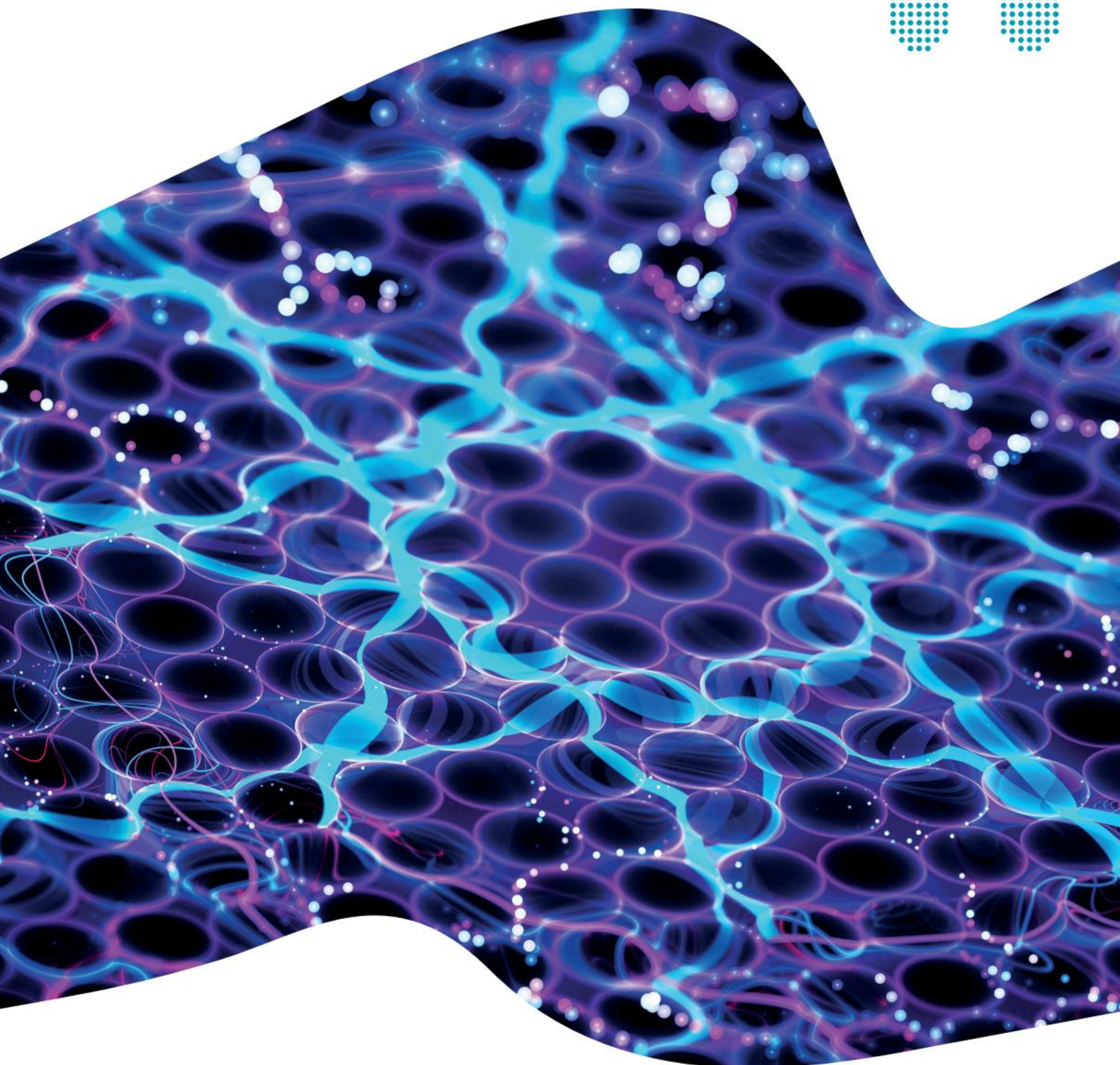
PEARSON

PHYSICS

WESTERN AUSTRALIA

11



2ND EDITION

PEARSON

PHYSICS 11

WESTERN AUSTRALIA 2ND EDITION

COORDINATING AUTHOR

Greg Moran

AUTHORS

Simon Carrello

Tracey Fisher

Kristen Hebden

Elke McKay

Pearson Australia
(a division of Pearson Australia Group Pty Ltd)
459–471 Church Street, Level 1, Building B
Richmond, Victoria 3121
PO Box 23360, Melbourne, Victoria 8012
www.pearson.com.au

First published 2025 by Pearson Australia
2028 2027 2026 2025
10 9 8 7 6 5 4 3 2 1

Project Leads: Misal Belvedere, Simon Tomlin
Senior Content and Learning Specialist: Fiona Cooke
Development Editors: Natalie Orr and Nicole Mount
Senior Delivery Manager: Michelle Thomas
Series Designer: Anne Donald
Cover Designer: David Doyle
Rights & Permissions Editor: Alice McBroom
Publishing Services Analyst: JitPin Chong
Illustrators: DiacriTech, QBS Learning, Newgen KnowledgeWorks
Editor: Newgen KnowledgeWorks
Proofreader: Newgen KnowledgeWorks
Indexer: Newgen KnowledgeWorks
Printed in Australia by Pegasus Media + Logistics

NATIONAL LIBRARY OF AUSTRALIA
A catalogue record for this book is available from the National Library of Australia

ISBN 978 0 6557 1911 3

Pearson Australia Group Pty Ltd ABN 40 004 245 943

Indigenous Australians
We respectfully acknowledge the traditional custodians of the lands upon which the many schools throughout Australia are located. We acknowledge traditional Indigenous Knowledge systems are founded upon a logic that developed from Indigenous experience with the natural environment over thousands of years. This is in contrast to Western science, where the production of knowledge often takes place within specific disciplines. As a result, there are many cases where such disciplinary knowledge does not reflect Indigenous worldviews, and can be considered offensive to Indigenous peoples.

Some of the images used in *Pearson Physics 11 Western Australia* 2nd edition might have associations with deceased Indigenous Australians. Please be aware that these images might cause sadness or distress in Aboriginal or Torres Strait Islander communities.

PEARSON
PHYSICS
WESTERN AUSTRALIA 2ND EDITION
11

Writing and development team

We are grateful to the following people for their time and expertise in contributing to the *Pearson Physics 11 Western Australia* 2nd edition.

Greg Moran
Head of Science
Past President STAWA
Coordinating Author

Simon Carrello
Physics and Chemistry teacher
Head of Science
Author

Tracey Fisher
Teacher
Former Lecturer, Research Scientist
Author

Kristen Hebden
Lecturer in Science and Technology Education, University of Melbourne
Author

Elke McKay
Head of Science
Author

The publisher would like to acknowledge the following authors for past contribution to *Pearson Physics 11 Western Australia* 1st edition content:

Doug Bail

Keith Burrows

Robert Chapman

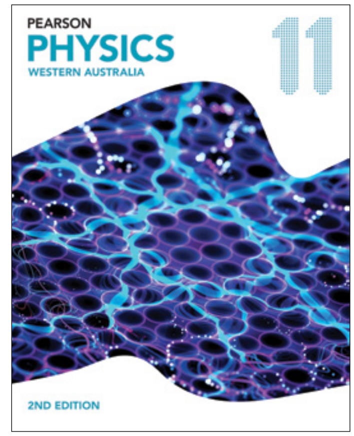
Ann Conibear

Carmel Fry

Geoff Lewis

David Madden

Gregory White



Contents

UNIT 2 Waves, nuclear and electrical physics

How to use this book

Pearson Physics 11 Western Australia

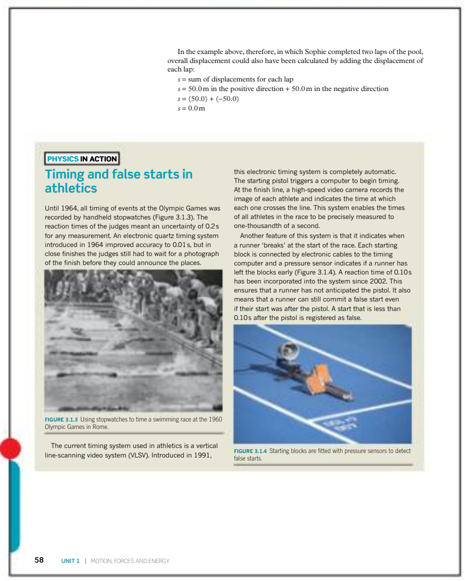
Pearson Physics 11 Western Australia 2nd edition has been written to the *WACE Physics ATAR Course, Year 11 Syllabus 2024*. Each chapter is clearly divided into manageable sections of work. Best practice literacy and instructional design are combined with high quality, relevant photos and illustrations. Explore how to use this book below.

Practical investigation

Chapter 1 covers most of the skills needed to successfully plan and conduct a practical investigation, addressing the Science Inquiry Skills content from the syllabus.

Chapter opening page

The chapter opening page links the syllabus to the chapter content. Science Understanding and Science as a Human Endeavour addressed in the chapter are clearly listed.

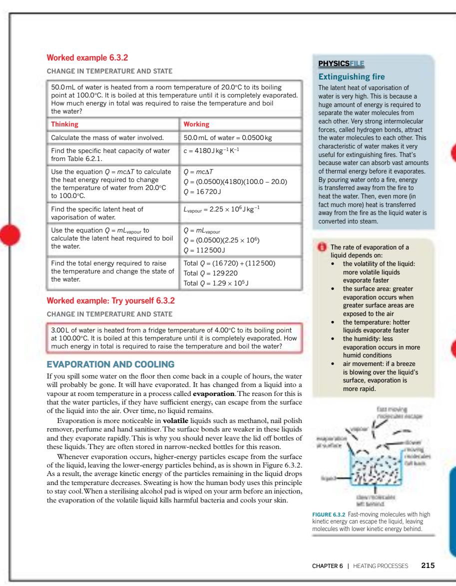


PHYSICS IN ACTION

Physics in Action boxes place physics in an applied situation or relevant context and encourage students to think about the development of physics and the use and influence of physics in society.

EXTENSION

Extension boxes include material that goes beyond the core content of the syllabus. They are intended for students who wish to expand their depth of understanding in a particular area.



Worked examples

Worked examples are set out in steps that show both thinking and working. This enhances student understanding by clearly linking underlying logic to the relevant calculations.

Each Worked example is followed by a Worked example: Try yourself. This mirror problem allows students to immediately test their understanding.

PHYSICSFILE

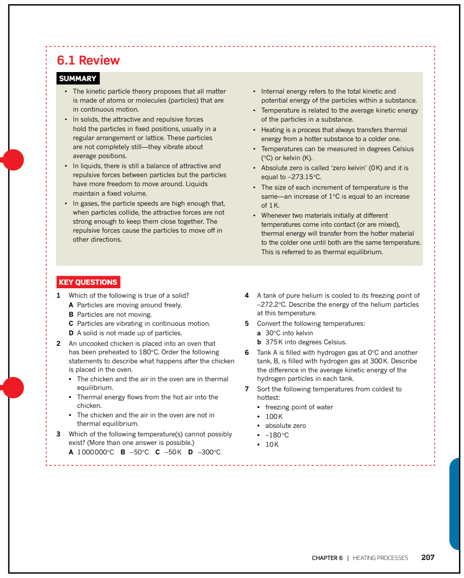
PhysicsFile includes a range of interesting information and real-world examples.

Highlight box

Highlight boxes focus students' attention on important information such as key definitions, formulae and summary points.

Section summary

Each section includes a summary to assist students in consolidating key points and concepts.



Section review questions

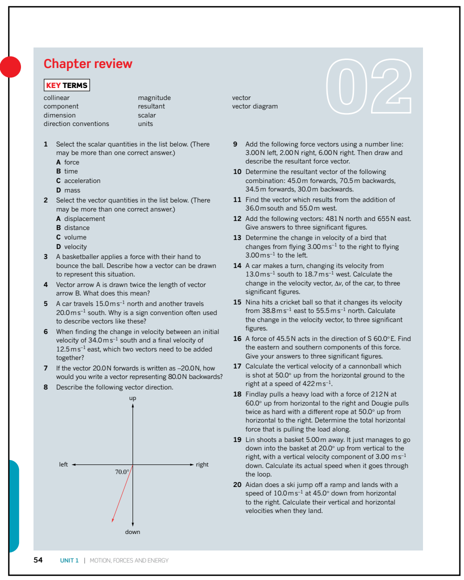
Each section finishes with questions to test students' understanding and ability to recall the key concepts of the section.

Glossary

Key terms are shown in bold and listed at the end of each chapter. A comprehensive glossary at the end of the book includes and defines all the key terms.

Chapter review

Each chapter finishes with a set of higher-order questions to test students' ability to apply the knowledge gained from the chapter.

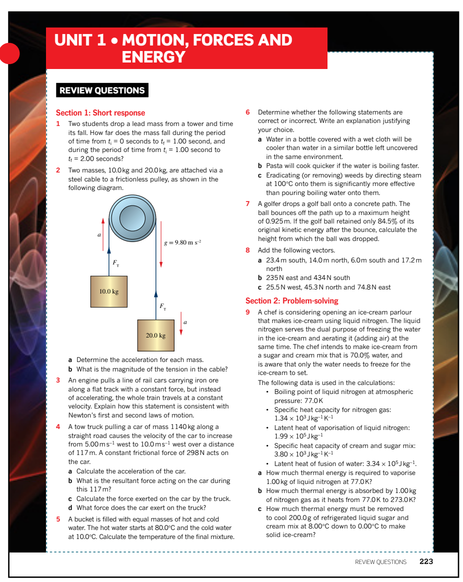


Answers

Numerical answers and key short response answers are included at the back of the book. Comprehensive answers and fully worked solutions for all section review questions, Worked examples: Try yourself, chapter review questions and unit review questions are provided via *Pearson Physics 11 Western Australia 2nd edition eBook*.

Unit review

Each unit finishes with a comprehensive set of exam-style questions, which assist students in drawing together their knowledge and understanding and applying it to this style of question.



CHAPTER 01

Practical investigation

This chapter covers most of the skills needed to successfully plan and conduct a practical investigation.

Section 1.1 is a guide to designing and planning an investigation, including how to write a hypothesis, and how to identify the variables. It explains validity, reliability, precision and accuracy, to assist in planning an investigation appropriately.

Section 1.2 is a guide to conducting investigations. It describes methods for accurately collecting and recording data to uncertainty errors. It explores presenting data using tables and graphs, to aid in selecting the most appropriate format for presenting the results.

Section 1.3 explains how to discuss an investigation and draw evidence-based conclusions that relate to the hypothesis and research question.

Practical Investigation Steps

The size and scope of a practical investigation can be initially quite daunting, but establishing a task list and timeline will help break it down into manageable steps. The entire task is expected to take between 7 and 10 hours.

Here are some steps that will need to be considered in a timeline:

- Determine the topic and type of investigation.
- Research and write down the theory on which the investigation is based.
- Determine an appropriate question to answer, and formulate a hypothesis.
- Identify the independent, dependent and controlled variables.
- Select equipment and resources needed for the investigation.
- Determine an appropriate procedure (methodology), considering validity, reliability and accuracy.
- Assess the risks and ethical issues, and identify measures to address these.
- Conduct the investigation and record all data obtained.
- Analyse and evaluate the data.
- Evaluate your methods. Suggest ways of improving or extending the investigation.
- Write an evidence-based conclusion. Describe the limitations of the study.
- Write the final report. (This should not be the focus of the investigation but rather an opportunity to communicate the investigation process and your conclusions.)

Some of these tasks are larger and will require more time than others. Many will overlap. Plan a realistic approach, consult with teachers to establish school-based time constraints and fix dates for the completion of each task. Allow time for reflection and to review your earlier work.

Science Inquiry Skills

- identify, research and construct questions for investigation; propose hypotheses; and predict possible outcomes
- design investigations, including the procedure to be followed, the materials required, and the type and amount of primary and/or secondary data to be collected; conduct risk assessments; and consider research ethics
- conduct practical work, including the manipulation of devices, safely, competently and methodically for the collection of valid and reliable data
- Represent data in meaningful and useful ways, including using appropriate *Système Internationale* (SI) units and symbols, and significant figures
- organise and analyse data to identify trends, patterns and relationships
- identify sources of random and systematic uncertainty and estimate their effect on measurement results
- state absolute uncertainties in values and calculate percentage uncertainty where appropriate
- combine uncertainties in calculations to determine the overall uncertainty in a measurement (addition, subtraction, multiplication and division)
- identify anomalous data and calculate the percentage difference between the experimental results and a currently accepted value
- select, synthesise and use evidence to make and justify conclusions
- interpret a range of scientific texts and evaluate processes and conclusions by considering the available evidence, and use reasoning to construct scientific arguments
- select, construct and use appropriate representations, including text and graphical representations of empirical and theoretical relationships, to communicate conceptual understanding, solve problems and make predictions
- select, use and interpret appropriate mathematical representations, including linear and non-linear graphs and algebraic relationships representing physical systems, to solve problems and make predictions
- relate gradients and axis intercepts of linear graphs to physical quantities
- apply dimensional analysis to determine the appropriate units for calculated quantities, e.g. a gradient in a graph
- use uncertainty bars to represent the uncertainty in a value on a graph and take into account when sketching a line of best fit
- communicate to specific audiences and for specific purposes using appropriate language and nomenclature

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.

1.1 Designing and planning the investigation

Taking the time to carefully plan and design a practical investigation before you begin will help you to maintain a clear and concise focus throughout. Preparation is essential. Ensure you understand the theory behind the investigation and prepare a detailed plan for the practical components of the investigation. This section is a guide to some of the key steps that should be taken when planning and designing a practical investigation (Figure 1.1.1).

FIGURE 1.1.1 There are many elements to a practical investigation, which may appear overwhelming to begin with. Taking a step-by-step approach will help the process and assist you in completing a worthwhile investigation.

DETERMINING THE TYPE OF INVESTIGATION

In your course, you will be expected to prepare a portfolio of the practical work you have undertaken across the range of physics areas studied. This portfolio will include evidence of a variety of science inquiry skills, including experimental design and the communication and evaluation of scientific data.

Several types of research methods are used in science. Physics investigations typically fall into two methodologies:

- analysing the slope of a linear graph
- determining the relationship between two continuous **variables**.

From your research on the topic you are investigating you may find that there is a mathematical relationship between the variables that you are exploring. It may be possible to use your algebra skills to write the mathematical relationship in the form of a linear equation, $y = mx + c$, where y is the dependent variable and x is the independent variable. A graph of the y and x variables will result in a linear graph with a slope equal to a constant, m, or collection of constants. By equating the value of the slope to these constants some meaningful analysis can be made, such as finding a value for the acceleration due to gravity or the specific heat capacity of a metal.

If, in your research, you find that there is no equation that links your variables, then you can investigate to determine what the relationship might be. By graphing your variables, you may find that the relationship is linear, or it may be inverse, exponential or even inverse squared. An investigation of this kind results in a calibration curve, which can be used to predict values of the dependent variable given particular values of the independent variable.

DEVELOPING RESEARCH QUESTIONS AND AIMS, FORMULATING HYPOTHESES AND IDENTIFYING VARIABLES

The research question, aim and hypothesis are interlinked. It is important to note that each of these can be refined as the planning of the investigation continues.

Formulating a question

A research question is a question that comes from an inquiring mind. When you are actively involved in developing your scientific understanding you will want to know what factors affect a variable, or what is the relationship between two variables. The research question poses the question that the investigation seeks to answer. For example: What is the relationship between voltage and current?

Before formulating a question, it is good practice to conduct a literature review of the topic to be investigated. You should become familiar with the relevant scientific concepts and key terms.

During this review, write down questions, correlations or equations as they arise.

Compile a list of possible ideas. Do not reject ideas that initially might seem impossible. Use these ideas to generate questions that are answerable.

Before constructing a hypothesis, formulate a question that needs an answer. This question will lead to a hypothesis when:

- the question is reduced to measurable variables
- a prediction is made based on knowledge and experience.

Evaluating your question

Once a question has been chosen, stop to evaluate the question before progressing. The question may need further refinement or even further investigation before it is suitable as a basis for an achievable and worthwhile investigation. A major planning point is to attempt something that is possible to complete in the time available or with the resources on hand. It might be a little difficult to create a particularly complicated device with the facilities available in the school laboratory.

To evaluate the question, consider the following:

- Relevance: Is the question related to the appropriate area of study?
- Clarity and measurability: Can the question lead to a clear hypothesis? If the question cannot lead to a specific hypothesis, it will be very difficult to complete the research.
- Time frame: Can the question be answered within a reasonable period of time? Is the question too broad?
- Knowledge and skills: Do you have a level of knowledge and a level of laboratory skills that will allow the question to be explored? Keep the question simple and achievable.
- Practicality: Are resources, such as laboratory equipment and materials, likely to be readily available? Keep things simple. Avoid investigations that require sophisticated or rare equipment. Equipment that is more-readily available includes timing devices, objects that could be used as projectiles, a tape measure and other common laboratory equipment.
- Safety and ethics: Consider the safety and ethical issues associated with the question you will be investigating. If there are issues, can these be addressed?
- Advice: Seek advice from your teacher about the question. Their input may prove very useful. Their experience may lead them to consider aspects of the question that you have not thought about.

Defining the aim of the investigation

An aim is a statement describing in detail what will be investigated to answer the research question. For example: The aim of the experiment is to investigate the relationship between the voltage and current in a circuit with constant resistance. Each aim should directly relate to the variables that will be referred to in the hypothesis. The aims do not need to include the details of the method.

Example

- Aim: The aim of the experiment is to investigate the relationship between mass and acceleration, when a constant force is applied.

Hypothesis

A hypothesis is a definite statement, based on previous knowledge and evidence or observations, that attempts to answer the research question. The hypothesis must relate the independent and dependent variables and describe the relationship between them. For example: Increasing the voltage supplied to a circuit with constant resistance increases the current proportionally.

Here are some further examples of hypotheses:

- For a constant force, if the mass is increased, the acceleration is decreased as an inverse relationship.
- If the value of the resistance of a circuit increases, the current flowing in the circuit will decrease as an inverse relationship.

- Assuming no heat loss to the surroundings, the temperature rise of a fixed mass of water is proportional to the time it is heated by a constant power source.
- As the height from which an object is dropped increases, the final velocity of the object will increase as a squared relationship.

There are no wrong or right hypotheses. You might formulate a hypothesis that a more experienced person will disagree with; however, the purpose of an investigation is to find the answer to a research question. If the answer to the question supports your hypothesis, then that is a positive result, as it will confirm your understanding of the concept. On the other hand, if your investigation does not support your hypothesis, then that is a useful result as well, as you can now say that your original understanding was not correct and you can change your understanding to a more scientific one. Some of you might notice that the following hypothesis will not be supported by the investigation:

- The greater the mass of a marble, the faster it will hit the ground, when dropped from the same height.

This doesn't mean that the hypothesis is wrong, but it may indicate that there was some misconception that you had that was not exposed in your literature review.

Formulating a hypothesis

A good hypothesis should:

- be a definite statement of the relationship
- include an independent and a dependent variable that is continuous and measurable
- be worded so that it can be tested in the experiment.

The hypothesis should also be falsifiable. This means that a negative outcome would disprove it. For example, the hypothesis that all apples are round cannot be proved beyond doubt, but it can be disproved, in other words, it is falsifiable. In fact, only one oval-shaped apple is needed to disprove this hypothesis. Unfalsifiable hypotheses cannot be tested by science. These include hypotheses on ethical, moral and other subjective judgements.

Variables

A good scientific hypothesis can be tested, that is, it can be supported or refuted through investigation. To be a testable hypothesis, it should be possible to measure both the change or treatment and the effect, or what will happen. The factors that can be changed, or are changed as a result of the experiment or investigation, are called the variables. An experiment or investigation determines the relationship between variables.

There are three categories of variables:

- The **independent variable** is the variable that is changed by the researcher. You must test only one independent variable in any investigation, otherwise it cannot be stated that the changes in the dependent variable are the result of changes in the independent variable.
- The **dependent variable** is the variable that may change in response to a change in the independent variable. This is the variable that will be measured or observed. You should measure only one dependent variable in any investigation. If you want to measure another dependent variable then you will need to do another investigation with another hypothesis.
- **Controlled variables** are all the variables that must be kept constant during the investigation otherwise the test cannot be fair.

Read the following example of a hypothesis.

If the cross-sectional area of a resistor is constant, the longer the wire, the greater the resistance as a linear relationship.

Identify the different variables.

- independent variable: length of wire
- dependent variable: resistance of the wire
- controlled variables: potential difference, material of the resistor, temperature of the resistor.

Completing a table like Table 1.1.1 will assist in evaluating the question or questions.

TABLE 1.1.1 Break the question down to determine the variables.

Research question	How does the power of a kettle affect the time taken to boil water?
Independent variable	the power of the kettle
Dependent variable	the time the kettle takes to boil water
Controlled variables	mass of the water, purity of the water, starting temperature of the water and kettle
Potential hypothesis	The greater the power of a kettle, the less time it will take to boil water, as an inverse relationship

Qualitative and quantitative variables

Variables are either qualitative or quantitative, with further subsets in each category.

- **Qualitative variables** can be observed but not measured; for example, describing a light globe as bright or dim. They can only be sorted into groups or categories such as brightness, type of construction material or type of device.
  - Nominal variables are categorical variables in which the order is not important; for example, the type of material or type of device.
  - Ordinal variables are categorical variables in which order is important and groups have an obvious ranking or level; for example, brightness (Figure 1.1.2).
- **Quantitative variables** can be measured. Length, area, weight, temperature and cost are all examples of quantitative data.
  - Discrete variables consist of only integer numerical values, not fractions; for example, the number of pins in a packet, the number of springs connected together, or the energy levels in atoms.
  - Continuous variables allow for any numerical value within a given range; for example, the measurement of temperature, length, mass and frequency.

In physics, you should choose continuous quantitative variables for both the independent and dependent variables. This will allow you to construct a line graph, and therefore determine the slope of the line, or the relationship between the variables.

FIGURE 1.1.2 When recording qualitative data, describe in detail how each variable will be defined. For example, if recording the brightness of light globes, light meters are a quantitative way to gather data.

WRITING THE METHODOLOGY

The methodology, or method, of your investigation is a step-by-step procedure. When detailing the method, ensure it enables you to conduct a valid, reliable and accurate investigation.

Validity

Validity refers to whether an experiment is in fact testing the hypothesis. Is the investigation obtaining data that is relevant to the question, or is it flawed?

To ensure an investigation is valid, it should be designed so that only one variable is changed at a time. The other variables must remain constant, so that meaningful conclusions can be drawn about the effect of the independent variable alone.

To ensure validity, you must carefully determine:

- the independent variable—the variable that will be changed, and how it will change
- the dependent variable—the variable that will be measured
- the controlled variables—the variables that must remain constant.

Reliability

Reliability refers to the idea that the experiment can be repeated many times and will obtain consistent results. You can maintain the investigation's reliability by:

- listing and defining the control variables and how they will be kept constant
- listing the detailed steps that you will take to conduct the experiment, describing what you will do and how you will measure and record data
- ensuring that there are enough changes of the independent variable. Typically, five changes over a wide range of the independent variable are considered sufficient.
- ensuring there are enough trials conducted for each value of the independent variable. Typically, you should conduct at least three trials repeating the experiment, then average the three measurements. This reduces random errors and allows systematic errors to be identified. If a reading differs too much from the rest (known as an outlier), discard it before averaging (Figure 1.1.3).

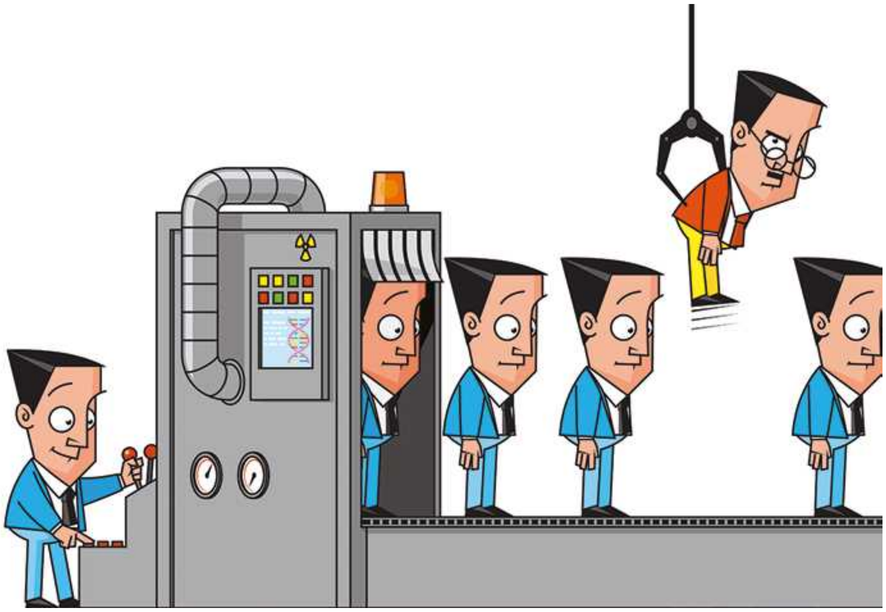


FIGURE 1.1.3 Replication increases the reliability of your investigation. It ensures that if anyone repeats the investigation they will obtain similar data.

Accuracy and precision

Precision refers to the extent to which the instrument can make repeated measures of the dependent variable that are the same for the same value of the independent variable. For example, if each measurement of the current in an electrical circuit is within 0.1 A of the others, then the device is more precise than a device for which there is a difference of 0.5 A. **Accuracy** refers to how close a measurement is to the true or accepted value.

You will need to consider if the instruments to be used are sensitive enough. Build some testing into your investigation to confirm the accuracy and reliability of the equipment and your ability to read the information obtained.

Reasonable steps to ensure the accuracy of the investigation include considering:

- the type of instrument that will be used to measure the independent and dependent variables.
- calibrating the measuring equipment by testing a standard.

Describe the materials and method in appropriate detail in your scientific reports. This should ensure that every measurement can be repeated and the same result obtained within reasonable margins of experimental uncertainty. (A margin of less than 5% is reasonable.)

Data analysis

How you will be analysing the data produced from your investigation must be considered when writing your method. A wide range of analysis tools are available. For example, tables can be used to organise data so that patterns can be established, and graphs can show relationships and comparisons. In fact, preparing an empty table showing the data that needs to be obtained will help in the planning of the investigation. See page 13 for more information on organising a data table.

Sourcing appropriate materials and technology

When designing your investigation, you will need to decide on the materials, technology and instrumentation that will be used to carry out your research. It is important to find the right balance between items that are easily accessible and those that will give you accurate results. As you move on to conducting your investigation, it will be important to take note of the quality of your chosen instruments and how this affects the accuracy and validity of your results.

Modifying the methodology

The methodology may need modifying as the investigation is carried out. The following actions will help to determine any issues in the methodology and how to modify it.

- Record everything.
- Be prepared to make changes to the approach.
- Note any difficulties encountered and the ways they were overcome. What were the failures and successes? Every test carried out can contribute to the understanding of the investigation as a whole, no matter how much of a disaster it may first appear.
- Do not panic. Go over the theory again, and talk to the teacher and other students. A different perspective can lead to a solution.

If the expected data is not obtained, don't worry. As long as it can be critically and objectively evaluated, with the limitations of the investigation identified and further investigations proposed, the work is worthwhile.

COMPLYING WITH ETHICAL AND SAFETY GUIDELINES

Ethical considerations

Some investigations require an ethics approval—consult with the teacher. In fact, when deciding on an investigation, identify all possible ethical considerations and evaluate whether those parts of the investigation are necessary or if there are ways you can reduce or mitigate them.

Occupational health and safety

While planning for an investigation, it is important to consider the potential risks to ensure the safety and yourself and others.

Everything we do has some risk involved. Risk assessments are performed to identify, assess and control hazards. A risk assessment should be performed for any situation, in the laboratory or outside in the field. Always identify the risks and control them to keep everyone safe. For example, carry out voltage–current experiments with low voltages (less than 6.0 V DC or 4 × 1.5 V batteries) coupled to resistors so that the currents in the circuits are of the order of milliamps. *At all times* avoid direct exposure to 240 V AC household voltages (Figure 1.1.4).

To identify risks, think about:

- the activity that will be carried out
- the equipment or chemicals that will be used.

The following hierarchy of risk controls is organised from most to least effective:

1. *Elimination:* Eliminate dangerous equipment, procedures or substances.
2. *Substitution:* Find different equipment, procedures or substances to use that will achieve the same result, but have less risk associated with them.
3. *Isolation:* Ensure there is a barrier between the person and the hazard. Examples include physical barriers such as guards in machines, or fume hoods to work with volatile substances.
4. *Engineering controls:* Modify equipment to reduce risks.
5. *Administrative controls:* Provide guidelines, special procedures, warning signs and information about safe behaviours for any participants.
6. *Personal protective equipment (PPE):* Wear safety glasses, lab coats, gloves and respirators etc. where appropriate. Provide these to other participants as needed.

FIGURE 1.1.4 When planning an investigation, you need to identify, assess and control hazards.

Science outdoors

Sometimes investigations and experiments will be carried out outdoors. Working outdoors has its own set of potential risks and it is equally important to consider ways to eliminate or reduce these risks.

Table 1.1.2 contains examples of risks associated with fieldwork outdoors.

TABLE 1.1.2 Risks associated with fieldwork outdoors.

Risks	Control measures
sunburn	wear sunscreen, a hat and sunglasses
hot or cold weather	wear clothing to protect against heat or cold
projectile launch	create barriers so that people know not to enter the area
trip hazards	minimise the use of cables (electrical, computer) and cover them up with matting be aware of tree roots, rocks etc.

First aid measures

Minimising the risk of injury reduces the chance of requiring first aid assistance. However, it is still important to have someone with first aid training present during practical investigations. Always tell the teacher or laboratory technician if an injury or accident happens.

Personal protective equipment

Everyone who works in a laboratory wears items that help keep them safe. This is called **personal protective equipment (PPE)** and includes:

- safety glasses
- shoes with covered tops
- disposable gloves when handling chemicals
- a disposable apron or a lab coat if there is risk of damage to clothing
- ear protection if there is risk to hearing.

1.1 Review

SUMMARY

- An aim is a statement describing in detail what will be investigated. For example: The aim of the experiment is to investigate the relationship between force, mass and acceleration.
- A hypothesis is a definite statement of the relationship between the independent and dependent variables based on previous knowledge and evidence or observations that attempts to answer the research question. For example: With the force kept constant, the acceleration decreases with increasing mass as an inverse relationship.
- Once a question has been chosen, stop to evaluate the question before progressing. The question may need further refinement or even further investigation before it is suitable as a basis for an achievable and worthwhile investigation. Make sure that it is possible to complete the activity in the time available and with the resources on hand. It might be a little difficult to create a particularly complicated device with the facilities available in the school laboratory.
- There are three categories of variables:
  - The independent variable is the variable that is changed by the researcher.
  - The dependent variable is the variable that may change in response to a change in the independent variable. This is the variable that will be measured or observed.
  - Controlled variables are all the variables that must be kept constant during the investigation so that it is a fair test.
- The methodology of your investigation is a step-by-step procedure. When detailing the methodology, ensure it complies as a valid, reliable and accurate investigation.
- It is also important to determine how many times the independent variable needs to be changed and how many trials need to be run for each change in the independent variable.
- Data analysis should be a consideration in the method. Determine how the data will be presented and analysed. A wide range of analysis tools could be used. For example, tables organise data so that patterns can be established and graphs can show relationships and comparisons.
- In every investigation you need to consider the risks and potentially hazardous situations, and act to minimise those risks.

KEY QUESTIONS

1 In a practical investigation a student changes the potential difference across a circuit by adding or subtracting batteries in series in the circuit.

a How could the potential difference be a discrete value?

b How could it be continuous?

2 In another experiment the student uses the following range of values to describe the brightness of a light: dazzling, bright, glowing, dim, off. What type of measurement is the variable 'brightness'?

3 Select the best hypothesis from the three options below. Give reasons for your choice.

A Hypothesis 1: If you increase the mass of the marble that you drop, the final velocity of the marble will increase linearly.

B Hypothesis 2: The greater the potential difference across a resistor, the greater the current through it.

C Hypothesis 3: Different metals will have different resistances.

4 Give the correct term that describes an experiment with each of the following conditions.

a The experiment addresses the hypothesis and aims.

b The experiment is repeated and consistent results are obtained.

c Appropriate, high-quality equipment is chosen and calibrated for the desired measurements.

5 A student wanted to find out how the tension in an elastic band affects the band's initial velocity when launched from their finger. State:

a the independent variable

b the dependent variable

c three controlled variables.

1.2 Conducting investigations, and recording and presenting data

Once the planning and design of a practical investigation is complete, the next step is to undertake the investigation and record the results. As with the planning stages, there are key steps and skills to keep in mind to maintain high standards and minimise potential error throughout the investigation (Figure 1.2.1).

This section will focus on the best methods for conducting a practical investigation, systematically generating, recording and processing data, and then presenting it in a concise and clear manner.

FIGURE 1.2.1 When carrying out your investigation try to maintain high standards to minimise potential errors.

CONDUCTING INVESTIGATIONS TO COLLECT AND RECORD DATA

For an investigation to be scientific, it must be objective and systematic. Ensuring familiarity with the methodology and protocols before beginning will help you to achieve this.

While working, keep asking questions: Is the work biased in any way? If changes are made, how will they affect the study? Will the investigation still be valid for the aim and hypothesis?

It is essential that during the investigation the following are recorded:

- all quantitative data collected
- the methods used to collect the data
- any incident, feature or unexpected event that may have affected the quality or validity of the data.

The data recorded is the **raw data**. Usually this data needs to be processed in some manner before it can be presented. If an error occurs in the processing of the data or you decide to present the data in an alternative format, the recorded raw data will always be available for you to refer back to.

IDENTIFYING ERRORS

All practical investigations have errors associated with them. Errors can occur for a variety of reasons. It is important that potential errors are considered when planning an investigation and that measures are taken to reduce them. This ensures the investigation is as accurate as possible. Any additional errors that occur during the collection of results should also be recorded.

There are two types of errors:

- systematic errors
- random errors.

Systematic errors

A **systematic error** is an error that is consistent and will occur again if the investigation is repeated in the same way.

Systematic errors are usually a result of instruments that are not calibrated correctly or methods that are flawed.

An example of a systematic error would be if a ruler mark indicating 5 cm from 0 cm was actually only 4.9 cm from 0 cm due to a manufacturing error or shrinkage of the wood. Another example would be if the researcher repeatedly used a piece of equipment incorrectly throughout the entire investigation. Figure 1.2.2 shows how traffic police reduce systematic errors in their data collection.

FIGURE 1.2.2 To avoid a systematic error, make sure that you are using measuring equipment correctly. Laser speed guns, for example, need to be placed on a stationary support so the aim point is held on a single target point for the duration of the read.

Random errors

Random errors occur in an unpredictable manner and are generally small. Random errors are typically caused by minor, unpredictable changes in experimental conditions that lead to fluctuations around the true value. An example of a random error could be electronic noise in the circuit of an electrical instrument.

Techniques for reducing error

Designing the method carefully, including selection and use of equipment, will help reduce errors.

Appropriate equipment

Use the equipment that is best suited to the type of data being collected to validate the hypothesis. Determining the appropriate units and scale for the data will help to select the correct equipment. Using the right unit and scale will ensure that measurements are more accurate and precise, thereby minimising systematic errors.

Significant figures represent precision and reliability of a measurement. The number of significant figures used depends on the scale of the instrument. It is important to record data to the number of significant figures available from the equipment or observation. Using more or fewer significant figures can be misleading.

Review the following examples to learn more about significant figures:

- 15 has two significant figures
- 3.5 has two significant figures
- 3.50 has three significant figures
- 0.037 has two significant figures
- 1401 has four significant figures.

To calculate gravitational potential energy (E_g), the formula is $E_g = mg\Delta h$.
If $g = 9.80\,\text{m s}^{-2}$, mass (m) = 7.50 kg, height (h) = 0.64 m (64 cm):

$$E_g = 9.80 \times 7.50 \times 0.64 = 47.09\,\text{J}$$

When reporting data, quote the result to the least number of significant figures found in the measured values. In this example, height is accurate to two significant figures while g and mass have three significant figures, so report the result to two significant figures, for example $E_g = 47\,\text{J}$.

Although digital scales can measure to many more than two figures and calculators can give 12 figures, be sensible and follow the significant figure rules.

Calibrated equipment

Some equipment, such as some motion sensors, needs to be calibrated before use to account for the temperature at the time. Before carrying out the investigation, make sure the instruments or measuring devices are properly calibrated and functioning correctly. For example, measure the temperature and apply a correction to the speed of sound to calibrate a motion sensor if necessary.

Correct use of equipment

Use the equipment properly. Ensure training has been completed and that you have practised using the equipment before beginning the investigation. Improper use of equipment can result in inaccurate data with large errors, and the validity of the data can be compromised.

Incorrect reading of measurements is a common misuse of equipment. Make sure all the equipment needed in the investigation can be used correctly and record the instructions in detail so they can be checked if the data doesn't appear correct.

RECORDING AND PRESENTING QUANTITATIVE DATA

Raw data is unlikely to be used directly to validate the hypothesis. However, raw data is essential to the investigation and plans for collecting the raw data should be made carefully. Consider the formulas or graphs that will be used to analyse the data at the end of the investigation. This will help to determine the type of raw data that needs to be collected in order to validate the hypothesis.

For example, to calculate take-off velocity for a vertical jump, three sets of raw data will need to be collected using a force platform: the athlete's standing body weight, the ground reaction force and the time during the vertical jump. The data can then be processed to obtain the take-off impulse.

Once you have determined the data that needs to be collected, prepare a table to record the data.

ANALYSING AND PRESENTING DATA

The raw data that has been obtained needs to be presented in a way that is clear, concise and accurate.

There are several ways to present data, including tables, graphs, flow charts and diagrams. The best way of visualising the data depends on its nature. Try multiple formats before making a final decision to create the best possible presentation.

Presenting raw and processed data in tables

Tables organise data into rows and columns, and can vary in complexity according to the nature of the data. Tables can be used to organise raw and processed data or to summarise results.

Data in the table should be ordered in columns. The first column should contain the independent variable (the one being changed). Subsequent columns should include the dependent variable results from all trials, with one column for each trial. The final column should then show the average of all of the trials for the dependent variable.

Tables should have the following features:

- a descriptive title that contains both the independent and depended variables
- column headings (including the unit and the uncertainty of measurement)
- aligned figures (align the decimal points)
- the independent variable placed in the left column
- the trials of the dependent variable placed in the right columns with the average column on the end
- an overarching heading for all columns showing the dependent variable results (including the average column).

You may sometimes need to process data to assist with producing a linear graph. This processed data can be added as an additional column to the right of the average column. An example of processed data would be if I^2 needed to be plotted on a graph instead of I. Any processed data should have appropriate units and uncertainty in the heading if the units and uncertainty remain the same, or alongside the data if they vary.

Look at the table in Figure 1.2.3, which has been used to organise raw and processed data about the effect of current on voltage.

The current through a resistor at different potential differences ← clear title

	Current 'I' (±0.01 A)				
Potential difference 'ΔV' (±0.01 V)	Trial 1	Trial 2	Trial 3	Average	I^2 (A^2)
1.50	0.31	0.34	0.32	0.32 ± 0.02	0.10 ± 0.01
2.00	0.42	0.45	0.41	0.43 ± 0.02	0.18 ± 0.02
2.50	0.50	0.51	0.52	0.51 ± 0.01	0.26 ± 0.01
3.00	0.62	0.65	0.58	0.62 ± 0.04	0.38 ± 0.05
3.50	0.70	0.71	0.72	0.70 ± 0.01	0.49 ± 0.01

← headings for each column with units and uncertainties of measurement

← consistent number of significant figures

↑ independent variable

↑ dependent variable trials with consistent number of significant figures

↑ averages calculated with uncertainty of averages displayed

processed data with processed uncertainties, correct significant figures and heading with appropriate units

FIGURE 1.2.3 A simple table listing the raw data obtained in the second, third and fourth columns with the common uncertainty in the overarching column heading, the average of each trial calculated in the fifth column with individual uncertainties, and the processed data, which is the average values squared along with their processed individual uncertainties, in the sixth column.

FIGURE 1.2.4 An analogue pressure scale.

A table of processed data usually presents the average values of trials, the **mean**. However, the mean on its own does not provide an accurate picture of the results. To report processed data more accurately, the uncertainty should be presented as well.

UNCERTAINTY

Measuring from an analogue scale

An analogue scale is a fixed scale from which measurements are taken. Examples of an analogue scale include rulers or pressure gauges, such as the one shown in Figure 1.2.4. When measuring with an analogue scale and the value falls between increments, an estimate should be made to one decimal place beyond the nearest increment. For example, if you measure the length of a block of wood using a ruler marked in centimetres and it aligns precisely with the 10 cm mark, the length is 10.0 cm. However, if the length is approximately halfway between the 10 and 11 cm marks, it should be recorded as 10.5 cm.

Types of uncertainties

When presenting a range of measurements for a particular value, it is essential to include both the mean and the associated **uncertainty** to accurately convey the results. In other words, the mean must be accompanied by an indication of the range of data obtained, reflecting variability in the measurements.

uncertainty = ± **(half the range of the values)**

For example, if the velocity, in $km\,h^{-1}$, of cars travelling down a certain road was:

46.0, 50.0, 55.0, 48.0, 50.0, 58.0 and 45.0

the average velocity would be:

$$\frac{46.0 + 50.0 + 55.0 + 48.0 + 50.0 + 58.0 + 45.0}{7} = 50.2857 = 50.3\,km\,h^{-1}$$

The uncertainty would therefore be half the maximum velocity minus the minimum velocity. In this case, 58.0 is the maximum velocity and 45.0 is the minimum velocity, meaning that the range is 13.0 and half the range is 6.50, so the uncertainty is $\pm 6.50\,km\,h^{-1}$.

This data should be presented as:

Average speed is $50.3 \pm 6.50\,km\,h^{-1}$.

This is called the **uncertainty of averages** as it is taken from the calculation of the mean. Make sure that the final value is expressed to the appropriate number of significant figures, and remember that the uncertainty should never exceed the number of decimal places of the measured value itself.

There is also uncertainty associated with using measuring devices. This is called the **uncertainty of measurement**. This type of uncertainty arises from the precision of the measuring device's scale. The uncertainty of measurement varies depending on the type of measuring device you are using.

- Some devices will have a given uncertainty; for instance, a measuring cylinder might have an uncertainty of ± 0.1 mL printed on it.
- If the uncertainty is not provided, it will depend on whether the measuring device is digital or analogue.
  - If you are measuring on a digital scale, the uncertainty is the smallest possible increment. For instance, a set of scales that gives a result of 100.01 g will have an uncertainty of measurement of ± 0.01 g.
  - If the measuring device is analogue (i.e. those with a fixed scale), the uncertainty of measurement is typically half of the smallest scale division. For instance, if you use a ruler with a millimetre scale and measure the length of an object as 25.0 mm, the uncertainty would be ± 0.5 mm. It is important to note that the uncertainty should have the same number of decimal places as the measurement, but not necessarily the same number of significant figures.

When stating a final value with its uncertainty, you may have two choices to make:

1 If the uncertainty of the measuring device is different from the uncertainty of the averages, you should use the larger of the two values for the average uncertainty. This ensures your reported data reflects the limitations of the instrument, not potentially overstating the precision of the average.
2 You should consider the nature of the investigation, the measuring devices used and the method of data collection in relation to the calculated uncertainty of the average. If it was difficult to take the measurements or you experienced issues with the measuring devices, then you may choose to use a reasonable percentage uncertainty to replace the uncertainty of the averages. For example, if the percentage uncertainty of the averages is around ± 1.5%, but you believe this does not adequately represent the precision of your data collection, you might opt for a higher percentage uncertainty, such as ± 5.0%. If you make this adjustment, make sure you include a note under your data table explaining your rationale.

Calculating percentage uncertainties

Uncertainties that are displayed in the same units as the measurement are known as **absolute uncertainties**. You can use these absolute uncertainties to calculate a percentage uncertainty using the equation below.

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measurement}} \times 100\%$$

Percentage uncertainties are helpful because they allow for the comparison and combination of uncertainties across different units.

Combining uncertainties

It is common to process data in a physics investigation. This may include altering a single set of data, like the column titled 'I^2' in Figure 1.2.3; or perhaps you may use two data sets to calculate a new value, for instance, combining a measured current and potential difference to calculate the power. As these are values calculated using measurements with their own uncertainties, it is crucial to combine the uncertainties to get an overall uncertainty for the calculated value.

The method for combining uncertainties depends on the type of mathematical operation used in the calculation: whether measurements are raised to a power, multiplied or divided, or added or subtracted.

Measurements raised to a power

If a measurement is raised to a power, such as squared or cubed, the percentage uncertainty associated with the measurement will be multiplied by that power. This is shown in step three of Worked example 1.2.1.

Multiplying or dividing measurements

When multiplying or dividing measurements, the total uncertainty is found by adding together the percentage uncertainties for each value. Since the measurements may not have the same units, it is important to use the percentage uncertainties rather than absolute uncertainties, which cannot be added directly. This is outlined in Worked example 1.2.1.

Worked example 1.2.1

COMBINING UNCERTAINTIES WHEN MULTIPLIED OR DIVIDED

A researcher wants to determine the density of a sample of an unknown substance that is in the shape of a cube. The following measurements are taken. • Length of each side of the cube: $\ell = 25.5 \pm 0.5$ mm • Mass of the cube: $m = 32.2 \pm 0.1$ g Calculate the density with the appropriate absolute uncertainty.	
Thinking	**Working**
Convert all absolute uncertainties to percentage uncertainties.	$\%\text{uncertainty} = \frac{\text{uncertainty}}{\text{measurement}} \times 100$ $\%\text{uncertainty}_{length} = \frac{(0.5)}{(25.5)} \times 100$ $\%\text{uncertainty}_{length} = \pm 1.96078\%$ $\%\text{uncertainty}_{mass} = \frac{(0.1)}{(32.2)} \times 100$ $\%\text{uncertainty}_{mass} = \pm 0.31056\%$
Calculate the density of the cube with the data converted to the SI units of kg and m^3.	$\rho = \frac{m}{V}$ $\rho = \frac{m}{\ell^3}$ $\rho = \frac{(32.2 \times 10^{-3})}{(25.5 \times 10^{-3})^3}$ $\rho = 1.26275$ $\rho = 1.26 \text{ kg m}^{-3}$
As the volume is found by cubing the length value, the percentage uncertainty for the volume is three times that of the length.	$\%\ \text{uncertainty}_{volume} = 3 \times (\pm 1.96078)$ $\%\ \text{uncertainty}_{volume} = \pm 5.88235\%$
Calculate the total percentage uncertainty. As the density is found by dividing the mass by the volume, you must add the percentage uncertainties of the mass and volume.	total % uncertainty = (0.31056) + (5.88235) total % uncertainty = ± 6.19291%
Convert the total percentage uncertainty to the absolute uncertainty of the answer.	absolute uncertainty = $\pm(1.26275) \times \frac{(6.19291)}{100}$ absolute uncertainty = ± 0.078201 kg m^{-3}
State the answer to the correct number of significant figures with the absolute uncertainty to the correct decimal place and the correct units.	Therefore, the density is 1.26 ± 0.08 kg m^{-3}.

Worked example: Try yourself 1.2.1

A student wanted to investigate the electrical power drawn by a kettle. The kettle had a measured resistance of 50.0 ± 0.1 Ω and was drawing a measured current of 4.21 ± 0.01 A.

Using the formula:

$P = I^2R$

where

P = power (W)

I = current (A)

R = resistance (Ω)

calculate the power drawn by the kettle and include the uncertainty with the value.

Adding or subtracting measurements

When adding or subtracting measurements, make sure that all measurements are in the same units. To find the total uncertainty of an addition or a subtraction, add together the absolute uncertainties of the individual measurements.

Evaluating investigations using uncertainties

Uncertainties provide a useful way of evaluating the accuracy and precision of your investigation's results. They indicate the potential variability or how scattered your values are in the measurement; a low total uncertainty suggests higher precision. Additionally, you can use the uncertainty to calculate possible minimum and maximum values, which could be useful for comparing your experimental results to a known values. If the known value falls within the range of uncertainty of the experimental results, this can be an indication of accuracy.

Other descriptive statistics measures

The mean and the uncertainty are statistical measures that help describe data accurately. Other statistical measures that can be used, depending on the data obtained, are:

- **mode:** the mode is the value that appears most often in a data set. This measure is especially useful to describe qualitative or discrete data. For example, the mode of the values 0.01, 0.01, 0.02, 0.02, 0.02, 0.03, 0.04 is 0.02.
- **median:** the median is the 'middle' value of an ordered list of values. For example, the median of the values 5, 5, 8, 8, 9, 10, 20 is 8. The median is particularly useful when the data range is wide or includes outliers (unusual results), which can make the mean less reliable.

Graphs

In general, tables provide more detailed data than graphs, but it is easier to observe trends and patterns in data in graphical form than in tabular form.

Graphs are used when two variables are being considered and one variable is dependent on the other. The graph shows the relationship between the variables.

Several types of graphs can be used, including line graphs, bar graphs and pie charts. The best one to use will depend on the nature of the data.

General rules to follow when making a graph (Figure 1.2.5) include the following:

- Keep the graph simple and uncluttered.
- Use a descriptive title that contains the independent and dependent variables.
- Represent the independent variable on the *x*-axis and the dependent variable on the *y*-axis.
- Make axes proportionate to the data.
- Clearly label axes with both the variable and the unit in which it is measured.

- Include error bars showing the uncertainty of each point. The error bar should extend above and below the plotted point by the uncertainty in the dependent variable and to the left and right of the plotted point by the uncertainty of the independent variable.
- If the *y*-axis intercept is important in your investigation, then the *x*-axis must not be broken (e.g. interrupted or truncated).
- Do not force the line of best fit through the origin of the graph (0, 0). If the line of best fit does not pass through the origin, it may indicate the presence of systematic errors in your investigation.

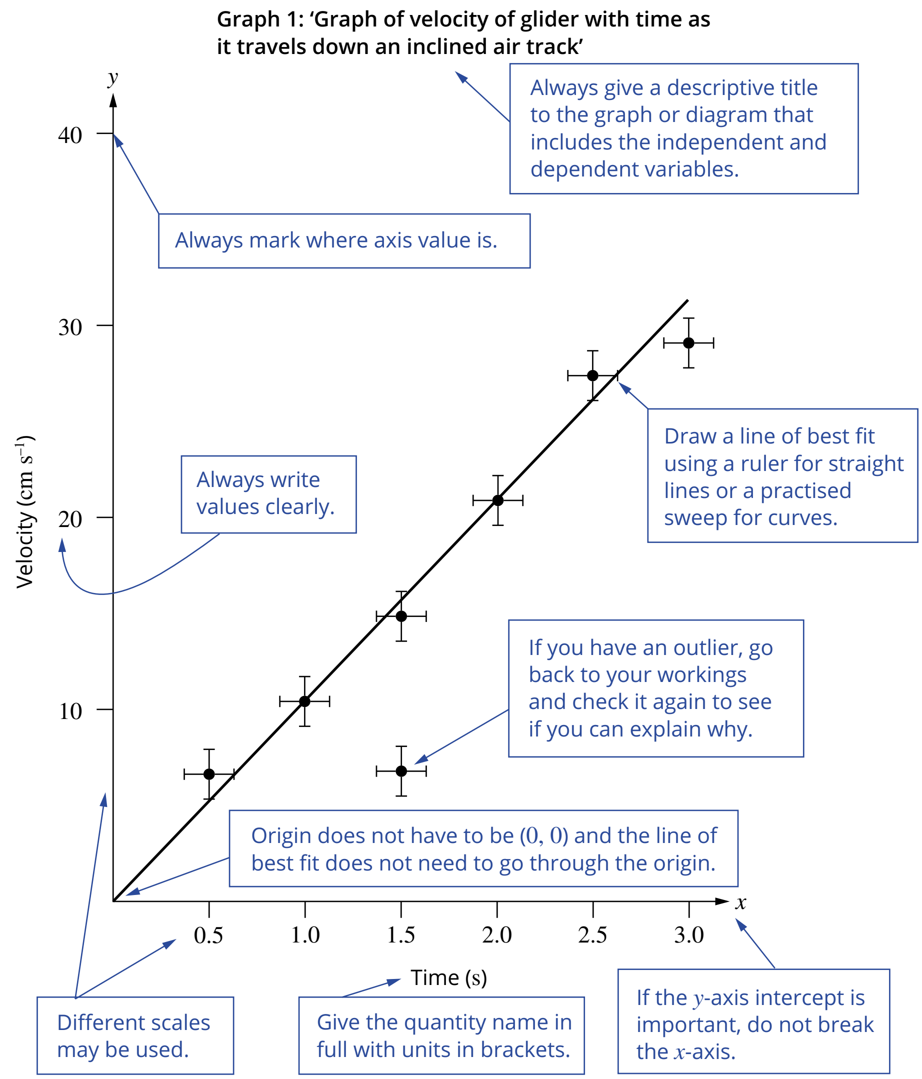


FIGURE 1.2.5 A graph shows the relationship between two variables.

Line graphs

Line graphs are a good way of representing continuous quantitative data. In a line graph, the values are plotted as a series of points with error bars on the graph. There are two ways of joining these points:

- A line can be ruled from each point to the next (Figure 1.2.6a). It shows the overall trend; it is not meant to predict the value of the points between the plotted data. These graphs are rarely used in physics.

- The points can be joined with a single smooth straight or curved line (Figure 1.2.6b). This creates a trend line, also known as a line of best fit or a curve of best fit. The line of best fit does not have to pass through every point but should go through as many error bars as possible. It is used when there is an obvious trend between the variables. These graphs are most commonly used in physics.

Outliers

Sometimes when the data is collected, there may be one point that does not fit with the trend and is clearly an error. This is called an **outlier**. An outlier is often caused by a mistake made in measuring or recording data, or from a random error in the measuring equipment. If there is an outlier, include it on the graph, but ignore it when adding a line of best fit (as in Figure 1.2.5, where the point (1.50, 6) is an outlier).

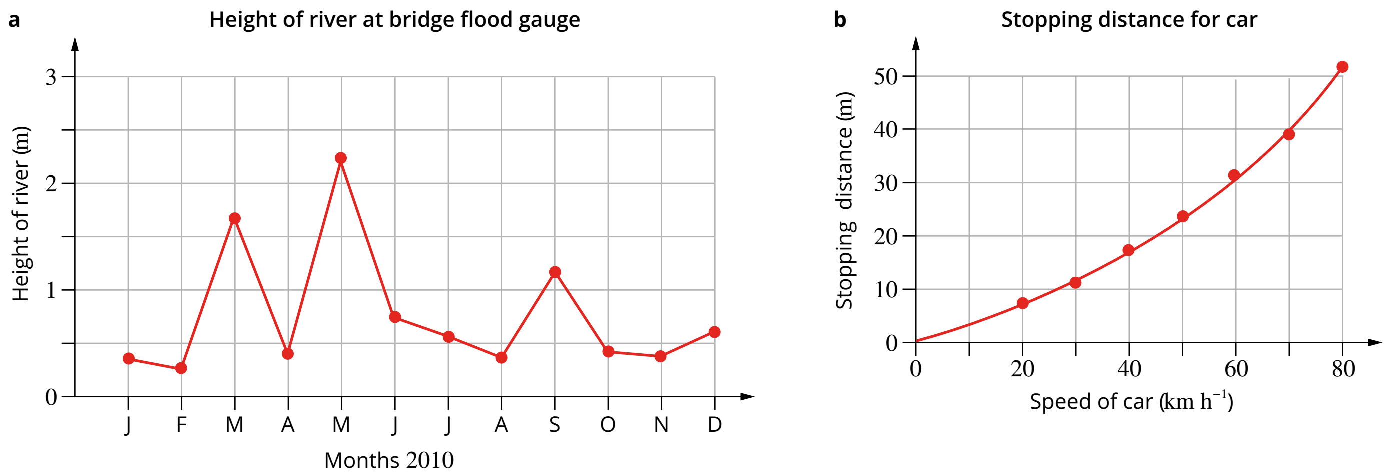


FIGURE 1.2.6 (a) The data in the graph is joined from point to point. (b) The data in graph is joined with a line of best fit, which shows the general trend.

1.2 Review

SUMMARY

- It is essential that during the investigation, the following are recorded:
  - all quantitative data collected
  - the methods used to collect the data
  - any incident, feature or unexpected event that may have affected the quality or validity of the data.
- A systematic error is an error that is consistent and will occur again if the investigation is repeated in the same way. Systematic errors are usually a result of instruments that are not calibrated correctly or methods that are flawed.
- Random errors occur in an unpredictable manner and are generally small. A random error could be, for example, the small fluctuations in the ambient temperature affecting a heat transfer investigation.
- The number of significant figures used depends on the scale of the instrument used. It is important to record data to the number of significant figures available from the equipment or observation.
- The simplest form of a table is a five-column format in which the first column contains the independent variable (the one being changed), the second to fourth columns contain the trials of the dependent variable (the one that may change in response to a change in the independent variable) and the final column contains the average of the trials.
- When there is a range of measurements of a particular value, the average must be accompanied by the uncertainty of averages. These uncertainties can be represented as absolute values or as a percentage of the measurement (percentage uncertainty).

1.2 Review *continued*

- When processing data, you must combine uncertainties:
  - If a measurement is raised to a power, the percentage uncertainty must be multiplied by the power value, i.e. if a measurement is squared, the percentage uncertainty is multiplied by two.
  - If measurements are multiplied or divided, the percentage uncertainties must be added together.
  - If measurements are added or subtracted, the absolute uncertainties must be added together.
- General rules to follow when making a graph include the following:
  - Keep the graph simple and uncluttered.
  - Use a descriptive title that contains the independent and dependent variables.
  - Represent the independent variable on the *x*-axis and the dependent variable on the *y*-axis.
  - Make axes proportionate to the data.
  - Clearly label axes with both the variable and the unit in which it is measured.
  - Include error bars showing the uncertainty of each point. The error bar should extend above and below the plotted point by the uncertainty in the dependent variable and to the left and right of the plotted point by the uncertainty of the independent variable.
  - If the *y*-axis intercept is important in your investigation, the *x*-axis must not be broken.
  - Do not force the line of best fit through the origin of the graph (0, 0). If the line of best fit does not pass through the origin, it can indicate that there may be systematic errors in your investigation.

KEY QUESTIONS

1 The masses of $1.00\,cm^3$ cubes of potato were recorded and the cubes placed in distilled water. After 60 minutes, the cubes were weighed again and the difference in mass was calculated. What type of error is involved:

a if the electronic scales were not tared properly?

b if the electronic scales were affected briefly by a power surge?

2 If using the quantities mass = 7.505 kg and speed = $1.40\,m\,s^{-1}$ in a calculation, what would be the appropriate number of significant figures in the answer?

3 For the data set 21.0, 28.0, 19.0, 19.0, 25.0, 24.0, determine:

a the mean

b the mode

c the median

d the uncertainty of averages.

4 Plot the following data set with error bars, assigning each variable to the appropriate axis on the graph.

Potential difference ± 5.00% (V)	Current ± 0.01 (A)
2.07	0.06
1.56	0.05
1.24	0.04
0.93	0.03
0.63	0.02

5 How can the general pattern (trend) of the data set in Question 4 be represented once the points are plotted?

6 Compare the error bars for the current in Question 4 to the error bars for the potential difference.

1.3 Discussing investigations and drawing evidence-based conclusions

Now that the chosen topic has been thoroughly researched and the investigation has been conducted and data collected, it is time to draw it all together. The final part of the investigation involves summarising the findings in an objective, clear and concise manner.

FIGURE 1.3.1 To discuss and conclude your investigation, use the raw and processed data.

EXPLAINING RESULTS IN THE DISCUSSION

The discussion is the part of the investigation where the evaluation and explanation of the investigation methods and results takes place. It is the interpretation of what the results mean.

The key sections of the discussion are:

- analysing and evaluating data
- evaluating the investigative method
- explaining the link between investigation findings and relevant physics concepts.

When writing the discussion, consider the message you want to convey to the audience. Statements should be clear and concise. By the time the discussion concludes, the audience must have a clear idea of the context, results and implications of the investigation.

ANALYSING AND EVALUATING DATA

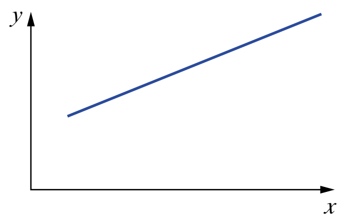
In the discussion, the findings of the investigation need to be analysed and interpreted.

- State whether a pattern, trend or relationship was observed between the independent and dependent variables. Describe what kind of pattern it was and specify the conditions under which it was observed.
- Were there discrepancies, deviations or anomalies in the data? If so, these should be acknowledged and explained.
- Identify any limitations in the data you have collected. Consider whether a larger sample size or further variations in the independent variable would lead to a stronger conclusion.

Trends in line graphs

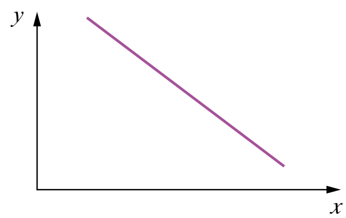
Graphs are drawn to show the relationship, or trend, between two variables, as shown in Figure 1.3.2.

- Variables that change in linear or direct proportion to each other produce a straight sloping trend line.
- Variables that change exponentially in proportion to each other produce a curved trend line.
- With an inverse relationship, one variable increases as the other variable decreases.
- When there is no relationship between two variables, one variable will not change even if the other changes.



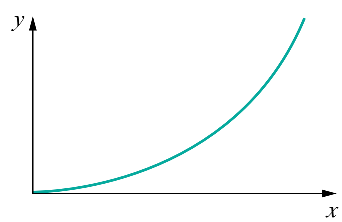
Positive directly proportional (or positive linear) relationship

- Variables change at the same rate (graph line is straight, slope is constant).
- Positive relationship as x increases, y increases.



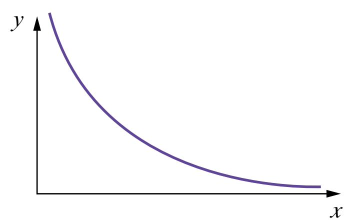
Negative directly proportional (or negative linear) relationship

- Variables change at the same rate (graph line is straight, slope is constant).
- Negative relationship as x increases, y decreases.



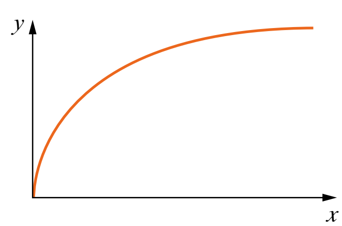
Exponential relationship

- As x increases, y increases slowly, then more rapidly.



Inversely proportional relationship

- As x increases, y decreases rapidly at first, then the rate of decrease slows down, approaching a value for y.



Logarithmic relationship, then levels off or plateaus (stops rising)

- As x increases, y increases rapidly at first, then slows, then does not increase at all as y reaches a maximum value.

No relationship between x and y

- As x increases, y remains the same.

FIGURE 1.3.2 Various relationships can exist between two variables.

Remember that the results may be unexpected. This does not make the investigation a failure. However, the findings must be related to the hypothesis, aims and method.

EVALUATING THE METHOD

It is important to discuss the limitations of the investigation method. Evaluate the method and identify any issues that could have affected the validity, accuracy, precision or reliability of the data. Sources of errors and uncertainty must also be evaluated in the discussion.

Once any limitations or problems in the methodology have been identified, recommend improvements on how the investigation could be conducted if repeated; for example, suggest how bias could be minimised or eliminated.

Bias

Bias may occur in any part of the investigation method, including sampling and measurements.

Bias is a form of systematic error resulting from the researcher's personal preferences or motivations. There are many types of bias, including:

- poor definitions of both concepts and variables (e.g. classifying cricket pitch surfaces as slow or fast without defining 'slow' and 'fast')
- incorrect assumptions (e.g. assuming that footwear type, model and manufacturer do not affect ground reaction forces, and as a result failing to control this variable during an investigation on slip risk on different indoor and outdoor surfaces)
- errors in the investigation design and methodology (e.g. taking a sample of a particular group of athletes that includes one gender more than the other in the group).

Some biases cannot be eliminated, but should still be addressed in the discussion.

Accuracy and precision

In the discussion, evaluate the degree of accuracy and precision of the measurements for each variable of the hypothesis. Address the uncertainties associated with these measurements and discuss their impact on the results.

When applicable compare the chosen method to alternative methods that could have been used, evaluating the advantages and disadvantages of the selected method and the effect on the results.

If your investigation involves comparing your results to a known accepted value, you should calculate a percentage difference. The percentage difference can be calculated by using the following formula:

$$\text{percentage difference} = \frac{\text{accepted value} - \text{experimental value}}{\text{accepted value}} \times 100$$

A smaller percentage difference indicates a more accurate result. For example, a percentage difference of less than 5% suggests that the difference between the accepted value and your experimental value is not significant.

Reliability

When discussing the results, indicate the range of the data obtained from replicates. Explain how the sample size was selected. Larger samples are usually more reliable, but time and resources might have been scarce. Discuss whether the results of the investigation have been limited by the sample size.

The control group is important to the reliability of the investigation. A control group helps identify whether an uncontrolled variable has been overlooked and may explain any unexpected results.

Error

Discuss any source of systematic or random error and suggest ways of improving the investigation.

FIGURE 1.3.3 Honest evaluation and reflection play important roles in analysing methodology.

DISCUSSING RELEVANT PHYSICS CONCEPTS

To make the investigation more meaningful, it should be explained within the right context; i.e. using related physics ideas, concepts, theories and models. Within this context, explain the basis for the hypothesis.

For example, if studying the impact of temperature on linear strain of a material (e.g. a rubber band), some of the contextual information to include in the discussion could be:

- the definition of linear strain
- the functions of linear strain
- the relationship between linear strain and temperature
- definitions of material behaviour (such as plastic and elastic)
- factors known to affect linear strain
- existing knowledge on the role of temperature on linear strain
- ranges of temperatures investigated and the reason they were chosen
- materials studied and the reasons for this choice
- methods of measuring the linear strain of a material.

Relating your findings to a physics concept

Once a context is established, you can use this as a framework to discuss whether the data supported or refuted the hypothesis. Ask questions such as:

- Was the hypothesis supported?
- Has the hypothesis been fully addressed? (If not, give an explanation of why this is so and suggest what could be done to either improve or complement the investigation.)
- Do the results contradict the hypothesis? If so, why? (The explanation must be plausible and must be based on the results and previous evidence.)

Providing a theoretical context also enables comparison of the results with existing relevant research and knowledge. After identifying the major findings of the investigation, ask questions such as:

- How does the data fit with the literature?
- Does the data contradict the literature?
- Do the findings fill a gap in the literature?
- Do the findings lead to further questions?
- Can the findings be extended to another situation?

Be sure to discuss the broader implications of the findings. Implications are the bigger picture. Outlining them for the audience is an important part of the investigation. Ask questions such as:

- Do the findings contribute to or impact the existing literature and knowledge of the topic?
- Are there any practical applications for the findings?

DRAWING EVIDENCE-BASED CONCLUSIONS

A conclusion is usually a paragraph that links the collected evidence to the hypothesis and provides a justified response to the research question.

Indicate whether the hypothesis was supported or refuted, citing the evidence that led to this conclusion. Do not provide irrelevant information. Only refer to the specifics of the hypothesis and the research question, and do not make generalisations.

Read the examples given for the following hypothesis and research question.

Hypothesis: An increase in temperature will cause an increase in linear deformation (change in length) before failure.

- Poor response to the hypothesis: Linear deformation has value y_1 at temperature 1 and value y_2 at temperature 2.
- Better response to the hypothesis: An increase in temperature from 1 to 2 produces an increase in linear deformation of z in the rubber band.

Research question: Does temperature affect the maximum linear deformation the material can withstand?

- Poor response to the research question: The results show that temperature does affect the maximum deformation of a material.
- Better response to the research question: Analysis of the results indicates that increasing the temperature from 1 to 2 in the rubber band correlates with an increase in the maximum linear deformation, which is consistent with existing knowledge on how temperature influences material deformation.

REFERENCES AND ACKNOWLEDGEMENTS

All the quotations, documents, publications and ideas used in the investigation need to be acknowledged in the 'references and acknowledgements' section to avoid plagiarism and to ensure authors are credited for their work. References and acknowledgements also give credibility to the study and allow the audience to locate information sources should they wish to study the topic further.

When referencing a book, include, in this order:

- author's surname and initials
- date of publication
- title
- publisher's name
- place of publication.

For example: Moran G. et al. (2024), *Pearson Physics 11*, Pearson Education, Melbourne, Victoria.

When referencing a website, include, in this order:

- author's surname and initials, or name of organisation, or title
- year website was written or last revised
- title of webpage
- date website was accessed
- website address.

For example: Wheeling Jesuit University/Center for Educational Technologies (2013), *NASA Physics Online Course: Forces and Motion*, accessed 16 June 2015, from http://nasaphysics.cet.edu/forces-and-motion.html.

1.3 Review

SUMMARY

- The discussion is the part of the investigation where the evaluation and explanation of the investigation methods and results takes place. It is the interpretation of what the results mean.
- In the discussion, the findings of the investigation need to be analysed and interpreted.
  - State whether a pattern, trend or relationship was observed between the independent and dependent variables. Describe what kind of pattern it was and specify under what conditions it was observed.
  - Were there discrepancies, deviations or anomalies in the data? If so, these should be acknowledged and explained.
  - Identify any limitations in the data collected. Perhaps a larger sample or further variations in the independent variable would lead to a stronger conclusion.
- It is important to discuss the limitations of the investigation method. Evaluate the method and identify any issues that could have affected the validity, accuracy, precision or reliability of the data. Sources of errors and uncertainty must also be stated in the discussion, and suggestions could be given as to how to reduce these errors.

1.3 Review *continued*

- When discussing the results, indicate the range of the data obtained from replicates. Explain how the sample size was selected. Larger samples are usually more reliable, but time and resources are likely to have been scarce. Discuss whether the results of the investigation have been limited by the sample size.
- To make the investigation more meaningful, it should be explained within the right context, including the related physics ideas, concepts, theories and models. Within this context, explain the basis for the hypothesis.
- Indicate whether the hypothesis was supported or refuted and the evidence that led to this conclusion. Do not provide irrelevant information or make generalisations.

KEY QUESTIONS

1 What relationship between the variables is indicated by a sloping linear graph?

2 What relationship exists if one variable decreases as the other increases?

3 What relationship exists if both variables increase or both decrease at the same rate?

4 What might cause a sample size to be limited in an investigation?

5 Consider this investigation hypothesis: An increase in the potential difference across a single resistor in an electric circuit will cause an increase in the current through the resistor.
Improve this hypothesis given the data below:
When the current was 0.03 A, the voltage was 0.93 V and when the current was 0.04 A, the voltage was 1.86 V.

Chapter review

01

KEY TERMS

absolute uncertainty
accuracy
controlled variable
dependent variable
independent variable
mean
median
mode
outlier
personal protective equipment (PPE)
precision
qualitative variable
quantitative variable
random error
raw data
reliability
significant figures
systematic error
uncertainty
uncertainty of averages
uncertainty of measurement
validity
variable

1 What is a hypothesis and what form does it take?

2 Consider the hypothesis provided below. What are the dependent, independent and controlled variables?
Hypothesis: Releasing an arrow in archery at an angle greater or smaller than 45° will result in a shorter flight displacement (range).

3 What is the dependent variable in each of the following hypotheses?

- **a** If you push an object with a fixed mass (e.g. shot-put) with a larger force, then the acceleration of that object will be greater.
- **b** As the vertical displacement of a falling object increases, the vertical acceleration of the falling object is constant.
- **c** A springboard diver rotates faster when in a tucked position than when in a stretched (layout) position.

4 List the following types of hazard controls from the most effective to the least effective.
substitution, personal protective equipment, engineering controls, administrative controls, elimination, isolation

5 The speed of a toy car rolling down an inclined plane was measured six times. The measurements obtained (in cm s^{-1}) were 7.0, 6.5, 6.8, 7.2, 6.5, 6.5.
What is the uncertainty of the average of these values?

6 Which of the statistical measurements of mean, mode and median is most affected by an outlier?

7 What relationship between variables is indicated by a curved trend line?

8 If you hypothesise that impact force is positive directly proportional to drop height, what would you expect a graph of the data to look like?

9 What is meant by the 'limitations' of the investigation method?

10 What is 'bias' in an investigation?

UNIT 1 Motion, forces and energy

An understanding of motion, forces, and mechanical and thermal energy is essential to appreciating how we do work, and what drives our global energy needs. In this unit, students explore the ways physics is used to describe, explain and predict how forces cause changes in motion. Students investigate how to model accelerated motion using equations, consider how Newton's laws explain the effect of forces on objects, and understand why objects fall at the same rate. Students also reflect on how energy transfers and transformations are pivotal to modern industrial societies. They explore how energy is conserved and how thermal energy affects the state and temperature of matter.

Contexts that students may investigate include technologies, such as accelerometers, motion sensors and air conditioners, as well as related areas of science and engineering, such as sports science, automobile engineering and road safety.

By the end of this unit, students will:

- develop an understanding of motion, forces, and mechanical and thermal energy
- describe linear motion in terms of position and time data
- examine the relationships between force, momentum and energy for interactions in one dimension
- explore how international collaboration, evidence from a range of disciplines and individuals, and the development of ICT and other technologies have contributed to developing understanding of motion, forces, mechanical and thermal energy, and associated technologies
- investigate how scientific knowledge is used to offer valid explanations and reliable predictions, and the ways in which it interacts with social, economic, cultural and ethical factors
- develop their understanding of motion, forces, and mechanical and thermal energy phenomena through laboratory experiences
- develop skills in relating graphical representations of data to quantitative relationships between variables
- continue to develop skills in planning, conducting, analysing and interpreting the results of primary and secondary investigations.

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.

CHAPTER 02

Scalars and vectors

Scalars and vectors are mathematical representations of quantities that are used in physics. An understanding of scalars and vectors is essential to learning concepts involving forces and motion.

By the end of this chapter, you will be able to distinguish between scalar and vector quantities. You will be able to use arrows to represent vectors and then add and subtract vectors in one and two dimensions. You will also be able to resolve vectors into their perpendicular components.

Science Understanding

Motion and forces

- distinguish between vector and scalar quantities
- addition and subtraction of vectors in one and two dimensions
- resolution of vectors into components and manipulation (addition and subtraction) of components

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.

2.1 Scalars and vectors

You come into contact with many physical quantities in the natural world every day. Time, mass and distance are all physical quantities. Each of these physical quantities has units to measure them; for example, seconds, kilograms and metres.

Some measurements only make sense if there is also a direction included. For example, a GPS system tells you when to turn and in which direction. Without both of these two instructions, the information is incomplete.

All physical quantities can be divided into two broad groups based on what information you need for the quantity to make logical sense. These groups are called **scalars** and **vectors**. Both of these types of measures will be investigated throughout this section.

SCALARS

There are a number of properties in nature that can be measured or determined, and described using only a number and a unit. For example, if the time taken for a student to travel to school is measured, you need the **magnitude** (size) and the **units** in order to understand the journey. It may take 90 minutes or one and a half hours, the number is important and the units are also important.

Quantities that require magnitude and units are called scalar quantities. Scalars do not need a direction to make logical sense.

Examples of scalar quantities are:

- time
- distance
- volume
- speed
- temperature.

VECTORS

A vector is a physical quantity that requires magnitude, direction and units to make logical sense.

Some physical quantities require magnitude, direction and units to make sense of them. For example, to describe a force applied on an object, you would need to state the magnitude, the direction of the force and the units to fully convey the information to a classmate. Such quantities are known as vectors.

Examples of vectors include:

- position
- displacement
- velocity
- acceleration
- force
- momentum.

These measures are discussed in more detail in the coming chapters.

VECTORS AS ARROWS

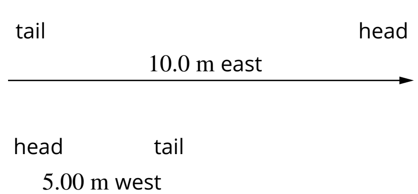


FIGURE 2.1.1 Two vector diagrams. As the top vector is twice as long as the bottom vector, it represents a measure twice the magnitude of the bottom vector. The arrowheads indicate that the vectors are in opposite directions.

A vector is a measurement that has a magnitude, a direction and a unit. A vector can be visually represented as an arrow.

Figure 2.1.1 shows two vector diagrams. In a **vector diagram**, the length of the arrow indicates the magnitude of the vector. The arrowhead shows the direction of the vector. The direction of the vector is always from its tail to its head.

A force is a push or a pull and the unit of measurement for force is the newton (N). If you push a book to the right, it will respond differently to if you push the book to the left. Therefore, a force is only described properly if a direction is included, and so force is considered to be a vector. Forces are described in more detail in Chapter 4. Force is an important concept to understand in physics, so many of the examples in this chapter refer to forces.

In most vector diagrams, the length of the arrow is drawn to scale so that it accurately represents the magnitude of the vector.

In the scaled vector diagram in Figure 2.1.2, a force $F = 4.00$ N to the left acting on the toy car is drawn as a 2.00 cm long arrow pointing to the left. In this example, a scale of 1 cm = 2 N force is used.

An exact scale for the magnitude is not always used. However, it is important that vectors are drawn relative to one another. For example, a vector of 50.0 m north should always be about half as long as a vector of 100.0 m north.

FIGURE 2.1.2 A force of 4.00 N to the left acts on a toy car.

Point of application of vectors

Vector diagrams may be presented slightly differently depending on what they are depicting. If the vector represents a force, the tail end of the arrow is drawn at the point where the force is applied to the object. If it is a displacement vector, attach the tail of the arrow to the position where the object starts. Friction vectors are drawn at the point where they act between an object and a surface.

Figure 2.1.3 shows a force applied by a foot to a ball (95.0 N east) and an opposing friction force (20.0 N west).

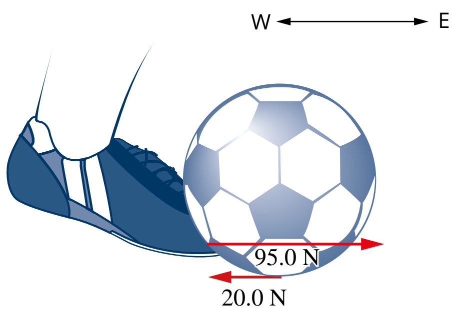


FIGURE 2.1.3 The force on the ball acts at the point of contact between the ball and the foot. The friction force acts between the ball and the ground. The kicking force, as indicated by the length of the arrow, is larger than the friction force.

DIRECTION CONVENTIONS

Vectors need a direction in order to make logical sense. However, for the description of a vector quantity to be useful, there needs to be a way of describing the direction so that everyone understands and agrees upon it.

Vectors in one dimension

For vector problems in one **dimension**, there are a number of **direction conventions** that can be used. For example:

- forwards or backwards
- up or down
- left or right.

You can also use more formal conventions such as:

- north or south
- east or west.

As you can see, for vectors in one dimension there are only two directions possible. The two directions must be in the same dimension or along the same line. The direction convention used should be presented graphically in all vector problems. This is shown in Figure 2.1.4. Arrows like these are placed near the vector diagram so that it is clear which convention is being used.

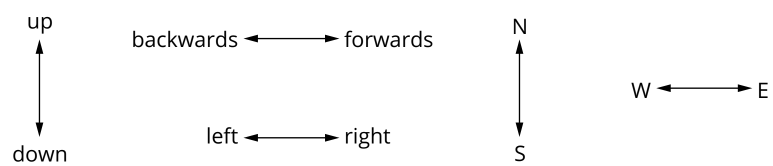


FIGURE 2.1.4 Some common one-dimensional direction conventions.

Sign convention

In calculations involving one-dimensional vectors, a sign convention can also be used to convert physical directions to the mathematical signs of positive and negative. For example, forwards can be positive and backwards can be negative, or right can be positive and left can be negative. A vector of 100 m up can be described as +100 m, provided the relationship between sign and direction conventions are clearly indicated in a legend or key. Some examples are provided in Figure 2.1.5.

The advantage of using a sign convention is that the signs of positive and negative can be entered into a calculator, while the words 'up' and 'right' cannot. This is useful when adding vectors together. This will be discussed in the next section.

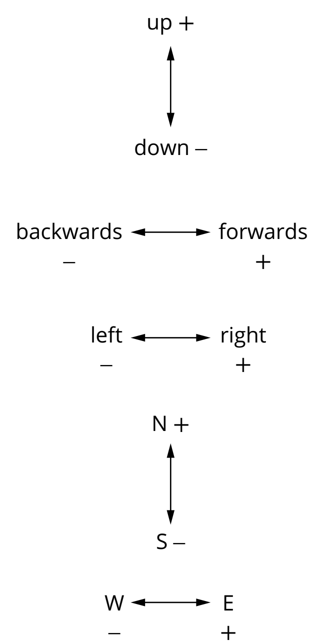
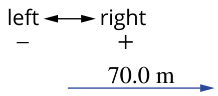


FIGURE 2.1.5 One-dimensional direction conventions can also be expressed as sign conventions.

Worked example 2.1.1

DESCRIBING VECTORS IN ONE DIMENSION



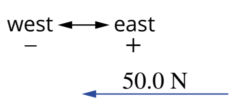
Describe the vector above using:

a the direction convention shown	
Thinking	**Working**
Identify the direction convention being used in the vector.	In this case, the vector is pointing to the right according to the direction convention.
Note the magnitude, unit and direction of the vector.	In this example, the vector is 70.0 m right.

b an appropriate sign convention.	
Thinking	**Working**
Convert the physical direction to the corresponding mathematical sign.	The physical direction of right is positive and left is negative. In this example, the arrow is pointing right, so the mathematical sign is +.
Represent the vector with a mathematical sign, magnitude and unit.	This vector is +70.0 m.

Worked example: Try yourself 2.1.1

DESCRIBING VECTORS IN ONE DIMENSION



Describe the vector above using:

a the direction convention shown

b an appropriate sign convention.

Vectors in two dimensions

When vectors are in one dimension, it is relatively simple to understand direction. However, some vectors will require a description in a two-dimensional plane. These planes could be:

- horizontal, which can be defined using north, south, east and west
- vertical, which can be defined in a number of ways including forwards, backwards, up, down, left and right.

The description of the direction of these vectors is more complicated. Therefore, a more detailed convention is needed for identifying the direction of a vector. There are a variety of conventions, but they all describe a direction as an angle from a known reference point.

Horizontal plane

For a horizontal, two-dimensional plane, the two common methods for describing the direction of a vector are:

- full circle (or true) bearing. A 'full circle bearing' describes north as zero degrees true. This is written as 0°T. In this convention, all directions are given as a clockwise angle from north. As an example, 95.0°T is 95.0° clockwise from north.
- quadrant bearing. An alternative method is to provide a 'quadrant bearing', where all angles are referenced from either north or south and are between 0° and 90.0° towards east or west. In this method, 30.0° T becomes N30.0°E, which can be read as 'from north 30.0° towards the east'.

Using these two conventions, north-west (NW) would be 315.0° T using a full circle bearing, or N 45.0°W using a quadrant bearing. Figure 2.1.6 demonstrates these two methods.

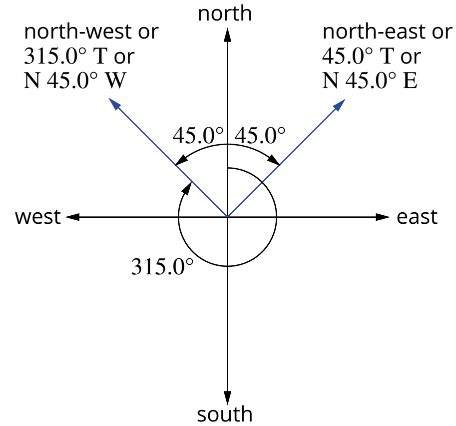


FIGURE 2.1.6 Two horizontal vector directions, viewed from above, using full circle bearings and quadrant bearings.

Vertical plane

For a vertical, two-dimensional plane the directions are referenced to vertical (upwards and downwards) or horizontal (left and right) and are between 0° and 90.0° up or down. For example, a vector direction can be described as '60.0° up from horizontal to the left'. The same vector direction could be described as '30.0° down from the vertical to the left'. The opposite direction to this vector would be '60.0° down from horizontal to the right'. This example is illustrated in Figure 2.1.7.

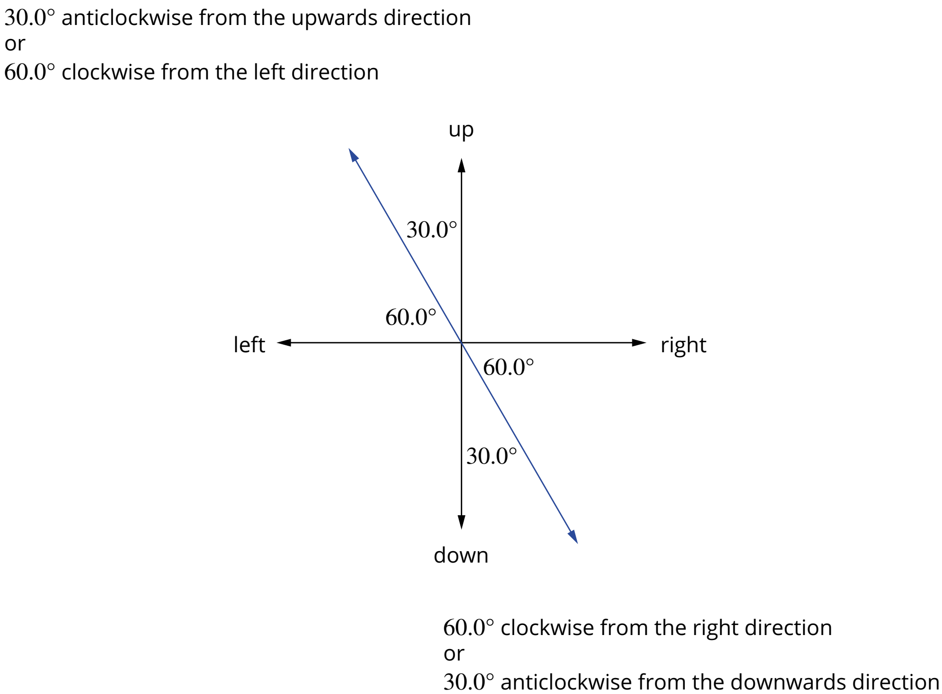


FIGURE 2.1.7 Two vectors in the vertical plane.

PHYSICSFILE

Orienteering

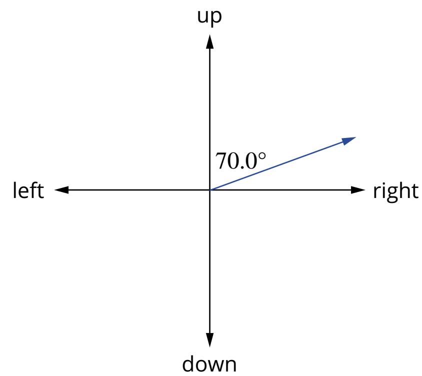
Orienteering is an adventure sport requiring participants to use a compass and map (Figure 2.1.8) to navigate from point to point. Finding your way from place to place involves determining angles measured from any of the cardinal points: north, south, east or west. The vectors between points are shown on the map but contestants need to determine the best way to get there, since the direct line might not be the fastest route. This sport is often timed and competitive and takes place in an unfamiliar, and sometimes challenging, environment. These courses can be followed individually or in teams. Not all orienteering courses have to be completed on foot, as some courses might be designed for mountain bikers or cross-country skiers depending on the environment. There are some permanent orienteering courses in popular spaces in WA such as Whiteman's Park and King's Park, which are free of charge and suitable for any age group. In addition, there are seriously competitive groups that meet on a regular basis.

FIGURE 2.1.8 A compass and map used in orienteering. The numbered points represent the order in which the course should be followed.

Worked example 2.1.2

DESCRIBING TWO-DIMENSIONAL VECTORS

Describe the direction of the following vector using an appropriate method.

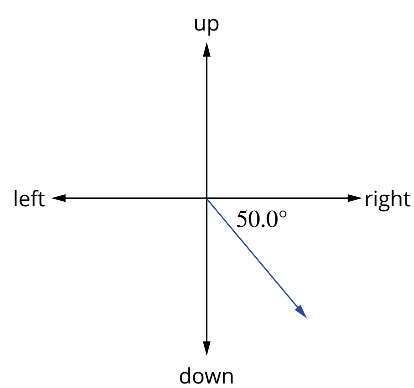


Thinking	Working
Choose the appropriate points to reference the direction of the vector. In this case using the vertical reference makes more sense, as the angle is given from the vertical.	The vector can be referenced to the vertical.
Determine the angle between the reference direction and the vector.	In this example, from vertical to the vector there is 70.0°.
Determine the direction of the vector from the reference direction.	From vertical, the vector is down to the right.
Describe the vector using the sequence: angle, up or down from the reference direction.	This vector is 70.0° down from vertical to the right.

Worked example: Try yourself 2.1.2

DESCRIBING TWO-DIMENSIONAL VECTORS

Describe the direction of the following vector using an appropriate method.



2.1 Review

SUMMARY

- Scalar quantities require a magnitude and a unit to make logical sense. No direction is required for scalar quantities.
- Distance, time, speed and mass are examples of scalar quantities.
- Vectors require magnitude, units and direction to make logical sense.
- Displacement, velocity, acceleration and force are examples of vectors.
- Arrows are used to represent vectors.
- The length of the arrow represents the magnitude of the vector.
- The direction the arrow is pointing indicates the direction of the vector.
- Vector arrows can be drawn to scale, or drawn with lengths that are relative to each other.
- Force vectors are drawn with their tails attached to the point of application of the force on the object.
- Displacement vectors are drawn from the start of the journey to the end of the journey.
- One-dimensional vectors use direction conventions and sign conventions to describe the direction of the vector. Examples include left and right, up and down, + and –.
- The direction of two-dimensional vectors in the horizontal plane can be described using a full circle bearing or a quadrant bearing. Vectors in the vertical plane can be described using angles measured up or down from the vertical or horizontal, to the right or to the left.

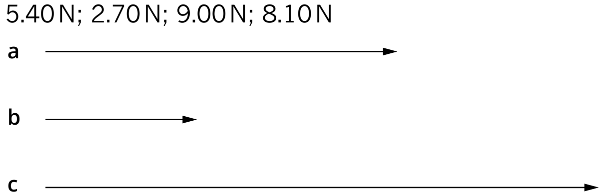
KEY QUESTIONS

1 What information is required to fully describe a scalar measure?

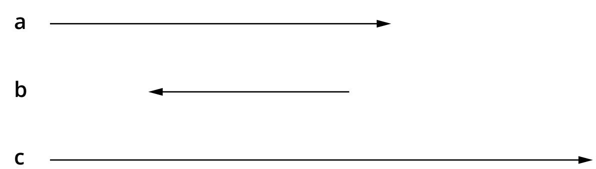
2 What information is required to fully describe a vector measure?

3 Classify each of the following as scalar or vector quantities.
  - **a** time
  - **b** force
  - **c** acceleration
  - **d** distance
  - **e** position
  - **f** displacement
  - **g** volume
  - **h** momentum
  - **i** speed
  - **j** velocity
  - **k** temperature

4 For the following, decide which of the vector magnitudes provided describes which vector diagram. Note: one of the vector magnitudes is not required.
5.40 N; 2.70 N; 9.00 N; 8.10 N



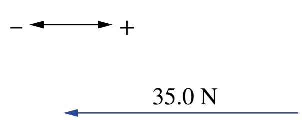
5 For the following, decide which of the vector magnitudes provided describes which vector diagram. Note: one of the vector magnitudes is not required.
10.80 N; –2.70 N; –5.40 N; 16.20 N



6 Give the opposing direction to each of the following one-dimensional descriptions.
  - **a** up
  - **b** north
  - **c** backwards
  - **d** down
  - **e** west
  - **f** negative

7 Why is it sometimes appropriate to rename direction conventions with a positive or negative sign—for example, + instead of north or – instead of left?

8 Describe the following vector using an appropriate convention.



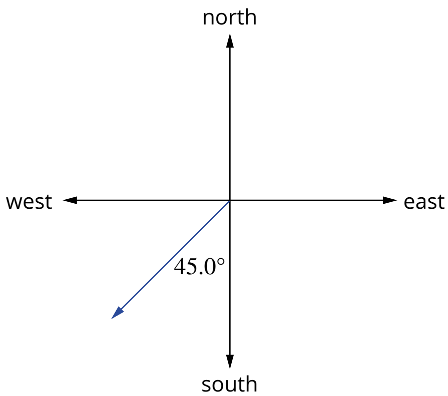
2.1 Review *continued*

9 Describe the following vectors using:

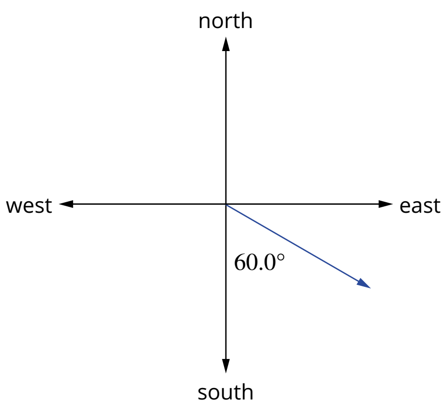
i full circle bearings

ii quadrant bearings.

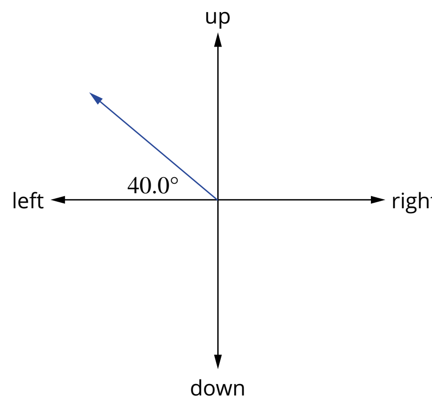
a



b



10 Describe the following vector using appropriate conventions.



2.2 Adding vectors in one and two dimensions

In real situations, more than one vector may act on an object. If this is the case, it is helpful to analyse the associated vector diagrams to find out the overall sum or combined effect of the vectors, known as the resultant vector.

When vectors are combined, it is called adding vectors. Adding vectors that are in one dimension means finding the one resultant vector that is the equivalent of any number of vectors in the same line. It is also possible to find the resultant of a number of vectors that are in two dimensions. The individual vectors can be in any direction, as long as they are all in the same plane.

Vectors can also be added in three dimensions, but this is beyond the realm of this course.

ADDING VECTORS IN ONE DIMENSION

When two or more vectors are in the same dimension, it means that the vectors are either pointing in the same direction or in the opposite direction to each other. They are **collinear** (in line with each other). For example, the displacements 10.0 m west, 15.0 m east and 25.0 m west are all in one dimension. They are all in the same or opposite direction to each other.

Graphical method of adding vectors

Vector diagrams, like those shown in Figure 2.2.1, are convenient for adding vectors. To combine vectors in one dimension, draw the first vector, then start the second vector with its tail at the head of the first vector. Continue adding arrows 'head to tail' until the last vector is drawn. The sum of the vectors, or the **resultant** vector, is drawn from the tail of the first vector to the head of the last vector.

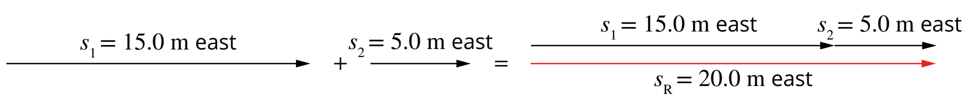


FIGURE 2.2.1 Adding vectors head to tail. This particular diagram represents the addition of 15.0 m east and 5.0 m east. The resultant vector, shown in red, is 20.0 m east.

Vectors are added head to tail. The resultant vector is drawn from the tail of the first vector to the head of the last vector.

In Figure 2.2.1, the two vectors s_1 (15.0 m east) and s_2 (5.0 m east) are drawn separately. The two vectors are then drawn with s_1 and s_2 connected head to tail. The resultant vector s_R is drawn from the tail of s_1 to the head of s_2. The magnitude (size) of the resultant vector can be deduced from the magnitudes of the separate vectors: 15.0 m + 5.0 m = 20.0 m.

Alternatively, vectors can be drawn to scale, for example: 1 m = 1 cm. The resultant vector is then directly measured from the scale diagram. The direction of the resultant vector is the same as the direction from the tail of the first vector to the head of the last vector.

Algebraic method of adding vectors

To add vectors in one dimension algebraically, a sign convention is used to represent the direction of the vectors (see Figure 2.2.2). When applying a sign convention, it is important to provide a key explaining the convention used.

The sign convention allows you to enter the signs and magnitudes of vectors into a calculator. The sign of the final magnitude gives the direction of the resultant vector.

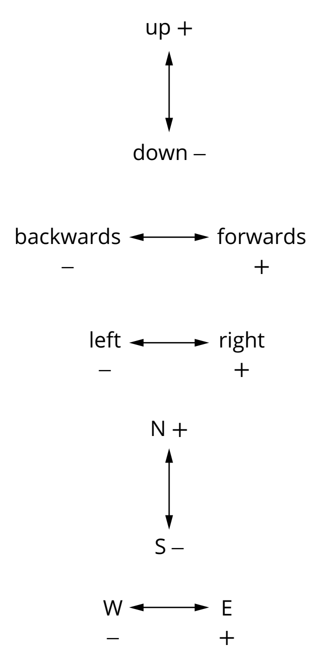


FIGURE 2.2.2 Common sign and direction conventions.

Worked example 2.2.1

ADDING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 2.2.2 on page 39 to determine the resultant vector of a student who walks 25.0 m west, 16.0 m east, 44.0 m west and then 12.0 m east.

Thinking	Working
Apply the sign conventions to change each of the directions to signs.	25.0 m west = –25.0 m 16.0 m east = +16.0 m 44.0 m west = –44.0 m 12.0 m east = +12.0 m
Add the magnitudes and their signs together.	Resultant vector = (–25.0) + (+16.0) + (–44.0) + (+12.0) = –41.0 m
Refer to the sign and direction conventions to determine the direction of the resultant vector.	Negative is west. ∴ Resultant vector = 41.0 m west

Worked example: Try yourself 2.2.1

ADDING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 2.2.2 on page 39 to determine the resultant force on a box that has the following forces acting on it: 16.0 N up, 22.0 N down, 4.0 N up and 17.0 N down.

ADDING VECTORS IN TWO DIMENSIONS

Adding vectors in two dimensions means that all of the vectors must be in the same plane (coplanar). The individual vectors can go in any direction within the plane, and can be separated by any angle. The examples in this section illustrate vectors in the horizontal plane, but the same strategies apply to adding vectors in the vertical plane.

The horizontal plane is the one that is looked down on from above. Examples include looking at a house plan or map placed on a desk. The direction convention that suits this plane best is the north, south, east and west convention, or the forwards, backwards, left and right convention. This is shown in Figure 2.2.3.

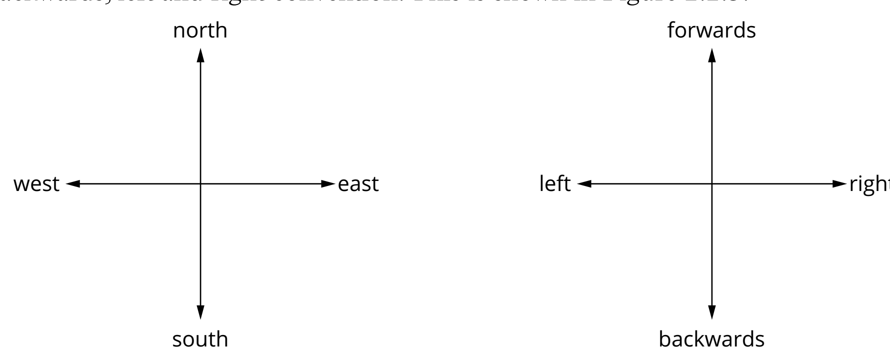


FIGURE 2.2.3 The direction conventions for the horizontal plane.

When two vectors are in the horizontal plane, the angles between them can be right-angled, acute or obtuse.

Graphical method of adding vectors

The magnitude and direction of a resultant vector can be determined by measuring an accurately drawn scaled vector diagram. There are two main ways to do this:

- head to tail method
- parallelogram method.

Head to tail method

To add vectors at right angles to each other using a graphical method, use an appropriate scale and then draw each vector head to tail. The resultant vector is the vector that starts at the tail of the first vector and ends at the head of the last vector. To determine the magnitude and direction of the resultant vector, measure the length of the resultant vector and compare it to the scale, then measure and describe the direction appropriately.

In Figure 2.2.4, two vectors, 30.0 m east and 20.0 m south, are added head to tail. The resultant vector, shown in red, is estimated to be about 36 m according to the scale provided. Using a protractor, the resultant vector is measured to be in the direction 34.0° south of east. This represents a direction of S 56.0° E when using quadrant bearings from south.

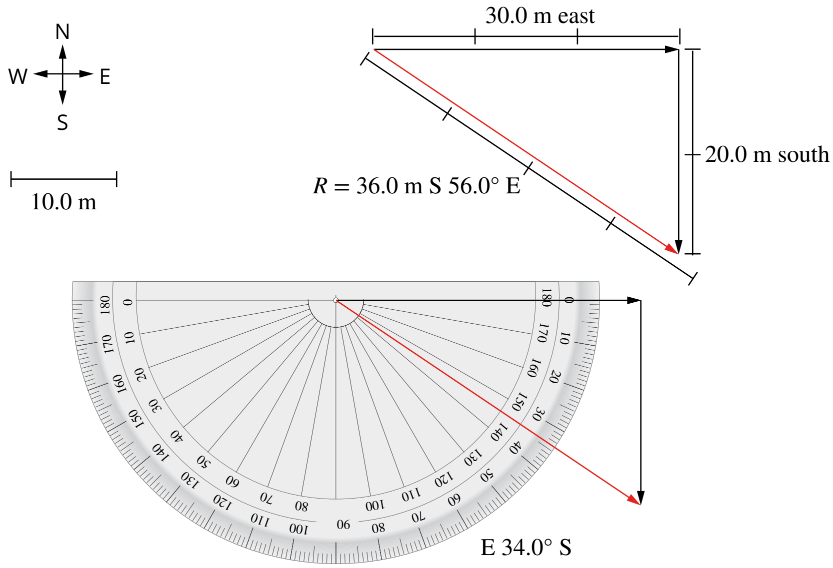


FIGURE 2.2.4 Adding two vectors at right angles, using the graphical method.

If the two vectors are at angles other than 90.0° to each other, the graphical method is ideal for finding the resultant vector. In Figure 2.2.5, the force vectors 15.0 N east and 10.0 N S 45° E are added head to tail. The magnitude of the resultant vector is estimated to be about 23 N. The direction of the resultant vector is measured by a protractor from east to be 18.0° towards the south, which should be written as E 18.0° S or S 72.0° E.

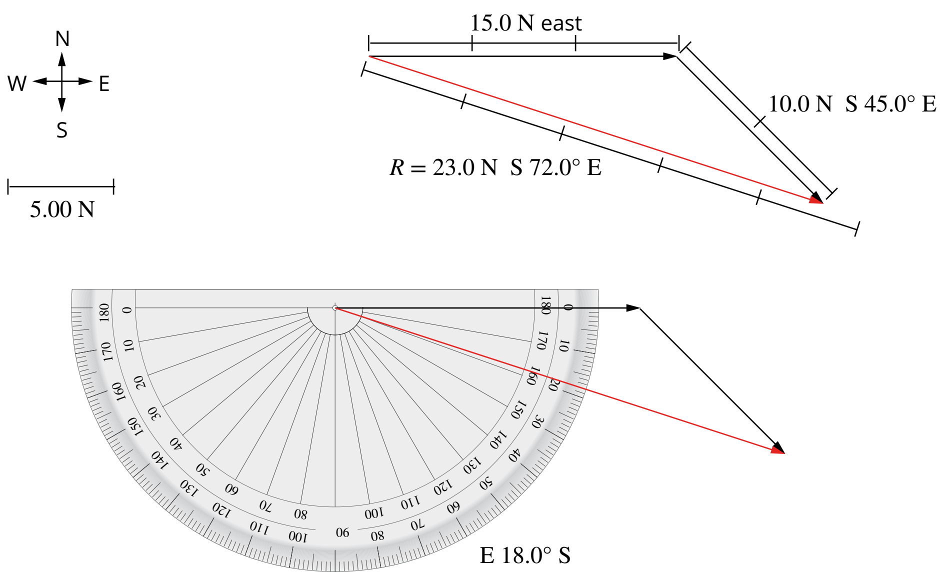


FIGURE 2.2.5 Adding two vectors not at right angles, using the graphical method.

Parallelogram method

An alternative method for determining a resultant vector is to construct a parallelogram of vectors. In this method, the two vectors to be added are drawn tail to tail. Next, a parallel line is drawn for each vector as shown in Figure 2.2.6. In this figure, the parallel lines have been drawn as dotted lines. The resultant vector is drawn from the tails of the two vectors to the intersection of the dotted parallel lines.

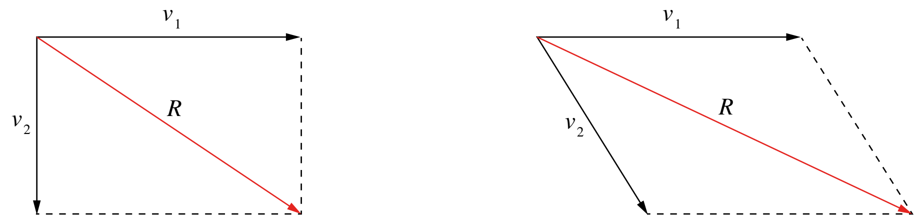
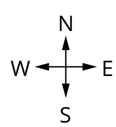


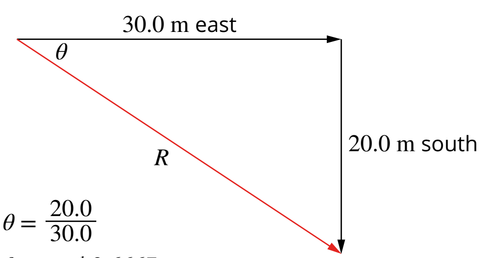
FIGURE 2.2.6 Parallelogram of vectors method for adding two vectors.

Geometric method of adding vectors

Graphical methods of adding vectors in two dimensions only give approximate results as they rely on comparing the magnitude of the resultant vector to a scale and measuring the direction with a protractor. A more accurate method to resolve vectors is to use Pythagoras' theorem and trigonometry. These techniques are referred to as geometric methods. Geometric methods can be used to calculate the magnitude of the resultant vector and its direction. However, Pythagoras' theorem and trigonometry can only be used for finding the resultant vector of two vectors that are at right angles to each other.

In Figure 2.2.7, two vectors, 30.0 m east and 20.0 m south, are added head to tail. The resultant vector, shown in red, is calculated using Pythagoras' theorem to be 36.1 m. The resultant vector is calculated to be in the direction S 56.3° E. This result is more accurate than the answer determined earlier in this section.





$$R^2 = 30.0^2 + 20.0^2$$
$$= 900 + 400$$
$$R = \sqrt{1300}$$
$$= 36.1 \text{ m}$$

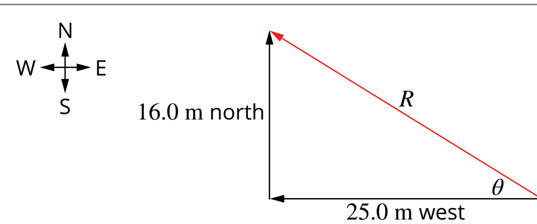
$$\tan\theta = \frac{20.0}{30.0}$$
$$\theta = \tan^{-1} 0.6667$$
$$= 33.7°$$

E 33.7° S (or S 56.3° E)

FIGURE 2.2.7 Adding two vectors at right angles, using the geometric method.

Worked example 2.2.2

ADDING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the resultant vector that represents a child running 25.0 m west and then 16.0 m north. Refer to Figure 2.2.2 on page 39 for sign and direction conventions if required.	
Thinking	**Working**
Construct a vector diagram showing the vectors drawn head to tail. Draw the resultant vector from the tail of the first vector to the head of the last vector.	

PHYSICSFILE

Pythagoras' theorem and trigonometric ratios

The Year 11 ATAR Physics syllabus assumes students will be able to:

- use Pythagoras' theorem, similarity of triangles and the angle sum of a triangle
- solve simple sine, cosine and tangent relationships in a right-angle triangle.

Pythagoras' theorem is $a^2 + b^2 = c^2$ where c is the hypotenuse (the longest side) and a and b are the two shorter sides of a right-angled triangle. The hypotenuse is easily recognised as it is directly across from (opposite) the right angle of the triangle.

Most students learn the mnemonic SOHCAHTOA in their maths classes. It is often pronounced soh-cah-toa and provides a way to remember the trigonometric ratios:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the resultant vector.	$R^2 = 25.0^2 + 16.0^2$ $= 625 + 256$ $R = \sqrt{881}$ $= 29.7\,\text{m}$
Using trigonometry, calculate the angle from the west vector to the resultant vector.	$\tan\theta = \frac{16.0}{25.0}$ $\theta = \tan^{-1} 0.640$ $= 32.6°$
Determine the direction of the vector relative to north or south.	$90.0° - 32.6° = 57.4°$ The direction is N57.4°W
State the magnitude and direction of the resultant vector.	$R = 29.7\,\text{m}$ N57.4°W

Worked example: Try yourself 2.2.2

ADDING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the resultant force when forces of 5.00 N east and 3.00 N north both act on a tree at the same time. Refer to Figure 2.2.2 on page 39 for sign and direction conventions if required.

PHYSICS IN ACTION

Surveying

Surveyors use technology to measure, analyse and manage data about the shape of the land and the exact location of landmarks and buildings. They take many measurements, including angles and distances, and use them to calculate more advanced data such as vectors, bearings, co-ordinates, elevations and maps. Surveyors typically use theodolites (see Figures 2.2.8 and 2.2.9), GPS survey equipment, laser range finders and satellite images to map the land in three dimensions.

Surveyors are often the first professionals on a building site to ensure that the boundaries of the property are correct. They also ensure that the building is built in the correct location. Surveyors must liaise closely with architects both before and during a building project as they provide position and height data for walls and floors.

FIGURE 2.2.8 Surveying the land with a theodolite.

FIGURE 2.2.9 Surveying equipment being used on a building site.

2.2 Review

SUMMARY

- Combining vectors is known as adding vectors.
- One-dimensional vector addition refers to vectors in a line, while two-dimensional vector addition refers to vectors on a plane.
- Adding vectors in one dimension can be done graphically by using vector diagrams. After adding vectors head to tail, the resultant vector can be drawn from the tail of the first vector to the head of the last vector.
- Adding vectors in one dimension can be done algebraically by applying a sign convention. Vectors with direction become vectors with either positive or negative signs.
- Adding vectors in two dimensions can be estimated graphically with a scale and a protractor.
- An alternate method of adding vectors in two dimensions is to construct a parallelogram of vectors.
- Adding vectors in two dimensions can be done more accurately using Pythagoras' theorem and the trigonometric ratios of a right-angled triangle.

KEY QUESTIONS

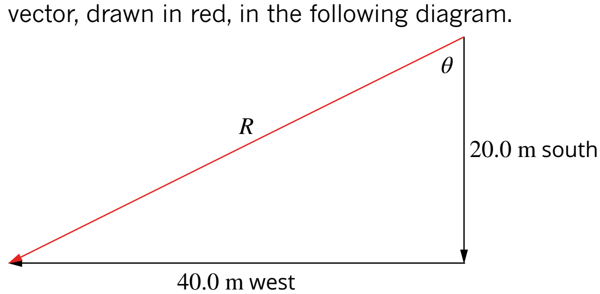
1 Every day, Monday to Friday, a man drives his car 3.00 km east to work and returns the same route home. He drives nowhere else.
 a What distance will he have driven in a week?
 b What is his displacement at the end of a week?

2 Add the following vectors to find the resultant vector: 3.00 m up, 2.00 m down and 3.00 m down.

3 Determine the resultant vector of a toy train that is made to move in these directions: 23.0 m forwards, 16.0 m backwards, 7.0 m forwards and 3.0 m backwards.

4 When adding vector B to vector A using the head to tail method, from what point, and to what point, is the resultant vector drawn?
 A from the head of A, to the tail of B
 B from the tail of B, to the head of A
 C from the head of B, to the tail of A
 D from the tail of A, to the head of B

5 Describe the magnitude and direction of the resultant vector, drawn in red, in the following diagram.



6 Forces of 2000 N north and 6000 N east act on an object. What is the resultant force?

7 Calculate the magnitude of the resultant vector when 30.0 m south and 40.0 m west are added.

8 Determine the resultant force acting on an object being pulled north by force 3000 N; south by 5000 N and east by 5000 N.

9 Determine the resultant vector of the following combination: 3350 N forwards, 6220 N backwards, 2235 N forwards and 634 N forwards.

2.3 Subtracting vectors in one and two dimensions

The previous section discussed combining or adding vectors. In physics, there are times when the difference between two vectors has to be determined. For example, a change in velocity is determined by the final velocity minus the initial velocity. In other words, you must subtract vectors. The best way to subtract one vector from another is to add the opposite vector.

SUBTRACTING VECTORS IN ONE DIMENSION

To find the difference between two vectors, you must subtract the initial vector from the final vector. To do this, work out which is the initial vector, then reverse its direction. These two vectors are then added: the final vector and the opposite of the initial vector.

This technique can be applied both graphically and algebraically.

Graphical method of subtracting vectors

Velocity is a quantity that gives an indication of how fast an object is moving in a certain direction. It is a vector because the direction is important when stating the velocity of an object. For example, the velocity of the tennis ball moving towards the racquet in Figure 2.3.1 is different from the velocity of the tennis ball as it leaves the racquet. The concept of velocity is covered in more detail in Chapter 3, but it is useful to use the example of velocity now when discussing the subtraction of vectors. The processes applied when subtracting velocity vectors work for all other vectors.

FIGURE 2.3.1 As velocity is a vector, direction is important. The tennis ball has a different velocity when it leaves the racquet from when it travelled towards the racquet.

To subtract velocity vectors in one dimension using a graphical method, determine which vector is the initial velocity and which is the final velocity. The final velocity is drawn first. The initial velocity is then drawn, but in the opposite direction to its original direction. The sum of these vectors, or the resultant vector, is drawn from the tail of the final velocity to the head of the reversed initial velocity. This resultant vector is the difference between the two velocities, or Δv.

In Figure 2.3.2, the two separate velocity vectors v_1 (9.00 m s^{-1} east) and v_2 (3.00 m s^{-1} east) are drawn separately. The initial velocity, v_1, is then drawn again in the opposite direction: $-v_1$ or 9.00 m s^{-1} west.

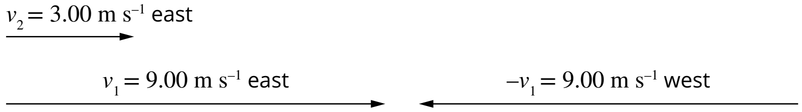


FIGURE 2.3.2 Subtracting vectors using the graphical method: the initial vectors.

PHYSICSFILE

Double negatives

When a negative number is multiplied by another negative number, the result is a positive number. A double negative is also illustrated when a negative number is subtracted from another number. The effect is to add the two numbers together. For example, (5) – (–2) = 7.

It is important to differentiate between the terms subtract, minus, take away or difference between, and the term negative. The terms subtract, minus, take away or difference between are processes, like add, multiply and divide. You will find these processes grouped together on your calculator. The term negative is a property of a number that means that it is opposite to positive. There is a separate button on your calculator (usually shown as ±) for this property.

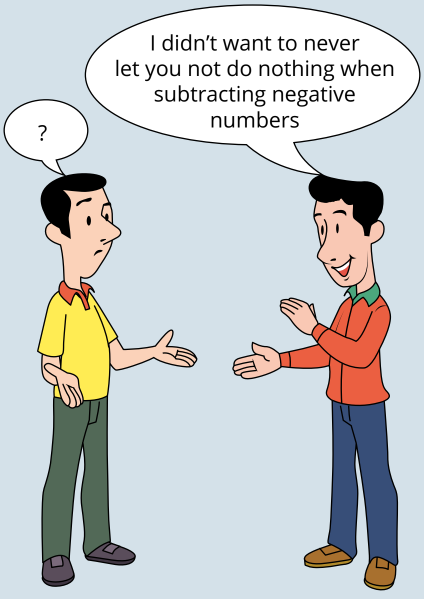


FIGURE 2.3.3 Double negatives can be confusing.

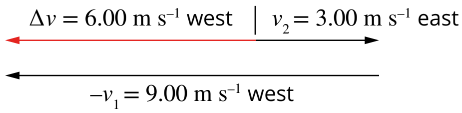


FIGURE 2.3.4 Subtracting vectors using the graphical method: the resultant vector.

> To find the difference between or change in vectors, subtract the initial vector from the final vector. Vectors are subtracted by adding the negative of a vector.

Figure 2.3.4 illustrates how the difference between the vectors is found. Firstly, the final velocity, v_2, is drawn. Then the opposite of the initial velocity, $-v_1$, is drawn head to tail. The resultant vector, Δv, is drawn from the tail of v_2 to the head of $-v_1$.

The magnitude of the resultant vector, Δv, can be calculated from the magnitudes of the two vectors. Alternatively, you could draw the vectors to scale and then measure the resultant vector against that scale—for example $1\,\text{m s}^{-1} = 1\,\text{cm}$.

The direction of the resultant vector, Δv, is the same as the direction from the tail of the final velocity, v_2, to the head of the opposite of the initial velocity, $-v_1$.

Algebraic method of subtracting vectors

To subtract velocity vectors in one dimension algebraically, a sign convention is used to represent the direction of the velocities. Some examples of one-dimensional directions include east and west, north and south, and up and down. These options are replaced by positive (+) or negative (–) signs when calculations are performed. To change the direction of the initial velocity, simply change the sign from positive to negative or from negative to positive.

The equation for finding the change in velocity is:

change in velocity = final velocity – initial velocity

$$\Delta v = v_2 - v_1$$

$$\Delta v = v_2 + (-v_1)$$

change in velocity = final velocity + the opposite of the initial velocity

The final velocity is added to the opposite of the initial velocity. Since the change in velocity is a vector, it will consist of a sign and a magnitude. The sign of the answer can be compared with the sign and direction convention (Figure 2.3.5) to determine the direction of the change in velocity.

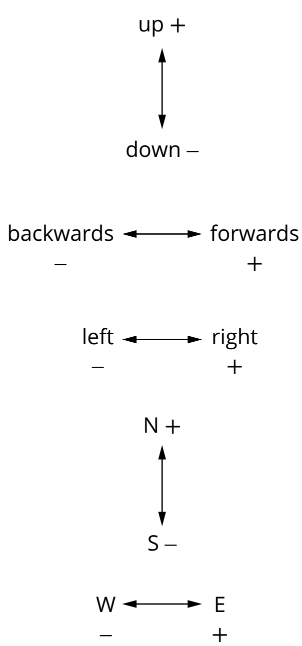


FIGURE 2.3.5 Sign and direction conventions.

Worked example 2.3.1

SUBTRACTING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 2.3.5 to determine the change in velocity of a plane as it changes from $255\,\text{m s}^{-1}$ west to $160\,\text{m s}^{-1}$ east.

Thinking	Working
Apply the sign and direction convention to change the directions to signs.	$v_1 = 255\,\text{m s}^{-1}$ west $= -255\,\text{m s}^{-1}$ $v_2 = 160\,\text{m s}^{-1}$ east $= +160\,\text{m s}^{-1}$
Reverse the direction of the initial velocity, v_1, by reversing the sign.	$-v_1 = 255\,\text{m s}^{-1}$ east $= +255\,\text{m s}^{-1}$
Use the formula for change in velocity to calculate the magnitude and the sign of Δv.	$\Delta v = v_2 + (-v_1)$ $= (+160) + (+255)$ $= +415\,\text{m s}^{-1}$
Refer to the sign and direction convention to determine the direction of the change in velocity.	Positive is east. $\therefore \Delta v = 415\,\text{m s}^{-1}$ east

Worked example: Try yourself 2.3.1

SUBTRACTING VECTORS IN ONE DIMENSION USING ALGEBRA

Use the sign and direction conventions shown in Figure 2.3.5 to determine the change in velocity of a rocket as it changes from $212\,\text{m s}^{-1}$ up to $2200\,\text{m s}^{-1}$ up.

SUBTRACTING VECTORS IN TWO DIMENSIONS

Changing velocity in two dimensions can occur when turning a corner. For example, walking at 3.00 m s^{-1} west, then turning to travel at 3.00 m s^{-1} north. Although the magnitude of the velocity is the same, the direction is different.

A change in velocity in two dimensions can be determined using either the graphical method or the geometric method described in the previous section. The initial velocity must always be reversed before it is added to the final velocity.

The two-dimensional direction conventions were introduced in the previous section and are shown here in Figure 2.3.6.

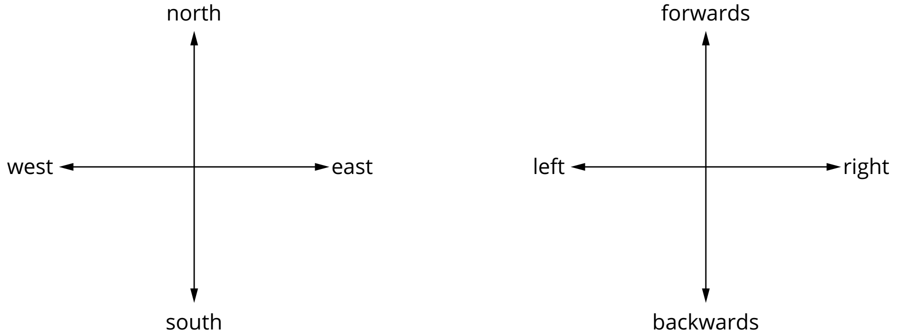


FIGURE 2.3.6 The direction conventions for the horizontal plane.

> When a change in a vector occurs, the magnitude and/or the direction of the vector can change.

Graphical method of subtracting vectors

To subtract vectors using a graphical method, choose a direction convention and a scale, and draw each vector.

Using velocity as an example, the steps to do this are as follows.

- Draw in the final velocity first.
- Draw the opposite of the initial velocity head to tail with the final velocity vector.
- Draw the resultant change in velocity vector, starting at the tail of the final velocity vector and ending at the head of the opposite of the initial velocity vector.
- Measure the length of the resultant vector and compare it to the scale to determine the magnitude of the change in velocity.
- Measure an appropriate angle to determine the direction of the resultant vector.

Figure 2.3.7 shows the velocity vectors for travelling 3.00 m s^{-1} west and then turning and travelling 3.00 m s^{-1} north. The opposite of the initial velocity is drawn as 3.00 m s^{-1} east.

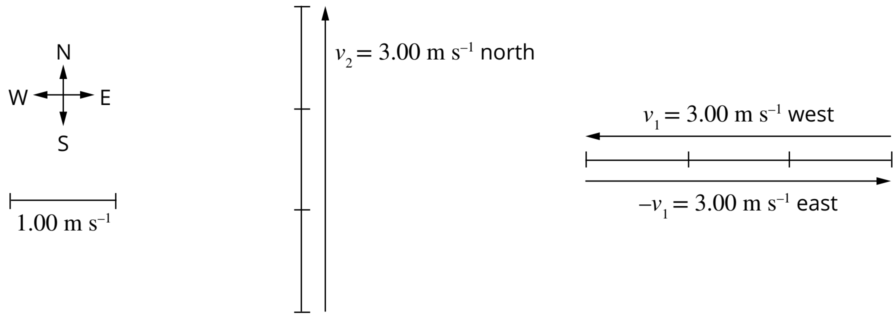


FIGURE 2.3.7 Subtracting two vectors at right angles, using the graphical method: the initial vectors.

To determine the change in velocity, the final velocity vector is drawn first, then from its head the opposite of the initial velocity is drawn. This is shown in Figure 2.3.8. The magnitude of the change in velocity (the resultant vector) is shown in red. It is estimated to be about 4.30 m s^{-1} according to the scale provided. Using a protractor, the resultant vector is measured to be in the direction N 45.0° E.

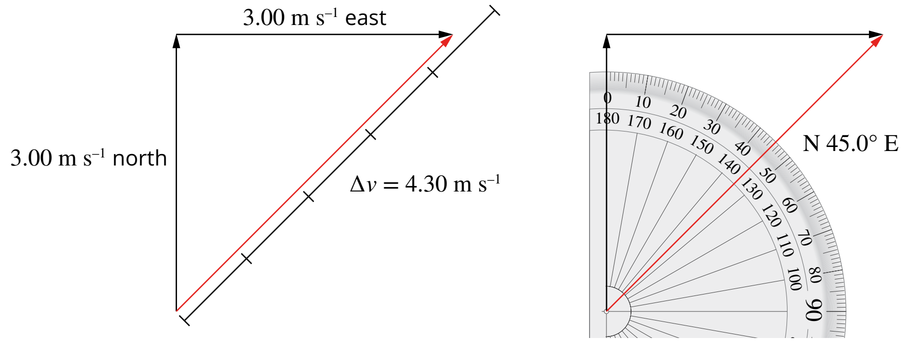


FIGURE 2.3.8 Subtracting two vectors at right angles, using the graphical method: the resultant vectors.

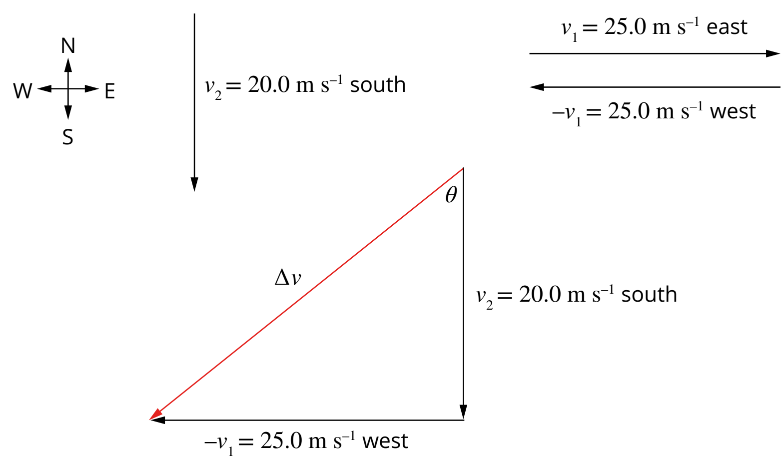
Geometric method of subtracting vectors

The graphical method of subtracting vectors in two dimensions only gives approximate results, as it relies on comparing the magnitude of the change in velocity vector to a scale and measuring its direction with a protractor.

As you saw earlier in the chapter when adding vectors, a more accurate method to subtract vectors is to use Pythagoras' theorem and trigonometry.

Figure 2.3.9 shows how to calculate the resultant velocity when changing from 25.0 m s^{-1} east to 20.0 m s^{-1} south. The initial velocity of 25.0 m s^{-1} east and the final velocity of 20.0 m s^{-1} south are drawn. Then the opposite of the initial velocity is drawn as 25.0 m s^{-1} west. The final velocity vector is drawn first, then from its head the opposite of the initial velocity is drawn. The resultant velocity vector, shown in red, is calculated to be 32.0 m s^{-1}. The resultant vector is calculated to be in the direction S 51.3° W.

The resultant vector is 32.0 m s^{-1} S 51.3° W.



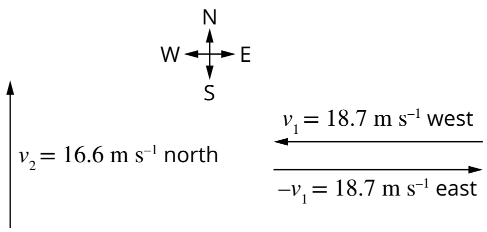
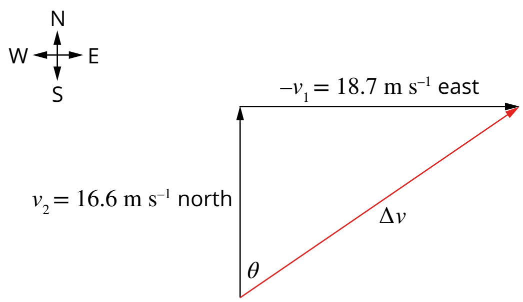
$$R^2 = 25.0^2 + 20.0^2$$
$$= 625 + 400$$
$$R = \sqrt{1025}$$
$$= 32.0 \text{ m s}^{-1}$$

$$\tan\theta = \frac{25.0}{20.0}$$
$$\theta = \tan^{-1} 1.25$$
$$= 51.3°$$
S 51.3° W

FIGURE 2.3.9 Subtracting two vectors at right angles, using the geometric method.

Worked example 2.3.2

SUBTRACTING VECTORS IN TWO DIMENSIONS USING GEOMETRY

<table>
<tr><td colspan="2">Determine the change in velocity of Clare's scooter as she turns a corner if she approaches it at 18.7 $m s^{-1}$ west and exits at 16.6 $m s^{-1}$ north.</td></tr>
<tr><td>Thinking</td><td>Working</td></tr>
<tr><td>Draw the final velocity vector, v_2, and the initial velocity vector, v_1, separately. Then draw the initial velocity in the opposite direction.</td><td>
</td></tr>
<tr><td>Construct a vector diagram drawing v_2 first and then from its head draw the opposite of v_1. The change of velocity vector is drawn from the tail of the final velocity to the head of the opposite of the initial velocity.</td><td>
</td></tr>
<tr><td>As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the change in velocity.</td><td>$R^2 = 16.6^2 + 18.7^2$
$= 275.26 + 349.69$
$R = \sqrt{625.25}$
$= 25.0\,m\,s^{-1}$</td></tr>
<tr><td>Calculate the angle from the north vector to the change in velocity vector.</td><td>$\tan\theta = \frac{18.7}{16.6}$
$\theta = \tan^{-1} 1.16$
$= 48.4°$</td></tr>
<tr><td>State the magnitude and direction of the change in velocity.</td><td>$\Delta v = 25.0\,m\,s^{-1}$ N 48.4° E</td></tr>
</table>

Worked example: Try yourself 2.3.2

SUBTRACTING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Determine the change in velocity of a ball as it bounces off a wall. The ball approaches at 7.00 $m s^{-1}$ south and rebounds at 6.00 $m s^{-1}$ east.

2.3 Review

SUMMARY

- To find the difference between, or change in vectors, subtract the initial vector from the final vector.
- Vectors are subtracted by adding the negative, or opposite, of a vector.
- Vector subtraction in one or two dimensions can be determined graphically using a scale and a protractor.
- Vector subtraction in one dimension can be determined algebraically by applying a sign and direction convention.
- Vector subtraction in two dimensions can be determined geometrically using Pythagoras' theorem and trigonometry.

KEY QUESTIONS

1 A car that was initially travelling at a velocity of $3.00\,m\,s^{-1}$ west is later travelling at $5.00\,m\,s^{-1}$ east. What is the difference between the two vectors?

2 Determine the change in velocity of a runner who changes from running on grass at $4.00\,m\,s^{-1}$ to the right to running in sand at $2.00\,m\,s^{-1}$ to the right.

3 A student throws a ball up into the air at $4.00\,m\,s^{-1}$. A short time later the ball is travelling back downwards to hit the ground at $3.00\,m\,s^{-1}$. Determine the change in velocity of the ball during this time.

4 Tom hits a tennis ball against a wall. If the ball travels towards the wall at $35.0\,m\,s^{-1}$ north and rebounds at $32.5\,m\,s^{-1}$ south, calculate the change in velocity of the ball.

5 Jamelia applies the brakes on her car and changes her velocity from $22.2\,m\,s^{-1}$ forwards to $8.20\,m\,s^{-1}$ forwards. Calculate the change in velocity of Jamelia's car.

6 A jet plane makes a turn after taking off, changing its velocity from $345\,m\,s^{-1}$ south to $406\,m\,s^{-1}$ west. Calculate the change in the velocity of the jet.

7 Yvette hits a golf ball that strikes a tree and changes its velocity from $42.0\,m\,s^{-1}$ east to $42.0\,m\,s^{-1}$ north. Calculate the change in the velocity of the golf ball.

8 A yacht tacks (changes course) during a race, changing its velocity from $7.05\,m\,s^{-1}$ south to $5.25\,m\,s^{-1}$ west. Calculate the change in the velocity of the yacht.

9 A cyclist travelling north at $40.0\,km\,h^{-1}$ does a U-turn at the halfway point of a race and slows to $25.0\,km\,h^{-1}$. Determine:

a the change in speed

b the change in velocity.

10 A motorcyclist travelling south turns around a roundabout and exits at the third exit (now heading west) maintaining a speed of $30.0\,km\,h^{-1}$. Determine their change in velocity.

2.4 Vector components

Sections 2.2 and 2.3 explored how vectors can be combined to find a resultant vector. In physics, there are times when it is useful to break one vector up into two vectors that are at right angles to each other. For example, if a force vector is acting at an angle up from horizontal, as shown in Figure 2.4.1, this vector can be considered to consist of two independent components, one vertical and one horizontal.

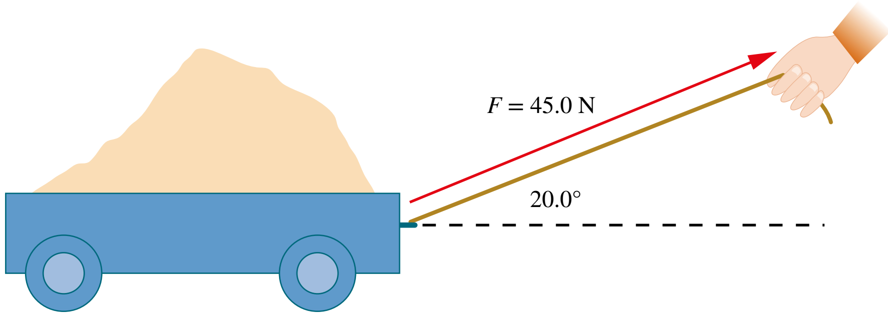


FIGURE 2.4.1 The pulling force acting on the cart has a component in the horizontal direction and a component in the vertical direction.

The components of a vector can be found using trigonometry.

FINDING PERPENDICULAR COMPONENTS OF A VECTOR

Vectors at an angle are more easily dealt with if they are broken up into perpendicular **components**, that is, two components that are at right angles to each other. These components, when added together, give the original vector. To find the components of a vector, a right-angled triangle is constructed with the original vector as the hypotenuse. This is shown in Figure 2.4.2. The hypotenuse is always the longest side of a right-angled triangle and is opposite the 90.0° angle. The other two sides of the triangle are each shorter than the hypotenuse and form the 90.0° angle with each other. These two sides are the perpendicular components of the original vector.

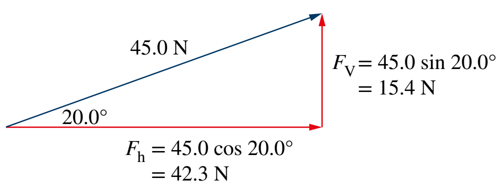


FIGURE 2.4.2 The perpendicular components (shown in red) of the original vector (shown in blue). The original vector is the hypotenuse of the triangle.

Geometric method of finding vector components

The geometric method of finding the perpendicular components of vectors is to construct a right-angled triangle using the original vector as the hypotenuse. This was illustrated in Figure 2.4.2. The magnitude and direction of the components are then determined using trigonometry. A good rule to remember is that no component of a vector can be larger than the vector itself. In a right-angled triangle, no side is longer than the hypotenuse. The original vector must be the hypotenuse and its components must be the other two sides of the triangle.

Figure 2.4.3 shows a force vector of 50.0 N (drawn in black) acting on a box in a direction 30.0° up from horizontal to the right. The horizontal and vertical components of this force must be found in order to complete further calculations.

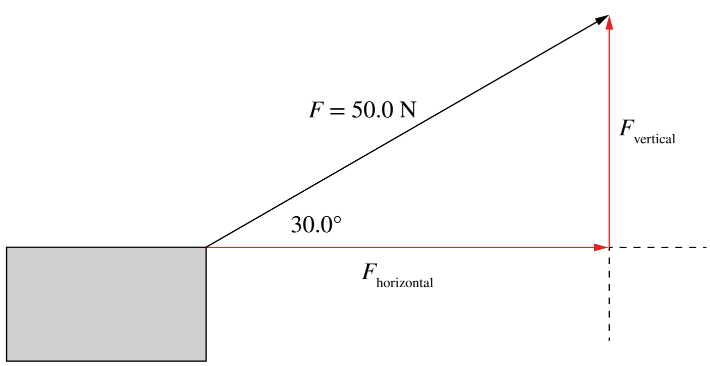


FIGURE 2.4.3 Finding the horizontal and vertical components of a force vector.

The horizontal component vector is drawn from the tail of the 50.0 N vector towards the right, with its head directly below the head of the original 50.0 N vector. The vertical component vector is drawn from the head of the horizontal component to the head of the original 50.0 N vector.

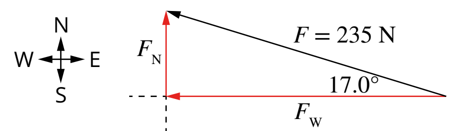
Using trigonometry, the horizontal component of the force is calculated to be 43.3 N horizontally to the right. The vertical component is calculated to be 25.0 N vertically upwards. The calculations are shown below:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\text{adj} = \text{hyp}\cos\theta$$
$$F_h = (50.0)(\cos 30.0°)$$
$$= 43.3\text{ N horizontal to the right}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\text{opp} = \text{hyp}\sin\theta$$
$$F_v = (50.0)(\sin 30.0°)$$
$$= 25.0\text{ N vertically upwards}$$

Worked example 2.4.1

CALCULATING THE PERPENDICULAR COMPONENTS OF A FORCE

Use the direction conventions to determine the perpendicular components of a 235 N force acting on a bike at a direction of 17.0° north of west.	
Thinking	**Working**
Draw F_W from the tail of the 235 N force along the horizontal direction, then draw F_N from the horizontal vector to the head of the 235 N force.	
Calculate the west component of the force F_W using $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\text{adj} = \text{hyp}\cos\theta$ $F_W = (235)(\cos 17.0°)$ $= 224.7$ N west
Calculate the north component of the force F_N using $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\text{opp} = \text{hyp}\sin\theta$ $F_N = (235)(\sin 17.0°)$ $= 68.7$ N north

Worked example: Try yourself 2.4.1

CALCULATING THE PERPENDICULAR COMPONENTS OF A FORCE

Use the direction conventions to determine the perpendicular components of a 3540 N force acting on a trolley at a direction of 26.5° down from horizontal to the left.

PHYSICSFILE

Projectile motion

When objects undergo projectile motion, such as when a ball leaves a tennis racquet, the vertical and horizontal motion are affected differently. So, problem solving begins with separating the initial velocity of the projectile into horizontal and vertical components. At all times during the flight of the ball, the force of gravity is pulling down on the ball, causing it to accelerate downwards until it hits something: another racquet, the net, or the ground. However, the ball will continue to move in the horizontal direction at the same constant velocity as there is no force in that direction, once the ball has left the racquet (Figures 2.4.4 and 2.4.5).

FIGURE 2.4.4 A tennis ball in motion accelerates downwards while maintaining a constant horizontal velocity.

FIGURE 2.4.5 By judging the perfect vertical and horizontal components of the velocity required, a tennis player can hit the perfect shot.

2.4 Review

SUMMARY

- A vector can be resolved into two perpendicular component vectors.
- Perpendicular component vectors are at right angles to each other.
- Any component vectors must be smaller in magnitude than the original vector.
- The hypotenuse of a right-angled triangle is the longest side of the triangle and the other two sides are each smaller than the hypotenuse.
- A right-angled triangle vector diagram can be drawn with the original vector as the hypotenuse and the perpendicular components drawn from the tail of the original to the head of the original.
- The perpendicular components can be determined using trigonometry.

KEY QUESTIONS

1 Rayko applies a force of 462 N on the handle of a mower in a direction of 35.0° down from horizontal to the right.
  a What is the downwards force applied?
  b What is the horizontal right force applied?

2 A force of 25.9 N acts in the direction of S 40.0° E. Find the perpendicular components of the force.

3 A ferry is transporting students to Rottnest Island. At one point in the journey the ferry travels at 18.3 $m\,s^{-1}$ N 75.6° W. Calculate its velocity in the northerly direction and in the westerly direction at that time.

4 Zehn walks 47.0 m in the direction of S 66.3° E across a hockey field. Calculate the change in Zehn's position down the field and across the field during that time.

5 A cargo ship has two tugs attached to it by ropes. One of the tugs is pulling directly north, while the other tug is pulling directly west. The pulling forces of the tugboats combine to produce a total force of 235 000 N in a direction of N 62.5° W. Calculate the force that each tug boat applies to the cargo ship.

6 Resolve the following forces into their perpendicular components around the north–south line. In part d, use the horizontal and vertical directions.
  a 108 N S 60.0° E
  b 60.0 N north
  c 312 N 165°T
  d 3.00×10^5 N 30.0° upwards from horizontal to the right.

7 What are the horizontal and vertical components of a 348 N force that is applied along a rope used to drag an object across a yard at 60.0° up from horizontal to the left?

8 A ball is hit from a racquet with a velocity of 30.0 $m\,s^{-1}$ at 50.0° up from horizontal to the right. Calculate the horizontal and vertical components of the velocity.

9 A person walks 445 m in a south-east direction. How far south have they travelled in this time?

10 A sprinter starts a race by pushing against the starting block with a force of 486 N. If the block is positioned at 70.0° up from horizontal to the left, what horizontal force does the sprinter apply to the block?

Chapter review

02

KEY TERMS

collinear
component
dimension
direction conventions
magnitude
resultant
scalar
units
vector
vector diagram

1 Select the scalar quantities in the list below. (There may be more than one correct answer.)
A force
B time
C acceleration
D mass

2 Select the vector quantities in the list below. (There may be more than one correct answer.)
A displacement
B distance
C volume
D velocity

3 A basketballer applies a force with their hand to bounce the ball. Describe how a vector can be drawn to represent this situation.

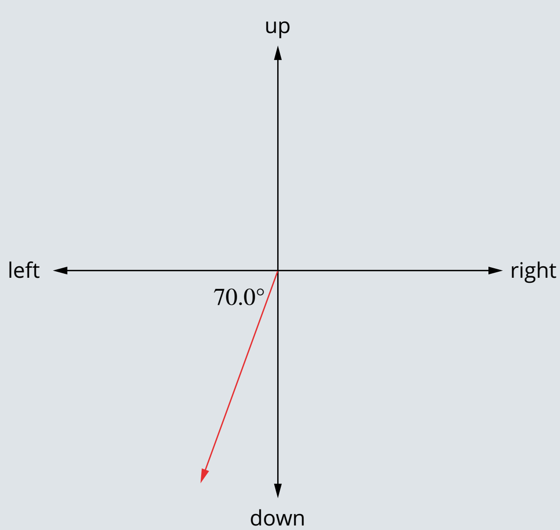
4 Vector arrow A is drawn twice the length of vector arrow B. What does this mean?

5 A car travels 15.0 m s^{-1} north and another travels 20.0 m s^{-1} south. Why is a sign convention often used to describe vectors like these?

6 When finding the change in velocity between an initial velocity of 34.0 m s^{-1} south and a final velocity of 12.5 m s^{-1} east, which two vectors need to be added together?

7 If the vector 20.0 N forwards is written as –20.0 N, how would you write a vector representing 80.0 N backwards?

8 Describe the following vector direction.



9 Add the following force vectors using a number line: 3.00 N left, 2.00 N right, 6.00 N right. Then draw and describe the resultant force vector.

10 Determine the resultant vector of the following combination: 45.0 m forwards, 70.5 m backwards, 34.5 m forwards, 30.0 m backwards.

11 Find the vector which results from the addition of 36.0 m south and 55.0 m west.

12 Add the following vectors: 481 N north and 655 N east. Give answers to three significant figures.

13 Determine the change in velocity of a bird that changes from flying 3.00 m s^{-1} to the right to flying 3.00 m s^{-1} to the left.

14 A car makes a turn, changing its velocity from 13.0 m s^{-1} south to 18.7 m s^{-1} west. Calculate the change in the velocity vector, Δv, of the car, to three significant figures.

15 Nina hits a cricket ball so that it changes its velocity from 38.8 m s^{-1} east to 55.5 m s^{-1} north. Calculate the change in the velocity vector, to three significant figures.

16 A force of 45.5 N acts in the direction of S 60.0° E. Find the eastern and southern components of this force. Give your answers to three significant figures.

17 Calculate the vertical velocity of a cannonball which is shot at 50.0° up from the horizontal ground to the right at a speed of 422 m s^{-1}.

18 Findlay pulls a heavy load with a force of 212 N at 60.0° up from horizontal to the right and Dougie pulls twice as hard with a different rope at 50.0° up from horizontal to the right. Determine the total horizontal force that is pulling the load along.

19 Lin shoots a basket 5.00 m away. It just manages to go down into the basket at 20.0° up from vertical to the right, with a vertical velocity component of 3.00 m s^{-1} down. Calculate its actual speed when it goes through the loop.

20 Aidan does a ski jump off a ramp and lands with a speed of 10.0 m s^{-1} at 45.0° down from horizontal to the right. Calculate their vertical and horizontal velocities when they land.

CHAPTER

03 Linear motion

Motion, from the simple to the complex, is a fundamental part of everyday life. The motion of a gymnast performing a floor routine is a complex form of motion. An Olympic snowboarder competing in a half-pipe event also exhibits a complex form of motion. Simpler examples include a skier travelling in a straight line down a ski run, a train pulling into a station and a swimmer completing a lap of a pool.

Science Understanding

Linear motion and force

- change (final – initial) in a variable is represented by the symbol Δ, for example $\Delta t = t_f - t_i$
- displacement is defined as the change in position of an object, including applying the relationship

$$s = \Delta x = x_f - x_i$$

- velocity is defined as the rate of change in displacement of an object, including applying the relationship

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

- uniformly accelerated motion (a is constant) is described in terms of relationships between measurable scalar and vector quantities, including displacement, velocity and time, including applying the relationships

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad v_f = v_i + a\Delta t \qquad s = v_i\Delta t + \frac{1}{2}a\Delta t^2 \qquad v_f^{\,2} = v_i^{\,2} + 2as$$

- motion can be represented graphically to describe linear motion, including determination, manipulation and use of gradients of curves and areas under graphs of
  - displacement–time
  - velocity–time
  - acceleration–time
- vertical motion is analysed by assuming the acceleration due to gravity is constant near Earth's surface.

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025.*

3.1 Displacement, speed and velocity

In order to analyse and communicate ideas about motion, it is important to understand the terms used to describe motion, even in its simplest form. In this section you will learn about some of the terms used to describe straight-line motion, such as position, distance, displacement, speed and velocity.

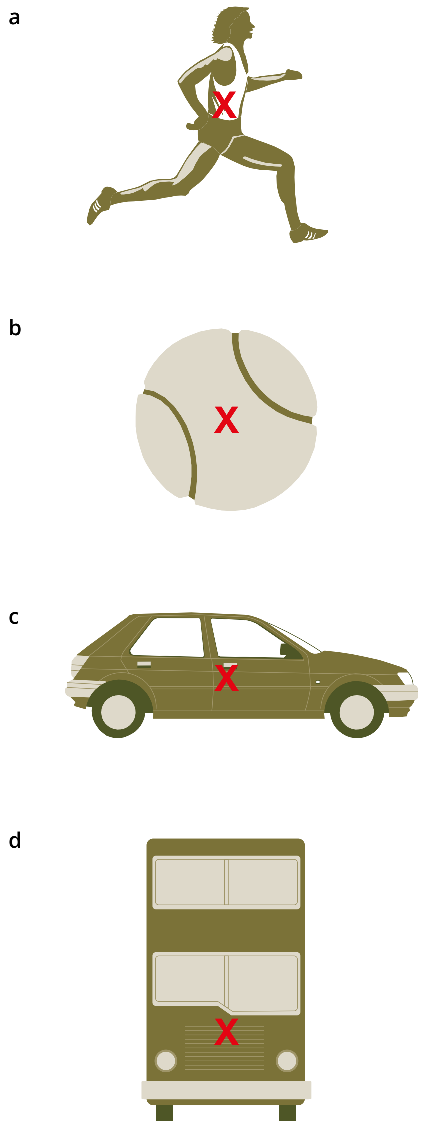


FIGURE 3.1.1 The centre of mass of each object is indicated by a cross.

CENTRE OF MASS

When analysing motion, things are often more complicated than they first appear. As a freestyle swimmer travels at a constant speed of 2.00 m s^{-1}, their head and torso move forwards at this speed. The motion of their arms, however, is more complex. At times their arms move forwards through the air faster than 2.00 m s^{-1}, and at other times they are actually moving backwards through the water.

It is beyond the scope of this course to analyse such a complex motion. However, the motion of the swimmer can be simplified by treating the swimmer as a simple object located at a single point called the **centre of mass** or centre of gravity. The centre of mass is the balance point of an object. For a person, the centre of mass is located near the waist. The centres of mass of some everyday objects are shown in Figure 3.1.1. The concept of centre of mass and centre of gravity is discussed in more detail in Year 12.

POSITION, DISTANCE AND DISPLACEMENT

Position, distance and displacement are three essential concepts for analysing straight-line motion and understanding how objects move.

Position

One important term to understand when analysing straight-line motion is **position**, which is given the symbol x.

- Position, x, describes the location of an object at a certain point in time with respect to the origin.
- Position is a vector quantity and therefore requires a direction.
- Change in position, Δx, is the difference between the final position and the initial position, $x_f - x_i$.

Consider a swimmer, Sophie, doing laps in a 50.0 m pool, as shown in Figure 3.1.2. To simplify her motion, Sophie is treated as a simple point object. The pool can be treated as a one-dimensional number line, with the starting block as the origin. The direction to the right of the starting block is taken to be positive.

Sophie's position as she is warming up behind the starting block in Figure 3.1.2a is $x = -10.0$ m. The negative sign indicates the direction from the origin; that is, to the left. Her position could also be given as $x = 10.0$ m to the left of the starting block.

At the starting block (Figure 3.1.2b), Sophie's position is $x = 0.0$ m. After swimming half the length of the pool, her centre of mass is at a position where $x = +25.0$ m or 25.0 m to the right of the origin (Figure 3.1.2c).

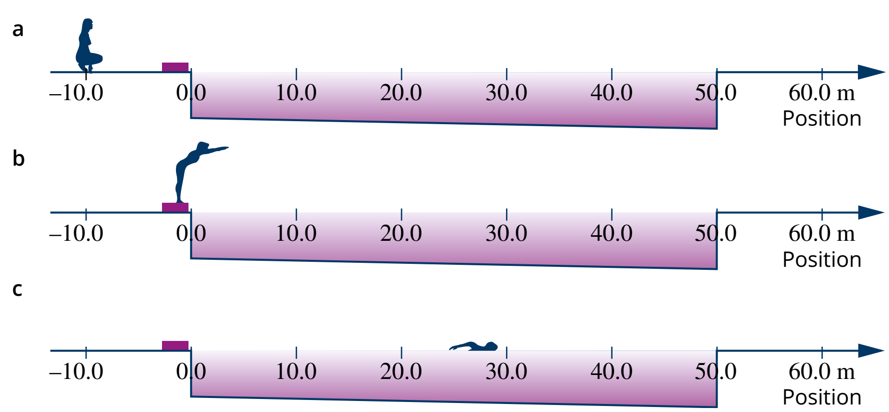


FIGURE 3.1.2 The position, x, of the swimmer is given with reference to the starting block. (a) While warming up, Sophie is at $x = -10.0$ m. (b) When she is on the starting block, her position is at $x = 0.0$ m. (c) After swimming for a short time, her centre of mass is at a position where $x = +25.0$ m.

Distance travelled

Position, x, describes where an object is at a certain point in time. However, **distance travelled**, d, is how far a body travels during a journey.

- Distance travelled, d, describes the length of the path covered during an object's entire journey.
- Distance travelled is a scalar quantity and is measured in metres (m).

For example, if Sophie completes three lengths of the pool, the distance, d, travelled during her swim will be 50.0 + 50.0 + 50.0 = 150.0 m.

The distance travelled is not affected by the direction of the motion; that is, the distance travelled by an object always increases as it moves, regardless of its direction. The tripmeter or odometer of a car or bike, for example, measures distance travelled. To better understand this, consider tying one end of a spool of string to your letterbox and letting it unravel behind you as you walk to school. The length of the unwound string would represent the distance you have travelled.

Displacement

Displacement, s, is defined as the **change in position**, Δx, of an object. Displacement considers only the initial or starting position, x_i, and the final or finishing position, x_f; the route taken between these two points has no effect on displacement. The sign of the displacement indicates the direction in which the position has changed from the start to the end.

- Change (final – initial) in a variable is represented by the symbol Δ.
- Displacement, s, is the change in position of an object in a given direction, $s = \Delta x$.
- Displacement = final position – initial position or $s = \Delta x = x_f - x_i$.
- Displacement is a vector quantity and is measured in metres (m).

Consider the example of Sophie completing one length of the pool. During her swim, the distance travelled is 50.0 m. Her final position, x_f, is +50.0 m and her initial position, x_i, is 0.0 m. Her displacement can therefore be shown as:

s = final position – initial position

$s = x_f - x_i$

$s = (+50.0) - (0.0)$ $s = +50.0$ m or 50.0 m in a positive direction.

Notice that **magnitude**, units and direction are required for a vector quantity. The distance will be equal to the magnitude of displacement only if the body is moving in a straight line and does not change direction. If Sophie swims two lengths, her distance travelled will be 100.0 m: 50.0 m out and 50.0 m back. However, her displacement during this swim will be:

s = final position – initial position

$s = (0.0) - (0.0)$

$s = 0.0$ m

Even though Sophie has swum 100.0 m, the displacement is zero because her initial and final positions are the same.

The above formula for displacement is useful if you already know the initial and final positions of a body's motion. An alternative method to determine total displacement, if you know the displacement of each section of the motion, is to sum or add up the individual displacements for each section of motion.

total displacement = sum of individual displacements, or $s = s_1 + s_2 + s_3 + \ldots$

It is important to remember that displacement is a vector and so, when adding displacements, you must obey the rules of vector addition (discussed in Chapter 2).

In the example above, therefore, in which Sophie completed two laps of the pool, overall displacement could also have been calculated by adding the displacement of each lap:

s = sum of displacements for each lap

s = 50.0 m in the positive direction + 50.0 m in the negative direction

$s = (50.0) + (-50.0)$

$s = 0.0\,\text{m}$

PHYSICS IN ACTION

Timing and false starts in athletics

Until 1964, all timing of events at the Olympic Games was recorded by handheld stopwatches (Figure 3.1.3). The reaction times of the judges meant an uncertainty of 0.2 s for any measurement. An electronic quartz timing system introduced in 1964 improved accuracy to 0.01 s, but in close finishes the judges still had to wait for a photograph of the finish before they could announce the places.

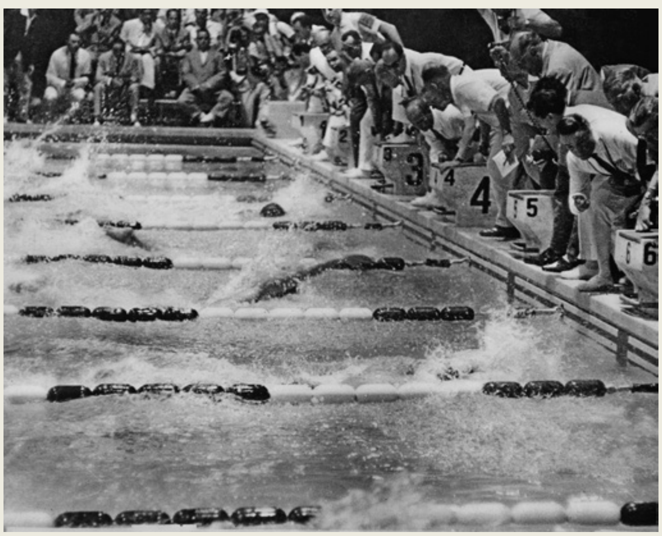


FIGURE 3.1.3 Using stopwatches to time a swimming race at the 1960 Olympic Games in Rome.

The current timing system used in athletics is a vertical line-scanning video system (VLSV). Introduced in 1991, this electronic timing system is completely automatic. The starting pistol triggers a computer to begin timing. At the finish line, a high-speed video camera records the image of each athlete and indicates the time at which each one crosses the line. This system enables the times of all athletes in the race to be precisely measured to one-thousandth of a second.

Another feature of this system is that it indicates when a runner 'breaks' at the start of the race. Each starting block is connected by electronic cables to the timing computer and a pressure sensor indicates if a runner has left the blocks early (Figure 3.1.4). A reaction time of 0.10 s has been incorporated into the system since 2002. This ensures that a runner has not anticipated the pistol. It also means that a runner can still commit a false start even if their start was after the pistol. A start that is less than 0.10 s after the pistol is registered as false.

FIGURE 3.1.4 Starting blocks are fitted with pressure sensors to detect false starts.

SPEED AND VELOCITY

For thousands of years, humans have tried to travel at ever greater speeds. This desire has contributed to the development of all sorts of competitive activities, as well as major advances in engineering and design. World records for some of these pursuits are given in Table 3.1.1.

TABLE 3.1.1 World record speeds for a variety of sports or modes of transport (as of June 2024).

Activity	World record speed ($m\,s^{-1}$)	World record speed ($km\,h^{-1}$)
street luge	45.522	163.88
maglev train	128	460
tennis serve	73.06	263.0
waterskiing (barefoot)	60.678	218.44
cricket delivery	44.81	161.3
electric formula 1	89.4	322

- Speed, v, is defined in terms of the rate, Δt, at which the distance, d, is travelled. Like distance, speed is a scalar quantity. A direction is not required when describing the speed of an object.
- Velocity, v, is defined in terms of the rate, Δt, at which displacement, s, changes and so is a vector quantity. A direction should always be given with a velocity.
- The SI unit for speed and velocity is metres per second ($m\,s^{-1}$), but kilometres per hour ($km\,h^{-1}$) is also commonly used.

Speed and **velocity** are both quantities that give an indication of how quickly the position of an object is changing. Both terms are in common use and are often assumed to have the same meaning. In physics, however, these two terms have different definitions.

Instantaneous speed and velocity

Instantaneous speed and instantaneous velocity give a measure of how fast something is moving at a particular point in time. The speedometer on a car or bike indicates instantaneous speed.

If a speeding car is travelling north and is detected on a police radar gun at $150\,km\,h^{-1}$, it indicates that this car's instantaneous speed is $150\,km\,h^{-1}$, while its instantaneous velocity is $150\,km\,h^{-1}$ north. Notice that the instantaneous speed is equal to the magnitude of the instantaneous velocity.

Average speed and velocity

Average speed and *average velocity* both give an indication of how fast an object is moving over a period of time.

$$\text{average speed, } v_{av} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{d}{\Delta t}$$

$$\text{average velocity, } v_{av} = \frac{\text{change in displacement}}{\text{time taken}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

A direction (such as north, south, up, down, left, right, positive, negative) must be given when describing a velocity. The direction will always be the same as that of the displacement.

For example, the average speed of a car that takes 30.0 minutes to travel 20.0 km from Perth to Sorrento is $40.0\,km\,h^{-1}$. However, this does not mean that the car travelled the whole distance at this speed. In fact, it is more likely that the car was moving at $60.0\,km\,h^{-1}$ for a significant amount of time, while for some of the time it may not be moving at all.

Like the relationship between distance and displacement, average speed will be equal to the magnitude of average velocity only if the body is moving in a straight line and does not change direction. In a race around a circular track like the velodrome shown in Figure 3.1.5, regardless of the average speed for one complete lap, the magnitude of the average velocity at the end of that lap will be zero because the displacement is zero.

FIGURE 3.1.5 Anna Meares won the UCI world championship in 2013. She rode 500 m in a world record time of 32.836 s. Her average speed was $55.6\,km\,h^{-1}$ but her average velocity was zero at the start–finish line.

EXTENSION

How police measure the speeds of cars

Road accidents cause the deaths of about 1200 people in Australia each year and many times this number are seriously injured. Numerous steps have been taken to reduce the number of road fatalities. Some of these include random alcohol and drug testing, speed cameras, mandatory wearing of bicycle helmets and the zero blood alcohol level for probationary drivers.

One of the main causes of road trauma is speeding. In their efforts to combat speeding motorists, police employ a variety of speed-measuring devices. One such device is shown in Figure 3.1.6.

FIGURE 3.1.6 Speed cameras on poles.

Speed camera radar

Camera radar units are usually placed in unmarked vehicles parked by the side of the road. These units emit a radar signal frequency of 24.15 GHz (2.415×10^{10} Hz). The radar antenna has a parabolic reflector that enables the unit to produce a directional radar beam that is 5° wide, allowing individual vehicles to be targeted. The radar range and field of vision for a camera is shown in Figure 3.1.7. The radar signal allows speeds to be determined by the Doppler effect, where the reflected radar signal from an approaching vehicle has a higher frequency than the original signal. Similarly, the reflected signal from a receding vehicle has a lower frequency. This change in frequency or 'Doppler shift' is processed by the unit and gives a measurement of the instantaneous speed of the target vehicle.

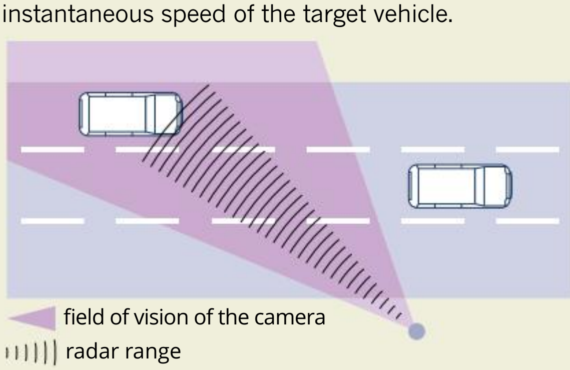


FIGURE 3.1.7 Diagram showing the visual range of a speed camera.

Camera radar units are capable of targeting a single vehicle up to 1.2 km away. In traffic, the units can distinguish between individual cars and take two photographs per second. The photographs and infringement notices are mailed to the offending motorists.

Laser speed guns

Speed guns are used by police to obtain an instant measure of the speed of an approaching or receding vehicle. The unit is usually handheld and is aimed directly at a vehicle using a target sight. It emits a pulse of infra-red radiation frequency of 331 THz (3.31×10^{14} Hz). As with camera radar units, the speed is determined by the Doppler shift produced by the target vehicle. The infra-red pulse is very narrow and directional, being just 0.17° wide. This allows vehicles to be targeted with great precision. Handheld units can be used at distances up to 800 m. If the vehicle's speed registers over the limit, police are likely to pull the driver over.

Fixed speed cameras

Fixed speed cameras obtain their readings by using a system of three strips with piezoelectric sensors in them across the road (see Figure 3.1.8). The strips respond to the pressure exerted as the car drives over them and create an electrical pulse that is detected by the unit. By knowing the precise distance between the strips and measuring the time that the car takes to travel across them, the speed of the car can be determined. This is actually measuring the average speed of the car, but by placing the strips close together the average speed gives a very good approximation of the instantaneous speed.

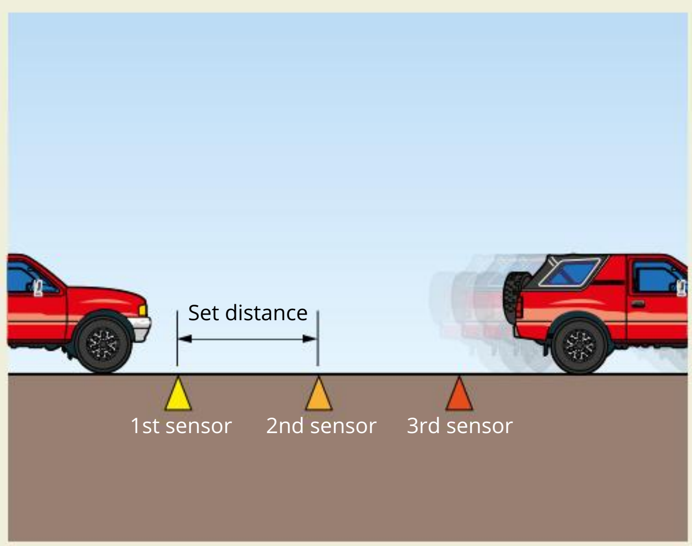


FIGURE 3.1.8 Fixed speed cameras record the speed of a car twice by measuring the time the car takes to travel over a series of three sensor strips embedded in the roadway.

Converting $km\,h^{-1}$ to $m\,s^{-1}$

You should be familiar with $100.0\,km\,h^{-1}$ as it is the speed limit for most freeways and country roads in Australia. Cars that maintain this speed would travel 100.0 km in 1.00 hour. Since there are 1000 m in 1.00 km and 3600 s in 1.00 hour (60.0 s × 60.0 min), this is the same as travelling 100 000 m in 3600 s.

$$\begin{aligned} 100.0\,km\,h^{-1} &= 100.0 \times 1000\,m\,h^{-1} \\ &= 100\,000\,m\,h^{-1} \\ &= \frac{100\,000}{3600}\,ms^{-1} \\ &= 27.8\,m\,s^{-1} \end{aligned}$$

So, $km\,h^{-1}$ can be converted to $m\,s^{-1}$ by multiplying by $\frac{1000}{3600}$ (or simply dividing by 3.60).

Converting $m\,s^{-1}$ to $km\,h^{-1}$

A champion Olympic sprinter can run at an average speed of close to $10.0\,m\,s^{-1}$. Each second, the athlete will travel approximately 10.0 m. At this rate, in 1.00 hour the athlete would travel 10.0 × 3600 = 36 000 m = 36.0 km.

$$\begin{aligned} 10.0\,m\,s^{-1} &= 10.0 \times 3600\,m\,h^{-1} \\ &= 36\,000\,m\,h^{-1} \\ &= \frac{36\,000}{1000}\,km\,h^{-1} \\ &= 36.0\,km\,h^{-1} \end{aligned}$$

So, $m\,s^{-1}$ can be converted to $km\,h^{-1}$ by multiplying by $\frac{3600}{1000}$ (or simply multiplying by 3.60).

When converting a speed from one unit to another, it is important to compare the speeds to ensure that your answers make sense. To do so, a good rule to remember is that the number in front of $km\,h^{-1}$ is always larger than the number in front of $m\,s^{-1}$. The diagram in Figure 3.1.9 summarises the conversion between units for speed.

PHYSICSFILE

Reaction time

Drivers are often distracted by loud music or phone calls. These distractions result in many accidents on the road. If cars are moving at high speeds, they can travel a considerable distance in the short time that the driver takes just to recognise a hazard and apply the brakes. This is known as the reaction distance, which adds to the stopping distance. Lower speeds and short reaction times are very important in helping all road users to avoid collisions. This is easy to understand given that distance is directly proportional to both speed and time, $d = v\Delta t$.

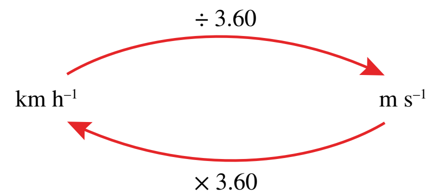


FIGURE 3.1.9 Rules for converting between $m\,s^{-1}$ and $km\,h^{-1}$.

PHYSICS IN ACTION

Alternative units for speed and distance

Metres per second is the standard unit for measuring speed as it is derived from the standard unit for distance (metres) and the standard unit for time (seconds). However, alternative units are often used that better suit certain applications.

The speed of a boat is usually measured in knots, where 1.00 knot = $0.510\,m\,s^{-1}$. This unit originated in the nineteenth century when the speed of sailing ships would be measured by allowing a rope, with knots tied at regular intervals, to be dragged by the water through a sailor's hands. By counting the number of knots that passed through the sailor's hands, and measuring the time taken for this to happen, the average speed formula could be applied to estimate the speed of the ship.

The speed of very fast jet planes, such as the one in Figure 3.1.10, can be measured in Mach numbers, which are related to the speed of sound. One Mach (referred to as Mach 1) is equal to the speed of sound, which is $340\,m\,s^{-1}$. Mach 2 is equal to $680\,m\,s^{-1}$, or twice the speed of sound.

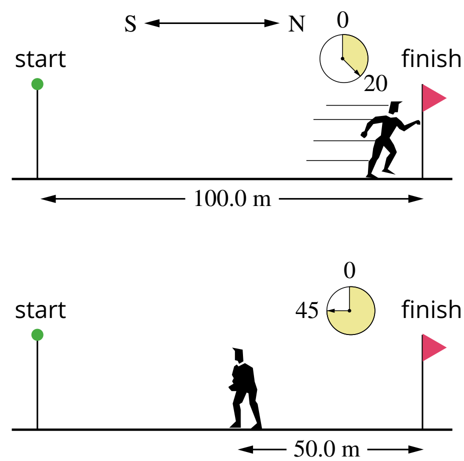
The light-year is an alternative unit for measuring distance. The speed of light in a vacuum is $3.00 \times 10^8\,m\,s^{-1}$, which is three hundred million metres every second. One light-year is the distance that light travels in 1 year, which is 3.15576×10^7 s. Because distances between objects in the universe are so large, astronomers use the light-year to measure distances in space. For example, it takes 4.24 years for light to travel 4.014×10^{16} m to us from Proxima Centauri, the nearest star to Earth other than our own star, Sol (the Sun). It is much easier to say that the distance from Earth to our nearest star is 4.24 light-years than it is to use units that work on a smaller scale. Light takes approximately 8.5 minutes to travel from the Sun to Earth, so it could be said that the Sun is 8.5 light-minutes away.

FIGURE 3.1.10 Modern fighter aeroplanes can fly at speeds above Mach 1.

Worked example 3.1.1

CALCULATING VELOCITY AND CONVERTING UNITS

Sam is an athlete performing a training routine by running back and forth along a straight stretch of running track. Sam jogs 100.0 m north in a time of 20.0 s, then turns and walks 50.0 m south in a further 25.0 s before stopping.

a Calculate Sam's velocity in $m\,s^{-1}$.	
Thinking	**Working**
Calculate the displacement. Remember that total displacement is the sum of individual displacements. Sam's total journey consists of two displacements: 100.0 m north and then 50.0 m south. Take north to be the positive direction.	s = sum of displacements s = 100.0 m north + 50.0 m south $s = (100.0) + (-50.0)$ $s = +50.0$ m or 50.0 m north 
Work out the total time taken for the journey.	$\Delta t = (20.0) + (25.0) = 45.0$ s
Substitute the values into the velocity equation.	Displacement, s, is 50.0 m north. Time taken, Δt, is 45.0 s. $v = \frac{s}{\Delta t}$ $v = \frac{(50.0)}{(45.0)}$ $v = 1.11111$
Velocity is a vector, so a direction must be given.	$v = 1.11\ m\,s^{-1}$ north

b Determine the magnitude of Sam's velocity in $km\,h^{-1}$.	
Thinking	**Working**
Convert from $m\,s^{-1}$ to $km\,h^{-1}$ by multiplying by 3.60.	$v = 1.11111\ m\,s^{-1}$ $v = (1.11111)(3.60)$ $v = 4.00000$
As the magnitude of the velocity is needed, the direction is not required in this answer.	magnitude of $v = 4.00\ km\,h^{-1}$

c What is Sam's speed in $m\,s^{-1}$?	
Thinking	**Working**
Calculate the distance. Remember that distance is the length of the path covered in the entire journey. The direction does not matter. Sam travels 100.0 m in one direction and then 50.0 m in the other direction.	$d = (100.0) + (50.0)$ $d = 150.0\,m$
Work out the total time taken for the journey.	$\Delta t = (20.0) + (25.0) = 45.0\,s$
Substitute the values into the speed equation.	Distance, d, is 150.0 m. Time taken, Δt, is 45.0 s. $v = \frac{d}{\Delta t}$ $v = \frac{(150.0)}{(45.0)}$ $v = 3.33333$ $v = 3.33\ m\ s^{-1}$

d What is Sam's speed in $km\,h^{-1}$?	
Thinking	**Working**
Convert from $m\,s^{-1}$ to $km\,h^{-1}$ by multiplying by 3.60.	$v = 3.33333\ m\ s^{-1}$ $v = (3.33333)(3.60)$ $v = 12.0000$ $v = 12.0\ km\ h^{-1}$

Worked example: Try yourself 3.1.1

CALCULATING VELOCITY AND CONVERTING UNITS

Sally is an athlete performing a training routine by running back and forth along a straight stretch of running track. Sally jogs 108.0 m west in a time of 20.0 s, then turns and walks 165.0 m east in a further 45.0 s before stopping.

a Calculate Sally's velocity in $m\,s^{-1}$.

b Calculate the magnitude of Sally's velocity in $km\,h^{-1}$.

c What is Sally's speed in $m\,s^{-1}$?

d What is Sally's speed in $km\,h^{-1}$?

PHYSICSFILE

Breaking the speed limit

Over the past 100 years, advances in engineering and technology have led to the development of faster machines. Cars, planes and trains can now move people at speeds that were thought to be both unattainable and life-threatening a century ago.

The 1 mile land-speed record is 1220 $km\,h^{-1}$ (339 $m\,s^{-1}$). This was set in 1997 in Nevada by Andy Green driving his jet-powered *Thrust* SSC.

The fastest combat jet is the MiG-25. In 1976 it reached a speed of 3800 $km\,h^{-1}$ (1056 $m\,s^{-1}$), which is more than three times the speed of sound.

In 2007, Markus Stoeckl of Austria set a new speed record for mountain biking. He reached a speed of 210 $km\,h^{-1}$ racing down a ski slope in Chile, pictured in Figure 3.1.11. This record was broken by Eric Barone in 2017 with a speed of 227.720 $km\,h^{-1}$.

FIGURE 3.1.11 Markus Stoeckl set a new speed record for mountain biking in 2007.

3.1 Review

SUMMARY

- Position, x, defines the location of an object with respect to a defined origin.
- Distance travelled, d, tells us how far an object has actually travelled. Distance travelled is a scalar quantity.
- Displacement, s, is a vector quantity and is defined as the change in position of an object, Δx, in a given direction: $s = \Delta x = x_f - x_i$
- The average speed of a body, v_{av}, is defined as the rate of change of distance and is a scalar quantity:
  $$v_{av} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{d}{\Delta t}$$
- The average velocity of a body, v_{av}, is defined as the rate of change of displacement and is a vector quantity:
  $$v_{av} = \frac{\text{displacement}}{\text{time taken}} = \frac{s}{\Delta t}$$
- To convert from m s^{-1} to km h^{-1}, multiply by 3.60.
- To convert from km h^{-1} to m s^{-1}, divide by 3.60.
- The SI unit for both speed and velocity is metres per second (m s^{-1}).

KEY QUESTIONS

1 A student jogs one lap of a 400.0 m track in 2.00 minutes. Calculate:

a their average speed

b their average velocity.

2 A person swims ten lengths of a 25.0 m pool. Which one or more of the following statements correctly describes their distance travelled and displacement?

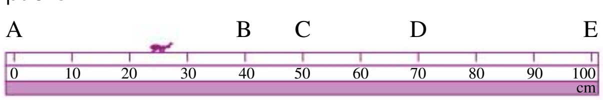
A Their distance travelled is zero.

B Their displacement is zero.

C Their distance travelled is 250.0 m.

D Their displacement is 250.0 m.

3 An ant is walking back and forth along a metre ruler, as shown in the figure below. Using the sign convention that right is positive and left is negative, determine both the displacement and the distance travelled by the ant as it moves along the following paths.



a A to B

b C to B

c C to D

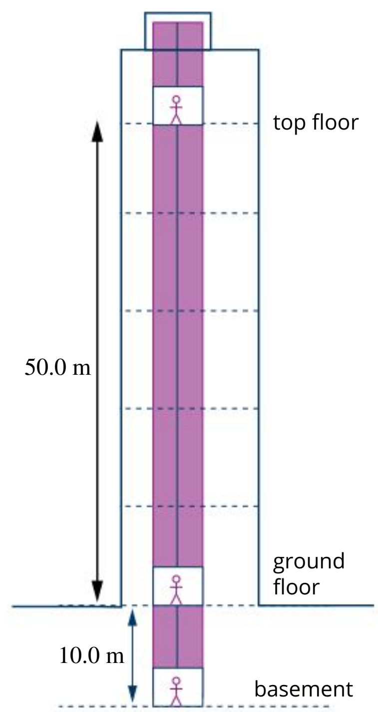
d C to E and then to D

4 During a training ride, a cyclist rides 50.0 km north then 30.0 km south.

a What is the distance travelled by the cyclist during the ride?

b What is the displacement of the cyclist for this ride?

5 A lift in a city building, shown in the figure below, carries a passenger from the ground floor down to the basement, then up to the top floor.



a What is the displacement of the lift as it travels from the ground floor to the basement?

b What is the displacement of the lift as it travels from the basement to the top floor?

c What is the distance travelled by the lift during this entire trip?

d What is the displacement of the lift during this entire trip?

6 A car travelling at a constant speed was timed over a distance of 400.0 m and was found to cover that distance in 12.0 s.

   a What was the car's average speed?

   b The driver was distracted when they encountered a hazard, which meant that their reaction time was 0.750 s before applying the brakes. How far did the car travel during this period of time?

7 A cyclist travels 25.0 km in 90.0 minutes.

   a What is their average speed in $km\,h^{-1}$?

   b What is their average speed in $m\,s^{-1}$?

8 Ali pushes a toy truck 5.00 m east, then stops it and pushes it 4.00 m west. The entire motion takes 10.0 s.

   a What is the truck's average speed?

   b What is the truck's average velocity?

9 Jackie rides a bicycle to school and travels 2.50 km south in 15.0 min.

   a Calculate Jackie's average speed in kilometres per hour ($km\,h^{-1}$).

   b What was Jackie's average velocity in metres per second ($m\,s^{-1}$)?

10 An athlete in training for a marathon runs 10.0 km north along a straight road before realising that they have dropped their drink bottle. The athlete turns around and runs back 3.0 km to find the bottle, then resumes running in the original direction. After running for 1.50 h, the athlete reaches a point 15.0 km from the starting position and stops.

   a What is the distance travelled by the athlete during the run?

   b What is the athlete's displacement during the run?

   c What is the average speed of the athlete in $km\,h^{-1}$?

   d What is the athlete's average velocity in $km\,h^{-1}$?

3.2 Acceleration

If you have been on a train as it pulled out of the station, you have experienced acceleration. If you have been in an aeroplane as it has taken off along a runway, you will have experienced a much greater acceleration. Astronauts and fighter pilots experience enormous accelerations that would make an untrained person lose consciousness. **Acceleration**, which is a measure of how quickly velocity changes, will be discussed in this section.

FINDING THE CHANGE IN VELOCITY AND SPEED

The velocity and speed of everyday objects are changing all the time. Examples of these are when a car moves away as the traffic lights turn green, when a tennis ball bounces or when you travel on a rollercoaster. If the initial and final velocity of an object are known, its **change in velocity** can be calculated.

To find the change, Δ, in any physical quantity, including speed and velocity, the initial value is taken away from the final value:

$$\Delta v = v_f - v_i$$

Change in speed is the final speed minus the initial speed:

$\Delta v = v_f - v_i$

where v_i is the initial speed ($m\,s^{-1}$)

v_f is the final speed ($m\,s^{-1}$)

Δv is the change in speed ($m\,s^{-1}$).

Since speed is a scalar, direction is not required.

Change in velocity is the final velocity minus the initial velocity:

$\Delta v = v_f - v_i$

where v_i is the initial velocity ($m\,s^{-1}$)

v_f is the final velocity ($m\,s^{-1}$)

Δv is the change in velocity ($m\,s^{-1}$).

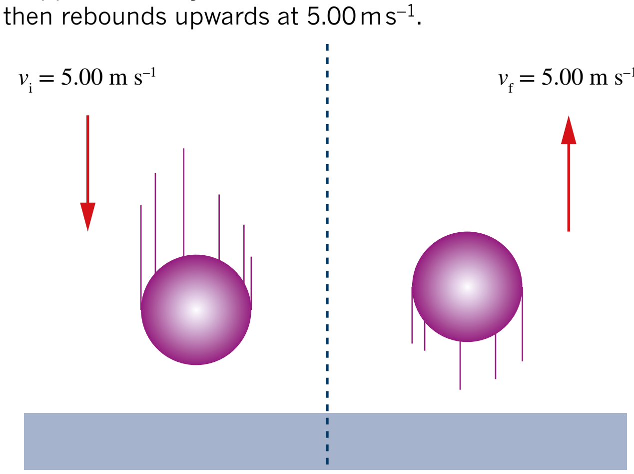
Since velocity is a vector, this should be done by performing a vector subtraction. As for all vectors, direction is required.

Vector subtraction was covered in detail in Section 2.3 on page 45.

Worked example 3.2.1

CHANGE IN SPEED AND VELOCITY 1

A golf ball is dropped vertically onto a concrete floor and strikes the floor at 5.00 $m\,s^{-1}$. It then rebounds upwards at 5.00 $m\,s^{-1}$.



a Calculate the change in speed of the ball.	
Thinking	**Working**
Find the values for the initial speed and the final speed of the ball.	$v_i = 5.00\,m\,s^{-1}$ $v_f = 5.00\,m\,s^{-1}$
Substitute the values into the change in speed equation: $\Delta v = v_f - v_i$	$\Delta v = v_f - v_i$ $\Delta v = (5.00) - (5.00)$ $\Delta v = 0.0\,m\,s^{-1}$

b Calculate the change in velocity of the ball.	
Thinking	**Working**
Velocity is a vector. Apply the sign convention up for positive and down for negative to replace the directions.	$v_i = 5.00\,m\,s^{-1}$ down $v_i = -5.00\,m\,s^{-1}$ $v_f = 5.00\,m\,s^{-1}$ up $v_f = +5.00\,m\,s^{-1}$
As the change in velocity equation is a vector subtraction equation, reverse the direction of v_i to get $-v_i$, then add the two vectors.	$v_i = -5.00\,m\,s^{-1}$, therefore $-v_i = +5.00\,m\,s^{-1}$
Substitute the values into the vector addition equation: $\Delta v = v_f + (-v_i)$	$\Delta v = v_f + (-v_i)$ $\Delta v = (+5.00) + (+5.00)$ $\Delta v = +10.0\,m\,s^{-1}$
Apply the sign convention to describe the direction.	$\Delta v = 10.0\,m\,s^{-1}$ up

Worked example: Try yourself 3.2.1

CHANGE IN SPEED AND VELOCITY 1

A golf ball is dropped onto a wooden floor and strikes the floor at $9.00\,m\,s^{-1}$. It then rebounds at $7.00\,m\,s^{-1}$.

a Calculate the change in speed of the ball.

b Calculate the change in velocity of the ball.

ACCELERATION

Consider the following information about the instantaneous velocity of a car that starts from rest as shown in Figure 3.2.1.

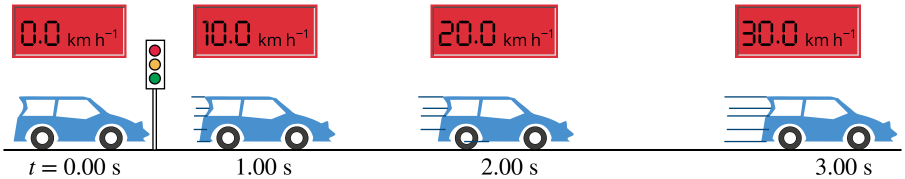


FIGURE 3.2.1 A car's acceleration as it increases in velocity from $0.0\,km\,h^{-1}$ to $30.0\,km\,h^{-1}$.

The velocity of the car pictured above increases by $10.0\,km\,h^{-1}$ each second. In other words, its velocity changes by $+10\,km\,h^{-1}$ per second. This is stated as an acceleration, $a = +10.0$ kilometres per hour per second or $+10.0\,km\,h^{-1}\,s^{-1}$. More commonly in physics, velocity information is given in metres per second.

The athlete in Figure 3.2.2 takes 3.00 s to come to a stop at the end of a race.

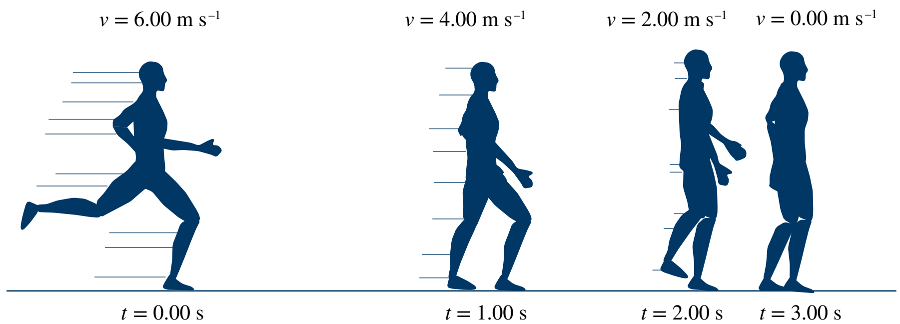


FIGURE 3.2.2 The velocity of the athlete changes by $-2.00\,\text{m s}^{-1}$ each second. Therefore, the acceleration, *a*, is $-2.00\,\text{m s}^{-2}$.

The velocity of the athlete changes by $-2.00\,\text{m s}^{-1}$ each second, so the acceleration is -2.00 metres per second per second. This is usually expressed as $a = -2.00$ metres per second squared or $a = -2.00\,\text{m s}^{-2}$.

A negative acceleration can mean that the object is slowing down in the direction of travel, as is the case with the athlete in Figure 3.2.2. A negative acceleration can also mean speeding up but in the opposite direction.

As acceleration is a vector quantity, vector diagrams can be used to calculate resultant accelerations of an object. Vector diagrams were covered in Chapter 2.

Average acceleration

As with speed and velocity, the average acceleration of an object can also be calculated. As all calculations of acceleration in this course are calculations of average acceleration, the av subscript can be assumed and so it can be omitted from your working.

Average acceleration, a_{av}, is the rate of change of velocity:

$$a = \frac{\text{change in velocity}}{\text{time taken}}$$

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v_f - v_i}{t_f - t_i}$$

where a is the acceleration (m s^{-2})

v_f is the final velocity (m s^{-1})

v_i is the initial velocity (m s^{-1})

$\Delta t = t_f - t_i$ is the time interval (s).

Worked example 3.2.2

CHANGE IN SPEED AND VELOCITY 2

A golf ball is dropped vertically onto a concrete floor and strikes the floor at $7.50\,\text{m s}^{-1}$. It then rebounds upwards at $7.50\,\text{m s}^{-1}$. The contact with the floor lasts for 25.0 milliseconds.

Calculate the average acceleration of the ball during its contact with the floor.

Thinking	Working
Note that the values you will need to calculate the average acceleration are initial velocity, final velocity and period of time. Use the convention that up is positive and down is negative. Convert 25.0 ms into s by multiplying by 10^{-3}, as the symbol m, for milli, represents 10^{-3}.	$v_i = -7.50\,\text{m s}^{-1}$ $-v_i = +7.50\,\text{m s}^{-1}$ $v_f = +7.50\,\text{m s}^{-1}$ $\Delta v = v_f + (-v_i)$ $\Delta v = (+7.50) + (+7.50)$ $\Delta v = +15.00\,\text{m s}^{-1}$ $\Delta t = 25.0\,\text{ms}$ $\Delta t = 25.0 \times 10^{-3}$ $\Delta t = 2.50 \times 10^{-2}\,\text{s}$

Substitute the values into the average acceleration equation.	$a = \frac{\text{change in velocity}}{\text{time taken}}$ $a = \frac{\Delta v}{\Delta t}$ $a = \frac{(+15.00)}{(2.50 \times 10^{-2})}$ $a = +600.00$ $a = +6.00 \times 10^{2}\,\text{m s}^{-2}$
Acceleration is a vector, so you must include a direction in your answer.	$a = 6.00 \times 10^{2}\,\text{m s}^{-2}$ up

Worked example: Try yourself 3.2.2

CHANGE IN SPEED AND VELOCITY 2

A netball is dropped vertically onto a court and strikes the surface at $9.00\,\text{m s}^{-1}$. It then rebounds upwards at $7.00\,\text{m s}^{-1}$. The contact time with the court is 35.0 milliseconds.

Calculate the average acceleration of the ball during its contact with the court.

PHYSICSFILE

Human acceleration

In the 1950s, the United States Air Force used a rocket sled to determine the effect of extremely large accelerations on humans. One of these sleds is shown in Figure 3.2.3. The aim was to find out the greatest accelerations that humans could safely withstand to help develop ejector seats for pilots.

The testing site consisted of an 800 m long railway track and a sled with nine rocket motors. One volunteer, Colonel John Stapp, was strapped into the sled and accelerated to speeds of over $1000\,\text{km h}^{-1}$ in a very short period of time. Water scoops were used to stop the sled abruptly in just 0.35 s. This equates to a deceleration of greater than $400\,\text{m s}^{-2}$. The effects of these massive accelerations are evident on his face (Figure 3.2.4).

Colonel John Stapp was a human guinea pig who suffered a great deal of discomfort so that other pilots would benefit. Safer ejector seats and non-human crash test dummies were developed because of these experiments.

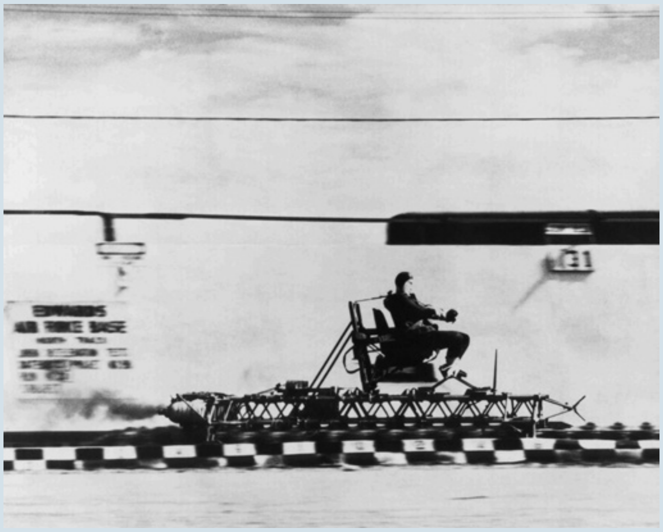


FIGURE 3.2.3 The rocket-powered sled used to test the effects of acceleration on humans.

FIGURE 3.2.4 Photos showing the distorted face of Colonel John Stapp.

3.2 Review

SUMMARY

- Change in speed is a scalar calculation:
  Δv = final speed – initial speed = $v_f - v_i$
- Change in velocity is a vector calculation:
  Δv = final velocity – initial velocity = $v_f - v_i$
- Acceleration is a vector. The acceleration of a body, *a*, is defined as the rate of change of velocity:

$$a = \frac{\text{change in velocity}}{\text{time taken}}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

- The standard unit of acceleration is metres per second per second ($m\,s^{-2}$).

KEY QUESTIONS

1 A radio-controlled car is travelling east at $10.0\,km\,h^{-1}$. After hitting some sand, it slows down to $3.00\,km\,h^{-1}$ east. Determine its change in speed.

2 A lump of Blu Tack is falling vertically at $5.00\,m\,s^{-1}$ and when it hits the floor it stops without rebounding. Calculate its change in velocity during the collision.

3 A ping pong ball is falling vertically at $6.00\,m\,s^{-1}$. As it hits the floor, it rebounds at $3.00\,m\,s^{-1}$ up. Calculate its change in velocity during the bounce.

4 While playing a 90 minute soccer match, Ashley is running north at $7.50\,m\,s^{-1}$. Ashley slides along the ground with 1.50 seconds remaining in the match and stops at the same time the referee ends the game. Calculate Ashley's average acceleration as they slide to a stop.

5 Dee launches a model rocket at $t = 0.00\,s$ vertically and it reaches a speed of $155\,km\,h^{-1}$ at $t = 3.50\,s$. What is the magnitude of its average acceleration in $km\,h^{-1}\,s^{-1}$?

6 A squash ball travelling east at $25.7\,m\,s^{-1}$ strikes the front wall of the court and rebounds at $15.2\,m\,s^{-1}$ west. The contact time between the wall and the ball is 0.0535 s. Calculate:

a the change in speed of the ball

b the change in velocity of the ball

c the average acceleration of the ball during its contact with the wall.

7 A greyhound starts from rest at $t = 0.00\,s$ and accelerates uniformly. Its velocity at $t = 1.25\,s$ is $8.08\,m\,s^{-1}$ south. Determine:

a the change in speed of the greyhound

b the change in velocity of the greyhound

c the acceleration of the greyhound.

8 How long does it take a vehicle travelling at $10.0\,m\,s^{-1}$ to reach $30.0\,m\,s^{-1}$ if it accelerates at $3.00\,m\,s^{-2}$?

9 A car travelling at $20.0\,m\,s^{-1}$ decelerates at $2.50\,m\,s^{-2}$. Calculate the time taken to stop.

10 A cyclist takes 4.00 s to slow down at $-3.00\,m\,s^{-2}$ and completely stop. Calculate the initial velocity of the cyclist.

3.3 Graphing position, velocity and acceleration over time

At times, even the motion of an object travelling in a straight line can be complicated. The object may travel forwards or backwards, speed up or slow down, or even stop. Where the motion remains in one dimension, however, the information can be more easily understood when presented in graphical form.

The main advantage of a graph compared with a table is that it allows the nature of the motion to be seen clearly. Information that is contained in a table is not as readily accessible or as easy to interpret as information presented graphically. This section examines position–time, velocity–time, and acceleration–time graphs.

POSITION–TIME (x–t) GRAPHS

A position–time graph indicates the position, x, of an object at any time, t, for motion that occurs over an extended time interval. However, the graph can also provide additional information.

Consider Sophie, shown in Figure 3.3.1, swimming laps of a 50.0 m pool. Her position–time data are shown in Table 3.3.1. The starting point is treated as the origin for this motion.

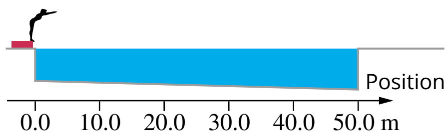


FIGURE 3.3.1 Swimmer standing at the end of a 50.0 m swimming pool.

TABLE 3.3.1 Positions and times of a swimmer completing 1.5 laps of a pool.

Time (s)	0.0	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	50.0	55.0	60.0
Position (m)	0.0	10.0	20.0	30.0	40.0	50.0	50.0	50.0	45.0	40.0	35.0	30.0	25.0

Analysis of Table 3.3.1 reveals several features of Sophie's swim. For the first 25.0 s, she swims at a constant rate. Every 5.00 s she travels 10.0 m in a positive direction, i.e. her velocity is $+2.00\,\mathrm{m\,s^{-1}}$. Then, from 25.0 s to 35.0 s, her position does not change. She seems to be resting, as she is stationary for this 10.0 s interval. Finally, from 35.0 s to 60.0 s, she swims back towards the starting point, in a negative direction. On this return lap, she maintains a more leisurely rate of 5.00 m every 5.00 s, so her velocity is $-1.00\,\mathrm{m\,s^{-1}}$. However, Sophie does not complete this lap, finishing 25.0 m from the start. This data is shown more conveniently on the position–time graph in Figure 3.3.2.

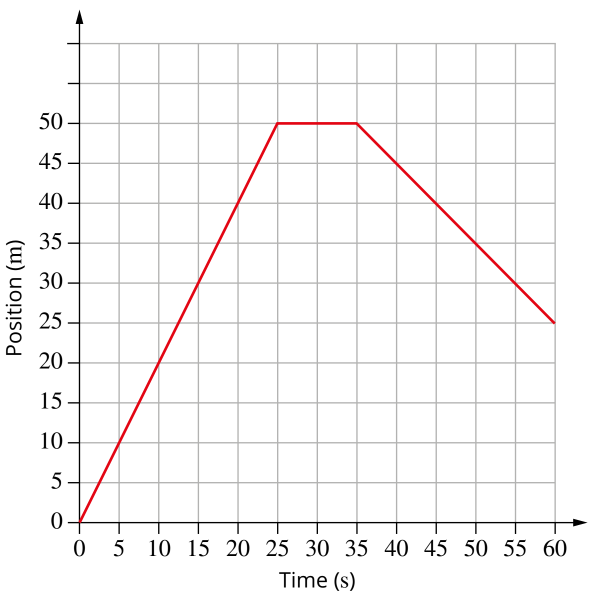


FIGURE 3.3.2 This position–time graph represents the motion of a swimmer travelling 50.0 m along a pool, then resting and swimming back towards the starting position. The swimmer finishes halfway along the pool.

The displacement, s, of the swimmer can be determined by comparing the initial and final positions. Her displacement between 20.0 s and 60.0 s is, for example:

$$s = \text{final position} - \text{initial position}$$
$$s = (25.0) - (40.0)$$
$$s = -15.0\,\text{m}$$

By further examining the graph, it can be seen that during the first 25.0 s the swimmer has a displacement of +50.0 m. Therefore, her average velocity is +2.00 m s^{-1}, i.e. 2.00 m s^{-1} to the right, during this time. This value can also be obtained by finding the gradient of this section of the graph.

A straight-line in a position–time graph (Figure 3.3.3) indicates a uniform velocity. The slope (gradient) of the line is equal to the velocity of the object.

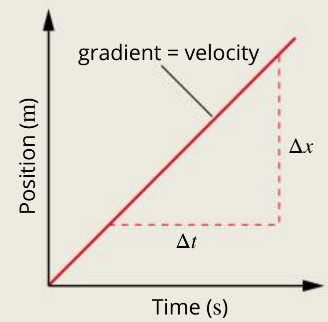


FIGURE 3.3.3 Position–time graph with gradient.

A positive velocity indicates that the object is moving in a positive direction and negative velocity indicates motion in a negative direction.

To confirm that the gradient of a position–time graph is a measure of velocity you can use **dimensional analysis**:

$$\text{gradient of } x\text{–}t \text{ graph} = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$$

The units of this gradient will be metres per second (m s^{-1}) so gradient is a measure of velocity. Note that the rise in the graph is the change in position, which is the definition of displacement; that is, $\Delta x = s$.

Non-uniform velocity

For motion with uniform (constant) velocity, the position–time graph will be a straight line, but if the velocity is non-uniform the graph will be curved. If the position–time graph is curved, the instantaneous velocity will be the gradient of the tangent to the line at the point of interest; the average velocity will be the gradient of the chord between two points. This is illustrated in Figure 3.3.4.

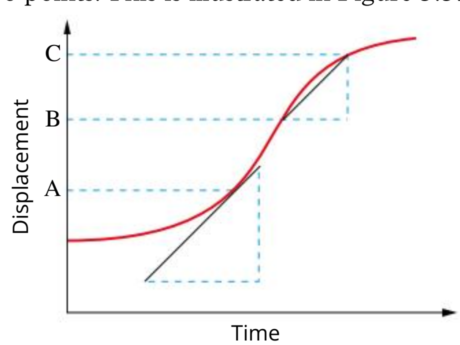
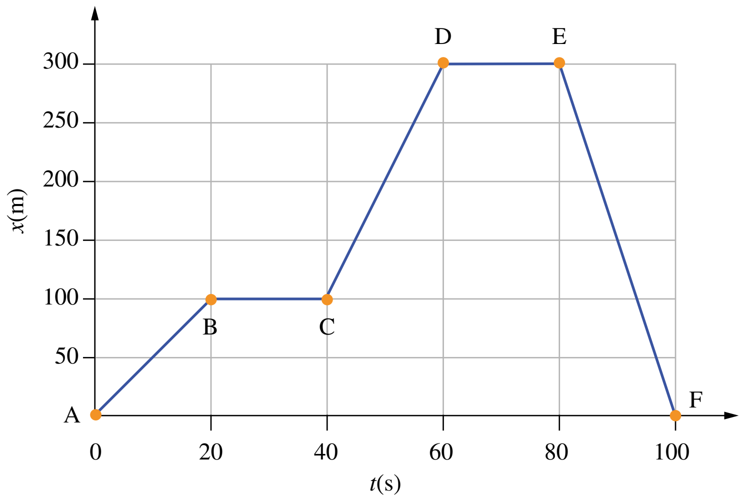


FIGURE 3.3.4 The instantaneous velocity at point A is the gradient of the tangent at that point. The average velocity between points B and C is the gradient of the chord between these points on the graph.

Worked example 3.3.1

ANALYSING A POSITION–TIME GRAPH

The motion of a cyclist is represented by the position–time graph below, with important features of the motion labelled A, B, C, D, E and F.



a What is the velocity of the cyclist between A and B?	
Thinking	**Working**
Determine the change in position (displacement) of the cyclist between A and B using: s = final position – initial position $s = \Delta x = x_f - x_i$	At A, $x_i = 0.0\,\text{m}$ At B, $x_f = 100.0\,\text{m}$ $s = (100.0) - (0.0)$ $s = +100.0\,\text{m}$ or $100.0\,\text{m}$ forwards (that is, away from the starting point)
Determine the time taken to travel from A to B. $\Delta t = t_f - t_i$	At A, $t_i = 0.0\,\text{s}$ At B, $t_f = 20.0\,\text{s}$ $\Delta t = (20.0) - (0.0)$ $\Delta t = 20.0\,\text{s}$
Calculate the gradient of the graph between A and B using: gradient of x–t graph $= \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$ Remember that $\Delta x = s$.	gradient $= \frac{\Delta x}{\Delta t}$ gradient $= \frac{(+100.0)}{(20.0)}$ gradient $= +5.00$
State the velocity, using: gradient of x–t graph = velocity Velocity is a vector so a direction must be given.	Since the gradient is $+5.00$, the velocity is $+5.00\,\text{m}\,\text{s}^{-1}$ or $5.00\,\text{m}\,\text{s}^{-1}$ forwards.

b Describe the motion of the cyclist between B and C.	
Thinking	**Working**
Interpret the shape of the graph between B and C.	The graph is flat between B and C, indicating that the cyclist's position is not changing during this time. So, the cyclist is not moving. If the cyclist is not moving, the velocity is $0\,\text{m}\,\text{s}^{-1}$.

You may confirm the result by calculating the gradient of the graph between B and C using: gradient of x–t graph = $\frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$ Remember that $\Delta x = s$.	gradient = $\frac{(0.00)}{(20.0)}$ gradient = 0.00 Since the gradient is 0.00, the velocity is $0.00\,\text{m}\,\text{s}^{-1}$.

Worked example: Try yourself 3.3.1

ANALYSING A POSITION–TIME GRAPH

Use the graph shown in Worked example 3.3.1 to answer the following questions.

a What is the velocity of the cyclist between E and F?

b Describe the motion of the cyclist between D and E.

VELOCITY–TIME (v–t) GRAPHS

A graph of velocity, v, against time, t, shows how the velocity of an object changes with time.

Analysing motion

A velocity–time graph is useful for analysing the motion of an object moving in a complex manner.

Consider the example of the girl in Figure 3.3.5. Aliyah is running back and forth along an aisle in a supermarket. A study of the velocity–time graph reveals that Aliyah is moving with a positive velocity, i.e. in a positive direction, for the first 6.0 s. Between the 6.0 s mark and the 7.0 s mark she is stationary, then she runs in the reverse direction, i.e. has a negative velocity, for the final 3.0 s.

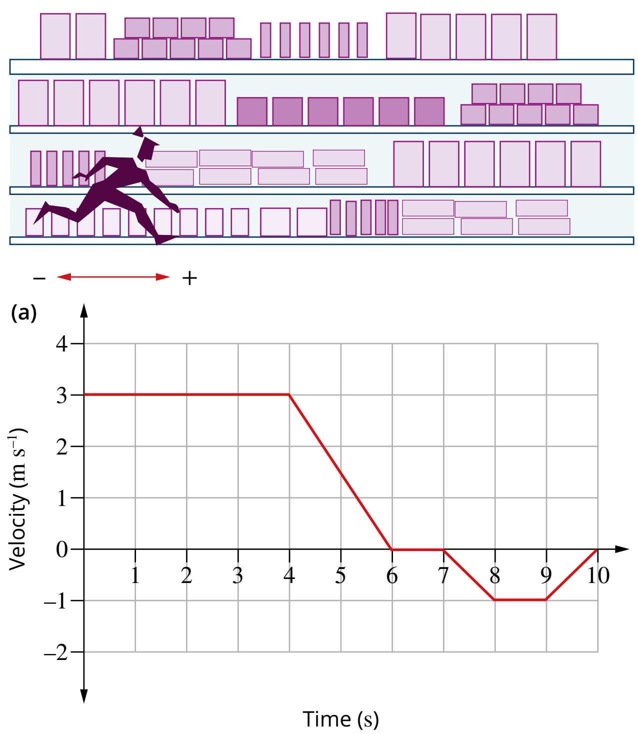


FIGURE 3.3.5 Diagram and v–t graph for a girl running along an aisle.

The graph shows Aliyah's velocity at each instant in time. She moves in a positive direction with a constant speed of $3.00\,\text{m}\,\text{s}^{-1}$ for the first 4.0 s. From 4.0 s to 6.0 s, she continues moving in a positive direction but slows down. At 6.0 s, she comes to a stop for 1.0 s. During the final 3.0 s, she accelerates in the negative direction for

1.0 s, then travels at a constant velocity of –1.0 m s^{-1} for 1.0 s. She then slows down and comes to a stop at 10.0 s. Remember that whenever the graph is below the time axis, velocity is negative, which indicates travel in the reverse direction. So, she is travelling in the reverse direction for the last 3.0 s of her journey.

Finding displacement

A velocity–time graph can also be used to find the displacement of the object under consideration, as shown in Figure 3.3.6.

Displacement, *s*, is given by the area under a velocity–time graph, i.e. the area between the graph and the time axis. It is important to note that an area below the time axis indicates a negative displacement, i.e. motion in a negative direction.

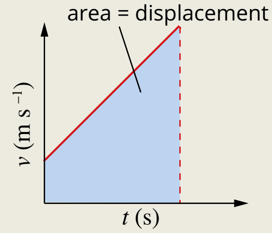


FIGURE 3.3.6 The area under a *v*–*t* graph gives displacement.

It is easier to see why the displacement is given by the area under the *v*–*t* graph when velocity is constant. For example, the graph in Figure 3.3.7 shows that in the first 6.0 s of motion, Aliyah moves with a constant velocity of +3.0 m s^{-1} for 4.0 s. Note that the area under the graph for this period of time is a rectangle. Her displacement, *s*, during this time can be determined by rearranging the formula for velocity:

$$v = \frac{s}{\Delta t}$$

$$\therefore s = v \times \Delta t$$

$$s = \text{height} \times \text{base}$$

$$s = \text{area under } v\text{–}t \text{ graph}$$

Aliyah then slows from 3.0 m s^{-1} to zero in the next 2.0 s. To understand why the displacement for this period of time is given by the triangular area under the graph requires more complicated mathematics known as calculus, which is beyond the scope of this book.

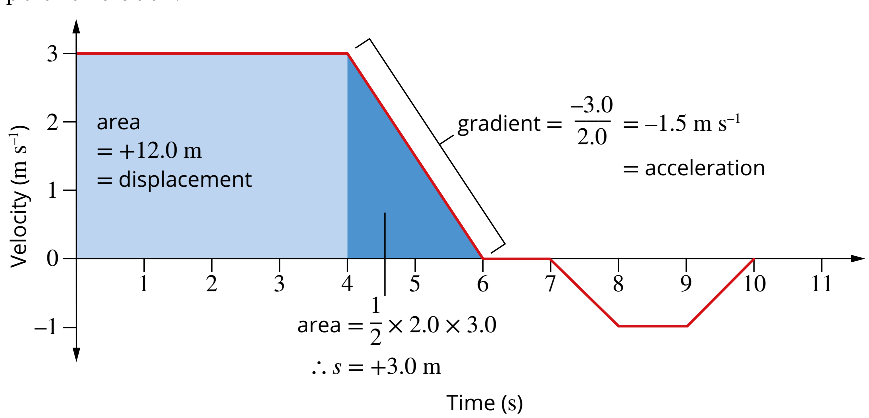


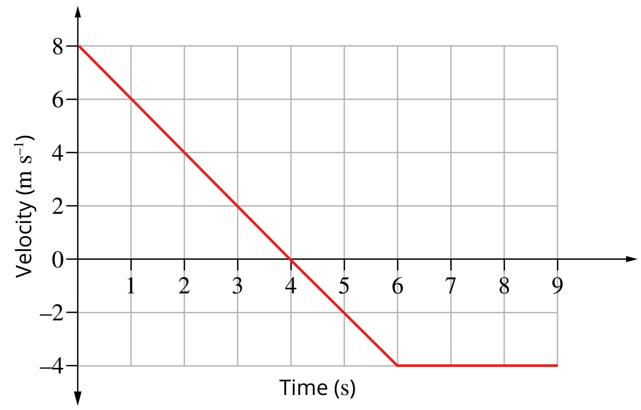
FIGURE 3.3.7 Area values as shown in a *v*–*t* graph.

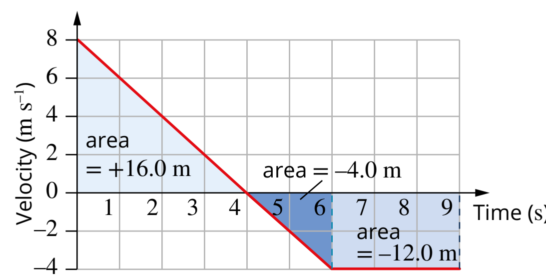
From Figure 3.3.7, the area under the graph for the first 4.0 s gives Aliyah's displacement during this time, i.e. +12.0 m. The displacement from 4.0 s to 6.0 s is represented by the area of the darker blue triangle and is equal to +3 m. The total displacement during the first 6 s is (+12.0 m) + (+ 3.0 m) = +15.0 m.

Worked example 3.3.2

ANALYSING A VELOCITY–TIME GRAPH

The motion of a radio-controlled car initially travelling east across a driveway in a straight line is represented by the graph below.



<table>
<tr><td colspan="2">a What is the displacement of the car during the first 4.0 s?</td></tr>
<tr><td>Thinking</td><td>Working</td></tr>
<tr><td>Displacement is the area under the graph. You must therefore find the area under the graph for the period of time for which you want to calculate the displacement.
As $s = v\Delta t$, or $s = \Delta t \times v$,
the base, b, is Δt and the height, h, is v.
Use $s = b \times h$ for squares and rectangles.
Use $s = \frac{1}{2}(b \times h)$ for triangles.</td><td>

The area from 0.0 to 4.0 s is a triangle, so:
$s = \frac{1}{2}(b \times h)$
$s = \frac{1}{2}(4.0)(+8.0)$
$s = +16.0\,\text{m}$</td></tr>
<tr><td>Displacement is a vector quantity, so a direction is needed.</td><td>displacement = 16.0 m east</td></tr>
</table>

<table>
<tr><td colspan="2">b What is the average velocity of the car for the first 4.0 s?</td></tr>
<tr><td>Thinking</td><td>Working</td></tr>
<tr><td>Identify the equation and variables, and apply the sign convention.</td><td>$v = \frac{s}{\Delta t}$
$s = +16.0\,\text{m}$
$\Delta t = 4.0\,\text{s}$</td></tr>
<tr><td>Substitute values into the equation:
$v = \frac{s}{\Delta t}$</td><td>$v = \frac{s}{\Delta t}$
$v = \frac{(+16.0)}{(4.0)}$
$v = +4.0000$</td></tr>
<tr><td>Velocity is a vector quantity, so a direction is needed.</td><td>$v = 4.0\,\text{m}\,\text{s}^{-1}$ east</td></tr>
</table>

Worked example: Try yourself 3.3.2

ANALYSING A VELOCITY–TIME GRAPH

Use the graph shown in Worked example 3.3.2 to answer the following questions.

a What is the displacement of the car from 4.0 to 6.0 s?

b What is the average velocity of the car from 4.0 to 6.0 s?

ACCELERATION FROM A VELOCITY–TIME (v–t) GRAPH

The acceleration of an object can also be found from a velocity–time graph, as shown in Figure 3.3.8.

The gradient of a velocity–time graph gives the average acceleration of the object over the time interval.

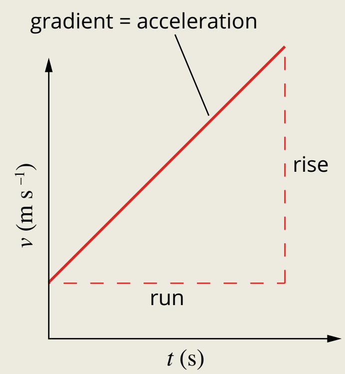


FIGURE 3.3.8 Gradient as displayed in a v–t graph.

Consider the motion of Aliyah in the 2.0 s interval between 4.0 s and 6.0 s on the graph in Figure 3.3.9. She is moving in a positive direction but slowing down from 3 m s^{-1} to rest.

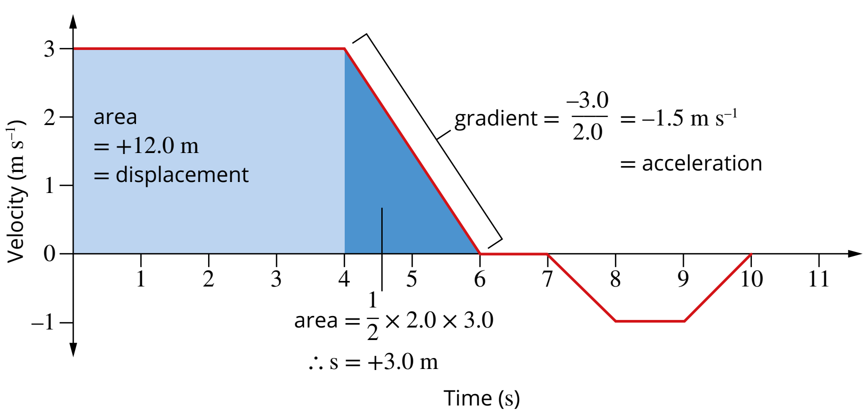


FIGURE 3.3.9 Acceleration as displayed in a v–t graph.

The gradient of the line from $t_i = 4.0$ s to $t_f = 6.0$ s is equal to her acceleration, as:

$$\text{gradient} = \frac{\Delta v}{\Delta t}$$

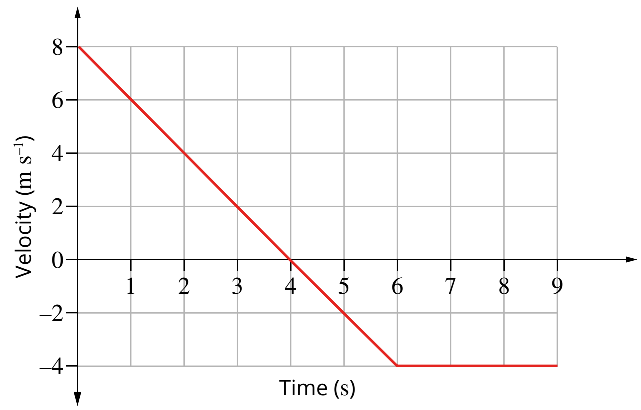
$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{(0.0) - (3.0)}{(2.0)} = -1.5\,\text{m s}^{-2}$$

Acceleration is the change in velocity divided by the period of time taken, which is equal to the gradient of the v–t graph. As can be seen from Figure 3.3.9 and the calculation above, the gradient of the line between 4.0 s and 6.0 s is $-1.5\,\text{m s}^{-2}$.

Worked example 3.3.3

FINDING ACCELERATION USING A VELOCITY–TIME GRAPH

Consider the motion of the radio-controlled car described in Worked example 3.3.2. The car initially travels east in a straight line across a driveway as shown by the graph below.



What is the acceleration of the car during the first 4.0 s?

Thinking	Working
Acceleration is the gradient of a v–t graph. Calculate the gradient using: $\text{gradient} = \frac{\text{rise}}{\text{run}}$ $a = \text{gradient} = \frac{\Delta v}{\Delta t}$	gradient from 0.0 to 4.0 s $= \frac{\text{rise}}{\text{run}}$ $a = \frac{\Delta v}{\Delta t}$ $a = \frac{v_f - v_i}{t_f - t_i}$ $a = \frac{(0.0) - (8.0)}{(4.0) - (0.0)}$ $a = -2.0000$ $a = -2.0\ \text{m s}^{-2}$
Acceleration is a vector quantity, so a direction is needed. Note: In this case, the car is moving in the easterly direction and slowing down.	$a = 2.0\ \text{m s}^{-2}$ west

Worked example: Try yourself 3.3.3

FINDING ACCELERATION USING A VELOCITY–TIME GRAPH

Use the graph shown in Worked example 3.3.3 to answer the following question.
What is the acceleration of the car during the period from 4.0 to 6.0 s?

DISTANCE TRAVELLED

A velocity–time graph can also be used to calculate the distance travelled by a moving object. The process of determining distance requires you to calculate the area under the v–t graph, as you would when calculating displacement. However, since distance travelled by an object always increases as the object moves, regardless of direction, you must add up all the areas between the graph and the time axis, regardless of whether the area is above or below the axis.

For example, Figure 3.3.10 shows the velocity–time graph of the radio-controlled car from Worked example 3.3.3. The area above the time-axis, which corresponds to motion in the positive direction, is +16.0 m, while the area below the axis, which corresponds to negative motion, consists of –4.0 m and –12.0 m. To calculate the total displacement, you would add up each displacement:

$$\text{total displacement } s = (+16.0) + (-4.0) + (-12.0)$$
$$s = (+16.0) + (-16.0)$$
$$s = 0.0\,\text{m}$$

To calculate the total distance, you would add up the magnitude of the areas, ignoring whether they are positive or negative:

$$\text{total distance } d = (16.0) + (4.0) + (12.0)$$
$$d = 32.0\,\text{m}$$

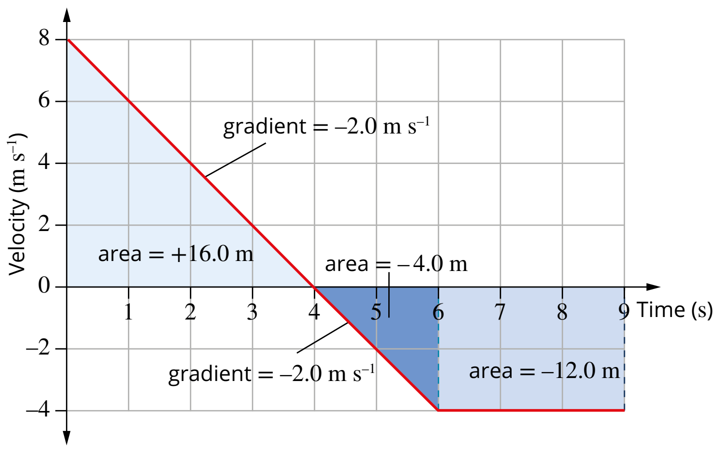


FIGURE 3.3.10 Both distance and displacement can be calculated by using the areas under the velocity–time graph.

Non-uniform acceleration

For motion with uniform (constant) acceleration, the velocity–time graph will be a straight line. For non-uniform acceleration, the velocity–time graph will be curved. If the velocity–time graph is curved, the instantaneous acceleration will be the gradient of the tangent to the curve at the point of interest; the average acceleration will be the gradient of the chord between two points. The displacement can still be calculated by finding the area under the graph; however, you will need to make some estimations.

PHYSICSFILE

Area under graphs

The calculation of the area under a graph is useful in many areas of physics.

Some examples include:

- power–time graphs, where the area represents the energy used over that period of time
- force–time graphs, where the area represents the impulse or change in momentum over a period time
- force–displacement graphs, where the area represents the work done or energy transferred while the forces are acting.

ACCELERATION–TIME (*a–t*) GRAPHS

An acceleration–time graph simply indicates the acceleration of the object as a function of time. The area under an acceleration–time graph is found by multiplying an acceleration, a, and the period of time, Δt, over which the acceleration changes. The area gives a change in velocity, Δv, value:

area = $\Delta v = a \times \Delta t$

In order to establish the actual velocity of the object, its initial velocity must be known. Figure 3.3.11 shows both Aliyah's velocity versus time (v–t) and acceleration versus time (a–t) graphs.

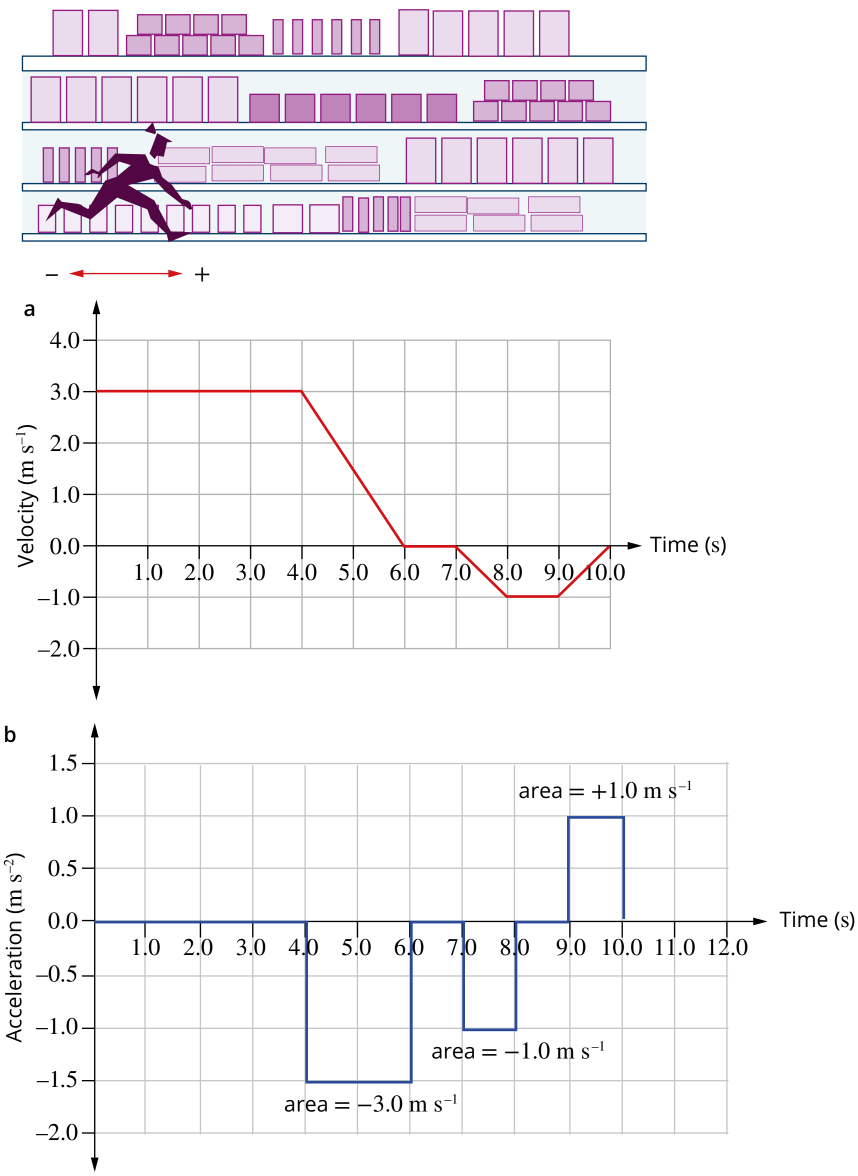


FIGURE 3.3.11 (a) Aliyah's velocity versus time (v–t) graph. (b) Aliyah's acceleration versus time (a–t) graph.

From $t_i = 4.0$ to $t_f = 6.0$ s, the area under the a–t graph shows that $\Delta v = -3.0\,\text{m}\,\text{s}^{-1}$. This indicates that she has slowed down by $3.0\,\text{m}\,\text{s}^{-1}$ during this period of time. Aliyah's v–t graph confirms this fact. Her initial speed is $3.0\,\text{m}\,\text{s}^{-1}$, so she must be stationary ($v = 0$) after 6.0 s. This calculation could not be made without knowing her initial velocity.

EXTENSION

Graphing in physics

Graphs in physics can be useful in solving problems as an alternative to using equations. An advantage of graphs is that they give a quick and easy picture of the relationship between the data that has been measured.

In this course, you will mainly analyse straight-line or linear graphs. A higher level of mathematical skills is required to analyse non-linear graphs or curves.

Differentiation is used to find the gradients of curves, where the intervals of change are infinitely small.

For a straight line, $\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$

If you consider a curve to be a series of infinitesimally small straight lines, then Δx and Δt are also extremely small. The gradient then becomes $\frac{\delta x}{\delta t}$, where the lower-case Greek letter delta, δ, denotes an infinitesimally small change. This is often written as $\frac{dx}{dt}$, i.e. the derivative of x with respect to t.

For a velocity–time graph, the gradient gives the acceleration—i.e. $a = \frac{dv}{dt}$ You have already seen that displacement can be found by calculating the area under a velocity–time graph, for example by breaking up the area under the graph into rectangles and triangles. Similarly, taking extremely small sections and adding them all together again gives the area under a non-linear graph, as shown in Figure 3.3.12. This is called integration, and for a curve given by a function *f(x)* can be written as:

$$\text{area} = \int_a^b f(x)dx$$

So, for a velocity–time graph, finding the area under the graph gives you the displacement:

$$s = \int_{t_1}^{t_2} v\,dt$$

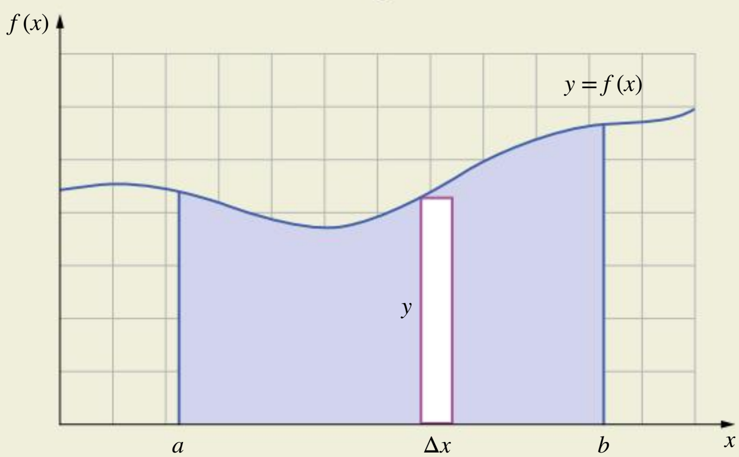


FIGURE 3.3.12 The small areas are added together by integration.

3.3 Review

SUMMARY

- A position–time (x–t) graph can be used to determine the location of an object at any given time. Additional information can also be derived from the graph:
  - displacement, s, is given by the change in position of an object
  - the velocity, v, of an object is given by the gradient of the position–time graph
  - if the position–time graph is curved, the gradient of the tangent at a point gives the instantaneous velocity, v.
- The gradient of a velocity–time (v–t) graph is the acceleration, a, of the object.
- The area under a velocity–time (v–t) graph is the displacement, s, of the object.
- The area under an acceleration–time graph (a–t) is the change in velocity, Δv, of the object.

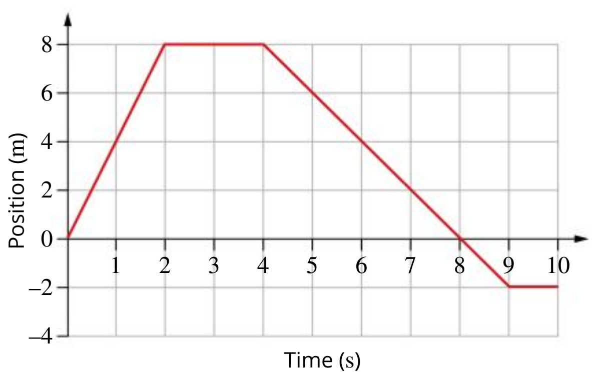
KEY QUESTIONS

1 Which of the following does the gradient of a position–time graph represent?

A displacement
B acceleration
C time
D velocity

The following information relates to questions 2–6.

The graph represents the straight-line motion of a radio-controlled toy car.



2 Describe the motion of the car in terms of its position.

3 Find out the position of the toy car after:

a 2.0 s
b 4.0 s
c 6.0 s
d 10.0 s.

4 When did the car return to its starting point?

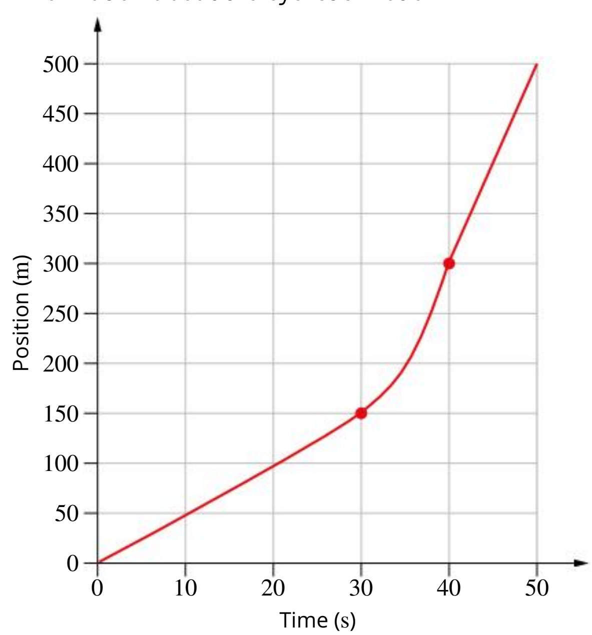
5 Calculate the velocity of the toy car:

a during the first 2.0 s
b at 3.0 s
c from $t_i = 4.0$ s to $t_f = 8.0$ s
d at 8.0 s
e from $t_i = 8.0$ s to $t_f = 9.0$ s.

6 During its total 10.0 s of motion, what was the car's:

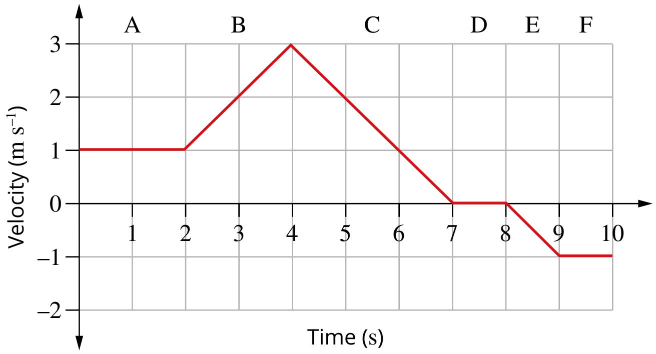
a distance travelled
b displacement?

7 The position–time graph for a cyclist travelling north along a straight road is shown. Calculate the following information about the cyclist's motion.



a What was the average speed of the cyclist during the first 30.0 s?
b What was the average velocity of the cyclist during the final 10.0 s?
c What was the average velocity of the cyclist for the whole trip?

8 The graph in the figure below shows the motion of a dog running along a footpath.



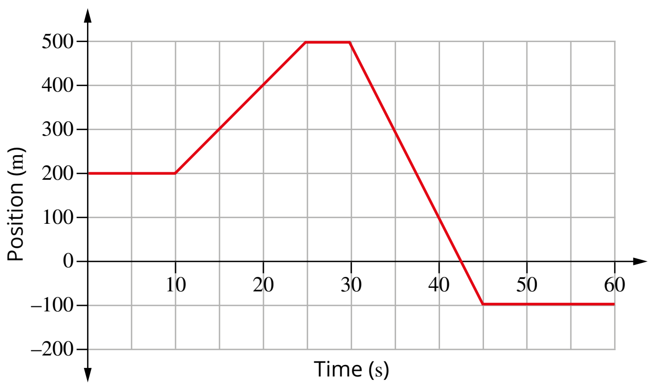
a What is the magnitude of the acceleration of the dog at $t = 1.0$ s?

b What is the magnitude of the acceleration of the dog at $t = 5.0$ s?

c What is the magnitude of the displacement of the dog for the first 7.0 s?

d What is the magnitude of the average velocity of the dog over the first 7.0 s?

9 The graph below shows the position of a motorbike along a straight stretch of road as a function of time. The motorcyclist starts 200.0 m north of a town.

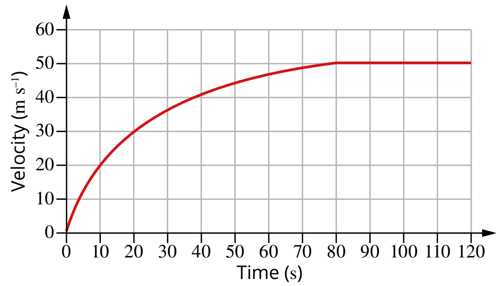


Calculate the instantaneous velocity of the motorcyclist at each of the following times:

a 15.0 s

b 35.0 s.

10 The straight-line motion of a high-speed train is shown in the graph below.



a How long does it take the train to reach its maximum speed?

b What is the acceleration of the train 10.0 s after starting?

c What is the acceleration of the train 40.0 s after starting?

d By counting squares, estimate the displacement of the train after 120.0 s.

3.4 Equations for uniform acceleration

A graph is an excellent way of representing motion as it provides a great deal of information that is easy to visualise and interpret. However, a graph is time-consuming to draw, and sometimes values can only be estimated rather than precisely calculated.

In the previous section, various graphs of motion were used to determine quantities such as displacement, velocity and acceleration. This section examines a more precise method of solving problems involving *constant* or *uniform acceleration*. This method involves the use of a series of equations that can be derived from the basic definitions developed earlier.

DERIVING THE EQUATIONS

Consider an object moving in a straight line with an initial velocity, v_i, and a uniform acceleration, a, for a time interval, Δt. As the variables v_i, v_f, and a are vectors, and the motion is limited to one dimension, the sign and direction convention of right as positive and left as negative can be used. After a period of time, Δt, the object has changed its velocity from an initial velocity of v_i and is now travelling with a final velocity of v_f. Its acceleration will be given by:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

If the initial time is t_i and the final time is t_f, then $\Delta t = t_f - t_i$. The above equation can then be rearranged as:

$$v_f = v_i + a\Delta t \qquad \text{(i)}$$

The average velocity of the object is:

$$v_{av} = \frac{s}{\Delta t}$$

When acceleration is uniform, average velocity, v_{av}, can also be found as the average of the initial and final velocities:

$$v_{av} = \frac{1}{2}(v_i + v_f)$$

This relationship is shown graphically in Figure 3.4.1.

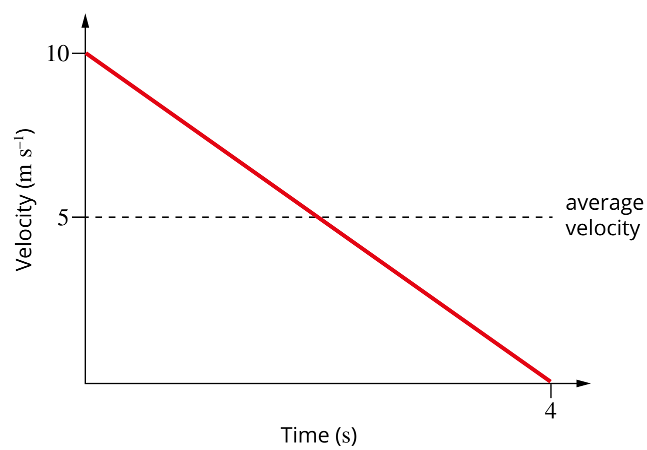


FIGURE 3.4.1 Uniform acceleration as displayed by a v–t graph.

By combining these two equations of average velocity, we get:

$$\frac{s}{\Delta t} = \frac{1}{2}(v_i + v_f)$$

This gives:

$$s = \frac{1}{2}(v_i + v_f)\Delta t \qquad \text{(ii)}$$

A graph describing constant acceleration motion is shown in Figure 3.4.2. For constant acceleration, the velocity is increasing by the same amount in each time interval, so the gradient of the v–t graph is constant. The displacement, which is equal to the area under the v–t graph, is given by the combined area of the rectangle and the triangle:

$$\text{area} = s = s_1 + s_2$$

$$s = (v_i \times \Delta t) + \frac{1}{2}(v_f - v_i) \times \Delta t$$

As $a = \frac{v_f - v_i}{\Delta t}$ then $v_f - v_i = a\Delta t$, and this can be substituted for $v_f - v_i$ in the equation above to give:

$$s = (v_i \times \Delta t) + \frac{1}{2}(a\Delta t)\Delta t$$

$$s = (v_i \Delta t) + \frac{1}{2}a\Delta t^2$$

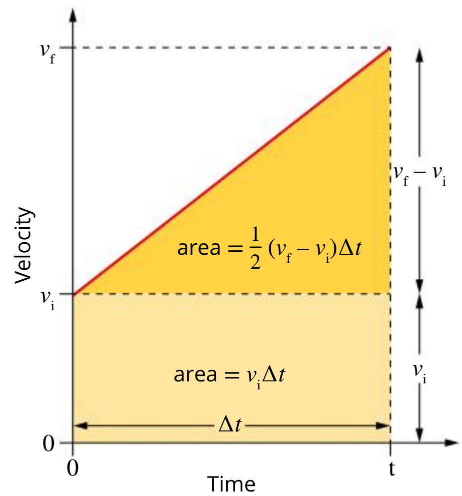


FIGURE 3.4.2 The area under a v–t graph broken up into a rectangle and a triangle.

$$s = v_i\Delta t + \frac{1}{2}a\Delta t^2 \qquad \text{(iii)}$$

Another way to calculate the area under the graph is to use the large area ($v_f\Delta t$) and subtract the triangle component ($\frac{1}{2}a\Delta t^2$). This will give you:

$$s = v_f\Delta t - \frac{1}{2}a\Delta t^2 \qquad \text{(iv)}$$

Rewriting equation (i) with Δt as the subject gives:

$$\Delta t = \frac{v_f - v_i}{a}$$

Now, if this is substituted into equation (ii):

$$s = \frac{1}{2}(v_f - v_i)\Delta t \qquad \text{(ii)}$$

$$s = \frac{v_i + v_f}{2} \times \frac{v_f - v_i}{a}$$

Multiplying the top line and bottom line gives:

$$s = \frac{v_f^2 - v_i^2}{2a}$$

Finally, transposing this gives:

$$v_f^2 = v_i^2 + 2as \qquad \text{(v)}$$

Equations (i) to (v) are commonly used to solve problems in which acceleration is constant. They are summarised below.

$$v_f = v_i + a\Delta t$$

$$s = \frac{1}{2}(v_i + v_f)\Delta t$$

$$s = v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$s = v_f\Delta t - \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2as$$

where s is the displacement (m)

v_i is the initial velocity (m s^{-1})

v_f is the final velocity (m s^{-1})

a is the acceleration (m s^{-2})

Δt is the time taken (s).

SOLVING PROBLEMS USING EQUATIONS

When solving problems using these equations, it is important to think about the problem and try to visualise what is happening. Follow the steps below.

Step 1 Draw a simple diagram of the situation.

Step 2 Write down the information that has been given in the question. You might like to use the word 'sifat' as a memory prompt to help you remember the list of variables in the order s, v_i, v_f, a and Δt. Use a sign convention to assign positive and negative values to indicate directions. Convert all units to SI form.

Step 3 Select the equation that matches your data. It should include three values that you know, and the one value that you want to solve.

Step 4 Write your preliminary answer to five or six significant figures, which may then be used in any subsequent questions, i.e. part (b), (c) etc. This will help to reduce rounding errors in subsequent answers.

Step 5 Use the appropriate number of significant figures in your final answer. If your answer is greater than 9999 or less than 0.001, provide your answer in scientific notation.

Step 6 Include units with the final answer and specify a direction if the quantity is a vector.

Worked example 3.4.1

USING THE EQUATIONS OF MOTION

A snowboarder in a race is travelling 10.0 m s^{-1} north as she crosses the finishing line. She then decelerates uniformly, coming to a stop over a distance of 20.0 m.

a Calculate her acceleration as she comes to a stop.

Thinking	Working
Write down the known quantities as well as the quantity you are finding. (The term 'sifat' may help you to recall them.) Apply the sign convention that north is positive and south is negative.	Take all the information that you can from the question: • constant acceleration, so use equations for uniform acceleration • 'coming to a stop' means that the final velocity is zero. $s = +20.0$ m $v_i = +10.0$ m s^{-1} $v_f = 0.00$ m s^{-1} $a = ?$ $\Delta t = ?$
Identify the correct equation to use.	$v_f^2 = v_i^2 + 2as$
Substitute known values into the equation and solve for a.	$v_f^2 = v_i^2 + 2as$ $a = \frac{v_f^2 - v_i^2}{2s}$ $a = \frac{(0.0)^2 - (+10.0)^2}{2(+20.0)}$ $a = \frac{(-100.0)}{(+40.0)}$ $a = -2.5000$
Use the sign convention to state the answer with its direction, units and the correct number of significant figures.	$a = 2.50$ m s^{-2} south

b How long does she take to come to a stop?	
Thinking	**Working**
Write down the known quantities as well as the quantity you are finding. (The term 'sifat' may help you to recall them.) Apply the sign convention that north is positive and south is negative.	Take all the information that you can from the question: • constant acceleration, so use equations for uniform acceleration • 'coming to a stop' means that the final velocity is zero. $s = +20.0\,\text{m}$ $v_i = +10.0\,\text{m}\,\text{s}^{-1}$ $v_f = 0.00\,\text{m}\,\text{s}^{-1}$ $a = -2.5000\,\text{m}\,\text{s}^{-2}$ $\Delta t = ?$
Identify the correct equation to use. Since you now know four values, any equation involving Δt will work.	$v_f = v_i + a\Delta t$
Substitute known values into the equation and solve for Δt.	$v_f = v_i + a\Delta t$ $\Delta t = \frac{v_f - v_i}{a}$ $\Delta t = \frac{(0.00) - (10.0)}{(-2.5000)}$ $\Delta t = 4.0000\,\text{s}$
State the answer with its units and the correct number of significant figures.	$\Delta t = 4.00\,\text{s}$

c What is the average velocity of the snowboarder as she comes to a stop?	
Thinking	**Working**
Write down the known quantities as well as the quantity that you are finding. Apply the sign convention that north is positive and south is negative.	Take all the information that you can from the question: • constant acceleration, so we only need to find the average of the final and initial speeds. $v_i = +10.0\,\text{m}\,\text{s}^{-1}$ $v_f = 0.00\,\text{m}\,\text{s}^{-1}$ $v_{av} = ?$
Identify the correct equation to use.	$v_{av} = \frac{1}{2}(v_f + v_i)$
Substitute known values into the equation and solve for v_{av}. Include units with the answer.	$v_{av} = \frac{1}{2}(v_f + v_i)$ $v_{av} = \frac{1}{2}(0.00 + 10.0)$ $v_{av} = 5.0000$
Use the sign convention to state the answer with its direction, units and the correct number of significant figures.	$v_{av} = 5.00\,\text{m}\,\text{s}^{-1}$ north

Worked example: Try yourself 3.4.1

USING THE EQUATIONS OF MOTION

A snowboarder in a race is travelling $15.5\,\text{m}\,\text{s}^{-1}$ east as she crosses the finishing line. She then decelerates uniformly until coming to a stop over a distance of 30.0 m.

a Calculate her acceleration as she comes to a stop.

b How long does she take to come to a stop?

c What is the average velocity of the snowboarder as she comes to a stop?

3.4 Review

SUMMARY

- The following equations can be used for situations in which there is a constant acceleration, where:
  s is the displacement (m)
  v_i is the initial velocity ($m\,s^{-1}$)
  v_f is the final velocity ($m\,s^{-1}$)
  a is the acceleration ($m\,s^{-2}$)
  Δt is the period of time (s).
- $v_f = v_i + a\Delta t$
- $s = \frac{1}{2}(v_i + v_f)\Delta t$
- $s = v_i\Delta t + \frac{1}{2}a\Delta t^2$
- $s = v_f\Delta t - \frac{1}{2}a\Delta t^2$
- $v_f^2 = v_i^2 + 2as$
- $v_{av} = \frac{s}{\Delta t} = \frac{v_i + v_f}{2}$
- A sign and direction convention for motion in one dimension needs to be used with these equations.

KEY QUESTIONS

1 A cyclist has a uniform acceleration as they roll down a hill. Their initial speed is $5.09\,m\,s^{-1}$. They travel a distance of 32.5 m and their final speed is $18.3\,m\,s^{-1}$. Which equation should be used to determine their acceleration?

A $v_f = v_i + a\Delta t$

B $s = \frac{1}{2}(v_i + v_f)\Delta t$

C $s = v_i\Delta t + \frac{1}{2}a\Delta t^2$

D $s = v_f\Delta t - \frac{1}{2}a\Delta t^2$

E $v_f^2 = v_i^2 + 2as$

2 A new-model hydrogen vehicle travels with a uniform acceleration on a racetrack. It starts from rest and covers 445 m in 16.0 s.

a Calculate the magnitude of its average acceleration during this time.

b What is the final speed of the car in $m\,s^{-1}$?

c What is the car's final speed in $km\,h^{-1}$?

3 An electric hybrid car starts from rest and accelerates uniformly in a positive direction for 3.10 s. It reaches a final speed of $19.9\,m\,s^{-1}$.

a Calculate the magnitude of the acceleration of the hybrid car.

b What is the magnitude of the average velocity of the hybrid car during this time?

c What is the distance travelled by the hybrid car during this time?

4 During its launch phase, a space rocket accelerates uniformly from rest to $167\,m\,s^{-1}$ upwards in 4.02 s, then enters a constant speed phase of $167\,m\,s^{-1}$ for the next 5.40 s.

a Calculate the acceleration of the rocket in its initial launch phase.

b Calculate the combined distance (in km) the rocket travels during both phases of the flight.

c What is the final speed of the rocket in $km\,h^{-1}$?

d What is the average speed of the rocket during the first 4.02 s?

e What is the average speed of the rocket during the total 9.42 seconds of motion?

5 While overtaking another cyclist, Charlie increases their speed uniformly from $4.12\,m\,s^{-1}$ to $6.07\,m\,s^{-1}$ east over a time interval of 0.508 s.

a Calculate the magnitude of Charlie's average acceleration during this time.

b How far does Charlie travel while accelerating?

c What is Charlie's average speed during this time?

6 A diver enters a diving pool headfirst while travelling at $18.0\,m\,s^{-1}$ downwards. The diver hits the water at $t_i = 0.00$ s and stops after a downwards displacement of 4.06 m. Consider the diver to be a single point located at their centre of mass and assume their acceleration through the water to be uniform.

a What is the magnitude of the average acceleration of the diver as the diver travels through the water?

b How long does the diver take to come to a stop?

c What is the velocity of the diver after they have dived through 2.00 m of water?

7 A car is travelling along a straight road at $75.0\,km\,h^{-1}$ east. In an attempt to avoid an accident, the motorist has to brake suddenly and stop the car.

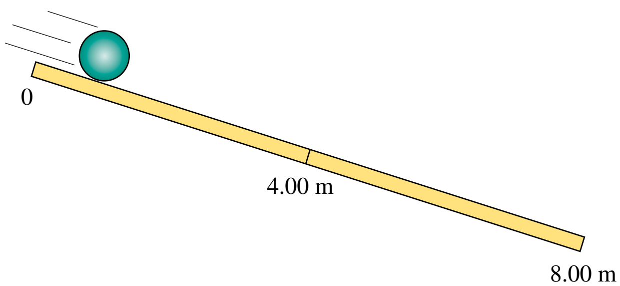
a What is the car's initial speed in $m\,s^{-1}$?

b If the reaction time of the motorist is 0.254 s, what is the displacement of the car before they are able to apply the brakes?

c Once the brakes are applied, the car has an acceleration of $-6.70\,m\,s^{-2}$. How far does the car travel while stopping?

d What is the total displacement of the car from the time the driver first notices the danger to when the car comes to a stop?

8 A billiard ball rolls from rest down a smooth ramp that is 8.00 m long. The acceleration of the ball is constant at $2.60\,m\,s^{-2}$.



a What is the velocity of the ball when it is halfway down the ramp?

b What is the final velocity of the ball at the bottom of the ramp?

c How long does the ball take to roll the first 4.00 m?

d How long does the ball take to travel the final 4.00 m?

9 A cyclist, Nolan, is travelling at a constant speed of $12.2\,m\,s^{-1}$ when they pass a stationary bus. The bus starts moving just as Nolan passes, and it accelerates uniformly at $1.50\,m\,s^{-2}$.

a When does the bus reach the same speed as Nolan?

b How long does the bus take to catch Nolan?

c What distance has Nolan travelled before the bus catches up?

3.5 Vertical motion

Until 500 years ago, it was widely believed that the heavier an object was, the faster it would fall. This was the theory proposed by Aristotle, and it lasted for 2000 years until the end of the Middle Ages. In the seventeenth century, the Italian scientist Galileo conducted experiments that showed that the mass of the object did not affect the rate at which it fell, as long as **air resistance** was not a factor.

It is now known that falling objects speed up because of gravity; however, many people still think that objects with a greater mass fall faster than objects of a lesser mass. This confusion often arises because they fail to consider the effects of air resistance. This section examines the motion of falling objects..

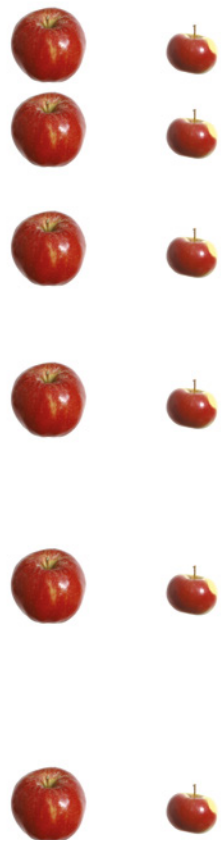


FIGURE 3.5.1 A stroboscopic image of a free-falling apple. The time elapsed between each image of the apple is the same but the vertical displacement increases during each period of time, which shows the apple is accelerating. Without air resistance, the two different mass apples accelerate at the same rate.

ANALYSING VERTICAL MOTION

Some falling objects are affected by air resistance more than others; for example, feathers and balloons. This is why these objects do not speed up much as they fall. However, if air resistance can be ignored, all bodies in **free fall** near the Earth's surface will move with an equal downwards acceleration. The stroboscopic image in Figure 3.5.1 clearly shows an apple accelerating as it falls, since the vertical displacement of the apple between each photograph increases. In a vacuum, this acceleration would be the same for a feather, a bowling ball, or any other object. The mass of the object does not matter if air resistance is removed.

At the Earth's surface, the acceleration due to gravity, g, is $-9.80\,\text{m}\,\text{s}^{-2}$, where the negative sign indicates a downwards direction. The acceleration of a body due to gravity is independent of its initial velocity and is the same whether the object has been thrown vertically upwards or is falling vertically downwards.

For example, a coin that is dropped from rest at $t = 0.00\,\text{s}$ will have an initial velocity of $0.00\,\text{m}\,\text{s}^{-1}$. At $t = 1.00\,\text{s}$ it will be falling with a velocity of $-9.80\,\text{m}\,\text{s}^{-1}$ and at $t = 2.00\,\text{s}$ with a velocity of $-19.6\,\text{m}\,\text{s}^{-1}$, and so on. As Δt increases with each second, the coin's velocity increases by $-9.80\,\text{m}\,\text{s}^{-1}$. The motion of a falling coin is illustrated in Figure 3.5.2.

However, if the coin was launched straight up at $t = 0.00\,\text{s}$ with an initial velocity of $+19.6\,\text{m}\,\text{s}^{-1}$, then at $t = 1.00\,\text{s}$ its velocity would be $+9.80\,\text{m}\,\text{s}^{-1}$ and at $t = 2.00\,\text{s}$ its velocity would be $0.00\,\text{m}\,\text{s}^{-1}$. In other words, with each second of the coin's upwards journey, its velocity would decrease by $-9.80\,\text{m}\,\text{s}^{-1}$. At the instant in time the velocity reaches $0.00\,\text{m}\,\text{s}^{-1}$, the motion of the coin changes from upwards to downwards, which means it would accelerate downwards as described in the previous paragraph. This point in time also represents the time at which the coin reaches its maximum vertical displacement. The motion of a coin thrown vertically upwards is shown in Figure 3.5.3.

So, regardless of whether the coin is falling vertically downwards or is flipped vertically upwards, its speed changes at the same rate. The speed of the falling coin *increases* by $-9.80\,\text{m}\,\text{s}^{-1}$ each second and the speed of the rising coin *decreases* by $-9.8\,\text{m}\,\text{s}^{-1}$ each second. That means that the acceleration of the coin due to gravity is $-9.80\,\text{m}\,\text{s}^{-2}$, or $9.80\,\text{m}\,\text{s}^{-2}$ downwards, in both cases.

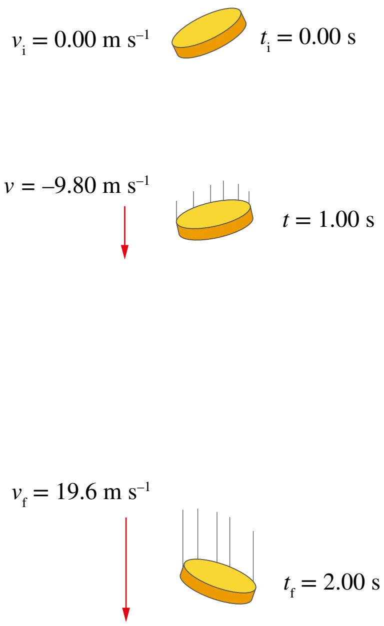


FIGURE 3.5.2 A falling coin.

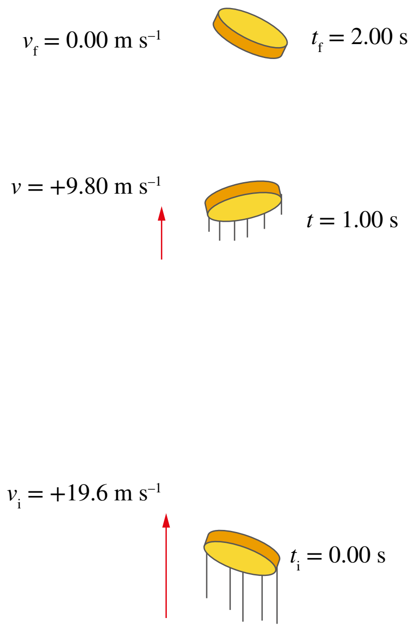


FIGURE 3.5.3 A coin thrown vertically upwards.

PHYSICSFILE

Galileo's experiment carried out on the Moon

In 1971, David Scott went to great lengths to show that Galileo's prediction was correct. As an astronaut on the Apollo 15 Moon mission, he took a hammer and a feather on the voyage. He stepped onto the lunar surface, held the feather and hammer at the same height and dropped them together. As Galileo had predicted 400 years earlier, in the absence of any air resistance, the two objects fell side by side as they accelerated towards the Moon's surface at exactly the same rate.

Popular physicist, musician and TV presenter Professor Brian Cox repeated a version of this experiment in the world's biggest vacuum chamber, the Space Simulation Chamber at NASA's Space Power Facility in Ohio. Professor Cox set up a mechanist that would drop a bowling ball and a feather at exactly the same time. When the air was removed from the chamber, they filmed the two different masses accelerating for over 9 metres at exactly the same rate.

FIGURE 3.5.4 Astronaut David Scott dropping a feather and a hammer on the Moon.

PHYSICS IN ACTION

Theories of motion: Aristotle and Galileo

Aristotle was a Greek philosopher who lived in the fourth century BCE. He was such an influential individual that his ideas on motion were generally accepted for nearly 2000 years. Aristotle didn't do experiments as we know them today, but simply *thought* about different bodies in motion to arrive at a plausible explanation for the way in which they moved.

Aristotle spent a lot of time classifying different animals and adopted a similar approach to his study of motion. His theory gave inanimate objects, such as rocks and rain, characteristics that were similar to living things. Aristotle then organised these objects into four terrestrial groups or elements: earth, water, air and fire (see Figure 3.5.5). He said that any object was a mixture of these elements in different proportions.

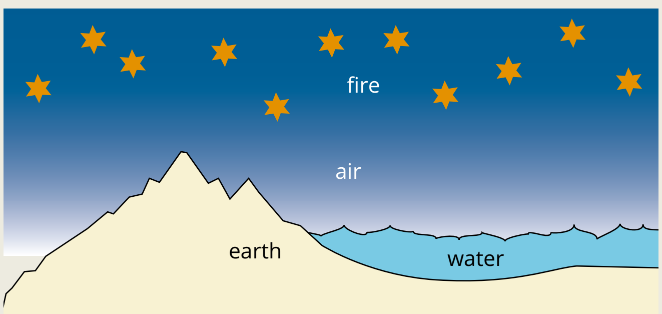


FIGURE 3.5.5 Aristotle's four elements of the universe: earth, water, air and fire.

According to Aristotle, a body would move because of a tendency that could come from inside or outside of the body. An internal tendency would cause 'natural' motion and result in a body returning to its proper place. For example, if a rock, which is an earth substance, is held in the air and released, then its natural tendency would be to return to the Earth, i.e. it falls to the ground. Similarly, fire was thought to head upwards, to return to its proper place in the universe, i.e. the Sun.

An external push that acts when something is thrown or hit was the cause of 'violent' motion according to the Aristotelian model. In other words, an external push acted to take a body away from its proper place. For example, when an apple is thrown into the air, a violent motion carries the apple away from the Earth, but then the natural tendency of the apple takes over and it returns to the ground.

Aristotle's theory worked quite well and could be used to explain the motion of many objects. However, there were also many examples that it could not successfully explain, such as why some solids floated while other solids sunk.

Aristotle explained the behaviour of a falling body by saying that its velocity depended on how much earth element it contained. This suggested that a 2.00 kg cat would fall twice as fast and in half the time as a 1.00 kg cat dropped from the same height. Many centuries later, Galileo Galilei (pictured in Figure 3.5.6) noticed that, at the start of a hailstorm, small hailstones arrived at the same time as large hailstones. This caused Galileo to doubt Aristotle's theory, and so he set about finding a better explanation for the motion of freely falling bodies.

FIGURE 3.5.6 Galileo Galilei.

A famous story in science is that of Galileo dropping different masses from the Leaning Tower of Pisa in Italy. This story may or may not be true, but Galileo did perform a very detailed analysis of falling bodies. Galileo used inclined planes because freely falling bodies moved too fast to analyse. He completed detailed experiments that showed conclusively that Aristotle was in fact incorrect.

By using a water clock to time balls as they rolled from rest down different inclines, he was able to show that the balls were accelerating, and that the distance they travelled was proportional to the square of the period of time, i.e. $d \propto \Delta t^2$.

Galileo found that this relationship also held true when he inclined the plane at larger and larger angles, allowing him to conclude that freely falling bodies actually fall with a uniform acceleration.

Since the acceleration of a free-falling body is constant, it is appropriate to use the equations that were studied in the previous section, under 'Equations for uniform acceleration'. It is necessary to specify whether up or down is positive when doing these problems, though you can simply follow the mathematical convention of regarding up as positive, which would mean the acceleration due to gravity would always be $-9.80\,m\,s^{-2}$. The variable for uniform acceleration, a, in these equations can be replaced by the variable for gravitational acceleration, g, in calculations involving vertical motion.

Worked example 3.5.1

VERTICAL MOTION

A construction worker accidentally knocks a brick from a building so that it falls vertically a distance of 47.0 m to the ground. Use $g = -9.80\,m\,s^{-2}$ and ignore air resistance when answering these questions.

a How long does the brick take to fall halfway, to 23.5 m?

Thinking	Working
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that up is positive and down is negative.	The brick starts at rest, so: $s = -23.5\,m$ $v_i = 0.00\,m\,s^{-1}$ $v_f = ?\,m\,s^{-1}$ $g = -9.80\,m\,s^{-2}$ $\Delta t = ?$
Identify the correct equation for uniform acceleration to use, but substitute the a for g.	$s = v_i\Delta t + \frac{1}{2}g\Delta t^2$
Substitute known values into the equation and solve for Δt. Think about whether the value seems reasonable.	$(-23.5) = (0.00)\Delta t + \frac{1}{2}(-9.80)\Delta t^2$ $(-23.5) = (-4.90)\Delta t^2$ $\Delta t = \sqrt{\frac{(-23.5)}{(-4.90)}}$ $\Delta t = 2.18996$ $\Delta t = 2.19\ s$

b How long does the brick take to fall all the way to the ground?

Thinking	Working
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that up is positive and down is negative.	$s = -47.0\,m$ $v_i = 0.00\,m\,s^{-1}$ $v_f = ?\,m\,s^{-1}$ $g = -9.80\,m\,s^{-2}$ $\Delta t = ?$
Identify the correct equation for uniform acceleration to use, but substitute the a for g.	$s = v_i\Delta t + \frac{1}{2}g\Delta t^2$
Substitute known values into the equation and solve for t. Think about whether the value seems reasonable. Notice that the brick takes 2.19 s to travel the first 23.5 m and only 0.91 s more to travel the final 23.5 m. This is because it is accelerating.	$(-47.0) = (0.00)\Delta t + \frac{1}{2}(-9.80)\Delta t^2$ $(-47.0) = (-4.90)\Delta t^2$ $\Delta t = \sqrt{\frac{(-47.0)}{(-4.90)}}$ $\Delta t = 3.09707$ $\Delta t = 3.10\,s$

PHYSICSFILE

Strength of gravity

The acceleration due to gravity, g, on Earth varies slightly from the accepted value of $-9.80\,m\,s^{-2}$ depending on the location. The reasons for this will be studied in Unit 3 Physics. On the Moon, the strength of gravity, g, is much weaker than on Earth, which causes falling objects to accelerate at the much lower rate of $-1.60\,m\,s^{-2}$. Other planets and bodies in the Solar System have different values of g depending on their mass and radius. The value of g at various locations in the Solar System is provided in Table 3.5.1.

TABLE 3.5.1 Acceleration due to gravity at different locations on Earth, and on other bodies in the Solar System.

Location	Acceleration due to gravity ($m\,s^{-2}$)
Perth	−9.794
South Pole	−9.832
Equator	−9.780
Moon	−1.600
Mars	−3.600
Jupiter	−24.600
Pluto	−0.670

In Unit 2 Physics you can assume that acceleration due to gravity is always $-9.80\,m\,s^{-2}$.

c What is the final velocity of the brick as it hits the ground?	
Thinking	**Working**
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) Apply the sign convention that up is positive and down is negative.	$s = -47.0\,\text{m}$ $v_i = 0.00\,\text{m s}^{-1}$ $v_f = ?\,\text{m s}^{-1}$ $g = -9.80\,\text{m s}^{-2}$ $\Delta t = 3.09707\,\text{s}$
Identify the correct equation to use. Since you now know four values, any equation involving *v* will work, but substitute the *a* for *g*.	$v_f = v_i + g\Delta t$
Substitute the known values into the equation and solve for v_f. Think about whether the value seems reasonable.	$v_f = (0.00) + (-9.80)(3.09707)$ $v_f = -30.3513$ $v_f = 30.4\,\text{m s}^{-1}$
Use the sign and direction convention to describe the direction of the final velocity.	$v = 30.4\,\text{m s}^{-1}$ downwards

Worked example: Try yourself 3.5.1

VERTICAL MOTION

A construction worker accidentally knocks a hammer from a building so that it falls vertically a distance of 60.0 m to the ground. Use $g = -9.80\,\text{m s}^{-2}$ and ignore air resistance when answering these questions.

a How long does the hammer take to fall halfway, to 30.0 m?

b How long does it take the hammer to fall all the way to the ground?

c What is the velocity of the hammer as it hits the ground?

Remember that when an object is thrown vertically up into the air, it will eventually reach a point where its velocity is zero for an instant in time, but not for any period of time, before returning back down. So, the vertical velocity of the object decreases as the object rises, is zero at the instant it achieves its maximum height, and then increases vertically downwards as the object falls. Throughout this motion, however, the object is still in the same gravitational field, so *g* remains at $-9.80\,\text{m s}^{-2}$. Knowing that the velocity of an object thrown into the air is zero at the top of its flight allows you to calculate the maximum height reached.

Worked example 3.5.2

MAXIMUM HEIGHT PROBLEMS

On winning a tennis match the victorious player, Corey, smashes the ball vertically into the air at $27.5\,\text{m s}^{-1}$. In the following questions ignore air resistance and use $g = -9.80\,\text{m s}^{-2}$.

a Determine the maximum height reached by the ball.	
Thinking	**Working**
Write down the known quantities and the quantity that you need to find. (The term 'sifat' may help you to recall them.) At the maximum height the velocity is zero. Apply the sign convention that up is positive and down is negative.	$s = ?\,\text{m}$ $v_i = +27.5\,\text{m s}^{-1}$ $v_f = 0.00\,\text{m s}^{-1}$ $g = -9.80\,\text{m s}^{-2}$ $\Delta t = ?$
Identify the correct equation to use, but substitute the *a* for *g*.	$v_f^2 = v_i^2 + 2gs$
Substitute known values into the equation and solve for *s*.	$s = \frac{v_f^2 - v_i^2}{2g}$ $s = \frac{(0.00)^2 - (+27.5)^2}{2(-9.80)}$ $s = +38.5841$ $s = +38.6\,\text{m}$ i.e. the ball reaches a height of 38.6 m above the racquet.

b Calculate the time that the ball takes to return to its starting position.	
Thinking	**Working**
To work out the time the ball is in the air, first calculate the time it takes to reach its maximum height. Write down the known quantities and the quantity that you need to find.	$v_i = +27.5\,\text{m s}^{-1}$ $v_f = 0.00\,\text{m s}^{-1}$ $g = -9.80\,\text{m s}^{-2}$ $s = 38.5481\,\text{m}$ $\Delta t = ?$
Identify the correct equation to use, but substitute the *a* for *g*.	$v_f = v_i + g\Delta t$
Substitute known values into the equation and solve for Δt.	$\Delta t = \frac{v_f - v_i}{g}$ $\Delta t = \frac{(0.00) - (+27.5)}{(-9.80)}$ $\Delta t = 2.80612$ to maximum height $\Delta t = 2.80612$ to return to the racquet total $\Delta t = 5.61224$ total $\Delta t = 5.61\,\text{s}$

Worked example: Try yourself 3.5.2

MAXIMUM HEIGHT PROBLEMS

On winning a cricket match, a fielder throws a cricket ball vertically into the air at 15.5 m s^{-1}. In the following questions, ignore air resistance and use $g = -9.80\,\text{m s}^{-2}$.

a Determine the maximum height reached by the ball.

b Calculate the time that the ball takes to return to its starting position.

3.5 Review

SUMMARY

- If air resistance can be ignored, all bodies falling freely near the Earth will move with the same constant acceleration.
- The acceleration due to gravity is represented by g and is equal to $-9.80\,\text{m}\,\text{s}^{-2}$ if the direction towards the centre of the Earth is considered to be negative.
- The equations for uniform acceleration can be used to solve vertical motion problems by substituting the variable a, for the constant g. It is necessary to specify and use a sign convention, such as up is positive and down is negative.

KEY QUESTIONS

For these questions, ignore the effects of air resistance and assume that the acceleration due to gravity is $-9.80\,\text{m}\,\text{s}^{-2}$ unless instructed otherwise.

1 A ball is thrown into the air. Describe how the velocity of the ball changes when it leaves the hand up until the instant before it hits the hand again.

2 Angus inadvertently drops an egg while baking a cake, and the egg falls vertically towards the ground. Which one of the following statements correctly describes how the egg falls?

- **A** The egg's acceleration increases.
- **B** The egg's acceleration is constant.
- **C** The egg's velocity is constant.
- **D** The egg's acceleration decreases.

3 Yvette is an Olympic trampolinist and is practising some routines. Which one or more of the following statements correctly describes Yvette's motion at the instant she is at the highest point of the bounce? Assume that her motion is vertical.

- **A** She has zero velocity.
- **B** Her acceleration is zero.
- **C** Her acceleration is upwards and downwards.
- **D** Her acceleration is always downwards.

4 A window cleaner working on the Bell tower accidently drops her mobile phone. The phone falls vertically towards the ground with an acceleration of $-9.80\,\text{m}\,\text{s}^{-2}$.

- **a** Determine the velocity of the phone after 3.04 s.
- **b** How fast is the phone moving after it has fallen 30.0 m?
- **c** What is the average velocity of the phone during a fall of 30.0 m?

5 A person tosses a marble straight up into the air at $5.18\,\text{m}\,\text{s}^{-1}$ and then catches it at the same height from which it was thrown. Ignore air resistance.

- **a** Is the acceleration of the marble on the way up the same as, less than, or greater than, its acceleration on the way down? Justify your answer.
- **b** Is the magnitude of the launch velocity of the marble the same as, less than or greater than the magnitude of its landing velocity? Justify your answer.

6 A rubber ball is bounced off a concrete floor so that it travels straight up into the air, reaching its highest point after 1.58 s.

- **a** What is the initial velocity of the rubber ball just as it leaves the ground?
- **b** What is the maximum height reached by the ball?

7 A book is knocked off a bench and falls vertically to the floor. If the book takes 0.400 s to fall to the floor, calculate the following descriptions of its motion.

- **a** What is the book's velocity the instant before it lands?
- **b** From what height did the book fall?
- **c** How far did the book fall during the first 0.200 s?
- **d** How far did the book fall during the final 0.200 s?

8 Jet the labrador is playing with a new toy. When Jet drops a tennis ball into a launcher, it shoots the ball into the air for him to catch. The ball travels vertically upwards into the air. Being a very clever dog, Jet notices that the ball takes 4.08 s to return to its starting position.

- **a** How long does the tennis ball take to reach its maximum height?
- **b** Calculate the velocity of the ball the instant it left the launcher.
- **c** What was the maximum height reached by the tennis ball?
- **d** What was the velocity of the ball as it returned to its starting point?

9 Two physics students conduct the following experiment from a very high bridge. Asuka drops a 1.57 kg shot put from a vertical height of 60.0 m and, at exactly the same time, Jordan throws a 109 g mass with an initial downwards velocity of 10.0 $m\,s^{-1}$ from a point 10.0 m above Asuka.

   a How long does it take Asuka's shotput to reach the ground?

   b How long does it take Jordan's 109 g mass to reach the ground?

10 At the start of a football match, the umpire bounces the ball off a rubber plate so that it travels vertically upwards and reaches a height of 15.0 m.

   a How long does the ball take to reach this maximum height?

   b One of the players is able to leap and reach to a height of 4.00 m with their hand. How long after the bounce should this player try to make contact with the ball as it is on its way down?

11 A stone is held out over the edge of a seaside cliff and thrown vertically upwards at $t_i = 0.00$ s with an initial velocity of 8.00 $m\,s^{-1}$. It lands in the sea below at $t_f = 3.00$ s. Calculate:

   a the maximum height above the top of cliff reached by the stone if it left the thrower's hand 1.90 m above the level of the cliff

   b the time taken by the stone to reach its maximum height

   c the height of the cliff above the sea.

Chapter review

03

KEY TERMS

acceleration
air resistance
centre of mass
change in position
change in velocity
dimensional analysis
displacement
distance travelled
free fall
magnitude
position
speed
velocity

For the following questions, ignore air resistance and use $g = -9.80\,m\,s^{-2}$ unless indicated otherwise.

1 A car travels at $95.0\,km\,h^{-1}$ along a freeway. What is its speed in $m\,s^{-1}$?

2 A cyclist travels at $15.3\,m\,s^{-1}$ during a sprint finish. What is this speed in $km\,h^{-1}$?

The following information relates to questions 3 and 4.

An athlete in training for a marathon runs 15.4 km north along a straight road before realising that they have dropped their drink bottle. The athlete turns around and runs back 5.7 km to find the bottle, then resumes running in the original direction. After running for 3.00 hours, the athlete reaches 20.2 km from their original starting position and stops.

3 Calculate the average speed of the athlete in $km\,h^{-1}$.

4 Calculate the average velocity in:

a $km\,h^{-1}$

b $m\,s^{-1}$.

5 A ping pong ball is falling vertically at $-6.00\,m\,s^{-1}$ as it hits the floor. It rebounds at $+4.50\,m\,s^{-1}$ up. What is its change in speed during the bounce?

6 A car is moving in a positive direction. It approaches a red light and slows down. Which of the following statements correctly describes its acceleration and velocity as it slows down?

A The car has positive acceleration and negative velocity.

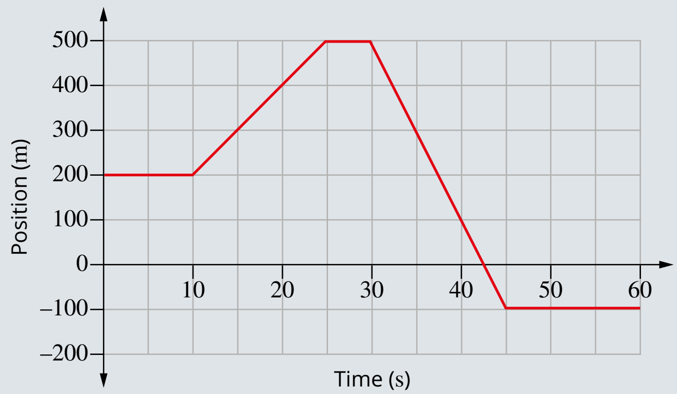
B The car has negative acceleration and positive velocity.

C Both the velocity and acceleration of the car are positive.

D Both the velocity and acceleration of the car are negative.

7 A skier is travelling along a horizontal ski run at a speed of $15.6\,m\,s^{-1}$. After falling over, the skier takes 2.55 s to come to rest. Calculate the average acceleration of the skier as they stop.

8 The following graph shows the position of a motorbike along a straight stretch of road as a function of time. The motorcyclist starts 200.0 m north of an intersection.



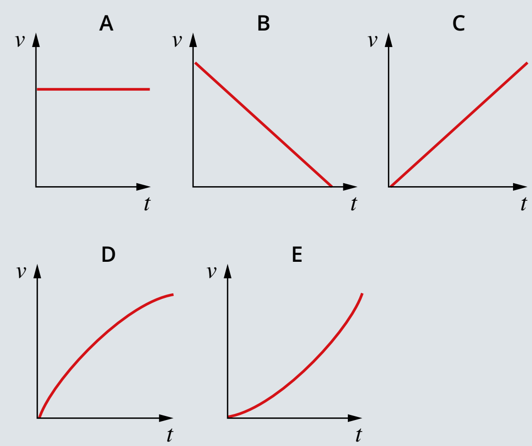
a At what time interval is the motorcyclist travelling in a northerly direction?

b At what time interval is the motorcyclist travelling in a southerly direction?

c At what time intervals is the motorcyclist stationary?

d At what time is the motorcyclist passing back through the intersection?

9 For each of the activities below, indicate which of the following velocity–time graphs best represents the motion involved.

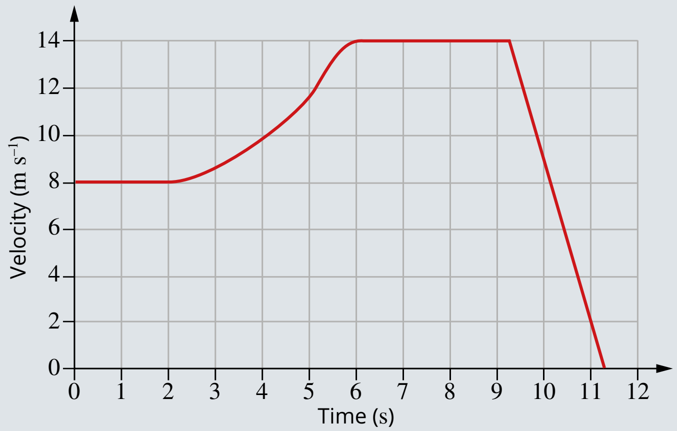


a A car comes to a stop at a red light.

b A swimmer is travelling at a constant speed.

c A motorbike starts from rest with uniform acceleration.

10 This velocity–time graph is for an Olympic road cyclist as they travel, initially north, along a straight section of the track.



a Estimate the displacement of the cyclist during the journey.

b Calculate the magnitude of the average velocity of the cyclist during this 11.3 s interval.

c Determine the acceleration of the cyclist at $t = 1.0$ s.

d Calculate the acceleration of the cyclist at $t = 10.0$ s.

e Which one or more of the following statements correctly describes the motion of the cyclist?

A They are always travelling north.

B They travel south during the final 2.0 s.

C They are stationary at $t = 8.0$ s.

D They return to the starting point after 11.0 s.

11 A car starts from rest and has a constant acceleration of 3.59 m s^{-2} west for 4.51 s. What is its final velocity?

12 A jet-ski starts from rest and accelerates uniformly east. If it travels 2.80 m in its first second of motion, calculate:

a its acceleration

b its velocity at the end of the first second

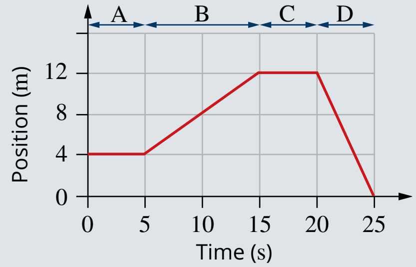
c the displacement of the jet-ski as it travels in its next one-second period of time from $t_i = 1.00$ s to $t_f = 2.00$ s.

13 A skater is travelling south along a horizontal skate rink at a speed of 10.3 m s^{-1}. After falling over, the skater travels in a straight line for 10.6 m before coming to rest. Calculate the answers to the following questions about the skater's movement.

a What is the average acceleration of the skater?

b How long does it take the skater to come to a stop?

14 The graph shows the position of Candice, who is dancing across a stage.



a What is Candice's starting position?

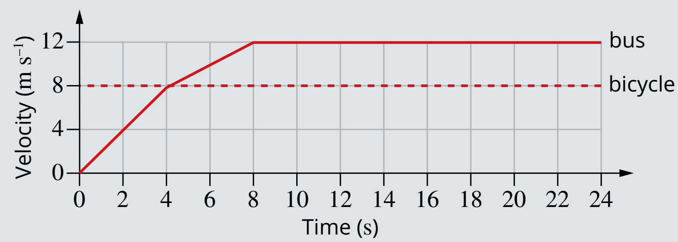
b In which of the sections (A–D) is Candice at rest?

c In which of the sections (A–D) is Candice moving in a positive direction. Determine the velocity during the section by looking at the graph, without using calculations. Explain how you arrived at the answer.

d In which of the sections (A–D) is Candice moving with a negative velocity and what is the magnitude of this velocity?

e Calculate Candice's average speed during the 25.0 s of motion.

15 The velocity–time graphs for a bus and a bicycle travelling along the same straight stretch of road are shown below. The bus is initially at rest and starts moving as the bicycle passes it.



a What is the magnitude of the initial acceleration of the bus?

b At what time does the bus overtake the bicycle? Determine the time by looking at the graph, without using calculations.

c How far has the bicycle travelled before the bus catches it?

d What is the magnitude of the average velocity of the bus during the first 8.0 s?

16 **a** Draw an acceleration–time graph for the bus discussed in Question 15.

b Use your acceleration–time graph to determine the change in velocity of the bus over the first 8.0 s.

17 A slingshot is used to launch a marble vertically into the air at 39.2 m s^{-1}. Discuss the velocity and acceleration of the marble as it travels to its maximum height. Indicate the time that it takes to reach the top. Consider up as positive.

18 A golfer mis-hits a golf ball straight up into the air. Which one of the following statements best describes the acceleration of the ball while it is in the air?

A The acceleration of the ball decreases as it travels upwards, becoming zero at the point in time it reaches its highest point.

B The acceleration is constant as the ball travels upwards, then reverses direction as the ball falls down again.

C The acceleration of the ball is greatest when the ball is at the highest point.

D The acceleration is constant for the entire time the ball is in the air.

19 Steph tosses a rock vertically into the air. Which of the options below correctly fills the blanks of the following statement about the rock's motion?

On its way upwards, the rock has ______________________ velocity and ____________________ acceleration. At the highest point, the rock has ___________________ velocity and ____________________ acceleration. On its way downwards, the rock has __________________ velocity and ____________________ acceleration.

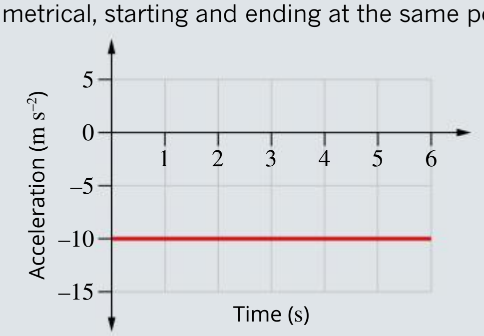
A upwards; upwards; zero; downwards; downwards; downwards

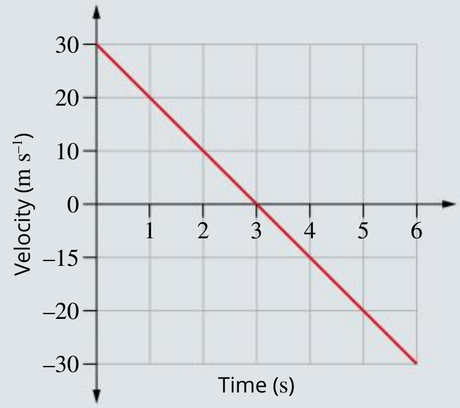
B upwards; downwards; zero; downwards; downwards; downwards

C upwards; upwards; zero; zero; downwards; downwards

D upwards; downwards; zero; zero; downwards; downwards

20 After winning a tennis match, Claire hits a tennis ball vertically into the air at 30.0 $m\,s^{-1}$. The *v*–*t* and *a*–*t* graphs for the tennis ball are shown below. Use the graphs or the equations for uniform acceleration to answer the following questions. Use $g = -10.0\ m\,s^{-2}$ for these questions. Assume the motion in question is symmetrical, starting and ending at the same point.





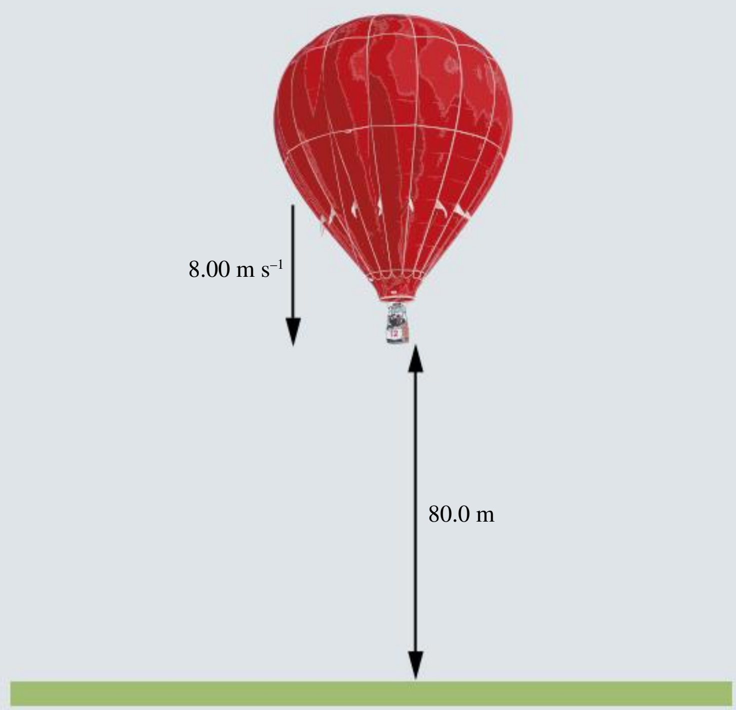
a What is the maximum height reached by the ball?

b What is the time that the ball takes to return to its starting position?

c What is the velocity of the ball 5.0 s after Claire hits it?

d What is the acceleration of the ball at its maximum height?

21 A hot-air balloon is 80.0 m above the ground and travelling vertically downwards at a constant –8.00 $m\,s^{-1}$ when one of the passengers, Tom, accidentally drops a coin over the side.



a How long does the balloon take to reach the ground?

b What is the velocity of the coin as it reaches the ground?

c How long after the coin reaches the ground does the balloon touch down?

The following information relates to questions 22 and 23.

During a game of minigolf, Renee putts a ball so that it hits an obstacle and rebounds vertically up into the air, reaching its highest point after 1.50 s.

22 What was the initial velocity of the ball the instant it is launched into the air?

23 Calculate the maximum height reached by the ball.

CHAPTER 04

Momentum and forces

In the seventeenth century, Sir Isaac Newton published three laws that explain why objects in our universe move as they do. These laws became the foundation of a branch of physics called mechanics—the science of how and why objects move. They have become commonly known as Newton's three laws of motion.

Using Newton's laws, this chapter will describe the relationship between the forces acting on an object and its motion. You will also learn about the relationship between force, period of time and change in momentum (impulse).

Science as a Human Endeavour

Safety for motorists and other road users has been substantially increased through application of Newton's laws and conservation of momentum by the development and use of passive safety devices, including:

- helmets
- seatbelts
- crumple zones
- airbags
- collapsible steering columns
- safety barriers.

Science Understanding

- momentum is a property of moving objects; it is conserved in a closed system and may be transferred from one object to another when a force acts over a time interval, including applying the relationships:

  $p = mv$

  $\Sigma mv_i = \Sigma mv_f$

- Newton's three laws of motion describe the relationship between the force or forces acting on an object, modelled as a point mass, and the motion of the object due to the application of the force or forces
- free body diagrams show the forces acting on objects, from descriptions of real-life situations involving forces acting in one or two dimensions
- the acceleration acting on an object can be determined from the summation (ΣF) of the forces (net force) acting on the object and mass of the object, including applying the relationship

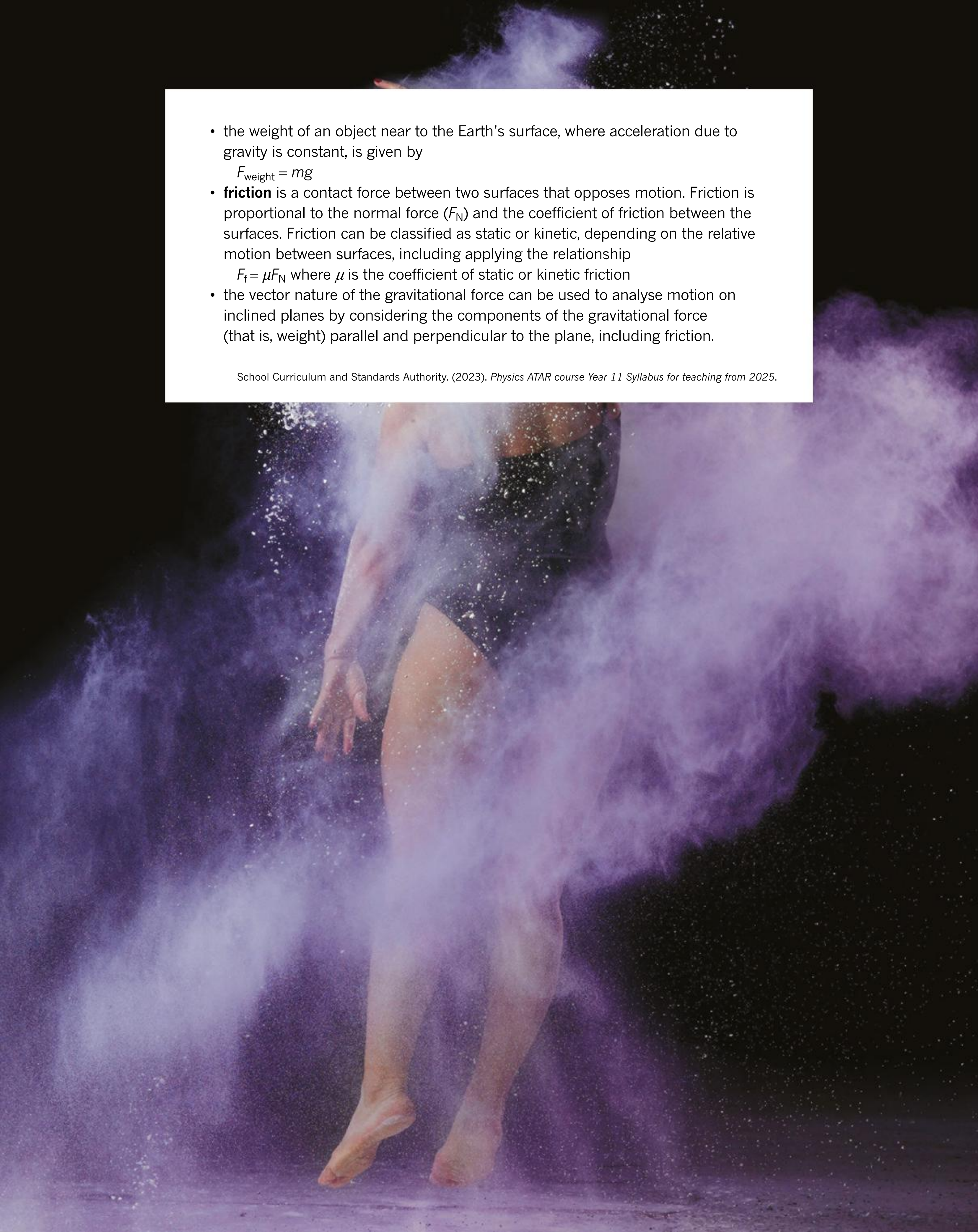
  $\Sigma F = ma$

- a change in momentum is caused by a force acting over a period of time and is referred to as impulse, including applying in one dimension the relationship

  $\Delta p = mv_f - mv_i = F\Delta t$

- the weight of an object near to the Earth's surface, where acceleration due to gravity is constant, is given by
  $F_{weight} = mg$
- **friction** is a contact force between two surfaces that opposes motion. Friction is proportional to the normal force (F_N) and the coefficient of friction between the surfaces. Friction can be classified as static or kinetic, depending on the relative motion between surfaces, including applying the relationship
  $F_f = \mu F_N$ where μ is the coefficient of static or kinetic friction
- the vector nature of the gravitational force can be used to analyse motion on inclined planes by considering the components of the gravitational force (that is, weight) parallel and perpendicular to the plane, including friction.

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.



4.1. Momentum and conservation of momentum

People experience the concepts of physics every day, even without having the understanding to explain them. For instance, you might have witnessed that a heavy object would be harder to stop compared to a lighter object if both are travelling at the same velocity. As another example, it would be harder for you to deflect or stop a basketball thrown towards you compared to a table tennis ball. This section will explain how these observations relate to the concept of momentum.

MOMENTUM

The **momentum** of an object relates to both its **mass** and its velocity. The rugby players colliding in Figure 4.1.1 have momentum due to their masses and velocities. The faster they run, the more momentum they will have. Additionally, a player with greater mass will have more momentum than a smaller, lighter player travelling at the same velocity. The more momentum an object has (due to its mass or velocity), the more momentum it has to lose before it stops.

FIGURE 4.1.1 Momentum is related to mass and velocity. The greater the mass or velocity, the harder it is to stop or start moving.

The momentum, p, of an object is the product of its mass, m, and its velocity, v.

$$p = mv$$

where p is momentum ($kg\,m\,s^{-1}$)
m is the mass of the object (kg)
v is the velocity of the object ($m\,s^{-1}$)

The greater an object's mass or velocity, the larger that object's momentum will be. As velocity is a vector quantity, momentum is also a vector and so it must have magnitude, direction and units. The direction of a momentum vector will always be the same as the direction of the velocity vector. For calculations of change in momentum in a single dimension, use the sign conventions of positive and negative.

As you will see in Section 4.3, force is equal to the rate of change of momentum. This means that any changes in momentum are caused by the action of an external net force.

Worked example 4.1.1

MOMENTUM

Calculate the momentum of a 64.0 kg student walking at 3.50 m s^{-1} east.	
Thinking	**Working**
Ensure that the variables are in their standard units.	$m = 64.0$ kg $v = 3.50$ m s^{-1} east
Apply the equation for momentum.	$p = mv$ $p = (64.0)(3.50)$ $p = 224$
Ensure that the final answer has the appropriate number of significant figures, the correct units and is in the same direction as the velocity.	$p = 224$ kg m s^{-1} east

Worked example: Try yourself 4.1.1

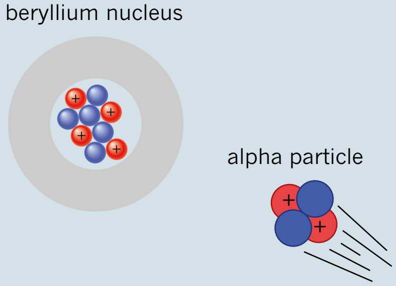
MOMENTUM

Calculate the momentum of a 1230 kg car driving at 16.7 m s^{-1} north.

PHYSICSFILE

The discovery of the neutron

The law of conservation of momentum was used to interpret the data from investigations that led to the discovery of the neutron. Because the neutron has no charge, it could not be investigated through the interactions of charged particles that had led to the discovery of the proton and electron. In 1932, James Chadwick investigated collisions between alpha particles and the element beryllium (shown in the figure below). However, the conservation of momentum calculations didn't add up. Chadwick knew that the law of conservation of momentum was true, so he reasoned that there was an unknown particle involved that had a mass close to the proton's mass, but without electric charge. Subsequent investigations confirmed his experiments and led to the naming of this particle as the neutron.



CONSERVATION OF MOMENTUM

The most significant feature of momentum is that it is **conserved** in any interaction or collision between objects. This means that the total (sum of) momentum in any system before a collision will be equal to the total (sum of) momentum in the system after the collision. This is known as the law of conservation of momentum and can be represented by the following relationship:

$$\sum p_i = \sum p_f$$

where $\sum p_i$ and $\sum p_f$ are the initial and final sum of the momentum of objects in a system, respectively.

Or:

$$\sum mv_i = \sum mv_f$$

To find the total momentum of objects in a system (either before or after a collision) simply find the momentum of each object, using their masses and velocities, and then add them together (making sure to perform the appropriate vector addition).

For collisions in one dimension, apply the sign convention of positive and negative directions to the velocities and then determine the answer to the problem. For collisions in two dimensions, resolve vectors describing motion into perpendicular components and then consider the conservation of momentum in each single dimension.

Momentum in one-dimensional collisions

If two objects are colliding in one dimension, then the following equation applies:

$$\sum mv_i = \sum mv_f$$

$$m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$$

where m_1 is the mass of object 1 (kg)

v_{i1} is the initial velocity of object 1 (m s^{-1})

v_{f1} is the final velocity of object 1 (m s^{-1})

m_2 is the mass of object 2 (kg)

v_{i2} is the initial velocity of object 2 (m s^{-1})

v_{f2} is the final velocity of object 2 (m s^{-1}).

PHYSICS IN ACTION

Elastic and inelastic collisions

Collisions can either be elastic or inelastic. In elastic collisions no kinetic energy is lost, while inelastic collisions will result in some kinetic energy being converted to another form, such as thermal energy. The loss of kinetic energy during inelastic collisions will result in a lower combination of final velocities. If the collision was 100% inelastic, all of the kinetic energy would be converted to thermal energy and both objects would be stationary immediately after the collision. If the collision was 100% elastic, then the final velocities would be maximum.

Most real-life collisions between two objects are inelastic. However, some large-scale interactions are perfectly elastic. An example of a perfectly elastic collision is a gravitational slingshot between a rocket and a planet, shown in Figure 4.1.2. You will explore this concept more in Section 5.5. It is impossible to predict the exact kinetic energy lost in any collision. Therefore, you can only apply the concept of conservation of momentum to predict the final velocity of objects with the assumption that the collision is perfectly elastic.

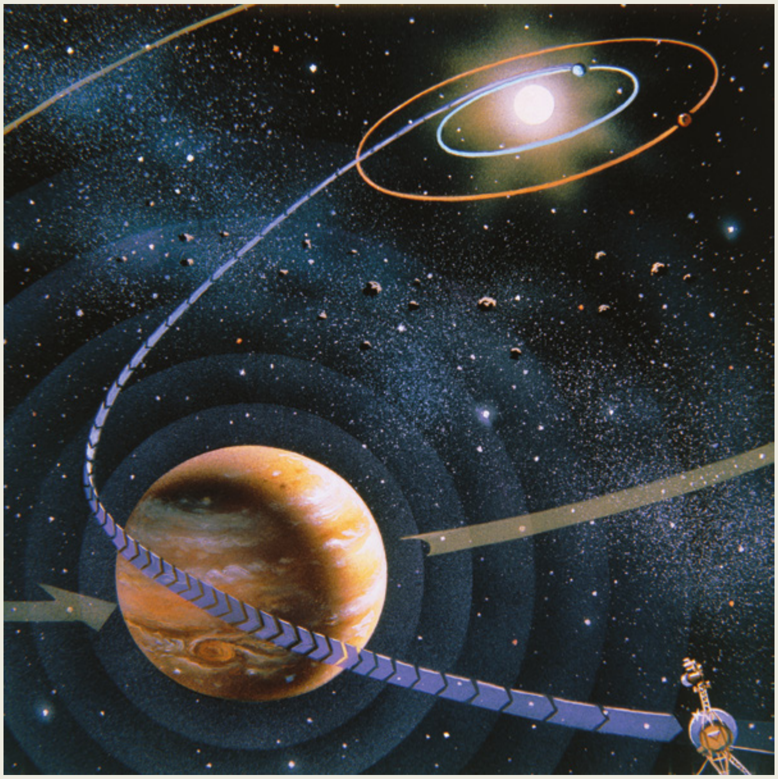


FIGURE 4.1.2 Artist's impression of the gravity assist flyby, also known as gravitational slingshot, of the spacecraft Voyager 2 around Jupiter.

Worked example 4.1.2

CONSERVATION OF MOMENTUM

A 2.50 kg mass is moving at 4.50 m s^{-1} west towards a 1.50 kg mass moving at 3.00 m s^{-1} east. Calculate the final velocity of the 2.50 kg mass if the 1.50 kg mass rebounds at 5.00 m s^{-1} west.

Thinking	Working
Identify the variables using subscripts. Ensure that the variables are in their standard units.	$m_1 = 2.50$ kg $v_{i1} = 4.50$ m s^{-1} west $v_{f1} = ?$ $m_2 = 1.50$ kg $v_{i2} = 3.00$ m s^{-1} east $v_{f2} = 5.00$ m s^{-1} west
Apply the sign convention to the variables. In this case east is positive and west is negative.	$m_1 = 2.50$ kg $v_{i1} = -4.50$ m s^{-1} $v_{f1} = ?$ $m_2 = 1.50$ kg $v_{i2} = +3.00$ m s^{-1} $v_{f2} = -5.00$ m s^{-1}

Apply the equation for conservation of momentum.	$\Sigma p_i = \Sigma p_f$ $m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$ $(2.50)(-4.50) + (1.50)(3.00) = 2.50v_{f1} + (1.50)(-5.00)$ $(2.50)v_{f1} = (-11.25) + (4.50) - (-7.50)$ $v_{f1} = \frac{(0.75)}{(2.50)}$ $v_{f1} = 0.300$
Ensure that the final answer has the appropriate number of significant figures, the correct units and apply the sign convention to describe the direction of the final velocity.	$v_{f1} = 0.300\,\mathrm{m\,s^{-1}}$ east

Worked example: Try yourself 4.1.2

CONSERVATION OF MOMENTUM

A 1250 kg wrecking ball is moving at 2.50 m s^{-1} north towards a 1550 kg wrecking ball moving at 4.00 m s^{-1} south. Calculate the final velocity of the 1550 kg ball if the 1250 kg ball rebounds at 3.50 m s^{-1} south.

PHYSICS IN ACTION

Conservation of momentum in sports

The law of conservation of momentum is observable in many human activities. In sport, however, the law is particularly evident (Figures 4.1.3 and 4.1.4). Consider the following sports:

- curling
- lawn bowls
- tenpin bowling
- bocce
- pool
- snooker.

All of these sports require the athlete to cause one object to collide with the other. The best players are able to control the initial velocity of the striking object so that the magnitude and direction of momentum for both the striking object and the target object after the collision result in a good score or a positional advantage for themselves or their team.

FIGURE 4.1.3 Curling is a sport that uses the conservation of momentum.

FIGURE 4.1.4 Controlling the direction and momentum of this bowling ball will enable the player to hit the remaining pins.

Momentum when masses combine

It is important to note that in the situations described in Worked example 4.1.2 (p. 107), the two objects remain separate from each other. However, it is possible for two objects to combine (stick together) when they collide. If two objects combine when they collide, then the equation is modified to:

$\sum p_i = \sum p_f$

$\sum mv_i = \sum mv_f$

$m_1 v_{i1} + m_2 v_{i2} = m_3 v_{f3}$

where m_1 is the mass of object 1 (kg)

v_{i1} is the initial velocity of object 1 ($m\,s^{-1}$)

m_2 is the mass of object 2 (kg)

v_{i2} is the initial velocity of object 2 ($m\,s^{-1}$)

m_3 is the combined mass of m_1 and m_2 (kg)

v_{f3} is the final velocity of combined mass of m_1 and m_2 ($m\,s^{-1}$).

Worked example 4.1.3

CONSERVATION OF MOMENTUM WHEN MASSES COMBINE

A 5.00 kg lump of clay is moving at 2.00 $m\,s^{-1}$ west towards a 7.50 kg mass of clay moving at 3.00 $m\,s^{-1}$ east. They collide to form a single, combined mass of clay. Calculate the final velocity of the combined mass of clay.	
Thinking	**Working**
Identify the variables using subscripts and ensure that the variables are in their standard units. Add m_1 and m_2 to get m_3.	m_1 = 5.00 kg v_{i1} = 2.00 $m\,s^{-1}$ west m_2 = 7.50 kg v_{i2} = 3.00 $m\,s^{-1}$ east m_3 = 12.50 kg v_{f3} = ?
Apply the sign convention to the variables. In this case east is positive and west is negative.	m_1 = 5.00 kg v_{i1} = –2.00 $m\,s^{-1}$ m_2 = 7.50 kg v_{i2} = +3.00 $m\,s^{-1}$ m_3 = 12.50 kg v_{f3} = ?
Apply the equation for conservation of momentum.	$\sum p_i = \sum p_f$ $m_1 v_{i1} + m_2 v_{i2} = m_3 v_{f3}$ $(5.00)(-2.00) + (7.50)(3.00) = (12.50)v_{f3}$ $v_{f3} = \frac{(-10.0)+(22.50)}{(12.50)}$ $v_{f3} = 1.00$
Ensure that the final answer has the appropriate number of significant figures, the correct units and apply the sign convention to describe the direction of the final velocity.	v_{f3} = 1.00 $m\,s^{-1}$ east

Worked example: Try yourself 4.1.3

CONSERVATION OF MOMENTUM WHEN MASSES COMBINE

An 80.0 kg rugby player is moving at 1.50 $m\,s^{-1}$ north when they tackle an opponent with a mass of 50.0 kg who is moving at 5.00 $m\,s^{-1}$ south. Calculate the final velocity of the two players.

Momentum in explosive collisions

It is also possible for one object to break apart into two objects in what is known as an 'explosive collision'. If an object breaks apart when an explosive collision occurs, then the equation is modified to:

$\sum p_i = \sum p_f$

$\sum mv_i = \sum mv_f$

$m_1v_{i1} = m_2v_{f2} + m_3v_{f3}$

where m_1 is the mass of object 1 (2 and 3 combined; kg)

v_{i1} is the initial velocity of object 1 ($m\,s^{-1}$)

m_2 is the mass of object 2 (kg)

v_{f2} is the final velocity of object 2 ($m\,s^{-1}$)

m_3 is the mass of object 3 (kg)

v_{f3} is the final velocity of object 3 ($m\,s^{-1}$).

Worked example 4.1.4

CONSERVATION OF MOMENTUM FOR EXPLOSIVE COLLISIONS

A 90.0 kg athlete holds a 1.00 x 10^3 g javelin. She approaches the line at 7.75 $m\,s^{-1}$ west and releases the javelin down the field. After throwing it, she continues with a velocity of 7.25 $m\,s^{-1}$ west. Calculate the velocity of the javelin just after she releases it.

Thinking	Working
Identify the variables using subscripts and ensure that the variables are in their standard units. Note that m_1 is the sum of the bodies, i.e. the athlete and the javelin.	$m_1 = 91.0$ kg $v_{i1} = 7.75\ m\,s^{-1}$ west $m_2 = 90.0$ kg $v_{f2} = 7.25\ m\,s^{-1}$ west $m_3 = 1.00$ kg $v_{f3} = ?$
Apply the sign convention to the variables. In this case, east is positive and west is negative.	$m_1 = 91.0$ kg $v_{i1} = -7.75\ m\,s^{-1}$ $m_2 = 90.0$ kg $v_{f2} = -7.25\ m\,s^{-1}$ $m_3 = 1.00$ kg $v_{f3} = ?$
Apply the equation for conservation of momentum for explosive collisions.	$\sum p_i = \sum p_f$ $m_1v_{i1} = m_2v_{f2} + m_3v_{f3}$ $(91.0)(-7.75) = (90.0)(-7.25) + (1.00)v_3$ $v_{f3} = \frac{(-705.25)-(-652.5)}{(1.00)}$ $v_{f3} = \frac{(-52.75)}{(1.00)}$ $v_{f3} = -52.75$
Ensure that the final answer has the appropriate number of significant figures, the correct units and apply the sign convention to describe the direction of the final velocity.	$v_{f3} = 52.8\ m\,s^{-1}$ west

Worked example: Try yourself 4.1.4

CONSERVATION OF MOMENTUM FOR EXPLOSIVE COLLISIONS

A 2.00 × 10^3 kg cannon fires a 10.0 kg cannonball. The cannon and the cannonball are initially stationary. After firing, the cannon recoils with a velocity of 8.15 $m\,s^{-1}$ north. Calculate the velocity of the cannonball just after it is fired.

PHYSICSFILE

Conservation of momentum in engines

If you release an inflated rubber balloon with its neck open, it will fly off around the room. In the figure, the momentum of the air to the left results in the movement of the balloon to the right. Momentum is conserved.

This is the principle upon which rockets and jet engines are based. Both rockets and jet engines employ a high-velocity stream of hot gases that are vented after the combustion of a fuel-air mixture. The hot exhaust gases have a very large momentum as a result of the high velocities involved, and can accelerate rockets and jets to high velocities as they acquire an equal momentum in the opposite direction. Rockets destined for space carry their own oxygen supply, while jet engines use the surrounding air supply.

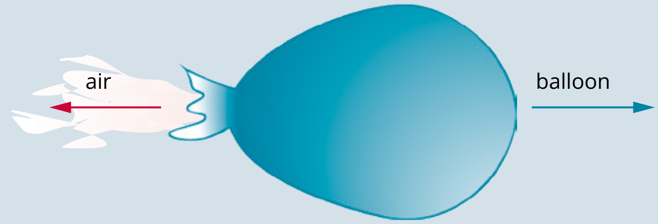


FIGURE 4.1.5 Conservation of momentum demonstrated by an open balloon.

4.1 Review

SUMMARY

- Momentum is the product of an object's mass and its velocity.
- Momentum is a vector quantity and is calculated using the equation $p = mv$.
- The law of conservation of momentum can be applied to situations in which:
  - two objects collide and remain separate
    $\Sigma p_i = \Sigma p_f$
    $\Sigma mv_i = \Sigma mv_f$
    $m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$
  - two objects collide and combine together
    $\Sigma p_i = \Sigma p_f$
    $\Sigma mv_i = \Sigma mv_f$
    $m_1v_{i1} + m_2v_{i2} = m_3v_{f3}$
  - one object breaks apart into two objects in an explosive collision
    $\Sigma p_i = \Sigma p_f$
    $\Sigma mv_i = \Sigma mv_f$
    $m_1v_{i1} = m_2v_{f2} + m_3v_{f3}$.

KEY QUESTIONS

1 Calculate the momentum of a 3.50 kg fish swimming at 2.50 m s^{-1} south.

2 Calculate the momentum of a 433 kg boat travelling at 22.2 m s^{-1} west.

3 Calculate the momentum of a 65.0 g tennis ball being served at 61.0 m s^{-1} south.

4 Which object has the greater momentum: a medicine ball of mass 4.50 kg travelling at 3.50 m s^{-1} or one of mass 2.50 kg travelling at 6.80 m s^{-1}?

5 A rower of mass 70.0 kg steps out of a stationary boat with a velocity of 2.50 m s^{-1} forwards onto the nearby riverbank. The boat has a mass of 400 kg and was initially at rest. With what velocity does the boat begin to move as the rower steps out?

6 A golf ball of mass 70.0 g is stationary on the ground when it is hit by a 545 g golf club travelling at 80.0 m s^{-1}. If the ball leaves the club at a speed of 75.0 m s^{-1}, with what speed does the club move just after hitting the ball?

7 A railway wagon of mass 2.50 tonnes moving along a horizontal track at 2.00 m s^{-1} runs into a stationary engine and is coupled to it. After the collision, the engine and wagon move off together at a slow velocity of 0.300 m s^{-1}. What is the mass of the engine alone?

8 A space shuttle of mass 1.00×10^4 kg (including fuel), initially at rest, burns 5.00 kg of fuel and oxygen in its rockets to produce exhaust gases ejected at a velocity of 6.00×10^3 m s^{-1}. Calculate the velocity that this exchange will give to the space shuttle.

4.2 Newton's first law

The previous section developed the concepts and ideas needed to describe the properties and motion of a moving body. In the following sections, however, rather than simply describing the motion, you will investigate the forces that cause the motion to occur. In addition, you will explore Newton's three laws of motion, which describe and predict the relationship between the forces acting on an object (modelled as a point mass) and its subsequent motion. In this section you will learn about Newton's first law of motion.

FORCE

In simple terms, a **force** can be thought of as a push or a pull. Forces exist in a wide variety of situations and are fundamental to the nature of matter and the structure of the universe. Consider each of the images in Figure 4.2.1. A force is acting in each situation.

FIGURE 4.2.1 (a) At the moment of impact, both the tennis ball and the racquet strings are distorted by the forces acting at this instant. (b) The rock climber is relying on the frictional force between their hands and feet and the rock face. (c) A continual force causes the clay to deform into the required shape. (d) The gravitational force between the Earth and the Moon is responsible for two high tides each day.

Some of the forces depicted in Figure 4.2.1 are applied directly to an object and some act on a body without touching it. Forces that act directly on a body are called **contact forces**, because the body will only experience the force while contact is maintained. Forces that act on a body at a distance are **non-contact forces**.

Contact forces are the easiest to understand and include the simple pushes that humans experience in their daily lives. Examples of these include the forces between colliding billiard balls and the forces that act between you and your chair as you sit reading this book. Friction and drag forces are also contact forces.

There is no such thing as a contact pull. A pull is always a push towards you. Even if you pull an object such as a rope towards yourself, the frictional force still occurs as a push on the object in the direction it moves. Non-contact forces occur when the object causing the push or pull is physically separated from the object that experiences the force. These forces are said to 'act at a distance'. Gravitation, magnetic and electric forces are examples of non-contact forces.

A force is a push on an object, measured in newtons (N). It is a vector quantity, as it attempts to change the velocity of an object in a certain direction. As a vector it requires a magnitude and directions to fully describe it.

The amount of force acting can be measured using the SI unit called the newton, which is given the symbol N. The unit, which will be defined later in the chapter, honours Sir Isaac Newton (1642–1727), who is considered to be one of the most significant physicists to have lived, and whose first law is the subject of this section. A force of one newton, 1 N, is approximately the force you have to exert when holding a 98.0 g mass against the downwards pull of gravity. In everyday life, this is about the same as holding a small apple.

If more than one force acts on a body at the same time, the vector sum of all the forces should be calculated. The vector sum of the forces is called the resultant or **net force**, $\sum F$. (Vectors were covered in detail in Chapter 2.) This net force is responsible for the motion of the object.

The net force acting on a body experiencing a number of forces acting simultaneously is given by the vector sum of all the individual forces:

$$\sum F = F_1 + F_2 + \ldots + F_n$$

NEWTON'S FIRST LAW

Inertia and Newton's first law are closely related; in fact, some people call Newton's first law the law of inertia. Inertia is the tendency of an object to maintain its velocity. This tendency is related to the mass of an object, so that the greater the mass, the harder it is to get it moving or to stop it from moving.

Newton's first law is a law that is often misunderstood due to a common misconception. People mistakenly think that an object that is moving at constant velocity must have a force causing it to move. This section will address this misconception and will enable you to understand how Newton's first law applies to all situations in which an object moves.

Newton's first law of motion can be stated as follows:

An object will maintain a constant velocity unless an unbalanced, external force acts on it.

This statement needs to be analysed in more detail by first examining some of the key terms used. The term 'maintain a constant velocity' implies that, if the object is moving, then it will continue to move with a velocity that has the same magnitude and direction. For example, if a car is moving at $12.0\,\text{m}\,\text{s}^{-1}$ south, then some time later it will still be moving at $12.0\,\text{m}\,\text{s}^{-1}$ south (Figure 4.2.2). It should also be noted that zero velocity can also be constant, so if the car is moving at $0\,\text{m}\,\text{s}^{-1}$, then some time later it will still be moving at $0\,\text{m}\,\text{s}^{-1}$.

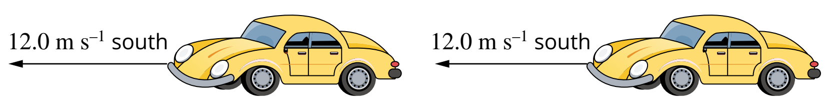


FIGURE 4.2.2 A car maintaining a constant velocity.

The term 'unless' is particularly important in Newton's statement. In effect, it tells you what must *not* happen for the motion to be constant instead of telling you what must happen. Rephrasing Newton's first law to remove the term 'unless' means that it could say:

An object will not maintain its velocity if an unbalanced, external force is applied.

In this context, *not* continuing with its velocity implies that the object is changing its velocity. A change in velocity means that the object is accelerating or decelerating.

The use of the term 'unbalanced' in relation to the acting force implies that there must be a net force acting on the object. If the forces are balanced, then the object's velocity will remain constant. If the forces are unbalanced, then the velocity will change, or will not remain constant. This is illustrated in Figure 4.2.3.

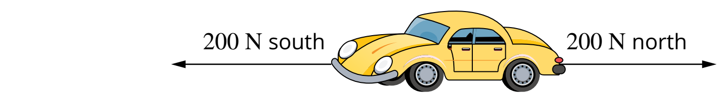


FIGURE 4.2.3 The forces on the top car are balanced, so it will maintain a constant velocity. The forces on the bottom car are unbalanced: it has a net force in the forwards direction, so its velocity will change (it will speed up).

PHYSICSFILE

The effects of forces

Applying a force can cause an object to maintain a constant velocity, speed up, to slow down, to start moving, to stop moving, or to change its direction (Figure 4.2.4). The effect depends on the direction of the force in relation to the direction of the velocity vector of the object experiencing the force. For example, the upwards thrust force on a rocket has the effect of speeding it up, whereas the upwards drag force on a parachute has the effect of slowing the descent of a skydiver. The effect of external forces is summarised in Table 4.2.1.

TABLE 4.2.1 The effect of the application of a force, depending on the relationship between the direction of the force and the velocity.

Relationship between velocity and force	Effect of force
force applied to object at rest	object starts moving
force in same direction as velocity	magnitude of velocity increases (object speeds up)
force in opposite direction to velocity	magnitude of velocity decreases (object slows down)
force perpendicular to velocity	direction of velocity changes (object turns)

In all cases, the effect of a force is to change the velocity of an object, whether it is a change in the magnitude of the velocity, the direction of the velocity, or both.

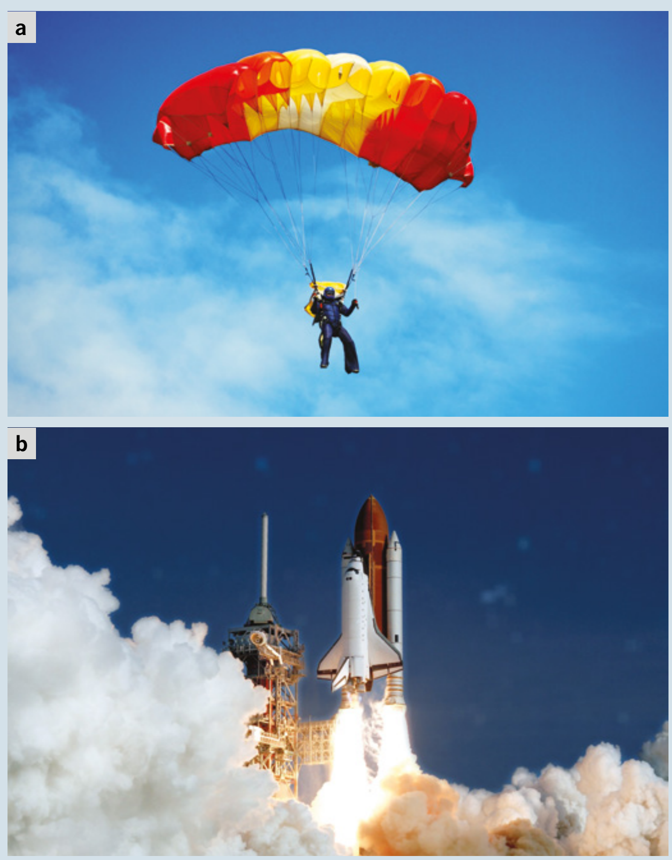


FIGURE 4.2.4 The upwards thrust force on a rocket has the effect of speeding it up, whereas the upwards drag force on a parachute has the effect of slowing the descent of the parascender.

The term 'external', in relation to forces, implies that the forces are not internal. When forces are internal, they will have no effect on the motion of the object. For example, if you are sitting in a car and push forwards on the steering wheel then the car will not move forwards due to this force. In order for you to push forwards on the steering wheel, you must push backwards on the seat. Both the steering wheel and the seat are attached to the car, therefore there are two forces acting on the car that are equal and in the opposite direction to each other, as shown in Figure 4.2.5. All internal forces must result in balanced forces on the object and therefore they will not change the velocity of the object.

FIGURE 4.2.5 This driver is applying internal forces on a car. These internal forces will balance and cancel each other.

Stating Newton's first law in different ways

Ludwig Wittgenstein, an Austrian-British philosopher, said that 'understanding means seeing that the same thing said different ways is the same thing'. To truly understand Newton's first law, you should be able to state it in different ways yet still recognise it as being consistent with Newton's first law.

All of the following statements are consistent with Newton's first law.

- An object will maintain a constant velocity unless an unbalanced, external force acts on it.
- A body will either remain at rest or continue with constant speed in a straight line (i.e. constant velocity) unless it is acted on by a net force.
- If a net force is applied, then the object's velocity will change.
- Net forces cause acceleration or deceleration.
- Constant velocity means no net force is applied.

PHYSICS IN ACTION

Terminal velocity

In Chapter 3 it was stated that, in the absence of air resistance, all objects accelerate towards the surface of Earth at a constant rate of $9.80\,m\,s^{-2}$. However, air resistance increases as the speed of an object increases, and it becomes very significant when objects are moving at high speeds. Newton's first law can be used to explain how air resistance causes a skydiver to experience terminal velocity (Figure 4.2.6). As the skydiver begins falling towards Earth, the only external force is the weight force. The weight force causes the skydiver to accelerate at $9.80\,m\,s^{-2}$. As the skydiver moves faster, air resistance, the force pushing upwards, increases. This reduces the magnitude of the net force, which decreases the skydiver's acceleration. Eventually the air resistance becomes so great that it exactly balances the weight force. According to Newton's first law, the skydiver maintains a constant velocity when all the external forces are balanced: this velocity is terminal velocity. Terminal velocity will be different for different objects because air resistance depends on the object's mass and surface area (i.e. size and shape). For example, terminal velocity for a sheet of paper is much less than for a skydiver.

FIGURE 4.2.6 When a skydiver reaches terminal velocity, weight force is in balance with air resistance.

INERTIA

Inertia is the resistance to a change in motion of an object. It is related to the mass of the object. As the mass of the object increases, the inertia increases and therefore:

- it becomes harder to start it moving if it is stationary, or
- it becomes harder to stop it moving if it is in motion, or
- it becomes harder to change its direction of motion.

You can experience the effect of inertia when you push a trolley in a supermarket. If the trolley is empty, it is relatively easy to start pushing it, or to stop when it is already moving. It is also easy to turn a corner. If you fill the trolley with heavy groceries, it becomes more difficult to make the trolley start moving when it is at rest, or to stop if it is already moving. It also requires more force to change the trolley's direction. This concept of momentum was explored in Section 4.1. You learned that the greater an object's mass or velocity, the greater its momentum, and consequently the harder it is to slow it down. Momentum has a direct relationship with its inertia.

Newton's first law and inertia

The connection between Newton's first law and inertia is very close. Due to inertia, an object will continue with its motion unless a net force acts on the object.

You experience the connection between Newton's first law and inertia if you are standing on public transport. Imagine standing on a train that is initially at rest and then starts moving forwards. If you are not holding on to anything, you may stumble backwards as though you have been pushed. However, you have not been pushed backwards; the train has started moving forwards and, since you have inertia, your mass resists the change in motion. According to Newton's first law, your body is simply maintaining its original state of being motionless until a net force acts to accelerate it. When the train later comes to a sudden stop, your body again resists the change by continuing to move forwards until a net force acts to bring it to a stop.

OBSERVING NEWTON'S FIRST LAW

An object that is in motion will eventually stop. It may not seem obvious, but this is a very good example of Newton's first law. The motion does not continue; therefore, a net force must be acting, in this case friction. Friction is the resistive force that exists between two surfaces in contact with one another. It is a force that always acts in the opposite direction to the motion of objects Friction will be further discussed in Section 4.4. Air resistance and the force due to gravity are other forces that are often overlooked. By ignoring the effect of these important forces, it can be easy to come to the incorrect conclusion that the natural state of any object is to be at rest. By considering all the external forces acting on an object, it becomes clear that the natural state of any object is to maintain whatever velocity it currently has.

PHYSICS IN ACTION

Galileo's law of inertia

Galileo Galilei (Figure 4.2.7) was born into an academic family in Pisa, Italy, in 1564. He made significant contributions to physics, mathematics and the scientific method through intellectual rigour and the quality of his experimental design. But, more than this, Galileo helped to change the way the universe is viewed.

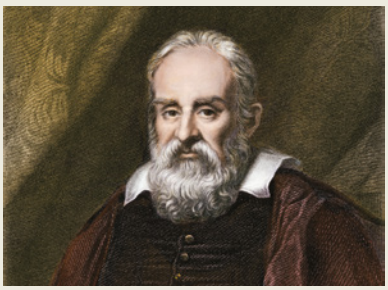


FIGURE 4.2.7 Galileo Galilei.

Galileo's most significant contributions were in astronomy. Through his development of the refracting telescope he discovered sunspots, lunar mountains and valleys, the four largest moons of Jupiter (now called the Galilean Moons, one of which is shown in Figure 4.2.8 below) and the phases of Venus. In mechanics, he demonstrated that projectiles moved with a parabolic path and that different masses fall at the same rate in a vacuum (the law of falling bodies).

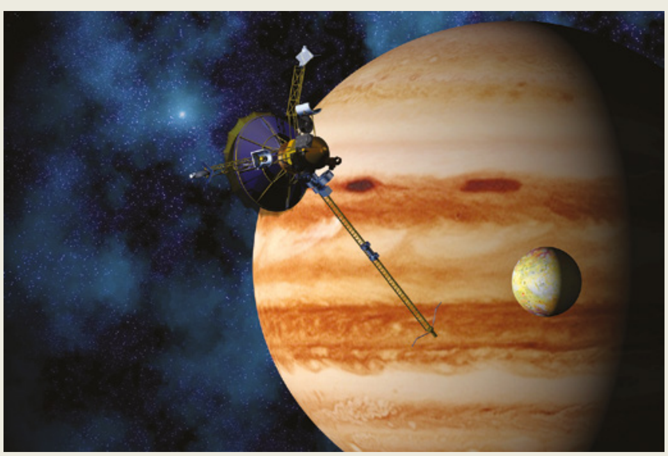


FIGURE 4.2.8 The major moons of Jupiter, including Io, are known as the Galilean Moons.

These developments were important because they changed the framework in which mechanics was understood. This framework had been in place since Aristotle constructed it in the fourth century BCE. Aristotle's thesis was based on the observation that a moving body's natural state is at rest, and the object will come to rest unless a force is applied to keep it moving. However, Galileo's experiments led him to believe that the natural state of an object is not at rest. He suggested instead that objects maintained their state of motion and he called this tendency inertia.

By the sixteenth century, the work of Aristotle and other Greek philosophers had become widely accepted and supported in the universities and at a political level. One might think that Galileo would have won praise from his peers for making such progress, but the Aristotelian view was so ingrained that Galileo actually lost his job as a professor of mathematics in Pisa in 1592.

Galileo did have some supporters and he was able to move from Pisa to Padua, where he continued teaching mathematics. In Padua, Galileo began to use measurements from carefully constructed experiments to strengthen his ideas. He entered into vigorous debate in which his ideas (founded on observation) were pitted against the philosophy of the past and the politics of the day. The most divisive debate involved the motion of the planets. The ancient Greek view, formalised by Ptolemy in the second century CE, was that Earth was at the centre of the solar system and that the planets, the Moon and the Sun were in orbit around it. This view was taught by the Roman Catholic Church and was also supported by common sense.

In 1630, Galileo published a book in which he debated the Ptolemaic view and the new Sun-centred model proposed by Copernicus. Based on his own observations, Galileo supported the Copernican view of the universe. Although the book was not censored and was allowed to circulate, Galileo was summoned to Rome to face the Inquisition for heresy (opposition to the Church). The finding went against Galileo and all copies of his book had to be burned. He was sentenced to permanent house arrest for the rest of his life.

Galileo died in 1642 in a village near Florence. He had become an influential thinker across Europe, and the scientific revolution he had helped to start accelerated in the freer Protestant countries in northern Europe. For its part, in 1979 the Roman Catholic Church under Pope John Paul II began an investigation into Galileo's trial, and in 1992 a Papal Commission reversed the Church's condemnation of him.

4.2 Review

SUMMARY

- A force is a push that acts to change the velocity of an object. Some forces act on contact while others can act at a distance.
- Force is a vector quantity whose SI unit is the newton (N).
- Newton's first law of motion states that an object will maintain a constant velocity unless an unbalanced, external force (i.e. net force) acts on it.
- Inertia is the tendency of an object to resist changes in motion.
- Inertia is related to mass; an object with a large mass will have a large inertia.

KEY QUESTIONS

1 A student observes a box sliding across a surface and slowing down to a stop. From this observation what can the student conclude about the forces acting on the box?

2 A car changes its direction as it turns a bend in the road while maintaining its speed of $16.0\,\text{m s}^{-1}$. From this, what can you conclude?

3 A bowling ball rolls along a smooth wooden floor at constant velocity. Ignoring the effects of friction and air resistance, which of the following options related to the force acting on the ball is correct?

A There must be a net force acting forwards to maintain the velocity of the ball.

B There must be no unbalanced force acting on the ball.

C The forwards force acting on the ball must be balanced by the friction that opposes the motion.

D More information is needed.

4 If a person is standing up in a moving bus that stops suddenly, the person will tend to fall forwards. Has a force acted to push the person forwards? Use Newton's first law to explain what is happening.

5 What horizontal force has to be applied to a wheelie bin if it is pushed on a horizontal path against a frictional force of 20.0 N at a constant velocity of $1.50\,\text{m s}^{-1}$?

6 A child is using a horizontal rope to pull a bodyboard across the sand at a constant velocity. A frictional force of 25.0 N also acts on the body board.

a What force must the child apply to the rope?

b The child's father then attaches a longer rope to the bodyboard because the short rope is uncomfortable to use. The rope now makes an angle of 30.0° to the horizontal. What is the horizontal component of the force that the child needs to apply in order to move the bodyboard with constant velocity?

c What is the tension force acting along the rope that the child must supply?

7 Passengers on commercial flights are required to be seated and have their seatbelts fastened when their plane is coming in to land. What would happen to a person who was standing in the aisle as the plane travelled along the runway during landing?

8 Consider the following situations and name the force that causes each object to travel along a path that is not a straight line.

a The Earth moves in a circle around the Sun with constant speed.

b An electron orbits the nucleus with constant speed.

c A cyclist turns a corner at constant speed.

d A hammer throw athlete swings a hammer in a circle with constant speed.

9 A magician performs a trick in which a cloth is pulled quickly from under a glass filled with water without causing the glass to fall over or the water to spill out.

a Explain the physics principles underlying this trick.

b Does using a full glass make the trick easier or more difficult? Explain.

10 Which of these objects would be most difficult to stop: a cyclist travelling at $50.0\,\text{km h}^{-1}$, a car travelling at $50.0\,\text{km h}^{-1}$ or a fully laden semitrailer travelling at $50.0\,\text{km h}^{-1}$? Explain your answer.

11 When flying at constant speed at a constant altitude, a light aircraft has a weight of 50.0 kN down, and the thrust produced by its engines is 12.0 kN to the east. What is the lift force required by the wings of the plane, and how large is the drag force that is acting?

4.3 Newton's second law

Newton's second law makes the quantitative connection between force, mass and acceleration. It also helps to resolve the misconception that many people have about the time taken for objects of different mass to fall to the ground. Many mistakenly believe that heavy objects will fall faster than lighter objects. Once again, air resistance acts to complicate the matter and results in the observation of different times for different masses to fall the same distance. However, the misconception that larger masses fall faster than lighter masses even when air is removed still persists.

Figure 4.3.1 depicts a famous experiment, mentioned in Section 3.5, which shows that objects fall together when the effect of air resistance is removed. A web search for 'hammer and feather on the Moon' will enable you to view a video of David Scott's 1971 experiment. Although the images are quite poor, you should be able to see both objects accelerating at the same rate.

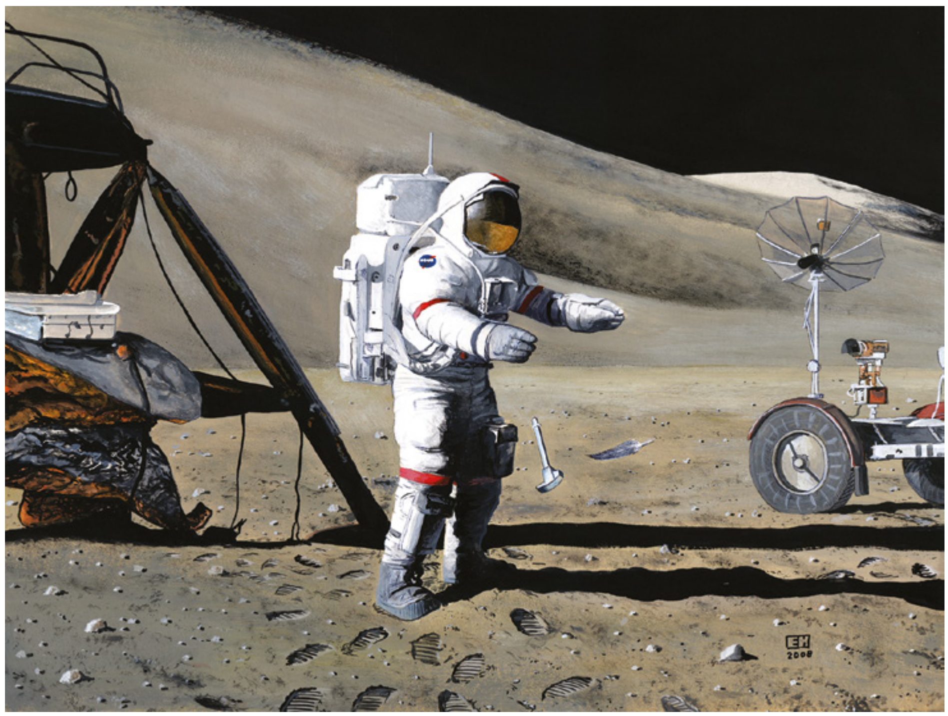


FIGURE 4.3.1 An artist's image of the famous hammer and feather experiment conducted on the Moon.

NEWTON'S SECOND LAW

Newton's second law of motion states the following:

The acceleration of an object is directly proportional to the net force on the object and inversely proportional to the mass of the object:

This can be displayed by the following relationship.

$\sum F = ma$

where

$\sum F$ is the net force applied to the object (N), which can be found by summing all forces acting on the object
m is the mass of the object (kg)
a is the acceleration of an object ($m\,s^{-2}$).

The unit of force is the units of mass and acceleration combined, or $kg\,m\,s^{-2}$. This unit was named the newton (N) in honour of Sir Isaac Newton. By definition, 1 **newton** is the force needed to accelerate a mass of 1 kg at $1\,m\,s^{-2}$.

One of the implications of Newton's second law is that, for a constant mass, a greater acceleration is achieved by applying a greater force. This is shown in Figure 4.3.2. Doubling the applied force will double the acceleration of the object. In other words, given a constant mass, the acceleration is proportional to the net force applied.

Notice also in Figure 4.3.2 that the acceleration of the object is in the same direction as the net force applied to it.

Newton's second law also explains how acceleration is affected by the mass of an object. For a given force, the acceleration of an object will decrease with increased mass. In other words, given a constant force, acceleration is inversely proportional to the mass of an object. This is shown in Figure 4.3.3.

The following derivation shows how Newton's second law relates to momentum. It shows that the net force, ΣF, is equal to the change in momentum, Δp, divided by the period of time, Δt, which is the rate of change of momentum:

$$\sum F = ma$$

$$\sum F = m\frac{(v_f - v_i)}{\Delta t}$$

$$\sum F = \frac{mv_f - mv_i}{\Delta t}$$

$$\sum F = \frac{\Delta p}{\Delta t}$$

This means that changes in momentum are caused by the action of a net force. This should seem reasonable, since you have now established that a net force causes the acceleration of an object, which in turn changes its velocity, and therefore its momentum.

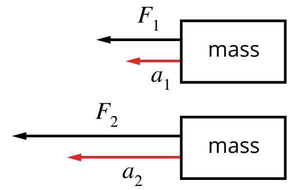


FIGURE 4.3.2 Given the same mass, a larger force will result in a larger acceleration. If the force is doubled, then the acceleration is also doubled.

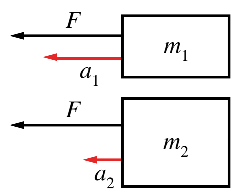


FIGURE 4.3.3 Given the same force, a larger mass will result in a lower acceleration. If the mass is doubled, then the acceleration is halved.

PHYSICSFILE

Maximising acceleration

Dragster race cars are designed to achieve the maximum possible acceleration in order to win a race in a straight line over a relatively short distance. According to Newton's second law, acceleration is increased by either increasing the applied force, or by reducing the mass of the object, or both. For this reason, dragster race cars are equipped with very powerful engines that produce an enormous forwards force, and have an aerodynamic shape to minimise air resistance. Additionally, there is not much else to the car, so this helps to minimise the mass.

Newton's second law also helps you to understand why a motorcycle can accelerate from the lights at a greater rate than a car or a truck (Figure 4.3.4). While the engines in a car or truck are usually more powerful than a motorcycle engine, the motorcycle has much less mass, which allows for greater acceleration.

FIGURE 4.3.4 The aerodynamic design of motorcycles and their lower mass enables them to accelerate faster than cars and trucks.

Observing Newton's second law

The equation $\Sigma F = ma$ enables you to calculate the force that causes a mass to accelerate. Mass is something that is easily experienced. You can measure the mass of an object on a balance. You can hold two masses in your hands and feel their effect. Similarly, you can observe acceleration, for example when a car accelerates away from the traffic lights. Force, on the other hand, is not something that you can see. However, you can see the *effect* of a force.

Worked example 4.3.1

CALCULATING THE FORCE THAT CAUSES AN ACCELERATION

Calculate the net force causing a 5.50 kg mass to accelerate at 3.75 m s^{-2} west.	
Thinking	**Working**
Ensure that the variables are in their standard units.	m = 5.50 kg a = 3.75 m s^{-2} west
Apply the equation for force from Newton's second law.	$\Sigma F = ma$ $\Sigma F = (5.50)(3.75)$ $\Sigma F = 20.625$
Ensure that the final answer has the appropriate number of significant figures, the correct units and give the direction of the net force, which is always the same as the direction of the acceleration.	ΣF = 20.6 N west

Worked example: Try yourself 4.3.1

CALCULATING THE FORCE THAT CAUSES AN ACCELERATION

Calculate the net force causing a 75.8 kg runner to accelerate at 4.05 m s^{-2} south.

The first equation for uniform acceleration, which is discussed in Chapter 3 can be combined with Newton's second law to calculate changes in time or velocity.

The first equation for uniform acceleration is:

$$v_f = v_i + a\Delta t$$

This can be rearranged to give:

$$a = \frac{v_f - v_i}{\Delta t}$$

Combining this with $\Sigma F = ma$ gives:

$$\Sigma F = \left(\frac{v_f - v_i}{\Delta t}\right)$$

Worked example 4.3.2

CALCULATING THE FINAL VELOCITY OF AN ACCELERATING MASS

Calculate the final velocity of a 225 kg scooter that accelerates for 2.00 s from rest due to a force of 2430 N north.	
Thinking	**Working**
Ensure the variables are in their standard units.	m = 225 kg Δt = 2.00 s v_i = 0 m s^{-1} ΣF = 2430 N north
Apply a variation of the equation for force from Newton's second law.	$\Sigma F = m\left(\frac{v_f - v_i}{\Delta t}\right)$ $(v_f - v_i) = \frac{\Sigma F \Delta t}{m}$ $v_f = m\frac{\Sigma F \Delta t}{m} + v_i$ $v_f = \frac{(2430)(2.00)}{(225)} + (0)$ $v_f = 21.6$

Ensure that the final answer has the appropriate number of significant figures, the correct units and give the direction of the final velocity as being the same as the direction of the force.	$v_f = 21.6\,m\,s^{-1}$ north

Worked example: Try yourself 4.3.2

CALCULATING THE FINAL VELOCITY OF AN ACCELERATING MASS

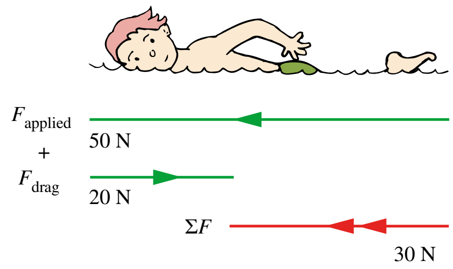
Calculate the final velocity of a 307 g fish that accelerates for 5.20 s from rest due to a force of 0.250 N left.

Forces do not always act alone. Mostly, more than one force will act on an object at any time. The overall effect of the forces depends on the direction of each of the forces. For example, some forces act together and some may oppose each other. When using Newton's second law, it is important to use the net, or resultant, force in the calculation. As forces are vectors, they can be added or combined using the techniques discussed in Chapter 2. Consider the following worked examples.

Worked example 4.3.3

CALCULATING THE ACCELERATION OF AN OBJECT WITH MORE THAN ONE FORCE ACTING ON IT

A swimmer whose mass is 75.0 kg applies a force of 50.0 N as they start a lap. The water opposes the swimmer's efforts to accelerate with a drag force of 20.0 N. What is the swimmer's initial acceleration?

Thinking	**Working**
Determine the individual forces acting on the swimmer, and apply the vector sign convention.	$F_1 = 50.0\,N$ forwards $F_1 = 50.0\,N$ $F_2 = 20.0\,N$ backwards $F_2 = -20.0\,N$
Determine the net force acting on the swimmer.	$\Sigma F = F_1 + F_2$ $\Sigma F = 50 + (-20)$ $\Sigma F = +30.0\,N$ or $30.0\,N$ forwards 
Use Newton's second law to determine the acceleration.	$\Sigma F = ma$ $a = \frac{\Sigma F}{m}$ $a = \frac{(+30.0)}{(75.0)}$ $a = 0.40000$
Ensure that the final answer has the appropriate number of significant figures, the correct units and the correct direction.	$a = 0.400\,m\,s^{-1}$ forwards

Worked example: Try yourself 4.3.3

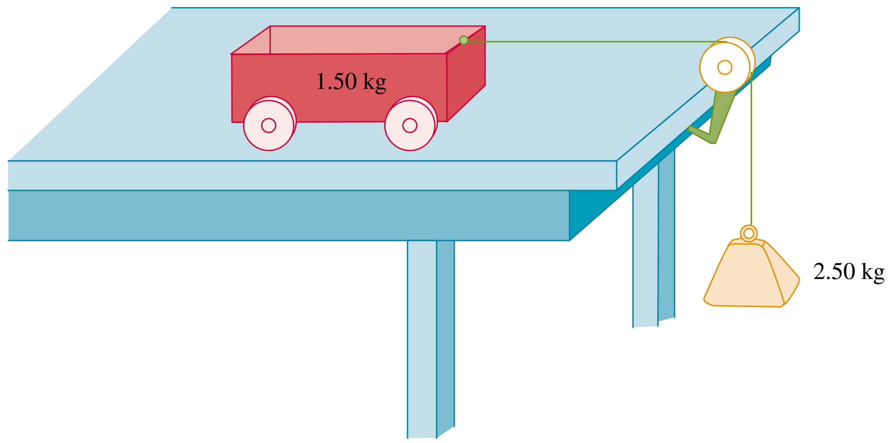
CALCULATING THE ACCELERATION OF AN OBJECT WITH MORE THAN ONE FORCE ACTING ON IT

A car with a mass of 905 kg applies a driving force of 3010 N as it starts moving. Friction and air resistance oppose the motion of the car with a force of 755 N. What is the car's initial acceleration?

Worked example 4.3.4

CALCULATING THE ACCELERATION OF A CONNECTED BODY

A 1.50 kg trolley cart is connected by a cord to a 2.50 kg mass as shown. The cord is placed over a pulley and the mass is allowed to fall under the influence of gravity.



a Assuming that the cart can move over the table without friction, determine the acceleration of the cart.

Thinking	Working
Recognise that the cart and the falling mass are connected. Direction conventions only act in one dimension. As this system acts in two dimensions, no sign convention is used for the acceleration due to gravity. The net force on the system can, however, be described as clockwise is positive (right and down) and anticlockwise is negative (up and left).	Relative to the system, clockwise is positive and anticlockwise is negative.
Write down the data that is given. Ignore the sign convention for the acceleration due to gravity.	$m_1 = 2.50\,kg$ $m_2 = 1.50\,kg$ $g = 9.80\,m\,s^{-2}$
Determine the forces acting on the system. The net force is positive as the net force causes the system to accelerate clockwise.	The only force acting on the combined system of the cart and mass is the weight of the falling mass. $\Sigma F = F_{weight}$ $\Sigma F = m_1 g$ $\Sigma F = (2.50)(9.80)$ $\Sigma F = 24.500\,N$ $\Sigma F = +24.5\,N$ on the system

Calculate the mass being accelerated.	This net force has to accelerate not only the cart but also the falling mass. $m_t = m_1 + m_2$ $m_t = (2.50) + (1.50)$ $m_t = 4.00\,\text{kg}$
Use Newton's second law to determine acceleration.	$a = \frac{\Sigma F}{m_t}$ $a = \frac{(+24.500)}{(4.00)}$ $a = +6.1250$
Ensure that the final answer has the appropriate number of significant figures, the correct units and the correct direction.	$a = 6.13\,\text{m s}^{-2}$ to the right

b If a frictional force of 8.50 N acts against the cart, what is the acceleration now?	
Thinking	**Working**
Write down the data that is given. No sign convention is used for the acceleration due to gravity. The net force on the system can, however, be described as clockwise is positive (right and down) and anticlockwise is negative (up and left).	$m_1 = 2.50\,\text{kg}$ $m_2 = 1.50\,\text{kg}$ $g = 9.8\,\text{m s}^{-2}$ $F_f = 8.50\,\text{N}$ left $F_f = -8.50\,\text{N}$
Determine the forces acting on the system The net force is positive as the net force causes the system to accelerate clockwise.	There are now two forces acting on the combined system of the cart and mass: the weight of the falling mass and friction. $\Sigma F = F_{weight} + F_f$ $\Sigma F = (+24.500) + (-8.50)$ $\Sigma F = 16.00\,\text{N}$ $\Sigma F = +16.00\,\text{N}$ on the system
Use Newton's second law to determine acceleration.	$a = \frac{\Sigma F}{m_t}$ $a = \frac{(+16.00)}{(4.00)}$ $a = +4.0000$
Ensure that the final answer has the appropriate number of significant figures, the correct units and the correct direction.	$a = 4.00\,\text{m s}^{-2}$ to the right

Worked example: Try yourself 4.3.4

CALCULATING THE ACCELERATION OF A CONNECTED BODY

A 0.600 kg trolley cart is connected by a cord to a 1.50 kg mass. The cord is placed over a pulley and the mass is allowed to fall under the influence of gravity in a similar way to the previous example.

a Assuming that the cart can move over the table unhindered by friction, determine the acceleration of the cart.

b If a frictional force of 4.20 N acts against the cart, what is the acceleration now?

THE FEATHER AND HAMMER EXPERIMENT

Why the experiment works on the Moon

When two objects with different mass fall under the influence of the force due to gravity in the absence of air resistance, they will both fall at the same rate. That is, their accelerations will be the same. They will cover the same displacement in the same period of time and will hit the ground at the same time if dropped from the same height. This experiment works on the Moon because there is no atmosphere and, therefore, no air resistance.

In a vacuum, all objects accelerate due to gravity at the same rate. It is a common misconception that the force due to gravity is the same on all objects. This is not true. In fact, the force due to gravity is larger on objects with a larger mass. This is because gravity acts on each particle of mass and a larger mass experiences a greater sum of gravity. Refer to Figure 4.3.5 to see how this works.

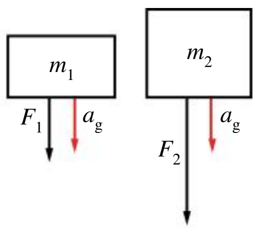


FIGURE 4.3.5 Given the same acceleration (a_g), a larger mass (m_2) must have a larger force (F_2) acting on it. If the mass is doubled, then the force is doubled.

If you consider the relationship between acceleration and mass, and the relationship between acceleration and force, then:

$$a = \frac{F}{m}$$

If m_2 is ten times the mass of m_1, then the force due to gravity on m_2 is ten times the force due to gravity on m_1. Consider the acceleration of both masses:

$$a_1 = \frac{F_1}{m_1}$$

If $F_2 = 10F_1$ and $m_2 = 10m_1$, then:

$$a_2 = \frac{F_2}{m_2}$$

$$a_2 = \frac{10F_1}{10m_1}$$

$$a_2 = \frac{F_1}{m_1}$$

$$a_2 = a_1$$

This proof shows that the ratio of force to mass is equal for all combinations of force and mass under the same effects due to gravity. This proves that all masses will experience the same acceleration if air resistance is removed.

Why the experiment does not work on Earth

When you see a feather floating down through the air, you know that it is accelerating at a rate far less than a hammer falling from the same height. From the previous section, you will know that the hammer and the feather have forces due to gravity acting on them that are proportional to their mass. They do not fall at the same rate because of the force of air resistance due to Earth's atmosphere. Remember, Newton's second law says the acceleration is proportional to the *net* force acting on an object, which means you must consider all the forces acting on an object to determine the resultant acceleration.

Air resistance is a force that results from air molecules colliding with the object. The faster the object moves, the greater the air resistance. In addition, the greater the surface area perpendicular to the direction of motion, the greater the air resistance. This force, which acts in the opposite direction to the motion of an object, is significant when compared with the weight of the feather, but insignificant when compared with the weight of the hammer. As a result, this force has a noticeable effect on the feather's acceleration but makes no noticeable difference to the hammer's acceleration if the experiment is performed on Earth.

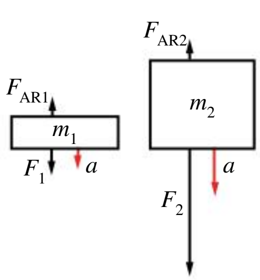


FIGURE 4.3.6 The effect of air resistance on an object depends on the surface area perpendicular to the motion and the speed of the object.

In Figure 4.3.6, the force of air resistance is denoted as F_{AR} and is the same size on both objects. The difference between the two objects is the downwards weight force (due to gravity).

Figure 4.3.6 also shows that due to the air resistance force the acceleration of the smaller mass is much less than the acceleration of the larger mass. Failing to consider the effect of air resistance is the origin of the common misconception.

4.3 Review

SUMMARY

- Newton's second law of motion states that the acceleration of an object is directly proportional to the net force on the object and inversely proportional to the mass of the object.
- Force can be calculated using the formula $\Sigma F = ma$
- This formula can be rewritten as follows:

$$\Sigma F = m\left(\frac{v_f - v_i}{\Delta t}\right) \text{ or } \Sigma F = \frac{\Delta p}{\Delta t}$$

- Force is difficult to perceive when it acts on objects, but we can perceive mass and acceleration.
- Different forces due to gravity act on different masses to cause the same acceleration.
- Air resistance is a force that acts to decrease the acceleration of objects moving through air.

KEY QUESTIONS

Use $g = -9.80\,\text{m s}^{-2}$ when answering the following questions.

1 Calculate the acceleration of a 23.9 kg mass when a net force of 158 N north acts on it.

2 Calculate the mass of an object if it accelerates at $9.20\,\text{m s}^{-2}$ east when a net force of 352 N east acts on it.

3 Calculate the final velocity of a stationary 55.9 kg mass when a net force of 56.8 N north acts on it for 3.50 seconds.

4 Calculate the acceleration of a 45.0 kg mass that has a net force of 441 N acting on it due to gravity.

5 Calculate the acceleration of a 90.0 kg mass that has a net force of 882 N acting on it due to gravity.

6 Calculate the final velocity of a 60.0 kg mass moving at $2.67\,\text{m s}^{-1}$ east, when a net force of 45.5 N west acts on it for 2.80 seconds.

7 Calculate the acceleration of a 60.9 g golf ball when a net force of 95.0 N south acts on it.

8 Calculate the mass of a train if it accelerates at $7.20\,\text{m s}^{-2}$ north when a net force of 565 000 N north acts on it. Give your answer to three significant figures.

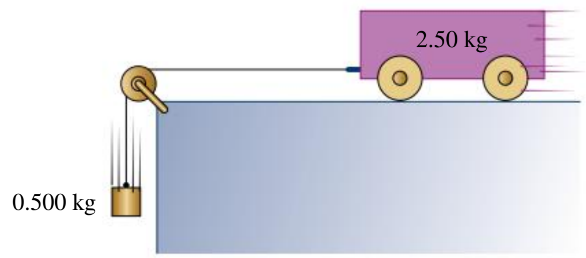
9 Calculate the final velocity of a stationary 3.00 g marble when a net force of 0.0823 N north acts on it for 0.0105 seconds.

10 Maru is paddling a canoe. The paddles are providing a constant driving force of 45.0 N south and the drag forces total 25.0 N north. The mass of the canoe is 15.0 kg and Maru has a mass of 50.0 kg.

a Find the net horizontal force acting on the canoe.

b Calculate the magnitude of the canoe's acceleration.

11 A 0.500 kg metal block is attached by a piece of string to a dynamics cart, as shown below. The block is allowed to fall from rest, dragging the cart along. The mass of the cart is 2.50 kg.



a If friction is ignored, what is the acceleration of the cart and the metal block?

b How fast will the cart and metal block be travelling after 0.500 s?

c If a frictional force of 4.30 N acts on the cart, what is the acceleration of the cart and metal block?

12 An empty truck of mass 2.00×10^3 kg has a top acceleration of $2.00\,\text{m s}^{-2}$. The mass of one box is 300.0 kg. How many boxes would the truck be carrying if the truck's top acceleration decreased to $1.25\,\text{m s}^{-2}$.

13 The thrust force of a rocket with a mass of 5.00×10^4 kg is 1.00×10^6 N. Neglecting air resistance, calculate its acceleration.

4.4 Newton's third law

Newton's first two laws of motion describe the motion of an object resulting from the forces that act on that object. Newton's third law of motion about actions and reactions is easily stated and seems to be widely known by students, but it is often misunderstood and misused. It is a very important law in physics as it assists with the understanding of the origin and nature of forces. Newton's third law is explored in detail in this section.

NEWTON'S THIRD LAW

Newton realised that all forces exist in pairs and that each force in the pair acts on a different object. Look at Figure 4.4.1, which shows a hammer hitting a nail on the head. Both the hammer and the nail experience a force at this point of contact. The nail experiences a downwards force as the hammer hits it, which causes the nail to penetrate into the wood. As it hits the nail, the hammer experiences an upwards force that causes the hammer to stop. These forces are known as an action–reaction pair and are shown in Figure 4.4.2.

FIGURE 4.4.1 A hammer hitting a nail is a good example of an action–reaction pair and Newton's third law.

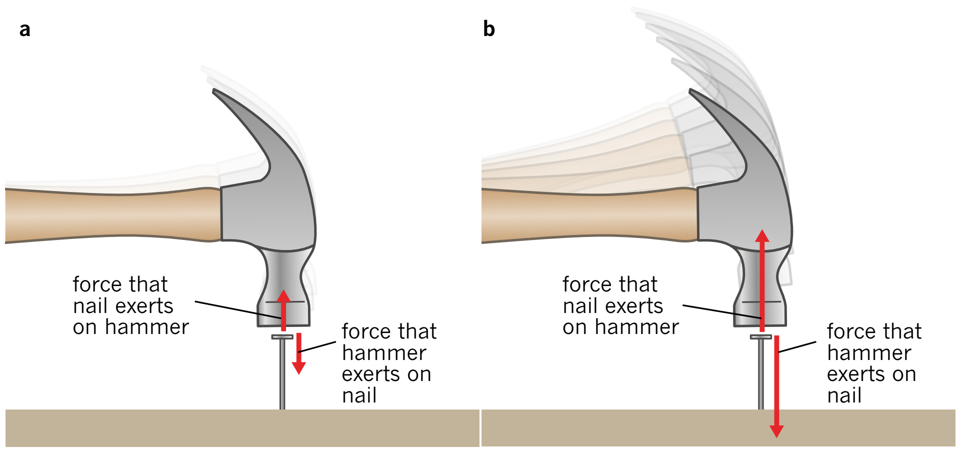


FIGURE 4.4.2 (a) As the hammer gently taps the nail, both the hammer and the nail experience small equal and opposite forces. (b) When the hammer hits the nail hard, both the hammer and the nail experience large equal and opposite forces.

It is important to note that, regardless of whether the hammer exerts a small or large force on the nail, the nail will exert exactly the same force back on the hammer.

Newton's third law of motion states the following:

> For every action (force), there is an equal and opposite reaction (force).

This means that when object A exerts a force, F, on object B, object B will exert an equal and opposite force, $-F$, on object A. It is important to recognise that the action force and the reaction force in Newton's third law act on different objects, so their effect will only be on the object on which they act. Newton's third law applies not only to forces between objects that are in direct contact, but also to non-contact forces, such as the force due to gravity between objects.

The main misconception that arises when considering Newton's third law is the belief that, if a large mass collides with a smaller mass, then the larger mass exerts a larger force on the smaller mass, and the smaller mass exerts a smaller force on the larger mass. This is not true. In fact, the force that the bus exerts on the car is equal to the force that the car exerts on the bus as they collide, but the effect of these forces differs depending on the mass of the objects involved. If you witnessed the collision between the car and the bus shown in Figure 4.4.3c, you would see the car undergoing a large deceleration while the bus undergoes only a small acceleration.

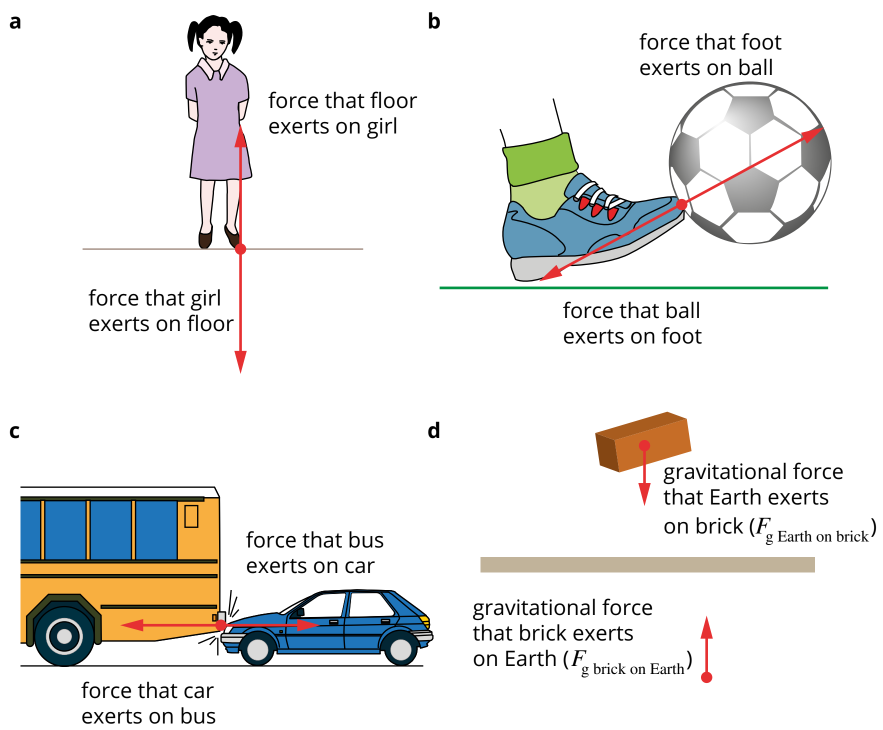


FIGURE 4.4.3 Some action–reaction pairs.

From Newton's second law, you know that an equal force acting on the bus's larger mass will result in a smaller acceleration. As a result, the occupants will not be as seriously affected. It follows that an equal force acting on the car's smaller mass will cause the car to undergo a large deceleration. The occupants may be seriously injured because of this. The forces acting on the bus and the car are equal in size, but the effect of the equal force on different masses will be different.

Identifying the action and reaction forces

When analysing a situation to determine the action–reaction pair of forces according to Newton's third law, it is helpful to be able to label the force vectors systematically. A good strategy for labelling force vectors is to use the capital letter F to represent the force and then to include a subscript consisting of the thing that is applying the force, the word 'on', and the thing on which the force is acting.

The equal and opposite force is then labelled with a capital F and a subscript with the objects in reverse. For example, the action and reaction force vector arrows shown in Figure 4.4.3 can be labelled as shown in Table 4.4.1.

TABLE 4.4.1 Labels of action and reaction force vectors shown in Figure 4.4.3.

	Action vector	Reaction vector
a	$F_{\text{girl on floor}}$	$F_{\text{floor on girl}}$
b	$F_{\text{foot on ball}}$	$F_{\text{ball on foot}}$
c	$F_{\text{car on bus}}$	$F_{\text{bus on car}}$
d	$F_{\text{brick on Earth}}$	$F_{\text{Earth on brick}}$

PHYSICSFILE

Combining Newton's second and third laws in the classroom

You can easily observe the effect of Newton's second and third laws in the classroom if you have two dynamics carts with wheels that are free to roll on a smooth surface (such as a bench or track). These carts generally have magnets attached to the ends. If you place two carts together so that similar magnetic poles are in contact, you will feel repulsion between the carts. Once you release the carts, you will observe that *both* carts roll backwards. This occurs due to the action–reaction pair described by Newton's third law. If the two carts have similar masses, they will accelerate apart at a similar rate. However, if one cart is heavier than the other, you will observe the lighter cart accelerates at a greater rate. This is because the forces acting on both carts are equal in magnitude and so, according to Newton's second law, the smaller mass will experience a greater acceleration.

Worked example 4.4.1

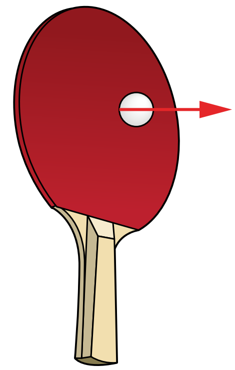
APPLYING NEWTON'S THIRD LAW

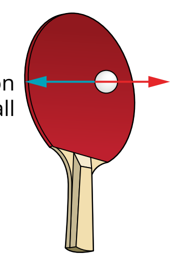
In the diagram, a table-tennis bat is in contact with a table-tennis ball, and one of the forces is given.

a Label the given force using the system
'*F* ______________________
on ______________________'.

b Label the reaction force to the given force using the system
'*F* ______________________
on ______________________'.

c Draw the reaction force on the diagram, showing its size and location.



Thinking	Working
Identify the two objects involved in the action–reaction pair.	the bat and the ball
Identify which object is applying the force and which object is experiencing the force for the force vector shown.	The force vector shown is a force from the bat on the ball.
Use the system of labelling action and reaction forces '*F* ______________________ on ______________________' to label the action force.	$F_{\text{bat on ball}}$
Use the system of labelling action and reaction forces '*F* ______________________ on ______________________' to label the reaction force.	$F_{\text{ball on bat}}$
Use a ruler to measure the length of the action force and construct a vector arrow of equal length in the opposite direction with its tail on the point of application of the reaction force.	

Worked example: Try yourself 4.4.1

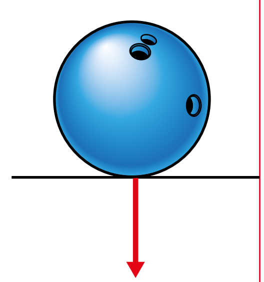
APPLYING NEWTON'S THIRD LAW

In the diagram, a bowling ball is resting on the floor, and one of the forces is given. Copy the diagram into your book and complete the following:

a Label the given force using the system '*F* _______ on _______'.

b Label the reaction force to the given force using the system '*F* _______ on _______'.

c Draw the reaction force on the diagram, showing its size and location.



NEWTON'S THIRD LAW AND MOTION

Newton's third law also explains the motion of walking around. Consider a person who is walking. Their foot pushes backwards on the ground with each step, which is an action force on the ground by the foot.

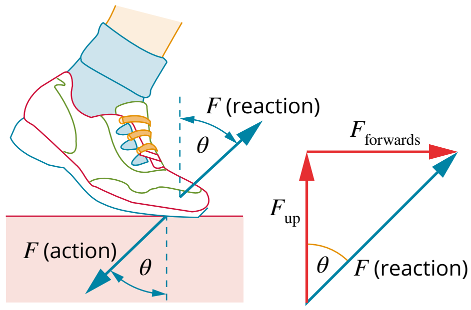


FIGURE 4.4.4 Walking relies on an action–reaction pair in which the foot pushes down and backwards with an action force. In response, the ground pushes upwards and forwards on you.

In response, the ground pushes forwards on the body via the foot. As shown in Figure 4.4.4, a component of the force of the foot on the ground acts downwards and another component pushes backwards horizontally along the surface of the ground. The force is transmitted because there is friction between the shoe and the ground. In other words, it is the ground pushing forwards on the body that moves a person forwards as they walk.

It is important to remember that in Newton's second law, $\Sigma F = ma$, the net force, ΣF, is the sum of the forces acting on the body. This does not include forces that are exerted by the body on other objects. When a foot pushes on the ground, this force is acting on the ground and may affect the ground's motion. If the ground is firm, this effect is usually not noticed, but if you run along a sandy beach, the sand is clearly pushed back by your feet.

The act of walking relies on there being some friction between the shoe and the ground. Without it, there is no grip and it is impossible to supply the action force to the ground, hence the ground cannot supply the reaction force needed to propel you forwards. Walking on smooth ice is a good example of this. Mountaineers use crampons (a rack of sharp spikes) attached to the soles of their boots in order to gain better grip in icy conditions.

Another key aspect is to remember that even though the action and reaction forces are equal and opposite, the accelerations imparted on the different objects are not. As a person, your mass is much smaller than the mass of the Earth, therefore you will witness a much larger acceleration compared to the Earth when you apply a force on the ground.

All motion can be explained in terms of action–reaction pairs. Table 4.4.2 gives some examples of the action–reaction pairs in familiar motions.

TABLE 4.4.2 Action–reaction pairs are responsible for all types of motion.

Motion	Action force	Reaction force
swimming	hand pushes backwards on water	water pushes forwards on hand
jumping	legs push down on Earth	Earth pushes up on legs
bicycle or car	tyre pushes backwards on ground	ground pushes forwards on tyre
jet aircraft and rockets	hot gas is forced backwards out of engine	gases push craft forwards
skydiving	force of gravitation on the skydiver from Earth	force of gravitation on Earth from skydiver

THE NORMAL FORCE

When an object, for example the rubbish bin shown in Figure 4.4.5, is allowed to fall under the influence of gravity, it is easy to see the effect of the force due to gravity. The action force is the force due to gravity of the Earth on the bin ($F_{weight} = mg$), so the net force on the bin is equal to the force due to gravity, and the bin therefore accelerates at $-9.80\,m\,s^{-2}$.

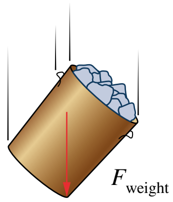


FIGURE 4.4.5 When the bin is in midair, there is an unbalanced force due to gravity acting on it, so it accelerates towards the ground.

When the bin is at rest on a table, as shown in Figure 4.4.6a, the force due to gravity is still acting between the Earth and the bin. Since the bin is at rest, there must be another force acting to balance the force due to gravity on the bin. This upwards force is provided by the table. Because gravity pulls down on the mass of the bin, the bottom of the bin will push down on the surface of the table and the table provides a reaction force on the bin that is equal and opposite, so it will push upwards on the bin as shown in Figure 4.4.6b.

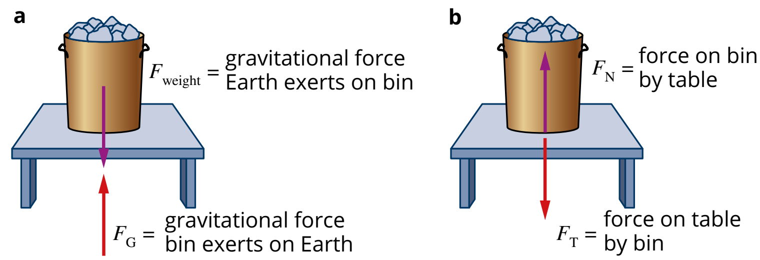


FIGURE 4.4.6 (a) Action–reaction gravitational forces between the bin and Earth. (b) Action–reaction contact forces between the bin and the table.

The magnitude and direction of the gravitational force on the bin by Earth is equivalent to the magnitude and direction of the force on the table by the bin. Therefore, the gravitational force on the bin by Earth (F_{weight} in Figure 4.4.6a) is balanced by the upwards contact reaction force on the bin by the table (F_N in Figure 4.4.6b). It is important to note that these two forces are not the pair of forces described in Newton's third law (Figure 4.4.7). This is because the two forces are both acting on the bin and according to Newton's third law, action–reaction pairs always act on different objects. The contact force provided by a surface that is perpendicular to another surface is called the normal reaction force. It is often abbreviated to normal force and represented by F_N.

The reaction force to the force due to gravity acting on the bin is in fact the force due to gravity of the bin acting on the Earth. This equal and opposite force, however, is tiny in comparison to the total mass of the Earth and so, from Newton's second law, the acceleration experienced by the Earth is negligible.

When you consider only the forces acting on the bin, you are left with the force due to gravity on the bin by the Earth and the normal force on the bin by the table. These two forces come from two different action–reaction pairs of forces.

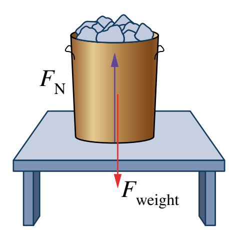


FIGURE 4.4.7 The effect of the two forces acting on the bin is zero acceleration. These are not an action–reaction pair because they are both acting on the same object, even though they are equal in magnitude and opposite in direction.

FRICTIONAL FORCES

Friction is a force that opposes movement. Suppose you want to push your textbook along the table. If you start to push the book very gently, you will find that the book does not move at first. You then increase the force that you apply. Suddenly, at a certain critical value, the book starts to move.

There is a maximum frictional force that resists the start of the slide of the book along the surface of the table. This force is called the **static friction force**, $F_{s.}$ Once the book begins to slide, a much lower force than F_s is needed to keep the book moving. This force is called the **kinetic** (or dynamic) **friction force** and is represented by F_K. The graph in Figure 4.4.8 shows how the force required to move an object changes as static friction is overcome.

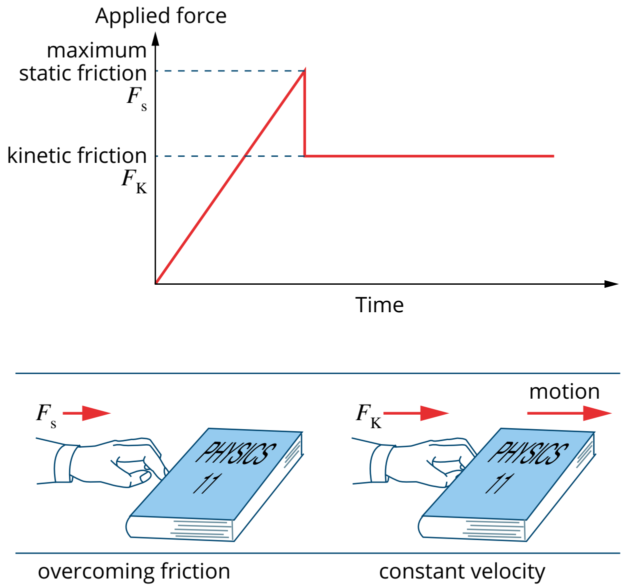


FIGURE 4.4.8 To get things moving, the static friction between an object and the surface must be overcome. This requires a larger force than is needed to maintain constant velocity.

This phenomenon can be understood when you consider that even the smoothest surfaces are quite jagged at the microscopic level. When the book is resting on the table, the jagged points of its bottom surface have settled into the valleys of the surface of the table, and this helps to resist attempts to slide the book. Once the book is moving, the jagged points of the surfaces do not have any time to settle into each other, so less force is required to keep the book moving.

Another factor that helps to explain friction arises from the forces of attraction between the atoms and molecules of the two different surfaces that are in contact. These surfaces produce weak bonds between their respective particles, so before one surface can move across the other, these bonds must be broken. This extra effort adds to the static friction force. Once there is relative motion between the surfaces, the bonds cannot re-form.

In everyday life, there are situations in which friction is desirable (e.g. walking) and others in which it is a definite problem. Consider the moving parts within the engine of a car. Friction can reduce an engine's fuel economy and cause it to wear out. Special oils and other lubricants are used in engines to prevent moving metal surfaces from touching. If the moving surfaces moved directly over each other, they would quickly wear, producing metal filings that could damage the engine. Instead, metal surfaces are separated by a thin layer of lubricant. These substances are chosen based on their viscosity (thickness). For example, low viscosity oils can be used in the engine, while heavier oils are needed in the gearbox and differential of the car where greater forces are applied to the moving parts. On the other hand, the lack of friction enables magnetic levitation trains to reach very high speeds, as shown in Figure 4.4.9.

FIGURE 4.4.9 This magnetic levitation train in China rides 1 cm above the track, so the frictional forces are negligible. The train is propelled by a magnetic force to a cruising speed of about 430 km h^{-1}.

At other times, having friction is essential. If there is any ice on the road when driving to snowfields, drivers are required to fit chains to their cars. When driving over a patch of ice, the chains break through the ice and the car is able to grip the road. Similarly, friction is definitely required within the car's brakes when the driver wants to slow down. In fact, modern brake pads are specially designed to maximise the friction between them and the brake drum or disc.

CALCULATING FRICTION

As mentioned earlier, the force of friction (F_f) will change depending on whether the object is moving or stationary. Additionally, different materials will have different resistive properties. These factors can be combined to create a value called the **coefficient of friction** (μ). An object with a higher coefficient of friction will apply a greater resistive force. Table 4.4.3 shows the coefficient of friction for a number of different surfaces.

TABLE 4.4.3 The static and kinetic coefficients of friction for different material combinations.

Material combination	Coefficient of static friction, μ_s	Coefficient of kinetic friction, μ_k
steel on steel	0.74	0.57
rubber on concrete (dry)	1.00	0.80
rubber on concrete (wet)	0.30	0.25
teflon on steel	0.04	0.04
wood on wood	0.50	0.30

The force of friction (F_f) is proportional to the normal force (F_N) and the coefficient of friction (μ) between the surfaces. The following relationship can be used to quantitatively calculate the force of friction.

$F_f = \mu F_N$

where F_f = the static or dynamic force of friction, measured in newtons (N)

μ = the coefficient of static or dynamic friction

F_N = normal force acting on an object, measured in newtons (N).

$F_N = -F_{weight} = -mg$

(F_{weight} is the force of gravity acting at the centre of mass of a body in newtons.)

The coefficient of friction can be experimentally determined by measuring the force needed to either begin to move an object (static coefficient of friction) or continue to move an object (kinetic coefficient of friction), and the normal force acting on the object. Rearranging the equation allows you to calculate the coefficient of friction.

$$\mu = \frac{F_f}{F_N}$$

As the coefficient of friction is a ratio of two forces, both measured in newtons, it is a dimensionless value. Therefore, it has no units.

Worked example 4.4.2

CALCULATING THE COEFFICIENT OF FRICTION

Oliver set up an experiment to calculate the coefficient of friction acting between a wooden block and a wooden desk. They placed a 2.50 kg block on the desk and applied a force until the block just started to move. Oliver then measured this force to be 15.0 N. Calculate the coefficient of friction and classify this as either the static or kinetic coefficient of friction.	
Thinking	**Working**
Calculate the normal force acting on the wooden block.	$F_N = -mg$ $F_N = -(2.50)(-9.80)$ $F_N = +24.500$ $F_N = 24.500$ N upwards
Calculate the coefficient of friction. Ensure the final answer has the appropriate number of significant figures.	$\mu = \frac{F_f}{F_N}$ $\mu = \frac{(15.0)}{(+24.500)}$ $\mu = 0.61224$ $\mu = 0.612$
Classify the coefficient of friction as either static or kinetic.	As this force just started the object's motion, it is the static coefficient of friction.

Worked example: Try yourself 4.4.2

CALCULATING THE COEFFICIENT OF FRICTION

Tate and Elise set up an experiment to calculate the coefficient of friction acting between a wooden block and a wooden desk. Elise placed a 5.00 kg block on the desk and applied a force until the block was in constant motion. Tate then measured this force to be 14.0 N. Calculate the coefficient of friction and classify this as either the static or kinetic coefficient of friction.

4.4 Review

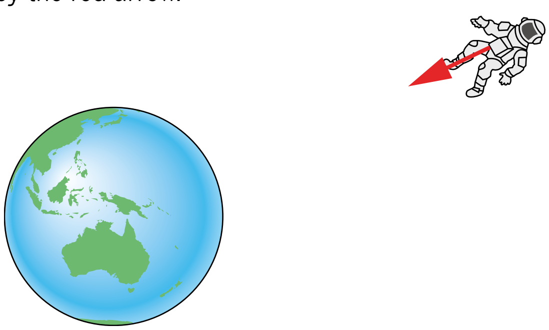
SUMMARY

- For every action (force), there is an equal and opposite reaction (force). This is known as Newton's third law of motion.
- If the action force is labelled systematically, the reaction force can be described by reversing the label of the action force.
- The action and reaction forces are equal and opposite even when the masses of the colliding objects are very different.
- The two equal and opposite forces making up an action–reaction pair act on different masses to cause different accelerations according to Newton's second law.
- When an object exerts a downwards force on a surface, there is an equal and opposite reaction that exerts an upwards force. This is called the normal force.
- Friction is a contact force between two objects that acts against motion. It is proportional to the normal force (F_N) and the coefficient of friction (μ) between the surfaces.
- The frictional force (F_f) can be calculated using the equation $F_f = \mu F_N$

KEY QUESTIONS

1 What forces act on the hammer and the nail when a heavy hammer hits a small nail? What are the comparative magnitude and direction of these forces?

2 In the figure below, an astronaut is orbiting the Earth and one of the forces acting on the astronaut is shown by the red arrow.



a Name the given force using the system '*F* _______ on_______'.

b Name the reaction force using the system '*F* _______ on _______'.

3 A swimmer completes a training drill in which they don't use their legs to kick, but only use their stroke (arms) to move down the pool. What force causes the swimmer to move forwards down the pool?

4 When an inflated balloon is released it will fly around the room. What is the force that causes the balloon to move?

5 Determine the reaction force involved when a ball is hit with a racquet with a force of 100.0 N west.

6 A 70.0 kg person is fishing in a 40.0 kg dinghy at rest on a still lake when, suddenly, they are attacked by a swarm of wasps. To escape, the person leaps into the water and exerts a horizontal force of 140 N north on the boat.

a What force does the boat exert on the person at the moment when they jump?

b With what acceleration will the boat move initially?

c If the force on the person lasted for 0.500 s, determine the initial speed attained by both the person and the boat.

7 An astronaut becomes untethered during a spacewalk and drifts away from the spacecraft. To get back to the spacecraft, they decide to throw their toolkit away. In which direction should they throw the toolkit?

8 Two students, Yara and Talia, are discussing the forces acting on a lunch box that is sitting on the laboratory bench. Yara states that the weight force and the normal force are acting on the lunch box and that since these forces are equal in magnitude but opposite in direction, they comprise a Newton's third law action–reaction pair. Talia disagrees, saying that these forces are not an action–reaction pair. Who is correct and why?

9 A skier has finished a race and is travelling on a flat path of snow. If the skier has a mass of 87.0 kg and the coefficient of kinetic friction between the skis and the snow is 0.0500, calculate the average force of friction acting on the skier.

10 An electric unicycle has a mass of 53.0 kg and is ridden on a road by a person with a mass of 75.0 kg. The unicycle is capable of accelerating from rest to 70.0 km h^{-1} in 5.05 seconds. If the coefficient of friction between rubber (unicycle's tyre) and asphalt (road) is 0.720, calculate the average force that needs to be applied by the unicycle's motor.

4.5 Change in momentum and impulse

Section 4.1 described the momentum of an object in terms of its velocity and its mass. For each of the different collisions described in that section, the momentum of the system was conserved. That is, when all the objects involved in the collision were considered, the total momentum before and after the collision was the same. But for each separate object, considered in isolation, momentum may not have been conserved. In the examples explored, an object experienced a change in its velocity due to the collision.

When an object changes its velocity, its momentum will also change. An increase in velocity means an increase in momentum, while a decrease in velocity means a decrease in momentum. Change in momentum, Δp, is also called **impulse**, I.

CHANGE IN MOMENTUM IN ONE DIMENSION

It is easy to change the velocity of an object. You can either run faster or run slower; you can press a little harder on the pedals of a bike or press a little softer. You can also bounce an object off a surface. For example, the basketball in Figure 4.5.1 experiences a change in velocity when it changes direction during the bounce. The cause of these changes in motion will be discussed in Section 4.6. First, consider impulse or change in momentum in one dimension.

The term 'impulse' means change in momentum. So, the impulse or change in momentum of an object moving in one dimension is calculated using the equation:

$$I = \Delta p$$

$$\Delta p = p_{final} - p_{initial}$$

$$\Delta p = mv_f - mv_i$$

where I is the impulse (kg m s^{-1})

Δp is the change in momentum (kg m s^{-1})

m is the mass (kg)

v_f is the final velocity (m s^{-1})

v_i is the initial velocity (m s^{-1}).

FIGURE 4.5.1 A bouncing basketball undergoes a change in momentum when it changes direction as it bounces.

As momentum is a vector quantity, the impulse or change in momentum is also a vector, so it is expressed in magnitude, units and direction.

Worked example 4.5.1

IMPULSE OR CHANGE IN MOMENTUM

A student rides a bike to school and approaches the bike rack at 8.20 m s^{-1} east. Calculate the impulse of the student during the time it takes to stop if the student and the bike have a combined mass of 80.0 kg and the student stops at the rack.

Thinking	Working
Ensure that the variables are in their standard units.	$m = 80.0$ kg $v_i = 8.20$ m s^{-1} east $v_f = 0.00$ m s^{-1}
Apply the sign convention to the velocity vector.	$m = 80.0$ kg $v_i = +8.20$ m s^{-1} $v_f = 0.00$ m s^{-1}
Apply the equation for impulse or change in momentum.	$\Delta p = mv_f - mv_i$ $\Delta p = (80.0)(0.00) - (80.0)(8.20)$ $\Delta p = (0.00) - (656.00)$ $\Delta p = -656.00$

Ensure that the final answer has the appropriate number of significant figures, the correct units and apply the sign convention to describe the direction of the impulse.	$\Delta p = 656\,\text{kg m s}^{-1}$ west

Worked example: Try yourself 4.5.1

IMPULSE OR CHANGE IN MOMENTUM

A student hurries to class after lunch, moving at $4.55\,\text{m s}^{-1}$ north. Suddenly the student remembers that they have forgotten their laptop and goes back to their locker at $6.15\,\text{m s}^{-1}$ south. If their mass is 75.0 kg, calculate the impulse of the student during the time it takes to turn around.

FIGURE 4.5.2 Changing momentum in two dimensions by changing direction.

CHANGE IN MOMENTUM IN TWO DIMENSIONS

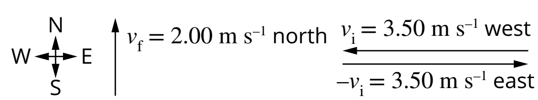
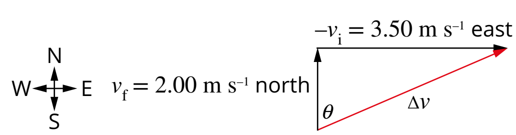
The velocity of an object can be changed not only by changing the magnitude of its velocity, but also by changing the direction of its motion. The velocity of the boat in Figure 4.5.2, for example, changes because the boat changes direction. As you saw in Chapter 2, a change in velocity in two dimensions can be calculated using geometry. The equation for impulse can be manipulated slightly to illustrate where the change in velocity is applied:

$$\Delta p = mv_f - mv_i$$
$$\Delta p = m(v_f - v_i)$$

Worked example 4.5.2

IMPULSE OR CHANGE IN MOMENTUM IN TWO DIMENSIONS

A 65.0 kg mass is moving at $3.50\,\text{m s}^{-1}$ west and then changes to $2.00\,\text{m s}^{-1}$ north. Calculate the change in momentum of the mass over the period of the change.

Thinking	Working
Identify the formula for calculating a change in velocity, Δv.	Δv = final velocity – initial velocity $\Delta v = v_f - v_i$ $\Delta v = v_f + (-v_i)$
Draw the final velocity vector, v_f, and the initial velocity vector, v_i, separately. Then draw the initial velocity in the opposite direction, which represents the negative of the initial velocity, $-v_i$.	
Construct a vector diagram drawing v_f first and then from its head draw the opposite of v_i. The change of velocity vector is drawn from the tail of the final velocity to the head of the opposite of the initial velocity.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the change in velocity.	$\Delta v^2 = 2.00^2 + 3.50^2$ $\Delta v^2 = 4.00 + 12.25$ $\Delta v = \sqrt{16.25}$ $\Delta v = 4.03113$ $\Delta v = 4.03\,\text{m s}^{-1}$

Calculate the angle from the north vector to the change in velocity vector.	$\tan\theta = \frac{3.50}{2.00}$ $\theta = \tan^{-1}(1.7500)$ $\theta = 60.2551$ $\theta = 60.3°$
State the magnitude and direction of the change in velocity.	$\Delta v = 4.03\,m\,s^{-1}$ N 60.3° E
Identify the variables using subscripts and ensure that the variables are in their standard units.	$m = 65.0\,kg$ $\Delta v = 4.03\,m\,s^{-1}$ N 60.3° E
Apply the equation for impulse or change in momentum.	$\Delta p = mv_f - mv_i$ $\Delta p = m(v_f - v_i)$ $\Delta p = m\Delta v$ $\Delta p = (65.0)(4.03113)$ $\Delta p = 262.023$
Ensure that the final answer has the appropriate number of significant figures, the correct units and apply the direction convention to describe the direction of the change in momentum.	$\Delta p = 262\,kg\,m\,s^{-1}$ N 60.3° E

Worked example: Try yourself 4.5.2

IMPULSE OR CHANGE IN MOMENTUM IN TWO DIMENSIONS

A 65.0 g pool ball is moving at $0.250\,m\,s^{-1}$ south towards a cushion and bounces off at $0.200\,m\,s^{-1}$ east. Calculate the impulse on the ball during the change in velocity.

4.5 Review

SUMMARY

- Change in momentum, Δp, is also known as impulse, I. It is a vector quantity.
- A change in momentum occurs when an object changes its velocity.
- The equation for impulse is: $I = \Delta p = mv_f - mv_i$
- Change in momentum in two directions can be calculated using geometry.

KEY QUESTIONS

1. Calculate the impulse of a 9.50 kg dog that changes its velocity from $2.50\,m\,s^{-1}$ north to $6.25\,m\,s^{-1}$ south.
2. Calculate the impulse of a 6050 kg truck as it changes from moving at $22.2\,m\,s^{-1}$ west to $16.7\,m\,s^{-1}$ east.
3. The velocity of an 8.00 kg mass changes from $3.00\,m\,s^{-1}$ east to $8.00\,m\,s^{-1}$ east. Calculate the change in momentum.
4. Calculate the change in momentum of a 250 g apple as it changes from rest to moving downwards at $9.80\,m\,s^{-1}$ after falling off a tree.
5. The momentum of a ball of mass 0.125 kg changes by $0.075\,kg\,m\,s^{-1}$ south. If its original velocity was $3.00\,m\,s^{-1}$ north, what is the final velocity?
6. A 45.0 kg mass moving at $45.0\,m\,s^{-1}$ west changes direction so that it moves at $45.0\,m\,s^{-1}$ north. Calculate the change in momentum of the mass over the period of the change.
7. A marathon runner with a mass of 70.0 kg is running with a velocity of $4.00\,m\,s^{-1}$ north, and then turns a corner to start running $3.60\,m\,s^{-1}$ west. Calculate the marathon runner's change in momentum.

4.6 Impulse and force

Section 4.3 on Newton's second law of motion discussed the quantitative connection between force, mass, time and change in velocity. This relationship is explored further in this section. The relationship between change in momentum, Δp, the period of time, Δt, and net force, ΣF, helps to explain the effects of collisions and how to minimise those effects. It is the key to providing safer environments, including in sporting contexts such as that shown in Figure 4.6.1.

FIGURE 4.6.1 When two footballers collide, they exert an equal and opposite force on each other.

Think about a stuntperson falling onto a foam mat. If they fell from the same height, perhaps off a roof, onto a concrete floor instead of a foam mat they would be injured. In both situations (with and without a foam mat) they would fall from the same height with the same acceleration due to gravity, hence their final velocity would be the same. They would have the same mass and have zero initial velocity. Yet the stuntperson is safer falling onto a foam mat.

CHANGE IN MOMENTUM (IMPULSE)

According to Newton's second law, a net force will cause a mass to accelerate. A larger net force will create a faster change in the velocity of the mass. The faster the change occurs the smaller the period of time, Δt, therefore the greater the net force that produced the change. Landing on a concrete floor changes the velocity of an object very quickly. The stuntperson is brought to an abrupt stop within a very short period of time. When landing on a foam mat, the change occurs over a greater period of time. Therefore, the force needed to produce the change is smaller and the stuntperson is less likely to be harmed.

Starting with the equation introduced in Section 4.3, the relationship between change in momentum, Δp, or impulse, I, and the variables of force, ΣF (often written just as F), and period of time, Δt, becomes:

$$F = \frac{\Delta p}{\Delta t}$$

$$F = \frac{mv_f - mv_i}{\Delta t}$$

$$F = \frac{m(v_f - v_i)}{\Delta t}$$

$$F\Delta t = m(v_f - v_i)$$

$$= I$$

where I is the impulse (kg m s^{-1}).

These equations illustrate that for a given change in momentum (Δp) or impulse (I), the product of force (F) and period of time (Δt) are constant. This relationship is key to understanding collisions. Worked examples 4.6.1 and 4.6.2, below, illustrate how this works.

Worked example 4.6.1

CALCULATING THE FORCE AND IMPULSE

A student drops a 105 g pool ball onto a concrete floor from a height of 2.00 m. Just before it hits the floor, the velocity of the ball is 6.26 m s^{-1} down. Before it bounces back up, there is an instant in time at which the ball's velocity is zero. The time it takes for the ball to change its velocity to zero is 5.02 milliseconds.

a Calculate the change in momentum of the pool ball.	
Thinking	**Working**
Ensure that the variables are in their standard units.	$m = 0.105$ kg $v_i = 6.26$ m s^{-1} down $v_f = 0$ m s^{-1}
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 0.105$ kg $v_i = -6.26$ m s^{-1} $v_f = 0$ m s^{-1}
Apply the equation for change in momentum.	$\Delta p = m(v_f - v_i)$ $\Delta p = (0.105)((0.00) - (-6.26))$ $\Delta p = 0.65730$
Ensure that the final answer has the appropriate number of significant figures, the correct units and refer to the sign and direction convention to determine the direction of the change in momentum.	$\Delta p = 0.657$ kg m s^{-1} up

b Calculate the impulse of the pool ball.	
Thinking	**Working**
Using the answer to part **a**, apply the equation for impulse.	$I = \Delta p$
Ensure that the final answer has the appropriate number of significant figures, the correct units and refer to the sign and direction convention to determine the direction of the impulse.	$I = 0.657$ kg m s^{-1} up

c Calculate the average force that acts to cause the impulse.	
Thinking	**Working**
Use the answer to part **b**. Ensure that the variables are in their standard units.	$I = 0.65730$ kg m s^{-1} $\Delta t = 5.02 \times 10^{-3}$ s
Apply the equation for force.	$F\Delta t = I$ $F = \frac{I}{\Delta t}$ $F = \frac{(0.65730)}{(5.02 \times 10^{-3})}$ $F = 130.936$

Ensure that the final answer has the appropriate number of significant figures, the correct units and refer to the sign and direction convention to determine the direction of the force.	F = 131 N up

Worked example: Try yourself 4.6.1

CALCULATING THE FORCE AND IMPULSE

A student drops a 56.0 g egg onto a table from a height of 60.0 cm. Just before it hits the table, the velocity of the egg is 3.43 m s^{-1} down. The egg's final velocity is zero as it smashes on the table. The time it takes for the egg to change its velocity to zero is 3.55 ms.

a Calculate the change in momentum of the egg.

b Calculate the impulse of the egg.

c Calculate the average force that acts to cause the impulse.

Worked example 4.6.2

CALCULATING THE FORCE AND IMPULSE (SOFT LANDING)

A student drops a 105 g pool ball onto a foam mattress from a height of 2.00 m. Just before it hits the foam mattress, the velocity of the ball is 6.26 m s^{-1} down. Before it bounces back up, there is an instant in time at which the ball's velocity is zero. The time it takes for the ball to change its velocity to zero is 0.360 seconds.

a Calculate the change in momentum of the pool ball.

Thinking	Working
Ensure that the variables are in their standard units.	m = 0.105 kg v_i = 6.26 m s^{-1} down v_f = 0.00 m s^{-1}
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	m = 0.105 kg v_i = −6.26 m s^{-1} v_f = 0.00 m s^{-1}
Apply the equation for change in momentum.	$\Delta p = m(v_f - v_i)$ $\Delta p = (0.105)((0.00) - (-6.26))$ $\Delta p = 0.65730$
Ensure that the final answer has the appropriate number of significant figures, the correct units and refer to the sign and direction convention to determine the direction of the change in momentum.	Δp = 0.657 kg m s^{-1} up

b Calculate the impulse of the pool ball.

Thinking	Working
Using the answer to part **a**, apply the equation for impulse.	$I = \Delta p$
Ensure that the final answer has the appropriate number of significant figures, the correct units and refer to the sign and direction convention to determine the direction of the impulse.	I = 0.657 kg m s^{-1} up

c Calculate the average force that acts to cause the impulse.	
Thinking	**Working**
Using the answer to part **b**, ensure that the variables are in their standard units.	$I = 0.65730\ \text{kg m s}^{-1}$ $\Delta t = 0.360\ \text{s}$
Apply the equation for force.	$F\Delta t = I$ $F = \frac{I}{\Delta t}$ $F = \frac{(0.65730)}{(0.360)}$ $F = 1.82583$
Ensure that the final answer has the appropriate number of significant figures, the correct units and refer to the sign and direction convention to determine the direction of the force.	$F = 1.83\ \text{N}$ up

Worked example: Try yourself 4.6.2

CALCULATING THE FORCE AND IMPULSE (SOFT LANDING)

A student drops a 56.0 g egg into a mound of flour from a height of 60.0 cm. Just before it hits the mound of flour, the velocity of the egg is $3.43\ \text{m s}^{-1}$ down. The egg's final velocity is zero as it sinks into the mound of flour. The time it takes for the egg to change its velocity to zero is 0.325 seconds.

a Calculate the change in momentum of the egg.

b Calculate the impulse of the egg.

c Calculate the average force that acts to cause the impulse.

From these worked examples you should notice several important things:

- The change in momentum and the impulse were always the same.
- Regardless of the surface that the object landed on, the impulse or change in momentum remained the same.
- The period of time was the main difference between the two different surfaces. Hard surfaces resulted in a short time to stop, and soft surfaces resulted in a longer time to stop.
- The effect of the period of time on the force was dramatic. A shorter time meant a greater force, while a longer time meant a much smaller force.

DETERMINING IMPULSE FROM A CHANGING FORCE

In the previous examples it was assumed that the force that acted to change the impulse over a period of time was constant during that time. This is not always the case in real situations. Often the force varies over the period of the impact, so there needs to be a way to determine the impulse as the force varies.

An illustration of this is when a tennis player strikes a ball with a racquet. At the instant the ball comes in contact with the racquet, the applied force will be small. As the strings distort and the ball compresses, the force will increase until the ball has been stopped. The force will then decrease as the ball accelerates away from the racquet. A graph of force against time is shown in Figure 4.6.2.

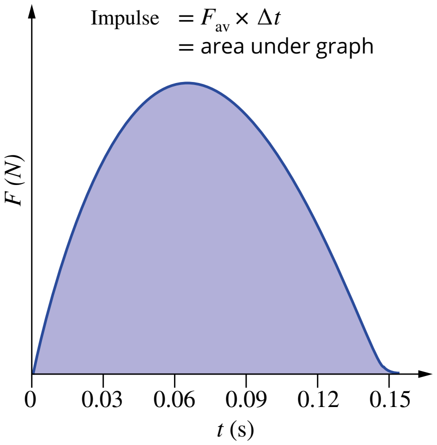


FIGURE 4.6.2 The forces acting on the tennis ball during its collision with the racquet are not constant.

The impulse, I, affecting the ball during any time interval will be the product of applied force, F (in newtons), and the period of time, Δt. The total impulse during the period of time the ball is in contact with the racquet will be:

$$I = F_{av}\Delta t$$

where F_{av} is the average force applied during the collision and Δt is the total period of time the ball is in contact with the racquet. In a graph showing force against time, the area under the line is the product of the height (force) and the width (time).

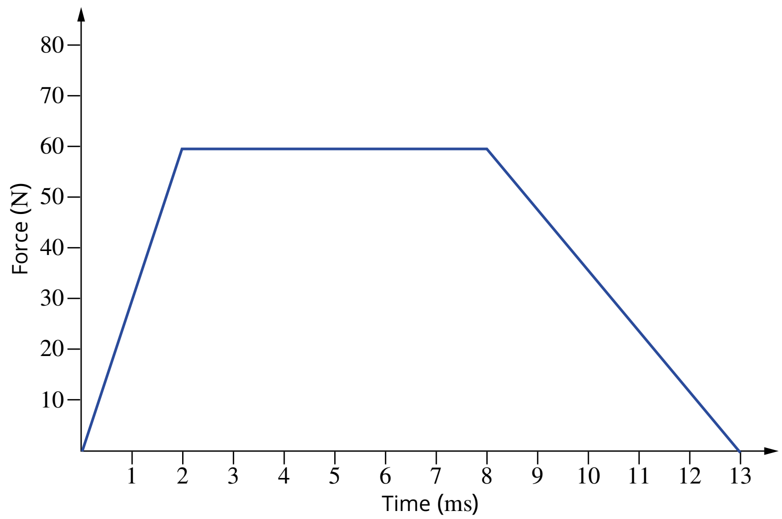
Thus, the total area under the line in a force against time graph is the total impulse for any collision, even those in which the force is not constant. Calculations of area under the line were explored in more detail in Section 3.4.

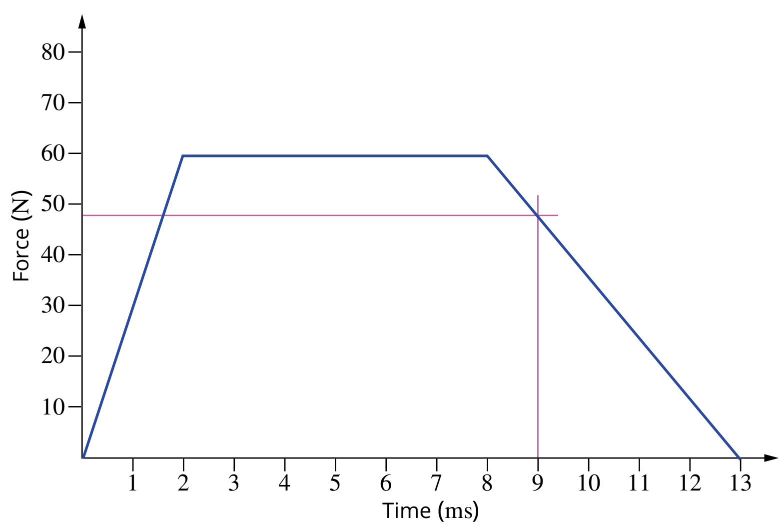
The concept of impulse is appropriate when dealing with forces during any collision since it links force and contact time as, for example, when a person's foot hits the ground or when a ball is hit by a bat or racquet. If applied to situations where contact is over an extended period of time, the average net force involved is used since the forces are generally changing (as the ball deforms, for example). The average net applied force can be found directly from the formula for impulse. The instantaneous applied force at any particular time during the collision must be read from a graph of force against time.

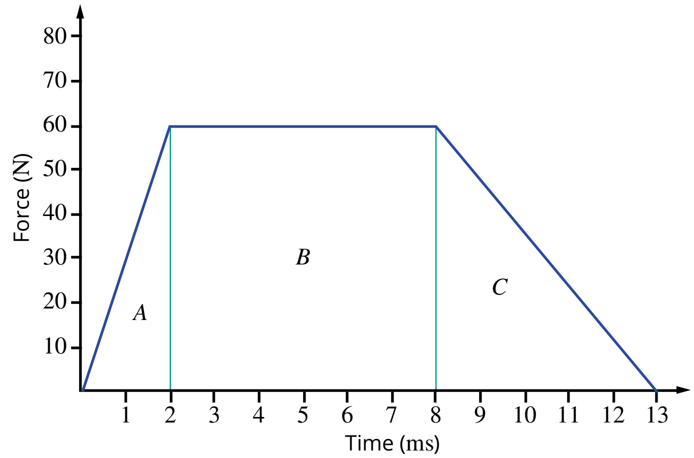
Worked example 4.6.3

CALCULATING THE TOTAL IMPULSE FROM A CHANGING FORCE

A student records the force acting on a rubber ball as it bounces off a hard concrete floor over a period of time. The graph shows the forces acting on the ball during its collision with the concrete floor.



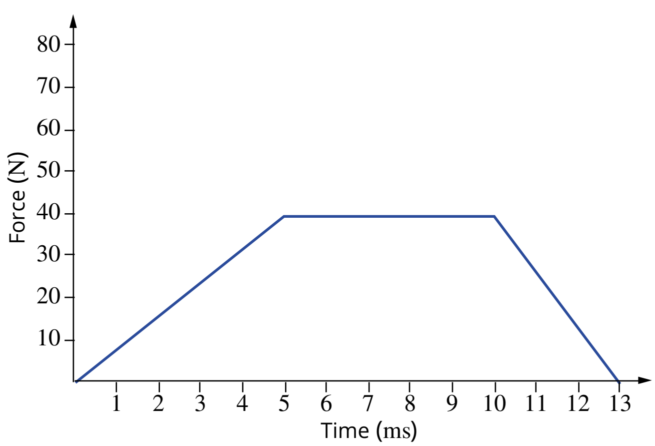
a Determine the force acting on the ball at a time of 9.0 ms.	
Thinking	**Working**
From the 9.0 ms point on the *x*-axis go up to the line of the graph, then across to the *y*-axis.	
The force is estimated by reading the intercept of the *y*-axis.	$F = 48\,\text{N}$

b Calculate the total impulse of the ball over the 13 ms period of time.	
Thinking	**Working**
Break the area under the line into sections for which you can calculate the area.	In this case, the graph can be broken into three sections: *A*, *B* and *C*. 
Calculate the area of the three sections *A*, *B* and *C* using the equations for area of a triangle and the area of a rectangle.	area = A + B + C area = $\left(\frac{1}{2}b \times h\right) + (b \times h) + \left(\frac{1}{2}b \times b\right)$ area = $\frac{1}{2}(2.0 \times 10^{-3})(60.0) + (6.0 \times 10^{-3})(60.0) + \frac{1}{2}(5.0 \times 10^{-3})(60.0)$ area = 0.060 + 0.360 + 0.150 area = 0.570
The total impulse is equal to the area.	I = area I = 0.57
Ensure that the final answer has the appropriate number of significant figures, the correct units and apply the sign and direction convention for motion in one dimension vertically.	$I = 0.57\ \text{kg m s}^{-1}$ up

Worked example: Try yourself 4.6.3

CALCULATING THE TOTAL IMPULSE FROM A CHANGING FORCE

A student records the force acting on a tennis ball as it bounces off a hard concrete floor over a period of time. The graph shows the forces acting on a ball during its collision with the concrete floor.



a Determine the force acting on the ball at a time of 4.0 ms.

b Calculate the total impulse of the ball over the 13.0 ms period of time.

PHYSICS IN ACTION

Car safety

Designing a successful car is a complex task. A vehicle must be reliable, economical, powerful, visually appealing, secure and safe. Public perception of the relative importance of these issues varies However, there is growing focus on car safety features thanks to government support of the Australasian New Car Assessment Program (ANCAP). ANCAP ensures that new cars meet minimum safety standards before they can be sold in Australia. Additionally, they rate the safety features of a car using a five-star system, with the purpose being to encourage manufacturers to exceed mandated levels of safety performance.

Vehicle safety is primarily about crash avoidance. Research shows potential accidents are avoided 99% of the time. The avoidance of vehicle crashes is mainly due to crash avoidance systems such as anti-lock brakes, automated emergency braking, adaptive cruise control and lane assist. When a collision does happen, passive safety features, such as airbags, come into operation. Understanding the theory behind accidents involves primarily an understanding of impulse and force.

The seatbelt

Whenever you enter a car, your first instinct is probably to reach for the seatbelt. Seatbelts started being included in cars in the 1950s, and in 1970 Victoria introduced the world's first seatbelt law.

The design of seatbelts works directly to try to mitigate the effects of Newton's first law. In order to get from one place to another at high speeds in a car, the occupants of the vehicle must themselves be travelling at high speeds within the car. You have seen from Newton's first law that a body in motion stays in motion unless acted on by an unbalanced (net) force. This means that if a car is involved in a collision, the occupants of the vehicle will keep moving, resulting in either ejection from the car or contact with the interior—causing serious injury or death. The seatbelt is designed to lock during severe deceleration and acts to oppose the occupants' motion. Seatbelts are a vital component of car safety—experts estimate that they decrease the risk of fatality by up to 50%.

The airbag

The introduction of seatbelts allowed many more people to survive car accidents. However, many of these survivors sustained serious injuries. So, although seatbelts saved lives, there was also an increase in serious injuries. A further safety device was required to minimise these injuries.

The airbag in a car is designed to inflate within a few milliseconds of the occurrence of a collision to reduce secondary injuries during the collision. The airbag is designed to inflate only when the vehicle experiences an impact with a solid object at 18–20 km h^{-1} or more. The required deceleration must be high, or accidental nudges with another car would cause the airbag to inflate. The car's computer control makes a decision within a few milliseconds to detonate the gas cylinders that inflate the airbag. The propellant detonates and inflates the airbag. According to Newton's first law, the driver continues to move towards the dashboard. For example, as the driver continues forwards into the airbag, the bag deflates, allowing the body to slow down over a longer time than would otherwise be possible as it moves towards the dashboard (Figure 4.6.3). The force is minimised so injury is reduced.

FIGURE 4.6.3 Airbags can prevent injuries by extending the period of time you take to stop.

Calculating exactly when the airbag should inflate, and for how long, is a difficult task. Many cars have been crash tested and the results painstakingly analysed. High-speed film demonstrates precisely why the airbag is so effective. During a collision the arms, legs and head of the occupants are restrained only by the joints and muscles. Enormous forces are involved because of the large deceleration. Looking back to the concept of impulse, the occupant's net change in momentum will be exactly the same as if they came to a halt themselves. However, the time over which the impulse takes place is reduced to milliseconds. From $I = F\Delta t$ it can be seen that this can increase the total force to a huge amount. The shoulders and hips can, in most cases, sustain the large forces for the short duration. However, the neck is the weak link. Victims of road accidents regularly receive neck and spinal injuries.

An airbag reduces the enormous forces the neck must withstand by extending the duration of the collision. This involves the direct application of the concept of impulse. A comparison of the forces applied to the occupant of a car with and without airbags is shown in Figure 4.6.4.

Airbags prevent the high forces caused by contact of the head with the interior of the car. The airbag ensures that the main thrust of the expansion is directed outwards instead of towards the occupant. The airbag's deflation rate, governed by the size of the holes in the rear of the airbag, provides the optimum deceleration of the head for a large range of impact speeds.

The airbag is not the answer to all safety concerns associated with a collision, but it is one of many safety features that form a chain of defence in a collision.

Other safety features in cars

Crumple zones, collapsible steering columns, helmets and safety barriers work on similar principles to airbags.

Humans are susceptible to damage from large forces, and so in the event of an accident, the primary goal of safety features is to reduce the amount of force acting on the person by extending the duration of the collision.

Crumple zones

Crumple zones are a structural feature in most cars (Figure 4.6.5). By providing a large distance between the driver and the front of the vehicle in the form of a long bonnet made out of malleable metals, engineers can extend the time period over which a crash takes place. Therefore, the total force applied is lowered before the impact has even reached the driver. Furthermore, much of the deformation which occurs to the car allows the energy of the crash to be dissipated into the metal rather than being delivered directly to the driver and passengers.

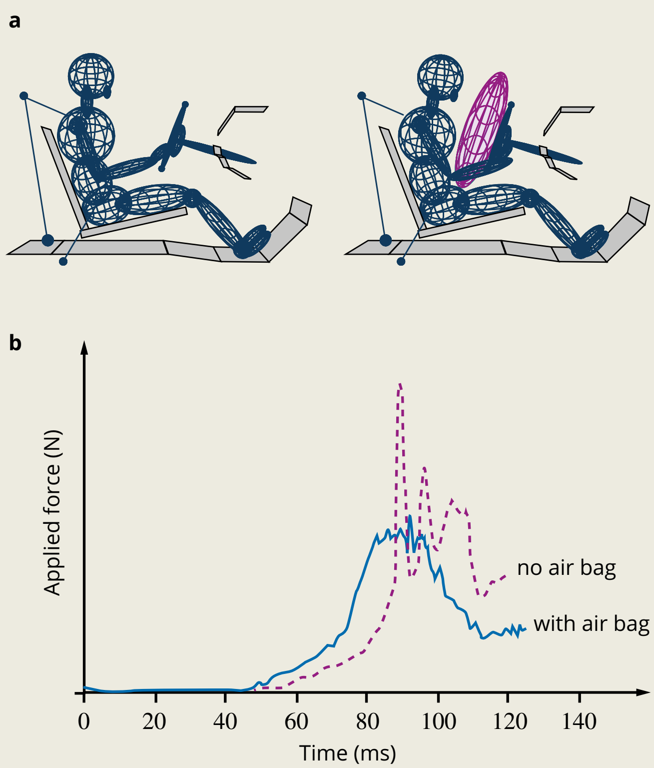


FIGURE 4.6.4 (a) The airbag extends the stopping time and distributes the force required to decelerate the mass of the driver or passenger over a larger area than a seatbelt. (b) The force withstood by the occupant of the car without an airbag is about double that felt with an airbag.

Car safety *continued*

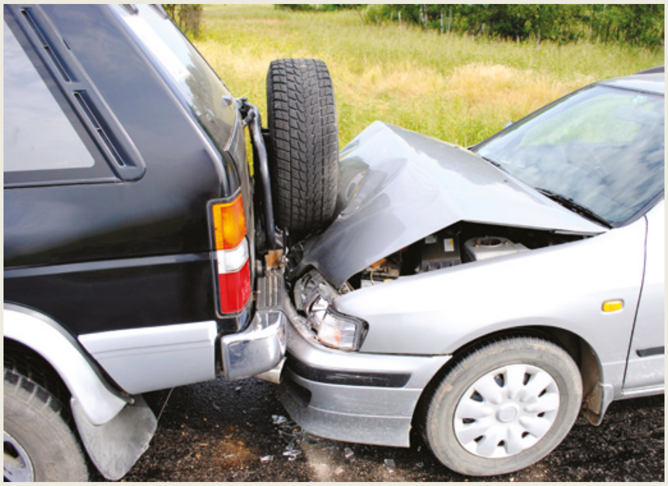


FIGURE 4.6.5 The crumple zone on this car has prevented serious injury to the car's occupants.

Collapsible steering columns

The driver of a vehicle is at a greater risk during a collision due to having a steering column right in front of them. In older vehicles, these steering columns were rigid and would transfer a large amount of the force from a collision into the driver. Collapsible steering columns are designed to have a breakable connection between the inner steering column and the outside housing. If a large enough force is applied, such as in a collision, the inner shaft will detach from the outer casing. This allows the column to collapse, absorbing energy from the impact and reducing the forces transferred to the driver.

Helmets

Helmets are similar in principle to crumple zones. The outer layer is typically made with a type of crushable foam. It controls the energy of an impact into the helmet itself rather than the user's head, extending collision time and spreading the energy of the collision over a greater area. Conservation of energy tells us that in a collision, all of the kinetic energy present has to either remain present or be transformed into another kind of energy. In the absence of any protective gear, this means that the very high kinetic energy of the collision will all be transferred into a very localised part of a rider's head. Helmets allow some of this kinetic energy to be spread out safely into the helmet to reduce the energy that reaches the head. Furthermore, helmets are usually designed with soft, padded interiors. This, in addition to the compressible foam, allows the helmet to move around a little in the event of an impact, therefore increasing the time of collision in the same way as airbags and crumple zones, and reducing the total force to which the rider is exposed.

Safety barriers

You may have seen traffic safety barriers at the side of roads and wondered what their purpose is—after all, a crash into a safety barrier will typically be anything but safe. They are designed to stop out-of-control vehicles running off the road or travelling into the path of other traffic (Figure 4.6.6). You know from Newton's first and second laws that a moving body needs an opposing force in order to stop it, and that in order to stop something moving very fast in a short amount of time the opposing force needs to be very large. Safety barriers are designed to withstand a high amount of force—this allows them to provide the normal force required back at vehicles in order to stop their motion. The sudden impact can be very damaging to the out-of-control vehicle, but ultimately the purpose of safety barriers is to prevent more widespread harm.

FIGURE 4.6.6 A steel safety barrier protecting motorists on a freeway bridge.

4.6 Review

SUMMARY

- Newton's second law describes the relationship between impulse, force and the period of time: $I = F\Delta t$
- The same mass changing its velocity by the same amount will have a constant change in momentum or impulse.
- The faster a mass changes its velocity, the greater the force required to change the velocity in that period of time.
- The slower a mass changes its velocity, the smaller the force required to change the velocity in that period of time.
- Forces can change during a collision.
- The impulse over a period of time can be found by calculating the area under the line on a force versus time graph.
- The period of time is the cause of the difference between the force provided by two different surfaces during a collision. Hard surfaces result in a short time to stop and soft surfaces result in a longer time to stop.
- The effect of the period of time on the force is dramatic. A shorter time means a greater force, while a longer time means a much smaller force.

KEY QUESTIONS

1 A 45.0 kg mass changes its velocity from 2.45 m s^{-1} east to 12.5 m s^{-1} east in a period of 3.50 s.
  a Calculate the change in momentum of the mass.
  b Calculate the impulse of the mass.
  c Calculate the force that causes the impulse of the mass.

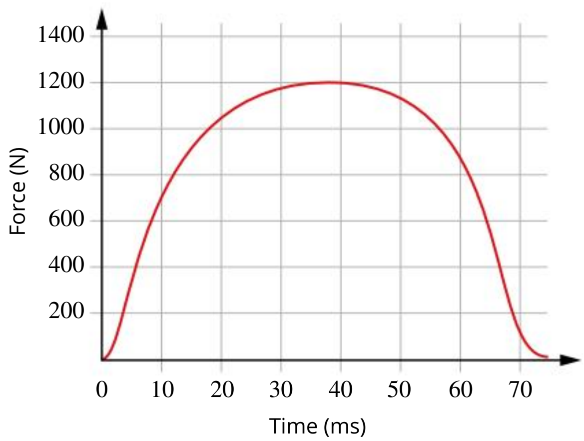
2 Using the concept of impulse, explain how airbags can reduce injuries during a collision.

3 A student catches a 75.0 g cricket ball with 'hard hands', which means catching the ball with hands held rigidly so that the ball will hit the palms hard. Just before the student catches the ball, its velocity is 15.6 m s^{-1} west. With hard hands, the velocity of the ball drops to zero in just 0.100 seconds.
  a Calculate the change in momentum of the cricket ball.
  b Calculate the impulse of the cricket ball.
  c Calculate the average force on the cricket ball.

4 The student from Question 3 now catches the same 75.0 g cricket ball, but this time with 'soft hands', which means catching the ball with hands held gently. Just before the student catches the ball, its velocity is 15.6 m s^{-1} west. With soft hands, the velocity of the ball drops to zero in 0.300 seconds. Calculate the average force on the cricket ball.

5 A 200.0 g cricket ball (at rest) is struck by a cricket bat. The ball and bat are in contact for 0.0500 s, during which time the ball is accelerated to a speed of 45.0 m s^{-1}.
  a What is the magnitude of the impulse the ball experiences?
  b What is the net average force acting on the ball during the contact time?
  c What is the net average force acting on the bat during the contact time?

6 The following graph shows the net vertical force generated as an athlete's foot strikes an asphalt running track.



  a Estimate the maximum force acting on the athlete's foot during the contact time.
  b Estimate the total impulse during the contact time.

7 A 25.0 g arrow buries its head 2.00 cm into a target on striking it. The arrow was travelling at 50.0 m s^{-1} just before impact.
  a What change in momentum does the arrow experience as it comes to rest?
  b What is the impulse experienced by the arrow?
  c What is the average force that acts on the arrow during the period of deceleration after it hits the target?

8 Crash helmets are designed to reduce the force of impact on the head during a collision.
  a Explain how their design reduces the net force on the head.
  b Would a rigid 'shell' be as successful? Explain.

4.7 Mass and weight

The difference between mass and weight is sometimes misunderstood because the terms are used interchangeably in everyday language. In physics, however, the two terms have different meanings. For instance, weight is a vector while mass is a scalar. The difference between these two terms is explained in this section.

MASS OF A BODY

Mass is a scalar quantity. In scientific contexts, mass is measured in kilograms (kg) and can be defined as 'the amount of matter in an object'.

MASS

The kilogram was originally defined as the mass of 1 L of water at 4°C. Since 1897 the measurement standard for the kilogram has been a cylindrical block of platinum–iridium alloy kept at the International Bureau of Weights and Measures in France (Figure 4.7.1). Australia has a copy of this standard mass at the CSIRO Division of Applied Physics in Sydney. At times it was returned to France to ensure that the mass remained accurate. To enable scientists around the world to standardise their mass measuring devices without needing to access the standard kilogram cylinder it was decided that a better standard should be developed. Since 2019 the standard kilogram is now defined in terms of the speed of light ($c = 299\,792\,458\,\text{m s}^{-1}$), and Planck's constant ($h = 6.62607015 \times 10^{-34}\,\text{J s}$), which combine to determine the mass–energy equivalent of the energy gained when an electron transitions in a specific isotope of caesium. Specifically, one kilogram is exactly equal to the change in mass of 1.4755214×10^{40} atoms of caesium-133 as each atom's outer electron transitions up to the hyperfine energy level.

FIGURE 4.7.1 Until recently all the mass in the world was compared to this small piece of platinum–iridium alloy held in a sealed vault in Paris.

PHYSICSFILE

Particle mass

There are several units used in physics that are not part of the SI system but are frequently used. One of these is the atomic mass constant (m_u), which is used to measure the tiny amounts of matter involved in nuclear reactions. the units of the atomic mass constant is the dalton (Da), named in honour of John Dalton the English Chemist and Physicist. Since 2019 the Bureau of Weights and Measures has only used the dalton to refer to masses of particles relative to the atomic mass constant. Prior to this date the term atomic mass unit (*amu*), which was shortened to (*u*), was used. The atomic mass constant (m_u) is equal to $\frac{1}{12}$ the mass of the carbon-12 isotope at rest and in its nuclear and atomic ground state. A mass of 1.00 Da is equal to 1.66×10^{-27} kg.

You have already encountered an understanding of mass in Section 4.3 with the introduction of Newton's second law and the effect of a force on a body. An object's mass, *m*, can be seen as a property by the amount of acceleration it undergoes for a given net force *F*. The greater the mass of an object, the less it will accelerate for that given force. Therefore, the concept of mass is directly tied with the *inertia* of that object. That is, the mass of an object can be seen as its resistance to change in velocity when a force is applied.

The more mass an object has, the greater the force required to make it accelerate. If the same force is applied to two different masses, the smaller mass will accelerate more than the greater mass. For this reason, mass can be seen as the property of a body that resists the change in velocity caused by a force.

If the above experiment is repeated on the Moon with the same horizontal force acting on the body on a frictionless surface, the same acceleration will result, as shown in Figure 4.7.2. This is because the mass of the body remains the same on the Earth and on the Moon. Mass is a property of the body and it is not affected by its environment.

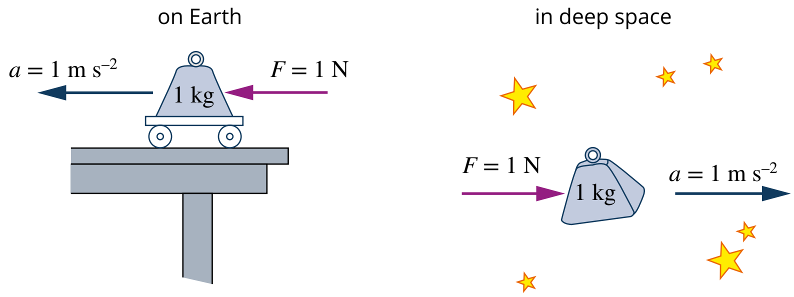


FIGURE 4.7.2 Mass is a property of the object and not of its surroundings.

GRAVITATIONAL FORCE

In the late 1500s, Galileo was able to show that all objects which are dropped near the surface of the Earth accelerate at the same rate, g, towards the centre of the Earth. The force that produces this acceleration is the force due to gravity. The force due to gravity is an attractive force that exists between all masses. In other words, it is a 'pulling' force that exists between everything that has a mass. It is one of the fundamental forces that acts over a distance (meaning that the two masses do not need to be in contact in order for the force to exist).

Gravitational forces result from a mass creating a gravitational field that spreads throughout the space around the mass. Any other mass that is within this field will experience a force towards the mass creating the field. The object that is in the gravitational field also has mass, so it too has a gravitational field around it that attracts the original mass with an equal and opposite force.

The gravitational field extends through space in an inverse squared relationship. This means that if you double the distance from the mass creating the field, then the force will be one-quarter the size.

Here on Earth, you are strongly affected by the Earth's gravitational field (and less strongly by fields from the Sun, the Moon and other objects in the solar system). Even if you were not on Earth, you could still measure the effect of the Earth's gravitational field. There is no place in the universe where the Earth's gravitational field will not reach. At the 'edge' of the universe it will be very small, but it can be calculated. The closer you or any mass is located to the Earth, the larger the gravitational force of attraction towards the Earth. At a height above the Earth's surface that is equal to the radius of the Earth, the force due to gravity on a mass will be one-quarter of that at the Earth's surface. At two Earth radii above the Earth's surface, the gravitational force will be one-ninth of the force experienced on the Earth's surface.

PHYSICSFILE

Inertial mass and gravitational mass

You may be wondering why there are two different definitions or indications of the mass of an object: its resistance to change velocity when a net force is applied to it (inertial mass), and the property of a body that the gravitational force acts on (gravitational mass). In 1907 Albert Einstein put forward an argument that these properties must be equal in his equivalence principal, which underpinned his work on general relativity. Einstein observed that an object subject to a constant gravitational field causing acceleration and one accelerating at a constant rate far from a gravitational field undergo identical physical laws.

A way to think about this is to imagine that you are in a windowless elevator freefalling to Earth. You would not be able to tell the difference between being in this elevator and being in a spaceship where there is no gravity.

Einstein used the equivalence principle to predict that clocks run at different rates in different gravitational fields, and that light rays bend in the presence of a gravitational field. Both of these predictions have been confirmed by subsequent experiments.

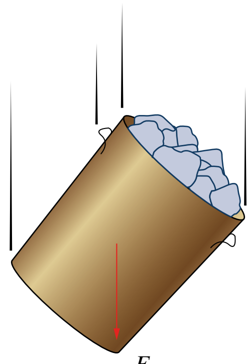


FIGURE 4.7.3 When the bin is in midair, there is an unbalanced force due to gravity acting on it so it accelerates towards the ground. This is the object's weight force. The vector representing weight is drawn from the centre of mass of the object and points downwards.

PHYSICSFILE

The force of gravity between Earth and the Moon

The Moon stays in orbit around Earth due to the attractive force of gravity that acts between the two bodies. The force required to maintain the Moon in an orbit of Earth is very large. If it could be replaced with a steel rod that connects Earth and the Moon, the rod would have to be over 700 km in diameter.

WEIGHT FORCE

In physics, the force on a body due to gravity is called the **weight** of a body, F_{weight}. Weight is a force, therefore it is a vector quantity. Like other forces, it is measured in newtons (N).

Figure 4.7.3 shows a bin falling through the air. As it falls, it accelerates vertically downwards due to the Earth's gravitational field strength, g, which near the surface of the Earth is $9.80\,\mathrm{N\,kg^{-1}}$ down.

As the weight of the bin is a vector, it can be represented with an arrow. An arrow representing weight is drawn downwards (towards the centre of the Earth) with its tail beginning at the object's centre of mass. Centre of mass is described in detail in Section 3.1. Stated simply, an object's centre of mass is the point where its mass can be considered to be 'concentrated' and where all external forces are applied. In an object of uniform density, there is as much mass above the centre of mass as there is below it, as much mass to the left as there is to the right, and as much mass in front as there is behind it.

The weight of a body F_{weight} (in N) is defined as the force of attraction on a body due to gravity and is calculated using the equation:

$$F_{weight} = mg$$

where F_{weight} is the force of gravity acting at the centre of mass of a body (N)

m is the mass of the body (kg)

g is the gravitational field strength ($\mathrm{N\,kg^{-1}}$), g is $-9.80\,\mathrm{N\,kg^{-1}}$ near the surface of the Earth.

Your mass and weight on the Moon

If you were ever lucky enough to travel to the Moon, you would notice that over the duration of the trip, the amount of matter that makes up your body would not change. There would still be as much of you present when you arrived on the Moon as there was when you left the Earth. Your mass wouldn't have changed because mass is a property of the matter and is not affected by its environment. However, you would notice when you stood on the floor of the Moon base that you were not pulled down as hard on the ground. In other words, your weight force would be much less on the Moon than it was on the Earth.

The Moon has a much smaller mass than the Earth. The Moon's mass is 7.35×10^{22} kg and the Earth's mass is 5.97×10^{24} kg. This means that the Moon's mass is about 81 times smaller than the Earth. This smaller mass, and the radius of the Moon, means that the Moon's gravitational field is much weaker than the Earth's gravitational field and therefore the force due to gravity on any object on the Moon is much less. As the force is less, the acceleration that an object experiences on the Moon will be less than here on Earth. Consider Figure 4.7.4, in which a 5 kg pumpkin is falling on the Earth and then on the Moon. The pumpkin has a much smaller weight force on the Moon than on the Earth due to the much smaller gravitational field.

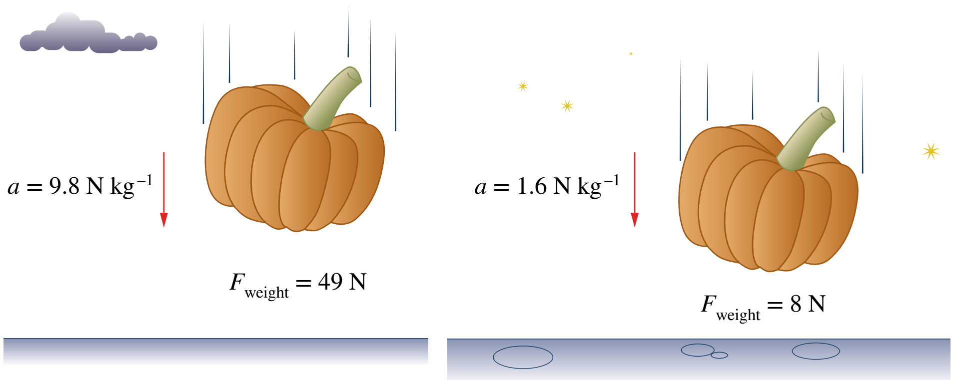


FIGURE 4.7.4 A 5 kg pumpkin falling on the Earth and on the Moon. The mass of the pumpkin is 5 kg no matter where it is, but the weight of the pumpkin is different.

The inclined plane

As discussed in Section 4.4, the normal force is the perpendicular force acting on an object when it is in contact with a surface. When the surface is horizontal, this normal force is equal and opposite to the force of gravity of the object, resulting in no net force and hence keeping the object stationary. This can be seen in Figure 4.7.5a.

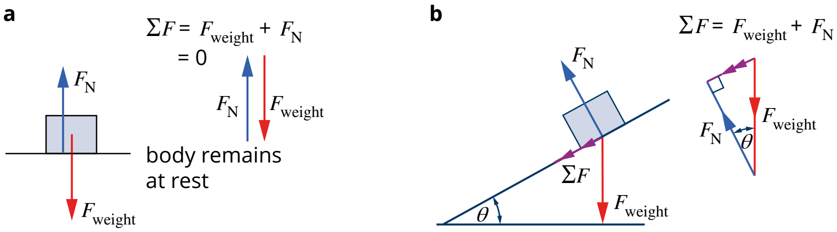


FIGURE 4.7.5 (a) Where the surface is perpendicular to the force due to gravity, the normal force (F_N) acts directly upwards. (b) On an inclined plane, F_N is at an angle to F_{weight} and is given by $F_N = mg\cos\theta$. If no friction acts, the force that causes the object to accelerate down the plane is $F = mg\sin\theta$.

It is possible that an object could be placed on a surface that is tilted so that it makes an angle, θ, to the horizontal (Figure 4.7.5b). This is called an **inclined plane.** In this case, the weight force remains the same: $F_{weight} = mg$ acting downwards. However, the normal force continues to act at right angles to the surface, and therefore at an angle to the weight force. This creates a net force down the slope and the object accelerates as predicted by Newton's second law.

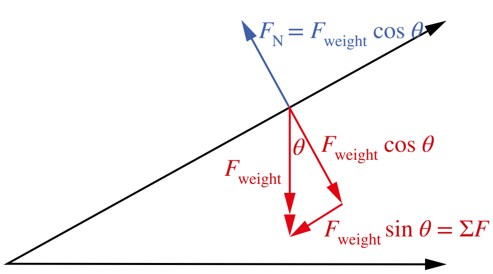


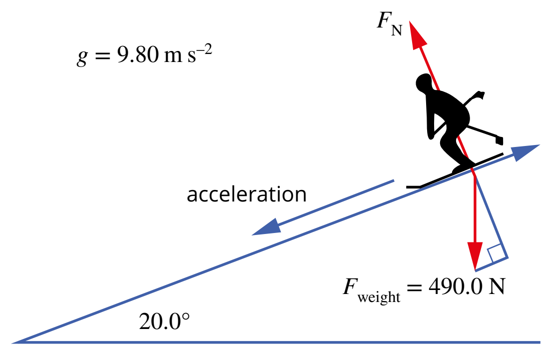
FIGURE 4.7.6 Block on an incline: the weight force can be resolved into a force perpendicular to the surface and a force parallel to the surface.

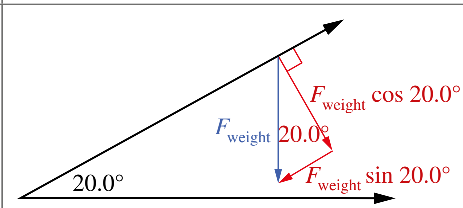
Another way of viewing the forces along the inclined plane is to resolve the weight vector into two components: one perpendicular (at right angles) to the slope, and one parallel to the slope, as shown in Figure 4.7.6. The component perpendicular to the surface is balanced by the normal force F_N. This perpendicular component has a magnitude of $F_{weight} \cos\theta$, where θ is the angle of inclination from horizontal. Given this, it can be seen that on an inclined plane, the normal force will be less than the weight of the body and will decrease as the angle of inclination increases. As indicated in Figure 4.7.5b, the component of the weight of the object directed down the slope is the force responsible for the acceleration down the slope. This component has a magnitude of $F_{weight} \sin\theta$.

Worked example 4.7.1

INCLINED PLANES

A skier with a mass of 50.0 kg is skiing down an icy slope that is inclined at 20.0° to the horizontal. Assume that friction is negligible and that the acceleration due to gravity is $-9.80\,\text{m s}^{-2}$.



a Determine the components of the weight of the skier perpendicular to the slope and parallel to the slope.	
Thinking	**Working**
Draw a sketch including the values provided.	
Resolve the weight into a component perpendicular to the slope.	The perpendicular component: $F_\perp = F_{weight} \cos 20.0°$ $F_\perp = 490 \cos 20.0°$ $F_\perp = 460.449$ $F_\perp = 460\,\text{N}$
Resolve the weight into a component parallel to the slope. This is the net force.	The parallel component: $\Sigma F = F_{weight} \sin 20.0°$ $\Sigma F = 490 \sin 20.0°$ $\Sigma F = 167.59$ $\Sigma F = 168\,\text{N}$

b Determine the normal force that acts on the skier.	
Thinking	**Working**
The normal force is equal in magnitude to the perpendicular component of the weight force.	$F_N = 460\,\text{N}$

c Calculate the acceleration of the skier down the slope.	
Thinking	**Working**
Apply Newton's second law. The net force along the plane is the component of the weight parallel to the slope.	$a = \frac{\Sigma F}{m}$ $a = \frac{(167.59)}{(50.0)}$ $a = 3.35180$
Ensure that the final answer has the appropriate number of significant figures, the correct units and determine the direction of the acceleration.	$a = 3.35\,\text{m s}^{-2}$ down the slope

Worked example: Try yourself 4.7.1

INCLINED PLANES

A skier of mass 85.0 kg travels down an icy slope inclined at 20.0° to the horizontal. Assume that friction is negligible and that the acceleration due to gravity is $-9.80\,\text{m s}^{-2}$.

a Determine the components of the weight of the skier perpendicular to the slope and parallel to the slope.

b Determine the normal force that acts on the skier.

c Calculate the acceleration of the skier down the slope.

Aside from rounding differences, the acceleration calculated in these Worked example and Try yourself questions were equal. That is because acceleration is independent of the mass of the object:

$$a = \frac{\Sigma F}{m} = \frac{mg\sin\theta}{m} = g\sin\theta$$

That is, acceleration down a frictionless inclined plane is affected only by the gravitational field strength and the angle of the plane.

STRATEGIES FOR SOLVING INCLINED PLANE PROBLEMS

Where forces on a body are given, Newton's laws can be applied.

Two questions can be asked:

1 Is the object described as stationary or travelling at constant velocity? In this case, $\Sigma F = 0$.

2 Is the object accelerating? In this case, $\Sigma F = ma$.

For coplanar forces that are not aligned, resolve forces into components. Once these forces have been resolved into two perpendicular directions—the components parallel and perpendicular to the inclined plane—the net force in each direction can be determined.

Typically, the components perpendicular to the incline will consist of the normal force, F_N, and the component of the weight force perpendicular to the incline, $F_{weight}\cos\theta$. The components parallel to the incline will consist of the frictional force (if present), F_f, and the component of the weight force parallel to the incline, $F_{weight}\sin\theta$.

Newton's second law can be used to find the acceleration of an object. The other equations of motion may then be used to find quantities such as displacement and final velocity.

4.7 Review

SUMMARY

- The standard mass is a 1 kg platinum–iridium cylinder, against which all other masses are compared.
- The mass of an object relates to its ability to resist changes in motion.
- Mass is a scalar quantity and is a property of a body that is not influenced by external environmental factors.
- Gravity is a force of attraction between masses that extends throughout space.
- Weight is a force due to gravity. Because it is a force, it is also a vector and requires a magnitude and direction.
- Mass is measured in kilograms and weight is measured in newtons.
- The mass of an object will be the same on the Earth and the Moon, but its weight will be different due to the decreased gravitational force on the Moon compared to the Earth.
- The centre of mass indicates the position at which the entire mass of a body is concentrated. At this point, all external forces are applied.
  - On an inclined surface, F_N is equal and opposite to the component of the weight force acting perpendicular to the plane: $F_N = F_{weight} \cos\theta$.
- The net force acting on an object on a plane inclined at an angle θ is $\Sigma F = F_{weight} \sin\theta$ when friction is negligible. It is the net force that causes acceleration down the incline.

KEY QUESTIONS

1 An object is placed in a spaceship and launched into space. As the object leaves the surface of the Earth, it has a mass of 50.0 kg. What will be the object's mass if it lands on Pluto?

2 Two students have a conversation. One of the students states that their weight is 60.0 kg. What is wrong with this statement?

3 Jarrah's mass is 75.0 kg. What is Jarrah's weight on Earth if $g = -9.80\ \text{N kg}^{-1}$?

4 A desk chair has a weight of 34.3 N on the surface of the Earth. Determine the mass of the chair. Use $g = -9.80\ \text{N kg}^{-1}$.

5 Determine the weight of the same chair in the previous question when it is on the surface of the Moon, where $g = -1.60\ \text{N kg}^{-1}$.

6 On the surface of the Earth, a geological hammer has a mass of 1.50 kg. Determine its mass and weight on Mars, where $g = -3.60\ \text{N kg}^{-1}$.

7 Would your weight be greater on Earth or on the Moon? Explain your answer.

8 Which of the following statements describes the forces acting on an object on a plane inclined at an angle?

A The normal force is always perpendicular to the surface.

B The normal force is equal in magnitude to the weight.

C The normal force and the weight cancel out.

D In the absence of friction, a component of the normal force causes the object to accelerate down the slope.

9 A skier with a mass of 112 kg is skiing down an icy slope that is inclined at 30.0° to the horizontal. Assume that friction and drag are negligible.

a Determine the component of the weight of the skier perpendicular to the incline.

b Determine the component of the weight of the skier parallel to the incline.

c Determine the skier's acceleration down the slope.

10 A smooth steel ramp 2.00 m long is inclined at 25.0° to the horizontal. A steel ball of mass 0.100 kg is rolled down the ramp followed by another steel ball of mass 0.200 kg.

a Determine the normal force acting on each ball.

b Determine the acceleration of each ball down the ramp.

The inclination of the ramp is now increased to 70.0° and the two balls are rolled down it once more.

c Determine the normal force acting on the two balls.

d Determine the acceleration of the two balls down the ramp.

e Compare your answers for parts **a** and **b** for the two balls. Then compare your answers for parts **a** and **b** with those of parts **c** and **d**. What can you say about the effects of increasing mass and angle on the normal force and the acceleration of an object on an inclined plane?

Chapter review

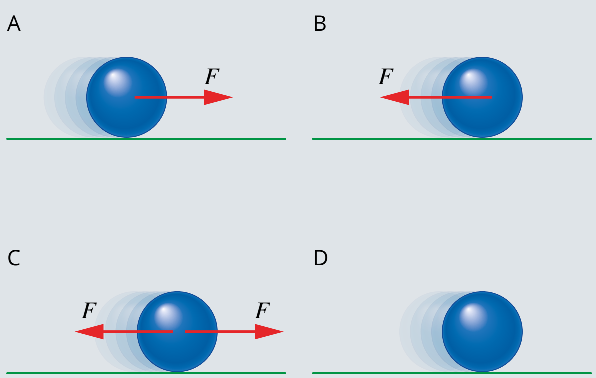
04

KEY TERMS

coefficient of friction
conserved
contact force
friction
force
impulse
inclined plane
inertia
kinetic friction force
mass
momentum
net force
newton
Newton's first law
Newton's second law
Newton's third law
non-contact force
static friction force
velocity
weight

1 A student is travelling to school on a train. When the train starts moving, the student notices that passengers tend to lurch towards the back of the train before grabbing a handrail to stop themselves from falling. Has a force acted to push the passengers backwards? Justify your answer.

2 A bowling ball rolls along a smooth wooden floor at constant velocity. Ignoring the effects of friction and air resistance, which of the following diagrams correctly indicates the forces acting on the ball?



3 Calculate the mass of an object if it accelerates at 9.20 m s^{-2} east when a force of 352 N east acts on it.

4 Calculate the acceleration of a 657 kg motorbike when a net force of 3550 N north acts on it.

The following information relates to questions 5–8.

Lachy is riding a bike, producing a forwards force of 155 N. The combined mass of Lachy and the bike is 90.0 kg.

5 If there is no friction or air resistance, what is the magnitude of the acceleration of Lachy and the bike?

6 If friction opposes the bike's motion with a force of 45.0 N, what is the magnitude of the acceleration of the bike?

7 What must be the magnitude of the force of friction if Lachy's acceleration is 0.600 m s^{-2}?

8 Lachy now carries an additional mass of 25.0 kg due to his school bag. What must be the new forwards force he produces in order to accelerate at 0.800 m s^{-2} if friction opposes the motion with a force of 35.0 N?

9 Calculate the final velocity of a 14.0 kg remote-controlled car moving at 3.75 m s^{-1} east, when a net force of 62.0 N west acts on it for 2.00 seconds.

10 A 65.0 kg student is standing on a 3.50 kg skateboard at rest when he steps off the board and exerts a horizontal force of 75.0 N south on the board. What force does the board exert on the student?

11 Calculate the change in momentum of a 155 kg rock whose velocity has changed from 6.50 m s^{-1} east to 3.25 m s^{-1} east in a period of 8.50 s.

12 Calculate the change in momentum of a 25.5 kg robot whose velocity changes from 6.40 m s^{-1} forwards to 2.25 m s^{-1} backwards.

13 An astronaut in a protective suit has a total mass of 154 kg and throws a 40.0 kg toolbox away from the space station. The astronaut and toolbox are initially stationary. After being thrown, the toolbox moves at 2.15 m s^{-1}. Calculate the velocity of the astronaut just after throwing the toolbox.

14 A 75.0 kg netball player moving at 4.00 m s^{-1} west changes direction during a game to 5.00 m s^{-1} north. Calculate the change in momentum of the player over the period of the change.

15 An athlete catches a 305 g volleyball by relaxing their elbows and wrists, allowing them to 'give' when catching the ball. Just before the athlete catches the ball, its velocity is 5.60 m s^{-1} west. The velocity of the ball drops to zero in 1.00 s. Calculate the average force exerted by the athlete on the volleyball.

16 Explain how crumple zones reduce the damage done to occupants of a car in the event of a collision. Is it better to have a completely rigid metal bonnet, a completely soft metal bonnet, or somewhere in between? Explain.

17 Young, a skateboarder with a mass of 70.0 kg, is riding along at 5.00 m s^{-1} east. Distracted by his mobile phone, he crashes head-first into a rigid metal pole and comes to a complete stop. Luckily, he is wearing a helmet that contains compressible foam and the collision takes place over 0.350 s.

a Calculate Young's momentum prior to the collision.

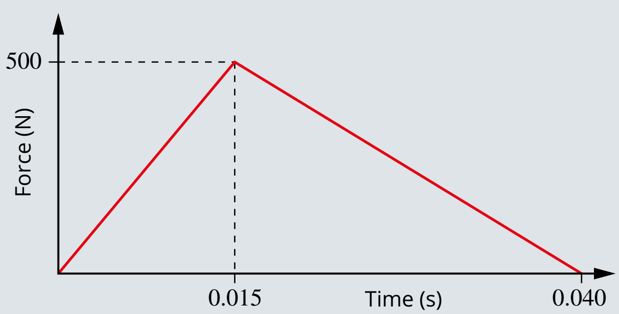
b Calculate the average force of the pole applied to Young's head.

c If Young had forgotten his helmet, and the collision took place over 7.00 ms instead, what would be the average force applied to his head?

d Apart from reducing the overall force applied to the head, how does a crash helmet protect its user in the event of a collision?

The following information applies to questions 18–20.

Jordy is playing softball and hits a ball with her softball bat. The force versus time graph for this interaction is shown below. The ball has a mass of 170 g.



18 Determine the magnitude of the change in momentum of the ball.

19 Determine the magnitude of the change in momentum of the bat.

20 Determine the magnitude of the change in velocity of the ball.

21 A pumpkin has a mass of 10.0 kg on Earth. What is its weight on Earth?

22 A skateboard has a weight of 20.6 N on Earth. What is its mass?

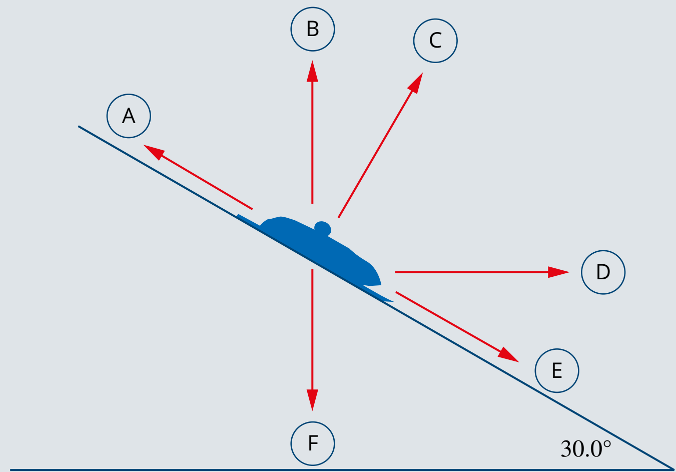
23 **a** What is the mass of an 85.0 kg astronaut on the surface of Earth where g is –9.80 N kg^{-1}?

b What is the mass of an 85.0 kg astronaut on the surface of the Moon where g is –1.60 N kg^{-1}?

c What is the weight of an 85.0 kg astronaut on the surface of Mars where g is –3.60 N kg^{-1}?

24 Given the figures in Question **23**, order the weight of a 1 kg object from greatest weight to least weight when it is on the Moon, on Mars and on Earth.

25 Kirsty is riding in a bobsled that is sliding down a snow-covered hill with a slope of 30° to the horizontal. The total mass of the sled and Kirsty is 100 kg. Initially the brakes are on and the sled moves down the hill with a constant velocity.



a Which one of the arrows (A–F) best represents the direction of the frictional force acting on the sled?

b Which one of the arrows (A–F) best represents the direction of the normal force acting on the sled?

c Calculate the net frictional force acting on the sled.

d Kirsty then releases the brakes and the sled accelerates. What is the magnitude of her initial acceleration?

e In a second run, Kirsty rides the bobsled down the same slope but with the brakes off, so friction can be ignored. The bobsled now has an extra passenger, so that its total mass is now 145 kg. How will this affect the acceleration of the bobsled?

26 Diya rolls a bowling ball down a smooth straight ramp. Choose the option below that best describes the way the ball will travel.

A with constant speed

B with constant acceleration

C with decreasing speed

D with increasing acceleration

27 A marble is rolled from rest down a smooth slide that is 2.50 m long. The slide is inclined at an angle of 30.0° to the horizontal.

a Calculate the acceleration of the marble.

b What is the velocity of the marble as it reaches the end of the slide?

28 Blake has a mass of 54.0 kg and they are riding their 3.00 kg skateboard down a 5.00 m long ramp that is inclined at an angle of 65.0° to the horizontal. Ignore friction when answering parts **a** to **d**.

- **a** Calculate the magnitude of the normal force acting on Blake and their skateboard.
- **b** What is the acceleration of Blake on their skateboard as they travel down the ramp?
- **c** What is the net force acting on Blake and their board when no friction acts?
- **d** If Blake's initial velocity is zero at the top of the ramp, calculate their final velocity as they reach the bottom of the ramp.
- **e** Blake now stands halfway up the incline while holding their board in their hands. Calculate the frictional force acting on Blake while they are standing stationary on the ramp.

29 A very high waterslide is 50.0 m tall and is inclined at an angle of 70.0° to the horizontal. It is known that riders reach a velocity of 105 km h^{-1} on this slide. Do not assume friction is negligible.

- **a** For a 70.0 kg teenager using the slide, calculate the net force on the teenager as they slide.
- **b** For the same teenager, calculate the magnitude of the average frictional force opposing the motion.
- **c** Calculate the average coefficient of friction between the teenager and the slide.

CHAPTER

05 Work, energy and power

Throughout this chapter you will be learning about the common thread of energy conversion that is present in so many daily activities, as well as some more extreme activities. Your own personal energy stores are burnt up climbing steps or running to catch a bus. In more thrill-seeking adventures, such as bungee jumping, gravitational potential energy is converted into kinetic and elastic potential energy. Even jumping from a plane, the laws of physics cannot be switched off.

At the end of this chapter, you will be able to define and use the terms *work*, *energy* and *power*. You will use force–displacement graphs to determine the amount of work done.

Science Understanding

- energy is associated with the motion of objects and their position relative to the surface of the Earth, including applying the relationships

  $E_k = \frac{1}{2}mv^2 \qquad E_p = mg\Delta h$

- total energy is conserved in isolated systems, including applying the relationship

  $\Sigma E_i = \Sigma E_f$

- energy is transferred from one object to another when a force is applied over a distance; this causes work to be done and changes the kinetic (E_k) and/or potential (E_p) energy of objects, including applying the relationships

  $W = Fs \qquad W = \Delta E$

- collisions may be elastic and inelastic; kinetic energy is conserved in elastic collisions, including applying the relationship

  $\Sigma \frac{1}{2} m{v_i}^2 = \Sigma \frac{1}{2} m{v_f}^2$

- power is the rate of doing work or transferring energy, including applying the relationships

  $P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.

5.1 Energy and work

The words 'energy' and 'work' are commonly used to describe a variety of everyday situations. However, these words take on quite specific definitions when used in a scientific context. They are two of the most important concepts in physics, allowing physicists to explain phenomena on a range of scales from collisions of subatomic particles to the interactions of galaxies.

ENERGY

Energy is the capacity to cause change. A moving car has the capacity to cause a change if it collides with something else. Similarly, a heavy weight lifted by a crane has the capacity to cause a change if it is dropped. Energy is a scalar quantity; it has magnitude but not direction.

FIGURE 5.1.1 A moving car has kinetic energy.

There are many different forms of energy. **Mechanical energy** is defined as the energy that a body possesses due to its position or motion. This category of energy can be broadly classified into two groups: kinetic energy and potential energy.

Kinetic energy is the energy associated with motion. Any moving object, like the moving car in Figure 5.1.1, has kinetic energy. In some forms of kinetic energy, the moving objects are not easily visible. An example of this is thermal energy, which is a type of kinetic energy related to the movement of particles. Table 5.1.1 lists some different types of kinetic energy and their associated moving objects.

TABLE 5.1.1 Types of kinetic energy and their associated moving objects.

Type of kinetic energy	Moving objects
translational kinetic	objects moving in a straight line
rotational kinetic	rotating objects
thermal	atoms, ions or molecules
sound	air molecules

Potential energy is energy associated with the position of objects relative to one another or within fields. For example, an object suspended by a crane has **gravitational potential energy** because of its position in the Earth's gravitational field. Some examples of potential energy are listed in Table 5.1.2.

TABLE 5.1.2 Types of potential energy and their causes.

Type of potential energy	Cause
gravitational	gravitational fields
chemical	relative positions of atoms
magnetic	magnetic fields
nuclear	forces within the nucleus of an atom
elastic	attractive forces between atoms

Unit of energy

The SI unit for energy, the joule (J), is named after the English scientist James Prescott Joule. He was the first person to show that kinetic energy could be converted into heat energy. The energy represented by 1 J is the equivalent to the energy needed to lift a 1 kg mass (e.g. 1 L of milk) through a height of 0.1 m or 10 cm. More commonly, scientists work in units of kilojoules ($1\,kJ = 1 \times 10^3\,J = 1000\,J$) or even megajoules ($1\,MJ = 1 \times 10^6\,J = 1\,000\,000\,J$).

WORK

Although in everyday life the word 'work' can take on a variety of meanings, in a scientific context work has a very specific meaning. In physics, when a force acts on an object and causes energy to be transferred or transformed, work is being done on the object. For example, if a weightlifter applies a force to a barbell to lift it, work has been done on the gravitational field via the barbell; chemical energy within the weightlifter's body has been transformed into the gravitational potential energy stored in the gravitational field due to the position of the barbell (Figure 5.1.2).

FIGURE 5.1.2 As a weightlifter lifts a barbell, chemical energy is transformed into gravitational potential energy.

Quantifying work

Work causes a change in energy, i.e. $W = \Delta E$.

More specifically, **work** is defined as the product of the force causing the energy change and the displacement of the object in the direction of the force during the energy change:

> $W = Fs$
> where W is work (J)
> F is force (N)
> s is the displacement in the direction of the force (m).

Since work corresponds to a change in energy, the SI unit of work is also the joule (J). The definition of work allows us to find a value for a joule in terms of other SI units.

Since $W = Fs$, $1\,J = 1\,N \times 1\,m = 1\,N\,m$.

A joule is equal to a newton-metre, that is, a force of 1 N acting over a distance of 1 m does 1 J of work.

Using the definition of a newton:

$1\,J = 1\,N \times 1\,m = 1\,kg\,m\,s^2 \times 1\,m = 1\,kg\,m^2\,s^{-2}$

This defines a joule in terms of fundamental units.

Although both force and displacement are vectors, work is a scalar unit. So, similar to energy, work has no direction.

PHYSICSFILE

Units of energy

A number of non-SI units for energy are still in use. When talking about the energy content of food, it is common to use a unit called a calorie (cal). One calorie is defined as the amount of heat required to increase the temperature of 1 g of water by 1°C. This equates to 4.2 J.

Electrical energy used in the home is often measured in kilowatt-hours (kWh). A kilowatt-hour is a very large unit of energy:

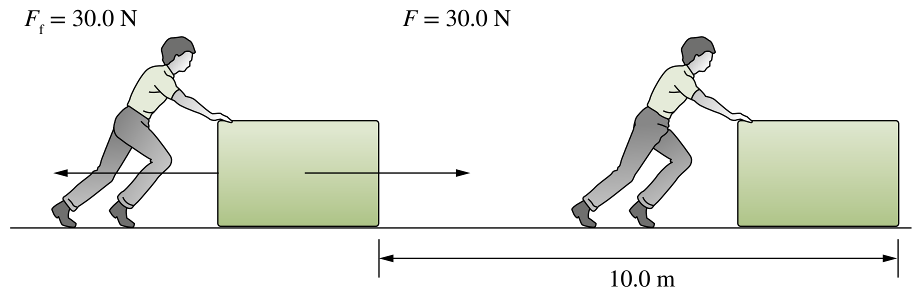
1 kWh = 3 600 000 J or 3.60 MJ.

Another, not-so-common unit of energy is the erg (from the Greek word *ergon* for energy). An erg is a very small unit of energy: 1 erg = 10^{-7} J.

Worked example 5.1.1

CALCULATING WORK

A person pushes a heavy box along the ground for 10.0 m with a horizontal force of 30.0 N. Calculate the amount of work done.



Thinking	Working
Recall the definition of work.	$W = Fs$
Substitute values into the definition.	$W = (30.0)(10.0)$ $W = 300.00$
Solve the problem, giving an answer with the appropriate units and significant figures.	$W = 3.00 \times 10^2$ J

Worked example: Try yourself 5.1.1

CALCULATING WORK

A person pushes a heavy wardrobe from one room to another by applying a force of 50.0 N for a distance of 5.00 m. Calculate the amount of work done.

Work and friction

The energy change produced by work is not always obvious. Consider Worked example 5.1.1, where 3.00×10^2 J of work was done on a box when it was pushed 10.0 m. A number of energy outcomes are possible for this scenario.

- In an ideal situation, where there was no friction, all of this work would be transformed into kinetic energy and the box would end up with a higher velocity than before it was pushed.
- In most real situations, where there is friction between the box and the ground, some of the work done would become heat and sound due to friction and the rest would become kinetic energy.
- In the limiting situation, where the force applied is exactly equal to the friction, the box would slide at a constant speed. This means that its kinetic energy would not change, so all of the work done would be converted into heat and sound due to friction.

Changing the displacement of a body is dependent on overcoming the force of friction.

A force with no work

The mathematical definition of work has some unusual implications. One is that if a force is applied to an object but the object does not move, then no work is done on the object.

This appears counterintuitive, that is, it goes against what you would probably expect. An example of this is shown in Figure 5.1.3. While picking up a heavy box requires work, holding the box at a constant height does no work on the box.

Assuming the box has a weight of 105 N and that it is lifted from the ground to a height of 1.20 m, the work done lifting it would be: $W = Fs = (105)(1.20) = 126$ J. In this case, energy is being transformed from chemical energy inside the person's body into gravitational potential energy stored in the gravitational field due to the position of the box.

However, when the box is held at a constant height, the definition of work gives: $W = Fs = (100)(0) = 0$ J. So, no work is being done on the gravitational field via the box. Although there would be energy transformations going on inside the person's body to keep their muscles working, the energy of the field due to the box does not change, and therefore no work has been done on the field.

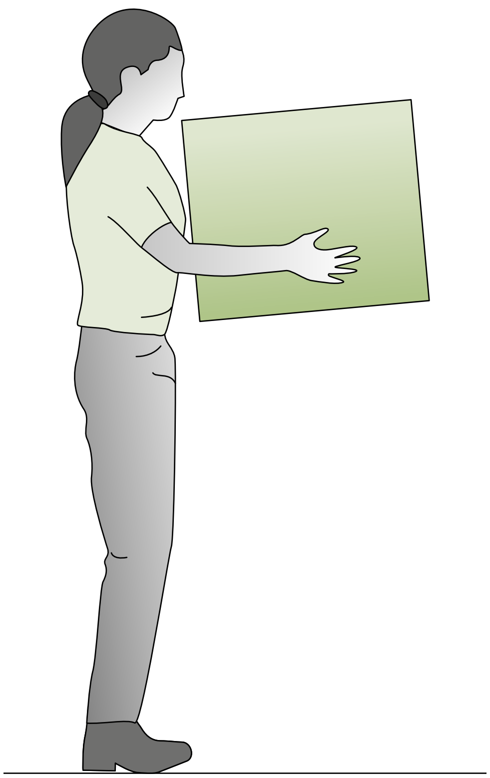


FIGURE 5.1.3 According to the definition of work, no work is done when a person holds a box at a constant height.

Work is done only if the net force causes a movement of one body in relation to other bodies.

Work and displacement at an angle

Sometimes, when a force is applied, the object does not move in the same direction as the force. For example, in Figure 5.1.4, when a person pushes a child in a pram, the direction of the force is at an angle downwards, although the pram itself moves horizontally forwards.

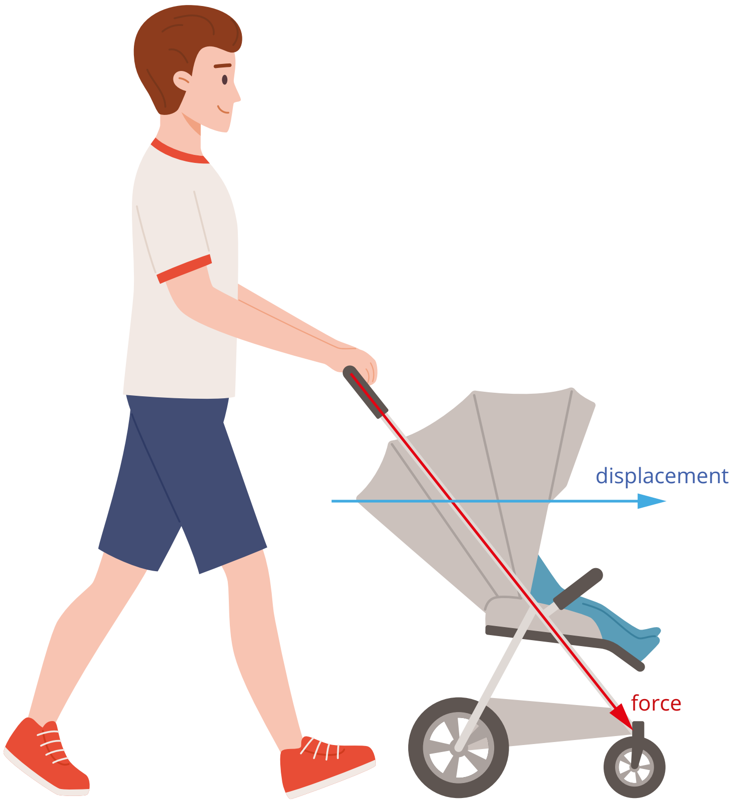


FIGURE 5.1.4 When a person pushes a pram, the force is applied at an angle to the displacement of the pram.

In this case, only the horizontal component of the push contributes to the work being done on the pram. The vertical component of this force pushes the pram downwards and is balanced by the normal reaction force from the ground.

In situations like this, work can be calculated using the general equation:

$W = Fs \cos \theta$
where θ is the angle between the force, F, and the displacement, s.

EXTENSION

Resolving forces

In the situation of a person pushing a pram, the general equation $W = Fs$ applies. The person's push can be resolved into a vertical component, $F \sin\theta$, and a horizontal component, $F \cos\theta$ (Figure 5.1.5). For more details and examples of vector resolution, see Chapter 2. Substituting the horizontal component into the general definition for work gives:

$$W = F\cos\theta \times s$$
$$= Fs\cos\theta$$

If the force applied is at right angles to the direction of displacement, then $\theta = 90°$, $\cos\theta = 0$ so $Fs\cos\theta = 0$, i.e. no work is done.

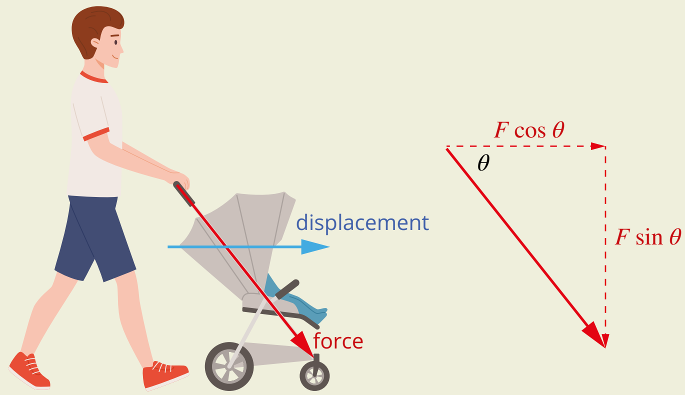
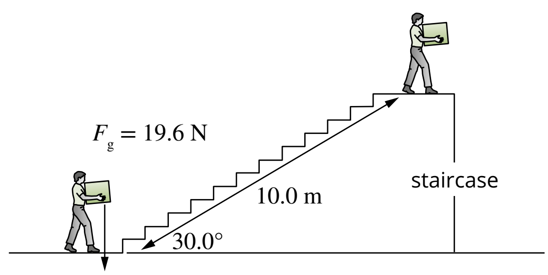


FIGURE 5.1.5 The force applied by the person pushing the pram can be resolved into a horizontal force and a vertical force.

Worked example 5.1.2

WORK WITH FORCE AND DISPLACEMENT AT AN ANGLE

A person carries a box weighing 19.6 N up a 10.0 m flight of stairs. Calculate the work done against gravity on the box.

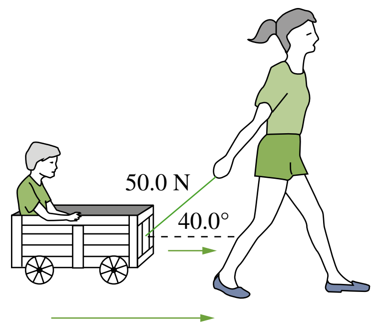


Thinking	Working
Determine values for F, s and θ. Note that the required component of the force is upwards, so the angle is not 30.0°. It is 90.0 – 30.0 = 60.0°.	Force applied to the box by the person: $F = 19.6$ N upwards Displacement: $s = 10.0$ m Angle between the force and displacement: $\theta = 60.0°$
Recall the work equation.	$W = Fs\cos\theta$
Substitute values into the work equation.	$W = (19.6)(10.0)\cos(60.0°)$ W = 98.0000
State the answer with appropriate units and significant figures.	$W = 98.0$ J

Worked example: Try yourself 5.1.2

WORK WITH FORCE AND DISPLACEMENT AT AN ANGLE

A child pulls their younger sibling along in a trolley for a distance of 30.0 m, as shown. Calculate the work done on the trolley.



FORCE–DISPLACEMENT GRAPHS

As its name suggests, a force–displacement graph illustrates the way a force changes with displacement. For a situation where the force is constant, this graph is simple. For example, in Figure 5.1.6, the force–displacement graph for a person picking up a box is a flat horizontal line showing that the force applied to the box is constant throughout the lift.

In contrast, an **elastic** object such as a spring obeys a relationship known as Hooke's law. Hooke's law describes how, the more you stretch a spring, the greater the force required to keep stretching it. The force–displacement graph for a spring is also a straight line, but this line shows the direct relationship described by Hooke's law (Figure 5.1.7). (Note: sometimes, you will see force–displacement graphs for elastic objects labelled as force–extension graphs. In this context, the term extension is the same as displacement.)

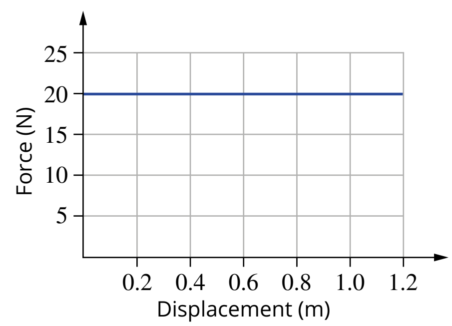


FIGURE 5.1.6 The force–displacement graph for a person picking up a box is a straight, horizontal line, indicating that the force applied to the box by the person is constant throughout the process.

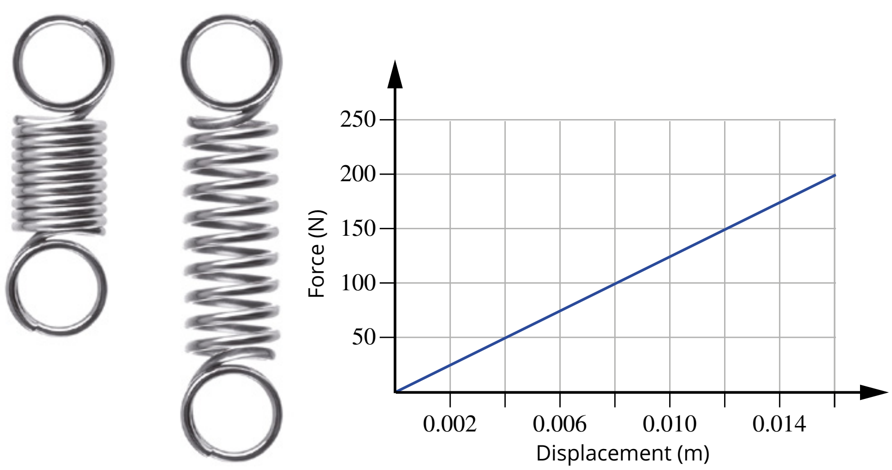


FIGURE 5.1.7 As a spring stretches, more force is required to keep stretching it. The force is proportional to the displacement.

Many everyday materials are only partially elastic. Their force–displacement graphs are relatively complex. For example, the force–displacement graph in Figure 5.1.8 for a sports shoe shows that the shoe is close to elastic for low displacements, but at high displacements the force is relatively constant.

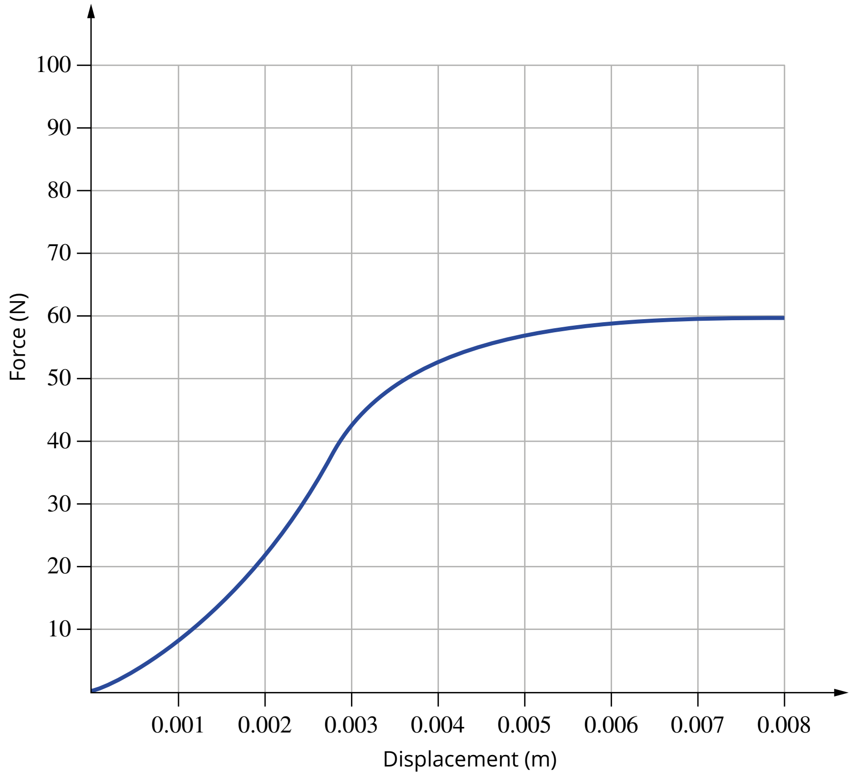


FIGURE 5.1.8 The force–displacement graph for a sports shoe is not a straight line; the change in force varies with how much the shoe has been stretched or compressed.

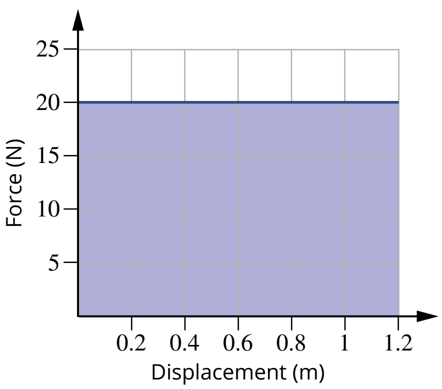


FIGURE 5.1.9 The area under a force–displacement graph gives the work done by the force.

Calculating work from a force–displacement graph

When a force changes with displacement, the amount of work done by the force can be calculated from the area under its force–displacement graph.

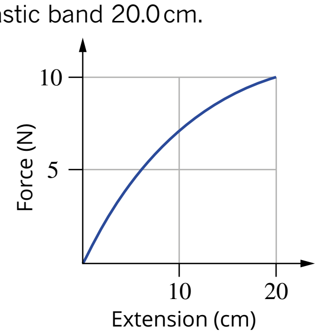
For a constant force, this is very simple. Considering the earlier example of a person lifting a box, the area can be found by counting the number of 'force times displacement' squares under the line. In the example in Figure 5.1.9, there are $6 \times 4 = 24$ of these squares. Since each square has an area of $5.0\,\text{N} \times 0.20\,\text{m} = 1.0\,\text{J}$, the total work done is 24.0 J. Alternatively, this area could be found by recognising that the area under the graph is a rectangle and multiplying length by width to find the total area. For Figure 5.1.9, this is $20.0\,\text{N} \times 1.20\,\text{m} = 24.0\,\text{J}$. Note that this second method is exactly the same as using the formula for work: $W = Fs = 20.0 \times 1.20 = 24.0\,\text{J}$. This relationship works in this case because the force is constant.

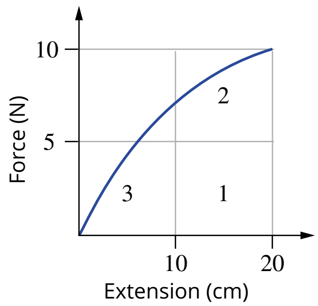
Similar strategies, such as counting grid squares or calculating the area of the shape under the graph, can also be used when the force varies with the displacement.

Worked example 5.1.3

WORK FROM THE AREA UNDER A FORCE–DISPLACEMENT GRAPH

Use the force–displacement graph for an elastic band to estimate how much work is done in stretching an elastic band 20.0 cm.

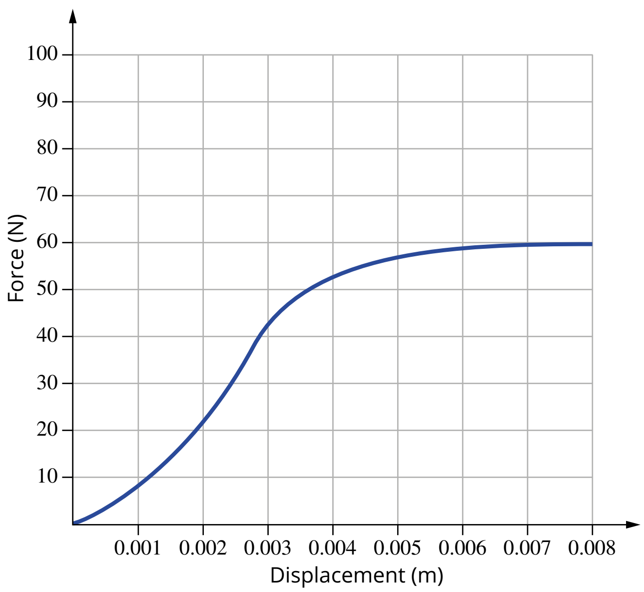


Thinking	Working
Calculate the work value of each grid square.	The dimensions of a grid square are: Force: 5.0 N, displacement: 10.0 cm = 0.10 m Area of 1 square = (5.0)(0.10) = 0.50 J
Count the number of grid squares under the curve. Only count grid squares that have more than half of their area under the curve. (If the curve cuts a square exactly in half, count every second one.)	 Number of squares = 3
Multiply the number of grid squares under the curve by the work value of each grid square.	$W = 3 \times (0.500)$ $W = 1.5000$
State the answer with appropriate units and significant figures.	$W = 1.50$ J

Worked example: Try yourself 5.1.3

WORK FROM THE AREA UNDER A FORCE–DISPLACEMENT GRAPH

While jogging, a person's shoes compress by an average of 3.00 mm with each step. Use the force–displacement graph for a sports shoe to estimate how much work is done on the shoe with each step.



5.1 Review

SUMMARY

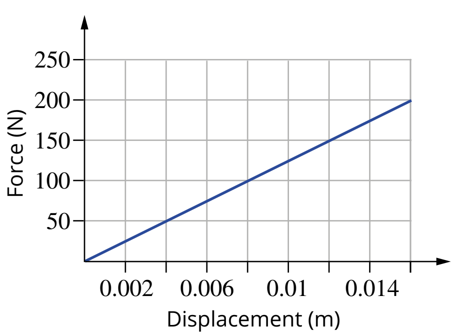
- Energy is the capacity to cause a change.
- Energy is conserved in an isolated system. It can be transferred or transformed, but not created or destroyed. $\Sigma E_i = \Sigma E_f$
- There are many different forms of energy. These can be broadly classified as either kinetic (associated with movement) or potential (associated with the relative positions of objects).
- Work is done when energy is transferred or transformed.
- Work is done when a force causes a change in kinetic energy (E_k) and/or potential energy (E_p) of an object.
- Work is the change in energy: $W = \Delta E$
- Work is the product of force and displacement: $W = Fs$
- When a force produces no displacement, or when the force and displacement are at right angles to each other, no work is done.
- Work is equal to the area under a force–displacement graph.
- A straight horizontal line in a force–displacement graph represents a constant force.
- The relationship between force and displacement for a perfectly elastic object is represented as a straight diagonal line in a force–displacement graph.

KEY QUESTIONS

1 When accelerating at the beginning of a ride, a cyclist applies a force of 525 N for a distance of 20.0 m. What is the work done by the cyclist on the bike?

2 In the case of a person leaning on a solid brick wall, explain why no work is being done.

3 A spring with this force–displacement graph is stretched as shown. Using the formula for the area of a triangle, calculate the work done to stretch the spring by 0.0160 m.

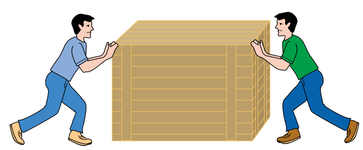


4 A cyclist does 2700 J of work when riding a bike at a constant speed for 150 m. Calculate the average force the cyclist applies over this distance.

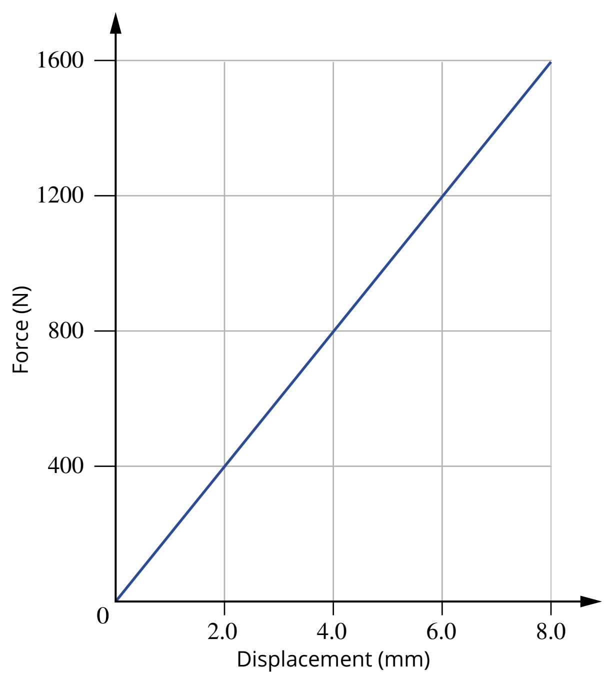
5 A rope at 40.0° to the horizontal is used to drag a heavy box along the ground for a distance of 5.00 m. Calculate the work done if the tension in the rope is 80.0 N.

6 Explain why the equation $W = Fs$ cannot be used to calculate the work done in compressing a spring.

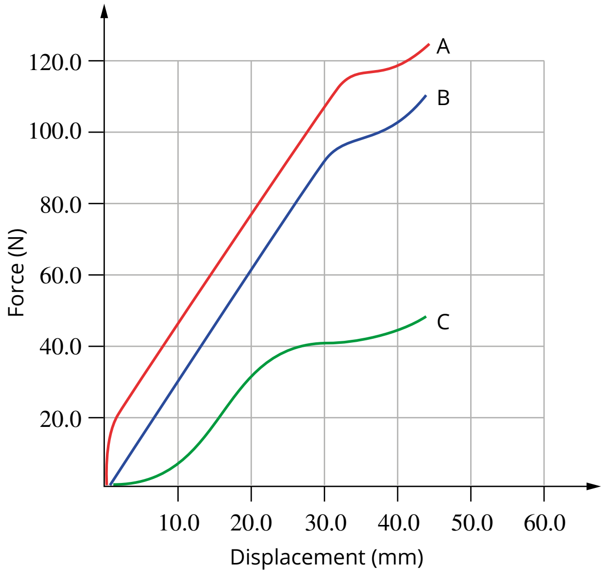
7 Two people push in opposite directions on a heavy box. One person applies 50.0 N of force, the other applies 40.0 N of force. There is 10.0 N of friction between the box and the floor which means that the box does not move. What is the work done by the person applying 50.0 N of force?



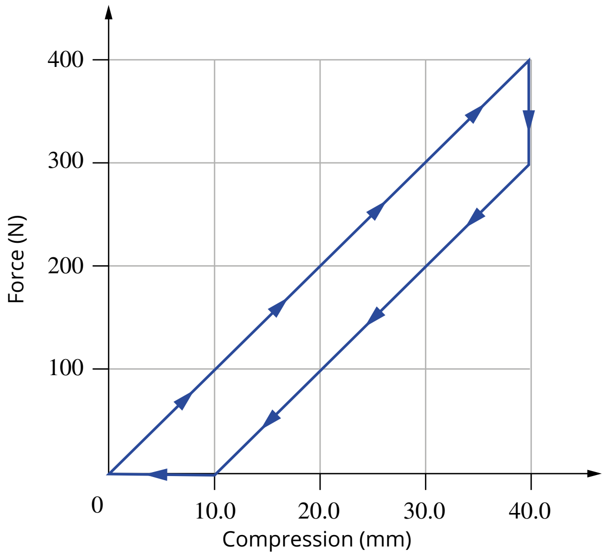
8 The strings of a graphite-head tennis racquet have the force–displacement graph shown. Calculate the work done when the strings displace by 6.0 mm.



9 Three different springs have the force–displacement graphs shown. Estimate the work done by stretching each of the springs by 40.0 mm.



10 The following diagram is a simplified representation of the forces acting on a basketball when it bounces. The upwards arrows show the force as the basketball compresses and the downwards arrows show the force as it rebounds.



a Calculate the work done on the basketball when it compresses by 40.0 mm.

b Calculate the work done by the ball as it decompresses from a compression of 40.0 mm.

c Explain why your answers to parts **a** and **b** differ.

5.2 Kinetic energy

Kinetic energy is energy that a body possesses due to its motion. Throwing a ball, rowing a canoe or launching a rocket ship, these all require energy as an inherent part of their motion.

Any object that moves, such as those shown in Figure 5.2.1, has kinetic energy. Many real-life energy interactions involve objects with kinetic energy. Some of these, like car collisions, have life-threatening implications. Hence, it is important to be able to quantify (find numerical values for) the kinetic energy of an object.

FIGURE 5.2.1 Any moving object, regardless of its size, has kinetic energy.

THE KINETIC ENERGY EQUATION

Kinetic energy is the energy of motion. It can be quantified by calculating the amount of work needed to give an object its velocity.

Consider the dynamics cart in Figure 5.2.2 of mass m, starting at rest (i.e. $v_i = 0$). It is pushed with force F, which acts while the cart undergoes a displacement, s, and gains a final velocity, v_f.

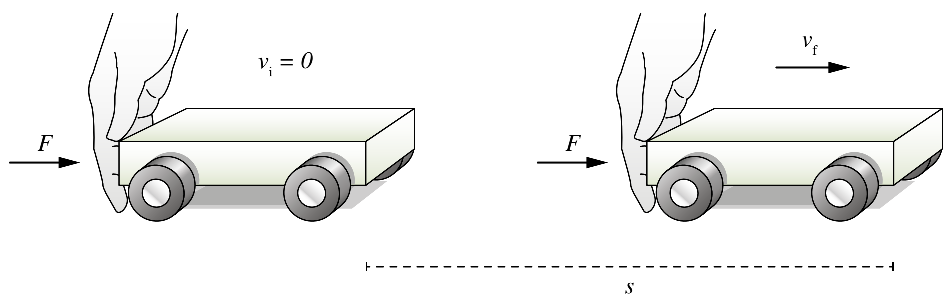


FIGURE 5.2.2 The kinetic energy of a dynamics cart can be calculated by considering the force, *F*, acting on it over a given displacement, *s*.

The work done by the force, W, causes a change in kinetic energy from its initial value $\frac{1}{2}mv_i^2$ to a new value of $\frac{1}{2}mv_f^2$.

The relationship between the work done and the change in kinetic energy can be written mathematically as:

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

where W is work (J)

m_f is mass (kg)

v_i is initial velocity ($m\,s^{-1}$)

v_f is final velocity ($m\,s^{-1}$).

This equation is known as the ‘work–energy theorem’.

In this situation, the cart was originally at rest (i.e. $v_i = 0$) so:

$$W = \frac{1}{2}mv_f^2$$

Assuming that no energy was lost as heat or sound and that all of the work is converted into kinetic energy, this equation gives us a mathematical definition for the kinetic energy of the cart in terms of its mass and its velocity, if its initial velocity is assumed to be zero:

$$E_k = \frac{1}{2}mv^2$$

where E_k is kinetic energy (J).

EXTENSION

Expressing the amount of work

Considering the scenario described in Figure 5.2.2, the work done by the force is given by the equation $W = Fs$. The force causes the cart to accelerate according to Newton's second law, $F = ma$.

Rearranging the equation of motion $v_f^2 = v_i^2 - 2as$ gives:

$$a = \frac{v_f^2 - v_i^2}{2s}$$

Combining this with $F = ma$ means that the force acting on the cart can be given by the equation:

$$F = m\left(\frac{v_f^2 - v_i^2}{2s}\right)$$

This equation can be transposed to find an expression for the amount of work (Fs) done on the cart:

$$F = \frac{m}{2s}(v_f^2 - v_i^2)$$

$$Fs = \frac{m}{2}(v_f^2 - v_i^2)$$

$$Fs = \frac{1}{2}m(v_f^2 - v_i^2)$$

Since $W = Fs$:

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Worked example 5.2.1

CALCULATING KINETIC ENERGY

A car with a mass of 1250 kg is travelling at 92.5 km h^{-1}. Calculate its kinetic energy at this speed.

Thinking	Working
Convert the car's speed to m s^{-1}.	$v = 92.5\text{ km h}^{-1}$ $v = \frac{92.5\text{ km}}{1\text{ h}}$ $v = \frac{92500\text{ m}}{3600\text{ s}}$ $v = 25.694\text{ m s}^{-1}$
Recall the equation for kinetic energy.	$E_k = \frac{1}{2}mv^2$
Substitute the values for this situation into the equation.	$E_k = \frac{1}{2}(1250)(25.694)^2$
State the answer with appropriate units and significant figures.	$E_k = 412\,627.8\text{ J}$ $E_k = 413\text{ kJ}$

Worked example: Try yourself 5.2.1

CALCULATING KINETIC ENERGY

A person crossing the street is walking at 5.15 km h^{-1}. If the person has a mass of 82.5 kg, calculate their kinetic energy.

APPLYING THE WORK–ENERGY THEOREM

The work–energy theorem can be seen as a definition for the *change* in kinetic energy produced by a force:

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = (E_k)_{final} - (E_k)_{initial} = \Delta E_k$$

Worked example 5.2.2

CALCULATING KINETIC ENERGY CHANGES

A 2.50 tonne truck that is travelling at 103 km h^{-1} slows to 62.5 km h^{-1} before turning a corner.

a Calculate the work done by the brakes to make this change.	
Thinking	**Working**
Convert the values into SI units.	$v_i = 103\,\text{km h}^{-1} = \frac{103\,\text{km}}{1\,\text{h}} = \frac{103\,000\,\text{m}}{3600\,\text{s}}$ $= 28.6111\,\text{m s}^{-1}$ $v_f = 62.5\,\text{km h}^{-1} = \frac{62.5\,\text{km}}{1\,\text{h}} = \frac{62\,500\,\text{m}}{3600\,\text{s}}$ $= 17.36111\,\text{m s}^{-1}$ $m = 2.50$ tonne $= 2500$ kg
Recall the work–energy theorem.	$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
Substitute the values for this situation into the equation.	$W = \frac{1}{2}(2500)(17.36111)^2$ $-\frac{1}{2}(2500)(28.6111)^2$
State the answer with appropriate units and significant figures.	$W = -646\,484$ J $W = -646$ kJ Note: the negative value indicates that the work has caused the kinetic energy to decrease.

b If it takes 50.0 m for this deceleration to take place, calculate the average force applied by the truck's brakes.	
Thinking	**Working**
Recall the definition of work.	$W = Fs$
Substitute the values for this situation into the equation. Note: The negative has been ignored as work is a scalar.	$(646\,484) = F(50.0)$
Transpose the equation to find the answer, given with the correct number of significant figures.	$F = \frac{(646\,484)}{(50.0)} = 12\,929.7$ N $F = 12.9$ kN

Worked example: Try yourself 5.2.2

CALCULATING KINETIC ENERGY CHANGES

As a bus with a mass of 10.0 tonnes approaches a school it slows from 60.0 km h^{-1} to 40.0 km h^{-1}.

a Calculate the work done by the brakes in the bus.

b The bus travels 40.0 m as it decelerates. Calculate the average force applied by the truck's brakes.

Notice that the definitions for kinetic energy and change in kinetic energy have been derived entirely from known concepts: the definition of work, Newton's second law and the equations of motion. This makes kinetic energy appear a redundant concept. However, using kinetic energy in calculations can often make analysis of physical interactions quicker and easier, particularly in situations where acceleration is not constant.

Worked example 5.2.3

CALCULATING SPEED FROM KINETIC ENERGY

The engine of a 1450 kg car can do 907 kJ of work in 10.0 s. Assuming all of this work is converted into kinetic energy, calculate the speed of the car after this time in km h^{-1}.

Thinking	Working
Recall the equation for kinetic energy.	$E_k = \frac{1}{2}mv^2$
Transpose the equation to make v the subject.	$v = \sqrt{\frac{2E_k}{m}}$
Substitute the values for this situation into the equation.	$v = \sqrt{\frac{2(907 \times 10^3)}{(1450)}}$ $v = 35.370$ $v = 35.4\ \text{m s}^{-1}$
To convert a speed from m s^{-1} to km h^{-1}, multiply the speed by 3.6. State the answer with appropriate units and significant figures.	$v = (35.370)(3.6)$ $v = 129.085$ $v = 129\ \text{km h}^{-1}$

Worked example: Try yourself 5.2.3

CALCULATING SPEED FROM KINETIC ENERGY

A 325 kg motorbike has 155 kJ of kinetic energy. Calculate the speed of the motorbike in km h^{-1}.

5.2 Review

SUMMARY

- All moving objects have kinetic energy.
- The kinetic energy of an object is equal to the work required to accelerate the object from rest to its final velocity.
- The kinetic energy of an object is given by the following equation:

  $E_k = \frac{1}{2}mv^2$
- The work–energy theorem defines work as *change* in kinetic energy:

  $W = Fs = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta E_k$

KEY QUESTIONS

1 The mass of a motorbike together with its rider is 232 kg. If the motorbike is travelling at 80.5 km h^{-1}, calculate its kinetic energy.

2 A 1580 kg car is travelling at 17.5 m s^{-1}. How much work would its engine need to do to accelerate the car to 28.2 m s^{-1}?

3 A cyclist has a mass of 72.0 kg and is riding a bicycle which has a mass of 9.00 kg. When riding at top speed on the bicycle, their kinetic energy is 5130 J. Calculate the top speed of the cyclist in km h^{-1}.

4 By how much is kinetic energy increased when the mass of an object is doubled?

5 By how much is kinetic energy increased when the velocity of an object is tripled?

6 Lin has eaten a 1.00 kJ sandwich while walking at a speed of 0.500 m s^{-1}. Lin then decides to burn the energy from the sandwich all at once. Assuming Lin's mass is 57.0 kg and all of the energy in the sandwich is converted into kinetic energy via the work done by Lin, how fast will Lin need to run?

7 What is the difference in kinetic energy between an 1850 kg car travelling at 65.0 km h^{-1} and one travelling at 60.0 km h^{-1}? What might this suggest about the dangers of driving 5 km faster than a 60 km per hour speed limit?

5.3 Gravitational potential energy

Gravitational potential energy is a measure of the amount of energy available to an object due to its position in a gravitational field. The gravitational potential energy of an object can be calculated from the amount of work that must be done against the gravitational field to get the object into its position.

Consider the weightlifter lifting a barbell in Figure 5.3.1. Assuming that the bar is lifted at a constant speed, the weightlifter must apply a lifting force equal to the force due to gravity on the barbell, F_g. The lifting force, F_l, is applied over a displacement, Δh, corresponding to the change in height of the barbell.

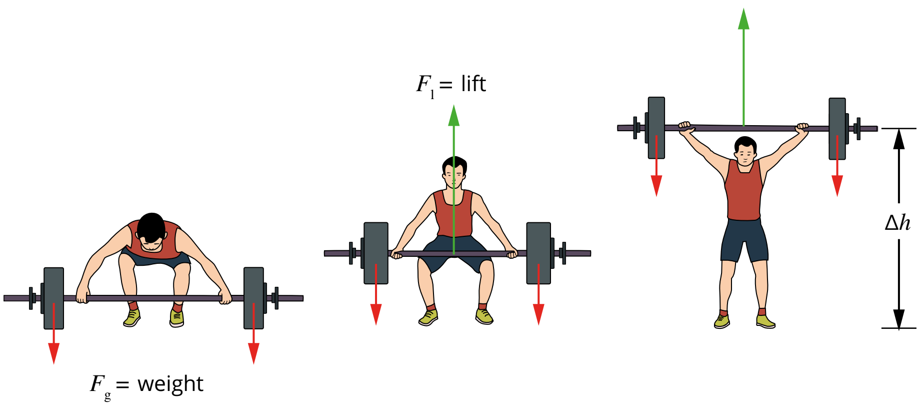


FIGURE 5.3.1 A weightlifter applies a constant force over a fixed distance to move the barbell in the gravitational field, increasing its potential energy.

The work done against gravity by the weightlifter is:

$W = Fs = F_g\Delta h$

Since the force due to gravity $F_g = mg$, the work done can be written as:

$W = mg\Delta h$

The work carried out in this example has resulted in the transformation of chemical energy within the weightlifter into gravitational potential energy due to the position of the barbell in the gravitational field, hence:

$\Delta E_p = mg\Delta h$

> Taking the surface of the Earth as the point where the gravitational potential energy is zero (that is, $E_p = 0$), the gravitational potential energy of an object, due to the work done against a gravitational field, is given by:
>
> $E_p = mg\Delta h$
>
> where E_p is the gravitational potential energy (J)
>
> m is the mass of the object (kg)
>
> g is the gravitational field strength (9.80 $N\,kg^{-1}$ on Earth)
>
> Δh is the change in height of the object (m).
>
> Note that the value of g is normally described as the acceleration due to gravity, measured in $m\,s^{-2}$. In this section, however, it is more appropriate to use g as the gravitational field strength. The value is the same, but the units are now $N\,kg^{-1}$. The two units are equivalent to each other, but since energy and work are scalar quantities there is no need to define a positive or negative direction for gravitational field strength. Gravitational field strength is used when considering factors affected by the gravitational field, while acceleration due to gravity is used for vertical motion in a gravitational field.

Worked example 5.3.1

CALCULATING GRAVITATIONAL POTENTIAL ENERGY

A weightlifter lifts a barbell that has a total mass of 80.0 kg from the floor to a height of 1.85 m above the ground. Calculate the gravitational potential energy of the barbell at this height.	
Thinking	**Working**
Recall the formula for gravitational potential energy.	$E_p = mg\Delta h$
Substitute the values for this situation into the equation.	$E_p = (80.0)(9.80)(1.85)$
State the answer with appropriate units and significant figures.	$E_p = 1450.4\text{ J} = 1.45\text{ kJ}$

Worked example: Try yourself 5.3.1

CALCULATING GRAVITATIONAL POTENTIAL ENERGY

A person doing their grocery shopping lifts a 5.85 kg grocery bag to a height of 32.9 cm. Calculate the gravitational potential energy of the bag at this height.

GRAVITATIONAL POTENTIAL ENERGY AND REFERENCE LEVEL

When calculating gravitational potential energy, it is important to carefully define the level that corresponds to $E_p = 0$. Often this can be taken to be the ground or sea level, but the zero potential energy reference level is not always obvious.

It does not really matter which point is taken as the zero potential energy reference level, as long as the chosen point is used consistently throughout a particular problem (Figure 5.3.2). If objects move below the reference level, then their energies will become negative and should be interpreted accordingly.

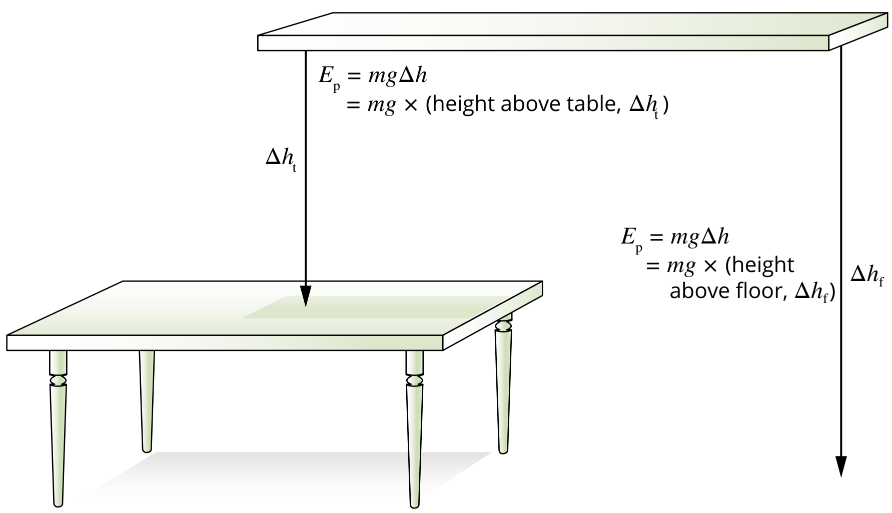


FIGURE 5.3.2 In this situation, the zero potential energy reference point could be taken as either the level of the table or the floor. If the floor is the reference point, the gravitational potential energy of the board is $mg\Delta h_f$; if the tabletop is the reference point, the gravitational potential energy of the board is $mg\Delta h_t$.

Worked example 5.3.2

CALCULATING GRAVITATIONAL POTENTIAL ENERGY RELATIVE TO A REFERENCE LEVEL

A weightlifter (m = 63.5 kg) lifts a 50.0 kg bar through a distance of 42.6 cm. Calculate the increase in gravitational potential energy of the bar with each lift. Use $g = 9.80\ \text{N kg}^{-1}$.

Thinking	Working
Recall the formula for gravitational potential energy.	$E_p = mg\Delta h$
Identify the relevant values for this situation. Only the mass of the bar is being lifted (the weightlifter's mass is a distractor). Assume that the ground is the zero potential energy level.	$m = 50.0\ \text{kg}$ $g = 9.80\ \text{N kg}^{-1}$ $\Delta h = 42.6\ \text{cm} = 0.426\ \text{m}$
Substitute the values for this situation into the equation.	$E_p = (50.0)(9.80)(0.426)$
State the answer with appropriate units and significant figures.	$E_p = 208.74$ $E_p = 209\ \text{J}$

Worked example: Try yourself 5.3.2

CALCULATING GRAVITATIONAL POTENTIAL ENERGY RELATIVE TO A REFERENCE LEVEL

A father picks up his baby from its bed. The baby has a mass of 6.35 kg and the mattress of the bed is 74.2 cm above the ground. When the father holds the baby in his arms, it is 125 cm off the ground. Calculate the increase in gravitational potential energy of the baby, taking g as $9.80\ \text{N kg}^{-1}$.

PHYSICSFILE

Newton's universal law of gravitation

The formula $E_p = mg\Delta h$ is based on the assumption that the Earth's gravitational field is constant. Newton's universal law of gravitation predicts that the Earth's gravitational field will decrease with altitude. However, this decrease only becomes significant many kilometres above the Earth's surface. For everyday purposes, the assumption of a constant gravitational field is valid.

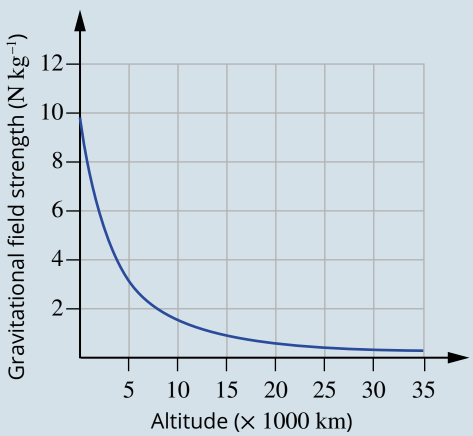


FIGURE 5.3.3 Earth's gravitational field strength decreases with altitude.

EXTENSION

The high jump

Science has long been used in sports to help athletes gain a competitive edge. The concept of gravitational potential energy is of obvious importance to a high jumper. Clearly, the high jumper must do enough work in their jump to gain sufficient gravitational potential energy to clear the bar.

The modern high jump technique known as the Fosbury flop gets the high jumper to bend their body as they go over the bar. This is illustrated in Figure 5.3.4.

When the technique is correctly performed, most of the mass of the jumper is actually lower than the bar throughout most of the jump. In other words, the centre of mass of the jumper passes below the bar while their body bends over it, as shown in Figure 5.3.5.

If the jumper's technique is right, they do not have to gain enough gravitational potential energy to lift all of their body higher than the bar for the time it takes to clear the bar. Without this technique, the world records for this event would probably be much lower than they currently are.

FIGURE 5.3.4 In the Fosbury flop technique, a high jumper must bend their body over the bar.

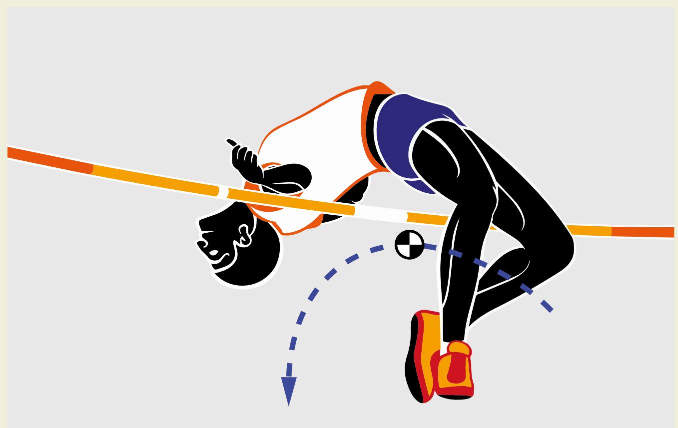


FIGURE 5.3.5 The path of the centre of mass of the high jumper (shown by the dashed curve) passes below the high-jump bar.

5.3 Review

SUMMARY

- Gravitational potential energy is the energy an object has due to its position in a gravitational field.
- The gravitational potential energy of an object, E_p, is given by the following:
  $E_p = mg\Delta h$
- Gravitational potential energy is calculated relative to a zero potential energy reference level, usually the ground or sea level.

KEY QUESTIONS

1 A 57.8 g tennis ball is thrown 8.32 m into the air. Use $g = 9.80\ \text{N kg}^{-1}$.
  a Calculate the gravitational potential energy of the ball at the top of its flight.
  b Calculate the gravitational potential energy of the ball when it has fallen halfway back down to Earth.

2 When climbing Mount Everest ($h = 8848$ m), a mountain climber stops to rest at North Base Camp ($h = 5150$ m). If the mountain climber has a mass of 65.0 kg, how much gravitational potential energy will they gain in the final section of the climb (i.e. from North Base Camp to the summit)? For simplicity, assume that g is constant at $9.80\ \text{N kg}^{-1}$.

3 An astronaut visiting Mars has a mass of 95.4 kg. The astronaut climbs 62.1 m up a hill and their gravitational potential energy increases by 21.9 kJ. What is the gravitational field strength on Mars?

4 A high jumper with mass 55.2 kg and height 1.75 m leaps with a jump of 550 J. The jumper needs the centre of their body (0.875 m) to clear a 1.90 m bar. Neglecting air resistance, and assuming all of this energy is converted into gravitational potential energy, does the jumper make it over the bar?

5 An eagle of mass 7.50 kg dives 153 m to catch a mouse. What is the eagle's change in potential energy?

6 If you were to lift a weight above your head and hold it still for several minutes, is any additional work against gravity done after the initial hoist? Why or why not?

5.4 Law of conservation of energy

In many situations, energy is transformed between kinetic and gravitational potential energy. For example, when a tennis ball bounces, as shown in Figure 5.4.1, much of its kinetic energy is converted into gravitational potential energy and then back into kinetic energy again.

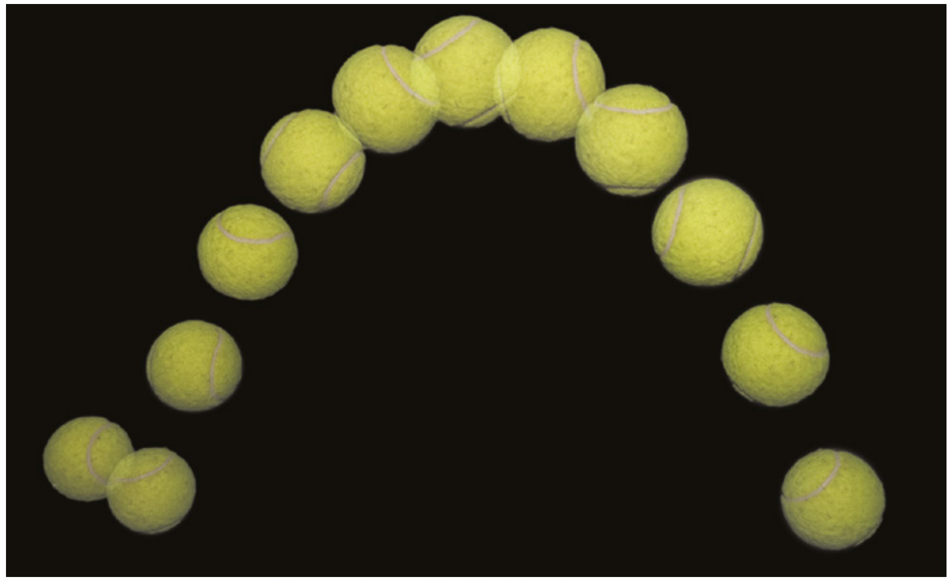


FIGURE 5.4.1 A bouncing tennis ball.

In analysing this type of situation, the concept of mechanical energy is useful. Mechanical energy is the energy that an object possesses due to its position and motion and is the sum of its potential energies and its kinetic energy.

CONSERVATION OF ENERGY

Recall from Section 5.1 that energy comes in many forms, such as heat, light, sound, chemical energy and electrical energy. It is a scalar quantity and is measured in joules (J). Energy is also associated with the motion and position of an object, and this energy is called the mechanical energy of the object. Mechanical energy is the sum of an object's potential and kinetic energies.

In the motion problems of this chapter, moving objects have been described as having kinetic energy. An object can also have stored energy, known as gravitational potential energy, due to its position. For instance, a building crane lifting a steel beam several stories high increases the gravitational potential energy stored in the gravitational field due to the position of the beam, which could be converted to kinetic energy if the lifting chain were to break and the beam were to accelerate under gravity.

The transfer of gravitational potential energy to kinetic energy is an illustration of the law of conservation of energy, a fundamental principle of nature. The law of **conservation of energy** states that energy is not created or destroyed, but can only change from one form to another, or in other words, **transform**. In the example above, as the gravitational potential energy of the gravitational field decreases, the kinetic energy of the beam increases. The total amount of mechanical energy remains constant.

While energy is never destroyed, it may dissipate in forms that are not easily recoverable. For instance, the kinetic energy of a vehicle is reduced as it encounters friction, causing the tyres to heat, or in the deformation of the bodywork should it collide with another object. The mechanical energy before and after a collision is only the same under ideal conditions, but in many cases, it is a useful approximation.

It is a common misconception to think that energy disappears when it converts from a more obvious form (such as kinetic energy). Consider the case of a car crashing into a tree. When the car stops and the tree remains stationary, it may seem as though all energy is lost because nothing is moving. In reality, all of the kinetic energy has been converted into other less noticeable forms, such as sound energy, heat energy, deformation energy, and so on. If you could collect all of this energy and compare it to the initial amount, it would be exactly equal. The energy would have been conserved.

MECHANICAL ENERGY

For a falling object, mechanical energy is calculated from the sum of its kinetic and gravitational potential energies:

$$E_m = E_k + E_p = \frac{1}{2}mv^2 + mgh$$

This is a useful concept in situations where gravitational potential energy is converted into kinetic energy or vice versa. For example, consider a tennis ball with a mass of 60.0 g that is dropped from a height of 1.00 m (Figure 5.4.2). Initially, its total mechanical energy would comprise the kinetic energy, which would be 0 J, and the gravitational potential energy that is stored at this height (taking $g = 9.80\,\text{N kg}^{-1}$):

$$E_p = mgh$$
$$E_p = (0.0600)(9.80)(1.00)$$
$$E_p = 0.588\,\text{J}$$

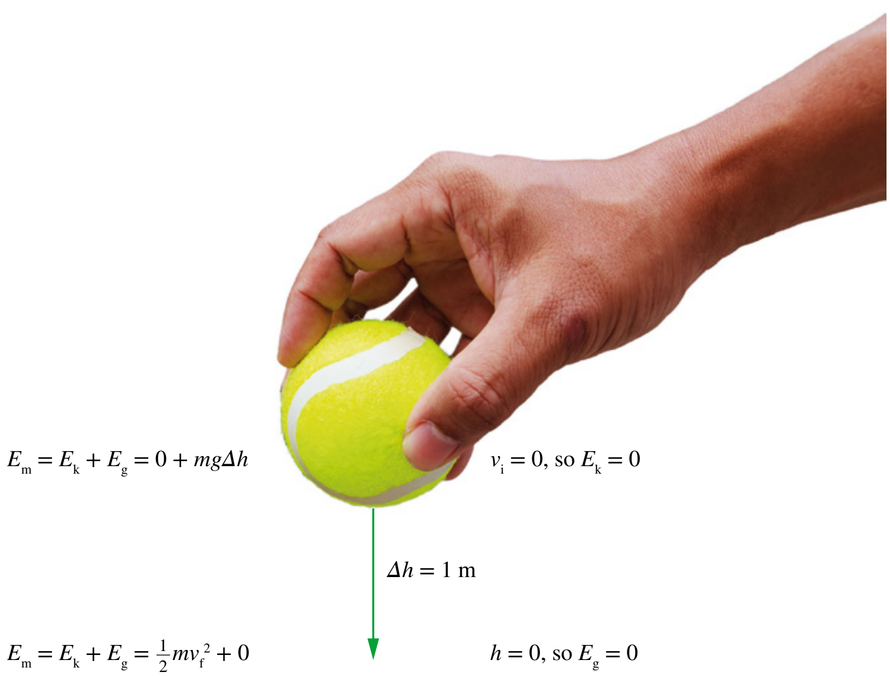


FIGURE 5.4.2 A falling tennis ball provides an example of conservation of mechanical energy.

At the instant the ball hits the ground, the total mechanical energy would comprise the gravitational potential energy available to it and the kinetic energy just prior to hitting the ground. The gravitational potential energy would be 0 J because the ball is at ground level. To calculate its kinetic energy, find the ball's velocity just before it hits the ground using one of the equations of motion:

$s = -1.00\,\text{m}$	$v_f^2 = v_i^2 + 2as$
$v_i = 0\,\text{m s}^{-1}$	$v_f^2 = 0^2 + 2(-9.80)(-1.00)$
$v_f = ?$	$v_f^2 = 19.6$
$a = -9.80\,\text{m s}^{-2}$	$v_f = 4.4272$
$\Delta t = ?$	$v_f = 4.43\,\text{m s}^{-1}$

PHYSICSFILE

Noether's theorem

It might seem obvious in context now that energy is always conserved, but historically, it took centuries of debate and experiment to conclude just exactly *which* quantity is conserved. The conversion of kinetic energy into heat and sound made it difficult for scientists to keep track of the energy and justify that it wasn't, say, velocity that was conserved in general.

While the conservation of energy was a foundational law of thermodynamics, and many experiments had shown it as an empirical fact by the early twentieth century, it wasn't until 1915 that the brilliant German mathematician Emmy Noether (1882–1935) provided a mathematical foundation and proof for general conserved quantities (such as energy and momentum) in theoretical physics. Her theorem has been described as one of the most important mathematical theorems ever proved in the development of modern physics.

FIGURE 5.4.3 Emmy Noether (1882–1935)

Therefore, the kinetic energy of the tennis ball just before it hits the ground is:

$E_k = \frac{1}{2}mv^2$

$E_k = \frac{1}{2} \times (0.0600)(4.43)^2$

$E_k = 0.588\,J$

Notice that at both the top and the bottom of the 1.00 m fall, the mechanical energy is the same.

At the top:

$E_m = E_k + E_p = 0 + 0.588 = 0.588\,J$

At the bottom:

$E_m = E_k + E_p = 0.588 + 0 = 0.588\,J$

In fact, mechanical energy is constant throughout the drop. Consider the tennis ball when it has fallen halfway to the ground. At this point, $h = 0.500\,m$ and $v = 3.13\,m\,s^{-1}$:

$E_m = E_k + E_p$

$E_m = \left(\frac{1}{2}(0.0600)(3.13)^2\right) + \left((0.0600)(9.80)(0.500)\right)$

$E_m = 0.294 + 0.294$

$E_m = 0.588\,J$

Notice that, at this halfway point, the mechanical energy is evenly split between kinetic energy (0.294 J) and gravitational potential energy (0.294 J).

Throughout the drop, the mechanical energy has been conserved.

PHYSICSFILE

Mechanical energy of a ball falling through the air

In reality, as the ball described above drops through the air, a very small amount of its energy is transformed into heat and sound due to friction. As a result, the ball won't quite reach a speed of $4.43\,m\,s^{-1}$ before it hits the ground. This means that mechanical energy is not entirely conserved. However, this effect can be considered negligible for many falling objects.

The principle of conservation of mechanical energy states the following:
Given that, in a system of bodies, there are no other forms of energy except kinetic and potential energy, the total mechanical energy of the system is constant.

Worked example 5.4.1

MECHANICAL ENERGY OF A FALLING OBJECT

A basketball with a mass of 602 g is dropped from a height of 1.25 m. Calculate its kinetic energy at the instant before it hits the ground.

Thinking	Working
Since the ball is dropped, its initial kinetic energy is zero.	$E_{k\ initial} = 0\,J$
Calculate the initial gravitational potential energy of the ball.	$E_{p\ initial} = mg\Delta h$ $E_{p\ initial} = (0.602)(9.80)(1.25)$ $E_{p\ initial} = 7.3745\,J$
Calculate the initial mechanical energy.	$E_{m\ initial} = E_{k\ initial} + E_{p\ initial}$ $E_{m\ initial} = 0 + (7.3745)$ $E_{m\ initial} = 7.3745\,J$
At the instant before the ball hits the ground, its gravitational potential energy is zero.	$E_{p\ final} = 0\,J$
Mechanical energy is conserved in this situation.	$E_{m\ initial} = E_{m\ final} = E_{k\ final} + E_{p\ final}$ $(7.3745) = E_{k\ final} + 0$ $E_{k\ final} = 7.3745$ $E_{k\ final} = 7.37\,J$

Worked example: Try yourself 5.4.1

MECHANICAL ENERGY OF A FALLING OBJECT

A 6.81 kg bowling ball is dropped from a height of 0.750 m. Calculate its kinetic energy at the instant before it hits the ground.

Using mechanical energy to calculate velocity

The speed of a falling object does not depend on its mass. This can be demonstrated using mechanical energy.

Consider an object with a mass, m, falling through a height, Δh. At the moment it begins to fall, its initial kinetic energy is zero. At the moment before it hits the ground, its final gravitational potential energy is zero. Therefore, using **conservation of mechanical energy**:

$$E_{m\ initial} = E_{m\ final}$$

$$E_{k\ initial} + E_{p\ initial} = E_{k\ final} + E_{p\ final}$$

$$0 + mg\Delta h = \frac{1}{2}mv_f^2 + 0$$

$$mg\Delta h = \frac{1}{2}mv_f^2$$

$$g\Delta h = \frac{1}{2}v_f^2$$

$$v_f^2 = 2g\Delta h$$

$$v_f = \sqrt{2g\Delta h}$$

This formula can be used to find the velocity of a falling object as it hits the ground. Note that the formula does not contain the mass of the falling object, so if air resistance is negligible, any object with any mass will have the same final velocity when it is dropped from the same height.

EXTENSION

Deriving a formula for velocity from the equations of linear motion

The velocity of a falling object formula can also be derived from the equations of linear motion, where $a = g$. Consider an object falling through a height, Δh, with negligible air resistance and an initial speed of $v_i = 0\ \text{m s}^{-1}$. Using the formula $v_f^2 = v_i^2 + 2as$:

$$v_f^2 = v_i^2 + 2as$$

$$v_f^2 = 0^2 + 2g\Delta h$$

$$v_f^2 = 2g\Delta h$$

$$v_f = \sqrt{2g\Delta h}$$

This formula is equivalent to the result achieved using the conservation of mechanical energy.

Worked example 5.4.2

FINAL VELOCITY OF A FALLING OBJECT

A basketball with a mass of 598 g is dropped from a height of 1.25 m. Calculate its speed at the instant before it hits the ground.	
Thinking	**Working**
Recall the formula for the velocity of a falling object.	$v_f = \sqrt{2g\Delta h}$
Substitute the relevant values into the formula and solve.	$v_f = \sqrt{2(9.80)(1.25)}$ $v_f = 4.9497$ $v_f = 4.95\,\text{m s}^{-1}$
Interpret the answer.	The basketball will be falling at a speed of $4.95\,\text{m s}^{-1}$ just before it hits the ground.

Worked example: Try yourself 5.4.2

FINAL VELOCITY OF A FALLING OBJECT

A 6.85 kg bowling ball is dropped from a height of 0.750 m. Calculate its speed at the instant before it hits the ground.

Using conservation of mechanical energy in complex situations

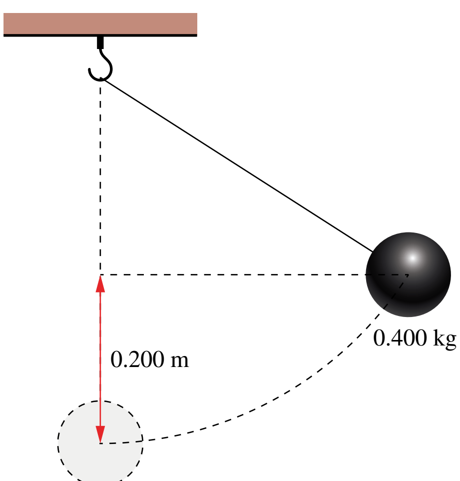


FIGURE 5.4.4 A falling pendulum provides an example of conservation of mechanical energy.

The concept of mechanical energy allows physicists to determine outcomes in non-linear situations where the equations of linear motion cannot be used. For example, consider a pendulum with a bob of mass 400 g displaced from its mean position such that its height has increased by 20.0 cm, as shown in Figure 5.4.4.

Since a falling pendulum involves gravitational potential energy being converted into kinetic energy, the conservation of mechanical energy applies to this situation. Therefore, the formula developed earlier for the velocity of a falling object can be used to find the velocity of the pendulum bob at its lowest point.

$$v_f = \sqrt{2g\Delta h}$$

$$v_f = \sqrt{2(9.80)(0.200)}$$

$$v_f = 1.9799$$

$$v_f = 1.98\,\text{m s}^{-1}$$

The speed of the pendulum bob will be $1.98\,\text{m s}^{-1}$ at its lowest point. However, unlike the falling tennis ball, the direction of the bob's motion will be horizontal instead of vertical at its lowest point. The equations of motion relate to linear motion and cannot be applied to this situation as the bob swings in a curved path.

Conservation of energy can also be used to analyse projectile motion, that is, when an object is thrown or fired into the air with some initial velocity. Since energy is not a vector, no vector analysis is required, even if the initial velocity is at an angle to the ground.

Worked example 5.4.3

USING MECHANICAL ENERGY TO ANALYSE PROJECTILE MOTION

A cricket ball (m = 151 g) is thrown upwards into the air at a speed of 15.0 m s^{-1}. Calculate the speed of the ball when it has reached a height of 8.20 m. Assume that the ball is thrown from a height of 1.55 m.	
Thinking	**Working**
Recall the formula for mechanical energy.	$E_m = E_k + E_p$ $E_m = \frac{1}{2}mv^2 + mg\Delta h$
Substitute in the values for the ball as it is thrown.	$E_{m\ initial} = \frac{1}{2}(0.151)(15.0)^2 + (0.151)(9.80)(1.55)$ $E_{m\ initial} = 16.9875 + 2.2937$ $E_{m\ initial} = 19.2812$ $E_{m\ initial} = 19.3\ J$
Use conservation of mechanical energy to find an equation for the final speed.	$E_{m\ initial} = E_{m\ final} = E_{k\ final} + E_{p\ final}$ $E_{m\ final} = \frac{1}{2}mv^2 + mg\Delta h$ $19.2812 = \frac{1}{2}(0.151)v^2 + (0.151)(9.80)(8.20)$
Solve the equation algebraically to find the final speed.	$\frac{1}{2}(0.151)v^2 = (19.2812) - (12.1344)$ $v^2 = 94.6601$ $v = 9.72934$ $v = 9.73\ m\ s^{-1}$
Interpret the answer.	The cricket ball will be moving at 9.73 m s^{-1} when it reaches a height of 8.20 m.

Worked example: Try yourself 5.4.3

USING MECHANICAL ENERGY TO ANALYSE PROJECTILE MOTION

An arrow with a mass of 35.2 g is fired into the air at 80.5 m s^{-1} from a height of 1.47 m. Calculate the speed of the arrow when it has reached a height of 30.0 m.

PHYSICSINACTION

Ballistics pendulum

The ballistics pendulum is an example of how the law of conservation of mechanical energy can be combined with an understanding of collisions to solve a practical problem. A ballistics pendulum is a device that can be used to measure the speed of a bullet fired from a gun or rifle. It consists of a block of wood hanging at a convenient height above the ground, as shown in Figure 5.4.5.

When a bullet is fired into the wooden block, an inelastic collision occurs. This means that much of the bullet's kinetic energy is converted into heat and sound and into changes made to the shape of the block. The conservation of mechanical energy does not apply for the impact of the bullet with the block.

However, the law of conservation of momentum still applies to the impact. This means that the block gains velocity from the bullet and it swings backwards and upwards, as shown in Figure 5.4.6. By measuring the change in height of the block and the masses of the bullet and block, the initial speed of the bullet can be calculated. Note that conservation of mechanical energy does occur when the block swings backwards and upwards, as no energy is converted into sound or heat during this part of its motion.

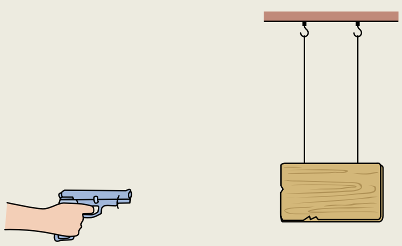


FIGURE 5.4.5 Using a ballistics pendulum combines an understanding of collisions and mechanical energy.

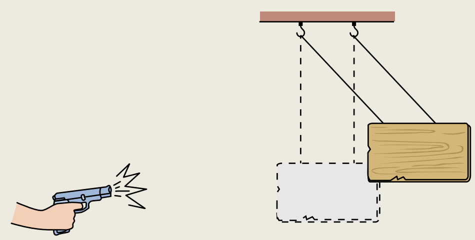


FIGURE 5.4.6 The change in height of a ballistics pendulum can be used to calculate the speed of the bullet fired into it.

Loss of mechanical energy

Mechanical energy is not conserved in every situation. For example, when a tennis ball bounces a number of times, each bounce is lower than the one before it, as shown in Figure 5.4.7.

While mechanical energy is largely conserved as the ball moves through the air, a significant amount of kinetic energy is transformed into heat and sound when the ball compresses and decompresses as it bounces. This means that the ball does not have as much kinetic energy when it leaves the ground as it did when it landed. Therefore, the gravitational potential energy it can achieve on the second bounce will be less than the gravitational potential energy it had initially, and so the second bounce is lower.

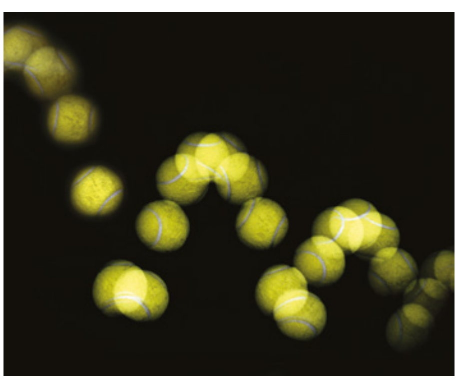


FIGURE 5.4.7 Mechanical energy is lost with each bounce of a tennis ball.

EXTENSION

Energy transformations in a bouncing ball

A bouncing ball involves forms of energy other than kinetic and gravitational potential energy. When the ball hits the ground, its gravitational potential energy is converted into elastic potential energy as it compresses. As the ball expands back to its original shape, some of the elastic potential energy is converted back into kinetic energy and some of it is converted into heat and sound. The amount of energy that is converted into heat and sound depends on the type of ball.

If you want the ball to reach a greater height than its original height, then instead of dropping the ball you could add to its energy by throwing it downwards with some velocity. Consider Figure 5.4.8a where a tennis ball ($m = 58.0\,g$) is thrown downwards at $4.00\,m\,s^{-1}$ from a height of 1.00 m above the ground.

Initially, the ball has 0.568 J of gravitational potential energy:

$E_p = mg\Delta h = (0.0580)(9.80)(1.00) = 0.568\,J$

and 0.464 J of kinetic energy:

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2}(0.0580)(4.00)^2$$

$$E_k = 0.464\,J$$

By the time it reaches the ground, the gravitational potential energy has been transformed into kinetic energy, giving it a total of 0.568 + 0.464 = 1.032 J of kinetic energy (Figure 5.4.8b). This is converted into elastic potential energy of 1.032 J (Figure 5.4.8c).

If 0.280 J of energy are lost as heat and sound as the ball expands, then the ball will have just 0.752 J of kinetic energy when it leaves the ground (Figure 5.4.8d). This means that it will rebound to a height of 1.32 m (Figure 5.4.8e):

$$\Delta h = \frac{E_p}{mg}$$

$$\Delta h = \frac{(0.752)}{(0.0580)(9.80)}$$

$$\Delta h = 1.3230$$

$$\Delta h = 1.32\ m$$

Even though some energy has been 'lost' in the bounce, the initial kinetic energy of the ball means that it ends up slightly higher than where it started.

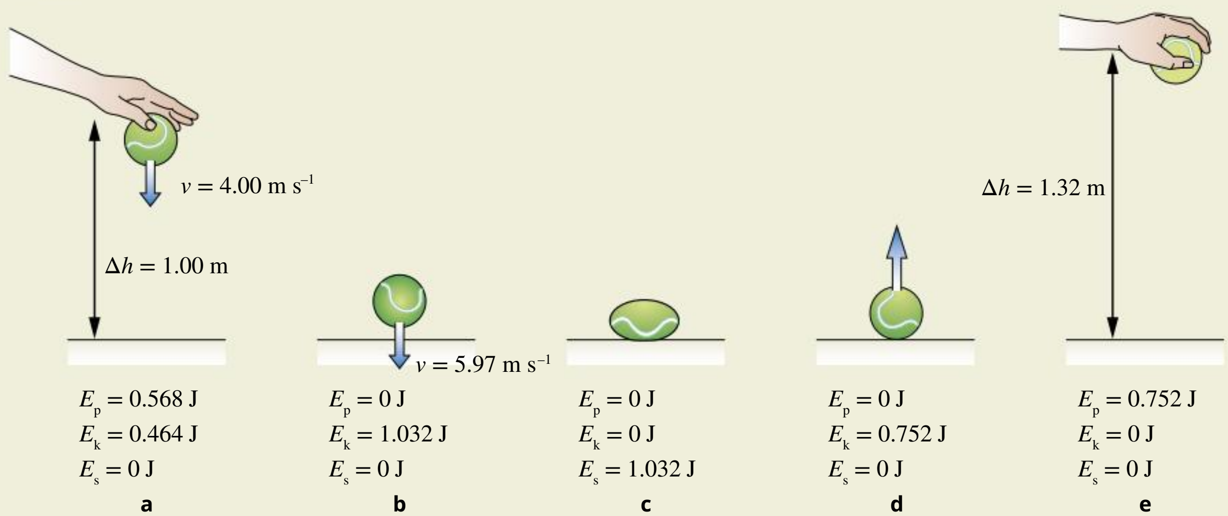


FIGURE 5.4.8 A tennis ball thrown downwards from a height.

EFFICIENCY OF ENERGY TRANSFORMATIONS

In the real world, energy transformations are never perfect as there is always some energy 'lost'. Because of this, for a system to continue operating (doing work), it must be constantly provided with energy. The percentage of energy that is effectively transformed by a device is called the **efficiency** of that device. A device operating at 45% efficiency is converting 45% of its supplied energy into the useful new form. The other 55% is 'lost' or transferred to the surroundings, usually as heat and/or sound. It is not truly lost since energy cannot be created or destroyed; rather, the form it becomes (heat and sound) is not useful.

The efficiency of a transformation from one energy form to another is expressed as:

$$\text{efficiency } (\eta) = \frac{\text{useful energy transformed}}{\text{total energy supplied}} \times 100\%$$

$$\text{efficiency } (\eta) = \frac{\text{energy output}}{\text{energy input}} \times 100\%$$

Table 5.4.1 shows the approximate efficiencies of some common objects.

TABLE 5.4.1 Efficiencies of some common objects or devices.

Device	Energy transfer	Efficiency (%)
electric motor	electric to kinetic	90
gas heater	chemical to thermal	75
incandescent light globe	electric to light	2
compact fluorescent light	electric to light	10
LED household light	electric to light	15
steam turbine	thermal to kinetic	45
coal-fired generator	chemical to electrical	30
high-efficiency solar cell	radiation to electrical	35
car engine	chemical to kinetic	25
open fireplace	chemical to thermal	15
human body	chemical to kinetic	25

Worked example 5.4.4

ENERGY EFFICIENCY

The energy input of a particular gas-fired power station is 1120 MJ. The electrical energy output is 301 MJ.

What is the efficiency of the power station?	
Thinking	**Working**
Recall the equation for efficiency. Substitute the values into the equation.	output = 301 MJ input = 1120 MJ $\text{efficiency } (\eta) = \frac{\text{energy output}}{\text{energy input}} \times 100\%$ $\text{efficiency } (\eta) = \frac{(301 \times 10^6)}{(1120 \times 10^6)} \times 100\%$
Solve the equation.	efficiency = 26.875% efficiency = 26.9%

Worked example: Try yourself 5.4.4

ENERGY EFFICIENCY

An electric kettle uses 23.3 kJ of electrical energy as it boils a quantity of water. The efficiency of the kettle is 18.0%.

How much electrical energy is expended in actually boiling the water?

PHYSICSFILE

Coefficient of restitution

The ‘bounce of the ball’ is an important factor in many sports. Physicists describe the ‘bounciness’ of balls using a concept known as the coefficient of restitution (COR). COR depends on both the ball and the surface it bounces on. For example, a tennis ball bouncing on grass has a different COR than one bouncing on clay. This is one reason why some tennis players prefer certain surfaces over others.

5.4 Review

SUMMARY

- The total energy of an isolated system is conserved in all circumstances.
- Mechanical energy is the sum of the potential and kinetic energies of an object.
- Mechanical energy is conserved in a falling object with negligible air resistance.
- Conservation of mechanical energy can be used to predict outcomes in a range of situations involving gravity and motion.
- The final velocity of an object falling with negligible air resistance from height Δh can be found using the equation $v = \sqrt{2g\Delta h}$.
- When a ball bounces, some mechanical energy is transformed into heat and sound.
- The efficiency of an energy transformation from one form to another is:

$$\text{efficiency } (\eta) = \frac{\text{useful energy transformed}}{\text{total energy supplied}} \times 100\%$$

$$\text{efficiency } (\eta) = \frac{\text{energy output}}{\text{energy input}} \times 100\%$$

KEY QUESTIONS

1 A piano with a mass of 189 kg is pushed off the roof of a five-storey apartment block. The piano falls 3.00 m for each storey (i.e. a total of 15.0 m).

a Calculate the piano's kinetic energy as it hits the ground.

b Calculate the piano's kinetic energy as it passes the windows on the second floor, having fallen 10.0 m.

2 A tennis ball is dropped from the roof of a five-storey apartment block. The tennis ball falls 3.00 m for each storey (i.e. a total of 15.0 m).

a Calculate the speed of the tennis ball as it hits the ground.

b Calculate the tennis ball's speed as it passes the windows on the second floor, having fallen 10.0 m.

3 A branch falls from a tree and hits the ground with a speed of 5.40 $m\,s^{-1}$. From what height did the branch fall?

4 A javelin with a mass of 825 g is thrown at an angle of inclination (i.e. the angle from the horizontal) of 40.0°. It is released at a height of 1.45 m with a speed of 28.5 $m\,s^{-1}$.

a Calculate the javelin's initial mechanical energy.

b Calculate the javelin's speed as it hits the ground.

5 A coal-fired generator has an efficiency of approximately 30.0%. If 2.50 kJ of energy is supplied to the generator, how much is converted into electrical energy?

6 A rubber ball is dropped from a height of 1.65 m and loses 20.0% of its mechanical energy as it hits the ground. To what height will it rebound?

7 A child rolls a ball up a frictionless hill. As the ball is travelling, does the kinetic energy of the ball increase or decrease? Does the gravitational potential energy increase or decrease? By what relative amount?

5.5 Elastic and inelastic collisions

FIGURE 5.5.1 Collisions often occur in sports.

Collisions occur when two or more objects hit each other and exchange momentum and energy. These are important interactions. Energy and momentum can help us to understand everyday collisions, such as those between soccer players (Figure 5.5.1), between a bat and a ball, or more serious collisions like car crashes. For physicists, studying collisions between subatomic particles may be the key to unlocking the secrets of the universe.

In Chapter 4 and the first part of this chapter, you have learnt that in all interactions, both energy and momentum are conserved. However, nature places no restrictions on the conversion of energy during collisions. During a collision, kinetic energy can be entirely conserved, or it can be converted into other types of energy—such as heat, sound and deformation of the objects involved.

ELASTICITY AND INELASTICITY

Recall from the previous section that kinetic energy (E_k) is the energy of motion of a body. It is calculated using:

$$E_k = \frac{1}{2}mv^2$$

In perfectly **elastic collisions**, kinetic energy is transferred between objects, with no energy transformed into heat, sound or deformation. In these cases, the relationship is stated as:

$$E_k \text{ (before)} = E_k \text{ (after)}$$

$$\Sigma \frac{1}{2}mv_i^2 = \Sigma \frac{1}{2}mv_f^2$$

In Chapter 4 you saw how momentum is always conserved in a collision. The total energy is also always conserved in a closed system. However, in general, *kinetic* energy is not conserved, and is usually converted to heat or sound. Such collisions are called **inelastic collisions**.

Perfectly elastic collisions do not exist in everyday situations, but they do exist in the interactions between atoms and subatomic particles. A collision between two billiard balls or the spheres in a Newton's cradle is nearly perfectly elastic, as only a minimal amount of kinetic energy is transformed into heat and sound energy. Collisions involving a bouncing basketball, a gymnast on a trampoline or a tennis ball being hit are moderately elastic, with about half of the system's kinetic energy being retained.

Perfectly inelastic collisions, on the other hand, occur when the colliding bodies stick together after impact, resulting in a loss of kinetic energy. Some examples include collisions between a meteorite and the Moon, or collisions involving two balls of plasticine. In these cases, most or all of the initial kinetic energy of the system is transformed into other forms of energy.

Worked example 5.5.1

ELASTIC OR INELASTIC COLLISION?

A car of mass 1.00×10^3 kg travelling west at $20.0\,m\,s^{-1}$ crashes into the rear of a stationary bus of mass 5.00×10^3 kg. The vehicles lock together on impact. Show calculations to test whether or not the collision is elastic.

Thinking	Working
Use conservation of momentum to find the final velocity of the joined vehicles after the collision.	$p_{i\,car} + p_{i\,bus} = p_{f\,car+bus}$ $mv_{i\,car} + mv_{i\,bus} = mv_{f\,car+bus}$ $(1.00\times10^3)(20)+(5.00\times10^3)(0) = (1.00\times10^3+5.00\times10^3)v_{f\,car+bus}$ $v_{f\,car+bus} = \frac{(2.00\times10^4)}{(6.00\times10^3)}$ $v_{f\,car+bus} = 3.3333\,m\,s^{-1}$ $v_{f\,car+bus} = 3.33\,m\,s^{-1}$
Calculate the initial kinetic energy before the collision for the bus and the car.	$E_{ki\,bus} = \frac{1}{2}mv^2$ $E_{ki\,bus} = \frac{1}{2}(5.00\times10^3)(0)^2$ $E_{ki\,bus} = 0\,J$ $E_{ki\,car} = \frac{1}{2}mv^2$ $E_{ki\,car} = \frac{1}{2}(1.00\times10^3)(20.0)^2$ $E_{ki\,car} = 2.00\times10^5\,J$ $E_{ki} = E_{k\,i\,bus} + E_{k\,i\,car}$ $E_{ki} = (0)+(2.00\times10^5)$ $E_{ki} = 2.00\times10^5\,J$ $E_{ki} = 200\,kJ$
Calculate the final kinetic energy of the joined vehicles.	$E_{kf} = \frac{1}{2}mv^2$ $E_{kf} = \frac{1}{2}(5.00\times10^3+1.00\times10^3)(3.3333)^2$ $E_{kf} = 33\,332.7\,J$ $E_{kf} = 33.3\,kJ$
Compare the kinetic energy before and after the collision to determine whether or not the collision is elastic.	The kinetic energy after the collision is significantly less than the kinetic energy before. The missing energy has been transformed to heat, sound and deformation of the vehicles. Therefore, this collision is inelastic.

Worked example: Try yourself 5.5.1

ELASTIC OR INELASTIC COLLISION?

A 162 g snooker ball with initial velocity $9.00\,m\,s^{-1}$ to the right collides with a stationary snooker ball of mass 145 g. After the collision, both balls are moving to the right and the 162 g ball has a speed of $3.00\,m\,s^{-1}$. Show calculations to test whether or not the collision is elastic.

EXTENSION

Spin

Although perfectly elastic collisions only occur on the sub-microscopic scale (e.g. between atoms or subatomic particles), some everyday collisions are nearly elastic, making their outcomes predictable. For example, a collision between two billiard balls generates very little heat or sound and is nearly elastic (Figure 5.5.2).

FIGURE 5.5.2 Pool, billiards and snooker are all games that involve nearly elastic collisions between balls.

This implies that when a moving billiard ball strikes a stationary billiard ball, all of the momentum and kinetic energy should ideally be transferred. As a result, the moving ball should stop, and the stationary ball should move off with a velocity close to the initial velocity of the ball that struck it (Figure 5.5.3).

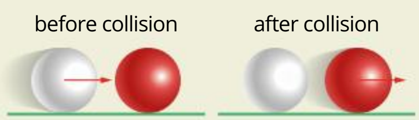


FIGURE 5.5.3 Since collisions between billiard balls are nearly elastic, both momentum and kinetic energy are transferred from the moving ball to the stationary ball.

In practice, however, a wider variety of outcomes is possible from a billiard-ball collision. Skilled billiards players can strike the cue ball (the white ball) in ways that can cause it to either 'follow' (i.e. continue rolling in its original direction after the collision; Figure 5.5.4) or to 'draw' (i.e. move backwards after the collision; Figure 5.5.5).

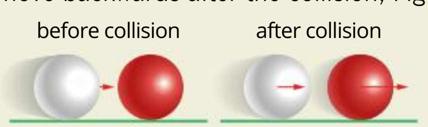


FIGURE 5.5.4 Billiards players can strike the cue ball so that, after the collision, it 'follows' the ball that it has struck.

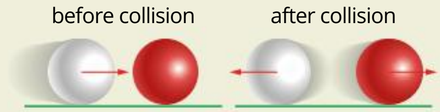


FIGURE 5.5.5 In a 'draw' shot, the player strikes the cue ball so that after the collision, it bounces back in the opposite direction.

These collisions might seem to violate the law of conservation of energy because it appears that kinetic energy is either created or destroyed. However, this is not the case. These sorts of shots are possible because there is more than one type of kinetic energy involved.

The energy that moves a ball from one spot on the table to another is known as translational kinetic energy. However, by hitting the cue ball slightly higher than its centre of mass, a billiards player can give the ball some topspin (Figure 5.5.6). This gives the ball some rotational kinetic energy, or energy of rotation, in addition to its translational kinetic energy. When the cue ball collides with another ball, some of the rotational kinetic energy is converted into translational kinetic energy, causing the cue ball to follow the ball it has struck.

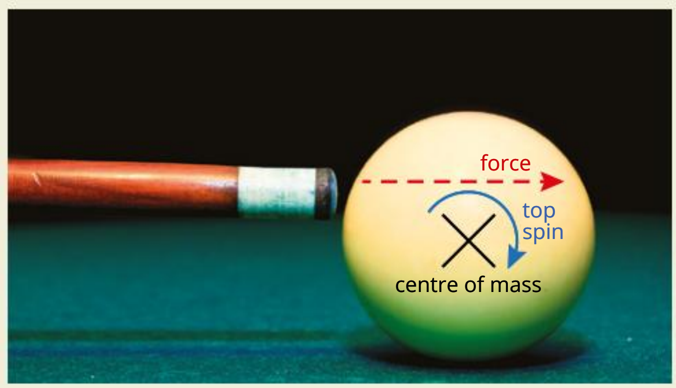


FIGURE 5.5.6 A billiards player can hit the cue ball slightly above its centre of mass to give it some topspin.

Similarly, by striking the cue ball below its centre of mass, the player can give a back pin to the ball. This rotational kinetic energy will cause the ball to move backwards after the collision.

Imparting spin to a ball is a common technique in many sports. The extra rotational energy that this gives the ball is not always obvious to other competitors, but it can cause the ball to move or bounce in unexpected ways.

5.5 Review

SUMMARY

- Kinetic energy is the energy of motion of a body:
  $E_k = \frac{1}{2}mv^2$
- The total energy of an isolated system is always conserved.
- For perfectly elastic collisions, the kinetic energy before the collision is equal to the kinetic energy after the collision.
  $\Sigma\frac{1}{2}mv_i^2 = \Sigma\frac{1}{2}mv_f^2$
- For inelastic collisions, the kinetic energy before the collision is greater than the kinetic energy after the collision; some of the initial kinetic energy is converted to other forms of energy such as heat or sound.

KEY QUESTIONS

1 Explain the difference between an elastic and an inelastic collision and why an inelastic collision does not violate the law of conservation of energy.

2 A car of mass 1500 kg travelling east at 12.0 $m\,s^{-1}$ collides head-on into a 2500 kg ute travelling at 16.0 $m\,s^{-1}$ in the opposite direction. The vehicles lock together on impact.

  a Calculate the velocity of the wreckage after the collision.

  b Determine whether the collision is elastic or inelastic.

3 In a Newton's cradle, one ball is dropped so that it has a velocity of 1.55 $m\,s^{-1}$ as it collides with the other four stationary balls. After the collision all five balls move together with the same speed. Use the law of conservation of momentum to determine the speed of the five balls after the collision and identify the collision as elastic or inelastic. Each ball has a mass of 42.5 g.

The following information applies to questions 4–7.

A 214 g toy truck with a springy bumper travelling at 0.300 $m\,s^{-1}$ collides with a 128 g toy car travelling in the same direction at 0.200 $m\,s^{-1}$. The toy car moves forward travelling at an increased speed of 0.300 s^{-1}.

4 Calculate the speed of the truck after the collision.

5 Calculate the total kinetic energy of the system before the collision.

6 Calculate the total kinetic energy of the system after the collision.

7 Complete the following statements by selecting the appropriate option from each set of brackets.

  a The total kinetic energy before the collision is [more than/less than/equal to] the total kinetic energy after the collision.

  b The kinetic energy of the system of toys [is/is not] conserved.

  c The total energy of the system of toys [is/is not] conserved.

  d The total momentum of the system of toys [is/is not] conserved.

  e The collision [is/is not] perfectly elastic because [kinetic energy/total energy/momentum] [is/is not] conserved.

5.6 Power

In physics, it is instructive to look not only at absolute amounts of quantities but also the rate at which these quantities change. Energy, in particular, is a quantity whose rate of change affects all aspects of society.

When considering energy changes, the rate at which work is done is often important. For example, if two cars have the same mass, the amount of work required to accelerate each car from stationary to $100\,\mathrm{km\,h^{-1}}$ will be the same. However, the fact that one car can do this more quickly than the other may be a significant factor for some drivers when choosing which car to buy.

Physicists use the concept of power to describe the rate at which work is done. Like work and energy, this word takes on a specific meaning in a scientific context.

DEFINING POWER

Power is a measure of the rate at which work is done. Mathematically:

$$P = \frac{W}{\Delta t}$$

Recall that when work is done, energy is transferred or transformed. So the equation can also be written as:

$$P = \frac{\Delta E}{\Delta t}$$

where P is the power (W)

ΔE is the energy transferred or transformed (J)

Δt is the time taken (s).

For example, a person running up a set of stairs does exactly the same amount of work as if they had walked up the stairs (i.e. $W = mg\Delta h$). However, the rate of energy change is faster for running up the stairs compared to walking. Therefore, the runner is applying more power than the walker (Figure 5.6.1).

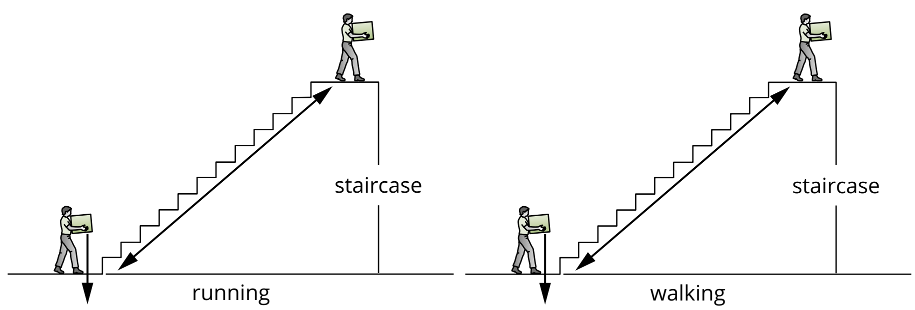


FIGURE 5.6.1 The runner and the walker both do the same amount of work, but the power output of the runner is higher than that of the walker.

Unit of power

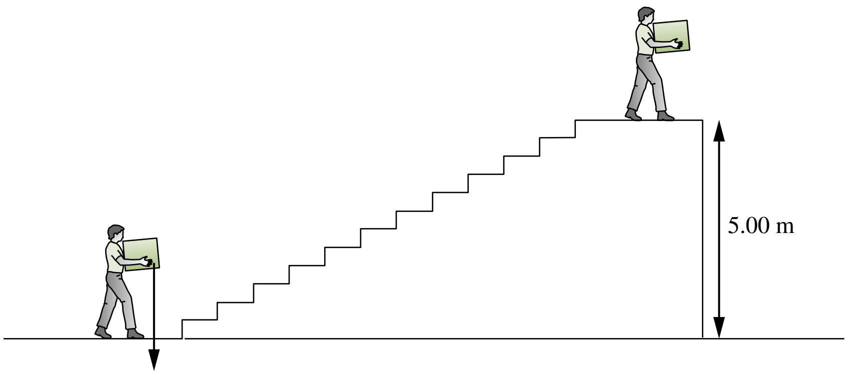
The unit of power is named after the Scottish engineer James Watt, who is most famous for inventing the steam engine. A watt (W) is defined as a rate of work of one joule per second, in other words:

$$1\,\mathrm{W} = \frac{1\,\mathrm{J}}{1\,\mathrm{s}} = 1\,\mathrm{J\,s^{-1}}$$

Worked example 5.6.1

CALCULATING POWER

Calculate the power required to carry a box with a mass of 2.00 kg up a 5.00 m staircase in 20.0 s. (Use $g = 9.80\,\text{m}\,\text{s}^{-2}$.)



Thinking	Working
Calculate the force applied.	$F_P = mg$ $F_P = (2.00)(9.80)$ $F_P = 19.6\,\text{N}$
Calculate the work done.	$W = Fs$ $W = (19.6)(5.00)$ $W = 98.0\,\text{J}$
Recall the formula for power.	$P = \frac{W}{\Delta t}$ or $P = \frac{\Delta E}{\Delta t}$
Substitute the appropriate values into the formula.	$P = \frac{(98.0)}{(20.0)}$
Solve.	$P = 4.90\,\text{W}$

Worked example: Try yourself 5.6.1

CALCULATING POWER

Calculate the power used by a weightlifter to lift a barbell that has a total mass of 50.5 kg from the floor to a height of 2.10 m above the ground in 1.45 s. (Use $g = 9.80\,\text{m}\,\text{s}^{-2}$.)

POWER, FORCE AND AVERAGE SPEED

In many everyday situations, a force is applied to an object to keep it moving at a constant speed, such as pushing a wardrobe across a carpeted floor or driving a car at a constant speed. In these situations, the power being applied can be calculated directly from the force applied and the speed of the object.

Since $P = \frac{W}{\Delta t}$ and $W = Fs$, then:

$$P = \frac{Fs}{\Delta t}$$

$$P = F \times \frac{s}{\Delta t}$$

Since $\frac{s}{\Delta t}$ is the definition of average speed, the power equation can be written as:

$$P = Fv_{av}$$

PHYSICSFILE

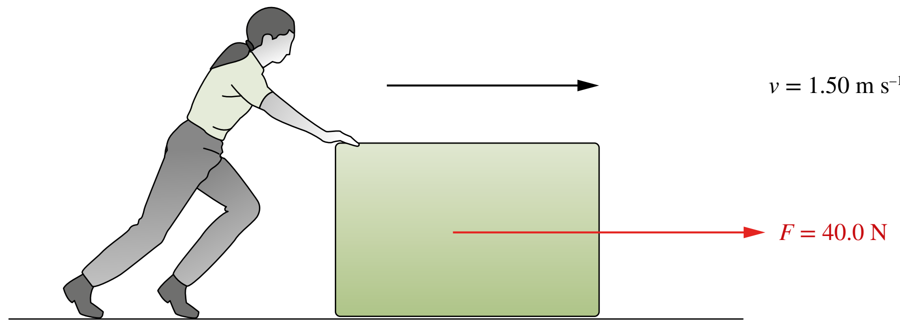
Horsepower

James Watt was a Scottish inventor and engineer. He developed the concept of horsepower as a way to compare the output of steam engines with that of horses, which were the other major source of mechanical energy available at the time. Although the unit of one horsepower (1 hp) has had various definitions over time, the most commonly accepted value today is around 750 W. This is actually a significantly higher amount than an average horse can sustain over an extended period of time.

Worked example 5.6.2

FORCE–VELOCITY FORMULATION OF POWER

A person pushes a heavy box along the ground at an average speed of 1.50 m s^{-1} by applying a force of 40.0 N. What amount of power does the person exert on the box?



Thinking	Working
Recall the force–velocity formulation of the power equation.	$P = Fv_{av}$
Substitute the appropriate values into the formula.	$P = (40.0)(1.50)$
Solve.	$P = 60.0$ W

Worked example: Try yourself 5.6.2

FORCE–VELOCITY FORMULATION OF POWER

Calculate the power required to keep a car moving at an average speed of 22.5 m s^{-1} if the force of friction (including air resistance) is 1250 N. Give your answer correct to three significant figures.

5.6 Review

SUMMARY

- Power is a measure of the rate at which work is done:
  $P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$
- The power required to keep an object moving at a constant speed can be calculated from the product of the force applied and its average speed: $P = Fv_{av}$.

KEY QUESTIONS

1. A 1610 kg car accelerates from 0.00 km h^{-1} to 100 km h^{-1} in 5.50 s. Calculate its average power output over this time.
2. A locomotive engine applies a force of 4.02 kN to keep a train moving at 22.0 m s^{-1}. Calculate the power output of the engine.
3. A 1780 kg car has an engine that uses 43.5 kW of power to maintain a constant speed of 80.0 km h^{-1}. Calculate the force being applied by this engine.
4. The motor of a crane has a maximum power output of 25.0 kW. At what average speed could it lift a concrete slab with a mass of 518 kg?
5. A box is pushed along a frictionless surface with a force of 15.0 N for 26.0 m in 14.5 s. Calculate the power output to push the box along.
6. A weightlifter lifts a 40.0 kg mass vertically upwards by 1.50 m. Given that this movement took 10.0 s to complete, what was the power generated by the weightlifter?
7. An 800.0 kg race car has an engine that can generate 120 kW of power. How long does it take the car to accelerate from 40.0 m s^{-1} to 55.0 m s^{-1}?

Chapter review

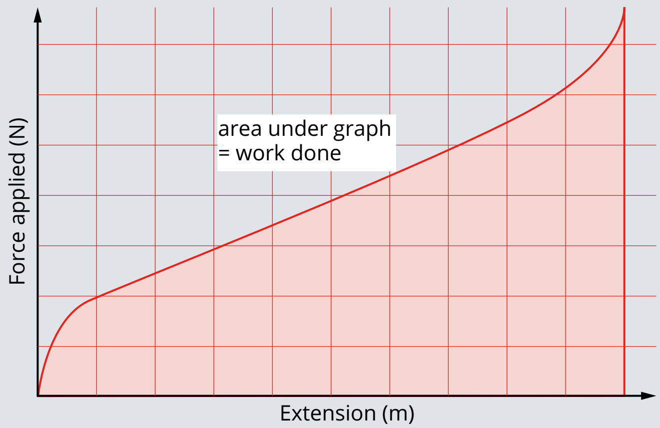
05

KEY TERMS

conservation of energy
conservation of mechanical energy
efficiency
elastic
elastic collision
gravitational potential energy
inelastic collision
kinetic energy
mechanical energy
power
transform
work

1 A car drives at a constant speed for 82.5 m. In order to overcome friction, its engine applies a force of 2.01 kN. Calculate the work done by the engine.

2 Estimate the total work shown in the following force–displacement graph, given that each grid square corresponds to 1.00 J.



3 A crane lifts a 235 kg load from the ground to a height of 30.0 m. What is the work done by the crane?

4 A person walks up a flight of 12 stairs. Each step is 240 mm long and 165 mm high. If the person has a mass of 60.0 kg and $g = 9.80\,\text{m}\,\text{s}^{-2}$, what is the total amount of work done against gravity?

5 If 4.25 kJ is used to lift a 50.0 kg object with a constant velocity, what is the theoretical maximum height to which the object can be raised?

6 A pram is pushed by the handle, which is at an angle of 35.0° to the horizontal (the direction of motion). If 1250 J of work is done pushing the pram 20.0 m, with what force was the pram pushed?

7 A person bowls a cricket ball with a mass of 162 g at a speed of 155 km h^{-1}. What is the kinetic energy of the cricket ball?

8 If a 1220 kg car has kinetic energy of 70.0 kJ, what is its speed?

9 The speed of an object is doubled. By how much does its kinetic energy increase?

10 A plumber (mass 88.3 kg) digs a ditch 42.5 cm deep. By how much does the plumber's gravitational potential energy change when they step from the ground down into the ditch?

11 A player kicks a soccer ball with a mass of 0.430 kg off the ground with a speed of 16.5 m s^{-1}. How fast will the ball be going when it hits the crossbar (the horizontal bar that connects the two vertical goal posts), which is 2.44 m above the ground?

12 A white billiard ball moving with a speed of 5.00 m s^{-1} strikes a stationary red billiard ball. After the collision, the white ball continues in the same direction at a speed of 1.00 m s^{-1}. The red ball rolls ahead of it (in the direction of the white ball's initial motion) with a speed of 4.00 m s^{-1}. Each ball has a mass of 163 g. Determine the energy of the objects both before and after the collision, and state whether the collision was elastic or inelastic.

13 A bullet of mass 20.0 g strikes a ballistics pendulum of mass 1.50 kg with speed v and becomes embedded in the pendulum. When the pendulum swings back, its height increases by 15.7 cm. For the questions that follow, assume that the initial gravitational potential energy of the pendulum was zero.

- **a** What was the gravitational potential energy of the pendulum at the top of its swing?
- **b** What was the kinetic energy of the pendulum when the bullet first became embedded in it?
- **c** What was the speed of the pendulum when it first started to swing?

14 A crane can lift a load of 5.20 tonnes vertically through a distance of 20.0 m in 5.00 s. What is the approximate power of the crane?

15 A Mini Cooper with a mass of 650 kg can accelerate from 0 to 100 km h^{-1} in 7.20 s. What is the average power output of the car over this time?

16 If the engine of a 1480 kg car uses 25.5 kW to maintain an average speed of 17.8 $m\,s^{-1}$, how much friction is acting on the car?

17 At the start of a 100 m race, a runner with a mass of 63.8 kg accelerates from a standing start to 8.00 $m\,s^{-1}$ in a distance of 20.0 m.

a Calculate the work done by the runner's legs.

b Calculate the average force that the runner's legs apply over this distance.

18 When moving around on the Moon, astronauts find it easier to use a series of small jumps rather than to walk. If an astronaut (with a mass of 129 kg including equipment) jumps to a height of 10.0 cm on the Moon, where the gravitational field strength is 1.62 $m\,s^{-2}$, by roughly how much does the astronaut's potential energy increase?

19 The efficiency of an appliance is known to be 80.0%. What energy was supplied if the output was 1250 J?

CHAPTER

06 Heating processes

Thermal energy is part of everyday experience. It's the thermal energy from the Sun that makes your world habitable. Humans can thrive in the climatic extremes of the Earth, from the outback deserts to ski slopes in winter. In this chapter, the nature of thermal energy is explored. Specifically, by the end of this chapter, you will have covered material relating to heating processes in the following areas:

- kinetic particle theory
- temperature
- internal energy
- specific heat capacity
- latent heat
- thermal equilibrium.

Science as a Human Endeavour

- passive solar design for heating and cooling of buildings
- the operation of a refrigerator and reverse-cycle air conditioner
- the use of the Sun for heating water
- engine cooling systems in cars

Science Understanding

- kinetic theory describes matter as consisting of particles in continuous motion, except at the temperature of absolute zero
- all substances have internal energy due to the motion and separation of their particles
- temperature is a measure of the average translational kinetic energy of particles in a system
- provided a substance does not change state, its temperature change is proportional to the amount of energy added to or removed from the substance; the constant of proportionality describes the heat capacity of the substance, including applying the relationship:

  $Q = mc\Delta T$

- change of state involves separating particles that exert attractive forces on each other; latent heat is the energy required to be added to, or removed from, a system to change the state of the system, including applying the relationship:

  $Q = mL$

- two systems in contact transfer energy between particles so that eventually the systems reach the same temperature; that is, they are in thermal equilibrium. This may involve changes of state as well as changes in temperature.

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.

6.1 Heat and temperature

In the sixteenth century, Sir Francis Bacon, an English essayist and philosopher, proposed a radical idea: that heat is motion. He went on to write that heat is the rapid vibration of tiny particles within every substance. At the time, his ideas were dismissed because the nature of particles wasn't fully understood. An opposing theory at the time was that heat was related to the movement of a fluid called 'caloric' that filled the spaces within a substance.

Today, it is understood that all matter is made up of small particles (atoms or molecules). Using this knowledge, it is possible to look more closely at what happens during heating processes.

This section starts by looking at the kinetic particle model, which states that the small particles (atoms or molecules) that make up all matter have kinetic energy. Therefore all particles are in constant motion, even in extremely cold solids. It was thought centuries ago that if a material was continually made cooler, there would be a point at which the particles would eventually stop moving. This coldest possible temperature is called **absolute zero** and will be discussed later in this section.

KINETIC PARTICLE MODEL

Some philosophers of the Middle Ages believed that heat was a fluid that filled the spaces between the particles of a substance and flowed from one substance to another. This is known as the 'caloric' theory. When caloric flowed from one substance into another, the first object cooled down and the second object heated up. Many attempts were made to detect caloric, but none were successful. It was assumed that caloric had no mass, odour, taste or colour. Scientists now know that caloric simply doesn't exist.

The best understanding of the behaviour of matter today depends on a model called the **kinetic particle model** (kinetic theory). A model is a representation that describes or explains the workings within an object, system or idea. This will generally include making some assumptions. The assumptions behind the kinetic particle model are as follows:

- All matter is made up of many very small particles (atoms or molecules).
- The particles are in constant motion.
- Overall, no kinetic energy is lost or gained during elastic collisions between particles.
- There are forces of attraction and repulsion between the particles in a material.
- The distances between particles in a gas are large compared with the size of the particles.

The kinetic theory applies to all states (or phases) of matter including the three you most commonly come across in your everyday activities: solids, liquids and gases.

Solids

Within a solid, the particles must be exerting attractive forces or bonds on each other for the matter to hold together in its fixed shape. There must also be repulsive forces, without which the attractive forces would cause the solid to collapse. In a solid, the attractive and repulsive forces hold these particles in fixed positions, usually in a regular arrangement or lattice (see Figure 6.1.1a). But the particles in a solid are not completely still; they vibrate around average positions. The forces on individual particles are sometimes predominantly attractive and sometimes repulsive, depending on their exact position relative to neighbouring particles.

Liquids

Within a liquid, there is still a balance of attractive and repulsive forces. Compared with a solid, the particles in a liquid have more freedom to move around each other and will therefore take the shape of the container (see Figure 6.1.1b). Generally, the liquid takes up a slightly greater volume than it would in the solid state. Particles collide but remain attracted to each other, so the liquid remains within a fixed volume but with no fixed shape.

Gases

In a gas, particles are in constant, random motion, colliding with each other and the walls of the container. The particles move rapidly in every direction, quickly filling the volume of any container, and occasionally colliding with each other (see Figure 6.1.1c). A gas has no fixed volume. The particle speeds are high enough that, when the particles collide, the attractive forces are not strong enough to keep the particles close together. The repulsive forces cause the particles to separate and move off in other directions.

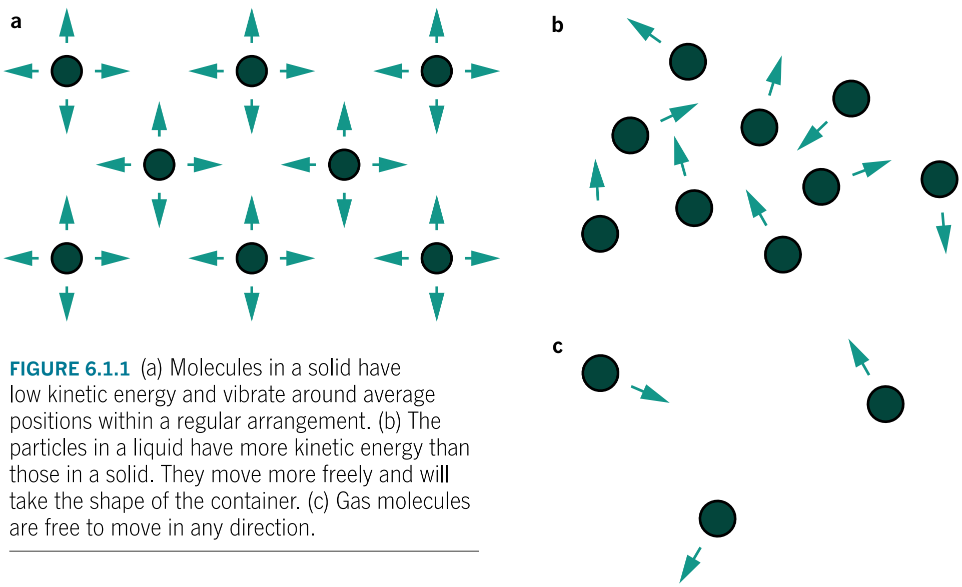


FIGURE 6.1.1 (a) Molecules in a solid have low kinetic energy and vibrate around average positions within a regular arrangement. (b) The particles in a liquid have more kinetic energy than those in a solid. They move more freely and will take the shape of the container. (c) Gas molecules are free to move in any direction.

PHYSICSFILE

States of matter: Plasma

The three phases (states) of matter that you normally come across are solid, liquid and gas. These are generally all that are discussed in secondary science.

There are in fact four phases of matter that are observable in everyday life, that is solid, liquid, gas and plasma. Plasma exists when matter is heated to very high temperatures and electrons are freed (ionisation). A gas that is ionised and has an equal number of positive and negative charges is called plasma. The interior of stars consists of plasma. In fact, most of the matter in the universe is plasma (Figure 6.1.2).

The fifth known state of matter is known as a Bose–Einstein condensate. This is a state of matter that exists very close to absolute zero, a point at which molecular motion almost stops. Atoms begin to clump together and matter exhibits many of the properties of a super fluid; that is, it flows without friction.

FIGURE 6.1.2 99.9 per cent of the visible universe is made up of plasma.

KINETIC PARTICLE MODEL, INTERNAL ENERGY AND TEMPERATURE

The kinetic particle model can be used to explain the idea of heat as a transfer of energy. **Heat** (measured in joules) is the transfer of **thermal energy** from a hotter body to a colder one. Heating is observed by the change in **temperature**, the change of state or the expansion of a substance.

When a solid substance is 'heated', the particles within the material gain either **kinetic energy** (i.e. they move faster) or **potential energy** (the energy stored in the bonds between particles).

The term heat refers to energy that is being transferred (moved). So it is incorrect to talk about heat contained in a substance. The term **internal energy** refers to the total kinetic and potential energy of the particles within a substance. Heating (the transfer of thermal energy) changes the internal energy of a substance by affecting the kinetic energy and/or potential energy of the particles within the substance. The movement of the whole object due to kinetic energy is ordered: the object moves back and forth and its behaviour can be modelled. In comparison, the internal energy of a system is associated with the rapid and chaotic motion of the particles, it concerns the behaviour of a large number of particles that all have their own kinetic and potential energy.

- Heating is a process that always transfers thermal energy from a hotter substance to a colder substance.
- Heat is measured in joules (J).
- Temperature is related to the average kinetic energy of the particles in the substance. The faster the particles move, the higher the temperature of the substance.

PHYSICS IN ACTION

Energy

Energy is a very important concept in the study of the physical world, and is a focus in all areas of scientific study. Later chapters investigate energy in more detail.

Energy is a measure of an object's ability to do work. For example, raising an object's temperature or lifting an object is referred to as doing work. Work is measured in joules. The symbol for joules is J. Figure 6.1.3 shows the amount of joules available from some energy sources.

Kinetic energy is the energy of movement. It is equal to the amount of work needed to bring an object from rest to its present speed or to return it to rest. Potential energy is stored energy. There are many forms of potential energy, for example gravitational, nuclear, elastic and chemical. Chemical potential energy is associated with the bonds between the particles of a substance. An increase in the potential energy of particles in a substance results in movement of the particles from their equilibrium positions.

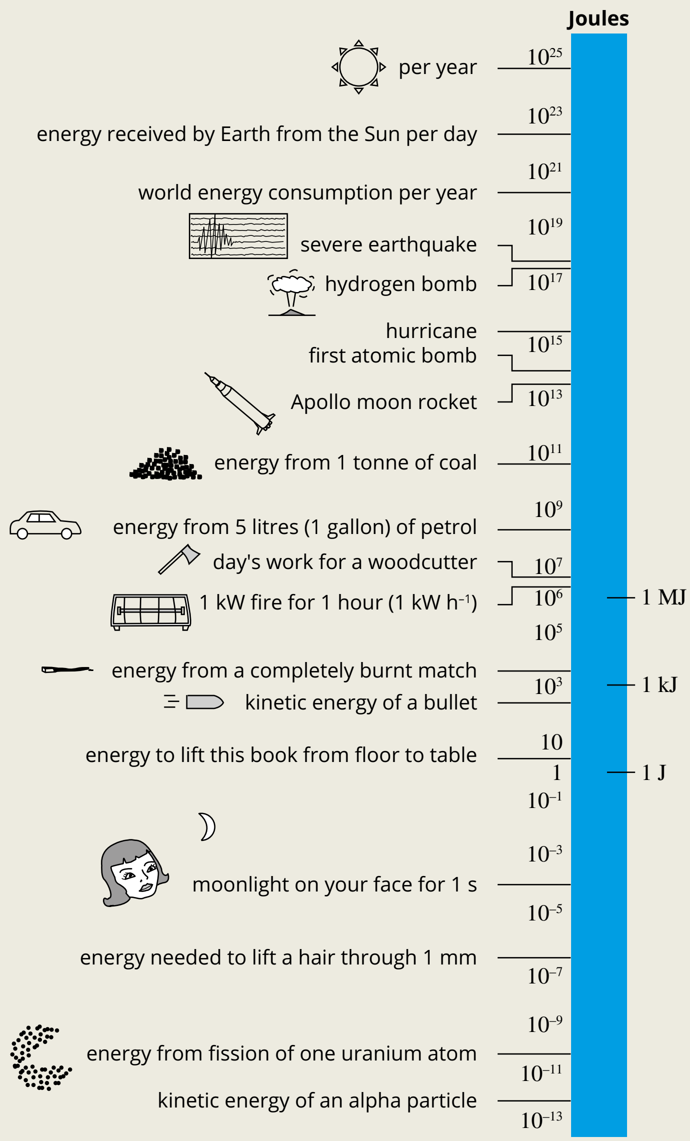


FIGURE 6.1.3 The comparative amounts of energy available from different sources.

Using the kinetic particle model, an increase in the total internal energy of the particles in a substance will result in an increase in temperature if there is a net gain in kinetic energy. Hot air balloons are an example of this process in action. The air in a hot air balloon is heated by a gas burner to a maximum of 120°C. The nitrogen (78%) and oxygen (21%) molecules in the hot air gain energy and so move a lot faster. The air in the balloon becomes less dense than the surrounding air, causing the balloon to float as seen in Figure 6.1.4.

Sometimes heating results only in the change of state or expansion of an object, and not a change in temperature. In these cases, the total internal energy of the particles has still increased but only the potential energy has increased, not the kinetic energy.

For instance, particles in a solid being heated will continue to be mostly held in place, due to the relatively strong interparticle forces. For the substance to change state from solid to liquid, it must receive enough energy to separate the particles from each other and disrupt the regular arrangement of the solid. During this 'phase change' process, the energy is used to overcome the strong interparticle forces, but not to change the overall speed of the particles. In this situation, the temperature does not change. This will be discussed in more detail in Section 6.3 'Latent heat'.

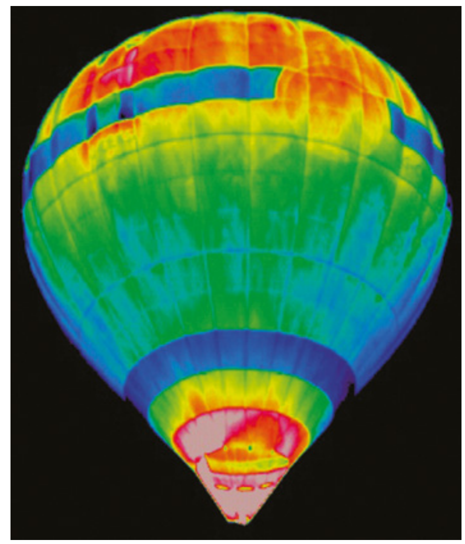


FIGURE 6.1.4 (a) Nitrogen and oxygen molecules gain energy when the air is heated, lowering the density of the air and causing the hot air balloon to rise off the ground. (b) A thermal image shows the temperature of the air inside the balloon, with the hotter areas showing as red.

MEASURING TEMPERATURE

Only four centuries ago, there were no thermometers and people described heating effects by vague terms such as hot, cold and lukewarm. In about 1593, Italian inventor Galileo Galilei made one of the first thermometers. His 'thermoscope' was not particularly accurate as it did not take into account changes in air pressure, but it did suggest some basic principles for determining a suitable scale of measurement. His work used two fixed points: the hottest day of summer and the coldest day of winter. A scale like this is referred to as an arbitrary scale, because the fixed points are randomly chosen.

Celsius and Fahrenheit scales

Two of the better known arbitrary temperature scales are the Fahrenheit and Celsius scales. Gabriel Fahrenheit of Germany invented the first mercury thermometer in 1714. While Fahrenheit is still used in the United States of America to measure temperature, the system used in most countries of the world is now the Celsius scale. Two well-known fixed points for the Celsius scale are the standard freezing point of water, 0 degrees Celsius, and the boiling point of water, 100 degrees Celsius (at 1 atmosphere pressure).

Kelvin scale

Absolute scales are different from arbitrary scales. For a scale to be regarded as 'absolute', it should have no negative values. The fixed points must be reproducible and have zero as the lowest value. The kelvin scale for measuring temperature is an example of an absolute scale.

In developing the absolute temperature scale, the triple point of water provided one reliable fixed point. This is a point where the combination of temperature and air pressure allows all three states of water to coexist. For water, the triple point is only slightly above the standard freezing point at approximately 0.01°C and is the standard fixed point temperature that is used to calibrate thermometers.

Absolute zero

Experiments indicate that there is a limit to how cold things can get. The kinetic theory suggests that if a given quantity of gas is cooled, its volume decreases. The volume can be plotted against temperature and results in a straight-line graph as shown in Figure 6.1.5. Extrapolating (extending) the line to where the volume is zero gives a theoretical value of absolute zero.

PHYSICSFILE

Unit conventions in physics

The unit for energy, the joule, is named after James Joule in recognition of his work. When a unit is named after a person, its symbol is usually a capital letter but the unit name is always lower case, e.g. joule (J) is named after James Joule, newton (N) is named after Isaac Newton, and kelvin (K) is named after Lord Kelvin.

Exceptions are degrees Celsius (°C) and degrees Fahrenheit (°F), which also include a degree symbol and have a capital letter for the unit.

Units not named after people usually have both the symbol and the name in lowercase, e.g. metre (m), second (s).

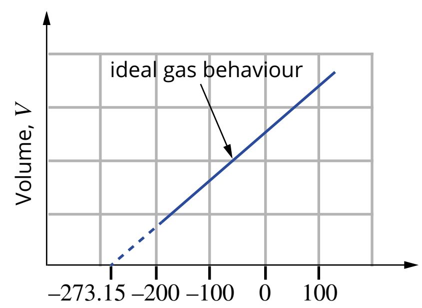


FIGURE 6.1.5 Gases have smaller volumes as they cool. This relationship is linear.

PHYSICSFILE

Close to absolute zero

As temperatures get close to absolute zero, atoms start to behave in weird ways. Since the French physicist Guillaume Amontons first proposed the idea of an absolute lowest temperature in 1699, physicists have theorised about the effects of such a temperature and how it could be achieved. The laws of physics dictate that absolute zero itself can be approached but not reached. The lowest temperature produced in a laboratory was achieved by a group of researchers from the QUANTUS Team in Bremen, Germany on 30 August 2021. The scientists cooled rubidium atoms to 38 picoKelvins (38×10^{-12} K). At this temperature, all elementary particles merge into a single state (a Bose–Einstein condensate), losing their separate properties and behaving as a single 'super atom', a state first proposed by Einstein 70 years earlier.

- Absolute zero = 0 K = –273.15°C
- All molecular motion ceases upon reaching absolute zero. This is the coldest temperature possible.

The absolute or **kelvin** temperature scale is based on absolute zero and the triple point of water. Refer to Figure 6.1.6 for a comparison of the kelvin and Celsius scales.

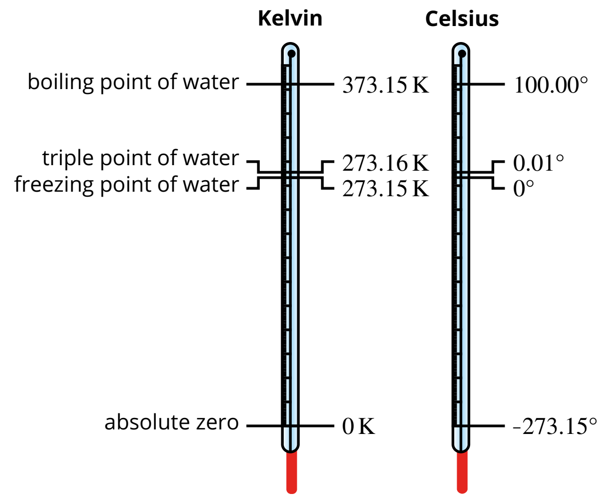


FIGURE 6.1.6 Comparison of the kelvin and Celsius scales. Note that there are no negative values on the kelvin scale.

- The freezing point of water (0°C) is equivalent to 273.15 K (kelvin). This is often approximated to 273 K.
- The size of each unit, 1°C or 1 K, is the same.
- The word 'degree' and the degree symbol are not used with the kelvin scale.
- To convert a temperature from degrees Celsius to kelvin: add 273.15.
- To convert a temperature from kelvin to degrees Celsius: subtract 273.15.
- 0°C (273.15 K) is the freezing point of water at standard atmospheric pressure.
- 100°C (373.15 K) is the boiling point of water at standard atmospheric pressure.
- Absolute zero is called 'zero kelvin' (0 K) and is equal to –273.15°C.

THERMAL EQUILIBRIUM AND ENERGY TRANSFER

Whenever two materials, initially at different temperatures, come into contact (or are mixed) then thermal energy will transfer from the hotter material to the colder one until both are the same temperature. For example, if a piece of frozen fruit is placed in a container of warm water, energy is transferred from the water to the fruit. The fruit gains energy and warms up. The water loses energy and cools down. Eventually the transfer of energy between the fruit and water will stop. This point is called **thermal equilibrium,** and the fruit and water will be at the same temperature. Two objects in thermal equilibrium with each other must be at the same temperature. This is referred to as the zeroth law of thermodynamics.

In this example, it's likely that the fruit will no longer be frozen. Moving to a temperature where thermal equilibrium occurs has involved not only a change in temperature but also a change of state. This is true of any thermal energy transfer where systems or objects in contact reach the same temperature. The energy transfer may involve changes of state as well as a change in temperature. Assuming no energy is lost from the systems, the total thermal energy will remain the same.

6.1 Review

SUMMARY

- The kinetic particle theory proposes that all matter is made of atoms or molecules (particles) that are in continuous motion.
- In solids, the attractive and repulsive forces hold the particles in fixed positions, usually in a regular arrangement or lattice. These particles are not completely still—they vibrate about average positions.
- In liquids, there is still a balance of attractive and repulsive forces between particles but the particles have more freedom to move around. Liquids maintain a fixed volume.
- In gases, the particle speeds are high enough that, when particles collide, the attractive forces are not strong enough to keep them close together. The repulsive forces cause the particles to move off in other directions.
- Internal energy refers to the total kinetic and potential energy of the particles within a substance.
- Temperature is related to the average kinetic energy of the particles in a substance.
- Heating is a process that always transfers thermal energy from a hotter substance to a colder one.
- Temperatures can be measured in degrees Celsius (°C) or kelvin (K).
- Absolute zero is called 'zero kelvin' (0 K) and it is equal to –273.15°C.
- The size of each increment of temperature is the same—an increase of 1°C is equal to an increase of 1 K.
- Whenever two materials initially at different temperatures come into contact (or are mixed), thermal energy will transfer from the hotter material to the colder one until both are the same temperature. This is referred to as thermal equilibrium.

KEY QUESTIONS

1 Which of the following is true of a solid?
  A Particles are moving around freely.
  B Particles are not moving.
  C Particles are vibrating in continuous motion.
  D A solid is not made up of particles.

2 An uncooked chicken is placed into an oven that has been preheated to 180°C. Order the following statements to describe what happens after the chicken is placed in the oven.
- The chicken and the air in the oven are in thermal equilibrium.
- Thermal energy flows from the hot air into the chicken.
- The chicken and the air in the oven are not in thermal equilibrium.

3 Which of the following temperature(s) cannot possibly exist? (More than one answer is possible.)
  A 1 000 000°C **B** –50°C **C** –50 K **D** –300°C

4 A tank of pure helium is cooled to its freezing point of –272.2°C. Describe the energy of the helium particles at this temperature.

5 Convert the following temperatures:
  a 30°C into kelvin
  b 375 K into degrees Celsius.

6 Tank A is filled with hydrogen gas at 0°C and another tank, B, is filled with hydrogen gas at 300 K. Describe the difference in the average kinetic energy of the hydrogen particles in each tank.

7 Sort the following temperatures from coldest to hottest:
- freezing point of water
- 100 K
- absolute zero
- –180 °C
- 10 K

6.2 Specific heat capacity

A small amount of water in a kettle will experience a greater change in temperature than a larger volume if heated for the same time. A metal object left in the sunshine gets hotter faster than a wooden object. Large heaters warm rooms faster than small ones.

These simple observations suggest that the mass, material, and the amount of energy transferred influence any change of temperature.

CHANGING TEMPERATURE

The temperature of a substance is a measure of the average kinetic energy of the particles inside the substance. To increase the temperature of the substance, the kinetic energy of its particles must increase. This happens when heat is transferred to that substance. The amount the temperature increases depends on several factors.

The greater the mass of a substance, the greater the energy required to change the kinetic energy of all the particles. So, the heat required to raise the temperature by a given amount is proportional to the mass of the substance.

$\Delta Q \propto m$

where ΔQ is the heat energy transferred in joules (J)

m is the mass of material being heated in kilograms (kg).

The more heat that is transferred to a substance, the more the temperature of that substance increases. The amount of energy transferred is therefore proportional to the change in temperature.

$\Delta Q \propto \Delta T$

where ΔT is the change in temperature in °C or K. In this book ΔT is defined as the highest temperature, T_h, minus the lowest temperature, T_l.

Heating experiments using different materials will confirm that these relationships hold true regardless of the material being heated. However, heating the same masses of different materials will show that the amount of energy required to heat a given mass of a material through a temperature change also depends on the nature of the material being heated. For example, a volume of water requires more energy to change its temperature by a given amount compared with the same volume of methylated spirits. For some materials, temperature change occurs more easily than for others.

The **specific heat capacity** of a material, c, is the amount of energy that must be transferred to change the temperature of 1 kg of the material by 1°C or 1 K.

Combining these observations, the amount of energy added to or removed from the substance is proportional to the change in its temperature, its mass and its **specific heat capacity** (provided a material does not change state). The specific heat capacity of a material changes when the material changes state.

As an equation:

$Q = mc\Delta T$

where Q is the heat energy transferred in joules (J)

m is the mass in kilograms (kg)

ΔT is the change in temperature $T_h - T_l$ in °C or K

c is the specific heat capacity of the material ($J\,kg^{-1}\,K^{-1}$).

Table 6.2.1 (on the next page) lists the specific heat capacities for some common materials. It also includes the average value for the human body, taking into account the various materials within the body and the percentage that each material contributes to the body's total mass.

TABLE 6.2.1 Approximate specific heat capacities of common substances.

Material	c ($J\,kg^{-1}\,K^{-1}$)
human body	2980
methanol	2140
air	1012
aluminium	897
glass	840
iron	449
copper	385
brass	370
lead	129
mercury	139.5
liquid water	4180

PHYSICSFILE

The mass of water

Since water is a familiar material, many of the examples in this section use it as the liquid being heated. The density of water decreases as the temperature increases. However, use 1 kg per litre for all calculations unless told otherwise.

Worked example 6.2.1

CALCULATIONS USING SPECIFIC HEAT CAPACITY

A hot water tank contains 135 L of water. Initially the water is at 20.0°C. Calculate the amount of energy that must be transferred to the water to raise the temperature to 70.0°C.

Thinking	Working
Calculate the mass of water. 1 L of water = 1 kg	Volume = 135 L So mass of water = 135 kg
ΔT = highest temperature – lowest temperature	$\Delta T = 70.0 - 20.0 = 50.0$°C
From Table 6.2.1, $c_{water} = 4180\,J\,kg^{-1}\,K^{-1}$. Use the equation $Q = mc\Delta T$.	$Q = mc\Delta T$ $Q = (135)(4180)(50.0)$ $Q = 28\,215\,000$ $Q = 2.82 \times 10^7\,J$

Worked example: Try yourself 6.2.1

CALCULATIONS USING SPECIFIC HEAT CAPACITY

A bath contains 75.0 L of water. Initially the water is at 50.0°C. Calculate the amount of energy that must be transferred from the water to cool the bath to 30.0°C.

Worked example 6.2.2

COMPARING SPECIFIC HEAT CAPACITIES

Different states of matter of the same substance have different specific heat capacities. What is the whole number ratio of the specific heat capacity of liquid water to that of ice?	
Thinking	**Working**
Table 6.2.1 lists the specific heat capacities of water in different states.	$c_{water} = 4180\,J\,kg^{-1}\,K^{-1}$ $c_{ice} = 2050\,J\,kg^{-1}\,K^{-1}$
Divide the specific heat of water by the specific heat of ice.	ratio $= \frac{c_{water}}{c_{ice}}$ ratio $= \frac{(4180)}{(2050)}$
Note that ratios have no units since the unit of each quantity is the same and cancels out.	ratio ≈ 2

Worked example: Try yourself 6.2.2

COMPARING SPECIFIC HEAT CAPACITIES

What is the whole number ratio of the specific heat capacity of liquid water to that of steam?

PHYSICSFILE

Specific heat capacity of water

One of the notable values in the table of specific heat capacities is the high value for water. It is 10 times, or an order of magnitude, higher than those of most metals listed. The specific heat capacity of water is higher than that of most common materials. As a result, water makes a very useful cooling and heat storage agent, and is used in areas such as generator cooling towers and car-engine radiators.

Life on Earth also depends on the specific heat capacity of water. About 70% of the Earth's surface is covered by water, and these water bodies can absorb large quantities of thermal energy without great changes in temperature. Oceans both heat up and cool down more slowly than the land areas next to them. This helps to maintain a relatively stable range of temperatures for life on Earth.

Scientists are now monitoring the temperatures of the deep oceans to determine how the ability of oceans to store large amounts of energy may affect climate change.

6.2 Review

SUMMARY

- When heat is transferred to or from a system or object, the temperature change depends upon the amount of energy transferred, the mass of the material(s) and the specific heat capacity of the material(s): $Q = mc\Delta T$

  where Q is the heat energy transferred in joules (J)

  m is the mass of material being heated in kilograms (kg)

  ΔT is the change in temperature (°C or K) defined as $T_{highest} - T_{lowest}$

  c is the specific heat capacity of the material ($J\,kg^{-1}\,K^{-1}$).
- A substance will have different specific heat capacities at different states (solid, liquid, gas).

KEY QUESTIONS

1 Equal masses of water and aluminium are heated through the same temperature range. Using the values of c from Table 6.2.1 on page 209, which material requires more energy to achieve this result?

2 Which has more thermal energy: 10.0 kg of iron at 20.0°C or 10.0 kg of aluminium at 20.0°C?

3 125 mL of water is heated to change its temperature from 15.0°C to 20.0°C. How much energy is transferred to the water to achieve this change in temperature?

4 155 mL of water is heated from 10.0°C to 50.0°C. What amount of energy is required for this temperature change to occur?

5 For a 1.00 kg block of aluminium, X J of energy are needed to raise the temperature by 10.0°C. How much energy, in J, is needed to raise the temperature by 20.0°C in terms of X?

6 Equal masses of aluminium and water absorb equal amounts of energy. What is the whole number ratio of the temperature rise of the aluminium to that of water?

7 Which one or more of the following statements about specific heat capacity is true?

A All materials have the same specific heat capacity when in solid form.

B The specific heat capacity of the liquid form of a material is different from those of the solid and gas forms.

C Good conductors of heat generally have high specific heat capacities.

D Specific heat capacity is independent of temperature.

8 If 4.00 kJ of energy is required to raise the temperature of 1.00 kg of paraffin by 2.00°C, how much energy (in kJ) is required to raise the temperature of 5.00 kg of paraffin by 1.00°C?

9 A cup holds 250 mL of water at 20.0°C. 10.5 kJ of heat energy is transferred to the water. What temperature does the water reach after the heat is transferred?

10 A block of iron is left to cool. After cooling for a short time, 13.2 kJ of energy has been transferred away from the block of iron and its temperature has decreased by 30.0°C. What is the mass of the block of iron?

6.3 Latent heat

If water is heated, its temperature will rise. If enough energy is transferred to the water, eventually the water will boil. The water changes state (from liquid to gas). The **latent heat** is the energy released or absorbed during a change of state. Latent means hidden or unseen. While a substance changes state, its temperature remains constant. The energy used in, for example, melting ice into water is hidden in the sense that the temperature doesn't rise while the change of state is occurring.

ENERGY AND CHANGE OF STATE

Figure 6.3.1 shows a heating curve for water, illustrating how the temperature of water changes as energy is added at a constant rate. Although the rate at which the energy is added is constant, the increase in temperature is not always constant. There are sections of increasing temperature, and sections where the temperature remains unchanged (the horizontal sections) while the material changes state. The temperature of the water remains constant during the change in state from ice to liquid water and again from liquid water to steam.

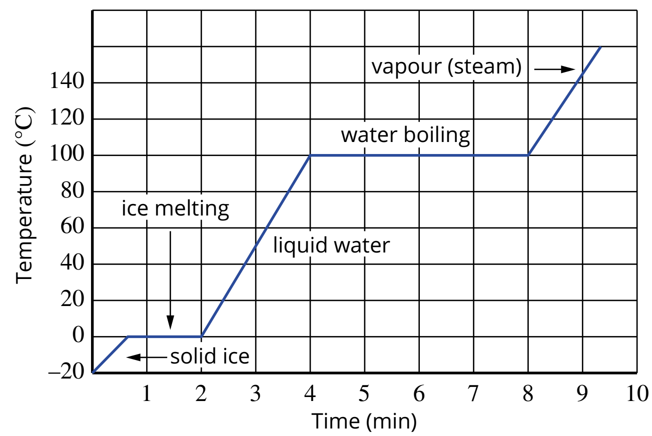


FIGURE 6.3.1 A heating curve for water.

Latent heat

The energy needed to change the state of a substance (e.g. solid to liquid, liquid to gas) is called latent heat. Latent heat is the 'hidden' energy that must be added or removed from a material in order for the material to change state.

The latent heat is calculated using the equation:

$Q = mL$

where Q is the heat energy transferred in joules (J)

m is the mass in kilograms (kg)

L is the latent heat (J kg^{-1}).

LATENT HEAT OF FUSION (MELTING)

As thermal energy is transferred to a solid, the temperature of the solid increases. The particles within the solid gain internal energy (as kinetic energy and some potential energy) and their speed of vibration increases. At the point where the solid begins to melt, the particles move further apart, reducing the strength of the bonds holding them in place. At this point, instead of increasing the temperature, the extra energy increases the potential energy of the particles, reducing the interparticle or intermolecular forces. No change in temperature occurs as all the extra energy supplied is used in reducing these forces between particles.

The amount of energy required to melt a solid is the same as the amount of potential energy released when the liquid re-forms into a solid. It is termed the **latent heat of fusion**.

The amount of energy required will depend on the particular solid.

For a given mass of a substance:

heat energy transferred = mass of substance × specific latent heat of fusion

$$Q = mL_{fusion}$$

where Q is the heat energy transferred in joules (J)

m is the mass in kilograms (kg)

L_{fusion} is the latent heat of fusion in $J\,kg^{-1}$.

It takes almost 80 times as much energy to turn 1 kg of ice into water (with no temperature change) as it does to raise the temperature of 1 kg of water by 1°C. It takes a lot more energy to overcome the large intermolecular forces within the ice than it does to simply add kinetic energy in raising the temperature.

The latent heats of fusion for some common materials are shown in Table 6.3.1.

TABLE 6.3.1 The latent heats of fusion for some common materials.

Substance	Melting point (°C)	L_{fusion} ($J\,kg^{-1}$)
water	0	3.34×10^5
oxygen	–219	1.37×10^4
lead	327	2.24×10^4
ethanol	–114	1.08×10^5
silver	961	1.04×10^5

Worked example 6.3.1

LATENT HEAT OF FUSION

How much energy must be removed from 2.50 L of water at 0.00°C to produce a block of ice at 0.00°C? Express your answer in kJ.

Thinking	Working
Cooling from liquid to solid involves the latent heat of fusion, where the energy is removed from the water. Calculate the mass of water involved.	1 L of water = 1 kg, so 2.50 L = 2.50 kg
Use Table 6.3.1 to find the latent heat of fusion for water.	$L_{fusion} = 3.34 \times 10^5\,J\,kg^{-1}$
Use the equation $Q = mL_{fusion}$	$Q = mL_{fusion}$ $Q = (2.50)(3.34 \times 10^5)$ $Q = 8.35 \times 10^5\,J$
Convert to kJ.	$Q = 8.35 \times 10^2\,kJ$

Worked example: Try yourself 6.3.1

LATENT HEAT OF FUSION

How much energy must be removed from 5.50 kg of liquid lead at 327°C to produce a block of solid lead at 327°C? Express your answer in kJ.

LATENT HEAT OF VAPORISATION (BOILING)

It takes much more energy to convert a liquid to a gas than it does to convert a solid to a liquid. This is because, to convert a liquid to a gas, the intermolecular bonds must be broken. During the change of state, the energy supplied is used solely in overcoming intermolecular bonds. The temperature will not rise until all the material in the liquid state is converted to a gas, assuming that the liquid is evenly heated. For example, when liquid water is heated to boiling point, a large amount of energy is required to change its state from liquid to steam (gas). The temperature will remain at 100°C until all the water has turned into steam. Once the water is completely converted to steam, then the temperature can start to rise again.

The amount of energy required to change a liquid to a gas is exactly the same as the potential energy released when the gas returns to a liquid. It is called the **latent heat of vaporisation**.

The amount of energy required will depend on the particular substance.

For a given mass of a substance:

heat energy transferred = mass of substance × latent heat of vaporisation

$Q = mL_{vapour}$

where Q is the heat energy transferred in joules (J)

m is the mass in kilograms (kg)

L_{vapour} is the latent heat of vaporisation ($J\,kg^{-1}$).

Note that, in just about every case, the latent heat of vaporisation of a substance will be different to the latent heat of fusion for that substance. Some latent heat of vaporisation values are listed in Table 6.3.2.

In many instances, it is necessary to consider the energy required to heat a substance and to also change its state. Problems like this are solved by considering the rise in temperature separately from the change of state.

TABLE 6.3.2 The latent heat of vaporisation of some common materials.

Substance	Boiling point (°C)	L_{vapour} ($J\,kg^{-1}$)
water	100	2.25×10^6
oxygen	−183	2.13×10^5
lead	1750	8.55×10^5
ethanol	78	8.46×10^5
silver	2193	2.63×10^6

Worked example 6.3.2

CHANGE IN TEMPERATURE AND STATE

50.0 mL of water is heated from a room temperature of 20.0°C to its boiling point at 100.0°C. It is boiled at this temperature until it is completely evaporated. How much energy in total was required to raise the temperature and boil the water?

Thinking	Working
Calculate the mass of water involved.	50.0 mL of water = 0.0500 kg
Find the specific heat capacity of water from Table 6.2.1.	$c = 4180\ \text{J kg}^{-1}\ \text{K}^{-1}$
Use the equation $Q = mc\Delta T$ to calculate the heat energy required to change the temperature of water from 20.0°C to 100.0°C.	$Q = mc\Delta T$ $Q = (0.0500)(4180)(100.0 - 20.0)$ $Q = 16\,720\ \text{J}$
Find the specific latent heat of vaporisation of water.	$L_{vapour} = 2.25 \times 10^6\ \text{J kg}^{-1}$
Use the equation $Q = mL_{vapour}$ to calculate the latent heat required to boil the water.	$Q = mL_{vapour}$ $Q = (0.0500)(2.25 \times 10^6)$ $Q = 112\,500\ \text{J}$
Find the total energy required to raise the temperature and change the state of the water.	Total $Q = (16\,720) + (112\,500)$ Total $Q = 129\,220$ Total $Q = 1.29 \times 10^5\ \text{J}$

Worked example: Try yourself 6.3.2

CHANGE IN TEMPERATURE AND STATE

3.00 L of water is heated from a fridge temperature of 4.00°C to its boiling point at 100.00°C. It is boiled at this temperature until it is completely evaporated. How much energy in total is required to raise the temperature and boil the water?

EVAPORATION AND COOLING

If you spill some water on the floor then come back in a couple of hours, the water will probably be gone. It will have evaporated. It has changed from a liquid into a vapour at room temperature in a process called **evaporation**. The reason for this is that the water particles, if they have sufficient energy, can escape from the surface of the liquid into the air. Over time, no liquid remains.

Evaporation is more noticeable in **volatile** liquids such as methanol, nail polish remover, perfume and hand sanitiser. The surface bonds are weaker in these liquids and they evaporate rapidly. This is why you should never leave the lid off bottles of these liquids. They are often stored in narrow-necked bottles for this reason.

Whenever evaporation occurs, higher-energy particles escape from the surface of the liquid, leaving the lower-energy particles behind, as is shown in Figure 6.3.2. As a result, the average kinetic energy of the particles remaining in the liquid drops and the temperature decreases. Sweating is how the human body uses this principle to stay cool. When a sterilising alcohol pad is wiped on your arm before an injection, the evaporation of the volatile liquid kills harmful bacteria and cools your skin.

PHYSICSFILE

Extinguishing fire

The latent heat of vaporisation of water is very high. This is because a huge amount of energy is required to separate the water molecules from each other. Very strong intermolecular forces, called hydrogen bonds, attract the water molecules to each other. This characteristic of water makes it very useful for extinguishing fires. That's because water can absorb vast amounts of thermal energy before it evaporates. By pouring water onto a fire, energy is transferred away from the fire to heat the water. Then, even more (in fact much more) heat is transferred away from the fire as the liquid water is converted into steam.

The rate of evaporation of a liquid depends on:

- the volatility of the liquid: more volatile liquids evaporate faster
- the surface area: greater evaporation occurs when greater surface areas are exposed to the air
- the temperature: hotter liquids evaporate faster
- the humidity: less evaporation occurs in more humid conditions
- air movement: if a breeze is blowing over the liquid's surface, evaporation is more rapid.

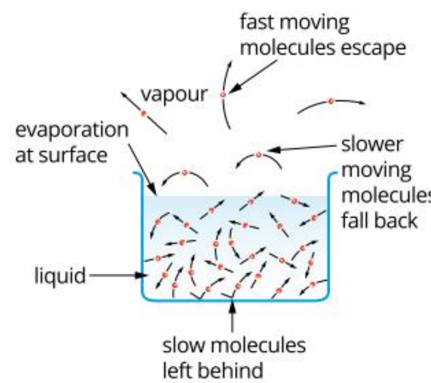


FIGURE 6.3.2 Fast-moving molecules with high kinetic energy can escape the liquid, leaving molecules with lower kinetic energy behind.

PHYSICS IN ACTION

The operation of refrigerators and reverse-cycle air conditioners

Most people are familiar with the cooling effect of getting out of a pool and into a breeze. It is due to evaporation and can make you feel cold even on a warm, sunny day. Refrigerators and air conditioners rely on the cooling effect of evaporation to chill food and cool rooms. Figure 6.3.2 illustrates what happens at the surface of a liquid during evaporation.

A molecule at the surface of the liquid gains energy, as thermal energy, from the surroundings. This energy is transformed to kinetic energy of the molecule. The extra kinetic energy allows the molecule to overcome the surface tension of the liquid and change state to a gas. If the molecule still has enough energy, it will stay in the gas state and escape the liquid entirely, 'evaporating'. As only those molecules with the highest kinetic energy escape from the liquid, the average kinetic energy of the molecules remaining in the liquid decreases. A lower average kinetic energy means a lower temperature.

Refrigerators and air conditioners make things cold by removing energy from them. Energy is pumped from the space being cooled to the outside air. As a result, modern refrigeration systems can be called 'heat pumps'. The cooling process is shown in Figure 6.3.3.

Inside a heat pump, a volatile liquid known as a refrigerant is circulated around a closed system of pipes by a pump. The pressure inside the evaporator pipes is reduced by an expansion valve (step 1), causing the refrigerant to evaporate. Energy is needed for this change of state to take place, i.e. the latent heat of vaporisation, so the system will absorb energy (step 2). As the gas evaporates, it absorbs energy from the surrounding air, making the air cooler.

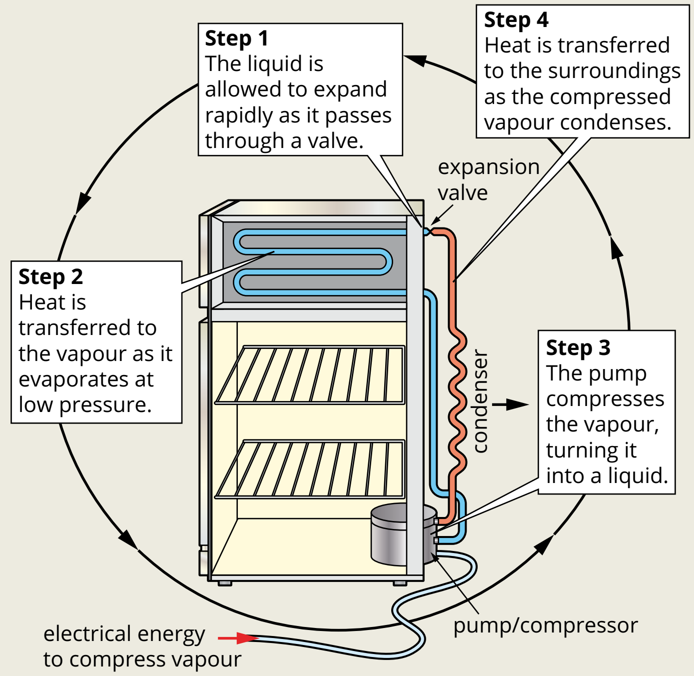


FIGURE 6.3.3 A refrigerator acts as a heat pump to remove energy from inside the refrigerator.

In the condenser pipes, the process is reversed. The refrigerant gas is compressed and condenses to a liquid again (step 3). The change of state from a gas to liquid releases energy. This energy is released outside the refrigerator and heats the surrounding air (step 4). Try putting your hand in the space behind your fridge at home. What do you notice?

Modern reverse-cycle air conditioners and heat-pump water heaters reverse this process to heat homes and water. Heat energy is picked up from outside the house (or from a power source) and released inside when the vapour condenses.

6.3 Review

SUMMARY

- When a solid material changes state, energy is needed to separate the particles by overcoming the attractive forces between the particles.
- Latent heat is the energy required to change the state of 1 kg of material at a constant temperature.
- In general, for any mass of material the energy required (or released) is $Q = mL$

  where Q is the energy transferred in joules (J)

  m is the mass in kilograms (kg)

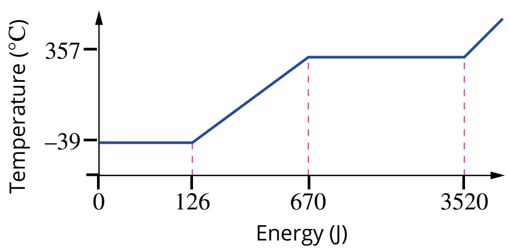
  L is the latent heat ($J\,kg^{-1}$).
- The latent heat of fusion, L_{fusion}, is the energy required to change 1 kg of a material between the solid and liquid states.
- The latent heat of vaporisation, L_{vapour}, is the energy required to change 1 kg of a material between the liquid and gaseous states.
- The latent heat of fusion of a material will be different to (and usually less than) the latent heat of vaporisation for that material.
- The rate of evaporation depends on the volatility, temperature and surface area of the liquid, and how much of a breeze there is.

KEY QUESTIONS

Refer to the values in Tables 6.3.1 and 6.3.2 on pages 213 and 214. You may also need to refer to Table 6.2.1 on page 209.

The following information applies to questions 1–5.

The graph below represents the heating curve for mercury, a metal that is a liquid at normal room temperature. Thermal energy is added to 10.0 g of solid mercury, initially at a temperature of –39.0°C, until all of the mercury has evaporated.



1 Why does the temperature remain constant during the first part of the graph?

2 What is the melting point of mercury, in degrees Celsius?

3 What is the boiling point of mercury, in degrees Celsius?

4 Referring to the graph, what is the latent heat of fusion of mercury?

5 Referring to the graph, what is the latent heat of vaporisation of mercury?

6 How much heat energy must be transferred away from 134 g of steam at 100.0°C to change it to a liquid?

7 How many kJ of energy are required to melt exactly 182 g of ice initially at –4.00°C? Assume no loss of energy to surroundings.

8 Explain why hot water in a spa-pool evaporates more rapidly than cold water in a pool.

9 A student spills some hand sanitiser onto a science lab floor. After a minute, most of the liquid is gone and the floor is cooler where the liquid was. Explain these observations.

6.4 Heating and cooling

Have you ever got into a bath only to find the water had gone cold? You probably then added hot water and waited, shivering, for the bath to get warm. In this section you will learn what happens to the energy from the hot water when it is added to the cold water, and how to calculate how much hot water you would need to add to get to the right temperature. As was explained in Section 6.1, when objects at different temperatures come into contact, thermal energy is transferred from the hotter object to the cooler object. Once the two objects reach the same temperature, they are said to be in thermal equilibrium.

This section focuses on solving heating problems involving the direct transfer of heat, and mixing substances that then reach thermal equilibrium.

THERMAL EQUILIBRIUM

Whenever two substances initially at different temperatures come into contact (or are mixed), thermal energy will transfer from the hotter material to the colder one until both are the same temperature. When both substances end up at the same final temperature, the mixture will have reached thermal equilibrium. The final temperature will be somewhere between the two original temperatures and will depend on the relative mass of each substance and their relative specific heat capacities. The final temperature will only be exactly halfway between the two original temperatures if the masses and the respective specific heat capacities of both substances are equal.

Assuming no loss or gain from the surrounding environment, the total energy would remain the same. This is an example of conservation of energy and is an essential concept when solving practical problems involving a transfer of energy.

Worked example 6.4.1

CALCULATING THERMAL EQUILIBRIUM 1

10.0 kg of water initially at 80.0°C is mixed with 30.0 kg of water initially at 20.0°C. What is the final temperature of the water once thermal equilibrium is reached?	
Thinking	**Working**
Total energy lost by hot water = total energy gained by cold water That is, the energy change, ΔQ, is equal for the hot and cold water. Use $\Delta Q = mc\Delta T$. Assume no loss of energy to the surrounding environment.	$\Delta Q_{hot} = \Delta Q_{cold}$ $m_{hot}\, c\Delta T_{hot} = m_{cold}\, c\Delta T_{cold}$
Since specific heat capacity of the water will be the same on both sides of the equation, the equation can be simplified.	$m_{hot}\, \Delta T_{hot} = m_{cold}\, \Delta T_{cold}$
Substitute the known values and simplify for the equilibrium temperature, T.	$(10.0)(80.0 - T) = (30.0)(T - 20.0)$ $800 - 10.0T = 30.0T - 600$ $800 + 600 = 30.0T + 10.0T$ $1400 = 40.0T$ $T = \frac{(1400)}{(40.0)}$ $T = 35.000$ $T = 35.0°C$
Do a quick intuitive check. Does the answer make sense?	As most of the water was colder, the final temperature should be closer to the temperature of the original colder water than to the temperature of the original hotter water.

Worked example: Try yourself 6.4.1

CALCULATING THERMAL EQUILIBRIUM 1

4.00 kg of water initially at 85.0°C is mixed with 3.00 kg of water initially at 25.0°C. What is the final temperature of the water once thermal equilibrium is reached?

Worked example 6.4.2

CALCULATING THERMAL EQUILIBRIUM 2

A 50.0 g piece of iron is heated over a flame for several minutes. The iron is then plunged into an insulated, closed container containing 1.00 L of cool water, originally at 15.0°C. When thermal equilibrium is reached, the temperature of the water is found to be 17.0°C. If no water changes state to become steam and there are no other energy losses, what was the temperature of the iron just before it was immersed in the water?

Thinking	Working
Convert all masses to standard units (kg).	Mass of iron = 50.0 g = 0.0500 kg mass of water = 1.00 kg (1.00 L of water has a mass of 1.00 kg)
Refer to Table 6.2.1 on page 209 for the relevant specific heat capacity (c) values.	$c_{iron} = 449\,J\,kg^{-1}\,K^{-1}$ $c_{water} = 4180\,J\,kg^{-1}\,K^{-1}$
total energy lost by iron = total energy gained by water That is, the energy change, ΔQ, is equal for the copper and the water.	$\Delta Q_{iron} = \Delta Q_{water}$ $m_{iron}\,c_{iron}\,\Delta T_{iron} = m_{water}\,c_{water}\,\Delta T_{water}$
Substitute the known values, expand and simplify to solve for the initial temperature of the iron.	$m_{iron}\,c_{iron}\,\Delta T_{iron} = m_{water}\,c_{water}\,\Delta T_{water}$ $(0.0500)(449) \times (T_{iron} - 17.0) = (1.00)(4180)(17.0 - 15.0)$ $22.45T_{iron} - 382 = 8360$ $22.45T_{iron} = 8742$ $T = \frac{(8742)}{(22.45)}$ $T_{iron} = 389.40$ $T_{iron} = 389°C$

Worked example: Try yourself 6.4.2

CALCULATING THERMAL EQUILIBRIUM 2

A 75.0 g piece of copper is heated over a flame for several minutes. The copper is then plunged into an insulated, closed container containing 0.500 L of cool water, originally at 20.0°C. When thermal equilibrium is reached, the temperature of the water is found to be 22.0°C. If no water changes state to become steam and there are no other energy losses, what was the temperature of the copper just before it was immersed in the water?

Solving 'heating' questions

Problems involving thermal energy often require calculations of both latent and specific heat as materials or systems change temperature and state during heating and/or cooling. They can require a number of steps and the process can at first seem quite complex. A flow diagram (Figure 6.4.1) often helps in understanding the steps and the form of heating or cooling required for each step.

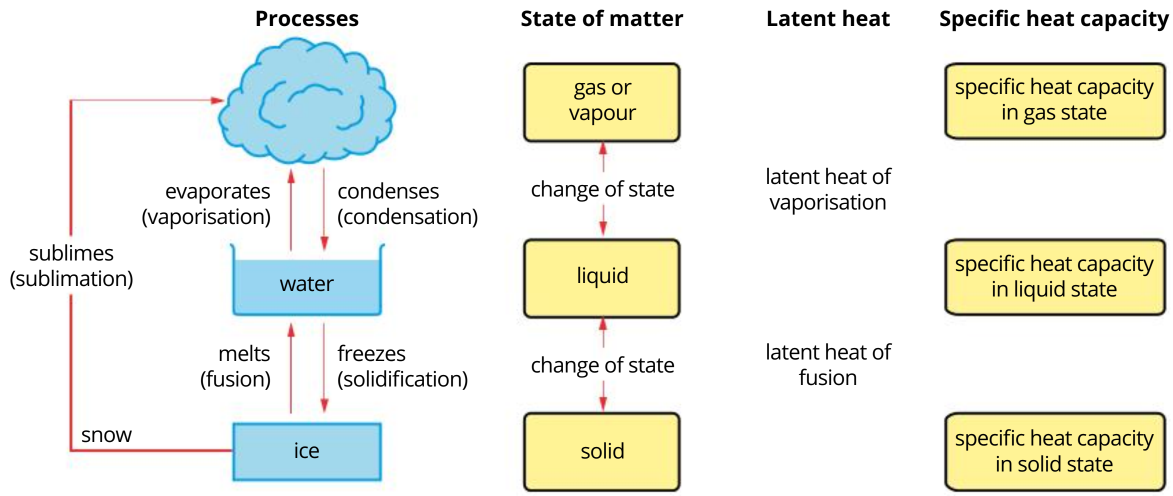


FIGURE 6.4.1 Solving heating and cooling questions involving changes of state.

Worked example 6.4.3

CHANGES OF STATE

Calculate the thermal energy required, in MJ, to convert 5.00 kg of ice at –20.0°C into steam at 100.0°C.	
Thinking	**Working**
Identify the steps involved in the process.	Step 1: Ice at –20.0°C to ice at 0.00°C Step 2: Ice at 0.00°C to water at 0.00°C Step 3: Water at 0.00°C to water at 100.0°C Step 4: Water at 100.0°C to steam at 100.0°C
Identify values for L and c for each step. Use Tables 6.2.1, 6.3.1 and 6.3.2 on pages 209, 213 and 214 to look up the values.	$c_{ice} = 2050\,J\,kg^{-1}\,K^{-1}$ $c_{water} = 4180\,J\,kg^{-1}\,K^{-1}$ $L_{fusion} = 3.34 \times 10^5\,J\,kg^{-1}$ $L_{vapour} = 2.25 \times 10^6\,J\,kg^{-1}$
Calculate the energy required for each step separately using the appropriate equation for specific heat or latent heat.	Step 1: Heating the ice $Q_1 = mc\Delta T = (5.00)(2050)(20.0 - 0.00) = 2.050 \times 10^5\,J$ Step 2: Melting the ice $Q_2 = mL_{fusion} = (5.00)(3.34 \times 10^5) = 1.670 \times 10^6\,J$ Step 3: Heating the water $Q_3 = mc\Delta T = (5.00)(4180)(100.0 - 0.00) = 2.090 \times 10^6\,J$ Step 4: Vapourising the water $Q_4 = mL_{vapour} = (5.00)(2.25 x 10^6) = 1.1250 \times 10^7\,J$
Add the energy required for each step together to find the total energy required.	$Q_T = Q_1 + Q_2 + Q_3 + Q_4$ $Q_T = (2.050 \times 10^5) + (1.670 \times 10^6) + (2.090 \times 10^6) + (1.1250 \times 10^7)$ $Q_T = 1.5215 \times 10^7$ $Q_T = 1.52 \times 10^7\,J$ $Q_T = 15.2\,MJ$

Worked Example: Try yourself 6.4.3

CHANGES OF STATE

Calculate the heat energy that must be lost, in J, to convert 5.00 kg of water vapour at 140.0°C into solid ice at 0.00°C.

6.4 Review

SUMMARY

- During a transfer of thermal energy, the materials in contact with each other will eventually come to thermal equilibrium.
- The equilibrium temperature will depend upon the amount of energy transferred, the mass of the individual materials involved and the specific heat capacity of each material.
- Heating questions involving changes of state can be solved by breaking the question into parts or steps where there is either heating or cooling (specific heat change), or a change of state (latent heat change).

KEY QUESTIONS

1 Which absorbs the most thermal energy: 10.0 kg of iron changing from 10.0°C to 20.0°C, or 10.0 kg of aluminium changing from 10.0°C to 20.0°C? The specific heat of iron is 449 J kg^{-1} K^{-1} and of aluminium is 897 J kg^{-1} K^{-1}.

2 10.0 kg of water at a temperature of 65.0°C is added to a bath containing 80.0 kg of water initially at 15.0°C. Calculate the final equilibrium temperature in degrees Celsius. The specific heat of water is 4180 J kg^{-1} K^{-1}.

3 A 20.0 kg block of copper at 100.0°C is put into a large pot containing 5.00 kg of water at 20.0°C. Assuming no energy is lost to the surrounding environment and no water changes state, calculate the final temperature of the mixture. Use the specific heat of water as 4180 J kg^{-1} K^{-1} and copper as 385 J kg^{-1} K^{-1}.

4 Experts recommend that the bath temperature for a newborn baby should be 36.0°C. A group of students want to check what mass of water at 45.0°C they should add to 12.0 kg of water at 19.0°C in order to reach a final temperature suitable for the baby. Assuming no heat is lost to the bath or the surroundings, calculate the mass of warm water required.

5 A large 598 kg iron rod is produced in an iron smelter at 1250°C. It is cooled before transport by placing it in a water tank containing 938 litres of water at 21.0°C. Calculate the final temperature of the water and the iron in the water tank, assuming no heat is lost to the tank or the surroundings and the specific heat capacity of iron is 449 J kg^{-1} K^{-1}. Assume no water changes state.

6 A 10.0 kg block of iron at 20.0°C and a 10.0 kg block of aluminium at 20.0°C were dropped into a trough of 100.0 litres of water at 12.0°C. Calculate the final equilibrium temperature, assuming no losses to the surrounding environment and no water changes state. Use c_{iron} = 449 J kg^{-1} K^{-1}, $c_{aluminium}$ = 897 J kg^{-1} K^{-1} and c_{water} = 4180 J kg^{-1} K^{-1}.

7 50.0 kg of water at 20.0°C is placed in a large electric urn and heated to boiling point. Later, when the urn is checked, it is found that the water had boiled at this temperature until it had completely vaporised. Calculate how much energy was required to heat and boil away all of the water. The specific heat of water is 4180 J kg^{-1} K^{-1} and the latent heat of vaporisation of water is 2.25×10^6 J kg^{-1}.

8 A chef uses a steamer to cook 3.00 kg of potatoes for a dish. The steam condenses on the potatoes and then the hot water continues to heat them up. Calculate the mass of steam at 100.0°C required to raise the temperature of the potatoes from 12.5°C to 85.0°C. The specific heat of water is 4180 J kg^{-1} K^{-1} and the specific heat of potatoes is 3430 J kg^{-1} K^{-1}. The latent heat of vaporisation of water is 2.25×10^6 J kg^{-1}. Assume all of the steam condenses and no heat is lost to the surrounds.

9 A jeweller wants to melt a 1.25 kg ingot of silver metal by heating it from 20.0°C to its melting point of 961.0°C, before pouring it into moulds. Calculate the energy required to carry out the procedure if the specific heat of solid silver is 235 J kg^{-1} K^{-1} and the latent heat of fusion of silver is 1.04×10^5 J kg^{-1}.

10 Steam cleaners use the extra energy contained in steam to break down grease and dirt from surfaces. A steam cleaner condenses 755 g of steam at 110.0°C onto a dirty surface which then ends up as water at 25.0°C. Calculate the energy transferred to the surface from the steam, assuming no energy is transferred to any other substance. The specific heat capacity of steam is 2030 J kg^{-1} K^{-1}, the specific heat capacity of water is 4180 J kg^{-1} K^{-1} and the latent heat of vaporisation of water is 2.25×10^6 J kg^{-1}.

Chapter review

06

KEY TERMS

absolute zero
evaporation
heat
internal energy
kelvin
kinetic energy
kinetic particle model
latent heat
latent heat of fusion
latent heat of vaporisation
potential energy
specific heat capacity
temperature
thermal energy
thermal equilibrium
volatile
work

1 According to the kinetic particle model, which of the following is true for particles of matter?
- **A** They are in continuous motion.
- **B** They have different sizes.
- **C** They have different shapes.
- **D** They are always floating.

2 What does the kelvin scale measure?

3 How does temperature differ from heat?

4 Convert:
- **a** 5°C to kelvin
- **b** 200 K to °C.

5 The kelvin scale is an example of an absolute scale. The Celsius scale is an arbitrary scale. List two key attributes of an absolute scale.

6 The Celsius scale has two fixed points—the freezing and boiling points of water—divided into 100 intervals. Explain why the Celsius scale cannot be considered an absolute scale.

7 The specific heat capacity of iron is approximately half that of aluminium. A cube of iron and a cube of aluminium, both at 80°C, are dropped into a thermally insulated jar that contains a mass of water, equal to that of the cubes, at 20°C. Thermal equilibrium is eventually reached. Describe the final temperatures of each of the metal cubes.

8 Two cubes, one silver and one iron, have the same mass and temperature. A quantity *Q* of heat is removed from each cube. Which one of the following properties causes the final temperatures of the cubes to be different?
- **A** density
- **B** specific heat capacity
- **C** latent heat of vaporisation
- **D** volume

9 A solid substance is heated but its temperature does not change. Explain what is occurring.

10 Which possesses the greater internal energy: 1.00 kg of water boiling at 100.0°C or 1 kg of steam at 100.0°C? Explain why.

11 A liquid is evaporating, causing the liquid to cool. Explain why the temperature of the liquid decreases.

12 A 2.00 kg unknown metal object requires 5.02×10^3 J of heat to raise its temperature from 20.0°C to 40.0°C. What is the specific heat capacity of the unknown metal in $J\,kg^{-1}\,K^{-1}$? Give your answer to the nearest whole number.

13 How many joules of energy are required to melt exactly 80.0 g of silver? ($L_{fusion} = 1.04 \times 10^5\,J\,kg^{-1}$)

14 Copper is considerably more expensive than iron but is the preferred material for hot water pipes around the home while galvanised iron is generally used for cold water. Explain why copper is the preferred material for hot water pipes.

15 100.0 mL of water at 60.0°C is placed into a copper cup, also at 60.0°C and weighing 200.0 g. Calculate the mass of ice required to cool the water to 20.0°C.

16 A barista uses steam to heat up 425 g of milk with a specific heat capacity of $3930\,J\,kg^{-1}\,K^{-1}$ from 4.00°C to 70.0°C. Calculate the mass of steam, in kilograms, required to heat the milk if the steam starts at 100.0°C and ends up at the equilibrium temperature of the milk.

17 A student has a part-time job in a shop making blended icy fruit drinks. A 468 g lemon juice drink at 20.0°C, with a specific heat capacity of $3850\,J\,kg^{-1}\,K^{-1}$, has ice at 0.00°C added to it. Calculate the mass of ice required for the mixture to end up at 3.00°C. The latent heat of fusion of ice is $3.34 \times 10^5\,J\,kg^{-1}$.

18 An airport worker uses a steam gun to melt the ice off the wheels of a plane at Australia's Antarctic base prior to departure. Calculate the mass of steam at 115.0°C required to convert 2.50 kg of ice at –12.5°C into water at 55.0°C. The specific heat capacity of steam is $2030\,J\,kg^{-1}\,K^{-1}$, of water is $4180\,J\,kg^{-1}\,K^{-1}$ and of ice is $2050\,J\,kg^{-1}\,K^{-1}$. The latent heat of fusion of water is $3.34 \times 10^5\,J\,kg^{-1}$ and the latent heat of vaporisation of water is $2.25 \times 10^6\,J\,kg^{-1}$.

19 1.50 kg of water at 22.0°C is poured on to an 18.0 kg iron barbeque hotplate at 545.0°C to cool it down. If all of the water is converted to steam at 100.0°C, calculate the final temperature of the hotplate. The specific heat capacity of water is $4180\,J\,kg^{-1}\,K^{-1}$ and of iron is $449\,J\,kg^{-1}\,K^{-1}$. The latent heat of vaporisation of water is $2.25 \times 10^6\,J\,kg^{-1}$.

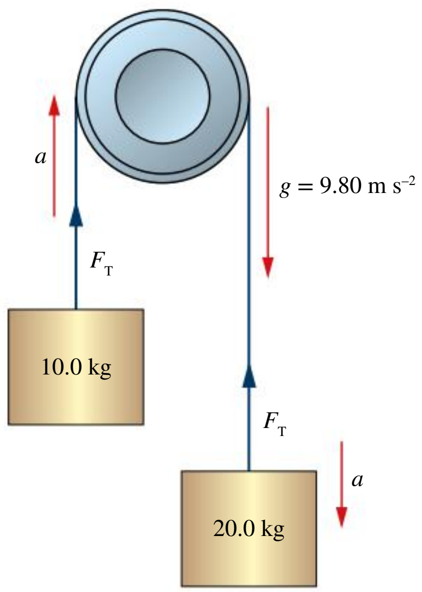
UNIT 1 • MOTION, FORCES AND ENERGY

REVIEW QUESTIONS

Section 1: Short response

1 Two students drop a lead mass from a tower and time its fall. How far does the mass fall during the period of time from $t_i = 0$ seconds to $t_f = 1.00$ second, and during the period of time from $t_i = 1.00$ second to $t_f = 2.00$ seconds?

2 Two masses, 10.0 kg and 20.0 kg, are attached via a steel cable to a frictionless pulley, as shown in the following diagram.



- **a** Determine the acceleration for each mass.
- **b** What is the magnitude of the tension in the cable?

3 An engine pulls a line of rail cars carrying iron ore along a flat track with a constant force, but instead of accelerating, the whole train travels at a constant velocity. Explain how this statement is consistent with Newton's first and second laws of motion.

4 A tow truck pulling a car of mass 1140 kg along a straight road causes the velocity of the car to increase from 5.00 m s^{-1} west to 10.0 m s^{-1} west over a distance of 117 m. A constant frictional force of 298 N acts on the car.

- **a** Calculate the acceleration of the car.
- **b** What is the resultant force acting on the car during this 117 m?
- **c** Calculate the force exerted on the car by the truck.
- **d** What force does the car exert on the truck?

5 A bucket is filled with equal masses of hot and cold water. The hot water starts at 80.0°C and the cold water at 10.0°C. Calculate the temperature of the final mixture.

6 Determine whether the following statements are correct or incorrect. Write an explanation justifying your choice.

- **a** Water in a bottle covered with a wet cloth will be cooler than water in a similar bottle left uncovered in the same environment.
- **b** Pasta will cook quicker if the water is boiling faster.
- **c** Eradicating (or removing) weeds by directing steam at 100°C onto them is significantly more effective than pouring boiling water onto them.

7 A golfer drops a golf ball onto a concrete path. The ball bounces off the path up to a maximum height of 0.925 m. If the golf ball retained only 84.5% of its original kinetic energy after the bounce, calculate the height from which the ball was dropped.

8 Add the following vectors.

- **a** 23.4 m south, 14.0 m north, 6.0 m south and 17.2 m north
- **b** 235 N east and 434 N south
- **c** 25.5 N west, 45.3 N north and 74.8 N east

Section 2: Problem-solving

9 A chef is considering opening an ice-cream parlour that makes ice-cream using liquid nitrogen. The liquid nitrogen serves the dual purpose of freezing the water in the ice-cream and aerating it (adding air) at the same time. The chef intends to make ice-cream from a sugar and cream mix that is 70.0% water, and is aware that only the water needs to freeze for the ice-cream to set.

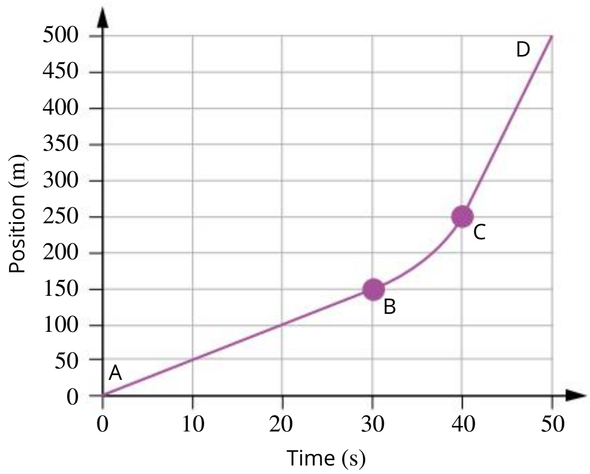
The following data is used in the calculations:

- Boiling point of liquid nitrogen at atmospheric pressure: 77.0 K
- Specific heat capacity for nitrogen gas: 1.34×10^3 J kg^{-1} K^{-1}
- Latent heat of vaporisation of liquid nitrogen: 1.99×10^5 J kg^{-1}
- Specific heat capacity of cream and sugar mix: 3.80×10^3 J kg^{-1} K^{-1}
- Latent heat of fusion of water: 3.34×10^5 J kg^{-1}.

- **a** How much thermal energy is required to vaporise 1.00 kg of liquid nitrogen at 77.0 K?
- **b** How much thermal energy is absorbed by 1.00 kg of nitrogen gas as it heats from 77.0 K to 273.0 K?
- **c** How much thermal energy must be removed to cool 200.0 g of refrigerated liquid sugar and cream mix at 8.00°C down to 0.00°C to make solid ice-cream?

d What mass of liquid nitrogen does the chef need per 200.0 g of liquid sugar and cream mix if they take the mix from the fridge at 8.00°C and cool it until it is solid at 0.00°C?

10 The following position–time graph depicts the motions of a cyclist travelling east along a straight road from points A to D.



a Describe each section of the cyclist's motion in terms of speed.

b What was the velocity of the cyclist for the first 30 s?

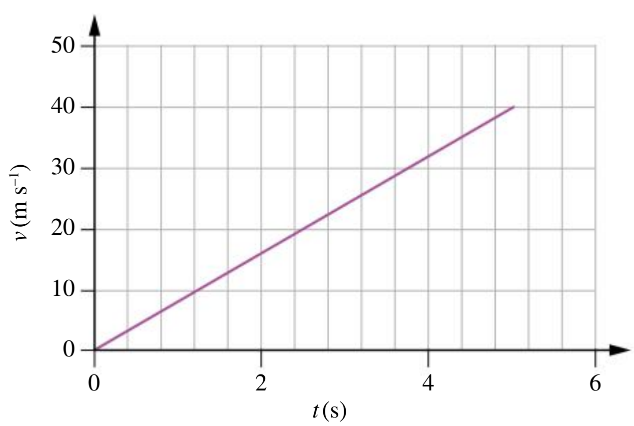
c What was the velocity of the cyclist for the final 10 s?

d Calculate the average velocity between points B and C.

e Calculate the average acceleration between points B and C.

f Calculate the average speed between points A and D.

11 The velocity–time graph for a car of mass 2070 kg travelling north along a straight stretch of road is shown below. The engine of the car provides a constant driving force. During the 5.0 s time period, the car encounters a constant combined frictional force of 412 N due to air resistance and the friction of the road on the tyres. At $t = 5.0$ s, the driver notices that their velocity is $v = 40.0\ \mathrm{m\,s^{-1}}$ north.



a How much kinetic energy (in MJ) does the car have at $t = 5.0$ s?

b What is the resultant force acting on the car?

c What force is provided by the car's engine during the 5.0 s time period?

d How much work is done by the car's engine during the 5.0 s time period?

e Determine the power output of the car's engine during the 5.0 s time period.

f How much energy is wasted due to air resistance and friction during the 5.0 s time period?

g Calculate the efficiency with which the energy provided by the car's engine is transformed into kinetic energy.

12 A baseball training device called a pitching machine uses compressed air to shoot baseballs at batters so they can practise hitting. The device has a mass of 1.08×10^2 kg and fires baseballs with a mass of 1.45×10^{-1} kg at a velocity of $17.0\ \mathrm{m\,s^{-1}}$. The barrel of the pitching machine is 45.0 cm long, and it can be assumed that compressed air acts on the baseball for the time that it is in the barrel.

a Calculate the magnitude of the average acceleration of a baseball down the barrel if it was initially at rest.

b Using Newton's second law, calculate the average force exerted by the compressed air as the baseball travels down the barrel.

c Calculate the momentum of the baseball as it leaves the barrel.

d Calculate the recoil velocity of the pitching machine.

e Calculate the average force of the compressed air from the change in momentum of the baseball.

f Calculate the average work done by the compressed air on the baseball.

g Compare your answer to part **f** with the kinetic energy gained by the baseball and comment on how realistic this comparison is.

Section 3: Comprehension

13 Speed skiing

Speed skiing is an extreme winter sport. It is usually conducted at high altitudes where there is less air and therefore less air resistance. The skier uses nothing more than gravity and their ability to minimise friction as they accelerate down a steep slope of 1000 to 1400 m to reach the highest velocity they possibly can. This can often exceed $200\ \mathrm{km\,h^{-1}}$. The track has a slope of around 400 m in length to allow the skiers to reach maximum velocity before passing through a set of light beams 100.0 m apart that control the timing device. Skiers usually reach their maximum velocity well before the first light beam. These light beams record the time the skier takes to travel the 100.0 m to a high degree of accuracy.

The equipment speed skiers use is highly specialised, having been developed to reduce friction and air resistance as much as possible. The skis, for example, are specially shaped to be longer, wider and heavier than those used for normal downhill and recreational skiing. Their extra weight comes from their larger size and the fact that they are made of both wood and steel. Their extra length and width helps to spread the skier's weight over a larger surface area, helping to reduce frictional forces between the skis and snow. Their shape gives the skis a low profile and, coupled with their weight and its distribution, helps to minimise wind resistance and to keep the tips of the skis on the snow.

Another specially designed piece of speed-skiing equipment is the helmet. This is made to match the skier's individual body size and the tuck position (a curled, compact posture with knees bent and body bent forward) they adopt as they travel down the slope. The design directs the wind from the top of their head straight down their back.

Speed skiers wear modified boots to allow them to bend their knees and lean well forward as they accelerate down the slope. This lowers their centre of gravity and provides a lower profile, thus reducing air resistance. The development of individually customised skin-tight polyurethane suits also minimises air resistance: both the tightness and the low interruption of airflow caused by their coating allows air to pass more freely over them. To help smooth the airflow around their calves, a dense foam insert extends from the knees of the suit to the top of the boots in an aerodynamic shape.

Skiers use specially designed poles to move off from the starting platform. Once on the slope, these poles become important in helping the skier to brace their arms into their sides. Custom-made for each skier, the poles are bent in such a way as to wrap around their body.

For a skier to reach maximum speed in less than 400 m, their initial acceleration is critical. The ski slope for speed skiing must be very steep at the start and should not level out until after the 100 m section where the timing takes place. There is also a lengthy braking region that allows the skier to stop safely over a long period of time.

a What aspect of the skier's motion is determined by measuring the time they take to travel 100.0 m?

b Suggest a reason why the system described above is used to measure velocity rather than a single radar gun at the start of the 100.0 m section.

c Speed skiers achieve acceleration 'using nothing more than gravity and their ability to overcome friction'. Explain how their acceleration is affected by these two factors.

d Why do skiers reach a maximum velocity and not continue to accelerate all the way down the slope?

e Speed skis are wider, longer and heavier than normal skis. How does this help to reduce friction?

f List four other design factors of speed-skiing equipment that reduce friction and state how each one achieves this effect.

g 'For a skier to reach maximum speed in less than 400 m, their initial acceleration is critical'. Explain why this is the case.

The following information relates to parts h–m.

The current world speed-skiing record for men is 254.958 km h^{-1}, achieved by Ivan Origone on 28 March 2016 in the French Alps. He started at an altitude of 2720 m and came to a halt at an elevation of 2285 m.

h If Origone reached this speed after skiing 340.0 m down the slope, how many seconds had passed from when he left the start?

i Calculate his average acceleration from the start to when he reached his maximum speed.

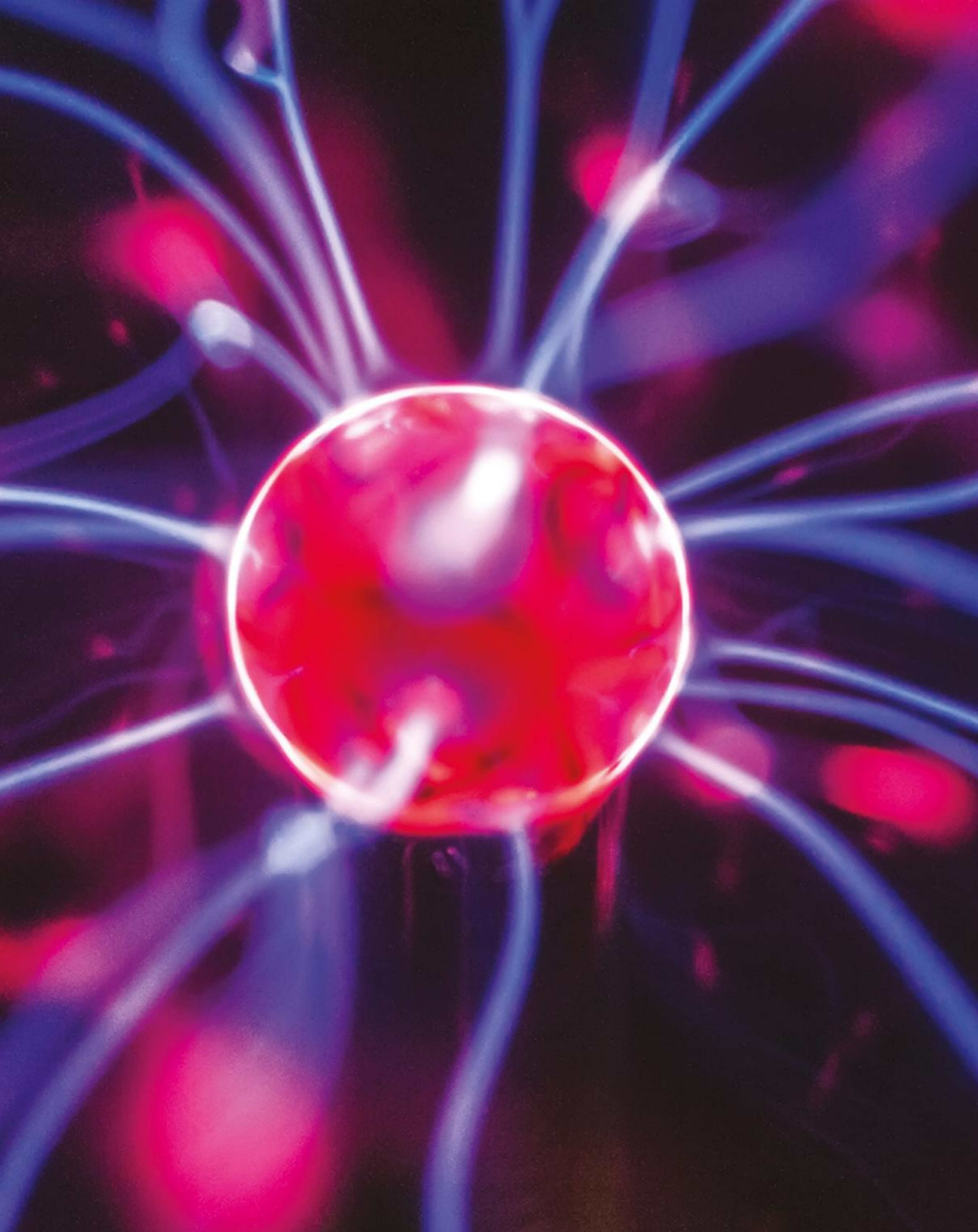
j Calculate how long it took him to travel between the two light beams in the 100.0 m section.

k Given his mass is 70.0 kg, calculate his kinetic energy at the start of the 100.0 m section.

l The energy conversion efficiency from potential to kinetic energy was 93.0%.

- **i** Calculate Origone's potential energy at the start of his run.
- **ii** At what altitude did he reach his maximum speed?

m Calculate the frictional force acting on him between the start and when he reached his maximum speed.



UNIT 2 Waves, nuclear and electrical physics

Students develop an understanding of waves which can be used to describe, explain and predict a wide range of phenomena. Students investigate the properties and interactions of a variety of waves, including waves on springs, water waves, sound and earthquake waves. They apply both the standard model and the nuclear model of the atom to investigate the cause of radioactivity and the properties of ionising radiation. They learn how nuclear reactions can cause transmutation of isotopes and how matter and energy are equivalent. They examine how electric fields cause the movement of electrical charge in circuits and use this to analyse, explain and predict electrical phenomena. Students also investigate how current flows, how work is done, and how energy is transformed into and out of the electric field. Household circuits and the safety devices we use to protect ourselves and our families from harm are also explored.

Contexts students may investigate include technologies related to nuclear energy, radiopharmaceuticals, seismic waves, musical instruments and electricity in the home; and related areas of science, such as nuclear fusion in stars, acoustics and nuclear medicine.

By the end of this unit, students will:

- explore the ways physics is used to describe, explain and predict the energy transfers and transformations that are pivotal to modern industrial societies
- investigate common wave phenomena in various media
- apply the nuclear model of the atom to investigate radioactivity
- learn how nuclear reactions convert mass into energy
- examine the movement of electrical charge in circuits and use this to analyse, explain and predict electrical phenomena
- understand how meeting world energy needs requires the international cooperation of multidisciplinary teams and relies on advances in ICT and other relevant technologies
- explore how science knowledge is used to offer valid explanations and reliable predictions, and the ways in which it interacts with social, economic, cultural and ethical factors
- develop skills in interpreting, constructing and using a range of mathematical and symbolic representations to describe, explain and predict energy transfers and transformations in wave interactions, nuclear reactions and electrical circuits
- develop their inquiry skills through primary and secondary investigations, including analysing wave behaviours and interactions, radioactive decay and a range of simple electrical circuits.

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.

CHAPTER 07

The nature of waves

Have you ever watched ocean waves heading toward the shore? For many people, their first thought when encountering a topic called ‘waves’ is to picture a water wave moving across the surface of an ocean. The wave may be created by some kind of disturbance, such as the action of wind on water or a boat as it moves through the water.

However, waves are everywhere: sound, visible light, radio waves, waves in the strings of an instrument, the wave of a hand and the ‘Mexican waves’ that move around a stadium. As you will learn in Year 12, waves are important in explaining electromagnetism and alternating current (AC) electrical power generation, which is essential for powering our society. Waves are also a fundamental concept in understanding the wave-particle duality of light and electromagnetic waves, as well as atomic and sub-atomic particles. They are even used by physicists to model vibrations and energy levels in matter. Another exciting phenomenon is gravity waves, whereby space-time itself vibrates, as predicted by Einstein and more recently measured by physicists through an extensive international collaboration.

Science Understanding

- waves are periodic oscillations that transfer energy from one point to another
- mechanical waves transfer energy through a medium; longitudinal and transverse waves are distinguished by the relationship between the directions of oscillation of particles relative to the direction of the wave velocity
- waves may be represented by displacement/time and displacement/distance wave diagrams and described in terms of relationships between measurable quantities, including period, amplitude, wavelength, frequency and velocity, including applying the relationships

  $$v = f\lambda \qquad T = \frac{1}{f}$$

- the mechanical wave model can be used to explain phenomena related to reflection and refraction, including echoes and seismic phenomena
- the superposition of waves in a medium may lead to the formation of standing waves and interference phenomena, including two parallel coherent speakers, standing waves in pipes and stretched strings and observation of beats, including applying the relationships for

  strings attached at both ends and pipes open at both ends $\lambda = \frac{2\ell}{n}$

  pipes closed at one end $\lambda = \frac{4\ell}{n}$

  where n = the number of the appropriate harmonic

  beat frequency $f_{beat} = |f_2 - f_1|$
- a mechanical system resonates when it is driven at one of its natural frequencies of oscillation; energy is transferred efficiently into systems under these conditions

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.

7.1 Longitudinal and transverse waves

Throw a stone into a pool or lake, and you will see circular waves form and move outwards from the source as ripples, as shown in Figure 7.1.1. Stretch a cord out on a table and wriggle one end back and forth across the table surface and another type of wave can be observed. Sound waves, water waves and waves in strings are all examples of **mechanical waves**. Mechanical waves need a medium to transmit energy. Electromagnetic waves, on the other hand, transmit energy through a vacuum. Mechanical waves are the focus of this chapter. Electromagnetic waves are discussed in greater detail in Year 12.

FIGURE 7.1.1 The ripples in a pond indicate a transfer of energy.

MECHANICAL WAVES

Watch a piece of driftwood, a leaf, or even a surfer, resting in the water as a smooth wave goes past. The object moves up and down but doesn't move forward with the wave. The movement of the object on the water reveals how the particles in the water move as the wave passes; that is, the particles in the water move up and down from an average position.

Any wave that needs a **medium** (such as water) through which to travel is called a mechanical wave. Mechanical waves can move over very large distances but the particles of the medium only have very limited movement.

In mechanical waves the particles of the matter vibrate back and forth or up and down about an average position, which transfers the energy from one place to another. For example, energy is given to an ocean wave by the action of the wind far out at sea. The energy is transported by waves to the shore but (except in the case of a tsunami event) most of the ocean water itself does not travel onto the shore.

> A wave involves the transfer of energy without the net transfer of matter.

Pulses versus periodic waves

A single wave **pulse** can be formed by giving a slinky spring or rope a single up and down motion as shown in Figure 7.1.2a. As the hand pulls upwards, the adjacent parts of the slinky will also feel an upward force and begin to move upward. The source of the wave energy is the movement of the hand.

If the up and down motion is repeated, each successive section of the slinky will move up and down, moving the wave forward along the slinky as shown in Figure 7.1.2b. Connections between each loop of the slinky cause the wave to travel away from the source, carrying with it the energy from the source.

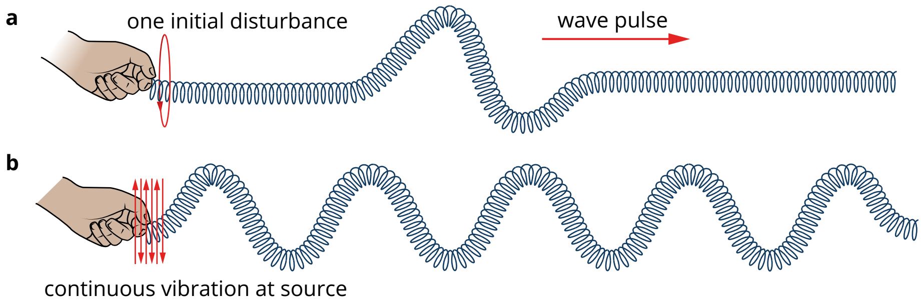


FIGURE 7.1.2 (a) A single wave pulse can be sent along a slinky by a single up and down motion. (b) A continuous or periodic wave is created by a regular, repeated movement of the hand.

In a continuous wave or periodic wave, continuous vibration of the source, such as that shown in Figure 7.1.2b, will cause the particles within the medium to **oscillate** about their average position in a regular, repetitive or periodic pattern. The source of any mechanical wave is this repeated motion or **vibration**. The energy from the vibration moves through the medium and constitutes a mechanical wave. Any wave that travels unimpeded through a medium is known as a **travelling wave**.

Transverse waves

When waves travel on water, or through a rope, spring or string, the particles within the medium can vibrate up and down in a direction perpendicular, or **transverse**, to the direction of motion of the wave energy (Figure 7.1.3). Such a wave is called a transverse wave. When the particles are displaced upwards from the average position, or resting position, they reach a maximum positive displacement called a **crest**. Particles below the average position fall to a maximum negative position called a **trough**.

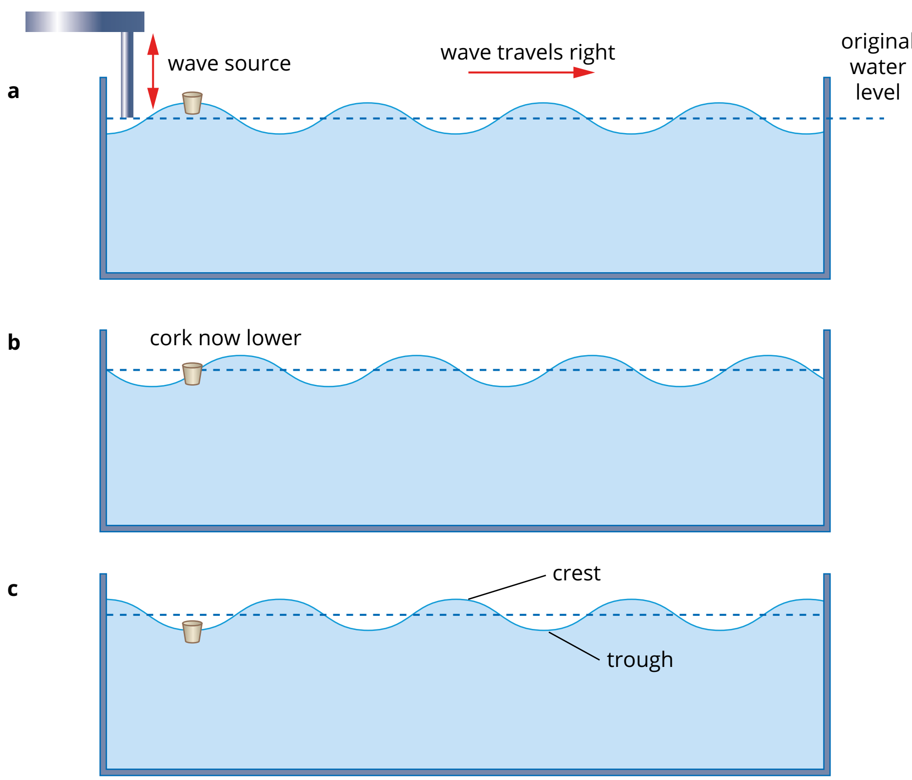


FIGURE 7.1.3 A continuous water wave moves to the right, showing an example of a transverse wave. As it does so, the up and down displacement of the particles, transverse to the wave motion, can be monitored using a cork. The cork simply moves up and down as the wave passes through its position.

A transverse wave oscillates or vibrates in a direction perpendicular to the direction of the wave.

Longitudinal waves

In a **longitudinal** mechanical wave, the vibrations of the particles within the medium are in the same direction, or parallel to, the direction of energy flow of the wave. You can demonstrate this type of wave with a slinky by moving your hand backwards and forwards in a line parallel to the length of the slinky, as shown in Figure 7.1.4a.

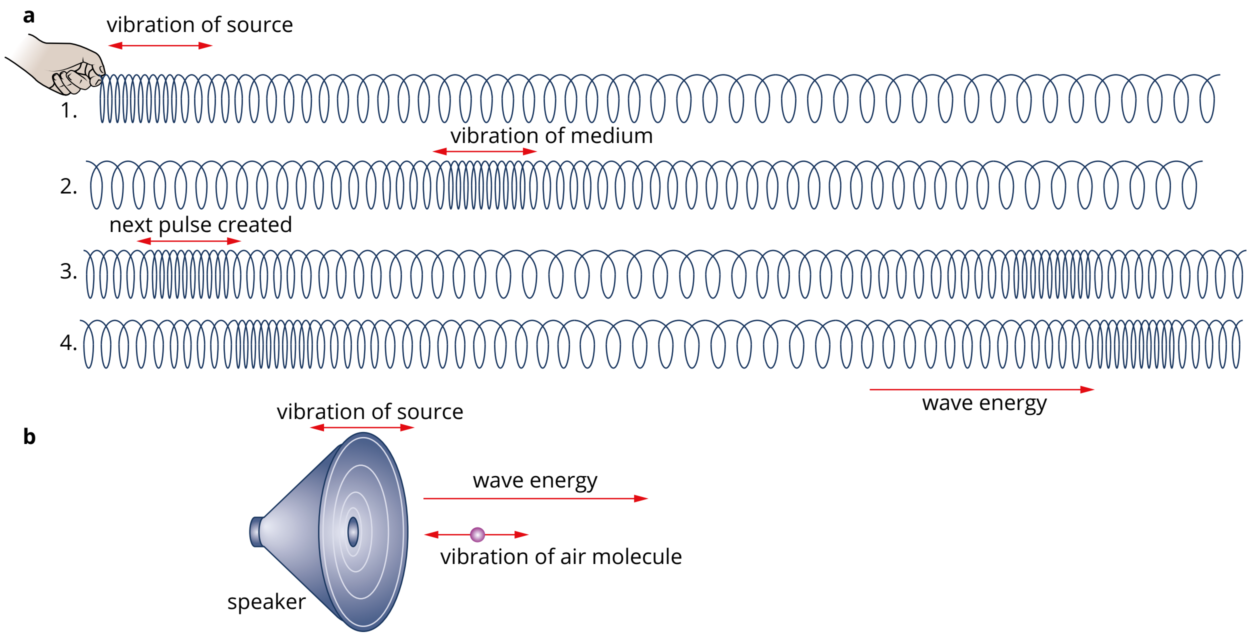


FIGURE 7.1.4 (a) When the direction of the vibrations of the medium and the direction of travel of the wave energy are parallel, a longitudinal wave is created. This can be demonstrated with a slinky. (b) Sound waves are longitudinal.

As you move your hand, a series of compressed and expanded areas form along the slinky. **Compressions** are those areas where the coils of the slinky come together. Expansions are regions where the coils are spread apart. Areas of expansion are called **rarefactions**. The compressions and rarefactions in a longitudinal wave correspond to the crests and troughs of a transverse wave.

An important example of a longitudinal wave is a sound wave, where small variations in **air pressure** carry the sound energy. As the cone of a loudspeaker vibrates (Figure 7.1.4b), the layer of air next to it is alternately pushed away and drawn back, creating a series of compressions (high pressure) and rarefactions (low pressure) in the air (demonstrated by the candle in Figure 7.1.5). This vibration is transmitted through the air as a sound wave, where the vibrations are in the direction of propagation. Like transverse waves, the individual particles undergo small vibrations back and forth about a mean position, while the wave itself can carry energy over very long distances. If the vibration was from a single point then the waves would tend to spread out spherically.

A longitudinal wave oscillates in the same direction as, or parallel to, the direction of the wave.

FIGURE 7.1.5 The motion of a flame in front of a loudspeaker is clear evidence of the continuous movement of air backwards and forwards as the loudspeaker creates a sound wave.

Measuring sound waves

It is important to remember that sound is a longitudinal wave. Sound waves are often measured by devices such as an oscilloscope, as shown in Figure 7.1.6. The sound waves are picked up by a microphone, then converted into an electrical signal that is then displayed as a transverse wave. Figure 7.1.7 shows the compressions (C) represented as a crest and the rarefactions (R) represented as a trough. For sound, the transverse wave display is only a convenient representation that makes it easier to determine properties such as amplitude, wavelength, frequency and period, discussed in the next section.

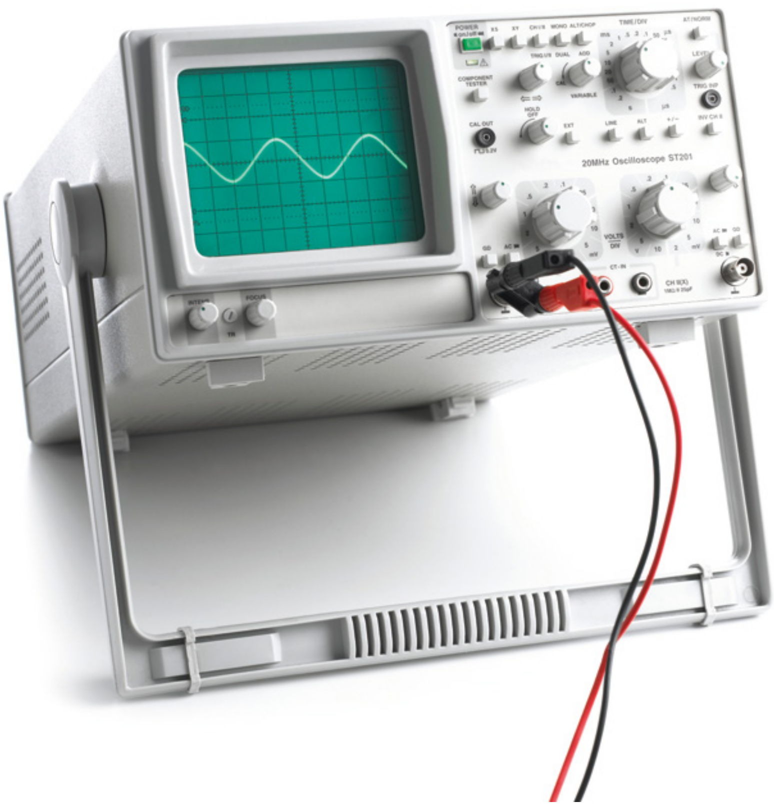


FIGURE 7.1.6 An oscilloscope measures the shape and parameters of electrical signals. Sound is converted to an electrical signal by a microphone and displayed on the oscilloscope.

PHYSICSFILE

Water waves

Water waves are often classified as transverse waves, but this is an approximation. In practical situations, transverse and longitudinal waves don't always occur in isolation. The breaking of waves on a beach produces complex wave forms which are a combination of transverse and longitudinal waves (Figure 7.1.8).

If you looked carefully at a cork bobbing about in gentle water waves you would notice that it doesn't move straight up and down but that it has a more elliptical motion. It moves up and down, and very slightly forwards and backwards as each wave passes. However, since this second aspect of the motion is so subtle, in most circumstances it is adequate to treat water waves as if they were purely transverse waves.

FIGURE 7.1.8 Even though this surfer rides forward on the wave, the water only moves in an elliptical motion as the wave passes.

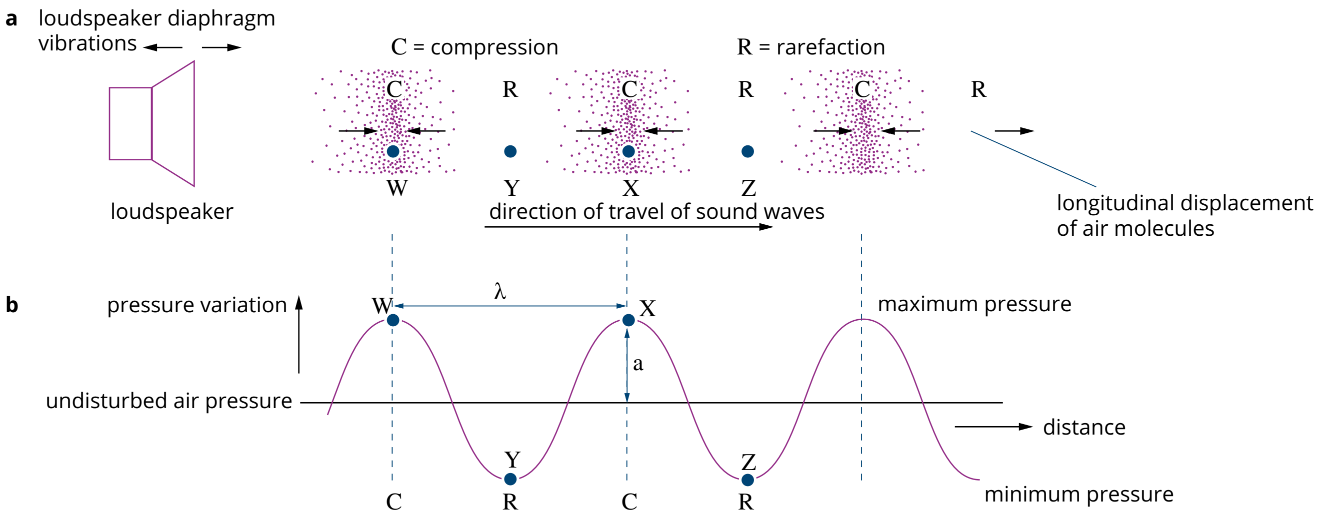


FIGURE 7.1.7 A longitudinal wave compared to a transverse wave. The compressions in a longitudinal wave are equivalent to a crest or peak in a transverse wave, and the rarefactions are equivalent to a trough.

7.1 Review

SUMMARY

- Vibrating objects transfer energy through waves travelling outwards from the source. Waves on water, on a string and sound waves in air are examples of mechanical waves.
- A wave may be a single pulse or it may be continuous or periodic (successive crests and troughs or compressions and rarefactions).
- A wave only transfers energy from one point to another. There is no net transfer of matter or material.
- The particles within the medium undergo small vibrations or oscillations about a mean position.
- Mechanical waves can be either transverse or longitudinal.
- In a transverse wave, the oscillations occur about a mean position and are perpendicular to the direction in which the wave energy is travelling. A wave in a string is an example of a transverse wave.
- In a longitudinal wave the particles or medium oscillate back and forth about a mean position. The oscillations are parallel to (along) the direction the wave energy is travelling. Sound is an example of a longitudinal wave.
- When longitudinal sound waves are converted to electrical signals by a microphone and displayed on an oscilloscope, the signal forms a transverse wave, where the compressions are represented as crests and the rarefactions as troughs.

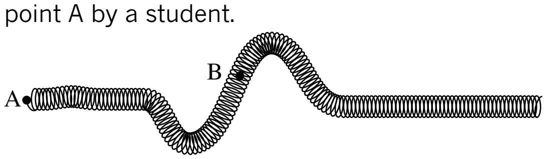
KEY QUESTIONS

1 Describe the motion of particles within a medium as a mechanical wave passes through the medium.

2 State whether the following statements are true or false. For the false statements, rewrite them so they become true.

- **a** Longitudinal waves occur when particles of the medium vibrate in the opposite direction to the direction of the wave.
- **b** Transverse waves are created when the direction of vibration of the particles is at right angles to the direction of the wave.
- **c** A longitudinal wave is able to travel through air.
- **d** The vibrating string of a guitar is an example of a transverse wave.

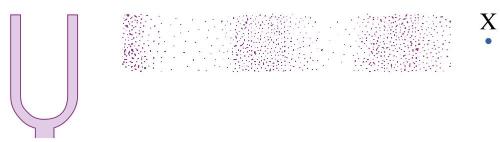
3 The diagram below represents a slinky spring held at point A by a student.



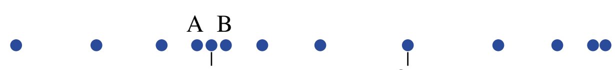
Draw an image of the pulse a short time after that shown in the diagram and determine the motion of point B. Will point B move upwards or downwards, or is it stationary?

4 Which of the following are everyday examples of mechanical waves?
light, sound, ripples on a pond, vibrations in a rope

5 Describe how the energy from the tuning fork in the diagram is transferred to point X. Justify your answer.



6 The diagram below shows dots representing the average displacement of air particles at one moment in time as a sound wave travels to the right.



Describe how particles A and B have moved from their equally spaced undisturbed positions to form the compression.

7 A mechanical wave may be described as transverse or longitudinal. In a *transverse* wave, how does the motion of the particles compare with the direction of travel of the wave?

8 Classify the waves described below as either longitudinal or transverse:

- **a** sound waves
- **b** a vibrating guitar string
- **c** slinky moved with an upward pulse
- **d** slinky pushed forwards and backwards.

7.2 Representing waves

In this section, you will explore how the displacement of particles within the wave can be represented using graphs. From these graphs, several key features of a wave can be identified:

- amplitude
- wavelength
- frequency
- period
- speed.

You will also learn how to do calculations using these features.

Transverse waves of different amplitudes and wavelengths can be seen in Figure 7.2.1.

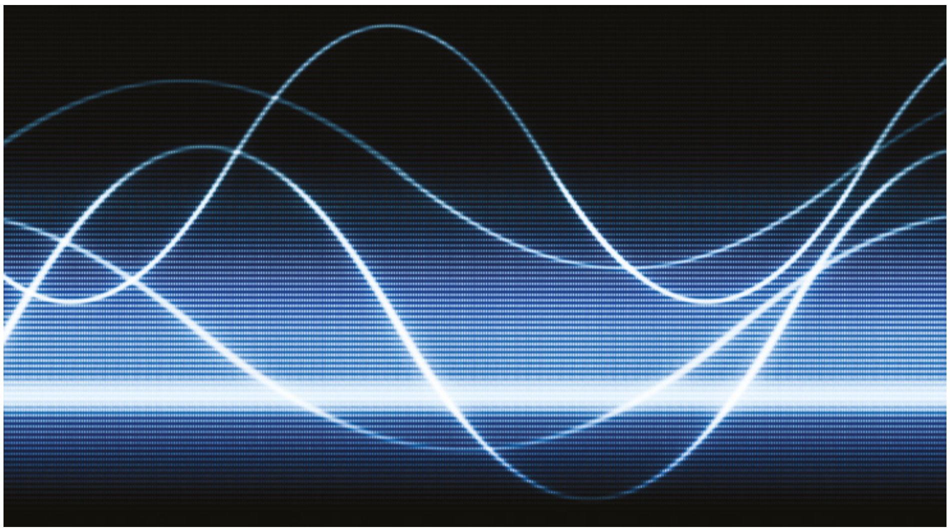


FIGURE 7.2.1 Waves can have different wavelengths, amplitudes, frequencies, periods and velocities, which can all be represented on a graph.

DISPLACEMENT–DISTANCE GRAPHS

The displacement–distance graph in Figure 7.2.2 shows the displacement of all particles along the length of a transverse wave at a particular point in time.

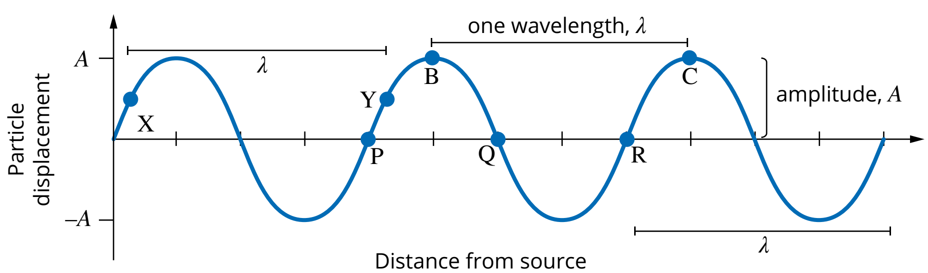


FIGURE 7.2.2 A sine wave representing the particle displacements along a wave over a distance.

Have a look back at Figure 7.1.2b on page 231, which shows a continuous wave in a slinky. This 'snapshot' in time shows the particles moving up and down **sinusoidally** about a central rest position. As a wave passes a given point, the particle at that point will go through a complete cycle before returning to its starting point. The whole wave spread along the length of the slinky has the shape of a sine or cosine function, which you will recognise from mathematics. A displacement–distance graph shows the position (displacement) of the particles along the slinky about a central position at a particular moment in time.

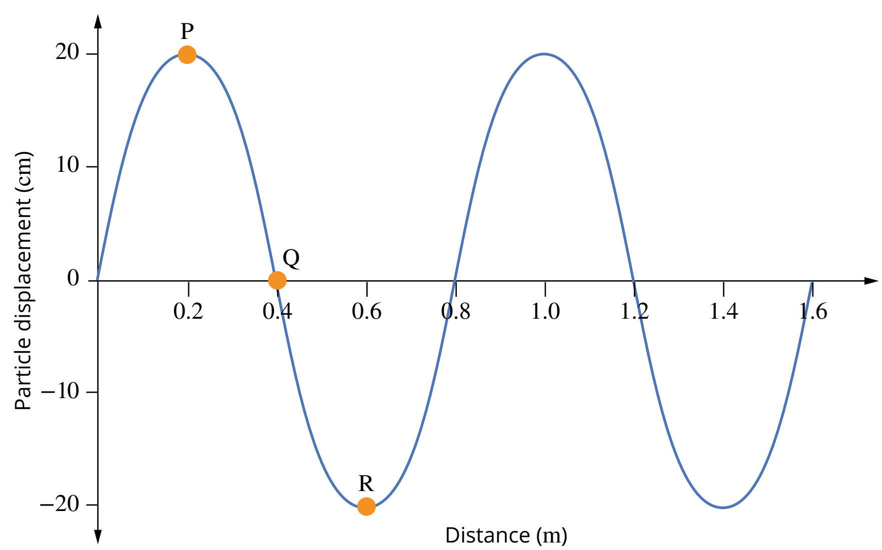
From a displacement–distance graph, the amplitude and wavelength of a wave are easily recognisable.

- The **amplitude** of a wave is the maximum displacement of a particle from the average or rest position and is indicated on the y-axis by A and $-A$. In other words, the amplitude is the maximum displacement from the x-axis to the top of a crest (A) or to the bottom of a trough ($-A$). Points B and C on the sinusoidal curve in Figure 7.2.2 show the positions of the crests.
- The **wavelength** of a wave is the distance between any two successive points in phase (e.g. points B and C, or X and Y in Figure 7.2.2). It is denoted by the Greek letter λ (lambda) and is measured in metres. Two particles on the wave are said to be in **phase** if they have the same displacements from the average position and are moving in the same direction. Points P and R in Figure 7.2.2 are two particles that are in phase, as are points B and C, and also X and Y. However, P and Q are not in phase as they are half a wavelength apart and are moving in opposite directions.
- The **frequency**, f, is the number of complete cycles that pass a given point per second and is measured in hertz (Hz). By drawing a series of displacement–distance graphs at various times, you can see the motion of the wave. By comparing the changes in these graphs, the travelling speed and direction of the wave can be found, as well as the direction of motion of the vibrating particles.

Worked example 7.2.1

DISPLACEMENT–DISTANCE GRAPHS

The displacement–distance graph below shows a snapshot of a transverse wave as it travels along a spring towards the right. Use the graph to determine the amplitude and the wavelength of this wave.

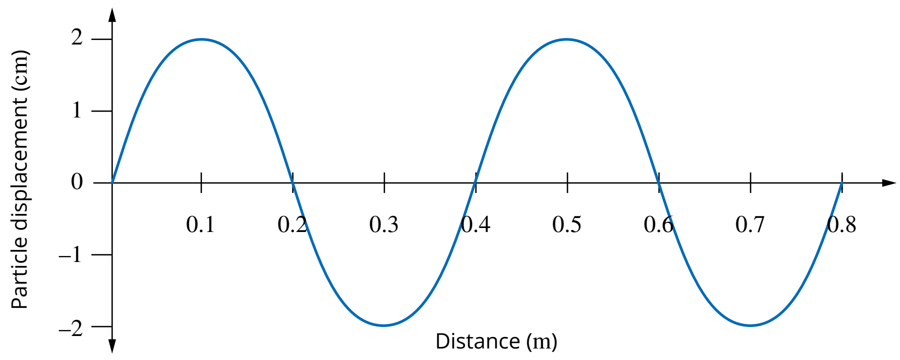


Thinking	Working
Amplitude on a displacement–distance graph is the distance from the average position (Q) to a crest (P) or a trough (R). Read the displacement of a crest or a trough from the vertical axis. Convert to SI units where necessary.	amplitude = 20.0 cm = 0.200 m
Wavelength is the distance for one complete cycle. Any two consecutive points in phase and at the same position on the wave could be used.	The first cycle runs from the origin through P, Q, R to intersect the horizontal axis at 0.80 m. This intersection is the wavelength. wavelength $\lambda = 0.80$ m

Worked example: Try yourself 7.2.1

DISPLACEMENT–DISTANCE GRAPHS

The displacement–distance graph below shows a snapshot of a transverse wave as it travels along a spring towards the right. Use the graph to determine the wavelength and the amplitude of this wave.



DISPLACEMENT–TIME GRAPHS

A displacement–time graph such as the one shown in Figure 7.2.3 tracks the position of one point over time as the wave moves through that point.

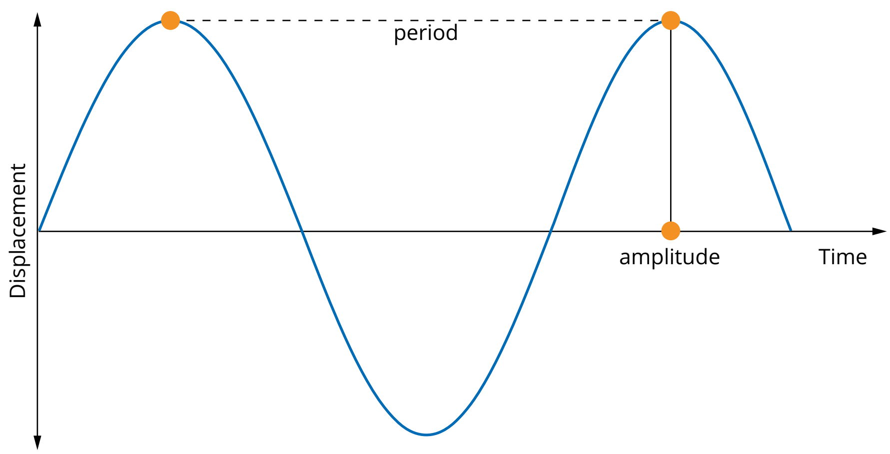


FIGURE 7.2.3 The graph of displacement versus time from the source of a transverse wave shows the movement of a *single point* on a wave over a period of time, as the wave passes through that point.

The displacement–time graph looks very similar to a displacement–distance graph of a transverse wave, so be careful to check the horizontal axis label.

Crests and troughs are shown the same way in both graphs. The amplitude is still the maximum displacement from the average or rest position of either a crest or a trough. But the distance between two successive points in phase in a displacement–time graph represents the **period** of the wave, T, measured in seconds.

The period is the time it takes for any point on the wave to go through one complete cycle (e.g. from crest to successive crest).

The period of a wave is inversely related to its frequency:

$$T = \frac{1}{f}$$

where T is the period of the wave (s)

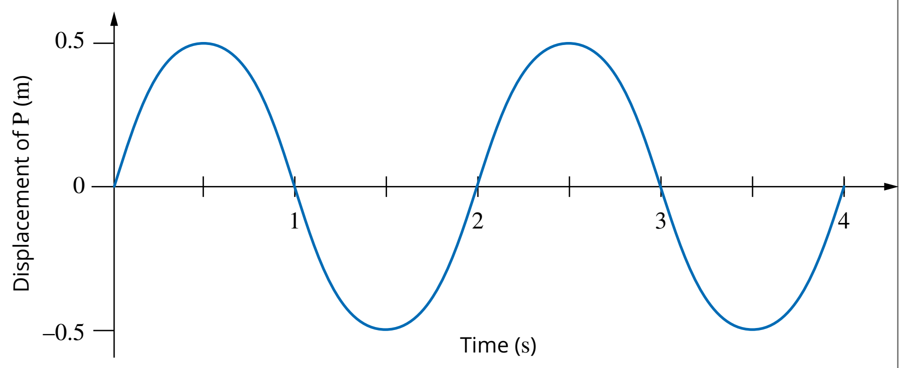
f is the frequency of the wave (Hz or s^{-1}).

The amplitude and period of a wave can be determined directly from a displacement–time graph.

Worked example 7.2.2

DISPLACEMENT–TIME GRAPHS

The displacement–time graph below shows the motion of a single part of a rope (point P) as a wave passes by travelling to the right. Use the graph to find the amplitude, period and frequency of the wave.

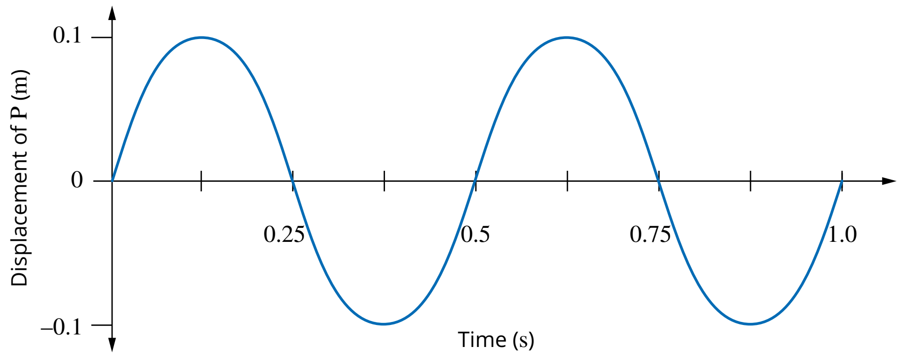


Thinking	Working
The amplitude on a displacement–time graph is the displacement from the average position to a crest or trough. Note the displacement of successive crests and/or troughs on the wave and carefully note units on the vertical axis.	Maximum displacement = 0.50 m Therefore, amplitude = 0.50 m
Period is the time it takes to complete one cycle and can be identified on a displacement–time graph as the time between two successive points that are in phase. Identify two points on the graph at the same position in the wave cycle, e.g. the origin and $T = 2.0$ s. Confirm by checking two other points, e.g. two crests or two troughs.	Period, $T = 2.0$ s
Frequency can be calculated using $f = \frac{1}{T}$, measured in hertz (Hz).	$f = \frac{1}{T}$ $f = \frac{1}{(2.0)}$ $f = 0.50000$
Express the answer with the correct number of significant figures and the appropriate unit.	$f = 0.50$ Hz

Worked example: Try yourself 7.2.2

DISPLACEMENT–TIME GRAPHS

The displacement–time graph below shows the motion of a single part of a rope as a wave passes travelling to the right. Use the graph to find the amplitude, period and frequency of the wave.



Worked example 7.2.2 and Worked example: Try yourself 7.2.2 represent travelling waves. In both cases, every particle on the rope would have a maximum displacement at some point in time as the wave travels down the rope, and similarly every particle would have a minimum displacement at some point in time.

THE WAVE EQUATION

There is a relationship between the speed of a wave and the other significant wave characteristics of frequency, period and wavelength.

Think back to the study of motion in Chapter 3. Speed is given by:

$$v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{d}{\Delta t}$$

This can be rewritten in terms of the distance of one wavelength (λ) in one period (T), which will be:

$$v = \frac{\lambda}{T}$$

and since

$$f = \frac{1}{T}$$

the relationship becomes:

$v = f\lambda$

where v is the speed (m s^{-1})

f is the frequency (Hz)

λ is the wavelength (m)

This is known as the wave equation and applies to both longitudinal and transverse mechanical waves.

Worked example 7.2.3

THE WAVE EQUATION 1

A longitudinal wave has a wavelength of 2.00 m and a speed of 344 m s^{-1}. What is the frequency, f, of the wave?	
Thinking	**Working**
The wave equation states that $v = f\lambda$. Knowing both v and λ, the frequency, f, can be found. Rewrite the wave equation in terms of f.	$v = f\lambda$ $f = \frac{v}{\lambda}$
Substitute the known values and solve.	$f = \frac{v}{\lambda}$ $f = \frac{(344)}{(2.00)}$ $f = 172.000$
Express the answer with the correct number of significant figures and the appropriate unit.	$f = 172$ Hz

Worked example: Try yourself 7.2.3

THE WAVE EQUATION 1

A longitudinal wave has a wavelength of 3.00 m and a speed of 1484 m s^{-1}. What is the frequency, f, of the wave?

Worked example 7.2.4

THE WAVE EQUATION 2

A longitudinal wave has a wavelength of 2.00 m and a speed of 344 m s^{-1}. Calculate the period, T, of the wave.	
Thinking	**Working**
Rewrite the wave equation in terms of T.	$v = f\lambda$ and $f = \frac{1}{T}$ $v = \frac{\lambda}{T}$ $T = \frac{\lambda}{v}$
Substitute the known values and solve.	$T = \frac{\lambda}{v}$ $T = \frac{(2.00)}{(344)}$ $T = 5.81395 \times 10^{-3}$
Express the answer with the correct number of significant figures and the appropriate unit.	$T = 5.81 \times 10^{-3}$ s

Worked example: Try yourself 7.2.4

THE WAVE EQUATION 2

A longitudinal wave has a wavelength of 3.00 m and a speed of 1484 m s^{-1}. Calculate the period, T, of the wave.

THE DOPPLER EFFECT

The **Doppler effect** is a phenomenon of waves that is observed whenever there is relative movement between the source of the waves and an observer. Named after Austrian physicist Christian Doppler, who proposed it in 1842, the Doppler effect only affects the *apparent* frequency of the wave. The actual frequency of the wave does not change. A common experience of the Doppler effect is in listening to the sound of a siren from an emergency vehicle as it approaches and passes by.

Suppose a wave source, such as an ambulance siren, is stationary relative to an observer, as shown in Figure 7.2.4a. The observer will receive and hear the disturbances (rarefactions and compressions in this example) at the same rate as the source creates them.

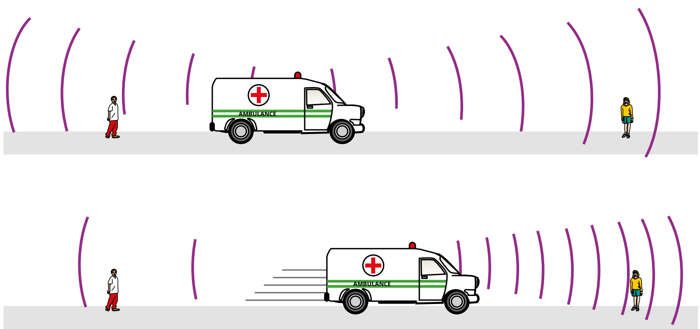


FIGURE 7.2.4 The Doppler effect. An object emitting a sound moving towards an observer (on the right) will emit sound waves closer together in its direction of travel and hence a higher frequency is heard by the observer. When the object is moving away from the observer (on the left), the sound waves are emitted further apart and hence a lower frequency is heard by the observer.

If, on the other hand, the wave source (the ambulance) were to travel towards the observer on the right in Figure 7.2.4b, then each consecutive disturbance would originate from a position a little closer than the previous one. The effective wavelength of the wave fronts is less, and therefore the frequency of the disturbances is higher than the originating frequency. As a result, the observer hears a higher frequency or pitch. Alternatively, if the source (the ambulance) is moving away from the observer (as shown on the left in Figure 7.2.4b), then each consecutive disturbance will originate from a distance a little further away than the one before, and so the effective wavelength of the wave fronts is greater. The disturbances will arrive at the observer less frequently—that is, less than the originating frequency—and so they will hear a lower pitched sound.

The net effect is that when the wave is moving towards an observer, the frequency of the wave will be higher than the frequency of the original source. When the wave is moving away from the observer, the frequency will be lower than the frequency of the original source. Therefore, because of the Doppler effect, a siren will appear to rise in frequency as the vehicle travels towards you and fall as it moves away.

For a mechanical wave, the Doppler effect may result from the motion of the source, the motion of the observer, or the motion of the medium the wave travels through. For light waves that don't require a medium, only the relative difference in speed between the observer and the source will contribute to the effect. As will be discussed in Year 12, the speed of light is constant to the observer, regardless of the speed of the observer or the source.

PHYSICSFILE

Police Doppler radar

A police radar gun emits a continuous microwave frequency, usually between 2 and 24 GHz, where a GHz is equal to 10^9 Hz. (Note that microwaves are electromagnetic radiation with a lower frequency than visible light.) The microwave travels towards the approaching car at constant frequency and wavelength. However, the waves reflected back to the radar gun from the approaching car are compressed due to the Doppler effect, and so the frequency is increased. The speed of the oncoming car is determined from the shift in the frequency.

FIGURE 7.2.5 Modern speed cameras use the Doppler effect to check motorists' speed in a variety of situations: at traffic lights, outside schools, on country roads, and in random locations.

EXTENSION

Doppler calculations

This study only requires a qualitative understanding of the Doppler effect as a wave phenomenon. However, as the relative motion between the observer and source is the cause of the change in apparent frequency (the frequency observed by the observer), by knowing what the relative motion is, the apparent frequency can be calculated.

In classical physics, where both the speed of the source and the observer are lower than the speed of the waves in the medium, and the source and observer are approaching each other directly, the apparent or observed frequency f (in Hz) is given by:

$$f = \left(\frac{v + v_0}{v - v_s}\right) f_0$$

where f_0 is the original frequency (Hz)

v is the speed of the waves in the medium ($m\,s^{-1}$).

v_0 is the speed of the observer relative to the medium ($m\,s^{-1}$). v_0 is positive if the observer is moving towards the source and negative if the observer is moving away.

v_s is the speed of the source relative to the medium ($m\,s^{-1}$). v_s is positive if the source is moving towards the observer and negative if the source is moving away.

As an approximation, if the speeds of the source and observer are small relative to the speed of the wave, then the approximate observed frequency is:

$$f = \left(1 + \frac{\Delta v}{v}\right) f_0, \text{ where } \Delta v = v_0 - v_s$$

and the approximate apparent change in frequency is

$$\Delta f = \left(\frac{\Delta v}{v}\right) f_0, \text{ where } \Delta f = f - f_0.$$

An interesting additional effect was predicted by British physicist Lord Rayleigh. He predicted that if the source is moving at double the speed of sound, a musical piece emitted by the source would be heard in correct time and frequency, but *backwards*. Try to establish whether his prediction is true mathematically using the formulas provided above.

7.2 Review

SUMMARY

- Waves can be represented by displacement–distance graphs and displacement–time graphs.
- From a displacement–distance graph, you can directly determine the amplitude and wavelength of the wave.
- From a displacement–time graph, you can directly determine the amplitude and period of the wave.
- The period of a wave has an inverse relationship to its frequency, according to the relationship:

$$T = \frac{1}{f}$$

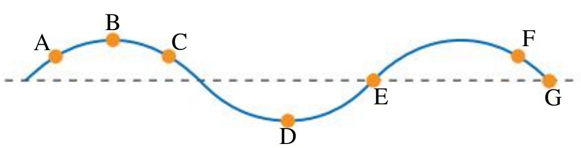
- The speed of a wave can be calculated using the wave equation:

$$v = f\lambda = \frac{\lambda}{T}$$

- The Doppler effect is a phenomenon that is observed whenever there is relative movement between the source of waves and an observer. It causes an apparent *increase* in frequency when the relative movement is *towards* the observer and an apparent *decrease* in frequency when the relative movement is *away* from the observer.
- For a mechanical wave, the total Doppler effect may result from the motion of the source, the motion of the observer, or the motion of the medium.

KEY QUESTIONS

1 Using the displacement–distance graph below, give the correct word or letters for the following:



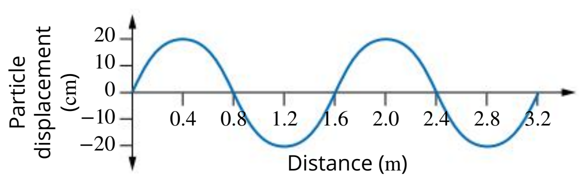
a two points on the wave that are in phase

b the name for the distance between these two points

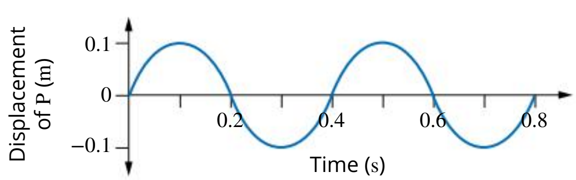
c the two particles with maximum displacement from their rest position

d the term for this maximum displacement.

2 Use the graph below to determine the wavelength and the amplitude of this wave.



3 This is the displacement–time graph for a particle P.



Calculate:

a the period of the wave

b the frequency of the wave.

4 Kathy measured 5.00 wavelengths of a wave passing a point each second. The amplitude was found to be 0.310 m and the distance between the successive crests was 1.35 m. What was the speed of the wave?

5 Which of the following is true and which is false? For the false statements, rewrite them to make them true.

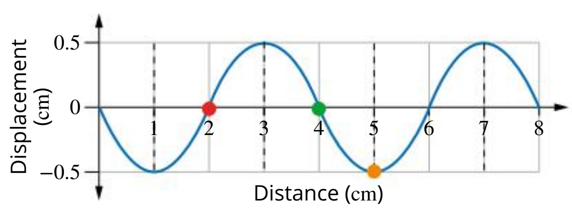
a The frequency of a wave is inversely proportional to its wavelength.

b The period of a wave is inversely proportional to its wavelength.

c The amplitude of a wave is not related to its speed.

d Only the wavelength of a wave determines its speed.

6 Consider the displacement–distance graph below.



a State the wavelength and amplitude of the wave.

b If the wave moves through one wavelength in 2.00 s, what is the speed of the wave?

c If the wave is moving to the right, which of the coloured particles is moving down?

7 Calculate the period of a wave with frequency 2.55×10^5 Hz.

8 A police car, travelling at 105 km h^{-1} along a straight road, has its siren sounding. The police car is pursuing another car travelling in the same direction, also at 105 km h^{-1}. There is no wind at the time. Would an observer in the car being pursued hear the siren from the police car at a higher, lower, or at the same frequency as it is emitted? Explain your answer.

9 An ambulance sounding its siren in still air moves towards you, then passes you and continues to move away in a straight path. How would the siren sound to you over this period of time?

7.3 Wave behaviours—reflection, refraction and diffraction

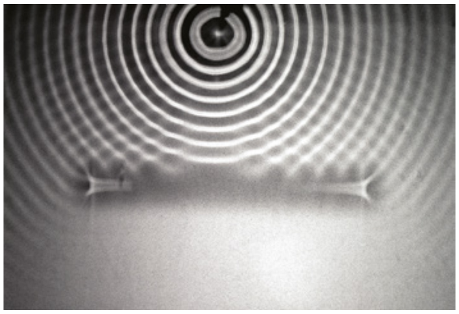


FIGURE 7.3.1 The reflection of spherical waves in a ripple tank when meeting a solid surface.

Mechanical waves transfer energy through a medium, but there will be times when that medium physically ends, such as when a water meets the edge of a pool or air meets a wall. A change in the physical characteristics in the same medium, such as density and temperature, can act like a change in medium. When the medium ends, or changes, the wave doesn't just stop. Instead, the energy that the wave is carrying will undergo three processes:

- some energy will be *reflected* (Figure 7.3.1)
- some energy will be *absorbed* by the new medium
- some energy will be *transmitted*.

The degree to which each occurs will depend upon the differences in properties between the original and new media. Energy transfers and transformations can be understood by investigating wave behaviours.

REFLECTION

When a transverse wave pulse reaches a hard surface, such as the fixed end of a rope, the wave is bounced back or **reflected**.

Reflection characteristics

When the end of the rope is fixed, the reflected pulse is inverted (Figure 7.3.2). So, for example, a wave crest would be reflected as a trough.

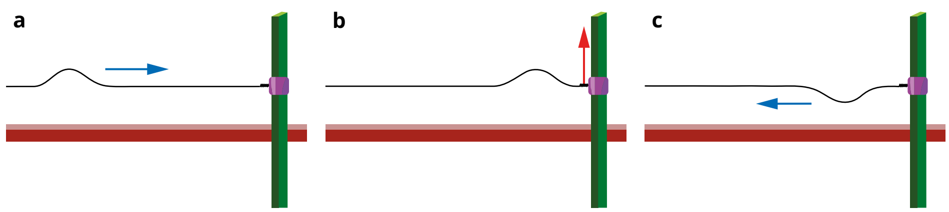


FIGURE 7.3.2 (a) A wave pulse moves along a string to the right and approaches a fixed post. (b) On reaching the end, the string exerts an upwards force on the fixed post. Due to Newton's third law, the fixed post exerts an equal and opposite force on the string which (c) inverts the wave pulse and sends its reflection back to the left on the bottom side of the string. There is a phase reversal on reflection from a fixed end.

This inversion can also be referred to as a 180° change of phase or, expressed in terms of the wavelength, λ, a phase shift of $\frac{\lambda}{2}$.

When a wave pulse hits the end of a rope that is free to move (known as a free boundary), the pulse returns with no change of phase (Figure 7.3.3). That is, the reflected pulse is the same as the incident pulse. A crest is reflected as a crest and a trough is reflected as a trough.

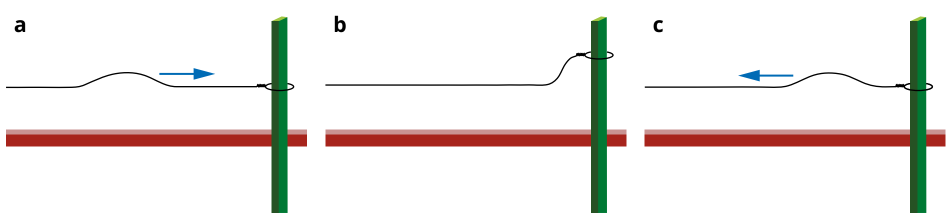


FIGURE 7.3.3 (a) A wave pulse moves along a string to the right and approaches a free end at the post. (b) On reaching the post the free end of the string is free to slide up the post. (c) No inversion happens, and the wave pulse is reflected back to the left on the same side of the string, i.e. there is no phase reversal on reflection from a free end.

When the transverse wave pulse is reflected, the amplitude of the reflected wave isn't quite the same as the original. Part of the energy of the wave is **absorbed** by the post, where it will be transformed into heat energy.

When a transverse wave pulse is sent from a light rope to a heavier rope, a large proportion of the wave pulse will be reflected back towards the source and a smaller proportion of it will be **transmitted** to the heavier rope, as shown in Figure 7.3.4. This is just the same as a wave pulse striking a wall. The more rigid and/or dense the wall, the more the wave energy will be reflected and the less it will be absorbed—but there will always be some energy that is absorbed by or transferred to the second medium. This explains why sound can travel through walls.

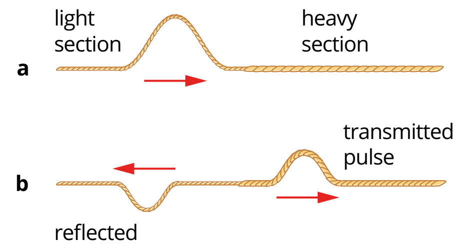


FIGURE 7.3.4 (a) A wave pulse travels along a light rope towards a heavier rope. (b) On reaching a change in density, the wave pulse will be partly reflected and partly transmitted. This is analogous to a change in medium.

Reflected wave fronts

Two- and three-dimensional waves, such as water waves, travel as **wave fronts**. When drawing wave fronts (Figure 7.3.5), it is common to show only the crests of the waves. When close to the source, wave fronts can show considerable curvature (Figure 7.3.5a) or may even be spherical when generated in three dimensions. Where a wave has travelled a long distance from its source, the wave front is nearly straight and is called a **plane wave**. A plane wave is shown in Figure 7.3.5b. Plane waves can also be generated by a long, flat source such as those often used in a ripple tank found in some school laboratories.

The direction of motion of any wave front can be represented by a line drawn perpendicular to the wave front and in the direction that the wave is moving, as shown by the blue arrows in Figure 7.3.5. This is called a **ray**. Rays can be used to study or illustrate the properties of two- and three-dimensional waves without the need to draw individual wave fronts.

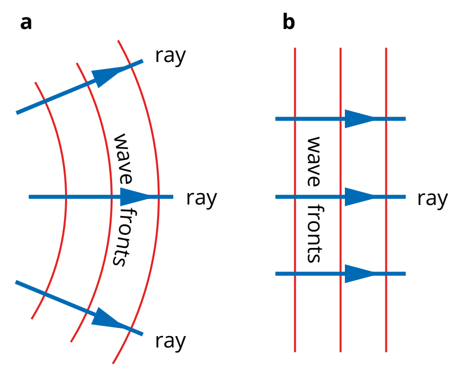


FIGURE 7.3.5 Rays can be used to illustrate the direction of motion of a wave. They are drawn perpendicular to the wave front of a two- or three-dimensional wave and in the direction of travel of the wave; (a) illustrates rays for circular wave fronts near a point source while (b) shows rays with plane wave fronts.

By using rays to illustrate the path of a wave front reflecting from a surface, it can be shown that for a two-dimensional or three-dimensional wave, the angle from the **normal** at which the incident wave's ray strikes a surface will equal the angle from the normal to the reflected wave's ray. The normal is the dotted line at 90°, i.e. perpendicular to the surface.

These angles of the incident and reflected waves from the normal are labelled θ_i and θ_r, respectively, in Figure 7.3.6.

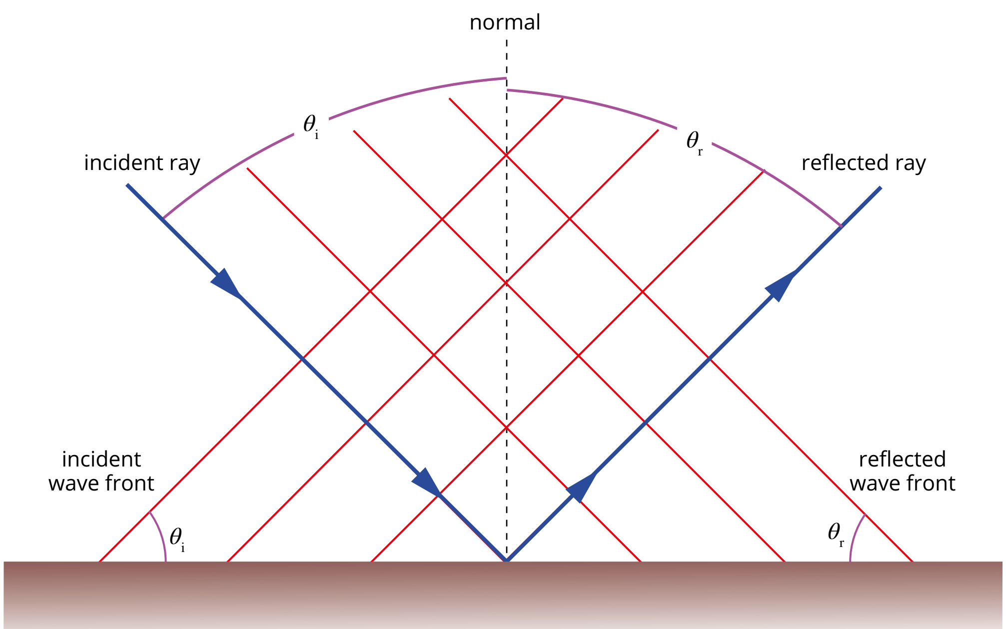


FIGURE 7.3.6 The law of reflection. The angle between the incident ray and the normal (θ_i) is the same as the angle between the normal and the reflected ray (θ_r).

This is referred to as the law of reflection.

> The law of reflection states that the **angle of reflection** (θ_r), measured from the normal, equals the **angle of incidence** (θ_i) measured from the normal:
> $\theta_i = \theta_r$.

The law of reflection is true for any surface whether it is straight, curved or irregular. For all surfaces, including curved or irregular surfaces, the normal is drawn perpendicular to the surface at the point of contact of the incident ray or rays.

When wave fronts meet an irregular, rough surface, the resulting reflection can be spread over a broad area. This is because each point on the surface may reflect the portion of the wave front reaching it in a different direction, as seen in Figure 7.3.7. This is referred to as **diffuse** reflection.

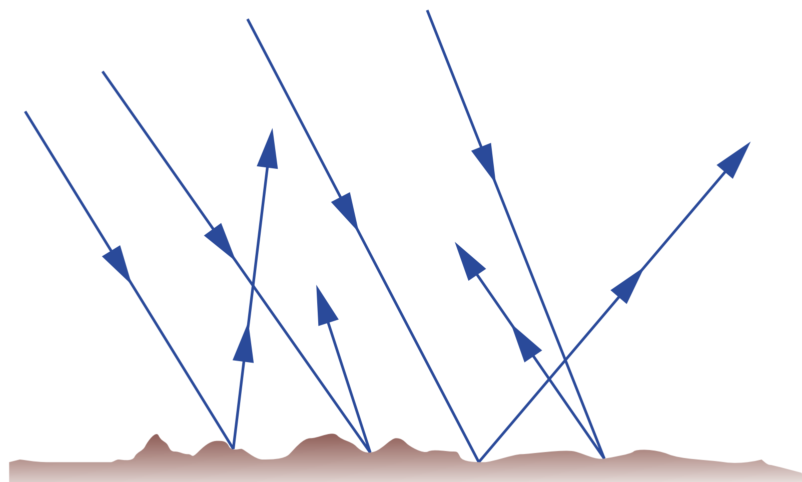


FIGURE 7.3.7 Reflection from an irregular surface. Each incident ray may be reflected in a different direction, depending upon how rough or irregular the reflecting surface is. The resulting wave will be diffuse (spread out).

PHYSICS IN ACTION

Sonic depth finder

An echo-sounding device, called a sonar, makes it possible to measure the depth of the sea (Figure 7.3.8). A sound wave is emitted from the device and is reflected by the seabed. The sonar device measures the time taken for the echo to return to the ship. This value, along with the speed of sound in water, can then be used to calculate the depth of water.

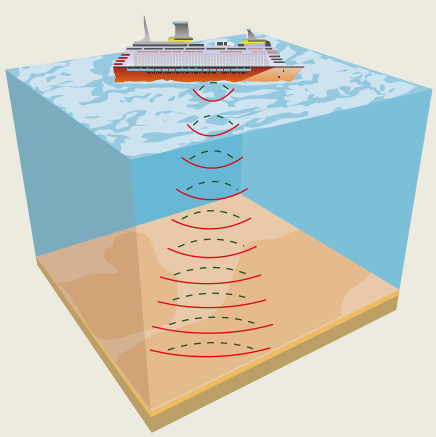


FIGURE 7.3.8 The ship sends a sound wave into the water below the ship. By measuring the time between the emitted sound (red waves) and its reflection (dashed lines), the ship's sonar can determine the depth of the water under the ship.

Echoes and reverberation

Echoes provide the most obvious evidence that sound waves are reflected. Like all waves, sound can be reflected when it strikes an obstacle.

When a sound wave that is reflected at a wall reaches us in about 0.10 seconds, it produces an echo that is heard as a separate sound to the original. Those 0.10 seconds are important as this is the time it takes for our ears to 'reset' before hearing a separate sound; any sound heard within this period of time will 'merge' with the previous sound. In smaller rooms where the period of time taken for the sound to reflect back to us is less than 0.10 seconds, the waves that are reflected multiple times off different walls will overlap and produce **reverberation**, which we hear as a longer, richer sound. Echoes typically occur in large rooms or caverns with bare walls and hard surfaces. You may have noticed this in a vacant house. Simple ways to reduce echoes include using acoustic foam sheets, rugs on the floor, and textiles on the wall to absorb the sound and objects in the room to diffuse the reflections of the sound waves, all of which play a major role in acoustic engineering.

REFRACTION

As discussed previously, when a wave hits a boundary between two media, some of the wave's energy is reflected, some is transmitted, and some is absorbed by the new medium. The velocity of the wave absorbed by the second medium will be affected by the properties of the new medium, which will also affect the wavelength of the wave in the new medium.

When a wave crosses the boundary from one medium to another at an angle other than zero degrees to the normal, and where there is a change in velocity and wavelength, the transmitted wave travels in a different direction to the original wave. This is referred to as **refraction**, and is the same phenomenon observed for light when a spoon appears broken at the interface between air and water in a glass. In Figure 7.3.9 the wave is travelling faster in the first medium and slower in the second medium, and so it changes direction.

Wave fronts refract because the part of the wave front that reaches the boundary first slows down (or speeds up), while the remainder of the wave front still in the original medium continues at its original speed.

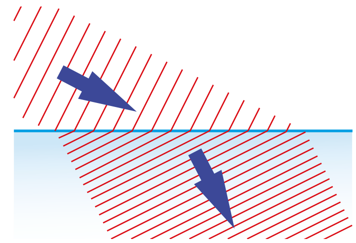


FIGURE 7.3.9 Refraction of wave fronts passing the boundary from one medium to another occurs when the speed of the wave differs between the two media. In this figure, the wave is travelling faster in the first medium and slower in the second, and so it changes direction.

The speed of sound

The speed of sound can be quite different in various media and depends on two main parameters: stiffness or **elasticity**, and inertia or density. The elasticity of a medium is a measure of the tendency of a medium to return to its original shape after it has been bent, compressed or stretched. A solid is significantly more elastic than a liquid, and a liquid is more elastic than a gas. When sound propagates through a medium in a series of compressions and rarefactions, it travels faster in a more elastic medium. Therefore, sounds travels fastest in solids and slowest in gases. If there is a change in state as a sound wave travels from one medium to the next, there will be a significant impact on the speed.

The density also has an effect on the speed. The more dense the medium is, the slower the speed. This is because the greater mass per unit volume makes it more difficult for a given force to move the medium, so vibrations take more time.

The speed of sound depends on the medium's stiffness or elasticity, called Young's modulus, E, and its density, ρ. It is given by the expression:

$$v = \sqrt{\frac{E}{\rho}}$$

where v is the speed of sound (m s^{-1})

E is Young's modulus (N m^{-2})

ρ is the density of the medium (kg m^{-3}).

The speed of sound for a selection of substances is given in Table 7.3.1.

TABLE 7.3.1 The speed of sound for a selection of substances.

State of matter	Substance	Speed of sound
solid	diamond	$12\,000\ \text{m s}^{-1}$
	copper	$3560\ \text{m s}^{-1}$
liquid	water	$1493\ \text{m s}^{-1}$
	mercury	$1450\ \text{m s}^{-1}$
	kerosene	$1324\ \text{m s}^{-1}$
gas	air (20°C)	$343\ \text{m s}^{-1}$
	air (0°C)	$331\ \text{m s}^{-1}$
	helium (0°C)	$972\ \text{m s}^{-1}$
	hydrogen (0°C)	$1286\ \text{m s}^{-1}$

Temperature also affects the speed of sound.

Increasing the temperature of a medium reduces its density without changing its elasticity, so it increases the speed that sound can travel through it.

Conversely, decreasing the temperature of a medium increases its density, therefore decreasing the speed that sound can travel through it. Refraction can therefore occur at the interface between a hot and a cold medium.

Refraction of sound

Figure 7.3.10 shows examples of refraction for sound waves. The normal, shown by the dashed line, is drawn at 90° to the boundary between the two media (the green line). The refraction of the wave is measured by comparing the **angle of incidence**, *i*, to the **angle of refraction**, *r*. The change in direction of the wave is described as being towards the normal, or away from the normal, when compared to the original direction of the sound.

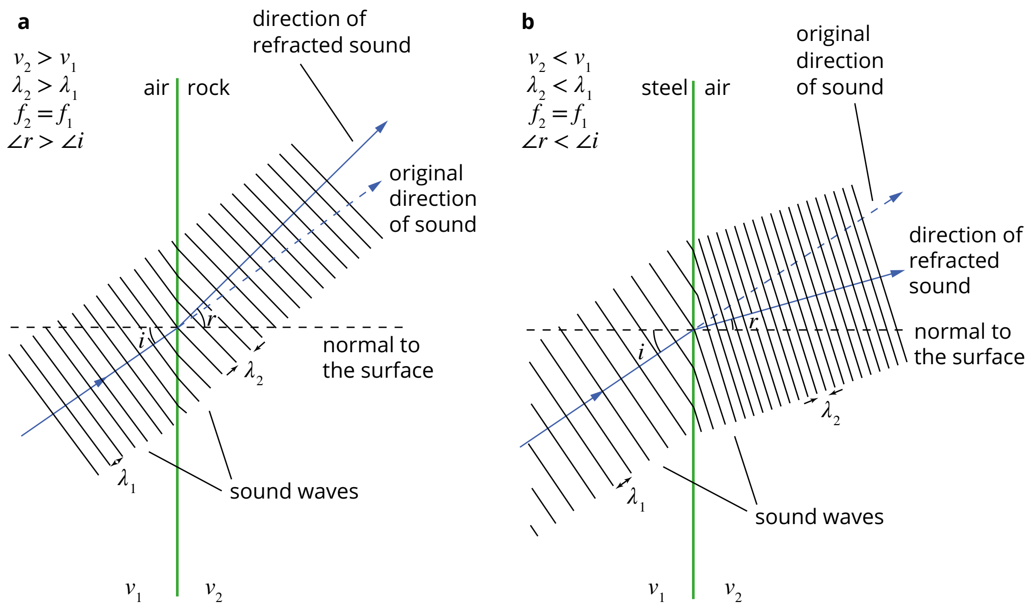


FIGURE 7.3.10 (a) The speed of the wave front in the second medium (the rock) is faster than in the first medium (air) and the wave front bends away from the normal. (b) The speed of the wave front in the second medium (air) is slower than in the first medium (steel) and the wave front bends towards the normal.

In Figure 7.3.10a the sound wave is travelling from air into rock. Since rock is a solid, the speed of sound in the rock, v_2, is greater than the speed of sound in air, v_1. The frequency of the waves ($f_1 = f_2$) and hence the time between wave fronts, T, remains constant in both media. Recalling the general expression, distance = velocity × time, the distance between the wave fronts is given by:

$$\lambda = vT$$

where $T = \frac{1}{f}$.

Therefore, since the velocity of the wave in the rock is greater ($v_2 > v_1$), the spacing between the wave fronts (the wavelength) will increase ($\lambda_2 > \lambda_1$). This will cause the angle of refraction, *r*, to increase and the direction of the wave front to bend away from the normal ($\angle r > \angle i$).

In Figure 7.3.10b the sound wave is travelling from steel into air. In this case, the speed of sound in air is less than in steel ($v_2 < v_1$). Therefore, the spacing between the wave fronts will decrease ($\lambda_2 < \lambda_1$), the angle of refraction, *r*, will decrease and the direction of the wave front will bend towards the normal ($\angle r < \angle i$).

The relationship can be stated mathematically using Snell's law, named after Dutch astronomer and mathematician Willebrord Snell:

$$\frac{\sin r}{\sin i} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$$

To summarise:

> When a wave front *speeds up* at the boundary between two media, the *wavelength increases* and the wave front will *bend away* from the normal.
>
> When a wave front *slows down* at the boundary between two media, the *wavelength decreases* and the wave front will bend *towards* the normal.
>
> The frequency of the wave is unaffected.
>
> A wave front reaching a boundary perpendicular to the normal will not be affected and will continue in the same direction, regardless of whether it is speeding up or slowing down.

Worked example 7.3.1

REFRACTION

A 514 Hz sound from a rock concert travels from cold air where the speed of sound is 331 $m\,s^{-1}$ into warmer air where the speed of sound is 355 $m\,s^{-1}$, and strikes the boundary between the two regions of air with an angle of incidence of 30.0°.

a Calculate the wavelength in each region of air.

Thinking	Working
The wavelength can be determined from $v = f\lambda$. Identify the variables to be used in the solution.	$f = 514$ Hz $v_{\text{cold air}} = 331\ m\,s^{-1}$ $v_{\text{warm air}} = 355\ m\,s^{-1}$
Rearrange to make λ the subject, $\lambda = \frac{v}{f}$, and solve for each speed of sound.	$\lambda_{\text{cold air}} = \frac{(331)}{(514)}$ $\lambda_{\text{cold air}} = 0.643969$ $\lambda_{\text{warm air}} = \frac{(355)}{(514)}$ $\lambda_{\text{warm air}} = 0.690661$
Express the answers with the correct number of significant figures and the appropriate unit.	$\lambda_{\text{cold air}} = 0.644$ m $\lambda_{\text{warm air}} = 0.691$ m

b Explain what will happen to the angle of refraction, and why.

The speed of sound in warm air is higher than in cold air due to the lower air density of the warmer air.	Since the velocity in warm air is higher than in cold air, the sound will refract away from the normal.

c Calculate the angle of refraction.	
The angle of refraction can be calculated using Snell's law. $\frac{\sin r}{\sin i} = \frac{v_{\text{warm air}}}{v_{\text{cold air}}}$	$\frac{\sin r}{\sin i} = \frac{v_{\text{warm air}}}{v_{\text{cold air}}}$ $\sin r = \frac{v_{\text{warm air}}}{v_{\text{cold air}}} \times \sin i$ $\sin r = \frac{(355)}{(331)} \times \sin 30.0°$ $\sin r = 0.536\,25$ $r = 32.4290$
Express the answer with the correct number of significant figures and the degree symbol.	$r = 32.4°$

Worked example: Try yourself 7.3.1

REFRACTION

Whales typically emit sounds between 10.0 and 40.0 Hz. (Humans can usually hear down to 20.0 Hz.) If a whale emits a 20.0 Hz sound in water towards the surface at an angle of 40.0° to the normal, the refracted wave emerges into air. The speed of sound in air is 343 m s^{-1} and the speed of sound in water is 1484 m s^{-1}.

a Calculate the wavelength of the sound in water and in air.

b Explain what will happen to the refracted wave, and why.

c Determine the angle of refraction.

It should be noted that whenever refraction occurs, a proportion of the wave is also reflected, as shown by the ray diagram in Figure 7.3.11. For example, when light is incident on a window, most of the light penetrates the window and is refracted into the glass, while approximately 4% of the light is reflected.

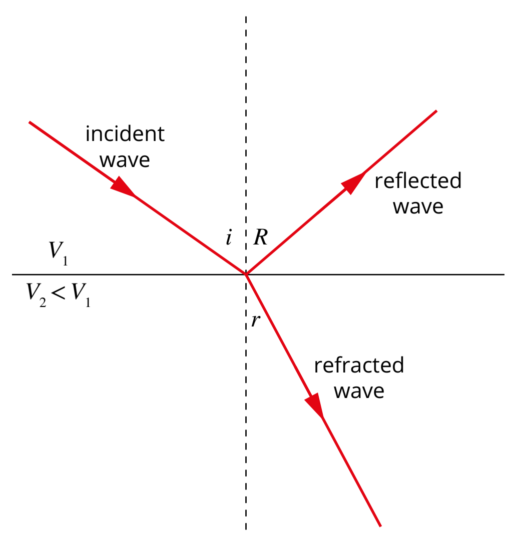


FIGURE 7.3.11 When a wave is incident on a new medium, some of the wave is refracted but a proportion is also reflected.

Total internal reflection

As discussed earlier in this section, when sound enters a medium where its speed is greater, the angle of refraction increases. It follows that as the incident angle gets larger then so will the reflected angle, as shown by rays a and b in Figure 7.3.12. At a particular incident angle known as the **critical angle**, θ_c, the refracted angle, r, will be 90°; that is, it will lie exactly along the interface between the two media, as shown by ray c in Figure 7.3.12. At incident angles greater than the critical angle, the ray is completely reflected (as shown by ray d in Figure 7.3.12). This is known as **total internal reflection**.

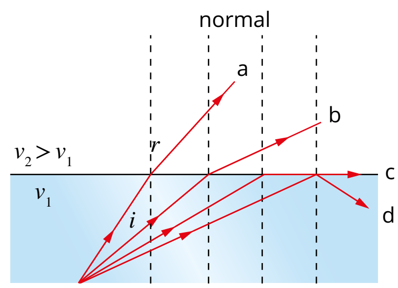


FIGURE 7.3.12 Rays a and b show the refracted angle, r, getting increasingly larger as the incident angle, i, increases. Ray c shows the critical angle where $i = \theta_c$ and the refracted angle, $r = 90°$. Ray d is incident at an angle greater than the critical angle and it reflects rather than refracts.

This can be shown mathematically through Snell's law. If the angle $r = 90°$, then $\sin r = 1$ and $i = \theta_c$ and Snell's law

$$\frac{\sin r}{\sin i} = \frac{v_2}{v_1} \text{ becomes } \sin \theta_c = \frac{v_1}{v_2}$$

Worked example 7.3.2

TOTAL INTERNAL REFLECTION

At what minimum incident angle would sound need to strike water from air if it is to reflect completely? The speed of sound in air is $344\,m\,s^{-1}$ and the speed of sound in water is $1450\,m\,s^{-1}$.	
Thinking	**Working**
The critical angle can be calculated using Snell's law: $\frac{\sin r}{\sin i} = \frac{v_2}{v_1}$ Identify the variables to be used in the solution.	When $\sin r = 1$, $\sin i = \sin \theta_c$ $v_1 = 344\,m\,s^{-1}$ $v_2 = 1450\,m\,s^{-1}$
Rearrange to make angle θ_c the subject and solve for the critical angle.	$\frac{\sin r}{\sin i} = \frac{v_2}{v_1}$ $\frac{1}{\sin \theta_c} = \frac{v_2}{v_1}$ $\sin\theta_c = \frac{v_1}{v_2}$ $\sin\theta_c = \frac{(344)}{(1450)}$ $\sin\theta_c = 0.237241$ $\theta_c = 13.7238$
Express the answer with the correct number of significant figures and the degree symbol.	$\theta_c = 13.7°$

Worked example: Try yourself 7.3.2

TOTAL INTERNAL REFLECTION

Sound is travelling through air and hits a steel wall. At what angle is the sound totally internally reflected? The speed of sound in steel is $5940\,m\,s^{-1}$ and the speed of sound in air is $346\,m\,s^{-1}$.

The critical angle is quite small if there is a large difference between the speed of sound in the first medium and the speed of sound in the second medium. In the example of sound going through air and striking water, the speed of sound in the second medium was five times that of the first, giving a critical angle of only 13.7°. Therefore, if the incident angle is greater than 13.7° the sound would be completely reflected.

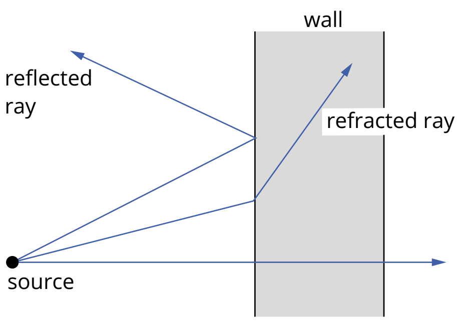


FIGURE 7.3.13 A wall acts as a good sound insulator, because sound transmitted through air towards a wall will only refract through a very narrow range of incident angles. At all incident angles greater than this critical angle sound will be reflected.

In Worked example: Try yourself 7.3.2, where the second medium is a solid wall of steel and the speed of sound in steel is 5940 m s^{-1}, over 17 times that of air, the critical angle is only 3.34°. This explains why liquids and solids are good reflectors of sound as the sound will only penetrate the media at a very narrow range of incident angles. Thus, it is common practice in acoustic engineering to use solid walls as sound insulators (Figure 7.3.13).

SEISMIC WAVES

When an earthquake occurs, it creates **seismic waves**. On 28 December 1989, an earthquake devastated the region in and around Newcastle, New South Wales. The earthquake was rated 5.6 on the Richter scale. It was not the most powerful earthquake recorded in Australia, but it caused the most damage.

There are three types of seismic waves produced in an earthquake. *Body waves* travel through the Earth and comprise two of the main wave types. The *primary* (P) waves are longitudinal waves (Figure 7.3.14a), and travel between 1.50 to 8.00 km s^{-1}. These P waves can move through liquids, such as molten material, and through solids. The blue grid pictured in image (a) shows the compression and rarefactions as the wave oscillates in the direction of the motion. The *secondary* (S) waves are transverse waves (Figure 7.3.14b). They do not travel through liquids and are slower than the P waves. The blue grid pictured in image (b) shows the displacement of the wave perpendicular to the direction of travel. The difference in speed between these two waves allows scientists to determine the location of the earthquake. The third type of wave is a *surface wave* (Figure 7.3.14c). It has a rolling motion and travels along the surface of the Earth. The blue grid pictured in image (c) shows it is a transverse wave, but the perpendicular displacement is only at the surface. This type typically causes the most damage.

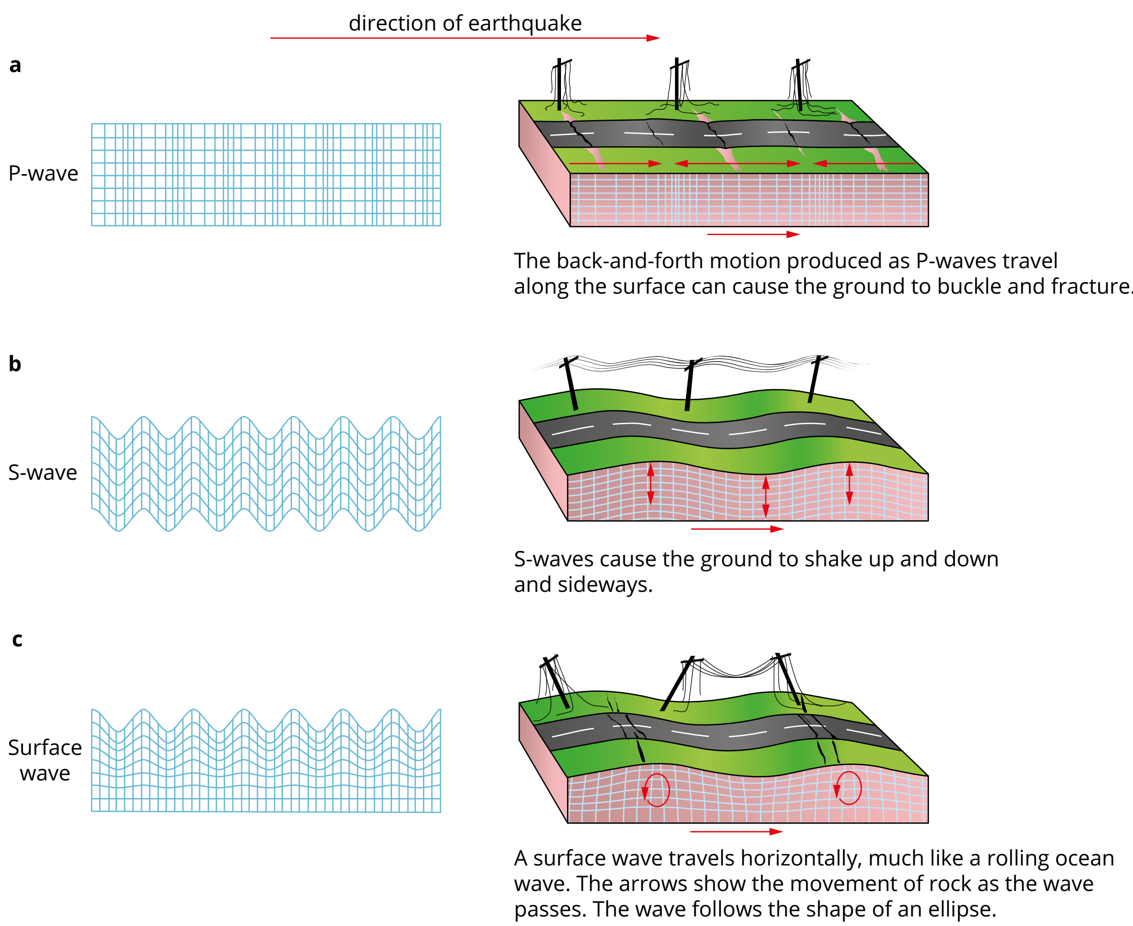


FIGURE 7.3.14 The three different seismic wave types, showing (a) a P wave (b) an S wave and (c) a surface wave.

Scientists can measure seismic waves using seismometers. From the information collected, they can determine the composition of the Earth. In fact, geophysicists have noted that both types of body waves, *P* and *S*, are detected close to the earthquake epicentre. However, while longitudinal *P* waves are detected at most places on the other side of the globe, the transverse *S* waves are not. There are large 'shadows' where transverse *S* waves are unable to propagate. As transverse *S* waves cannot travel through a liquid, this implies that the Earth's outer core is liquid. This is also supported by the fact that the molten core refracts the longitudinal *P* waves. Figure 7.3.15 shows where in the Earth's surface *P* and *S* waves are detected following an earthquake.

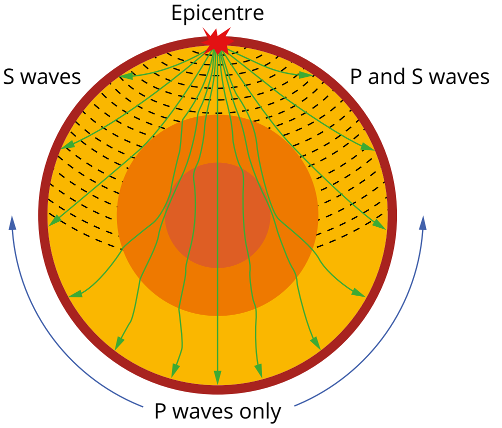


FIGURE 7.3.15 Geophysicists have determined that the Earth has a liquid outer core by observing the behaviour of seismic waves travelling through the Earth. Transverse *S* waves cannot travel through a liquid and are blocked by the liquid outer core while longitudinal *P* waves can be observed travelling through all of the Earth's interior.

DIFFRACTION OF SOUND

If you call someone from around the corner of a building they can still hear you even though they can't see you. This well-known ability of sound to travel around corners provides further evidence that sound is wave-like in nature. Reflection alone cannot account for all the indirect sounds. In addition, higher frequency sounds can be heard more clearly if the listener is directly in front of the source, whereas low frequency sounds can be heard quite clearly from a wide range of angles. For example, if you drive past someone who has their car stereo volume up very loud, the dominant sounds you will hear are the bass or low frequency sounds. This phenomenon is known as **diffraction**.

Diffraction effects in water as a plane wave passes through a narrow aperture are shown in Figure 7.3.16a and b. The interference effects shown in Figure 7.3.16b are discussed in Section 7.4.

a **b**

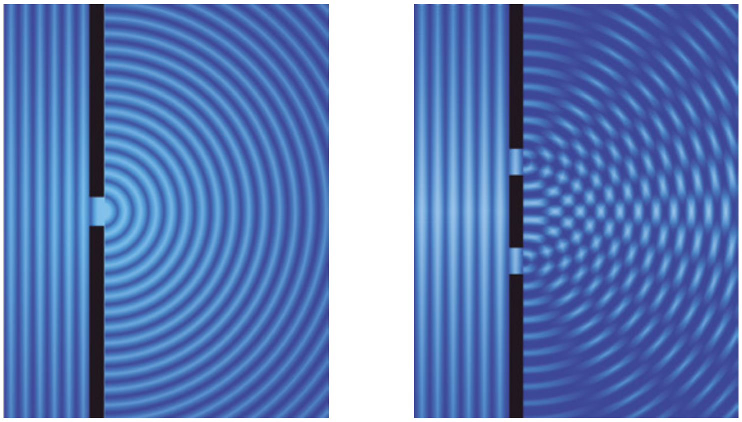


FIGURE 7.3.16 (a) A plane wave undergoes diffraction after passing through an aperture. (b) Interference effects occur due to diffraction from both of the slits.

Diffraction is the bending of waves as they pass the edge (or edges) of an obstacle or pass through an aperture.

Diffraction and wavelength

How much a sound wave spreads out when passing an obstacle, or going through an aperture, will depend upon the size of the wavelength of the sound, λ, in relation to the width of the aperture or obstacle, d. If the aperture width is equal to, or much less than, the wavelength, diffraction effects become significant, as shown in Figure 7.3.17a and b, where there is significant spreading of the waves. However, if the aperture width, or the barrier width, is much larger than the wavelength then the amount of diffraction is very limited, as shown in Figure 7 3.17c and d, where there is minimal disturbance to the wave.

Significant diffraction will occur when the wavelength is equal to, or larger than, the width of the obstacle or aperture.

Diffraction is therefore more significant at longer wavelengths and is the reason why it is difficult to pinpoint the exact source of long wavelength (low frequency) sound. The wavelengths of sound within normal hearing range are between about 2.00 cm and 20.0 m. A typical human voice has a wavelength of around 1.00 m, so voices diffract easily through doorways and around large obstacles.

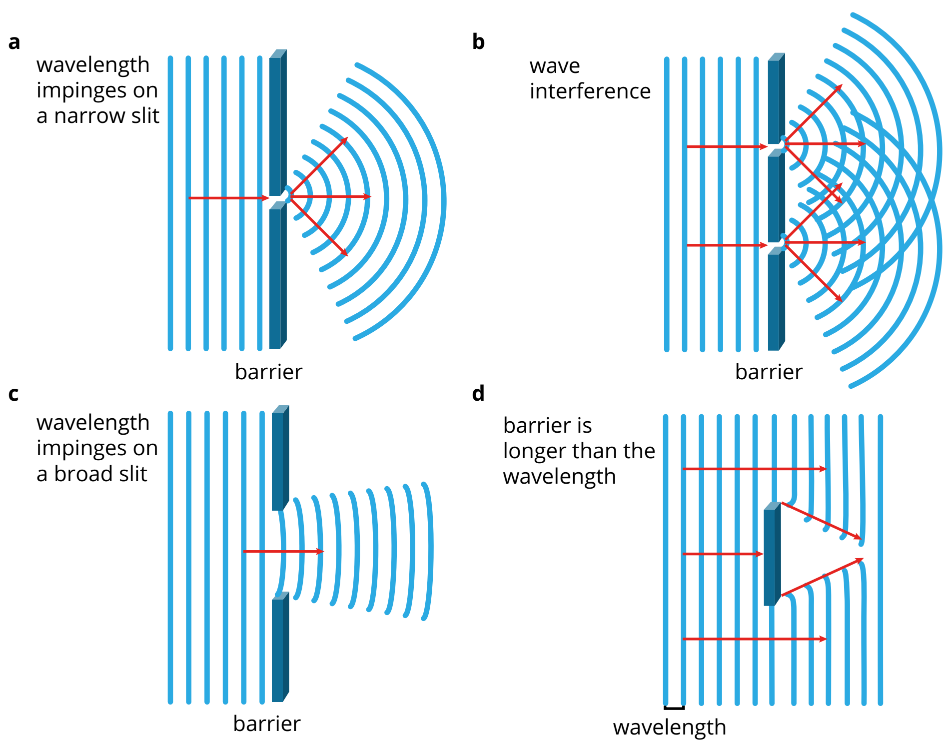


FIGURE 7.3.17 The amount of diffraction depends on the size of the gap or aperture and the wavelength. When the aperture is much smaller than the wavelength, significant diffraction, or spreading out of the waves, will occur ((a) and (b)). A large aperture (c) or a wide barrier (d) relative to the wavelength will cause little diffraction. Interference effects occur due to the superposition of two diffracted waves (b).

Diffraction and applications

As discussed previously, the wave equation is $v = f\lambda$, which gives f inversely proportional to λ. Therefore, high frequency (short wavelength) sounds are diffracted less and are more directional, making it easier to hear them from a particular direction. Ultrasound (with frequencies greater than 20 000 Hz) are used for sonar and ultrasonic motion detectors because its short wavelength means that diffraction is very limited. The sonar beam tends to travel to and from an object with only a small degree of spread. Figure 7.3.18 shows a dolphin locating its prey within a narrow range using sonar.

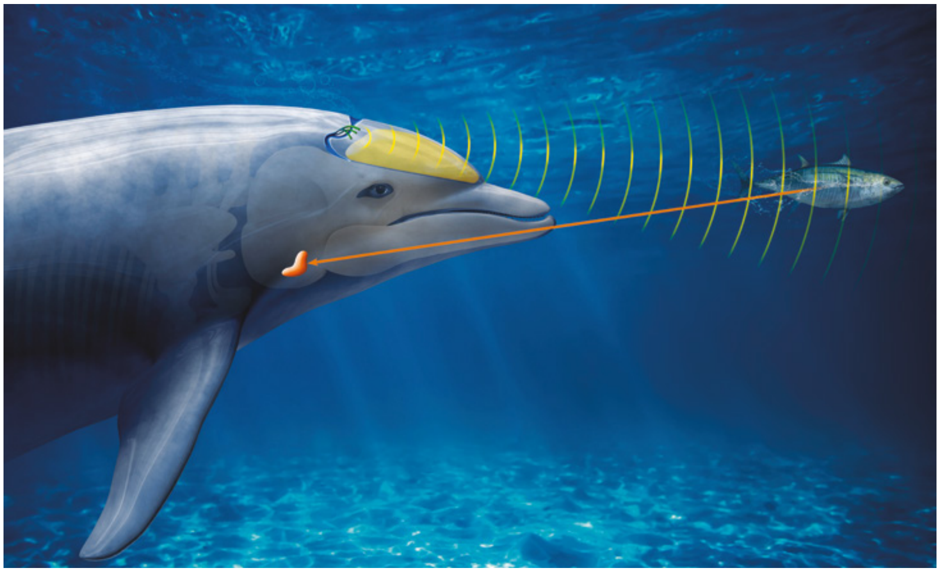


FIGURE 7.3.18 An artist's impression of how a dolphin accurately locates its prey using echolocation or sonar.

Lower frequency sounds, with larger wavelengths, are significantly diffracted and therefore will readily fill a room. Therefore, when setting up a stereo surround sound system, one correctly placed subwoofer (low-frequency speaker) is usually adequate, whereas the mid and high-frequency speakers need to be placed around the listener.

Worked example 7.3.3

DIFFRACTION

When high frequency sound waves, for example of 9550 Hz, strike an obstacle such as a person's head, they leave a distinct sound shadow region on the opposite side of the head from the speaker in which little of the sound can be heard. If one ear is closer to the sound source than the other, the ear closer to the source will hear the higher frequency sounds as louder.	
a Assuming the speed of sound is 346 m s^{-1}, calculate the wavelength of the 9550 Hz sound.	
Thinking	**Working**
The formula is $v = f\lambda$. Identify the variables to be used in the solution.	$f = 9550$ Hz $v = 346$ m s^{-1}
Rearrange to make λ the subject, $\lambda = \frac{v}{f}$, and solve for the speed of sound.	$\lambda = \frac{v}{f}$ $\lambda = \frac{(346)}{(9550)}$ $\lambda = 0.036230$
Express the answer with the correct number of significant figures and the appropriate unit.	$\lambda = 0.0362$ m

b Use this calculation to explain why this frequency leaves a sound shadow.	
Comment on the diffraction of the sound around a person's head when the width of their head is approximately 20 cm.	λ = 3.32 cm is much less than 20 cm. Therefore, diffraction effects are minimal.

PHYSICSFILE

Hearing loss

As a person gets older, or if they have been continually exposed to excessive noise, such as overloud earphones, they tend to lose the ability to hear some of the higher frequency sounds. These sounds are highly directional and help your brain determine which direction a sound is coming from. The loss of these higher frequencies also makes it more difficult to distinguish sounds clearly in a crowded room. In addition, people with high-frequency hearing loss often complain they can hear the sounds but not understand the words.

Worked example: Try yourself 7.3.3

DIFFRACTION

In ultrasound imaging, the speed of sound is 1540 m s^{-1}. The resolution of an image depends on the wavelength of the sound, with a smaller wavelength (higher frequency) enabling more detail to be seen with minimal diffraction effects. High-frequency sound (5.00 to 10.0 MHz) can resolve more detail but has limited penetration depth, whereas low-frequency sound (2.00 to 5.00 MHz) can penetrate deeper into tissue structures but has lower resolution.

a Assuming the speed of sound in human body tissue is 1540 m s^{-1}, calculate the wavelength of the 2.00 MHz sound.

b Use this calculation to explain why this frequency would not be appropriate to image a vein wall that is 0.50 mm thick.

7.3 Review

SUMMARY

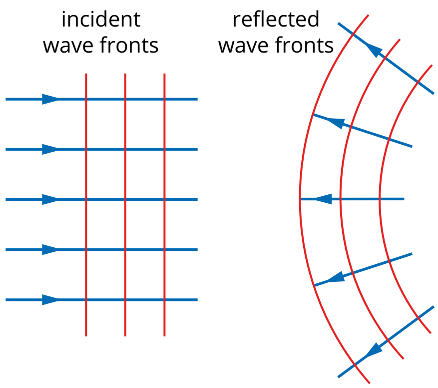
- A wave reaching the boundary between two materials through which it can travel will always be partly reflected, partly transmitted and partly absorbed.
- A wave has been reflected if it bounces back after reaching a boundary or surface.
- Waves reflect with a 180°, or $\frac{\lambda}{2}$, phase change from fixed boundaries. That is, crests reflect as troughs and troughs reflect as crests.
- Waves reflect with no phase change from free boundaries. That is, crests reflect as crests and troughs as troughs.
- When a wave is reflected from a surface, the angle of reflection will equal the angle of incidence.
- Refraction occurs when a wave changes speed as it passes from one medium to another.
- The refraction of the wave is measured by comparing the angle of incidence, i, to the angle of refraction, r.
- When a wave front speeds up at the boundary between two media, the wavelength increases and the wave front will bend away from the normal.
- When a wave front slows down at the boundary between two media, the wavelength decreases and the wave front will bend towards the normal.
- Wave frequency is unaffected by refraction.
- Total internal reflection occurs when the refracted angle is 90° to the normal.
- The incident angle at which total internal reflection occurs is called the critical angle.
- Diffraction, or spreading out of waves, around an obstacle or through an aperture occurs when λ is equal to or greater than the width of the aperture.
- Diffraction allows sound to be heard around corners and is more prominent at low frequency.

KEY QUESTIONS

1 A wave travels along a rope and reaches the fixed end of the rope. What occurs next?

2 Which one or more of the following properties of a wave can change when the wave is reflected: frequency, amplitude, wavelength or speed?

3 The following diagram shows a wave before and after being reflected from an object.

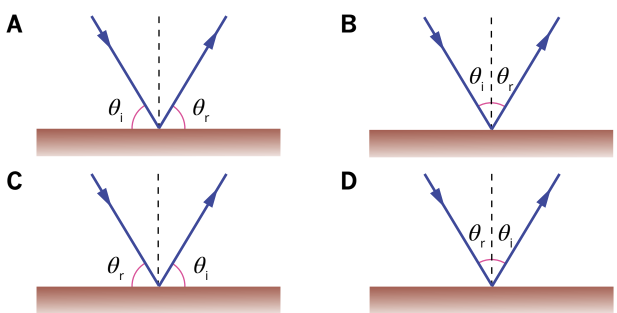


What is the shape of the object?

A flat **B** concave

C convex **D** parabolic

4 In which diagram are the angles of incidence and angles of reflection correctly labelled?



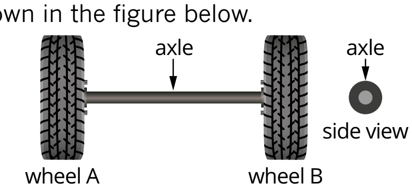
5 A geophysicist studying seismic waves determines that part of the interior of the Earth is molten. Explain why this statement can be made.

6 When water waves travelling through deep water reach a region of shallow water they are refracted. Which of the properties of water waves given below always changes when the waves reach shallower water? (More than one correct answer is possible.)

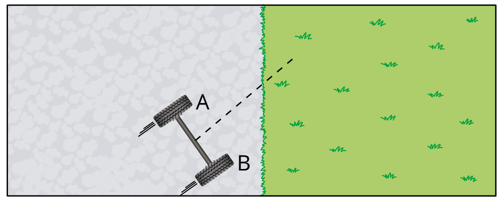
A frequency **B** wavelength

C direction **D** speed

7 A popular analogy used to model refraction involves two wheels (A and B) attached to either end of an axle as shown in the figure below.



Rolling these wheels from a surface where they move faster (like on concrete) to where they move slower (like on grass) models the process of refraction (shown in the figure below).



Choose the correct words from those in brackets to complete the description of how this model works.

As wheel B rolls onto the grass it [speeds up/slows down]. Since wheel A is now moving [faster/slower] than wheel B, the wheels change [direction/speed]. When wheel A rolls onto the grass the wheels' direction [continues/stops] changing.

8 The velocity of sound increases by about $0.600\,m\,s^{-1}$ for each degree increase in temperature. The speed of sound in air at 0.00°C is $331\,m\,s^{-1}$. A flute plays a note of 886 Hz at 20.0°C. A short distance above the ground there is a sudden increase in temperature to 30.0°C.

a Calculate the speed of sound in air at 20.0°C.

b Calculate the speed of sound in air at 30.0°C.

c If the sound travels through cold air and is incident on the warm air at an angle of 50.0°, describe what will happen to the refracted angle.

d Calculate the refracted angle.

e At what angles must the sound be incident on the warm air layer for the sound to be reflected rather than refracted?

9 A flute and a tuba are being played at the same time. The flute is producing a note with a frequency of 2012 Hz and the tuba is producing a note with a frequency of 125 Hz at the same volume. A listener outside the door of the auditorium complains that the tuba is drowning out the flute. How can this be so?

Questions 10–12 relate to seismic waves.

10 An average *P* wave has a speed of $6.00\,km\,s^{-1}$. If it has a period 0.200 seconds, calculate the wavelength and the frequency of the *P* wave.

11 The difference in arrival time at a seismometer between a *P* wave (t_P) and an *S* wave (t_S) is given by $\Delta t = t_P - t_S$. Each wave travels the same distance. Derive an expression for Δt in terms of the distance travelled, the velocity of the *P* wave (v_P) and the velocity of the *S* wave (v_S).

12 Use your answer to Question 11 to calculate the distance from the seismometer to the epicentre of the earthquake if $v_S = 3.45\,km\,s^{-1}$ and $v_P = 8.00\,km\,s^{-1}$. The difference in the time for arrival of the waves is 9.00 seconds.

7.4 Wave interactions—superposition, interference and resonance

Wave interactions, such as superposition, interference and resonance, play crucial roles in various physical phenomena and technological applications. Understanding these interactions helps in grasping the behaviour of waves in different environments.

SUPERPOSITION AND INTERFERENCE

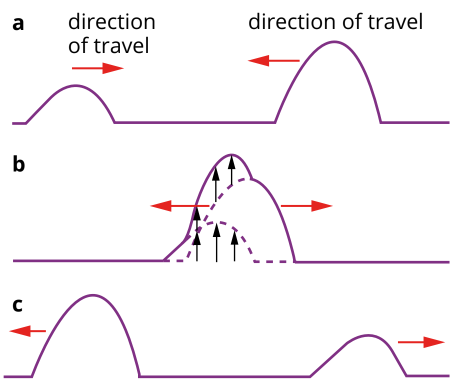


FIGURE 7.4.1 (a) As two wave pulses approach each other superposition occurs. (b) The occurrence of constructive interference. (c) After the interaction, the pulses continue unaltered; they do not permanently affect each other.

The sounds produced by musical instruments and the human voice are the product of the interaction of many waves. Imagine two transverse mechanical waves travelling towards each other along a string, as shown in Figure 7.4.1a. When the crest of one wave coincides with the crest of the other, the resulting displacement of the string is the vector sum of the two individual displacements (Figure 7.4.1b). The amplitude at this point is increased and the shape of the string resembles a combination of the two pulses. After they interact, the two pulses continue unaltered (Figure 7.4.1c). The resulting pattern is a consequence of the principle of **superposition**. The combination, or superposition, of the separate waves is called **interference**. In this case, as the displacement of the two waves was in the same direction, the two waves were added together, and **constructive interference** occurred.

When a pulse with a positive displacement meets one with a negative displacement as shown in Figure 7.4.2b the resulting displacement of the string is the vector sum of the two individual displacements; in this case a negative displacement adds to a positive displacement to produce a wave of smaller magnitude. This is called **destructive interference**. Once again, the pulses emerge from the interaction unaltered (Figure 7.4.2c).

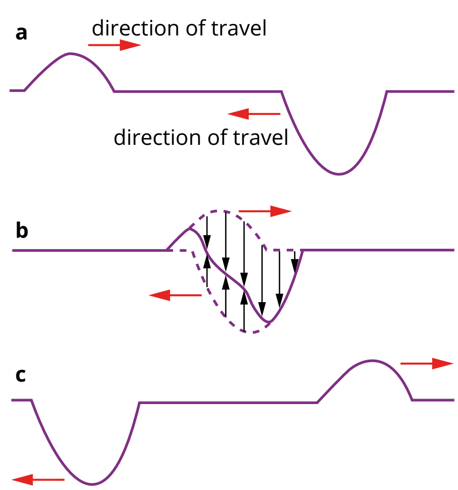


FIGURE 7.4.2 (a) As two wave pulses approach each other superposition occurs. (b) Superposition of waves in a string showing destructive interference. (c) As in constructive interference, the waves do not permanently affect each other.

When two waves meet and combine, there will be places where constructive interference occurs and places where destructive interference occurs. Although the wave pulses interact when they meet, passing through each other does not permanently alter the shape, amplitude or speed of either pulse. Just like transverse waves, longitudinal waves will also be superimposed as they interact. The terms superposition and interference are often used interchangeably.

A special case of constructive interference is where two waves of the same amplitude and wavelength, exactly **in phase**, add to together to give a wave of twice the amplitude as shown in Figure 7.4.3a. Complete destructive interference occurs when two waves are exactly opposite in phase (that is, one has a positive amplitude and the other a negative amplitude) and the two waves add together to give zero displacement (Figure 7.4.3b). The interference process can be used to explain the complex pattern seen in Figure 7.3.16b on page 253 where the interference or superposition of two diffraction patterns gives regions of complete destructive and constructive interference.

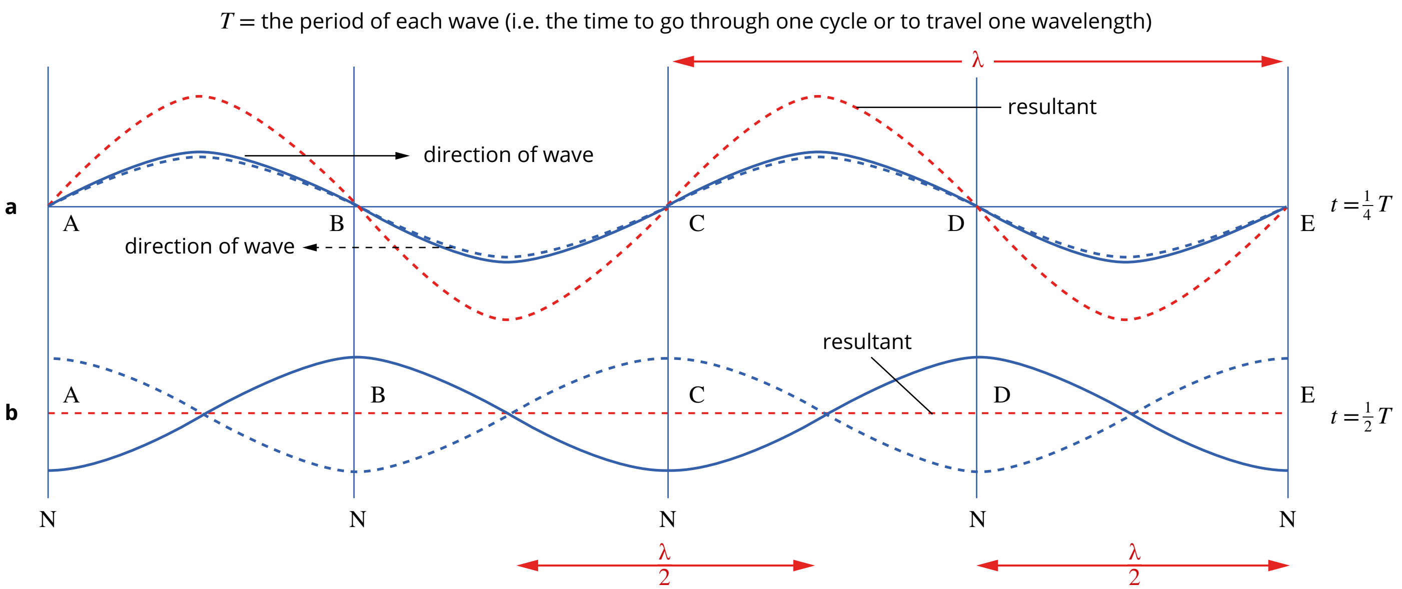


FIGURE 7.4.3 (a) Complete constructive interference occurs when two waves of the same amplitude that are exactly in phase add together to give a wave of twice the amplitude. (b) Complete destructive interference occurs when the two waves that are 180° or $\frac{\lambda}{2}$ out of phase add together to completely cancel out and give a wave of zero amplitude.

The effects of superposition and interference can be seen in many everyday examples. The ripples in the pond in Figure 7.4.4 were caused by raindrops hitting a pond. Where two ripples meet, a complex interference pattern is seen, where regions of constructive and destructive interference result from the superposition of the two waves. After this the ripples continue unaltered. Similarly, in a crowded room, all the sounds reaching your ear are superimposed, so that one complex sound wave arrives at the eardrum.

Superposition is important both theoretically and practically in the formation of complex sounds. Consider two single-frequency sound waves, or pure tones, one of which is twice the frequency of the other. The two individual waves are added together to give a more complicated resultant sound wave, as shown in Figure 7.4.5. Where one sound wave has a greater amplitude, as in the example illustrated, it will be the predominant sound heard. The quieter, higher frequency sound will combine with the louder one to create the sound that we hear. Note that for Figure 7.4.5, a transverse wave is used to depict the sound wave. The crests represent compressions (areas of high pressure) and the troughs represent rarefactions (areas of low pressure). The complex sounds that you emit when you speak or sing, or the square waves on a signal generator, can be modelled by superimposing various sine waves of different frequencies and amplitudes.

FIGURE 7.4.4 The ripples from raindrops striking the surface of a pond behave independently regardless of whether they cross each other or not. Where the ripples meet, a complex wave will be seen as the result of the superposition of the component waves. After interacting, the component waves continue unaltered.

PHYSICSFILE

The cocktail party effect and hearing loss

In a crowded room, individual sound waves will interfere with each other repeatedly, but it is still possible to distinguish which person is speaking. If you know the person's voice, then you know that their voice will sound the same. To discern one person's speech amid all the sounds in the room, your brain uses an innate ability to 'undo' the superposition of waves by selecting one person's voice and suppressing all the other noise. The 'cocktail party effect' also highlights the ability to hear your own name over the noise of a group of people talking.

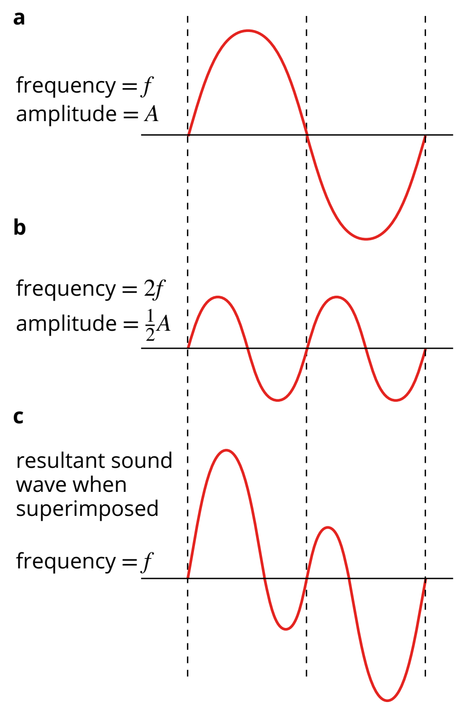


FIGURE 7.4.5 Two sound waves, one twice the frequency of the other, produce a complex wave of varying amplitude when they are superimposed.

PHYSICSFILE

Noise-cancelling headphones

Noise-cancelling headphones use interference to help reduce unwanted ambient noise, as shown in Figure 7.4.6. The circuitry in the headphones analyses the sound (Figure 7.4.6a) and in less time than a human can detect, it produces a new sound or wave form (Figure 7.4.6b) that counteracts the unwanted sound. The new sound is an inversion (out of phase by 180°) of the unwanted sound, so when the two waves are superposed (Figure 7.4.6c), they cancel each other out completely, as shown in Figure 7.4.6d.

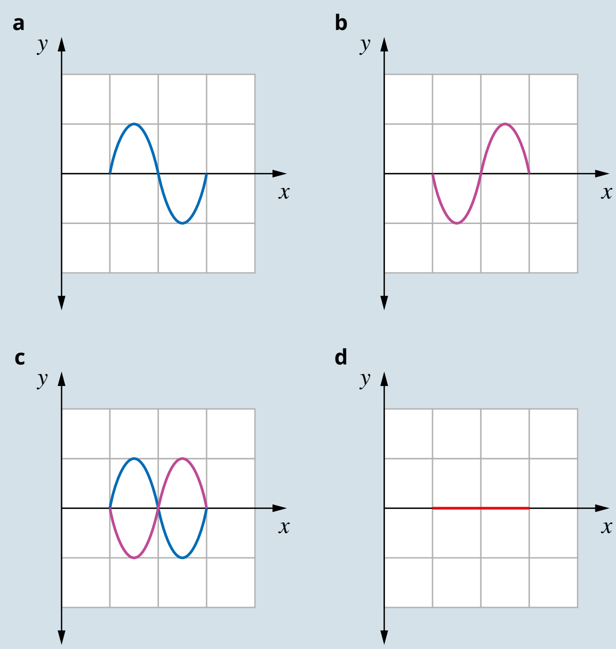


FIGURE 7.4.6 (a) The ambient (external noise) sound wave is detected by the microphone in the headphones. (b) The circuitry produces an inverted wave form of the ambient wave. (c) The sum of these two waves (d) cancels out the unwanted noise.

BEATS

An interesting phenomenon occurs when two sound waves of equal amplitude, but slightly different frequency, are added together. They superpose or interfere to form a resulting sound with a regular pulsation, known as a beat.

Consider the two waves in Figure 7.4.7a, wave 1 with a frequency of $f_1 = 20.0\,\text{Hz}$ and period of $T_1 = 0.0500\,\text{s}$, shown in blue, and wave 2 at the slightly different frequency of $f_2 = 18.0\,\text{Hz}$ and period of $T_2 = 0.0556\,\text{s}$, shown in orange. At $t = \text{X}$, both waves are in phase and their maximum positive amplitudes line up. Constructive interference occurs, and there is a double maximum in the resultant amplitude, shown in Figure 7.4.7b, which produces a maximum in the loudness of the sound. The maximum positive amplitudes of wave 1 and wave 2 line up again at point Z where $t = 0.50\,\text{s}$, so this is the next instance of double maximum amplitude in the resultant wave.

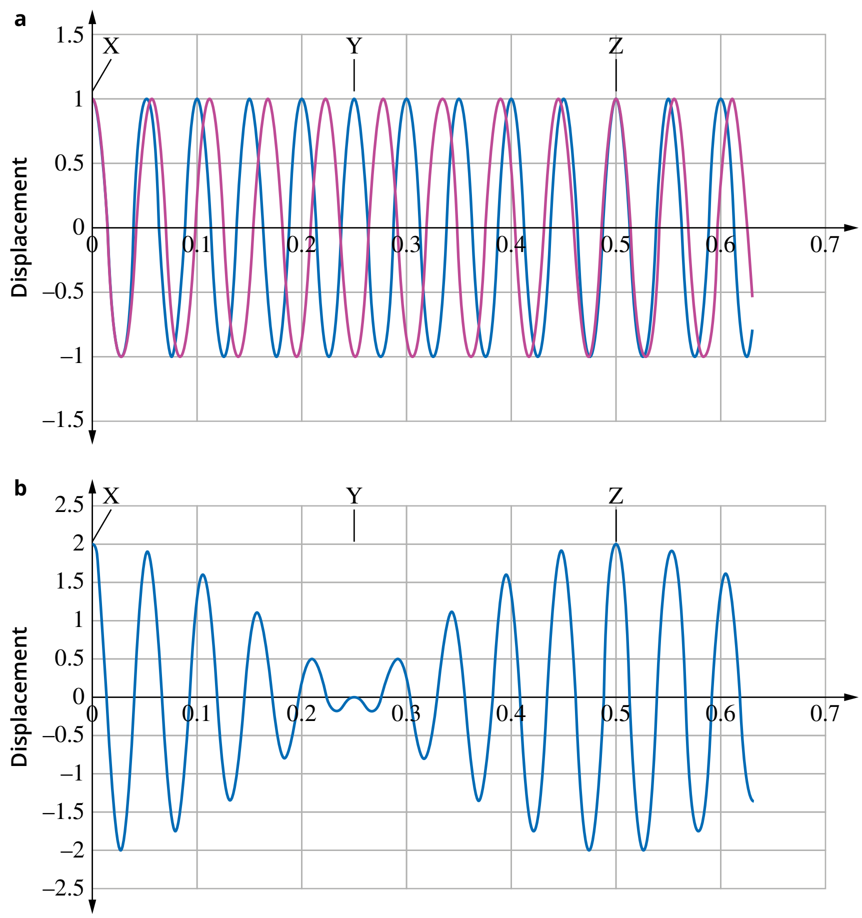


FIGURE 7.4.7 (a) Wave 1, shown in blue, has a frequency of $f_1 = 20.0\,\text{Hz}$ and wave 2, shown in orange, has a slightly different frequency of $f_2 = 18.0\,\text{Hz}$. (b) Superposition or interference of these two waves of slightly different frequencies, f_1 and f_2, give rise to a resultant wave with an oscillating amplitude.

Wave 1 has 10 whole cycles between $t = \text{X}$ and $t = \text{Z}$, with each cycle having a period of $T_1 = 0.0500\,\text{s}$.

Wave 2 has 9 whole cycles between $t = \text{X}$ and $t = \text{Z}$, with each cycle having a period of $T_2 = 0.0556\,\text{s}$.

Now compare wave 1 and wave 2 at $t = \text{Y}$, where $t = 0.25\,\text{s}$.

Wave 1 has 5 complete cycles with a positive amplitude crest at that time:

number of cycles, $n_1 = \dfrac{t}{T_1} = \dfrac{0.2500}{0.0500} = 5$ cycles.

Wave 2 has 4.5 complete cycles and has a negative maximum trough at that time:

number of cycles, $n_2 = \dfrac{t}{T_2} = \dfrac{0.025}{0.0556} = 4.500$ cycles.

The waves are out of phase, as the positive amplitude crest of wave 1 lines up with the negative amplitude trough of wave 2. Destructive interference occurs at that time, so there is a minimum in the loudness of the sound, as shown in the resultant wave in Figure 7.4.7b. Over time, you would hear a regular maximum and minimum in the loudness of the sound. The frequency at which the loudness of the sound oscillates is known as the **beat frequency**, f_{beat}. If the beat frequency increases to above 10.0 Hz, then the human ear cannot discern between two successive loud maximums and so the sounds merge to produce one continuous tone.

The beat frequency is given by:

$$f_{beat} = |f_2 - f_1|$$

In Figure 7.4.7, we can see that $f_{beat} = |20.0 - 18.0| = 2.0$ Hz. This means that there will be 2.00 maximum loud sounds every second, the first occurring at $t = 0.50$ s and the second at $t = 1.00$ s. (Note that the mathematical notation used in this equation, shown as | |, denotes the absolute value, so the answer is always positive. You will get the same beat frequency of 2.0 Hz if 20.0 Hz is subtracted from 18.0 Hz, i.e. $|18.0 - 20.0| = |20.0 - 18.0| = 2.0$ Hz.)

Beats can be used to tune a musical instrument against a standard tone. For example, key A4 on a piano is designed to have a frequency of 440.0 Hz. If a piano tuner has a tuning fork or an app that emits exactly 440.0 Hz, and the piano is slightly out of tune, then beats will be heard. The piano tuner can then adjust the piano string so that no beat frequency is heard, i.e. there is no difference between the two frequencies.

You can investigate this phenomenon yourself using a tone-generating app on two different devices with slightly different frequencies, or by using two tuning forks and placing a small amount of Blu Tack on one of the tines of one tuning fork.

Worked example 7.4.1

SUPERPOSITION AND INTERFERENCE

Clare has two different tuning forks that produce sound waves with a frequency of 349.00 Hz and 354.00 Hz, respectively.	
a Determine the beat frequency heard by Clare when they are sounded together.	
Thinking	**Working**
Determine the correct formula: $f_{beat} = \|f_2 - f_1\|$	$f_{beat} = \|354.00 - 349.00\|$ $f_{beat} = 5.0000$
Express the answer with the correct number of significant figures and the appropriate unit.	$f_{beat} = 5.00$ Hz

b If Clare slightly increases the frequency of the 349.00 Hz tuning fork, describe what she would expect to hear when sounding the two forks together.	
The difference in frequency between the two forks becomes smaller.	Clare would hear fewer beats per second.

Worked example: Try yourself 7.4.1

SUPERPOSITION AND INTERFERENCE

A piano tuner is trying to tune a middle C key that is producing sound waves with an incorrect frequency of 263.80 Hz. They have a device that produces the correct frequency for middle C of 261.63 Hz.

a Determine the beat frequency that the piano tuner hears.

b Tightening the piano string would increase the frequency of the key. Describe what the piano tuner would hear if they tightened the string.

RESONANCE

You may have heard about singers who supposedly can break glass by singing particularly high notes. Figure 7.4.8 shows a glass being broken in this way. All objects that can vibrate tend to do so at a specific frequency known as their **natural frequency** or **resonant frequency**. **Resonance** is when an object is exposed to vibrations at a frequency equal to their resonant frequency. The vibration from one object causes a strong vibration in another. If the amplitude of the vibrations becomes too great, the object can be destroyed.

FIGURE 7.4.8 A glass can be destroyed by the vibrations caused by a singer emitting a sound of the same frequency as the resonant frequency of the glass.

A swing pushed once, and left to swing or oscillate freely, is an example of an object vibrating at its natural frequency. The frequency at which it moves backwards and forwards depends entirely on the design of the swing, mostly on the length of its supporting ropes. In time, the oscillations will fade away as the energy is transferred to the supporting frame and the air.

If you watch a swing in motion, it is possible to determine its natural oscillating frequency. It is then possible to push the swing at exactly the right time so that you match its natural oscillation. The additional energy you add by pushing will increase the amplitude of the swing rather than work against it. Over time, the amplitude will increase and the swing will go higher and higher; this increase in amplitude is called resonance. The swing can only be pushed at one particular rate to get this increase in amplitude (i.e. to get the swing to resonate). The frequency with which you push is the **forcing frequency**. If you push at a rate that is faster or slower, the forcing frequency that you are providing will not match the natural frequency of the swing and you will be fighting against the swing rather than assisting it.

Other examples of resonant frequency that you may have encountered are blowing air across the mouthpiece of a flute or drawing a bow across a string of a violin in just the right place (Figure 7.4.9). In each case, a clearly amplified sound is heard when the frequency of the forcing vibration matches a natural resonant frequency of the instrument.

In musical instruments and loudspeakers, resonance is a desired effect. The sounding boards of pianos and the enclosures of loudspeakers are designed to enhance and amplify particular frequencies.

Resonance, however, can be undesirable in mechanical systems, such as car exhaust systems. It can also cause screws and ropes to loosen with spectacular and tragic consequences. On 16 April 1850, almost 200 soldiers died when a bridge in Angers, France, collapsed because of the resonance caused by the synchronised oscillations from the soldiers marching in step. Nowadays, care is taken to design mechanical systems that prevent resonance.

FIGURE 7.4.9 The sound box of a stringed instrument is tuned to resonate for the range of frequencies of the vibrations being produced by the strings. When a string is plucked or bowed, the airspace inside the box vibrates in resonance with the natural frequency, amplifying the sound.

For resonance to occur, the forcing frequency must match the natural frequency.

Two very significant effects occur when this happens.

1. The amplitude of the oscillations within the resonating object will increase dramatically.
2. The maximum possible energy from the source creating the forced vibration is transferred to the resonating object.

7.4 Review

SUMMARY

- The principle of superposition states that when two or more waves interact, the resultant displacement or pressure at each point along the wave will be the vector sum of the displacements or pressures of the component waves.
- The process of superposition can also be referred to as interference.
- Constructive interference occurs when the superposition of waves gives an increase in amplitude in the positive and/or negative direction.
- Complete constructive interference occurs when two waves of the same amplitude that are in phase combine to give a wave of double the amplitude.
- Destructive interference occurs when the superposition of two waves results in a reduction in amplitude.
- Complete destructive interference occurs when two waves of equal amplitude, exactly 180° or $\frac{\lambda}{2}$ out of phase, combine to give a wave with zero displacement.
- Beats occur when two sounds of slightly different frequencies are sounded together.
- Beats can be calculated using the formula $f_{beat} = |f_2 - f_1|$
- Resonance occurs when the frequency of a forcing vibration equals the natural or resonant frequency of an object.
- Two special effects occur with resonance:
  - the amplitude of vibration increases
  - the maximum possible energy from the source is transferred to the resonating object.

KEY QUESTIONS

1 Which of the following statements about wave pulses are true and which are false? For the false statements, rewrite them so they are true.

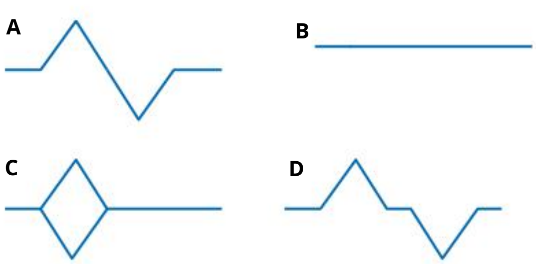
a The displacement of the resultant pulse is equal to the sum of the displacements of the individual pulses.

b As the pulses pass through each other, the interaction permanently alters the characteristics of each pulse.

c After the pulses have passed through each other, they will have the same characteristics as before the interaction.

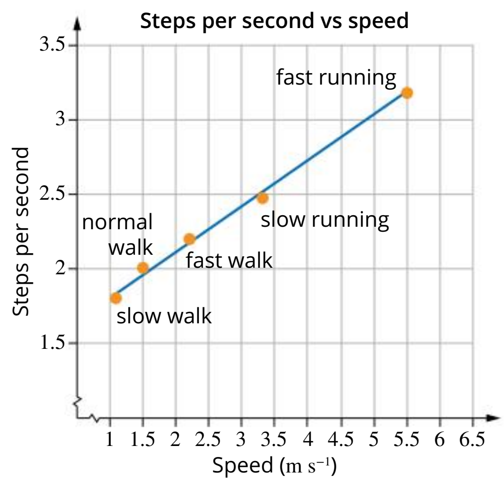
2 Two triangular wave pulses head towards each other at $1.00\,m\,s^{-1}$. Each pulse is 2.00 m wide.

What will the superposition of these two pulses look like in 3.00 s?



3 Explain why resonance can result in significant damage to buildings.

4 A footbridge over a river has a natural frequency of oscillation from side to side of approximately 1 Hz. When pedestrians walk at a pace that will produce an oscillation in the bridge close to the natural frequency of the footbridge, resonance will occur. The graph below displays relevant data about pedestrians walking or running. A pedestrian completes 1 cycle of their motion every 2 steps. Which activity of the pedestrians is most likely to cause damage to the footbridge over time? Explain your answer.



5 Calculate the beat frequency for two waves with frequencies of 524.05 Hz and 525.32 Hz, respectively.

6 Two tuning forks are sounded at the same time to produce a beat frequency of 6.34 Hz. If one tuning fork has a frequency of 480.25 Hz and the other has a lower pitch, calculate the frequency of the lower-pitched tuning fork.

7.5 Standing waves and harmonics

Drawing a bow across a violin string causes the string to vibrate between the fixed bridge of the violin and the finger of the violinist (Figure 7.5.1). The simplest vibration will have maximum amplitude at the centre of the string, halfway between bridge and finger. This is a very simple example of a transverse **standing wave**.

FIGURE 7.5.1 Transverse standing waves can form along a violin string when the string is bowed by the violinist.

Standing waves are an important phenomenon of the superposition of waves. They occur when two waves of the same amplitude and frequency are travelling in opposite directions towards each other in the same string. Usually, one wave is the reflection of the other. Standing waves are responsible for the wide variety of sounds associated with speech and music.

STANDING WAVES IN A STRING

In Section 7.4 it was shown that when a wave pulse reaches a fixed end, it is reflected back 180° out of phase. That is, crests are reflected as troughs and troughs are reflected as crests.

Imagine creating a series of waves in a rope by shaking it vigorously. As the rope continues to be shaken, waves will travel in both directions. The new waves travelling down the rope will interfere with those being reflected back along the rope. This kind of motion will usually create quite a random pattern with the waves quickly dying away. Oscillating the rope at just the right frequency, however, will create a new wave that interferes with the reflection in such a way that the two superimposed waves create a single, larger amplitude standing wave.

It is called a standing wave because the wave doesn't appear to be travelling along the rope. The rope simply seems to oscillate up and down with a fixed pattern. This situation contrasts with a standard travelling wave, in which a transverse wave is created where every point on the rope would have a maximum displacement at some time as the wave travels along the rope.

In Figure 7.5.2a–d, two waves (drawn in blue) are shown travelling in opposite directions towards each other along a rope. One of the waves is a string of pulses (shown as a solid line) and the other is its reflection (shown as a dashed line). The two waves superimpose when they meet. Since the amplitude and frequency of each is the same, the end result, as shown in part e, is a standing wave. At the points where complete destructive interference occurs, the two waves totally cancel each other out and the rope will remain still. These are called **nodes**. Where the rope oscillates with maximum amplitude, complete constructive interference is occurring. These points on the standing wave are called **antinodes**.

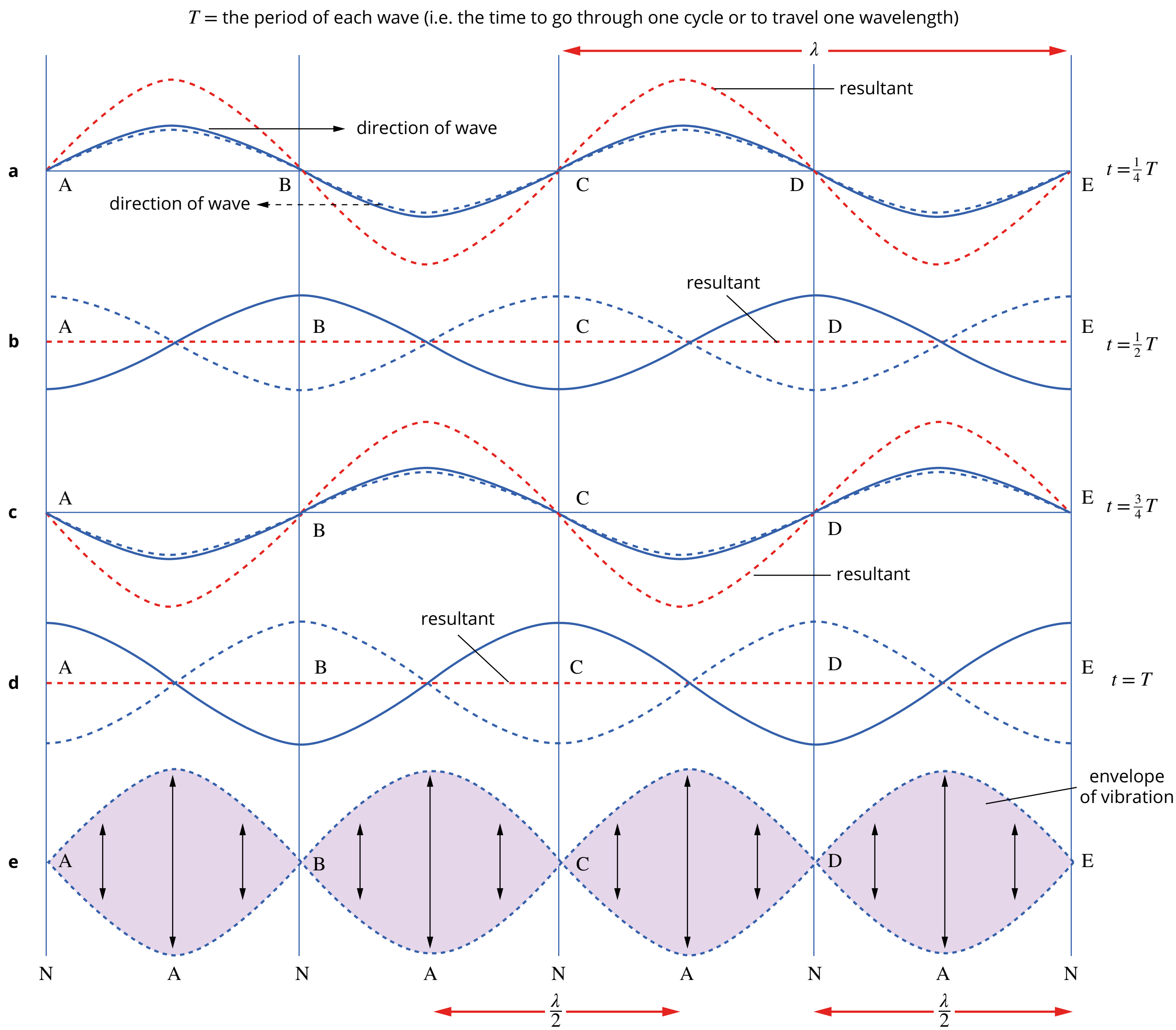


FIGURE 7.5.2 A standing wave created in a rope from two waves travelling in opposite directions, each with the same amplitude and frequency.

In Figure 7.5.2a at $t = \frac{1}{4}T$, the two waves are completely superimposed (red dotted line) and complete constructive interference is occurring, resulting in a wave twice the original amplitude.

In Figure 7.5.2b at $t = \frac{1}{2}T$, a further period of time equal to $\frac{T}{4}$ has passed by and the waves each have moved $\frac{\lambda}{4}$, which means that they have moved a total of $\frac{\lambda}{2}$ in relation to each other. The waves are completely out of phase, complete destructive interference is occurring, and the resulting displacement is zero.

As more time goes by, the waves continue to move past each other.

In Figure 7.5.2c at $t = \frac{3}{4}T$, the waves constructively interfere in the opposite direction, and in Figure 7.5.2d at $t = T$ the waves completely cancel out again.

In Figure 7.5.2e the cycles shown in a–d form a standing wave. A standing wave swings between maximum positive and negative displacements, creating antinodes (A) that lie halfway between the stationary nodes (N). Regardless of the position of the component waves, these nodes stay in the same place as the displacement at these points is always zero. Successive nodal points lie $\frac{\lambda}{2}$ apart, as do successive antinodal points.

Nodes and antinodes in a standing wave remain in a fixed position for a particular frequency of vibration. Figure 7.5.3 illustrates a series of possible standing waves in a rope, with both ends fixed, corresponding to three different frequencies. The lowest frequency of vibration, seen in part a, produces a standing wave with one antinode in the centre of the rope. The ends are fixed so they will always be nodal points. Assuming the tension in the rope is kept the same, the patterns shown in parts b and c are produced at twice and three times the original frequency, respectively.

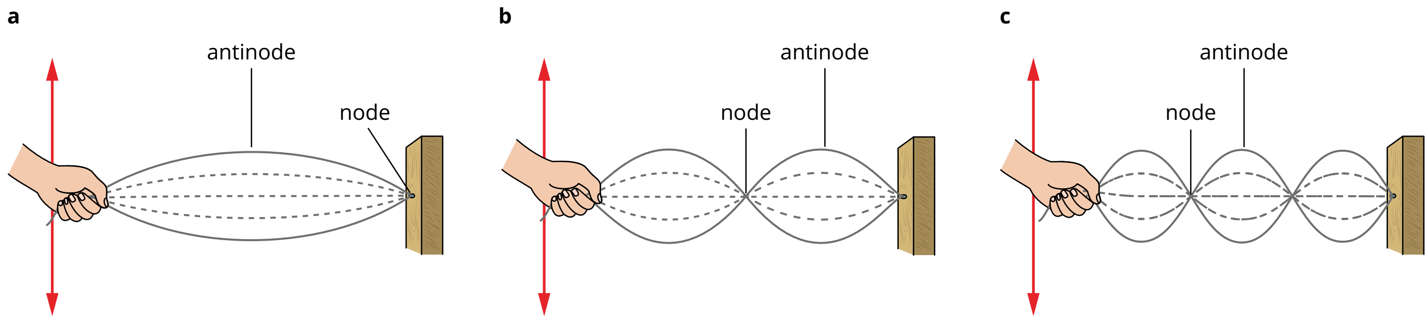


FIGURE 7.5.3 A rope vibrated at three different resonant frequencies, illustrating the standing waves produced at each frequency.

The rope could also vibrate at a frequency four times that of the original, and so on. The frequencies at which standing waves are produced are called the resonant frequencies of the rope.

It is important to note that the formation of a standing wave does not mean that the string or rope itself is stationary. It will continue to oscillate as further wave pulses travel up and down the rope. It is the relative position of the nodes and antinodes that remain unchanged.

It is also important to note that standing waves are not a natural consequence of every wave reflection.

Standing waves don't only exist in the everyday world but also occur at the sub-atomic level, and are the reason electrons exist in electron shells at fixed energies. This concept is covered in detail in Year 12.

> Standing waves are only produced by the superposition of two waves of equal amplitude and frequency, travelling in opposite directions.
>
> Standing waves are a result of resonance and occur only at the natural frequencies of vibration, or resonant frequencies, of the particular medium.

HARMONICS IN A STRING

Stringed musical instruments are examples of strings fixed at both ends. A large variety of waves of different frequencies/wavelengths are created. They travel along the string in both directions and reflect from the fixed ends, undergoing a phase change. Most of these vibrations will interfere in a random fashion and die away. However, those corresponding to resonant frequencies of the string will form standing waves and remain.

The resonant frequencies produced in this complex vibration of multiple standing waves are called **harmonics**. The lowest and simplest form of vibration, with one antinode (Figure 7.5.3a), is called the **fundamental** frequency. Higher-level harmonics (Figure 7.5.3b and c) are referred to by musicians as **overtones**.

The fundamental frequency usually has the greatest amplitude, so it has the greatest influence on the sound. The amplitude generally decreases for each subsequent harmonic. Usually, all possible harmonics are produced in a string simultaneously, and the instrument and the air around it also vibrate to create the complex mixture of frequencies heard as an instrumental note.

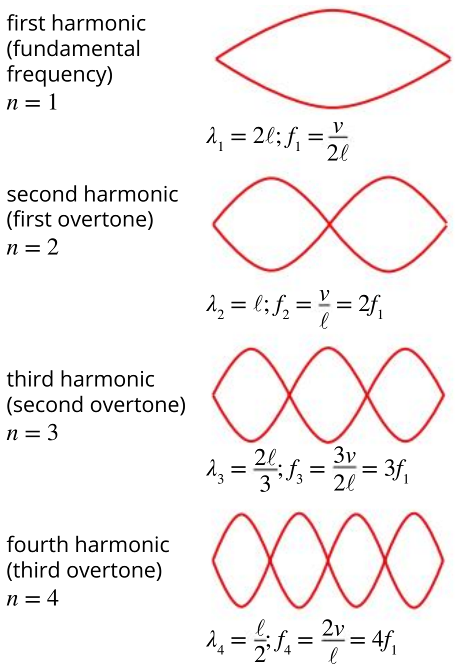


FIGURE 7.5.4 The first four resonant frequencies, or harmonics, in a stretched string fixed at both ends. The ends are fixed so they will always be nodal points.

The resonant frequencies or harmonics in a string of length ℓ can be calculated from the relationship between the length of the string and the wavelength, λ, of the corresponding standing wave. Refer to Figure 7.5.4 to assist with understanding the following derivations.

For a string fixed at both ends:

The **first harmonic**, or fundamental frequency, has one antinode in the centre of the string and a node at each end, therefore there is only half a wavelength on the string.

$\ell = \frac{1\lambda_1}{2}$ therefore $\lambda_1 = \frac{2\ell}{1}$

The second harmonic (first overtone) will have two antinodes and three nodes, therefore there is one wavelength on the string.

$\ell = \frac{2\lambda_2}{2}$ therefore $\lambda_2 = \frac{2\ell}{2}$

The third harmonic (second overtone) will have three antinodes and four nodes, therefore there are $1\frac{1}{2}$ wavelengths on the string.

$\ell = \frac{3\lambda_3}{2}$ so $\lambda_3 = \frac{2\ell}{3}$

And so, in general, for any harmonic:

$$\ell = \frac{n\lambda_n}{2}$$

$$\lambda_n = \frac{2\ell}{n}$$

where λ_n is the wavelength (m)

ℓ is the length of the string (m)

n is the number of the harmonic, which is also the number of antinodes (i.e. 1, 2, 3, 4…)

Using the wave equation $v = f\lambda$ gives the relationship between frequency, velocity and string length.

For the first harmonic, or fundamental frequency:

$\lambda_1 = 2\ell$ and $v = f_1\lambda_1$ and so $f_1 = \frac{v}{\lambda_1} = \frac{v}{2\ell}$

For the second harmonic (first overtone):

$\lambda_2 = \ell$ and $v = f_2\lambda_2$ so $f_2 = \frac{v}{\lambda_2} = \frac{v}{\ell}$ and $f_2 = 2f_1$

For the third harmonic (second overtone):

$\lambda_3 = \frac{2\ell}{3}$ and $v = f_3\lambda_3$ so $f_3 = \frac{v}{\lambda_3} = \frac{3v}{2\ell}$ and $f_3 = 3f_1$

And so, in general:

$$f_n = \frac{nv}{2\ell} \text{ and } f_n = nf_1$$

where n is the number of the harmonic

f is the frequency of the wave (Hz)

v is the velocity of the wave (m s^{-1})

ℓ is the length of the string (m)

It should also be noted that the resonant frequencies of a string correspond to its tension and mass per unit length. Tightening or loosening the string will change the wavelengths and resonant frequencies for that string (i.e. the instrument will need tuning by adjusting the tension of the string). Heavier strings of a particular length will have different resonant frequencies than lighter strings of the same length and tension. For example, in a guitar the deeper notes are produced by the thicker strings.

Worked example 7.5.1

FUNDAMENTAL FREQUENCY

A violin string, fixed at both ends, has a length of 22.0 cm. It is vibrating at its fundamental frequency of vibration, at a frequency of 880.0 Hz.

a Calculate the wavelength of the fundamental frequency.

Thinking	Working
Identify the length of the string (ℓ) in metres and the harmonic number (n).	ℓ = 22.0 cm = 0.220 m $n = 1$
Recall that for any frequency, $\lambda = \frac{2\ell}{n}$. Substitute the values from the question and solve for λ.	$\lambda = \frac{2\ell}{n}$ $\lambda = \frac{2 \times (0.220)}{1}$ $\lambda = 0.44000$
Express the answer with the correct number of significant figures and the appropriate unit.	$\lambda = 0.440$ m

b Calculate the wavelength of the second harmonic.

Thinking	Working
Identify the length of the string (ℓ) in metres and the harmonic number (n).	ℓ = 22.0 cm = 0.220 m $n = 2$
Recall that for any frequency, $\lambda = \frac{2\ell}{n}$. Substitute the values from the question and solve for λ.	$\lambda = \frac{2\ell}{n}$ $\lambda = \frac{2 \times (0.220)}{2}$ $\lambda = 0.220000$
Express the answer with the correct number of significant figures and the appropriate unit.	$\lambda = 0.220$ m

Worked example: Try yourself 7.5.1

FUNDAMENTAL FREQUENCY

A guitar string, fixed at both ends, has a length of 48.0 cm. It is vibrating at its fundamental frequency of vibration, which is 345.0 Hz.

a Calculate the wavelength of the fundamental frequency.

b Calculate the wavelength of the third harmonic.

PHYSICSFILE

Surface waves

Seismic surface waves travel along the boundary between materials, such as the Earth's crust and upper mantle. One type of surface wave is called the Rayleigh wave, or ground roll. These are surface waves that travel as ripples with a motion like that of waves on the surface of water, although the restoring force is elastic rather than gravitational as it is for water waves. A phenomenon known as free oscillation of the Earth is the result of the superposition between two such surface waves travelling in opposite directions creating a surface standing wave.

The first observations of free oscillations of the Earth were made during the 1960 Chile earthquake. Since then thousands of harmonics have been identified.

WIND INSTRUMENTS AND AIR COLUMNS

Longitudinal standing waves are also possible in air columns. These create the sounds associated with wind instruments. Blowing over the hole of a flute (Figure 7.5.5) or the reed of a saxophone produces vibrations that correspond to a range of frequencies that create standing waves in the tube.

FIGURE 7.5.5 Blowing over the mouthpiece of a flute and controlling the length of the flute with the keys enables a particular note to be produced.

The compressions and rarefactions of the sound waves, confined within the tube, reflect from both open and closed ends. This creates the right conditions for resonance and the formation of standing waves. The length of the pipe will determine the frequency of the sounds that will resonate.

The open end of a pipe corresponds to the fixed end of a string in that the reflected wave is fully inverted (i.e. undergoes a 180° change of phase). This means a compression is reflected as a rarefaction and a rarefaction is reflected as a compression. At a closed end, there is no change of phase, so a compression is reflected as a compression and a rarefaction as a rarefaction.

OPEN-ENDED AIR COLUMNS

Air columns open at both ends are referred to as 'open-ended' air columns. The flute is a typical example of a pipe open at both ends in which an air column can be made to vibrate. Another example includes the muffler in a car exhaust system.

> A compression or rarefaction in a longitudinal sound wave will reflect from an open end with a phase change of $\frac{\lambda}{2}$.
>
> Thus, the harmonics in the pressure variation for the wave will be very similar in nature to those of a string fixed at both ends.

Figure 7.5.6a shows the variation in pressure for the various harmonics formed in the pipe. The antinode, which is the region of maximum pressure variation, is shown as maximum positive and negative amplitude. The nodes, which are the regions of minimum pressure variation, are shown as zero displacement. Figure 7.5.6b shows the corresponding **particle displacement** for each of the pressure-variation harmonics.

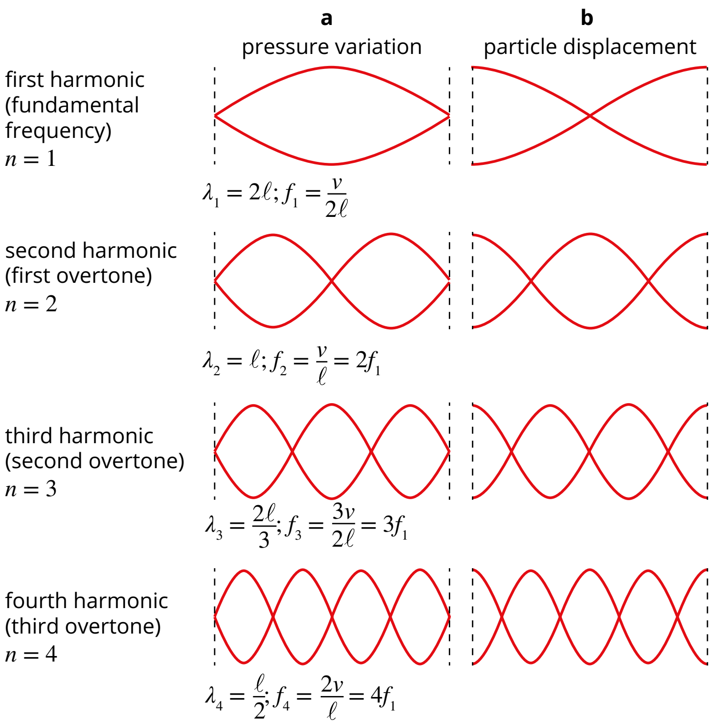


FIGURE 7.5.6 (a) the variation in pressure for each harmonic when standing waves are formed in a pipe open at both ends. (b) The corresponding particle displacement for each harmonic.

Where there is an antinode in pressure variation, there is maximum pressure variation and therefore minimum particle displacement. Similarly where there is a node in pressure variation, there is an antinode (maximum) in particle displacement, as shown in Figure 7.5.7.

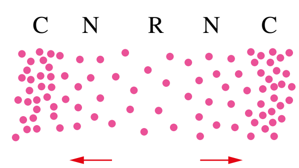


FIGURE 7.5.7 The diagram illustrates a sound wave showing compressions (C), rarefactions (R) and regions of normal pressure (N). The arrows indicate the particle displacement. In the regions of maximum pressure variation (C) and (R), the particle displacement will be at a minimum. In the region of minimum pressure variation (N), the particle displacement will be maximum.

> An antinode in pressure variation corresponds to a node in particle displacement.
> A node in pressure variation corresponds to an antinode in particle displacement.

When analysing the harmonics of the pressure wave in a pipe open at both ends, the same reasoning can be used as for a string fixed at both ends. Refer to Figure 7.5.6 throughout the following analysis.

$n = 1$: The first harmonic, or fundamental frequency, has one pressure antinode (compression) in the centre of the pipe and a node at each end, therefore there is only half a wavelength in the pipe.

$\ell = \frac{1\lambda_1}{2}$ and $\lambda_1 = \frac{2\ell}{1}$. Using $v = f_1\lambda_1$, then $f_1 = \frac{v}{\lambda_1} = \frac{v}{2\ell}$.

Conversely the particle displacement has a maximum (antinode) at the end of the pipe and a minimum (node) in the centre.

$n = 2$: The second harmonic or first overtone will have two pressure antinodes and three nodes, therefore there is only one wavelength in the pipe.

$\ell = \frac{2\lambda_1}{2}$, therefore $\lambda_2 = \ell = \frac{2\ell}{2}$. Using $v = f_2\lambda_2$, then $f_2 = \frac{v}{\lambda_2} = \frac{2v}{2\ell}$ and $f_2 = 2f_1$.

The particle displacement has three antinodes—one at each end and one in the middle—and two nodes.

$n = 3$: The third harmonic or second overtone will have three pressure antinodes and four nodes, therefore there are 1.5 wavelengths in the pipe.

$\ell = \frac{3\lambda_3}{2}$ so $\lambda_3 = \frac{2\ell}{3}$. Using $f_3 = \frac{v}{\lambda_3} = \frac{3v}{2\ell}$ and $f_3 = 3f_1$.

The particle displacement has three nodes and four antinodes. And so in general,

> For a harmonic in a pipe open at both ends
>
> $$\ell = \frac{n\lambda_n}{2}$$
>
> $$\lambda_n = \frac{2\ell}{n}$$
>
> where λ is the wavelength (m)
>
> ℓ is the length of the string (m)
>
> n is the number of the harmonic, which is also the number of pressure antinodes (i.e. 1, 2, 3, 4…)

It should also be noted that, due to the air pressure from the sound wave in the tube, the standing wave produced will extend slightly beyond the end of the tube. The actual length that should be used will therefore be a little longer than the tube itself. The distance that the sound wave will extend beyond the end of the tube depends partially on the diameter of the tube itself and also on the surrounding air pressure. Since the discrepancy is small, for the purposes of this study the assumption will be made that the length of the tube coincides with the length of the standing wave.

Worked example 7.5.2

OPEN-ENDED AIR COLUMNS

A flute has an effective length of 35.0 cm. It can be thought of as an open-ended air column. The speed of sound is 346 m s^{-1}.	
a Calculate the wavelength of the second harmonic.	
Thinking	**Working**
Identify length of the air column (ℓ) in metres and the harmonic number (n).	$\ell = 35.0$ cm $\ell = 0.350$ m $n = 2$
Recall that for any frequency, $\lambda_n = \frac{2\ell}{n}$. Substitute the values from the question and solve for λ.	$\lambda_n = \frac{2\ell}{n}$ $\lambda_2 = \frac{2 \times (0.350)}{2}$ $\lambda = 0.35000$
Express the answer with the correct number of significant figures and the appropriate unit.	$\lambda = 0.350$ m

b Determine the frequency of the second harmonic.	
The frequency can be calculated using $f_2 = \frac{v}{\lambda_2}$.	$f_2 = \frac{(346)}{(0.350)}$ $f_2 = 988.571$
Express the answer with the correct number of significant figures and the appropriate unit.	$f_2 = 989$ Hz

c Determine the frequency of the fourth harmonic.	
Recall $f_n = nf_1$. Determine the fundamental frequency from the second harmonic frequency, then determine the fourth harmonic frequency from the fundamental.	$f_n = nf_1$ $f_1 = \frac{f_2}{2}$ $f_1 = \frac{(988.571)}{2}$ $f_1 = 494.286$ $f_n = nf_1$ $f_4 = 4 \times (494.286)$ $f_4 = 1977.143$
Express the answer with the correct number of significant figures and the appropriate unit.	$f_4 = 1980\,\text{Hz}$

Worked example: Try yourself 7.5.2

OPEN-ENDED AIR COLUMNS

A traditional Japanese bamboo flute has an effective length of 42.8 cm. It can be thought of as an open-ended air column. The speed of sound is $346\,\text{m}\,\text{s}^{-1}$.
a Calculate the wavelength of the third harmonic of the flute.

b Determine the frequency of the third harmonic.

c Determine the frequency of the fifth harmonic.

AIR COLUMNS CLOSED AT ONE END

In this context, a 'closed air column' means that the pipe or tube is closed at one end and remains open at the other. Some examples of closed air columns are the human vocal tract, the ear canal, ported loudspeakers and car engine manifolds.

In both strings and fully open pipes, the reflection of the wave is the same at both ends. The *open end* of the air column reflects a sound wave with a change of phase. However, in a pipe closed at one end it's different. At the *closed end*, there will be no change of phase for the reflected sound wave. Here the reflected waves will interfere constructively with the incoming waves so there will be a maximum in pressure variation (a compression). However, the air particle movement (displacement) will be minimal in this region.

The result is that the pressure variation established in a tube closed at one end will have a node at the open end and an antinode at the closed end, as shown in Figure 7.5.8a. Conversely, there will be a maximum in particle displacement at the open end and a minimum at the closed end, as shown in Figure 7.5.8b.

> For a tube closed at one end the pressure variation will have an antinode at the closed end and a node at the open end.
>
> Particle displacement will be a minimum at the closed end and a maximum at the open end.

For the following analysis refer to Figure 7.5.8a.

For the first harmonic, or fundamental frequency, only a quarter of a wavelength will fit into the air column, therefore it will have a wavelength four times the length of the effective air column.

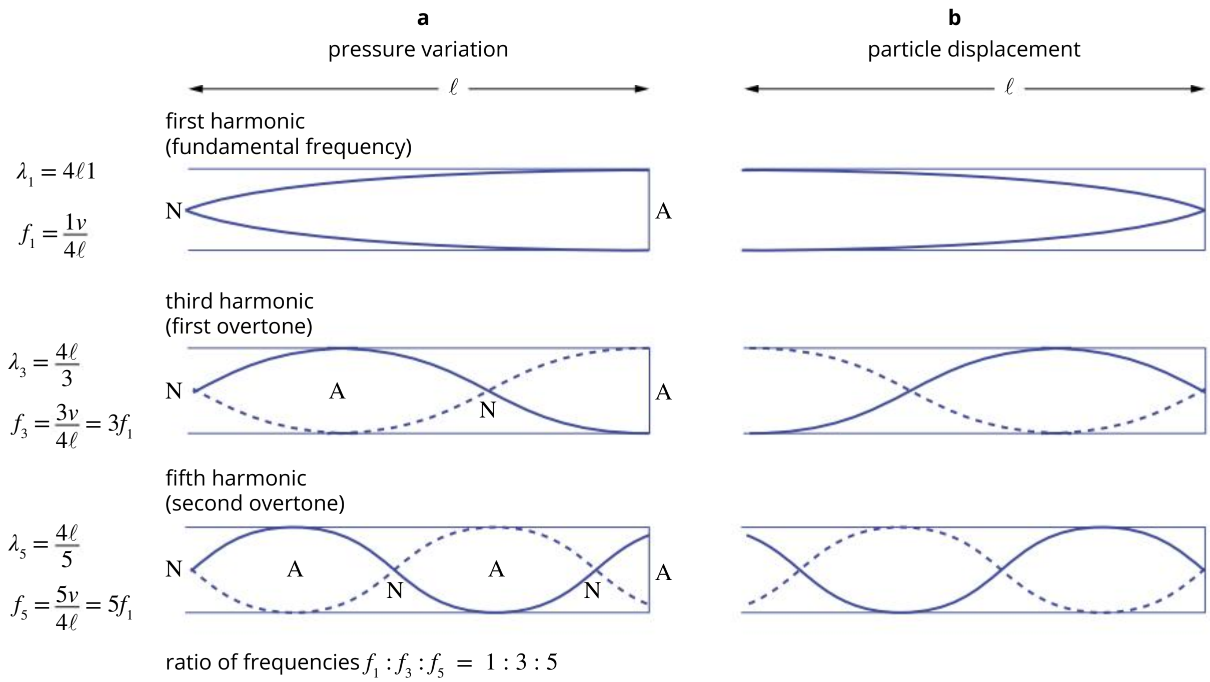


FIGURE 7.5.8 (a) The lower harmonics for a pipe closed at one end. Only odd-numbered harmonics are possible, since only these satisfy the condition of having a node in pressure variation at the open end and an antinode at the closed end. (b) The equivalent particle displacement. The particles have maximum displacement at the open end and minimum displacement at the closed end.

The first harmonic ($n = 1$), or fundamental frequency, will have:

$\ell = \frac{1\lambda_1}{4}$ and $\lambda_1 = 4\ell 1$. Using $f = \frac{v}{\lambda}$ then $f_1 = \frac{1v}{4\ell}$

$n = 2$ cannot form because this would result in a pressure antinode at each end.

For the next harmonic, three quarters of a wavelength can fit in the air column; therefore, it will have a wavelength $\frac{4}{3}$ the length of the pipe. This is the $n = 3$ harmonic.

$\ell = \frac{3\lambda_3}{4}$ gives $\lambda_3 = \frac{4\ell}{3}$ and $f_3 = \frac{v}{\lambda_3} = \frac{3v}{4\ell}$

The next harmonic that can form will have five quarters of a wavelength that will fit in the air column, therefore:

$\ell = \frac{5\lambda_5}{4}$ and $\lambda_5 = \frac{4\ell}{5}$ and $f_5 = \frac{v}{\lambda_5} = \frac{5v}{4\ell}$. This is the $n = 5$ harmonic.

In general, for a pipe closed at one end:

$$\ell = \frac{n\lambda_n}{4}$$

and

$$\lambda_n = \frac{4\ell}{n}$$

where λ is the wavelength in metres (m)

ℓ is the length of the pipe in metres (m)

n is the number of the harmonic; odd-number integers only (i.e. 1, 3, 5...)

The frequency of the harmonic is given by:

$$f_n = \frac{nv}{4\ell}$$

where n is the number of the harmonic (i.e. 1, 3, 5…)

Notice that only odd-numbered harmonics will form since the conditions necessary for a standing wave to form are only met when there is a pressure antinode at the closed end of the pipe and a pressure node at the open end. The ratio of the wavelengths of the harmonics will be 1 : 3 : 5 : ... That means the wavelength of the third harmonic is $\frac{1}{3}$ the length of the fundamental frequency (first harmonic), the fifth harmonic is $\frac{1}{5}$, and so on.

As in open columns, the air pressure from the sound wave in the tube will cause the standing wave produced to extend slightly beyond the end of the tube. The actual length that should be used will therefore be a little longer than the tube itself. Since the discrepancy is small, for the purposes of this study the assumption will be made that the length of the tube coincides with the length of the standing wave.

Worked example 7.5.3

AIR COLUMN CLOSED AT ONE END

The ear canal, from the outer ear to the eardrum, can be thought of as a tube closed at one end (by the eardrum) and open at the other. It is approximately 3.00 cm long in an adult.

a Calculate the wavelength of the fundamental frequency of the adult ear canal.

Thinking	Working
Identify the length of the air column (ℓ) in metres and the harmonic number (n).	$\ell = 3.00\text{ cm}$ $\ell = 0.0300\text{ m}$ $n = 1$
Recall that for any frequency for closed tubes, $\lambda_n = \frac{4\ell}{n}$. Substitute the values from the question and solve for λ.	$\lambda_n = \frac{4\ell}{n}$ $\lambda_1 = \frac{4 \times (0.0300)}{1}$ $\lambda_1 = 0.120000$
Express the answer with the correct number of significant figures and the appropriate unit.	$\lambda_1 = 0.120\text{ m}$

b Assuming that the speed of sound through air is 346 m s^{-1}, what frequency does this wavelength correspond to?

Thinking	Working
Identify the speed of the sound (v) in m s^{-1} and the wavelength (λ) from the previous question.	$v = 346\text{ m s}^{-1}$ $\lambda_1 = 0.120000\text{ m}$
Recall the wave equation $v = f\lambda$. Rearrange to find f.	$f = \frac{v}{\lambda_1}$ $f_1 = \frac{(346)}{(0.120000)}$ $f_1 = 2883.333$
Express the answer with the correct number of significant figures and the appropriate unit.	$f_1 = 2880\text{ Hz}$

PHYSICS IN ACTION

Music and musical scales

Musical instruments generally produce standing waves that are whole-number multiples of a fundamental frequency. A superposition of these waves generally produces a sound that is harmonious. Harmonies are an important part of music. For example, a ratio of 2 : 1 in frequency is called an octave. Middle C has a frequency of 261.6 Hz. If you go up an octave, the next C is at 523.3 Hz, which is double the frequency of middle C.

This answer explains why sounds of around 3 kHz are generally heard best by adult humans. The length of the ear canal means that sounds of around this frequency are best amplified by resonance forming a standing wave. There are, however, many other factors which influence the range of frequencies a person is able to hear.

Worked example: Try yourself 7.5.3

AIR COLUMN CLOSED AT ONE END

Tom notices that he can hear a note coming from his open bottle of water as the wind blows over it. The water bottle is acting like an air column closed at one end and is 12.5 cm long. Assume that the standing wave does not extend beyond the end of the bottle.

a Calculate the wavelength of the third harmonic of the water bottle.

b Calculate the frequency of the third harmonic assuming the speed of sound is $346\ \text{m s}^{-1}$.

While the discussion in this section has been of two-dimensional standing waves, standing waves may also form in three dimensions, such as in a section of the Earth's crust. Standing waves will form as a result of resonance in any wave form.

7.5 Review

SUMMARY

- Standing or stationary waves occur as a result of resonance at the natural frequency of vibration.
- Standing waves are produced by the superposition of waves of equal amplitude and frequency/wavelength travelling in the opposite direction.
- Points on a standing wave that remain still are known as nodes.
- Points of maximum vibration (in a string) or pressure variation (in a tube) on a standing wave are known as antinodes.
- The standing wave frequencies are referred to as harmonics. The simplest mode is referred to as the fundamental frequency.
- The harmonics above the fundamental frequency are also the called the first overtone, second overtone, third overtone and so on.
- Within a string fixed at both ends, the wavelength of the standing waves corresponding to the various harmonics is:
  $$\lambda = \frac{2\ell}{n}$$
  and the frequency is:
  $$f = \frac{nv}{2\ell}$$
  All harmonics may be present.
- Longitudinal standing waves can also form in air columns.
- At the open end a phase change occurs and a pressure variation node forms.
- If a pipe is closed at one end, there is no phase change and a pressure variation antinode is formed at the closed end.
- An antinode in the standing wave (maximum pressure variation) corresponds to a minimum in particle displacement.
- A node in the standing wave (minimum pressure variation) corresponds to a maximum in particle displacement.
- For a pipe open at both ends, the pressure variation is similar to that of a string fixed at both ends, therefore:
  $$\lambda_n = \frac{2\ell}{n}$$
  and the frequency is:
  $$f_n = \frac{nv}{2\ell}.$$

- For a pipe with one end closed, an antinode forms at the closed end and a node forms at the open end:

$$\lambda_n = \frac{4\ell}{n}$$

and the frequency is:

$$f_n = \frac{nv}{4\ell}.$$

- Expressed this way, only the odd numbered harmonics, $n = 1, 3, 5...$, can be formed.

KEY QUESTIONS

1 A transverse standing wave is produced using a rope. Is the standing wave actually standing still? Explain your answer.

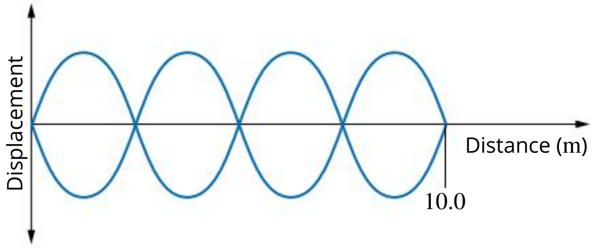
2 Describe how superposition and interference are related to the formation of standing waves in a stretched slinky spring.

3 What is the wavelength of the fundamental mode of a standing wave on a string 0.400 m long and fixed at both ends?

4 Calculate the length of a string fixed at both ends when the wavelength of the fourth harmonic is 0.750 m.

5 A standing wave is produced in a rope fixed at both ends by vibrating the rope with four times the frequency that produces the fundamental or first harmonic. How much larger or smaller is the wavelength of this standing wave compared to that of the fundamental or first harmonic?

6 A standing wave pattern in a string is shown over a distance of 10.0 m.



What is the length of the rope that would generate the first harmonic if a standing wave of the same wavelength shown in the diagram was produced?

7 The fundamental frequency of a violin string is 352 Hz and the velocity of the waves along it is 387 m s^{-1}. What is the wavelength of the new fundamental when a finger is pressed to shorten the string to $\frac{2}{3}$ of its original length?

8 A metal string (at constant tension) of length 50.0 cm is plucked, creating a wave pulse. The speed of the transverse wave created is 304 m s^{-1}. Both ends of the string are fixed.

a Calculate the fundamental frequency.

b Calculate the frequency of the second harmonic.

c Calculate the frequency of the third harmonic.

9 A flute can be considered an open-ended air column. For a flute of effective length 45.0 cm:

a What is the wavelength of the fundamental frequency?

b What is the wavelength of the second harmonic?

c Calculate that the frequency of the third harmonic, assuming that the speed of sound in the flute is 346 m s^{-1}.

10 An organ pipe is an air column closed at one end with an effective length of 75.0 cm. The speed of sound inside the pipe is 346 m s^{-1}.

a Calculate the frequency of the fundamental note produced by the pipe.

b Calculate the frequency of the third harmonic.

c Calculate the frequencies of the next two harmonics, after the third harmonic, that the pipe can produce.

11 **a** A pipe produces a fundamental frequency of 453 Hz and subsequent resonant frequencies of 906 Hz, 1359 Hz and 1812 Hz. Is this an open-ended or closed pipe? Explain why.

b A pipe produces a fundamental wavelength of 3.00 m and subsequent resonances at 1.00 m and 0.600 m. Is this an open-ended or closed pipe? Explain why.

7.6 Applications of wave properties

Waves are fundamental phenomena observed in various contexts, from natural occurrences to engineered applications. Understanding wave properties enables us to harness them in diverse fields, including music, medicine and industry.

PHYSICSFILE

Symphony orchestras

In a symphony orchestra, shown in Figure 7.6.1, a large variety of instruments is required to produce the complex sounds in a piece of classical music. These range from small instruments like the piccolo and the flute, which produce high-frequency sounds, to mid-frequency instruments such as the cello, and finally to the basses, which produce deep sounds.

The positioning of the instruments is similar in most orchestras and allows the sounds to superpose to produce the complex range of sounds required. You will recall that high-frequency sounds reflect and low-frequency sounds are more likely to diffract. The theatre is designed to ensure each member of the audience receives the full frequency range of sounds from the orchestra.

FIGURE 7.6.1 The instruments in the West Australian Symphony Orchestra (WASO) are placed so that the sounds superpose to produce the best listening experience for the audience.

APPLICATIONS OF WAVES—MUSIC AND MUSICAL INSTRUMENTS

One of the most common applications of wave properties is in the realm of music. Musical instruments exploit the principles of wave mechanics to produce harmonious sounds. By mastering these principles, musicians can create a wide array of tones and melodies.

Guitar tuning

A typical acoustic guitar, as shown in Figure 7.6.2, comprises six strings that are connected to a bridge, then a wooden body. The vibrating strings cause the bridge, and consequently the front of the body, to vibrate. These vibrations resonate with the air in the body cavity, causing the sound to amplify. The length of the guitar can vary depending on the model, but the length of each string that vibrates on the guitar will be the same. A guitarist can press on the frets to change the length of each string, which changes the resonant frequency of the string and therefore the pitch it produces.

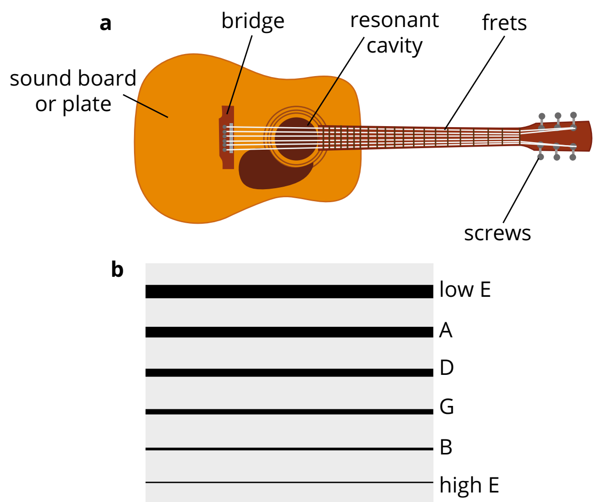


FIGURE 7.6.2 (a) A guitar consists of six strings, a sound plate and a resonating cavity. The tension in the strings can be adjusted by tightening or loosening the screws at the end of the guitar. (b) The names of the strings on a standard guitar which refer to musical notes.

As the length of each string (ℓ) is the same, the fundamental wavelength $\left(\frac{\lambda_1}{2\ell}\right)$ for each string will also be the same. To change the fundamental frequency without pressing on the frets, the speed of the waves along the string must be changed. This can be done in two ways: by adjusting the tension (T) using the screws at the end of the arm, or by changing the thickness or density of the string, which changes the mass per unit length (μ). The relationship is described by the following formula:

$$v = \sqrt{\frac{T}{\mu}}$$

where μ is the mass per unit length (kg m^{-1})

T is the tension in the string (N)

v is the speed of the wave on the string (m s^{-1}).

On an acoustic guitar, the higher strings are often made of a lighter material such as low-density nylon and the lower strings are made of either high-density nylon or steel. The guitarist tunes each string to the desired note, or frequency, by turning the corresponding screw at the end of the string. Tightening the string will increase the tension and loosening the string will decrease the tension. The standard fundamental frequencies for a tuned guitar are given in Table 7.6.1.

TABLE 7.6.1 The standard fundamental frequencies for a tuned guitar.

Name of string	Frequency (Hz)
high E	329.63
B	246.94
G	196.00
D	146.83
A	110.00
low E	82.41

Once the guitar is tuned, the guitarist can play a variety of notes on a single string by pressing the string behind the frets to shorten its length. It should be noted that the dominant frequency will be the fundamental frequency, but the overtones or harmonics will also be heard, which gives the guitar its rich sound.

Wind instruments

Wind instruments do not have strings of different masses that can be tensioned or released to produce different frequencies. To understand how they work, first recall from Section 7.5 that the resonant frequency of sound from a pipe can be changed by varying the length of the pipe or by accessing different harmonics.

In woodwind instruments such as the saxophone, shown in Figure 7.6.3a, the effective length of the pipe can be changed by covering and uncovering holes along the length of the pipe. In bugles or ceremonial trumpets where the length is fixed, however, the pitch of the note produced must come from the range of harmonics available as a direct result of that fixed length, which severely restricts the number of notes that can be played. In the case of orchestral trumpets, shown in Figure 7.6.3b, valves are used to connect additional curved lengths of pipe. The coiled lengths of pipe in trombones, French horns and trumpets allow access to a greater total length and thus a greater potential range of notes. Part of the skill of a woodwind or brass musician is controlling which mode of vibration dominates the final sound.

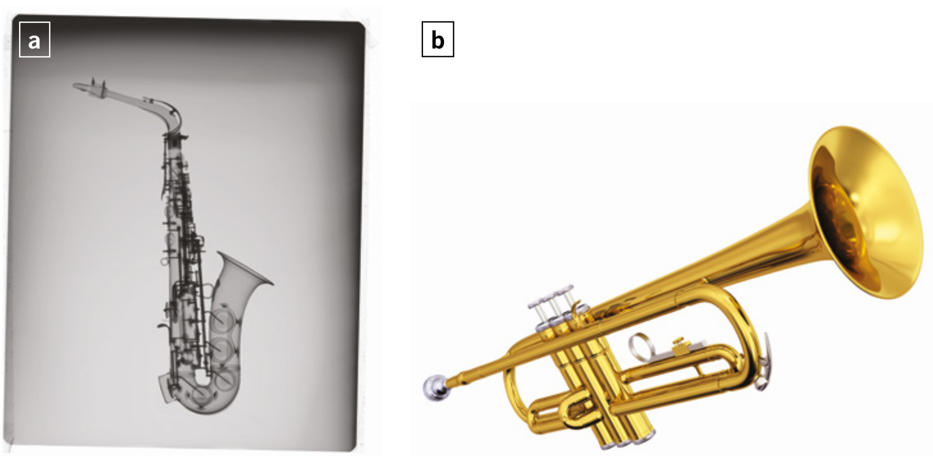


FIGURE 7.6.3 (a) An X-ray image of a saxophone where the pitch can be changed by depressing different keys. (b) A trumpet where pitch can be changed either by depressing the keys or by blowing into the mouthpiece in different ways.

PHYSICSFILE

Wind instrument: didjeridu

The didjeridu, shown in Figure 7.6.4, is a First Nations wind instrument believed to have originated in the north-west region of the Northern Territory and Western Australia, and was made from bamboo. It then migrated to north-eastern Arnhem Land where it is known as the yidaki by the Yolngu people. Its use has now spread to many other areas of Australia, with its name varying based on the cultural language of the region. The resonant frequencies of the sound produced depend on the length of the pipe and the shape of the musician's mouth.

FIGURE 7.6.4 A First Nations wind instrument, commonly known as the didjeridu. It has different names depending on the region.

APPLICATIONS OF WAVES—ULTRASOUND

Sound is an example of energy travelling in the form of a wave. Humans can hear sound waves in the frequency range of 20 to 20 000 Hz. Frequencies above 20 000 Hz are called ultrasound. Ultrasounds are used in medical applications in two different ways: as a diagnostic tool and for treatment.

Diagnostic tool and imaging

The most common use of ultrasound is diagnostic, primarily for producing images of internal parts of the body, and relies on the reflection of sound waves. To minimise effects on cells within the body, low-intensity waves are used. For example, Figure 7.6.5 shows how ultrasound can be directed through the amniotic fluid within the womb to build a picture of a developing foetus. It can also be used to detect the presence of foreign bodies and cancerous cells, as well as to guide a needle for tumour treatment, biopsy or cortisone injection.

FIGURE 7.6.5 An ultrasound image of a second trimester foetus.

In diagnostic ultrasound, sound waves travel into the tissue and are reflected back. The intensity of the reflected waves depends on the density of the structure in the body. Structures with higher densities, such as bone and dense foreign bodies, reflect more waves and appear white, whereas water, fluid and urine reflect no waves and appear black. Other tissues within the body will have a greyscale appearance in between these two extremes. In Figure 7.6.5, for example, the amniotic fluid surrounding the foetus appears black and the grey-white structures of the foetus indicate bone and other tissue.

Clinical ultrasound frequencies range between 1.00 and 20.0 MHz. Given the average speed of sound in the human body is 1540 m s^{-1}, the corresponding wavelength range, calculated using $\lambda = \frac{v}{f}$, is from 1.54×10^{-3} m to 7.70×10^{-5} m. The resolution of the ultrasound image (i.e. the level of detail) is therefore higher using the high-frequency waves because they have shorter wavelengths. However, penetration depth into the body decreases with increasing frequency, so the clinician has to compromise depending on the depth required. Interestingly, tissue harmonics can occur, where the reflected wave resonates at multiples of the incident frequency. For example, a 4.00 MHz wave can produce reflected frequencies at 4.00, 8.00 and 12.0 MHz. Clinicians can use this effect to build more accurate and detailed images.

Ultrasound as a treatment tool

Ultrasound can also be used as a therapy, ranging from the treatment of kidney stones to enabling surgeons to perform delicate brain surgery. The therapeutic application of ultrasound uses high-intensity sound waves on the cells of the body. Recall that sound waves are just mechanical vibrations in the particles of the carrying medium. During therapeutic ultrasound procedures, these vibrations are introduced into the cells of the body. At low intensities, the vibrations are hardly more energetic than they would normally be. However, if the sound waves are intense enough, they can heat targeted regions deep inside the body and damage or destroy cells in those areas.

Heat treatments

An example of the use of the heating effects of ultrasound is the treatment of sports injuries in muscles and joints as shown in Figure 7.6.6. To avoid tissue damage, intensity values must be kept below $30\,000\,\text{W}\,\text{m}^{-2}$. Ultrasound exposures of 5 to 10 minutes are often included in physiotherapy programs. The heating effect is believed to increase metabolism at the treated site and accelerate healing. This method is most effective in bone and denser muscles, which absorb sound waves more efficiently. Mild heating by ultrasound can also be used to treat blockages of the middle ear region. Additionally, ultrasound is being investigated as a treatment for arthritis, although the results remain inconclusive.

FIGURE 7.6.6 High-intensity ultrasound is used to treat some sports injuries.

Destructive effects of ultrasound

If the intensity of the sound waves that are sent into the body is high enough, they can be used to destroy certain cells. The intense vibrations cause overheating and large stresses, which can rupture cell membranes. This property can be used beneficially, for example in the treatment of gallstones (in the gall bladder) or kidney stones. High-intensity ($\sim 10^5\,\text{W}\,\text{m}^{-2}$) ultrasound waves physically break up the stones, and their component particles are then washed away by the normal removal processes of the body. Previously the removal of these items would have involved surgery.

Some types of tumorous cells can also be treated with ultrasound. Again, an intense beam of sound waves is used with the intention of breaking the cells apart. When this process is applied to a very concentrated area, surgeons have a very effective cauterising (cutting and sealing) tool. Neurosurgeons use extremely narrow beams of sound waves with an intensity of around $2.50 \times 10^5\,\text{W}\,\text{m}^{-2}$ to 'cut out' brain tumours.

APPLICATIONS OF WAVES—INDUSTRY

Ultrasound waves can be used in a variety of industries from cleaning, welding plastic, cutting and forming to diagnosing the quality of materials and structures. Ultrasonic waves are usually generated using transducers made from piezoelectric materials. In these materials, electrical energy is converted to mechanical energy, producing ultrasound waves from 50.0 kHz to 10.0 MHz. Similarly, piezoelectric materials can be used to detect ultrasonic pulses, making them versatile for both generating and sensing ultrasound.

Ultrasonic cleaning

An ultrasonic bath, as shown in Figure 7.6.7, can be used to clean medical and laboratory equipment, as well as everyday items such as glasses and jewellery. High-frequency ultrasound waves, typically 40.0 kHz, are used to produce tiny bubbles in a process known as cavitation. These bubbles form and collapse millions of times per second, generating enormous heat and pressure that dislodge and break up the dirt and contaminants on the items being cleaned.

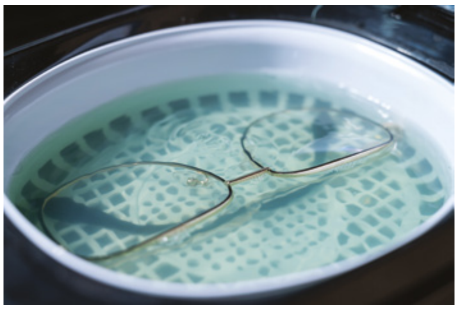


FIGURE 7.6.7 In an ultrasonic bath, ultrasound waves dislodge dirt and clean the glasses.

Ultrasonic welding

In ultrasonic welding, 20.0–40.0 kHz ultrasound waves with an amplitude of 1.00 to 25.0 μm are used to create mechanical vibrations, which generate heat. This heat is sufficient to melt or soften thermoplastics and is therefore used to weld thermoplastic structures.

Diagnosis—defect detection

Ultrasonic transducers and detectors are used in various industries as a non-destructive method for detecting defects inside materials such as concrete, steel and industrial structures. Figure 7.6.8 shows an operator looking for defects, though remote operation is also possible. This is particularly useful in hazardous environment, such as high-temperature areas or around nuclear reactors.

FIGURE 7.6.8 An operator using an ultrasound transducer to check for defects within the pipe.

PHYSICSFILE

Light from sound—sonoluminescence

Ultrasound waves can create small gas bubbles that are briefly suspended in liquid before undergoing periodic collapse many millions of times per second. This produces an enormous amount of energy. In certain circumstances these bubbles emit light when they collapse.

7.6 Review

SUMMARY

- Stringed musical instruments use the principles of resonance on a string to produce harmonics.
- Woodwind and brass musical instruments rely on resonance in pipes and depend on whether they are open or closed at one end.
- Ultrasound frequencies are above 20 000 Hz.
- Low-intensity ultrasound can be used for diagnostic purposes in medical imaging.
- High-intensity ultrasound can be used for heat treatment and for removing unwanted cells such as cancers and kidney stones.
- In industry, ultrasound waves can be used for a variety of applications, such as cleaning, welding of thermoplastics, and defect detection in industrial materials like metal and concrete, and structures such as pipes.

KEY QUESTIONS

For questions 1 to 5, use the information found in Table 7.6.1 and consider the length of the guitar strings to be 651 mm.

1 Calculate the speed of the wave on the low E string as a low E note is played.

2 Calculate the wavelength and the frequency of the D string when it is shortened to two thirds of its regular length.

3 Determine how much the low E string needs to be shortened to match the frequency of the high E string.

4 Guitarists sometimes use the third harmonic of the low E string when tuning the B string.

- **a** Calculate the frequency of the third harmonic of the low E string.
- **b** Use resonance and the information in Table 7.6.1 to explain why this works.

5 Consider the fundamental frequency of each string on the guitar. Describe, using appropriate equations from the text:

- **a** how the speed of vibration differs in the strings
- **b** why the thickness of the strings needs to change from the higher strings to the lower strings.

6 Compare and contrast ultrasound waves used for diagnostic purposes and ultrasound waves used for treatment purposes.

7 Explain the compromise made in ultrasound between depth and resolution.

8 Why is ultrasound effective in cleaning?

Chapter review

07

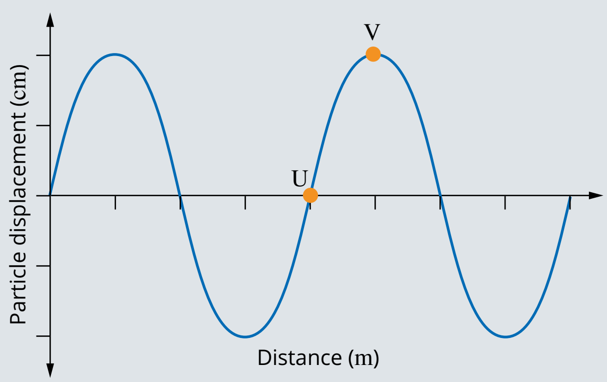
KEY TERMS

absorb
air pressure
amplitude
angle of incidence
angle of reflection
angle of refraction
antinode
beat frequency
compression
constructive interference
crest
critical angle
destructive interference
diffraction
diffuse
Doppler effect
echo
elasticity
first harmonic
forcing frequency
frequency
fundamental
harmonic
interference
in phase
longitudinal
mechanical wave
medium
natural frequency
node
normal
oscillate
overtone
particle displacement
period
phase
plane wave
pulse
rarefaction
ray
reflect
refraction
resonance
resonant frequency
reverberation
seismic wave
sinusoidal
standing wave
superposition
total internal reflection
transmit
transverse
travelling wave
trough
vibration
wave front
wavelength

1 Imagine that you watch from above as a stone is dropped into water. Describe the movement of the particles on the surface of the water.

2 Describe the similarities and differences between transverse and longitudinal waves.

3 At the moment in time shown on the graph, in what directions are the particles U and V moving?



4 The source of waves in a ripple tank vibrates at a frequency of 10.0 Hz. If the wave crests formed are 30.0 mm apart, what is the speed of the waves (in $m\,s^{-1}$) in the tank?

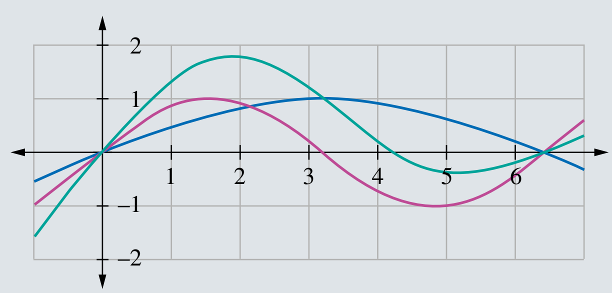
5 A submarine's sonar sends out a signal with a frequency of 32.0 kHz. If the wave travels at 1450 $m\,s^{-1}$ in seawater, calculate the wavelength of the signal.

6 A motorbike is able to produce a long, steady sound. Because of the surrounding trees, you are unable to see the motorbike, but you can hear the sound from it rising in frequency, then falling. Which one or more of the following options best explains the motion of the motorbike relative to you?

A The bike travelled towards you.
B The bike travelled away from you.
C The bike travelled past you.
D The bike travelled towards you, then away from you.

7 If you decreased the wavelength of the sound made by a loudspeaker, what effect would this have on the frequency and the velocity of the sound waves?

8 The following graph shows three wave forms. Two of the wave forms superimpose to form the third:



Which wave is the result of the superposition of the other two?

9 Using ideas about the movement of particles in air, explain how you know sound waves only carry energy and not matter from one place to another.

10 A sound wave is emitted from a piano and heard by a person who is 50 m from the piano. The person made several statements once they heard the sound. Which one or more of the following statements made by the person would be correct? Explain your answers.
   A Hearing a sound wave tells me that air particles have travelled from the piano to me.
   B Air particles carried energy with them as they travelled from the piano to me.
   C Energy has been transferred from the piano to me.
   D Energy has been transferred from the piano to me by the oscillation of air particles.

11 Describe the concept of resonance and why it would need to be considered when designing structures like buildings or bridges.

The following information relates to questions 12–13.

A signal generator is attached to a device that produces vibrations down a string fixed at one end and free to move at the other. The string is kept at constant tension. The effective length of the string is 85.0 cm. The speed of the vibrations along the string is 445 m s^{-1}.

12 What is the lowest frequency of vibration that will produce a standing wave in the string?

13 What is the frequency of vibration of the third harmonic?

14 An earthquake causes a footbridge to oscillate up and down with a fundamental frequency once every 4.05 s. The motion of the footbridge can be considered to be like that of a string fixed at both ends. What is the frequency of the second harmonic for this footbridge?

15 The velocity of waves in a particular string at constant tension is 78.0 m s^{-1}. The string is fixed at both ends. If a particular frequency of a standing wave formed in the string is 428 Hz, how far apart would two adjacent antinodes be?

16 Reflection is possible from which of the following shaped surfaces if each surface is reflective? (More than one answer may be correct.)
   A flat surface
   B concave (curved in) surface
   C uneven surface
   D convex (curved out) surface

17 Resonance occurs when the frequency of a forcing vibration exactly equals the natural frequency of vibration of an object. Which one of the following responses relating to the effects of resonance is true? Explain your answer.
   A The amplitude of vibration will decrease.
   B The amplitude of vibration will increase.
   C The frequency of vibration will increase.
   D The frequency of vibration will decrease.

18 The third harmonic of an open pipe produces a frequency of 408 Hz. Assuming the speed of sound is 346 m s^{-1}, calculate the length of the pipe.

19 A signal generator connected to a speaker produces sound waves that are directed into a tube closed at one end. The effective length of the tube is 85.0 cm, and the speed of sound is 346 m s^{-1}.
   a What is the lowest frequency of sound that will produce resonance in the tube?
   b What frequency of sound will cause the tube to resonate at its third harmonic?

20 If a sound wave travels through water into air at an angle, what would you expect to happen to the angle of refraction relative to the angle of incidence? Explain your answer.

21 Under what conditions does total internal reflection occur?

22 Ming has two tuning forks that produce a beat frequency of 4.00 Hz when sounded together. Ming is certain that one of the tuning forks is accurate and is equal to 427 Hz. Determine all of the possible values of the other tuning fork.

CHAPTER

08 Particles in the nucleus

Radioactive emission is important in medicine for treating and detecting cancers, and for many imaging purposes. It also has many industrial applications, including power generation (covered in Chapter 9). This chapter examines the instability in the nucleus that leads to radioactive emissions. You will learn the properties of alpha, beta and gamma radiation, including balancing nuclear equations and predicting radiation types. You will also understand the importance of the rate of decay and half-lives of radioactive substances in determining appropriate nuclides for medical and industrial applications, how a radiation dose is measured, and the safe limits for doses of different types of radiation.

Science as a Human Endeavour

- radioisotopes are used as diagnostic tools and for tumour treatment in medicine

Science Understanding

- the nuclear model of the atom describes the atom as consisting of an extremely small nucleus which contains most of the atom's mass, and is made up of positively charged protons and uncharged neutrons surrounded by negatively charged electrons
- nuclear stability is the result of the strong nuclear force which operates between nucleons over a very short distance and opposes the electrostatic repulsion between protons in the nucleus
- some nuclides are unstable and spontaneously decay, emitting alpha, beta (+/–) and/or gamma radiation over time until they become stable nuclides
- protons and neutrons are made of fundamental particles called quarks
- beta minus (β^-) and beta plus (β^+) decay can be explained by the transformation of quarks in the nucleus
- in nuclear reactions, energy, momentum and charge are conserved; one example is through the emission of neutrinos or antineutrinos in beta plus (β^+) and beta minus (β^-) decay
- each species of radionuclide has a half-life which indicates the rate of decay, including applying the relationship

  $$N = N_0\left(\frac{1}{2}\right)^n$$

- alpha, beta and gamma radiation have different natures, properties and effects
- the measurement of absorbed dose and dose equivalence enables the analysis of health and environmental risks, and includes applying the relationships

  $$\text{absorbed dose} = \frac{E}{m}, \text{ dose equivalent} = \text{absorbed dose} \times \text{quality factor}$$

- alpha and beta decay are examples of spontaneous transmutation reactions, while artificial transmutation is a managed process that changes one nuclide into another

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.

8.1 Atoms, isotopes and radioisotopes

Many people mistakenly think that they never come into contact with radioactive materials, or the radiation that these materials produce. However, the Earth is a radioactive planet, and it is impossible to avoid exposure to some radioactivity. Human senses cannot detect the radiation from radioactive atoms and human beings are biologically adapted to cope with the background level of radiation; however, high-energy radiation in larger than normal doses can be damaging to living tissue. Radiation and radioactive elements can also be used in a variety of applications that are beneficial, like medicine. These radioactive atoms, or radioisotopes, will be discussed in this section.

THE STANDARD MODEL

To understand radiation and radioactivity, it is necessary to know about the structure of atoms. The **Standard Model** states that all matter is composed of one or more of the 12 fundamental or elementary particles. A fundamental particle is, to the best of our knowledge, not made of other smaller particles. All these particles are called **fermions** and are divided into two groups of six particles called **quarks** (blue) and **leptons** (green), shown in Figure 8.1.1. Most of the particles described by the Standard Model are not observed in everyday situations. They are made in high-energy particle collisions within machines called **particle accelerators**.

	Charge			
Quarks	$+\frac{2}{3}$	up u 0.004	charm c 1.5	top t 176
Quarks	$-\frac{1}{3}$	down d 0.008	strange s 0.15	bottom b 4.7
Leptons	–1	electron e^- 5×10^{-4}	muon μ^- 0.1	tau τ^- 1.8
Leptons	0	electron neutrino ν_e $< 1 \times 10^{-8}$	muon neutrino ν_μ $< 1 \times 10^{-4}$	tau neutrino ν_τ $< 1 \times 10^{-2}$

Note: The number below the symbol for each particle is the mass and is given relative to the mass of a proton, ~1 GeV/c^2.

FIGURE 8.1.1 Fermions are the fundamental particles found in the universe.

The Standard Model of particle physics also explains three of the four fundamental forces in the universe (i.e. electromagnetic force, the strong nuclear force and the weak nuclear force) in terms of an exchange of particles called **gauge bosons**. In this model, the fundamental forces that act at a distance are mediated (transmitted) by these force-carrying particles. The gauge bosons (or just bosons) discovered so far are the **photon** (electromagnetic force), the **gluon** (strong nuclear force), and the Z, W^- and W^+ bosons (weak nuclear force).

Most particles have what is called an **antiparticle**. Each antiparticle has the same properties as its corresponding particle, such as mass, spin and lifetime. However, their electric charges and another characteristic called the **quantum number** have the same magnitude with the opposite sign. For example, a positron has the same mass as an electron but a positive charge. Similarly, antiquarks have the same mass as quarks but carrying opposite fractional charges.

There are two conventions used to indicate that a particle is an antiparticle.

- The antiparticles of uncharged particles are denotated by placing a bar above the symbol for the normal matter particle. For example, an electron neutrino has the symbol ${}^{0}_{0}\nu_e$ and the antielectron neutrino has the symbol ${}^{0}_{0}\bar{\nu}_e$.
- The antiparticle of a charged particle is given the symbol of the particle but with the opposite sign. For example, the antiparticle of the muon (μ^-) is the antimuon (μ^+).

All **hadrons** are made of quarks held together by the strong nuclear force. Hadrons fall into two categories: **baryons** and **mesons**.

Protons and neutrons are composite particles made up of three quarks held together by the strong nuclear force, which is mediated by gluons. All of the particles that comprise three quarks are collectively known as baryons. It is interesting to note that the combined masses of the quarks within a proton or neutron are much smaller than the total mass of those particles. This is because most of the mass of a proton or neutron is attributed to the energy stored in the strong interactions mediated by gluons. In fact, gluons, which act as exchange particles for the strong nuclear force, contribute just over 99% of the mass of a proton or neutron, while the quarks themselves contribute less than 1%.

Specifically, a proton is composed of two up quarks and one down quark, whereas a neutron consists of two down quarks and one up quark, as shown in Figure 8.1.2.

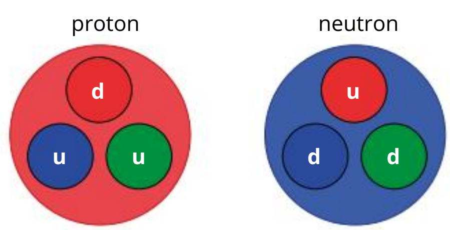


FIGURE 8.1.2 Protons and neutrons are baryons, which are all made of three quarks.

The fractional charges of the quarks combine to give the proton a charge of +1 and the neutron a charge of 0. Using the charges of the quarks found in Figure 8.1.2, the sum of the charges for each particle can be shown as below.

$$\text{Proton} = \text{uud} = \frac{2}{3} + \frac{2}{3} - \frac{1}{3} = \frac{3}{3} = 1$$

$$\text{Neutron} = \text{udd} = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$$

All hadrons that are made up of two quarks, one is a quark and the other is an antiquark, are called mesons. For example, the pion-plus (π^+) meson consists of an up quark and an anti-down quark, while the kaon-plus (K^+) meson consists of an up quark and an anti-strange quark. These mesons live long enough for their tracks to be seen in a detector.

Leptons, such as electrons and **electron neutrinos**, do not interact via the strong nuclear force and exist as individual particles. Charged leptons, like the electron, interact via the **electromagnetic force**, while **neutral** leptons, such as neutrinos, rarely interact at all. Electrons are never found within the nucleus of atoms but can be ejected from an unstable nucleus during radioactive decay. This apparent contradiction can be explained by the action of the weak nuclear force, which is mediated by the W^+ and W^- bosons. Radioactive decay of this kind is called beta decay and will be covered in more detail in Section 8.2.

EXTENSION

Discovery of other subatomic particles

Scientific understanding of the atom has changed greatly in the past 100 years. At one time, atoms were thought to be like miniature billiard balls: solid and indivisible. The word 'atom' comes from the Greek *atomos*, meaning indivisible. That idea was forever changed when the first subatomic particles were discovered. The electron, the proton and then the neutron were discovered between 1897 and 1932.

Since World War II, further research has uncovered about 300 other subatomic particles. These include pi-mesons, mu-mesons, kaons, tau leptons and neutrinos.

Figure 8.1.3 shows a photograph taken at CERN (the European Organization for Nuclear Research) in Switzerland. It shows hundreds of charged subatomic particles spilling out from the collision of a high-energy oxygen nucleus with a lead nucleus in the target. Figure 8.1.4 shows an underground tunnel with a blue tube stretching into the distance. It is part of the particle accelerator at CERN. The accelerator can speed up protons from rest to 99.99995% of the speed of light in under 20 seconds.

For many years, physicists found it difficult to make sense of this array of subatomic particles. It was known that one family of particles called the leptons had six members: electron, electron neutrino, muon, muon neutrino, tau and tau neutrino.

Then, in 1964, Murray Gell-Mann proposed a simple theory. He suggested that most subatomic particles were composed of even more fundamental particles called quarks. Currently, it is accepted that there are six different quarks: up, down, charmed, strange, top and bottom. The latest quark to be identified was the top quark, whose existence was confirmed in 1995. A proton consists of two up quarks and one down quark, while a neutron consists of one up quark and two down quarks. Subatomic particles that consist of quarks are known as hadrons. Leptons, such as electrons, are indivisible point particles and are not composed of quarks.

A significant amount of effort and money has been directed to testing Gell-Mann's theory, both theoretically and experimentally. This has involved the construction of larger and larger particle accelerators, such as Fermilab in Chicago and CERN in Geneva. Figure 8.1.5 shows particle tracks from a proton–proton collision seen by the Large Hadron Collider detector at CERN. CERN was the site of the discovery of the Higgs boson in 2012, a fundamental particle needed to explain the property of mass. Australia built its own particle accelerator—a synchrotron—next to Monash University in Victoria. This facility began operating in 2007 and is still used in many types of research today.

While the current theory suggests that quarks and leptons are the ultimate fundamental particles that make up matter and cannot be further divided, scientific theories and models can change as new experimental data are obtained. Are quarks and leptons made of smaller particles again? Time will tell.

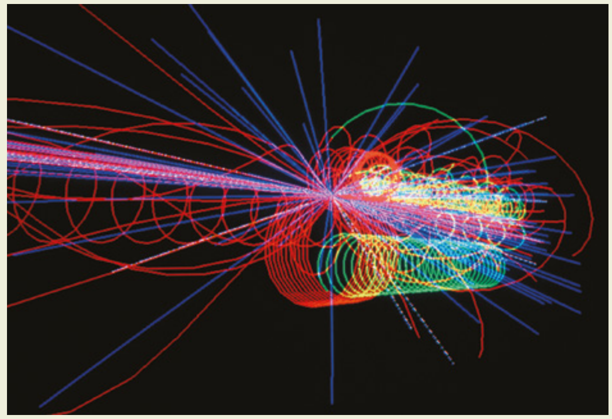


FIGURE 8.1.3 Collision of subatomic particles at CERN (the European Organization for Nuclear Research) in Switzerland.

FIGURE 8.1.4 Particle accelerator at CERN in Switzerland.

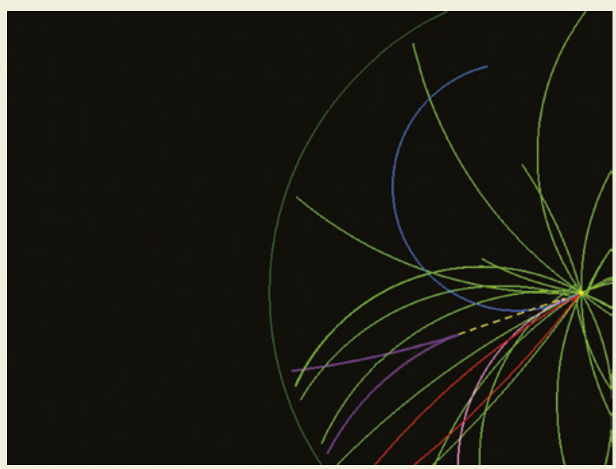


FIGURE 8.1.5 Collision between protons in the Large Hadron Collider at CERN.

THE STRUCTURE OF ATOMS

If an atom is **radioactive**, it will spontaneously emit **radiation** from its **nucleus**. Figure 8.1.6 shows this radiation being emitted in the form of particles and electromagnetic energy.

Atoms are made up of positively charged **protons** and neutral **neutrons**, which are found in the nucleus and are collectively called **nucleons**. Negatively charged **electrons** surround the nucleus and are much lighter in mass. The diameter of the nucleus is about 10 000 to 100 000 times smaller than that of the atom. Almost the entire mass of an atom is concentrated in its nucleus; the total mass of its electrons is less than one thousandth of the mass of the atom.

Figure 8.1.7 shows the structure of a typical atom, with an expanded view of the nucleus. Radioactive decay and nuclear reactions result from interactions within or between nuclei.

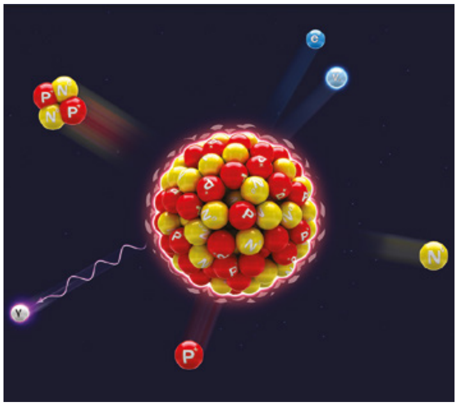


FIGURE 8.1.6 Radiation is spontaneously emitted from a radioactive nucleus.

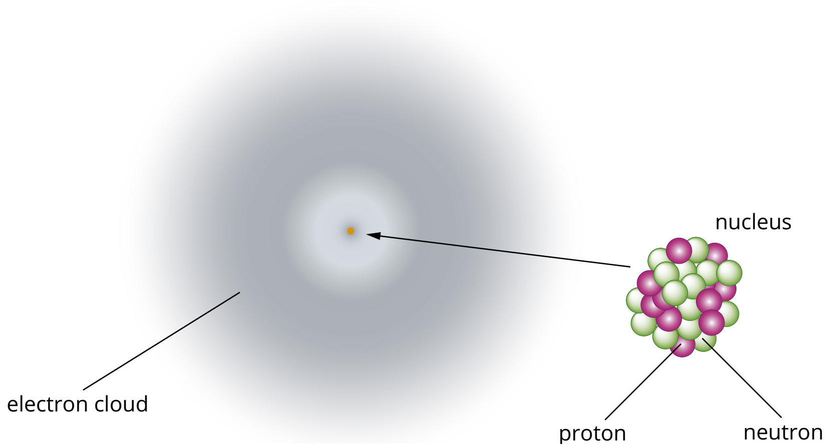


FIGURE 8.1.7 The nucleus of an atom occupies about 10^{-12} of the volume of the atom, yet it contains more than 99% of its mass. Atoms are mostly empty space. (Note: this atom is not drawn to scale.)

A particular atom can be identified by using atomic symbols that have the format shown in Figure 8.1.8.

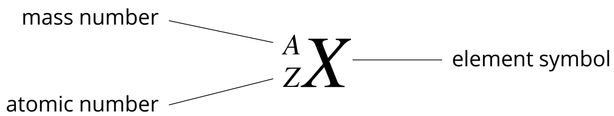


FIGURE 8.1.8 Atomic notation.

In atomic notation, the **mass number** (A) is the total number of protons and neutrons in the nucleus.

The **atomic number** (Z) is the number of protons in the nucleus.

The number of neutrons (N) is given by $N = A - Z$.

Atoms with the same number of protons belong to the same element. For example, if an atom has six protons in its nucleus (i.e. $Z = 6$), then the atom must be carbon. The number of neutrons does not affect which element the atom is, but it does affect the mass of the atom. Figure 8.1.9 shows how the size of the nucleus depends on the mass number. The more protons and neutrons there are in a nucleus, the heavier and larger it is.

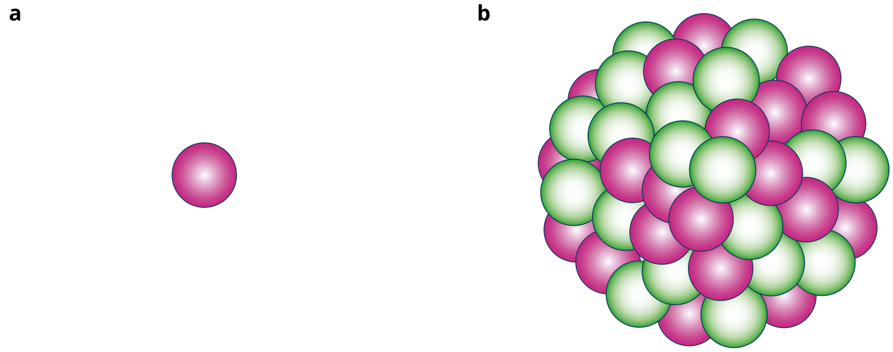


FIGURE 8.1.9 (a) and (b) are both nuclei, however the hydrogen atom (a) has a very different sized nucleus to a uranium atom (b). (Note: the nuclei are not drawn to scale.)

In an electrically neutral atom, the number of electrons is equal to the number of protons. For example, any neutral atom of uranium ($Z = 92$) has 92 protons in the nucleus and 92 electrons in the electron cloud.

Worked example 8.1.1

WORKING WITH ATOMIC NOTATION

How many protons, neutrons, nucleons and electrons are there in $^{197}_{79}Au$?	
Thinking	**Working**
The number at the bottom is the atomic number, *Z*. This is the number of protons.	atomic number, $Z = 79$ This nuclide has 79 protons.
The number at the top is the mass number, *A*. This indicates the number of particles in the nucleus, i.e. the number of nucleons.	mass number, $A = 197$ This nuclide has 197 nucleons.
The number of neutrons, *N*, is the difference between the mass number, *A* (the number of nucleons), and the atomic number, *Z* (the number of protons).	$N = A - Z$ $= 197 - 79$ $= 118$ This nuclide has 118 neutrons.
In an electrically neutral atom, the number of protons = the number of electrons.	The nuclide has 79 protons, so the atom will have 79 electrons in the electron cloud.

PHYSICSFILE

Neutron stars

In the universe, there are objects whose density is almost equal to that of nuclear matter. These objects are called neutron stars. They are huge balls with a diameter of around 10 to 20 kilometres and are made only of neutrons, essentially forming a gigantic atomic nucleus without protons. If a one-litre juice carton was filled up with this type of matter, it would weigh approximately 1×10^{14} kg, whereas a litre of lead weighs only 11.3 kg.

Worked example: Try yourself 8.1.1

WORKING WITH ATOMIC NOTATION

How many protons, neutrons, nucleons and electrons are there in $^{241}_{92}U$?

ISOTOPES

All atoms of a particular element will have the same number of protons but may have a different number of neutrons. For example, lithium exists naturally in two different forms. One form has three protons and three neutrons, while the other has three protons and four neutrons. These different forms of lithium are called **isotopes** of lithium and they are illustrated in Figure 8.1.10.

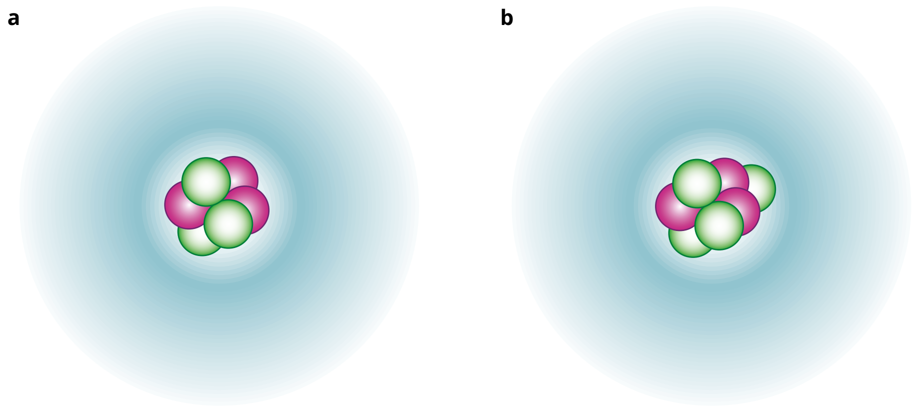


FIGURE 8.1.10 Two different isotopes of lithium: (a) ${}^{6}_{3}Li$ and (b) ${}^{7}_{3}Li$.

Isotopes are atoms that have the same number of protons but different numbers of neutrons. While isotopes have the same chemical properties, they have different physical properties, such as density and volume.

The term **nuclide** is used when referring to a particular nucleus. For example, lithium-6 is a nuclide that has three protons and three neutrons.

There are three isotopes of hydrogen: the nuclide with one proton is called hydrogen, the nuclide with one proton and one neutron is called **deuterium**, and the nuclide with one proton and two neutrons is called **tritium**.

Worked example 8.1.2

WORKING WITH ISOTOPES

Consider the isotope of molybdenum, ${}^{95}_{42}Mo$. Work out the number of protons, nucleons and neutrons in this isotope.	
Thinking	**Working**
The number at the bottom is the atomic number, Z. This is the number of protons.	atomic number, $Z = 42$ This nuclide has 42 protons.
The number at the top is the mass number, A. This indicates the number of particles in the nucleus, i.e. the number of nucleons.	mass number, $A = 95$ This nuclide has 95 nucleons.
Subtract the atomic number, Z, from the mass number, A, to find the number of neutrons, N.	$N = A - Z$ $N = 95 - 42$ $N = 53$ This isotope has 53 neutrons.

Worked example: Try yourself 8.1.2

WORKING WITH ISOTOPES

Consider the isotope of thorium, ${}^{230}_{90}Th$. Work out the number of protons, nucleons and neutrons in this isotope.

PHYSICSFILE

Mass number and atomic mass

Mass number referred to in this text is the total number of protons and neutrons in an isotope. An element can have several stable isotopes and many unstable isotopes in a sample of the substance. The most stable isotopes usually account for most of the mass of the sample. For example, chlorine has many possible isotopes, but the two most common stable isotopes are found in the following ratios: 24.2% chlorine-37 and 75.8% chlorine-35.

The isotope chlorine-35 has a mass number of 35, and the isotope chlorine-37 has a mass number of 37. However, the atomic mass of chlorine listed on the periodic table is 35.45. This value represents the weighted average of the mass numbers of the more commonly found isotopes.

PHYSICSFILE

Heavy water

A compound of oxygen and deuterium has the same chemical properties as ordinary water. However, the molecular mass of ordinary water is approximately 18 (16 for oxygen + 1 for each hydrogen atom), whereas the molecular mass of water containing deuterium is 20 (16 for oxygen + 2 for each deuterium atom). Thus, water that contains deuterium has a higher density (by about 11%) and is commonly known as ‘heavy water’.

RADIOISOTOPES

Most of the atoms that make up the world around us are stable; their nuclei have not altered in the billions of years since they were formed. These atoms will stay unchanged in the future. There are about 270 stable isotopes in nature. Tin ($Z = 50$) has ten stable isotopes, while aluminium ($Z = 13$) has just one.

There are also naturally occurring isotopes that are unstable. In a process known as **spontaneous transmutation**, an unstable nucleus may become more stable by spontaneously emitting a particle and transforming into a different element or isotope. Unstable atoms are radioactive and an individual radioactive isotope is known as a **radioisotope**. For example, carbon has two stable isotopes: carbon-12 and carbon-13. Carbon also has one naturally occurring isotope that is unstable: carbon-14. The nucleus of a carbon-14 atom may spontaneously decay into a different substance, emitting high-energy particles in the process. A known radioactive substance is identified by the radiation warning symbol shown in Figure 8.1.11.

FIGURE 8.1.11 This symbol is used to label and identify a radioactive source.

The periodic table is shown in Figure 8.1.12. Every isotope of every element with an atomic number greater than that of bismuth ($Z = 83$) is radioactive, as is technetium 99m ($Z = 43$), which is artificially produced. Apart from technetium, the first 92 elements are naturally occurring.

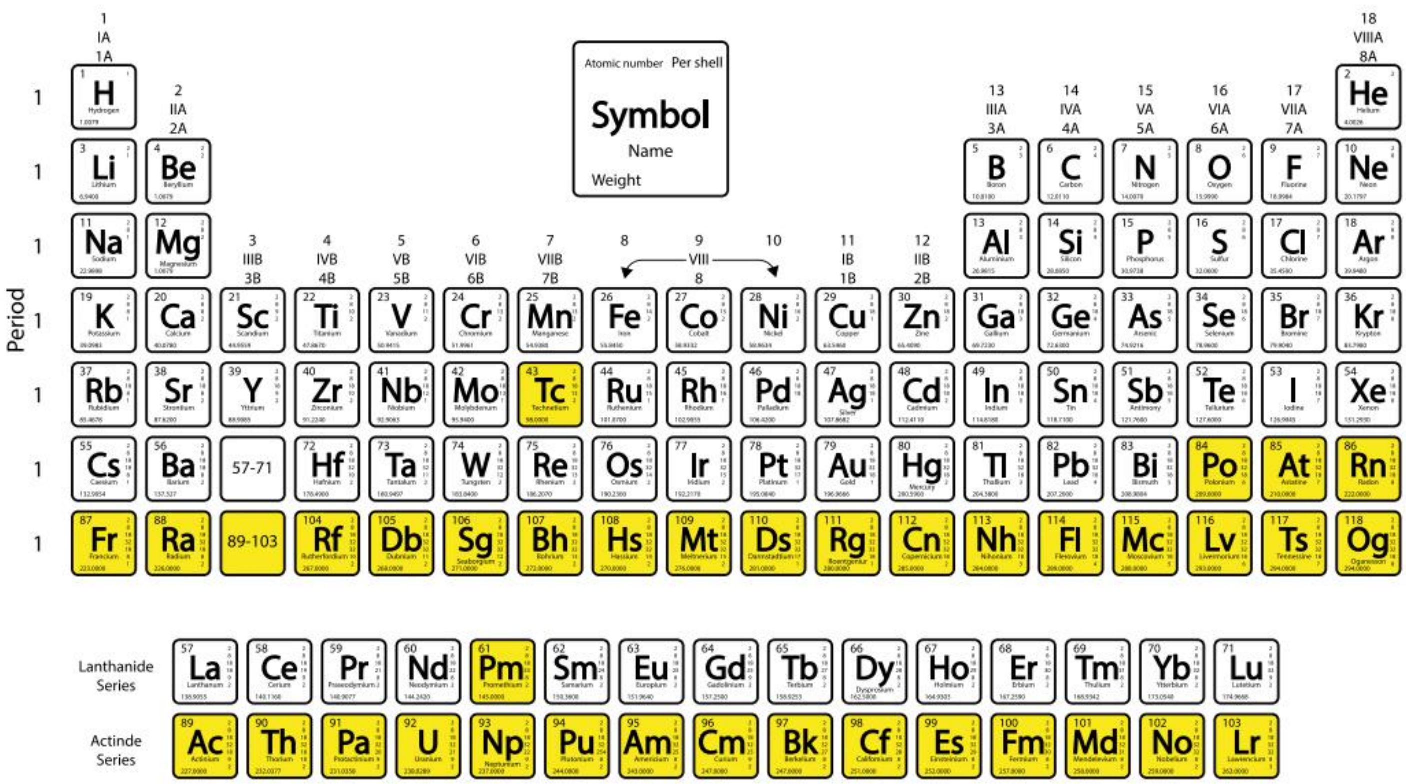


FIGURE 8.1.12 The periodic table of the elements. All isotopes of the elements shaded in yellow are radioactive.

Most of the elements found on Earth have naturally occurring radioisotopes; there are about 200 of these natural radioisotopes. During the twentieth century, an enormous number of radioisotopes were also artificially produced in a process known as **artificial transmutation**. This is a managed process that changes the number of protons, thus changing the identity of the nuclide. Artificial radioisotopes are produced in nuclear reactors or particle accelerators. Most of the radioisotopes used in industry, medicine and for scientific research are artificially produced.

ARTIFICIAL TRANSMUTATION: HOW RADIOISOTOPES ARE MANUFACTURED

Radioisotopes have many medical and industrial applications, such as for the diagnosis of cancer. Most of the radioisotopes that are used in these applications are synthesised by artificial transmutation. There are now about 3000 different artificial radioisotopes. In the periodic table, every element with an atomic number greater than 92 (i.e. that appears after uranium) is radioactive and is produced artificially. The periodic table in Figure 8.1.12 includes recently discovered elements nihonium (Nh), moscovium (Mc), tennessine (Ts) and oganesson (Og), which were recognised formally by IUPAC, the International Union for Pure and Applied Chemistry, in 2016. You may see some older versions of the periodic table that have the elements' temporary names of ununtrium (Uut), ununpentium (Uup), ununseptium (Uus) and ununoctium (Uuo), given before they were officially recognised by IUPAC. These names come from the Latin for their atomic numbers.

One of the ways that artificial radioisotopes are manufactured is by neutron absorption. In this method, a sample of a stable isotope is placed inside a nuclear reactor and bombarded with neutrons. For example, artificial radioisotopes for medical and industrial uses are manufactured in the core of the Lucas Heights reactor at the ANSTO (Australian Nuclear Science and Technology Organisation) facility in Sydney (Figure 8.1.13). This is Australia's only nuclear reactor facility and has been operating since 1958. The original reactor was replaced by the OPAL (Open Pool Australian Light-water) reactor in 2007. Nuclear reactors will be discussed in Chapter 9.2.

When one of the bombarding, or irradiating, neutrons collides with a nucleus of the stable isotope, the neutron is absorbed into the nucleus. This may create an unstable isotope of the same element.

FIGURE 8.1.13 The nuclear reactor at ANSTO, Lucas Heights, Sydney.

PHYSICSFILE

Oganesson (Og)

The element with the highest atomic number, and highest atomic mass discovered so far, is oganesson (Og), originally named as ununoctium (Uuo). In 2006, three atoms of this element were made in a particle accelerator when calcium-48 nuclei were bombarded with californium-249 nuclei. The 20 protons from calcium combined with the 98 protons from californium to make just a few nuclei of Og. Oganesson is highly unstable and so it decays very rapidly with a half-life of less than 1 millisecond. You will learn more about half-lives in Section 8.4.

One of the most widely used radioisotopes is cobalt-60. It does not exist in nature but is artificially produced in the core of a nuclear reactor by bombarding stable cobalt-59 with neutrons. Figure 8.1.14 shows a large cobalt-59 nucleus with a small neutron speeding towards it. The arrow indicates that the product of the collision is a radioactive cobalt-60 nucleus.

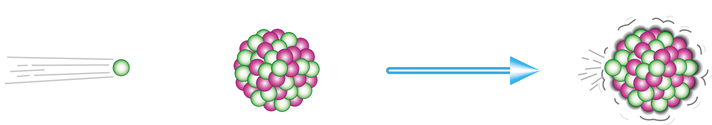


FIGURE 8.1.14 A neutron colliding with ^{59}Co nuclei to form ^{60}Co.

STABILITY OF NUCLEI AND WHY RADIOACTIVE NUCLEI ARE UNSTABLE

In Figure 8.1.15, stable and radioactive isotopes have been plotted according to their number of protons (atomic number, Z) and their number of neutrons (N). The stable nuclides that exist in nature are indicated by purple squares. The radioisotopes that emit radiation are indicated by the black plus and minus signs and α symbols, which will be discussed in the next section.

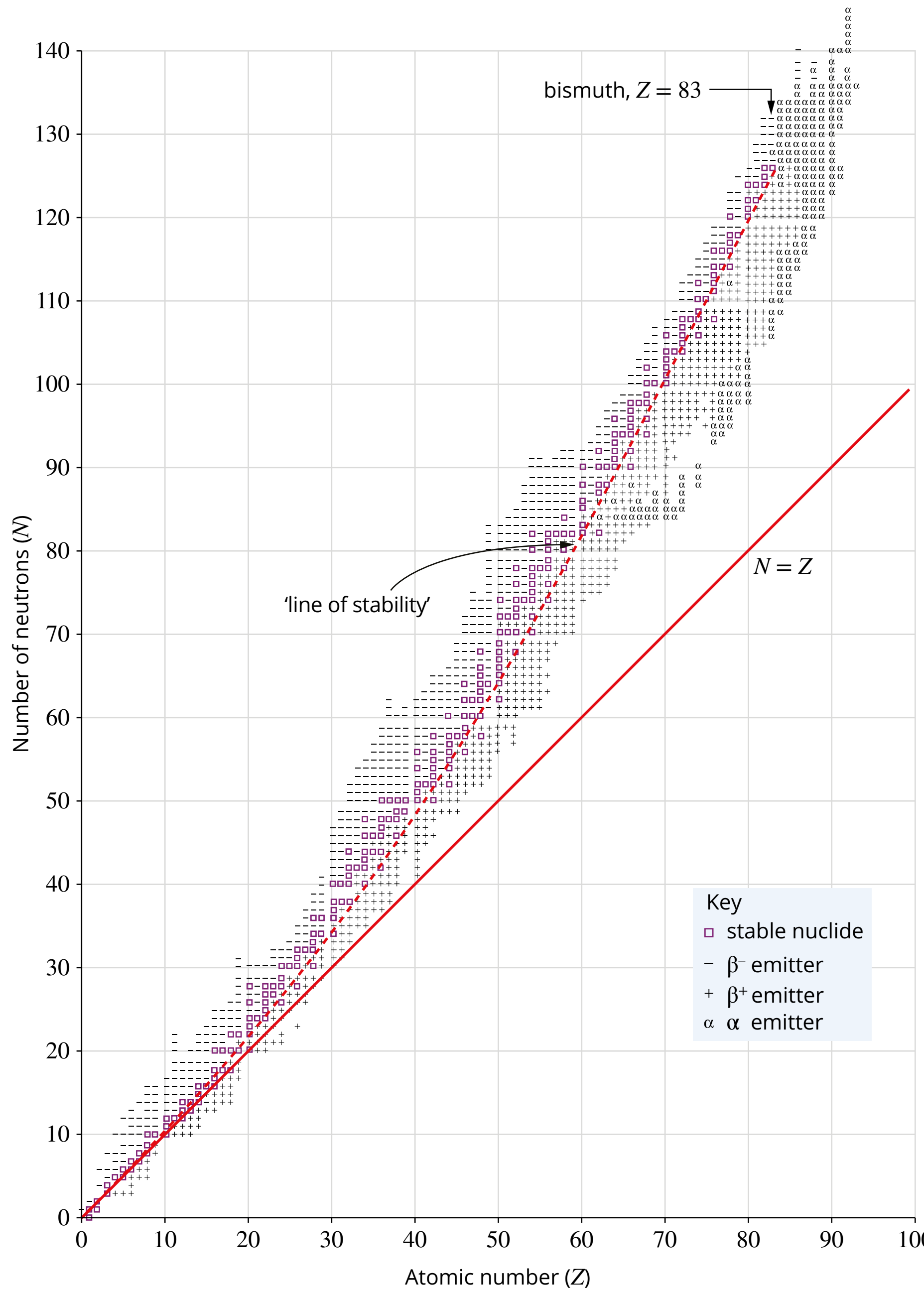


FIGURE 8.1.15 A chart showing stable and radioactive isotopes, plotted according to their number of protons (atomic number) and number of neutrons.

Within the nucleus, protons are very close to other protons, which might seem surprising as protons exert strong **electrostatic forces** of repulsion on one another. Electrostatic forces act between charged particles even over relatively large distances. In the nucleus, this means that each proton strongly repels every other proton; that is, the electrostatic force is trying to make the nucleus break apart. As the nucleus does not disintegrate, however, there must be other factors at play. Another force, known as the **strong nuclear force**, mediated by gluons, is also acting to hold the nucleus together.

Specifically, the same strong nuclear force that holds the quarks together in the nucleus also acts between every nucleon, regardless of its charge. This force acts like 'nuclear glue', which is why the boson involved is called a gluon. Gluons ensures that neutral neutrons are attracted to nearby neutrons and protons, while positively charged protons are attracted to nearby neutrons and protons. Although this force only acts over relatively short distances, it is significantly stronger than the electromagnetic force that is repelling the charged particles.

In a stable nucleus, there is a delicate balance between the repulsive electric force and the attractive strong nuclear force. For example, bismuth-209, the heaviest stable isotope, has 83 protons and 126 neutrons. Here, the electrostatic repulsion between the protons is balanced by the strong nuclear forces mediated by gluons, keeping the nucleus stable. In contrast, bismuth-211, with two extra neutrons disrupts the balance between these opposing forces. The nucleus of ^{211}Bi is unstable and emits an alpha particle (a charged $^{4}_{2}$He nucleus) to become more stable.

From Figure 8.1.15 (page 296) it is evident that there is a 'line of stability' (indicated by the curved red dashed line on the graph) along which the stable nuclei are clustered. Nuclei away from this line are unstable.

For small nuclei with atomic numbers up to about 20, the ratio of neutrons to protons in stable nuclei is close to one. However, as the nuclei become bigger, this ratio increases for stable nuclei. Zirconium ($Z = 40$) has a neutron-to-proton ratio of about 1.25, while for mercury ($Z = 80$) the ratio is close to 1.66. This indicates that for higher numbers of protons, nuclei must have even more neutrons to remain stable. These neutrons dilute the repelling forces that exist between the extra protons.

Elements with more protons than bismuth ($Z = 83$) simply have too many repulsive electric charges in the nucleus. Additional neutrons are unable to dilute and stabilise these nuclei, and so all these elements are unstable and radioactive. Figure 8.1.16 illustrates stable and unstable nuclei.

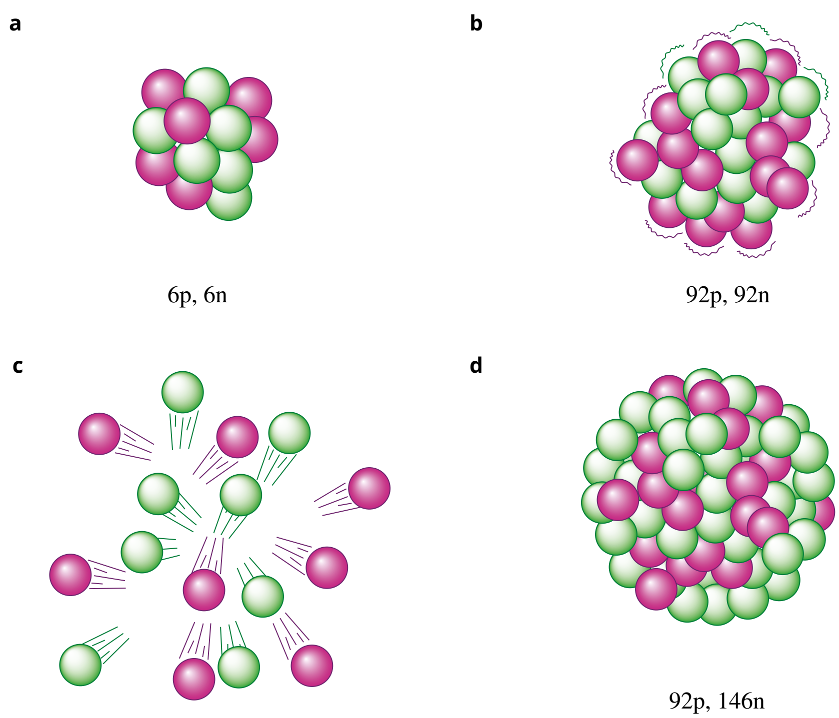


FIGURE 8.1.16 Stable and unstable nuclei. (a) A small nucleus such as carbon-12 is stable. This is because the electrostatic force of repulsion that acts between the protons is overcome by the strong nuclear force of attraction. (b) and (c) A large nucleus with equal numbers of protons and neutrons cannot exist. The electrostatic forces of repulsion between the protons overcome the strong nuclear forces. (d) Additional neutrons increase the stability of large nuclei. The extra neutrons increase the influence of the strong nuclear force mediated by the gluons that hold the nucleus together.

8.1 Review

SUMMARY

- All matter in the universe is made up of fundamental particles called fermions, which are subdivided into quarks and leptons.
- There are six quarks that experience the strong nuclear force, mediated by gluons. These quarks combine to form hadrons and cannot exist alone. Hadrons include baryons, made of three quarks; and mesons, made of two quarks.
- Matter–antimatter pairs have similar properties, such as mass, spin and lifetime, but their electric charge and quantum numbers have the same magnitude with the opposite sign.
- Protons and neutrons are composite particles made up of three smaller particles called quarks, held together by the strong nuclear force.
- Electrons are leptons which are fundamental particles not affected by the strong nuclear force.
- The nucleus of an atom consists of positively charged protons and neutral neutrons. Collectively, protons and neutrons are known as nucleons. Negatively charged electrons surround the nucleus.
- The nucleus of the atom is extremely small but contains most of the atom's mass.
- The atomic number, Z, is the number of protons in the nucleus. The mass number, A, is the number of nucleons in the nucleus, i.e. the combined number of protons and neutrons. Elements are represented as ${}^{A}_{Z}X$. The number of neutrons, $N = A - Z$.
- Isotopes of an element have the same number of protons but different numbers of neutrons. Isotopes of an element are chemically identical to each other but have different physical properties.
- An unstable isotope called a radioisotope, may spontaneously decay by emitting a particle from the nucleus. This is called spontaneous transmutation if the number of protons, and hence the identity of the element, is changed.
- Artificial isotopes are produced by a process called artificial transmutation, which changes the number of protons, thus changing the identity of the nuclide. This process commonly takes place because of neutron bombardment in the core of a nuclear reactor.
- Nuclear stability is the result of the strong nuclear force occurring between nucleons over a very short distance, which opposes the electrostatic repulsion occurring between protons.

KEY QUESTIONS

1 What is the collective term for protons and neutrons?

2 Compare:

- **a** protons and neutrons
- **b** nucleons and electrons
- **c** particles and antiparticles

3 How many protons and how many neutrons are in the ${}^{197}_{79}Au$ nuclide?

4 How many nucleons are there in the ${}^{235}_{92}U$ nuclide?

5 Determine the number of protons, neutrons and nucleons in the following nuclides. You may need to refer to the periodic table in Figure 8.1.12 on page 294.

- **a** chlorine-35 (Cl-35)
- **b** plutonium-239 (Pu-239)

6 Which one or more of the following nuclides have seven neutrons in the nucleus? You may need to refer to the periodic table in Figure 8.1.12 on page 294.

A carbon-12 **B** carbon-13
C carbon-14 **D** nitrogen-14

7 How is the number of electrons in a neutral atom found from information given in the periodic table?

8 Explain the meaning of the term isotope.

9 Krypton-84 is stable, but krypton-89 is radioactive. Imagine that you have just one atom of each isotope.

- **a** Are their atomic numbers and mass numbers the same or different? Justify your answers.
- **b** Compare the way these atoms would interact chemically with other atoms.

10 What is the difference between a stable isotope and a radioisotope?

11 Can a natural isotope be radioactive? If so, give an example of such an isotope.

12 Why does the number of neutrons become greater than the number of protons in a nucleus as the elements get heavier?

8.2 Radioactivity

Around the beginning of the twentieth century, scientists such as Marie Curie, pictured in Figure 8.2.1, were investigating the newly discovered radioactive substances polonium and radium. Ernest Rutherford and Paul Villard found that there were three different types of emission from these mysterious substances. They named them alpha, beta and gamma radiation.

Further experiments showed that the alpha and beta emissions were actually particles expelled from the nucleus. Gamma radiation was found to be high-energy **electromagnetic radiation** (like visible light but of significantly higher energy) expelled from the nucleus. The term radioactive decay refers to the process that emits these particles and radiation from a nucleus.

The origin and nature of these radiations will be discussed in this section.

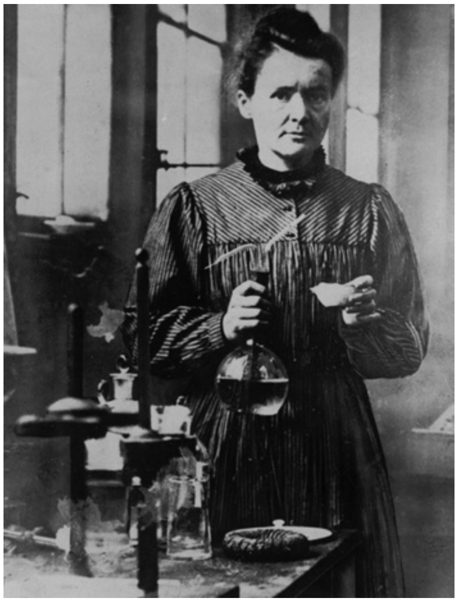


FIGURE 8.2.1 Marie Curie, pioneer of research into radioactivity.

ALPHA (α) DECAY

When a heavy unstable nucleus undergoes radioactive decay, it may eject an **alpha particle**. This is a very stable, positively charged particle that consists of two protons and two neutrons, with no orbiting electrons. An alpha particle, symbol ${}^{4}_{2}\alpha$, is identical to a helium nucleus and can also be written as ${}^{4}_{2}\text{He}$ or ${}^{4}_{2}\text{He}^{2+}$. In this text, the symbol ${}^{4}_{2}\alpha$ will be used when referring to a highly energetic alpha particle, while the symbol ${}^{4}_{2}\text{He}$ will be used when the alpha particle has transferred most of its energy to surrounding particles.

Uranium-238 is radioactive and may decay by emitting an alpha particle from its nucleus. Figure 8.2.2 shows the unstable nucleus of uranium-238 ejecting an alpha particle. This process is represented in a nuclear equation that shows the changes occurring in the nuclei. Electrons are not considered in these equations, only the nucleons (protons and neutrons). The nuclear equation for the alpha decay of uranium-238 is:

$$ {}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha + \text{energy} $$

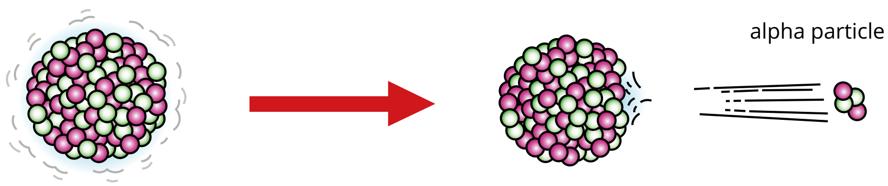


FIGURE 8.2.2 Alpha emission from uranium-238.

The **parent nucleus** ${}^{238}_{92}\text{U}$ has spontaneously emitted an alpha particle (α) and has changed into a completely different element, ${}^{234}_{90}\text{Th}$. Thorium-234 is called the **daughter nucleus**. The energy released is mostly kinetic energy carried by the fast-moving alpha particle.

When an atom changes into a different element, it is said to undergo a **nuclear transmutation**. In nuclear transmutations, electric charge is conserved. This results in the conservation of atomic number, i.e. the number of protons. The sums of atomic numbers on both sides of a nuclear equation must be equal. In the uranium decay equation, the atomic number, or the number of protons, is conserved: 92 = 90 + 2. The mass number is also conserved: 238 = 234 + 4.

> In any nuclear reaction, including radioactive decay, atomic and mass numbers are conserved. Energy is released during these decays.

Worked example 8.2.1

ALPHA DECAY

A polonium-212 nucleus is known to decay to a new element through the emission of an alpha particle. Determine the new element, write its symbol and write the decay equation.

Thinking	Working
From the periodic table, polonium-212 has 84 protons. Therefore its atomic number, *Z*, is 84 and its mass number, *A*, is 212.	It can be written as ${}^{212}_{84}\text{Po}$.
The initial nucleus $\left({}^{212}_{84}\text{Po}\right)$ is written on the left-hand side of the equation. The unknown nucleus $\left({}^{A}_{Z}\text{X}\right)$ is the result the of alpha decay, and is written on the right-hand side, along with the alpha particle $\left({}^{4}_{2}\alpha\right)$.	${}^{212}_{84}\text{Po} \rightarrow {}^{A}_{Z}\text{X} + {}^{4}_{2}\alpha$
Charge must be conserved, so the total number of protons, *Z*, must be the same.	$84 = Z + 2$ $Z = 82$
The number of protons and neutrons, *A*, must also be the same.	$212 = A + 4$ $A = 208$
For the new element, $Z = 82$. From the periodic table, this is lead.	${}^{208}_{82}\text{Pb}$
The decay equation can now be determined.	${}^{212}_{84}\text{Po} \rightarrow {}^{208}_{82}\text{Pb} + {}^{4}_{2}\alpha$

Worked example: Try yourself 8.2.1

ALPHA DECAY

A radium-224 nucleus is known to decay to a new element through the emission of an alpha particle. Determine the new element, write the appropriate symbol and the decay equation.

BETA (β) DECAY

Many radioactive materials emit **beta particles**. There are two different types of beta particles: beta minus (${}^{0}_{-1}\beta^-$) and beta plus (${}^{0}_{+1}\beta^+$).

Beta minus (β^-)

Beta-minus radioactive decay occurs when an electron (e^-) is created and emitted from the nucleus of a radioactive atom, rather than an existing electron being ejected from the electron cloud. This high-energy electron is identical in every way to other electrons; however, when it is ejected from a nucleus, it is called a beta-minus particle, which can be written as ${}^{0}_{-1}\beta^-$.

The atomic number of −1 indicates that the beta particle (the electron) has a single negative charge. The mass number of zero indicates that its mass is far less than that of a proton or a neutron. In this text, the symbol ${}^{0}_{-1}\beta^-$ will be used when the electron is highly energetic, and the symbol e^- will be used when the particle has transferred most of its energy to surrounding particles.

PHYSICSFILE

Radioactive lamps

The wicks or mantles used in old-style camping lamps, as shown in Figure 8.2.3, are slightly radioactive. They contain a radioisotope of thorium, an alpha-particle emitter. They have not been banned from sale so far because they contain only small amounts of the radioisotope and can be used safely by taking simple precautions such as washing hands and avoiding inhalation or ingestion.

However, a scientist from the Australian National University in Canberra has called for these mantles to be banned because they tend to crumble and turn to dust as they age. If this dust were inhaled, alpha particles could settle in someone's lung tissue, possibly causing cancers to form.

FIGURE 8.2.3 An old radioactive gas-light mantle.

Typically, beta-minus decay occurs if a nucleus has too many neutrons to be stable. In this process, a neutron (udd) spontaneously changes into a proton (uud). This involves the **weak nuclear force** and occurs through the conversion of a down quark in the neutron into an up quark. This conversion emits a W^- boson, which then decays into an **antielectron neutrino** (${}^{0}_{0}\bar{v}_e$) and a beta-minus particle (${}^{0}_{-1}\beta^-$).

Feynman diagrams are a system invented by the American physicist Richard Feynman to visualise and simplify complex nuclear reactions. In these diagram, straight lines represent matter particles (fermions), while wavy lines represent exchange particles (bosons). Figure 8.2.4 is an example of a Feynman diagram for beta-minus decay of a neutron.

In the Feynman diagram to the right, the neutron (udd) continues through time until one of the down quarks decays into an up, transforming the neutron into a proton. The process also produces a W^- boson, which then decays into a beta-minus particle and an antielectron neutrino. In Feynman diagrams, real matter is represented by arrows pointing upwards, while antimatter is indicated by arrows pointing downwards. This does not mean that antimatter goes backwards in time; the direction of the arrows simply distinguishes the two forms of matter.

An example of an isotope that undergoes beta-minus decay is carbon-14. While the other isotopes of carbon, carbon-12 and carbon-13, are both stable, carbon-14 is unstable due to its excess neutrons. To become stable, carbon-14 undergoes beta-minus decay. One of its neutrons changes into a proton, emitting a beta-minus particle and an antielectron neutrino in the process. Nitrogen-14 is then formed, and energy is released. The beta-minus decay of carbon-14 is shown in Figure 8.2.5.

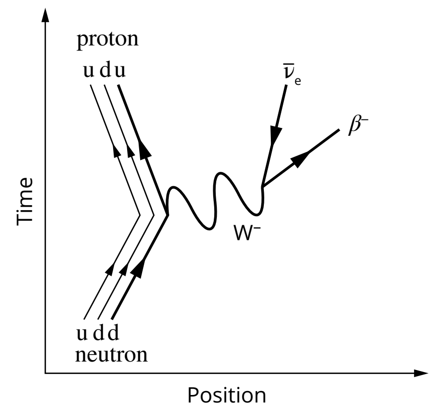


FIGURE 8.2.4 The Feynman diagram for the decay of a neutron.

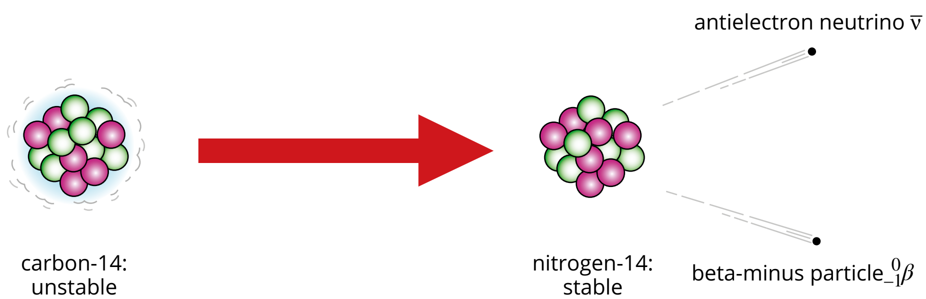


FIGURE 8.2.5 The beta-minus decay of carbon-14.

The nuclear equation for this decay is:

$${}^{14}_{6}C \rightarrow {}^{14}_{7}Th + {}^{0}_{-1}\beta^- + {}^{0}_{0}\bar{v}_e + \text{energy}$$

The transformation taking place inside the nucleus is:

$${}^{0}_{1}n \rightarrow {}^{1}_{1}p^+ + {}^{0}_{-1}\beta^- + {}^{0}_{0}\bar{v}_e$$

Notice that in all these equations, the atomic numbers and the mass numbers are conserved. The antielectron neutrino has no charge and has so little mass that both its atomic and mass numbers are zero.

Worked example 8.2.2

BETA-MINUS DECAY

A bismuth-212 nucleus is known to decay to a new element through the emission of a beta-minus particle. Determine the new element, write the appropriate symbol and the decay equation.

Thinking	Working
From the periodic table, bismuth-212 has 83 protons. Therefore its atomic number, Z, is 83 and its mass number, A, is 212.	It can be written as $^{212}_{83}\text{Bi}$.
The initial nucleus is $^{212}_{83}\text{Bi}$ and is written on the left-hand side of the equation. The unknown nucleus is a result of beta-minus decay and is written on the right-hand side along with the beta-minus particle and an antielectron neutrino.	$^{212}_{83}\text{Bi} \rightarrow {}^{A}_{Z}\text{X} + {}^{0}_{-1}\beta^- + {}^{0}_{0}\bar{v}_e$
Charge must be conserved, so the total number of protons, Z, must be the same.	$83 = Z - 1$ $Z = 84$
The number of protons and neutrons, A, must also be the same.	$212 = A + 0$ $A = 212$
For the new element, $Z = 84$. From the periodic table, this is polonium.	$^{212}_{84}\text{Po}$
The decay equation can now be determined.	$^{212}_{83}\text{Bi} \rightarrow {}^{212}_{84}\text{Po} + {}^{0}_{-1}\beta^- + {}^{0}_{0}\bar{v}_e$

Worked example: Try yourself 8.2.2

BETA-MINUS DECAY

An astatine-219 nucleus is known to decay to a new element through the emission of a beta-minus particle. Determine the new element, write its symbol and write the decay equation.

Beta plus (β^+)

Beta-plus radioactive decay occurs when a nucleus has too many protons (p^+). In this case, a proton may spontaneously change into a neutron (n), emitting a positively charged beta particle and an electron neutrino. This process involves the weak nuclear force and occurs through the conversion of an up quark in a proton into a down quark, forming a neutron. It also requires the emission of a W^+ boson, which then decays into an electron neutrino ($^{0}_{0}\nu_e$) and a beta-plus particle ($^{0}_{+1}\beta^+$). A Feynman diagram for beta-positive decay of a proton is shown in Figure 8.2.6.

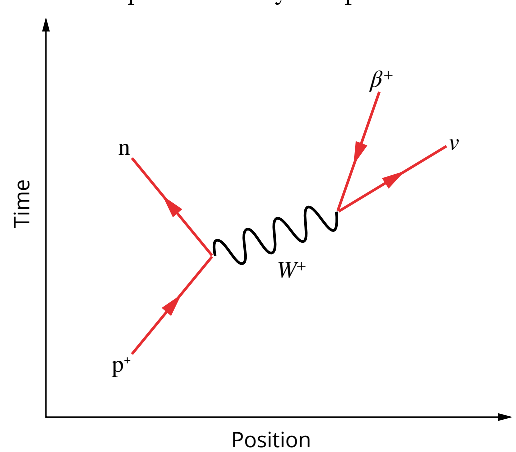


FIGURE 8.2.6 The Feynman diagram for the decay of a proton into a neutron.

The positively charged beta particle (${}^{0}_{+1}\beta^{+}$) is also called a **positron**. Positrons (e^{+}) are the antiparticles of electrons (e^{-}) ; that is, they have the same properties as electrons, but their electrical charge is positive rather than negative. Note that there is no need for a bar above the antimatter positron as the charge is opposite to the electron's normal negative charge, indicating it is the antimatter form.

An example of beta-plus decay is given by:

$$ {}^{12}_{7}\text{N} \rightarrow {}^{12}_{6}\text{C} + {}^{0}_{+1}\beta^{+} + {}^{0}_{0}\nu_{e} $$

The transformation taking place inside the nucleus is:

$$ {}^{1}_{1}\text{p} \rightarrow {}^{1}_{0}\text{n} + {}^{0}_{+1}\beta^{+} + {}^{0}_{0}\nu_{e} $$

Worked example 8.2.3

BETA-PLUS DECAY

Carbon-10 decays by radioactive emission to form boron-10. The equation is:

$$ {}^{10}_{6}\text{C} \rightarrow {}^{10}_{5}\text{B} + X + Y $$

Determine the atomic number and mass number for *X*, and identify the type of radiation emitted. Also identify particle *Y* that is emitted with radiation *X*.

Thinking	Working
Balance the mass numbers.	The mass numbers of 10 are already balanced, so the mass number of *X* is zero.
Balance the atomic numbers.	$6 = 5 + Z$ $Z = 6 - 5 = +1$
X has an atomic number of −1 and a mass number of zero.	*X* is a beta-plus particle: ${}^{0}_{+1}\beta^{+}$
Y must be an electron neutrino, which is always emitted with a beta-plus particle.	*Y* is an electron neutrino particle: ${}^{0}_{0}\nu_{e}$

Worked example: Try yourself 8.2.3

BETA-PLUS DECAY

Protactinium-230 decays by radioactive emission to form thorium-230. The equation is:

$$ {}^{230}_{91}\text{Pa} \rightarrow {}^{230}_{90}\text{U} + X + Y $$

Determine the atomic number and mass number for *X*, and identify the type of radiation emitted. Also identify particle *Y* that is emitted with radiation *X*.

GAMMA (γ) DECAY

After a radioisotope has emitted an alpha or beta particle, the resulting daughter nucleus usually has excess energy. The protons and neutrons in the daughter nucleus then rearrange slightly and offload this excess energy by releasing a **gamma ray**, ${}^{0}_{0}\gamma$.

Gamma rays are high-energy electromagnetic radiation, so they have no mass, are uncharged, and travel at the speed of light ($3.00 \times 10^{8}\,\text{m}\,\text{s}^{-1}$).

A common example of a gamma-ray emitter is iodine-131. It decays initially by beta emission and then by gamma emission to form xenon-131, as shown in Figure 8.2.7. The xenon nuclide is in an excited state, as indicated by the asterisk, and loses energy by emitting a gamma ray.

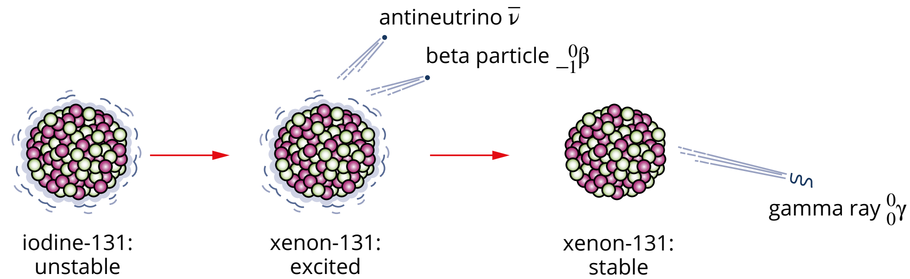


FIGURE 8.2.7 The gamma and beta decay of iodine-131.

The sequence of equations for this decay is:

$$^{131}_{53}\text{I} \rightarrow {}^{131}_{54}\text{Xe}^{\star} + {}^{0}_{-1}\beta^{+}$$

$$^{131}_{54}\text{Xe}^{\star} \rightarrow {}^{131}_{54}\text{Xe} + {}^{0}_{0}\gamma$$

Since gamma rays carry no charge and have no mass, they have no effect when balancing the atomic or mass numbers in a nuclear equation.

Worked example 8.2.4

RADIOACTIVE DECAY

An excited helium-3 atom decays by radioactive emission to form helium-3. The equation is: $^{3}_{2}\text{He}^{*} \rightarrow {}^{3}_{2}\text{He} + X$ Determine the atomic and mass numbers for *X*, and complete the reaction by identifying the type of radiation being emitted.	
Thinking	**Working**
Balance the mass numbers.	The mass numbers of 3 are already balanced, so the mass number of *X* is zero.
Balance the atomic numbers.	The atomic numbers of 2 are already balanced, so the atomic number of *X* is zero.
X has an atomic number of zero and a mass number of zero, therefore *X* is a gamma ray.	$^{3}_{2}\text{He}^{*} \rightarrow {}^{3}_{2}\text{He} + {}^{0}_{0}\gamma$

Worked example: Try yourself 8.2.4

RADIOACTIVE DECAY

After beta-minus decay from boron to carbon-12, the carbon-12 atom is in an excited state and decays further to a more stable form of carbon-12. The equation is:

$$^{12}_{6}\text{C}^{*} \rightarrow {}^{12}_{6}\text{C} + X$$

Determine the atomic and mass numbers for *X*, and complete the reaction by identifying the type of radiation emitted.

CONSERVATION LAWS

In nuclear reactions and particle interactions, several **conservation laws** apply. These laws state that certain quantities remain unchanged before and after a reaction occurs. Examples include the conservation of mass–energy, momentum, charge and baryon number. These laws are fundamental to our understanding of physics and are especially crucial in the study of particle physics.

Conservation of mass–energy

The law of conservation of energy (covered in Section 5.4) also applies to particle interactions and nuclear decays. You must therefore consider two things when studying nuclear reactions:

- the total energy contained in the rest masses of the particles
- the kinetic energy before and after the event.

A simple example is the decay of a stationary neutron to produce a proton, an electron and an antielectron neutrino, as shown in Figure 8.2.8.

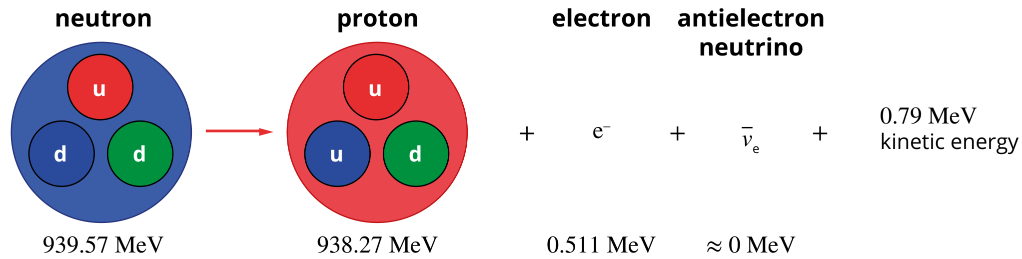


FIGURE 8.2.8 The decay of a stationary neutron is shown to illustrate the conservation of mass–energy. The total energy of the system is conserved.

The products of the decay of a stationary neutron have a total mass less than that of the original neutron. Since the neutron is originally stationary, the difference in mass is converted into the kinetic energy of the decay products. The energy of the mass of the particles (in MeV) is determined using $E = \Delta m \times 931$, where Δm is the rest mass, which is measured in the units of dalton (Da) and 1 Da = 1.66×10^{-27} kg. This relationship will be discussed further in Section 9.1. In high-energy particle physics, the entire rest mass of particles is often expressed in the equivalent units of energy. The electron-volt (eV), or the mega-electron volt (MeV), are the preferred units for such tiny amounts of energy, where 1 eV = 1.602×10^{-19} J.

The example shown in Figure 8.2.8 therefore obeys the law of conservation of energy, as the energy stored within the mass of the neutron before the decay is equal to the sum of the energies stored within the mass of the products, plus their kinetic energies, after the decay.

Worked example 8.2.5

CONSERVATION OF ENERGY

A proton with 2.785 MeV of kinetic energy undergoes beta-plus decay to become a neutron with the ejection of an electron neutrino. Calculate the total kinetic energy that the three product particles share.

Thinking	Working
Determine the energy of the proton from its mass (Da).	mass of proton ($^1_1$p) = 1.007 277 Da where 1 Da = 931 MeV
Convert to energy (MeV).	$E_p = (1.007\,277)(931)$ $E_p = 937.774\,89$ MeV
Add the kinetic energy of the proton to find the total energy before the decay.	$E_{total} = 937.774\,89 + 2.785 = 940.559\,89$ MeV

Calculate the energy of the products.	The neutron, electron neutrino and positron have these rest energies: neutron mass ($^1_0$n) = 1.008 665 Da E_n = (1.008 665)(931) = 939.067 12 MeV electron neutrino mass ($^0_0\nu_e$) = 1.289 × 10^{-10} Da E_v = (1.289 × 10^{-10})(931) = 1.200 059 × 10^{-7} MeV beta-plus mass ($^{\ 0}_{+1}\beta^+$) = 0.000 548 580 Da $E_{\beta+}$ = (0.000 548 580)(931) = 0.510 727 98 MeV
Calculate the total rest energy of the products.	$E_{products} = E_n + E_v + E_{\beta+}$ $E_{products}$ = (939.067 12) + (1.200 059 × 10^{-7}) + (0.5107 279 8) $E_{products}$ = 939.577 85 MeV
Calculate the difference in energy between reactants and products to find the kinetic energy of the products.	$E_k = E_{total} - E_{products}$ E_k = (940.559 89) – (939.577 85) E_k = 0.982 04 E_k = 0.982 MeV

Worked example: Try yourself 8.2.5

CONSERVATION OF ENERGY

A uranium-238 nucleus decays into thorium-234 with the release of an alpha particle. Show that 5.30 MeV of energy must also be produced. Use the following masses: uranium-238 = 238.0508 Da, thorium-238 = 234.0436 Da and the alpha particle mass = 4.001 506 Da.

THE RANGE OF ENERGIES FOR THE DIFFERENT RADIATIONS

The range of energies for the different radiations discussed in this section are as follows. (Recall that the **electronvolt** (eV), or the megaelectronvolt (MeV), are the preferred units for tiny amounts of energy, where 1 eV = 1.602 × 10^{-19} J.)

- Alpha particles typically have energies between 5–10 million electronvolts or megaelectronvolts (5–10 MeV).
- Beta particles are usually ejected with energies up to a few million electronvolts or MeV.
- Gamma rays normally have less than 1 MeV of energy. For example, the gamma rays emitted by the radioactive isotope gold-198 have a maximum energy of 412 000 eV or 412 kiloelectronvolts (412 keV), or 6.60 × 10^{-6} J.

An increase in the energy of alpha or beta particles will result in an increase in their speed of ejection. However, as a gamma ray is electromagnetic radiation, it always travels at the speed of light and so its speed does not depend on its energy. As will be discussed in the Year 12 Physics course, any increase in energy of a gamma ray will increase the frequency of the radiation. Low-energy radiation, such as infra-red and radio waves, has a much lower energy (less than 1 eV) and therefore is considered harmless.

CONSERVATION OF MOMENTUM

The law of conservation of momentum is covered in Section 4.1 and applies to all particle interactions and nuclear decays. The initial momentum of the stationary neutron shown in Figure 8.2.8 is zero. Therefore, the momentum carried away by the decay products must be in opposite directions (+ and –), so that they sum to equal zero. This ensures that the law of **conservation of momentum** is obeyed, with the total momentum before (zero) equal to the total momentum after (zero overall).

In beta decay, the vector sum of the momenta of the beta-minus particle and the antielectron neutrino will be equal and opposite to the momentum of the recoiling proton, ensuring that momentum is conserved.

CONSERVATION OF CHARGE

There are other quantities that must be conserved in particle interactions, along with energy and momentum. Electric charge is one such quantity, meaning that the total charge present before an event must equal the total charge after the event.

For example, in the decay of a neutral neutron at rest, the total charge of the products must balance to produce an overall charge of zero. The neutron has a charge of zero, the proton has a charge of +1, the beta-minus particle has a charge of −1 and the antielectron neutrino has no charge. Thus, the total charge of the products is zero. Therefore, charge is conserved.

EXTENSION

How radiation is detected

Our bodies cannot detect alpha, beta or gamma radiation. Therefore, devices have been developed to detect and measure radiation.

A common detector is the Geiger counter. Geiger counters are used:

- by geologists when searching for radioactive minerals such as uranium
- to monitor radiation levels in mines
- to measure the level of radiation after a nuclear accident; for example, Fukushima, Japan, in 2011
- to check the safety of nuclear reactors
- to monitor radiation levels in hospitals and factories.

A Geiger counter consists of a Geiger–Muller tube filled with argon gas, as shown in Figure 8.2.9.

A voltage of about 400 V is maintained between the positively charged central electrode and the negatively charged aluminium tube. When radiation enters the tube through the thin mica window, the argon gas becomes ionised and releases electrons. These electrons are attracted towards the central electrode and ionise more argon atoms along the way. For an instant, the gas between the electrodes becomes ionised enough to conduct a pulse of current between the electrodes. This pulse is registered as a count. The counter is often connected to a small loudspeaker so that the count is heard as a 'click' (Figure 8.2.10).

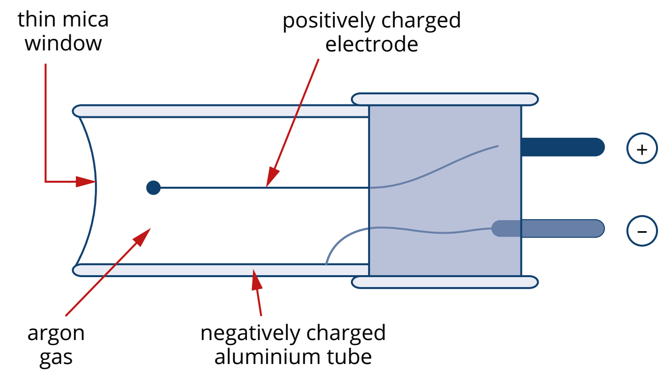


FIGURE 8.2.9 A schematic diagram of a Geiger counter, used for detecting ionising radiation.

FIGURE 8.2.10 A scientist using a Geiger counter to measure radiation levels.

PHYSICS IN ACTION

How technetium is produced

Technetium-99m is the most widely used radioisotope in nuclear medicine. It is used for diagnosing and treating cancer. However, this radioisotope decays relatively quickly and so usually needs to be produced close to where it will be used. Technetium-99m is produced in small nuclear generators that are located in hospitals around the country (Figure 8.2.11). In this process, the radioisotope molybdenum-99, obtained from the Lucas Heights reactor, Sydney, is used as the parent nuclide. Molybdenum-99 decays by beta-minus emission to form a relatively stable (or metastable) isotope of technetium, technetium-99m, as shown below:

$$^{99}_{42}\text{Mo} \rightarrow {}^{99m}_{43}\text{Tc} + {}^{0}_{-1}\beta^{-} + {}^{0}_{0}\bar{\nu}_e$$

Technetium-99m is flushed from the generator using a saline solution. The radioisotope is then diluted and attached to an appropriate chemical compound before being administered to the patient as a tracer. Technetium-99m is purely a gamma emitter. This makes it very useful as a diagnostic tool for locating and treating cancer. Its decay equation is:

$$^{99m}_{43}\text{Tc} \rightarrow {}^{99}_{43}\text{Tc} + {}^{0}_{0}\gamma$$

FIGURE 8.2.11 Technetium generators are used in hospitals that require radioisotopes. The generator has a thick lead shield that absorbs the beta and most of gamma radiation.

8.2 Review

SUMMARY

- Radioactive isotopes may decay by emitting alpha, beta or gamma radiation from their nuclei.
- An alpha particle (α) consists of two protons and two neutrons and is emitted from the nuclei of some radioisotopes. It is identical to a helium nucleus and can be written as ${}^{4}_{2}\text{He}$.
- A beta-minus particle (${}^{0}_{-1}\beta^{-}$) is an electron (e^{-}) that has been emitted from the nucleus of a radioactive atom when a neutron transmutes into a proton. An antielectron neutrino (${}^{0}_{0}\bar{v}_e$) is always emitted alongside a beta-minus particle.
- A beta-plus particle (${}^{0}_{+1}\beta^{+}$) or positron (e^{+}) is a positively charged electron that has been emitted from the nucleus of a radioactive atom when a proton transmutes into a neutron. An electron neutrino (${}^{0}_{0}v_e$) is always emitted alongside a beta-plus particle.
- A gamma ray (${}^{0}_{0}\gamma$) is high-energy electromagnetic radiation that is emitted from the nuclei of radioactive atoms. It has no mass and no charge.
- In any nuclear reaction, both the atomic and mass numbers are conserved, along with energy, momentum and charge.
- Mass and energy are interchangeable according to Einstein's famous equation, $E = mc^2$. Particle masses are often quantified as an equivalent amount of energy. The total energy in an interaction includes both rest mass and kinetic energy.
- Particle masses are expressed in daltons (Da), and energy calculations in high-energy particle physics should always be expressed in eV, or MeV, rather than joules (J).

KEY QUESTIONS

1 Determine the nature of the unknown, *X*, for the following transmutation:

$${}^{218}_{86}\text{Rn} \rightarrow {}^{214}_{84}\text{Po} + X$$

2 In the following nuclear reaction, *Y* represents a beta particle. What type of beta particle is it?

$${}^{214}_{82}\text{Pb} \rightarrow {}^{214}_{83}\text{Bi} + Y$$

3 What type of decay occurs when a nucleus has too many protons?

4 What is a positron?

5 What is the nature of the alpha, beta-minus, beta-plus and gamma radiation?

6 What are the mass numbers of the six stable nuclides of calcium (atomic number 20)? Use Figure 8.1.15 on page 296 to answer this question.

7 Where in an atom do alpha, beta and gamma radiation originate?

8 For the unknown nuclides X and Y in each of these decay equations, determine the atomic number and mass number, and use the periodic table to identify the unknown elements.

a ${}^{235}_{92}\text{U} \rightarrow {}^{4}_{2}\alpha + \text{X} + {}^{0}_{0}\gamma$

b ${}^{228}_{88}\text{Ra} \rightarrow \text{Y} + {}^{0}_{-1}\beta^{-} + {}^{0}_{0}\gamma$

9 Carbon-14 decays by beta-minus emission to form nitrogen-14. The equation for this is:

$${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\beta^{-} + {}^{0}_{0}\bar{v}_e + \text{energy}$$

a How many protons and neutrons does the nitrogen atom have?

b What particle (nucleon) on the left side of the equation has transformed into what particle(s) on the right side of the equation?

10 What is missing in each of the following decay equations?

a ${}^{45}_{20}\text{Ca} \rightarrow {}^{45}_{21}\text{Sc} + ?$

b ${}^{150}_{70}\text{Yb} \rightarrow {}^{146}_{68}\text{Er} + ?$

11 Show that charge is conserved in beta-plus decay.

12 A stationary helium-6 nucleus decays into a lithium-6 nucleus, releasing a beta-minus particle. Use the data below to show that the total kinetic energy of the products is 2.97 MeV.

mass of He-6 nucleus = 6.01889 Da

mass of Li-6 nucleus = 6.01515 Da

mass of beta-minus particle = 0.000548580 Da

mass of electron neutrino = 1.289×10^{-10} Da

8.3 Properties of alpha, beta and gamma radiation

In the early experiments with radioactivity, emissions from a sample of radium were directed through a magnetic field as shown in Figure 8.3.1. The emissions followed three distinct paths, which suggested that there were three different forms of radiation being emitted. The emissions each had different charges, masses and speeds. These properties will be discussed in this section.

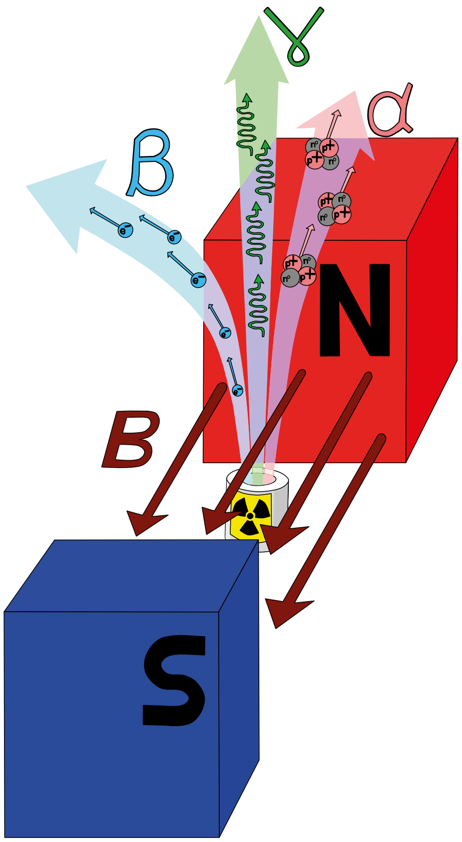


FIGURE 8.3.1 Applying a magnetic field shows that there are three different types of emissions from a radium source. A charged particle travelling through a magnetic field will experience a force acting on it, at right angles to the direction of travel. This will cause the path of the charged particle to bend. The positively charged alpha particle will be deflected at right angles in one direction, and the negatively charged beta particle will be deflected in the opposite direction, also at right angles. The gamma ray, being uncharged, will pass through the magnetic field without its path being changed. (The details of this are covered in Year 12 Physics.)

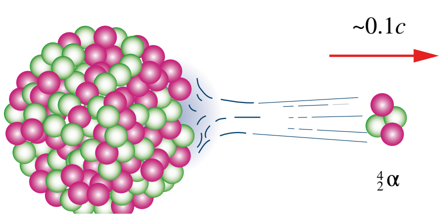


FIGURE 8.3.2 The speed and structure of an alpha particle being emitted from a nucleus.

SPEED AND CHARGE

Alpha (α) particles

Alpha particles consist of two protons and two neutrons. This means that they are relatively heavy and slow moving. Alpha particles are emitted from the nucleus at speeds of up to 20 000 km s^{-1} (2.0×10^7 m s^{-1}), just less than 10% of the speed of light (Figure 8.3.2). Alpha particles have a double positive charge.

Beta-minus ($^{0}_{-1}\beta^{-}$) and beta-plus ($^{0}_{+1}\beta^{+}$) particles

Beta particles are fast-moving electrons ($^{0}_{-1}\beta^{-}$) carrying a negative charge, or positrons ($^{0}_{+1}\beta^{+}$) carrying a positive charge.

Beta-minus and beta-plus particles have the same mass and are much lighter than alpha particles. As a result, they leave the nucleus at much higher speeds, even up to 90% of the speed of light (c), as shown in Figure 8.3.3 for beta-minus decay.

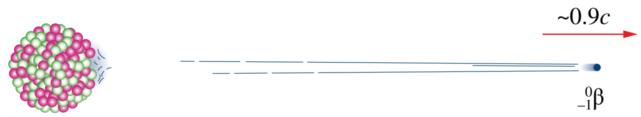


FIGURE 8.3.3 The speed and structure of a beta-minus particle being emitted from a nucleus.

Gamma (γ) rays

Gamma rays are electromagnetic radiation with very high frequency. Figure 8.3.4 shows where gamma rays lie along the electromagnetic spectrum. They have no rest mass and travel at the speed of light: $3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$ (Figure 8.3.5). Gamma rays have no electric charge.

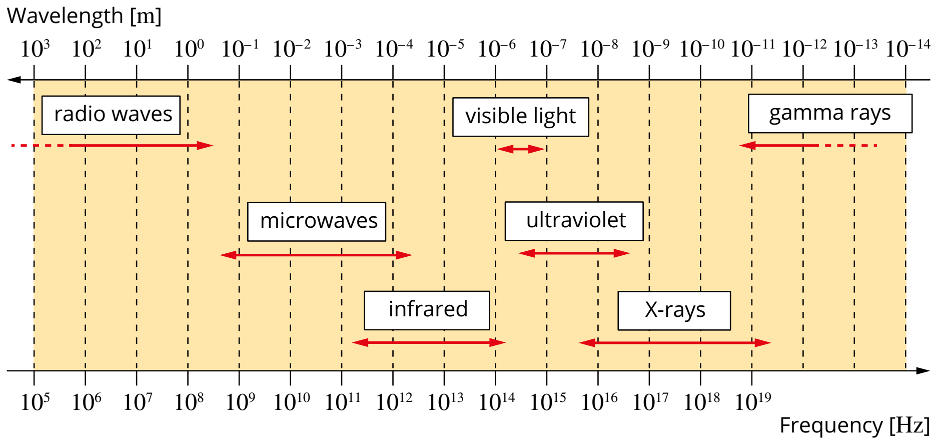


FIGURE 8.3.4 The electromagnetic spectrum contains many different types of radiation that differ in their wavelength and frequency. Gamma rays have very high frequencies and very short wavelengths, making them very energetic and highly penetrating.

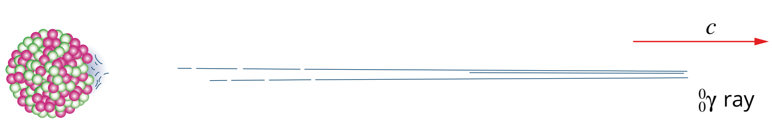


FIGURE 8.3.5 The speed and nature of gamma radiation.

PENETRATION ABILITY AND IONISATION

Alpha, beta and gamma particles have widely different penetration ability in air and through solid objects due to the nature of the particles and their ionisation capability. Understanding their penetration ability allows the design of adequate shielding for radiation.

Alpha (α) particles

Alpha particles have relatively slow speed (approximately a tenth of the speed of light, or $0.1c$), relatively heavy mass and a double charge. When an alpha particle travels through air, its slow speed and double positive charge cause it to interact with just about every atom that it encounters. The alpha particle dislodges electrons from many thousands of these atoms, turning them into ions, it ionises them. Each interaction slows the alpha particle down a little, and eventually it will be able to pick up some free electrons to become a helium atom. All this takes place within one or two centimetres in air. As a result, the air becomes quite ionised. The alpha particles are said to have a high **ionising ability**. Since the alpha particles do not get very far in the air, they have poor **penetrating ability**.

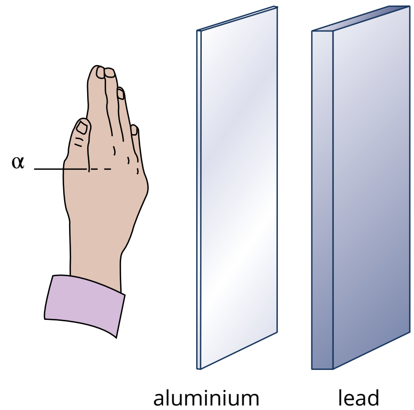


FIGURE 8.3.6 The penetrating ability of alpha radiation.

Alpha particles only travel a few centimetres in air before losing their energy, and will be completely absorbed by thin card or a human hand (Figure 8.3.6).

An example of an isotope that emits alpha radiation (or undergoes alpha decay) is the isotope of americium $^{241}_{95}\text{Am}$. Americium can be found in ionisation smoke detectors (Figure 8.3.7). The alpha particles are easily absorbed within the detector.

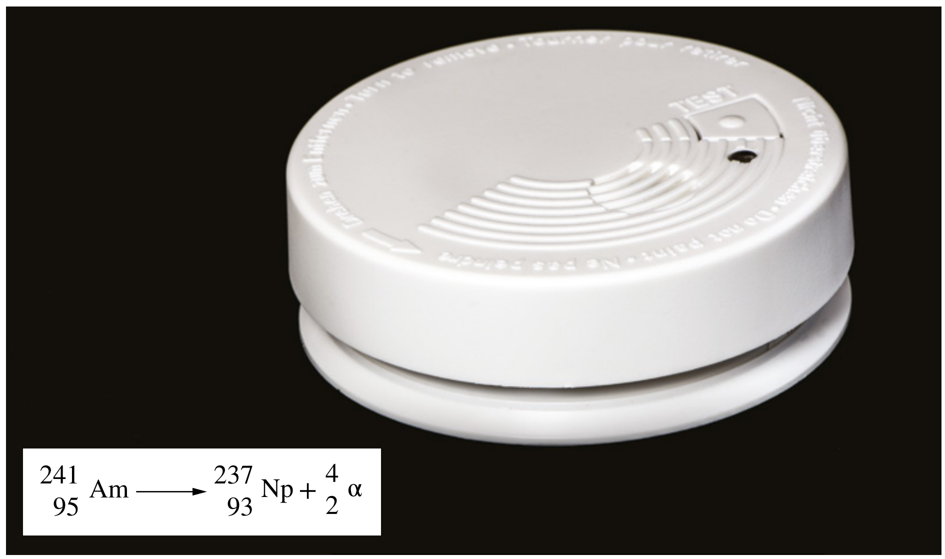


FIGURE 8.3.7 A domestic smoke detector contains a radioactive alpha emitter.

Beta-minus ($^{0}_{-1}\beta^{-}$) and beta-plus ($^{0}_{+1}\beta^{+}$) particles

Beta particles are faster (approximately $0.9c$) than alpha particles and have a smaller charge (+1 or −1). Beta-minus particles have a negative charge and are repelled by the electron clouds of the atoms they interact with. This means that when a beta-minus particle travels through matter, it experiences a large number of glancing collisions. It loses less energy per collision than an alpha particle. As a result, beta-minus particles do not ionise atoms as readily and are more penetrating than alpha particles. Beta-plus particles, or the positrons that they become, are unstable and will annihilate if they strike an electron. They also have a weak penetrating power.

Beta-minus particles will travel a few metres through air and through a human hand. Typically, a sheet of aluminium about 1 mm thick will stop them, as shown in Figure 8.3.8.

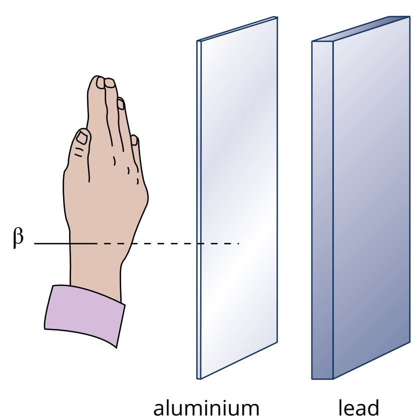


FIGURE 8.3.8 The penetrating ability of beta radiation.

Gamma (γ) rays

Gamma rays have no charge and move at the speed of light. They are the most highly penetrating form of radiation. Gamma rays have a low probability of interacting with matter. This only occurs if they happen to collide directly with a nucleus or electron. The low density of an atom makes this relatively unlikely. Gamma rays pass through matter very easily, making them highly penetrating but with a very poor ionising ability.

Gamma rays can travel an almost unlimited distance through air and can even penetrate through a human hand, an aluminium sheet and a few centimetres of lead (Figure 8.3.9). To fully block gamma rays, a significantly thicker layer of lead is required. Even a metre of concrete would not completely absorb a beam of gamma rays.

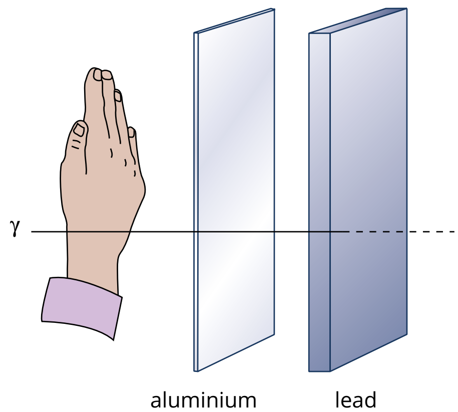


FIGURE 8.3.9 The penetrating ability of gamma radiation.

PHYSICS IN ACTION

Monitoring the thickness of sheet metal

Beta-minus particles can be used to monitor the thickness of rolled sheets of metal and plastic during manufacture, as shown in Figure 8.3.10. A beta-minus particle source is placed under the newly rolled sheet and a detector is placed on the other side. If the sheet being made is too thick, fewer beta-minus particles will penetrate and the detector count will fall. This information is instantaneously fed back to the rollers and the pressure is increased until the correct reading is achieved and hence the right thickness of metal is attained.

Alpha particles or gamma rays would not be appropriate for this task.

Alpha particles have a very poor penetrating ability, so none would pass through the metal. Gamma rays usually have a high penetrating ability and so a thin metal sheet would not stop them. In addition, workers would need to be shielded from gamma radiation.

The penetrating properties of beta rays make them ideal for this job. The thickness of photographic film and plastic sheets is also monitored in this way.

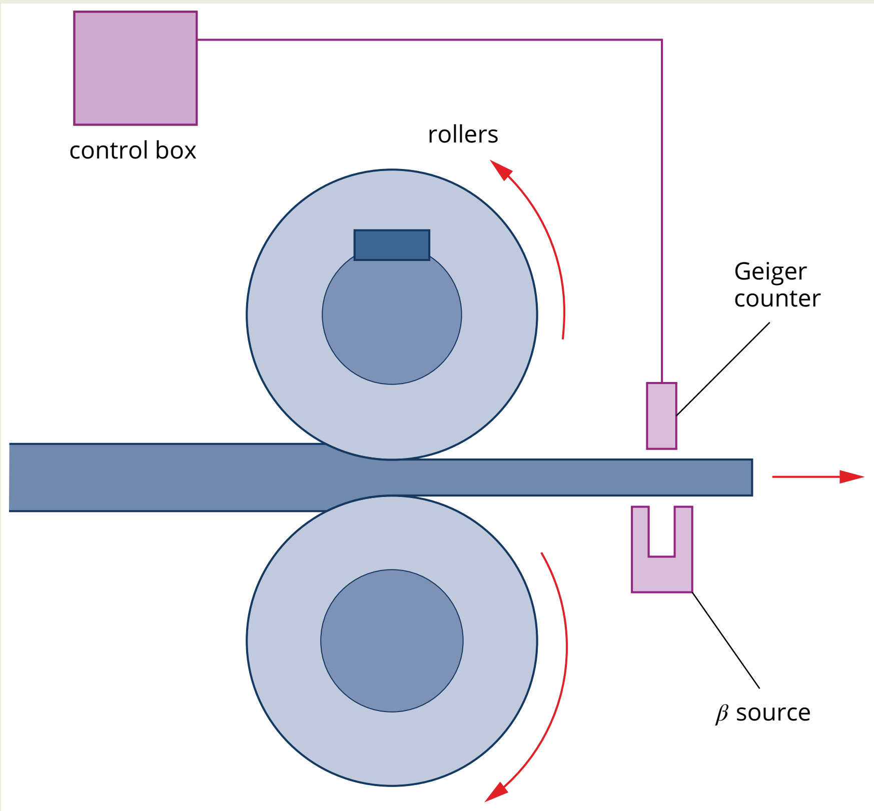


FIGURE 8.3.10 Beta emitters are used to monitor metal-sheet thickness.

PHYSICSFILE

Gamma knife therapy

Gamma knife therapy is often used to target brain tumours without the need to make a surgical incision. Beams of gamma radiation (up to 201) placed at different angles around the head are programmed to intersect at the position of the tumour. Only the tumour being targeted receives a significant radiation dose, whilst the surrounding tissue is unharmed by weaker individual beams.

FIGURE 8.3.11 A doctor prepares a patient for gamma knife therapy.

SUMMARY OF RADIATION TYPES AND PROPERTIES

The properties of alpha, beta and gamma radiation are summarised in Table 8.3.1.

TABLE 8.3.1 A comparison of the properties of alpha, beta and gamma radiation.

	alpha (α) particle	beta (β) particles	gamma (γ) ray
mass	heavy	light	none
speed	up to 20 000 $km\,s^{-1}$ or about 10% of the speed of light	about 90% of the speed of light	the speed of light
charge	+2	–1 or +1	0
ionising ability	very high	low	very low
range in air	a few centimetres	1 or 2 m	many metres
penetration in matter	~10^{-2} mm	a few mm	high

8.3 Review

SUMMARY

- Alpha ($^{4}_{2}\alpha$) particles are ejected from the nucleus at around 10% of the speed of light. They have a double positive charge and are relatively heavy. Alpha particles have high ionisation ability and poor penetrating power.
- Beta particles ($^{0}_{-1}\beta^{-}$ and $^{0}_{+1}\beta^{+}$) are ejected from the nucleus at up to 90% of the speed of light. They are much lighter than alpha particles. Beta-minus particles ($^{0}_{-1}\beta^{-}$) have a single negative charge. Beta-plus particles ($^{0}_{+1}\beta^{+}$) have a single positive charge. Beta radiation has moderate ionising and penetrating ability.
- Gamma (γ) rays are high-energy electromagnetic radiation and so travel at the speed of light and have no charge. They have high penetrating power and much lower ionisation ability.
- Alpha and beta particles can be accelerated by an electric field and their paths can be bent by a magnetic field. The speed and direction of gamma rays are unaffected by electric and magnetic fields.

KEY QUESTIONS

1 Identify the type of radiation (i.e. alpha, beta or gamma) that:
  a can easily penetrate aluminium foil
  b is ejected when a neutron decays into a proton
  c travels relatively slowly, at typically around 10% of the speed of light
  d travels at speeds of up to 90% of the speed of light
  e has no charge.

2 Which type of radiation (alpha, beta or gamma) is unaffected by a magnetic field?

3 Which type(s) of radiation (alpha, beta or gamma) could penetrate human skin but not 1 mm of aluminium?

4 Which type(s) of radiation (alpha, beta or gamma) would be used to treat brain cancer, where the radiation needs to penetrate the skull and reach the site of the tumour?

5 A radioactive source is emitting alpha, beta-minus and gamma radiation into the air. Which type(s) of radiation would a Geiger counter held about 20 cm from the source most likely detect?

6 Where in the atom do the following types of radiation originate?
  a alpha
  b beta
  c gamma

7 List the types of radiations—alpha, beta-minus and gamma—in order of decreasing penetrating power.

8 Briefly explain why alpha particles have a very poor penetrating ability.

9 A radiographer inserts a radioactive wire into a breast cancer to destroy the cancerous cells close to the wire. Should this wire be an alpha, beta-minus or gamma emitter? Explain your reasoning.

10 Explain why beta-minus particles, not alpha particles or gamma rays, are the best to use for monitoring the thickness of metal sheets.

8.4 Half-life and decay series

Different radioisotopes will emit radiation and will decay at very different rates. A **Geiger counter** held close to a small sample of polonium-218 will initially detect a very high level of radiation. Half an hour later, this count rate will have dropped to almost zero.

In contrast, a similar sample of uranium-235 will show a low count rate on a Geiger counter held close to it. However, over time, the count rate remains unchanged, even if measured decades later. Figure 8.4.1 compares the activity of both of these radioisotopes.

The half-life of a radioisotope describes how long it takes for half of the atoms in a given mass to decay. The count rate is the activity of the sample. These ideas will be studied in this section.

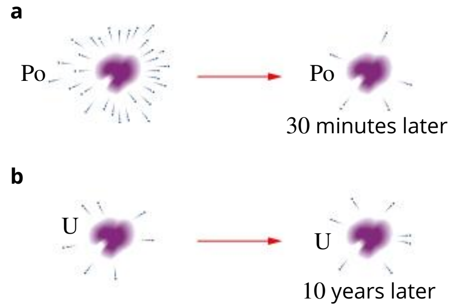


FIGURE 8.4.1 The activity of (a) polonium-218 and (b) uranium-235.

HALF-LIFE

All radioisotopes are unstable but some are more unstable than others. In the previous example, polonium-218 is more unstable than uranium-235. One way of determining this instability is by measuring the **half-life** ($t_{1/2}$) of the radioisotope.

> The half-life ($t_{1/2}$) of a radioisotope is the time that it takes for half of the nuclei of the sample radioisotope to decay.

The half-life of polonium-218 is about 3.10 minutes. Consider a sample of polonium that initially contains 100 million undecayed polonium-218 nuclei, as shown in Figure 8.4.2. Over the first 3.10 minutes about half of these will have decayed, leaving around 50 million polonium-218 nuclei. Over the next 3.10 minutes, half of these 50 million polonium-218 nuclei will decay, leaving approximately 25 million of the original radioactive nuclei. The process continues as time passes.

> The number of nuclei remaining after a particular number of half-lives can be found mathematically using:
>
> $$N = N_0\left(\frac{1}{2}\right)^n$$
>
> where N is the number of radioactive nuclei remaining
> N_0 is the initial number of radioactive nuclei
> n is the number of half-lives elapsed.
>
> The number of half-lives in a period of time can be found using:
>
> $$n = \frac{T}{t_{1/2}}$$
>
> where n is the number of half-lives elapsed
> T is the period of time that the radioactive nuclei has decayed
> $t_{1/2}$ is the half-life of the radioactive nuclei.

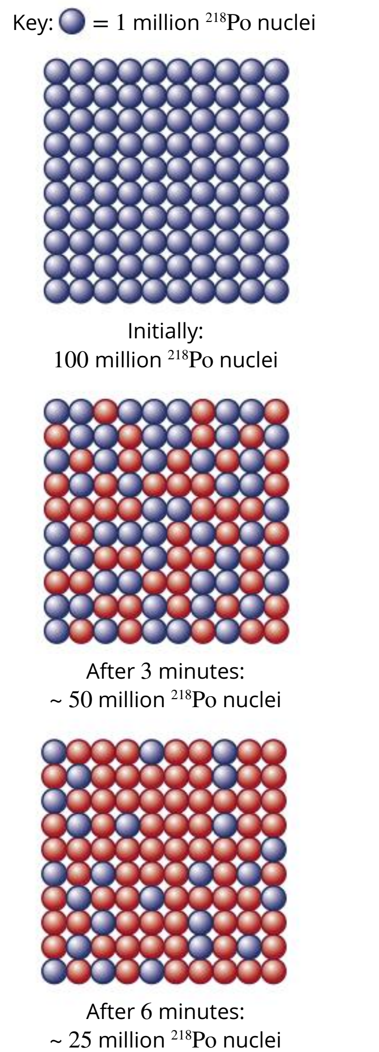


FIGURE 8.4.2 The decay of polonium-218 (blue dots) over two half-lives, showing how only one-quarter (25%) of the original radioisotope remains after two half-lives.

As time passes, a smaller and smaller proportion of the original radioisotope remains in the sample, until eventually the amount of decay is negligible. The graph in Figure 8.4.3, known as a decay curve, shows this.

Even a very small radioactive sample will contain billions of atoms. It is important to know that although the behaviour of such a large sample of nuclei can be mathematically predicted, the behaviour of one particular nucleus cannot. It has a 50% chance of decaying in each half-life. Also, the half-life of a radioisotope is constant and cannot be changed by chemical reactions, heat and so on. Half-life is solely determined by the instability of the nuclei of the radioisotope.

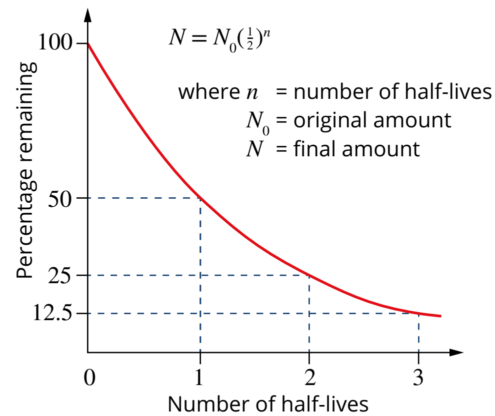


FIGURE 8.4.3 A decay curve for a radioisotope.

Worked example 8.4.1

HALF-LIFE

A sample of the radioisotope thorium-234 contains 8.00×10^{12} nuclei. The half-life of thorium-234 is 24.0 days. How many thorium-234 nuclei will remain in the sample after 120.0 days?

Thinking	Working
Calculate how many half-lives 120 days corresponds to.	$n = \frac{(120.0)}{(24.0)}$ $n = 5.00$ half-lives
Substitute $N_0 = 8.00 \times 10^{12}$ and $n = 5.00$ into the equation. Calculate the number of nuclei remaining.	$N = N_0\left(\frac{1}{2}\right)^n$ $N = (8.00 \times 10^{12})\left(\frac{1}{2}\right)^{5.00}$ $N = 2.5000 \times 10^{11}$ $N = 2.50 \times 10^{11}$ nuclei

Worked example: Try yourself 8.4.1

HALF-LIFE

A sample of the radioisotope sodium-24 contains 4.00×10^{10} nuclei. The half-life of sodium-24 is 15.0 hours. How many sodium-24 atoms will remain in the sample after 150.0 hours?

ACTIVITY

A Geiger counter can be used to record the number of radioactive decays occurring in a sample each second. It measures the **activity** of the sample.

Activity is measured in becquerels, Bq.

1 Bq = 1 disintegration per second

$$A = \frac{N}{t}$$

where A is the activity

N is the number of disintegrations

t is the time.

Over time, the activity of a sample of a radioisotope will decrease. More and more of the radioactive nuclei will have decayed and at some point it will no longer emit radiation.

If a sample of polonium-218 ($t_{1/2} = 3.10$ minutes) has an initial activity of 2000 Bq, then after one half-life its activity will be 1000 Bq. After a further 3.10 minutes, the activity of the sample will have reduced to 500 Bq, and after a further 3.10 minutes it will be 250 Bq and so on.

Uranium-235 has a half-life of 700 000 years. Its activity will remain virtually constant for decades and will certainly not change over 3 minutes.

The activity of a radioactive sample is directly related to the number of remaining nuclei that are still emitting radiation. Therefore, the equation used to determine the activity of nuclei after a number of half-lives is very similar to the one used to calculate the remaining number of a radioisotope.

The activity of the nuclei remaining after a number of half-lives (n) can be found mathematically using:

$$A = A_0\left(\frac{1}{2}\right)^n$$

where A is the activity of the radioactive nuclei remaining

A_0 is the initial activity of the radioactive nuclei

n is the number of half-lives elapsed.

Worked example 8.4.2

HALF-LIFE AND ACTIVITY

Technetium-99m is a metastable compound with a half-life of 6.00 hours, commonly used for radioactive imaging of bones, the brain and the lungs. The sample of technetium-99m has an initial activity of 4.00 kBq. Estimate its activity after 2.00 complete days.

Thinking	Working
Calculate how many half-lives 2.00 days corresponds to. Convert the number of days to hours first.	2.00 days = (2.00)(24.0) = 48.0 hours $n = \frac{(48.0)}{(6.00)} = 8.00$ $n = 8.00$ half-lives
Substitute the initial activity $A_0 = 4.00$ kBq and the number of half-lives, $n = 8.00$, into the equation. Calculate the final activity	$A = A_0\left(\frac{1}{2}\right)^n$ $A = 4000 \times \left(\frac{1}{2}\right)^{8.00}$ $A = 15.6250$ $A = 15.6$ kBq

You will notice that due to the short half-life the activity has significantly reduced after 8.00 half-lives or 2.00 days, making it ideal for medical purposes.

Worked example: Try yourself 8.4.2

HALF-LIFE AND ACTIVITY

A sample of strontium-90 has an initial activity of 4.00 kBq. Calculate its activity after 6.00 months using Table 8.4.1.

COMMON RADIOISOTOPES AND THEIR APPLICATIONS

The half-lives of some common radioisotopes are shown in Table 8.4.1. The half-life of a radioisotope will determine what it is used for. For example, the most commonly used medical tracer, technecium-99m, has a short half-life of just 6 hours. The short half-life means that radioactivity does not remain in the body any longer than necessary. On the other hand, the radioisotope used in a smoke detector, americium-241, is chosen because of its long half-life, 461 years. The smoke detector can continue to function for a very long time, as long as the battery is replaced each year.

TABLE 8.4.1 Some common radioisotopes, their half-lives and applications.

Isotope	Emission	Half-life	Application
Natural			
polonium-214	$^{4}_{2}\alpha$	0.000 164 s	nothing at this time
strontium-90	$^{0}_{-1}\beta^{-}$	28.8 years	destroys cancer tissue in bone marrow; treats eye diseases; used in agricultural studies
radium-226	$^{4}_{2}\alpha$	1602 years	used in medical equipment, and as a tracer in geochemical processes, especially in marine environments
carbon-14	$^{0}_{-1}\beta^{-}$	5730 years	used to measure the age of organic material up to 50 000 years old
uranium-235	$^{4}_{2}\alpha$	703 million years	fuel for nuclear power plants and for nuclear reactors that run naval ships and submarines
uranium-238	$^{4}_{2}\alpha$	4.50 billion years	nuclear fuel; dating of sediments in a marine environment or from a dry lake bed
thorium-232	$^{4}_{2}\alpha$	13.9 billion years	fertile fuel, so can only be used in conjunction with a fissile material in Gen IV reactors (see Chapter 9.2)
Artificial			
technetium-99m	$^{0}_{-1}\beta^{-}$	6.01 h	used to image the brain, thyroid, lungs, liver, spleen, kidney, gall bladder and to detect infection
sodium-24	$^{0}_{-1}\beta^{-}$	15 h	salt solutions injected into the blood stream to locate obstructions
iodine-131	$^{0}_{-1}\beta^{-}$	8.03 days	used to diagnose and treat various diseases associated with the human thyroid
phosphorus-32	$^{0}_{-1}\beta^{-}$	14.3 days	used in labs to label DNA and proteins; used to track the uptake of fertiliser from roots to leaves in plant studies
cobalt-60	$^{0}_{0}\gamma$	5.27 years	one of the most precise and advanced forms of radiation used to target brain tumours (Gamma Knife radiosurgery); gamma sterilisation of orthopaedic implants
americium-241	$^{4}_{2}\alpha$	432.5 years	smoke detectors
plutonium-239	$^{4}_{2}\alpha$	24 110 years	nuclear fuel; rock dating; powers batteries for some heart pacemakers

DECAY SERIES

Generally, when a radionuclide decays, its daughter nucleus is not completely stable and is itself radioactive. This daughter nucleus will then undergo further decay. Eventually a stable isotope is reached and the sequence ends. This is known as a **decay series**. An example of a decay series is shown in Figure 8.4.4. This particular series shows the decay of uranium-238 (shown at the top of the chart). The uranium-238 has a long half-life of 4.5×10^9 years but eventually decays into thorium-234 by alpha emission. Thorium-234 has a short half-life of only 24 days, decaying into protactinium 234 by beta decay, and so on until the final product is lead-206 (shown at the bottom of the chart).

The Earth is 4.5 billion years old, which is old enough to have only four naturally occurring decay series that remain active. These are:

- the uranium series, in which uranium-238 eventually becomes lead-206
- the actinium series, in which actinum-235 eventually becomes lead-207
- the thorium series, in which thorium-232 eventually becomes lead-208
- the neptunium series, in which neptunium-237 eventually becomes bismuth-209.

(Since neptunium-237 has a relatively short half-life, it is no longer present in the crust of the Earth, but the rest of its decay series is still continuing.)

Geologists analyse the proportions of the radioactive elements in a sample of rock to gain a reasonable estimate of the rock's age. This is known as rock dating.

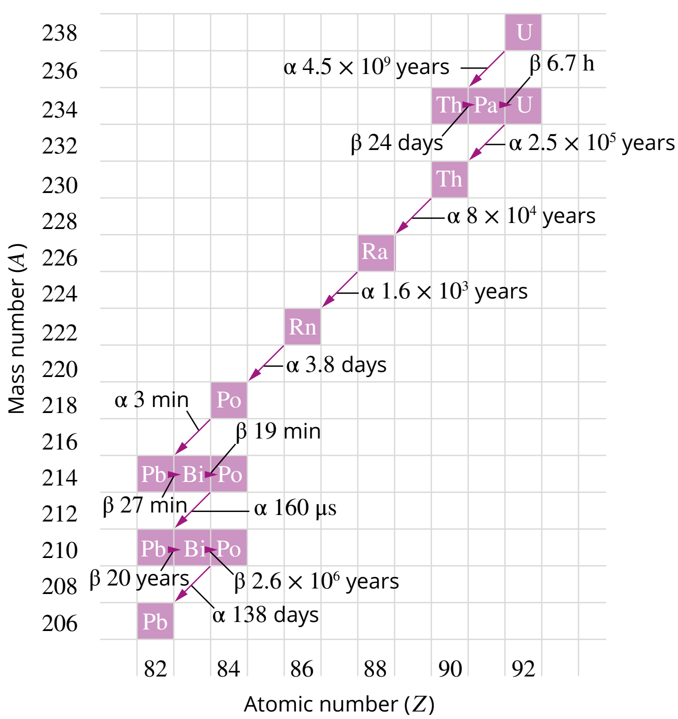


FIGURE 8.4.4 The uranium decay series. The half-life and emissions are indicated on each of the decays as radioactive uranium-238 is gradually transformed into stable lead-206. Mining companies find significant quantities of lead at uranium mines.

EXTENSION

Radiocarbon dating

Radiocarbon dating (or carbon dating) is a technique used by archaeologists to determine the ages of fossils and ancient objects that were made from plant matter. This method involves measuring and comparing the proportion of two isotopes of carbon, carbon-12 and carbon-14, in the specimen.

Carbon-12 is a stable isotope but carbon-14 is radioactive. Carbon-14 only exists in trace amounts in nature. Carbon-12 atoms are about 1 000 000 000 000 (10^{12}) times more common than carbon-14 atoms.

Carbon-14 has a half-life of 5730 years and decays by beta-minus emission to nitrogen-14. Its decay equation is:

$${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\beta^{-} + {}^{0}_{0}\bar{\nu}_e$$

Both carbon-12 and carbon-14 can combine with other atoms in the environment. For example, they both combine with oxygen to form carbon dioxide. Plants and animals take in carbon-based molecules from the air and food. This means that all living organisms contain the same percentage of carbon-14. In the environment, the production of carbon-14 is matched by its decay and so the proportion of carbon-14 atoms to carbon-12 remains constant.

After an organism dies, the amount of carbon-14 it contains will decrease as these atoms decay to form nitrogen-14 and are not replaced from the environment. The number of atoms of carbon-12 does not change as carbon-12 is a stable atom. So, over time, the proportion of carbon-14 to carbon-12 atoms decreases.

The proportion of carbon-14 to carbon-12 in a dead organism can be compared with that found in living organisms and the approximate age of the specimen can be determined from the half-life of carbon-14.

Consider this example. The count rate from a 1.00 gram sample of carbon that has been extracted from an ancient wooden spear is 10.0 Bq. A 1.00 gram sample of carbon from a living piece of wood has a count rate of 40.0 Bq. We can assume that this was also the initial count rate of the spear. For its count rate to have reduced from 40.0 to 10.0 Bq, the spear must have gone through two half-lives of carbon-14 (40.0 → 20.0 → 10.0). Since carbon-14 has a half-life of 5730 years, the spear must be twice that, about 11 500 years old.

In 1988, scientists used carbon-dating techniques to show that the Shroud of Turin was probably a medieval forgery. It had been claimed that the Shroud of Turin was the piece of cloth that was the burial shroud of Jesus Christ (Figure 8.4.5). Carbon-dating tests on small samples of the cloth established that there was a high probability that it was made between 1260 and 1390.

FIGURE 8.4.5 The Shroud of Turin.

Radiocarbon dating is an important aid to anthropologists who are interested in finding out about the migration patterns of early people—including the Australian Aborigines. This technique is very powerful since it can be applied to the remains of ancient campfires. It is accurate and reliable for samples up to about 60 000 years old. Carbon dating cannot be used to date dinosaur bones as they are more than 60 million years old, but it can be used to determine the age of more recently extinct mammoth fossils, like that shown in Figure 8.4.6.

FIGURE 8.4.6 A fossilised mammoth analysed by carbon dating.

8.4 Review

SUMMARY

- The rate of decay of a radioisotope is measured by its half-life, $t_{1/2}$. This is the time that it takes for half of the radioisotope to decay.
- The activity of a sample indicates the number of emissions per second. Activity is measured in becquerels (Bq), where 1 Bq is equal to 1 emission per second.
- The number of atoms of a radioisotope will decrease over time. Over one half-life, the number of atoms of a radioisotope will halve.
- The half-life equation can be used to calculate the number (N) or activity (A) of a radioisotope remaining after a number of half-lives (n) has passed:

$$N = N_0\left(\frac{1}{2}\right)^n,\ A = A_0\left(\frac{1}{2}\right)^n$$

- When a radionuclide decays, its daughter nucleus is usually radioactive as well. This daughter will then decay to a grand-daughter nucleus, which may also be radioactive, and so on. This is called a decay series.

KEY QUESTIONS

1 What is meant by the activity of a radioisotope?

2 Technetium-99m has a half-life of 6.00 hours. A sample of the radioisotope originally contains 8.50×10^{10} atoms. How many technetium-99m nuclei remain after 2.00 full days?

3 Iodine-131 has a half-life of 8.00 days. A sample of the radioisotope initially contains 2.40×10^{12} iodine-131 nuclei. Estimate how many iodine-131 nuclei remain after 24.0 days.

4 Radioactive materials are considered to be relatively safe when their activity has fallen below 0.1% of the initial value.

a How many half-lives does this take?

b Plutonium-239 is a by-product of nuclear reactors. Its half-life is 24 110 years. How long does the plutonium-239 have to be stored as nuclear waste before it is considered safe to handle?

5 If a particular atom in a radioactive sample has not decayed during the previous half-life, what is the percentage chance it will decay in the next half-life?

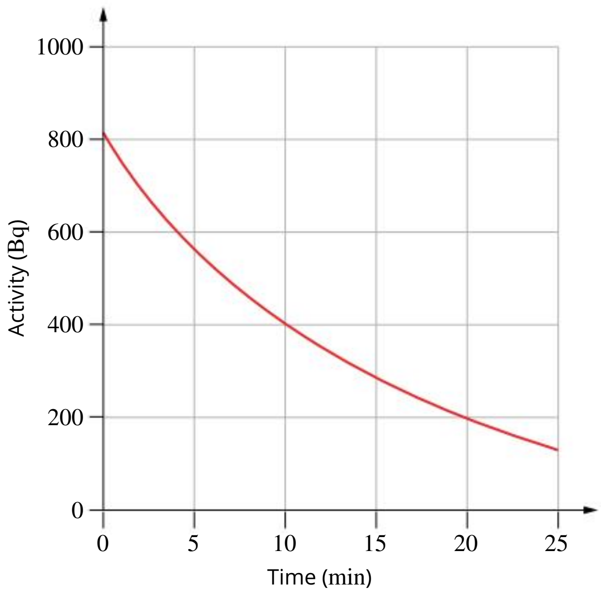
6 A hospital in Broome requires the activity of the radioisotope technetium-99m to be 12.0 μBq upon arrival. The dose has to be delivered from Sydney and the delivery takes at least 24.0 hours. Given that the half-life of technetium-99m is 6.00 hours, what must the activity be when the technetium-99m leaves Sydney to ensure that the dose meets the required activity in Broome? Give your answer in Bq.

7 The activity of a radioisotope changes from 6.00 kBq to 0.375 kBq over a period of 60.0 minutes. Determine the half-life of this radioisotope.

8 A Geiger counter is used to measure the radioactive emissions from a certain radioisotope. The activity of the sample is shown in the graph.

a Estimate the half-life of the radioisotope, according to the graph.

b What would you expect the activity of the sample to be after 40.0 minutes have elapsed?



9 According to Figure 8.4.4 on page 319, what type of decay does lead-210 undergo and what is its half-life?

10 In the uranium decay series shown in Figure 8.4.4, page 319, $^{234}_{92}U$ decays to produce stable $^{206}_{82}Pb$. How many alpha and beta-minus decays have occurred?

8.5 Radiation doses and effects on humans

FIGURE 8.5.1 A dosimeter used to monitor gamma radiation exposure.

We are exposed to low levels of ionising radiation, known as background radiation, throughout our lives. However, exposure to high levels of ionising radiation from alpha, beta and gamma sources is harmful to humans and other living organisms. People who work with radiation in fields such as medicine, mining, nuclear power plants and industry must be able to closely monitor the radiation dose to which they are exposed. Similarly, radiologists who administer courses of radiation treatment to cancer patients need to be able to measure the radiation dose that they are applying. Figure 8.5.1 shows a dosimeter used by doctors, radiologists and technicians who work with gamma radiation to monitor their exposure levels. In this section, you will learn about radiation doses, how they are measured, the effects of high exposure and medical applications of radiation.

IONISING RADIATION

The electromagnetic spectrum consists of a variety of types of electromagnetic radiation. From low energy to high energy, these are: radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays, and are shown in Figure 8.5.2. **Non-ionising radiation** includes radio waves, microwaves, infrared radiation, visible light and UV-A radiation. Every day, people are exposed to significant amounts of such radiation without serious consequences.

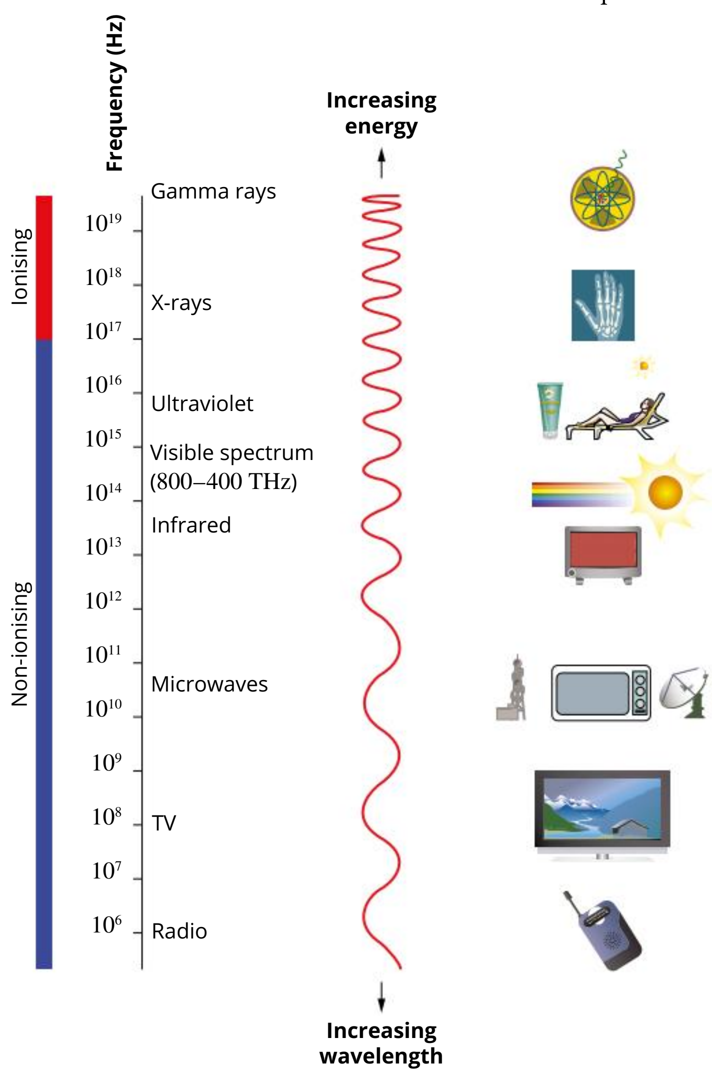


FIGURE 8.5.2 The electromagnetic spectrum consists of ionising and non-ionising radiation.

Some forms of electromagnetic radiation are harmful to living things. These ionising radiations are found at the high-energy end of the electromagnetic spectrum, starting at UV-C, which is blocked by the ozone layer, and include gamma and X-rays. **Ionising radiation** is electromagnetic radiation with a frequency above 2.00×10^{16} Hz. When ionising radiation interacts with the tissue in an organism, it creates ions, which can damage tissue or lead to the development of cancerous tumours. Figure 8.5.3 shows an X-ray heading towards a water molecule. The ionising radiation has enough energy to break the bonds within the water molecule and create a pair of ions.

The Earth itself is radioactive and we are all exposed to a low level of ionising radiation from many different sources, as shown in Figure 8.5.4. Almost 90% of our annual exposure is from the surrounding environment. This radiation is called **background radiation**, and this background level of exposure is not a significant problem to our health.

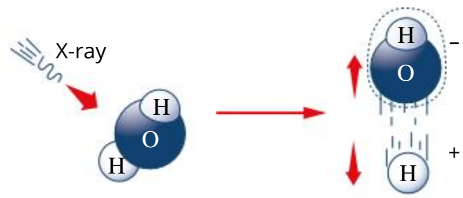


FIGURE 8.5.3 Water molecule being ionised.

MEASURING RADIATION EXPOSURE

Exposure to high-energy radiation is harmful to living tissue. The energy of the radiation acts to break apart molecules and ionise atoms in the body's cells. It may lead to long-term problems such as cancer and deformities in future generations. Extremely high levels of radiation exposure can cause death, and this can happen within just a few hours. It is therefore important to be able to measure the amount of exposure a person has had.

Absorbed dose

The severity of radiation exposure depends on the amount of radiation energy that has been absorbed (E) and the mass of tissue involved (m).

The radiation energy absorbed per kilogram of tissue is known as the **absorbed dose** (AD).

$$\text{absorbed dose} = \frac{\text{energy absorbed by the tissue}}{\text{mass of tissue}}$$

$$AD = \frac{E}{m}$$

Absorbed dose is measured in joules per kilogram ($J\,kg^{-1}$) or grays (Gy), i.e. $1\,Gy = 1\,J\,kg^{-1}$.

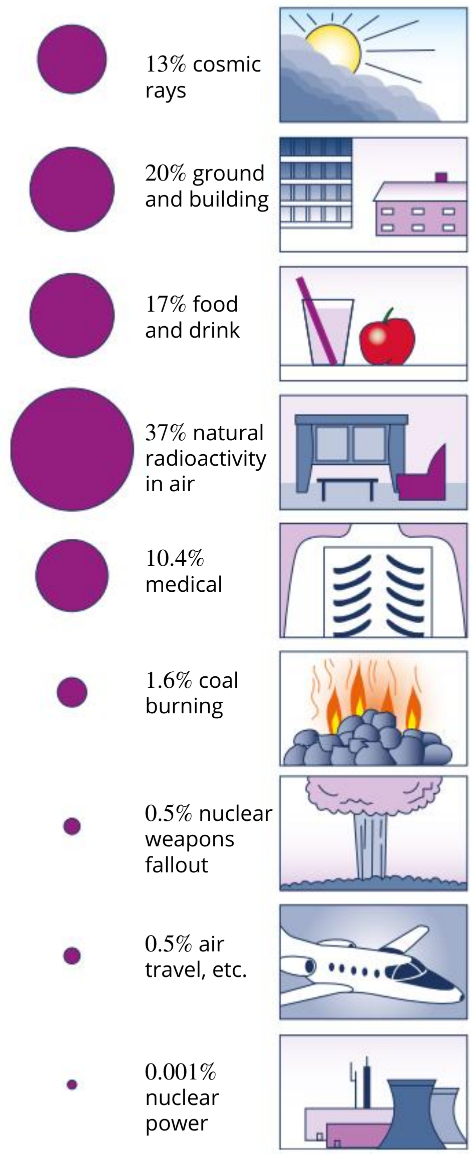


FIGURE 8.5.4 Everyone is exposed to radiation from the environment.

Worked example 8.5.1

ABSORBED DOSE

A cancer tumour of mass 155 g is exposed to 0.325 J of radiation energy. Calculate the absorbed dose (AD) in grays. Assume that all the radiation is absorbed by the tumour.	
Thinking	**Working**
Convert the mass into kg.	155 g = 0.155 kg
Use $AD = \frac{E}{m}$ to calculate the absorbed dose.	$AD = \frac{(0.325)}{(0.155)}$ AD = 2.096 AD = 2.10 Gy

Worked example: Try yourself 8.5.1

ABSORBED DOSE

A cancer tumour is exposed to 0.585 J of radiation energy. The absorbed dose is 3.50 Gy. Calculate the mass of the tumour. Assume that all the radiation is absorbed by the tumour.

Dose equivalent

Absorbed dose is not widely used when measuring radiation dose. It does not take into consideration the type of radiation involved. **Dose equivalent** (DE) does, and it is the most common way in which radiation doses are measured.

Alpha particles are the most ionising form of radiation. This is because of their low speed, high charge and large mass (discussed in Section 8.3). Alpha particles interact with, and ionise, almost every atom that lies in their path. This means that an absorbed dose of alpha radiation is about 20 times more damaging to human tissue than an equal absorbed dose of beta or gamma radiation.

The weighting of the biological impact of radiation is called the **quality factor**, QF, or weighting factor. A list of quality factors is shown in Table 8.5.1.

TABLE 8.5.1 The quality factors for various types of radiation.

Radiation	Quality factor
α particles	20
neutrons* (10 keV)	10
β particles	1
γ rays	1
X-rays	1

*Radiation from neutrons is only found around nuclear reactors and neutron bomb explosions.

Gamma rays and X-rays have relatively low ionising powers. They have no charge and move at the speed of light, so they fly straight past most atoms and interact only occasionally as they pass through. Gamma rays and X-rays would cause only slight damage to living cells. Beta-minus particles are considered to be as damaging as gamma rays and X-rays. This is reflected in their low quality factor of 1.

Neutrons are a more damaging form of radiation and have a weighting factor of 10. Neutrons are present in fission reactions which take place in nuclear reactors and bombs (fission will be covered in Chapter 9.1). The high quality factor explains why accidents at nuclear reactors are so dangerous to anyone present.

The dose equivalent takes into account the absorbed dose and the type of radiation. This gives a more accurate picture of the effect of the radiation on tissue.

dose equivalent = absorbed dose × quality factor

This can be abbreviated to:

$$DE = AD \times QF$$

Dose equivalent is measured in Sieverts (Sv).

An absorbed dose of just 0.0500 Gy of alpha radiation is equally as damaging to a person as an absorbed dose of 1.00 Gy of beta radiation. While alpha particles carry less energy than beta particles, each alpha particle does far more damage. For the 0.0500 Gy alpha particle, the dose equivalent $DE = 0.0500 \times 20 = 1.00$ Sv; and for the 1.00 Gy beta particles, $DE = 1.00 \times 1 = 1.00$ Sv. In each case, the dose equivalent is 1.00 Sv, and 1.00 Sv of any radiation causes the same amount of damage.

Worked example 8.5.2

DOSE EQUIVALENT

Calculate the dose equivalent (in µSv) from various types of radiation if the absorbed dose is 0.500 µGy.	
Thinking	**Working**
The quality factor for each type of radiation can be found in Table 8.5.1 (page 324). $1\,\mu Gy = 1 \times 10^{-6}\,Gy$	$QF_\alpha = 20$ $QF_\beta = 1$ $QF_\gamma = 1$
a Calculate the dose equivalent (in µSv) if the source is emitting alpha particles.	
The dose equivalent $DE_\alpha = AD \times QF_\alpha$	$DE_\alpha = (0.500 \times 10^{-6})(20)$ $DE_\alpha = 1.0000 \times 10^{-5}$ $DE_\alpha = 10.0\,\mu Sv$

b Calculate the dose equivalent (in µSv) if the source is emitting beta particles.	
Thinking	**Working**
The dose equivalent $DE_\beta = AD \times QF_\beta$	$DE_\beta = (0.500 \times 10^{-6})(1)$ $DE_\beta = 0.50000 \times 10^{-6}$ $DE_\beta = 0.500\,\mu Sv$

c Calculate the dose equivalent (in µSv) if the source is emitting gamma rays.	
Thinking	**Working**
The dose equivalent $DE_\gamma = AD \times QF_\gamma$	$DE_\gamma = (0.500 \times 10^{-6})(1)$ $DE_\gamma = 0.50000 \times 10^{-6}$ $DE_\gamma = 0.500\,\mu Sv$

Worked example: Try yourself 8.5.2

DOSE EQUIVALENT

Calculate the dose equivalent (in mSv) from various radiation sources if the absorbed dose is 1.25 mGy.

a Calculate the dose equivalent (in mSv) if the source is emitting alpha particles.

b Calculate the dose equivalent (in mSv) if the source is emitting beta particles.

c Calculate the dose equivalent (in mSv) if the source is emitting gamma rays.

Worked example: 8.5.3

TREATING TUMOURS

A 12.5 g cancer tumour absorbs 2.50×10^{-3} J of energy from an applied radiation source. Calculate the dose equivalent if the source is an alpha emitter using information from Table 8.5.1.	
Thinking	**Working**
Convert mass from grams to kg.	$m = \dfrac{(12.5)}{(1000)}$ $m = 0.012500\,kg$

Calculate the absorbed dose (AD) using the energy and the mass.	$AD = \frac{E}{m}$ $AD = \frac{(2.50 \times 10^{-3})}{(0.012500)}$ AD = 0.20000 Gy
Calculate the dose equivalent (DE) using the quality factor for alpha particles of 20.	DE = AD × QF DE = (0.20000)(20) DE = 4.0000 DE = 4.00 Sv

Worked example: Try yourself 8.5.3

TREATING TUMOURS

A 28.5 g cancer tumour absorbs 5.00×10^{-3} J of energy from an applied radiation source. Calculate the dose equivalent if the source is an alpha emitter, using information from Table 8.5.1.

People who work in occupations that involve ongoing exposure to levels of ionising radiation usually pin a small radiation-monitoring device to their clothing. This is usually a thermoluminescent dosimeter (TLD), as pictured in Figure 8.5.5. TLDs contain a disc of lithium fluoride encased in plastic. Lithium fluoride can detect beta and gamma radiation, as well as X-rays and neutrons. TLDs are a cheap and reliable method for measuring radiation doses.

Dosimeters like these are used by personnel in nuclear power plants, radiotherapy departments at hospitals, airport security gates and uranium mines.

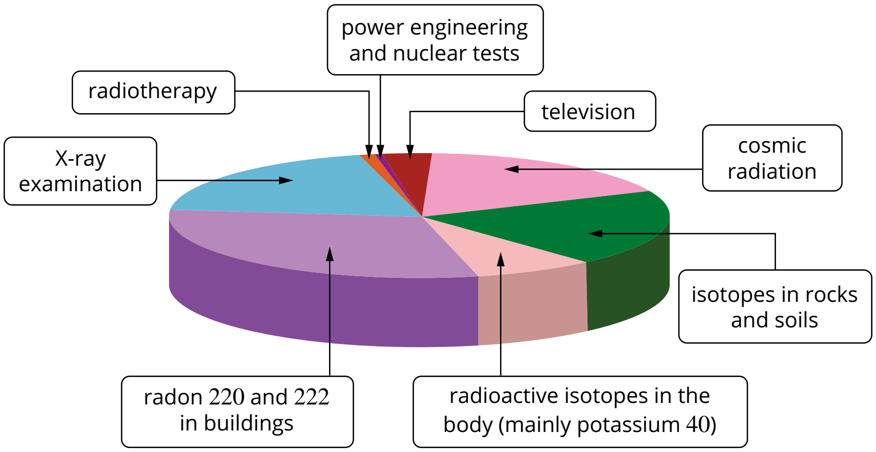
FIGURE 8.5.5 Thermoluminescent dosimeters are used by doctors, radiologists and scientists who work with radiation to monitor their exposure levels.

BACKGROUND RADIATION

It is important to appreciate that 1.00 Sv is a massive dose of radiation. It would not be fatal, but it would certainly lead to a severe case of radiation sickness. However, we are exposed to background radiation continually and it is not harmful to us.

In Australia, the average annual background radiation dose is about 1.50 mSv (millisievert) or 1500 μSv. The average in the world is 2.40 mSv, and it is 3.00 mSv in North America. Figure 8.5.6 shows the origin of background radiation. In India, there is a village situated above natural uranium and thorium deposits. The average equivalent dose for its inhabitants is 15.0 mSv per year, three times the recommended equivalent dose.

Share of the different sources of radiation
The average equivalent of the dose absorbed by a person in a year is equal to a few millisieverts.



Natural sources	*DE* (mSv)	**Artificial sources**	*DE* (mSv)
cosmic radiation	0.3–0.5	X-ray examination	0.5
isotopes in rocks and soils	0.4–0.5	radiotherapy	0.01
radioactive isotopes in the body (mainly potassium 40)	0.2	power engineering and nuclear tests	0.01
radon 220 and 222 in buildings	0.6–0.8	television	0.08
total	1.5–2.0	total	0.6

FIGURE 8.5.6 Pie chart showing common sources of background radiation.

Table 8.5.2 can be used to estimate your own dose over the past year.

TABLE 8.5.2 Annual radiation doses in Australia for different radiation sources.

Radiation source	Average annual dose (μSv)	Local variations
cosmic radiation	300	plus 200 μSv for each round-the-world flight plus 20 μSv for each 10° of latitude plus 150 μSv if you live 1000 m above sea level
rocks, air and water	1350	plus 1350 μSv if you live underground plus 1350 μSv if your house is made of granite minus 140 μSv if you live in a weatherboard house
radioactive foods and drinks	350	plus 1000 μSv if you have eaten food affected by the Fukushima fallout
manufactured radiation	60	plus 60 μSv if you live near a coal-burning power station plus 30 μSv from nuclear testing in the Pacific

EFFECT OF RADIATION EXPOSURE ON HUMANS

High levels of radiation exposure can have an adverse effect on human tissue. Radiation exposure can be chronic (occurs over a long period of time), as would be expected by a worker in a nuclear reactor plant, or acute (occurs all at once), such as in a nuclear reactor accident or explosion. Immediate effects of radiation

PHYSICSFILE

Iodine-131 and the thyroid

On 11 March 2011, a catastrophic earthquake and tsunami hit Japan, killing tens of thousands of people and severely damaging the nuclear power station at Fukushima. Radioactive materials, including caesium-137 and iodine-131, escaped. These materials have half-lives of 30 years and 8 days respectively.

Our bodies need iodine for the healthy functioning of the thyroid gland, which maintains proper metabolism. Foods rich in iodine include seafood, vegetables and salt. However, our bodies cannot tell the difference between normal iodine and radioactive iodine. To prevent people in Japan from absorbing radioactive iodine into their thyroid glands, they were issued with iodine tablets (Figure 8.5.7). Taking an iodine tablet each day ensured that the thyroid gland was saturated with iodine and so any radioactive iodine ingested by eating contaminated food would not be taken into the body and deposited in the thyroid.

FIGURE 8.5.7 Children in Japan receiving iodine tablets to prevent their body ingesting radioactive iodine after the Fukushima nuclear accident in 2011.

Many victims of the Chernobyl nuclear disaster in 1986 died of thyroid cancer years after the accident. They ingested radioactive iodine and this accumulated in the thyroid gland, eventually leading to cancer.

exposure include a drop in the white blood cell count, nausea, fatigue, hair-loss and skin reddening. Radiation has the potential to cause DNA damage and cause cells to become cancerous. In addition, different radioactive elements can target different parts of the body; for example, iodine affects the thyroid and strontium-90 affects the bone and bone marrow.

Table 8.5.3 shows some typical levels of radiation dosages and their effects, as put out by the World Nuclear Association. The effects discussed are assuming the whole body has been exposed. Dosages of radiation for cancer treatment can be significantly higher but these are specifically targeted at the cancerous cells.

TABLE 8.5.3 Effect of radiation dosage on the human body.

Radiation doses	Effects
1.5 mSv	typical background exposure in Australia
9 mSv year^{-1}	exposure by airline crew on the New York–Tokyo route
100 mSv year^{-1}	highest annual safe level; above this the probability of cancer is assumed to increase with the dose 4 months on the International Space Station, 350 km above the Earth
350 mSV year^{-1}	criterion for relocating people after the Chernobyl accident
700 mSv year^{-1}	suggested threshold for maintaining evacuation after a nuclear incident
1000 mSv short term, whole body	threshold for causing radiation sickness and nausea, but not death
5000 mSv whole body	fatal for 50% of those exposed
10 000 mSv whole body	acute radiation poisoning, death within a few weeks

MEDICAL APPLICATIONS OF RADIATION

Rapidly dividing cancerous cells are more susceptible to radiation than healthy tissue, so radiation is often used in cancer treatment. In addition, sources such as cobalt-60 can be used to specifically target only the cancerous tissue, minimising the damage to healthy tissue (Figure 8.5.8).

FIGURE 8.5.8 A cancer patient about to receive radiation therapy.

Typical exposures to the targeted area for various medical procedures are given in Table 8.5.4.

TABLE 8.5.4 Typical radiation exposures for various medical procedures.

Medical exposures	30 μSv for a chest X-ray 300 μSv for a pelvic X-ray 5000 μSv for a CT scan 40 000 000 μSv for a course of radiotherapy using cobalt-60

Detecting cancer with radioactive tracers

Cancers that form on the skin can often be detected by a simple external examination. However, to diagnose cancerous growths at specific sites inside the body, a variety of radioisotopes tagged to particular drugs are used. The radioisotope is known as a radioactive **tracer**. These drugs, radiopharmaceuticals, can be administered by swallowing (ingestion), inhalation or injection. In Figure 8.5.9 a gamma-ray camera is being used to perform a bone scan. This patient has been injected with the radioisotope technetium-99m. This commonly used isotope is a gamma emitter with a half-life of only 6 hours. The camera detects the emitted gamma rays and produces an image on a computer screen.

FIGURE 8.5.9 A cancer patient receiving a bone scan with a gamma-ray camera.

The radioisotope used in the radiopharmaceutical depends on the site of the suspected tumour. The body naturally distributes different elements to different organs. For example, iodine is sent to the thyroid gland by the liver. So if a radiopharmaceutical containing radioactive iodine is ingested, most of this iodine will end up in the thyroid.

When the tracer has reached the target organ, a radiation scan is taken with a gamma-ray camera. An unusual pattern on the scan indicates a possible cancerous tumour. The radioisotopes used for this type of diagnosis need to be gamma-ray emitters so that the radiation has enough penetrating ability to pass out of the body to reach the detector, called a gamma-ray camera. The isotope should have a relatively short half-life so that the patient is not subjected to any unnecessary long-term exposure to radiation.

Radioactive tracers are also used to monitor other bodily functions. Some examples are shown in Table 8.5.5.

TABLE 8.5.5 Some radioactive tracers and their target organs.

Radioactive tracer	Function monitored
iodine-123	function of thyroid gland
xenon-133	function of lungs
phosphorus-32	blood flow through body
iron-59	level of iron uptake by spleen
technetium-99m	blood flow in brain, lungs and heart function of liver metabolism of bones

PHYSICS IN ACTION

Scintigraphy

The radioactive isotopes of an element have chemical properties that are identical to those of the non-radioactive isotopes. This feature is used by putting radioactive isotopes of elements into various compounds and, in this way, obtaining tracers.

Because a tracer continuously emits radiation, it is possible to observe what happens to it, to see the path along which it is moving, and the reactions in which it becomes involved.

Tracers are used, for example, in medicine (for example, in a scintigraphic examination) and in industry (for example, when testing the water-tightness of a water system). Figure 8.5.10 shows a scintigram of the human central nervous system (CNS), showing the brain at the top with the spinal cord running below it. The image was obtained by injecting a radioactive tracer into the subject which concentrates in the tissues of the CNS. Gamma rays emitted from the tracer are resolved by a gamma-ray camera's crystal scintillator as flashes of light and are processed electronically to give a map of distribution of radioactivity in the CNS.

FIGURE 8.5.10 Scintigram of the human central nervous system.

8.5 Review

SUMMARY

- Alpha, beta, gamma and high-energy electromagnetic radiation are ionising and are harmful to biological tissues at high levels.
- Exposure to background ionising radiation is a natural part of our existence.
- Unnecessary exposure to high levels of ionising radiation can be dangerous and should be avoided.
- Absorbed dose (AD) is a measure of the radiation dose per kilogram of irradiated tissue. AD is measured in grays (Gy).

  $$AD = \frac{\text{energy absorbed by tissue (J)}}{\text{mass of tissue (kg)}}$$
- The quality factor (QF) of radiation is a number that indicates the relative damaging effect of the particular radiation.
- Dose equivalent (DE) gives a measure of the biological damage that a dose of radiation causes. $DE = AD \times QF$ and is measured in sieverts (Sv).
- High doses of targeted radiation can be used to destroy cancerous cells.
- Radiation from radioisotopes can be used as a diagnostic tool in medicine and industry.

KEY QUESTIONS

1 Which of the following is the most dangerous exposure to radiation?

A 1 Gy of alpha radiation
B 1 Gy of beta radiation
C 1 Gy of gamma radiation
D All are equally damaging.

2 A geologist receives a dose of 250 μSv. Which of the following is the most damaging dose?

A 250 μSv of alpha radiation
B 250 μSv of beta radiation
C 250 μSv of gamma radiation
D All doses are equally damaging.

3 An 82.5 kg tourist absorbs a gamma radiation dose of 235 μGy on a return flight from London.

a Calculate the dose equivalent received by the tourist on the flight.
b Calculate the amount of radiation energy that has been absorbed by the tourist.

4 Rank the following radiation doses from the most damaging to the least damaging.

A 250 μGy of gamma radiation
B 20 μGy of alpha radiation
C 50 μGy of beta radiation
D 30 μGy of neutron radiation

5 When in space, astronauts receive a radiation dose of about 1000 μSv per day. The normal annual background dose on Earth is about 2 mSv.

a Roughly how many days does it take for astronauts to exceed the normal background dose?
b In June 2024, Oleg Kononenko, a Russian cosmonaut, set a new record for the most cumulative time spent in space. The record was previously 879 days. Estimate how much radiation in millisieverts (mSv) the former record-holder was exposed to during this time.

6 To treat cancer of the uterus, a radioactive source is implanted directly into the affected region. If the uterus receives a dose of $0.450\,\text{Gy}\,\text{h}^{-1}$ from the source, how many hours should it be left there to deliver a dose of 36.0 Gy?

7 Which of the following is the most appropriate for use as a radioactive tracer so that a brain tumour can be detected by a gamma-ray camera?

A radon-222, α emitter, half-life = 3.82 days
B sulfur-35, β emitter, half-life = 87.4 days
C cobalt-60, γ emitter, half-life = 5.27 years
D technetium-99m, γ emitter, half-life = 6.01 hours

Chapter review

08

KEY TERMS

absorbed dose
activity
alpha particle
antielectron neutrino
antiparticle
artificial transmutation
atomic number
background radiation
baryon
beta particle
conservation laws
conservation of momentum
daughter nucleus
decay series
deuterium
dose equivalent
electromagnetic force
electromagnetic radiation
electron
electron neutrino
electronvolt
electrostatic force
fermion
Feynman diagram
gamma ray
gauge boson
Geiger counter
gluon
hadron
half-life
ionising ability
ionising radiation
isotope
lepton
mass number
meson
neutral
neutron
non-ionising radiation
nuclear transmutation
nucleon
nucleus
nuclide
parent nucleus
particle accelerator
penetrating ability
photon
positron
proton
quality factor
quantum number
quark
radiation
radioactive
radioisotope
spontaneous transmutation
Standard Model
strong nuclear force
tracer
tritium
weak nuclear force

1 Briefly outline two key differences between electrons and quarks.

2 Which of the following correctly describes the strong nuclear force? Explain your choice.

A The strong nuclear force attracts protons and neutrons.

B The strong nuclear force attracts neutrons to protons.

C The strong nuclear force attracts protons and neutrons to each other.

D The strong nuclear force attracts electrons to the nucleus.

3 How many protons and neutrons are present in the $^{45}_{20}Ca$ nuclide?

4 Use the periodic table in Figure 8.1.12 on page 294 to determine the number of protons, neutrons and nucleons in cobalt-60.

5 Determine the nature of the unknown, X, for the following transmutation:

$$^{60m}_{27}Co \rightarrow ^{60}_{27}Co + X$$

(60m means the nuclide is metastable and has a higher level of stability than very short-lived isotopes. The mass number is still 60.)

6 What type of radiation does potassium-48 (atomic number 19) emit? Use Figure 8.1.15 on page 296 to answer this question.

7 Identify each of these radiation types:

a $^{0}_{-1}A$

b $^{1}_{1}B$

c $^{4}_{2}C$

d $^{1}_{0}D$

e $^{0}_{0}E$

f $^{0}_{1}F$

8 Some nuclei can be made unstable by firing neutrons into them. The neutron is captured, and the nucleus becomes unstable. The nuclear equation when the stable isotope boron-10 transmutates by neutron capture into a different element, X, by emitting alpha particles is:

$$^{10}_{5}B + ^{1}_{0}n \rightarrow X + ^{4}_{2}He$$

Identify the unknown element, X, and give its atomic and mass numbers.

9 Identify each of the unknown particles X and Y in the following nuclear transmutations.

a $^{14}_{7}N + ^{4}_{2}\alpha \rightarrow ^{17}_{8}O + X$

b $^{27}_{13}Al + Y \rightarrow ^{27}_{12}Mg + ^{1}_{1}H$

10 Find the values of x and y in each of these radioactive decay equations.

a $^{208}_{81}Ti \rightarrow ^{x}_{y}Pb + ^{0}_{-1}\beta^{-} + ^{0}_{0}\bar{\nu}_{e}$

b $^{180}_{80}Hg \rightarrow ^{x}_{y}Pt + ^{4}_{2}\alpha$

11 Fluorine-18 is a radioisotope that is used for detecting tumours. It is formed when radioactive neon-18 decays by positron emission. Fluorine-18 in turn also decays by positron emission. The equations are as follows:

$$^{18}_{10}\text{Ne} \rightarrow {}^{a}_{b}X + {}^{0}_{+1}\beta^{+} + {}^{0}_{0}\nu_e$$

$$^{a}_{b}X \rightarrow {}^{c}_{d}Y + {}^{0}_{+1}\beta^{+} + {}^{0}_{0}\nu_e$$

Determine the values of *a*, *b*, *c*, *d* and identify *X* and *Y*, which are the resulting daughter nuclei.

12 The radioisotope nitrogen-12 decays by emitting a positron and an electron neutrino. The decay equation for nitrogen-12 is:

$$^{12}_{7}\text{N} \rightarrow X + {}^{0}_{+1}\beta^{+} + {}^{0}_{0}\nu_e$$

Identify *X*.

13 A stable isotope of neon has 10 protons and 10 neutrons in each nucleus. Every proton is repelling all the other protons. Why is the nucleus stable?

14 Determine which type of radiation, out of alpha, beta, and gamma:

a is the fastest

b has the greatest penetrating power.

15 Health workers who deal with radiation to treat cancer must wear a lead vest to protect their vital organs from exposure. Outline the reason for wearing a lead vest.

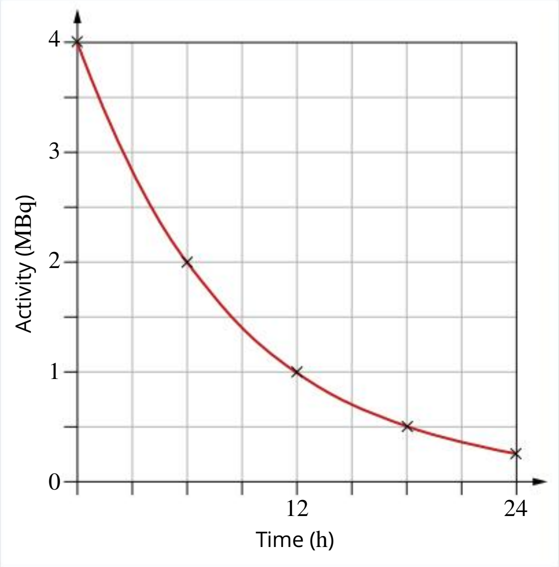
16 A nuclear physicist was bombarding a sample of beryllium-7 with a beam of electrons in an effort to smash the electrons into the beryllium nuclei. Why would it be quite difficult for a collision between the electrons and the nuclei to occur?

17 A radioactive isotope *X* has a half-life of 20.0 minutes. A sample starts with 6.50×10^{14} atoms of the isotope. What amount of the original isotope will remain after the 20.0 minutes?

18 Radioisotope *Y* has a half-life of 3.00 hours. A sample starts with about 5.60×10^{15} atoms of the radioisotope. Estimate how many atoms of *Y* will remain after 9.00 hours.

19 Uranium-235 has a half-life of 700 000 000 years (700 million years), while uranium-238 has a half-life many times longer, of 4.5×10^{9} (4.5 billion) years. In samples of 1.00 kg of each of these pure radioisotopes, which would have the greater activity? Explain your answer.

20 The decay curve for a sample of the radioisotope technetium-99m is shown. It has an initial activity of around 4.0×10^{6} Bq.



a What is the activity of this sample after one half-life?

b Referring to the graph, estimate the half-life of technetium-99m.

c If the sample is produced in a hospital at 4 pm, roughly what is its activity when it is used at 10 am the next day?

21 Protactinium-234 is a radioactive element with a half-life of 70.0 s. If a sample of this radioisotope contains around 6.00×10^{10} nuclei, approximately how many nuclei of this element will remain after 140.0 s?

22 Radiotherapy treatment of brain tumours involves irradiating the target area with radiation from an external source. Why is cobalt-60 (a gamma emitter with a half-life of 5.3 years) generally used as the radiation source for this treatment?

23 An airline pilot of mass 87.5 kg absorbs a gamma radiation dose of 355 mGy during a return flight to New York. Calculate the dose equivalent that has been received, in mSv.

24 In a major incident in a nuclear reactor, a 78.5 kg employee received a full-body absorbed radiation dose of 5.65 Gy. The radiation was gamma rays.

a Calculate the amount of energy that was absorbed during this exposure.

b Calculate the dose equivalent for this person.

25 A worker in an X-ray clinic takes an average of 10 X-ray photographs each working day and receives an annual radiation dose equivalent of 7900 μSv.

a Estimate the dose, in μSv, that the worker receives from each X-ray photograph. (Assume they work for 5 days per week for 45 weeks a year.)

b How many times greater than the normal background radiation dose is the worker's annual dose?

CHAPTER

09 Fission and fusion

This chapter looks at typical nuclear fission and fusion reactions, the forces that act within the nucleus, energy transfer and important transformation phenomena in stars and in the production of nuclear energy. It also examines the benefits and risks of using nuclear power as an energy source for society.

Science as a Human Endeavour

- nuclear power stations employ a variety of safety mechanisms to prevent nuclear accidents, including shielding, cooling systems and radiation monitors
- the management of nuclear waste is based on the knowledge of the behaviour of radiation
- international research teams are developing Generation IV fission reactors and fusion reactors as long-term solutions to meeting the world's energy needs

Science Understanding

- Einstein's mass/energy relationship relates the binding energy of a nucleus to its mass defect, including applying the relationship
  $\Delta E = \Delta mc^2$
- Einstein's mass/energy relationship also applies to all energy changes and enables the energy released in nuclear reactions to be determined from the mass change in the reaction, including applying the relationship
  $\Delta E = \Delta mc^2$
- alpha and beta decay are examples of spontaneous transmutation reactions, while artificial transmutation is a managed process that changes one nuclide into another
- neutron-induced nuclear fission is a reaction in which a heavy nuclide captures a neutron and then splits into smaller radioactive nuclides with the release of energy
- a fission chain reaction is a self-sustaining process that may be controlled to produce thermal energy, or uncontrolled to release energy explosively if its critical mass is exceeded
- nuclear fusion is a reaction in which light nuclides combine to form a heavier nuclide, with the release of energy
- more energy is released per nucleon in nuclear fusion than in nuclear fission because a greater percentage of the mass is transformed into energy
- the early universe was composed mostly of hydrogen, and all other elements that are present in today's universe, including the human body, were created via nuclear processes inside stars and during the destruction of stars

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.

9.1 Nuclear fission and energy

FIGURE 9.1.1 An atomic bomb explosion and its associated mushroom cloud.

In 1905, Albert Einstein theorised that mass, m, and energy, E, are equivalent through the equation $E = mc^2$. This led to the realisation that vast amounts of energy lie unharnessed within the nuclei of atoms. The ramifications of Einstein's work and the discovery of nuclear fission were realised in 1945 with the explosion of the first atomic bomb in the desert near Alamogordo in New Mexico, USA (Figure 9.1.1). In this section, nuclear fission and the energy that it can unleash will be explored.

INSIDE THE NUCLEUS

The current understanding of the basic properties and structure of the nucleus is the result of intense scientific investigation in the early part of the twentieth century. Physicists such as Becquerel, Rutherford, Chadwick, Geiger, Marsden and Harkins were instrumental in the development of the model of the nucleus that exists today. These renowned scientists are shown in Figure 9.1.2.

FIGURE 9.1.2 (a) Henri Becquerel, (b) Ernest Rutherford, (c) James Chadwick, (d) Hans Geiger, (e) Ernest Marsden and (f) William Harkins.

Recall from Chapter 8 that there is a strong nuclear force acting within the nucleus to overcome the electrostatic repulsion between protons. This force holds the nucleons, which are composed of quarks, together through the exchange of gluons.

An example of how the electrostatic and strong nuclear forces act in a nucleus is shown in Figure 9.1.3. In this example, proton A both attracts and repels proton B, but, at short distances, the attraction due to the strong nuclear force is much greater than the repulsion due to the electrostatic force. Proton A also both attracts and repels proton C, but because of the greater distance between them, the force of repulsion is larger than the force of attraction. However, proton A and proton C do not fly apart because of the strong attractive forces exerted on them by adjacent nucleons.

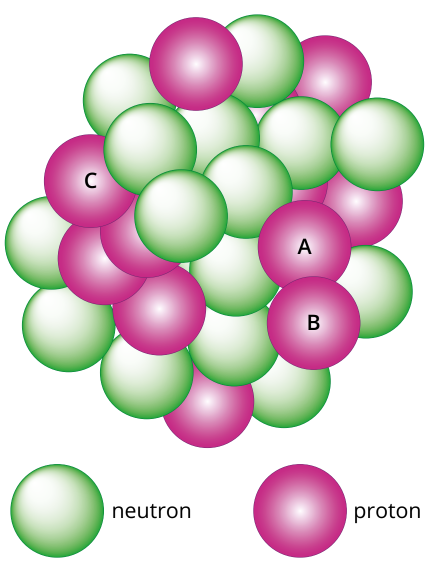


FIGURE 9.1.3 The interaction between the electrostatic and strong nuclear forces acting in a nucleus.

NUCLEAR FISSION

The discovery of the neutron by James Chadwick in 1932 enabled scientists to explore the behaviour of larger atomic nuclei. Up until then, physicists such as Enrico Fermi had been firing alpha particles at target nuclei and analysing the results. Chadwick found that with larger target nuclei, the positive alpha particles were too strongly repelled from the positively charged nuclei and collisions did not occur.

The advantage of firing a neutron in such experiments is that it is neutral and so it is not repelled by any target nucleus. The bombarding neutron can be absorbed into the nucleus of the target atom, as shown in Figure 9.1.4. This makes neutrons a very useful form of radiation for experimentation, as they can artificially transmutate target isotopes into different isotopes.

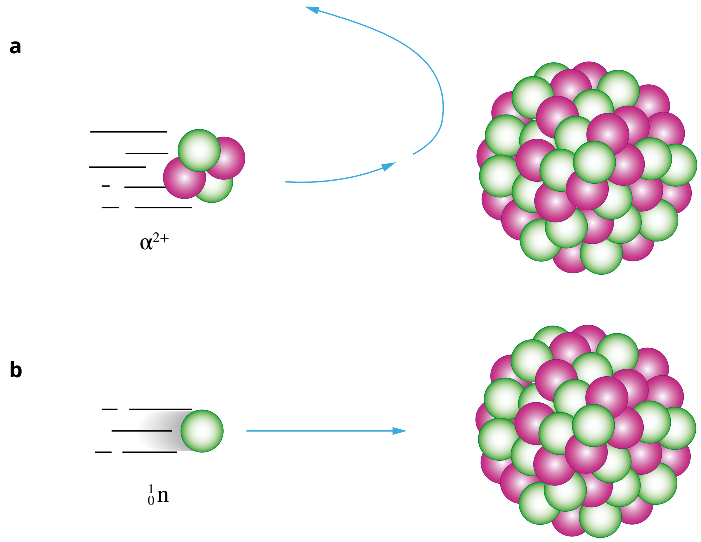


FIGURE 9.1.4 (a) Charged α-particles are repelled by a nucleus. (b) Uncharged neutrons are able to smash into a nucleus and be absorbed.

Nuclear **fission** occurs when an atomic nucleus splits into two or more pieces. This is usually triggered or induced by the absorption of a neutron, as shown in Figure 9.1.5. Nuclides that are capable of undergoing nuclear fission after absorbing a neutron are said to be **fissile**. Fissile nuclides are all elements with high atomic numbers, very few of which exist in nature.

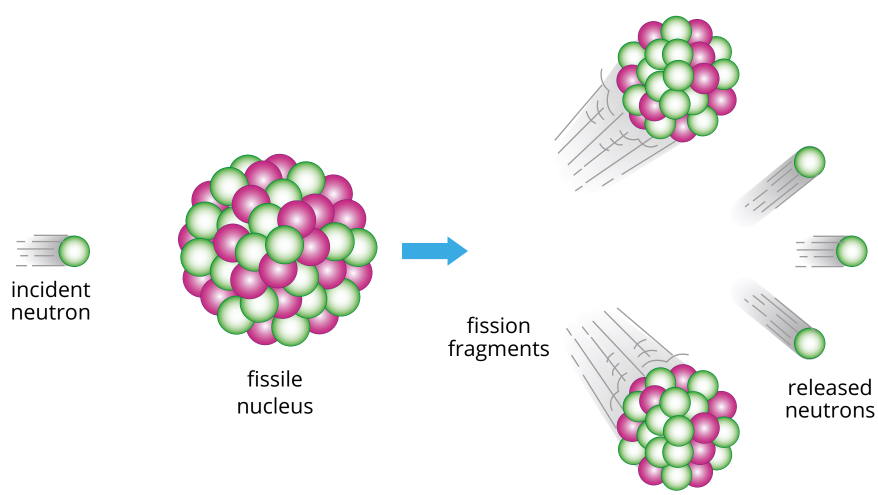


FIGURE 9.1.5 Nuclear fission is the splitting of a nucleus after a bombarding neutron has been absorbed.

PHYSICSFILE

Strong nuclear force

The existence of the strong nuclear force was first proposed by Japanese theoretical physicist Hideki Yukawa in 1935. However, the properties of this force are so complex that it took until 1975 for physicists to develop a mathematical model that could successfully describe it.

Uranium-235 and plutonium-239 are fissile and can be made to split when bombarded by a slow-moving neutron (around $2.2\,km\,s^{-1}$). Uranium-238 and thorium-232 require a very high-energy neutron (around 1 MeV with a speed faster than $14\,000\,km\,s^{-1}$) to induce fission, and so they are regarded as fissionable, but non-fissile.

Uranium is one of the heaviest naturally occurring elements; it has several isotopes and is found in most rocks. It is believed to have formed in supernovas around 6.6 billion years ago. Uranium's isotopes have very long half-lives and the energy generated from their radioactive decay is considered to be the main source of heat in the Earth's core, which results in convection currents and continental drift.

RELEASE OF NEUTRONS DURING FISSION

Uranium-235 and plutonium-239 are the fissile nuclides most commonly used in nuclear reactors and nuclear weapons. When a uranium-235 or a plutonium-239 nucleus absorbs either a slow-moving or fast-moving neutron, it becomes unstable and spontaneously undergoes fission. However, fission is more likely to be induced by a slow-moving neutron because it is more easily captured by the target nucleus.

When a uranium-235 nucleus undergoes fission, it splits into two smaller nuclei and releases a number of neutrons. Figure 9.1.6 shows one possible outcome for the nuclei produced during the fission of uranium-235, but many others can occur due to the random nature of the nucleus split. In this example three neutrons are released, along with a krypton-91 and a barium-142 nuclide. Typically, either two or three neutrons are released in each fission reaction. For uranium-235, an average of 2.47 neutrons per fission has been calculated.

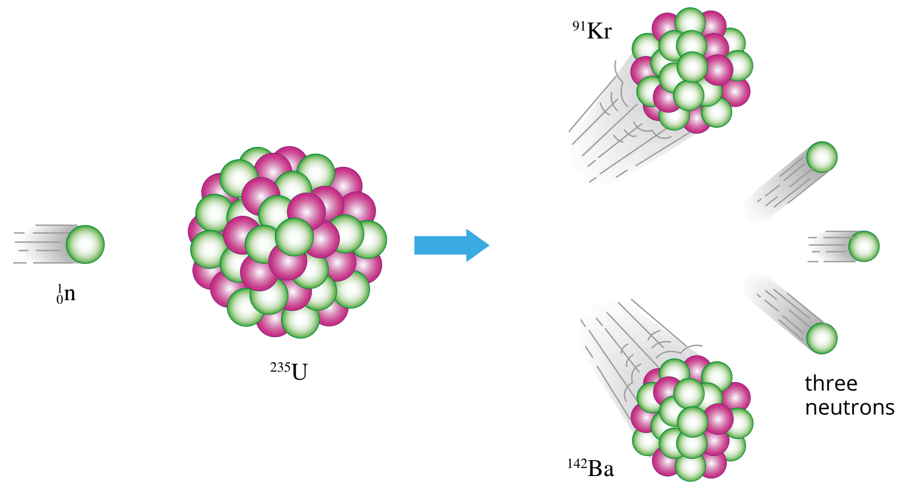


FIGURE 9.1.6 One possible outcome for the neutron-induced fission of uranium-235.

The equation for this reaction is:

$$^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{91}_{36}Kr + {}^{142}_{56}Ba + 3\,{}^{1}_{0}n + \text{energy}$$

Krypton-91 and barium-142 are known as **fission fragments** or **daughter nuclei**. Three neutrons are also freed from this uranium nucleus when it splits. Note that in the same way as for radioactive decay, both the atomic number, Z, and mass number, A, are conserved in these nuclear reactions. For the reaction equation shown, the atomic numbers on either side of the arrows add up to 92 and the mass numbers add up to 236.

Plutonium-239 will also undergo fission in a variety of ways. It releases an average of 2.89 neutrons per fission, slightly more than uranium-235, but it does not undergo fission as easily.

Many of the products of the nuclear fission process are themselves radioactive and form a lethal cocktail of decaying isotopes. It is these radioactive fission fragments that comprise the bulk of the high-level waste produced by nuclear reactors. The management of the waste depends on the type of radiation emitted as the products decay and is discussed in Section 9.2.

MEASURING ENERGY IN ELECTRONVOLTS AND MASS IN DALTONS

Atomic nuclei, subatomic particles and radioactive emissions have such small amounts of energy that the joule, the SI unit typically used to measure energy, is inappropriate for them. As mentioned in Section 8.2, the mass of subatomic particles is often stated in megaelectronvolts or MeV, which is their rest mass–energy equivalent. This unit is far better suited to expressing the very small amounts of energy involved in this area of physics.

An **electronvolt** is the energy that an electron would gain if it were accelerated by a voltage of 1 volt and is equal to 1.60×10^{-19} J.

To convert from eV to joules: multiply by 1.60×10^{-19} J.

To convert from joules to eV: divide by 1.60×10^{-19} J.

In nuclear physics, energies are typically measured in millions of electronvolts, otherwise known as megaelectronvolts (MeV).

The mass of subatomic particles is also incredibly small, so the unit of kilograms is likewise seldom used. Instead, mass is determined relative to the **atomic mass constant** (m_u), which has the unit dalton (Da), so the m_u is 1 Da = 1.66×10^{-27} kg. The energy equivalent of the mass is then calculated by $\Delta E = \Delta m \times 931$ MeV.

The atomic mass constant (m_u) is defined as 1/12 of the mass of an unbound neutral atom of carbon-12, in its nuclear and electronic ground state, and at rest. The unit of the atomic mass constant is the dalton (Da).

The mass of 1 Da = 1.66×10^{-27} kg.

To convert from kg to daltons: divide by 1.66×10^{-27} kg.

The mass–energy equivalence equation converts mass in Da to energy in MeV using: $\Delta E = \Delta m \times 931$.

ENERGY RELEASED DURING NUCLEAR REACTIONS

It is well established that the mass of any nucleus is always less than the mass of its individual nucleons. Two separate protons and two separate neutrons will have slightly more total mass than a helium nucleus.

Albert Einstein, pictured in Figure 9.1.7, provided the explanation of the origins of this missing mass. He showed that *mass* and *energy* were not completely independent quantities. Indeed, mass can be converted into energy and energy can be converted into mass.

If you wanted to separate a helium nucleus into four free nucleons, you would need to add energy to overcome the **binding energy** of the nucleus (discussed further in Section 9.3). The free nucleons will have more energy and so, according to Einstein, will have greater mass. The difference between the total mass of the individual nucleons and the actual mass of the nucleus is known as the **mass defect**, Δm.

The energy released as a result of a mass defect (mass decrease) is given by Einstein's famous equation:

$\Delta E = \Delta mc^2$

where ΔE is energy (J)

Δm is the mass defect (the decrease in mass, in kg)

c is the speed of light = 3.00×10^8 m s^{-1}

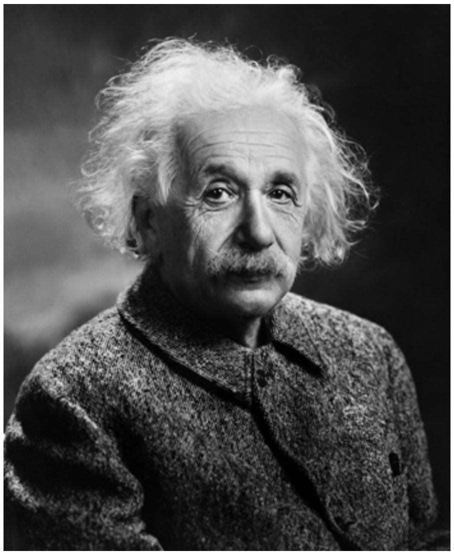


FIGURE 9.1.7 Albert Einstein.

The chemical reactions that you have probably performed at school typically release only a few electronvolts of energy. Compared with this, an enormous amount of energy is released during nuclear reactions. This has made nuclear energy a major focus of scientific research over the past century.

As mentioned in Section 8.2, radioactive decay can release energy measured in megaelectronvolts (MeV). For instance, alpha particle decay typically releases between 5 and 10 MeV of energy. Nuclear fission releases significantly much more energy, generally around 200 MeV. This energy is mainly in the form of the kinetic energy of the fission fragments and neutrons, as well as the emission of gamma radiation.

During any fission reaction, the combined mass of the incident neutron and the target nucleus is always slightly greater than the combined mass of the fission fragments and the released neutrons. For example, as shown in Figure 9.1.8 below, the mass of the incident neutron and the uranium-235 nucleus is greater than the combined masses of the fission products; that is, the barium-142 and krypton-91 nuclides, and the three neutrons. This missing mass is converted into energy according to the equation $\Delta E = \Delta mc^2$. In this case, around 3.2×10^{-11} J of energy is released.

Only a very small proportion of the original mass of the nuclei is converted into usable energy; typically around 0.1%. For example, if you started with a 1.000 kg block of pure uranium-235, where every isotope underwent fission completely, around 0.999 kg would remain unconverted. This is due to the small amount of mass converted into energy during the fission process.

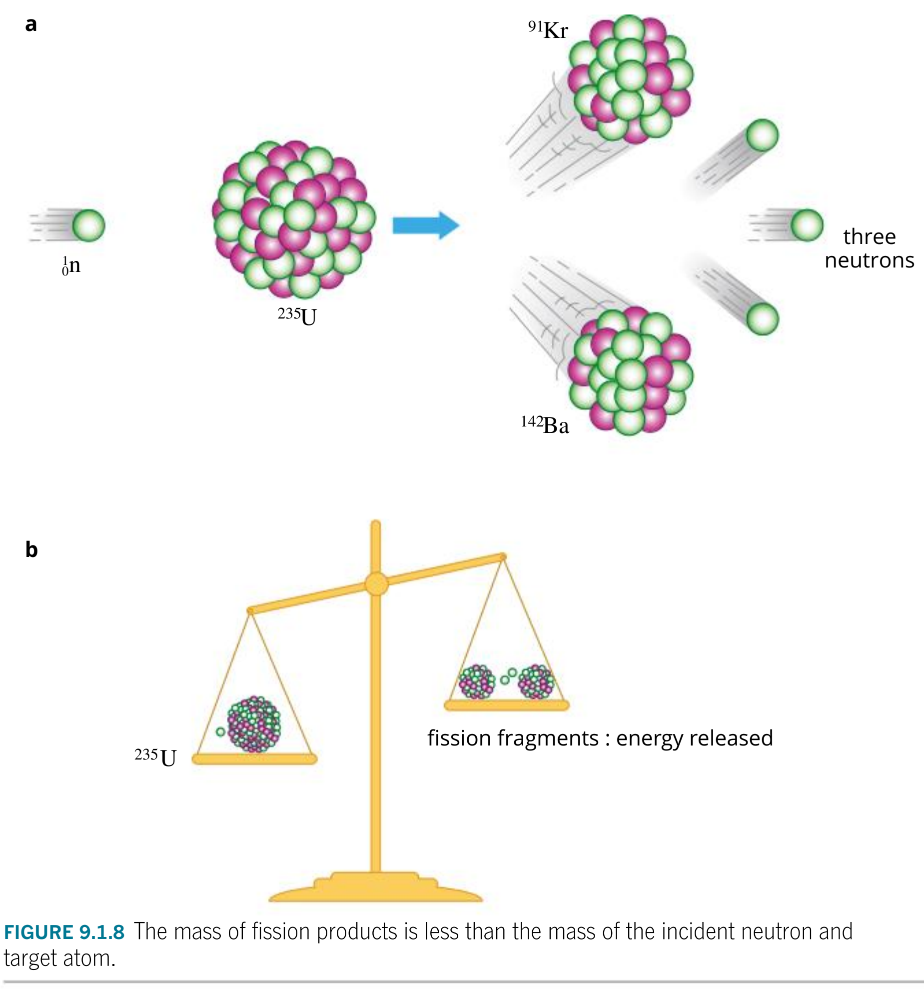


FIGURE 9.1.8 The mass of fission products is less than the mass of the incident neutron and target atom.

Worked example 9.1.1

FISSION USING EINSTEIN'S EQUATION, KILOGRAMS AND JOULES

Plutonium-239 is a fissile material. When a plutonium-239 nucleus is struck by and absorbs a neutron, it can split in many different ways. Consider the example of a nucleus that splits into barium-145 and strontium-93 and releases some neutrons.

The nuclear equation for this is:

$$^{1}_{0}\text{n} + ^{239}_{94}\text{Pu} \rightarrow ^{145}_{56}\text{Ba} + ^{93}_{38}\text{Sr} + a^{1}_{0}\text{n} + \text{energy}$$

a How many neutrons are released during this fission process, i.e. what is the value of *a*?	
Thinking	**Working**
Analyse the mass numbers (A).	Reactants: $(1) + (239) = 240$ Products: $(145) + (93) + (a) = 238 + (a)$ $a = (240) - (238)$ $a = 2$ 2 neutrons are released during this fission.

b During this single fission reaction, there is a loss of mass (a mass defect) of 3.06×10^{-28} kg. Use Einstein's famous equation to calculate the amount of energy that is released during the fission of a single plutonium-239 nucleus. Answer in both MeV and joules.	
Thinking	**Working**
The energy released during the fission of this plutonium nucleus can be found by using $\Delta E = \Delta mc^2$.	$\Delta E = \Delta mc^2$ $\Delta E = (3.06 \times 10^{-28})(3.00 \times 10^{8})^2$ $\Delta E = 2.75400 \times 10^{-11}$ $\Delta E = 2.75 \times 10^{-11}$ J
To convert J into eV, divide by 1.60×10^{-19}. Remember that 1 MeV = 10^6 eV.	$\Delta E = \frac{(2.75400 \times 10^{-11})}{1.60 \times 10^{-19}}$ $\Delta E = 1.72125 \times 10^{8}$ $\Delta E = 172.125 \times 10^{6}$ eV $\Delta E = 172$ MeV

c The combined mass of the plutonium nucleus and bombarding neutron is 3.99×10^{-25} kg. What percentage of this initial mass is converted into the energy produced during the fission process?	
Thinking	**Working**
Use the relationship: percentage of initial mass converted into energy (%m): $\%m = \frac{\text{mass defect}}{\text{initial mass}} \times \frac{100}{1}$	Percentage of initial mass converted into energy: $\%m = \frac{\text{mass defect}}{\text{initial mass}} \times \frac{100}{1}$ $\%m = \frac{(3.06 \times 10^{-28})}{(3.99 \times 10^{-25})} \times 100$ $\%m = 0.0766917$ $\%m = 0.0767\%$ This is a very small percentage loss in mass.

Worked example: Try yourself 9.1.1

FISSION USING EINSTEIN'S EQUATION, KILOGRAMS AND JOULES

Plutonium-239 is a fissile material. When a plutonium-239 nucleus is struck by and absorbs a neutron, it can split in many different ways. Consider the example of a nucleus that splits into lanthanum-143 and rubidium-94 and releases some neutrons.

The nuclear equation for this is:

$$ {}^{1}_{0}\text{n} + {}^{239}_{94}\text{Pu} \rightarrow {}^{143}_{57}\text{La} + {}^{94}_{37}\text{Rb} + a\,{}^{1}_{0}\text{n} + \text{energy} $$

a How many neutrons are released during this fission process, i.e. what is the value of *a*?

b During this single fission reaction, there is a loss of mass (a mass defect) of 4.58×10^{-28} kg. Use Einstein's famous equation to calculate the amount of energy that is released during fission of a single plutonium-239 nucleus. Give your answer in both MeV and joules.

c The combined mass of the plutonium nucleus and bombarding neutron is 2.86×10^{-25} kg. What percentage of this initial mass is converted into the energy produced during the fission process?

While it is important to recognise Einstein's revolutionary equation, the reality of modern nuclear physics is that it is rarely used. The unit of mass preferred by nuclear physicists is the dalton ($1\,\text{Da} = 1.66 \times 10^{-27}$ kg), and the preferred unit of energy is the MeV ($1\,\text{MeV} = 10^6$ electronvolts). The equation for the mass–energy equivalence used most frequently is therefore:

> $\Delta E = \Delta m \times 931$
>
> where ΔE is energy (MeV)
>
> Δm is the mass defect (the decrease in mass in Da)
>
> 931 is the conversion factor.

In this textbook, the masses for nuclei and atomic particles will mostly be presented in Da, and energies will mostly be in MeV. Mass–energy conversions, in which a mass defect is converted into energy, will also use the equation shown above. One advantage of presenting masses in Da is that the value is very similar to the mass number (protons + neutrons) of the nucleus. Using MeV as the unit of energy allows the values can be written without scientific notation, unlike the equivalent values in joules.

Worked example 9.1.2

FISSION USING MASS–ENERGY EQUIVALENCE, DALTONS AND MeV

Consider the fission reaction from Worked example 9.1.1, in which a mass defect equivalent to 0.184 939 76 Da is converted into energy.

Use the equation of mass–energy equivalence to calculate the amount of energy that is released during the fission of a single plutonium-239 nucleus in MeV and in joules.

Thinking	Working
The energy released during the fission of this plutonium nucleus can be found by using $\Delta E = \Delta m \times 931$.	$\Delta E = \Delta m \times 931$ $\Delta E = (0.18493976) \times 931$ $\Delta E = 172.17892$ $\Delta E = 172$ MeV
Convert MeV into eV, then multiply by 1.60×10^{-19}. Remember that 1 MeV = 10^6 eV.	$\Delta E = 172.17892 \times 10^6$ eV $\Delta E = 172.17892 \times 10^6 (1.60 \times 10^{-19})$ $\Delta E = 2.7548627 \times 10^{-11}$ $\Delta E = 2.75 \times 10^{-11}$ J

Worked example: Try yourself 9.1.2

FISSION USING MASS-ENERGY EQUIVALENCE, DALTONS AND MeV

Consider the fission reaction from Worked example 9.1.1, in which a mass defect equivalent to 0.275 903 6 Da is converted to energy.

Use the equation of mass–energy equivalence to calculate the amount of energy that is released during the fission of a single plutonium-239 nucleus in MeV and in joules.

PHYSICS IN ACTION

Enrico Fermi, Lise Meitner and nuclear fission

Enrico Fermi, pictured in Figure 9.1.9, was born in Italy in 1901. He completed his doctorate and post-doctorate work in Physics at the University of Pisa and in Germany. Fermi had emigrated to the USA by the time the nuclear age dawned in the 1930s. The neutron had just been discovered in 1932, which enabled scientists to fire neutral particles at atomic nuclei for the first time. Fermi was at the forefront of this research.

Fermi bombarded uranium-238 atoms with neutrons and found that uranium-238 nuclei absorbed the neutrons and formed a radioactive isotope of uranium. This isotope then decayed by emitting a beta-minus particle to become neptunium, which then emitted another beta-minus particle to become plutonium, two completely undiscovered elements. Fermi had successfully produced the world's first artificial and transuranic (i.e. after uranium) elements. The nuclear reactions for this process are:

$$^{1}_{0}n + ^{238}_{92}U \rightarrow ^{239}_{92}U$$

$$^{239}_{92}U \rightarrow ^{239}_{93}Np + ^{0}_{-1}\beta^{-} + ^{0}_{0}\bar{\nu}_e$$

$$^{239}_{93}Np \rightarrow ^{239}_{94}Pu + ^{0}_{-1}\beta^{-} + ^{0}_{0}\bar{\nu}_e$$

In 1938, following on from Fermi's work, two German scientists, Otto Hahn and Fritz Strassmann, were also bombarding uranium ($Z = 92$) in an attempt to produce some transuranic elements ($Z > 92$). They found that, rather than producing larger elements, they were getting isotopes of barium ($Z = 56$). Hahn wrote to his colleague Lise Meitner, pictured in Figure 9.1.10, about this unexpected result. She then discussed this with her nephew Otto Frisch, a nuclear physicist, and realised that the bombarding neutrons were causing the uranium nuclei to split. If barium ($Z = 56$) was one of the products, then krypton ($Z = 36$) must be another. This was found to be the case. It was Frisch who coined the term 'fission' and Meitner who proposed that energy would be released during this process.

After the start of World War II, Enrico Fermi was commissioned by President Roosevelt to design and build a device that would sustain the fission process in the form of a chain reaction. In 1942, Fermi succeeded in this task. A squash court at the University of Chicago was used as the site for the world's first nuclear reactor. It produced less than 1 W of power—not even enough to power a small light globe! This sounds like a bit of a failure, but in fact, achieving fission for the first time was a very important breakthrough. The reactor was later modified to produce about 200 W. Fermi died of cancer in 1954. One year after his death, the element with atomic number 100 was artificially produced and named fermium, Fm, in his honour.

FIGURE 9.1.9 Enrico Fermi.

FIGURE 9.1.10 Lise Meitner.

9.1 Review

SUMMARY

- Within a nucleus, forces of attraction and repulsion are acting. The long-distance electrostatic force of repulsion acts between the protons. A short-distance strong nuclear force of attraction acts between every nucleon.
- Nuclear fission can be induced by striking a fissile nucleus with a neutron to produce two smaller daughter fragments and two or three neutrons. A relatively large amount of energy is released during this process.
- When fission occurs, the mass of the fission fragments is always less than the mass of the original particles. This decrease in mass is called the mass defect (Δm).
- When the mass defect is given in kilograms (kg), the energy released is given by Einstein's equation, $\Delta E = \Delta mc^2$, where c is the speed of light ($3.00 \times 10^8\,\text{m s}^{-1}$) and the energy is measure in joules (J).
- When the mass defect is given in daltons (Da), the mass–energy equivalence can be given by $\Delta E = \Delta m \times 931$, where the energy is measured in megaelectronvolts (MeV).

KEY QUESTIONS

1 Describe the strong nuclear force and what it acts on.

2 Why is the waste produced by nuclear fission reactors considered dangerous?

3 Consider one particular proton in the nucleus of a gold atom. Describe the forces that the proton experiences from other nucleons.

4 Convert 5.85 MeV into joules.

5 Convert 6.37×10^{-15} J into eV.

6 Which of the nuclides below are fissile and which are non-fissile?

cobalt-60, uranium-235, uranium-238, plutonium-239

7 Determine the number of neutrons (x) released in this fission reaction:

$$^{1}_{0}\text{n} + ^{235}_{92}\text{U} \rightarrow ^{148}_{57}\text{La} + ^{85}_{35}\text{Br} + x^{1}_{0}\text{n}$$

8 The mass defect during the process in question 7 is 0.127 7108 Da.

a Calculate the energy (in MeV) released per uranium-235 atom in this fission process.

b Express this energy in joules.

9 Einstein said that mass and energy are equivalent. In one particular nuclear reaction, there is a decrease in mass of 3.48×10^{-28} kg. Use Einstein's equation to calculate the energy this represents in both joules and electronvolts.

10 During the radioactive decay of an excited cobalt-60 nucleus to a stable cobalt-60 nucleus, a gamma ray of 1.33 MeV is released. Calculate the mass defect in Da and in kg for this gamma decay.

11 Determine the value of the unknown mass number, x, and atomic number, y, in the fission reaction below.

$$^{1}_{0}\text{n} + ^{x}_{94}\text{Pu} \rightarrow ^{130}_{54}\text{Xe} + ^{106}_{y}\text{Xr} + 4^{1}_{0}\text{n}$$

9.2 Chain reactions and nuclear reactors

In the years following the discovery of nuclear fission, intensive scientific and military research was conducted to explore and harness fission energy. In this section, you will learn about the work scientists undertook to go from the fission of a single nucleus releasing a few MeV to a chain reaction that would release vast amounts of energy. You will investigate the use of fission reactions in both the creation of destructive nuclear weapons and the development of nuclear reactors for electricity generation. This will include various fuels used for the fission reactions and the methods for storing radioactive waste products.

CHAIN REACTIONS AND CRITICAL MASS

The concept of chain reactions is fundamental to understanding how nuclear fission can lead to both powerful explosions and controlled energy production.

Chain reactions

Nuclear fission weapons, developed in World War II, are capable of causing death and devastation on a massive scale. This was tragically demonstrated by the bombing of Hiroshima and Nagasaki in 1945.

During World War II, the United States and Great Britain established a massive top-secret research project, named the Manhattan Project, to design and build the first atomic bombs. The scientists working on this project knew that nuclear energy could be released from a single fissile nucleus, but the problem they faced was how to obtain this energy from a vast number of fissile nuclei and hence create an explosive device.

When uranium-235 undergoes fission, it releases two or three neutrons each time. Each of these neutrons is then able to cause fission in another uranium-235 nucleus, which in turn will also release another two or three neutrons. Within a very short time, the number of released neutrons and fission reactions escalates in a process known as a **chain reaction**.

Figure 9.2.1 shows two neutrons being released during the fission reaction. Each of these neutrons in turn causes fission, releasing yet more neutrons. The number of nuclei undergoing fission doubles with each generation, and after five nuclear generations, 16 neutrons are produced. Within a small fraction of a second an enormous number of nuclei have undergone fission. Only a miniscule amount of energy (of the order of 10^{-13} J) is released by each fission reaction, but in this uncontrolled chain reaction there are so many fission reactions occurring in such a short time that a massive explosion results. In just 1 kg of uranium-235, so many reactions occur that about 8×10^{13} J of energy is released in just over one-millionth of a second.

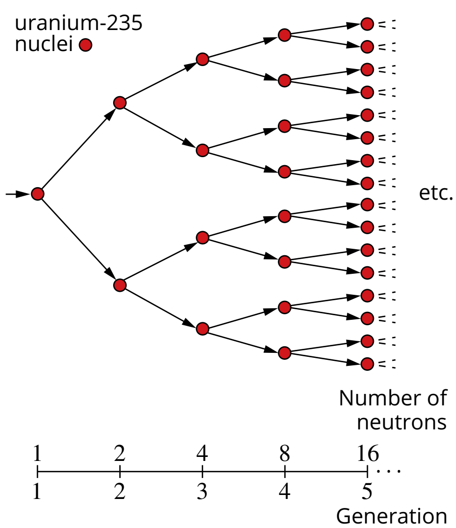


FIGURE 9.2.1 An uncontrolled chain reaction of nuclei.

Nuclear fuel

The Earth contains many naturally occurring radioisotopes. In the 4.5 billion years since the Earth was formed these have been decaying to form more stable isotopes. Uranium is one such element. Its two most common isotopes are uranium-238 and uranium-235. These two isotopes have half-lives of 4.50 billion years and 703 million years, respectively, and so uranium-235 has been decaying at a faster rate than uranium-238. This means that far less uranium-235 remains in the Earth's crust, so that the uranium that is mined from the ground today consists of:

- 99.3% uranium-238—the non-fissile isotope
- 0.7% uranium-235—the readily fissile isotope.

This means that a chain reaction cannot occur in a sample of uranium taken from the ground because the proportion of fissile uranium-235 is far too low. To be useful as a nuclear fuel, the ore has to be enriched. This involves increasing the proportion of uranium-235 relative to uranium-238, a very difficult and expensive process. The slightly different masses of these two isotopes enables them to be separated. The three common enrichment methods are the ultracentrifuge, and electromagnetic and gaseous diffusion separation techniques.

Nuclear weapons require fissile material that has been enriched to over 90% purity. The bomb that was dropped over Hiroshima contained 40 kg of 95% pure uranium-235. Nuclear reactors require fissile material enriched to only about 4% uranium-235.

Critical mass

In their efforts to produce an explosion, the scientists working on the Manhattan Project had to establish a nuclear chain reaction in a sample of nuclear fuel. They found that the explosive ability of a sample of fissile material depended on its purity, shape and size.

In a sample of nuclear material where the concentration of uranium-235 or plutonium-239 is too low, a chain reaction cannot be established. This is because the neutrons have only a small chance of being absorbed by fissile nuclei and causing a further fission reaction. The chain reaction will die out. The fuel used in nuclear fission weapons is enriched to a high degree of purity so that a chain reaction can be easily sustained.

The shape of the nuclear fuel is an important factor in its explosive ability. A 20 kg sample of enriched uranium-235 in the shape of a sphere will spontaneously explode, whereas 20 kg of uranium-235 flattened into a sheet will not. The flat piece has a very large surface area and so an enormous number of neutrons are able to escape from the uranium into the air. As these neutrons do not cause further fission reactions, the chain reaction will die out. In the spherical piece of uranium, however, the surface area is much smaller and a greater proportion of neutrons remain in the uranium to cause more fission reactions, sustaining the chain reaction. Figure 9.2.2 shows how a greater proportion of neutrons are retained within a spherical shape than in a flat shape.

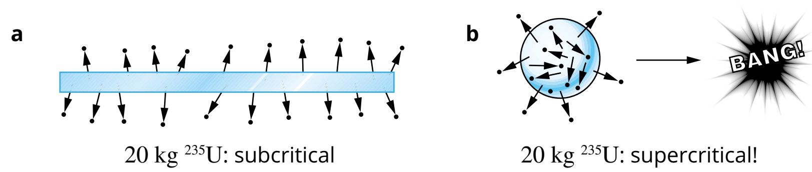


FIGURE 9.2.2 (a) A large proportion of neutrons escape the flat piece of uranium. (b) A sufficient proportion of neutrons remain to maintain the chain reaction—leading to an explosion.

The minimum amount of enriched fissile material required to sustain a continuous chain reaction, when arranged in the shape of a sphere, is known as the **critical mass.**

The explosive ability of a fissile material also depends on its physical size. For example, a piece of uranium-235 the size of a marble will not explode, but a piece the size of a grapefruit most definitely will. The marble-sized piece has a higher surface-area-to-volume ratio, resulting in a greater proportion of neutrons escaping into the air and causing the chain reaction to die out. This is a **subcritical mass**. In contrast, the larger piece has a lower surface-area-to-volume ratio, allowing more neutrons to be retained and sustain the chain reaction within the material. This is a **supercritical mass**, which can lead to a nuclear explosion.

NUCLEAR REACTORS

Since the 1950s, it has been possible to control nuclear fission within a nuclear reactor to produce electrical power for domestic use. Currently, over 440 nuclear power plants across more than 30 countries are in operation, producing approximately 10% of the world's electricity. France leads in this field, with 69.0% of its total electricity coming from 58 nuclear reactors. Although Australia has significant uranium resources, it primarily exports them due to a legislated ban on nuclear power; instead, Australia favours renewable energy sources as solar and wind. Nonetheless, Australia operates one Open Pool Australian Lightwater (OPAL) reactor in Lucas Heights, Sydney, dedicated to producing radioisotopes for medical applications.

A nuclear power plant generates electricity in much the same way as a coal-burning power plant. The primary difference is how the heat is produced and collected. Most power stations in Australia generate electricity by burning coal, oil or gas to produce heat that creates the steam used to turn the generator turbines. A nuclear power station simply has a different way of producing heat, by nuclear fission.

Thermal nuclear reactor

A thermal nuclear reactor generates energy through the fission of uranium-235, the isotope that is most likely to undergo fission when it is hit by slow moving, or thermal, neutrons.

As they tried to harness the energy from the nuclear fission reactions, nuclear reactor designers had to overcome three major difficulties. The first was that neutrons released from a uranium-235 nucleus undergoing fission are travelling at very high speeds of around 20 000 $km\,s^{-1}$. Uranium-235 is most fissile when irradiated by slow-moving neutrons. Thus, these emitted neutrons needed to be slowed down. Second, the fission of each uranium-235 nucleus releases an average of 2.47 neutrons. This can lead to a chain reaction that will result in an explosion. A way had to be found to absorb some of these emitted neutrons and maintain a steady chain reaction. Third, the heat generated in the reactor by the fission process had to be somehow collected and used to create steam to drive the turbine and generate electricity.

There are many different varieties of thermal nuclear reactors, but they all include the following design elements:

- fuel rods—long, thin rods containing pellets of enriched uranium
- a moderator—a material that slows the neutrons
- control rods—material that absorbs neutrons
- a coolant—a liquid to absorb heat energy that has been produced by nuclear fission
- radiation shield—a thick concrete wall that prevents neutrons escaping from the reactor.

Nuclear fuel rods

Uranium-235 is used as nuclear fuel as it is readily fissile with slow-moving neutrons. However, this isotope comprises only 0.7% of naturally occurring uranium, which is not enough to sustain a chain reaction. The predominant (99.3%) isotope, uranium-238, is effectively non-fissile. Therefore, the proportion of uranium-235 in the ore has to be increased. That is, the uranium ore has to be enriched by increasing the uranium-235 content before it can be used as reactor-grade fuel. Uranium enriched for use in a thermal nuclear reactor contains around 96% uranium-238 and 4% uranium-235.

The proportion of uranium-235 is increased to around 4% for fuel rods. The enriched uranium, in pellet form, is then packed into a thin aluminium tube, known as a **fuel rod**. This is usually 3 to 5 m long. A large nuclear reactor has over 1000 fuel rods in its core. A fuel rod will eventually become depleted of uranium-235. This means that, over time, the concentration of uranium-235 falls to a level where it cannot sustain the fission chain reaction. Each fuel rod needs to be replaced every 4 years or so, and a typical 1000 MW reactor produces around 25 tonnes of spent fuel each year.

While uranium-238 is not readily fissile, it is classified as 'fertile' because a uranium-238 nucleus can capture a fast neutron from a uranium-235 disintegration and form plutonium-239, which is itself fissile. This reaction yields a similar amount of energy per fission as does uranium-235, and releases sufficient neutrons to sustain a chain reaction.

The initial neutrons given off by the uranium-235 have therefore been able to sustain their own chain reaction while at the same time 'breeding' plutonium-239 to be further used as a fuel. These reactors can be configured to produce more fissionable fuel than they use and are referred to as **fast breeder reactors**.

PHYSICSFILE

Australia's nuclear reactor

Australia's only nuclear reactor is located at Lucas Heights, a suburb of Sydney. The HIFAR (High Flux Australian Reactor) operated here from 1958 to 2007 (Figure 9.2.3) but has now been replaced by a new research reactor known as the OPAL reactor (Figure 9.2.4). About 7 kilograms of uranium enriched to 20% uranium-235 is immersed in a pool of water almost 13 metres deep. Radioisotopes that are used in industry and in the nuclear medicine departments of Australia's hospitals are synthesised here. Another important process that is carried out at Lucas Heights is the irradiation of silicon chips, which creates high-conductivity silicon for the computer industry.

FIGURE 9.2.3 The original housing of the HIFAR nuclear research reactor at Lucas Heights, Sydney decommissioned in 2007.

FIGURE 9.2.4 Inside the nuclear research reactor at Lucas Heights, Sydney.

Moderators

In a nuclear reactor, the problem of fast-moving neutrons is overcome by including a material that slows down, or moderates, the speed of the free neutrons. This is known as a **moderator**. It has been found that substances whose nuclei are small will slow the neutrons down to speeds at which they can be captured by a fissile nucleus. When the emitted neutrons collide with these small nuclei, they lose most of their kinetic energy and so slow down. After many collisions, the neutrons have been slowed down to about $2\,km\,s^{-1}$ and have less than 1 eV of energy.

Some materials that are commonly used as moderators are:

- graphite, consisting of carbon atoms
- normal water, H_2O
- heavy water, containing at least one atom of deuterium in place of hydrogen
- carbon dioxide, CO_2.

Each of these materials works well as a moderator because it slows the neutrons without absorbing a significant number of them. **Heavy water** is the most effective moderator, but is also the most expensive. Water is the cheapest material, but absorbs more neutrons than the others and so reduces the extent of the chain reaction. Graphite is less effective than water because carbon nuclei are heavier than hydrogen nuclei. To lose the same amount of energy, the emitted neutrons have to collide with about 120 carbon nuclei, but only about 25 water molecules.

Control rods

As shown in Figure 9.2.1 on page 345, the number of neutrons released during the uncontrolled fission chain reaction grows exponentially. These neutrons go on to cause more fission reactions, releasing enormous amounts of energy in a split second. However, to generate electricity in a nuclear reactor, a steady and controlled release of energy is required. This is achieved using **control rods**, commonly made of cadmium and boron steel, which are able to absorb neutrons, preventing them from participating in the reaction. This controls the number of neutrons involved in the fission chain reaction of uranium-235. Typical reactors can have up to 50 clusters with 20 individual control rods in each cluster. The rate of the chain reaction is thus controlled by raising or lowering the control rods. To reduce the energy output of the reactor, or even shut it down completely, operators lower the control rods further into the core. This absorbs more neutrons, thereby reducing or stopping the chain reaction.

Coolant

The fission reaction in the reactor core produces an enormous amount of heat energy, resulting in the reactor core being typically at a temperature of 500 to 1500°C. This heat energy is removed from the core of the reactor by pipes that contain a **coolant** with a high specific heat capacity, typically liquid sodium, water, carbon dioxide gas or heavy water. A **heat exchanger** then transfers this energy into pipes containing water. This water is converted into high-pressure steam that is used to rotate the turbines that drive the generator.

Radiation shield

The core of the reactor is encased in a protective **radiation shield** consisting of layers of concrete, steel, graphite and lead with a total thickness of about 2 metres. The function of this shield is to prevent neutrons and gamma rays from escaping the reactor core, thus protecting the workers at the plant from harmful radiation. The layers of graphite in the shield reflect escaping neutrons back into the core to take part in the chain reaction. The workers at a nuclear power plant are continually monitored to ensure that they are not exposed to unacceptably high levels of radiation, although their allowed dose is much higher than that of the general population.

Putting it all together

Figure 9.2.5a is a schematic diagram showing how all of the individual components of a nuclear reactor are combined. The **core** of a thermal nuclear reactor consists of the moderating material with fuel rods and control rods placed in it. These rods could be inserted into holes drilled into a pile of graphite several metres thick, or immersed into a volume of water or heavy water. The reactor that blew up at Chernobyl had about 1600 fuel rods and over 200 control rods in its graphite core.

Figure 9.2.5b shows the energy transformations that occur in a nuclear reactor in order to produce electricity. The primary difference between this and a gas- or coal-fired generator is in the way the heat is produced. A nuclear reactor uses the fission process and a gas- or coal-fired generator uses the combustion of the fossil fuel. A typical 1000 MW power plant consumes about six million tonnes of black coal each year, or about 25 tonnes of enriched uranium that has been obtained from around 75 000 tonnes of ore.

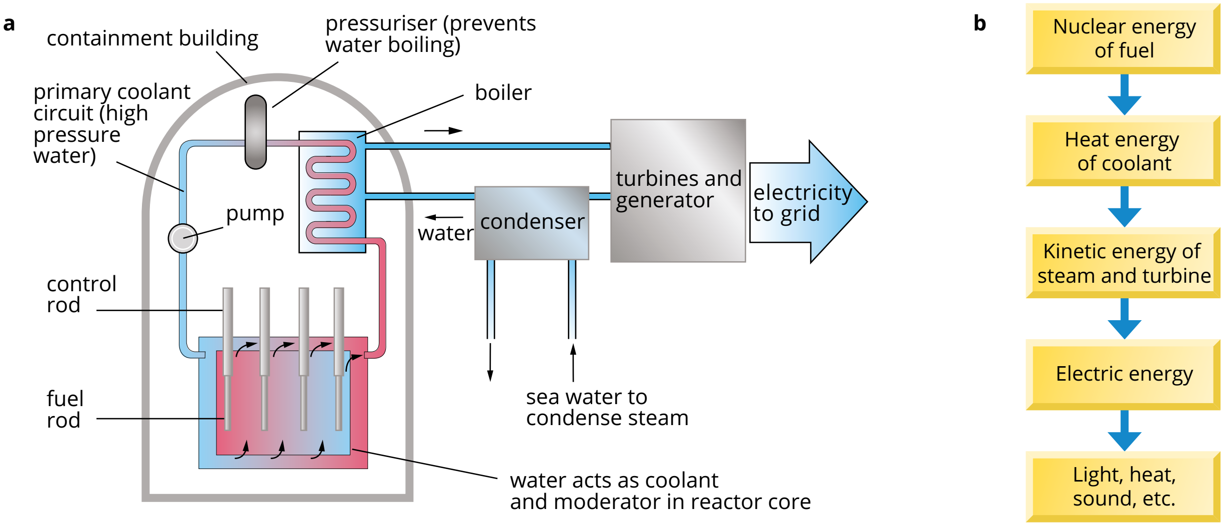


FIGURE 9.2.5 (a) A schematic diagram of a nuclear reactor. (b) Energy transformations involved as a reactor is used to produce electricity.

PHYSICSFILE

Nuclear disasters: Fukushima

The biggest earthquake in Japan's history (magnitude 9) occurred on 11 March 2011 and triggered a massive tsunami along the east coast. Tragically, about 20 000 people were killed by the tsunami.

Japan has 52 nuclear reactors. When the earthquake hit, 11 reactors at four nuclear power plants all shut down automatically. The Fukushima nuclear power plant, consisting of six reactors, was located close to the coast and protected by sea walls designed to withstand a 5.7 metre tsunami. However, the 15 metre wave that arrived about an hour after the earthquake completely inundated the reactors, causing loss of power to the electrical switching systems and disabling the backup generators. The pumps that pushed water through the reactors stopped working, causing the reactors to overheat. Hydrogen explosions caused fires and damaged the containment vessels. This resulted in the release of the radioisotopes iodine-131 and caesium-137 into the atmosphere. Over the next 4 to 6 days, a total of around 940 PBq (9.40×10^{17} Bq) of radiation was detected.

Water leaked from Reactor 1, leaving fuel rods exposed, thus triggering their meltdown. As pictured in Figure 9.2.6, one hydrogen explosion caused the roof of Fukushima's Reactor 1 to collapse, releasing radioactive materials into the atmosphere. The Japanese government created a 20 km exclusion zone around the power plant, forcing 80 000 people from their homes.

Nuclear accidents are rated according to the International Nuclear and Radiological Event scale from 1 to 7, with 7 being the most serious. Both the Fukushima and Chernobyl disasters (which occurred in Ukraine in 1986) were rated at 7.

FIGURE 9.2.6 A hydrogen explosion caused the roof of Reactor 1 at Fukushima to collapse, releasing radioactive materials into the atmosphere.

Management of nuclear waste

A major problem facing the nuclear power industry is the disposal of the unstable radioactive waste. There are about 400 nuclear power plants generating electricity around the world, producing large quantities of radioactive waste with long half-lives. A typical 1000 MW reactor will produce about 25 tonnes of spent fuel rods annually. The safe disposal of this high-level waste is of concern to many people around the world.

Figure 9.2.7 shows the inside of the storage bunker for radioactive waste at Nieuwdorp in the Netherlands. All radioactive waste from across the Netherlands is collected and recorded then crushed into small packages and placed in 200-litre drums. This material is eventually decanted with concrete and placed in barrels for long-term storage. These barrels are then stored in a bunker secure against burglary and terrorist attacks.

FIGURE 9.2.7 The radioactive waste storage bunker in Nieuwdorp, Netherlands.

Radioactive waste products are classified as low-level, intermediate-level or high-level waste.

- Low-level waste is generated primarily from hospitals, industry and laboratories and consists mostly of tools, clothing, used wrapping material and other items that have been contaminated with radionuclides with short half-lives. Low-level waste solids are usually compacted or incinerated, then buried in shallow pits on land or at sea.
- Intermediate-level waste typically consists of reactor components, chemical sludges, and contaminated materials from reactors that have been decommissioned. Intermediate-level wastes are solidified in bitumen or concrete, then buried or stored in deep trenches.
- High-level waste is waste from contaminated reactor parts, as well as liquid waste from the reprocessing of spent fuel rods. This waste contains highly radioactive fission fragments and **transuranic** elements, and so requires special shielding during handling and transport. As can be seen in the graph in Figure 9.2.8, high-level waste remains radioactive for an exceedingly long time and needs to be stored permanently. By way of comparison, the activity of one tonne of uranium ore is only 8×10^{11} Bq.

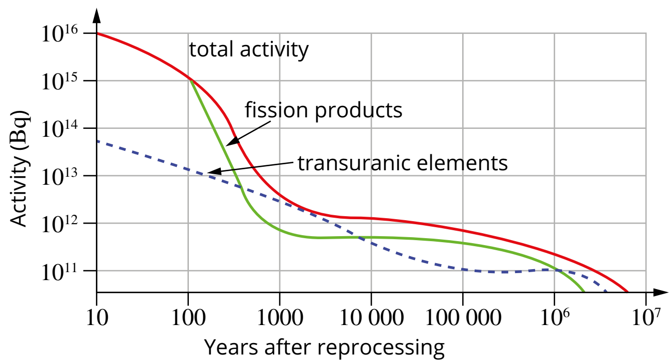


FIGURE 9.2.8 The activity of 1 tonne of high-level nuclear waste.

In the USA, Canada and a few other countries, spent fuel rods are permanently stored in cooling ponds, like the one shown in Figure 9.2.9, to protect people from radioactive emissions. The uranium-235 and uranium-238 nuclei in these fuel rods have half-lives of 703 000 and 4.5 billion years, respectively. Around 40% of spent fuel rods produced worldwide are permanently stored in this way. However, in Japan, Russia and Europe, these fuel rods are reprocessed and the fissile material is reused as nuclear fuel. and are stored permanently in underground bunkers.

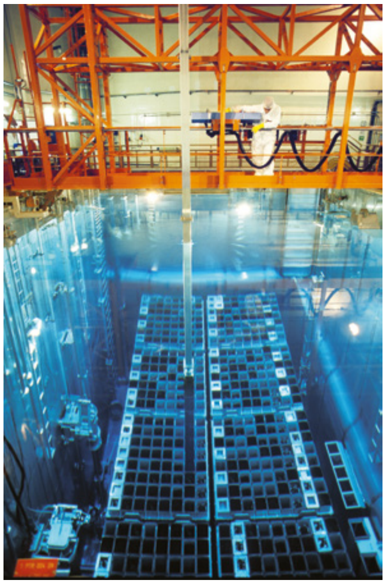


FIGURE 9.2.9 Spent fuel rods in a cooling pond at a nuclear power station.

PHYSICS IN ACTION

Generation IV fission reactors

The first-generation reactors were developed in the 1950s and 1960s using natural uranium that was not enriched. Generation II reactors followed in the 1970s to improve competitiveness in energy generation and included notable reactors like Three-Mile Island and Chernobyl reactors. Generation III reactors focused on enhancing safety and structural strength, with the European Pressurised Reactor (EPR) being a prime example. Most of the reactors in operation today are Generation II or III, but according to Chinese state media, China began work on the first fourth-generation reactor with a high-temperature, gas-cooled modular pebble bed (HTR-PM) in December 2023, which would make it the first Generation IV reactor to enter commercial operation.

To be classified as Generation IV, a reactor must be more fuel-efficient than current systems. It must also be designed to prevent major accidents, such as those caused by earthquakes, as seen at Fukushima in 2011. Finally, the technology should minimise the potential for nuclear weapons production from the fuel, which is mostly uranium-238. The Generation IV International Forum (GIF) focuses on six reactor designs that aim to deliver safe, secure, sustainable, competitive and versatile nuclear technology, shown in Figure 9.2.10. Like Generation II and III reactors, the non-reusable products of the fission reactions from Generation IV will still require safe disposal.

Gas-cooled fast reactor (GFR) Like other helium-cooled reactors previously operated or under development, GFRs will be high temperature units – typically 800–850°C. They use reactor technology similar to that of the VHTR, suitable for power generation, thermochemical hydrogen production or other thermal processes.	**Lead-cooled fast reactor (LFR)** The LFR is a flexible fast reactor that can use depleted uranium or thorium fuel matrices, and burn actinides from LWR fuel. Liquid metal (Pb or Pb-Bi eutectic) cooling is done at atmospheric pressure by natural convection.	**Molten salt reactor (MSR)** There are now two variants: one is a fast reactor with fissile material dissolved in circulating salt fuel; the other with solid particle fuel in graphite, with the salt serving only as a coolant.
Sodium-cooled fast reactor (SFR) The SFR uses liquid sodium as the reactor coolant, which allows for high power density with a small volume of coolant, at low pressure. This was the technology that interested GIF initially and it remains important despite the need for a sealed cooling system.	**Supercritical water-cooled reactor (SCWR)** This is a very high-pressure water-cooled reactor that operates above the thermodynamic critical point of water (374°C, 22 MPa), achieving a thermal efficiency about one third higher than that of current light water reactors (LWRs), from which the design evolves.	**Very high-temperature gas reactor (VHTR)** This is a graphite-moderated, helium-cooled reactor. The Chinese HTR-PM demonstration belongs to this reactor family and its successful start-up will pave the way for a commercial version of the VHTR.

FIGURE 9.2.10 Different types of GIF reactors. This short list of proposed technologies focuses on the most promising and those most likely to meet the requirements of the Generation IV criteria.

9.2 Review

SUMMARY

- A nuclear reactor uses fuel rods that contain uranium-235 enriched to between 3% and 4% as its fuel.
- The core of the reactor contains a material called a moderator, such as graphite or water. This material slows neutrons down, enabling them to be captured by the uranium-235 nucleus and bring about a fission reaction.
- The rate at which the fission reactions occur is determined by control rods. These are raised and lowered in the core and contain a material that absorbs neutrons, such as cadmium or boron, and thereby controls the chain reaction.
- The coolant is a liquid that flows through the core of the reactor, extracting heat from it and transferring this, through a heat exchanger, to water that is converted to steam to drive the generator turbines.
- Uranium-238 absorbs neutrons and undergoes transmutation to produce plutonium-239 as one of its daughter nuclei.
- Plutonium-239 is used as a fuel in fast breeder reactors as it undergoes fission when struck with fast-moving neutrons.
- A chain reaction may be established when a fission reaction occurs that generates one or more neutrons per fission.
- Critical mass is the minimum amount of enriched fissile material in the shape of a sphere required to sustain a chain reaction.
- Many of the fission products from a nuclear reactor have very long half-lives. This makes the long-term safe storage of this waste difficult.

KEY QUESTIONS

1 Which one of the following isotopes is most suitable as a fuel for a nuclear reactor?

A uranium-238
B uranium-235
C uranium-234
D plutonium-239

2 The uranium ore that is dug from the ground contains two different isotopes, uranium-235 and uranium-238. Explain why uranium in this form is not immediately suitable as a fuel for a nuclear reactor.

3 The uranium that is used as the fuel for a nuclear reactor has been enriched so that its uranium-235 content is around:

A 0.7%
B 4%
C 10%
D 95%

4 Describe the function of the moderator in a nuclear reactor and give an example of the material that is used.

5 Describe the function of the control rods in a nuclear reactor and give an example of the material that is used.

6 In a chain reaction that results in an explosion, approximately 10^{24} uranium-235 nuclei undergo fission in just over 1 µs. What conditions would be necessary for this to occur?

7 The critical mass of uranium-235 is about 1 kg, but a 5 kg piece of uranium-235 that is flattened like a sheet is not capable of exploding. Explain why.

8 Explain why lead ($Z = 82$) would be unsuitable for use as a moderator.

9 Describe the effect on the operation of a nuclear reactor if the number of neutrons released per fission is:

a equal to one
b less than one
c greater than one.

10 The fissile material that is used in a nuclear reactor is uranium-235. Describe the effect on the nuclei of this isotope when it is bombarded with:

a fast neutrons
b slow neutrons.

11 Approximately 97% of the uranium in the fuel rods of nuclear reactors is uranium-238. When struck by a fast neutron, a uranium-238 nucleus is more likely than a uranium-235 nucleus to absorb the neutron.

a In what way does this change the uranium nuclei?
b Why does this lead to problems in disposing of the nuclear waste from the reactor?

12 During the fission of plutonium-239, the average number of neutrons released is 2.91 per fission. This is higher than the average of 2.47 released during the fission of uranium-235. How is this of benefit in a fast breeder reactor?

13 The neutron bombardment of uranium-238 triggers two successive beta decays before reaching the final product of plutonium-239. The neutron-induced fission of plutonium-239 randomly produces many pairs of nuclei, with one of these being xenon-134 and zircon-103.

a Write equations for the three steps between uranium-238 and plutonium-239.
b Write an equation for the neutron-induced fission of plutonium-239 to xenon-134 and zircon-103.

14 During the lifetime of a reactor, the control rods need to be gradually removed over a period of months to maintain the energy production at a constant rate. Explain why this is necessary.

9.3 Nuclear fusion

Nuclear fusion is a process that has been occurring inside the Sun and other stars for billions of years. **Fusion** involves the combining of small nuclei, such as hydrogen and helium, to form a larger nucleus. The amount of energy released per nucleon is greater with fusion than with fission, and there is no radioactive waste produced. Scientists are working on experimental fusion reactors, such as the International Thermonuclear Experimental Reactor (ITER) in France. Originally scheduled to open in 2016 at a cost around 5 billion euros (8 billion AUD), its price has since quadrupled, and its start-up has been pushed back to 2025.

Fusion achieving 'target gain' on 5 December 2022 at the National Ignition Facility (NIF) in the USA, shown in Figure 9.3.1, was the result of over 60 years of research using high-energy lasers. Target gain is when more energy is released than is required to initiate the fusion process. Another three successful ignitions followed in 2023, and on 12 February 2024 approximately 5.2 MJ of energy was produced, more than double the input laser energy. NIF is not designed to produce power but is used for experimental research.

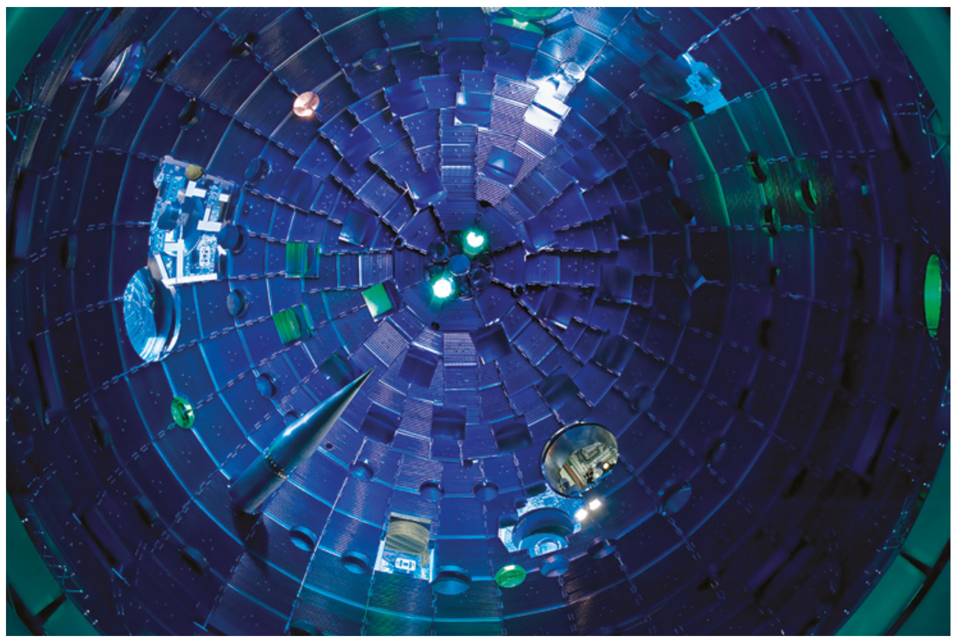


FIGURE 9.3.1 The experimental fusion reactor at the National Ignition Facility in the USA.

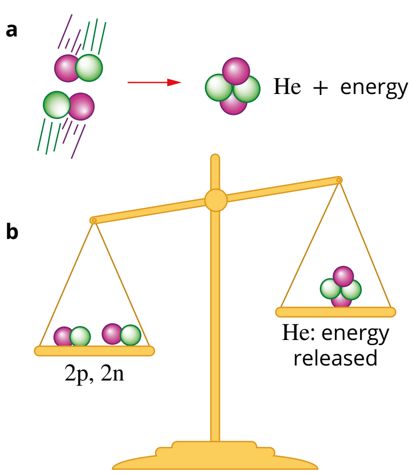


FIGURE 9.3.2 (a) When two isotopes of hydrogen fuse to form a helium nucleus, energy is released. (b) The loss in mass, Δm, can be used to calculate the energy emitted.

In the study of nuclear reactions, Einstein's idea of mass–energy equivalence is used to explain why energy is released during the splitting of a nucleus. When fission occurs, the mass of the particles in the fission fragments is lower than that of the parent nucleus. Einstein's famous equation, $\Delta E = \Delta mc^2$, or the mass–energy equivalence equation, $\Delta E = \Delta m \times 931$, can be used to calculate the amount of energy related to this missing mass. These equations also apply to fusion reactions. It is important to note that less than 1% of matter is converted into energy in all of the energy transformations discussed here.

ENERGY AND MASS CHANGES IN NUCLEAR FUSION

Nuclear fusion occurs when two light nuclei are combined (fuse) to form a larger nucleus. The example of nuclei fusing to form a helium atom is shown in Figure 9.3.2a.

In fusion reactions, as in fission, the atomic numbers and mass numbers on either side of the equation are conserved. For example, the fusion of two hydrogen-2 (deuterium) nuclei is shown below:

$$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + \text{energy}$$

In this reaction, the atomic numbers add up to two on both sides, and the mass numbers add up to four on both sides. However, the total mass of the reactants (on the left-hand side) will be greater than the total mass of the products (on the right-hand side). This mass difference is represented by the unbalanced scales shown in Figure 9.3.2b. The difference in mass is called the mass defect, Δm, and the energy created by this missing mass can be calculated in joules or MeV as shown below.

$$\Delta E = \Delta mc^2$$

where ΔE is energy (J)

Δm is the mass defect (in kilograms, kg)

c is the speed of light $= 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$.

or

$$\Delta E = \Delta m \times 931$$

where ΔE is energy (MeV)

Δm is the mass defect (in daltons, Da)

931 is the mass–energy equivalent.

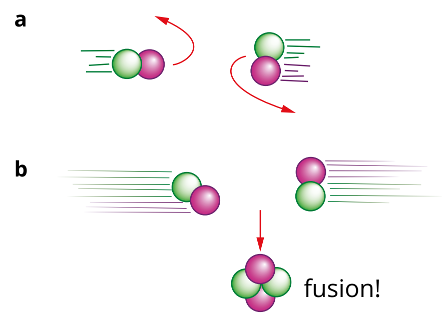


FIGURE 9.3.3 (a) Slow-moving nuclei do not have enough energy to fuse together. The electrostatic forces cause them to be repelled from each other. (b) If the nuclei have sufficient kinetic energy, they will overcome the repulsive forces and move close enough together for the strong nuclear force to come into effect. At this point, fusion will occur and energy will be released.

Nuclear fusion is very difficult to achieve. The main problem is that nuclei are positively charged. They exert a strong electrostatic force of repulsion on each other; that is, they repel each other strongly. Therefore, it is not easy to force the nuclei together. Remember that the electrostatic force is a long-range force, whereas the strong nuclear force of attraction only acts at much shorter distances.

As two nuclei approach each other, the electrostatic force will cause them to decelerate. Slow-moving nuclei with relatively small amounts of kinetic energy will not be able to get close enough for the strong nuclear force to come into effect. Fusion will not happen, as can be seen in Figure 9.3.3a.

If the nuclei travel towards each other at higher speeds, as shown in Figure 9.3.3b, they may have enough kinetic energy to overcome the repulsive force. The nuclei can now get close enough for the strong nuclear force to start acting. If this happens, fusion will occur.

The graph in Figure 9.3.4 shows the effect of the electrostatic force and the strong nuclear force on the potential energy of a pair of deuterium ($^2_1$H) nuclei. Imagine a deuterium nucleus at the origin of the graph, and another deuterium nucleus approaching from the right and moving towards the origin. At large separation distances, the electrostatic force dominates and the nuclei repel each other, as shown to the right of the energy barrier in the graph. However, if the approaching nucleus can get over the potential energy barrier by doing work with its kinetic energy, then the nuclei can get close enough for the strong nuclear force to start acting. At small separation distances, the strong nuclear force dominates and the nuclei can fuse together, but they need an enormous amount of kinetic energy to get to this point. Temperatures in the hundreds of millions of degrees are required (as in the Sun).

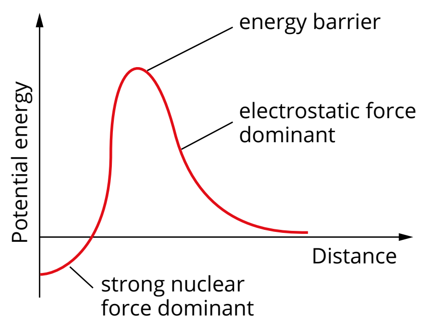


FIGURE 9.3.4 If two hydrogen-2 (deuterium) nuclei are to get close enough for the strong nuclear force to act, they must overcome the energy barrier presented by the electrostatic force.

PHYSICSFILE

Hydrogen bomb

In 1952, a fusion reaction was used to power the world's first hydrogen bomb. It had five times the destructive power of all the conventional bombs that were dropped during the whole of World War II. The high temperature achieved by a fissile fuel explosion was used to initiate the fusion reaction. In other words, an atomic bomb was used as the fuse for the hydrogen fusion bomb.

FIGURE 9.3.5 The hydrogen bomb dropped at Bikini Atoll in 1956.

Worked example 9.3.1

FUSION

Two deuterium nuclei undergo fusion to produce one tritium nucleus, a proton and energy according to the equation below. Calculate the energy released in this reaction in both MeV and joules. Use the following data in your calculations: mass of deuterium = 2.013 553 Da, mass of tritium = 3.016 049 Da, and mass of a proton = 1.007 276 Da. $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{1}H + ^{1}_{1}H + \text{energy}$

Thinking	Working
Determine the mass of the reactants (m_r).	mass of reactants: $m_r = 2 \times$ mass of deuterium $m_r = (2)(2.013\,553)$ $m_r = 4.027\,106$ Da
Determine the mass of the products (m_p).	mass of products: $m_p =$ mass of tritium + mass of proton $m_p = (3.016\,049) + (1.007\,276)$ $m_p = 4.023\,325$ Da
Determine the mass defect (Δm).	mass defect = mass of reactants – mass of products $\Delta m = m_r - m_p$ $\Delta m = (4.027\,106) - (4.023\,325)$ $\Delta m = 0.003\,781\,0$ Da
Determine the energy equivalent.	$\Delta E = \Delta m \times 931$ MeV $\Delta E = (0.003\,781\,0)(931)$ $\Delta E = 3.5201$ $\Delta E = 3.52$ MeV
Convert to joules.	$\Delta E = (3.5201 \times 10^{6})(1.60 \times 10^{-19})$ $\Delta E = 5.6322 \times 10^{-13}$ $\Delta E = 5.63 \times 10^{-13}$ J

Worked example: Try yourself 9.3.1

FUSION

One of the possible nuclear fusion reactions in a star involves the fusion of two helium-3 nuclei to produce a helium-4 nucleus, two protons and energy according to the equation below. Calculate the energy, in joules and MeV, released in this reaction. Use the following data in your calculations: mass of helium-3 nucleus = 3.014 932 Da, mass of helium-4 nucleus = 4.001 505 Da, and mass of a proton = 1.007 276 Da. $^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + 2^{1}_{1}H + \text{energy}$

BINDING ENERGY

As seen earlier in this chapter, the total mass of a stable nucleus is slightly less than the combined mass of the individual protons and neutrons. This mass defect, Δm, in daltons is converted to energy using $\Delta E = \Delta m \times 931$, which is known as the binding energy of the nucleus. The binding energy indicates how much energy is needed to separate the nucleus into individual protons and neutrons.

Each nucleus has its own binding energy value. It quantifies the stability of the nucleus, with a higher binding energy indicating a more tightly bound and stable nucleus. A binding-energy-per-nucleon graph, as shown in Figure 9.3.6, allows a comparison of nuclear stabilities.

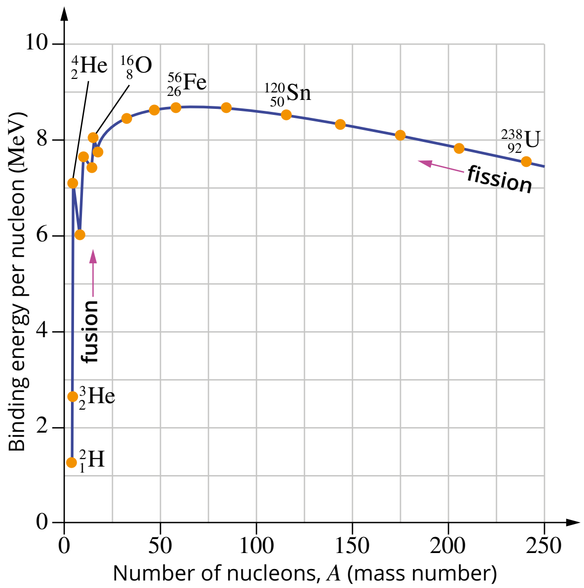


FIGURE 9.3.6 The graph of binding energy per nucleon.

Figure 9.3.6 can be analysed to better understand fission and fusion.

- Small nuclei have lower binding energy per nucleon values, indicating that they are easier to break apart compared to larger nuclei. Helium-4 has a relatively high value indicating that it is stable.
- The binding energy per nucleon increases dramatically for very small nuclei. As they fuse together, the binding energy per nucleon increases. This is the energy released during fusion.
- Elements with mass numbers between 40 and 80 have nuclei that are tightly bound. It takes more energy to break these nuclei apart. These are the most stable nuclei. As you can see from the graph, these elements have the highest binding energy per nucleon. The higher the binding energy per nucleon, the more stable the nucleus.
- Larger nuclei have lower binding energy per nucleon values, indicating that they are less stable.
- If a large nucleus such as uranium splits into two fragments, the binding energy per nucleon of the fragments again increases. This is the energy released during fission.
- Iron (Fe), with a mass number of 56, has the most stable nucleus.
- Nuclei smaller than iron undergo fusion and release energy. Nuclei larger than iron undergo fission and release energy.

Worked example 9.3.2

BINDING ENERGY

Calculate the average binding energy per nucleon for the carbon-12 nucleus in MeV and joules. Use the following data in your calculations: mass of a carbon-12 nucleus = 11.993 417 Da, mass of a proton = 1.007 276 Da and mass of a neutron = 1.008 664 Da.

Thinking	Working
Determine the total mass of the nucleons in a carbon-12 nucleus (m_t).	m_t = mass of 6 neutrons + mass of 6 protons $m_t = 6 \times (1.008664) + 6 \times (1.007276)$ $m_t = (6.051984) + (6.043656)$ $m_t = 12.095640$ Da
Determine the mass defect.	Δm = mass of nucleons − actual mass of nucleus $\Delta m = (12.095640) - (11.993417)$ $\Delta m = 0.102223$ Da
Determine the binding energy in MeV.	$\Delta E = \Delta m \times 931$ $\Delta E = (0.102223)(931)$ $\Delta E = 95.1696$ MeV
Determine the binding energy per nucleon in MeV.	$\Delta E = \frac{\text{binding energy}}{\text{number of nucleus}}$ $\Delta E = \frac{(95.1696)}{(12)}$ $\Delta E = 7.930801$ $\Delta E = 7.93$ MeV per nucleon
Determine the binding energy per nucleon in J.	$\Delta E = (7.930801 \times 10^{6})(1.60 \times 10^{-19})$ $\Delta E = 1.268928 \times 10^{-12}$ $\Delta E = 1.27 \times 10^{-12}$ J

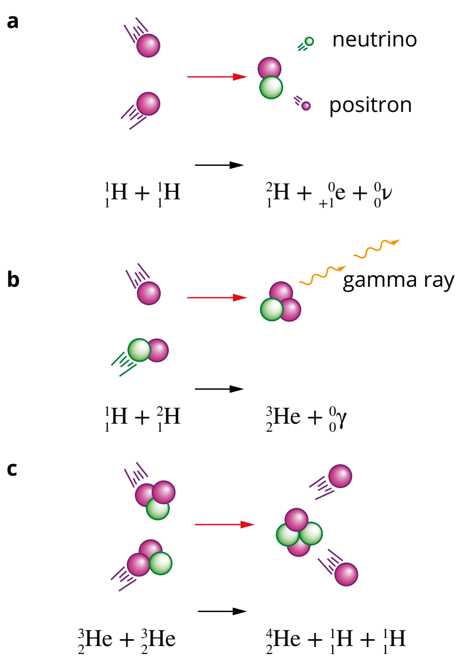


FIGURE 9.3.7 The three main fusion reactions taking place inside in the Sun.

Worked example: Try yourself 9.3.2

BINDING ENERGY

Calculate the average binding energy per nucleon for the uranium-235 nucleus in MeV and joules. Use the following data in your calculations: mass of a uranium-235 nucleus = 234.993 462 Da, mass of a proton = 1.007 276 Da and mass of a neutron = 1.008 664 Da.

FUSION IN THE SUN AND SIMILAR STARS

In the Sun, many different fusion reactions are taking place, as shown in Figure 9.3.7. Each second, about 657 million tonnes of hydrogen isotopes fuse to form about 653 million tonnes of helium, resulting in a mass defect of about 4 million tonnes and releasing an enormous amount of energy. A small fraction of this energy reaches Earth and sustains life. This sequence of fusion reactions has been occurring inside the Sun for the past 5 billion years and is expected to last for another 5 billion years or so.

NUCLEAR FUSION REACTORS

You might recall from studying radioactivity that for each nuclear fission reaction, about 200 MeV of energy is released. This is a lot of energy relative to other nuclear reactions, but fission typically involves nuclei with around 240 nucleons.

In nuclear fusion, less energy is emitted than in fission, at around 20 to 25 MeV. However, this energy is released from a reaction involving just a few nucleons. The energy per nucleon is greater than that for fission reactions because a greater percentage of the mass is transformed into energy. For scientists working on experimental fusion reactors, this is very important as massive amounts of energy could be obtained from fusion reactions between small, readily available nuclides, with no radioactive by-products.

Since the 1950s, a great deal of research has been devoted to recreating nuclear fusion in a laboratory. However, major technical problems have been encountered in trying to initiate ongoing fusion reactions on Earth. Replicating the reactions that take place on the Sun is extraordinarily difficult. This is because extremely high densities, temperatures and pressures are required. Fusion researchers currently use two isotopes of hydrogen—deuterium and tritium—as fuel. Deuterium can be extracted in vast quantities from lakes and oceans, but tritium is radioactive, with a half-life of 12.3 years, and must be artificially produced. The nuclear reactions used in current fusion reactors are as follows:

$$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \rightarrow ^{3}_{2}\text{He} + ^{1}_{0}\text{n}$$

$$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \rightarrow ^{3}_{1}\text{H} + ^{1}_{1}\text{H}$$

$$^{2}_{1}\text{H} + ^{3}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{1}_{0}\text{n}$$

The main obstacle to the development of a successful fusion reactor is the extremely high temperature that must be achieved before fusion can commence. Temperatures of over 100 million degrees Celsius are needed to trigger a self-sustaining fusion reaction and this must then be contained inside the reactor. Current fusion reactors have achieved temperatures of about 100 million degrees Celsius, but only for very short periods.

PHYSICSFILE

Tunnel effect

The temperature at the surface of the Sun is about 5500°C, hot enough to vaporise any known material.
The Sun is much, much hotter at the core, where temperatures reach around 15 million degrees. But this is well below the hundreds of millions of degrees needed to sustain nuclear fusion. The fact that fusion occurs at these low temperatures cannot be explained by classical physics. Quantum mechanics must be used, in which protons are treated as waves rather than particles. This process is called the tunnel effect and is beyond the scope of this course.

PHYSICS IN ACTION

Fusion-reactor research

A commercial nuclear fusion reactor is likely still four or five decades away. Research is currently focused on devices such as tokamaks, which are doughnut-shaped reactors that use magnetic fields to confine the plasma of fusion reactants away from the reactor walls.
Those tokamaks, like the Joint European Torus (JET) in the UK, can achieve temperatures of hundreds of millions of degrees. The design of the reactor and shape of a tokamak is shown in Figure 9.3.8 (on the next page).

The largest project in the field of nuclear fusion research is being conducted at the International Thermonuclear Experimental Reactor (ITER) in France. This experiment is a joint venture between Europe, China, India, Japan, Russia, the USA and South Korea. Although the goal was to initiate plasma experiments by 2020, the project continued to advance despite challenge posed by the COVID-19 pandemic. In July 2020, representatives from all the ITER nations came together, albeit socially distanced, in a ceremony hosted by President Emmanuel Macron, to commence the assembly of the reactor components. During the following two years, until June 2022, meetings were conducted virtually to further the project's progress.

In March 2022, the JET tokamak, using the same material for walls as were planned for the ITER, achieved the first ever confined plasma of fusion reactants. This milestone showed that it is possible to sustain a high-fusion reaction using the same deuterium–tritium fuel mix intended for ITER and future devices. JET set a new record with a 69.26 megajoule pulse of energy in October 2023 but ceased operating at the end of that year, after 40 years of research. Meanwhile, the assembly of ITER continues, but the manufacturing of components and delivery to France from the various countries is costly and time-consuming. In June 2024, the ITER council met to review the project's progress and now anticipates that ITER will be fully operational by 2034. The ITER facility, spanning 42 hectares, is shown in Figure 9.3.9 (on the next page).

Nuclear fusion may be the ultimate energy solution. The fuel for nuclear fusion, deuterium, can be readily obtained from seawater. Moreover, vast quantities of energy are released during the fusion process, yet a relatively small amount of radioactive waste is created.
The radioactive waste consists mostly of the reactor parts that have suffered neutron irradiation.

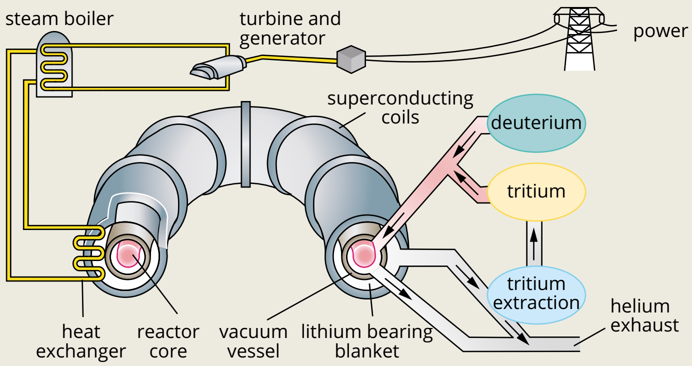


FIGURE 9.3.8 Design of experimental tokamak reactor.

FIGURE 9.3.9 The ITER facility in France as of June 2024.

FROM THE BIG BANG TO THE FIRST STARS

Perhaps the biggest question of all is, 'Where do we come from?' Humans have always sought to answer this question, but today our understanding is largely based on the Big Bang theory. At the moment of the Big Bang, all the matter and energy in our present universe was contained in some infinitely small, incredibly dense region, which rapidly expanded. Based on insights from nuclear and particle physics, the Big Bang theory suggests that energy was converted into matter in the early universe. As the universe expanded and cooled, this matter began to condense into galaxies after millions of years, leading to the formation of the cosmos as we know it.

The Standard Model of particle physics has allowed us to piece together a picture of how matter formed and evolved. As the universe cooled after the Big Bang, fundamental particles and later composite particles formed. The strong nuclear force and the electromagnetic force were thus able to create nuclei, and then atoms. Next, gravity constructed the universe as we know it by pulling matter together into structures, such as galaxies made of stars, and planets, such as Earth.

In the first few seconds, while the temperature remained over a billion kelvin (K), quarks combined to form protons and neutrons. These protons and neutrons were then forced close enough to fuse together, forming hydrogen, helium and lithium nuclei (Figure 9.3.10). A little while later, the temperature dropped below that needed for fusion to occur, and so no further nuclei were formed.

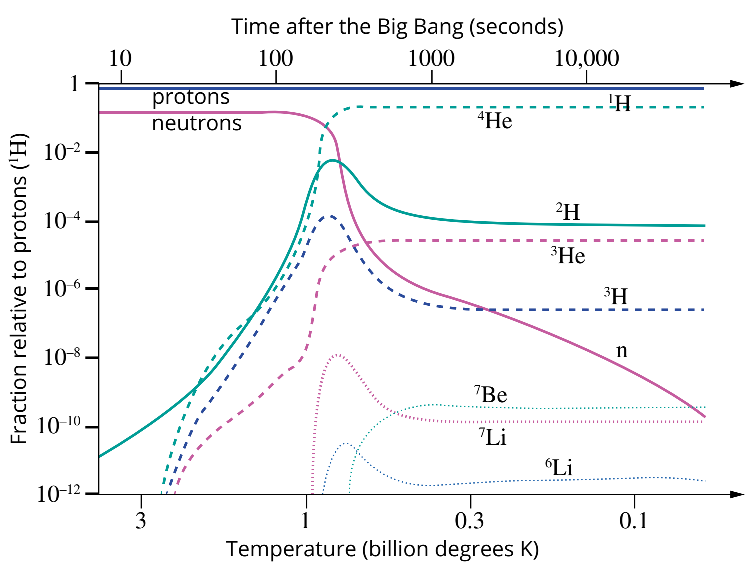


FIGURE 9.3.10 Most of the matter in the universe came into existence in the first few minutes after the Big Bang. Hydrogen (protons, ^{1}H) and helium (^{4}He) were by far the most dominant nuclei formed. Many neutrons were formed initially, but rapidly decayed into protons and electrons.

The first stars

It wasn't until about one billion years after the Big Bang that things started to get more interesting. By this time, the average temperature had fallen to just 15 K (i.e. *very* cold) and, because of the slight unevenness in the early universe, matter started to clump together. Within those clumps, gravitational forces slowly but surely pulled vast numbers of atoms together in an almighty crunch. The collapse of all these atoms created the first stars. The gravitational energy released during the collapse raised the temperature to millions of degrees Celsius, causing hydrogen fusion to start up again, raising the temperature even more, and creating more helium and lithium atoms.

A universe consisting of just hydrogen, helium and a little lithium could not produce life, which is dependent on a multitude of heavier elements, such as carbon. It took the incredibly massive energy of supernova explosions of the early stars to produce the rest of the elements of the periodic table, elements from which life would finally start to evolve some 10 billion years after the Big Bang.

9.3 Review

SUMMARY

- Nuclear fusion is the combining of light nuclei to form heavier nuclei.
- For fusion to occur, nuclei must have enough energy to overcome the electrostatic forces and get close enough for the strong nuclear force to take effect. The energy required to achieve this is called the energy barrier.
- The mass of the product nuclei is less than the combined mass of the parent nuclei. This mass difference accounts for the energy released in the reaction according to Einstein's equation, $\Delta E = \Delta mc^2$, or the mass–energy equivalence, $\Delta E = \Delta m \times 931$.
- The binding energy indicates how much energy is needed to separate the nucleus into individual protons and neutrons.
- If very small nuclei fuse together, their binding energy per nucleon increases.
- If a large nucleus, such as uranium, splits into two fragments, the binding energy per nucleon in the fragments increases.
- Nuclear fusion is the process producing energy in the Sun and other stars.
- The amount of energy released per nucleon is greater for fusion than it is for fission.
- Very high temperatures are needed to overcome the repulsive forces between the nuclei in order to trigger and sustain a nuclear fusion reaction. This is the main obstacle to the development of fusion reactors.
- Fusion creates very little radioactive waste.
- The light elements hydrogen, helium and lithium formed after the Big Bang. All other heavier elements were created in fusion reactions within stars and during supernova explosions.

KEY QUESTIONS

1 How does fusion differ from fission?

2 The fusion reaction that is most promising for use in nuclear fusion reactors is: ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$.
Why is energy released during this process?

3 Compare the amount of energy released per nucleon during a single nuclear fission reaction with the amount of energy released per nucleon for a single fusion reaction.

4 During the process of nuclear fusion, mass is lost and this appears as energy. What is the approximate percentage of mass lost during a typical fusion reaction?

5 Consider the fusion reaction shown below.
$${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{a}_{b}X + {}^{1}_{0}\text{n}$$
a Determine the values of *a* and *b* and hence the symbol of the unknown element *X*.
b 33.0 MeV of energy is released. Determine the mass defect in Da and in kg.

6 Two slow-moving protons are travelling directly towards each other. Explain whether or not the protons will collide and fuse together.

7 Two fast-moving protons are travelling directly towards each other. The protons collide and fuse together. Explain why this happens.

8 The following fusion reaction is taking place in the Sun.
$${}^{3}_{2}\text{He} + X \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{1}\text{p} + {}^{1}_{1}\text{p}$$
During each fusion reaction, 23.0 MeV of energy is released.
a What are the atomic and mass numbers of particle *X* and what is its symbol?
b Convert 23.0 MeV into joules.
c Determine the mass defect for this fusion process in kg.

9 What happens to the binding energy and the stability of two hydrogen-2 nuclei when they are fused together to form helium-4?

10 What happens to the number of nucleons during the fusion reaction below?
$${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{p}$$

Chapter review

09

KEY TERMS

atomic mass constant
binding energy
chain reaction
control rod
coolant
core
critical mass
daughter nucleus
fast breeder reactor
fissile
fission
fission fragments
fuel rod
fusion
heat exchanger
heavy water
mass defect
moderator
radiation shield
subcritical mass
supercritical mass
transuranic

1 What does the term 'fissile' mean?

2 Are all atoms fissile? Give examples to support your answer.

3 Consider one particular proton in the nucleus of a zinc atom. Describe the forces that the proton experiences from other nucleons.

4 Neutrons and alpha particles can both be used to trigger nuclear fission. Explain why neutrons are better than alpha particles for inducing fission.

5 Einstein said that mass and energy are equivalent. In one particular fission reaction, a decrease in mass of 3.48×10^{-28} kg occurs.

a Express the energy equivalent of this mass in joules.

b Express the energy equivalent of this mass in MeV.

6 Determine the value of the unknown mass number x and atomic number y in this fission reaction:

$${}^{1}_{0}\text{n} + {}^{x}_{94}\text{Pu} \rightarrow {}^{130}_{54}\text{Xe} + {}^{106}_{y}\text{Xr} + 4{}^{1}_{0}\text{n}$$

7 Determine the number of neutrons (x) released in the following fission reaction:

$${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{127}_{50}\text{Sn} + {}^{102}_{42}\text{Mo} + x{}^{1}_{0}\text{n}$$

8 A typical fusion reaction is: ${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{1}\text{He} + {}^{1}_{1}\text{H}$
Why are high temperatures such as 100 million degrees Celsius needed for this reaction to occur?

9 Consider the following fission reaction of uranium-235:

$${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{141}_{55}\text{Cs} + {}^{93}_{37}\text{Rb} + 2{}^{1}_{0}\text{n}$$

During this reaction, there is a mass defect of 0.003 006 024 Da. How much energy in MeV is produced per reaction?

10 Consider the following fusion reaction:

$${}^{1}_{1}\text{H} + {}^{3}_{2}\text{He} \rightarrow {}^{4}_{2}\text{He} + {}^{0}_{+1}\beta^{+} + {}^{0}_{0}\bar{\nu}_{e}$$

Hydrogen and helium-3 are fused together and a helium-4 nucleus is created along with a positron and a neutrino. 21.0 MeV of energy is released.

a How does the combined mass of the hydrogen and helium-3 nucleus compare with the combined mass of the helium-4 nucleus, positron and neutrino?

b Where has the energy that was released come from?

c Convert the energy into joules.

d What is the mass defect of this fusion reaction in daltons?

11 Compare the waste and the energy per nucleon produced by fusion reactors with those of fission reactors.

12 What happens to the binding energy per nucleon and the stability of the nucleus when a uranium-235 nucleus splits apart to form two smaller nuclei?

13 The binding energy per nucleon for iron (mass number 56) is higher than for other elements. What does this mean for the stability of iron nuclei?

14 Which form of electromagnetic radiation is released from the nucleus of radioactive atoms?

15 Plutonium is an element that does not exist naturally on Earth. How can this element be available for use in nuclear reactors?

16 Explain the primary function of each of the following components of a nuclear reactor, and state what material they are made of:

a the coolant

b the moderator

c the control rods.

17 In a tokomak fusion reactor a proton fuses with a deuterium nucleus to form a helium nucleus and release a gamma ray. The energy created during this process is 20.0 MeV.

a Write the equation for this reaction.

b How much energy is released per reaction in joules?

c Calculate the mass defect for this reaction in daltons.

18 A series of fusion reactions that occur in the Sun begins with two protons fusing to form deuterium and a positron. A deuterium nucleus and another proton can then fuse to form a helium-3 nucleus and a gamma ray. Two helium-3 nuclei can fuse to form an alpha particle and two protons with the release of 12.98 MeV of energy per alpha particle formed.

a Write equations for the three fusion reactions stated above.

b Considering the last reaction of these three, calculate the average power, in megawatts, that could be produced by a controlled reactor, which fuses 100 g of helium-3 into alpha particles and protons in one day. Use the following masses in your calculations: helium-3 nucleus = 3.014 93 Da, helium-4 nucleus = 4.001 51 Da and proton = 1.007 28 Da, where 1 Da = $1.660\,54 \times 10^{-27}$ kg.

CHAPTER

10 Electrical physics

Electricity is a term that is used to categorise a set of phenomena that can occur when charged particles move. Such phenomena can include lightning, static electricity, electrical power and electric heating. Charged particles produce an electric field, and the motion of these charged particles creates a magnetic field. Due to the interactions of these fields, electricity can be explained by James Clerk Maxwell's theories on electromagnetism.

This chapter will focus on the fundamental concepts of electricity. This includes introducing the idea of charge and charged particles. Additionally, there will information on the concepts of current and potential difference, explaining how these features are used to generate work in an electrical circuit. There will also be details on how electrical circuits are constructed and depicted, as well as an exploration of the dangers of electricity and the kinds of safety devices used in domestic (household) electrical circuits to avoid accidents.

Science as a Human Endeavour

- there is an inherent danger involved with the use of electricity that can be reduced by using various safety devices, including fuses, residual current devices (RCD), circuit breakers, earth wires and double insulation.

Science Understanding

- there are two types of electric charge, positive and negative
- the energy available to charges moving in an electrical circuit is measured using electric potential difference (ΔV), which is defined as the change in potential energy per unit charge between two defined points in the circuit, including applying the relationship
  $$\Delta V = \frac{W}{q}$$
- energy is required to separate positive and negative charge carriers; charge separation produces an electrical potential difference that drives current in circuits
- electric current is carried by discrete charge carriers; charge is conserved at all points in an electrical circuit, including applying the relationship
  $$I = \frac{q}{\Delta t}$$
- energy is conserved in the energy transfers and transformations that occur in an electrical circuit
- electrical circuits enable electrical energy to be transferred and transformed into a range of other useful forms of energy, including thermal and kinetic energy and light
- power is the rate at which energy is transformed by a circuit component; power enables quantitative analysis of energy transformations in the circuit, including applying the relationship
  $$P = \frac{W}{\Delta t} = \Delta VI$$
- resistance is defined as the ratio of potential difference across the component to the current in the component. This includes applying the relationship
  $$R = \frac{\Delta V}{I}$$
- conductors can be classified as ohmic or non-ohmic

- circuit analysis and design involve calculating the potential difference across, the current through, and the power supplied to components in series, parallel and compound circuits. This includes applying the relationships

series components, I = constant, $\Delta V_T = \Delta V_1 + \Delta V_2 + \Delta V_3 \ldots$

$R_T = R_1 + R_2 + R_3 \ldots$

parallel components, V = constant, $I_T = I_1 + I_2 + I_3 \ldots$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots$$

School Curriculum and Standards Authority. (2023). *Physics ATAR course Year 11 Syllabus for teaching from 2025*.

10.1 Behaviour of charged particles

All matter in the universe is made up of tiny particles. These particles have a property called **charge** that can be either positive, negative or neutral. Usually, the numbers of positive and negative charges balance out so that you are completely unaware of them. However, when significant numbers of these charged particles become separated or move relative to each other, it results in **electricity**.

In order to understand electricity, it is important to first understand the way charged particles interact with each other.

EXISTENCE OF CHARGE CARRIERS

Matter is comprised of small particles called atoms. Atoms themselves are made from a combination of three subatomic particles: the proton which is positively charged, the neutron which is neutral and the electron which is negatively charged. Each atom contains a nucleus at its centre, which is comprised of protons and neutrons. The nucleus, which is positively charged due to the protons, is surrounded by negatively charged electrons. A model of an atom is shown in Figure 10.1.1.

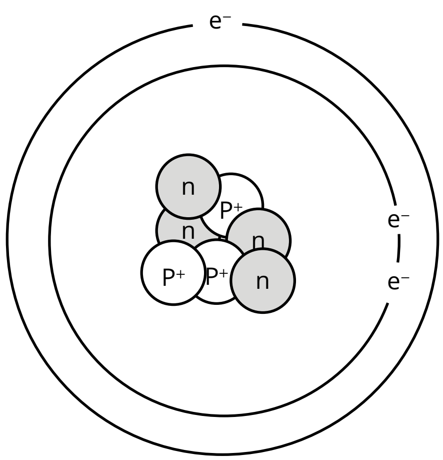


FIGURE 10.1.1 A simple Bohr diagram of an atom.

Simple models of the atom, such as the Bohr model, show the electrons orbiting the nucleus in distinct shells or energy levels. This is because particles that have the same charge will repel each other, while particles with opposite charges will attract each other. In an atom, the negatively charged electrons are held in place by their attraction to the positively charged nucleus, as well as their repulsion from other electrons. This is an important rule to remember when thinking about the interaction of charged particles.

Charge	Positive	Negative
Positive	repel	attract
Negative	attract	repel

The Bohr model of the atom proposes that electrons are held in specific energy levels (shells) surrounding the nucleus. In neutral atoms, the number of electrons is exactly the same as the number of protons, balancing their charges and leaving the atom electrically neutral. It is difficult to remove a proton from the nucleus of an atom, but the outermost electrons are held most loosely and can become delocalised, meaning they are not bonded to a particular atom and are free to move. When a potential difference is applied, these delocalised electrons can carry an electric charge.

PHYSICSFILE

Electron models

The way in which an electron moves around the nucleus of an atom is more complex than the Bohr model would suggest. An individual electron is so small that its exact position at any point in time is impossible to measure. Recent models of the structure of the atom, notably Schrödinger's model, describe an electron in terms of the probability of finding it in a certain location. In diagrams of atoms, this is often represented as clouds or bubbles of space around the nucleus, rather than as points or solid spheres. Atoms are mostly empty space since the nucleus occupies about 10^{-12} of the volume of the atom, yet it contains more than 99% of its mass.

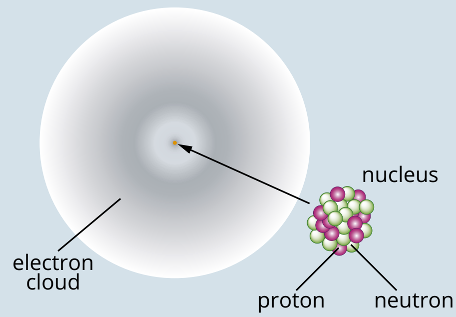


FIGURE 10.1.2 The nucleus of an atom occupies about 10^{-12} of the volume of the atom, yet it contains more than 99% of its mass. Atoms are mostly empty space.

When electrons move from one object to another, each object is said to have gained a **net charge**. The object that loses the electrons will have a net positive charge, since it will now have fewer negative electrons than positive protons. The object that gains electrons will have a net negative charge. When an atom has gained or lost electrons, you say it has been **ionised** or has become an **ion**.

The understanding that the movement of electrons, rather than protons, creates electrical effects is a relatively new discovery. Unfortunately, this means that many of the rules and conventions used when talking about electricity refer to electric current as the movement of positive charge carriers.

PHYSICSFILE

Bohr model

The Bohr model describes each electron as occupying a discrete energy level around the nucleus. Moving electrons between different energy levels involves absorbing or emitting energy. The Bohr model is also referred to as the shell model.

An excess of electrons (more electrons than protons) causes an object to be negatively charged, and a deficit in electrons (fewer electrons than protons) means the object is positively charged.

MEASURING CHARGE

In order to measure the actual amount of charge on a charged object, a 'fundamental' unit would be the size of the charge on one electron or one proton. This fundamental charge is often referred to as the **elementary charge** and is given the symbol e. A proton therefore has a charge of $+e$ and an electron has a charge of $-e$.

The *Système International* (SI) unit of charge is known as the **coulomb** (symbol C). It is named after Charles-Augustin de Coulomb, who was the first scientist to measure the forces of attraction and repulsion between charges.

A coulomb is quite a large unit of charge: +1 coulomb (1 C) is equivalent to the combined charge of 6.20×10^{18} protons. The size of the elementary charge is very small: the charge on a single proton is $+1.60 \times 10^{-19}$ C and the charge on a single electron is -1.60×10^{-19} C. The letter q is used to represent the quantity of charge.

The elementary charge, e, of a proton is equal to $+1.60 \times 10^{-19}$ C.

The elementary charge, $-e$, of an electron is equal to -1.6×10^{-19} C.

Worked example 10.1.1

AMOUNT OF CHARGE ON A GROUP OF ELECTRONS

Calculate the charge, in coulombs, carried by 6.00 billion electrons.	
Thinking	**Working**
Express 6.00 billion in scientific notation.	1.00 billion = 10^9 6.00 billion = 6.00×10^9
Calculate the charge, q, in coulombs, by multiplying the number of electrons by the charge on an electron (-1.60×10^{-19} C). Be sure to present your answer to the correct number of significant figures.	$q = (6.00 \times 10^9)(-e)$ $q = (6.00 \times 10^9)(-1.60 \times 10^{-19}\text{ C})$ $q = -9.60 \times 10^{-10}$ C

Worked example: Try yourself 10.1.1

AMOUNT OF CHARGE ON A GROUP OF ELECTRONS

Calculate the charge, in coulombs, carried by 4.00 million electrons.

Worked example 10.1.2

NUMBER OF ELECTRONS IN A GIVEN AMOUNT OF CHARGE

The net charge on an object is $-3.00\,\mu$C ($1\,\mu$C = 1 microcoulomb = 10^{-6} C). Calculate the number of extra electrons on the object.	
Thinking	**Working**
Express $-3.00\,\mu$C in scientific notation.	$q = -3.00\,\mu$C $q = -3.00 \times 10^{-6}$ C
Find the number of electrons by dividing the charge on the object by the charge on an electron (-1.60×10^{-19} C). Be sure to present your answer to the correct number of significant figures.	$n_e = \frac{q}{-e}$ $n_e = \frac{(-3.00 \times 10^{-6})}{(-1.60 \times 10^{-19})}$ $n_e = 1.875 \times 10^{13}$ $n_e = 1.88 \times 10^{13}$ electrons

Worked example: Try yourself 10.1.2

NUMBER OF ELECTRONS IN A GIVEN AMOUNT OF CHARGE

The net charge on an object is $-4.80\,\mu$C ($1\,\mu$C = 1 microcoulomb = 10^{-6} C). Calculate the number of extra electrons on the object.

PHYSICSFILE

Separating positive and negative charges

Electrons can be transferred (moved) from one object to another by simply rubbing two objects made from different materials together. A good example of this is if you rub a balloon against your hair and then slowly move the balloon away. You will notice that your hair seems to stick to the balloon. This is because electrons are rubbed off your hair and transferred onto the balloon. This causes the balloon to gain a net negative charge and your hair to gain a net positive charge, which means the balloon and your hair are attracted to each other.

ELECTRICAL CONDUCTORS AND INSULATORS

Electrons move much more easily than protons. They also move more freely in some materials than in others.

In the atoms of **metals**, the outermost electrons are only very slightly attracted to their respective nuclei. As a consequence, metals are good **conductors** of electricity. In conductors, loosely held electrons can effectively 'jump' from one atom to another and move freely throughout the material. This can be seen in Figure 10.1.3.

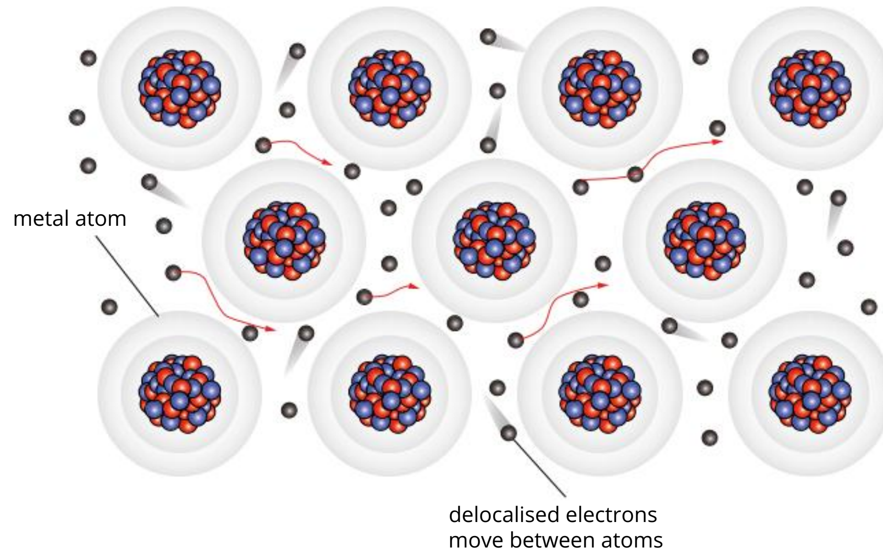


FIGURE 10.1.3 Electrons moving through a conductor. The outermost electrons are free to move throughout the lattice of positive ions.

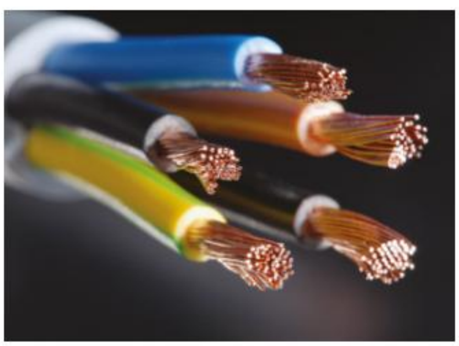


FIGURE 10.1.4 These copper wires conduct electricity by allowing the movement of charged particles.

Copper is an example of a very good conductor. For this reason, it is used in telecommunications, and electrical and electronic products (see Figure 10.1.4).

In comparison, the outer electrons in **non-metals** are very tightly bound to their respective nuclei or are held within chemical bonds and cannot readily move from one atom to another. Non-metals do not conduct electricity very well and are known as **insulators**. A list of common conductors and insulators is provided in Table 10.1.1.

TABLE 10.1.1 Some common conductors and insulators.

Good conductors	Good insulators
all metals, especially silver, gold, copper and aluminium, and any ionic solution	plastics polystyrene dry air glass porcelain cloth (dry)
Moderate conductors	**Moderate insulators**
water earth semiconductors, e.g. silicon, germanium skin	wood paper damp air ice, snow

PHYSICS IN ACTION

Lightning

Lightning is one of nature's greatest spectacles (see Figure 10.1.5). No wonder it was so long thought of as the voice of the gods. In the mid-eighteenth century, Benjamin Franklin showed that lightning is basically the same sort of electrical phenomenon as can be achieved by rubbing a glass rod with wool or rubbing a balloon on your hair.

FIGURE 10.1.5 Lightning bolts over a city skyline.

A typical lightning bolt transfers 10 or more coulombs of negative charge (over 60 billion billion electrons) in approximately one thousandth of a second. A moderate thundercloud with a few flashes per minute generates several hundred megawatts of electrical power, the equivalent of a small power station.

It is thought that, during a thunderstorm, charge is transferred in collisions between the tiny ice crystals that form as a result of the cooling of upwards-flowing moist air and the larger, falling hailstones. As a result of small temperature differences between the crystals and hailstones, the crystals become positively charged and the hailstones negatively charged. The crystals carry their positive charge to the top of the cloud while the negative charge accumulates in the lower region. There is normally also a second smaller positively charged region at the bottom owing to positive charges attracted up from the ground towards the negative region, as seen in Figure 10.1.6.

There will be strong electric fields between these regions of opposite charge. If they become sufficiently strong, electrons can be stripped from the air molecules (they become ionised). The strong negative charge of the lower region of the cloud will induce positive charges on tall objects on the ground. Because of the electric field, ions and delocalised (free) electrons will gain kinetic energy and collide with more molecules, starting an 'avalanche of charges'. This may lead to a discharge, which can form a conductive path for lightning. This is the flash seen either within the cloud or between the Earth and the cloud. Most flashes are within the cloud; only a relatively small number actually strike the ground.

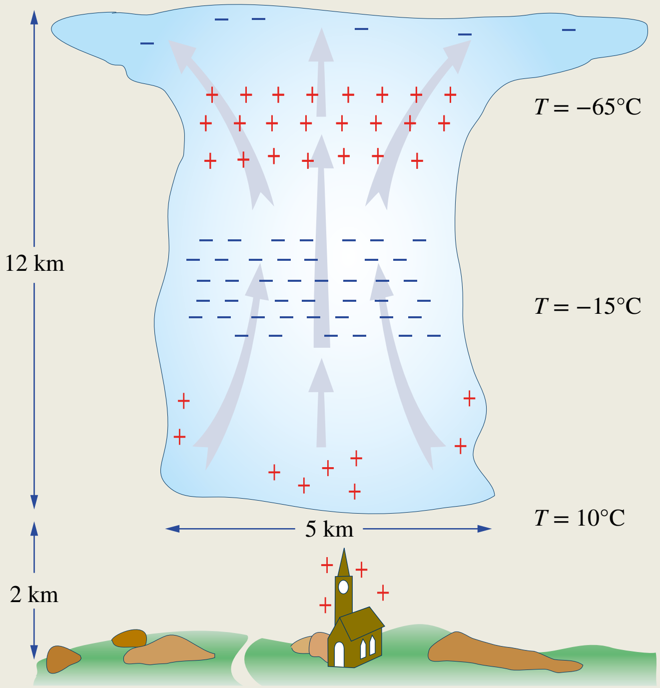


FIGURE 10.1.6 A thundercloud can be several kilometres wide and well over 10 km high. Strong updrafts drive the electrical processes that lead to the separation of charge that generate flashes of lightning.

EXTENSION

Semiconductors

Some materials, such as silicon, are known as semimetals or metalloids. Their properties are somewhere between those of metals and non-metals. For example, the electrons in a silicon atom are not as tightly bound to the nucleus as those in a non-metal. However, they are not as easy to remove as the electrons in a metal. Hence, silicon and elements like it are known as semiconductors.

Silicon's ability to conduct electricity can be adjusted by adding small amounts of other elements such as boron, phosphorus, gallium or arsenic in a process known as doping. Adding another substance contributes delocalised electrons, which can greatly increase the conductivity of silicon within electronic devices. This makes silicon very useful in the construction of computer chips like the one shown in Figure 10.1.7. As the world continues to become more reliant on computers, there is a growing need for semiconductors. Companies are continuing to develop computer chips that are smaller, faster and cheaper. These chips contain more transistors, which allows for the chip to work at a faster rate. This growing competition has resulted in an observation called Moore's law, which states that the number of transistors in a dense integrated circuit doubles approximately every two years.

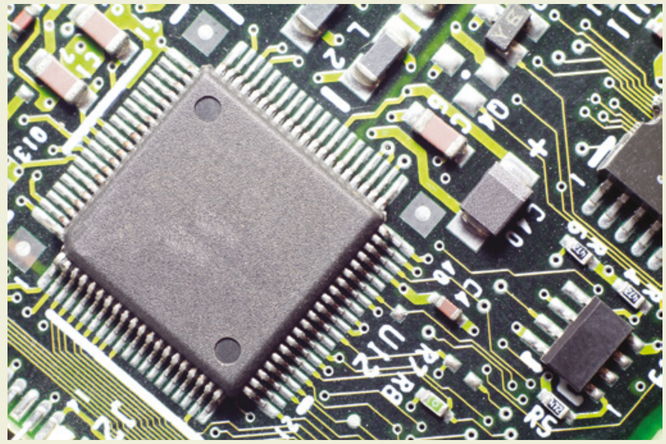


FIGURE 10.1.7 Silicon has been used in the construction of computer chips since the 1950s.

10.1 Review

SUMMARY

- Like charges repel; unlike charges attract.
- When an object loses electrons, it develops a positive net charge; when it gains electrons, it develops a negative net charge.
- The letter q is used to represent the quantity of charge. The SI unit of charge is the coulomb (C).
- The elementary charge (e), the charge on a proton, is equal to $+1.60 \times 10^{-19}$ C. The elementary charge, $-e$, of an electron is -1.60×10^{-19} C.
- Electrons move easily through conductors, but not through insulators. This is because the outermost electrons in materials that are good conductors are weakly attracted to the nucleus and are free to move, whereas electrons in insulators are more strongly held to the nucleus of the atom, or are involved in bonding atoms together.

KEY QUESTIONS

1. Plastic strip A, when rubbed, is found to attract plastic strip B. Strip C is found to repel strip B. What will happen when strips A and C are brought together?
2. Calculate how many electrons make up a charge of -5.00 C.
3. Calculate the charge, in coulombs, of 4.20×10^{19} protons.
4. Explain why electric circuits often consist of copper wires that are coated in protective plastic.
5. Explain, using the Bohr model, how a metallic wire can conduct electricity.

10.2 Energy in electric circuits

In this section, the concept of electrical potential energy will be explored, as well as the force that causes electrons to move in a circuit.

Free, delocalised electrons won't move around a circuit unless they are forced to. This force is provided by the electric field created between the positive and negative terminals of a source like a battery (Figure 10.2.1). Inside every battery, a chemical reaction is taking place where electrons are transferred from one reactant to another. These reactants are held at separate terminals. The chemical reactions convert chemical energy into potential energy as electrons are pushed from the positive terminal onto the negative terminal. This results in a separation of charge, which causes an energetic electric field in the space surrounding the terminals. The potential energy stored in this electric field is known as **electrical potential energy**, or just electrical potential. When a circuit connects two ends of the battery, the electric field travels through the wires at the speed of light, carrying the potential energy with it. It is the electric field that provides the force on the delocalised electrons within the wires that causes them to move as an electric current. The electrical potential energy stored in the field is transferred by the delocalised electrons in energy transformation devices like motors or light globes and is converted into other forms of energy such as kinetic energy, heat or light.

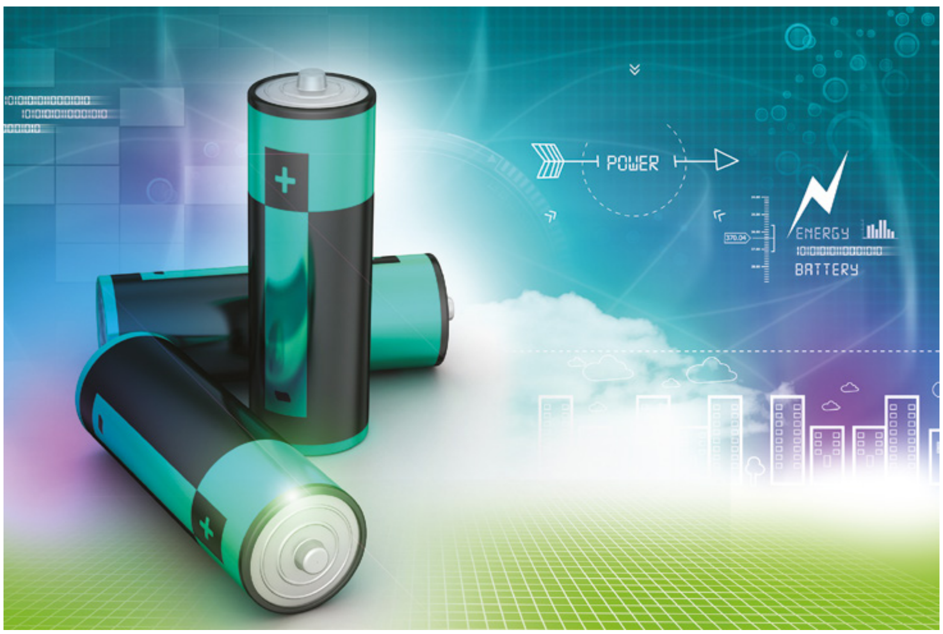


FIGURE 10.2.1 Chemical energy is stored and converted to electrical potential energy by batteries.

ENERGY AND POTENTIAL DIFFERENCE

Chemical energy is stored inside a battery as certain combinations of metals and compounds that will react in a reduction–oxidation (redox) reaction. In this type of reaction, the chemical energy is **transformed** into electrical potential energy. This potential energy is stored in the electric field that exists as a result of the separation and build-up of charge on the two terminals of the battery. This can be visualised as the coiling of invisible springs that exist between like charges that have been forced too close together. One terminal (the negative terminal) has a concentration of negative charges; the other terminal (the positive terminal) has a concentration of positive charges. When the battery is isolated (i.e. not connected in a circuit), the build-up of charge will oppose the redox reaction and so the reaction stops and the unused battery can maintain its reactants for years. Once the battery is connected to a device by wires, the redox reactions will continue for some time and will maintain the difference in charge between the two terminals.

PHYSICSFILE

Cells and batteries

A single cell generates electricity by converting chemical energy to electrical potential energy. A commercial battery can be a single cell or multiple cells connected together. Often a series of cells are packaged in a way that makes it look like a single device, but inside is a battery (series) of cells connected together, such as the module phone battery in this figure. The term 'battery' actually refers to a series of electric cells connected together (see Figure 10.2.2) but in everyday language, the terms 'battery' and 'cell' are often used interchangeably.

FIGURE 10.2.2 A mobile phone battery. The term battery actually refers to a group of electric cells connected together.

EXTENSION

Fields

Field theory says that there are certain properties of matter that create regions around an object in which matter with the corresponding property will experience a force. These forces are usually described as 'acting at a distance'. Mass creates a gravitational field in which other masses will experience an attractive force called gravity. Similarly, the poles of a magnet will create a field that will either attract or repel the pole of another magnet.

Electric fields form around any charged object. In these fields, other objects of charge will experience attractive or repulsive forces. Electric fields can be represented by arrows that show the direction that a small positive charge would move if placed in the field. The direction of the field will be away from a positively charged plate towards a negatively charged plate. An electron placed in an electric field would experience a force that is away from the negative plate and towards the positive plate, which is in the opposite direction of the field direction. This is shown in Figure 10.2.3.

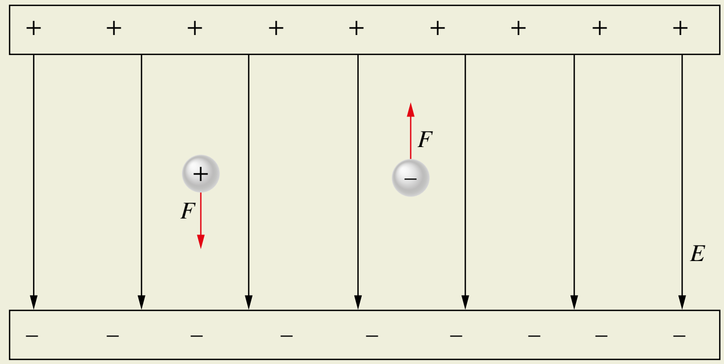


FIGURE 10.2.3 An electric field is created between two charged plates. The direction of the field is shown by the arrows labelled *E*, and the force on a positive particle and a negative particle is represented by the force vectors labelled *F*.

The shape of the electric field around some charged objects is shown in Figure 10.2.4. Electric and magnetic fields are actually unified into one type of field called an electromagnetic field. This implies that there is a connection between magnetism and electricity. One feature of electromagnetic fields is that they spread at the speed of light. The concept of fields will be developed further in Unit 3, in Year 12.

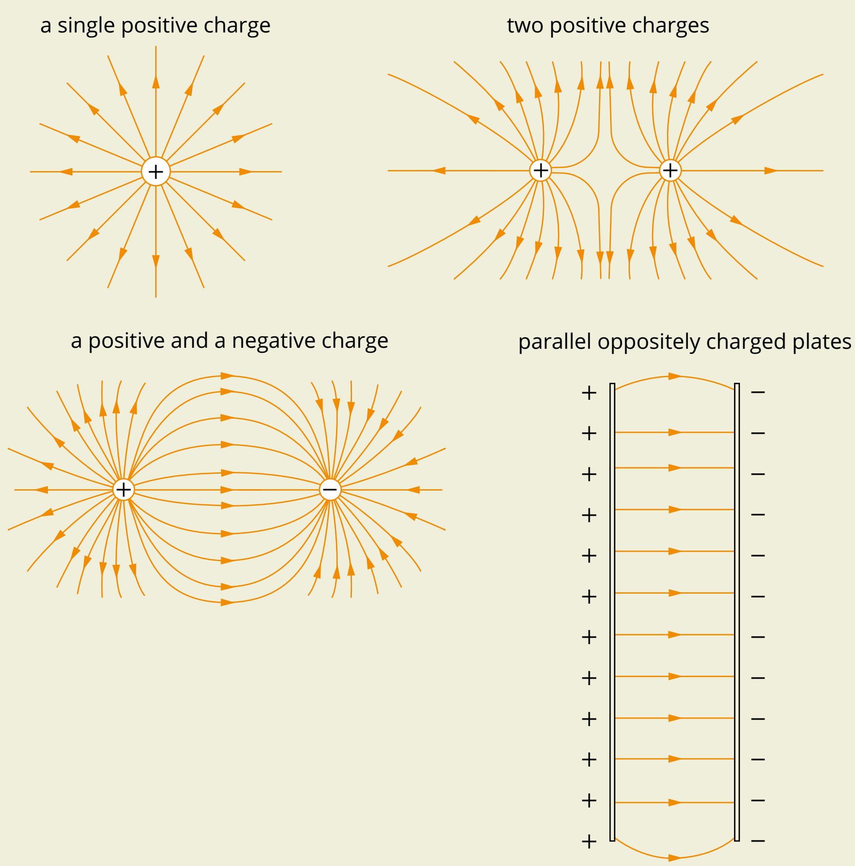


FIGURE 10.2.4 Electric fields can be represented by lines showing the direction of the field. The closeness of the lines indicates the strength of the field.

The difference in electrical potential energy between the two terminals of a battery can be quantified as a difference in the electrical potential energy per unit charge. This is commonly called **potential difference** (ΔV) and is measured in **volts** (V).

It is this potential difference at the terminals of the battery that provides the energy to a circuit. The energy is stored in the electric field and then transferred via delocalised electrons to different components in the circuit. At each component the energy is transformed into a different type of energy. For example, the energy could be transformed into light and heat if the component is a light bulb. If the component is a fan, the energy is transformed into kinetic energy (motion), with some heat and sound.

Energy transfers and transformations in a torch

A torch is a simple example of how energy is transformed and transferred within a circuit. In the torch shown in Figure 10.2.5, chemical energy in the battery is transformed into electrical potential energy in the electric field. There are two batteries connected in series so a bigger potential difference is available. Energy can be transferred via the electrons to the light bulb once the end terminals of the batteries have been connected to the torch's circuit: that is, when the torch is switched on. Once the connection is made in the switch, the electric field spreads through the wire at the speed of light. The electric field applies a force on every charged object within the wire, including protons in each nucleus and electrons around each atom. The positively charged nuclei are held in place by the lattice structure of metals, while the inner electrons are held strongly by their attraction to the nucleus, and so they will not move. The delocalised outer electrons, however, are only loosely held and so they will move in the opposite direction to the electric field. As they move, they interact with the matter in the device and **transfer** the energy from the field into other forms. In the case of a torch, they transform electrical potential energy in the electric field into the kinetic energy of the atoms within the filament wire in the bulb, which is then emitted as heat and light.

The energy changes can be summarised as:

$$\text{chemical energy} \xrightarrow{\text{transformed}} \text{electrical potential energy}$$

$$\text{electrical potential energy} \xrightarrow[\text{via free electrons}]{\text{transformed}} \text{kinetic energy (filament atoms)}$$

$$\text{kinetic energy (filament atoms)} \xrightarrow{\text{transformed}} \text{heat energy + light}$$

Eventually, when most of the chemicals within the battery have reacted, the battery is no longer able to provide enough electrical potential energy to power the torch. This is because the chemical reaction has slowed, and electrons are not being driven to the negative terminal in sufficient numbers. The torch stops working and the batteries are said to have gone flat.

Quantifying potential difference

As with other forms of energy, it is useful to be able to quantify the amount of potential difference in a given situation. Potential difference is formally defined as the amount of electrical potential energy given to each coulomb of charge. As an equation, it is:

$$\Delta V = \frac{E}{q}$$

where ΔV is potential difference (V)

E is electric potential energy (J)

q is charge (C).

Since energy is measured in joules and charge in coulombs, the potential difference is measured in joules per coulomb ($J\,C^{-1}$). This quantity has been assigned a unit, the volt (V) to honour Alessandro Volta, who invented the first battery. A potential difference of $1\,J\,C^{-1}$ is equal to 1 V. When a battery is labelled 9 V, this means that the battery provides 9 joules of energy to each coulomb of charge.

PHYSICSFILE

Voltage and potential difference

In the past, the term 'voltage' has been used interchangeably with the term 'potential difference', with both volts and potential difference sharing the symbol, V. However, this is confusing because the volt is the unit of measurement for potential difference. In this textbook, for the quantity of potential difference, we will use the symbol ΔV. The unit, volts, has the symbol V, which is not in italics. The context usually makes it clear which meaning is intended. This textbook will use the more correct term, potential difference, ΔV.

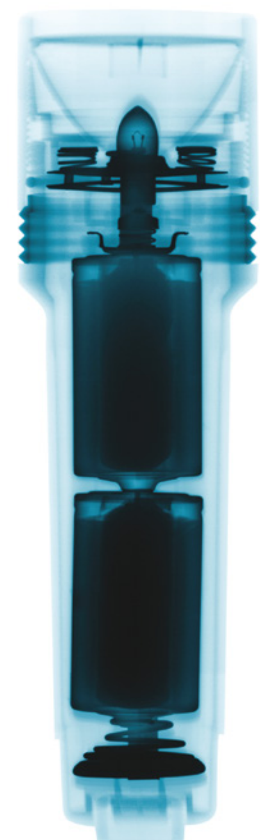


FIGURE 10.2.5 An X-ray image of the internal structure of a torch. The bulb and two batteries are clearly visible.

PHYSICSFILE

Birds on a wire

Birds can sit on power lines and not get electrocuted even though the wires are not insulated.

For a current to flow through a bird on a wire, there would have to be a potential difference between its two feet. Since the bird has both feet touching the same wire, even a wire that might carry a very high potential (voltage), there is no potential difference between the bird's feet. If the bird stood on the wire and touched any other object, such as the ground or another wire, then it would get a big electric shock. This is because there would be a potential difference between the wire and the other object, causing a current to flow.

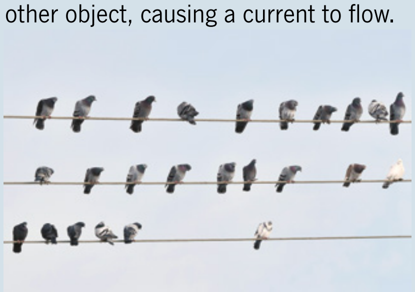


FIGURE 10.2.6 There is no potential difference between each bird's feet.

Worked example 10.2.1

USING THE DEFINITION OF POTENTIAL DIFFERENCE

Calculate the amount of electrical potential energy carried by 5.00 C of charge at a potential difference of 10.0 V.

Thinking	Working
Recall the formula for potential difference.	$\Delta V = \frac{E}{q}$
Rearrange this formula to make energy the subject.	$E = \Delta V q$
Substitute in the appropriate values and solve.	$E = (10.0)(5.00)$ $E = 50.000$ $E = 50.0\text{ J}$

Worked example: Try yourself 10.2.1

USING THE DEFINITION OF POTENTIAL DIFFERENCE

A car battery can provide 3650 C of charge at 12.0 V. How much electrical potential energy is stored in the battery?

MEASURING POTENTIAL DIFFERENCE: THE VOLTMETER

Potential difference is usually measured by a device called a **voltmeter**.

When a circuit component is connected to a complete circuit, the component transforms electrical potential energy from the field to do work, and so the field decreases in energy across the component. Therefore, the electrical potential energy in the field before the component is higher than the electrical potential energy in the field after the component. Voltmeters are wired into circuits to measure the change in electrical potential between two points as current passes through a particular component. This means that one wire of the voltmeter is connected to the circuit before the component, and the other wire is connected to the circuit after the component. This is called connecting the voltmeter 'in parallel'.

In Figure 10.2.7, the voltmeter is connected to the circuit on either side of the light globe to measure the potential difference (voltage drop) across the light globe. It is important to connect the voltmeter with its positive terminal closest to the positive terminal of the power supply and its negative terminal connected closest to the negative terminal of the power supply.

FIGURE 10.2.7 A voltmeter measures the potential difference (in this case, 6.23 V) across a light globe.

Analogies for potential difference

Analogies (comparisons) help us to understand concepts that cannot be seen directly. By analysing how an analogy functions, you can make comparisons to the concept you are studying. There are a number of popular analogies for electrical potential difference, including pumps that cause water pressure in a pipe, and pedals pushing a bike chain around the cogs. A particularly useful analogy compares a source of electrical potential energy to an escalator at a shopping centre. Just as a cell or battery uses energy to push electrons to a higher energy state, an escalator uses energy to lift people to a higher level, increasing their gravitational potential energy. This analogy effectively links the gain in electrical potential energy in a cell to the gain in gravitational potential energy in the shopping centre, as in both cases the energy is stored in a field ready to do work via the electrons or the people.

10.2 Review

SUMMARY

- Electric potential difference measures the difference in electric potential energy available per unit charge.
- The difference in electrical potential between the two terminals of a battery can be quantified as a difference in the electrical potential energy per unit charge using the following equation:

  $E = \Delta Vq$ or $\Delta V = \dfrac{E}{q}$
- In a circuit, the energy required for charge separation is provided by a cell or battery. The chemical energy within the cell is transformed into electric potential energy stored in an electric field.

KEY QUESTIONS

1 Under what conditions will charge flow between two bodies linked with a rod? Choose the correct response from the following options.

A The potential difference between the bodies is not zero and the rod is made of a conducting material.

B The potential difference between the bodies is not zero and the rod is made of an insulating material.

C The potential difference between the bodies is equal to zero and the rod is made of a conducting material.

D The potential difference between the bodies is equal to zero and the rod is made of an insulating material.

2 Calculate the potential difference of a battery that gives a charge of 10.0 C if it delivers:

a 40.0 J of energy

b 15.0 J of energy

c 20.0 J of energy.

3 A charge of 5.00 C flows from a battery through an electric water heater and delivers 111 J of heat. What was the potential difference of the battery?

4 How much charge must have flowed through a 12.0 V car battery if 2.00 kJ of energy was delivered to the starter motor?

10.2 Review *continued*

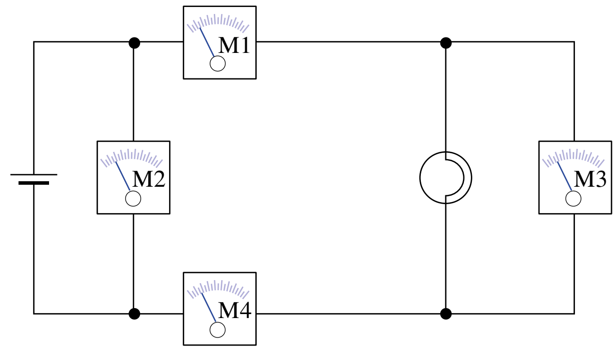
5 A light globe that is connected to 240.0 V of mains power uses 3.60 kJ of electric potential energy in 1.00 minute.

a What type(s) of energy has the electrical energy been transformed into?

b Calculate the charge that flows through the globe.

6 The electrical energy obtained from a battery can be compared to the energy of water stored in a hydroelectric dam in the mountains. In this analogy, to what could the electrical potential difference of the battery be likened?

7 Andrea wishes to measure the potential difference across a light globe and has set up a circuit as shown below.



In which positions (M1, M2, M3 or M4) should Andrea place a voltmeter?

10.3 Electric current and circuits

A flow of electric charge is called electric **current**. Current can occur as delocalised (free) electrons moving in a wire or as ions moving in solution. This section explores current as it flows through wires in electric circuits.

Electric circuits are involved in much of the technology used every day and are responsible for many familiar sights (Figure 10.3.1). To construct electric circuits, you must know about the different components of a circuit and be able to interpret circuit diagrams.

FIGURE 10.3.1 Electric circuits are responsible for lighting up entire cities.

ELECTRIC CIRCUITS

An **electric circuit** is a path made of conductive material, through which charges can flow in a closed loop. This flow of charges is called electric current. The most common conductors used in circuits are metals, such as copper wire. The charges that flow around the circuit within the wire are negatively charged delocalised electrons. The movement of electrons in the wire is called **electron flow**.

Electric fields created by the separation of charge in a cell can travel through air, but air is a very poor conductor of electricity. Therefore, no electrons will leave the negative end of a battery and travel to the positive end if the potential difference is small and the air gap is large. If, however, a substance like copper is placed in between the two charged ends of a battery, the electric field will concentrate and travel through the metal in preference to spreading through the air. The electric field spreads through the circuit at the speed of light and causes any delocalised electrons in the copper wire to move in the opposite direction to the electric field. It is interesting to note that while the electric field propagates through the circuit at the speed of light, the delocalised electrons drift through the circuit at the speed of snails, that is millimetres per minute.

A simple example of an electric circuit is shown in Figure 10.3.2. The light bulb is in contact with the positive terminal of the battery; a copper wire joins the negative terminal of the battery to one end of the filament in the light bulb. This arrangement forms a closed loop that allows delocalised electrons that are already in the circuit to flow away from the negative terminal towards the positive terminal of the battery. The battery is the source of energy and provides the electric field that spreads through the circuit. The light bulb converts this energy into heat and light energy via the delocalised electrons when the circuit is connected.

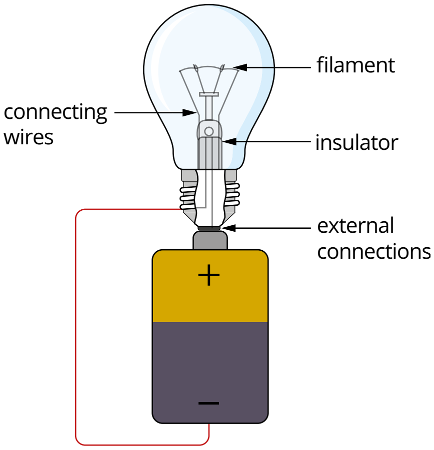


FIGURE 10.3.2 When there is a complete conduction path from the positive terminal of a battery to the negative terminal, a current flows.

If a switch is added to the circuit in Figure 10.3.2, the light bulb can be turned off and on. When the switch is closed (in the 'on' position), the electric field spreads through the wires and the circuit is complete. The current flows in a loop, in response to the electric field, along a path made by the conductors.

When the switch is open (in the 'off' position), there is a break in the circuit, which means that there is an air gap between the terminals of the switch. Although the field is established through the air gap, no current can flow through the gap if the potential difference is low enough and the separation is large enough. Therefore current can no longer flow in the circuit. This is what happens when you turn off the switch for a lamp or TV. A circuit where the conducting path is broken is often called an open circuit.

> Current will flow in a circuit only when the circuit forms a continuous (closed) loop from one terminal of a power supply to the other terminal.

One common misconception about current is that charges are used up or lost when a current flows around a circuit. In fact, the charge-carrying electrons are conserved at all points in a circuit. The electrons do work on components of the circuit by transferring energy to them, so it is actually the energy that is 'lost' as it is transferred and transformed.

REPRESENTING ELECTRIC CIRCUITS

A number of different components can be added to an electric circuit. A circuit diagram is a graphical representation of all the elements in a circuit.

Common symbols for electronic components

It is not necessary to draw detailed pictures of the components in a circuit because simple symbols are much clearer. The common symbols used to represent the electrical components in electric circuits are shown in Figure 10.3.3.

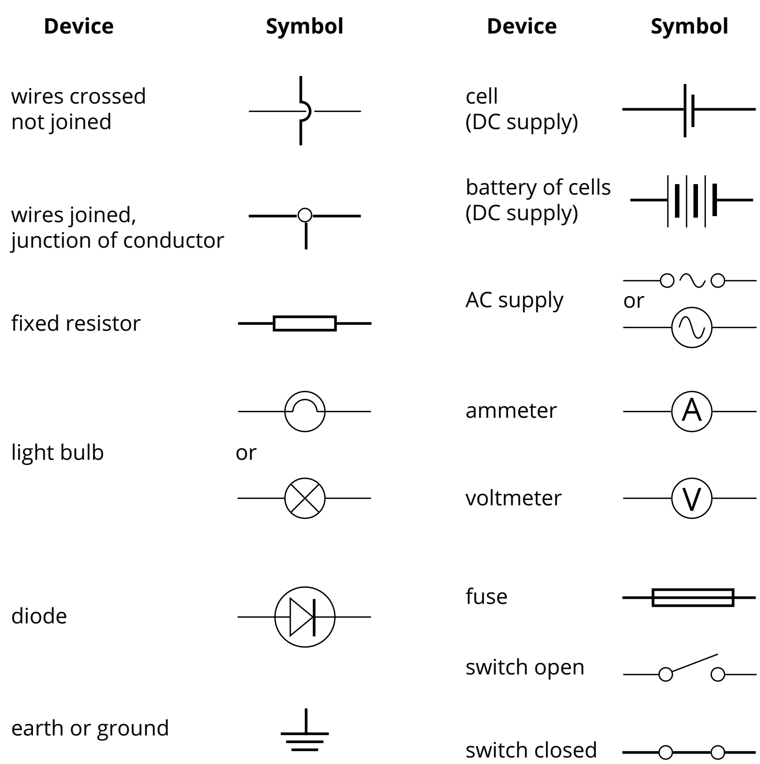


FIGURE 10.3.3 Some commonly used electrical devices and their symbols.

Circuit diagrams

When building anything, it is important that the builder has a clear set of instructions from the designer. This is as much the case for electric circuits as it is for a tall building or a motor vehicle.

Circuit diagrams are used to clearly show how the components of an electric circuit are connected. They simplify the physical layout of the circuit into a diagram that is recognisable by anyone who knows how to interpret it. You can use the list of common symbols for electrical components (Figure 10.3.3) to interpret any circuit diagrams.

The circuit diagram in Figure 10.3.4b shows how the components of the torch shown in Figure 10.3.4a are connected in a circuit. The circuit can be traced by following the straight lines representing the connecting wires.

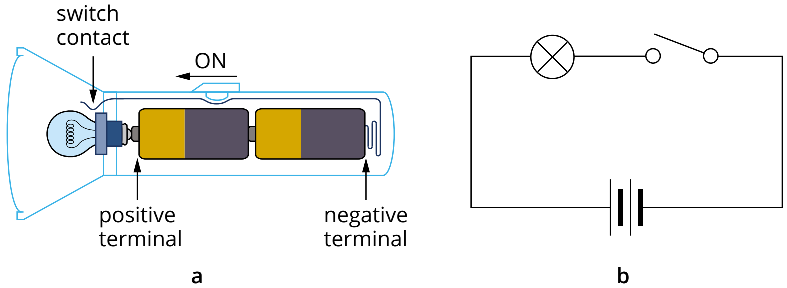


FIGURE 10.3.4 (a) A light bulb and batteries connected by conductors in a torch constitute an electric circuit. (b) The torch's circuit can be represented by a simple circuit diagram.

Conventional current vs electron flow

The study of electrical currents preceded the discovery of the electron by J.J. Thomson in 1897. Initially, it was incorrectly thought that the charges that flowed in circuits were positive. Based on this, scientists traditionally talked about electric current as if current flowed from the positive terminal of the battery to the negative terminal. This convention is still used today, even though it is now known that it is actually the negative charges (electrons) that flow around a circuit.

In a circuit diagram, current is indicated by a small arrow and the symbol I. This is called **conventional current** or just current. The direction of conventional current is opposite to the direction of electron flow (Figure 10.3.5).

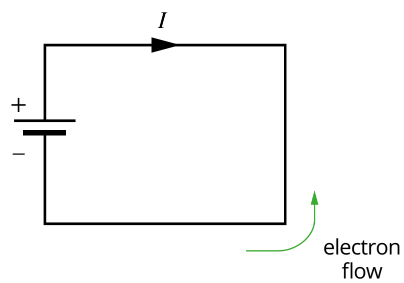


FIGURE 10.3.5 Conventional current (I) and electron flow are in opposite directions.

Conventional current (or current), I, flows from the positive terminal to the negative terminal of a power supply.

Electron flow (or electron current) refers to the flow of electrons from the negative terminal to the positive terminal of a power supply.

The amount of charge is the same in both cases for the same circuit.

QUANTIFYING CURRENT

In common electrical circuits, a current consists of electrons flowing within a copper wire (Figure 10.3.6). This current, I, can be defined as the amount of charge that passes through a point in the conductor per second.

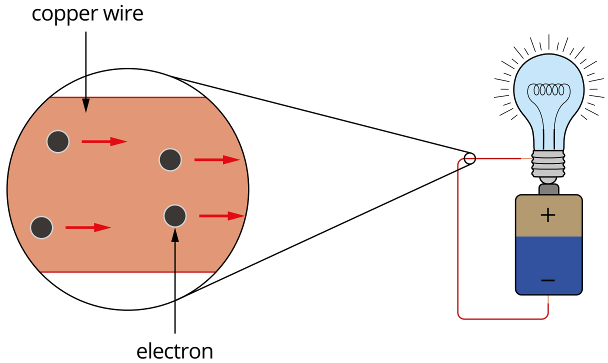


FIGURE 10.3.6 The sum of the charges of the delocalised electrons that pass through a point per second gives a measure of the current. Because these electrons do not leave the wire, current is conserved in all parts of the circuit.

An equation to express this is:

$$I = \frac{q}{\Delta t}$$

where I is the current in amperes (or amps, A)

q is the amount of charge in coulombs (C)

Δt is the period of time that has passed, in seconds (s).

One ampere is equivalent to one coulomb per second ($C\,s^{-1}$).

Current is a flow of charge and the charge is carried by electrons. The charge that flows is equal to the number of electrons (n_e) that flow through a particular point in the circuit multiplied by the charge on one electron ($q_e = -1.60 \times 10^{-19}$ C). When this total charge is divided by the period of time (Δt) that has elapsed in seconds, this gives you the current in amperes (A). This makes the equation:

$$I = \frac{q}{\Delta t} = \frac{n_e q_e}{\Delta t}$$

A typical current in a circuit powering a small DC motor would be about 50.0 mA. Even with this seemingly small current, approximately 3.13×10^{17} electrons flow past any point on the wire each second.

Worked example 10.3.1

USING $I = \frac{q}{\Delta t}$

Calculate the number of electrons that flow past a particular point each second in a circuit that carries a current of 0.500 A.	
Thinking	**Working**
Rearrange the equation $I = \frac{q}{\Delta t}$ to make q the subject.	$I = \frac{q}{\Delta t}$ so $q = I\Delta t$
Calculate the amount of charge that flows past the point in question by substituting the values given.	$q = (0.500)(1.00)$ $q = 0.500$ C
Find the number of electrons by dividing the charge found by the charge on one electron (1.60×10^{-19} C). Be sure to present your answer to the correct number of significant figures.	$n_e = \frac{q}{q_e}$ $n_e = \frac{(0.500)}{(1.60 \times 10^{-19})}$ $n_e = 3.12500 \times 10^{18}$ $n_e = 3.13 \times 10^{18}$ electrons

Worked example: Try yourself 10.3.1

USING $I = \frac{q}{\Delta t}$

Calculate the number of electrons that flow past a particular point each second in a circuit that carries a current of 0.750 A.

Measuring current: The ammeter

Current is commonly measured by a device called an **ammeter**. Figure 10.3.7 shows an ammeter connected along the same path taken by the current flowing through the light bulb. This is referred to as connecting the ammeter 'in series'. Series circuits are covered in more detail in Section 10.5. The positive terminal of the ammeter is connected so that it is closest to the positive terminal of the power supply. The negative terminal of the ammeter is closest to the negative terminal of the power supply.

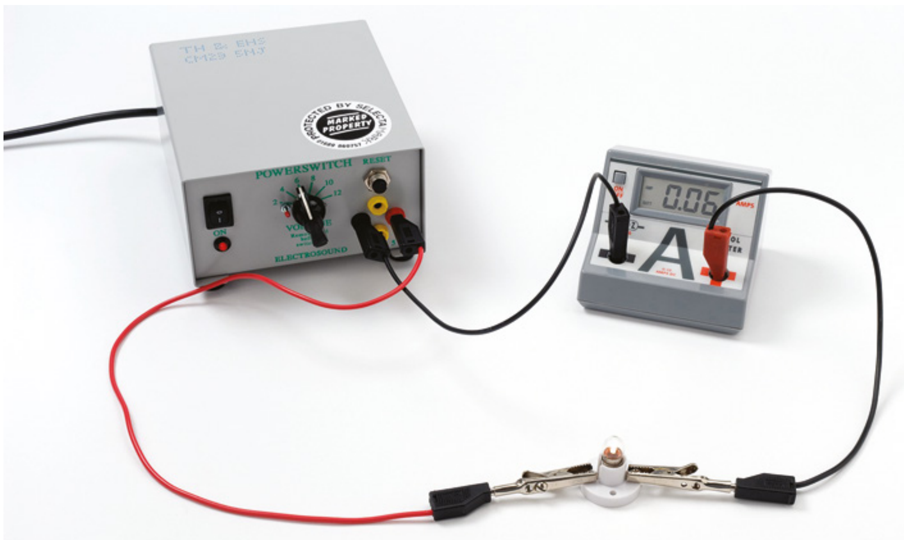


FIGURE 10.3.7 A digital ammeter (labelled with an A) measures current in a circuit.

Measuring the current is possible because charge is conserved at all points in a circuit. This means that the current that flows into a light bulb is the same as the current that flows out of the light bulb. An ammeter can therefore be connected before or after the bulb in series to measure the current. Table 10.3.1 lists some typical values for electric current in common situations.

TABLE 10.3.1 Typical values for electric current.

Situation	Current
lightning	10 000 A
starter motor in car	200 A
fan heater	10 A
toaster	3 A
light bulb	400 mA
pocket calculator	5 mA
nerve fibres in body	1 μA

WORK DONE BY A CURRENT IN A CIRCUIT

In electrical circuits, electrical potential energy is converted into other forms of energy. When energy is changed from one form to another, work is done. (Work is covered in more detail in Chapter 5.) The amount of energy provided by a circuit can be calculated using the definitions for potential difference $\Delta V = \frac{W}{q}$ and current $I = \frac{q}{\Delta t}$.

Rearranging the definition of potential difference gives:

$W = \Delta V q$

Using the definition of current:

$q = I \Delta t$

PHYSICSFILE

Multimeters

The internal circuitry of a voltmeter is significantly different to an ammeter. Electricians and scientists who work with electrical circuits often find it inconvenient to keep collections of voltmeters and ammeters, so they have found a way to bundle up the circuitry for both meters into a single device known as a multimeter.

This is much more convenient because the multimeter can be switched easily from being an ammeter to being a voltmeter if needed. However, when a multimeter is switched from one mode to another, it is important to make a corresponding change to the way it is connected to the circuit being measured. An ammeter is connected in series and a voltmeter is connected in parallel. In fact, if a multimeter is working as an ammeter, and it is connected in parallel like a voltmeter, it may draw so much current that its internal circuitry will be burnt out and the multimeter will be destroyed.

FIGURE 10.3.8 A digital multimeter can be used as either an ammeter or a voltmeter.

Therefore:

$W = \Delta VI\Delta t$

where W is the work done by the current (J)

ΔV is the potential difference (V)

I is the current (A)

Δt is the period of time (s).

Since the work done by the current W is the same as the energy provided by the current E, this can also be written as:

$E = \Delta VI\Delta t$

This gives you a practical way to calculate the energy used in a circuit from measurements you can make with a multimeter.

Worked example 10.3.2

USING $E = \Delta VI\Delta t$

A potential difference of 12.0 V is used to generate a current of 755 mA to heat some water for 5.00 minutes. Calculate the energy transferred to the water during that time.	
Thinking	**Working**
Convert the quantities to SI units.	$I = 755\,\text{mA} = 755 \times 10^{-3}\,\text{A}$ $I = 0.755\,\text{A}$ $\Delta t = (5.00)(60.0) = 300\,\text{s}$
Substitute values into the equation and calculate the amount of energy in joules. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$E = \Delta VI\Delta t$ $E = (12.0)(0.755)(300)$ $E = 2718$ $E = 2720\,\text{J}$

Worked example: Try yourself 10.3.2

USING $E = \Delta VI\Delta t$

A potential difference of 12.0 V is used to generate a current of 1750 mA to heat water for 7.50 minutes. Calculate the energy transferred to the water in that time.

Rate of doing work: Power

If purchasing a new kettle, you might want to consider how quickly different kettles boil water. Printed on all appliances is a rating for the power of that device. **Power** is a measure of how fast energy is converted by the appliance. In other words, power is the rate at which energy (E) is transformed by the components within the device. This can also be described as the rate at which work (W) is done. As an equation:

$$P = \frac{\text{work done}}{\text{time}} = \frac{W}{\Delta t} = \frac{E_{\text{transformed}}}{\Delta t} = \frac{\Delta VI\Delta t}{\Delta t} = \Delta VI$$

where P is the power in joules per second (J s^{-1}). One joule per second is 1 watt (W).

The more powerful an appliance is, the faster it can do a given amount of work. In other words, an appliance that draws more power can do the same amount of work in a shorter amount of time. Therefore, if you want something done quickly, you need an appliance with a higher power rating.

By rearranging the expression above, you can calculate the energy transformations in a circuit by measuring potential difference across and current through circuit components and the period of time over which the current flows.

Given $P = \frac{W}{\Delta t} = \Delta VI$

Then work done, $W = \Delta VI\Delta t$.

The power dissipated by those components can be calculated in watts (W), and the work done is given in joules (J).

Worked example 10.3.3

USING $P = \Delta VI$

An appliance running on 235 V draws a current of 4.50 A. Calculate the power used by this appliance.	
Thinking	**Working**
Identify the relationship needed to solve the problem.	$P = \Delta VI$
Identify the required values from the question, substitute and calculate. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$P = (235)(4.50)$ $P = 1057.5$ $P = 1060\text{ W}$

Worked example: Try yourself 10.3.3

USING $P = \Delta VI$

An appliance running on 125 V draws a current of 7.50 A. Calculate the power used by this appliance.

ANALOGIES FOR ELECTRIC CURRENT

Since you cannot see the movement of electrons in a wire, it is sometimes helpful to use analogies or 'models' to visualise or explain the way an electric current behaves. It is important to remember that no analogy is perfect, so there may be situations where the electric current does not act as you would expect from the analogy.

Water model

A very common model is to think of electric current as water being pumped up to a high tank that empties into a closed pipe system that is already full of water, as shown in Figure 10.3.9 (on the next page). It is important to understand that water cannot be lost in this system, just as electrons cannot be lost in a circuit. The pressure created by a pump pushes water up to a higher potential energy level, just as the chemical reactions push electrons to a higher potential energy level as it creates the electric field. The electric field created by the battery pushes all the delocalised electrons through the wires, and the gravitational field pulls the water down through the pipe. Since the water cannot be lost, the same amount of water flows in every part of a pipe, just as the electric current is the same in every part of a wire. Light bulbs in an electric circuit are like turbines: the turbine converts the energy stored in the gravitational field via the flowing water into the kinetic energy of the turbine, and a light bulb converts electrical energy stored in the electric field into heat and light.

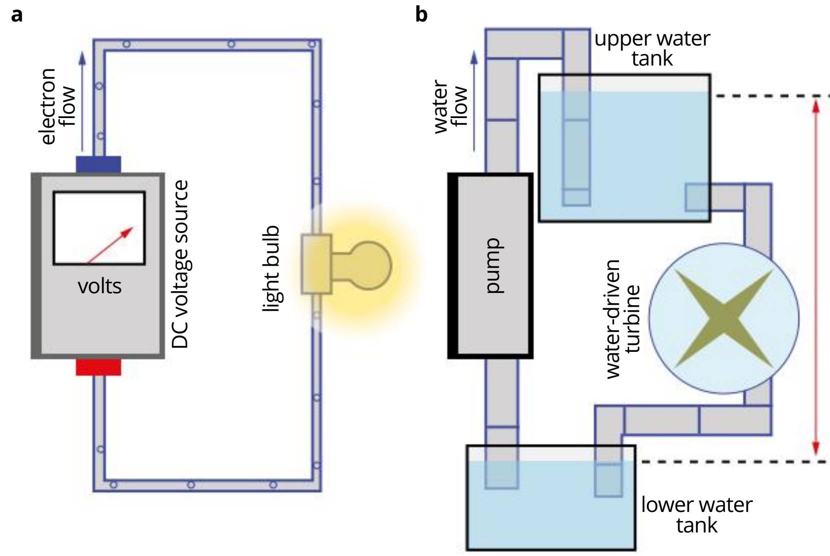


FIGURE 10.3.9 An electric current flowing in a circuit (a) can be compared to water flowing through a pipe system (b).

This model explains the energy within a circuit quite well.

- The power supply transfers energy to the electric field by forcing the electrons towards the negative end of the battery.
- The electrical energy in the electric field is transferred via the electrons in the components in the circuit and is converted into other forms.

10.3 Review

SUMMARY

- Current will flow in a circuit only when the circuit forms a continuous (closed) loop from one terminal of a power supply to the other terminal.
- When an electric current flows, delocalised (free) electrons all around the circuit move towards the positive terminal at the same time, due to the electric field spreading at the speed of light. This is called electron flow.
- Conventional current in a circuit flows from the positive terminal to the negative terminal.
- Current, I, is defined as the amount of charge, q, that passes through a point in a conductor per second. It has the unit amperes or amps (A), which are equivalent to coulombs per second. The equation for this is:

$$I = \frac{q}{\Delta t} = \frac{n_e q_e}{\Delta t}$$

- Current is measured with an ammeter connected along the same path as the current flowing (in series) within the circuit.
- Work is the measure of energy transfer. In an electrical circuit, work is done by charge carriers (electrons) when energy is transformed. Work can be defined and quantified using the relationships:

$$W = \Delta V q = \Delta V I \Delta t$$

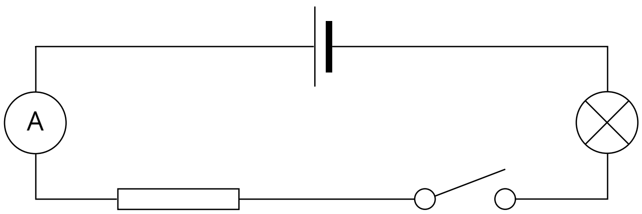
- Power is the rate at which energy is transformed in a circuit component. It is defined and quantified by the relationships:

$$P = \frac{W}{\Delta t} = \Delta V I$$

KEY QUESTIONS

1 What are the requirements for current to flow around a circuit?

2 List the electrical devices shown in the circuit diagram.



3 Why do scientists refer to conventional current as flowing from positive to negative?

A Protons flow from the positive terminal of a battery to the negative terminal.

B Electrons flow from the positive terminal of a battery to the negative terminal.

C Originally, scientists thought that charge carriers were positive.

D Charges flow in both directions in a wire; conventional current refers to just one of the flows.

4 Calculate the current flowing in a light bulb through which a charge of 30.0 C flows in:

a 10.0 seconds

b 1.00 minute

c 1.00 hour.

5 A car headlight may draw a current of 5.00 A. Calculate the charge flowing through it in:

a 1.00 second

b 1.00 minute

c 1.00 hour.

6 Using the values given in Table 10.3.1 (page 383), calculate the amount of charge that would flow through a:

a pocket calculator in 10.0 min

b car starter motor in 5.00 s

c light bulb in 1.00 hour.

7 1.00×10^{20} electrons flow past a point in 4.00 seconds. Calculate:

a the amount of charge, in coulombs, that moves past the point in this time

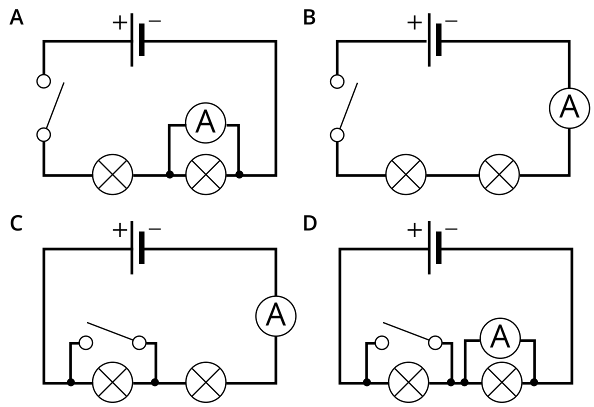
b the current, in amps.

8 A total of 3.20 C of charge flows past a point in a circuit over a period of 10.0 seconds. Calculate:

a the number of electrons that move past the point in this time

b the current, in amps.

9 Which of the circuits shown in the figure below would enable you to measure the current passing through both light bulbs when the switch is closed?



10 A freezer has a power rating of 475 W and is designed to be connected to 235 V. Calculate:

a the work performed by the freezer in 5.50 minutes

b the current flowing through the freezer.

10.4 Resistance

Resistance is an important concept because it links the ideas of potential difference and current. **Resistance** is a measure of how hard it is for current to flow through a particular material. As conductors allow current to pass through easily, they are said to have low resistance. Insulators have high resistance because they 'resist' or limit the flow of charges through them.

For a particular object or material, the amount of resistance can be quantified (given a numerical value). This means that the performance of electrical circuits can be studied, and predicted, with a high degree of confidence.

- Resistance is a measure of how hard it is for current to flow through a particular material.
- The unit for resistance is ohms (Ω).

RESISTANCE TO THE FLOW OF CHARGE

Energy is required to create and maintain an electric current. For electrons to move from one place to another, they need to first be separated from their atoms and then given energy to move. In some materials (i.e. conductors), the amount of energy required for this is negligible (almost zero). In insulators, a much larger amount of energy is required.

Once the electrons are moving through the material, energy is also required to keep them moving at a constant speed. Consider an electron travelling through a piece of copper wire. While it is common to imagine the wire as an empty pipe or hose through which electrons flow, a piece of copper wire is actually full of tightly packed copper ions arranged in a lattice structure. As an electron moves through the wire, it frequently 'bumps' into these copper ions. The electron needs constant 'energy boosts' to keep it moving in the right direction. This is why an electrical device stops working as soon as the energy source (e.g. battery) is disconnected.

PHYSICSFILE

Electron movement

Even without current, delocalised (free) electrons move around a piece of metal due to thermal effects. The delocalised electrons are rushing around at random with great speed. The net speed of an electron in one dimension through a wire, however, is quite slow. In the figure below, the dotted line AB shows the random motion path of an electron due to thermal effects. The solid line AB' shows the path of the same electron when an electric field is present. The two paths are similar and the combined effect of countless electrons moving together in this way results in a significant net movement of charge, so there is a net flow in the direction indicated.

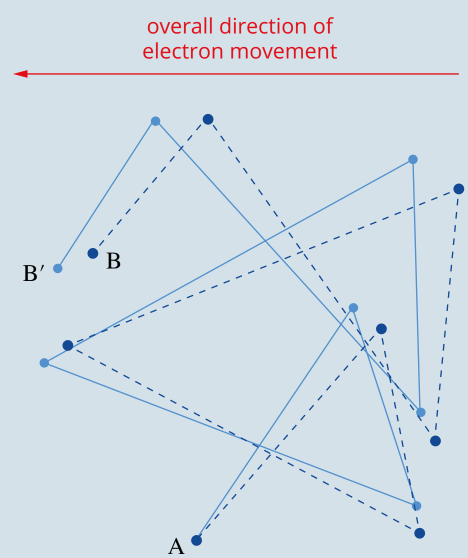


FIGURE 10.4.1 Path AB shows the random motion of an electron due to thermal effects. Path AB' shows the path of the same electron when an electric current is flowing in the direction indicated.

EXTENSION

Variables that affect resistance

Effect of cross-sectional area and length on resistance

Understanding the way electrons move through a wire can help you make some predictions about the resistance of different objects.

For example, in a longer piece of wire, the electrons bump into more ions along the way, so more energy is needed for the electrons to travel from one end to the other. In other words, a longer piece of wire provides greater 'resistance' to the flow of electric current.

Similarly, a thicker piece of wire allows more electrons to flow through it at the same time, much like a dual-lane highway allows more traffic flow than a single lane. The cross-sectional area of the wire (its area when viewed from the end) is the measurable variable that reflects the thickness of the wire. The greater the cross-sectional area of the wire, the lower its resistance will be.

Calculating the effect of length and area on resistance

The relationship between the resistance of a conductor and its length and thickness follows a mathematical relationship. There is a directly proportional relationship between resistance and length: doubling the length of the conductor doubles its resistance. There is an inversely proportional relationship between resistance and the cross-sectional area of the conductor. These relationships are captured in the equation:

$$R = \frac{\rho L}{A}$$

where R is resistance (Ω), L is length (m), A is cross-sectional area (m^2) and ρ is **resistivity** (Ω m), a property of the material from which the conductor is made.

Temperature and resistance

Another factor that affects the resistance of a material is its temperature. The temperature of an object is a measure of the average kinetic energy of its particles. The temperature of a solid is an indication of how quickly its particles are vibrating.

Increasing the temperature of a piece of copper wire means that the copper atoms will vibrate back and forth more quickly. This makes it more likely that an electron will collide with an atom as it moves past. Therefore, increasing the temperature of the wire also increases the resistance of the wire.

Similarly, current passing through a conductor can cause it to heat up. Think again of an electron moving through a copper wire: when the electron collides with a copper atom, it loses some of its kinetic energy. However, due to this collision, the copper atom gains kinetic energy, causing it to vibrate more quickly. An increase in the kinetic energy of the copper means that its temperature has increased, so the copper wire heats up.

This is one of the reasons why personal computers contain cooling fans, as shown in Figure 10.4.2. Electrical components are packed very tightly together on the computer motherboard. Cooling the components and the conductors that connect them prevents the computer from overheating. It also reduces the resistance of the components and helps them to run more efficiently. In modern devices, liquid cooling has also been utilised. This involves a closed-loop system using a liquid such as water. Water blocks are placed on hot parts of the computer, allowing the water to absorb the heat energy. The water is then pumped away and cooled by fans, exhausting the heat away from the computer. This process uses thermal transfer principles that are outlined in Chapter 6.

FIGURE 10.4.2 The cooling fan in this computer motherboard circulates air around the electrical components to cool them down.

Mathematically, the relationship is expressed as follows:

$$R = R_0\,[1 + \alpha\,(T - T_0)]$$

where R is the resistance of the conductor (Ω) at temperature T(K), R_0 is the resistance (Ω) at temperature T_0(K) and α is the temperature coefficient of resistance

(K^{-1}), which is a property of each material that describes how much changing temperature impacts the resistance of the material. Temperature coefficient of resistance values are shown for some common conducting materials in Table 10.4.1.

TABLE 10.4.1 Temperature coefficient of resistance values for some common conducting materials.

Substance	copper	tungsten	nickel	iron	steel
$\alpha\ (K^{-1})$	3.9×10^{-3}	4.5×10^{-3}	6.0×10^{-3}	5.0×10^{-3}	3.0×10^{-3}

PHYSICS IN ACTION

Incandescent light bulbs

The complex relationship between electric current and temperature is put to use in a very common application: the incandescent light bulb (Figure 10.4.3).

FIGURE 10.4.3 An incandescent bulb produces light when its filament heats up.

An incandescent light bulb consists of a thin piece of curled or bent wire, called a filament, in a glass bulb. Often the bulb is evacuated (has the air removed) or filled with an inert (unreactive) gas so that the metal filament does not corrode. The wire is usually made of tungsten or another metal with a high melting point.

When an electric current passes through the filament, it heats up. This in turn increases the resistance of the filament, causing it to heat up further. The filament quickly becomes so hot that it starts to glow, radiating heat and light.

Traditionally, most household lighting was provided by incandescent light bulbs. However, this form of lighting is very inefficient. Only a small amount of the energy that goes into an incandescent light bulb is transformed into light: over 95% of the energy is lost as heat.

An early alternative to incandescent light bulbs were fluorescent lighting, which were adopted in commercial settings. Fluorescent lights come in the form of tubes or bulbs; however, people often use fluorescent tube to refer to both types. Fluorescent tubes are low-pressure mercury vapour gas discharge tubes. Exciting the mercury with an electrical current produces ultraviolet light, which then causes phosphorous on the coating of the tube to glow. Fluorescent tubes are much more energy efficient in comparison to incandescent bulbs and produce a larger number of lumens per watt (i.e. they produce very bright light). However, they are more expensive than incandescent bulbs.

FIGURE 10.4.4 Examples of fluorescent tubes, fluorescent bulbs and LED bulbs. All have been alternatives to incandescent lightbulbs.

A more recent development in lighting is LEDs (light-emitting diodes). LEDs are comprised of a semiconductor that emits light as current flows through it. LEDs are much more efficient and are being used in preference to incandescent light bulbs and fluorescent tubes. Examples of these three light sources are shown in Figure 10.4.4. Since 2007, Australian state, territory and federal governments have planned to phase out the use of incandescent bulbs, which has resulted in huge reductions in electrical energy lost as heat.

Many household electrical heating devices such as toasters and bar heaters (which have a visible heating element that glows) work on a similar principle to the incandescent light bulb; although, in these situations, light is the unwanted or wasted energy.

OHM'S LAW

Georg Ohm (1789–1854) discovered that if the temperature of a resistor was kept constant, the current flowing through it was directly proportional to the potential difference across it: mathematically, $I \propto \Delta V$. This relationship is known as Ohm's law. This relationship means that if the potential difference across the resistor is doubled, for example, then the current flowing through the resistor must also double. If the potential difference is tripled, the current would also triple.

Ohm's law is usually written as:

> $\Delta V = IR$
>
> where ΔV is the potential difference in volts (V)
>
> I is current in amps (A)
>
> R is the constant of proportionality called resistance, in ohms (Ω).
>
> This equation can be transposed to give a quantitative (mathematical) definition for resistance: $R = \frac{\Delta V}{I}$

If an identical potential difference produces two different sizes of current when separately connected to two light bulbs, then the resistance of the two light bulbs must differ. A higher current would mean a lower resistance of the light bulb, according to Ohm's law. This is because, when a conductor provides less resistance, more current can flow.

Worked example 10.4.1

USING OHM'S LAW TO CALCULATE RESISTANCE

When a potential difference of 3.00 V is applied across a piece of resistor wire, 5.00 A of current flows through it. Calculate the resistance of the wire.

Thinking	Working
Ohm's law is used to calculate resistance.	$\Delta V = IR$
Rearrange the equation to find R.	$R = \frac{\Delta V}{I}$
Substitute in the known values. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$R = \frac{(3.00)}{(5.00)}$ $R = 0.600\,\Omega$

Worked example: Try yourself 10.4.1

USING OHM'S LAW TO CALCULATE RESISTANCE

An electric bar heater draws 10.0 A of current when connected to a 245 V power supply. Calculate the resistance of the element in the heater.

OHMIC AND NON-OHMIC CONDUCTORS

Conductors that obey Ohm's law are known as **ohmic** conductors. Ohmic conductors are usually called **resistors**.

An ohmic conductor can be identified by measuring the current that flows through the conductor when different potential differences are applied across it.

Worked example 10.4.2

USING OHM'S LAW TO CALCULATE RESISTANCE, CURRENT AND POTENTIAL DIFFERENCE

The table below shows measurements for the potential difference and corresponding current for an ohmic conductor.

ΔV (V)	0.00	2.00	4.00	ΔV_2
I (A)	0.00	0.250	I_1	0.750

Determine the missing results, I_1 and ΔV_2

Thinking	Working
Determine the factor by which potential difference has increased from the second column to the third column.	$\frac{(4.00)}{(2.00)} = 2$ The potential difference has doubled.
Apply the same factor increase to the current in the second column, to determine the current in the third column (I_1). Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$I_1 = 2 \times (0.250)$ $I_1 = 0.500\,\text{A}$
Determine the factor by which current has increased from the second column to the fourth column.	$\frac{(0.750)}{(0.250)} = 3$ The current has tripled.
Apply the same factor increase to the potential difference in the second column, to determine the potential difference in the fourth column (ΔV_2). Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$\Delta V_2 = 3 \times (2.00)$ $\Delta V_2 = 6.00\,\text{V}$

Worked example: Try yourself 10.4.2

USING OHM'S LAW TO CALCULATE RESISTANCE, CURRENT AND POTENTIAL DIFFERENCE

The table below shows measurements for the potential difference and corresponding current for an ohmic conductor.

ΔV (V)	0.00	3.00	9.00	ΔV_2
I (A)	0.00	0.200	I_1	0.800

Determine the missing results, I_1 and ΔV_2

The data from an experiment measuring the current and potential difference for a device is usually plotted on an I–ΔV graph. If the conductor is ohmic, this graph will be a straight line, as can be seen in Figure 10.4.5.

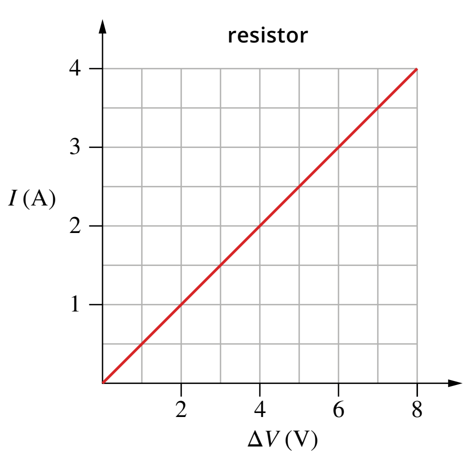


FIGURE 10.4.5 As the resistance of an ohmic conductor is constant, the I–ΔV graph is a straight line.

The resistance of the ohmic conductor (or resistor) can be found from the gradient of the I–ΔV graph. Ohm recognised that the gradient was equal to the inverse of the resistance:

$$\frac{1}{R} = \frac{\text{rise}}{\text{run}} = \frac{(4.0 - 1.0)}{(8.0 - 2.0)} = \frac{3.0}{6.0}$$

$$\therefore\ R = \frac{6.0}{3.0} = 2.0\,\Omega$$

However, not all conductors are ohmic. The I–ΔV graphs for **non-ohmic** conductors are not straight lines (see Figure 10.4.6). Light bulbs and diodes are examples of non-ohmic conductors.

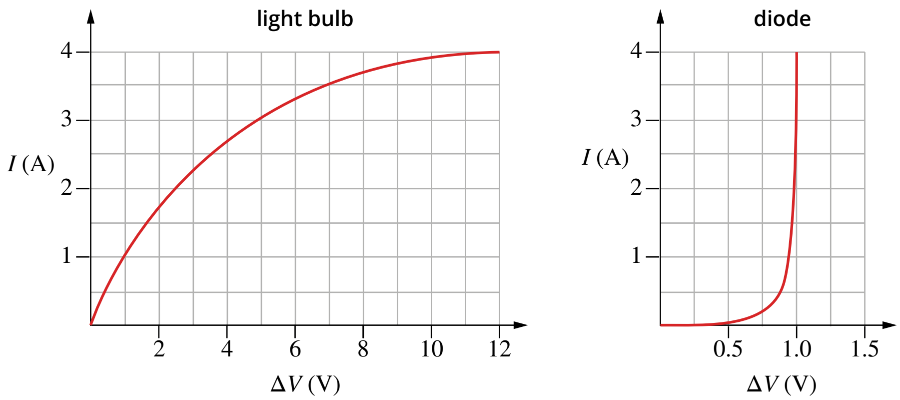


FIGURE 10.4.6 The I–ΔV graph for a non-ohmic resistor is not a straight line.

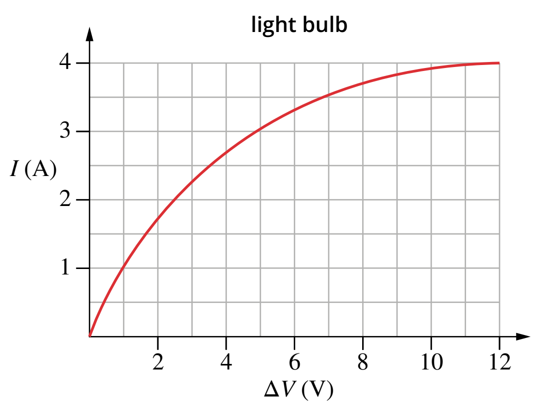
Using I–ΔV graphs to determine resistance

The inverse of resistance is defined as the ratio $I/\Delta V$. For an ohmic conductor, this value will be a constant regardless of the potential difference across the conductor. However, the resistance of a non-ohmic conductor will vary. The resistance of a non-ohmic conductor for a particular potential difference can be found by determining the current flowing through the conductor at this value.

Worked example 10.4.3

CALCULATING RESISTANCE FOR A NON-OHMIC CONDUCTOR

A light globe has the I–ΔV characteristics shown in the graph. Calculate the resistance of the light globe when the potential difference is 5.00 V.

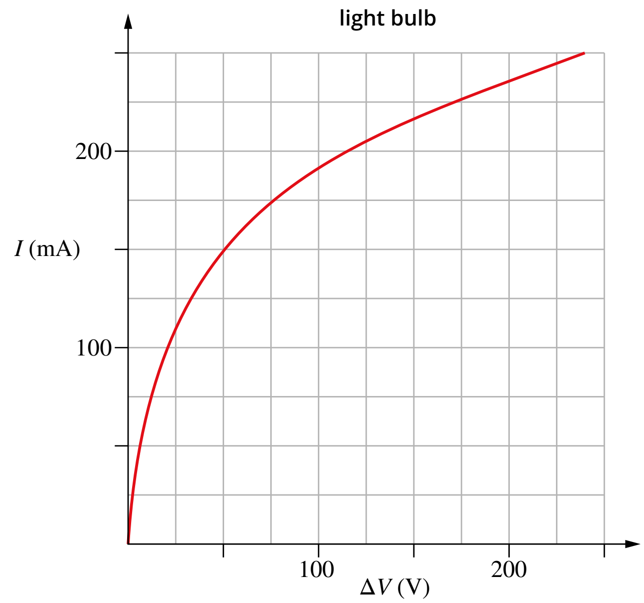


Thinking	Working
From the graph, determine the current at the required potential difference.	At $\Delta V = 5.00\,\text{V}$, $I = 3.00\,\text{A}$
Substitute these values into Ohm's law to find the resistance. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$R = \frac{\Delta V}{I}$ $R = \frac{(5.00)}{(3.00)}$ $R = 1.6667$ $R = 1.67\,\Omega$

Worked example: Try yourself 10.4.3

CALCULATING RESISTANCE FOR A NON-OHMIC CONDUCTOR

A 240 V, 60.0 W incandescent light globe has the I–ΔV characteristics shown in the graph. Calculate the resistance of the light globe when the potential difference is 175 V.



RESISTORS IN SIMPLE CIRCUITS

Ohmic resistors are often used to control the amount of current in a particular circuit. Resistors can be manufactured to produce a relatively constant resistance over a range of temperatures. A colour-coding system is used on resistors to explain the amount of resistance they provide, including a percentage tolerance (precision). Figure 10.4.7 shows a resistor that uses the colour-coding system.

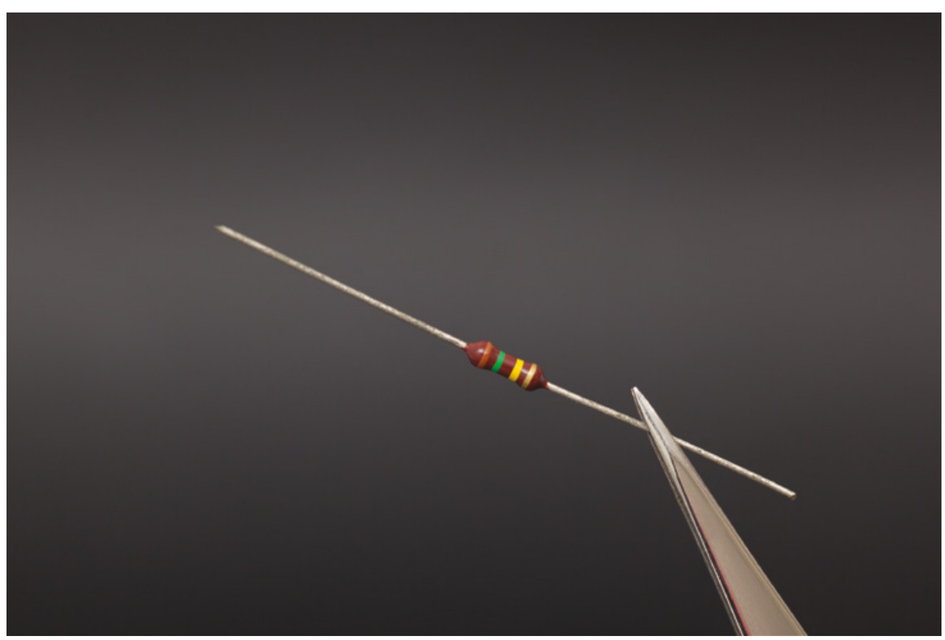


FIGURE 10.4.7 Common resistors are electrical devices with a known resistance. The coloured bands indicate the resistor's resistance and tolerance.

PHYSICSFILE

Colour-coded resistors

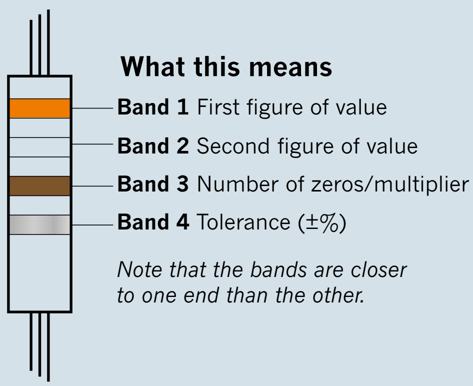
A resistor is typically a small piece of equipment which does not allow enough room to clearly print information about the resistor in the form of numbers. A colour-coding system is used on many resistors to convey detailed information in a small space about the resistance and tolerance of the resistor. Figure 10.4.8 below explains how to interpret the colour-coding system.

Resistor colour code

Band colour	Value
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Purple	7
Grey	8
White	9
Gold	0.1
Silver	0.01

Tolerance colour code

Band colour	±%
Brown	1
Red	2
Gold	5
Silver	10
None	20



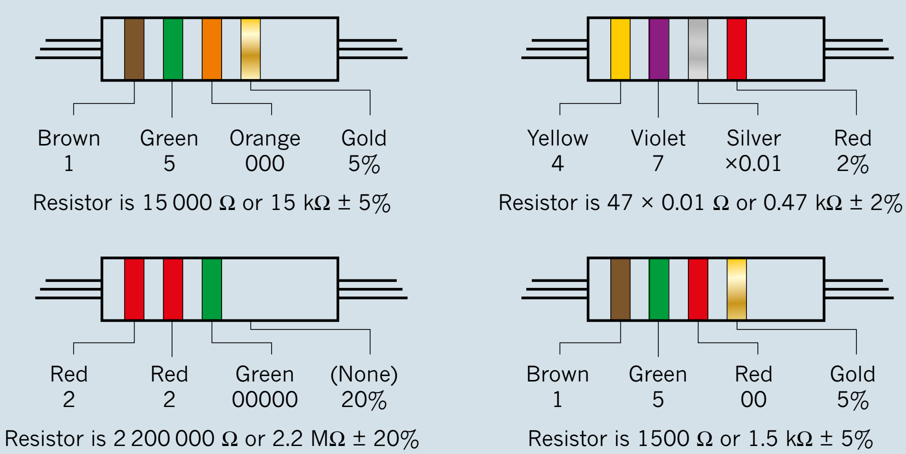


FIGURE 10.4.8 Examples of resistor colour-coding

Ohm's law can be used to determine the current flowing through a resistor when a particular potential difference is applied across it. Similarly, if the current and resistance are known, the potential difference across the resistor can be calculated.

Worked example 10.4.4

USING OHM'S LAW TO FIND CURRENT

A 115 Ω resistor is connected to a 12.0 V battery. Calculate the current (in mA) that would flow through the resistor.	
Thinking	**Working**
Recall Ohm's law.	$\Delta V = IR$
Rearrange the equation to make I the subject.	$I = \frac{\Delta V}{R}$
Substitute in the values for this problem and solve. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$I = \frac{(12.0)}{(115)}$ $I = 0.10435$ $I = 0.104\ \text{A}$ $I = 104\ \text{mA}$

Worked example: Try yourself 10.4.4

USING OHM'S LAW TO FIND CURRENT

The element of a bar heater has a resistance of 25.0 Ω. Calculate the current (in mA) that will flow through this element if it is connected to a 245 V supply.

Worked example 10.4.5

USING OHM'S LAW TO FIND POTENTIAL DIFFERENCE

A current of 0.250 A flows through a 22.0 Ω resistor. Calculate the potential difference across the resistor. Give your answer correct to one decimal place.	
Thinking	**Working**
Recall Ohm's law.	$\Delta V = IR$
Substitute in the known values and solve. Ensure that you present your answer to the correct number of decimal places and use the appropriate units.	$\Delta V = (0.250)(22.0)$ $\Delta V = 5.5000$ $\Delta V = 5.50\ \text{V}$

Worked example: Try yourself 10.4.5

USING OHM'S LAW TO FIND POTENTIAL DIFFERENCE

The globe of a torch has a resistance of 5.70 Ω when it draws 748 mA of current. Calculate the potential difference across the globe.

10.4 Review

SUMMARY

- Resistance is a measure of how hard it is for current to flow through a particular material. Resistance is measured in ohms (Ω).
- The resistance of a material depends on its length, cross-sectional area and temperature. Resistance can be determined using the following formulas:

$$R = R_0[1 + \alpha(T - T_0)]$$

$$R = \frac{\rho L}{A}$$

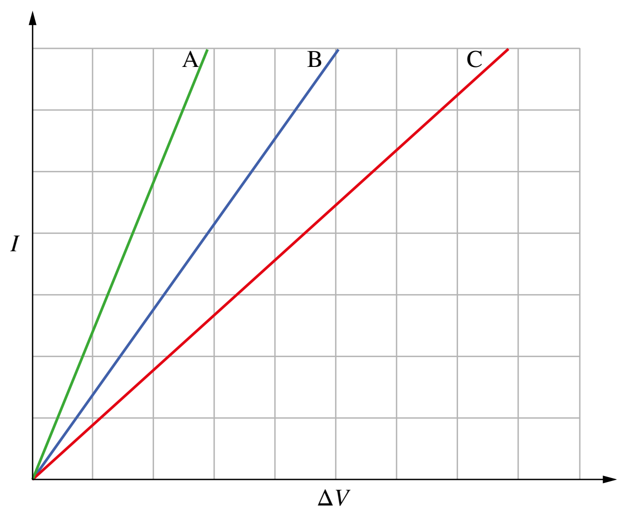
- Ohm's law describes the relationship between current, potential difference and resistance:

$$\Delta V = IR$$

- Ohmic conductors have a constant resistance. The resistance of non-ohmic conductors varies for different potential differences.

KEY QUESTIONS

1 An experiment is conducted to gather data about the relationship between current and potential difference for three ohmic devices, labelled A, B and C. The data is used to plot an I–ΔV graph for each device, as shown below.



a For a given potential difference, list the devices in order of highest current to lowest current.

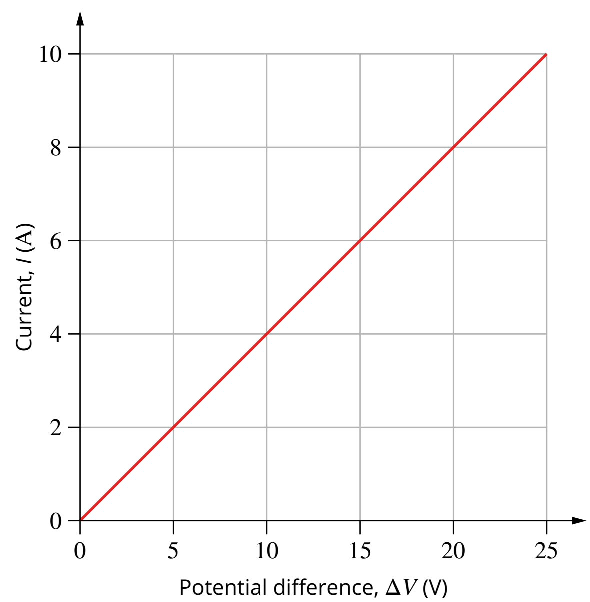
b List the devices in order of highest resistance to lowest resistance.

2 The table below shows measurements for the potential difference and corresponding current for an ohmic conductor.

ΔV (V)	0.00	2.00	3.00	ΔV_2
I (A)	0.00	0.250	I_1	0.600

Determine the missing results, I_1 and ΔV_2

3 A student obtains the graph of the I–ΔV characteristics of a piece of resistance wire shown below.



a Explain whether this piece of wire is ohmic or non-ohmic.

b What current flows in this wire at a potential difference of 7.50 V?

c What is the resistance of this wire?

4 A student finds that the current through a resistor is 3.50 A when a potential difference of 2.50 V is applied to it.

a What is the resistance?

b The potential difference is then doubled and the current is found to increase to 7.00 A. Is the resistor ohmic or not?

10.4 Review *continued*

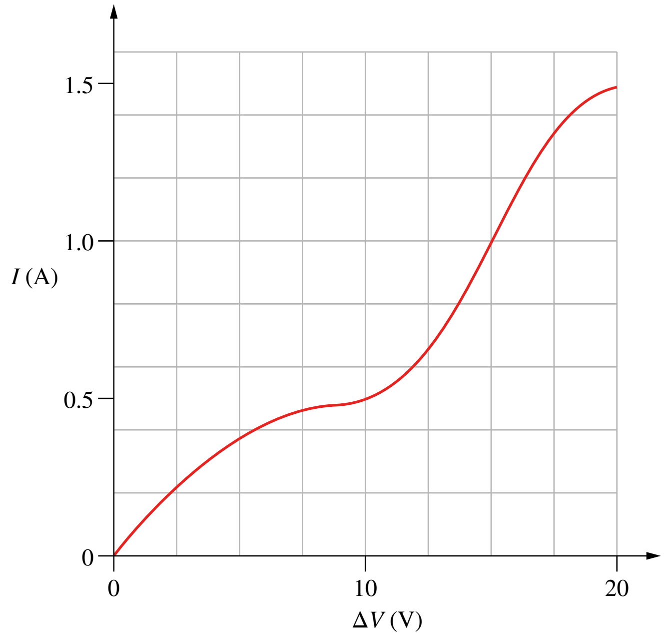
5 Ras and Rachel are trying to find the resistance of an electrical heating device. They find that at 5.00 V it draws a current of 200.0 mA and at 10.0 V it draws a current of 500.0 mA. Ras says that the resistance is 25.0 Ω, but Rachel maintains that it is 20.0 Ω. Who is right and why?

6 Mo has an ohmic resistor with a potential difference of 5.00 V applied across it. Mo measures the current as 45.0 mA and then increases the potential difference to 8.00 V. What current will Mo measure now?

7 Li finds that when the potential difference across an ohmic resistor is increased from 6.00 V to 10.0 V, the current increases by 2.00 A.
   - **a** What is the resistance of this resistor?
   - **b** What current does it draw at 10.0 V?

8 The resistance of a piece of wire is found to be 0.800 Ω. Calculate the resistance of:
   - **a** a piece of the same wire twice as long
   - **b** a piece of wire of twice the diameter.

9 A strange electrical device has the I–ΔV characteristics shown in the graph below.



   - **a** Is it an ohmic or non-ohmic device? Explain.
   - **b** What current is drawn when a potential difference of 10.0 V is applied to it?
   - **c** What potential difference would be required to double the current drawn at 10.0 V?
   - **d** Calculate the resistance of the device at:
      - **i** 10.0 V
      - **ii** 20.0 V.

10.5 Series and parallel circuits

Electric circuits are the basis of much of our modern society. This section introduces a range of circuits, from simple series circuits to the complex parallel wiring systems that make up a modern home. Electric circuits can be used to perform energy transfers and transformations through devices such as light bulbs, thermistors, light-dependent resistors and light-emitting diodes.

When a circuit contains more than one resistor, Ohm's law alone is not sufficient to predict the current flowing through and the potential difference across each resistor. Additional concepts such as Kirchhoff's rules and the idea of equivalent resistance can be used to analyse these complex, multi-component circuits.

No matter how complex a circuit, it can always be broken up into sections in which the circuit elements are in series or parallel. This section investigates the difference between these two types of circuits.

RESISTORS IN SERIES

Some circuits contain more than one electrical component. When these components are connected one after another in a continuous loop, this is called a **series circuit**. Components connected in this way are said to have been connected 'in series'. The circuit shown in Figure 10.5.1 shows a resistor and a light bulb connected in series with an electric cell.

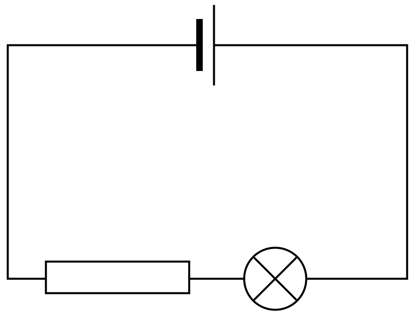


FIGURE 10.5.1 This circuit has a resistor and light bulb connected in series.

Series circuits are very easy to construct, but they have some disadvantages. As every component is connected one after the other, all components are dependent on each other. If one component is removed or breaks down, the circuit is no longer a closed loop and it won't work. This is referred to as an open circuit and the current stops. Figure 10.5.2 shows how removing a globe from a series circuit interrupts the entire circuit and breaks the current flow. Due to this characteristic, series circuits with more than one component are not commonly used in the home.

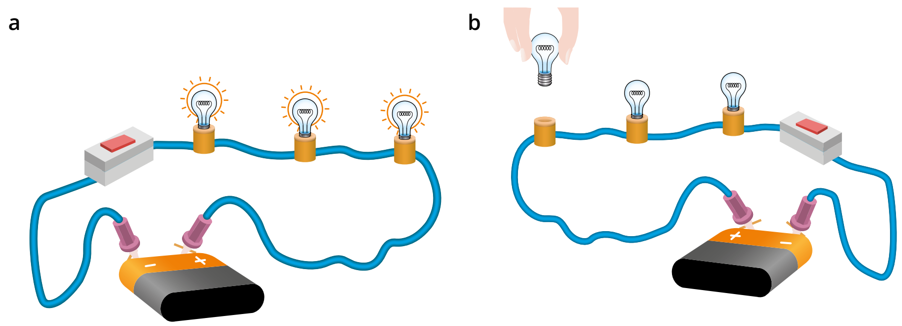


FIGURE 10.5.2 (a) There is no break in the circuit, so the circuit is a closed loop. (b) When one light bulb is removed there is an open circuit.

Conservation of charge

When analysing a series circuit, it is important to understand that the same amount of current flows in every part of the circuit. An example of a series circuit is shown in Figure 10.5.3. Since electric charges are not created or destroyed within an electric circuit, the current flowing out of the cell must be the same as the current flowing through the bulb, which is also the same as the current flowing through the resistor. This current also flows unchanged back into the cell.

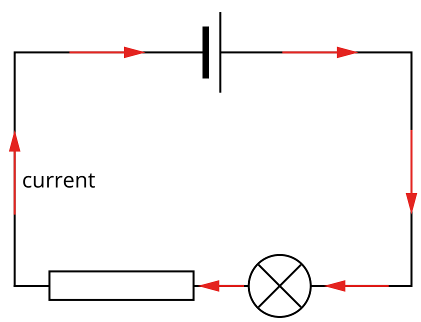


FIGURE 10.5.3 In a series circuit, the same current flows through each component.

> The current in a series circuit is the same in every part of the circuit.

Remember that, by convention, the current is represented as flowing from the positive terminal of the cell to the negative terminal. Electrons move in the opposite direction.

Kirchhoff's loop rule

Kirchhoff's loop rule says that the sum of the potential differences across all the elements around any circuit (loop) must be zero. This means that the total potential drop (reduction in electrical potential) around a closed circuit must be equal to the total potential gain in the power source. For example, if a battery provides 9.00V to a circuit, then the sum of the potential drop across each of the components must add to 9.00V.

This rule is essentially another version of the law of conservation of energy.

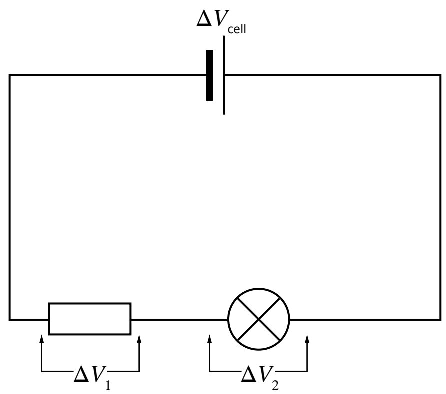


FIGURE 10.5.4 Kirchhoff's loop rule.

> The energy given to the charges (electrical potential energy gained) must be equal to the energy lost by the charges (electrical potential energy drop). In a series circuit, the energy loss will be spread across the different components.

Figure 10.5.4 shows how the electrical potential difference provided by the cell, ΔV_{cell}, is shared across a resistor and a bulb. In this series circuit, the sum of the potential drops across the resistor and the bulb (i.e. $\Delta V_1 + \Delta V_2$) will be equal to the potential difference provided by the cell, ΔV_{cell}, assuming that the resistance of the wires is negligible.

There are a number of ways to visualise the energy changes in this circuit. One common analogy is to think of the charges as water being pumped around an elevated water course. The water gains potential energy as it is pumped higher, and as it flows back down the potential energy is converted into other forms. The diagram in Figure 10.5.5 shows how the analogy works with the energy changes that occur in a circuit. The battery acts as a 'pump' that lifts electrons up to a higher energy level, and so they gain electrical potential energy. As the electrons pass through components in the circuit, their electrical potential energy is 'lost' as it is transformed into other forms.

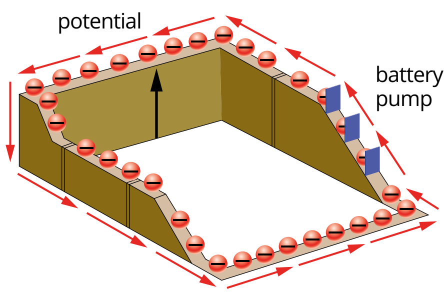


FIGURE 10.5.5 An analogy for analysing a circuit: the battery acts as a 'pump' that transfers electrical potential energy to the field as it lifts electrons to a higher level. The electrons 'lose' electrical potential energy as they drop down to lower energy levels through the various components in the circuit.

The change in electrical energy available to electrons can also be represented graphically, as shown in Figure 10.5.6. The electric potential energy available in the field changes as electrons move around the circuit. Some of this energy is lost as the electrons pass through the resistor. The remaining energy is lost as the electrons pass through the bulb. In this circuit, the bulb has more resistance than the resistor.

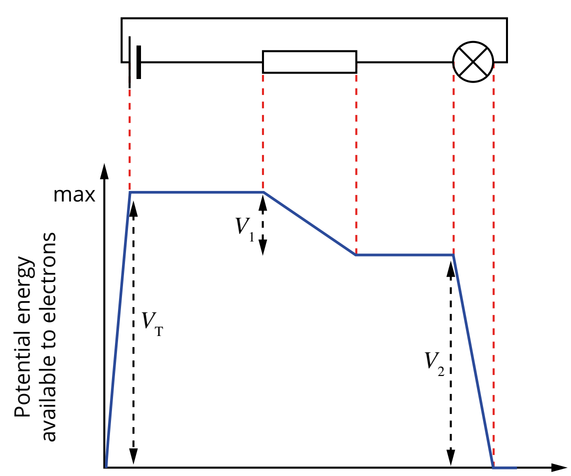


FIGURE 10.5.6 The electric potential energy of an electron changes as it moves around the circuit.

Equivalent series resistance

Consider the circuit in Figure 10.5.7. If the resistance of the fixed resistor is R_1, the resistance of the bulb is R_2, and the current flowing through both of them is I, then Ohm's law gives:

$\Delta V_1 = IR_1$

and

$\Delta V_2 = IR_2$

The total potential difference drop across the two components is:

$\Delta V_{Total} = \Delta V_1 + \Delta V_2 = IR_1 + IR_2 = I \times (R_1 + R_2)$

This equation shows the relationship between the potential difference supplied by the cell and the potential differences of the bulb and resistor. The last part of the equation also shows that the bulb and resistor can be replaced with a single equivalent resistor, without changing the current in the circuit. The single equivalent resistor needs to have a total resistance of $R_1 + R_2$.

Using Ohm's law, it is possible to show the relationship between the potential difference supplied by the cell, ΔV_{Total}, the current flowing in the circuit, I, and the resistances of the two components R_1 and R_2.

In general, a number of individual resistors connected in series can be replaced by an equivalent **effective resistance** (also called the total resistance, R_T) equal to the sum of the individual resistances. Figure 10.5.8 shows how two resistors can be replaced with a single one.

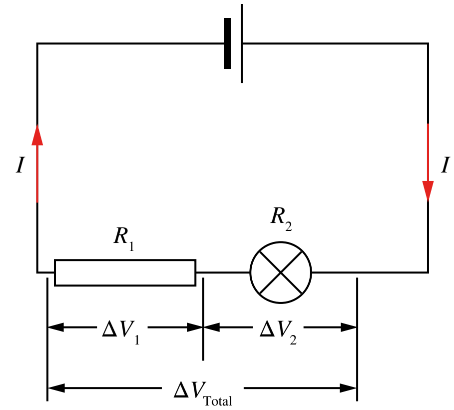


FIGURE 10.5.7 Ohm's law can be used to show the total potential difference is the sum of the potential differences across the individual components.

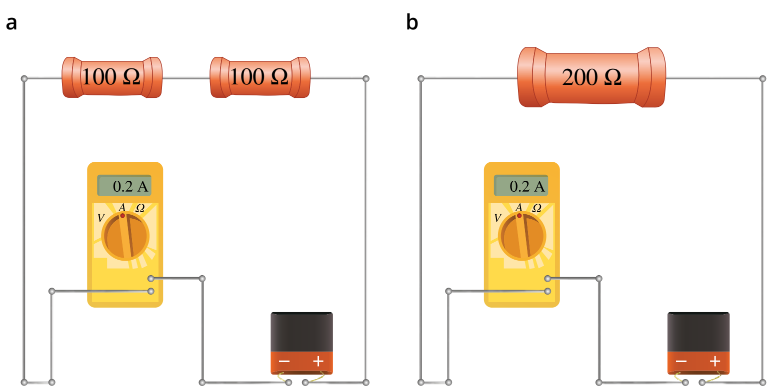


FIGURE 10.5.8 Two 100 Ω resistors (a) can be replaced with a single equivalent 200 Ω resistor (b) to have the same effect in a circuit.

$R_T = R_1 + R_2 + \ldots + R_n$

where R_T is the equivalent effective series resistance and $R_1, R_2, \ldots, R_n$ are the individual resistances.

Equivalent resistances can be used in circuit analysis to simplify a complicated circuit diagram so that current and potential difference can be determined.

Worked example 10.5.1

CALCULATING AN EQUIVALENT SERIES RESISTANCE

A 125 Ω resistor is connected in series with a 695 Ω resistor and a 1.20 kΩ resistor. Calculate the equivalent series resistance.	
Thinking	**Working**
Recall the formula for equivalent series resistance.	$R_T = R_1 + R_2 + \ldots + R_n$
Substitute in the given values for resistance. Convert kΩ to Ω. Solve to find the equivalent series resistance. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$R_T = (125) + (695) + (1200)$ $R_T = 2020\,\Omega$

Worked example: Try yourself 10.5.1

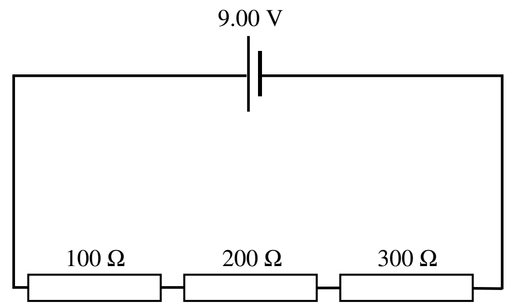
CALCULATING AN EQUIVALENT SERIES RESISTANCE

A string of Christmas lights consists of 20 light bulbs connected in series. Each bulb has a resistance of 8.40 Ω. Calculate the equivalent series resistance of the Christmas lights.

Worked example 10.5.2

USING EQUIVALENT SERIES RESISTANCE FOR CIRCUIT ANALYSIS

Use an equivalent series resistance to calculate the current flowing in the series circuit below, and the potential difference across each resistor.

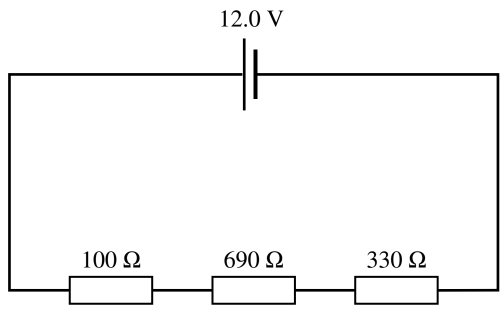


Thinking	Working
Recall the formula for equivalent series resistance.	$R_T = R_1 + R_2 + R_3 + \ldots + R_n$
Find the equivalent (total) resistance in the circuit.	$R_T = (100) + (200) + (300) = 600\,\Omega$
Use Ohm's law to calculate the current in the circuit. Whenever calculating current in a series circuit, use R_T and the potential difference of the power supply. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$I = \frac{\Delta V}{R}$ $I = \frac{(9.00)}{(600)}$ $I = 0.0150\,\text{A}$
Use Ohm's law to calculate the potential difference across each separate resistor. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$\Delta V = IR$ Therefore $\Delta V_1 = (0.0150)(100) = 1.50\,\text{V}$ $\Delta V_2 = (0.0150)(200) = 3.00\,\text{V}$ $\Delta V_3 = (0.0150)(300) = 4.50\,\text{V}$
Use the loop rule to check the answer.	$\Delta V_T = \Delta V_1 + \Delta V_2 + \Delta V_3$ $\Delta V_T = (1.50) + (3.00) + (4.50)$ $\Delta V_T = 9.00\,\text{V}$ Since this is the same as the potential difference provided by the cell, the answer is acceptable.

Worked example: Try yourself 10.5.2

USING EQUIVALENT SERIES RESISTANCE FOR CIRCUIT ANALYSIS

Use an equivalent series resistance to calculate the current flowing in the series circuit below and the potential difference across each resistor.



RESISTORS IN PARALLEL

One of the disadvantages of series circuits is that if a switch is opened or a device disconnected, the circuit is broken and current stops flowing. In practical situations you often want to switch devices on and off independently. **Parallel circuits** allow you to do this.

The circuit diagram in Figure 10.5.9 shows a simple parallel circuit. Even if switch A is open (as shown), lamp B will still light up as it is part of an unbroken complete circuit including the battery. Similarly, if switch A is closed and switch B is opened, current will light up lamp A and not lamp B. Alternatively, both switches could be closed to light up both lamps or both switches could be opened to switch both lamps off.

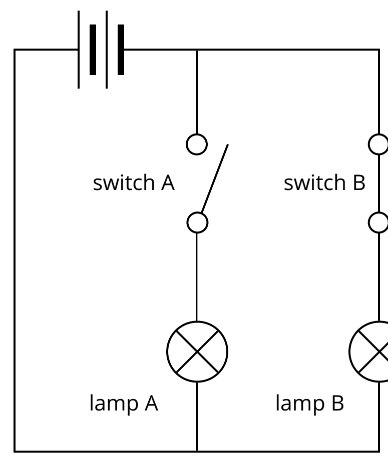


FIGURE 10.5.9 In this parallel circuit, lamp A will be off and lamp B will be on.

In a series circuit, all the components are in the same loop and therefore the same current flows through each component. In comparison, each loop of a parallel circuit acts like an independent circuit with its own current. However, since electric charges are not created or destroyed within an electric circuit, the current that flows out of the cell must be the same as the current that flows into it.

Consider again the shopping centre analogy outlined in Section 10.2. When a certain number of people are on the first floor, they can get to the ground floor by either going down the lift or walking down the stairs. One way is easier than the other, so this represents a difference in resistance between two parallel paths. Most people will take the path of least resistance (the lift) and fewer people will take the path of greater resistance (the stairs). The rate at which people enter the lift plus the rate at which they enter the stairs combines to equal the rate at which people leave the first floor. When the people re-join, the rate at which people leave the lift and stairs combine to be the rate at which people enter the ground floor. This is the same rate as it was for people leaving the first floor. The same occurs with charges flowing in a parallel circuit. Figure 10.5.10 shows how the charges go through one globe or the other. This means that while the current before and after the components remains constant, in the parallel section, the current is divided between each branch. The readings on both ammeters, A_1 and A_2, will be the same.

The current in the main part of a parallel circuit is the sum of the currents in each branch of the circuit.

$$I_T = I_1 + I_2 + \ldots + I_n$$

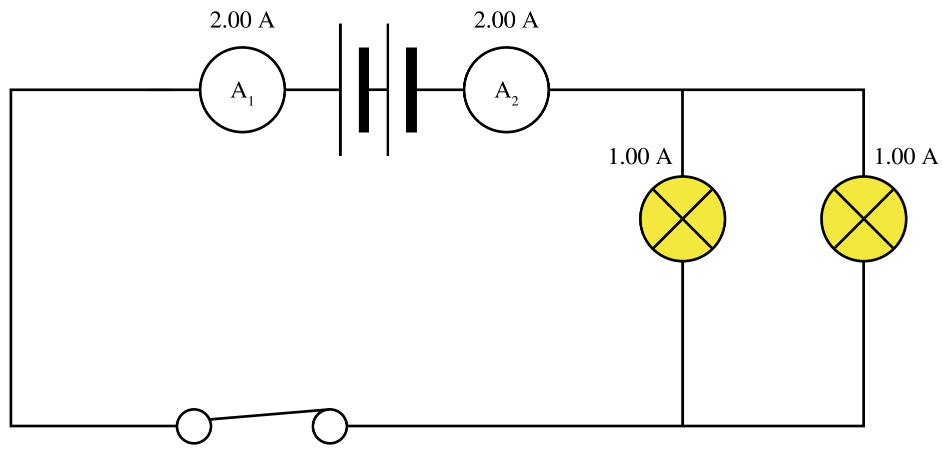


FIGURE 10.5.10 Charges flowing in this parallel circuit will flow down one of two paths.

Unlike a series circuit, in a parallel circuit the potential difference is not shared between resistors; the potential difference is the same across each branch. This is because, while the charges take different pathways, they have the same change in energy no matter which path they take. The electrical potential in the wire before the parallel circuit components is the same no matter the path, and the electrical potential in the wire after the parallel components is also the same, so the difference in electrical potential must also be the same no matter which path the current takes. An advantage of parallel circuits is that globes connected in this way are brighter than if they were connected in series. The potential energy of the charges is not divided and shared between the globes.

Using the shopping centre analogy, the people start on the first floor and walk along until some walk towards the lift and some walk towards the stairs, with both groups still on the first floor. In the circuit, electrons experience the same electrical potential whether they flow along one parallel branch or the other. As the people get to the ground floor, via the lift or the stairs, they lose energy from the field (in this case, gravitational potential energy). In the circuit, electrical potential energy is transferred from the electric field, via the electrons, into other forms of energy. When the people reach the ground floor via either the lift of the stairs, they are once again at the same level as each other and they can re-join and continue walking together. In the electric circuit, the electrons passing from the resistors are at the same potential as each other (but are at a lower potential compared to the electrons before the resistors), and they can continue to move along the circuit.

The potential difference is the same across each branch of a parallel circuit.

Kirchhoff's junction rule

Parallel circuits involve **junctions** where current can flow in a variety of directions. The behaviour of current at these points is predicted by Kirchhoff's junction rule:

The total amount of current flowing into a junction must be the same as the total current flowing out of the junction.

This rule is just an extension of the idea of conservation of charge; that is, charges cannot be created or destroyed. Although the number of electrons flowing into a junction might be very large, the same number of electrons must flow out again, as electrons are not created or destroyed in the junction. This is illustrated in Figure 10.5.11. Kirchhoff's junction rule explains how current splits in a parallel circuit. It explains why the current in the main part of the circuit is equal to the sum of the currents in each parallel branch.

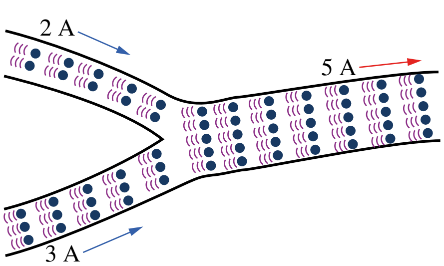


FIGURE 10.5.11 The current flowing into any junction must be equal to the current flowing out of it.

Equivalent parallel resistance

When additional resistors are added into a series circuit the total resistance of the circuit increases. As the potential difference across the resistors remains the same, increased resistance means that less current flows through the circuit.

In contrast, adding an additional resistor in parallel means that more current flows through the circuit because another path for the charges to take has been added. The potential difference across each resistor in the parallel combination is the same, so this means that the total resistance for the circuit decreases, such that it is always less than the value of the smallest resistor in the parallel pathways.

Consider the shopping centre analogy once again. If there were only two ways to get to the ground floor from the first floor, then there would be a limit to the rate at which people could get to the ground level. However, if another lift were added, then more people could get to the ground in the same period of time, so the rate at which people could get to the ground floor would be greater. There would be less 'resistance' for people trying to get to the ground floor.

To represent the addition of resistors in a mathematical way, such that the overall resistance decreases, you can use the following relationship:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

where R_T is the equivalent effective resistance and $R_1, R_2, \ldots, R_n$ are the individual resistances.

Worked example 10.5.3

CALCULATING AN EQUIVALENT PARALLEL RESISTANCE

A 125 Ω resistor is connected in parallel with a 355 Ω resistor. Calculate the equivalent parallel resistance.	
Thinking	**Working**
Recall the formula for equivalent effective resistance.	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$
Substitute in the given values for resistance.	$\frac{1}{R_T} = \frac{1}{(125)} + \frac{1}{(355)}$
Solve for R_T. Ensure that you present your answer to the correct number of significant figures and use the appropriate units. Check that this value is less than the smallest resistor in the parallel branch.	$\frac{1}{R_T} = 0.010817$ $R_T = \frac{1}{(0.010817)}$ $R_T = 92.448$ $R_T = 92.4\ \Omega$

Worked example: Try yourself 10.5.3

CALCULATING AN EQUIVALENT PARALLEL RESISTANCE

A 25.0 Ω resistor is connected in parallel with a 57.5 Ω resistor. Calculate the equivalent parallel resistance.

Notice that in the previous worked example, the equivalent effective resistance was smaller than the smallest individual resistance. This is because adding a resistor in parallel provides an additional pathway for current. Since more current flows, the resistance of the circuit has been effectively reduced.

> ℹ The effective (total) resistance of a set of resistors connected in parallel will always be smaller than the smallest resistor in the set.
>
> In a parallel circuit:
>
> $R_{\text{Total}} < R_{\text{smallest resistor}}$

PHYSICSFILE

Kirchhoff's contributions

Both the junction rule discussed here, and the loop rule described earlier, were first discovered by the German physicist Gustav Kirchhoff (1824–87). Kirchhoff discovered the rules that underpin our understanding of how electric circuits work and also made important contributions in the fields of spectroscopy, thermochemistry and the study of blackbody radiation. He worked with Robert Bunsen, the German chemist who developed the Bunsen burner.

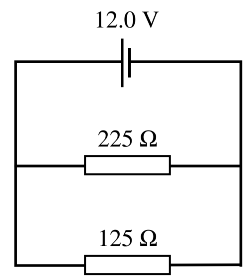
FIGURE 10.5.12 Gustav Kirchhoff discovered the rules that underpin our understanding of how electric circuits work.

If you consider the smallest resistor in any parallel combination, say, the 25.0 Ω resistor in Worked example: Try yourself 10.5.3, the addition of the 57.5 Ω resistor in parallel with it allows the current an extra pathway and therefore it is easier for the current to flow through the combination. The effective resistance of the pair must be less than the 25.0 Ω alone.

Worked example 10.5.4

USING EQUIVALENT PARALLEL RESISTANCE FOR CIRCUIT ANALYSIS

Find the equivalent parallel resistance to calculate the current flowing out of the 12.0 V cell in the parallel circuit shown. Then find the current flowing through each resistor.

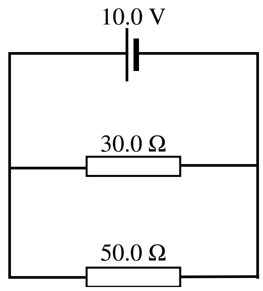


Thinking	Working
Recall the formula for equivalent parallel resistance.	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$
Substitute in the given values for resistance and solve for R_T. Ensure that you present your answer to the correct number of significant figures and use the appropriate units. Check that this value is less than the smallest resistor in the parallel branch.	$\frac{1}{R_T} = \frac{1}{(125)} + \frac{1}{(225)}$ $\frac{1}{R_T} = 0.012444$ $R_T = \frac{1}{(0.01244)}$ $R_T = 80.357$ $R_T = 80.4\,\Omega$
Use Ohm's law to calculate the current in the circuit. To calculate I, use the potential difference of the power supply and the total resistance.	$I_{\text{circuit}} = \frac{\Delta V}{R}$ $I_{\text{circuit}} = \frac{(12.0)}{(80.357)}$ $I_{\text{circuit}} = 0.14933$ $I_{\text{circuit}} = 0.149\text{ A}$
Use Ohm's law to calculate the current through each separate resistor. Remember that the potential difference across each resistor is the same as the potential difference of the power supply, 12.0 V in this case. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	125 Ω resistor: $I_{125} = \frac{\Delta V}{R} = \frac{(12.0)}{(125)} = 0.0960\text{ A}$ 225 Ω resistor: $I_{225} = \frac{\Delta V}{R} = \frac{(12.0)}{(225)} = 0.0533\text{ A}$
Use the junction rule to check the answers.	$I_{\text{circuit}} = I_{125} + I_{225}$ $0.14933 = 0.0960 + 0.0533$ This is correct, so the answers are acceptable.

Worked example: Try yourself 10.5.4

USING EQUIVALENT PARALLEL RESISTANCE FOR CIRCUIT ANALYSIS

Find the equivalent parallel resistance to calculate the current flowing out of the 10.0 V cell in the parallel circuit shown. Then find the current flowing through each resistor.



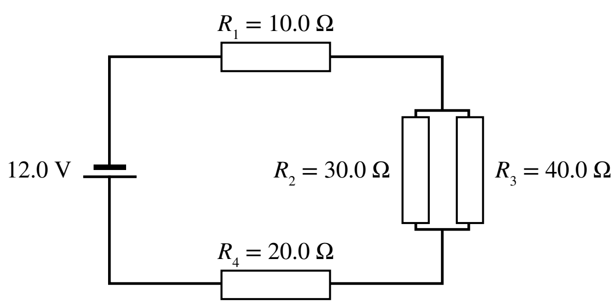
COMPLEX CIRCUIT ANALYSIS

Some circuits combine elements of both series wiring and parallel wiring. A general strategy for analysing these circuits is to reduce the complex circuit to a single equivalent resistance to determine the total current drawn by the circuit. It is then possible to step back through the process of simplification to analyse each section of the circuit as needed.

Worked example 10.5.5

COMPLEX CIRCUIT ANALYSIS

Calculate the potential difference across and current through each resistor in the circuit below.



Thinking	Working
Find an equivalent resistance for the parallel resistors. The effective resistance of these should be less than the smaller resistor, that is, less than 30.0 Ω.	$\frac{1}{R_{2,3}} = \frac{1}{R_2} + \frac{1}{R_3}$ $\frac{1}{R_{2,3}} = \frac{1}{30.0} + \frac{1}{40.0}$ $R_{2,3} = 17.1429$ $R_{2,3} = 17.1\,\Omega$
Find an equivalent series resistance for the circuit as the circuit can now be thought of as three resistors in series: 10.0 Ω, 17.1 Ω and 20.0 Ω.	$R_T = (10.0) + (17.1429) + (20.0)$ $R_T = 47.1429$ $R_T = 47.1\,\Omega$

Use Ohm's law to calculate the current in the circuit. Use the potential difference of the power supply and the total resistance to do this calculation.	$\Delta V = IR$ $I = \frac{\Delta V}{R}$ $I = \frac{(12.0)}{(47.1429)}$ $I = 0.25455$ $I = 0.255\,A$
Use Ohm's law to calculate the potential difference across each resistor (or parallel group of resistors) in series. (Note that the potential difference across R_2 is the same as that across R_3 as they are in parallel.) Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$\Delta V = IR$ $\Delta V_1 = (0.25455)(10.0) = 2.5455 = 2.54\,V$ $\Delta V_{2,3} = (0.25455)(17.1429) = 4.3636 = 4.36\,V$ $\Delta V_4 = (0.25455)(20.0) = 5.0909 = 5.09\,V$ Check: $(2.5455) + (4.3636) + (5.0909) = 12.0000\,V$ This confirms that the loop rule holds for this circuit.
Use Ohm's law where necessary to calculate the current through each resistor. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$I_1 = I_4 = 0.25455\,A$ $I = \frac{\Delta V}{R}$ $I_2 = \frac{(4.3636)}{30.0} = 0.14545 = 0.145\,A$ $I_3 = \frac{(4.3636)}{40.0} = 0.10909 = 0.109\,A$ Check: $(0.14545) + (0.10909) = 0.25455\,A$ This confirms that the junction rule holds for this section.

PHYSICSFILE

Identical resistors in parallel

Where identical resistors are placed in parallel, the total resistance of the combination can be found by simply dividing the value of one of the resistors by the number of resistors.

For example, three 12.0 Ω resistors connected in parallel would have an effective total resistance of 4.00 Ω.

$R_T = 12.0 \div 3 = 4.00\,\Omega$.

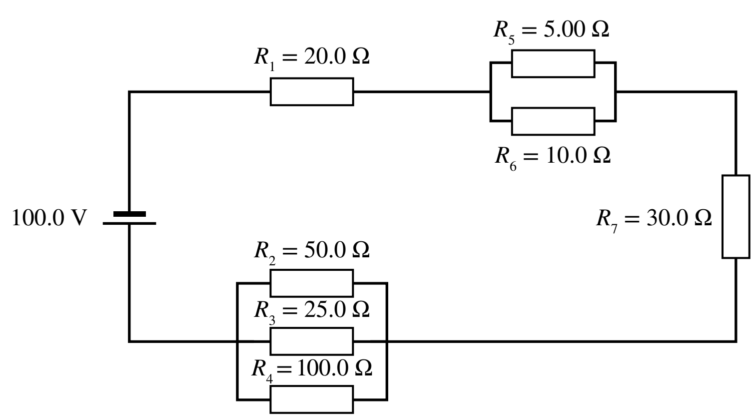
The three 12.0 Ω resistors in parallel could therefore be replaced with a single 4.00 Ω resistor.

Similarly, the equivalent resistance of two 10.0 Ω resistors placed in parallel would be 5.00 Ω.

Worked example: Try yourself 10.5.5

COMPLEX CIRCUIT ANALYSIS

Calculate the potential difference across and the current through each resistor in the circuit below.



PHYSICS IN ACTION

Superconductors

In 1908, Dutch physicist Kamerlingh Onnes (1853–1926) was the first to liquefy helium (Figure 10.5.13). This occurred at 4.2 K (–269°C). Onnes was the first to liquefy a number of gases. He was also the first person to achieve the then world-record lowest temperature of 1.5 K.

Onnes began investigating the resistance of pure metals at very low temperatures. Some scientists believed at the time that the resistance of metals would greatly increase or even become infinite near absolute zero (–273°C). Other scientists, Onnes among them, believed that the electrical resistance would eventually drop to nil.

In 1911, Onnes immersed a solid wire of mercury into liquid helium and found that at 4.2 K its resistance was indeed zero. He called this the *superconducting state*, a new state of matter. Theoretically, once an electric current was started in a loop maintained at superconducting temperatures, it would circulate indefinitely. Onnes was awarded the Nobel Prize in Physics in 1913.

Other metals were soon found to become superconductors at extremely low temperatures; for example, aluminium at 1.2 K and lead at 7.9 K. However, not all metals can become superconducting. Perhaps surprisingly, the excellent electrical conductor copper does not become superconducting at any temperature.

The mechanism by which a metal's electrical resistance drops to zero was not understood at the time. In 1957, three American physicists, John Bardeen, Leon Cooper and Robert Schrieffer, published a paper that explained the phenomenon. The explanation is now known as the BCS theory, and required quantum mechanics, which was unknown in Onnes' era. Bardeen, Cooper and Schrieffer were awarded the 1972 Nobel Prize in Physics.

Superconductivity hit the headlines again in the late 1980s when some ceramic compounds were discovered to become superconducting at relatively high temperatures. Despite the fact that copper by itself is not a superconductor, most high-temperature superconductors (HTS) are compounds of copper. An example is the compound yttrium barium copper oxide, $YBa_2Cu_3O_7$, which becomes superconducting at liquid nitrogen temperatures (77 K or –196°C). This new class of ceramic superconductors generated a great deal of interest. Currently, the record for the highest temperature for a superconductor stands at 135 K (–138°C) using the compound mercury barium calcium copper oxide, $HgBa_2Ca_2Cu_3O_8$. The mechanism by which this class of ceramic materials becomes superconducting is not well understood and so they are currently a very active area of both theoretical and experimental research.

The eventual goal is to discover a material that will become superconducting at room temperature. Such a material would have many applications if it didn't require all the equipment and expense to maintain extremely low or high temperatures. Currently, researchers have been investigating lanthanum decahydride (LaH_{10}), which has the highest accepted superconducting temperature of 250 K or –23°C. However, lanthanum decahydride has to be at an elevated pressure of 200 GPa to reach this superconducting phase at these temperatures.

Superconductors have made it possible to produce extremely powerful magnets. Such magnets find applications in magnetic resonance imaging (MRI) machines used in medical scanning. Superconductors are also used in maglev (magnetic levitation) trains that float above the rails and therefore do not experience friction and clunking from the rails, and in the Large Hadron Collider, where they are used to bend extremely high-speed protons around the beamline.

FIGURE 10.5.13 The Dutch physicist Kamerlingh Onnes.

RESISTORS AND POWER

A particular combination of resistors will draw different amounts of power depending on whether the resistors are wired in series or parallel. In general, since resistors in parallel circuits will draw more current than resistors in series circuits, parallel circuits use more power than series circuits containing the same resistors.

Recall from Section 10.3 that the equation for power is:

$P = \Delta VI$

where P is the power (W)

ΔV is the potential difference (V)

I is the current (A).

Worked example 10.5.6

COMPARING POWER IN SERIES AND PARALLEL CIRCUITS

Consider a 145 Ω and a 315 Ω resistor wired in parallel with a 12.0 V cell. Calculate the power drawn by these resistors. Compare this to the power drawn by the same two resistors when wired in series.	
Thinking	**Working**
Calculate the equivalent resistance for the parallel circuit.	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$ $\frac{1}{R_T} = \frac{1}{(145)} + \frac{1}{(315)}$ $R_T = 99.293$ $R_T = 99.3\ \Omega$
Calculate the total current drawn by the parallel circuit.	$\Delta V = IR$ $I = \frac{\Delta V}{R}$ $I = \frac{(12.0)}{(99.3)}$ $I = 0.12085$ $I = 0.121\ \text{A}$
Use the power equation to calculate the power drawn by the parallel circuit. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$P = \Delta VI$ $P = (12.0)(0.12085)$ $P = 1.45020$ $P = 1.45\ \text{W}$
Calculate the equivalent resistance for the series circuit.	$R_T = R_1 + R_2 + \ldots + R_n$ $R_T = (145) + (315)$ $R_T = 460\ \Omega$
Calculate the total current drawn by the series circuit.	$\Delta V = IR$ $I = \frac{\Delta V}{R}$ $I = \frac{(12.0)}{(460)}$ $I = 0.026087$ $I = 0.0261\ \text{A}$

Use the power equation to calculate the power drawn by the series circuit. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	$P = \Delta VI$ $P = (12.0)(0.026\,087)$ $P = 0.313\,043$ $P = 0.313\,\text{W}$
Compare the power drawn by the two circuits.	$\frac{P_{\text{parallel}}}{P_{\text{series}}} = \frac{(1.450\,20)}{(0.313\,043)} = 4.632\,58 = 4.63$ The parallel circuit draws 4.63 times as much power as the series circuit.

Worked example: Try yourself 10.5.6

COMPARING POWER IN SERIES AND PARALLEL CIRCUITS

Consider a 225 Ω and an 875 Ω resistor wired in parallel with a 12.0 V cell. Calculate the power drawn by these resistors. Compare this to the power drawn by the same two resistors when wired in series.

In Worked example 10.5.6 and Worked example: Try yourself 10.5.6, Ohm's law and the power equation were used separately to determine the power of a circuit component. You can combine the two equations to derive two useful equations:

$P = \Delta VI$ and $\Delta V = IR$

so

$P = (IR) \times I$

$P = I^2R$

Similarly

$P = \Delta VI$ and $I = \frac{\Delta V}{R}$

so

$P = \Delta V\left(\frac{\Delta V}{R}\right)$

$P = \frac{\Delta V^2}{R}$

PHYSICS IN ACTION

High power–low power

Simple heaters of various sorts often have a 'three heat' switch. An electric blanket will usually have 'low', 'medium' and 'high' settings, for example, as shown in Figure 10.5.14. Rather than making three different heating elements, the manufacturer can use two elements in different series and parallel combinations to obtain the three heat settings. If the two elements are placed in series, the total resistance is relatively high and therefore the power will be a minimum, as $P = \frac{\Delta V^2}{R}$. For the medium setting, one of the elements will be used on its own. The high setting is then achieved by placing both elements in parallel.

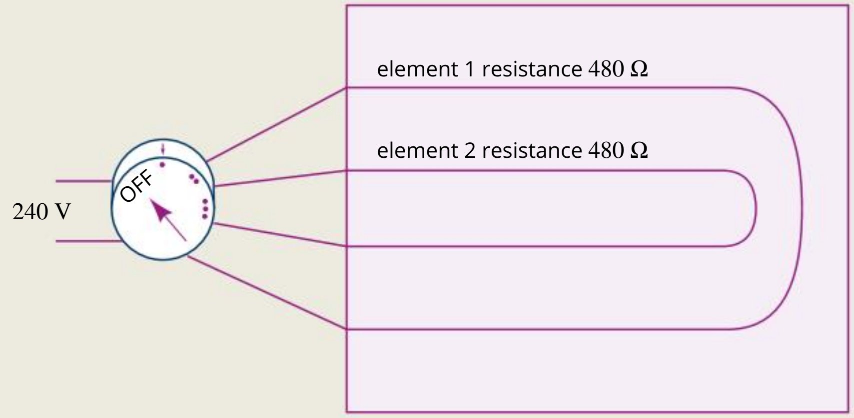


FIGURE 10.5.14 An example of the use of series and parallel combinations of resistors to achieve three heat settings for an electric blanket.

It is a simple matter to work out the relative power being used for the three settings. If it is assumed that the resistance of both elements is the same (R) and does not change appreciably with temperature, the effective resistance in the three cases will be given by:

Low heat (two elements in series): $R_T = R + R = 2R$

Medium heat (one element only): $R_T = R$

High heat (both in parallel): $R_T = \frac{1}{2}R$

As the power is inversely proportional to the resistance $P = \frac{\Delta V^2}{R}$, if you call the high setting 100%, then the other two will be 50% and 25%.

To obtain the various heat settings, the following circuits are used:

- Not connected to power: OFF
- Resistors connected in series: low heat, $R = 960\,\Omega$, $I = 0.25\,\text{A}$, $P = 60\,\text{W}$
- Only one resistive element is connected: medium heat, $R = 480\,\Omega$, $I = 0.5\,\text{A}$, $P = 120\,\text{W}$
- Resistors connected in parallel: high heat, $R = 240\,\Omega$, $I = 1.0\,\text{A}$, $P = 240\,\text{W}$

PHYSICS IN ACTION

Parallel connections in the home

All household appliances and lights are connected in parallel. This is done for two reasons.

Figure 10.5.15 shows a TV, air conditioner, heater and washing machine connected in series. In Australia, each of these devices is designed to operate at 240 V.

A circuit designed like the one in Figure 10.5.15 poses many problems. Firstly, all of the devices need to be switched on for the circuit to operate.

Secondly, the 240 V supplied to the circuit needs to be shared among all the components. Each component in the circuit would receive far less than the 240 V they require to operate. Also, as more and more devices are added to the circuit, the share of the 240 V would become even smaller. This system could never be practical.

The circuit diagram in Figure 10.5.16 shows how the same devices could be connected in parallel.

Each device in the parallel circuit receives the same potential difference, 240 V. Each device can be independently switched on or off without affecting the others and more devices can be added to this system without affecting the operation of the others. The only practical issue with devices in a parallel circuit such as this is the total current they draw. This cannot exceed the capacity of the protection device in the circuit. This aspect of electric circuits will be dealt with in detail in the next section of this book.

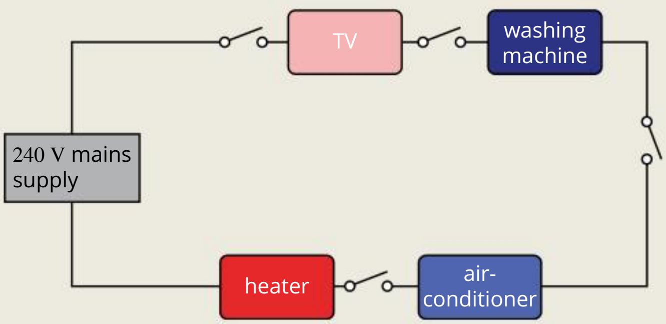


FIGURE 10.5.15 Household appliances connected in series.

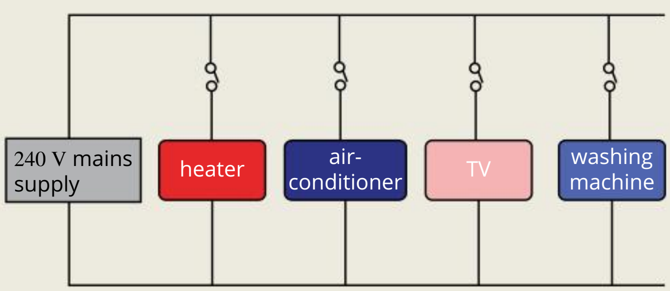


FIGURE 10.5.16 Household appliances connected in parallel.

10.5 Review

SUMMARY

- When resistors are connected in series:
  - the current through each resistor is the same
  - the sum of the potential differences is equal to the potential difference provided to the circuit
  - the equivalent effective resistance R_T is equal to the sum of the individual resistances: $R_T = R_1 + R_2 + R_3 + \ldots + R_n$
- Parallel circuits allow individual components to be switched on and off independently.
- When resistors are connected in parallel:
  - the potential difference across each resistor is the same
  - the current is shared between the resistors
  - the equivalent effective resistance is given by the equation: $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$
- Complex circuit analysis may require the calculation of both equivalent series and equivalent parallel resistances.
- A parallel circuit generally draws more power than a series circuit using the same resistor.
- Power can be calculated using the following equation:

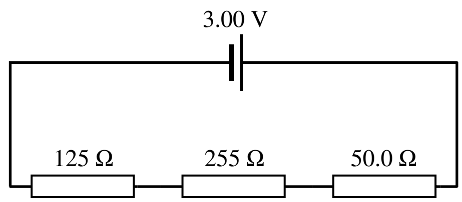
$$P = \frac{\Delta V^2}{R},\ P = I^2R \text{ and } P = \Delta VI$$

KEY QUESTIONS

1 Two 20.0 Ω resistors are connected in series with a 6.00 V battery. Which one of the following options is the potential difference across each resistor?

A 0.300 V **B** 3.00 V
C 6.00 V **D** 12.0 V

2 Consider the series circuit below.



a Calculate the current flowing in the circuit.
b Calculate the potential difference across the 125 Ω resistor in the circuit.

3 Two equal resistors are connected in parallel and are found to have an equivalent resistance of 68.0 Ω. Calculate the resistance of each resistor.

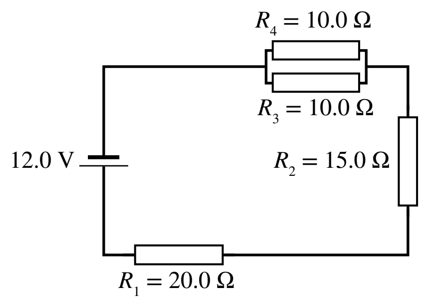
4 A 20.0 Ω resistor and a 10.0 Ω resistor are connected in parallel to a 5.00 V battery. Give your answers correct to two decimal places.

a Calculate the current drawn from the battery.
b Calculate the current flowing through the 20.0 Ω resistor.
c Calculate the current flowing through the 10.0 Ω resistor.

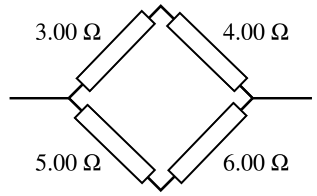
5 A 40.0 Ω resistor and a 60.0 Ω resistor are connected in parallel to a battery, with 315 mA flowing through the 40.0 Ω resistor.

a Calculate the potential difference of the battery.
b Calculate the current flowing through the 60.0 Ω resistor.

6 Calculate the potential difference across, and the current through, each resistor in the circuit below.



7 Calculate the equivalent resistance of the combination of resistors shown below.



8 Four 20.0 Ω light bulbs are connected to a 10.0 V battery. Calculate the total power output of the circuit if the light bulbs are connected:

a in series
b in parallel.

9 Why are household circuits wired in parallel?

A to reduce the amount of expensive copper wire used
B to reduce the amount of current drawn by the household
C to allow appliances to be switched on and off independently
D to reduce the amount of electrical energy used by the household

10.6 Electrical safety

Homes, schools and workplaces are filled with all sorts of electrical appliances. You use these appliances every day. A scientific understanding of electricity can help you to understand how they work and how to make sure you use them safely and effectively.

ELECTRICITY IN THE HOME

In our homes, appliances like the washing machine, TV, rechargeable power tools, and other conveniences such as lighting, hot water and Wi-Fi all rely on electricity.

Circuits in the home

The wiring for a house is much more complicated than the relatively simple series and parallel circuits considered so far. Figure 10.6.1 shows the basic structure of the electrical wiring in a house. Most appliances and power points are wired in parallel to allow them to be switched on and off independently of each other.

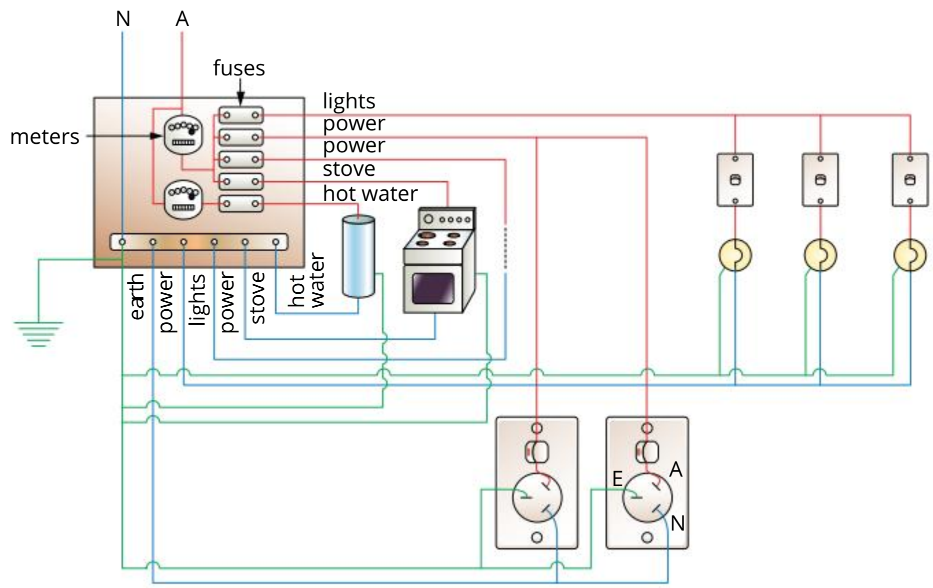


FIGURE 10.6.1 A household wiring diagram includes active (A), neutral (N) and earth (E) wires.

Power points in Australia have three pins. Each of these pins is connected to a different wire. Figure 10.6.1 shows the arrangement of the active, neutral and **earth** pins on a typical power point. The wires carrying the electric current to and from the appliance are known as the active wire (usually red or brown) and the neutral wire (usually black or blue). The third wire is an important safety feature called the earth wire (usually green or green and yellow). Figure 10.6.2 shows the corresponding active, neutral and earth pins in a power cord. This wiring pattern is an Australian Standard.

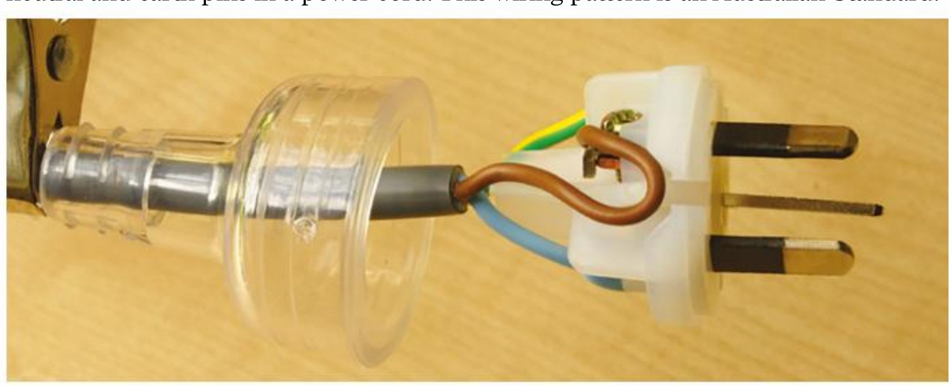


FIGURE 10.6.2 This three-pin plug shows the correct colour code for each pin.

The purpose of household electrical circuits is to enable electrical energy to be transferred to electrical appliances, where it is transformed into a range of other useful forms of energy. For example, an electric oven converts electrical energy into heat, whereas fans convert electrical energy into kinetic energy. Power points give users the option of connecting their own appliances and therefore choosing the type of energy produced.

Figure 10.6.3 shows that, before being distributed to various parts of the house, the active wires pass through meters that measure the amount of electrical energy supplied to the household.

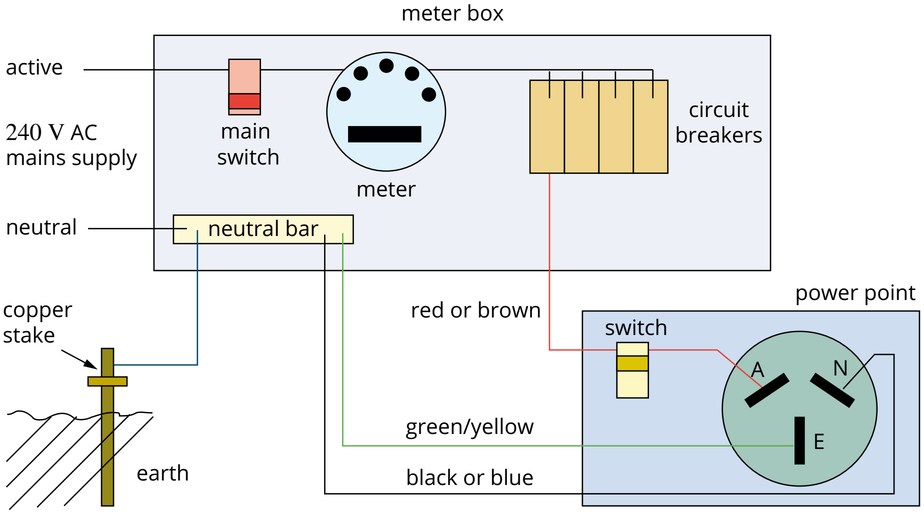


FIGURE 10.6.3 The active wire passes through the meter box before being connected to power points throughout the house. The circuit is completed by the neutral wire, returning through the neutral bar.

AC/DC

The electrical current that comes out of a power point is very different to the current that comes from a battery or electric cell (Figure 10.6.4). A power point provides alternating current (AC) whereas a car battery provides direct current (DC).

FIGURE 10.6.4 A power point and a car battery.

For a start, the average potential difference between the active and neutral pins of an Australian household power point is 240V. This is much higher than most electric cells or batteries can provide. In fact, 240V is the equivalent of putting twenty 12.0V car batteries together in series.

Secondly, because of the way household electrical energy is generated, it is delivered as an **alternating current** (AC). This means that the electrons in the wire oscillate backwards and forwards in the wire. In comparison, a battery provides **direct current** (DC), which means the electrons travel in one direction only.

Fortunately, most AC systems used in the household can be modelled using simple DC circuits.

Electricity bills

The work done, or energy consumed by a household appliance, is measured in joules. It is the power, P, in watts multiplied by the time period, Δt, in seconds.

$E = P\Delta t$

If you have a look at your home electricity bill, you'll see it is measured in **kilowatt-hours** (kWh). This gives a convenient number, without scientific notation, that is easy to write on your bill.

To calculate how much an appliance costs to run, multiply its power consumption in kW by the number of hours it runs for. Then take this number in kWh and multiply it by the cost of electricity per kWh. Worked example 10.6.1 shows how to calculate the cost of running some household appliances.

Worked example 10.6.1

CALCULATING THE COST OF ELECTRICITY

A 2450 W air conditioner runs for 5.00 hours. Assume the price for household electricity is 28.0109 cents per kWh. How much would it cost (to the nearest cent) to run this air conditioner for 5.00 hours?

Thinking	Working
Convert the power consumption of the appliance to kW.	$\frac{(2450)}{(1000)} = 2.45\,\text{kW}$
Use the appropriate equation to multiply the power of the appliance in kW by the number of hours for which it operates.	$E = P\Delta t$ $E = (2.45)(5.00)$ $E = 12.250\,\text{kWh}$
Multiply the number of kWh by the cost per kWh. Ensure that you present your answer to the correct number of significant figures and use the appropriate units.	cost = (12.250)(0.280 109) cost = 3.436 34 cost = \$3.44

Worked example: Try yourself 10.6.1

CALCULATING THE COST OF ELECTRICITY

A 2540 W iron is used for 2.50 hours. Assume the price for household electricity is 28.0109 cents per kWh. How much would it cost (to the nearest cent) to use this iron for 2.50 hours?

PHYSICSFILE

Kilowatt-hours and joules

The energy unit of kWh can be converted to joules (J) by multiplying the number of kWh by 3 600 000. Therefore,

$1\,\text{kWh} = 3.60 \times 10^6\,\text{J}$ or 3.60 MJ.

This value is calculated as follows:

$1\,\text{kWh} = 1000\,\text{W} \times 60\text{ minutes} \times 60\text{ seconds}$

$= 3\,600\,000\text{ watts} \times \text{seconds}$

$= 3.60 \times 10^6\,\text{J}$

ELECTRICAL SAFETY DEVICES

Household electrical wires can carry large amounts of energy. This means that they have the potential to do a lot of harm. The inherent danger associated with the use of electricity can be reduced using various safety devices, understanding the risks associated with electrical circuits, and always taking care when using electrical appliances of any sort.

Fuses and circuit breakers

Since wires heat up when current passes through them, there is a limit to how much current the wires in a house or building can safely carry. Household wiring systems are designed to prevent wires from becoming **overloaded**. Appliances that draw a lot of current, such as ovens, hot-water systems and air conditioners, are put on separate circuits to lights and power points.

Despite these precautions, overloading can still occur, most often due to a **short circuit**. A short circuit occurs when an electric circuit contains very little resistance. This can occur in an electrical appliance when the insulation between the active and neutral wires becomes damaged and these wires are in direct contact. In household circuits, short circuits are always dangerous situations. Large amounts of current mean that wires will heat up, causing insulation to melt or catch alight.

An electric current will always take the path of least resistance. For example, the globe in Figure 10.6.5a is on because the current has no alternative but to pass through the high resistance of the bulb. The globe in Figure 10.6.5b does not work because the closed switch provides a zero-resistance alternative pathway for the current, causing a short circuit.

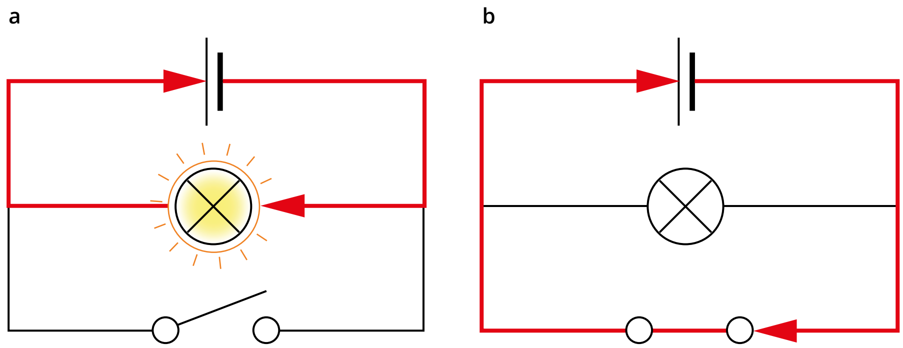


FIGURE 10.6.5 The globe in (a) functions normally, while (b) does not light up because it is a short circuit.

PHYSICSFILE

Power board overload

Another common reason for circuit overloading is the overuse of power boards and double adaptors. Most power boards are designed to carry a maximum of 7.5 A or 10 A of current. This value is written on them. If too many high-current appliances, such as heaters, kettles and irons, are plugged into a power board, it can overheat. This may cause the insulation around the wires to melt, causing a short circuit or even a fire. Figure 10.6.6 shows the overuse of power boards. This can result in more current being drawn than the maximum rating of the power board, causing it to overload. This can result in the wires and components overheating and possibly causing a fire.

FIGURE 10.6.6 Overloading of power boards.

Every domestic electric circuit contains either a **fuse** or a **circuit breaker**. A lot of the time these devices are built into circuit boards or small electrical appliances. A fuse will melt when too much current flows through it, breaking the circuit. A circuit breaker detects the magnetic field of an excessive current and automatically switches the circuit off. The function of both of these components is to interrupt the flow of current if it exceeds a certain value. Unlike a fuse, a circuit breaker can be easily reset after it has been activated, whereas a fuse needs to be physically replaced once it has melted through. Both fuses and circuit breakers can be chosen for different amounts of current, since some appliances such as ovens and hot water systems typically draw much larger amounts of current than regular power points.

Earth wires

Many household electrical appliances such as kettles, toasters and ovens have metal cases. If the active wire inside the appliance becomes loose and touches the case, then the whole case becomes electrically live. If anyone touches the case, the current will flow through their body, with possibly fatal consequences.

To prevent this, an earth wire is permanently connected to the metal case of the appliance, as shown in Figure 10.6.7. When the appliance is plugged in, this wire is connected via the household wiring system to the Earth. This means that, if the active wire touches the case, a short circuit will be created and current will immediately flow directly to the Earth. The large amount of current that flows in this situation should trip the fuse or circuit breaker, alerting users of the appliance to the problem.

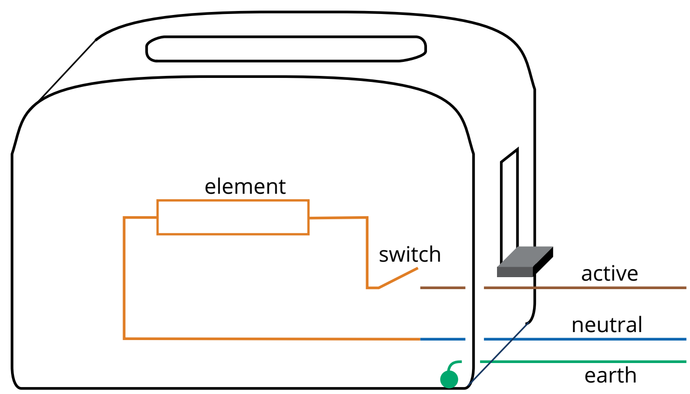


FIGURE 10.6.7 The earth wire inside a metal toaster is permanently connected to the casing.

Double insulation

Double insulation is another way of protecting against the possibility of a loose active wire inside an appliance. This process involves using two insulating barriers to protect users. Often this is done by making the case of the appliance out of plastic. This acts as an insulating layer in addition to the plastic insulation surrounding the active wire inside the appliance. Double-insulated appliances do not need an earth wire, so their electrical plugs only have two pins. They also usually have a special symbol on their cases, indicating that they are double insulated, as shown in Figure 10.6.8.

FIGURE 10.6.8 The double square symbol indicates that this appliance has double insulation.

Residual current devices

Residual current devices, also known as RCDs or earth leakage systems, detect any difference between the current in the active wire and the current in the neutral wire. In a properly operating circuit, these two currents should be exactly the same, but in opposite directions. The most likely reason for a difference between the active and neutral currents is that some current is going to earth through a fault or, in a worse case, through a person. If this happens, the RCD is able to switch off the supply in about 20 milliseconds, hopefully preventing any serious harm.

ELECTRIC SHOCK

Despite all the safety features of modern electrical systems and appliances, each year a number of Australians are killed in electrical accidents. The effect of **electric shock** depends on several factors including:

- the amount of current passing through the body
- the duration of the current flow
- the path the current takes through the body.

When attending to a victim of electrocution, it is important to first check if they are still in contact with the electrical source. First aid should only be administered when it is safe to approach the victim. This is because if they are still in contact with the current, it can pass through the body of any other person who touches them, causing another dangerous electric shock.

The amount of current that will flow through the body when in strong contact with a 240V source is well above the level that can cause death. Anything that provides a more effective electrical contact, such as having wet hands or bare feet, lowers the resistance and increases the current, potentially to life-threatening levels. Although rubber boots and gloves will increase resistance and lower the current, these should never be used as a form of protection in place of other sensible electrical safety precautions.

Since our bodies are relatively poor conductors, electrical energy passing through our bodies is quickly converted into heat and can cause terrible internal and external burns. Table 10.6.1 shows the likely effect on the human body of a half-second electric shock at different currents.

TABLE 10.6.1 The effect of a half-second electric shock on the body.

Current (mA)	Effect on the body
1	able to be felt
3	easily felt
10	painful
20	muscles paralysed, cannot let go
50	severe shock
90	mild breathing difficulties
150	severe breathing difficulties
200	death likely
500	serious burning, breathing stops, death inevitable

The duration of electrocution also affects the severity of the electric shock. The longer the electrocution is, the greater amount of electrical energy enters the body. Table 10.6.2 shows the likely effect on the human body of a 50 mA shock for different time periods.

TABLE 10.6.2 The effect of time on the severity of a shock.

Time (s)	Effect on the body
less than 0.2	noticeable but usually not dangerous
0.2–4	significant shock, possibly dangerous
more than 4	severe shock, possible death

The path that the current takes through the body is also important in determining its effect. Since our bodies are controlled by electrical impulses along the nerves, any current that flows into the body from an external source may interfere with our vital functions. In particular, any current flowing from one arm to the other may cause the chest muscles to contract and breathing to stop. Current through the heart regions can cause the muscles to become uncoordinated and heart function to stop. A brief current of about 80 mA is sufficient to cause fibrillation (irregular contraction of the heart muscle) if it flows directly through the heart. This is the cause of most electrical fatalities.

10.6 Review

SUMMARY

- Electrical energy use in the home is usually measured in kilowatt-hours, where $1\,\text{kWh} = 1000\,\text{W} \times 1\,\text{h}$
- The effect of electrocution depends on the amount of current passing through the body, its duration and the path it takes through the body.
- The danger associated with the use of electricity in the home can be managed by using fuses, circuit breakers, double insulation, earth wires and residual current devices.

KEY QUESTIONS

1 How does a fuse increase household electrical safety?
  - **A** by breaking the flow of current if it becomes too high
  - **B** by breaking the flow of current if a difference is detected between the currents in the active and neutral wires
  - **C** by taking current to the Earth if the metal casing of the appliance becomes live
  - **D** by providing an extra layer of electrical insulation

2 How does double insulation increase household electrical safety?
  - **A** by breaking the flow of current if it becomes too high
  - **B** by breaking the flow of current if a difference is detected between the currents in the active and neutral wires
  - **C** by taking current to the Earth if the metal casing of the appliance becomes live
  - **D** by providing an extra layer of electrical insulation

3 Convert 10.0 kWh into J.

4 One of the values given in the information below is incorrect. Use your knowledge of kWh to determine which value it is.
A 750 W air conditioner uses 0.750 kWh of energy in 1.00 hour. A typical price for household electricity is 28.0109 cents per kWh. Therefore, this air conditioner would cost approximately $10 to run for 5.00 hours.

5 Why are there only two cables coming to the house from the street and yet power points always have three connections?

6 The function of a fuse is to burn out, and thus break the supply of current if the circuit is overloaded. Why is it always placed in the active wire rather than the neutral one, given that this function could be fulfilled if it was in either?

7 What is the function of the 'earth stake' that will normally be found near a meter box?

8 An appliance was mistakenly wired between the active and earth instead of between the active and neutral. Why is it a very dangerous thing to do, even though the appliance will appear to work normally?

9 How much current would flow through a person with dry hands and a total contact resistance of 100.0 kΩ when they touch a 240.0 V live wire?

Chapter review

10

KEY TERMS

alternating current
ammeter
charge
circuit breaker
conductor
conventional current
coulomb
current
direct current
earth
effective resistance
electric circuit
electric current
electric shock
electrical potential energy
electricity
electron flow
elementary charge
fuse
insulator
ion
ionised
junction
kilowatt-hour
metal
net charge
non-metal
non-ohmic
ohmic
overload
parallel circuit
potential difference
power
residual current device (RCD)
resistance
resistivity
resistor
series circuit
short circuit
transfer
transform
volt
voltmeter

1 Approximately how many electrons make up a charge of −3.00 C?

2 What will be the approximate charge on 4.20×10^{19} protons?

3 Which charged particles are moving when an electric current flows in a circuit made from metallic wires?

A negatively charged electrons

B positively charged electrons

C positively charged protons

D both negative and positive charges

4 An alpha particle consists of two protons and two neutrons. Calculate the charge on an alpha particle.

5 Calculate the current that flows when 0.230 C of charge passes a point in a circuit each minute.

6 Compare the meaning of the terms 'conventional current' and 'electron flow'.

7 A current of 1.60 A flows for 115 seconds. Calculate:

a the amount of charge, in coulombs, that moves past a point in this time

b the number of electrons that move past a point in this time.

8 A current of 0.0400 A flows for a certain amount of time. In this time 5.00×10^{18} electrons move past a point. Calculate:

a the amount of charge, in coulombs, that moves past the point

b the amount of time that the current is flowing.

9 A phone battery has a potential difference of 3.80 V. If 2.00 C of charge is drawn from the battery. What amount of energy would this provide?

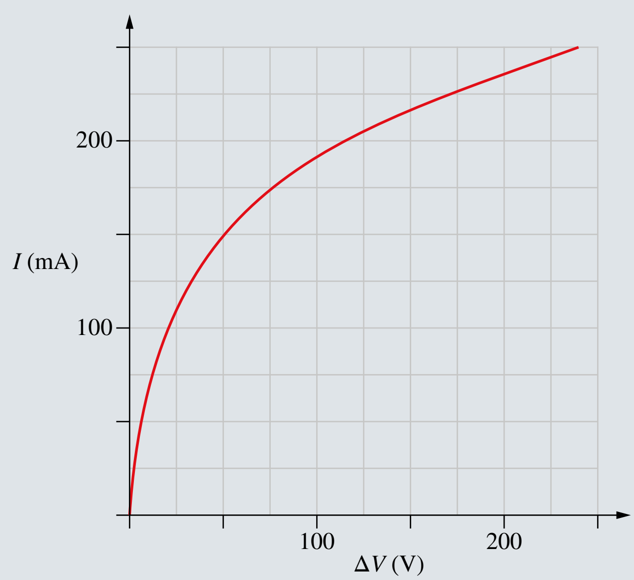
10 A battery does 2.00 J of work on a charge of 0.500 C to move it from point A to point B. Calculate the potential difference between the two points A and B.

11 How much power does an appliance use if it does 2520 J of work in 30.0 minutes?

12 A battery gives a single electron 1.40×10^{-18} J of energy. Calculate the potential difference supplied by the battery to two decimal places.

13 A 235 V appliance consumes 2130 W of power. The appliance is left on for 2.00 hours. What current flows through the appliance?

14 Calculate the resistance at 50.0 V of the non-ohmic conductor with the I–ΔV characteristics shown in the graph below.



15 Two resistors, R_1 and R_2, are wired in series. Which of the following gives the equivalent series resistance for these two resistors?

A $R_T = R_1 + R_2$

B $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$

C $R_T = R_1 - R_2$

D $R_T = R_1 \times R_2$

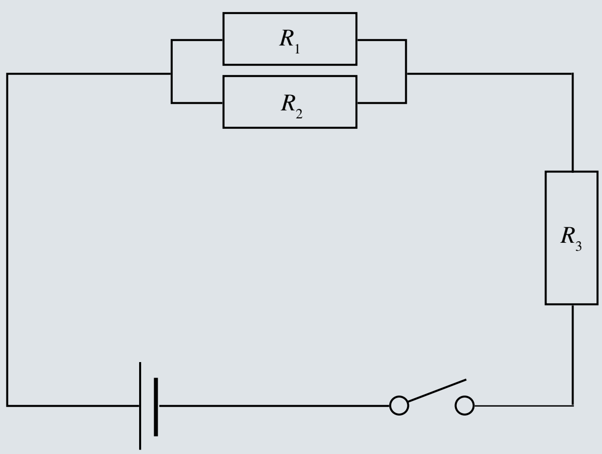
16 An electrical circuit is constructed as shown below. Use the information given about the circuit diagram to answer the following questions.

The electric cell provides 3.00 V.

The total resistance R_T of the circuit is 8.50 Ω.

R_2 has a resistance of 15.0 Ω.

The total resistance of resistors R_1 and R_2 is 5.00 Ω.



a Find the value of R_3.

b Find the current through R_3.

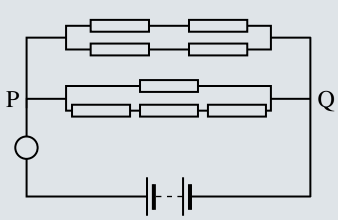
c Find the potential difference across the parallel pair R_1 and R_2.

d Find the current through R_2.

e Find the current through R_1.

f Find the value of R_1.

17 Eight equal-value resistors are connected between points P and Q in the circuit shown below. The value of each of these resistors is 20.0 Ω.



a The circle in the circuit diagram represents either an ammeter or a voltmeter. Identify which type of meter this should be.

b Calculate the total equivalent resistance of the circuit. Assume that the resistance of the meter and power source can be ignored.

18 Explain how an earth wire improves electrical safety in the home.

19 Sketch a circuit diagram showing how four 10.0 Ω resistors can be connected using a combination of series and parallel wiring to have a total equivalent resistance of 10.0 Ω.

20 A circuit consists of a 12.0 V battery and three 20.0 Ω light bulbs. The bulbs are initially connected in series.

a Calculate the power output of the circuit.

b The circuit is changed so that the bulbs are connected in parallel. Calculate the power output of the circuit.

c Compare the power drawn in the parallel circuit with that of the series circuit.

21 Which of the following would be most likely to cause serious electrocution harm to a human being?

A a high voltage spark from a Van de Graaff generator; duration = 1 ms

B 3.00 mA current; duration = 0.500 s

C 50.0 mA current; duration = 0.100 s

D 50.0 mA current; duration = 4.50 s

22 A 3.00 kW heating unit runs for 4.00 hours. If household electricity costs 28.0109 cents per kWh, how much does it cost to run the heater for this time?

23 The best definition of power from the following choices is:

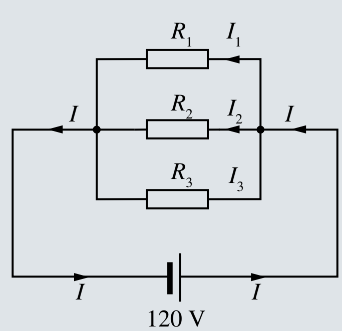
A the total amount of energy consumed by a circuit component

B the current drawn by a circuit component each second

C the rate at which potential difference is supplied to a circuit component

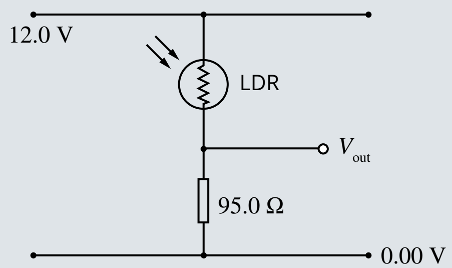
D the rate at which energy is transformed by a circuit component

24 Consider the following circuit where three resistors R_1, R_2 and R_3 are connected in parallel. Assume that R_1 = 112 Ω, R_2 = 235 Ω and R_3 = 605 Ω. The battery provides a potential difference of 125 V.



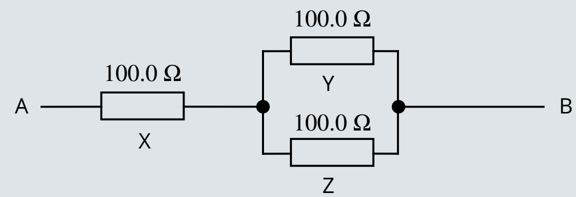
a Calculate the total resistance, R_T, in the circuit.
b Calculate the current, I, in the circuit.
c Determine the branch currents I_1, I_2 and I_3.
d What is the power output, P, of the battery?
e Calculate the total power consumed by all of the resistors in the external circuit.

25 In the simple light-dependent resistor (LDR) light-detector circuit shown below, ΔV_{out} is to be used to activate an alarm when the ambient light reaches a certain level. At this particular light level, the resistance of the LDR is 205 Ω. The alarm activates whenever ΔV_{out} is above the trigger level.



a What is the value of ΔV_{out} at which the alarm should activate?
b Will the alarm activate when the light is above or below the level of concern? Explain your answer.
c When it is very dark, what would you expect ΔV_{out} to become?

26 Three 100.0 Ω resistors are connected as shown. The maximum power that can safely be dissipated in any one resistor is 25.0 W.



a What is the maximum potential difference that can be applied between points A and B?
b What is the maximum power that can be dissipated in this circuit?

27 Why is the electric shock received when a finger touches a live wire likely to be less severe than the electric shock received by a person who touches a live wire with a pair of uninsulated pliers?

28 It is said that a fuse protects property, and a safety switch or residual current device protects lives. Explain why this statement is true.

UNIT 2 • WAVES, NUCLEAR AND ELECTRICAL PHYSICS

REVIEW QUESTIONS

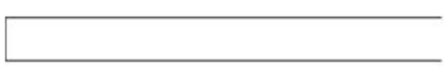
Section 1: Short response

1 Copy the axes below into your workbook to complete parts **a** and **b**.

a On your axes draw a labelled displacement–distance graph of a particle oscillating with a wavelength of 2.00 m and an amplitude of 3.00 cm. You must show two complete cycles.

b On another set of axes draw a labelled displacement–time graph for 1.25 cycles of a transverse wave with a period of 4.00 s and amplitude of 1.50 cm. Label this wave 'A'. On the same axis draw another transverse wave, also with a period of 4.00 s, but with an amplitude of 2.00 cm and out of phase from wave A by 90°. Label this wave 'B'.

2 **a** The closed organ pipe shown below is resonating at its fifth harmonic. Draw the displacement wave occurring in the tube for this harmonic and label the nodes with the letter 'N' and antinodes with the letter 'A'.



b If the length of the pipe in part **a** is 1.14 m and the speed of sound is 346 m s^{-1}, what is the fundamental frequency of vibration of this pipe?

3 Two speakers on the stage of an auditorium are 1.50 m apart and both are facing the people seated in the auditorium. The sounds from each of them are in phase and of the same amplitude. Jin is seated 3.60 m directly in front of one of the speakers and is noticing an audible drop on certain notes from the speakers. Determine three possible frequencies that could be causing this observation.

4 **a** Nori struck a tuning fork with a small wooden mallet and recorded the sound using an app on the phone. When listening to the recording on the phone, Nori noticed that the tuning fork was also making a sound, even before it was struck with the mallet. Name the phenomenon and explain why it occurred.

b Nori was able to slightly change the frequency of the sound made by the tuning fork on the app so that it was 4.0 Hz higher than the normal frequency of the fork. Nori then played the tuning fork and the slightly altered sound recording at the same time. Describe the resulting sound that Nori heard and name this phenomenon. Describe what Nori would hear when increasing the difference in frequency from 4.0 Hz to 15.0 Hz.

5 Explain each of the following.

a The difference between a fuse and a residual current device (RCD) in terms of their operation and the kind of protection that they provide.

b A short circuit in a household electrical appliance may trigger the circuit breaker in its circuit.

c Some household electrical appliances have plugs with three prongs and some have only two, yet both are safe.

6 Write a balanced nuclear equation for each of the following scenarios.

a A lithium-7 nucleus is bombarded with a high-speed proton resulting in the production of two identical particles.

b Gold-185 emits an alpha particle.

c Thallium-218 undergoes beta-minus decay.

7 **a** Two slow-moving protons are travelling directly towards each other. Will the protons collide and fuse together? In your answer make reference to the forces acting on the protons and the energy barrier.

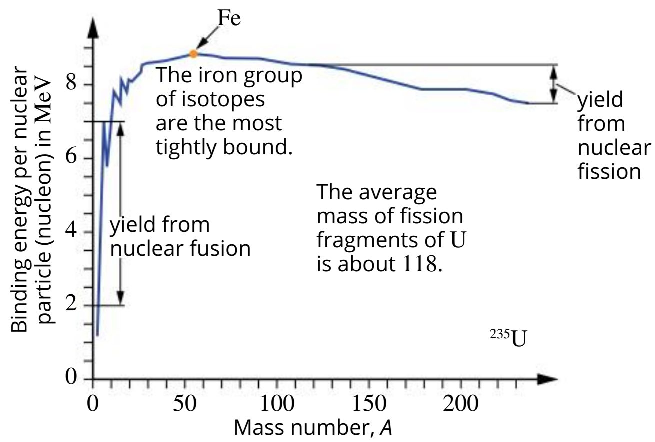
b Two fast-moving protons are travelling directly towards each other. The protons collide and fuse together. Discuss the forces acting on the protons and make reference to the energy barrier in your answer.

8 An electron and a positron collide and annihilate each other, producing two photons.

a Explain the source of the energy of the photons.

b Assuming the electron and positron each had minimal kinetic energy, calculate the combined energy (in joules and MeV) of the photons produced.

9 Use the following graph to answer the questions below.



a Explain what is meant by binding energy per nucleon.

b The element iron is considered to be the most stable of all the nuclei. Using the graph, explain why this is the case.

c Fission of uranium-235 results in fission fragments of average mass number around 118. Referring to the binding energy per nucleon for the fuel and the fragments, explain why there is a net energy release in a fission reaction.

10 A group of students perform an experiment in which an electric motor is used to lift a 244.9 g weight through 2.00 m, thus increasing its potential energy by 4.80 J. They will determine the efficiency of the motor from measurements of the rate at which the weight is lifted. In the experiment, when the voltage was 6.00 V a current of 0.250 A was measured, the weight took 5.05 s to rise the 2.00 m. Calculate the efficiency of the motor.

Section 2: Problem solving

11 The slow neutron bombardment of uranium-235 produces many fission products via a series of reactions. One such reaction yields caesium-140 and rubidium-93 as products.

a Write an equation for this fission reaction.

b Calculate the energy, in both electron volts and joules, which would be released from the fission of one uranium-235 atom in this reaction.

Particle	Mass (kg)	Mass (Da)
U-235	3.90221×10^{-25}	235.07295
Cs-140	2.32338×10^{-25}	139.96265
Rb-93	1.54301×10^{-25}	92.95241
neutron	1.67493×10^{-27}	1.00899

c Calculate the energy in joules that would be released from the fission of 1.00 kg of uranium-235 atoms if the reaction products were only those given above.

d The Trojan nuclear power station in Oregon produces 9.76×10^{13} J of electrical energy per day. In theory, what mass of U-235, in kg, would be needed to produce this energy?

e This power station in fact uses around 2.78 kg of U-235 per day. Discuss the reasons for the difference in the mass of U-235 that is theoretically required (part **d**) and the mass that is actually used.

12 A Geiger counter measures the radioactive disintegrations from a sample of a certain radioisotope. The count rate is recorded in the table.

Activity (Bq)	800	560	400	280	200	140
Time (min)	0.0	5.0	10.0	15.0	20.0	25.0

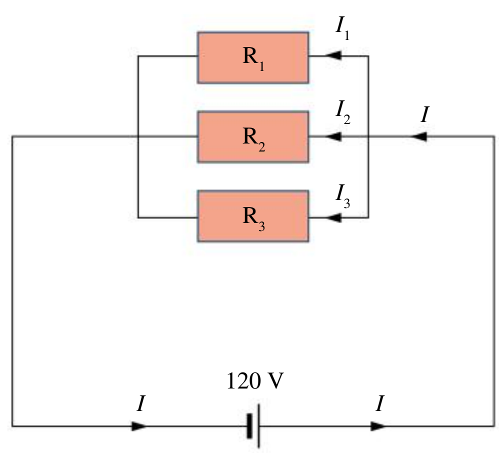
a Plot a graph of activity versus time.

b Use your graph to estimate the activity of the sample after 13 minutes.

c What is the half-life of this element?

d Determine the activity of the sample after 30 minutes.

13 Three resistors, $R_1 = 100.0\,\Omega$, $R_2 = 200.0\,\Omega$ and $R_3 = 600.0\,\Omega$, in a circuit are connected in parallel with a 120 V battery.



a Calculate the total resistance in the circuit.

b Determine the total current in the circuit.

c Calculate the branch currents I_1, I_2 and I_3.

d Determine the power output of the battery.

e Calculate the total power consumed by all the resistors in the circuit.

14 The didjeridu is a musical wind instrument originated in the north of Western Australia near the Northern Territory border. It is a straight, hollow pipe that was originally made of bamboo but now is often manufactured from the branch of a eucalypt tree. It varies in length but is usually around 1.5 to 2.0 m long. The didjeridu acts like an air column open at both ends and has no holes, keys or valves like orchestral wind instruments; the different harmonics are simply produced by overblowing (blowing more strongly).

a Draw the pressure standing wave produced when the instrument is blown in such a way that the fifth harmonic can be heard.

b Given that the speed of sound in desert air is 336 m s^{-1}, calculate the frequency of the fifth harmonic produced from a 1.50 m long didjeridu played in the desert.

c A second didjeridu has a fundamental frequency of 106 Hz. Which of the two instruments is longer? Justify your answer using relevant calculations.

Section 3: Comprehension

15

We live in a world of radiation

Radiation is all around us all day, every day. It sustains our lives. We can see things because our eyes detect, and our brain analyses, the radiation we call light. Infra-red radiation, whether from the sun or from a glowing fire, keeps us warm. We often heat up food with electromagnetic radiation in the form of microwaves. Radio waves allow us to communicate sound or pictures over huge distances. Ultra-violet radiation can be used for sterilising medical equipment. All living things rely on some form of radiation for their existence.

During the last century, scientists recognised a type of radiation known as ionising radiation. This naturally occurring radiation comes from many sources, including outer space, the Sun, the rocks and soil beneath our feet, the buildings we live in, the air we breathe, the food and drink we ingest, and even our own bodies. These sources combine to give us our naturally occurring background radiation dose. In Australia the average background radiation dose is approximately 1.5 mSv per year.

Cosmic radiation dose rates at different altitudes

supersonic aircraft	15 000 m	13.00 μSv per hour
international air travel	8000 m	3.70 μSv per hour
La Paz, Bolivia (world's highest capital city)	3900 m	0.23 μSv per hour
Mexico City	2240 m	0.09 μSv per hour
sea level	0 m	0.03 μSv per hour

1000 microsieverts (μSv) = 1 millisievert (mSv)

1000 millisieverts = 1 sievert (Sv)

Radiation doses

Ionising radiation and radioactive materials are widely used in medicine, industry, agriculture, environmental studies, pollution control and research. These uses benefit each of us individually and the Australian community as a whole.

Humans have increased their radiation dose through a variety of activities. One is living indoors. By surrounding ourselves with bricks, stone and mortar, we increase the concentration of a radioactive gas called radon in the air we breathe. Radon arises naturally from the radioactive decay of uranium and thorium, normally present in rocks, soil, bricks, mortar, tiles and concrete.

Another source of radiation is the medical use of electromagnetic radiation, such as X-rays in radiography and tomography, and the use of radioisotopes and radioactivity in nuclear medicine. Some therapeutic uses of radiation can give a dose to certain organs many times higher than our annual background radiation dose.

Small extra doses of radiation occur in a number of ways. The further from the Earth's surface you go, the less shielding the atmosphere provides you from cosmic rays. On a mountain top the air may be cleaner, but the radiation dose is higher. Air travel increases radiation dose; astronauts receive even higher doses. Fallout from atmospheric nuclear testing in the 1950s and 1960s is still present in the environment. Many industries such as the fertiliser, mining and building industries release otherwise locked-in radioactivity into the environment by burning coal and other fossil fuels. This is especially true of coal-fired power stations, which release radioactive materials from the structure of the solid coal into the atmosphere as the coal is burnt.

Other common but minor sources of radiation are some older luminescent clocks, watches, compasses, gunsights and exit signs, certain paints and pigments, dental porcelain, fire alarms, smoke detectors and televisions.

Although some radiation is capable of travelling large distances, it may be stopped by appropriate absorbers. The light from distant stars travels fantastic distances from other galaxies but may be stopped by a piece of paper. Radio waves are also capable of travelling great distances but may be absorbed by materials such as metals. Like light, ionising radiation travels in straight lines until absorbed or deflected. The material used to absorb ionising radiation depends on the type and energy of the radiation.

Cancer risk

Risk estimates for cancer following an exposure to ionising radiation are the subject of ongoing detailed studies. Existing estimates are based largely on information of the cancer rates in the survivors of the atom bombs at Hiroshima and Nagasaki and a few other groups of people subjected to large doses given quickly.

Present evidence indicates that the damaging after-effects of radiation exposure are greatly reduced when the dose is delivered in small amounts spread over a long time period. Nevertheless, for the purposes of radiation protection, it is assumed that any radiation dose, however small, can have some effect, and so protective measures are put in place for radiation workers. International studies of large groups of workers in the nuclear industry (who receive low doses spread over several years) generally agree with existing risk estimates.

Medical uses

Doses received by patients are classified separately because these exposures to radiation are justified on the grounds that the prescribed exposures pose a lesser threat to a person's welfare than does the risk of going undiagnosed (e.g., by avoiding having an X-ray or a CT scan) or not treating a disease using nuclear medicine. No limits are set for diagnostic or therapeutic radiation exposures, except that they should be as low as possible after considering the risk and benefit factors. In nuclear medicine, the doses from diagnostic procedures are typically in the range of 1000 to 10 000 μSv (average 3300 μSv). Doses from X-ray studies are generally slightly less.

When a nuclear medicine examination is proposed for a pregnant woman, care is taken to determine if the examination is necessary for a medical condition that requires prompt therapy. For these diagnostic examinations, the risk to the mother of not performing the examination must be greater than the radiation risk to the foetus. If an examination is performed, extra care is taken to ensure that the risk to the mother and foetus is kept as low as possible.

Medical radiation sources

The average dose of radiation from medical procedures in Australia is about 800 μSv per person per year. Typical radiation doses for various medical diagnostic procedures using radioisotopes are listed in the tables below. All figures are based on data from Radiation Protection in Australasia, 2000.

Typical doses for X-rays

Conventional X-rays:	
dental	5 to 10 μSv
chest	20 μSv
leg or foot	20 μSv
skull	70 μSv
mammography	400 μSv
barium meal	2500 μSv
intestine	3000 μSv

CT scans (computerised tomography):	
head	2600 μSv
abdomen	13 000 μSv

Typical doses for other types of medical diagnostic scans

kidney scan	1400 μSv
liver scan	1700 μSv
thyroid scan	2600 μSv
lung scan	2600 μSv
bone scan	4600 μSv
soft tumours	40 000 μSv

Examples of alpha, beta and gamma emitters

α emitter:

americium-241, used in smoke detectors

β emitter:

carbon-14, used in carbon dating

γ emitter:

technetium-99m, used as a diagnostic medical radioisotope

Average annual radiation dose

The average annual radiation dose per person per year is approximately 1500 μSv, plus any exposure from medical procedures.

Some naturally occurring radiation sources

50 μSv from travel and power stations, such as air travel and coal-fired power stations.

300 μSv from cosmic rays; if you live 1000 metres above sea level, add 200 μSv, more if higher.

400 μSv from food and drink, mostly from naturally occurring radioactive potassium-40 and polonium-210. Some foods concentrate more radioactivity than others, although generally not enough to make a significant difference to this total.

800 μSv from terrestrial radiation. Long-lived radioactive materials like uranium and thorium occur in the environment. They emit ionising radiation that contributes 600 μSv a year to your average terrestrial radiation dose. This radiation comes from rocks and soils, and from building materials like bricks, mortar, concrete and tiles. Radon and thoron are naturally occurring radioactive gases. Both these gases are present in the air you breathe. This part of your average terrestrial radiation dose (200 μSv a year) therefore derives from the decay of the alpha-emitting radon gas in your lungs. In the open, these gases are diluted by the wind mixing them in the atmosphere. Indoors they may concentrate in still air.

Deduct 10 per cent if you live in a wooden house.

Deduct 20 per cent if you live in a tent.

Deduct 50 per cent or more if you live in the open.

Add 10 per cent or more if you live in a granite building.

Add 100 per cent or more if you keep doors and windows shut.

Add 100 per cent or more if you use bore water (water extracted from a borehole or well), especially in a hot shower. Bore water, having been underground, often contains radon. When the water emerges from the bore, radon is released and the release is enhanced when the water is heated or broken into droplets. As a result, the radon release is most significant in a hot shower.

Source: ANSTO – Australian Nuclear Science and Technology Organisation.

a What is background radiation and where does it come from?

b How do coal-fired power stations add to the background radiation?

c Why would astronauts on the International Space Station be exposed to higher doses of radiation than passengers in a commercial jet aircraft?

d Radiation can be human-made or naturally occurring. All forms of radiation can also be divided into the two categories: ionising or non-ionising. Create a table in your workbook using the headings shown below and compare the two categories of radiation by completing the table.

Type of radiation	Description	Name three types	Name one source for each
ionising			
non-ionising			

e Radon is a colourless, odourless and tasteless inert gas, and is therefore chemically inactive and easily inhaled. It occurs naturally in the decay chain of uranium, through thorium, to lead. Uranium and thorium are among the most commonly found radioactive elements on Earth, with both having extremely long half-lives, so radon will be around for a long time yet. Radon is formed from the decay of uranium-238 in a six-step process to radon-222.

- **i** The first three steps are from uranium-238 to uranium-234 via alpha, then two beta decays. Write the three decay equations for the decay series of uranium-238 to uranium-234.
- **ii** The next three steps in the decay process all result in alpha-particle emission as uranium-234 decays to radon-222. Write a single equation to represent the decay of uranium-234 to radon-222.

f Given what you know about radon, would it be safer for miners if uranium was mined underground or open cut? Explain.

g The article makes the argument that using bore water increases your radiation dose. Explain why this would be the case.

h All living things undergo a carbon cycle as they recycle carbon in various compound forms. When they die, this stops and the amount of carbon at that time remains constant. The age of fossilised material can be determined using carbon dating.

- **i** Write an equation for the beta-minus decay of carbon-14.
- **ii** Write equations for the transition that occurs in the carbon-14 nucleus to produce nitrogen-14 at the nucleon level and at the quark level.
- **iii** Explain what carbon dating is and how this is a successful means of determining the age of fossils.

i A Qantas A380 flight from Perth to Tokyo (one-way) takes 13 hours. An 84.0 kg person makes five return flights each year. Using information in the article, calculate the dose of radiation, in Sv, that this person would receive as a result of making these flights.

APPENDIX A SI units

THE STANDARD UNITS OF MEASUREMENT

Accurate and easy measurement of quantities is essential in both everyday life and scientific investigation. Over the centuries, many different systems of measuring physical quantities have been developed. For example, length can be measured in chains, fathoms, furlongs, yards, feet, rods and microns. Some units were based on parts of the body. The cubit was defined as the distance from the elbow to the fingertip, and so the amount of cloth that you obtained from a tailor depended on the physical size of the person selling it to you.

The metric system was established by the French Academy of Science at the time of the French Revolution (1789–1815) and is now used in most countries. This system includes units such as the metre, litre and kilogram. Countries of the British Empire adopted the British Imperial system of the mile, gallon and pound. These two systems developed independently, and their dual existence created problems in areas such as trade and scientific research. In 1960, an international committee set standard units for fundamental physical quantities. This system was an adaptation of the metric system and is known as the *Système International d'Unités* (International System of Units) or SI system of units.

TABLE A.1 The SI units identify the seven fundamental quantities whose basic value is defined to a high degree of accuracy.

Fundamental quantity	SI unit	SI unit symbol
mass	kilogram	kg
length	metre	m
time	second	s
electric current	ampere	A
temperature	kelvin	K
luminous intensity	candela	cd
amount of substance	mole	mol

Mass

The kilogram was originally defined as the mass of 1 L of water at 4°C. Since 1897 the measurement standard for the kilogram has been a cylindrical block of platinum–iridium alloy kept at the International Bureau of Weights and Measures in France. Australia has a copy of this standard mass at the CSIRO Division of Applied Physics in Sydney. At times it is returned to France to ensure that the mass remains accurate. To enable scientists around the world to standardise their mass measuring devices without needing to access the standard kilogram cylinder, it was decided that a better standard should be developed. Since 2019 the standard kilogram is now defined in terms of the speed of light ($c = 299\,792\,458\,\text{m}\,\text{s}^{-1}$), and Planck's constant ($h = 6.626\,070\,15 \times 10^{-34}\,\text{J}\,\text{s}$), which combine to determine the mass-energy equivalent of the energy gained when an electron transitions in a specific isotope of caesium. Specifically, one kilogram is exactly equal to the change in mass of $1.475\,521\,4 \times 10^{40}$ atoms of caesium-133 as each atom's outer electron transitions up to the hyperfine energy level.

Length

The metre was originally defined in 1792 as one ten-millionth of the distance from the equator to the North Pole (approximately 10 000 km). This definition has changed a number of times since. In 1983, to give a more accurate value, the metre was redefined as the distance that light in a vacuum travels in $\frac{1}{299\,792\,458}$ of a second. This standard can be reproduced all over the world, as light travels at a constant speed in a vacuum.

PHYSICSFILE

Metric system

The metric system was originally developed in France and is known as the *Système International* (SI). It was adopted in France in 1840 as the official system of units, although it had been developing in that country since 1545. It has remained in use ever since and has gradually been adopted by most other countries. It has been modified a little over the years and now, in Australia, we use SI units that have been standardised by the International Standards Organisation (ISO) since the 1960s. Some countries such as France, Italy and Spain use an earlier form of the metric system that is slightly different. The USA still measures almost everything in the old imperial units such as pounds for mass and feet for distance but, even there, scientists use the SI system of units.

There are two major advantages of using the metric system. It is easier to use than other systems because derived units are straightforward and various sizes of units are created using multiples of 10. The other very big advantage is the international nature of the standards and units. All units are standardised, making comparisons straightforward.

Time

Up to 1967, time had always been based on the apparent motion of the heavens. The second was once defined in terms of the motion of the Sun. Until 1960, one second was defined as $\frac{1}{60}$ of $\frac{1}{60}$ of $\frac{1}{24}$ of an average day in 1900. This reflected the rate of the Earth's rotation on its axis; however, its rotation is not quite uniform. In 1967, a more accurate definition was adopted that was not based on the motion of the Earth. One second is now defined as the time required for a caesium-133 atom to undergo 9 162 631 770 vibrations. These vibrations are stimulated by an electric current and are extremely stable, allowing this standard to be reproduced all over the world.

DERIVED UNITS

As well as the seven fundamental quantities, a wide variety of other physical quantities can be measured. You may have already encountered some of these, such as frequency, velocity, energy and density. A derived quantity is defined in terms of the fundamental quantities. For example, the SI unit for area is square metres (m^2).

TABLE A.2 Some derived SI quantities and their units.

Quantity	SI unit	SI unit symbol	Equivalent unit
velocity	metres per second	$m\,s^{-1}$	not applicable
acceleration	metres per second per second	$m\,s^{-2}$	not applicable
frequency	hertz	Hz	s^{-1}
force	newton	N	$kg\,m\,s^{-2}$
energy/work	joule	J	$kg\,m^2\,s^{-2}$

NON-SI UNITS

There are several units used in physics that are not part of the SI system but are frequently used. One of these is the atomic mass constant (m_u), which is used to measure the tiny amounts of matter involved in nuclear reactions. The units of the atomic mass constant is the dalton (Da), named in honour of John Dalton the English chemist and physicist. Since 2019 the Bureau of Weights and Measures has only used the dalton to refer to masses of particles relative to the atomic mass constant. Prior to this date the term atomic mass unit (*amu*), which was shortened to (*u*), was used. The atomic mass constant (m_u) is equal to $\frac{1}{12}$ the mass of the carbon-12 isotope at rest and in its nuclear and atomic ground state. A mass of 1.00 Da is equal to 1.66×10^{-27} kg.

Another non-SI unit is the electronvolt (eV), which is also used for very small measurements of energy. An electronvolt is the energy that an electron would gain if it were accelerated by a potential difference of 1 volt, and is equal to 1.60×10^{-19} J.

The unit called the astronomical unit (au) is also accepted due to the very large distances involved in astronomy. One astronomical unit is equal to the average distance from Earth to the Sun. 1 au is equal to 149 597 870 700 m.

APPENDIX B Understanding measurement

MEASUREMENT AND UNITS

In every area of physics, we have attempted to quantify the phenomena we study. In practical demonstrations and investigations, we generally make measurements and process those measurements in order to come to some conclusions. Scientists have a number of conventional ways of interpreting and analysing data from their investigations. There are also conventional ways of writing numerical measurements and their units.

Correct use of unit symbols

The correct use of unit symbols removes ambiguity, as symbols are recognised internationally. The symbols for units are not abbreviations and should not be followed by a full stop unless they are at the end of a sentence.

Capital letters are not used for the names of any units. For example, we write newton for the unit of force, while we write Newton if referring to someone with that name. Capital letters are only used for the *symbols* of the units that are named after people. For example, the unit of energy is the joule, and the symbol is J. The joule is named after James Joule, who was famous for studies of energy conversions. The exception to this rule is 'L' for litre. We do this because a lower-case 'l' looks like the numeral '1'. The unit of distance is the metre, and the symbol is m. The metre is not named after a person and so it starts with a lower-case m. The unit metre is often misspelled as 'meter', especially in the USA. However, in countries that have converted to the metric system, the correct spelling is 'metre'. Meanwhile, a meter is a device that can be used to measure things, like a water meter, or an electricity meter.

The product of a number of units is shown by separating the symbol for each unit with a dot or a space. Most teachers prefer a space but a dot is perfectly correct. The division or ratio of two or more units can be shown in fraction form, using a slash, or using negative indices. Most teachers prefer negative indices. Prefixes should not be separated by a space.

TABLE B.1 Some examples of the use of symbols for derived units.

Preferred	Correct also	Wrong
$\mathrm{m\,s^{-1}}$	$\mathrm{m{\cdot}s^{-1}}$ or $\mathrm{m/s}$	$\mathrm{ms^{-1}}$
$\mathrm{m\,s^{-2}}$	$\mathrm{m{\cdot}s^{-2}}$ or $\mathrm{m/s^2}$	$\mathrm{ms^{-2}}$
$\mathrm{kg\,m^{-3}}$	$\mathrm{kg{\cdot}m^{-3}}$ or $\mathrm{kg/m^3}$	$\mathrm{kgm^{-3}}$
µm	not applicable	µ m
N m	N·m	Nm

Units named after people can take the plural form by adding an 's' when used with numbers not equal to one. Never do this with the unit symbols. It is acceptable to say 'two newtons' but wrong to write 2 Ns. It is also acceptable to say 'two newton'.

Numbers and symbols should not be mixed with words for units and numbers. For example, twenty metres and 20 m are correct while 20 metres and twenty m are incorrect.

FIGURE B.1 A scientific calculator.

Scientific notation

To overcome confusion or ambiguity, measurements are often written in scientific notation. Quantities are written as a number between one and ten and then multiplied by an appropriate power of ten. Note that 'scientific notation', 'standard notation' and 'standard form' all have the same meaning.

Examples of some measurements written in scientific notation are:

$$0.0540\,\text{m} = 5.40 \times 10^{-2}\,\text{m}$$
$$245.7\,\text{J} = 2.457 \times 10^{2}\,\text{J}$$
$$2080\,\text{N} = 2.08 \times 10^{3}\,\text{N}$$

You should be routinely using scientific notation to express numbers. This also involves learning to use your calculator intelligently. Scientific and graphics calculators can be put into a mode whereby all numbers are displayed in scientific notation. It is useful when doing calculations to use this mode rather than frequently attempting to convert to scientific notation by counting digits on the calculator display. It is quite acceptable to write all numbers in scientific notation, although most people prefer not to use scientific notation when writing numbers between 0.01 and 1000.

An important reason for using scientific notation is that it removes ambiguity about the precision of some measurements. For example, a measurement recorded as 240 m could be a measurement to the nearest metre; that is, somewhere between 239.5 m and 240.5 m. It could also be a measurement to the nearest ten metres; that is, somewhere between 235 m and 245 m. Writing the measurement as 240 m does not indicate which level of precision is intended. If the measurement was taken to the nearest metre, it would be written in scientific notation as 2.40×10^2 m. If it was taken to the nearest ten metres only, it would be written as 2.4×10^2 m.

PREFIXES AND CONVERSION FACTORS

Conversion factors should be used carefully. You should be familiar with the prefixes and conversion factors in Table B.2. The most common mistake made with conversion factors is multiplying rather than dividing. Some simple strategies can save you this problem. Note that the table gives all conversions as a multiplying factor.

TABLE B.2 Prefixes and conversion factors.

Multiplying factor		Prefix	Symbol
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	µ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p

Do not put spaces between prefixes and unit symbols. It is important to give the symbol the correct case (capital or lowercase). There is a big difference between 1 mm and 1 Mm.

There is no space between prefixes and unit symbols. For example, one-thousandth of an ampere is given the symbol mA. Writing it as m A is incorrect. The space would mean that the symbol is for a derived unit, a metre ampere.

Worked example B1

The diameter of a cylindrical piece of copper rod was measured at 24.8 mm with a vernier calliper. Its length was measured at 35 cm with a tape measure.
a Find the area of cross-section in m^2.
b Find the volume of the copper rod in m^3.
Answer
a The area of cross-section is πr^2. The radius is calculated by dividing the diameter by two. Hence the radius is 12.4 mm. To calculate the area in m^2, first halve the diameter and convert it to metres. The radius is $\frac{24.8}{2} = 12.4\,\text{mm} = 12.4 \times 10^{-3}\,\text{m}$. The radius is not written in scientific notation. This is not necessary. All you need to do is multiply by the appropriate factor. The conversion factor for mm to m is 10^{-3}. Just multiply by the conversion factor and don't bother to rewrite the result in scientific notation. This is because it is only going to be used in a calculation and is not a final result. The area of cross-section is $\pi r^2 = \pi(12.4 \times 10^{-3})^2 = 4.830\,51 \times 10^{-4} = 4.83 \times 10^{-4}\,\text{m}^2$.
b The volume is $\pi r^2 h$, where h is the length of the cylinder. The length is $35\,\text{cm} = 35 \times 10^{-2}\,\text{m}$. Hence the volume is $\pi(12.4 \times 10^{-3})^2(35 \times 10^{-2}) = 1.690\,68 \times 10^{-4} = 1.7 \times 10^{-4}\,\text{m}^3$.

Worked example B2

a A car is traveling at 115 km h^{-1}. How fast is this in m s^{-1}?
b Convert 35 miles per hour to metres per second. A mile is approximately 1600 m.
Answer
a 115 km h^{-1} is 1105×10^3 metres per 3600 s. $\frac{110 \times 10^3}{3600} = 31.944\,44$ Hence 115 km h^{-1} = 31.9 m s^{-1}.
b 35 miles per hour is 35×1600 metres per 3600 s. $\frac{35 \times 1600}{3600} = 15.5556$ Hence 35 mph = 16 m s^{-1}.

DATA

Physicists and physics students collect, analyse and interpret experimental data. In fact, you will do this when you conduct your practical investigation. Working with data requires a good understanding of the meaning and limitations of measurement.

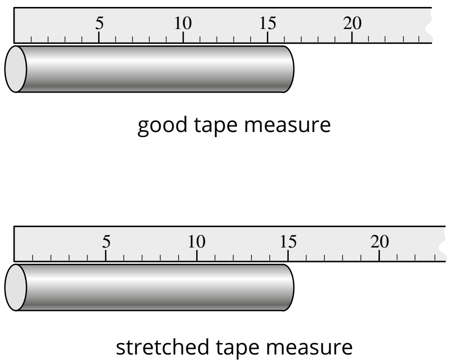


FIGURE B.2 The diagram shows that a correctly manufactured tape measure correctly measures the cylinder to be 16 cm long, while the stretched tape measure gives a wrong measurement of 15 cm. The stretched tape measure is inaccurate.

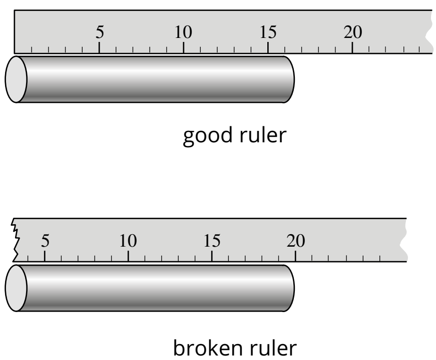


FIGURE B.3 The diagram shows that an undamaged ruler correctly measures the cylinder to be 16 cm long, while the broken ruler gives a wrong measurement of 19 cm. The broken ruler is inaccurate but equally as precise as the unbroken ruler.

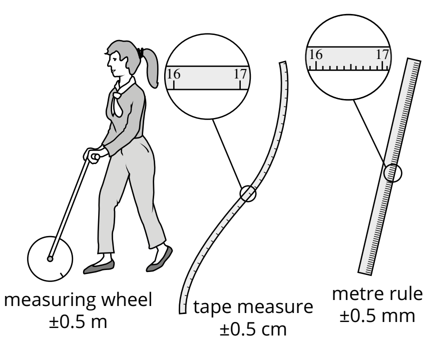


FIGURE B.4 The measuring wheel has low precision and only measures to the nearest metre. It has an uncertainty of 0.5 m. The tape measure has more precision and has an uncertainty of 0.5 cm or 0.005 m. The metre rule has an uncertainty of 0.5 mm or 0.0005 m.

Accuracy and precision

Two very important aspects of any measurement are accuracy and precision. Accuracy and precision are not the same thing. The distinction between the two ideas is only hard to grasp because the two words are defined in a similar way in the dictionary. We often hear the words used together, and in general conversation they tend to be used interchangeably.

Instruments are said to be *accurate* if they truly measure what they are supposed to measure. For example, if a tape measure is correctly manufactured it can be used to measure lengths accurately to the nearest centimetre.

Imagine that the tape measure is accidentally stretched during the manufacturing process, as shown in Figure B.2. It would still be used to measure length to the nearest centimetre but all measurements would be wrong. It would be inaccurate.

Suppose an accurate ruler had 3 cm snapped off the end, as shown in Figure B.3. It would now give readings all too large by 3 cm if no allowance were made for the missing piece. This ruler measure would be inaccurate.

In these two examples, the tape measure or ruler is used to measure to the nearest centimetre but is inaccurate. Inaccurate means just plain wrong. Instruments are said to be *precise* if they can differentiate between slightly different quantities. Precision refers to the fineness of the scale being used.

Consider the metre rule, the tape measure and the measuring wheel used to mark out sports fields. All three measure distance. All three can be accurate. The metre rule is more *precise* because it measures to the nearest millimetre, the tape measure has less precision due to measuring only to the nearest centimetre, while the wheel measures only to the nearest metre (Figure B.4).

The tape measure is a more precise instrument than the measuring wheel. Suppose two distances of 2673 mm and 2691 mm are being measured with these two instruments. Each distance would be measured as 3 m, to the nearest metre, by the wheel. They would be measured differently as 2.67 m and 2.69 m, to the nearest centimetre, by the tape measure. The tape measure is more precise because it has a finer scale. You might also say that it has greater resolution. The measuring wheel has such low precision that it can't be used to measure which of the two distances is greater or smaller. Measuring instruments with less precision give measurements that are less certain. The uncertainty in the measurement is due to a coarser scale. The measuring wheel gives less certain measurements than the tape measure even though both instruments may be equally accurate.

All measurements have some amount of *uncertainty*, due to the precision of the instrument that does the measuring. (Note that in Chapter 1 the uncertainty due to collecting a range of data was analysed. This section deals with uncertainty due to precision.) The uncertainty is generally one half of the finest scale division on the measuring instrument. The measuring wheel has an uncertainty of 0.5 m. The metre rule has an uncertainty of 0.5 mm. The tape measure has an uncertainty of 0.5 cm. An electronic balance set to measure grams to two decimal places has an uncertainty of 0.005 g.

Sometimes this uncertainty is referred to as error. It is not error, in that it is not a mistake or something wrong. All measuring instruments have limited precision and, in general, the uncertainty is half of the smallest scale division on the instrument.

The uncertainty is the measure of the precision of an instrument. It is not related to accuracy. A micrometer screw gauge, which measures length to the nearest one-hundredth of a millimetre and hence is very precise, may not be accurate. Usually they are, but if one has been badly manufactured or bent by being over-tightened repeatedly it most likely will be inaccurate. But its precision will still be ±0.000 005 m, or half of one-hundredth of a millimetre.

The uncertainty gives the range in which a measurement falls. If you measured the length of a stick with a metre rule then you would get a measurement 'plus or minus' half a millimetre.

Any stick between 127.5 mm and 128.5 mm long would be measured as 128 mm to the nearest millimetre (refer to Figure B.5). We would record this as 128 ± 0.5 mm.

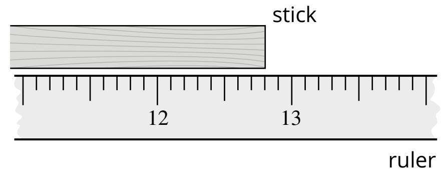


FIGURE B.5 A stick anywhere between 127.5 mm and 128.5 mm would be recorded as having a length of 128 mm if measured by a metre rule with a scale division of 1 mm. Conversely, a measurement recorded as 128 mm could be of an object of length anywhere between 127.5 mm and 128.5 mm.

When using an analogue scale, you might think that you can 'judge by eye' fractions of a scale division and hence get greater precision than half a scale division. You should be able to judge to the nearest half a scale division. You might think you can judge to the nearest tenth of a division. You can't. Research shows that despite the fact that people try to judge the spaces between scale divisions to better than half a division, as soon as this is done, inconsistent measurements are obtained. That is, different people get different measurements of the same thing.

The best judgement you can definitely claim is one half of a scale division. The uncertainty we will still assume, however, is a full half-scale division. Hence, you might measure another stick, one that has a length somewhere between 154 mm and 155 mm, as 154.5 ± 0.5 mm.

Of course, you don't have the option of adding an extra decimal place containing a 0 or a 5 if you are using a digital instrument.

The uncertainty can be recorded as the *absolute* uncertainty as we have done above. The absolute uncertainty is the actual uncertainty in the measurement. In this case it is 0.5 mm. It is often useful to write the uncertainty as a *percentage*: 0.5 mm is 0.32% of 154.5. Hence, the above length would be recorded as 154.5 mm ± 0.32%.

Percentage uncertainty is also called *relative* uncertainty. It is the size of the uncertainty relative to the size of the measured quantity.

PHYSICSFILE

Many people use the term 'error' to refer to uncertainty and many other things. The problem with referring to uncertainty as error is that it is not actually error. Things that are a normal consequence of the limitations of measuring instruments must happen, and are not mistakes. If they are not mistakes or 'something gone wrong' then it makes no sense to call them errors.

Errors are the factors that limit the accuracy of your results. For example, if you perform a calorimetry experiment and do not use a good enough insulator, you will get inaccurate results due to heat losses to the environment. This will contribute to the error in your measurement. Suppose you measured the refraction of light in glass but did not place the protractor in the correct place when measuring angles. This would also cause error.

Many different things can contribute to experimental error. Some are unavoidable. Some are factors in the design of the experiment. Good experimental design seeks to eliminate or at least minimise potential sources of error.

Never quote 'human error' as a source of error. Your data should be examined carefully, and mistakes eliminated or at least ignored. So-called human errors, or lack of care, have no place in your experimental work. If you make mistakes, then you should repeat the measurements.

Estimating the uncertainty in a result

An experiment or a measurement exercise is not complete until the uncertainties have been analysed. Chapter 1 explained how uncertainties were treated when a range of data had been collected and then averaged out. It is also important to explain how uncertainty due to the precision of instruments affects results.

The following three processes are used for estimating uncertainty in calculations due to the precision of instruments. They are demonstrated in Worked example B3.

- When adding or subtracting data, add the absolute uncertainties.
- When multiplying or dividing data, add the percentage uncertainties.
- When raising data to power *n*, multiply the percentage uncertainty by *n*.

In Worked example B3, the analysis of uncertainty reveals the *precision* of an experimental result.

Worked example B3

You might have measured the specific heat capacity of a metal. You could have calculated your result using:

$$c_{metal} = = \frac{c_{water} m_{water} \Delta T_{water}}{m_{metal} \Delta T_{metal}}$$

Suppose you had the following data included in your table.

Quantity		Absolute uncertainty	Percentage uncertainty
c_{water}	$4180\,J\,kg^{-1}\,K^{-1}$	$5\,J\,kg^{-1}\,K^{-1}$	0.120
m_{water}	$72.5 \times 10^{-3}\,kg$	$0.05 \times 10^{-3}\,kg$	0.069
ΔT_{water}	5°C	1°C*	20
m_{metal}	$87.3 \times 10^{-3}\,kg$	$0.05 \times 10^{-3}\,kg$	0.057
ΔT_{metal}	72°C	1°C*	1.389

*Note that the ΔT values have an absolute uncertainty of 1°C because they are calculated by subtracting one temperature measurement from another.

You would calculate as follows:

$$c_{metal} = 241\,J\,kg^{-1}\,K^{-1}$$
$$\text{Uncertainty (\%)} = 0.120 + 0.069 + 20 + 0.057 + 1.389$$
$$= 21.6\%$$

Hence, you would obtain the following result:

$$c_{metal} = 241\,J\,kg^{-1}\,K^{-1} \pm 21.6\%$$
$$c_{metal} = 241 \pm 52\,J\,kg^{-1}\,K^{-1}$$

Once you have done all of this you can consider the relative success of your measurement exercise.

Your result is:

$$189\,J\,kg^{-1}\,K^{-1} \leq c_{metal} \leq 293\,J\,kg^{-1}\,K^{-1}$$

If measurements by other people, such as the constants published in data books, fall within this range then you can conclude that your experiment is consistent with established values. That is, within the precision of your technique, there are probably no significant errors although the final measurement is rather imprecise in this case. We might say that it is accurate within the limitations of the equipment.

PHYSICSFILE

In some classes, students are instructed to quote all results to two decimal places or to three significant figures. You should be able to see from Worked example B3 that these rules are not absolutely correct when applied to real data. For ordinary calculations in assignments, tests and examinations, you might just give your answers to three figures.

If a calculation is done in several stages then you should not round off any intermediate results. This will add rounding error to your calculations. Use the memory on your calculator so that there is no rounding until the end of your calculation.

You are also now in a position to refine the experiment by reducing the larger uncertainties. In this case, the largest uncertainty was in the temperature change for the water. Hence, it would not be very helpful to measure the masses to greater precision because the limit to precision in this activity would be the temperature differences. Getting greater precision in the temperature changes would be a useful refinement.

You could consider ways of getting larger temperature changes in the water and hence obtain a smaller percentage uncertainty in the temperature change. Alternatively, you might consider ways of measuring the temperatures to greater precision.

If your measurement range does not include the result you expect, you should think about the origin of the errors. In other words, if you are sure that c_{metal} is less than $189\,J\,kg^{-1}\,K^{-1}$ or more than $293\,J\,kg^{-1}\,K^{-1}$ then there must be some error in your experimental technique or more uncertainty than you realised.

When reviewing an experiment or a measurement exercise, it is a good idea to consider both errors and uncertainties.

Significant figures

The number of significant figures in a measurement is simply the number of digits used when the number is written in scientific notation. (Note: Significant figures were explained in Chapter 1.) Your calculator usually has eight or ten digits in the display of the answer for a calculation, but most of them are meaningless. You must round off your answer appropriately.

Consider the result of the experiment described in Worked example B3. It would make no sense to quote the result to two decimal places (or five significant figures) when clearly the precision of the experiment gives less than three significant figures.

Calculated results never have more significant figures than the original data and might have fewer than the original data. If you are not doing a full analysis of the uncertainties, it is customary to give your answers to the same number of significant figures as the least precise piece of data. For example, in Worked example B3, the least precise data is the change in temperature of the water, which has only a single digit. The value for the specific heat might then be quoted simply as $2 \times 10^2\,J\,kg^{-1}\,K^{-1}$, but doing the full calculation of the uncertainty in the result is much more informative.

GRAPHICAL ANALYSIS OF DATA

A major problem with doing a calculation from just one set of measurements is that a single incorrect measurement can significantly affect the result. Scientists like to take a large amount of data and observe the trends in that data. This gives more precise measurements and allows scientists to recognise and eliminate problematic data.

Physicists commonly use graphical techniques to analyse a set of data. In this section, the basic techniques that they use will be outlined and a general method for using a set of data that fits a known mathematical relationship will be developed.

Linear relationships

Some relationships studied in physics are linear, that is they can be represented by a straight line, while others are not. It is possible to manipulate non-linear data so that a linear graph reveals a measurement. Linear relationships and their graphs are fully specified with just two numbers: gradient, m, and vertical axis intercept, c. In general, linear relationships are written:

$$y = mx + c$$

The gradient, m, can be calculated from the coordinates of two points on the line:

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

where (x_1, y_1) and (x_2, y_2) are any two points on the line. Don't forget that m and c have units. Omitting these is a common error.

PHYSICSFILE

Graphs

When analysing data from a linear relationship, it is first necessary to obtain a graph of the data and an equation for the line that best fits the data. This line of best fit is often called the trendline or the regression line. The entire process can be done on paper but most people will use a computer spreadsheet, the capabilities of a scientific or a graphics calculator, or some other computer-based process. In what follows, it is not assumed that you are using any particular technology.

If you are plotting your graph manually on paper then proceed as follows:

1. Plot each data point on clearly labelled, unbroken axes.
2. Identify and label but otherwise ignore any suspect data points.
3. Draw, by eye, the 'line of best fit' for the points. The points should be evenly scattered either side of the line.
4. Locate the vertical axis intercept and record its value as 'c'.
5. Choose two points on the line of best fit to calculate the gradient. Do not use two of the original data points as this will not give you the gradient of the line of best fit.
6. Write $y = mx + c$, replacing x and y with appropriate symbols, and use this equation for any further analysis.

If you are using a computer or a graphics calculator then proceed as follows:

1. Plot each data point on clearly labelled, unbroken axes.
2. Identify suspect data points and create another data table without the suspect data.
3. Plot a new graph without the suspect data. Keep both graphs, as you don't actually discard the suspect data, but do eliminate it from the analysis.
4. Plot the line of best fit—the regression line. The manner in which you do this depends on the model of calculator or the software being used.
5. Compute the equation of the line of best fit that will give you values for m and c.
6. Write $y = mx + c$, replacing x and y with appropriate symbols, and use this equation for any further analysis.

Worked example B4

Some students used a computer with an ultrasonic detector to obtain the speed–time data for a falling tennis ball. They wished to measure the acceleration of the ball as it fell. They assumed that the acceleration was nearly constant and that the relevant relationship was $v_f = v_i + at$, where v_f is the speed of the ball at any given time, v_i was the speed when the measurements began, a is the acceleration of the ball and t is the time since the measurement began.

Their computer returned the following data:

Time (s)	Speed ($m\,s^{-1}$)
0.0	1.25
0.1	2.30
0.2	3.15
0.3	4.10
0.4	5.25
0.5	6.10
0.6	6.95

Find their experimental value for acceleration.

Solution

The data is assumed to be linear, with the relationship $v_f = v_i + at$, which can be thought of as being $v_f = at + v_i$, which makes it clear that putting v_f on the vertical axis and t on the horizontal axis gives a linear graph with gradient a and vertical intercept v_i. A graph of the data is shown in Figure B.6.

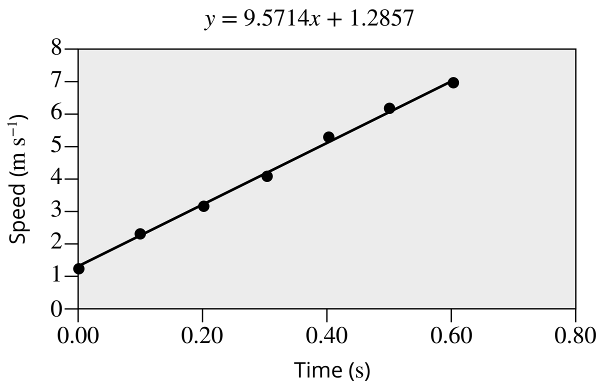


FIGURE B.6 Speed–time profile for a falling tennis ball.

This graph of the data was created on a computer spreadsheet. The line of best fit was created mathematically and plotted. The computer calculated the equation of the line. Graphics calculators can also do this.

A scientific calculator, graphics calculator or spreadsheet gives the regression line as $y = 9.5714x + 1.2857$. If this is rearranged and the constants are suitably rounded, the equation is $v_f = 1.3 + 9.6t$. This indicates that the ball was moving at $1.3\,m\,s^{-1}$ at the commencement of data collection and accelerating at $9.6\,m\,s^{-2}$.

Manipulating non-linear data

Suppose you were examining the relationship between two quantities B and d and had good reason to believe that the relationship between them is

$$B = \frac{k}{d}$$

where k is some constant value. Clearly, this relationship is non-linear and a graph of B against d will not be a straight line. By thinking about the relationship it can be seen that in 'linear form':

$$\begin{array}{ccccc} B & = & k & \frac{1}{d} & \\ \uparrow & & \uparrow & \uparrow & \\ y & = & m & x & + c \end{array}$$

A graph of B (on the vertical axis) against $\frac{1}{d}$ (on the horizontal axis) will be linear. The gradient of the line will be k and the vertical intercept, c, will be zero. The line of best fit would be expected to go through the origin because, in this case, there is no constant added and so c is zero.

In the above example, a graph of the raw data would just show that B gets larger as d gets smaller. It would be impossible to determine the mathematical relationship just by looking at a graph of the raw data.

A graph of raw data will not give the mathematical relationship between the variables, but it can give some clues. The shape of the graph of raw data may suggest a possible relationship. Several relationships may be tried and then the best is chosen. Once this is done, it is not proof of the relationship but, possibly, strong evidence.

When an experiment involves a non-linear relationship, the following procedure is followed:

1. Plot a graph of the original raw data.
2. Choose a possible relationship based on the shape of the initial graph and your knowledge of various mathematical and graphical forms.
3. Work out how the data must be manipulated to give a linear graph.
4. Create a new data table.

Then follow the steps given in the Physics file on page 438. It may be necessary to try several mathematical forms to find one that seems to fit the data.

Worked example B5

Some students were investigating the relationship between current and resistance for a new solid-state electronic device. They obtained the data shown in the table.

According to the theory they had researched, the students believed that the relationship between I and R is $R = dI^3 + g$, where d and g are constants.

By appropriate manipulation and graphical techniques, find their experimental values for d and g. The following steps should be used:

a Plot a graph of the raw data.

b Work out what you would have to graph to get a straight line.

c Make a new table of the manipulated data.

d Plot the graph of manipulated data.

e Find the equation relating I and R.

Current, I (A)	Resistance, R (Ω)
1.5	22
1.7	39
2.2	46
2.6	70
3.1	110
3.4	145
3.9	212
4.2	236

Solution

a Figure B.7 shows the graph obtained using a spreadsheet.

It might be argued that the second piece of data is suspect. The rest of this solution supposes the students chose to ignore this piece of data.

b You can see what to graph if you think of the equation like this:

$$\begin{array}{ccccc} R & = & d\ I^3 & + & g \\ \uparrow & & \uparrow\ \uparrow & & \uparrow \\ y & = & m\ \ x & + & c \end{array}$$

A graph of R on the vertical axis and I^3 on the horizontal axis would have a gradient equal to d and a vertical axis intercept equal to g.

c The data is manipulated by finding the cube of each of the values for current.

Current cubed, I^3 (A^3)	Resistance, R (Ω)
3.38	22
10.65	46
17.58	70
29.79	110
39.30	145
59.32	212
74.09	236

d The graph in Figure B.8 was obtained from the spreadsheet.

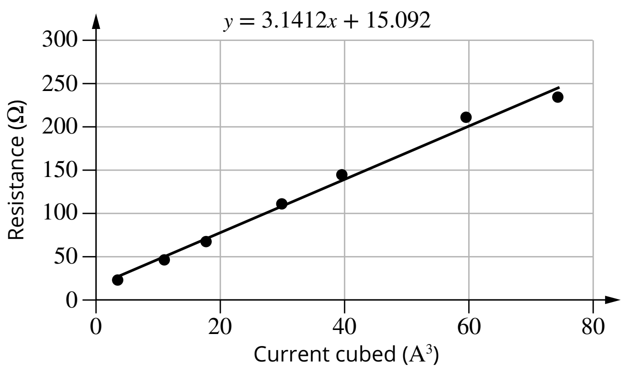


FIGURE B.8 Current–resistance characteristic (manipulated data).

e The regression line has the equation $y = 3.1x + 15.1$, so the equation relating I and R is $R = 3.1I^3 + 15.1$. Hence, the value of d is $3.1\,\Omega\,A^{-3}$ and the value of g is $15.1\,\Omega$.

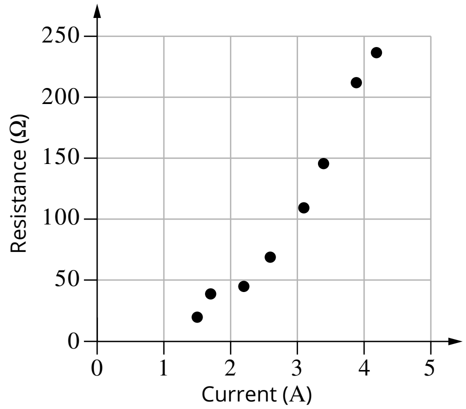


FIGURE B.7 Current–resistance graph of device.

APPENDIX C Mathematical skills for physics ATAR

1. Transforming decimal notation to scientific notation

Scientists use scientific notation to handle very large and very small numbers.

For example, instead of writing 0.000 000 035, scientists would write 3.5×10^{-8}.

A number in *scientific notation* (also called standard form or power of ten notation) is written as:

$$a \times 10^n$$

where a is a number greater than or equal to 1 and less than 10, that is $1 \le a < 10$

n is an integer (a positive or negative whole number) and n is the power that 10 is raised to and is called the index value.

To transform a very large or very small number into scientific notation:

1 Write the original number as a decimal number greater than or equal to 1 but less than 10.

2 Multiply the decimal number by the appropriate power of 10.

The index value is determined by counting the number of places the decimal point needs to be moved to form the original number again.

If the decimal point is moved n places to the left, n will be a positive number. For example:

$$51 = 5.1 \times 10^1$$

If the decimal point is moved n places to the right, n will be a negative number. For example:

$$0.51 = 5.1 \times 10^{-1}$$

You will notice from these examples that when large numbers are written in scientific notation, the 10 has a positive index value. When very small numbers are written in scientific notation, the 10 has a negative index value.

Practice questions

1 Match each number with its correct scientific notation.

Number	Scientific notation
0.002	2×10^3
2000	1.234×10^{-1}
0.1234	2×10^{-3}
12.34	1.234×10^1
123.4	1.234×10^2

2 Write 7.009×10^{-4} using decimal notation.

2. Identifying significant figures

When giving an answer to a calculation it is important to take note of the number of significant figures that you use.

You should give an answer that is as accurate as possible. However, an answer can't be more accurate than the data or the measuring device used to calculate it. For example, if a digital set of scales that measures to the nearest gram shows that an object has a mass of 56 g, then the mass should be recorded as 56 g, not 56.0 g. This is because you do not know whether it is 56.0 g, 56.1 g, 56.2 g or 55.8 g.

56 is a number with two significant figures. Recording to three significant figures (e.g. 56.0 g or 55.8 g) would not be scientifically 'honest'.

If this mass of 56 g is used to calculate another value it would also not be 'honest' to give an answer that has more than two significant figures.

Determining the number of significant figures to give in an answer depends on what kind of calculation you are doing.

If you are multiplying or dividing, use the smallest number of significant figures provided in the initial values.

If you are adding or subtracting, use the smallest number of decimal places provided in the initial values.

Working out the number of significant figures

The following rules should be followed to avoid confusion in determining how many significant figures are in a number.

1. All non-zero digits are always significant. For example, 21.7 has three significant figures.
2. All zeroes between two non-zero digits are significant. For example, 3015 has four significant figures.
3. A zero to the right of a decimal point and following a non-zero digit is significant. For example, 0.5700 has four significant figures.
4. Any other zero is not significant, as it will be used only for locating other digits in their correct decimal places. For example, 0.005 has just one significant figure as the two zeros to the right of the decimal point are only there to place the digit (5) in the thousandth position.

Practice questions

1. Which of the following is written to two significant figures?
   A 30.1
   B 0.00040
   C 0.5
   D 5.12
2. Sam multiplied 1.22 by 1.364. Which of the options below shows the result of this multiplication with the correct number of significant figures?
   A 1.66
   B 1.664
   C 1.65
   D 1.7
3. How can 12000 be written to three significant figures?
   A 120
   B 120×10^2
   C 1.2000×10^5
   D 1.20×10^4
4. Alex is getting ready to go for a bike ride. Alex's mass is 65.3 kg. The bicycle has a mass of 12.92 kg.
   a Which of the following is the correct calculation, with the correct number of significant figures, of the combined mass of Alex and the bicycle?
      A 78
      B 78.2
      C 78.22
      D 78.3
   b Using the combined mass calculated in part (a) above, and the formula $F_{net} = ma$, calculate the force Alex needs to apply to achieve an acceleration of $1.250\,m\,s^{-2}$. Give your answer to the correct number of significant figures.

3. Calculating percentages

Scientists use percentages to express a ratio or fraction of a quantity.

To express one quantity as a percentage of another, use the second quantity to represent 100%.

For example, expressing 6 as a percentage of 24 is like saying '6 is to 24 as x is to 100':

$$\frac{6}{24} = \frac{x}{100}$$
$$x = \frac{6}{24} \times 100$$
$$= 25\%$$

To calculate a percentage of a quantity, the percentage is expressed as a decimal then multiplied by the quantity.

For example, to calculate 40% of 20:

$$x = \frac{40}{100} \times 20$$
$$x = 0.4 \times 20$$
$$= 8$$

Practice questions

1 What is 9 as a percentage of 12?
A 25%
B 50%
C 75%
D 30%

2 What is 25% of 24?
A 6.6
B 6.0
C 5.0
D 0.50

3 Which of the following values expresses 15.0 as a percentage of 125?
A 8.00%
B 5.00%
C 12.0%
D 0.125%

4. Converting between percentages and fractions

To write a percentage as a fraction, divide the percentage by 100.

For example:

$$25\% = \frac{25}{100} = \frac{1}{4}$$

$\frac{25}{100}$ is not the simplest form of this fraction. If you divide both the numerator and the denominator by 25 (their highest common factor) then the fraction simplifies to $\frac{1}{4}$.

Whenever you give a fraction as an answer, always try and simplify it by dividing the numerator and denominator by the highest common factor.

To write a fraction as a percentage, multiply the fraction by 100%. In many cases it is easier to convert the fraction to a decimal number first.

For example:

$$\frac{1}{4} = 0.25 \times 100 = 25\%$$

The value of the fraction or percentage has not changed. It is just being represented in a different way.

Practice questions

1 Choose the option that expresses $\frac{1}{5}$ as a percentage.
  A 25%
  B 20%
  C 30%
  D 50%

2 Match each percentage with its corresponding fraction.

Percentage	Fraction
0.2%	$\frac{7}{20}$
2.5%	$\frac{7}{40}$
17.5%	$\frac{1}{40}$
35%	$\frac{111}{250}$
44.4%	$\frac{1}{500}$

5. Changing the subject of an equation

Scientists use equations to represent relationships between variables. In an equation like $A = \pi r^2$, A is called the subject of the equation.

Sometimes the subject of the equation has to be changed in order to express the relationship in a more useful way. For example, if you need to find the radius of a circle, you will want r to be the subject of the equation above.

To change the subject of a simple equation, transpose the equation to leave the new subject on its own. In the example above, the equation needs to read

$r = \ldots$

Keep the equation balanced by performing the same operation to both sides of the equation to cancel operations being performed on the desired subject. Inverse operations (the opposite operation; for example, dividing is the inverse of multiplying) will allow cancelling.

For example, make r the subject of the equation $A = \pi r^2$.

1 Divide both sides of the equation by π.

$$A = \pi r^2$$
$$\frac{A}{\pi} = \frac{\pi r^2}{\pi}$$

The π in the numerator and denominator on the right side of the equation cancel out, giving

$$\frac{A}{\pi} = \frac{\not{\pi} r^2}{\not{\pi}}$$
$$\frac{A}{\pi} = r^2$$

2 To cancel the squaring operation of r, take the square root of both sides of the equation.

$$\sqrt{\frac{A}{\pi}} = r^2$$

$$\sqrt{\frac{A}{\pi}} = \sqrt{r^2}$$

The square and square root on the right side of the equation cancel out, giving

$$\frac{A}{\pi} = r$$

3 Rewrite the equation swapping the right-hand side with the left-hand so that r is now the subject of the equation

$$r = \sqrt{\frac{A}{\pi}}$$

Practice questions

1 Rearrange the formula $A = \frac{2}{3}R$ to make R the subject.

A $R = \frac{2A}{3}$

B $R = \frac{A}{3}$

C $R = \frac{3A}{2}$

D $R = 6A$

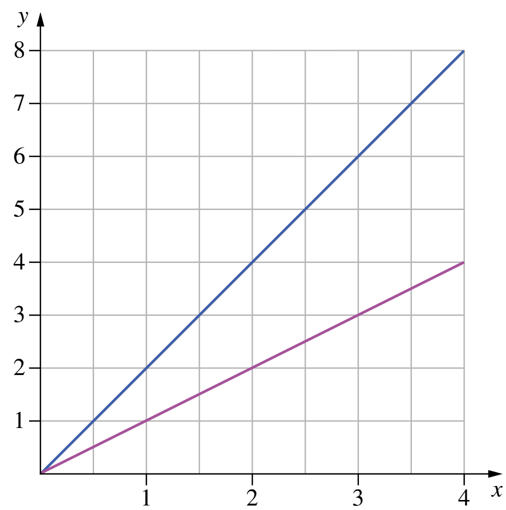
2 Rearrange the formula $y = 3 \times \sqrt{\frac{p}{q}}$ to make p the subject.

6. Interpreting the slope of a linear graph

Scientists often represent a relationship between two variables as a graph. For directly proportional relationships, the variables are connected by a straight line, where the slope (or gradient) of the line represents the constant of proportionality between the two variables.

The slope or gradient of the line is defined as the ratio of change between two points in the vertical axis (Δy), divided by the change between two points in the horizontal axis (Δx). In other words, it measures the rate at which one variable (the dependent variable) changes with respect to the other (the independent variable).

The graph below has two straight lines with different slopes. The steeper slope (blue line) indicates that the rate of change is higher. This means the change is happening more quickly. On the other hand, the flatter slope (purple line) indicates that the rate of change is lower. This means the change is happening more slowly.



Practice questions

1 On a graph with two sloped lines, what does the steeper sloped line indicate?
  A a faster rate of change
  B a slower rate of change
  C the same rate of change
  D a much slower rate of change

2 The rate of change of a straight line on a graph is given by the:
  A *y*-intercept
  B *x*-intercept
  C gradient
  D area under the graph

7. Understanding mathematical symbols

Part of the language of science is using symbols to represent quantities or to give meanings. For example, the four symbols <, >, ≤ and ≥ are known as 'inequalities'.

The following mathematical symbols are commonly used in science.

Symbol	Meaning	Example	Explanation
$<$	less than	$2 < 3$	2 is less than 3
$>$	greater than	$6 > 1$	6 is greater than 1
$\leq$	less than or equal to	$2x \leq 10$	$2x$ is less than or equal to 10
$\geq$	greater than or equal to	$3y \geq 12$	$3y$ is greater than or equal to 12
$\sqrt{\ }$	square root	$\sqrt{4} = 2$	The square root of 4 is 2
Δ	change in (difference between)	Δt	change in t (time)
$\approx$	approximately equal to	$\pi \approx 3.14$	π is approximately equal to 3.14
Σ	summation	$\Sigma E = E_k + E_p$	The addition of the kinetic and potential energy

Practice questions

1 The symbol that means 'less than' is:
  A $<$
  B $>$
  C $\leq$
  D $\geq$

2 Which of these symbols is an inequality?
  A $\approx$
  B Δ
  C $\sqrt{\ }$
  D $\leq$

8. Understanding the difference between discrete and continuous data

Quantitative data forms the backbone of science. Scientists are constantly working with data when they are measuring, recording, analysing, and interpreting it.

Quantitative data consists of numerical values that can either be discrete or continuous.

Discrete data consists of distinct and separate values. For example, the number of students in a class can be represented by discrete data, as it includes a specific set of possible values.

Continuous data is usually measured in some way and can have an infinite number of values within a given range. For example, your height or weight represents continuous data.

The easiest way to distinguish between the two types of quantitative data is to ask, 'Is the data measured or counted?' If it is counted, the data set is discrete. If it is measured, the data set is continuous.

Practice questions

1 Which one of these data sets is continuous?
- **A** the number of cars parked in a street
- **B** the temperature of the air over a 24-hour period
- **C** the number of students at a school
- **D** the number of nails used to build a fence

2 Which one of these data sets is discrete?
- **A** the number of cars parked in a street
- **B** the temperature of the air over 24 hours
- **C** the heights of a team of footballers
- **D** the mass of a team of netballers

9. Calculating the mean, median and range of a data set

When handling data, scientists often look for ways to describe patterns in the data. Common terms used when analysing a set of data include the mean, median and range.

Mean: the *average* value in the data set. To calculate the mean, sum all the values in the data set and then divide this total by the number of data values.

Median: the *middle* value in an ordered data set. To calculate the median, arrange the data set in ascending order and then count the number of data values. If the number of values is odd, the median is the middle value. If the number of values is even, calculate the median by adding the two middle values and dividing by 2, i.e. by calculating the average of the two middle numbers.

Range: the *spread* of values in the data set. To calculate the range, take the largest data value and then subtract the smallest data value.

Practice questions

1 The following set of data is recorded:
44, 17, 21, 26, 42, 18
Find the
- **a** mean
- **b** median
- **c** range.

2 The mass in kilograms of each student in a class of 25 students is recorded below. The combined mass of all the students is 1340 kg.
Students' weights: 67, 60, 41, 52, 39, 60, 42, 55, 55, 50, 46, 62, 48, 48, 56, 64, 55, 56, 59, 61, 41, 63, 53, 62, 45
Find the
- **a** mean
- **b** median
- **c** range.

10. Solving simple algebraic equations

To solve an equation means to find the values that make the equation true. Scientists manipulate equations and substitute in known variables in order to solve for the variable required. It is good practice to substitute numbers into variables using brackets, and to record your answer to more significant figures than required before you express your final answer to the correct number of significant figures with the correct units.

For example, you can solve

$$a = \frac{F_{net}}{m}$$

where F_{net} is the net force on the car, which is 2410 N
m is the mass of the car, which is 1208 kg
a is the acceleration of the car in m s^{-2}, which is unknown.

$$a = \frac{(2410)}{(1208)}$$
$$a = 1.99503$$
$$a = 2.00\ \text{m s}^{-2}$$

Practice questions

1 Solve the equation $\Delta v = IR$ if $I = 3.00$ A, and $R = 9.00\,\Omega$.
 A $\Delta v = 3.00$ V
 B $\Delta v = 27.00$ c
 C $\Delta v = 12$ V
 D $\Delta v = 27.0$ V

2 Solve the equation and find the value of Q if $Q = mc\Delta T$, $m = 1.20$ kg, $c = 4180\ \text{J kg}^{-1}\,\text{K}^{-1}$ and $\Delta T = 30.0$°C.
 A $Q = 1.50480 \times 10^5$ J
 B $Q = 150480$
 C $Q = 150000$
 D $Q = 1.50 \times 10^5$ J

11. Completing calculations with more than one operation

Scientists often deal with complex calculations that can involve numerous operations within the one calculation. The order in which these operations are performed can affect the result of the calculation.

For example, the calculation $2 + 3 \times 4$ will give an incorrect answer of 20 if you calculate the $2 + 3$ part first, but it will give the correct answer of 14 if you calculate the 3×4 part first.

Scientists and mathematicians have agreed on a set order in which operations are carried out so that calculations are consistent. You can remember this order using the acronym 'BIMDAS':

- brackets
- indices (powers, square roots, etc.)
- multiplication and division
- addition and subtraction.

The operations present in a calculation are performed in the order shown in the list. If there are multiple instances of division and multiplication, or addition and subtraction, work from left to right.

For example, the 3×4 part in the original example would always be performed first, since multiplication is higher in the list than addition.

When dealing with scientific notation, it is important to keep each individual number complete. Using the calculator's *EXP* button, or the $\times 10^x$ button, keeps the number and power of 10 together as one number and avoids the problems of using the number times 10^x. If your calculator does not have an *EXP* or $\times 10^x$ button, check the user manual. It may be that it is just labelled differently on your calculator. Alternatively, remember to always use brackets to keep the terms in the denominator together, as shown below.

If you used a calculator and entered $3.01 \times 10^{21} \div 6.02 \times 10^{23}$, the answer would come out as 5.00×10^{43}. This is not the correct answer: what you actually calculated was $\frac{3.01 \times 10^{21}}{6.02} \times 10^{23}$. The correct answer is obtained by entering $\frac{3.01 \times 10^{21}}{(6.02 \times 10^{23})}$. This time, the answer comes out correctly as 5.00×10^{-3}.

Practice questions

1 What is 3.4×10^{-4} divided by 1.7×10^{-3}?

A 2.0×10^{-7}
B 2.0
C 20.0
D 0.20

2 Substitute $m = 1.40$, $d = 3.90$ and $c = 2.70$ into $W = 6m - 4(d + c)$ and solve for W.

A –4.50
B –18.0
C 34.8
D 26.7

12. Understanding the relationship between data, graphs and algebraic rules

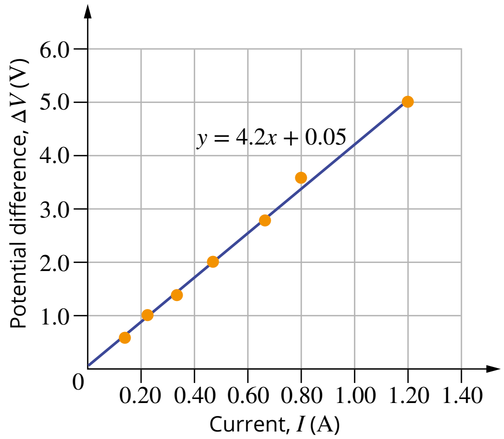
Scientists use graphs to analyse the data they collect from experiments. All graphs tell a story. The shape of the graph shows the relationship between the variables, and this relationship can be written algebraically and numerically. The horizontal axis is known as the x-axis and the vertical axis is known as the y-axis.

Once the algebraic rule is known, the values for one variable can be substituted and the values for the other variable can be calculated. These values can also be determined by reading them from the graph.

For example, when investigating how current and potential difference vary across a light bulb, the following data was collected:

Current, I (A)	Potential difference, ΔV (V)
0.14	0.6
0.22	1.0
0.33	1.4
0.47	2.0
0.66	2.8
0.80	3.6
1.20	5.0

Graphing this data produced:



The numerical values from the experiment are listed in the table and plotted on the graph. The algebraic relationship between the variables is given by the equation of the line:

$$y = 4.2x + 0.05$$

The value of the y-intercept is approximately zero, so assuming that the y-intercept is zero, and labelling the x-axis as current and the y-axis as potential difference, the relationship can be written as:

$$\text{potential difference} = 4.2 \times \text{current}$$

Using the appropriate symbols this can also be written as: $\Delta V = 4.2I$

Practice questions

1 If a graph had L on the y-axis and B on the x-axis and the equation of the straight line was $y = 3.7x$, what is the algebraic form of the graph?

A $L = 3.7x$
B $y = 3.7x$
C $y = 3.7B$
D $L = 3.7B$

2 If a relationship was written as $m = 5.9L$, what shape would the graph be, and which variable would be plotted on which axis?

A The graph would be non-linear, with m on the y-axis and L on the x-axis.
B The graph would be non-linear, with m on the x-axis and L on the y-axis.
C The graph would be linear, with m on the y-axis and L on the x-axis.
D The graph would be linear, with m on the x-axis and L on the y-axis.

13. Recognising and using ratios

A ratio is the relationship between two numbers of the same kind. It could be the quantities in a recipe, the division of profits from a sale, or the number of different types of the same thing.

Scientists use ratios to compare quantities. This might be the numbers of atoms of different elements in a compound, or the number of primary and secondary windings of a transformer.

You can also use the principle of ratios to solve problems. For example, if 1 reaction produces 2.5 MeV of energy, then how much energy does 34 reactions produce?

The reaction-to-energy ratio of 1 : 2.5 should remain constant as the number of reactions increases. So, you need to find the factor that 1 needs to be multiplied by to give 34:

$$1 \times 34 = 34$$

You then multiply the energy amount by the same factor:

$$2.5 \times 34 = 85\,\text{MeV}$$

You may also see ratios expressed as fractions.

Practice questions

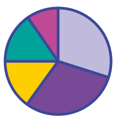
1 If the primary coil of a transformer has 2400 windings and the secondary coil has 600 windings, what is the simplest ratio of primary to secondary windings?

A 2400 : 600
B 24 : 6
C 12 : 3
D 4 : 1

14. Understanding pie charts, frequency graphs, and histograms

It is essential in science to collect data and arrange it in an orderly way. Tables are often used to organise data, which can then be displayed in a graph.

Pie charts



A pie chart is a circle that is divided into sectors. Each sector represents one item in the data set and is shown as a percentage or fraction of the total data set.

Frequency graphs and histograms

Frequency graphs and histograms are another way of representing data visually.

If data is discrete (i.e. can be counted), each column in a column graph will represent one category, e.g. 'apples' or 'strawberries'. Often these columns have a gap between them.

If the data is continuous (i.e. can be measured), such as the heights of the students in that class, each column will represent a range of possible heights, e.g. 140 to 160 cm, and there will be no gaps between the columns. These are called histograms.

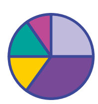
Practice questions

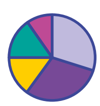
1 Choose the pie chart that correctly shows the data from Emmanuel's poll of soccer fans.

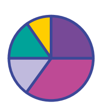
Number of matches watched	10	12	15	18	21
Relative frequency (%)	25	35	15	15	10

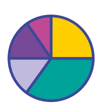
Matches watched:

- = 10
- = 18
- = 12
- = 21
- = 15

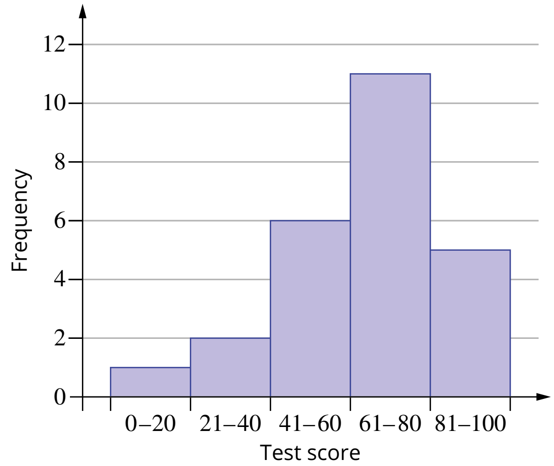
A 

B 

C 

D 

A class of Year 11 Chemistry students had a test. A histogram of their test scores is shown below.



a How many students scored over 80?
b How many students scored below 40?
c How many students are in this class?

15. Understanding the graphical representation of a sine curve

The sine curve is a mathematical curve that describes a smooth repetitive oscillation. It is relevant to the physics topics of sound, AC electricity, simple harmonic motion, waves and many others.

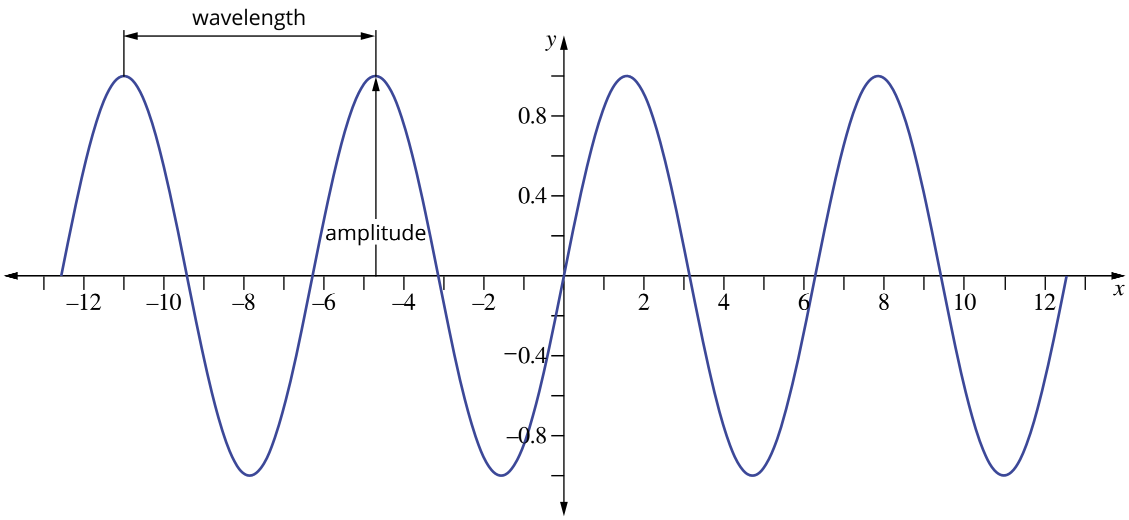
The amplitude of the curve is the distance from the midpoint of the curve to the highest peak or lowest trough, or half the distance from the lowest to the highest point.

The wavelength of the curve is the length of one complete wave.

The period of the curve is the time measurement for one complete cycle of the wave.

Practice questions

1 What is the amplitude of this sine curve?



A 1.0
B 2.0
C 4.0
D 8.0

2 Sine curves in science can be applied to:
A AC electricity
B simple harmonic motion
C sound waves
D all of the above

16. Understanding sine, cosine and tangent relationships in right-angled triangles

Vector problems in physics (and other applications) often involve using trigonometric relationships to find the unknown side of a right-angled triangle.

You will recall that:

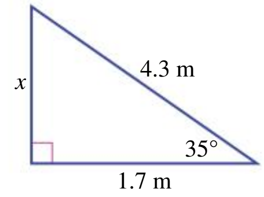
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}},\ \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}},\ \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

The acronyms for each of these rules are:

Equations	$\sin\theta = \frac{O}{H}$	$\cos\theta = \frac{A}{H}$	$\tan\theta = \frac{O}{A}$
Acronym	SOH	CAH	TOA

To solve a trigonometry problem, the appropriate formula needs to be identified and solved for the unknown side or angle of a triangle.

In the triangle below, side *x* is opposite the angle shown as 35°. The hypotenuse, which is always the side opposite the right angle, is 4.3 m long.



θ and the length of the hypotenuse are known. To find the length of the opposite side, use the SOH acronym:

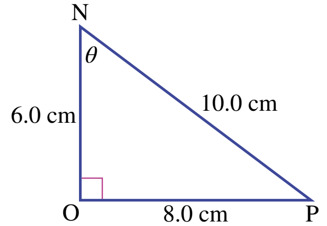
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 35° = \frac{x}{(4.3)}$$
$$x = (4.3)(\sin 35°)$$
$$x = 2.46637$$
$$x = 2.5\,\text{m}$$

You may need to solve a trigonometry problem by finding the angle given two sides in a right triangle. In this case you can use the $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ buttons on your calculator to determine the angle. These buttons are usually found as the second function of the sin, cos, and tan button. Using the triangle above you can calculate the other acute angle using the $\sin^{-1}$ function.

$$\theta = \sin^{-1}\frac{\text{opposite}}{\text{hypotenuse}}$$
$$\theta = \sin^{-1}\frac{(1.7)}{(4.3)}$$
$$\theta = \sin^{-1}(0.395349)$$
$$\theta = 23.28773$$
$$\theta = 23°$$

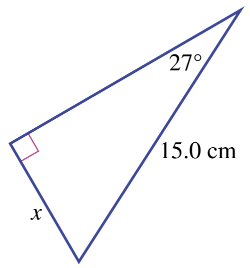
Practice questions

1 On the following right-angled triangle, label the sides as opposite (O), adjacent (A) and hypotenuse (H) in relation to the angle θ.



2 What is the angle θ shown in the right triangle in question 1?
A 39°
B 53°
C 37°
D 31°

3 What is the length of side x in the following triangle?



A 6.8 cm
B 13.3 cm
C 7.6 cm
D 12.3 cm

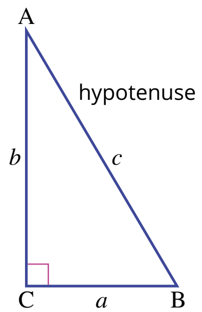
17. Understanding Pythagoras' theorem and similar triangles

Vector problems in physics (and other applications) often involve finding the side lengths of a right-angled triangle and other geometrical unknowns.

Pythagoras' theorem

For any right-angled triangle, Pythagoras' theorem states that the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two shorter sides.

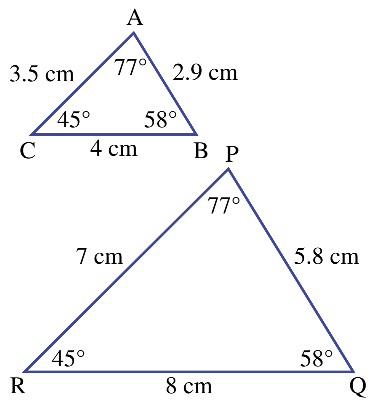
$$a^2 + b^2 = c^2$$



Similar figures

Similar figures are figures that have the same shape but are not necessarily the same size.

In similar triangles, all pairs of corresponding angles are equal, and all pairs of matching sides are in the same ratio. For example, the triangles shown below are similar triangles.

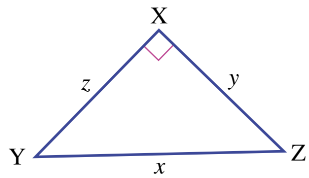


The corresponding angles in these triangles are equal. The corresponding sides in these triangles are in the same ratio. The ratio can be shown as:

$$\frac{PQ}{AB} = \frac{5.8}{2.9} = 2$$
$$\frac{QR}{BC} = \frac{8}{4} = 2$$
$$\frac{PR}{AC} = \frac{7}{3.5} = 2$$

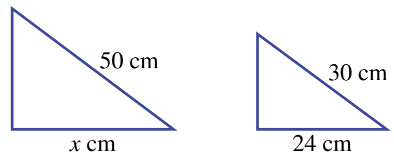
Practice questions

1 For the following triangle, select the correct statement of Pythagoras' theorem.



A $z^2 + x^2 = y^2$
B $z^2 - y^2 = x^2$
C $x^2 + y^2 = z^2$
D $z^2 + y^2 = x^2$

2 These two triangles are similar. Find the value of *x*.



18. Using units in an equation to check for dimensional consistency

Scientists know that each term in an equation represents a quantity. The units used to measure that quantity are not used in the calculations. Units are only indicated on the final line of the solved equation.

For example, this is the equation for the area (A) of a rectangle of length (L) and width (W):

$$A = L \times W$$

If L has a value of 7 m and W has a value of 4 m, it is written:

$$\begin{aligned} A &= L \times W \\ &= 7 \times 4 \\ &= 28\,\text{m}^2 \end{aligned}$$

Note that the units are left out of the actual calculation on the second line and only included at the end, after the numerical answer.

You can use units to check the dimensional consistency of the answer. In the example above, the two quantities of L (length) and W (width) both have to be expressed in consistent units, in this case metres (m), to give an answer that is expressed in square metres ($\text{m} \times \text{m} = \text{m}^2$).

If you had made a mistake and used the formula $A = L + W$ instead, the answer would be expressed in metres only. This is not the correct unit to express area, so you would know that was wrong.

Practice questions

1 Which formula has the correct dimensions for calculating volume in m^3?

A m × m
B m × s
C m × m × s
D m × m × m

2 Which of these shows the correct substitution into $P = 2L + 2W$ using consistent units for $L = 3.5$ m and $W = 240$ cm?

A $P = 2 \times 3.5 + 2 \times 240$
B $P = 2 \times 3.5 + 2 \times 24$
C $P = 2 \times 35 + 2 \times 240$
D $P = 2 \times 3.5 + 2 \times 2.40$

3 By using the equation $p = mv$ as a guide, select the correct units in which to measure momentum, p.

A m s
B m s^{-1}
C kg m s^{-1}
D kg m s^{-2}

19. Understanding inverse and inverse square relationships

Some relationships in science involve one quantity in a relationship increasing and the other quantity decreasing proportionally. This is an inverse relationship.

Thicker wires have lower resistance. This means that, as the cross-sectional area of a wire increases, the value of its resistance decreases. This is an inverse relationship, and it can be written as:

$$R \propto \frac{1}{A} \quad \text{or} \quad R \times A$$

(This shows it is a fixed amount that doesn't change even if R and A change.)

An inverse square relationship is similar, but one quantity increases as the square of the other quantity decreases. Wires with a larger radius will have a lower resistance following an inverse square law.

$$R \propto \frac{1}{r^2} \quad \text{or} \quad R \times r^2$$

Practice questions

1 Which of these rules represents an inverse relationship?

A $B \propto \frac{1}{L}$

B $y \propto d$

C $X \propto \frac{1}{c^2}$

D $K \propto t^2$

2 Which of these rules represents an inverse square relationship?

A $B \propto \frac{1}{L}$

B $y \propto d$

C $X \propto \frac{1}{c^2}$

D $K \propto t^2$

20. Understanding lines of best fit

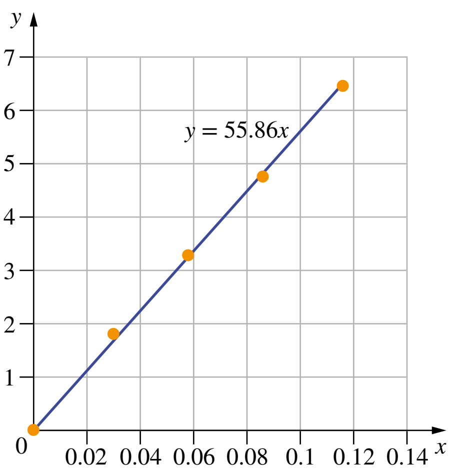
When scientists observe that the points on a graph seem to form a straight line, then a line of best fit (or trendline) can be drawn through them. Computer programs such as Excel can fit a trendline to a data set; lines of best fit can also be drawn by hand onto a printed or drawn graph.

The line of best fit should pass as close as possible to as many of the points as possible (i.e. it should 'fit' the data closely). It may not pass exactly through any of the points, but once the line of best fit is drawn, the points should be spaced equally on each side, above and below the line. There should be no points very far away from the line, unless they are considered to be unreliable. Unreliable points are called 'outliers' and can be disregarded for the purposes of creating a line of best fit.

You should never 'force' your line of best fit to pass through the origin. If it doesn't pass through the origin on your graph, even though it theoretically should, it may indicate systematic errors in your experiment. A line that passes through the y-axis (at $x = 0$), but above $y = 0$, may indicate that the instrument used to measure the y-variable was giving a value that was too high, or it may indicate the instrument that you used to measure the x-variable was giving a reading that was too low.

The gradient of the line of best fit (never use data points), and the y-intercept of the line, can be determined to find the relationship between the variables using the general equation for a straight line: $y = mx + c$.

For example:



Practice questions

1 Are the following statements true or false?

- **a** All the points on a scatter plot must lie on the line of best fit.
- **b** The line of best fit may pass through none of the points.
- **c** The points of the scatter plot should lie close to the line of best fit.
- **d** The line of best fit must pass through the origin.

2 Are the following statements true or false?

- **a** A trendline is the same as a line of best fit.
- **b** A line of best fit can only be drawn using a computer program such as Excel.
- **c** Outliers should be included as normal points when considering where to draw a line of best fit.
- **d** Data point must never be used to determine the gradient of the line of best fit.

Mathematical skills for physics ATAR

Answers to practice questions

1. Transforming decimal notation to scientific notation

1

0.002	2×10^{-3}
2000	2×10^{3}
0.1234	1.234×10^{-1}
12.34	1.234×10^{1}
123.4	1.234×10^{2}

0.002: Move the decimal point three places to the right, which gives an index of –3, so 0.002 is written as 2×10^{-3}.
2000: Move the decimal point three places to the left, which gives an index of +3, so 2000 is written as 2×10^{3}.
0.1234: Move the decimal point one place to the right, which gives an index of –1, so 0.1234 is written as 1.234×10^{-1}.
12.34: Move the decimal point one place to the left, which gives an index of +1, so 12.34 is written as 1.234×10^{1}.
123.4: Move the decimal point two places to the left, which gives an index of +2, so 123.4 is written as 1.234×10^{2}.

2 The –4 index shows the decimal point has been moved four places to the right. Move it back four places to the left to give 0.000 700 9.

2. Identifying significant figures

1 B 0.000 40

2 A 1.66
When two numbers are multiplied, use the smallest number of significant figures in the initial values to give your answer. In Sam's multiplication, the answer is 1.66408, but as 1.22 has three significant figures, the correct answer is 1.66.

3 D 1.20×10^4
The zeroes to the right of the decimal point are significant. When writing an answer to a correct number of significant figures, you may need to use scientific notation. Scientific notation starts with a non–zero digit, and zeros to the right of the decimal point are significant.

4 **a** B 78.2
When 65.3 is added to 12.92, the answer is 78.22, but only if it is assumed that 65.3 is actually 65.30. When adding or subtracting with significant figures, use the smallest number of significant figures provided in the initial values. As there is no way of knowing the accuracy of beyond 65.3, the answer should only be given to three significant figures, which is one decimal place, 78.2.

b When multiplying or dividing, use the smallest number of significant figures in the initial values to give your answer.
$F_{net} = ma$
$F_{net} = 78.2 \times 1.250$
$F_{net} = 97.7500$
$F_{net} = 97.8\,N$
Since the mass value only has three significant figures, the answer should also only have three significant figures. Therefore, even though the calculated answer is 97.7500, the answer to three significant figures is 97.8 N.

3. Calculating percentages

1 C 75%

$$\frac{9}{12} = \frac{x}{100}$$
$$x = \frac{9}{12} \times 100$$
$$= 75\%$$

2 B 6.0

$$x = \frac{25}{100} \times 24$$
$$= 0.25 \times 24$$
$$= 6.0$$

3 C 12.0%

$$\frac{15.0}{125} = \frac{x}{100}$$
$$x = \frac{15.0}{125} \times 100$$
$$= 12.0\%$$

4. Converting between percentages and fractions

1 B 20%
To write a fraction as a percentage, multiply the fraction by 100.

$$\frac{1}{5} = 0.2 \times 100$$
$$= 20\%$$

2

Percentage	Fraction
0.2%	$\frac{1}{500}$
2.5%	$\frac{1}{40}$
17.5%	$\frac{7}{40}$
35%	$\frac{7}{20}$
44.4%	$\frac{111}{250}$

To change a percentage to a fraction, divide by 100, multiply both lines by the same multiple of 10 to remove the decimal, then simplify.

$$0.2\% = \frac{0.2}{100} = \frac{2}{1000} = \frac{1}{500}$$
$$2.5\% = \frac{2.5}{100} = \frac{25}{1000} = \frac{1}{40}$$
$$17.5\% = \frac{17.5}{100} = \frac{175}{1000} = \frac{35}{200} = \frac{7}{40}$$
$$35\% = \frac{35}{100} = \frac{7}{20}$$
$$44.4\% = \frac{44.4}{100} = \frac{444}{1000} = \frac{222}{500} = \frac{111}{250}$$

5. Changing the subject of an equation

1 C $R = \frac{3A}{2}$

Multiply both sides of the equation by 3

$3A = 3 \times \frac{2}{3}R$

$3A = 2R$

Divide both sides of the equation by 2

$\frac{3A}{2} = R$

Rewrite the equation so it reads

$R = \frac{3A}{2}$

2 $p = \frac{y^2 q}{9}$

Divide both sides of the equation by 3

$\frac{y}{3} = \sqrt{\frac{p}{q}}$

Square both sides

$\left(\frac{y}{3}\right)^2 = \frac{p}{q}$

Expand the brackets on the left

$\frac{y^2}{9} = \frac{p}{q}$

Multiply both sides by q

$\frac{y^2 q}{9} = p$

Rewrite the equation so it reads

$p = \frac{y^2 q}{9}$

6. Interpreting the slope of a linear graph

1 A The steepness of the slope indicates the rate of change. A line with a steeper slope indicates a faster rate of change.

2 C The gradient or slope of a linear graph indicates the rate of change.

7. Understanding mathematical symbols

1 A $<$. One way is to remember this is that the smaller end of the shape points toward the smaller number. For example, $3 < 6$ means 3 is less than 6.

2 D $\leq$. The symbols $<$, $>$, $\leq$ and $\geq$ are all inequalities.

8. Understanding the difference between discrete and continuous data

1 B The temperature of the air over a 24–hour period.
If the data can be counted, the data set is discrete. If the data can be measured, the data set is continuous. The temperature of the air can be measured with a thermometer, so the data set is continuous.

2 A The number of cars parked in a street.
If the data can be counted, the data set is discrete. If the data can be measured, the data set is continuous. The number of cars parked in a street can be counted, so the data set is discrete.

9. Calculating the mean, median and range of a data set

1 **a** $44 + 17 + 21 + 26 + 42 + 18 = 168$

$\frac{168}{6} = 28$

b Place the numbers in ascending order: 17, 18, 21, 26, 42, 44
As there is an even number of values, add the two middle values and divide by 2.

$\frac{21 + 26}{2} = 23.5$

c The highest value minus the lowest value is the range.
$44 - 17 = 27$

2 **a** $\frac{1340}{25} = 53.6$ kg

b 55 kg

c $67 - 39 = 28$ kg

10. Solving simple algebraic equations

1 D $\Delta V = 27.0$ V
Substitute the values and solve the equation.
$\Delta V = (3.00)(9.00)$
$= 27.0$ V

2 D $Q = 1.50 \times 10^5$ J
Substitute the values and solve the equation.
$Q = (1.20)(4180)(30.0)$
$= 15048.0$
$= 1.50 \times 10^5$ J

11. Completing calculations with more than one operation

1 D 0.20
The correct calculation, using brackets, is:

$\frac{3.4 \times 10^{-4}}{(1.7 \times 10^{-3})} = 0.20$

2 B -18.0
$W = 6 \times 1.40 - 4 \times (3.90 + 2.70)$
$= 6 \times 1.40 - 4 \times 6.60$
$= 8.40 - 26.40$
$= -18.0$

12. Understanding the relationship between data, graphs and algebraic rules

1 D $L = 3.7B$
Substituting L for y and B for x gives $L = 3.7B$.

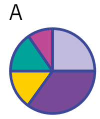
2 C The graph would be linear, with m on the y-axis and L on the x-axis.
The graph would be linear as the equation is written in the form $y = mx + 0$ with m on the y-axis and L on the x-axis.

13. Recognising and using ratios

1 D 4 : 1
The primary to secondary ratio is 2400 : 600. Dividing both sides of the ratio by 600 simplifies it to 4 : 1.

14. Understanding pie charts, frequency graphs and histograms

1 A



The largest sector of the pie chart is purple, representing the percentage of people who watched 12 matches (35%). The next in size is blue (10 matches, 25%), then green (18 matches, 15%) and yellow (15 matches, 15%). The smallest sector is red (21 matches, 10%).

2 **a** 5
b 3
c 25
The height of the 81–100 column is 5. This shows five students scored over 80.
The heights of the 0–20 and 21–40 columns are 1 and 2. This shows that three students scored below 40.
If you add up the heights of all the columns (1 + 2 + 6 + 11 + 5 = 25), it tells you that there are 25 students in the class.

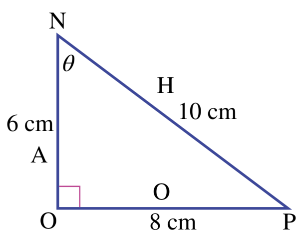
15. Understanding the graphical representation of a sine curve

1 A 1.0. The amplitude of the curve is the distance from the midpoint (0) to the highest peak (1.0) or the lowest trough (–1.0). You could also work out distance from the lowest point to the highest point (1.0 + 1.0 = 2.0) and halve that figure. The amplitude of this sine curve is 1.0.

2 D all of the above. Sine curves have many applications in physics including those mentioned here.

16. Understanding sine, cosine and tangent relationships in right-angled triangles

1



The hypotenuse is the longest side and the one opposite the right angle, so the 10 cm side is labelled H.
The opposite side is the side opposite the θ, so the 8 cm side is labelled O.
The adjacent side is the side next to the angle θ, so the 6 cm side is labelled A.

2 B 53°
First, identify the correct trigonometric formula to use. You know the hypotenuse is 10.0 cm long, and the side opposite the angle θ is 8.0 cm. You want to find the angle θ, so use the inverse SOH or inverse sine formula.

$$\theta = \sin^{-1}\frac{\text{opposite}}{\text{hypotenuse}}$$
$$\theta = \sin^{-1}\frac{8.0}{10.0}$$
$$\theta = \sin^{-1}(0.80000)$$
$$\theta = 53.1301$$
$$\theta = 53°$$

3 A 6.8 cm
First, identify the correct trigonometric formula to use. You know the angle 27°, and the hypotenuse is 15.0 cm long. You want to find the length of x, the side opposite the angle, so use the SOH or sine formula.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 27° = \frac{x}{15}$$
$$x = 15 \times \sin 27°$$
$$x = 6.80956$$
$$x = 6.8\,\text{cm}$$

17. Understanding Pythagoras' theorem and similar triangles

1 D $z^2 + y^2 = x^2$
The hypotenuse, which is the longest side of a right-angled triangle, is opposite the right angle. In this case it is side x.
x^2 is the sum of the squares of the other two sides: $x^2 = z^2 + y^2$

2 $x = 40\,\text{cm}$
Identify the matching sides. These will be in the same ratio. The hypotenuse of the first triangle is 50 cm. The hypotenuse of the second triangle is 30 cm. The ratio can be written as $\frac{50}{30}$.
The other matching sides will have the same ratio.

$$\frac{x}{24} = \frac{50}{30}$$
$$x = \frac{50 \times 24}{30}$$
$$x = 40\,\text{cm}$$

18. Using units in an equation to check for dimensional consistency

1 D m × m × m

2 D $P = 2 \times 3.5 + 2 \times 2.40$
Both units must be consistent.
Since 100 cm = 1 m the consistent units must be either
$L = 3.5$ m and W = 2.40 cm
or
$L = 350$ cm and $W = 240$ cm
The only correct combination is $P = 2 \times 3.5 + 2 \times 2.40$.

3 C kg m s^{-1}
The unit for momentum is taken from the unit for mass (kg) multiplied by the unit for velocity (m s^{-1}). Therefore, momentum is measured in kg m s^{-1}.

19. Understanding inverse and inverse square relationships

1 A $B \propto \frac{1}{L}$
An inverse relationship contains a term in the form $\frac{1}{x}$.

2 C $X \propto \frac{1}{c^2}$
An inverse square relationship contains a term in the form $\frac{1}{x^2}$.

20. Understanding lines of best fit

1 **a** false
b true
c true
d false

2 **a** true
b false
c false
d true

Answers

Chapter 1 Practical investigation

1.1 Designing and planning the investigation

1.1 Review

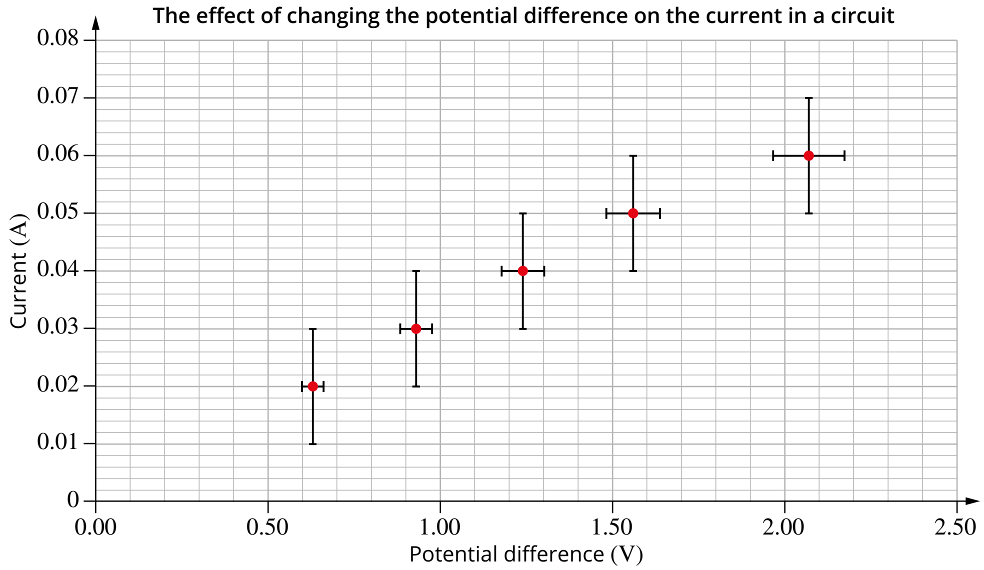
1 **a** if the potential difference is measured by counting the number of batteries
b if the potential difference is measured with a voltmeter
2 qualitative
3 A
4 **a** valid **b** reliable **c** accurate
5 **a** the tension in the elastic band
b the initial launch velocity of the elastic band
c The same elastic band is used, the elastic band is held in the same way, the elastic band is launched in the same direction, the elastic band is placed on the finger in the same way.

WE 1.1.1 886 ± 4 W

1.2 Conducting investigations, and recording and presenting data

1.2 Review

1 **a** a systematic error **b** a random error
2 three significant figures
3 **a** 22.7 **b** 19.0 **c** 22.5 **d** 4.5
4



5 as a line of best fit on the graph
6 The error bars for the current are all the same magnitude, which is due to the absolute uncertainty for the current being a constant ± 0.01 A. The error bars for the potential difference increase as the potential difference increases. Although the percentage uncertainty of the potential difference remains constant at 5.00%, the absolute uncertainty grows larger for larger potential difference values.

1.3 Discussing investigations and drawing evidence-based conclusions

1.3 Review

1 a proportional relationship between two variables
2 an inversely proportional relationship
3 a directly proportional relationship
4 time restraints and limited resources
5 As the potential difference across a single resistor increases, the current through the resistor will increase in a positive directly proportional relationship.

Chapter 1 Review

1 A hypothesis is a prediction, based on evidence and prior knowledge, to answer the research question. A hypothesis often takes the form of a definite statement that proposes the relationship between two or more variables.
2 dependent variable: flight displacement
independent variable: release angle
controlled variable: (any of) release velocity, release height, landing height, air resistance (including wind)
3 **a** the acceleration of the object
b the vertical acceleration of the falling object
c the rate of rotation of the springboard diver
4 elimination, substitution, isolation, engineering controls, administrative controls, personal protective equipment
5 ±0.4 cm s^{-1} **6** the mean **7** an exponential relationship
8 This graph should show a straight line with a positive gradient.
9 'Limitations' of the investigation method refers to any issues that could have affected the validity, accuracy, precision or reliability of the data, plus any sources of error or uncertainty.
10 'Bias' is a form of systematic error resulting from a researcher's personal preferences or motivations.

Chapter 2 Scalars and vectors

2.1 Scalars and vectors

WE 2.1.1 **a** 50.0 N west **b** −50.0 N
WE 2.1.2 This vector is 50.0° down from horizontal to the right.

2.1 Review

1 A magnitude (size) and units **2** A magnitude, units and a direction
3 **a** scalar **b** vector **c** vector **d** scalar **e** vector **f** vector **g** scalar **h** vector **i** scalar **j** vector **k** scalar
4 9.00 N **5** −2.70 N
6 **a** down **b** south **c** forwards **d** up **e** east **f** positive
7 + and − can be entered into calculators to do calculations with vectors. **8** −35.0 N
9 **a** **i** 225.0° T **ii** S 45.0° W **b** **i** 120.0° T **ii** S 60.0° E
10 40.0° up from horizontal to the left

2.2 Adding vectors in one and two dimensions

WE 2.2.1 19.0 N down **WE 2.2.2** R = 5.83 N, N 59.0° E

2.2 Review

1 a 30.0 km b zero each day and each week
2 2.00 m down 3 11.0 m forwards 4 D
5 R = 44.7 m, S 63.4° W 6 R = 6325 N, N 71.6° E 7 50.0 m
8 5385 N S 68.2° E 9 1 N backwards

2.3 Subtracting vectors in on and two dimensions

WE 2.3.1 Δv = 1988 m s^{-1} up **WE 2.3.2** Δv = 9.22 m s^{-1} N 40.6° E

2.3 Review

1 8.00 m s^{-1} east 2 2.00 m s^{-1} left 3 7.00 m s^{-1} downwards
4 67.5 m s^{-1} south 5 14.0 m s^{-1} backwards
6 Δv = 533 m s^{-1} N 49.6° W 7 Δv = 59.4 m s^{-1} N 45.0° W
8 Δv = 8.79 m s^{-1} N 36.7° W 9 a 15.0 km h^{-1} b 65.0 km h^{-1}
10 Δv = 42.4 kmh^{-1} N 45.0° W

2.4 Vector components

WE 2.4.1 1580 N downwards

2.4 Review

1 a 265 N downwards b 378 N right
2 19.8 N south and 16.6 N east
3 4.55 ms^{-1} north and 17.7 ms^{-1} west
4 Zehn is 18.9 m south and 43 m east of his starting point.
5 208 000 N west
6 a 54.0 N south, 93.5 N east
b 60.0 N north
c 293 N south, 107 N east
d 1.50 × 10^5 N up, 2.60 × 10^5 N horizontal
7 horizontal component = 174 N, vertical component = 301 N
8 vertical = 23.0 m s^{-1}, horizontal = 19.3 m s^{-1}
9 315 m 10 166 N

Chapter 2 Review

1 B and D 2 A and D
3 The arrow points in the direction of the 'push' of the hand.
4 It has twice the magnitude of B.
5 Words cannot be used with a calculator.
6 34.0 m s^{-1} north and 12.5 m s^{-1} east need to be added together.
7 + 80.0 N
8 70.0° down from horizontal to the left or 20.0° up from vertical to the left
9 5.00 N right 10 21.0 m backwards 11 65.7 m S 56.8° W
12 R = 813 N, N 53.7° E 13 6.00 m s^{-1} left
14 Δv = 22.8 m s^{-1} N 55.2° W 15 Δv = 67.7 m s^{-1} N 35.0° W
16 22.8 N south 17 323 m s^{-1} 18 379 N to the right
19 v = 3.19 m s^{-1}
20 7.07 m s^{-1} down, 7.07 m s^{-1} to the right

Chapter 3 Linear motion

3.1 Displacement, speed and velocity

WE 3.1.1 a 0.877 m s^{-1} east b 3.16 km h^{-1}
c 4.20 m s^{-1} d 15.1 km h^{-1}

3.1 Review

1 a 3.33 m s^{-1} b 0.00 m s^{-1} 2 B and C
3 a distance travelled = 40.0 cm
b distance travelled = 10.0 cm
c distance travelled = 20 cm
d distance covered = 80.0 cm
4 a d = 80.0 km b s = +20.0 km or 20.0 km north
5 a –10.0 m or 10.0 m downwards b +60.0 m or 60.0 m upwards
c The total distance travelled is 70.0 m
d +50.0 m or 50.0 m upwards
6 a 33.3 m s^{-1} b 25.0 m
7 a 16.7 km h^{-1} b 4.63 m s^{-1}
8 a 0.900 m s^{-1} b +0.100 m s^{-1}, or 0.100 m s^{-1} east
9 a 10.0 km h^{-1} b 2.78 m s^{-1} south
10 a 21.0 km b +15.0 km, or 15.0 km north
c 14.0 km h^{-1} d +10.0 km h^{-1}, or 10.0 km h^{-1} north

3.2 Acceleration

WE 3.2.1 a –2.00 m s^{-1} b 16.0 m s^{-1} up **WE 3.2.2** 457 m s^{-2} up

3.2 Review

1 –7.00 km h^{-1} 2 +5.00 m s^{-1} or 5.00 m s^{-1} up
3 9.00 m s^{-1} up 4 5.00 m s^{-2} south 5 44.3 m s^{-2} up
6 a –10.5 m s^{-1} b 40.9 m s^{-1} west c 764 m s^{-2} west
7 a 8.08 m s^{-1} b 8.08 m s^{-1} south c 6.46 m s^{-2} south
8 6.67 s 9 8.00 s 10 12.0 m s^{-1}

3.3 Graphing position, velocity and acceleration over time

WE 3.3.1 a –15.0 m s^{-1} or 15.0 m s^{-1} b The cyclist is not moving.
WE 3.3.2 a 4.0 m west b 2.0 m s^{-1} west
WE 3.3.3 2.0 m s^{-2} west

3.3 Review

1 D
2 The car initially moves in a positive direction and travels 8.0 m in 2.0 s. It then stops for 2.0 s. The car then reverses direction for 5.0 s, passing back through its starting point after 8.0 s. It travels a further 2.0 m in a negative direction before stopping after 9.0 s.
3 a +8.0 m b +8.0 m c +4.0 m d –2.0 m
4 t = 8.0 s
5 a +4.0 m s^{-1} b 0.0 c –2.0 m s^{-1}
d –2.0 m s^{-1} e –2.0 m s^{-1}
6 a 18.0 m b –2.0 m
7 a +5.00 m s^{-1} b 20.0 m s^{-1} north c 10.0 m s^{-1} north
8 a 0.0 m s^{-2} b –1.0 m s^{-2} c 10.5 m d 1.5 m s^{-1}
9 a 20.0 m s^{-1} north b 40.0 m s^{-1} south
10 a 80.0 s b +2.0 m s^{-2} (answers may vary slightly)
c +0.40 m s^{-2} (answers may vary slightly) d 4.9 × 10^3 m

3.4 Equations for uniform acceleration

WE 3.4.1 a 4.00 m s^{-2} west b 3.87 s c 7.75 m s^{-1} east

3.4 Review

1 E
2 a 3.48 m s^{-2}; no direction is required.
b 55.6 m s^{-1} c 2.00 × 10^2 km h^{-1}
3 a 6.42 m s^{-2} b 9.95 m s^{-1} c 30.8 m
4 a 41.5 m s^{-2} b 1.24 km c 601 km h^{-1}
d 83.5 m s^{-1} e 131 m s^{-1}
5 a 3.84 m s^{-2} b 2.59 m c 5.10 m s^{-1}
6 a 39.9 m s^{-2} up b 0.451 s c 22.0 m s^{-1} down
7 a 20.8 m s^{-1} b 5.29 m forwards c 32.4 m forwards
d 37.7 m
8 a 4.56 m s^{-1} down the ramp b 6.45 m s^{-1} down the ramp
c 1.75 s d 0.728 s
9 a 8.13 s b 16.3 s c 198 m

3.5 Vertical motion

WE 3.5.1 a 2.47 s b 3.50 s c 3.43 m s^{-1} downwards
WE 3.5.2 a +12.3 m b 3.16 s

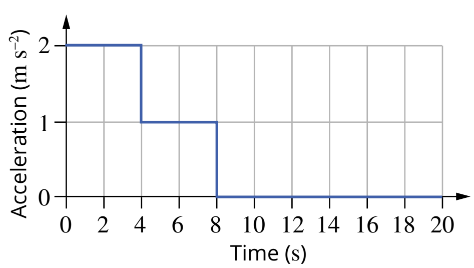
3.5 Review

1 The upwards velocity will decrease by –9.80 m s^{-1} for every second that passes until the ball reaches its highest point. At this point in time the velocity is zero, after which the velocity will increase by –9.80 m s^{-1} for every second of its downwards motion until the instant before it hits the hand.
2 B 3 A and D

4 a 29.8 m s^{-1} downwards
  b 24.2 m s^{-1} downwards
  c 12.1 m s^{-1} downwards
5 a The acceleration of the marble on the way up is the same as its acceleration on the way down. The acceleration of a falling object is due to gravity and it is constant, no matter the direction of vertical travel (upwards or downwards).
  b The magnitude of the launch velocity of the marble is the same as the magnitude of its landing velocity. The flight is symmetrical, so the magnitudes of the starting and landing velocities are the same, but in opposite directions. This only applies if the marble returns to the same vertical position from which it left.
6 a 15.5 m s^{-1} upwards b 12.2 m upwards
7 a 3.92 m downwards b 0.784 m downwards
  c 0.196 m downwards d 0.588 m downwards
8 a 2.04 s b 20.0 m s^{-1} upwards
  c 20.4 m upwards d 20.0 m s^{-1} downwards
9 a 3.50 s b 2.89 s
10 a 1.75 s b 3.25 s
11 a 5.17 m upwards b 0.816 s
  c The height of cliff is 18.2 m above the sea

Chapter 3 Review

1 26.4 m s^{-1} 2 55.1 km h^{-1} 3 10.5 km h^{-1}
4 a 6.73 km h^{-1} north b 1.87 m s^{-1} north
5 –15.0 m s^{-1} 6 B 7 –6.12 m s^{-2}
8 a 25.0 s b 45 s
  c from 0.00 to 10.0 s, from 25.0 to 30.0 s, and from 45.0 to 60.0 s
  d 42.5 s
9 a B b A c C
10 a 1.1×10^2 m north b 1.0×10^1 m s^{-1} north c zero
  d 7.0 m s^{-2} south e A
11 16.2 m s^{-1} west
12 a 5.60 m s^{-2} east b 5.60 m s^{-1} east c 8.40 m east
13 a 5.00 m s^{-2} north b 3.20 s
14 a 4.0 m b A and C
  c B with a velocity of 0.80 m s^{-1}
  d D and is travelling at 2.4 m s^{-1} e 0.80 m s^{-1}
15 a 2.0 m s^{-2} b 10.0 seconds c 8.0×10^1 m d 7.0 m s^{-1}
16 a



  b 12 m s^{-1}
17 The marble has a positive initial velocity that changes to a final velocity of zero at the highest point. It slows down by –9.80 m s^{-1} each second, so it will take 4.00 s to reach 0.00 m s^{-1} at the instant in time it reaches the top of its journey. Its acceleration is constant at –9.80 m s^{-2} due to gravity.
18 D 19 B
20 a 45 m b 6.0 s c 20.0 m s^{-1} down
21 a 10.0 s b 38.8 m s^{-1} down c 6.86 s
22 14.7 m s^{-1} up 23 11.0 m up

Chapter 4 Momentum and forces

4.1 Momentum and conservation of momentum

WE 4.1.1 20 500 kg m s^{-1} north
WE 4.1.2 0.839 m s^{-1} north
WE 4.1.3 1.00 m s^{-1} south
WE 4.1.4 1630 m s^{-1} south

4.1 Review

1 8.75 kg m s^{-1} south 2 9610 kg m s^{-1} west
3 3.97 kg m s^{-1} south
4 15.75 kg m s^{-1}
  The second ball has the greater momentum.
5 The boat moves backwards at 0.438 m s^{-1}
6 v_{f2} = 70.4 m s^{-1} 7 14 200 kg
8 3.00 m s^{-1} in the direction opposite to that of the exhaust gases.

4.2 Newton's first law

4.2 Review

1 The box has changed its velocity so the student can use Newton's first law to conclude that an unbalanced (net) force must have acted on the box to slow it down.
2 Even though the car has maintained its speed, the direction has changed, which means the velocity has changed. Using Newton's first law, it can be concluded that an unbalanced (net) force has acted on the car to change the direction of its velocity.
3 B
4 No, horizontal force acts on the person. According to Newton's first law of motion, the bus slows down, but the standing passenger will continue to move with constant velocity unless acted on by an unbalanced (net) force; usually the passenger will lose their balance and stumble forwards.
5 20.0 N
6 a 25.0 N b 25.0 N
  c 29.0 N at an angle of 30.0° to the horizontal
7 The plane slows down as it travels along the runway because of the large retarding forces acting on it. The passengers wearing seatbelts would have drag forces provided by the seatbelt and would slow down at the same rate as the plane. A passenger standing in the aisle, if they were not hanging on to anything, would have no drag forces acting on them and so would tend to maintain their original velocity and stumble towards the front of the plane.
8 a gravitational force of attraction between the two masses
  b electrical force of attraction between the negative electron and the positive nucleus
  c friction between the tyres and the road
  d tension in the wire
9 a If the cloth is pulled quickly, the force on the glass acts for a short time only. This force does not overcome the tendency of the glass to stay where it is, i.e. its inertia.
  b Using a full glass makes the trick easier because the force of pulling the cloth away will have less effect on the glass due to its greater mass. The inertia of the full glass is greater than that of an empty glass.
10 The fully laden semitrailer will find it most difficult to stop. Its large mass means that more force is required to bring it to a stop.
11 lift = 50.0 kN up, and drag = 12.0 kN west

4.3 Newton's second law

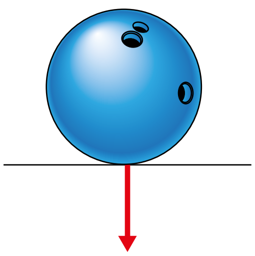
WE 4.3.1 307 N south
WE 4.3.2 4.23 m s^{-1} left
WE 4.3.3 2.49 m s^{-2} forwards
WE 4.3.4 a 7.00 m s^{-2} to the right b 5.00 m s^{-2} to the right

4.3 Review

1 6.61 m s^{-2} north **2** 38.2 kg **3** 3.56 m s^{-2} north
4 9.80 m s^{-2} down **5** 9.80 m s^{-2} down **6** 0.547 m s^{-1} east
7 1560 m s^{-2} south **8** 78 500 kg **9** 0.288 m s^{-2} north
10 **a** 20.0 N south **b** 0.308 m s^{-2} south
11 **a** 1.63 m s^{-2} anticlockwise **b** 0.817 m s^{-1} anticlockwise
c 0.200 m s^{-2} anticlockwise
12 4 boxes **13** 10.2 m s^{-2} upwards

4.4 Newton's third law

WE 4.4.1



WE 4.4.2 As this force continued the object's constant motion, it is the kinetic coefficient of friction.

4.4 Review

1 There is a force on the hammer by the nail, and a force on the nail by the hammer. These two forces are equal in magnitude and opposite in direction.
2 **a** $F_{\text{Earth on astronaut}}$ **b** $F_{\text{astronaut on Earth}}$
3 the force on the hand by the water
4 the force on the balloon by the escaping air **5** 100.0 N east
6 **a** 140 N in the opposite direction to person as they leap from the boat.
b 3.50 m s^{-2} in the opposite direction to the person
c speed of the person: v_f = 1.00 m s^{-1}
speed of the boat: v_f = 1.75 m s^{-1}
7 in the opposite direction from the spacecraft
8 Talia is correct. For every action–reaction pair the action force is a force from object B on object A, and the reaction force is a force from object A on object B. That is, the two forces act on different objects. In this case, both the weight force and the normal force are acting on the same object: the lunch box.
9 42.6 N against the skier's motion
10 F_{needed} = 1.40 × 10^2 N in the direction of the unicycle's travel

4.5 Change in momentum and impulse

WE 4.5.1 803 kg m s^{-1} south
WE 4.5.2 0.0208 kg m s^{-1} N 38.7° E

4.5 Review

1 83.1 kg m s^{-1} south **2** 235 000 kg m s^{-1} east
3 40.0 kg m s^{-1} east **4** 2.45 kg m s^{-1} down
5 2 40 m s^{-1} north **6** 2864 kg m s^{-1} N 45.0° E
7 377 kg m s^{-1} S 42.0° W

4.6 Impulse and force

WE 4.6.1 **a** 0.192 kg m s^{-1} up **b** 54.1 N up
WE 4.6.2 **a** 0.192 kg m s^{-1} up **b** 0.591 N up
WE 4.6.3 **a** 32 N **b** 0.36 kg m s^{-1} up

4.6 Review

1 **a** 452 kg m s^{-1} east
b 452 kg m s^{-1} east
c 129 N east
2 Airbags are designed to increase the duration of the collision, which changes the momentum of a person's head during a car accident. Increasing the duration of the collision decreases the force, which reduces the severity of injury.
3 **a** 1.17 kg m s^{-1} east **b** 1.17 kg m s^{-1} east **c** 11.7 N east
4 3.90 N east
5 **a** 9.00 kg m^{-1}
b 1.80 × 10^2 N in the direction of the ball's travel
c 180 N in the opposite direction to the ball's travel
6 **a** 1200 N **b** 66 N s
7 **a** 1.25 kg m s^{-1} opposite in direction to its initial velocity
b 1.25 kg m s^{-1} opposite in direction to its initial velocity
c 1.56 × 10^3 N in the opposite direction to the initial velocity of the arrow
8 **a** The crash helmet is designed so that the stopping time is increased by the collapsing shell during impact. This will reduce the force, as impulse = $F\Delta t = \Delta p$.
b No. A rigid shell would reduce the stopping time, therefore increasing the force.

4.7 Mass and weight

WE 4.7.1 **a** 285 N **b** 783 N
c 3.35 m s^{-2} down the slope

4.7 Review

1 50.0 kg **2** 60.0 kg is the student's mass, not weight.
3 735 N downwards **4** 3.50 kg **5** –5.60 N down
6 The mass of the hammer remains constant at 1.50 kg.
The weight of the hammer on Mars is 5.40 N down.
7 The weight of any object will be less on the Moon compared with its weight on the Earth as gravity is weaker on the Moon, due to its smaller mass.
8 A **9** **a** 951 N **b** 549 N **c** 4.90 m s^{-2}
10 **a** ball 1: 0.888 N, ball 2: 1.78 N
b ball 1: 4.14 m s^{-2}, ball 2: 4.14 m s^{-2}
c ball 1: 0.335 N, ball 2: 0.670 N
d ball 1: 9.21 m s^{-2}, ball 2: 9.21 m s^{-2}

Chapter 4 Review

1 No, a force has not pushed the passengers backwards. Since the passengers have inertia, as the train has started moving forwards the passengers' masses resist the change in motion. According to Newton's first law, their bodies are simply maintaining their original state of being motionless until an unbalanced force acts to accelerate them.
2 D **3** 38.3kg **4** 5.40 m s^{-2} north **5** 1.72 m s^{-2}
6 1.22 m s^{-2} **7** 101 N **8** 127 N **9** 5.11 m s^{-2} west
10 The reaction of the board acting on the student results in a force of 75.0 N north.
11 504 kg m s^{-1} west **12** 221 kg m s^{-1} backwards
13 –0.558 m s^{-1} **14** 4.80 × 10^2 kg m s^{-1} N 38.7° E **15** 1.71 N east
16 By designing the bonnet of the car to be long and to crumple, a collision will deform this metal and slow the car down before the impact reaches the occupants, thereby reducing the total force applied to the occupants.
A frame made out of metal that is of medium rigidity is best for this purpose.
17 **a** 102 kg m s^{-1} **b** 1000 N **c** 500 N
d A crash helmet spreads out the area of the head over which the force is applied, which reduces the risk of serious injury.
18 10.0 kg m s^{-1} **19** 10.0 kg m s^{-1} **20** 58.8 m s^{-1}
21 98.0 N downwards **22** 2.10 kg
23 **a** 85.0 kg **b** 85.0 kg **c** 306 N down
24 The object's greatest weight is when it is on Earth. The second greatest weight is on Mars, and its least weight is when it is on the Moon.
25 **a** A **b** C **c** 490 N up the hill **d** 4.90 m s^{-2}
e Acceleration is not affected by mass if there is no friction.
26 B

27 a $4.90\,m\,s^{-2}$ **b** $4.95\,m\,s^{-1}$ down the ramp
28 a 236 N **b** $8.88\,m\,s^{-2}$ down the ramp
c 506 N down the ramp **d** $9.42\,m\,s^{-1}$ down the ramp
e 506 N up the ramp
29 a 10^2 N down the slide **b** 85.0 N up the slide **c** 0.361

Chapter 5 Work, energy and power

5.1 Energy and work

WE 5.1.2 1150 J **WE 5.1.3** 0.050 J

5.1 Review

1 10.5 kJ
2 The person exerts a force on the wall but the wall has no displacement (s = 0), so no work is done.
3 1.600 J **4** 18.0 N **5** 306 J
6 The equation $W = Fs$ applies to situations where the applied force is constant. Since a spring obeys Hooke's law, the force required to compress a spring is not constant.
7 Since the box does not move, no work is done.
8 3.60 J **9** W_A 3.00 J W_B 2.40 J W_C 1.00 J
10 a 8.00 J **b** 4.50 J
c As the basketball bounces, some energy is lost as heat and sound so the work done when the ball rebounds is less than the work done when the ball compresses.

5.2 Kinetic energy

WE 5.2.1 84.4 J **WE 5.2.2 a** –772 kJ **b** 19.3 kN
WE 5.2.3 $111\,km\,h^{-1}$

5.2 Review

1 58.0 kJ **2** 386 kJ **3** $40.0\,km\,h^{-1}$
4 Doubling the mass causes E_k to increase by a factor of 2 as well.
5 The kinetic energy of the object is increased by a factor of 9.
6 $5.92\,ms^{-1}$
7 So car 1 has an extra 44.6 kJ of energy just by going $5\,km\,h^{-1}$ faster. Energy differences such as these become very significant in the event of a collision.

5.3 Gravitational potential energy

WE 5.3.1 18.9 J **WE 5.3.2** 31.6 J

5.3 Review

1 a 4.71 J **b** 2.36 J **2** 10^6 J **3** $3.70\,N\,kg^{-1}$
4 The jumper has not jumped with enough energy to equal the change in gravitational potential energy and will not clear the bar.
5 The eagle's potential energy has decreased by 11.2×10^3 J.
6 No. In physics, work against gravity is defined as force exerted over a displacement. Holding the weight above your head might require effort and energy, but you will not be doing any actual work against gravity.

5.4 Law of conservation of energy

WE 5.4.1 50.1 J
WE 5.4.2 The bowling ball will be falling at $3.83\,m\,s^{-1}$ just before it hits the ground.
WE 5.4.3 The arrow will be moving at $76.9\,m\,s^{-1}$ when it reaches a height of 30.0 m.
WE 5.4.4 4.19 kJ

5.4 Review

1 a 27.8 kJ **b** 18.5 kJ
2 a $17.1\,m\,s^{-1}$ **b** $14.0\,m\,s^{-1}$ **3** 1.49 m
4 a 336 J **b** $28.6\,m\,s^{-1}$ **5** 750 J **6** 1.32 m
7 The ball is moving upwards, hence its gravitational potential energy is increasing. Using conservation of energy, its kinetic energy must therefore be decreasing by the exact same amount.

5.5 Elastic and inelastic collisions

WE 5.5.1 The kinetic energy after the collision is significantly less than the kinetic energy before the collision. The collision is inelastic.

5.5 Review

1 An elastic collision is one in which the total kinetic energy of the objects involved before the collision is exactly equal to the total kinetic energy of the objects after the collision. An inelastic collision is one where the total kinetic energy of the objects after the collision is less than before the collision. The law of conservation of energy places no restrictions on which type of energy is present; even though kinetic energy is lower after a collision, overall energy is still conserved.
2 a $5.50\,m\,s^{-1}$ west **b** 60 500 J
3 Since $E_{ki} > E_{kf}$, the collision is inelastic. **4** $0.240\,ms^{-1}$
5 1.22×10^{-2} J **6** 1.19×10^{-2} J
7 a The total kinetic energy before the collision is *more than* the total kinetic energy after the collision.
b The kinetic energy of the system of toys *is not* conserved.
c The total energy of the system of toys *is* conserved.
d The total momentum of the system of toys *is* conserved.
e The collision *is not* perfectly elastic because *kinetic energy is not* conserved.

5.6 Power

WE 5.6.1 683 W **WE 5.6.2** 28.1 kW

5.6 Review

1 113 kW **2** 88.4 kW **3** 1.95 kN **4** $4.92\,m\,s^{-1}$
5 26.9 W **6** 58.8 W **7** 4.75 s

Chapter 5 Review

1 166 kJ **3** 69.1 kJ **4** 97.020 J **5** 8.67 m
6 76.3 N **7** 150.156 **8** $10.7\,m\,s^{-1}$ **10** 368 J
11 $15.0\,ms^{-1}$ **12** 1.3855 J
13 a 2.34 J
b The gain in gravitational potential energy of the pendulum (2.34 J) is equal to the kinetic energy of the pendulum as it starts to swing upwards, so the pendulum had 2.34 J of kinetic energy.
c $1.75\,ms^{-1}$
14 204 kW **15** 34.8 kW **16** 1.43×10^3 N
17 a 2041.6 J **b** 102 N **18** 20.9 J **19** 1.56 kJ

Chapter 6 Heating and cooling

6.1 Heat and temperature

6.1 Review

1 C
2
- The chicken and the air in the oven are not in thermal equilibrium.
- Thermal energy flows from the hot air into the chicken.
- The chicken and the air in the oven are in thermal equilibrium.

3 C and D
4 The temperature of the gas is just above absolute zero so the particles have very little energy.
5 a 303 K **b** 102°C
6 The average kinetic energy of the hydrogen molecules in tank B is greater than the average kinetic energy of the hydrogen molecules in tank A.
7 absolute zero, 10 K, –180°C, 100 K, freezing point of water

6.2 Specific heat capacity

WE 6.2.1 6.27×10^6 J **WE 6.2.2** ratio ≈ 2

6.2 Review

1 water **2** aluminium **3** 2610 J
4 2.59×10^4 J **5** $2x$ J

6 The temperature of the aluminium is 5 times that of the water.
7 B
8 To raise the temperature of 5.00 kg, you will need five times as much energy, i.e. 5 × 2.00 kJ = 10.0 kJ to raise the temperature of 5.00 kg of paraffin by 1.00°C.
9 final temperature = 30.0°C **10** 0.980 k

6.3 Latent heat

WE 6.3.1 1.23×10^2 kJ **WE 6.3.2** 7.95×10^6 J

6.3 Review

1 The mercury is changing state from solid to liquid.
2 –39.0 °C **3** 357.0°C **4** 1.26×10^4 J kg^{-1}
5 2.85×10^5 J kg^{-1} **6** 3.02×10^5 J **7** 62.3 kJ
8 Hot water molecules have more thermal energy than cold water molecules and so are able to leave the surface of the spa-pool water at a greater rate. Therefore hot water will evaporate from the spa-pool faster than cold water.
9 The volatile hand sanitiser liquid has evaporated. As the liquid evaporates it cools the remaining liquid. Heat flows from the floor into the cool liquid thus cooling the floor.

6.4 Heating and cooling

WE 6.4.1 59.3°C **WE 6.4.2** 167°C **WE 6.4.3** 1.54×10^7 J

6.4 Review

1 aluminium **2** 20.6°C **3** 41.5°C **4** 22.7 kg
5 99.8°C **6** 12.2°C **7** 1.29×10^8 J **8** 0.323 kg
9 4.06×10^5 J **10** 1.95×10^6 J

Chapter 6 Review

1 A
2 temperature – the average kinetic energy of particles in a substance
3 Heat refers to the thermal energy that is transferred between objects, whereas temperature directly related to the average kinetic energy of the particles within a substance.
4 **a** 278 K **b** –73°C
5 The fixed points must be reproducible under any conditions. The starting point of the scale must be zero, with no negative values.
6 0°C is not the lowest value on the Celsius scale—negative values are possible. The freezing and boiling points of water are not fixed but vary with changing pressure.
7 As thermal equilibrium is reached, the cubes must be at the same temperature.
8 B
9 The substance is changing state—in this case, it is melting.
10 Both have the same kinetic energy as their temperatures are the same; however, the steam has more potential energy due to its change in state. Therefore the steam has greater internal energy.
11 The higher energy particles are escaping from the surface of the liquid, leaving behind the lower energy particles. The result is that the average kinetic energy of the remaining particles decreases, thus the temperature drops.
12 126 J kg^{-1} K^{-1} **13** 8.32×10^3 J
14 Copper requires less thermal energy to heat it than iron, so copper will cool the water travelling through it less than iron.
15 0.0474 kg **16** 0.0464 kg **17** 3.4654×10^5 m
18 0.597 kg **19** 66.9°C

Unit 1 Review

1: Short responses

1 14.7 m
2 **a** 3.27 ms^{-2} clockwise **b** 131 N
3 The constant force applied by the engine is equal and opposite to the combined resistance forces of, for example, air resistance and friction between the wheels and the track. The net resultant force on the engine and carriages is zero and so, according to Newton's first law, the engine and the carriages will continue their constant straight-line motion. According to Newton's second law, with zero net force acting on the mass the acceleration will also be zero, and so the engine and carriages will not change their velocity.
4 **a** 0.321 ms^{-2} west **b** 365 N west **c** 663 N west
d 663 N east **5** 45.0°C
6 **a** correct **b** incorrect **c** correct
7 after the bounce: $9.0650 \times m_{gb}$
during the bounce: $E_{ki} = 10.7278 \times m_{gb}$
before the bounce: 1.09 m
8 **a** 1.8 m north **b** 494 N E61.6°S c 69.6 N N49.4°E

2: Problem-solving

9 **a** 1.99×10^5 J **b** 2.63×10^5 J
c 5.28×10^4 J **d** 0.114 kg of nitrogen
10 **a** For the first 30.0 s the cyclist travels 150 m east at a constant speed. They accelerate for the next 10.0 s, travelling a further distance of 100 m. The cyclist then travels at a higher constant speed for the next 10.0 s, moving another 250 m during this time period.
b 5.0 m s^{-1} east **c** 25.0 m s^{-1} east **d** 10.0 m s^{-1} east
e 2.0 m s^2 east **f** 10.0 m s^{-1}
11 **a** 1.66 MJ **b** 1.66×10^4 N north
c 1.70×10^4 N north **d** 1.70×10^6 J
e 3.39×10^5 W **f** 4.12×10^4 J **g** efficiency = 97.6%
12 **a** 321 ms^{-2} **b** 46.6 N **c** 2.47 kgms^{-1}
d -2.28×10^{-2} ms^{-1} **e** 46.6 N as calculated in part b
f 21.0 J **g** 21.0 J

3: Comprehension

13 **a** Measuring the time the skier takes to travel 100 m gives their average velocity over that distance.
b A radar gun only measures the skier's velocity at a given point in time. Timing the run over 100.0 m gives an average velocity in case there are some variations.
c The skier accelerates down the slope due to a component of gravity, $g \sin\theta$, acting down the plane, where θ is the angle of the slope with respect to the horizontal. The frictional force of the skis against the snow counteracts this component, reducing the net force and thus the acceleration of the skier.
d Air resistance and friction act against the motion of the skier down the slope. The skier reaches a terminal velocity when this resistance is equal and opposite to $mg \sin\theta$.
e The design of the skis acts to reduce friction by spreading the weight of the skier and equipment over a larger surface area due to the equation $\text{pressure} = \frac{\text{force}}{\text{area}}$. Lower pressure reduces the depth of the ski's penetration into the snow, thereby minimising the frictional force between the skis and the snow.
f

Design factor	How it reduces friction
shape of the skis	low profile to reduce wind resistance
helmet design	directs the wind from the top of the head down the back while in the tuck position, minimising air resistance
shape of the boots	low profile to reduce wind resistance
skin-tight polyurethane suit	allows air to pass over the skier easily, reducing wind resistance

g Acceleration is the rate of change of velocity. The faster the skiers accelerate, the quicker they will reach terminal speed. This ensures that they are travelling at their maximum velocity when they break the first light beam that starts the timer.
h 9.60 s **i** 7.38 m s^{-2} **j** 1.41 s **k** 1.76×10^5 J
l **i** 1.89×10^5 J **ii** altitude = 2445 m **m** 38.9 N

Chapter 7 The nature of waves

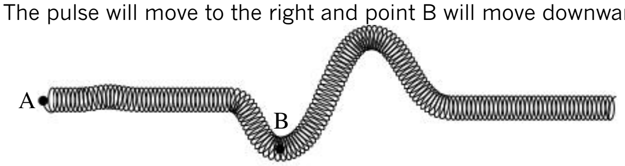
7.1 Longitudinal and transverse waves

7.1 Review

1 The particles oscillate back and forth or up and down around a central or average position and pass on the energy carried by the wave. They do not move along with the wave.

2 **a** False: Longitudinal waves occur when particles vibrate backwards and forwards about a mean position, parallel to the direction of the wave.
b true **c** true **d** true

3 The pulse will move to the right and point B will move downwards.



4 sound, ripples on a pond, vibrations in a rope

5 The tuning fork vibrates back and forth, creating a series of compressions and rarefactions in the air as the energy is transferred.

6 The energy pushes particle A to the right, the energy pushes particle B to the left. This causes a compression in the area around particles A and B.

7 In a transverse wave the motion of the particles is at right angles (i.e. perpendicular) to the direction of travel of the wave itself.

8 longitudinal: a and d
transverse: b and c

7.2 Representing waves

WE 7.2.1 amplitude = 2.0 cm = 0.020 m
wavelength $\lambda = 0.40$ m

WE 7.2.2 amplitude = 0.10 m
period, $T = 0.50$ s
$f = 2.0$ Hz

WE 7.2.3 495 Hz

WE 7.2.4 2.02×10^{-3} s

7.2 Review

1 **a** C and F **b** wavelength **c** B and D **d** amplitude

2 wavelength $\lambda = 1.60$ m; amplitude = 20.0 cm

3 **a** 0.40 s **b** 2.5 Hz **4** 6.75 m s^{-1}

5 **a** true
b false: The period of a wave is *directly proportional* to its wavelength.
c true
d false: The wavelength *and frequency* of a wave determine its speed.

6 **a** wavelength = 4.0 cm; amplitude = 0.50 cm
b 2.0 cm s^{-1} or 0.020 m s^{-1} **c** red **7** 3.92×10^{-6} s

8 As the speed of each vehicle is the same and there is no relative motion of the medium, the frequency observed would be the same as that of the source.

9 The siren would sound lower in pitch.

7.3 Wave behaviours—reflection, refraction and diffraction

WE 7.3.1 **a** 8.58 m
b Since the speed in air is slower than in water, the angle will refract significantly towards the normal.
c 8.54°

WE 7.3.2 3.34°

WE 7.3.3 **a** 7.70×10^{-4} m
b The wavelength of the ultrasound is 0.770 mm.

7.3 Review

1 The wave is reflected and there is a 180° change in phase.

2 amplitude **3** C **4** B

5 The *P*-waves are longitudinal waves and can travel through both solid and molten substances. The *S*-waves are transverse waves and can only travel through solids. Therefore, because they cannot travel through the centre of Earth, the core (or part of it) must be molten. Given the main composition of Earth is rocks, then the core must contain molten rock.

6 B, C and D **7** *slows down, faster, direction, stops*

8 **a** 343 m s^{-1} **b** 349 m s^{-1}
c The refracted angle will increase. **d** $r = 51.2$°C
e greater than 79.4°

9 The higher frequency sound of the flute corresponds to a shorter wavelength so it will be diffracted less and will be more directional. Therefore, it will not be heard as well outside the door of the auditorium. The tuba undergoes a lot more diffraction and so will be louder outside the door.

10 frequency: 5.00 Hz
wavelength: 1.20×10^3 m

11 $\Delta t = d\left(\frac{1}{v_s} - \frac{1}{v_P}\right)$

12 54.6 km

7.4 Wave interactions—superposition, interference and resonance

WE 7.4.1 **a** 2.17 Hz
b The piano string has the higher frequency of the two sounds, so increasing the frequency would result in a greater difference between the two, so the piano tuner would hear more beats per second.

7.4 Review

1 **a** true
b false: As the pulses pass through each other, the interaction *does not* permanently alter the characteristics of each pulse.
c true

2 B

3 If a building is subjected to forces with a forcing frequency matching its natural oscillating frequency, the building will oscillate with increasing amplitude as resonance occurs. This could continue until the structure can no longer withstand the internal forces and fails.

4 52.0°

5 Normal walking results in a frequency of 1 Hz or 1 cycle per second, i.e. two steps per second. This frequency may result in an increase in the amplitude of oscillation of the bridge over time, which could damage the structure.

6 1.27 Hz **7** 480.25 Hz

7.5 Standing waves and harmonics

WE 7.5.1 **a** 0.960 m **b** 0.320 m

WE 7.5.2 **a** 0.285 m **b** 1210 Hz **c** 2020 Hz

WE 7.5.3 **a** 0.160 m **b** 2160 Hz

7.5 Review

1 It is only the pattern made by the amplitude along the rope that stays still at the nodes. The rope is still moving, especially at the antinodes.

2 A transverse wave moving along a slinky spring is reflected at a fixed end with a phase change. The interference that occurs during the superposition of this reflected wave and the original wave creates a standing wave. This standing wave consists of locations called nodes, where the movement of the spring is cancelled out, and antinodes where maximum movement of the spring occurs. Nodes always occur at the ends.

3 0.800 m **4** 1.50 m

5 It will have a wavelength $\frac{1}{4}$ of the fundamental wavelength.
6 2.5 m **7** 0.733 m
8 **a** 304 Hz **b** 608 Hz **c** 912 Hz
9 **a** 0.900 m **b** 0.450 m **c** 1150 Hz
10 **a** 115 Hz **b** 346 Hz **c** 807 Hz
11 **a** Each of the subsequent frequencies is odd and even multiples of the fundamental frequency, therefore all harmonics are formed and the pipe is open ended.
b The next resonance wavelength is $\frac{1}{3}$ that of the fundamental and the following is $\frac{1}{5}$ of the fundamental. This means the frequencies would be 3 times the fundamental and 5 times the fundamental, therefore the pipe is closed at one end.

7.6 Applications of wave properties

7.6 Review

1 107 $m\,s^{-1}$ **2** 2.20×10^2 Hz **3** 0.163 m
4 **a** 247 Hz
b This value is close to the fundamental frequency of the B string at 246.94 Hz. This means that the third harmonic of the low E string should cause the B string to resonate, as the forcing frequency is very close to the natural frequency of the B string.
5 **a** The wavelength of the fundamental frequency for each string remains the same. The fundamental frequency decreases for each string as the guitarist moves from the higher strings to the lower strings. Therefore, by the equation $v = f_1\lambda_1$, the speed on the string must also decrease. Thus, from the relationship $v = \sqrt{\frac{T}{\mu}}$, the mass per unit length must increase (as it is an inverse relationship) or the tension must decrease.
b The mass per unit length can increase by using a low-density material such as nylon for the higher strings and denser steel for the lower strings. Guitarists tune guitars by adjusting the tension in the strings.
6 Ultrasound for imaging uses low-intensity waves and relies on the reflection of those waves from tissue in the body to build an image while minimising damage to the cells. Ultrasound for heat treatment, on the other hand, depends on heating effects due to increased vibration within the body tissue. For the removal of kidney stones or tumours, the intensity of the waves used is sufficiently high to damage or destroy the unwanted cells.
7 The higher the frequency of the ultrasound wave, the smaller the wavelength, resulting in a higher resolution image than a low-frequency wave. However, higher frequency waves have less depth penetration, so a clinician may need to compromise the image quality depending on the depth required. Images closer to the surface of the body will have higher resolution than those deeper in the body.
8 Ultrasound is effective in cleaning as high-frequency waves agitate the water and produce cavitation effects, creating and destroying low-pressure bubbles millions of times a second. This process generates pressure waves that work to dislodge dirt.

Chapter 7 review

1 The particles on the surface of the water move up and down as the waves radiate outwards, carrying energy away from the point on the surface of the water where the stone entered the water.
2 Similarities: both are waves, both carry energy away from the source, both are caused by vibrations.
Differences: transverse waves involve particle displacement at right angles to the direction of travel of the wave; longitudinal waves involve particle displacement parallel to the direction of travel of the wave.
3 U is moving down and V is momentarily stationary (and will then move downwards).
4 0.300 $m\,s^{-1}$ **5** 0.0453 m **6** C and D
7 The frequency must increase. **8** the green wave
9 Sound waves are longitudinal mechanical waves where the particles only move back and forth around a mean position, parallel to the direction of travel of the wave. When these particles move in the direction of the wave, they collide with adjacent particles and transfer energy to the particles in front of them. This means that kinetic energy is transferred between particles in the direction of the wave through collisions. Therefore, the particles cannot move along with the wave from the source as they lose their kinetic energy to the particles in front of them.
10 C and D
11 All objects/materials have a resonant frequency. If the object is made to vibrate at this frequency by a forcing frequency, the amplitude of the object's vibrations will increase with time. If a building or bridge was subjected to forces caused by the wind, or an earthquake, that made it vibrate
12 131 Hz **13** 393 Hz **14** 0.494 Hz **15** 0.0911 m
16 All of the options are correct. **17** B **18** 1.27 m
19 **a** 102 Hz **b** 305 Hz
20 The angle of refraction from the normal to the refracted ray would decrease relative to the angle of incidence.
21 Total internal reflection occurs when the wave goes from a slower-speed medium to a higher-speed medium. As the wave passes from the slower to the faster medium, the refracted angle increases. At the critical angle, the angle of refraction is exactly 90.0°, causing the refracted waves to travel along the interface between the two media. For any angles greater than the critical angle, the wave will reflect back into the first medium.
22 431 Hz or 423 Hz

Chapter 8 Particles in the nucleus

8.1 Atoms, isotopes and radioisotopes

WE 8.1.1 241 nucleons, 149 neutrons, 92 protons, 92 electrons
WE 8.1.2 90 protons, 230 nucleons, 140 neutrons

8.1 Review

1 'nucleons'.
2 **a** Both protons and neutrons are hadrons (or baryons), composed of three quarks.
Both protons and neutrons are nucleons that reside in the nucleus.
Both protons and neutrons are affected by the strong nuclear force.
Protons are positively charged; neutrons have no charge.
Protons and neutrons have similar masses.
b Nucleons are found in nucleus; electrons surround the nucleus.
Nucleons are composite particles made of three quarks; electrons are leptons and are fundamental particles.
Nucleons interact with the strong nuclear force; electrons do not.
c Particles and antiparticles have the same mass and magnitude of charge.
Particles and antiparticles have the opposite sign, e.g. an electron (–1) and a positron (+1).
3 79 protons, 118 neutrons
4 235 nucleons
5 **a** Chlorine-35 has 17 protons, 18 neutrons, 35 nucleons.
b Plutonium-239 has 94 protons, 145 neutrons, 239 nucleons.
6 B and D
7 The number of electrons in a neutral atom is the same as the number of protons, which is given by the atomic number.
8 Isotopes are atoms with the same number of protons but different numbers of neutrons.
9 **a** Their atomic numbers are the same as they are both krypton. Their mass numbers (84 and 89) are different as they are isotopes and have different numbers of neutrons.
b There would be no difference in their chemical interactions with other atoms.
10 A radioisotope is an unstable isotope. A radioisotope will eventually spontaneously emit radiation from its nucleus in the form of alpha particles, beta particles or gamma rays.
11 Yes, uranium

12 Larger nuclei require more neutrons than protons to balance the long-range electrostatic force of repulsion between protons.

8.2 Radioactivity

WE 8.2.1 $^{224}_{88}\text{Ra} \rightarrow {}^{220}_{86}\text{Rn} + {}^{4}_{2}\alpha$

WE 8.2.2 $^{219}_{85}\text{At} \rightarrow {}^{219}_{86}\text{Rn} + {}^{0}_{-1}\beta^{-} + {}^{0}_{0}\bar{v}_e$

WE 8.2.3 *X* is a beta-plus particle: $^{0}_{+1}\beta^{+}$
Y is an electron neutrino particle: $^{0}_{0}v_e$

WE 8.2.4 $^{12}_{6}\text{C}^* \rightarrow {}^{12}_{6}\text{C} + {}^{0}_{0}\gamma$

WE 8.2.5 2.53 MeV

8.2 Review

1 an alpha particle **2** beta-minus **3** beta-plus
4 positively charged electron **5** Alpha is a helium nucleus. Beta is a positively or negatively charged electron ejected from the nucleus. Gamma is electromagnetic radiation.
6 40, 42, 43, 44, 46 and 48. **7** the nucleus
8 **a** *X* = 90, *X* is thorium. **b** *Y* = 89, *Y* is actinium.
9 **a** 7 protons and 7 neutrons
b A neutron has changed into a proton, an electron and an antineutrino.
10 **a** beta-minus particle **b** alpha particle
11 charge before decay of proton = charge after decay
12 2.97 MeV

8.3 Properties of alpha, beta and gamma radiation

8.3 Review

1 **a** gamma **b** beta-minus **c** alpha
d beta-minus **e** gamma
2 gamma **3** beta-minus **4** gamma
5 beta and gamma radiation
6 **a** nucleus **b** nucleus **c** nucleus
7 gamma, beta, alpha
8 Alpha particles travel through air at a relatively low speed and have a double positive charge, which means they readily ionise the air. Their charge, ionising ability and their relatively slow speed make them very easy to stop. This means that they have a very poor penetrating ability.
9 The wire should be a beta-minus emitter, since the irradiation needs to be confined to a relatively small area. Alpha radiation does not have sufficient penetrating power, while gamma radiation would irradiate adjacent healthy cells.
10 Alpha particles will all be stopped by the metal sheet. Gamma rays will all penetrate the metal sheet. Differences in the thickness of the metal sheet will not affect the count rates of these two. Some beta-minus particles will pass through thin metal, so for a set metal thickness there is a set rate of beta-minus particles that should make it to the other side.

8.4 Half-life and decay series

WE 8.4.1 3.91×10^7 nuclei **WE 8.4.2** 3.95 kBq

8.4 Review

1 the count rate or the number of decays each second
2 3.32×10^8 **3** 3.00×10^{11} **4** **a** 10 **b** 241 100 years
5 50% **6** 9.60×10^{-5} Bq **7** 15.0 minutes
8 **a** 10.0 minutes **b** 50.0 Bq **9** beta decay, 20 years
10 seven alpha and four beta-minus decays

8.5 Radiation dose and its effect on humans

WE 8.5.1 0.167 kg
WE 8.5.2 **a** 25.0 mSv **b** 1.25 mSv **c** 1.25 mSv
WE 8.5.3 3.51 Sv

8.5 Review

1 A **2** D **3** **a** 235 μSv **b** 0.0194 J
4 B, D, A, C **5** **a** 2.00 days **b** 879 mSv
6 80.0 hours **7** D

Chapter 8 Review

1 Any of the points below:
Both are fundamental particles, but quarks are affected by the strong nuclear force and electrons are not.
Electrons are fundamental particles that can exist alone, whereas quarks exist in groups (of three called baryons or quark–antiquark pairs called mesons).
Electrons surround the nucleus, whereas quarks make up nucleons inside the nucleus of atoms.
Electrons have an integer charge of –1, whereas quarks have fractional charges of 1/3, 2/3, etc.
2 C **3** 20 protons and 25 neutrons
4 27 protons, 33 neutrons and 60 nucleons **5** *X* is a gamma ray.
6 beta-minus
7 **a** beta-minus **b** proton **c** alpha
d neutron **e** gamma **f** beta-plus
8 *X* is lithium. **9** **a** *X* is a proton. **b** *Y* is a neutron.
10 **a** *x* = 208 *y* = 82 **b** *x* = 176 *y* = 78
11 *a* = 18 *b* = 9 *c* = 18 *d* = 8
X is fluorine, F. *Y* is oxygen, O.
12 carbon-12
13 Electromagnetic forces are balanced by the strong nuclear force acting between all nucleons in close proximity.
14 **a** gamma **b** gamma
15 Gamma rays, often used in cancer treatments, are the most energetic and have the greatest penetrating power. If the vest were made of a material thinner than lead, it would stop only alpha particles and some beta particles. Because lead is very dense, however, it is able to prevent even the most energetic gamma rays from causing damage to the cells of the workers' vital organs.
16 The bombarding electrons will be strongly repelled by the electron clouds of the atoms as they are all negatively charged. The small mass of the bombarding electrons also makes them relatively easy to repel compared to, for example, a proton.
17 3.25×10^{14} **18** 7.00×10^{14}
19 There would be roughly the same number of nuclei of U-238 and U-235 in the 1 kg sample. U-235 has the greater activity, as more of these nuclei will decay in the same time interval. U-238 has a half-life that is around 6.5 times longer than that of U-235, so will decay at a slower rate.
20 **a** 2.0 MBq **b** 6.0 hours **c** 5.0×10^5 Bq
21 1.50×10^{10}
22 The long half-life of cobalt-60 means that the source will not need to be replaced for many years and the activity will be constant over the course of each treatment. The gamma rays of cobalt-60 also have a strong penetrating power, so they are able to penetrate the skull and reach the tumour site.
23 355 mSv **24** **a** 444 J **b** 5.65 Sv
25 **a** 3.51 μSv **b** 5.3 times

Chapter 9 Fission and fusion

9.1 Nuclear fission and energy

WE 9.1.1 **a** Three neutrons are released during this fission.
b 257 MeV **c** 0.160%
WE 9.1.2 4.10×10^{-11} J

9.1 Review

1 The strong nuclear force is a very strong force of attraction that acts between quarks within nucleons (protons and neutrons) and between nucleons themselves. It only acts over relatively short distances and weakens as the distance between nucleons increases in larger nuclei.

2 The decay products of the nuclear fission process comprise many different, often highly radioactive isotopes. This is what makes up the waste.
3 As the proton is positive, it will experience the electromagnetic force of repulsion only from the other protons. However, it will also experience the strong nuclear force that attracts it to all other nucleons, protons and neutrons. This attractive force between protons is far stronger than the repulsive force until the nucleus becomes very large.
4 9.36×10^{-13} J
5 3.98×10^{4} eV
6 fissile: uranium-235 and plutonium-239
non-fissile: uranium-238 and cobalt-60
7 3
8 **a** 119 MeV
b 1.90×10^{-11} J
9 196×10^{6} MeV
10 2.37×10^{-30} kg
11 $x = 239$
$y = 40$

9.2 Chain reactions and nuclear reactors

9.2 Review

1 B
2 There is not a high enough concentration of fissile uranium-235 to sustain a chain reaction.
3 B
4 The moderator slows neutrons down, which allows them to induce fission in the nuclear fuel.
5 Control rods absorb neutrons and maintain a controlled chain reaction.
6 The mass of material must exceed the critical mass and it must have the correct shape to sustain a chain reaction
7 As a result of the flat shape, a high proportion of the neutrons emitted in the fission reaction will escape.
8 The lead nucleus is too heavy, so the incident neutron will be absorbed rather than slowed down. Thus, it would not be able to cause further fissions.
9 **a** The one emitted neutron could sustain a chain reaction if it caused a further fission.
b This could not sustain the chain reaction as at least one neutron is needed to cause further fission.
c This would sustain a chain reaction if at least one neutron caused further fission after each reaction.
10 **a** Uranium-235 nuclei do not readily absorb fast-moving neutrons and so the neutron will not cause fission but will be scattered.
b Slow neutrons are likely to be absorbed by the uranium-235 nucleus and cause fission. Uranium-235 is fissile and does not need much extra energy from the neutron to induce fission.
11 **a** The uranium-238 is fertile, i.e. non-fissile, but will become fissionable if it absorbs a fast neutron with a lot of extra energy. The urianium-238 nucleus then becomes unstable uranium-239, which decays into plutonium-239.
b Plutonium-239 is highly radioactive with a half-life of 24 000 years, so will need to be stored for a long time.
12 Only one neutron is needed to sustain a chain reaction, leaving the remaining neutrons to breed more plutonium.
13 **a** ${}^{238}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{239}_{92}\text{U}$
${}^{239}_{92}\text{U} \rightarrow {}^{239}_{93}\text{Np} + {}^{0}_{-1}\beta^{-} + {}^{0}_{0}\bar{\nu}_e$
${}^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + {}^{0}_{-1}\beta^{-} + {}^{0}_{0}\bar{\nu}_e$
b ${}^{239}_{94}\text{Pu} + {}^{1}_{0}\text{n} \rightarrow {}^{134}_{54}\text{Xe} + {}^{103}_{40}\text{Zr} + 3{}^{1}_{0}\text{n}$
14 Over time, the number of fissile nuclei in the fuel rods becomes depleted, resulting in a reduced number of fission reactions and hence fewer mobile neutrons in the core. In order to maintain a chain reaction, the control rods must be gradually withdrawn over time to avoid absorbing too many of the neutrons and reducing the chain reaction even more.

9.3 Nuclear fusion

WE 9.3.1 2.06×10^{-12} J
WE 9.3.2 1.21×10^{-12} J

9.3 Review

1 Fusion is the joining together of two small nuclei to form a larger nucleus. Fission is the splitting apart of one large nucleus into smaller fragments.
2 The mass of the products is less than the mass of the reactants. The mass difference is related to the energy released via $\Delta E = \Delta mc^2$.
3 The amount of energy released per reaction is greater for fission than for fusion. However, since the nuclei used for fission are much larger, the energy released per nucleon during a single nuclear fission reaction is less than the amount for a single fusion reaction.
4 less than 1%
5 **a** $a = 4$
$b = 2$
X is helium, ${}^{4}_{2}\text{He}$
b 5.88×10^{-29} kg
6 Electrostatic forces of repulsion are acting on the protons. If the protons are moving slowly, they will not have enough energy to overcome these repulsive forces. In other words, they will not get close enough for the attractive strong nuclear force to act and so they will not fuse together.
7 Electrostatic forces of repulsion are acting on the protons, but they are travelling fast enough to overcome these forces. The protons will get close enough for the attractive strong nuclear force to take effect and they will fuse together. These protons have overcome the energy barrier.
8 **a** X is ${}^{3}_{2}\text{He}$
b 3.68×10^{-12} J
c 4.09×10^{-29} kg
9 The binding energy per nucleon increases and the nucleus becomes more stable.
10 The number of nucleons is conserved as there are five nucleons on each side of the reaction.

Chapter 9 Review

1 A nuclide is fissile if it can undergo fission, splitting into two smaller fragments when hit by a neutron.
2 No, only a few nuclides (e.g. uranium-235 and plutonium-239) are fissile.
3 The strong nuclear force causes the proton to be attracted to all other nucleons. It will also experience a smaller electrostatic force of repulsion between itself and other protons.
4 Neutrons are uncharged and are not repelled by the nucleus.
5 **a** 3.13×10^{-11} J **b** 197 MeV **6** $y = 40$ **7** $x = 7$
8 The nuclei are all positively charged and so repel each other. They need a massively large amount of energy to overcome these forces and get close enough for the strong nuclear force to take effect. 100 million degrees add either Celcius or the units provides the required energy for this to occur.
9 2.80 MeV
10 **a** The combined mass of the hydrogen and helium-3 nuclei is greater than the combined mass of the helium-4 nucleus, positron and neutrino. The difference is the mass defect.
b from the lost mass (or mass defect) via $\Delta E = \Delta m \times 931$
c 3.36×10^{-12} J **d** 0.0226 Da
11 Fission reactors create a great deal more waste. Fusion releases more energy per nucleon than fission.
12 When uranium-235 splits into smaller nuclei, the binding energy per nucleon of each product increases. This means the smaller nuclei are more stable.

13 The higher the binding energy, the more stable the nucleus. This is because higher binding energy means that it takes more energy to completely separate particles in the nucleus. Iron, therefore, has the most stable nuclei of all the elements.
14 Gamma rays are released from the nucleus of radioactive atoms.
15 Uranium-238, which is fertile, comprises 98.3% of the uranium resources found on Earth, which means it can absorb fast neutrons and undergo transmutation to produce plutonium-239.
16 **a** The coolant transfers the heat from the reactor to the heat exchanger.
b The moderator slows down, or moderates, the speed of the neutrons.
c The control rods work by absorbing excess neutrons, thus limiting the number of neutrons that can continue the chain reactions.
17 **a** ${}^{1}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{0}_{0}\gamma$
b 3.20×10^{-12} J
c 0.0215 Da
18 **a** ${}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} \rightarrow {}^{2}_{1}\text{H} + {}^{0}_{+1}\beta^{+} + {}^{0}_{0}\nu_e$
${}^{1}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{0}_{0}\gamma$
${}^{3}_{2}\text{He} + {}^{3}_{2}\text{He} \rightarrow {}^{4}_{2}\text{He} + 2{}^{1}_{1}\text{H}$
b 237 MW

Chapter 10 Electrical physics

10.1 Behaviour of charged particles

WE 10.1.1 -6.40×10^{-13} C
WE 10.1.2 3.00×10^{13} electrons

10.1 Review

1 They will attract as they will be oppositely charged.
2 3.12×10^{19}
3 6.72 C
4 Copper is a good conductor of electricity. Plastic is a good insulator. The plastic coating is used to insulate copper wiring to prevent charge leaving the circuit.
5 The Bohr model of the atom proposes that electrons are held in specific energy levels or shells. In metals, the outermost electrons are held more loosely, so can become delocalised. When a potential difference is applied, these electrons are free to move and carry the charge through the wire.

10.2 Energy in electric circuits

WE 10.1.1 4.38×10^{4} J

10.2 Review

1 A **2** **a** 4.00 V **b** 1.50 V **c** 2.00 V **3** 22.2 V
4 167 C **5** 15.0 C
6 the gravitational potential energy of the water
7 M2 or M3.

10.3 Electric current and circuits

WE 10.3.1 4.69×10^{18} electron
WE 10.3.2 9450 J **WE 10.3.3** 938 W

10.3 Review

1 A continuous conducting loop (closed circuit) must be created from one terminal of a power supply to the other.
2 Cell, light bulb, open switch, resistor and ammeter **3** C
4 **a** 3.00 A **b** 0.500 A **c** 8.33×10^{-3} A
5 **a** 5.00 C **b** 3.00×10^{2} C **c** 1.80×10^{4} C
6 **a** 3.00 C **b** 1.00×10^{3} C **c** 1440 C
7 **a** 16.0 C **b** 4.00 A
8 **a** 2.00×10^{-19} electrons **b** 0.320 A **9** B
10 **a** 157 000 J **b** 2.02 A

10.4 Resistance

WE 10.4.1 24.5 Ω **WE 10.4.2** $I_1 = 0.600$ A $\Delta V_2 = 12.0$ V
WE 10.4.3 778 Ω **WE 10.4.4** 9.80×10^{3} mA
WE 10.4.5 4.26 V

10.4 Review

1 **a** A, B, C **b** C, B, A **2** $I = 0.375$ A, $\Delta V_2 = 4.80$ V
3 **a** The wire is ohmic, shown by the linear graph
b 3.00 A **c** 2.50 Ω
4 **a** 0.714 Ω **b** The resistor is ohmic.
5 They are both right. The resistance of the device is different for different voltages. Therefore, the device is non-ohmic.
6 72.0 mA
7 **a** 2.00 Ω **b** 5.00 A **8** **a** 1.60 Ω **b** 0.200 Ω
9 **a** It is non-ohmic, as the I–ΔV relationship is non-linear.
b 0.500 A **c** 15.0 V **d** **i** 20.0 Ω **ii** 13.3 Ω

10.5 Series and parallel circuits

WE 10.5.1 168 Ω
WE 10.5.2 $I = 0.0107$ A
$\Delta V_1 = 1.07$ V
$\Delta V_2 = 7.38$ V
$\Delta V_3 = 3.53$ V
WE 10.5.3 17.4 Ω
WE 10.5.4 $I_{circuit} = 0.533$ A
$I_{30} = 0.333$ A
$I_{50} = 0.200$ A
WE 10.5.5 $\Delta V_1 = 29.6$ V
$\Delta V_{2,3,4} = 21.2$ V
$\Delta V_{5,6} = 4.92$ V
$\Delta V_7 = 44.4$ V
$I_1 = I_7 = 1.478873$ A
$I_2 = 0.423$ A
$I_3 = 0.845$ A
$I_4 = 0.211$ A
WE 10.5.6 Parallel circuit: $P = 0.90$ W, series circuit: $P = 0.144$ W. This parallel circuit draws over 6 times as much power as the series circuit.

10.5 Review

1 B **2** **a** 6.98×10^{-3} A **b** 0.872 V **3** 136 Ω
4 **a** 0.750 A **b** 2.50 A **c** 0.500 A
5 **a** 12.6 V **b** 0.210 A (or 210 mA)
6 $I_1 = 0.300$ A
$\Delta V_1 = 6.00$ V
$\Delta V_2 = 4.50$ V
$\Delta V_3 = 1.50$ V
$I_3 = I_4 = 0.150$ A
7 4.28 Ω **8** **a** 1.25 W **b** 20.0 W **9** C

10.6 Electrical safety

WE 10.6.1 $1.78

10.6 Review

1 A **2** D **3** 3.60×10^{7} J
4 This air conditioner would cost (0.750)(5.00)(0.280109) = $1.0504 or approximately $1 to run for 5.00 hours. Therefore, the figure $10 in the statement is incorrect.
5 The neutral and earth are common.
6 It is much safer to place the fuse in the active circuit because it cuts off the supply to the device, so that it is not active.
7 The earth wire ensures that the neutral and earth conductors are at zero potential.
8 The outer casing of the appliance could become live.
9 2.40 mA

Chapter 10 Review

1 1.90×10^{19} electrons **2** 6.72 C **3** A
4 3.20×10^{-19} C **5** 3.83×10^{-3} A

6 Conventional current represents the flow of charge around a circuit as if the moving charges were positive, which means the direction is from the positive terminal to the negative terminal. In reality, the moving particles in a metal wire are negatively charged electrons. Electron flow describes the movement of these electrons from the negative terminal to the positive terminal.

7 **a** 184 C **b** 1.15×10^{21} electrons

8 **a** 0.800 C **b** 20.0 seconds

9 7.60 J **10** 4.00 V **11** $P = 1.40$ W

12 8.75 V **13** 9.06 A **14** 333 Ω

15 A **16** **a** 3.50 Ω **b** 0.353 A **c** 1.76 V **d** 0.118 A **e** 0.235 A **f** 7.50 Ω

17 **a** ammeter **b** 8.57 Ω

18 The earth wire is usually connected to the metal casing of an electrical appliance. If the insulation around the wire inside the appliance becomes degraded, the casing of the appliance could become live and dangerous to touch. In this situation, the earth wire provides an alternative low-resistance path to the Earth, protecting users of the appliance from electrocution.

19 The circuit will need to have either two pairs of series resistors connected in parallel, or two pairs of parallel resistors connected in series.

20 **a** 2.40 W **b** 21.6 W
c The parallel circuit draws 9 times more power.

21 D **22** $3.36 **23** D

24 **a** 67.4 Ω **b** 1.85 A **c** 0.207 A **d** 232 W **e** 232 W

25 **a** 4.20 V
b above; as light increases, the resistance of the LDR decreases, hence ΔV_{out} rises
c ΔV_{out} approaches zero, as the LDR has increased resistance and therefore the voltage drop across the LDR approaches 12 V.

26 **a** 75.0 Ω
b 37.5 W

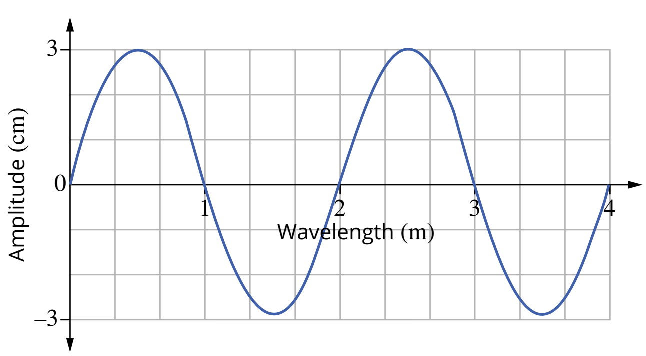
27 The pliers would be made from a more conductive material than the skin of the person's finger, even though they are insulated. As a result, the current passing through them would be higher than the current passing through a person's finger.

28 A fuse will melt when a high current flows in a circuit. Without the fuse the heat generated from a high current could be enough to start a fire and burn the house down. A safety switch switches off a circuit when the current in the active and neutral wires is not equal, thus preventing possible electrocution. This process is much faster than the time it takes a fuse to melt, as well as being more sensitive to smaller changes in current, and so it can directly save a person's life by reducing the duration of an electric shock.

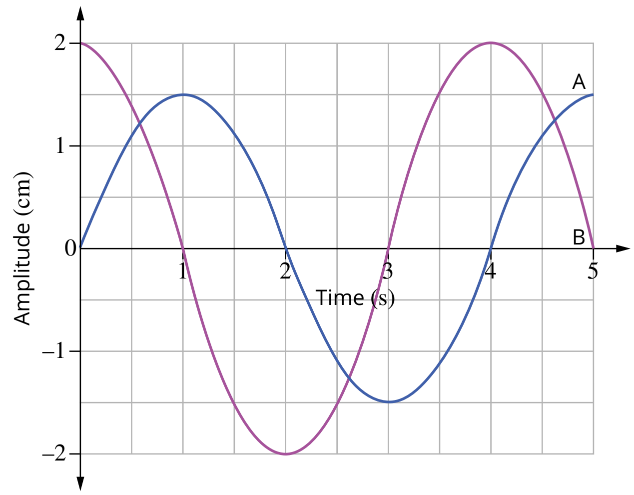
Unit 2 Review

1: Short responses

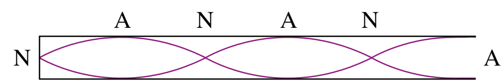
1 **a**



b



2 **a**



b 4.56 m

3 the first three frequencies, $f = 576.7$ Hz or 1730 Hz or 2883 Hz

4 **a** The phenomenon is called resonance.
b Nori would hear the sound increasing and decreasing in loudness at a rate of 4 cycles per second. This phenomenon is called beats.

5 **a** A fuse protects against overload current in the total circuit. It prevents overheating of the wiring due to excess current as this poses a fire hazard. An RCD detects an imbalance between current entering and leaving a device, which suggests there is some earth leakage with that current flowing to earth. Both will shut down the circuit.
b A short circuit is a fault in the circuit that connects the active and neutral wires, effectively bypassing the load in the circuit. This means there is a greatly reduced resistance due to the absence of a load, causing a high current to flow. This condition will trigger the circuit breaker.
c Plugs with three prongs have an active, a neutral and an earth pin. The connection of an earth wire is required when there is any possibility that the active lead could contact the metal casing on the outside of an appliance and risk electrocution of the user as they become the contact to earth. Some smaller devices are double insulated and so the active wire cannot deliver charge to any part of the device that a user can touch. In this case, the earth wire is not needed, and the plug can safely have only two prongs.

6 **a** ${}^{7}_{3}\text{Li} + {}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\alpha + {}^{4}_{2}\alpha$
b ${}^{185}_{79}\text{Au} \rightarrow {}^{4}_{2}\alpha + {}^{181}_{77}\text{Ir}$
c ${}^{218}_{81}\text{Tl} \rightarrow {}^{218}_{82}\text{Pb} + {}^{0}_{-1}\beta^{-} + {}^{0}_{0}\bar{\nu}_{e}$

7 **a** Electrostatic forces of repulsion act on the protons. Since they are moving slowly, they do not have enough energy to overcome this force to get close enough for the strong nuclear force to come into effect and hence they will not fuse. These protons have not overcome the energy barrier.
b Electrostatic forces of repulsion act on the two protons initially, but the protons have enough energy to approach each other despite these forces. Once they get close enough, the strong nuclear force takes effect, enabling the nucleons to fuse. These protons have overcome the energy barrier.

8 **a** The energy of the photons results from the conversion of the mass of the electron and positron to energy.
b in joules: 1.64×10^{-13} J
in MeV: 1.02 MeV

9 **a** The binding energy of a nucleus is the energy that would be needed to break the nucleus into its component nucleons. The binding energy per nucleon is this total value divided by the number of nucleons in the nucleus.

b From the graph, it can be seen that iron atoms have the highest binding energy per nucleon. Iron atoms require the most energy per nucleon to break up their nucleus, therefore they are the most stable.

c The energy per nucleon for uranium is about 7.5 MeV and the binding energy per nucleon for fragments of mass number 118 is about 8.5 MeV. That means that when the smaller fragments are formed, they are more tightly bound and the difference in energy is released in the fission reaction. This is about 1 MeV for each nucleon.

10 power input to motor: 1.5000 W
power output: 0.950 50 W
efficiency = 63.4%

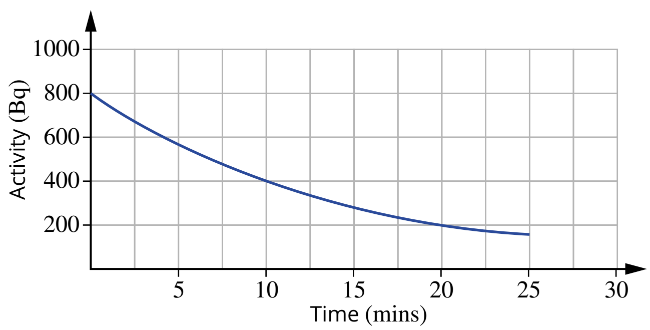
2: Problem-solving

11 **a** $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{55}Cs + ^{93}_{37}Rb + 3^{1}_{0}n$

b 2.08×10^{-11} J **c** 5.34×10^{13} J **d** 1.83 kg

e The conversion of energy released in the reaction to the final generation of electricity is not 100% efficient. The energy released in the fission reactions as heat must first be used to heat up water to produce steam, which then drives the generators. There are many opportunities for energy losses in this system.

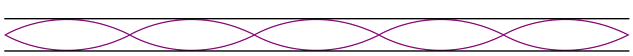
12 **a**



b 320 Bq
c $t_{1/2} \approx 10$ min
d 150 Bq

13 **a** 60.0Ω
b 2.00A
c 0.200A
d 240 W
e 240 W

14 **a**



b 5.60×10^{2} Hz
c 1.58 m
The second didjeridu is longer.

3: Comprehension

15 **a** Background radiation is the naturally occurring radiation that is around us every day. It can come from the Sun, outer space, materials in the Earth's crust and the food we eat, to name a few.

b The burning of coal (as well as other fossil fuels) releases radioactive materials into the atmosphere that are normally locked into the structure of the solid coal.

c Astronauts are further outside of the protective atmospheric layers of the Earth so less radiation is absorbed before reaching them. The atmosphere becomes less dense the higher you go, so the shielding effect the atmosphere has on the incident radiation is less for an astronaut in space than a passenger on a commercial jet aircraft.

d

Type of radiation	Description	Name three types	Name one source for each
ionising	This is a particle or electromagnetic radiation with enough energy per particle of photon to remove electrons from an atom or molecule and produce an ion.	alpha particles	nuclear decay
		beta particles	nuclear decay
		gamma rays	cosmos, from the hottest objects such as neutron stars, pulsars and black holes
		X-rays	cosmic radiation or X-ray machines
		high-frequency ultraviolet	solar radiation and electric arcs
non-ionising	This is electromagnetic radiation with insufficient energy per photon to ionise an atom or molecule.	low-frequency radio waves	starlight two-way radio, TV and radio stations
		microwaves	mobile phones, microwave ovens
		infrared	any object emitting heat (above zero K)
		visible light	Sun, gas discharge tubes, light bulbs, LEDs
		low-frequency ultraviolet	Sun

e **i** $^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}\alpha$

$^{234}_{90}Th \rightarrow ^{234}_{91}Pa + ^{0}_{-1}\beta^{-} + ^{0}_{0}\bar{\nu}_e$

$^{234}_{91}Pa \rightarrow ^{234}_{92}U + ^{0}_{-1}\beta^{-} + ^{0}_{0}\bar{\nu}_e$

ii $^{234}_{92}U \rightarrow ^{222}_{86}U + 3^{4}_{2}\alpha$

f Radon is a radioactive gas that is part of the uranium decay chain, and it poses significant health risks when inhaled. In underground mining, radon can accumulate in confined spaces, increasing the risk of miners inhaling this harmful gas. Open-cut mining, being an exposed method, allows radon to disperse more easily into the atmosphere, thereby reducing the concentration of radon in the air that miners breathe. Thus, the risk of radon exposure and associated health hazards is lower in open-cut mining compared to underground mining.

g Bore water comes up from underground where it has been in contact with rock containing uranium that decays to release radon. The radon will be dissolved in the water under pressure and released when the water comes to the surface. If the water is heated the radon will be less soluble and come out of solution more readily.

h **i** $^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}\beta^{-} + ^{0}_{0}\bar{\nu}_e$ **ii** $^{1}_{0}n \rightarrow ^{1}_{1}H + ^{0}_{-1}\beta^{-} + ^{0}_{0}\bar{\nu}_e$

$d^{-\frac{1}{3}} \rightarrow u^{+\frac{2}{3}} + ^{0}_{-1}\beta^{-} + ^{0}_{0}\bar{\nu}_e$

iii Carbon dating relies on measuring the concentration of carbon-14 atoms relative to the total carbon in a once-living fossil. The principle behind this method is that the total amount of carbon in the fossil will remain constant but the relative concentration of carbon-14 decreases over time due to due to its radioactive decay. By knowing the half-life and measuring the current concentration of carbon-14, scientists can determine the age of the fossil.

i 5.8×10^{-4} Sv

Glossary

A

absolute uncertainty The uncertainty in a measurement expressed in the same units as the measured value. This is different to the fractional uncertainty, and the percentage uncertainty, which are dimensionless.

absolute zero The coldest possible temperature, where all particle motion has ceased.

absorb Taking up and storing energy, such as radiation, light or sound, without it being reflected or transmitted. During absorption the energy may change from one form into another. When radiation strikes the electrons in an atom, the electrons move to a higher orbit or state of excitement by absorption of the radiation's energy.

absorbed dose The amount of ionising radiation absorbed per kilogram of irradiated material, measured in grays (Gy).

acceleration The rate of change of velocity. Acceleration is a vector quantity. The SI unit for acceleration is $m\,s^{-2}$.

accuracy The degree to which an experimentally determined value is close to that of the accepted value. A measure of the accuracy is the percentage difference, which is the accepted value minus the experimental value as a percentage of the accepted value.

activity The number of nuclei of a radioactive substance that decay each second, measured in becquerels (Bq).

air pressure The force per unit area exerted by air on an object; related to the density or the number of particles.

air resistance The retarding force (drag) caused by collisions between air and moving objects.

alpha (α) particle A particle consisting of two protons and two neutrons; ejected from the nucleus of a radioactive nuclide.

alternating current In an alternating current (AC), electrons oscillate backwards and forwards around a mean position, as opposed to direct current (DC). Household power supplies usually operate at 240V AC.

ammeter A device used to measure the current in a circuit. The ammeter is connected in series.

amplitude The maximum displacement of a particle from the average or rest position.

angle of incidence The angle an incident ray makes to the normal of the surface that it strikes.

angle of reflection The angle a reflected ray makes to the normal of the surface it strikes. Equal to the angle of incidence.

angle of refraction The angle a refracted ray makes to the normal of the surface when it has travelled from one medium to another.

antielectron neutrino A neutral subatomic lepton particle related to the electron, that interacts very weakly with other matter, the antimatter particle of neutrino.

antiparticle A particle that has equivalent mass, but opposite charge, and opposite quantum properties of normal matter.

antinode Areas in a standing wave where complete constructive interference is happening.

artificial transmutation The changing of one element or isotope into another. This happens during radioactive decay and during neutron bombardment in a nuclear reactor.

atomic mass constant The standard unit of mass used with nuclear particles. It has the symbol m_u and is equal to one-twelfth the mass of a carbon-12 nucleus in the ground state and unbound. The unit of the atomic mass constant is the dalton (Da). One dalton is equal to 1.66×10^{-27} kg.

atomic number The number of protons in a nucleus.

B

background radiation The low level of ionising radiation that exists in the environment as a result of the Earth being radioactive, nuclear testing, and nuclear accidents.

baryon Particle comprised of three quarks bound together by gluons. Protons and neutrons are baryons.

beat frequency The alternating loud and soft sound that results from two sound waves of the same amplitude, but slightly different frequency, sounding together.

beta (β) particle An electron or positron; ejected from the nucleus of a radioactive nuclide.

binding energy Energy required to split a nucleus into its separate nucleons.

C

centre of mass A single point in an object where the mass can be considered to be 'concentrated' for the purposes of analysing motion.

chain reaction A series of nuclear fissions that may be controlled or uncontrolled.

change in position Final position minus the initial position.

change in velocity Final velocity minus the initial velocity.

charge A property of matter that causes electric effects. Protons have positive charge, electrons have negative charge and neutrons have no charge.

circuit breaker A device that automatically switches off an excessive current by detecting the magnetic field associated with it.

coefficient of friction The constant of proportionality between the force of friction and the normal force acting upwards on the surface of the object, due to the object's weight force.

collinear Lying on the same straight line.

component The components of a force are two vectors at right angles to each other that when added together will be equivalent to the original force.

compression Area of high pressure in a wave.

conductor A substance, body or system that readily conducts heat, electricity, sound or light.

Conservation laws Laws that state certain quantities remain unchanged before and after a reaction occurs.

conservation of energy The energy in a system before an interaction is exactly equal to the energy in the system after the interaction.

conservation of mechanical energy The total mechanical energy in a system (i.e. the potential and kinetic energies) remains constant.

conservation of momentum The sum of the momentum of the bodies in a system before an interaction is exactly equal to the sum of the momentum of the bodies in the system after the interaction.

conserved When a quantity that exists before an interaction is exactly equal to the quantity that exists after the interaction.

constructive interference The process where two or more waves combine or superpose to reinforce each other. This occurs where the displacement of the individual waves is in the same direction so the amplitude is increased.

contact force Push force that exists when one object or material is touching another. Friction, drag and normal reaction forces are contact forces.

control rod Material, commonly boron, steel or cadmium, that absorbs neutrons in a nuclear reactor.

controlled variable A variable that must be kept constant during an investigation.

conventional current A flow of positive electric charge. Conventional current is in the opposite direction to electron flow.

coolant A substance, commonly water, carbon dioxide or liquid sodium, used to transfer thermal energy from the core of a nuclear reactor.

core Part of a nuclear reactor where nuclear fission occurs, and thermal energy is produced.

coulomb The SI unit of charge; 1 C is equivalent to the combined charge of 6.25×10^{18} protons.

crest The maximum positive displacement reached when particles in a transverse wave are displaced upwards from the average position, or resting position.

critical angle For refraction, the incident angle at which total internal reflection occurs. That is, the refracted angle is exactly 90° from the normal and lies along the interface between the two media.

critical mass The minimum amount of enriched fissile material in the shape of a sphere that leads to a sustained fission reaction.

current The net flow of electric charge. Current is measured in amperes (A) where $1\,A = 1\,C\,s^{-1}$. By convention, electric current is assumed to flow from positive to negative.

D

daughter nucleus A nucleus on the product side of a nuclear equation; it results when a nucleus undergoes fission or radioactive decay.

decay series A sequence of radioactive decays that results in the formation of a stable isotope.

dependent variable The variable that may change in response to a change in the independent variable. On a graph, the dependent variable is plotted on the vertical axis.

destructive interference The process in which two or more waves combine or superimpose to reduce the amplitude. This occurs where the displacement of the individual waves is in the opposite direction.

deuterium An isotope of hydrogen with one proton and one neutron.

diffraction The process affecting light and other wave forms that causes a wave to spread out as it passes through a narrow aperture or past an edge.

diffuse Spread out; for example, a wave reflecting off an irregular surface.

dimension Space can be considered to consist of three length dimensions. These length dimensions are arranged at 90° to each other with their point of intersection being the origin. The position of an object can be defined in relation to its position along each of the three dimensions. Typically, these three dimensions are labelled x, y and z. However, up-down, left-right and backward-forward are also appropriate.

dimensional analysis Using the units in a graph or formula to check that the derived term is correct.

direct current In a direct current (DC), electrons travel in one direction only, as opposed to alternating current (AC). Batteries and electric cells provide direct current.

direction conventions Standardised systems for describing the direction in which an object is travelling. The use of cardinal points of a compass (N, S, E and W) is an example of a direction convention.

displacement An object's change in position, relative to its starting position and final position. Displacement does not consider the route the object took to change position, only where it started and where it ended. Displacement is a vector quantity. It is measured in metres (m) and given the symbol s.

distance travelled How far an object travels during a particular motion or journey. Distance is a scalar value. Direction is not required when expressing magnitude. It is measured in metres (m) and given the symbol d.

Doppler effect A change in the observed frequency of a wave, such as sound or light, that occurs when the source and observer are in motion relative to each other.

dose equivalent A measure of the biological damage inflicted on a tissue due to absorption of a defined quantity of radiation. Dose equivalent measurements take into account the nature of the radiation applied. It is measured in sieverts (Sv).

E

earth The third wire (usually green or green and yellow) in electrical devices that acts as an important safety feature by carrying excess current due to a device malfunction directly into the Earth.

echo The reflection of sound from a distant surface that reaches the ear after more than 0.1 seconds, and is therefore heard as a separate sound to the original sound.

effective resistance A single resistance that could be used to replace a number of individual resistors for the purpose of circuit analysis.

efficiency The percentage of energy that is effectively transformed by a system.

elastic Objects that obey Hooke's law, typically springs, rubber bands and tendons.

elastic collision Collision in which kinetic energy is conserved.

elasticity A measure of the tendency of a medium to return to its original shape after it has been bent, compressed or stretched.

electric circuit An electric circuit is comprised of a source, conducting wires and circuit components. The source provides the electrical potential difference, the wires provide the free electrons in a continuous connection between each end of the source, and the circuit component provides the place for the electrical energy to be converted into a useful form.

electric current The flow of charged particles.

electric shock Also known as electrocution, in which excess electricity flows into the human body due to a device malfunction or electrical accident.

electrical potential energy Potential energy due to the separation of charge in part of an electric circuit. The energy is stored in the electric field.

electricity A form of energy resulting from the existence of charged particles (electrons or protons). Electricity is fuelled by the work done in separating opposite charges and the electric field that results, in which the energy is stored. Free electrons in conductors then transfer the energy from the field to the circuit components.

electromagnetic force The attractive or repulsive force experienced by electric or magnetic objects due to other electric or magnetic objects.

electromagnetic radiation A wide range of frequencies (or wavelengths) that can be created by accelerating charges, resulting in a rapidly changing magnetic field and electric field travelling out from the source.

electron A negatively charged particle in the outer region of an atom; it can move from one object to another, creating an electrostatic charge. When electrons move in a conductor, they constitute an electric current. The electron is one of the six leptons.

electron flow The net flow of electrons. Although electric current is assumed to flow from positive to negative, delocalised electrons in the conductors physically move from negative to positive.

electron neutrino A neutral subatomic lepton particle that interacts very weakly with other matter. It is associated with the electron and is the counterpart to the antielectron neutrino.

electronvolt (eV) A small unit of energy. One electronvolt (1 eV) is the energy an electron would gain when accelerated across a potential difference of one volt: $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.

electrostatic force A force that acts between charged particles and can act over relatively large distances.

elementary charge The magnitude of the charge on an electron or proton: $e = 1.6 \times 10^{-19}\,\text{C}$.

evaporation The changing of a liquid into a gas, often under the influence of heat (at a temperature below the boiling point).

F

fast breeder reactor A nuclear fission reactor in which some neutrons from the fission of uranium-235 are absorbed by non-fissile uranium-238. After absorbing a neutron the U-238 undergoes two beta-minus decays to transmutate into the fissile plutonium-239 isotope. The term 'fast' refers to the fact that fast neutrons are more effectively absorbed by U-238 than slow neutrons, and so a moderator is not required.

fermion Particle that include quarks and leptons, and the particles that are composed of odd numbers of quarks.

Feynman diagram A way to visualise particle interactions designed by Richard Feynman, in which quarks, leptons and bosons are represented.

first harmonic Also known as the fundamental; the longest resonant wavelength in a string or a pipe. For a string or a pipe open at both ends, it is half a wavelength and consists of a node in pressure at each end and an antinode in the middle. For a pipe closed at one end it is a quarter of a wavelength and consists of a node in pressure at the open end and an antinode at the closed end.

fissile Capable of undergoing nuclear fission after capturing low-energy neutrons.

fission When a nucleus splits into two or more pieces, usually after bombardment by neutrons.

fission fragments Nuclides formed during nuclear fission; these are usually radioactive.

forcing frequency The frequency of the force applied to an oscillating substance or object

force A vector quantity which measures the magnitude and direction of a push or a pull. It is measured in newtons (N).

free fall A motion whereby gravity is the only force acting on a body.

frequency A measure of the rate at which something occurs, for example the number of vibrations or cycles that are completed per second or the number of complete waves that pass a given point per second. Measured in hertz (Hz).

friction a contact force that opposes motion, usually applies at the surfaces in contact between two substances.

fuel rod Long, thin rod of enriched uranium used in a nuclear reactor.

fundamental The lowest and simplest form of standing wave vibration, with one node and one antinode.

fuse A circuit device that melts when too much current flows through it, breaking the circuit and protecting the other circuit components.

fusion A process taking place inside stars in which small nuclei are forced together to make larger nuclei. Energy is released.

G

gamma (γ) ray High-energy electromagnetic radiation ejected from the nucleus of a radioactive nuclide.

gauge boson Force mediating particle used in the Standard Model that is equivalent to forces that act at a distance via fields. There are four gauge bosons: gluon (strong nuclear), W (weak nuclear), Z (weak nuclear), and the photon (electromagnetic).

Geiger counter A device for measuring radioactive emissions.

gluon Force mediating particle used in the Standard Model that is equivalent to the strong nuclear force.

gravitational potential energy Energy stored in the gravitational field due to an object's position in the gravitational field. Measured in joules (J).

H

hadron Particle comprised of quarks bound together by gluons. Hadrons may have three quarks, or a quark and an antiquark.

half-life The time taken for half of the nuclei of a radioactive isotope to decay.

harmonic The resonant frequencies produced when standing waves are formed in a string or air column.

heat The energy in transfer from a hotter object to a cooler one, which increases the kinetic and/or potential energy of the particles in the cooler object.

heat exchanger Part of a nuclear reactor where heat drawn from the reactor core is used to turn water into steam.

heavy water Water that has a higher than normal proportion of water molecules that contain deuterium.

I

impulse The change in momentum of an object is also called the impulse of an object. The impulse is calculated by the final momentum minus the initial momentum.

independent variable The variable that is selected and deliberately changed by the researcher. On a graph, the independent variable is plotted on the horizontal axis.

inclined plane A surface that is tilted so that it makes an angle to the horizontal.

inelastic collision Collision in which kinetic energy is not conserved.

inertia A property of an object, related to its mass, that opposes changes in motion.

in phase When two waves occupy the same medium and each wave is at the same point in their period, or distance along a wave, relative to the origin.

insulator A material or an object that does not easily allow heat, electricity, light or sound to pass through it. Air, cloth and rubber are good electrical insulators; feathers and wool are good thermal insulators.

interference This occurs when two or more waves occupy the same place at the same time. The resultant displacement of the wave particles is the vector addition of the amplitudes of the individual waves.

internal energy The total kinetic and potential energy of the particles within a substance.

ion Atom of a chemical element in which the number of electrons and protons is not equal and therefore the atom is electrically charged. If extra electrons are present, the ion has a negative charge. If electrons are missing, the ion has a positive charge.

ionised A condition in which an atom has become positively charged by losing one or more electrons from its outer energy level.

ionising ability The ability of particles or radiation to ionise matter.

ionising radiation Radiation with enough energy to alter the molecular structure of matter by displacing one or more electrons from an atom and thus creating electrically charged ions.

isotope Atoms with the same number of protons but with different numbers of neutrons.

J

junction A point in an electric circuit from which current can flow in or out in more than one direction.

K

kelvin An absolute temperature scale based on the triple point of water.

kilowatt hour (kW h) Unit of energy equivalent to 3.6 megajoules. The equivalent amount of energy to that used by a 1000W device turned on for one hour. It is the unit of measure of electricity usage that is measured by electricity meters and appears on electricity bills.

kinetic energy The energy of a moving body, measured in joules (J).

kinetic friction force The frictional force that acts between two moving surfaces.

kinetic particle model A model that states that the small particles (atoms or molecules) that make up all matter have kinetic energy, which means that all particles are in constant motion, even in solids.

L

latent heat The 'hidden' energy needed to change the state of a substance at the same temperature, i.e. the energy is not seen as a change in temperature.

latent heat of fusion The energy required to change 1 kg of solid to a liquid at its melting point.

latent heat of vaporisation The energy required to change 1 kg of liquid to a gas at its boiling point.

lepton A fundamental particle that does not interact with the strong nuclear force. There are six leptons: electron, electron neutrino, muon, muon neutrino, tau, and tau neutrino. Each of these particles has an antimatter particle.

longitudinal A longitudinal wave is one in which the vibration of the particles within the medium are parallel to the direction of energy flow of the wave.

M

magnitude The size or extent of something, with no need for direction. In physics, this is usually a quantitative measure expressed as a number of a standard unit.

mass An amount of matter. One kilogram of mass is equal to the increase in mass of $1.475\,521\,4 \times 10^{40}$ atoms of cesium-133 as each atom's outer electron transitions up to the hyperfine energy level. Mass can also be defined by the amount of matter that would result in an acceleration of $1\,\text{m}\,\text{s}^{-2}$ when a force of 1 N is applied in a frictionless environment.

mass defect The change in mass of the reactants to the products of a nuclear reaction. The mass defect is converted to energy.

mass number The number of nucleons (protons and neutrons) in a nucleus.

mean The average value that is calculated by taking the sum of all values and then dividing by the total number of values.

mechanical energy The energy that a body possesses due to its position or motion. Kinetic energy, gravitational energy and elastic potential energy are all forms of mechanical energy.

mechanical wave A wave that transfers energy through a medium.

median The middle piece of data when a data set is listed in order.

medium The material or substance through which a mechanical wave moves.

meson Particle comprised of a quark and an antiquark, bound together by gluons. Pions and kaons are examples of mesons.

metal Material in which some of the electrons are only loosely attracted to their atomic nuclei. The properties of metals include: high strength, good electrical and thermal conductivity, lustre, malleability and ductility.

mode The most common piece of data in a data set.

moderator A material, usually graphite or water, that slows neutrons in a nuclear reactor.

momentum The product of an object's mass and velocity. Objects with larger momentum require a larger force to stop them in the same time that an object with smaller momentum takes to stop. It is given by the equation $p = mv$.

N

natural frequency The specific frequency at which an object will tend to vibrate due to its dimensions and properties.

net charge When the number of positive and negative charges in an object is not balanced.

net force The vector sum of all the individual forces acting on a body.

neutral No electric charge, or a situation in which positive and negative charges are balanced.

neutron An uncharged subatomic particle found in the nucleus, consisting of two down quarks and one up quark.

newton SI unit of force. One newton (1 N) is defined as the force required to make a mass of 1 kg accelerate at $1\,\text{m}\,\text{s}^{-2}$.

Newton's first law States that an object will maintain a constant velocity unless an unbalanced, external force acts on it.

Newton's second law States that force is equal to the rate of change of momentum. This can be processed mathematically to: the acceleration of an object is directly proportional to the force on the object and inversely proportional to the mass of the object.

Newton's third law States that for every action (force), there is an equal and opposite reaction (force).

node Areas in a standing wave where complete destructive interference is occurring, and the two waves totally cancel each other out.

non-contact force A force that acts at a distance and does not require the bodies to physically touch each other. Strong nuclear, weak nuclear, gravitational and electromagnetic forces are non-contact forces.

non-ionising radiation Radiation that does not have enough energy to break the molecular bonds within molecules and to alter the number of electrons in an atom. Lower forms of energy in the electromagnetic spectrum such as radio waves, microwaves, visible light and UVA radiation are non-ionising.

non-metal Material in which all of the electrons are either strongly attracted to their atomic nuclei, or are strongly held in bonding interactions.

non-ohmic Not behaving according to Ohm's law; resistance changes depending on the potential difference.

normal An imaginary line at 90°, i.e. perpendicular, to a surface.

nuclear transmutation The changing of one element into another by nuclear decay or bombardment.

nucleon A particle located in the nucleus of an atom.

nucleus The central part of an atom.

nuclide The range of atomic nuclei associated with a particular atom, which is defined by its atomic number, and the various isotopes of that atom as identified by the mass number.

O

ohmic A resistor that follow's Ohm's law; i.e. has a linear relationship between the current it draws and the potential difference across it.

oscillate The movement of particles about their average position in a regular, repetitive or periodic pattern.

outlier A value that lies outside the main group of data of which it is a part. Outliers in data could be caused by errors in the experiment.

overload When an unsafe amount of current flows through a wire; for example, when too many electrical appliances are connected to the same power point.

overtone A harmonic (resonant frequency) that is higher than the natural frequency.

P

parallel circuit A circuit that contains junctions; the current drawn from the battery, cell or electricity supply splits before it reaches the components and re-joins afterwards.

parent nucleus A nucleus on the reactant side of a nuclear equation that when struck by a neutron undergoes fission or simply decays by natural means.

particle accelerator A device used to accelerate charged particles using electric and magnetic fields. Particle physicists use these to collide particles in their search for new matter.

particle displacement The measure of the displacement a particle moves about its equilibrium position during the propagation of a wave. In a longitudinal wave this motion is parallel to the direction of propagation of the wave. In a transverse wave it is perpendicular to the direction of travel.

penetrating ability A measure of how easily radiation passes through matter.

period The time interval for one vibration or cycle to be completed.

personal protective equipment (PPE) Equipment such as safety glasses and disposable gloves used to protect people working in a laboratory.

phase A point in the period, or distance along a wave, relative to the origin.

photon A wave-like particle that carries electromagnetic energy. In the Standard Model it is a particle that mediates the electromagnetic force.

plane wave A wave that has a straight wave front.

position The location of an object with respect to a reference point. Position is a vector quantity.

positron The antimatter pair of the electron. This means it shares the same mass as an electron but has opposite properties, such as its electromagnetic charge and spin.

potential difference The difference in electric potential between two points in a circuit; measured by a voltmeter when placed across a circuit. A battery creates the potential difference across a circuit, which drives the current. The unit of potential difference (ΔV) is the volt (V).

potential energy Energy that can be considered to be 'stored' within the field due to an object's position within the field, composition or molecular arrangement.

power The rate at which work is done; a scalar quantity measured in watts (W).

proton A positively charged subatomic particle found in the nucleus consisting of two up quarks and one down quark.

precision The degree to which the measurements recorded for repeated trials are similar to each other. Measurements with high precision are very similar to each other. A measure of the precision is the uncertainty of averages, with a small value indicating high precision.

pulse A single movement, vibration or undulation.

Q

qualitative variable A variable that can be observed but not measured.

quality factor The number used to indicate the weighting of the biological impact of each type of radiation.

quantitative variable A variable that can be measured.

quantum number According to the quantum theory, particles have properties referred to as quantum numbers, one of which is 'spin'.

quark Fundamental particles that only interact via the strong nuclear force. There are six types of quark: up, down, top, bottom, charmed, and strange. Each quark has an antimatter particle.

R

radiation Rays or particles that carry energy. Also, the process by which energy is emitted by an object or system, transmitted through an intervening medium or space, and absorbed by another object or system.

radiation shield A thick concrete wall that prevents neutrons escaping from a nuclear reactor.

radioactive Something that spontaneously emits radiation in the form of alpha particles, beta-plus or beta-minus particles, and/or gamma rays.

radioisotope An isotope of a chemical element that emits radioactivity due to having an unstable combination of neutrons and protons in the nucleus.

random error An error in measurement that occurs in an unpredictable manner.

rarefaction An area of decreased pressure within a longitudinal sound wave.

raw data The actual measurements taken directly during an investigation without being processed in any way.

ray A line drawn perpendicular to a wave front and in the direction that the wave energy is moving. (Also a narrow beam of light.)

reflection To change the direction of a wave as it strikes a surface and is bounced back.

refraction The bending of the direction of travel of a ray of light, sound, or other wave as it enters a medium of differing refractive index (optical density).

reliability The consistency of the results obtained from an experiment or collection of data. Reliable results are also repeatable, meaning another scientist performing the same analysis will come up with the same results.

residual current device (RCD) A device that can detect a difference in the current flowing in the active and neutral wires and switch off the current if it detects a difference. Current can be lost to the earth wire if a fault occurs, this helps to prevent electrocution.

resistance A measure of how much an object or material resists the flow of current; the ratio of the potential difference across a circuit component and the current flowing through it: $R = V/I$. Resistance is measured in ohms (Ω).

resistivity A measure of how much a material resists the flow of current.

resistor A circuit component, often used to control the amount of current in a circuit by providing a constant resistance. Resistors are ohmic conductors, i.e. they obey Ohm's law.

resonance The state of a system in which an abnormally large vibration is produced in response to an external vibration. Resonance occurs when the forcing frequency is the same, or nearly the same, as the natural vibration frequency of the system.

resonant frequency The natural frequency at which an object tends to vibrate.

resultant One vector that is the sum of two or more vectors.

reverberation This is a reflection of sound from a nearby surface that reaches the ear in less than 0.1 seconds, and combines with the original sound. It often sounds like a longer sound.

S

scalar A physical quantity that is represented by magnitude and units only. Mass, time and speed are examples of scalar quantities.

seismic wave Vibrations within the earth caused by phenomena such as earthquakes, explosions, volcanoes and landslides.

series circuit When circuit components are connected one after another in a continuous loop so that the same current passes through each component.

short circuit The situation in which a good conductor is inadvertently placed across a battery or the circuit component, and an excessive current flows, which may cause damage.

significant figures The numbers in a measurement or calculation that convey meaning and precision.

sinusoidal In the shape of a sine wave.

specific heat capacity The amount of energy that must be transferred to change the temperature of 1 kg of a material by 1°C or 1K.

speeds The ratio of distance travelled to time taken. Speed is a scalar quantity. The SI unit for speed is $m\,s^{-1}$.

spontaneous transmutation The changing of one element into another in a natural process involving radioactive decay.

Standard Model A system of categorising subatomic particles including quarks and leptons, which can be used to describe the vast array of particles observed in particle accelerator collisions.

standing wave Also called a stationary wave, the periodic disturbance in a medium resulting from the combination of two waves of equal frequency and intensity travelling in opposite directions. The waves do not appear to be travelling across the surface, rather they oscillate up and down in one position.

static friction force The maximum force of friction that occurs between two surfaces, which occurs at the start of motion.

strong nuclear force A short-range but powerful force of attraction that acts between all the nucleons in the nucleus. The strong nuclear force acts on quarks and binds them together in hadrons. It also acts to bind protons and neutrons together within atomic nuclei.

subcritical mass A quantity of fissile material that is too small to sustain a chain reaction.

supercritical mass A quantity of fissile material that is large enough to sustain a chain reaction.

superposition When two or move waves travel in a medium, the resulting wave at any moment is the sum of the displacements associated with the individual waves.

systematic error An error that is consistent and will occur again if the investigation is repeated in the same way.

T

temperature A measure of the average translational kinetic energy of the particles in a substance. Temperature can be measured in degrees Celsius (°C) or kelvin (K).

thermal energy Energy that contributes to an object's temperature or state.

thermal equilibrium For two bodies in thermal contact, the condition in which the two reach the same temperature and there is no further net transfer of thermal energy.

total internal reflection Occurs when the angle of incidence exceeds the critical angle for refraction. Light or waves are reflected back into the medium; there is no transmission of light.

tracer A radioactive isotope with a short half-life that is injected into or ingested by a patient to monitor biological processes in the body.

transfer The conversion of energy from one system to another. Also the movement of particles or objects from one location to another.

transform To change from one form to another; for example, to change energy from electrical potential energy to kinetic energy.

transmitted Light, heat or sound, etc. that has passed through a medium.

transuranic Elements with atomic numbers greater than uranium ($Z = 92$). All of these elements are unstable and radioactively decay into lighter elements.

transverse Lying or extending across something. The vibrations of a transverse wave are at right angles to the direction of travel of the energy in a wave.

travelling wave A wave that travels, or transfers its energy, unimpeded through a medium and is not confined to a given space. Every point on the wave has maximum displacement at some point in time. Similarly, each point also has minimum displacement at some point.

tritium An isotope of hydrogen with one proton and two neutrons.

trough The maximum negative displacement reached when particles in a transverse wave are displaced downwards from the average position, or resting position.

U

uncertainty The description of the range of data obtained; the maximum variance from the mean.

uncertainty of averages A method used to calculate the uncertainty of the calculated average value from multiple trials in an experiment. A reliable method is to use half the range of the values, calculated by halving the largest value minus the smallest value. If the uncertainty of averages is different to the instrument uncertainty, then the larger of the two uncertainties should be used.

uncertainty of measurement When measurements are made using scales, such as metre rules, or using digital devices, such as electronic balances, it is scientifically responsible to indicate the uncertainty in the measurement. Instrumental uncertainty for scaled measuring devices is determined by halving the smallest increment in the scale. The instrument uncertainty for digital devices is determined by placing the number 1 in the last column of the digital display.

units Properties related to physical measurements. Units can be fundamental, such as metres (m), seconds (s) or kilograms (kg). Units can also be derived by combining fundamental units; for example, metres per second ($m\,s^{-1}$).

V

validity The reasonableness of the results received from an experiment or collection of data. Valid results meet all the requirements of the criteria of the scientific method.

variable A factor or condition that can change.

vector A physical quantity that requires magnitude, units and a direction in order to be fully defined. Velocity, acceleration and force are examples of vector quantities.

vector diagram A system of adding vectors where each vector is drawn head-to-tail, with the resultant vector drawn from the tail of the first vector to the head of the last vector.

velocity The ratio of displacement to time taken. Velocity is a vector quantity. The SI unit for velocity is $m\,s^{-1}$.

vibration A repeated side-to-side motion.

volatile Liquids with weak surface bonds that evaporate rapidly.

volt The unit of electrical potential. One volt is equal to one joule of potential energy given to one coulomb of charge in a source of potential difference.

voltmeter A device used to measure the electrical potential difference between two points in a circuit.

W

W^- Force mediating particle used in the Standard Model that is equivalent to the weak nuclear force. This particle is involved in beta-minus decay.

W^+ Force mediating particle used in the Standard Model that is equivalent to the weak nuclear force. This particle is involved in beta-plus decay.

wave front The set of maximum amplitude points on neighbouring two-dimensional waves that are in phase. Wave fronts generally form a continuous line, curve or surface.

wavelength The distance between one peak or crest of a wave of light, heat or other energy and the next corresponding peak or crest (symbol: λ).

weak nuclear force Includes the force mediating particles W^-, W^+, and Z used in the Standard Model that are equivalent to the weak nuclear force. These particles are involved in beta decay.

weight The force of attraction on a body due to gravity near the surface of the planet.

work The transfer of energy as a result of the application of a force; measured by multiplying the force and the displacement of its point of application along the line of action. Measured in joules (J).

Z

Z Force mediating particle used in the Standard Model that is equivalent to the weak nuclear force.

Index

E

F

G

H

Attributions

The following abbreviations are used in this list: t = top, b = bottom, l = left, r = right, c = centre.

123RF GB Limited: Alexlmx, p. 373r; Denisbelitsky, p. 138; Januaryxii, p. 270; Kadmy, p. 43r; Liudmilachernetska, p. 168; Lublubachka, p. 112tl; Masuti, p. 108r; Sahua, p. 180r; Sean Nel, p. 121; Stefanholm, p. 259; Tharinsinlapachai, p. 205t; Tonobalaguer, p. 281.
AAP Images: Tracey Nearmy, p. 348b.
Adobe Stock: Ilya Akinshin, p. 167b; AM, p. 172l; Yuri Bizgaimer, p. 162; Carbouval, p. 7; Cutimage, p. 373l; Destina, p. 172r; George Dolgikh, p. 383r; Dreaming Andy, p. 415br; Eaumstocker, p. 14; Ezume Images, p. 6; Fotomatrix, p. 390tr; Generalfmv, p. 291; Deyan Georgiev, p. 390cr; Itestro, p. 235; Francisco Javier Gil, p. 394; Leungchopan, p. 379; Manfredxy, p. 312; Marcel, p. 182; George Rudy, p. 194l; Roman Samokhin, p. 390br; Ian Scott, p. 415bl; Somchaisom, p. 390l; Studioloco, 367t; Stuporter, p. 172c; T-REX, p. 372; Tadamichi, p. 21.
Agence France-Presse: AFP/BIPM, p. 150.
Alamy Images: ANP, p. 359; Cisfo/Mauritius images GmbH, p. 263l; Colport, p. 406; Natalya Danko, pp. 102-104; Everett Collection Historical, pp. 336br, 355; GL Archive, pp. 33tbl, 336cl; Granger, Historical Picture Archive, p. 336cr; History and Art Collection, p. 348t; Ian Dagnall Computing, p. 339; Interfoto Personalities, p. 409; ITAR-TASS Photo Agency, p. 320b; Tom Koene, p. 350; Military Collection, p. 69r; North Wind Picture Archives, p. 118t; Oxford Science Archive/Heritage Images/ The Print Collector, p. 343l; Photo Researchers, Science History Images, p. 183r; Photo12/Ann Ronan Picture Library, p. 320t; Pictorial Press Ltd, p. 343r; Science History Images, p. 343r; Science Photos, p. 244; Travelscape Images, p. 241; US Air Force Photo, p. 69l; Wales, p. 228; WENN Rights Ltd, p. 63.
Keith Burrows: pp. 414, 418.
Dale Mann: p. 232.
Getty Images: p. 295; AFP/Stringer, pp. 58l, 322; The Asahi Shimbun, p. 328l; Michael Dodge, p. 140; Julian Finney, p. 143; Bryn Lennon, p. 59; Skynesher, p. 329.
Mike Babcock: p. 389.
NASA: pp. 91, 120.
National Library of New Zealand: p. 336bc.
Pearson Australia: Alice McBroom, p. 11b.
School Curriculum and Standards Authority, Government of Western Australia: Extracts from WACE Physics ATAR Course Year 11 Syllabus 2024, reproduced by permission, pp. 1-2, 29, 31, 55, 103-104, 161, 201, 227, 229, 287, 235, 365-366.
Science Photo Library: pp. 98, 233l, 300; Claus Lunau, p. 255; CMS Experiment, Cern, p. 290b Rana Dias/Caia Image, pp. 160-161; Patrick Dumas/Look at Sciences, p. 417; Giphotostock, p. 90; Gustoimages, pp. 180l, 375; David A. Hardy, p. 107; Patrick Landmann, p. 351; Lawrence Livermore National Laboratory, p. 354; Dr Najeeb Lyyous, p. 280r; David De Martin, p. 203; L. Medard/Eurelios, p. 290t; David Parker, pp. 290c, 308; Public Health England, p. 326; Sakkmesterke, pp. cover, 201, 334-335; Victor De Schwanberg, p. 287; Dr Gary Settles, pp. 52t, 188; Alan Sirulnikoff, p. 9; Tek Image, pp. 390-391; Trevor Clifford Photography, pp. 11t, 376r, 383l; US Department of Energy, p. 336t; Nick Veasey, p. 279l.
Shutterstock: p. 349; Agsandrew, p. 223-225; Galyna Andrushko, p. 112tr; APChanel, p. 43l; Artsiom P, p. 114b; Peter Bernik, p. 53; Joggie Botma, p116; Ivan Cholakov, p. 61; Coryn, p. 376l; Reid Dalland, p. 279r; Dave Hewison Photography, p. 60; Demarcomedia, p. 370; Designua, p. 254; Borodin Denis, p. 114c; Nicolas Datiche/ Sipa, p. 307; EpicStockMedia, p. 233r; Everett Collection, p. 112br; Stephen Girimont, p. 118b; Angelo Gilardelli, p. 128; Gl0ck, p. 148b; Gabor Havasi, p. 36; HTU, pp. 226-227, 424-428; Ibreakstock, p. 3; Brian A Jackson, p. 164; Jag CZ, p. 230; JRC-Stop Motion, 183l; Lassedesignen, p. 55; Liza1979, p. 336; Joana Lopes, p. 23; Antonov Maxim, pp. 165, 166; Sergey Merkulov, p. 310; Mutation, p. 58r; Orion Production, p. 328r; Pavel L Photo and Video, p. 313; Pia Unseentown, p. 263r; Pistolseven, p. 265; Pixelparticle, p. 365; Plati Photography, p. 432; Rawpixel.com, p. 137; Reshetnikov Art, p. 282t; Jamie Roach, p. 108l; Fouad A. Saad, p. 253; N. Sakarin, p. 282b; Joao Seabra, p. 194r; Sideways Design, p. 279c; Sirtravelalot, p. 45; Robbie Stephenson/JMP, p. 105; Thieury, p. 146; Thirteen, p. 112bl; Tanawit Thirintong, p. 294; Joe Toth/BPI, p. 192; Lee Yiu Tung, pp. 28-29; Adam Vilimek, pp. 52b; Violetkaipa, p. 371; Vpvhunter, p. 148t; YAO23, p. 30-31; YegoroV, p. 134.
The West Australian Symphony Orchestra: Daniel James Grant, p. 280.
Thermal Image UK: p. 205b.