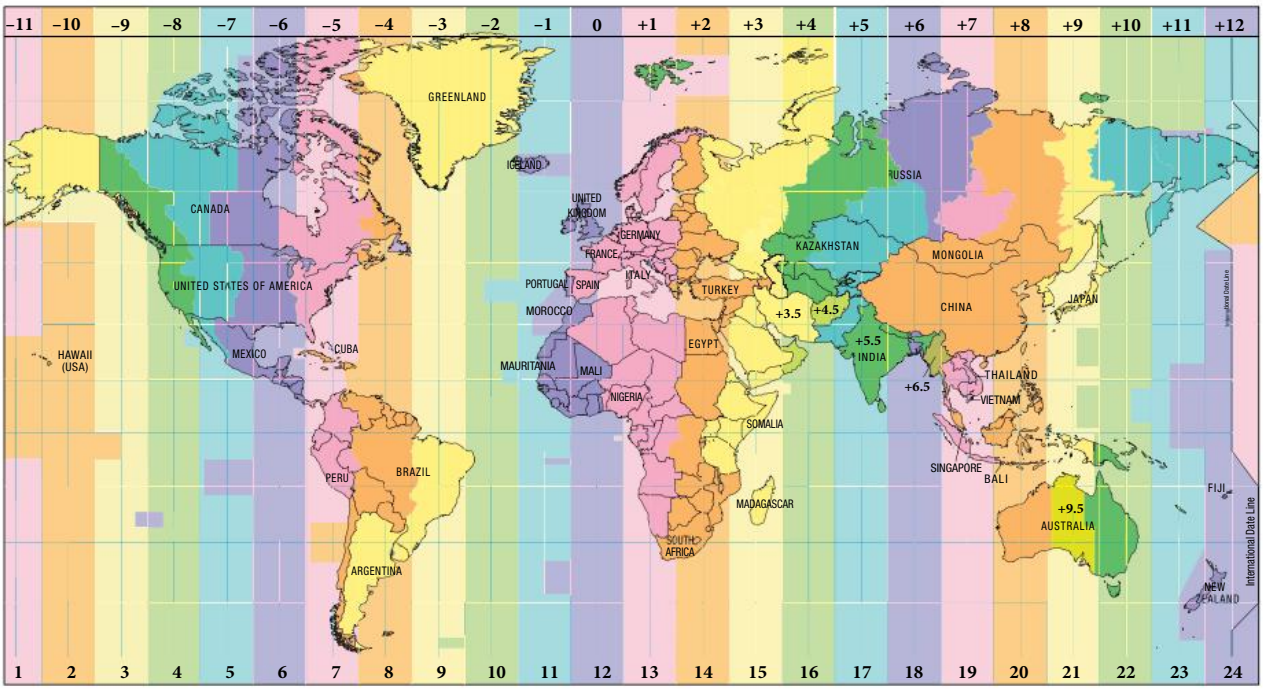
**10** What time zone is China in? (Hint: look at the colour of the country.)

**11** Is China ahead of or behind the UK? By how much?

## KEY IDEAS

- There are 24 time zones.
- A time zone map shows how many hours ahead or behind a time zone is from the time zone '0', which is UTC or GMT (Greenwich Mean Time).
- Australia covers three time zones: Western Standard Time (WST), Eastern Standard Time (EST) and Central Standard Time (CST). CST is only a half-hour behind EST instead of 1 hour.
- The **International Date Line (IDL)** is where the date changes.

# EXERCISE 8F Time zones




UNDERSTANDING AND FLUENCY

**1** Refer to the time zones shown at the top of the map above to answer these questions.

- **a** List the countries that are in time zone '0'.
- **b** List all the countries that are in the same time zone as you are.
- **c** How many time zones are used in North America?
- **d** How many time zones are used in China?
- **e** Which country covers the largest number of time zones? How many time zones is this?
- **f** How many time zones are used in Australia?
- **g** Why is Central Australia a different colour to its zone?

**2** Which time zone is each of these countries in?

- **a** India
- **b** South Africa
- **c** Mexico
- **d** Italy
- **e** New Zealand
- **f** Nigeria



## EXAMPLE 8F-1 Working out time zone differences

What is the time difference between Nigeria and Cuba?

| THINK | WRITE |
|---|---|
| **1** Find the time zone for Nigeria by looking at the colour (pink). | Nigeria is in UTC +1. |
| **2** Find the time zone for Cuba (orange). | Cuba is in UTC –4. |
| **3** Which country is ahead? | Nigeria is ahead of Cuba. |
| **4** Work out how many hours Cuba is behind Nigeria. Subtract –4 from +1. | (+1) – (– 4) = 1 + 4 = 5<br>Nigeria is 5 hours ahead of Cuba. |

**3** What is the time difference between these countries?

- **a** Madagascar and Vietnam
- **b** Mongolia and Peru
- **c** Argentina and Morocco
- **d** Mexico and Fiji
- **e** Turkey and Greenland
- **f** Japan and India

**4** What is the time difference between these Australian cities?

- **a** Perth and Canberra
- **b** Melbourne and Darwin
- **c** Adelaide and Brisbane
- **d** Hobart and Sydney
- **e** Brisbane and Perth
- **f** Darwin and Hobart

## EXAMPLE 8F-2 Calculating world times

Find the time in Buenos Aires, the capital of Argentina, when it is 3 pm in Perth.

| THINK | WRITE |
|---|---|
| **1** Find the time zone for Argentina (yellow). | Argentina is in UTC −3. |
| **2** Find the time zone for Perth (orange). | Perth is in UTC +8. |
| **3** Work out how many hours Argentina is behind Perth. | (+8) − (−3) = 8 + 3 = 11<br>Argentina is 11 hours behind Perth. |
| **4** Subtract 11 hours from 3 pm to find the time in Buenos Aires. | 3 pm − 11 hours = 4 am<br>It is 4 am in Buenos Aires when it is 3 pm in Perth. |

**5** Time '0' is in London. Find the time in each country when it is 4 pm in London.

- **a** Peru
- **b** France
- **c** Madagascar
- **d** Vietnam
- **e** Japan
- **f** Australia (Sydney)

**6** What is the time:

**a** in Greenland when it is 1.30 pm in Peru?

**b** in Kazakhstan when it is 11.15 am in Argentina?

**c** in Mali when it is 10.45 pm in Singapore?

**d** in China when it is 4.40 am in New Zealand?

**e** in India when it is 4.26 pm in China?

**f** in Mexico when it is 3.10 am in Thailand?

**7** When you travel around the world, it is nice to call home from time to time. What time would you have to call from the following countries to be phoning home at 8 pm?

**a** Egypt **b** Fiji **c** Mongolia

**d** Morocco **e** Spain **f** New Zealand

**8** **a** What are the three time zones in Australia called?

**b** List the capital cities in each time zone.

**c** If it is 10 am in Darwin, what time is it in each of these places?

**i** Adelaide **ii** Canberra **iii** Broome

**d** If it is 11 pm in Perth, what time is it in each of these places?

**i** Alice Springs

**ii** Broken Hill

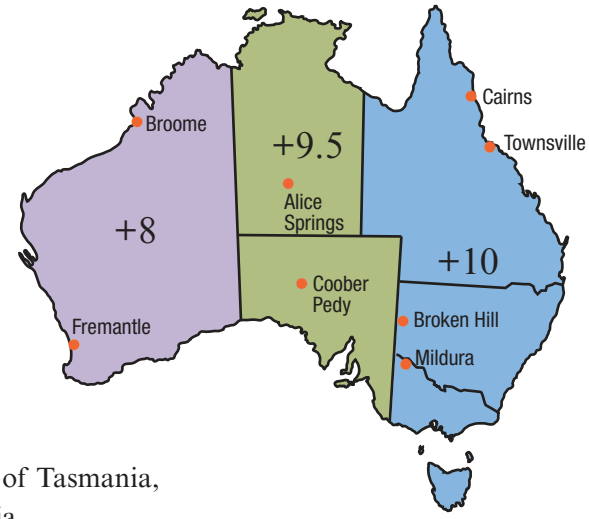
**iii** Cairns

**e** If you wanted to call your friends in Mildura at 7 pm their time, what time would you call if you were in:

**i** Sydney?

**ii** Coober Pedy?

**iii** Fremantle?




**9** Consider the cities of Hobart, the capital of Tasmania, and Perth, the capital of Western Australia.

**a** Which time zone is Hobart in?

**b** Which time zone is Perth in?

**c** How many time zones difference is this?

**d** Is Hobart ahead or behind Perth?

**e** Copy and complete this sentence, crossing out the incorrect word.
To find the time in Perth, *add*/*subtract* ____ hours from the time in Hobart.

**f** If it is 4 pm in Hobart, what time would it be in Perth?

**g** How many hours separate Hobart and the countries listed in question **2**?

**h** If it is 8 pm in Hobart, what time would it be in each of the countries listed in question **2**?



**10** Why are there 24 different time zones?

**11** How does the world map show that New Zealand is the furthest ahead in time and Hawaii (USA) is the furthest behind?

**12** **a** In which time zone is Honolulu, Hawaii?

**b** In which time zone is Townsville?

**c** Is Honolulu ahead or behind Townsville in time? By how much?

**d** Count the shortest number of time zones between Honolulu and Townsville. (Hint: cross the International Date Line.)

**e** Copy and complete the following sentences, crossing out the incorrect words.
This is like Honolulu being *ahead/behind* in time, but *ahead/behind* a day.
To find the time in Honolulu, *add/subtract* _____ hours from the time in Townsville, or *add/subtract* _____ hours and subtract a day.

**f** Find the time and day in Honolulu when in Townsville it is:
**i** 4 pm Thursday **ii** 9 am Monday **iii** 11 pm Saturday.

**13** Write three of your own questions that relate to the world time zones map on page 439.

**14** Why is Australia one of the first countries to experience the New Year?

**15** List the advantages and disadvantages of being a large country and having only a single time zone. Which large country does this?

**16** You are watching a football game at 7.30 pm EST.

**a** What time would it be in Perth?

**b** If the game goes for about 3 hours, when would it be safe to call a friend in Perth to discuss the final scores, assuming that it was broadcast from 7.30 pm WST?

**17** Michael Jackson's memorial was held at 10 am on 7 July 2009 in Los Angeles (west coast of the USA).

**a** Which time zone is LA found in?

**b** If it was broadcast live, what time would it be in:
**i** Australian EST? **ii** Australian WST? **iii** Australian CST?
**iv** London? **v** Singapore? **iv** New York (east coast of the USA)

**18** The 2010 World Cup was held in South Africa.

**a** What time zone is this?

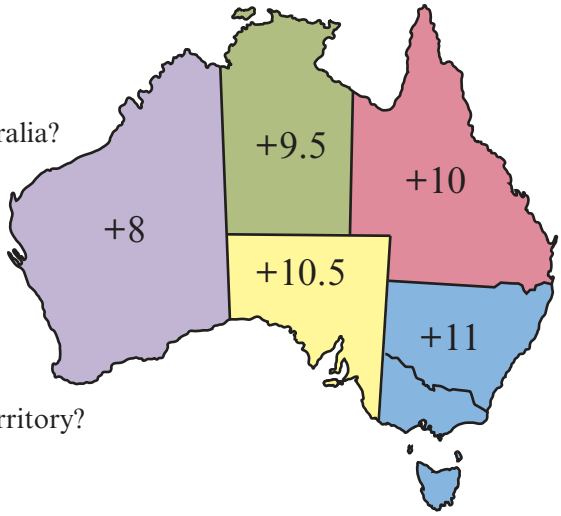
**b** If a game starts at 5.30 pm in South Africa, what time would it be in each of these countries?
**i** Germany **ii** Argentina
**iii** Italy **iv** Australia (at the place you live)

**c** If you wanted to watch a game but could not get access to the television between 10 pm and 7 am, what time period in South Africa would the game you watch have to be played?

**d** The 2014 World Cup was held in Brazil. What time would it be for us if the game started at 10 am in Rio de Janeiro (on the east coast of Brazil)?

CHALLENGE

**19** Daylight saving time is where people put their clocks forward 1 hour in summer to have an 'extra' hour of daylight in the evening. In Australia, Australian Capital Territory, New South Wales, South Australia, Tasmania and Victoria observe daylight saving time. The Northern Territory, Queensland and Western Australia do not. This diagram shows the different time zones in summer when daylight saving time is observed.




**a** In summer, how many hours is:
- **i** Victoria ahead of Queensland?
- **ii** South Australia ahead of Western Australia?
- **iii** Australian Capital Territory ahead of Northern Territory?
- **iv** Tasmania ahead of Western Australia?

**b** In winter, if it is 6 pm in South Australia, what time is it in:
- **i** Western Australia?
- **ii** Northern Territory?
- **iii** Victoria?
- **iv** Queensland?

**c** Repeat part **b**, but for summer.

**d** What is special about the time difference between South Australia and Queensland in summer and winter?

**e** Find the time in Darwin when it is:
- **i** 4 pm on 27 January in Melbourne
- **ii** 3 am on 16 July in Cairns
- **iii** midday on 4 December in Perth
- **iv** 9.30 am on 25 December in Sydney
- **v** 5.45 pm on 30 June in Hobart
- **vi** 11 am on 30 November in Adelaide.

**f** Americans use the saying 'Spring forward, fall back' to help remember which way to turn their clocks. How does it help?

**20** Zoë and her friend Wes want to take a holiday in Thailand. Use your knowledge of time zones to help them out.



**a** Zoë's flight leaves Brisbane at 1.00 pm and arrives in Thailand at 7.00 pm. How long will the flight be?

**b** Wes has to take a different flight. His flight leaves at 1445. If his flight takes the same time, what time will he land in Thailand?

**c** On the way home they plan to stop in Bali. If they leave Thailand at 1000 and arrive in Bali at 1530, how long is this flight?

**d** What time will Zoë and Wes land home if they leave Bali at 1330 and their flight is 8 hours long?

**Reflect**

Why do we have time zones?

# 8G Understanding mass

## Start thinking!



For this task, you will need to work in pairs and have access to 1-kg, 500-g and 100-g weights as well as scales capable of weighing up to at least 2 kg.

1 Collect five objects of varying masses from around the room. Try to choose objects that you think each have a mass of under 2 kg.

2 Estimate the mass of each object and record this in a table in your workbook (Estimate 1).

| Object | Estimate 1 | Estimate 2 | Mass (g) | Mass (kg) |
|---|---|---|---|---|
| | | | | |



3 For each object, use the weights to help refine your estimates. Place the object in one hand and a weight in the other. Does it feel heavier or lighter? By how much? Record your new estimate in the table (Estimate 2).

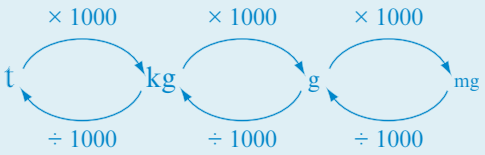
4 Use the scales to help you make an accurate mass measurement for each object and record this, in grams, in your table.

5 How many grams in a kilogram?

6 Use your knowledge of converting to larger units to write a sentence describing how to convert grams to kilograms.

## KEY IDEAS

- Common units of mass are milligrams (mg), grams (g), kilograms (kg) and tonnes (t).
  1000 mg = 1 g
  1000 g = 1 kg
  1000 kg = 1 t
- Mass can be converted to another unit of mass by multiplying or dividing by the conversion factor.
- To convert to a larger unit, divide by the conversion factor. (There will be fewer of the larger unit.)
- To convert to a smaller unit, multiply by the conversion factor. (There will be more of the smaller unit.)



# EXERCISE 8G Understanding mass

UNDERSTANDING AND FLUENCY

**1** List these animals in order from lightest to heaviest.

**a**



**b**



**c**



**d**



**e**



**2** For each animal in question **1**, which unit you would use to measure mass: milligrams, grams, kilograms, or tonnes?

## EXAMPLE 8G-1 Converting mass units in one step

Convert:

**a** 820 g into kg **b** 12.4 g into mg.

**THINK**

**a** To convert to a larger unit, divide by the conversion factor of 1000. (1000 g = 1 kg)

**b** To convert to a smaller unit, multiply by the conversion factor of 1000. (1000 mg = 1 g)

**WRITE**

820 g = (820 ÷ 1000) kg
= 0.82 kg

12.4 g = (12.4 × 1000) mg
= 12 400 mg

**3** Convert these mass units.

**a** 1.2 kg into grams **b** 6000 mg into grams
**c** 72 kg into tonnes **d** 1 g into milligrams
**e** 450 g into kilograms **f** 3.5 t into kilograms
**g** 750 mg into grams **h** 9.8 g into milligrams
**i** 8.13 kg into grams **j** 2045 g into kilograms
**k** 0.93 kg into grams **l** 145 kg into tonnes

## EXAMPLE 8G-2 Converting mass units in two steps

Convert 67 000 mg into kilograms.

| THINK | WRITE |
|---|---|
| **1** First convert from milligrams to grams. To convert to a larger unit, divide by the conversion factor of 1000. (1000 mg = 1 g) | 67 000 mg = (67 000 ÷ 1000) g<br>= 67 g |
| **2** Next convert from grams to kilograms. To convert to a larger unit, divide by the conversion factor of 1000. (1000 g = 1 kg) | = (67 ÷ 1000) kg<br>= 0.067 kg |
| **3** Write your answer. | 67 000 mg = 0.067 kg |

**4** Convert these mass units.

**a** 1 kg into milligrams
**b** 2.5 t into grams
**c** 0.45 kg into milligrams
**d** 0.0006 t into grams
**e** 34 500 g into tonnes
**f** 129 000 000 mg into kilograms
**g** 85 g into tonnes
**h** 12345 mg into kilograms
**i** 8 460 000 000 mg into tonnes
**j** 3.3 t into milligrams
**k** 0.004 kg into milligrams
**l** 2 mg into kilograms

PROBLEM SOLVING AND REASONING

**5** Estimate the mass of these objects and compare your results with those of a classmate.

**a** an MP3 player
**b** a pen
**c** your full school bag
**d** a normal-sized cat
**e** this book
**f** a brick



**6** Look at this docket from a delicatessen.

| | |
|---|---|
| 1.1 kg sausages @ $4.59/kg | 5.05 |
| 250 g of roast beef @ $17.99/kg | 4.50 |
| 500 g Bulgarian feta @ _____/kg | 6.99 |
| 100 g of olives @ $21.99/kg | _____ |
| 725 g of chicken breast fillets @ _____/kg | 8.70 |

- **a** How is the price of sausages calculated?
- **b** Calculate the price of 100 g of olives to fill in the docket.
- **c** Calculate the price of Bulgarian feta per 1 kg. (Hint: how many times does 500 g fit into 1 kg?)
- **d** Find the price per kg of chicken breast fillets by following these steps.
  - **i** Express 725 g in kilograms.
  - **ii** Divide the price by the mass to find the price per kilogram.
- **e** Find the total of the docket.

> **NOTE** To find the price, multiply mass by price per kilogram. Because you are doing it the opposite way, divide price by mass.

**7** Look at the tin of tomatoes shown at right.




- **a** Net mass refers to the mass of the contents of the container. What is the net mass of this tin?
- **b** Gross mass is the mass of the container plus contents. If you measured the gross mass as 450 g, what is the mass of the tin itself?
- **c** If you know that the mass of the empty tin is actually 75 g and the gross mass is 450 g, what does this mean that the net mass is?
- **d** Is this under or over the advertised net mass?

**8** If one egg has a mass of 60 g, what is the mass of each of the following quantities of eggs?

- **a** half a dozen eggs
- **b** a dozen eggs
- **c** six cartons of a dozen eggs
- **d** Might there be other factors to consider when determining the mass of six cartons of eggs? Explain.

**9** Rob needs to give his dog some antibiotics after she has had an operation. The instructions say that for every 5 kg of body mass, the dog needs half a tablet.




- **a** How many antibiotic tablets would Rob's dog need?
- **b** How many antibiotic tablets would you give a dog of the following mass?
  - **i** 5 kg
  - **ii** 10 kg
  - **iii** 25 kg
  - **iv** 17.5 kg
- **c** A friend gives her dog 1.5 tablets. What is the mass of her dog?

**10** If you go bungee jumping, your mass will be measured before you are put into a harness and allowed to jump.

**a** Why is it important that an accurate mass measurement is obtained before letting you jump?

**b** If you put on a big, heavy jacket after being measured, could this affect your jump? What might happen?



**11** A classmate adds up the following masses: 1.3 kg, 325 g, 0.8 kg, 75 g and 100 g. He obtains an answer of 502.1 kg. Give him some advice on how to answer the question correctly, and state the right answer.

**12** A friend needs to find out the mass of one sheet of paper, but does not have a scale sensitive enough to measure something this light. Fortunately, you know a trick she can use to find the mass. You measure the mass of a ream (500 sheets) of paper and find that it has a mass of 2.25 kg.

**a** Write the mass of a ream of paper in grams.

**b** Copy and complete the following sentence.
If 500 sheets = _____ g, you need to _____ by 500 to find the mass of one sheet.

**c** Complete this calculation to find the mass of one sheet of paper.
_____ g _____ 500 = _____ g

**d** Explain in your own words how you can find the mass of a very light object.



CHALLENGE

**13** Can you think of a method that can help you to find the mass of something too heavy for a set of scales, such as an elephant?

**14** Density is a measure of mass per volume. For example, water has a density of 1 kg per 1 L or 1 kg/L.
Use the equation:
density = mass ÷ volume.



**a** Find the density of each unknown liquid using the mass and volume listed in this table.

| | Mass | Volume |
|---|---|---|
| **i** | 500 g | 500 mL |
| **ii** | 2 kg | 500 mL |
| **iii** | 750 g | 250 mL |
| **iv** | 125 g | 1 L |

**b** Why might it be important to make sure that, before comparing densities, the same units are used for the mass measurements and the volume measurements?

**c** Express all your answers to part **a** in g/mL.
This is easiest if you convert units before dividing.

**d** Which liquid has a higher density, **i** or **ii**?

**15** Your friend buys a beautiful gold necklace. However, she is suspicious that it might not be pure gold and is not sure how to check.
She already owns a locket she knows is pure gold, which has a mass of 96.5 g and a volume of 5 $cm^3$.



**a** What is the density of the locket?

The necklace has a mass of 125 g and a volume of 9.6 $cm^3$.

**b** What is the density of the necklace?

**c** Do the densities match? Which has a lower density?

**d** What might this mean? Remember that your friend wants to know if the necklace is pure gold.

**Reflect**

Why is it important to make sure you are using the correct unit of mass?

# 8H Understanding temperature




## Start thinking!

For this task, you will need to work in pairs and have access to a thermometer.

1 What is the major interval on the thermometer?

2 What is the minor interval on the thermometer?

3 Draw a table and estimate the temperature of five different places in the room. Include one place in the sun, one in the shade, and under your armpit or your knee.

4 Use the thermometer to record the temperature at your five places.

5 Which place was the hottest? What temperature did you record?

6 Which place was the coldest? What temperature did you record?

7 What was the difference in temperature between these two places?

8 Was there a difference between the temperature in the sun and in the shade? Why do you think this is?

9 If official temperatures are recorded in the shade, comment on the temperature you feel on a hot, sunny day.

## KEY IDEAS

- Temperature measures how hot or cold something is relative to a reference point.
- Temperature is most commonly measured in degrees Celsius (°C).
- The reference points for °C are the freezing point of water (0°C) and the boiling point of water (100°C) at sea level.
- Some thermometers measure temperature in degrees Fahrenheit (°F).
- To find the difference between two temperatures, subtract the lower temperature from the higher temperature.

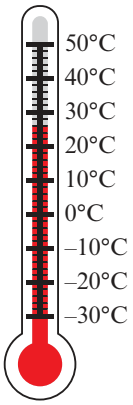
**NOTE** Subtracting a negative number is the same as adding a positive number.

# EXERCISE 8H Understanding temperature

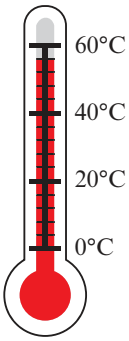
UNDERSTANDING AND FLUENCY

**1** Read the scales on these thermometers.

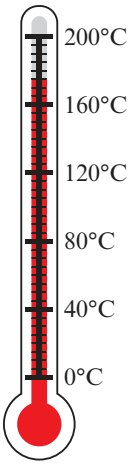
**a**



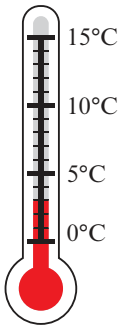

**b**



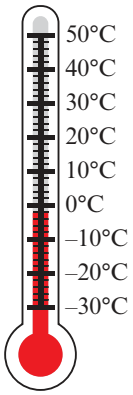

**c**




**d**




**e**




**2** Choose which part from question **1** matches the temperature of each situation described below.

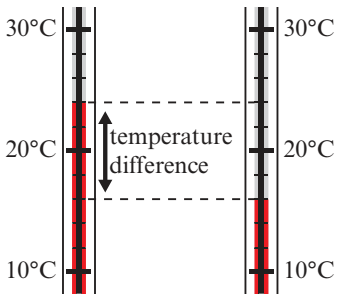
- **a** a piece of metal sitting in the sun on a hot day
- **b** inside an oven
- **c** a very cold winter's night
- **d** warm spring day
- **e** inside your refrigerator

## EXAMPLE 8H-1 Calculating the difference between two positive temperatures

Calculate the temperature difference between 24°C and 16°C.

### THINK

**1** Imagine the measurements on a thermometer scale. The higher temperature is 24 degrees above 0 and the lower temperature is 16 degrees above 0.




**2** Calculate the number of degrees between the two temperatures by subtracting the lower temperature from the higher temperature.

### WRITE

24°C means 24°C above 0°C.
16°C means 16°C above 0°C.

difference in temperature
= 24°C − 16°C
= 8°C



**3** Calculate the temperature difference between each of these pairs.

- **a** 22°C and 85°C
- **b** 17°C and 44°C
- **c** 30°C and 102°C
- **d** 9°C and 21°C
- **e** 42°C and 51°C
- **f** 66°C and 81°C
- **g** 101°C and 47°C
- **h** 29°C and 92°C

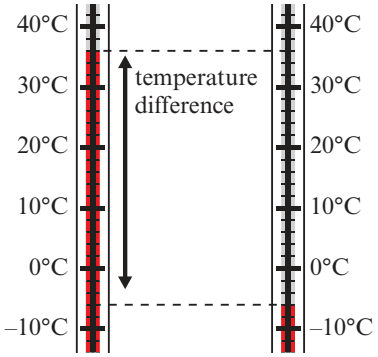
## EXAMPLE 8H-2

### Calculating the difference between temperatures when one is negative

Find the difference in temperature between day and night in the desert, if it hits 36°C during the day and drops to −6°C at night.

**THINK**

**1** Imagine the measurements on a thermometer scale. The higher temperature is 36 degrees above 0 and the lower temperature is 6 degrees below 0.




**2** Calculate the number of degrees between the two temperatures.

**3** Write your answer.

**WRITE**

36°C means 36°C above 0°C.
−6°C means 6°C below 0°C.

difference in temperature
= 36°C − (−6°C)
= 36°C + 6°C
= 42°C

The difference in temperature between day and night in the desert is 42°C.

**4** Calculate the difference in temperature between each of these pairs.

- **a** −20°C and 50°C
- **b** −31°C and 23°C
- **c** −6°C and 25°C
- **d** −15°C and 33°C
- **e** −8°C and 42°C
- **f** −19°C and 5°C
- **g** −23°C and 39°C
- **h** −35°C and 11°C

**5** Many thermometers today are digital. Look at the following thermometers and suggest where they may be useful.

**a**



**b**



**c**



**d**



**6** Wind chill is where the wind makes it feel colder than the temperature recorded on a thermometer. It can be very noticeable in areas that get very cold in winter, such as Canada and Russia. For example, if 18°C is recorded using a thermometer and there is a −5°C wind chill, the apparent temperature (what it feels like) is 13°C.
Calculate the apparent temperature for each of these.

**a** 15°C with a −5°C wind chill
**b** 0°C with a −10°C wind chill
**c** 25°C with a −7°C wind chill
**d** 12°C with a −12°C wind chill
**e** 4°C with a −16°C wind chill
**f** −25°C with a −15° wind chill

**7** The coldest temperature recorded on Earth was −89.2°C at Vostok, Antarctica. The hottest temperature recorded on Earth was 56.7°C in Death Valley, USA.
Find the range of temperatures on Earth (the difference between these two temperatures).



**8** Temperature is often considered to be relative. Room temperature is usually considered to be about 21°C. Discuss the difference in 'feel' of 21°C if you have:

**a** just left a sauna at 40°C
**b** just walked out of a freezer room at −20°C.

PROBLEM SOLVING AND REASONING

**9** The coldest inhabited place on Earth is Oymyakon in Russia. Look at this table and answer the questions below.

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | Year |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Average high °C (°F) | −40.7 (−41) | −35.0 (−31) | −21.9 (−7) | −3.6 (26) | 9.2 (49) | 19.9 (68) | 22.2 (72) | 18.2 (65) | 8.9 (48) | −8.9 (16) | −32.5 (−27) | −41.9 (−43) | −8.8 (16) |
| Average low °C (°F) | −49.2 (−57) | −47.0 (−53) | −41.2 (−42) | −24.0 (−11) | −4.8 (23) | 4.0 (39) | 5.8 (42) | 1.6 (35) | −4.7 (24) | −20.6 (−5) | −41.3 (−42) | −48.9 (−56) | −22.5 (−9) |

**a** What is the average high temperature for April?
**b** What is the average high temperature for the year?
**c** Which month has the highest average high temperature for the year? What is the temperature?
**d** Which month has the lowest average high temperature for the year? What is the temperature?
**e** What is the average low temperature for the year?
**f** Which month has the highest average low temperature for the year? What is the temperature?
**g** Which month has the lowest average low temperature for the year? What is the temperature?

**10** The hottest inhabited place on Earth is Wyndham, Western Australia.

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | Year |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Average high °C (°F) | 36.9 (98) | 35.9 (97) | 36.0 (97) | 36.0 (97) | 33.6 (92) | 31.0 (88) | 31.1 (88) | 33.6 (92) | 36.7 (98) | 38.9 (102) | 39.5 (103) | 38.2 (101) | 35.6 (96) |
| Average low °C (°F) | 26.4 (80) | 25.9 (79) | 25.5 (78) | 23.6 (74) | 20.7 (69) | 17.7 (64) | 16.9 (62) | 18.9 (66) | 22.8 (73) | 25.8 (78) | 27.2 (81) | 27.0 (81) | 23.2 (74) |

**a** What is the average high temperature for the year?
**b** Which month has the highest average high temperature for the year? What is the temperature?
**c** Which month has the lowest average high temperature for the year? What is the temperature?
**d** What is the average low temperature for the year?
**e** Which month has the highest average low temperature for the year? What is the temperature?
**f** Which month has the lowest average low temperature for the year? What is the temperature?

**11** Use the tables in questions **9** and **10** to answer these questions. For parts **a**–**c**, use the year averages.
**a** What is the difference between the average high temperatures for Wyndham and Oymyakon?

**b** What is the difference between the average low temperatures?

**c** Which town has a larger range of temperatures?

**d** Which is warmer: a night in July in Wyndham (62°F) or a day in July in Oymyakon (22.2°C)?

**e** Where would you rather live and why?

CHALLENGE

**12** Different situations require different scales on thermometers. Draw a thermometer with a scale suited to record temperatures:

**a** in your lounge room

**b** inside your fridge

**c** inside your oven

**d** inside your kettle.



**13** The Fahrenheit temperature scale uses units of degrees Fahrenheit (°F). This scale was once used in Australia and can be found on a lot of thermometers (a few countries, such as the USA, still use it).
To convert from °C to °F, the formula $C = \frac{5}{9} \times (F - 32)$ can be used, where $C$ is the temperature in degrees Celsius and $F$ is the temperature in degrees Fahrenheit.

**a** Convert these temperatures to degrees Celsius.

**i** 100°F **ii** 50°F **iii** 32°F

**iv** 212°F **v** 0°F **vi** 75°F

**b** If the formula is rearranged, temperatures in degrees Celsius can be converted to degrees Fahrenheit. Use the formula $F = \frac{9}{5}C + 32$ to convert these temperatures to degrees Fahrenheit.

**i** 0°C **ii** 25°C **iii** 50°C

**iv** 15°C **v** −5°C **vi** 20°C

**14** Find the following, giving your answer in degrees Celsius.

**a** the temperature difference between 13°F and 102°F

**b** the apparent temperature if it is 5°F with a −14°F windchill

**c** the temperature difference between −5°F and 45°C

**d** the apparent temperature if it is 38°C but humidity increases the feeling of this by 10°F

**Reflect**

Why is recorded temperature not always a good indicator of how hot or cold something feels?

# CHAPTER REVIEW

## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

| | | | |
|---|---|---|---|
| scales | convert | timetable | daylight savings |
| scale marks | time | timeline | units of mass |
| intervals | units | time zones | temperature |
| estimate | 24-hour time | Greenwich Mean Time | degrees Celsius |
| accuracy | elapsed time | International Date Line | degrees Fahrenheit |

## MULTIPLE-CHOICE



8A **1** What measurement is shown?

**A** 100 **B** 50
**C** 354 **D** 390

8B **2** Which is the best estimate for the mass of a cat?

**A** 2.9865 kg **B** 3294 g
**C** 30 kg **D** 3 kg

8C **3** What is the number of hours in 1000 seconds closest to?

**A** 1 **B** 17
**C** 0.3 **D** 3 600 000

8C **4** Which is the largest measurement?

**A** 130 minutes **B** 2.25 hours
**C** 4000 seconds
**D** 2 hours 20 minutes

8D **5** What is the time difference between 11.00 am and 5.31 pm?

**A** 6.31 hours
**B** 4 hours 31 minutes
**C** 5 hours 29 minutes
**D** 6 hours 31 minutes

8D **6** What is the date 41 days after 25 December?

**A** 30 January **B** 4 February
**C** 2 February **D** 14 November

Use this flight timetable for questions **7** and **8**.

| | Flight A | Flight B |
|---|---|---|
| Leaves Melbourne | 6 September, 17:00 | 7 September, 14:00 |
| Arrives Sydney | 6 September, 18:25 | 7 September, 15:30 |
| Departs Sydney | 6 September, 19:40 | 7 September, 18:00 |
| Arrives Honolulu | 6 September, 09:25 | 7 September, 07:45 |

8E **7** How long does Flight A's first leg take?

**A** 1.25 hours
**B** 1 hour 30 minutes
**C** 16 hours 25 minutes
**D** 1 hour 25 minutes

8F **8** What is the date and time in Melbourne when Flight B lands in Honolulu?

**A** 7 September, 07:45
**B** 8 September, 03:45
**C** 6 September, 11:45
**D** 7 September, 27:45

8G **9** Which is the largest measurement?

**A** 50 000 g **B** 5000 kg
**C** 0.5 t **D** 5 000 000 mg

8H **10** Which pair of measurements has the largest range?

**A** 50°C and 15°C
**B** −10°C and 5°C
**C** −19°C and 11°C
**D** −14°C and −1°C

## SHORT ANSWER

8A **1** Record the measurement on these scales.

a




b




8B **2** Which units of measurement would you use to measure each of the following?

a length of a truck

b time taken to run a marathon

c mass of a small bird

d distance between Melbourne and Sydney

8B **3** Estimate then measure the length of this object.



8C **4** Convert the following.

a 2 hours into minutes

b 180 seconds into minutes

c 30 minutes into hours

d 180 seconds into hours

e 4.1 hours into hours and minutes

f 225 minutes into hours and minutes

8D **5** Calculate the time elapsed between:

a 4.30 pm and 9.00 pm

b 9.24 am and 11.58 pm

c 0745 and 1420

d 1530 and 9.27 pm.

8D **6** Find the time 75 minutes after the first times mentioned in question **5**.

8E **7** Consider the weekday train timetable on page 433.

a If you took the 12.15 pm train from Traralgon, what time would you get to Flinders Street Station?

b If you wanted to be in Richmond by midday, what is the latest time you could leave Warragul?

c How much longer does the 14:07 IC train take than the train that leaves at 1.15 pm?

d If you wanted to be at Southern Cross by 10 am, how late could you leave Trafalgar?

8F **8** Consider the world map on page 439.

a What is the time difference between your time zone and New Zealand?

b If it is midday in the UK, what time is it in each of these countries?

i India ii Japan

iii South Africa iv Cuba

c To call New York at 7 pm New York time, what time would you call?

d What time is it in the UK when it is midday in each of these countries?

i China ii Mexico

iii Italy iv Madagascar

8G **9** Convert the following.

a 34 g into mg b 280 g into kg

c 0.3 t into kg d 1.7 kg into mg

e 42 mg into kg f 5.2 t into g

8H **10** Calculate the temperature difference between these pairs.

a 5°C and 31°C

b −5°C and 31°C

c −20°C and −5°C

d −45°C and 17°C

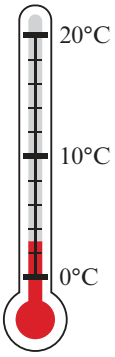
8H **11** Use the formulas listed on page 455 to convert:

a 120°F into °C b 38°C into °F

c 45°F into °C d −4°C into °F.

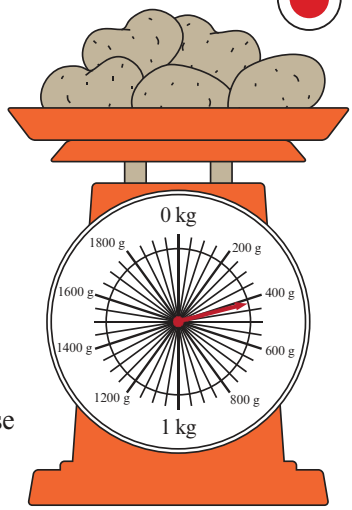
## NAPLAN-STYLE PRACTICE

**1** The thermometer shown here measures the temperature in a cool room.
What temperature does it show?




[ ] °C

**2** What is the best estimate of the mass recorded on the scales?




- ◯ 400 g
- ◯ 450 g
- ◯ 407 g
- ◯ 420 g

**3** Which of these is best for measuring a mobile phone?

- ◯ 30-cm ruler
- ◯ 1-m ruler
- ◯ trundle wheel
- ◯ microscope

**4** Which of these measurements would be the most realistic for a mobile phone?

- ◯ 8.23678 cm
- ◯ 0.9 m
- ◯ 2 cm
- ◯ 7.5 cm

**5** How many minutes in a day?

- ◯ 24
- ◯ 1440
- ◯ 60
- ◯ 3600

**6** What is 3.32 pm written in 24-hour time?

[ ]

**7** What is the time difference between 9.30 am and 1.15 pm?

- ◯ 45 minutes
- ◯ 4 hours 15 minutes
- ◯ 3 hours 45 minutes
- ◯ 3 hours 15 minutes

**8** What time will it be 75 minutes after the time shown on the clock below?




- ◯ 5.12 pm
- ◯ 3.22 pm
- ◯ 3.62 pm
- ◯ 5.52 pm

**9** What time will it be 45 minutes before the time shown on the clock above?

- ◯ 3.37 pm
- ◯ 3.52 pm
- ◯ 3.42 pm
- ◯ 4.12 pm

**10** What is the time difference between the times from questions **8** and **9**?

[ ]

Questions **11**–**13** refer to this timetable.

| | Tuesday | Thursday | Saturday |
|---|---|---|---|
| Melbourne | 0805 | 0805 | 0840 |
| North Shore | 0947 | 0947 | 0947 |
| Ararat | 1149 | 1149 | 1149 |
| Horsham | 1256 | 1256 | 1256 |
| Dimboola | 1322 | 1322 | 1322 |
| Nhill | 1344 | 1344 | 1344 |
| Bordertown | 1404 | 1404 | 1404 |
| Murray Bridge | 1557 | 1604 | 1557 |
| Adelaide | 1745 | 1745 | 1745 |

**11** What time does the Thursday train from Melbourne arrive in Murray Bridge?

[ ]

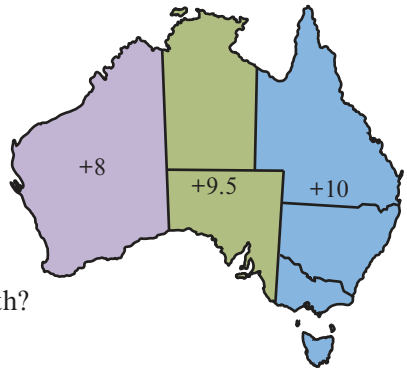
**12** If Murray Bridge is half an hour behind Melbourne, what time is it in Melbourne when the train arrives in Murray Bridge?

- ◯ 1604
- ◯ 1534
- ◯ 4.34 pm
- ◯ 1557

**13** How long is the train trip from Melbourne to Adelaide on Tuesday?

- ◯ 10 hours 10 minutes
- ◯ 9 hours 40 minutes
- ◯ 9 hours 10 minutes
- ◯ 9 hours 35 minutes

Questions **14–17** refer to this map of Australia.




**14** How many hours behind Hobart is Perth?

**15** What is the time in Adelaide when it is 5.50 am in Brisbane?

**16** What is the time difference between Perth and Sydney?

**17** Nikita takes a flight from Perth to Sydney. If the flight leaves at 10.00 am and lasts for 4 hours, what time does it arrive in Sydney?

- ☐ 6.00 pm
- ☐ 4.00 pm
- ☐ 2.00 pm
- ☐ 12.00 pm

**18** What is 42 500 g written in kilograms?

☐ kg

**19** A mass of 450 g is added to 1.5 kg. What is the total mass?

☐ kg

**20** What is the temperature difference between 17°C and –4°C?

☐ °C

**21** Ava's grandmother asks Ava what the temperature is in degrees Fahrenheit. The method for converting from degrees Celsius to degrees Fahrenheit is to multiply the number of degrees Celsius by 1.8 and then add 32 to the result. If the temperature is a sunny 25°C, what should Ava tell her grandmother?

- ☐ 77°F
- ☐ 99°F
- ☐ 51.4°F
- ☐ 82.6°F

## ANALYSIS

A recipe for cupcakes calls for 1 cup of milk, but you don't have any cup measurements. You do know that 1 cup is equivalent to 250 mL, which is roughly equivalent to 250 g.

**a** If you want to make a double batch, what mass of milk will you need?

**b** You need to have 60 cupcakes. If the recipe only makes 12 cupcakes, how many batches will you need?

**c** Calculate the mass of milk needed, giving your answer in both grams and kilograms.

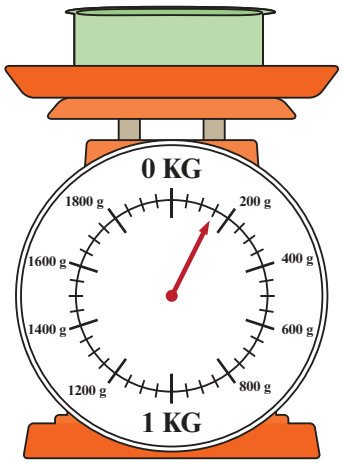
**d** The scales show the mass of an empty container ready to pour the milk into. Copy and draw where the scale needle should sit after you pour in the milk.

After making the mixture, you need to bake the cupcakes for 15 minutes. However, you discover that the oven scale is in degrees Fahrenheit.

**e** If the oven needs to be set to 200°C, what is this in degrees Fahrenheit?

**f** If the oven clock reads 14:52, what time will it show when the cupcakes are ready to come out?

**g** When the cupcakes come out of the oven, they have a temperature of about 180°C, but you can't eat one until they are 35°C. By how much must they drop in temperature?



# CONNECT

## Planning a trip

To go on an overseas trip requires considerable planning and a strong ability to work with time, mass and temperature.

Over the school holidays, your family decides to go away for four weeks. What type of things do you need to consider when planning a holiday?




### Your task

To plan your trip, follow these steps.

1 Where would you like to visit? Consider:
   - time of year and temperature
   - time zones for making and receiving calls from home
   - where you would stay.
2 What flights would be required? Consider:
   - length and cost of flights
   - times of arrivals and departures in the correct time zone.
3 What luggage would you need? Consider:
   - what to pack
   - restrictions on the mass of luggage you can take.
4 Would you use public transport? Consider:
   - getting to and from the airports – you may like to include timetables
   - using transport between cities at your destination.
5 What activities would you like to do? Consider:
   - time of year and day they may occur
   - mass and temperature restrictions.
6 What would be the costs for each of the above categories?













Complete the 8 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a PowerPoint presentation
- a video
- a booklet
- other (check with your teacher).

# 9 LENGTH, AREA AND VOLUME


**9A** Understanding length

**9B** Perimeter

**9C** Understanding area

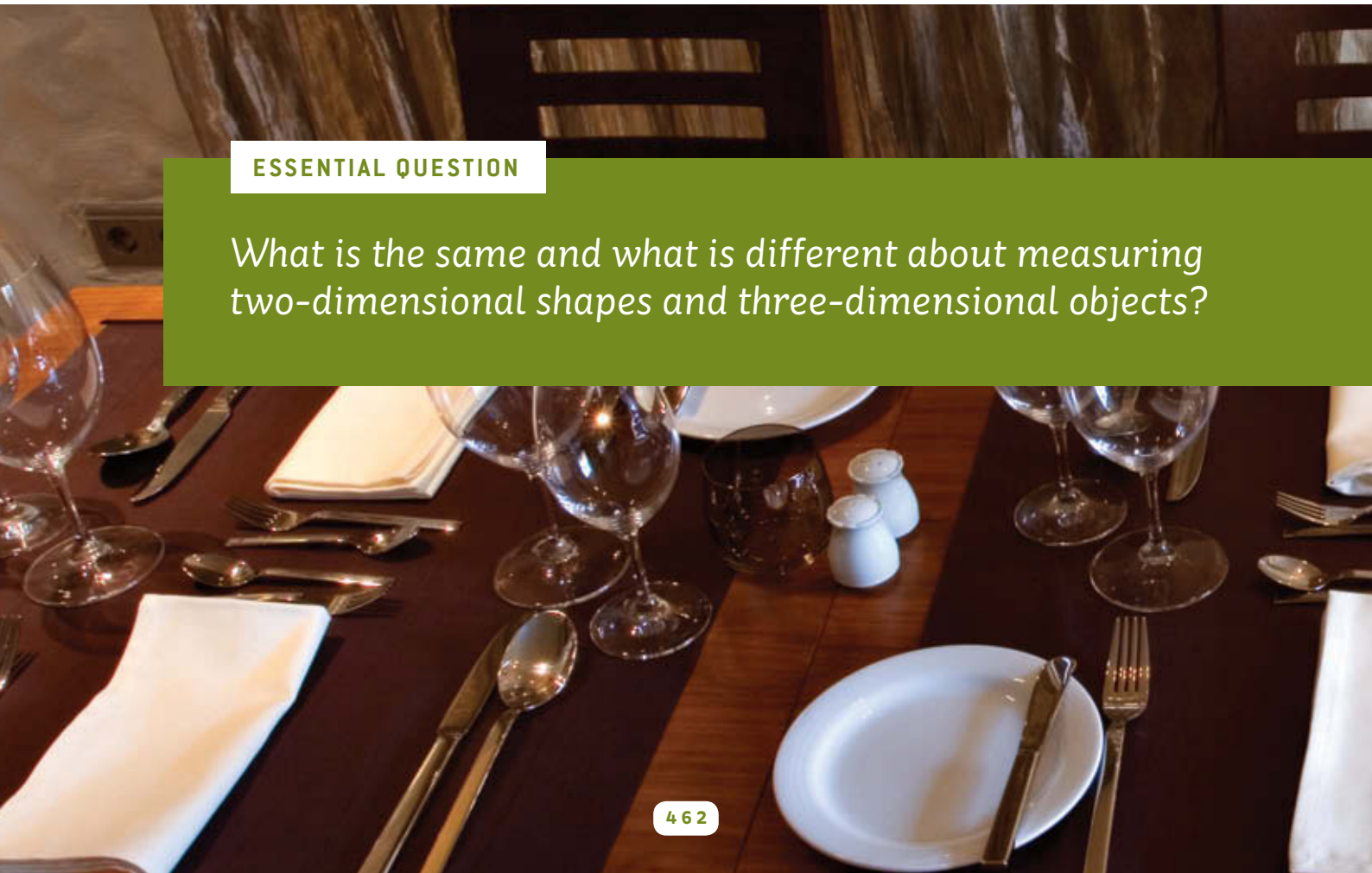
**9D** Area of a rectangle

**9E** Area of a parallelogram

**9F** Area of a triangle

**9G** Surface area

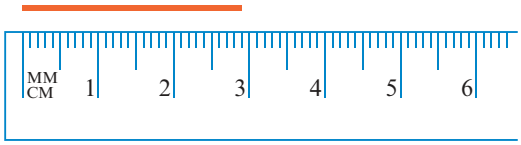
**9H** Volume and capacity

**ESSENTIAL QUESTION**

*What is the same and what is different about measuring two-dimensional shapes and three-dimensional objects?*

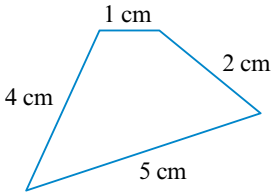
9A **1** Use the scale on the ruler to measure the length of the line.




What is the length of this line?

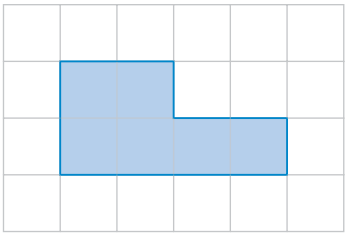
**A** 24 mm **B** 29 mm
**C** 34 mm **D** 39 mm

9B **2** To trace around this shape, what distance would the tip of your pencil travel?



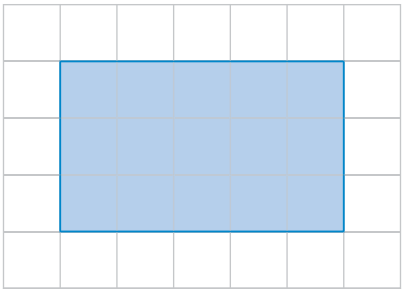

**A** 6 cm
**B** 8 cm
**C** 10 cm
**D** 12 cm

9C **3** What is the area of this shape?



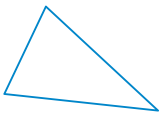
**A** 6 square units **B** 7 square units
**C** 8 square units **D** 9 square units

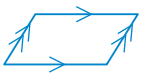
9D **4** This rectangle is drawn on 1-cm grid paper. What is its area?



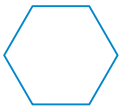
**A** 5 cm$^2$ **B** 8 cm$^2$
**C** 15 cm$^2$ **D** 16 cm$^2$

9E **5** Which of these shapes is a parallelogram?

**A**


**B**


**C**

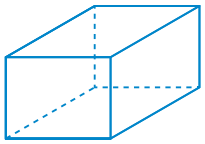

**D**


9E **6** Which of these statements about a parallelogram is *not* true?

**A** A parallelogram has four sides.
**B** Opposite sides have equal length.
**C** Opposite sides are parallel.
**D** All sides are different lengths.

9F **7** How many sides does a triangle have?

Questions **8** and **9** refer to this object.

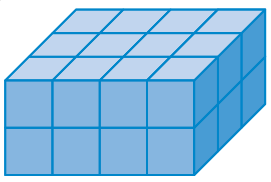


9G **8** How many faces does this object have?

9G **9** Which face is the same size and shape as the front face?

**A** back face **B** top face
**C** bottom face **D** right face

9H **10** This object is made up of blocks. Each block is 1 cm long, 1 cm wide and 1 cm high. Choose the statement that correctly describes the dimensions of this object.



**A** length 4 cm, width 5 cm, height 1 cm
**B** length 4 cm, width 3 cm, height 2 cm
**C** length 2 cm, width 3 cm, height 2 cm
**D** length 4 cm, width 4 cm, height 3 cm

# 9A Understanding length

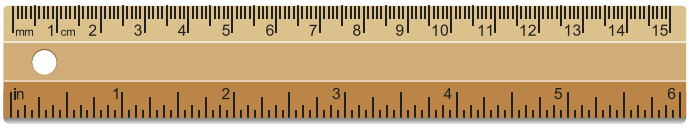
## Start thinking!

**Length** can be measured in millimetres (mm), centimetres (cm), metres (m) or kilometres (km).

**1** Look around the room. Find an object that has a length dimension (length, **width** or height) of 1 mm.

**2** Look for an object that has a length of 1 cm and another that has a length of 1 m. Visualising these unit lengths will help you estimate the length of things.

**3** Look at your ruler. How many millimetres are equivalent to 1 cm?




**4** Look at a tape measure or metre ruler.
**a** How many centimetres are equivalent to 1 m? **b** How many millimetres are equivalent to 1 m?

**5** How many metres are equivalent to 1 km? (Think about the prefix 'kilo'. What does 'kilo' mean?)

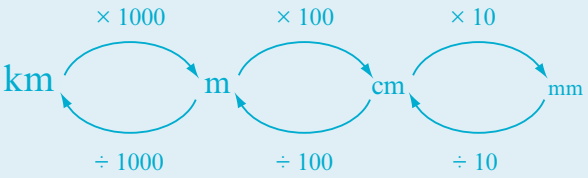
**6** Complete these statements.
**a** _____ mm = 1 cm **b** _____ cm = 1 m **c** _____ m = 1 km

**7** Discuss with a partner how to convert a length measurement from centimetres to millimetres. How is this different from converting millimetres to centimetres? Would this method work for other units?

**8** Explain how to convert a length measurement to: **a** a smaller unit **b** a larger unit.

## KEY IDEAS

- Metric units of length are millimetres (mm), centimetres (cm), metres (m) and kilometres (km).
- If you start by estimating a length, you can decide whether your length measurement is reasonable.
- A unit of length can be converted to another unit of length by multiplying or dividing by the conversion factor.
- Multiply by the conversion factor to convert to a smaller unit.
- Divide by the conversion factor to convert to a larger unit.



# EXERCISE 9A Understanding length

UNDERSTANDING AND FLUENCY

**1** Estimate the length of each line segment.

**a** _____

**b** _____

**c** _____

**2** Use a ruler to accurately measure the length of each line segment in question **1**.

**3** Complete each statement by using the words *multiply* or *divide*.

**a** When converting a length measurement to a smaller unit, you need to _____ by the conversion factor.

**b** When converting a length measurement to a larger unit, you need to _____ by the conversion factor.

## EXAMPLE 9A-1 Converting units of length

Complete these length conversions.

**a** 5.4 km = _____ m **b** 900 mm = _____ cm

**THINK**

**a** Multiply by 1000 as you are converting to a smaller unit. (1 km = 1000 m.)

**b** Divide by 10 as you are converting to a larger unit. (10 mm = 1 cm.)

**WRITE**

**a** 5.4 km = (5.4 × 1000) m
= 5400 m

**b** 900 mm = (900 ÷ 10) cm
= 90 cm

**4** Complete these length statements.

**a** 8 km = _____ m

**b** 3 m = _____ cm

**c** 700 mm = _____ cm

**d** 500 cm = _____ m

**e** 16 cm = _____ mm

**f** 2000 m = _____ km

**g** 3.5 m = _____ cm

**h** 9 mm = _____ cm

**i** 1.34 km = _____ m

**j** 0.47 cm = _____ mm

**k** 84 m = _____ km

**l** 51.2 cm = _____ m

**m** 0.06 km = _____ m

**n** 3.8 m = _____ cm

**o** 7.9 mm = _____ cm

**p** 4310 mm = _____ km

**5** Complete these length statements.

**a** 4 km = _____ m = _____ cm

**b** 12 000 mm = _____ cm = _____ m

**c** 73 m = _____ cm = _____ mm

**d** 8400 cm = _____ m = _____ km

**e** _____ mm = 250 cm = _____ m

**f** _____ cm = 362 m = _____ km

**6** Convert these measurements to the units shown in brackets.

**a** 55 cm (mm)

**b** 27 000 m (km)

**c** 41 mm (cm)

**d** 6.2 m (cm)

**e** 3.6 km (m)

**f** 89 cm (m)

**g** 1.7 km (cm)

**h** 500 mm (m)

**i** 7000 cm (km)

**j** 3.65 m (mm)

**k** 0.94 km (mm)

**l** 1800 mm (km)

**7** Some students made errors when they were converting some length measurements. Explain the error in each case and give the correct answer.

**a** 55 cm = 5.5 mm

**b** 24 km = 2400 m

**c** 0.66 m = 60.6 cm

**d** 1.73 mm = 0.0173 cm

**e** 420 cm = 42 m

**f** 800 m = 0.08 km

**8** Put the length measurements in each list into ascending order (from smallest to largest).

**a** 5 cm, 15 mm, 60 mm

**b** 2 m, 190 cm, 210 mm

**c** 0.7 km, 701 m, 7001 cm

**d** 82 mm, 8 cm, 0.8 m

**e** 3 km, 3000 mm, 300 m, 30 cm

**f** 5500 mm, 0.005 km, 5.7 m, 560 cm

**9** Without using a ruler, draw a line segment 7 cm long. Have a classmate use a ruler to check how close your line segment is to 7 cm.

**10** Give three examples where the length would be best measured in each of these units.

**a** mm **b** cm **c** m **d** km

**11** **a** Without using a ruler, estimate the length of the caterpillar in:

**i** centimetres

**ii** millimetres.

**b** Compare your answers to part **a** with a classmate. Discuss how you each got your answers.

**c** Use a ruler to accurately measure the length of the caterpillar. Think of some reasons why you and your classmate may not get the same measurement.



UNDERSTANDING AND FLUENCY

**12** Here are some life-size photos of objects. First estimate and then measure the lengths indicated.

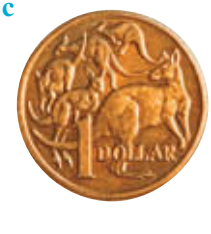
**a**



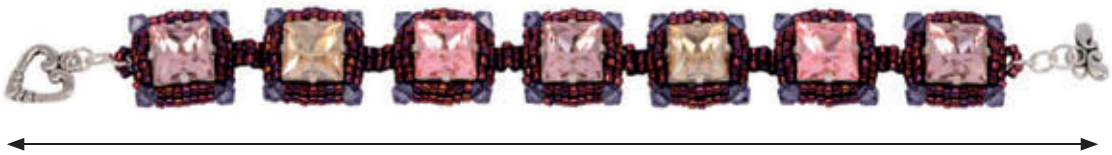
**b**



**c**




**d**



**13** Without using a ruler, estimate each of these.

**a** length of your pen
**b** height of your desk
**c** width of your fingernail
**d** height of the door
**e** width of the palm of your hand
**f** length of the room

**14** Discuss your answers to question **13** with a classmate. Do the measurements sound reasonable? Use a ruler or tape measure to check how close your estimates are to the actual measurements.

**15** Which unit of length (mm, cm, m or km) would be most suitable for measuring each of these?

**a** distance to the Moon
**b** length of a bracelet
**c** height of a tennis net
**d** thickness of a CD
**e** depth of a fish pond
**f** length of a cricket pitch

PROBLEM SOLVING AND REASONING

**16** The length of a swimming pool is 50 m.

**a** How far would you swim if you completed 25 laps?
**b** Would this distance be more or less than 1 km?
**c** How many laps would you swim to complete a distance of 2 km?

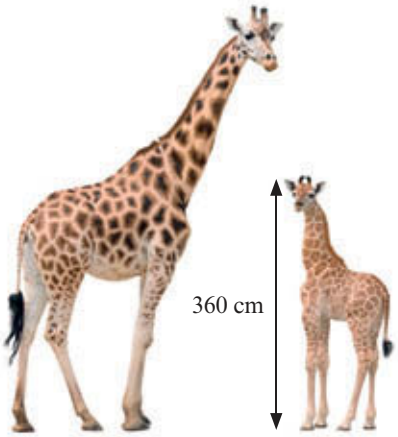
**17** A ream of paper contains 500 sheets. Calculate the thickness of a single sheet of paper. State your answer in millimetres.




**18** A group of people are lining up to buy tickets for a concert. Estimate how long the queue will be if there are 85 people in the line. Explain how you reached your answer.

**19** Estimate the following length measurements using the information supplied in each photo.

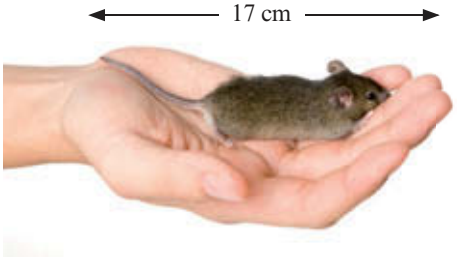
**a** height of the adult giraffe (cm)




**b** height of the pelican (m)



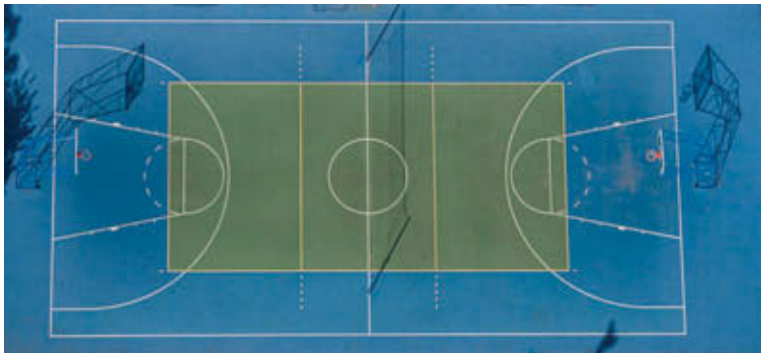

**c** length of the mouse's tail (mm)




**d** length of the snake (m)




**20** Different instruments can be used to measure length; for example, a ruler, tape measure or trundle wheel.



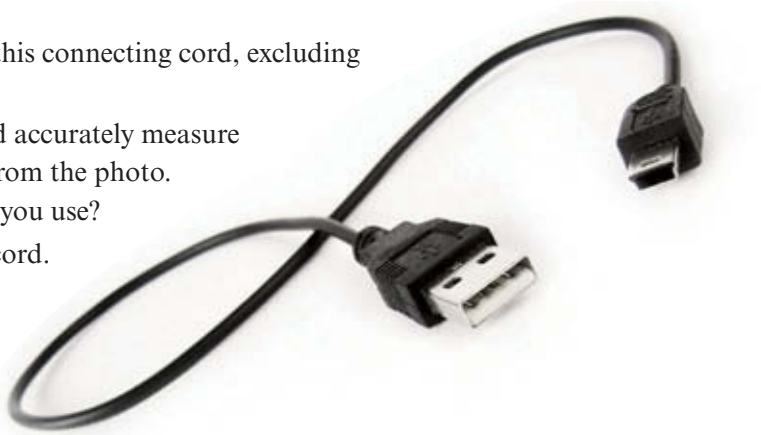
**a** Choose something in your classroom such as a desk, table, chair or the floor. Use an appropriate instrument to take length measurements. Draw a diagram and show all the measurements you found. Remember to include the unit you used.

**b** Choose something outside in the school grounds that could be measured in metres or perhaps kilometres. Again, measure it using an appropriate instrument. Draw a diagram and show all the measurements you found, including the units.

**c** Show your results for parts **a** and **b** to a classmate. Explain your choice of measuring instrument and length unit used. Were these the best choices?

CHALLENGE

**21** **a** Estimate the length of this connecting cord, excluding the connectors.

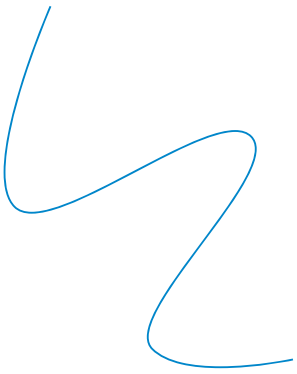


**b** Describe how you could accurately measure the length of the cord from the photo. What equipment could you use?

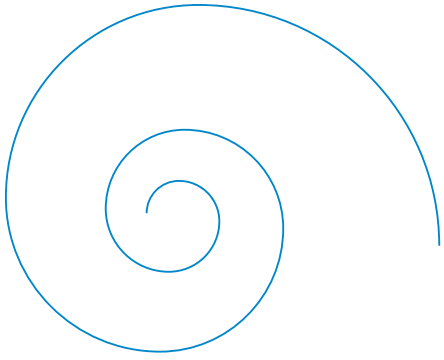
**c** Find the length of the cord. Compare your result to the estimate you made in part **a**.

**22** Estimate the length of each of these. Then use string or cotton and a ruler to measure their length.

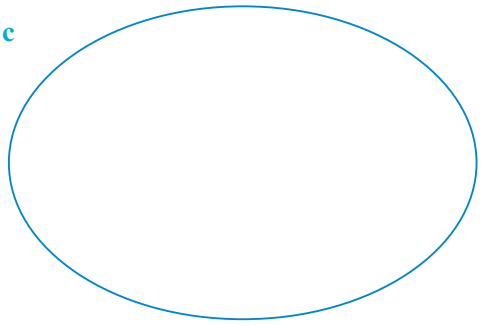
**a**



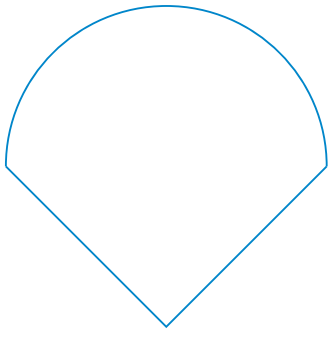
**b**



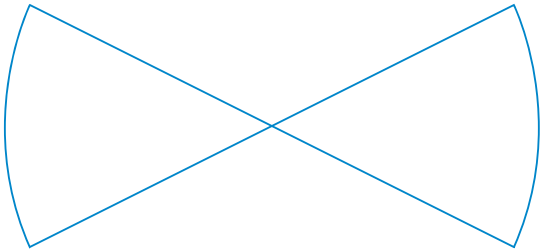
**c**



**d**



**e**



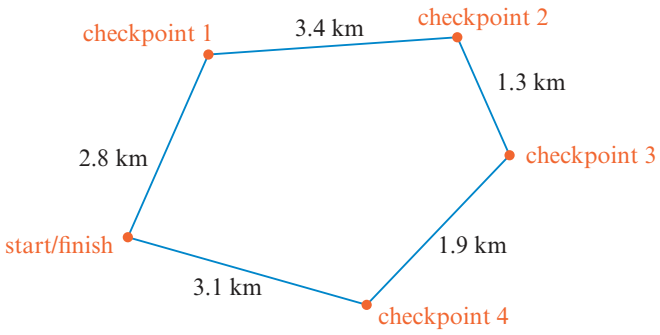
**Reflect**

How do you decide which instrument and unit to use when measuring length?

# 9B Perimeter

## Start thinking!

The course for a charity walk is shown. The walkers start and finish at the same place.






1 What is the distance from the start to the:
   - **a** first checkpoint?
   - **b** second checkpoint?
   - **c** third checkpoint?
   - **d** fourth checkpoint?

2 Calculate the total distance around the course from start to finish. Explain how you worked this out.

3 Compare your answer to question **2** with that of some classmates. Explain why they may be different.

4 The total distance around the course from start to finish is also called the **perimeter** of the course. Why are the distances you calculated in question **1** not considered to be the perimeter?

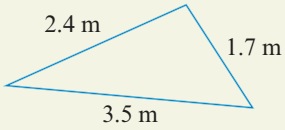
## KEY IDEAS

- Perimeter is the distance around the outside edge of a **two-dimensional** (2D) shape.
- Since perimeter is a length measurement, common units are millimetres, centimetres, metres and kilometres.
- To calculate the perimeter, add the length of each side together.
- All length measurements must be in the same unit before being added.

# EXERCISE 9B Perimeter

## EXAMPLE 9B-1 Calculating perimeter

Calculate the perimeter of this shape.




**THINK**

1. There are three sides to the shape so there are three lengths to be added. Write the calculation.
2. Check whether all the measurements are in the same unit (yes).
3. Add all the lengths together and include the appropriate unit.

**WRITE**

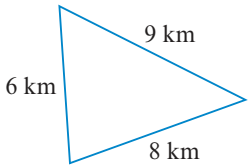
perimeter = 1.7 m + 2.4 m + 3.5 m

= 7.6 m

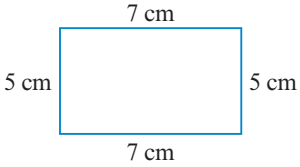
UNDERSTANDING AND FLUENCY

**1** Calculate the perimeter of each shape.

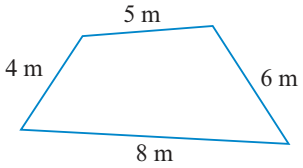
**a**



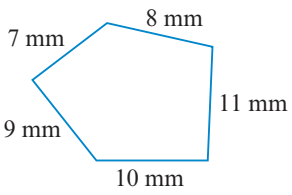

**b**



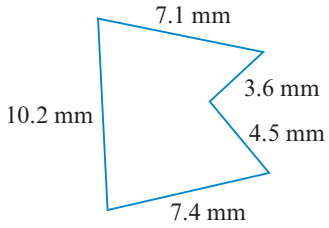

**c**



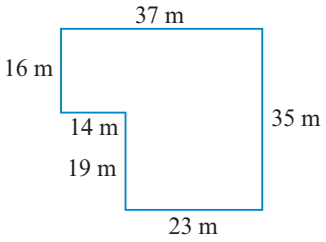

**d**




**e**

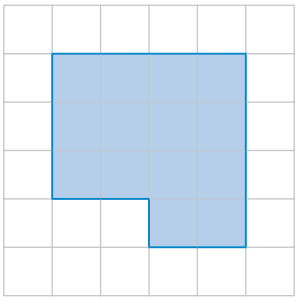



**f**

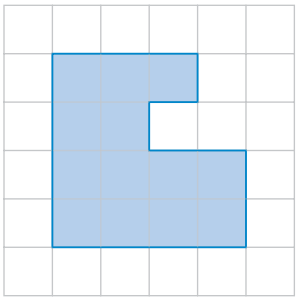


**2** Calculate the perimeter of these shapes, which are shown on 1-cm grid paper.

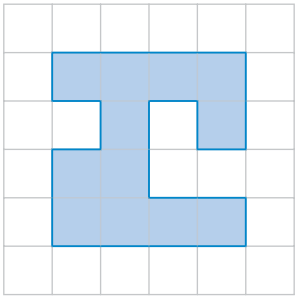
**a**



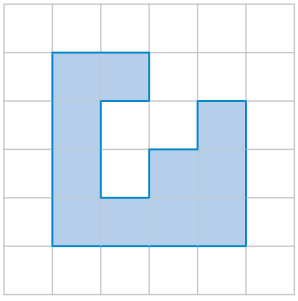
**b**



**c**



**d**

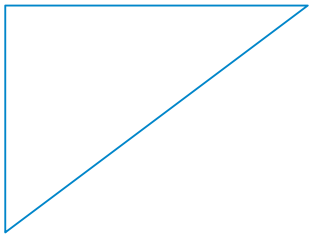


**3** Calculate the perimeter of each shape by using a ruler to measure the lengths of the sides.

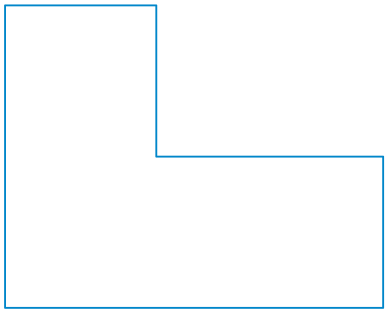
**a**



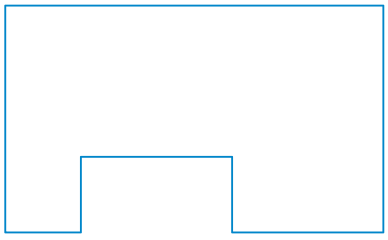
**b**



**c**

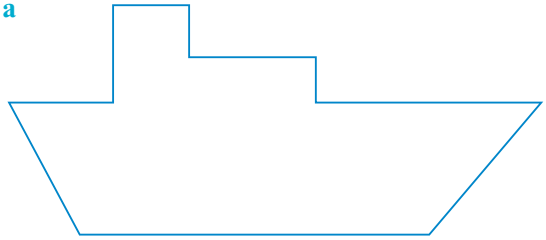


**d**

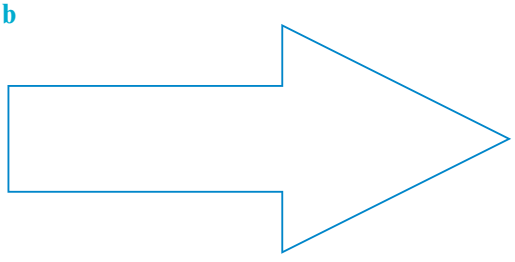


**4** Find the perimeter of each shape by using a ruler to measure each side to the nearest millimetre.

**a**

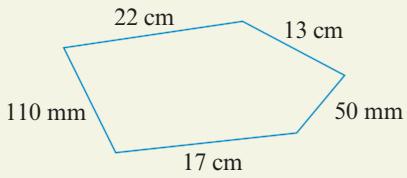


**b**



## EXAMPLE 9B-2 Calculating perimeter after converting units

Calculate the perimeter of this shape in centimetres.




### THINK

1. There are five sides to the shape so there are five lengths to be added. Write the calculation.
2. Check whether all the measurements are shown in the same unit (no). Express them in centimetres.
3. Add all the lengths together and include the appropriate unit.

### WRITE

perimeter = 22 cm + 13 cm + 50 mm
+ 17 cm + 110 mm

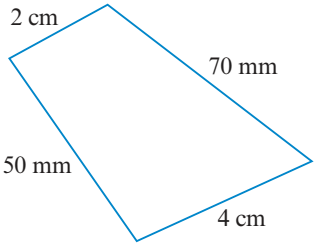
= 22 cm + 13 cm + 5 cm
+ 17 cm + 11 cm

= 68 cm

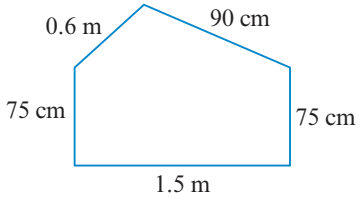
UNDERSTANDING AND FLUENCY

**5** Calculate the perimeter of each shape in centimetres.

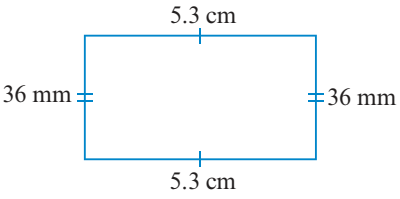
**a**



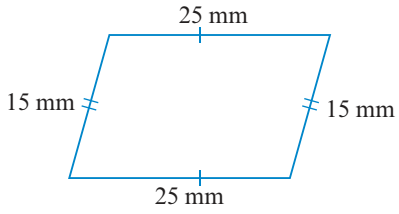

**b**



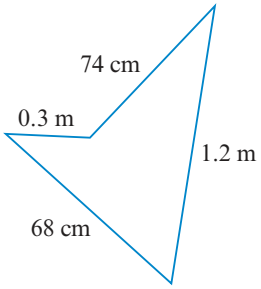

**c**



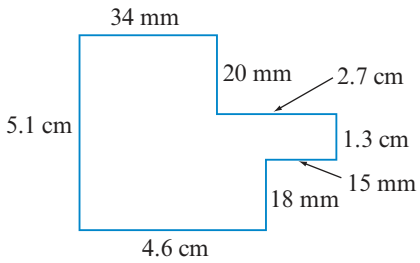

**d**




**e**

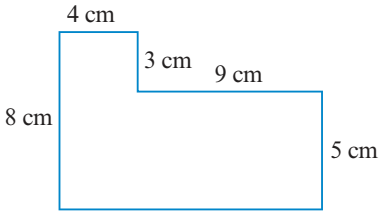



**f**

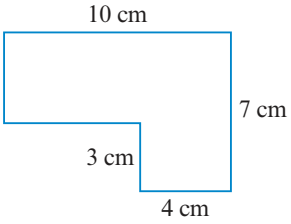


**6** Find the lengths of all the sides and then calculate the perimeter of each shape.

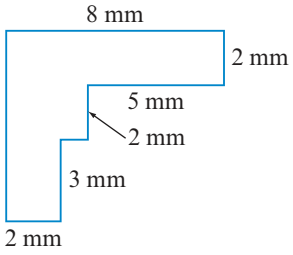
**a**



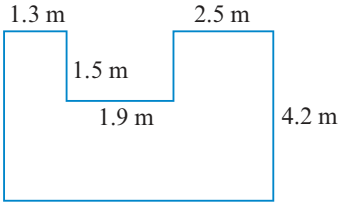

**b**



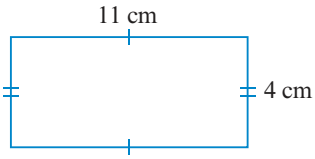

**c**




**d**




**7** A **rectangle** has length 11 cm and width 4 cm. The sides of this rectangle are marked with either | or || symbols.




**a** Explain what these symbols mean.

**b** Calculate the perimeter of this rectangle.

**c** Instead of adding the four sides, is there a quicker way to calculate the perimeter of a rectangle? Describe this shortcut.

**d** If $l$ is used to represent the length of a rectangle and $w$ is used to represent the width of a rectangle, write a formula for finding the perimeter of a rectangle.

**8** Use the formula to find the perimeter of these rectangles.

**a** length 5 cm, width 3 cm

**b** length 12 m, width 10 m

**c** length 47 mm, width 35 mm

**d** length 100 cm, width 24 cm

**e** length 6 m, width 4.5 m

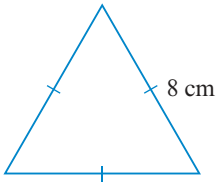
**f** length 3.2 cm, width 1.9 cm

**g** length 8.62 m, width 425 cm

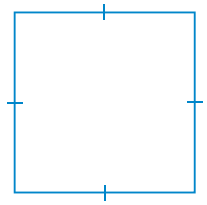
**h** length 15 mm, width 1.2 cm

**9** Calculate the perimeter of each shape. Look for a shortcut to use in each case.

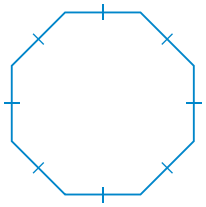
**a**



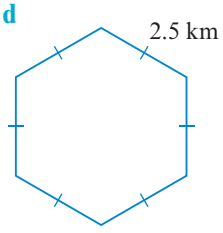

**b**




**c**




**d**

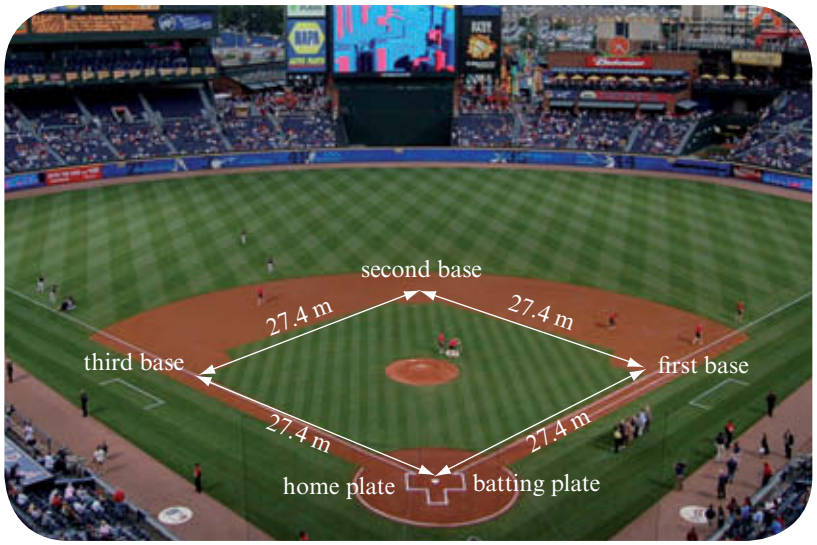



**10** For each shape in question **9**, write a formula for the perimeter using $l$ to represent the length of a side.

PROBLEM SOLVING AND REASONING

**11** In baseball, a batter must run from the batting plate to first base, then second base, third base and finally home. Use the measurements shown to answer the following.

- **a** How far does a batter run to reach first base after hitting the ball?
- **b** How far does a batter run to reach third base after hitting the ball?
- **c** How far does a batter run to reach the home plate when scoring a home run?
- **d** Which one out of parts **a**, **b** and **c** is the same as the perimeter of the baseball diamond?




**12** A volleyball court is 18 m long and 9 m wide.

- **a** Draw a diagram of the court showing its measurements and the net.
- **b** Calculate its perimeter.
- **c** To warm up, each team jogs five laps around their half of the court. That is, the players jog along the outside lines and along the net on their side of the court. How far does a player jog during the warm-up?

**13** The photo shown has a perimeter of 52 cm. What is the width of the photo?

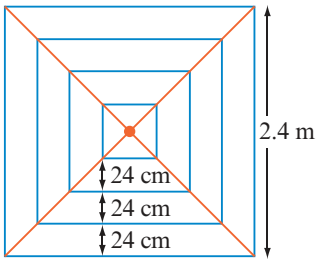



**14** A **square** drink coaster has a perimeter of 342 mm. What is the length of the drink coaster?

**15** A gardener plans to dig a rectangular herb garden with a border 36 m long. List five sets of possible dimensions for the herb garden.

CHALLENGE

**16** A Hills hoist clothesline has four lines arranged in squares. Use the measurements shown on the diagram (looking at the clothesline from above) to calculate the total length of line used.




**17** A rectangular swimming pool of length 7.5 m and width 4.6 m is to have a safety fence built around it. If the fence will be 2 m from the edge of the pool, calculate the total length of fencing needed (include the gate).

**Reflect**

Explain to a friend how to calculate the perimeter of a shape.

# 9C Understanding area

## Start thinking!




A square with side length 1 cm has area 1 cm².

The amount of space enclosed by the outside edge of a 2D shape is called the **area**.

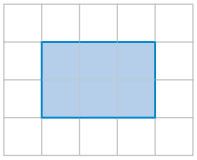
Common units used for area are square millimetres (mm²), square centimetres (cm²), square metres (m²) and square kilometres (km²).



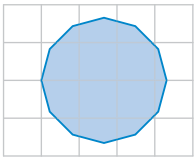


1 Predict which item has the larger area: the postage stamp or the 50-cent coin.

2 The outline of the stamp is traced on to 1-cm grid paper.
   **a** What is the area of the stamp in square centimetres?
   **b** How did you work this out?



3 The outline of a 50-cent coin is traced on to 1-cm grid paper.
   **a** What is the area of the coin in square centimetres?
   **b** How did you work this out?



4 Did you find an exact or approximate value for the area of the coin? Give a reason.

5 Which has the larger area: the stamp or the coin? Was your prediction correct?

## KEY IDEAS

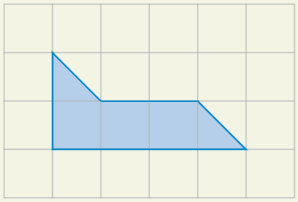
- Area is the amount of space enclosed by a 2D shape.
- Common units of area are square millimetres (mm²), square centimetres (cm²), square metres (m²) and square kilometres (km²).
- The area of a shape can be found by counting the number of whole square units covered by the shape as well as any half square units covered.
- For an irregular shape, the area can be approximated by counting all the square units that are covered and all the square units that are *at least* half covered by the shape.



# EXERCISE 9C Understanding area

## EXAMPLE 9C-1 Finding area by counting squares

Find the exact area of this shape drawn on 1-cm grid paper.



**THINK**

1 Count the number of whole squares and half squares within the shape.

2 Write the area of the shape. Each square in the grid has an area of 1 $cm^2$.

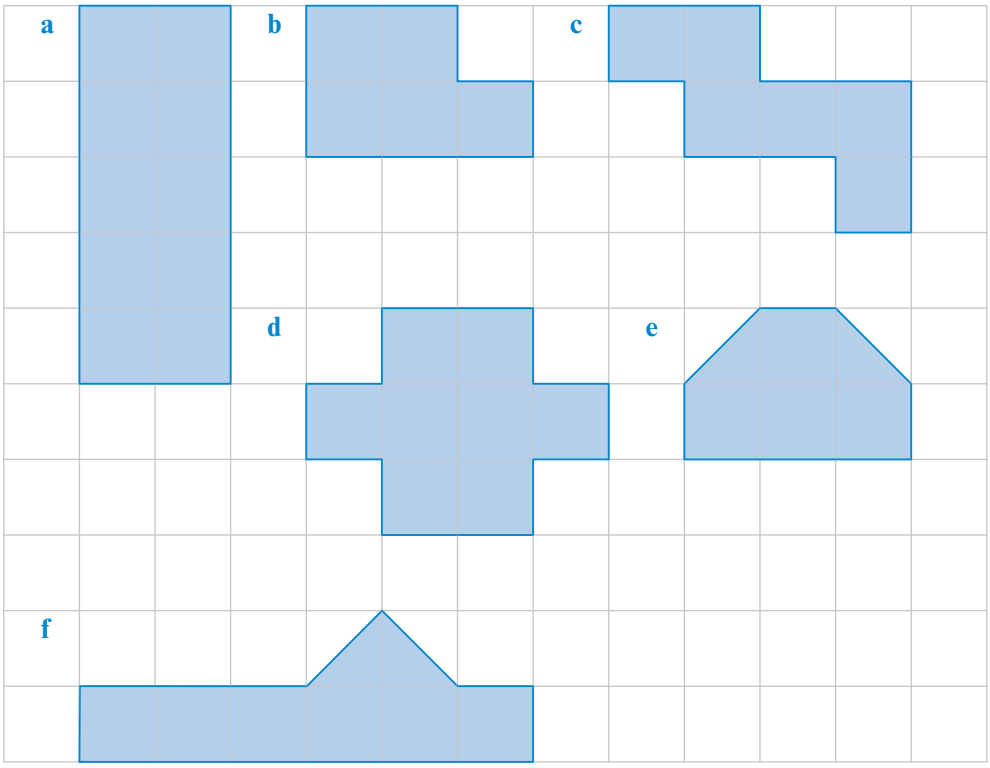
**WRITE**

3 whole squares + 2 half squares = 4 whole squares

area = 4 $cm^2$

UNDERSTANDING AND FLUENCY

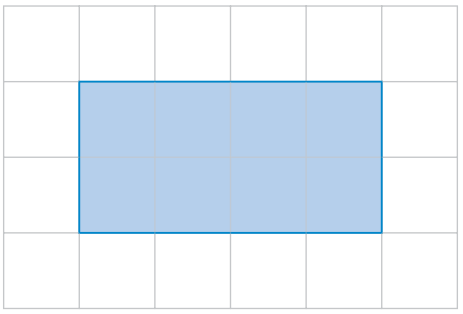
**1** Find the exact area of these shapes drawn on 1-cm grid paper.



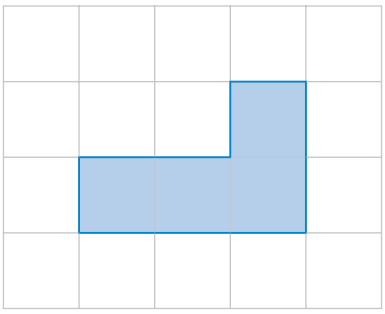
UNDERSTANDING AND FLUENCY

**2** These shapes are shown on 1-cm grid paper.

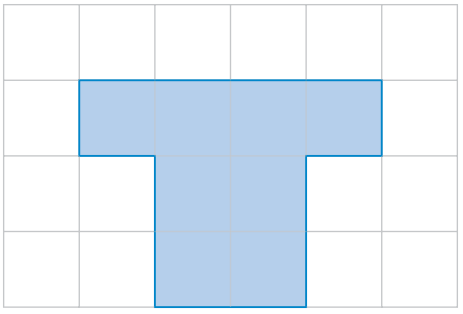
**i**



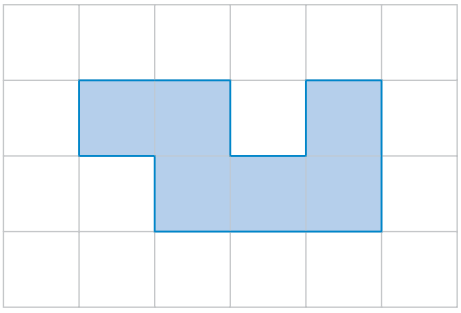
**ii**



**iii**



**iv**



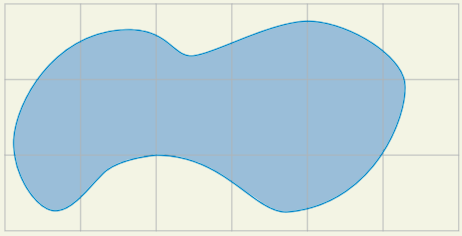
**a** Find the area of each shape.

**b** Find the perimeter of each shape.

**c** Explain the difference between finding the area and finding the perimeter of each shape.

## EXAMPLE 9C-2 Estimating area

Estimate the area of this shape drawn on 1-cm grid paper.

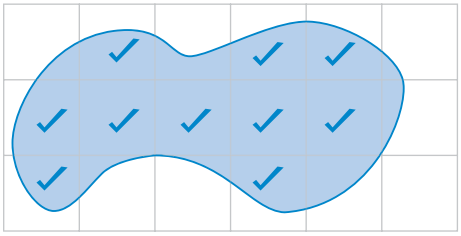


### THINK

**1** Some squares are not totally enclosed in the shape. Count the squares that are completely enclosed and those that are at least half covered by the shape. Tick the squares to be counted.

**2** Count the number of squares that are ticked and write your answer for the area. To show your answer is approximate, use the symbol ≈, which means 'is approximately equal to'.

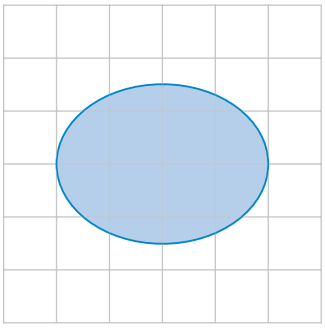
### WRITE



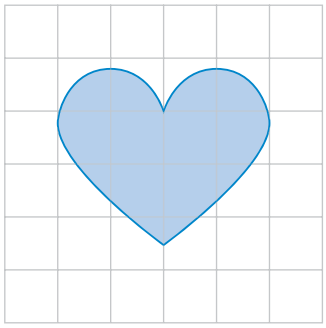
area ≈ 10 cm$^2$

**3** Estimate the area of each shape drawn on 1-cm grid paper.

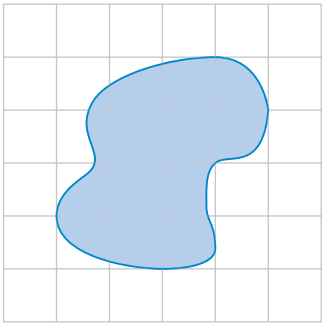
**a**



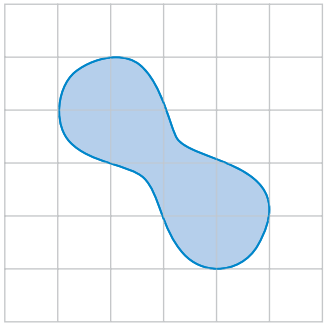
**b**



**c**



**d**



**4** Work with a partner to answer these questions.

**a** **i** Draw a square that has an area of 1 $mm^2$.

**ii** Give two examples where area would best be measured in $mm^2$.

**b** **i** Draw a square that has an area of 1 $cm^2$.

**ii** Give two examples where area would best be measured in $cm^2$.

**c** **i** Use some sheets of newspaper, scissors and sticky tape to make a square that has an area of 1 $m^2$.

**ii** Give two examples of situations where area would best be measured in $m^2$.

**d** **i** Describe the size of 1 $km^2$.

**ii** Give two examples of situations where area would best be measured in $km^2$.



**5** Use your answers to question **4** to estimate the area of these.

**a** top of your desk

**b** front cover of your textbook

**c** one wall of your classroom

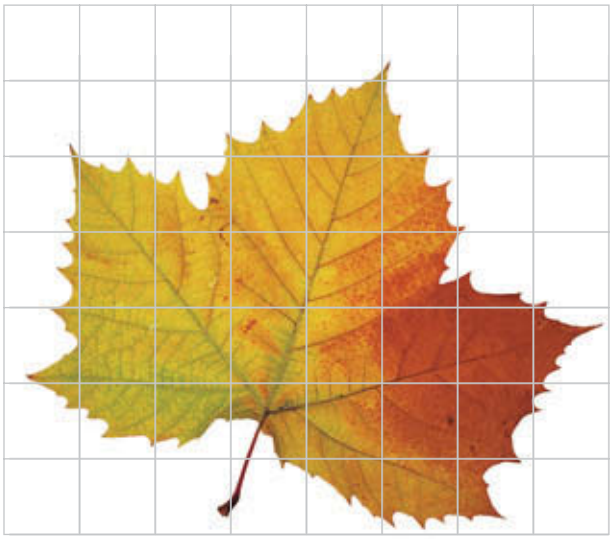
**d** surface of your thumb nail

**e** sole of your shoe

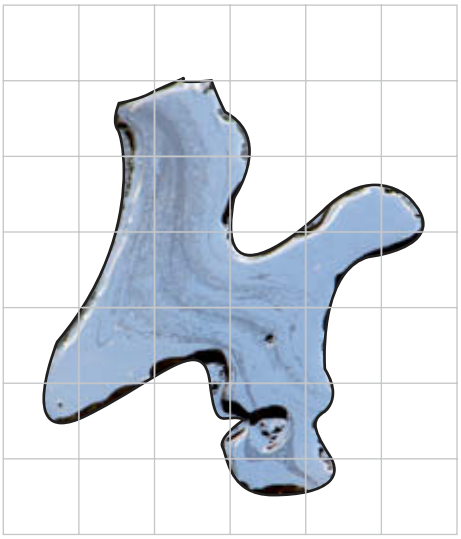
**f** floor of your classroom

**6** Find the area covered by each of these (shown actual size). A 1-cm grid has been placed over each photo.

**a** a leaf



**b** oil spot on a garage floor



**c** dog pawprints



**d** a coin



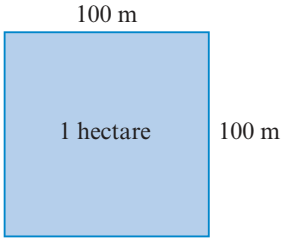
**7** Which unit of area ($mm^2$, $cm^2$, $m^2$ or $km^2$) would be best for measuring the area of these?

**a** basketball court
**b** wristwatch face
**c** national park
**d** five-dollar note
**e** surface of a skateboard ramp
**f** family-size pizza

**8** Another unit of area is the hectare (ha). This unit is often used for land area. One hectare has the area of a square with side lengths of 100 m. That is, 1 ha = 10 000 $m^2$ or 100 ha = 1 $km^2$.

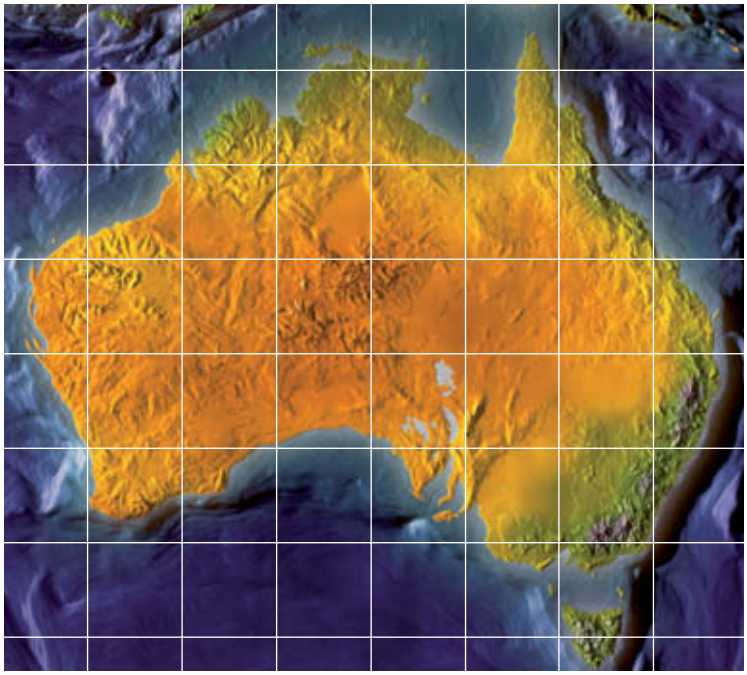
**a** A dairy farm covers an area of 8 $km^2$. What is the area of the farm in hectares?

**b** A 40-ha property is advertised for sale. What is the land area of the property in $m^2$?



CHALLENGE

**9** A grid has been placed over a map of Australia where each square in the grid covers 250 000 km$^2$.



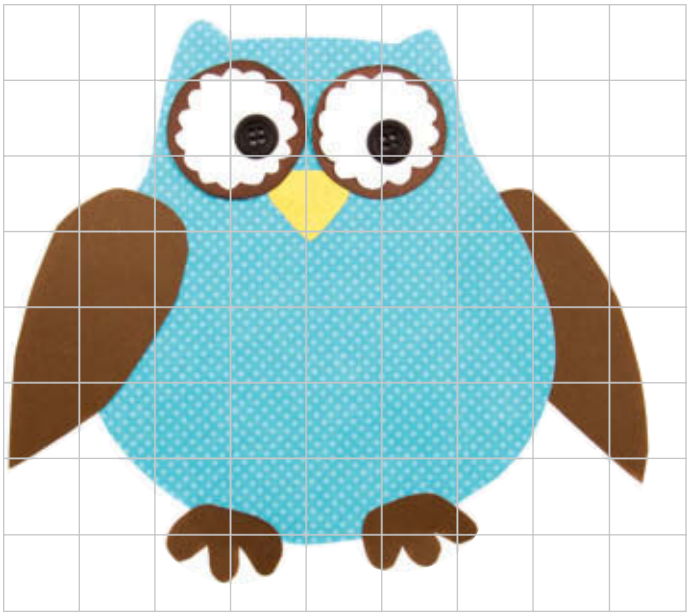
**a** Find the approximate area of Australia in km$^2$.

**b** Express the area of Australia in hectares.

**c** The official area of Australia (including islands) is 7 692 024 km$^2$. Compare your approximation with this. Explain why the values are different and suggest how you could improve the accuracy of your approximation.

**10** Explain why there are situations where you can find the exact area and others where you can only find the approximate area. Provide examples to support your answer.

**11** Use the 1-cm grid to estimate the area of this cartoon character.



**Reflect**

What is the difference between measuring length and measuring area?

# 9D Area of a rectangle

## Start thinking!

1 Using 1-cm grid paper, draw three different rectangles. In each case make the measurements for the length and width a whole number of centimetres.

2 Copy and complete this table.

| Rectangle | Length (cm) | Width (cm) | Area ($cm^2$) |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |

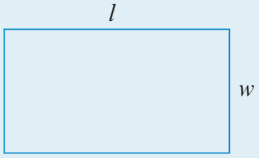
3 Describe a shortcut you could use to calculate the area of each rectangle without counting the squares in the grid.

4 Use the words length, width and area to write a rule for calculating the area of a rectangle.

5 When would this rule be more useful than counting the squares in a grid?

## KEY IDEAS

- Rule for area of a rectangle: Area = length × width
- Instead of using words, **pronumerals** can be used to write a formula for calculating the area of any rectangle.
- Formula for area ($A$) of a rectangle, where $l$ represents the length measurement and $w$ represents the width measurement: $A = l \times w$ (or $A = lw$)

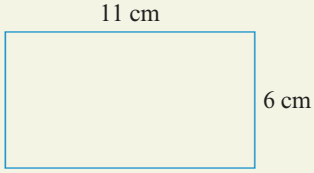



- The measurements for $l$ and $w$ must be in the same unit before substituting them into the formula.

# EXERCISE 9D Area of a rectangle

## EXAMPLE 9D-1 Calculating area of a rectangle using a formula

Calculate the area of this rectangle.

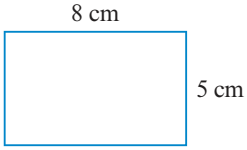



| THINK | WRITE |
|---|---|
| **1** Identify the shape (rectangle). Write the formula for its area. | $A = l \times w$ |
| **2** Identify the measurements for $l$ and $w$. Check they are in the same unit (yes). | $l = 11$ cm, $w = 6$ cm |
| **3** Substitute the values for $l$ and $w$ into the formula. | $A = 11 \text{ cm} \times 6 \text{ cm}$ |
| **4** Calculate the result and include the appropriate unit. | $= 66 \text{ cm}^2$ |

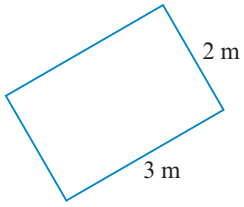
UNDERSTANDING AND FLUENCY

**1** Calculate the area of each rectangle.

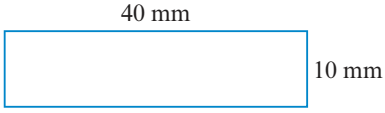
**a**



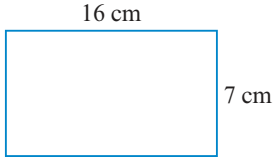

**b**




**c**



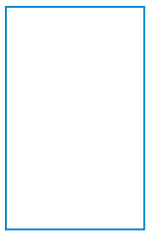

**d**




**e**




**f**

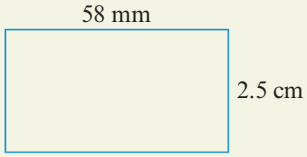


**2** Find the area of the rectangles with these dimensions.

**a** length 9 cm, width 7 cm
**b** length 13 mm, width 4 mm
**c** length 6 m, width 1 m
**d** length 10.4 cm, width 5 cm
**e** length 14.2 mm, width 9.7 mm
**f** length 17.03 m, width 5.4 m

## EXAMPLE 9D-2 Calculating area of a rectangle after converting units

Calculate the area of this rectangle.




### THINK

**1** Identify the shape (rectangle). Write the formula for its area.

**2** Identify the measurements for $l$ and $w$. Check that they are in the same units (no). Convert $l$ to centimetres so that both $l$ and $w$ are in the same units.

**3** Substitute the values for $l$ and $w$ into the formula.

**4** Calculate the result and include the appropriate unit.

**NOTE** w could have been converted to millimetres and the area found to be 1450 mm².

### WRITE

$A = l \times w$

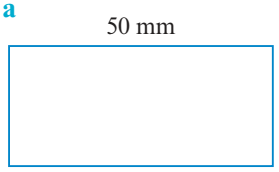
$l = 58 \text{ mm} = 5.8 \text{ cm}$
$w = 2.5 \text{ cm}$

$A = 5.8 \text{ cm} \times 2.5 \text{ cm}$
$= 14.5 \text{ cm}^2$

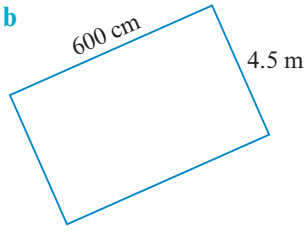
UNDERSTANDING AND FLUENCY

**3** Calculate the area of each rectangle.

**a**



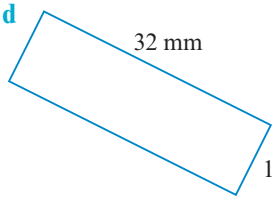

**b**




**c**



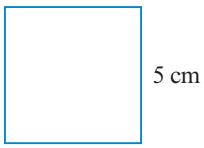

**d**




**e**




**f**

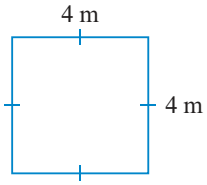



**4** Find the area of each rectangle with these dimensions.

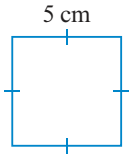
**a** length 3 m, width 100 cm
**b** length 8 cm, width 50 mm
**c** length 4 km, width 280 m
**d** length 600 mm, width 30 cm
**e** length 3.1 cm, width 20 mm
**f** length 8.4 cm, width 0.05 m

**5** Calculate the area of each square.

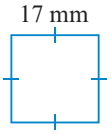
**a**



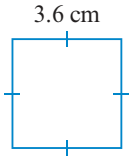

**b**




**c**




**d**




**6** Write a formula for the area of a square of length $l$. Use your working from question **5** to help you.

**7** Find the area of the squares with these side lengths.

**a** 7 m **b** 15 cm **c** 11 km **d** 60 mm

**e** 6.4 cm **f** 180 m **g** 0.7 mm **h** 0.53 km

**8** The shape shown in Figure A can be divided into two rectangles (Figure B).

**Figure A**

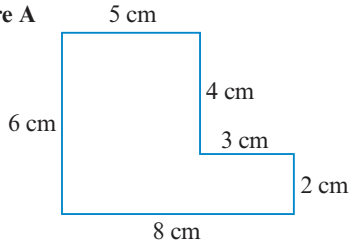
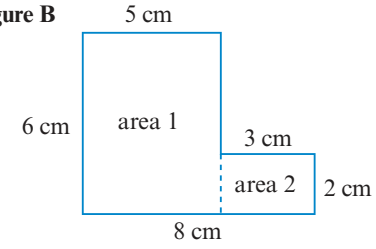



**Figure B**




**a** Calculate the area of the rectangle labelled as area 1.

**b** Calculate the area of the rectangle labelled as area 2.

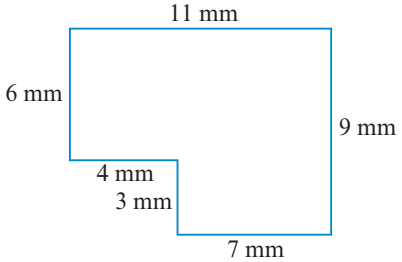
**c** What is the total area of the shape?

**d** Is there another way of dividing the shape into two rectangles? Draw a diagram to show how this could be done.

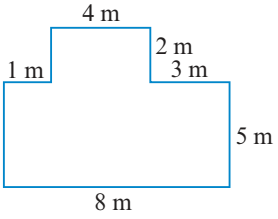
**e** Calculate the area of the shape using your diagram from part **d**.

**9** Copy each diagram and show how each shape can be divided into two or more rectangles.

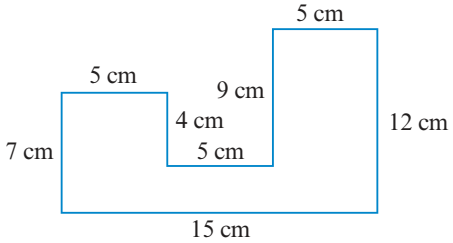
**a**



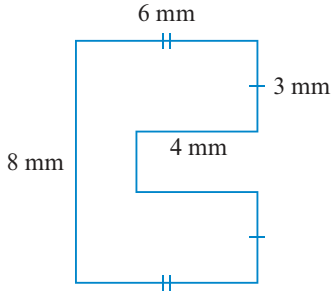

**b**




**c**

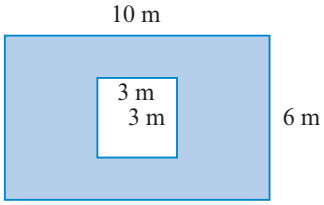



**d**



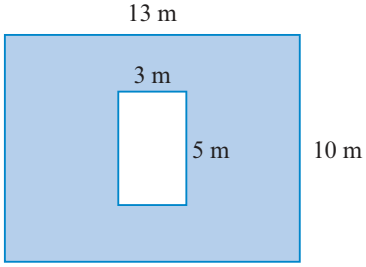
**10** Find the area of each shape in question **9**.

**11** In questions **9** and **10**, the shapes are divided into separate rectangles and the area of each rectangle added together to find the total area of the shape. Explain how you could calculate the area of the shaded region in this shape using subtraction instead. What is the area?

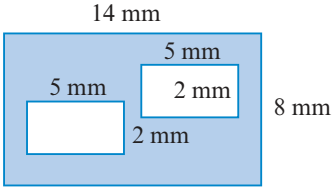



**12** Find the area of the shaded region in each of these.

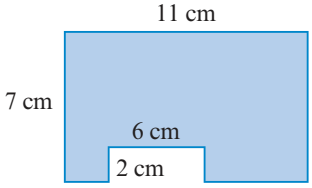
**a**



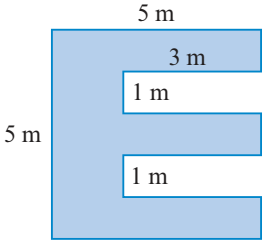

**b**




**c**

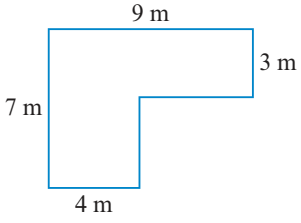



**d**

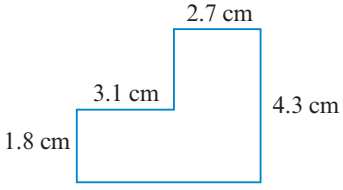



**13** Find **i** the area and **ii** the perimeter of each shape.

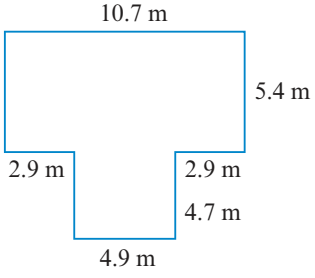
**a**



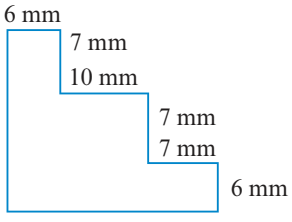

**b**




**c**




**d**



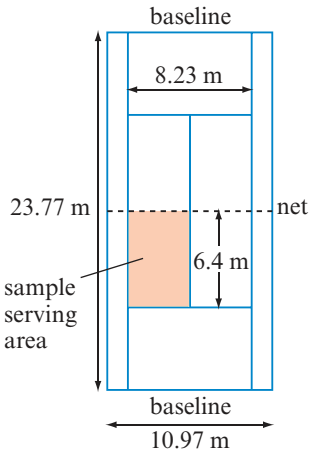

**14** Write three sets of possible dimensions for rectangles with these areas.

**a** 48 $cm^2$ **b** 15 $m^2$

**15** How many sets of possible dimensions are there for a square that has an area of 16 $mm^2$? List the possibilities.

PROBLEM SOLVING AND REASONING

**16** A tennis court has a playing area that is divided by lines drawn on the ground.




**a** Use the dimensions shown to calculate each of these.

- **i** the length from the baseline to the net
- **ii** the width of the doubles playing area (the full court)
- **iii** the width of the singles playing area (the outer sidelines are not used in singles matches)
- **iv** the playing area used in a doubles match
- **v** the playing area used in a singles match
- **vi** the area of the court that a player can serve the tennis ball into

**b** When it starts to rain, a waterproof cover is placed over the playing surface. Suggest how a cover or covers could be designed to be quickly and effectively used.

**c** Draw a diagram with the dimensions labelled to show how the court could be covered.

**d** What amount of waterproof material would be required for the cover?

**17** What is the width of the screen of this iPhone (shown at near right), if its area is 3750 $mm^2$?

**18** If the area of the screen on the BlackBerry phone (shown at far right) is 3000 $mm^2$, what is its length?




**19** The perimeter of a rectangle with a length of 14 cm is 46 cm. What is its area?

**20** The perimeter of a square is 80 m. What is its area?

CHALLENGE

**21** The perimeter of a rectangle is 64 cm. What is its area? Explain how you worked out your answer.

**22** To mark out the boundary of a new vegetable garden, I have been given a piece of string 12 m long. List the dimensions of five possible rectangular vegetable gardens I could make. Which of these five would have the largest area?

**23** Does a rectangle exist where the perimeter has the same value (measured in cm) as the area (measured in $cm^2$)? Explain your answer.

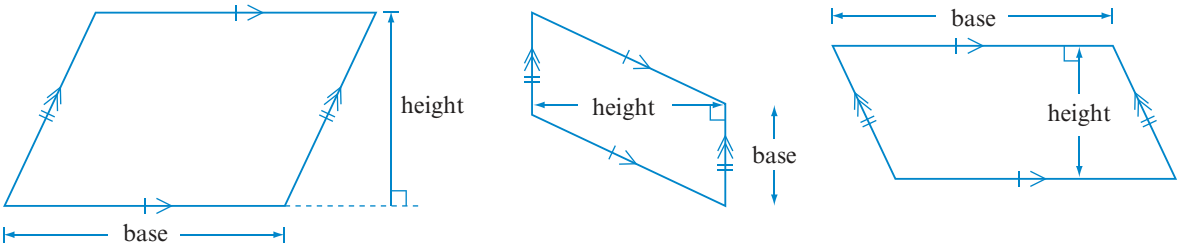
**Reflect**

How would you explain to a friend the difference between the area and the perimeter of a rectangle?

# 9E Area of a parallelogram

## Start thinking!

A **parallelogram** is a four-sided shape with two pairs of parallel sides and opposite sides equal in length. The dimensions **base** and **height** are always at right angles to each other.

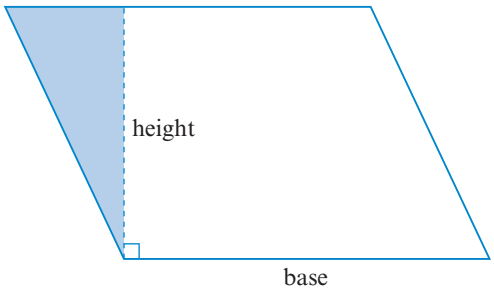



**1** Copy this parallelogram on to a sheet of paper and carefully cut around the shape.

**2** Cut out the shaded triangle and place it at the opposite end of the parallelogram to form a rectangle.

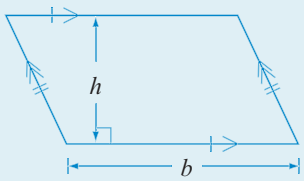
**3** Draw a diagram to show how you changed the parallelogram into a rectangle. Label the diagram with base and height.

**4** Explain how you can use the dimensions of the rectangle to work out the area of the parallelogram.




## KEY IDEAS

- Rule for area of a parallelogram: Area = base × height
- Formula for area ($A$) of a parallelogram, where $b$ represents the base length and $h$ represents the height: $A = b \times h$ (or $A = bh$)



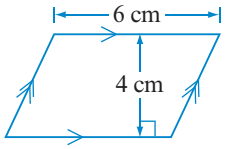

- The base and height are perpendicular (at right angles) to each other.
- The measurements for $b$ and $h$ must be in the same unit before substituting into the formula.

# EXERCISE 9E Area of a parallelogram

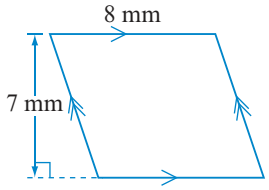
UNDERSTANDING AND FLUENCY

**1** Copy each parallelogram and label the base and the height.

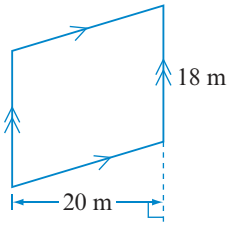
**a**



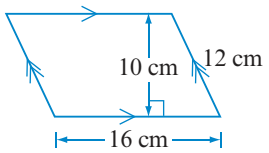

**b**



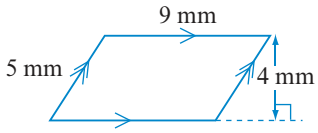

**c**



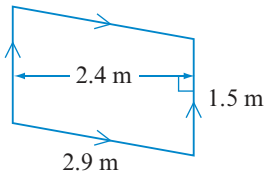

**d**




**e**

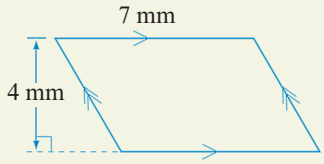



**f**




## EXAMPLE 9E-1 Calculating area of a parallelogram using a formula

Calculate the area of this parallelogram.




| THINK | WRITE |
|---|---|
| **1** Identify the shape (parallelogram). Write the formula for the area of a parallelogram. | $A = b \times h$ |
| **2** Identify the measurements for $b$ and $h$. Check that they are in the same unit (yes). | $b = 7$ mm<br>$h = 4$ mm |
| **3** Substitute the values for $b$ and $h$ into the formula. | $A = 7 \text{ mm} \times 4 \text{ mm}$ |
| **4** Calculate the result and include the appropriate unit. | $= 28 \text{ mm}^2$ |

**2** Calculate the area of each parallelogram in question **1**.

**3** Find the area of each parallelogram with these dimensions.

**a** base length 5 cm, height 3 cm

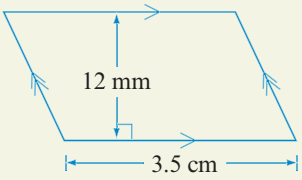
**b** base length 10 mm, height 17 mm

**c** base length 4 m, height 1.5 m

**d** base length 2.5 cm, height 4.1 cm

## EXAMPLE 9E-2 Calculating area of a parallelogram after converting units

Calculate the area of this parallelogram in $cm^2$.

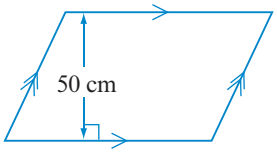



| THINK | WRITE |
|---|---|
| **1** Identify the shape (parallelogram). Write the formula for the area of a parallelogram. | $A = b \times h$ |
| **2** Identify the measurements for $b$ and $h$. Both measurements need to be in centimetres so convert the height measurement. | $b = 3.5$ cm<br>$h = 12$ mm $= 1.2$ cm |
| **3** Substitute the values for $b$ and $h$ into the formula. | $A = 3.5$ cm $\times$ 1.2 cm |
| **4** Calculate the result and include the appropriate unit. | $= 4.2$ $cm^2$ |

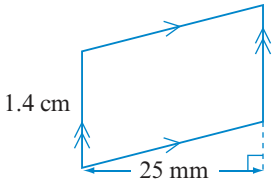
UNDERSTANDING AND FLUENCY

**4** Calculate the area of each parallelogram in $cm^2$.

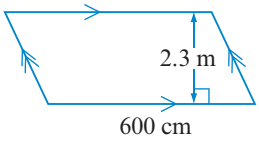
**a**



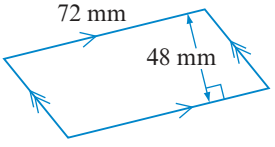

**b**



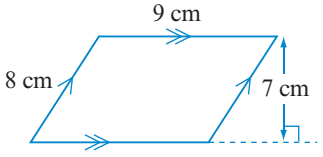

**c**



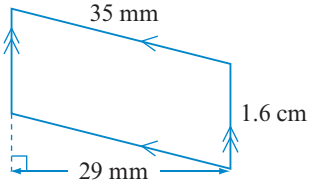

**d**




**e**




**f**

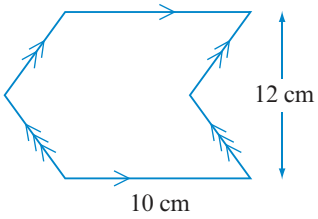



**5** Find the area of each parallelogram with these dimensions.

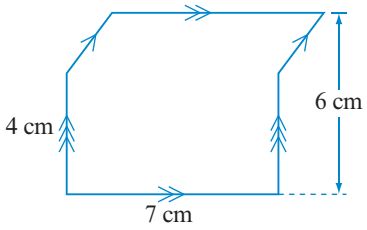
- **a** base length 30 mm, height 5 cm
- **b** base length 58 m, height 200 cm
- **c** base length 0.06 m, height 40 mm
- **d** base length 27.4 cm, height 27.4 mm

**6** Copy each diagram and show how each shape can be divided into two or more rectangles or parallelograms.

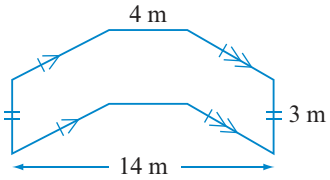
**a**



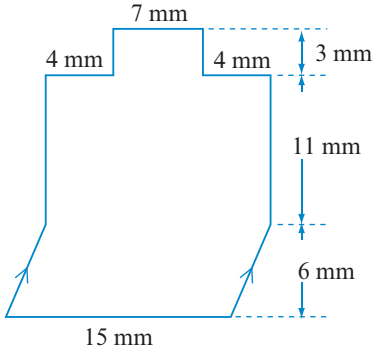

**b**




**c**



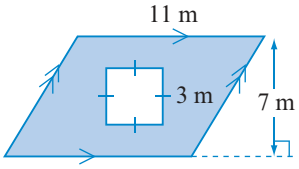

**d**



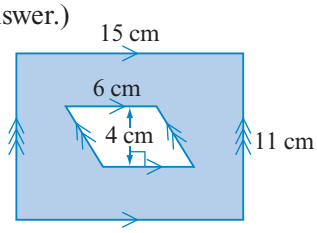

**7** Find the area of each shape shown in question **6**.

**8** Find the area of the shaded region in each of these. (Consider subtracting areas as well as adding areas to obtain your answer.)

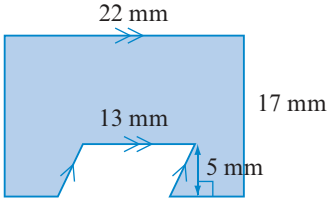
**a**



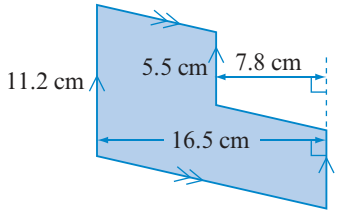

**b**




**c**

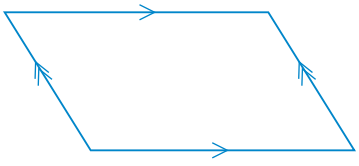



**d**

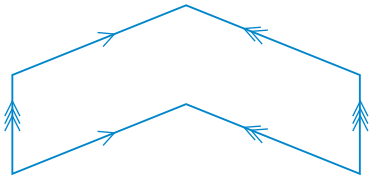



**9** Find **i** the area and **ii** the perimeter of each shape. Use a ruler to obtain the necessary measurements.

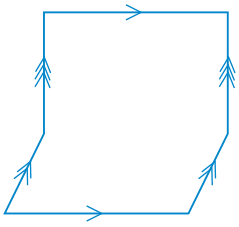
**a**



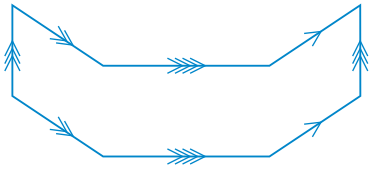
**b**



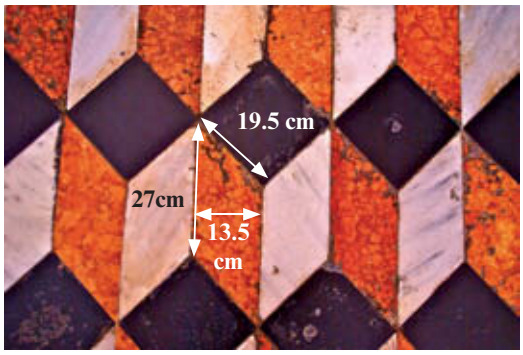
**c**



**d**



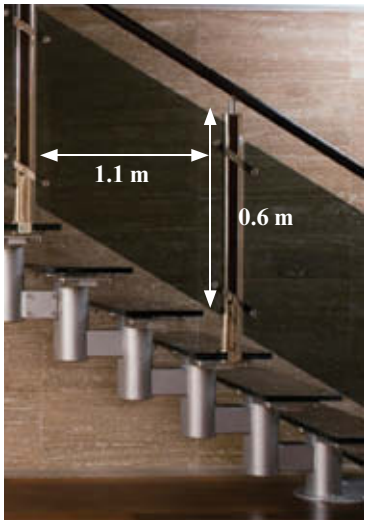
**10** Use the measurements shown on this photo of floor tiles to calculate the area of a red tile.



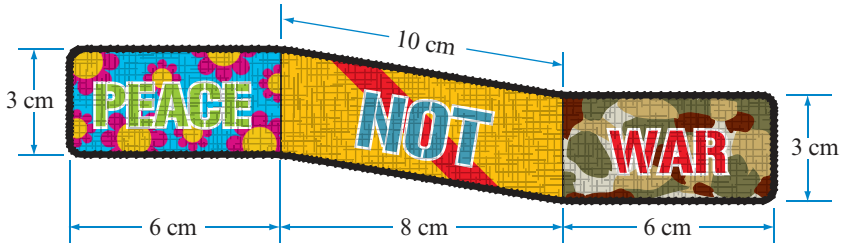

**11** A stair rail is supported by five identical sections of glass panels.

**a** Use the dimensions shown in the photo to calculate the area of glass needed for all sections.

**b** If the glass was to be 10 cm higher in each panel, what area of glass would be needed?




**12** A badge for a jacket is made by sewing three pieces of fabric as shown. The centre piece is in the shape of a parallelogram and the other two pieces are rectangles.




**a** What area of fabric is needed for the parallelogram?

**b** What area of fabric is needed for each rectangle?

**c** Calculate the total area of fabric needed for this badge.

**d** Black cord is sewn around the outer edge of the badge to make it stand out on the jacket. Calculate how much black cord is needed.

**13** Write three sets of possible dimensions for parallelograms with these areas.

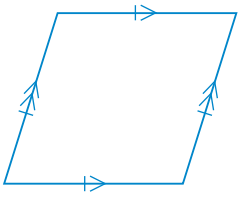
**a** 24 $cm^2$ **b** 14 $mm^2$

**14** The area of a parallelogram is 20 $cm^2$. If the height is 4 cm, how long is the base?

**15** The area of a parallelogram is 36 $cm^2$. If the base is 45 mm long, what is the height?

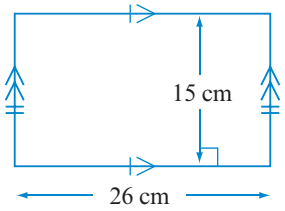
PROBLEM SOLVING AND REASONING

**16** The perimeter of the **rhombus** shown is 40 cm.



- **a** Explain how the area of the rhombus could be calculated. Do you have enough information to calculate the area?
- **b** Would the area of the rhombus be larger or smaller than 100 $cm^2$? Explain your answer.

**17** Consider the rectangle shown.




- **a** Calculate its area using the formula for the area of a rectangle ($A = l \times w$).
- **b** Calculate its area using the formula for the area of a parallelogram ($A = b \times h$).
- **c** Explain why you obtain the same answer in each case.

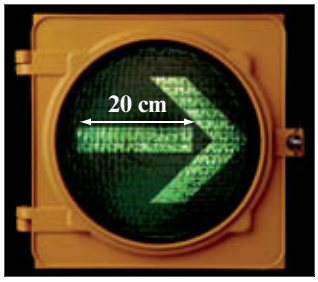
CHALLENGE

**18** The marble-tiled floor of the church of San Giorgio Maggiore in Venice, Italy (completed in 1610) is made up of just two different 2D shapes. When you walk over the floor you experience the optical illusion of viewing **three-dimensional (3D)** objects.



- **a** What are the two shapes used for the tiles?
- **b** What 3D object appears in the photo at right as a result of the optical illusion?
- **c** Use the dimensions shown in the photo for question **10** to calculate the area of the three tiles (black, red, white) that make up the illusion of one 3D object.

**19** While walking along a street, you notice that the shape of the green arrow in a traffic light is formed using a rectangle and two parallelograms. Estimate the area of the green arrow in the traffic light shown below using the length measurement provided.



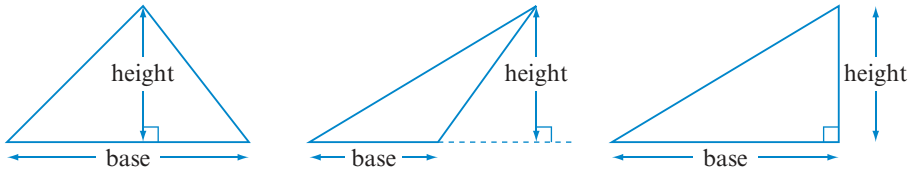

**Reflect**

Explain why there is a connection between the area of a parallelogram and the area of a rectangle.

# 9F Area of a triangle

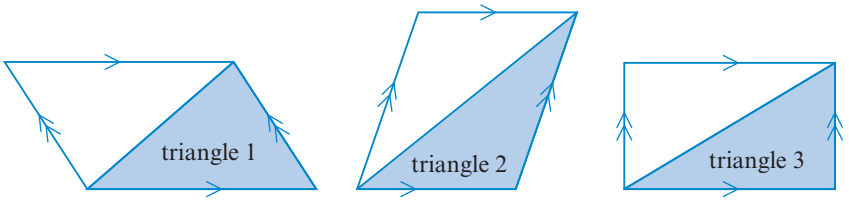
## Start thinking!

A **triangle** has three sides. The dimensions base and height are always at right angles to each other.




The area of a triangle can be compared to the area of a parallelogram or rectangle (as a rectangle is a type of parallelogram).

**1** Copy each parallelogram shown below. A triangle has been formed inside each shape.



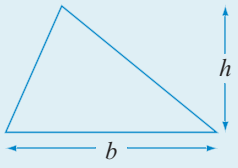

**2** Label the base and the height on each diagram.

**3** How does the area of the triangle compare to the parallelogram in each case?

**4** If the rule for the area of a parallelogram is base × height, write a rule for the area of a triangle.

## KEY IDEAS

- Rule for area of a triangle: Area $= \frac{1}{2} \times$ base $\times$ height
- Formula for area ($A$) of a triangle, where $b$ represents the base measurement and $h$ represents the height: $A = \frac{1}{2} \times b \times h$ (or $A = \frac{1}{2}bh$)



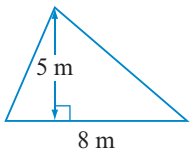

- The base and height must be perpendicular to each other.
- The measurements for $b$ and $h$ must be in the same unit before substituting into the formula.

# EXERCISE 9F Area of a triangle

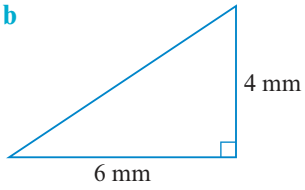
UNDERSTANDING AND FLUENCY

**1** Copy each triangle and label the base and the height.

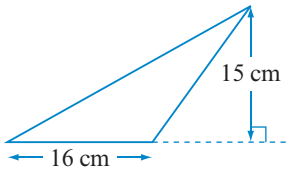
**a**



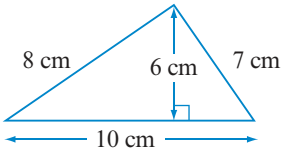

**b**



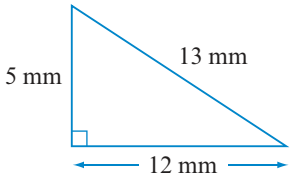

**c**



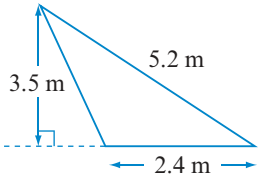

**d**




**e**

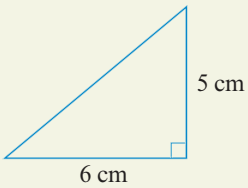



**f**




## EXAMPLE 9F-1 Calculating area of a triangle using a formula

Calculate the area of this triangle.




| THINK | WRITE |
|---|---|
| **1** Identify the shape (triangle). Write the formula for the area of a triangle. | $A = \frac{1}{2} \times b \times h$ |
| **2** Identify the measurements for $b$ and $h$. Check that they are in the same unit (yes). | $b = 6$ cm<br>$h = 5$ cm |
| **3** Substitute the values for $b$ and $h$ into the formula. | $A = \frac{1}{2} \times 6 \text{ cm} \times 5 \text{ cm}$ |
| **4** Calculate the result and include the appropriate unit. | $= 15 \text{ cm}^2$ |

**2** Calculate the area of each triangle in question **1**.

**3** Find the area of the triangles with these dimensions.

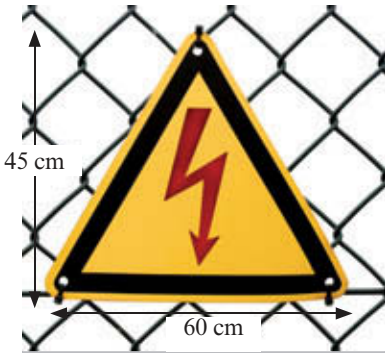
**a** base 12 cm, height 9 cm
**b** base 24 mm, height 4 mm
**c** base 5 m, height 3 m
**d** base 8.6 cm, height 7 cm
**e** base 14.2 mm, height 9.7 mm
**f** base 17.03 m, height 15.4 m

**4** **a** Draw diagrams of three different triangles where the base measurement is 10 cm and the height measurement is 6 cm.

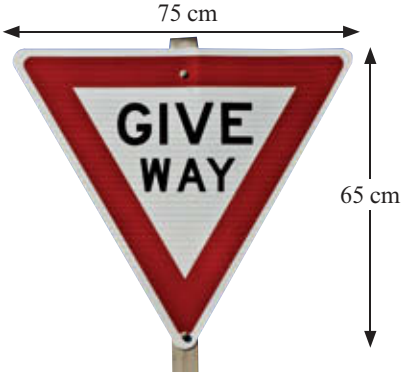
**b** Calculate the area of each triangle.

**5** Calculate the area of metal required for each sign.

**a**

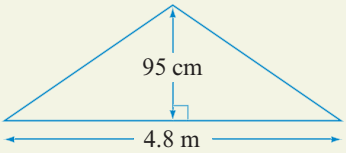



**b**




## EXAMPLE 9F-2 Calculating area of a triangle after converting units

Calculate the area of this triangle.




### THINK

**1** Identify the shape (triangle). Write the formula for the area of a triangle.

**2** Identify the measurements for $b$ and $h$. Check that they are both in the same units (no). Convert $h$ to metres.

**3** Substitute the values for $b$ and $h$ into the formula.

**4** Calculate the result and include the appropriate unit.

### WRITE

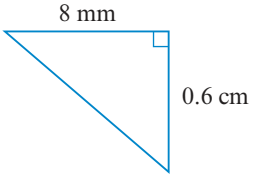
$A = \frac{1}{2} \times b \times h$

$b = 4.8$ m
$h = 95$ cm $= 0.95$ m

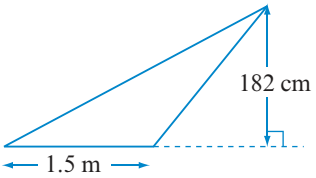
$A = \frac{1}{2} \times 4.8 \text{ m} \times 0.95 \text{ m}$
$= 2.28 \text{ m}^2$

**6** Calculate the area of each triangle.

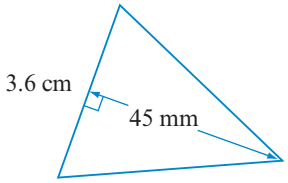
**a**



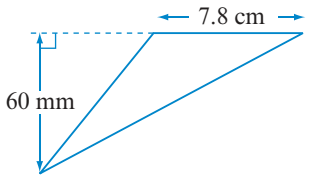

**b**



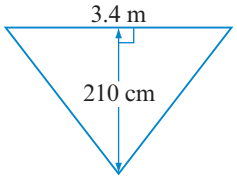
**c**



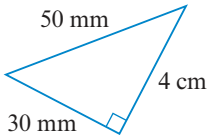

**d**




**e**



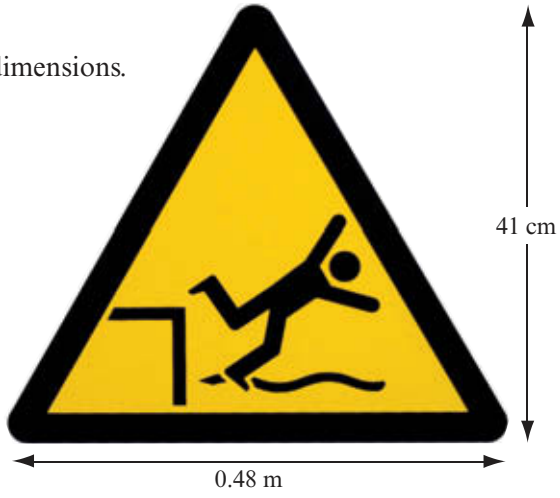

**f**




**7** Find the area of each triangle with these dimensions.

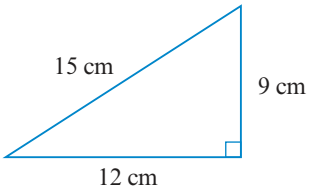
- **a** base 8 cm, height 30 mm
- **b** base 16 mm, height 3 cm
- **c** base 4 m, height 50 cm
- **d** base 310 cm, height 10 m
- **e** base 0.7 cm, height 20 mm
- **f** base 2.8 m, height 97 cm

**8** Calculate the area of metal required for this sign.



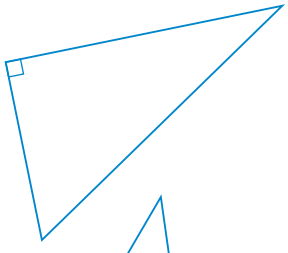

**9** **a** A student calculates the area of the triangle shown at right to be 90 cm$^2$. Is this correct? Explain your answer. If it is not correct, work out what the answer should be.

**b** Calculate the perimeter of this triangle.

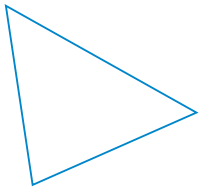



**10** Calculate **i** the area and **ii** the perimeter of each triangle. Use a ruler to obtain the necessary measurements.

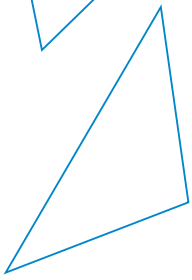
**a**



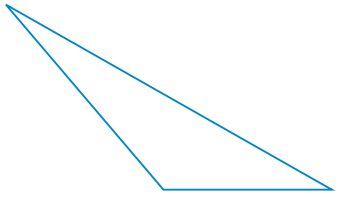
**b**



**c**



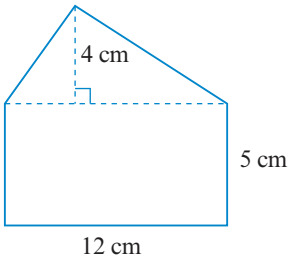
**d**



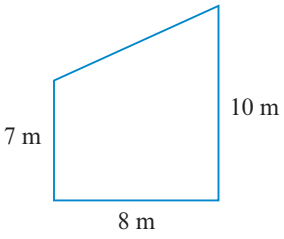
UNDERSTANDING AND FLUENCY

**11** Calculate the area of each composite shape.

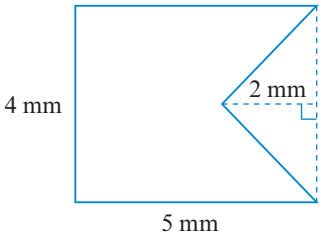
**a**



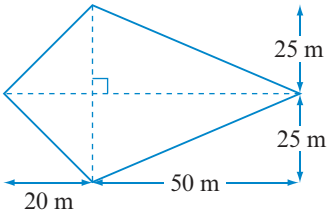

**b**



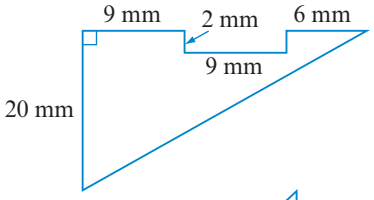

**c**



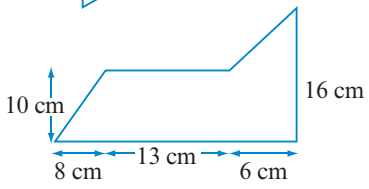

**d**




**e**




**f**



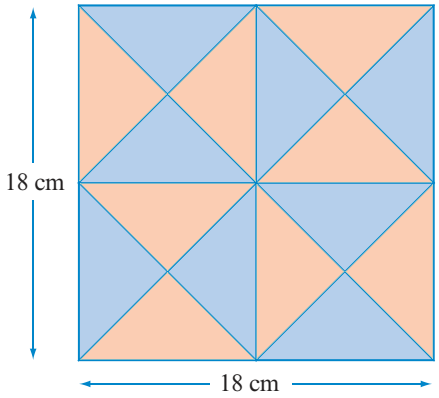

**12** Calculate the amount of sailcloth needed for the two triangular sails on this boat.



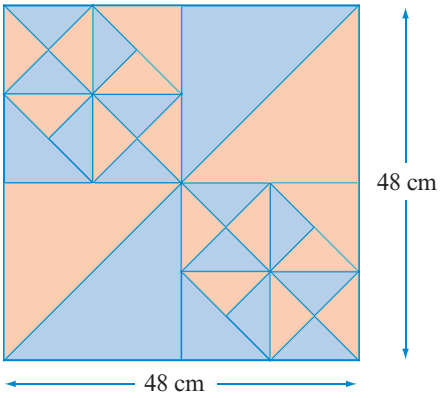

PROBLEM SOLVING AND REASONING

**13** Calculate the area shaded orange in each shape below.

**a**




**b**



**14** Write three sets of possible dimensions for triangles with these areas.

**a** 32 $cm^2$ **b** 40 $m^2$

**15** The area of a triangle is 20 $cm^2$. If the base is 10 cm long, what is the height?

**16** If the total area that has been enamelled on these earrings is 270 $mm^2$, calculate how wide each earring is at the top.



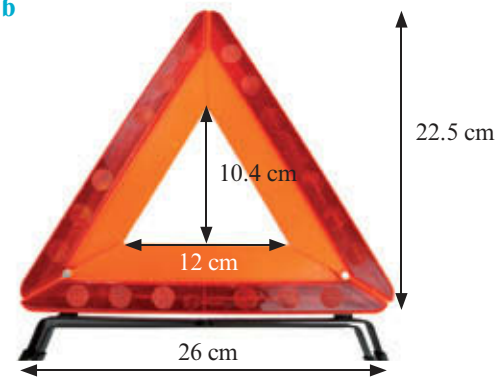

CHALLENGE

**17** Use your understanding of how to find the area of a triangle to calculate the area of metal needed for each of these signs.

**a**




**b**




**18** Estimate the total area of fabric needed to make the sail used by this windsurfer.



**Reflect**

What is important to remember about the base and height of a triangle?

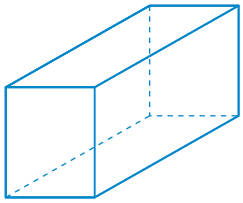
# 9G Surface area

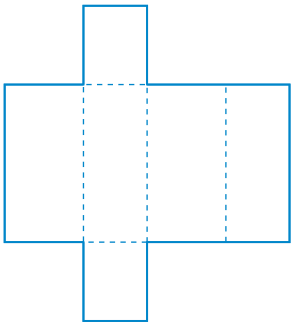


## Start thinking!

The **surface area** of a 3D object is the total area of its surface. Drawing a **net** of the object helps us to identify the shapes that make up the surface.

For example, the **rectangular prism** and its net can be drawn for this tissue box.



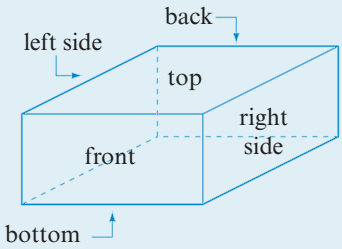


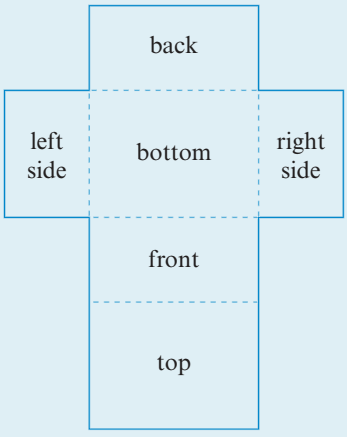
1 How many faces does the rectangular prism have?

2 Explain how the net of the prism can help you work out the surface area.

## KEY IDEAS

- The surface area of a 3D object is the total area of the outer surface of the object.
- The surface area of a prism is the sum of the areas of each face of the prism.
- A rectangular prism has six faces. There are three pairs of identical faces.
- Drawing the net of a prism can help you see all the faces of the prism and allow you to calculate the surface area more easily.



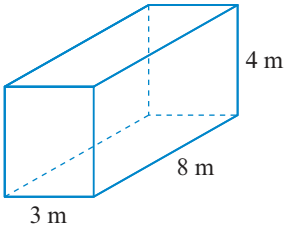



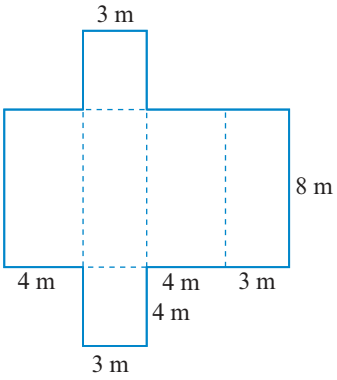
# EXERCISE 9G Surface area

UNDERSTANDING AND FLUENCY

**1** A rectangular prism and its net are shown.

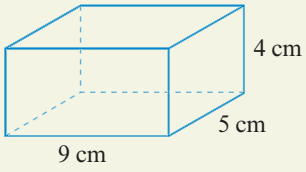
- **a** How many faces does this rectangular prism have?
- **b** Describe the shape of each face.
- **c** Calculate the area of each face of the prism.
- **d** Use your answer from part **c** to calculate the surface area of the prism.




## EXAMPLE 9G-1 Calculating surface area of a rectangular prism

Calculate the surface area of this rectangular prism.



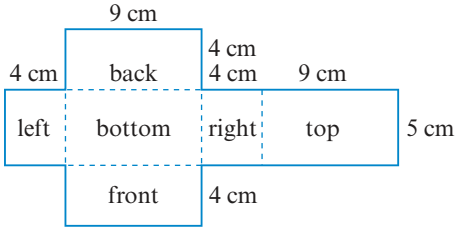

### THINK

**1** Work out how many faces the prism has. You can draw a net of the prism to clearly see all the faces.

**2** Find the area of each face before adding. Check that all six faces have been considered.

**3** Substitute the appropriate measurements into the formula for the area of a rectangle and simplify.

### WRITE

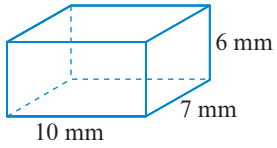



There are six faces. Each face is rectangular in shape.
Surface area
= area of front face + area of back face
+ area of top face + area of bottom face
+ area of left-side face + area of right-side face

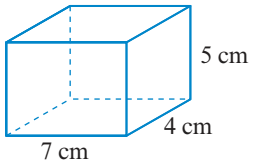
= 9 cm × 4 cm + 9 cm × 4 cm + 9 cm × 5 cm +
9 cm × 5 cm + 5 cm × 4 cm + 5 cm × 4 cm
= 36 $cm^2$ + 36 $cm^2$ + 45 $cm^2$ + 45 $cm^2$ + 20 $cm^2$ + 20 $cm^2$
= 202 $cm^2$

**2** Calculate the surface area of each rectangular prism.

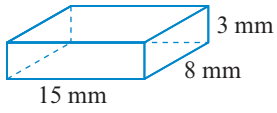
**a**



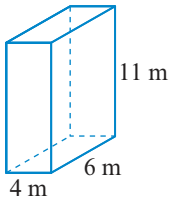

**b**



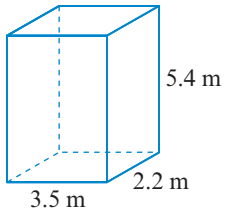

**c**



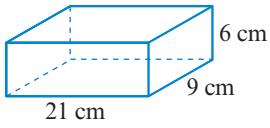

**d**



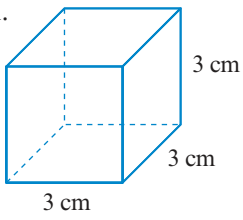

**e**

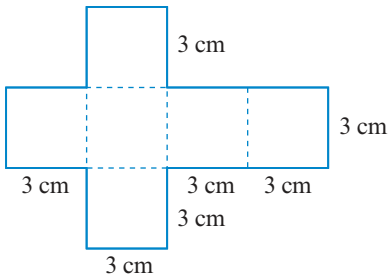



**f**




**3** A cube and its net are shown.




**a** How many faces does this cube have?

**b** Describe the shape of each face.

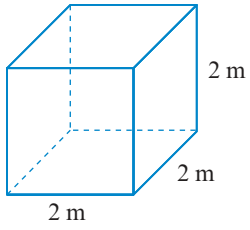
**c** Calculate the area of each face of the cube.

**d** Use your answer from part **c** to calculate the surface area of the cube.

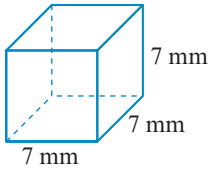
**e** Is there a shortcut that can be used to calculate the surface area of a cube? Explain your answer.

**4** Calculate the surface area of each cube.

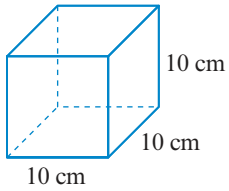
**a**



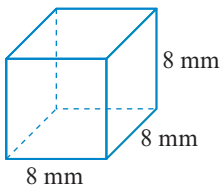

**b**



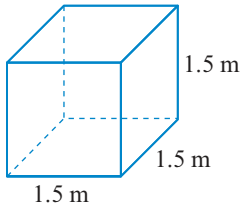

**c**



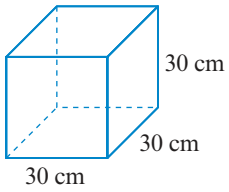

**d**



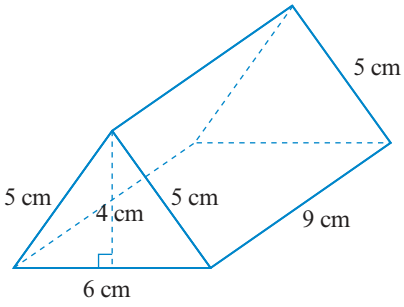

**e**

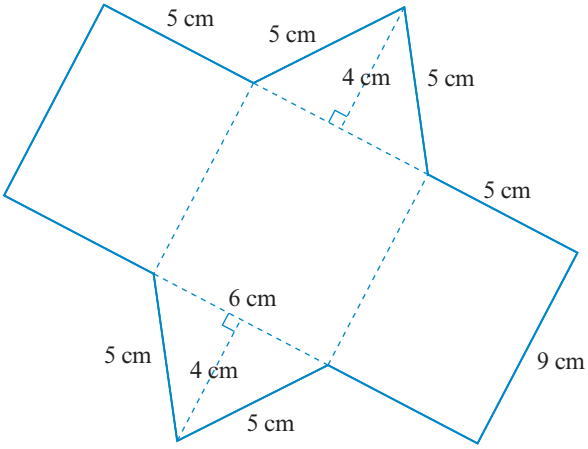



**f**



**5** A triangular prism and its net are shown below.




**a** How many faces does this triangular prism have?

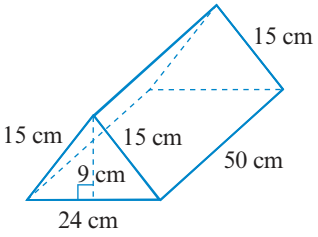
**b** Describe the shape of each face.

**c** Calculate the area of each face of the triangular prism.

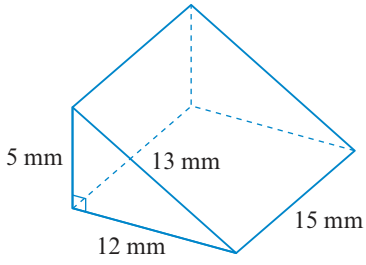
**d** Use your answer from part **c** to calculate the surface area of the triangular prism.

**6** Calculate the surface area of each triangular prism.

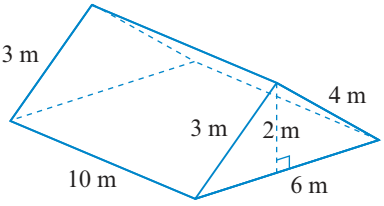
**a**



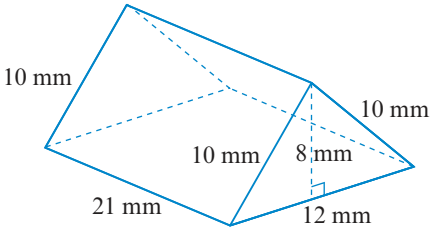

**b**



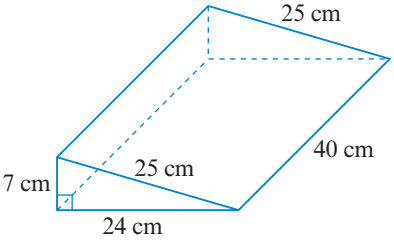

**c**



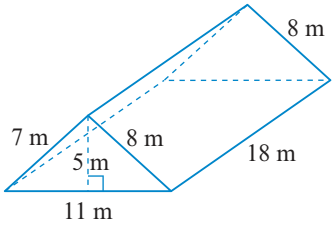

**d**



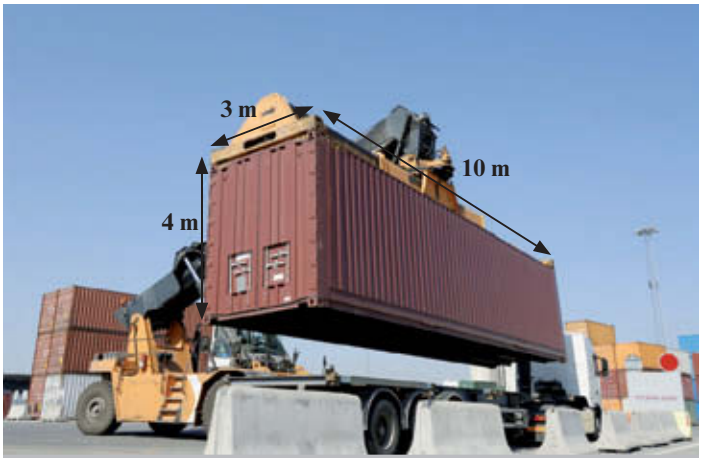

**e**




**f**



**7** Calculate the area of metal sheeting needed for this shipping container.

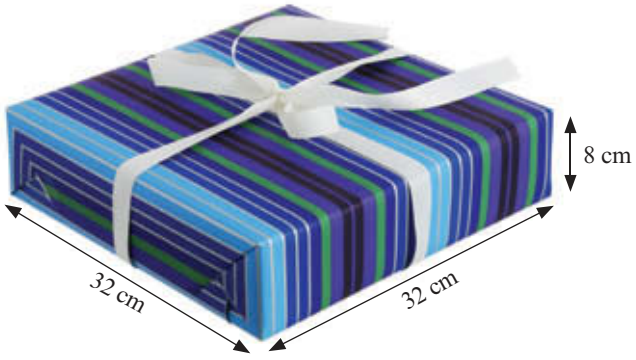



**8** Calculate the surface area of each of these.

**a**




**b**



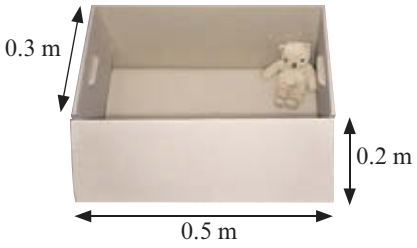

**c**




**d**




**9** This toy box is to be covered with coloured contact paper. What area of paper will be needed to cover both the inside and outside surfaces of the box?



PROBLEM SOLVING AND REASONING

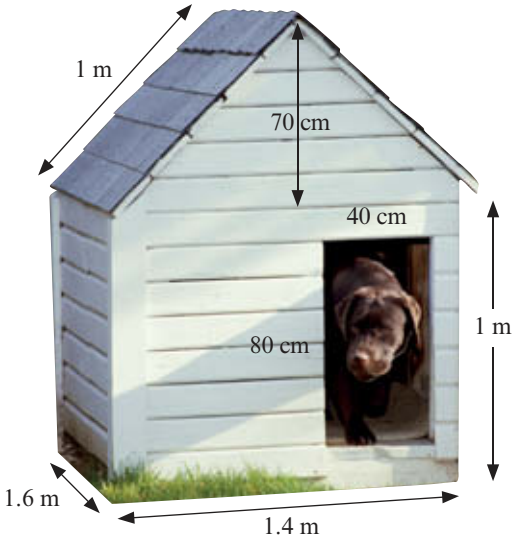
**10** A cube has a surface area of 150 $cm^2$. What is the side length of the cube?

**11** A sculpture shaped as a cube has a surface area of 24 $m^2$. What are the dimensions of the sculpture?

**12** The outside walls and the roof of the dog kennel are to be painted.

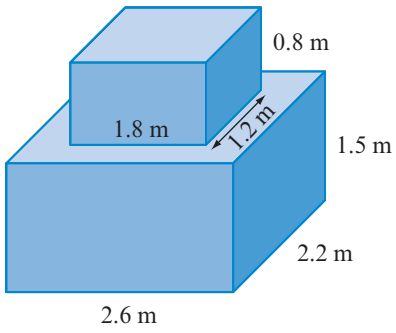
**a** Calculate the area of the kennel to be painted (in $m^2$).

**b** If a small can of paint covers 8 $m^2$, how many cans of paint will be needed?



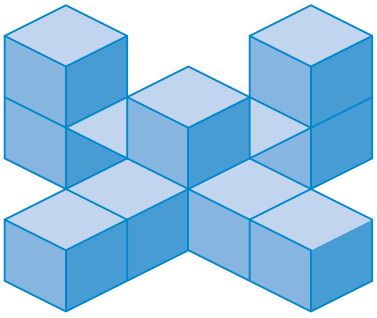

CHALLENGE

**13** Another sculpture is constructed in the shape of a rectangular box with a smaller box placed on top. The dimensions of the boxes are shown in the diagram. To finish the sculpture, the artist must work out the surface area to be painted and hence how much paint is needed. Calculate the surface area of the sculpture. (Ignore the surface sitting on the ground as the viewing public will not see this.)

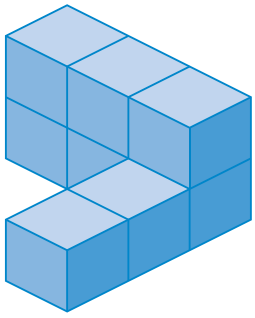



**14** These objects are made of 1-cm blocks. Work out their surface area (including the base).

**a**



**b**



**Reflect**

Explain how area can still relate to 3D objects as well as 2D shapes.

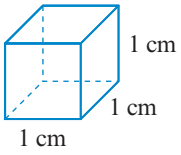
# 9H Volume and capacity




## Start thinking!

Besides surface area, other measurements that relate to 3D objects are **volume** and **capacity**. The volume of a 3D object is the amount of space it occupies.

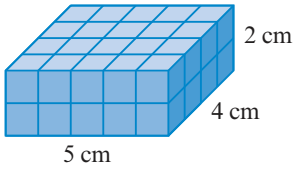
Common units used for volume are cubic millimetres ($mm^3$), cubic centimetres ($cm^3$) and cubic metres ($m^3$). Cubic units are used since the standard units of volume are based on cubes.




A cube with a side length of 1 cm has a volume of 1 $cm^3$.

**1** Describe the dimensions of a cube that would represent:
**a** a cubic centimetre **b** a cubic metre **c** a cubic millimetre.

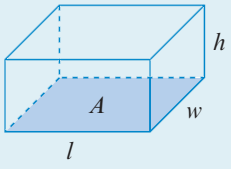
**2** This rectangular prism is made up of 40 cubes (two layers of 20 cubes). As each cube has a volume of 1 $cm^3$, the volume of the prism is 40 $cm^3$.




**a** How many cubes are in one layer? **b** How many layers are there?
**c** Use your answers to parts **a** and **b** to work out the volume of the prism.
**d** What is the length, width and height of the prism?
**e** We can use a shortcut to calculate the volume. If the base of the prism has an area of length × width, show how you can use the shortcut of multiplying the base area by the height to find the volume of this prism.

## KEY IDEAS

- Volume is the amount of space that a 3D object occupies.
- Common units of volume are cubic millimetres ($mm^3$), cubic centimetres ($cm^3$) and cubic metres ($m^3$).
- **Volume of any prism**
  Rule: volume = base area × height
  Formula: $V = A \times h$ (or $V = Ah$)
- **Volume of a rectangular prism**
  Rule: volume = base area × height
  = length × width × height
  Formula: $V = l \times w \times h$ (or $V = lwh$)

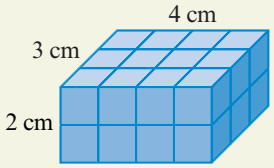



- The base and height of a prism must be perpendicular (at right angles) to each other.
- The measurements for $l$, $w$ and $h$ must be in the same unit before substituting into the formula.
- Capacity is the amount of fluid that a container can hold.
- Common units of capacity are millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML).

# EXERCISE 9H Volume and capacity

## EXAMPLE 9H-1 Finding volume by counting cubes

Find the volume of this rectangular prism.




**THINK**

1. Work out the number of cubes in one layer.
2. Multiply by the number of layers to calculate the total number of cubes in the prism.
3. Write the volume of the prism. Each cube has a side length of 1 cm, so its volume is 1 $cm^3$.

**WRITE**

$4 \times 3 = 12$ cubes in one layer

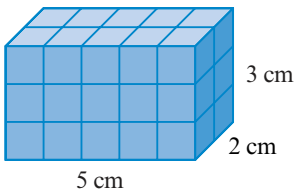
$12 \times 2 = 24$ cubes in prism

volume = 24 $cm^3$

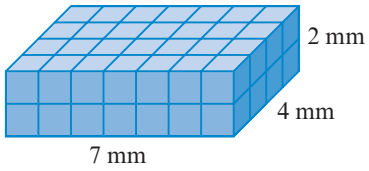
UNDERSTANDING AND FLUENCY

**1** Find the volume of each rectangular prism.

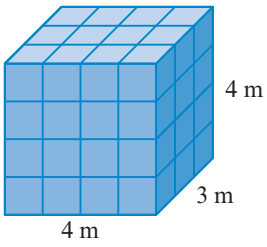
**a**



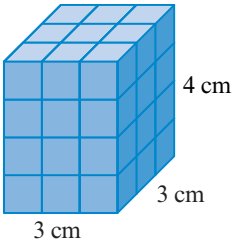

**b**



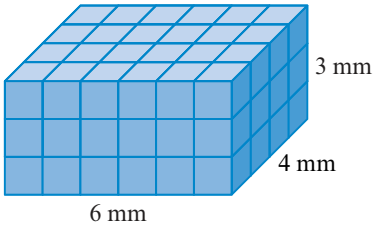

**c**



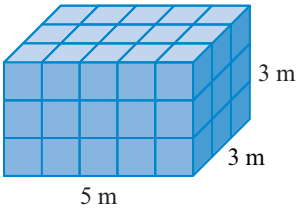

**d**




**e**

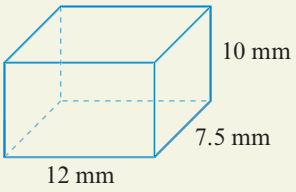



**f**



## EXAMPLE 9H-2 Calculating volume of a rectangular prism using a formula

Calculate the volume of this rectangular prism.

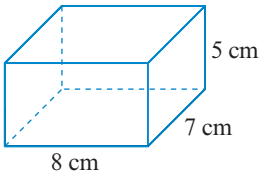



| THINK | WRITE |
|---|---|
| **1** Identify the 3D object (rectangular prism) and write the appropriate formula for volume. | $V = l \times w \times h$ |
| **2** Identify the measurements for $l$, $w$ and $h$. Check that they are all in the same unit (yes). | $l = 12$ mm, $w = 7.5$ mm, $h = 10$ mm |
| **3** Substitute the values for $l$, $w$ and $h$ into the formula. | $V = 12 \text{ mm} \times 7.5 \text{ mm} \times 10 \text{ mm}$ |
| **4** Calculate the result and include the appropriate unit. | $= 900 \text{ mm}^3$ |

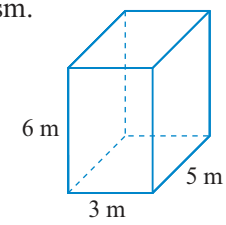
UNDERSTANDING AND FLUENCY

**2** Calculate the volume of each rectangular prism.

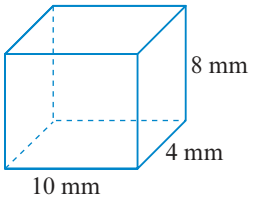
**a**



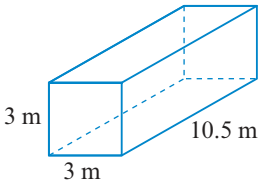

**b**



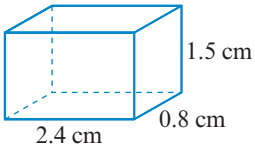

**c**



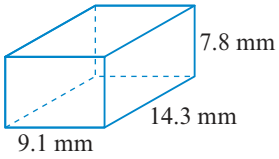

**d**




**e**

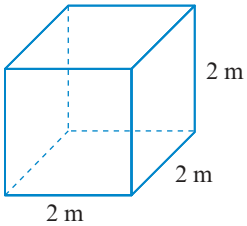



**f**

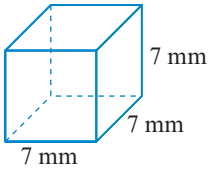



**3** Calculate the volume of each cube.

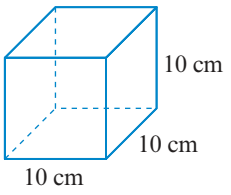
**a**



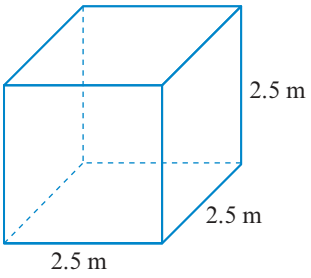

**b**



**c**




**d**




**4** Calculate the volume of each object.

**a**




**b**




**c**



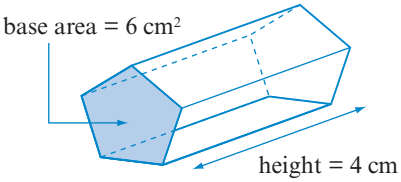

**d**




**5** The object at right has its base area and height shown.

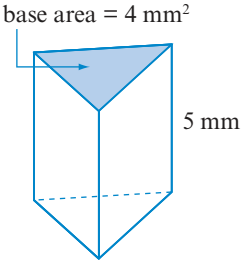
**a** The base shown is not the bottom surface of the object and the height is not a vertical measurement. Does this matter when calculating the volume? Explain.

**b** Use the rule for the volume of any prism to calculate the volume of this prism.

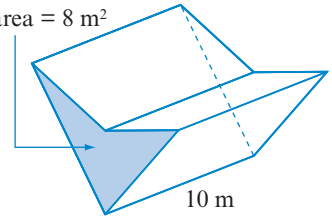



**6** Calculate the volume of these prisms. The base area for each prism is shown.

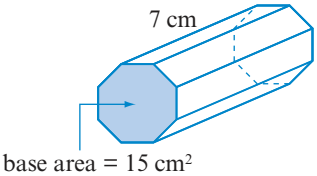
**a**



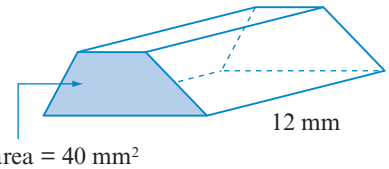

**b**




**c**

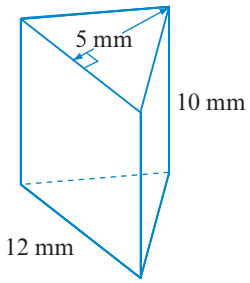



**d**

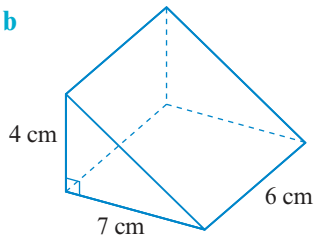


**7** Calculate the volume of each triangular prism. (Hint: what shape is the base of each prism?)

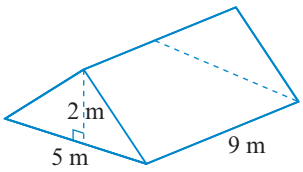
**a**



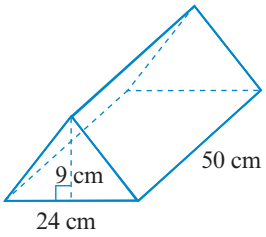

**b**



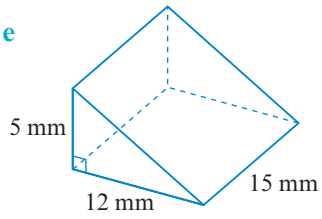

**c**



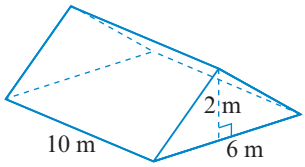

**d**




**e**




**f**




**8** With a partner, list two examples where amounts of liquid could be measured in the following capacity units.

**a** mL **b** L **c** kL **d** ML

**9** Use the conversion diagram to complete these statements.

**a** 8 L = _____ mL

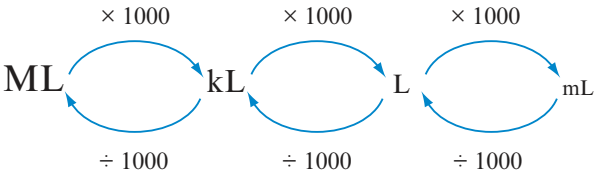
**b** 15 kL = _____ L

**c** 70 mL = _____ L

**d** 6.4 ML = _____ kL

**e** 2500 L = _____ kL

**f** 3 kL = _____ mL




**10** The units of volume and capacity are related. A container with an inside volume of 1 $cm^3$ holds 1 mL of liquid. This means that a container with an inside volume of 1000 $cm^3$ holds 1 L and a container with an inside volume of 1 $m^3$ holds 1 kL.

Copy and complete the table by working out the missing value for each container.

| | Inside volume of container | Capacity of container |
|---|---|---|
| **a** | 14 $cm^3$ | _____ mL |
| **b** | _____ $cm^3$ | 3 L |
| **c** | 5.8 $m^3$ | _____ kL |
| **d** | 750 $cm^3$ | _____ L |
| **e** | _____ $m^3$ | 1000 kL |
| **f** | _____ $m^3$ | 6.4 ML |

**11** Soil has been dug from a garden to form a rectangular hole of length 5 m, width 2 m and depth 50 cm. How many cubic metres of soil have been removed?

**12** A delivery of 3 $m^3$ of concrete is to be spread to form a driveway that is 7.5 m long and 3.2 m wide. How thick will the layer of concrete be?

PROBLEM SOLVING AND REASONING

**13** A sugar cube has the dimensions 1 cm by 1 cm by 1 cm.

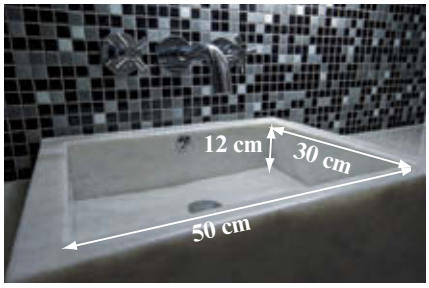
- **a** Explain what 1 cm by 1 cm by 1 cm means.
- **b** What is the volume of one sugar cube?
- **c** How many sugar cubes would fit into a box with dimensions 12 cm by 10 cm by 7 cm?
- **d** Describe the dimensions of a box that could exactly fit 1000 sugar cubes in it. Is there more than one answer? Explain.
- **e** Give five possible sets of dimensions for a box that could exactly fit 240 sugar cubes in it.
- **f** How many sets of dimensions are possible for the smallest box that could fit 20 sugar cubes? List them.

**14** Consider this container of juice.




- **a** Calculate the volume of the container.
- **b** Use your answer to part **a** to calculate the capacity of the container.
- **c** Is the capacity of juice shown on the labelling of the container the same as your calculated value? If not, give reasons why this might be.

**15** Find how many litres of water this rectangular sink will hold if filled to the top.




**16** List the most suitable volume or capacity unit for measuring these amounts.

- **a** the space taken up by a train carriage
- **b** the liquid in a tea cup
- **c** the chocolate in a chocolate bar
- **d** the water that a bath can hold

CHALLENGE

**17** Provide an estimate of each amount described in question **16**.

**18** The fuel tank of a car has a capacity of 50 L. How big is the tank? Suggest some possible dimensions if the tank is in the shape of a rectangular prism.

**19** List two examples of different objects that have a volume close to the following amounts. Explain your choices.

- **a** 10 $cm^3$
- **b** 10 $m^3$
- **c** 10 $mm^3$

**Reflect**

What is the difference between volume and capacity?

# CHAPTER REVIEW

## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

two-dimensional (2D) shape
three-dimensional (3D) object
units of measurement
length
width
perimeter
area
rectangle
$A = l \times w$
parallelogram
base
height
perpendicular
$A = b \times h$
rhombus
triangle
$A = \frac{1}{2} \times b \times h$
surface area
net
prism
volume
capacity
$V = A \times h$
$V = l \times w \times h$

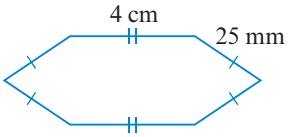
## MULTIPLE-CHOICE

9A **1** Which length measurement is equivalent to 123 cm?

**A** 1230 mm **B** 12.3 m
**C** 0.123 km **D** 12 300 m

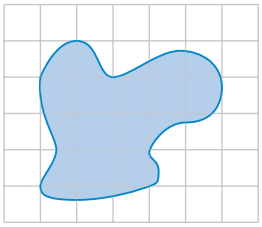
9B **2** The perimeter of this object is:

**A** 6.5 cm
**B** 108 mm
**C** 18 cm
**D** 29 mm




9C **3** A 1-cm grid has been drawn over the shape below. The area of the shape is closest to:

**A** 8 cm$^2$
**B** 14 cm$^2$
**C** 20 cm$^2$
**D** 22 cm$^2$

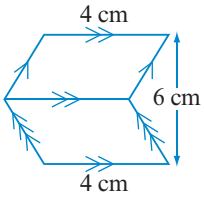


9D **4** The area of a rectangle with length 9 cm and width 3 cm is:

**A** 6 cm$^2$ **B** 12 cm$^2$
**C** 24 cm$^2$ **D** 27 cm$^2$

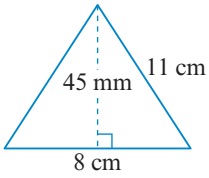
9E **5** The area of this shape is:

**A** 14 cm$^2$ **B** 24 cm$^2$
**C** 49 cm$^2$ **D** 144 cm$^2$



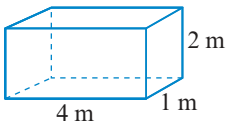

9F **6** The base length and height of this triangle are:

**A** 8 cm, 4.5 cm
**B** 8 cm, 11 cm
**C** 11 cm, 4.5 cm
**D** 4.5 cm, 8 cm




9G **7** The surface area of this rectangular prism is:

**A** 8 m$^2$ **B** 14 m$^2$
**C** 28 m$^2$ **D** 56 m$^2$




9H **8** Which pair of volume and capacity measurements are equivalent?

**A** 40 mL and 4 cm$^3$
**B** 1 m$^3$ and 1 L
**C** 250 mL and 2.5 L
**D** 500 cm$^3$ and 0.5 L

## SHORT ANSWER

9A **1** Convert these units.

a 34 cm into mm

b 170 cm into m

c 45 mm into cm

d 8.7 km into m

9A **2** Which unit of measurement would be the best choice to measure each of these?

a the length of a pencil

b the length of a football oval

c the distance from your house to school

d the length of a fingernail

9B **3** Estimate, then measure, the perimeter of this shape.

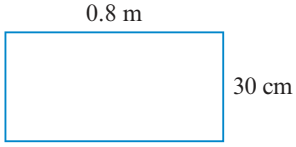


9B **4** Calculate the perimeter of each rectangle.

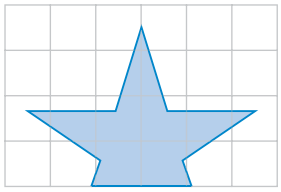
a




b



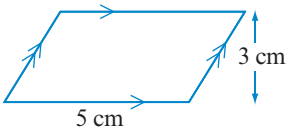

9C **5** A 1-cm grid has been drawn over this shape. Find the approximate area of the shape.



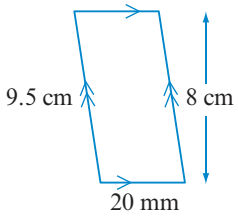
9D **6** Calculate the area of each rectangle shown in question **4**.

9E **7** Calculate the area of each parallelogram.

a

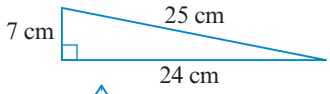



b

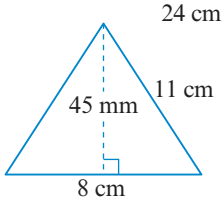



9F **8** Calculate the area of each triangle.

a

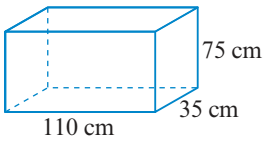



b

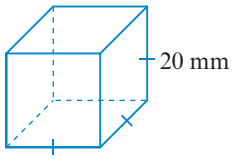



9G **9** Calculate the surface area of each object.

a



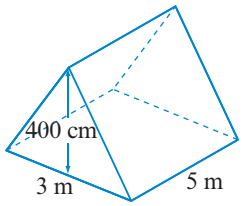

b



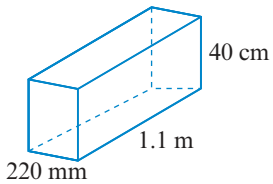

9H **10** Calculate the volume of each object shown in question **9**.

9H **11** Calculate the volume of each object.

a




b




9H **12** Find the volume of a rectangular prism if its capacity is 0.75 L. Suggest a set of dimensions for the prism.

## NAPLAN-STYLE PRACTICE

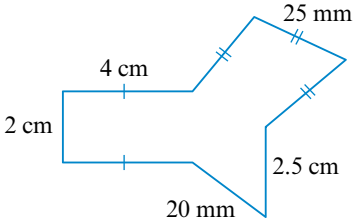
**1** Kyle knows that it is exactly 798 m from his house to school. Convert this distance to kilometres.

☐ km

**2** Which length measurement is the largest?

- ◯ 6500 mm
- ◯ 5600 cm
- ◯ 7.5 m
- ◯ 0.006 km

**3** What is the perimeter of this shape?



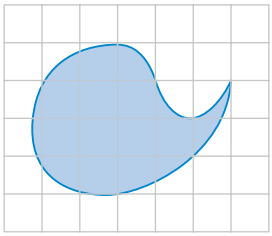

- ◯ 53.5 mm
- ◯ 13 cm
- ◯ 22 cm
- ◯ 107.5 mm

**4** What is the perimeter of a square of length 8 mm?

☐ mm

**5** A 1-cm grid has been placed over this shape.

What is the approximate area of the shape?

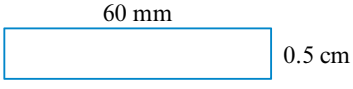


☐ $cm^2$

**6** What is the area of a square of length 8 mm?

☐ $mm^2$

Questions **7** and **8** refer to this rectangle.




**7** What is the area of the rectangle?

- ◯ 30 $mm^2$
- ◯ 3 $cm^2$
- ◯ 30 $cm^2$
- ◯ 121 $mm^2$

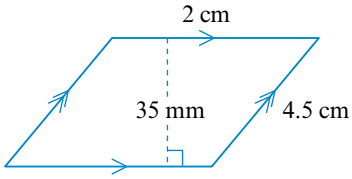
**8** What is the perimeter of the rectangle?

- ◯ 30 mm
- ◯ 13 cm
- ◯ 30 cm
- ◯ 121 mm

**9** A rectangle has a length of 30 mm and an area of 24 $cm^2$. What is the width of the rectangle in centimetres?

☐ cm

Questions **10**–**12** refer to this shape.




**10** What is the name of the shape?

- ◯ rectangle
- ◯ triangle
- ◯ parallelogram
- ◯ square

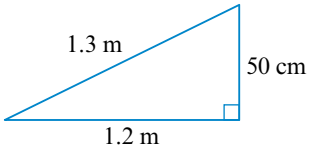
**11** What is the perimeter of the shape?

- ◯ 65 mm
- ◯ 9 cm
- ◯ 110 mm
- ◯ 13 cm

**12** What is the area of the shape?

- ◯ 130 $mm^2$
- ◯ 7 $cm^2$
- ◯ 9 $cm^2$
- ◯ 1575 $mm^2$

Questions **13** and **14** refer to this shape.




**13** What is the perimeter of the shape?

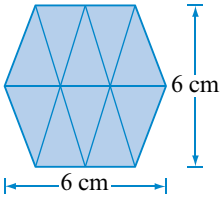
- ◯ 2.5 m
- ◯ 3 m
- ◯ 7.5 m
- ◯ 52.5 m

**14** What is the area of the shape?

- ◯ 30 $m^2$
- ◯ 60 $m^2$
- ◯ 300 $cm^2$
- ◯ 3000 $cm^2$

**15** All of the triangles in this shape are identical.

What is the area of the entire shape?




- ◯ 3 $cm^2$
- ◯ 30 $cm^2$
- ◯ 45 $cm^2$
- ◯ 60 $cm^2$

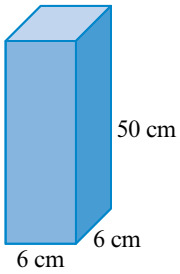
**16** The area of a square is 49 $cm^2$. What is its perimeter?

_____ cm

**17** Which shape has the largest area?

- ◯ rectangle with length 70 mm and width 3 cm
- ◯ triangle with base length 10 cm and height 40 mm
- ◯ square with length 5 cm
- ◯ parallelogram with base length 8 cm and height 30 mm

Questions **18**–**20** refer to this object.




Ella bought a vase for her new house.

**18** What is the surface area of the outside of the vase?

- ◯ 62 $cm^2$
- ◯ 300 $cm^2$
- ◯ 1236 $cm^2$
- ◯ 1800 $cm^2$

**19** What is the volume of the vase?

- ◯ 50 $cm^3$
- ◯ 62 $cm^3$
- ◯ 300 $cm^3$
- ◯ 1800 $cm^3$

**20** What is the amount of water that this vase could hold?

- ◯ 1.8 L
- ◯ 18 L
- ◯ 180 L
- ◯ 1800 L

Questions **21** and **22** refer to a new storage room of length 6 m, width 4 m and height 3 m.

The storage room needs to be painted. There is a door to the room with dimensions 200 cm by 150 cm that has already been painted.

**21** What is the area of the walls, floor and ceiling that would need to be painted?

_____ $m^2$

**22** What volume of goods could be stored in the room?

_____ $m^3$

## ANALYSIS

For a friend's birthday, you decide to buy a portable DVD player. There are three within your price range, with slightly different screen dimensions.

Option 1: height 8 cm by width 16 cm

Option 2: height 9 cm by width 15 cm

Option 3: height 110 mm by width 140 mm

**a** The shop assistant tells you that all three have the same screen perimeter. Find the perimeter of each screen. Is the shop assistant correct?

**b** A friend tells you that if the screens have the same perimeter, they must have the same area. Calculate the area of each screen and explain why this is not true.

**c** You decide to select the DVD player that has the widest screen dimension. What disadvantage does this have compared to the other two options?

**d** If the DVD player is only 3 cm thick, what is the volume of the smallest box that you could put it in?

**e** The DVD player comes in a box that measures 100 mm by 200 mm by 50 mm. What is the volume of this box?

**f** You will need to wrap the present. Calculate the surface area of the box so that you know the minimum amount of paper you will need.

**g** Why might you need a bit more wrapping paper than the minimum? Think about how you wrap objects.

# CONNECT

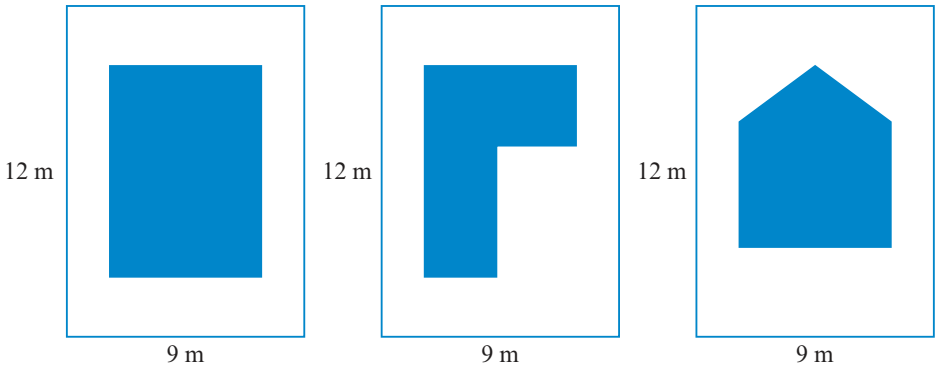
## Planning a swimming pool



To build an in-ground swimming pool requires planning and some careful mathematical calculations.

What measurements are needed when planning the building of a backyard swimming pool?

A rectangular section of land of length 12 m and width 9 m has been cleared for an in-ground swimming pool. This land also includes space for paving around the pool and a safety fence. The floor of the pool will be horizontal. This means that the depth of the water will be the same across the whole pool. The concrete walls and floor of the pool are to be painted blue.




Possible plans for a swimming pool (aerial view)

### Your task

Your task is to design your own swimming pool and calculate each of these measurements:

- the amount of paving required
- the length of safety fencing needed
- the amount of soil to be removed to form the hole for the pool
- the area of the walls and floor of the pool to be painted
- the amount of paint needed to paint the pool
- the amount of water needed to fill the pool
- the amount of plastic material needed for the pool cover
- any other measurements you think are needed.

Another option for the design of an in-ground swimming pool is to have the depth of the pool vary from one end to the other, as shown in this diagram.



Side view of pool

How would this change affect your measurement calculations? Include all necessary working to justify your answer.

Complete the 9 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a PowerPoint presentation
- a scale model
- a video
- other (check with your teacher).












# 10 STATISTICS AND PROBABILITY


**10A** Collecting data

**10B** Interpreting data

**10C** Dot plots, column graphs and bar graphs

**10D** Pie graphs

**10E** Line graphs and scatterplots

**10F** Stem-and-leaf plots

**10G** Summary statistics

**10H** Describing probability

**10I** Theoretical probability

**10J** Experimental probability




**ESSENTIAL QUESTION**

*How can you make use of data?*

Questions **1**–**3** refer to this table, which shows the number of TV shows watched by Year 7 students per week.

| Number of TV shows watched | Number of students |
|---|---|
| 0 | 1 |
| 1 | 4 |
| 2 | 7 |
| 3 | 16 |
| 4 | 17 |
| 5 | 14 |
| 6+ | 11 |

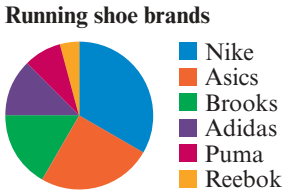
10A **1** How many students watch two TV shows per week?

10A **2** How many students watch more than five TV shows per week?

10A **3** How many people have been surveyed?

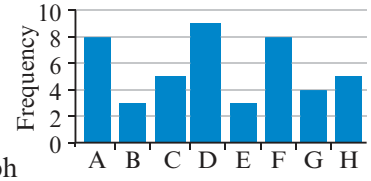
**A** 70 **B** 5 **C** 21 **D** 17

10B **4** This pie graph shows brands of running shoes used by a group of joggers.



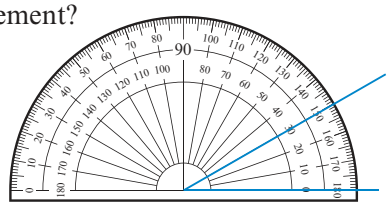

Which is the most popular?

10C **5** What type of graph is this?



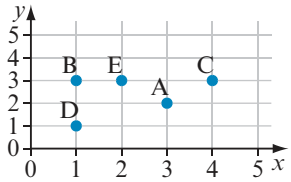

**A** pie graph
**B** column graph
**C** bar graph **D** line graph

10D **6** A protractor is placed as shown to measure an angle. What is the angle measurement?



**A** 150°
**B** 90°
**C** 180°
**D** 30°

Questions **7** and **8** refer to this Cartesian plane, which shows a number of points.




10E **7** What are the coordinates of the point A?

**A** (2, 3) **B** (3, 1)
**C** (3, 2) **D** (4, 3)

10E **8** Which point has the coordinates (1, 3)?

**A** A **B** B **C** C **D** D

Questions **9**–**11** refer to this data set.
1, 5, 3, 7, 6, 2, 6, 9, 4

10G **9** When the data are put in order, what is the middle number?

10G **10** Which number appears the most times?

10G **11** What is the range of the numbers?

10H **12** How would you describe flipping a coin and getting a head?

**A** likely **B** unlikely
**C** impossible **D** even chance

10I **13** What is 0.5 as a fraction?

Questions **14** and **15** refer to this spinner.



10I **14** What is the probability of spinning pink?

**A** 3 **B** $\frac{1}{4}$ **C** $\frac{1}{8}$ **D** $\frac{3}{8}$

10I **15** What is the probability of spinning blue?

**A** 3 **B** $\frac{1}{4}$ **C** $\frac{1}{8}$ **D** $\frac{3}{8}$

10J **16** If you flipped a coin 100 times, how many tails would you expect to get?

**A** 0.5 **B** 20 **C** 50 **D** 100

# 10A Collecting data



## Start thinking!

When you collect **data** it is important to group together data that are of a similar type.

1 Create a table with the headings shown.

2 Fill in information about yourself and three classmates on the following data.

| Data | Me | | | |
|---|---|---|---|---|
| | | | | |

Hair colour, height, eye colour, favourite animal, number of people in your family, age (in years), favourite song, how much you like music, how many pets you have, how much you are enjoying today, length of your pencil-case in centimetres, number of phones in your house, and three other questions that you can think of.

3 Look at the data you have collected. Can you sort them into two different types? How would you describe these two groups?

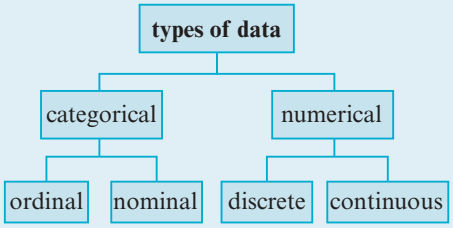
4 Data that can be counted or measured are called **numerical data**. Highlight the rows with numerical data in one colour.

5 Data that can be put into categories are called **categorical data**. Highlight the rows with categorical data in a different colour.

6 Look at the rows you have highlighted as numerical. There are two types of numerical data. How might you separate your numerical data into two groups?

## KEY IDEAS

- Data can be either numerical or categorical.
- There are two types of numerical data:
  **discrete data** are data that can be counted in whole numbers
  **continuous data** are data that can be measured.
- There are two types of categorical data:
  **ordinal data** are data that can be put in order
  **nominal data** are data that can be put into unrelated categories.




- To record data you can create a **frequency table** (sometimes called a **frequency distribution table**) to tally the **frequency** (the total number) of each group.
- A smaller selection from a larger group is called a **sample**. The larger group is called the **population**.
- When there is a large number of data, they may be grouped into **classes** for clearer presentation.

# EXERCISE 10A Collecting data

## EXAMPLE 10A-1 Classifying data

Classify these data.

a your height

b opinion on a local issue, from strongly disagree to strongly agree

**THINK**

**a** **1** Does the data deal with numbers or categories? Height is described using numbers, so it is numerical data.

**2** Can this data be measured or counted? Height can be measured and is not limited to whole numbers, so it is continuous data.

**b** **1** Does the data deal with numbers or categories? There are no numbers, so it is categorical data.

**2** Can these categories be put in order or are they unrelated? The categories can be put in order, so it is ordinal data.

**WRITE**

**a** The data is numerical, continuous.

**b** The data is categorical, ordinal.

UNDERSTANDING AND FLUENCY

**1** **a** Classify these data as either numerical (N) or categorical (C).

- **i** weight
- **ii** favourite animal
- **iii** nationality
- **iv** time on the Internet per week
- **v** hair colour
- **vi** number of pages in a book
- **vii** earring length
- **viii** how much people like dogs
- **ix** head size

**b** Classify all the numerical data as discrete (D) or continuous (C).

**c** Classify all the categorical data as nominal (N) or ordinal (O).

UNDERSTANDING AND FLUENCY

**2** Classify these data.

- **a** length of a tablet
- **b** number of people at a video store
- **c** your favourite colour
- **d** amount of time spent at a computer per week
- **e** how much people like exercise
- **f** movie genre
- **g** size of meals you can get at McDonald's
- **h** number of spots on a leopard.

**3** List two of your own examples for each of these data categories.

- **a** numerical, discrete
- **b** numerical, continuous
- **c** categorical, ordinal
- **d** categorical, nominal

## EXAMPLE 10A-2 Creating a frequency table

Alexis asked people what their favourite chocolate bar was and got the following results.

Time Out, Crunchie, Picnic, Mars Bar, Crunchie, Milkybar, Aero, Mars Bar, Mars Bar, Milkybar, Time Out, Aero, Mars Bar, Picnic, Picnic, Picnic, Aero, Mars Bar, Time Out, Mars Bar

- **a** What type of data is this?
- **b** Create a frequency table of the data.
- **c** How many people were surveyed?
- **d** Which is the most popular type of chocolate bar?

### THINK

**a** The data are not numbers but categories, so it is categorical data. There is no order of categories, so it must be nominal data.

**b** **1** Create a table with three columns headed: Chocolate, Tally and Frequency.

**2** Enter the types of chocolate into the first column.

**3** Draw a tally line for each chocolate bar in the appropriate row of the tally column.

**4** Write the final number for each chocolate bar in the frequency column.

**c** Add a row at the bottom of the table and count the total number of people surveyed.

**d** Identify which chocolate has the highest frequency.

### WRITE

**a** categorical, nominal data

**b**

| Chocolate | Tally | Frequency |
|---|---|---|
| Time Out | \|\|\| | 3 |
| Crunchie | \|\| | 2 |
| Picnic | \|\|\|\| | 4 |
| Mars Bar | 𝍸 \| | 6 |
| Milkybar | \|\| | 2 |
| Aero | \|\|\| | 3 |
| Total | | 20 |

**c** 20 people were surveyed.

**d** The most popular chocolate bar is the Mars Bar.

UNDERSTANDING AND FLUENCY

**4** Look at this frequency table and answer the following questions.

| Movie genre | Tally | Frequency |
|---|---|---|
| Action | 𝍸 \|\| | 7 |
| Comedy | 𝍸 𝍸 𝍸 | 15 |
| Horror | \|\| | 2 |
| Animated | 𝍸 𝍸 \|\| | 12 |

**a** What type of data is this?

**b** What is:

**i** most popular?

**ii** least popular?

**5** A survey of the number of siblings class members have gave the following results.

1 2 1 0 2 2 4 1 2 1 1 3 1 2 3 0 1 2 1 1 2 2 8 1 2 1

**a** What type of data is this?

**b** Create a frequency table of the data.

**c** How many people were surveyed?

**d** What was the most common number of siblings?

PROBLEM SOLVING AND REASONING

**6** Create a frequency table using this photograph. Use the table to state the most common and the least common colour. Why might you get different results from your classmates?



**7** When someone is planning a survey, it is important that they ask appropriate questions. Your friend Yanni wants to find out the most popular types of music to play on the local school radio. He wrote these three questions.

1 Do you prefer rock or pop?
2 What is your favourite type of music?
3 Should the radio play hip-hop music?

**a** Answer each of these questions.

**b** How useful do you think these answers would be to Yanni?

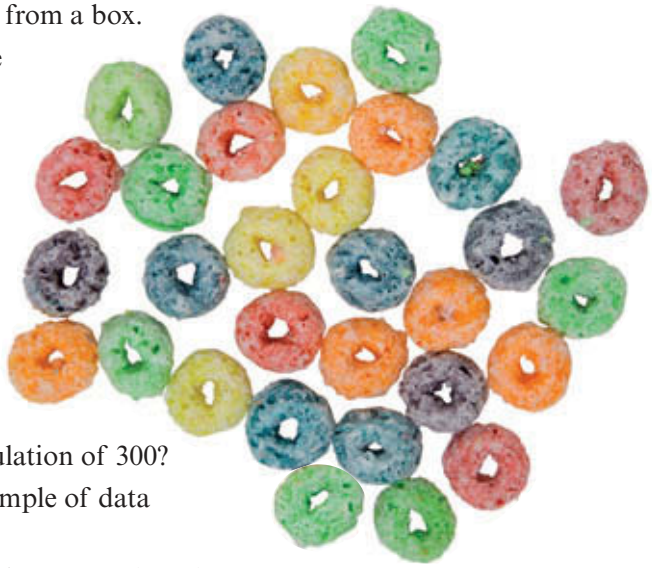
**c** If someone answers yes to the third question, does it mean that hip-hop is their favourite kind of music? Explain your answer.

**d** Pick the best question from the three and explain why the other two questions are not appropriate.

**8** Rewrite the following questions so that they provide fair answers.

| | Topic | Question |
|---|---|---|
| **a** | Deciding what to provide for a school lunch | Do you like pizza? |
| **b** | Finding the most popular colour | Do you prefer blue, pink or black? |
| **c** | Investigating languages spoken at home | Which language do you think is most popular? |

**9** Ryan grabbed a handful of Fruit Loops from a box.



**a** Create a frequency table showing the frequency of the different colours of Fruit Loops in the handful.

**b** What is the most common colour? What is the least common colour?

A smaller selection from a larger group is called a sample. The larger group is called the population.

**c** There are five pink Fruit Loops in this sample of 30. How many might you expect in the box if it has a population of 300?

**d** What is the advantage of taking a sample of data rather than the entire population?

**e** What might be a disadvantage of taking a sample rather than the entire population?

**10** For each of the following data surveys, state:

**i** the type of data **ii** whether it is a sample or a population.

**a** surveying birthday months in your class by asking everybody in your class

**b** surveying favourite sport in Year 7 by asking everybody in your class

**c** surveying height in your class by asking four of your classmates

**d** surveying number of languages spoken at school by asking each person in the school

**e** surveying number of people in several of your classmates' families by asking each of them

**f** surveying opinions on whole school policy by asking 10 people from each year level

**11** A Year 7 group recorded their heights in centimetres and the results are shown below.

156 161 150 157 155 167 158 165 145 153 159 159
162 172 158 156 139 160 151 157 156 159 166 161

**a** Create a frequency table for the data.

Having such a large number of different heights can make frequency tables big and not very useful. A better way to show the data is to group the data into classes. This allows us to see the spread of the data more clearly.

| Height (cm) | Tally | Frequency |
|---|---|---|
| 135–139.5 | | |
| 140–144.5 | | |
| 145–149.5 | | |
| 150–154.5 | | |
| 155– | | |
| 160– | | |
| | | |
| | | |

**b** Copy and fill in this frequency table. You will need to complete the first column before you enter the data.

**c** What is the most common height class?

**d** What is the least common height class?

**e** What is the advantage of grouping data into classes?

PROBLEM SOLVING AND REASONING

**12** The frequency table in question **11** groups the students into classes with a range of 5 cm, but you can use whatever size class is best.

- **a** Redraw the frequency table but this time use class intervals of 10 cm.
- **b** What are the advantages and disadvantages of having larger classes?

**13** The AFL ladder shown displays the position of the top eight football teams at the end of a season. The position on the ladder is indicated by 1st, 2nd, 3rd, etc. These are called **ordinal numbers**.

| AFL 2013 FINAL LADDER | |
|---|---|
| 1 | Hawthorn |
| 2 | Geelong |
| 3 | Fremantle |
| 4 | Sydney Swans |
| 5 | Richmond |
| 6 | Collingwood |
| 7 | Port Adelaide |
| 8 | Carlton |

- **a** Why do you think they are called ordinal numbers?
- **b** Can you explain why position on a ladder is categorical data and not numerical data, even though it involves numbers? Can you count or measure these numbers?
- **c** Provide another example of categorical data that involves numbers.

**14** Conduct a survey on a topic of your choice.

- **a** Ask at least four questions so that you can collect data of each type (discrete, continuous, nominal, ordinal) and present the results in a frequency table.
- **b** Think about whether you will be taking a sample or surveying the entire population.
- **c** What can you conclude from your survey?

CHALLENGE

**15** A survey was conducted to investigate the ages of customers at an EB Games store. The results are listed.

14 29 16 41 18 22 34 65 52 21 24 37 42 30 17 17 19 20 25 21
33 83 22 18 17 21 19 22 27 39 54 15 18 23 29 30 31 28 17

- **a** There are too many different numbers to create a normal frequency table. Choose a class size that allows you to see a pattern.
- **b** Create a frequency table with your chosen class size.
- **c** What is the most common age range to shop at EB Games?
- **d** What is the least common age range to shop at EB Games?
- **e** What was the most common age (from the original data) to shop at EB Games? Does this age fit in the most common age range?
- **f** How many people were surveyed?

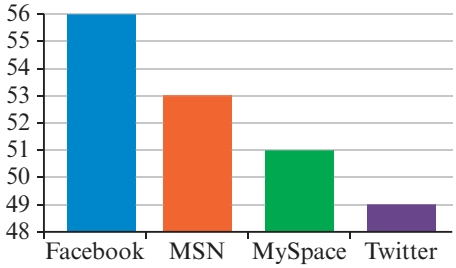


**Reflect**

Why are tables useful?

# 10B Interpreting data

## Start thinking!




A graph is an effective way to display data in an easy-to-read format.

**1** What data is the **column graph** showing?

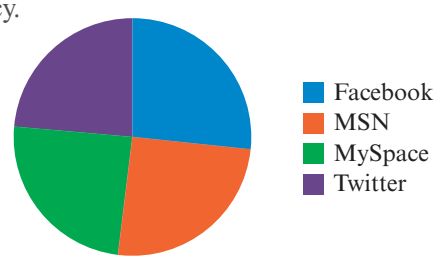
Look at the vertical axis of the column graph. This shows the frequency of the categories.

**2** What is the purpose of the lines across the graph?

**3** How many people use Facebook?

**4** Make a table from the graph that shows each category and its frequency.

**5** Add a row to the table. Add all the frequencies to show how many people were surveyed.




**6** What data is the **pie graph** showing?

**7** What are the differences between the two graphs?

**8** The two graphs show the same data. Why do the categories seem roughly equal in the pie graph but very different in the column graph?

## KEY IDEAS

- A good graph is easy to read and displays data clearly.
- All graphs should include:
  - a title
  - axes that are clearly labelled (including any units)
  - a regular scale on both axes
  - a legend or key, if different parts of the graph are in different colours.
- A column graph shows the frequency of categorical data, which are represented in vertical columns with space between them.
- A **histogram** is similar to a column graph, but the columns represent classes of data. There is no space between columns and the class boundaries are placed on the lines between the columns.
- A **bar graph** shows the frequency of categorical data and is presented in horizontal bars.
- A pie graph shows the frequency of categorical data (size of sector) as a fraction of the total pieces of data (circle).
- A **line graph** shows continuous data where points are plotted and joined by a line.
- An **outlier** is a piece of data that is much higher or lower than the other data in a set.

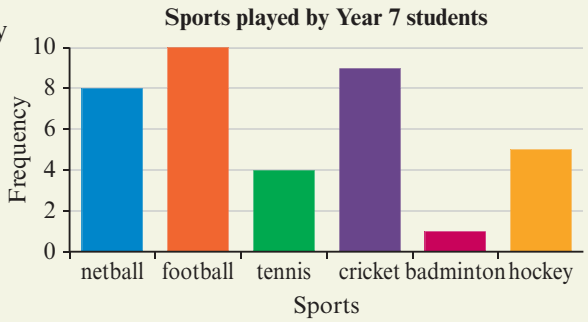
**NOTE** watch for graphs that are not clear. In particular, graphs that do not have an even scale (such as in the Start thinking! task) can be misleading.

# EXERCISE 10B Interpreting data

## EXAMPLE 10B-1 Reading a column graph

This column graph shows the results of a survey of Year 7 students and the sports they play.

a What is the most common sport played and how many students play it?
b What is the least common sport played and how many students play it?
c How many sports are in the survey?
d How many students were surveyed?




### THINK

a The most common sport has the tallest column (football). Look at how high the column reaches on the vertical axis (10).

b The least common sport has the shortest column (badminton). Look at how high the column reaches on the vertical axis (1).

c Count the number of columns.

d Find the frequency of each column and add them together.

### WRITE

a Football is the most common sport, with 10 students playing it.

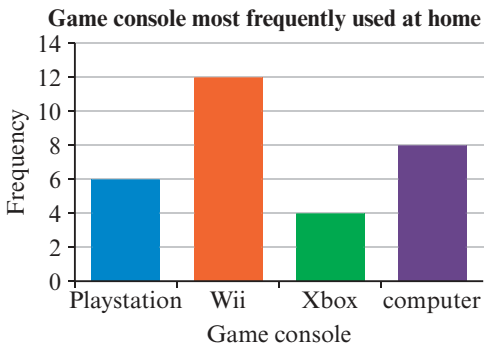
b Badminton is the least common sport, with one student playing it.

c There are six types of sports.

d total number of students surveyed
= 8 + 10 + 4 + 9 + 1 + 5
= 37

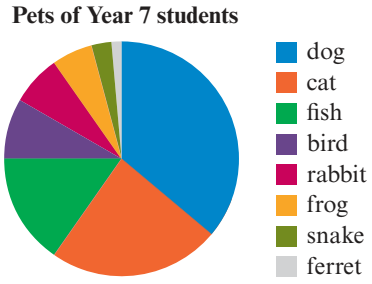
UNDERSTANDING AND FLUENCY

1 A class of Year 7 students were surveyed about what game consoles they played at home.
   a What is the most common console and how many students play it?
   b What is the least common console and how many students play it?
   c How many different game consoles are in the survey?
   d How many students were surveyed?



UNDERSTANDING AND FLUENCY

**2** This pie graph shows the results of a survey of Year 7 students who have pets at home.



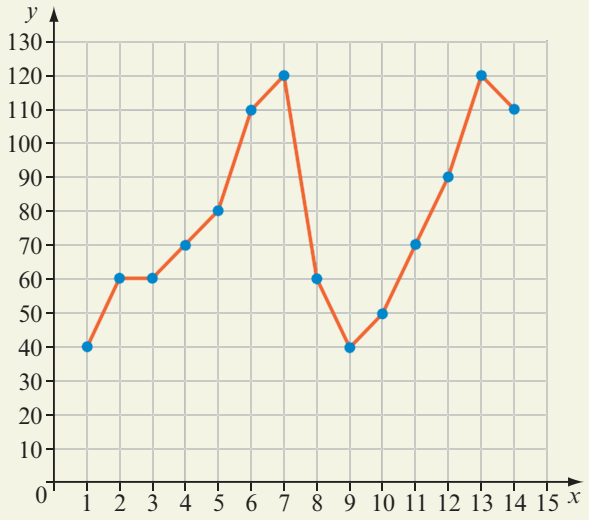



**a** What is the most common pet?
**b** What is the least common pet?
**c** How many different pets are in the survey?

## EXAMPLE 10B-2 Reading a line graph

The line graph shows the number of customers per day in a new swim and surf store.

**a** What is the largest number of customers per day?
**b** On which day(s) does this occur?
**c** How many customers were in the shop on day 3?
**d** Which day had 90 customers?




### THINK

**a** Look for the highest vertical dot(s). There are two. Look across to the vertical scale at the left to read off the value.

**b** There are two points. Look at the horizontal scale to read off the values.

**c** Look for day 3 on the horizontal axis and trace up to the dot, then across to the left to read off the vertical scale.

**d** Look for 90 on the vertical scale, and trace across to the dot, then down to read off the horizontal scale.

### WRITE

**a** The largest number of customers per day is 120.

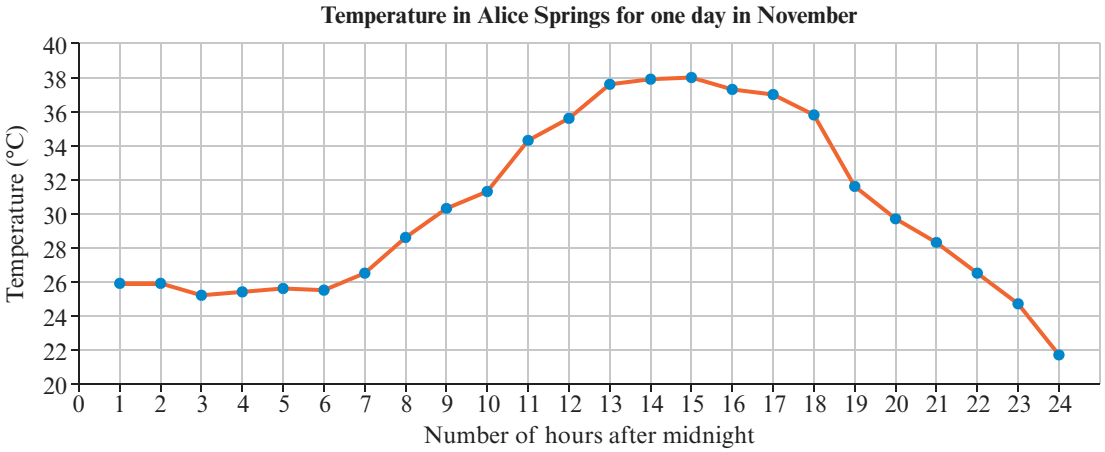
**b** 120 customers shop on days 7 and 13.

**c** On day 3 there are 60 customers.

**d** Day 12 had 90 customers.

UNDERSTANDING AND FLUENCY

**3** This line graph shows the temperature over a 24-hour period from midnight in Alice Springs one day in November.




**a** What is the hottest temperature recorded? At what time does it occur?

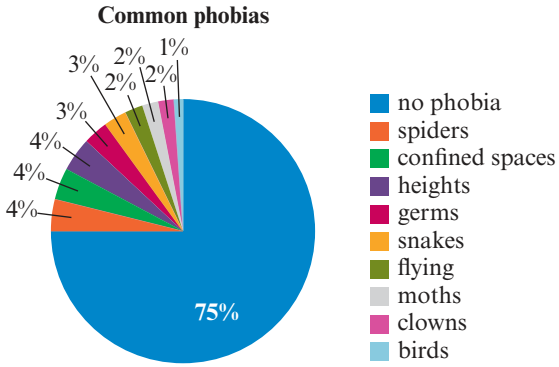
**b** What is the coolest temperature recorded? At what time does it occur?

**c** What was the temperature at 7.00 am?

**d** At what time after lunch did the temperature drop below 30°C?

PROBLEM SOLVING AND REASONING

**4** Consider this graph.



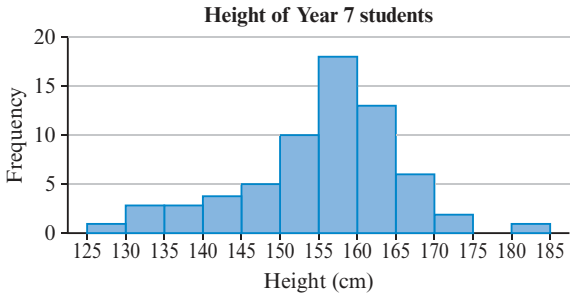

**a** What type of graph is it?

**b** What is it showing?

**c** What conclusions can you make from this graph?

**d** Can you tell how many people have been surveyed from this graph? Why or why not?

**5** The heights of a group of Year 7 students were measured and the results shown in this histogram.




**a** What type of data is this?

**b** How many centimetres are in each height class?

**c** The smallest height class is 125–130 cm. How many students are in this height class?

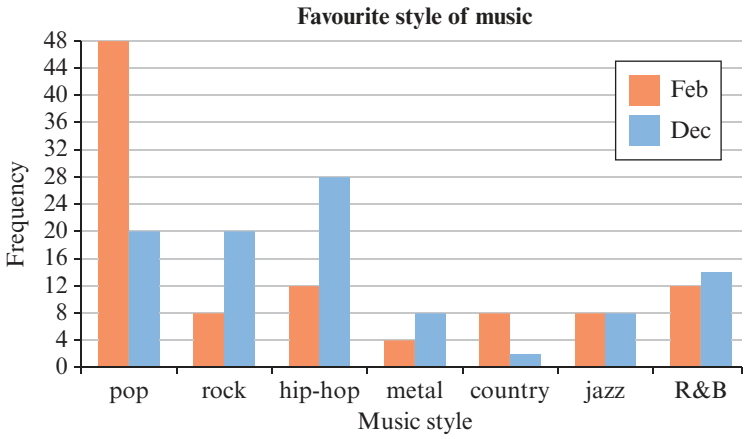
**d** What is the most common class? How many students are in this height class?

**e** What is the tallest height class recorded?

**f** Why is there no column for the height class 175–180 cm?

**g** How would you find out how many students were measured?

**6** This column graph shows favourite music styles for Year 7 students at the start and end of the year.




**a** What type of data is this?

**b** What was the most popular style of music at the beginning of the year? How many students chose it?

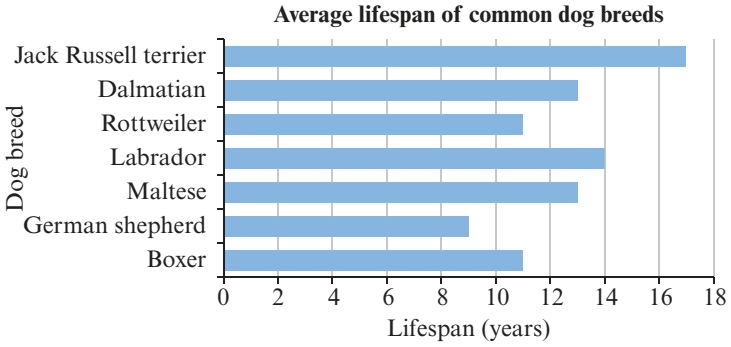
**c** What was the most popular style of music at the end of the year? How many students chose this?

**d** Which style of music improved its popularity the most over the year?

**e** What was the least popular style of music throughout the year? Consider just the smallest column.

**f** If you had to purchase two different CDs to play at a whole school function in the middle of the year, which two styles of music would you pick? Why?

**7** Look at the bar graph shown.

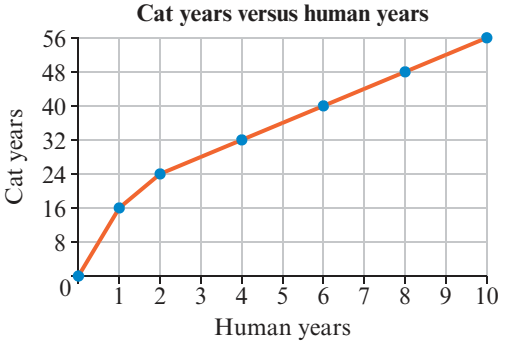



**a** What has been surveyed?

**b** How is this graph different from a column graph?

**c** Write two sentences to describe what the graph shows.

**8** This graph shows the age of a cat in 'cat years' over a period of 10 human years.




**a** How old is a cat in 'cat years' after 1 year?

**b** How old is a cat in 'cat years' after 2 years?

From 2 years onwards, for every human year a cat ages 4 'cat years'.



**c** How old is a cat in 'cat years' after 10 years?

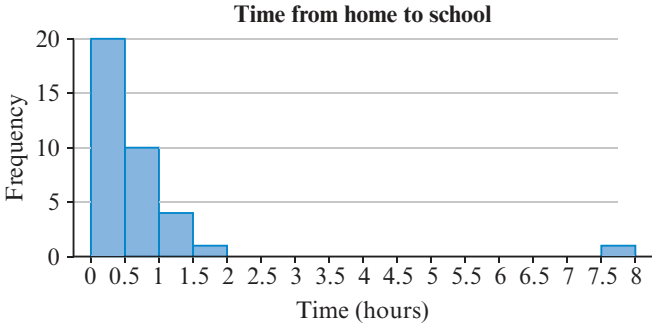
**d** How old would a cat be in 'cat years' after 11 years?

**e** How old would a cat be in 'cat years' after 20 years?

**f** An old wives' tale is that for every 1 human year, both cats and dogs age 7 years. How is this different from what you see on the graph?

CHALLENGE

**9** Year 7 students were surveyed on how long it takes them to get to school. This histogram displays the results.




**a** How much time does the width of each column represent?

**b** How many students get to school in:

**i** half an hour or less?

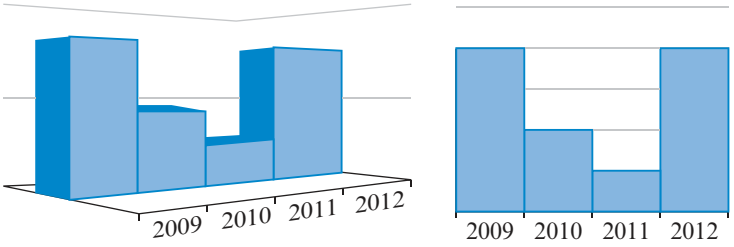
**ii** more than half an hour but less than an hour?

**iii** an hour or less?

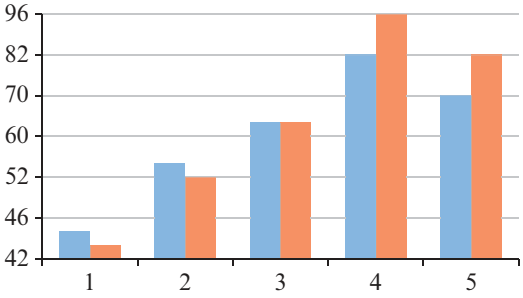
**c** One piece of this data is called an outlier. Which piece of data do you think it is? How long does it take this student to get to school?

**d** Can you think of a reason why it may take a student this long?

**10** A type of graph that is commonly used but can be very misleading is the **3D graph**. Use the 2D column graph which is displaying the same data to help explain why.




**11** Name at least five things wrong with this graph.




**Reflect**

How can you decide if a graph is misleading?

# 10C Dot plots, column graphs and bar graphs



## Start thinking!

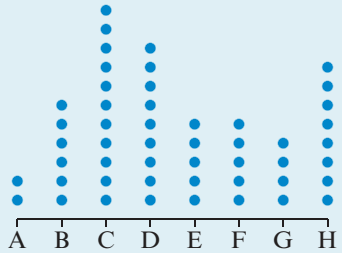
When presenting data you need to think about the best way to display it.
Often, different graphs suit different types of data. Even when you could use many different graphs, it is important to choose the graph which best represents the data.

1. How many pets do you have at home?
2. Have a class member draw a number line on the board, starting at 0 and finishing at maybe 10.
3. Go to the board and draw a dot above the number of pets that you have. If there is already a dot in this position, draw your dot above this.
4. Why do you think this graph is called a **dot plot**?
5. What is the lowest number of pets anyone has? This is the smallest number with a dot above it.
6. What is the largest number of pets anyone has? This is the biggest number with a dot above it.
7. What is the most common number of pets people have? How do you know?
8. What other graph looks like this?

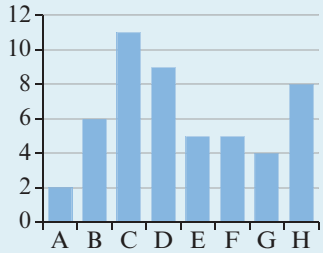
## KEY IDEAS

- When displaying data, choose a graph that is clear and informative.
- Draw your axes first. Check your minimum and maximum values and make sure your scale is appropriate. All values should be easy to read.
- Always include a title, axis labels and a legend if necessary.
- Dot plots should only be used for data with small frequencies.
- Long category labels are better suited to bar graphs rather than column graphs.

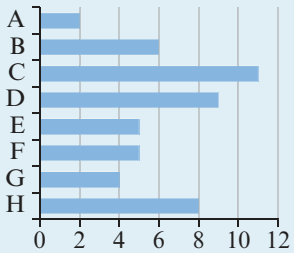
Dot plot




Column graph




Bar graph



# EXERCISE 10C Dot plots, column graphs and bar graphs

**1** A group of music students were asked what instrument they played. Use this data to construct a dot plot.

piano, saxophone, violin, guitar, guitar, saxophone, trombone, piano, clarinet, bass, drums, clarinet, saxophone, guitar, cello, violin, flute, trombone, flute, saxophone, guitar, violin, piano, guitar

## EXAMPLE 10C-1 Drawing a graph

Look at the data in this table.

**a** How many different categories are there?
**b** What is the minimum value?
**c** What is the maximum value?
**d** Choose a scale that covers the maximum value and allows all values to be seen easily.
**e** Create a column graph from the data.

| Email service | Frequency |
|---|---|
| Gmail | 16 |
| Hotmail | 10 |
| Yahoo | 12 |
| AOL | 2 |

### THINK

**a** Count the number of email services.

**b** Look at the frequency column for the smallest number.

**c** Look at the frequency column for the largest number.

**d** The vertical scale should cover the minimum and maximum values and must allow all values to be read easily. One option is a maximum of 20 with intervals of 2. The horizontal scale must fit four categories and allow for spaces between the columns.

**e** Using a ruler, draw each column accurately. Remember to include a title and axis labels.

### WRITE

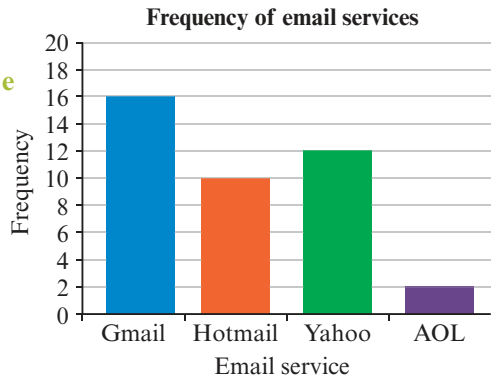
**a** There are four different categories.

**b** The minimum value is 2.

**c** The maximum value is 16.

**d** Use vertical scale from 0 to 20 with intervals of 2.
Use horizontal scale to fit four columns.

**e**



UNDERSTANDING AND FLUENCY

**2** Look at the data in the table on the right.

**a** How many categories are there?

**b** What is the minimum value?

**c** What is the maximum value?

**d** Choose a scale that covers the maximum value and allows all values to be seen easily.

**e** Create a column graph from the data.

| Clothing store | Frequency |
|---|---|
| Jay Jay's | 18 |
| Supre | 6 |
| Jetty Surf | 27 |
| Sportsgirl | 9 |
| Cotton On | 12 |

**3** Draw a bar graph using the data in this table.

| State/Territory | Number on a tour |
|---|---|
| Australian Capital Territory | 4 |
| New South Wales | 11 |
| Northern Territory | 9 |
| Queensland | 4 |
| South Australia | 8 |
| Tasmania | 13 |
| Victoria | 10 |
| Western Australia | 7 |

## EXAMPLE 10C-2 Choosing and drawing a graph

Use the data in the table to draw a graph.
Explain your choice.

| Favourite cartoon | Frequency |
|---|---|
| *Powerpuff girls* | 15 |
| *The Simpsons* | 35 |
| *SpongeBob SquarePants* | 54 |
| *Pokémon* | 20 |
| *Ben 10 Alien Force* | 5 |

### THINK

**1** Choose a graph. Since there are categories and the names are long, it is best to use a bar graph.

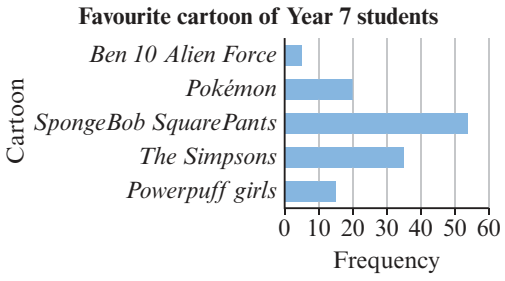
**2** Look at the frequency column of the table. The highest number is 54 and the lowest is 5. Choose a scale that allows you to show all numbers. One option is from 0 to 60 with intervals of 10.

**3** Draw the vertical axis long enough to fit the five categories and the horizontal axis long enough for the scale.

**4** Using a ruler, draw each bar accurately. Remember to include a title and axis labels.

### WRITE

A bar graph is best because the category names are long.



**4** Use the data in the table on the right to draw a graph. Explain your choice.

| Birth month | Frequency |
|---|---|
| January | 17 |
| February | 31 |
| March | 28 |
| April | 20 |
| May | 16 |
| June | 25 |
| July | 18 |
| August | 33 |
| September | 54 |
| October | 42 |
| November | 21 |
| December | 26 |

**5** Use the table below to draw a graph. Explain your choice.

| Hair colour | Frequency |
|---|---|
| Black | 13 |
| Blonde | 8 |
| Brown | 15 |
| Red | 4 |
| Dyed | 7 |

**6** Year 7 students were surveyed on what type of cars their family had and the results are shown below.

| | | | | | | |
|---|---|---|---|---|---|---|
| Holden | Toyota | Ford | Audi | Ford | Ford | Volvo |
| Holden | Holden | Hyundai | BMW | Holden | Toyota | Audi |
| Ford | Ferrari | Toyota | Ford | Ford | BMW | Hyundai |
| Volvo | Volvo | Toyota | Holden | Ford | Hyundai | Ford |
| Holden | Hyundai | | | | | |

**a** Create a bar graph to show the data.

**b** What is the most common type of car?

**c** What is the least common type of car?



**7** The table below right shows the number of pairs of shoes owned by a group of young women. To display data with class intervals, use a histogram.

**a** What is the size of the class interval?

**b** What data values would fit in the class interval 5–<10?

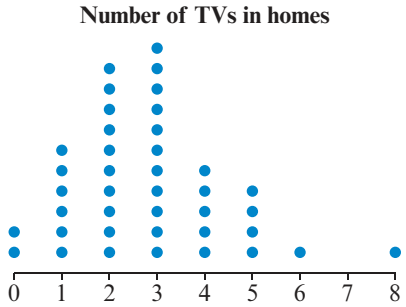
**c** Draw a vertical axis to contain the maximum frequency number.

**d** Draw a horizontal axis up to 30. The horizontal scale should match the class interval size. In this case, it should increase by 5.

**e** Finish the histogram by drawing the columns. Make sure the columns are between the scale marks, not on top of them. If you are uncertain, check the histogram on page 531 (10B question **9**).

| Number of pairs | Frequency |
|---|---|
| 0–<5 | 2 |
| 5–<10 | 5 |
| 10–<15 | 18 |
| 15–<20 | 19 |
| 20–<25 | 16 |
| 25–<29 | 8 |

**8** Sarah conducted a survey on the number of televisions in homes and produced a dot plot. Change the dot plot into a column graph.




**9** A group of Year 7 students were asked the age of their oldest sibling. The results are shown below.

14 17 15 20 14 16 15 15 16 18 19 18 21
26 12 13 15 16 14 17 17 18 14 16 15 15

**a** Construct a dot plot to represent this data.

**b** Describe the dot plot in at least two sentences.

**10** Data was collected on the average monthly maximum temperatures (in degrees Celsius) for Hobart and Darwin.

| | J | F | M | A | M | J | J | A | S | O | N | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hobart | 23.1 | 22.2 | 20.7 | 17.0 | 13.9 | 11.4 | 10.5 | 12.2 | 14.9 | 17.4 | 19.4 | 21.9 |
| Darwin | 32.4 | 32.2 | 32.7 | 33.5 | 32.6 | 31.2 | 30.6 | 31.7 | 33.0 | 34.1 | 34.2 | 33.6 |

**a** Draw a column graph to represent this data. Remember to use a legend.

**b** What are the highest and lowest average maximum temperatures for Hobart and Darwin?

**c** Are there any patterns? Describe the graph in at least two sentences.

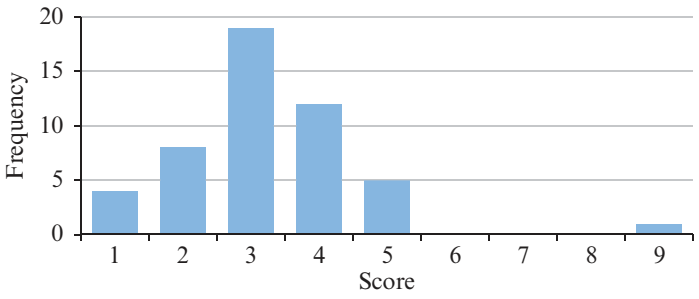
**11** Your brother started an exercise program and made note of the number of push-ups that he could do in one session. The results are shown in the table.

**a** Draw a column graph to represent this data.

**b** Comment on any pattern you see.

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of push-ups | 5 | 10 | 15 | 25 | 45 | 50 | 55 | 55 | 60 | 60 |

**12** Salwa produced this column graph.




**a** What is the value of the outlier (the piece of data far away from the rest)?

**b** Salwa realised that she misread the outlier when plotting the graph. It should have been 3. Redraw the column graph with the outlier changed to its correct value.

PROBLEM SOLVING AND REASONING

**13** How much water we use has become a big issue in recent years because of drought. The table below summarises average water usage per person from all states and territories in 2001 and 2005.



**a** Draw a bar graph to represent the data. Remember to include a legend.

**b** Comment on any patterns you see.

| State/territory | 2001 water usage (kL) | 2005 water usage (kL) |
|---|---|---|
| ACT | 120 | 95 |
| NSW | 95 | 85 |
| NT | 160 | 150 |
| Qld | 140 | 125 |
| SA | 110 | 95 |
| Tas. | 125 | 140 |
| Vic. | 95 | 80 |
| WA | 185 | 175 |

**14** Conduct a survey (or use your results from **10A** question **14**).

**a** Place your results in a frequency table.

**b** Create a column graph, a bar graph and a dot plot to represent your data.

**c** Which graph represents the data the best? Why?

CHALLENGE

**15** Data was collected on the length of a tree's shadow over time.

**a** Create a column graph showing this data.

**b** Over what time period do you think the data was taken?

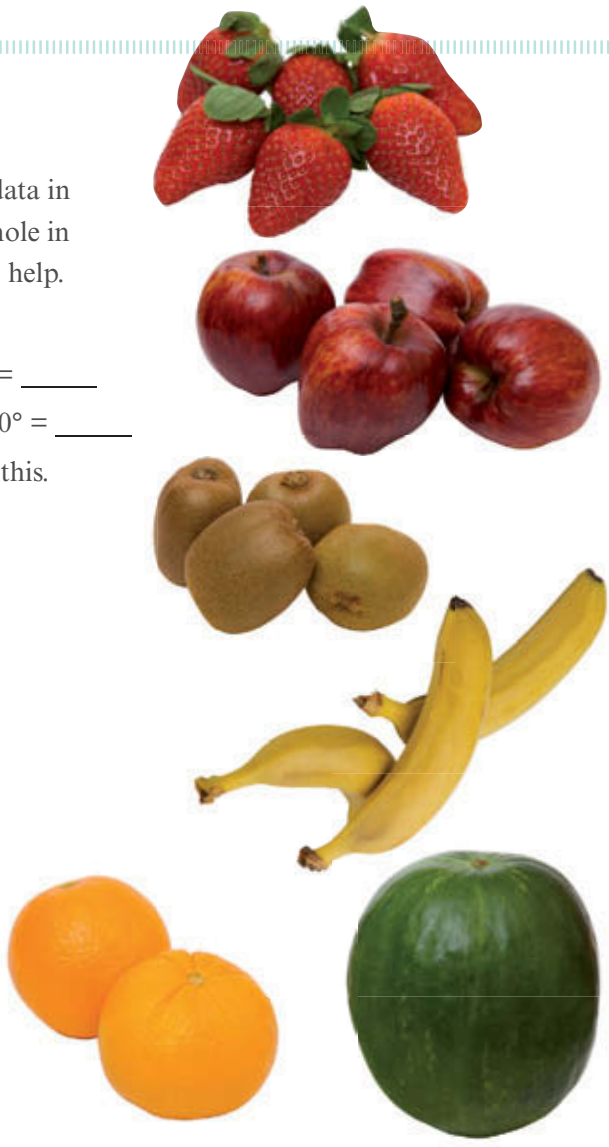
| Time (hours) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| Length (m) | 8.5 | 8 | 7 | 4.5 | 0.5 | 1 | 4.5 | 6 | 7 |



**Reflect**

What do you look for when deciding which graph is best to use?

# 10D Pie graphs



## Start thinking!

Pie graphs (also called pie charts) are an effective way to display data in an easy-to-read format. It is important to look at the data as a whole in order to draw a pie graph. You can use the properties of circles to help.

1 How many degrees in a circle?

2 How many degrees in $\frac{1}{2}$ a circle? Copy and complete: $\frac{1}{2} \times 360° =$ _____

3 How many degrees in $\frac{1}{4}$ of a circle? Copy and complete: $\frac{1}{4} \times 360° =$ _____

4 How many degrees in $\frac{1}{6}$ of a circle? Write an equation to show this.

5 Copy and complete the following sentence.

To find the angle size of a sector, _____ the fraction by 360°.

6 Look at the fruit in the photo and copy and complete this frequency table for each type of fruit.

| Fruit | Tally | Frequency | Fraction | Angle of sector |
|---|---|---|---|---|
| Strawberry | 𝍸 \| | | $\frac{6}{20}$ | $\frac{6}{20} \times 360° = 108°$ |
| Apple | | | $\frac{4}{20}$ | |
| Kiwifruit | | | | |
| Banana | | | | |
| Orange | | | | |
| Watermelon | | | | |
| Total | | 20 | | |

## KEY IDEAS

- Pie graphs represent different categories of data as sectors of a circle.
- To draw a pie graph you must either have data in a frequency table or as percentages.
- Always start with the largest sector and work clockwise from '12 o'clock' to the smallest sector.
- To find the angle size of each sector, multiply the percentage (or fraction of the total) by 360°.
- Always include a title and a legend. Percentages or numbers on the graph are also useful.
- Pie graphs are easily drawn with software such as Microsoft Excel.

# EXERCISE 10D Pie graphs

## EXAMPLE 10D-1 Drawing a pie graph using software

Use Microsoft Excel or other software to create a pie graph of this data.

| Book title | Frequency |
|---|---|
| *Harry Potter* | 8 |
| *Deltora Quest* | 4 |
| *Chronicles of Narnia* | 2 |
| *Spiderwick Chronicles* | 6 |

### THINK

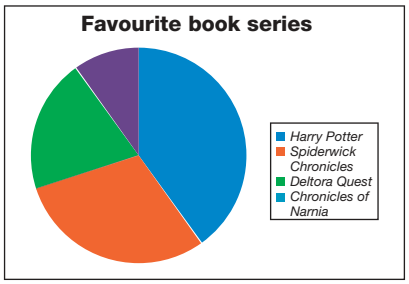
1 Open a new spreadsheet and enter the categories into column A and the data into column B in order from largest to smallest.

2 Highlight all data and create a pie graph using the appropriate tool bars. See your teacher if you need help with this.

3 Add a Chart title and any other information you need to finish your pie graph.

### WRITE

| | A | B |
|---|---|---|
| 1 | Harry Potter | 8 |
| 2 | Spiderwick Chronicles | 6 |
| 3 | Deltora Quest | 4 |
| 4 | Chronicles of Narnia | 2 |




UNDERSTANDING AND FLUENCY

1 Use this frequency table to create a pie graph.

| Candidate | Number of votes |
|---|---|
| Will | 15 |
| Caleb | 25 |
| Teagan | 20 |
| Kyle | 40 |
| Emma | 30 |
| Ella | 35 |
| Andy | 35 |

**NOTE** Remember to put the categories in order from largest to smallest and include a legend.

## EXAMPLE 10D-2 Drawing a pie graph by hand

Create a pie graph of this data.

**Favourite ice cream store**

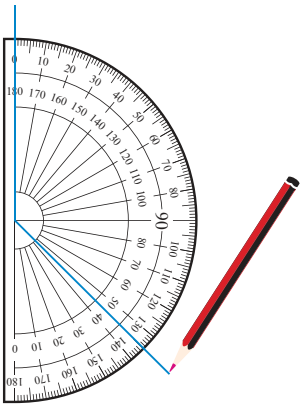
| Store | Frequency |
|---|---|
| Cold Rock | 15 |
| Trampoline | 12 |
| New Zealand Natural | 7 |
| Baskin Robbins | 4 |
| Wendy's | 2 |

### THINK

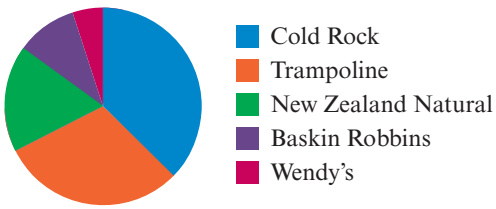
1. Add two columns to the table headed Fraction and Angle of sector.
2. Find the total of the frequency column and use this to complete the Fraction column.
3. Multiply each fraction by 360° to find the angle of each sector.
4. Draw a vertical line for the start of your pie graph. Using a protractor, measure the first angle (135°) in a clockwise direction.
5. Continue to add sectors to the pie graph until it is complete.
6. Include a title and a legend to complete the pie graph.

### WRITE

| Store | Frequency | Fraction | Angle of sector |
|---|---|---|---|
| Cold Rock | 15 | $\frac{15}{40}$ | $\frac{15}{40} \times 360° = 135°$ |
| Trampoline | 12 | $\frac{12}{40}$ | $\frac{12}{40} \times 360° = 108°$ |
| New Zealand Natural | 7 | $\frac{7}{40}$ | $\frac{7}{40} \times 360° = 63°$ |
| Baskin Robbins | 4 | $\frac{4}{40}$ | $\frac{4}{40} \times 360° = 36°$ |
| Wendy's | 2 | $\frac{2}{40}$ | $\frac{2}{40} \times 360° = 18°$ |



**Favourite ice cream stores**




**2** Calculate the angles and use a protractor to draw a pie graph of the data shown in Example 10D-2 above.

UNDERSTANDING AND FLUENCY

**3** The ingredients of Sanitarium's Berry and Yoghurt Coated Flakes Muesli are listed below. Create a pie graph to represent the percentage of each ingredient in this muesli.

68% cereals, 11% raisins, 8% berry fruit pieces, 3.5% yoghurt coated cornflakes, 0.5% freeze-dried fruit, 9% others (including flavours, colours, vitamins)

**4** Year 7 students were surveyed on their favourite drink. The results are shown in the box at right.

Coke, Pepsi, Sprite, Big M, Coke, Coke, Fanta, Sprite, fruit juice, Sprite, Pepsi, Coke, fruit juice, Coke, Big M, Pepsi, Sprite, Big M, Coke, iced tea, Coke, Big M, iced tea, Big M

**a** Create a frequency table of this data.

**b** Create a pie graph. Remember to put them in order of highest to lowest frequency.

**c** Which drink is most popular?

**d** Which drink is least popular?

**5** A group of Year 7 students were surveyed on the brand of their mobile phone. The study found that 41% had Nokias, 17% had Sony Ericssons, 15% had LGs, 10% Samsungs, 9% Motorolas and the remainder had iPhones.

**a** What percentage of students had an iPhone?

**b** Create a pie graph showing the mobile phone brands that Year 7 students own.

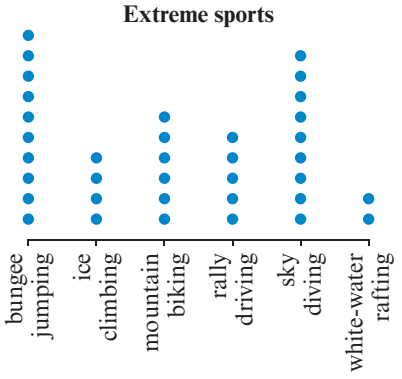
PROBLEM SOLVING AND REASONING

**6** Use this dot plot to complete these tasks.

**a** Draw a frequency table of the data.

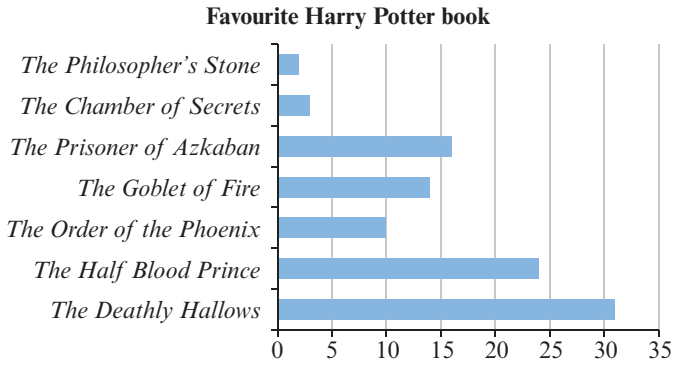
**b** Create a pie graph. Remember to put the data in order of largest to smallest frequency.

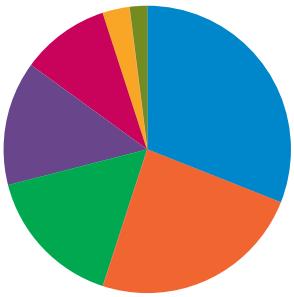
**c** Discuss the differences and similarities between the dot plot and the pie graph. Which do you think is the better display?






**7** A group of Year 7 students were asked which Harry Potter book was their favourite. The results are in the bar graph below, and in the pie graph. Unfortunately, the pie graph is missing its labels.



**a** Use the bar graph to write a legend for the pie graph.

**b** Write at least *three* sentences discussing the survey and the differences between the two graphs.

**8** A group of students was surveyed on the number of hours that they watch TV per week. The results are below.

1 4 3 8 10 3 5 7 21 13 18 5 0

5 2 9 6 10 16 12 12 8 9 7 7 2

| Class interval | Tally | Frequency |
|---|---|---|
| 0–4 | | |
| 5–9 | | |
| 10–14 | | |
| 15–19 | | |
| 20–24 | | |

**a** Copy and complete this frequency table.

**b** Create a pie graph.

**c** Comment on your pie graph in at least *two* sentences.

**9** Thanh uses 1 L of pineapple juice, 500 mL of apple juice, 300 mL of orange juice and 200 mL of tropical juice to make a bowl of punch.

**a** Write all of these amounts in millilitres.

**b** What is the total volume of juice used?

**c** Write each type of juice as a fraction (or percentage) of the whole.

**d** Create a pie graph showing the different juices that make up the punch.

**10** The table on the right shows the favourite Pokémon characters of a group of Year 7 students.

| Favourite Pokémon | Frequency |
|---|---|
| Charizard | 3 |
| Flareon | 8 |
| Jirachi | 4 |
| Lapras | 3 |
| Ninetails | 1 |
| Pikachu | 5 |
| Piplup | 6 |

**a** How many students were surveyed?

**b** Copy the table and add a column. In this column, calculate the percentage of students that like each Pokémon character listed.

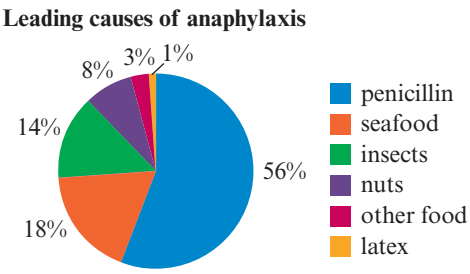
**c** Create a pie graph to show the percentage of students that like each Pokémon character.

**d** Write two sentences describing what you find.



PROBLEM SOLVING AND REASONING

**11** This pie graph shows the leading causes of anaphylaxis.

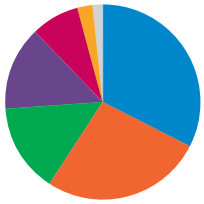



**a** Complete the following table.

| Cause | Percentage | Number affected in 1000 sufferers |
|---|---|---|
| penicillin | 56% | 56% × 1000 = 560 |
| seafood | 18% | 18% × 1000 = |
| insects | 14% | |
| nuts | | |

**b** Create a column graph to represent the numbers of people affected by each category in a group of 1000 anaphylaxis sufferers.

**12** This pie graph is missing all of its labels. Make up a topic and give the graph a title and labels for each sector.



**13** Conduct a survey on a topic of your choice and create a pie graph to represent your results.

CHALLENGE

**14** Some Year 7 students were asked what their favourite flavour ice cream was. The results are below.

chocolate strawberry toffee strawberry chocolate other vanilla chocolate other cookies'n'cream toffee strawberry other chocolate toffee cookies'n'cream chocolate toffee chocolate vanilla other cookies'n'cream toffee strawberry

**a** Create a frequency table.

**b** Create a pie graph.

**c** What is the most popular flavour of ice cream?

**d** What does 'other' mean?

**e** Is vanilla the least popular flavour of ice cream? Think carefully about this!



**Reflect**

Why are fractions and percentages important when dealing with pie graphs?

# 10E Line graphs and scatterplots

## Start thinking!

You can plot coordinates on a Cartesian plane to draw line graphs and scatterplots.

**1** Consider these two graphs.

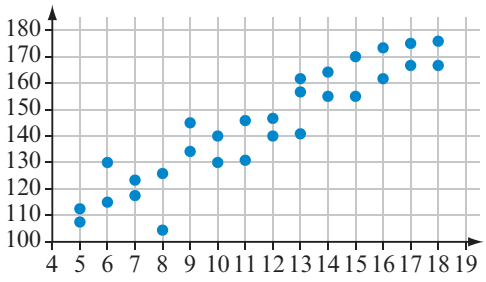



**Figure A**

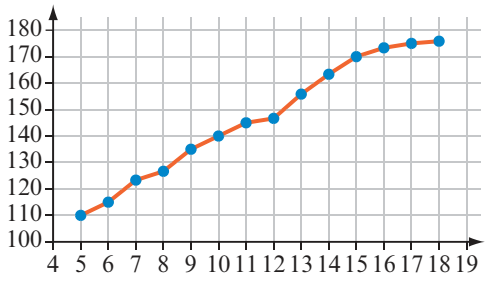



**Figure B**

**a** How are coordinates used in these graphs?

**b** What are the similarities between these graphs?

**c** What are the differences between these graphs?

**d** Which one do you think is a line graph? Why?

**e** Write a sentence describing how you can identify a line graph from a scatterplot.

Both graphs show the relationship between age and height. One graph shows how the height of one person increases as they get older. The other graph shows the heights of different people at different ages.

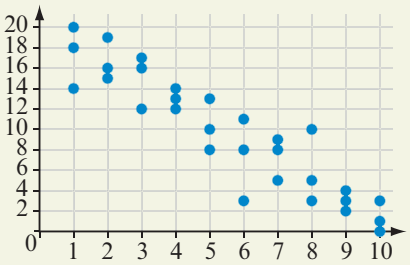
**2** Which graph do you think shows the height of one person changing? Why?

## KEY IDEAS

- A line graph is nearly always used to display how one thing changes over time. This means that the horizontal axis is nearly always time. The points are joined with a line.
- A scatterplot is used to look at the relationship between two things; for example, mass and height. The horizontal axis is not usually time (though it can be). The points are not joined.
- Points on a graph can only be joined up if they are related and represent continuous data.
- A relationship between two variables is called **bivariate data** and can be positive or negative.
- A **positive relationship** between two variables is where, as one variable increases, the other also increases. The graph moves upwards as you move right along the horizontal axis.
- A **negative relationship** between two variables is where, as one variable increases, the other decreases. The graph moves downwards as you move right along the horizontal axis.

# EXERCISE 10E Line graphs and scatterplots

## EXAMPLE 10E-1 Identifying graphs




Identify this graph as either a line graph or a scatterplot.

**THINK**

Check whether the coordinate points are joined or not.

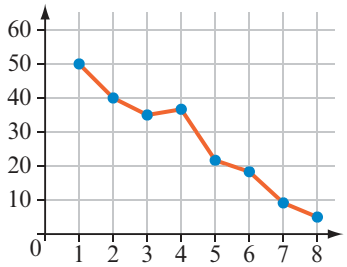
**WRITE**

The graph is a scatterplot as the coordinate points are not joined.

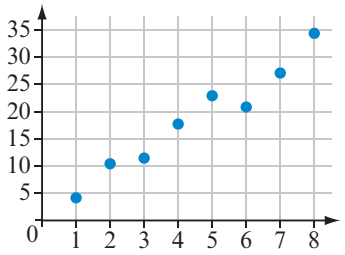
UNDERSTANDING AND FLUENCY

**1** Identify each graph as either a line graph or a scatterplot.

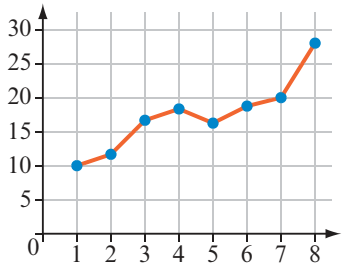
**a**



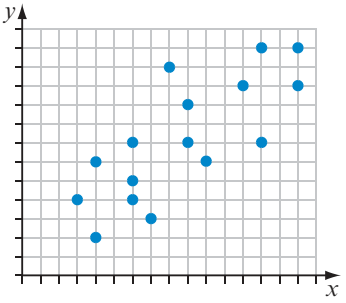

**b**



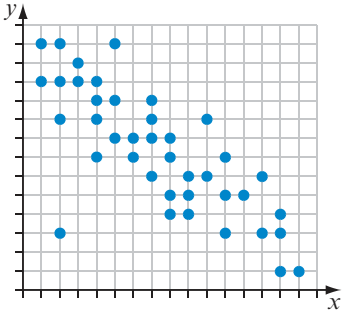

**c**



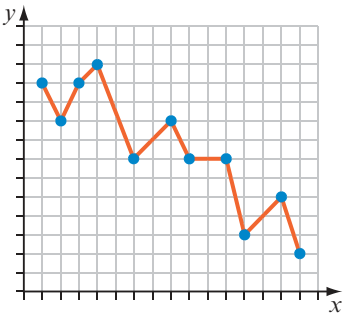

**d**




**e**




**f**



## EXAMPLE 10E-2 Describing graphs

Describe the relationship shown between the two variables in the graph from Example 10E-1.

**THINK**

Observe whether the dots move upwards or downwards as you move right along the horizontal axis ($x$-axis). Since they move downwards, it is a negative relationship.

**WRITE**

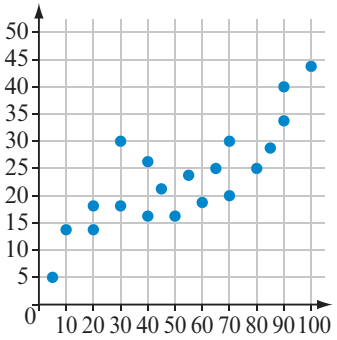
The graph shows a negative relationship.

UNDERSTANDING AND FLUENCY

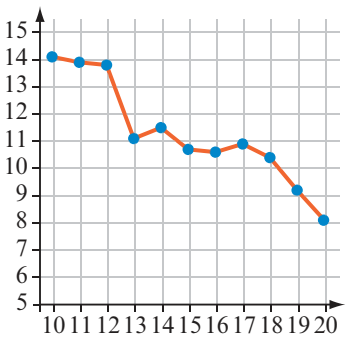
**2** Describe the relationships in question **1** as either positive or negative.

**3** Describe the relationship between the two variables in each graph.

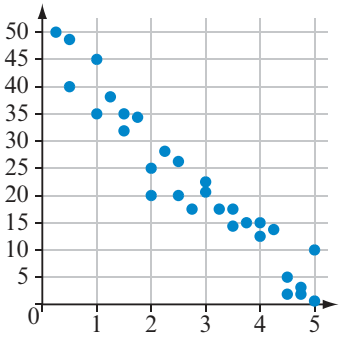
**a**



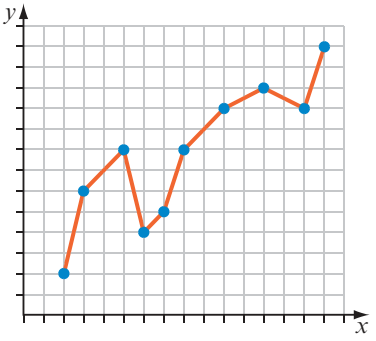

**b**



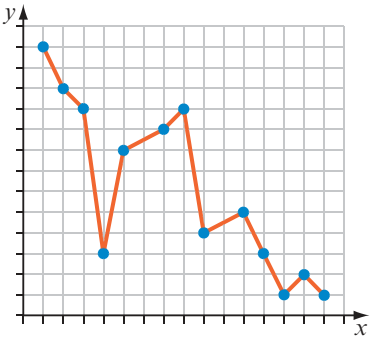

**c**



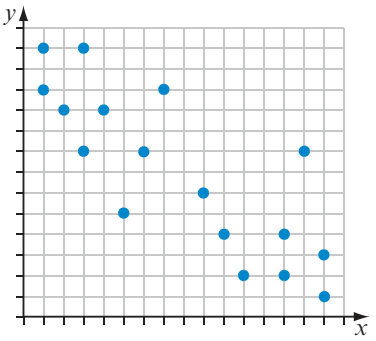

**d**




**e**




**f**





**4** Would you use a line graph or a scatterplot to graph the following relationships?

- **a** number of heart beats per minute and number of minutes spent exercising
- **b** hours spent studying and test score achieved
- **c** height of a candle and length of time it is left burning
- **d** height of a person and length of their legs

**5** Describe the relationships listed in question **4** as either positive or negative.

**6** The data in this table was recorded over a two-week period.

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Weight of puppy (g) | 100 | 110 | 125 | 123 | 131 | 142 | 150 | 156 | 164 | 177 | 181 | 184 | 190 | 195 |

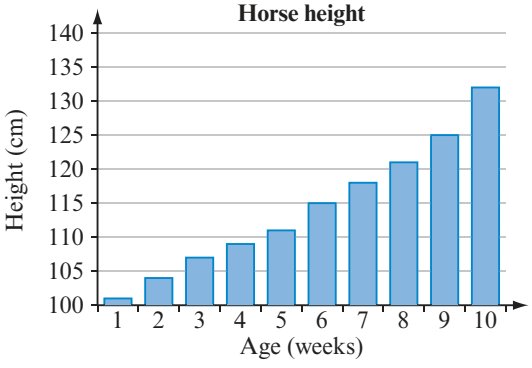
- **a** What are the variables?
- **b** Would you draw a line graph or a scatterplot to represent this data?
- **c** Draw the graph that best represents the data. Remember to include axis labels and scales.
- **d** Would you describe this relationship as positive or negative?

**7** The data in this table was gathered from a group of 12 people.

| Number of people in family | 3 | 4 | 4 | 3 | 5 | 6 | 5 | 4 | 7 | 3 | 8 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Time spent on computer per week (hours) | 12 | 8 | 6 | 14 | 4 | 3 | 5 | 5 | 2 | 15 | 2 | 10 |

- **a** What are the variables?
- **b** Would you draw a line graph or a scatterplot to represent this data?
- **c** Draw the graph that best represents the data. Remember to include axis labels and scales.
- **d** Would you describe this relationship as positive or negative?

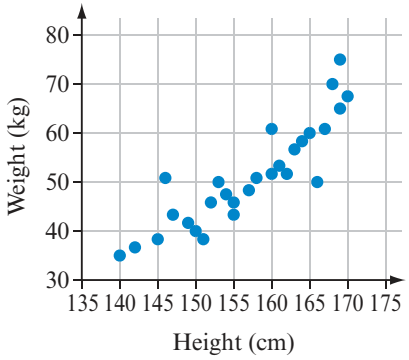
**8** This column graph shows the height of a particular horse over a 10-week period from birth. This information is better shown in a graph using coordinates.



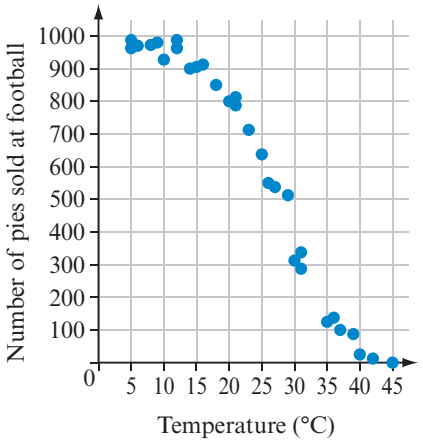

- **a** Would you draw a line graph or a scatterplot to represent this data? Explain.
- **b** Draw your chosen graph using the information in the column graph.
- **c** Would you join the points together? Explain.

**9** Look at the graphs below and write a statement about what they show. For example, the scatterplot in the Start thinking! task shows a positive relationship where, generally speaking, the older a person is, the taller they are.

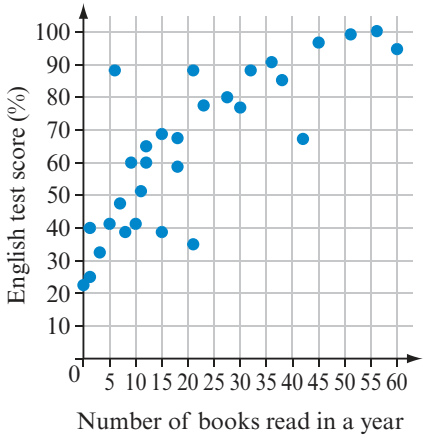
**a**




**b**




**c**






**10** Scatterplots are useful to see if there is a relationship between two variables. Consider height and arm span.

**a** Do you think there would be a relationship between these two variables? If so, do you think it would be positive or negative?

**b** Copy and complete this sentence to describe what you would predict.

As height increases, I think that arm span will _____.

**c** Gather data on the height and arm span of each person in the class (draw up an appropriate table to do this).

**d** Draw a scatterplot with all the data that you have.

**e** What did you find? Write a sentence to describe this.

**f** Imagine that a new person with a height of 155 cm joins the class. What would you predict that their arm span would be?

**11** Pick another topic to investigate within your class.

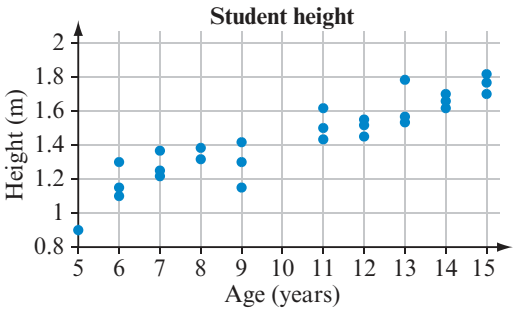
**a** Make a prediction about what you expect to find about the relationship.

**b** Gather the data and draw a scatterplot.

**c** Write a sentence describing what you find.

CHALLENGE

**12** Scatterplots can be used to make predictions. Look at the scatterplot on the right.




**a** What is the height of a person at 5 years?

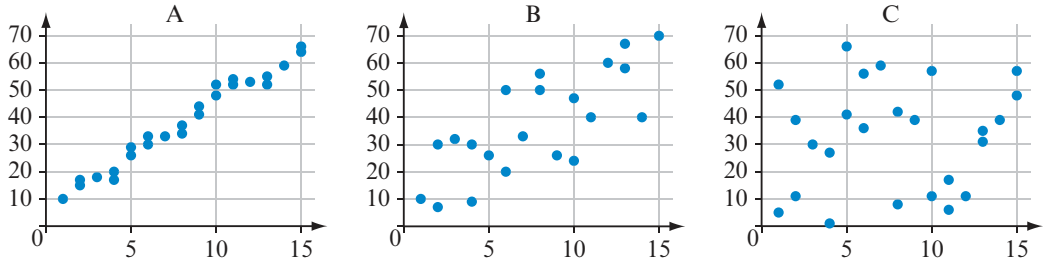
**b** What height would a person be at age 10? Explain.

**c** According to the graph, what height would a person be at age 30? Explain.

**d** Explain why it is more likely to be accurate when you make predictions about data within the set rather than outside the set. (Hint: would you really expect someone to be the height you predicted at age 30?)



**13** Another way to describe a scatterplot is to look at the strength of the relationship between the two variables. Look at the three scatterplots below.




**a** Which scatterplot would you say shows a strong relationship? Why?

**b** Which scatterplot would you say shows a weak relationship? Why?

**c** The remaining scatterplot shows no association between the two variables. How can you tell?

**d** How does the strength of a relationship affect how possible it is to make a prediction from a graph?

**Reflect**

How are bivariate relationships important?

# 10F Stem-and-leaf plots

## Start thinking!

**Stem-and-leaf plots (stem plots)** use a short cut to display data. Instead of writing whole numbers, stem-and-leaf plots split each number or piece of data into two: a stem and a leaf.

**1** Pair up with a classmate and decide who will be person A and who will be person B. At the same time, each read aloud your row of numbers (shown below) as quickly as you can.

Person A: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40

Person B: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 1, 2, 3, 4, 5, 6, 7, 8, 9, 20, 1, 2, 3, 4, 5, 6, 7, 8, 9, 30, 1, 2, 3, 4, 5, 6, 7, 8, 9, 40



**2** Who finished first? Why do you think this is?

**3** Explain how you are both counting to 40, but in different ways.

**4** When you are counting, how is it an advantage to say the tens value (for example 10, 20, 30, etc.) only once?

**5** How would you split the following numbers into two? Copy the numbers and draw a line between the digits to split them into two.

15 23 18 34 52 22 29 17 48 30

**6** Copy the following and write each number split into a stem and a leaf into the table. The first few are done for you (15, 23, 18).

| Stem | Leaf |
|---|---|
| 1 | 5 8 |
| 2 | 3 |

**7** Stem-and-leaf plots are generally written with their leaves in ascending order (from smallest to largest). Rewrite your stem-and-leaf plot so that the leaves are in order.

## KEY IDEAS

- Stem-and-leaf plots (also called stem plots) are another way of displaying data.
- Each piece of data is split into two parts: a stem and a leaf.
- The stem is written once in the left column, then the leaves are placed on the right, from smallest to largest.
- All stems between a minimum and maximum value must be shown, even if there is no data point.
- A key must be included with the stem-and-leaf plot so that the reader knows the original numbers.
- The numbers 21, 25, 26, 30, 32, 39, 39, 43 and 47 are shown in the stem-and-leaf plot on the right.

Key: 2 | 1 = 21

| Stem | Leaf |
|---|---|
| 2 | 1 5 6 |
| 3 | 0 2 9 9 |
| 4 | 3 7 |

# EXERCISE 10F Stem-and-leaf plots

## EXAMPLE 10F-1 Reading a stem-and-leaf plot

Look at this stem-and-leaf plot.

a What does the key mean?
b What is the largest number in the plot?
c What is the smallest number in the plot?
d How many numbers are there?
e What is the most common number?

Key: 1 | 9 = 19

| Stem | Leaf |
|---|---|
| 0 | 2 5 8 |
| 1 | 0 7 7 9 |
| 2 | 3 6 |

### THINK

a The key shows an example of how a number is represented in the plot. It helps you to read the numbers in the plot.

b Since the numbers in the plot are from smallest to largest, the largest number is the last one. The stem is 2 and the leaf is 6 so the number is 26 (not 6).

c The smallest number is the first number in the plot. 0 | 2 represents 2.

d Count the number of leaves. This is how many numbers there are.

e Look to see if there are repeated leaves in each stem. There are two leaves of 7 with a stem of 1.

### WRITE

a The key shows a stem and leaf for one number. A stem of 1 and a leaf of 9 represents 19.

b The largest number is 26.

c The smallest number is 2.

d There are nine numbers.

e The most common number is 17.

UNDERSTANDING AND FLUENCY

**1** Look at this stem-and-leaf plot.

Key: 2 | 1 = 21

| Stem | Leaf |
|---|---|
| 0 | 1 5 6 |
| 1 | 2 8 9 9 |
| 2 | 1 4 8 |
| 3 | 0 1 4 |
| 4 | 3 |

a What does the key mean?
b What is the largest number in the plot?
c What is the smallest number in the plot?
d How many numbers are there?
e What is the most common number? Explain why it is *not* 9.

UNDERSTANDING AND FLUENCY

**2** Look at this stem-and-leaf plot.

Key: 5 | 1 = 51

| Stem | Leaf |
|---|---|
| 5 | 1 2 9 8 |
| 6 | 4 6 7 8 2 |
| 7 | 1 1 5 8 6 |
| 8 | 3 3 4 9 2 2 1 3 |
| 9 | |
| 10 | 8 |

**a** Redraw the stem-and-leaf plot so that the leaves are in order.

**b** What is the smallest number?

**c** What is the largest number?

**d** What is the most common number?

**e** Add the number 112 to the plot.

(Hint: you may need to draw another stem.)

## EXAMPLE 10F-2 Drawing a stem-and-leaf plot

Arrange this set of data into a stem-and-leaf plot.

12, 17, 23, 46, 8, 34, 33, 15, 28, 22, 42, 12, 4, 35, 39, 27, 30, 28, 13, 10, 25, 28, 18, 15

### THINK

**1** Find the smallest and largest numbers so you know what numbers to use for the stems: the smallest number is 4 and the largest number is 46.
4 has a stem of '0' and 46 has a stem of '4'.
Write all stems up to and including 4.

**2** Fill in each leaf next to the correct stem.

**3** Rewrite the stem-and-leaf plot so that the leaves are in order. Add a key so that the plot can be read.

### WRITE

| Stem | Leaf |
|---|---|
| 0 | 8 4 |
| 1 | 2 7 5 2 3 0 8 5 |
| 2 | 3 8 2 7 8 5 8 |
| 3 | 4 3 5 9 0 |
| 4 | 6 2 |

Key: 1 | 0 = 10

| Stem | Leaf |
|---|---|
| 0 | 4 8 |
| 1 | 0 2 2 3 5 5 7 8 |
| 2 | 2 3 5 7 8 8 8 |
| 3 | 0 3 4 5 9 |
| 4 | 2 6 |

**3** Arrange each set of data into a stem-and-leaf plot. Remember to include a key.

**a** 11, 14, 18, 19, 23, 23, 26, 31, 38, 42, 42, 43

**b** 13, 18, 22, 46, 54, 59, 31, 36, 42, 19, 4, 28

**c** 54, 34, 33, 65, 39, 27, 35, 51, 42, 49, 37, 53

**d** 41, 37, 53, 29, 31, 59, 67, 23, 41, 59, 29, 41

UNDERSTANDING AND FLUENCY

**4** The number of people at a local skate park over the course of a month is displayed in this stem-and-leaf plot.



| Stem | Leaf |
|---|---|
| 1 | 4 8 |
| 2 | 2 5 5 6 9 |
| 3 | 1 1 2 4 5 6 6 7 7 7 |
| 4 | 4 4 8 9 |
| 5 | 1 2 3 |
| 6 | 0 0 1 6 |
| 7 | 4 8 |
| 8 | |
| 9 | 8 |

**a** Add an appropriate key.

**b** What was the smallest number of people at the park during the month?

**c** What was the largest number of people?

**d** How many days in this month?

PROBLEM SOLVING AND REASONING

**5** The ages of people sitting in a sushi restaurant at lunch were recorded and are displayed below.



15 23 29 45 19 65 18 29 31 34 22
24 19 33 25 32 54 39 28 19 26

**a** Draw a stem-and-leaf plot to represent this data. Remember to include a key.

**b** Find the youngest and oldest people in the restaurant.

**c** What is the most common age?

**d** What is the most common age bracket? (Hint: consider which stem this would be.)

**6** The ages of people that use Facebook on a daily basis are recorded below.

16 18 20 21 29 34 25 15 17 19 13 14 31
14 14 18 23 27 36 54 45 46 76 53 41 22
46 29 30 18 21 24 41 33 15 16 25 12 10

**a** Create a stem-and-leaf plot to display this data. Remember to include a key.

**b** What is the age of the youngest Facebook user?

**c** What is the age of the oldest Facebook user?

**d** What is the most common age bracket to use Facebook?

**e** Make up another two stem-and-leaf plots to represent the ages of people that you think use:

**i** Myspace

**ii** Twitter.

**7** Feathertail gliders (tiny possums native to Australia) were weighed to monitor their health. The results are shown in this stem-and-leaf plot.

Key: 10 | 1 = 10.1 g

| Stem | Leaf |
|---|---|
| 10 | 1 4 8 |
| 11 | 2 3 6 6 |
| 12 | 1 2 4 5 5 6 9 |
| 13 | 1 2 4 4 4 6 8 8 8 8 |
| 14 | 0 1 |
| 15 | |
| 16 | 1 |



**a** What is the lightest mass?

**b** What is the heaviest mass?

**c** Will said that the most common mass recorded was 138 g. Explain where he went wrong and provide the correct answer.

**d** Is there an outlier in the data? If so, what is the heaviest 'normal' piece of data?

**e** Can you think of a reason why one feathertail glider might be significantly heavier than the rest?

**f** Another five feathertail gliders were found and had their masses (in grams) recorded below. Redraw the stem-and-leaf plot, adding in these masses.

14.3 12.9 13.1 13.6 11.0

**8** At the release of the latest Spiderman movie, the approximate number of people lined up at cinemas an hour before the opening session was recorded across the country. The results are below.

120 250 360 80 150 430 570 220 160 130 170 240 220
60 110 140 200 130 120 100 180 220 260 130 150 100

**a** Copy and complete the stem-and-leaf plot.

Key: 1 | 2 = 120

| Stem | Leaf |
|---|---|
| 0 | 6 8 |
| 1 | |
| 2 | |

**b** How is the key different from what you have seen before?

**c** What was the smallest number of people waiting?

**d** What was the largest number of people waiting?

**9** The schedule shows a list of the times a tram arrives at a certain stop in Melbourne on a Sunday.

| Sunday | |
|---|---|
| AM | |
| 6 | |
| 7 | 16 46 |
| 8 | 16 46 |
| 9 | 16 46 |
| 10 | 12 30 43 56 |
| 11 | 08 18 30 42 54 |
| PM | |
| 12 | 06 18 30 42 54 |
| 1 | 06 18 30 42 54 |
| 2 | 06 18 30 42 54 |
| 3 | 06 18 30 42 54 |
| 4 | 06 18 30 42 54 |
| 5 | 06 18 30 42 54 |
| 6 | 06 18 30 42 54 |
| 7 | 04 19 34 |
| 8 | 04 35 |
| 9 | 05 35 |
| 10 | 04 31 |
| 11 | 01 31 59 |

**a** How is this different from a normal stem-and-leaf plot?

**b** What is the earliest time the tram leaves from this stop?

**c** What is the last tram from this stop?

**d** How many trams are there per hour from midday to 7 pm?

**10** Sam drew this stem-and-leaf plot. Find three things that she needs to fix. Redraw the stem-and-leaf plot correctly.

| Stem | Leaf |
|---|---|
| 2 | 1 4 9 2 |
| 3 | 3 9 8 3 5 |
| 4 | 1 1 6 7 5 3 1 |
| 5 | 0 6 3 |
| 8 | 7 8 |
| 9 | 9 4 2 2 |

**11** Conduct your own survey and record the results in a stem-and-leaf plot. Discuss what you find in at least two sentences. Why must the data you collect be numerical?

CHALLENGE

**12** You can use stem-and-leaf plots to compare different data sets. The plot to the right is called a back-to-back stem-and-leaf plot. It shows the number of text messages sent in a week by a group of Year 7 students.

| Leaf Boys | Stem | Leaf Girls |
|---|---|---|
| 9 8 6 3 3 | 1 | 1 2 3 6 |
| 8 8 7 5 4 | 2 | 0 4 5 5 6 7 |
| 9 9 7 6 5 4 4 4 3 | 3 | 1 8 8 9 |
| | 4 | 2 6 |
| 1 | 5 | |
| 8 | 6 | |
| | 7 | 9 |

**a** What are the two groups on the stem-and-leaf plot?

**b** What is the maximum number of text messages sent in a week? Which group does it belong to?

**c** What is the minimum number of text messages sent by a boy in a week?

**d** Do both sides of the stem-and-leaf plot have the numbers in order? Explain why they look different.

**13** Two of the greatest goal kickers in AFL history are Jason Dunstall and Tony Lockett. The number of goals they kicked in each year they played is listed in the table.

| Player | Goals |
|---|---|
| Dunstall | 36, 77, 94, 132, 138, 83, 82, 145, 123, 101, 66, 102, 21, 54 |
| Lockett | 19, 77, 79, 60, 117, 35, 78, 65, 127, 132, 53, 56, 110, 121, 37, 109, 82, 3 |

**a** Display this data in a back-to-back stem-and-leaf plot.

**b** What was the highest number of goals kicked in a year? Who kicked this?

**c** What was the lowest number of goals kicked in a year? Who kicked this?

**d** Who kicked the largest number of goals in total?

**e** Who do you think was a better goal kicker? Explain.

**Reflect**

How are stem-and-leaf plots useful?

# 10G Summary statistics

## Start thinking!



You often hear people talking about the **average** of something. What does this mean? There are three different measures of the centre of a data set. When people talk about the average they are usually talking about the **mean** of a set.

Here is a list of the runs scored by Don Bradman in each innings of the last year of his Test cricket career (1948).

132 127 201 57 138 0 38 89 7 30 33 173 0

**1** How many innings did Bradman bat in?

**2** To calculate the mean of a data set, you add all the numbers together and divide by how many numbers you have.

**a** Copy and complete the boxed calculations.

**b** Use a calculator to find the mean of Bradman's runs in 1948 (to the nearest whole number).

$$\text{mean} = \frac{\text{sum of terms}}{\text{number of terms}}$$
$$= \frac{132 + 127 + \ldots}{13}$$
$$= \frac{\ldots}{13}$$

**3** The **median** is the middle number of the set when the numbers are listed in order.

**a** Write the runs scored in order from smallest to largest.

**b** Find the middle number in this list. You might want to count from both the start and the end.

**4** Find the most common number. This is called the **mode**.

## KEY IDEAS

- Mean, median, mode and range are **summary statistics** as they are used to provide a summary of the data.
- Sets of data are described by their centre and their spread.
- There are three main measures of centre.

  The mean is the average of the set. To find the mean, add all the numbers together, then divide by how many numbers are in the set.

  The median is the middle number of the set (when the numbers are in order).

  The mode is the most common number in the set.
  The mode can be more than one number.
- One measure of spread is the **range**. This is the difference between the largest and smallest numbers.

**NOTE** Putting the numbers in order makes it much easier to find both the median and the mode.

# EXERCISE 10G Summary statistics

## EXAMPLE 10G-1 Finding the mean

Year 7 students were surveyed on how many hours they spend on the Internet a week. The results are below.

2 6 10 5 7 5 10 23 4

What is the average (mean) time spent on the Internet?

**THINK**

Add all the numbers together and divide by how many students were surveyed. There is a total of nine students.

**WRITE**

$$\text{mean} = \frac{2 + 6 + 10 + 5 + 7 + 5 + 10 + 23 + 4}{\text{number of terms}}$$

$$= \frac{72}{9}$$

$$= 8$$

**1** Find the mean for each data set.

**a** 4, 6, 1, 9, 4, 3, 5, 8

**b** 6, 4, 6, 7, 9, 4, 8, 3, 8, 5

**c** 2, 4, 5, 3, 8, 7, 3, 2, 4, 2

**d** 2, 3, 5, 9, 8, 1, 2, 1, 2, 4, 3, 2

**e** 11, 10, 8, 12, 6, 13, 14, 12, 5, 9

**f** 22, 29, 24, 37, 15, 42, 26, 16, 23

## EXAMPLE 10G-2 Finding the median

Use the data from Example 10G–1 to find the median time.

**THINK**

**1** Copy the list and arrange the numbers in order.

**2** Count to the middle number of the ordered list of numbers.

**WRITE**

2, 4, 5, 5, 6, 7, 10, 10, 23

2, 4, 5, 5, (6), 7, 10, 10, 23

The median number of hours spent on the Internet is 6.

**2** Find the median for each data set.

**a** 4, 8, 5, 9, 13, 11, 7
**b** 4, 8, 5, 6, 7, 5, 1, 2, 3
**c** 16, 18, 15, 3, 7, 8, 9, 14, 6
**d** 7, 8, 4, 6, 7, 9, 4, 7, 7, 8, 2
**e** 11, 16, 18, 19, 13, 9, 12, 16, 13, 17, 13, 18, 21
**f** 18, 6, 2, 3, 17, 38, 9, 4, 45, 4, 8, 7, 9

## EXAMPLE 10G-3 Finding the mode

Use the data from Example 10G-1 to find the modal time(s).

**THINK**

The mode is the most common number(s) in your list.

**WRITE**

The modes are 5 and 10 hours per week.

**3** Find the mode for each data set.

**a** 1, 8, 6, 4, 5, 7, 4, 2, 4, 7, 8
**b** 3, 4, 8, 6, 7, 6, 1, 3, 6, 4
**c** 1, 6, 4, 8, 5, 4, 9, 4, 6, 11, 19
**d** 7, 6, 8, 7, 6, 8, 5, 4, 5, 7, 8, 6, 4
**e** 8, 1, 19, 5, 4, 16, 13, 17, 19, 13, 15, 6
**f** 10, 16, 18, 5, 17, 19, 21, 26, 27, 13, 15, 14, 11

## EXAMPLE 10G-4 Finding the range

Use the data from Example 10G-1 to find the range of times.

**THINK**

Subtract the lowest number from the highest number.

**WRITE**

range = 23 − 2
= 21
There is a range of 21 hours in this survey.

UNDERSTANDING AND FLUENCY

**4** Find the range for each data set.

**a** 5, 4, 9, 7, 6, 2, 1, 6, 3
**b** 6, 8, 4, 0, 9, 7, 8, 2, 3, 4, 8
**c** 4, 8, 6, 5, 6, 7, 5, 2, 5, 9, 4, 5
**d** 9, 6, 11, 18, 5, 6, 13, 12, 7, 18, 19, 14
**e** 21, 46, 23, 18, 16, 24, 39, 48, 51, 13, 11, 46, 18
**f** 49, 23, 78, 45, 22, 46, 48, 61, 31, 75, 46, 23, 24

**5** Consider this data set: 4, 7, 5, 1, 7, 8, 10.

**a** Put the numbers in order from smallest to largest.
**b** How many numbers are there?
**c** What is the mode?
**d** What is the median?
**e** What is the mean?
**f** What are the smallest and largest numbers?
**g** What is the range of numbers?

UNDERSTANDING AND FLUENCY



**6** Data was collected on the number of young in a number of litters of fat-tailed dunnarts. The results are below.

7 8 6 7 6 7 9 8 15 7 8

**a** What is the average (mean) number of young in a litter?

**b** Put the numbers in order from smallest to largest.

**c** What is the middle (median) number of young?

**d** What is the most common (mode) number of young?

**e** What is the range of the number of young?

**7** Information was collected on the number of online games students play in a normal week. The results are below. (Round your answers to two decimal places where appropriate.)

0 4 2 1 1 3 2 1 0 0 2 1 0
1 0 2 1 1 0 0 2 1 1 1 0 0

**a** Find the three measures of centre (mean, median and mode).

**b** Find the range.

**8** Find the mean, median, mode and range for each data set. (Round your answers to two decimal places where appropriate.)

**a** 4, 8, 2, 5, 3, 7, 5, 7, 7, 7, 1, 2, 6

**b** 1, 1, 1, 2, 5, 6, 8, 4, 5, 8, 6, 9, 6

**c** 1, 2, 3, 4, 5, 6, 7, 8, 9

**d** 4, 7, 5, 8, 3, 6, 9, 9, 1, 0, 3, 5, 4, 7, 2

**e** 4, 8, 3, 7, 11, 15, 4, 9, 5

**f** 17, 13, 11, 7, 5, 3, 2

**g** 11, 17, 13, 9, 13, 5, 18

**h** 8, 5, 8, 6, 8, 1, 93, 5, 5, 5, 8

PROBLEM SOLVING AND REASONING



**9** A group of Year 7 students were asked how much money they spend on their mobile phones every month. The results (in dollars) are:

20, 50, 30, 10, 30, 20, 20, 15, 20, 30, 20, 20, 25.

**a** Find the mean, median, mode and range.

**b** Describe the set in at least two sentences, using your answers to part **a**.

**10** After finishing the survey in question **9**, students were then asked how much pocket money they earned per week. The results (in dollars) were:

5, 5, 10, 20, 5, 5, 0, 5, 0, 5, 10, 5, 100.

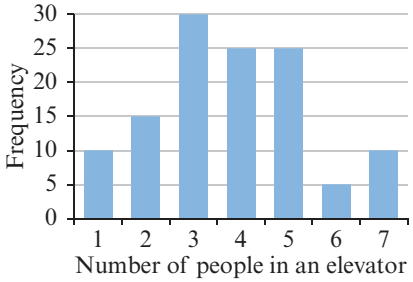
**a** Find the mean, median and mode.

**b** What is the difference between the mean and median?

**c** Explain why the mean is not a good indicator of the average in this data set.

**d** Would any of these students be able to afford to pay for their phone bill from question **9** using just their pocket money? Explain.

**11** Sometimes data is presented to us in displays instead of in raw form like a list. Look at this column graph.




**a** What is the mode?

**b** What is the highest value on the horizontal axis? The smallest?

**c** Use your answers from part **b** to find the range.

**d** Write the data from this graph as a list of numbers and use the list to find the mean and median.

**e** Use the titles on the axes to explain what these statistics actually mean.

**12** Match each summary statistic to the correct data set.

| Summary statistic | Data set |
|---|---|
| **a** mean = 9 | **A** 15, 8, 4, 7, 3, 10, 1 |
| **b** mode = 4 | **B** 11, 4, 6, 8, 3 |
| **c** median = 7 | **C** 3, 17, 8, 6, 14, 8, 7 |
| **d** range = 8 | **D** 1, 7, 4, 5, 9, 4, 2, 3, 3, 4, 10 |

**13** Two friends were comparing trains and how late they ran. Derrick thought that his train was the worst – once it ran 20 minutes late. Brendan thought that his train was always late, but could only remember it being 10 minutes late at worst. They decided to record how many minutes late the trains were for two weeks and compare. The results are below.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Derrick: | 5 | 3 | 1 | 4 | 2 | 2 | 5 | 15 | 20 | 3 | 2 | 4 | 2 | 3 |
| Brendan: | 4 | 6 | 8 | 5 | 6 | 7 | 7 | 9 | 8 | 8 | 7 | 9 | 9 | 9 |

**a** Whose train had the worst single time? How late was this?

**b** Find the mean number of minutes late for each train.

**c** Whose train do you think is the worst? Explain, using your results from parts **a** and **b**.

**14** Consider this data set: 1, 1, 4, 5, 7, 7, 8, 9.

**a** How many numbers are there in the set?

**b** What would you write down as the middle number?

To find the median when there is an even number of numbers, find the average of the two middle numbers.

**c** What are the two middle numbers in this set?

**d** What is the average of these two numbers? This is the median.

**15** Find the median of each data set, which contains an even number of items. If the numbers are not in order, make sure you order them first!

**a** 1 3 3 4 6 8 9 9

**b** 2 2 5 5 6 7 8 9

**c** 7 1 4 9 7 3 5 5

**d** 9 7 5 8 6 3 2 7

PROBLEM SOLVING AND REASONING



**16** Some zoologists were arguing about the number of spots that cheetahs have. They collected the data shown below. One said that the average number of spots was 466. Another said that it was 456. A third said that it was 472. Use your knowledge of statistics to discuss why all three are correct.

450 495 413 456 517 472 436 485 476 499 486 444 456 424 481

**17** A recent census found there is an average of 1.97 children in every family.

**a** Explain why this doesn't mean that every family has 1.97 children.

**b** Which measure of centre do you think they used to get this number? Why?

**c** Conduct a survey in your class to find the number of children in each family.

**d** Calculate the average number of children in each family in your class.

**e** Do the class results match the average found in the census? Can you think of a reason why it might differ?

**18** When you collect your own data, it is called primary data. If you use somebody else's data, it is called secondary data. Find and use secondary data to investigate at least one of the following.

- the most common country of birth for a Year 7 class in China compared to a Year 7 class in Australia
- the relationship between wealth or education and the health of populations from different countries
- the mean (average) areas set aside for parkland, manufacturing, retail and residential dwellings to compare land use in your local area
- other (check with your teacher).

CHALLENGE

**19** Create data sets that have the following statistics.

**a** mean of 10, range of 8

**b** mode of 6, median of 5

**c** mean of 7, median of 6

**20** The number of points scored by a junior basketball team in each match over a season is recorded in the stem-and-leaf plot shown.

Key: 1 | 5 = 15

| Stem | Leaf |
|---|---|
| 1 | 1 5 6 |
| 2 | 0 0 1 2 3 4 4 5 |
| 3 | 0 1 2 3 6 7 7 |
| 4 | 1 1 1 3 |
| 5 | 2 5 |
| 6 | 7 |

**a** What is the range? Remember to use the whole numbers and not just the leaves.

**b** What is the mode?

**c** What is the median?

**d** What is the mean?

**e** How is calculating summary statistics from a stem-and-leaf plot different from doing it from a raw list of data?

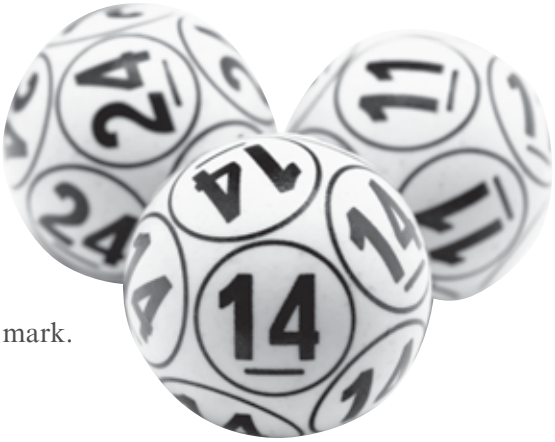


**Reflect**

What are the advantages and disadvantages of each of the three measures of centre?

# 10H Describing probability

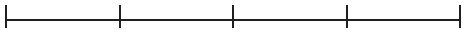
## Start thinking!



You often hear people talk about the 'chances' of something happening, or how likely something is. Both words and numbers are used to describe the **probability** of an **event** occurring.

1 Put these terms in order of likeliness:
likely, impossible, certain, unlikely, even chance

2 Copy the scale and place the terms in order, each one over a scale mark.



3 Give an example for each term.

4 Compare your examples with a classmate's.

A probability is often represented by a decimal number.

5 If something that is certain has a probability of 1, what does something that is impossible have a probability of?

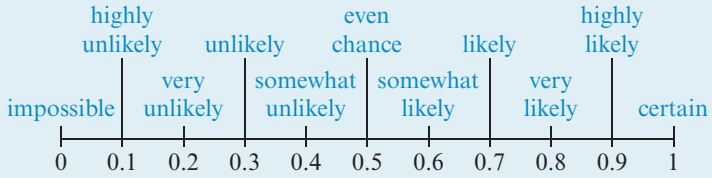
6 If 0 is the start of the **probability scale** and 1 is the end, what decimal number is halfway in between?

7 Would a probability of 0.25 represent something likely or unlikely? Why?

8 On the bottom of your scale marks, fill in the decimal numbers in the correct places: 0.25, 1, 0.75, 0.5, 0.

## KEY IDEAS

- We can describe the probability of an event occurring using words or numbers.




- A probability must have a value in the range 0 (impossible) to 1 (certain). This can be displayed on a probability scale.

- The **sample space** of an event is a list of all the different **outcomes** possible and is written within curly brackets.

# EXERCISE 10H Describing probability

## EXAMPLE 10H-1 Listing sample space

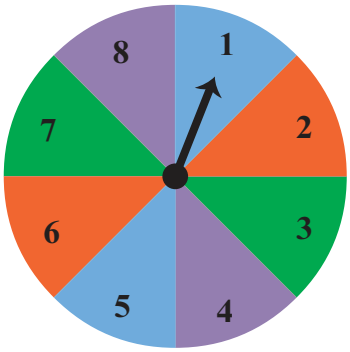
List the sample space for each of these.

a flipping a coin
b rolling a die
c drawing a card from a deck and recording the suit

| THINK | WRITE |
|---|---|
| a There are only two possible outcomes. Place these within curly brackets. | a {head, tail} |
| b There are six possible outcomes. Place these within curly brackets. | b {1, 2, 3, 4, 5, 6} |
| c Although there are 52 cards in a deck, there are only four different suits. Place these within curly brackets. | c {hearts, diamonds, clubs, spades} |

UNDERSTANDING AND FLUENCY

1 List the sample space for each of these.
   a A baby is born and the gender is recorded.
   b This spinner is spun and the colour recorded.
   c This spinner is spun and the number recorded.
   d People are asked which month they were born.
   e Selecting a random coin from a wallet containing at least one of every Australian coin.
   f Surveying traffic lights and recording the colour.
   g People are asked how many days a week they work.
   h People are asked what their favourite season is.
   i Naming one of the colours from a rainbow.
   j Picking a vowel to play a game.






## EXAMPLE 10H-2 Describing probability

Describe the probability of each event using one of the ten descriptor terms used in the Key ideas (page 562).

a Rolling a die and obtaining a six.
b Flipping a coin and obtaining a tail.
c May following April.
d Selecting a non–picture card from a deck of cards.

### THINK

a Rolling a six is not very likely, but it's not impossible.
b Flipping a tail is as likely as flipping a head.
c May always follows April.
d There are more non-picture cards than picture cards in a deck (nine compared to four), so this is a likely event.

### WRITE

a Rolling a die and obtaining a six is very unlikely.
b Flipping a coin and obtaining a tail is an even chance.
c May following April is certain.
d Selecting a non-picture card from a deck of cards is likely.

UNDERSTANDING AND FLUENCY

**2** Describe the probability of each event using one of the ten descriptor terms used in the Key ideas.

a Growing 20 cm overnight.
b Rolling a 1, 2, 3, 4 or 5 on a die.
c A woman giving birth to triplets.
d Rain on a random winter's day.
e Selecting a red card from a deck.
f The $10 note being blue.
g A baby born being a girl.
h Selecting a draw four card from a UNO deck.
i You pick the winning option out of three.
j Having at least one hot day in summer.

**3** Match each event to the most appropriate probability.

a a pregnant rabbit having babies
b winning a coin toss
c every person in your class having a sibling
d living to 100 years old
e finding $5 on the street
f Tuesday following Monday

A 0.1
B highly likely
C 1
D 0.5
E unlikely
F somewhat likely



**4** Provide an example of an event that is:

a somewhat unlikely
b very likely
c impossible
d highly likely
e very unlikely
f certain.

## EXAMPLE 10H-3 Using a probability scale

Place these events on a probability scale with ten divisions.

**a** flipping a coin and getting a head
**b** an event, B, with a probability of 0.7
**c** rolling a die and getting a 13
**d** getting a pack of Clinkers that are all green inside
**e** an event, E, with a probability of 0.9

### THINK

Draw a probability scale with ten divisions and mark in all numbers.

**a** Getting a head is as likely as getting a tail, so it has a probability of 0.5. Place it above 0.5 on the scale.

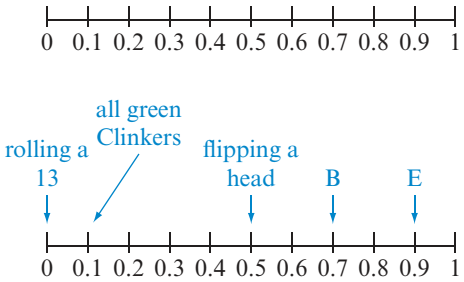
**b** Place B above 0.7.

**c** A die has the numbers 1, 2, 3, 4, 5 and 6. It is impossible to get 13, so it has a probability of 0. Place it above 0 on the scale.

**d** It is highly unlikely that the Clinkers in a pack will be all green inside. Place it above 0.1 on the scale.

**e** An event that is highly likely has a probability of 0.9. Place E above 0.9 on the scale.

### WRITE




UNDERSTANDING AND FLUENCY

**5** Place these events on a probability scale with ten divisions.
- **a** Flipping four coins and obtaining four tails.
- **b** You don't pick the winning option out of four.
- **c** An event, C, with a probability of 0.6.
- **d** Rolling a 7 on a standard die.
- **e** A baby being born on a weekend.
- **f** You select an ace from a deck of cards.
- **g** The sun rising tomorrow.
- **h** A random day in summer being hot.
- **i** Rolling an odd number on a standard die.
- **j** Flipping two coins and obtaining at least one head.

**6** Place the events from question **2** on a probability scale with ten divisions.

**7** There are many other terms that can be used to describe probability.
- **a** Name at least five.
- **b** Draw a probability scale and place these terms on it.



**8** Look at the picture of lollies. Think of at least three different ways to group the objects and list the sample space for each.

**9** Give an example to match each sample space.

**a** {clubs, diamonds, hearts, spades}

**b** {Saturday, Sunday}

**c** {1, 2, 3}

**d** {win, lose}

**e** {red, orange, yellow, green, blue, indigo, violet}

**f** {mammal, bird, amphibian, reptile, fish}

**10** For the sample spaces listed in question **9**, write in fraction form the probability of the outcomes below. Assume that each outcome is equally likely.

**a** diamond **b** Sunday **c** 3

**d** win **e** blue **f** bird

**11** The number of elements in a sample space is just the number of different possible outcomes. For example, when flipping a coin you can get heads or tails. There are two different possibilities. Find the number of elements in the sample space of the following.

**a** randomly choosing a letter in the alphabet

**b** randomly choosing a day of the week

**c** the outcome of the game 'noughts and crosses'

**d** randomly picking a student from your class

**e** randomly drawing a letter from the word TEXT

**f** randomly selecting a ball out of a bag containing 8 red, 7 green and 10 blue coloured balls

**12** State which of the two events mentioned is most likely. Describe each event in both words and numbers.

**a** Australia winning the World Cup or Essendon winning the next AFL grand final

**b** tomorrow being sunny or rainy

**c** getting an email or a text message between 6 pm and 7 pm tonight

**d** rolling an even number on a die or flipping a coin twice and getting two heads

**e** having eggs and bacon or fruit and muesli for breakfast

**f** rolling a double or triple 6 when throwing dice



**13** Probabilities are also described as percentages.

**a** Write 0.5 as a percentage.

**b** Write 0.3 as a percentage.

PROBLEM SOLVING AND REASONING

**c** Copy and complete these sentences.

An event with the probability of 0.5 has a _____% chance of occurring.

An event with the probability of 0.3 has a _____% chance of occurring.

An event with the probability of 0.9 has a _____% chance of occurring.

An event that is likely has a probability of _____, which means it has a _____% chance of occurring.

An event that is highly unlikely has a probability of _____, which means it has a _____% chance of occurring.

An event that is certain has a probability of _____, which means it has a _____% chance of occurring.

**14** Probabilities can also be put in to fraction form.

**a** Write 0.1 as a fraction. **b** Write 0.7 as a fraction.

**c** Copy and complete these sentences.

An event with the probability of 0.1 has 1 chance out of 10 of occurring.

An event with the probability of 0.7 has _____ chances out of 10 of occurring.

An event with the probability of 0.9 has _____ chances out of 10 of occurring.

An event that is unlikely has a probability of _____, which means it has _____ chances out of 10 of occurring.

An event that is likely has a probability of _____, which means it has _____ chances out of 10 of occurring.

An event that is impossible has a probability of _____, which means it has _____ chances out of 10 of occurring.

**15** Can something have a 200% chance of occurring? Explain.

**16** Not all outcomes are equally likely. For example, you are equally likely to roll a 1 as you are to roll a 5 on an ordinary die, but you are more likely to select a 'B' from the word 'BABY' than you are to select an 'A'.

**a** For the parts in question **11**, state whether each outcome is equally likely or not.

**b** Provide three examples which have:

**i** equally likely outcomes

**ii** not equally likely outcomes.

CHALLENGE

**17** The events A, B, C, D and E are evenly spread along a probability scale.

**a** Use the clues provided to place them in the right order.

**b** Which event might be described as impossible?

**c** Which event might be described as having a probability of 0.5?

**d** Describe a situation that might match events A to E.

**Clues**

E is less likely to occur than B but more likely to occur than A.

A is more likely to occur than C.

D is certain to occur.

**Reflect**

How is knowledge of percentages, decimals and fractions useful when discussing probabilities?

# 10I Theoretical probability

## Start thinking!



Often you may want to know in advance what the chances are of getting a **favourable outcome** – the result that you want. If you have enough information you can calculate the probability of an event occurring.

1 How many jellybeans are there in total?

2 How many pink jellybeans are there?

3 How many yellow jellybeans are there?

4 Copy and complete these sentences.

**a** There are _____ opportunities to pick a pink jellybean out of a total of _____. Written as a decimal number, the probability of selecting a pink jellybean is _____.

**b** There is _____ opportunity to pick a yellow jellybean out of a total of _____. Written as a decimal number, the probability of selecting a yellow jellybean is _____.

When calculating probability, look for the number of favourable outcomes (what you want) out of the total number of possible outcomes (what you have).

5 Copy and complete:

probability = number of _____ outcomes out of _____ outcomes = $\frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

6 Use this formula to find the probability of selecting:

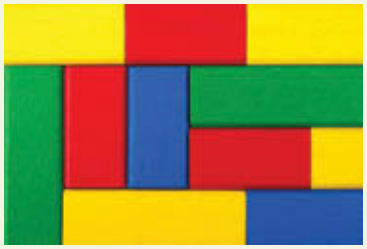
**a** a blue jellybean **b** a green jellybean.

## KEY IDEAS

- For the theoretical probability of an event occurring:
  $\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
- To calculate probability, follow these steps.
  1 Make a list of all the possible outcomes.
  2 Make a list of all the favourable outcomes (the outcomes that you want).
  3 Write the probability as a fraction using the above formula.
  4 Simplify the fraction if possible.
- Two events are **complementary** when their probabilities add to 1.

# EXERCISE 10I Theoretical probability

## EXAMPLE 10I-1 Using the probability formula



Consider selecting a block from this pile of blocks.

a What is the total number of outcomes?
b What is the number of yellow blocks?
c What is the probability of selecting a yellow block?

**THINK**

a Count the number of blocks in the pile.

b Count the number of favourable outcomes (yellow blocks).

c Use the probability formula to find the probability of selecting a yellow block from this pile.

**WRITE**

a There are 11 blocks in this pile.

b There are four yellow blocks in this pile.

c $\text{Pr(event)} = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

$= \frac{4}{11}$

UNDERSTANDING AND FLUENCY

1 Consider the pile of blocks in Example 10I-1.
   a What is the total number of outcomes?
   b What is the number of blue blocks?
   c What is the probability of selecting a blue block?

2 Consider selecting a block from this building of blocks.



   a What is the total number of outcomes?
   b What is the number of red blocks?
   c What is the probability of selecting a red block?
   d What is the number of green blocks?
   e What is the probability of selecting a green block?
   f What is the number of triangular blocks?
   g What is the probability of selecting a triangular block?

## EXAMPLE 10I-2 Finding probability



A six-sided die is rolled.

a What are the possible outcomes?
b What is the probability of rolling a 4?
c What is the probability of rolling an even number?
d What is the probability of rolling a number greater than 4?

**THINK**

a List all the possible results.

b There is one chance out of six of rolling a 4.

c There are three chances out of six (2, 4, 6) of rolling an even number.

d There are two chances out of six (5, 6) of rolling a number greater than 4.

**WRITE**

a $\{1, 2, 3, 4, 5, 6\}$

b $\Pr(4) = \frac{1}{6}$

c $\Pr(\text{even number}) = \frac{3}{6}$
$= \frac{1}{2}$

d $\Pr(>4) = \frac{2}{6}$
$= \frac{1}{3}$

UNDERSTANDING AND FLUENCY

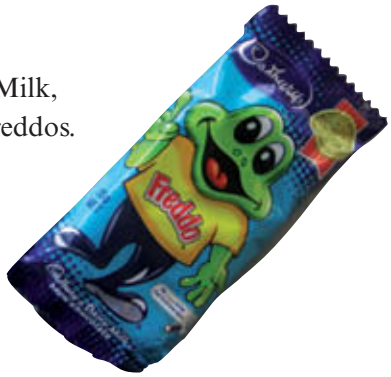
**3** A coin is flipped.
- **a** What are the possible outcomes?
- **b** What is the probability of:
  - **i** getting a head?
  - **ii** getting a tail?

**4** A six-sided die is rolled.
- **a** What are the possible outcomes?
- **b** What is the probability of:
  - **i** rolling a 2?
  - **ii** rolling an odd number?
  - **iii** rolling a number less than 5?

**5** A card is selected from a deck and the suit is recorded.
- **a** What are the possible outcomes?
- **b** What is the probability of:
  - **i** selecting a club?
  - **ii** selecting a red card?
  - **iii** selecting a green card?



**6** A letter is selected at random from the word PROBABILITY.
- **a** What are the possible outcomes?
- **b** What is the probability of:
  - **i** selecting a Y?
  - **ii** selecting a B?
  - **iii** selecting a vowel?




**7** A bag of 20 mixed Freddo Frogs contains six Dairy Milk, six Milky Top, four Strawberry and four Crunchie Freddos.

- **a** How many Freddos are there in total?
- **b** What is the probability of:
  - **i** picking a standard Dairy Milk Freddo?
  - **ii** picking a Crunchie Freddo?
  - **iii** picking a Freddo without a gooey centre?



**8** A flower is randomly selected from this bunch.

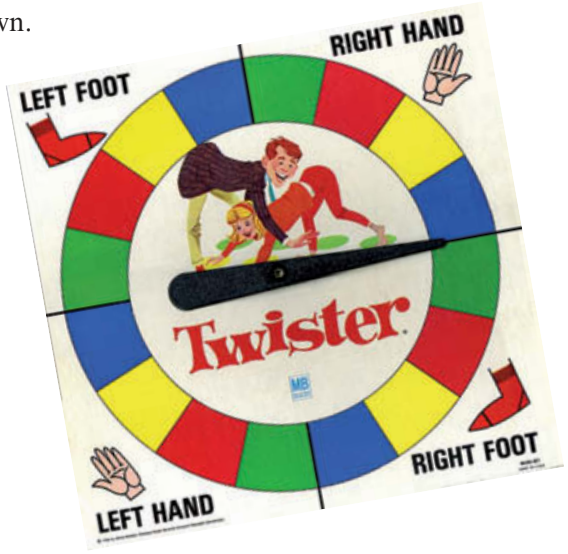
- **a** How many flowers are there in total?
- **b** How many roses are there?
- **c** How many lilies are there?
- **d** What is the probability of selecting a lily?
- **e** What is the probability of selecting a white flower?

**9** A normal deck of playing cards has four suits containing 13 cards each: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king. There are a total of 52 cards. A card is randomly drawn from the deck.

- **a** How many picture cards are there in each suit?
- **b** How many picture cards are there in the entire deck?
- **c** What is the probability of selecting a picture card?
- **d** How many aces are there in the deck?
- **e** What is the probability of selecting an ace?
- **f** What is the probability of selecting the queen of spades?
- **g** How many even numbers are there in each suit?
- **h** What is the probability of selecting an even number from the deck?
- **i** What is the probability of selecting a prime number from the deck?

**10** The spinner from the game Twister is shown.

- **a** How many different colours are there?
- **b** How many different body parts are there?
- **c** How many different outcomes are there in total?
- **d** What is the probability of:
  - **i** spinning a yellow?
  - **ii** spinning a left foot?
  - **iii** spinning a blue right hand?
  - **iv** spinning a green or red?
  - **v** spinning a right foot or blue?



**11** Tess looked at the jellybeans on page 568 and said that because there are four different colours of jellybeans, the probability of selecting a green jellybean is $\frac{1}{4}$. Can you explain her mistake?

**12** A pack of UNO cards has four suits: red, yellow, green and blue. Each suit contains the cards 0–9, Reverse, Skip and Draw Two. There are two copies of all cards except the 0s. In addition, there are four standard Wild Cards and four Draw Four Wild Cards. This gives a total of 108 cards.







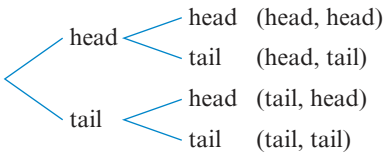
- **a** How many 0 cards are there in a pack?
- **b** How many 6 cards are there in a pack?
- **c** What is the probability of selecting a 0?
- **d** What is the probability of selecting a 6?
- **e** What is the probability of selecting a Reverse?
- **f** How many Wild Cards (of any kind) are there?
- **g** What is the probability of selecting a Wild Card?
- **h** What is the probability that a card selected will make another player draw cards?

**13** Make up at least three questions that you can ask a classmate about probability involving a deck of cards.

**14** Tree diagrams are a useful way of showing probabilities of events with more than one 'step'. If you flip two coins, there are four possible outcomes (where H represents flipping a head and T represents flipping a tail): HH, HT, TH and TT. This can be shown on a tree diagram.

- **a** What is the probability of flipping two heads?
- **b** What is the probability of flipping exactly one head?
- **c** Describe both these events with a term from the probability scale and explain why it is more likely you will flip one head.

tree diagram



PROBLEM SOLVING AND REASONING

**15** This table shows the outcomes when two dice are rolled.



| | | Die 2 | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Die 1 | 1 | 1, 1 | 1, 2 | 1, 3 | 1, 4 | 1, 5 | 1, 6 |
| | 2 | 2, 1 | 2, 2 | | | | |
| | 3 | 3, 1 | | | | | |
| | 4 | 4, 1 | | | | | |
| | 5 | 5, 1 | | | | | |
| | 6 | 6, 1 | | | | | |

**a** Copy and complete this table.

**b** How many outcomes are there?

**c** How many outcomes include a 6?

**d** What is the probability of:

**i** throwing at least one 6?

**ii** throwing a double 6?

**iii** throwing a double number?

CHALLENGE

**16** A 10-sided die with the numbers 0–9 is rolled.

**a** What is the probability of:

**i** rolling an odd number?

**ii** rolling an even number?

**iii** rolling a prime number?

**b** Draw a table or tree diagram to show the possible outcomes if the die was rolled twice.

**c** Use your answer to part **b** to help you find the probability of throwing a double 0.

**17** Two events are complementary when their probabilities add to 1. One example of complementary events is rolling a 6 on a die (Pr (6) = $\frac{1}{6}$) and not rolling a 6 (Pr (not 6) = $\frac{5}{6}$).

**a** Copy and complete these equations.

**i** 0.4 + _____ = 1 **ii** $\frac{1}{3}$ + _____ = 1 **iii** 0.5 + _____ = 1

**b** What is the complementary event to rolling a 4 on a die?

**c** What is the complementary event to flipping a tail on a coin?

**d** What is the complementary event to rolling an even number on a die?

**e** What is the complementary event to rolling a 1 or 2 on a die?

**f** Find the probabilities (in fraction form) for the events mentioned in parts **b**–**e**.

**Reflect**

How does knowledge of sample space help when calculating probabilities?

# 10J Experimental probability

## Start thinking!

The **experimental probability** of an event occurring is found when you conduct an experiment and count the number of times a favourable outcome occurs.



1 What are the possible outcomes of flipping a coin?
2 What is the probability of getting a tail?
3 How many tails would you expect to get if you flipped a coin:
   **a** 10 times? **b** 100 times?
4 Would you expect to get exactly this number of tails? Why or why not?
5 Draw a frequency table in your book with two columns, heads and tails.
6 With a partner, flip a coin 10 times and tally your results. This is experiment 1, with 10 **trials**.
7 How many tails and how many heads did you get?
8 Does this match what you predicted? Why or why not?
9 From the results of this experiment, calculate the probability of flipping a tail using the formula:

$$\text{experimental probability} = \frac{\text{number of tails flipped}}{10}$$
$$= ?$$

10 How does this compare to the theoretical probability of flipping a tail?
11 With a partner, now flip a coin 100 times and tally your results. This is experiment 2, with 100 trials.
12 How many tails and how many heads did you get?
13 Does this match what you predicted? Why or why not?

## KEY IDEAS

- Experimental probability is the probability of an outcome based on an experiment instead of theory. It is also called **relative frequency**.
- An experiment is when you carry out a series of trials and record the outcomes.
- A **successful trial** is one where you get a favourable outcome.
- To calculate experimental probability, use the formula: $\Pr(\text{success}) = \frac{\text{number of successful trials}}{\text{total number of trials}}$
- A probability can be written as a fraction, a decimal or a percentage.
- In an individual experiment, the experimental probability may not match the theoretical probability. However, as the number of trials in an experiment increases, the experimental probability gets closer to the theoretical probability. This is called the **long-term probability**.
- **Simulation** involves using technology or simple devices like coins or dice to simulate real events.

# EXERCISE 10J Experimental probability

## EXAMPLE 10J-1 Finding expected number

A coin was flipped 50 times and 27 heads were obtained.

a How many trials are in this experiment?
b What is the theoretical probability of getting a head?
c What number of heads you would expect to get?

**THINK**

a State the number of times the coin is flipped.

b Divide the number of favourable outcomes (1) by the total number of outcomes (2).

c You would expect $\frac{1}{2}$ of the results to be heads. Find $\frac{1}{2}$ of 50.

**WRITE**

a number of trials
$= 50$

b theoretical probability of obtaining a head
$= \frac{1}{2}$ (or 0.5)

c expected number of heads
$= \frac{1}{2} \times 50$
$= 25$

UNDERSTANDING AND FLUENCY

1 A coin was flipped 20 times and 13 tails were obtained.
   a How many trials are in this experiment?
   b What is the theoretical probability of obtaining a tail?
   c What is the expected number of tails?

2 A die was rolled 60 times and six 1s were obtained.
   a How many trials are in this experiment?
   b What is the theoretical probability of obtaining a 1?
   c What is the expected number of 1s?



## EXAMPLE 10J-2 Finding experimental probability

Consider the experiment described in Example 10J-1.

a Calculate the experimental probability of obtaining a head in this experiment.
b How does this compare to the theoretical probability?
c If you flipped a coin 5000 times, would you expect the experimental probability for obtaining a head to increase, decrease or stay the same? Explain.

### THINK

a Divide the number of heads obtained (27) by the total number of coin flips (50).

b Theoretical probability is $\frac{1}{2}$ or 0.5.

c You would expect the experimental probability to get closer to the theoretical probability so would expect the experimental probability of obtaining a head to decrease.

### WRITE

a experimental probability of obtaining a head $= \frac{27}{50}$ (or 0.54)

b Experimental probability is higher than theoretical probability.

c Expect the experimental probability of getting a head to decrease. The more times the coin is flipped, the closer the experimental probability gets to the theoretical probability.

UNDERSTANDING AND FLUENCY

**3** Consider the experiment listed in question **1**.
- **a** Calculate the experimental probability of getting a tail.
- **b** How does this compare to the theoretical probability?
- **c** If you flipped a coin 2000 times, would you expect the experimental probability for obtaining a tail to increase, decrease or stay the same? Explain.

**4** Consider the experiment listed in question **2**.
- **a** Calculate the experimental probability of getting a 1.
- **b** How does this compare to the theoretical probability?
- **c** If you rolled a die 6000 times, would you expect the experimental probability of obtaining a 1 to increase, decrease or stay the same? Explain.

**5** Roll a die 36 times and record your results in this table.
- **a** What is the theoretical probability of rolling a 1?
- **b** What experimental probability did you obtain for rolling a 1?
- **c** Explain why this may be different.
- **d** What would you expect to happen if you repeated the experiment 1000 times?

| Number | Expected frequency | Actual frequency |
|---|---|---|
| 1 | 6 | |
| 2 | 6 | |
| 3 | 6 | |
| 4 | 6 | |
| 5 | 6 | |
| 6 | 6 | |

**6** Another experiment is to roll two dice and add the values.

| | | Die 2 | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Die 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | 2 | 3 | 4 | | | | |
| | 3 | | | | | | |
| | 4 | | | | | | |
| | 5 | | | | | | |
| | 6 | | | | | | |



**a** Copy and complete the table.

**b** Create a second table to show all the possible different outcomes and their theoretical probability.

**c** How many chances do you have to roll a total of 6?

**d** What is the theoretical probability of rolling a total of 6?

**e** Add a column to your second table to calculate the theoretical frequency of each total for 36 rolls. This is done by multiplying the theoretical probability by 36.

**f** How many times in 36 rolls would you expect to roll a total of 6?

**g** Add another column to your table for the experimental frequency of each total.

**h** Perform the experiment and tally the results.

**i** What do you find?

**7** A classmate performs the experiment from question **5** and obtains these results.

| Number | Expected frequency | Actual frequency | Theoretical probability | Relative frequency (Experimental probability) |
|---|---|---|---|---|
| 1 | 6 | 3 | $\frac{6}{36}$ | $\frac{6}{36}$ |
| 2 | 6 | 5 | $\frac{6}{36}$ | $\frac{5}{36}$ |
| 3 | 6 | 7 | $\frac{}{36}$ | $\frac{}{36}$ |
| 4 | 6 | 5 | | |
| 5 | 6 | 5 | | |
| 6 | 6 | 11 | | |

**a** Copy and complete the table, calculating the theoretical probability and the relative frequency of throwing each number.

**b** What is the theoretical probability of rolling a 6? Express in simplest form.

**c** What is the relative frequency of rolling a 6?

**d** Is this bigger or smaller than the theoretical probability?

It is possible to get skewed results from such a small experiment. However, sometimes people can specifically design things like dice to get the outcomes that they want. To find out if this die is unfair, another 3600 trials were conducted.

**e** If the die is fair, what would you expect to see?

The results are given in the table at right.

| Number | Expected frequency | Actual frequency |
|---|---|---|
| 1 | 600 | 285 |
| 2 | 600 | 576 |
| 3 | 600 | 745 |
| 4 | 600 | 467 |
| 5 | 600 | 512 |
| 6 | 600 | 1015 |

**f** Would you say that this is a fair die? Why or why not?

**g** What explanation can you give for the low frequency of rolling a 1?

**8** Another common **probability experiment** is to flip three coins. There are eight possible outcomes.

| Outcome | Theoretical probability |
|---|---|
| 0 heads, 3 tails | |
| 1 head, 2 tails | |
| 2 heads, 1 tail | |
| 3 heads, 0 tails | |

- **a** Copy the tree diagram from page 572 and extend it so that it represents flipping three coins.
- **b** Write down the eight outcomes and use it to fill in this probability table.
- **c** Why are there only four different outcomes listed in the table when the tree diagram shows eight outcomes?
- **d** Add a column to your table to predict the number of each outcome you would expect if you completed 40 trials.
- **e** Using three coins, perform the experiment and record your results in an additional column with the heading 'Actual frequency'.
- **f** Calculate the experimental probabilities for each outcome.
- **g** Write at least two sentences comparing your results to the theoretical results.
- **h** What would you expect to happen to the difference between the theoretical and experimental probabilities if you performed 1 000 000 trials?

**9** A 10-sided die is pictured.




- **a** Create a table listing the possible outcomes and their theoretical probabilities when this die is thrown.
- **b** What is the theoretical probability of:
  - **i** throwing an even number?
  - **ii** throwing an odd number?
- **c** Add a column to your table and use it to record your predictions about the frequency of each outcome if you threw this die 100 times.
- **d** Throw a 10-sided die 100 times and record your results in an additional column. If you do not have access to a 10-sided die, you could simulate it by choosing something else with 10 possible outcomes. This could just be having 10 different counters or pieces of paper that you select randomly.
- **e** Discuss your results and make a statement about the long-term predictions for the experimental probabilities.



PROBLEM SOLVING AND REASONING

**10** Flipping a coin is a common way to simulate the outcomes of the birth of a baby as a girl or a boy.

**a** Assign the birth of a girl to either heads or tails.

**b** Flip a coin 10 times and record your results.

**c** Find the relative frequency of the birth of a girl for your experiment. Is this higher or lower than you expected?

**d** Repeat the experiment another nine times. Do you get the same result each time? Why do you think this is?

**e** A classmate does a similar simulation, but in each experiment she flips the coin 100 times. Her results are shown below.

| Experiment | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Relative frequency for birth of girl | 0.51 | 0.52 | 0.49 | 0.50 | 0.50 | 0.53 | 0.51 | 0.48 | 0.47 | 0.51 |

**f** How are her results different from yours?

**g** Is there more or less variation in the relative frequency? Why do you think this is?

**11** Design your own experiment that you can perform with a classmate. Make a table of your theoretical predictions and record your experimental results. Write at least three sentences describing what you find.

CHALLENGE

**12** The chocolate company Mars often run a promotion where you can win a Mars Bar. One out of every six wrappers gives you a free bar. You can use your knowledge of probability to make predictions about winning and to simulate experiments.

**a** What is the theoretical probability of winning a Mars Bar?

**b** What is the theoretical probability of not winning a Mars Bar?

**c** How many Mars Bars would you expect to have to buy in order to win five free Mars Bars?




**d** What is a probability experiment that has six possible outcomes that can be used to simulate buying Mars Bars?

To use a die to simulate buying a Mars Bar, assign one of the outcomes as success or a 'win'. Any outcome will do – for now, use '1'.

**e** Roll the die 30 times and record the number of 1s. How many do you get?

**f** For better results, simulations should always be repeated. Repeat this experiment another four times and record the number of 1s obtained.

**g** What is the average number of 1s that you obtain over the five simulations?

**h** Did the simulations reflect your predictions? Explain.

**Reflect**

How does relative frequency differ from theoretical probability?

# CHAPTER REVIEW

## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

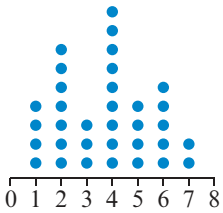
numerical data
categorical data
continuous data
discrete data
ordinal data
nominal data
frequency distribution tables
sample
population
class
frequency
graph
title
axes
scale
legend
dot plot
bar graph
column graph
pie graph
line graph
scatterplot
stem-and-leaf plot
back-to-back stem-and-leaf plot
measure of centre
measure of spread
average
mean
median
mode
range
probability
event
probability scale
sample space
outcome
favourable outcome
theoretical probability
experimental probability
relative frequency
trial
success
long-term probability
simulation

## MULTIPLE-CHOICE

10A **1** Which of these is an example of numerical discrete data?

**A** your height
**B** coming first in a race
**C** number of pets that you have
**D** colour of your hair

10C **2** What is the most common number in this dot plot?




**A** 3 **B** 4
**C** 7 **D** 8

10C **3** How many people were surveyed in the dot plot above?

**A** 34 **B** 7 **C** 8 **D** 30

10C 10D 10E **4** Students who prefer blue make up 25% of the class. In a pie graph showing favourite colours, what would be the angle for the blue sector?

**A** 25° **B** 90° **C** 180° **D** 360°

10F **5** The key to a stem-and-leaf plot is 1|2 = 12. What is the leaf of 48?

**A** 4 **B** 8 **C** 12 **D** 48

10G **6** What is the median of the data set 5, 3, 4, 7, 9, 16, 5, 6, 8?

**A** 5 **B** 6 **C** 7 **D** 9

10H **7** How could the probability of a day in summer being hot be described?

**A** impossible **B** even chance
**C** very likely **D** certain

10I **8** What is the probability of selecting a heart from a deck of cards?

**A** $\frac{1}{13}$ **B** $\frac{1}{52}$ **C** $\frac{1}{2}$ **D** $\frac{1}{4}$

10J **9** If a die is rolled 120 times, how many times would you expect to obtain a four?

**A** 4 **B** 20 **C** 30 **D** 120

## SHORT ANSWER

10A **1** Diana surveyed her class to see who could roll their tongues. Y represents 'yes' and N represents 'no'.
Y N N Y Y N N Y Y Y N N Y

a What kind of data is this?
b Put the data into a frequency table
c How many people were surveyed?
d What was the most common attribute?

10C **2** Draw the data in the frequency table in these forms.

a a dot plot b a bar graph

| Favourite holiday activity | Frequency |
|---|---|
| Swimming at the beach | 11 |
| Camping | 4 |
| Watching TV | 7 |
| Reading a book | 2 |
| Hiking | 5 |
| Snow-skiing | 4 |
| Other | 3 |

10D **3** Draw the data from question **2** as a pie graph. Which graph displays the data the best? Why?

10E **4** Graph the data below and describe the relationship in the scatterplot.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *a* | 1 | 5 | 3 | 8 | 6 | 8 | 10 | 4 | 4 | 9 | 2 | 3 |
| *b* | 18 | 10 | 14 | 6 | 8 | 3 | 2 | 20 | 12 | 10 | 25 | 16 |

10F **5** Consider this stem-and-leaf plot.
Key: 1 | 5 = 15

| Leaf Boys | Stem | Leaf Girls |
|---|---|---|
| 6 4 2 | 0 | 9 |
| 8 7 6 4 4 2 | 1 | 1 5 5 6 |
| 9 9 9 9 6 | 2 | 4 6 6 8 9 |
| 3 2 | 3 | 1 2 2 5 9 |
| | 4 | 2 2 2 |

a What is the most common value? Does this occur for boys or girls?
b What is the highest value? Does this occur for boys or girls?
c How many people were surveyed?

10F **6** Create a stem-and-leaf plot using this data. Remember to use a key.
13 16 45 32 62 17 22 28 55 9 12 38 42 51 59 31 16 13 12 25 30 41 20 5 23 33

10G **7** a Calculate the mean, median, mode and range for this set of data.
7, 1, 8, 12, 71, 6, 14, 10, 2, 2, 3, 5, 9
b Which is the best indicator of centre for this data?

10H **8** For each of these, list the sample space and describe the outcomes in brackets using a term from the probability scale.

a flipping a coin (head)
b rolling a die (odd number)
c selecting a card from a deck and recording the suit (diamond)
d randomly selecting a counter from 12 red, 7 blue and 1 white (red counter).

10I **9** Calculate the probability of:

a rolling a die and getting a 3
b flipping a coin twice and getting tails both times
c selecting a consonant from the word REVISION
d drawing an ace from a deck of cards.

10J **10** An experiment was conducted and the results in this table were obtained.

| Outcome | Frequency |
|---|---|
| A | 28 |
| B | 48 |
| C | 24 |

a How many trials were there?
b Add a column to the table and calculate the relative frequency for each outcome. Express these values as decimal numbers.
c In the long term, what would you expect the probability for each outcome to be?
d Can you think of what the experiment might have been?

## NAPLAN-STYLE PRACTICE

**1** What type of data is hair colour?

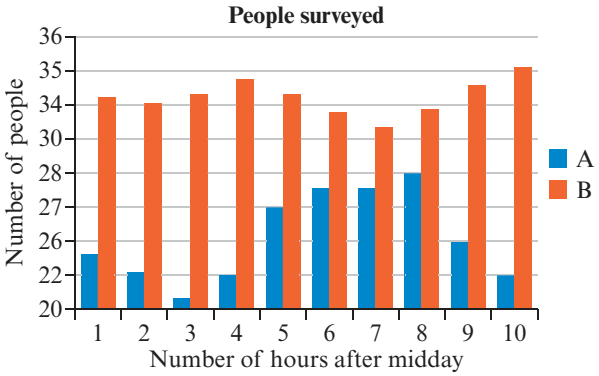
- ☐ categorical, ordinal
- ☐ numerical, discrete
- ☐ numerical, continuous
- ☐ categorical, nominal

**2** This frequency table is missing a value. If 50 students were surveyed, how many students like green eyes?

☐

| Favourite eye colour | Frequency |
|---|---|
| Brown | 13 |
| Hazel | 9 |
| Green | |
| Blue | 17 |
| Total | 50 |

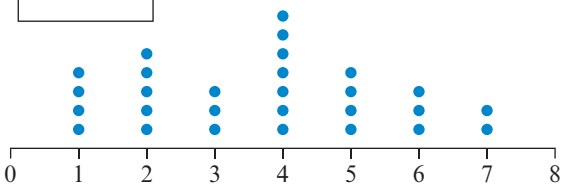
**3** Which of the following characteristics of the graph below does not need fixing?



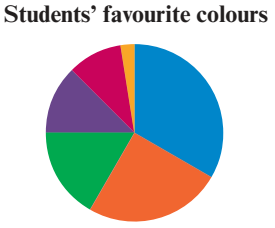

- ☐ the horizontal scale
- ☐ the vertical scale
- ☐ the legend
- ☐ the title

**4** How many people were surveyed for this dot plot?

☐



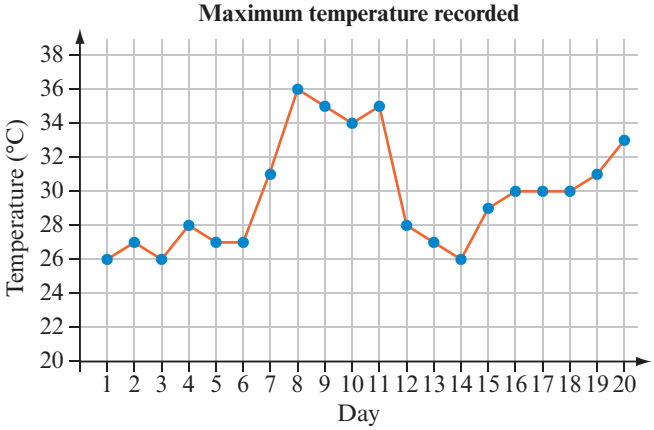

**5** The pie graph shows the favourite colour of students. Approximately what percentage of students like pink?



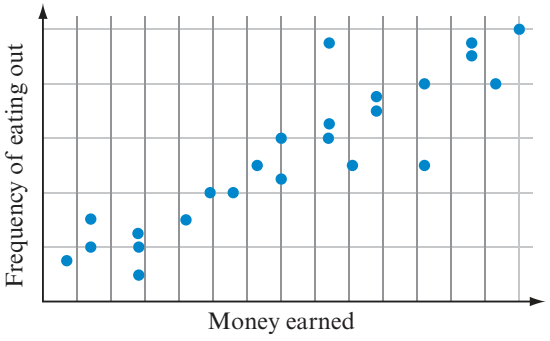

10% ☐ 30% ☐ 70% ☐ 95% ☐

**6** What temperature is shown on day eight of the line graph?

☐ °C




**7** A scatterplot has been drawn that shows the relationship between the money earned and the frequency of eating out.




What does the scatterplot show?

- ☐ People who earn more money don't eat out very often.
- ☐ Eating out a lot makes you poor.
- ☐ People with less money don't eat out.
- ☐ People who earn more money tend to eat out more often than people who earn less money.

**8** What is the largest number shown in this stem-and-leaf plot?

Key: 1 | 1 = 11

| Stem | Leaf |
|---|---|
| 1 | 1 4 5 7 |
| 2 | 2 3 4 5 9 9 |
| 3 | 1 9 |
| 4 | 0 3 3 3 5 6 |
| 5 | 0 0 4 5 8 8 |

- ◯ 8
- ◯ 9
- ◯ 5
- ◯ 58

Questions **9** and **10** refer to this data set:

4, 7, 5, 9, 1, 3, 4, 11, 9, 4, 4, 8, 9

**9** The mean, median and mode for this set are:

- ◯ 4, 5 and 6
- ◯ 6, 5 and 4
- ◯ 5
- ◯ 6

**10** What is the range of the data set?

**11** Which data set has a mode of 8?

- ◯ 8, 3, 8, 5, 7, 3
- ◯ 3, 6, 4, 8, 5, 6, 3, 7, 8, 1, 2, 8
- ◯ 2, 4, 6, 8, 10, 12, 14
- ◯ 7, 3, 6, 8, 5, 2, 6, 9

**12** Which of these situations would be best described as 'likely'?

- ◯ the sun rising tomorrow
- ◯ rolling a die and getting an even number
- ◯ selecting a non-picture card from a deck of cards
- ◯ winning Tattslotto

Questions **13**–**15** refer to a box containing Lindt chocolate balls. There are four milk chocolate balls, three white chocolate, four hazelnut and three dark chocolate.

**13** What is the sample space for the box of chocolates?

- ◯ {4, 3, 4, 3}
- ◯ {12}
- ◯ {milk, hazelnut, white, dark}
- ◯ {chocolate, box}

**14** What is the probability (written as a fraction) of selecting a white chocolate from the box?

**15** What is the probability of selecting a chocolate that is not dark?

- ◯ $\frac{3}{14}$
- ◯ $\frac{1}{2}$
- ◯ $\frac{1}{4}$
- ◯ $\frac{11}{14}$

Questions **16**–**17** refer to this experiment.
A fair die was rolled 100 times and a 6 was obtained 20 times.

**16** What is the experimental probability of obtaining a 6? Write your answer as a decimal number.

**17** What would you expect to happen to the experimental probability if the experiment was repeated 1000 times?

- ◯ increase
- ◯ decrease
- ◯ stay the same
- ◯ become 1

## ANALYSIS

A survey was conducted on the number of movies people had watched in the past month. The results are below.

1 5 3 2 7 1 4 2 3 6 2 5 7
13 2 2 1 0 4 2 5 2 1 0 5 1

**a** What type of data is this?

**b** How many people were surveyed?

**c** Place the results in a frequency table.

**d** Draw a graph to show the data and state why this is the best graph to use.

**e** Calculate summary statistics on the data (mean, median, mode, range).

**f** What is the probability that a random person selected from the group has watched:

- **i** two movies in the past month?
- **ii** more than five movies in the past month?

Write these probabilities as a fraction and a decimal number, and describe them using a term.

# CONNECT

## Surveying people



When people want to find out information, they often do a survey. From individuals to companies to governments, it is the best way to get information and perform statistical and probability calculations. You are going to conduct a survey on a topic of your choice.

What things do you have to think about before you conduct a survey or questionnaire?



### Your task

For your investigation into conducting a survey, follow these steps.

- Decide on a topic to survey and formulate at least five questions.
- Perform the survey and collect the information.
- Present the data in:
  - frequency tables
  - at least one of each type of graph mentioned in this unit.
- Calculate summary statistics on the numerical data in the survey.
- Calculate the probability of at least three different outcomes for the data you have collected.
- Make a prediction about the long-term probability; that is, what kind of results you think that you'd see if you surveyed a much larger group.
- Write a page detailing the main discoveries of your survey and form a conclusion about what you found.

You might need

- a ruler
- a protractor
- a calculator.









Complete the 10 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a statistical report
- a PowerPoint presentation
- a Web page
- other (check with your teacher).

# Answers

## CHAPTER 1 WHOLE NUMBERS

### 1 Are you ready?

**1** D **2** B **3** A
**4** **a** 41 **b** 153 **5** C
**6** **a** 77 **b** 75 **7** **a** 53 **b** 36
**8** **a** 128 **b** 504 **9** **a** 420 **b** 7500
**10** C **11** **a** 12 **b** 530
**12** D **13** B **14** 81
**15** C **16** D **17** B

### 1A Understanding place value

#### 1A Start thinking!

1. The second number (97 302) is the smallest, as 302 is smaller than 531.
2. The third number (100 012) is the largest, as 100 000 is larger than 97 000.
3. The 1 is in the 100 000 column, whereas the 9 is in the 10 000 column and has a smaller value.

#### Exercise 1A Understanding place value

**1** **a** 50 + 6 **b** 200 + 30 + 8
**c** 4000 + 700 + 50 + 1
**d** 10 000 + 2000 + 600 + 40 + 9
**e** 8000 + 500 + 7
**f** 60 000 + 3000 + 40 + 4

**2** **a** even **b** even **c** odd
**d** odd **e** odd **f** even

**3** **a** 90 **b** 900 **c** 9
**d** 9000 **e** 90 **f** 900 000

**4** **a** 90 **b** 8
**c** **i** 10 **ii** 6 **iii** 500

**5** **a** 62 000 **b** 978 **c** 304
**d** 250 112 **e** 12 543 **f** 9026

**6** **a** three hundred and sixty-two
**b** seven thousand, two hundred and fifteen
**c** forty-five thousand, seven hundred and thirty-three
**d** two hundred and thirty-four thousand, six hundred and one
**e** six million, four hundred and twenty thousand and fifty-eight
**f** fifty-five thousand, five hundred and fifty-five

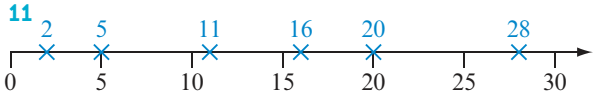
**7** **a** 439, 4562, 45 629
**b** 25 786, 35 218, 53 176
**c** 6754, 67 454, 67 554 **d** 9013, 9103, 9130
**e** 24 007, 24 071, 24 701
**f** 613 538, 613 548, 613 583

**8** **a** 8530 > 7503 **b** 46 249 < 64 249
**c** 317 294 > 37 294 **d** 709 < 4503
**e** 5678 < 5876 **f** 10 462 > 10 248
**g** 360 > 306 **h** 8245 < 8254
**i** 214 007 > 210 007 **j** 989 000 < 998 000

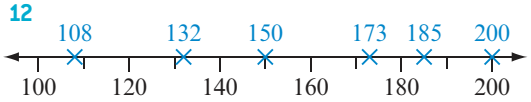
**9** **a** 9, 15, 28, 40, 58, 72, 88
**b** 805, 806, 846, 856, 890
**c** 625, 6105, 9472, 9902, 10 417
**d** 234, 2347, 2374, 23 074, 23 704

**10** **a** 8000, 1800, 870, 807, 87
**b** 3999, 3909, 3099, 399, 309
**c** 75 561, 75 516, 75 156, 75 126
**d** 2 567 291, 1 967 219, 256 291

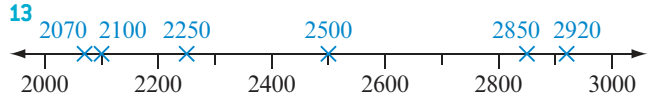
**11**




**12**



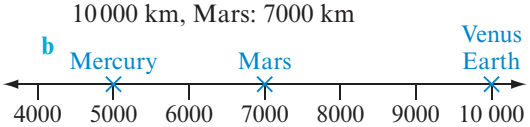

**13**




**14** **a** **i** 200 **ii** 300 **iii** 300
**iv** 200 **v** 300
**b** **i** 6000 **ii** 5000 **iii** 6000
**iv** 6000 **v** 5000
**c** If the number is 250 or larger, it is approximated as 300.
If the number is 5500 or larger, it is approximated as 6000.

**15** **a** 800 **b** 50 **c** 100 **d** 7000
**e** 40 000 **f** 600 **g** 9000 **h** 20 000
**i** 200 000 **j** 1000 **k** 70 000 **l** 100 000

**16** **a** Mercury: 5000 km, Venus: 10 000 km, Earth: 10 000 km, Mars: 7000 km
**b**




**c** Mars is slightly larger than Mercury but Earth and Venus are nearly the same size and twice that of the smaller two.

**17** **a** **i** 3000 km **ii** 400 000 km
**b** The distance from the Earth to the Moon is approximately 100 times greater than the diameter of the Moon. The diameter of the Moon is approximately 3000 km. Multiply this by 100 to get 300 000. This is approximately the distance from the Earth to the Moon.

**18** The digit 1 appears 38 times. It appears once in the numbers 1 to 9, eleven times in the numbers 10 to 19, eight times in the numbers 20 to 99 (that is, 21, 31, 41, 51, 61, 71, 81 and 91) and eighteen times in the numbers 100 to 112.
This makes a total of 1 + 11 + 8 + 18 = 38.

**19** **a** two different two-digit numbers; 35, 53
**b** four different two-digit numbers; 55, 53, 35, 33

**20** **a** six different two-digit numbers; 74, 72, 47, 42, 27, 24
**b** nine different two-digit numbers; 22, 24, 27, 42, 44, 47, 72, 74, 77

**21** **a** 9631 **b** 1111 **c** 9316
**d** 3691, 3916, 3961, 6139, 6193, 6319
**e** 9993 **f** 9993

## 1B Adding whole numbers

### 1B Start thinking!

**1** 769 **3** 769 kg

### Exercise 1B Adding whole numbers

**1** **a** 80 **b** 300 **c** 7000
**d** 180 **e** 380 **f** 300
**g** 38 **h** 77 **i** 90
**j** 70 **k** 100 **l** 468

**2** **a** 70 **b** 900 **c** 6000
**d** 80 **e** 70 **f** 100
**g** 90 **h** 99 **i** 66
**j** 180 **k** 400 **l** 609

**3** **a** 958 **b** 692 **c** 1219
**d** 1502 **e** 5119 **f** 13 699
**g** 2715 **h** 135 470

**4** **a** 1119 **b** 453 **c** 7598
**d** 5251 **e** 2660 **f** 5839
**g** 17 189 **h** 43 723

**5** **a** 1569 **b** 1804 **c** 1413
**d** 1597 **e** 960 **f** 1515
**g** 11 508 **h** 7889 **i** 4959

**6** **a** 138 849 **b** 92 203 **c** 349 908
**d** 686 461 **e** 85 157 **f** 30 555

**7** **a** 11 374 **b** 102 409
**c** 75 701 **d** 905 264

**9** **a** **i** Add 32 and 45 and 59 together.
**ii** What is 32 plus 45 plus 59?
**iii** Find the result of 32 + 45 + 59.
**iv** Find the sum of 32, 45 and 59.
**b** One possible answer is: Find the total of 32, 45 and 59.
**c** 136 km

**10** **a** **i** 750 km **ii** 1591 km
**iii** 226 km in stage 1. It may not be the hardest day of cycling, as travelling up and down hills for a shorter distance may be more difficult.
**iv** 626 km **v** 3430 km

**11** **a** 38 000 **b** 39 174
**c** the estimation

**12** One possible answer is: 50 + 3 = 53.

**13** **a** One possible answer is: 233 + 153 = 386.
**b** One possible answer is: 214 + 172 = 386.

**14** One possible answer is:
367 + 1289 + 3551 = 5207.

**15** **a** 49 **b** 296 **c** 120
**d** 257 **e** 200 **f** 200

**16** **a** 105 kg **b** 19 kg **c** 124 kg
**d** One possible answer is:
51 kg + 62 kg + 37 kg = 150 kg

**17** **a** 55
**b** (1 + 10) + (2 + 9) + (3 + 8) + (4 + 7) + (5 + 6)
= 11 + 11 + 11 + 11 + 11
= 55

**18** 210

**19** **a** **i** 45 **ii** 190
**b** There will be one number that is not part of a pair and will need to be added separately.
**c** **i** 35 **ii** 150 **iii** 315

## 1C Subtracting whole numbers

### 1C Start thinking!

**1** 342 **3** 342 cm

### Exercise 1C Subtracting whole numbers

**1** **a** 20 **b** 100 **c** 3000
**d** 140 **e** 210 **f** 690
**g** 21 **h** 23 **i** 52
**j** 120 **k** 235 **l** 112

**2** **a** 60 **b** 500 **c** 4000
**d** 10 **e** 109 **f** 515
**g** 15 **h** 24 **i** 50
**j** 32 **k** 42 **l** 435

**3** **a** 233 **b** 371 **c** 265
**d** 272 **e** 6162 **f** 815
**g** 6277 **h** 577

**4** **a** 425 **b** 452 **c** 222
**d** 287 **e** 439 **f** 2122
**g** 1877 **h** 1656

**5** **a** 6112 **b** 5315 **c** 1747
**d** 6928 **e** 3563 **f** 4571
**g** 1385 **h** 9727

**6** **a** 8181 **b** 370 595 **c** 175 128
**d** 73 390 **e** 4451 **f** 607 779
**g** 919 592 **h** 99 187

**8** **a** **i** 30 **ii** 32
**b** **i** 600 **ii** 579
**c** **i** 5100 **ii** 4871
**d** **i** 18 000 **ii** 19 880
**e** **i** 120 **ii** 122
**f** **i** 960 **ii** 1271

**9** **a** **i** Subtract 2740 from 6650.
**ii** What is 6650 minus 2740?
**iii** Find the result of 6650 − 2740.
**iv** Find the difference between 6650 and 2740.
**b** One possible answer is:
6650 take away 2740.
**c** 3910 m

**10** **a** 16 742 km **b** 16 112 km
**c** The Singapore stop over is the shortest by 630 km.
**11** **a** Washington Monument: 659 m, Eiffel Tower: 528 m, Empire State Building: 447 m, Sydney Tower: 519 m, Petronas Towers: 376 m, Taipei 101: 319 m, Q1: 505 m, Eureka Tower: 531 m
**b** Taipei 101 and the Q1 give a total height of 832 m.
**c** Washington Monument, Empire State Building and Eureka Tower give a total height of 847 m.
**12** **a** Mount Everest is 6620 m higher than Mount Kosciuszko.
**b** 4148 m higher, 2152 m lower, 12 323 m lower
**c** 16 471 m **d** 8772 m
**13** 193
**15** **c** You are taller than the wallaby but shorter than the emu.
**16** **a** **i** 645 200 000 km **ii** 1 365 691 000 km
**iii** 50 291 000 km **b** 929 392 000 km
**c** **i** 720 491 000 km **ii** 1 315 400 000 km
**iii** 2 203 392 000 km **d** 78 340 000 km
**17** One possible answer is: 95 − 52 = 43.
**18** **a** One possible answer is: 259 − 91 = 168.
**b** One possible answer is: 578 − 410 = 168.
**19** **a** One possible answer is: 486 − 200 = 286.
**b** 429 − 143 = 286.

## 1D Multiplying whole numbers

### 1D Start thinking!

**1** 162 **2** 162 kg **3** 4536 kg

### Exercise 1D Multiplying whole numbers

**1** **a** 592 **b** 186 **c** 135
**d** 182 **e** 960 **f** 7236
**g** 3192 **h** 1124 **i** 18 474
**j** 21 525 **k** 120 795 **l** 3 717 747
**2** **a** 50 **b** 320 **c** 6390
**d** 516 230 **e** 700 **f** 278 400
**g** 4800 **h** 10 300 **i** 85 000
**j** 936 000 **k** 15 000 **l** 7 020 000
**3** **a** 120 **b** 420 **c** 180
**d** 6000 **e** 16 800 **f** 91 000
**g** 40 000 **h** 123 000 **i** 712 000
**j** 57 600 **k** 500 000 **l** 355 150
**4** **a** 480 **b** 120 **c** 840
**d** 3720 **e** 10 850 **f** 126 900
**g** 41 600 **h** 128 400 **i** 4 495 800
**j** 344 000 **k** 2 233 000 **l** 3 084 000
**5** **a** 65 × (40 + 8) **b** 283 × (50 + 4)
**c** 415 × (20 + 3) **d** 1059 × (10 + 8)
**6** **a** 570 **b** 1488 **c** 1472
**d** 6205 **e** 4551 **f** 12 936
**g** 38 318 **h** 40 986 **i** 111 468
**j** 235 425 **k** 2 454 192 **l** 1 019 994
**7** **a** 3120 **b** 15 282
**c** 9545 **d** 19 062
**9** **a** **i** Multiply 37 and 28 together.
**ii** What is 37 times 28?
**iii** Find the result of 37 × 28.
**iv** Find the product of 37 and 28.
**v** Find 37 lots of 28.
**b** One possible answer is: 37 by 28.
**c** $1036
**10** **a** 538 × (100 + 20 + 4)
**b** 361 × (200 + 50 + 3)
**c** 4825 × (600 + 20 + 7)
**d** 9141 × (300 + 80 + 2)
**11** **a** 43 250 **b** 300 155 **c** 157 872
**d** 585 625 **e** 256 452 **f** 197 262
**g** 486 948 **h** 5 259 232 **i** 997 425
**j** 14 745 990 **k** 61 518 675 **l** 75 884 094
**12** **a** 66 712 **b** 91 333
**c** 3 025 275 **d** 3 491 862

| First number × second number | Product | Number of zeros in the first number | Number of zeros in the second number | Number of zeros in the product |
|---|---|---|---|---|
| 10 × 10 | 100 | 1 | 1 | 2 |
| 100 × 10 | 1000 | 2 | 1 | 3 |
| 1000 × 10 | 10 000 | 3 | 1 | 4 |
| 10 000 × 10 | 100 000 | 4 | 1 | 5 |

**c** The total number of zeros in the product is the sum of the number of zeros in the original calculation.
**d** **i** 1000 **ii** 10 000 **iii** 100 000
**iv** 1 000 000
**15** **a** **i** 100 000 **ii** 400 000
**b** **i** 3000 **ii** 70 000 **iii** 600 000
**iv** 2 000 000 **v** 5000 **vi** 400 000
**vii** 80 000 000 **viii** 900 000
**16** **a** **i** 6 **ii** 1000 **iii** 6000
**b** **i** 8000 **ii** 90 000 **iii** 120 000
**iv** 4 500 000 **v** 56 000 **vi** 2 400 000
**vii** 210 000 000 **viii** 16 000 000
**17** **a** 80 000 **b** 162 000 000
**c** 5 600 000 **d** 120 000 000
**18** **a** 1800 **b** 6790 **c** 401 600
**d** 79 300 **e** 35 800 **f** 17 000
**g** 891 000 **h** 6 055 000
**19** **a** **i** 48 000 **ii** 180 000
**iii** 2 400 000 **iv** 360 000 000
**b** **i** 48 462 **ii** 202 275
**iii** 2 388 224 **iv** 355 868 293
**20** **a** 4320 beats per hour **b** 103 680 per day
**21** **b** Answer to part a multiplied by 3.
**c** Answer to part b multiplied by 60.
**d** Answer to part c multiplied by 24.
**22** **a** 800 students **b** 733 students
**23** 2576 menus
**24** **a** 1040 m **b** 18 000 m **c** 8760 hours

**d** **i** 1848 months **ii** 56 210 days
**iii** 1 349 040 hours
**iv** 80 942 400 minutes
**v** 4 856 544 000 seconds

**25** **a** Answers given for a person turning 13 years old.
**i** 156 months **ii** 4745 days
**iii** 113 880 hours
**iv** 6 832 800 minutes
**v** 409 968 000 seconds

**26** 1 200 000 000 km

## 1E Dividing whole numbers

### 1E Start thinking!

**1** 4 **2** $4 \times 5 = 20$
**3** 3 remainder 2

### Exercise 1E Dividing whole numbers

**1** **a** **i** 9 **ii** 2 **iii** 4 **iv** 1
**b** **i** 17 **ii** 7 **iii** 2 **iv** 3
**c** **i** 30 **ii** 5 **iii** 6 **iv** 0

**2**

| | quotient | quotient × divisor | remainder |
|---|---|---|---|
| $7 \div 3$ | 2 | $2 \times 3 = 6$ | $7 - 6 = 1$ |
| $19 \div 5$ | 3 | $3 \times 5 = 15$ | $19 - 15 = 4$ |
| $33 \div 4$ | 8 | $8 \times 4 = 32$ | $33 - 32 = 1$ |
| $62 \div 15$ | 4 | $4 \times 15 = 60$ | $62 - 60 = 2$ |

**3** **a** 6 remainder 1 **b** 4 remainder 3
**c** 5 remainder 2 **d** 2 remainder 5
**e** 4 remainder 6 **f** 6 remainder 2
**g** 7 remainder 3 **h** 8 (or 8 remainder 0)

**4** **a** 134 remainder 2 **b** 126
**c** 57 remainder 1 **d** 245 remainder 4
**e** 381 **f** 1401 remainder 5
**g** 33 529 remainder 1 **h** 31 816 remainder 3

**5** **a** 7 **b** 245 **c** 94 **d** 417

**6** **a, b**

| First number ÷ second number | first number second number | Quotient | Number of zeros in the first number | Number of zeros in the second number | Number of zeros in the quotient |
|---|---|---|---|---|---|
| $10 \div 10$ | $\frac{10}{10}$ | 1 | 1 | 1 | 0 |
| $100 \div 10$ | $\frac{100}{10}$ | 10 | 2 | 1 | 1 |
| $1000 \div 10$ | $\frac{1000}{10}$ | 100 | 3 | 1 | 2 |
| $10\,000 \div 10$ | $\frac{10\,000}{10}$ | 1000 | 4 | 1 | 3 |
| $100 \div 100$ | $\frac{100}{100}$ | 1 | 2 | 2 | 0 |
| $1000 \div 100$ | $\frac{1000}{100}$ | 10 | 3 | 2 | 1 |
| $10\,000 \div 100$ | $\frac{10\,000}{100}$ | 100 | 4 | 2 | 2 |

**c** The total number of zeros in the quotient is the difference in the number of zeros in the original calculation.

**d** **i** 1000 **ii** 1000 **iii** 1 **iv** 10

**7** **a** **i** 10 **ii** 50
**b** **i** 20 **ii** 6 **iii** 90 **iv** 4
**v** 80 **vi** 30 **vii** 50 **viii** 600

**8** **a** **i** 3 **ii** 10 **iii** 30
**b** **i** 30 **ii** 400 **iii** 20 **iv** 50
**v** 20 **vi** 20 **vii** 100 **viii** 20

**9** **a** **i** 20 **ii** 300 **iii** 20 **iv** 20
**b** **i** 19 **ii** 330 **iii** 18 **iv** 22

**10** **a** 25 remainder 17 **b** 41 remainder 1
**c** 34 **d** 213 remainder 7
**e** 38 remainder 24 **f** 752 remainder 10
**g** 1413 **h** 3225

**11** **a** 16 **b** 134 **c** 1473 **d** 694

**12** **a** **i** Divide 165 by 15.
**ii** How many times does 15 go into 165?
**iii** Find the result of $165 \div 15$.
**iv** Find the quotient of 165 divided by 15.
**b** One possible answer is: How many groups of 15 in 165?
**c** 11 sweets in each bag.

**13** 24 minutes each

**14** **a** 4 minutes **b** $4 \times 68 = 272$

**15** 30 laps

**16** **a** 54 **b** 95

**17** **a** 16 bags of rice **b** six left over
**c** One possible answer is: 75 food parcels.

**18** **a** 10 **b** about 11

## 1F Powers and square roots

### 1F Start thinking!

**1** **a** 2 **b** 3 **c** 8

**2** **a** 3 to the power of 5
**b** 3 **c** 5 **d** 5 **e** 243
**f** $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

### Exercise 1F Powers and square roots

**1** **a** base **b** index

**2** **a** $5^9$ **b** $4^7$ **c** $11^3$ **d** $20^5$
**e** $9^8$ **f** $300^4$

**3** **a** 5 to the power of 9 **b** 4 to the power of 7
**c** 11 to the power of 3
**d** 20 to the power of 5
**e** 9 to the power of 8
**f** 300 to the power of 4

**4** **a** $7^3 \times 9^4$ **b** $4^6 \times 2^3$
**c** $3^2 \times 5^6$ **d** $8^4 \times 13^2$
**e** $2^4 \times 6^2 \times 7^2$ **f** $19^2 \times 23^4 \times 31$

**5** **a** $3 \times 3 = 9$
**b** $2 \times 2 \times 2 \times 2 \times 2 = 32$
**c** $7 \times 7 \times 7 = 343$ **d** $5 \times 5 = 25$
**e** $9 \times 9 \times 9 \times 9 = 6561$
**f** $4 \times 4 \times 4 = 64$
**g** $1 \times 1 \times 1 \times 1 \times 1 = 1$
**h** $6 \times 6 \times 6 = 216$

**6**

| Index form | Base | Index or power | Expanded form | Basic numeral |
|---|---|---|---|---|
| $2^4$ | 2 | 4 | $2 \times 2 \times 2 \times 2$ | 16 |
| $6^3$ | 6 | 3 | $6 \times 6 \times 6$ | 216 |
| $7^2$ | 7 | 2 | $7 \times 7$ | 49 |
| $5^3$ | 5 | 3 | $5 \times 5 \times 5$ | 125 |

**7** $3^5$

**8** **a** $2^3, 3^2, 5^4, 4^5$ **b** $1^{50}, 7^6, 50^3, 6^7$
**c** $10^3, 9^4, 6^5, 3^{10}$ **d** $12^2, 8^3, 4^6, 3^8$

**9** **a** $2 \times 2 \times 2 \times 4 \times 4 = 128$
**b** $3 \times 3 \times 3 \times 3 \times 5 \times 5 = 2025$
**c** $8 \times 8 \times 10 \times 10 \times 10 = 64\,000$
**d** $3 \times 3 \times 3 \times 3 \times 3 \times 1 \times 1 \times 1 \times 1 = 243$
**e** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 = 3136$
**f** $10 \times 10 \times 3 \times 3 \times 3 = 2700$
**g** $9 \times 9 \times 9 \times 9 \times 11 \times 11 = 793\,881$
**h** $6 \times 6 \times 3 \times 3 = 324$

**10** **a** 89 **b** 1 000 025 **c** 0
**d** 63 **e** 4 **f** 48
**g** 59 **h** 84

**11** **a**

| Index form | Expanded form | Basic numeral | Number of zeros in the basic numeral |
|---|---|---|---|
| $10^1$ | 10 | 10 | 1 |
| $10^2$ | $10 \times 10$ | 100 | 2 |
| $10^3$ | $10 \times 10 \times 10$ | 1000 | 3 |
| $10^4$ | $10 \times 10 \times 10 \times 10$ | 10 000 | 4 |
| $10^5$ | $10 \times 10 \times 10 \times 10 \times 10$ | 100 000 | 5 |

**b** The index is the same as the number of zeros in the basic numeral.
**c** **i** 100 000 000 **ii** 1 000 000
**iii** 1 000 000 000 000 000 **iv** 10 000 000 000
**d** 1
**e** **i** $10^2$ **ii** $10^4$ **iii** $10^0$ **iv** $10^7$

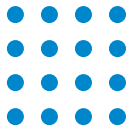
**12** **a** 2 rows and 2 columns



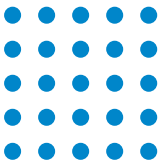
**b** 3 rows and 3 columns



**c** 4 rows and 4 columns



5 rows and 5 columns



**d** because the counters form the shape of a square
**e** 5 square numbers **f** 36, 49, 64, 81, 100

**13** **a**

| Index form | Expanded form | Basic numeral |
|---|---|---|
| $1^2$ | $1 \times 1$ | 1 |
| $2^2$ | $2 \times 2$ | 4 |
| $3^2$ | $3 \times 3$ | 9 |
| $4^2$ | $4 \times 4$ | 16 |
| $5^2$ | $5 \times 5$ | 25 |

**b** They are the same.
**c** By raising the base to a power of 2, the result is a square number.
**d** **i** $9^2 = 81$ **ii** $4^2 = 16$
**iii** $10^2 = 100$ **iv** $7^2 = 49$

**14** **a** 121 **b** 625 **c** 10 000 **d** 1024
**e** 10 **f** 28 **g** 130 **h** 75

**15** **a** 5 **b** 9 **c** 8 **d** 3
**e** 1 **f** 10 **g** 2 **h** 6

**16** **a** 5 **b** $\sqrt{25} = 5$
**c** **i** $\sqrt{16} = 4$ **ii** $\sqrt{81} = 9$ **iii** $\sqrt{4} = 2$
**iv** $\sqrt{100} = 10$

**17** **a** 3 **b** 6 **c** 8 **d** 1
**e** 7 **f** 11 **g** 12 **h** 20

**18** **a** 12 **b** 35 **c** $\sqrt{3721}$ **d** $\sqrt{88\,804}$

**19** Finding the square root of a number is the opposite operation to finding the square of a number.

**20** **a**

| Number of days | Amount of growth | Amount of growth in index form | Number of times larger than original size |
|---|---|---|---|
| 1 | 3 | $3^1$ | 3 |
| 2 | $3 \times 3$ | $3^2$ | 9 |
| 3 | $3 \times 3 \times 3$ | $3^3$ | 27 |
| 4 | $3 \times 3 \times 3 \times 3$ | $3^4$ | 81 |
| 5 | $3 \times 3 \times 3 \times 3 \times 3$ | $3^5$ | 243 |

**b** **i** 729 **ii** 2187 **iii** 4 782 969
**c** **i** 3 days **ii** 5 days **iii** 8 days

**21** **a** 2 **b** $2^1, 2^2$ **c** $2^3$
**d** $2^5$ or 32 times bigger

**22** **a** 64 **b** $4^3$

**23** The boss should pay Taylor \$100 per week, making a total of \$400. If she used the other method she would have to pay Taylor \$5 + \$25 + \$125 + \$625 = \$780.

## 1G Order of operations

### 1G Start thinking!

**2** Uses the first letter of each operation.
**3** 14

### Exercise 1G Order of operations

**1** **a** 11 **b** 1 **c** 21 **d** 59
**e** 19 **f** 8 **g** 15 **h** 60
**i** 4 **j** 37 **k** 67 **l** 4

**2** **a** 16 **b** 6 **c** 24 **d** 3
**e** 12 **f** 86

**4** **a** 11
**b** **i** 19 **ii** 7 **iii** 317 **iv** 10
**v** 53 **vi** 40

**5** **a** 10 **b** 10 **c** They are the same.
**d** No. One example is: $5 + 4 = 9$, $4 + 5 = 9$.
**e** Yes. $8 - 2$ is not the same as $2 - 8$.
**f** No. One example is: $3 \times 6 = 18$, $6 \times 3 = 18$.
**g** Yes. One example is: $10 \div 5 = 2$; $5 \div 10 = \frac{1}{2}$
**h** The operations of addition and multiplication obey the commutative law.

**6** **a** 14 **b** 14 **c** 14
**d** They are the same. Numbers may be added in any order.
**e** 37 and 13 are easier to add first as they give 50. $25 + 37 + 13 = 75$.

**7** **a** 90 **b** 90 **c** 90
**d** They are the same. Numbers may be multiplied in any order.

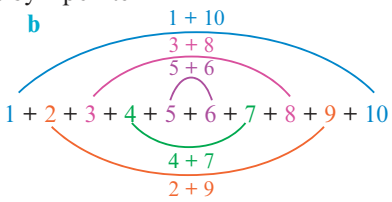
**e** As we can multiply numbers in any order, it is easier to multiply 25 and 4 together first. That is, $25 \times 4 = 100$. Then multiply by 58 to give 5800.

**8** no

**9** **a** 27 **b** 27 **c** They are the same.
**d** Yes, as you get the same answer for each calculation.
**e** It can split one of the numbers up to make multiplication easier.
**f** **i** 490 **ii** 1292 **iii** 4182 **iv** 9222

**10** **a** Sharks: $10 \times 6 + 15 \times 1$,
Visitors: $9 \times 6 + 23 \times 1$
**b** Sharks: 75, Visitors: 77
**c** Visitors by 2 points

**11** **a** 55 **b**




**c** 11 **d** 5 **e** 55

**12** **a** $21 \times 10 = 210$ **b** $41 \times 20 = 820$
**c** $60 \times 20 + 30 = 1230$ **d** $50 \times 10 + 25 = 525$

**14** **a** $8 \times 6 + 4 \times 10 + 7 \times 5$ **b** $123

## 1H Multiples and factors

### 1H Start thinking!

**1** **a**

| Number | 3 × ? |
|---|---|
| 3 | 3 × 1 |
| 6 | 3 × 2 |
| 9 | 3 × 3 |
| 12 | 3 × 4 |
| 15 | 3 × 5 |
| 18 | 3 × 6 |
| 21 | 3 × 7 |
| 24 | 3 × 8 |

**b** 120, multiply 3 by 40
**c** One possible answer is: 30, 36, 54, 99, 246.

**2** **a** One possible answer is: 2 and 18. (Others are: 1 and 36, 3 and 12, 6 and 6.)
**b** yes; 1 and 36, 3 and 12, 6 and 6
**c** 1, 2, 3, 4, 6, 9, 12, 18, 36

**3** A multiple of a number is the answer obtained when that number is multiplied by another whole number. A factor of a number is a number that divides exactly into the given number.

### Exercise 1H Multiples and factors

**1** **a** 2, 4, 6, 8, 10, 12 **b** 9, 18, 27, 36, 45, 54
**c** 5, 10, 15, 20, 25, 30 **d** 4, 8, 12, 16, 20, 24
**e** 8, 16, 24, 32, 40, 48 **f** 7, 14, 21, 28, 35, 42

**2** **a** 10, 20, 30, 40, 50, 60, 70, 80
**b** 6, 12, 18, 24, 30, 36, 42, 48
**c** 9, 18, 27, 36, 45, 54, 63, 72
**d** 5, 10, 15, 20, 25, 30, 35, 40
**e** 11, 22, 33, 44, 55, 66, 77, 88
**f** 30, 60, 90, 120, 150, 180, 210, 240

**3** 24, 32, 40, 48, 56, 64

**4** 56, 63, 70, 77, 84, 91, 98

**5** **a** multiples of 3:
3, 6, 9, (12), 15, 18, 21, (24), 27, 30, …
multiples of 4:
4, 8, (12), 16, 20, (24), 28, 32, 36, 40, …
**b** 12, 24
**c** No. If the lists were extended, more common multiples would be found.
**d** 12, 24, 36, 48, 60, 72

**6** **a** 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
**b** 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
**c** multiples of 4:
4, 8, 12, 16, (20), 24, 28, 32, 36, (40), …
multiples of 5:
5, 10, 15, (20), 25, 30, 35, (40), 45, 50, …
**d** 20, 40, 60, 80

**7** **a** multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95
multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96
**b** 30, 60, 90 **c** 30

**8** **a** 10 **b** 28 **c** 18 **d** 24
**e** 40 **f** 70 **g** 60 **h** 24

**9** **a** 12 **b** 60 **c** 36 **d** 24
**e** 30 **f** 90 **g** 168 **h** 60

**10** **a** $15 = 1 \times 15$ or $3 \times 5$
**b** $18 = 1 \times 18$ or $2 \times 9$ or $3 \times 6$
**c** $32 = 1 \times 32$ or $2 \times 16$ or $4 \times 8$
**d** $28 = 1 \times 28$ or $2 \times 14$ or $4 \times 7$
**e** $49 = 1 \times 49$ or $7 \times 7$
**f** $50 = 1 \times 50$ or $2 \times 25$ or $5 \times 10$

**11** **a** 1, 3, 5, 15 **b** 1, 2, 3, 6, 9, 18
**c** 1, 2, 4, 8, 16, 32 **d** 1, 2, 4, 7, 14, 28
**e** 1, 7, 49 **f** 1, 2, 5, 10, 25, 50

**12** **a** 1, 2, 3, 6 **b** 1, 2, 4, 5, 10, 20
**c** 1, 2, 4, 7, 8, 14, 28, 56
**d** 1, 3, 9, 27, 81 **e** 1, 3, 9, 27
**f** 1, 2, 4, 5, 10, 20, 25, 50, 100

**13** **a** 1, 2 **b** 1, 3, 9, 27
**c** 1, 2, 5, 10, 25, 50
**d** 1, 2, 4 **e** 1, 3 **f** 1, 2, 4, 8
**g** 1, 3 **h** 1, 2, 4 **i** 1, 7

**14** **a** Factors of 24:
(1), (2), (3), (4), (6), 8, (12), 24
Factors of 36:
(1), (2), (3), (4), (6), 9, (12), 18, 36
**b** 1, 2, 3, 4, 6, 12
**c** Yes, as both lists are finite.

**15** **a** 1, 2, 3, 6, 9, 18 **b** 1, 3, 5, 9, 15, 45
**c** Factors of 18: (1), 2, (3), 6, (9), 18
Factors of 45: (1), (3), 5, (9), 15, 45
**d** 1, 3, 9

**16** **a** 1, 2, 3, 4, 6, 12
**b** 1, 2, 3, 5, 6, 10, 15, 30
**c** 1, 2, 3, 6 **d** 6

**17** **a** 8 **b** 3 **c** 6 **d** 5
**e** 4 **f** 15 **g** 7 **h** 50

**18** **a** 4 **b** 5 **c** 8 **d** 3
**e** 6 **f** 10 **g** 2 **h** 3

**19** **a** LCM = 60, HCF = 6
**b** LCM = 70, HCF = 5
**c** LCM = 48, HCF = 4
**d** LCM = 18, HCF = 1

**20** **a** 21 **b** 11 **c** multiples

**21** **a** 5.24 am, 5.36 am, 5.48 am, 6.00 am
**b** **i** 5.36 am **ii** 6.12 am **c** 17th train

**22** **a** One possible answer is: three rows of six pieces.
**b** Six ways are possible: one row of 18 pieces, two rows of 9 pieces, three rows of 6 pieces, six rows of 3 pieces, nine rows of 2 pieces, eighteen rows of 1 piece.
**c** factors

**23** Some suggestions are: four bags of 30 cookies, twelve bags of 10 cookies, six bags of 20 cookies.

**24** **a** 12 **b** 12, 24, 36, 48, 60, 72
**c** No, as the list of common multiples can extend indefinitely.

**25** **a** 9 **b** 1
**c** Every lowest common factor is 1 since 1 is a factor of every number.

**26** **a** 24 minutes
**b** Imogen will have jogged four laps and Olivia, three laps.
**c** Imogen will have jogged 15 laps and Olivia, 11 laps.
**d** three times

**27** **a** 1 **b** 2 **c** 4 **d** 6 **e** 16 **f** 12

## 1I Prime and composite numbers

### 1I Start thinking!

**1** **a**

| Number | Factors |
|---|---|
| 2 | 1, 2 |
| 3 | 1, 3 |
| 5 | 1, 5 |
| 7 | 1, 7 |
| 11 | 1, 11 |
| 13 | 1, 13 |
| 17 | 1, 17 |
| 19 | 1, 19 |

**b** 2

**2** **a, b**

| Number | Factors | Number of factors |
|---|---|---|
| 4 | 1, 2, 4 | 3 |
| 6 | 1, 2, 3, 6 | 4 |
| 8 | 1, 2, 4, 8 | 4 |
| 9 | 1, 3, 9 | 3 |
| 10 | 1, 2, 5, 10 | 4 |
| 12 | 1, 2, 3, 4, 6, 12 | 6 |
| 14 | 1, 2, 7, 14 | 4 |
| 15 | 1, 3, 5, 15 | 4 |
| 16 | 1, 2, 4, 8, 16 | 5 |
| 18 | 1, 2, 3, 6, 9, 18 | 6 |
| 20 | 1, 2, 4, 5, 10, 20 | 6 |

**c** More

**3** Prime numbers have only two factors, 1 and the number itself. Composite numbers have more than two factors.

**4** The number 1 has only one factor so it is not one of the prime numbers (which have two factors) and it is not one of the composite numbers (which have more than two factors).

### Exercise 1I Prime and composite numbers

**1**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ~~1~~ | 2 | 3 | ~~4~~ | 5 | ~~6~~ | 7 | ~~8~~ | ~~9~~ | ~~10~~ |
| 11 | ~~12~~ | 13 | ~~14~~ | ~~15~~ | ~~16~~ | 17 | ~~18~~ | 19 | ~~20~~ |
| ~~21~~ | ~~22~~ | 23 | ~~24~~ | ~~25~~ | ~~26~~ | ~~27~~ | ~~28~~ | 29 | ~~30~~ |
| 31 | ~~32~~ | ~~33~~ | ~~34~~ | ~~35~~ | ~~36~~ | 37 | ~~38~~ | ~~39~~ | ~~40~~ |
| 41 | ~~42~~ | 43 | ~~44~~ | ~~45~~ | ~~46~~ | 47 | ~~48~~ | ~~49~~ | ~~50~~ |
| ~~51~~ | ~~52~~ | 53 | ~~54~~ | ~~55~~ | ~~56~~ | ~~57~~ | ~~58~~ | 59 | ~~60~~ |
| 61 | ~~62~~ | ~~63~~ | ~~64~~ | ~~65~~ | ~~66~~ | 67 | ~~68~~ | ~~69~~ | ~~70~~ |
| 71 | ~~72~~ | 73 | ~~74~~ | ~~75~~ | ~~76~~ | ~~77~~ | ~~78~~ | 79 | ~~80~~ |
| ~~81~~ | ~~82~~ | 83 | ~~84~~ | ~~85~~ | ~~86~~ | ~~87~~ | ~~88~~ | 89 | ~~90~~ |
| ~~91~~ | ~~92~~ | ~~93~~ | ~~94~~ | ~~95~~ | ~~96~~ | 97 | ~~98~~ | ~~99~~ | ~~100~~ |

**2** **a** 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
**b** 12, 14, 15, 16, 18 **c** four; 2, 3, 5, 7
**d** one; 2 **e** 73

**3** **a** 1, 2, 3, 4, 6, 8, 12, 24
**b** 1, 7, 49 **c** 1, 73 **d** 1, 5, 25, 125
**e** 1, 37 **f** 1, 2, 41, 82

**4** 73 and 37 are prime numbers. The rest are composite numbers.

**5** **a** composite; even number
**b** prime; only two factors
**c** prime; only two factors
**d** composite; ends in a five
**e** prime; only two factors
**f** composite; ends in a zero
**g** composite; factors 1, 7, 29, 203
**h** composite; multiple of 3

**6** **a** 7 + 17 = 24 **b** 13 + 23 = 36
**c** 71 + 11 = 82 **d** 61 + 83 = 144

**7** **a** 3 × 7 = 21 **b** 5 × 11 = 55
**c** 2 × 13 = 26 **d** 5 × 23 = 115

**8** 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199

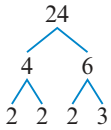
**9** **a** False, as 2 is an even number and it is prime.
**b** False, as 9 is odd but is a composite number.
**c** True, as each number has only two factors.
**d** False, as 9 is a composite number but is odd.
**e** False, as 2 + 3 = 5 is odd.
**f** True, as a composite number has more than two factors and therefore if two prime numbers are multiplied, the result is a composite number.
**g** True, as the number 1 has only one factor.
**h** True, as there are fewer prime numbers than composites.

**10** **a** **i** 2 **ii** 4 = 2 × 2
**b** **i** 2, 5 **ii** 10 = 2 × 5
**c** **i** 2, 3 **ii** 18 = 2 × 3 × 3
**d** **i** 5, 7 **ii** 35 = 5 × 7
**e** **i** 2 **ii** 16 = 2 × 2 × 2 × 2

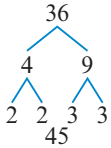
**f** **i** 2, 5 **ii** $20 = 2 \times 2 \times 5$
**g** **i** 3 **ii** $27 = 3 \times 3 \times 3$
**h** **i** 2, 5 **ii** $40 = 2 \times 2 \times 2 \times 5$

**11** **a** $4 = 2^2$ **b** $10 = 2 \times 5$
**c** $18 = 2 \times 3^2$ **d** $35 = 5 \times 7$
**e** $16 = 2^4$ **f** $20 = 2^2 \times 5$
**g** $27 = 3^3$ **h** $40 = 2^3 \times 5$

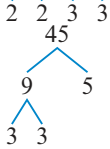
**12** **a** 2 and 3. They are both prime numbers.
**b** 2 and 4; 2 is a prime number, 4 is a composite number
**c** Factors only branch from composite numbers.
**d** 2, 3 **e** $2^4 \times 3$

**13** **a**


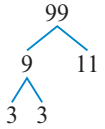
prime factors: 2, 3
$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

**b**


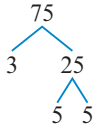
prime factors: 2, 3
$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

**c**


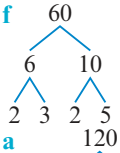
prime factors: 3, 5
$45 = 3 \times 3 \times 5 = 3^2 \times 5$

**d**


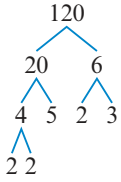
prime factors: 3, 11
$99 = 3 \times 3 \times 11 = 3^2 \times 11$

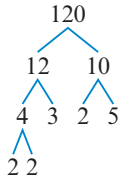
**e**


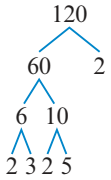
prime factors: 3, 5
$75 = 3 \times 5 \times 5 = 3 \times 5^2$

**f**


prime factors: 2, 3, 5
$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

**14** **a**



**b**



**c**


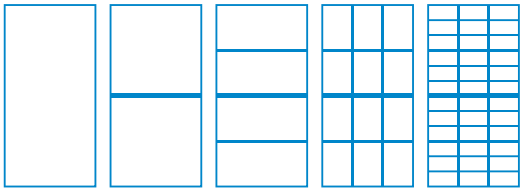
$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$

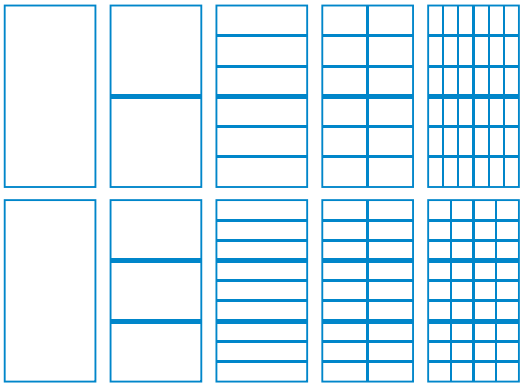
**15** **a** **i** 2, 7 **ii** $28 = 2 \times 2 \times 7 = 2^2 \times 7$
**b** **i** 2, 7 **ii** $56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7$
**c** **i** 2, 11 **ii** $44 = 2 \times 2 \times 11 = 2^2 \times 11$
**d** **i** 2, 5
**ii** $80 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5$
**e** **i** 2, 3, 11
**ii** $132 = 2 \times 2 \times 3 \times 11 = 2^2 \times 3 \times 11$
**f** **i** 2, 13 **ii** $52 = 2 \times 2 \times 13 = 2^2 \times 13$
**g** **i** 2, 5 **ii** $250 = 2 \times 5 \times 5 \times 5 = 2 \times 5^3$
**h** **i** 2, 3, 5
**ii** $90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$
**i** **i** 2, 3
**ii** $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$
**j** **i** 2, 3, 5, 7 **ii** $210 = 2 \times 3 \times 5 \times 7$
**k** **i** 2, 5
**ii** $100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$
**l** **i** 2, 5
**ii** $400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^4 \times 5^2$

**16** **a**


**b** Cut each piece of the pizza into three equal pieces.
**c** No. The pizza could have been cut into three equal pieces first and then each piece could be cut in half to make six pieces.

**17** **a** $36 = 2 \times 2 \times 3 \times 3$. This means the cake can be cut in half to produce two pieces, then each piece is cut in half again, then each of these pieces is cut into equal thirds and then each of these pieces into equal thirds again.

**b**


**c** yes


**18** **a** 5 and 7, 11 and 13, 17 and 19
**b** four: 29 and 31, 41 and 43, 59 and 61, 71 and 73
**c** seven: 101 and 103, 107 and 109, 137 and 139, 149 and 151, 179 and 181, 191 and 193, 197 and 199

## 1 Chapter review

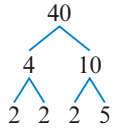
### MULTIPLE-CHOICE

| | | | | |
|---|---|---|---|---|
| **1** B | **2** C | **3** D | **4** A | **5** C |
| **6** D | **7** B | **8** A | **9** B | **10** C |

### SHORT ANSWER

**1** **a** false **b** true **c** true **d** false
**2** **a** 57, 547, 5347, 53 047
**b** 87 056, 87 506, 87 605
**3** **a** 400 **b** 9000 **c** 600 **d** 10 000

4 a 9267 b 402 253
5 a One possible answer is: 355 + 123 = 478.
b One possible answer is: 146 + 332 = 478.
6 a 4183 b 333 578 c 7843 d 41 333
7 a 2945 b 253
8 a 1998 b 2 631 915 c 108 416
d 544 036
9 a 38 900 b 2 472 000 c 680 000
d 6 000 000 e 4800 f 93 100
10 a 81 029 remainder 1 b 53 remainder 2
c 327 d 1521
11 a 700 b 40
12 a $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
b $5 \times 5 \times 5 \times 3 \times 3 = 1125$
13 $10^4$
14 a $7^2 = 49$ b $6^2 = 36$
15 a 5 b 25
16 a 28 b 60 c 31 d 11
17 24, 28, 32, 36, 40, 44
18 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
19 a LCM = 36 b LCM = 90
20 a HCF = 6 b HCF = 3
21 Some possible answers are provided.
a 7, 14, 21 b 6, 8, 10 c 4, 9, 16
22 a composite, as it has more than two factors
b Prime factors are 2 and 5.




c $40 = 2^3 \times 5$

### NAPLAN-STYLE PRACTICE

1 6048 2 603, 623, 630, 632, 662
3 $529 4 63 5 375 6 161
7 1204 minutes or 20 hours 4 minutes
8 181 9 800 × 300 10 55 grams
11 $25 000 12 15 km 13 $4 \times 4 \times 4$
14 10 15 17 16 $4^2 \times 6^2$
17 $3 + 9 \times 5$ 18 54 19 168
20 12 minutes 21 51
22 $2 \times 2 \times 2 \times 3 \times 3$ 23 3

### ANALYSIS

a

| Item | Quantity | Price per item | Total cost per item |
|---|---|---|---|
| Party pie 24 pack | 4 | $4 | $16 |
| Allen's mixed lollies | 3 | $3 | $9 |
| Chips 100 g | 4 | $2 | $8 |
| M&Ms 250 g | 2 | $4 | $8 |
| Freddos 20 pack | 2 | $5 | $10 |
| Cocktail sausages 2 kg pack | 1 | $6 | $6 |
| Frozen pizza | 1 | $10 | $10 |
| Mudcake | 3 | $4 | $12 |

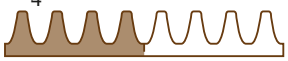
b $79 c $21 d chips
e Cocktail sausages. This is different from part d as he only bought one of these, but four packets of chips.
f 120 lollies
g 11 friends (12 people in total)
h three Freddos each with four left over
i four groups of 3, three groups of 4, two groups of 6
j $12 = 2 \times 2 \times 3$. He can cut the pizza into halves, then halve each piece and then cut each of these pieces into thirds.

## 1 Connect

For feedback on this open-ended task, see your teacher.

# CHAPTER 2 FRACTIONS AND RATIOS

## 2 Are you ready?

1 a $\frac{1}{4}$ b $\frac{5}{8}$
2 
3 a proper fraction b mixed number
c proper fraction d proper fraction
e improper fraction f mixed number
4 C 5 D 6 A
7 a $\frac{2}{11}, \frac{5}{11}, \frac{7}{11}, 1, \frac{13}{11}$ b $\frac{21}{17}, \frac{19}{17}, 1, \frac{8}{17}, \frac{3}{17}, \frac{2}{17}$
8 a $\frac{13}{15}$ b $\frac{17}{23}$ c $\frac{2}{9}$ d $\frac{11}{21}$
9 B
10 a 8 b 4
11 a 6 b 12
12 a 49 b 125 c 16 d 6
13 a B b D

## 2A Understanding fractions

### 2A Start thinking!

1 The numerator in a proper fraction is less than the denominator and the fraction is less than one. The numerator in an improper fraction is greater than the denominator and the fraction is greater than one.
2 Improper fractions are a fraction only. Mixed numbers are a combination of a whole number and a fraction.
3 Both improper fractions and mixed numbers represent a number larger than 1.
4 David has eaten $\frac{7}{8}$ of a pizza
5 proper fraction
6 Xavier has eaten $\frac{11}{8}$ or $1\frac{3}{8}$ of a pizza

### Exercise 2A Understanding fractions

1 a $\frac{5}{8}$ b $\frac{3}{5}$ c $\frac{9}{16}$ d $\frac{9}{16}$
2 a $\frac{3}{8}$ b $\frac{2}{5}$ c $\frac{7}{16}$
3 B. The rectangle is divided into three equal sections and one of the sections is shaded. The other two rectangles are not divided into three equal sections.

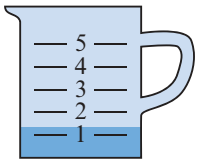
**4** **a** i $\frac{5}{4}$ ii $1\frac{1}{4}$ **b** i $\frac{11}{4}$ ii $2\frac{3}{4}$
**c** i $\frac{4}{3}$ ii $1\frac{1}{3}$ **d** i $\frac{12}{5}$ ii $2\frac{2}{5}$

**5** proper fractions: $\frac{7}{8}, \frac{16}{31}, \frac{13}{22}$
improper fractions: $\frac{5}{2}, \frac{17}{2}, \frac{29}{15}$
mixed numbers: $2\frac{3}{5}, 1\frac{5}{9}, 12\frac{1}{7}$

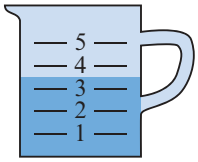
**6** **a** $\frac{11}{9}$ **b** $\frac{19}{6}$ **c** $\frac{53}{9}$ **d** $\frac{28}{11}$
**e** $\frac{54}{5}$ **f** $\frac{35}{4}$ **g** $\frac{17}{7}$ **h** $\frac{37}{3}$
**i** $\frac{37}{8}$ **j** $\frac{53}{10}$ **k** $\frac{85}{12}$ **l** $\frac{34}{15}$

**7** **a** $1\frac{2}{5}$ **b** $8\frac{2}{3}$ **c** $2\frac{7}{9}$ **d** $4\frac{3}{10}$
**e** $9\frac{3}{8}$ **f** $6\frac{10}{13}$ **g** $3\frac{1}{7}$ **h** $4\frac{1}{4}$
**i** $8\frac{1}{6}$ **j** $4\frac{6}{11}$ **k** $13\frac{2}{5}$ **l** $55\frac{1}{2}$

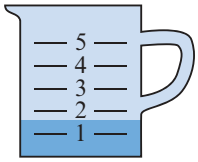
**8** $1\frac{1}{3}$



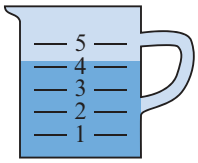

**9** **a** $3\frac{1}{2}$




**b** $1\frac{3}{5}$

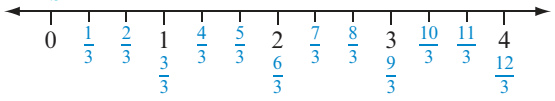



**c** $4\frac{1}{4}$

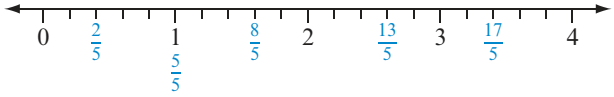



**10** **a** $\frac{18}{7} > \frac{17}{7}$ **b** $\frac{26}{3} < \frac{28}{3}$
**c** $\frac{38}{11} > \frac{28}{11}$ **d** $\frac{49}{5} < \frac{51}{5}$

**11** **a** $\frac{1}{3}$
**b**




**12**




**13** **a** $6\frac{3}{7}$ or $\frac{45}{7}$ **b** $7\frac{6}{7}$ or $\frac{55}{7}$
**c** $8\frac{4}{7}$ or $\frac{60}{7}$ **d** $9\frac{2}{7}$ or $\frac{65}{7}$

**14** **a** $\frac{3}{8}$ **b** $\frac{1}{8}$ **c** $\frac{7}{8}$ **d** $\frac{3}{8}$

**15** **a** $\frac{17}{25}$ **b** $\frac{7}{12}$ **c** $\frac{23}{50}$

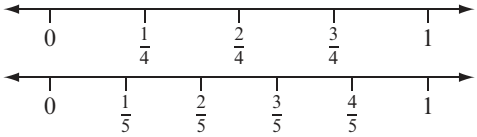
**16** **a** $\frac{17}{60}$ **b** $\frac{31}{60}$ **c** $\frac{119}{60}$ **d** $5\frac{11}{60}$

**17** **a** i $\frac{44}{8}$ ii $5\frac{4}{8}$ (or $5\frac{1}{2}$)
**b** No, because there will only be 40 slices.
**c** nine slices **d** $\frac{1}{9}$ **e** 37 slices

**18** **a** 4 **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **19** $\frac{1}{4}$

**20** **a** i $\frac{1}{30}$ ii $\frac{1}{100}$ iii $\frac{1}{4}$ iv $\frac{1}{10}$ v $\frac{1}{3}$
**b** $\frac{1}{3}$ **c** $\frac{1}{15}$

**21** Line up two number lines, one divided into quarters and the other into fifths and compare the lengths.




## 2B Equivalent fractions

### 2B Start thinking!

**1** The same amount of cake is iced.

**2**

| Cake | Total number of equal pieces | Number of pieces that have been iced | Fraction of the cake that has been iced |
|---|---|---|---|
| A | 5 | 2 | $\frac{2}{5}$ |
| B | 10 | 4 | $\frac{4}{10}$ |
| C | 15 | 6 | $\frac{6}{15}$ |
| D | 20 | 8 | $\frac{8}{20}$ |

**3** Total number of equal pieces: 25, number of pieces that have been iced: 10, fraction of the cake that has been iced: $\frac{10}{25}$.

**4** They each represent the same (or equivalent) amount of cake.

### Exercise 2B Equivalent fractions

**1** **a** $\frac{1}{3}$ **b** $\frac{4}{7}$ **c** $\frac{1}{4}$ **d** $\frac{1}{2}$ **e** $\frac{2}{6}$ **f** $\frac{3}{8}$

**2** **a** $\frac{2}{3}$ **b** $\frac{3}{7}$ **c** $\frac{3}{4}$ **d** $\frac{1}{2}$ **e** $\frac{4}{6}$ **f** $\frac{5}{8}$

**3** Some possible answers are given.
**i** **a** $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}$ **b** $\frac{8}{14}, \frac{12}{21}, \frac{16}{28}$ **c** $\frac{10}{40}, \frac{8}{32}, \frac{2}{8}$
**d** $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ **e** $\frac{1}{3}, \frac{4}{12}, \frac{6}{18}$ **f** $\frac{6}{16}, \frac{12}{32}, \frac{30}{80}$
**ii** **a** $\frac{10}{15}, \frac{8}{12}, \frac{4}{6}$ **b** $\frac{6}{14}, \frac{15}{35}, \frac{9}{21}$ **c** $\frac{9}{12}, \frac{60}{80}, \frac{6}{8}$
**d** $\frac{10}{20}, \frac{20}{40}, \frac{100}{200}$ **e** $\frac{2}{3}, \frac{8}{12}, \frac{12}{18}$ **f** $\frac{10}{16}, \frac{20}{32}, \frac{50}{80}$

**4** **a** $\frac{4}{9} = \frac{16}{36}$ **b** $\frac{24}{8} = \frac{12}{4}$ **c** $2\frac{5}{11} = \frac{81}{33}$
**d** $\frac{42}{70} = \frac{6}{10}$ **e** $\frac{144}{48} = \frac{36}{12}$ **f** $3\frac{2}{8} = \frac{234}{72}$
**g** $\frac{2}{3} = \frac{6}{9} = \frac{60}{90}$ **h** $\frac{5}{6} = \frac{15}{18} = \frac{20}{24}$ **i** $\frac{2}{5} = \frac{10}{25} = \frac{6}{15}$
**j** $5\frac{1}{13} = \frac{198}{39} = \frac{330}{65}$ **k** $\frac{1}{10} = \frac{2}{20} = \frac{3}{30} = \frac{8}{80}$
**l** $\frac{8}{7} = \frac{24}{21} = \frac{40}{35} = \frac{88}{77}$ **m** $3\frac{3}{4} = \frac{90}{24} = \frac{45}{12} = \frac{270}{72}$
**n** $\frac{1}{12} = \frac{6}{72} = \frac{8}{96} = \frac{12}{144} = \frac{15}{180} = \frac{20}{240}$
**o** $\frac{2}{3} = \frac{4}{6} = \frac{10}{15} = \frac{20}{30} = \frac{50}{75} = \frac{400}{600}$

**5** **a** $\frac{5}{8}$ **b** $\frac{1}{3}$ **c** $\frac{7}{9}$
**d** $\frac{5}{8}$ **e** $\frac{9}{8}$ or $1\frac{1}{8}$ **f** $\frac{58}{47}$ or $1\frac{11}{47}$
**g** $2\frac{2}{5}$ **h** $7\frac{4}{9}$ **i** $4\frac{7}{8}$
**j** $5\frac{11}{12}$ **k** $6\frac{1}{5}$ **l** $3\frac{8}{27}$

**6** $\frac{11}{17}$ **7** $\frac{1}{6}$

**8** One possible answer is: $\frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \frac{20}{35}$.

**9** One possible answer is: $\frac{10}{18}, \frac{15}{27}, \frac{20}{36}$.

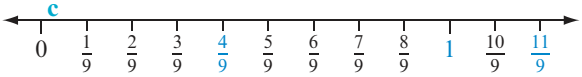
**10** One possible answer is: $\frac{14}{6}, \frac{21}{9}, \frac{28}{12}$.

**11** One possible answer is: $1\frac{6}{10}, 1\frac{9}{15}, 1\frac{12}{20}$.

**12 a** One possible answer is: $\frac{4}{9}$.

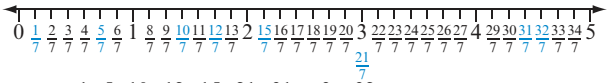
**b** One possible answer is: $\frac{11}{9}$.

**c**




When looking at the number line, fractions smaller than 1 will be positioned to the left of 1. Fractions that are larger than 1 will be positioned to the right of 1.

**13 a**

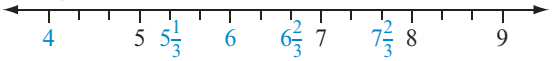



**b** $\frac{1}{7}, \frac{5}{7}, \frac{10}{7}, \frac{12}{7}, \frac{15}{7}, \frac{21}{7}, \frac{31}{7}, (4\frac{3}{7}), \frac{32}{7}$

The numerators are in ascending order.

**14** Compare the numerators and place them in ascending order (from smallest to largest).

**15 a, b**




**c** Largest fraction is furthest right.

**d** Smallest fraction is furthest left.

**16 a** > **b** < **c** > **d** <

**e** < **f** > **g** < **h** >

**17** $\frac{1}{3}, \frac{6}{18}, \frac{16}{48}, \frac{18}{54}, \frac{20}{60}, \frac{52}{156}$

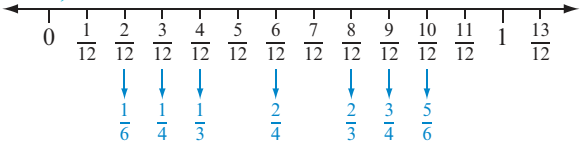
**18 a** $\frac{3}{6}$ **b** $\frac{12}{16}$ **c** $\frac{45}{7}$

**d** $\frac{7}{8}$ **e** $1\frac{11}{20}$ **f** $\frac{180}{14}$

**g** $3\frac{7}{9}$ **h** $\frac{17}{4}$ **i** $\frac{37}{6}$

**19 a** $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{2}{4}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

**b, c**




**20 a** $\frac{1}{3}, \frac{2}{5}, \frac{4}{3}, \frac{7}{5}, \frac{8}{3}, \frac{16}{5}$ **b** $\frac{2}{6}, \frac{5}{8}, \frac{9}{8}, \frac{7}{6}, \frac{5}{4}, \frac{11}{4}$

**c** $\frac{2}{9}, \frac{5}{6}, \frac{11}{9}, \frac{4}{3}, \frac{9}{6}, \frac{7}{3}$ **d** $\frac{9}{20}, \frac{8}{15}, \frac{11}{10}, \frac{5}{4}, \frac{10}{6}, \frac{6}{3}$

**21 a** $\frac{5}{2}, \frac{12}{5}, \frac{10}{6}, \frac{8}{5}, \frac{3}{2}, \frac{7}{6}$ **b** $\frac{11}{12}, \frac{5}{6}, \frac{7}{12}, \frac{3}{8}, \frac{2}{6}, \frac{1}{8}$

**c** $\frac{3}{2}, \frac{10}{7}, \frac{9}{8}, \frac{7}{8}, \frac{6}{7}, \frac{3}{4}$ **d** $\frac{11}{7}, \frac{13}{9}, \frac{4}{3}, \frac{5}{7}, \frac{1}{3}, \frac{2}{9}$

**22** One possible answer is given.

**a** $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}$ **b** $3\frac{4}{5}, 2\frac{6}{7}, 3\frac{1}{4}$ **c** $\frac{11}{35}, \frac{12}{35}, \frac{13}{35}$

**d** $\frac{6}{9}, \frac{8}{9}, \frac{10}{9}$ **e** $\frac{56}{22}, \frac{60}{22}, \frac{70}{22}$ **f** $\frac{13}{36}, \frac{15}{36}, \frac{17}{36}$

**23** second test

**24** Melanie; $\frac{21}{35}$ compared to $\frac{20}{35}$

**25** One possible answer is: 3, 6, 4.

**26** One possible answer is: 13, 4, 2.

**27 a** Each horizontal strip is equal to one unit.

**b** $\frac{1}{2}$ **c** $\frac{1}{3}$

**d** Each segment of a particular horizontal strip is equal.

**e** $\frac{2}{8}, \frac{3}{12}, \frac{5}{20}, \frac{6}{24}$

**f** One possible answer is: $\frac{1}{2}, \frac{2}{3}, \frac{3}{7}, \frac{4}{9}$.

## 2C Adding and subtracting fractions

### 2C Start thinking!

**1** The denominators are the same.

**2** The sum of the numerators of the fractions that are being added equals the numerator of the answer.

**3**

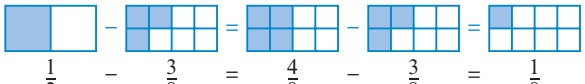


$\frac{3}{8}$

**4** Keep the denominator and add or subtract the numerators.

**5** Convert them to improper fractions. Check that they are written with a common denominator and add the numerators.

**6**



$\frac{1}{2} - \frac{3}{8} = \frac{4}{8} - \frac{3}{8} = \frac{1}{8}$

**7** Convert the fractions so that they have a common denominator.

### Exercise 2C Adding and subtracting fractions

**1 a** $\frac{3}{5}$ **b** $\frac{6}{7}$ **c** $\frac{2}{3}$ **d** $\frac{11}{18}$

**e** $\frac{19}{37}$ **f** $1\frac{2}{23}$ **g** $2\frac{3}{11}$ **h** $\frac{11}{13}$

**i** $\frac{11}{25}$ **j** $\frac{6}{7}$ **k** $\frac{17}{21}$ **l** $1\frac{1}{13}$

**m** $\frac{2}{3}$ **n** $\frac{1}{27}$ **o** $\frac{16}{33}$ **p** $1\frac{8}{17}$

**q** $\frac{1}{3}$ **r** $\frac{3}{8}$

**2 a** $5\frac{3}{7}$ **b** $\frac{2}{3}$ **c** $7\frac{3}{5}$ **d** $3\frac{2}{3}$

**e** $4\frac{2}{11}$ **f** $3\frac{2}{5}$ **g** $7\frac{4}{9}$ **h** $\frac{5}{6}$

**i** $1\frac{2}{3}$ **j** $9\frac{2}{5}$ **k** $\frac{7}{12}$ **l** $3\frac{7}{8}$

**3 a** $1\frac{1}{6}$ **b** $\frac{7}{20}$ **c** $1\frac{5}{24}$ **d** $\frac{1}{18}$

**e** $1\frac{5}{12}$ **f** $\frac{2}{15}$ **g** $1\frac{1}{14}$ **h** $\frac{7}{30}$

**i** $\frac{11}{21}$ **j** $1\frac{11}{30}$ **k** $1\frac{1}{66}$ **l** $\frac{1}{40}$

**m** $\frac{15}{28}$ **n** $1\frac{3}{35}$ **o** $\frac{29}{30}$ **p** $\frac{5}{24}$

**q** $\frac{29}{40}$ **r** $\frac{45}{92}$

**4 a** 9 **b** 32 **c** 38 **d** 15

**e** 12 **f** 30

**5 a** $1\frac{5}{9}$ **b** $3\frac{5}{32}$ **c** $\frac{23}{38}$ **d** $\frac{7}{15}$

**e** $3\frac{5}{12}$ **f** $1\frac{17}{30}$

**6 a** Dean **b** $\frac{1}{28}$ of an hour

**7 a** $2\frac{1}{3}$ **b** $2\frac{1}{20}$ **c** $2\frac{1}{8}$ **d** $\frac{17}{24}$

**e** $5\frac{17}{30}$ **f** $\frac{19}{40}$

**8 a** $4\frac{1}{6}$ **b** $2\frac{19}{21}$ **c** $5\frac{13}{20}$ **d** $1\frac{13}{15}$

**e** $9\frac{1}{10}$ **f** $6\frac{1}{12}$ **g** $2\frac{8}{15}$ **h** $3\frac{5}{12}$

**i** $5\frac{5}{21}$ **j** $7\frac{1}{3}$ **k** $1\frac{3}{8}$ **l** $1\frac{7}{8}$

**9** **a** $5\frac{13}{42}$ **b** $8\frac{19}{24}$ **c** $6\frac{16}{21}$ **d** $1\frac{67}{70}$
**e** $5\frac{31}{45}$ **f** $4\frac{1}{4}$ **g** $7\frac{15}{44}$ **h** $\frac{2}{7}$
**i** $4\frac{97}{150}$

**10** $10\frac{7}{15}$ minutes

**11** **a–c**

| Day | Number of hours and minutes worked | Number of minutes worked | Number of hours worked |
|---|---|---|---|
| Monday | 3 h 45 min | 225 | $3\frac{3}{4}$ |
| Tuesday | 1 h 50 min | 110 | $1\frac{5}{6}$ |
| Wednesday | 5 h 30 min | 330 | $5\frac{1}{2}$ |
| Thursday | 7 h | 420 | 7 |
| Friday | 10 h 30 min | 630 | $10\frac{1}{2}$ |
| Saturday | 8 h 10 min | 490 | $8\frac{1}{6}$ |

**d** $36\frac{3}{4}$ hours

**12** **a** 4 **b** $\frac{5}{8}$

**13** **a** $\frac{103}{120}$ **b** $\frac{17}{120}$ will go towards savings.

**14** $14\frac{7}{10}$ cm **15** $\frac{2}{5}$

**16** **a** $5\frac{4}{15}$ km **b** $1\frac{1}{15}$ km

**17** **a** **i** $\frac{1}{4}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{16}$ **iv** $\frac{1}{32}$

The fraction is multiplied by $\frac{1}{2}$ each time and the answer is the final fraction of the problem.

**b** $\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\frac{1}{16}-\frac{1}{32}-\frac{1}{64}=\frac{1}{64}$ and
$\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\frac{1}{16}-\frac{1}{32}-\frac{1}{64}-\frac{1}{128}=\frac{1}{128}$

**18** **a** $\frac{11}{7}, \frac{14}{7}$ (or 2), $\frac{17}{7}, \frac{20}{7}$; increases by $\frac{3}{7}$ each time.

**b** $14\frac{1}{5}, 15\frac{3}{5}$, 17, $18\frac{2}{5}$; increases by $1\frac{2}{5}$ each time.

**c** $\frac{10}{3}, \frac{7}{3}, \frac{4}{3}, \frac{1}{3}$; decreases by $\frac{3}{3}$ (or 1) each time.

**d** $6\frac{2}{15}, 3\frac{13}{15}, 1\frac{9}{15}$ (or $1\frac{3}{5}$); decreases by $2\frac{4}{15}$ each time.

**19** **a** $\frac{5}{10}$ or $\frac{1}{2}$ **b** $\frac{7}{10}$ **c** $\frac{9}{22}$ **d** $\frac{9}{14}$
**e** $\frac{5}{24}$ **f** $\frac{8}{15}$ **g** $\frac{7}{24}$ **h** $\frac{9}{80}$

**20** One possible answer is given.
**a** $\frac{3}{6}$ and $\frac{5}{10}$ **b** $\frac{12}{24}$ and $\frac{4}{32}$ **c** $\frac{4}{15}$ and $\frac{2}{30}$

**21** One possible answer is: $\frac{1}{6}$ and $\frac{2}{3}$.

**22** **a** $\frac{1}{2}-(\frac{5}{8}-\frac{2}{5})=\frac{11}{40}$ **b** $\frac{19}{28}-(\frac{3}{7}+\frac{3}{14})=\frac{1}{28}$

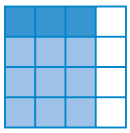
## 2D Multiplying fractions

### 2D Start thinking!

**1** **a** Two of the eight sections have been shaded.
$\frac{2}{8}=\frac{1}{4}$

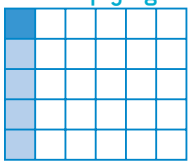
**2** **a** $\frac{1}{8}$ **b** $\frac{1}{8}$ **c** $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$

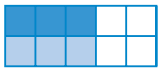
**3** **a, b** 
**c** $\frac{3}{8}$

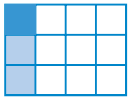
**4** **a, b** 
**c** $\frac{3}{16}$

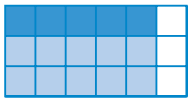
**5** To multiply fractions, multiply the numerators together and multiply the denominators together.

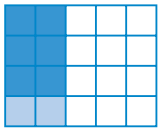
### Exercise 2D Multiplying fractions

**1** **a–c** 
**d** $\frac{1}{30}$ **e** $\frac{1}{30}$ **f** $\frac{10}{30}$ or $\frac{1}{3}$
**g** a multiplication symbol

**2** **a** $\frac{1}{2}\times\frac{3}{5}=\frac{3}{10}$ 

**b** $\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$ 

**c** $\frac{1}{3}\times\frac{5}{6}=\frac{5}{18}$ 

**d** $\frac{3}{4}\times\frac{2}{5}=\frac{6}{20}$ 

**3** **a** $\frac{1}{10}$ **b** $\frac{2}{21}$ **c** $\frac{6}{55}$ **d** $\frac{4}{27}$
**e** $\frac{3}{16}$ **f** $\frac{66}{91}$ **g** $\frac{8}{25}$ **h** $\frac{7}{45}$
**i** $\frac{10}{81}$ **j** $\frac{20}{99}$ **k** $\frac{21}{80}$ **l** $\frac{40}{117}$

**5** **a** $\frac{2}{9}$ **b** $\frac{4}{11}$ **c** $\frac{6}{13}$ **d** $\frac{1}{34}$
**e** $\frac{4}{15}$ **f** $\frac{4}{9}$ **g** $\frac{9}{16}$ **h** $\frac{12}{49}$
**i** $\frac{4}{13}$ **j** $\frac{4}{15}$ **k** $1\frac{1}{3}$ **l** $\frac{2}{5}$
**m** $\frac{5}{17}$ **n** $1\frac{1}{20}$ **o** $\frac{1}{15}$

**6** **a** $\frac{9}{11}$ **b** $4\frac{5}{7}$ **c** $1\frac{1}{20}$ **d** 14
**e** $2\frac{8}{9}$ **f** $13\frac{3}{5}$ **g** $18\frac{1}{5}$ **h** $13\frac{3}{4}$

**7** **a** 9 **b** 30 **c** 32 **d** 40
**e** 21 **f** 76 **g** $3\frac{1}{5}$ **h** $3\frac{1}{5}$
**i** $\frac{3}{8}$ **j** $\frac{5}{6}$ **k** 40 **l** $\frac{4}{9}$

**8** smaller; larger

**9** **a** $\frac{6}{17}$ **b** $\frac{5}{26}$ **c** $35\frac{1}{5}$ **d** $10\frac{5}{6}$
**e** $36\frac{4}{5}$ **f** $63\frac{63}{80}$ **g** $58\frac{1}{2}$ **h** 45

**10** **a** Count the number of red vespas and divide by the total number of vespas.
**b** 12 **c** 3
**d** $\frac{\text{number of red vespas}}{\text{total number of vespas}}=\frac{3}{12}=\frac{1}{4}$
**e** $\frac{1}{4}$ of 12 equals 3
**f** $\frac{1}{4}\times 12=3$, multiplication

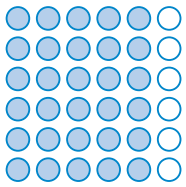
**11** **a** $\frac{1}{20}$ **b** $\frac{9}{28}$ **c** $\frac{14}{99}$
**d** $1\frac{1}{44}$ **e** 66 **f** $18\frac{1}{3}$

**12** **a** **i** $2\frac{1}{2}$ hours **ii** $\frac{5}{2}$ **b** $12\frac{1}{2}$ hours

**13** **a** $8\frac{1}{4}$

**b** $6\frac{12}{72}$ is incorrect; the whole numbers were multiplied rather than converting the mixed numbers to improper fractions. $6\frac{12}{17}$ is incorrect; the denominators were added.

**14** **a** 45 minutes **b** 8 months **c** \$64
**d** 24 weeks **e** 48 seconds **f** 75 L
**g** 6 days **h** \$450 **i** $4\frac{1}{3}$ hours
**j** $15\frac{3}{4}$ km **k** 100 months **l** 160 cents

**15** **a**  **b** 24 **c** 3

**16** \$3792 **17** $2\frac{1}{16}$ km

**18** **a** Boris: 48 minutes, Roger: 168 minutes (or 2 hours 48 minutes)

**b** 2 hours

**19** One possible answer is: $\frac{3}{14}$ and $\frac{10}{3}$.

**20** **a** $\frac{32}{35}$ **b** $1\frac{11}{14}$ **c** $28\frac{13}{42}$

## 2E Dividing fractions

### 2E Start thinking!

**1** **a** 4 **b** 8 **c** 12

**2** Each answer is obtained by multiplying the whole number by the denominator, 4.

**3** 5, 10, 15. Each answer is obtained by multiplying the whole number by the denominator; that is, 5.

**4** **a** 4 **b** 8 **c** 12 **d** 5 **e** 10 **f** 15

**5** **a** multiplying **b** multiplying, 5

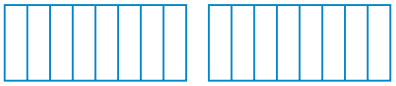
### Exercise 2E Dividing fractions

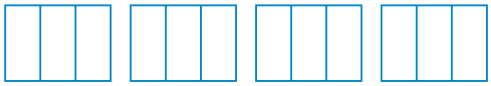
**1** **a** How many lots of $\frac{1}{8}$ are there in 2?

**b** How many lots of $\frac{1}{3}$ are there in 4?

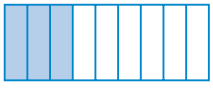
**c** How many lots of $\frac{1}{12}$ are there in $\frac{1}{2}$?

**d** How many lots of $\frac{1}{9}$ are there in $\frac{1}{3}$?

**2** **a** 16 

**b** 12 

**c** 6 

**d** 3 

**4** **a** 16 **b** 12 **c** 6 **d** 3

**5** **a** $\frac{9}{4}$ **b** $\frac{3}{11}$ **c** $\frac{5}{2}$ **d** $\frac{4}{27}$
**e** 100 **f** $\frac{1}{8}$ **g** $\frac{1}{34}$ **h** $\frac{31}{65}$

**6** **a** $\frac{5}{16}$ **b** $2\frac{1}{2}$ **c** $\frac{11}{12}$ **d** $\frac{17}{36}$
**e** $\frac{5}{8}$ **f** $2\frac{1}{4}$ **g** $\frac{3}{4}$ **h** $1\frac{1}{8}$
**i** $1\frac{1}{5}$ **j** $\frac{1}{6}$ **k** $\frac{14}{25}$ **l** $2\frac{2}{3}$
**m** $3\frac{6}{7}$ **n** $\frac{4}{9}$ **o** $\frac{5}{6}$ **p** $\frac{3}{52}$
**q** $7\frac{1}{3}$ **r** $\frac{12}{260}$ or $\frac{3}{65}$

**7** **a** 42 **b** 12 **c** $13\frac{1}{2}$ **d** $6\frac{1}{2}$
**e** $\frac{1}{27}$ **f** $\frac{1}{12}$ **g** $\frac{1}{4}$ **h** $\frac{4}{9}$
**i** $2\frac{6}{7}$

**8** **a** $4\frac{3}{8}$ **b** $2\frac{9}{56}$ **c** $5\frac{1}{15}$ **d** $2\frac{34}{55}$
**e** $1\frac{4}{5}$ **f** $12\frac{1}{2}$ **g** $1\frac{1}{4}$ **h** 2
**i** $3\frac{1}{3}$ **j** $\frac{7}{15}$ **k** $\frac{10}{11}$ **l** $\frac{2}{5}$

**9** **a** $1\frac{2}{33}$ **b** $2\frac{11}{27}$ **c** $1\frac{3}{22}$ **d** $\frac{3}{28}$
**e** $2\frac{2}{5}$ **f** $\frac{1}{15}$ **g** $1\frac{1}{2}$ **h** $\frac{63}{86}$
**i** $\frac{13}{15}$

**10** Larger number, since we are dividing by a value that is less than 1.

**11** **a** $2\frac{1}{4}$ cups dates, 3 tspns bicarb soda, 180 g butter, 2 cups sugar, 6 eggs, 3 cups SR flour

**b** $1\frac{1}{8}$ cups dates, $1\frac{1}{2}$ tspns bicarb soda, 90 g butter, 1 cup sugar, 3 eggs, $1\frac{1}{2}$ cups SR flour

**c** **i** $\frac{3}{8}$ cup dates, $\frac{1}{2}$ tspns bicarb soda, 30 g butter, $\frac{1}{3}$ cup sugar, 1 egg, $\frac{1}{2}$ cup SR flour

**ii** $\frac{9}{16}$ cup dates, $\frac{3}{4}$ tspns bicarb soda, 45 g butter, $\frac{1}{2}$ cup sugar, $1\frac{1}{2}$ eggs, $\frac{3}{4}$ cup SR flour

**12** 6 **13** 8 **14** 20 **15** $\frac{3}{8}$ **16** 72

**17** **a** 75

**b** No, not able to maintain same pace and will need to include rest/meal breaks.

**18** **a** 11 bags **b** $\frac{1}{3}$ kg

**19** Some possible answers are: $\frac{1}{7} \div \frac{1}{8}$, $\frac{1}{5} \div \frac{2}{21}$, $\frac{4}{5} \div \frac{2}{3}$.

**20** Some possible answers are: $\frac{1}{5} \div \frac{1}{15}$, $\frac{1}{7} \div \frac{1}{28}$, $\frac{1}{9} \div \frac{1}{45}$.

**21** Some possible answers are:
$1\frac{2}{7} \div 3\frac{1}{4}$, $2\frac{1}{6} \div 4\frac{2}{5}$, $1\frac{1}{4} \div 2\frac{2}{3}$.

**22** Some possible answers are: $\frac{5}{3} \div \frac{8}{3}$, $\frac{1}{4} \div \frac{2}{5}$, $\frac{3}{4} \div \frac{6}{5}$.

**23** **a** $\frac{2}{3}$

**b** Misread question and calculated $4\frac{1}{14} \div 2\frac{5}{7}$ to obtain $\frac{1}{6}$. Converted mixed numbers to improper fractions, replaced the division sign with a multiplication sign but did not turn the second fraction upside down to obtain $\frac{57}{14} \times \frac{19}{7} = \frac{1083}{98}$.

**24** **a** $2\frac{1}{8}$ **b** $7\frac{1}{2}$ **c** $1\frac{1}{7}$ **d** $\frac{1}{9}$ **e** $\frac{32}{45}$ **f** $\frac{2}{3}$
**g** 3 **h** $1\frac{1}{3}$

**25** **a** $2\frac{11}{12}$ **b** $1\frac{17}{25}$ **c** $3\frac{1}{2}$ **d** $\frac{9}{41}$ **e** 3 **f** 1

## 2F Powers and square roots of fractions

### 2F Start thinking!

**1** **a** 4 **b** 25 **c** 81 **d** 100
Multiplied each number by itself.

**2** **a** $\frac{1}{4}$ **b** $\frac{1}{25}$ **c** $\frac{1}{81}$ **d** $\frac{1}{100}$

**3** **a** $\frac{1}{4}$ **b** $\frac{1}{25}$ **c** $\frac{1}{81}$ **d** $\frac{1}{100}$

**4** **a** $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ or $\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$

**b** $\left(\frac{1}{5}\right)^2 = \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$ or $\left(\frac{1}{5}\right)^2 = \frac{1^2}{5^2} = \frac{1}{25}$

**5** **a** 2 **b** 5 **c** 9 **d** 10
To calculate a square root, find the number that, when squared, gives the number under the square root sign.

**6** **a** $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{1}{9}$ **d** $\frac{1}{10}$

**7** **a** $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$ **b** $\sqrt{\frac{1}{25}} = \frac{\sqrt{1}}{\sqrt{25}} = \frac{1}{5}$

### Exercise 2F Powers and square roots of fractions

**1** **a** $\frac{4}{25}$ **b** $\frac{9}{64}$ **c** $\frac{81}{49}$ **d** $\frac{100}{9}$
**e** $\frac{16}{25}$ **f** $\frac{121}{36}$ **g** $\frac{4}{121}$ **h** $\frac{25}{49}$

**2** **a** $6\frac{19}{25}$ **b** $39\frac{1}{6}$ **c** $2\frac{46}{49}$ **d** $12\frac{1}{4}$
**e** $7\frac{1}{9}$ **f** $1\frac{57}{64}$ **g** $2\frac{2}{49}$ **h** $8\frac{8}{49}$

**3** **a** $\frac{2}{9}$ **b** $\frac{5}{8}$ **c** $\frac{3}{11}$ **d** $\frac{7}{8}$
**e** $\frac{8}{12}$ or $\frac{2}{3}$ **f** $\frac{3}{9}$ or $\frac{1}{3}$ **g** $\frac{4}{5}$ **h** $\frac{6}{10}$ or $\frac{3}{5}$

**4** **a** $2\frac{2}{5}$ **b** 2 **c** $2\frac{1}{3}$ **d** $2\frac{2}{3}$
**e** $1\frac{4}{5}$ **f** $1\frac{5}{6}$ **g** $2\frac{3}{4}$ **h** $1\frac{1}{9}$

**5** **a** $\frac{1}{2}$ **b** $\frac{5}{6}$ **c** $\frac{4}{5}$ **d** $\frac{3}{4}$
**e** $\frac{2}{3}$ **f** $2\frac{2}{5}$ **g** $\frac{2}{3}$ **h** 4

**6** Simplifying the fraction first made it possible to find the square roots of the numerators and denominators.

**7** **a** $\frac{29}{25}$ or $1\frac{4}{5}$ **b** $\frac{11}{6}$ or $1\frac{5}{6}$ **c** $\frac{22}{45}$
**d** $\frac{83}{36}$ or $2\frac{11}{36}$ **e** $\frac{13}{64}$ **f** $\frac{51}{20}$ or $2\frac{11}{20}$

**8** **a** $\frac{1}{7}$ **b** $\frac{15}{8}$ or $1\frac{7}{8}$ **c** $\frac{91}{20}$ or $4\frac{11}{20}$
**d** $\frac{11}{20}$ **e** $\frac{1}{10}$ **f** $\frac{1}{4}$

**9** **a** 8 **b** 64 **c** 343 **d** $\frac{1}{8}$
**e** $\frac{1}{64}$ **f** $\frac{8}{343}$ **g** $\frac{27}{1331}$ **h** $\frac{64}{125}$

**10** The power indicates how many lots of the fraction to multiply together. If the power is 4, then multiply four lots of the fraction together. If the power is 5, then multiply five lots of the fraction together and so on. Alternatively, can raise the numerator to the given power and then raise the denominator to that power.

**11** **a** $\frac{27}{64}$ **b** $\frac{8}{125}$ **c** $\frac{64}{27}$ or $2\frac{10}{27}$
**d** $\frac{729}{343}$ or $2\frac{43}{343}$ **e** $11\frac{25}{64}$ **f** $42\frac{7}{8}$
**g** $181\frac{26}{27}$ **h** $18\frac{45}{512}$

**12** **a** $\frac{1}{16}$ **b** $\frac{16}{81}$ **c** $\frac{1}{1024}$
**d** $\frac{32}{3125}$ **e** $\frac{1}{729}$ **f** $\frac{729}{64}$ or $11\frac{25}{64}$
**g** $\frac{81}{256}$ **h** $\frac{1}{100\,000\,000}$

**13** **a** $\frac{14\,641}{625}$ **b** $\frac{10\,000}{81}$ **c** $\frac{3125}{32}$
**d** $\frac{3125}{243}$ **e** $161\frac{280}{729}$ **f** $1178\frac{1721}{4096}$
**g** $3\frac{526}{625}$ **h** $1\frac{2465}{4096}$

**14** **a** $\left(\frac{3}{7}\right)^4, \left(\frac{3}{7}\right)^3, \left(\frac{3}{7}\right)^2, \frac{3}{7}$
**b** The fraction raised to the highest power is the smallest fraction because the denominator is larger than the numerator.

**15** **a** $\frac{6}{5}, \left(\frac{6}{5}\right)^2, \left(\frac{6}{5}\right)^3, \left(\frac{6}{5}\right)^4$
**b** The fraction raised to the highest power is the largest fraction because the numerator is larger than the denominator.

**16** Proper fractions raised to powers of 2 or more will be smaller than the original fraction. Improper fractions raised to powers of 2 or more will be larger than the original fraction.

**17** $\frac{9}{100}$ m$^2$

**18** **a** **i** $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ **ii** $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$
**iii** $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$ **iv** $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$
**b** **i** $\frac{1}{4}$ **ii** $\frac{1}{9}$ **iii** $\frac{4}{9}$ **iv** $\frac{9}{16}$
**c** The answers are the same.

**19** **a** No, since both the numerator and denominator must be squared.
**b** No, since both the numerator and denominator must be squared.
**c** Yes, since both the numerator and denominator are squared.
**d** No, since the sum within the pair of brackets must first be calculated and then squared. Remember to use the correct order of operations.

**20** Kate is correct because the mixed number must first be converted to an improper fraction and then squared.

**21** Both the process for finding the solution and the answer are incorrect because the mixed number must first be converted to an improper fraction and then the square root found.

**22** **a** Find the square root of the area.
**b** $1\frac{2}{3}$ km **c** $6\frac{2}{3}$ km **d** \$36 000

**23** **a** $\frac{59}{9}$ or $6\frac{5}{9}$ **b** $\frac{208}{343}$ **c** $\frac{29}{18}$ or $1\frac{11}{18}$
**d** $\frac{53}{9}$ or $5\frac{8}{9}$ **e** $\frac{81}{50}$ or $1\frac{31}{50}$ **f** $\frac{7}{6}$ or $1\frac{1}{6}$

## 2G Understanding ratios

### 2G Start thinking!

**1** **a** 7, 10 **b** 15, 2

**2** Total number of people is same in each part of question 1.

3 Some possible answers include: the number of people wearing ties compared to the number of people not wearing ties, the number of people with dark-coloured hair compared to the number of people with light-coloured hair.
4 a 7:10 5 15:2 6 1:4
7 Darker shade of pink compared to the shade produced in question 6.
8 3:2 would produce a darker shade as there is more red paint.
2:3 would produce a slightly lighter shade as there is less red paint.
9 The order of each amount (part) of paint is important because the first number relates to the red paint and the second relates to the white, and you need the right proportion of each to produce the correct shade.

### Exercise 2G Understanding ratios

1 a 12:5 b 4:43 c 6:5:3 d 15:8
2 a i 4:5 ii 4:9 b i 3:5 ii 3:8
c i 7:8 ii 7:15 d i 5:8 ii 5:13
3 a 132:207 b 16:35 c 18:11
d 85:121 e 12:39 f 25:17
4 a 83:67 b 5:1 c 247:209
d 3:4:9 e 28:23 f 1:4
5 7:1
6 a 5:1 b 4:2:5 c 13:5:82
d 1:1 e 3:5
8 a i 3:2 ii 1:3 iii 5:1
b Ratios cannot be written for amounts in different units.
c Amounts need to be in same unit so need to know how many teaspoons make up a tablespoon, or how many mL was in each.
9 a 1:3 b 3:7 c 2:1:4
10 a Masses are in different units.
b Convert one of the values so that the measurements are in the same unit.
c The bigger unit (tonnes) was converted to the smaller unit (kilograms).
d 4000:67
11 a Yes; both quantities are of same kind (distance).
b No; quantities are not of same kind. First is a capacity in mL and other is a mass in kg.
c Yes; both quantities are of same kind (amount of money) although they are shown in different units.
d No; quantities are not of same kind. First is a distance in km and the other is an amount of time in hours.
12 a 10:49 b 23:240 c 19:360
d 25:199 e 157:5200 f 239:1700
g 51:60 h 11:36 i 49:1000
j 300:47 k 13 500:8729 l 23:5400
13 a 173:200 b 27:200 c 173:27
14 a 87:100 b 13:100 c 87:13
15 a 3:2 b 2:5 c 3:5
d Parts b $\left(\frac{2}{5}\right)$ and c $\left(\frac{3}{5}\right)$ can be written as fractions as they are expressed as part of the whole.
e Part a cannot be written as a fraction as it is comparing shaded to non-shaded sections.
f whole
16 a 4:9 b $\frac{4}{9}$ c $\frac{5}{9}$
17 a 5:3 b 8 hours c $\frac{5}{8}$
18 The ratio is 63:158. Goals can be converted to points (1 goal = 6 points) so that the scores can be compared.

## 2H Working with ratios

### 2H Start thinking!

1 Tray A: 2:5, Tray B: 4:10, Tray C: 6:15, Tray D: 8:20
2 10:25, 12:30, 14:35
3 Tray A: $\frac{2}{5}$, Tray B: $\frac{4}{10}$, Tray C: $\frac{6}{15}$, Tray D: $\frac{8}{20}$
4 Each fraction represents fraction of lasagne eaten from whole tray; that is, number of slices eaten out of total number of slices in tray.
5 They are equivalent fractions.
6 multiplied, divided

### Exercise 2H Working with ratios

1 a 7:2 b 2:3 c 13:16 d 3:8
e 11:6 f 8:9 g 5:1 h 1:6
i 2:5:2 j 5:3:1 k 3:5:7 l 3:7:5
2 a 3:2 = 6:4 = 15:10 b 4:5 = 20:25 = 16:20
c 30:210 = 10:70 = 2:14
d 144:54 = 24:9 = 8:3
e 165:121 = 150:110 = 15:11
f 147:189 = 49:63 = 7:9
3 a 3:5 b 5:1 c 1:6 d 7:3
e 5:36 f 67:120 g 11:42 h 31:9
i 1:26 j 21:19 k 18:625 l 13:237
4 a 2:1 b 1:1 c 1:2:2 5 8:7
6 a 12 b 4:3
7 a 2:1:3 b $\frac{1}{3}, \frac{1}{6}, \frac{1}{2}$ 8 16:9
9 a 1945 b i 54:61 ii 3:4 c 389:440
10 a 36:19 b 7:20
11 Some possible answers are:
12:14, 60:70, 30:35.
12 Some possible answers are:
4:5, 240:300, 12:15.
13 Write each ratio in its simplest form.
They both simplify to 5:8.
14 a Yes, since the ratios can be expressed in the same simplest form.
b Yes, since the ratios can be expressed in the same simplest form.
c No, since the ratios cannot be expressed in the same simplest form.
d No, since the ratios cannot be expressed in the same simplest form.
15 a 10 b 5 c 6 d 3 e 2 f 8
16 12 17 a 10 b 25

**18** **a** 2 cups water, 4 cups flour, 2 tsp yeast, 2 tsp salt
**b** 6 cups water, 12 cups flour, 6 tsp yeast, 6 tsp salt
**c** $\frac{1}{2}$ cup water, 1 cup flour, $\frac{1}{2}$ tsp yeast, $\frac{1}{2}$ tsp salt

**19** 450 mL **20** $15\frac{1}{4}$ **21** 35 750

## 2 Chapter review

### MULTIPLE-CHOICE

**1** D **2** B **3** D **4** A **5** A **6** D
**7** B **8** B **9** C **10** C **11** C

### SHORT ANSWER

**1** **a** $\frac{19}{8}$ **b** $5\frac{1}{2}$ **c** $1\frac{8}{9}$ **d** $\frac{16}{3}$
**2** **a** $\frac{4}{5}$ **b** $\frac{1}{9}$ **c** $\frac{1}{2}$ **d** $\frac{3}{7}$
**3** $\frac{1}{4}, \frac{2}{7}, \frac{11}{20}, \frac{3}{5}, 1\frac{2}{3}, \frac{9}{2}$
**4** **a** $\frac{5}{9}$ **b** $\frac{2}{11}$ **c** $\frac{1}{6}$ **d** $\frac{7}{10}$
**5** **a** $5\frac{6}{7}$ **b** $\frac{4}{5}$ **c** $1\frac{1}{8}$ **d** $4\frac{7}{15}$
**6** **a** $\frac{8}{35}$ **b** $\frac{1}{6}$ **c** $\frac{20}{33}$ **d** $1\frac{1}{15}$
**7** **a** $1\frac{1}{5}$ **b** $\frac{17}{35}$ **c** $8\frac{1}{4}$ **d** $5\frac{23}{24}$
**8** **a** $1\frac{1}{8}$ **b** $1\frac{1}{4}$ **c** $4\frac{2}{3}$ **d** $\frac{3}{4}$
**9** **a** $1\frac{2}{3}$ **b** $\frac{2}{3}$ **c** 9 **d** $1\frac{5}{8}$
**10** **a** $\frac{9}{16}$ **b** $\frac{25}{36}$ **c** $\frac{27}{1000}$ **d** $\frac{1}{16}$
**11** **a** $\frac{5}{7}$ **b** $\frac{4}{3}$ **c** $\frac{1}{3}$ **d** $1\frac{1}{3}$
**12** **a** 4:3 **b** 3:2 **c** 1:14 **d** 5:2
**13** **a** 2:3 **b** 3:4 **c** 5:8 **d** 14:1
**14** 56

### NAPLAN-STYLE PRACTICE

**1** $\frac{25}{7}$ **2** 7 **3** $4\frac{4}{5}$
**4** third arrow from the left **5** $\frac{10}{24}$
**6** $\frac{2}{5}, 1\frac{3}{4}, \frac{9}{4}, 2\frac{7}{10}, \frac{16}{5}$ **7** $\frac{3}{2}$ or $1\frac{1}{2}$
**8** $\frac{1}{4}$ **9** $\frac{2}{5}$ **10** $7\frac{7}{15}$ **11** $\frac{8}{15}$
**12** $2\frac{1}{4}$ cups **13** $\frac{18}{5}$ or $3\frac{3}{5}$ **14** $\frac{1}{4}$
**15** $\frac{9}{10}$ L **16** $\frac{15}{22}$ **17** $\frac{9}{49}$ **18** $\frac{81}{16}$
**19** $\frac{4}{5}$ **20** $\frac{7}{12}$ **21** 5:7 **22** 98:121
**23** 4:7 **24** 6:10 **25** 60 minutes
**26** 2:3 **27** 63

### ANALYSIS

**1** **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** 3:2 **d** 25
**e** $\frac{1}{12}$, 25 families
**f** Correct about one pet, because $\frac{5}{10} = \frac{1}{2}$, but not about two pets, because $\frac{3}{10} \neq \frac{1}{3}$.
**2** **a** \$2 **b** 25 **c** \$24
**d** **i** $\frac{12}{5}$ **ii** $2\frac{2}{5}$ **e** \$8
**f** \$84, 42 weeks **g** 14 weeks **h** 1:3

## 2 Connect

For feedback on this open-ended task, see your teacher.

# CHAPTER 3 DECIMALS AND PERCENTAGES

## 3 Are you ready?

**1** **a** B **b** C **c** C **2** D
**3** **a** 0.7 **b** 0.01 **4** **a** 600 **b** 20
**5** **a** B **b** 67 136 **6** **a** 3515 **b** B
**7** **a** B **b** 314
**8** **a** C **b** A **c** 0.3 **d** C
**9** **a** D **b** $\frac{75}{100}$ or $\frac{3}{4}$ **c** $\frac{31}{100}$
**10** **a** D **b** $\frac{70}{100}$ **11** **a** A **b** 138

## 3A Understanding decimals

### 3A Start thinking!

**1**

| Thousands 1000 | Hundreds 100 | Tens 10 | Ones 1 | Decimal point . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ |
|---|---|---|---|---|---|---|---|
| 5 | 2 | 1 | 8 | . | 3 | 4 | 7 |

**2** It becomes smaller (by a factor of 10).
**3** **a** $5 \times 1000 + 2 \times 100 + 1 \times 10 + 8 \times 1 + 3 \times \frac{1}{10} + 4 \times \frac{1}{100} + 7 \times \frac{1}{1000}$
$= 5000 + 200 + 10 + 8 + \frac{3}{10} + \frac{4}{100} + \frac{7}{1000}$
**b** Number of decimal places matches number of zeros in denominator.
**4** **a** **i** 4 **ii** 7
**b** three

### Exercise 3A Understanding decimals

**1** **a** eight ones or 8
**b** eight thousandths or $\frac{8}{1000}$
**c** eight tenths or $\frac{8}{10}$
**d** eight ten-thousandths or $\frac{8}{10\,000}$
**e** eight tens or 80
**f** eight hundredths or $\frac{8}{100}$
**g** eight hundreds or 800
**h** eight hundredths or $\frac{8}{100}$
**i** eight ten-thousandths or $\frac{8}{10\,000}$
**j** eight tenths or $\frac{8}{10}$
**k** eight thousands or 8000
**l** eight tens or 80
**2** **a** $28.61 = 2 \times 10 + 8 \times 1 + 6 \times \frac{1}{10} + 1 \times \frac{1}{100}$
**b** $304.128 = 3 \times 100 + 0 \times 10 + 4 \times 1 + 1 \times \frac{1}{10} + 2 \times \frac{1}{100} + 8 \times \frac{1}{1000}$
**c** $927.835 = 9 \times 100 + 2 \times 10 + 7 \times 1 + 8 \times \frac{1}{10} + 3 \times \frac{1}{100} + 5 \times \frac{1}{1000}$
**d** $16.0058 = 1 \times 10 + 6 \times 1 + 0 \times \frac{1}{10} + 0 \times \frac{1}{100} + 5 \times \frac{1}{1000} + 8 \times \frac{1}{10\,000}$
**e** $84.761 = 8 \times 10 + 4 \times 1 + 7 \times \frac{1}{10} + 6 \times \frac{1}{100} + 1 \times \frac{1}{1000}$

**f** $62.1852 = 6 \times 10 + 2 \times 1 + 1 \times \frac{1}{10} + 8 \times \frac{1}{100} + 5 \times \frac{1}{1000} + 2 \times \frac{1}{10\,000}$

**g** $821.793 = 8 \times 100 + 2 \times 10 + 1 \times 1 + 7 \times \frac{1}{10} + 9 \times \frac{1}{100} + 3 \times \frac{1}{1000}$

**h** $71.085 = 7 \times 10 + 1 \times 1 + 0 \times \frac{1}{10} + 8 \times \frac{1}{100} + 5 \times \frac{1}{1000}$

**i** $0.0018 = 1 \times \frac{1}{1000} + 8 \times \frac{1}{10\,000}$

**j** $3.8024 = 3 \times 1 + 8 \times \frac{1}{10} + 0 \times \frac{1}{100} + 2 \times \frac{1}{1000} + 4 \times \frac{1}{10\,000}$

**k** $8236.12 = 8 \times 1000 + 2 \times 100 + 3 \times 10 + 6 \times 1 + 1 \times \frac{1}{10} + 2 \times \frac{1}{100}$

**l** $183.049 = 1 \times 100 + 8 \times 10 + 3 \times 1 + 0 \times \frac{1}{10} + 4 \times \frac{1}{100} + 9 \times \frac{1}{1000}$

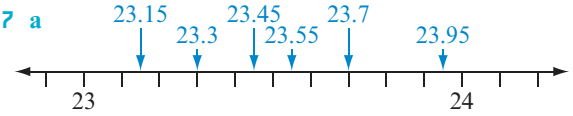
**3** **a** 5785.195 **b** 496.283 **c** 0.006
**d** 5.703 **e** 36.07 **f** 40.1007

**4** **a** 3 **b** 5 **c** 2 **d** 6 **e** 0
**f** 1 **g** 8 **h** 1 **i** 4 **j** 4
**k** 6 **l** 5

**5** **a** > **b** < **c** > **d** < **e** >
**f** > **g** > **h** > **i** >

**6** **a** 0.146, 0.238, 0.328, 0.461, 0.641, 0.823
**b** 1.234, 2.143, 2.341, 2.431, 3.412, 4.132
**c** 32.861, 36.182, 36.218, 36.812, 38.162, 38.261
**d** 18.1524, 18.2145, 18.2154, 18.4125, 18.4152, 18.4251

**7** **a**




**b** 23 **c** 24
**d** If the tenths digit is 0, 1, 2, 3 or 4, round to the whole number part of the number. If the tenths digit is 5, 6, 7, 8 or 9, add 1 to the whole number and discard the remaining digits (the decimal place digits).

**8** **a** The digit in the third decimal place is an 8. When rounding to two decimal places, if the digit in the third decimal place is 5 or more, then round up to the next digit in the second decimal place. This means that 4.5382 is closer to 4.54 than it is to 4.53.
**b** second, third, doesn't, one
**c** 4.538
**d** Look at the digit in the fourth decimal place (the next place value). Since it is a 2, the digit in the third decimal place doesn't change.
**e** Write the number and draw or imagine a vertical line after the required place value. Look at the digit to the right of this line. If the digit is 0, 1, 2, 3 or 4, then the digit in the required place does not change. If the digit is 5, 6, 7, 8 or 9, then add 1 to the digit in the required place value. Discard the digits after the line and write the answer.

**9** **a** 3.547 **b** 3.5465 **c** 3.55

**10** **a** 82.63 **b** 59.112 **c** 0.2634
**d** 0.27 **e** 15.000 **f** 2.175 499
**g** 0.219 **h** 165.5455 **i** 43.806 25

**11** **a** 21.89 m **b** 22.81 m
**c** The largest value in the highest place value represents the largest distance.

**12** **a** **i** $\frac{3}{10}$ **ii** 0.3 **b** **i** $\frac{5}{10}$ **ii** 0.5
**c** **i** $\frac{8}{10}$ **ii** 0.8

**13** **a** 0.3
**b** There are 100 small squares; 30 are shaded.
**c** $\frac{30}{100}$
**d** 0.30 and 0.3 are the same, as trailing zeros can be ignored.

**14**

**a**

| Ones 1 | . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ |
|---|---|---|---|
| 0 | . | 3 | 0 |

**b**

| Ones 1 | . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ |
|---|---|---|---|
| 0 | . | 4 | 7 |

0.47 = 4 tenths + 7 hundredths or 47 hundredths

**c**

| Ones 1 | . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ |
|---|---|---|---|---|
| 0 | . | 1 | 6 | 3 |

0.163 = 1 tenth + 6 hundredths + 3 thousandths or 163 thousandths

**d** 681 thousandths = 6 tenths + 8 hundredths + 1 thousandth = 0.681

**15** **a** 0.7 **b** 0.9 **c** 0.33 **d** 0.531
**e** 0.27 **f** 0.11 **g** 0.2978 **h** 0.804
**i** 0.80 **j** 0.063

**16**

| Decimal number | Number of decimal places | Value of digit in first decimal place | Value of digit in second decimal place | Value of digit in third decimal place | Fractional form |
|---|---|---|---|---|---|
| 0.275 | 3 | $\frac{2}{10}$ | $\frac{7}{100}$ | $\frac{5}{1000}$ | $\frac{275}{1000}$ |
| 0.58 | 2 | $\frac{5}{10}$ | $\frac{8}{100}$ | $\frac{0}{1000}$ | $\frac{58}{100}$ |
| 0.634 | 3 | $\frac{6}{10}$ | $\frac{3}{100}$ | $\frac{4}{1000}$ | $\frac{634}{1000}$ |
| 0.983 | 3 | $\frac{9}{10}$ | $\frac{8}{100}$ | $\frac{3}{1000}$ | $\frac{983}{1000}$ |
| 0.71 | 2 | $\frac{7}{10}$ | $\frac{1}{100}$ | $\frac{0}{1000}$ | $\frac{71}{100}$ |
| 0.407 | 3 | $\frac{4}{10}$ | $\frac{0}{100}$ | $\frac{7}{1000}$ | $\frac{407}{1000}$ |
| 0.026 | 3 | $\frac{0}{10}$ | $\frac{2}{100}$ | $\frac{6}{1000}$ | $\frac{26}{1000}$ |

**e** The number of decimal places matches the number of zeros in the denominator of the corresponding fraction.

**17** **a** $\frac{127}{1000}$ **b** $\frac{401}{1000}$ **c** $\frac{3}{100}$
**d** $\frac{289}{10\,000}$ **e** $\frac{2001}{10\,000}$ **f** $\frac{107}{10\,000}$

**18** **a** $12.021 = 1 \times 10 + 2 \times 1 + 0 \times \frac{1}{10} + 2 \times \frac{1}{100} + 1 \times \frac{1}{1000}$
$12.201 = 1 \times 10 + 2 \times 1 + 2 \times \frac{1}{10} + 0 \times \frac{1}{100} + 1 \times \frac{1}{1000}$
$12.210 = 1 \times 10 + 2 \times 1 + 2 \times \frac{1}{10} + 1 \times \frac{1}{100} + 0 \times \frac{1}{1000}$

b 0 tenths, 0 hundredths, 0 thousandths
c A different value would result. The zero holds the position of the digits in the correct place value.
d Zeros are needed so that the correct decimal number can be represented with all digits in the right positions according to place value.
e Trailing zeros can be left out as they do not change the value of the decimal.
f 12.021, 12.201, 12.210

19 He has failed to put a zero in the tenths place: 42 thousandths means 4 hundredths and 2 thousandths.
20 Because the digit in the next place value is a 5, meaning the digit in the third place value must have 1 added to it.
21 a 200 b 170 c 168
d 168.2 e 168.25 f 168.247
22 Some examples are: 5.821, 5.820, 5.817, 5.818.
23 a 0.01 b 0.001 c 0.000 01

## 3B Adding and subtracting decimals

### 3B Start thinking!

1 a $13 b $14
2 According to estimates, $10 will not be enough to pay for either order.
3 a yes
b Either the chips, garden salad or whiting will reduce the value below $10 as each costs more than $1.65.
4 a no
b He added up the total in cents, not dollars.
c He added the calamari ring and onion ring as cents, not as dollars.
d $6.50 + $0.80 + $0.80 + $2.50 + $1.70
e $12.30

### Exercise 3B Adding and subtracting decimals

1 a 90 b 310 c 200 d 290
e 150 f 1210 g 940 h 8100
2 a 88.21 b 316.47 c 192.273
d 292.065 e 149.601 f 1208.2163
g 944.911 h 8102.391
3 a 36.012 b 1519.797 c 66.017
d 3855.1036 e 177.9932 f 15.971
4 a 60 b 30 c 400 d 20
e 1470 f 110 g 10 h 1230
5 a 56.23 b 34.72 c 394.9
d 23.22 e 1476.68 f 113.27
g 15.588 h 1233.6118
6 a 123.926 b 7.487 c 15.47
d 25.14 e 6.675 f 10.338
8 a $43 b $7
c $42.51, change is $7.49
d only 49 cents away from actual cost
9 a 42.6 b 42.1
c Natasha received the highest score by 0.5.
10 17.6°C
11 Oliver 0.09 m, Sam 0.13 m, Than 0.14 m
12 Align place values correctly and leave zero place values correctly in place. Correct answer is 125.243.
13 66.68 m
14 a 12.49 L b three tins
15 Total monthly rainfall of 64.68 mm is 1.02 mm below September average.
16 7.5 kg
17 a Day 1: 187.2 km; Day 2: 148.6 km; Day 3: 249.8 km; Day 4: 144.8 km; Day 5: 251.5 km
b 28 071.5 c Day 4, 144.8 km
d Day 5, 251.5 km e 981.9 km
18 a 22.09 kg b more than 2.09 kg
19 a Total mass is 239.52 kg, so lift capacity has not been reached.
b Extra mass of 25.48 kg will take total mass to maximum capacity of lift.
20 a 5.47 km (or 5470 m)
b 1.4 km, 1.8 km, 1.96 km, 2.31 km
c 7.47 km (or 7470 m)
21 a i $453.98, $394.99, $329.99, $79.99, $51.46, $53.24, $48.24
ii $407.52 iii $127.20 iv $48.24
b Elisa has $48.24 in bank at end of month; not enough to purchase iPod nano.
22 a 283.15 b 62.369 c 15.413
d 617.58 e 0.23 f 158.7
23 Check totals are correct and correct signs are included.

## 3C Multiplying decimals

### 3C Start thinking!

1 0.9; counting graduations on scale
2 2.0
3 4 × 0.6 =, 8 × 0.25 =
4 a 0.9 b 2.0 c 1.8 d 2.75
5 a 9 b 200 c 18 d 275
6 Digits in original calculations and answers are same, just place values are different.
7 same 8 8.4 m

### Exercise 3C Multiplying decimals

1 a 5 b 7 c 6 d 6 e 6 f 6
g 8 h 7 i 8 j 10 k 9 l 8
2 a 0.12 b 0.24 c 0.42
d 0.072 e 0.0036 f 0.49
g 0.96 h 0.000 24 i 0.000 000 7
j 0.000 15 k 0.0084 l 0.0088
3 a 26.6 b 39.2 c 3.15
d 9.12 e 36.0072 f 19.68
g 8.277 h 359.8 i 79.44
4 a 69.16 b 456.48 c 1258.32
d 28.644 e 130.974 f 964.73
g 47.7776 h 24.311 92 i 2251.1775

**5** **a** 32.88 **b** 86.76 **c** 198.15
**d** 622.08 **e** 0.185 68 **f** 7.4684
**g** 12.4 **h** 3.702 **i** 1041.56

**7** **a** $60 **b** $77.48
**c** Estimate is $17.48 lower.

**8** **a** $9.74 **b** $6.08 **c** $9.73
**d** $8.28 **e** $8.97 **f** $10.09

**9** **a** All these calculations involve the same digits.
**b** The digits represent different place values in the different numbers.
**c, d**

| Original calculation | Estimated answer | Answer |
|---|---|---|
| 38 × 21 | 40 × 20 = 800 | 798 |
| 3.8 × 21 | 4 × 20 = 80 | 79.8 |
| 38 × 2.1 | 40 × 2 = 80 | 79.8 |
| 3.8 × 2.1 | 4 × 2 = 8 | 7.98 |
| 380 × 210 | 400 × 200 = 80 000 | 79 800 |

**e** Number of decimal places in answer equals total number of decimal places in original calculation.

**10** **a** **i** 20 **ii** 200 **iii** 2000 **iv** 20 000
**b** The original place value of 2 is 2 or 2 ones. After each calculation, place value of 2 increases by 10. That is, digit 2 moves to left by one place. Number of places the digit 2 is moved from original position equals number of zeros after the 1 in each number.
**c** **i** one, left **ii** two, left
**iii** three, left **iv** four, left
**d** **i** 52.634 **ii** 526.34
**iii** 5263.4 **iv** 52 634
**e** Decimal point moves to right (or digits move to left). Number of places moved equals number of zeros after the 1.

**11** **a** 6000 **b** 4200 **c** 1.284
**d** 1572.1 **e** 0.4 **f** 7.02
**g** 61.0 **h** 150.0 **i** 310.0
**j** 6481.3 **k** 480 000 **l** 128 468 000
**m** 4.56 **n** 100.09 **o** 1003.01

**12** **a** $713.40 **b** option 1 **c** $68.40

**13** **a** $258 **b** $147.60

**14** 8 mm

**15** **a** 9086 **b** 297 600 **c** 104 688
**d** 33 984 **e** 97 500 **f** 12 150
**g** 511 299 **h** 152 334 **i** 47 634.3
**j** 568 480

**16** **a** $110 600 **b** $198 940 **c** $88 340

**17** **a** 12.0807 **b** 36.62
**c** 12.958 **d** 0.481 74

**18** **a** **i** 0.04 **ii** 0.16
**iii** 0.0625 **iv** 0.000 064
**b** The number of decimal places in the answer is twice the number of decimal places in the number to be squared.

## 3D Dividing a decimal by a whole number

### 3D Start thinking!

**1** **a** no **b** 14 remainder 1 (or $14\frac{1}{4}$)

**2** **a** $14.25
**b** To enable division calculation to continue until there is no remainder.
**c** No, there still would have been a remainder.
**d** Yes, but answer would contain trailing zeros. Exact decimal answer (without remainder) was obtained after only two decimal places so remaining trailing zeros in dividend are not required.

**3** **a** **i** 3.5 **ii** 4.375 **iii** 7.25 **iv** 170.8
**b** **i** one **ii** three **iii** one **iv** none

### Exercise 3D Dividing a decimal by a whole number

**1** **a** 6.23 **b** 13.721 **c** 9.102 **d** 23.14
**e** 11.115 **f** 1.6398 **g** 2.9217 **h** 4.15
**i** 3.51 **j** 0.0256 **k** 0.1208 **l** 0.728 04

**2** **a** 2.5 **b** 6.75 **c** 12.6 **d** 4.35
**e** 2.05 **f** 0.4062 **g** 0.603 75 **h** 39.05
**i** 14.0575 **j** 1.193 75 **k** 1.145 **l** 3.2375

**3** **a** $1.28 **b** $217.50 **c** $1.28 **d** $0.75
**e** $11.80 **f** $7.25 **g** $1.30 **h** $1.80
**i** $2.53 **j** $123.33

**4** **a** $3.88 **b** $4.98 **c** $3.98 **d** $11.97
**e** $6.98 **f** $7.98 **g** $1.62 **h** $4.27
**i** $0.17 **j** $2.22

**5** $186.25 **6** $135.90 **7** $24.65

**8** **a** $2.49
**b** 2-L container, as it is cheaper per L.

**9** $18.90 ÷ 6 = $3.15. Whole pizza is better value by $0.35 per slice.

**10** **a** $2.75
**b** If using it five days per week, weekly option is cheaper by 20 cents per day.
**c** If plan to travel less than five days per week, buying daily tickets will cost less than a weekly ticket.

**11** 5 kg bag is the better buy as the cost is equivalent to $1.26 per kg, a saving of 19 cents per kg compared to loose potatoes.

**12** **a** $8.99 ÷ 12 = $0.75. Carton of eggs is better value.
**b** $0.15
**c** Some people want to buy less than 12 eggs.

**13** **a** $3.73 **b** $1.56 **c** $0.53 **d** $1.50

**14** **a** Simon $0.80, Julia $0.66 **b** Julia

**15** **a** Barilla $0.93, Leggos $0.82; Leggos is better value
**b** two, Barilla ($7.46 compared to $8.18)

**16** $357 122.89; divide dividend by 18, then round to two decimal places.

**17** Less than 1 L, as divisor (180) is larger than dividend (100); that is, dividing the smaller amount by the larger amount.

**19** packing boxes at $8.13 an hour

**21** **a** $2.10 **b** $21.00
**c** Cost of one mango found by 6.30 ÷ 3. Multiply by 10 for cost of 10 mangoes.

**22** **a** $7.00 **b** $6.00 **c** $6.50 **d** $8.50

**23** **a** $12.40 **b** $25.44 **c** $3.73 **d** $0.68

**24** **a** Crinkle cut chips $2.54, small Pringles $2.85, large Pringles $2.81, Red Rock Deli $2.25
**b** Red Rock Deli **c** taste, texture
**25** Ava at 6.08 km/h
**26** 21

## 3E Dividing a decimal by a decimal

### 3E Start thinking!

**1** **a** 20 **b** 20
**c** Advantages include that it gives accurate results; disadvantages include it is time-consuming to draw number lines.
**2** **a** **i** 20 **ii** 20 **iii** 20 **iv** 20
**b** Each answer is 20. Dividend and divisor are each reduced by same factor so calculations are equivalent.
**c** 20
**d** 20. The pattern continues as the divisor and dividend have both been reduced by the same factor.
**e** yes
**3** **a** 60 ÷ 3 = 20 **b** 6 ÷ 0.3 = 20
**c** 0.6 ÷ 0.03 = 20 **d** 0.06 ÷ 0.003 = 20

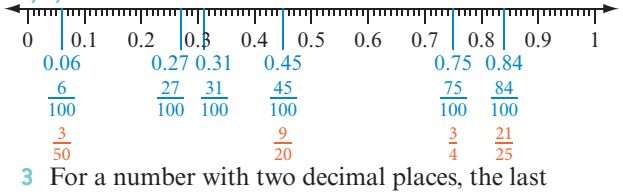
### Exercise 3E Dividing a decimal by a decimal

**1** **a** Multiply by 10; 0.5 × 10 = 5
**b** Multiply by 100; 0.04 × 100 = 4
**c** Multiply by 10; 0.3 × 10 = 3
**d** Multiply by 10; 0.7 × 10 = 7
**e** Multiply by 100; 0.02 × 100 = 2
**f** Multiply by 10; 0.8 × 10 = 8
**g** Multiply by 1000; 0.006 × 1000 = 6
**h** Multiply by 10; 0.4 × 10 = 4
**i** Multiply by 100; 0.09 × 100 = 9
**2** **a** 623.56 ÷ 8 **b** 9840 ÷ 8
**c** 1956.74 ÷ 6 **d** 4.62 ÷ 3
**e** 420 ÷ 4 **f** 0.5 ÷ 2
**g** 4 620 850 ÷ 9 **h** 0.56 ÷ 2
**i** 2108 ÷ 5
**3** **a** 158.51 ÷ 5 **b** 1648.2 ÷ 4
**c** 1.9584 ÷ 3 **d** 3.6505 ÷ 7
**e** 5059.56 ÷ 2 **f** 304.56 ÷ 8
**g** 10 473.24 ÷ 6 **h** 51 250 ÷ 4
**i** 322.2 ÷ 9
**4** **a** 19.3 **b** 281 **c** 84.7
**d** 119.7 **e** 162.8 **f** 185.3
**g** 34 150 **h** 47.1 **i** 364.8
**5** **a** 11.905 **b** 43.375 **c** 5412.5
**d** 0.82 **e** 3187.895 **f** 642.175
**g** 188.25 **h** 2282.45 **i** 14.325
**6** **a** 31.702 **b** 412.05 **c** 0.6528
**d** 0.5215 **e** 2529.78 **f** 38.07
**g** 1745.54 **h** 12 812.5 **i** 35.8
**7** **a** 9 **b** 6 **c** 7
**d** 200 **e** 3 **f** 800
**g** 90 **h** 30 **i** 7100
**8** **a** 8.40 ÷ 0.60 **b** 84 ÷ 6 **c** 14
**9** 86
**10** **a** **i** 12.00 ÷ 0.50 **ii** 120 ÷ 5 **iii** 24
**b** **i** 12.00 ÷ 0.20 **ii** 120 ÷ 2 **iii** 60
**c** **i** 12.00 ÷ 0.80 **ii** 120 ÷ 8 **iii** 15
**d** **i** 12.00 ÷ 1.20 **ii** 120 ÷ 12 **iii** 10
**11** **a** 240 000 ÷ 3000 = 80
**b** 24 000 ÷ 300 = 80
**c** 2400 ÷ 30 = 80 **d** 240 ÷ 3 = 80
**e** 24 ÷ 0.3 = 80 **f** 2.4 ÷ 0.03 = 80
**g** 0.24 ÷ 0.003 = 80 **h** 0.024 ÷ 0.0003 = 80
**12** **a** **i** 90 **ii** 90 **iii** 90
**b** **i** 72 ÷ 0.8 = 90 **ii** 7.2 ÷ 0.08 = 90
**iii** 0.72 ÷ 0.008 = 90
**c** 0.99 ÷ 0.11 = 9 = 0.099 ÷ 0.011 = 9.9 ÷ 1.1 = 99 ÷ 11
**13** **a** **i** 5 **ii** 50 **iii** 500 **iv** 5000
**b** The answer increases by a power of 10.
**c** The answer is a value larger than the dividend.
**d** larger
**14** **a** A, B, E, G
**b** Dividend and divisor in each problem is multiplied (or divided) by same factor.
**15** **a** no
**b** Check if dividend and divisor have both been multiplied (or divided) by same factor.
**16** **a** $2.40 **b** $2.75 **c** 1.2-kg bag
**17** **a** 144 pieces **b** yes, 0.1 m remaining
**18** 45.5 L
**19** Divide cost by volume. Orange mineral water is the better buy ($1.68 per L compared to $1.80 per L for cola).
**20** **a** **i** 3876.52 **ii** 387.652
**iii** 38.7652 **iv** 3.876 52
**b** Decimal point is moved left (or digits move right). Number of places moved equals number of zeros after the 1 in divisor.
**c** **i** one, left **ii** two, left
**iii** three, left **iv** four, left
**d** **i** 0.678 543 **ii** 9.53 **iii** 28.894
**iv** 0.157 21 **v** 0.2934 **vi** 0.702
**vii** 68.9421 **viii** 0.150 95 **ix** 0.0436
**21** **a** 0.562 78 **b** 96.3 **c** 52.47
**d** 0.8326 **e** 6.8256 **f** 2.825
**g** 5.21 **h** 0.8921
**22** **a** 15 960 **b** 22 **c** 2096.96
**d** 4 **e** 500 **f** 42.6768
**23** 32.4 ÷ 4.05 = 8

## 3F Converting between fractions and decimals

### 3F Start thinking!

**1, 2, 4**




**3** For a number with two decimal places, the last decimal place is hundredths.

4 $\frac{84}{100} = \frac{21}{25}$, $\frac{75}{100} = \frac{3}{4}$, $\frac{6}{100} = \frac{3}{50}$, $\frac{45}{100} = \frac{9}{20}$

5 A given fraction can have many equivalent fractions. These are fractions that represent the same value but have different numerators and denominators.

6 Some possible answers are:
$\frac{1}{2} = \frac{5}{10} = \frac{10}{20} = \frac{50}{100} = \frac{4}{8} = \frac{6}{12}$

7 The fourth decimal place is ten-thousandths, so $0.1587 = \frac{1587}{10\,000}$.

### Exercise 3F Converting between fractions and decimals

**1** **a** $\frac{25}{100}$ **b** $\frac{50}{100}$ **c** $\frac{35}{100}$ **d** $\frac{88}{100}$
**e** $\frac{23}{100}$ **f** $\frac{8}{100}$ **g** $\frac{12}{100}$ **h** $\frac{95}{100}$

**2** **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{7}{20}$ **d** $\frac{22}{25}$
**e** $\frac{23}{100}$ **f** $\frac{2}{25}$ **g** $\frac{3}{25}$ **h** $\frac{19}{20}$

**3** **a** $\frac{1}{5}$ **b** $\frac{7}{10}$ **c** $\frac{13}{100}$ **d** $\frac{13}{50}$
**e** $\frac{9}{1000}$ **f** $\frac{213}{500}$ **g** $\frac{64}{125}$ **h** $\frac{18}{25}$
**i** $\frac{1}{2500}$ **j** $\frac{14}{25}$ **k** $\frac{3}{8}$ **l** $\frac{7}{25}$

**4** **a** 0.03 **b** 0.17 **c** 0.9
**d** 0.2010 **e** 0.375 **f** 0.001 56
**g** 0.07 **h** 0.921 **i** 0.042
**j** 0.0003 **k** 0.01981 **l** 0.0086

**5** **a** 0.875 **b** 0.75 **c** 0.8 **d** 1.125
**e** 14.25 **f** 1.875 **g** 6.6 **h** 7.25
**i** 0.15 **j** 0.04 **k** 0.14 **l** 0.225

**6** **a** Any decimal number with a finite number of decimal places.
**b, c** 0.2727… (the pattern continues indefinitely); you never get to a stage where there is no remainder.
**d** 27 **e** $0.2\dot{7}$ or $0.\overline{27}$

**7** **a** T **b** R **c** R **d** T **e** R **f** R
**g** R **h** T **i** T **j** R **k** T **l** R

**8** **a** $0.\dot{8}$ **b** $0.\overline{21}$ **c** $0.\overline{467}$
**d** $0.127\dot{4}$ **e** $8.3\dot{1}6$ **f** $1.\dot{3}$
**g** $9.10\overline{246}$ **h** $0.\overline{461538}$

**9** **a** $0.\dot{2}$ **b** $0.8\dot{3}$ **c** $0.\overline{857143}$
**d** $0.\overline{72}$ **e** $1.1\dot{6}$ **f** $0.\dot{6}$
**g** $0.7\dot{3}$ **h** $4.41\dot{6}$

**10** 0.3 is greater than $\frac{1}{4}$ (which is equivalent to 0.25).

**11** **a** 10 **b** $\frac{2}{10}$ **c** 0.2
**d** apple: $\frac{2}{10} = 0.2$; cola: $\frac{1}{10} = 0.1$; watermelon: $\frac{2}{10} = 0.2$; strawberries and cream: $\frac{3}{10} = 0.3$

**12** **a** $\frac{1}{5} = \frac{2}{10}$ **b** $\frac{3}{4} = \frac{75}{100} = 0.75$
**c** $\frac{7}{20} = \frac{35}{100} = 0.35$ **d** $\frac{4}{5} = \frac{8}{10} = 0.8$
**e** $\frac{11}{50} = \frac{22}{100} = 0.22$ **f** $\frac{2}{4} = \frac{50}{100} = 0.50$
**g** $\frac{13}{20} = \frac{65}{100} = 0.65$ **h** $\frac{1}{2} = \frac{50}{100} = 0.50$
**i** $\frac{19}{50} = \frac{38}{100} = 0.38$ **j** $\frac{1}{8} = \frac{125}{1000} = 0.125$
**k** $\frac{4}{125} = \frac{32}{1000} = 0.032$
**l** $\frac{53}{2500} = \frac{212}{10\,000} = 0.0212$

**13** Denominators are factors of a suitable power of 10.

**14** **a, e, g, i, l;** $\frac{3}{5} = \frac{6}{10} = 0.6$, $\frac{5}{25} = \frac{20}{100} = 0.2$, $\frac{497}{500} = \frac{994}{1000} = 0.994$, $\frac{7}{25} = \frac{28}{100} = 0.28$, $\frac{103}{125} = \frac{824}{1000} = 0.824$. Others cannot be written as decimals without first dividing, as denominators are not factors of a suitable power of 10.

**15** **a** $8.605 = 8 \times 1 + 6 \times \frac{1}{10} + 5 \times \frac{1}{1000}$
**b** decimal part **c** $\frac{605}{1000} = \frac{121}{200}$ **d** $8\frac{121}{200}$

**16** **a** $5\frac{83}{100}$ **b** $3\frac{451}{500}$ **c** $\frac{9}{20}$ **d** $8\frac{7}{20}$
**e** $2\frac{1}{1250}$ **f** $3\frac{9}{10}$ **g** $12\frac{22}{55}$ **h** $1\frac{353}{500}$
**i** $40\frac{3}{8}$ **j** $1\frac{73}{100}$ **k** $3\frac{3}{20}$ **l** $26\frac{421}{500}$

**17** **a** $4\frac{1}{4}$ m and $1\frac{4}{5}$ m (or 4.25 m and 1.8 m)
**b** $6\frac{1}{20}$ m (or 6.05 m) **c** yes

**18** **a** **i** $0.\overline{142857}$ **ii** $0.\overline{285714}$
**iii** $0.\overline{428571}$ **iv** $0.\overline{571428}$
**b** Same digits are used in given order but with different starting point. Starting digit in recurring pattern for each fraction is in ascending order. That is, 1, 2, 4, 5.
**c, d** $0.\overline{714285}$ and $0.\overline{857142}$

**19** $\frac{1}{9} = 0.\dot{1}$, $\frac{2}{9} = 0.\dot{2}$, $\frac{3}{9} = 0.\dot{3}$, $\frac{4}{9} = 0.\dot{4}$, $\frac{5}{9} = 0.\dot{5}$, $\frac{6}{9} = 0.\dot{6}$, $\frac{7}{9} = 0.\dot{7}$, $\frac{8}{9} = 0.\dot{8}$. Numerator of each fraction becomes recurring digit.

**20** Denominators of 3 and 6 produce recurring fractions.

**21** $\frac{28}{36} = 0.\dot{7}$, $\frac{38}{46} \approx 0.826$. Lou performed better this week.

**22** Forces and motion: 0.8; Solids, liquids and gases: $0.8\dot{4}$; Separating mixtures: 0.875; Science at work: $0.\dot{8}$

**23** Some possible answers are: $\frac{76}{100}$, $\frac{4}{5}$, $\frac{81}{100}$.

**24** Some possible answers are: 0.25, 0.3, 0.35, 0.395.

**25** $3\frac{162}{200} = 3\frac{81}{100}$

**26** **a** 0.08, $\frac{1}{10}$, $\frac{16}{34}$, $\frac{15}{25}$, 0.62, $\frac{14}{16}$, 1.05, $\frac{22}{20}$
**b** 0.28, $\frac{62}{90}$, $\frac{58}{75}$, 0.82, $\frac{37}{17}$, 2.21, $5\frac{12}{20}$, $5.\dot{6}$

## 3G Understanding percentages

### 3G Start thinking!

**1** **a** 23 **b** 65
**2** **a** 23% **b** 65%
**3** **a** 77% **b** 35%
**4** **a** **i** 23% **ii** 77% **b** Add to 100%.

### Exercise 3G Understanding percentages

**1** **a** 52% **b** 50% **c** 36%
**2** **a** 48% **b** 50% **c** 64%
**3** **a** $\frac{7}{100}$ **b** $\frac{19}{100}$ **c** $\frac{43}{100}$ **d** $\frac{57}{100}$
**e** $\frac{3}{100}$ **f** $\frac{63}{100}$ **g** $\frac{11}{100}$ **h** $\frac{91}{100}$
**i** $\frac{79}{100}$ **j** $\frac{21}{100}$ **k** $\frac{99}{100}$ **l** $\frac{33}{100}$

**4** **a** $\frac{16}{25}$ **b** $\frac{9}{50}$ **c** $\frac{27}{100}$ **d** $\frac{4}{5}$
**e** $\frac{1}{20}$ **f** $\frac{19}{20}$ **g** $\frac{1}{10}$ **h** $\frac{9}{20}$
**i** $\frac{3}{5}$ **j** $\frac{3}{4}$ **k** $\frac{7}{20}$ **l** $\frac{22}{25}$

**5** **a** 23% **b** 78% **c** 35% **d** 99%
**e** 13% **f** 7% **g** 82% **h** 4%

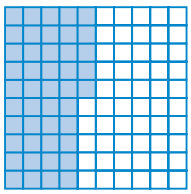
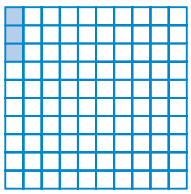
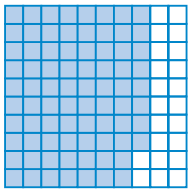
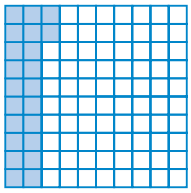
**6** **a** 45% **b** 22% **c** 75% **d** 68%
**e** 60% **f** 20% **g** 60% **h** 92%
**i** 36% **j** 25% **k** 80% **l** 20%

**7** **a** **i** 14 sections shaded of total 20 sections
**ii** $\frac{14}{20}$ **iii** $\frac{70}{100}$ **iv** 70%
**b** **i** 5 sections shaded of total 10 sections
**ii** $\frac{5}{10}$ **iii** $\frac{50}{100}$ **iv** 50%
**c** **i** 1 section shaded of total 4 sections
**ii** $\frac{1}{4}$ **iii** $\frac{25}{100}$ **iv** 25%

**8** **a** 76% **b** 40% **c** 75%
**d** 30% **e** 35% **f** 6%

**9** **a** **i** 20% **ii** 10% **iii** 30%
**b** **i** 80% **ii** 90% **iii** 70%
**c** Express number of pieces eaten out of total number of pieces as equivalent fraction with denominator of 100.

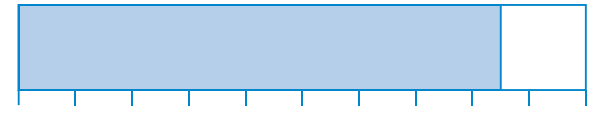
**10** **a** 55% **b** 82% **c** 25% **d** 33%

**11** **a**  **b** 
**c**  **d** 

**12** **a** see part c
**b** **i** 10 is factor of 100 so denominator of 10 can easily be expressed as denominator of 100.
**ii** Each cm will represent 10%.
**iii** Each mm will represent 1%.
**c** 15% = $\frac{15}{100}$ so 15 mm out of 100 mm (or 1.5 cm out of 10 cm) needs to be shaded.



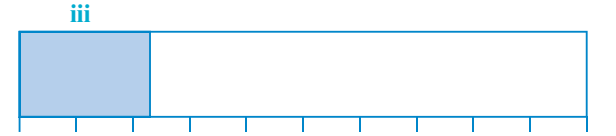

**d** **i**



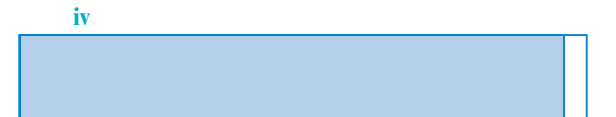

**ii**




**iii**




**iv**




**13** **a** C **b** A **c** D **d** B

**14** **a** 40% **b** 90% **c** 4% **d** 20%

**15** **a** 40% **b** 30% **c** 65% **d** 75%

**16** **a** 25% **b** 45% **c** 70%

**17** Percentage is calculated by finding volume of liquid in each container as a fraction of total volume that container holds. However, 50% of a large container is still going to be more that 50% of a small container.

**18** **a** three times whole amount
**b** **i** 2 **ii** 3 **iii** 4 **iv** 1.5

**19** **a** 4.5 **b** 525%

**20** Multiply denominator by 100 (change any mixed number to improper fraction first).

**21** **a** $\frac{1}{700}$ **b** $\frac{1}{450}$ **c** $\frac{1}{125}$ **d** $\frac{1}{200}$
**e** $\frac{7}{1000}$ **f** $\frac{3}{400}$ **g** $\frac{2}{325}$ **h** $\frac{1}{120}$
**i** $\frac{33}{800}$ **j** $\frac{23}{300}$ **k** $\frac{37}{200}$ **l** $\frac{41}{400}$
**m** $\frac{12}{125}$ **n** $\frac{61}{700}$ **o** $\frac{67}{600}$ **p** $\frac{23}{550}$

## 3H Converting between fractions, decimals and percentages

### 3H Start thinking!

**1** 10 **2** 3 **3** $\frac{3}{10}$ **4** $\frac{30}{100}$

**5** **a** 30% **b** 0.3

**6** The percentage value is the same as the number of hundredths in the decimal.

**7**

| Balloon colour | Fraction of total | Fraction with a denominator of 100 | Percentage | Decimal |
|---|---|---|---|---|
| Red | $\frac{3}{10}$ | $\frac{30}{100}$ | 30% | 0.3 |
| Blue | $\frac{2}{10}$ | $\frac{20}{100}$ | 20% | 0.2 |
| Yellow | $\frac{1}{10}$ | $\frac{10}{100}$ | 10% | 0.1 |
| Green | $\frac{4}{10}$ | $\frac{40}{100}$ | 40% | 0.4 |

**8** A percentage and a decimal number that are equivalent have same digits. Both can be related to a fraction with denominator 100. Percentage amount is same as numerator and decimal amount is number of hundredths or result of dividing numerator by 100.

**9** All descriptions are equivalent but they are expressed in different forms: fraction (Stephanie), percentage (Maria) and decimal (Ben).

### Exercise 3H Converting between fractions, decimals and percentages

**1** **a** 0.46 **b** 0.13 **c** 0.99
**d** 0.25 **e** 0.2 **f** 0.5
**g** 0.05 **h** 0.08 **i** 0.01

**2** **a** 0.2384 **b** 0.1965 **c** 0.467
**d** 0.0309 **e** 5.674 **f** 0.00467
**g** 0.128 95 **h** 0.7328 **i** 2.005
**j** 0.1092 **k** 4.0404 **l** 0.000 101

**3** **a** 28% **b** 88% **c** 15% **d** 62%
**e** 46% **f** 72% **g** 9% **h** 4%

**4** **a** 51.8% **b** 902% **c** 10.5%
**d** 70% **e** 2150% **f** 841%
**g** 710% **h** 0.9% **i** 300.1%
**j** 500% **k** 17.654% **l** 0.0045%

**5** **a** 37.5% **b** 25% **c** 43.75%
**d** 52.5% **e** 87.5% **f** 31.25%
**g** 4.8% **h** 11.25%

**6** **a** 33.33% **b** 42.86% **c** 83.33%
**d** 81.82% **e** 55.56% **f** 71.43%
**g** 61.54% **h** 58.33%

**8** **a** $\frac{2}{5}$ **b** **i** 40% **ii** 60%
**c** **i** 0.4 **ii** 0.6

**9**

| Fraction | Decimal | Percentage |
|---|---|---|
| $\frac{1}{2}$ | 0.5 | 50% |
| $\frac{1}{4}$ | 0.25 | 25% |
| $\frac{3}{4}$ | 0.75 | 75% |
| $\frac{1}{3}$ | 0.3 | 33.3.% |
| $\frac{1}{8}$ | 0.125 | 12.5% |
| $\frac{5}{8}$ | 0.625 | 62.5% |
| $\frac{1}{5}$ | 0.2 | 20% |
| $\frac{2}{5}$ | 0.4 | 40% |
| $\frac{3}{5}$ | 0.6 | 60% |

**10** **a** 72% **b** 76.6.%
**c** second test, as percentage is higher

**12** **a** $\frac{1}{8}$ **b** 0.125 **c** 12.5%

**13** **a** 24.4% **b** $\frac{61}{250}$ **c** 0.244

**14** **a** $\frac{47}{100}$ **b** $\frac{53}{100}$

**15** **a** $\frac{71}{100}$, 0.71
**b** **i** 0.09% **ii** 0.0009 **iii** $\frac{9}{10\,000}$
**c** 99.853%; values may be rounded and there may be other substances in Sun.

**16** 20%

**17** Some examples are provided.
**a** $\frac{101}{1000}, \frac{105}{1000}, \frac{1075}{10\,000}$
**b** 72%, 72.65%, 73%, 73.25%
**c** 0.887, 0.8888, 0.888 987, 0.888 999 999

**18** **a** 0.122 **b** 0.2825 **c** 0.057
**d** 0.2024 **e** 0.1726 **f** 0.0835
**g** 0.755 **h** 0.218

**19** Both methods of working correct.

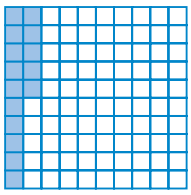
**20** Nick

**21** **a** $\frac{6}{31}, \frac{3}{7}$, 0.4231, 0.249, 12.8%, $\frac{9}{7}$%
**b** 388%, $3\frac{7}{8}$, 3.87, $385\frac{2}{3}$%, 3.826, $3\frac{9}{11}$

## 3I Calculating percentages

### 3I Start thinking!

**1** **a**



**b** 15 **c** 15

**2** **a** Draw two 10 × 10 grids and shade 15 squares in each grid.
**b** 30

**3** **a** Draw six 10 × 10 grids and shade 15 squares in each grid.
**b** 90

**4** 90; same result **5** $90 **6** $510

### Exercise 3I Calculating percentages

**1** **a** 195 **b** 12 **c** 104 **d** 40
**e** 7.5 **f** 18 **g** 39 **h** 12
**i** 100 **j** 480 **k** 200 **l** 132

**2** **a** $4.80 **b** $76.50 **c** $55
**d** $198.90 **e** $700 **f** $168.30
**g** $1485 **h** $3.98 **i** $8.95

**3** **a** 4.5 **b** 3.85 **c** 90.96 **d** 30.6
**e** 27.82 **f** 288.75 **g** 50.88 **h** 25.2
**i** 80.94 **j** 21.12 **k** 12.375 **l** 7.956

**4** **a** $68\frac{3}{5}$ **b** $67\frac{1}{5}$ **c** $57\frac{3}{5}$ **d** $14\frac{2}{5}$
**e** $67\frac{1}{5}$ **f** $807\frac{1}{2}$ **g** $11\frac{3}{5}$ **h** $43\frac{3}{4}$
**i** $141\frac{3}{4}$ **j** $83\frac{3}{10}$ **k** $294\frac{2}{5}$ **l** $101\frac{1}{4}$

**5** **a** 5980 **b** 520

**6** **a** 44 **b** 73.5 **c** 45.6
**d** 400 **e** 10.68 **f** 450.8
**g** 148.5 **h** 308 **i** 1134

**7** **a** $659.70 **b** $1539.30 **c** no

**8** **a** $114 **b** $456

**9** **a** Each percentage is greater than 100%.
**b** Final amount is greater than original amount.

**10** **a** Ripper Reptiles: $108, Enclosed: $162.50
**b** Ripper Reptiles: $492, Enclosed: $487.50
**c** Enclosed

**11** **a** water: 29.25 kg, protein: 8.1 kg, fat: 4.5 kg, minerals and vitamins: 2.7 kg, carbohydrates: 0.45 kg
**b** water: 40.95 kg, protein: 11.34 kg, fat: 6.3 kg, minerals and vitamins: 3.78 kg, carbohydrates: 0.63 kg
**c** water: 63.7 kg, protein: 17.64 kg, fat: 9.8 kg, minerals and vitamins: 5.88 kg, carbohydrates: 0.98 kg

**12** **a** \$3.75 **b** \$1.30 **c** \$7.35
**d** \$9.20 **e** \$1.75 **f** \$1.20

**13** **a** **i** \$119.90 **ii** \$1318.90
**b** **i** \$20.90 **ii** \$229.90
**c** **i** \$49.90 **ii** \$548.90
**d** **i** \$0.30 **ii** \$3.29
**e** **i** \$3.00 **ii** \$32.99
**f** **i** \$5.40 **ii** \$59.40

**14** Move decimal point one place to left of original price value to obtain GST amount.

**15** **a** **i** \$69.60 **ii** \$510.40
**b** **i** \$1125 **ii** \$3375
**c** **i** 2.11 km **ii** 40.09 km
**d** **i** 0.86 L **ii** 7.74 L
**e** **i** 22.2 kg **ii** 125.8 kg
**f** **i** 35.2 kg **ii** 140.8 kg

**16** \$55.99

**17** Prices may be increased by any amount. Reduction is limited because 100% reduction means \$0.

**18** **a** 15 cents **b** \$2.65
**c** $1\frac{3}{50}$ **d** 106%

**19** **a** marked price should be \$6.24 (rounded incorrectly)
**b** marked price should be \$2.58 (rounded to the next 5 cents)
**c** marked price should be \$3.61 (rounded incorrectly)
**d** marked price should be \$4.63 (no increase calculated)
**e** marked price should be \$1 (calculated 25% of 80 dollars, not 80 cents)

**20** **a** halve **b** triple **c** quadruple

**21** 1%: find 10% of amount and then 10% of this result (or move decimal point two places to left); 50%: halve original amount

**22** Answers are the same.

**23** Add 10% to 100%, then calculate 110% of pre-tax amount to determine final amount.

**24** Some possible answers are provided.
**a** First calculate 10% of amount and then 5% of amount. Add two results together.
**b** First calculate 10% of amount and then 1% of amount. Add 10% result and three lots of 1% result.
**c** Halve amount to calculate 50% and then add 5% of original amount.
**d** Subtract two lots of 10% of amount.

## 3 Chapter review

### MULTIPLE-CHOICE

| | | | |
|---|---|---|---|
| **1** C | **2** B | **3** B | **4** B |
| **5** A | **6** C | **7** A | **8** A |
| **9** B | **10** D | **11** D | **12** C |
| **13** D | **14** A | **15** B | **16** D |

### SHORT ANSWER

**1** **a** hundredths **b** hundredths
**c** millionths **d** tenths

**2** **a** **i** five **ii** 1589.23
**b** **i** five **ii** 12.71
**c** **i** four **ii** 0.93
**d** **i** seven **ii** 152.88

**3** 18.0958 12.8905 12.5089 12.0985 10.9852

**4** **a** 120.46 **b** 9.532
**c** 1.693 **d** 4.2885

**5** **a** 1263 **b** 153.298
**c** 210 **d** 609.4

**6** **a** 0.21 **b** 0.036 **c** 0.088 **d** 0.0032

**7** **a** 2.58 **b** 0.468 **c** 46.8 **d** 8.037

**8** **a** 7.894 **b** 12.563 59
**c** 12.456 **d** 189.7656

**9** **a** 9.327 **b** 15.76
**c** 29.726 **d** 40.931 25

**10** **a** \$1.56, \$1.80
**b** 500-g pack, cost per 100 g is cheaper

**11** **a** 12.4 **b** 3.75 **c** 21.55 **d** 0.162

**12** **a** $\frac{3}{4}$ **b** $\frac{9}{50}$ **c** $\frac{489}{1000}$ **d** $\frac{3}{1000}$

**13** **a** 0.87 **b** 0.117 **c** 0.3 **d** 0.07

**14** **a** 0.64 **b** 0.38 **c** 0.75 **d** 0.375

**15** **a** **i** $\frac{49}{100}$ **ii** $\frac{49}{100}$ **b** **i** $\frac{22}{100}$ **ii** $\frac{11}{50}$
**c** **i** $\frac{75}{100}$ **ii** $\frac{3}{4}$ **d** **i** $\frac{5}{100}$ **ii** $\frac{1}{20}$

**16** 46%

**17** **a** 56% **b** 8% **c** 179% **d** 27.48%

**18** **a** 0.78 **b** 0.465 **c** 0.09 **d** 0.106 79

**19** **a** 60% **b** 87.5% **c** 100% **d** 275%

**20** **a** 45 **b** 150 **c** 135 **d** 552.5
**e** 89.94 **f** 142.27

**21** **a** 1496 km **b** 330.125 g
**c** \$6.96 **d** 72.9 L

### NAPLAN-STYLE PRACTICE

**1** hundredths **2** 164.51
**3** 1.26, 1.623, 1.86, 1.88 **4** 6.528
**5** D **6** 6.65 **7** 0.693
**8** \$17.97 **9** 2.74 **10** \$59.90
**11** 100 for \$7.48 **12** 2 L for \$5.92
**13** 0.55 **14** $\frac{7}{8}$ **15** 10 **16** \$1.24
**17** \$3.40 **18** D **19** 12.5% **20** 70
**21** 65% **22** 0.5923
**23** $\frac{4}{5}$, 0.76, 45%, $\frac{1}{8}$, 0.12, 9%
**24** \$217 **25** \$37.49 **26** \$36.75 **27** \$1.20

### ANALYSIS

**a** \$39.30 **b** 13, \$0.43 left over
**c** \$35.88 **d** \$26.91 **e** 0.375
**f** \$53.82 **g** \$20.1825 **h** four, \$20.18
**i** \$33.64, \$5.66 left over **j** about \$1.42

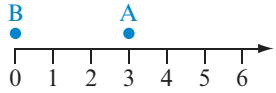
## 3 Connect

For feedback on this open-ended task, see your teacher.

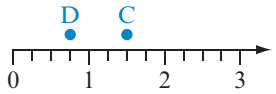
# CHAPTER 4 INTEGERS AND THE CARTESIAN PLANE

## 4 Are you ready?

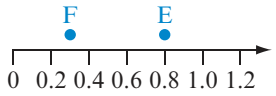
**1** 6 **2** B
**3** A is at 10; B is at 0; C is at 25.
**4** **a** 5 > 3 **b** 0 < 9
**5** A **6** B **7** D
**8** D **9** 2 **10** 0
**11**




**12**




**13**




**14** D **15** 50 km/h

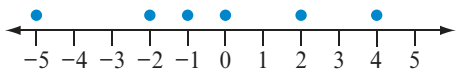
## 4A Understanding negative numbers

### 4A Start thinking!

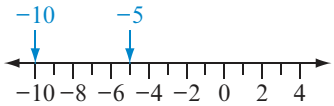
**1** Negative numbers have a negative (minus) sign in front.
**2** −27 is larger than −60.
**3** **a** Some examples are: +5, 29, +102.
**b** Some examples are: −8, −26, −354.
**4** Some examples are: +19, 0, −83

### Exercise 4A Understanding negative numbers

**1** **a** Some examples are given.
**i** −2, −4, −5 **ii** 2, 3, 4 **b** 0
**c**




**d** **i** 4 **ii** 2 **iii** 0 **iv** 4
**v** 2 **vi** −2 **vii** 4 **viii** −1
**e** **i** 1 **ii** 0 **iii** −4 **iv** −1
**v** −2 **vi** −3 **vii** −1 **viii** −4
**2** **a** −4 < 6 **b** −5 > −9 **c** 7 > 0
**d** 0 > −3 **e** −2 < −1 **f** −8 > −9
**g** 6 > −6 **h** −7 < −1 **i** −4 < −3
**3** **a** −25 **b** −27 **c** −75
**d** 0 **e** −81 **f** −300
**g** −853 **h** −2000 **i** −4444
**4** **a** −40 < 30 **b** 0 > −22 **c** −65 < −55
**d** 78 > −87 **e** 127 > −134
**f** −500 < 0 **g** −120 > −170
**h** 248 > −300 **i** −362 < −326
**5** **a** −6, −2, 0, 4, 6 **b** −15, −13, −3, 4, 10
**c** −12, −11, −2, −1, 1
**d** −18, −9, −5, 5, 10, 16
**e** −45, −20, −1, 0, 3, 35
**f** −77, −37, −17, −7, 7, 57
**6** **a** −10°C
**b** **i** higher **ii** higher
**iii** lower **iv** higher
**c** A larger number is higher on the scale.
**d** **i** Some examples are: −8°C, −2°C, 12°C.
**ii** Some examples are: −18°C, −12°C, −20°C.
**7** −4, −3, −2, −1, 0, +1, +2, +3, +4, +5, +6
**8** One possible answer is: −39, −33, −27.
**9** One possible answer is: +22, +3, 0, −9, −15.
**10** **a** −3 **b** +360 **c** −2 **d** −15
**e** +2230 **f** −28
**11** **a** **i** 6 units to the left of 0
**ii** 8 units to the right of 0
**iii** 2 units to the right of 0
**iv** 2 units to the left of 0
**v** 4 units to the left of 0
**vi** 4 units to the right of 0
**b** +2 and −2, −4 and +4
**c** **i** +9 **ii** −7 **iii** +25 **iv** −10
**v** −99 **vi** +53
**d** Some examples are: −24 and +24, −77 and +77, −3 and +3.
**12** **a** two
**b** **i** +3 **ii** −4 **iii** −5 **iv** +6
**c** **i** four blue counters
**ii** three red counters
**iii** seven blue counters
**iv** eight red counters
**v** two red counters **vi** five blue counters
**vii** nine red counters **viii** ten blue counters
**d** Since +1 and −1 are opposite integers (same distance from zero), one blue counter and one red counter 'cancel' each other out and you are left with zero.
**e** Two blue counters and two red counters form two zero pairs. These counters 'cancel' each other out. The three red counters left over represent −3.
**f** **i** +2 **ii** −1 **iii** +1 **iv** −4
**v** 0 **vi** 0
**13** **a**




**b** −10°C **c** −10°C, −7°C, −5°C
**d** cooler
**14** **a** nine levels **b** Level 5 **c** Level −3
**d** You go up three levels.
**e** You go down two levels.
**f** Higher; Level 1 is located higher on the scale than Level −1.
**g** 3 and −3, 2 and −2, 1 and −1
**h** So that you can see how many levels you are above or below ground level.
**15** **a** +100 **b** $20 **c** −20
**16** **a** +5 **b** −30 **c** 35 m **d** −3800
**17** **a** Paulo: −85; Kamilla: +98; Amad: +135; Jessica: −172
**b** Amad **c** Jessica
**18** One example is given in each case.
**a** five blue counters and two red counters
**b** five blue counters and seven red counters
**c** one blue counters and seven red counters

**d** eight blue counters and four red counters
**e** three blue counters and three red counters
**f** six blue counters and eleven red counters

## 4B Adding integers

### 4B Start thinking!

**1 a** eight blue counters **b** +8
**c** (+6) + (+2) = +8
**2 a** +4 **b** (+6) + (−2) = +4
**3 a** (−4) + (+3) = −1 **b** (−4) + (−3) = −7
**c** (−3) + (+5) = +2 **d** (−5) + (+5) = 0
**4 a** positive **b** negative
**c** positive, negative (or negative, positive)

### Exercise 4B Adding integers

**1 a** (−6) + (−1) = −7 **b** (−8) + (+3) = −5
**c** (+7) + (−4) = +3 **d** (−3) + (−6) = −9
**e** (+2) + (+4) = +6 **f** (−5) + (+3) = −2
**2 a** +5 **b** −9 **c** +2 **d** −5
**e** +3 **f** −2 **g** −3 **h** −6
**i** 0 **j** +1 **k** 0 **l** 0
**3 a** +8 **b** +5 **c** −8 **d** +10
**e** −2 **f** −8 **g** −4 **h** +5
**i** 0 **j** 0 **k** 0 **l** 0
**4** If the positive number is further from zero than the negative number, the sum will be positive. If the negative number is further from zero than the positive number, the sum will be negative. If the positive number and the negative number are the same distance from zero (opposite numbers), the sum will be zero.
**5 a** negative **b** zero **c** positive
**d** negative **e** positive **f** negative
**g** zero **h** negative **i** positive
**j** negative **k** positive **l** zero
**6 a** (−8) + (+3) = −5 **b** (+7) + (−4) = +3
**c** (−6) + (−1) = −7 **d** (−1) + (+4) = +3
**e** (+5) + (+3) = +8 **f** (−2) + (−7) = −9
**7 a** right, positive **b** left, negative
**8 a** +1 **b** +7 **c** −2 **d** −7
**e** −9 **f** +3 **g** −3 **h** +2
**i** 0 **j** +1 **k** 0 **l** +1
**m** 0 **n** +4 **o** −5
**9 a** +8 **b** +5 **c** −8 **d** +10
**e** −2 **f** −8 **g** −4 **h** +5
**i** 0 **j** 0 **k** 0 **l** 0
**11 a** −5 **b** +13 **c** −13 **d** −23
**e** −29 **f** +12 **g** −59 **h** −96
**i** −27 **j** −22 **k** +2 **l** +108
**12 a** (−18) + (+98) = +80 **b** 80°C
**13 a** (−8) + (+5) = −3 **b** −3°C
**14 a** −26 **b** (−26) + (+40) = +14 **c** $14
**15 a** −50 **b** (−50) + (+35) = −15 **c** $15
**16** Some examples are: −6 and +2, −18 and −3, 0 and −4.
**17** Some examples are: −9, −13, −7, −45.
**18 a** +4 **b** −7 **c** −1 **d** +10

## 4C Subtracting integers

### 4C Start thinking!

**1 a** four blue counters **b** +4
**c** (+6) − (+2) = +4
**2 a** one red counter **b** −1
**c** (−4) − (−3) = −1

### Exercise 4C Subtracting integers

**1 a** +2 **b** −3 **c** −6 **d** +3
**2 a** +4 **b** −3 **c** −7
**d** +1 **e** −3 **f** +5
**g** 0 **h** 0 **i** 0
**3** no, (−2) + (0) = −2
**4 a** +4 **b** −3 **c** −1 **d** +2
**5 a** −5 **b** +4
**6** By adding zero pairs we can introduce counters of the second colour so that they can be used for the subtraction. The number of zero pairs to be added is the same as the number of counters of the second colour that are to be removed.
**7 a** +6 **b** −7 **c** −6
**d** +9 **e** −5 **f** +4
**g** +8 **h** −8 **i** +10
**8 a** −4 **b** +3
**9** The number of zero pairs to be added is the same as the number of extra counters of the same colour that are to be removed.
**10 a** −3 **b** +5 **c** −1
**d** +4 **e** −2 **f** +5
**g** +1 **h** +4 **i** −2
**11** You need to add zero pairs when there are no counters of the correct colour to take away.
**12 a** −3 **b** +5 **c** −1 **d** 0
**e** +4 **f** −9 **g** +7 **h** −9
**i** −1 **j** +2 **k** −5 **l** +2
**m** 0 **n** −6 **o** +4
**13 a** (−6) − (−4) = −2 **b** (+7) − (+3) = +4
**c** (+2) − (−8) = +10 **d** (+2) − (+6) = −4
**e** (−3) − (−4) = +1 **f** (−1) − (+5) = −6
**14 a** left, negative **b** right, positive
**15 a** +5 **b** +1 **c** −10 **d** +6
**e** −8 **f** −2 **g** −3 **h** 0
**i** −4 **j** +5 **k** +10 **l** −6
**m** −1 **n** +3 **o** −6
**16 a** −3 **b** +5 **c** −1 **d** 0
**e** +4 **f** −9 **g** +7 **h** −9
**i** −1 **j** +2 **k** −5 **l** +2
**m** 0 **n** −6 **o** +4
**17** They are the same.
**18 a** −5 **b** −11 **c** −49
**d** +9 **e** −80 **f** +65
**g** −10 **h** −71 **i** +300
**19 a** (−15) − (+4) = −19 or (+4) − (−15) = +19
**b** −19°C or 19°C
**20 a** (+14) − (+20) = −6 **b** −6°C
**21 a** +74 **b** (−31) − (+74) **c** $105
**22** Some examples are: (−1) − (+2) = −3, (+4) − (+6) = −2, (−3) − (−2) = −1.

**23** Some examples are: +16, +21, +32.
**24** **a** −5 **b** +2 **c** −8 **d** 7

## 4D Simplifying addition and subtraction of integers

### 4D Start thinking!

**1** **a** **i** +8 **ii** +8 **b** right
**c** You move in the same direction along the number line (to the right) when adding a positive number or subtracting a negative number.
**d** They both produce the same result.
**2** **a** **i** +4 **ii** +4 **b** left
**c** You move in the same direction along the number line (to the left) when subtracting a positive number or adding a negative number.
**d** They both produce the same result.
**3** **a** positive **b** –
**c** – **d** negative, +

### Exercise 4D Simplifying addition and subtraction of integers

**1** the same; different
**2** **a** – **b** + **c** – **d** +
**e** – **f** +
**3** **a** –3 + 5 **b** 6 – 5 **c** –2 – 4
**d** 5 + 7 **e** 7 + 1 **f** 4 – 9
**4** **a** 2 **b** 4 **c** –4 **d** –5
**e** –1 **f** 5 **g** 8 **h** –1
**i** +11 **j** –8 **k** –2 **l** –7
**5** **a** –9 **b** 9 **c** –9 **d** 4
**e** –1 **f** 2
**6** **a** 2 **b** 1 **c** –6 **d** 12
**e** 8 **f** –5
**7** **a** 5 **b** –3 **c** 6 **d** –7
**e** –4 **f** –4 **g** –5 **h** –6
**i** 0 **j** –3 **k** –14 **l** 5

**8** **a**

| + | −2 | −1 | 0 | +1 | +2 |
|---|---|---|---|---|---|
| −2 | −4 | −3 | −2 | −1 | 0 |
| −1 | −3 | −2 | −1 | 0 | +1 |
| 0 | −2 | −1 | 0 | +1 | +2 |
| +1 | −1 | 0 | +1 | +2 | +3 |
| +2 | 0 | +1 | +2 | +3 | +4 |

**b**

| + | −5 | −3 | −2 | 2 | 4 |
|---|---|---|---|---|---|
| 7 | 2 | 4 | 5 | 9 | 11 |
| 5 | 0 | 2 | 3 | 7 | 9 |
| 0 | −5 | −3 | −2 | 2 | 4 |
| −2 | −7 | −5 | −4 | 0 | 2 |
| −4 | −9 | −7 | −6 | −2 | 0 |

**9** **a** 2 **b** –74 **c** –4
**d** –21 **e** –10 **f** –191
**g** 348 **h** –72 **i** –3000
**11** **a** **i** –6 **ii** –6 **iii** –6
**b** **i** –6 **ii** –6 **iii** –6
**c** –6, –6 **d** the same **e** –6

**12** **a**

| | | |
|---|---|---|
| 4 | −3 | 2 |
| −1 | 1 | 3 |
| 0 | 5 | −2 |

**b**

| | | |
|---|---|---|
| 1 | 0 | −4 |
| −6 | −1 | 4 |
| 2 | −2 | −3 |

**c**

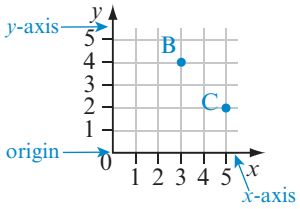
| | | |
|---|---|---|
| 6 | −6 | −9 |
| −18 | −3 | 12 |
| 3 | 0 | −12 |

**13** **a** 2 **b** –5 **c** 0 **d** –9 **e** –1 **f** –2
**14** **a** 6°C **b** 5°C
**c** Tue 9°C, Wed 5°C, Thu 4°C, Fri 6°C, Sat 5°C
**d** Tue
**15** **a** +182 **b** (+182) – (+200) or 182 – 200
**c** –18 **d** Raj owes $18.
**16** third level below ground level; 7 – 10 = –3
**17** 62 m; (+50) – (–12) = 50 + 12
**18** 1 m; (–2) + (+3) = –2 + 3
**19** **a** deposit **b** withdrawal
**c** Lily has money in her account.
**d** Lily owes money to the bank.
**e** +$28 **f** withdrawal of $9
**20** Some possible answers are: –3 and 3, 5 and –5, 20 and –20.
**21** **a** 7 **b** 3 **c** –4
**d** –10 **e** –1 **f** –10
**g** –3 **h** –15 **i** –14
**22** **a** –8 **b** 9 **c** 4 **d** 14
**e** –15 **f** –5 **g** –11 **h** 3

## 4E Introducing the Cartesian plane

### 4E Start thinking!

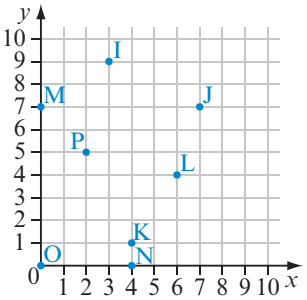
**1** 3 units
**2, 3**




**5** Move 5 units along the $x$-axis from the origin and then 2 units up parallel to the $y$-axis.

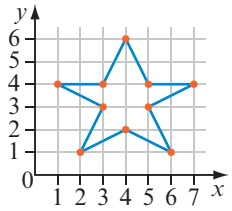
### Exercise 4E Introducing the Cartesian plane

**1** The $x$-coordinate comes first. One possible answer is: $x$ comes before $y$ in the alphabet.
**2** A(3, 5), B(6, 8), C(1, 2), D(9, 6), E(8, 3), F(2, 4), G(2, 7), H(5, 3)
**3** (0, 0)
**4** **a** F and G **b** E and H
**c** B **d** D
**5** **i** **a** 3 units right and 9 units up
**b** 7 units right and 7 units up
**c** 4 units right and 1 unit up
**d** 6 units right and 4 units up
**e** 0 units right and 7 units up
**f** 4 units right and 0 units up
**g** 0 units right and 0 units up (at the origin)
**h** 2 units right and 5 units up
**ii**



**6** **a** K and N; M and O
**b** J and M; N and O
**c** M and O **d** N and O
**e** O **f** L

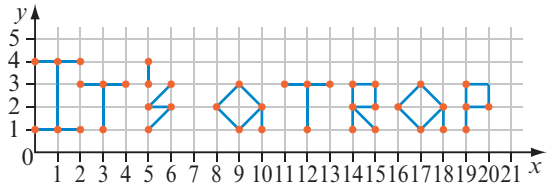
**7** star



**8** One possible answer is: eyes at (3, 5) and (5, 5); nose at (4, 4); mouth to start at (2, 3) then join to (4, 2) and then to (6, 3).

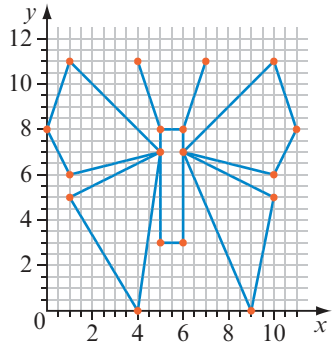
**9** **a** The email password is TICKLES.
**b** (6, 4) (2, 1) (5, 4) (1, 2) (2, 4) (6, 4) (5, 4) (2, 2) (2, 5) (4, 1) (6, 1) (5, 4) (3, 3)

**10**



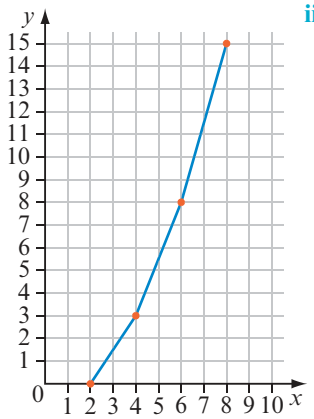
It's a trap

**11**



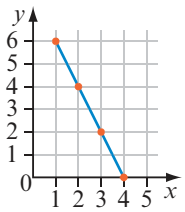
butterfly

**12** **a** **i** (2, 0), (4, 3), (6, 8), (8, 15)
**ii**



**iii** curved line

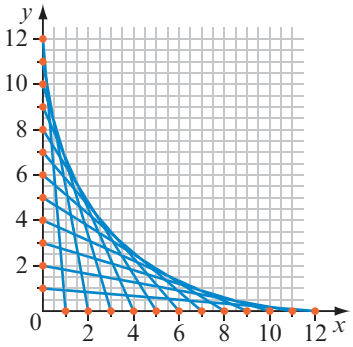
**b** **i** (1, 6), (2, 4), (3, 2), (4, 0)
**ii**



**iii** straight line

**13** **a** (12, 6) (10, 7) (6, 6) (4, 5) (3, 5) (3, 7) (1, 11) (1, 13) (4, 14) (5, 16) (7, 16) (7, 17) (10, 17) (10, 16) (12, 15)
**c** One possible answer is: Perth (3, 7), Adelaide (14, 5), Melbourne (16, 3), Sydney (19, 6), Brisbane (21, 10), Darwin (7, 17), Canberra (18, 5).

**15**

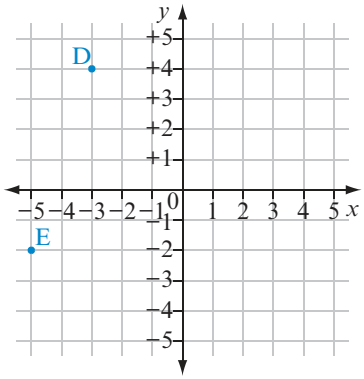


## 4F Negative numbers and the Cartesian plane

### 4F Start thinking!

**2** **a** 3 units left and 4 units up
**b** (–3, 4) **c** quadrant 2

**3** **a**




**c** quadrant 3

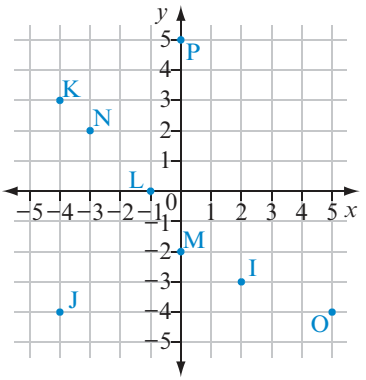
### Exercise 4F Negative numbers and the Cartesian plane

**1** A(2, 5), B(3, –2), C(4, 0), D(–5, –4), E(–3, 3), F(0, 2), G(2, –4), H(0, –3)

**2** A is in quadrant 1; B is in quadrant 4; C is on the *x*-axis; D is in quadrant 3; E is in quadrant 2; F is on the *y*-axis; G is in quadrant 4; H is on the *y*-axis.

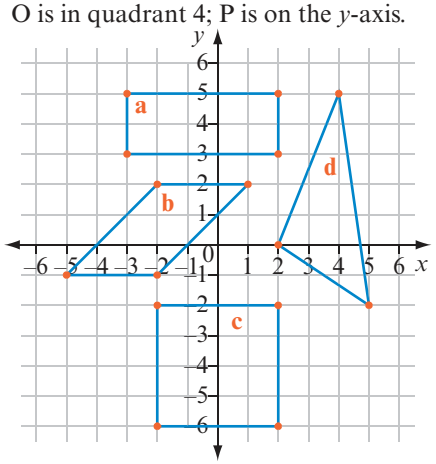
**3** **i** **a** 2 units right and 3 units down
**b** 4 units left and 4 units down
**c** 4 units left and 3 units up
**d** 1 unit left and 0 units up or down
**e** 0 units right or left and 2 units down
**f** 3 units left and 2 units up
**g** 5 units right and 4 units down
**h** 0 units right or left and 5 units up

**ii**




**4** I is in quadrant 4; J is in quadrant 3; K is in quadrant 2; L is on the *x*-axis; M is on the *y*-axis; N is in quadrant 2; O is in quadrant 4; P is on the *y*-axis.

**5**



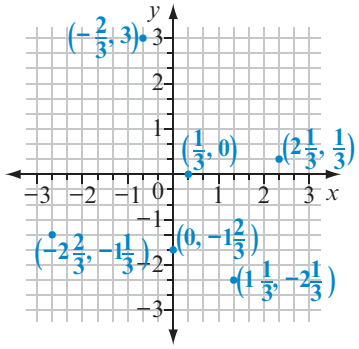

**a** rectangle **b** parallelogram
**c** square **d** triangle

**6 a i** quadrant 3 **ii** quadrant 1
**b** quadrants 2 and 4

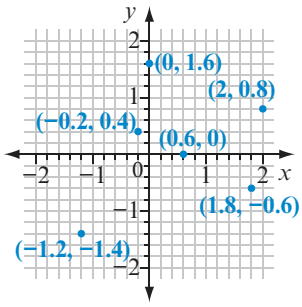
**7 a** A($1\frac{1}{2}$, 2), B(–2, $-3\frac{1}{2}$), C($-3\frac{1}{2}$, $-1\frac{1}{2}$), D($-4\frac{1}{2}$, $2\frac{1}{2}$), E($-2\frac{1}{2}$, $3\frac{1}{2}$), F($4\frac{1}{2}$, $3\frac{1}{2}$), G($\frac{1}{2}$, 0), H(0, $4\frac{1}{2}$), I($3\frac{1}{2}$, –2), J($4\frac{1}{2}$, $-\frac{1}{2}$)

**b** A(–0.4, 0.6), B(0.4, 0.4), C(–0.8, –0.6), D(0.8, 0), E(0.5, –0.7), F(0, –0.5), G(–0.9, 0.3), H(0.2, –0.2), I(0.3, –0.9), J(–0.5, –0.3)

**8 a**



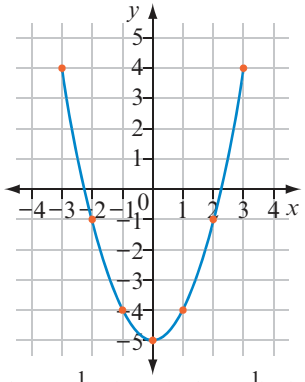

**b**




**9** Compare your answer with those of your classmates.

**10 a i** (–3, 4), (–2, –1), (–1, –4), (0, –5), (1, –4), (2, –1), (3, 4)

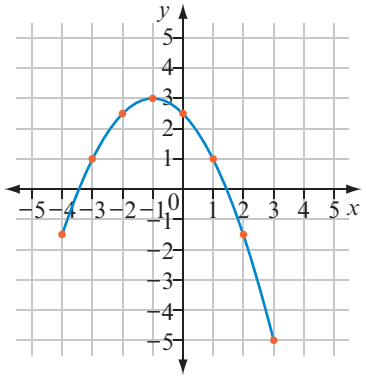
**ii**




**iii** curved line

**b i** (–4, $-1\frac{1}{2}$), (–3, 1), (–2, $2\frac{1}{2}$), (–1, 3), (0, $2\frac{1}{2}$), (1, 1), (2, $-1\frac{1}{2}$), (3, –5)

**ii**



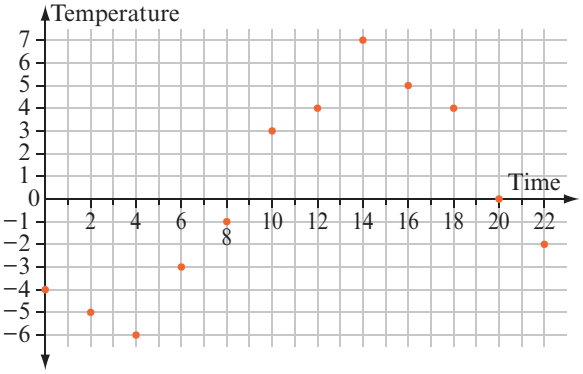

**iii** curved line

**11 b** Negative numbers not needed. Scale should be from 0 to 22 on the horizontal axis.

**c** Scale should go from –6 to 7 on the vertical axis.

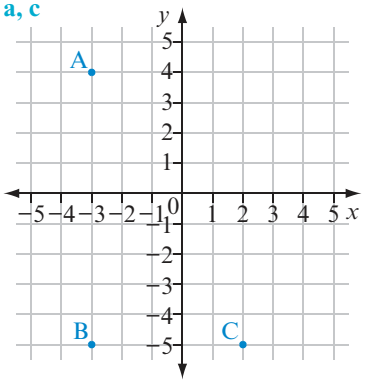
**d** (0, –4), (2, –5), (4, –6), (6, –3), (8, –1), (10, 3), (12, 4), (14, 7), (16, 5), (18, 4), (20, 0), (22, –2)

**a, e**



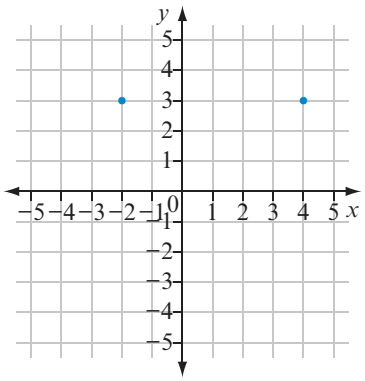
**f** –6°C at 4 am **g** 7°C at 2 pm

**12 a, c**




**b** 9 units
**d** 5 units
**e** D(2, 4)

**13 a** 6 units
**b** One possible answer is: (4, –3) and (–2, –3).
**c** No; other possible answers are: (4, 9) and (–2, 9); (1, 6) and (1, 0).




## 4G Interpreting graphs

### 4G Start thinking!

**1 a** time **b** distance from home

**2** Kurt leaves home and travels 240 m in 2 minutes (120 m each minute). He stops at the shop for 2 minutes to buy a magazine. He then takes 4 minutes to walk 120 m to his friend's house (30 m each minute) but finds that he is not there. He then jogs 360 m in 2 minutes to return home (180 m each minute).

### Exercise 4G Interpreting graphs

**1 a i** 0 m **ii** 120 m **iii** 240 m
**b i** 120 m **ii** 120 m
**c** Kurt travelled the same distance in each minute, so he was travelling at a constant speed.

**2 a i** 240 m **ii** 240 m **iii** 240 m
**b i** 0 m **ii** 0 m
**c** Kurt was stationary.

**3 a i** 240 m **ii** 270 m **iii** 300 m
**iv** 330 m **v** 360 m
**b** 30 m
**c** Kurt travelled the same distance in each minute, so he was travelling at a constant speed.

**4 a** 360 m **b** 2 minutes

**5** D; Kurt travels 180 m each minute compared to 120 m in section A, 0 m in section B and 30 m in section C.

**6** Section B is horizontal; it has no slope. This means that Kurt is not moving.

**7 a** B: 0 m; D: 180 m **b** D
**c** The steeper the slope, the higher the speed.

**8 a i** 0 km **ii** 2 km **iii** 2 km
**iv** 3 km **v** 4 km
**b** one hour into her journey
**c** 1 hour **d** 2 km **e** A
**f** Sarah's distance from home has not changed.
**g** Sarah leaves home and travels 2 km in 1 hour. She stops at her friend's house for 1 hour. She then takes 2 hours to slowly walk 2 km to the music store.

**9 a** 2 km/h **b** 0 km/h **c** 1 km/h

**10 a** 120 m/min **b** 0 m/min **c** 180 m/min

**11 a** 2 km **b** 4 h
**c** from home to school; steeper line
**d** 4 km

**12 a** 3 km **b** $6\frac{1}{2}$ hours after leaving home
**c** 1 km **d** 1 h **e** 4 pm
**f** from dental clinic to home; 4 km/h
**g** Nina leaves home and travels 3 km in 1 hour to reach school. She stays at school for $5\frac{1}{2}$ hours before taking 1 hour to walk 1 km to the dental clinic. After a 1-hour dental appointment, she takes $\frac{1}{2}$ hour to travel 2 km home.

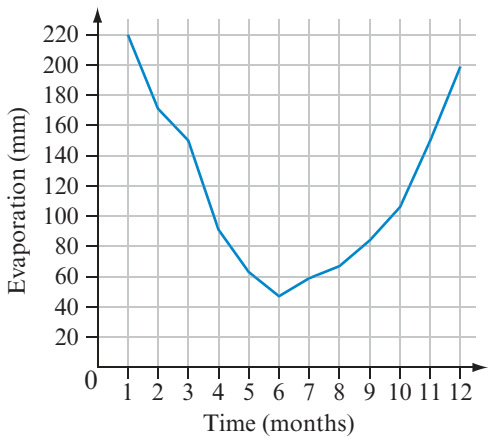
**13 a** false **b** true **c** false **d** true

**16 a** Tom **b** Sue

**17** No; you can't be in different places at the same time.

**18 a** –10°C **b** –5°C
**c** after 3 min **d** 35°C
**e** from 3 to $3\frac{1}{2}$ min; from 5 to $5\frac{1}{2}$ min
**f** from $3\frac{1}{2}$ to 5 min and from $5\frac{1}{2}$ to 6 min
**g** 1 min
**h** Temperature of the frozen soup after 3 minutes of heating rose by 15°C. The heating was paused for 30 seconds for the soup to be stirred. It was then heated for $1\frac{1}{2}$ minutes to increase the temperature by 30°C. Heating was again paused for 30 seconds for the soup to be stirred. After another one minute of heating, the temperature of the soup rose 10°C to reach 45°C.

**20 a** 220 mm **b** 150 mm
**d**



**e** January, 220 mm **f** June, around 47 mm
**g** Between March and April; steepest line
**h** Higher evaporation during hotter, dry months. Factors such as temperature, humidity and wind affect the amount of evaporation.

## 4 Chapter review

### MULTIPLE-CHOICE

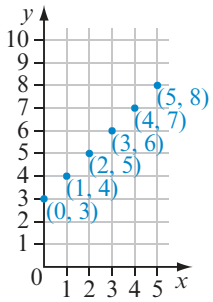
**1** C **2** A **3** D **4** D
**5** B **6** C **7** B **8** A

### SHORT ANSWER

**1** **a** false **b** true **c** true **d** false
**2** **a** –10, –5, 0, 2, 12, 25 **b** –12, –6, –3, 0, 6, 9
**3** **a** –86 **b** +615 **c** +8840 **d** –35
**4** **a** –9 **b** +15 **c** –2 **d** –1
**5** **a** –8 **b** +1 **c** –14 **d** +8
**6** **a** 2 **b** –7 **c** –3 **d** –5
**e** 0 **f** –40
**7** 78°C **8** 38 m
**9** **a**

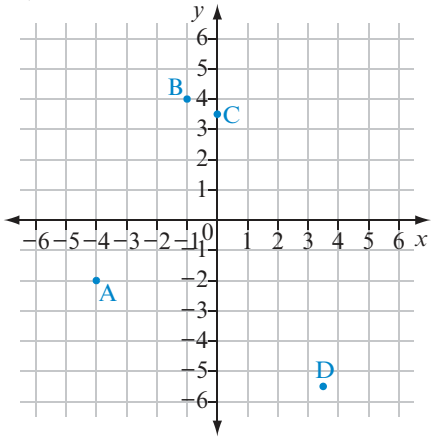
| *x*-coordinate | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| *y*-coordinate | 3 | 4 | 5 | 6 | 7 | 8 |
| coordinates | (0, 3) | (1, 4) | (2, 5) | (3, 6) | (4, 7) | (5, 8) |

**b**




**10** A(3, 1), B(–3, 3), C(4, –2), D(1, 0), E(–2, –3), F(0, –2)

**11**



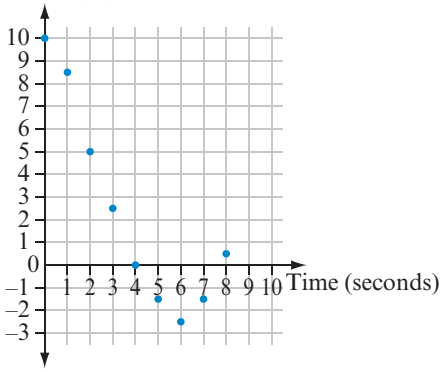

**12** **a** 4 km **b** A, 4 km/h
**c** A: journey from home to school
B: Ben at school
C: journey from school to shop
D: Ben at shop with friends
E: journey from shop to home

### NAPLAN-STYLE PRACTICE

**1** –5 **2** 2 **3** –20 **4** 4°C
**5** Level 1 above the surface
**6** Level 3 below the surface
**7** –6 **8** –1 **9** –6°C **10** 14
**11** –16 **12** $20 **13** 8 **14** (4, 3)
**15** B **16** 2 **17** D **18** false
**19** G **20** (6, –3) **21** H
**22** B and E **23** A

### ANALYSIS

**a** **i** 10 m **ii** –2 m **iii** 0 **iv** –8 m
**b** **i** 12 m **ii** 6 m **c** +5 m **d** –6 m
**e** (0, 10), (1, 8.5), (2, 5), (3, 2.5), (4, 0), (5, –1.5), (6, –2.5), (7, –1.5), (8, 0.5)
**f** Height (m)




**g** The seagull dived to the water, plunged under the water and then came back to the surface.

## 4 Connect

For feedback on this open-ended task, see your teacher.

# CHAPTER 5 ALGEBRA AND EQUATIONS

## 5 Are you ready?

**1** **a** 7 **b** 9 **c** 13 **d** 5
**2** **a** 4 **b** 1 **c** 9 **d** 0
**3** **a** 12 **b** 30 **c** 48
**4** **a** 4 **b** 1 **c** 7
**5** C
**6** **a** 4 + 7 **b** 6 × 5 **c** 8 – 3 **d** 12 ÷ 2
**7** B **8** 6
**9** **a** 5 **b** 6 **c** 24 **d** 11
**10** **a** true **b** false
**11** B **12** 45 **13** 2
**14** **a** C **b** 2

## 5A Understanding rules

### 5A Start thinking!

**1** **b** Piece of dough that comes out is three times as long as piece that goes into machine.
**2** **a** Variables can have different values.
**b** C
**3**

| IN | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|
| OUT | 30 | 60 | 90 | 120 | 150 | 180 |

**4** **a** 300 cm **b** 80 cm

**5** **a** One example is:

| IN | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| OUT | 20 | 40 | 60 | 80 |

**b** OUT = IN × 2

**6** A rule is useful when describing relationship between two or more variables because it can be used to work out the value of one variable for different values of the other variables.

### Exercise 5A Understanding rules

**1** **a** 13 **b** 10 **c** 27 **d** 33
**e** 16 **f** 20 **g** 18 **h** 9

**2** **a** 6, 7, 8, 9, 10 **b** 4, 6, 8, 10, 12
**c** 2, 5, 8, 14, 22 **d** 10, 12, 14, 16, 18
**e** 1, 4, 11, 14, 20 **f** 0, 8, 40, 64, 88

**3** **a** 11, 14, 17, 20, 23 **b** 16, 23, 30, 37, 44
**c** 0, 10, 20, 30, 40 **d** 20, 24, 28, 32, 36
**e** 15, 22, 31, 42, 55 **f** 0, 7, 16, 27, 40

**4** **a** 13, 15, 19, 1, 21, 9 **b** 5, 7, 11, 9, 29, 17
**c** 24, 32, 48, 2, 7, 4 **d** 3, 4, 6, 16, 56, 32
**e** 8, 10, 14, 6, 26, 14 **f** 12, 16, 24, 4, 14, 8

**5** **a** 6, 14, 20, 5, 8, 12 **b** 9, 13, 17, 6, 7, 8
**c** 3, 9, 15, 9, 10, 14 **d** 0, 1, 2, 25, 40, 50

**6** **a** 12 **b** 4

**7** **a** Add 6 to IN number to get OUT number.
**b** Multiply IN number by 3 to get OUT number.
**c** Subtract 9 from IN number to get OUT number.
**d** Multiply IN number by 7 to get OUT number.
**e** Divide IN number by 8 to get OUT number.
**f** Add 15 to IN number to get OUT number.
**g** Subtract 14 from IN number to get OUT number.
**h** Divide IN number by 11 to get OUT number.

**8** **a** C **b** A **c** D **d** B

**9** **i** **a** OUT = IN + 7 **b** OUT = IN – 1
**c** OUT = IN × 4 **d** OUT = IN ÷ 2
**e** OUT = IN + 2 **f** OUT = IN × 2
**ii** **a** OUT = (IN + 3) × 2
**b** OUT = IN × 4 + 1
**c** OUT = (IN – 2) × 3
**d** OUT = IN ÷ 5 – 2

**10** **a**

| IN (number of apples) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| OUT (amount of juice in mL) | 100 | 200 | 300 | 400 | 500 | 600 | 700 |

**b** number of apples and amount of juice
**c** **i** 600 mL **ii** 1300 mL **iii** 2000 mL
**d** **i** four apples **ii** nine apples
**iii** 30 apples

**11** **a** multiply, 2
**b** number of muesli bars, total cost
**c** **i** $4 **ii** $10 **d** **i** $22 **ii** $76
**e** Table was easier to use because answers were already worked out.
**f** Rule was easier to use because it would take a long time to work out all values up to 38.
**g** **i** three muesli bars **ii** 17 muesli bars

**12** **a** 3, 6, 9, 12, 15, 18
**b** The number of triangles and the number of toothpicks.
**c** **i** 15 toothpicks **ii** 42 toothpicks
**d** Multiply the number of triangles by 3 to get the number of toothpicks.
**e** OUT = IN × 3
**f** **i** 360 toothpicks **ii** 79 triangles

## 5B Using pronumerals

### 5B Start thinking!

**1** **a** 5 **b** 8
**2** 4, 5, 6, 7, 8, 9
**3** number of pencils in case and total number of pencils
**4** OUT = IN + 3 **5** B

**6**

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $y$ | 4 | 5 | 6 | 7 | 8 | 9 |

**7** 14 pencils in total

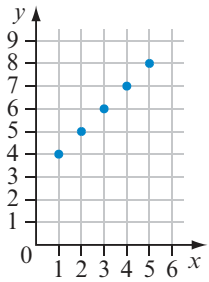
### Exercise 5B Using pronumerals

**1** **a** $b = a + 8$ **b** $b = a - 5$ **c** $b = 6a$
**d** $b = \frac{a}{3}$ **e** $b = 2a$ **f** $b = a + 1$
**g** $b = \frac{a}{4}$ **h** $b = a - 9$ **i** $b = 11a$
**j** $b = 6 + a$ **k** $b = 3 - a$ **l** $b = \frac{4}{a}$
**m** $b = 2a + 5$ **n** $b = 6a - 2$ **o** $b = 3a + 7$

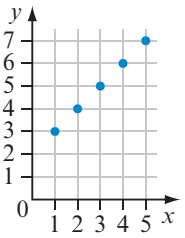
**2** **a** $b = a + 7$ **b** $n = m - 4$ **c** $y = 5x$
**d** $f = \frac{e}{3}$

**3** **a** $y = x + 2$ **b** $t = 3p$ **c** $d = c - 6$
**d** $n = 7k$ **e** $y = \frac{x}{2}$ **f** $h = m + 4$

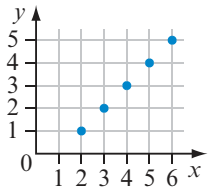
**4** **a**



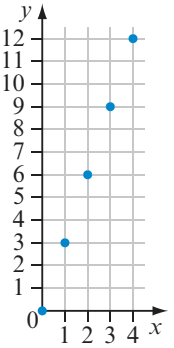

**b**




**c**




**d**




**5** **a** $y = x + 3$ **b** $y = x + 2$
**c** $y = x - 1$ **d** $y = 3x$

**6** **a**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 5 | 6 | 7 | 8 | 9 |

**b**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | 2 | 4 | 6 | 8 |

**7** **a** $y = x + 4$ **b** $y = 2x$

**8** **a** 22, 23, 24, 25, 26 **b** $n = x + 4$

**c** So that you know what the variables are and what the relationship is describing.

**9** **a** number of people, number of eggs

**b** $p$ represents number of people and $e$ represents number of eggs.

**c**

| $p$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $e$ | 2 | 4 | 6 | 8 | 10 |

**d** $e = 2p$

**10** If $a$ represents amount of money she originally had and $m$ represents amount of money she now has then $m = a + 20$.

**11** **a** amount of money Laurence spends during the week

**b** amount of money Laurence saves at end of week

**c**

| $m$ | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| $t$ | 45 | 40 | 35 | 30 | 25 |

**d** Both $m$ and $t$ can change depending on the amount Laurence spends.

**12** **a** number of hamburgers bought

**b** cost of buying hamburgers

**c**

| $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $c$ | 5 | 10 | 15 | 20 | 25 |

**14** **a** $y = 2x + 3$ **b** $y = 5x - 2$ **c** $y = 3x + 4$

**15** **a** Numbers above $x$ values show difference between each consecutive number.

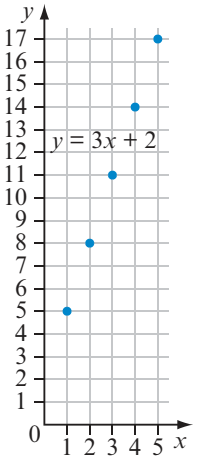
**b** Numbers below $y$ values show difference between each consecutive number.

**c** Difference between consecutive $y$ values is identical. This difference is same as number multiplied to $x$ in formula.

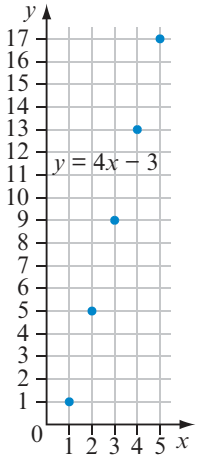
**d** Finding differences in $y$ values gives starting point to finding formula: gives value of number multiplied to $x$.

**16** **a** $y = 3x + 2$ **b** $y = 4x - 3$ **c** $y = 2x + 6$
**d** $y = 7x + 1$ **e** $y = 3x - 8$ **f** $y = 5x - 4$

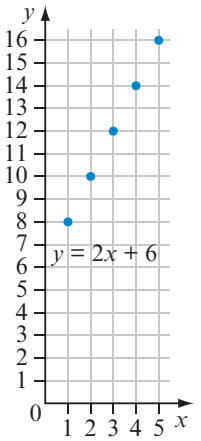
**17** **a**



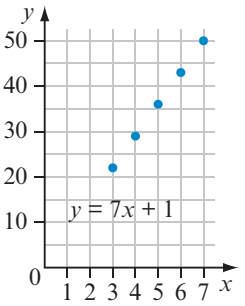

**b**



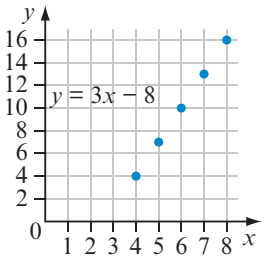

**c**



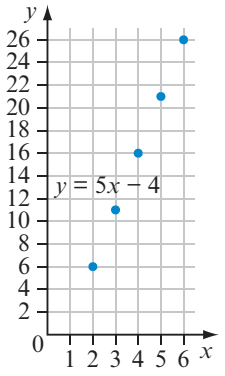

**d**




**e**




**f**




**18** **a** number of diamonds in track

**b** number of concrete slabs needed to make track

**c** Design would have five diamonds in track.

## 5C Terms, expressions and equations

### 5C Start thinking!

**1** **a** cost = 1 × \$15 + \$10 = \$25

**b** cost = 3 × \$15 + \$10 = \$55

**2** cost of hiring the canoe for $n$ hours without booking fee

**3** $15n$ is cost of hiring canoe for $n$ hours and 10 is booking fee. Expression gives total cost.

**4** **a** $15n$ **b** **i** $n$ **ii** 15

**5** **a** 10 **b** It does not have a pronumeral.

**6** 2 **7** 2 hours

### Exercise 5C Terms, expressions and equations

**1** **a** False, it is an equation as it has an equals sign.
**b** true **c** true **d** False, constant is 3.
**e** true **f** true
**g** False, it is an equation as it has an equals sign.
**h** true **i** true
**j** False, it is an expression as there is no equals sign.
**k** true **l** true

**2** **a** equation **b** expression **c** equation
**d** equation **e** expression **f** expression
**g** equation **h** expression **i** equation
**j** expression

**3** $d = 3f + 7$, $a + b + c = 11$, $k = 8m$

**4** **a** **i** 2 **ii** $x$ **iii** 5 **iv** 6
**b** **i** 3 **ii** $a, b$ **iii** 2, 4 **iv** 3
**c** **i** 2 **ii** $m, n$ **iii** 7 **iv** 4
**d** **i** 2 **ii** $k$ **iii** 11 **iv** 2

**e** **i** 2 **ii** $c, d$ **iii** 4, 7 **iv** none
**f** **i** 1 **ii** $y$ **iii** 13 **iv** none
**g** **i** 2 **ii** $f$ **iii** 1 **iv** 5
**h** **i** 4 **ii** $x, w, y$ **iii** 6, 1, 3 **iv** 1
**i** **i** 3 **ii** $p, m$ **iii** 8, 1 **iv** 10

**5** **a** $a + 7$ **b** $\frac{b}{2}$ **c** $c - 5$
**d** $3d$ **e** $8 - e$ **f** $2f$
**g** $\frac{4}{g}$ **h** $h + 9$ **i** $k + m + n$

**6** **a** 2 **b** 1 **c** 2
**d** 1 **e** 2 **f** 1
**g** 1 **h** 2 **i** 3

**7** One example is: $a + 2b + 3c + 6$

**8** **a** $a + 8$ **b** $a - 15$ **c** $3a$ **d** $\frac{a}{7}$

**9** **a** $4n + 5$ **b** $\frac{n}{3} + 7$ **c** $2n - 9$ **d** $6n - 1$

**10** Variable would be written as $x$ rather than $n$ but operations performed on variable would be unchanged.

**11** **a** $x + 3$ **b** $(x + 3) \times 4$ or $4 \times (x + 3)$
**c** $4(x + 3)$

**12** **a** $k + 7$ **b** $(k + 7) \div 3$ **c** $\frac{k + 7}{3}$

**13** **a** $7(x + 6)$ **b** $2(x - 3)$ **c** $5(x + 1)$
**d** $8(x - 2)$ **e** $\frac{x + 4}{7}$ **f** $\frac{x - 5}{11}$
**g** $\frac{9 + x}{2}$ **h** $\frac{1 - x}{6}$

**14** **a** $5(x + 4)$ **b** $2(x - 11)$ **c** $\frac{x + 6}{7}$
**d** $\frac{x - 3}{4}$ **e** $3(10 - x)$ **f** $\frac{6 - x}{9}$

**16** **a** $x + 5 = 11$ **b** $x - 3 = 4$ **c** $6x = 18$
**d** $\frac{x}{2} = 5$ **e** $2x = 24$ **f** $\frac{x}{2} = 9$

**18** Equations and formulas are of the form: expression = expression. They contain terms and an equals sign. Formulas are a special type of equation where there is more than one variable.

**20** **a** $n + 10$ **b** $3n$ **c** $\frac{n}{2}$ **d** $n - 4$

**21** **a** **i** $2p$ **ii** $52p$ **b** $p + w$

**22** **a** One example is: let $a$ represent cost of an apple, $b$ represent cost of a banana, $c$ represent cost of an orange.
**b** **i** $10a$ **ii** $7b$ **iii** $6c$
**c** $10a + 7b + 6c$ **d** $w = 10a + 7b + 6c$

**23** **a** $g + b$ **b** $3(g + b)$ **c** $g + b + 1$
**d** $3g + 2b + 4$ **e** $b - 3$ **f** $\frac{g + b}{2}$

**25** **a** $a + 4 = 12$ **b** $20 - b = 12$
**c** $6c = 12$ **d** $\frac{d}{5} = 12$
**e** $e - 7 = 12$ **f** $5f + 2 = 12$

**26** **a** $x + 5$ **b** $x + 1$ **c** $x - 4$
**d** $x$ **e** $2x + 7$ **f** $3x + 1$

**27** $y \times y = y^2$

## 5D Evaluating expressions

### 5D Start thinking!

**1** **a** 12 points **b** 3 points
**c** 15 points **d** 38 points

**2** **a** $6g$ **b** $b$ **3** $6g + b$

**4** $g$ represents the number of goals and $b$ represents the number of behinds. As the numbers of goals and behinds are variables (the values can change) they can be represented by pronumerals.

**5** $6 \times 2 + 3$ **6** 15

**7** They are the same.

**8** **a** 38 **b** 60 **c** 73 **d** 132

### Exercise 5D Evaluating expressions

**1** **a** 11 **b** 4 **c** 18 **d** 3
**e** 38 **f** 15

**2** **a** 17 **b** 4 **c** 40 **d** 1
**e** 13 **f** 0

**3** **a** 8 **b** 14 **c** 17 **d** 26
**e** 5 **f** 35

**4** **a** 7 **b** 19 **c** 3 **d** 31
**e** 39 **f** 35

**5** **a** 6 **b** 9 **c** 30 **d** 0
**e** 17 **f** 2 **g** 48 **h** 1
**i** 12 **j** 30 **k** 5 **l** 10

**6**

| $x$ | $x + 6$ | $5x$ | $13 - x$ | $6x + 7$ | $3(x - 1)$ | $\frac{x + 4}{2}$ | $4x - 5$ | $\frac{x}{2} + 8$ | $\frac{120}{x}$ |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 8 | 10 | 11 | 19 | 3 | 3 | 3 | 9 | 60 |
| 4 | 10 | 20 | 9 | 31 | 9 | 4 | 11 | 10 | 30 |
| 6 | 12 | 30 | 7 | 43 | 15 | 5 | 19 | 11 | 20 |
| 8 | 14 | 40 | 5 | 55 | 21 | 6 | 27 | 12 | 15 |
| 10 | 16 | 50 | 3 | 67 | 27 | 7 | 35 | 13 | 12 |
| 12 | 18 | 60 | 1 | 79 | 33 | 8 | 43 | 14 | 10 |

**7** $a \times a$

**8** **a** 9 **b** 25 **c** 14 **d** 17
**e** 36 **f** 53

**9** **a** 4 **b** 52 **c** 118 **d** 36.4
**e** 29.8 **f** 2.8 **g** 1 **h** 2

**10** **a** 7 **b** 1 **c** 13 **d** 2
**e** 2 **f** 1 **g** 10 **h** 33
**i** 3

**11** **a** $5r + 2d$ **b** \$25 **c** \$54

**12** **a** Let $m$ represent number of months and $y$ represent number of years.
**b** $m = 12y$
**c** **i** 120 **ii** 204 **iii** 102 **iv** 9

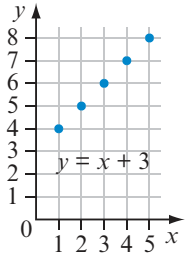
**13** **a** $l = 8n$
**b** **i** 16 **ii** 40 **iii** 64
**iv** 88 **c** 1848

**14** **a** **i** 5 **ii** 4 **iii** 2.1 **iv** 5.5
**b** 20 m/s

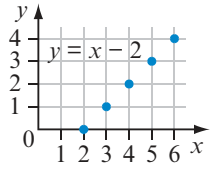
**15** **a** **i** 15°C **ii** 30°C **iii** 0°C **iv** 35°C
**b** **i** 68°F **ii** 212°F **iii** 77°F **iv** 140°F
**c** 5°C **d** 50°F = 10°C, so 15°C is higher

**16** **a** 4, 5, 6, 7, 8 **b** 0, 1, 2, 3, 4
**c** 0, 4, 8, 12, 16 **d** 12, 6, 4, 3, 2
**e** 1, 3, 5, 7, 9 **f** 0, 1, 4, 9, 16

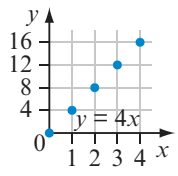
**17** **a**



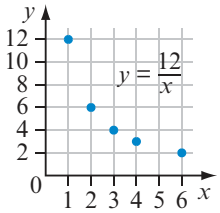

**b**



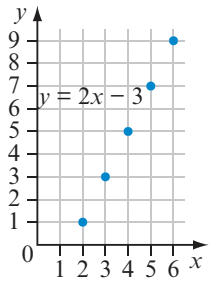
**c**



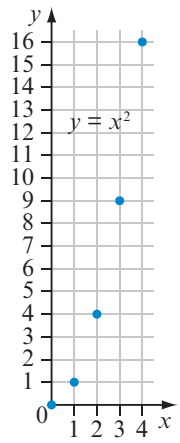

**d**




**e**




**f**

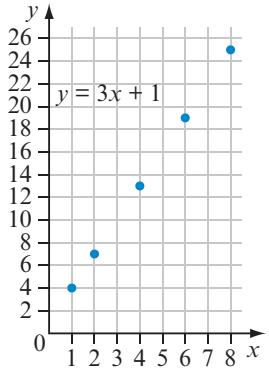



**18** **b** Parts a, b, c, and e show a linear relationship; d and f are non-linear.

**19** **a** **i** 4, 7, 13, 19, 25

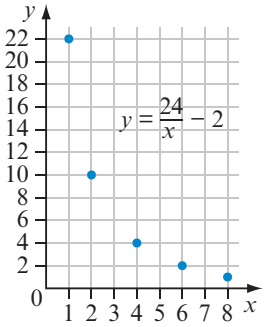
**b** **i** 22, 10, 4, 2, 1

**ii**




**iii** linear

**ii**

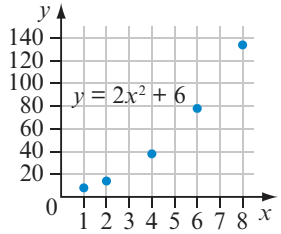



**iii** non-linear

**c** **i** 8, 14, 38, 78, 134

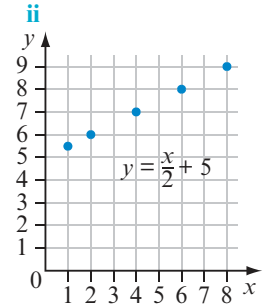
**d** **i** 5.5, 6, 7, 8, 9

**ii**




**iii** non-linear

**ii**




**iii** linear

**20** The variables $x$ and $y$ must have a power of 1. If the variables are part of a fraction they must also be in the numerator not the denominator.

## 5E Strategies for solving an equation

### 5E Start thinking!

**1** $x + 3$ **2** \$8 **3** $x + 3 = 8$

**4** Find value of $x$ that makes equation true.

**5** No, because $4 + 3 \neq 8$ **6** $x = 5$

**7** Check left side of equation equals right side of equation.

### Exercise 5E Strategies for solving an equation

**1** **a** true **b** false **c** false **d** true
**e** true **f** false

**2** **a** yes **b** no **c** no **d** yes
**e** yes **f** no **g** yes **h** no
**i** yes

**3** **a** $a = 5$ **b** $b = 6$ **c** $c = 8$ **d** $d = 18$
**e** $e = 10$ **f** $f = 16$ **g** $g = 9$ **h** $h = 7$
**i** $i = 4$ **j** $j = 8$ **k** $k = 10$ **l** $l = 11$
**m** $m = 4$ **n** $n = 7$ **o** $x = 54$ **p** $p = 0$
**q** $q = 11$ **r** $r = 11$ **s** $s = 9$ **t** $t = 8$
**u** $u = 2$

**5** To find value of pronumeral that makes equation true.

**6** **a** $x + 6 = 13$ **b** $x - 4 = 8$ **c** $9x = 54$
**d** $\frac{x}{2} = 5$ **e** $x + 3 = 14$ **f** $x - 7 = 5$
**g** $2x = 18$ **h** $\frac{x}{3} = 4$ **i** $x + 8 = 21$
**j** $5x = 30$

**7** **a** $x = 7$ **b** $x = 12$ **c** $x = 6$ **d** $x = 10$
**e** $x = 11$ **f** $x = 12$ **g** $x = 9$ **h** $x = 12$
**i** $x = 13$ **j** $x = 6$

**8** B **9** 3

**10** **a** $x = 5$ **b** $x = 2$ **c** $x = 4$
**d** $x = 3$ **e** $x = 1$ **f** $x = 2$
**g** $x = 12$ **h** $x = 7$ **i** $x = 8$

**11** Possible examples are: $4x = 24$, $x + 7 = 13$, $2x - 3 = 9$.

**12**

| Guess $x$ | LS $4x + 11$ | RS 35 | Check Is LS = RS? |
|---|---|---|---|
| 3 | 23 | 35 | no |
| 9 | 47 | 35 | no |
| 5 | 31 | 35 | no |
| 7 | 39 | 35 | no |
| 6 | 35 | 35 | yes |

$x = 6$

**13** **a** $x = 11$ **b** $x = 8$ **c** $x = 13$
**d** $x = 16$ **e** $x = 23$ **f** $x = 19$

**14**

| Guess $x$ | LS $5x + 3$ | RS 9 | Check Is LS = RS? |
|---|---|---|---|
| 1 | 8 | 9 | no |
| 2 | 13 | 9 | no |
| 1.5 | 10.5 | 9 | no |
| 1.3 | 9.5 | 9 | no |
| 1.2 | 9 | 9 | yes |

**15** **a** $x = 9.5$ **b** $x = 2.4$ **c** $x = 4.8$ **d** $x = 7.1$

**16** **a** $15n$ **b** $15n = 60$ **c** $n = 4$
**d** four t-shirts

**17** **a** $x + 17$ **b** $x + 17 = 48$ **c** $x = 31$
**d** 31 people

**18** **a** $\frac{x}{6}$, where $x$ is the number of Smarties
**b** $\frac{x}{6} = 8$ **c** $x = 48$ **d** 48 Smarties

**19** **a** $5 - s$, where $s$ is cost of sausage in bread
**b** $5 - s = 3$ **c** $s = 2$ **d** \$2

**21** **a** $x = 3$ **b** $x = 2$ or $x = 5$

## 5F Using flowcharts

Note that for some flowchart answers, missing numbers only are given (listed from left to right).

### 5F Start thinking!

**1** B → put on socks → D → put on runners → A → tie laces → C

**2** **a** 2 **b** × 3 and + 5
**c** × 3 is performed first because you follow direction of arrows.
**d** 11

**3** **a** 4 → (× 2) 8 → (+ 6) 14 **b** 4 ← (÷ 2) 8 ← (− 6) 14

**4** **a** Numbers in each flowchart are same but operations used are different depending on direction you follow along flowchart.
**b** When backtracking, the operation used is opposite or inverse of the one going forward at that step. For example, ÷ 2 is the inverse operation to × 2 and –6 is the inverse operation to + 6.

### Exercise 5F Using flowcharts

**1** **a** 22 **b** 12 **c** 6 **d** 10, 30
**e** 35, 36 **f** 4, 1

**2** **a** Start with 4, multiply by 6 and subtract 2.
**b** Start with 9, add 3 and divide by 4.
**c** Start with 11, subtract 5 and multiply by 7.
**d** Start with 2, add 8 and multiply by 3.
**e** Start with 5, multiply by 7 and add 1.
**f** Start with 8, divide by 2 and subtract 3.

**3** **a** 3 → (× 2) 6 → (+ 8) 14 **b** 2 → (+ 10) 12 → (÷ 3) 4
**c** 11 → (− 5) 6 → (× 4) 24 **d** 8 → (÷ 4) 2 → (+ 1) 3
**e** 20 → (+ 5) 25 → (− 9) 16 **f** 1 → (× 6) 6 → (− 6) 0
**g** 35 → (÷ 7) 5 → (− 5) 0 **h** 18 → (× 2) 36 → (÷ 9) 4

**4** **a** 21, 12, 6 **b** 6, 30, 31 **c** 2, 1, 8
**d** 15, 5, 20

**5** **a** 13, 26, 22, 2 **b** 14, 20, 4, 0

**6** **a** 48 **b** 9, 29 **c** 15, 9 **d** 2, 8
**e** 8, 16 **f** 18, 6

**7** **a** 4 **b** 5 **c** 5 **d** 14, 18
**e** 1, 7 **f** 20, 12

**8** **a** **i** + 3 **ii** – 3
**iii** Inverse operation to + 3 is – 3.
**b** **i** – 3 **ii** + 3
**iii** Inverse operation to – 3 is + 3.
**c** **i** × 3 **ii** ÷ 3
**iii** Inverse operation to × 3 is ÷ 3.
**d** **i** ÷ 3 **ii** × 3
**iii** Inverse operation to ÷ 3 is × 3.

**9** **a** – 2 **b** ÷ 6 **c** × 5 **d** + 9
**e** ÷ 3 **f** –1

**10** **a** subtract **b** add
**c** divide **d** multiply

**11** **a** 3 → (× 4) 12 → (− 2) 10 ← (÷ 4) ← (+ 2)
**b** 7 → (+ 8) 15 → (÷ 3) 5 ← (− 8) ← (× 3)
**c** 5 → (− 4) 1 → (+ 6) 7 ← (+ 4) ← (− 6)
**d** 8 → (÷ 2) 4 → (× 5) 20 ← (× 2) ← (÷ 5)

**12** **a** 5 → (+ 4) 9 → (− 2) 7 ← (− 4) ← (+ 2)
**b** 2 → (× 6) 12 → (+ 3) 15 ← (÷ 6) ← (− 3)
**c** 11 → (− 4) 7 → (× 5) 35 ← (+ 4) ← (÷ 5)
**d** 12 → (÷ 3) 4 → (+ 7) 11 ← (× 3) ← (− 7)
**e** 1 → (× 8) 8 → (− 5) 3 ← (÷ 8) ← (+ 5)
**f** 9 → (+ 7) 16 → (÷ 4) 4 ← (− 7) ← (× 4)

**13** **a** 6 → (− 2) 4 → (× 5) 20 → (+ 3) 23 ← (+ 2) ← (÷ 5) ← (− 3)
**b** 7 → (+ 5) 12 → (÷ 2) 6 → (− 5) 1 ← (− 5) ← (× 2) ← (+ 5)
**c** 3 → (× 8) 24 → (− 3) 21 → (÷ 7) 3 → (+ 5) 8 ← (÷ 8) ← (+ 3) ← (× 7) ← (− 5)
**d** 4 → (÷ 2) 2 → (+ 9) 11 → (× 3) 33 → (− 6) 27 ← (× 2) ← (− 9) ← (÷ 3) ← (+ 6)

**14** **a** 6 → (× 5) 30 → (− 7) 23 ← (÷ 5) ← (+ 7)
Mystery number is 6.
**b** 14 → (÷ 2) 7 → (+ 11) 18 ← (× 2) ← (− 11)
Mystery number is 14.
**c** 4 → (+ 8) 12 → (× 3) 36 ← (− 8) ← (÷ 3)
Mystery number is 4.
**d** 52 → (− 10) 42 → (÷ 6) 7 ← (+ 10) ← (× 6)
Mystery number is 52.
**e** 15 → (× 2) 30 → (÷ 3) 10 → (+ 12) 22 ← (÷ 2) ← (× 3) ← (− 12)
Mystery number is 15.
**f** 2 → (+ 13) 15 → (× 4) 60 → (− 5) 55 ← (− 13) ← (÷ 4) ← (+ 5)
Mystery number is 2.

**16** **a** 70 → (÷ 2) 35 → (− 19) 16 **b** $16

**17** **a** 15 → (+ 2) 17 → (− 3) 14 → (÷ 2) 7 → (+ 5) 12 ← (− 2) ← (+ 3) ← (× 2) ← (− 5)
**b** 15 animals

**18** **a** 5 → (+ 4) 9 → (+ 6) 15 → (× 2) 30 → (− 14) 16 ← (− 4) ← (− 6) ← (÷ 2) ← (+ 14)
**b** five people

## 5G Building expressions using flowcharts

### 5G Start thinking!

**1** **a** $a$ **b** + 3 **c** $a + 3$

**2** **a** $b \xrightarrow{+7} b+7$ **b** $c \xrightarrow{-2} c-2$

**3** **a** $d$ **b** $\times 5$ **c** $d \xrightarrow{\times 5} 5d$

**4** **a** $e$ **b** $\div 4$ **c** $e \xrightarrow{\div 4} \frac{e}{4}$

**5** **a** $f$ **b** $\times 3$ **c** $3f$ **d** $+8$

**6** $g \xrightarrow{\times 2} 2g \xrightarrow{-5} 2g-5$

**7** Start with the variable and apply each operation in order. The resulting expression after each operation is performed is shown in the next box of the flowchart.

## Exercise 5G Building expressions using flowcharts

**1** **a** $x \xrightarrow{+5} x+5$ **b** $x \xrightarrow{-4} x-4$

**c** $x \xrightarrow{\times 9} 9x$ **d** $x \xrightarrow{\div 3} \frac{x}{3}$

**e** $x \xrightarrow{\times 15} 15x$ **f** $x \xrightarrow{-61} x-61$

**2** **a** $2a$, $2a+3$ **b** $5b$, $5b-6$ **c** $\frac{c}{7}$, $\frac{c}{7}+2$

**d** $d-5$, $4(d-5)$ **e** $3e$, $\frac{3e}{4}$ **f** $f+8$, $\frac{f+8}{2}$

**3** **a** $\times 6$, $-2$ **b** $\div 7$, $+3$ **c** $+4$, $\times 9$

**d** $\times 11$, $+8$ **e** $-5$, $\div 2$ **f** $\div 3$, $-1$

**4** **a** $\times 7$, $+1$ **b** $\times 3$, $-5$ **c** $\times 4$, $+8$

**d** $\div 5$, $-7$ **e** $\div 2$, $+4$ **f** $\div 9$, $-1$

**g** $-3$, $\times 4$ **h** $+10$, $\times 2$ **i** $-2$, $\times 7$

**j** $+4$, $\div 7$ **k** $-3$, $\div 2$ **l** $+9$, $\div 5$

**5** **a** $x \xrightarrow{\times 7} 7x \xrightarrow{+1} 7x+1$

**b** $x \xrightarrow{\times 3} 3x \xrightarrow{-5} 3x-5$

**c** $x \xrightarrow{\times 4} 4x \xrightarrow{+8} 4x+8$

**d** $x \xrightarrow{\div 5} \frac{x}{5} \xrightarrow{-7} \frac{x}{5}-7$

**e** $x \xrightarrow{\div 2} \frac{x}{2} \xrightarrow{+4} \frac{x}{2}+4$

**f** $x \xrightarrow{\div 9} \frac{x}{9} \xrightarrow{-1} \frac{x}{9}-1$

**g** $x \xrightarrow{-3} x-3 \xrightarrow{\times 4} 4(x-3)$

**h** $x \xrightarrow{+10} x+10 \xrightarrow{\times 2} 2(x+10)$

**i** $x \xrightarrow{-2} x-2 \xrightarrow{\times 7} 7(x-2)$

**j** $x \xrightarrow{+4} x+4 \xrightarrow{\div 7} \frac{x+4}{7}$

**k** $x \xrightarrow{-3} x-3 \xrightarrow{\div 2} \frac{x-3}{2}$

**l** $x \xrightarrow{+9} x+9 \xrightarrow{\div 5} \frac{x+9}{5}$

**6** **a** $x \xrightarrow{+4} x+4 \xrightarrow{\times 5} 5(x+4)$

**b** $x \xrightarrow{\div 3} \frac{x}{3} \xrightarrow{-2} \frac{x}{3}-2$

**c** $x \xrightarrow{\times 4} 4x \xrightarrow{-3} 4x-3$

**d** $x \xrightarrow{+6} x+6 \xrightarrow{\div 8} \frac{x+6}{8}$

**e** $x \xrightarrow{\times 9} 9x \xrightarrow{+1} 9x+1$

**f** $x \xrightarrow{\div 4} \frac{x}{4} \xrightarrow{+6} \frac{x}{4}+6$

**g** $x \xrightarrow{-5} x-5 \xrightarrow{\div 9} \frac{x-5}{9}$

**h** $x \xrightarrow{-1} x-1 \xrightarrow{\times 3} 3(x-1)$

**i** $x \xrightarrow{\times 5} 5x \xrightarrow{\div 7} \frac{5x}{7}$

**j** $x \xrightarrow{\div 6} \frac{x}{6} \xrightarrow{-4} \frac{x}{6}-4$

**k** $x \xrightarrow{+5} x+5 \xrightarrow{\times 9} 9(x+5)$

**l** $x \xrightarrow{-1} x-1 \xrightarrow{\div 3} \frac{x-1}{3}$

**7** No, 7 is added first and then the whole expression is multiplied by 4 to give $4(x+7)$.

**8** Expressions in boxes are given.

**a** $a$, $4a$, $4a+7$ **b** $b$, $\frac{b}{3}$, $\frac{b}{3}+5$

**c** $c$, $8c$, $\frac{8c}{5}$ **d** $d$, $d+4$, $\frac{d+4}{3}$

**e** $e$, $e-1$, $5(e-1)$ **f** $f$, $f+8$, $7(f+8)$

**g** $g$, $2g$, $2g-9$ **h** $h$, $3h$, $\frac{3h}{11}$

**9** **a** $4a+7$ **b** $\frac{b}{3}+5$ **c** $\frac{8c}{5}$

**d** $\frac{d+4}{3}$ **e** $5(e-1)$ **f** $7(f+8)$

**g** $2g-9$ **h** $\frac{3h}{11}$

**10** **a** $+2$, $\times 7$, $-3$ **b** $\times 9$, $-3$, $\div 5$

**c** $\times 6$, $+1$, $\times 2$ **d** $-4$, $\div 3$, $+9$

**e** $\times 5$, $\div 6$, $-11$ **f** $\div 4$, $+3$, $\times 8$

**11** **a** $a+5$, $2(a+5)$, $2(a+5)+3$

**b** $5b$, $5b-4$, $6(5b-4)$

**c** $3c$, $3c+1$, $\frac{3c+1}{7}$ **d** $8d$, $\frac{8d}{5}$, $\frac{8d}{5}+2$

**e** $\frac{e}{9}$, $\frac{e}{9}-4$, $3(\frac{e}{9}-4)$ **f** $f+8$, $\frac{f+8}{13}$, $\frac{f+8}{13}-4$

**g** $g-2$, $3(g-2)$, $3(g-2)+5$

**12** Inverse operations listed from left to right are: $\div 2$, $-7$, $\times 5$.

**13** Inverse operations are listed from left to right.

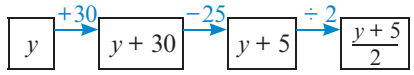
**a** $-5, \div 2, -3$ **b** $\div 5, +4, \div 6$

**c** $\div 3, -1, \times 7$ **d** $\div 8, \times 5, -2$

**e** $\times 9, +4, \div 3$ **f** $-8, \times 13, +4$

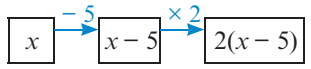
**g** $+2, \div 3, -5$

**15 a**




**b** $\frac{y+5}{2}$ **c** \$29

**16 a**



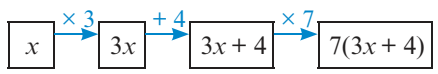

**b** $2(x-5)$ **c** 12

**17** **a** $\times 3$ **b** $-5$ **c** $+6$

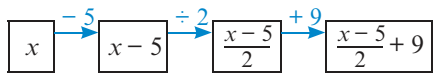
**d** $\times 3$ **e** $\div 8$ **f** $-3$

**g** $+7$ **h** $\times 8$ **i** $\times 6$

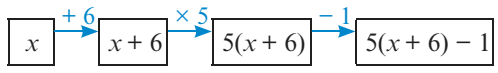
**18 a**



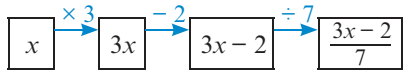

**b**



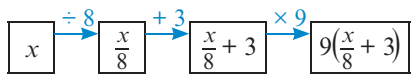

**c**



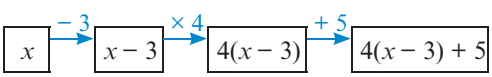

**d**



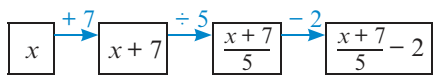

**e**



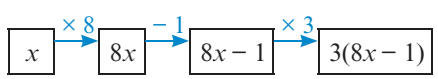

**f**



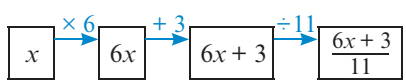

**g**




**h**




**i**




**19** Inverse operations are listed from left to right.

**a** $\div 3, -4, \div 7$ **b** $+5, \times 2, -9$

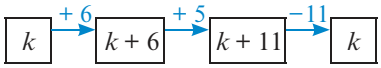
**c** $-6, \div 5, +1$ **d** $\div 3, +2, \times 7$

**e** $\times 8, -3, \div 9$ **f** $+3, \div 4, -5$

**g** $-7, \times 5, +2$ **h** $\div 8, +1, \div 3$

**i** $\div 6, -3, \times 11$

**20** One example is:

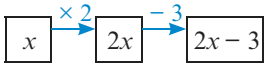



## 5H Solving equations using backtracking

### 5H Start thinking!

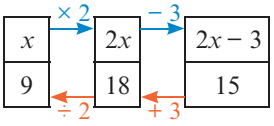
**1** number of mice in a cage **2** $2x - 3$

**3**




**4 a** seven mice **b** 23 mice

**5, 6**




**7** 9 **8** $x = 9$

**9** The equation $2x - 3 = 15$ represents the final number of mice in the cage. The solution, $x = 9$, is the original number of mice in the cage.

### Exercise 5H Solving equations using backtracking

**1 a** **i** $4a + 9 = 17$ **ii** $a = 2$

**b** **i** $\frac{b}{5} - 1 = 3$ **ii** $b = 20$

**c** **i** $7(c - 3) = 35$ **ii** $c = 8$

**d** **i** $2d - 3 = 5$ **ii** $d = 4$

**e** **i** $\frac{4e}{3} = 8$ **ii** $e = 6$

**f** **i** $\frac{f+20}{7} = 15$ **ii** $f = 85$

**2 a** $\frac{x}{4} + 3 = 10$, $x = 28$ **b** $9x - 7 = 11$, $x = 2$

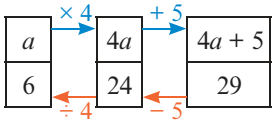
**c** $\frac{x+5}{3} = 6$, $x = 13$ **d** $\frac{2x}{5} = 8$, $x = 20$

**e** $6(x - 1) = 18$, $x = 4$ **f** $\frac{x-9}{2} = 1$, $x = 11$

**g** $\frac{x}{5} - 1 = 1$, $x = 10$ **h** $3(x + 7) = 27$, $x = 2$

**3** Build a flowchart for the expression involving a pronumeral, write equivalent value of expression underneath and work backwards using inverse operations to find missing value for pronumeral.

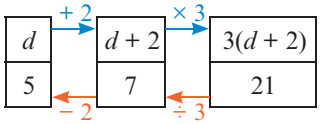
**4 a**




$a = 6$

**b** $b = 7$ **c** $c = 3$

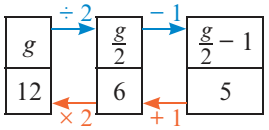
**d**




$d = 5$

**e** $e = 9$ **f** $f = 8$

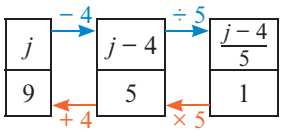
**g**




$g = 12$

**h** $h = 15$ **i** $i = 36$

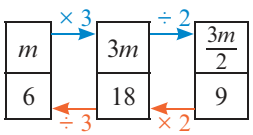
**j**




$j = 9$

**k** $k = 5$ **l** $l = 13$

**m**




$m = 6$

**n** $n = 10$ **o** $x = 16$

**5** **a** $x = 4$ **b** $x = 9$ **c** $x = 10$ **d** $x = 14$

**e** $x = 1$ **f** $x = 15$ **g** $x = 27$ **h** $x = 3$

**i** $x = 7$ **j** $x = 9$ **k** $x = 6$ **l** $x = 11$

**m** $x = 22$ **n** $x = 22$ **o** $x = 2$

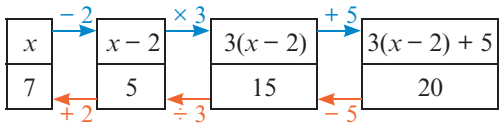
**7** **a** **i** $2(x + 4) - 1 = 17$ **ii** $x = 5$

**b** **i** $\frac{5x}{3} + 2 = 12$ **ii** $x = 6$

**c** **i** $\frac{x-4}{3} + 5 = 7$ **ii** $x = 10$

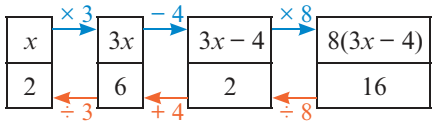
**d** **i** $\frac{2x+7}{5} = 3$ **ii** $x = 4$

**8 a**



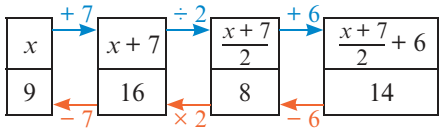

$3(x - 2) + 5 = 20$, $x = 7$

**b**



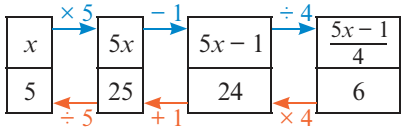

$8(3x - 4) = 16$, $x = 2$

**c**



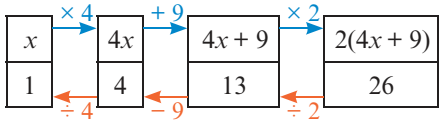

$\frac{x+7}{2} + 6 = 14$, $x = 9$

**d**



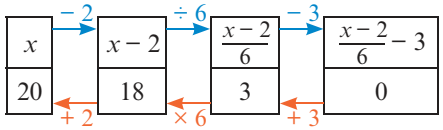

$\frac{5x-1}{4} = 6$, $x = 5$

**e**




$2(4x + 9) = 26$, $x = 1$

**f**




$\frac{x-2}{6} - 3 = 0$, $x = 20$

**9 a** B **b** $x = 2$ **c** $3 \times 2 + 1 = 7$

**10 a** D **b** $x = 19$ **c** $\frac{19+5}{3} = 8$

**11 a** $2x + 5 = 23$ **b** $\frac{x}{3} - 2 = 5$

**c** $6(x + 4) = 30$ **d** $\frac{x-7}{2} = 3$

**e** $2x - 1 = 9$ **f** $\frac{3x}{4} = 9$

**12 a** $x = 9$ **b** $x = 21$ **c** $x = 1$

**d** $x = 13$ **e** $x = 5$ **f** $x = 12$

**13 a** $2x + 6 = 40$ **b** $x = 17$, \$17

**14** $\frac{x}{2} + 20 = 59$, $x = 78$, \$78

**16 a** $x = 2$ **b** $x = 10$ **c** $x = 6$

**d** $x = 14$ **e** $x = 16$ **f** $x = 3$

**g** $x = 8$ **h** $x = 19$ **i** $x = 5$

**j** $x = 0$ **k** $x = 10$ **l** $x = 1$

**18** $5(3x + 4) - 7 = 43$, $x = 2$

**19 a** $x = 6$ **b** $x = 15$

## 5I Solving equations using a balance model

### 5I Start thinking!

**1 a** Left side of scales would go up.

**b** Remove 2 g from right side. **c** $x + 1 = 3$

**2 a** Left side of scales would go up.

**b** Remove 3 g from right side. **c** $x = 2$

**3 a** $x = 2$ **b** $x = 2$ **c** $x = 2$ **d** $x = 2$

**4** The solutions are the same

**5** $-3$ **6** $x + 3 - \mathbf{3} = 5 - \mathbf{3}$; $x = 2$

**7** Decide what operation needs to be undone and perform the inverse operation on both sides of equation.

### Exercise 5I Solving equations using a balance model

**1 a** Yes, scales are level. **b** $x + 5 = 7$

**c** Remove 3 g. **d** $x + 2 = 4$

**e** Remove 5 g from right side. **f** $x = 2$

**2 a** $-5$ **b** $+3$ **c** $\div 8$ **d** $\times 2$

**3 a** $a = 7$ **b** $b = 11$ **c** $c = 4$ **d** $d = 15$

**4 a** $x = 5$ **b** $x = 6$ **c** $x = 8$ **d** $x = 30$

**e** $x = 8$ **f** $x = 17$ **g** $x = 12$ **h** $x = 7$

**i** $x = 19$ **j** $x = 3$ **k** $x = 27$ **l** $x = 180$

**6 a** $x = 6$ **b** $x = 3$ **c** $x = 4$ **d** $x = 2$

**e** $x = 7$ **f** $x = 5$ **g** $x = 1$ **h** $x = 8$

**7 a** $\times 2$, $+7$ **b** $-7$, $\div 2$

**c**
$$2x + 7 = 13$$
$$2x + 7 - 7 = 13 - 7$$
$$2x = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

**d** They show the equivalent equations obtained after performing an operation on both sides of an equation.

**e** $x = 3$

**8 a** $+1$, $\times 3$ **b** $+1$ **c** $\times 3$

**d**
$$\frac{x}{3} - 1 = 4$$
$$\frac{x}{3} - 1 + 1 = 4 + 1$$
$$\frac{x}{3} = 5$$
$$\frac{x}{3} \times 3 = 5 \times 3$$
$$x = 15$$

**e** The equivalent equations produced are same.

**9 a** $x = 10$ **b** $x = 18$ **c** $x = 6$

**d** $x = 28$ **e** $x = 27$ **f** $x = 24$

**g** $x = 33$ **h** $x = 42$

**10 a** $\div 4$, $+5$ **b** $\div 4$ **c** $+5$

**d**
$$4(x - 5) = 8$$
$$\frac{4(x-5)}{4} = \frac{8}{4}$$
$$x - 5 = 2$$
$$x - 5 + 5 = 2 + 5$$
$$x = 7$$

**11 a** $\times 9$, $-12$ **b** $\times 9$ **c** $-12$

**d**
$$\frac{x+12}{9} = 2$$
$$\frac{x+12}{9} \times 9 = 2 \times 9$$
$$x + 12 = 18$$
$$x + 12 - 12 = 18 - 12$$
$$x = 6$$

**12 a** $x = 5$ **b** $x = 19$ **c** $x = 11$

**d** $x = 11$ **e** $x = 7$ **f** $x = 3$

**g** $x = 10$ **h** $x = 16$

**13 a** $x = 1$ **b** $x = 14$ **c** $x = 6$

**d** $x = 11$ **e** $x = 42$ **f** $x = 9$

**g** $x = 8$ **h** $x = 29$ **i** $x = 4$

**j** $x = 30$ **k** $x = 10$ **l** $x = 21$

**m** $x = 10$ **n** $x = 12$ **o** $x = 2$

**p** $x = 27$

**14** Let $x$ represent variable.
- **a** $4x = 64$, $x = 16$, 16 tables
- **b** $15x + 6 = 81$, $x = 5$, 5 hours
- **c** $2x + 12 = 20$, $x = 4$, 4 extra toppings
- **d** $24x + 105 = 369$, $x = 11$, 11 weeks

**15** $2(x + 5) = 136$, $x = 63$, \$63

**17** **a** $x = 4$ **b** $x = 11$ **c** $x = 16$

## 5 Chapter review

### MULTIPLE-CHOICE

**1** B **2** A **3** D **4** B
**5** B **6** A **7** D

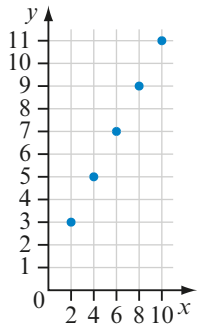
### SHORT ANSWER

**1** **a** 6, 10, 14, 18, 22 **b** 0, 3, 6, 9, 12

**2** OUT = IN – 4

**3** **a** $y = x + 1$

**b**




**c** Linear, as points lie in straight line.

**4** **a** **i** 2 **ii** $x$, $y$ **iii** 7, 3 **iv** none
**b** **i** 2 **ii** $a$ **iii** 1 **iv** 3
**c** **i** 3 **ii** $d$, $c$ **iii** 5, 2 **iv** 9
**d** **i** 4 **ii** $e$, $t$, $s$ **iii** 1, 2, 6 **iv** 4

**5** **a** $7x = 21$ **b** $\frac{x}{3} = 6$
**c** $x + 7 = 10$ **d** $x - 2 = 17$

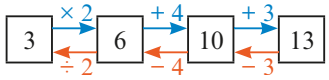
**6** **a** 7 **b** 6 **c** 10 **d** 2

**7** **a** 14 **b** 5 **c** 18 **d** 2

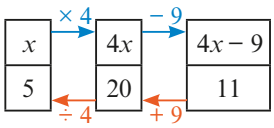
**8** **a** $a = 7$ **b** $b = 31$ **c** $c = 3$ **d** $d = 10$

**9** **a** $x = 11$ **b** $x = 4.5$

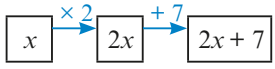
**10**



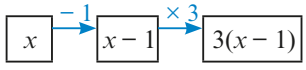

**11** $x = 5$



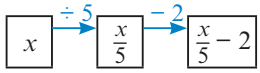

**12** **a**



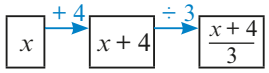

**b**




**c**




**d**




**13** $3(2x - 4) = 6$, $x = 3$

**14** **a** $x = 4$ **b** $x = 3$ **c** $x = 6$ **d** $x = 7$

**15** **a** $x = 12$ **b** $x = 48$ **c** $x = 4$ **d** $x = 13$
**e** $x = 37$ **f** $x = 60$

### NAPLAN-STYLE PRACTICE

**1** \$45 **2** 3 **3** $s = \frac{n}{5}$ **4** B
**5** $m = b + 5$ **6** \$62 **7** A **8** 7
**9** $4a + 2c$ **10** \$14 **11** $2(x + 5) - 3$
**12** true **13** 6 **14** $5a + 3 = 13$
**15** \$2 **16** $2x + 4 = 12$ **17** 4 kg

### ANALYSIS

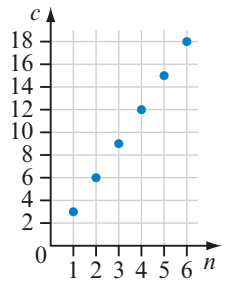
**a** number of ice creams, total cost

**b** Total cost is three times number of ice creams.

**c** One answer could be: Let $n$ represent number of ice creams and $c$ represent total cost.

**d** $c = 3n$

**e**

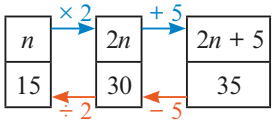



**f** \$24

**g** $2n + 5$ represents total cost in dollars, where $n$ is number of ice creams. n is multiplied by 2 (as each ice cream costs \$2) and then upfront fee of \$5 is added to give total cost.

**h** \$21 **i** $2n + 5 = 35$

**j**




**k** 15 ice creams

## 5 Connect

For feedback on this open-ended task, see your teacher.

# CHAPTER 6 LINES, ANGLES AND LOCATION

## 6 Are you ready?

**1** B **2** C
**3** **a** D **b** D **c** C
**4** B **5** **a** C **b** A **c** D
**6** **a** B **b** C **c** B
**7** C **8** **a** C **b** A **c** D

## 6A Lines, rays and segments

### 6A Start thinking!

**2** No

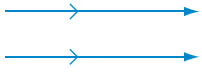
**3** The grey lines have arrow heads but the red lines do not.

## Exercise 6A Lines, rays and segments

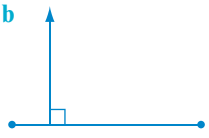
**1**

| Term | Example | Definition |
|---|---|---|
| point | X | an exact position in space |
| ray | X, R | a line that has a definite starting point and continues in one direction indefinitely |
| segment | P, Q | a part of a line that has a definite beginning and end |
| parallel lines | | two straight lines the same distance apart over their entire length |
| perpendicular lines | | lines that meet at a 90° angle |
| angle | | a shape formed by two lines or rays that meet at a point |
| circle | | a line with every point the same distance from the centre |
| right angle | | an angle of 90° |

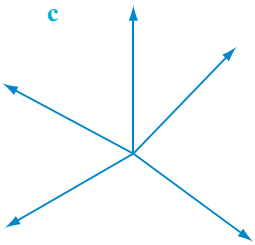
**2 a**



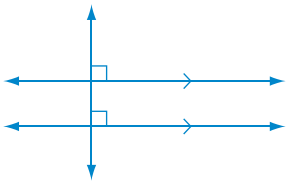
**b**



**c**

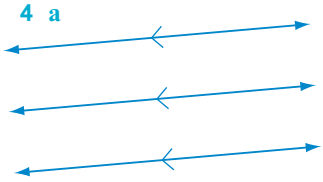


**d**

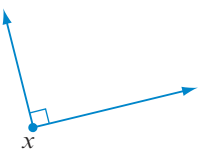


**3 a** two parallel line segments
**b** two perpendicular rays
**c** a ray parallel to a segment
**d** a ray perpendicular to a segment
**e** two parallel lines perpendicular to another two parallel lines
**f** a ray and a segment that are parallel and both cross a line.

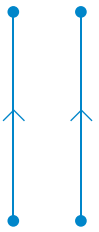
**4 a**



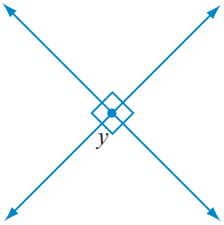
**b**



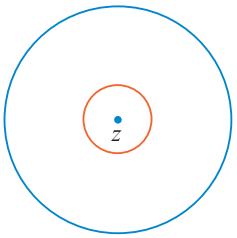

**c**



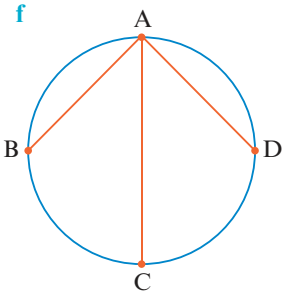
**d**




**e**

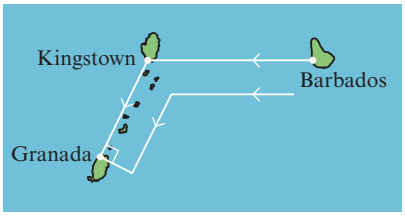



**f**




**5 a** **i** yes **ii** no **b** **i** yes **ii** no
**6 a** segment **b** ray **c** segment

**10**




## 6B Types of angles

### 6B Start thinking!

**1–3**



acute
between 0° and 90°

right
90°

obtuse
between 90°and 180°



straight
180°

reflex
between 180° and 360°

revolution
360°

### Exercise 6B Types of angles

**1 a** acute **b** right **c** revolution
**d** reflex **e** acute **f** obtuse
**g** acute **h** straight **i** reflex

**2 a** 67° **b** 13° **c** 2° **d** 81°
**e** 45° **f** 33° **g** 56° **h** 29°

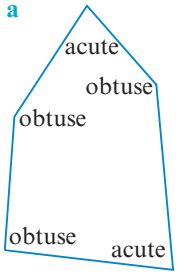
**3 a** 22° **b** 159° **c** 80° **d** 83°
**e** 31° **f** 45° **g** 99° **h** 90°

**4** **a** **i** 44° **ii** 134° **b** **i** 78° **ii** 168°
**c** **i** 5° **ii** 95° **d** **i** 63° **ii** 153°
**e** **i** 57° **ii** 147° **f** **i** 26° **ii** 116°
**g** **i** 18° **ii** 108° **h** **i** 85° **ii** 175°

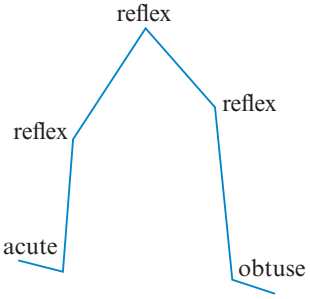
**5** **a** E and F, A and G **b** B and F, C and D

**6** **a** 130° **b** 67° **c** 99° **d** 19°
**e** 66° **f** 84° **g** 30° **h** 21°
**i** 132° **j** 37°

**7** **a**



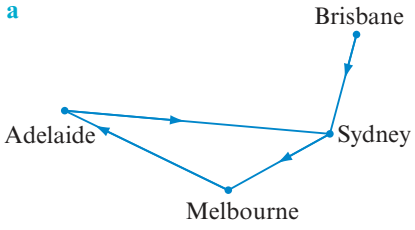

**b**




**c** Door has four right angles.

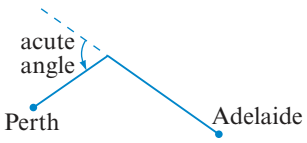
**8** **d** In the opposite direction. This angle will be 360° minus the angle for part c.

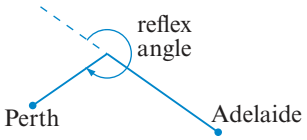
**9** **a**




**b** acute, acute, obtuse

**10**




**11** **a** right angles **b** revolutions
**c** right angles

**12** 270°; reflex angle

**13** **a** **i** complementary **ii** 34°
**b** **i** supplementary **ii** 93°
**c** **i** supplementary **ii** 34°
**d** **i** complementary **ii** 30°

**14** Answers will vary but should add to 90°.

**15** Answers will vary but should add to 180°.

**16** No, the largest acute angle is less than 90°, therefore two acute angles must add to less than 180°.

**17** Answers will vary but should add to 360°.

## 6C Measuring and drawing angles

### 6C Start thinking!

**1** **a** 90° **b** 180° **c** 360°

**2** Estimates should be close to the values given for question 4.

**3** Use the scale that starts at 0 on the first arm of the angle.

**4** **a** 80° **b** 160° **c** 40° **d** 110°

### Exercise 6C Measuring and drawing angles

**1** **a** 2 **b** 4 **c** 2

**2** **a** right, 0, 90
**b** right, straight, 90, 180
**c** straight, revolution, 180, 360

**3** **a** A is at 30° or 150°; B is at 75° or 105°; C is at 102° or 78°; D is at 124° or 56°; E is at 165° or 15°; F is at 180° or 0°
**b** **i** 150° **ii** 105° **iii** 78° **iv** 56° **v** 15°

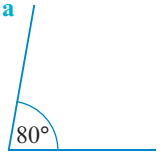
**4** **a** 84° **b** 40° **c** 64° **d** 15°
**e** 79° **f** 35° **g** 52° **h** 8°

**6** **a** 137° **b** 172° **c** 150° **d** 120°

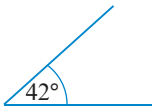
**7** **a** acute, 44° **b** obtuse, 173° **c** acute, 13°
**d** acute, 76° **e** obtuse, 131° **f** obtuse, 92°
**g** acute, 34° **h** obtuse, 115° **i** acute, 55°
**j** obtuse, 159°

**9** The protractor was read from the opposite direction. The answer should be 106°.

**10** **a**




**b**



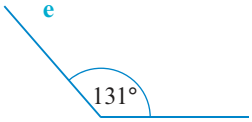

**c**



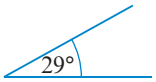

**d**



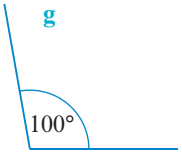

**e**



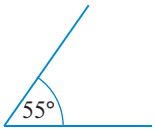

**f**




**g**



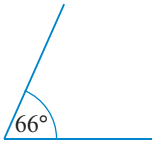

**h**



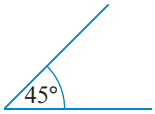

**i**



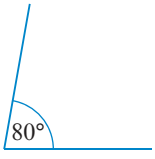

**j**



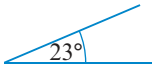

**12** **a**



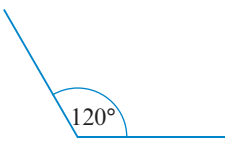

**b**




**c**




**d**



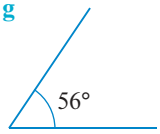
**e**



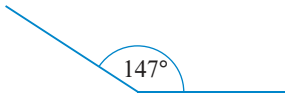

**f**



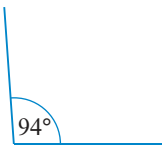

**g**



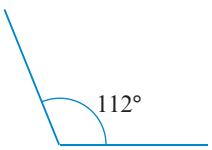

**h**




**i**




**j**




**13** A and C will be suitable.

**14** **a** top right pocket **b** right centre pocket

## 6D Angles around a point

### 6D Start thinking!

**1** $a = 60°$; $b = 120°$; $c = 60°$; $d = 120°$

**2** **a** 180° **b** 180° **c** 180° **d** 180° **e** 360°

**3** Possible pairs are: $a$ and $b$; $c$ and $d$; $a$ and $d$; $b$ and $c$.

**4** Opposite angles are equal. **5** yes

### Exercise 6D Angles around a point

**1** **a** 360° **b** identical **c** 180°

**2** **a** $a = 100°$ **b** $b = 27°$
**c** $c = 38°$ **d** $d = 133°$
**e** $e = 21°$; $f = 159°$ **f** $g = 64°$; $h = 116°$

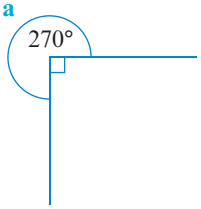
**3** **a** $a = 215°$ **b** $b = 300°$
**c** $c = 270°$ **d** $d = 150°$
**e** $e = 20°$ **f** $f = 109°$

**4** **a** $d = 20°$; $e = 160°$; $f = 20°$
**b** $x = 136°$; $y = 44°$; $z = 136°$
**c** $q = 108°$; $r = 72°$; $s = 72°$ **d** $c = 90°$
**e** $m = 107°$; $n = 73°$; $p = 107°$
**f** $g = 29°$; $h = 29°$; $k = 151°$

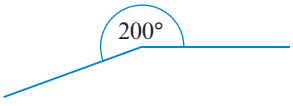
**5** **a** $a = 50°$; $b = 50°$; $c = 50°$; $d = 80°$
**b** $e = 142°$; $f = 135°$ **c** $g = 60°$; $h = 60°$
**d** $i = 30°$ **e** $j = 127°$
**f** $k = 137°$; $l = 43°$; $m = 137°$
**g** $n = 22°$; $o = 70°$; $p = q = 88°$
**h** $r = 147°$ **i** $s = 35°$; $t = 270°$

**6** **a** $a = 270°$ **b** $b = 332°$ **c** $c = 229°$
**d** $d = 300°$ **e** $e = 265°$ **f** $f = 186°$

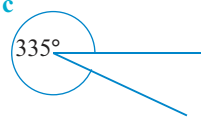
**7** **a**




**b**




**c**



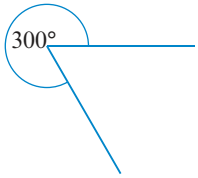

**d**



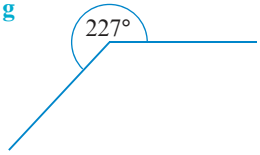

**e**



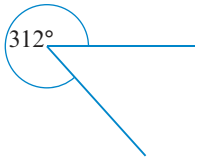

**f**




**g**



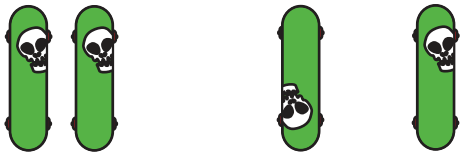

**h**




**8** 60°

**9** **a** 8 **b** 45° **c** 10 **d** 72°

**10** **a** start end **b** end **c** end



**11** 125°

## 6E Angles and parallel lines

### 6E Start thinking!

**2** A number of angles are the same.

**3** **a** $c$ and $f$

**4** **a** $b$ and $f$; $c$ and $g$; $d$ and $h$

**5** **a** $d$ and $f$

### Exercise 6E Angles and parallel lines

**1** **a** alternate **b** co-interior
**c** corresponding **d** corresponding
**e** alternate **f** co-interior

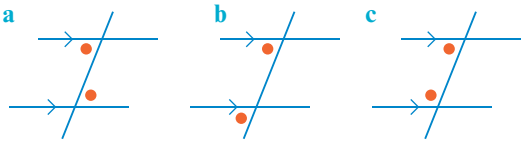
**2** **a** equal **b** supplementary
**c** equal **d** equal
**e** equal **f** supplementary

**3** **a** $f$ **b** $d$ **c** $e$ **d** $e$ **e** $d$ **f** $e$

**4** **a** $b$ **b** $e$ **c** $g$ **d** $d$ **e** $c$ **f** $h$

**5** **a** 125° **b** 110° **c** 131° **d** 60°
**e** 150° **f** 138°

**6** **a** **b** **c**



**7** **a** $n$ and $q$, $o$ and $p$ **b** $n$ and $p$, $o$ and $q$
**c** $l$ and $p$, $m$ and $q$, $n$ and $r$, $o$ and $s$

**8** **a** co-interior **b** corresponding
**c** alternate

**9** **a** 125° **b** 55° **c** 71° **d** 71°
**e** 67° **f** 123°

**10** They are not parallel, as the two angles are not supplementary (they add to 190°).

**11** **a** $a = 71°$, alternate angles
**b** $b = 122°$, co-interior angles
**c** $c = 65°$, alternate angles
**d** $d = 65°$, co-interior angles
**e** $e = 35°$, alternate angles

**f** $f = 121°$, corresponding angles
**g** $g = 133°$, corresponding angles
**h** $h = 160°$, alternate angles

**12** 35° **13** 133°

**14** **a** $a = b = c = 64°$ **b** no
**c** Move the top left corner up so that $c$ (and hence $b$) increases by 2°.

## 6F Understanding bearings

### 6F Start thinking!

**1** **a** 180° **b** 270° **c** 180° **d** 360°

**2** north-west (NW); south-east (SE); south-west (SW)

**3** **a** 45° **b** 135° **c** 90° **d** 270°

### Exercise 6F Understanding bearings

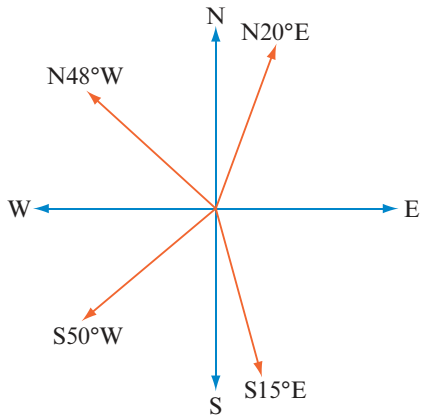
**1** **a** **i** W **ii** S **iii** NE
**b** **i** yacht **ii** palm tree **iii** periscope
**c** **i** yacht **ii** yacht
**iii** steamship (and sailboat)

**2** **a** true bearing **b** compass bearing
**c** true bearing **d** compass bearing

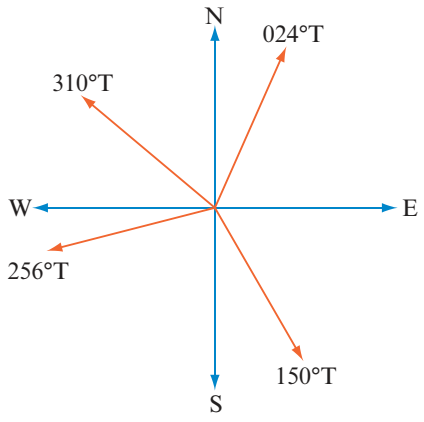
**3** **a** A is N30°E, B is N68°E, C is S52°W, D is S9°E.
**b** A is 030°T, B is 068°T, C is 232°T, D is 171°T

**4** **a** 000°T **b** 270°T **c** 180°T
**d** 045°T **e** 135°T **f** 315°T

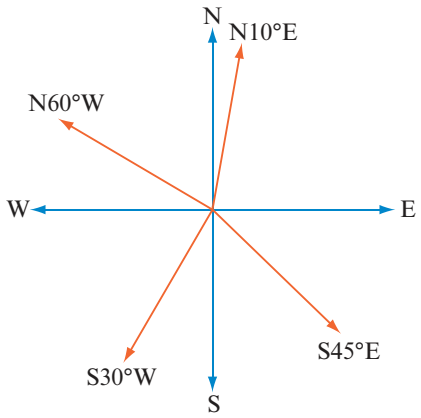
**5**



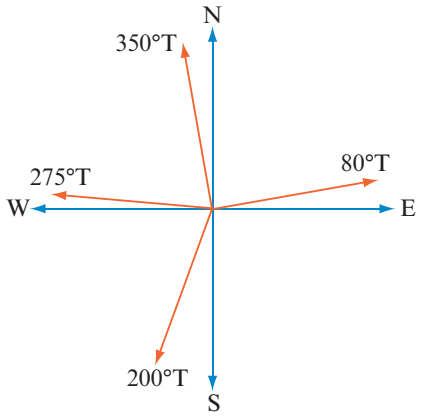

**6**




**7**




**8**




**9** **a** N10°E **b** S80°W **c** S70°E
**d** N30°W **e** S34°E **f** N59°W
**g** S35°W **h** N84°E

**10** **a** 050°T **b** 102°T **c** 342°T
**d** 190°T **e** 140°T **f** 089°T
**g** 239°T **h** 333°T

**11** **a** **i** switched S and E **ii** S30°E
**b** **i** measured anticlockwise from north instead of clockwise
**ii** 205°T
**c** **i** measured from west instead of north
**ii** N80°W
**d** **i** forgot that true bearings have three digits and a 'T'
**ii** 060°T

**12** **c** S and E **e** 150°T, S30°E

**13** **a** 080°T, N80°E **b** 240°T, S60°W
**c** 040°T, N40°E **d** 162°T, S18°E
**e** 342°T, N18°W

**14** **a** south **b** 180° **c** 180°
**d** They are the same.
**e** add or subtract 180°
**f** a: 260°T; b: 060°T; c: 220°T
**g** a: S80°W; b: N60°E; c: S40°W
**h** swap N/S and E/W

**15** **a** 015°T **b** 140°T **c** 010°T

**16** **a** 172°T **b** N9°W **c** 093°T **d** N87°W

**18** Monkey Forest

## 6G Understanding scale

### 6G Start thinking!

1 200 cm 2 2 m 3 100 4 B

### Exercise 6G Understanding scale

1 a 1:300 b 1:50 c 1:20 000 d 1:4000

2 a $\frac{1}{800}$ b $\frac{1}{70}$ c $\frac{1}{200\,000}$ d $\frac{1}{5000}$

3

| | scale factor | ratio scale | 1 cm on the map represents ___ in the real world | measurement in meaningful units |
|---|---|---|---|---|
| a | $\frac{1}{500}$ | 1:500 | 500 cm | 5 m |
| b | $\frac{1}{250}$ | 1:250 | 250 cm | 2.5 m |
| c | $\frac{1}{10}$ | 1:10 | 10 cm | 10 cm |
| d | $\frac{1}{100\,000}$ | 1:100 000 | 100 000 cm | 1 km |

4 a i 10 ii 1 000 000
b 1:1 000 000 c $\frac{1}{1\,000\,000}$

5 b 10 c 10 m d 1000 cm e 1:1000
f $\frac{1}{1000}$ g C h 10 m

6 a 10 m b 5 m
c 2.5 m d 10 cm e 1 km

7 a 5 cm b 5000 cm c 50 m d 35 m

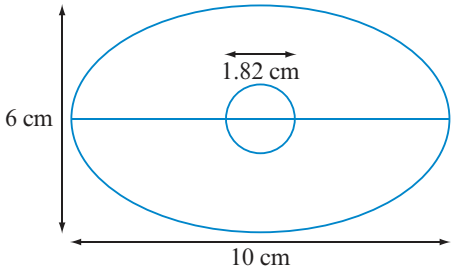
8 a 80 times bigger b 5 cm
c 4 m d 1.4 m

9 a 30 km b 120 km c 285 km d 2 cm

10 a 2 km; 8 km; 19 km; 30 cm
b 1 km; 4 km; 9.5 km; 60 cm
c 5 km; 20 km; 47.5 km; 12 cm
d 600 km; 2400 km; 5700 km; 0.1 cm or 1 mm

11 a 7.35 cm b 1.864 cm c 75 m by 35 m

12 a 10 cm b 6 cm c see part e
d 0.91 cm
e




13 a 500 km b 50 000 000
c 1:50 000 000

14 a 625 km b 1025 km c 2450 km
d 825 km

15 a i Hobart and Darwin
ii approximately 3500 km
b i Canberra and Sydney
ii approximately 225 km

16 approximately 34 000 km

17 a 3 km; yes

18 a i map is 1000 times bigger than real life
ii $\frac{1}{1000}$ is reduction, 1000 is enlargement
b i map is 100 times bigger than real life
ii 1:100 is reduction, 100:1 is enlargement
d i enlargement ii reduction
iii enlargement iv reduction
v enlargement
e i 0.5 cm or 5 mm ii 10 cm
iii 0.1 mm iv 50 km
v 0.2 cm or 2 mm

19 a 10 b 1 mm c 4.8 mm d 25 cm
e i a: 1000, b: 0.1 mm, c: 0.48 mm, d: 25 m
ii a: 100 000, b: 0.0001 mm, c: 0.000 48 mm, d: 2.5 km
f No; the drawings would need to be extremely large (25 m and 2.5 km) for these scale factors. Scale factors like these would be appropriate for drawing microscopic items.

20 a 1:10 b 1.79 m c 13 cm

21 a $\frac{1}{50}$ b 1:50 c 2.6 cm d 4.1 m

## 6 Chapter review

### MULTIPLE-CHOICE

1 A 2 C 3 B 4 D
5 C 6 D 7 A 8 B

### SHORT ANSWER

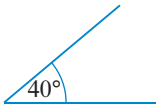
1 a D b C c F d E e A f B

2 a acute b obtuse c right d reflex

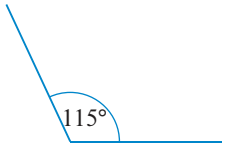
3 a i 67° ii 11° iii 35°
b i 53° ii 138° iii 171°

4 a 15° b 135° c 90° d 216°

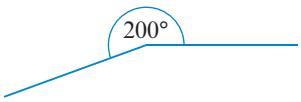
5 a




b



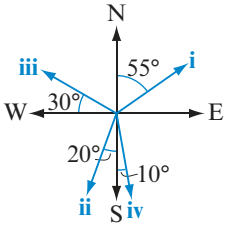

c




6 a 320° b 145° c 25°
d $d = 57°$; $e = f = 123°$

7 a $g$ b $d$ c $d$ d 77° e 103° f 103°

8 a i N55°E ii S20°W iii 300°T iv 170°T
b




9 a $\frac{1}{200\,000}$ b 2 km c 26.2 km d 2.2 cm

10 a 168 000 cm b $\frac{1}{15\,000}$ c 1:15 000
d 4.41 km

### NAPLAN-STYLE PRACTICE

1 B

2 A pair of lines that meet at right angles are called perpendicular.

3 Line A is parallel to ray D.

**4** 180° **5** 90° **6** obtuse **7** 113°
**8** 61° **9** 280° **10** 29° **11** 137°
**12** 43° **13** 43° **14** *a* and *e* **15** *c* and *f*
**16** *c* and *e* **17** 125° **18** 55° **19** 55°
**20** 55° **21** 075°T **22** 198°T **23** N65°W
**24** S80°E **25** 4 km **26** 2.5 cm **27** 4 cm
**28** Sally thinks she is 4 cm further north on the map than she actually is.

ANALYSIS

**a** N64°E **c** S60°W or 240°T
**d** straight angle **e** 1:1000

## 6 Connect

For feedback on this open-ended task, see your teacher.

# CHAPTER 7 SHAPES AND OBJECTS

## 7 Are you ready?

**1** 90° **2** A **3** 4 **4** B
**5** D **6** 6 **7** B **8** A
**9** A **10** B **11** C **12** A

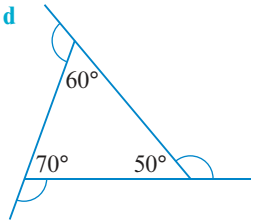
## 7A Classifying triangles

### 7A Start thinking!

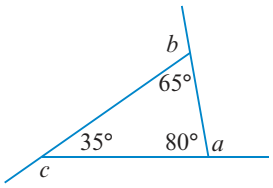
**1** The sides have equal length.
**2** Answers will vary, but should be similar to: The three sides of an equilateral triangle are equal. The three angles of an equilateral triangle are equal and are 60°.
**3** B is isosceles; two sides have the same marks on them and are equal.
**4** C

### Exercise 7A Classifying triangles

**1** **a** isosceles **b** equilateral **c** scalene
**d** isosceles **e** scalene **f** scalene
**2** **a** right **b** acute **c** acute
**d** acute **e** obtuse **f** right
**3** **a** acute-angled scalene triangle
**b** obtuse-angled isosceles triangle
**c** obtuse-angled scalene triangle
**d** equilateral triangle
**e** right-angled isosceles triangle
**f** acute-angled isosceles triangle
**4** **a** $a = 80°$ **b** $b = 82°$ **c** $c = 91°$
**d** $d = 50°$ **e** $e = 67°$ **f** $f = 33°$
**5** **a** $x = y = 60°$ **b** $l = 30°, m = 120°$
**c** $c = 60°$
**6** **a** $a = 53°$ **b** $a = b = c = 60°$
**c** $a = b = 73°$ **d** $a = b = 45°$
**e** $a = 25°; b = 130°$ **f** $a = b = 60°$
**9** One possible answer is ΔCBA.
**10** 90 or more **11** 33°
**13** **a** It is outside the triangle. **b** 130°
**c** 130°. They add to the size of the exterior angle.
**d**




**e** 110° and 120°
**f** 110° and 120°. They add to the size of the exterior angle.
**g** exterior, opposite
**14** **a** $x = 105°$ **b** $x = 120°$ **c** $x = 115°$
**d** $x = 40°$ **e** $x = 95°$ **f** $x = 25°$
**16** **a** 360°
**b** One possible answer is:




**i** $a = 100°; b = 115°; c = 145°$
**ii** $a + b + c = 360°$
**c** The exterior angles of a triangle add to 360°.
**17** **a** $x = 30°$ **b** $x = 10°$ **c** $x = 10°$
**d** $x = 10°$ **e** $x = 12°$ **f** $x = 20°$

## 7B Classifying quadrilaterals

### 7B Start thinking!

**1** quad means four
**2** rectangle, kite, square, rhombus, trapezium, parallelogram
**3** sample answer:

| Properties | Rectangle | Kite | Square | Rhombus | Trapezium | Parallelogram |
|---|---|---|---|---|---|---|
| Two pairs of opposite sides are parallel | ✓ | | ✓ | ✓ | | ✓ |
| Opposite sides equal | ✓ | | ✓ | ✓ | | ✓ |
| All sides are equal | | | ✓ | ✓ | | |
| Opposite angles equal | ✓ | | ✓ | ✓ | | ✓ |
| All angles are 90° | ✓ | | ✓ | | | |
| Two pairs of adjacent sides are equal | | | ✓ | ✓ | | |
| One pair of opposite sides is parallel | | | | | ✓ | |
| One pair of opposite angles is equal | | ✓ | | | | |

### Exercise 7B Classifying quadrilaterals

**1** **a** parallelogram; two pairs of parallel lines, opposite sides are equal
**b** kite; adjacent sides are equal, no parallel lines
**c** square; four equal sides, 90° angles
**d** trapezium; one pair of parallel sides, no equal sides
**e** rectangle; 90° angles, two pairs of equal, parallel sides
**f** rhombus; four equal sides, opposite sides are parallel

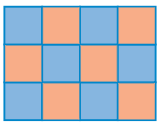
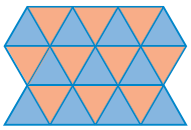
**2** **a** $a = 112°$ **b** $b = 135°$ **c** $c = 142°$
**d** $d = 253°$ **e** $e = 59°$ **f** $f = 79°$
**3** **a** irregular quadrilateral; $a = 70°$
**b** parallelogram; $b = 53°$
**c** trapezium; $c = 35°$
**d** irregular quadrilateral; $d = 145°$
**e** irregular quadrilateral; $e = 65°$
**f** rhombus; $f = 105°$
**4** **a** $a = 130°$ **b** $b = 90°$ **c** $c = 80°$
**5** **a** $d = 60°$; $e = 130°$ **b** $c = 80°$
**c** $x = 150°$; $y = 30°$; $z = 150°$ **d** $k = 45°$
**e** $m = 90°$ **f** $f = 135°$
**g** $x = w = 111°$ **h** $e = f = g = 90°$
**i** $s = t = 118°$
**7** **a** rhombus **b** trapezium
**c** rectangle **d** parallelogram
**e** square **f** kite
**8** **a** false **b** false **c** false **d** false
**9** trapezium, rectangle, rhombus, parallelogram
**10** **a** yes **b** no **c** yes
**e** A rhombus has all sides equal, opposite sides parallel and opposite angles equal. A square has these characteristics with the special case of all angles equal 90°.
**11** **a** two **b** 180° **c** $2 \times 180° = 360°$
**d** A quadrilateral can always be split into two triangles by a diagonal line and therefore has 360°.
**e** The angles in a quadrilateral add to 360°.
**f** Some of the angles within the triangles are in the centre of the quadrilateral and not on the outside.
**12** **b** revolution **c** 360°; yes
**d** All quadrilaterals have an angle sum of 360°.
**14** **a** $x = 35°$ **b** $x = 80°$ **c** $x = 15°$
**d** $x = 37°$ **e** $x = 17°$ **f** $x = 12°$
**15** **c** 360°. It is the same.
**d** One possible answer is: The exterior angles on a triangle and a quadrilateral both add to 360°.

## 7C Identifying 2D shapes

### 7C Start thinking!

1 straight-sided shapes and curved shapes
2 Polygons have straight sides.
3 Shape A has five equal sides and shape D has eight unequal sides.
4 Shape A is a regular polygon. The sides are equal in length.

### Exercise 7C Identifying 2D shapes

**1** **a** hexagon, heptagon, octagon, nonagon, decagon, dodecagon
**b** octagon
**2, 4** **a** polygon; regular; 10 sides; decagon
**b** polygon; irregular; six sides; hexagon
**c** not a polygon
**d** polygon; regular; three sides; triangle
**e** polygon; regular; five sides; pentagon
**f** not a polygon
**g** polygon; irregular; four sides; quadrilateral
**h** polygon; irregular; eight sides; concave octagon
**i** not a polygon
**3** **a** regular and convex
**b** irregular and convex
**c** irregular and concave
**d** regular and convex
**e** irregular and concave
**f** irregular and convex
**5** **a** polygon; regular; eight sides; octagon
**b** polygon; irregular; five sides; pentagon
**c** polygon; irregular; five sides; concave pentagon
**d** polygon; regular; nine sides; nonagon
**e** polygon; regular; six sides; hexagon
**f** polygon; irregular; 10 sides; concave decagon
**6** Two possible examples are: irregular quadrilateral, irregular pentagon.
**7** Some examples are:
**a**  **b**  **c** 
**8** **a** irregular hexagon
**b** irregular concave decagon
**c** irregular concave octagon
**d** irregular pentagon
**e** irregular concave decagon
**f** irregular concave heptagon
**10** **a** One name describes the points or vertices and the other name describes how many sides it has.
**b** **i** four-pointed star or concave octagon
**ii** six-pointed star or concave dodecagon
**iii** three-pointed star or concave hexagon
**c** The number of points is always half the number of sides.
**11** **a** Angles in a triangle add to 180°.
**b** 10 sides **c** eight **d** 1440°
**12** **a** **i** three **ii** 540° **b** **i** four **ii** 720°
**c** **i** five **ii** 900° **d** **i** seven **ii** 1260°
**13** **a** Yes; triangles and quadrilaterals have straight sides.
**b** triagon and tetragon
**14** **a** regular hexagons in the honeycomb and triangles, quadrilaterals, pentagons and hexagons in the Water Cube
**b** Regular hexagons fit together perfectly in the honeycomb. More than one shape is needed to fit irregular shapes together.
**15** **a** regular hexagonal, irregular
**b** **i**  **ii** 
**16** **a** squares, triangles
**18** **a** 108° **b** 120° **c** 128.57° **d** 140°
**19** **a** $a = 120°$ **b** $b = 41°$ **c** $c = 146°$

## 7D Identifying 3D objects

### 7D Start thinking!

2 Pyramids and prisms

3 A has a triangular base; B has a rectangular base; C has an octagonal base; D has a pentagonal base; E has a square base; and F has a nonagonal base.

4

| Object | Base shape | Does it end in a point? | Prism or pyramid? |
|---|---|---|---|
| A | triangle | no | prism |
| B | rectangle | no | prism |
| C | octagon | no | prism |
| D | pentagon | yes | pyramid |
| E | square | yes | pyramid |
| F | nonagon | yes | pyramid |

### Exercise 7D Identifying 3D shapes

**1** **a** prism **b** prism **c** pyramid
**d** prism **e** pyramid **f** pyramid

**2** **a** heptagon **b** square **c** triangle
**d** triangle **e** decagon **f** rectangle

**3** **a** heptagonal prism
**b** square-based prism or cube
**c** triangular pyramid or tetrahedron
**d** triangular prism **e** decagonal pyramid
**f** rectangular pyramid

**4** **a** pentagonal prism **b** hexagonal pyramid
**c** heptagonal pyramid **d** decagonal prism
**e** nonagonal prism **f** octagonal pyramid

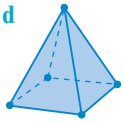
**5** **a** rectangular prism
**b** tetrahedron (triangular pyramid)
**c** sphere

**7** **a** yes; it has flat faces, straight edges and vertices
**b** no; it has some curved sides
**c** yes; it has flat faces, straight edges and vertices

**8** **a** cylinder, sphere, cone
**b** They have some curved sides.
**c** Because they have some curved sides.

**9** **a** **iii** cone and sphere
**b** **iii** cylinder and trapezoidal prism
**c** **iii** rectangular prism and cylinders

**11** **a** five faces
**b** eight edges
**c** five vertices
**d**



**12**

| Shape | F | E | V |
|---|---|---|---|
| heptagonal prism | 9 | 21 | 14 |
| cube | 6 | 12 | 8 |
| tetrahedron | 4 | 6 | 4 |
| triangular prism | 5 | 9 | 6 |
| decagonal pyramid | 11 | 20 | 11 |
| rectangular pyramid | 5 | 8 | 5 |
| pentagonal prism | 7 | 15 | 10 |
| hexagonal pyramid | 7 | 12 | 7 |
| heptagonal pyramid | 8 | 14 | 8 |
| decagonal prism | 12 | 30 | 20 |
| nonagonal prism | 11 | 27 | 18 |
| octagonal pyramid | 9 | 16 | 9 |

**13** Each face is flat and all edges are straight.

**14** Some possible answers are: yes, each face is a hexagon or a pentagon and all edges are straight; or no, the faces are slightly curved because of the pressure in the ball.

**15** **a–c**

| | Tetrahedron | Cube (hexahedron) | Octahedron |
|---|---|---|---|
| $F$ | 4 | 6 | 8 |
| polygon | triangle | square | triangle |
| $V$ | 4 | 8 | 6 |
| $E$ | 6 | 12 | 12 |
| $F+V-E$ | 2 | 2 | 2 |

| | Dodecahedron | Icosahedron |
|---|---|---|
| $F$ | 12 | 20 |
| polygon | pentagon | triangle |
| $V$ | 20 | 12 |
| $E$ | 30 | 30 |
| $F+V-E$ | 2 | 2 |

**d** $F + V - E = 2$

**16** **a** yes **b** The rule stays the same.

**17** **a** 8 **b** 5 **c** 14 **d** 12

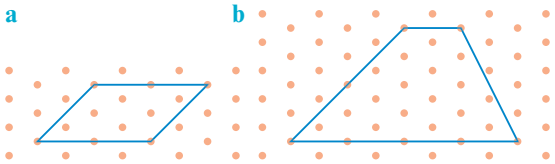
## 7E Drawing 2D shapes and 3D objects

### 7E Start thinking!

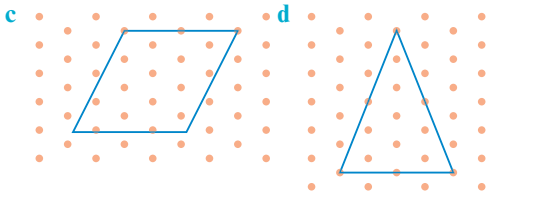
2 Triangles: equilateral, isosceles, scalene.
Quadrilaterals: parallelogram, rhombus, rectangle, square, kite, trapezium.

### Exercise 7E Drawing 2D shapes and 3D objects

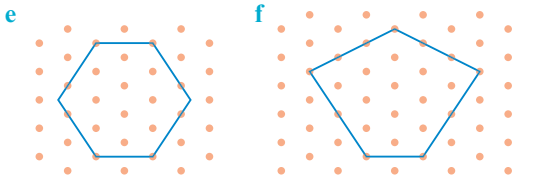
**1** **a** **b**



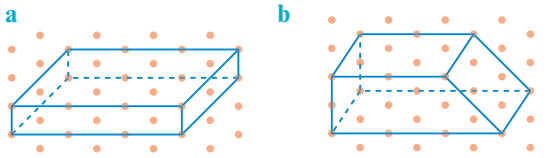
**c** **d**



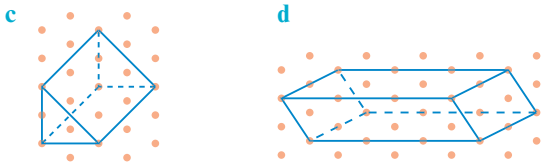
**e** **f**



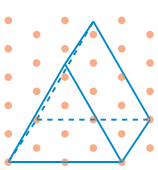
**2** **a** **b**



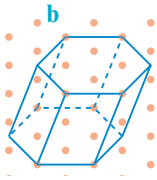
**c** **d**



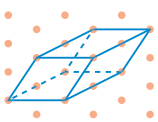
**3** **a**



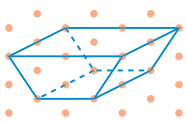
**b**



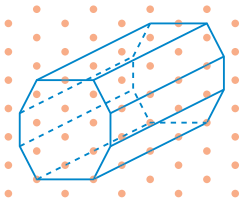
**c**



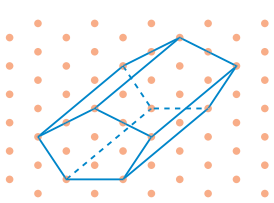
**d**



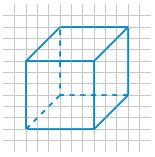
**e**



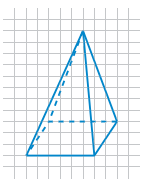
**f**



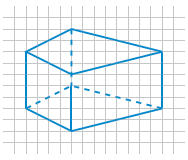
**4** **a**



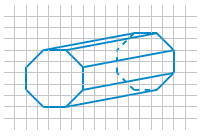
**b**



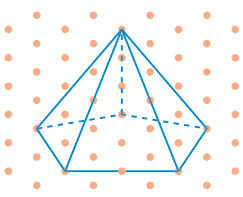
**c**



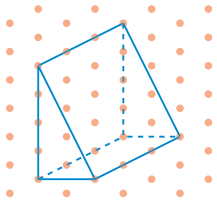
**d**



**e**

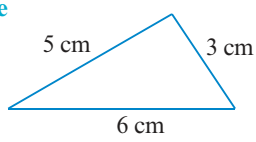


**f**



**5** **a** 6 cm is the longest

**b–e**

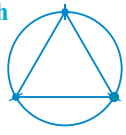



**f** no

**7** **b** Because we are making a triangle with three equal sides.

**c** 360° **d** 120°

**e–h**



**g** 120°

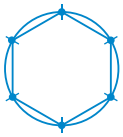
**j** The sides of these triangles are not equal.

**8** Because the length of any two sides of a triangle must add up to be more than the length of the third side.

**9** **a** six sides **b** six sectors **c** 60°

**d** circle, sides, protractor, circumference, erase

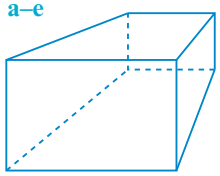
**e**



**10** Check with your teacher. Internal angles for each sector are:

**a** 45° **b** 72° **c** 40° **d** 36°

**12** **a–e**

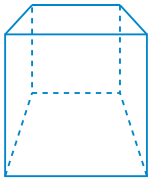


**f** The rectangle that represents the back face of the rectangular prism is smaller in size and the edges between the front and back faces are not parallel.

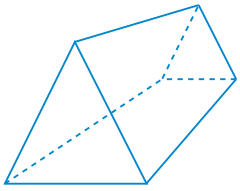
**g** They could be viewed as having a vanishing point already.

**13** Sample answers are shown.

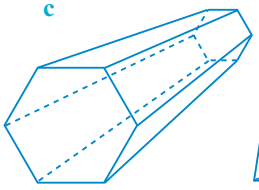
**a**



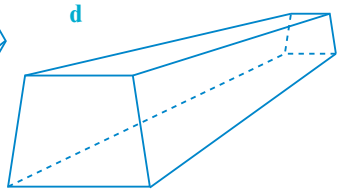
**b**



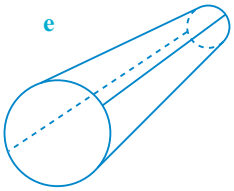
**c**



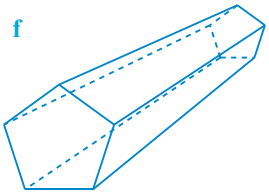
**d**



**e**



**f**



## 7F Planning and constructing 3D objects

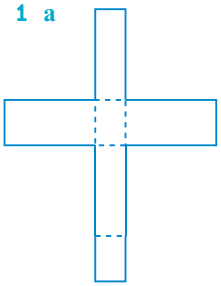
### 7F Start thinking!

**1** six **2** one **3** square

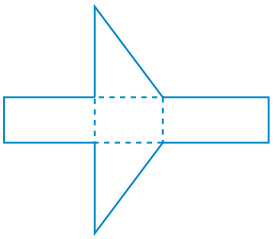
**5** No, there would be no sides.

### Exercise 7F Planning and constructing 3D objects

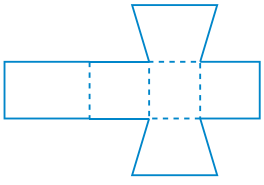
**1** **a**



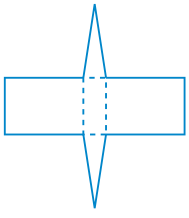
**b**



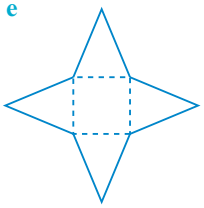
**c**



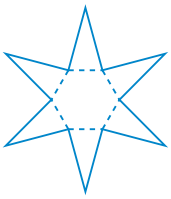
**d**



e

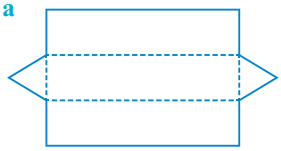


f

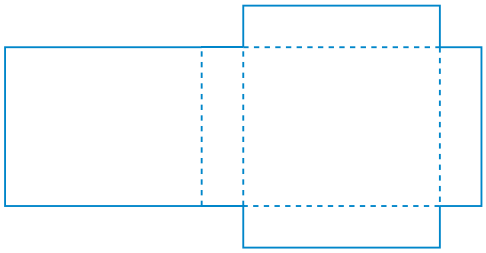


2 Sample answers are shown.

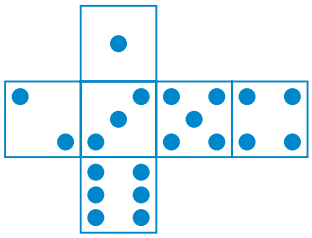
a



b

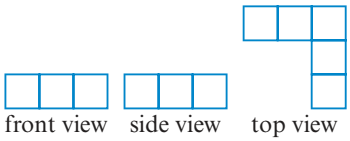


c

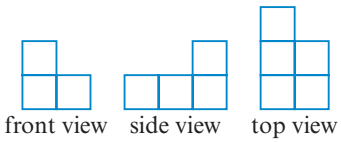


4 b 20, equilateral triangles

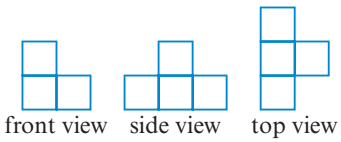
5 a



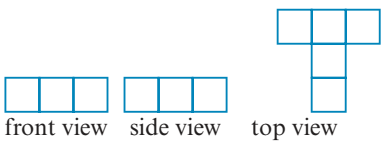

b



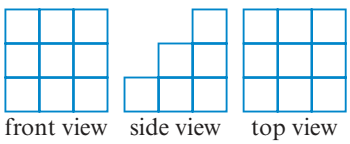

c



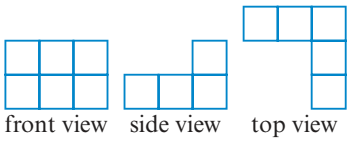

d




e




f

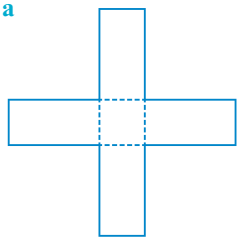



6 a rectangular prism
b tetrahedron (triangular pyramid)
c cylinder

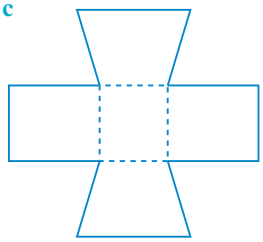
8 two

9 Sample answers are shown.

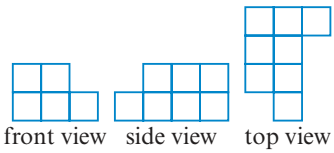
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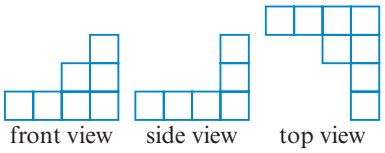
c



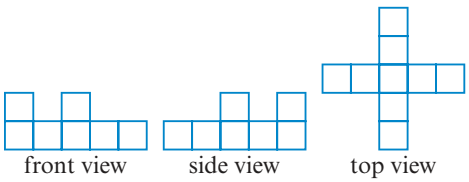
10 a



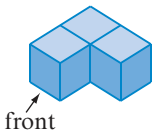

b



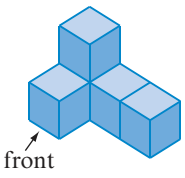

c



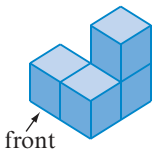

12 a



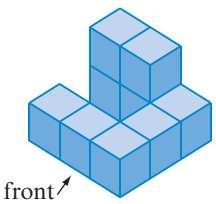

b



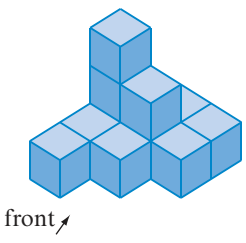

c




d




13




## 7G Symmetry of 2D shapes and 3D objects

### 7G Start thinking!

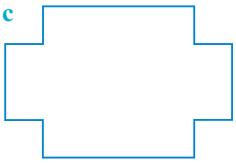
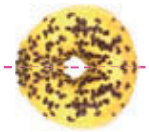
1 along the length and through the centre of the ladybird

2 The mirror forms a line and the object is reflected on the other side of the line.

4 a yes b yes c yes d four

### Exercise 7G Symmetry of 2D shapes and 3D objects

**1** **a** yes; horizontal line of symmetry through the centre
**b** yes; diagonal line of symmetry through the centre
**c** not symmetrical
**d** yes; vertical line of symmetry through the centre
**e** not symmetrical
**f** not symmetrical

**2** **a** 
**b** 
**c** 
**d** 
**e** 
**f** 

**3** **a** two **b** one **c** two **d** zero
**e** four **f** six

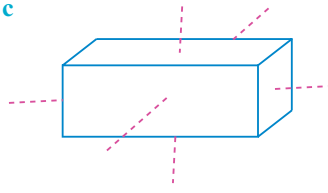
**4** **a, b**

A B C D E F
G H I J K L
M N O P Q R
S T U V W X
Y Z

**c** F, G, J, L, N, P, Q, R, S, Z
**d** H, I, O, X
**e**

a b c d e f
g h i j k l
m n o p q r
s t u v w x
y z

**h** Some axes of symmetry will change.

**5** **a** four **b** five **c** two
**6** **c** three times **d** three **e** eight
**f** H, two; I, two; O, two; S, two; X, two; Z, two
**7** **a** three **b** two **c** five
**8** **a** One side of the animal does match the other side; however, the individual fine markings are different.
**9** **a** two **b** yes
**c** 
**10** nine axes of symmetry
**12** Although they can happen together, they are different. For example, the queen of diamonds has no axes of symmetry but has a rotational symmetry order of two.
**13** Yes, if the shape can only be turned once to get back to its original position.
**14** B: horizontal line through 8 on the *y*-axis; C: vertical line through 8 on the *x*-axis, horizontal line through 5.5 on the *y*-axis; D: vertical line through 13 on the *x*-axis, horizontal line through 5 on the *y*-axis.
**15** **a** horizontal axis of symmetry
**b** One example is BED.
**16** **a** vertical symmetry through the letter U
**b** no **c** One example is 'dob'.
**17** rotational symmetry

## 7H Describing transformations

### 7H Start thinking!

**3** Shapes are the same size but in different orientations and positions.
**5** yellow shape; no change in orientation

### Exercise 7H Describing transformations

**1** **a** 2 units down, 3 units right
**b** 90°, anticlockwise or 270°, clockwise
**c** vertical
**2** **a** reflection **b** rotation **c** translation
**d** translation **e** reflection **f** rotation
**3** **a** reflection across a horizontal mirror line
**b** rotation 90° anticlockwise from two spaces above top right corner
**c** translation 5 units right and 3 units down
**d** rotation 90° clockwise or 270° anticlockwise from one space below bottom left corner
**e** reflection across a diagonal mirror line
**f** translation 4 units left and 4 units down
**4** **a** reflection across a horizontal mirror line
**b** rotation 90° clockwise or 270° anticlockwise from bottom left corner
**c** translation 3 units left and 1 unit up
**d** translation 2 units right and 4 units down
**e** reflection across a vertical mirror line
**f** rotation 180° clockwise or 180° anticlockwise from bottom right corner
**5** **a** (2, 12) **b** 1, 10 **c** C **d** (4, 7)
**e** They've counted 6 from the right corner of the shape instead of the left corner.
**f** **i** 2 units to the right and 5 units down
**ii** 8 units to the right and 10 units down
**iii** 6 units to the left and 3 units down
**iv** 8 units to the left and 10 units up
**v** 3 units to the left and 2 units up
**vi** 3 units to the right and 5 units up

**6** **a** Shape A has rotated 90° clockwise around the topmost point of A to form shape B; shape A has rotated 180° around the same point to make shape C; and shape A has rotated 270° clockwise or 90° anticlockwise around the same point to form shape D.

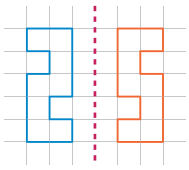
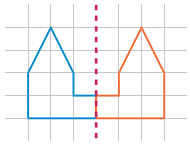
**7** **a**  **b** 

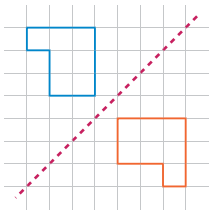
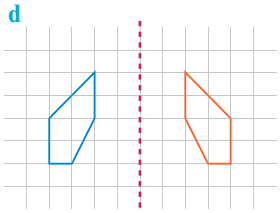
**8** Original shape is D, final image is A.

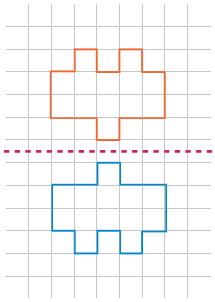
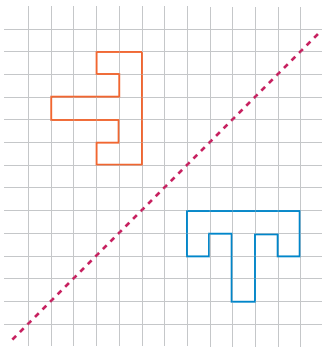
**9** Turning 90°clockwise results in the same position as turning 270° anticlockwise because there are 360° in a full circle.

**10** **b** 270° anticlockwise **f** 180° anticlockwise

**11** A′ (green) is the result of a rotation 90° around the point in the lower right corner of the original (blue) shape. A″ (purple) is the result of a rotation 90° around the point outside and below the original shape.
A‴ (orange) is the result of a rotation of 90° around the point inside the centre of the original shape.

**12** **a**  **b** 

**c**  **d** 

**e**  **f** 

**13** **a** (8, 5) **b** (4, 1) **c** (10, 12)

**14** no

**18** The shape has been translated 3 units up and 3 units right, then rotated 90° clockwise from the bottom right corner, then reflected in a horizontal mirror line.

## 7I Performing transformations

### 7I Start thinking!

**1** translation, rotation, reflection

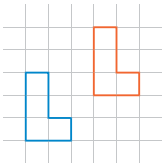
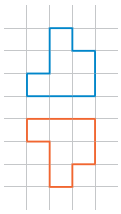
**2** The image is marked with a dash or prime (′).

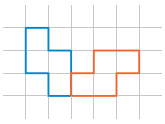
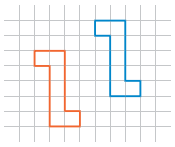
**4** Move it.

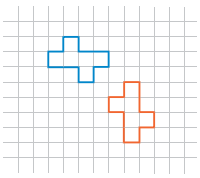
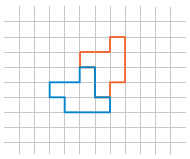
**5** When a shape or object is moved without turning or changing size.

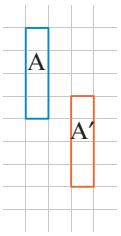
**6, 7** One possible answer is: I translated the shape 13 units up and 7 units right.

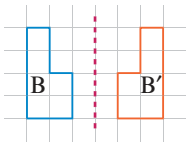
### Exercise 7I Performing transformations

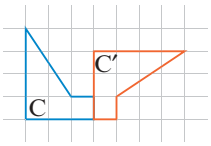
**1** **a**  **b** 

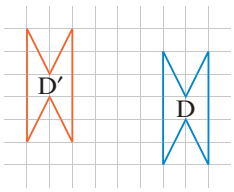
**c**  **d** 

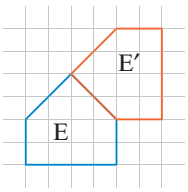
**e**  **f** 

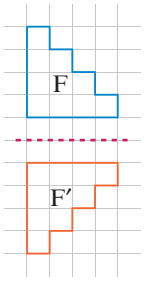
**2** **a** 


**b** 

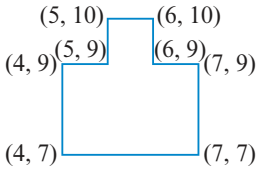

**c** 


**d** 

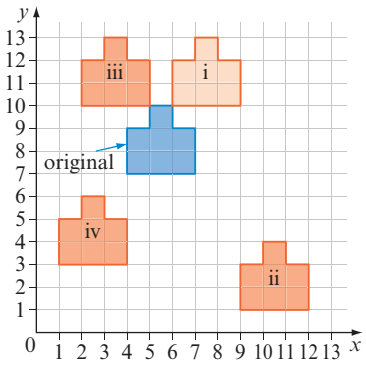

**e** 


**f** 

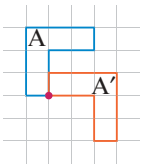
**3 a**



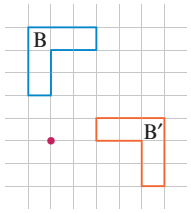

**b**



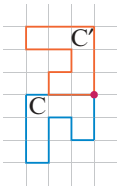

**4 a**



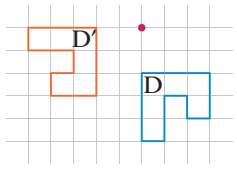

**b**



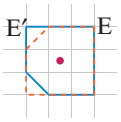

**c**



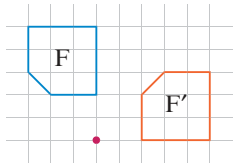

**d**



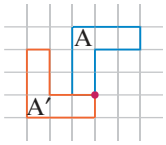

**e**



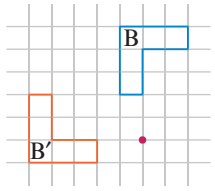

**f**



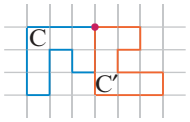

**5 a**



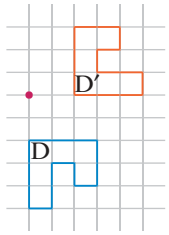

**b**



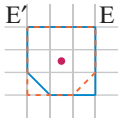

**c**



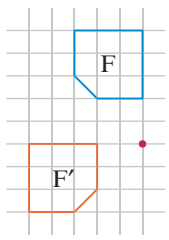

**d**



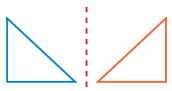

**e**




**f**



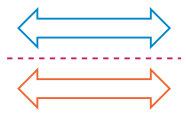

**6 a**



**b**



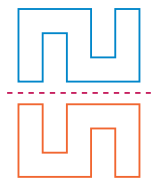
**c**



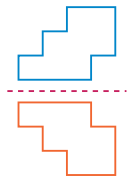
**d**



**e**



**f**



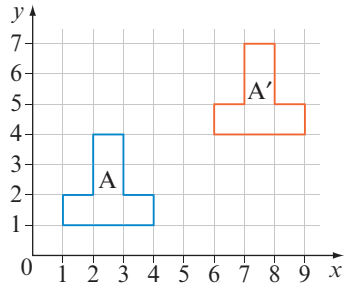
**7** One possible answer is:

**ii** 3 units right, 8 units down then 2 units right, 2 units up

**iii** 4 units right, 2 units up then 6 units left, 1 unit up

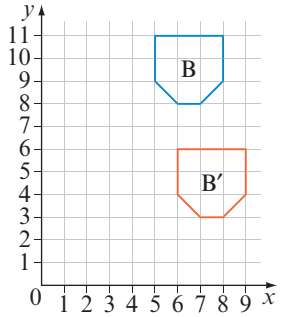
**iv** 5 units left, 1 unit down then 2 units right, 3 units down

**8 a**



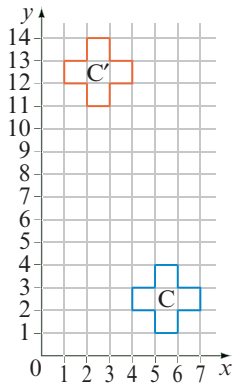

5 units right, 3 units up

**b**



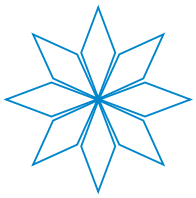

1 unit right, 5 units down

**c**

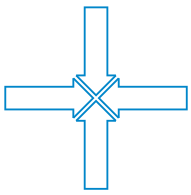



3 units left, 10 units up

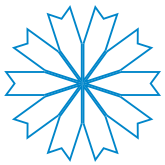
**10** **a**



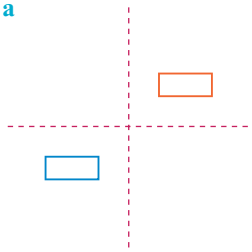
**b**



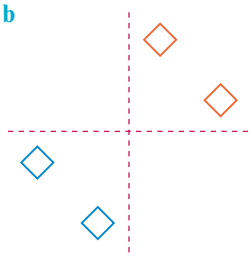
**c**



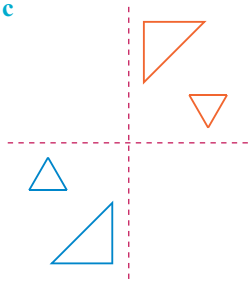
**12** **a**



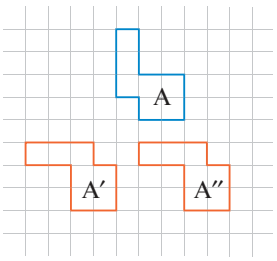
**b**



**c**



**14** **a,b**




## 7 Chapter review

### MULTIPLE-CHOICE

**1** A **2** B **3** C **4** C **5** C

### SHORT ANSWER

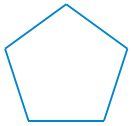
**1** **a** $x = 60°$ **b** $x = 30°$

**2** **a** $x = 86°$ **b** $x = 115°$

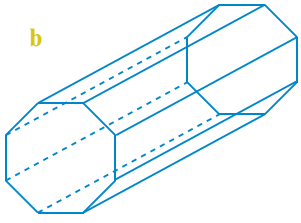
**3** **a** irregular quadrilateral
**b** concave heptagon **c** regular octagon
**d** regular hexagon

**4** Shape I is a triangular prism and shape J is a pentagonal pyramid.

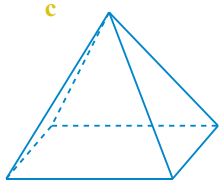
**5** **a**



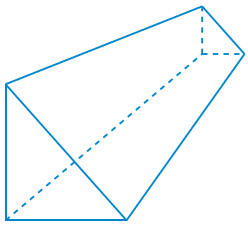
**b**



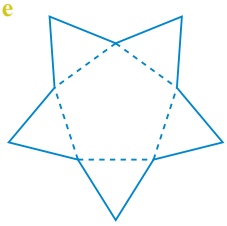
**c**



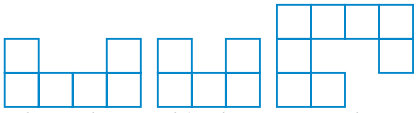
**d**



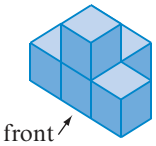
**e**



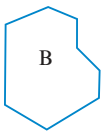
**6**



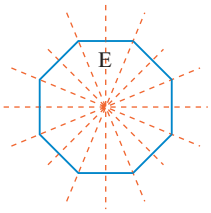

**7**



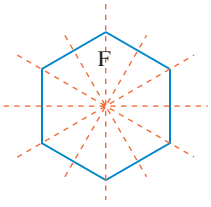

**8**




no axes of symmetry




eight axes of symmetry

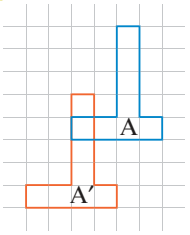



six axes of symmetry

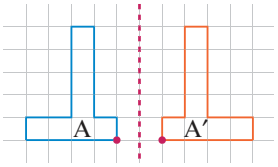
**9** **a** B: rotated 90° clockwise from bottom right corner
C: rotated 90° clockwise from bottom right corner then reflected in horizontal mirror line
D: reflected in horizontal mirror line
E: translated 1 unit up and 3 units left
F: rotated 180° clockwise from bottom left corner
G: rotated 90° clockwise from bottom right corner then translated 3 units down

**b** A: rotated 90° anticlockwise from bottom left corner
C: reflected in horizontal mirror line
D: rotated 90° anticlockwise from bottom left corner then reflected in horizontal mirror line
E: rotated 90° anticlockwise from bottom left corner then translated 1 unit up and 3 units left
F: rotated 90° anticlockwise from bottom left corner then rotated 180° anticlockwise from new bottom left corner
G: translated 3 units down

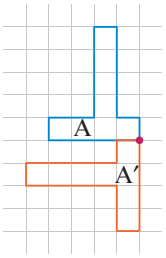
**10 a, b**




**c**




**d**

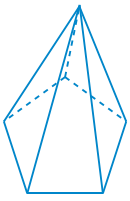



## NAPLAN-STYLE PRACTICE

**1** isosceles **2** 140° **3** rhombus
**4** 100° **5** irregular hexagon
**6** 10 **7** D **8** pentagon
**9** pentagonal prism **10** trapezoidal prism
**11** 8 **12** hexagonal pyramid
**13** B **14** none **15** two
**16** 5 units right and 4 units down
**17** 90 **18** reflected in horizontal mirror line
**19** 3 units right and 4 units down

## ANALYSIS

**a** five-pointed star
**b** 5
**c** one regular pentagon and five isosceles triangles
**d** pentagonal pyramid
**e**



## 7 Connect

For feedback on this open-ended task, see your teacher.

## CHAPTER 8 TIME, MASS AND TEMPERATURE

### 8 Are you ready?

**1** 24°C **2** B **3** 60 **4** 24
**5** 5.45 **6** 2.21 pm **7** B **8** 55
**9** 6.58 am **10** 4 minutes
**11** B **12** B **13** 17°C **14** D

## 8A Reading scales

### 8A Start thinking!

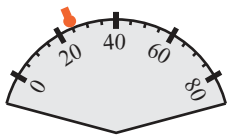
**1** 10°C **2** 20°C and 30°C
**3** 28°C; it is nearly at 30°C
**4** four marks **5** five intervals
**6** 10, five, 10 ÷ 5 = 2°C **7** 28°C
**8** It is on the fourth minor mark.

### Exercise 8A Reading scales

**1 a** 10 units **b** two **c** 5 units
**2 a** 20 units **b** four **c** 5 units
**3 a** 1 cm **b** 5 mm **c** 1 mm
**d** 5.2 cm or 52 mm
**4 a** 375 mL **b** 98 cm **c** 25°C
**5 a** from question 1:

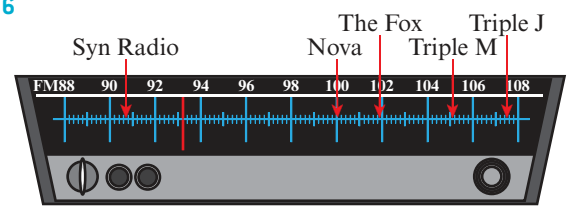



from question 2:

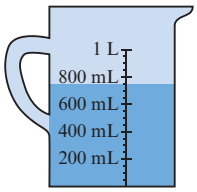



**b** No, on both scales 25 falls on a minor mark.

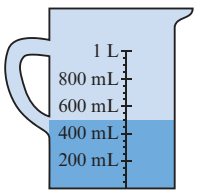
**6**



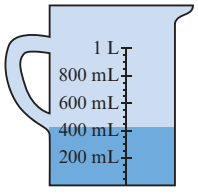

**7 a** 1-L jug
**b**




**c** 500 mL
**d**




**e i** 425 mL
**ii**



iii Because minor increments are in 50 mL not 25 mL, have to estimate without being able to match exact interval marking.

**8** a 1150 g b grams c 950 g
d no; 1150 g − 200 g = 950 g

**9** a 20 km/h b 10 km/h c 5 km/h
d 70 km/h e 55 km/h f 35 km/h
g 


**10** a 5 cm b 16.2 cm
c By subtracting the first measurement from the second.
d 11.2 cm or 112 mm e 5.6 cm
f Phone will fit but about 1.2 cm of it will show above top of pocket.

**12** a 1060 millibars b very dry
c 1010 millibars
d changing, rain clouds forming
e 960–970 millibars

**13** a Because shape of flask changes with its height.
b 250 mL

## 8B Estimation and accuracy

### 8B Start thinking!

**1** about 8–9 cm **2** 8.5 cm

**4** b Because there is human error in taking measurements.

**6** b They should. There is less room for error when minor marks are further apart.
c These results probably not more accurate; minor scale is larger.

**7** a no; iPod is too small
b yes, car is appropriate size
c no, walking track is too large

**8** To be accurate you need to have a measuring device that measures in units appropriate to object.

### Exercise 8B Estimation and accuracy

**1** a cm; 30-cm ruler or measuring tape
b kg; bathroom scales c s; stopwatch
d L or mL; 2-L water jug
e m; measuring tape, trundle wheel
f min, h; watch g g; kitchen scales

**2** a i mm
ii small ruler or measuring tape
b i mL
ii jug with appropriate mL marks
c i minutes ii watch
d i g ii kitchen scales
e i cm ii measuring tape or 1-m ruler
f i tonnes ii truck scales

**3** a 4.4 cm, 44 mm b 3 cm, 30 mm
c 3 cm (30 mm) long, 1.3 cm (13 mm) wide
d 3.9 cm, 39 mm
e 15.5 cm (155 mm) long, 0.4 cm (4 mm) wide
f 1.8 cm (18 mm) wide, 2.5 cm (25 mm) long
g 1.5 cm (15 mm) wide, 5 cm (50 mm) long

**4** a i 13.0005
ii suggest too great a level of accuracy
b i 13 ii larger than the rest
c i 437.5
ii much larger than the rest, decimal point in the wrong position
d i 700 ii smaller than the rest
e i 18.2
ii not the same degree of accuracy

**6**

| | Time | Msss | Temperature |
|---|---|---|---|
| small measurements | stopwatch, atomic clock | 2-g balance scales | digital thermometer |
| medium measurements | wall clock | bathroom scales, kitchen scales | basic thermometer |
| large measurements | calendar | weigh station | oven temperature scale |

**7** Possible answers are:
a i 30 m ii trundle wheel
b i 5 hours ii stopwatch
c i 120 g ii kitchen scales
d i 30°C ii thermometer

**8** a not reasonable; suggests too great a level of accuracy
b reasonable
c not reasonable; should be in grams
d not reasonable; too cold for a warm summer's day

**9** Possible answers are:
a 28 m c 100 g
d between 30°C and 35°C

**11** Temperature can be negative and has only one unit (°C), as opposed to g, kg and s, min, etc.

**12** a i 73.5 kg ii 73–74 kg
b i 18°C ii 17–19°C
c i 102.5 g ii 100–105 g
d i 4.13 ii 4.12–4.13
e i 35 mL ii 34–36 mL

**13** a i yes ii only as an estimate
b i yes ii no; too accurate
c i no ii no; too large
d i yes ii yes
e i yes ii yes

**14** Your estimate is only as good as minor intervals. You can be more accurate with smaller minor intervals, but with larger minor intervals there is too much room for error.

15 By measuring twice you make sure you have the correct measurement and have not made a mistake.

16 a 100 mL b 50 mL c 25 mL
d 500 mL e 475 mL f 525 mL
g 475–525 mL

17 a 0.5 kg, 73–74 kg b 0.5°C, 17.5–18.5°C
c 2.5 g, 100–105 g d 0.5 min, 4.12–4.13
e 0.5 mL, 34.5–35.5 mL

## 8C Understanding time

### 8C Start thinking!

1 a i 60 ii 120 iii 180
b Multiply number of minutes by 60.

2 a i 60 ii 120 iii 180
b Multiply number of hours by 60.

3 To convert to smaller unit, multiply by conversion factor.

4 a i 1 ii 2 iii 3
b Divide number of seconds by 60.

5 a i 1 ii 2 iii 3
b Divide number of minutes by 60.

6 To convert to larger unit, divide by conversion factor.

### Exercise 8C Understanding time

1 a 180 minutes b 300 minutes
c 600 minutes d 2 minutes
e 10 minutes f 100 minutes

2 a 600 seconds b 300 seconds
c 3600 seconds d 2 hours
e 30 hours f 4 hours

3 a 0.5 or $\frac{1}{2}$ hour b 0.75 or $\frac{3}{4}$ hour
c $\frac{1}{3}$ hour d 0.25 or $\frac{1}{4}$ minute
e $\frac{1}{10}$ minute f $\frac{4}{5}$ minute

4 a 10 800 seconds b 36 000 seconds
c 21 600 seconds d 5400 seconds
e 4 hours f 2.5 hours

5 a 1440 minutes b 168 hours
c 86 400 seconds d 8784 hours
e 604 800 seconds f 720 hours

6 a 0230 b 1615 c 1348 d 2311

7 a 7.25 pm b 12.00 pm
c 10.18 pm d 9.58 am

8 a four thirty-five or twenty-five to five
b five past eleven c ten past nine pm
d two-thirty am e twelve minutes to two
f nineteen minutes past six pm

9 a minutes b hours or days or weeks
c seconds d months
e years f days or weeks

10 a 2 hours 21 minutes
b 10 hours 3 minutes c 5 hours 40 minutes
d 8 hours 29 minutes
e 18 hours 59 minutes
f 33 hours 18 minutes

11 a 2 hours 37 minutes
b 7 hours 31 minutes
c 2 hours 3.45 minutes
d 1 hour 33.25 minutes
e 211 hours 57 minutes
f 3055 hours 52 minutes

12 a $\frac{1}{4}$ hour b 0.25 hour
c 1.25 hours = 1 hour 15 minutes. Decimals can be written as fractions out of 100, but minutes are written as fractions of 60 (of an hour).

14 a 120 weeks b 2.31 years
c 60 weeks d 1.15 years
e 26 weeks f 4.62 hours
g 4 hours 37 minutes
h In NSW and Qld: 100 weeks, 1.92 years, 50 weeks, 0.96 years, 26 weeks, 3.85 hours, 3 hours 51 minutes SA: 75 weeks, 1.44 years, 37.5 weeks, 0.72 years, 26 weeks, 2.88 hours, 2 hours 53 minutes Tas: 50 weeks, 0.96 years, 25 weeks, 0.48 years, 26 weeks, 1.92 hours, 1 hour 55 minutes WA: 25 weeks, 0.48 years, 12.5 weeks, 0.24 years, 26 weeks, 0.96 hours, 58 minutes

16 a *Madagascar*: 1 hour 22 minutes
*Nim's Island*: 1 hour 32 minutes
*Journey to the Centre of the Earth*: 1 hour 28 minutes
*Night at the Museum*: 1 hour 42 minutes
b *Nim's Island* and *Night at the Museum*
c *Madagascar* and *Journey to the Centre of the Earth*

17 a C, iv b B, i c D, iii d A, ii

18 

19 a 
b 
c 
d 

20 a $\frac{1}{60}$ b 5
c In relation to hour hand, each major interval represents an hour. An hour is equal to 60 minutes, and 60 ÷ 5 = 12 minutes.

**d** i



ii



iii



iv



**21** Estimate where hour hand is and from this, estimate where minute hand is. The time is approximately 9.50.

## 8D Working with time

### 8D Start thinking!

**8** Subtract first timer's time from second timer's time.

**10** Subtract smaller time from larger time to find time difference.

### Exercise 8D Working with time

**1** **a** 5 hours **b** 15 hours **c** 3 hours
**d** 5 hours **e** 9 hours
**f** 5 hours 30 minutes

**2** **a** 2 hours 18 minutes **b** 5 hours 13 minutes
**c** 7 hours 54 minutes **d** 19 hours 27 minutes
**e** 3 hours 3 minutes **f** 7 hours 45 minutes

**3** **a** 7.30 pm **b** 5.15 pm **c** 2245
**d** 9.15 pm **e** 11.20 am **f** 1715

**4** **a** 44 **b** 126 **c** 124
**d** 209 **e** 141 **f** 188

**5** **a** 23 January **b** 2 July **c** 18 February
**d** 11 March **e** 30 June **f** 19 March

**6** **a** 11 hours **b** $\frac{1}{2}$ hour **c** 10.5 hours
**d** 9.5 hours, 10.75 hours, 10.5 hours
**e** 41 hours 15 minutes **f** 83 km/h
**g** 39 hours 15 minutes **h** 87 km/h

**7** **a** 7 April 2000 **b** 10 April 2002
**c** 7 years old
**d** 3 years 216 days (about 3 years 7 months)
**e** 4 years old **f** 16 years old

**9** **a** 2 km **b** 12 km **c** 2 hours **d** 5 km/h

**10** **a** They are total times for the relay time, not individual times.
**b** Subtract the time of the previous person from their time.
**c**

| | Trial 1 | Trial 2 | Trial 3 | Trial 4 |
|---|---|---|---|---|
| First person | 15.1 | 14.9 | 16.5 | 15.1 |
| Second person | 17.7 | 17.1 | 12.4 | 15.8 |
| Third person | 13.5 | 16.9 | 16.2 | 12.8 |
| Fourth person | 13.7 | 14.7 | 17.5 | 15.2 |

**d** Trial 1: third person; Trial 2: fourth person; Trial 3: second person; Trial 4: third person
**e** No; Trial 1 had two people faster than anyone in Trial 2.
**f** Trial 3: second person; Trial 4: third person; Trial 1: third person and fourth person.

**11** **a** 3 hours 40 minutes
**b** 22 days **c** 11 days
**d** just less than 37 minutes every day
**e** standby-time

**12** **a** 1 hour 32 minutes; 4 hours 45 minutes; 1 hour 51 minutes
**b** 9 hours 15 minutes; 7 hours 43 minutes; 2 hours 58 minutes; 1 hour 7 minutes

**13** **a** 3.22.58 pm, 6 September 2009
**b** 8.55.42 pm, 16 March 2011
**c** 5.00.24 pm, 13 May 1998

**14** **a** 127:13:53:00 **b** 114:01:39:00
**c** 432:07:00:00

## 8E Reading time

### 8E Start thinking!

**1** **a** Traralgon
**b** Melbourne (Southern Cross Station)
**c** Where the timetable is blank, it means the train does not stop at that station.

**2** **a** 8404, 8408, 8410, 8414, 8418, 8422, 8424, 8426, 8430, 8432, 8434
**b** The first five leave in the morning.
**c** four: 8404, 8408, 8410, 8414

**3** **a** 8.07 am **b** 10.21 am
**c** 2 hours and 14 minutes

**4** Latest train they can take from Traralgon leaves at 9.20 am.

### Exercise 8E Reading time

**1** **a** nine **b** seven **c** 7.07 am
**d** 15:56 or 3.56 pm **e** 14:48 or 2.48 pm

**2** **a** 18 every day **b** 11 every day
**c** They are peak hour trains. **d** six
**e** 7.12 pm from Traralgon, arrives 9.31 pm into Southern Cross.
**f** 2 hours 19 minutes **g** 2 hours 13 minutes
**h** 1 hour 56 minutes **i** 23 minutes faster

**3** **a** 14:20 or 2.20 pm **b** 14:48 or 2.48 pm
**c** 6.12 pm
**d** **i** 37 minutes **ii** 17 minutes
**iii** 41 minutes **iv** 1 hour 36 minutes

**4** **a** nine **b** 6.00 pm
**c** Ten, 6.00 pm **d** *Kroll Show*
**e** *Please Marry My Boy* **f** 1 hour
**g** *Nine's Live Thursday Night Football*, 2 hours
**h** Go! at 6.00 pm or Nine at 7.00 pm
**i** two
**j** There are three $\frac{1}{2}$-hour episodes.
**k** $\frac{1}{2}$ hour

**6** **a** Yes, they stop at fewer stops.
**b** 5.38 am weekday train

**8** **a** 10 buses
**b** They are the times that the train goes to the city.
**c** The bus does not stop here.
**d** 9.50 am **e** 9.17 am
**f** No, you would get there earlier and take the 10:07 train.
**g** 8.20 am
**h** **i** 40 minutes **ii** 51 minutes **iii** 42 minutes
**i** differences in traffic
**9** **a** 1989 **b** DVDs **c** 22 years **d** 90 years
**f** Answers will vary, but may include that it is easy to see the order of time.

## 8F Time zones

### 8F Start thinking!

1 night
3 Because the sun only shines on half of the planet at one time, and it rises and sets at different times as the Earth rotates.
4 The bands match the numbers at the top (and bottom) of the map and the colour on a country (or part of a country) usually matches the band that it is in.
5 Some countries may not want to split their country into different time zones (e.g. Greenland, China). Others may want to have the same time zone as adjacent countries (e.g. Argentina, Spain).
6 24 7 24
8 Each time zone is one hour long.
9 Any two of: Iceland, Portugal, Morocco, Mauritania, Mali.
10 UTC +8 11 ahead by 8 hours

### Exercise 8F Time zones

**1** **a** UK, Iceland, Portugal, Morocco, Mauritania, Mali
**b** Answers will vary depending on state.
**c** six time zones **d** one time zone
**e** Russia; eight time zones
**f** three time zones
**g** It is $\frac{1}{2}$ hour ahead or behind instead of an hour.
**2** **a** $+5\frac{1}{2}$ **b** +2 **c** −6 **d** +1
**e** +12 **f** +1
**3** **a** Vietnam is 4 hours ahead of Madagascar
**b** Mongolia is 13 hours ahead of Peru
**c** Morocco is 3 hours ahead of Argentina
**d** Fiji is 18 hours ahead of Mexico
**e** Turkey is 5 hours ahead of Greenland
**f** Japan is $3\frac{1}{2}$ hours ahead of India
**4** **a** Canberra is 2 hours ahead of Perth
**b** Melbourne is $\frac{1}{2}$ hour ahead of Darwin
**c** Brisbane is $\frac{1}{2}$ hour ahead of Adelaide
**d** Hobart and Sydney share the same time zone
**e** Brisbane is 2 hours ahead of Perth
**f** Hobart is $\frac{1}{2}$ hour ahead of Darwin
**5** **a** 11 am **b** 5 pm **c** 7 pm
**d** 11 pm **e** 1 am **f** 2 am
**6** **a** 3.30 pm **b** 7.15 pm **c** 3.45 pm
**d** 12.40 am **e** 1.26 pm **f** 2.10 pm
**7** For EST:
**a** 12 pm **b** 10 pm **c** 6 pm
**d** 10 am **e** 11 am **f** 10 pm
For CST:
**a** 12.30 pm **b** 10.30 pm **c** 6.30 pm
**d** 10.30 am **e** 11.30 am **f** 10.30 pm
For WST:
**a** 2 pm **b** 12 am **c** 8 pm
**d** 12 pm **e** 1 pm **f** 12 am
**8** **a** Eastern Standard Time, Central Standard Time and Western Standard Time
**b** EST: Hobart, Melbourne, Canberra, Sydney and Brisbane; CST: Adelaide and Darwin; WST: Perth
**c** **i** 10 am **ii** 10.30 am **iii** 8.30 am
**d** **i** 12.30 am **ii** 1 am **iii** 1 am
**e** **i** 7 pm **ii** 6.30 pm **iii** 5 pm
**9** **a** +10 **b** +8 **c** two time zones
**d** ahead **e** subtract 2 **f** 2 pm
**g** **i** $4\frac{1}{2}$ **ii** 8 **iii** 16
**iv** 9 **v** 2 **vi** 9
**h** **i** 3.30 pm **ii** 12 pm **iii** 4 am
**iv** 11 am **v** 10 pm **vi** 11 am
**10** There are 24 hours in a day.
**11** Hawaii is at very start of map (UTC – 10) and New Zealand is at end of map (UTC +12). Because IDL is after this, can tell it shows one whole day of times.
**12** **a** −10 **b** +10
**c** behind by 20 hours **d** four time zones
**e** ahead, behind; subtract 20, add 4
**f** **i** 8 pm Wed **ii** 1 pm Sun
**iii** 3 am Sat
**14** Because it is close to the International Date Line.
**15** Answers will vary but may include the following. Advantage: don't have to think about what time to call people. Disadvantage: may be very dark or light at wrong times. China has only one time zone.
**16** **a** 5.30 pm **b** 12.30 am
**17** **a** −8
**b** **i** 4 am, 8 July **ii** 2 am, 8 July
**iii** 3.30 am, 8 July **iv** 6 pm, 7 July
**v** 2 am, 8 July **vi** 1 pm, 7 July
**18** **a** +2
**b** **i** 4.30 pm **ii** 12.30 pm **iii** 4.30 pm
**iv** WST: 11.30 pm; CST: 1.00 am; EST: 1.30 am
**c** WST: 1 am to 4 pm; CST: 2.30 am to 5.30 pm; EST: 3 am to 6 pm
**d** WST: 9 pm; CST: 10.30 pm; EST: 11 pm
**19** **a** **i** 1 hour **ii** 2.5 hours **iii** 1.5 hours
**iv** 3 hours
**b** **i** 4.30 pm **ii** 6 pm **iii** 6.30 pm
**iv** 6.30 pm
**c** **i** 3.30 pm **ii** 5 pm **iii** 6.30 pm
**iv** 5.30 pm

**d** In winter, Queensland is ahead, but in summer, South Australia is ahead.
**e** **i** 2.30 pm **ii** 2.30 am **iii** 1.30 pm
**iv** 8 am **v** 5.15 pm **vi** 10 am
**f** Turn clocks forward in spring, and backwards in autumn (which Americans call 'fall').

**20** **a** 9 hours **b** 8.45 pm
**c** 4 hours 30 min **d** 11.30 pm

## 8G Understanding mass

### 8G Start thinking!

**5** 1000
**6** To convert grams to kilograms, divide by 1000.

### Exercise 8G Understanding mass

**1** ladybird, fairy-wren, frog, turtle, orca
**2** ladybird: mg, fairy-wren: g, frog: g, turtle: kg, orca: t
**3** **a** 1200 g **b** 6 g **c** 0.072 t
**d** 1000 mg **e** 0.450 kg **f** 3500 kg
**g** 0.75 g **h** 9800 mg **i** 8130 g
**j** 2.045 kg **k** 930 g **l** 0.145 t
**4** **a** 1 000 000 mg **b** 2 500 000 g
**c** 450 000 mg **d** 600 g
**e** 0.0345 t **f** 129 kg
**g** 0.000 085 t **h** 0.012 345 kg
**i** 8.46 t **j** 3 300 000 000 mg
**k** 4000 mg **l** 0.000 002 kg
**6** **a** They multiply mass by price per kg.
**b** \$2.20 **c** \$13.98/kg
**d** **i** 0.725 kg **ii** \$12/kg **e** \$27.44
**7** **a** 400 g **b** 50 g **c** 375 g **d** under
**8** **a** 360 g **b** 720 g **c** 4.32 kg or 4320 g
**d** the mass of the egg carton
**9** **a** 2 tablets
**b** **i** $\frac{1}{2}$ tablet **ii** 1 tablet
**iii** $2\frac{1}{2}$ tablets **iv** $1\frac{1}{2}$-2 tablets **c** 15 kg
**10** **a** They need to make sure the rope stretches exactly the right amount.
**b** Yes, this could make your rope stretch too much and it could make you hit the ground.
**11** He needs to convert all masses to the same units before adding them; 2.6 kg.
**12** **a** 2250 g **b** 2250, divide
**c** 2250 g ÷ 500 = 4.5 g
**d** Measure a large number of the objects and divide this total mass by the number of objects to find the individual mass.
**13** A possible answer could involve placing an elephant on a boat and marking the line of water, then removing the elephant and repeating the activity with smaller items of known weight until the marked line is reached. The sum of the items will give the total mass of the elephant.
**14** **a** **i** 1 g/mL **ii** 0.004 kg/mL or 4 g/mL
**iii** 3 g/mL **iv** 125 g/L or 0.125 g/mL
**b** Because you cannot tell if one is heavier or lighter unless it is in the same units.
**c** **i** 1 g/mL **ii** 4 g/mL
**iii** 3 g/mL **iv** 0.125 g/mL
**d** part ii
**15** **a** 19.3 g/cm$^3$ **b** 13.02 g/cm$^3$
**c** No. Necklace has lower density.
**d** Necklace is not pure gold. It contains a lighter metal mixed with gold.

## 8H Understanding temperature

### Exercise 8H Understanding temperature

**1** **a** 26°C **b** 56°C **c** 175°C **d** 3°C
**e** −2°C
**2** **a** b **b** c **c** e **d** a **e** d
**3** **a** 63°C **b** 27°C **c** 72°C **d** 12°C
**e** 9°C **f** 15°C **g** 54°C **h** 63°C
**4** **a** 70°C **b** 54°C **c** 31°C **d** 48°C
**e** 50°C **f** 24°C **g** 62°C **h** 46°C
**5** **a** testing the temperature of a baby
**b** testing the temperature when cooking hot food
**c** testing body temperature
**d** recording room and outside temperature
**6** **a** 10°C **b** −10°C **c** 18°C **d** 0°C
**e** −12°C **f** −40°C
**7** 145.9°C
**8** **a** It would feel cold. **b** It would feel warm.
**9** **a** −3.6°C **b** −8.8°C **c** July, 22.2°C
**d** December, −41.9°C **e** −22.5°C
**f** July, 5.8°C **g** January, −49.2°C
**10** **a** 35.6°C **b** November, 39.5°C
**c** June, 31.0°C **d** 23.2°C
**e** November, 27.2°C **f** July, 16.9°C
**11** **a** 44.4°C **b** 45.7°C
**c** Oymyakon (13.7°C) **d** a day in Oymyakon
**13** **a** **i** 38°C **ii** 10°C **iii** 0°C
**iv** 100°C **v** −18°C **vi** 24°C
**b** **i** 32°F **ii** 77°F **iii** 122°F
**iv** 59°F **v** 23°F **vi** 68°F
**14** **a** 31.7°C **b** –22.8°C **c** 65.6°C **d** 43.6°C

## 8 Chapter review

### MULTIPLE-CHOICE

**1** D **2** D **3** C **4** D **5** D
**6** B **7** D **8** B **9** B **10** A

### SHORT ANSWER

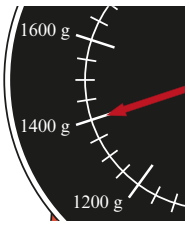
**1** **a** 5 **b** 0.49
**2** **a** metres **b** hours **c** grams
**d** kilometres
**3** 6.7 cm
**4** **a** 120 minutes **b** 3 minutes
**c** $\frac{1}{2}$ hour **d** 0.05 hour
**e** 4 hours 6 minutes **f** 3 hours 45 minutes
**5** **a** 4 hours 30 minutes **b** 14 hours 34 minutes
**c** 6 hours 35 minutes **d** 5 hours 57 minutes

**6** **a** 5.45 pm **b** 10.39 am
**c** 0900 **d** 1645
**7** **a** 2.19 pm **b** 9.53 am
**c** 8 minutes longer **d** 0740
**8** **a** WST: 4 hours; CST: $2\frac{1}{2}$ hours; EST: 2 hours
**b** **i** 5.30 pm **ii** 9.00 pm **iii** 2.00 pm **iv** 7.00 am
**c** WST: 8 am; CST: 9.30 am; EST: 10 am
**d** **i** 4 am **ii** 6 pm **iii** 11 am **iv** 9 am
**9** **a** 34 000 mg **b** 0.280 kg
**c** 300 kg **d** 1 700 000 mg
**e** 0.000 042 kg **f** 5 200 000 g
**10** **a** 26°C **b** 36°C **c** 15°C **d** 62°C
**11** **a** 49°C **b** 100°C **c** 7°C **d** 25°F

### NAPLAN-STYLE PRACTICE

**1** 3 **2** 420 g **3** 30-cm ruler
**4** 7.5 centimetres **5** 1440
**6** 1532 **7** 3 hours 45 minutes
**8** 5.52 pm **9** 3.52 pm **10** 2 hours
**11** 1604 or 4.04 pm **12** 4.34 pm
**13** 10 hours 10 minutes **14** 2 hours
**15** 5.20 am **16** 2 hours **17** 4.00 pm
**18** 42.5 **19** 1.95 **20** 21 **21** 77°F

### ANALYSIS

**a** 500 g **b** five batches
**c** 1.25 kg or 1250 g
**d** 

**e** 392°F
**f** 15:07
**g** 145°C

## 8 Connect

For feedback on this open-ended task, see your teacher.

## CHAPTER 9 LENGTH, AREA AND VOLUME

### 9 Are you ready?

**1** B **2** D **3** A **4** C **5** B
**6** D **7** 3 **8** 6 **9** A **10** B

## 9A Understanding length

### 9A Start thinking!

**3** 10
**4** **a** 100 **b** 1000 **5** 1000
**6** **a** 10 **b** 100 **c** 1000
**7** To convert from centimetres to millimetres, multiply by 10. To convert from millimetres to centimetres, divide by 10. Multiplying when converting to a smaller unit and dividing when converting to a larger unit also works for other length units.
**8** **a** Multiply by conversion factor.
**b** Divide by conversion factor.

### Exercise 9A Understanding length

**2** **a** 5 cm **b** 9.7 cm **c** 12.4 cm
**3** **a** multiply **b** divide
**4** **a** 8000 **b** 300 **c** 70
**d** 5 **e** 160 **f** 2
**g** 350 **h** 0.9 **i** 1340
**j** 4.7 **k** 0.084 **l** 0.512
**m** 60 **n** 380 **o** 0.79
**p** 0.004 31 km
**5** **a** 4000, 400 000 **b** 1200, 12
**c** 7300, 73 000 **d** 84, 0.084
**e** 2500, 2.5 **f** 36 200, 0.362
**6** **a** 550 mm **b** 27 km **c** 4.1 cm
**d** 620 cm **e** 3600 m **f** 0.89 m
**g** 170 000 cm **h** 0.5 m **i** 0.07 km
**j** 3650 mm **k** 940 000 mm **l** 0.0018 km
**7** **a** Divided by 10 instead of multiplying by 10 to convert from cm to mm. Correct answer is 55 cm = 550 mm.
**b** Multiplied by 100 instead of multiplying by 1000 to convert from km to m. Correct answer is 24 km = 24 000 m.
**c** Inserted a zero before multiplying by 100 to convert from m to cm. Correct answer is 0.66 m = 66 cm.
**d** Divided by 100 instead of dividing by 10 to convert from mm to cm. Correct answer is 1.73 mm = 0.173 cm.
**e** Divided by 10 instead of dividing by 100 to convert from cm to m. Correct answer is 420 cm = 4.2 m.
**f** Divided by 10 000 instead of dividing by 1000 to convert from m to km. Correct answer is 800 m = 0.8 km.
**8** **a** 15 mm, 5 cm, 60 mm (or 15 mm, 50 mm, 60 mm)
**b** 210 mm, 190 cm, 2 m (or 210 mm, 1900 mm, 2000 mm)
**c** 7001 cm, 0.7 km, 701 m (or 70.01 m, 700 m, 701 m)
**d** 8 cm, 82 mm, 0.8 m (or 80 mm, 82 mm, 800 mm)
**e** 30 cm, 3000 mm, 300 m, 3 km (or 30 cm, 300 cm, 30 000 cm, 300 000 cm)
**f** 0.005 km, 5500 mm, 560 cm, 5.7 m (or 500 cm, 550 cm, 560 cm, 570 cm)
**11** **c** 9.5 cm, 95 mm
**12** **a** 3.9 cm **b** 5.1 cm or 51 mm
**c** 2.5 cm or 25 mm **d** 14.5 cm or 145 mm
**15** **a** km **b** cm **c** m or cm
**d** mm **e** cm **f** m
**16** **a** 1250 m or 1.25 km
**b** more **c** 40
**17** 0.104 mm

**18** If the distance from one person to the next is estimated as 0.5 m, then the queue would be approximately 85 × 0.5 = 42.5 m.

**19** **a** 6 m **b** 1.25 m
**c** 60 mm **d** 3.5 m

**21** **b** Use string to trace along the curve of the cord, then lay the string straight and measure its length with a ruler.
**c** 17 cm

**22** **a** 11 cm **b** 23 cm **c** 16 cm
**d** 12.4 cm **e** 20.5 cm

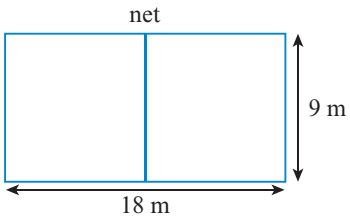
## 9B Perimeter

### 9B Start thinking!

**1** **a** 2.8 km **b** 6.2 km **c** 7.5 km **d** 9.4 km
**2** 12.5 km
**4** Perimeter is total distance around a shape. Measuring needs to start and finish at same position on the shape.

### Exercise 9B Perimeter

**1** **a** 23 km **b** 24 cm **c** 23 m
**d** 45 mm **e** 32.8 mm **f** 144 m
**2** **a** 16 cm **b** 18 cm **c** 24 cm **d** 22 cm
**3** **a** 14 cm **b** 12 cm **c** 18 cm **d** 18 cm
**4** **a** 186 mm or 18.6 cm **b** 170 mm or 17 cm
**5** **a** 18 cm **b** 450 cm **c** 17.8 cm
**d** 8 cm **e** 292 cm **f** 22.4 cm
**6** **a** 42 cm **b** 34 cm **c** 30 mm **d** 22.8 m
**7** **a** Sides marked with same symbol have same length.
**b** 30 cm **c** Twice length plus twice width.
**d** $P = 2l + 2w$ or $P = 2(l + w)$
**8** **a** 16 cm **b** 44 m **c** 164 mm **d** 248 cm
**e** 21 m **f** 10.2 cm **g** 25.74 m or 2574 cm
**h** 54 mm or 5.4 cm
**9** **a** 24 cm **b** 12 m **c** 88 mm **d** 15 km
**10** **a** $P = 3l$ **b** $P = 4l$ **c** $P = 8l$ **d** $P = 6l$
**11** **a** 27.4 m **b** 82.2 m **c** 109.6 m **d** part c
**12** **a**




**b** 54 m **c** 180 m
**13** 9 cm **14** 85.5 mm
**15** Some possible answers are: 10 m by 8 m, 9 m by 9 m, 16 m by 2 m.
**16** 26.88 m **17** 40.2 m

## 9C Understanding area

### 9C Start thinking!

**2** **a** 6 cm²
**b** by counting number of grid squares covered by stamp (6); each square has area 1 cm²
**3** **a** approx. 8 cm²
**b** by counting number of whole squares covered by coin (4) and estimating area partly covered (equivalent to approx. 4 squares); each square has area 1 cm²
**4** Approximate value, since if some of the 1-cm grid squares are not fully in the shape then you need to estimate how much of the square should be counted.
**5** Coin has larger area.

### Exercise 9C Understanding area

**1** **a** 10 cm² **b** 5 cm² **c** 6 cm²
**d** 8 cm² **e** 5 cm² **f** 7 cm²
**2** **a** **i** 8 cm² **ii** 4 cm² **iii** 8 cm² **iv** 6 cm²
**b** **i** 12 cm **ii** 10 cm **iii** 14 cm **iv** 14 cm
**c** Area is space inside shape; perimeter is total distance around edge of shape.
**3** **a** 8 cm² **b** 10 cm² **c** 12 cm² **d** 8 cm²
**6** **a** 24 cm² **b** 10 cm² **c** 11 cm² **d** 12 cm²
**7** **a** m² **b** mm² **c** km² **d** cm²
**e** m² **f** cm²
**8** **a** 800 ha **b** 400 000 m²
**9** **a** 28 × 250 000 = 7 000 000 km²
**b** 700 000 000 ha
**11** 41 (or 42) cm²

## 9D Area of a rectangle

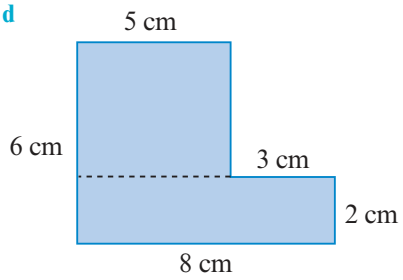
### 9D Start thinking!

**3** Multiply length by width.
**4** area = length × width
**5** When no grid is shown with the rectangle.

### Exercise 9D Area of a rectangle

**1** **a** 40 cm² **b** 6 m² **c** 400 mm²
**d** 112 cm² **e** 532 mm² **f** 9.03 m²
**2** **a** 63 cm² **b** 52 mm² **c** 6 m²
**d** 52 cm² **e** 137.74 mm² **f** 91.962 m²
**3** **a** 10 cm² or 1000 mm²
**b** 27 m² or 270 000 cm²
**c** 0.084 km² or 84 000 m²
**d** 5.76 cm² or 576 mm²
**e** 0.035 m² or 35 000 mm²
**f** 25 cm² or 2500 mm²
**4** **a** 3 m² or 30 000 cm² **b** 40 cm² or 4000 mm²
**c** 1.12 km² or 1 120 000 m²
**d** 180 000 mm² or 1800 cm²
**e** 6.2 cm² or 620 mm² **f** 42 cm²
**5** **a** 16 m² **b** 25 cm² **c** 289 mm²
**d** 12.96 cm²
**6** $A = l \times l$ or $A = l^2$
**7** **a** 49 m² **b** 225 cm² **c** 121 km²
**d** 3600 mm² **e** 40.96 cm² **f** 32 400 m²
**g** 0.49 mm² **h** 0.2809 km²
**8** **a** 30 cm² **b** 6 cm² **c** 36 cm²

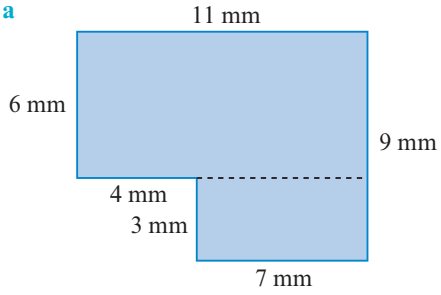
**d**



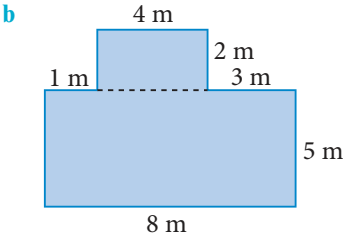

**e** $20\text{ cm}^2 + 16\text{ cm}^2 = 36\text{ cm}^2$

**9** One possible answer is shown.

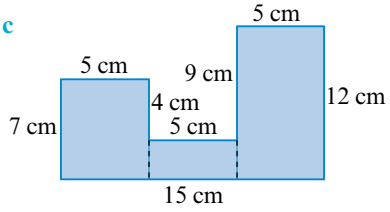
**a**



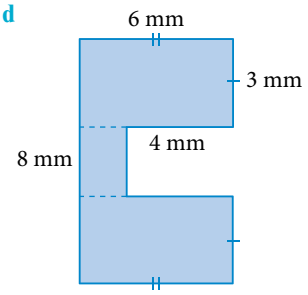

**b**




**c**




**d**




**10** **a** $87\text{ mm}^2$ **b** $48\text{ m}^2$ **c** $110\text{ cm}^2$
**d** $40\text{ mm}^2$

**11** Find the area of the larger rectangle and subtract the area of the smaller rectangle. Area of shaded region is $51\text{ m}^2$.

**12** **a** $115\text{ m}^2$ **b** $92\text{ mm}^2$ **c** $65\text{ cm}^2$
**d** $19\text{ m}^2$

**13** **a** **i** $43\text{ m}^2$ **ii** 32 m
**b** **i** $17.19\text{ cm}^2$ **ii** 20.2 cm
**c** **i** $80.81\text{ m}^2$ **ii** 41.6 m
**d** **i** $292\text{ mm}^2$ **ii** 86 mm

**14** Product of length and width should equal area. Some possible answers are given.
**a** length 12 cm, width 4 cm; length 8 cm, width 6 cm; length 16 cm, width 3 cm
**b** length 5 m, width 3 m; length 15 m, width 1 m; length 7.5 m, width 2 m

**15** One set only: square of length 4 mm.

**16** **a** **i** 11.885 m **ii** 10.97 m
**iii** 8.23 m **iv** $260.7569\text{ m}^2$
**v** $195.6271\text{ m}^2$ **vi** $26.336\text{ m}^2$
**d** More than $261\text{ m}^2$. Allow extra for material to cover the court surface past the outer lines, particularly past the baselines where players may run during rallies.

**17** 50 mm **18** 60 mm

**19** $126\text{ cm}^2$ **20** $400\text{ m}^2$

**21** Many answers are possible. Length and width measurements of rectangle must add to 32 cm.

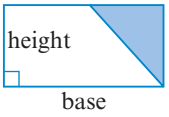
**22** Many answers are possible. Length and width measurements of rectangular garden must add to 6 m.

**23** Yes; one possible rectangle has length 4 cm and width 4 cm. Perimeter is 16 cm and area is $16\text{ cm}^2$.

## 9E Area of a parallelogram

### 9E Start thinking!

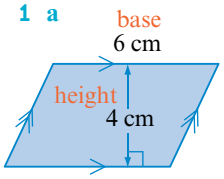
**3**



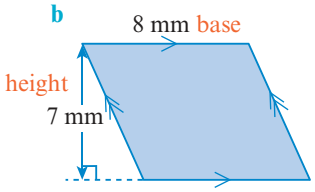

**4** Base of parallelogram is equivalent to length of rectangle and height of parallelogram is equivalent to width of rectangle. Using rule for area of a rectangle gives rule for area of a parallelogram: area = base × height.

### Exercise 9E Area of a parallelogram

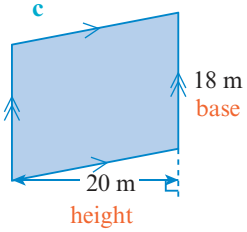
**1** **a**



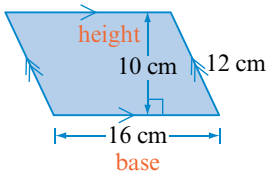

**b**



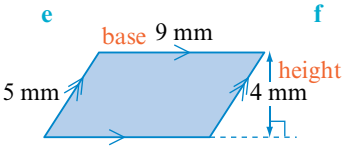

**c**



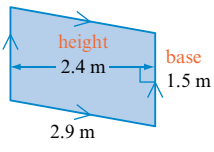

**d**




**e**

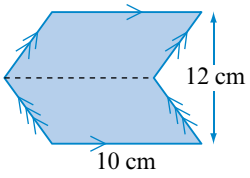



**f**

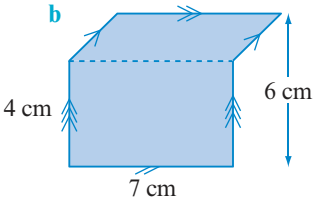



**2** **a** $24\text{ cm}^2$ **b** $56\text{ mm}^2$ **c** $360\text{ m}^2$
**d** $160\text{ cm}^2$ **e** $36\text{ mm}^2$ **f** $3.6\text{ m}^2$

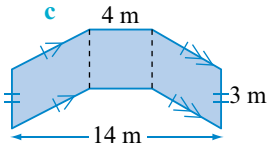
**3** **a** 15 cm² **b** 170 mm² **c** 6 m²
**d** 10.25 cm²
**4** **a** 4000 cm² **b** 3.5 cm² **c** 138 000 cm²
**d** 34.56 cm² **e** 63 cm² **f** 4.64 cm²
**5** **a** 1500 mm² or 15 cm² **b** 116 m²
**c** 2400 mm² **d** 75.076 cm² or 7507.6 mm²
**6** **a**



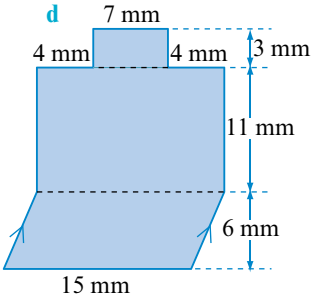

**b**




**c**




**d**

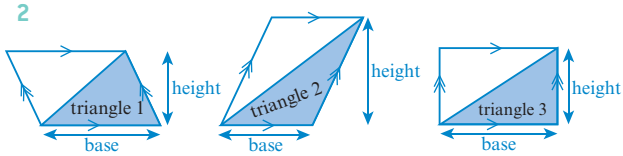



**7** **a** 120 cm² **b** 42 cm² **c** 42 m²
**d** 276 mm²
**8** **a** 68 m² **b** 141 cm² **c** 309 mm²
**d** 141.9 cm²
**9** **a** **i** 6.3 cm² **ii** 11.4 cm
**b** **i** 6 cm² **ii** 12.6 cm
**c** **i** 6.48 cm² **ii** 10.2 cm
**d** **i** 5.44 cm² **ii** 12.4 cm
**10** 364.5 cm²
**11** **a** 3.3 m² **b** 3.85 m²
**12** **a** 24 cm² **b** 18 cm² **c** 60 cm² **d** 50 cm
**13** Product of base and height should equal the area. Some possible answers are given.
**a** base 12 cm, height 2 cm; base 6 cm, height 4 cm; base 8 cm, height 3 cm
**b** base 7 mm, height 2 mm; base 1 mm, height 14 mm; base 4 mm, height 3.5 mm
**14** 5 cm **15** 8 cm
**16** **a** Multiply base by height. Side length of rhombus is 10 cm so base measurement is 10 cm. However, height measurement is not given.
**b** Height would be less than side length of rhombus so area of rhombus would be smaller than 100 cm².
**17** **a** 390 cm² **b** 390 cm²
**c** A rectangle is a special parallelogram where height of parallelogram equals width of rectangle and base of parallelogram is equal to length of rectangle. In both cases, the two dimensions (length and width in a rectangle, and base and height in a parallelogram) are at right angles to each other.
**18** **a** square and parallelogram
**b** rectangular prisms **c** 1109.25 cm²
**19** approx. 236 cm²

## 9F Area of a triangle

### 9F Start thinking!

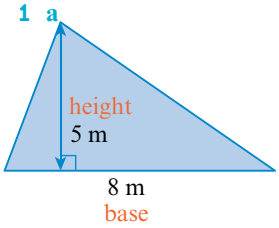
**2**



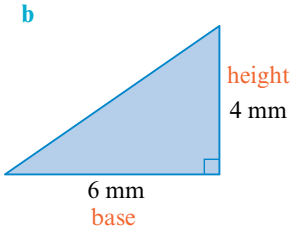

**3** Area of triangle is half area of matching parallelogram.
**4** area = $\frac{1}{2}$ × base × height

### Exercise 9F Area of a triangle

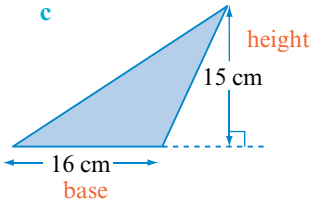
**1** **a**



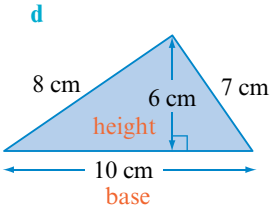

**b**



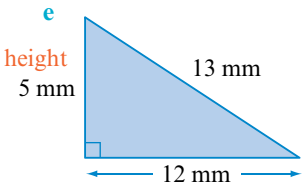

**c**



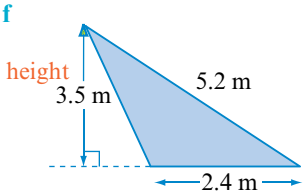

**d**




**e**




**f**




**2** **a** 20 m² **b** 12 mm² **c** 120 cm²
**d** 30 cm² **e** 30 mm² **f** 4.2 m²
**3** **a** 54 cm² **b** 48 mm² **c** 7.5 m²
**d** 30.1 cm² **e** 68.87 mm² **f** 131.131 m²
**4** **b** 30 cm²
**5** **a** 1350 cm² **b** 2437.5 cm²
**6** **a** 24 mm² or 0.24 cm²
**b** 13 650 cm² or 1.365 m²
**c** 8.1 cm² or 810 mm²
**d** 23.4 cm² or 2340 mm²
**e** 3.57 m² or 35 700 cm²
**f** 6 cm² or 600 mm²
**7** **a** 12 cm² or 1200 mm²
**b** 240 mm² or 2.4 cm²
**c** 1 m² or 10 000 cm²
**d** 155 000 cm² or 15.5 m²
**e** 70 mm² or 0.7 cm²
**f** 1.358 m² or 13 580 cm²
**8** 984 cm²
**9** **a** Not correct. The base measurement is 12 cm and the height is 9 cm so the area should be 54 cm².
**b** 36 cm

**10** **a** **i** 4.44 $cm^2$ or 444 $mm^2$
**ii** 10.6 cm or 106 mm
**b** **i** 2.76 $cm^2$ or 276 $mm^2$
**ii** 7.7 cm or 77 mm
**c** **i** 3.28 $cm^2$ or 328 $mm^2$
**ii** 9.3 cm or 93 mm
**d** **i** 2.76 $cm^2$ or 276 $mm^2$
**ii** 10.5 cm or 105 mm

**11** **a** 84 $cm^2$ **b** 68 $m^2$ **c** 16 $mm^2$
**d** 1750 $m^2$ **e** 222 $mm^2$ **f** 248 $cm^2$

**12** 45 $m^2$

**13** **a** 162 $cm^2$ **b** 1152 $cm^2$

**14** Product of base and height should be double the area. Some possible answers are given.
**a** base 16 cm, height 4 cm; base 2 cm, height 32 cm; base 8 cm, height 8 cm
**b** base 20 m, height 4 m; base 5 m, height 16 m; base 40 m, height 2 m

**15** 4 cm **16** 15 mm

**17** **a** 2880 $cm^2$ **b** 230.1 $cm^2$

**18** approx. 4 $m^2$

## 9G Surface area

### 9G Start thinking!

**1** six

**2** Find area of each face (the area of each of the six rectangles in the net), then add all areas.

### Exercise 9G Surface area

**1** **a** six **b** Each face is rectangular.
**c** 32 $m^2$, 32 $m^2$, 24 $m^2$, 24 $m^2$, 12 $m^2$, 12 $m^2$
**d** 136 $m^2$

**2** **a** 344 $mm^2$ **b** 166 $cm^2$ **c** 378 $mm^2$
**d** 268 $m^2$ **e** 76.96 $m^2$ **f** 738 $cm^2$

**3** **a** six **b** Each face is square.
**c** Each face has area 9 $cm^2$.
**d** 9 + 9 + 9 + 9 + 9 + 9 = 54 $cm^2$
**e** Since there are six identical faces, the surface area of a cube is six times the area of one face.

**4** **a** 24 $m^2$ **b** 294 $mm^2$ **c** 600 $cm^2$
**d** 384 $mm^2$ **e** 13.5 $m^2$ **f** 5400 $cm^2$

**5** **a** five
**b** There are two triangular faces and three rectangular faces.
**c** 12 $cm^2$, 12 $cm^2$, 45 $cm^2$, 45 $cm^2$, 54 $cm^2$
**d** 168 $cm^2$

**6** **a** 2916 $cm^2$ **b** 510 $mm^2$ **c** 142 $m^2$
**d** 768 $mm^2$ **e** 2408 $cm^2$ **f** 523 $m^2$

**7** 164 $m^2$

**8** **a** 1678 $cm^2$ **b** 3072 $cm^2$ **c** 580.2 $cm^2$
**d** 13.5 $cm^2$

**9** 0.94 $m^2$ **10** 5 cm

**11** length = 2 m, width = 2 m, height = 2 m

**12** **a** 9.86 $m^2$ **b** two cans

**13** 24.92 $m^2$

**14** **a** 50 $cm^2$ **b** 30 $cm^2$

## 9H Volume and capacity

### 9H Start thinking!

**1** **a** cube with side length 1 cm
**b** cube with side length 1 m
**c** cube with side length 1 mm

**2** **a** 5 × 4 = 20 cubes **b** two layers
**c** two layers of 20 cubes = 40 cubes, so volume = 40 $cm^3$
**d** length = 5 cm, width = 4 cm, height = 2 cm
**e** volume = base area × height = (5 × 4) × 2 = 40 $cm^3$

### Exercise 9H Volume and capacity

**1** **a** 30 $cm^3$ **b** 56 $mm^3$ **c** 48 $m^3$
**d** 36 $cm^3$ **e** 72 $mm^3$ **f** 45 $m^3$

**2** **a** 280 $cm^3$ **b** 90 $m^3$ **c** 320 $mm^3$
**d** 94.5 $m^3$ **e** 2.88 $cm^3$
**f** 1015.014 $mm^3$

**3** **a** 8 $m^3$ **b** 343 $mm^3$ **c** 1000 $cm^3$
**d** 15.625 $m^3$

**4** **a** 4500 $cm^3$ **b** 5940 $cm^3$ **c** 3240 $cm^3$
**d** 108 $cm^3$

**5** **a** No, it is only important that base and height are at right angles.
**b** 24 $cm^3$

**6** **a** 20 $mm^3$ **b** 80 $m^3$ **c** 105 $cm^3$
**d** 480 $mm^3$

**7** **a** 300 $mm^3$ **b** 84 $cm^3$ **c** 45 $m^3$
**d** 5400 $cm^3$ **e** 450 $mm^3$ **f** 60 $m^3$

**9** **a** 8000 mL **b** 15 000 L **c** 0.07 L
**d** 6400 kL **e** 2.5 kL
**f** 3 000 000 mL

**10** **a** 14 mL **b** 3000 $cm^3$ **c** 5.8 kL
**d** 0.75 L **e** 1000 $m^3$ **f** 6400 $m^3$

**11** 5 $m^3$ **12** 0.125 m or 12.5 cm

**13** **a** length = 1 cm, width = 1 cm, height = 1 cm
**b** 1 $cm^3$ **c** 840
**d** One possible set of dimensions for box is 10 cm by 10 cm by 10 cm. Box must have dimensions in whole numbers of cm, where length × width × height = 1000 $cm^3$.
**e** List five sets of dimensions in whole numbers of cm where length × width × height = 240 $cm^3$. Some possible answers are: 12 cm by 10 cm by 2 cm, 12 cm by 5 cm by 4 cm, 10 cm by 6 cm by 4 cm.
**f** four sets: 20 cm by 1 cm by 1 cm, 10 cm by 2 cm by 1 cm, 5 cm by 4 cm by 1 cm, 5 cm by 2 cm by 2 cm

**14** **a** 1020 $cm^3$ **b** 1.02 L
**c** No. Manufacturers often don't fill containers completely, they allow some space.

**15** approx. 18 L

**16** **a** $m^3$ **b** mL **c** $cm^3$ **d** kL

**18** inside volume of tank = 50 000 $cm^3$. Suggest dimensions in cm for a rectangular prism where length × width × height = 50 000 $cm^3$. One possible answer is 100 cm by 50 cm by 10 cm.

## 9 Chapter review

### MULTIPLE-CHOICE

1 A 2 C 3 B 4 D
5 B 6 A 7 C 8 D

### SHORT ANSWER

1 a 340 mm b 1.7 m c 4.5 cm d 8700 m
2 a cm b m c km d mm
3 132 mm or 13.2 cm
4 a 60 m b 220 cm or 2.2 m
5 6 $cm^2$
6 a 125 $m^2$ b 2400 $cm^2$ or 0.24 $m^2$
7 a 15 $cm^2$ b 16 $cm^2$ or 1600 $mm^2$
8 a 84 $cm^2$ b 18 $cm^2$ or 1800 $mm^2$
9 a 29 450 $cm^2$ b 2400 $mm^2$
10 a 288 750 $cm^3$ b 8000 $mm^3$
11 a 30 $m^3$ b 96 800 $cm^3$
12 Volume is 750 $cm^3$. Some possible sets of dimensions are: 25 cm × 6 cm × 5 cm, 75 cm × 5 cm × 2 cm, 30 m × 5 cm × 5 cm.

### NAPLAN-STYLE PRACTICE

1 0.789 2 5600 cm 3 22 cm 4 32
5 12 6 64 7 3 $cm^2$ 8 13 cm
9 8 10 parallelogram 11 13 cm
12 7 $cm^2$ 13 3 m 14 3000 $cm^2$
15 30 $cm^2$ 16 28
17 square with length 5 cm
18 1236 cm 19 1800 $cm^3$
20 1.8 L 21 105 22 72

### ANALYSIS

a Option 1: 48 cm, Option 2: 48 cm, Option 3: 500 mm or 50 cm. Screen perimeters are not the same.
b Option 1: 128 $cm^2$, Option 2: 135 $cm^2$, Option 3: 154 $cm^2$.
c Option 1 has widest screen dimension but has smallest screen area.
d 384 $cm^3$ e 1000 $cm^3$ f 700 $cm^2$
g More paper needed so can overlap edges of paper when wrapping.

## 9 Connect

For feedback on this open-ended task, see your teacher.

# 10 STATISTICS AND PROBABILITY

## 10 Are you ready?

1 7 2 11 3 A 4 Nike
5 B 6 D 7 C 8 B
9 5 10 6 11 8 12 D
13 $\frac{1}{2}$ 14 C 15 D 16 C

## 10A Collecting data

### Exercise 10A Collecting data

1 a i N ii C iii C iv N v C
vi N vii N viii C ix N
b i C iv C vi D vii C ix C
c ii N iii N v N viii O
2 a numerical, continuous
b numerical, discrete
c categorical, nominal
d numerical, continuous
e categorical, ordinal
f categorical, nominal
g categorical, ordinal
h numerical, discrete
3 Sample answers are given.
a number of rooms in your house, number of meals you have a day
b length of fingernails, volume of water you drink
c how much you like chocolate, how much you like winter
d favourite movie, gender
4 a categorical, nominal
b i comedy ii horror
5 a numerical, discrete
b

| Number of siblings | Tally | Frequency |
|---|---|---|
| 0 | \|\| | 2 |
| 1 | 𝍸 𝍸 \| | 11 |
| 2 | 𝍸 \|\|\|\| | 9 |
| 3 | \|\| | 2 |
| 4 | \| | 1 |
| 5 | | 0 |
| 6 | | 0 |
| 7 | | 0 |
| 8 | \| | 1 |

c 26 d 1
7 c No, it just means that they think the radio should play hip-hop.
d 'What is your favourite type of music?' is the best question. 'Do you prefer rock or pop?' only gives you two options, and the third question doesn't even show if a person likes hip-hop.
8 Sample answers are given.
a What is your favourite food?
b What is your favourite colour?
c What language do you speak at home?
9

| Colour | Tally | Frequency |
|---|---|---|
| Pink | 𝍸 | 5 |
| Blue | 𝍸 \| | 6 |
| Green | 𝍸 \|\| | 7 |
| Purple | \|\|\| | 3 |
| Yellow | \|\|\|\| | 4 |
| Orange | 𝍸 | 5 |

b green, purple c 50
d It is quicker and easier.
e You might not get an accurate view of the population.

**10** **a** **i** categorical, nominal **ii** population
**b** **i** categorical, nominal **ii** sample
**c** **i** numerical, continuous **ii** sample
**d** **i** numerical, discrete **ii** population
**e** **i** numerical, discrete **ii** population
**f** **i** categorical, ordinal **ii** sample

**11** **a**

| Height (cm) | Tally | Frequency | Height (cm) | Tally | Frequency |
|---|---|---|---|---|---|
| 139 | \| | 1 | 156 | \|\|\| | 3 |
| 140 | | 0 | 157 | \|\| | 2 |
| 141 | | 0 | 158 | \|\| | 2 |
| 142 | | 0 | 159 | \|\|\| | 3 |
| 143 | | 0 | 160 | \| | 1 |
| 144 | | 0 | 161 | \|\| | 2 |
| 145 | \| | 1 | 162 | \| | 1 |
| 146 | | 0 | 163 | | 0 |
| 147 | | 0 | 164 | | 0 |
| 148 | | 0 | 165 | \| | 1 |
| 149 | | 0 | 166 | \| | 1 |
| 150 | \| | 1 | 167 | \| | 1 |
| 151 | \| | 1 | 168 | | 0 |
| 152 | | 0 | 169 | | 0 |
| 153 | \| | 1 | 170 | | 0 |
| 154 | | 0 | 171 | | 0 |
| 155 | \| | 1 | 172 | \| | 1 |

**b**

| Height (cm) | Tally | Frequency |
|---|---|---|
| 135–139.5 | \| | 1 |
| 140–144.5 | | 0 |
| 145–149.5 | \| | 1 |
| 150–154.5 | \|\|\| | 3 |
| 155–159.5 | 𝍸 𝍸 \| | 11 |
| 160–164.5 | \|\|\|\| | 4 |
| 165–169.5 | \|\|\| | 3 |
| 170–174.5 | \| | 1 |

**c** 155–159.5 cm **d** 140–144.5 cm
**e** You can see patterns over a large range of data.

**12** **a**

| Height (cm) | Tally | Frequency |
|---|---|---|
| 130–139.5 | \| | 1 |
| 140–149.5 | \| | 1 |
| 150–159.5 | 𝍸 𝍸 \|\|\|\| | 14 |
| 160–169.5 | 𝍸 \|\| | 7 |
| 170–179.5 | \| | 1 |

**b** One possible answer is that larger class sizes are good for larger ranges of data but can hide patterns if they are made too large.

**13** **a** Because they indicate an order.
**b** You cannot count or measure the data and you can't find the average of position.
**c** A possible answer is clothing or shoe size.

**15** One possible answer is given below.
**a** class size of 10

**b**

| Age | Tally | Frequency |
|---|---|---|
| 10–19 | 𝍸 𝍸 \|\| | 12 |
| 20–29 | 𝍸 𝍸 \|\|\|\| | 14 |
| 30–39 | 𝍸 \|\| | 7 |
| 40–49 | \|\| | 2 |
| 50–59 | \|\| | 2 |
| 60–69 | \| | 1 |
| 70–79 | | 0 |
| 80–89 | \| | 1 |

**c** 20–29 **d** 70–79 **e** 17; no **f** 39

## 10B Interpreting data

### 10B Start thinking!

1 number of users on different social networking sites
2 To help you read frequency accurately.
3 56

4

| Social networking site | Frequency |
|---|---|
| Facebook | 56 |
| MSN | 53 |
| MySpace | 51 |
| Twitter | 49 |

5 209
6 number of users on different social networking sites
7 Pie graph sectors look roughly even but column graph frequencies look very different.
8 Vertical scale does not start from 0 on column graph.

### Exercise 10B Interpreting data

**1** **a** Wii, 12 **b** Xbox, 4 **c** 4
**d** 30

**2** **a** dog **b** ferret **c** 8

**3** **a** 38°C; at 3 pm
**b** 21.5°C; at midnight at end of 24-hour period
**c** 26.5°C **d** 8 pm

**4** **a** pie graph **b** phobias people have
**c** Most people don't have a phobia. Most common phobias involve spiders, confined spaces and heights. Fear of clowns is the least common phobia in the graph.
**d** No; only know percentage of people for each category, not numbers.

**5** **a** numerical, continuous **b** 5 cm
**c** 1 **d** 155–160 cm, 18 **e** 180–185 cm
**f** No students in this category from this survey.
**g** Find frequency of each column and add them together.

**6** **a** categorical, nominal
**b** pop music; 48 **c** hip-hop; 28
**d** hip-hop **e** country music
**f** One possible answer is hip-hop and pop because they are the most commonly liked.

**7** **a** average lifespan of common dog breeds
**b** The bars are horizontal instead of vertical.
**c** One possible answer is the average lifespan of these dogs ranges from 9 years for a German shepherd to 17 years for a Jack Russell. The 'average' lifespan seems to be about 12–13 years old.

**8** **a** 16 **b** 24 **c** 56 **d** 60 **e** 96
**f** Graph is not just a straight line. Except in first few years, myth would make the animal out to be older than it actually is.

**9** **a** half an hour
**b** **i** 20 **ii** 10 **iii** 30
**c** The student that takes between 7.5 and 8 hours to get from home to school.
**d** A possible answer is that the student's home is in a different country or state.

**10** One possible answer is the angle and shape of column graph make 2009 column appear bigger than 2012 column, when they are actually the same size.

**11** Possible answers include: vertical scale is uneven, vertical scale does not start at zero, no title, no legend and no labels on axes.

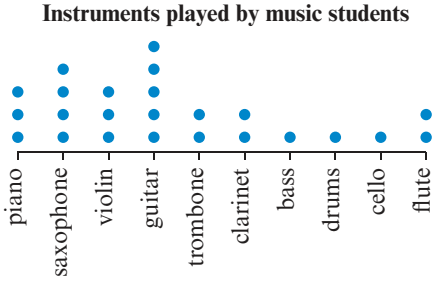
## 10C Dot plots, column and bar graphs

### 10C Start thinking!

**4** Because you use dots to represent the data.
**8** column graph

### Exercise 10C Dot plots, column and bar graphs

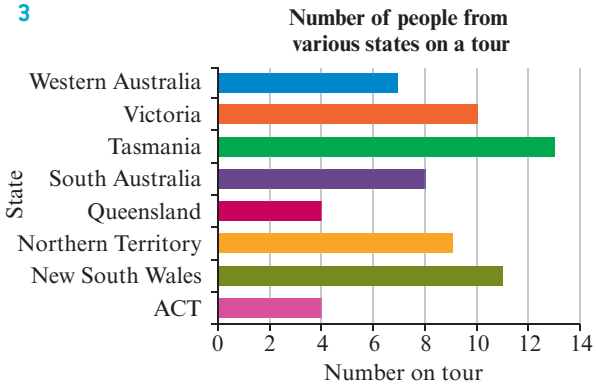
**1**



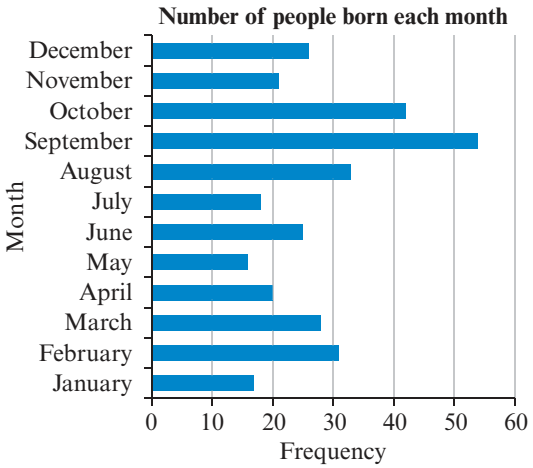

**2** **a** 5 **b** 6 **c** 27
**d** One possible answer is: a scale from 0 to 30 with intervals of 5.
**e**



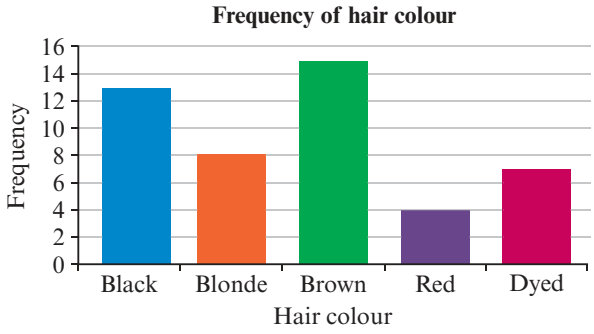

**3**



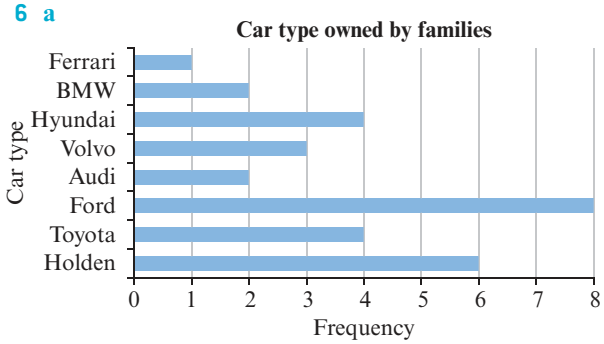

**4** A bar graph is better because titles wouldn't fit on a column graph.




**5** A column graph is better because titles are short and it is easier to read.



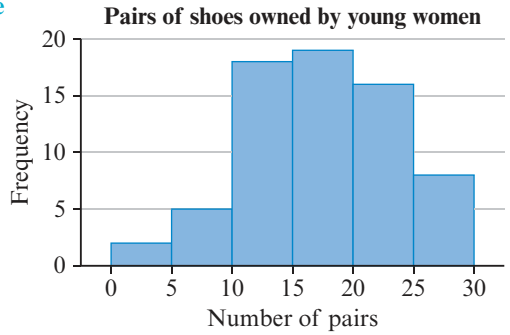

**6** **a**



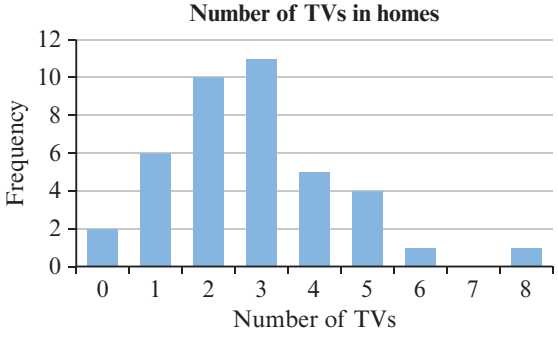

**b** Ford **c** Ferrari

**7** **a** 5 **b** 5, 6, 7, 8, 9

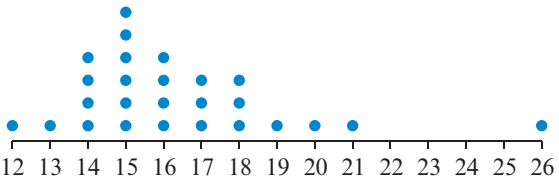
**c–e**




**8**

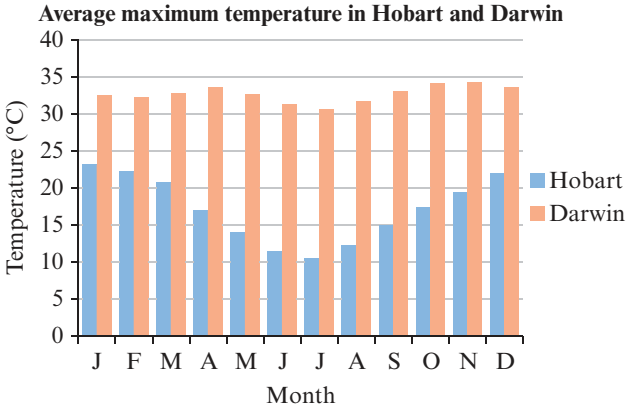



**9 a**




**b** One possible answer is that most Year 7 students have their oldest sibling within a few years of their age. In this data the most common age is 15 and the maximum age is 26.

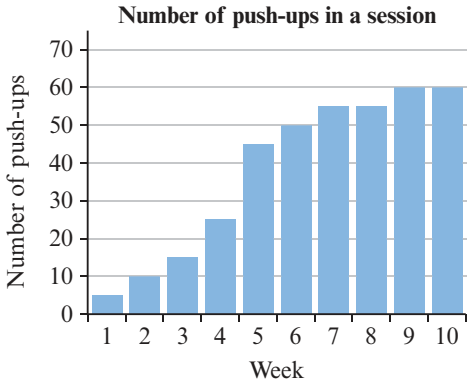
**10 a**




**b** Hobart: 23.1°C and 10.5°C. Darwin: 34.2°C and 30.6°C.

**c** One possible answer is Hobart is generally much colder than Darwin and experiences much more variation. There is not a big range of values for Darwin (3.6°C) but there is for Hobart (12.6°C).

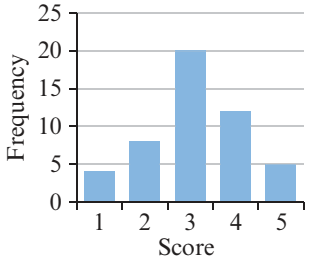
**11 a**



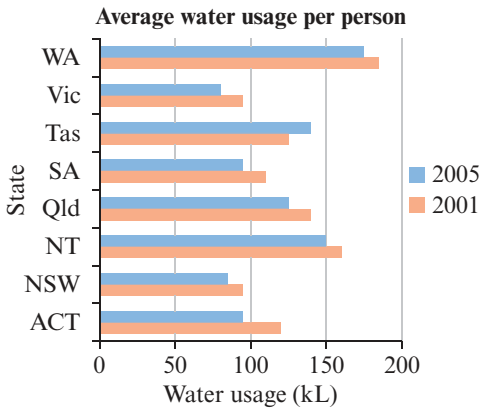

**b** Number of push-ups this person could do increased over time.

**12 a** 9

**b**

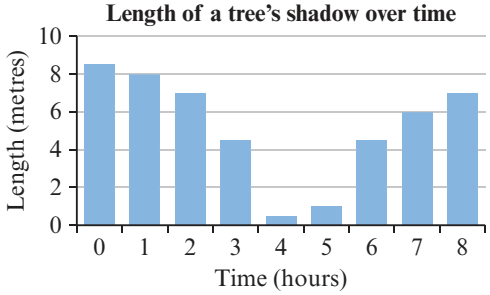



**13 a**




**b** In general states were using less water in 2005 than 2001 with exception of Tas. The big water users are WA and NT, and water savers are Vic. and NSW.

**15 a**




**b** Probably taken over period before and after midday, because shadow decreases then increases.

## 10D Pie graphs

### 10D Start thinking!

1 360° 2 180° 3 90°

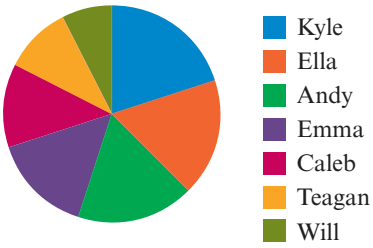
4 $\frac{1}{6} \times 360° = 60°$ 5 multiply

6

| Fruit | Tally | Frequency | Fraction | Angle of sector |
|---|---|---|---|---|
| Strawberry | 𝍸 \| | 6 | $\frac{6}{20}$ | $\frac{6}{20} \times 360° = 108°$ |
| Apple | \|\|\|\| | 4 | $\frac{4}{20}$ | $\frac{4}{20} \times 360° = 72°$ |
| Kiwifruit | \|\|\|\| | 4 | $\frac{4}{20}$ | $\frac{4}{20} \times 360° = 72°$ |
| Banana | \|\|\| | 3 | $\frac{3}{20}$ | $\frac{3}{20} \times 360° = 54°$ |
| Orange | \|\| | 2 | $\frac{2}{20}$ | $\frac{2}{20} \times 360° = 36°$ |
| Watermelon | \| | 1 | $\frac{1}{20}$ | $\frac{1}{20} \times 360° = 18°$ |
| Total | | 20 | | |

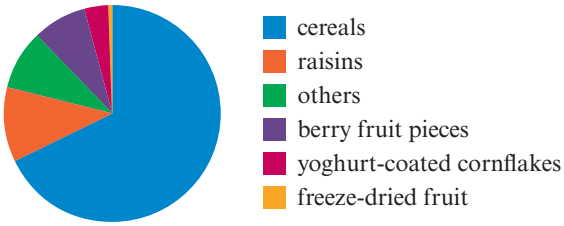
### Exercise 10D Pie graphs

1 **Votes for candidate**




2 144°, 108°, 72°, 36°

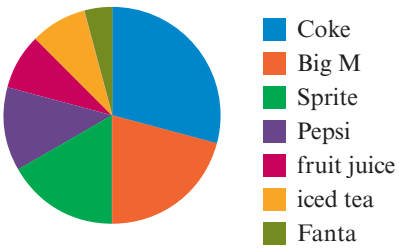
3 **Muesli ingredients**




4 a

| Favourite soft drink | Frequency |
|---|---|
| Coke | 7 |
| Pepsi | 3 |
| Sprite | 4 |
| Big M | 5 |
| Fanta | 1 |
| Fruit juice | 2 |
| Iced tea | 2 |

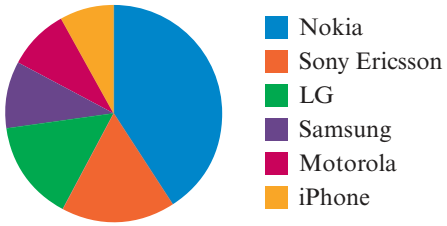
b **Favourite soft drink**




c Coke d Fanta

5 a 8%

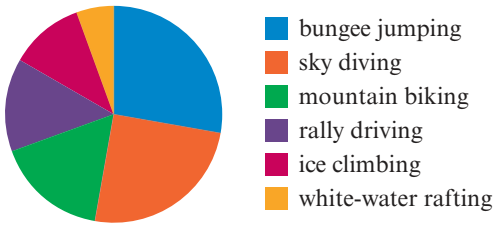
b **Mobile phone brands owned by Year 7 students**




6 a

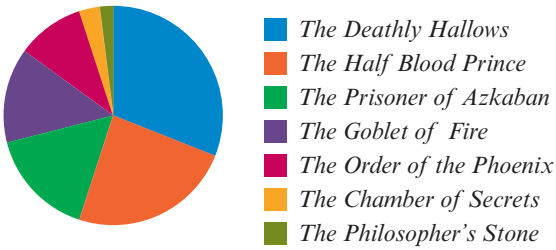
| Extreme sports | Frequency |
|---|---|
| Bungee jumping | 10 |
| Ice climbing | 4 |
| Mountain biking | 6 |
| Rally driving | 5 |
| Sky diving | 9 |
| White-water rafting | 2 |

b **Extreme sports**




7 a

**Favourite Harry Potter book**

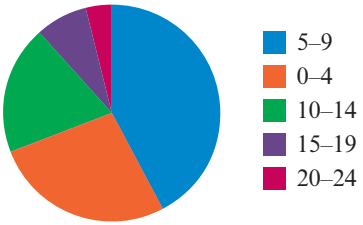



b One possible answer is bar graph has books in order of sequence but pie graph has books in order of popularity. In general, students preferred later books (last book, *The Deathly Hallows*, was most popular) to earlier books (first book, *The Philosopher's Stone*, was least popular).

8 a

| Class interval | Tally | Frequency |
|---|---|---|
| 0–4 | 𝍸 \|\| | 7 |
| 5–9 | 𝍸 𝍸 \| | 11 |
| 10–14 | 𝍸 | 5 |
| 15–19 | \|\| | 2 |
| 20–24 | \| | 1 |

**b** **Hours of TV watched per week**




**c** One possible answer is that many students watch between 0 to 9 hours of TV per week.

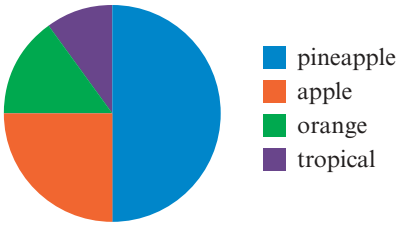
**9 a** 1000 mL pineapple juice, 500 mL apple juice, 300 mL orange juice and 200 mL tropical juice

**b** 2000 mL or 2 L

**c**

| Juice | Volume | Fraction | Percentage |
|---|---|---|---|
| Pineapple | 1000 mL | $\frac{1000}{2000} = \frac{1}{2}$ | 50% |
| Apple | 500 mL | $\frac{500}{2000} = \frac{1}{4}$ | 25% |
| Orange | 300 mL | $\frac{300}{2000} = \frac{3}{20}$ | 15% |
| Tropical | 200 mL | $\frac{200}{2000} = \frac{1}{10}$ | 10% |

**d** **Juices in a punch**

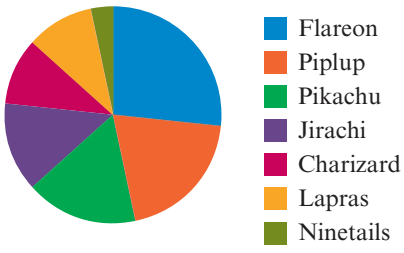



**10 a** 30

**b**

| Favourite Pokémon | Frequency | Percentage |
|---|---|---|
| Charizard | 3 | 10% |
| Flareon | 8 | 27% |
| Jirachi | 4 | 13% |
| Lapras | 3 | 10% |
| Ninetails | 1 | 3% |
| Pikachu | 5 | 17% |
| Piplup | 6 | 20% |

**c** **Favourite Pokémon**

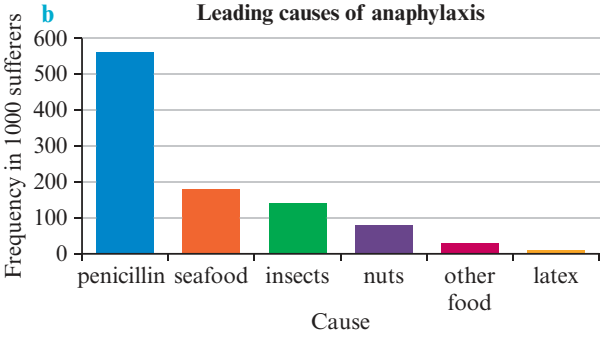



**d** One possible answer includes that Flareon is the most popular, with 8 students out of 30 (27%) liking it best. Least popular was Ninetails, with only 1 student (3%) preferring it.

**11 a**

| Cause | Percentage | Number affected in 1000 sufferers |
|---|---|---|
| Penicillin | 56% | 56% × 1000 = 560 |
| Seafood | 18% | 18% × 1000 = 180 |
| Insects | 14% | 14% × 1000 = 140 |
| Nuts | 8% | 8% × 1000 = 80 |
| Other food | 3% | 3% × 1000 = 30 |
| Latex | 1% | 1% × 1000 = 10 |

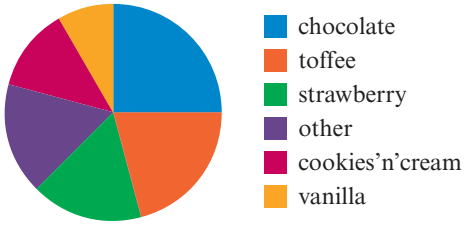
**b** **Leading causes of anaphylaxis**




**14 a**

| Ice cream flavour | Tally | Frequency |
|---|---|---|
| Chocolate | 𝍸 \| | 6 |
| Strawberry | \|\|\|\| | 4 |
| Toffee | 𝍸 | 5 |
| Vanilla | \|\| | 2 |
| Cookies'n'cream | \|\|\| | 3 |
| Other | \|\|\|\| | 4 |

**b** **Favourite ice cream flavours**




**c** chocolate

**d** flavours other than those mentioned

**e** No. Flavours with only one vote would appear in 'other'.

## 10E Line graphs and scatterplots

### 10E Start thinking!

**1 a** Each point on these graphs has a pair of coordinates and represents a piece of data.

**b** They both have labelled axes, and both have points that tend to increase in value as you move along the $x$-axis.

**c** Second graph has only one $y$-value for each $x$-value, whereas first graph sometimes has more than one value; in second graph points are connected by a line.

**d** Second graph: points are joined by a line.

**e** In a line graph points are joined by a line and there is only one $y$-value for every $x$-value. Points in a scatterplot are not joined and there can be multiple values for any $y$- or $x$-value.

2 Line graph, because scatterplot often has more than one value for a given value, so it could not reflect one person's height.

### Exercise 10E Line graphs and scatterplots

1 a line graph b scatterplot c line graph
d scatterplot e scatterplot f line graph

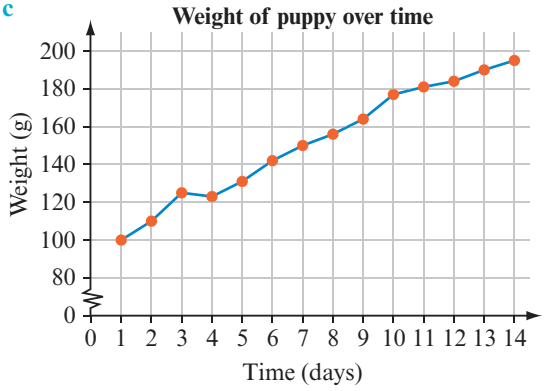
2 a negative b positive c positive
d positive e negative f negative

3 a positive scatterplot b negative line graph
c negative scatterplot d positive line graph
e negative line graph f negative scatterplot

4 a line graph b scatterplot
c line graph d scatterplot

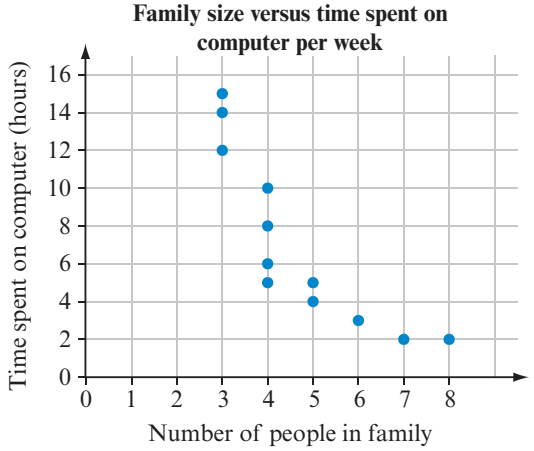
5 a positive b positive
c negative d positive

6 a day, weight b line graph
c



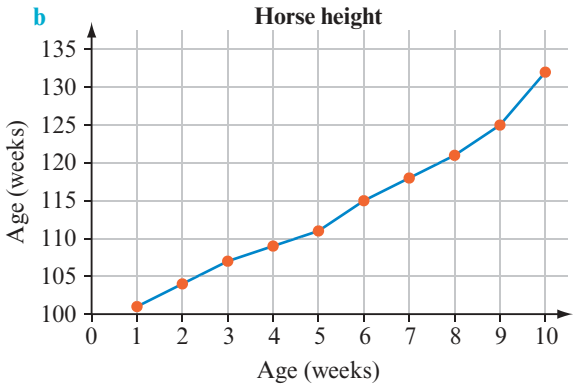

d positive

7 a family size, time on computer
b scatterplot
c




d negative

8 a line graph
b




c Yes; data is taken on one individual and it is continuous.

9 a The taller somebody is, the heavier they are.
b As temperature increases, fewer pies are sold.
c The more books you read, the better you will perform on an English test.

10 a probably; positive b increase

12 a 0.9 m or 90 cm
b around 1.4 m; obtained this value by choosing a point between the data for age 9 and age 11
c around 2.8 m; obtained this answer by extending in a straight line from the data points provided
d There might be other factors that affect the data outside of the set you have. In this case, for example, you stop growing at some point.

13 a A, as dots form a reasonably straight line and there is a clear positive relationship.
b B, as there seems to be a positive relationship but with lots of variation.
c There does not seem to be a positive or negative relationship in C; points are randomly scattered around the graph.
d The stronger the association, the more confident you can be with your predictions.

## 10F Stem-and-leaf plots

### 10F Start thinking!

2 Person B should be finished first because they have less to say.

3 You are both counting to 40 but Person B is only saying each tens place value once.

4 It is quicker.

5 1 | 5 2 | 3 1 | 8 3 | 4 5 | 2 2 | 2 2 | 9 1 | 7 4 | 8 3 | 0

6

| Stem | Leaf |
|---|---|
| 1 | 5 8 7 |
| 2 | 3 2 9 |
| 3 | 4 0 |
| 4 | 8 |
| 5 | 2 |

7

| Stem | Leaf |
|---|---|
| 1 | 5 7 8 |
| 2 | 2 3 9 |
| 3 | 0 4 |
| 4 | 8 |
| 5 | 2 |

## Exercise 10F Stem-and-leaf plots

**1 a** 2 | 1 on stem-and-leaf plot represents number 21.
**b** 43 **c** 1 **d** 14
**e** 19. You need to write the number with both its stem and leaf.

**2 a** Key: 5 | 1 = 51

| Stem | Leaf |
|---|---|
| 5 | 1 2 8 9 |
| 6 | 2 4 6 7 8 |
| 7 | 1 1 5 6 8 |
| 8 | 1 2 2 3 3 3 4 9 |
| 9 | |
| 10 | 8 |

**b** 51 **c** 108 **d** 83
**e** Key: 5 | 1 = 51

| Stem | Leaf |
|---|---|
| 5 | 1 2 8 9 |
| 6 | 2 4 6 7 8 |
| 7 | 1 1 5 6 8 |
| 8 | 1 2 2 3 3 3 4 9 |
| 9 | |
| 10 | 8 |
| 11 | 2 |

**3 a** Key: 1 | 1 = 11

| Stem | Leaf |
|---|---|
| 1 | 1 4 8 9 |
| 2 | 3 3 6 |
| 3 | 1 8 |
| 4 | 2 2 3 |

**b** Key: 1 | 3 = 13

| Stem | Leaf |
|---|---|
| 0 | 4 |
| 1 | 3 8 9 |
| 2 | 2 8 |
| 3 | 1 6 |
| 4 | 2 6 |
| 5 | 4 9 |

**c** Key: 2 | 7 = 27

| Stem | Leaf |
|---|---|
| 2 | 7 |
| 3 | 3 4 5 7 9 |
| 4 | 2 9 |
| 5 | 1 3 4 |
| 6 | 5 |

**d** Key: 2 | 3 = 23

| Stem | Leaf |
|---|---|
| 2 | 3 9 9 |
| 3 | 1 7 |
| 4 | 1 1 1 |
| 5 | 3 9 9 |
| 6 | 7 |

**4 a** One possible answer is: 1 | 4 = 14
**b** 14 **c** 98 **d** 31

**5 a** Key: 1 | 5 = 15

| Stem | Leaf |
|---|---|
| 1 | 5 8 9 9 9 |
| 2 | 2 3 4 5 6 8 9 9 |
| 3 | 1 2 3 4 9 |
| 4 | 5 |
| 5 | 4 |
| 6 | 5 |

**b** 15 and 65 **c** 19 **d** 20–29

**6 a** Key: 1 | 5 = 15

| Stem | Leaf |
|---|---|
| 1 | 0 2 3 4 4 4 5 5 6 6 7 8 8 8 9 |
| 2 | 0 1 1 2 3 4 5 5 7 9 9 |
| 3 | 0 1 3 4 6 |
| 4 | 1 1 5 6 6 |
| 5 | 3 4 |
| 6 | |
| 7 | 6 |

**b** 10 **c** 76 **d** 10–19
**e** Possible answers may show younger users for MySpace and older users for Twitter.

**7 a** 10.1 g **b** 16.1 g
**c** They didn't read the key properly. It is 13.8 g.
**d** Yes. 14.1 g is the heaviest 'normal' piece of data.
**e** One possible answer is possum could be pregnant.
**f** Key: 10 | 1 = 10.1 g

| Stem | Leaf |
|---|---|
| 10 | 1 4 8 |
| 11 | 0 2 3 6 6 |
| 12 | 1 2 4 5 5 6 9 9 |
| 13 | 1 1 2 4 4 4 6 6 8 8 8 8 |
| 14 | 0 1 3 |
| 15 | |
| 16 | 1 |

**8 a** Key: 1 | 2 = 120

| Stem | Leaf |
|---|---|
| 0 | 6 8 |
| 1 | 0 0 1 2 2 3 3 3 4 5 5 6 7 8 |
| 2 | 0 2 2 2 4 5 6 |
| 3 | 6 |
| 4 | 3 |
| 5 | 7 |

**b** Each leaf represents values of 10 rather than 1.
**c** 60 **d** 570

**9 a** Leaves have two digits instead of one.
**b** 7.16 am **c** 11.59 pm **d** 5

**10** Plot needs a key, stem should not skip numbers and leaves need to be in order.
Key: 1 | 2 = 12

| Stem | Leaf |
|---|---|
| 2 | 1 2 4 9 |
| 3 | 3 3 5 8 9 |
| 4 | 1 1 1 3 5 6 |
| 5 | 0 3 6 |
| 6 | |
| 7 | |
| 8 | 7 8 |
| 9 | 2 2 4 9 |

**11** Data must be numerical because a stem-and-leaf plot is used to plot numbers.

**12** **a** boys and girls **b** 79, girls **c** 13
**d** Yes; both groups start from stem and work outward. For girls, this is left to right. For boys, this is right to left.

**13** **a** Key: 1 | 9 = 19

| Leaf Dunstall | Stem | Leaf Lockett |
|---|---|---|
| | 0 | |
| | 1 | |
| 1 | 2 | |
| 6 | 3 | 5 7 |
| | 4 | |
| 4 | 5 | 3 6 |
| 6 | 6 | 0 5 |
| 7 | 7 | 7 8 9 |
| 3 2 | 8 | 2 |
| 4 | 9 | |
| 2 1 | 10 | 9 |
| | 11 | 0 7 |
| 3 | 12 | 1 7 |
| 8 2 | 13 | 2 |
| 5 | 14 | |

**b** 145, Dunstall **c** 3, Lockett
**d** Lockett (1360)

## 10G Summary statistics

### 10G Start thinking!

**1** 13

**2** **a** $\frac{\text{sum of terms}}{\text{number of terms}} = \frac{132 + 127 + 201 + 57 + 138 + 0 + 38 + 89 + 7 + 30 + 33 + 173 + 0}{13} = \frac{1025}{13}$
**b** 79

**3** **a** 0, 0, 7, 30, 33, 38, 57, 89, 127, 132, 138, 173, 201
**b** 57

**4** 0

### Exercise 10G Summary statistics

**1** **a** 5 **b** 6 **c** 4
**d** 3.5 **e** 10 **f** 26

**2** **a** 8 **b** 5 **c** 9
**d** 7 **e** 16 **f** 8

**3** **a** 4 **b** 6 **c** 4
**d** 6, 7, 8 **e** 13, 19 **f** no mode

**4** **a** 8 **b** 9 **c** 7
**d** 14 **e** 40 **f** 56

**5** **a** 1, 4, 5, 7, 7, 8, 10 **b** 7 **c** 7
**d** 7 **e** 6 **f** 1, 10 **g** 9

**6** **a** 8 **b** 6, 6, 7, 7, 7, 7, 8, 8, 8, 9, 15
**c** 7 **d** 7 **e** 9

**7** **a** mode = 1, median = 1, mean = 1.04
**b** 4

**8** **a** mean = 4.92, median = 5, mode = 7, range = 7
**b** mean = 4.77, median = 5, modes = 1, 6, range = 8
**c** mean = 5, median = 5, no mode, range = 8
**d** mean = 4.87, median = 5, modes = 4, 5, 7, 9, range = 9
**e** mean = 7.33, median = 7, mode = 4, range = 12
**f** mean = 8.29, median = 7, no mode, range = 15
**g** mean = 12.29, median = 13, mode = 13, range = 13
**h** mean = 13.82, median = 6, modes = 5, 8, range = 92

**9** **a** mean = \$23.85, median = \$20, mode = \$20, range = \$40
**b** Most students spend about \$20 on their mobile phones every month, though one student surveyed spent \$50. Minimum spent was \$10.

**10** **a** mean = \$13.46, median = \$5, mode = \$5
**b** \$8.46
**c** Only two students earn more than 'mean'. The student who gets \$100 has had a big effect on results. Most students earn much less than the mean.
**d** Most would not be able to, or if they could, would have no money left over for anything else; the median pocket money is \$5 a week which is about \$20 a month, which is what most students spend on their mobile phone per month.

**11** **a** 3 **b** 7, 1 **c** 6
**d** Write out all numbers (ten 1s, fifteen 2s, etc). Find middle number for median (4). Find sum and then divide by number of numbers to find mean (3.79).

**12** **a** C **b** D **c** A **d** B

**13** **a** Derrick, 20 minutes late
**b** Derrick: 5.07 minutes, Brendan: 7.29 minutes
**c** One possible answer is that Brendan's train is worse because on average it runs later than Derrick's, even though Derrick's has had two very late departures.

**14** **a** 8 **c** 5, 7 **d** 6

**15** **a** 5 **b** 5.5 **c** 5 **d** 6.5

**16** mean = 466, median = 472 and mode = 456. All three are measures of centre.

**17** **a** You cannot have 1.97 children, and families vary. Some have one, some have four, etc.
**b** Mean, because of number of decimal places.
**e** One possible answer is that because it is such a small sample it may not match the population average.

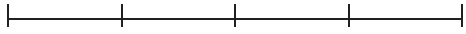
**20** **a** 56 **b** 41 **c** 31 **d** 31.88
**e** One possible answer is that it is easier to locate mode.

## 10H Describing probability

### 10H Start thinking!

1 impossible, unlikely, even chance, likely, certain.

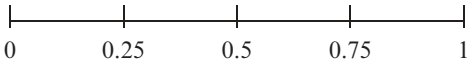
2 impossible unlikely even chance likely certain



5 0 6 0.5

7 Unlikely; it has a less-than-even chance.

8 impossible unlikely even chance likely certain



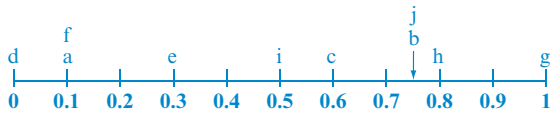

### Exercise 10H Describing probability

1 a {male, female}
b {blue, green, purple, red}
c {1, 2, 3, 4, 5, 6, 7, 8}
d {Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec}
e {5c, 10c, 20c, 50c, $1, $2}
f {red, amber, green} g {1, 2, 3, 4, 5, 6, 7}
h {spring, summer, autumn, winter}
i {red, orange, yellow, green, blue, indigo, violet}
j {A, E, I, O, U}

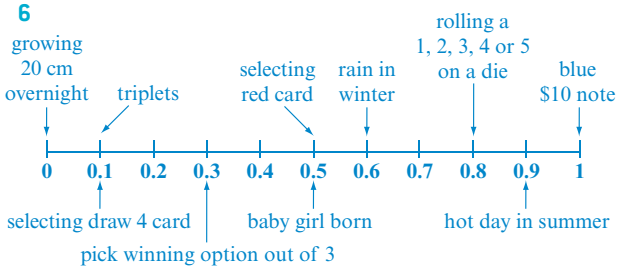
2 Possible answers given below.
a impossible b very or highly likely
c very or highly unlikely d likely
e even chance f certain
g even chance h very or highly unlikely
i unlikely j highly likely

3 a B b D c F d A e E f C

5




6




7 Some possible answers include: probable, doubtful, good chance, Buckley's chance, most likely.

9 Some possible answers are given.
a drawing card from deck and recording suit
b picking day of weekend
c number of points for goal in basketball
d result of game
e picking colour of rainbow
f picking type of animal

10 a $\frac{1}{4}$ b $\frac{1}{2}$ c $\frac{1}{3}$ d $\frac{1}{2}$ e $\frac{1}{7}$ f $\frac{1}{5}$

11 a 26 b 7 c 3 (win, lose, draw)
d Answers will vary.
e 3 (T, E and X are the different letters to choose from)
f 3

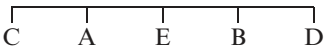
13 a 50% b 30%
c 50, 30, 90, 0.7, 70, 0.1, 10, 1, 100

14 a $\frac{1}{10}$ b $\frac{7}{10}$ c 7, 9, 0.3, 3, 0.7, 7, 0, 0

15 No. The chance of something happening must be between 0% and 100%.

16 a equally likely: parts a, b, d; not equally likely: parts c, e, f

17 a




b C c E

## 10I Theoretical probability

### 10I Start thinking!

1 10 2 3 3 1

4 a 3, 10, 0.3 b 1, 10, 0.1

5 favourable, total

6 a $\frac{4}{10} = \frac{2}{5}$ b $\frac{2}{10} = \frac{1}{5}$

### Exercise 10I Theoretical probability

1 a 11 b 2 c $\frac{2}{11}$

2 a 21 b 6 c $\frac{6}{21} = \frac{2}{7}$
d 5 e $\frac{5}{21}$ f 3 g $\frac{3}{21} = \frac{1}{7}$

3 a {heads, tails} b i $\frac{1}{2}$ ii $\frac{1}{2}$

4 a {1, 2, 3, 4, 5, 6}
b i $\frac{1}{6}$ ii $\frac{1}{2}$ iii $\frac{4}{6} = \frac{2}{3}$

5 a {club, diamond, heart, spade}
b i $\frac{1}{4}$ ii $\frac{1}{2}$ iii 0

6 a {P, R, O, B, A, I, L, T, Y}
b i $\frac{1}{11}$ ii $\frac{2}{11}$ iii $\frac{4}{11}$

7 a 20
b i $\frac{6}{20} = \frac{3}{10}$ ii $\frac{4}{20} = \frac{1}{5}$ iii $\frac{16}{20} = \frac{4}{5}$

8 a 12 b 11 c 1 d $\frac{1}{12}$ e $\frac{1}{2}$

9 a 3 b 12 c $\frac{12}{52} = \frac{3}{13}$ d 4
e $\frac{4}{52} = \frac{1}{13}$ f $\frac{1}{52}$ g 5 (2, 4, 6, 8, 10)
h $\frac{20}{52} = \frac{5}{13}$ i $\frac{16}{52} = \frac{4}{13}$ (2, 3, 5, 7)

10 a 4 b 4 c 16
d i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{1}{16}$ iv $\frac{1}{2}$ v $\frac{7}{16}$

11 Although there are four different outcomes, each outcome is not equally likely. There are different numbers of each coloured jellybean.

12 a 4 b 8 c $\frac{4}{108} = \frac{1}{27}$ d $\frac{8}{108} = \frac{2}{27}$
e $\frac{8}{108} = \frac{2}{27}$ f 8 g $\frac{8}{108} = \frac{2}{27}$ h $\frac{12}{108} = \frac{3}{27}$

14 a $\frac{1}{4}$ b $\frac{1}{2}$
c a: unlikely, b: even chance

**15 a**

| Die 1 \ Die 2 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1, 1 | 1, 2 | 1, 3 | 1, 4 | 1, 5 | 1, 6 |
| 2 | 2, 1 | 2, 2 | 2, 3 | 2, 4 | 2, 5 | 2, 6 |
| 3 | 3, 1 | 3, 2 | 3, 3 | 3, 4 | 3, 5 | 3, 6 |
| 4 | 4, 1 | 4, 2 | 4, 3 | 4, 4 | 4, 5 | 4, 6 |
| 5 | 5, 1 | 5, 2 | 5, 3 | 5, 4 | 5, 5 | 5, 6 |
| 6 | 6, 1 | 6, 2 | 6, 3 | 6, 4 | 6, 5 | 6, 6 |

**b** 36 **c** 11

**d** **i** $\frac{11}{36}$ **ii** $\frac{1}{36}$ **iii** $\frac{6}{36} = \frac{1}{6}$

**16 a** **i** $\frac{5}{10} = \frac{1}{2}$ **ii** $\frac{5}{10} = \frac{1}{2}$ **iii** $\frac{4}{10} = \frac{2}{5}$

**b**

| First roll of die \ Second roll of die | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0, 0 | 0, 1 | 0, 2 | 0, 3 | 0, 4 | 0, 5 | 0, 6 | 0, 7 | 0, 8 | 0, 9 |
| 1 | 1, 0 | 1, 1 | 1, 2 | 1, 3 | 1, 4 | 1, 5 | 1, 6 | 1, 7 | 1, 8 | 1, 9 |
| 2 | 2, 0 | 2, 1 | 2, 2 | 2, 3 | 2, 4 | 2, 5 | 2, 6 | 2, 7 | 2, 8 | 2, 9 |
| 3 | 3, 0 | 3, 1 | 3, 2 | 3, 3 | 3, 4 | 3, 5 | 3, 6 | 3, 7 | 3, 8 | 3, 9 |
| 4 | 4, 0 | 4, 1 | 4, 2 | 4, 3 | 4, 4 | 4, 5 | 4, 6 | 4, 7 | 4, 8 | 4, 9 |
| 5 | 5, 0 | 5, 1 | 5, 2 | 5, 3 | 5, 4 | 5, 5 | 5, 6 | 5, 7 | 5, 8 | 5, 9 |
| 6 | 6, 0 | 6, 1 | 6, 2 | 6, 3 | 6, 4 | 6, 5 | 6, 6 | 6, 7 | 6, 8 | 6, 9 |
| 7 | 7, 0 | 7, 1 | 7, 2 | 7, 3 | 7, 4 | 7, 5 | 7, 6 | 7, 7 | 7, 8 | 7, 9 |
| 8 | 8, 0 | 8, 1 | 8, 2 | 8, 3 | 8, 4 | 8, 5 | 8, 6 | 8, 7 | 8, 8 | 8, 9 |
| 9 | 9, 0 | 9, 1 | 9, 2 | 9, 3 | 9, 4 | 9, 5 | 9, 6 | 9, 7 | 9, 8 | 9, 9 |

**c** $\frac{1}{100}$

**17 a** **i** 0.6 **ii** $\frac{2}{3}$ **iii** 0.5

**b** rolling a 1, 2, 3, 5 or 6

**c** flipping a head

**d** rolling an odd number on a die

**e** rolling a 3, 4, 5 or 6 on a die

**f** $\frac{5}{6}, \frac{1}{2}, \frac{1}{2}, \frac{4}{6} = \frac{2}{3}$

## 10J Experimental probability

### 10J Start thinking!

**1** {head, tail} **2** $\frac{1}{2}$ or 0.5

**3** **i** 5 **ii** 50

**4** Probably not; it is a prediction only. It is possible to get outcomes that don't reflect theory exactly.

### Exercise 10J Experimental probability

**1 a** 20 **b** $\frac{1}{2}$ or 0.5 **c** 10

**2 a** 60 **b** $\frac{1}{6}$ **c** 10

**3 a** $\frac{13}{20}$ **b** It is higher.

**c** Decrease; would expect it to get closer to theoretical probability.

**4 a** $\frac{6}{60} = \frac{1}{10}$ **b** lower

**c** Increase; would expect it to get closer to theoretical probability.

**5 a** $\frac{1}{6}$

**d** Would expect experimental probability of rolling a 1 to get closer to theoretical probability.

**6 a**

| Die 1 \ Die 2 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

**b, e**

| Outcome | Theoretical probability | Theoretical frequency |
|---|---|---|
| 2 | $\frac{1}{36}$ | 1 |
| 3 | $\frac{2}{36} = \frac{1}{18}$ | 2 |
| 4 | $\frac{3}{36} = \frac{1}{12}$ | 3 |
| 5 | $\frac{4}{36} = \frac{1}{9}$ | 4 |
| 6 | $\frac{5}{36}$ | 5 |
| 7 | $\frac{6}{36} = \frac{1}{6}$ | 6 |
| 8 | $\frac{5}{36}$ | 5 |
| 9 | $\frac{4}{36} = \frac{1}{9}$ | 4 |
| 10 | $\frac{3}{36} = \frac{1}{12}$ | 3 |
| 11 | $\frac{2}{36} = \frac{1}{18}$ | 2 |
| 12 | $\frac{1}{36}$ | 1 |

**c** 5 **d** $\frac{5}{36}$ **f** 5

**7 a** theoretical probability for each: $\frac{6}{36}$

relative frequency: 3: $\frac{7}{36}$, 4: $\frac{5}{36}$, 5: $\frac{5}{36}$, 6: $\frac{11}{36}$

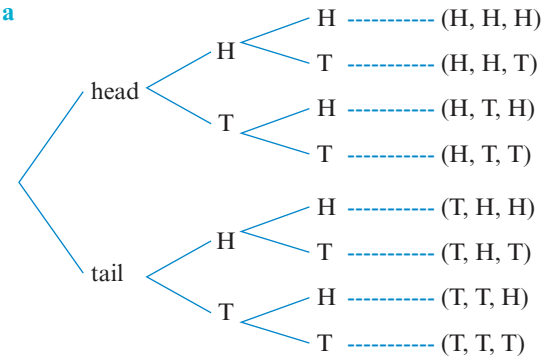
**b** $\frac{1}{6}$ **c** $\frac{11}{36}$ **d** bigger

**e** roughly 600 sixes

**f** Die is not fair; 6 rolled too many times.

**g** 1 is on other side of die to 6, and therefore would be rolled least.

**8 a**




**b, d** TTT, TTH, THT, THH, HTT, HTH, HHT, HHH

| Outcome | Theoretical probability | Theoretical frequency |
|---|---|---|
| 0 heads, 3 tails | $\frac{1}{8}$ | 5 |
| 1 head, 2 tails | 3 | 15 |
| 2 heads, 1 tail | 3 | 15 |
| 3 heads, 0 tails | $\frac{1}{8}$ | 5 |

**c** Because some outcomes are the same. For example, both TTH and THT have two tails and one head.

**h** The difference should become smaller.

**9 a**

| Outcome | Theoretical probability | Theoretical frequency |
|---|---|---|
| 0 | $\frac{1}{10}$ | 10 |
| 1 | $\frac{1}{10}$ | 10 |
| 2 | $\frac{1}{10}$ | 10 |
| 3 | $\frac{1}{10}$ | 10 |
| 4 | $\frac{1}{10}$ | 10 |
| 5 | $\frac{1}{10}$ | 10 |
| 6 | $\frac{1}{10}$ | 10 |
| 7 | $\frac{1}{10}$ | 10 |
| 8 | $\frac{1}{10}$ | 10 |
| 9 | $\frac{1}{10}$ | 10 |

**b** **i** $\frac{5}{10} = \frac{1}{2}$ **ii** $\frac{5}{10} = \frac{1}{2}$

**10 g** Should be less variation in relative frequency because there is a greater number of trials, and therefore results should be closer to theoretical probability.

**12 a** $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** about 30

**d** rolling a die

## 10 Chapter review

### MULTIPLE-CHOICE

**1** C **2** B **3** A **4** B **5** B

**6** B **7** C **8** D **9** B

### SHORT ANSWER

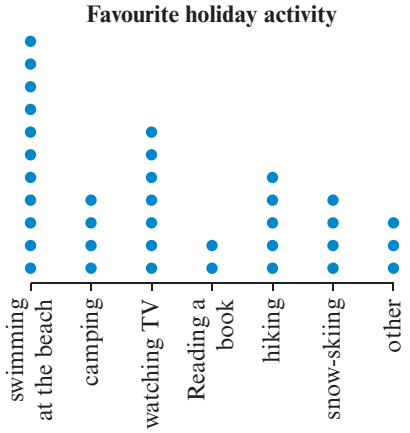
**1 a** categorical, nominal

**b**

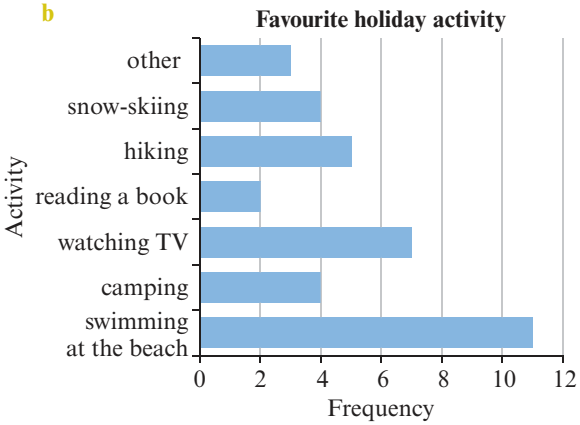
| Answer | Frequency |
|---|---|
| Y | 7 |
| N | 6 |

**c** 13 **d** Y: tongue rolling

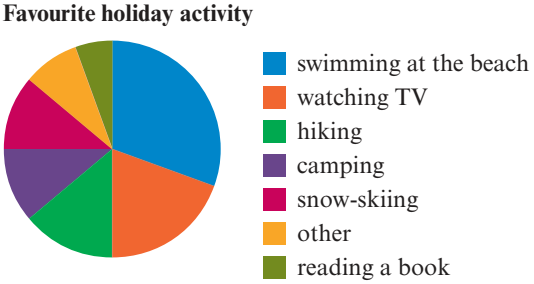
**2 a**



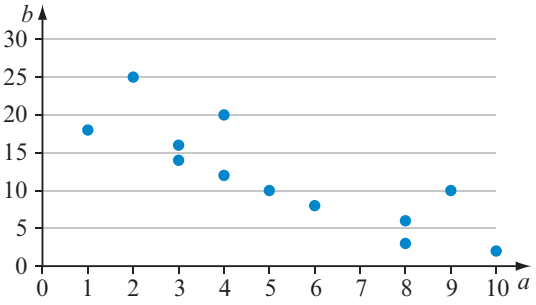

**b**




**3** One possible answer is shown.




**4** It shows a negative relationship.




**5 a** 29, boys **b** 42, girls **c** 34

**6** Key: 1 | 3 = 13

| Stem | Leaf |
|---|---|
| 0 | 5 9 |
| 1 | 2 2 3 3 6 6 7 |
| 2 | 0 2 3 5 8 |
| 3 | 0 1 2 3 8 |
| 4 | 1 2 5 |
| 5 | 1 5 9 |
| 6 | 2 |

**7 a** mean = 11.5, median = 7, mode = 2, range = 70.

**b** Median is best measure of centre; mode is not near centre, and mean is skewed by a single large data point.

**8 a** {head, tail}; even chance

**b** {1, 2, 3, 4, 5, 6}; even chance

**c** {club, diamond, heart, spade}, unlikely

**d** {red, blue, white}, somewhat likely

**9 a** $\frac{1}{6}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{13}$

**10** **a** 100

**b** A: $\frac{28}{100} = 0.28$, B: $\frac{48}{100} = 0.48$, C: $\frac{24}{100} = 0.24$

**c** A: 0.25, B: 0.50, C: 0.25

**d** One possible answer is: flipping two coins.

## NAPLAN-STYLE PRACTICE

**1** categorical, nominal **2** 11

**3** horizontal scale **4** 28

**5** 10% **6** 36

**7** People with more money tend to eat out at restaurants more than people with less money.

**8** 58 **9** 6, 5 and 4

**10** 10 **11** 3, 6, 4, 8, 5, 6, 3, 7, 8, 1, 2, 8

**12** selecting a non-picture card from a deck of cards

**13** {milk, hazelnut, white, dark}

**14** $\frac{3}{14}$ **15** $\frac{11}{14}$ **16** 0.2 **17** decrease

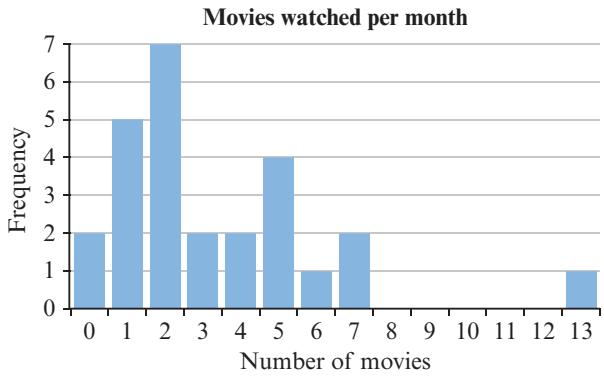
## ANALYSIS

**a** numerical, discrete **b** 26

**c**

| Number | Frequency |
|---|---|
| 0 | 2 |
| 1 | 5 |
| 2 | 7 |
| 3 | 2 |
| 4 | 2 |
| 5 | 4 |
| 6 | 1 |
| 7 | 2 |
| 13 | 1 |

**d** One possible answer is a column graph, because it allows you to see range accurately. A bar graph or a dot plot would also be appropriate.




**e** mean = 3.3, median = 2, mode = 2, range = 13

**f** **i** $\frac{7}{26} = 0.27$; unlikely

**ii** $\frac{4}{26} = \frac{2}{13} = 0.15$; very unlikely

## 10 Connect

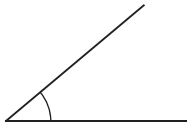
For feedback on this open-ended task, see your teacher.

# GLOSSARY

## A

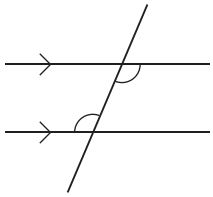
**accuracy** describes the closeness of a measurement to a given exact measurement. The higher the degree of accuracy, the closer the measurement is to the given exact measurement.

**acute angle** angle greater than 0° but less than 90°.

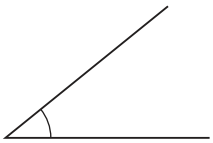


**allied angles** *see* co-interior angles

**alternate angles** pair of angles not adjacent to (next to) each other on opposite sides of the transversal between two lines. If the two lines are parallel, the alternate angles are equal.



**angle** formed when two lines meet at a point.



**area** amount of space enclosed by a two-dimensional (2D) shape. Common units are $mm^2$, $cm^2$, $m^2$, $km^2$, ha.

**associative law** order in which three or more numbers are added or multiplied is not important. For example, $(4 + 5) + 6 = 9 + 6 = 15$ and $4 + (5 + 6) = 4 + 11 = 15$ and $(4 + 6) + 5 = 10 + 5 = 15$; $(4 \times 5) \times 6 = 20 \times 6 = 120$ and $4 \times (5 \times 6) = 4 \times 30 = 120$ and $(4 \times 6) \times 5 = 24 \times 5 = 120$.

**average** for a list of scores, it is the value obtained by adding all of the scores together and dividing by the number of scores.

**axis of symmetry** line drawn on a shape or object to cut it into two symmetrical halves (each half is a reflection of the other). There may be more than one axis of symmetry. Also known as line of symmetry.

## B

**backtracking** method of following a flow chart from the final result back to the starting number using inverse operations.

**balance method** strategy for solving equations by using inverse operations to make equivalent equations. An equation remains balanced (or equivalent) by performing the same operation on both sides of the equation.

**bar graph** graph where the frequency of categorical data is presented in bars or columns. Category names are placed underneath the bars if they are drawn vertically or beside the bars if they are drawn horizontally.

**base** 1 for a value expressed in index form, the base is the number which is repeatedly multiplied. For example, $2^4$ has a base of 2 and can be written as $2 \times 2 \times 2 \times 2$.

2 in a 2D shape, it is a nominated side of the shape. The base and height are perpendicular to each other. *See* height for diagram.

3 in a 3D object, it is a nominated face of the object. The base and height are perpendicular to each other.

**basic numeral** result of a repeated multiplication. For example, $2^4 = 16$ where 16 is the basic numeral.

**bearing** direction from one position to another.

**best buy** found by comparing a number of purchase options to find the one that is the best value for money.

**bivariate** relationship between two variables.

## C

**cancelling (simplifying)** dividing the numerator and the denominator of a fraction by the same factor (usually the highest common factor) to produce an equivalent fraction.

**capacity** amount of liquid that a three-dimensional object (container) can hold. Common units are mL, L, kL, ML.

**Cartesian coordinates** pair of numbers that describe the position of a point on the Cartesian plane. For example, (3, 5) indicates an $x$-coordinate of 3 and a $y$-coordinate of 5. The $x$-coordinate is always listed first.

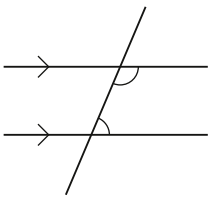
**Cartesian plane** number plane or region formed by a pair of horizontal and vertical axes that allows any point to be described.

**categorical data** data that can be put into categories.

**classes** groupings of data when there is a large spread of data.

**coefficient** number that appears in front of a variable showing that the number and the pronumeral are to be multiplied together. For example, the coefficient of $5m$ is 5.

**co-interior angles (allied angles)** pair of angles on the same side of the transversal between two lines. If the two lines are parallel, the co-interior angles are supplementary (add to 180°).



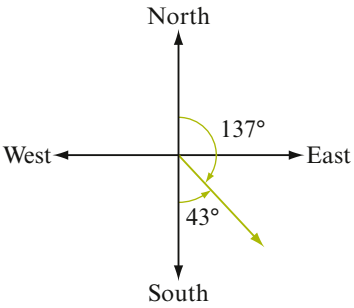
**column graph** graph where the frequency of categorical data is presented in columns.

**common multiples** multiples that appear in the multiples list for two or more different whole numbers. For example, multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, … and multiples of 4 are 4, 8, 12, 16, 20, 24,… so the first three common multiples of 3 and 4 are 12, 24 and 36.

**commutative law** order in which two numbers are added or multiplied is not important. For example, $3 + 4 = 7$ and $4 + 3 = 7$; $3 \times 4 = 12$ and $4 \times 3 = 12$.

**compass** 1 instrument to measure direction showing four major or cardinal points (north, east, south, west).

2 short for 'pair of compasses', a drawing instrument that has an arm with a sharp point and an arm that can fit a pencil, joined together by a flexible hinge.




**compass bearing** direction from one position to another and described by the number of degrees from north or south towards east or west. For example, S43°E represents a direction that is 43° east of due south.

**complementary** two probability events are complementary when their probabilities add to 1.

**complementary angles** angles that add to make a right angle (90°).

**composite number** counting number that has more than two factors. For example, 8 has factors of 1, 2, 4 and 8.

**concave polygon** polygon with at least one interior angle that is greater than 180°.

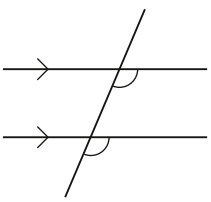
**cone** 3D object that tapers from a circular base to a point.

**constant** numerical value in an algebraic expression or equation. It is a term without any pronumeral component. For example, the constant in the expression $3b - 4c + 10$ is 10.

**continuous data** type of numerical data that can be measured; for example, height.

**convex polygon** polygon with interior angles that are all less than 180°.

**corresponding angles** pair of angles on the same side of the transversal and in corresponding positions on the two lines (both above a line or both below a line). If the two lines are parallel, the corresponding angles are equal.



**cylinder** 3D object with a uniform circular cross-section.

## D

**data** individual facts or pieces of information that can be collected. Data can be numerical or categorical.

**decimal fractions** *see* decimal numbers

**decimal numbers (decimal fractions)** have a whole number component and a decimal (or fractional) component. The digits to the left of the decimal point indicate the whole number and the digits to the right of the decimal point indicate a value less than one; for example, 3.456.

**decimal place** position of a digit after the decimal point. Each decimal place represents a different fractional value. The first decimal place is the first digit to the right of the decimal point. The number of digits after the decimal point is the number of decimal places. For example, 5.249 has three decimal places.

**decimal point** dot that separates the whole number component and the decimal component of a decimal number; for example, 3.456.

**decimals** *see* decimal numbers

**denominator** bottom number of a fraction. For example, the denominator of $\frac{2}{3}$ is 3.

**difference** result of a subtraction. For example, $10 - 6 = 4$ where 4 is the difference.

**discrete data** type of numerical data that can be counted; for example, number of siblings.

**distributive law** same answer is obtained whether you add the numbers inside a pair of brackets first before multiplying by another number or multiply each number inside the brackets by the number outside the brackets before adding. For example, $3(2 + 5) = 3 \times 7 = 21$ and $3(2 + 5) = 3 \times 2 + 3 \times 5 = 6 + 15 = 21$.

**dividend** number that is being divided by another number (divisor). For example, 6.84 is the dividend, 2 is the divisor and 3.42 is the quotient in $6.84 \div 2 = 3.42$.

**divisor** number that divides into another number; for example, in $15 \div 3 = 5$, 3 is the divisor.

**dot plot** graph that presents every piece of data in the data set as a dot above a matching number or category on a horizontal scale. Suitable for discrete numerical data and categorical data.

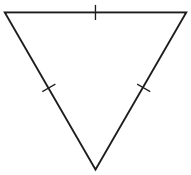
## E

**edge** line segment that join two faces of a polyhedron. *See* polyhedron

**elapsed time** difference between the start time and the end time. It specifies a period of time.

**equation** collection of two or more algebraic terms separated by mathematical operation symbols and containing an equals sign; for example, $2d + 5 = 10$.

**equilateral triangle** triangle with all three sides equal in length. All three angles are also equal in size.



**equivalent equations** equations that have the same solution.

**equivalent fractions** different fractions that have the same value. An equivalent fraction is formed by multiplying or dividing the numerator and the denominator of a fraction by the same number. For example, $\frac{2}{3} = \frac{4}{6}$.

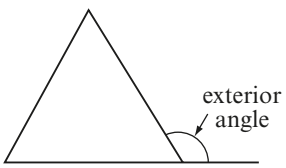
**evaluating** calculating the numerical value of a term or expression after substitution. For example, evaluating $3x + 4$ when $x = 2$ gives 10.

**event** a specified result that may be obtained in a probability experiment.

**experimental probability** probability of an event based upon the results of a probability experiment. Also known as relative frequency.

**expression** collection of two or more algebraic terms separated by mathematical operation symbols like +, −, × and ÷. An expression does not have an equals sign. For example, $3b - 4c + 10$ is an expression.

**exterior angle** angle formed outside a triangle where a side has been extended. The exterior angle and the internal or interior angle directly adjacent to it are supplementary (add to 180°).



## F

**face** flat surface of a polyhedron. Each face is a polygon. *See* polyhedron

**factors** whole numbers that divide exactly into a given whole number. For example, factors of 8 are 1, 2, 4 and 8.

**favourable outcome** outcome within the sample space that we want to occur.

**formula** relationship or rule between two or more variables. A formula contains an equals sign; for example, $y = 3x + 7$.

**frequency** number of times that a particular value occurs in a data set.

**frequency distribution table** *see* frequency table

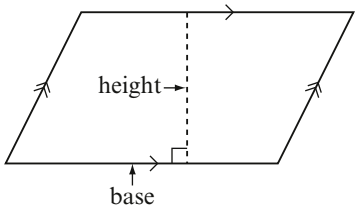
**frequency table (frequency distribution table)** table organised to show data by recording the frequency of each value in the data set.

## G

**graph** effective way to display data in an easy-to-read format and should include a title, clearly labelled axes, a regular or even scale and a legend or key (where appropriate).

## H

**height** length measurement from the base to the top or end of a shape or object. The height is perpendicular to the base.




**highest common factor (HCF)** largest factor that is common to two or more given numbers. For example, factors of 8 are 1, 2, 4, 8 and factors of 12 are 1, 2, 3, 4, 6, 12 so HCF of 8 and 12 is 4.

**histogram** graph used to display the frequency of grouped continuous data where the horizontal scale boundaries are located at the edges of the columns, rather than the centre.

## I

**image** result of completing a transformation on a shape or object. If original shape is labelled A, then the image (new shape) is labelled A′.

**improper fraction** fraction where the numerator is larger than (or equal to) the denominator; for example, $\frac{9}{7}$ or $\frac{6}{6}$.

**index (power)** for a value expressed in index form, the index (or power) indicates the number of times the base is written as a repeated multiplication. For example, $2^4$ has an index of 4 and can be written as $2 \times 2 \times 2 \times 2$. (Plural of index is indices.)

**index form (index notation)** shorter form of writing a repeated multiplication. A number in index form is written with a base and an index (power). For example, $2^4$ is written in index form.

**index notation** *see* index form

**integers** whole numbers that are positive, negative or zero.

**internal angle (interior angle)** angle inside a two-dimensional shape. In triangles, the sum of the internal angles is 180°. In quadrilaterals, the sum of the internal angles is 360°.

**International Date Line (IDL)** line of longitude that separates two time zones (UTC +12 and UTC −12) where the date changes by one day. If you travel from west to east across the IDL, you gain one day.

**interval** distance between marks on a line which form part of a scale. A major interval is the distance between two major marks on a scale.

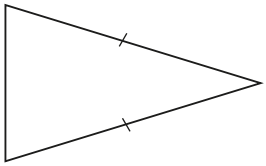
**inverse operation** operation that undoes the effect of a previous operation. For example, multiplication and division are inverse operations and addition and subtraction are inverse operations.

**irregular polygon** polygon that is not regular. It does not have all sides equal in length or all angles equal in size.

**irregular quadrilateral** four-sided shape that has no special properties relating their sides or angles

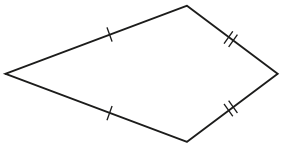
**isometric transformation** type of transformation (such as translation, rotation or reflection) applied to a shape or object that does not change its shape or size.

**isosceles triangle** triangle with two sides equal in length. The angles opposite the equal sides are also equal.



## K

**kite** quadrilateral (four-sided shape) with two pairs of adjacent sides equal in length, one pair of opposite angles equal in size and no parallel sides.



## L

**length** length measurement from one end to the other of a shape or object. In a rectangle, it is the side measurement that is not defined as the width. Common units are mm, cm, m, km.

**line graph** graph that displays the relationship between two variables where points are plotted and joined by a line. Often used to show data collected over time (horizontal axis labelled as time).

**linear relationship** relationship between two variables where the coordinate points describing this relationship lie in a straight line when plotted on the Cartesian plane.

**long-term probability** obtained after performing a large number of trials in a probability experiment so that the experimental probability is close to the theoretical probability.

**lowest common denominator (LCD)** lowest common multiple of the denominators in two or more fractions. For example, $\frac{1}{3}$ and $\frac{1}{2}$ have a LCD of 6.

**lowest common multiple (LCM)** lowest (first) multiple that is common in the multiples list of two or more numbers. For example: multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, … and multiples of 4 are 4, 8, 12, 16, 20, 24,… so LCM of 3 and 4 is 12.

## M

**map scale** measurement on a map shown as an interval between two scale marks to indicate the matching real-life distance.

**mean** a measure of the centre of a data set. It is calculated by adding all of the data values together and dividing by the number of values. Also known as the average.

**median** a measure of the centre of a data set. For data placed in ascending order (smallest to largest), it is the middle number of the data set if there is an odd number of values, and the average of the two middle numbers if there is an even number of values.

**mirror line** line in which a shape or object is reflected to create its exact mirror image.

**mixed number** made up of a whole number component and a proper fraction component; for example, $3\frac{4}{5}$.

**mode** a measure of the centre of a data set. It is the value that occurs most frequently in the data set. There can be more than one mode or no mode.

**multiples** result of multiplying a given number by whole numbers. For example, multiples of 4 are 4, 8, 12, 16, … ($4 \times 1$, $4 \times 2$, $4 \times 3$, $4 \times 4$, …)

## N

**negative numbers** numbers that are less than zero; for example, −25.

**negative relationship** relationship between two variables where, as one variable increases, the other decreases. On a scatterplot, this is seen as a pattern of dots/points moving downwards from left to right.

**net** two-dimensional plan that can be folded to form a three-dimensional object.

**nominal data** type of categorical data where the categories are unrelated; for example, eye colour.

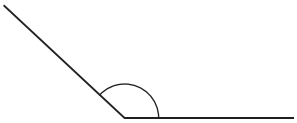
**non-linear relationship** relationship between two variables where the coordinate points describing this relationship do not lie in a straight line when plotted on the Cartesian plane.

**numerator** top number of a fraction. For example, the numerator of $\frac{2}{3}$ is 2.

**numerical data** data that can be counted or measured.

## O

**obtuse angle** angle greater than 90° but less than 180°.



**opposite integers** a positive integer and a negative integer that are the same distance either side of zero on a number line; their sum is zero.

**order of operations** specific order in which operations are performed when there is more than one operation in a calculation. The order is: perform operations inside Brackets first, then Indices (powers and roots), then Division and Multiplication (left to right) and finally Addition and Subtraction (left to right). It can also be remembered as BIDMAS (or BODMAS).

**ordinal data** type of categorical data that can be placed in categories in a specific order; for example, a rating system from 1–5.

**ordinal numbers** describe position when data are placed in order; for example, 1st, 2nd, 3rd, etc.

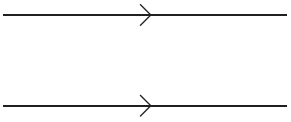
**origin** point where the $x$- and $y$-axes cross on a Cartesian plane. Coordinates are (0, 0).

**outcomes** possible results in a probability experiment. For example, when rolling a die, there are six possible outcomes: 1, 2, 3, 4, 5 and 6.

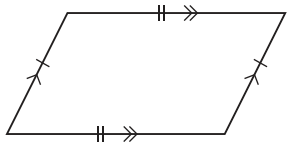
**outlier** extreme piece of data that is much higher or lower than the rest of the data in the data set.

## P

**parallel** lines (or rays or segments) that are drawn side by side and are the same distance apart so they never touch are parallel. Matching arrowheads are used to indicate that lines are parallel.



**parallelogram** quadrilateral (four-sided shape) with two pairs of opposite sides that are parallel and equal in length, and two pairs of opposite angles that are equal in size. Rectangles, squares and rhombuses are also classified as parallelograms.

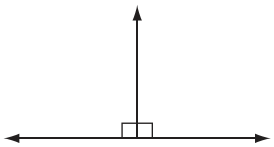


**per cent** for every one hundred (or per hundred). For example, 20 per cent means 20 for every 100.

**percentage** shows how part of an amount is related to the total of an amount. It is an amount out of 100 (or per hundred) and can be expressed as a fraction with a denominator of 100. Symbol for percentage is %.

**perimeter** distance around the outside edge or boundary of a two dimensional (2D) shape. Common units are mm, cm, m, km.

**perpendicular** two lines (or rays or segments) that meet to form a right angle are perpendicular.



**perpendicular bisector** line that is perpendicular to a line segment and intersects it at a point that divides the segment in half.

**perspective drawings** drawings of an image as it is seen by the eye. Objects are drawn smaller as their distance from the observer increases, making use of the vanishing point. The vanishing point is typically located on the horizon.

**pie graph** graph where the size of each sector in a circle represents the frequency of each data category as a fraction of the total number of pieces of data in the data set.

**place value** value of a digit in a number depending on its position. For example, the digit 3 in 2316 represents 3 hundreds or 300 and in 15.34 it represents 3 tenths or $\frac{3}{10}$.

**placeholder zeros** zeros between non-zero digits in a number. For example, 3.904 has one placeholder zero.

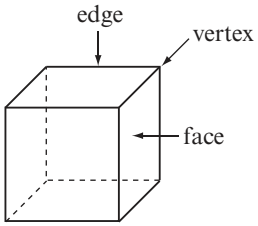
**place-value notation** where a number is written as the sum of its place-value components. For example, 3479 = 3000 + 400 + 70 + 9.

**point** exact position (shown as a dot) in space.

**polygon** general name for a two-dimensional shape with straight sides.

**polyhedra** plural of polyhedron. *See* polyhedron

**polyhedron** object made up of flat surfaces or faces (each face is a polygon) with vertices and straight edges. For example, a cube is a polyhedron but a cylinder is not.




**population** complete set of individuals or things that we are seeking information about.

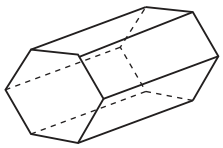
**positive relationship** relationship between two variables where, as one variable increases, the other also increases. On a scatterplot, this is seen as a pattern of dots/points moving upwards from left to right.

**power (index)** *see* index

**prime factors** factors that are prime numbers.

**prime number** counting number that has exactly two factors (itself and one). For example, 5 has factors of 1 and 5.

**prism** object with two ends that are identical polygons and joined by straight edges.



**probability** chance of an event occurring. The event can be very likely and have a high probability or not likely and have a low probability. Words or numbers can be used to describe the chance of an event occurring.

**probability experiment** where trials are performed to obtain data to predict the chances of an event occurring.

**probability scale** number line from 0 to 1 that can be used to indicate the probability of an event. An impossible event has a probability of 0. A certain event has a probability of 1.

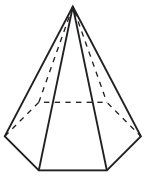
**product** result of a multiplication. For example, $10 \times 3 = 30$ where 30 is the product.

**profit** money made on the sale of an item; the difference between the buying price and the selling price.

**pronumeral** letter or symbol that takes the place of a number.

**proper fraction** fraction where the numerator is smaller than the denominator; for example, $\frac{4}{5}$.

**pyramid** object with an end that is a polygon and triangular sides (faces) that meet at a point (vertex).



## Q

**quadrants** four different regions on a Cartesian plane formed by the intersection of the $x$- and $y$-axes. They are numbered in order from the top right quadrant in an anticlockwise direction.

**quadrilateral** two-dimensional shape with four straight sides.

**quotient** result of a division. For example, $28 \div 4 = 7$ where 7 is the quotient.

## R

**range** difference between the highest and lowest value in a data set.

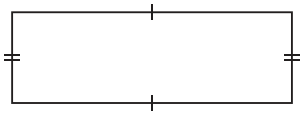
**ratio** comparison of two or more quantities of the same kind. For example, when comparing the number of red jellybeans to blue jellybeans, the ratio is 2:3.

**ratio scale** scale factor written as a ratio. It compares map distances to real-life distances (in the same unit). For example, a ratio scale of 1:1000 indicates that every 1 cm on the map represents 1000 cm in real life.

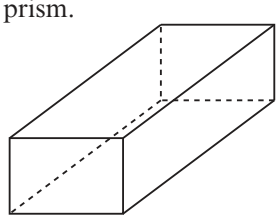
**ray** part of a line that starts at a point and continues forever.

**reciprocal** found by first writing a number as a proper or improper fraction and then turning it upside down. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

**rectangle** quadrilateral (four-sided shape) with two pairs of opposite sides that are parallel and equal in length, and four angles of 90°. A rectangle is also classified as a parallelogram.



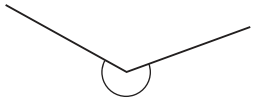
**rectangular prism** prism with three pairs of identical faces (total of six faces). A cube is also a rectangular prism.



**recurring decimal** decimal that has an endless (infinite) number of decimal places and a repeating (recurring) pattern of digits after the decimal point. It can be written in abbreviated form. For example, 0.333 33K = $0.\dot{3}$ and 5.476 767 6K = $5.4\overline{76}$.

**reflection** transformation where a shape or object is reflected (flipped) in a mirror line to produce its exact mirror image.

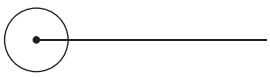
**reflex angle** angle that is greater than 180° but less than 360°.



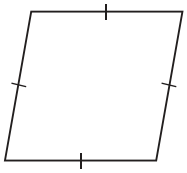
**regular polygon** polygon with all sides equal in length and all angles equal in size.

**relative frequency** *see* experimental probability

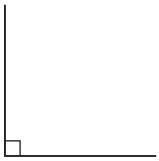
**revolution** angle that is exactly 360°. Also known as a perigon.



**rhombus** quadrilateral (four-sided shape) with two pairs of opposite sides that are parallel, all four sides equal in length and two pairs of opposite angles that are equal in size. A rhombus can also be classified as a parallelogram.



**right angle** angle that is exactly 90°. Often represented by a small box drawn where the lines meet.



**rotation** transformation where a shape or object is rotated (turned) around a point. Movement is described as an angle of rotation in either a clockwise or anti-clockwise direction from a given point.

**rotational symmetry** property of a shape or object where it looks the same after a given amount of rotation. The number of times that a figure can be rotated and still look the same before it returns to its original position is called the order of rotational symmetry.

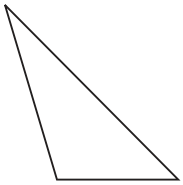
## S

**sample** small selection from a population.

**sample space** list of all the different outcomes possible for a probability experiment. For example, sample space when rolling a die is {1, 2, 3, 4, 5, 6}.

**scale factor** indicates how many times larger or smaller the distance on a map or diagram is compared to the matching real-life distance. For example, a scale factor of $\frac{1}{1000}$ indicates that the distance on the map is 1000 times smaller than in real life.

**scalene triangle** triangle with three sides of different lengths. All three angles are different in size.



**scatterplot** graph that displays a relationship between two variables where points are plotted but not joined.

**segment (line segment)** part of a line between two points.

**set of axes** two axes ($x$-axis and $y$-axis) of the Cartesian plane. (Plural of axis is axes.)

**simplifying (cancelling)** for fractions, *see* cancelling

**simulation** using technology or simple devices like coins or dice to simulate real-life events. Makes performing a large number of trials in a probability experiment easier.

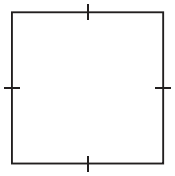
**solution** value of the pronumeral that makes an equation a true statement; that is, the left side and the right side of the equation are equal.

**solve** find the value of the pronumeral that makes an equation a true statement; that is, the left side and the right side of the equation are equal.

**solving by inspection** mental strategy to solve simple equations. To inspect or use observation to work out the solution to an equation in your head.

**sphere** 3D object shaped like a ball.

**square** quadrilateral (four-sided shape) with all four sides equal in length, two pairs of opposite sides that are parallel and four angles of 90°. A square is also classified as a rectangle.



**square number** number that can be represented in a square pattern. For example, 4 and 9 are square numbers.




**square root** number that when multiplied by itself produces a result that can be represented in a square pattern. For example, the square root of 9 (or $\sqrt{9}$) is 3.

**stem-and-leaf plots (stem plots)** a display of data where each piece of data is split into two parts: the last digit becoming the leaf and the other digits becoming the stem. The stem is written once in the left column of the plot and the leaves are listed in numerical order beside the appropriate stem.

**straight angle** angle that is exactly 180°.



**substitution** replacement of a pronumeral with a given number. For example, substituting $x = 2$ in $3x + 4$ gives $3 \times 2 + 4$.

**successful trial** probability trial where a favourable outcome is obtained.

**sum** result of an addition. For example, 4 + 5 = 9 where 9 is the sum.

**summary statistics** information such as the mean, median, mode and range that provide a summary of the data set.

**supplementary angles** angles that add to make a straight angle (180°).

**surface area** total area of the surface of an object. Surface area of a prism is the sum of the areas of each face of the prism. Common units are $mm^2$, $cm^2$, $m^2$.

**symmetrical (symmetric)** property of a shape or object where it can be cut in half and each half is a reflection of the other.

## T

**terminating decimal** decimal that 'terminates' or stops after a set number of decimal places; for example, 0.625.

**terms** components of an algebraic expression or equation and separated by a mathematical operation symbol. For example, the expression $3b - 4c + 10$ has three terms.

**tessellation** pattern of shapes that has no overlaps or gaps. Regular tessellations are made from identical, regular polygons. Semi-regular tessellations are made from a number of regular polygons. Irregular tessellations are made from any shape.

**three-dimensional (3D)** object with three dimensions such as length, width and height.

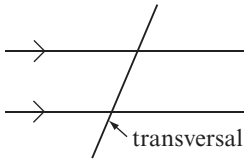
**3D graph** graph where the frequency of data is presented as three-dimensional (3D) blocks or rectangular prisms instead of two-dimensional columns.

**time zones** the Earth has been divided into 24 time zones based on each 15° of longitude. Sometimes boundaries vary to line up with country and state borders.

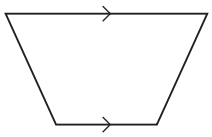
**trailing zeros** zeros to the right of the last non-zero digit after the decimal point in a decimal number. For example, 3.900 has two trailing zeros.

**translation** transformation where a shape or object is translated (moved) in a straight line without turning or changing size. Movement is often described as the number of units up or down and left or right.

**transversal** line that crosses (intersects) a pair or set of lines.




**trapezium** quadrilateral (four-sided shape) with one pair of opposite sides that are parallel.



**travel graph** graph that displays the relationship between distance and time.

**trials** performed to obtain results in a probability experiment. For example, rolling a die 50 times in a probability experiment is completing 50 trials.

**triangle** two-dimensional shape with three straight sides.

**true bearing** direction from one position to another and described by the number of degrees from north in a clockwise direction. The angle is written as three digits followed by T (or true). For example, 090°T is due east, 180°T is due south, 270°T is due west.

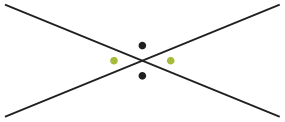
**two-dimensional (2D)** shape with only two dimensions such as length and width.

## V

**vanishing point** point in a perspective drawing to which parallel lines appear to converge and disappear into the horizon.

**variables** quantities that can have varying or different values. A variable can be represented with words or a symbol.

**vertically opposite angles** pair of equal angles not adjacent to (next to) each other, formed where two lines intersect.



**vertex (plural vertices)** corner point where three or more edges of a polyhedron meet. *See* polyhedron

**vinculum** horizontal line separating the numerator from the denominator. For example, in 3/4, the vinculum is the horizontal line between 3 and 4. Also represents a division sign.

**volume** amount of space that a three-dimensional object occupies. Common units are $mm^3$, $cm^3$, $m^3$.

## W

**width** length measurement from one end to the other of a shape or object. In a rectangle, it is the side measurement that is not defined as the length. Common units are mm, cm, m, km.

## X

***x*-axis** horizontal number line that forms part of the Cartesian plane.

***x*-coordinate** first of a pair of numbers (coordinates) that describes a point on the Cartesian plane. It indicates how far the point is from the origin in a horizontal direction.

## Y

***y*-axis** vertical number line that forms part of the Cartesian plane.

***y*-coordinate** second of a pair of numbers (coordinates) that describes a point on the Cartesian plane. It indicates how far the point is from the origin in a vertical direction.

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